QED Self-Energy Corrections to Fermions at Finite Temperature and Chemical Potential

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QED Self-Energy Corrections to Fermions

at Finite Temperature and Chemical Potential

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ABSTRACT

Calculation of the one-loop QED self-energy for fermions in the presence of a background characterized by a temperature T and a chemical potential μ is performed using the real time propagator formalism of Niemi and Semenoff. Two general cases are studied: (1) chirally invariant (massless) QED; (2) QED with broken chiral symmetry (massive QED). For the chirally invariant case a dispersion relation is obtained in the form of two coupled non-linear integral equations which are solved numerically. The dispersion relation is shown to be similar to that for a free particle of mass $[(\alpha/2\pi)(\mu^2+\pi^2T^2)]^{1/2}$. For the case of broken chiral symmetry, the finite T,μ mass shift is calculated. Two simple limits, T=0, large μ (neutron stars) and T≠0, μ=0 are studied in detail. For the neutron star case it is shown that although the effective mass of an electron is approximately 14 m_e, this effect is probably not important in the evolution of neutron stars. For the T≠0,μ=0 case, discrepancies between results obtained here, and those published in the literature are discussed and an explanation for these differences is given. The renormalization procedure is also discussed and it is shown that in order to give a direct physical interpretation to the self-energy corrections, the renormalization condition must be specified at a μ,T dependent point.

For my father, who has

waited so patiently.

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Chapter 1

Introduction

When one considers an interacting relativistic field theory, conventionally there is a term in the lagrangian which describes the interaction. For example, the addition of the term $-e\bar{\psi}\not{A}\psi$ to the free field QED lagrangian appears to describe very well how photons interact with electrons, and leads to a variety of experimentally verified phenomena such as the Lamb shift, the Casimir effect and prediction of the magnetic moment of the muon.[1] There are however, other types of interactions of interest, such as many-body effects. Even though no new term appears in the lagrangian, the effect is just as real.

Such a many-body system will be discussed here in an effort to answer the following question: Given a particle and a background parameterized by some temperature T and chemical potential μ (hereafter referred to as the thermal background), how can the interaction between the particle and the background be characterized? Only one aspect of this rather general problem will be investigated here, namely the effective mass of a fermion in the presence of a thermal background. The technique for studying this is finite temperature and density (FTD) field theory.

The study of FTD field theories began with Matsubara's discovery thirty years ago of the relationship between the quantum mechanical evolution operator and the density operator[2] (see Chapter 2 for details). This non-relativistic theory has since been applied to a wide range of physical systems including superconductivity, superfluidity, condensed nuclear matter, and electron-phonon interactions.[3-4] In contrast, only in the last

decade has there been any interest in relativistic versions of FTD field theories. This has been due largely to the discovery that broken gauge symmetries can be restored at sufficiently high temperatures.[5-6] This discovery spurred several lines of research including: studies of phase transitions,[7,8] the plasmon effect for Yang-Mills theories,[9] construction of the thermodynamic potential,[9,10] stellar evolution and the early universe,[8,11] renormalization techniques for FTD theories,[12] and a FTD path integral formulation.[13]

More recently, self-energy corrections to FTD propagators have been investigated. Bechler has studied all one-loop FTD diagrams for QED and obtained a spectral representation. Ueda[15] has suggested a renormalization scheme for the FTD self-energy which removes a gauge invariance problem first raised by Dolan and Jackiw[6] and by Weinberg.[16] Using a $T\neq0, \mu=0$ background, Weldon has calculated the one loop vacuum polarization[17] for QED and the dispersion relation for a chirally invariant (zero rest mass) fermion interacting with a gauge field[18] which results in an effective fermion mass of $O(T)$. Extension to the $\mu\neq0$ case has also been done by Boal and Levinson[19] with results that are similar to those found in ref. 18. Peressutti and Skagerstam[20] have calculated the effective mass of an electron in the presence of a $T\neq0$, $\mu=0$ background and found it to be of $O(T^2)$. Discussion of the disagreement between this result and the results of ref. 18 and 19 can be found in Levinson[21] and Chapter 5 of this thesis. Cambier, Primack & Sher[22] and Dicus et al.,[23] using calculations similar to those in ref. 20, have investigated the effect of the FTD electron mass shift on primordial nucleosynthesis and found the effects to be unimportant.

The calculations in refs. 17-20,22,23 use a representation for the FTD propagators introduced in ref. 6, known as the real time formalism (in contrast to Matsubara's imaginary time formalism). Niemi and Semenoff[24] have pointed out that the applicability of this formalism is extremely limited but can be extended by giving the FTD propagators a matrix structure and introducing a new set of Feynman rules. The matrix structure for the propagators and the Feynman rules are identical to those obtained by Ojima,[25] and by Umezama, Matsumoto and Tachiki[4] in formulating Thermo Field Dynamics.

In this thesis the real time (matrix) propagator formalism is used to obtain the one loop QED correction to the dispersion relation for a chirally invariant spin 1/2 fermion in the presence of a thermal background. It should be emphasized that the primary thermodynamic variables are temperature and chemical potential. The broken chiral symmetry phase of the theory is also studied and the effective mass calculated. A summary of this work can be found in refs. 19 and 21. Certain aspects of this problem have already been investigated,[14,18,20,26-28] but the literature cannot be condered either complete or even consistent. In an effort to remedy this situation, the calculations done here will emphasize two aspects of the problem:

1. Previous calculations rarely if ever include finite μ effects. While they turn out to be of a similar nature to the finite T effects when $\mu>0$ (particle and background of opposite sign for net charge), they can be different when $\mu<0$ (particle and background with the same sign for net charge).

2. The calculations done here disagree in some respects with those of other workers - the main differences being in the high temperature fermion mass shift and the renormalization procedure for obtaining the physical mass. An attempt is made here not only to produce, what is hoped to be, a correct one loop calculation, but also discuss the differences between the results presented here and those found elsewhere.

The remainder of this work is organized as follows: Chapter 2 contains a derivation of the FTD propagators for a scalar field in both the imaginary time and real time formalisms, and a discussion of the relative merits of the two. In order to give the treatment some unity, both formalisms are derived from a path integral representation of the generating functional. Spinor and gauge field propagators will then be written down in analogy with the scalar field case. In Chapter 3 the general structure of the self-energy and the renormalized FTD propagator are considered from the standpoint of Lorentz invariance. Some subtleties regarding the renormalization procedure required to obtain the physical mass are also discussed. Chapter 4 contains the details of the calculation of the one loop FTD dispersion relation for a chirally invariant fermion, and a discussion of the effective mass for these particles. Chapter 5 contains the details of the calculation of the FTD mass shift for a fermion with broken chiral symmetry. As an application, the mass shift for an electron in a neutron star is calculated. The results obtained in Chapter 5 are also compared with those given in refs. 18 and 20, and the differences discussed. Chapter 6 contains a summary of the conclusions. An appendix on the FTD chain approximation is also included.

In the interest of clarity, the following notation will be used consistently throughout the thesis:

1) quantities with a zero subscript (e.g. Σ_0) denote $T=\mu=0$ terms;

2) quantities with a β subscript (e.g. Σ_β) denote FTD terms;

3) quantities with no subscript denote the sum of the $T=\mu=0$ and FTD terms (e.g. $\Sigma = \Sigma_0+\Sigma_\beta$);

4) matrices in FTD space are denoted by capital letters (with or without latin superscript indices) and a bar over the letter.

Chapter 2

FTD Propagators

A. Path Integral Formulation

We will use the path integral formulation to derive the FTD propagators, but before doing this we briefly review its application to the T=μ=0 case. A much more detailed derivation can be found in ref. 29 and the second listing in ref. 1.

The Feynman path integral for the vacuum-to-vacuum transition amplitude for a scalar field is

$$F = \langle 0|0\rangle = N \int D[\pi]\, D[\phi]\, e^{i\int d^4x[\hat{\pi}\dot{\hat{\phi}}-\hat{\mathcal{H}}(\pi,\phi)]} \tag{2.1}$$

where $|0\rangle$ is the vacuum or ground state, $\mathcal{H}$ is the hamiltonian density appropriate for a field theory characterized by the lagrangian density

$$\mathcal{L}_0 = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2} m_0^2 \phi^2$$

and π is the momentum density conjugate to the field;

$$\pi = \frac{\partial\mathcal{H}}{\partial\dot{\phi}} .$$

In order to make (2.1) calculable perturbatively we make F a functional of a c-number variable J; $F \to F[J]$ where

$$F[J] = N \int D[\pi]D[\phi]\, e^{i\int d^4x(\hat{\pi}\dot{\hat{\phi}}-\hat{\mathcal{H}}+J\hat{\phi})} = \langle 0|e^{i\int d^4x J\hat{\phi}}|0\rangle . \tag{2.2}$$

Performing the gaussian integration over $D[\pi]$ has the simple effect of changing the normalization constant from N to N';

$$F[J] = N' \int D[\phi]\, e^{i\int d^4x(\hat{\mathcal{L}}_0+J\hat{\phi})} . \tag{2.3}$$

In order to do the $D[\phi]$ integration, we first rewrite the argument of the exponential in terms of its Fourier transform, and then, making the change of variable

$$\phi'(p) = \phi(p) + \Delta_0(p)\, J(p)$$

leads to

$$F[J] = e^{-\frac{i}{2}\int d^4p J(p)\Delta_0(p)J(-p)} \int D[\phi] e^{i\int d^4x \hat{\mathcal{L}}_0}$$

$$= F[0] e^{-\frac{i}{2}\int d^4x d^4y J(x)\Delta_0(x-y)J(y)} \quad . \tag{2.4}$$

The function

$$\Delta_0(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip\cdot(x-y)}}{p^2-m_0^2+i\varepsilon} \tag{2.5}$$

is the scalar particle propagator.

From (2.3) it is easy to see that

$$\frac{-1}{F[0]} \frac{\delta^2 F[J]}{\delta J(x)\delta J(y)}\bigg|_{J=0} = \frac{N'}{F[0]} \int D[\phi]\hat{\phi}(x)\hat{\phi}(y) e^{i\int d^4x \hat{\mathcal{L}}_0} \quad .$$

Furthermore, the rhs only has meaning when $t_1 > t_2$,[30] so

$$\frac{-1}{F[0]} \frac{\delta^2 F[J]}{\delta J(x)\delta J(y)}\bigg|_{J=0} = \frac{\langle 0|T(\hat{\phi}(x)\ \hat{\phi}(y))|0\rangle}{\langle 0|0\rangle}$$

$$= i\Delta_0(x-y) \tag{2.6}$$

where T is the time ordering symbol and the second line in (2.6) follows from (2.4). We conclude from (2.6) that $F[J]$ is the generating functional for the scalar particle propagator. In the preceeding discussion, only free fields have been treated. The extension to interacting fields is well known,[29] and of course, does not effect results (2.5) and (2.6). This is also true for FTD theories.

Turning now to FTD theories, we define the FTD Feynman propagator for a scalar field in analogy with the T=μ=0 theory;

$$i\Delta(x-y) \equiv \frac{\mathrm{Tr}\{\hat{\rho}_G \, T[\hat{\phi}(x)\,\hat{\phi}(y)]\}}{\mathrm{Tr}\,\hat{\rho}_G} \tag{2.7}$$

where $\hat{\rho}_G = e^{-\beta\hat{H}}$ is the quantum mechanical density operator, $\hat{H}$ is the system hamiltonian operator, and β is a Lorentz invariant defined as the inverse temperature as measured in the reference frame of the heat bath. Taking a functional approach

$$\hat{\rho}_G \to \hat{\rho}_G[J] = e^{-\beta\hat{H}}\, \mathrm{T} e^{i\int d^4x J\hat{\phi}}$$

gives the relation

$$i\Delta(x-y) = \frac{-1}{Z[0]} \frac{\delta^2 Z[J]}{\delta J(x)\delta J(y)}\bigg|_{J=0} \tag{2.8}$$

where

$$Z[J] = \mathrm{Tr}\,\hat{\rho}_G[J] \ .$$

Noting that

$$\mathrm{Tr}\,\hat{A}[\phi] = \int D[\phi]\langle\phi|\hat{A}|\phi\rangle$$

$Z[J]$ can be written as

$$\int D[\phi]\,\langle\phi|e^{-\beta\hat{H}}\,\mathrm{T}e^{i\int d^4x J\hat{\phi}}|\phi\rangle \ . \tag{2.9}$$

The $|\phi\rangle$'s form a complete set of states and are eigenvectors of the operator ϕ with similarly labelled eigenvalues, i.e.

$$\hat{\phi}|\phi\rangle = \phi|\phi\rangle \ . \tag{2.10}$$

If support of the field variables for $\phi = \phi(t,\vec{x})$ is extended to the complex time domain then

$$\langle\phi(t,\vec{x})|e^{-\beta\hat{H}} = \langle\phi(t-i\beta,\vec{x})| \ . \tag{2.11}$$

Hence,

$$Z[J] = \int D[\phi] \; < \phi(t-i\beta,\vec{x}) | T e^{i\int d^4x J\hat{\phi}} | \phi(t,\vec{x}) > . \qquad (2.12)$$

A consequence of (2.10) and (2.12) is that only those states for which $\phi(t-i\beta,\vec{x}) = \phi(t,\vec{x})$ will contribute to the integral. In other words, we are concerned only with states that are periodic (anti-periodic for fermions) in the complex time variable. The space of the field is thus spanned by integrating over all 3-space and from t to t-iβ in the time direction. Writing this as a contour integral in the complex t-plane

$$Z[J] = \int_{t\,c} D[\phi(t)] \; < \phi(t-i\beta,\vec{x}) | T e^{i\int_c dt \int d^3\vec{x} J\hat{\phi}} | \phi(t,\vec{x}) > . \qquad (2.13)$$

The notation $\int_{t\,c} D[\phi(t)]$ is interpreted as

$$\int \Pi \; d\phi(t_1,\vec{x}_1) \; d\phi(t_2,\vec{x}_2) \ldots$$

with the t_i's constrained to lie on the contour and the $\vec{x}_i$'s unconstrained.

As emphasized in ref. 31, the choice of contour for evaluating (2.13) is not unique - the only requirements on it being

$$c_i - c_f = i\beta \qquad (2.14)$$

$$\mathrm{Im}t_1 > \mathrm{Im}t_2 \qquad (2.15)$$

where c_i and c_f are respectively the initial and final points on the contour, and t_1 occurs before t_2 when moving along the contour. Figs. 1 and 2 show two contours which are consistent with requirements (2.14) and (2.15). If the contour c_I is chosen, the imaginary time formalism is obtained, whereas contour c_R leads to real time formalism.

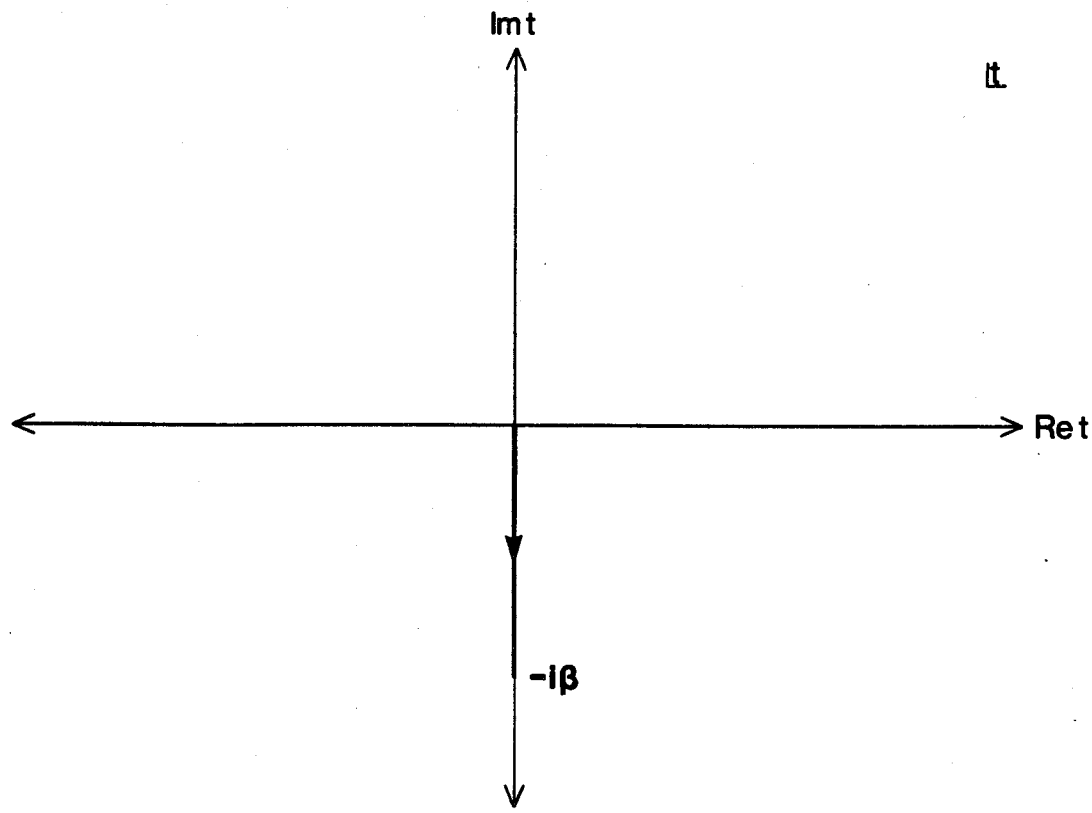

Fig. 1. Contour used to obtain the imaginary time formalism.

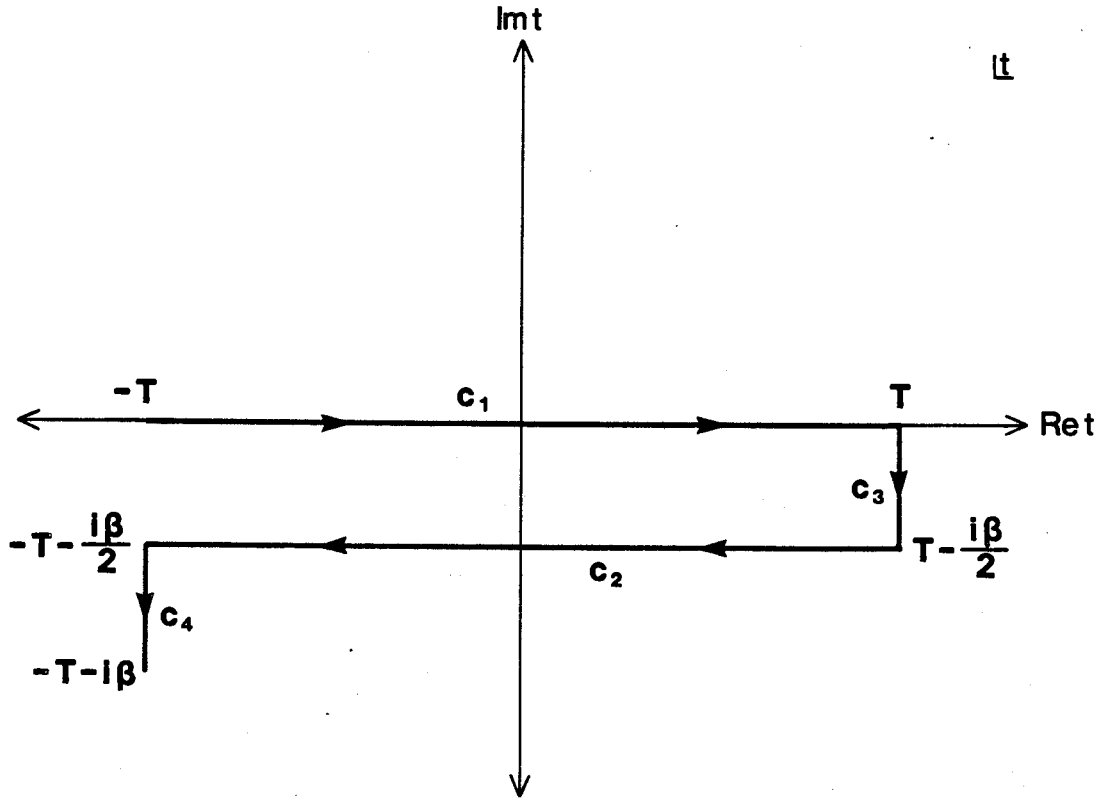

Fig. 2. Contour used to obtain the real time formalism.

B. Imaginary time formalism

Using c_I, (2.13) becomes

$$Z[J] = \int_{t\ c_I} D[\phi(t)] \langle \phi(t-i\beta,\vec{x}) | T e^{i\int_0^{-i\beta} dt \int d^3\vec{x} J\phi} | \phi(t,\vec{x}) \rangle . \qquad (2.16)$$

Eq. (2.16) is very similar to (2.2) the only difference being that now the integration in the exponent is finite in t and the $D[\phi]$ integration is periodic. Following steps similar to the $T=\mu=0$ case we find

$$Z[J] = Z[0]\ e^{-\frac{1}{2}\int_0^\beta d\tau_x d\tau_y \int d^3\vec{x} d^3\vec{y} J(x) i\Delta(x-y) J(y)}$$

where

$$d\tau = -idt$$

and $\Delta(x-y)$ is given by a spatial Fourier transform and a temporal Fourier series, viz.

$$\Delta(x-y) = \frac{1}{\beta} \sum_n \int \frac{d^3p}{(2\pi)^3} \frac{e^{i\vec{p}\cdot(\vec{x}-\vec{y}) - i\omega_{bn}(\tau_x-\tau_y)}}{\omega_{bn}^2 - \vec{p}^2 - m_0^2} \qquad (2.17)$$

$$\omega_{bn} = \frac{2\pi i n}{\beta} \qquad n = 1, 2, \ldots$$

As before

$$\frac{-1}{Z[0]} \frac{\delta^2 Z[J]}{\delta J(x)\ \delta J(y)} \bigg|_{J=0} = i\Delta(x-y) .$$

Thus, the only difference between the $T=\mu=0$ and FTD cases is that the 0 component of the momentum in the FTD propagator is a discrete function of energy rather than a continuous one. While all of our derivations have been for a free scalar field, the results still hold for interacting fields, including the correspondence between $T=\mu=0$ and FTD propagators. This leads us to the following important conclusion:

FTD imaginary time Feynman rules are obtained from their T=μ=0 counterparts by the following substitutions;[13]

$$\int \frac{d^4p}{(2\pi)^4} \to \frac{1}{\beta} \sum_n \int \frac{d^3\vec{p}}{(2\pi)^3} \quad ,$$

$$p_o \to \omega_n$$

and $$(2\pi)^4 \, \delta^4(p_1+p_2+\ldots) \to (2\pi)^3 \, \beta\delta(\vec{p}_1+\vec{p}_2+\ldots) \, \delta(\omega_{n1}+\omega_{n2}+\ldots).$$

Similar results hold for fermion and gauge fields

$$iS(p) = \frac{i}{(\omega_{fn}+i\mu)\gamma^o - \vec{p}\cdot\vec{\gamma}-m_o} \qquad \omega_{fn} = \frac{(2n+1)\pi i}{\beta} \tag{2.18}$$

$$iD_{\mu\nu}(p) = \frac{-ig_{\mu\nu}}{\omega_{bn}^2 - \vec{p}^2} \tag{2.19}$$

C. Real time formalism

The real time formalism is obtained from (2.13) by using contour c_R as shown in fig. 2. Following procedures that are analogous to the $T=\mu=0$ case above we find

$$Z[J] = e^{-\frac{i}{2}\int_{c_R} d^4x d^4y J(x)\Delta(x-y)J(y)} \tag{2.20}$$

where c_R is the contour composed of segments c_1, c_3, c_2, and c_4 in that order.

The limit $T\to\infty$ $(-T \to -\infty)$ is now taken and, by virtue of Riemann's lemma, results in a decoupling of the two-point functions for adjacent segments (in time) of the contour. That is, two-point functions that connect points along c_1 or c_2 to points along c_3 or c_4 vanish. Hence

$$Z[J] = e^{-\frac{i}{2}\int_{c_1,c_2} J\Delta J \int_{c_3,c_4} J\Delta J} .$$

The second integral is uninteresting here and is simply absorbed into the normalization leaving

$$Z[J] = e^{-\frac{i}{2}\int_{c_1,c_2} J\Delta J} . \tag{2.21}$$

There are actually four separate terms in the exponent of (2.21); one each to connect points on the same contour segment and two terms which connect points on different segments. Explicitly

$$Z[J] = e^{-\frac{i}{2}\left[\int_{c_1} dt \int_{c_1} dt' \int d^3\vec{x} \int d^3\vec{y} J\Delta J + \int_{c_2 c_2} + \int_{c_1 c_2} + \int_{c_2 c_1}\right]} .$$

These four terms can be arranged in the form of a matrix product

$$(J_1 \; J_2) \begin{pmatrix} \Delta^{11} & \Delta^{12} \\ \Delta^{21} & \Delta^{22} \end{pmatrix} \begin{pmatrix} J_1 \\ J_2 \end{pmatrix} .$$

It is now easy to see that the original field actually has two components; a type 1 field (corresponding to the c_1 segment) and a type 2 "ghost field" (corresponding to the c_2 segment). The four components of the matrix propagator connect the two components of the field.[32]

The Feynman rules for this theory are the same as for the $T=\mu=0$ theory with the modification that every $T=\mu=0$ diagram generates a set of FTD diagrams that are distinguished from each other by creating all possible combinations of type 1 and type 2 vertices (which determine the appropriate component of the propagator matrix for each line). There is further the proviso that external lines must be of type 1. We give a very simple example of the application of these rules below. A much more sophisticated example can be found in ref. 34.

The self-energy graph for a $T=\mu=0$ ϕ^3 theory (including external legs) is shown in fig. 3a, while its FTD extension is shown in fig. 3b. The numbers at each vertex determine which component of the propagator matrix is appropriate for each diagram. Thus, for example, the self energy insertion for the second diagram in fig. 3b is calculated by integrating the product of two 12 components of the propagators.

Explicit results for scalar, fermion, and photon propagators are shown below.

For scalars

$$i\bar{\Delta}^{ab}(p) = \bar{U}(\theta_p)\begin{pmatrix} i\Delta_0(p) & 0 \\ 0 & -i\Delta_0^*(p) \end{pmatrix}\bar{U}(\theta_p) \tag{2.22}$$

$$\bar{U}(\theta_p) = \begin{pmatrix} ch\theta_p & sh\theta_p \\ sh\theta_p & ch\theta_p \end{pmatrix} \quad sh^2\theta = \frac{1}{e^{\beta|p\cdot u|}-1} \quad \Delta_0(p) = \frac{1}{p^2-m_0^2+i\varepsilon}\,.$$

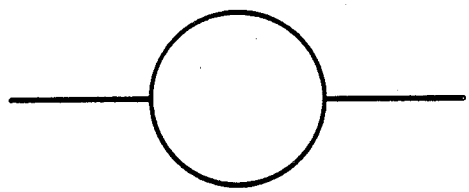

Fig. 3a. One-loop T=μ=0 self-energy for a ϕ^3 theory.

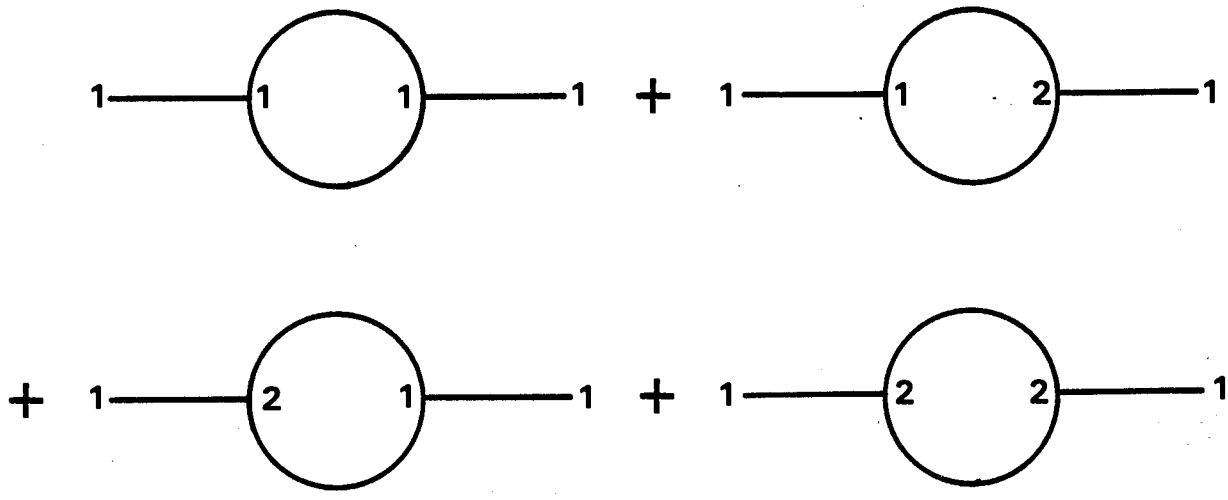

Fig. 3b. One-loop FTD self-energy for a ϕ^3 theory.

For photons (Feynman gauge):

$$[i\bar{D}^{\mu\nu}(p)]^{ab} = -g^{\mu\nu}\,\bar{U}(\theta_p)\begin{pmatrix} iD_0(p) & 0 \\ 0 & -iD_0^*(p) \end{pmatrix}\bar{U}(\theta_p) \tag{2.23}$$

$$D_0(p) = \frac{1}{p^2+i\varepsilon} \quad .$$

For fermions:[35]

$$i\bar{S}^{ab}(p) = \theta(p_0)\,\bar{V}(p,\mu)\begin{pmatrix} iS_0(p) & 0 \\ 0 & -iS_0^*(p) \end{pmatrix}\bar{V}(p,\mu)$$

$$+\,\theta(-p_0)\,\bar{V}^\dagger(p,-\mu)\begin{pmatrix} iS_0(p) & 0 \\ 0 & -iS_0^*(p) \end{pmatrix}\bar{V}^\dagger(p,-\mu) \tag{2.24}$$

$$\bar{V}(p,\pm\mu) = \begin{pmatrix} \cos\psi_p^\pm & -e^{\mp\beta\mu/2}\sin\psi_p^\pm \\ e^{\pm\beta\mu/2}\sin\psi_p^\pm & \cos\psi_p^\pm \end{pmatrix}$$

$$\sin^2\psi_p^\pm = \frac{1}{e^{\beta[|p\cdot\mu|\pm\mu]}+1} \qquad S_0(p) = \frac{1}{\not{p}-m_0+i\varepsilon}$$

The quantity u^α in the preceeding equations is the 4-velocity of the heat bath normalized by $u_\alpha u^\alpha = 1$. Thus the quantity $p\cdot u$ is a Lorentz invariant equal to p_0 as measured in the rest frame of the heat bath.

By multiplying out the matrices in (2.21)-(2.24) and noting that

$$\lim_{\varepsilon\to 0}\frac{\varepsilon}{x^2+\varepsilon^2} = \pi\delta(x)$$

the propagators can be written as

$$i\bar{\Delta}^{ab}(p) = i\bar{\Delta}_0^{ab}(p) + i\bar{\Delta}_\beta^{ab}(p) \tag{2.25a}$$

where

$$i\bar{\Delta}_0^{ab} = \begin{pmatrix} i\Delta_0(p) & 0 \\ 0 & -i\Delta_0^*(p) \end{pmatrix} \tag{2.25b}$$

and

$$i\bar{\Delta}_\beta^{ab} = 2\pi\delta(p^2-m_0^2)\begin{pmatrix} \mathrm{sh}^2\theta_p & \frac{1}{2}\,\mathrm{sh}2\theta_p \\ \frac{1}{2}\,\mathrm{sh}2\theta_p & \mathrm{sh}^2\theta_p \end{pmatrix} \tag{2.25c}$$

$$[i\bar{D}^{\mu\nu}(p)]^{ab} = [i\bar{D}_o^{\mu\nu}(p)]^{ab} + [i\bar{D}_\beta^{\mu\nu}(p)]^{ab} \tag{2.26a}$$

where

$$[i\bar{D}_o^{\mu\nu}(p)]^{ab} = -g^{\mu\nu}\begin{pmatrix} iD_o(p) & 0 \\ 0 & -iD_o^*(p) \end{pmatrix} \tag{2.26b}$$

and

$$[i\bar{D}_\beta^{\mu\nu}]^{ab} = -2\pi g^{\mu\nu}\,\delta(p^2)\begin{pmatrix} sh^2\theta_p & \frac{1}{2}\,sh2\theta_p \\ \frac{1}{2}\,sh2\theta_p & sh^2\theta_p \end{pmatrix} \tag{2.26c}$$

$$i\bar{S}^{ab}(p) = i\bar{S}_o^{ab}(p) + \theta(p_o)i\bar{S}_{\beta+}^{ab} + \theta(-p_o)i\bar{S}_{\beta-}^{ab} \tag{2.27a}$$

where

$$i\bar{S}_o^{ab}(p) = \begin{pmatrix} iS_o(p) & 0 \\ 0 & -iS_o^*(p) \end{pmatrix} \tag{2.27b}$$

and

$$i\bar{S}_{\beta\pm}^{ab}(p) = -2\pi(\not{p}+m_o)\,\delta(p^2-m_o^2)\begin{pmatrix} \sin^2\psi_p^{\pm} & \pm\frac{1}{2}\,e^{-\beta\mu/2}\,\sin2\psi_p^{\pm} \\ \mp\frac{1}{2}\,e^{\beta\mu/2}\,\sin2\psi_p^{\pm} & \sin^2\psi_p^{\pm} \end{pmatrix}. \tag{2.27c}$$

Notice that the propagators have now separated into temperature independent and temperature dependent pieces.

D. Comparison of the Formalisms

Historically, the imaginary time formalism has been used much more widely than the real time formalism - partly because it was discovered first and partly because the older version[6] of the real time formalism did not give correct results beyond the one loop level. There are however, some disagreeable aspects to the imaginary time formalism, which arise from the special connection between time and temperature. In this formalism Lorentz covariance and manifest Lorentz invariance are lost as can be seen by inspecting (2.17)-(2.19). More importantly, the formalism as it stands is only defined at certain points in the complex ω-plane. Thus, in order to study dynamical aspects of a system, an analytic continuation to the whole real axis must be performed. This is not always easy or unambigious. There is a further technical difficulty that multiple sums over ω in higher order diagrams can be difficult to perform.

In contrast, the real time formalism maintains Lorentz covariance and manifest Lorentz invariance and is defined for all real ω. There is the additional feature that the real time propagators separate into $T=\mu=0$ and FTD parts. The great disadvantage of this formalism is that a single $T=\mu=0$ diagram can generate many FTD diagrams whereas there is a one to one correspondence between $T=\mu=0$ and FTD diagrams when the imaginary time formalism is used.

Chapter 3

Structure of the Self-Energy and Renormalization

Before actually calculating the self-energy corrections, we find it necessary to investigate its general structure and the attendant renormalization procedure.

A. Self-energy for chirally invariant fermions[36]

For $T=\mu=0$ QED, the bare fermion propagator is

$$S_0(K) = \frac{1}{\not{K}+i\varepsilon} \quad . \tag{3.1}$$

Using the chain approximation,[1] we find that the full propagator is

$$\mathcal{S}_0(K) = \frac{1}{\not{K}-\Sigma_0(K)+i\varepsilon} \tag{3.2}$$

where

$$\Sigma_0(K) = -a_0\not{K} \quad . \tag{3.3}$$

The structure of (3.3) follows from two observations:

1) $\Sigma_0(K)$ is a Lorentz invariant

2) Chiral symmetry holds to all orders of perturbation theory.

From the first observation we can deduce that Σ_0 can be written as a linear combination of all Lorentz invariants found in the theory, of which there are only two; $\not{K}$ and K^2. This also implies that the coefficients (such as a_0) must also be Lorentz invariants. From the second observation $K^2=0$ always and Σ_0 reduces to (3.3).

For FTD QED the 11 component of the full propagator is

$$\mathcal{S}^{11}(K) = \frac{1}{\not K - \Sigma(K) + i\varepsilon} - \left(\theta(K_o)\sin^2\psi_K^+ + \theta(-K_o)\sin^2\psi_K^-\right) \times \left[\frac{1}{\not K - \Sigma + i\varepsilon} - \frac{1}{\not K - \Sigma^* - i\varepsilon}\right] \tag{3.4}$$

where

$$\Sigma = \Sigma_o + \Sigma_\beta$$

with

$$\Sigma_\beta(K) = -a_\beta \not K - b \not u \ . \tag{3.5}$$

and we have set $\mu=0$ for simplicity. We see that Σ has an extra term which is related to the heat bath 4-velocity, u^α.[37] For the remainder of this discussion it is sufficient to consider only the first term in (3.4). Dropping the other term leaves

$$\mathcal{S}^{11}(K) = \frac{(1+a_\beta)\not K + b \not u}{D} \tag{3.6}$$

where

$$D = (1+a_\beta)^2 \ (\omega^2 - k^2) + 2(1+a_\beta)b\omega + b^2 \tag{3.7}$$

and we have introduced the Lorentz invariant functions

$$\omega = K \cdot u$$

$$k = [\omega^2 - K^2]^{1/2}$$

which have the simple interpretation of the particle's energy and 3-momentum in the rest frame of the heat bath. Eq. (3.7) also assumes that he $T=\mu=0$ wave function regularization has been performed, hence, a_o is not present in (3.7).

For both the $T=\mu=0$ and the FTD theories, the excitations of the system (or, in the language of field theory, the physical masses) are governed by a dispersion relation which yields the poles of the propagator. The structure of (3.3) insures that the poles are always located at $K^2=0$ for the $T=\mu=0$

case. On the other hand, the poles of the FTD propagtor are determined by setting D=0 which gives the dispersion relation

$$\omega = k - \frac{b}{1+a_\beta} \quad . \tag{3.8}$$

The solution of (3.8) is not $K^2=0$, and this shift in the location of the pole is entirely due to the presence of the second term in (3.5).

B. Self-energy for fermions without chiral symmetry

Turning now to the massive case, the $T=\mu=0$ renormalized fermion propagator is

$$\mathscr{S}_0(K) = \left(\not{K} - m_B - \Sigma_0(K)\right)^{-1} \tag{3.9}$$

where

$$\Sigma_0(K) = - a_0(K^2) - d_0(K^2) \tag{3.10}$$

and m_B is the bare mass. Note that Σ_0 now has a K^2 dependence. Eq. (3.9) can also be written as

$$\mathscr{S}_0(K) = \frac{\not{K} + m_B - d_0(K^2)}{D}$$

where

$$D(K) = K^2 - \left(m_B - d_0(K^2)\right)^2$$

and we have absorbed the wave function renormalization constant, $(1+a_0)$, as it is unimportant in what follows. If we now choose

$$d_0(m_0^2) = \delta m_0 \ , \tag{3.11}$$

with m_0, the physical mass, related to m_B by

$$m_B = m_0 + \delta m_0$$

then

$$D(m_0^2) = 0 \ ,$$

and we can interpret m_0 as the physical mass. What we wish to emphasize here is not so much that renormalization condition (3.11) is defined at the $T=\mu=0$ mass, but rather that it is defined at the physical mass. While there is no distinction between the two here, the difference will be important for FTD theories.

For the FTD fermion full propagator ($\mu=0$), the 11 component is

$$\frac{1}{\not K - m_B - \Sigma(K) + i\epsilon} + \text{(other terms)} \tag{3.12}$$

where

$$\Sigma_\beta(K) = -a_\beta \not K - b\not u - d_\beta \ . \tag{3.13}$$

As before, Σ_β is a Lorentz invariant, hence a_β, b and d_β must also be Lorentz invariants. Keeping only the first term in (3.12) for the discussion here leaves

$$\mathcal{S}^{11}(K) = \frac{(1+a_\beta)\not K + b\not u + (m_B - d_o - d_\beta)}{D} \tag{3.14}$$

where

$$D(K) = (1+a_\beta)^2 K^2 + b^2 + 2(1+a_\beta)\, bK\cdot u - (m_B - d_o - d_\beta)^2 \ . \tag{3.15}$$

In analogy with the T=0 theory we define the physical mass m, in the following way;

$$\mathrm{Re}D(K)\big|_{K^2=m^2} = 0 \ . \tag{3.16}$$

Two popular methods for obtaining the physical mass are:

1) define it by[38]

$$m = m_B + \Sigma(K)\big|_{K^2=m_o{}^2} \ , \tag{3.17}$$

2) first invoke condition (3.11) and then require[39]

$$\mathrm{Re}\ D_1(K)\big|_{K^2=m^2} = 0 \ , \tag{3.18}$$

where

$$D_1(K) = (1+a_\beta)^2 K^2 + b^2 + 2(1+a_\beta)\, bK\cdot u - (m_o - d_\beta)^2 \ . \tag{3.19}$$

Neither of these methods gives a mass which satisfies (3.16) as can be seen by simply substituting (3.17) and (3.18) into (3.16). For the first method we find D(m) is a complicated expression in a_β, b, and d_β, but should approach zero if these constants are small. In other words, (3.9)

provides a first approximation to m for sufficiently small temperatures. For the second method we find

$$\mathrm{ReD}(m) = d_0(m^2) - d_0(m_0^2) \neq 0 \ . \tag{3.20}$$

Equation (3.20) clearly illustrates the source of the problem. Doing the T=0, and T≠0 renormalizations separately has the effect of specifying the renormalization condition for the T=0 term at a point which is off-shell. It is not surprising then, that when the self-energy is evaluated on-shell, the T=0 mass counterterm no longer cancels the bare mass infinity. In order to insure the infinities properly cancel we must use

$$d_0(m^2) = \delta m_0 \tag{3.21}$$

as our renormalization condition.

A few comments regarding (3.21) are in order:

1. The above discussion does not imply that (3.21) is the only valid FTD renormalization condition. As with T=μ=0 theories, one can renormalize at any convenient point. However, just as in the T=μ=0 case, it is true for FTD theories that only certain renormalization schemes give quantities which can be readily interpreted in a physical manner. Eq. (3.21) is such a condition.
2. Although the renormalization point specified in (3.21) is T dependent (since m is T dependent), δm_0 has no T dependence. This result has been shown[11,12,40] to hold to all orders of perturbation theory.
3. Since the renormalization prescription varies with temperature, it makes it difficult to compare quantities calculated at different temperatures. This is not a problem here since the mass shift depends only on one temperature. It would, however, be a problem when doing thermodynamics

or any calculation which involved two or more values of T such as differentiating with respect to T. It is quite likely that this problem can be avoided using FTD renormalization group techniques.[41]

The technique for actually carrying out this calculation is straightforward. One proceeds in the same way as method 2 described above employing (3.18) and (3.19). The only difference is that in doing the loop integrals one must set $K^2 = m^2$ instead of $K^2 = m_0^2$. Unfortunately, this usually leads to an implicit equation for m.

Chapter 4

Dispersion Relation for Chirally Invariant Fermions

In order to give an explicit expression for dispersion relation (3.8), the coefficients a_β and b from (3.5) must be calculated. Using the well-known identity

$$\frac{1}{4}\,\mathrm{Tr}(\not{a}\not{b}) = a\cdot b$$

we find[42]

$$\frac{1}{4}\,\mathrm{Tr}(\not{K}\mathrm{Re}\Sigma_\beta) = -a_\beta(\omega^2-k^2) - b\omega \tag{4.1}$$

$$\frac{1}{4}\,\mathrm{Tr}(\not{K}\mathrm{Re}\Sigma_\beta) = -a_\beta\omega - b\ . \tag{4.2}$$

Combining (4.1), (4.2) and (3.8) gives

$$\omega = k + \frac{1/4\ \mathrm{Tr}(\not{K}\mathrm{Re}\Sigma_\beta)}{k + 1/4\ \mathrm{Tr}(\not{u}\mathrm{Re}\Sigma_\beta)} \tag{4.3}$$

for the dispersion relation. The physical mass is obtained from (4.3) by imposing the additional condition that the particle be at rest relative to the heat bath. Hence

$$m = \lim_{k\to 0}\ \omega \tag{4.4}$$

Figure 4 shows the one loop FTD self-energy diagrams that can be used to calculate Σ. Alternatively, we can use (A.17), (A.18) and (A.12) directly (provided we set $m_0 = 0$) and obtain

$$\mathrm{Re}\Sigma(K) = \mathrm{Re}\Sigma_+(K) = e^2\theta(K_0)\ \mathrm{Im}\int \frac{d^4p}{(2\pi)^4}\,\gamma_\mu\left[i\bar{D}_0^{\mu\nu}(p) + i\bar{D}_\beta^{\mu\nu}(p)\right]^{11} \times$$
$$\left[i\bar{S}_0(K-p) + \theta(K_0-p_0)\ i\bar{S}_{\beta+}(K-p) + \theta(p_0-K_0)\ i\bar{S}_{\beta-}(K-p)\right]^{11}\gamma_\nu\ . \tag{4.5}$$

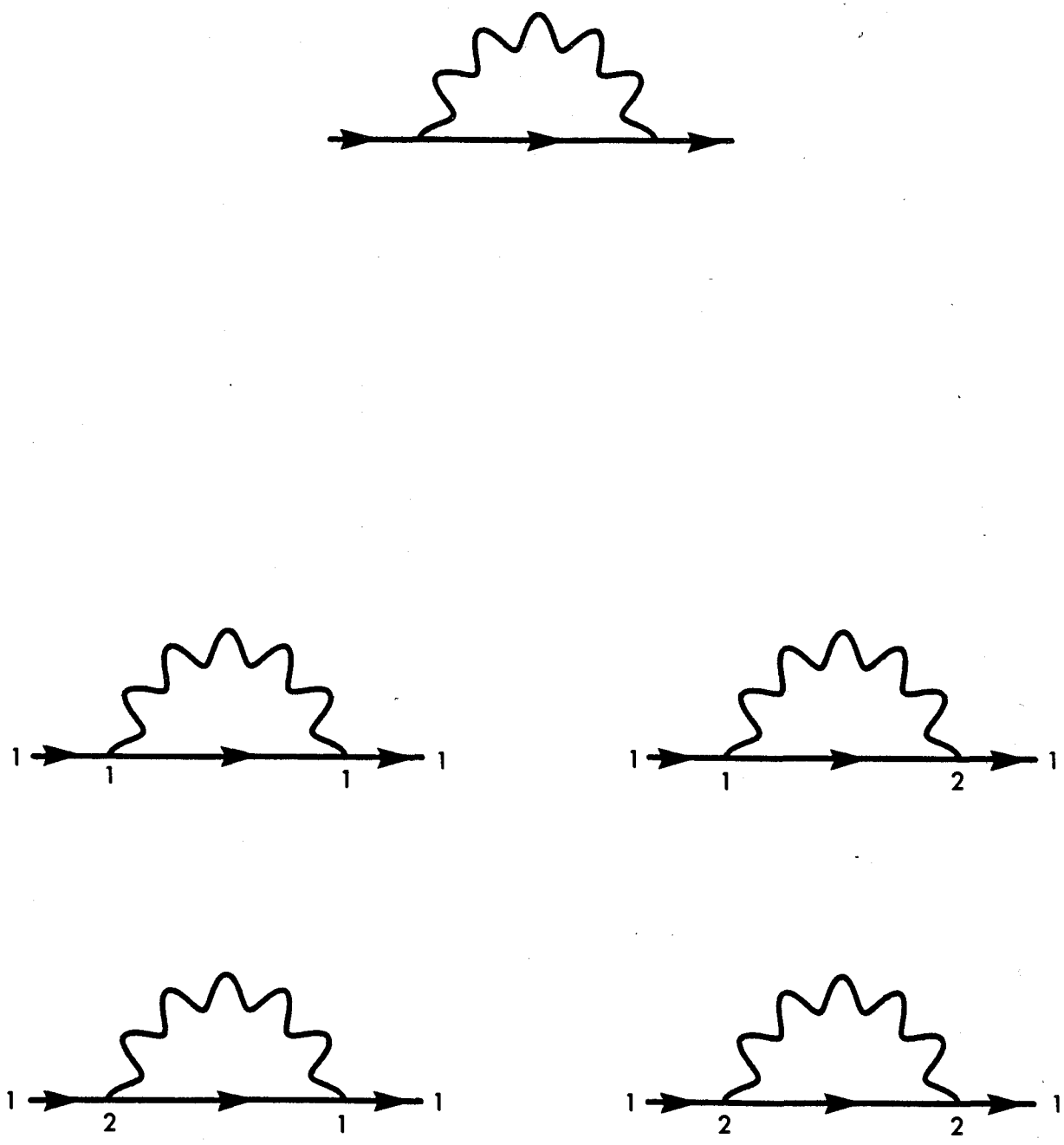

Fig. 4. The one-loop FTD fermion self-energy (including the $T=\mu=0$ term) and its expansion in terms of labelled vertex diagrams. The $T=\mu=0$ counterterm is not shown.

The $T=\mu=0$ part of the propagators are now split into two parts, viz.

$$D_0(p) = P\,\frac{1}{p^2} - i\pi\delta(p^2)$$

$$S_0(p) = \not{p}\left(P\,\frac{1}{p^2} - i\pi\delta(p^2)\right) \tag{4.6}$$

where P stands for the principal value. There will be terms in (4.5) which include the δ-functions of (4.6) but these are the $T=\mu=0$ part of the self-energy, $\Sigma_0(K)$. Dropping these terms we are left with

$$\mathrm{Re}\Sigma_\beta = \frac{\alpha}{\pi^2}\,\theta(K_0)\int d^4p\,\Big\{\frac{\not{p}}{(p+K)^2}\,\delta(p^2)\big(\theta(-p_0)\sin^2\psi_p^+ + \theta(p_0)\sin^2\psi_p^-\big) + \frac{\not{K}-\not{p}}{(K+p)^2}\,\delta(p^2)\,\mathrm{sh}^2\theta_p\Big\} \tag{4.7}$$

where $$\alpha \equiv \frac{e^2}{4\pi}\ .$$

A change of variables: $p \to p+K$ for the first term and $p \to -p$ for the second term has been made. $\frac{1}{4}\mathrm{Tr}(\not{u}\mathrm{Re}\Sigma_\beta)$ and $\frac{1}{4}\mathrm{Tr}(\not{K}\mathrm{Re}\Sigma_\beta)$ can now be evaluated. We find

$$\frac{1}{4}\mathrm{Tr}(\not{K}\mathrm{Re}\Sigma_\beta) = \frac{\alpha}{2\pi}\int_0^\infty dq\Big\{\Big[2q + \frac{K^2}{2k}\Big(\ell n\Big(\frac{\omega_+}{\omega_-}\Big) + \frac{1}{2}\,[J_1(q) - J_2(q)]\Big)\Big]n_f(q+\mu) + \Big[2q - \frac{K^2}{2k}\Big(\ell n\Big(\frac{\omega_+}{\omega_-}\Big) - \frac{1}{2}\,[J_1(q) + J_2(q)]\Big)\Big]n_f(q-\mu) + \Big[4q - \frac{K^2}{2k}\,J_1(q)\Big]n_b(q)\Big\} \tag{4.8}$$

$$\frac{1}{4}\mathrm{Tr}(\not{u}\mathrm{Re}\Sigma_\beta) = \frac{\alpha}{2\pi k}\int_0^\infty dq\Big\{q\,\Big[\ell n\Big(\frac{\omega_+}{\omega_-}\Big) - \frac{1}{2}\,(J_1(q) + J_2(q))\Big]n_f(q+\mu) + q\,\Big[\ell n\Big(\frac{\omega_+}{\omega_-}\Big) + \frac{1}{2}\,(J_1(q) - J_2(q))\Big]n_f(q-\mu) + \Big[2q\,\ell n\Big(\frac{\omega_+}{\omega_-}\Big) - qJ_2(q) - \omega J_1(q)\Big]n_b(q)\Big\} \tag{4.9}$$

where $q = |\vec{p}|$, $\omega_\pm = \frac{1}{2}(\omega \pm k)$ and

$$J_1(q) = \ell n\left(\frac{q+\omega_+}{q+\omega_-}\right) - \ell n\left(\frac{q-\omega_+}{q-\omega_-}\right) .$$

$$J_2(q) = \ell n\left(\frac{q+\omega_+}{q+\omega_-}\right) + \ell n\left(\frac{q-\omega_+}{q-\omega_-}\right) . \qquad (4.10)$$

The distribution functions $n_{f,b}$ are the usual

$$n_{f,b}(x) = [\exp(\beta x) \pm 1]^{-1} .$$

Substituting (4.8) and (4.9) into (4.3) gives the promised implicit equation for ω. However, we can gain some insight into the solution of (4.3) by investigating the regime where T and/or μ are sufficiently large. There the distribution functions in (4.9) and (4.10) will effectively cut off the integration at $q \sim T \pm \mu$. Thus, for the high T/μ limit we need only keep terms in (4.9) and (4.10) which are highest order in q. Noting that $J_{1,2}(q) \sim 1/q$ for large q gives

$$\frac{1}{4}\,\mathrm{Tr}(\not{K}\mathrm{Re}\Sigma_\beta) = \frac{\alpha}{2\pi}\int_0^\infty dq\; 2q\left[n_f(q+\mu) + n_f(q-\mu) + 2n_b(q)\right] \qquad (4.11)$$

$$= M^2$$

$$\frac{1}{4}\,\mathrm{Tr}(\not{k}\mathrm{Re}\Sigma_\beta) = \frac{\alpha}{2\pi k}\int_0^\infty dq\; q\;\ell n\left(\frac{\omega_+}{\omega_-}\right)\left[n_f(q+\mu) + n_f(q-\mu) + 2n_b(q)\right] \qquad (4.12)$$

$$= \frac{1}{2k}\,\ell n\left(\frac{\omega_+}{\omega_-}\right) M^2$$

where

$$M^2 \equiv \frac{\alpha}{\pi}\int_0^\infty dq\; q\left[n_f(q+\mu) + n_f(q-\mu) + 2n_b(q)\right] \qquad (4.13)$$

$$= \frac{\alpha}{2\pi}\left[\mu^2 + \pi^2\, T^2\right] .$$

The dispersion relation can now be written in the form

$$\omega = k + \frac{M^2}{k}\left[1 + \left(1 - \frac{\omega}{k}\right)\frac{1}{2}\ln\left(\frac{\omega_+}{\omega_-}\right)\right] . \qquad (4.14)$$

Taking the $k \to 0$ limit we find

$$\lim_{k\to 0} \omega = M \qquad (4.15)$$

indicating that M is the physical mass.

In order to give the reader a better idea of the meaning of (4.15), eq. (4.14) has been graphed in fig. 5 along with the dispersion relations for a free massless fermion ($\omega=k$) and a free massive fermion with rest mass M ($\omega = (M^2+k^2)^{1/2}$). As can be seen clearly from the graph, the dispersion relation for a massless fermion interacting with a thermal background closely resembles the dispersion relation for a free massive particle. Although the two dispersion relations are identical only in the limits $k \to \infty$ and $k \to 0$, they never vary from each other by more than 10%. In this sense M is a mass.

Figure 6 shows the results of a numerical calculation using (4.8) and (4.9) for a typical value of μ and T and its counterpart obtained from (4.11) and (4.12). As can be seen, the inclusion of terms of smaller power in q do not effect the dispersion relation very much - even for small values of T and μ. In all cases studied ($-1000 \leq \mu \leq 1000$, $0 \leq T \leq 1000$; arbitrary units for T and μ), the difference between the two dispersion relations is never more than 4% and the difference in the effective mass is negligible (<0.2%).

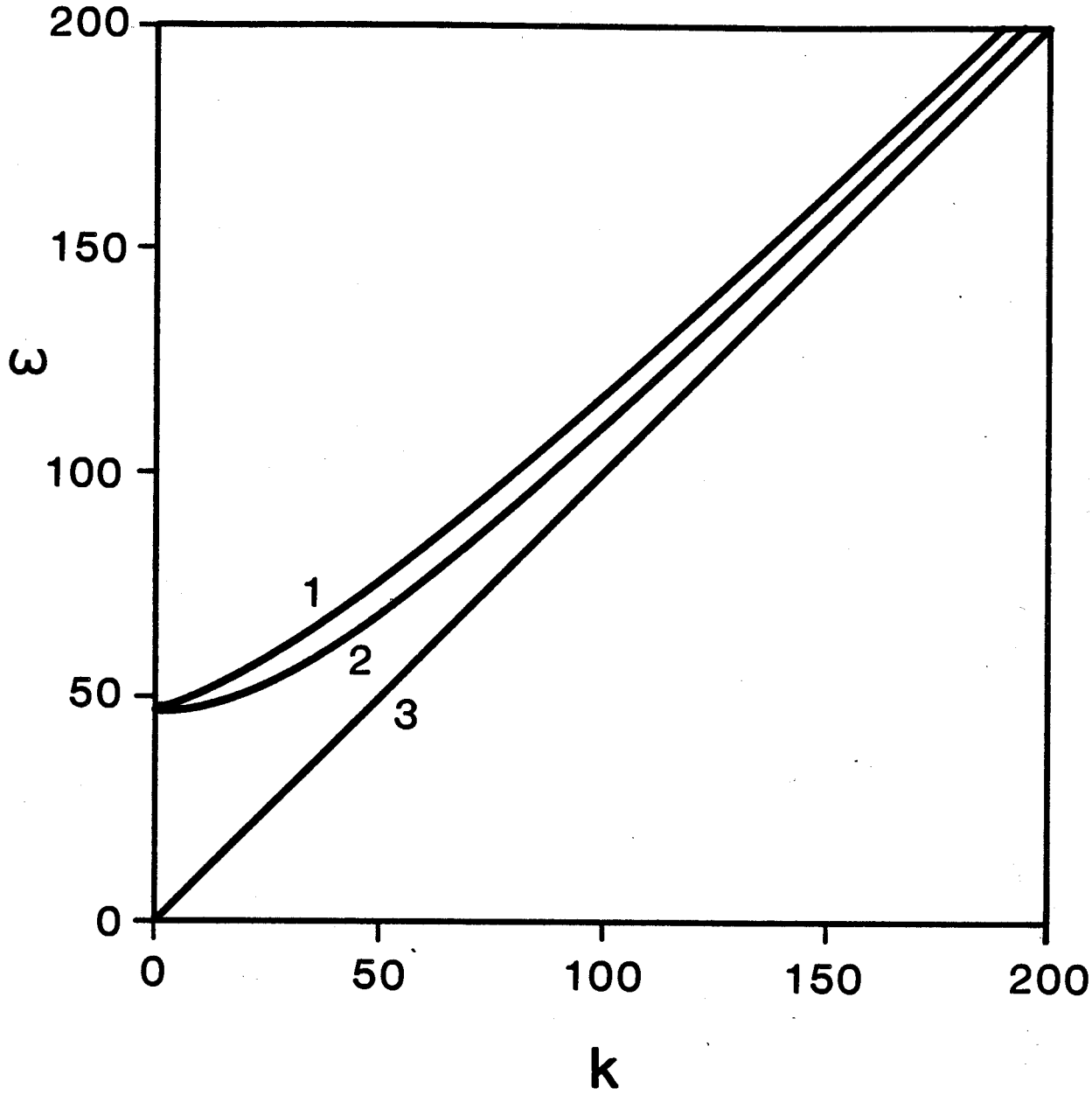

Fig. 5. Dispersion relation for: (1) fermion interacting with a thermal background; (2) free fermion of mass M; (3) free massless fermion. The units on the ω and k axes are arbitrary.

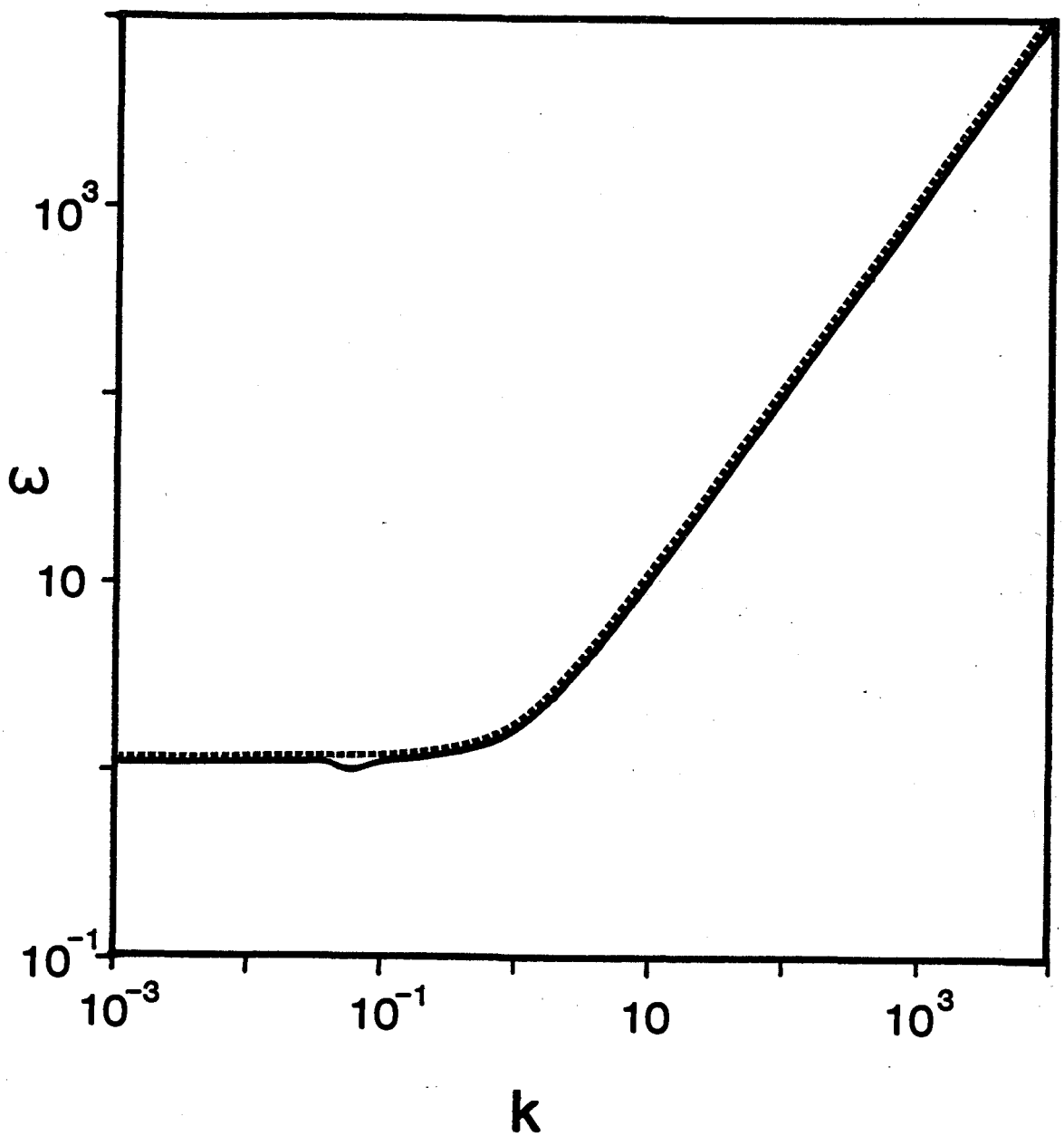

Fig. 6. Comparison of dispersion relations calculated using (4.8) and (4.9) (solid line) and (4.11) and (4.12) (broken line). The bump in the solid line has been exagerated for the purpose of illustration. Units for ω and k are arbitrary.

Chapter 5

Calculation of the Mass Shift for Fermions without Chiral Symmetry

A. General calculation

We begin by rewriting (3.19) in the following way;

$$(1+a_\beta)^2(\omega^2-k^2) + 2(1+a_\beta)b\omega + b^2 - c^2 = 0 \tag{5.1}$$

where

$$c = m_B - d_0(m^2) - \mathrm{Re}d_\beta$$

$$= m_0 - \mathrm{Re}d_\beta \quad . \tag{5.2}$$

Writing (4.1) in the form of a dispersion relation gives

$$\omega = \frac{-b \pm \left[c^2 + (1+a_\beta)^2\, k^2\right]^{1/2}}{1+a_\beta} \quad . \tag{5.3}$$

This expression is rather complicated - especially when one remembers that a_β, b and c are, of necessity, functions of ω and k. Rather than attempt to solve (5.3) for the complete dispersion relation, we will content ourselves with calculating the FTD mass shift which we define by

$$\delta m_\beta \equiv \lim_{k\to 0} \omega - m_0 \quad . \tag{5.4}$$

Applying this definition to (5.3) we find

$$\delta m_\beta = \lim_{k\to 0} \left[\frac{1}{4} \mathrm{Tr}(\not{k}\mathrm{Re}\Sigma_\beta) + \frac{1}{4} \mathrm{Tr}(\mathrm{Re}\Sigma_\beta)\right] \quad . \tag{5.5}$$

In order to calculate δm_β we proceed in a manner similar to the chiral symmetric case. Once again we use either the diagrams in fig. 4 or (A.17), (A.18) and (A.12) to obtain an expression for $\mathrm{Re}\Sigma(K)$ which is formally identical to (4.5). Dropping the $T=\mu=0$ piece leaves

$$\mathrm{Re}\Sigma_\beta = \frac{\alpha}{\pi^2}\,\theta(K_o)\int d^4p\;\frac{\not p+2m_o}{(p+K)^2}\,\delta(p^2-m_o^2)\left(\theta(-p_o)\sin^2\psi_p^+ + \theta(p_o)\sin^2\psi_p^-\right)$$

$$+\frac{\not K-\not p-2m_o}{(K+p)^2-m_o^2}\,\delta(p^2)\,\sinh^2\theta_p \quad . \tag{5.6}$$

Using (5.6) we find

$$\frac{1}{4}\,\mathrm{Tr}(\not K\mathrm{Re}\Sigma_\beta) = \frac{\alpha}{2\pi k}\int_o^\infty dq\,\{[\omega(L_1^+ + L_1^-) + q(L_1^+ - L_1^-)]n_b(q)$$

$$+ q[L_2^+\,n_f(r-\mu) - L_2^-\,n_f(r+\mu)]\} \tag{5.7}$$

$$\frac{1}{4}\,\mathrm{Tr}(\mathrm{Re}\Sigma_\beta) = \frac{\alpha m_o}{\pi k}\int_o^\infty dq\,\left\{\frac{q}{r}\left[L_2^+ n_f(r-\mu) + L_2^- n_f(r+\mu)\right] - [L_1^+ + L_1^-]n_b(q)\right\} \tag{5.8}$$

where

$$L_1^\pm = \pm\ln\left(\frac{q(\omega+k) \pm 1/2(K^2-m_o^2)}{q(\omega-k) \pm 1/2(K^2-m_o^2)}\right) \qquad L_2^\pm = \pm\ln\left(\frac{r\omega+qk \pm 1/2(K^2+m_o^2)}{r\omega-qk \pm 1/2(K^2+m_o^2)}\right)$$

$$r = \left[q^2 + m_o^2\right]^{1/2} .$$

We now take the $k \to 0$ limit and find

$$\frac{\delta m_\beta}{m_o} = I_B + I_{F+} + I_{F-} \tag{5.9}$$

where

$$I_B = \frac{4\alpha w}{\pi}\int_o^\infty dx\; x\,\frac{1 + 2x^2 + 2(w-1/w) - w^2}{4x^2w^2 - (w^2-1)^2}\,\frac{1}{e^{x/t}-1} \tag{5.10}$$

$$I_{F+} = \frac{2\alpha}{\pi}\int_o^\infty dx\,\frac{x^2}{y}\,\frac{y-2}{2yw-w^2-1}\,\frac{1}{e^{(y+v)/t}+1} \tag{5.11}$$

$$I_{F-} = \frac{2\alpha}{\pi}\int_o^\infty dx\,\frac{x^2}{y}\,\frac{y+2}{2yw+w^2+1}\,\frac{1}{e^{(y-v)/t}+1} \tag{5.12}$$

and $\qquad w = \dfrac{\omega}{m_o} \qquad t = \dfrac{1}{\beta m_o} \qquad v = \dfrac{\mu}{m_o} \qquad y = (x^2+1)^{1/2}$.

Once again we have an implicit equation for m which cannot be solved easily. However, in the limit where μ and/or T are sufficiently large

compared to m_0, m_0 can be neglected and we recover (4.13). For small values of μ and T (5.9) can be solved iteratively.

Figure 7 shows the results of a numerical solution to (5.9) for various values of μ. The small T region is shown in more detail in fig. 8, and it can be seen that for $\mu<0$ it is possible to have a negative mass shift, although its magnitude is small. We now investigate two special cases of particular interest; (1) $T\neq0$, $\mu=0$ and (2) $T<0$, $\mu>>m_0$, (neutron stars).

B. Finite temperature, zero chemical potential case

As mentioned in the introduction, this calculation has already been performed several times. For the sake of comparison, fig. 9 shows numerical solutions for the results obtained here and in refs. 18 and 20.
We see that our results agree with ref. 20 for $T << m_0$ but disagree at higher temperatures; the most serious disagreement being for $T >> m_0$, where $\delta m_\beta \sim T^2$ in ref. 20 and we find $\delta m_\beta \sim T$ which agrees with ref. 18. The source of the discrepancy is not hard to find. In evaluating the self-energy, Peressutti and Skagerstam ignore the effect of the heat bath 4-velocity, effectively setting $\frac{1}{4}\,\mathrm{Tr}(\not{u}\,\mathrm{Re}\Sigma_\beta) = 0$ (or, alternatively, b=0). While the Σ_β term proportional to u^α is relatively unimportant at low temperatures, it becomes increasingly important as the temperature increases, and is the most important term for $T >> m_0$. At these temperatures $m >> m_0$ and we get agreement with calculations for massless fermions as we would expect.
As a final point we note that the small effects of the finite temperature electron mass on early universe helium production calculated in ref. 22 and 23 which assumed $\delta m_\beta \sim T^2$ will be reduced even further.

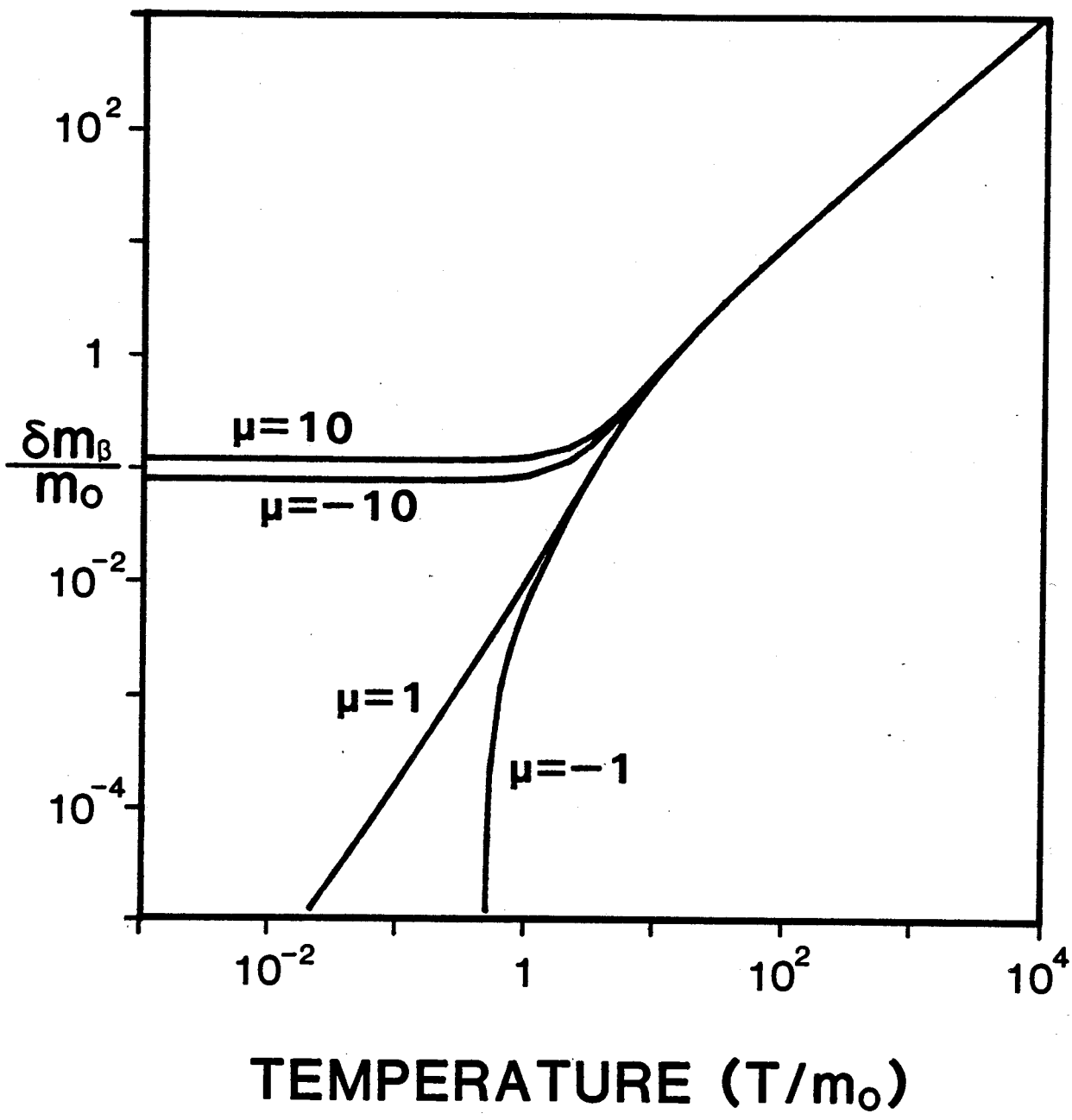

Fig. 7. FTD mass shift as a function of temperature for various values of μ.

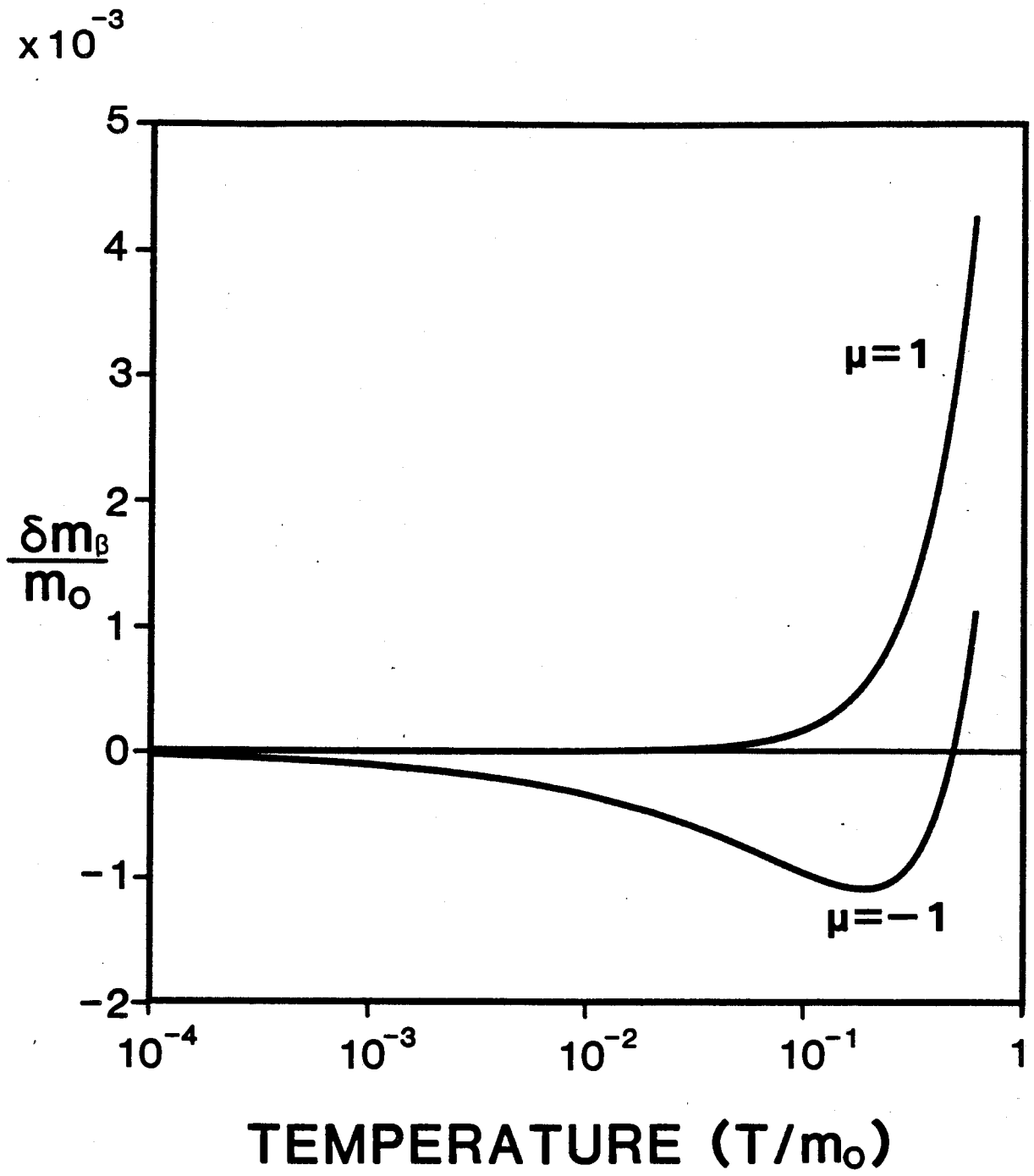

Fig. 8. Region of negative values for the FTD mass shift.

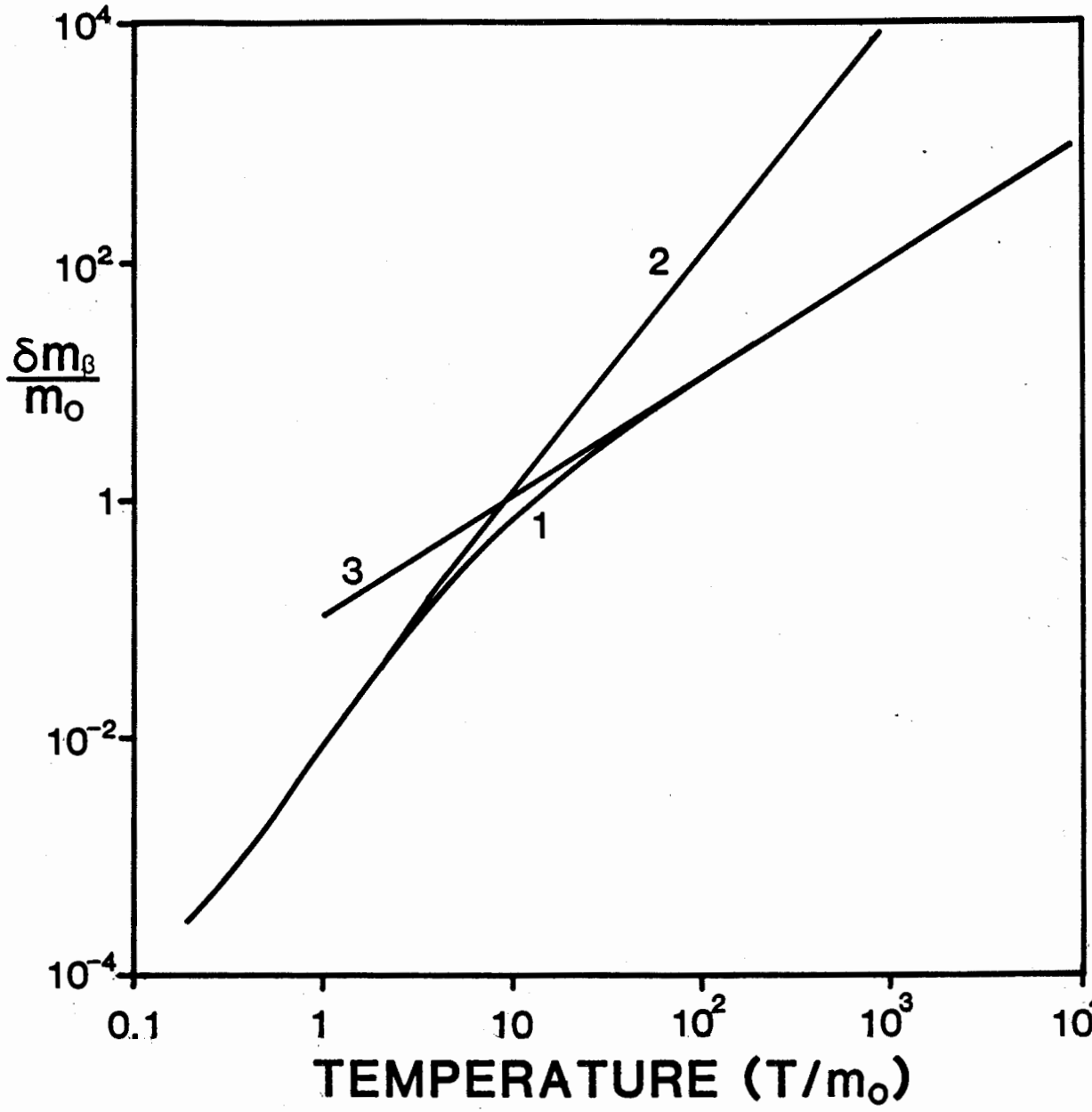

Fig. 9. Comparison of FTD mass shift calculations performed in: (1) this thesis, (2) ref. 20, and (3) ref. (18). Since ref. 18 deals with massless fermions, it can only be compared to the other two calculations in the large T regime.

C. Neutron stars[43]

We now investigate what effect the mass shifts discussed above might have on the evolution of neutron stars. For our purposes a neutron star can be parametrized thermodynamically by T=0 and an electron chemical potential (μ_e), much larger than the T=0 electron mass (m_e). From (4.13) we then find for the effective electron mass

$$M = \sqrt{\frac{\alpha}{2\pi}}\,\mu_e \ . \tag{5.13}$$

We now adopt a simplified model for neutron star evolution in order to see what effect (5.13) has on it. Construction of a detailed model is beyond the scope of this thesis, and in any event, will prove to be unnecessary.

For the model, a uniform gas of electrons, protons and neutrons will be chosen for the neutron star matter (clearly such a model will not be valid at high densities where coulomb and strong interaction effects will become important). Chemical potentials μ_e, μ_p, and μ_n, which include the T=μ=0 mass, are assigned respectively to the electrons, protons and neutrons present with number densities n_e, n_p and n_n. For a Fermi gas, these quantities can be related by $\mu^2 = m_0^2 + (3\pi^2 n)^{2/3}$. If $\mu_e + \mu_p$ exceeds μ_n, then electrons will be captured by protons until the number densities change such that $\mu_e + \mu_p = \mu_n$. Hence, for each value of n_n, there will be a value of μ_e which satisfies the β-stability condition.

Assuming local electrical neutrality so that $n_e = n_p$, then the chemical potential equality yields

$$\mu_e + \left(\mu_e^2 + m_p^2 - m_e^2\right)^{1/2} = \left((3\pi^2 n_n)^{2/3} + m_n^2\right)^{1/2} \ . \tag{5.14}$$

The solution of this equation for μ_e as a function of n_n is shown in

fig. 10. At small n_n, $\mu_e \sim m_n - m_p$, while for large n_n (but not so large that the neutrons are relativistic)

$$\mu_e = (3\pi^2 n_n)^{2/3}/2m_n \ . \tag{5.15}$$

Both of these limits are obvious from fig. 10. This figure includes a region in which μ_e exceeds m_π, although obviously $e^- \to \pi^- + \nu_e$ would be allowed in this region (see ref. 44).

From fig. 10 it can be seen that over much of the neutron density range of interest in the formation of a neutron star, μ_e is large compared to m_e and the large μ expression for M should be reasonably accurate. The mass shift is then found to be substantial compared to m_e, as is also shown on fig. 10, but small compared to μ_e ($M/\mu_e \sim 1/30$). The larger electron mass will lead to an earlier onset of electron capture in the formation of the neutron star, and hence an increase in the rate of neutrino emission. Since the neutrino mass is changed only by the weak interaction, its mass shift would be very small and would not compensate for the increased electron mass. Similarly, the abundance of electrons in the neutron star core would be lowered.

We can get some insight into the importance of the mass shift on observational quantities by looking at the particle energy,

$$E^2 = m^2 + p_F^2$$

where $p_F^2 = \mu_e^2 - m_e^2$ is the Fermi momentum. Making use of (5.13) we find

$$E^2 \simeq \mu^2\left(1 + \frac{\alpha}{2\pi}\right)$$

from which we conclude that FTD effects induce a shift in the energy of about 0.1%. The smallness of the correction can be understood physically in the following way. Since FTD effects induce mass shifts of $O(\alpha\mu)$ the chemical

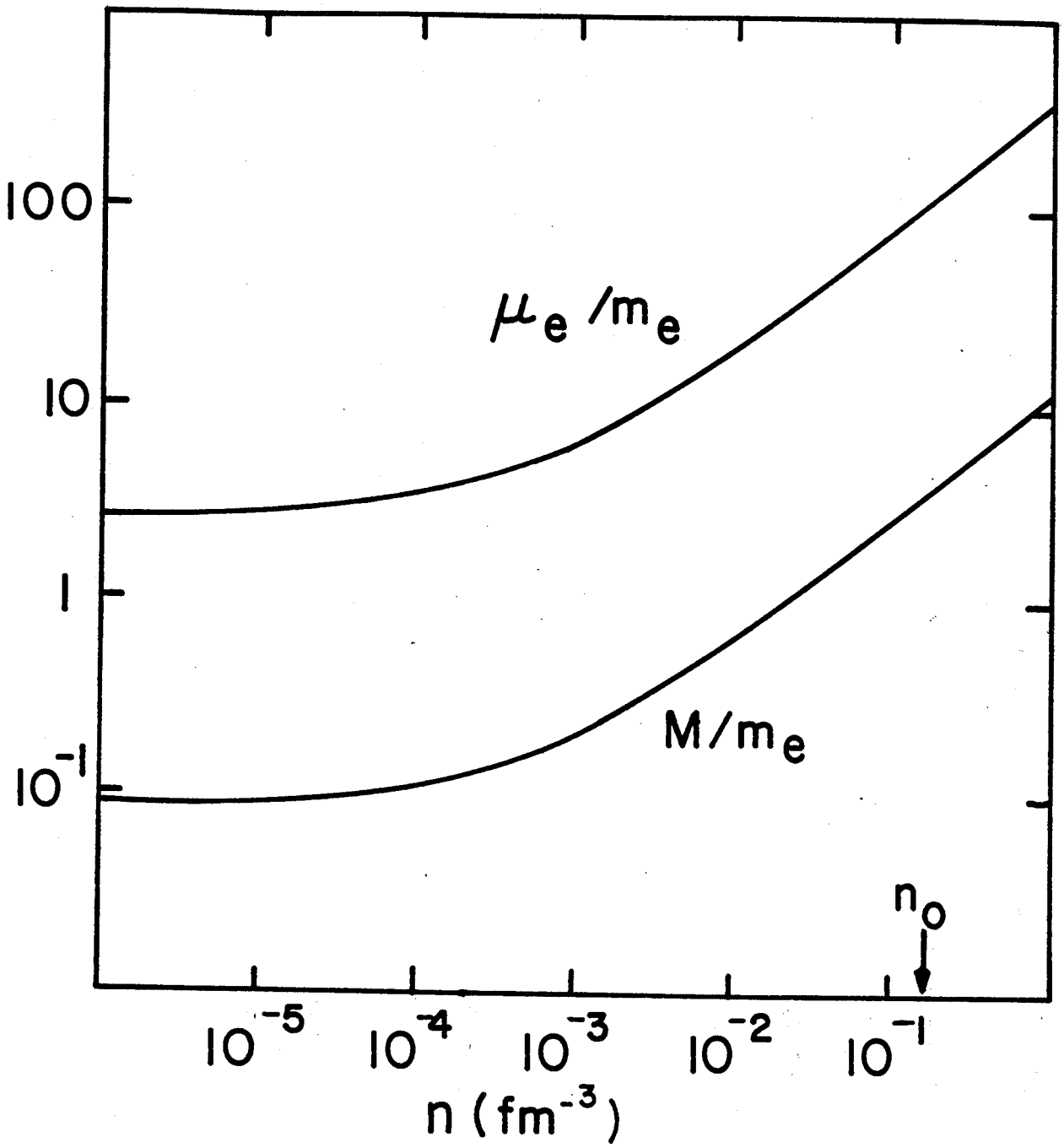

Fig. 10. Expected value of the electron chemical potential and electron mass shift shown as a fuction of neutron number density. Normal nuclear matter density (n_0) is indicated for comparison.

potential must be of $O(m_o/\alpha)$ before the effect is important. However, for such a large chemical potential the electron 3-momentum is so large that the rest mass (even with FTD effects included) is small by comparison.

Chapter 6

Conclusions

In this thesis, the one loop FTD corrections to the self-energy of spin 1/2 fermions, in the context of QED, have been studied. We have found that the interaction between a fermion and a thermal background can be characterized reasonably well by a single parameter, m, in the sense that the dispersion relation for such a particle closely resembles the dispersion relation for a free particle of mass m. In this way m can be considered the effective mass. Several specific cases have been studied and we have found

$$m \sim \alpha[\pi^2 T^2 + \mu^2]^{1/2} \tag{6.1}$$

for chirally invariant fermions and for fermions with broken chiral symmetry provided

$$\mu^2 + T^2 \gg m_0^2 \ . \tag{6.2}$$

When $\mu^2 + T^2 \ll m_0^2$ we have found

$$m \sim \frac{\alpha}{m_0} (\pi^2 T^2 + \mu^2) \ . \tag{6.3}$$

Some workers have claimed that (6.3) applies even when (6.2) is true, but we have shown this to be a consequence of ignoring the heat bath 4-velocity. Early universe calculations which have employed (6.3) in the high T regime to obtain finite temperature corrections to the helium abundance are apparently incorrect but not in a significant way since the small effect calculated will only be reduced even further.

The FTD renormalization procedure has also been analyzed and we have shown that in order to calculate the physical mass, the $T=\mu=0$ part of the theory must be renormalized at a T,μ-dependent point. Finally, in the

appendix, we have shown that despite its more complicated analytic structure (vis a vis the $T=\mu=0$ theory), the chain approximation still works and allows a simple physical interpretation of the self-energy.

Appendix A: The FTD Chain Approximation

The chain approximation is a well-known technique in $T=\mu=0$ field theories for demonstrating that one of the effects of the interaction lagrangian is to cause the pole of the complete propagator to shift away from the bare propagator value. The QED chain approximation is expressed diagrammatically in fig. 11a and algebraically by

$$\mathscr{S}_0(K) = S_0(K) + S_0(K)\,\Sigma_0(K)\,S_0(K) + S_0(K)[\Sigma_0(K)\,S_0(K)]^2 + \ldots \qquad (A.1)$$

where $\mathscr{S}_0$ is the full propagator, S_0 is the bare propagator and Σ_0 is the self energy insertion given by

$$-i\Sigma_0(K) = -e^2 \int \frac{d^4p}{(2\pi)^4}\,\gamma_\mu(-g^{\mu\nu})iD_0(p)\;iS_0(K-p)\gamma_\nu\ . \qquad (A.2)$$

The solution is the well-known Dyson equation

$$\begin{aligned}\mathscr{S}_0 &= [\not K - m_B - \Sigma_0(K) + i\varepsilon]^{-1} \\ &= (1+a_0)^{-1}\,[\not K - m_0]^{-1} \qquad (A.3)\end{aligned}$$

[cf (3.2)-(3.3)]. The real miracle of (A.3) is that by summing the series in (A.1) we go from a complicated analytic structure (poles of all orders located at m_B) to a much simpler one (single simple pole at m_0) with a simple physical interpretation for δm_0 (mass shift).

The more complicated structure of the FTD propagators raises the question of whether this approximation scheme is still useful when $T,\mu \neq 0$. Specifically, does the chain approximation to the complete matrix propagator have an analytic structure analogous to its $T=\mu=0$ counterpart with a correspondingly simple interpretation for the self-energy matrix $\overline{\Sigma}^{ab}$? It is shown here that the simple analytic structure of $\mathscr{S}_0$ is preserved at finite temperature, and that $\overline{\Sigma}^{ab}$ can be interpreted physically. In the process of demonstrating this, we will obtain

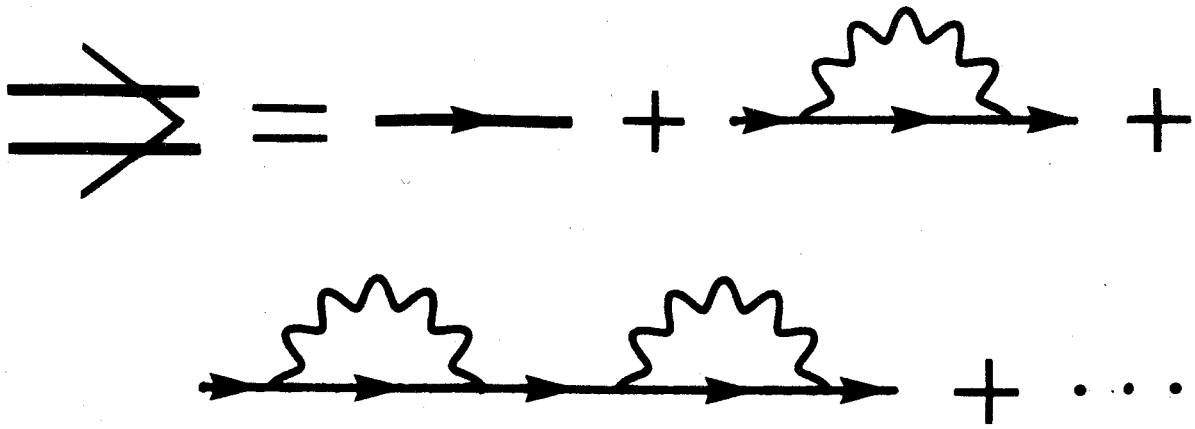

Fig. 11a. Diagrammatic equation for the T=μ=0 chain approximation.

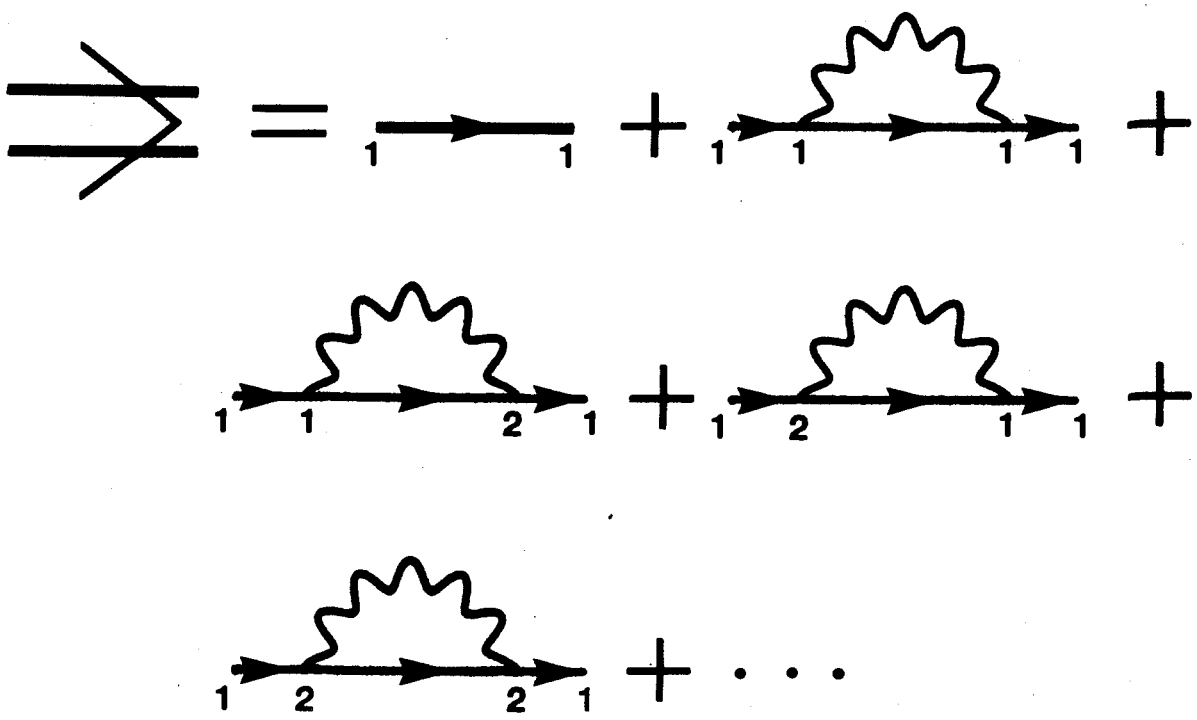

Fig. 11b. Diagrammatic equation for the FTD chain approximation.

expressions for the real and imaginary parts of the self-energy.

The FTD chain approximation is given diagrammatically in fig. 11b (see sect. 2 for details) and algebraically by

$$\bar{\mathcal{S}}^{ab} = \bar{S}^{ab} + \bar{S}^{ac}\,\bar{\Sigma}^{cd}\,\bar{S}^{db} + \ldots \tag{A.4}$$

where

$$-i\bar{\Sigma}^{cd}(K) = -e^2 \int \frac{d^4p}{(2\pi)^4}\,\gamma^\mu\; i\bar{D}^{cd}_{\mu\nu}(p)\; i\bar{S}^{cd}(K-p)\gamma^\nu \tag{A.5}$$

and $i\bar{S}^{ab}$ and $i\bar{D}^{ab}_{\mu\nu}$ are defined in (2.20) and (2.21) respectively. Proceeding in a manner similar to the $T=\mu=0$ case, we right multiply (A.4) by $\bar{\Sigma}^{bc}\,\bar{S}^{cd}$ and then subtract the resulting equation from (A.4). The resulting <u>matrix</u> equation is[45]

$$\bar{\mathcal{S}}(K) = [\bar{S}^{-1}(K) - \bar{\Sigma}(K)]^{-1} \ . \tag{A.6}$$

Writing $\Sigma(K)$ as

$$\theta(K_0)\;\bar{V}^{-1}(K,\mu)\;\bar{T}_+(K)\;\bar{V}^{-1}(K,\mu) + \theta(-K_0)\big(\bar{V}^\dagger(K,-\mu)\big)^{-1}\;\bar{T}_-(K)\big(\bar{V}^\dagger(K,-\mu)\big)^{-1} \tag{A.7}$$

will give the desired analytic properties for $\bar{\mathcal{S}}$ if

$$\bar{T}_\pm = \begin{pmatrix} \Sigma_\pm & 0 \\ 0 & -\Sigma^*_\pm \end{pmatrix} \ . \tag{A.8}$$

Furthermore, the interpretation of $\bar{T}_\pm$ is then clear since (A.6) will only contain terms like

$$\bar{V}\begin{bmatrix} \dfrac{1}{\not K - m_B - \Sigma_+ + i\varepsilon} & 0 \\ 0 & \dfrac{-1}{(\not K - m_B - \Sigma^*_+ - i\varepsilon)} \end{bmatrix}\bar{V} \ . \tag{A.9}$$

In order to prove (A.8) we first note the identity

$$\theta(K_0) = \theta(K_0)[\theta(K_0-p_0) + \theta(p_0-K_0)] \ . \tag{A.10}$$

Combining (A.5), (A.7) and (A.10) gives

$$\bar{T}_+(K) = -ie^2\theta(K_0) \int \frac{d^4p}{(2\pi)^4}\,\bar{V}(K,\mu)\big[\theta(K_0-p_0)\bar{M}_+(p,K-p) + \theta(p_0-K_0)\bar{M}_-(p,K-p)\big]\,\bar{V}(K,\mu) \tag{A.11a}$$

$$\bar{T}_-(K) = -ie^2\theta(-K_o)\int\frac{d^4p}{(2\pi)^4}\,\bar{V}^\dagger(K,-\mu)\big[\theta(K_o-p_o)\bar{M}_+(p,K-p) + \theta(p_o-K_o)\bar{M}_-(p,K-p)\big]\,\bar{V}^\dagger(K,-\mu) \qquad (A.11b)$$

where $\bar{M}^{ab}_{\pm}(p,K-p) = \gamma^\mu\, i\bar{D}^{ab}_{\mu\nu}(p)\big[i\bar{S}^{ab}_o(K-p) + i\bar{S}^{ab}_{\beta\pm}(K-p)\big]\gamma^\nu$.

For convenience we introduce the following notation

$$\bar{\Sigma}_{\pm_1}(K) = -ie^2\theta(\pm K_o)\int\frac{d^4p}{(2\pi)^4}\,\theta(K_o-p_o)\bar{M}_+(p,K-p) \qquad (A.12a)$$

$$\bar{\Sigma}_{\pm_2}(K) = -ie^2\theta(\pm K_o)\int\frac{d^4p}{(2\pi)^4}\,\theta(p_o-K_o)\bar{M}_-(p,K-p) \qquad (A.12b)$$

$$\bar{T}_{+(1,2)}(K) = \bar{V}(K,\mu)\bar{\Sigma}_{+(1,2)}(K)\,\bar{V}(K,\mu) \qquad (A.13a)$$

$$\bar{T}_{-(1,2)}(K) = \bar{V}^\dagger(K,-\mu)\bar{\Sigma}_{-(1,2)}(K)\,\bar{V}^\dagger(K,-\mu)\ . \qquad (A.13b)$$

Concentrating on $\bar{T}_{+_1}$, it is given explicitly by

$$-ie^2\theta(K_o)\int\frac{d^4p}{(2\pi)^4}\,\theta(K_o-p_o)$$

$$\times\Big[M^{11}_+\cos^2\psi^+_K + e^{\beta\mu/2}\cos\psi^+_K\sin\psi^+_K(M^{12}_+-M^{21}_+e^{-\beta\mu}) - M^{22}_+\sin^2\psi^+_K$$
$$(M^{11}_++M^{22}_+)e^{\beta\mu/2}\cos\psi^+_K\sin\psi^+_K + M^{12}_+e^{\beta\mu}\sin^2\psi^+_K + M^{21}_+\cos^2\psi^+_K$$
$$- (M^{11}_++M^{22}_+)e^{-\beta\mu/2}\cos\psi^+_K\sin\psi^+_K + M^{12}_+\cos^2\psi^+_K + M^{21}_+e^{-\beta\mu}\sin^2\psi^+_K\Big]$$
$$- M^{11}_+\sin^2\psi^+_K + e^{\beta\mu/2}\cos\psi^+_K\sin\psi^+_K(M^{12}_+-M^{21}_+e^{-\beta\mu}) + M^{22}_+\cos^2\psi^+_K\ \Big] \qquad (A.14)$$

The matrix elements of M_+ are

$$M^{11}_+ = \gamma^\mu(\not{K}-\not{p}+m_o)\gamma_\mu(D_o\mathrm{ch}^2\theta_p - D^*_o\mathrm{sh}^2\theta_p)\big(\Delta_o\cos^2\psi^+_{K-p} + \Delta^*_o\sin^2\psi^+_{K-p}\big) \qquad (A.15a)$$

$$M^{12}_+ = \gamma^\mu(\not{K}-\not{p}+m_o)\gamma_\mu(D^*_o-D_o)(\Delta_o-\Delta^*_o)e^{-\beta\mu/2}\,\mathrm{ch}\theta_p\,\mathrm{sh}\theta_p\cos\psi^+_{K-p}\sin\psi^+_{K-p} \qquad (A.15b)$$

$$M^{21}_+ = \gamma^\mu(\not{K}-\not{p}+m_o)\gamma_\mu(D_o-D^*_o)(\Delta_o-\Delta^*_o)e^{\beta\mu/2}\,\mathrm{ch}\theta_p\,\mathrm{sh}\theta_p\cos\psi^+_{K-p}\sin\psi^+_{K-p} \qquad (A.15c)$$

$$M^{22}_+ = \gamma^\mu(\not{K}-\not{p}+m_o)\gamma_\mu(D^*_o\mathrm{ch}^2\theta_p - D_o\mathrm{sh}^2\theta_p)\big(\Delta^*_o\cos^2\psi^+_{K-p} + \Delta_o\sin^2\psi^+_{K-p}\big)\ . \qquad (A.15d)$$

The abbreviations $D_o = D_o(p)$ and $\Delta_o = \Delta_o(K-p)$ have been used.

Since $M_+^{22} = (M_+^{11})^*$ one of the conditions of (A.8) is satisfied. The other condition

$$T_{+1}^{12} = T_{+1}^{21} = 0$$

can be restated as

$$\mathrm{Im}(\Sigma_+^{11}) = -i\Sigma_{+1}^{12}\, e^{\beta\mu/2}\, \mathrm{cotan}2\psi_K^+ \quad . \tag{A.16}$$

By making use of the following identities,

$$-\int dp_o\left[D_o\Delta_o + D_o^*\Delta_o^*\right]f(p) = \int dp_o\, \frac{\pi^2}{qr}\left[\delta(p_o-q)\delta(K_o-p_o-r) + \delta(p_o+q)\delta(K_o-p_o+r)\right]f(p)$$

$$\int dp_o\left[D_o\Delta_o^* + D_o^*\Delta_o\right]f(p) = \int dp_o\, \frac{\pi^2}{qr}\left[\delta(p_o+q)\delta(K_o-p_o-r) + \delta(p_o-q)\delta(K_o-p_o+r)\right]f(p)$$

where $q = |\vec{p}|$ and $r = [(\vec{K}-\vec{p})^2 + m_B^2]^{1/2}$

(A.16) is shown to be true. Similar arguments can be made for T_{+2}, T_{-1} and T_{-2}.

Using (A.14)-(A.16) we also find

$$\mathrm{Re}\Sigma_+ = \mathrm{Re}\Sigma_- = \mathrm{Re}\Sigma_{+1}^{11} + \mathrm{Re}\Sigma_{+2}^{11} \tag{A.17}$$

$$\mathrm{Im}\Sigma_\pm = \sec 2\psi_K^\pm\, \mathrm{Im}\left(\Sigma_{\pm 1}^{11} + \Sigma_{\pm 2}^{11}\right) . \tag{A.18}$$

References and Footnotes

1. J.D. Bjorken and S.D. Drell, Relativistic Quantum Mechanics (McGraw Hill, New York, 1964).

   C. Itzykson and J.-B. Zuber, Quantum Field Theory (McGraw Hill, New York, 1980).

2. T. Matsubara, Prog. Theor. Phys. 14, 351 (1955).

3. H. Umezawa, H. Matsumoto, and M. Tachiki, Thermo Field Dynamics and Condensed States (North-Holland, Amsterdam, 1982).

4. A.L. Fetter and J.D. Walecka, Quantum Theory of Many-Particle Systems (McGraw Hill, New York, 1971).

5. D.A. Kirzhnitz and A.D. Linde, Phys. Lett. 42B, 471 (1972).

6. L. Dolan and R. Jackiw, Phys. Rev. D9, 3320 (1974).

7. B.J. Harrington and A. Yildiz, Phys. Rev. D11, 779 (1975).

8. A.D. Linde, Rep. Prog. Phys. 42, 389 (1979).

9. M.B. Kislinger and P.D. Morley, Phys. Rev. D13, 2765 (1976).

10. B.A. Freedman and L.D. McLerran, Phys. Rev. D16, 1130 (1977) and Phys. Rev. D16, 1147 (1977).

11. P.D. Morley and M.B. Kislinger, Phys. Rep. C51, 63 (1979).

12. M.B. Kislinger and P.D. Morley, Phys. Rev. D13, 2771 (1976).

13. C.W. Bernard, Phys. Rev. D9, 3312 (1974).

14. A. Bechler, Ann. Phys. (N.Y.) 135, 19 (1981).

15. Y. Ueda, Phys. Rev. D23, 1383 (1981).

16. S. Weinberg, Phys. Rev. D9, 3357 (1974).

17. H.A. Weldon, Phys. Rev. D26, 1394 (1982).

18. H.A. Weldon, Phys. Rev. D26, 2789 (1982).

19. D.H. Boal and E.J. Levinson, Neutron Stars and the Electron Mass at Finite Chemical Potential, MSU preprint.

20. G. Peressutti and B.-S. Skagerstam, Phys. Lett. 110B, 406 (1982).

21. E.J. Levinson, Self-Energy Corrections for Fermions in the Presence of a Thermal Background, TRIUMF preprint (August 1984).

22. J.-L. Cambier, J.R. Primack and M. Sher, Nucl. Phys. B209, 372 (1982).

23. D.A. Dicus, E.W. Kolb, A.M. Gleeson, E.C.G. Sudarshan, V.L. Teplitz, and M.S. Turner, Phys. Rev. D26, 2694 (1982).

24. A.J. Niemi and G.W. Semenoff, Ann. Phys. (N.Y.) 152, 105 (1984).

25. I. Ojima, Ann. Phys. (N.Y.) 137, 1 (1981).

26. J.F. Donoghue and B.R. Holstein, Phys. Rev. D28, 340 (1983).

27. J. Hyung Yee, Photon Propagators at Finite Temperature, ICTP preprint IC/82/81 (July 1982).

28. A. Palanques-Maestre and R. Tarrach, Electrons in a Heated Vacuum, University of Barcelona preprint UBFT-FP-3-83.

29. C. Nash, Relativistic Quantum Fields (Academic Press, London, 1978). P. Ramond, Field Theory - A Modern Primer, (Benjamin/Cummings, Reading MA, 1981).

30. E.S. Abers and B.W. Lee, Phys. Rep. C9, 1 (1973).

31. R. Mills, Propagators for Many Particle Systems, (Gordon and Breach, N.Y., 1969).

32. This result is consistent with the axiomatic field theory result that a free field FTD propagator must include a ghost field (cf. ref. 33). It is also consistent with non-relativistic many-body theory where the necessity fo a matrix formulation has been known for a long time (cf. ref. 31).

33. H. Araki and E.J. Woods, J. Math. Phys. 4, 637 (1963);
H. Araki and W. Wyss, Helv. Phys. Acta 37, 136 (1964);
R. Haag, N.W. Hugenholtz, and M. Winnink, Comm. Math. Phys. 5, 215 (1967).

34. A.J. Niemi and G.W. Semenoff, Nucl. Phys. B230 [FS10], 181 (1984).

35. The fermion propagator shown here differs from the one in refs. 3, 24, 25, and 34, but the difference is only important for $\mu \neq 0$. The author is grateful to G. Semenoff for making (2.24) available prior to publication.

36. This subsection borrows heavily from ref. 18.

37. There is another term in (3.5) ~ $[\not{K}, \not{A}]$, but it is only important for higher order calculations.

38. This definition was originally introduced in ref. 15 in a different context and has since been used in refs. 20, 27, and 28 for studying self-energy corrections.

39. See, for example, refs. 22, 23, and 28.

40. H. Matsumoto, I. Ojima, and H. Umezawa, Perturbation and Renormalization in Thermo Field Dynamics, Univ. of Alberta preprint [to appear in Ann. Phys. (N.Y.)].

41. H. Matsumoto, Y. Nakano, and H. Umezawa, Renormalization Group at Finite Temperature, Univ. of Alberta preprint.

42. Only the real part of Σ will be treated here. For a more careful treatment that includes the imaginary part of Σ, see ref. 21.

43. For a review of neutron star properties, see J.M. Irvine, Neutron Stars (Oxford, 1978).

44. See, for example, J.M. Irvine, Rep. Prog. Phys. 38, 1835 (1975) and references therein, as well as N.K. Glendenning, Phys. Lett. 114B, 392, (1982).

45. For the remainder of this appendix, matrix indices will usually be suppressed for the sake of clarity.