THE DEVELOPMENT OF PROBABILITY CONCEPTS IN ELEMENTARY SCHOOL CHILDREN (K-7)

by

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THE DEVELOPMENT OF PROBABILITY CONCEPTS IN ELEMENTARY SCHOOL CHILDREN (K-7)

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ABSTRACT

The work of Piaget on the origin of the concept of chance in children provided much of the impetus for studying the development of probabilistic thinking.

In recent years there has been an increased effort to include the study of probability in the elementary mathematics curriculum. Researchers, however, do not agree upon the age at which children develop an understanding of probability. This study is a partial replication of an experiment which investigated the level of probabilistic understanding of elementary school children.

A total of 48 children, from Kindergarten to Grade 7, in a Delta public school participated in the study. Individual sessions of approximately 30 minutes, involving 36 tasks, were conducted independently with each subject. For each task, the subject was presented with a pair of spinners with specific probabilities. The spinners were circles of varying radii which were divided into sectors of equal central angles. The area of the spinner was proportional to the number of its sectors. The sectors were

coloured blue or yellow. The children were asked to choose the spinner which was more likely to produce the payoff colour (blue). The choice of the subject and the outcome of the task were recorded by the experimenter. For certain preselected tasks, the students were asked to explain verbally the reason for their selections.

The percent of correct scores increased with grade level. The percent of correct responses within each grade level decreased with the level of difficulty of the task.

Five specific strategies were used by the subjects: 1) reliance upon the pay-off colour (blue); 2) reliance upon the non-payoff colour (yellow); 3) the physical setup of the spinner board and the pointer; 4) the physical size or area of the spinner; and 5) a comparison between the pay-off colour and the nonpay-off colour.

The study indicates that most children in elementary school do not have a verbal understanding of probability concepts even though the mean percent of correct responses for each grade level was above chance level. The choice of spinner was usually based upon features other than proportion. The most common strategy was to choose the spinner with the greatest number of payoff elements.

Two of the 48 subjects demonstrated an understanding of probability concepts.

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Chapter 1

INTRODUCTION

In the elementary school mathematics curriculum, the topic of probability has historically served as a possible extension to core areas such as problem solving, fractions, and ratios. Often paired with the study of statistics, the exploration of probability theory has been left to interested university students. This has produced a significant void in the general mathematics education of those students who do not continue their formal education after high school graduation (Reys, 1978).

Recently, there has been some movement by curriculum developers to include probability as a distinct area of study in the elementary school curriculum. At present, the mathematics curriculum in British Columbia is undergoing a major revision. The curriculum is in the draft stage and the Data Analysis Strand consists of an outline of topics and intended learning outcomes in probability and statistics for grades 1 through 7. Furthermore, the December 1984 information circular distributed by the Ministry of

Education stated "Probability and Statistics will be essential parts of the mathematics curriculum at all levels." This interest in developing probability as an area of study is reflected in recent editions of mathematics textbooks in which specific sections, and often entire chapters, are devoted to the study of probability.

Although the decision to introduce the concept of probability into elementary schools has been made, little research has been completed which might guide its introduction into the curriculum. It is this paucity of relevant information that has prompted the present investigation.

The Definition of Probability

Probability is a "mathematical theory of processes involving uncertainty" (Bullock & Stallybrass, 1982, p. 497). The probability of an event can be expressed mathematically as a proportion which represents the likelihood, or chance, of the occurrence of a specific event. Probability, therefore, can be viewed as the combination of two concepts, chance and proportion. It is important to be aware of these elements for it is possible for a person to have an understanding of one or the other, but fail to understand the complex whole.

To illustrate; it is possible to have an understanding of the chance of the occurrence of an event while being unable to express it as a proportion. When a die is thrown into the air there is an equal chance that any number on the die will come up. Most children will tell you that there is a possibility for the number they have chosen to come up, but they may not be able to articulate the proportion. Similarly, it is conceivable for a person to recognize and state a simple fraction, such as one-third of the circle is red, but be unable to apply this as an expression of the chance of an event occurring. The child may not understand that there is a one chance out of three that the pointer, or indicator, will land on red. This means that if children are aware of the element of chance without understanding and applying proportion, they do not have an understanding of probability. Moreover, if a child can figure out the proportion of elements but is unable to apply this as an expression of the chance of a specific event occurring, that child does not understand probability.

The Relevance of Probability in the Curriculum

There are three important reasons for including the topic of probability in the mathematics curriculum. First, probability is a term which is often used in daily life and therefore should be defined and explained to students so they have some understanding of the concept and its application. The weather forecaster may indicate the probability of precipitation for the next twenty-four hours is 75 percent, or the chance of winning a lottery is one in a million. Second, some research has shown that the ability to solve probability problems increases only marginally through the levels of the education system, while also suggesting that probability concepts are unlikely to develop incidently or through maturation (Reys, 1978). If we accept these findings, and we want students to become familiar with the concept of probability, the mathematics curriculum should be revised to include the opportunity for students to develop probabilistic understanding. Third, probability exercises force students to examine the options within a problem, the likely and the unlikely. The notion of a single correct answer to a mathematics question is altered and a discussion of possible outcomes is encouraged. Carefully prepared lessons present the opportunity for students to systematically probe possible solutions to a problem.

The Problem

The development of the idea of chance and probability in children was initially investigated by Piaget and Inhelder (1951/1975). They concluded that children must progress through a series of stages--first developing a recognition and understanding of the reversibility of operations and then the concept of chance. Studies involving comparisons of method (Yost, Seigel & Andrews, 1962), the use of reinforcement (Goldberg, 1966), the relative significance of verbal and non-verbal techniques (Carlson, 1970), and children's understanding of probability (Davies, 1965; Hoemann & Ross, 1971) have stemmed directly from the work of Piaget and Inhelder (1951/1975). Other researchers (Fischbein, Pampu, & Manzat, 1970; Perner, 1979a, 1979b) have investigated alternative models of the development of chance and probability concepts in children.

Researchers in the area of children's probabilistic thinking have reached differing conclusions as to the age at which probabilistic thinking is in evidence. Piaget and Inhelder (1951/1975), and Fischbein et al. (1970) concluded that formal probabilistic thinking is not in evidence until the child is nine or ten years old, and even then Fischbein contended they must have a period of instruction to understand the concept. Other researchers (Davies, 1965; Goldberg, 1966; Yost et al., 1962) found that children as young as four years old were capable of forming systematic probabilistic judgments.

Various reasons have been offered to explain the discrepancy between these results (Hoemann & Ross, 1971). One of the explanations for the difference between these findings is the *magnitude estimation* hypothesis. This hypothesis states that children can select the task with the better probability of occurring, not by calculating the proportion of elements, but by looking at the relative physical area or the absolute number of elements in the specific task. This explanation is particularly valid in experiments where the subject is given a choice between two tasks, and the total number of elements (denominator) or the number of the pay-off elements (numerator) in each task are equal. The subject has only to select the task with the greatest number, or magnitude, to also, unwittingly, choose the task with the greatest probability.

Recently, Falk, Falk, and Levin (1980) devised a sophisticated experimental procedure which addressed the factor of magnitude estimation, and they appear to have resolved the issue of the development of probability concepts. They found that the children have the *potential* for discriminating between probabilities around the age of six. The present research is a partial replication of one of the probability experiments of Falk et al. (1980) in which a series of paired spinners were used to investigate the level of students' probabilistic understanding.

The experimental design required choices similar to the *decision-making* tasks of Yost et al. (1962). In a decision-making task, the subject is offered a choice of two items, and is asked to choose the item which fulfills specific criteria previously outlined by the experimenter. For this experiment the subject was presented with a choice of two spinners, and was asked to decide which spinner to twirl to provide the best chance of achieving the pay-off colour. For the purpose of this experiment, a spinner is a circle of a specific radius which has been proportionally segmented and coloured to represent a certain probability. Pivoted at the centre, and unattached to each spinner, is an indicator which can be twirled to land, without bias, on a segment of the circle.

Purpose

The purpose of this study was to analyze the development of probabilistic understanding at various age levels in elementary school in order to: 1) determine a general age range at which children have developed an understanding of the concept of probability; 2) identify and discuss any recurring strategies which the subjects use to help solve probability problems; 3) suggest an age level at which probability concepts and activities could be introduced in the mathematics curriculum.

Description of the Study

In the first chapter, a definition of probability is provided and the relevance of probability in the curriculum is outlined. In addition, the background to the problem is discussed and the purpose of the study identified.

In Chapter 2, the literature is reviewed. The work of

Piaget and Inhelder is outlined, other studies based upon Piaget's research are discussed, and alternative hypotheses are presented.

The research methodology is outlined in Chapter 3. The subjects, the apparatus, and the research design are described, and the data collection procedure is discussed.

In Chapter 4, the results of the experiment are tabulated. Grade level results and individual results are described.

The results of the study are discussed in Chapter 5. Methodological implications are outlined, and topics for further research are suggested.

Chapter 2

REVIEW OF THE LITERATURE

The work of Piaget and Inhelder will be discussed at the outset of this chapter because their research on the development of the notion of chance in children was the first to be published and consequently prompted additional studies in the field. Following this, other investigations related to probabilistic thinking in children will be discussed.

Stages in the Development of Children's Concept of Chance

Piaget and Inhelder (1951/1975) were the first to attempt an explanation of the development of the idea of chance in children. Their studies of the development of children's thought led them to believe that the notion of chance is not innate, but comes from a search for order and its causes. They held that children progress through a series of stages in which they develop a recognition and understanding of operations. The stage development hypothesis means they gradually understand and accept that some operations obey set laws and always follow a pattern, while others do not. Piaget and Inhelder explain that,

> Logical and arithmetical operations constitute systems of actions interrelated in a rigorous way and always reversible, this reversible aspect rendering deduction possible. Transformations occurring by chance, on the contrary, are not interrelated in a rigorous way and the most probable systems that they form are essentially irreversible. While logical and arithmetical operations lead to and end in the construction of groupings or groups fortuitous transformation remain nonreducible to this type of structure. Finally, induction lies between operative deduction and nondeducible fortuitous transformations: induction comprising those steps by which one sifts through experimentation separating what is fortuitous from what is deducible, while at the same time preparing for deduction itself (p. 212).

In Piaget and Inhelder's hypothesis, the preoperational stage, prior to about age six, is characterized by a failure to differentiate between the possible and the necessary, between chance and nonchance. The child has no well-defined concept of cause and effect, nothing is certain or uncertain, and therefore there is no understanding of what constitutes chance.

In the concrete operations stage, from age 6 to about age 12, the child recognizes the element of chance. Children discover there are areas of knowledge which are certain. There are things they can know, but there are also other areas which are uncertain, where they must guess. They can complete simple probability tasks, but are unable to perform tasks requiring combinatoric operations and proportions. They do not have the conceptual strategies for defining all possible outcomes of an event. They need to see or handle objects to define their actual relationship.

In the final stage, beginning at about age 12, there is a synthesis of chance and deductive operations. Children can "tie" together concrete operations and at the same time hypothesize what the outcome might be. Children develop a system for finding combinations, and the Laws of Large Numbers provide a basis for making predictions. This means they begin to see a relationship developing after many trials have been completed.

Through these stages there is a gradual recognition of the reversibility of operations. Since Piaget and Inhelder claimed the discovery of chance comes after an understanding of reversible operations, it is only after children have the mental organization to recognize operations that are reversible that they will be able to identify those that are not reversible and therefore fortuitous.

Piaget and Inhelder's work was divided into three parts; chance in physical reality, random drawings and combinatoric operations.

In the first part, they studied the notion of random mixture and irreversibility. For the first experiment, they used a rectangular box which rested on a pivot, enabling it to be tilted like a seesaw. Eight white balls and eight red balls were lined up at one end of the box and were separated into colour groups by a divider. The box was slowly tipped back and forth so the balls were gradually mixed. The subjects' reaction to the mixing of the elements were recorded. Random mixing of elements leads to the study of distributional form, since the final positions of the elements in mix represent a distribution. The distributions which Piaget and Inhelder studied were centered distributions (ie. the normal curve), and uniform distributions. Piaget and Inhelder used the distribution of sand grains as an example. A centered distribution was formed when the grains of sand poured out of a small hole in a funnel, and a uniform distribution is represented when the grains of sand are spread over a flat surface.

Following this, Piaget and Inhelder investigated the child's ability to dissociate what can be due to chance and what is due to non-fortuitous elements. "In reality it is precisely this combination of what today is commonly called uncertain and what is not uncertain (or only weakly so) which most frequently gives rise to the intervention of the intuition of probabilities, both in the laboratory and in life" (p. 57).

In the second part of their work they investigated random drawings. They examined children's notion of chance and miracle in the game of heads and tails, and in drawing pairs of marbles from a jar. They also studied the quantification of probabilities in random drawings.

Finally, they analyzed the development of operations of combination, permutation, and arrangement, and concluded with their hypothesis of three stages in the development of the idea of chance.

Piaget and Inhelder's experiments were presented to the children as a game. In their experiments they used the clinical method, in which an experimenter worked with each child individually, asking on-going questions about the activity. The questions and the child's responses were recorded verbatim. For example, in an experiment using a single spinner and a 16-sectored circle (equal proportions) with 8 colours (colours of opposite sections are identical) the following exchange occurred:

> "Do we know where the bar will stop?" *"On the red."* "Is that certain or not?" *"Certain."* (We do the experiment: green.) "Where now?" *"On the yellow."* "Why?" *"Because that's a wonderful yellow."* (Experiment: blue.) *"Oh, no, on the blue."* "Where now?" *"Red."* (Experiment: red.) "And next?" *"Again red."* "Why? . . . ." The same kind of answers, without motives. We put the magnets in place: *"The bar got tired."* (Piaget & Inhelder, 1951/1975, p.62)

All responses were classified at the end of the experiment. The child in the above example would be placed in the first stage of development, or preoperational stage, as the subject does not use logical considerations and is predicting solely on the basis of personal feeling for a colour.

Decision-making Tasks

The work of Piaget and Inhelder prompted a number of studies in the field of probabilistic thinking in children. Yost, Siegel, and Andrews (1962) maintained that Piaget's techniques underestimated the ability of young children. They cited five factors that they believed might account for Piaget and Inhelder's negative results: 1) Piaget and Inhelder relied heavily on verbal skills; the child had to understand terms such as "most likely", "predict", and "expect"; they had to demonstrate their understanding of the concept by verbalization; 2) there was no control for subjects' colour preference; 3) the memory aids (small tokens which represented the payoff colour) proved to be a distracting influence as they were not randomized; 4) there were no tangible rewards or positive verbal reinforcement given for correct responses to provide motivation; and, 5) no provision was made for statistical treatment of results, as the conclusions were based upon individual responses which could not be compared, rather than response frequencies for repeated comparable events.

Yost et al. (1962) attempted to take these factors into consideration by designing a two-choice decision-making task in which the child could make non-verbal responses, the influence of colour preference was controlled, the memory aids were randomized, and the child would receive a reward for a correct response. They compared their decision-making task with a one choice Piagetian style task. They hypothesized that children exhibit a greater understanding of probability under the decision-making condition than under the Piaget-styled condition. Two groups of 10 subjects, ranging in age from 4 years 10 months to 5 years 8 months, were individually presented with 24 trials of both tasks. All subjects performed at or above chance level at both tasks; however, no group's mean score represented consistent use of probability concepts in their responses. Yost et al. (1962) claimed that the introduction of reinforcement into the experiment, as well as statistical controls gave validity to their conclusion that 4-year-old children do have some understanding of probability.

Effect of Reinforcement

Goldberg (1966) investigated the effect of reinforcement on the performance of subjects in Piaget's single task model and Yost et al.'s two task decision-making model. In both cases, the knowledge of whether their decision was correct or incorrect was the only reinforcement for the subjects.

Goldberg (1966) found the subjects in the decision-making model made more correct responses than those in the Piagetian model. This meant that under conditions of equal reinforcement the subjects performed better in the decision-making model. Searching for an alternative explanation for the superior performance of subjects in the decision-making task, Goldberg found colour preference, or confusion of colour preference and colour expectation can strongly influence the outcome of the experiment. The absence of a control for colour preference in the Piagetian model was cited as a possible explanation for the poor performance of the subjects on these probability tasks. However, Hoemann and Ross (1971) inferred from Goldberg's results, if preschool children can be swayed by colour preference then the strength of their grasp of probability concepts is questionable.

Verbal and Nonverbal Techniques

Davies (1965) supported Piaget's interpretation of the acquisition of probability concepts as a developmental progression. However, in support of Yost et al. (1962), she found that children do have an understanding of probability concepts at an early age, but they cannot verbalize them. In a study of 112 subjects ranging from age 3 to 9 years, the mean age for acquisition of verbal ability to describe probability was 7 years 4 months, but 100% of the subjects did not pass the verbal test until 9 years of age.

Carlson (1970) compared Piaget's (1951/1975) verbal technique and Yost et al.'s (1962) nonverbal technique. He found that verbal and nonverbal techniques assess different aspects of development and consequently should not be compared. In contrast to Davies (1965), the general age brackets suggested by Piaget for the development of probabilistic thinking were supported. In addition, Carlson (1970) suggested that the procedure used by Yost et al. (1962) may not adequately assess a child's concept of probability. He suggested that correct responses appeared to be dependent upon an understanding of simple empirical frequencies rather than an operative system that allows prediction of distributional form.

Role of Dominant Features

Falk, Falk, and Levin (1980) claimed that the results of Yost et al. (1962), Goldberg (1966), and Davies (1965) contradicted those of Piaget and Inhelder (1951/1975) because of differences in the mathematical features of the problems that were presented. After analyzing the problems used in the studies, Falk et al. found that some investigators included certain elements in their tasks that others overlooked. For example, Yost et al. (1962) and Davies (1965) compared only complementary probabilities (e.g. 1/4 with 3/4, 1/3 with 2/3). These always included the same number of elements and therefore the correct answers could be obtained by looking at absolute numbers of the payoff colour. Hoemann and Ross (1982) also suggested that, while the studies of Yost et al., Davies, and Goldberg have in common the above chance level of performance of very young children on probability tasks, they do not show any evidence that their tasks do indeed measure understanding of probability. They cited an example; if a child, who likes black jelly beans, was offered a choice of two equal handfuls of jelly beans, one which has four black and one other colour or one which has one black and four other colour, the child would likely pick the hand with the most black without using the concept of probability.

Development of Probability Concepts

Hoemann and Ross (1971) carried out a series of studies to find out whether probability concepts are required for above-chance performance in probability tasks. They investigated two types of probability tasks; 1) the decision-making task, where the choice is between two alternate spinners, and 2) a single spinner task, where a prediction is made based upon the coloured elements on the spinner. In all the experiments the subjects were presented with spinners that were divided into relative proportions of black and white. Hoemann and Ross devised two types of instructions, one requiring a probability judgment and the other requiring a magnitude estimation. For example, for the single spinner prediction task, the probability instruction asked the subjects where they thought the pointer will land when it stopped, on white or black. For the magnitude estimation instruction, the subjects were asked to state which colour was the most, white or black.

Hoemann and Ross found that in the two-choice, alternative odds task the two types of instructions gave the same results. They concluded that comparing proportions and estimating odds was not required for a correct answer in this two-choice task, and therefore an understanding of probability concepts was not required for above-chance performance. However, they pointed out that this does not mean that two-choice tasks fail to measure probability concepts, only that researchers who use the two-choice tasks must demonstrate that the task measures what it is designed to measure.

For the single-spinner prediction task the results showed a difference between the two types of instructions. The errors in the probability instruction were significantly higher that of the magnitude estimation instruction. Hoemann and Ross concluded that an understanding of probability concepts did contribute to above-chance performance in the prediction task.

Hoemann and Ross concluded that successfully choosing the task with the more favourable odds did not give an index of probability knowledge, particularly if direct magnitude comparison can take place. They indicated that not all the tasks that nominally are probability tasks require the use of probability concepts. They also acknowledged the various definitions of probability and concluded that magnitude discrimination, for them, was not considered to be a probability concept, but rather a necessary step before understanding probability. Recalling Piaget and Inhelder (1951/1975), they concluded that the "discovery of chance" does not occur until the onset of concrete operations when a child is about 6-8 years old, but magnitude discrimination is almost completely mastered by this time. They also concluded that the 4-year-old children showed no evidence of a concept of probability.

Chapman (1975) supported Piaget and Inhelder's (1951/1975) view that proportionality concepts and the ability to deal logically with abstract relations do not develop before formal operations. He found that even 10 to 11 year old children do not discriminate proportions for the probabilistic reasoning task. Chapman also compared children's performance on one-container and two-container tasks. Previously, Goldberg (1966) and Yost et al. (1962) had found that performance of preschool children on two-container tasks were significantly better than performance on one-container tasks. However, Chapman (1975), did not find any significant difference between performances of first grade children on one-container and two-container tasks.

Alternative Hypotheses

Perner (1979a) offered an "Alternative Events" hypothesis to counter Piaget and Inhelder's (1951/1975) claim that young children's inability to deal with probabilistic tasks stems from difficulty with part-whole relationships (the "Part-Whole" hypothesis). Perner (1979a) suggested that young children have difficulty understanding truly alternative events. He claimed that once they can evaluate and compare alternatives they will arrive at an initial understanding of probability. This hypothesis, although similar in form to Piaget and Inhelder's (1951/1975), does not assume that children understand part-whole relationships before they understand probability. Perner tested his "Alternative Events" hypothesis and Piaget and Inhelder's "Part-Whole" hypothesis in two experiments involving part-whole related spinners and disjoint spinners. Neither hypothesis was supported.

In a second study, Perner (1979b) sought to explain why some studies found performance differences between single-spinner and double-spinner tasks, yet others did not. Perner conducted two experiments. In the first experiment, he hypothesized that preferential priming, that is asking children to express their preference between sets of colours, would induce the subjects to choose the correct spinner in the double-spinner task, but the incorrect event in the single-spinner task. The preferential priming was completed before the experiment and the answers given by the subject during the preferential priming were neither right or wrong.

Perner found no evidence that preferentially primed subjects performed better on the double-spinner task than on the single-spinner task. Perner concluded that preferential priming did not seem to account for the performance differences in the two tasks.

In the second experiment, Perner (1979b) hypothesized that subjects would be more likely to attend to features such as number and area the more these features varied. Focusing on variation in magnitude, Perner used seven different disks to test this hypothesis. The hypothesis was not supported by the data.

Perner concluded that reported differences between single-spinner and double-spinner tasks are due to idiosyncratic experimental methods and cannot be replicated.

As previously mentioned, Piaget and Inhelder's (1951/1975) research has formed the impetus for much of the investigation into the development of probabilistic thinking in children. By focusing on the concept of chance, they have provided a conceptual framework for discussion of the systematic development and understanding of laws and causality.

Fischbein (1975) explored what he called the "primary intuition" of chance. He claimed that the intuition of chance is evident before the concept of chance is developed. Fischbein suggested that the intuition of chance exists before the ages of 6 to 7 years, in preschool children. He claimed "there certainly exists a primary, pre-operational intuition constructed out of the day-to-day experience of the child and complementary to the intuition of necessity" (p.71).

Fischbein (1975) hypothesized two kinds of intuition: primary intuitions, which are contained within the individual and do not require instruction; and secondary intuitions, which are formed through education. He stated that preschool children have an intuition of chance, but it is distorted by: 1) subjectivism, where the child sees the random event of an object as having a "will" of its own; 2) passive induction, where children base their decision on the event immediately before, not on the basis of all previous events; 3) a belief that random events are controlled by the operator; and 4) changes in the experiment which are unnecessary and can confuse the subject (e.g. subjective preference, preceding outcomes). Fischbein contended that if preschool children work with a small number of possibilities and therefore limited possible outcomes, they can reason correctly.

Fischbein, in support of Piaget and Inhelder, acknowledged that a systematic understanding of probability does not appear until a much later age (11 to 12 years).

But he also contended that attaining an understanding of probability could occur at a younger age (9 to 10 years) if there is elementary instruction. Fischbein, Pampu, and Manzat (1971) stated that 9 to 10 year olds have the conceptual framework to complete these problems, but it must be nurtured through training.

Fischbein (1975) concluded that the development of chance is a progressive phenomenon which begins with the intuition of chance in preschool children. He contended that probabilistic thinking is an important element of our scientific education which cannot be entrusted to primary intuitions. He stated:

> (But) in order for this requirement of an efficient scientific culture to be met, it is necessary to train, from early childhood, the complex intuitive base relevant to probabilistic thinking; in this way a genuine and constructive balance between the possible and the determined can be achieved in the working of intelligence (p.131).

Falk et al. (1980) also supported the inclusion of probability instruction at a very young age. They suggested that one method of helping to develop probabilistic concepts in young children is to have them play probability games.

Gender Differences

Some probability studies have included investigations of the differences in task success between male and female subjects. Davies (1966) found no significant difference at any age level in a non-verbal test. However, on a verbal test there was one significant difference in favor of girls at age 7. Other probability studies which have shown significant gender differences (Chapman, 1975; Perner, 1979b; Ross, 1966) favoured boys. In all the above studies, the gender differences were so small that generalizations concerning sex differences in performance on probabilistic tasks cannot be made.

Conclusion

Most recently, one of the emphases of the research in probabilistic thinking in children has been to encourage the exposure to and instruction of probability concepts in the elementary school (Falk et al., 1980; Fischbein, 1975; Fischbein et al., 1971; Reys, 1978). This has evidently provided the impetus for some publishing companies to include units on probability in the upper intermediate texts, and introductory sections on probability in programs for primary children.

With the exception of Quebec, which articulated the inclusion of probability as an enrichment unit in 1980, the field of probability has been conspicuously absent from most of the elementary mathematics provincial curriculum guides until very recently. At present, the newly revised provincial curriculum guides for mathematics in British

Columbia, Alberta, and Manitoba have included guidelines for the introduction of probability and statistics to elementary students.

In summary, the development of probabilistic thinking in children is a relatively new area of research. Piaget and Inhelder (1951/1975) provided much of the impetus for early explorations in the field. Recently, the importance of including probabilistic thinking as a topic in mathematics education (Falk et al. 1980; Fischbein, 1975; Reys, 1978) has been explored. Additional research in this area needs to be pursued so that educators can use appropriate methods to provide interesting and relevant instruction in probability.

Chapter 3
METHODOLOGY

This study is a partial replication of one of the experiments developed by Falk, Falk, and Levin (1980), who studied the development of probability concepts in children from the ages of 4 to 11 years. The reasons for replicating their study were as follows: 1) the experimental design was well-defined and provided substantial information about the details of the experiment; 2) major issues arising from other research were addressed, for example, the factor of magnitude estimation; 3) the subjects were from a different education system and consequently the findings of Falk et al. (1980) were not directly applicable to our educational situation; and 4) the subjects were not randomly selected and represented a potentially biased sample, namely those who were interested in playing the "lottery game".

The following is an outline of the methodology used in this experiment, including the changes that were made to the procedures used by Falk et al. (1980).

The Subjects

A total of 48 subjects from a single elementary school participated in the experiment. The elementary school has a total population of approximately 650 students and is located in an upper-middle socioeconomic class suburban neighbourhood.

The subjects of the study consisted of 24 boys and 24 girls. They were chosen according to grade level, rather than on the basis of age, because the main purpose of this research was to study the development of probability concepts in children at the various grades in elementary school. To select the sample, the students at each grade level were divided into two groups on the basis of gender. Four boys and four girls were randomly selected from each grade list from kindergarten to grade seven. The first three boys and three girls selected at each grade level were contacted to participate in the study. The fourth selection for each gender at each grade level was reserved as an alternate.

Some parental resistance to the research was encountered. A few parents were concerned with the ethical and moral implications of exposing their children to "gambling" and refused to allow their children to take part. These parents were assured that the intention of the research was to study the cognitive development of probability concepts in children and their children were replaced by the alternate students at that grade level.

The research of Falk, Falk, and Levin (1980) comprised two experiments. The first experiment involved 36 children, 20 girls and 16 boys, from the age of 5 to 11 years. For the second experiment the 25 children, 15 girls and 10 boys, were between the ages of 4 and 7 years. The children were unsolicited volunteers from an upper-middle class neighbourhood near the Hebrew University of Jerusalem.

The Apparatus

Falk, Falk, and Levin (1980) initially designed three distinct types of apparatus to assess the performance of the subjects on probability comparison activities in different dimensions: 1) pairs of spinners of different radii divided into proportional yellow and blue sectors (one-dimensional); 2) pairs of plastic containers with different numbers of yellow and blue wooden beads in each (two-dimensional); and 3) pairs of wooden spinning tops of different volumes divided into sections of yellow and blue (three-dimensional). Using Pearson's coefficient of correlation, Falk et al. (1980) found no significant difference between the results obtained from each apparatus (p. 192). This meant that each set of apparatus was found to be reliable as a independent measure of children's ability to compare probabilities. Because no statistical difference was found between the results of the different kinds of materials, the spinner apparatus was selected for use in their second experiment because it had produced the shortest average session time.

Based upon the findings of Falk et al. (1980), the decision was made to use spinners in the present study. The spinners were round circles of varying radii (see Table 1) which were divided into sectors of equal central angles. The area of the spinner was proportional to the number of its sectors. This ensured that the sectors were of equal area. The apparatus was designed so that two of the largest spinners would fit onto one half of a piece of standard railway board (36 cm wide by 56 cm long). Thirty-six pairs of cardboard spinners were constructed. The sectors on each spinner were coloured blue or yellow. These colours were chosen because Falk, Falk, and Levin (1980) had used blue and yellow as their sector colours, and the colours offered a good contrast, but were not too bright. The sectors of the spinner were coloured by alternating blue with yellow. If the number of sectors were not equal, the colours were arranged to present as great a contrast as possible. For example, if the spinner had four yellow sectors and six blue sectors, the pattern would be yellow, blue, blue, yellow, blue, yellow, blue, blue, yellow, blue. In addition to adjusting the pattern of colours on the spinner, the

TABLE 1

Spinner Sizes

Number of Sectors	Radius of Spinner (cm)	Central Angle of Each Sector (degrees)
2	5.0	180.0
3	6.1	120.0
4	7.1	90.0
5	7.8	72.0
6	8.5	60.0
7	9.2	51.4
8	9.8	45.0
9	10.4	40.0
10	11.0	36.0
11	11.5	32.7
12	12.0	30.0
13	12.5	27.7
14	13.0	25.7
15	13.5	24.0
16	14.0	22.5

orientation of the spinner was shifted. For example, for a set of spinners, each with three sectors, two blue and one yellow, one spinner would have the yellow sector at the "top" of the spinner board, while the other would be rotated to either side or to the "bottom" of the spinner board.

A seven millimetre hole was made in the centre of each spinner. The cardboard spinner pairs were stacked in groups of 18 on a wooden board (45 cm wide by 65 cm long). A piece of wooden doweling (6 mm in diameter by 8 cm long) was pushed through the central hole in each spinner and anchored in a drilled hole in the wooden board (see Figure 1). A small pointer, made from a wooden popsicle stick, was used as the spinner's indicator. The popsicle stick was cut to seven centimetres and then filed to a point at one end. The opposite end was filed to conform to the inner concave curve of a metal washer. This end was glued to the metal washer and easily slipped over the doweling.

The Design

For each task the subject was presented with a pair of spinners of specific probabilities and asked to choose the spinner which he or she would like to twirl in order to produce an element of the *payoff colour* (POC). The probability of the payoff colour for each spinner was the ratio of the number of elements of the POC to the total number of elements in the spinner. This meant that the two

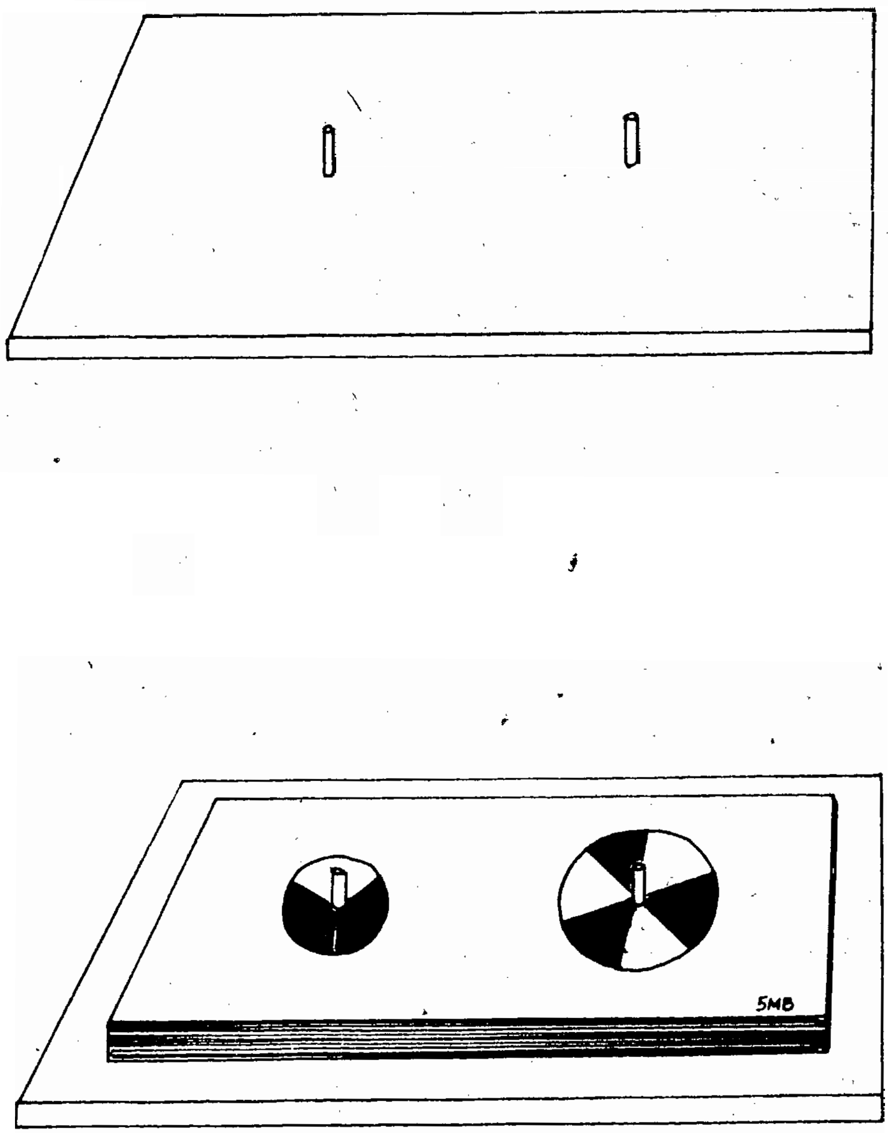

Figure 1. Diagram of spinner board apparatus.

spinners in each spinner set could differ in three ways: 1) the total number of elements; 2) the number of payoff elements; and 3) the number of non-payoff elements. Table 2 shows the relation between the number of payoff elements and the total number of elements. This produced a potential for nine different kinds of tasks. However, three of these combinations (Cell D, Cell F, Cell G) are impossible as the two variables, POC and the total number, are not independent of each other. For example, the number of POC cannot be smaller on the correct side if the total number of elements are equal (Cell F). Six different tasks remained. Another task (Cell I), where the total number is equal on both spinners and the POC is equal on both spinners, produces identical tasks.

With the addition of the number of non-payoff elements as a variable, the number of potential tasks increases to 18 (see Table 3). Most of these potential tasks, however, are impossible because the elements are not independent of each other. As a result, only two different tasks were added, for a total of eight task categories.

After this initial categorization, the tasks were divided into three levels of difficulty. The level of difficulty for each task was determined by the relation of the proportion of the spinner to the fraction 1/2. Easy tasks were those in which one of the two proportions was larger than 1/2 and the other smaller than 1/2. Medium

TABLE 2

Classification of Tasks by Number of POC Elements and Total Number of Elements

		Total Number of Elements		
		Greater on correct side	Smaller on correct side	Equal
POC Elements	Greater on correct side	Cell A	Cell B	Cell C
	Smaller on correct side	Cell D[1]	Cell E	Cell F[1]
	Equal	Cell G[1]	Cell H	Cell I

[1] Tasks in these cells are impossible.

TABLE 3

Classification of Tasks by Number of POC Elements, NPOC Elements, and Total Number of Elements

		TOTAL NUMBER OF SECTORS								
		Greater on Correct Spinner			Smaller on Correct Spinner			Equal		
NUMBER OF NPOC SECTORS[1]		>	<	=	>	<	=	>	<	=
NUMBER OF POC SECTORS[1]	>	CELL 1 (9)	CELL 2 (3)	CELL 3 (3)	X	CELL 4 (3)	X	X	CELL 7 (3)	X
	<	X	X	X	X	CELL 5 (9)	X	X	X	X
	=	X	X	X	X	CELL 6 (3)	X	X	X	CELL 8 (3)

[1] on correct spinner (#) number of items X impossible

tasks offered a comparison where one proportion was 1/2 and the other differed from 1/2. Difficult tasks presented proportions on the spinners which were both larger or both smaller than 1/2. In the easy and medium tasks, the proportions were designed using the minimal number of elements that satisfied all the requirements of the task. In the difficult tasks the proportions were constructed so that the tasks became increasingly difficult, with the most difficult task having the smallest difference of ratios between the elements. The experiment included 12 easy, 10 medium, and 14 difficult tasks. The tasks are outlined in Table 4.

Each child was presented with all 36 tasks. The tasks were presented in one sitting with a break at the conclusion of 18 tasks. The tasks were presented in a random order to each student. This was accomplished by shuffling all of the spinner boards at the conclusion of each experimental session.

The Procedure

The experiment was held in a small seminar room in a classroom complex. The apparatus was set up for the duration of the experiment. Each child was met at his or her classroom and escorted by the experimenter to the seminar room. During this time the child was familiarized with the basic purpose of the experiment and any

TABLE 4

Composition of Tasks

Category	Level of Difficulty	Item Identifier	Composition (B-Y) (B-Y)
1	Easy	1EA	(4-3) (1-2)+
1	Easy	1EB	(6-4) (2-3)
1	Easy	1EC	(6-3) (4-2)+
1	Medium	1MA	(5-3) (2-2)
1	Medium	1MB	(4-4) (2-3)+
1	Difficult	1DA	*(4-6) (1-2)
1	Difficult	1DB	(3-5) (1-2)+
1	Difficult	1DC	(3-6) (1-3)
1	Difficult	1DD	(3-4) (2-3)+
2	Easy	2E	(5-3) (2-4)
2	Medium	2M	*(5-2) (3-3)+
2	Difficult	2D	(7-3) (5-4)
3	Easy	3E	*(2-3) (1-3)+
3	Medium	3M	(3-2) (2-2)
3	Difficult	3D	(4-2) (3-2)+
4	Easy	4E	(4-1) (2-4)
4	Medium	4M	(5-2) (4-4)+
4	Difficult	4D	*(3-4) (2-6)
5	Easy	5EA	(2-1) (5-6)+
5	Easy	5EB	(3-2) (4-6)
5	Easy	5EC	*(2-6) (3-9)+
5	Medium	5MA	(2-2) (3-5)
5	Medium	5MB	(2-1) (3-3)+
5	Difficult	5DA	(3-1) (5-2)
5	Difficult	5DB	(4-2) (7-4)+
5	Difficult	5DC	(3-2) (7-5)
5	Difficult	5DD	(2-1) (4-3)+
6	Easy	6E	(3-2) (3-4)
6	Medium	6M	(3-3) (3-5)+
6	Difficult	6D	*(2-3) (2-4)
7	Easy	7E	*(4-1) (3-2)+
7	Medium	7M	(6-4) (5-5)
7	Difficult	7D	(2-6) (1-7)+
8	Easy	8E	(1-1) (1-1)
8	Medium	8M	*(2-3) (2-3)
8	Difficult	8D	(5-3) (5-3)

+ reverse order on actual spinner board
* item selected for specific questioning

apprehensions he or she may have had with regards to the experiment were eased. Upon arrival at the seminar room, the child was asked to sit at a large rectangular table opposite the experimenter. The spinner board, the incentive board (see Figure 2), and a tape recorder microphone were arranged on the table. The purpose of this apparatus was explained to child and any questions regarding the apparatus were answered.

Each child was invited to play a preliminary game to familiarize the child with the incentive procedure and the idea that the game involved chance. In the game the child was asked to pick a number from one to six on a over-sized die. The child was told to throw the die in the air to see if the die would land with the number that he or she selected showing on top. The child was given three tosses of the die.

Each time the number the child selected came up, the child was given a small plastic token. The plastic tokens were used as "steps" on a trail toward an item of food for "Hotdog Day". Hotdog Day is a monthly fund raising event at the school sponsored by the Parent Consultative Committee. The Hotdog Day chart offered three items: a hotdog, a donut and a drink (see Figure 2). Each item had a trail of spaces for the tokens. When the token spaces were filled the child was given a credit for that item for the following Hotdog Day. This procedure was followed for all children in

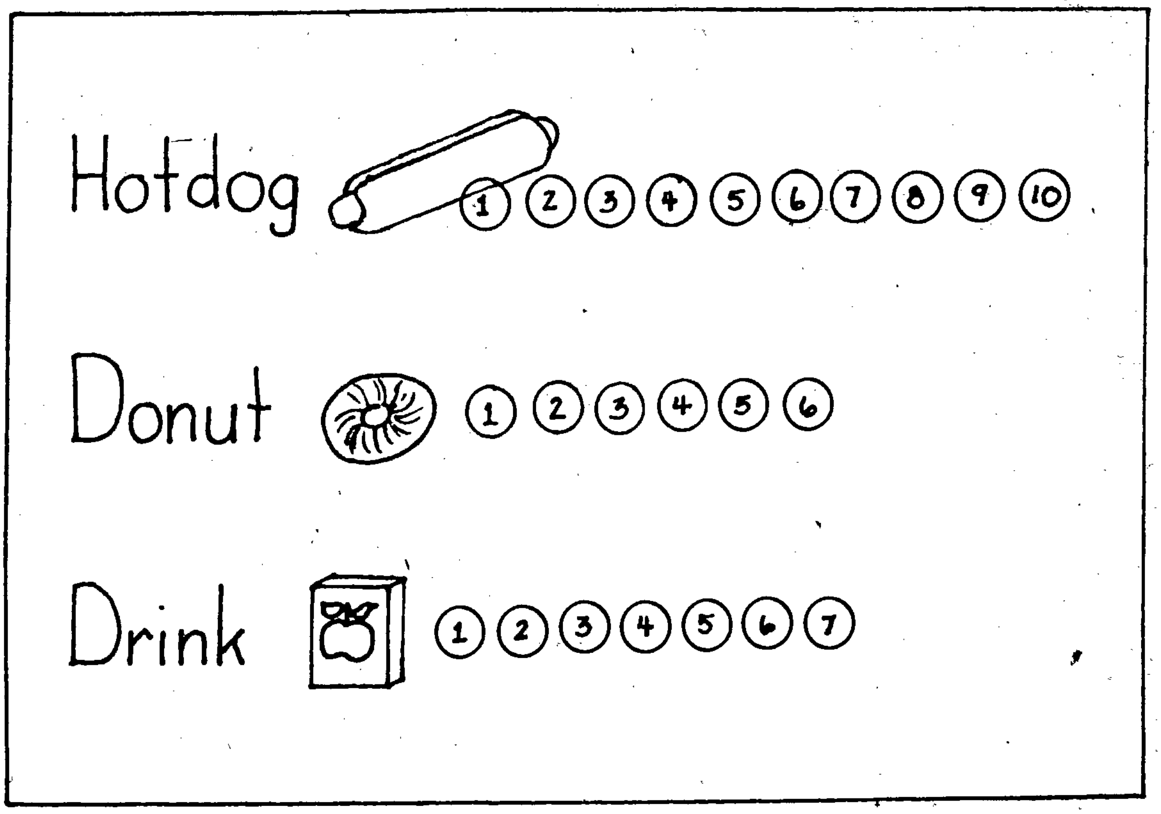

Figure 2. Incentive board.

Grades 1 through 7.

A different incentive was used for the kindergarten children because their short day did not permit them to take part in Hotdog Day. The kindergarten children were given a choice of ten different "Scratch-n-Sniff" stickers. The chosen sticker was placed at the end of a trail of token spaces. If the number the kindergarten child selected came up, the child was given a token to place along the trail. When the trail was filled with tokens the child was given the Scratch-n-Sniff sticker.

When the preliminary game was finished, the purpose of the experiment was reiterated. The spinner board was explained to the child and the apparatus was demonstrated. Each child was given the opportunity to flick the pointer a couple of times in order to familiarize himself or herself with the apparatus. At the outset of each trial during the experiment, a token was placed to the side of the spinner board. The subject was initially informed and subsequently reminded throughout the experiment that the payoff colour was blue. A blue block was used as a visual reminder and was placed at the top of the spinner apparatus in full view of the subject. The child was then asked to choose the spinner which he or she would expect to produce an element of the required colour (blue). After making a choice, the child was asked to spin the pointer of that spinner. If the pointer landed on a blue segment, the token was placed along the selected trail. If the pointer did not land on a blue segment, the token remained for the next item. The spinner the student chose, as well as the outcome of spinning the pointer were recorded by the experimenter on a form (see Figure 3).

One record form was used for each student. The gender and the grade level of the student was recorded. Each item was listed by code. Adjacent to the code a narrow bar was coloured black to indicate right side of the spinner board (experimenter's right) as the correct choice, or left uncoloured to indicate the left side of the spinner board (experimenter's left) as the correct choice. The subject's selection of the spinner (experimenter's right or left) and the outcome of the spinner were recorded. The third column was used for scoring at the conclusion of the experiment.

Each session was audio-recorded to obtain an accurate account of all verbal comments made by the students. This provided an explanation for the choices made by individuals.

In order to enable a comparison between the grade groups, and between the individual responses within grade levels, one task was chosen from each of the eight categories for specific questioning. The choice to limit specific questioning to eight items was necessary in order to keep the individual sessions to a manageable time (30 minutes). The main factor when selecting the items was representation of the levels of difficulty. The items were

Subject: M F Grade: K 1 2 3 4 5 6 7

		Choice	Decision
1 E A			
1 E B			
1 E C			
1 M A			
1 M B			
1 D A			
1 D B			
1 D C			
1 D D			
2 E			
2 M			
2 D			
3 E			
3 M			
3 D			
4 E			
4 M			
4 D			

		Choice	Decision
5 E A			
5 E B			
5 E C			
5 M A			
5 M B			
5 D A			
5 D B			
5 D C			
5 D D			
6 E			
6 M			
6 D			
7 E			
7 M			
7 D			
8 E			
8 M			
8 D			

Total Correct.... ________ .... ________%

Easy Correct.... ________ .... ________%

Medium Correct.. ________ .... ________%

Difficult Correct... ________ .... ________%

Figure 3. Record form.

selected from the eight categories without bias. The selected items were 1DA, 2M, 3E, 4D, 5EC, 6D, 7E, and 8M. These task items are defined in Table 4. The selected tasks were marked with a small asterisk by on the spinner board to facilitate identification by the experimenter during the sessions.

As the selected tasks were completed by the student and the results were recorded by the experimenter, the student was asked why he or she made that particular choice. At the end of each session the audiotapes were coded and subsequently transcribed. The transcriptions were mounted by category and grade level on large pieces of cardboard. This enabled a ready comparison between and within the grade groups.

The Analysis

The data collected was analyzed by determining: 1) the percent of correct responses of all tasks by grade; 2) the percent of correct responses by grade and category; 3) the percent of correct responses by grade and the level of difficulty; and 4) individual responses from the verbal discussions.

Chapter 4

RESULTS

The results for each grade, task category, and level of difficulty were calculated by compiling the number of correct responses for each grouping. This raw score was transformed into a percent correct score by dividing the actual number of correct responses by the total possible number of correct responses. The verbal responses made by each student were also assessed.

Group Results

The group results were calculated from subject response on 31 of the 36 items. Five items were not included in these results because they represented a choice between two spinners with equal proportions. An analysis of these tasks will conclude the section on group results.

For the 31 tasks, the percent of correct response by all subjects was 77%. This is above chance level, as the upper threshold for percent correct by chance is about 53%

(n of items = 31, n of subjects = 48, α = 0.05).

The group results were evaluated in three ways: 1) by grade level; 2) by task category (see Table 5); and 3) by level of difficulty: easy, medium, and difficult.

Grade Level Results

For each grade the percent of correct responses for each category and for the total of each category were calculated. These results are presented in Table 6.

The total percent correct scores for each grade are above chance level. The upper threshold for percent correct by chance is about 57% (n of items = 31, n of students = 6, α = 0.05). The percent correct scores tend to increase through the grades. The Grade Three average of 75% and the Grade Six average of 79% show some deviation from the upward trend, with scores lower than the previous grade, but these do not represent a substantial shift from the trend. When the adjacent grade groups are paired the improvement through the grade levels is evident. Table 7 shows the percent of correct responses for the adjacent grade pairs (K-1; 2-3; 4-5; 6-7). Response by grade and paired grades are graphically represented in Figure 4.

There is a spread of 20% between the highest success rate (Grade Seven; 84%) and the smallest success rate

TABLE 5

Category Definition For Task Items

Category 1 -	POC greater on correct spinner NPOC greater on correct spinner Total number greater on correct spinner
Category 2 -	POC greater on correct spinner NPOC less on correct spinner Total number greater on correct spinner
Category 3 -	POC greater on correct spinner NPOC equal Total number greater on correct spinner
Category 4 -	POC greater on correct spinner NPOC less on correct spinner Total number less on correct spinner
Category 5 -	POC less on correct spinner NPOC less on correct spinner Total number less on correct spinner
Category 6 -	POC equal NPOC less on correct spinner Total number less on correct spinner
Category 7 -	POC greater on correct spinner NPOC less on correct spinner Total number equal
Category 8 -	POC equal NPOC equal Total number equal

TABLE 6

Percent of Correct Response by Category and Grade

Grade	Category 1	2	3	4	5	6	7	Total
	Number of Tasks							
	8	3	3	3	8	3	3	31
K	79	78	67	67	46	56	61	64
1	75	89	78	72	63	89	72	74
2	75	83	94	94	58	78	94	77
3	75	89	89	94	52	72	94	75
4	46	100	89	83	81	100	94	78
5	63	94	94	83	77	94	100	81
6	60	94	89	94	75	83	89	79
7	65	100	100	84	88	89	94	84
Total	67	91	88	84	67	83	87	77

TABLE 7

Percent Correct Response by Category and Grouped Grades

Grade	Category 1	2	3	4	5	6	7	Total
	Number of Tasks							
	8	3	3	3	8	3	3	31
K-1	77	83	72	69	54	72	67	69
2-3	75	86	92	94	55	75	94	76
4-5	54	97	92	83	79	97	97	80
6-7	63	97	94	89	81	86	92	82
Total	67	91	88	84	67	83	88	77

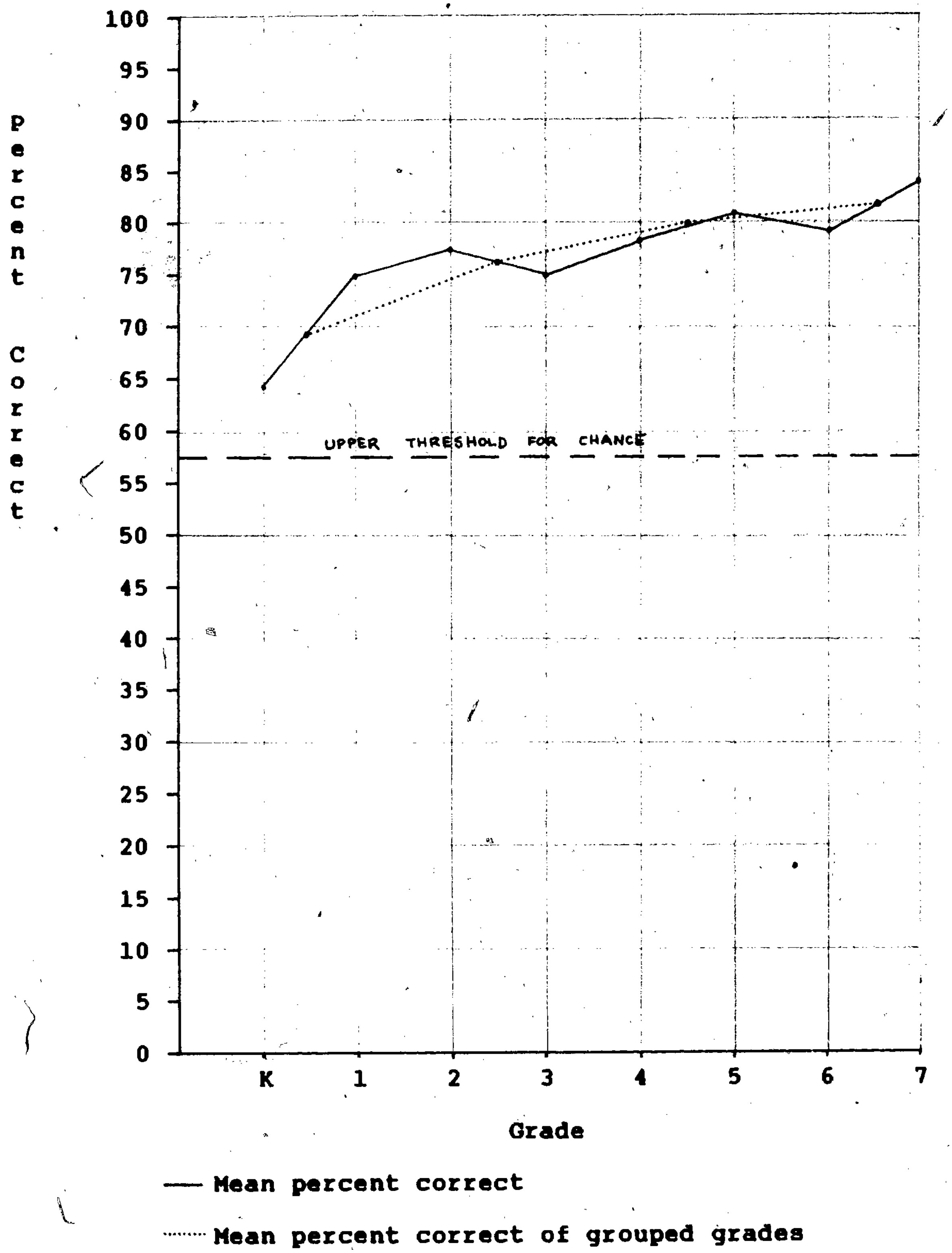

Figure 4. Percent correct by grade level and grouped grades.

(Kindergarten; 64%). The greatest difference between adjacent average success scores is 10% between Kindergarten and Grade One. This is the same as the difference between the Grade One average score (74%) and the Grade Seven average score (84%). In this sample, there was a greater improvement in task understanding between Kindergarten and Grade One than between any other adjacent grade levels.

Grade Results for Specific Categories

The 36 tasks were divided into eight categories according to three criteria: 1) the total number on the correct spinner; 2) the relative number of the pay-off colour (POC) on the correct spinner; 3) the relative number of the nonpay-off colour (NPOC) on the correct spinner. The categories have been defined in Table 5.

The categories with the greatest average success percentage were Category Two (91%) and Category Three (88%). In both of these groups the POC and total number were greatest on the correct spinner. The category with the next highest average success percentage was Category Seven (87%) where the POC was greatest on the correct spinner, but the total number was the same. In these three groups the POC was greater on the correct side, and the NPOC on the correct side was less than or equal to the NPOC on the incorrect side. In these categories the students could choose the correct spinner simply on the basis of the greatest area (or number of sectors) devoted to the POC.

The lowest average success percent occurred in Category One (67%) even though it fulfilled the main characteristic of the categories which evoked the highest correct response; specifically, the POC was greater on the correct spinner.

But Category One had an additional feature; the NPOC was greater on the correct spinner. This means that if a child chose the spinner on the basis of the area or number of sectors devoted to the POC, the choice would be correct. However, if the decision was based on the NPOC, and the spinner with the least number of sectors devoted to the NPOC was selected, the choice would be incorrect.

Another feature of the results in Category One was the relative percent correct of the Kindergarten subjects compared to the other grades. Kindergarten subjects had the highest percent correct (79%), with Grades One, Two, and Three each with 75% (see Figure 5).

However, the percent correct response for Grades Four to Seven dropped considerably with the Grade Four group producing the smallest percent of correct responses out of all groups of items in the experiment (46%). Although there is improvement with the Grade Five scores (63%), the Grade Six and Seven percent of correct responses remain in the same range (Grade Six, 60%; Grade Seven, 65%). The upper threshold for chance level of correct response is 64% (*n* of items = 8, *n* of subjects = 6, $\alpha = 0.05$). This

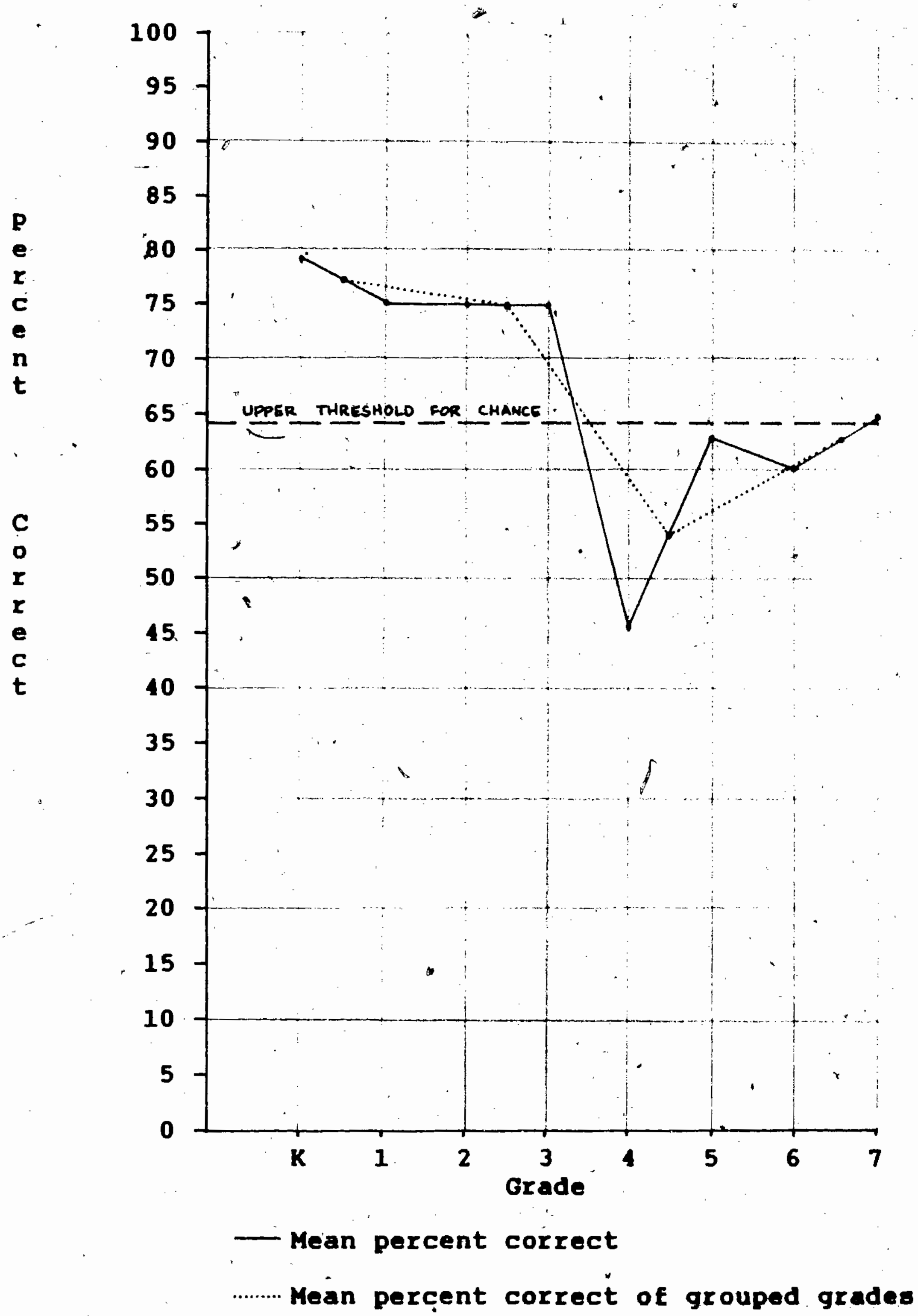

Figure 5. Percent correct in Category One by grade level and grouped grades.

suggests that the primary children may have been single-minded in their responses...looking for the spinner with the greatest number of POC while not even considering the NPOC. It also suggests that the majority of Grade Four to Seven subjects did not calculate the proportion of POC to NPOC to determine a response.

When the audio-taped comments were reviewed and paired with the results of each item, the primary strategy seems to have consisted of a comparison of the relative number of sectors on each spinner devoted to the POC or the NPOC. Students would look at the dominant colour and make their choice. If the dominant colour was blue, the choice would usually be the spinner with the greater number of blue sectors. For example, student MC (grade 2), on task 1MA, reported: " 'Cause this one had more spaces of blue than this one."

If the dominant colour was yellow, the choice would often be the spinner with the least yellow, as it was the non-payoff colour. For example, student SS (grade 4), on task 1DD, reported: "Less yellow triangles than on the other side."

Within Categories Two and Three, the percent of correct responses generally increased through the grade levels. There are small discrepancies in the grade to grade development, but an overall trend of improvement is evident.

Category Four shows improvement in the percent of correct responses from Kindergarten to Grade Three. This is followed by moderate fluctuations in the percent of correct responses from Grade Four to Seven.

In Category Five there is a definite split in the data (see Figure 6). In this category the Kindergarten to Grade Three groups show the lowest percent of correct responses of all categories. As in Category One, the upper threshold for chance level of correct response is 64% (*n* of items = 8, *n* of subjects = 6, = 0.05). There is a 29% jump in the percent of correct responses from Grade Three to Grade Four, with marginal decreases in Grades Five and Six. The average percent correct for the Kindergarten to Grade Three group was 55% and for the Grade Four to Grade Seven group, 80%.

Category Five was the only category where the POC was smallest on the correct spinner. The NPOC and the total number were also smallest on the correct spinner. This would seem to reinforce the hypothesis that primary children exclusively examined the number of sectors, or area, devoted to the POC. If the sectors, or area, devoted to the NPOC had been taken into account the choice would have been the alternative spinner that is, selecting the spinner with the smallest number of NPOC would have produced the correct choice.

In Category Six the percent correct does not show a smooth increase from grade to grade. There is a large

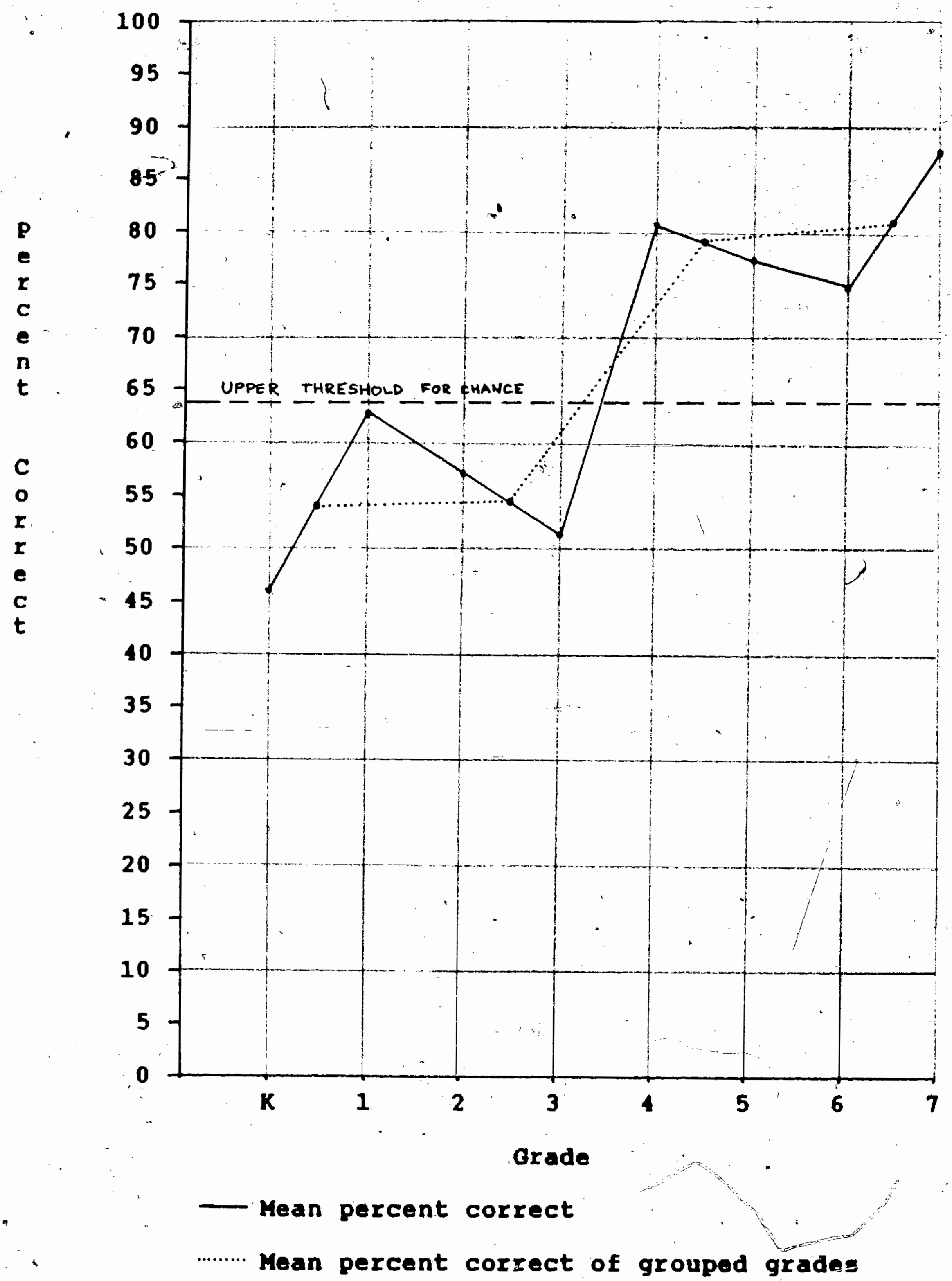

Figure 6. Percent correct response in Category Five by grade and grouped grades.

increase in the percent correct from Kindergarten to Grade One, but then the scores seem to fluctuate.

Category Seven shows an increase in percent correct responses from Kindergarten to Grade Two, with fluctuations thereafter. The upper threshold for chance level of correct response is 74% (*n* of items = 3, *n* of subjects = 6, $\alpha = 0.05$).

Grade Results for Levels of Difficulty

The tasks were also grouped according to level of difficulty within each of the categories. The tasks were classified as easy, medium, and difficult. As is shown in Table 8, the total percent correct for the easy tasks was slightly better (87%) than that for medium tasks (84%). However, there is a large drop of 19% between the percent correct of the medium and difficult tasks, indicating that the difficult tasks were relatively more challenging. With the exception of the easy and medium scores at the Kindergarten and Grade One level, there is a tendency for the total number of correct responses to decrease with the difficulty of the task.

Because the level of difficulty of the task was determined by the relation of the proportion of the POC on the spinner to the fraction 1/2, the difference between the proportion of the POC on each spinner can be used as a measure of the difficulty of the task. One can assume that the smaller the difference between the proportions on the spinners, the more difficult the task. For example, on item 1EB (an easy task), one spinner had 6 blue and 4 yellow (0.60 blue), and the other, 2 blue and 3 yellow (0.40 blue), a difference of 0.20; on item 6D (a difficult task), one spinner had 2 blue and 3 yellow (0.40 blue), and the other, 2 blue and 4 yellow (0.33 blue), a difference of 0.07. The correlation between the percent of correct response on each item, and the difference between the proportion of the POC on each spinner was calculated by means of the Pearson product-moment. The results of the Pearson product-moment calculations showed a relationship between the percent of correct response and the difference between the proportion of the POC on each spinner, $r = 0.47$, $p < .01$.

The percent of correct responses by grade for the easy, medium, and difficult categories do not demonstrate any trend from grade to grade (Table 8). However, when the grades are grouped in pairs (K-1; 2-3; 4-5; 6-7) the percent of correct responses monotomically increases through the grades (see Table 9).

The difference between the percent correct of the Kindergarten group and the Grade Seven group on the easy tasks was 26%, and on the medium tasks, the difference was 24%. For the difficult tasks the difference between the Kindergarten and Grade Seven percent correct scores was the

TABLE 8

Percent Correct Response by Level of Difficulty and Grade

	Category			
	Easy	Medium	Difficult	Total
	Number of Tasks			
Grade	9	9	13	31
K	69	70	56	64
1	76	80	69	74
2	96	83	60	77
3	83	82	65	75
4	89	87	64	78
5	93	89	68	81
6	94	85	63	79
7	94	94	71	84
Total	87	84	65	77

TABLE 9

Percent of Correct Response of Paired Grades by Level of Difficulty

	Category			
	Easy	Medium	Difficult	Total
	Number of Tasks			
Grade	9	9	13	31
K-1	72	75	63	69
2-3	90	82	63	76
4-5	91	88	66	80
6-7	94	90	67	82
Total	87	84	65	77

smallest at 14%. Thus, there was less improvement between groups for the difficult tasks than for either the medium or easy tasks. The decreasing difference between the Kindergarten and the Grade Seven results on the tasks is even more striking when the grades are grouped (Table 9). The difference between the percent of correct responses on the easy tasks for the K-1 group and 6-7 group remains the greatest at 22%, the difference between the groups on the medium tasks decreased slightly to 15%, but the difference between the groups on the difficult tasks was only 4%.

Grade Results for Items

The percent of correct response were calculated for each item and the results are shown in Table 10. Item 2E (5b, 3y vs. 2b, 4y) generated the highest percent of correct response (94%). This is not surprising as the POC was greater on the correct spinner and the NPOC was lesser on the correct spinner.

Item 1DB (3b, 5y vs. 1b, 2y) had the smallest percent of correct response (15%). The reason for this is unclear. The item with next smallest percent of correct response was item 5DB, where the percent of correct response was 42%. The difference between the proportion of the POC of the spinners in Item 1DB was small (0.05), but other spinner sets had a smaller difference between the proportion of the POC and achieved a higher percent correct response. For

TABLE 10

Frequency of Correct and Incorrect Response and Percent Correct Response on Items

Item	Percent Correct
1EA	79
1EB	90
1MA	92
1MB	85
1DA	52
1DB	15
1DC	67
1DD	58
2E	94
2M	88
2D	92
3E	88
3M	92
3D	83
4E	81
4M	85
4D	85
5EA	88
5EB	88
5MA	67
5MB	79
5DA	63
5DB	42
5DC	56
5DD	58
6E	85
6M	81
6D	81
7E	90
7M	83
7D	90

example item 1DD, had a difference between the proportion of the POC on the spinners of 0.03, and a percent correct response of 59%, and item 5DC, had a difference between the proportion of the POC on the spinners of 0.02, and a percent correct response of 56%. A possible explanation for the choice of the incorrect spinner on Item 1DB is the amount of area devoted to the NPOC on the correct spinner. The correct spinner is much larger (radius; 9.8 cm) than the incorrect spinner (radius; 6.1 cm), and with the greater number of NPOC, there appears to be relatively more area devoted to the POC.

Individual Responses

Individual responses to items in the experiment were interpreted by examining the audio-tape transcriptions. The individual responses to each item were examined by grade in order to detect similar comments. Distinct response patterns were indicated and these were recorded in tabular form (see Tables 11 to 18). In this study, the individual verbal responses seem to fall into six categories according to the feature the subject used to make the spinner choice. These were:

1. Reliance on the POC, that is, the subjects referred to some feature of the POC (blue) as the reason for their selection, for example:

> AF (Grade 2), on item 7E, reported: "It had more blues. I just see if there's very many and if there aren't then I go the the other side and look and if there's more then I take that side."

> LE (Grade 6), on item 2M, stated: "This one had more blues. I go for the one that has more blues."

2. Reliance on the NPOC, that is, the subjects referred to some feature of the NPOC (yellow) as the reason for their choice of spinner, for example:

> SG (Grade 1), on item 2M, reported: "Because this one has less yellow than this one."

> KM (Grade 6), on item 6D, stated: "Um-m ...This one has more yellows than this one. This one has three yellow pieces and that one has four yellow pieces."

3. The physical setup of the spinner board and spinner, that is, the subjects referred to the arrangement of the sectors of the spinner, for example, if two blues side by side or, the placement of the pointer (which was started in the same location throughout the experiment). For example:

> AH (Grade 4), on item 6H, responded: "Well, because when you started it, it was in the yellow and like usually spinners go all the way around and it would come all the way around again and I'd have a better chance of getting blue because they're right together."

> JS (Grade 5), on item 1DA, stated: "These ones are more spread out and these ones aren't. The blues are more spread out and the yellows are all in one place.

> 'Cause if the yellows are closer together than you only have a chance on one side with the blue and if they're all spread out then you have a better chance."

4. The physical size or area of the spinner, that is, the student referred to the relative size of one spinner compared to the other, for example:

> DS (Grade 7), on item 3E, stated: "It's smaller (circle) and it's thicker. Well, I don't know if it's much thicker, but it's smaller."

> BN (Kindergarten), on item 1DA, responded: " 'Cause. This one's small. This one's bigger than it."

5. A comparison of the POC and the NPOC between spinners, that is, the student made an observation about the relationship of the POC and the NPOC on one spinner and compared this to the other spinner, for example:

> SW (Grade 4), on item 4D, reported: "There were only three blue and two yellows here and one here (yellow) and one here (yellow). And that one had three yellows and three yellows and only two blues."

> KD (Grade 3), on item 7E, stated: " 'Cause it has three blues and, (pause - counting mentally) 4 blues actually, and one yellow and this has three blues and two yellows."

6. Comments that were not related to the POC, the NPOC, the physical setup or the area of the spinner, for example:

> LE (Grade 7), on item 5EC, responded: " 'Cause I usually always get blue on this side. It's my lucky side."

SG (Kindergarten), on item 1DA, stated: " 'Cause I like it. I like big ones."

JB (Grade 1), on item 5EC, stated: "It would go on blue. (Experimenter: "Why?") 'Cause I want one of those things (points to hotdog on Incentive Board)."

Tables 11 to 18 show the range of responses for each item at each grade level. In four or five instances, a subject provided a response which combined the elements of the categories, and in these cases the first response was used.

The number of responses at each grade level does not always equal the number of subjects at that grade level due to the reluctance of subjects to supply a verbal response, or the failure of the researcher to ask about that item. The reluctance to respond to questions was most common at the Kindergarten level. Although subjects at other grade levels did not always provide a verbal response, there would be a shrug of the shoulders as if to say "I don't know".

For six of the eight items, the absolute number of the POC was the reason most often used by the greatest percentage of subjects.

On item 8M, the vast majority of subjects made a comparison between the two spinners on the spinner board. In this item both sides were identical in size and number of sectors, and the only difference between the tasks was the orientation of each spinner on the board.

On item 6H, the absolute number of the NPOC was used most often to determine the choice. This was an appropriate strategy because the POC was equal on both spinners, while the NPOC was greater on one spinner.

A thorough review of the verbal responses revealed that only two students demonstrated a sound understanding of the concept of probability. These students showed evidence of calculating proportion, as well as an understanding of the element of chance. For example:

> JD (Grade 6), on item 2M, stated: "This one's half and this one's over half so you have a better chance with the one over half."
>
> JD (Grade 6), on item 5EC, responded: "One, two, three, ... twelve. So you have twelve. Three out of twelve. One, two, three, four and two out of eight. So they both equal one-quarter, so it doesn't really matter which one you decide to pick so I'll pick the one with less yellows."
>
> CB (Grade 7), on item 3E, stated: "Two -fifths and one quarter and have the same denominator. Change them to twentieths and this (pointed to the spinner with two blue and three yellow) will have more twentieths."
>
> CB (Grade 7), on item 2M, commented: "This one was seven-fourteenths and this one was ten-fourteenths (and so) this one had a higher chance of getting it."

After choosing many items by correctly computing the proportion of the blue and yellow sectors on each spinner and not experiencing success at getting the POC, student CB (Grade 7) decided, "Most of the ones I have been calculating have been wrong so that's why I picked this one" (1HA). In this instance the student purposely chose the wrong spinner in the hope of a "chance" success.

TABLES 11 to 18

Categorization of Individual Tasks

A - Reliance on the POC
B - Reliance on the NPOC
C - Physical setup of the spinner board and spinner
D - Physical size or area of the spinner
E - A comparison of the POC and the NPOC
F - Comments that were not related to the POC, the NPOC, the physical setup, or the area of the spinner

TABLE 11
Categorization of Item 1DA

	A	B	C	D	E	F	Total
K	3			2			5
1	3				1	1	5
2	3		1			1	5
3	4				1		5
4	2	1		1	1		5
5		1	1	1	1	1	5
6	1	2			1	1	5
7		1		1	1	2	5
Total	16	5	2	5	6	6	40

TABLE 12
Categorization of Item 2M

	A	B	C	D	E	F	Total
K	2					2	4
1	2	1		2		1	6
2	2				2	1	5
3	3	1			1		5
4		1	2		1		4
5	2		3		1		6
6	2		2		1		5
7	1	2	1		1		5
Total	14	5	8	2	7	4	40

TABLE 13
Categorization of Item 3E

	A	B	C	D	E	F	Total
K	3						3
1	4					1	5
2	6						6
3	5			1			6
4	1						1
5	2			1	2		5
6	2		1		1		4
7	1			2	1		4
Total	24	0	1	4	4	1	34

TABLE 14
Categorization for Item 4D

	A	B	C	D	E	F	Total
K	2					1	3
1	5					1	6
2	4					2	6
3	4			1	1		6
4	1	1	1		1	1	5
5	1	2			2	1	6
6	2			1	1	2	6
7	4	1			1		6
Total	23	4	1	2	6	8	44

TABLE 15
Categorization of Item 5EC

	A	B	C	D	E	F	Total
K	4					1	5
1	4	1	1				6
2	2	1	1			1	5
3		3		1	1	1	6
4	1	1		1			3
5	2	1	1		2		6
6				1	4	1	6
7	1	1			3		5
Total	14	8	3	3	10	4	42

TABLE 16
Categorization of 6D

	A	B	C	D	E	F	Total
K	1			1		1	3
1		2			1	2	5
2	2		2			2	6
3		3		1			4
4		2	2				4
5		1		2	3		6
6	1	3	1			1	5
7		2	1			2	5
Total	4	13	6	4	4	8	39

TABLE 17
Categorization of Item 7E

	A	B	C	D	E	F	Total
K	3					2	5
1	2	1	1	1		1	6
2	3	1			1		5
3	2	1	1		2		6
4		3		1	1		5
5	1	2	1		1		5
6	1				4		5
7	1		1		2		4
Total	13	8	4	2	11	3	41

TABLE 18
Categorization of Item 8M

	A	B	C	D	E	F	Total
K					2	1	3
1	1		1	1	1	2	6
2				1	3	2	6
3	1				4	1	6
4			3		2	1	6
5	1	1			1	1	4
6			1	1	3	1	6
7					5		5
Total	3	1	5	3	21	9	42

<u>Items with Spinners of Equal Proportion</u>

In order to investigate student strategies for choosing one spinner over the other when the spinners were of equal proportion, five tasks were designed. These were 1EC, 5EC, 8E, 8M, and 8D. Table 19 shows the frequency of students in each grade who chose the left hand side or the right hand side for each category with spinners of equal proportion. The responses were categorized according to the left hand or right hand of the experimenter.

For items 8E, 8M, and 8D, the subjects were shown spinner sets which were identical in size, and in which the number of sectors devoted to the POC and the NPOC each were equal. The only difference between the spinners was the orientation of the sectors. On all three tasks, the majority of students chose the right hand side over the left hand side. The individual responses for item 8M indicate that half of the given responses made a comparison between the two spinners in the set (see Table 19). Most students did acknowledge that the spinners were the same, but the reasons for choosing one spinner over the other were varied. For example, student SG (Kindergarten) stated: "I could get blue. I thought it would come on that one. (Experimenter: "Why?") 'Cause." Student CW (Grade 1) responded: " 'Cause that one was nice and straight and that one was crooked" (referring to the orientation of the sectors). Student JS (Grade 5) stated: "Well, they're both the same. I just

TABLE 19

Frequency of Response and Percent of Response on Items of Equal Proportion

	Grade								
Item	K	1	2	3	4	5	6	7	Total[a]
1EC									
Right[b]	4	3	4	5	4	4	2	3	29 (60)
Left	2	3	2	1	2	2	4	3	19 (40)
5EC									
Right	2	2	3	3	6	2	5	4	27 (56)
Left	4	4	3	3	0	4	1	2	21 (44)
8E									
Right	2	2	2	2	2	3	4	4	21 (44)
Left	4	4	4	4	4	3	2	2	27 (56)
8M									
Right	2	1	3	3	2	2	0	2	15 (31)
Left	4	5	3	3	4	4	6	4	33 (69)
8D									
Right	3	3	2	2	2	4	1	2	19 (40)
Left	3	3	4	4	4	2	5	4	29 (60)

[a]Numbers in parentheses indicate the percent of responses for that choice.

[b]Refers to the right hand side of the experimenter - subject's left.

went for this one because it's my right arm." LE (Grade 6) stated: "They're both the same. But whenever I spun this one I got blue and whenever I spun that I got yellow, so I spun this one."

Item 1EC had the same proportion of POC and NPOC, but the number of sectors on each spinner varied (experimenter's left: 4 blue, 2 yellow; experimenter's right: 6 blue, 3 yellow). The total size of the spinner also varied to correspond to the total number of sectors. For this item the POC was greater than the NPOC on both spinners and the majority of students chose the side with the greater number of POC. Individual comments were not available for this item.

In item 5EC, for each spinner the POC was less than the NPOC, and the number of sectors varied (experimenter's right: 3 blue, 9 yellow; experimenter's left: 2 blue, 6 yellow). The majority of students (56%) chose the spinner on the experimenter's right hand side of the spinner board. The main reason given for the choice was the greater number of POC (blue) sectors on the larger spinner. For example, student KD (Grade 3), stated: "It's more bigger and it has three blues and this only has two and it's smaller than this one." Student PH (Grade 7), responded: " 'Cause there's three blues and I thought that they'd have more chance than trying to land on just two." Two students recognized that the proportions were the same on both spinners. For example, student CB (Grade 7) responded: "They're the same - (eenie, meenie, points with finger). They're exactly the same so it doesn't matter."

Gender Differences

For the items where a correct or incorrect answer was possible (31 items), there was no significant difference between the scores of males and females, $F(1,14) = 0.64$.

Chapter 5

DISCUSSION

The purpose of this research was to investigate the level of probabilistic understanding at the various levels in the elementary school in order to: 1) establish a general age range (grade level) at which children have an understanding of the concepts of probability; 2) outline and discuss any recurring strategies which the subjects have used to help solve probability problems; and 3) suggest a grade level at which probability concepts and activities could be introduced in the mathematics curriculum.

Summary of Method

A total of 48 students from Kindergarten to Grade Seven in a Delta public school participated in the study.

Individual sessions of approximately 30 minutes, involving 36 tasks, were conducted with each subject. For each task the subject was presented with a pair of spinners of specific probabilities and asked to choose the one where the pay-off colour (POC) would come up when the pointer was spun. The choice of the subject and the outcome of the task were recorded by the experimenter. For certain preselected tasks additional information regarding the subject's choice was solicited by asking for the reasons behind the selection.

The session scores were tabulated, and the percent of correct response for each item at each grade level was tabulated. The percent of correct response for each level of difficulty (easy, medium, difficult) was also determined.

Discussion

Strategy

Although most of the percent correct responses were above chance, the verbal response to questioning indicated that specific features of the spinner served as indicators of success for the students, rather than a calculation of ratio. Five specific strategies were used by the subjects: 1) reliance on the pay-off colour; 2) reliance on the nonpay-off colour; 3) the physical setup of the spinner board and the pointer; 4) the physical size or area of the spinner; 5) comparison between the pay-off colour and the nonpay-off colour. Similar to the results of Falk, Falk, and Levin (1980), many of the children in Kindergarten and

Grade One stated the placement of the pointer, or their favourite colour, or sheer will made the pointer stop at the pay-off colour. These students tended to concentrate on one aspect of the task. Older students often made comparisons between the elements of the task. They considered both the POC and the NPOC, but they looked at the relative number of elements of each colour, not the proportion. There were only two students who calculated the proportion of the elements of the two colours.

Falk, Falk, and Levin (1980) found that a few subjects integrated the numbers of the POC and the NPOC by using a strategy which involved the computation of difference between the numbers of elements of the two colours rather than their ratio. Although this study was designed to accommodate the difference strategy, no students demonstrated that technique to select a spinner.

The Role of Conservation

Falk, Falk, and Levin (1980) found that understanding ratio and proportion is presupposed by the principle of conservation. This means that the size, and number of objects within the set can be different, but the proportion can remain unchanged. The understanding of conservation was investigated through pairs of spinners with equal proportions.

In the items where the spinners were identical, some students at each grade recognized that they were both the same. However, when the spinners had the same proportion, but were different in number of POC and NPOC, many students did *not* recognize that the items had the same probability. An understanding of the conservation of size, and number in probability activities was not evident in most students.

Learning Probability

Falk et al. (1980) state that there is a *potential* for discriminating between probabilities at about the beginning of Grade One. Although the results from this study show no specific grade level emerging as the definitive point for beginning the instruction of probability concepts, the results indicate that Grade One students did perform at an above chance level (74%) on 31 items, although their ability to verbalize the reason for their choice of spinner was limited. The upper threshold for percent correct by chance is about 57% (n of items = 31, n of subjects = 6, $\alpha = 0.05$).

Falk et al. suggest that practice playing probability games and "acquiring experience with the operation of random processes may promote the existing potential" (p. 199). They also suggest that exposure to probability activities will enhance the ability of children to articulate the explanation of their choices.

The results of this study support the conclusion of

Falk et al. that Grade 1 would be an appropriate level to begin introducing probability tasks through games and group activities.

Limitations

This study was limited to one large elementary school (about 650 students) in a middle-class neighbourhood. The results of this study cannot be applied without consideration of this variable. The results could vary according to socio-economic status and school size.

During the sampling procedure, eight students from the initial random selection decided not to participate. The reasons for not participating were varied. Two specific reasons indicated were: 1) the student's lack of ability in mathematics; and 2) the concern that probability tasks encourage gambling.

Two main problems occurred during the data collection. The recording of individual verbal response was to be ongoing throughout the experiment, with the experimenter asking questions about each item. However, after completing a couple of sessions it was evident that the sessions were exceeding the time constraint and specific questioning had to be limited to specific tasks. The selected tasks were marked, but one task (3E) was inadvertently left out. This was not noticed until the first day of the study was completed and consequently resulted in a loss of verbal response data for task 3E at the grade seven and the grade four level. Second, one tape was unknowingly recorded over, eliminating the verbal response of one kindergarten subject.

Suggestions for Further Study

Four specific areas which could be addressed in further studies are: 1) the physical set up of the spinner board, specifically the arrangement of the colour sectors; 2) the level of probabilistic understanding of students over the age of thirteen, for example, in junior secondary school; 3) the strategies used with spinners of equal proportion, but different numbers of POC and NPOC; and 4) the individual verbal responses.

In this study each individual spinner was composed of equal size sectors. The sectors were outlined so the subject could clearly see how many sectors were coloured blue and how many were coloured yellow. The arrangement of the colours for each spinner was not controlled. When it was possible the colours were alternately spaced. However, a number of subjects used the arrangement of the sectors as a reason for their choice. These subjects chose the spinner where the sectors that were coloured the same were packed together. It would be interesting to investigate whether grouping the same-coloured sectors together would change the results of the study.

The oldest students in this study were 13 years old, in

Grade 7. There was no indication of an age range, or grade level where probabilistic understanding was evident. Further study with older children who have not been exposed to probability instruction would provide additional data to isolate an age range where an understanding of probability is evident in the majority of students. Alternate strategies, such as the "difference method" noted by Falk, Falk, and Levin might also be investigated.

In this study, two items had spinners with an equal proportion of POC and NPOC which were represented by a different number sectors. Most of the students did not recognize that the probability of achieving the POC on each spinner in the item was equal. An evaluation of the strategies used by individual students on the items with spinners of equal proportion compared to those spinners with unequal proportions was not possible as individual verbal responses were not available for all items. Falk, Falk, and Levin (1980) predicted "...that a child who seeks the set with more POC elements would select the expanded fraction, whereas a child who prefers the set with few NPOC elements would select the reduced fraction" (p. 197).

Students were asked to explain their reasons for selecting particular spinners in several items in order for the experimenter to understand the student's perception of the task. The results show a disparity between the verbal explanation and the achievement on individual items. Verbal achievement seems to lag behind performance achievement as the students performed above chance level on all tasks, but often the explanation for the choice of spinner was unclear. Without individual verbal responses, it is possible for the data to misrepresent the level of understanding of the task by the students. A more detailed discussion with the students on each of the tasks could provide a better understanding of the reason for the shifts in strategy. Falk et al. (1980) suggest that the individual verbal responses must be secondary to the performance on items because the lack of the ability to clearly articulate a reason for a choice does not mean the subject does not understand the task. However, it remains that the verbal responses do indicate the students who use irrelevant principles, such as pointer placement, as the reason for their selection.

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APPENDIX A.

LETTER OF INTENT

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February 26, 1985.

Dear Parents,

I am about to begin a research project at Gray Elementary School. The purpose of this letter is to ask you to give permission for your child to participate in this project. The project has been approved by the Delta School District (#37), and by Mr. Archibald, the school principal. It has also been carefully examined and approved by Simon Fraser's University's Committee on Ethics.

The purpose of this project is to learn about the development of probability concepts in elementary school children. Probability concepts will be included at all levels of the revised Mathematics curriculum. This research will explore children's acquisition of this concept. Each pupil participating in this research will be excused from class for about 30 minutes. I will be working individually with each pupil. The pupil will be shown two spinners (similar to those used in board games). The student will be asked to point to the spinner which has the best chance of producing the 'pay-off colour'. The spinner will then be spun and the results will be recorded. A series of trials involving different pairs of spinners will be initiated.

Participation in this project will have no bearing on your child's regular classwork or grades. Your child can withdraw his/her participation, partially or fully, at any time. The confidentiality of response of individual students will be strictly maintained.

Please fill the blanks below and have your child return this letter to the classroom teacher tomorrow. If you would like more information about this project, please phone me at Gray Elementary before 9 am or after 3 pm.

Thank you very much for your kind consideration.

Yours sincerely,

Mrs. S. Higginbottom

Dr. T. O'Shea

I will permit my child, ______________________________, to participate in this project. ________ YES ________ NO

____

Signature of Parent or Guardian

APPENDIX B.

ITEM LEVEL RESPONSES

TABLE 20
Codes for Item Level Responses

Column	Variable	Code
1, 2	Student Number	01 to 48
3	Gender	1 = male 2 = female
4	Grade	0 = kindergarten 1 to 7 = grades 1 to 7
5, 6, 7	Age (months)	
8, 9	1EA, 1EB	0 = incorrect choice 1 = correct choice
10	1EC	2 = left-hand side 3 = right-hand side
11 to 16	1MA, 1MB, 1DA 1DB, 1DC, 1DD	0 = incorrect choice 1 = correct choice
17 to 19	2E, 2M, 2D	0 = incorrect choice 1 = correct choice
20 to 22	3E, 3M, 3D	0 = incorrect choice 1 = correct choice
23 to 25	4E, 4M, 4D	0 = incorrect choice 1 = correct choice
26, 27	5EA, 5EB	0 = incorrect choice 1 = correct choice
28	5EC	2 = left-hand side 3 = right-hand side
29 to 34	5MA, 5MB, 5HA 5HB, 5HC, 5HD	0 = incorrect choice 1 = correct choice
35 to 37	6E, 6M, 6D	0 = incorrect choice 1 = correct choice
38 to 40	7E, 7M, 7D	0 = incorrect choice 1 = correct choice
41 to 43	8E, 8M, 8D	2 = left-hand side 3 = right-hand side

1	2524116113110001111111111111131110111111111323
2	2624119013000000111111011113111111111111111322
3	27141161131100001110110111131110111111111222
4	281412011311001011111111111113001010111111222
5	29141110120100001111111111111311111111111111232
6	3024122112110001111011110113011100111101233
7	3125126113110001111111011112010111111111111233
8	3215126113110011111110111112111111111111111322
9	3315127112111010111111111111131110011111111223
10	3425131013100010101111111110211111111111111323
11	3515127112110000111111111111131100111111111223
12	36251211131100111111111100112001001011111332
13	3716141002111001110101110113101111001110322
14	38261431131100111111111111113100001111101323
15	392614511311101111111111111311001111111111322
16	40261431121100101111101111121111010111111222
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19	432715111211001111111111011121111111110111323
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22	46271560121111111111110101131101010111111322
23	47171581131100001111111111111311110111111110332
24	48171571131100111111111111111211011011111111333
25	01100640121111111111110101002111000100001222
26	02100681121110101110101011121010101111110222
27	032007001311111111101110011120000000010111222
28	042006811311101111111111111120000000101111333
29	0510064103111001101011111111311000000100000223
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31	0711081002101010101010000113101111111011223
32	08210861021100011111101111131111111110111222
33	09110791131110111011100110121100101110111322
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37	1312087112111000011111111101201001110110222
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39	1512097113111011111111111111112010000110111323
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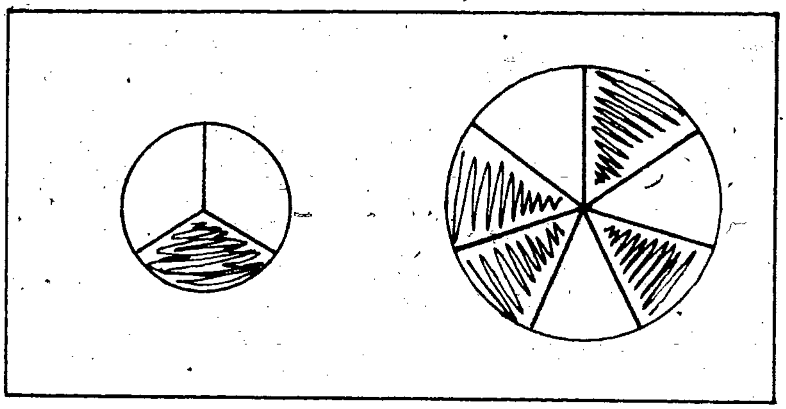
43	19131091131110111111111111110131101001111111222
44	20131080021100001011001111112111011100111222
45	212310711311101011111111111120110001111111333
46	222309811311111111111111111103001000111111232
47	23230991131111111111111111010020100000000101322
48	24131060131010110111111111111311111011111111233

APPENDIX C.

PHYSICAL COMPOSITION OF TASKS



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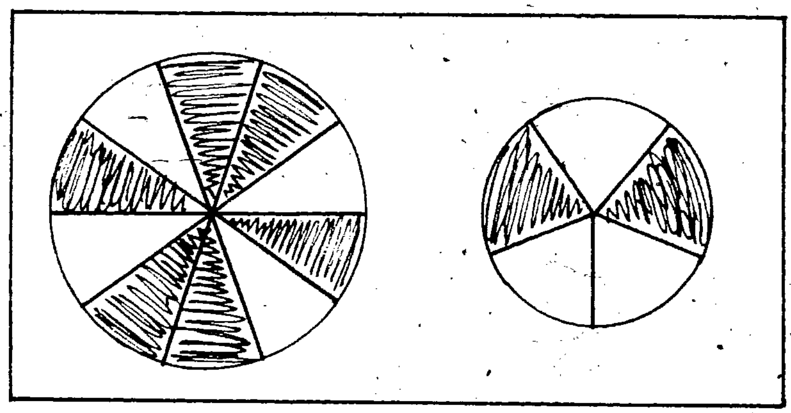

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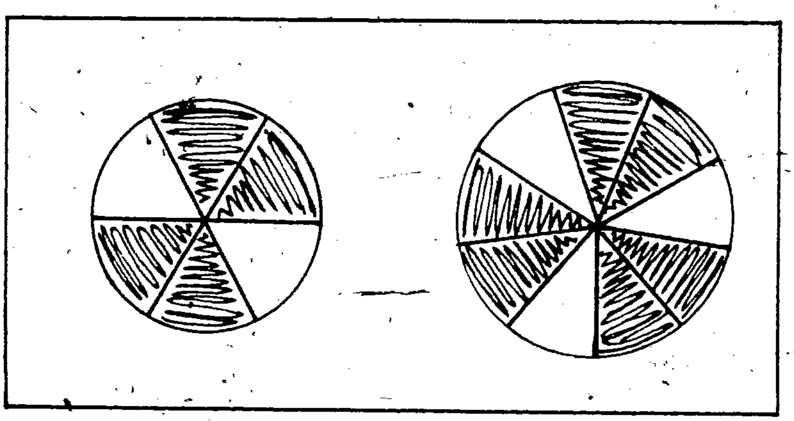

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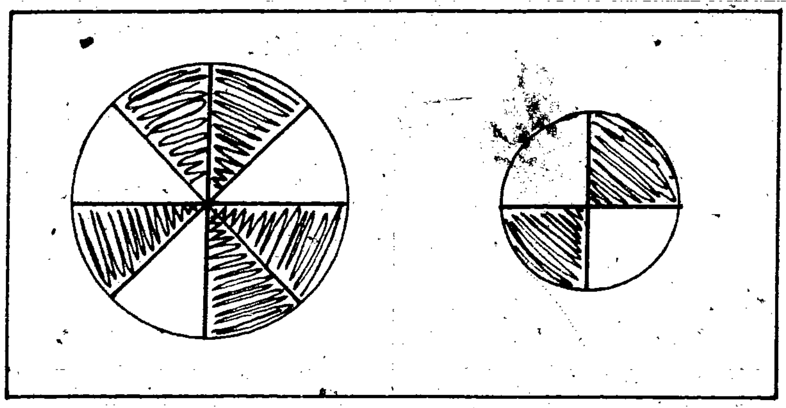

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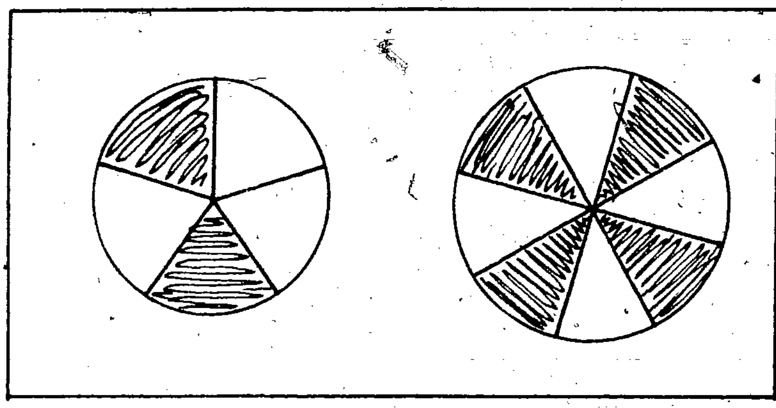

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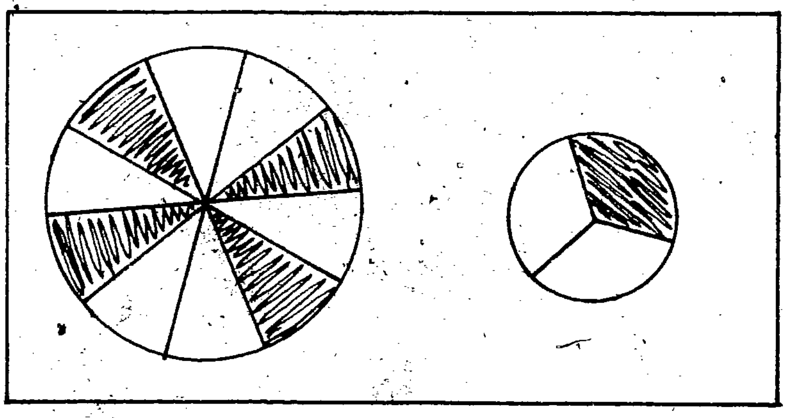

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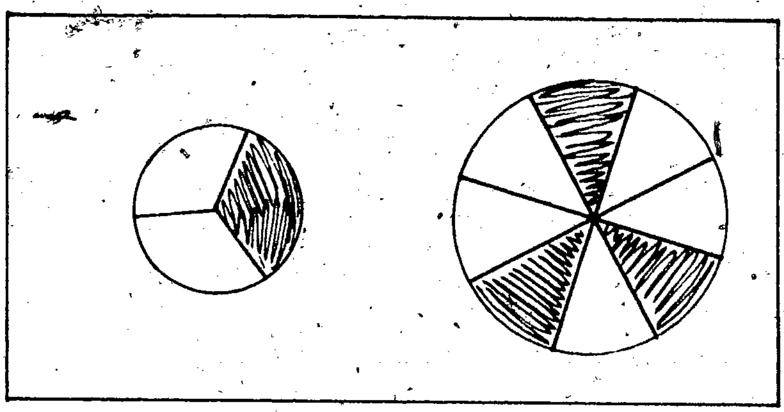

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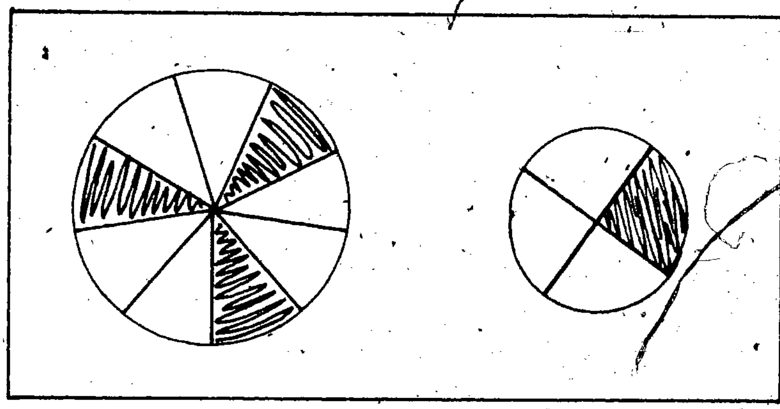

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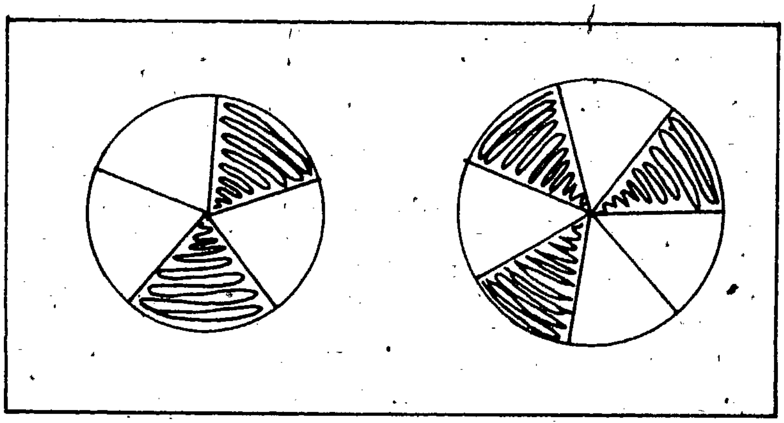

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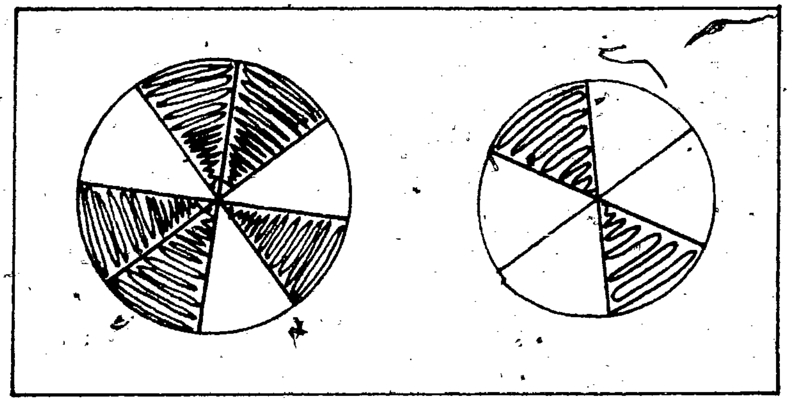

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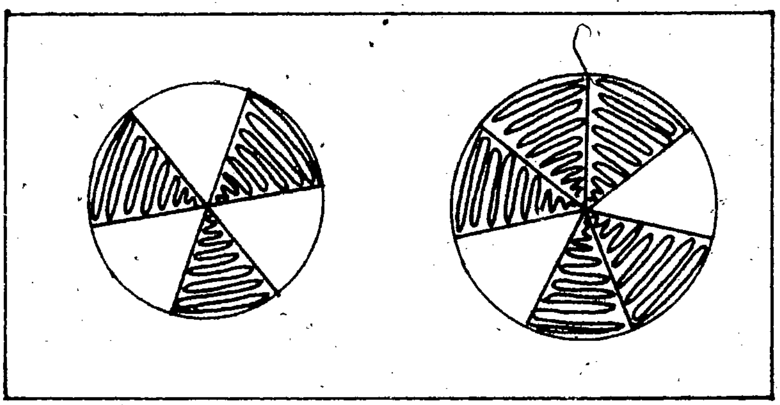

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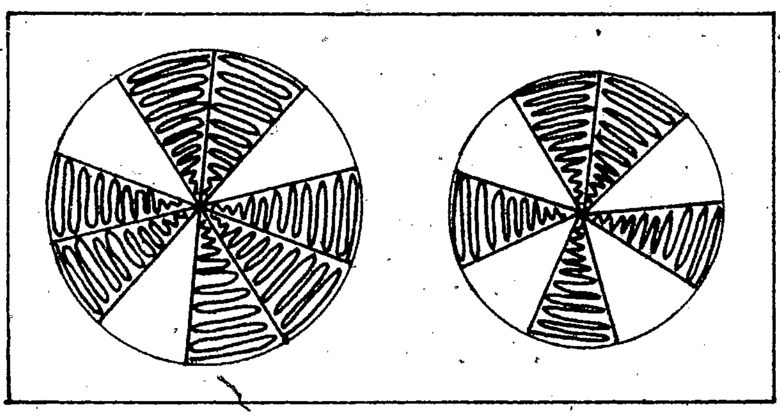

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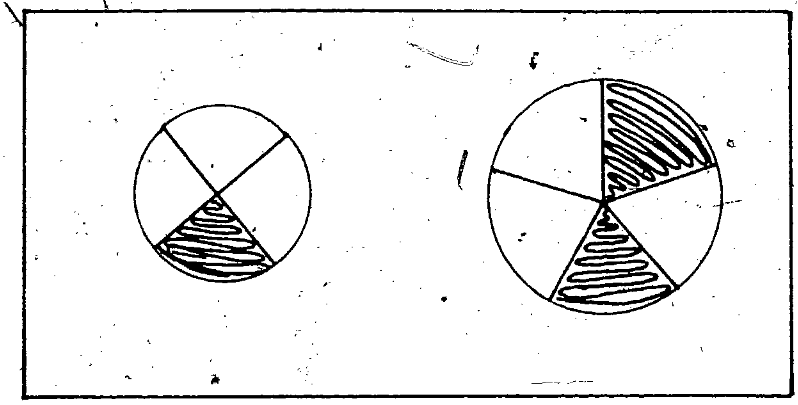

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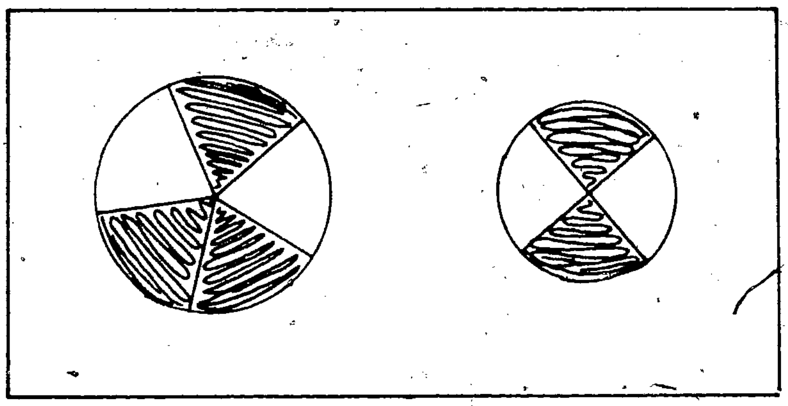

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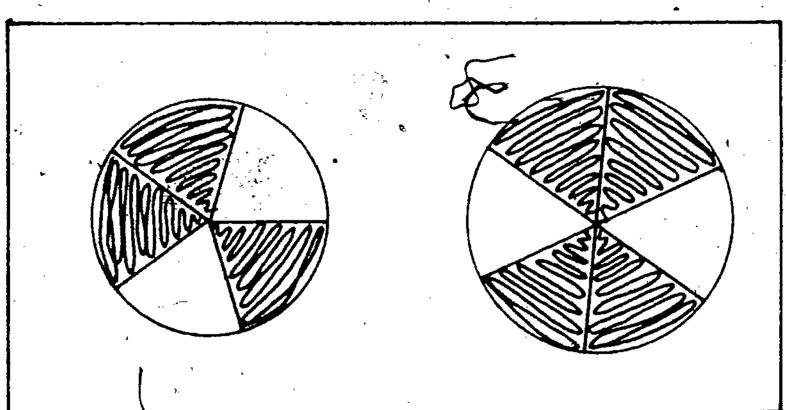

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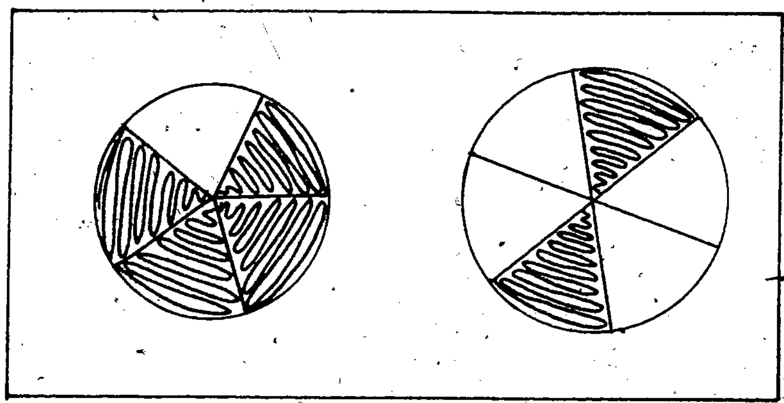

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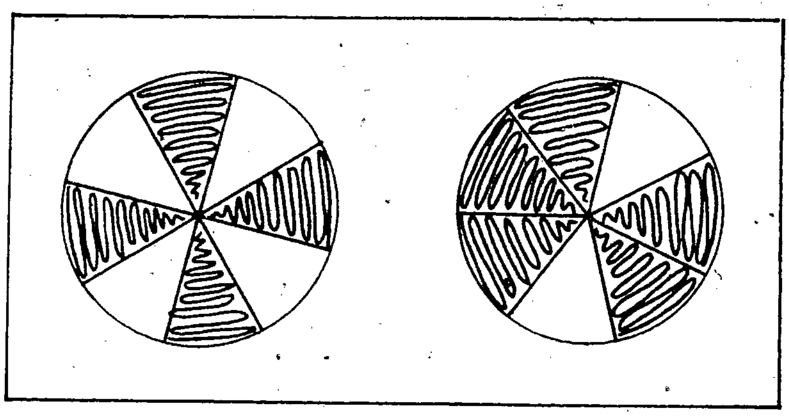

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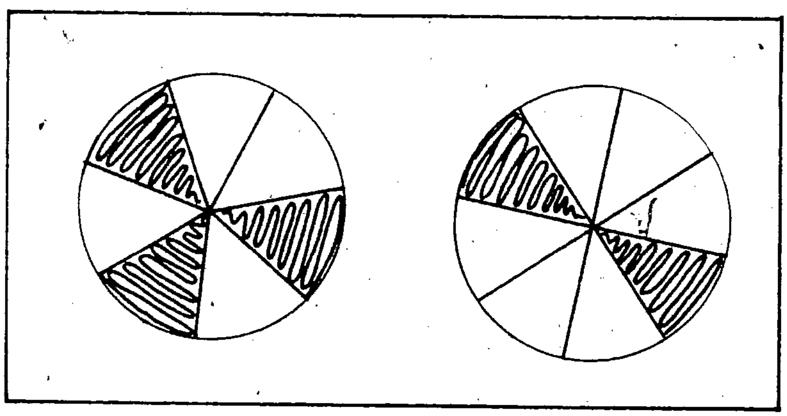

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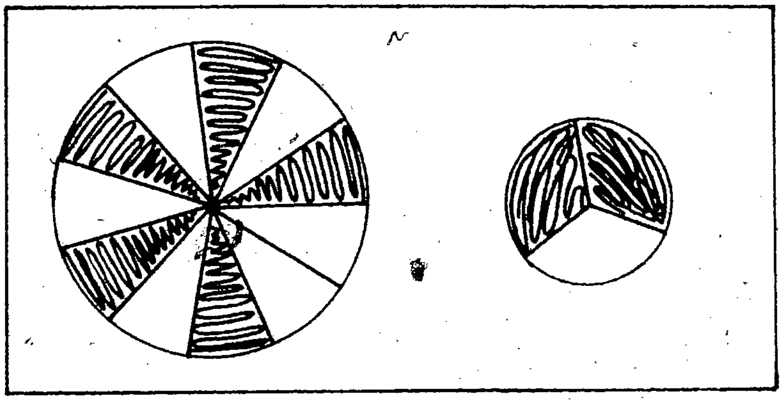

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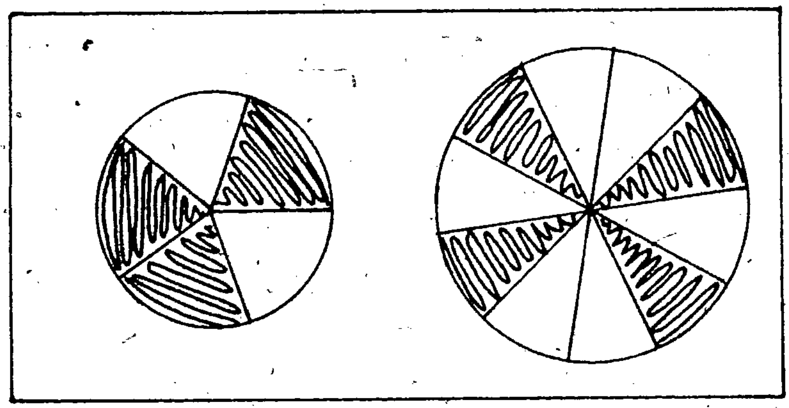

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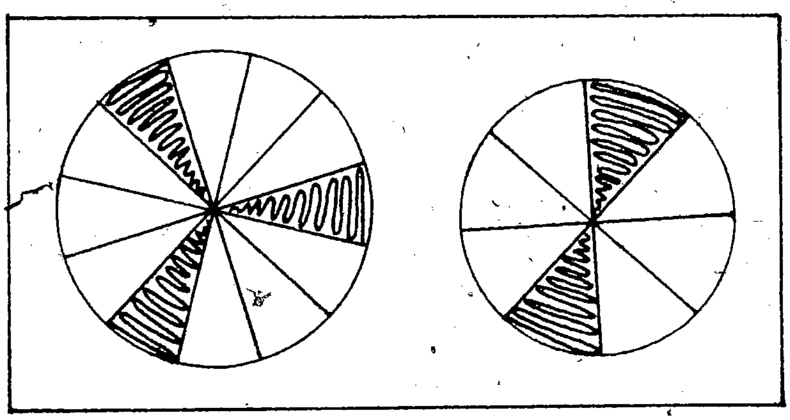

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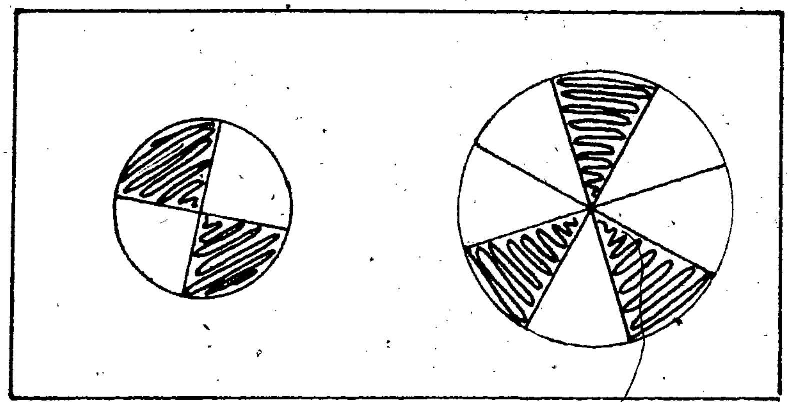

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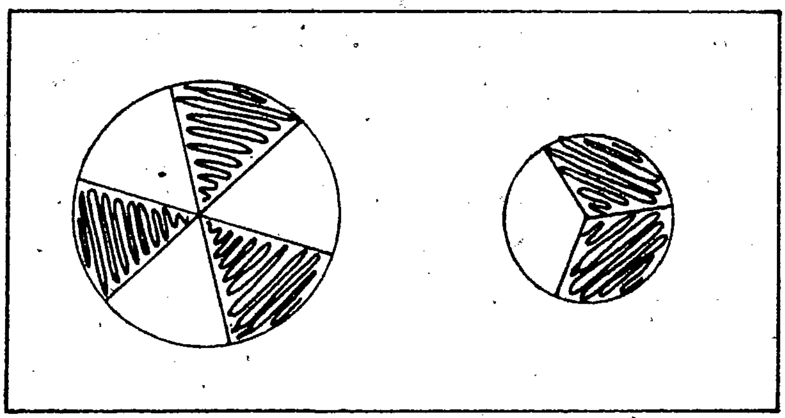

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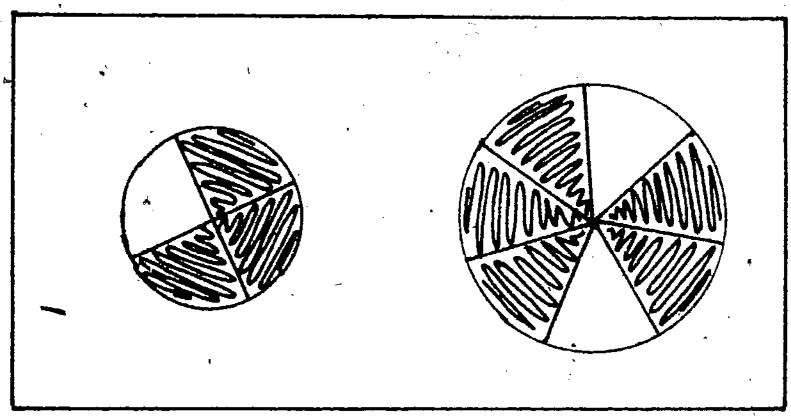

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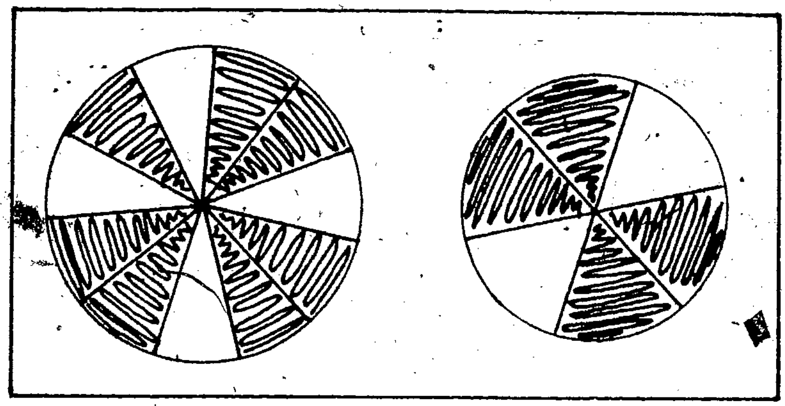

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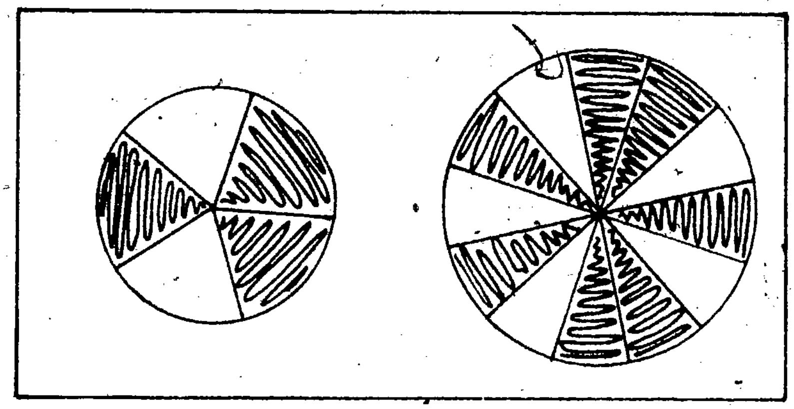

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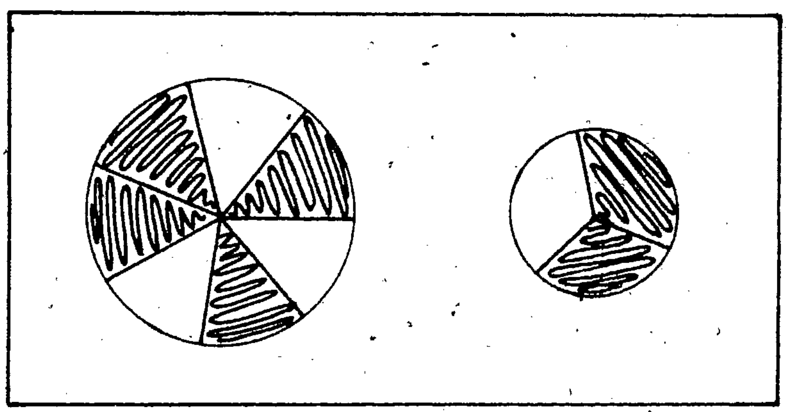

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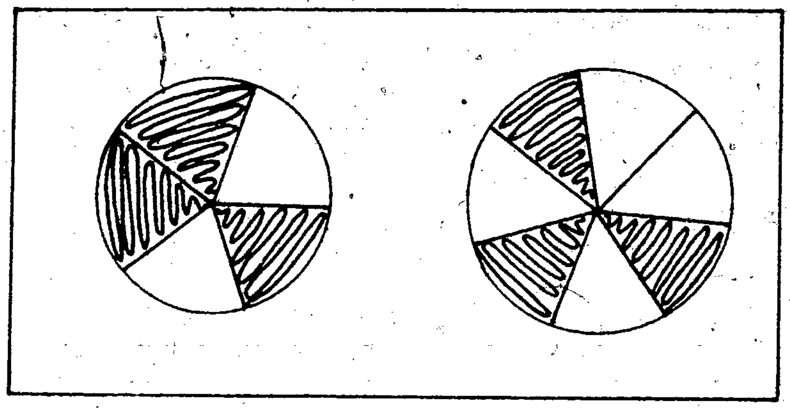

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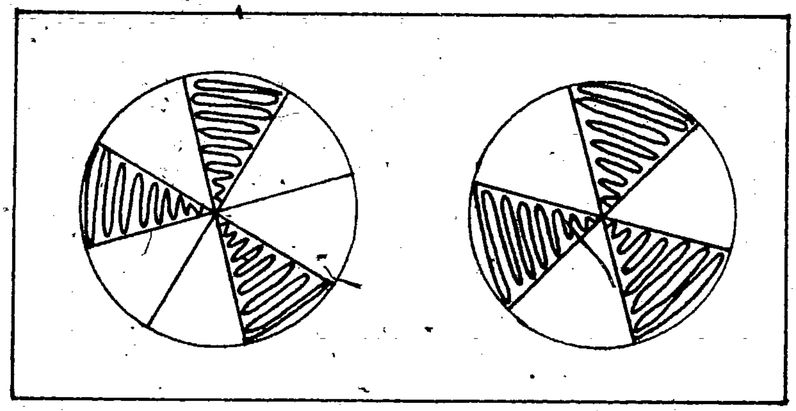

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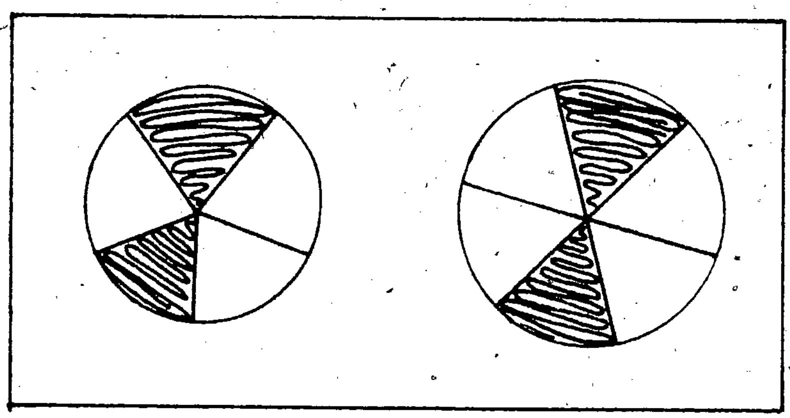

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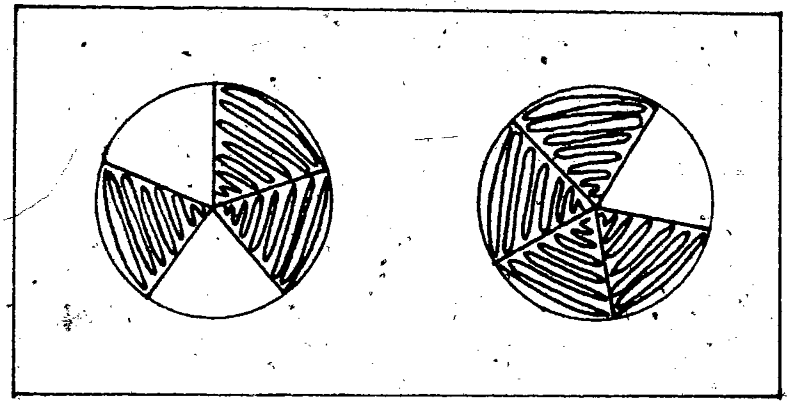

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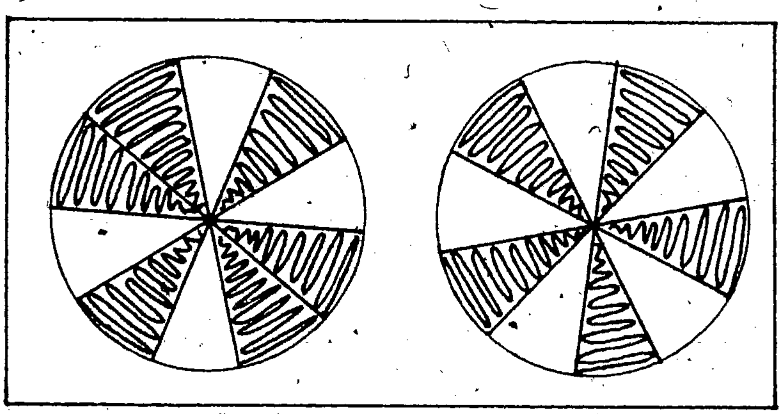

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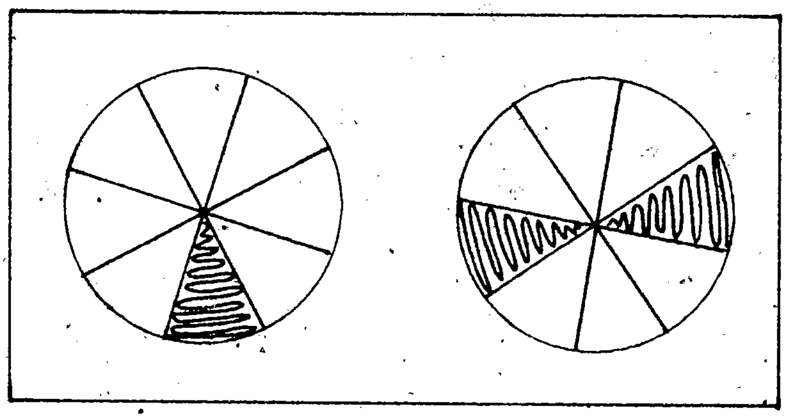

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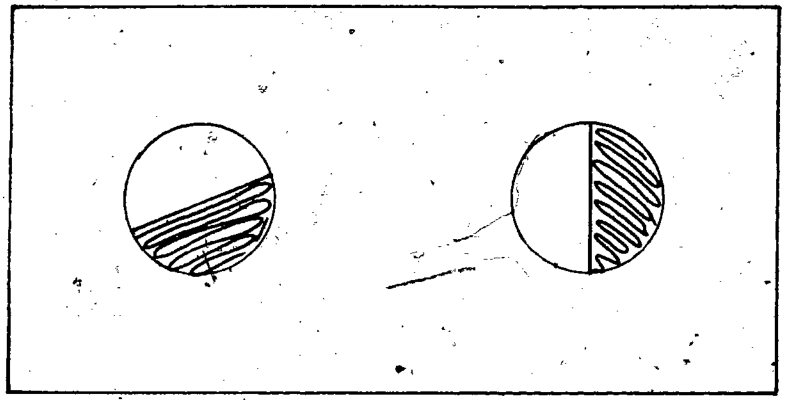

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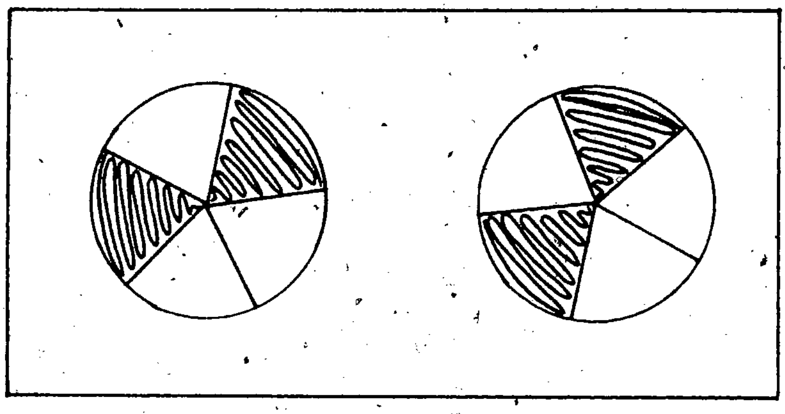

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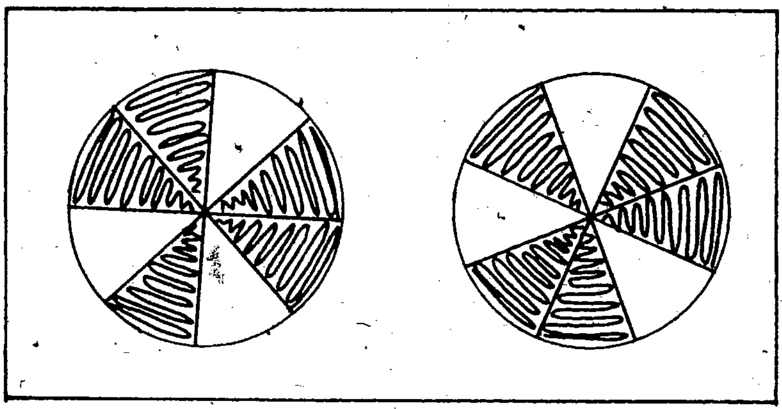

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