AN ALTERNATIVE SEMI-STRONG FORM TEST OF THE EFFICIENT MARKET HYPOTHESIS BY A TRANSFER FUNCTION APPROACH

by

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AN ALTERNATIVE SEMI-STRONG FORM TEST

OF THE EFFICIENT MARKET HYPOTHESIS

BY A TRANSFER FUNCTION APPROACH

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Abstract

This paper tests the semi-strong form of the Efficient Market Hypothesis (EMH) using the daily return series of the value-weighted Standard and Poor 500 Index (SP500 series). If the SP500 series is efficient in its semi-strong form, its series values cannot be forecasted by the use of any publicly available series. This means that, at any time t, the SP500 series has fully and instantaneously adjusted to information embedded in "all" series which are publicly available at that time.

In testing the null hypothesis of inability to forecast the SP500 series, the Box-Jenkins Transfer Function (TF) approach is employed. Specifically, the publicly available daily return series of the value-weighted New York and American Stock Exchange Combined Index (NYAM series) is used as input series for the TF in an attempt to forecast the output SP500 series. In addition, the martingale property of stock return is used as a supplement to the TF in testing the EMH. The acceptance of the martingale hypothesis for the SP500 series may indicate that the Standard and Poor 500 Index is efficient.

Our conclusion is that the semi-strong form efficiency of the SP500 series is not rejected. The identification of a zero order TF with no input lead does not allow the rejection of the null hypothesis of "inability to forecast". The identification of a white noise process, with slight evidence of a Thursday

effect, for the SP500 series leads to the rejection of the martingale hypothesis. Since the rejection of the martingale hypothesis may not necessarily imply market inefficiency, it provides us with consistent results to support our previous conclusion. The semi-strong form of the EMH is not rejected, however, it does not necessarily imply its unambiguous acceptance.

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Introduction

The Efficient Market Hypothesis (hereafter, EMH), according to Fama (1970), consists of three forms. These three forms of EMH are concerned with whether prices fully reflect particular subsets of available information. We summarize them as follows:

Strong form: prices will adjust fully and instantaneously to all available information.

Semi-strong form: prices will adjust fully and instantaneously to all 'publicly' available information.

Weak form: prices will adjust fully according to information on past prices.

The categorization of the EMH into weak, semi-strong and strong form will serve the purpose of allowing us to identify the level of information subset at which the hypothesis holds true or breaks down.

Previous tests have employed equilibrium models as an indication of market efficiency. Fama, Fisher, Jensen and Roll (1969) used the Market Model [1] in an attempt to test the semi-strong form of EMH. Specifically, they studied the adjustment of stock returns to information concerning stock splits by searching for unusual behavior of the Market Model residuals of the split securities. They claimed that their

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[1]For a description of the Market Model, see appendix A

results were consistent with the hypothesis that the stock market is efficient in its semi-strong form. Fama (1965) tested the weak form of market efficiency by employing the Constant Expected Return Model[2] and concluded that the weak form of EMH is a reasonable description of the world. Jaffe (1974) used the two Parameters Capital Asset Pricing Model (CAPM) [3] to study the returns on insider trading in an attempt to test the strong form of EMH. He concluded with the rejection of the strong form.

Because of the employment of equilibrium models, Fama (1970) claimed that, a test of market efficiency is a simultaneous test of market equilibrium. Since any anomalous evidence regarding market efficiency resulting from the use of these models will violate the implicitly assumed equilibrium conditions and signal a disequilibrium market as well. Moreover, it is also a joint test of the hypothesis that the underlying model employed by the test is not a valid description of the process for securities' equilibrium expected return formation.

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[2]The Constant Expected Return Model assumes that the expected return from holding securities is constant over time. i.e. $E(\tilde{R}_{jt}|\phi_{t-1})=E(\tilde{R}_j)$ where $E(\tilde{R}_{jt}|\phi_{t-1})$ is the expected return of security j at time t given information at time t-1, $E(\tilde{R}_j)$ is the expected return of security j which is a constant over all time t. If the expected return on a security is constant, then any serial correlation in past data on returns is an indication of market failure. The hypothesis that expected returns are constant can also be written as follows: $E[\tilde{R}_{jt}-\bar{R}|\phi_{t-1}]=0$ where $\tilde{R}_{jt}$ is the actual return of security j at time t, $\bar{R}$ is the assumed constant expected return and ϕ_{t-1} is the information set available at time t-1. If this model correctly specifies the process that generated expected return, it requires that the difference between actual and expected returns be uncorrelated with any past information in order to achieve market efficiency. If this is not the case, it would be possible to exploit past information to earn economic profits.

[3]For a description of Sharpe (1964)-Lintnen (1965) version of the two Parameters Capital Asset Pricing Model, see Appendix B

Hence, when a particular test fails to support the EMH, the researcher will face a dilemma of deciding whether this reflects a true violation of market efficiency, poor assumptions about the nature of market equilibrium and/or misspecification of the testing model.

Grossman (1976;1978) shows that if information is costly, then markets cannot be informationally efficient. In reality, "all" information cannot be costlessly available, it follows that, at least, the strong form of EMH will break down assuming Grossman's view. Note that, it may also be true that some public information cannot be available in a costless fashion. The cost of processing and manipulating of public information in the desired manner may well be non-trivial. This leads us to suspect the validity of the semi-strong form of EMH. We, thereby, focus on the testing of the semi-strong form.

This paper proposes a test of the EMH by a transfer function approach (hereafter, TF). We attempt to forecast the daily return series of the value-weighted Standard and Poor 500 Index (hereafter,SP500 series) by using the publicly available daily return series of the value-weighted New York and American Stock Exchange Combined Index (hereafter, NYAM series) as input series to the TF. If the SP500 series is efficient in its semi-strong form, its series values cannot be forecasted by the use of any publicly available series including the NYAM series. This means that, at any time t, the SP500 series has fully and instantaneously adjusted to information embedded in "all" series which are publicly available at that time. Since the TF is a forecasting model and does not assume equilibrium in markets, such a test is an independent test of market efficiency capable of rejecting the EMH unambiguously. As a result, ambiguity will not arise when the TF fails to support the EMH because this indicates an unambiguous violation of market efficiency. In addition, a univariate time series model is identified for the SP500 series in order to test the martingale hypothesis of stock return which is used as a supplementary test to the TF. Samuelson's (1965) martingale property claims that if the stock market is efficient, then the rates of return on stocks will follow a financial martingale --- i.e. that the expected rate of return on stock conditional on past realized rates of return is always equal to its unconditional expectation which, in turn, is a constant. Although the conclusion that can be drawn from the financial martingale about market efficiency is rather weak, it provides us with implications about the TF results.

Like the methodologies employed by previous tests of the EMH, the TF is not capable ~~of~~ accepting any one of the three forms of EMH unambiguously unless the corresponding information set is exhausted by the test. An explanation of this phenomenon, an interpretation of the LeRoy (1973) and Ohlson (1977) papers about the financial martingale, together with a description of the precise relationship between the TF and market efficiency will be discussed in the "Problem Statement" section. A review of some selected researches dealing with the weak, the semi-strong as well as the strong forms of EMH are highlighted in the "Literature Review" section. The "Research Statement" section is concerned with the particular methods, namely, the Box-Jenkins TF procedures and the Intervention Model, used to test the semi-strong form of EMH via the SP500 series. A discussion of the data, their analysis and the results of our research will be given in the "Procedure" and "Results" sections. The "Conclusion" section gives the implications of our finding.

Problem statement

If traders have diverse information about the returns on risky assets, then the competitive equilibrium prices will aggregate their information. Grossman (1976) considers an economy with one risky and one risk free asset where consumers have constant absolute risk aversion utility functions. It is shown that, in this economy where traders have diverse information sources, the equilibrium price produces allocations as if each trader had all the economy's information, i.e., the equilibrium price will fully reflect all the economy's information. This was demonstrated for a market with two types of traders, "informed" and "uninformed". The "informed" traders obtain information about the true underlying probability distribution which generates a future price, based on this information they take a position in the market. When all "informed" traders take the same action, current market prices are affected. The "uninformed" traders who do not collect the information know that current prices reflect the information of the "informed" traders. By observing the current prices, they can form a belief about a future price. Grossman (1978) obtains similar results when he extends his analysis by considering an economy with many risky assets where no special assumption is made about trader utility functions.

It follows that when information is costly and the price system still reveals all the information in the economy, each trader can do as well by just observing the market prices and information collectors cannot earn a return on their information. It, therefore, eliminates all private incentive for information gathering. If no one collects information, the price system will convey no information and an equilibrium will not exist. Grossman and Stiglitz (1976) argue that only markets with noise* will exist in equilibrium when information is costly. The market never fully adjusts. Prices never fully reflect all the information possessed by the informed individuals. The price system can be maintained only when it is noisy enough so that traders who collect information can hide it from other traders. Grossman and Stiglitz (1976) suggests that this capital market inefficiency exists in order to provide the revenue required to compensate the informed for purchasing the information. Thus, it is impossible to have an informationally efficient market in an economy where information is costly. Only an imperfect information equilibrium would exist.

Assuming Grossman's (1976;1978) view about capital market efficiency, it follows that, at least, the strong version of the EMH will break down since, in reality, "all" information cannot be costlessly available. It may also be true that some subsets of public information may not be free in a strict sense. The cost of processing and manipulating these subsets of public information may be non-trivial. We, therefore, suspect the validity of the semi-strong form of EMH. We are interested to

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* Market where prices are not perfect aggregators of diverse information

see whether prices do in fact fully reflect all publicly available information and, thereby, conduct a test on the semi-strong form.

Denote by I, II and III the strong form, semi-strong form and weak form of EMH respectively. Let θ_I, θ_{II} and θ_{III} be respectively the information set associated with I, II and III. Note that $\theta_{III} \subset \theta_{II} \subset \theta_I$. We now offer the following comments relating to any single test of I, or II, or III, using some components of information in θ_I, or θ_{II} or θ_{III}.

a) A test using some components of information in $\theta_I-\theta_{II}$ [5] and rejecting I does not imply rejection of II or III.

b) A test using some components of information in $\theta_{II}-\theta_{III}$ and rejecting II does not imply rejection of III, but it does imply rejection of I.

c) A test using some components of information in θ_{III} and rejecting III implies rejection of I and II as well.

d) If a test, using some components of information in $\theta_I-\theta_{II}$, does not reject I, it does not prevent other tests using other components of information in $\theta_I-\theta_{II}$ from rejecting I. Thus, accepting I requires non-rejection of I based upon all components of θ_I. It follows that accepting I implies acceptance of II and III.

e) If a test, using some components of information in $\theta_{II}-\theta_{III}$, does not reject II, it does not prevent other tests using other components of information in $\theta_{II}-\theta_{III}$ from rejecting

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[5] $\theta_I-\theta_{II}$ denotes information contained in θ_I but not in θ_{II} where $\theta_{II} \subset \theta_I$

II. Thus, accepting II requires non-rejection of II based upon all components of θ_{II}. It follows that accepting II implies acceptance of III, but it does not imply acceptance of I.

f) If a test, using some components of information in θ_{III}, does not reject III, it does not prevent other tests using other components of information in θ_{III} from rejecting III. Thus, accepting III requires non-rejection of III based upon all components of θ_{III}. Note that accepting III does not imply acceptance of either II or I.

From condition (d), (e) and (f), acceptance of the various forms of EMH requires the exhaustion of their associated information sets. Previous researches can be singled out as event studies (e.g. earnings announcements, information contents of stock split etc.) and do not satisfy conditions (d) through (f). It follows that non-rejection of I, II or III resulting from previous tests do not lead to unambiguous acceptance of I, II or III. Conducting a research methodology that is capable of exhausting θ_I, θ_{II} or θ_{III} in the testing of I, II or III respectively will be difficult and timely, if not impossible. It follows that a methodology that is capable of rejecting I, II or III unambiguously would provide unambiguous implication about market efficiency.

Ball (1978) examines the evidence contained in 20 previous studies regarding stock price reaction to earnings announcements. He finds that the post-announcenment risk adjusted abnormal returns are systematically non-zero in the period following earnings announcements in a fashion inconsistent with market efficiency. Ball offers several explanations concerning these anomalies which include i) systematic experimental error, ii) market imperfections and iii) the failure of the two parameter CAPM.

1. Systematic experimental error

Ball (1978) suggests the following possible sources of systematic experimental error.

a. Failure to collect the actual date of securities earnings announcements in the studies would result in including some securities pre-announcement and at-announcement excess return. This introduces bias in the direction of the cited anomaly. However, he argues that this bias is substantially smaller than the anomaly, and could not be the source.

b. If securities' relative risks are not independent of earning or dividend yields, then, risk controlling would be difficult when earnings or dividend yields vary across time or across securities. Ball suggests that it is implausible for any systematic bias in estimating relative risks to cause large under-predictions of securities' average returns to result in a large excess return.

c. The effect of errors in estimating securities' relative risks, which can arise from sample variation in securities returns or from using a mean-variance inefficient market portfolio as a proxy for aggregate wealth[6]. However, Ball argues that errors in estimating risks cannot explain the large post-announcements excess return.

2. Market imperfections

One explanation of the anomaly is that the security market is slow in adjusting to earnings announcements since transactions cost inhibits the market in the process of restoring equilibrium after earnings announcements. However, market imperfection does not seem to be in line with the anomaly. If the slow market reaction is explained in terms of transactions costs, the post-announcement excess return should persist up to, but not beyond, the level of marginal transactions cost. The cross-sectional variation in excess return is not consistent with this source of market imperfection.

3. Failure of the two-parameters model

The observed anomalies of the 20 studies cited by Ball (1978) may be due to the failure of the two parameter CAPM used in those tests. The two parameter model, when applied to portfolios of common stocks, misspecifies the process generating securities' expected returns, and allows earnings and dividends variables to proxy for the underlying determinants of equilibrium expected returns. The market portfolio proxy used in these experiments may not be

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[6]See Roll (1977) and the assumption embedded in the CAMP from Appendix B

mean-variance efficient or they do not represent the true composition of the market portfolio.[7] Ball (1978) considers the effect of omitted variables in a model in which securities' expected returns are a function of various independent parameters of their return distributions. The earnings variables will have a tendency to proxy for those omitted variables (when they are not independent) and explain the differences in securities' rate of return which are not predicted by the misspecified model used. Ball (1978) argues that the misspecification of the model used in previous studies is more consistent with the observed anomalies than the other two sources of errors.

Systematic experimental error and transactions costs can explain the sign but not the magnitude of postannouncement excess return. Failure of the two-parameter model is consistent with both the sign and magnitude of the observed anomalies (i.e. earnings acting as a proxy for omitted variable or other misspecification effects). Because a perfect model of the determination of securities' equilibrium expected returns is not available, we cannot verify Ball's claims directly. Giving alternative explanations of the anomaly rather than rejecting the notion of market efficiency displays a commonly held allegiance within the finance community to the EMH.

Previous tests of the EMH have all employed equilibrium models of one form or another (the most widely used model is the

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[7]see Roll (1977), P148-157

two parameters CAPM) and a model that describe the process of securities' equilibrium expected return formation may be misspecified. It follows that previous tests of EMH are actually a joint tests of a) market efficiency, b) market equilibrium c) the validity of the model in determining assets' equilibrium expected return formation. Any anomalous evidence regarding market efficiency resulting from the use of these models would imply market inefficiency, market disequilibrium and/or model misspecification. Thus, previous tests cannot reject the EMH unambiguously. Moreover, they cannot accept EMH free of ambiguity (due to non-exhaustion of the information set). A methodology that is capable of rejecting the EMH has to be devised.

We propose a test of the semi-strong form of the EMH by a transfer function approach. The TF is a forecasting model and does not assume equilibrium markets. If the TF fails to support the EMH, it indicates an unambiguous violation of market efficiency independent of market equilibrium. Note that, misspecification of the TF cannot be used to explain anomalous results regarding market efficiency. The necessary condition to reject the EMH is one where the TF parameters are highly significant. A highly significant TF parameter[8] suggests a higher likelihood for a causal relationship to exist between the input and the output series. Although the TF identified may not be the best one available, a causal relationship implies a rejection of the EMH as will be seen later. However, if such a

[8] We refer to the W_0 parameter of the TF

test does not contradict the EMH, we face the problem of determining whether the market is truly efficient or whether the test has failed to identify a relevant input series.

The TF has the following general form:

$$Y_t = f(X_{1t}, X_{2t}, \ldots, X_{nt}) + N_t \qquad \text{(P1)}$$

where Y_t is the output series,
X_{nt} is the n^{th} publicly avaiable input (or causor) series,
N_t is the noise component which requires an appropriate fitting of the ARIMA Model to transform it into white noise[9]

The most commonly encountered TF for socio-economic time series are the 0 and 1^{st} order TF while higher order TF are rare and seldom identified. For the 0 and 1^{st} order TF, we write Eqn. (P2) and Eqn.(P3) as follows:

$$Y_t = W_o X_{t-b} + N_t \qquad \text{(P2)}$$

where b denotes the no. of period(s) the input series X_t is leading the output series Y_t
W_0 is the 0-order TF parameter.

$$Y_t = W_o/(1-\delta_1 B)\ X_{t-b} + N_t \qquad \text{(P3)}$$

where W_o & δ_1 are the 1^{st} order TF parameters.

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[9] a white noise series a_t consists of a series of random shocks, each distributed normally and independently about a zero mean with constant variance, σa^2, i.e. $E(a_t) = 0$, $E(a_t, a_{t+i}) = 0$ for all $i \neq 0$ and $E(a_t)^2 = \sigma a^2$.

B is the backward shift operator.[10]

Eqn. (P2) corresponds to a 0-order TF while Equation (P3) corresponds to a 1st-order TF. If b>0, the models postulate that the publicly available input series leads the output series by b days, which implies that, at time t, information embedded in the input series b days earlier has a correlation with the output series. This, in turn, would imply that there is a "possible" transition periods of b days for the output series to reflect publicly available information. As a result, the output series would not be considered efficient in its semi-strong form.

However, the identification of a leading input series alone cannot serve as a sufficient basis for the rejection of the semi strong form of EMH because most socio-economic time series, including the stock prices (or returns) series, exhibit correlation between series that is due to other exogenous forces in the economy rather than a causal relationship. Hence, a leading series does not necessarily imply a causal relationship, whereas, the significance of the TF parameter may indicate the presence of a causal relationship. Therefore, the condition to reject the semi-strong form of EMH is for W_0 in Eqn. (P2) not to equal zero (with statistical significance) and $b \geq 1$, [11]i.e., the

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[10]The backward shift operator allows us to move backward in time by applying it to a time series, such that, $B(Y_t)=Y_{t-1}$. This expression does not mean "B multiplies Y_t", but rather means that "B operates on Y_t to shift it backward one point in time". Hence, $B^n(Y_t)= Y_{t-n}$ and $B^nB^m(Y_t)=Y_{t-n-m}$. The backward shift operator obeys all the laws of exponents that are routinely used in polynomial algebra.
[11]in this case b is the set of positive integer:{1,2......}

input series leads the output series by at least 1 day.[12] However, failure to identify a relevant leading input series, when one does exist, will result in $W_o=0$ and/or $b=0$ for Eqn. (P2). In such an event, it would still be incorrect to conclude that the output series is efficient (semi -strong form).

In the case of 1st order TF (Eqn. P3), the significance of the W_o parameter, [13] regardless of whether the input series possesses a lead or not, will be used to reject the semi -strong form of EMH. Since the 1st order TF indicates that the information of the input series has an exponentially decaying effect on the output series for all future periods, identification of a 1st order TF with highly significant W_o parameter (even though with no lead) suggests a higher likelihood for a causal relationship to exist, thus, it can be used to reject the semi-strong form of EMH. To see this point clearly, note that $1/(1-\delta_1 B)$ is the convergence of an infinite series $(1+\delta_1 B+\delta_1{}^2B^2+\delta_1{}^3B^3+\ldots)$, rewriting Eqn. (P3) using this fact, and with $b=0$ (no lead), we have Eqn. (P4):

$$Y_t=W_o(1+\delta_1 B+\delta_1{}^2B^2+\ldots)X_t+N_t \qquad \text{(P4)}$$

Since the back-shift operator, B, obeys all the laws of exponents that are routinely used in polynomial algebra, Eqn. (P4) becomes Eqn.(P5):

---

[12]Note that, significance of the W_0 parameter may be a statistical aberration,i.e., there is always the chance of making a Type I error. Thus, the higher is the significance level of the W_0 parameter, the higher would be the likelihood for a causal relationship to exist.
[13] for a 1st order TF, the δ_1 parameter must be significant. If this is not the case, Eqn.(P3) will be reduced to a 0-order TF

$$Y_t = W_o X_t + \delta_1 W_o X_{t-1} + \delta_1^2 W_o X_{t-2} + \ldots + N_t \quad \text{(P5)}$$

It is clear from Eqn. (P5) that, at time t, previous values of the publicly available input series can be used to forecast the current value of the output series. Thus, current value of the output series does not reflect all the information embedded in previous values of the publicly available input series, when the information set is increasing through time, i.e., $I_{t-1} \subset I_t$, which captures the concept of "learning without forgetting". As a result, the output series is not considered efficient in its semi-strong form. Even though the input series does not possess a lead, the identification of a 1st order TF with significant W_o parameter behaves as if it were a zero order TF with input leads.

In testing the null hypothesis of inability to forecast the output series, with any publicly available input series, if it is efficient in its semi-strong form, we use H1 and H2 as follows:

H1 : Parameter W_o and/or time lag b of Eqn. (P2) are/is statistically insignificant.

H2 : Parameter W_o of Eqn.(P3) is statistically insignificant.

Our goal is to identify at least one publicly available input series with significant W_o parameter that leads the output series for at least 1 day for a 0-order TF, or to identify at least one publicly available input series with significant

W_o parameter (with or without lead) for a 1st order TF. If successful, the semi-strong form of EMH can be rejected unambiguously.

The financial martingale property [14] of stock returns is used as a supplementary test to the TF. Samuelson (1965) derives the martingale property for future prices by ignoring risk aversion and the interest factor. His fundamental assumption is that a future price is to be set by competitive bidding at the now expected level of terminal spot price which can be written as Eqn. (P6):

$$Y(T,t)=E[X_{t+T}|I_t] \qquad (P6)$$

where $Y(T,t)$ is the future price that will prevail T periods from time t and quoted at time t.

X_{t+T} is the actual spot price that will prevail T periods from time t.

I_t is the information set which contains all present and previous periods values of spot price, i.e. X_t, X_{t-1}, X_{t-2}, ...

It follows from Eqn. (P6) that

$$\begin{aligned} E[Y(T-1,t+1)|I_t] &= E[E(X_{t+T}|I_{t+1})|I_t] \\ &= E[X_{t+T}|I_t] \\ &= Y(T,t) \qquad (P7) \end{aligned}$$

Thus, the sequence of future prices, [Y(T,t),

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[14] Note that the financial martingale property considered here should not be mistaken as the mathematical martingale. Let X_n denotes a random variable at time n and I_n denotes an information set at time n. The mathematical martingale states that $E[X_{n+1}|I_n]=X_n$ alongside with the conditions that (i) $F_n \subset F_{n+1}$ (ii) X_n is measurable F_n and (iii) $E(|X_n|)$ is less than ∞. Interested readers should refer to Malliaris (1982), p.16-21

$Y(T-1,t+1), \ldots, \quad Y(1,t+T-1), \quad Y(0,t+T)]$ is said to be a martingale in the sense of having unbiased price changes. Note that Eqn.(P7) can be rewritten as:

$$Y(T-1,t+1)-Y(T,t)=\Delta Y(T,t) \qquad (P8)$$

$$\text{and } E[\Delta Y(T,t)]=0 \qquad (P9)$$

From (Eqn. 9), successive periods price difference, i.e., $\Delta Y(T,t)$, are shown to be uncorrelated. This means that there is no way of making an expected profit by extrapolating past changes in the futures price. The market quotation $Y(T,t)$ already contains all that can be known about the future. This martingale property of zero expected capital gain is then expanded into a more general case of a constant mean percentage gain per unit time. If money has to be tied up in holding the $Y(T,t)$ contract and assuming a positive riskless rate of interest, the martingale property of zero expected gain does not stand to earn even the opportunity cost of foregone safe interest. Therefore, Samuelson (1965) replaces the simple axiom of zero expected gain by a slightly more general axioms -- the axiom of Present-Discounted Expected Value which posits that

$$Y(T,t)=\lambda^{-T}E(X_{t+T}|I_t) \qquad (P10)$$

where λ is the assumed constant risk free discount factor (or interest rate).

If the sequence of future price follows Eqn. (P10), Samuelson's (1965) Theorem of Mean Percentage Price Drift states that

$$E[Y(T-n,t+n)|I_t]=\lambda^{n}Y(T,t) \qquad (P11)$$

Eqn.. (P11) denotes that expected successive future price change would amount to a positive rate equal to $\lambda-1$ (λ is one plus the assumed constant riskless interest rate) which is just enough to compensate for the foregone safe interest by tying up money in holding the future contract. Eqn. (P11) implies that the market has already discounted all knowable future information so that the present discounted futures price sequence is a martingale.

Although Samuelson's (1965) paper establishes the martingale property for futures pricing rather than for an equity asset, a share of a stock may be regarded as a sequence of future claims due to mature at successive intervals. Thus the martingale property applied to stock prices may be used as a measure of capital market efficiency. Since Samuelson derives his martingale property by assuming a constant and exogenously given riskless rate of interest, holding a future contract for one period should entitle an expected return equal to the riskless rate of interest, which is a constant. When applying Samuelson's martingale property to stock prices, it implies that the expected single period holding return of stocks would be a constant, if the stock market is efficient. Thereby, in testing the martingale property, we use Eqn. (P12):

$$Y_t = W_M I_{M,t} + W_T I_{T,t} + W_W I_{W,t} + W_{Th} I_{Th,t} + W_F I_{F,t} + NOISE \qquad (P12)$$

where Y_t is the returns series whose efficiency is to be examined, it is the output series used in the TF models,

$I_{M,t}=1$ for Monday and 0 otherwise,

$I_{T,t}=1$ for Tuesday and 0 otherwise,

$I_{W,t}=1$ for Wednesday and 0 otherwise,

$I_{Th,t}=1$ for Thursday and 0 otherwise,

$I_{F,t}=1$ for Friday and 0 otherwise,

W_M, W_T, W_W, W_{Th} & W_F are parameters associated with Monday, Tuesday, ..., & Friday respectively.

Note that, Eqn.(P12) is an Intervention Model (the intervention models will be discussed in the "Research Statement" section), and Y_t is a martingale if either one of the following two conditions is met:

a) There is no significant difference amongst the estimated values of the W_i's parameters in Eqn. (P12) and all the W_i's parameters are statistically significant, where i=M,T,W Th and F, and the resulting residuals of Eqn. (P12) is white noise. In this case, the expected value of the Y_t series would be constant at W, where W is the estimated values of the W_i's parameters when there is no significant difference amongst them. The residuals of Eqn. (P12) must be white noise since when taking expectation on both sides, Eqn. (P12) becomes:

$$E(Y_t)=E(WI_t)+E(a_t) \qquad (P13)$$

where I_t is a series of one's, i.e.,

$I_t=1$ for Monday through Friday,

W is the estimated value
for all the W_i's
parameters which is the same
under condition (a).

thus, $E(Y_t)=W+0$

$=W$ which is a constant. (P14)

When the residuals of Eqn. (P12) is not white noise, $E(Y_t) \neq$ constant.

b) All the estimated parameters of Eqn. (P12) are insignificant and the residuals of Eqn. (P12) is white noise. Under this condition,

$$E(Y_t)=E(a_t)=0, \qquad (P15)$$

i.e., the expected value of Y_t is constant at 0.

Therefore, we state the martingale hypothesis, H3, as follows:

H3: The univariate ARIMA model for the Y_t series satisfies either condition (a) or (b) concerning the martingale property.

In testing the martingale hypothesis of H3, we employ the univarite ARIMA time series modelling strategies. [15] Note that rejection of the martingale hypothesis does not necessarily imply the rejection of market efficiency. With risk aversion and

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[15]The univariate ARIMA Model will be discussed in the Research Statement Section

a stochastic rate of interest, LeRoy (1973) shows that under a specific set of assumptions concerning investor utility function (risk aversion) and the dividends process, the martingale property does not hold, i.e., the expected return of stocks is not a constant. Ohlson (1977) shows that the martingale property holds when investors have constant relative risk aversion and the percentage change in dividends is stationary. Thus, rejection of H3 may be due to other factors in the economy (e.g. investors' risk attitudes, dividends payout strategies etc.) rather than market inefficiency. Although the conclusion that can be drawn from the financial martingale property about market efficiency is rather weak, it can provide us with implications and consistency about the TF results.

Literature Review

Tests of the weak form of EMH have been conducted by both Alexander (1961;1964) and Fama & Blume (1966) via the filter rules. The filter rules state that if price of a security moves up at least *Y* percent, buy and hold the security until its price moves down at least *Y* percent from a subsequent high, at which time simultaneously sell and go short. The short position is maintained until price rises at least *Y* percent above a subsequent low, at which time one covers the short position and goes long. Moves less than *Y* percent in either directions are ignored (a *Y* percent filter).

If the capital market is efficient in its weak form (security prices cannot be predicted by the trend embedded in its "past" prices) and if the market sets price so that expected returns are positive (the positive expected return model is the equilibrium model assumed), excess profit cannot be extracted by employing the filter rules, since the filter rules implicitly claim that upward price movements tend to persist and to be followed by downward movements, which also tend to persist and to be followed by upward movements and so on. This implies that current security prices do not fully reflect past information (past security prices) and that their current prices are predictable from their past trend. If the market correctly uses past available information and if it set prices so that expected returns are positive, then, the best rule would be to buy and hold.

Alexander (1961;1964) reports extensive tests of filter rules using daily data on price indexes from 1897 to 1959 and filters from 1 to 50%. He concludes that the buy and hold strategy consistently outperforms the filter rules when transaction costs are taken into consideration. Further evidence is provided by Fama & Blume (1966), who compare the profitability of various filters to a buy and hold strategy for daily data on the individual stock of the Dow-Jones Industrial Average that run from about the end of 1957 to September 26, 1962. They conclude that for the most part their evidence is in favor of buy and hold, and they reject the hypothesis that there is any important information in past prices that the markct neglects in setting current prices.

Fama (1965) has conducted another piece of research concerning the weak form of EMH. He studies the sample autocorrelation of daily returns for each of the 30 Dow-Jones Industrial series, for time periods that vary slightly from stock to stock but usually run from about the end of 1957 to September 26,1962. Fama claims that, market efficiency (weak form), in combination with the assumption that equilibrium expected returns are constant through time, implies that the autocorrelations of the returns on any security are zero for all values of the autocorrelation lags. When the true autocorrelation is zero, the sampling distribution of the sample autocorrelation with lag τ, is approximately normal, with approximate mean and standard deviation depicted by Eqn. (R1) and Eqn. (R2) respectively.

$$E[\gamma(\tilde{R}_{jt},\tilde{R}_{j,t-\tau})] \simeq \frac{-1}{(T-\tau)} \qquad (R1)$$

$$\sigma[\gamma(\tilde{R}_{jt},\tilde{R}_{j,t-\tau})] \simeq \sqrt{\frac{-1}{(T-\tau)}} \qquad (R2)$$

where $\gamma(\tilde{R}_{jt},\tilde{R}_{j,t-\tau})$ is the sample autocorrelation of security j return with lag τ.

T is the number of returns data in the sample.

Fama calculates the sample autocorrelations of returns, for each of the thirty stocks in the Dow-Jones Industrial, with lags from one to ten days. Eqn. (R1) and (R2) are used to estimate the t-values for the calculated autocorrelations. Of the 30 sample autocorrelations between successive daily returns, 11 have t-values that are greater than 2 and 9 of these 11 are positive. Overall, 22 of the 30 sample autocorrelations between successive daily returns are positive. There seems to be positive autocorrelation between successive daily returns. Fama argues that the 11 stocks with t-value greater than 2 may be regarded as extreme in the sense that they are low probability events if the true autocorrelations are zero. The 22 positive sample autocorrelations are regarded as close in absolute values to zero. Fama concludes that even though the true autocorrelation may be non-zero, it would be close enough to zero so that market efficiency in its weak form is a reasonable description of the world.

Fama, Fisher, Jensen & Roll (hereafter, FFJR) (1969) attempt to test the semi-strong form of EMH by employing the market model to study the adjustment of stock prices to information concerning stock splits. The FFJR sample includes all 940 stock splits involving 622 different common stocks on the New York Stock Exchange from 1927-1959 when the split was at least 5 new shares for 4 old shares, and where the security was listed for at least 12 months before and after the split. Since any information in a split is likely to be company-specific, they search for unusual behavior of the market model's [Eqn. (R3)] residuals for each of the 622 different securities in the sample.

$$\tilde{R}_{jt} = \alpha_j + \beta_j \tilde{R}_{mt} + \tilde{\epsilon}_{jt} \qquad \text{(R3)}$$

where $\tilde{R}_{jt}$ is security j return at time t,

$\tilde{R}_{mt}$ is the market return at time t,

α_j & β_j are the model parameters for security j and assumed to capture the market-wide factor that affects security j's returns.

$\tilde{\epsilon}_{jt}$ is the model residual of security j at time t which is assumed to be company specific (specific to security j).

To estimate α_j and β_j of Eqn. (R3), FFJR use all of the monthly return data available for security j during the 1926-1960 period. They compute the market model residuals for each security for the period from 29 months before to 30 months after any split of the security. For a given split, they define month 0 as the month in which the effective date of a split occurs, month 1 and month -1 as respectively the month immediately following and preceding the split month, and so on. The average residual for month s, with s measured relative to the split month, is defined as

$$\bar{e}_s = \frac{\sum_{j=1}^{N_s} e_{js}}{N_s} \qquad \text{(R4)}$$

where e_{js} is the sample Market Model residual for security j in month s,

N_s is the number of split in month s.

$-29 \le s \le 30$ (60 months surrounding the split month).

They also examine the cumulative effects of abnormal return behavior for the same 60 months surrounding the split month by studying the cumulative average residual according to Eqn. (R5).

$$U_s = \sum_{k=-29}^{s} \bar{e}_k \qquad \text{(R5)}$$

The average residual $\bar{e}_s$ is interpreted as the average deviation, in month s relative to the split month, of the returns of split stocks from their normal relationships with the market. The cumulative average residual U_s is interpreted as the cumulative deviation from month - 29 to month s; it shows the cumulative effects of the returns of split stocks from their normal relationships with the market. FFJR further subdivide the split stocks into the "dividend increase" and "dividend decrease" categories. The former category includes all split stocks that experience a dividend increase after the split, while the latter includes split stocks associated with dividend decrease. The increase or decrease of dividend is justified by comparing to the dividend payments of the New York Stock Exchange as a whole. Therefore, u_S^+, u_S^- and U_S^+, U_S^- are defined as the average and cumulative average residuals for splits followed by "increased" and "decreased" dividends respectively. FFJR find that the average residuals in the 29 months prior to the split are uniformly positive for all splits and for both dividend classes. They argue that the split itself cannot account for the behavior of the residuals as far as 2 years in advance of the split date. Rather, they suggest that splits tend to occur when firms have experienced unusual increases in earning during the years immediately preceding a split, which accounts for the positive average residuals of splitting shares in months preceding the split. When all splits are examined together, the largest average residuals occur in the three or four months immediately preceding the split, but after the split, the average residuals are randomly distributed about 0. Equivalently, the cumulative average residuals rise up to the split month and experience no further systematic movement thereafter. Evidence also suggests that during the first year after the split, the cumulative average residual changes by less than 0.001 and that the total change during the $2^1/_2$ years following the split is less than 0.01. FFJR suggest that the informational effects of actual and anticipated dividend increases may be used to explain the behavior of common stock returns in the months immediately surrounding a split. A split is normally expected to associate with increases in future dividends, the evidence that the cumulative average residual for both dividend classes rises sharply in the few months before the split (the earliest time information about a splitting stock may reach the market before the effective date of the split) is consistent with the hypothesis that the market recognizes that splits are usually associated with higher dividend payments. However, returns behavior subsequent to the split would be substantially different in cases where dividend increases materialize and in cases where it does not. By examining the stocks in the "decrease" dividend class, FFJR find that the average and cumulative average residuals rise in the few months before the split but then decrease sharply in the few month after the split, when the anticipated dividend increase is not forthcoming. When a year has passed after the split, the cumulative average residual falls back to about where it was five months prior to the split, which is probably about the earliest time reliable information concerning a possible split may reach the market. When a dividend increase is not forthcoming, the effects of the split are completely wiped away and the split stocks return to their normal relationship with the market. For the stocks in the "increase" dividend class, there exists no further systematic movements in the cumulative average residuals for the year after the split and the average residuals are randomly distributed about zero. FFJR suggest that once the information effects of associated dividend changes are properly considered, a split has no net effect on common stock returns. Thus, the market apparently makes an unbiased forecast of the implication of a split for future dividends, and these forecasts are fully reflected in the price of the security by the end of the split month. The results of FFJR are consistent with the hypothesis that the stock market is efficient in its semi-strong form, at least, with respect to its ability to adjust to the information implicit in a split.

Charest (1978 a) investigates the behavior of the New York Stock Exchange (hereafter, NYSE) stocks around split information events from the 1947-67 period. These include 606 split proposals, 435 split approvals and 1252 split realizations. From the CRSP monthly file for the 1947-67 period, Charest samples all NYSE stocksplit realizations involving at least a 5:4 ratio and five years of presplit quotations. The split proposals and approvals information follows from the Wall Street Journal Dividend News Columns where the announcement date of the proposal or approval is assumed to be the date of the Journal. Charest assumes Eqn. (R6) as the return generating process that describes NYSE stocks in the 1947-67 period.

$$\tilde{R}_{jt} = \tilde{\gamma}_{0t} + \tilde{\gamma}_{1t}\beta_{jt} + \tilde{\epsilon}_{jt} \qquad \text{(R6)}$$

where $\tilde{R}_{jt}$ is the return of stock j in month t,

β_{jt} is the systematic risk or beta for stock j in month t,

$\tilde{\epsilon}_{jt}$ is the disturbance term of security j in month t which has a zero mean and independent of $\tilde{\gamma}_{0t}$ and $\tilde{\gamma}_{1t}$,

$\tilde{\gamma}_{0t}$ is the return on the minimum variance zero-beta portfolio,

$\tilde{\gamma}_{1t}$ is the return premium per unit of beta.

He estimates the residual of stock j in period t by Eqn. (R7).

$$\hat{\epsilon}_{jt} = R_{jt} - (\hat{\gamma}_{0t} + \hat{\gamma}_{1t}\hat{\beta}_{jt}) \qquad \text{(R7)}$$

The Fisher's Arithmetic Index is used as a proxy for the market rates of return, R_{mt}, that is used for direct estimation of stock j's beta according to Eqn. (R8).

$$\hat{\beta}_{jt} = \frac{COV(R_{jt},R_{mt})}{\sigma^2(R_{mt})} \qquad \text{(R8)}$$

where $COV(R_{jt},R_{mt})$ is the sample covariance between the return of stock j & the Fisher's Index at time t.

$\sigma^2(R_{mt})$ is the sample variance of the market proxy's return (the Fisher's Index).

Eqn. (R8) is applied to a series of 61 successive R_{jt} and corresponding R_{mt} in such a way to obtain, for stock j and month t relative to split event month zero, two competing beta estimates: $\hat{\beta}_{1jt}$ and $\hat{\beta}_{2jt}$. Method 1 uses past data -- the 61-month series of stock and market returns ending with month t. Method 2 uses symmetrical data, 30 months on each side of month t. Two corresponding vectors of residuals, $\hat{\epsilon}_{1jt}$ and $\hat{\epsilon}_{2jt}$ are obtained using Eqn. (R7) with Method 1 and Method 2 respectively. Charest labels Eqn. (R9) as the sample's average residual for month t relative to month zero (the split event month).

$$AR_t = \frac{1}{N} \sum_{j=1}^{N} \hat{\epsilon}_{jt} \qquad (R9)$$

where N is the number of splitting stocks in the sample. The average residuals (hereafter, AR) is used to measure the average percent abnormal return experienced by the sampled stocks in any month t relative to the split event. The sample's cumulative average residual over m months (say t+1 to t+m) is given by Eqn. (R10).

$$CAR = \sum_{\tau=t+1}^{\tau=t+m} AR_\tau \qquad (R10)$$

The cumulative average residuals (hereafter, CAR) is used to measure the average percent abnormal returns experienced by the sample stocks over a number of months relative to information event month zero. Charest's results for split realizations conform to those of FFJR in spite of differences in residual assessment methods and limited overlapping of test periods. AR in the pre-split realization month are positive and increasing. By month zero, the CAR is about 30% and the largest AR occurs in month -2, -3 and -4, the most likely months of split proposal announcement. The adjustment of stock returns to split realization appears to have essentially ended by month zero. Twenty-four months after the split realization, the CAR corresponding to method 1 is 30.86%, hardly 0.5% above the CAR for month zero. Charest suggests that such small changes in CAR appears to be consistent with the semi-strong form of EMH. In order to determine whether the abnormal returns from systematic monthly investments in split stocks over the 1947-1967 period would have been statistically different from zero, Charest constructs both an equally weighted and a value weighted portfolio consists of stocks corresponding to his five trading rules. His trading rules are outlined as follows:

A : all stocks whose information event date occurs in month t.

B : all stocks whose information event date occurs in month t, t-1, t-2.

C : all stocks whose information event date occurs in month t, t-1, t-2, t-3, t-4 and t-5.

D : all stocks whose information event date occurs in month t, t-1, t-2, ... t-11.

E : all stocks whose information event date occurs in month t, t-1, t-2, ... t-23.

Charest finds that 4 out of 5 trading rules and 9 out of 10 portfolio tests produce t-values of less than 2. Only rule E yields a significant t-value of 2.15 when method 2 residuals under an equal-weighted portfolio strategy are considered but the significance disappears under a value-weighted strategy, or when method 1 residual are used. The average monthly abnormal gain is less than 1/4% per month. Charest concludes that his results are generally consistent with the semi-strong form of EMH in that trading based on split realization events seems to offer no significant excess returns. In determining whether significant excess returns are more likely to arise when trading is based on earlier split information (i.e. split proposal and approval) rather than on split realization, Charest examines the

AR and CAR of the 606 split proposal and 435 approval events respectively. For both the approval and proposal samples, the abnormal return adjustment up to month zero which is in line with the results for the split realization sample. However, the 20 t-values corresponding to abnormal excess return based on split approvals, with both methods, all 5 trading rules and both the equal and value weighted investment strategies, are all within the bound of ±2 and are otherwise insignificant. On the other hand, the tests based on split proposals indicate some degree of apparent market inefficiency. It is especially the case with trading rule B where splitting stocks are held for 3 months beyond the proposal month. Its t-values exceed 2 for all methods and portfolio investment strategies, ranging from 2.25 to 3.08. Rule E, with a 24-month stock holding period, shows conflicting evidence. Its t-values exceed 2 for both the equal and value weighted strategies with method 2 but fail to show significant evidence when method 1 is used. Charest suggests that the stock market appears on average not to have fully anticipated, by the end of split proposal month the implications of split information for future stock returns. The market does not take for granted that a split proposal will eventually be approved while experience indicates it should. The market does not seem to have learned from experience, this would imply that the market does not "see" far ahead and does not realize what a "proposal" means for the future. Charest concludes that there is no significant excess return for trading systems triggered in split approval or realization methods. But investing monthly in stocks for 3 months beyond split proposals (Rule B) produces significant excess returns across all calculation methods and investment strategies. The market acts as if it needs approval information, two or three months away, to complete its adjustment. In this case, the market would not be semi-strong form efficient.

Watts (1978) studies the semi-strong form of EMH by determining whether significance abnormal returns can be observed after public announcements of quarterly earnings when valid significance tests are used and the proxy bias is reduced using the steps Ball (1978)[16] outlines. Watts suggests that information in quarterly earnings announcements could be measured by the difference between the actual quarterly earnings and a measure of quarterly expected earnings. If the market is efficient in its semi-strong form, stock prices changes should simultaneously reflect the information of public earnings announcements. In order to test the efficiency of the market with respect to quarterly earnings announcements, an estimation of the unexpected earnings and abnormal returns are required. Watts estimated the unexpected earnings as the forecast errors corresponding to three time-series models which can be used to describe the behavior of quarterly earnings. These three time-series models are outlined as follows:

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[16] refer to Ball (1978), p. 115-116

1. The Watts-Griffin Model

$$E(Z_t) = Z_{t-1} + (Z_{t-4} - Z_{t-5}) - \theta a_{t-1} - \gamma a_{t-4} + \theta\gamma a_{t-5} + \delta \qquad \text{(R11)}$$

where Z_t is the earning of quarter t,

E is an expectation operator,

θ is a moving average parameter,

γ is a seasonal moving average parameter,

δ is a constant,

a_t is a serially uncorrelated error term.

In this model, quarterly earnings have both seasonal and quarter to quarter dependency.

2. The Brown-Rozeff Model

$$E(Z_t) = Z_{t-4} + \phi(Z_{t-1} - Z_{t-5}) - \gamma a_{t-4} + \delta \qquad \text{(R12)}$$

where ϕ is an autoregressive parameter.

This model incorporates both a seasonal and an adjacent quarter component.

3. The Foster Model

$$E(Z_t) = Z_{t-4} + \phi(Z_{t-1} - Z_{t-5}) + \delta \qquad \text{(R13)}$$

The Foster Model incorporates seasonality and dependency in adjacent quarterly earnings, it constrains the seasonal moving average parameter in the Brown-Rozeff Model, γ, to be zero.

In order to estimate abnormal returns, for each quarter and each forecast model, Watts splits the firms under investigation into two portfolios: the firms with positive forecast errors for the quarter and those with negative forecast errors. The weights applied to the securities in each portfolio are calculated to make β (the systematic risk) of each portfolio equal to 1. Since both portfolios have equal risk, Watts suggests that abnormal returns can be measured as the difference in return between the two portfolios. In order to weight the securities within the portfolios, the Market Model is estimated for each firm in the sample using the 60 monthly rates of return immediately preceding each quarter. The market rate of return used is an arithmetic average of monthly rates of return of firms listed on the NYSE. With the estimations of beta for each security using the Market Model, each quarter, the securities in the positive and negative portfolios are further subdivided equally into low beta and high beta portfolios. The mean beta for each of the resulting 4 portfolios is calculated and the weights λ and $1-\lambda$ that applies to the low and high portfolios in order to produce portfolios with unity beta are calculated as follows for both the positive and negative portfolios:

$$\lambda\beta_{low} + (1-\lambda)\beta_{high} = 1 \qquad \text{(R14)}$$

The rates of return on the portfolios are calculated for weeks subsequent to the announcement of the firm's earnings to determine whether abnormal returns are earned after an earnings announcement. In so doing, Watts defines R_{jtw} as the rate of return on the shares of firm j in week w relative to the announcement of earning of firm j for calendar quarter t. The rate of return for week w on each of the 4 portfolios are calculated according to Eqn. (R15):

$$R^{b}_{stw} = \frac{1}{N_{sb}} \sum_{j=1}^{N_{sb}} R_{jtw} \quad \text{(R15)}$$

where N_{sb} is the number of securities in the portfolio for quarter t,

b represents high or low beta portfolio,

s represents positive or negative portfolio.

The rates of return on the positive and negative forecast error portfolios for quarter t are determined by applying the corresponding weights calculated according to Eqn. (R14) to the corresponding returns of the high and low beta portfolio calculated according to Eqn. (R15). The measure of abnormal returns in week w relative to the announcement of the earning of quarter t (DR_{tw}) is defined as:

$$DR_{tw} = R_{+tw} - R_{-tw} \quad \text{(R16)}$$

where R_{+tw} is the return of the positive portfolio,

R_{-tw} is the return of the negative portfolio.

Watts posits that DR_{tw} is the average rate of return on a trading strategy that involves no outlay and has a zero beta. If the market is efficient and the Capital Asset Pricing Model is correct, $E(DR_{tw}) = 0$ for all periods for which the strategy is feasible (i.e. long in the positive portfolio and short sale the negative portfolio). The abnormal returns for each quarter q after the earning announcement, defined as DR_{tq}, is calculated by summing the corresponding 13 weeks that make up the quarter. The estimate of the abnormal returns in quarter q relative to the earning announcement is calculated as the mean of the time series of DR_{tq} according to Eqn. (R17) :

$$\overline{DR}_q = \sum_{t=1}^{T} \frac{DR_{tq}}{T} \qquad (R17)$$

While the estimate of the standard error of abnormal returns in quarter q relative to the earnings announcement is depicted by Eqn. (R18).

$$S(\overline{DR}_q) = \frac{S(DR_{tq})}{\sqrt{T}} \qquad (R18)$$

where $S(DR_{tq})$ is the estimated standard deviation of the time series DR_{tq}.

Watts tests the null hypothesis of no abnormal return in quarter q relative to the earnings announcement, i.e. $\overline{DR}_q = 0$, by using the t statistic labels by Eqn.. (R19).

$$t(\overline{DR}_q) = \frac{\overline{DR}_q}{S(\overline{DR}_q)} \qquad (R19)$$

The sample employed by Watts consists of 73 firms which meet the following criteria:

a) All the firm's quarterly and annual earnings per share number are available in Moodys' over the period January, 1950 to December, 1957 and all the firm's quarterly and annual earnings per share announcements are available in the Wall Street Journal Index (WSJI) in the period January, 1950 through December 1969.

b) The firm's shares are listed on the NYSE for the period July

2,1962 through September 27,1968.

c) The firm's share price relatives are available on the Wells Fargo Bank daily file for the period July 2, 1962 through July 11 , 1969.

d) The firm's fiscal year is constant in period January, 1958 through December, 1969.

All of the earnings per share numbers are adjusted for stock splits and stock dividends; Watts uses the earnings for the 75 quarters from January, 1950 through September, 1968 in his study. The first 51 quarters of earnings data are used to estimate the Brown-Rozeff, Foster and Watts-Griffin models for each of the 73 firms. These estimated models are then used to forecast the earnings of the next 3 quarters, 52-54. Each of the models is re-estimated using observations 1-54 and earnings are forecasted for the following 3 quarters, 5-57. This procedure is continued until forecasts are obtained for quarters 52-75. These forecasts and the actual earnings are then used to obtain 24 earnings forecast errors for each firm under each of the 3 forecast models. Abnormal returns (DR_{tw}) are calculated for each week in the period from 25 weeks before the earnings announcement through 65 weeks after the announcement for all 24 quarters for all three sets of forecast errors. The abnormal returns for quarters relative to the week of the earnings announcement ($\overline{DR}_q$) and the related t statistics are calculated. For the quarter of the earnings announcement (q=0), it is found that the estimated $\overline{DR}_0$ ranges from 0.04 for the Watts-Griffin model to 0.053 for the Foster Model. Watts suggests that if the quarterly earnings at the beginning of each quarter during the sample period is known, an abnormal return of 4 to 5 percent could have been earned over the quarter. The t statistics for the estimated DR_0 are significant at 0.1% level for all 3 models. Watts also finds evidence suggesting abnormal returns after quarterly earnings announcement. The hypothesis that abnormal returns are zero in the first quarter after announcement can be rejected at 0.1% level using the Foster model and at 5% level using the Brown-Rozeff and Watts-Griffin models. The estimated abnormal returns for that quarter are 0.021, 0.0180 and 0.012 for the Foster, Watts-Griffin and Brown-Rozeff model respectively. The hypothesis that abnormal returns are zeros in the second quarter is also rejected at the 5% level for the Brown-Rozeff Model but cannot be rejected at any reasonable level for the other 2 models. After the second quarter, the estimated abnormal returns are effectively zero. Watts argues that his results could not be due to measurement error in estimating the securities beta or to change in the securities, since the abnormal returns are too large to be explained by measurement error. However, he suggests that not everyone could have earned abnormal returns by following the strategy implicit in the abnormal returns measure, since the transactions costs involved are more than sufficient to eliminate those profits. The only individual who could earn abnormal returns by using the trading rule are brokers since they can avoid some of the direct transaction costs. Watts concludes that " while the abnormal returns do not represent a gross inefficiency, they do represent a profit opportunity foregone, an inefficiency" [17]

Mishkin (1981 b) tests whether information on the past listing of short-term interest rates and inflation rates are used efficiently in the market for long term bonds. The tests involved examining whether the stochastic process that bond market participants believe these variables to follow is the same as the true stochastic process for the variables. Letting $(R_t - R^*)$ denote the excess return on long term bonds, Mishkin argues that the excess return could be expressed as a function of the unanticipated change $(X_t - X_t^e)$ in a variable according to Eqn. (R20).

$$R_t - R_t^* = \delta + (X_t - X_t^e)a + \epsilon_t \qquad \text{(R20)}$$

where δ is the constant liquidity premium,

a is the coefficient on the unanticipated movement of a prespecified variable,

ϵ_t is the independent error term,

R_t^* is the short term interest rate at time t-1, i.e. γ_{t-1}, which is assumed to be a rational expectation of a 1 period ahead long term rate.

R_t is the long term interest rate, at time t.

Assuming that the stochastic process for X_t can be written as:

$$X_t = b_0 + \sum_{i=1}^{n} b_i X_{t-i} + \mu_t \qquad \text{(R21)}$$

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[17] Watts (1978), p142

where μ_t is the random error term.

then the expectation of X_t, denoted as X_t^e which is a one-period ahead optimal forecast based upon all past available information, can be written as Eqn. (R22).

$$X_t^e = b_o + \sum_{i=1}^{n} b_i X_{t-i} \qquad \text{(R22)}$$

Substituting Eqn. (R21) and (R22) into Eqn. (R20), gives the observable equation:

$$R_t - \gamma_{t-1} = (X_t - b_o - \sum_{i=1}^{n} b_i X_{t-i})\alpha + \delta + \epsilon_t \qquad \text{(R23)}$$

Eqn. (R23) is labelled as the efficient markets model for the long term intereste rate R_t. Mishkin suggests that Eqn. (R23) and (R21) can be stacked into one regression system and be estimated by nonlinear least squares methods imposing the restrictions that the b_i coefficients in Eqn. (R21) and (R22) are equal for all i. The data employed by Mishkin in testing the efficient market system of Eqn. (R23) and (R21) comes from the following sources:

1. Quarterly long term U.S. government bond rate (R_t) is obtained from the CRSP file.
2. 90 days Treasury bill rate (γ_t) is obtained from the Board of Governors of the Federal Reserve System.
3. The quarterly rate of the Consumer Price Index is collected from Business Statistics and Survey of Current Business.

Using thirty quarterly observations from September 1969 to

December 1976, the bond return and Treasury bill rate data are used to estimate Eqn.. (R21) and (R23) system in order to determine whether the short term interest rate is used efficiently in the long-term bond market. Substitute X_t in Eqn. (R21) and (R23) by γ_t (the quarterly T-bill rate), Mishkin estimates the following system of equations:

$$R_t - R_{t-1} = \delta + a(\gamma_t - b_0 - \sum_{i=1}^{6} b_i \gamma_{t-i}) + \epsilon_t \qquad \text{(R24)}$$

$$\gamma_t = b_0 + \sum_{i=1}^{6} b_i \gamma_{t-i} + \mu_t \qquad \text{(R25)}$$

Mishkin finds that the coefficients on the unanticipated movement of the bill rate (a) is significantly different from zero at the 1% level, thus indicating that movements in the short-term interest rate embody relevant information to the pricing of long term bonds. Moreover, the sign of this coefficient is negative, indicating that an unanticipated rise in the bill rate is accompanied by higher long term rates with a resulting lower bond return. The likelihood ratio statistic also indicates that there is very little evidence in the bond market data supporting irrationality of interest rate forecasts. In order to test whether information about inflation is used efficiently by the bond market, Mishkin substitutes X_t in Eqn. (R21) and (R23) by π_t, where π_t is the quarterly Consumer Price Index inflation rate calculated from the change in the log of the Index (seasonally adjusted) from the last month of the previous quarter to the last month of the current quarter t. The following system of equation is obtained:

$$R_t - \gamma_{t-1} = \delta + a(\pi_t - b_o - \sum_{i=1}^{6} b_i \pi_{t-i}) + \epsilon_t \qquad \text{(R26)}$$

$$\pi_t = b_o + \sum_{i=1}^{6} b_i \pi_{t-i} + \mu_t \qquad \text{(R27)}$$

It is found that the efficient market model system, [Eqn. (R26) and (R27)] with data running from 1959-1969, yields the expected result that an unanticipated rise in inflation is associated with higher long rates and lower bond returns (sign of a is negative) but the a coefficient on unanticipated inflation is not as significant as the coefficient on unanticipated interest rate movements. The likelihood ratio test rejects the rationality restrictions at the 1% level. Mishkin suggests that this rejection is due to the evidence that the sum of the coefficients on the lagged inflation rates ($\gamma_{t-1},.....,\gamma_{t-6}$) in the autoregression model of inflation [Eqn. (R27)] is positive and greater than one, indicating that a rise in inflation would persist. This evidence suggests that information about inflation over the period 1959-1969 was not used efficiently. However, Mishkin argues that this is an unusual historical period, with inflation starting at a low level and persistently rising. By repeating the analysis using a longer time horizon (from 1954-1976), the inefficiency disappears. Mishkin suggests that the bond market may have had rational inflation forecast when a longer time horizon is taken into account. He concludes that the bond market does exhibit rational forecasting behavior and is efficiently exploiting publicly available information.

Frenkel (1977) examines the efficiency of the foreign exchange market to see whether the forward market for foreign exchange is efficient during the German Hyperinflation. According to the generally accepted theory, an efficient forward market for foreign exchange requires that the forward rate be an unbiased predictor of future spot rates. In examining the efficiency of the market, Frenkel regresses the logarithm of the current spot exchange rate, log S_t, on the logarithm of the one-month forward exchange rate prevailing at the previous month, log F_{t-1}.

$$\log S_t = a + b \log F_{t-1} + \mu \qquad \text{(R28)}$$

Frenkel demonstrates that, if the German foreign exchange market is efficient, the constant term (a) of Eqn. (R28) should not differ significantly from zero, and that the slope coefficient (b) should not differ significantly from unity. The error term, u, should be serially uncorrelated. Eqn. (R28) is estimated over the period February 1921 to August 1923 (31 months), since data on the German Mark-Pound Sterling forward exchange rate are available only from February 1921. Frenkel finds that the constant term does not differ significantly from zero at the 95% confidence level, however, the slope coefficient is somewhat above unity -- 1.09, at the 95% level. The Durbin-Watson statistic, with a value of 1.89, indicates that the residuals are not serially correlated. Frenkel suggests that transactions costs may be used to explain the evidence that the slope coefficient is slightly above unity. He further posits that in an efficient market F_{t-1} summarizes all the information concerning the expected value of S_t that is available at time t-1. One of the items of information available at t-1 is the stock of information available at t-2, and if the market is efficient, that information will be contained in F_{t-2}. If F_{t-1} summarizes all available information including those contained in F_{t-2}, then, adding F_{t-2} as an explanatory variable to equation (R28) will not affect the R^2 and will have a coefficient that is not significantly different from zero. As a result, Frenkel regresses log S_t on log F_{t-1} and log F_{t-2} according to Eqn. (R29):

$$\log S_t = a + b \log F_{t-1} + c \log F_{t-2} + \mu \qquad \text{(R29)}$$

The regression results support the EMH. The constant term is not significant with t-value of 1.7, the slope coefficient associated with F_{t-1}, which is slightly above unity (1.1), is significant with t-value of 13.8. However, the coefficient associated with F_{t-2} is not significant. The D.W. statistics, with a value of 1.91, indicates that the residuals are not serially correlated. Moreover, the coefficient of determination (R^2) remains the same. Frenkel suggests that during the hyperinflation, expectations may have behaved rationally and that one may use data from the foreign exchange market to measure expectations. He concludes that even during the turbulent periods of the German hyperinflation, the foreign exchange market remains remarkably efficient.

Hansen & Hodrick (1980) conduct tests of the efficiency of the foreign exchange market with flexible exchange which began essentially in March 1973, and for the period of generalized floating rates following World War I. They suggest that the results obtained by Frenkel (1977) may not literally be true since they are able to find several currencies whose past forecast errors may help to predict current forecast errors. Letting $s_t = \ln(S_t)$ and $f_{t,k}=\ln(F_{t,k})$ where S_t and $F_{t,k}$ are the levels of the spot exchange rate and the k-period forward exchange rate determined at time t. Hansen and Hodrick suggest that $s_{t+k} - f_{t,k}$ is an approximate measure of the rate of return to speculation in the foreign exchange market, it follows that their simple efficient markets hypotheses is that :

$$f_{t,k} = E(s_{t+k}|I_t) \qquad \text{(R30)}$$

where $E(\cdot|I_t)$ signifies the mathematical expectation conditioned on the information set available at time t.

They indicate that rejection of Eqn. (R30) does not necessarily imply the rejection of the efficiency or rationality of the foreign exchange market, since risk aversion in equilibrium would imply that the forward exchange rate equals the conditional expectation of the future spot rate plus a risk premium. The data for flexible exchange rates are obtained from the Board of Governors of the Federal Reserve System for seven currencies: the Canadian dollar, the deutsche mark, the French franc, the U.K. pound, the Swiss franc, the Japanese yen and the Italian lira. While the floating rates following World War I are taken from Einzing (1937) where weekly observations on the spot exchange rates and the 1-month forward rates are available after

November, 1921. All exchange rates are expressed in U.S. cents per unit of foreign currency. The hypothesis that $f_{t,k} = E(s_{t+k}|I_t)$ implies that forecast error $(s_{t+k} - f_{t,k})$ is uncorrelated with information available at time t. As a result, past forecast errors cannot help to predict current forecast error. Their tests employ a new and asymptotically more powerful technique to estimate various periods' forecast errors to be used in testing the null hypothesis. In order to test their claim that past forecast errors cannot be used to predict current forecast error, they regress the forecast error on a constant and two lagged forecast errors, using weekly data and a 3-month or 13-week forward rate:

$$s^i_{t+13} - f^i_t = a^i + b_{i1}(s^i_t - f^i_{t-13}) + b_{i2}(s^i_{t-1} - f^i_{t-14}) + \mu^i_t \qquad \text{(R31)}$$

where i = 1, ..., 7 currencies.

They test the joint hypothesis that a_i, b_i, and b_{i2} in Eqn. (R31) are all zero. Evidence suggests that the joint hypothesis is rejected for the deutsche mark-U.S. dollar exchange rate. Although the constant term is insignificant, the two lagged forecast errors have significant levels smaller than 0.02. For the Swiss franc and Italian lira, they find individual coefficient estimates that have significant levels less than 0.1. The other four currencies provide no strong evidence against the null hypothesis. By regressing the forecast error of a currency on lagged values of its own forecast error and four other currencies' lagged forecast errors, as in Eqn. (R32),

$$s^i_{t+13} - f^i_t = a_i + \sum_{j=1}^{5} b_{ij}(s^j_t - f^i_{t-13}) + \mu^i_t \qquad \text{(R32)}$$

where $i = 1, \ldots, 5$ currencies.

the null hypothesis that all coefficients in the regression are zero is rejected for the Canadian dollar, the deutsche mark, and the Swiss franc at all significance levels smaller than 0.06. Hansen and Hodrick suggests that the multicountry test appears to be a more powerful test of the efficiency hypothesis because they are able to reject the hypothesis for two other countries except at very low significant levels. Their results indicate that lagged forecast errors for some currencies have explanatory power in predicting the current forecast errors and the foreign exchange market may not be efficient. They re-estimate Eqn. (R32) for a period running from June 25, 1974 to January 16, 1979. In ~~the joint~~ test of all six coefficients, only the Canadian dollar has a marginal significant level below 0.1, and it is below 0.01. In the deutsche mark, the U.K. pound and the Swiss franc regressions, there are individual coefficients with significance levels less than 0.1. This suggest evidence in favor of the null hypothesis. When the multicountry test is expanded to include the Japanese yen and the Italian lira, the null hypothesis is rejected for the Canadian dollar except at extremely low significance levels. However, the significance level for the duetsche mark has fallen to 0.001 and for the Swiss franc to 0.09. The coefficient for the lagged Japanese yen is significantly different from zero in four countries at the 0.04 level and in the fifth at the 0.08 level. Hansen & Hodrick suggest that failure of the efficiency hypothesis is also manifest in relative high R^2 statistic for the Canadian dollar and the deutsche mark. In examining the floating rates which occurred after World War I, tests of the efficiency hypothesis for three currencies relative to the U.K. pound are conducted according to Eqn. (R33):

$$s^i_{t+4} - f^i_t = a_i + b_{i1}(s^i_t - f^i_{t-4}) + b_{i2}(s^i_{t-1} - f^i_{t-5}) + \mu^i_t \qquad \text{(R33)}$$

The forecast interval is four rather than 13 as in Eqn. (R31). The results indicate that the constant term is significantly different from zero at the 0.01 level for the entire sample. Hansen & Hodrick conclude that the logarithm of the forward exchange rate is an underestimate of the logarithm of the subsequently observed spot rate, it is not an optimal predictor of the future rate for some currencies. This evidence is inconsistent with the efficiency hypothesis. However, they emphasize that they are testing a joint hypothesis involving efficient use of information along with a particular model of market equilibrium and suggest that an alternative model of market equilibrium may be consistent with their empirical results.

Lorie & Niederhoffer (1968) analyze data on insider[18] tradings and study their subsequent effects on stock prices which is essentially a test on the strong form of EMH. Their study is based on data published by the Securities and Exchange Commission for all listed companies in its Official Summary of

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[18]according to Lorie and Niederhoffer (1968), insiders are officers, directors, and owners of 10% or more of the common stock of the companies listed on the New York and American Stock Exchanges.

Stock Transactions. The Summary is a complete record of transactions of at least 100 shares in the stock of their own companies by the directors, officers and owners of 10% or more of the outstanding shares. By analyzing data of insider trades from January, 1950 to December, 1960 of a stratified random sample of 105 NYSE companies, Lorie & Niederhoffer (hereafter, L & N) deduce the statistical properties of insider trading. From frequency distribution of the number of net buyers or sellers, L & N find that there are three or more net buyers in approximately 5-9% of all months when there are non-option and non-gift transactions and there are three or more net sellers approximately 4% of the time. These numbers allow them to construct control bands that filter out unusual instances of insider trading for further scrutiny. In their entire sample of 3973 purchases and 3277 sales, the odds in favour of a purchase followed by a purchase is found to be 3 times as great as a purchase followed by a sale. Furthermore, the odds in favor of a sale after a sale is twice as great as after a purchase. L & N suggest that a purchase is more likely after a purchase than after a sale. One purchase indicates that other purchases are likely to follow and that the first purchase tells more than subsequent ones. A change in direction from selling to buying indicates the new fact that future purchases are to be expected, whereas a sale followed by a sale merely confirms the preceding expectations concerning the direction of insider activities. Therefore, the change in direction of activity is seem to be important in deducing insiders' expectation concerning their stocks. In order to determine whether insiders possess special information, i.e., buy before the announcement of good news and sell before bad news, L & N analyze insider transactions before large price change[19] in three ways. First, they analyze the last transaction in the six months before a large change. If the last transaction was a purchase, the chance of a large increase is 71 out of 100. If the last transaction sold stock, the chance of a large decrease is 53 out of 100. The odds in favor of a large increase is 2.5/1 after a purchase and 1.1/1 after a sale. The second kind of analysis concerns the number of purchases and sales in the 6 months prior to the large price change. The odds in favor of a large increase is found to be 2.2 times as great when the number of purchases is greater. The third analysis deals with the volume of purchases and sales in the six months prior to the large price changes. This evidence is much weaker than the other two kinds of analysis, but again the skill of insiders in forecasting large price changes is demonstrated. L & N further suggest that if insiders do possess special kinds of information, then, the number of different insider purchasers or sellers in a month would serve as a measure of the extent of interest by insiders in a stock. In order to test this claim, they study the relationship between intensive insider trading and subsequent price movements in stocks. L & N choose 30 stocks at random that satisfy the following criteria:

1. They are purchased by 3 or more insiders within one month.
2. They are sold by no insiders in a month of intensive

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[19]The large price change is defined as changes of 8% or more.

purchasing.

3. At least 2 purchasers increase their holdings by more than 10%

Calculation of month-end prices from January 1961 to June 1964 are made for these stocks and for the Dow Jones Industrials on all 315 occasions where there are 2 or more insiders buying or selling. During this period, intensive buying and selling are not useful predictors of stock performance in the subsequent six months. L & N then study a sample of stocks for a period for which the exact date of purchase and sales are available. For each transaction, the price on the exact day of the transaction and the price 6 trading days into the next month are determined.[20] Percentage changes in price are computed over the next 6 months. These changes are then compared to changes in the Dow Jones Industrial Average over the same period. L & N find that data based on this procedure indicate a strong relationship between insider trading and price movements. There appears to be an opportunity for investors to profit from knowledge of insider trading. When the number of buyers exceeds the number of sellers by at least two, it is found that the probability is about 0.6 that the stock would outperform the Dow Jones Industrial during the six months after the sixth trading day following the end of the trading month, and when sellers exceed buyers by 2 or more, the probability is about 0.64 that the stock would perform worse than the Dow Jones Industrials in the following 6 month. L & N

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[20]The earliest an investor could be confident of finding out about insider trading is assumed to be 6 trading days after the end of the month.

argue that, if intensive insider buying and selling are independent of future price movements, the probability that such relative frequencies would occur by chance is substantially less than 1/100,000. Insider transactions are slightly more successful when measured from the actual date of the transaction than when measured from 6 tradings days after the end of the month. L & N further suggest that during 1963 and 1964, the stock market rose substantially and more volatile companies would be expected to perform better than the average during a rising market. It is possible that the companies associated with intensive inside trading in the study are more volatile than other companies. To test this theory, L & N measure the volatility of each stock in which there was intensive trading in 1963 or 1964. The measure of volatility they choose is the beta coefficient of the Market Model, using the Fisher Index as a proxy for the market. The median beta coefficient for all companies on the stock exchange is approximately 0.95. L & N argue that if the companies in which insiders made intensive transactions are more volatile than the market, more than half of the beta coefficients for these companies should be greater than 0.95. The results show that during 1963, 41 have beta coefficient greater than 0.95 and 43 have coefficient less than 0.95. L & N conclude that the higher rates of return for those companies with intensive insider trading apparently are not attributable to their volatility. Proper and prompt analysis of data on insider trading can be profitable. When insiders accumulate a stock intensively, the stock can be expected to outperform the market during the next six months. Insiders tend to buy more often then usual before large price increases and to sell more than usual before price decreases. These evidences indicate the possibility of special information possessed by insiders so that stock prices do not fully reflect all available information.

Scholes (1972) investigates secondary offerings, many of which are issued by insiders, to determine whether insiders possesses inside information of an adverse nature. Secondary distributions [21] are chosen instead of primary distributions because Scholes suggests that many of the variables that affect the firm's prospect could be held constant, so that, secondary distributions are the result of decisions that are independent of the factors affecting company operation. Daily price data of 345 secondary distributions for listed NYSE firms from July 1961 to December 1965 is used by Scholes in conducting the analysis. Monthly data for the period 1947-1965, which consists of 1,207 secondary distributions, is used to confirm the analysis of the daily data sample. To isolate the movement in security prices

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[21] Secondary distributions are initiated not by the company but by one or more shareholders to whom the future proceeds from the sale of the secondary distribution will accrue. The distributions are underwritten on a principal or agency basis by an investment banking group that buys the entire block of stock from the selling shareholders. The shares are then sold to subscribers after normal trading hours at the `subscription price', set at or near the closing price of the shares in the open market on the day of the sale. There are two types of secondary distributions, registered and unregistered. The Securities and Exchange Commission requires that a distribution be registered if the shares involved in the sale represent a control relationship to the user. If a distribution is registered, there is a waiting period of 20 days, from the day of registration, before actual sale can take place

associated with market-wide information from possible insider information, Scholes employs the Market Model:

$$\tilde{R}_{it} = \alpha_i + \beta_i \tilde{R}_{mt} + \tilde{\mu}_{it} \qquad \text{(R34)}$$

where $\tilde{R}_{it}$ is the return for period t on the i^{th} security,
$\tilde{R}_{mt}$ is the average return on a market portfolio,
α_i & β_i are the Market Model's parameters to be estimated,
$\tilde{\mu}_{it}$ is the random disturbance term for period t.

Scholes estimates the parameters of the Market Model using 100 days of return data on each security in the sample around the day of the secondary but excluding the 6 observations prior to, and 7 observations including and subsequent to the day of the secondary. An estimated prediction error, $\hat{E}_{it}$, is computed for a period of 25 days prior to and 14 days subsequent to the distribution according to Eqn. (R35):

$$\hat{E}_{it} = R_{it} - [\hat{a}_i + \hat{b}_i R_{mt}] \qquad \text{(R35)}$$

where R_{it} is the actual return for security i on day t,
R_{mt} is the return on the Standard & Poor Composit Index for day t,
$\hat{a}_i$ & $\hat{b}_i$ are the estimated coefficients of Eqn. (R34).

Each security's prediction errors are then used to compute an average prediction error for each day relative to the distribution day according to Eqn. (R36) where the distribution day is defined as day 0 (d=0):

$$\bar{E}_d = \frac{1}{N} \sum_{i=1}^{N} \hat{E}_{id} \qquad \text{(R36)}$$

where i= 1, ..., N, the number of securities in the sample.

The average error measures the average estimated percentage deviation of the returns of the securities in the sample from their normal relationship to the market. Scholes suggests that using the average error and its standard error, the significance of the effects of secondaries on market prices can be estimated. An abnormal performance index (API) is also constructed and is labelled as Eqn. (R37).

$$API_D = \frac{1}{N} \sum_{i=1}^{N} \left[\prod_{\tau=d}^{D} (1 + \hat{E}_{i\tau}) \right] \qquad (R37)$$

This index traces out the value of \$1.00 invested in equal amounts in each of the N securities in the sample at time τ and held until the end of period D, after abstracting from general market effects on returns. In the absence of any abnormal returns, Scholes demonstrates that API_D would be approximately equal to 1.0. From the abnormal performance index, the marginal rate of return from holding a portfolio from period D to period $D+\tau$ calculated as $[API_{D+\tau}/API_D]-1$, and this can enable the calculation of returns on this portfolio for various holding periods. Scholes suggests that the methodology described can provide a means of estimating the average effect of the sale of large-block distributions on security prices. The estimated prediction errors are abnormal returns not accounted for by the security's normal relationship to the market as described by the Market Model. By taking the average of the prediction errors for each day relative to the distribution day, the abnormal return on each day associated with the sale of the large block distributions can be estimated. The abnormal performance index further enables the estimation of the cumulative abnormal performance, through time, of a portfolio of secondaries purchased at the start of the period of interest and held through the end of the period of interest. By applying the methodology to the daily sample of 345 secondary distributions and assuming that $1.00 is invested in the portfolio of secondaries 25 days prior to the distribution day, Scholes finds that the abnormal performance index falls from an initial level of 1.0 to a final value of 0.977, 14 days subsequent to the distribution, a decline of 2.2%. The absolute value of the average error is greater on each of the 6 days including and subsequent to day 0 than on any other single day. On the day of the secondary the average error is found to be equal to -0.5%. Scholes demonstrates that the cause of the observed abnormal return cannot be attributed to either the price effect or the size of the distribution. In order to confirm the results obtained from daily samples, Scholes uses monthly samples covering the period 1947-1964 during which there are 1,207 distributions. The average prediction error is found to be -2.15% in the month of the secondary. The value of the abnormal performance index is 1.01 at the end of the month of the secondary, and at the end of month 1 and month 5. It stands at 1.00 at the end of month 18 after the secondary. No inducement in the form of an abnormal return is realized over the 18-month period subsequent to the distribution. Scholes concludes that the examination of abnormal return on both a daily and month basis shows a permanent average 2% loss associated with the sale of a secondary distribution. Scholes further suggests that the likelihood that a sale contained adverse information is very different among the vendor categories due to the degree of contact of their day to day operations with the operations of the firms they sell. Scholes ranks the 5 vendors categories, in decreasing order concerning their possession of possible adverse information, as follows: 1) corporations, 2) investment companies, 3) banks and insurance companies, 4) estates and trusts, 5) individuals.

On the day of the secondary, the vendor is not generally known. If the announcement of a secondary distribution conveys information to the market, then, on the day of the sales, all the average errors will be expected to be negative and of about the same magnitude. If the value of the information exceeds the expected value of information contained in secondaries, the price will fall further to a new equilibrium price. Scholes computes the abnormal performance index and the average predicting errors for each of the 5 vendor categories. It is found that the average errors at day 0 are indeed of approximately the same order of magnitude for all groups. After the distribution, the absolute magnitude of the abnormal return is largest for corporations, followed by mutual funds, and smallest for banks, estates, and individuals, which is what he has expected on the basis of a prior classification for likelihood of adverse information, Scholes finds that, from the analysis of the category of corporations, the sale of a corporate officer does contain information of significant value.

They sell when the security has experienced positive abnormal returns and is considered to be overvalued in the market. He concludes that there appears to be significant differences by vendor and information.

In order to determine the effect on the return of stock between registered and non-registered distribution, Scholes computes the abnormal performance index and average error for both groups of distributions by employing his daily sample which consists of 73 registered secondary distribution out of a total of 345. He finds that the average abnormal return on the day of the distribution is -0.6% for non-registered and the performance index falls from a level of 0.989 on the day of the secondary to 0.975 10 days subsequent to day 0, a return of -1.4%. For the registered secondaries the average error on day 0 is -0.099%, and the performance index falls from a level of 0.992 on day 0 to 0.988 on day 10, a return of -0.4%. The average errors are the largest on day -20 and -19 which are -0.26% and -0.41% respectively. Scholes suggests that this is the announcement date of the registered secondary. From 20 days to 1 day prior to the secondary the performance index drops from a level of 1.005 to 0.99, a return of -1.3%. However, for the same period, the performance index of the non-registered secondaries falls only by -0.4%. Scholes suggests that this provides evidence that the market conveys information efficiently which causes the largest drop of the abnormal performance index at different time intervals, relative to day 0, for the registered and non-registered secondaries. He rejects the hypothesis that the residuals fall becauses of selling pressure and concludes that the drop in residual is due to the market's belief that the issuer possesses inside information of an adverse nature.

Jaffe (1974) attempts to estimate the profitability of insider trades by examining the performance of a security subsequent to specific types of insider trades in that security, which he calls insider trading events. His initial sample consists of the 200 largest securities on the Chicago Research in Security Prices (CRSP) tape. Insiders' transactions for each of the 200 securities are observed in 5 separate months during the period from 1962 to 1968. While insiders may transact without special information, Jaffe suggests that their large transactions would more likely be based on inside information. Thus a sample of large transactions is constructed by including all transactions from the initial sample with values greater than $20,000. This subsample contains 370 trades, representing 39% of the initial sample of 952. Finally, 4 intensive trading samples are constructed by including companies with at least 3, 4, 5, and 6 respectively more purchasers then sellers (or more sellers than purchasers) for each month. These samples include all intensive trading companies listed on the CRSP tape during the month from April to October 1961; from December 1961 to November 1962; from January 1964 to March 1965; from May to December 1965; and from September 1966 to March 1967. First of all, Jaffe uses Eqn. (R38), which is consistent with the two -parameter CAPM of Sharpe and Lintner, to measure the abnormal performance of a security.

$$R_{jt} = \tilde{\gamma}_{0t} + \tilde{\gamma}_{1t}\beta_j + \tilde{e}_{jt} \qquad \text{(R38)}$$

where $\beta_j = \dfrac{COV(\tilde{R}_{jt},\tilde{R}_{mt})}{Var(\tilde{R}_{mt})}$.

$\tilde{R}_{jt}$ = rate of return on security j during period t,

$\tilde{R}_{mt}$ = rate of return on the market portfolio in period t,

$COV(\tilde{R}_{jt},\tilde{R}_{mt})$ = covariance between $\tilde{R}_{jt}$ & $\tilde{R}_{mt}$,

$Var(\tilde{R}_{mt})$ = Variance of $\tilde{R}_{mt}$,

$\tilde{\gamma}_{0t}$ & $\tilde{\gamma}_{1t}$ = market determined parameters,

$\tilde{e}_{jt}$ = the disturbance of the j^{th} security at time t.

The disturbance in Eqn. (R38) serves as a measure of the abnormal performance of a security. Since it is assumed that insiders possess more special information concerning their own security than concerning the market as a whole, Jaffe studies the residuals of securities subsequent to insider trading events. He defines U_m as the mean residual over all securities in the sample for month m:

$$U_m = \frac{\sum_{j=1}^{N} \hat{U}_{jm}}{N} H_j \qquad \text{(R39)}$$

where N=number of securities in the sample.

$\hat{U}_{jm}$ = estimated residual for security j in month m according to Eqn. (R38) (month 0 in Eqn. (R39) is the event month, month 1 is the following month, and so on),

H_j = 1 if the event of the j^{th} security is a purchase or set of purchases,
= -1 if the event is a sale.

Next, Jaffe defines the cumulative average residual, CU_m, as:

$$CU_m = \sum_{k=-15}^{m} U_k \qquad \text{(R40)}$$

For each sample, Jaffe constructs 3 portfolios including the securities of all companies with events representing 1, 2 and 8 months holding periods respectively. The performance of portfolio t in the month from t to t+1 is defined as follows:

$$\hat{e}_{t,t+1} = \frac{\sum_{i=1}^{S} (\hat{e}_{i,t+1} H_i)}{S} \qquad \text{(R41)}$$

where $\hat{e}_{t,t+1}$ = residual of portfolio t in month t +1,
$\hat{e}_{i,t+1}$ = residual of the i^{th} security of portfolio t in month t+1;
S = number of securities in portfolio t;
H_i = 1 if event of the i^{th} security is a purchase,
= -1 if event is a sale.

A measure of the variability of the performance of portfolio t, called $\hat{SD}_t$, is defined as the computed standard deviation of the residual of portfolio t, using data during the period from month (t-49) to month t:

$$\hat{SD}_t = \sqrt{\frac{1}{49} \sum_{j=1}^{50} \left(\hat{e}_{t,t-j+1} - \frac{1}{50} \sum_{i=1}^{50} \hat{e}_{t,t-i+1}\right)^2} \qquad \text{(R42)}$$

The standardized residual for portfolio t at time t+1 is defined as $\hat{se}_{t,t+1}$:

$$\hat{se}_{t,t+1} = \frac{\hat{e}_{t,t+1}}{\hat{SD}_t} \qquad \text{(R43)}$$

As a different portfolio is formed for each calendar month, for a given holding period, portfolio t is just one of many portfolios. The average standardized residual across all of these portfolios, $\overline{sr}$ is defined as:

$$\overline{sr} = \frac{1}{n} \sum_{t=51}^{401} \hat{s}e_{t,t+1}D_t \qquad (R44)$$

where D_t = 1 when there is at least one security in portfolio t,
= 0 when there is no security,
n = number of months in which the portfolio has at least one security.

Jaffe uses data from January 1935 to June 1968 (month 1 is January 1935 and month 402 is June 1968) to estimate γ_{0t} and γ_{1t} of Eqn. (R38). In Eqn. (R44), t begins at 51 because 50 months of past data are required to estimate a portfolio's residual variance [Eqn. (R42)]. Jaffes tests whether $\overline{sr}$ is significantly different from zero by the following t-statistic:

$$t = \frac{\overline{sr}}{s/\sqrt{n}} \qquad (R45)$$

where s = estimate of the standard deviation of each standardized portfolio.

In any month an individual trader is classified as a purchaser if the number of days during the month in which he buys stock is greater than the number of days in which he sells. If the reverse is true, he is classified as a seller. For each company in the initial sample, a month is classified as a month of net purchasers or a month of net sellers depending on whether the number of purchasers is greater or less than the number of sellers. Months of net purchasers and net sellers are defined by Jaffe as insider trading events. Events are excluded if companies are not listed on the CRSP tape for 50 months before and 10 months after the event. This restriction assures sufficient data to form portfolios and to calculate residual variances. For the initial sample, Jaffe finds that the cumulative average residuals actually fall by approximately 2% in the 15 months prior to events. Cummulative average residuals rise approximately one-half of 1% in the 15 months following trading. The most rapid rise occurs in the few months after trading, suggesting that insiders can forecast residuals in the near future better than residuals in the distant future. The t-values corresponding to the hypothesis that the expected value of the standardized residuals equals zero are 1.93, 2.24 and 1.32 for 1, 2 and 8 month holding periods respectively. Though these results suggest that insiders do possess and exploit special information, Jaffe further examines the sample of large transactions to confirm these results. However, the results do not suggest that large transactions contain more information than small transactions. The resulting t-values are 1.99, 2.09 and 1.14 for a 1, 2 and 8 month holding period respectively. Jaffe then studies the 4 intensive trading samples and the resulting high t-values suggest that insiders trade successfully. The t-values are respectively 3.65, 4.73 and 5.23 for 1, 2 and 8 month holding periods corresponding to the event when there are at least 3 more buyers (sellers) than sellers (buyers) in a month. Average residuals are large, rising 0.0507 in 8 months. The events corresponding to at least 4 more buyers (sellers) than sellers (buyers) have resulting t-values of 3.06, 3.16 and 4.69 for a 1, 2, and 8 month holding period respectively. The cumulative average residuals again rise over 5% in 8 months, suggesting that insiders possess special information. When the event is 5 more buyers (sellers) than sellers (buyers), the t-values are respectively 2.97, 2.76 and 3.18 for a 1, 2 and 8 month holding period. The cumulative average residuals are of similar magnitude than the previous event (4). When the event is 6 more buyers (sellers) than sellers (buyers), the t-values are 1.6, 0.77 and 1.05 for a 1, 2 and 8 month holding period respectively. The cumulative residuals and the t-value are small. These findings indicate successful trading by insiders but do not suggest that profit to insiders is an increasing function of the intensity of trading. Under the Securities and Exchange Act of 1933-34, insiders must return all profit from a purchase and subsequent sale (or vice versa) occurring within 6 months of each other. Therefore, an insider must be prepared to retain his new acquisition for at lest 6 months in order to profit from his inside information. As a result, with consideration given to transactions cost (assume to be 2% for a round lot transactions), Jaffe studies the first 3 intensive trading samples corresponding to an 8-month holding period. The results indicate that the t-values for 8-month periods are statistically large (3.26, 2.92 and 1.84 for the 3 intensively trading sample respectively in order of increasing intensity). Jaffe suggests that insiders earn approximately 3% profit in the 8 months after transaction, indicating that transaction costs only diminish profit by 40%. Jaffe concludes that insiders do possess special information, after adjustment for transactions cost, the intensive trading samples with 8-month holding periods are still earning statistically large returns. This evidence is not consistent with the strong form of EMH.

Chiras and Manaster (1978) examine the informational content of option prices and the efficiency of the options exchange market by employing the Black-Scholes-Merton option pricing model [Eqn. (R46)] to calculate the implied standard deviation (ISD) [22]

$$W = e^{-yt}XN(d_1) - e^{-rt}CN(d_2) \qquad \text{(R46)}$$

where $d_1 = \dfrac{\ln(X/C) + (r - y + 1/2\nu^2)t}{\nu t^{1/2}}$,

$d_2 = d_1 - \nu t^{1/2}$,

W = the current option price for a single share of stock,

X = the current stock price,

C = the exercise price of the option,

e = the base of natural logarithms,

t = the time remaining until expiration of the option,

r = the continuous risk-free rate of interest for the period t,

ν = the standard deviation of returns on the stock during the period t,

$N(\cdot)$ = cummulative normal density function of $(\cdot)$,

y = the continuous dividend yield on the stock.

Chiras and Manaster (hereafter, C & M) suggest that dividend on

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[22] An implied standard deviation equates an observed option price with the price calculated from a particular option pricing formula.

some stocks may be substantial and can have a significant effect on the valuation of options whose stocks make such payments during the life of the options. Since the Black-Scholes model assumes no such dividend payment, Eqn. (R46) which assumes dividend payments to be continuous over time, is used to calculate the ISD. During the period encompassed by their study, there is an average of 6.3 option prices recorded per stock for each observation date. Each of these options had a different ISD. C & M suggest that the ISDs must be combined in order to produce a single estimate of future standard deviation of returns for each stock. The equation used to obtain the weight implied standard deviation (WISD) of the options on one stock for each observation date is as follows:

$$WISD = \frac{\sum_{j=1}^{N} ISD_j \frac{\partial W_j}{\partial \nu_j} \frac{\nu_j}{W_j}}{\sum_{j=1}^{N} \frac{\partial W_j}{\partial \nu_j} \frac{\nu_j}{W_j}} \qquad (R47)$$

where N = the number of options recorded on a particular stock for the observation date,

ISD_j = the implied standard deviation of option j for the stock,

$\frac{\partial W_j}{\partial \nu_j} \frac{\nu_j}{W_j}$ = the price elasticity of option j with respect to its implied standard deviation ν.

C & M posit that in an efficient market prices will fully reflect all available information. Thus, estimated variances calculated from option prices should reflect not only the informational content of stock price history but also any other variable information. They suspect that the WISD values would reflect future standard deviations more accurately than do the historic sample standard deviations. As a result, they state their hypothesis as: "standard deviations inferred from option prices have been better predictors of standard deviations of future stock returns than standard deviations obtained from historical stock returns." In testing their hypothesis, they run the following 3 regression equations:

$$SDFUT_{i,t} = a_h + B_h \, SDHIST_{i,t} \qquad (R48)$$

$$SDFUT_{i,t} = a_0 + B_0 \, WISD_{i,t} \qquad (R49)$$

$$SDFUT_{i,t} = a_c + B_{ch} \, SDHIST_{i,t} + B_{c0} WISD_{i,t} \qquad (R50)$$

where $WISD_{i,t}$ = the weighted implied standard deviation of returns for stock i at time t,

$SDHIST_{i,t}$ = the sample standard deviation of returns for stock i from time t-20 to time t,

$SDFUT_{i,t}$ = the sample standard deviation of returns for stock i from time t to time t+20,

a_h, B_h, a_0, B_0, a_c, B_{ch}, B_{c0} = regression coefficients.

The $SDHIST_s$ and the $WISD_s$ are compared to determine which predictor is superior during the test period. The combination of $SDHIST_s$ and $WISD_s$ are also compared to determine whether each predictor provides unique information or contains only information already captured by the others. Their testing period begins two months after the start of the listed option trading and lasts for 22 months, i.e. with 23 monthly observations beginning June 1973 and ending April ,1975. Data are recorded for the last trading day of each month for each option on stocks whose options are traded on the Chicago Board Options Exchange (CBOE) as of June 29, 1973. All the necessary data for Eqn. (R46) are taken directly from the Wall Street Journal and the Moody's Handbook of Common Stocks. The monthly price ratios of each underlying stocks are taken from the CRSP tapes for each of the 23 stocks in the sample. By employing their procedure to their sample, the results indicate that historical standard deviations explain approximately 26% of the future standard deviations of stock returns and the corresponding R^2 for the weighted implied standard deviations obtained from option prices is 0.32. The increase is 23%. From June 1973 to February 1974, evidence indicates that there is little difference between the predictive characteristics of the WISD and the SDHIST values. Beginning in March 1974, less than a year after the start of listed option trading, the option implied standard deviations show a sudden increase in predictive ability and begin to explain more of the future standard deviation of stock returns. The average value of R^2 in this period increases to 0.39 as compared to 0.21 in the previous period. The regression using SDHIST does not indicate any trend in predictive ability over time. Evidence suggests that the predictive abilities of option implied standard deviations are continuing to improve during the entire period under study and that the option market becomes more efficient as traders gain more experience. The null hypothesis that the regression coefficient equals zero is rejected at the 5% level in each month for both Eqn. (R48) and Eqn. (R49). The t-value shows a tendency to increase over time.

When considering Eqn. (R50), C & M find that the R^2 values are substantially higher than corresponding values associated with Eqn. (R49) for only 3 months (July, August and October 1973). After October 1973, the multiple regression model does not produce substantially better R^2 values than those obtained by use of the option implied standard deviations. The t-values for the coefficients of WISD are at a substantially higher value than those of SDHIST. After February 1974, the t-values associated with the coefficients of $SDHIST_S$ deteriorate sharply relative to those of $WISD_S$ suggesting that the use of SDHIST value add no information that is not already contained in the values of WISD. The analysis indicates that the weighted implied standard deviation is a better predictor for future standard deviation of stock returns. C & M argue that although the t-statistic for the coefficient of historical standard deviations is significant, any additional information contained does not appear to adequately reward the extra effort required to include them in the analysis. They conclude that the weighted implied standard deviations have been substantially better predictors of future standard deviations than have the historical ones. C & M then develop a trading strategy which uses WISDs to test the efficiency of the CBOE and the accuracy of the evaluation model. They suggest that if the assumptions of the option model are correct and the option market is efficient, then all options on the same stock at a given time would be expected to have identical ISD values. However, the ISDs values are found to differ widely. They explain the differences in terms of non-simultaneous data, improper model specification and/or market inefficiency. To determine whether the CBOE is inefficient, their trading strategy examines whether arbitrary profit can be exploited from the CBOE by creating a risk-free portfolio of hedges which eliminates the effect of stock price movements. Hedges are constructed between options on the same underlying stock such that a riskless trading process is obtained. Of the 118 hedges 93 are profitable and the anticipated gain is 17.45% per month. The portion of the anticipated gain which is realized is 57%, leaving 43% attributable to model error. The gain on the short positions is 134% of their anticipated returns and the gain on the long positions is -24% of their anticipated return. C & M show that the gains are positive for 78.8% of the hedges and for 95% of the monthly portfolios. They conclude that during the period covered by their data, June 1973 to April 1975, the prices of options on the CBOE provide the opportunity to earn economic profits, and therefore, the CBOE market is inefficient.

Gibbons & Hess (1981) studies the day of the week effects by conducting tests on the S & P500 index, the value and equally weighted CRSP type indexes. Their overall sample periods running from July 2, 1962--December 28, 1978. To test the day of the week effect, Gibbons & Hess (hereafter, G & H) use the following equation:

$$\tilde{R}_{it} = \alpha_{1i}D_{1t} + \alpha_{2i}D_{2t} + \alpha_{3i}D_{3t} + \alpha_{4i}D_{4t} + \alpha_{5i}D_{5t} + \tilde{\nu}_{it} \qquad \text{(R51)}$$

where $\tilde{R}_{it}$ = the return of index (or security) i in period t,

$\tilde{\nu}_{it}$ = the disturbance term for index (or security) i in period t,

D_{1t} = 1 for Monday and 0 otherwise (a dummy variable),

D_{2t} = 1 for Tuesday and 0 otherwise,

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D_{5t} = 1 for Friday and 0 otherwise.

$\tilde{\nu}_{it}$ is assumed to be independently and identically distributed with mean 0 and constant variance. G & H suggest that the coefficients of Eqn. (R51) are the mean returns for Monday through Friday. For all sample periods, except the November 29, 1974 to December 28, 1978 period, the hypothesis of equality across all the regression coefficient is rejected for each of the 3 indexes employed. The results indicate that Monday consistently offers a negative return, while Tuesday's return appears to be slightly low. Moreover, the returns on Wednesday and Friday appear to be somewhat higher than Tuesday or Thursday. For the overall sample, the average annual return on Monday ranges from -33.5% (the S and P500) to -26.8% (the equally weighted index). The estimated autocorrelation at lag 1 for the S & P500 and the value-weighted indexes are about 0.2 and that for the equally-weighted index is about 0.4. G & H suggest that these autocorrelations may be explained by non-trading of securities. In order to overcome the possible problem of nontrading, individual securities are tested with Eqn. (R51) serving as the underlying statistical model. G & H select the firms of the Dow Jones 30 for the tests since these securities are actively traded. The sample means for each day of the week are obtained for periods running from July 3, 1962 to December 28, 1978, from July 3, 1962 to October 27, 1970 and from October 30, 1970 to December 28, 1978. They find that Monday effects are not limited to a few securities, for the overall period and the first subperiod, all 30 securities have a negative mean on Monday. In order to ensure that these results are not due to inappropriate statistical assumptions concerning the underlying model of Eqn. (R51), i.e., the assumption that the covariance matrix is the same for all days of the week, the data is adjusted for heteroscedasticity. G & H find that the heteroscedasticity adjustment has no important impact on the previous results. They suggest that one possible interpretation of their findings may be market inefficiency. Such a conclusion assumes that the market attempts to price securities to yield the same expected return for all days of the week. However, the constant expected return model may not be a valid description of capital market equilibrium and conclusion based on this model may be misleading. Thus, G & H conclude that their results provide strong evidence of varying equilibrium returns across days of the week which may be independent of market inefficiency.

In summary, the weak form of the EMH is generally considered to be a reasonable description of capital market efficiency. Alexander(1961;1964), Fama & Blume(1966) and Fama(1965) have all suggested that securities are priced efficiently with respect to the weak form information set. However, their researches still face the problem of non-exhaustion of the set of information concerning past prices across securities. On the other hand, the strong form of the EMH is generally considered to be an incorrect description of capital market efficiency. Examples are the researches conducted by L&N(1968) and Jaffe(1974) concerning insider tradings. L&N and Jaffe have all concluded with the rejection of the strong form. However, L&N have assumed the Market Model during some point of their study and Jaffe has assumed the CAPM. Thus, rejection of the strong form may be due to market disequilibrium and/or model misspecification, and may be independent of market inefficiency.

The most contradicting results follow from the researches dealing with the semi-strong form of the EMH. Considering the semi-strong form alone, one half of the studies cited in the section suggest its rejection, while the remaining half hold an opposite viewpoint. Examples are the two independent studies conducted by FFJR(1969) concerning the information content of stock splits and by Charest(1978a) concerning capital market efficiency with respect to stock split proposals, approvals and realizations. FFJR provide evidence to support the semi-strong form while Charest suggests that the stock market is not semi-strong form efficient with respect to the information contents of split proposals. However, note that FFJR's study does not exhaust the information set corresponding to the semi-strong form. Thus, non-rejection of the semi-strong form does not necessarily imply its unambiguous acceptance. On the other hand, Charest has assumed the two parameters CAPM in his study; thus, rejection of the semi-strong form may be due to market disequilibrium, misspecification of the CAPM and/or market inefficiency. It follows that Charest's study does not suggest unambiguous violation of the semi-strong form.

Since unambiguous rejection or acceptance of the semi-strong form of the EMH is not provided by previous researches, it motivates us to devise a methodology that is capable of rejecting the EMH unambiguously via the TF. A study concerning the 'day of the week' effect is also cited in the section to justify the inclusion of this effect in our study.

Research Statement

The objective of this research, as indicated previously, is to test the semi-strong form of the EMH by a TF approach. Specifically, the efficiency of the Standard and Poor 500 Index will be examined. The data employed are the most recently available daily returns data of the value-weighted New York & American Stock Exchange Combined Index (the input series) and those of the Standard & Poor 500 Index (the output series), which run from January 3, 1984 to December 21, 1984. The daily returns data, which include all distributions, i.e., dividend payments and stock splits, are taken from the computer tapes available from the Center for Research in Security Prices at the University of Chicago (CRSP tapes).

Previous research indicates that small stocks normally show a higher risk adjusted average return. Thus, it may be possible that small stocks are more responsive to some public information than the larger stocks in the S&P500 Index, since, speculators may move more quickly to buy or sell the smaller stocks than the less risky stocks in the S&P500 Index. It follows that it is possible for the existence of a transition period before the information can be 'transmitted' to the SP500 series. Thus, information affecting the NYAM series(including small stocks) may be used to forecast the SP500 series. As a result, we attempt to use the NYAM series as input to predict the output SP500 series.

In order to identify a TF model for the output SP500 series accurately, the time series of concern must be continuous. If this is not the case, and the output and/or the input series exhibit(s) seasonal variations, inaccurate univariate time series model(s) for the input and/or output series may arise during the initial steps of the TF modelling process. This, in turn, may bias the subsequent 'Identification' process and result in an inappropriate or inadequate TF model.[23] Thus, conclusions regarding market efficiency based on this inappropriate TF model may be misleading and incorrect.

The problem of discontinuity is not concerned with the 'five-day work-week' of the stock market. Rather, it has to do with the public holiday(s), i.e., we are interested in continuity of 'trading' days rather than 'calender' days. Fortunately, the problem of unavailability of data due to public holiday(s) can be overcome by the introduction of an intervention model to both series prior to the identification of the subsequent TF component. The most commonly encountered intervention models are the zero order and first order models which are analogous to Eqn. (P2) and Eqn. (P3) [24] that describe the zero order and first order TF. The only difference is that,

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[23]To see this point clearly, consider an output series which possesses a 5-day(or weekly) seasonal cycles. If there is one missing observation per week during the model building period, a 5-day cycle may be identified wrongly as a 4-day cycle during the univariate analysis. Thus, the subsequent CCF analysis may be biased since the incorrect univariate model would be applied. This, in turn, may lead to the identification of an incorrect TF component and the subsequent noise component. It follows that the TF identified may be biased and inappropriate.

[24]Refer to p.14.

in attempting to estimate the series levels for the periods of missing observations, the input series of the intervention model takes on ~~a value~~ of 1 for periods of unavailable data and 0 otherwise. For the zero and first order Intervention Models, we write Eqn. (1) and Eqn. (2) respectively as:

$$Y_t = WI_t + NOISE \qquad (1)$$

$$Y_t = \frac{WI_t}{(1-\delta_1 B)} + NOISE \qquad (2)$$

where Y_t is the time series to be modelled.
I_t is the intervention time series (input series) of the intervention model.
$I_t=1$ for periods of missing observations
$=0$ otherwise.
B = the backward shift operator.
NOISE is the noise residuals of the Y_t series.

The zero-order intervention model posits that the series level subsequent to the periods of unavailable data will not be distorted while the first-order intervention model assumes that the series level subsequent to periods of unavailable data will change gradually and permanently with the total change in the j^{th} subsequent period amounted to $\sum_{i=1}^{j} W_0\delta_1^{\,i}$, i.e., the net change in each j^{th} subsequent period is $W_0\delta_1^{\,j}$. Depending on the series levels subsequent to periods of missing observations, Eqn. (1) or (2) may be used to estimate the missing data.

In order to build a more reliable univariate model for both the input and output series in the initial step of the TF modelling process, we isolate the effects of possible differences in returns associated with each day of the week by a zero-order intervention model before analyzing the series residuals. A zero-order intervention model is justified because there is no a priori reason that the returns series will behave abnormally, say, after a Monday, Tuesday, ... , etc. This same univariate model associated with the SP500 series will be used to test the martingale hypothesis that expected returns are constant for Monday through Friday.

After the effects of unavailable data and possible returns differences have been properly isolated by an appropriate intervention model, we can proceed with the Box-Jenkins procedures for the identification of the TF for the output SP500 series. The Box-Jenkins procedure for time series model building, as suggested by Box & Jenkins (1976), involves seven steps which can be outlined as follows:

1. Preliminary Univariate Analysis -- Univariate models must be built for the output and input series. If modeling indicates that either series is non-stationary, the series must be differenced appropriately to eliminate between-series correlations due only to drift or trend.
2. Transfer Function Identification -- The Integrated Autoregressive Moving Average (ARIMA) model for the input series is inverted and applied to prewhiten both the input and output series. The Cross Correlation Function (CCF)[25] between these prewhitened series is used to identify a transfer function model for the relationship between the input and output series. Prewhitening is necessary to ensure that the CCF obtained is not contaminated by within-series

[25] For a description of the CCF, see appendix C.

correlation. By prewhitening, we mean that the univariate ARIMA model for the input series identified in step (1) is inverted and applied to both series.

3. Noise Component Identification -- Parameters for the transfer function component are estimated. Residuals from this estimation are used to identify an ARIMA model for the noise component.
4. Estimation -- Parameters for the fully identified tentative model are estimated. If the parameters of either component, i.e., the TF or noise component, are not statistically significant and otherwise acceptable, a new component must be identified, i.e., return to step 2. By otherwise acceptable, we mean that noise component parameters must lie within bounds of stationarity-invertibility, i.e., within -1 and 1; TF parameters must lie within the bounds of system stability, e.g., δ_1 for first-order TF such that $-1<\delta_1<1$.
5. Noise Component Diagnosis -- If residuals of the tentative model are not white noise, a new noise component must be identified, i.e., return to step 3.
6. Transfer Function Diagnosis -- If residuals of the tentative model are correlated with the prewhitened input series, a new TF component must be identified, estimated and diagnosed, i.e., return to step 2. The TF component has been specified to account for all process variance common to the input and output series. If the TF component is statistically inadequate, a large portion of the output variance will show up as model residuals. Thus, a

statistically adequate TF component will be independent of the noise component. The CCF estimated between the prewhitened input series and the model residual can serve as an indication of the independency. If the TF and noise components are not independent, there will be spikes at the low-order lags of the CCF.

7. Interpretation of the Final Adequate Model -- The final adequate TF model identified must have practical interpretation depending on its form.

Since a univariate time series model must be identified for both the input and output series in the initial step (step(1)) of the procedure, and the univariate model identified for the output SP500 series will also be used to test the martingale hypothesis (H3), we outline the univariate modelling process as follows:

1. Identification -- The estimated autocorrelation function (ACF)[26] of the time series is used to determine whether the series is stationary, if non-stationarity is indicated, e.g., by an ACF which fails to die out quickly, it will be necessary to difference and/or transform the series appropriately prior to identifying a tentative model. If the time series is itself (or after appropriate differencing) stationary, then, the ACF and PACF (Partial Autocorrelation Function)[27] can be used to identify an ARIMA (p,d,q) model for it. E.g., autoregressive processes are characterized by

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[26] For a description of the ACF, see appendix C.
[27] For a description of the PACF, see appendix C.

decaying ACFs and spiking PACFs. An ARIMA (p,0,0) process is expected to have exactly p non-zero spikes in the first p lags of its PACF, all successive lags of the PACF are expected to be zero. Moving average processes are characterized by spiking ACFs and decaying PACFs, e.g., an ARIMA (0,0,q) process is expected to have exactly q non-zero spikes in the first q lags of its ACF, all successive lags of the ACF are expected to be zero. An ARIMA (p,0,q) process is expected to have both decaying ACF and PACF.

2. Estimation -- Parameter estimates must be statistically significant and must lie within the bounds of stationarity-invertibility, i.e., between -1 and 1. Any parameter whose estimated value is not significantly different than zero should be dropped from the tentative model. If the estimated parameters of the tentative model do not satisfy the stationarity-invertibility conditions, then the tentative model must be rejected and a new model must be identified, i.e., return to step 1.
3. Diagnosis -- The model residuals of the tentative model must be white noise. i) It must be independent at a first and second lag, thus, the ACF for the model residuals must have no statistically significant values (or spikes) at the first 2 lags. ii) The residuals must be distributed as white noise as shown by a statistically insignificant Q-statistic[28]. If

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[28]The Q-statistic is given by the formula:

$$Q = N \sum_{i=1}^{k} [ACF(i)]^2.$$

The Q-statistic is distributed approximately chi-square with degree of freedom equal to k-p-q-P-Q, where k = # of lags in the

the tentative model meets the white noise requirements, it is accepted, if not, a new model must be identified, i.e., return to step (1).

The final form of the TF model identified for the SP500 series will be used to test against the null hypothesis (H1) or (H2):

H1: Parameter W_0 and/or time lag b of the final <u>0-order</u> TF model are/is statistically insignificant.

H2: Parameter W_0 of the final <u>1st</u> <u>order</u> TF model is statistically insignificant.

The univariate ARIMA model identified for the SP500 series will be used to test against the martingale hypothesis, (H3):

H3: The expected value of the SP500 series is a constant, i.e., the univariate ARIMA model built for the SP500 series satisfies either condition (a) or (b) concerning the martingale property.[29]

If (H1) or (H2) is rejected, the semi-strong form of EMH is rejected while rejection of (H3) would imply the rejection of the martingale property. Note that, non-rejection of (H1) or (H2) does not necessarily imply the acceptance of the semi-strong form of EMH, since it is possible that some other input series, besides the NYAM series, in the set of publicly available information, which have not been used by the market,

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[28](cont'd) ACF, p & q are respectively the order of the autoregressive and moving average process identified, P & Q are respectively the order of the seasonal autoregressive and moving average process identified.

[29]Refer to p.21-22 of the 'Problem Statement' Section.

may be able to reject the null hypothesis via the TF test.

Procedure

The first 238 daily returns data of the value-weighted Standard & Poor 500 Index (SP500 series) and that of the New York & American Stock Exchange Combined Index (NYAM series) from January 3, 1984 to December 21, 1984 are used to build the TF model, while the last 10 observations of each series are reserved for comparison between ex-post forecasts generated from the best model chosen and the actual values of the SP500 series.

The time series plots for the SP500 and NYAM series, from January 3 to December 7, 1984, are shown respectively in Fig. 1a & 1b. From January 3, 1984 to December 7, 1984 (the model building period), there are 6 missing observations due to public holidays, (Feb. 20, April 20, May 28, July 4, Sept. 3 & Nov. 22). If there exists a seasonality component for one or both of the univariate time series and/or the final TF model identified, these missing observations need to be properly accounted for by some estimation technique. This technique should also be capable of isolating the effects of possible difference of returns between each day of the week by the univariate series before proceeding with the identification procedure. The estimation technique employed is the Intervention Model.

Eqn. (3) and (4) are the zero-order intervention models used for the estimation of the 6 missing observations for the NYAM and SP500 series respectively.[30]

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[30]Eqn. (2) is also used to estimate the series level corresponding with the 6 missing observations. For Eqn. (2), the estimated value of the W parameter and its associated t-value

$NYAM_t = WI_t + NOISE$ (3)

$SP500_t = WI_t + NOISE$ (4)

where I_t = 1 for period t with missing observation,

= 0 otherwise.

W = estimated value for the missing series levels.

NOISE = unmodelled noise residuals corresponding to the univariate time series indicated.

In order to employ Eqn.(3) and Eqn.(4), the 6 missing observations are initially assumed to be zeros. The justification of initially assigning zero values for the missing observations follows from the inherent assumption of the CRSP tape in calculating daily returns. Let P_{t-1} be the price (or index level) of the NYAM series at the end of day t-1, P_{t+1} be the price of the NYAM series at the end of day t+1, D_{t+1} be the dividend payment received on day t+1 and let day t be a public holiday. The CRSP tape assumes that the return of day t+1 is equal to $(P_{t+1} + D_{t+1} - P_{t-1})/P_{t-1}$, however, the actual return on day t+1 is $(P_{t+1} + D_{t+1} - P_t)/P_t$, P_t is not observable (or available) and is approximated by P_{t-1} and D_t is equal to 0

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[30](cont'd) for the NYAM series are respectively -0.0002 and 0.09 while those for the SP500 series are respectively -0.0006 and 0.28. The estimated value of the δ_1 parameter and its associated t-value are respectively 0.1967 and 0.05 for the NYAM series, while those for the SP500 series are respectively -0.0159 and 0.0080. Note that, the insignificant of the δ_1 parameter implies that Eqn. (2) is reduced to a 0-order intervention model which justifies our use of the 0-order models labelled by Eqn. (3) and (4). The insignificance of all the estimated parameters, for both the NYAM and SP500 series, corresponding to the use of Eqn. (2) imply identical results with the use of Eqn. (3) and (4), i.e., the missing series levels are estimated to be 0.

since there is presumably no dividend payment on a public holiday. As a consequence, the implicit return on day t is equal to $(P_t + D_t - P_{t-1})/P_{t-1}$ which is equal to 0 since P_t is assumed to be equal to P_{t-1} and $D_t = 0$. Thus, if the W parameter in Eqn. (3) or (4) is significant, it means that the 6 missing observations are statistically different from 0. Evidence suggests that the W parameter corresponding to Eqn. (3) and (4) are not significant. The missing observations are estimated to be -0.006 for the NYAM series with associated t-value = 0.2, while those for the SP500 series are estimated to be -0.0003 with t-value = 0.09. Thus, all the missing observations are assumed to be 0. They are not excluded from the data set because of the possibility for a seasonal variation.

In attempting to isolate the 'day of the week' effect before proceeding with the prewhitening and identification process, we examine Eqn. (5) and Eqn. (6) corresponding to the output SP500 series and the input NYAM series respectively, with missing series levels estimated to be zero:

$$SP500_t = W_M I_{M,t} + W_T I_{T,t} + W_W I_{W,t} + W_{Th} I_{Th,t} + W_F I_{F,t} + NOISE \quad (5)$$

$$NYAM_t = W_M I_{M,t} + W_T I_{T,t} + W_W I_{W,t} + W_{Th} I_{Th,t} + W_F I_{F,t} + NOISE \quad (6)$$

where $I_{M,t} = 1$ for Monday & 0 otherwise,

$I_{T,t} = 1$ for Tuesday & 0 otherwise,

$I_{W,t} = 1$ for Wednesday & 0 otherwise,

$I_{Th,t} = 1$ for Thursday & 0 otherwise,

$I_{F,t}$ = 1 for Friday & 0 otherwise,

W_M, W_T, W_W, W_{Th} & W_F are the parameters to be estimated which correspond to the estimated series levels for Monday, Tuesday, Wednesday, Thursday & Friday respectively.

The estimated parameter values and their associated t-values for Eqn. (5) and Eqn. (6) are shown respectively in Table 1 and Table 2. From Table 1 & 2, we notice that all the W_i parameters are statistically insignificant, except for W_{Th} which shows a slight evidence of a Thursday effect with t-values equal to 1.78 and 1.61 respectively for Eqn. (5) and (6). The residuals of Eqn. (5) are stationary and are not different than white noise as shown by the ACF & PACF in Fig. 2a & 2b, having no significant spike at any lag. The ACF & PACF (Fig. 3a & 3b) of the residuals corresponding to Eqn. (6) indicate that the NYAM series, considering the 'day of the week' effect, are stationary. However, they are not a white noise process since significant spikes show at lag 1 of both the ACF & PACF. Since the prewhitening process requires the input series to be white noise, we model the non-white residuals of Eqn. (6) as Eqn. (7).

$$(1-\underset{(3.25)}{\underset{0.2034}{\phi_1}}B)NYAM_t = \underset{(1.84)}{\underset{0.0019}{W_{Th}}}I_{Th,t} + a_t \qquad (7)$$

Note that only the W_{Th} parameter is retained in Eqn.(7) because of a 'slight' Thursday effect shown by Eqn.(6). The estimated values for the W_{Th} and ϕ_1 parameters are shown directly below Eqn.(7) with t-values in parentheses. The residuals of Eqn.(7) as indicated by its ACF & PACF (Fig. 4a & 4b) are not different than white noise with Q-stat=19.2. Thus, the output SP500 series is prewhitened as Eqn.(8):

$$(1-\phi_1 B)SP500_t = W_{Th}I_{Th,t} + a_t \qquad (8)$$

The CCF between the noise components of Eqn.(7) & Eqn.(8) is examined in order to identify a TF model for the SP500 series. The CCF, as shown by Fig.4c, has a sigle significant spike at lag 0. This signifies the appropriateness of a 0-order TF with no input lead, thus, Eqn.(9) is run:

$$SP500_t = W_0 NYAM_t + W_{Th}I_{Th,t} + NOISE \quad (9)$$

The residuals of Eqn.(9), as indicated by its ACF & PACF (Fig.5a & 5b) are not white noise with significant spike at lag 2. Many different models have been examined. The best model obtained is an AR(1) model, thus, the residuals of Eqn.(9) is modelled as Eqn.(10):

$$SP500_t = W_0 NYAM_t + W_{Th}I_{Th,t} + a_t/(1-\phi_1 B) \quad (10)$$

where a_t indicates that the residuals are white noise.

The ϕ_1 parameter associated with Eqn.(10) is highly significant with t-value of 9.9 and estimated value of -0.0678. The residuals of Eqn.(10), as shown by their ACF & PACF (Fig.6a & 6b), are not different than white noise with Q-stat=20.1. However, Fig.6c shows that the CCF between the white noise residuals and the prewhitened input series of Eqn.(10) shows a significant spike at lag 3 indicating that the residuals of Eqn.(10) at time t are dependent upon the input series at time t-3. This implies that the input series at time t-3 may help forecasting the output series at time t. Thus, Eqn.(10) is not adequate, we examine Eqn. (11).

$$SP500_t = W_0 NYAM_t + W_1 NYAM_{t-3} + W_{Th} I_{Th,t} + a_t/(1-\phi_1 B) \quad (11)$$

The residuals of Eqn. (11), as shown by its ACF & PACF (Fig. 7a & 7b), are not different than white noise with no significant spikes at any lag and a Q-statistic value of 19.2. The CCF (Fig. 7c) between the white noise residuals of Eqn. (11) and Eqn. (7) (the prewhitened input series) indicates that Eqn. (11) may be an appropriate TF model for the SP500 series due to the absence of any significant spike at lower order lags. However, the estimated parameter values of Eqn. (11) together with their corresponding t-values, as shown in Table 3, indicates that the W_{Th} parameter is not statistically significant while all other parameters are highly significant with the estimated value of ϕ_1 (-0.0673) being well within the bound of system stability. So, the Thursday effect is ruled out and assumed to have no significant effect on the TF model (Eqn. (11)), at least, for bivariate analysis during the model building period. This evidence, coupled with the absence of a seasonal component in Eqn. (11), suggests the use of the original 'raw' data series free of any estimation for the missing observations and possible 'day of the week' effects. Thus, the Box-Jenkins procedure is re-iterated by ignoring the missing data and the Thursday effect.

Once again, the 'raw' SP500 series is stationary and is not different than white noise, as shown by its ACF & PACF in Fig. 8a & 8b, with a Q-stat value of 20.0. Eqn. (12) is an appropriate representation of the SP500 series while Eqn. (13) is the NYAM series.

$$SP500_t = a_t \quad (12)$$

$$NYAM_t = NOISE \quad (13)$$

The ACF & PACF (Fig. 9a & 9b) of the NYAM series (Eqn. (13)) indicates that it is stationary but non-white, with significant spike at lag 1 of both figures. This indicates the possibility of a first order autoregressive process, thus, we examine Eqn.(14).

$$(1-\phi_1 B)NYAM_t = a_t \quad (14)$$

The residuals of Eqn.(14) as indicated by its ACF & PACF (Fig.10a & 10b), are not different than white noise with Q-stat=14.7. Thus, Eqn.(12) is prewhitened as Eqn.(15):

$$(1-\phi_1 B)SP500_t = a_t \quad (15)$$

The CCF between the noise components of Eqn.(14) & Eqn.(15) is examined in order to indentify a TF model for the SP500 series. The CCF, as shown by Fig.10c, has a single significant spike at lag 0. This indicates the appropriateness of a 0-order TF with no input lead. Therefore, we examine Eqn.(16).

$$SP500_t = W_0 NYAM_t + NOISE \quad (16)$$

The residuals of Eqn.(16), as shown by its ACF & PACF (Fig.11a & 11b), are not white noise with significant spike at lag 5 and a Q-stat value of 33.2. Attempting to fit a 5-day seasonal cycle to the residuals of Eqn.(16) is not successful.[31] Many different models have been examined, the best

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[31] The ϕ_5 parameter corresponding to a 5-day seasonal model is not significant with t-value=0.39 and estimated value=-0.0032.

one obtained is Eqn.(17):

$$SP500_t = W_0 NYAM_t + a_t/(1-\phi_1 B) \quad (17)$$

The estimated value of the ϕ_1 parameter is -0.0687 with a t-value of 9.95. The residuals of Eqn.(17). as indicated by its ACF & PACF (Fig.12a & 12b), are not different than white noise with Q-stat=16.9. However, the CCF (Fig.12c) between the white noise residuals and the prewhitened input series of Eqn.(17) shows a significant spike at lag 3. Eqn.(17) is not adequate and Eqn.(18) is examined.

$$SP500_t = W_0 NYAM_t + W_1 NYAM_{t-3} + a_t/(1-\phi_1 B) \quad (18)$$

The ACF & PACF of the residuals from Eqn. (18)(Fig.13a & 13b) show that the residuals are not different than white noise with Q-stat = 16.0. The absence of a significant spike at lower order lag of the CCF (Fig. 13c) between the residuals of Eqn. (18) and the prewhitened input series (Eqn. (14)) signifies that Eqn. (18) may be an appropriate TF model for the SP500 series. Note that, the two input components $NYAM_t$ and $NYAM_{t-3}$ have no significant correlation, as indicated by the ACF & PACF of the 'raw' NYAM series (Fig. 9a & 9b), which has no significant spike at lag 3. Thus, no significant dependency exists between $NYAM_t$ and $NYAM_{t-3}$. The estimated parameter values for Eqn. (18), together with their corresponding t-values, are shown in Table 4(page 104). The t-values for all the estimated parameters are highly significant and the estimated value for ϕ_1 (-0.0676) lies well within the bound of system stability. Eqn. (18) is an adequate zero order TF model for the SP500 series.

In testing the martingale hypothesis, we use Eqn. (5) by ignoring the missing observations.

$$SP500_t = W_M I_{M,t} + W_T I_{T,t} + W_W I_{W,t} + W_{Th} I_{Th,t} + W_F I_{F,t} + NOISE \quad (5)$$

The ACF & PACF of the residuals of Eqn.(5) show exactly the same pattern as those in Fig.2a & 2b when missing observations are not ignored. Thus, the residuals of EQn.(5) are not different than white noise. Its estimated parameter values and their respective t-values are shown in Table 5. Table 5 indicates that all the estimated parameters, except for W_{Th}, are insignificant. As a result, the W_M, W_T, W_W and W_F parameters are dropped from Eqn.(5). We have Eqn.(19).

$$SP500_t = W_{Th} I_{Th,t} + a_t \quad (19)$$

The estimated value for the W_{Th} parameter is 0.0020 with t-value=1.77. Note that this t-value is considered to be significant by a one-tail test at the 5% level and can serve as slight evidence for a Thursday effect. Eqn.(19) is a white noise process identified for the SP500 series.

Results

A zero-order TF model with 0 lead (Eqn.18) is identified for the SP500 series. Eqn.(18) suggests that information embedded in $NYAM_{t-3}$ can help forecasting the current value of SP500 at time t. This indicates a violation of the 'weak' form. However, note that, although the $NYAM_{t-3}$ is 'past' information, it only captures the 'residual effect' after the 0-order TF has been properly modelled. Without the 0-order component, the $NYAM_{t-3}$ may not have any significant forecasting value. Note that, as mentioned in the 'Problem Statement' section, a statistically significant parameter may be an aberration. Statistical test provides only supporting evidence for causality. Although a highly significant parameter may indicate a higher likelihood for a causal relationship, there is always the possibility of making a Type I error. Further justification that the $NYAM_{t-3}$ component cannot be used to reject the null hypothesis can be seen from Fig.10c. The CCF between the prewhitened NYAM and SP500 series shows no significant spike at lag 3 which might indicate that the $NYAM_{t-3}$ component in Eqn.(18) is a statistical aberration.

By Eqn.(18), the semi-strong form of the EMH (H1) cannot be rejected. Eqn.(18) suggests that majority of the information embedded in the publicly available NYAM series at time t has been reflected in the SP500 series. In attempting to reverse the causal relationship of Eqn.(18), i.e., by using the SP500 series as input and the NYAM series as output, the CCF (Fig.14) between the white noise SP500 (input) series (Eqn.12) and the stationary NYAM (output) series (Eqn.13) shows significant spikes at lag -1 and 0. This indicates that $NYAM_{t-1}$ has a correlation with $SP500_t$, thus, the causality cannot be reversed. The NYAM series should be used as input series.

The ex-post forecast conducted by using Eqn. (18), from Dec. 10 to Dec. 21, 1984, is shown in Table 6 while the plot of the ex-post forecast vs. actual values is shown in Fig. 15. With consideration given to the 'day of the week' effect, the SP500 series is identified as a whtie noise process as indicated by Eqn. (19). Eqn. (19) does not satisfy the conditions concerning the martingale property[32], thus, the martingale hypothesis, H3, is rejected. Note that, Eqn. (19) indicates slight evidence of the 'day of the week' effect suggesting that, on average, Thursday will provide a higher rate of return. Although this particular effect does not have any significant impact on the TF model, the effect exists in the univariate time series. The evidence that both input and output series show a higher return on Thursday may warrant this effect to be cancelled out for bivariate TF analysis. In any case, the 'day of the week' effect appears to exist in both the SP500 and NYAM series for the period from Jan. 3 to Dec. 7, 1984, although it will not bias the TF model when it is ignored.

As previously mentioned, the rejection of H3 does not necessarily imply market inefficiency. LeRoy (1973) demonstrates

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[32]Refer to p.21-22 of the 'Problem Statement' Section

that the martingale property need not hold when stochastic risk free interest rate is accounted for. By assuming that investors exhibit constant absolute risk aversion and earnings on stock conform to a first order autoregressive process, he shows that expected returns on stock will not conform to the martingale process even though the market is efficient. Thus, H3 may be rejected because the martingale misspecifies the true underlying expected returns generating process. The rejection may be independent of market efficiency.

Conclusion

The semi-strong form of the EMH is tested by a TF approach and supplement with a test of the martingale property concerning stock price returns. Before considering our empirical results, note that:

1. Non-rejection of the semi-strong form of EMH by the TF approach is consistent with either acceptance or rejection of the martingale process. Non-rejection of the semi-strong form of EMH by the TF approach implies that the output SP500 series is efficient with respect to the input NYAM series. However, it does not prevent the SP500 series from being inefficient with respect to other publicly available input series. Consider the case where the SP500 series is not semi-strong form efficient, i.e., other publicly available input series besides NYAM can be used to forecast the SP500 series. Under this scenario, and assuming that the martingale process is a correct description of equilibrium expected returns, the martingale property would be rejected. Next, consider the case where the SP500 series is truly semi-strong form but not strong-form efficient. Non-rejection of the semi-strong form by the TF approach would result in rejection of the martingale hypothesis because the market is not truly efficient. Thirdly, consider the case where the SP500 series is strong-form efficient. This scenario would imply non-rejection of the semi-strong form of EMH by the TF approach and acceptance of the martingale hypothesis based on the assumption that expected

return formation is correctly specified by Samuelson's martingale. However, this scenario may also imply rejection of the martingale process if the process misspecifies equilibrium expected returns.

2. Rejection of the semi-strong form of the EMH by the TF approach is consistent with rejection of the martingale hypothesis. Under this scenario, the SP500 series is definitely inefficient in its semi-strong form, thus, the martingale property would also be rejected even though Samuelson's martingale is a correct description of the world.

3. Rejection of the semi-strong form of EMH by the TF approach is not consistent with the acceptance of the martingale hypothesis. Under this scenario, market efficiency in its semi-strong form is rejected on one hand and the joint hypothesis of market efficiency and a 'correct' martingale process is accepted on the other. Although there exists the possibility that a misspecified 'martingale model' couples with market inefficiency resulting in a constant expected return, this possibility is unlikely. Thus, generally speaking, this scenario is considered as inconsistent.

From Eqn.(19), p.96, our finding for a Thursday effect implies that the expected return from holding stock for a single period is not a constant. Thus, the martingale hypothesis of constant expected return(H3) is rejected. Our finding for a Thursday effect, in the absence of a Monday effect, is not in conformity with previous research findings. The studies conducted by Gibben & Hess(1981) and French(1980) do not suggest the existence of a Thursday effect. However, they both find a Monday effect, i.e., Monday consistently offers a lower return on average. Thus, our Thursday effect may be a statistical aberration. Acceptance of the effect may imply a Type I error, particularly, since none of the other studies have found a Thursday effect. This may warrant future research to look more specifically into the 'day of the week' effect before our finding can be justified. Also note that, the t-value associated with the W_{Th} parameter of Eqn.(19) is only marginally significant. Dropping the W_{Th} parameter from Eqn.(19) will result in a white noise series for the SP500 series. Under this scenario, H3 is not rejected.

Previous research indicates that small stocks normally show a higher risk adjusted average return. Thus, it may be possible that small stocks are more responsive to some public information than the larger stocks in the S&P500 Index, since, speculators may move more quickly to buy or sell the smaller stocks than the less risky stocks in the S&P500 Index. It follows that it is possible for the existence of a transition period before the information can be 'transmitted' to the SP500 series. Thus, information affecting the NYAM series(including small stocks) may be used to forecast the SP500 series. As a result, we attempt to use the NYAM series as input to predict the output SP500 series by a TF approach. If a 0-order TF with input lead is identified, it suggests that there is a possible transition period for information embedded in the publicly available NYAM series to be reflected in the SP500 series. Thus, the semi-strong form of the EMH is rejected(H1). If a first order TF can be identified(with or without input lead), it suggests that the SP500 series does not reflect fully and instantaneously to the information embedded in the publicly available NYAM series.[33] Thus, the semi-strong form of the EMH can be rejected(H2). However, note that, the significance of the TF parameter(W_0) may be a statistical aberration and there is always the possibility of making a Type I error. Thus, a highly significant TF parameter may suggest a higher likelihood for a causal relationship.

Eqn. (18), p.108, is the adequate TF model for the assessment of the Standard & Poor 500 Index's daily return. It is a 0-order TF with no input lead and suggests non-rejection of the semi-strong form of the EMH(H1). However, from Eqn.(19), the identification of a white noise process, with slight evidence of a Thursday effect, for the univariate SP500 series leads to the rejection of the martingale hypothesis. Samuelson (1965), when deriving the martingale property, assumes a constant and exogenously given rate of interest which might not be a reasonable real world assumption. Thus, the rejection of the martingale hypothesis(H3) may result from the expected return generating process being misspecified by the martingale property. Therefore, rejection of the martingale hypothesis cannot be used solely to reject market efficiency. However, note that, one might argue that day to day interest rate fluctuations

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[33]Refer to Eqn.(P5), p.17.

may be minimal so that the daily riskless interest rate is approximately constant through time. Under this scenario of a constant daily interest rate, non-conformity to the martingale property may imply market inefficiency. However, without definite knowledge of whether the daily interest rate that prevails in the market is truly constant through time, rejection of the martingale property may not provide any implication about market inefficiency.

The results suggest that the semi-strong form of EMH is not rejected (H1) via the identification of a zero order TF model. However, the martingale hypothesis (H3) is rejected. As previously mentioned, non-rejection of the EMH by the TF is consistent with rejection of the martingale hypothesis. Thus, our findings do not provide us with inconsistent results. These results indicate that the Standard & Poor 500 Index is efficient with respect to the New York & American Stock Exchange Combined Index, however, it does not prevent the SP500 series from being inefficient with respect to other publicly available input series. Thus, non-rejection of the semi-strong form of the EMH does not necessarily imply its acceptance.

Table 1

Estimated parameter values and their corresponding t-values of Eqn. (5).

Eqn. (5): $SP500_t = W_M I_{M,t} + W_T I_{T,t} + W_W I_{W,t} + W_{Th} I_{Th,t} + W_F I_{F,t} + NOISE$

PARAMETER	ESTIMATED VALUE	t-VALUE
W_M	-0.0014	1.28
W_T	0.0005	0.42
W_W	-0.0012	1.10
W_{Th}	0.0020	1.78
W_F	0.0002	0.17

Table 2

Estimated parameter values and their corresponding t-values of Eqn. (6).

Eqn. (6): $NYAM_t = W_M I_{M,t} + W_T I_{T,t} + W_W I_{W,t} + W_{Th} I_{Th,t} + W_F I_{F,t} + NOISE$

PARAMETER	ESTIMATED VALUE	t-VALUE
W_M	-0.0015	1.47
W_T	0.0004	0.35
W_W	-0.0011	1.04
W_{Th}	0.0016	1.61
W_F	0.0005	0.54

Table 3

Estimated parameter values and their corresponding t-values of Eqn. (11).

Eqn. (11): $SP500_t = W_0 NYAM_t + W_1 NYAM_{t-3} + W_{Th} I_{Th,t} + a_t/(1-\phi_1 B)$

PARAMETER	ESTIMATED VALUE	t-VALUE
W_0	1.0996	146.43
W_1	-0.0222	3.06
W_{Th}	0.00007	0.59
ϕ_1	-0.0673	9.86

Table 4

Estimated parameter values and their corresponding t-values of Eqn. (18).

Eqn. (18): $SP500_t = W_0 NYAM_t + W_1 NYAM_{t-3} + a_t/(1-\phi_1 B)$

PARAMETER	ESTIMATED VALUE	t-VALUE
W_0	1.0995	146.57
W_1	-0.0255	3.51
ϕ_1	-0.0676	9.9

Table 5

Estimated parameter values and their corresponding t-values of Eqn. (5) -- ignoring missing observations.

Eqn. (5): $SP500_t = W_M I_{M,t} + W_T I_{T,t} + W_W I_{W,t} + W_{Th} I_{Th,t} + W_F I_{F,t} + NOISE$

PARAMETER	ESTIMATED VALUE	t-VALUE
W_M	-0.0015	1.30
W_T	0.0005	0.41
W_W	-0.0012	1.09
W_{Th}	0.0020	1.77
W_F	0.0002	0.17

Table 6

Ex-post forecast vs. actual values of the SP500 series from Dec. 10 to Dec. 21, 1984 using Eqn. (18).

Eqn. (18): $SP500_t = W_0 NYAM_t + W_1 NYAM_{t-3} + a_t/(1-\phi_1 B)$

DATE	EX-POST FORECAST	ACTUAL VALUE
DEC. 10	0.0035	0.0035
11	0.0020	0.0015
12	-0.0026	-0.0027
13	-0.0041	-0.0050
14	0.0066	0.0054
17	0.0060	0.0057
18	0.0260	0.0275
19	-0.0063	-0.0057
20	-0.0042	-0.0047
21	-0.0045	-0.0052

FIG.1A

SP500 DAILY RETURN
FROM JAN. 3 TO DEC. 7,1984

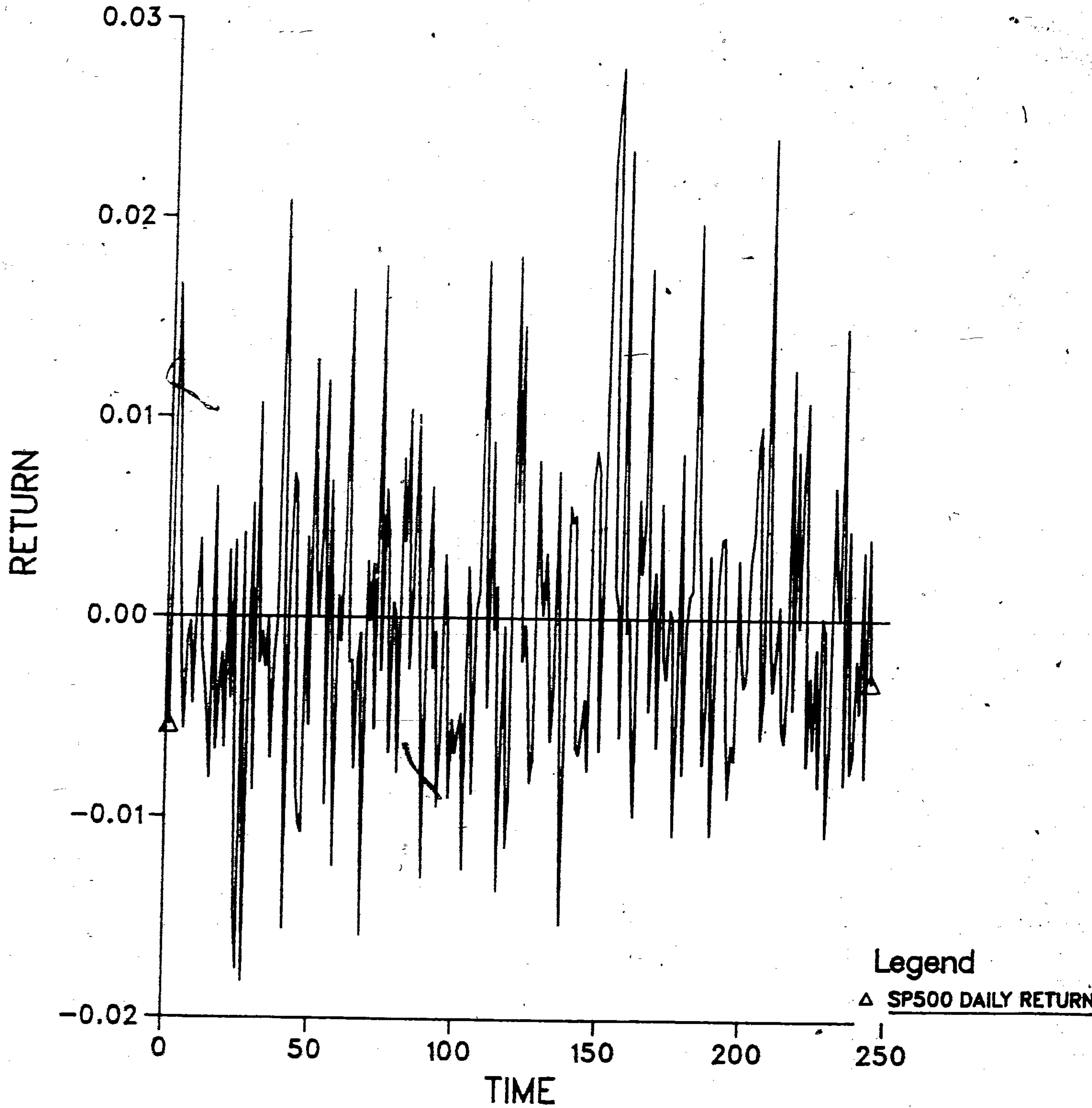


FIG.1B
NYAM DAILY RETURN
FROM JAN. 3 TO DEC. 7,1984

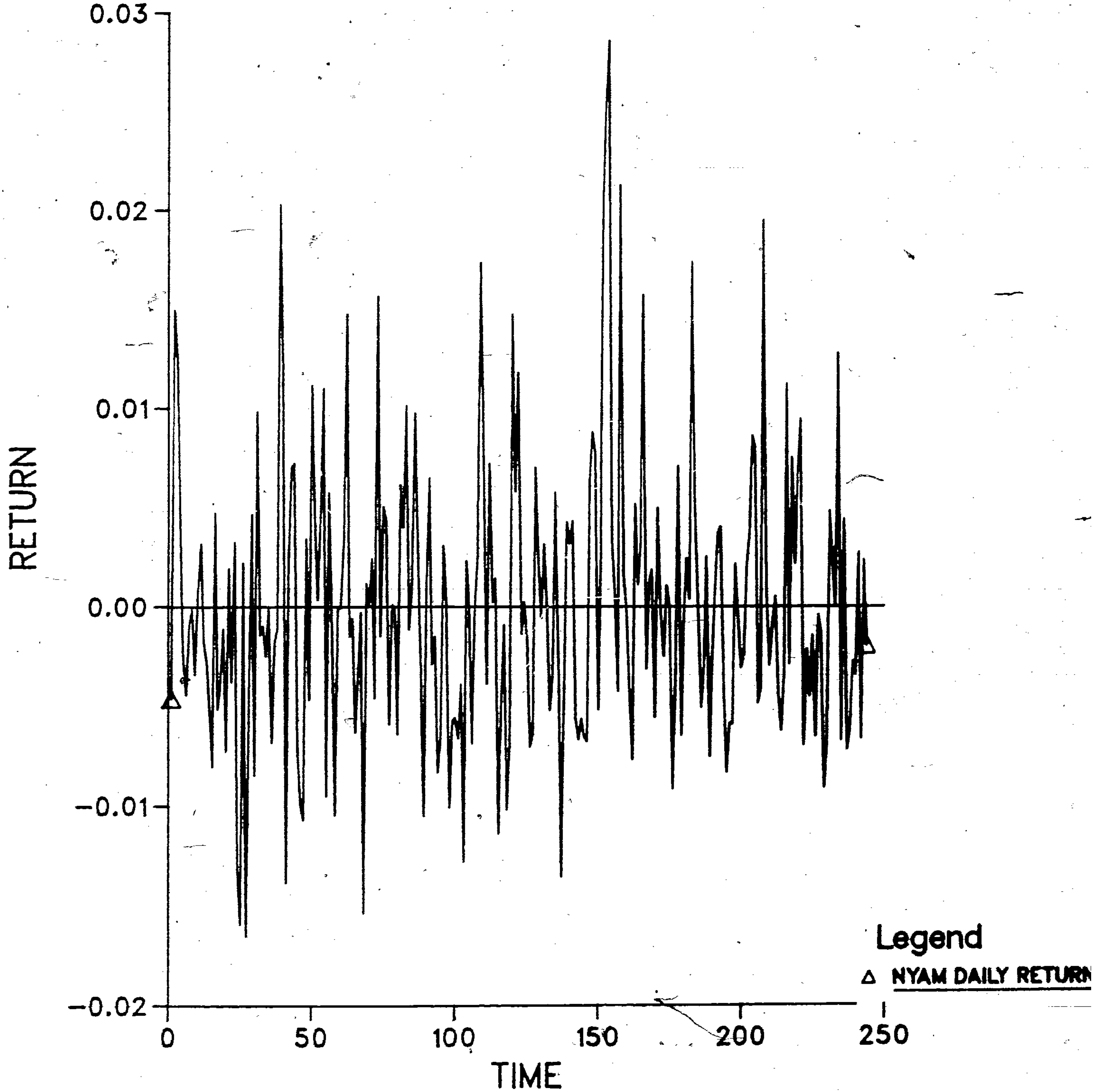


FIG.2A : ACF OF THE SP500 SERIES WITH ESTIMATION FOR MISSING OBSERVATIONS(EQN.(5))

Eqn. (5): $SP500_t = W_M I_{M,t} + W_T I_{T,t} + W_W I_{W,t} + W_{Th} I_{Th,t} + W_F I_{F,t} + NOISE$

```
             -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8 1.0
             +----+----+----+----+----+----+----+----+----+----+
                                          I
  1   0.12                             +  IXXX
  2   0.02                             +  I  +
  3   0.00                             +  I  +
  4  -0.06                             + XI  +
  5   0.08                             +  IXX+
  6  -0.06                             +XXI  +
  7  -0.10                             XXXI  +
  8   0.01                             +  I  +
  9  -0.05                             + XI  +
 10   0.03                             +  IX +
 11   0.07                             +  IXX+
 12   0.07                             +  IXX+
 13   0.10                             +  IXX+
 14  -0.04                             + XI  +
 15  -0.12                             XXXI  +
 16  -0.09                             +XXI  +
 17   0.01                             +  I  +
 18  -0.02                             +  I  +
 19   0.07                             +  IXX+
 20  -0.02                             + XI  +
 21  -0.10                             +XXI  +
 22  -0.07                             +XXI  +
 23   0.00                             +  I  +
 24  -0.03                             + XI  +
 25   0.00                             +  I  +
 26  -0.03                             + XI  +
 27   0.01                             +  I  +
 28  -0.06                             +XXI  +
 29  -0.01                             +  I  +
 30   0.07                             +  IXX+
 31   0.11                             +  IXXX
 32  -0.04                            +  XI   +
 33   0.00                            +   I   +
 34   0.01                            +   I   +
 35  -0.03                            +  XI   +
 36   0.02                            +   I   +
```

FIG.2B : PACF OF THE SP500 SERIES WITH ESTIMATION FOR MISSING OBSERVATIONS(EQN.(5))

Eqn. (5): $SP500_t = W_M I_{M,t} + W_T I_{T,t} + W_W I_{W,t} + W_{Th} I_{Th,t} + W_F I_{F,t} + NOISE$

```
            -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
              +----+----+----+----+----+----+----+----+----+----+
                                       I
  1    0.12                         +  IXXX
  2    0.00                         +  I  +
  3   -0.00                         +  I  +
  4   -0.06                         + XI  +
  5    0.09                         +  IXX+
  6   -0.08                         +XXI  +
  7   -0.09                         +XXI  +
  8    0.03                         +  IX +
  9   -0.05                         + XI  +
 10    0.03                         +  IX +
 11    0.06                         +  IXX+
 12    0.08                         +  IXX+
 13    0.06                         +  IXX+
 14   -0.06                         + XI  +
 15   -0.12                         XXXI  +
 16   -0.08                         +XXI  +
 17    0.05                         +  IX +
 18   -0.02                         + XI  +
 19    0.10                         +  IXXX
 20   -0.01                         +  I  +
 21   -0.11                         XXXI  +
 22   -0.09                         +XXI  +
 23    0.00                         +  I  +
 24   -0.07                         +XXI  +
 25    0.01                         +  I  +
 26    0.03                         +  IX +
 27    0.04                         +  IX +
 28   -0.07                         +XXI  +
 29   -0.01                         +  I  +
 30    0.03                         +  IX +
 31    0.07                         +  IXX+
 32   -0.08                         +XXI  +
 33    0.06                         +  IXX+
 34    0.05                         +  IX +
 35   -0.04                         + XI  +
 36   -0.03                         + XI  +
```

FIG.3A : ACF OF THE NYAM SERIES CONSIDERING THE DAY OF THE WEEK EFFECT(EQN.(6))

Eqn. (6): $NYAM_t = W_M I_{M,t} + W_T I_{T,t} + W_W I_{W,t} + W_{Th} I_{Th,t} + W_F I_{F,t} + NOISE$

```
              -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                +----+----+----+----+----+----+----+----+----+----+
                                         I
  1   0.20                            +  IXX+XX
  2   0.05                            +  IX +
  3   0.03                            +  IX +
  4  -0.02                            + XI  +
  5   0.08                            +  IXX+
  6  -0.05                            + XI  +
  7  -0.11                            XXXI  +
  8   0.00                            +  I  +
  9  -0.05                            + XI  +
 10   0.01                            +  I  +
 11   0.07                            +  IXX+
 12   0.07                            +  IXX+
 13   0.09                            +  IXX+
 14  -0.03                            + XI  +
 15  -0.14                            XXXI  +
 16  -0.11                            XXXI  +
 17  -0.01                            +  I  +
 18  -0.03                            + XI  +
 19   0.06                            +  IX +
 20  -0.03                            + XI  +
 21  -0.09                            +XXI  +
 22  -0.06                           +  XI   +
 23   0.00                           +   I   +
 24  -0.04                           +  XI   +
 25  -0.02                           +  XI   +
 26  -0.04                           +  XI   +
 27   0.01                           +   I   +
 28  -0.06                           + XXI   +
 29  -0.01                           +   I   +
 30   0.06                           +   IXX +
 31   0.11                           +   IXXX+
 32  -0.01                           +   I   +
 33   0.03                           +   IX  +
 34   0.02                           +   I   +
 35  -0.03                           +  XI   +
 36   0.00                           +   I   +
```

FIG.3B : PACF OF THE NYAM SERIES CONSIDERING THE DAY OF THE WEEK EFFECT(EQN.(6))

Eqn. (6): $NYAM_t = W_M I_{M,t} + W_T I_{T,t} + W_W I_{W,t} + W_{Th} I_{Th,t} + W_F I_{F,t} + NOISE$

```
             -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
               +----+----+----+----+----+----+----+----+----+----+
                                        I
  1    0.20                         +   IXX+XX
  2    0.01                         +   I   +
  3    0.02                         +   IX  +
  4   -0.03                         +  XI   +
  5    0.09                         +   IXX+
  6   -0.09                         +XXI    +
  7   -0.09                         +XXI    +
  8    0.04                         +   IX  +
  9   -0.05                         +  XI   +
 10    0.02                         +   IX  +
 11    0.07                         +   IXX+
 12    0.07                         +   IXX+
 13    0.04                         +   IX  +
 14   -0.06                         +XXI    +
 15   -0.14                         XXXI    +
 16   -0.08                         +XXI    +
 17    0.04                         +   IX  +
 18   -0.02                         +  XI   +
 19    0.11                         +   IXXX
 20   -0.03                         +  XI   +
 21   -0.09                         +XXI    +
 22   -0.08                         +XXI    +
 23    0.01                         +   I   +
 24   -0.08                         +XXI    +
 25    0.00                         +   I   +
 26    0.02                         +   IX  +
 27    0.05                         +   IX  +
 28   -0.07                         +XXI    +
 29    0.01                         +   I   +
 30    0.02                         +   I   +
 31    0.06                         +   IXX+
 32   -0.07                         +XXI    +
 33    0.08                         +   IXX+
 34    0.04                         +   IX  +
 35   -0.04                         +  XI   +
 36   -0.04                         +  XI   +
```

FIG.4A : ACF OF THE RESIDUALS OF EQN.(7)

Eqn. (7): $(1-\phi_1 B)NYAM_t = W_{Th}I_{Th,t} + a_t$

```
                 -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                   +----+----+----+----+----+----+----+----+----+----+
                                            I
  1    0.00                             +   I  +
  2    0.00                             +   I  +
  3    0.03                             +   IX +
  4   -0.05                             + XI   +
  5    0.10                             +   IXX+
  6   -0.05                             + XI   +
  7   -0.11                             XXXI   +
  8    0.04                             +   IX +
  9   -0.06                             + XI   +
 10    0.01                             +   I  +
 11    0.05                             +   IX +
 12    0.05                             +   IX +
 13    0.08                             +   IXX+
 14   -0.02                             + XI   +
 15   -0.12                             XXXI   +
 16   -0.09                             +XXI   +
 17   -0.02                             +   I  +
 18   -0.04                             + XI   +
 19    0.08                             +   IXX+
 20   -0.03                             + XI   +
 21   -0.07                             +XXI   +
 22   -0.04                             + XI   +
 23    0.02                             +   I  +
 24   -0.04                             + XI   +
 25   -0.01                             +   I  +
 26   -0.05                             + XI   +
 27    0.03                             +   IX +
 28   -0.06                             +XXI   +
 29   -0.02                             +   I  +
 30    0.05                             +   IX +
 31    0.11                             +   IXXX
 32   -0.05                             + XI   +
 33    0.03                             +   IX +
 34    0.02                             +   IX +
 35   -0.04                             + XI   +
 36    0.01                             +   I  +
```

FIG.4B: PACF OF THE RESIDUALS OF EQN.(7)

Eqn. (7): $(1-\phi_1 B)NYAM_t = W_{Th}I_{Th,t} + a_t$

```
L
          -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
            +----+----+----+----+----+----+----+----+----+----+
                                     I
  1   0.00                        +  I  +
  2   0.00                        +  I  +
  3   0.03                        +  IX +
  4  -0.05                        + XI  +
  5   0.10                        +  IXX+
  6  -0.05                        + XI  +
  7  -0.11                        XXXI  +
  8   0.03                        +  IX +
  9  -0.04                        + XI  +
 10  -0.00                        +  I  +
 11   0.05                        +  IX +
 12   0.07                        +  IXX+
 13   0.06                        +  IXX+
 14  -0.03                        + XI  +
 15  -0.12                        XXXI  +
 16  -0.11                        XXXI  +
 17   0.03                        +  IX +
 18  -0.04                        + XI  +
 19   0.10                        +  IXXX
 20   0.01                        +  I  +
 21  -0.07                        +XXI  +
 22  -0.10                        +XXI  +
 23   0.01                        +  I  +
 24  -0.08                        +XXI  +
 25  -0.02                        +  I  +
 26   0.00                        +  I  +
 27   0.07                        +  IXX+
 28  -0.06                        + XI  +
 29  -0.01                        +  I  +
 30   0.01                        +  I  +
 31   0.08                        +  IXX+
 32  -0.07                        +XXI  +
 33   0.06                        +  IX +
 34   0.06                        +  IXX+
 35  -0.03                        + XI  +
 36  -0.03                        + XI  +
```

FIG.4C : CCF BETWEEN THE PREWHITENED INPUT SERIES(EQN.7) AND THE OUTPUT SERIES(EQN.8)

Eqn. (7): $(1-\phi_1 B)NYAM_t = W_{Th} I_{Th,t} + a_t$

Eqn. (8): $(1-\phi_1 B)SP500_t = W_{Th} I_{Th,t} + a_t$

```
            -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
              +----+----+----+----+----+----+----+----+----+----+
                                       I
 -36    0.02                        +  IX +
 -35   -0.04                        + XI  +
 -34    0.02                        +  I  +
 -33    0.02                        +  I  +
 -32   -0.05                        + XI  +
 -31    0.11                        +  IXXX
 -30    0.06                        +  IX +
 -29   -0.02                        +  I  +
 -28   -0.07                        +XXI  +
 -27    0.02                        +  IX +
 -26   -0.04                        + XI  +
 -25    0.00                        +  I  +
 -24   -0.04                        + XI  +
 -23    0.01                        +  I  +
 -22   -0.04                        + XI  +
 -21   -0.08                        +XXI  +
 -20   -0.03                        + XI  +
 -19    0.08                        +  IXX+
 -18   -0.04                        + XI  +
 -17    0.03                        +  IX +
 -16   -0.08                        +XXI  +
 -15   -0.12                        XXXI  +
 -14   -0.03                        + XI  +
 -13    0.09                        +  IXX+
 -12    0.06                        +  IXX+
 -11    0.05                        +  IX +
 -10    0.01                        +  I  +
  -9   -0.06                        +XXI  +
  -8    0.02                        +  IX +
  -7   -0.10                        XXXI  +
  -6   -0.05                        + XI  +
  -5    0.09                        +  IXX+
  -4   -0.05                        + XI  +
  -3    0.01                        +  I  +
  -2    0.01                        +  I  +
  -1    0.01                        +  I  +
   0    0.99                        +  IXX+XXXXXXXXXXXXXXXXXXXXXX
   1   -0.00                        +  I  +
   2   -0.00                        +  I  +
   3    0.03                        +  IX +
   4   -0.06                        + XI  +
```

```
 5   0.09      +  IXX+
 6  -0.05      + XI  +
 7  -0.10      XXXI  +
 8   0.04      +  IX +
 9  -0.05      + XI  +
10   0.01      +  I  +
11   0.06      +  IX +
12   0.04      +  IX +
13   0.09      +  IXX+
14  -0.02      +  I  +
15  -0.13      XXXI  +
16  -0.08      +XXI  +
17   0.02      +  I  +
18  -0.03      + XI  +
19   0.08      +  IXX+
20  -0.04      + XI  +
21  -0.08      +XXI  +
22  -0.05      + XI  +
23   0.02      +  I  +
24  -0.03      + XI  +
25  -0.02      +  I  +
26  -0.04      + XI  +
27   0.03      +  IX +
28  -0.05      + XI  +
29  -0.02      +  I  +
30   0.05      +  IX +
31   0.10      +  IXXX
32  -0.05      + XI  +
33   0.03      +  IX +
34   0.02      +  I  +
35  -0.04      + XI  +
36   0.02      +  IX +
```

FIG.5A: ACF OF THE RESIDUALS OF EQN.(9)

Eqn. (9): $SP500_t = W_0 NYAM_t + W_{Th} I_{Th,t} + NOISE$

```
           -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
            +----+----+----+----+----+----+----+----+----+----+
                                     I
  1   0.03                        +  IX +
  2   0.12                        +  IXXX
  3  -0.00                        +  I  +
  4   0.00                        +  I  +
  5   0.10                        +  IXX+
  6   0.08                        +  IXX+
  7   0.01                        +  I  +
  8   0.01                        +  I  +
  9  -0.04                        + XI  +
 10  -0.05                        + XI  +
 11  -0.14                       X+XXI  +
 12  -0.10                        XXXI  +
 13   0.08                        +  IXX+
 14  -0.11                        XXXI  +
 15  -0.10                        +XXI  +
 16  -0.11                        XXXI  +
 17  -0.02                        + XI  +
 18  -0.05                        + XI  +
 19   0.00                        +  I  +
 20   0.02                        +  IX +
 21  -0.05                        + XI  +
 22   0.01                        +  I  +
 23   0.04                        +  IX +
 24  -0.05                        + XI  +
 25   0.03                        +  IX +
 26   0.02                        +  IX +
 27  -0.02                        + XI  +
 28  -0.04                        + XI  +
 29  -0.07                        +XXI  +
 30   0.03                       +   IX  +
 31   0.05                       +   IX  +
 32  -0.02                       +   I   +
 33  -0.02                       +   I   +
 34  -0.02                       +  XI   +
 35   0.01                       +   I   +
 36   0.07                       +   IXX +
```

FIG.5B: PACF OF THE RESIDUALS OF EQN.(9)

Eqn. (9): $SP500_t = W_0 NYAM_t + W_{Th} I_{Th,t} + NOISE.$

```
          -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
            +----+----+----+----+----+----+----+----+----+----+
                                     I
  1    0.03                       +  IX +
  2    0.12                       +  IXXX
  3   -0.01                       +  I  +
  4   -0.01                       +  I  +
  5    0.10                       +  IXXX
  6    0.08                       +  IXX+
  7   -0.02                       + XI  +
  8   -0.01                       +  I  +
  9   -0.04                       + XI  +
 10   -0.05                       + XI  +
 11   -0.15                      X+XXI  +
 12   -0.10                       +XXI  +
 13    0.13                       +  IXXX
 14   -0.09                       +XXI  +
 15   -0.12                       XXXI  +
 16   -0.05                       + XI  +
 17    0.06                       +  IXX+
 18   -0.05                       + XI  +
 19   -0.01                       +  I  +
 20    0.07                       +  IXX+
 21   -0.03                       + XI  +
 22   -0.03                       + XI  +
 23    0.03                       +  IX +
 24   -0.03                       + XI  +
 25   -0.01                       +  I  +
 26   -0.04                       + XI  +
 27   -0.05                       + XI  +
 28   -0.05                       + XI  +
 29   -0.07                       +XXI  +
 30    0.01                       +  I  +
 31    0.07                       +  IXX+
 32   -0.05                       + XI  +
 33   -0.05                       + XI  +
 34    0.04                       +  IX +
 35    0.03                       +  IX +
 36    0.03                       +  IX +
```

FIG.6A: ACF OF THE RESIDUALS OF EQN.(10)

Eqn. (10): $SP500_t = W_0 NYAM_t + W_{Th} I_{Th,t} + a_t/(1-\phi_1 B)$

```
              -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                +----+----+----+----+----+----+----+----+----+----+
                                         I
  1   0.10                            +  IXX+
  2   0.04                            +  IX +
  3  -0.07                            +XXI  +
  4  -0.05                            + XI  +
  5  -0.01                            +  I  +
  6   0.09                            +  IXX+
  7  -0.02                            + XI  +
  8   0.01                            +  I  +
  9   0.03                            +  IX +
 10  -0.03                            + XI  +
 11  -0.10                            +XXI  +
 12  -0.12                            XXXI  +
 13  -0.01                            +  I  +
 14  -0.07                            +XXI  +
 15  -0.06                            + XI  +
 16  -0.08                            +XXI  +
 17  -0.04                            + XI  +
 18  -0.05                            + XI  +
 19   0.06                            +  IX +
 20   0.04                            +  IX +
 21  -0.07                            +XXI  +
 22   0.01                            +  I  +
 23   0.05                            +  IX +
 24   0.00                            +  I  +
 25   0.10                            +  IXX+
 26  -0.05                            + XI  +
 27   0.00                            +  I  +
 28  -0.02                            +  I  +
 29   0.03                            +  IX +
 30   0.05                            +  IX +
 31  -0.06                            +XXI  +
 32  -0.10                            XXXI  +
 33  -0.06                            + XI  +
 34  -0.10                            +XXI  +
 35   0.03                           +   IX  +
 36  -0.00                           +   I   +
```

FIG.6B: PACF OF THE RESIDUALS OF EQN.(10)

Eqn. (10): $SP500_t = W_0 NYAM_t + W_{Th} I_{Th,t} + a_t/(1-\phi_1 B)$

```
           -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
             +----+----+----+----+----+----+----+----+----+----+
                                      I
   1   0.10                        +  IXX+
   2   0.03                        +  IX +
   3  -0.08                        +XXI  +
   4  -0.04                        + XI  +
   5   0.01                        +  I  +
   6   0.09                        +  IXX+
   7  -0.05                        + XI  +
   8   0.01                        +  I  +
   9   0.05                        +  IX +
  10  -0.04                        + XI  +
  11  -0.10                        +XXI  +
  12  -0.11                        XXXI  +
  13   0.02                        +  IX +
  14  -0.08                        +XXI  +
  15  -0.08                        +XXI  +
  16  -0.07                        +XXI  +
  17  -0.01                        +  I  +
  18  -0.05                        + XI  +
  19   0.04                        +  IX +
  20   0.05                        +  IX +
  21  -0.09                        +XXI  +
  22   0.01                        +  I  +
  23   0.05                        +  IX +
  24  -0.02                        +  I  +
  25   0.07                        +  IXX+
  26  -0.10                        +XXI  +
  27   0.00                        +  I  +
  28  -0.05                        + XI  +
  29  -0.00                        +  I  +
  30   0.05                        +  IX +
  31  -0.11                        XXXI  +
  32  -0.11                        XXXI  +
  33  -0.05                        + XI  +
  34  -0.08                        +XXI  +
  35   0.04                        +  IX +
  36  -0.03                        + XI  +
```

FIG.6C: CCF BETWEEN THE PREWHITENED INPUT SERIES(EQN 7) AND THE WHITE NOISE RESIDUALS OF EQN.(10)

Eqn. (7): $(1-\phi_1 B)NYAM_t = W_{Th}I_{Th,t} + a_t$

Eqn. (10): $SP500_t = W_0 NYAM_t + W_{Th}I_{Th,t} + a_t/(1-\phi_1 B)$

```
             -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
               +----+----+----+----+----+----+----+----+----+----+
                                        I
-36   0.08                           +  IXX+
-35  -0.05                           + XI  +
-34  -0.03                           + XI  +
-33  -0.04                           + XI  +
-32  -0.02                           +  I  +
-31  -0.03                           + XI  +
-30  -0.02                           +  I  +
-29  -0.01                           +  I  +
-28   0.07                           +  IXX+
-27   0.01                           +  I  +
-26   0.10                           +  IXXX
-25  -0.07                           +XXI  +
-24   0.08                           +  IXX+
-23   0.01                           +  I  +
-22  -0.07                           +XXI  +
-21  -0.04                           + XI  +
-20  -0.09                           +XXI  +
-19   0.06                           +  IXX+
-18   0.13                           +  IXXX
-17   0.03                           +  IX +
-16   0.02                           +  IX +
-15  -0.04                           + XI  +
-14   0.04                           +  IX +
-13   0.11                           +  IXXX
-12  -0.03                           + XI  +
-11   0.03                           +  IX +
-10   0.03                           +  IX +
 -9   0.07                           +  IXX+
 -8   0.05                           +  IX +
 -7   0.07                           +  IXX+
 -6  -0.00                           +  I  +
 -5  -0.05                           + XI  +
 -4  -0.09                           +XXI  +
 -3  -0.05                           + XI  +
 -2  -0.04                           + XI  +
 -1  -0.06                           +XXI  +
  0   0.00                           +  I  +
  1   0.02                           +  I  +
  2  -0.06                           +XXI  +
  3  -0.18                          X+XXI  +
```

```
 4   -0.10     +XXI  +
 5   -0.10     XXXI  +
 6   -0.07     +XXI  +
 7    0.00     +  I  +
 8   -0.11     XXXI  +
 9   -0.07     +XXI  +
10    0.06     +  IXX+
11    0.02     +  I  +
12    0.17     +  IXX+X
13    0.09     +  IXX+
14   -0.08     +XXI  +
15   -0.02     +  I  +
16    0.01     +  I  +
17    0.12     +  IXXX
18    0.02     +  I  +
19    0.01     +  I  +
20    0.03     +  IX +
21   -0.08     +XXI  +
22   -0.03     + XI  +
23   -0.03     + XI  +
24    0.07     +  IXX+
25    0.11     +  IXXX
26    0.06     +  IX +
27   -0.08     +XXI  +
28   -0.03     + XI  +
29   -0.00     +  I  +
30    0.08     +  IXX+
31    0.10     +  IXX+
32   -0.06     +XXI  +
33   -0.12     XXXI  +
34   -0.06     + XI  +
35   -0.09     +XXI  +
36    0.07     +  IXX+
```

FIG.7A: ACF OF THE RESIDUALS OF EQN.(11)

Eqn. (11): $SP500_t = W_0 NYAM_t + W_1 NYAM_{t-3} + W_{Th} I_{Th,t} + a_t/(1-\phi_1 B)$

```
                -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                  +----+----+----+----+----+----+----+----+----+----+
                                           I
  1    0.07                             +  IXX+
  2    0.03                             +  IX +
  3   -0.09                             +XXI  +
  4   -0.07                             +XXI  +
  5   -0.04                             + XI  +
  6    0.07                             +  IXX+
  7   -0.04                             + XI  +
  8    0.01                             +  I  +
  9    0.06                             +  IXX+
 10   -0.01                             +  I  +
 11   -0.10                             XXXI  +
 12   -0.11                             XXXI  +
 13    0.01                             +  I  +
 14   -0.04                             + XI  +
 15   -0.06                             + XI  +
 16   -0.07                             +XXI  +
 17   -0.02                             + XI  +
 18   -0.07                             +XXI  +
 19    0.06                             +  IX +
 20    0.04                             +  IX +
 21   -0.03                             + XI  +
 22    0.04                             +  IX +
 23    0.05                             +  IX +
 24   -0.03                             + XI  +
 25    0.08                             +  IXX+
 26   -0.05                             + XI  +
 27    0.04                             +  IX +
 28   -0.02                             +  I  +
 29    0.03                             +  IX +
 30    0.03                             +  IX +
 31   -0.06                             + XI  +
 32   -0.12                             XXXI  +
 33   -0.05                             + XI  +
 34   -0.12                             XXXI  +
 35    0.02                            +   IX   +
 36   -0.00                            +   I    +
```

FIG.7B: PACF OF THE RESIDUALS OF EQN.(11)

Eqn. (11): $SP500_t = W_0 NYAM_t + W_1 NYAM_{t-3} + W_{Th} I_{Th,t} + a_t/(1-\phi_1 B)$

```
         -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
           +----+----+----+----+----+----+----+----+----+----+
                                    I
 1   0.07                       +  IXX+
 2   0.02                       +  IX +
 3  -0.09                       +XXI  +
 4  -0.06                       + XI  +
 5  -0.03                       + XI  +
 6   0.07                       +  IXX+
 7  -0.06                       + XI  +
 8   0.00                       +  I  +
 9   0.08                       +  IXX+
10  -0.02                       + XI  +
11  -0.11                       XXXI  +
12  -0.10                       +XXI  +
13   0.04                       +  IX +
14  -0.06                       + XI  +
15  -0.11                       XXXI  +
16  -0.07                       +XXI  +
17  -0.00                       +  I  +
18  -0.10                       +XXI  +
19   0.03                       +  IX +
20   0.05                       +  IX +
21  -0.04                       + XI  +
22   0.02                       +  IX +
23   0.04                       +  IX +
24  -0.02                       +  I  +
25   0.07                       +  IXX+
26  -0.09                       +XXI  +
27   0.04                       +  IX +
28  -0.04                       + XI  +
29  -0.01                       +  I  +
30   0.03                       +  IX +
31  -0.08                       +XXI  +
32  -0.13                       XXXI  +
33  -0.06                       + XI  +
34  -0.12                       XXXI  +
35   0.02                       +  IX +
36  -0.04                       + XI  +
```

FIG.7C: CCF BETWEEN THE PREWHITENED INPUT SERIES(EQN 7) AND THE WHITE NOISE RESIDUALS OF EQN.(11)

Eqn. (7): $(1-\phi_1 B)NYAM_t = W_{Th}I_{Th,t} + a_t$

Eqn. (11): $SP500_t = W_0 NYAM_t + W_1 NYAM_{t-3} + W_{Th}I_{Th,t} + a_t/(1-\phi_1 B)$

```
             -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
               +----+----+----+----+----+----+----+----+----+----+
                                        I
 -36   0.09                        +   IXX+
 -35  -0.04                        + XI   +
 -34  -0.04                        + XI   +
 -33  -0.04                        + XI   +
 -32  -0.03                        + XI   +
 -31  -0.02                        + XI   +
 -30  -0.01                        +  I   +
 -29  -0.02                        +  I   +
 -28   0.10                        +   IXX+
 -27   0.02                        +  I   +
 -26   0.11                        +   IXXX
 -25  -0.09                        +XXI   +
 -24   0.09                        +   IXX+
 -23   0.01                        +  I   +
 -22  -0.08                        +XXI   +
 -21  -0.05                        + XI   +
 -20  -0.09                        +XXI   +
 -19   0.05                        +   IX +
 -18   0.12                        +   IXXX
 -17   0.02                        +   IX +
 -16   0.04                        +   IX +
 -15  -0.04                        + XI   +
 -14   0.05                        +   IX +
 -13   0.10                        +   IXXX
 -12  -0.06                        +XXI   +
 -11   0.02                        +   IX +
 -10   0.05                        +   IX +
  -9   0.08                        +   IXX+
  -8   0.06                        +   IXX+
  -7   0.07                        +   IXX+
  -6  -0.02                        +  I   +
  -5  -0.05                        + XI   +
  -4  -0.12                        XXXI   +
  -3  -0.07                        +XXI   +
  -2  -0.03                        + XI   +
  -1  -0.07                        +XXI   +
   0   0.00                        +  I   +
   1   0.02                        +  I   +
   2  -0.07                        +XXI   +
   3  -0.00                        +  I   +
```

```
 4  -0.05          + XI   +
 5  -0.10          +XXI   +
 6  -0.06          +XXI   +
 7   0.00          +  I   +
 8  -0.10          XXXI   +
 9  -0.08          +XXI   +
10   0.03          +  IX  +
11   0.02          +  I   +
12   0.16          +  IXX+X
13   0.08          +  IXX+
14  -0.08          +XXI   +
15   0.00          +  I   +
16   0.04          +  IX  +
17   0.13          +  IXXX
18  -0.03          + XI   +
19  -0.03          + XI   +
20   0.03          +  IX  +
21  -0.09          +XXI   +
22  -0.01          +  I   +
23  -0.03          + XI   +
24   0.05          +  IX  +
25   0.10          +  IXXX
26   0.05          +  IX  +
27  -0.10          XXXI   +
28  -0.05          + XI   +
29  -0.01          +  I   +
30   0.08          +  IXX+
31   0.08          +  IXX+
32  -0.08          +XXI   +
33  -0.12          XXXI   +
34  -0.04          + XI   +
35  -0.10          XXXI   +
36   0.08          +  IXX+
```

FIG.8A: ACF OF THE RESIDUALS OF EQN.(12)

Eqn.(12): $SP500_t = a_t$

```
             -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
               +----+----+----+----+----+----+----+----+----+----+
                                        I
   1    0.12                        +  IXXX
   2    0.04                        +  IX +
   3   -0.02                        + XI  +
   4   -0.07                        +XXI  +
   5    0.09                        +  IXX+
   6   -0.06                        +XXI  +
   7   -0.08                        +XXI  +
   8   -0.02                        + XI  +
   9   -0.06                        +XXI  +
  10    0.05                        +  IX +
  11    0.09                        +  IXX+
  12    0.06                        +  IXX+
  13    0.07                        +  IXX+
  14   -0.06                        + XI  +
  15   -0.08                        +XXI  +
  16   -0.07                        +XXI  +
  17    0.00                        +  I  +
  18   -0.02                        +  I  +
  19    0.05                        +  IX +
  20   -0.04                        + XI  +
  21   -0.10                        XXXI  +
  22   -0.08                        +XXI  +
  23    0.03                        +  IX +
  24   -0.01                        +  I  +
  25    0.00                        +  I  +
  26   -0.04                        + XI  +
  27   -0.03                        + XI  +
  28    0.00                        +  I  +
  29    0.02                        +  IX +
  30    0.11                        +  IXXX
  31    0.00                       +   I   +
  32   -0.03                       +  XI   +
  33   -0.00                       +   I   +
  34   -0.01                       +   I   +
  35   -0.02                       +  XI   +
  36   -0.02                       +  XI   +
```

FIG.8B: PACF OF THE RESIDUALS OF EQN.(12)

Eqn.(12): $SP500_t = a_t$

```
          -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
            +----+----+----+----+----+----+----+----+----+----+
                                     I
 1   0.12                         +  IXXX
 2   0.02                         +  IX +
 3  -0.03                         + XI  +
 4  -0.06                         +XXI  +
 5   0.11                         +  IXXX
 6  -0.08                         +XXI  +
 7  -0.07                         +XXI  +
 8  -0.00                         +  I  +
 9  -0.04                         + XI  +
10   0.04                         +  IX +
11   0.09                         +  IXX+
12   0.05                         +  IX +
13   0.04                         +  IX +
14  -0.06                         +XXI  +
15  -0.08                         +XXI  +
16  -0.06                         + XI  +
17   0.03                         +  IX +
18  -0.02                         + XI  +
19   0.08                         +  IXX+
20  -0.03                         + XI  +
21  -0.11                         XXXI  +
22  -0.09                         +XXI  +
23   0.04                         +  IX +
24  -0.05                         + XI  +
25   0.01                         +  I  +
26   0.00                         +  I  +
27  -0.00                         +  I  +
28  -0.02                         +  I  +
29   0.02                         +  I  +
30   0.08                         +  IXX+
31  -0.03                         + XI  +
32  -0.02                         +  I  +
33   0.04                         +  IX +
34   0.02                         +  I  +
35  -0.05                         + XI  +
36  -0.04                         + XI  +
```

FIG.9A: ACF OF THE RESIDUALS OF EQN.(13)

Eqn.(13): $NYAM_t$ = NOISE

```
                  -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                    +----+----+----+----+----+----+----+----+----+----+
                                             I
   1   0.20                               +  IXX+XX
   2   0.06                               +  IX +
   3   0.00                               +  I  +
   4  -0.04                               + XI  +
   5   0.09                               +  IXX+
   6  -0.05                               + XI  +
   7  -0.08                               +XXI  +
   8  -0.03                               + XI  +
   9  -0.06                               + XI  +
  10   0.04                               +  IX +
  11   0.08                               +  IXX+
  12   0.06                               +  IXX+
  13   0.04                               +  IX +
  14  -0.05                               + XI  +
  15  -0.09                               +XXI  +
  16  -0.08                               +XXI  +
  17  -0.02                               + XI  +
  18  -0.04                               + XI  +
  19   0.05                               +  IX +
  20  -0.03                               + XI  +
  21  -0.10                               XXXI  +
  22  -0.07                              + XXI   +
  23   0.01                              +   I   +
  24  -0.02                              +  XI   +
  25  -0.01                              +   I   +
  26  -0.06                              +  XI   +
  27  -0.03                              +  XI   +
  28   0.00                              +   I   +
  29   0.03                              +   IX  +
  30   0.10                              +   IXXX+
  31   0.02                              +   I   +
  32  -0.01                              +   I   +
  33   0.01                              +   I   +
  34   0.01                              +   I   +
  35  -0.04                              +  XI   +
  36  -0.01                              +   I   +
```

FIG.9B: PACF OF THE RESIDUALS OF EQN.(13)

Eqn.(13): $NYAM_t$ = NOISE

```
          -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
            +----+----+----+----+----+----+----+----+----+----+
                                     I
  1   0.20                        +  IXX+XX
  2   0.02                        +  I  +
  3  -0.01                        +  I  +
  4  -0.04                        + XI  +
  5   0.11                        +  IXXX
  6  -0.09                        +XXI  +
  7  -0.07                        +XXI  +
  8   0.00                        +  I  +
  9  -0.04                        + XI  +
 10   0.04                        +  IX +
 11   0.08                        +  IXX+
 12   0.04                        +  IX +
 13   0.01                        +  I  +
 14  -0.06                        +XXI  +
 15  -0.08                        +XXI  +
 16  -0.07                        +XXI  +
 17   0.02                        +  I  +
 18  -0.03                        + XI  +
 19   0.10                        +  IXX+
 20  -0.04                        + XI  +
 21  -0.11                        XXXI  +
 22  -0.06                        +XXI  +
 23   0.03                        +  IX +
 24  -0.07                        +XXI  +
 25   0.02                        +  I  +
 26  -0.00                        +  I  +
 27  -0.00                        +  I  +
 28  -0.00                        +  I  +
 29   0.02                        +  IX +
 30   0.06                        +  IXX+
 31  -0.03                        + XI  +
 32  -0.01                        +  I  +
 33   0.03                        +  IX +
 34   0.02                        +  I  +
 35  -0.07                        +XXI  +
 36  -0.01                        +  I  +
```

FIG.10A: ACF OF THE RESIDUALS OF EQN.(14)

Eqn.(14): $(1-\phi_1 B)NYAM_t = a_t$

```
              -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                +----+----+----+----+----+----+----+----+----+----+
                                         I
  1   0.00                            +  I  +
  2   0.02                            +  IX +
  3  -0.00                            +  I  +
  4  -0.06                            +XXI  +
  5   0.12                            +  IXXX
  6  -0.05                            + XI  +
  7  -0.08                            +XXI  +
  8  -0.01                            +  I  +
  9  -0.06                            +XXI  +
 10   0.04                            +  IX +
 11   0.07                            +  IXX+
 12   0.04                            +  IX +
 13   0.04                            +  IX +
 14  -0.05                            + XI  +
 15  -0.06                            +XXI  +
 16  -0.07                            +XXI  +
 17   0.00                            +  I  +
 18  -0.05                            + XI  +
 19   0.07                            +  IXX+
 20  -0.02                            + XI  +
 21  -0.09                            +XXI  +
 22  -0.05                            + XI  +
 23   0.03                            +  IX +
 24  -0.03                            + XI  +
 25   0.01                            +  I  +
 26  -0.06                            + XI  +
 27  -0.03                            + XI  +
 28   0.01                            +  I  +
 29   0.00                            +  I  +
 30   0.11                            +  IXXX
 31  -0.00                            +  I  +
 32  -0.02                            +  I  +
 33   0.02                            +  I  +
 34   0.01                            +  I  +
 35  -0.04                            + XI  +
 36  -0.02                            +  I  +
```

FIG.10B: PACF OF THE RESIDUALS OF EQN.(14)

Eqn.(14): $(1-\phi_1 B)NYAM_t = a_t$

```
              -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                +----+----+----+----+----+----+----+----+----+----+
                                         I
   1   0.00                           +  I  +
   2   0.02                           +  IX +
   3  -0.00                           +  I  +
   4  -0.06                           +XXI  +
   5   0.12                           +  IXXX
   6  -0.05                           + XI  +
   7  -0.08                           +XXI  +
   8  -0.00                           +  I  +
   9  -0.05                           + XI  +
  10   0.02                           +  I  +
  11   0.07                           +  IXX+
  12   0.06                           +  IX +
  13   0.03                           +  IX +
  14  -0.04                           + XI  +
  15  -0.07                           +XXI  +
  16  -0.09                           +XXI  +
  17   0.01                           +  I  +
  18  -0.04                           + XI  +
  19   0.09                           +  IXX+
  20  -0.00                           +  I  +
  21  -0.10                           +XXI  +
  22  -0.09                           +XXI  +
  23   0.02                           +  IX +
  24  -0.07                           +XXI  +
  25   0.01                           +  I  +
  26  -0.01                           +  I  +
  27  -0.00                           +  I  +
  28  -0.00                           +  I  +
  29   0.00                           +  I  +
  30   0.07                           +  IXX+
  31  -0.01                           +  I  +
  32  -0.02                           +  I  +
  33   0.02                           +  IX +
  34   0.03                           +  IX +
  35  -0.06                           +XXI  +
  36  -0.03                           + XI  +
```

FIG.10C: CCF BETWEEN THE PREWHITENED INPUT SERIES(EQN 14) AND THE OUTPUT SERIES(EQN 15)

Eqn.(14): $(1-\phi_1 B)NYAM_t = a_t$
Eqn.(15): $(1-\phi_1 B)SP500_t = a_t$

```
           -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
             +----+----+----+----+----+----+----+----+----+----+
                                      I
-36  -0.02                         + XI  +
-35  -0.03                         + XI  +
-34   0.01                         +  I  +
-33   0.01                         +  I  +
-32  -0.03                         + XI  +
-31  -0.01                         +  I  +
-30   0.12                         +  IXXX
-29   0.01                         +  I  +
-28   0.00                         +  I  +
-27  -0.03                         + XI  +
-26  -0.06                         +XXI  +
-25   0.02                         +  I  +
-24  -0.02                         +  I  +
-23   0.03                         +  IX +
-22  -0.06                         + XI  +
-21  -0.10                         XXXI  +
-20  -0.02                         + XI  +
-19   0.06                         +  IXX+
-18  -0.04                         + XI  +
-17   0.01                         +  I  +
-16  -0.07                         +XXI  +
-15  -0.06                         + XI  +
-14  -0.06                         + XI  +
-13   0.06                         +  IX +
-12   0.05                         +  IX +
-11   0.07                         +  IXX+
-10   0.04                         +  IX +
 -9  -0.07                         +XXI  +
 -8  -0.02                         +  I  +
 -7  -0.07                         +XXI  +
 -6  -0.06                         + XI  +
 -5   0.11                         +  IXXX
 -4  -0.07                         +XXI  +
 -3  -0.02                         +  I  +
 -2   0.04                         +  IX +
 -1   0.01                         +  I  +
  0   0.99                         +  IXX+XXXXXXXXXXXXXXXXXXXXXX
  1  -0.00                         +  I  +
  2   0.02                         +  I  +
  3  -0.00                         +  I  +
  4  -0.08                         +XXI  +
```

```
  5   0.12          +  IXXX
  6  -0.06          + XI  +
  7  -0.07          +XXI  +
  8   0.00          +  I  +
  9  -0.06          +XXI  +
 10   0.05          +  IX +
 11   0.07          +  IXX+
 12   0.04          +  IX +
 13   0.05          +  IX +
 14  -0.05          + XI  +
 15  -0.07          +XXI  +
 16  -0.06          +XXI  +
 17   0.01          +  I  +
 18  -0.03          + XI  +
 19   0.06          +  IX +
 20  -0.03          + XI  +
 21  -0.08          +XXI  +
 22  -0.06          +XXI  +
 23   0.03          +  IX +
 24  -0.03          + XI  +
 25   0.01          +  I  +
 26  -0.04          + XI  +
 27  -0.02          + XI  +
 28   0.01          +  I  +
 29   0.00          +  I  +
 30   0.11          +  IXXX
 31  -0.00          +  I  +
 32  -0.02          + XI  +
 33   0.01          +  I  +
 34  -0.00          +  I  +
 35  -0.02          + XI  +
 36  -0.02          + XI  +
```

FIG.11A: ACF OF THE RESIDUALS OF EQN.(16)

Eqn. (16): $SP500_t = W_0 NYAM_t + NOISE$

```
            -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
              +----+----+----+----+----+----+----+----+----+----+
                                       I
  1   0.02                          +  I  +
  2   0.14                          +  IXXX
  3   0.02                          +  IX +
  4  -0.06                          +XXI  +
  5   0.15                          +  IXX+X
  6   0.03                          +  IX +
  7   0.06                          +  IX +
  8   0.00                          +  I  +
  9  -0.08                          +XXI  +
 10  -0.05                          + XI  +
 11  -0.12                          XXXI  +
 12  -0.09                          +XXI  +
 13   0.05                          +  IX +
 14  -0.11                          XXXI  +
 15  -0.12                          XXXI  +
 16  -0.04                         +  XI   +
 17  -0.07                         + XXI   +
 18   0.02                         +   IX  +
 19  -0.09                         + XXI   +
 20   0.07                         +   IXX +
 21   0.01                         +   I   +
 22  -0.05                         +  XI   +
 23   0.03                         +   IX  +
 24  -0.04                         +  XI   +
 25   0.06                         +   IX  +
 26   0.07                         +   IXX +
 27  -0.11                         +XXXI   +
 28  -0.03                         +  XI   +
 29  -0.08                         + XXI   +
 30   0.10                         +   IXX +
 31  -0.01                         +   I   +
 32   0.01                         +   I   +
 33  -0.05                         +  XI   +
 34   0.03                         +   IX  +
 35   0.07                         +   IXX +
 36  -0.06                         +  XI   +
```

FIG.11B: PACF OF THE RESIDUALS OF EQN.(16)

Eqn. (16): $SP500_t = W_o NYAM_t + NOISE$

```
         -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
           +----+----+----+----+----+----+----+----+----+----+
                                    I
 1   0.02                        +  I  +
 2   0.14                        +  IXXX
 3   0.02                        +  I  +
 4  -0.08                        +XXI  +
 5   0.16                        +  IXX+X
 6   0.04                        +  IX +
 7   0.02                        +  I  +
 8  -0.02                        + XI  +
 9  -0.07                        +XXI  +
10  -0.07                        +XXI  +
11  -0.10                        XXXI  +
12  -0.09                        +XXI  +
13   0.08                        +  IXX+
14  -0.08                        +XXI  +
15  -0.14                        XXXI  +
16   0.02                        +  I  +
17   0.01                        +  I  +
18   0.00                        +  I  +
19  -0.09                        +XXI  +
20   0.10                        +  IXXX
21   0.04                        +  IX +
22  -0.08                        +XXI  +
23  -0.02                        +  I  +
24   0.00                        +  I  +
25   0.02                        +  IX +
26   0.00                        +  I  +
27  -0.14                       X+XXI  +
28  -0.05                        + XI  +
29  -0.05                        + XI  +
30   0.09                        +  IXX+
31  -0.04                        + XI  +
32   0.03                        +  IX +
33  -0.07                        +XXI  +
34   0.06                        +  IXX+
35   0.12                        +  IXXX
36  -0.09                        +XXI  +
```

FIG.12A: ACF OF THE RESIDUALS OF EQN.(17)

Eqn. (17): $SP500_t = W_o NYAM_t + a_t/(1-\phi_1 B)$

```
              -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                +----+----+----+----+----+----+----+----+----+----+
                                         I
  1    0.10                           +  IXXX
  2    0.03                           +  IX +
  3   -0.03                           + XI  +
  4   -0.11                           XXXI  +
  5    0.04                           +  IX +
  6    0.05                           +  IX +
  7   -0.02                           +  I  +
  8    0.01                           +  I  +
  9   -0.01                           +  I  +
 10   -0.03                           + XI  +
 11   -0.08                           +XXI  +
 12   -0.08                           +XXI  +
 13   -0.05                           + XI  +
 14   -0.05                           + XI  +
 15   -0.12                           XXXI  +
 16   -0.01                           +  I  +
 17   -0.03                           + XI  +
 18    0.03                           +  IX +
 19   -0.05                           + XI  +
 20    0.02                           +  I  +
 21   -0.02                           +  I  +
 22    0.03                           +  IX +
 23    0.02                           +  IX +
 24    0.01                           +  I  +
 25    0.04                           +  IX +
 26    0.07                           +  IXX+
 27   -0.06                           + XI  +
 28    0.03                           +  IX +
 29   -0.05                           + XI  +
 30    0.02                           +  I  +
 31   -0.08                           +XXI  +
 32   -0.06                           + XI  +
 33   -0.13                           XXXI  +
 34    0.05                          +   IX  +
 35    0.01                          +   I   +
 36   -0.03                          +  XI   +
```

FIG.12B: PACF OF THE RESIDUALS OF EQN.(17)

Eqn. (17): $SP500_t = W_0 NYAM_t + a_t/(1-\phi_1 B)$

```
           -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
            +----+----+----+----+----+----+----+----+----+----+
                                     I
  1   0.10                        +  IXXX
  2   0.02                        +  I  +
  3  -0.04                        + XI  +
  4  -0.10                        XXXI  +
  5   0.07                        +  IXX+
  6   0.04                        +  IX +
  7  -0.04                        + XI  +
  8   0.01                        +  I  +
  9   0.01                        +  I  +
 10  -0.02                        + XI  +
 11  -0.09                        +XXI  +
 12  -0.07                        +XXI  +
 13  -0.03                        + XI  +
 14  -0.05                        + XI  +
 15  -0.14                        XXXI  +
 16   0.01                        +  I  +
 17  -0.02                        + XI  +
 18   0.02                        +  I  +
 19  -0.08                        +XXI  +
 20   0.05                        +  IX +
 21  -0.02                        +  I  +
 22   0.02                        +  I  +
 23  -0.01                        +  I  +
 24   0.00                        +  I  +
 25   0.02                        +  IX +
 26   0.03                        +  IX +
 27  -0.09                        +XXI  +
 28   0.03                        +  IX +
 29  -0.06                        +XXI  +
 30   0.01                        +  I  +
 31  -0.12                        XXXI  +
 32  -0.04                        + XI  +
 33  -0.13                        XXXI  +
 34   0.06                        +  IXX+
 35  -0.02                        +  I  +
 36  -0.05                        + XI  +
```

FIG.12C: CCF BETWEEN THE PREWHITENED INPUT SERIES(EQN 14) AND THE WHITE NOISE RESIDUALS OF EQN.(17)

Eqn.(14): $(1-\phi_1 B)NYAM_t = a_t$
Eqn.(17): $SP500_t = W_0 NYAM_t + a_t/(1-\phi_1 B)$

```
          -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
            +----+----+----+----+----+----+----+----+----+----+
                                     I
-36  -0.06                        +XXI  +
-35   0.10                        +  IXXX
-34  -0.09                        +XXI  +
-33  -0.03                        + XI  +
-32  -0.06                        + XI  +
-31  -0.03                        + XI  +
-30  -0.01                        +  I  +
-29  -0.03                        + XI  +
-28  -0.01                        +  I  +
-27   0.03                        +  IX +
-26   0.16                        +  IXX+X
-25   0.00                        +  I  +
-24  -0.03                        + XI  +
-23   0.07                        +  IXX+
-22  -0.11                        XXXI  +
-21   0.05                        +  IX +
-20  -0.09                        +XXI  +
-19  -0.07                        +XXI  +
-18   0.14                        +  IXX+X
-17   0.07                        +  IXX+
-16   0.07                        +  IXX+
-15  -0.01                        +  I  +
-14  -0.01                        +  I  +
-13   0.12                        +  IXXX
-12  -0.01                        +  I  +
-11  -0.01                        +  I  +
-10   0.10                        +  IXX+
 -9   0.00                        +  I  +
 -8   0.09                        +  IXX+
 -7   0.07                        +  IXX+
 -6  -0.06                        +XXI  +
 -5  -0.01                        +  I  +
 -4  -0.14                        XXXI  +
 -3  -0.03                        + XI  +
 -2  -0.05                        + XI  +
 -1  -0.05                        + XI  +
  0   0.00                        +  I  +
  1   0.00                        +  I  +
  2  -0.02                        +  I  +
  3  -0.20                      XX+XXI  +
```

```
 4   -0.08          +XXI  +
 5   -0.08          +XXI  +
 6   -0.07          +XXI  +
 7   -0.02          +  I  +
 8   -0.12          XXXI  +
 9   -0.05          + XI  +
10    0.02          +  I  +
11    0.09          +  IXX+
12    0.10          +  IXX+
13    0.14          +  IXXX
14   -0.09          +XXI  +
15    0.01          +  I  +
16    0.03          +  IX +
17    0.08          +  IXX+
18    0.09          +  IXX+
19   -0.04          + XI  +
20    0.02          +  IX +
21   -0.12          XXXI  +
22   -0.06          + XI  +
23    0.02          +  IX +
24    0.11          +  IXXX
25    0.08          +  IXX+
26   -0.04          + XI  +
27   -0.03          + XI  +
28   -0.04          + XI  +
29    0.07          +  IXX+
30    0.11          +  IXXX
31   -0.04          + XI  +
32   -0.10          XXXI  +
33   -0.10          +XXI  +
34   -0.08          +XXI  +
35    0.06          +  IX +
36   -0.02          + XI  +
```

FIG.13A: ACF OF THE RESIDUALS OF EQN.(18)

Eqn. (18): $SP500_t = W_0 NYAM_t + W_1 NYAM_{t-3} + a_t/(1-\phi_1 B)$

```
             -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
               +----+----+----+----+----+----+----+----+----+----+
                                        I
 1   0.08                            +  IXX+
 2   0.01                            +  I  +
 3  -0.05                            + XI  +
 4  -0.13                            XXXI  +
 5   0.01                            +  I  +
 6   0.03                            +  IX +
 7  -0.04                            + XI  +
 8   0.03                            +  IX +
 9  -0.00                            +  I  +
10   0.01                            +  I  +
11  -0.08                            +XXI  +
12  -0.08                            +XXI  +
13  -0.02                            +  I  +
14  -0.03                            + XI  +
15  -0.12                            XXXI  +
16   0.00                            +  I  +
17  -0.03                            + XI  +
18  -0.00                            +  I  +
19  -0.04                            + XI  +
20   0.05                            +  IX +
21   0.04                            +  IX +
22   0.03                            +  IX +
23  -0.00                            +  I  +
24   0.00                            +  I  +
25   0.01                            +  I  +
26   0.10                            +  IXX+
27  -0.04                            + XI  +
28   0.01                            +  I  +
29  -0.05                            + XI  +
30   0.01                            +  I  +
31  -0.10                            +XXI  +
32  -0.06                            + XI  +
33  -0.15                           X+XXI  +
34   0.03                           +   IX   +
35   0.01                           +   I    +
36  -0.06                           +  XI    +
```

FIG.13B: PACF OF THE RESIDUALS OF EQN.(18)

Eqn. (18): $SP500_t = W_0 NYAM_t + W_1 NYAM_{t-3} + a_t/(1-\phi_1 B)$

```
           -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
             +----+----+----+----+----+----+----+----+----+----+
                                      I
  1   0.08                         +  IXX+
  2   0.00                         +  I  +
  3  -0.06                         + XI  +
  4  -0.13                         XXXI  +
  5   0.03                         +  IX +
  6   0.03                         +  IX +
  7  -0.06                         +XXI  +
  8   0.02                         +  IX +
  9   0.00                         +  I  +
 10   0.01                         +  I  +
 11  -0.10                         +XXI  +
 12  -0.06                         + XI  +
 13   0.00                         +  I  +
 14  -0.04                         + XI  +
 15  -0.15                        X+XXI  +
 16   0.00                         +  I  +
 17  -0.03                         + XI  +
 18  -0.03                         + XI  +
 19  -0.09                         +XXI  +
 20   0.07                         +  IXX+
 21   0.04                         +  IX +
 22  -0.01                         +  I  +
 23  -0.03                         + XI  +
 24   0.02                         +  IX +
 25   0.02                         +  IX +
 26   0.06                         +  IX +
 27  -0.08                         +XXI  +
 28   0.02                         +  IX +
 29  -0.05                         + XI  +
 30  -0.00                         +  I  +
 31  -0.13                         XXXI  +
 32  -0.04                         + XI  +
 33  -0.15                        X+XXI  +
 34   0.02                         +  IX +
 35  -0.02                         + XI  +
 36  -0.09                         +XXI  +
```

FIG.13C: CCF BETWEEN THE PREWHITENED INPUT SERIES(EQN 14) AND THE WHITE NOISE RESIDUALS OF EQN.(18)

Eqn.(14): $(1-\phi_1 B)NYAM_t = a_t$
Eqn.(18): $SP500_t = W_0 NYAM_t + W_1 NYAM_{t-3} + a_t/(1-\phi_1 B)$

```
            -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
              +----+----+----+----+----+----+----+----+----+----+
                                       I
 -36   -0.07                        +XXI  +
 -35    0.10                        +  IXXX
 -34   -0.08                        +XXI  +
 -33   -0.04                        + XI  +
 -32   -0.07                        +XXI  +
 -31   -0.03                        + XI  +
 -30   -0.00                        +  I  +
 -29   -0.03                        + XI  +
 -28   -0.01                        +  I  +
 -27    0.05                        +  IX +
 -26    0.17                        +  IXX+X
 -25    0.00                        +  I  +
 -24   -0.04                        + XI  +
 -23    0.06                        +  IXX+
 -22   -0.12                        XXXI  +
 -21    0.05                        +  IX +
 -20   -0.08                        +XXI  +
 -19   -0.08                        +XXI  +
 -18    0.12                        +  IXXX
 -17    0.07                        +  IXX+
 -16    0.09                        +  IXX+
 -15   -0.02                        +  I  +
 -14   -0.02                        +  I  +
 -13    0.11                        +  IXXX
 -12   -0.03                        + XI  +
 -11   -0.02                        + XI  +
 -10    0.11                        +  IXXX
  -9    0.01                        +  I  +
  -8    0.11                        +  IXXX
  -7    0.08                        +  IXX+
  -6   -0.08                        +XXI  +
  -5   -0.01                        +  I  +
  -4   -0.16                       X+XXI  +
  -3   -0.05                        + XI  +
  -2   -0.03                        + XI  +
  -1   -0.06                        +XXI  +
   0   -0.00                        +  I  +
   1   -0.00                        +  I  +
   2   -0.03                        + XI  +
   3    0.00                        +  I  +
```

```
 4   -0.03          + XI  +
 5   -0.07          +XXI  +
 6   -0.06          +XXI  +
 7   -0.02          + XI  +
 8   -0.11          XXXI  +
 9   -0.07          +XXI  +
10   -0.01          +  I  +
11    0.09          +  IXX+
12    0.07          +  IXX+
13    0.14          +  IXXX
14   -0.09          +XXI  +
15    0.03          +  IX +
16    0.05          +  IX +
17    0.07          +  IXX+
18    0.05          +  IX +
19   -0.08          +XXI  +
20    0.01          +  I  +
21   -0.13          XXXI  +
22   -0.04          + XI  +
23    0.03          +  IX +
24    0.08          +  IXX+
25    0.06          +  IX +
26   -0.04          + XI  +
27   -0.05          + XI  +
28   -0.04          + XI  +
29    0.06          +  IXX+
30    0.10          +  IXXX
31   -0.05          + XI  +
32   -0.12          XXXI  +
33   -0.08          +XXI  +
34   -0.08          +XXI  +
35    0.05          +  IX +
36    0.00          +  I  +
```

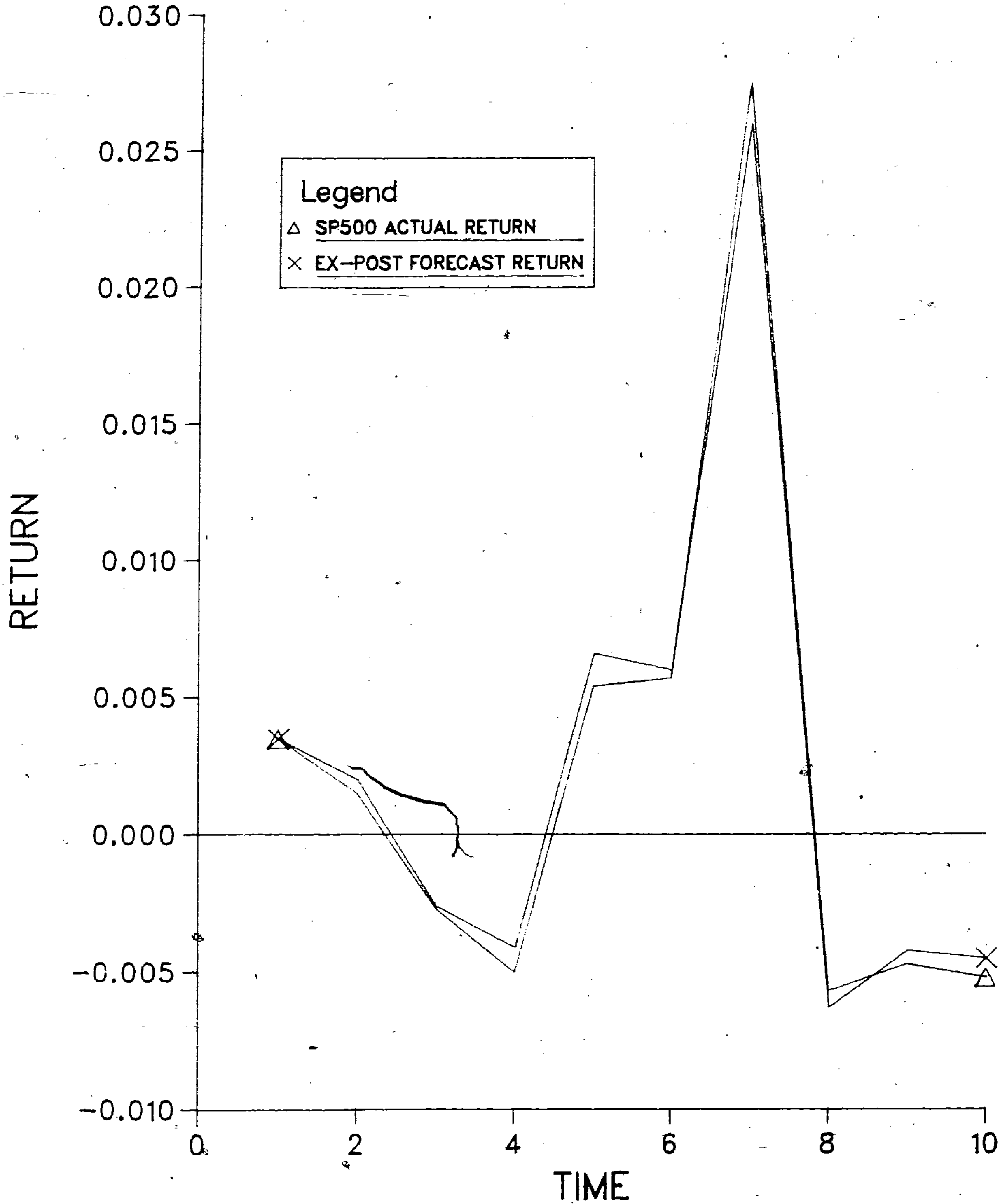
FIG.14: CCF BETWEEN THE WHITE NOISE SERIES(EQN 12) AND THE OUTPUT SERIES(EQN 13) WHEN ATTEMPTING TO USE THE SP500 AS INPUT

Eqn.(12): $SP500_t = a_t$
Eqn.(13): $NYAM_t = NOISE$

```
           -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
             +----+----+----+----+----+----+----+----+----+----+
                                      I
-36  -0.02                         + XI  +
-35  -0.03                         + XI  +
-34   0.00                         +  I  +
-33   0.01                         +  I  +
-32  -0.02                         +  I  +
-31   0.02                         +  I  +
-30   0.10                         +  IXXX
-29   0.02                         +  I  +
-28   0.00                         +  I  +
-27  -0.03                         + XI  +
-26  -0.04                         + XI  +
-25  -0.00                         +  I  +
-24  -0.02                         + XI  +
-23   0.02                         +  IX +
-22  -0.08                         +XXI  +
-21  -0.09                         +XXI  +
-20  -0.03                         + XI  +
-19   0.05                         +  IX +
-18  -0.03                         + XI  +
-17  -0.01                         +  I  +
-16  -0.08                         +XXI  +
-15  -0.08                         +XXI  +
-14  -0.04                         + XI  +
-13   0.06                         +  IX +
-12   0.06                         +  IX +
-11   0.08                         +  IXX+
-10   0.04                         +  IX +
 -9  -0.06                         + XI  +
 -8  -0.02                         +  I  +
 -7  -0.08                         +XXI  +
 -6  -0.05                         + XI  +
 -5   0.09                         +  IXX+
 -4  -0.06                         + XI  +
 -3   0.00                         +  I  +
 -2   0.05                         +  IX +
 -1   0.19                         +  IXX+XX
  0   0.99                         +  IXX+XXXXXXXXXXXXXXXXXXXXXX
  1   0.13                         +  IXXX
  2   0.04                         +  IX +
  3  -0.02                         + XI  +
  4  -0.05                         + XI  +
```

```
 5   0.09          +  IXX+
 6  -0.06          +XXI  +
 7  -0.08          +XXI  +
 8  -0.04          + XI  +
 9  -0.06          + XI  +
10   0.05          +  IX +
11   0.09          +  IXX+
12   0.07          +  IXX+
13   0.05          +  IX +
14  -0.06          +XXI  +
15  -0.08          +XXI  +
16  -0.07          +XXI  +
17  -0.01          +  I  +
18  -0.03          + XI  +
19   0.05          +  IX +
20  -0.04          + XI  +
21  -0.11          XXXI  +
22  -0.06          +XXI  +
23   0.02          +  IX +
24  -0.01          +  I  +
25   0.00          +  I  +
26  -0.06          + XI  +
27  -0.03          + XI  +
28   0.00          +  I  +
29   0.03          +  IX +
30   0.11          +  IXXX
31   0.00          +  I  +
32  -0.02          + XI  +
33   0.00          +  I  +
34   0.00          +  I  +
35  -0.03          + XI  +
36  -0.02          +  I  +
```

FIG.15
ACTUAL VS EX–POST FORECASTS
OF SP500 DAILY RETURN
FROM DEC.10 TO DEC.21, 1984
USING EQN.18



Appendix A: The Market Model

The Market Model assumes that individual security returns are linearly related to the returns on a market portfolio. The model asserts that

$$\tilde{R}_{it}=\alpha_i+\beta_i\tilde{R}_{mt}+\tilde{\mu}_{it} \qquad \text{(A1)}$$

where $\tilde{R}_{it}$=return for period t on the i^{th} security,

$\tilde{R}_{mt}$=average return on a market portfolio of all assets on the exchange or a representative sample of all securities,

α_i&β_i=model parameters to be estimated by least squares regression.

μ_{it}=the disturbance term for period t on the i^{th} security, with $E(\mu_{it})=0$, $E(\mu_{it},\mu_{jt})=0$ for $i \neq j$ and $E(\mu_{it},\mu_{jt})=\sigma_u^2$ for $i = j$.

Movements in security prices are associated with market-wide information that differentially affects the value of securities. In order to control for the differential effects of market wide information on individual security returns, the Market Model presumes that the systematic part of a security's return can be captured by its normal relationship to the returns on the market portfolio. Any returns not accounted for by a security's normal relationship to the market will be impounded in the disturbance term, $\tilde{\mu}_{it}$, which presumably captures the effects of company-specific influences. Thus, in studying a security's reaction to various kinds of information, researchers search for unusual behavior of the disturbance term, μ_{it}, which is specific to the particular security for the time period under study.

The Market Model is usually used in conjunction with the two parameters asset pricing model. Note that, the β_i parameter estimated is the same under both the Market Model and the two parameters CAPM[34] and β_i is estimated to be:

$$\beta_i = \frac{COV(\tilde{R}_{it}\tilde{R}_{mt})}{VAR(\tilde{R}_{mt})} \qquad (A2)$$

where $COV(\tilde{R}_{it}\tilde{R}_{mt})$= covariance between the returns on security i and the market portfolio,

$VAR(\tilde{R}_{mt})$= variance of the market portfolio's returns.

The β_i parameter estimated by the Market Model during a particular "estimation period" is assumed to be constant throughout a subsequent "testing period". Thus, this estimated β_i parameter is sometimes used as an estimate for the β_i parameter of the CAPM during the "testing period" in an attempt to capture the behavior of the disturbance term. Although the Market Model is not an equilibrium model, its simplicity facilitates portfolio analysis problems without assuming away the existence of interrelationships among securities, and there is considerable evidence that it can capture a large part of such interrelationships.

---

[34]See Appendix B

Appendix B: The two parameters CAPM

The CAPM developed independently by Sharpe (1964), Lintner (1965), and Mossin (1966) describes the relative prices of securities or how portfolios of securities should be priced. The earliest Sharpe-Lintner version is based on very strong assumptions:

1. Risk-averse investors choose portfolios based on single period means and variances of returns.
2. There is unlimited lending and borrowing at a riskless rate of interest, R_F, and no restriction on short selling.
3. There are homogeneous expectations.
4. Markets are frictionless, competitive, and all assets are perfectly divisible.

With these assumptions, it is shown that expected returns on assets must follow the relationship described by the Security Market Line (SML) which is labelled as Eqn.(B1).

$$E(\tilde{R}_i)=R_F+[E(\tilde{R}_m)-R_F]\beta_i \qquad \text{(B1)}$$

where $E(\tilde{R}_i)$=expected returns on the i^{th} asset,

R_F=the riskless rate of interest,

$E(\tilde{R}_m)$=the expected return on a market portfolio that includes all assets weighted by their market values.

$$\beta_i = \frac{COV(\tilde{R}_j \tilde{R}_m)}{VAR(\tilde{R}_m)}$$

which is referred as the systematic risk of security i.

With Eqn.(B1), the actual random returns of security i for period t, $\tilde{R}_{it}$, can be written in terms of Eqn.(B2).

$$\tilde{R}_{it} = R_{Ft} + [\tilde{R}_{mt} - R_{Ft}]\beta_{it} + \mu_{it} \qquad (B2)$$

where μ_{it} = random disturbance term for security i in period t, μ_{it} is i.i.d. with 0 mean, 0 covariance & constant variance.

The systematic part of a security's return is presumed to be captured by its normal relationship to the returns on the market portfolio, $\tilde{R}_{mt}$. Any returns not accounted for by a security's normal relationship to the market will be impounded in the disturbance, μ_{it}, which presumably captures the effects of company-specific influences. Thus, researchers look for unusual behavior of the μ_{it} term of Eqn.(B2) in an attempt to study whether the i^{th} security is priced efficiently. The basic SML equation implies that a security will only earn an expected return that exceeds the riskless rate of interest to the extent that the return on the security is correlated with the return on the market. In an efficient capital market, fluctuations in security returns that are not correlated with the market can be diversified away by holding a sufficient number of assets, and the market pays no premium for diversifiable risks. Note that, the Market Model would be identical to the CAPM if α_i in Eqn.(A1) is equal to $(1-\beta_{it})R_{Ft}$, i.e., Eqn.(A1) and (B2) would then be identical.

Appendix C: The ACF, PACF and CCF

a) The ACF

The ACF is defined as:

$$ACF(k)=COV(Y_t \; Y_{t+k})/VAR(Y_t) \qquad (C1)$$

The ACF(k) is a measure of correlation between Y_t and Y_{t+k}, i.e., it is the correlation coefficient estimated between the time series (lag-0) and its k^{th} lag (lag-k). By definition, ACF (0)=1; a time series is always perfectly correlated with itself. Also, by definition, ACF(k)=ACF(-k); the ACF(k) is the same whether the series is lagged forward or backward. Because the ACF is symmetrical about lag-0, only the positive half of the ACF need to be examined . When each time the series is lagged, one pair of observations is lost form the estimate of ACF(k). thus, with N observations, ACF(1) is estimated from N-1 pairs of observation, ..., and ACF(k) from N-k pairs of observations. As the value of k increases, confidence in the estimate of ACF(k) diminishes.

In theory, each time series process has an unique ACF, and if two processes have the same ACF, they are identical. Consider an ARIMA (0,0,0) or white noise process:

$$Y_t=a_t+\theta_o \qquad (C2)$$

Eqn.(C2) is expected to have an uniformly zero ACF. This follows from the definition of white noise. For all k, $COV(a_t,a_{t+k})=0$.

An ARIMA (0,1,0) process written as

$$(1-B)Y_t=a_t+\theta_o \qquad (C3)$$

is expected to have an ACF that is positive and dies out slowly from lag to lag, i.e., ACF(1)≈ACF(2)≈...≈ACF(k). In general, the ACF of a nonstationary process is expected to have a relatively high positive value for ACF(1) and successive lags of the ACF are expected to die out slowly.

An ARIMA (0,0,1) process written as

$$Y_t=(1-\theta_1 B)a_t \qquad (C4)$$

is expected to have nonzero ACF(1). All other lags of the ACF are expected to be zero. Note that

$$\begin{aligned} COV(Y_t\ Y_{t+1}) &= E(Y_t\ Y_{t+1}) \qquad (C5)\\ &= E[(a_t-\theta_1 a_{t-1})(a_{t+1}-\theta_1 a_t)]\\ &= E(a_t a_{t+1}-\theta_1 a_t{}^2-\theta_1 a_{t-1}a_{t+1}+\theta_1{}^2 a_{t-1}a_t)\\ &= -\theta_1 E(a_t{}^2)\\ &= -\theta_1 \sigma a^2 \end{aligned}$$

$$\begin{aligned} VAR(Y_t) &= E(Y_t{}^2) \qquad (C6)\\ &= E[(a_t-\theta_1 a_{t-1})^2]\\ &= E(a_t{}^2-2\theta_1 a_t a_{t-1}+\theta_1{}^2 a_{t-1}{}^2)\\ &= E(a_t{}^2)+\theta_1{}^2 E(a_{t-1}{}^2)\\ &= \sigma a^2(1+\theta_1{}^2) \end{aligned}$$

Thus, $E[ACF(1)]=COV(Y_t\ Y_{t+1})/VAR(Y_t) \qquad (C7)$

$$\begin{aligned} &= -\theta_1 \sigma a^2/\sigma a^2(1+\theta_1{}^2)\\ &= -\theta_1/(1+\theta_1{}^2) \end{aligned}$$

Using the same procedures,

$$E(Y_t\ Y_{t+2})=E[(a_t-\theta_1 a_{t-1})(a_{t+2}-\theta_1 a_{t+1})] \quad \text{(C8)}$$

$$=E(a_t a_{t+2}-\theta_1 a_t a_{t-1}-\theta_1 a_{t-1} a_{t+2}+$$

$$\theta_1{}^2 a_{t-1} a_{t+1})$$

$$=0,$$

thus, $E[ACF(2)]=0/\sigma a^2(1-\theta_1{}^2)=0$

Through the same procedure, it can be shown that ACF(3), ACF(4),....,ACF(k) are all expected to be zero.

An ARIMA (0,0,2) process written as

$$Y_t=(1-\theta_1 B-\theta_2 B^2)a_t \quad \text{(C9)}$$

is expected to have nonzero values for ACF(1) and ACF(2). The values of ACF(3) and all successive ACF(k) are expected to be zero. Note that,

$$E(Y_t\ Y_{t+1})=E[(a_t-\theta_1 a_{t-1}-\theta_2 a_{t-2})$$

$$(a_{t+1}-\theta_1 a_t-\theta_2 a_{t-1})] \quad \text{(C10)}$$

$$=E(a_t a_{t+1}-\theta_1 a_t{}^2-\theta_2 a_t a_{t-1}-$$

$$\theta_1 a_{t-1} a_{t+1}+\theta_1{}^2 a_{t-1} a_t+$$

$$\theta_1\theta_2 a_{t-1}{}^2-\theta_2 a_{t-2} a_{t+1}+$$

$$\theta_2\theta_1 a_{t-2} a_t+\theta_2{}^2 a_{t-2} a_{t-1})$$

$$=-\theta_1 E(a_t{}^2)+\theta_2\theta_1 E(a_{t-1}{}^2)$$

$$=\sigma a^2\theta_1(\theta_2-1)$$

$$E(Y_t{}^2)=E[(a_t-\theta_1 a_{t-1}-\theta_2 a_{t-2})^2] \quad \text{(C11)}$$

$$=E(a_t{}^2-2\theta_1 a_t a_{t-1}-\theta_2 a_t a_{t-2}+$$

$$\theta_1{}^2 a_{t-1}{}^2+2\theta_1\theta_2 a_{t-1} a_{t-2}-$$

$$\theta_2 a_t a_{t-2}+\theta_2{}^2 a_{t-2}{}^2)$$

$$=\sigma a^2(1+\theta_1{}^2+\theta_2{}^2)$$

Thus, $E[ACF(1)]=\sigma a^2\theta_1(\theta_2-1)/\sigma a^2(1+\theta_1{}^2+\theta_2{}^2)$

$$=\theta_1(\theta_2-1)/(1+\theta_1^2+\theta_2^2)$$

For the second lag of the ACF,

$$E(Y_t\ Y_{t+2})=E[(a_t-\theta_1 a_{t-1}-\theta_2 a_{t-2})(a_{t+2}-\theta_1 a_{t+1}-\theta_2 a_t)] \quad (C12)$$

$$=-\theta_2 E(a_t^2)$$

$$=-\theta_2 \sigma a^2$$

This gives:

$$E[ACF(2)]=-\theta_2 \sigma a^2/\sigma a^2(1+\theta_1^2+\theta_2^2)$$

$$=-\theta_2/(1+\theta_1^2+\theta_2^2)$$

For the third lag of the ACF,

$$E(Y_t\ Y_{t+3})=E[(a_t-\theta_1 a_{t-1}-\theta_2 a_{t-2})(a_{t+3}-\theta_1 a_{t+2}-\theta_2 a_{t+1})] \quad (C13)$$

$$=0$$

Therefore, E[ACF(3)]=0. Continuing this procedure, it can be shown that an ARIMA (0,0,q) process is expected to have nonzero values for ACF(1), ..., ACF(q) while ACF(q+1) and all successive lags are expected to be zero.

Consider an ARIMA (1,0,0) process written as

$$(1 - \phi_1 B)Y_t=a_t \quad (C14)$$

Eqn.(C14) is expected to have an ACF that decays exponentially beginning with lag 1. Note that,

$$E(Y_t\ Y_{t+1})=E[(Y_t)(\phi_1 Y_t+a_{t+1})] \quad (C15)$$

$$=E(\phi_1 Y_t^2+Y_t a_{t+1})$$

$$=\phi_1 \sigma_Y^2$$

Thus, $E[ACF(1)] = \phi_1 \sigma_Y^2/\sigma_Y^2 = \phi_1$

For lag 2 of the ACF,

$$E(Y_t\ Y_{t+2})=E[(Y_t)(\phi_1 Y_{t+1}+a_{t+2})] \qquad (C16)$$

$$=E[(Y_t)(\phi_1(\phi_1 Y_t+a_{t+1})+a_{t+2})]$$

$$=E(\phi_1^2 Y_t^2+\phi_1 Y_t a_{t+1}+Y_t a_{t+2})$$

$$=\phi_1^2 \sigma_Y^2$$

Thus, $E[ACF(2)]=\phi_1^2\sigma_Y^2/\sigma_Y^2 = \phi_1^2$

By the same procedure, $E[ACF(3)]=\phi_1^3$, ..., $E[ACF(k)]=\phi_1^k$. The expected ACF(k) grows smaller and smaller from lag to lag until after 3 or 4 lags, ACF(k) is approximately zero.

An ARIMA(2,0,0)process written as

$$(1-\theta_1 B-\theta_2 B^2)Y_t=a_t \qquad (C17)$$

is expected to have an ACF that decays exponentially beginning with the first lag. It can be shown that its expected ACF is given by:

$$ACF(k)=\theta_1 ACF(k-1)+\theta_2 ACF(k-2) \qquad (C18)$$

In the general case, an ARIMA (p,0,0) process is expected to have an ACF that decays from lag to lag with the rate of decay determined by the values of θ_1, θ_2, ..., θ_p.

If the estimated ACF is zero for all lags, it can be inferred that the time series is generated by an ARIMA (0,0,0) process. If the estimated ACF(1) is large and positive but dies out slowly from lag to lag, the process is nonstatinary and the series must be differenced. If the estimated ACF(1) is nonzero but ACF(2) and all successive lags are zero, the time series is generated by an ARIMA (0,0,1) process. Finally, if the estimated ACF dies out exponentially from lag to lag, the time series probably is generated by an ARIMA (1,0,0) process.

b) The PACF

PACF(k) is a measure of correlation between time series observations k units apart after the correlation at intermediate lags has been controlled or "partialed out" . PACF(k) is estimated from a solution of the Yule-Walker equation system, it is shown that the solution gives the following identities:

1) PACF(1)=ACF(1) (C19)

2) $$PACF(2)=\frac{ACF(2)-[ACF(1)]^2}{1-[ACF(1)]^2} \quad (C20)$$

3) $$PACF(3)=\{ACF(3)+ACF(1)[ACF(2)]^2+[ACF(1)]^3-2ACF(1)ACF(2) - [ACF(1)]^2-ACF(3)\}/\{1+2[ACF(1)]^2ACF(2) - [ACF(2)]^2-2[ACF(1)]^2\} \quad (C21)$$

and so forth.

An ARIMA (1,0,0) process whose ACF is expected to be $ACF(k)=\phi_1^k$ is expected to have a nonzero PACF(1) while PACF(2) and all successive lags are expected to be zero:

$$PACF(1)=\phi_1$$

$$PACF(2)=\phi_1^2-\phi_1^2/1-\phi_1^2=0$$

$$PACF(3)=\frac{\phi_1^3+\phi_1\phi_1^4+\phi_1^3-2\phi_1\phi_1^2-\phi_1^2\phi_1^3}{1+2\phi_1^2\phi_1^2-\phi_1^4-2\phi_1^2}$$

$$=\frac{2\phi_1^3+\phi_1^5-2\phi_1^3-\phi_1^5}{1+\phi_1^4-2\phi_1^2}=0$$

Successive lags of PACF(k) are also expected to be zero.

An ARIMA (2,0,0) process whose ACF is expected to be

$$ACF(1) = \phi_1 / 1-\phi_2$$

$$ACF(2) = (\phi_1^2 / 1-\phi_2) + \phi_2$$

$$ACF(3) = \frac{\phi_1(\phi_2 + \phi_1^2)}{1 - \phi_2} + \phi_1\phi_2$$

is expected to have nonzero values of PACF(1) and PACF(2) while PACF(3) and all successive lags are expected to be zero. Thus,

$$PACF(1) = \phi_1 / 1-\phi_2$$

$$PACF(2) = \frac{\phi_2(\phi_2 - 1)^2 - \phi_1\phi_2}{(1-\phi_2)^2 - \phi_1^2}$$

$$PACF(3) = 0$$

Successive lags are all expected to be zero. In general, an ARIMA (p,0,0) process is expected to have nonzero values for PACF(1), ..., PACF(p) while PACF(p+1) and all successive lags are expected to be zero.

An ARIMA (0,0,1) process whose ACF is expected to be:

$$ACF(1)=\frac{-\theta_1}{1+\theta_1^2}$$

$$ACF(2)=.....=ACF(k)=0,$$

has a decaying PACF, i.e., all PACF(k) are expected to be nonzero:

$$\mathrm{PACF}(1) = \frac{-\theta_1}{1+\theta_1^2}$$

$$\mathrm{PACF}(2) = \frac{-\theta_1^2}{1+\theta_1^2\theta_1^4}$$

$$\mathrm{PACF}(3) = \frac{-\theta_1^3}{1+\theta_1^2+\theta_1^4+\theta_1^6}$$

Successive lags of the expected PACF grows smaller and smaller in absolute value. In general, the PACF of an ARIMA (0,0,q) process is expected to decay at a rate determined by the values of $,\theta_1,\ldots,\theta_q$.

Autoregressive process are characterized by decaying ACFs and specifying PACFs. An ARIMA (p,0,0) process is expected to have exactly p nonzero spikes in the first p lags of its PACF. All successive lags of the PACF are expected to be zero. Moving average processes are characterized by spiking ACFs and decaying PACFs. An ARIMA (0,0,q) process is expected to have exactly q nonzero spikes in the first q lags of its ACF. All successive lags of the ACF are expected to be zero. An ARIMA (p,0,q) process is expected to have both decaying ACF and PACF.

c) <u>C.C.F.</u>

The CCF is used to identify between-series correlation, the patterns of between-series correlation are then used to identify a transfer function relationship between two time series. Note that, two nonstationary time series will always be correlated due to common patterns of drift or trend. To eliminate between-series correlations due only to drift or trend, the time series must be made stationary by appropriate differenings prior to estimation of the CCF.

By convention, X_t is referred to as the input series, or causor, and Z_t is referred to as the output series, or effector. Given two stationary time series, X_t & Z_t, the CCF for lags $\pm k$ is given by the formulae:

$$\mathrm{CCF}(+k) = \frac{\sum_{t=1}^{N-k} (X_t-\overline{X})(Z_{t+k}-\overline{Z})}{\sqrt{\sum_{t=1}^{N} (X_t-\overline{X})^2 \sum_{t=1}^{N} (Z_{t+k}-\overline{Z})^2}} \quad \text{(C22)}$$

$$\mathrm{CCF}(-k) = \frac{\sum_{t=1}^{N+k} (X_{t-k}-\overline{X})(Z_t-\overline{Z})}{\sqrt{\sum_{t=1}^{N} (X_{t-k}-\overline{X})^2 \sum_{t=1}^{N} (Z_t-\overline{Z})^2}} \quad \text{(C23)}$$

Eqn.(C22) and (C23) give the Pearson product-moment correlation coefficient between two time series separated by $\pm$ k observations. When k=0, Eqn.(C22) and (C23) are identical. When $k \neq 0$, Eqn.(C22) gives the positive half of the CCF by lagging the Z_t series forward in time, and Eqn.(C23) gives the negative half of the CCF by lagging the X_t series forward in time. Note that, the CCF need not be symmetrical about lag 0, thus, in general, $\mathrm{CCF}(+k) \neq \mathrm{CCF}(-k)$. As a convention, it is assumed that any estimate of CCF (+k) smaller in absolute value than 2 standard error is 0.

The CCF measures not only the strength but also the direction. When "X_t cause Z_{t+b}", evidence of the relationship is found at CCF (+b), in the positive half of the CCF. When the reverse is true, evidence of the relationship is found at CCF (-b) in the negative half of the CCF.

Consider the 0-order relationship:

$$Z_t=W_oX_{t-b}+N_t \qquad \text{(C24)}$$

Eqn.(C24) is expected to have a nonzero value of CCF (+b), all other lags of the CCF are expected to be zero. Defining CCF (+k) as Eqn.(C25).

$$CCF(+k)=\frac{COV(X_{t-k}Z_t)}{\sqrt{VAR(X_t)VAR\ (Z_t)}} \qquad \text{(C25)}$$

Note that,

$$COV((X_{t-k}Z_t)=E[(X_{t-k})(W_oX_{t-b}+N_t)] \qquad \text{(C26)}$$

$$=E[(W_oX_{t-k}X_{t-b}+X_{t-k}N_t)]$$

Assuming $E(X_{t-k}N_t)=0$, $COV(X_{t-k}Z_t)= W_oE(X_{t-k}X_{t-b})$

Now, assuming that X_t is a white noise process, [35]

$$COV((X_{t-k}Z_t)=W_o\sigma_t^2 \text{ whenever } b=k$$
$$=0 \text{ otherwise} \qquad \text{(C27)}$$

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[35]the input series, X_t, must be white noise so that the CCF estimate is not contaminated by within series correlations, or autocorrelation. When the causor series is not white noise, the CCF will reflect both between-series and within-series dependencies.

Thus, the expected CCF is:

$$CCF(k)=\frac{W_o\sigma_x^2}{\sigma_x\sigma_z}$$

$$\begin{aligned} &= W_o\sigma_x/\sigma_z \quad \text{whenever } b=k \\ &= 0 \quad \text{otherwise} \end{aligned} \qquad (C28)$$

Consider the 1st order transfer function relationship of Eqn.(C29).

$$Z_t=\frac{W_o}{1-\delta_1 B}X_{t-b}+N_t \qquad (C29)$$

The CCF for a 1st order transfer function is expected to be zero until CCF (b). Successive positive lags, CCF(b+1), CCF(b+2), ..., CCF(b+n), decay exponentially to zero. Since when b>k, $COV(X_{t-k}Z_t)=0$. When b=k, $COV(X_{t-b}Z_t)=W_o\sigma_x^2$. When k=b+1, $COV(X_{t-b-1}Z_t)=W_o\delta_1\sigma_x^2$. In general, when k=b+n, $COV(X_{t-b-n}Z_t)=W_o\delta_1^n\sigma_x^2$. Thus,

$CCF(k) = 0$ for $k<b$

$CCF(k) = W_o\sigma_x/\sigma_z$ for $k=b$

$CCF(k) = W_o\delta_1^n\sigma_x/\sigma_z$ for $k=b+n$

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