OPTIMAL LOAD-FREQUENCY CONTROL OF A MULTIMACHINE POWER SYSTEM WITH THE VARYING POWER DEMAND

by

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DEDICATION

To my wife

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ABSTRACT

Optimal modal load-frequency control (LFC) of an interconnected multi-machine power system is considered in this thesis. Two considerations has been given to this problem which sets this thesis apart from other studies involving the LFC. First, we make a practical assumptions that the power demand is time varying and unknown, and that the system state is not available for feedback purposes. This is in contrast to a number of past studies which treat the power demand as a constant, and use state feedback for control purposes. Second, a (sub)optimal modal control strategy is adopted for obtaining (sub)optimum performance as well as systematic control over the location of the system's eigenspectrum to achieve good transient response.

For the sake of our study, we consider an interconnected single area multi-machine power system. A mathematical model of the system is derived with the power demand modeled as an unknown disturbance. A sequential design strategy is used for designing an optimal control law which would assign the eigenvalues of the closed loop system to desired locations, and at the same time would minimize a quadratic cost functional. This optimal modal controller is designed in a systematic fashion by selecting the weights in the cost functional so that a single real or a complex conjugate pair of poles are assigned at each stage. Once the appropriate weights are computed the control strategy which would achieve the pole placement is computed and the next round of the sequential design would then take place. Since this controller is based on state feedback, the unknown input observer (UIO) theory is then used to correctly estimate the system's state in spite of the time varying and unknown power demand. Finally, a supplementary control law based on the estimate of the power demand is designed in order to correct for the effect of load changes on the power system, and maintain the system's frequency as well as the tie line power at the scheduled values.

Simulation studies are used to illustrate the effectiveness of the proposed load frequency control strategy.

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NOMENCLATURE

CONTROL PARAMETERS

LFC	Load Frequency Control
LQR	Linear Quadratic Regulator
UIO	Unknown Input Observer
A	System Matrix
B	Control Input Matrix
C	Output Matrix
$\mathbf{x}$	State Variable
$\mathbf{y}$	Output Vector
K	Control Gain
$\mathbf{u}$	Control Input
J	Performance Index
R	Control Input Weighting Matrix
Q	State Variable Weighting Matrix
H	Hamiltonian Matrix
$\mathbf{p}$	Unknown Input (Load)
λ_i	ith Eigenvalue
V_i	ith Eigenvector associated with
V	Eigenvector Matrix
Λ	Eigenvalue Matrix
C^+	Pseudo Inverse of C
$\hat{\mathbf{x}}$	State Variable Estimate
S_i	ith Subsystem
Γ	Unknown Input Matrix

t	Time
α,σ,ν,ϕ	Real Part of the Eigenvalues
β,γ	Imaginary Part of the Eigenvalues

PLANT PARAMETERS

SG	Speed Governer
r	Speed Regulation due to the SG
K_g, t_g	Gain and Time Constant of SG
K_t, t_t	Gain and Time Constant of Turbine
ΔX_e	Main Piston Position Change due to SG
ΔP_c	Generating Power Increase due to SG
ΔP_{gt}	Generating Power Increase due to Turbine
ΔP_g	Generating Power Increase
ΔP_{ge}	Generating Power Increase due to Exciter
ΔP_d	Load Change
f	Frequency
$\Delta\delta$	Torque Angle Change
W_{kin}	Kinetic Energy of the System
e	Exciter Voltage
u	Generator Terminal Voltage
P_{tie}	Tie Power
t°	Nominal Synchronizing coefficient
h	Inertia Constant
d	Rate of Load Change due to Freq Change
P_{du}	Rate of Load Change due to Voltage Change
...(s)	Laplace Transform
X_{12}	Transmission Impedance
$(\ldots)^t$	Transpose

CHAPTER 1

INTRODUCTION

Interconnection in large electric power systems is intended to make electric energy generation and transmission more economical and reliable. However, with highly interconnected power grid many new dynamic power system problems have emerged, low frequency oscillation, load frequency instabilities, to name a few Yu(1973).

The economical aspect of the large scale power system interconnections is manifested through the remarkable reduction of spinning reserve or the stand by generating capacity for maintenance or emergency use. The reliability of the interconnected system is also enhanced by the capability of transferring power from one area to others within the system. But in the meantime multiple interconnections of multi areas make the system much more vulnerable to instability. First of all, the reduction in spinning reserve of the individual areas and secondly complexity of the multi area interconnections can be considered as the main reasons for instability.

CHAPTER 1. INTRODUCTION

There are variety of control problems that need to be addressed for efficient and safe operation of interconnected power systems. Also, as new problems are emerging, additional more sophisticated control is necessary for stable operation of the system. As an example, voltage collapse phenomenon is a problem that has arisen due to the fact that power transmission lines are being used almost at their full capacity in recent times.

One of the well known control problems in interconnected power systems is that of the Load Frequency Control (LFC). This problem is the subject of our study. The purpose of LFC is supplying a time varying load while maintaining scheduled tie line powers and system frequency levels at the nominal values. In this thesis, we shall concentrate on modern control approaches to LFC. Basically four category of control will be discussed. These are: optimal control, pole placement, optimal modal control and decentralized control of power systems.

Modelling is a basic part of the modern control design. It is obvious that without a proper model, we can not be successful in controlling the behavior of any system. Generally, for application of modern control concepts, dynamical systems are described in state space form. In linear system, time response of the system is in terms of eigenvalues and eigenvectors of the system matrix. To achieve desired response of a system without expenditure of high control effort, optimal control is often employed, where a performance index or cost function for the system is defined. Minimizing the cost function will result in the optimal control law. Often the cost function is defined as a weighted quadratic function of state variables and the control inputs. This the so called Linear Quadratic Regulator (LQR) strategy.

The dynamic aspects of LFC were first considered by Elgerd and Fosha (1970) using optimal control theory. In that work a two area system (which is the simplest multi area system) was modelled. The cost function that was to be optimized was in terms of frequency and the tie power deviations. Optimal state feedback law was then used for LFC purposes. Since the original work of Elgerd and Fosha (1970), a number of other optimal control approaches has been proposed by other researchers. While in Elgerd's work, linear feedback controller is a function of all the state variables, Calovic (1977) proposes a PI controller law in which the proportional part as well as the integral part is only a function of the output variables. A similar approach to Elgerd's work, was also proposed by Nanda and Kothari (1987). In this approach a proportional and integral control strategy was used. Proportional control is a function of all state variables of the system and the integral part has only output terms.

Another approach to LFC using modern control theory concepts has been through use of eigenvalue/eigenvector placement. In these approaches the transient response can be better manipulated by appropriate placement of eigenvalue/eigenvector, however the optimality is lost. In Porter and D'azzo (1977) work, an approach based on the entire eigenstructure assignment is proposed.

Chow (1989), in his paper lists four different ways of pole placement for the power systems :

a): Direct pole placement Algorithm (Mayne and Murdoch (1970))

b) Indirect Pole placement Algorithm (Solheim (1972)) where by Q selection, we can shift the eigenvalues and minimize the quadratic cost function of the system. This approach falls under the next category of approaches to be discussed as well.

c) Projective Output feedback Design (Hopkins, Medanic and Perkins (1981)) where uses the outputs (which are available) feedback instead of using state variables (which has to be reconstructed)

d) Low Order Optimal Design (Medanic (1988)) where instead of full state observer only local dominant modes are used, and pole placement design is to improve only the local dominant modes. Thereby, some modes will shift from their open loop values.

On the other hand, Hsu and Huang (1990) present Eigenstructure Assignment Control (EAC) in power systems. In this approach, the objectives are to change the both eigenvalues and the eigenvectors of the system. It is well known that linear system solution can be described in terms of eigenvalues and the eigenvectors of the system, so changing them to desired values will completely control the response of the system. However, Shapiro (1975) shows that it is not possible to assign all of the eigenvectors and only some of them can be set to predefined vectors.

There is another class of approaches that are combination of eigenvalue assignment and optimization together for control of the power systems. Yu (1983) presents the LQR design with dominant eigenvalue shift, which uses sensitivity analysis of eigenvalues with respect to the elements of a diagonal Q matrix to shift the dominant eigenvalue of the system. Habibullah (1974) uses the canonical form of state space model and finds the similarity transformation to place the eigenvalues of the system.

Finally, since power systems are among few large scale systems, and due to their geographically distributed nature, they are ideal candidate for decentralized, and hierarchical control applications. In a decentralized control scheme, the feedback control law in each area is computed on the basis of measurements taken in that area only. The advantages of this operating philosophy are apparent in providing cost saving in data communications and in reducing the scope of controlled area. Christensen (1987) formulates the LFC problem as a parameter optimization problem. It is based on finding proportional and integral gain of a PI controller to minimize the system transient and the control action such that the steady state, dynamic limit, and area decentralization are met. A two-level LFC scheme was introduced by Miniesy and Bohn(1971), where a local closed loop feedback for each plant is calculated in the first level and the control law is supplemented with an open loop global control, calculated in the second level. However the algorithm in the second level is based on a linear search, which makes it computationally complicated. In Saif and Villaseca (1986) the Interaction prediction approach is used which is computationally much simpler and the nonlinear effects of the interaction between systems do not affect the main calculations. More recently, Aldeen and Marsh (1991) method proposes the use of observer in each step of load disturbance and a PI controller with proportional and integral terms of area control error (ACE) to compensate for the steady state error of the frequency and the tie power deviation.

A common feature of most of the optimal control approaches as well as the others is that:

1) load is assumed to be known and constant, which usually is not true and fluctuations of the consumption specially in emergency situations can not be ignored.

2) state variables have been used in control design which makes estimation necessary. Because the load is generally an unknown variable, we need a special estimator which can reconstruct state variables of the system in a varying load demand system.
3) transient response of the system can not be easily adjusted and the only means for getting a better response is through trial and error.
4) No systematic way of selecting the weight in the cost function is given in the optimal control based techniques.

In this thesis, we shall address the above issues as follows: Optimal modal control strategy (Saif(1989)) shall be employed to address the third and fourth issues. Optimal modal controllers are a class of optimal controllers where in addition to minimizing a suitably selected performance measure, they can also assign all the modes (or a subset of them) of the system to desired locations. A number of researchers have considered this problem in recent years. Amin (1985) and the Medanic (1988) works are capable of placing the real part of the eigenspectrum, while Saif's approach (1989) is capable of placing both the real and the imaginary part of the closed loop eigenvalues. Saif's approach is computationally more attractive because it is based on aggregating the system to a first, or a second degree one for placing a real or a complex conjugate pairs respectively, rather than the original (possibly high dimensional) system's equations which has to be dealt with in the other approaches. In this approach, for large scale systems, it is possible to assign a subset of the closed loop poles without altering the remaining ones. Thus we can decompose the system into subsystems of order one or two (depending on the real or imaginary eigenvalues respectively), and then we try to find the proper weighting matrix elements for the reduced order subsystems. The results obtained from these single subsystems would then enable us to arrive at the final solution of the original problem.

The above controllers would require the state of the system. Thus, there is a need to address the first and second issues in this thesis as well. If the load of the system was known, we would be able to estimate the state variables of the system and thereby realize the control input. While conventional observers can be used to estimate state variables of the system with known inputs, this approach would not be practical in LFC problem.

To address the decentralized state estimation task siljak(1978), Siljak and Vukcevic(1978) and Sundarereshan(1977) proposed the local estimators which have to communicate with one another. Ozguner(1977) addresses the problem of designing observers for a class of multilevel hierarchical systems with two time scale property. Another approach based on the design of unknown input observer UIO (Guan and Saif(1991)) was proposed by Saif and Guan(1992). For a class of interconnections, this approach can result in a totally decentralized estimators for large scale systems.

In this thesis, centralized as well as decentralized optimal modal control as well as estimation will be employed for LFC problem. The thesis consists of four chapters and one appendix. Chapter One is an overview to general ideas about linear optimal control (LOC), linear quadratic control (LQR) and different electrical power systems stabilizers with introducing related works and approaches.

Chapter Two addresses the modelling of the electrical two-area system with supplementing the exciter loop for the generators to take into consideration the effect of the exciters on the response of the system.

Chapter Three addresses the theoretical background for the optimal linear quadratic control. In this approach the special technique used in changing the state variable weighting matrix to accomplish the LQR is introduced.

In chapter Four optimal LQR technique is applied to two interconnected generators, and different simulation tests are carried out.

Finally in chapter Five the effectiveness of the LQR approach in enhancing stability of the systems is discussed and some advantages of that over other methods are reviewed. The appendix provides the main system data which is not in the main text.

CHAPTER 2

MODELLING

In order to study or alter the behavior of a dynamic system via feedback control a proper mathematical model is essential. There are various kinds of power system dynamics: high or low frequency oscillations, large or small system disturbances and large or small electrical power systems. Generally, there are a number of system components that are important to the dynamic study of the power systems such as the hydraulic and steam turbines, synchronous generator and the excitation system. For each of them, several basic models are recommended, and can be adapted for the studies of specific problems. Among the basic component models, that of the synchronous generator is probably the most important and complicated.

The selection of the synchronous generator model for power system dynamic studies depends not only on the nature of the problems itself, but also on the computational facilities and control techniques available. Yu (1983) gives the first, second, third and higher order synchronous generator models.

First order model is based on Park's equations which are fundamental equations for the synchronous machine. In the second order model the torque relation is described by differential equation. Third order model system takes into consideration the change in flux linkage of the field winding as well, while in high order model not only the field winding voltage relation, but also, the armature and damper winding voltage relations must be described by differential equations.

Like generators, we have different models for governors, turbines and exciters, whose dynamical model become more complex as a function of the degree of accuracy, specific design and the dynamical study performed. For example in transient stability study we have to use high order models, because the behavior of the system in the first cycles of the transient response is the main concern. Since our concern in this thesis is to study a two-area system with unknown load, in steady state, we'll use an extension of the model recommended by Elgerd (1971). Here we add an excitation loop to account for the effect of both megawatt and megavar control on the two-area system.

We have to mention also that the model we develop applies to small deviations around a nominal steady state. We use the model proposed by Elgerd mainly due to the fact that, known inputs (such as voltage and exciter voltage) and unknown inputs (such as the load and tie line power) are separated and the system matrices are independent of these values. In addition to that, most of the outputs (or state variables) are measurable, such as power, frequency, etc.

This model can be put into state space formulation, which is suitable for modern control application. It is assumed that a sudden change in the power demand will affect all the systems in the area simultaneously, and thus frequency deviation is the same every where in the system.

2.1) Power System Model

In this model we use the simple time delay transfer functions for the response of the speed governor, turbine and the exciter.

2.1.a) Speed Governor : (SG)

Speed governor regulates the synchronous speed of the generator, which is translated into frequency of the output electrical power and should be maintained to its nominal value. Speed governor performs its control via main steam control valve, where any change of its piston position change will increase or decrease the amount of steam flow into the turbine and therefore accelerate or decelerate the main turbine shaft speed. Main piston position change is performed by the change in hydraulic oil pressure of the piston. The relationship between hydraulic oil pressure change to that of main piston position change is given through the following transfer function:

$$\Delta x_e(s) = \Delta p_c \quad \frac{k_g}{1+st_g} \tag{2.1.1}$$

k_g, t_g : gain and time constants of (SG)

Δp_c : hydraulic oil pressure change

Δx_e : main piston position change for steam r : speed regulation due to governor action

Later we will deduct the effect of speed regulation from above equation.

2.1.b) Turbine : $g_t(s)$

Turbine converts the mechanical power of steam into the electrical power in the stator winding circuit. Exciter winding is located on the main turbine shaft and provides the electromagnetic field, necessary to induce voltage in the stator winding circuit. The dynamics of this subsystem is described by:

$$\Delta p_{gt} = \frac{k_t}{1 + st_t} \Delta x_e \tag{2.1.2}$$

Δp_{gt} : generating power change by turbine.

k_t, t_t : gain and time constants of a non-reheat turbine

2.1.c) Excitation : $g_e(s)$

Exciter regulates the stator winding voltage, and therefore can be regarded as one of our inputs, which can control output electrical power. Because of the exciter electrical nature its response is faster than speed governor, and that makes it desirable in feedback control. Usually generator voltage is compared to the nominal value and the difference (voltage error) is given to the exciter circuit as its input, and so exciter voltage increases or decreases to compensate for the negative or positive voltage error. The excitation system is described by:

$$\Delta e = \frac{k_e}{1 + st_e} \quad \Delta u \tag{2.1.3}$$

k_e, t_e: gain and time constants of exciter

u : absolute value of generator terminal voltage

e : absolute value of exciter voltage

2.1.d) Control Area (External System) : $g_p(s)$

_ Control area is regarded as that part of the electrical system which is to be controlled. Usually, the boundaries of the control areas coincide with those of the individual power systems belonging to the network, but its concept is really a relative one. For example the eastern and western power blocks in the US each contain many individual control areas. In general, the difference between generation and load demand of the system is absorbed in :

1) change in kinetic energy of the system

2) change in load consumption due to the change in frequency (d)

3) change in tie power . So

$$\Delta p_g - \Delta p_d = \frac{d}{dt} w_{kin} + d\Delta f + \Delta p_{tie} \qquad (2.1.4)$$

where:

p_g, p_d : generating and demanding power

and we can write :

$$w_{kin} = \left(\frac{\overset{\circ}{f} + \Delta f}{\overset{\circ}{f}}\right)^2 \overset{\circ}{w}_{kin} \approx \left(1 + 2\frac{\Delta f}{\overset{\circ}{f}}\right)\overset{\circ}{w}_{kin} \qquad (2.1.5)$$

where :

$$\Delta f = f - \overset{\circ}{f} \qquad (2.1.5a)$$

f and $\overset{\circ}{f}$: frequency and its nominal value

w_{kin} and $\overset{\circ}{w}_{kin}$: kinetic energy of the system and its nominal value .

If we differentiate the above equation and define $\overset{\circ}{w}_{kin}$ as the inertia of the system (h), then we get:

$$\frac{d}{dt}w_{kin} = 2\frac{\overset{\circ}{w}_{kin}}{\overset{\circ}{f}}\frac{d}{dt}\Delta f = 2\frac{h}{\overset{\circ}{f}}\frac{d}{dt}\Delta f \qquad (2.1.6)$$

by substituting (2.1.6) into (2.1.4) we get :

$$\Delta p_g - \Delta p_d = 2\frac{h}{\overset{\circ}{f}}\frac{d}{dt}(\Delta f) + d\Delta f + \Delta p_{tie} \qquad (2.1.7)$$

If the line losses are neglected ,the incremental tie power can be written in the form :

$$\Delta p_{tie} = \overset{\circ}{t}(\Delta\delta_1 - \Delta\delta_2) \qquad (2.1.8)$$

where $\overset{\circ}{t}$ is the synchronizing coefficient and δ_1 , δ_2 are the torque angles of the two machines.

Now by substituting the laplace transforms of the (2.1.8) in (2.1.7) and remembering that the frequency is the time derivative of the torque angle , we arrive at :

$$(\Delta p_g - \Delta p_d - \Delta p_{tie})g_p(s) = \Delta F(s) \qquad (2.1.9)$$

where :

$$g_p(s) = \frac{k_p}{1 + st_p} \qquad (2.1.9a)$$

h : inertia constant

d : rate of the load change of load due to the change of frequency

where $k_p = \frac{1}{d}$, $t_p = 2\frac{h}{\overset{\circ}{f}d}$

2.1.e) Generating Power Change due to the Exciter : Δp_{ge}

To notice the effect of exciter on the generating power of the system, we recall the generating power change equation in (2.1.9) :

$$p_g = \frac{ue}{x_s}\sin\delta \qquad (2.1.9b)$$

where u and e are voltage terminal and exciter voltages respectively, x_s is stator winding impedance and δ is the torque angle. By ignoring the change of p_g due to the change of u and by using (2.1.3), differentiation of the above equation gives:

$$\Delta p_{ge} = \bar{p}_{ge}\frac{k_e}{1+t_e s}\Delta u + p_{g\delta}\Delta\delta \qquad (2.1.9c)$$

where:

$$\bar{p}_{ge} \equiv \frac{\partial p_g}{\partial e} \quad , \quad p_{g\delta} \equiv \frac{\partial p_g}{\partial \delta}$$

by rearranging the above equation:

$$\Delta\dot{p}_{ge} = -\frac{1}{t_e}\Delta p_{ge} + \frac{k_e}{t_e}\bar{p}_{ge}\Delta u + \frac{p_{g\delta}}{t_e}\Delta\delta + p_{g\delta}\Delta f \qquad (2.1.10)$$

2.1.f) Total Generating Power Change : Δp_g

Now total generating power change of the system can be formulated as the difference between turbine generating power change and the generating power change due to the exciter. Exciter provides part of the generating power change needed to compensate for the load change. Therefore,

$$\Delta p_g = \Delta p_{gt} - \Delta p_{ge} \qquad (2.1.11)$$

2.1.g) Tie Power between systems : p_{tie}

Tie power is one of the main parameters of the interconnected electrical power systems that should be controlled and maintained in the nominal range of the individual systems. We know :

$$p_{tie} = \frac{u_1 u_2}{x_{12}} \sin(\delta_1 - \delta_2) \tag{2.1.12}$$

where u_1 , u_2 are terminal voltages of the two end machines and δ_1 , δ_2 are respective torque angles and x_{12} is the interconnecting tie line impedance. So we can write:

$$\Delta p_{tie} = \frac{\partial p_{tie}}{\partial u_1} \Delta u_1 + \frac{\partial p_{tie}}{\partial u_2} \Delta u_2 + \frac{\partial p_{tie}}{\partial(\delta_1 - \delta_2)} \Delta(\delta_1 - \delta_2) \tag{2.1.13}$$

If we define:

rate of tie power change by system No. 1 voltage change as:

$$p_{tu_1} \equiv \frac{\partial p_{tie}}{\partial u_1} = \frac{\overset{\circ}{u}_2}{x_{12}} \sin(\overset{\circ}{\delta}_1 - \overset{\circ}{\delta}_2)$$

rate of tie power change by system No. 2 voltage change as:

$$p_{tu_2} \equiv \frac{\partial p_{tie}}{\partial u_2} = \frac{\overset{\circ}{u}_1}{x_{12}} \sin(\overset{\circ}{\delta}_1 - \overset{\circ}{\delta}_2)$$

rate of tie power change by torque angle difference change as:

$$p_{t\delta} \equiv \frac{\partial p_{tie}}{\partial(\delta_1 - \delta_2)} = \frac{\overset{\circ}{u}_1 \overset{\circ}{u}_2}{x_{12}} \cos(\overset{\circ}{\delta}_1 - \overset{\circ}{\delta}_2)$$

then eq (2.1.13) becomes:

$$\Delta p_{tie} = p_{tu_1} \Delta u_1 + p_{tu_2} \Delta u_2 + p_{t\delta} \Delta \delta_1 - p_{t\delta} \Delta \delta_2 \tag{2.1.14}$$

here "o" means nominal values.

2.1.h) Rate of Load Change due to the change of Voltage : p_{du}

As we considered "d", rate of the load change due to the change of the frequency, we have to consider the rate of the load change due to the change of the voltage (p_{du}). For example, electrical load of motors is dependent on both voltage and frequency. We can define it as:

$$p_{du} \equiv \frac{\partial p_d}{\partial u}$$

2.2) State Space Formulation of the power system

2.2.a) Equations

At this point, we shall put the previous equations describing dynamic operation of the interconnected system into a state space formulation. This formulation is suitable for computer studies as well as application of modern control concepts.

1a) Frequency Equation System 1 :

$$\Delta\delta_1 = \frac{1}{s}\Delta F_1 \tag{2.2.1}$$

1b) Frequency Equation System 2 :

$$\Delta\delta_2 = \frac{1}{s}\Delta F_2 \tag{2.2.2}$$

2a) Control Area Equation System 1 (Eq 2.1.9) :

$$\Delta F_1 = k_{p1}\frac{1}{1+st_{p1}}(\Delta p_{g1} - \Delta p_{d1} - \Delta p_{tie1} - p_{du1}\Delta u_1) \tag{2.2.3}$$

2b) Control Area Equation System 2 :

$$\Delta F_2 = k_{p2}\frac{1}{1+st_{p2}}(\Delta p_{g2} - \Delta p_{d2} - a_{12}\Delta p_{tie2} - p_{du2}\Delta u_2) \tag{2.2.4}$$

a_{12} :transmission coefficient of Δp_{tie1} into system 2

3a) Speed Governor System 1 (Eq 2.1.1) :

$$\Delta x_{e1} = \frac{k_{g1}}{1+st_{g1}}\left(\Delta p_{c1} - \frac{1}{r_1}\Delta f_1\right) \tag{2.2.5}$$

3b) Speed Governor System 2 :

$$\Delta x_{e2} = \frac{k_{g2}}{1+st_{g2}}\left(\Delta p_{c2} - \frac{1}{r_2}\Delta f_2\right) \tag{2.2.6}$$

4) Tie Power between Systems (Eq 2.1.11) :

$$\Delta p_{tie1} = p_{tu_1}\Delta u_1 + p_{tu_2}\Delta u_2 + p_{t\delta}\Delta\delta_1 - p_{t\delta}\Delta\delta_2 \tag{2.2.7}$$

5a) Generating Power Change in System 1 due to the Exciter in System 1 :

$$\Delta p_{ge1} = \overline{p}_{ge1}\frac{k_{e1}}{1+t_{te1}s}\Delta u_1 + p_{g\delta 1}\Delta\delta_1 \tag{2.2.8}$$

5b) Generating Power Change in System 2 due to the Exciter in System 2 :

$$\Delta p_{ge2} = \overline{p}_{ge2}\frac{k_{e2}}{1+t_{te2}s}\Delta u_1 + p_{g\delta 2}\Delta\delta_2 \tag{2.2.9}$$

6a) Total Generating Power Change in System 1 (Eq 2.1.11) :

$$\Delta p_{g1} = \Delta p_{gt1} - \Delta p_{ge1} \tag{2.2.10}$$

6b) Total Generating Power Change in System 2 :

$$\Delta p_{g2} = \Delta p_{gt2} - \Delta p_{ge2} \tag{2.2.11}$$

7a) Turbine in System 1 :

$$\Delta p_{gt1} = \frac{k_{t1}}{1 + st_{t1}} \Delta x_{e1} \tag{2.2.12}$$

7b) Turbine in System 2 :

$$\Delta p_{gt2} = \frac{k_{t2}}{1 + st_{t2}} \Delta x_{e2} \tag{2.2.13}$$

2.2.b) State Equations :

Now we write each of the above equations in state space form :

State Equation No.1 :

$$\delta_1 = f_1$$

State Equation No.2 :

$$\delta_2 = f_2$$

State Equation No.3 :

$$\Delta \dot{x}_{e1} = \frac{1}{t_{g1}} \left(-\Delta x_{e1} + k_{g1} \Delta p_{c1} - \frac{k_{g1}}{R_1} \Delta f_1 \right)$$

State Equation No.4 :

$$\Delta \dot{x}_{e2} = \frac{1}{t_{g2}} \left(-\Delta x_{e2} + k_{g2} \Delta p_{c2} - \frac{k_{g2}}{R_2} \Delta f_2 \right)$$

State Equation No.5 :

$$\Delta \dot{p}_{ge1} = -\frac{1}{t_{e1}} \Delta p_{ge1} + \frac{1}{t_{e1}} \overline{p}_{ge1} k_{e1} \Delta u_1 + \frac{1}{t_{e1}} p_{g\delta 1} \Delta \delta_1 + p_{g\delta 1} \Delta f_1$$

State Equation No.6 :

$$\Delta\dot{p}_{ge2} = -\frac{1}{t_{e2}}\Delta p_{ge2} + \frac{1}{t_{e2}}\overline{p}_{ge2}k_{e2}\Delta u_2 + \frac{1}{t_{e2}}p_{g\delta 2}\Delta\delta_2 + p_{g\delta 2}\Delta f_2$$

State Equation No.7 :

$$\Delta\dot{f}_1 = -\frac{1}{t_{p1}}\Delta f_1 + \frac{k_{p1}}{t_{p1}}(\Delta p_{g1} - \Delta p_{d1} - p_{tu2}\Delta u_2 - p_{t\delta}\Delta\delta_1 + p_{t\delta}\Delta\delta_2 - (p_{tu1} + p_{du1})\Delta u_1)$$

State Equation No.8 :

$$\Delta\dot{f}_2 = -\frac{1}{t_{p2}}\Delta f_2 + \frac{k_{p2}}{t_{p2}}(\Delta p_{g2} - \Delta p_{d2} - a_{12}p_{tu1}\Delta u_1 - a_{12}p_{t\delta}\Delta\delta_1 + a_{12}p_{t\delta}\Delta\delta_2 - (a_{12}p_{tu2} + p_{du2})\Delta u_2)$$

State Equation No.9 :

$$\Delta\dot{p}_{g1} = -\frac{1}{t_{t1}}\Delta p_{g1} + \frac{k_{t1}}{t_{t1}}\Delta x_{e1} + \left(\frac{1}{t_{e1}} - \frac{1}{t_{t1}}\right)\Delta p_{ge1} - p_{g\delta 1}\Delta f_1 - \frac{1}{t_{e1}}\overline{p}_{ge1}k_{e1}\Delta u_1 - \frac{1}{t_{e1}}p_{g\delta 1}\Delta\delta_1$$

State Equation No.10:

$$\Delta\dot{p}_{g2} = -\frac{1}{t_{t2}}\Delta p_{g2} + \frac{k_{t2}}{t_{t2}}\Delta x_{e2} + \left(\frac{1}{t_{e2}} - \frac{1}{t_{t2}}\right)\Delta p_{ge2} - p_{g\delta 2}\Delta f_2 - \frac{1}{t_{e2}}\overline{p}_{ge2}k_{e2}\Delta u_2 - \frac{1}{t_{e2}}p_{g\delta 2}\Delta\delta_2$$

2.2.c) State Space Model:

According to the above equations, we can construct the state space model of two area system as:

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} + \Gamma\mathbf{p} \qquad (2.2.14)$$

where:(index 1 and 2 refer to machine No. 1 or No. 2 respectively)

State Variable :

$$\mathbf{x} \equiv [\Delta f_1 \quad \Delta f_2 \quad \Delta\delta_1 \quad \Delta\delta_2 \quad \Delta p_{g1} \quad \Delta p_{g2} \quad \Delta p_{ge1} \quad \Delta p_{ge2} \quad \Delta x_{e1} \quad \Delta x_{e2}]^t \qquad (2.2.15)$$

Control Input :

$$\mathbf{u} \equiv [\Delta p_{c1} \quad \Delta p_{c2} \quad \Delta u_1 \quad \Delta u_2]' \tag{2.2.16}$$

where unknown inputs :

$$\mathbf{p} \equiv [\Delta p_{d1} \quad \Delta p_{d2}]' \tag{2.2.17}$$

Unknown Input matrix :

$$\Gamma = \begin{bmatrix} -\frac{k_{p1}}{t_{p1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{k_{p2}}{t_{p2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.2.18}$$

Known Input matrix :

$$B = \begin{pmatrix} 0 & 0 & -(p_{tu1}+p_{du1})\dfrac{k_{p1}}{t_{p1}} & -p_{tu2}\dfrac{k_{p1}}{t_{p1}} \\ 0 & 0 & -a_{12}p_{tu1}\dfrac{k_{p2}}{t_{p2}} & -(a_{12}p_{tu2}+p_{du2})\dfrac{k_{p2}}{t_{p2}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -k_{e1}\dfrac{\overline{p}_{ge1}}{t_{e1}} & 0 \\ 0 & 0 & 0 & -k_{e2}\dfrac{\overline{p}_{ge2}}{t_{e2}} \\ 0 & 0 & \overline{p}_{ge1}\dfrac{k_{e1}}{t_{e1}} & 0 \\ 0 & 0 & 0 & \overline{p}_{ge2}\dfrac{k_{e2}}{t_{e2}} \\ \dfrac{k_{g1}}{t_{g1}} & 0 & 0 & 0 \\ 0 & \dfrac{k_{g2}}{t_{g2}} & 0 & 0 \end{pmatrix} \tag{2.2.19}$$

System matrix :

$$A=\begin{bmatrix}
-\frac{1}{t_{p1}} & 0 & -\frac{k_{p1}}{t_{p1}}p_{t\delta} & \frac{k_{p1}}{t_{p1}}p_{t\delta} & \frac{k_{p1}}{t_{p1}} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{t_{p2}} & 0 & -a_{12}\frac{k_{p2}}{t_{p2}}p_{t\delta} & a_{12}\frac{k_{p2}}{t_{p2}}p_{t\delta} & \frac{k_{p2}}{t_{p2}} & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-p_{g\delta1} & 0 & -\frac{p_{g\delta1}}{t_{e1}} & 0 & -\frac{1}{t_{t1}} & 0 & -\frac{1}{t_{t1}}+\frac{1}{t_{e1}} & 0 & \frac{k_{t1}}{t_{t1}} & 0 \\
0 & -p_{g\delta2} & 0 & -\frac{p_{g\delta2}}{t_{e2}} & 0 & -\frac{1}{t_{t2}} & 0 & -\frac{1}{t_{t2}}+\frac{1}{t_{e2}} & 0 & \frac{k_{t2}}{t_{t2}} \\
p_{g\delta1} & 0 & \frac{p_{g\delta1}}{t_{e1}} & 0 & 0 & 0 & -\frac{1}{t_{e1}} & 0 & 0 & 0 \\
0 & p_{g\delta2} & 0 & \frac{p_{g\delta2}}{t_{e2}} & 0 & 0 & 0 & -\frac{1}{t_{e2}} & 0 & 0 \\
-\frac{k_{g1}}{R_1 t_{g1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{t_{g1}} & 0 \\
0 & -\frac{k_{g2}}{R_2 t_{g2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{t_{g2}}
\end{bmatrix} \tag{2.2.20}$$

If we ignore the variation of systems terminal voltages, or in other words let both control inputs Δu_1 and Δ_2 equal to zero, we arrive at the two area system model of Reddoch (1971) where:

$$\mathbf{x}=[\Delta p_{tie},\Delta f_1,\Delta p_{g1},\Delta x_{e1},\Delta f_2,\Delta p_{g2},\Delta x_{e2}]' \tag{2.2.21}$$

$$\mathbf{u}=[\Delta p_{c1},\Delta p_{c2}]' \tag{2.2.22}$$

$$\mathbf{p}=[\Delta p_{d1},\Delta p_{d2}] \tag{2.2.23}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{t_{g1}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{t_{g2}} \end{bmatrix} \tag{2.2.24}$$

$$\Gamma = \begin{bmatrix} 0 & \frac{-f^{\circ}}{2H_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-f^{\circ}}{2H_2} & 0 & 0 \end{bmatrix} \tag{2.2.25}$$

In fiure 1a, two-area system model and in figure 1b, elements of a single area power system has been shown.

SUMMARY

In this chapter we described the main components of the two area system, containing two interconnected generators. Each component input-output characteristic was formulated, and then the equations of the two area system were constructed and rewritten into state space model. The next step is to design a controller capable of maintaining the system stability and also providing desirable transient behavior with a unknown varying load demand. In the next chapter we address the LFC problem, where we shall give theoretical background in the design of modal controllers and estimators for the systems with unknown inputs.

Figure 1a- Two-Area System Model

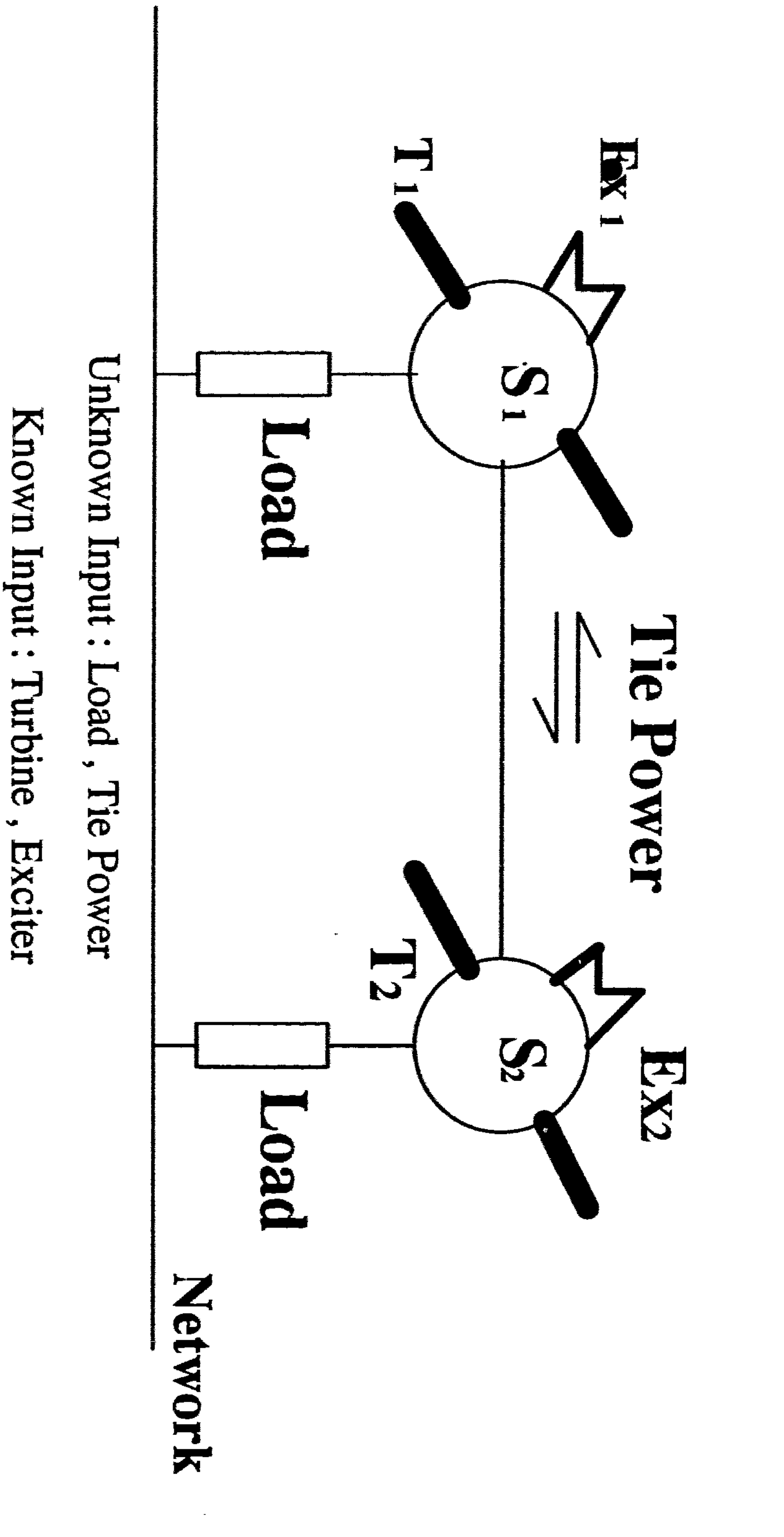
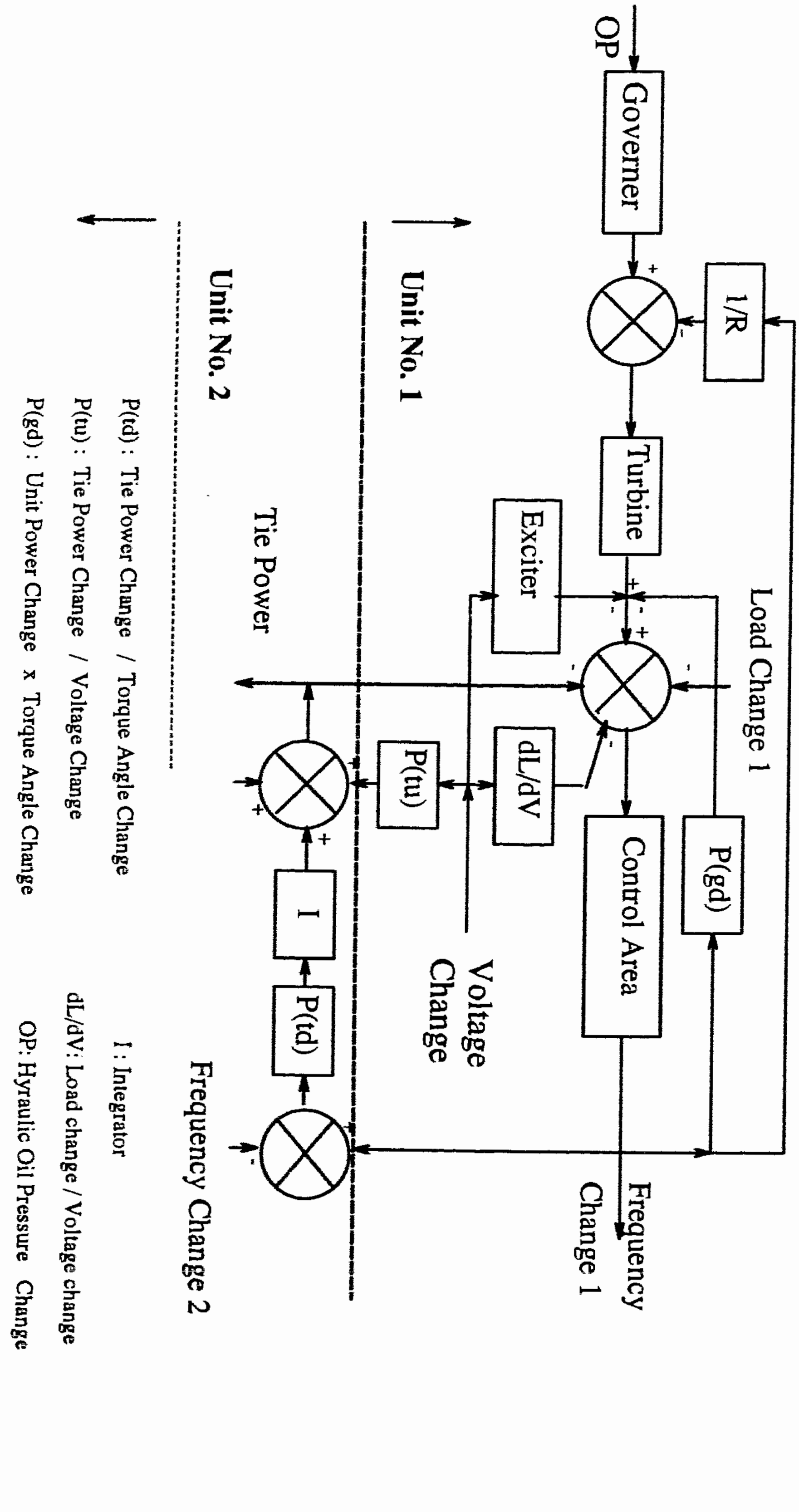


Figure 1b- Elements of a Single Area Power System



CHAPTER 3

BACKGROUND AND RESEARCH OVERVIEW

In this chapter we shall study control techniques and implement them for the LFC to ensure a stable and reliable operation under varying load demand. The control approach to be used is based on LQR theory.

The linear quadratic regulator theory (LQR) is one of the most powerful techniques for designing multivariable control systems, and has several desirable properties such as good sensitivity and robustness behavior. The LQR problem is a multiobjective optimization task, namely the regulation of the state trajectories and minimizing the control efforts (Saif 1989). The elements of the weights on the states (Q) and the controls (R) are indicators of the relative importance of each of them with respect to others. It is well known that the transient behavior of the closed loop system can be modified by changing these matrices. Unfortunately, there is no systematic way of controlling the transient behavior of the system through selection of appropriate Q and R. As a result, selection of these weights has been a problem for long time (Saif and Villaseca (1986)). In general Q and R elements are chosen as diagonal positive (semi) definite matrices, but as we will see later these assumptions are not necessary.

The controller that we shall supply in this study will combine the minimization of the cost functional of the system and pole placement of the eigenvalues simultaneously. This is achieved through proper selection of Q and R, to satisfy both objectives. The control design is achieved in a sequential manner. The sequential procedure amounts to aggregating the system into smaller subsystems whose controller design is simpler to solve and the individual solutions are added up to find the overall optimal control law and weights.

The controller obtained using the above procedure would require the availability of the states of the system. As a result, estimation of the state is necessary for implementation of this controller. It should be noted however, that standard estimation technique based on Luenburger Observer or Kalman Filter assume that the inputs to the system are completely known. However, as we have seen in the previous chapter this need not be the case. It is clear from Chapter 2 that the two area system is influenced by the controlled inputs which are obviously known as well as the load demand which is an unknown input to the system. Therefore, a special type of estimator need to be used. The unknown input observer (UIO) is such an estimator. This is another unseperable part of the control system design.

In the following discussion, we shall briefly discuss the controller as well as the estimator design. It should be noted that we shall not attempt to prove various results, and only the necessary material for LFC is covered here. More details could be found in Saif (1989) and references given there.

3.1) Optimal Modal Linear Quadratic Regulator (LQR)

Consider the following linear dynamical system:

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \quad , \quad \mathbf{x}(0) = \mathbf{x}_0 \tag{3.1.1}$$

where $\mathbf{x} \in R^n$ and $\mathbf{u} \in R^m$ are state variable and control input of the system. The standard LQR problem objective is to find an optimal control law which would minimize the following quadratic cost functional subject to (3.1.1).

$$J = \frac{1}{2}\int_0^\infty (\| \mathbf{x} \|_Q^2 + \| \mathbf{u} \|_R^2)dt \tag{3.1.2}$$

where Q and R are weighting matrices for state and input variables respectively.

The optimal control law for the above problem is given by Yu (1983))

$$\mathbf{u} = -R^{-1}B'p\mathbf{x} \tag{3.1.3}$$

where p is the symmetric positive semidefinite solution of the Algebric Matrix Riccati Equation (AMRE).

$$PA + A'P - PBR^{-1}B'P + Q = O \tag{3.1.4}$$

An alternative solution for AMRE can be found by defining Hamiltonian matrix H as :

$$H = \begin{bmatrix} A & -BR^{-1}B' \\ -Q & -A' \end{bmatrix} \tag{3.1.5}$$

if H has no eigenvalue with zero real part and (A, B) is stabilizable, then the solution of the AMRE can be obtained as:

$$p = ba^{-1} \tag{3.1.6}$$

where nxn matrices 'a' and 'b' are given by :

$$V_i = \begin{bmatrix} a_i \\ b_i \end{bmatrix}$$

$$a = [a_1 \cdots a_n]$$

$$b = [b_1 \cdots b_n]$$

where V_i is an eigenvector (or generalized eigenvector) associated with stable eigenvalues of H and 'n' is the order of system matrix A.

3.1.1) Real Shifting of Eigenvalues:

It can be shown that for any fixed R>0, by properly selecting of Q one can place the entire closed-loop eigenspectrum to the left of any vertical line defined in the left hand side. This can be done as follows:

We know that the closed-loop eigenspectrum of the system is given by

$$A_c = A - BR^{-1}B'p \tag{3.1.7}$$

Now if we select:

$$\hat{Q} = Q - 2\varepsilon P^- \tag{3.1.8}$$

where Q is any initial weight matrix, P^- is the unstable solution of (AMRE) and ε is a real number . Then

$$\Lambda(\hat{A}_c) = \Lambda(A - BR^{-1}B'\hat{P}) = \Lambda(A_c) - 2\varepsilon \tag{3.1.9}$$

Furthermore, the optimal control law that achieves this placement is given by

$$\mathbf{u} = -R^{-1}B'\hat{P}\mathbf{x} = -K\mathbf{x} \tag{3.1.10}$$

where $\hat{A}_c$ is the new closed loop system matrix, K is the control gain and $\hat{P}$ is the stable solution of the AMRE obtained from the following Hamiltonian matrix,

$$\hat{H} = \begin{bmatrix} A + \varepsilon I & -BR^{-1}B' \\ -\hat{Q} & -(A' + \varepsilon I) \end{bmatrix} \tag{3.1.11}$$

Thus we can conclude that by appropriate selection of ε, the real part of the entire closed-loop eigenvalues can be shifted to any desired values in the left hand plane.

3.1.2) Placing a Subset of Poles:

In the previous section, we discussed the idea of shifting the entire eigenvalues. To shift a subset of the eigenvalues the following approach which uses aggregation of the system can be used. Consider the reduced-order (aggregated) $2l$x$2l$ Hamiltonian system (l<n)

$$\tilde{H} = \begin{bmatrix} \tilde{A} & -\tilde{B}R^{-1}\tilde{B}' \\ -\tilde{Q} & -\tilde{A}' \end{bmatrix} \tag{3.1.12}$$

here " $(\cdot\tilde{\cdot}\cdot)$ " refers to aggregated values such that :

$$\Lambda(\tilde{A}) \subset \Lambda(A) = \{\lambda_1 \cdots \lambda_l \cdots \lambda_n\}$$

For a choice of $\tilde{Q}$,suppose Q in (3.1.4) is selected as

$$Q = C'\tilde{Q}C \tag{3.1.13}$$

where C is an $l$xn, full rank matrix given by:

$$C = [I_l \mid 0]M^{-1} \tag{3.1.14}$$

where M is modal matrix of A with its first l columns being the eigenvectors $\{V_1 \cdots V_l\}$ corresponding to $\{\lambda_1 \cdots \lambda_l\}$. Then for the given choice of Q, the eigenvalues of H are those of $\tilde{H}$ plus (n-1) eigenvalues $\overline{\Lambda(A) \cap \Lambda(\tilde{A})}$ and their corresponding mirror images about the imaginary axis and

$$\tilde{A} = CAC^{+} \quad , \quad \tilde{B} = CB \tag{3.1.15}$$

where $C^+ = C'(CC')^{-1}$: Pseudo Inverse of C (Aoki 1968)

In summary, with the help of above results, we are able to :

1) find the optimum control gain.

2) shift all eigenspectrum to the left of the imaginary axis by the amount of ε.

3) With the aid of the previous results one can shift all or a subset of the closed loop eigenvalues of the system. This subset of eigenvalues can have as few as one real pole or a pair of complex conjugate poles in which case through repeated application of the above procedures all of the eigenvalues can be assigned to different locations in the left hand plane.

Next algorithm gives us the procedure to assign the closed-loop eigenvalues of the systems.

3.1.3) Optimal Modal Controller Design Algorithm (M. Saif 1989)

Step 1

Let $A_i = A$, the sequential procedure starts at stage (i=0):

a) If a real open pole λ_o is to be placed at λ_d, use transformation given in (3.1.15) to obtain $\hat{A}_i$ and $\hat{B}_i$.

b) If a complex conjugate pair $-\alpha \pm j\beta$ is to be assigned to $-\sigma \pm j\gamma$, use the transformation below:

Suppose $\tilde{A}$ in (3.1.14) be given by :

$$\tilde{A} = \begin{bmatrix} -\alpha + j\beta & 0 \\ 0 & -\alpha - j\beta \end{bmatrix} \tag{3.1.16}$$

In order to work with real matrices consider the transformation L given by :

$$L = \begin{bmatrix} .5 & j.5 \\ .5 & -j.5 \end{bmatrix} \tag{3.1.17}$$

where now $\tilde{A}$ is given by :

$$\tilde{A} = L^{-1}CAC^{+}L = \begin{bmatrix} -\alpha & \beta \\ -\beta & -\alpha \end{bmatrix} \tag{3.1.18}$$

and accordingly:

$$\tilde{B} = L^{-1}CB \tag{3.1.19}$$

c) If a complex conjugate pair $-\alpha \pm j\beta$ is to be assigned to two distinct real locations $-\nu$ and $-\mu$, use the same transformation b) to obtain $\hat{A}_i$ and $\hat{B}_i$.

<u>Step 2</u>

a) For a value of $\hat{Q}_i$, construct $\hat{H}_i$ in (3.1.12), and obtain the unstable solution to the AMRE ($\hat{P}^{-}$) from (3.1.6).

b) Find the appropriate $\hat{Q}_i = qI$ to achieve the desired imaginary part γ and the new real part of the eigenvalue δ using the following relations:

$$q = \frac{[2(\delta^2 + \beta^2 - \gamma^2 - \alpha^2)]}{(e_1 + e_3)} \tag{3.1.20}$$

$$(\delta^2 + \gamma^2)^2 = q(e_1 + e_3)(\alpha^2 + \beta^2) + q^2(e_1 e_3 - e_2^2) + (\alpha^2 + \beta^2)^2 \tag{3.1.21}$$

$$E = \begin{bmatrix} e_1 & e_2 \\ e_2 & e_3 \end{bmatrix} = \tilde{B}R^{-1}\tilde{B}' \tag{3.1.22}$$

Using the value of $\hat{Q}_i$ obtained perform Step 2 a).

c) Find the appropriate $\hat{Q}_i = qI$ to achieve one of the desired real poles (say $-\nu$) and the other one ϕ by:

$$q = \frac{[(\phi^2 + \nu^2) - 2(\alpha^2 - \beta^2)]}{e_1 + e_3} \tag{3.1.23}$$

$$\phi^2\nu^2 = q(e_1 + e_3)(\alpha^2 + \beta^2) + q^2(e_1e_3 - e_2^2) + (\alpha^2 + \beta^2)^2 \tag{3.1.24}$$

<u>*Step 3*</u>

Calculate the value of ε in theorem 2 as

a) $\varepsilon = (|\lambda_d| - |\lambda_c|)/2$; where $\pm\lambda_c$ are the eigenvalues of $\hat{H}_i$ with $\hat{Q}_i$.

b) $\varepsilon = (|\sigma| - |\delta|)/2$.

c) Go to Step 5.

<u>*Step 4*</u>

a, b) Calculate the appropriate $\tilde{Q}_i$ for proper pole placement according to (3.1.8).

<u>*Step 5*</u>

a, b) Find the stable solution of the AMRE ($\tilde{P}$) corresponding to the Hamiltonian system $\tilde{H}_i$ given in (3.1.11).

c) Find the stable solution of the AMRE ($\hat{P}$) corresponding to Hamiltonian system $\tilde{H}_i$ in (3.1.12).

<u>*Step 6*</u>

Let $\hat{Q}_\epsilon$ be the desired value of the weighting matrix that accomplishes the pole placement for the aggregated system, then the value of this weighting matrix is

a, b)
$$\hat{Q}_\epsilon = \tilde{Q}_i + 2\varepsilon\tilde{P} \tag{3.1.25}$$

c) $$\hat{Q}_{\epsilon} = \hat{Q}_i \tag{3.1.26}$$

Step 7

Calculate $\hat{K}_i$ given by

a, b) $$\hat{K}_i = R^{-1}\hat{B}_i'\tilde{P} \tag{3.1.27}$$

c) $$\hat{K}_i = R^{-1}\hat{B}_i'\hat{P} \tag{3.1.28}$$

Step 8

Use the following to obtain the desired weighting matrix, and the optimal feedback gain for the original higher dimensional system,

a) $$Q_{\epsilon} = C'\hat{Q}_{\epsilon}C \tag{3.1.29}$$

$$K_i = \hat{K}_i C \tag{3.1.30}$$

b, c) $$Q_{\epsilon} = C'L^{-\prime}\hat{Q}_{\epsilon}L^{-1}C \tag{3.1.31}$$

$$K_i = \hat{K}_i L^{-1}C \tag{3.1.32}$$

Step 9

Let

$$A_{i+1} = A_i - BK_i \tag{3.1.33}$$

Step 10

If all the eigenvalues are placed, stop and find Q_d as

$$Q_d = \sum_i Q_{\epsilon} \tag{3.1.34}$$

and the optimal gain

$$K^{*} = \sum_{i} K_i \tag{3.1.35}$$

otherwise let i=i+1, and go to Step 1.

Remarks

In pole placement algorithm mentioned earlier , we have four possibilities:

b.1 : real to real change ; real pole ==> real pole

$$-\alpha \Rightarrow -\hat{\alpha}$$

b.2 : complex shift change ; complex pole ==> complex pole (equal imaginary part)

$$-\alpha \pm j\beta \Rightarrow -\hat{\alpha} \pm j\beta$$

b.3 : complex to real change ; complex pole ==> two real poles

$$-\alpha \pm j\beta \Rightarrow -\mu, -\nu$$

b.4 : complex to complex change ; complex pole ==>complex pole

$$-\alpha \pm j\beta \Rightarrow -\sigma \pm j\gamma$$

Both cases (b-1) and (b-2), can be done unconditionally. (except $\alpha = 0$). In case (b-3), we can do it if two necessary conditions are satisfied. By using equations (3.1.23) and (3.1.24), we construct the equation of fourth degree in ϕ

$$a_2\phi^4 + b_2\phi^2 + c_2 = 0 \tag{3.1.36}$$

with the following coefficients:

$$a_2 = \frac{(e_1e_3 - e_2^2)}{(e_1 + e_3)^2} \tag{3.1.37}$$

$$b_2 = -\nu^2 + \alpha^2 + \beta^2 + 2a_2(\nu^2 - 2\alpha^2 + 2\beta^2) \tag{3.1.38}$$

$$c_2 = (\alpha^2+\beta^2)^2 + (\nu^2 - 2\alpha^2 + 2\beta^2)(\alpha^2+\beta^2) + a_2(\nu^2 - 2\alpha^2 + 2\beta^2) \tag{3.1.39}$$

The necessary conditions are:

If $a_2 \neq 0$

$$\text{a:} \quad \Delta = b_2^2 - 4a_2c_2 \geq 0$$

$$b: \quad \Delta' = ((-b_2 \pm \Delta^{1/2})/2a_2) > 0 \tag{3.2.37}$$

If $a_2 = 0$

$$\frac{c_2}{b_2} > 0 \quad (b_2 \neq 0) \tag{3.1.40}$$

In case (b-4), also we can find the similar conditions. From (3.1.20) and (3.1.21), and with the same procedures, we can find the coefficients of the fourth degree equation and the necessary conditions to have a solution:

$$a_1 = 1 - 4\frac{(e_1e_3 - e_2^2)}{(e_1+e_3)^2} \tag{3.1.41}$$

$$b_1 = 2\gamma^2 - 2(\alpha^2+\beta^2) - 8(e_1e_3 - e_2^2)\frac{(\beta^2 - \gamma^2 - \alpha^2)}{(e_1+e_3)^2} \tag{3.1.42}$$

$$c_1 = \gamma^4 - 2(\alpha^2+\beta^2)(\beta^2-\gamma^2-\alpha^2) - (\alpha^2+\beta^2)^2 - 4(\beta^2-\gamma^2-\alpha^2)^2(e_1e_3 - e_2^2) \tag{3.1.43}$$

The necessary conditions are :

If $a_1 \neq 0$

$$\text{a :} \quad \Delta = b_1^2 - 4a_1c_1 \geq 0$$

$$b: \quad \Delta^{'} = ((-b_1 \pm \Delta^{1/2})/2a_1) > 0 \tag{3.1.44}$$

If $a_1 = 0$,the only condition is :

$$\frac{c_1}{b_1} > 0 \quad (b_1 \neq 0) \tag{3.1.45}$$

Inequalities for Δ and $\Delta^{'}$ in both cases are complicated and can't give a explicit inequality in terms of parameters, but apparently for $\gamma > \beta$ there will be no solution.

This will conclude the controller design using an optimal modal approach. However, as can be seen from (3.1.3), the control law designed using this scheme requires the availability of the power system state for feedback purposes. This however is rarely the case in practice. As a result an estimator capable of estimating the state of the system in the face of unknown load demand variation is needed. The unknown input observer (UIO) is such a estimator and will be presented in the next section. Again, only the necessary material for implementation of the estimator for LFC purpose will be covered here. Interested reader should refer to Saif and Guan (1992), and Guan and Saif (1991) and references cited there for more detail.

3.2 Estimator Design

Recall from (2.2.14) that the two area system can be described using the following state space formulation:

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} + \Gamma\mathbf{p}$$

$$\mathbf{y} = C\mathbf{x}$$

The output of the system is usually considered to be frequency and the tie power but as we will explain it in the chapter four, to realize LFC, area control error of the system should be taken as output of the system. In the last section, we ignored the effect of the unknown input **p** (load), and our basic assumption was that state vector **x** is available and equation (3.1.2) gave the proper control input to find optimum performance index J (3.1.1b). With the presence of **p**, we can't use normal observers and we need a unknown input observer which, with different load **p**, still can estimate state vector **x**. Suppose our time invariant system can be represented as:

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} + \Gamma\mathbf{p} \tag{3.2.1}$$

$$\mathbf{y} = C\mathbf{x} = [0 \quad I]\mathbf{x} \tag{3.2.2}$$

where $\mathbf{x} \in R^n, \mathbf{u} \in R^q, \mathbf{p} \in R^m, \mathbf{y} \in R^p$ are the state, known input, unknown input and output of the system respectively . Special form assumed for matrix C defined in (3.2.2) is not a restrictive assumption, since as long as C is a full rank matrix, there exists a similarity transformation that if applied to the system, will result the desired output matrix. The existence condition for (UIO) states that for designing a stable observer it is necessary that:

$$rank(C\Gamma) = m \quad with \quad m \le p \tag{3.2.3}$$

3.2.1 Case 1 (Number of unknown inputs are less than the outputs (m<p))

Partition (3.2.1) and (3.2.2) as follows:

$$\dot{\mathbf{x}} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} \mathbf{x} + \begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} \mathbf{u} + \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{pmatrix} \mathbf{p} \tag{3.2.4}$$

$$\begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{pmatrix} = \begin{pmatrix} 0 & I_{(p-m)x(p-m)} & 0 \\ 0 & 0 & I_{mxm} \end{pmatrix} \mathbf{x} \tag{3.2.5}$$

where:

$A_1 \in R^{(n-p)xn}$, $A_2 \in R^{(p-m)xn}$, $A_3 \in R^{mxn}$

$B_1 \in R^{(n-p)xq}$, $B_2 \in R^{(p-m)xq}$, $B_3 \in R^{mxq}$

$\Gamma_1 \in R^{(n-p)xm}$, $\Gamma_2 \in R^{(p-m)xm}$, $\Gamma_3 \in R^{mxm}$

and the state vector **x** is partitioned as:

$$\mathbf{x} = \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{pmatrix} = \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{y}_1 \\ \mathbf{y}_2 \end{pmatrix} \tag{3.2.6}$$

and $x_1 \in R^{n-p}$ is the vector whose estimate is required. From the structure of the matrix C and the necessary condition assumed above, it can be shown easily:

$$rank \begin{pmatrix} \Gamma_2 \\ \Gamma_3 \end{pmatrix} = m \tag{3.2.7}$$

From (3.2.7), without loss of any generality, we can assume D_3 is nonsingular, therefore the following operator can be defined:

$$T \equiv \begin{pmatrix} I & 0 & -\Gamma_1\Gamma_3^{-1} \\ 0 & I & -\Gamma_2\Gamma_3^{-1} \\ 0 & 0 & I \end{pmatrix} \tag{3.2.8}$$

postmultipling (3.2.4) with operator T, yields :

$$\begin{pmatrix} \dot{\mathbf{x}}_1 - \Gamma_1\Gamma_3^{-1}\dot{\mathbf{y}}_2 \\ \dot{\mathbf{y}}_1 - \Gamma_2\Gamma_3^{-1}\dot{\mathbf{y}}_2 \\ \dot{\mathbf{y}}_2 \end{pmatrix} = \begin{pmatrix} A_1 - \Gamma_1\Gamma_3^{-1}A_3 \\ A_2 - \Gamma_2\Gamma_3^{-1}A_3 \\ A_3 \end{pmatrix}\mathbf{x} + \begin{pmatrix} B_1 - \Gamma_1\Gamma_3^{-1}B_3 \\ B_3 \end{pmatrix}\mathbf{u} + \begin{pmatrix} 0 \\ 0 \\ \Gamma_3 \end{pmatrix}\mathbf{p} \tag{3.2.9}$$

As a result of performing this operation, it is clear that in (3.2.9) the unknown inputs enter only through the third row and the first two rows are independent of any unknown inputs. Thus by defining:

$$\overline{A}_i \equiv A_i - \Gamma_i\Gamma_3^{-1}A_3 \tag{3.2.10}$$

$$\overline{B}_i \equiv B_i - \Gamma_i\Gamma_3^{-1}B_3 \tag{3.2.11}$$

The first two rows of (3.2.9) can be written as:

$$\dot{\mathbf{x}}_1 - \Gamma_1\Gamma_3^{-1}\dot{\mathbf{y}}_2 = \overline{A}_1\mathbf{x} + \overline{B}_1\mathbf{u} \tag{3.2.12}$$

$$\dot{\mathbf{y}}_1 - \Gamma_2\Gamma_3^{-1}\dot{\mathbf{y}}_2 = \overline{A}_2\mathbf{x} + \overline{B}_2\mathbf{u} \tag{3.2.13}$$

By partitioning $\overline{A}_i$ as :

$$\overline{A}_i = [\overline{A}_{i1} \quad \overline{A}_{i2} \quad \overline{A}_{i3}] \tag{3.2.14}$$

and some calculations, we arrive at the following theorem:

Theorem: If the pair $\{\overline{A}_{11}, \overline{A}_{12}\}$ is observable, the state of the dynamical system given in (3.2.1) can be estimated by using the UIO. The estimate of the state variable is given by:

$$\hat{\mathbf{x}} = \begin{pmatrix} I \\ 0 \end{pmatrix}\mathbf{w} + \begin{pmatrix} N \\ I \end{pmatrix}\mathbf{y} \tag{3.2.15}$$

where $\mathbf{w}$ is the vector satisfying:

$$\dot{\mathbf{w}} = F\mathbf{w} + EC\mathbf{x} + L\mathbf{u} \tag{3.2.16}$$

and the remaining parameters are given as:

$$H = (\bar{A}_{13} - M\bar{A}_{23}) + (\bar{A}_{11} - M\bar{A}_{21})(\Gamma_1 - M\Gamma_2)\Gamma_3^{-1} \tag{3.2.17}$$

$$G = (\bar{A}_{12} - M\bar{A}_{22}) + (\bar{A}_{11} - M\bar{A}_{21})M \tag{3.2.18}$$

$$F = \bar{A}_{11} - M\bar{A}_{21} \tag{3.2.19}$$

$$L = \bar{B}_1 - M\bar{B}_2 \tag{3.2.20}$$

$$E = [G \quad H] \tag{3.2.21}$$

$$N = [M \quad (\Gamma_1 - M\Gamma_2)\Gamma_3^{-1}] \tag{3.2.22}$$

In the above equations, by properly selecting the estimator's gain, one can assign the poles of the F to appropriate locations. By combining the (3.2.16) and the (3.2.1), we arrive at:

$$\begin{pmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{w}} \end{pmatrix} = \begin{pmatrix} A & 0 \\ EC & F \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \mathbf{w} \end{pmatrix} + \begin{pmatrix} B \\ L \end{pmatrix} \mathbf{u} + \begin{pmatrix} D \\ 0 \end{pmatrix} \mathbf{p} \tag{3.2.23}$$

The equations (3.2.23) and (3.2.15) are the main equations of the UIO which describe its function. In those cases where m=p, it is not possible to assign the eigenspectrum of the observer to arbitrary locations, although a stable observer with fixed eigenvalues may be possible.

3.2.2 Case II (Equal number of unknown inputs and the outputs):

In this case, we assume that p = m. Let's rewrite (3-2-1) in the following partitioned form:

$$\begin{pmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{y}} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{y} \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} \mathbf{u} + \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \end{pmatrix} \mathbf{p} \tag{3.2.24}$$

Note that in the above representation D_2 is full rank square matrix and therefore invertible.

Theorem : The eigenspectrum of the UIO can not be arbitrarily assigned if p = m. In this case an asymptotically state observer with fixed eigenspectrum of the form (3.2.30) would exist, if and only if the following matrix is stable (negative eigenvalues).

$$F = A_{11} - \Gamma_1 \Gamma_2^{-1} A_{21} \tag{3.2.25}$$

In this case the estimator dynamics is given as in (3.2.16) and the state variable estimate is given by:

$$\hat{\mathbf{x}}_1 = \mathbf{w} + M\mathbf{y} \tag{3.2.26}$$

with

$$L = B_1 - \Gamma_1 \Gamma_2^{-1} B_2 \tag{3.2.27}$$

$$E = \Sigma A \Gamma \tag{3.2.28}$$

where

$$\Sigma = [I \quad -\Gamma_1 \Gamma_2^{-1}] \quad , \quad \Gamma = (\Gamma_1 \Gamma_2^{-1} \quad I)' \tag{3.2.29}$$

This concludes the design of the UIO. It should be noted that such estimators are useful in large scale system studies. It is possible in certain large scale systems to design a totally decentralized estimation scheme by treating the interconnections of the systems as unknown inputs, Saif and Guan (1992)

Remarks:

We have to point out here that the two area system modelled in (2-3-14) to (2-3-20) has two unknown inputs: ΔP_{D1},ΔP_{D2} , so even if we have only three outputs of the systems, by the use of the UIO, we are able to estimate all ten state variables and thus feedback controller is completed. For simplicity in our simulation, the state equations are so arranged such that the state variables of each power system be together, i.e:

$$\mathbf{x} = [\Delta f_1 \quad \Delta\delta_1 \quad \Delta P_{G1} \quad \Delta P_{GE1} \quad \Delta X_{E1} \quad \Delta f_2 \quad \Delta\delta_2 \quad \Delta P_{G2} \quad \Delta P_{GE2} \quad \Delta X_{E2}]' \qquad (3.2.30)$$

and A and B are also appropriately rearranged. To find a similar C as in (3.2.2), we will assume that the last three state variables are the outputs:

$$C_{(3x10)} = \begin{bmatrix} & 1 & 0 & 0 \\ 0_{(3x7)} & 0 & 1 & 0 \\ & 0 & 0 & 1 \end{bmatrix} \qquad (3.2.31)$$

In the next chapter LQR and UIO techniques and their applicability are illustrated in the simulation of the electrical two area system load frequency control in response to varying load demand.

CHAPTER 4

TWO AREA SYSTEM CONTROL

This chapter presents simulated results of two area system control by the LQR theory discussed in the earlier chapter. The two area system selected is the same one considered in Elgerd and Fosha (1970) work. The per unit(pu) of megawatt is 2000 MW.

As we have seen in chapter two, the dynamics of the two-area system can be written in state space model which is the basic formulation in modern control theory. In that model we had known inputs as well as unknown inputs. In chapter three we discussed the LQR technique which is a powerful tool to stabilize the linear systems. This technique is able to optimize the cost function and place the eigenvalues of the system to the desired locations. For realizing the control input we needed all state variables, so we discussed the UIO to estimate the states of the system with unknown inputs. For the purpose of the simulation, two similar generator with a connecting tie line has been considered. The simulation was carried on a digital computer using a sampling interval of t=0.01 sec. Two generators in the system supply different loads which are assumed to be unknown. Terminal voltage is assumed to be constant and so the tie power becomes a function of torque angle only. Two different initial conditions are applied to the systems and the load is changed for one of them and the response of the systems are studied. To demonstrate the performance of LQR and UIO theory two different tests will be conducted.

Part 1: a) decentralized system with 4% step load in one of the and 2% step load for the other

b) decentralized system with 2% triangle load in one of them and 4% step load for the other

Part 2: centralized two area system with a change of 10% load change in one of them.

In each of the experiments first control gain will be found by using optimal modal LQR theory discussed in chapter three, then a corrective control signal is added to compensate for the unknown input and finally the estimation of the state variables of the system will be carried out to realize the control law. System parameters are given in this chapter and the data for the generators are given in Appendix.

4.1 Modified Optimal Modal Controller

To accomplish the task of regulating the frequency and the tie power deviations, the optimal modal LQR discussed in chapter three needs to be modified. The controller that will be used here will be a proportional plus integral controller which would eliminate the steady state error due to step load change. For more detail on the PI optimal modal controller the reader is referenced to Saif(1992).

We use area control error ACE (Elgerd (1970) and Nanda and Kothari (1987)), which is a combination of the measurable variables of the system as our output. Area control error defined as the sum of the frequency and tie power change or

$$ACE = \Delta p_{tie} + B\Delta f \tag{4.1.1}$$

where B is the coefficient determined by the parameters of the system. The reason for chosing area control error as a control measure is described below, where we show that if the steady state error of the ACE of the two areas approach zero, tie power and the frequency deviation also will approach zero individually which is exactly the objectives of the LFC. In the steady state :

$$\Delta p_{tie1} + B1\Delta f_1 = 0 \tag{4.1.2}$$

$$\Delta p_{tie2} + B2\Delta f_2 = 0 \tag{4.1.3}$$

but we know:

$$\Delta p_{tie1} = -a_{12}\Delta p_{tie2} \tag{4.1.4}$$

where a_{12} is the transformation ratio of the tie power between machine one and two, and is dependent on the different (Pu) values of the systems. The equations (4.1.2), (4.1.3) and (4.1.4) will result in:

$$\Delta p_{tie1} = \Delta p_{tie2} = \Delta f_1 = \Delta f_2 = 0 \tag{4.1.5}$$

which is exactly the objectives of the load frequency control.

The control law to be used is defined as:

$$\mathbf{u} = -\underline{K}_{\mathbf{p}}\mathbf{x} - K_{\mathrm{I}}\int ACE\,dt \tag{4.1.6}$$

Define :

$$\dot{\mathbf{z}} \equiv -\mathbf{y} \tag{4.1.7}$$

where $\mathbf{y}$ is assumed to be the area control error (ACE). Augmenting the above equation with the (4.1.1) gives a PI controller design where:

$$\dot{\overline{\mathbf{x}}} = \overline{A}\mathbf{x} + \overline{B}\mathbf{u} + \overline{\Gamma}\mathbf{p} \tag{4.1.8}$$

$$\overline{\mathbf{y}} = \overline{C}\overline{\mathbf{x}} \tag{4.1.9}$$

and, $\overline{\mathbf{x}} = \begin{bmatrix} \mathbf{x} \\ \mathbf{z} \end{bmatrix}$, $\overline{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}$, $\overline{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$, $\overline{\Gamma} = \begin{bmatrix} \Gamma \\ 0 \end{bmatrix}$, $\overline{C} = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}$

To ensure that the above set of equations (4.1.3) and (4.1.4) is stable and steady state error in response to the step load change becomes zero, the following conditions should be satisfied: (Davison (1971))

(i) the pair $(\overline{A}, \overline{B})$ is stabilizable.

(ii) the matrix $\begin{bmatrix} \overline{A} & \overline{B} \\ \overline{C} & 0 \end{bmatrix}$ is of full row rank.

Differentiating eq (4.1.8) gives:

$$\ddot{\overline{\mathbf{x}}} = \overline{A}\dot{\overline{\mathbf{x}}} + \overline{B}\dot{\mathbf{u}} \tag{4.1.10}$$

It is now desired to obtain the control law $\dot{\mathbf{u}}$ such that the following performance measure in minimized subject to eq (4.1.10)

$$J = \frac{1}{2}\int_0^\infty (\| \dot{\overline{\mathbf{x}}} \|_Q^2 + \| \dot{\mathbf{u}} \|_R^2) dt \tag{4.1.11}$$

the optimal control law becomes:

$$\begin{aligned}\mathbf{u} &= -R^{-1}\overline{B}'\overline{P}\overline{\mathbf{x}} = K\overline{\mathbf{x}} = [K_P \quad K_I]\overline{\mathbf{x}} \\ &= K_P\mathbf{x} + K_I\int \dot{\mathbf{z}}dt \end{aligned} \tag{4.1.12}$$

This control law can perfectly regulate the tie line power deviation as well as the frequency deviation that would result as the load changes in a step like fashion. However, if we assume a general time varying load profile, perfect regulation would not be possible. To compensate for the unknown input effects on the system we add a Corrective part to the control input solutions of LQR, that will be discussed now.

Corrective Control Unknown input observer is able to estimate the state variables of the system, but compensating for the time varying loads is necessary to achieve better regulation. The following method is used to correct for effects of the time varying loads.

Consider the power system model described as:

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} + \Gamma\mathbf{p} \tag{4.1.13}$$

If we define :

$$\mathbf{u} = K\mathbf{x} + \mathbf{u}^* \tag{4.1.14}$$

in which $\mathbf{u}^*$ is a corrective control signal to account for the unknown inputs. Our system now becomes :

$$\begin{aligned}\dot{\mathbf{x}} &= A\mathbf{x} + B(K\mathbf{x} + \mathbf{u}^*) + \Gamma\mathbf{p} \\ &= (A + BK)\mathbf{x} + (B\mathbf{u}^* + \Gamma\mathbf{p})\end{aligned} \tag{4.1.15}$$

The first part of the equation on the right hand side is found from (4.1.12), so it is stable. Next it is desired to find $\mathbf{u}^*$ such that the unknown effect is compensated. In order to achieve that, lets define a performance measure:

$$\min_{\mathbf{u}} \quad J \equiv \| B\mathbf{u}^* + \Gamma\mathbf{p} \|^2 \tag{4.1.16}$$

Expanding J, we have

$$\begin{aligned} J &= (B\mathbf{u}^* + \Gamma\mathbf{p})'(B\mathbf{u}^* + \Gamma\mathbf{p}) \\ &= (\mathbf{u}^{*\prime}B' + (\Gamma\mathbf{p})')(B\mathbf{u}^* + \Gamma\mathbf{p}) \\ &= \mathbf{u}^{*\prime}B'B\mathbf{u}^* + \mathbf{u}^{*\prime}B'\Gamma\mathbf{p} + \mathbf{p}'\Gamma'B\mathbf{u}^* + (\Gamma\mathbf{p})'(\Gamma\mathbf{p}). \end{aligned} \tag{4.1.17}$$

Taking the derivative,

$$\frac{\partial J}{\partial \mathbf{u}^*} = 2B'B\mathbf{u}^* + 2B'\Gamma\mathbf{p} \tag{4.1.18}$$

The minimum occurs at $\frac{\partial J}{\partial \mathbf{u}^*} = 0$ which implies :

$$B'B\mathbf{u}^* = -B'D\mathbf{v}$$

$$\mathbf{u}^* = -(B'B)^{-1}B'D\mathbf{v} \tag{4.1.19}$$

Thus our overall control will be :

$$\mathbf{u} = K\mathbf{x} - (B'B)^{-1}B'\Gamma\mathbf{p}. \tag{4.1.20}$$

If we can get a good estimate of $\mathbf{p}$ $(\hat{\mathbf{p}})$,then we can find the control input (4.1.20). To find the estimate of unknown inputs, assuming that it is smooth enough, we use the discretized state eqation of the system. Suppose our system is discretized into :

$$\mathbf{x}_{k+1} = A^*\mathbf{x}_k + B^*\mathbf{u}_k + \Gamma^*\mathbf{p}_k \tag{4.1.21}$$

Thus we have :

$$\mathbf{p}_k = \Gamma^{*+}(\mathbf{x}_{k+1} - A^*\mathbf{x}_k - B^*\mathbf{u}_k) \qquad (4.1.22)$$

where D^{*+} is the psudo inverse of D^*.

Algorithm:

In this section we summarize the all optimal modal LQR and UIO design procedure, which will be used in our simulation. First consider equation (3.2.31):

$$\begin{pmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{w}} \end{pmatrix} = \begin{pmatrix} A & 0 \\ EC & F \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \mathbf{w} \end{pmatrix} + \begin{pmatrix} B \\ L \end{pmatrix} \mathbf{u} + \begin{pmatrix} \Gamma \\ 0 \end{pmatrix} \mathbf{p} \qquad (4.1.23)$$

where $\mathbf{w}$ is a variable such that the state variable estimate can be found by:(eq 3.2.28)

$$\hat{\mathbf{x}} = \begin{pmatrix} I \\ 0 \end{pmatrix} \mathbf{w} + \begin{pmatrix} N \\ I \end{pmatrix} \mathbf{y} \qquad (4.1.24)$$

and our control input is calculated by (4.1.20) as:

$$\mathbf{u} = K\mathbf{x} - (B'B)^{-1} B'\Gamma\hat{\mathbf{p}} \qquad (4.1.25)$$

where $\hat{\mathbf{p}}$ is the estimate of the unknown input $\mathbf{p}$ that should be found. This estimation is done by the discretized form of equation (4.28).

$$\begin{pmatrix} \mathbf{x} \\ \mathbf{w} \end{pmatrix}_{(k+1)} = A_d \begin{pmatrix} \mathbf{x} \\ \mathbf{w} \end{pmatrix}_{(k)} + B_d\mathbf{u}_k + \Gamma_1\hat{\mathbf{p}}_k + \Gamma_2\mathbf{p}_k \qquad (4.1.26)$$

Now we are able to itemize the algorithm as follows:

1) Define the initial conditions of state variables, unknown inputs and their estimations.(first iteration k = 1)
2) Find the descrete formulation of the equation (4.1.26)
3) Find the new value of $\mathbf{x}(k+1)$ from (4.1.26).

4) Find the estimate of $\mathbf{x}$(k+1) from (4.1.24)
5) Find the control input $\mathbf{u}$(k) from (4.1.25)
6) Find the estimate of unknown input $\mathbf{p}$(k) from (4.1.26), using the estimate of $\mathbf{x}$(k+1), estimate of $\mathbf{x}$(k), $\mathbf{u}$(k) found earlier.
7) Initialize the next step unknown input $\mathbf{p}$(k+1) with $\mathbf{p}$(k)
8) Start the next iteration. Go to step 3.

Selection of the time steps depends on several factors such as acceptable accuracy and the unknown input waveform, however we have to keep in mind that smaller time steps requires more computation and memory.

4.2) Simulation Studies

We are now ready to apply the previous concepts to LFC of a two area system. We shall consider two control strategies: centralized and decentralized control.

4.2.1 Decentralized Control

For the state equations, we can use the same equations as before, the only difference is that we have to define load and tie power between two systems as unknown inputs. State equations for the single generator in the two-area system are determined as follows.

$$\dot{\mathbf{x}}_i = A_i\mathbf{x}_i + B_i\mathbf{u}_i + \Gamma_i\mathbf{p}_i \tag{4.2.1}$$

$$\mathbf{x}_i = [\Delta f \quad \Delta\delta \quad \Delta p_g \quad \Delta p_{ge} \quad \Delta x_e]_i^t \tag{4.2.2}$$

$$\mathbf{u}_i = [\Delta p_c \quad \Delta u]_i^t \tag{4.2.3}$$

$$\mathbf{p}_i = \Delta p_{d_i} + \Delta p_{tie_i} \tag{4.2.4}$$

where index i refers to ith subsystem. The state equations from chapter two become (for simplicity we have dropped the index i):

$$\dot{\delta} = f \tag{4.2.5}$$

$$\Delta\dot{x}_e = \frac{1}{t_g}\left(-\Delta x_e + k_g \Delta p_c - \frac{k_g}{R}\Delta f\right) \tag{4.2.6}$$

$$\Delta\dot{p}_{ge} = -\frac{1}{t_e}\Delta p_{ge} + \frac{1}{t_e}\bar{p}_{ge}k_e\Delta u + \frac{1}{t_e}p_{g\delta}\Delta\delta + p_{g\delta}\Delta f \tag{4.2.7}$$

$$\Delta\dot{f} = -\frac{1}{t_p}\Delta f + \frac{k_p}{t_p}(\Delta p_g - \Delta p_d - p_{du}\Delta u - \Delta p_{tie}) \tag{4.2.8}$$

$$\Delta\dot{p}_g = -\frac{1}{t_t}\Delta p_g + \frac{k_t}{t_t}\Delta x_e + \left(\frac{1}{t_e} - \frac{1}{t_t}\right)\Delta p_{ge} - p_{g\delta}\Delta f - \frac{1}{t_e}\bar{p}_{ge}k_e\Delta u - \frac{1}{t_e}p_{g\delta}\Delta\delta \tag{4.2.9}$$

where **x**, **u** are state vector and known input vector respectively and **p** is unknown input vector.

In this experiment two identical power systems (thermal) was considered. The numerical values describing each area model is given in appendix A. It is assumed that tie power is measureable and the two areas have similar control laws. The open loop eigenvalues of each area power system are located at:

$$\textit{Openloop} \quad \textit{eigenvalues} = \begin{pmatrix} -13.2768 \\ -0.7608 + 2.9832i \\ -0.7608 - 2.9832i \\ -1.0817 \end{pmatrix}$$

Each area's model was augmented as described in section 4.1 and the optimal modal controller was then designed. It was decided to retain the modes of the system and place the augmented mode pole for better speed of response. The desired closed loop poles decided upon were:

$$Desired \quad closedloop \quad eigenvalues = \begin{pmatrix} -1.7679+2.9833i \\ -1.7679-2.9833i \\ -13.2817 \\ -1.0830 \\ -1.0315 \end{pmatrix}$$

The optimal control law as well as the open loop system are given in this chapter. Notice that since two identical areas are assumed, all the aforementioned values are the same for each area. Next, an estimator with eigenvalues located at (-1, -1.1, -1.2) was designed for each area. The numerical values of the estimators' parameters are given in this chapter. Finally the control system designed was tested under various conditions. As an example, we increased the load 2 by 4% while increasing other load by 2%. Initial conditions for the state variables are set at zero, but estimation initial values are all at 2 and those for the load estimation are at 4. figure 1-2 show the load estimation of the two areas, and as can be noticed the estimates follow the actual values rapidly. The speed of estimator depends on its eignvalues locations and the time steps of simulation. In figure 3-5 estimations of the two systems frequencies and the generation power change of system 1 are given. Again the estimators follow the actual values in a short time. Figure 6-7 are the response of the closed and open loop system 1, and shows that generation and main piston position approach to the load demand. The same response for the system 2 is shown in figure 8-9. In the open loop system, generation and the main piston position would not approach the load but it will be shared between the systems. Transient response time of the closed loop system can be reduced by lowering the real part of the eigenvalues, but on the other hand the overshoots of the system are increased too, so a compromise between these two factors should be made. In fig10 and fig11 frequency deviations of the system are shown. While in closed loop system the frequency deviations approach zeros, in open loop system responses they indicate steady state errors which depend on the loads of the systems. Tie power change in closed loop system approaches zero (figure 12), which is as desired. When each area, in steady state supplies its own load (figure 6-9) the tie power would approach zero.

In the second experiment, the load of the system No. 2 remains the same but the load of system No. 1 is changed to a 2% triangle form. Load and state variable estimators still can follow the actual loads perfectly (figure 13-17). Generating power change and piston position change (figure 18-19) of the closed loop show an steady state error. There are two reasons to this: first, the corrective control signal is not capable of compensating the load change effect on the system (B and D of the system are nearly orthogonal) and second, PI controller gives an steady state error to ramp functions. Interaction of the systems results in the similar responses in system 2 (figure 20-21). Frequency deviation of the systems also experience a steady state error. This same effect is true for the tie power change, however the open loop system response is much worse.

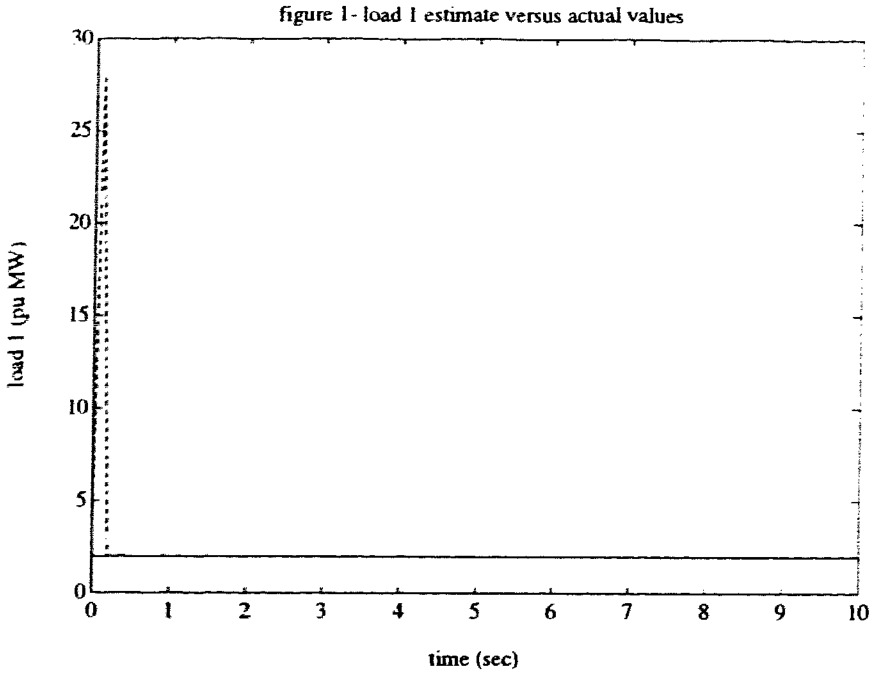


figure 1- load 1 estimate versus actual values
load 1 (pu MW)
time (sec)

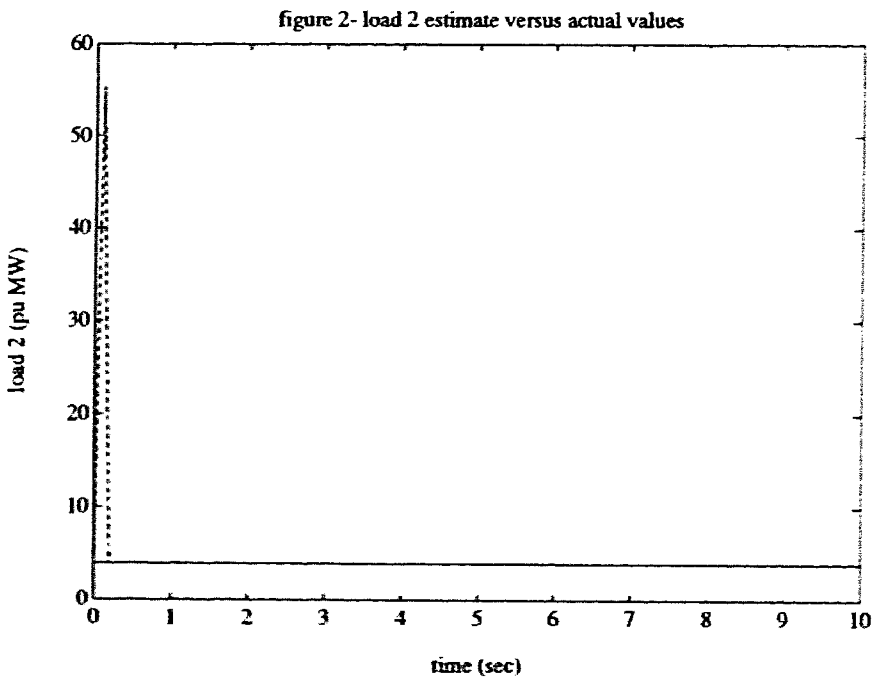


figure 2- load 2 estimate versus actual values
load 2 (pu MW)
time (sec)

figure 3- generating power change estimate versus actual values

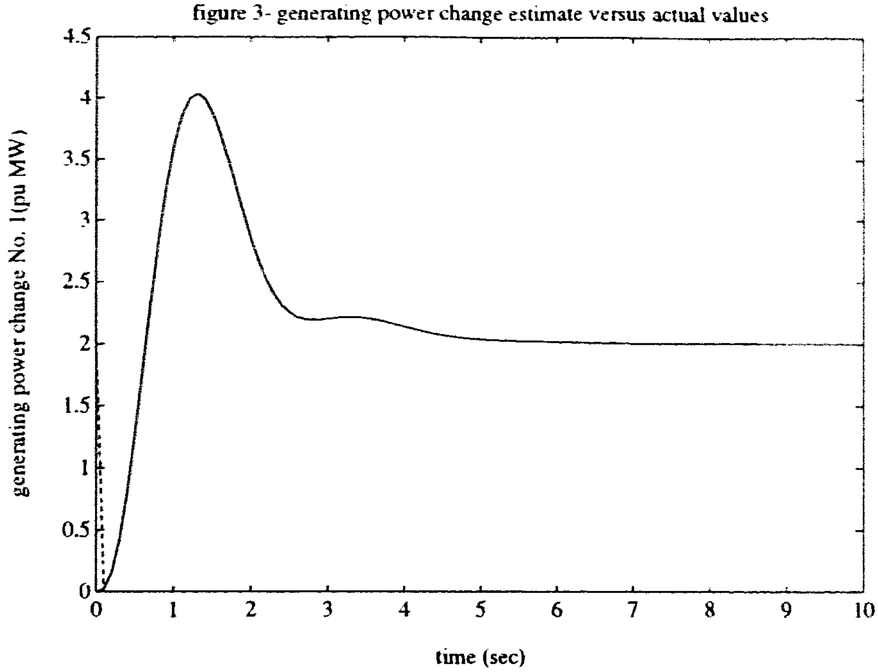


figure 4- frequency change No. 2 estimate versus actual values

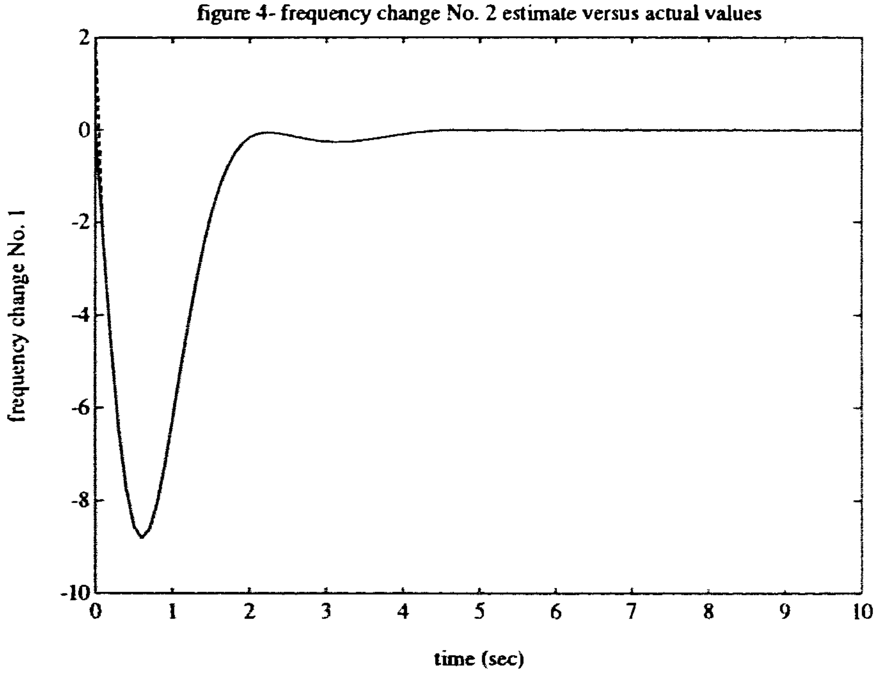


figure 5- frequency change No. 1 estimate versus actual values

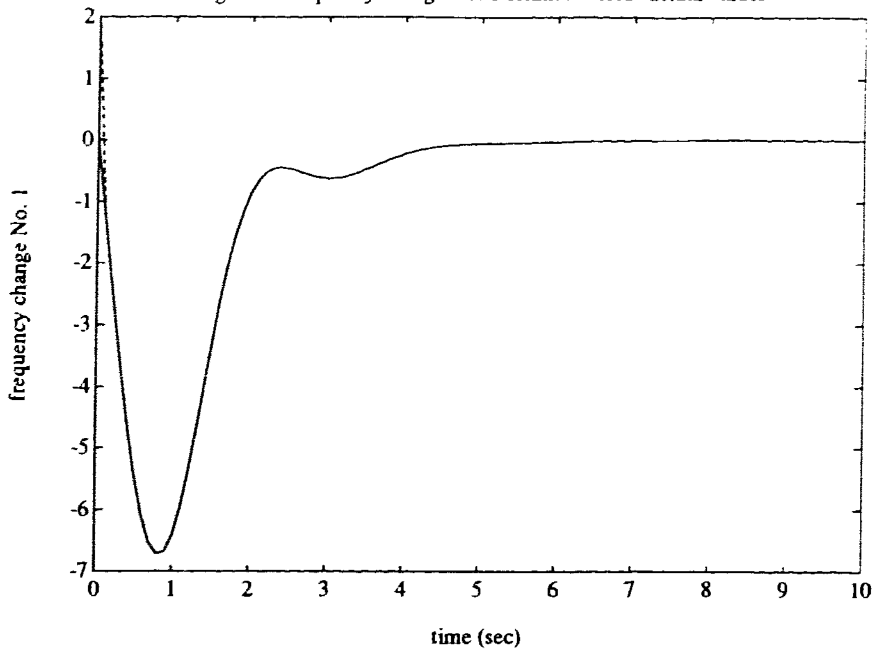


figure 6- closed loop versus open loop values

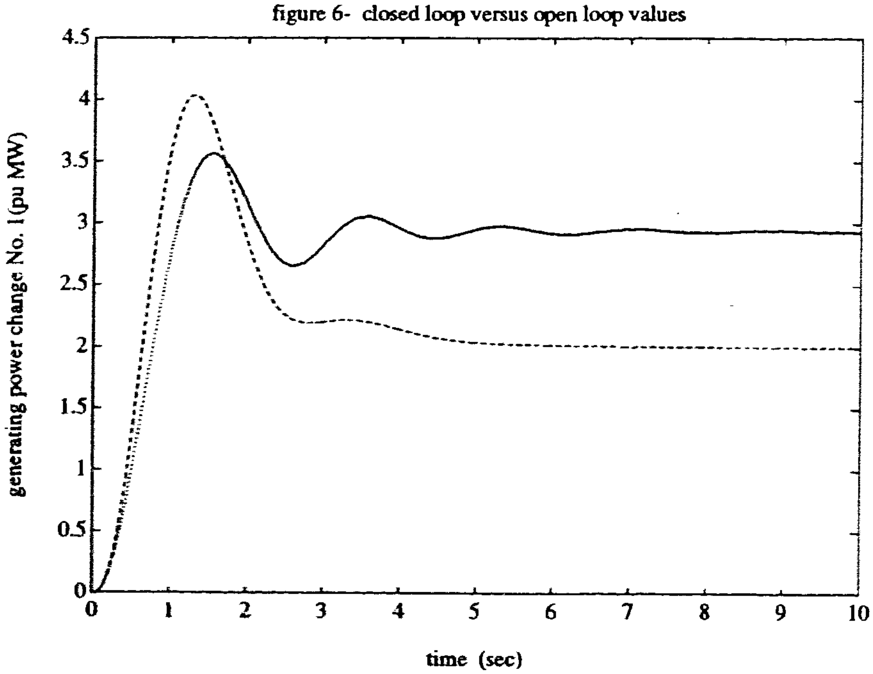


figure 7- closed loop versus open loop values

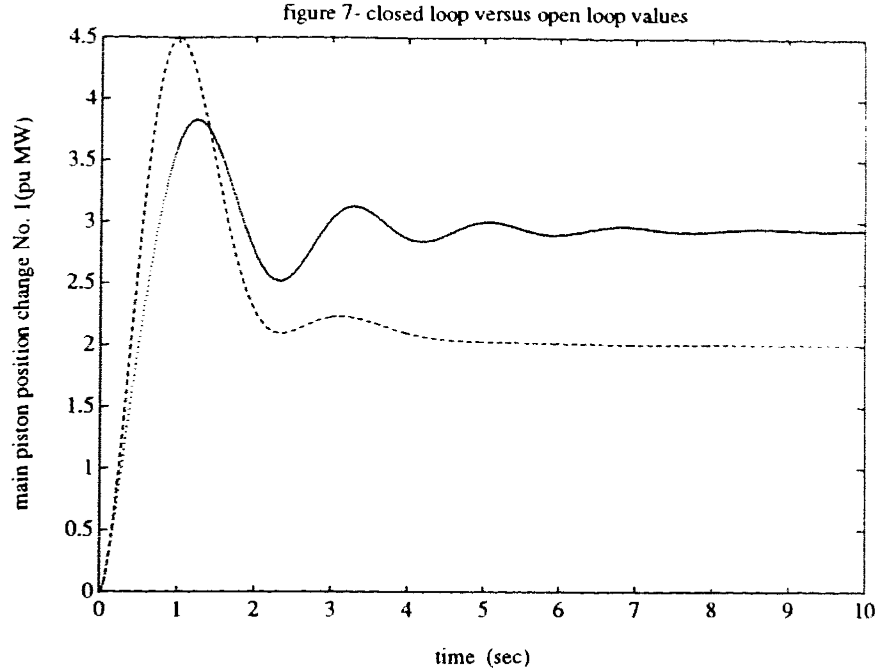
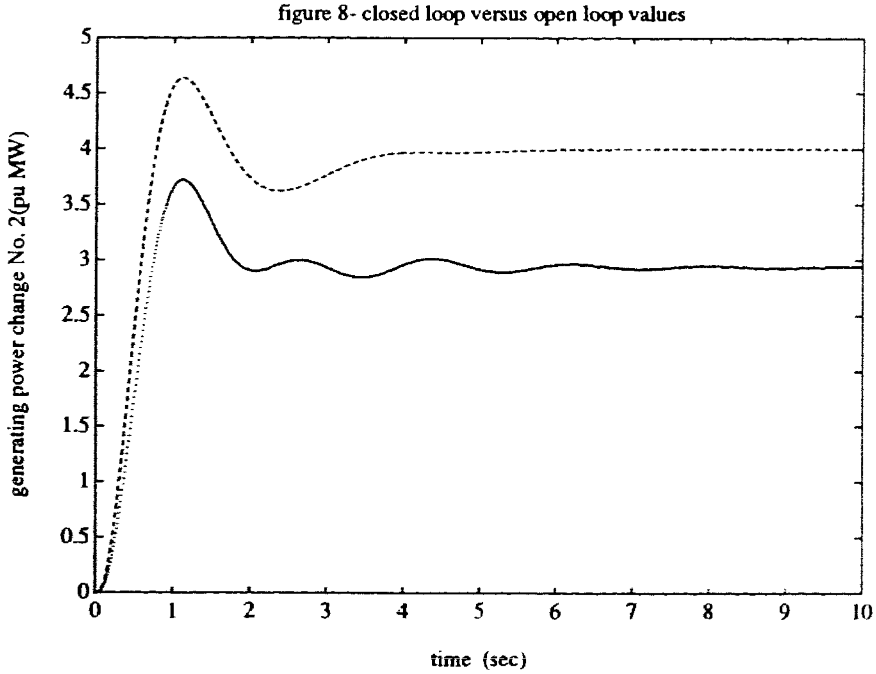


figure 8- closed loop versus open loop values



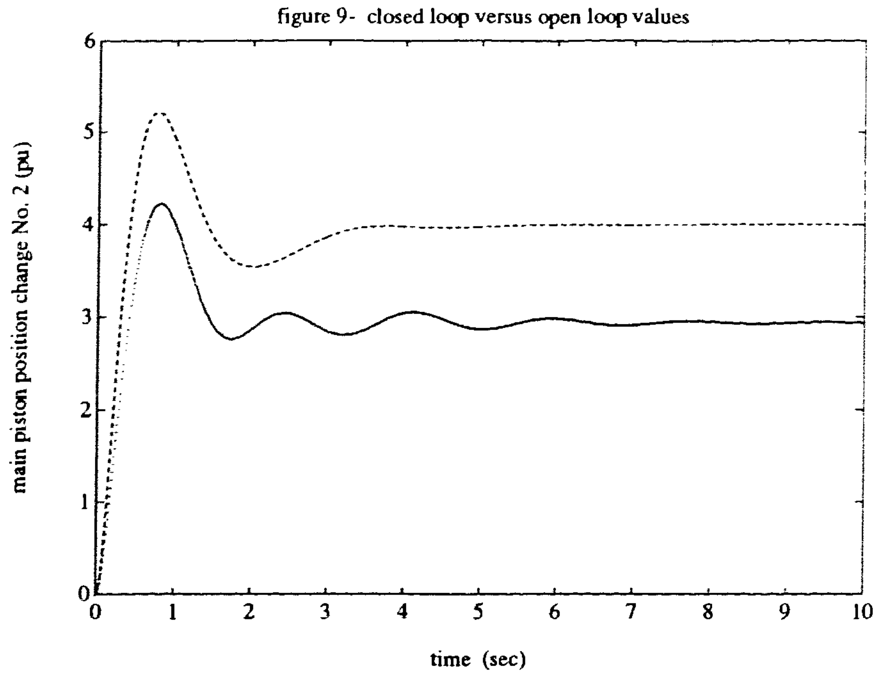


figure 9- closed loop versus open loop values
main piston position change No. 2 (pu)
0
1
2
3
4
5
6
0
1
2
3
4
5
6
7
8
9
10
time (sec)

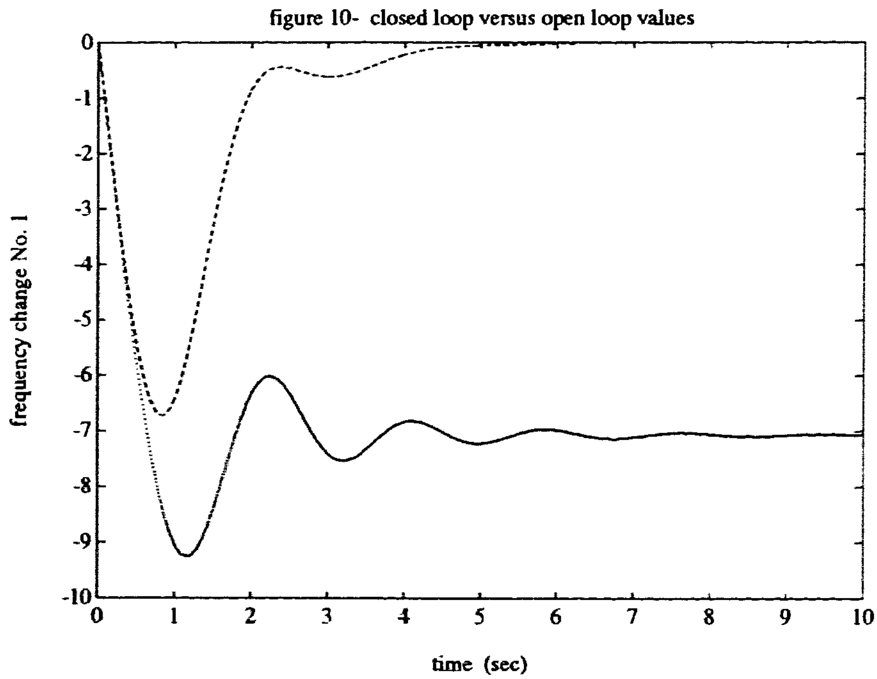


figure 10- closed loop versus open loop values
frequency change No. 1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
0
1
2
3
4
5
6
7
8
9
10
time (sec)

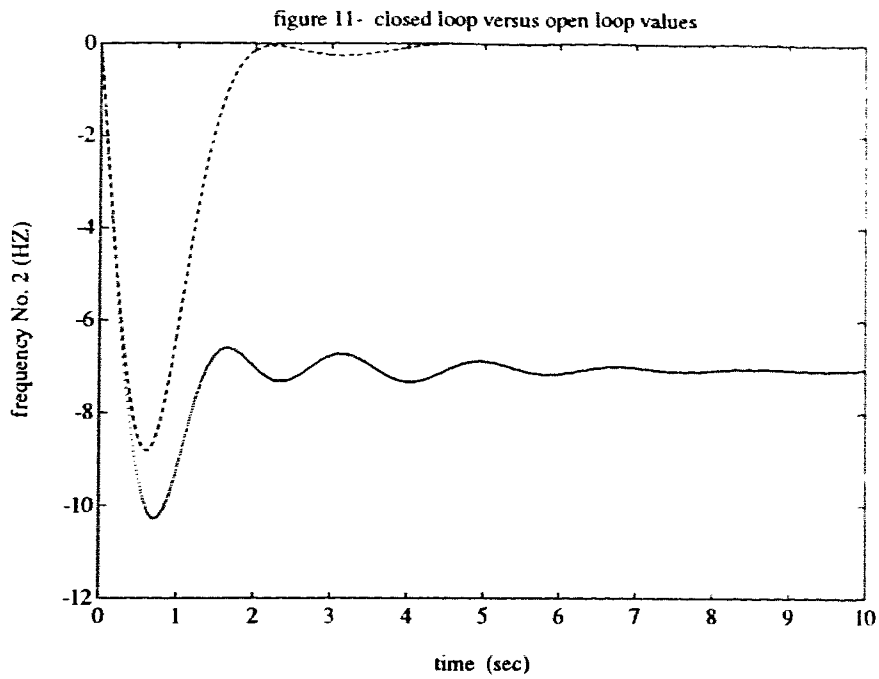


figure 11- closed loop versus open loop values
frequency No. 2 (HZ)
0
-2
-4
-6
-8
-10
-12
0
1
2
3
4
5
6
7
8
9
10
time (sec)

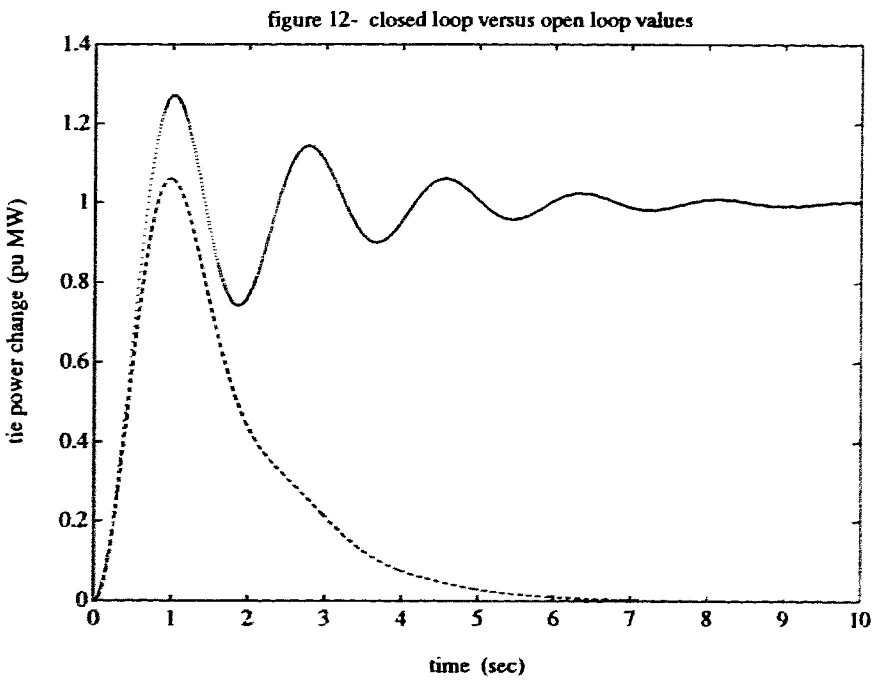


figure 12- closed loop versus open loop values
tie power change (pu MW)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8
9
10
time (sec)

figure 13- load 1 estimate versus actual values

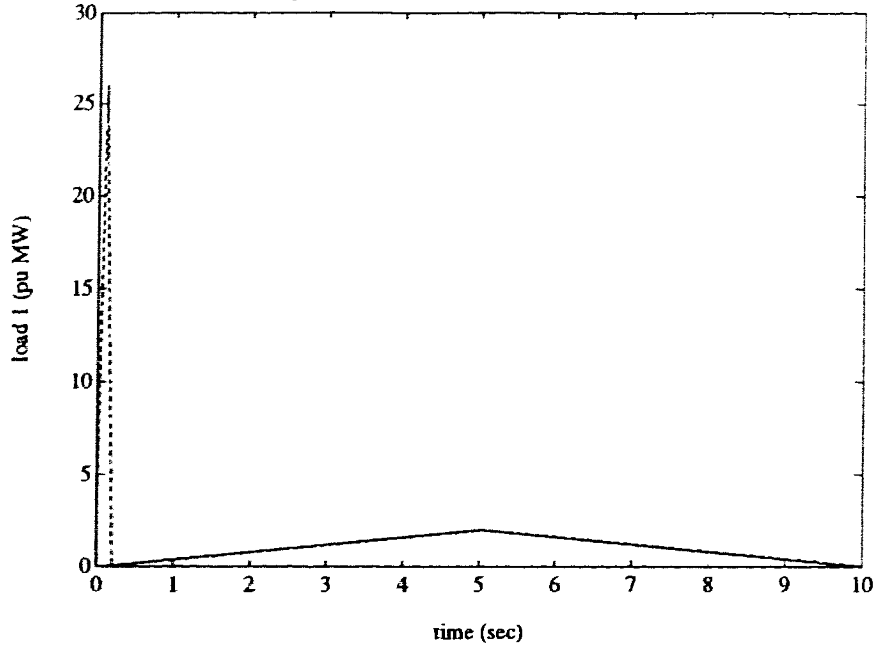


figure 14- load 2 estimate versus actual values

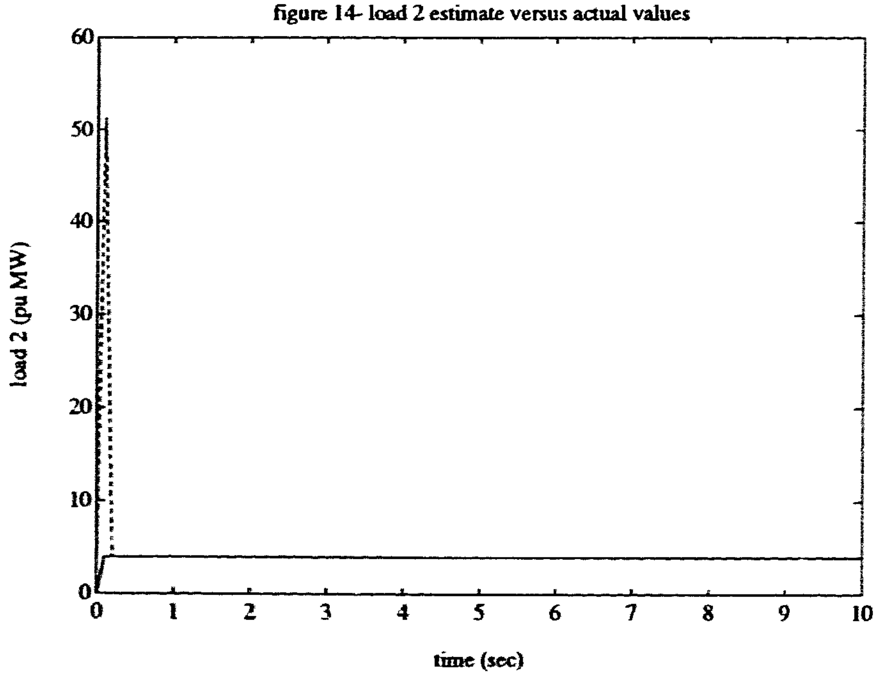


figure 15- generating power change estimate versus actual values

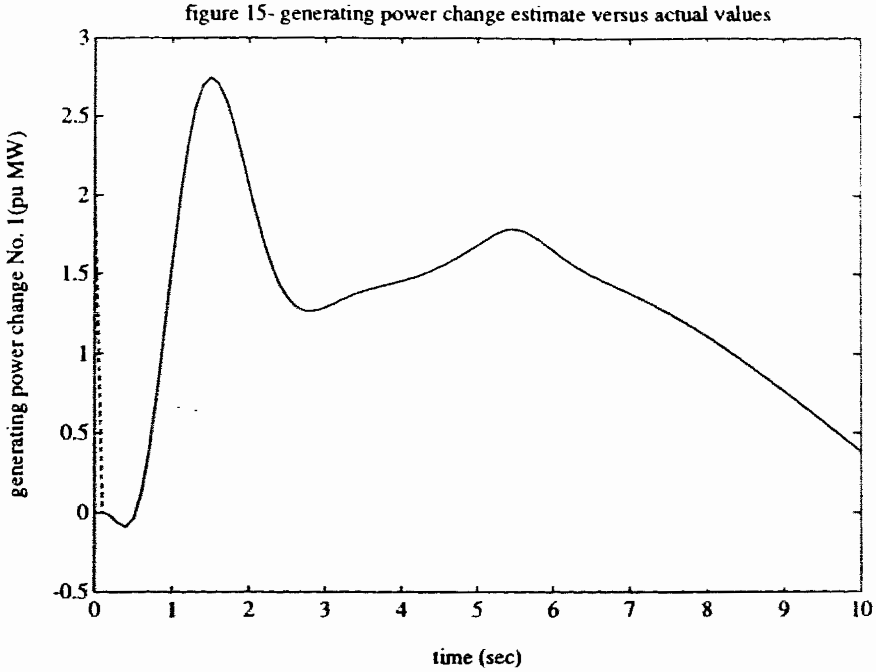


figure 16- frequency change No. 2 estimate versus actual values

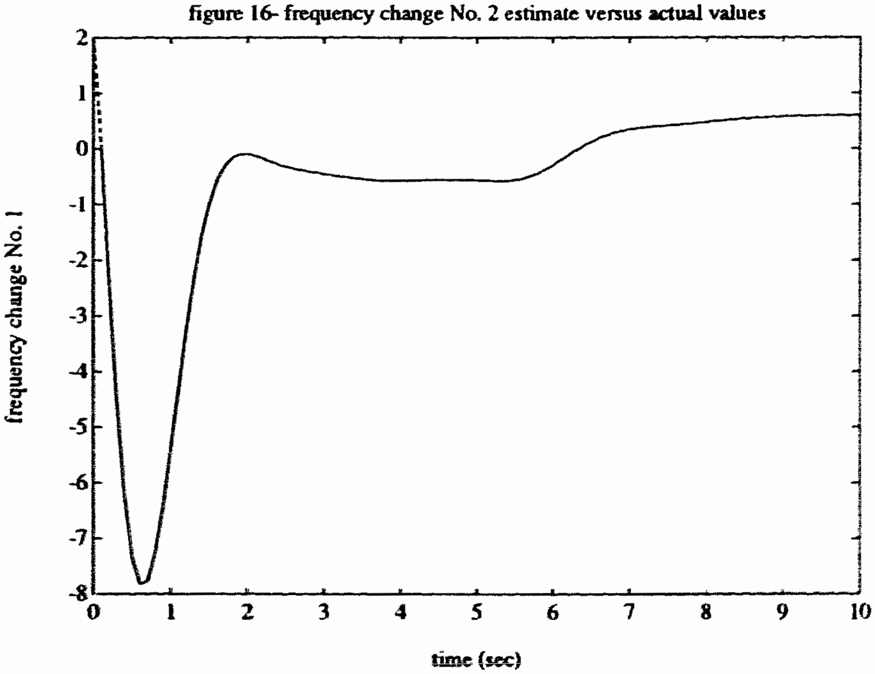


figure 17- frequency change No. 1 estimate versus actual values

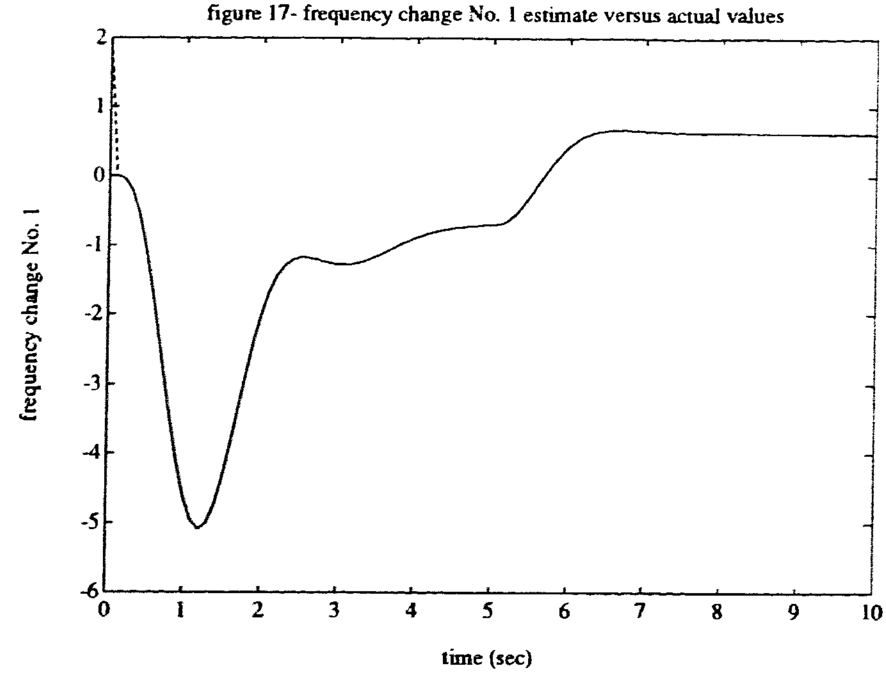
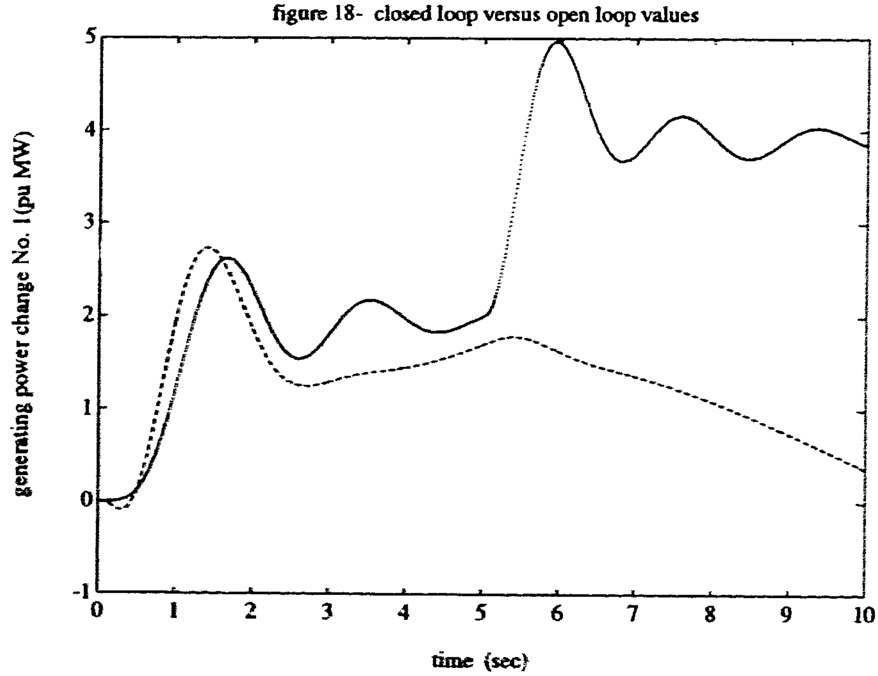


figure 18- closed loop versus open loop values



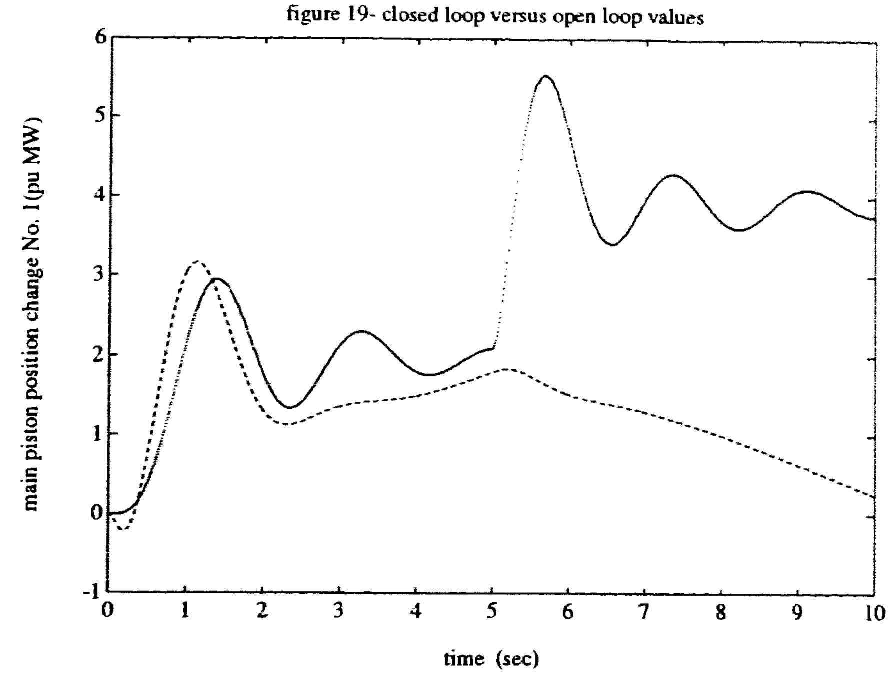


figure 19- closed loop versus open loop values
main piston position change No. 1(pu MW)
time (sec)
6
5
4
3
2
1
0
-1
0 1 2 3 4 5 6 7 8 9 10

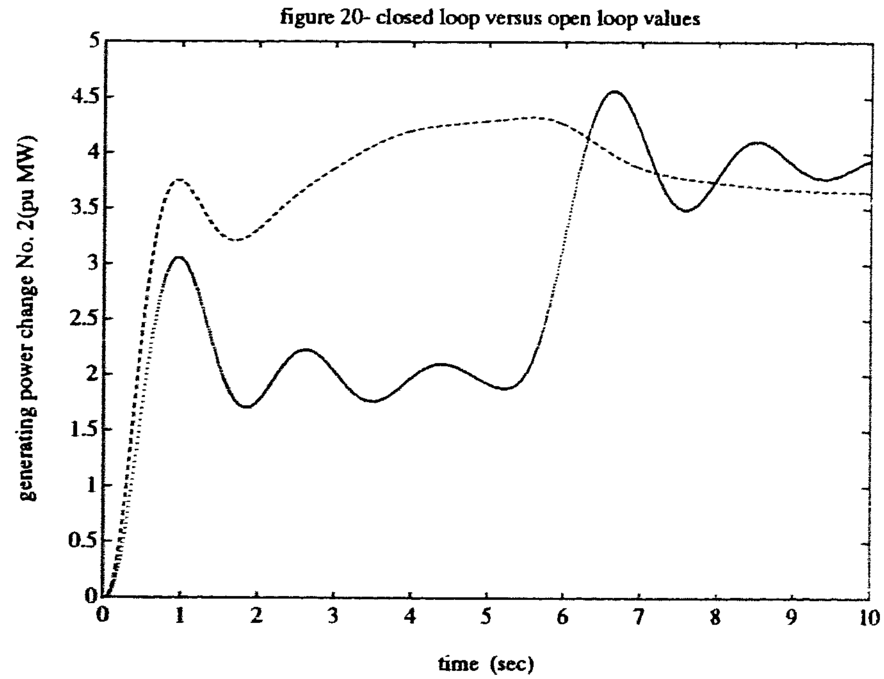


figure 20- closed loop versus open loop values
generating power change No. 2(pu MW)
time (sec)
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0 1 2 3 4 5 6 7 8 9 10

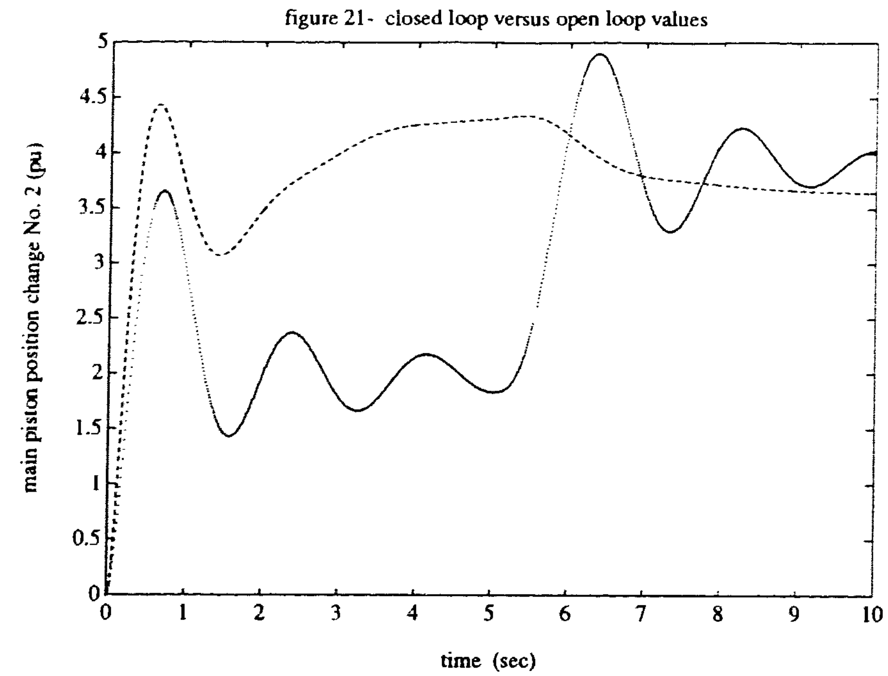


figure 21- closed loop versus open loop values
main piston position change No. 2 (pu)
time (sec)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
1
2
3
4
5
6
7
8
9
10

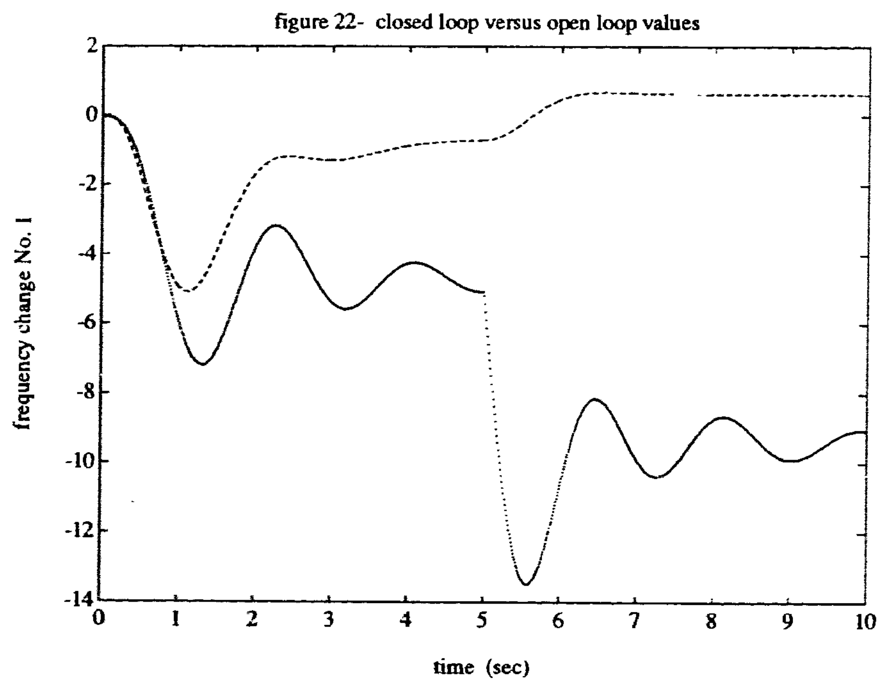


figure 22- closed loop versus open loop values
frequency change No. 1
time (sec)
2
0
-2
-4
-6
-8
-10
-12
-14
0
1
2
3
4
5
6
7
8
9
10

figure 23- closed loop versus open loop values

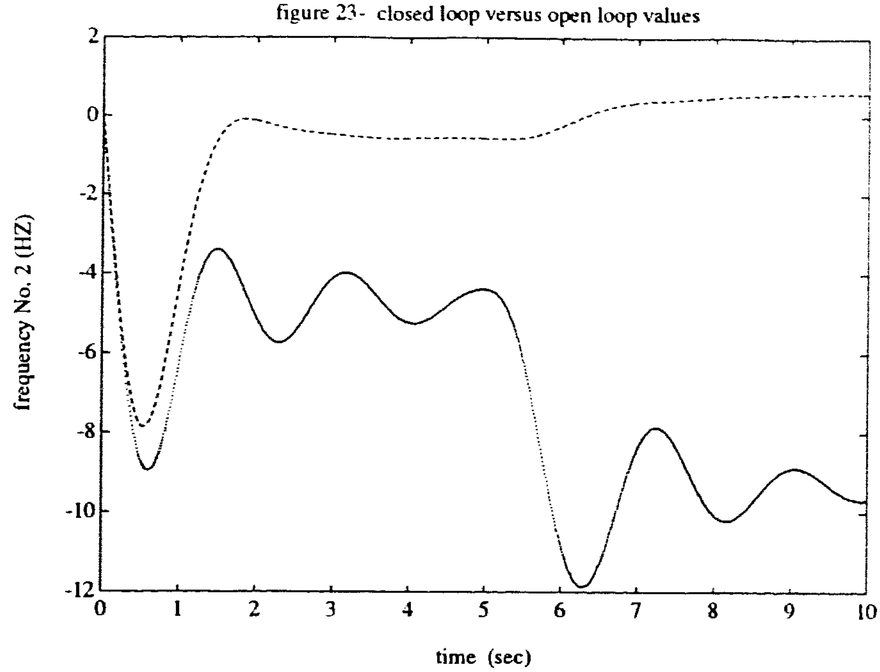
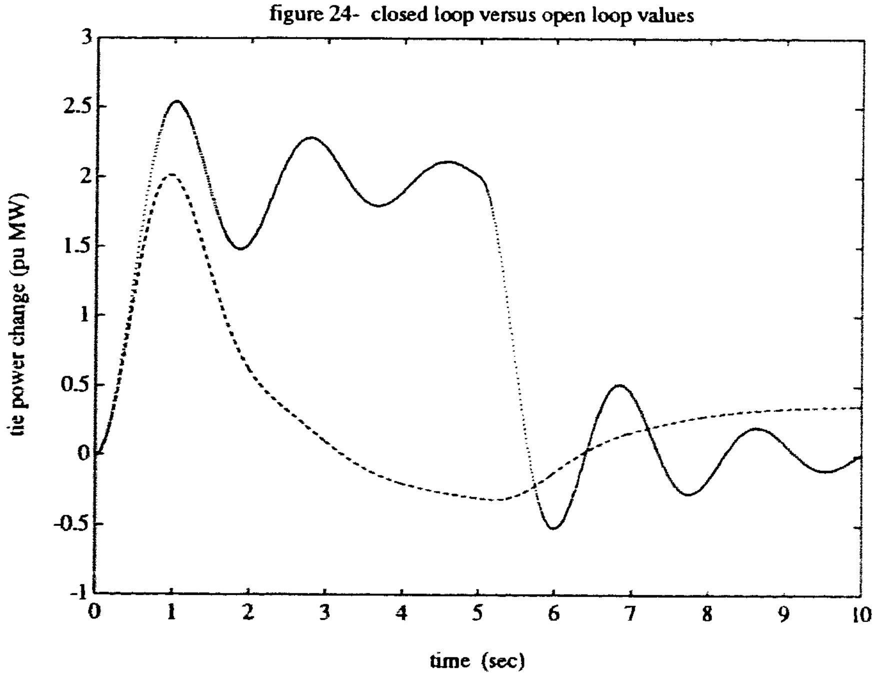


figure 24- closed loop versus open loop values



4.2.2 Centralized Electrical System Control

In this part two area system that was discussed in chapter two is simulated. Both areas are the same as decentralized experiment, but in this case all estimation and control laws are globally found. The same method of defining of PI controller and the corrective control signal used in decentralized case, will be applied here too. Output vector is defined as the area control error of the two systems and the corrective control signal compensates for the unknown loads of the two systems. Estimation as well as control law is found for both systems simultaneously, so a central feedback controller provides the LFC for the system. Numerical values of the two area system are given at the end of this chapter.

The open loop eigenvalues of the two area system are:

$$eigenvalues = \begin{pmatrix} -13.2644 \\ -.4967+3.5220i \\ -.4967-3.5220i \\ -1.6223 \\ -13.2895 \\ -1.2953+2.5123i \\ -1.2953-2.5123i \end{pmatrix}$$

Two area model is augmented with the same procedure described in 4.1 and the optimal modal controller was designed. To ensure a better performance for the closed loop system the eigenvalues close to imaginary axis are shifted to the left. The new eigenvalues of the system are:

$$eigenvalues = \begin{pmatrix} -13.2694 \\ -1.5+3.5217i \\ -1.5-3.5217i \\ -1.6235 \\ -1.0316 \\ -13.2944 \\ -1.3014+2.5136i \\ -1.3014-2.5136i \\ -1.0742 \end{pmatrix}$$

Next, an estimator with eigenvalues located at (-1.1, -1.2, -1.3, -1.4, -1.5) was designed for the system. All data about the estimation and the control law are given at the end of this chapter. Finally the following experiment is carried out to test applicability of the controller.

In this test, the load of system 2 is increased by 10% and system 1 is operating no-load. In figure 13-17 estimates of the load and some of the state variables estimation are given. Similar to the decentralized case, estimators approach actual values in a short time. In contrast to the decentralized experiment, availability of the tie power is not necessary, because tie power is one of the state variables of the system. Generating power and the main piston position change for the system 1 are given in figure 18-19, which shows they approach zero (no-load). Open loop system again shares the load between two systems. The same variables for the system 2 are given in figure 20-21, and as is expected, approach the load of the system 2 in the closed loop system. In figure 22-23, frequencies of both areas are given. Closed loop system satisfies the LFC objective and retains the frequency level in steady state. Finally, tie power change in fig24, shows that all LFC objectives are met in closed loop system.

figure 25- load 1 estimate versus actual values

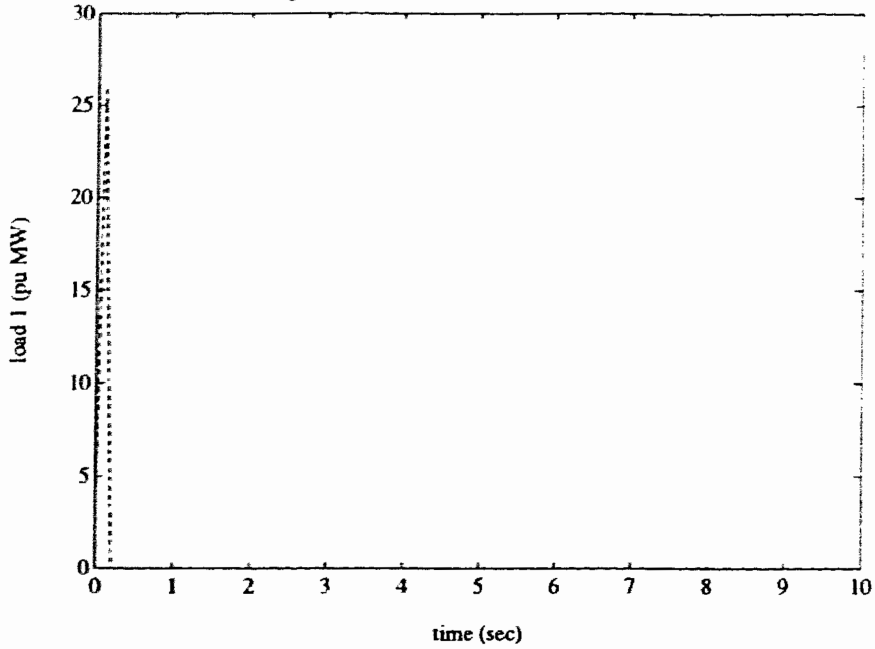


figure 26- load 2 estimate versus actual values

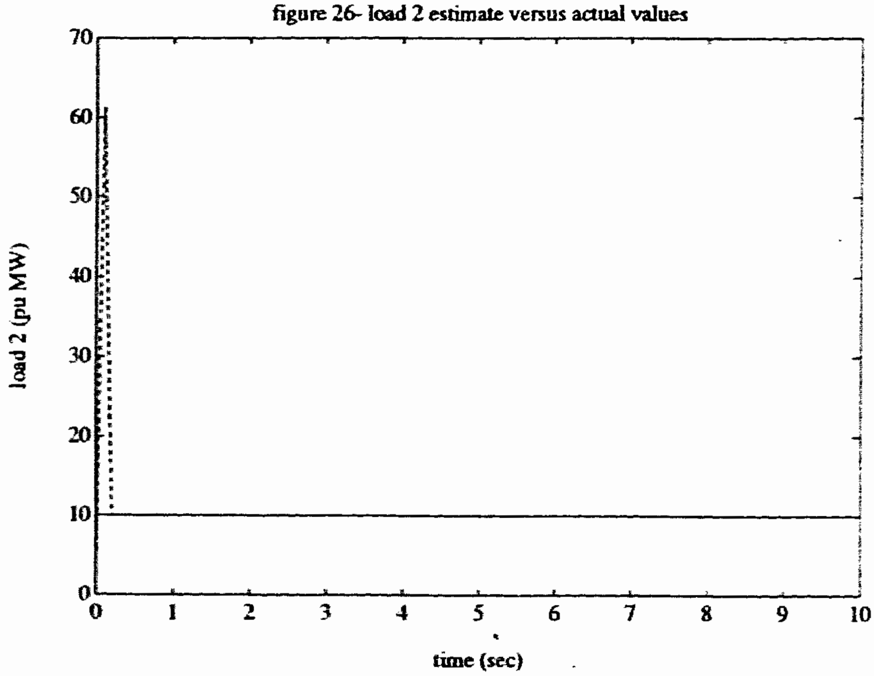


figure 27- generating power change estimate versus actual values

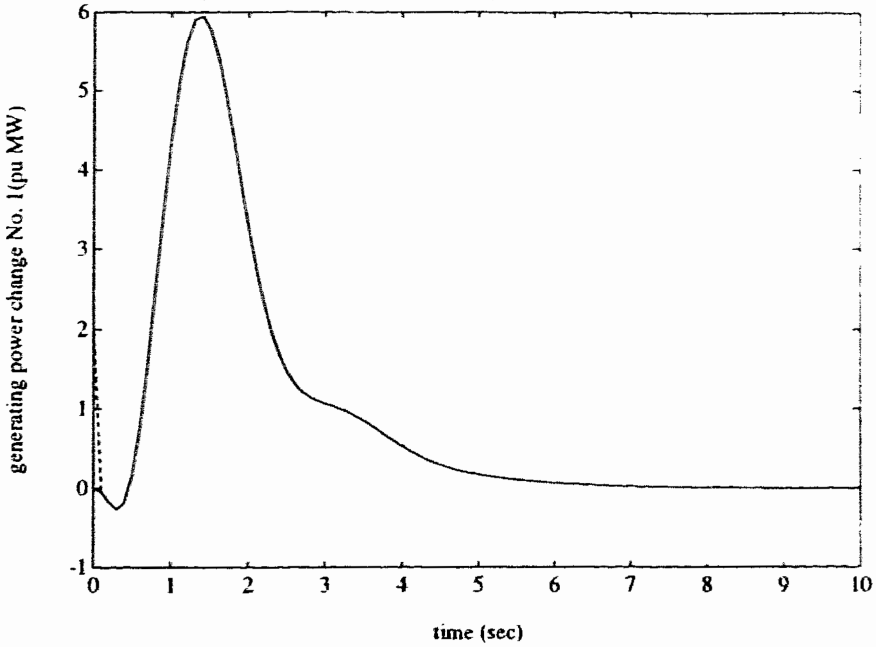
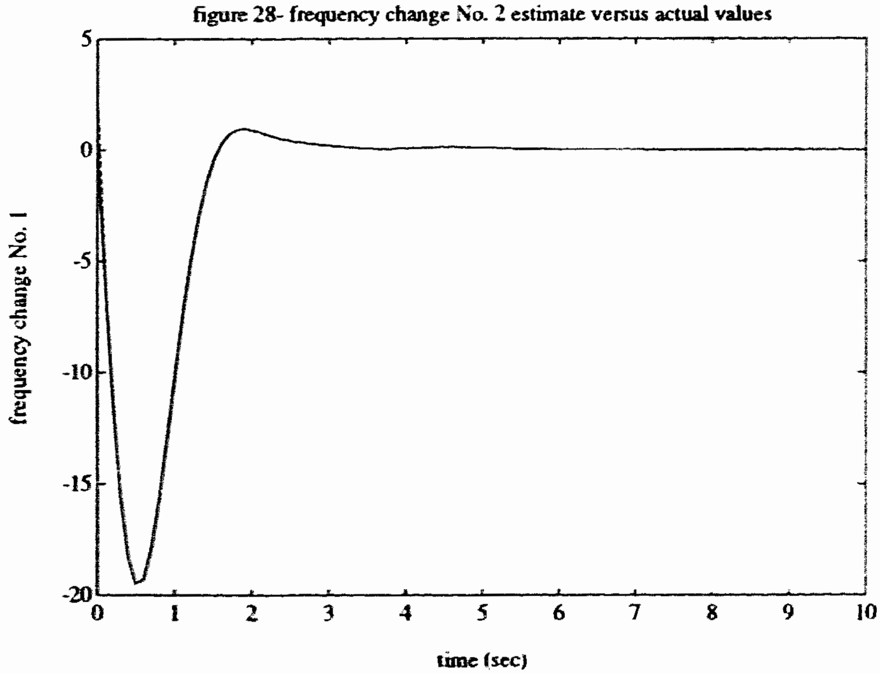


figure 28- frequency change No. 2 estimate versus actual values



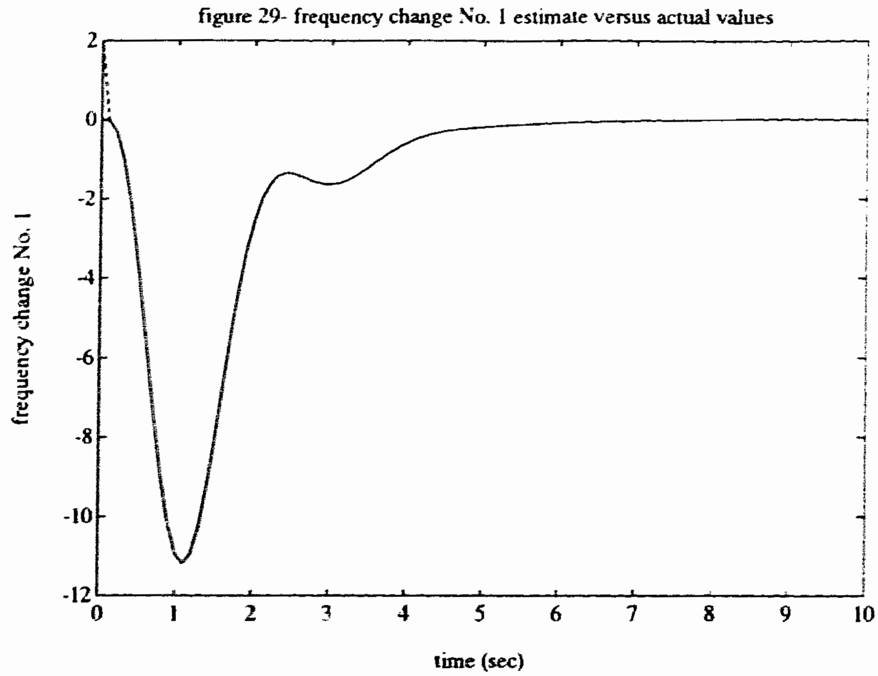


figure 29- frequency change No. 1 estimate versus actual values
frequency change No. 1
time (sec)
2
0
-2
-4
-6
-8
-10
-12
0
1
2
3
4
5
6
7
8
9
10

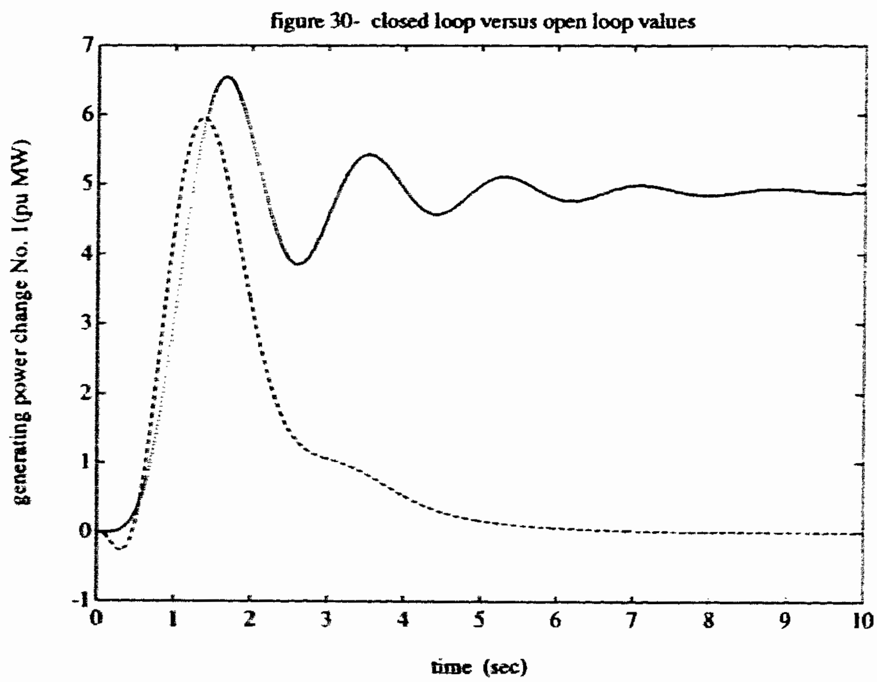


figure 30- closed loop versus open loop values
generating power change No. 1 (pu MW)
time (sec)
7
6
5
4
3
2
1
0
-1
0
1
2
3
4
5
6
7
8
9
10

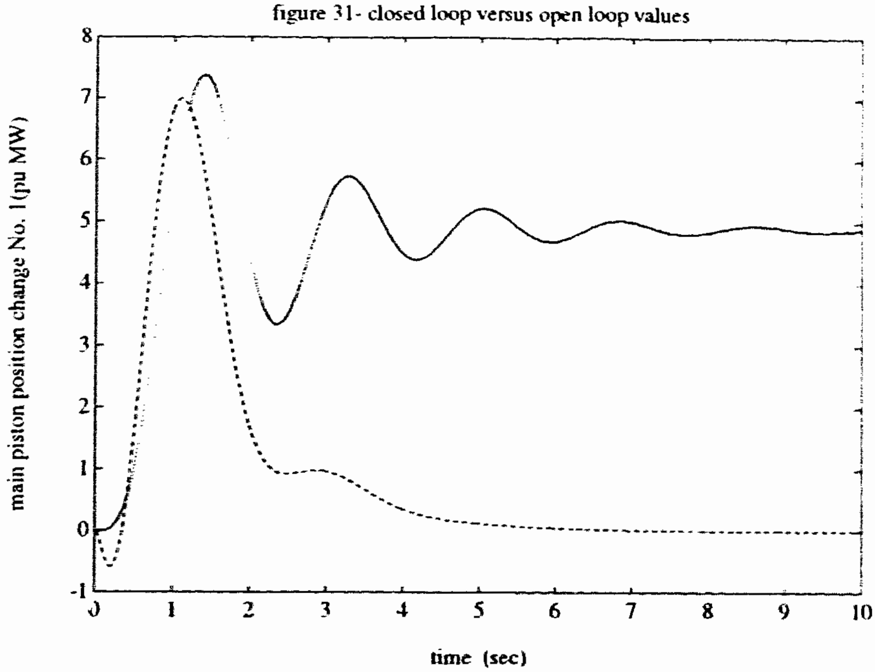


figure 31- closed loop versus open loop values
main piston position change No. 1(pu MW)
time (sec)

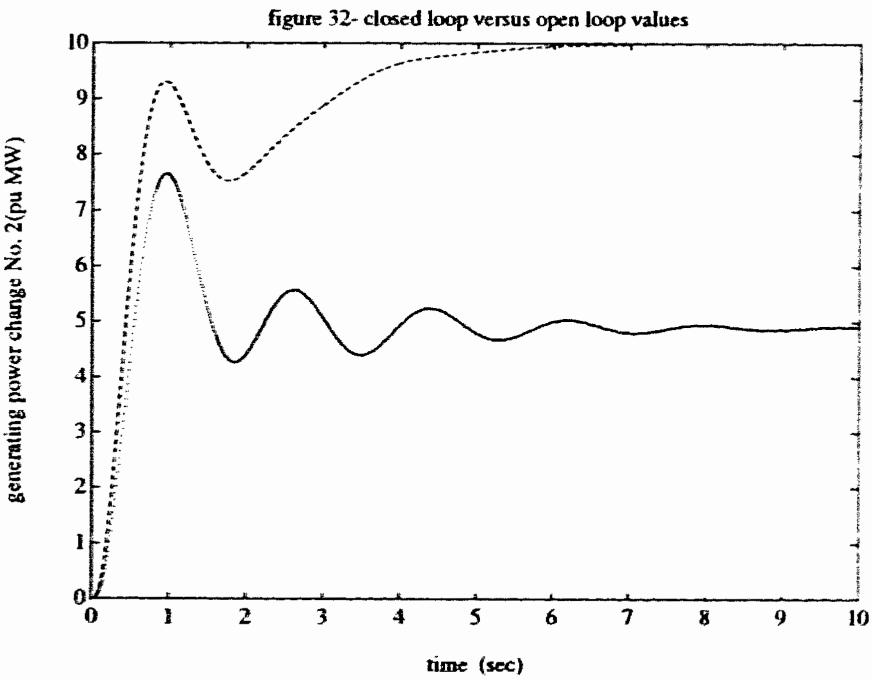


figure 32- closed loop versus open loop values
generating power change No. 2(pu MW)
time (sec)

figure 33- closed loop versus open loop values

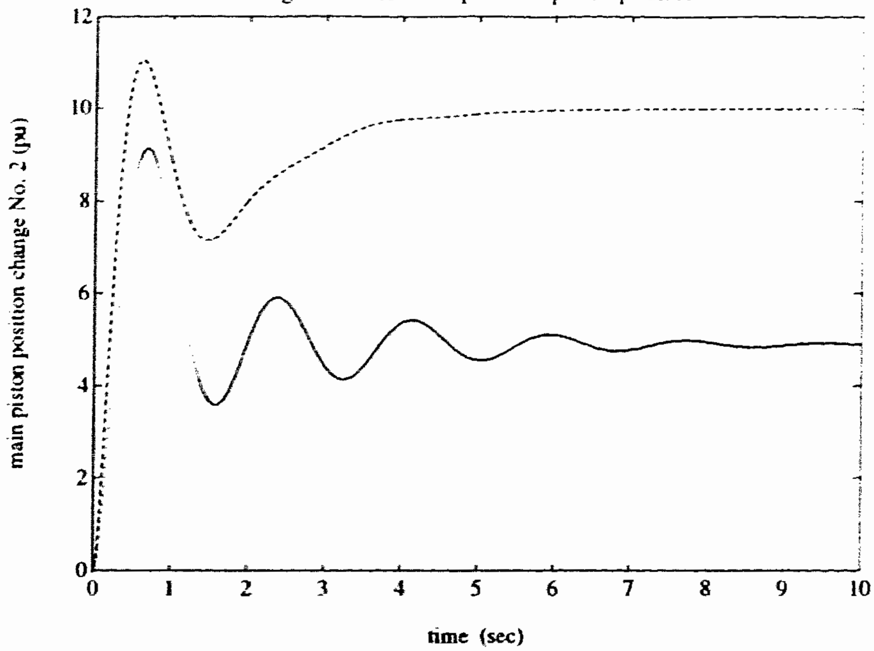


figure 34- closed loop versus open loop values

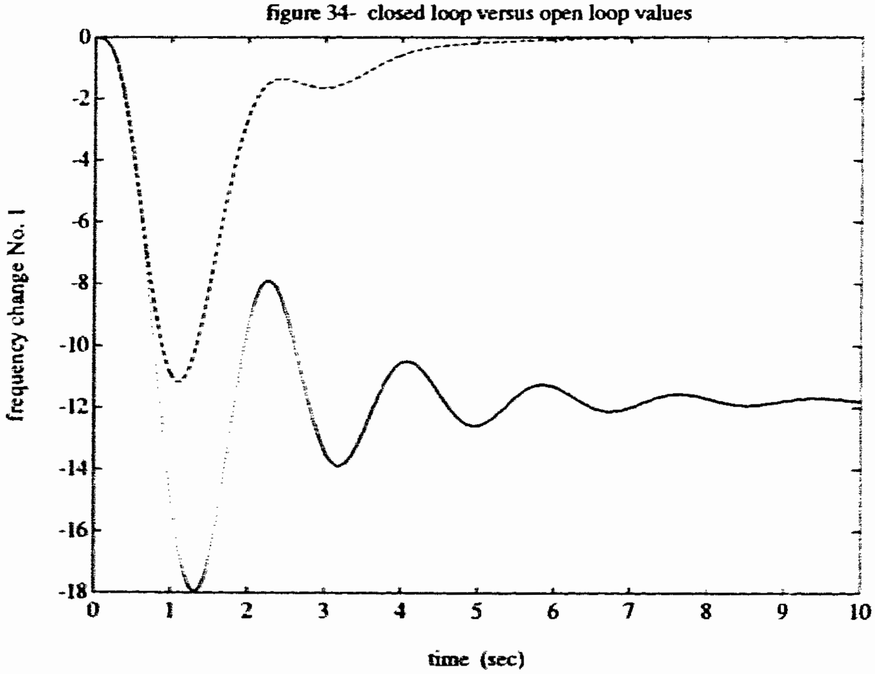


figure 35- closed loop versus open loop values

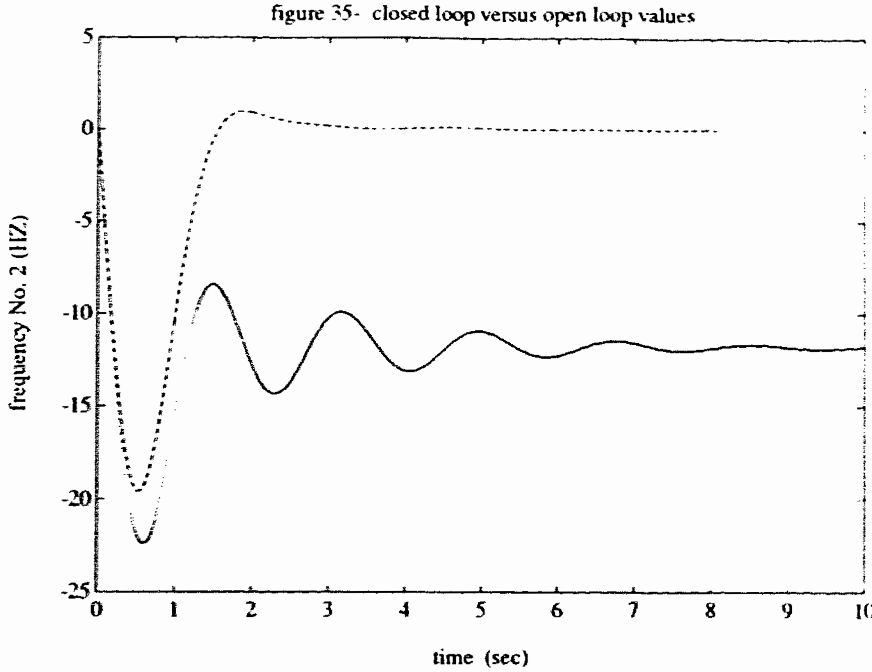
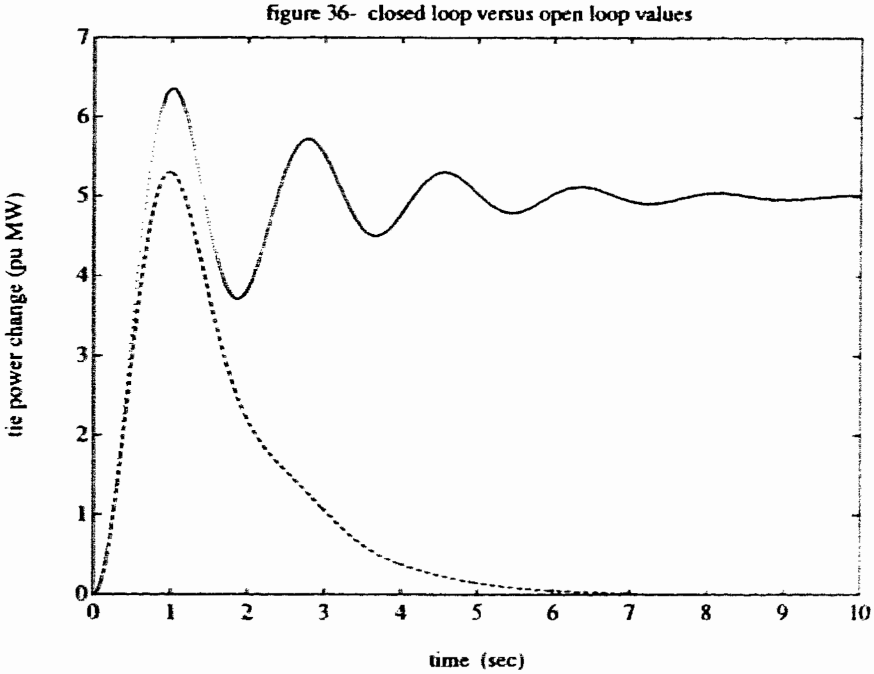


figure 36- closed loop versus open loop values



Centralized System Parameters

The parameters of the generator using the earlier equations are :

$$A = \begin{bmatrix} 0 & .545 & 0 & 0 & -.545 & 0 & 0 \\ -6 & -.05 & 6 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3.33 & 3.33 & 0 & 0 & 0 \\ 0 & -5.21 & 0 & -12.5 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & -.05 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3.33 & 3.33 \\ 0 & 0 & 0 & 0 & -5.21 & 0 & -12.5 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 12.5 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 12.5 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ -6 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -6 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$estimator\_eigenvalues = \begin{bmatrix} -1 \\ -1.2 \\ -1.4 \\ -1.6 \\ -1.8 \end{bmatrix}$$

$$Q = \begin{bmatrix} 2.0645 & -.5513 & -.595 & .595 & .5513 & -.1834 & .1834 & -.0618 & .0618 \\ -.5513 & .8205 & .2920 & -.1448 & -.6116 & .0692 & -.03 & .2285 & -.1465 \\ -.5950 & .2920 & .3695 & -.2656 & -.1448 & .0926 & -.0650 & .2076 & -.1498 \\ .595 & -.1448 & -.2656 & .3695 & .2920 & -.0650 & .0926 & -.1498 & .2076 \\ .5513 & -.6116 & -.1448 & .2920 & .8205 & -.03 & .0692 & -.1465 & .2285 \\ -.1834 & .0692 & .0926 & -.0650 & -.03 & .0248 & -.0174 & .0398 & -.0245 \\ .1834 & -.03 & -.0650 & .0926 & .0692 & -.0174 & .0248 & -.0245 & .0398 \\ -.0618 & .2285 & .2076 & -.1498 & -.1465 & .0398 & -.0245 & .2155 & -.1833 \\ .0618 & -.1465 & -.1498 & .2076 & .2285 & -.0245 & .0398 & -.1833 & .2155 \end{bmatrix}$$

$$K = \begin{bmatrix} -1.0534 & .7960 & .6262 & -.3255 & -.3695 & .16 & -.08 & .3498 & -.1823 \\ 1.0534 & -.3695 & -.3255 & .6262 & .7960 & -.08 & .16 & -.1823 & .3498 \end{bmatrix}$$

$$N = \begin{bmatrix} -1.11o8 & .8051 & 0 & 0 \\ -3.5341 & 1.5229 & 0 & 0 \\ 3.3216 & -.2490 & 0 & 0 \\ 3.3216 & -.2490 & 0 & 0 \\ .1219 & 3.7088 & 0 & 0 \\ 2.0708 & -2.9763 & 0 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} 13.8849 & -10.0635 \\ 44.1767 & -19.0360 \\ -41.5205 & 3.1130 \\ -1.5243 & -46.3606 \\ -25.8856 & 37.2037 \end{bmatrix}$$

$$F = \begin{bmatrix} .5062 & -.4391 & -.2151 & .1171 & .3182 \\ 2.3361 & -1.397 & -.7609 & .1159 & .6020 \\ -.9434 & 1.3130 & -2.5278 & .1831 & -.0984 \\ .9477 & .0482 & .3032 & -2.4049 & 1.4660 \\ -2.3335 & .8186 & .2924 & -.5827 & -1.1765 \end{bmatrix}$$

$$E = \begin{bmatrix} -13.0007 & 9.3791 & -5.3483 & 3.7031 \\ -43.3252 & 17.6577 & -17.7960 & 7.9726 \\ 32.9138 & -.2246 & 17.7179 & -1.1829 \\ 4.1322 & 37.4375 & .8142 & 19.8009 \\ 24.1392 & -36.7416 & 10.9219 & -14.7945 \end{bmatrix}$$

Decentralized System Parameters

The parameters of the generator are :

$$A = \begin{bmatrix} 0 & .545 & 0 & 0 \\ -6 & -.05 & 6 & 0 \\ 0 & 0 & -3.33 & 3.33 \\ 0 & -5.21 & 0 & -12.5 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 12.5 \end{bmatrix}$$

$$C = [1 \quad 1 \quad 0 \quad 0]$$

$$D = \begin{bmatrix} 0 \\ -6 \\ 0 \\ 0 \end{bmatrix}$$

$$eigenvalues = \begin{bmatrix} -13.2768 \\ -0.7608+2.9832i \\ -0.7608-2.9832i \\ -1.0817 \end{bmatrix}$$

$$Q = \begin{bmatrix} 2.1231 & -.2166 & -.7372 & -.2176 & 1.4988 \\ -.2166 & .2942 & .2832 & .0542 & -.4059 \\ -.7372 & .2832 & .5493 & .1419 & -.4779 \\ -.2176 & .0542 & .1419 & .0403 & -.1088 \\ 1.4988 & -.4059 & -.4770 & -.1088 & 1.7185 \end{bmatrix}$$

$$K = [-1.6369 \quad .5318 \quad .9529 \quad .2424 \quad -1.3109]$$

$$N = \begin{bmatrix} -1.0028 & 0 \\ 3.6049 & 0 \\ 5.7862 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} 12.5352 \\ -45.0609 \\ -72.3278 \end{bmatrix}$$

$$K = [.4265 \quad .3007 \quad .08 \quad .1675]$$

$$F = \begin{bmatrix} .4569 & .3964 & -.1730 \\ -2.6424 & -1.4249 & .6218 \\ -2.6363 & -2.2872 & -2.3320 \end{bmatrix}$$

$$E = \begin{bmatrix} -12.6175 & -4.7653 \\ 46.3596 & 18.0893 \\ 56.8647 & 30.6405 \end{bmatrix}$$

$$estimator\_eigenvalues = \begin{bmatrix} -1 \\ -1.1 \\ -1.2 \end{bmatrix}$$

SUMMARY

In this chapter simulation of the two area system and decentralized system has been carried out. Control input is of the PI structure where state variables are the proportional part and the area control error is used for the integral part. Least square error technique is used to partially compensate for the unknown load. The responses of the systems show that load frequency objectives are met and UIO is capable of estimating state variables disregarding the unknown loads.

CHAPTER 5

SUMMARY AND CONCLUSIONS

This final chapter provides a summary of the thesis and outlines the major conclusions to be drawn.

5-1 Summary

Chapter one gave the general definitions of LFC, LOC and LQR. A literature review of some traditional and existing methods were briefly described. In addition, the need for UIO as a part of control scheme was emphasized.

In chapter two the modelling of the electrical two-area system was fully described. Modelling started with formulating different parts of the system and then they were combined to construct the state equations of the system. Load or interconnecting effect was defined as unknown input in the systems.

Chapter three addressed the general ideas of LQR. The approach of changing state variable weighting matrix Q to adopt the desired LQR was introduced. Aggregation to decompose the problem into first or second order system was used to facilitate the procedure of the control. The UIO technique to estimate the state variables and load, needed for control input was described.

In chapter four simulation tests on the two-area system were carried out. Simulation had two parts. In the first part by using LQR method explained in chapter three, simulation tests of the decentralized system for step and triangle load change was carried out, and the difference between open loop and closed loop systems were explained. Second part demonstrates the responses of the centralized two-area system to the step and triangle load change.

5-2 Conclusions

In this thesis, an approach for LFC based on optimal modal LQR was introduced. In this approach , the proper state variable weighting matrix is ffound analyticly to ensure robust and stable response of the system in varying power demand. The accomplishments are listed as follows:

a) Aggregation: By using this method we can divide the systems into subsystems and find appropriate control input for each of those subsystems to get the desired LQR. Overall control input and weighting matrices are the combination of those of the individual subsystems. Without using this technique we had to deal with complete order system which involves a lot more calculations.

b) Unknown Input Observer: As we know in most of the real systems unknown inputs are present. They can be load, noise or even the own system parameter changes, so the need for the UIO is inevitable. The approach used for UIO eliminates the unknown input from observer equations and enables us to use conventional observer to reconstruct state variables. Necessary condition for UIO existence is that the number of the unknown inputs are less than that of the outputs. However in some cases as we did in two-area systems, we can combine different unknown inputs into smaller groups which can help us to satisfy the necessary requirement.

c) Optimal Modal Linear Quadratic Regulator Control: The main purpose of this controller is improving system response in different operating conditions. This approach is a multi-objective controller, which minimizes the cost function of the system and places the eigenvalues of the system simultaneously. While pole placement tries to restrict the transient time of the system response, optimization improves the transient response and the control effort needed for the control.

5-3 Toward the Future

In the text we made the necessary assumptions for each of the techniques. Now the questions can be raised as follows:

a) Unknown Input Observer : The number of unknown inputs is crucial for the UIO, so if the number of them can not satisfy the necessary condition, UIO can not be used. The question for changing the necessary condition with at least a more relaxed one is standing.

b) Linear Quadratic Regulator Control: The questions regarding LQR are in three areas;

1) The choice of quadratic cost function is the usual selected one in modern approach. But the search for other cost functions that ensure a better response for the system can be promising.

2) Compensation of the unknown input effects for the control procedure is one other necessary objective. Otherwise we will not be able to compensate for its effect on the system, and this can cause the deteriorating of the LQR solutions. This part of the research also is the important part of the control.

3) Necessary conditions for the feasibility of the LQR were outlined in chapter three and as it was mentioned there, those necessary conditions do not lend themselves to an explicit equation. This is the main shortcoming of this approach. Changing the control input weighting matrix R, is another possibility that can help solve the problem.

APPENDIX

TWO AREA SYSTEM DATA

System Parameters : For the purpose of simulation a two-area system with the following constants has been considered . It is supposed here that we have three state variables as our output.

Terminal Voltages : (Kv)

$$u_1 = 400$$

Nominal Frequency : (Hz)

$$f^{\circ} = 60$$

Source Impedances : (ohm)

$$x_{s1} = 0.15$$

Transmission Line Impedance : (ohm)

$$x_{12} = 1$$

APPENDIX. TWO AREA SYSTEM DATA

Transmission Factor :

$$a_{12} = -1$$

Exciter Voltages : (Kv)

$$E_1 = 0.6$$

Torque Angles : (Rad)

$$\delta_1 = \pi/6$$

$$\delta_2 = \pi/3$$

Speed Regulations : (Hz/Mw)

$$r_1 = 2.4$$

Turbine Time Constants : (sec)

$$T_{t1} = 0.25$$

Speed Governer Gain :

$$K_{G1} = 20$$

Turbine Gains :

$$K_{t1} = 0.05$$

Generators Time Constants : (sec)

$$T_{G1} = 0.20$$

Exciter Gain :

$$K_{E1} = 20$$

Exciter Time Constants : (sec)

APPENDIX. TWO AREA SYSTEM DATA

$$T_{E1} = 0.05$$

Inertia Constants : (sec)

$$H_1 = 7$$

Load Freq Regulations : (Mw/Hz)

$$D_1 = 0.008$$

Load Voltage Requlations : (Mw/Kv)

$$P_{du1} = 0.005$$

References:

M. Aldeen, J.F. Marsh, (1990), *Observability, Controllability and Decentralized Control of Interconnected Power Systems*, Computers & Elect. Eng Vol. 16, No. 4, pp. 207-220

M. Aldeen, J.F. Marsh, (1991), *Decentralised Proportional-Plus-Integral Design Method for Interconnected Power Systems*, IEE Proceedings-C, Vol. 138, No. 4

A.T. Alexanderidis, and G.D.Galanos, (1987), *Optimal pole placement for multi-input controllable systems*, IEEE Trans., CAS-34, pp 1602-1604

M.H.Amin, (1985), *Optimal pole shifting for continuous multivariable linear systems*, Int. J. Control, Vol 1, pp. 701-707

B. D. O.Anderson, and J.B.Moore (1971), *Linear Optimal Control*, Prince Hall, Englewood Cliffs, N.J.

A.N. Andry, Jr. , E.Y. Shapiro, J.C.Chung, (1983 , *Eigenstructure Assignment for Linear Systems*, IEEE Trans. Vol AES-19, pp 711-729

M. Aoki, (1968), *Control of large scale dynamic systems by aggregation*, IEEE Trans., AC-13, pp 246-253

C. T. Chen, (1984), *Linear system theory and design*, (New York: HRW Publishing).

J.H. Chow, J.J. Sanchez-gasca, (1989), *Pole placement Design of PSS*, IEEE Trans Vol.4., No.1, pp 271-277

G.S. Christensen, M. E. El-Hawary and S. A. Soliman, (1987), *Optimal Control Applications in Electric Power Systems*, Plenum Press

E. J. Davison, and H. W. Smith, (1971), *Pole Placement in Linear Time-Invariant Multivariable Systems with Constant Disturbances*, Automatica, Vol. 7, pp. 489-498

REFERENCES

O.I. Elgerd, (1970), *Optimum Megawatt-Frequency Control of Multiarea Electric Energy Systems*, IEEE Trans, Vol. pas-89, No. 4, pp. 556-563.

O.I. Elgerd, (1981), *Control of Electric Power Systems*, McGraw Hill Co.

O.I. Elgerd, (1971), *Electric Energy Systems Theory*, Mcgraw-Hill Book Company,

Y.Guan, and M.Saif, (1991), *A Novel Approach to the Design of Unknown Input Observer*, IEEE Trans Vol. 36, No. 5, pp 632-635,

B. Habibullah and Yao-nan Yu, (1974), *Physically Realizable wide range Optimal Controllers for Power Systems*, IEEE Trans. Power Appar. Syst, pp 1498-1506

W.E. Hopkins, J.Medanic and W.R. Perkins, (1981), *Output Feedback Poleplacement in the Desin of Suboptimal Linear Quadratic Regulators*, Int.J.Ctr., Vol 34, pp 593-612

Pei-Hwa Huang, Yuan-Yih Hsu, (1990), *Eigenstructure Assignment in a Longitudal Power System via Excitation Control*, IEEE Trans Vol 5., No.1, pp 96-102

C.D. Johnson, (1975), *On observers for linear systems with unknown and inaccessible inputs.* Int. J. Contr., Vol. 21, pp. 825-831

M.A.Johnson, and M.J.Grimble, (1987), *Recent Trends in Linear Optimal Quadratic Multivariable Control Systems Design*, IEE Proceedings D, Vol. 134

N. Kobayashi and T. Nakamizo, (1982), *An observer design for linear systems with unknown inputs*, Int. J. Contr . Vol 35, pp. 605-619

M.L.Kothari, J.Nanda, (1988), *Application of Optimal Control Strategy to Automatic Generation Control of a Hydrothermal System*, IEE Proceedings, Vol 135, Pt.d, No 4

P. Kudva, N. VIswanadham, and A. Ramakrishna., (1980), *Observers for linear systems with unknown inputs*, IEEE Trans, Automat Contr, vol AC-25 , pp 113-115

S. Mankin, and Y. Shinohara, (1975), *Application of Linear Optimal Regulator Technique to Control of a Nuclear Reactor Plant, J. Nucl. Sci.* Technology, Vol. 12, pp 727-731

D.Q. Mayne and P.Muroch, (1970), *Modal Control of Linear Time Invariant Systems*, Int.J. Ctr, Vol 11., pp 223-227

J.Medanic, H. S.Tharp, and W. R.Perkins, (1988), *Pole placement by performance criterion modification*, IEEE Trans., AC-33, pp. 469-472

REFERENCES

J.Medanic, (1979), *Design of Low Order Optimal Dynamic Regulators for the Linear Time Invariant Systems*, Conference on Isnformation Sciences and Systems, John Hopkins Univ.

J.I. Meditch and G.H. Hostetter, (1974), *Observers for systems with unknown and inaccessible inputs*, Int. J. Contr, Vol 19, pp 473-480

R.J. Miller and R.Mukundan, (1982), *On designing reduced order observers for linear time invariant systems subject to unknown inputs*, Int. J. Contr., Vol 35, pp 183-188

S.M. Miniesy, E.V. Bohn, (1971), *Two Level Control Of Interconnected Power Plants*, IEEE Trans. On Power Apparatus and Systems, Vol. PAS-90, pp 2742-2748,

V.R.Moorthi and R.P.Aggarwal, (1972), *Suboptimal and near-optimal control of a load-frequency-control system*, Control & Science, Vol 119 pp 223-229

B.Porter, and J. J. D'Azzo, (1977), *Algorithm for the synthesis of state-feedback regulators by entire eigenstructure assignment*, Electron. Lett., Vol 13, (8), pp 230-231

T. Reddoch, P. Julich, T. Tan and E. Tacker, (1971), *Models and Performance Functional for Load Frequency Control in Interconnected Power Systems*, IEEE Conf. Decision and Control, Florida

M.Saif, (1989), *Optimal Linear Regulator Pole Placement by Weight Selection, Int. J. Of Control Vol. 50, pp. 399-414,*

M.Saif, (1989), *Optimal Modal Control of A Power Reactor*, Control Theory and Advanced Technology, Vol No. 3, pp 249-264

M.Saif, (1989), *A Novel Approach for Optimal Control of A Pressurized Water reactor, IEEE Trans, Nucl. Sci., Vol. NS-36, pp. 1317-1325*

M.Saif and Y.Guan (1992), *Decentralized Estimation in large scale interconnected dynamical systems*, Automatica, Vol 28, No 1., pp 215-219

M. Saif, F. Villaseca, (1986), *Hierarchical Load-Frequency Control of Hydrothermal Systems*, Proceedings of the 1986 North American Power Symposium, pp 164-170

M. Saif, (1993), *Robust Servo Design with Applications*, IEE Proceedings- Part D, Vol. 140, No. 2, pp. 87-92,

K.M. Sobel, E.Y. Shapiro, (1985), *Eigenstructure Assignment: A Tutorial-Part 1 Theory, Part 2 Applications*, Proceedings of American Control Conference, pp. 456-467

REFERENCES

O.A.Solheim, (1972), *Design of Optimal Control Systems with Prescribed Eigenvalues*, Int J. Of Control, Vol 15, pp 143-160

S. Srinathkumar, (1978), *Eigenvalue/Eigenvector Assignment Using Output Feedback*, IEEE Trans Vol.AC-23, pp 79-81

S. H. Wang, E. J.Davison, and P. Dorato, (1975), *Observing the states of systems with unmeasurable disturbances*, IEEE Trans, Automat. Contr., Vol AC-20, pp. 716-717

Yao-Nan Yu, (1983), *Electric Power System Dynamics*, Academic Press