USING GRAPHICS CALCULATORS AND COMPUTERS TO TEACH TRANSFORMATIONS OF FUNCTIONS AND RELATIONS IN MATHEMATICS 11

by

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Using Graphics Calculators and Computers to Teach Transformations of Functions and Relations in Mathematics 11

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ABSTRACT

The purpose of this study was to investigate a technologically-inexperienced teacher's attempts to use three different modes of technology; graphics calculators, computer lab, and a single computer with a projection unit, and to examine his and his students' impressions of teaching and learning with them in order to determine if they can be used successfully and if their use is desirable.

The seven week study involved three classes of Mathematics 11 students, each using a different mode of technology solely within the classroom to study a chapter on transformation of functions. During that time the teacher kept a log that recorded his and the students' reactions to using technology, and the relevant contents of this log are presented to show the participants' impressions of each mode. At the conclusion of the classroom work all students completed a written questionnaire with six students giving additional taped interviews.

The results of the study indicate that none of the three modes is relatively superior with respect to increasing student achievement on the prescribed curriculum as measured using paper and pencil tests. Students enjoyed working with all forms of technology, but preferred a mode that they could operate individually. A comparison of the three modes indicates that some devices may be superior for a particular topic, but any device that allows the students to see a display of a graph enhances learning. The success of the computer as a teaching tool, however, is largely dependent on the quality of the software.

The study contains numerous implications for teaching with technology, and offers many suggestions for planning and teaching with each mode. The conclusions reveal that a technologically-inexperienced teacher can use any of the three modes successfully, and that such a methodology is desirable. The recommendation is made that schools considering the use of technology should give priority to graphics calculators. The study also suggests that the current secondary mathematics curriculum is out-dated with respect to technology,

that there is a need for provincial leadership in assisting teachers to maximize the potential offered by technology, and that the support resources for teachers are inadequate.

DEDICATION

To my family:

To my parents, for their continual support and encouragement,

To Deretta, for her understanding,

To Christie and Steven, may they experience the satisfaction of meeting a challenge.

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To those at Simon Fraser University who created and taught the Master of Science (Education) program for secondary mathematics teachers: Sandy Dawson, Harvey Gerber, and Tom O'Shea, I extend my sincere thanks and appreciation for a challenging and rewarding experience.

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CHAPTER ONE

INTRODUCTION

In Canada today, as in many other countries in the world, virtually all aspects of our lives are touched in some way by the increasingly rapid advances being made in technology. From space shuttles to sophisticated children's toys, from orbiting telecommunication satellites to car telephones, we are in the midst of an explosion of technological advances. The education system too shows the impact of today's scientific leaps with schools making daily use of modern equipment such as laser printers, photo-copying machines, fax machines, computers and modems.

Yet the majority of these technological wonders in schools are used *outside* the classroom. *Inside* the classroom, specifically the mathematics classroom, there is little everyday use of technology as a teaching tool with the exception of the basic scientific calculator. Possible reasons for this imbalance could be the cost of the equipment, lack of teacher knowledge of how and when to use the equipment available, and uncertainty about the effect of the new technology on student achievement.

Present literature on the subject of technology in the schools, and specifically in the mathematics classroom, offers considerable reading with regard to why technology should be used in schools, what the benefits of technology to education are, and why technology is important to mathematics education in particular. On the other hand, few articles discuss the related issue of what type of technology is best suited to the mathematics classroom and to today's mathematics teacher and how and when the technology should be used.

Evidence regarding the importance and relevance today of technology in schools is supplied by the National Council of Teachers of Mathematics' *Curriculum and Evaluation*

Standards for School Mathematics (1989) which makes some very pointed recommendations about the use of technology in mathematics classrooms. The NCTM's document suggests that each mathematics classroom should be equipped with a computer for the purposes of demonstrations, that each student should have access to a computer and a graphics calculator for individual work (p. 124), and that graphics calculators and computers should definitely be used to teach the concept of functions (p. 155).

Further support for the use of technology in general in the mathematics classroom is supplied by the NCTM's *Professional Standards For Teaching Mathematics* (1991) in which Standard Three states: "The teacher of mathematics should promote classroom discourse in which students use a variety of tools to reason, make connections, solve problems and communicate..." (p. 45). This document also asserts that "educational research findings from cognitive psychology and mathematics education indicate that learning occurs as students actively assimilate new information and experiences and construct their own meaning" (p. 2).

Additional support for the position of using graphics calculators in the classroom is provided by Dion (1990) who conjectures that "precalculus students benefit from an intuitive understanding of functions gained through the use of graphics calculators" (p. 564). The virtue of computers and their value in teaching graphing-related topics is mentioned by DiFazio (1990) who states that "graphics software can be used in a tremendous variety of ways to supplement instruction in mathematics" (p. 440).

The above publications provide ample evidence of the generally accepted value of graphics calculators and computers in today's schools. But while there seems to be an abundance of opinions that calculators and computers are a valuable weapon in the mathematics teachers' arsenal of methods, not enough is known about how or where to use the technology available. Kelly (1991) points out that while the NCTM and various provincial bodies are clamoring for teachers to implement technology, there is very little material to suggest how all of these modern marvels should be used (p. 2).

Dion (1990) states, "several years after the introduction of graphics calculators, no consensus has been reached on how these calculators should be used in secondary school and undergraduate mathematics" (p. 564). She further concludes that "graphics calculators offer an insight into the nature of functions that was previously unavailable to students" (p. 567). She does not mention what type of insights one might expect.

The Royal Commission on Education for British Columbia (Sullivan, 1988) acknowledges the fact that technology is a factor in today's educational setting by stating "it is apparent that the school curriculum and, indeed, the nature of the learning process itself is being transformed by such technology" (p.14). However, just how the learning process is being transformed is a question that researchers are only beginning to look at, and consequently it is a question that would seem to be very relevant in terms of investigation by todays' practicing mathematics teachers.

Barrett and Goebel (1990) state that "the microcomputer has not had the impact that many people predicted, however. There appear to be two primary reasons for this. First, many schools still do not have a computer in each mathematics classroom. Second, those teachers who have a computer to use in front of their classes have had trouble defining its role in the classroom" (p.205).

As the literature referred to thus far indicates, the issue of graphics calculators and computers as teaching aids is a current one, and one that still has many unanswered questions.

Personal Background

My interest in using computers and graphics calculators as teaching tools was stimulated by the arrival in the school of a class set (30) of graphics calculators. As I investigated the possibilities for their use, I realized that there was other equipment in the school that could also be used for teaching mathematics; the computers in the computer lab, and the overhead projection device combined with a single computer and an overhead projector. Although the computer lab was not new to the school, and in fact had been at my disposal to a limited degree for several years, because I had no idea of when or how to use it as a teaching tool, I, like many other teachers, had ignored it. Now that I was beginning to have the feeling that I was surrounded by the potential of technology, I thought it a logical step to try to determine how to use that potential to benefit my students.

There were many other reasons for my desire to explore the use of computers and graphics calculators, one of them being the fact that after 20 years of teaching mathematics I was looking for something different in order to help maintain my own interest in the profession. On a similar note I was also curious to determine if the students might likewise enjoy a change in the normal routine of their mathematics classes, and to see if they found any one of the three modes they were going to use more interesting than another. I wondered too if using some form of technology would provide a vehicle for answering one of my perennial questions, that of how to challenge the more capable students. Another major concern I have about teaching mathematics, and teaching any subject area, is the question of how to encourage students to think independently and draw conclusions as opposed to the time honored system of memorizing facts and feeding them back on tests. I queried whether computers and graphics calculators might help answer that concern. Finally, I was curious about what impact the use of this technology might have on student achievement. All of these thoughts combined to help me decide to take a leap into what was for me the void of technology.

Purpose of the Study

The major purpose of the study is to investigate, using a descriptive approach, students' and a teacher's impressions of and reactions to the use of three different types of technology for teaching a unit on transformations of functions and relations in Mathematics 11. The three modes of technology referred to are those that are most likely to exist in B.C. secondary schools at present, specifically a computer lab with one computer for every one or two students, a regular classroom equipped with a single computer connected to an overhead projection device, and a class set of hand held graphics calculators. Specifically, the study examines the questions of whether it is possible for an experienced teacher who is inexperienced with computers and graphics calculators to use those devices as teaching tools successfully and whether it is desirable to teach using this technology. A third question is to determine which of the three modes of technology being employed is best for teaching a given topic. A final question is to determine if any conclusions can be reached regarding how the different technologies compare with respect to their effect on student achievement.

Whether a teacher can use the modes successfully or if technology is even desirable as a teaching tool will be examined from several points of view. Both the students' and teacher's views will be examined with respect to how the technology affected interest, how difficult it was to use, how it affected achievement, and how it contributed as an aid to teaching and learning. In addition, the effect the technology had on meeting the lesson objectives, specifically learning the content of the chapter being studied, promoting discussions about mathematics among students and aiding in having students *doing* math as opposed to simply being passive observers will be investigated. These criteria will also be used to judge which mode is "best". The study should also provide a picture from the students' points of view of how they would compare learning using some modern technology to learning in their usual mathematics classroom.

Significance of the Study

It is hoped that this study will have immediate and useful application for present mathematics teachers. It will provide implications for teachers with regard to the planning and instruction of certain topics in Mathematics 11 using modern technology. Specifically it will indicate what parts of the lesson plans used were not successful, what general types of changes need to be made to those lesson plans, and will provide a teacher with some information about how to plan successful lessons involving technology. It should show which topics in the Mathematics 11 unit covered in this study are best served by which (if any) of the three technologies and what effect the various technologies have on student interest and achievement. It is also hoped that teachers who have access to only one of the three modes of technology investigated here will be able to obtain some idea of what to do and what not to do by reading the lesson observations presented in Chapter Four. The review of the literature in Chapter Two may also provide interested teachers with some further ideas about how and when to use technology as a teaching aid.

Limitations to the Study

As might be expected with a study being carried out on a relatively small sample, there are some limitations to the research. The fact that there was only one teacher involved could lead to a problem of bias with respect to the teacher favoring one type of technology or one class, but knowing that this is a potential problem should help minimize it, if not eliminate it. Related to the issue of a small sample is the realization that this study was being carried out in one school only, and as a result the conclusions drawn might only reflect upon other schools with a similar type of student body. The physical setting of the computer lab itself was a problem for the class using the lab because there were not enough computers for each student. As a result the students were forced to share a computer and work in pairs in fairly cramped and stuffy quarters. This overcrowding could have affected their progress with and opinions of computers. (1)

The method of determining which groups got which technology was pre-determined on the basis of when the computer lab was available, when the graphics calculators were available, and when the overhead projection device was available, and this could be considered a limitation because the selection process was not random. There is also a possibility that students may have shared methods among the three classes and group contamination could have resulted. With respect to which students were placed in which class, it was possible that the classes may have had differing academic abilities because the classes were loaded by a computer program in a manner that was not entirely random (for example students involved in the French Immersion Program were intentionally programmed into many of the same blocks, and as a result many of them often tended to end up in the same non-French speaking classes).

The modes of technology that the students were using were available to them primarily during their regular classroom periods only. This limited accessibility resulted in the students generally being unable to use them outside of their regular class time and consequently they were denied the opportunity to experiment with them at their leisure. Such a constraint could have negated some of the potential of the devices.

Finally, the achievement tests that the students wrote were paper and pencil tests that measured their progress on the prescribed curriculum. The graphing devices were intentionally not permitted for the tests for two reasons: (1) the learning objectives outlined in the Curriculum Guide did not require them, and (2) each class had a different device and common tests were needed in order to make some comparisons with regard to achievement. This type of paper and pencil testing could be a limitation to the study because it did not measure any gains in the students' understanding of the technological devices nor the students' ability to apply the technology to solving problems, consequently the full impact of the devices may not have been measured. The tests have had no validity tests done on them, but they were consistent for all groups and were intentionally similar to tests given on the same units in the previous year.

Structure of the Thesis

Chapter One describes the general topic of the thesis, namely technology and the mathematics classroom. Some literature is cited to show that although some research on the topic has been done, important questions still remain to be answered. My reasons for being interested in this particular topic are given, and lead into an outline of the purpose of the study. The significance of the project is also outlined, as are its limitations. The chapter concludes with an outline of the central theme of each of the five chapters.

Chapter Two examines studies that have been done on similar topics,and presents the conclusions from those studies in addition to outlining the questions that those studies leave unanswered.

A setting for the study is sketched in Chapter Three with the presentation of information about several topics related to the planning and preparation of the lessons. These topics include information about the school in which the study took place and the types of students that attended that school so that readers will be able to judge the appropriateness of the results of this study to their own unique situations. It also contains specific data on the students in the three classes in which the various modes of technology were employed. General information about how the three different modes of technology were used in the classroom is given. A detailed list of the learning objectives that directed the lessons in the units studied is given in addition to showing how these objectives relate to the intended learning outcomes of the British Columbia Mathematics 11 curriculum. Problems that arose in the planning of the lesson and decisions that had to be made with respect to how to use the technology are expanded upon. The chapter concludes with a brief outline of the evaluation questionnaire administered to all students involved in the project at the culmination of the study, and an explanation of how students were selected for the taped interviews and where the questions they were asked originated.

What actually happened during the lessons forms the basis for Chapter Four. This chapter follows the three modes of technology independently, and presents a picture of what actually transpired in each of the three classes as the students worked their way through the course. The decision was made to discuss each mode of technology individually so that a reader can follow an uninterrupted dialogue of the proceedings for that mode. The observations reported in this chapter include my observations of what the students were doing and how they reacted to the various graphing devices being used, together with my examination of my reactions to what was taking place in the classroom. The chapter concludes with the text of the evaluation questionnaires given to the students at the end of the project. For the portion of the questionnaire in which students were asked to rate a statement on a scale from one to five about the use of their particular mode of technology, the means of the responses for each class are reported. For the open-ended questions section of the questionnaire, a summary of the most frequent responses is reported. Finally, some of the common themes from the students' comments in the taped interviews are outlined.

The final chapter focuses on answering the questions raised in Chapter One. To reiterate, the major purpose of the study and the one from which the questions referred to in Chapter One are derived, is to investigate students' and a teacher's impressions of and reactions to using three different types of technology as teaching and learning devices in a section of the Mathematics 11 course. Chapter Five contains a report on this investigation, a report that examines the observations I made while teaching the classes and the students' responses to their questionnaires. The chapter considers how technology affected student achievement, how the students reacted to the use of technology, and draws some conclusions with regard to the effect using technology had on the teacher involved. Conclusions are also reached about how the three modes compared with an attempt to suggest which mode is "best". Changes that might be made to the lessons to improve them are also outlined. The thesis concludes with some specific implications for planning and instruction and some comments about whether a "technologically inexperienced" teacher can successfully use some or all forms of the technology used in this study and whether such implementation is desirable.

CHAPTER TWO

LITERATURE REVIEW

"Computers and calculators change what is feasible and what is important. They make the difficult easy and the infeasible possible." This statement by the Mathematical Sciences Education Board (MSEB) (cited in Foley, 1992, p. 144), is indicative of the thoughts of many mathematics educators who predict that the nature of mathematics teaching and learning in secondary schools is on the threshold of some major changes, with technology being the driving force behind those changes. The concept of school mathematics that should emerge will require a new vision of what school mathematics means. The rapid advances being made in technology, especially with graphics calculators and their increased affordability, is making it possible for the majority of secondary students to experience technology one-on-one.

An examination of the literature regarding the use of two major modes of technology--computers and graphics calculators--reveals some studies that have centered on only one of these technologies, and others that have used both. In the majority of cases the studies have many conclusions that are applicable to both modes, and in those cases I have generalized these conclusions to both modes. In cases where only one form has been used and the study's conclusions are specific to that mode only, it has been so noted. The literature also suggests that while there is a distinction between computers and graphics calculators, a more significant distinction exists between interactive and non-interactive modes of technology, and these distinctions are acknowledged in this review.

Why Use New Technology?

The question of why the newer forms of technology should be used to teach secondary school mathematics has multiple answers. Perhaps one of the most prominent reasons is found in the NCTM's Curriculum and Evaluation Standards (1989) which states that a major goal for students in mathematics is to make and test their own conjectures about the relationships between quantities (p. 84). This shift in focus will transform school mathematics into an investigative and exploratory subject, one with a potentially revitalized curriculum. Students will have the opportunity to explore new mathematical relationships that may not have surfaced otherwise, as typified by the student who discovered the relationship between the slopes of perpendicular lines using a graphing calculator, then continued his explorations to determine if there might also be a relationship between lines intersecting at any angle (Burrill, 1992). [If students are able to conjecture about their own ideas and to evaluate those ideas themselves, then they are playing a more active role in learning mathematics and are becoming more independent learners who do not rely solely on the teacher for ideas.] Currently this goal of exploring and conjecturing is not being met in the majority of classrooms, largely because of the difficulty students have working symbolically. But technological advances such as improved software and graphics calculators have the potential to reduce the obstacle of symboic manipulation.

Many educators believe that in discovering relationships for themselves, students will learn from understanding rather than from the traditional way of learning mathematics in which many students simply performed and often memorized meaningless symbolic manipulations. In the early 1980s many of those involved in mathematics education predicted that mathematics classrooms would change to resemble computer labs, with computers managing instruction of traditional curriculum, but this view never fully materialized. Recent advances in software and improvements in calculators have forced another change in thinking. The new technology allows a change in direction that promotes the implementation of a discovery approach to mathematical instruction.

Ruthven (1992) suggests students often treat the symbols as objects themselves, not as representations of the elements of some problem, and they learn, or try to learn, to manipulate meaningless symbols. He claims the power of the new technology lies in allowing students to first explore mathematical relations through numeric or graphic representations, a discovery process that aids students in formulating the nature of a relation more clearly and fully. Students are able to "see" and connect graphic images, symbolic expressions, and sets of related numerical values to compose mathematical pictures in their minds. Demana and Waits (1990) support this view, suggesting that analyzing a problem situation through both algebraic and geometric representations deepens students' understandings about the problem. For this particular study it means students will have the opportunity to make their own generalizations about transformations of functions and relations based on geometric evidence. It has been further hypothesized that being able to move among the three different representations--numeric, symbolic and graphic--develops a background of experiences for students that allows them to associate rules with graphs so that they will have a firm foundation for later work with graphically-introduced calculus concepts (Hector, 1992).

Ruthven expands his conclusions about technology and understanding to make further suggestions about the potential of the new devices to provide a medium for thinking and learning. A small research study done by Ruthven (cited in Ruthven, 1992) showed graphics calculators have the capacity to promote cognitive growth. In the study, two groups were compared, one with graphics calculators, one without. After almost a year in their courses, both groups were asked to write equations for six given graphs (the reverse of the function carried out by the graphics calculator and one of the goals of the unit under investigation in the current study). The technology group scored significantly higher than the other group. Specifically, they were better at simply recognizing what type of graph it was, such as quadratic or sine. They were also superior at extracting key information from a graph, recognizing the relationship between these features and their symbolization, and consequently better at writing the precise equation for the graph. Ruthven also suggests that the study gives evidence that the use of a "trial and improve" strategy can help ignite the critical insight needed to elevate a student's thinking to a more direct analytic approach. He further speculates that the use of technology, in this case in the form of graphics calculators, can favorably influence both the approaches students take to mathematics problems and their achievement. Finally, he suggests that this influence may depend as much on how the technology was used as much as the fact that it was used.

A project carried out by Montgomery Community College (Pennsylvania) in which all students in the class were equipped with graphics calculators showed that in the first year of the project involving a college algebra class, 72 percent of the class using the calculators received a C grade or better, a higher average than for previous similar courses not using graphics calculators taught by the same professor (Long, 1993). The suggestion from this project is that using this form of technology can increase student achievement, although the report did not indicate how achievement was measured.

A project carried out by some Vancouver teachers, in which their students used graphics calculators to study the same material as did the students in this study, is of special interest because of the similarity of subject matter (Gatley, 1990). They concluded that their students learned the relationships between the functions more quickly than if they had not had the technological tools, and that they reached a skill level that allowed them to voluntarily give up calculator use to answer some questions because they could picture the function mentally more quickly than previous classes that did not have access to this technology.

The new technology has another major asset, and that is its ability to make the mathematics associated with real-world problems accessible to secondary school students. Technology removes the drudgery of creating symbolic or graphic representations of these problems, reduces the need for contrived problems, and allows students to explore and solve realistic and interesting applications. It also makes realistic problems accessible to students earlier in their development because they can overcome their lack of ability with algebraic techniques. Particle motion problems are examples of a type of problem that formerly required analytical calculus methods and were consequently beyond the scope of most secondary students (Demana & Waits, 1993). Now technology can simulate the motion of the particle visually, allowing the secondary student to investigate the problem. It should be noted, however, that in this particular example, the students and in all probability the teacher, would have to extend their capabilities with the calculator in order to perform the simulation. With technology, problem solving takes on a new perspective. "It is now feasible to recommend 'investigating' a problem rather than focus on 'solving' a problem. It is in such investigations that much of the creative work of modelling is accomplished" (Dance, Jeffers, Nelson, & Reinthaler, 1992, p. 120).

The ease with which students can obtain graphs with the new technology is another obvious benefit. Students are able to quickly and accurately obtain a graph, then zoom in for greater detail, zoom out for a larger view, and compare several graphs simultaneously on one screen. In Sweden, researchers stated that the use of graphic calculators and computers is school mathematics enabled them to "reduce the routine work and teach for understanding" (Brolin & Bjork, 1992, p. 231). Another advantage to the time saved by obtaining the graphs with a graphing tool is that it provides more time for other investigations. Demana and Waits (1990) suggest that the ability of graphing devices to graph numerous functions quickly also enables students to establish common properties of classes of functions, which is one of the goals in the present Mathematics 11 course in British Columbia.

Another advantage to using graphing tools to study functions and graphing was determined in a study that reviewed two hundred papers on that topic (Leinhardt, Zalavsky, & Stein, cited in Hector, 1992). The report noted that students who are introduced to graphing through a hand-drawn table of values approach have a narrow focus that causes them to overlook global characteristics of a function. The graphing devices allowed students to explore more functions and enabled them to generalize their observations, but it was noted that the teacher had to be prepared to play a role in drawing the characteristics to the students' attention. Although the study did not examine any classes beyond grade nine, the implications from the report have value for all secondary grades.

The Calculator and Computer Precalculus Project (C^2PC), which involves the same three modes of technology used in the present study, shares many of the conclusions and observations of the other studies cited in this chapter (Demana & Waits, 1990). This study does, however, reveal an additional advantage to using technology. Students using technology were more motivated to ask and answer questions about properties of a function when they were generated by a graph.

The support for using technology as tool for exploration and conjecturing is not unanimous, however. An alternate view, though a minority one, is that the tools should not be used to teach, just to give answers. Bagget and Ehrenfeucht (1992) support this view and outline their position on the use of technology by stating that mathematics classes should follow a pattern of letting the teacher explain, letting the students think, and letting the computer or calculator do the mindless work. In short, technology should be used solely as a computational tool, either numerically or symbolically.

All of the previously mentioned studies are related to interactive modes of technology. The combination of a single classroom computer connected to an overhead projector is a non-interactive mode of technology that also enables students to benefit from technology. This particular mode does not provide many of the benefits of the interactive modes, but a three year project in Montana, the IMPACT Project, indicated that this particular form of technology can be successfully integrated into the classroom for presentations (Billstein & Anderson, 1989).

The literature strongly supports the use of the new tools of technology in the teaching of mathematics in general, and functions in particular. But if there is so much support for using this technology, why is it not currently widely used in the schools? What are the issues underlying the apparent dichotomy between theory and practice?

Background to the Issues

Before investigating these issues, it is pertinent to note that the development of the use of graphics calculators and computers in secondary school mathematics parallels a similar development that started in the mid-1970s with respect to basic four-function and scientific calculators. In 1974 the NCTM produced a policy statement that urged the use of calculators, a statement that prompted a considerable amount of research about the effects of such a policy (cited in Hembree & Dessart, 1992). Hembree and Dessart (1992) compared, analyzed, and summarized many of the resulting studies carried out in the United States dealing with the use of basic calculators. Their summary indicates the majority of states in the United States have recommended calculators be used for instruction in high schools. By 1987, 42 percent of the states had produced guidelines for aiding integration of calculators into mathematics instruction, but those guidelines were not universally implemented at the school level, with only a small minority of teachers reporting a substantial change to their instructional practices. It would appear then that although students have basic calculators, neither the curriculum nor teaching practices have changed significantly to reflect their availability. Their findings suggest that integrating a new technology with a curriculum is not easily achieved, and that it takes time to invoke a major change in educational practice. The study indicates that many issues need to be studied and acted upon soon if educators wish to learn from the past and accelerate the implementation of the new technology in order to see the impact of the new technology reflected in the schools faster than was the impact of the four-function calculators.

Issues Raised by New Technology

Many of the issues regarding the use of new technology to teach mathematics in secondary schools are not new, but have existed for many years prior to the recent surge of interest in technology. These issues, however, did not receive an extensive amount of attention because they were concerned primarily with the use of computers as teaching tools, and only a minority of teachers or students were using computers for teaching and learning mathematics. With the emergence of the relatively inexpensive yet powerful graphics calculators, the classroom environment has gone through, or can undergo, a radical change that needs to be reflected in what is taught and how it is taught. As a result the issues that had formerly been of interest only to computer users became important to an increasing number of people. Comments Burrill (1992):

> Easy-to-use graphing calculators present a dramatic new challenge in teaching mathematics....These tools have changed the very nature of the problems important to mathematics and the methods used to investigate those problems. Calculators

change activities in the classroom, raise questions about the mathematics that should be taught and suggest issues that must be considered in designing curricula and assessment strategies. (p. 15)

One of the groups charged with the responsibility of investigating these issues, the NCTM, has made the use of technology one of its priorities, but as yet does not have a detailed plan as to how to achieve its objective. Consequently it is reconvening its 1986 technology task force to make practical recommendations. The head of this task force, Bill Masalski, states, "We need to look at the appropriate use for technology. We have not defined what that is yet" (Hill, 1993, p. 24).

In a summary of the Sixth International Congress on Mathematical Education, Shumway (cited in Dick, 1992) argues that graphics calculators have broad implications for the mathematics curriculum and teaching strategies. He summarizes their findings as follows:

> •Calculators must be required for all teaching, homework, and testing in mathematics.
>
> •Substantial changes and redirection of the curriculum must be made to de-emphasize numerical and symbolic computation and emphasize earlier, deeper, conceptual learning.
>
> •Teaching strategies must de-emphasize drill and practice and focus on examples, nonexamples, and proofs. (p. 145)

As a result of the recent wave of interest regarding graphics calculators, many of the research articles written today are concerned with graphics calculators more than with computers, but many of the issues dealt with in these articles are pertinent to both modes of instruction when they are used as interactive tools.

Curriculum

One of the largest issues, or combination of issues, facing mathematics educators today is the curriculum and how to implement it with the use of technology. Important decisions need to be made with respect to what topics should be taught, in what order they should be taught, and how they should be taught using the new devices. These issues apply to school mathematics in general, and therefore they can be applied to the limited scope of this study, that is to the topic of transformations of functions and relations.

With respect to what topics should be taught, the new technology makes it possible for students to quickly execute many operations that previously were tedious or difficult, and this may make some topics obsolete and allow the introduction of others. As was the case with four-function calculators, educators are now faced with the question of whether many of the skills we now teach are no longer necessary. For example, if students can now graph parabolas using a mode of technology, do we still need to teach the skills relative to their generating a graph by hand? In the words of the executive director of MSEB, "Some topics used to be very important to teach. Now, because of computers and calculators, other topics are suddenly important" (Steen, cited in Hill, 1993, p. 24). Much research is needed to determine what topics in the current curriculum can be deleted to make room for higher order investigation activities. In the meantime, in the view of Kelly (1993), we can continue to use the devices for graphical explorations, knowing that this approach is helping to build an intuitive understanding of critical mathematics concepts.

The MSEB has further suggested that the new technology virtually compels a re-ordering of traditional topics and asks, "What orders yield optimal learning, and what is the relation between the stage of introduction and ultimate understanding?" (cited in Burrill, 1992, p. 17). The suggestion for teachers using the guided discovery approach with technology is to carefully select a sequence of visual experiences that will help students understand or discover a given mathematics concept or idea (Demana & Waits, 1990).

In a project being carried out in Michigan, all students have access to a graphing calculator at all times (Long, 1993). A new textbook has been written in which the order of topics has been rearranged to reflect new possibilities available with technology. The instructional methodology in the text reflects the power of technology with fewer problems, applications spread throughout the textbook, considerable small group explorations, and analyzing of graphs. The type of textbook used in this project is typical of many of the newer texts that are now being published.

Regarding the issue of methodology and technology, the MSEB comments, "Computers and calculators have changed not only what mathematics is important, but how mathematics should be taught" (cited in Kelly, 1993, p. 11). Kelly (1993) predicts that in the future, instead of using technology for instruction, students in the high technology classrooms will use these tools predominantly for investigation. Much of the literature seems to suggest that one way in which technology should be employed is in discovery learning, but this raises another issue, that of how much to explain, how much to leave for discovery, and how long to wait for discovery.

Another issue created by the application of technology to the learning of mathematics is in deciding how much of their work students should record. The question concerns how much written work a teacher needs to see to diagnose an incorrect problem solving strategy.

Technology and Understanding

Many students see mathematics as a form of "magic" and there is a possibility they will view doing mathematics with the new technology as simply an extension of that magic. The question of whether students using technology are finding solutions without understanding the "why" behind the solutions is an issue that is presently unresolved. Teachers will have to accept the fact that as with many other questions surrounding the use of technology, there is no answer to this question, and they will have to continue to work with the new technology to the best of their ability while research into the solutions continues.

A related issue concerns the effect technology will have on symbolic manipulations. A curriculum organized around technology will probably result in students being less proficient than traditional students on purely symbolic computations (Dick, 1992). But how much and what type of symbolic manipulation skill development is adequate for students to still use symbolic representations effectively? Heid (1988) suggests a substantial amount. When considering this issue it must be remembered that the goals of mathematics education are changing. In a curriculum based on technology the primary skills to be emphasized are those in interpreting and translating information presented in numeric, symbolic, or graphic form. The topic of translating among these three forms is one that takes on an increasing importance in a technology-driven curriculum, and consequently it too needs additional research.

New Skills Required

The technology of computers and graphics calculators requires new skills, or makes some old skills more important, for both students and teachers. The skills required to operate the graphics calculators are similar to the skills required to use the computer software, but these skills now appear more important because they affect more people than they did prior to the graphics calculator explosion.

The first skill students need to acquire in order to use the power of technology to produce useful graphs is to be able to input data correctly into the device. Students also need to be able to estimate a reasonable domain and range for a given problem and to be able to choose appropriate scales for the axis. Formerly, textbooks provided questions whose domain and range were usually [-10,10]. With the new technology this restriction need no longer apply. Approximation and rounding skills have new significance, and the students' ability to judge the reasonableness of approximated numerical solutions found using technology, compared to the exact solutions they were accustomed to in textbooks, also becomes more important.

The positioning of the graph on the screen is another new skill that needs to be addressed. Graphical evidence can lead to misinterpretations, particularly by inexperienced users (Goldenberg, cited in Dick, 1992). Students also need to be made aware of the limitations of the graphs. In addition to the problems created by not having a complete graph on the screen, the "hole" in the graph of a discontinuous function may appear as a missing pixel, a jagged jump, or not at all, and students need to be alerted to these possibilities.

<u>Technology and Testing</u>

Hill (1993) identifies two major hurdles to overcome in order for a new technology-based curriculum to be widely implemented: teacher development and student assessment. With regard to the latter, the new curriculum ideas based on technology, and the old testing practices, are a misfit. New methods of assessment, perhaps even different types of questions that reflect students' possession of technology, are required. Different strategies such as interviews rather than multiple choice questions have been suggested (Heid, Matras & Sheets, 1990).

Another concern with regard to testing is illustrated by the fact that the most common reason given by teachers in the United States for not using calculators of any kind in their classrooms is that they are not allowed to be used on standardized tests (Wilson & Kirkpatrick, cited in Hopkins, 1992). The dilemma facing the teachers is that if the tests do not allow the same technology the students have been using, then the test questions may be different from those the students in that class have been doing. This situation presents a problem for teachers who wish to have their classes use the newer technology. The consequences are that either the classes stop using the technology or keep using it and risk poorer performances on the tests. In their summary of studies of the use of technology in mathematics classrooms, Hembree and Dessart (1992) suggest that the policy of using technology for instruction but not for testing should be eliminated. The overall suggestion from the literature is that the curriculum needs to change to fit technology, and the tests need to change to fit that new curriculum.

New Technology and the Teacher

Maximizing the potential of technology in mathematics education will require teachers to change their roles. They will find it necessary to give up some of their traditional control of the classroom and become more flexible in order to create an atmosphere that encourages students to explore, experiment, conjecture, and evaluate. To achieve such an environment teachers will need to become discussion leaders and catalysts for self-directed student learning. It will be their responsibility to ensure that the students are able to cope with the responsibility that the discovery approach will force upon them. They will find it necessary to serve as facilitators for small and large group discussions.

The use of the new technology results in students asking many "what if" questions. Questions such as these provide spontaneous opportunities for teaching, learning, and student explorations. This is a prime example of when teachers need to be prepared to exercise their flexibility and let go of their traditional control in order to take advantage of the opportunities.

Ruthven (1992) found that the teachers' attitudes toward the new technology had an effect on the students. In his study, two of the participating teachers had strong reservations about using the graphics calculators, and the percentage of students in those classes having similar feelings about technology was considerably higher than in the other classes. Similarly it has been suggested (Dick, 1992) that to maximize the use of graphics calculators, students need to view their use as a routine method of solution, not as an occasional extra. The teachers can help promote this attitude by making extensive, but not exclusive, use of the technology themselves.

In a project with graphics calculators in secondary schools, Ruthven (1992) focussed much of the program on teachers working together, as it is his contention that much educational innovation fails because it ignores the role of the teacher. He required teachers in the project to meet twice a year for three days, local groups to meet occasionally, and teachers to visit each others' classrooms. Other educators echo the call for in-service for teachers in order to adequately prepare them to use the technological tools. Bright, Lamphere and Usnick (1992), writing about the Statewide In-Service Program on Calculators in Mathematics Teaching they were involved with, emphasize that the need for in-service training is critical.

<u>New Technology and the Student</u>

Ruthven (1992) had students initially use graphics calculators simply to replace various mental and written methods, just as they would use scientific calculators . As their confidence increased, some students began to use the devices in more creative manners, and eventually were using them to find alternate approaches to solving a problem. After one term nearly all the students were using the device, including its graphing capabilities, confidently and spontaneously. A small number, although proficient with the new calculators, preferred to use their old, non-graphics calculators due to a lack of confidence with the new ones. An even smaller number were reluctant to use calculators of any type because they felt they were losing control over the mathematics they were doing.

Gradually these groups began to realize the problems inherent in their position, such as a loss of time, distraction from the main objective of the problem, and an increased chance of error. As they gradually increased their use of the graphics calculators, their reluctance to lose control had a positive side-effect in that they tended to interpret their calculator-based results particularly critically.

In a study in which the students worked in pairs in a computer lab to explore mathematical ideas, Heid et al. (1990) made a number of observations. The students were required to accept more responsibility for their own learning, they had to adjust to the teachers not telling them the answers, and they had to become comfortable with the idea that there may be many correct solutions to a problem, that every person does not have to do a problem the same way. All of these perceptions would apply to classes using any form of technology that put the emphasis on student exploration and discovery. This study also found that when the students worked in pairs, they sometimes worked together and served as resources for each other, and at other times they engaged in a friendly competition.

Comparing Computers and Graphics Calculators

The theme of the literature with respect to why students in secondary schools should be using the new technology is centered around the students being able to experience the power of mathematics themselves, through exploring, experimenting and conjecturing using the technology. These experiences can not be sporadic in nature, but rather need to be continual. Students need to have access to the tools on a regular basis, both in and out of class. The occasional trip to the computer lab gives students the impression the computer is not really an integral part of learning mathematics, but rather a supplementary activity, as would the occasional bringing to class of a class set of graphics calculators. In his project with secondary school students and graphics calculators in Britain, Ruthven (1992) credits the fact that the students had unrestricted access to the calculators as being one of the keys to the success of the project. Therefore the two criteria for evaluating the potential of a technological tool seem to be the power to do the job, and accessibility.

Relatively easy-to-use computer programs have been available for years, and in general the available software, although not spectacular for the most part, is rapidly improving (Billstein & Anderson, 1989). The quality of the software is the key to the ability of a computer enhanced program to provide students with the experiences required by the NCTM's Standards. Software programs are now turning away from computational skills games and towards allowing students to construct their own problems and explore and discover mathematics properties on their own. Yet the expectation from the 1980s of mathematics classes being conducted in a computer lab with all students sitting at their own computer, even with improved software, has not materialized, mainly because of cost. It is not economically feasible for schools to acquire enough desktop computers to give students regular access.

Recently, graphics calculator prices have dropped to a level that makes them affordable to many students, or for group purchases by a school. And while their prices have dropped, their power has increased. According to Kelly (1993), as graphing calculators become more powerful and have larger screens and virtually unlimited memory, the distinction between calculator and computer will fade, and the need for class sets of computers, even as investigating tools, will diminish. Does this mean that teachers should stop experimenting with computer algebra systems? Definitely not, say Demana and Waits (1992), they should still be used, but teachers should keep questioning their role in mathematics classes. Much more research is needed relative to the entire area of technology and mathematics instruction. In the meantime teachers should promote inexpensive, easy-to-use available technology such as graphics calculators.

The executive director of MSEB argues that prior to college, computers are helpful but not essential (Steen, cited in Hill, 1993). Schwartz (the developer of Geometric Supposer), strongly disagrees with Steen:

> That's a narrow view. I think computers are a necessary tool for all math curriculum - starting at age zero. A computer is so flexible, so supportive of different scenarios. With graphing calculators, there's a lot of overhead to learning because you are driving it from an idiotic keyboard. As an interface, it's crummy. I would much rather have three kids on one computer that one to each calculator. (cited in Hill, 1993, p.24)

Overall, however, the literature does tend to suggest that because of its power, its relatively low price and resulting accessibility, the graphics calculator is the more desirable of the two technologies for use in the secondary school mathematics classroom at the present time. The president of the NCTM was quoted as believing the graphing calculator "has really made more of an impact than the computer...because every child can have one" (Lindquist, cited in Hill 1993, p.24).

The preponderance of literature relative to the interactive technologies (calculators and individual computers) as compared to the non-interactive technology (a single computer and an overhead projector) is an indicator that the latter is not widely established, or even seen as having the potential to be a major factor in the new curriculum. The prime concern about this mode is that students are unable to solve problems and discover relationships for themselves except where computers are available for their own use. Many schools in Britain are now planning to equip each mathematics classroom with a single computer, but this computer is primarily used for classroom demonstration by the teacher. Few students are able to access the computer, which means not many students are able to take advantage of the potential of the technology.

The literature suggests that a single computer and an overhead projector in a classroom is acceptable as a means of demonstrating, but the interactive modes of technology are preferable for everyday mathematics classes. The literature also supports the position that a technology that allows continual and individual use, such as the graphics calculator, is the mode that best accommodates the goals of a technology-driven curriculum.

CHAPTER THREE

METHODOLOGY

This chapter provides a setting for the study by presenting information about several topics related to the planning and preparation of the lessons. These topics include information about the school in which the study took place and the types of students that attend that school. In addition there is some specific data on the students in the three classes in which the various modes of technology (the overhead projector combined with a single classroom computer, the computer, and the graphics calculator) were employed. This is followed by a detailed list of the particular learning objectives pursued in the units studied, and how these objectives relate to the intended learning outcomes of the British Columbia Mathematics 11 curriculum. General information about how the three different modes of technology are to be used in the classroom is also given. Problems that arose in the planning of the lessons and decisions that had to be made are expanded upon. The chapter concludes with the questions used in the written questionnaire given to all students and the conditions surrounding the administering of the questionnaire.

The School

The study was conducted at Handsworth Secondary School in North Vancouver, British Columbia, a school with a population of approximately 965 students from Grades 8 to 12 (September, 1991). The school tends to cater to a university-bound student body, with approximately 80% of the graduating class each year proceeding to some form of post secondary education.

The school is located in a relatively affluent urban neighborhood, an area in which the assessed property values are 67% above the provincial average and the average income of a private household is 40% above the provincial average (approximate percentages from the 1986 census). Twenty-five percent of the adult population within the Handsworth boundaries have a university degree, compared to 9.5% for the province. The school houses approximately 11.0% ESL students, compared to 19.1% for the province, with the majority of those speaking Persian (Farsi), Chinese, and Korean, but has no First Nation students. The population tends to be stable, with only 3.6% of the student population transferring in to the school during the previous school year, and 2.0% transferring out.

The Mathematics Department at Handsworth offers the usual range of courses to the students, including Mathematics 8, 9, 9A, 10, 10A, 11, 11A and 12, although only about 6% of the students in Grade 9 elect the 'A' course and about 7% in Grade 10. There are two locally developed courses in the school, one being Mathematics 7/8 (a much slower paced version of Mathematics 8 that enrolls from 4% to 8% of the Grade 8's), and Calculus 12 (a bridging course intended for students who plan to take post-secondary calculus that enrolls about 40% of the Grade 12's). Students electing Mathematics 12 have two options, they can select the regular Mathematics 12, or can opt for Mathematics 12E (enriched) with the understanding that this more extensive course will prepare them for scholarship examinations in June. Enrollment for the two Mathematics 12 courses usually runs about 30% of the Grade 12 population for each course, resulting in well over half of the graduating class taking Mathematics 12 in some form.

One very interesting aspect of the mathematics culture at Handsworth, and one that is most definitely pertinent to this study, is the extent to which Introductory Mathematics 11 is subscribed to by the students. Each year about 20% of the Mathematics 10 students, usually those whose marks are in the C- or C range, choose to take Introductory

Mathematics 11 instead of the regular Mathematics 11 in order to get a better background with which to tackle Mathematics 11, which they will then confront when they are in Grade 12. Almost all of these students do actually move on to Mathematics 11 as Grade 12's, hence each Mathematics 11 class usually contains from 4 to 8 Grade 12 students, which can affect the tone of the class. In addition, approximately 10% of the Grade 11 students select Mathematics 11A, which is a slight increase in the 'A' program from Grade 10. The average class size for the Mathematics 11 classes was 27.

To portray a more detailed picture, in the academic sense, of the type of student that attends Handsworth Secondary, the table in the Appendix A contains the letter grade distribution for all mathematics courses for the first term (December, 1991, the term just prior to the undertaking of the study). This table also indicates the total number of students enrolled in each of the mathematics courses operating in the school, and gives the average class size for each (about 27 for Mathematics 11).

Handsworth also operates on a "vanishing" timetable, a form of the Flex-Mod type of organization in which the day is divided into 20 modules, or mods, of 20 minutes each. Students are scheduled into classes that can run for 2 or 3 or 4 mods, meaning 40, 60 or 80 minutes. In Mathematics 11 each class meets twice for three mods and twice for two mods each week. What really makes the system different from most schools is that there is no scheduled lunch break, and, as in a university, students can find themselves with breaks of perhaps two hours or even more during the school day. The negative aspect to this apparent gift of free study time is that students may find themselves on some other day with 4 or even 5 hours of continuous classes, which can affect their performance in the later classes.

The Sample

The study involved three different Mathematics 11 classes, with each class using a different form of what is currently considered modern technology. In one class, students experienced the use of a singe MacIntosh Plus computer connected to a Kodak Data-Show overhead projection device as a means of viewing two software packages, Zap-a-Graph (Pitre, 1990) and Master-Grapher (Waits, Demana & New, 1988). The Zap-a-Graph program was used the majority of the time because it adequately served the needs of the graphing tasks required to investigate the Functions and Relations Unit of the Mathematics 11 curriculum. This particular software allows one to graph lines, parabolas, cubics, circles and more by selecting the function or relation from a menu, and further allows one to translate, stretch or dilate the graph on the screen by entering a horizontal or vertical factor. It also permits one to view on the screen simultaneously an original graph and a multitude of transformed graphs, which is an asset when discussing the sections on transformations. The program is not without its faults, for example, when trying to identify coordinates of points on a curve one cannot be sure whether or not the cursor is exactly on the curve, however the merits of the program exceed the limitations. The Master-Grapher program was used occasionally to help overcome some of the weaknesses of the Zap-a-Graph program.

A second class had their classes in the MacIntosh Lab, which housed 20 networked MacIntosh Plus computers, with one of the computers connected to an overhead projection device. The software these students used almost entirely was Zap-a-Graph because it was networked in the lab and Master-Grapher was not.

In the final class, each student used a Texas Instruments TI-81 Graphics Calculator. These were for in-school use only and students were not allowed to take them home, although they could have access to them before and after school. I had at my disposal a similar calculator designed to be used with an overhead projector. This particular calculator was used because the school district purchased a class set of these calculators for each secondary school in the district and therefore these were the ones that were readily available.

The question of which class was to use which graphing device was answered by purely practical circumstances. When the timetable for the school year was being developed, a request was made for the computer lab to be made available for one Mathematics 11 class, hence the students who were programmed into that class by the computer loading of the school's timetable automatically became the Computer Lab class. With regard to the computer-overhead projection device, the school has only one such device and consequently it must be shared. Of the two remaining Mathematics 11 classes, using the overhead device for one of the classes caused less conflict with other teachers than using it with the other class, so the lesser conflict class became the Overhead Projector class. By default then, the third class became the Graphics Calculator class with the other four mathematics teachers agreeing not to use the department's single class set of calculators at the times they were needed for the third class.

Both the Graphics Calculator class and the Overhead Projector class took all of their classes in the regularly assigned mathematics classroom. The graphics calculators were stored in the mathematics office, which was directly across the hall from the classroom where they were required, so access to them was easy. The overhead projector model of the graphics calculator was stored in the classroom so it was always immediately available. The computer-overhead device was not as conveniently located, and had to be retrieved from and returned to a room on a different floor, which was a minor irritation but not a real problem for use with a specific class, but certainly would be if one wished to use it spontaneously with some other class.

The computer lab class moved to the computer lab, a relatively short distance away, for the duration of the study. The computer lab did not have enough computers for all students to have access to their own, therefore students were required to work in pairs at a computer. This resulted in a minor space problem as there was not adequate space for two senior students to sit comfortably at each terminal. The room was also long and narrow which meant an adjustment by the teacher and the students in order to hear each other and maintain cooperative visual contact.

Some background information about each of these classes will help interpret the observations presented in Chapter 4 about how each of the three classes reacted to their particular graphing device. With respect to the ages of the students, in the computer lab class about 20% of the class were Grade 12 students who had taken Introductory Mathematics 11 the previous year. In each of the other two classes the figure was approximately 14%.

The first term Mathematics 11 letter grade distribution for each class shows the relative achievements of each class at the beginning of the study (see Table 1).

Table 1

First Term Letter Grade Distribution For Each Class

Class	No. of students	Percent receiving given letter grade in first term					
		A	B	C+	C	C-	D
Overhead Projector	24	13	33	33	13	8	0
Computer Laboratory	31	7	27	7	36	13	10
Graphics Calculator	28	7	36	18	25	14	0

Planned Use of Technology

The manner in which the graphing devices were to be used in the lessons depended partially on the particular device being used and on the topic being covered. In general the devices were to be used to generate graphs of functions quickly and accurately for several purposes. They were to be used to enable students to solve "real-world" problems by analyzing the graphs that represented the problems. In addition they were to enable students to generate their own conclusions with respect to relationships between types of equations and their corresponding graphs. The graphics calculator and computer lab groups were to enter their own data, but the computer-overhead group would be restricted to giving input data to the teacher or to whoever was operating the computer.

Further, the graphing devices were to be used in combination with investigation activities in which students were to be given equations of various functions and asked to use their particular device to obtain accurate graphs of those functions. Then they were to draw their own conclusions with respect to what type of equation resulted in which particular graph. A similar use of the graphing tools was to be followed when the students investigated transformations of relations and the importance of order of transformations. The graphing calculator and computer lab classes were to use their respective devices in a manner parallel to the first set of activities, while the overhead class was to use the same activity package as the other two classes but would again work through the activities as a class, with students suggesting input values for the computer.

When the intention of the lesson plan was for students to be working on an investigation type of activity, students were to work in groups or individually. Consequently some assignments were to be handed in as group projects, with everyone in the group receiving a common group mark, while other assignments were to be handed in and graded on an individual basis. It was also to be possible that for some activities the students would work in their groups and discuss their ideas with each other, and then hand in individual reports with their own specific conclusions.

Finally, the graphing devices were to provide students with a reliable means of checking graphs they had done, whether those graphs were ones they developed in the traditional way, or were the result of their conjectures and speculations. For occasional lessons it was even to be possible that the graphing devices would not be used at all, depending upon the objectives of the lesson.

As suggested previously, students were to work either individually or in groups, with the decision as to which route to take depending upon the particular activity or the mood of the class, with groups being used as often as possible in order to foster a cooperative environment. In the Computer Lab class, students were to work in groups of two (a necessity because the lab did not have enough computers for students to work at their own terminal), while in the Graphics Calculator and Overhead Projector classes groups of three or four were to be formed. In each class, whether the groups formed were to have two, three, or four members, students were to be selected for the groups by the teacher in order to create, as much as possible, a balance among group members with respect to mathematical ability (as demonstrated by previous test results) and gender. In the computer lab class an attempt was also to be made to divide the students with computer experience among the pairings.

In order to be able to reflect on and analyze the use of the three types of technology in teaching Mathematics 11 it was necessary to keep some type of record of what was happening in each of the classes and what the students' and teacher's reactions were to the lessons. The procedure used to record this information was that of a written log of each individual lesson, maintained by the teacher. As each lesson progressed, a brief written account of what was actually happening in the classroom was kept. With respect to the log, of particular interest was the students' reactions to the graphing device they were using as revealed by their actions and comments and by the teacher's observations of their behavior. It was also useful to record, when possible, the students' comments about how they were learning, not merely just what they were learning.

In addition to noting what the students were doing and saying, it was appropriate to record what the teacher was thinking and doing. For example notes were taken with regard to what appeared to work about the lesson plan, from the points of view of promoting interest and of encouraging learning, and what did not work (relative to the graphing device). If some aspect of the lesson plan did not seem to yield the result hoped for, then a speculative note as to what might have been the problem was made. Notes were made of what changes could be made in the lesson plans with a view to eliminating whatever pitfalls may have been encountered. Also recorded were changes made in mid-lesson as a reaction to something that was happening during the lesson.

Lesson Objectives

The material covered in this study relates to Intended Learning Outcomes (ILO's) 11.18, 11.23, 11.25-11.28 in the British Columbia Mathematics Curriculum Guide (1988) for Mathematics 11. The lessons prepared for this research program covered all of the above ILO's but not necessarily in the order that they appear in the curriculum guide. The content was organized into 17 distinct topics, many of which may took more than one lesson, or period, to complete. The objectives of those topics are listed below, with a detailed example of one lesson appearing in Appendix B.

Topic 1:

1. To review the definitions of function, domain and range and to review what the graph of a function represents.

2. To emphasize that decisions about what the domain and range should be must be made for each new question.
3. To practice changing the domain and range on the particular graphing device their class is using.
4. To have students realize that not all functions have linear graphs, and to draw conclusions about the type of equation and the shape of its graph (for linear, quadratic and inverse variation equations).
5. To define a quadratic function as one whose equation is $y = ax^2 + bx + c$.
6. To use graphing devices to determine maximum or minimum values of quadratic functions.

<u>Topic 2:</u>

1. To make students aware that there are many other functions whose number pairs form graphs different from those studied thus far and whose equations have a different form. Eight different types of functions are examined.
2. To relate the form of an equation to its graph.

<u>Topic 3</u>

1. To explore the graph of a basic parabola ($y = x^2$) and identify vertex, axis of symmetry, x and y intercepts and direction of opening.
2. To draw conclusions about the effect an additive constant q has on $y = x^2$, that is, how to sketch the graph of $y = x^2 + q$ by translating the graph of $y = x^2$.
3. To draw conclusions about the effect an additive constant q has on $y = f(x)$ that is, how to sketch the graph of $y = f(x) + q$ by translating the graph of $y = f(x)$. For all lesson topics, $y = f(x)$ refers to the basic eight functions graphed in Topic 2.

Topic 4

1. To determine how the graph of $y = (x - p)^2$ differs from that of $y = x^2$ or $y = x^2 + q$ and to draw conclusions about the effect the constant p has on $y = x^2$, that is, how to sketch the graph of $y = (x - p)^2$ by translating the graph of $y = x^2$.
2. To draw conclusions about the effect a constant p has on the graph of $y = f(x)$, that is, how to sketch the graph of $y = f(x - p)$ by translating the graph of $y = f(x)$

Topic 5

1. To draw conclusions about the effect a constant a has on the graph of $y = x^2$, that is, how to sketch the graph of $y = ax^2$ by stretching or compressing the graph of $y = x^2$.
2. To draw conclusions about the effect a constant a has on the graph of $y = f(x)$, that is how to sketch the graph of $y = af(x)$ by stretching or compressing the graph of $y = f(x)$.
3. To determine coordinates (x,y) from $y = af(x)$ when given coordinates (x,y) from $y = f(x)$.

Topic 6

1. To clarify the difference the placement of the negative sign makes to the graphs of functions of the form $y = f(x)$ (that is $y = -f(x)$ and, $y = f(-x)$) and to graph those functions without using a table of values or a graphing device when given the graph of $y = f(x)$.

Topic 7

1. To combine the conclusions from the previous lessons with regard to the effects of the constants a, p and q on the graphs of functions of the type

$y = af(x-p)+q$ and to use those new conclusions, along with the knowledge of the shapes of the basic eight functions, in order to sketch the graphs of certain functions by transforming a given basic graph.

Topic 8

1. To review the topics studied thus far in this chapter in preparation for a test. Graphing devices will not be allowed for the test.

Topic 9

1. To test students' understanding of the objectives of topics 1-7.

Topic 10

1. To examine how the concept of the maximum/minimum value of a quadratic function (as intuitively explored in lesson 1) can be applied to word problems.
2. To solve maximum/minimum word problems for which the equation is given by reading the required information from a graph, a graph obtained by using a graphing device (determining the equation for a problem is a valid topic but it is not on the mathematics curriculum for quadratic functions, would take time not available and is covered in Mathematics 12, therefore it is omitted for some problems).

Topic 11

1. To recognize that $x^2 + y^2 = r^2$ determines a circle with center $(0,0)$ and radius r and to sketch the graph of the circle without using a table of values or a graphing device.

Topic 12

1. To draw conclusions about the effect p and q have on the graph of $x^2 + y^2 = r^2$, that is, to sketch the graph of $(x-p)^2 + (y-q)^2 = r^2$ by translating the graph of $x^2 + y^2 = r^2$

2 To discover that (p, q) are the coordinates of the center of the circle.

Topic 13

1. To draw conclusions about the effect a and b have on the graph of $x^2 + y^2 = r^2$, that is, to sketch the graph of $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = r^2$ by stretching and/or compressing the graph of $x^2 + y^2 = r^2$.

Topic 14

1. To sketch the graphs of functions that combine the transformations studied in lessons 12 and 13 by transforming a given basic function

Topic 15

1. To graph quadratic inequalities in two variables without using a table of values or a graphing device.

Topic 16

1. To review topics 10-15 in preparation for a test.

Topic 17

1. To test the students' understanding of topics 10-15.

Problems in Planning

As the lesson plans for the technology-aided units were being constructed, several problems and concerns arose, and decisions had to be made that seemed to center around two central themes. The first set of problems evolved from my experience, or more properly, lack of experience, with computers and graphics calculators, and the second set centered around decisions that had to be made regarding when and how to use the graphing devices at my disposal.

With respect to the first of the two main problems, my previous experience with computers had been chiefly limited to using one as a word processor, although I had spent a few classes in the previous school year experimenting in a very limited fashion with teaching several lessons to my Mathematics 11 and Calculus classes using a single computer and an overhead projector. Until this particular study, I had never used the computer lab for any classes, and had never taught a class in which each student was using a graphics calculator. I had toyed with my own graphics calculator and had attended a couple of workshops on their use, but I still considered myself very much a neophyte with regard to their potential as a teaching tool.

My lack of experience with the graphing devices I was going to be using was a problem for a variety of reasons. First of all there was the intangible feeling of unease, of wondering how many students knew more about the devices than I did and whether I would be able to respond to any questions relative to the devices that students might hurl my way. I was also plagued by a suspicion that there were probably certain short-cuts or "tricks" one could employ with the computer or graphics calculator, and at this time I was not privy to that sort of inside information.

A more concrete problem that my lack of experience caused when trying to plan lessons was the very real fact that it took much longer to plan a lesson because of the time it took to experiment with the devices in order to learn their capabilities and their limitations. It was also very time consuming trying to find function equations that would demonstrate the concept in question clearly on the calculator or computer screen. For example if the goal of the lesson was to show what effect adding some constant 'q' to an equation had on the graph of that function, care had to be taken to find an equation that had enough integral number pairs that would appear as integers on the screen so that transformations could be easily observed by students when doing their investigation type activities.

With regard to the second of the two themes of problems encountered during lesson planning, namely the decisions that had to be made regarding when and how to use the technology, a very broad and, in my opinion, extremely important question is involved. That question revolves around trying to marry the present British Columbia Mathematics Curriculum to the potential for executing mathematics provided by modern technology in the form of computers and graphics calculators. This question is perhaps parallel to one many years ago regarding when and how to use the basic four function calculator, a question that still causes debate in some schools.

The general question is whether British Columbia's mathematics curriculum is out of date. With respect to this study the question is whether or not the present curriculum is asking teachers to teach topics or skills that modern technology has made obsolete. Specifically, the question becomes to what degree graphing devices should be used to teach the present content of Mathematics 11. At this time there does not appear to be any agreement as to the answers to these questions, and in fact conversations I have had with fellow mathematics teachers leads me to conclude that some teachers not only do not have answers to these questions, they have not yet realized that the questions exist. The problem that exists then for any teacher who wishes to embark on a voyage of discovery with modern graphing devices is trying to decide when to use the technology and to what degree. As I tried to wrestle with the question I found myself wondering how much I was being restricted by a possibly out of date set of values that had their roots in a traditional type of curriculum.

Another decision that had to be made was with regard to homework. Most of the students did not have access to the graphing device they were working with (with the exception of some of the Computer Lab class who could access the lab during their unstructured time) and consequently it was a very real problem trying to decide whether or not to assign questions to be done in the traditional way in the midst of using modern technology.

A decision about course content had to be made with regard to the topic of maximum and minimum word problems. This is a topic that is covered in the Mathematics 11 curriculum only to the extent that students are expected to read the maximum and minimum values from given graphs. With the use of graphing devices it is possible to do more problems, ones for which the graphs are not already given, consequently a decision of whether to extend this topic had to be made.

The software being used in the Computer Lab class forced another decision because it was written so that the computer operator needed to know what type of relation the equation represented in order to enter that equation. The problem with this situation was that when students were asked to graph a new type of equation, they either had to be told what type of relation it was, which would seem to take some of the interest out of the activity, or they could be encouraged to scroll though the options in order to find an equation that fit the form of the one they were trying to graph. The latter plan may seem to be the one most suited to an investigation activity, but there was a concern that many students would become quickly frustrated and discouraged, so the question was not a simple one to resolve.

A concern in the Overhead Projector class centered around the question of how to maintain the students' interest if only the teacher and a very few students know how to input data into the computer. Even if all of the students were capable of entering data, how could their expertise be used and still avoid the boredom that can result from someone else doing the majority of the work?

Quizzes presented a challenge for all of the classes with regard to what type of questions to ask. Again the dilemma of breaking from the traditional curriculum thrust its way into the lesson planning. The problem of how to give individual quizzes to the Computer Lab class, where students were working in pairs and the pairs were situated very close to one another, had to be addressed. A problem also arose in the Overhead Projector class because students were unable to enter their own data into the computer, so they could only use the computer as an aid for a quiz if someone else entered the required data.

One of the advantages of the investigation type of activities is their open-ended nature. Students can vary their speed of working through the problems and the more inquisitive and able students can go beyond the boundaries of the original activity. The investigation activities planned for the lessons in the unit in this study required students to draw their own conclusions from the graphs displayed by the graphing device. This type of activity is ideal for the Graphics Calculator and Computer Lab classes, but creates a problem in the Overhead Projector class. The question of how much time a teacher should allow for students to consider the graphs displayed on the overhead and to reach some conclusions must be answered. Similarly the teacher must decide whether or not to wait until everyone has written down some conjecture, or to move on after most of the students have made a written effort. In fact, the more general question of whether or not the overhead projector mode even lends itself to individual student investigations must be addressed.

Evaluation Questionnaires

In order to solicit students' opinions regarding the use of graphing devices as an aid to learning mathematics, all students involved in the study were given a written questionnaire at the conclusion of the units in which the technology was used. The questionnaire had two parts, the first part having nine questions (eight for the Overhead Projector class) in which students were asked to indicate on a scale from 1 to 5 their opinions about a given statement. The second part asked for written responses to four open-ended questions. More details about the questionnaire, including the full text of the questionnaire, are given in Chapter 4.

To obtain more in-depth information from the students, and to seek clarification about some of the points raised by the students in the open-ended portion of the questionnaire, verbal interviews were conducted with two randomly selected students per class. The questions used in the interviews were composed after both sections of the completed questionnaires had been read and informally analyzed. With respect to the questions in which students were asked to rate their opinions on a scale from 1 to 5, the arithmetic means were used as a basis for forming further questions that attempted to probe why the students had answered as they had. For example, a majority of the students in each class indicated via these questions that using technology made learning easier, consequently a natural question for the taped interview was to ask how they thought it made learning easier. Other questions were formulated based on comments students had made on the written answer portion of the questionnaires. One such question arose from the students' expression of the point of view that using a graphics calculator or a computer actually restricts learning, a point of view that made a follow-up question about how that might be possible seem like a natural for the individual interviews. A final source of questions for these interviews came from my own curiosity about some components of the study, questions that were perhaps too awkward to put into a short questionnaire. For example, the type of instruction used in conjunction with the computers and graphics calculators required students to form their own conclusions from their data much more than they had in the previous units, as a result I wanted to know what students though about this style of learning.

For each interview I had prepared a list of 16 to 19 questions (Appendix C), depending upon the technology that class had used, and I used these questions as a framework around which the interview was built. The order of questions was not strictly followed as often students would volunteer answers that would take us in different but interesting directions, and I felt it was useful to let the students fully express their opinions. The questions were roughly the same for each group, although there were some variations that were a result of the different mode of technology that a particular class had been working with, or were a result of some written response to the questionnaire that was pertinent to a particular class.

In order to randomly choose students from each class, yet still guarantee some sort of a mixture regarding mathematical ability, students in each class were ranked according to their mathematics mark in the first term of Mathematics 11 and assigned a number corresponding to their ranking. A random number table was then used to select one student from the top half of the ranked class list, and one from the bottom half. Students selected in this manner were then asked if they would agree to be interviewed, and all six readily agreed. The taped interviews took about 30 minutes each and were conducted during the students' unstructured (non-class) time in their regular mathematics classroom.

CHAPTER FOUR

RESULTS

The intention of this chapter is to relate what actually happened in each of the three classes being used in this study as the students and the teacher worked their way through the objectives of a particular unit of Mathematics 11. The reactions and behaviors of both the students and the teacher as observed by the teacher are described on the following pages. The observations are organized by method of technology, so that a reader may choose any of the three technological devices used in this study and follow the progress of the class that experimented with that device as it advanced through the unit. The chapter concludes with the full text of the questionnaire that all students were given at the conclusion of their units of study involving their respective graphing devices in addition to a summary of their responses.

For each method of technology, the objectives of each of the major topics are restated in general terms prior to the discussion of the lessons relevant to that topic (see Chapter Three for a detailed list of objectives). Following the restatement of the objectives is the description of what happened in that class with regard to the teaching and learning of those objectives using that particular type of graphing technology. Most of the topics required more than one lesson, so the observations have been organized by lesson under separate headings such as Lesson OP 1 (the first lesson in the overhead projector group) or Lesson Lab 12 (the twelfth lesson in the computer lab) or Lesson Calc 5 (the fifth lesson in the graphics calculator class).

For each of the three classes involved in the study, I was the only teacher the students had for the duration of the project, consequently I have decided to report the observations from the lessons using the first person singular.

THE OVERHEAD PROJECTOR CLASS

Topic 1: Defining a Quadratic Function

The general objectives for this topic were to review the definitions of function, domain and range; to review what the graph of a function represents; to enable students to realize that not all functions have graphs that are linear; and to define a quadratic function.

Lesson OP 1. At the beginning of the class the students were divided into previously determined teacher-selected groups of four (each group had a male/female mix as well as a mathematical ability mix). All students were given four "real-life" problems illustrating different types of functions, and asked to graph the indicated functions. The intention was to use the graphing device in order to quickly obtain the graph, then to compare the shape of the graph to its equation to see if any conclusions could be reached regarding characteristics of an equation and its graph.

Prior to using the computer to show graphs of the four functions, the groups were instructed to determine the domain and range for each function. This task caused considerable discussion among group members, with apparent confusion as to what the domain and range should be for a "real-world" type of problem. The students' lack of ability on this topic resulted in far less being accomplished in class than had been planned. So although the computer can graph an equation very quickly, setting a usable scale still takes a large portion of a class period.

After the groups had agreed among themselves on scales to use for the first two questions, a class discussion followed with regard to what scales to use on the computer.

It was somewhat surprising to observe that only a few students were involved in suggesting scales, and that there seemed to be very little enthusiasm for the class in spite of the fact that we were about to use the computer and the overhead projection device (it may be pertinent to note that during three prior classes this school year this group had been exposed to the computer and the overhead, so it is possible that any novelty factor may have worn off).

The class finished only two of the four problems set aside for the lesson, so the homework assigned was for each person to determine the scales that could be used for the last two problems, an assignment that could be done without computers.

Two problems with the particular software being used arose during the lesson, one that could have been avoided with careful planning and one that was unavoidable. The first problem was one that all teachers should learn to avoid in their undergraduate years and resulted from an ill-chosen attempt to save time in lesson planning. That problem was the unnecessary confusion that results if a teacher fails to work through any new questions or materials before using them with a class. Had this basic rule been followed the students would not have been given equations with excessively long coefficients (the software used accepted only five characters for coefficients). The lesson plans for the graphics calculator and computer lab classes were changed as a result of the lesson learned with the overhead group. The other problem with the software is that it is necessary to know what type of function a particular equation represents in order to enter the equation into the computer, and at this stage of the unit the students do not know this. Consequently I had to make use of information the students did not yet have in order to enter the equation, with the result that I felt as though I was some mystical agent performing magic that was beyond the students' comprehension.

As the lesson progressed I realized I was so caught up in using the technology I had lost sight of the overall objective of the lesson and I was not directing the lesson toward a specific conclusion.

Lesson OP 2. The discussion about homework seemed to be lagging, chiefly because some students had considerable difficulty determining scales for the two remaining questions from last class. As a result, the lesson plan was changed to allow students time to work in their groups in order for all students to have some idea of a domain and range so that they could feel more involved in the subsequent discussions, and would be more interested in the computer graphs when they appeared on the screen.

Questions from the students as they continued to work in their groups on the four problems presented to them last period also pointed out a problem many of them had with one particular symbolism in the assignment, namely using *t*^2 for t^2. The situation resulted from my unfamiliarity with the word processor resulting in the need to use *t*^2 on the problem sheet I had prepared for them. This problem could have been avoided if I had either learned more about the word processor or had simply informed the students ahead of time about the symbolism. As a result of the experience with this class, the confusion was avoided for the other two classes by simply pointing out the symbolism to the students before they tackled the problem.

The computer/overhead combination did promote class interest and discussion as the class began to discuss the results of their group deliberations regarding the domain and range for the last two problems. Groups were chosen randomly and asked to put their domains and ranges on the board, then these were entered into the computer and the graphs seen almost immediately. The unexpected bonus from the teaching standpoint was that by chance the first scales suggested by the students resulted in no graph appearing on the screen, which led to looks of bewilderment and cries of "why?" and a subsequent short and useful discussion.

The question of what homework to assign proved to be a difficult one. Ideally the students should have been asked to do a couple of graphs for homework, but because they did not have access to a computer, they were not assigned any. They could have been assigned a couple to be done in the traditional way, but because I had the idea that this unit should be done using technology, no assignment was given.

Lesson OP 3. As a result of the confusion over the topic of finding appropriate scales for "real-life" problems, it was necessary to spend the third lesson doing more of the same type of questions. Consequently, in this lesson a slow and deliberate example of determining suitable scales by calculating number pairs in the traditional manner was done before the computer was used to draw the graph. Finally the students seemed to be grasping the idea of what constitutes an acceptable domain and range, and demonstrated this new knowledge by reacting to computer-generated graphs put on the overhead and suggesting repeated changes to the scale in order to obtain complete graphs. The computer did not dominate today's lesson, but was used at the beginning and the end of the class to graph functions once the class had made some decisions regarding domain and range for the problems in question.

Lesson OP 4. Today I used overhead transparencies I had prepared of the graphs of two functions in order to show students non-computer generated graphs for problems, and in so doing felt like I was breaking the stranglehold on teaching methods held over me by the computer. Displaying graphs such as these seems reasonable because students are still expected to be able to draw correct graphs of functions without the use of a computer. As the curriculum changes it is possible that a skill such as drawing graphs may be deleted or lessened, but at the present time it is still required.

The computer was used for a substantial part of today's lesson as the class was again graphing "real-world" problems found in their text book. Students were working in their groups and suggesting various domains and ranges for the problems. It was very easy to input differing suggestions from various groups, observe the resulting graphs on the overhead and decide which group had the most reasonable scale. The students liked the fact that once they suggested a domain and a range, the computer could generate a graph with this input almost immediately and consequently there was instant feedback from their suggestions. This procedure created student interest, but the interest faded as the procedure became repetitive, and both the students and myself lost an earlier sense of excitement. Even though the students were telling me what data to input into the computer, there did not seem to be a sense of class involvement in the generating of the graphs. (A mitigating factor in the lack of interest problem may be because it is an 8:30 a.m. Monday class.) Towards the end of the class students were given two more word problems from the text, problems that could be solved graphically or algebraically, and instructed that they could use the computer to do a graph if they wished. The majority of the class tried to solve the problems algebraically, and only one student tried to use the computer (perhaps because many of them were not sure of how to use the computer, even though knew they could receive any help they needed) and after working for a while she concluded it was easier to solve it algebraically.

<u>Lesson OP 5</u>. This lesson was a short one, with some of it taken with students drawing graphs of their two homework questions on the board, which they did very well. For the remainder of the class students did a quiz that required them to find the maximum or minimum value of a function, and this they did very well even though the concepts of maximum and minimum had not been taught. Students appeared to have picked up the concepts of maximum and minimum intuitively from looking at many graphs. The

students had no computer help for the quiz, and in order to solve the question some students made a graph by determining number pairs and other students tried to solve it algebraically.

My plan had been to start Topic 2 today, but because the students had to do the quiz without any computer help they took longer than I had anticipated, consequently the new topic was not started. As a result each student was given one problem from the text to graph for homework. These problems provided real world examples of the eight types of functions to be covered in this chapter.

Topic 2: Other Types of Functions

The objectives of this topic were to make students aware that there are many other functions whose number pairs form graphs different from those studied thus far and whose equations have a different form from one another; and to relate the form of eight different types of equations to their graphs.

Lesson OP 6. Students drew the graphs of the "real-world" problems that had been done for homework (one graph per group) on the board. Some of the graphs done on large pieces of paper by the Calculator class were also put up, and these were used to generate a discussion about different types of graphs for different types of equations.

The next objective of the lesson was to learn the eight different shapes of graphs that represent eight particular types of functions. The students worked in groups and generated the graphs by determining number pairs, then the overhead and computer were used to check the students' graphs. In ten minutes we had accurate graphs of all eight functions on the screen, and we were able to quickly and accurately correct the students' graphs. In addition, I was able to use one of the computer graphs to explain the concept of asymptotes, and to further answer a question about why one student's calculator showed error when she tried to calculate $f(0)$ for $f(x)=\frac{1}{x}$. Today's use of the overhead and computer gave me a feeling of satisfaction. With the students making changes to their graphs as the correct ones appeared on the overhead from the computer, the entire process of correcting the graphs took very little time with the result that no one felt the class was dragging.

One change that helped today's class run smoothly was a change in software. Instead of using Zap-a-Graph I switched to Master Grapher for this period only because Master Grapher allowed me to type in any function without first having to know what type of function it was. For most other topics I preferred Zap-a-Graph, but for this particular topic Master Grapher suited my needs better. I had decided to make this switch after encountering some problems with a similar situation in lesson OP 1.

Topic 3: Graphing $y = f(x) + q$

The three main objectives of this topic were to examine the graph of $y = x^2$; to determine how $y = x^2 + q$ differs from $y = x^2$; and to draw conclusions about how any of the basic eight functions graphed last class are affected by adding a constant q, that is how $y = f(x) + q$ differs from $y = f(x)$.

Lesson OP 7. An investigation sheet was used in which the students were asked to note features such as vertex, axis of symmetry and coordinates of intercepts as the graphs of the parabolic equations on their sheets were displayed on the overhead. There was a good flow to the lesson today as the students took to the idea of having the computer quickly generate the graphs, and they used the time the computer saved them to enter into discussions among themselves about the features they were investigating. After we had looked at a couple of graphs produced by the computer, the class enjoyed the game of trying to guess where the next graph might be located, with several guessing correctly.

During the latter part of the period the students worked on questions from their text, questions that did not require a graphing device.

Lesson OP 8. Today's objective was to generalize the conclusions regarding $y = x^2 + q$ to $y = f(x) + q$. It was first necessary to explain the meaning of $y = f(x) + q$ to the class, then we used the computer and overhead to examine the graphs of several functions whose equations were on their investigation sheet. The students copied into their notes the graphs that appeared on the screen, and after finishing the second set of examples a couple of students asked if all the graphs were simply going to be shifted up or down, so they had quickly seen the concept. It took only ten minutes to show enough examples that the class was able to formalize their conclusions and begin working on the questions in the text. I again felt comfortable with the computer and overhead. They were doing quickly and accurately what I had hoped they would do, and there was no feeling of tediousness that often accompanies a graphing assignment.

Topic 4: Graphing $y = f(x - p)$

Students were to sketch the graph of $y = f(x - p)$ without the aid of a graphing device by translating the graph of $y = f(x)$, where $f(x)$ is any one of the eight basic functions studied thus far or any other function whose graph is given.

Lesson OP 9. The lesson began with students successfully doing a quiz in which they were asked to sketch three graphs for functions such as $y = \sqrt{x - 4}$ by translating the graph of $y = \sqrt{x}$, without using a graphing device.

The focus of today's lesson was to have students explore the graphs of functions of the type $y = (x - p)^2$ with me entering the equations from the investigation sheet into the computer and having the students observe the resulting graphs on the overhead screen. As we began working through the equations on the sheet, I thought that it might be more interesting for the students if they first tried to predict the position of a new graph before it was displayed on the screen, so we followed that procedure and the students reacted with considerable interest. We first reviewed what effect the amount 3 had in $y = x^2 + 3$, then wondered what $y = (x - 3)^2$ might look like. I had expected most students to guess correctly, but they suggested a variety of answers (some voluntarily and some through direct questions from me) and each answer came with some sort of supporting rationale. It was fun to finally plot the graph on the overhead and to hear the "Oh!" and "Yeah!" and so on from the class. Then I asked them to guess at the graph for $y = (x + 2)^2$, and some wanted to shift left and some right. The students then spontaneously began discussing the topic among themselves and tried to explain to one another why each thought he or she was correct. There was a rewarding amount of interest on the part of the students when the computer generated graph finally appeared on the screen. After a couple of examples most of the students seemed able to make the generalization regarding horizontal shifting.

My surprise today was that most students did not have a feel for what $(x - 3)$ would do to the position of a graph. The students also indicated that they thought that the graphs of $y = (x - 3)^2$ and $y = x^2 - 6x + 9$ might be different, so I was able to use the computer and overhead to show them that the graphs for both were the same, and then I was able to explain why algebraically. When the students were guessing as to what the graph of $y = x^2 - 6x + 9$ might look like, some wanted to move the graph up 9, while others wondered what effect the $-6x$ would have.

<u>Lesson OP 10</u>. The intention of this lesson was to progress from graphing $y = (x - p)^2$ to graphing $y = f(x - p)$ in a manner similar to that used last class. I had to be out of the room for a short time because of the Fermat contest, therefore I asked a student who was familiar with computers to enter the equations from the investigation sheet into the computer. As a result of this switch in roles the student operating the computer

was more involved than usual in the class while the rest of the class seemed unaffected, although there was less enthusiasm than there was yesterday. After I returned to the class I let her continue at the computer, but felt that I had to interject occasionally to explain certain points about a graph, which made me realize that for all students the graphs are not necessarily self-explanatory. Again the students seem to be having no trouble with the concept of horizontal shifting, and with the time saved by having the computer do the graphing examples the students were able to use class time to work on their assignment. However many of the students were still uncomfortable with using only three or four points with which to sketch a graph, even if they were simply copying a graph from the overhead, and they continued to want to draw very accurate graphs when all that was required was a sketch to show relative position and shape.

Topic 5: Graphing $y = af(x)$

Students were to sketch the graph of $y = af(x)$ by stretching or compressing the graph of $y = f(x)$ without the aid of a graphing device.

Lesson OP 11. For the first part of the class we found we did not need to use the computer and overhead as we were going over the homework, much of which included questions from a supplementary sheet. These supplementary questions were obtained from a text book titled *Pre-Calculus Mathematics - A Graphing Approach* by Demana and Waits (1990) and were intended for students who had a graphing device at their disposal. Most of the questions asked students to apply what they had learned more than the questions from the student text did, but did not require a graphing device to obtain an answer. In order to review some of the work from the last period, I used the computer and overhead to show the graph of a function on the screen, then the students were asked to transform the graph and they did so quite easily.

To teach today's lesson about graphing $y = af(x)$, I graphed $y = x^2$ on the overhead using the computer, then before showing the graph of $y = 3x^2$ on the screen I asked the students to guess where they thought the graph would be, and this generated a good discussion in spite of the fact that this was an 8:30 a.m. Monday class. The students guessed up, down, wider, and skinnier, and then the correct graph was shown. After repeating this for two more examples the students entered a conclusion into their notebooks and then began to work on questions from their text and from the supplementary sheets.

Lesson OP 12. In order to correct the homework from last period, I sketched graphs on the board rather than use the computer and overhead. I followed this strategy because I did not have the overhead and computer connected and I thought it might be more appropriate to do the questions the way the students were expected to do them. I also found it easier to explain the homework questions on the board rather than with the computer. As the period progressed, several students encountered difficulties with a question asking them to graph $y = \frac{3}{x}$ so I connected the equipment because I wanted the students to see quickly an accurate graph of this particular function, plus I wanted them to be able to see an accurate relationship between the graph they were trying to get and the basic graph of $y = \frac{1}{x}$ The class spent the remainder of the period working on the assigned questions, and the questions were such that the computer was not required.

Topic 6: Graphing $y = -f(x)$, and $y = f(-x)$

In this section the objective was to clarify the difference the placement of the negative sign makes to the graphs of functions of the form $y = -f(x)$, and $y = f(-x)$ when compared to $y = f(x)$.

Lesson OP 13. The students were given an investigation sheet with which to explore this topic. I put the first pair of graphs on the overhead using the computer and we very quickly got neat, accurate large graphs and the translation appeared obvious. I then verbally explained why the graphs looked the way they did. The students seemed to grasp the concept quickly so I had them sketch the graphs for the next few examples and then we checked their graphs with the computer. Today I verbalized more than in previous classes and did not give them as much time to discover the concepts as I had in previous periods, and the students asked fewer questions. Today's use of the overhead and computer seemed to serve a short and specific purpose. The students spent the remainder of the class doing a short assignment from their text that did not require the computer.

Topic 7: Graphing $y = af(x - p) + q$

Students were to combine the conclusions from the previous lessons about the effects of the constants a, p, and q on the graphs of functions of the form $y = af(x - p) + q$ in order to sketch, without the aid of a graphing device, graphs of these functions by transforming a basic graph of the form $y = f(x)$.

Lesson OP 14. The students were occupied all period working in their groups on the investigation sheet for this topic. The essence of the investigation was to determine in which order or orders to do the transformations in order to obtain the correct graph. The computer was not used at all because the students were expected to do each individual transformation according to rules learned earlier. I had originally planned to use the computer to check the students' graphs, however as this was an assignment to be handed in next period and I wanted to be sure all students obtained their answers without using the computer, I altered my original plan and asked them to check their final graphs by taking some number pairs from their graph and checking to see if those numbers satisfied the equation. Most of the class found this particular investigation to be difficult, and the difficulty had nothing to do with not being able to use the computer, rather the problem was in trying to organize their work and to form some conclusions.

Lesson OP15. The students were given the first ten minutes of the class to discuss in their groups their conclusions from last day's assignment. After the assignments were handed in we discussed their conclusions, then they worked on questions from their textbook. The computer was not needed for today's class.

Topic 8: Review

Students were to review the concepts of the unit in preparation for a unit test on Topics 1 to 15 for which they will not be permitted to use a graphing device.

Lesson OP 16. The students finished the questions they had started in the last class, then they worked on some review questions from the textbook. Again the computer was not needed.

Topic 9: Unit Test

The objective of this test was to determine to what level the students have met the objectives of Topics 1 to 7 of this unit.

Lesson OP 17. Today the students wrote a unit test that did not require the use of a computer.

Topic 10: Maximum-Minimum Word Problems

The students are to solve a maximum-minimum word problem for which the equation is given by using a graphing device to obtain a graph for the problem and then reading the appropriate information from the graph.

Lesson OP 18. The plan had been to do some maximum-minimum word problems this period, but we spent longer discussing last day's test than I had planned, consequently we only started the topic of word problems. We were able to do only one problem, and the students did it via a table of values approach. We then discussed the idea of the maximum or minimum value of a function by referring to the table of values they had established. There was no time to use the computer to graph the function.

Lesson OP 19. We began the lesson by looking at a computer-generated graph of the problem we examined last period, and related the maximum value from the graph to the table of values we had calculated last class. The connection appeared to be clear to the students.

For the rest of the period the students worked on problems from a supplementary sheet they were given and from the textbook. For each problem I allowed the groups time to discuss among themselves which variable was the input and which was the output and to speculate as to reasonable limits for them. Groups were then asked to suggest their opinions and I tried them in the computer until we obtained a complete graph for the problem. Once the graph appeared on the screen, the students had no difficulty in answering the problem. After doing a couple of problems this way, I decided that the process was too slow so we changed schemes to a much faster one in which we started graphing the given equation on the default computer scale first, and then adjusted the domain and range until we could see a complete graph.

The students knew how to determine the maximum and minimum value for a quadratic function if the equation was in the form $y = a(x-p)^2 + q$, but one question I accidentally gave them asked them to find the maximum or minimum value for $y = 7 - 2x^2$ without using the computer. Most of the students were unable to do it, but an "A" student explained to the class that the maximum was 7 at $x = 0$ because the graph was merely the graph of $y = x^2$ flipped and shifted up with no horizontal shift. She had applied some of the ideas from the last unit.

Other students wanted to know how to determine the maximum and minimum values algebraically, something I had not shown them because it is not required in the Mathematics 11 curriculum, so I went through the procedure with them.

Lesson OP 20. The computer and overhead were used throughout the lesson today as we solved maximum-minimum word problems. For homework the students had been asked to set up appropriate scales for the axes for graphs for some of the problems, and as the lesson progressed I heard students exclaiming "That's what I got!" in a triumphant tone, or "I only went to 5", and so on as they reacted to the computer graphs on the overhead. There was some interest on the part of the students as we did the problems the students had started for homework because the students were giving me suggestions as to how to change the scale on the computer to conform with what they had in their notebooks, and we changed scales on a trial and error basis that did not take too long. We all enjoyed it when a first graph would appear as two straight lines, or not appear at all, and the students would tell me how to change the scale to get a graph that was similar to one they had in their notebooks. As we did the first couple of questions I felt it was tedious to do the graphs, but as both the students and myself got the knack of changing the scales on the graphs, and as the students read out values to plug into the computer, the feeling changed and the time went quickly.

The class seemed to be more involved in their work and exhibited a friendlier, more cooperative attitude than usual today. There were also a significant number of students absent this period, resulting in a smaller class with a more "club-like" atmosphere.

Once we agreed on a final graph for a problem the students copied that graph into their notebooks using the x and y intercepts and maximum or minimum points as references, and that procedure, which I had reservations about at the beginning of the period, took only seconds and was done willingly by the class. At the end of the period I had a sense that the computer and overhead had been a definite asset to the learning situation in the classroom.

Topic 11: Graphing Circles

The students were to recognize that equations of the form $x^2 + y^2 = r^2$ determine a circle, and were to sketch the graph of the circle without using a graphing device.

Lesson OP 21. We spent the first 15 minutes of the period finishing the maximum-minimum word problem topic by correcting some questions from the textbook. For one of the questions I tried to show an algebraic solution on the blackboard, but the students were still puzzled so I obtained the graph for the function on the computer and once it was displayed on the screen I was able to utilize it as an aid in explaining a solution, and the students had greater success understanding the solution with the graph to refer to.

The topic of graphing circles was introduced this period, and the method I chose to use did not involve the use of a computer. One student, however, who had missed the last class, sat at the computer for 15 minutes and worked on the assignment the class had done last period. He had minimal computer experience but had no trouble with the program after we went through one example together.

Lesson OP 22. This period we worked on questions from the textbook related to graphs of circles. The computer was no advantage in solving these questions, consequently it was not used.

Topic 12: Graphing $(x-p)^2+(y-q)^2=r^2$

Students were to sketch the graph of $(x-p)^2+(y-q)^2=r^2$ by translating the graph of $x^2+y^2=r^2$ and to determine that the center of the circle is given by (p,q).

Lesson OP 23. To explore today's topic the students were given an investigation sheet with several equations of the form $(x-p)^2+(y-q)^2=r^2$. For each equation the students were instructed to sketch their guess as to the location of the graph (using the transforming techniques they had learned in the previous unit) and then I entered the equation into the computer and the students compared their graphs to the one on the screen. I thought it was great to see such an accurate graph on the screen almost instantly, although the students were not quite as enthusiastic, but it was 8:30 a.m. Monday after Spring Break.

For the graph of $(x-6)^2+y^2=9$ everyone guessed correctly. For the graph of $x^2+(y+6)^2=9$ almost everyone was wrong. In questioning students I discovered that they were thinking of the shifting rules they had memorized in the last unit where they did the opposite with x and the same with y, for example they graphed $y=(x-2)^2+4$ by shifting right 2 and up 4.

Then a student asked, "What if it was $(x-6)^2-y^2=9$?" We discussed this question, a few students expressed some opinions, then I entered the equation into the computer and we saw the graph. Again I felt excited that we could get the correct graph so quickly.

It took less time than I had anticipated to do the examples, so the students used the extra time to do some additional questions from the textbook that required more thinking on their part.

Topic 13: Graphing $\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=r^2$

The students were to sketch the graph of an equation of the form $\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=r^2$ by stretching or compressing the graph of an equation of the form $x^2+y^2=r^2$.

Lesson OP 24. The students were given an investigation sheet with six questions that required them to sketch graphs of the form $\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=r^2$. For each equation the students guessed first and did a quick sketch, then I showed the correct graph on the overhead via the computer and we discussed why it was where it was. After the six examples the students wrote down their own rule for the transformations (most favored saying "you do the opposite"). Even though the computer gave us a quick and efficient way of getting the correct graphs with which to base our conclusions, the students seemed to be treating the use of the computer as routine, and although they worked conscientiously, they did not show any signs of increased enthusiasm.

Topic 14: Graphing $\left(\frac{x-p}{a}\right)^2+\left(\frac{y-q}{b}\right)^2=r^2$

The students were to sketch the graph of an equation of the type $\left(\frac{x-p}{a}\right)^2+\left(\frac{y-q}{b}\right)^2=r^2$ by applying the rules of transformations of graphs in the correct order to the graph of an equation of the type $x^2+y^2=r^2$ without the aid of a graphing device or a table of values.

Lesson OP 25. Today's topic was the final one in the series on transformations, with the major objective being to decide on the order in which the individual transformations should be applied when shifts were combined with stretches and compressions. In part one of the lesson we investigated the graphs of equations such as $(x+3)^2+(2y)^2=16$. For each example the students first identified the individual transformations involved, then different groups were assigned different orders and asked to sketch where they thought the graph should be according to the order they had. Ideally I would have preferred to have had all groups do all possible orders and then show their resulting graphs on the board, but by this time in the unit I was interested in saving time and the investigations were beginning to get repetitious so I did not follow this scheme. When all groups had finished I randomly selected one order and followed that order in graphing the equation on the computer. When that graph appeared on the overhead screen, I asked which groups agreed with it, and when all groups indicated they did, we concluded that the order did not matter for the first example. As a further check we picked different points from our final graph and substituted those coordinates into the equation to verify that the final graph was correct. The students were not confused by this type of question, although one "A" student did ask why, when we graphed $\frac{x^2}{5}+y^2=16$ we changed only the x coordinate by a factor of 5 and not the y coordinate, but when we graphed $y=5x^2$ we did change the y coordinate by a factor of 5. I tried to answer his question algebraically.

Part two of the lesson involved graphing equations in which a shift and a stretch or compression were applied to the same variable, such as $\left(\frac{x-2}{3}\right)^2+[2(y+3)]^2=16$ Again all of the groups identified all of the transformations that were involved, then different groups were asked to try one order each and check their own resulting graph by substituting a number pair from their graph into the original equation to see if their graph was correct. We then discussed everyone's findings, checked a couple with the computer, and formed some conclusions with regard to the correct order of transformations. With regard to this particular form of equation, I was not totally sure in my own mind of the correct order, so while the students were working on their sketches, I quickly tried a couple of different orders with the computer (but did not display them on the overhead), examined the results, and then logically reasoned why the correct order was the one it was.

I had originally planned to do only part one of todays' lesson this period, but the combination of using the computer and not having the students put their sketches on the board for part two enabled us to complete both parts in one class.

Topic 15: Graphing Inequalities

The students were to sketch the graphs of quadratic inequalities in two variables of the form $\left(\frac{x-p}{a}\right)^2 + \left(\frac{y-q}{b}\right)^2 \leq r^2$ by transforming the graph of $x^2 + y^2 \leq r^2$ without the use of a graphing device.

Lesson OP 26. It was my opinion, with my limited knowledge of the software, that the computer would not be of value for this topic, so it was not used.

Topic 16: Review

The students were to review Topics 10 to 15 in preparation for a unit test on those topics.

Lesson OP 27. We worked on some questions relative to last day's assignment on inequalities, discussed the topics that will be tested next class, then did some review questions from the textbook. I did not feel the computer was needed for any of these activities therefore we did not use it.

Topic 17: Unit Test

The students were to write a test, without the aid of a graphing device, that will determine to what degree they have learned the concepts in Topics 10 to 15.

Lesson OP 28. The students wrote the unit test without the aid of the computer and overhead projector.

The Computer Lab Class

Prior to beginning the unit using the computer lab, the class and myself spent one period in the lab with the computer teacher, during which time he showed us how to log on and off the computer, how to open the Zap-a-Graph program, and he allowed the students time to play with the program while he and I circulated through the class answering various questions about operating the program. Upon seeing their first graph, virtually all students reacted with an "Oh!", and many times thereafter the expression "Oh cool!" was heard as they saw some new feature of the program. The students were on task all period and we (the students and myself) were so involved with what we were doing that we did not even realize the period was over, and were quite surprised when the computer teacher informed us that our time was up.

Topic 1: Defining a Quadratic Function

The general objectives for this topic were to review the definitions of function, domain and range; to review what the graph of a function represents; to enable students to realize that not all functions have graphs that are linear; and to define a quadratic function.

Lesson Lab 1. "Mr. Bowles, are we going on the computers today?" "Yes." "Oh cool." This was the exchange with one of the students in the class as we assembled in our regular classroom prior to having our inaugural lesson in the computer lab, and it reflected the enthusiasm many of the students appeared to have. Before moving to the computer lab, the students were divided into pairs. I had previously determined the pairings and had attempted to achieve a balance between male and female, experienced and inexperienced computer users, and high and low mathematics achievers.

All students were given four "real-life" problems, illustrating different types of functions, and asked to graph the indicated functions. The intention was for each pair of students to use the computer to quickly obtain graphs of the functions, then to have them compare the graphs they obtained with their equation to see if they could draw any conclusions about the characteristics of an equation and its corresponding graph. The students were given explicit verbal and written instructions to determine a suitable domain and range before attempting to obtain a graph for a particular equation, but most students ignored these instructions and immediately started entering coefficients and as a result obtained graphs that were not meaningful representations of the problems. This was a problem I had not expected.

Another unexpected situation arose as one of the two printers in the room started to chatter, and I discovered that some of the experienced computer students were printing their graphs using the printer, and soon some other students were asking if they too could print their graphs on the printer. I had asked the students to do a neat but quick sketch of their graphs in their notebooks, but obviously some students did not want to follow those instructions. I felt uncomfortable about the problem because I had not anticipated it and had therefore did not have an answer ready. My instinct was to say no because I thought if all of the groups printed it would take too long (I was not really sure if this was the case) plus in many cases the graphs that students wanted to print were incorrect and I did not want to waste computer paper. Of the graphs that were printed before I had a chance to tell the students not to use the printer, all had incorrect domain and range, so I felt my decision not to use the printer was a good one.

The students were on task all period, and both they and I seemed to enjoy the hour. I was excited by the fact that some of the students' questions were new challenges to me, as using the computers led them to ask questions they had not asked in other years because the computer brings out different problems, some related to the computer software and some to the course content. I found the change in questions refreshing.

<u>Lesson Lab 2</u> The students encountered some difficulty today as they continued to work on the second problem from the set assigned last class because they were trying to graph $y = \frac{3000000}{x}$, but the software will not accept coefficients with more than five characters and similarly the x and y axis will not accept values with more than five characters. As a result students had to measure the x axis in 1000's, and this procedure caused considerable confusion with the result that the conclusions that I wanted the students to obtain regarding $y = \frac{k}{x}$ being a hyperbola were getting lost in a sea of mechanical questions. The thinking and the mathematics students are forced to do because of this software limitation are good, but in a crowded curriculum a teacher must carefully select the times and topics for "side-trips".

An unexpected bonus came today as a direct result of using the computers. As students began to graph the hyperbola, they looked at some of the other students' computer screens and noticed that all groups' hyperbolas did not look the same. In fact different scales on the axes resulted in different looking hyperbolas, and students were looking at each others' screens and asking one another who was correct. This provided me with an ideal opportunity to explain that graphs representing the same function may not necessarily look identical if the axes have different scales.

Today's class was only 30 minutes, and the students were on task all of the time, but I was still feeling a sense of frustration because it was taking so long to do the questions that were intended only as data to be used to enable students to reach conclusions about types of equations and their graphs. I also experienced a feeling of lack of control in that I could not see all 15 screens at once and I did not know exactly what each pair of students was doing, but perhaps this was not so different from students working in their notebooks in a regular classroom. The facts that the screens were so visible and the methodology of using computers was so new to me were tending to make me overly conscious of keeping abreast of every students' progress.

Lesson Lab 3. Students began arriving early for class asking, "Can we start?" "Oh good." Then they started helping each other to graph numbers three and four from their problem sheet (their homework had been to decide on a domain and range for each of these problems). The interest shown at the beginning of the class seemed to continue as the students maintained interest throughout the period and appeared to be adapting well to the use of computers.

The questions that students asked during the class were related both to the use of the software and to the lesson content. A weak but determined student, when starting to graph a problem, decided she wanted a "bigger picture" so she easily changed the scale on her computer, an accomplishment I felt was a victory for her and for computer methodology. In general, as I watched the students working I observed by the end of the period that the idea of changing scale to see the entire graph was understood by the majority of the class.

An ongoing problem was that of trying to get and maintain the students' attention for any type of class discussion. The long narrow room seemed to make it more difficult for me to get their attention, then, once I had their attention, their fingers seemed to want to get back to punching the keyboards. To that point, class discussions did not have the student involvement a teacher would have hopee for.

The dilemma of which questions to assign for homework arose today. I had wanted them to do some questions from the text using the computers to generate the graphs. I attempted to solve the dilemma by having them do the questions requiring the computer in class, and assigning questions not requiring the computer to be done at home. In addition, as an experiment, I assigned two more questions that were computer oriented and asked the students to come in to the computer lab during their unstructured time to do them. One result of this compromise was that fewer questions were assigned than I had originally planned .

<u>Lesson Lab 4</u>. Most of todays' class was spent discussing last day's homework and working on the remainder of the assignment. I found it difficult to discuss their homework questions because of the problem of getting and maintaining their attention, and the suspicion persisted that they may be looking at me, but their brains are still locked on their particular computer. My experiment with regard to assigning a couple of questions to be done in their unstructured time in the computer lab did not meet with success, as many of the students had not gone to the lab and consequently had not done the questions.

On the positive side, the class remained on task all period, and seemed to be enjoying trying to graph "real world" problems. I enjoyed going around the class helping individuals with their questions, but I still had the nagging feeling, in spite of the fact that everyone was on task all period, that I did not have as much control over the class as I was used to having .

One of the questions being done in class today gave an equation for a height versus time function and asked the students questions about height and time but did not ask them to draw a graph to solve the problem, as some previous questions had done. Several students asked, "Do we have to draw a graph? It doesn't say we have to."

Lesson Lab 5. "Are we still on the computers? Oh no! I hate those things. They're monsters." These were the comments made before class from a girl who ironically had made a significant content breakthrough the previous period, but reflect a minority opinion as most of the class appeared to be enjoying using the computers as evidenced by their continuing to be on task while working with the computers.

Today students did a quiz working in pairs in which they were asked to solve a "real world" problem. I had constructed several different problems to avoid the possibility of students copying from their neighbors. The students were not instructed to use the computer to solve the problem, but every group did (with some individual exceptions for some parts of the question, but even those students got the graph first). One student, whose quiz question was a quadratic function, looked at her first attempt at graphing the function and saw a straight line (she had a very small domain) and commented: "I don't know what it should look like. I guess we could change the scale." (They did.) "Oh good - look." (A more appropriate graph appeared.) Another student, who already had a good graph for his particular function, when asked, "How's it going?", answered, "Not so good." I replied, "But the graph looks good", to which he responded, "Yeah, but now what do we do?"

During the last portion of the period each pair of students was trying to graph problems from the text, problems that contained several different types of functions, some of which were new to the students. Again the limitations of Zap-a-Graph were a problem, as students were confronted with the problem of having to input coefficients larger than the software would accept. This time though, many students tried different things, such as dividing all numbers in the equation by ten in order to fit the software's restrictions.

Another restriction imposed by Zap-a-Graph was that students needed to know the name of the particular function in order to graph it, for example, was it a rational or a cubic function? Usually they did not know the name, consequently they either played with the menu trying to solve their dilemma, or simply gave up.

Topic 2: Other Types of Functions

The objectives were to make students aware that there are many other functions whose number pairs form graphs different from those studied thus far and whose equations have a different form from one another; and to relate the form of eight different types of equations to their graphs.

Lesson Lab 6. The first part of the period was spent discussing the graphs to the "real-world" problems the students had worked on at the end of the previous period. Some groups drew their graphs on the board, and we used the large paper graphs from the calculator class for the groups who had been unable to get a graph of their own. Those who did not have a graph had functions whose names they needed to know in order to use the computer, and since they did not know the names of the functions they simply gave up rather than trying to get a graph by determining number pairs.

The second part of the period was spent having each pair of students use their computer to obtain the graphs of eight different types of functions. I had hoped to use the Master-Grapher software today because some of the students experienced problems last period using the Zap-a-Graph software in doing a similar exercise, but it was unavailable in the computer lab because it was not networked, so we had to continue with the Zap-a-Graph program. As a means of helping, I mentioned to them that they would have to scroll through the menus and try to find equations that looked like the ones they were trying to graph. Some of the students enjoyed trying to find the correct form, while others got very frustrated. One C- student discovered how to graph $y = \sqrt{x}$ and $y = \sqrt{x-3}$ (graphing a square root function in the Zap-a-Graph program requires you to use the "Transform" menu and I had not yet explained this to the students) and he and his partner had an animated discussion about his discovery. When I asked him how he had found the correct method of obtaining the graphs, he explained that liked to play with the different menus in the program, and when he had some graphs on the screen, he was wandering through the menus and under the heading "Transformations" he found "Square Root" so he tried it and it gave him the correct graph. His discovery was not a mathematical one, but he had the thrill of discovering something for himself and obviously was very proud of his accomplishment. A Grade 12 girl who did not have a positive attitude toward mathematics found working with the computer a significant help, and she commented, "If it wasn't for the computer I wouldn't have a clue how to get the graphs."

The work done by another student pointed out an advantage to the problem generated by needing to know the name of the function as she tried to graph $y = x^3$. From the menu she had selected the form $y = a(\text{base})^{(ax + b)}$ as the basic form to graph $y = x^3$, and she wanted to enter 3 for a and x for b but the computer program would not let her. She could not figure out why not, so I used the opportunity to explain to her the difference between the two types of functions.

<u>Topic 3: Graphing</u> $y = f(x) + q$

The three main objectives of this topic were to examine the graph of $y = x^2$; to determine how $y = x^2 + q$ differs from $y = x^2$; and to draw conclusions about how any of the basic eight functions graphed last class are affected by adding a constant q, that is how $y = f(x) + q$ differs from $y = f(x)$.

Lesson Lab 7 . Today's lesson started by correcting the eight types of functions they had graphed last period. To expedite the procedure I used the Master Grapher program in the computer that was connected to the overhead projection device (this program is not available to me on the network) in order to display the correct graphs. The students were very attentive as the correct graphs were displayed. For some of the graphs I found it helpful to draw one on the board, using the graph from the computer as a model, in order to make some further point about a certain feature of a graph.

The new work today was to have the students explore features such as vertex and axis of symmetry for the graph of a basic parabola $y = x^2$, then investigate the differences between the graphs of $y = x^2$ and $y = x^2 + q$ by using the equations given on their investigation sheet. The students used the computers and quickly got the correct graphs and had enough time left in the class to start discussing among themselves some possible conclusions about an equation and its graph. During this investigation the students asked me very few questions, and seemed able to easily form their own conclusions.

Lesson Lab 8 . We began by using the overhead projector with the computer to check a couple of the graphs they had done last class. Most of the students were watching and listening and not playing with their computers, and, although the class discussion was better than it was a week ago, I was still not as satisfied with the degree of participation by this group as I was with a group in my regular classroom.

I had been questioning my decision to have students make neat sketches of the graphs they produced with the computer, but today I felt somewhat vindicated as the graphs served a useful purpose. One student, who was struggling with the conclusions from last days' graphs, asked for help, and we used her graphs as a reference in order to answer her questions. We could have reproduced the graphs on the computer, but that would have taken a little more time and more importantly would have robbed her partner of the use of the computer.

Some of the graphing questions in the textbook were to be done without the aid of the computer, and the students followed these instructions. A few of the weaker students decided on their own to use the computer to check their graphs after they had finished the questions.

As one boy was graphing $y = 2^x$ with his computer he asked me why his graph stopped at $x = -4$, which the software appeared to show. By way of explanation I suggested we change the y scale and I started to do so, but the student stopped me and said enthusiastically, "No, let me." After he changed the scale, the graph seemed to stop at $x = -8$, so the boy decided to change the scale again, and this time the graph stopped at $x = -12$. The student then drew the conclusion himself about the y values getting closer and closer to 0 without reaching 0, a concept I had unsuccessfully tried to teach him earlier.

<u>Topic 4: Graphing</u> $y = f(x-p)$

Students were to sketch the graph of $y = f(x-p)$ without the aid of a graphing device by translating the graph of $y = f(x)$, where $f(x)$ is any one of the eight basic functions studied thus far or any other function whose graph is given.

<u>Lesson Lab 9</u>. Today the period began with students very successfully doing a quiz, without the use of the computers, based on last day's work on vertical transformations. In order to pursue today's topic of horizontal transformations, students again were given an investigation sheet and directed to work their way through the given examples in order to derive their own conclusions. The Zap-a-Graph software accepts different forms of an equation for some functions, consequently some students used $y = x^2 - 4x + 4$ instead of $y = (x-2)^2$ and asked if that was permissible. This question

led us into a discussion about different forms of an equation, and the students were somewhat surprised to see on their computers that different forms of an equation could have the same graph.

All groups seem to be getting the required conclusions regarding horizontal shifts, although there was some early confusion for a few students who thought $(x-3)$ should result in a shift to the left. One student who could not accept the conclusion investigated further himself by putting the graph on the computer screen, then selecting number pairs from the "Table of Values" option under the "Analyze" menu and checking them algebraically in the equation.

The students were on task all period, and generally the topics of discussion in the groups were related to "What did you get for your conclusions?", rather than "What does your graph look like?". The tone of class discussions also appeared to be improving, and I found it easier to get their attention for short lessons.

<u>Lesson Lab 10</u>. "Mr. Bowles, why? I can see how to do it (shifting the graphs) but why does it work? I can't do math unless I understand why." These were the comments from two "C" students as we discussed the conclusions from last days' investigation sheet. All of the students could do the shifting correctly, but the questions asked by these two students raised a question that was also bothering me, and that was whether I was relying on the graphing devices to show "what" was happening, but was not following up with an explanation of "why" it was happening. They were specifically interested in why changing $\sqrt{x}$ to $\sqrt{x-5}$ obeyed the same transformation rules as changing x^2 to $(x-5)^2$.

One of the questions students tried today asked students to draw the graph of $y=\frac{1}{x^2+1}$ then to obtain the graph of $y=\frac{1}{(x-3)^2+1}$ by applying what they had learned about horizontal shifting. I expected the students to obtain the first graph by using their computer, then to sketch the second graph without using the computer, but many students tried to obtain the second graph also using the computer and encountered numerous problems, such as expanding the denominator then not knowing how to enter the resulting equation. Only a few students realized that all that was required was a horizontal shift of 3. Commented one "B" student, "This computer is more trouble than it is worth!" After doing the first two questions of this type, students began to see the pattern so that when the question asked students to sketch the graph of $y = \frac{1}{x^2 + 10x + 26}$, two students (C+, C-) both saw that to graph it all you had to do was change the equation to $y = \frac{1}{x^2 + 10x + 25 + 1}$ or $y = \frac{1}{(x+5)^2 + 1}$ and then shift the original. This question was not part of the original assignment because I thought it might be too difficult for some of the students, but many tried it and I was pleasantly surprised with the results.

I realized at the end of this class that we were not covering as much material in class-time as I had anticipated. It was taking longer to do the investigations than I thought it would, but not as long as I suspected it would have to do the same investigation activity in a non-graphing device class.

Topic 5: Graphing $y = af(x)$

Students were to be able to sketch the graph of $y = af(x)$ by stretching or compressing the graph of $y = f(x)$ without the aid of a graphing device.

Lesson Lab 11. For the first 25 minutes of the class we found we did not need to use the computers as we were going over the homework, much of which included questions from a supplementary sheet. These questions were obtained from a text book titled *Pre-Calculus Mathematics - A Graphing Approach* by Demana and Waits (1990) and were intended for students who had a graphing device at their disposal. Most of the questions asked students to apply what they had learned more than the questions from the student text did, but did not require a graphing device to obtain an answer. The questions were also of a different nature than the ones in the students' text and were more relevant to the technological situation of the students in my class. This particular set of questions was intended for use during the remainder of the unit.

Once we got on to today's topic it took less than five minutes for the students to obtain five accurate graphs from their computers with which to make conclusions about the graphs of $y = af(x)$. In order to help students arrive at some sort of conclusion I encouraged them to find the y values for $x = \pm 2$ for each graph so they would have some y values to compare between graphs. In order to do this, some students used the cursor and some used the "Table of Values" feature under the "Options" menu. Now that I was becoming more comfortable with the use of the computer, I was more aware of the time it took to do the investigation activities and consequently today I imposed a time limit for arriving at some conclusions. Discussions revealed that all groups could see that a made the graph "skinnier or fatter", but only a few saw that it was by a factor of a. In discussing the conclusions with the class, I used the white-board instead of the computer and overhead because I was able to write number pairs on the board and leave them there for comparison to other number pairs, something I could not do with the computer. While we were discussing the conclusions one student commented, "There is no way I could have learned that without the computer." She then went on to explain that she needed to see the graphs in order to understand the conclusions, and that she could not conceive taking the time to get the graphs without a computer.

Lesson Lab 12. Before the class started at 8:30 a.m. many students were in the lab working on the questions from their text and from their supplementary sheet concerning last days' topic. During the period their assignment was to continue working on these questions The students found that plotting a transformed equation was of no use to them unless they also plotted the original equation on the same screen for comparison. Most of the questions the students did today did not require a computer, although some of the class did use the computer to check their graphs. Again I used the white-board to explain difficulties with the questions because I found it much easier to show the transformation of points using the board than using the graph on the computer screen.

Topic 6: Graphing $y = -f(x)$ and $y = f(-x)$

In this section the objective was to clarify the difference the placement of the negative sign makes to the graphs of functions of the form $y = -f(x)$ and $y = f(-x)$ when compared to $y = f(x)$.

Lesson Lab 13. The students were given an investigation sheet and instructed to work through the examples using their computers in order to observe the changes in the graphs as the negative signs were put in different places and then to generalize the effect of the placement of the negative sign for any function.

One C- student told me after class she was excited because she did today's work without the computer. She had reasoned that if x was multiplied by -1 then she just had to multiply all x coordinates by -1 and get a new graph, similarly for y. She felt very pleased because she had reasoned it out and not used the computer (which she says just gives her a graph but does not tell her where the values come from). When I asked her how she knew she was right (expecting her to tell me she checked with the computer) she just went through her multiplication of x and y reasoning again. She based her conclusions on her work with quadratic and cubic functions only, and when I asked her if she thought her reasoning would remain true for other functions such as square root or exponential, she simply replied that she had not considered those but that it did not matter, regardless of where the x was in the equation, she was going to multiply the x coordinates by -1.

Some students were confused when graphing $y=(-x)^2$ or $y=-x^3$ and $y=(-x)^3$. For the cubics they could not tell which axis the graph had been flipped over, but when I encouraged them to analyze the equations some were able to reason what was happening and why. After the students had finished their investigations, they worked through some questions from their textbook and from their supplementary sheet, questions that did not require the use of the computer.

Topic 7: Graphing $y=af(x-p)+q$

Students were to combine the conclusions from the previous lessons about the effects of the constants a,p, and q on the graphs of functions of the form $y=af(x-p)+q$ in order to sketch, without the aid of a graphing device, graphs of these functions by transforming a basic graph of the form $y=f(x)$.

Lesson Lab 14. The students were occupied all period working in their pairs on the investigation sheet for this topic. The purpose of the investigation was to determine in which order or orders to do the transformations in order to obtain the correct graph. The computers were not used at all because the students were expected to do each individual transformation according to rules learned earlier. I had originally planned to allow the students to use the computers to check their graphs, however as this was an assignment to be handed in at the end of the period and I wanted to be sure all students obtained their answers without using the computer, I altered my original plan and asked them to check their final graphs by taking some number pairs from their graphs and checking to see if those numbers satisfied the equations. The students were having some difficulty with the assignment so I asked them to hand in only part of it at the end of the period.

Lesson Lab 15. At the beginning of the class I returned the assignments they had handed in at the end of the previous period, and we discussed their conclusions. The students did not tend to play with the computers to any degree that hampered the class discussion, as was the case in earlier lessons, but I still found the physical setting of this particular long rectangular room a distraction.

In marking the portion of the assignment that they did hand in I determined that many of the students found this particular investigation to be difficult, and the difficulty had nothing to do with not being allowed to use the computers, rather their problems were in trying to organize their work and to form some conclusions. After we discussed the work they had done so far, they finished the investigation sheet, we discussed their conclusions to those sections, and they proceeded to work on some questions from their textbook. The computers were available but not needed.

One student, who had missed a couple of classes, spent most of the period trying to review and sort out the past few days' work. He said he worked the rules out by reasoning and then used the computer to verify what he had concluded.

Topic 8: Review

Students were to review the concepts of Topics 1 to 15 in preparation for a unit test for which they will not be permitted to use a graphing device.

Lesson Lab 16. For the first few minutes of the class the students were finishing the textbook questions that they had started last period, then we discussed questions arising from them. The remainder of the period was spent with the class working on review questions from their textbooks. The computers were not used for any questions, with one exception. One of the questions asked the students to solve a "real-world" problem, and a few students opted to solve it by generating a graph with the computer.

Lesson Lab 17. More review questions were done this period, utilizing the textbook and the sheet of supplementary questions handed out several periods previously. A further period for review was not actually needed, but due to this school's particular timetable this was a short class and there was not enough time to do the test or begin the next unit, consequently it became a review period by default.

Topic 9: Unit Test

The objective of this test was to determine to what level the students had met the objectives of Topics 1 to 7 of this unit.

Lesson Lab 18. The students wrote the unit test this period, and since the test did not require the use of a computer, and in the interests of test validity, the students wrote the test in their regular mathematics classroom.

Topic 10: Maximum-Minimum Word Problems

The students were to solve a maximum-minimum word problem for which the equation was given by using a graphing device to obtain a graph for the problem and then reading the appropriate information from the graph.

Lesson Lab 19. After discussing last day's test we began the lesson on maximum-minimum word problems by examining an introductory problem via a table of values approach. We discussed the idea of the maximum or minimum value of a function by referring to the table of values they had established, then graphed the function on their computers in order to compare the table of values with the graph. The final step was to realize that the maximum-minimum value of the function could simply be read from the graph.

The class spent the remainder of the period graphing functions, for which they were given the equations, on their computers and answering the questions using their computer generated graphs. For the "real-world" problems part of the challenge for the students was to get complete graphs on their screens, ones that had realistic domains and ranges. By chance I stumbled on an excellent method of illustrating to the students which scales were appropriate and which were not, and how to find them, when a "C" student happened to sit at the computer that was connected to the overhead projection device. I was able to show the entire class her graphing attempts on the screen and have her tell the class how she was going to change the scales if they needed changing. She enjoyed the involvement and the class benefitted by seeing the attempted solutions of a peer. Another advantage for me was that I was then able to walk around the class helping individuals.

When one student asked, "I can see where the minimum is, but where is the maximum?", I was able to use the graph on his screen to answer his question.

Lesson Lab 20. For homework they had been given some questions from the textbook that asked them to determine the maximum-minimum values for given graphs or equations in the form $y = a(x-p)^2 + q$. But one of the questions gave them an equation in a slightly different form, $y = 7 - 2x^2$, and many students could not do it. I used the white-board to review how we could transform the graph of $y = x^2$ in order to get a solution.

The students spent the rest of the period doing more maximum-minimum word problems by entering the given equations into their computers then examining the graphs to answer the questions. I continued the scheme I discovered last period and had a student operate the computer that was connected to the overhead and used her graphs as teaching examples when they were appropriate. For example, as I wandered around the class examining the students' work I noticed that many of them were using all four quadrants for some problems that only required one, so I used the student's graph on the overhead to review the concept of a complete graph.

During the period the students were able to do enough problems by using the computers to satisfy me that they understood the concept involved and did not require any more time on this topic.

Topic 11: Graphing Circles

The students were to recognize that equations of the form $x^2 + y^2 = r^2$ determine a circle, and were to sketch the graph of the circle without using a graphing device.

Lesson Lab 21. For the first 15 minutes the students did a few maximum-minimum word problems from their textbook as a review with no apparent difficulty.

The remainder of the period was spent exploring the graph of $x^2 + y^2 = r^2$ without using the computers, as I prefer an alternate method that does not require a graphing device.

Lesson Lab 22. We discussed the homework questions regarding the graphs of circles, did a few more questions and the students successfully wrote a circles quiz. Today's class was held in our regular class-room and not in the computer lab because I thought the computers would be of no advantage for today's lesson.

Topic 12: Graphing $(x-p)^2 + (y-q)^2 = r^2$

Students were to sketch the graph of $(x-p)^2 + (y-q)^2 = r^2$ by translating the graph of $x^2 + y^2 = r^2$ and were to determine that the center of the circle is given by (p,q).

Lesson Lab 23. The students explored this topic by working through an investigation sheet that gave them several equations of the type $(x-p)^2 + (y-q)^2 = r^2$ and asked them to first sketch their guess as to the location of the graph, then to check their graph with their computer. The students worked diligently in their pairs, with comments like "Yup", "Hmm" and "Ah" as they looked at their quickly produced computer graphs and compared them to their guesses. After they had finished the investigation sheet we discussed their conclusions as a class and they proceeded to work on the questions in their textbook. The students had finished the investigations more quickly than I had anticipated, consequently they were finished their homework questions in class so I encouraged them to experiment with their computers to see if they could obtain the graph of a circle that had been stretched. I mentioned to them that they would have to use the Transform menu, but I did not tell them anymore than that and after ten minutes they were getting frustrated.

Topic 13: Graphing $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = r^2$

The students were to sketch the graph of an equation of the form $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = r^2$ by stretching or compressing the graph of an equation of the form $x^2 + y^2 = r^2$.

Lesson Lab 24. The students worked through an investigation sheet that required them to use their computers to obtain graphs for the given equations, compare those graphs to the graph of $x^2 + y^2 = r^2$, and to draw conclusions about the significance of a and b in transforming the graph. I went through question 1 with them with a student operating the computer that was connected to the overhead, and explained how to use the "Transform" menu. Most of the students did the remaining five questions quickly, arrived at the correct conclusions, and were able to do most of their assignment in class, an assignment that did not require the computers.

Topic 14: Graphing $\left(\frac{x-p}{a}\right)^2 + \left(\frac{y-q}{b}\right)^2 = r^2$

The students were to sketch the graph of an equation of the type $\left(\frac{x-p}{a}\right)^2 + \left(\frac{y-q}{b}\right)^2 = r^2$ by applying the rules of transformations of graphs in the correct order to the graph of an equation of the type $x^2 + y^2 = r^2$ without the aid of a graphing device.

Lesson Lab 25. The students began the lesson by investigating the order of transformations for equations such as $(x-2)^2 + (3y)^2 = 16$ in which each variable was affected by only one transformation. They first identified the two possible orders, then used their computers to follow each of those orders. To determine which graph was correct they selected a point from each graph and substituted its coordinates into the equation. After all students had completed at least the first equation, I reviewed how to do a particular order with the computer and how to leave the first graph on the screen for comparison with the next order by having a student operate the computer connected to the overhead projector. When the students had completed the first three examples they were able to form conclusions in their pairs about the correct order of transformations for this form of equation and were able to do some questions from the textbook that asked them to apply their new knowledge without using the computer.

The next step was to investigate the correct order for equations such as $\left(\frac{x-2}{3}\right)^2 + [2(y+3)]^2 = 16$. In order to simplify the investigations (we were running out of time in the period and I was beginning to tire of investigating the same type of thing) the students examined two orders only: all shifts first; or all expansions and compressions first (the software makes this breakdown easier). They determined in their pairs which order was correct by examining their graphs as they did in the first part of today's lesson and all students were able to get the correct conclusion.

Topic 15: Graphing Inequalities

The students were to sketch the graphs of quadratic inequalities in two variables of the form $\left(\frac{x-p}{a}\right)^2 + \left(\frac{y-q}{b}\right)^2 \le r^2$ by transforming the graph of $x^2 + y^2 \le r^2$ without the use of a graphing device.

Lesson Lab 26 . We discussed last day's homework and found we did not need the computers for that task, then proceeded to the lesson on inequalities, a topic that I had decided could also be adequately done without the computers.

Topic 16: Review

The students were to review Topics 10 to 15 in preparation for a unit test on those topics.

Lesson Lab 27. We discussed some questions from last day's assignment on inequalities and the topics that would be covered on the test they were to write next period, then the students did some review questions from their textbook. None of these activities involved the use of the computer.

Topic 17: Unit Test

The students were to write a test, without the aid of a graphing device, that would determine to what degree they have learned the concepts in Topics 10 to 15.

Lesson Lab 28. The students wrote the unit test, and because the computers were not used, they wrote it in their regular mathematics classroom.

The Graphics Calculator Class

Topic 1: Defining a Quadratic Function

The general objectives for this topic were to review the definitions of function, domain and range; to review what the graph of a function represents; to enable students to realize that not all functions have graphs that are linear; and to define a quadratic function.

Lesson Calc 1. At the beginning of the class the students were divided into groups of three or four, groups that I had pre-selected (each group had a male/female and a mathematical ability mix), and each person in a group was given a graphics calculator. It was my intention to have this class work both individually and in groups, depending upon the topic. All students were given four "real-world" problems, illustrating different types of functions, and asked to graph the indicated functions. This class had used the graphics calculators during three classes earlier in the year, so they were slightly familiar with their operation and most had the ability to obtain the graph of a simple function. The intention of today's lesson was to have the students use the graphics calculators to quickly obtain the four graphs, then have the groups discuss among themselves any conclusions they might reach about the characteristics of an equation and the appearance of its graph.

We did the first problem together, that is I used a graphics calculator connected to the overhead projector as a means of displaying graphs, while the students worked in their groups with their individual calculators. The students appeared to be at ease working with the calculators, as was I, and students who had forgotten how to use them were given instructions from other students in their group, with no urging from me. Progress was slower than I had anticipated, and we were able to graph only the first problem, a problem that involved large coefficients. The students suggested two ways of fitting a useable graph onto the screen; either by dividing the data by 1000 and keeping the original screen, or by changing the domain and range. The students seemed comfortable with the idea of idea of resetting the calculator in order to change the domain and range.

Using the graphics calculators today gave me a sense of "differentness" about the lesson that picked up my enthusiasm.

Lesson Calc 2. The homework had been to determine a suitable domain and range for each of the remaining three problems, and the students had difficulty with the assignment, consequently we spent the first 15 minutes of the class discussing domain and range. In order to promote the students' attention during the discussion, I did not hand out the graphics calculators until the discussion was over. The students were attentive during the discussion and continued to be on task, working with the calculators, for the remainder of the period. Even though the students were working diligently, it was taking longer than anticipated to graph the four functions. Once the students determined a useful domain and range for a function, the graphics calculator filled its intended role by quickly and accurately providing a graph of the function.

As the students worked on the idea of changing the scale on their calculators in order to obtain a complete graph, many of them asked the same question, "Why did the graph look one way, then later (after changing scale in the calculator) it looked different but the equation is the same?" This question provided the basis for an interesting class discussion.

The third problem, the bullet question, was their first encounter with a parabola using the graphics calculator. The equation, $h(t) = 0.44t^2 - 39.7t + 1039$, caused problems for some students. Some ignored the question entirely, while others did not change the scale adequately with the result that no graph at all showed on their screen. For those who persisted in changing scale and did obtain a complete graph, a common question was, "Is it supposed to look like this?" This question provided a good lead into a review discussion of what a graph represents. After all students had a correct graph on their screen, their comments and questions indicated confusion between the shape of the graph and the path of the bullet, suggesting they still do not fully understand what a graph represents.

As the period progressed I found myself spending what seemed to be an inordinate amount of time taking notes about what the students were doing, and I wondered if I was neglecting my teaching.

Lesson Calc 3. As the students filed into the classroom, they automatically got into their groups, picked up their graphics calculators, and began working. This enthusiastic attitude continued all period with the students remaining on task. There appeared to be no technical problems with respect to the students' operation of the calculators as they finished off graphing the four functions given in the first period, and began to discuss their conclusions with respect to the different equations and their shapes.

Toward the end of the previous period, a few students plotted an equation without changing the domain and the range, with the result that no graph appeared on the calculator screen. In order to discuss this problem with the entire class, I used the overhead graphics calculator and worked through the question by having the students suggest how they had attempted to graph the equation. As I worked on the overhead calculator, students worked on the same problem on their own calculator, with the result that all students were participating in finding a solution to the problem of how to obtain a correct graph and all were involved in the discussion.

Number scales do not appear on the graphics calculator, but this did not seem to be a problem as all students wrote the scales in their notebook when they copied the graph (as they had been instructed to do). One problem that did occur as students copied the graphs into their notebooks was the compulsion of a few to plot too many points when all they were trying to do was retain a neat, approximate sketch that emphasized the shape of the graph and its location on the coordinate plane. Some of these students stated that they were accustomed to plotting many number pairs when drawing graphs and did not feel comfortable unless many points were present, so I had to spend some time with these students emphasizing the value of approximating in general.

"Why does one parabola go up and one go down?" This question was asked by several students today as they looked for conclusions from the four graphs done thus far, and it formed the basis for a useful discussion.

Lesson Calc 4. In today's class we discussed the definition of a quadratic function, then spent the remainder of the period working on questions from the text. The students worked in their groups, and I wandered about the class checking on the progress of the various groups. As common problems arose, I answered them on the board or on the overhead graphics calculator, whichever was the most appropriate method for the question raised. As the students worked their way through the assignment, one student asked, "Is there a way to do this (solve a problem by graphing) without using the calculator?" I thought this was an insightful question until I discovered that she was having difficulty getting a correct graph with her calculator and was only looking for a faster way to do the question.

I had to omit three of the eight questions from the assignment because some questions were taking longer than I anticipated. However it should be noted that when I planned the lesson I had inserted more questions than I would have for a non-graphing device class because I thought the graphing devices would allow for a greater speed in doing questions, an anticipated benefit that had not yet materialized.

Lesson Calc 5. After giving students an opportunity to ask questions about last days' work (there were almost no questions), the students wrote a quiz to see if they were able to obtain a useful graph for a given function by changing the domain and range and to read information from the graph. The students worked individually and had the use of a graphics calculator. They were also asked to indicate if they answered the question about maximum and minimum by using the graphics calculator or by using algebra, and a heavy majority indicated they used the graph generated by the calculator. The results of the quiz were very good, but many of the students who used the graphics calculators gave answers that contained too many decimal places .

The remainder of the class time was spent introducing Topic 2 by having each group draw a graph of a "real-world " problem on a 1m by 1m piece of paper . Each group was given a different problem and asked to draw a correct graph for that problem. No instructions were given about whether or not to use the graphics calculator, but all groups used it to first obtain a graph that seemed to them to be correct, and then they used that graph as a model to draw the larger one. The groups were able to produce their large graphs much more quickly than I had anticipated by using the graphics calculators.

Topic 2: Other Types of Functions

The objectives were to make students aware that there are many other functions whose number pairs form graphs different from those studied thus far and whose equations have a different form from one another; and to relate the form of eight different types of equations to their graphs.

Lesson Calc 6. Work on this topic had started toward the end of the last period when each group was asked to produce a large graph of one particular function. At the beginning of today's class each group taped its large graph on the wall, and the majority of the period was spent discussing the shapes of these graphs and the domains and ranges that had been selected. The class was attentive and involved during the discussion (the graphics calculators had not been handed out) and as the graphs were discussed I gave the groups a mark for their graph. All groups had used the graphics calculators to obtain their graphs, and all groups had the correct shape for their particular graph, however four of the nine groups had domains and ranges that were unreasonable. For example, one group placed a 500 kg man and then a -200 kg man on the end of a diving board, another group had a tidal wave 500 m high hitting a beach at a speed of 250 km/h, while a third group was investing money at 50% interest.

For the remaining ten minutes of the class the graphics calculators were handed out and the students were instructed to obtain and record in their notebooks graphs of eight different types of equations. With the next mathematics period for this class two days away, students were informed they could come in after school or on their breaks if they wished to borrow a graphics calculator in order to complete the assignment. Eight of the students stayed behind after the class to continue working with the calculators.

<u>Lesson Calc 7</u>. One-third of the class had not completed their homework because they had not come in to use the graphics calculators and felt that if they did not have such a calculator they could not do the graphs. The students were displaying a perception similar to my early one and that is that if a graphics calculator is not available, a question can not be done. I had previously changed my attitude and I spent a few vigorous minutes trying to change theirs. In order to correct the homework that had been done, students went to the board and neatly sketched their graphs, so that we soon had all eight graphs visible. Once all the graphs were on the board, I was able to use them to explain certain features of each graph to the class. I could have used the calculator and the overhead to demonstrate the correct graphs, but the board graphs were much larger and it was easier to explain features such as asymptotes on a large graph.

At the end of the period students were beginning to work on an investigation sheet for Topic 3 using the graphics calculators.

Topic 3: Graphing $y = f(x) + q$

The three main objectives of this topic were to examine the graph of $y = x^2$; to determine how $y = x^2 + q$ differs from $y = x^2$; and to draw conclusions about how any of the basic eight functions graphed last class were affected by adding a constant q, that is how $y = f(x) + q$ differs from $y = f(x)$.

Lesson Calc 8. The discussion regarding vertex and axis of symmetry for a parabola of the form $y = x^2$, based on the investigation the students had started last class, went very quickly, with the students having no trouble with the concepts. The students continued with the investigation sheet, and when they saw the graphs of $y = x^3$ and $y = x^3 + 4$ on their screens a common reaction was, "Oh neat!" Some students did comment that they were having difficulty determining if the graphs of $y = x^3$ and $y = x^3 + 4$ were the same size and shape because of the small screen on their graphics calculators. I had planned to show some of the graphs from their assignment on the overhead using the TI-81 overhead calculator, but as I looked at the good quality of the students' work I decided that was not necessary and I simply sketched large graphs on the board for the few questions they asked. I sensed that I was becoming more comfortable with when and how to use the graphics calculator overhead device. The students remained on task working on the investigation sheet all period, which was significant as today's class happened to be a potentially distracting combination of last period Friday and Valentine's Day.

Topic 4: Graphing $y = f(x - p)$

Students were to sketch the graph of $y = f(x - p)$ without the aid of a graphing device by translating the graph of $y = f(x)$, where $f(x)$ is any one of the eight basic functions studied thus far or any other function whose graph is given.

Lesson Calc 9. The students successfully did a quiz today in which they were asked to sketch a graph by first drawing one of the eight basic graphs and then translating it without using the graphing calculators. The students were involved the remainder of the period working in their groups on an investigation activity that had them exploring the graphs of functions of the form $y = f(x - p)$. The use of the TI-81 led to an unexpected discussion as students graphed $y = \frac{1}{x-3}$ and the calculator showed a two-part vertical line between the branches of the hyperbola. The students were curious about this part of the graph and consequently we entered into a discussion about why this happened and how this calculator plots its points, and ultimately into a discussion of why we could also have a problem when we try to interpret the graph at $x = 3$.

Lesson Calc 10. The students' homework assignment, one that did not require the use of a graphing device, caused no problems. The work done in class today, which was questions from the textbook and from a supplementary sheet, was aimed at giving the students an opportunity to apply the concepts they had discovered last class. "Can we check with the calculator ?", was a common question today, and that seemed to be the primary function of the graphing calculator this period as the students used them to verify that the graphs they had sketched by using their transformation rules were correct.

Some students, in using a calculator to graph $y = \frac{1}{x^2+1}$, accidentally graphed $y = \frac{1}{x^2} + 1$ and questioned their results. They discussed the problem in their group and came to the realization that the position of the constant made a significant difference to the graph. Other students were finding their graphs were too small and demonstrated relative ease at changing the scale in order to make the graph more useful.

Topic 5: Graphing $y = af(x)$

Students were to sketch the graph of $y = af(x)$ by stretching or compressing the graph of $y = f(x)$ without the aid of a graphing device.

Lesson Calc 11. For the first part of the class we found we did not need to use the calculators as we were going over the homework, much of which included questions from a supplementary sheet. These questions were obtained from *Pre-Calculus Mathematics - A Graphing Approach* by Demana and Waits (1990) and were intended for students who had a graphing device at their disposal. Most of the questions asked students to apply what they had learned more than the questions from the student text did, but did not require a graphing device to obtain an answer. The questions were also of a different nature than the ones in the students' text and were more relevant to the technological situation of the students in my class. For example one of the questions gave us (3,4) as a point on the graph of $y = f(x)$ and asked what point must therefore be on the graph of $y = f(x)+2$? Although the students had no trouble shifting a graph, this type of question caused a certain amount of difficulty. This particular set of questions was applicable for the remaining topics in the unit.

Today's lesson revolved around graphing $y = af(x)$, and for the first time I put a time limit on the investigations in an attempt to make sure a certain amount of content got covered. I had found that it was too easy to allow students to "play" with the graphics calculator, which is not necessarily without value, but I was starting to feel some pressure about covering the curriculum. As students worked in groups on their investigations, most got the idea of a negative value for a causing the graph to flip over the x axis, and of $|a| \geq 1$ making the parabola "thinner", but most missed the idea of the y values changing by a factor of a. As I observed the students working I noticed that when they used the cursor to determine the coordinates of points, they got values with so many decimals that it was difficult for them to make any comparisons among the y values, and as a result they found it difficult to draw conclusions about the magnitude of the transformation. I had suggested they find y values for $x = \pm 1$ and $x = \pm 2$, however with a standard scale x does not equal exactly 1, consequently there were a confusing number of decimals shown on the calculator, whereas if $x = 2$ was used at least the x values contain no decimals, which reduces the confusion.

Lesson Calc 12. Today the students continued working on questions from their text and questions from the supplementary sheet that pertained to the conclusions reached last class, and although the graphing calculators were available, the students found they were not needed.

Topic 6: Graphing $y = -f(x)$ and $y = f(-x)$

In this section the objective was to clarify the difference the placement of the negative sign makes to the graphs of functions of the form $y = -f(x)$ and $y = f(-x)$ when compared to $y = f(x)$.

Lesson Calc 13. "Can we get a calculator? I want to check something." This comment came from a student after she had been working for a few minutes on the first part of her investigation sheet which asked her to speculate about how how the graph of $y = f(-x)$ would be related to the graph of $y = f(x)$. She had done some reasoning in her mind and wanted to use the calculator to verify her suspicions.

As the students worked farther into the investigations, they were required to graph some specific equations, and several students asked, "Should we graph the original as well?". I had neglected to include this instruction in my directions, but it made immediate sense for the students to have a graph to compare the new one to so I instructed the class also graph $y = f(x)$ for each new question.

"Whoa!" exclaimed a student after he graphed $y = (-x)^2$ on his graphics calculator. He had expected to see a different graph from the one that appeared on his screen. After he thought about it he realized that the graph appeared as it did because $y = x^2$ is symmetric about the y axis, but seeing the graph quickly appear in front of him made him stop and think about what he thought should have happened and what actually happened.

Students were asked to first sketch their guess as to where the graph of each new equation on the investigation sheet would be, then to use their graphics calculator to check their guess. Some students followed the instructions, but others said they just wanted to think about the position of the graph and not sketch it because they did not want to sketch an incorrect graph.

One student asked for help with the graph of $y = (-x)^3$ because the graph of this function seemed to flip but the graph of $y = (-x)^2$ did not. I asked him if he thought the calculator had given him a correct graph and he replied: "Yeah, (pause) of course." He indicated that he believed the calculator, but still did not understand why it gave him the answer it did. I attempted to give him an algebraic explanation, but I am not sure I was successful.

Another student who was graphing $y = (-x)^3$ and $y = -x^3$ also tried graphing $y = x^{-3}$ and was puzzled by the resulting graph, consequently we had a concentrated discussion about the graph of $y = x^{-3}$.

The students finished the period by doing questions from their textbook and their supplementary sheet, questions that did not require the graphics calculators.

Lesson Calc 14. Most of this period was spent discussing last day's homework and having the students successfully write a quiz on the content of the last class. In the last five minutes of the class the students began investigating next day's work. The graphics calculators were not needed for any of today's work, and the students did not ask for them.

Topic 7: Graphing $y = af(x-p)+q$

Students were to combine the conclusions from the previous lessons about the effects of the constants a, p and q on the graphs of functions of the form $y = af(x-p)+q$ in order to sketch, without the aid of a graphing device, graphs of these functions by transforming a basic graph of the form $y = f(x)$.

Lesson Calc 15. The class was occupied all period working in their groups on the investigation sheet for this topic. The essence of the investigation was to determine in which order or orders to do the transformations in order to obtain the correct graph. The calculators were not used at all because the students were expected to do each individual transformation according to rules learned earlier. I had originally planned to use the calculators to check the students' graphs, however as this was an assignment to be handed in next period and I wanted to be sure all students obtained their answers without using the calculators, I altered my original plan and asked them to check their final graphs by taking some number pairs from their graph and checking to see if those numbers satisfied the equation. Most of the class found this particular investigation to be difficult, and the difficulty had nothing to do with not being able to use the calculators, rather the problem was in trying to organize their work and to form some conclusions.

Lesson Calc 16. We discussed the students' conclusions with regard to last day's assignment, and several students showed their work on the board because some of the them had some excellent solutions that were different from one another's. Once we had finished discussing the conclusions, the students involved themselves with the questions in their textbooks, questions that did not require the use of graphing calculators.

Topic 8: Review

Students were to review the concepts of Topics 1 to 15 in preparation for a unit test for which they will not be permitted to use a graphing device.

Lesson Calc 17. The students spent the period doing review questions from their textbooks. They did not use graphics calculators since they could not use them on the test. However, one of the questions was a "real-world" problem that they could only solve if they graphed the equation for the problem. All of the students who tried this question were unable to solve it because they did not think of the strategy of using a graph (and the question did not instruct them to graph). Once we discussed the possibility of solving it by graphing, students obtained a calculator, graphed the problem and solved it.

Topic 9: Unit Test

The objective of this test was to determine to what level the students had met the objectives of Topics 1 to 7 of this unit.

Lesson Calc 18. During this period the students wrote a unit test without the aid of a graphics calculator.

Topic 10: Maximum-Minimum Word Problems

The students were to solve a maximum-minimum word problem for which the equation was given by using a graphing device to obtain a graph for the problem and then reading the appropriate information from the graph.

Lesson Calc 19. The class started with a discussion of last day's test, then we continued on to the work on maximum-minimum word problems. To introduce the topic I gave the students a problem that asked them to determine a maximum revenue, but gave them no guide-lines as to a method of solution. Most students set up a table of values for the problem and obtained the correct answer. I then asked them whether there might be another method of solving the problem, and the idea of writing an equation was suggested. After we derived an equation for the problem we decided to use the graphics calculators to get a graph of the equation. Most of the class remembered they needed to change the domain and range on the calculator in order to get a meaningful graph. Once all the students had complete graphs on their calculators we compared the number pairs on the graph with those from their earlier table of values, and realized how we could solve this type of problem from a graph.

Lesson Calc 20. We started the period by reviewing what we had concluded last class and then looked at another example of how to read the solution to a problem from its graph. The remainder of the period was spent solving a few word problems from a supplementary sheet that gave the students the equation for the problem and asked them questions related to the maximum and minimum of the function. The students solved the problems by graphing the equations with their graphics calculators and interpreting the graphs. Unlike during the first few classes of this unit, the students had little difficulty obtaining a complete graph for the various equations. Their strategy for obtaining a complete graph was to enter the equation, graph it, then look at it to see what changes must be made to get a complete graph. This trial and error technique was easy for them to do and was surprisingly fast. For each question, the students sketched in their notebooks the graph they obtained with their calculators, indicating the scale on each axis and the general shape of the graph in order to have a record of how they got their solution to the problem.

Lesson Calc 21. The students' homework was to set up labelled coordinate axes for six word problems from their problem sheet, and to guess at what reasonable domains and ranges might be. Some students had difficulty, as they had earlier in the unit, but once they got a graphics calculator in their hands they were able to get a complete graph with no trouble. One student, who, when doing her homework, had wanted to have correct domains and ranges for all six of her questions found number pairs for all problems and actually ended up constructing relatively accurate graphs for each of the problems and solving all the problems. When she came in to today's class she asked for a graphics calculator with which to check her graphs. Her graphs were correct but she agreed that the calculator was much faster and that she would not repeat her method again.

The students continued to use the calculators during the remainder of the class in order to solve more of the problems from the sheet. I did not assign any homework because what we were doing relied so heavily on the calculator, and I felt a repeat of last day's type of homework assignment was not a productive use of the students' time.

Lesson Calc 22. The plan had been to start the lesson on circles today, but because students were unable to do any homework questions on maximum-minimum word problems due to their not having access to the graphics calculators, more time was spent on this topic today. One student asked, "Will we have a TI-81 for the test? I thought questions 5 to 9 were for homework and I tried one with my normal calculator and it took forever and I still didn't get it. It is totally hard (without the TI-81)." I reassured the student that any questions on the test requiring a graph would contain the graph, and that questions such as the ones they were doing now would be tested via quizzes, if they were to be tested. Some students who finished the problems early were given two challenging bonus questions related to the topic under discussion.

Topic 11: Graphing Circles

The students were to recognize that equations of the form $x^2 + y^2 = r^2$ determine a circle with center $(0,0)$ and radius r, and were to sketch the graph of the circle without using a graphing device.

Lesson Calc 23. The topic of graphing circles was introduced without using the graphics calculators, and the questions in the textbook related to circles were such that the calculators were not needed for them either, consequently the calculators were not made available today. After the students had finished the questions in the textbook, they were given a short quiz on the material and they showed that they understood the concepts.

Topic 12: Graphing $(x-p)^2 + (y-q)^2 = r^2$

Students were to sketch the graph of $(x-p)^2 + (y-q)^2 = r^2$ by translating the graph of $x^2 + y^2 = r^2$ and were to determine that the center of the circle is given by (p,q).

Lesson Calc 24. The students were to explore this topic by following an investigation sheet that asked them sketch their guesses to the graphs of several equations of the form $(x-p)^2 + (y-q)^2 = r^2$ by using the rules they had learned in the previous unit in order to shift a graph of the form $x^2 + y^2 = r^2$. A couple of students asked if they had to solve the equation for y before they could use their graphics calculator, and since the answer to that question was "Yes" I showed them on the blackboard how to accomplish that. The students demonstrated a lot of interest during the explanation.

I used the calculator attached to the overhead projector to go through the various "tricks" of the TI-81 in graphing this type of equation, including the use of the "Y-Vars", "Zoom 5" and "Box" functions. I enjoyed demonstrating these aspects of the calculator to the students, and they were interested in them and adapted to them fairly well. There were some errors in some of their graphs due to a faulty entering of data, but these were problems that were easily remedied with an explanation from me.

It took the full period for the class to do the four examples on the investigation sheet because of the time it took to master the techniques of using the TI-81. The students enjoyed the time it took, though, with considerable "oohing" and "aahing" as the graphs appeared on their screens.

<u>Topic 13: Graphing</u> $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = r^2$

The students were to sketch the graph of an equation of the form $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = r^2$ by stretching or compressing the graph of an equation of the form $x^2 + y^2 = r^2$.

<u>Lesson Calc 25</u>. The first 15 minutes of the period were used to discuss the conclusions from last period's examples of translating circle graphs. I had originally planned to have the students work on a textbook assignment related to the translating topic, but changed my plans when I realized that those questions did not require graphics calculators and therefore could be done at home, whereas today's new topic of stretching and compressing circle graphs did require the calculators. Consequently I gave the students the investigation sheet for the new topic in which they were directed to first guess at the graph for an equation of the type $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = r^2$, then to use the graphics calculator to check their graph to see if they were correct.

The students did not appear to have any difficulty today changing the equations to make *y* the subject, but the intricacies of the TI-81 resulted in the students graphing only two or three of the equations on the investigation sheet. One of the factors slowing down work with the TI-81 is the awkwardness of finding number pairs for the graphs in order to compare them to see the effect of a stretch or compression. After we all played with the "Box" function for a couple of questions we decided it would be more efficient and accurate enough if we used the cursor to get the coordinates from the calculator, rounded those numbers to the nearest digit, checked those numbers to see if they satisfied the equation, then worked with those numbers. This method was faster and not as awkward as using the "Box" function to determine coordinates, and proved to be of adequate accuracy.

Although we did not finish the investigation sheet, I assigned the remaining questions from the ones we had started at the beginning of the period for homework, and planned to continue today's investigation next period.

Lesson Calc 26. The students worked in their groups with the graphics calculators to finish last day's investigation sheet on stretching and compressing. The activity in the groups was on-task, with the students helping one another to get the graphs and determine points on the graphs, but they did have some difficulty trying to put their conclusions into words. One of the topics of discussion in the groups revolved around how to change the equation from its given form into one in which *y* was the subject, an excellent algebraic exercise, and a topic I had worked on briefly with them a couple of periods previously. Eventually we discussed the conclusions about the stretching and compressing as a class, and the students proceeded to do some non-calculator questions from their textbook.

Topic 14: Graphing $\left(\frac{x-p}{a}\right)^2 + \left(\frac{y-q}{b}\right)^2 = r^2$

The students were to sketch the graph of an equation of the type $\left(\frac{x-p}{a}\right)^2 + \left(\frac{y-q}{b}\right)^2 = r^2$ by applying the rules of transformations of graphs in the correct order to the graph of an equation of the type $x^2 + y^2 = r^2$ without the aid of a graphing device.

Lesson Calc 27. The original plan had been to have the students working in their groups and to have each student write down all the possible orders of transformations for each equation, then use the graphics calculator to determine which of the orders was correct. But as they were working I decided I wanted to move through the topic more quickly (I had a feeling that this unit had taken enough time) so after everyone had written down only one order and checked it with their calculator, I asked those students who had used an order that gave them a correct graph to write their particular order on the board. We then used those lists on the board to arrive at a class conclusion about the correct order of transformations.

One student asked, "What if the equation was $x^2 - y^2 = 16$ instead of $x^2 + y^2 = 16$?" I suggested he answer the question by graphing the new equation with the graphics calculator, which he did, and when he saw the resulting hyperbola his comment was, "Cool".

Topic 15: Graphing Quadratic Inequalities

The students were to sketch the graphs of quadratic inequalities in two variables of the form $\left(\frac{x-p}{a}\right)^2 + \left(\frac{y-q}{b}\right)^2 \le r^2$ by transforming the graph of $x^2 + y^2 \le r^2$ without the use of a graphing device.

Lesson Calc 28. Today's topic was covered without using the graphics calculators by relying on the conclusions learned in previous lessons with regard to the transformation of graphs. We also discussed the topics that would be on the unit test they were to write the following period, and they worked on an assignment that covered both today's new work and the review.

Topic 16: Review

The students worked on review questions as part of Lesson Calc 28.

Topic 17: Unit Test

The students were to write a test, without the aid of a graphing device, that would determine to what degree they have learned the concepts in Topics 10 to 15.

Lesson Calc 29. The students wrote their unit test without using the graphics calculators.

Evaluation Questionnaires

All students involved in the study were given a written questionnaire at the conclusion of the units in which the technology was used in order to examine their opinions about learning mathematics with the graphing devices they had been using. The students were given the questionnaires after the unit tests on the chapters relevant to the study were completed, marked, returned and discussed. The questionnaire had two parts, the first part having nine questions (eight for the overhead class) in which students were asked to indicate on a Likert scale from 1 (strongly disagree) to 5 (strongly agree) their opinions about a given statement. The second part asked for written responses to four open-ended questions. The questionnaires were administered to the students at the beginning of one of their regular mathematics classes, and they were informed that their responses were anonymous. Students were given unlimited time to complete their responses, with most students taking about 15 minutes.

Questionnaire - Part I: Summary of Responses - All Classes

The questions used on part 1 of the questionnaire are shown in Table 2, and the arithmetic mean of the responses (1 indicated strongly disagree and 5 indicated strongly agree) for each of the three classes is indicated in the columns on the right. (OP: Overhead Projector class; LAB: Computer Lab class; CAL: Graphics Calculator class.)

This part of the questionnaire was modified for the computer lab and overhead projector classes by replacing "graphics calculators" with "computers" in each of the questions. In addition, item 9 was omitted from the overhead projector class questionnaire as they had almost no experience actually operating the computer.

Questionnaire - Part II: Written Responses

The open ended portion of the questionnaire contained four questions requiring written responses, and was the same for all classes (with the exception of the substitution of "computer" for "graphing calculator"):

1. What did you like most about using a graphing calculator to assist you in learning mathematics?
2. What did you like least about using a graphing calculator to assist in learning mathematics?
3. What would you change in the way the graphing calculator was used?
4. Other comments?

The comments written by students on the open-ended component were analyzed by listing each of the different responses, then determining the frequency of those responses to try to determine some common themes among the students' opinions. Some of the more

Table 2

Arithmetic Means of Student Responses to Part 1 of the Questionnaire

	OP	LAB	CAL
1. When I was told, before we started the units in which we used the graphing calculator, that we were to be using one for the next units of mathematics, I believed using the calculator would:			
a) help me improve my mark in mathematics.	3.1	3.2	3.3
b) make mathematics easier for me to understand.	3.8	3.3	4.1
c) make mathematics more enjoyable.	3.5	3.7	4.0
2. As a result of my use of a graphing calculator in this course, I believe this equipment should be used more in the teaching and learning of mathematics.	4.1	4.0	4.1
3. I feel I could learn mathematics just as well without a graphing calculator.	2.7	2.9	3.1
Answer only one of number 4 or number 5. If you had previously worked with graphing calculators, answer number 4. If you had no or very little prior experience with them, answer number 5.			
4. Before starting these recently completed graphing calculator-aided units in mathematics, I enjoyed working with graphing calculators.	3.8	3.8	4.5
5. Before starting these recently completed graphing calculator-aided units in mathematics, I felt threatened by graphing calculators.	2.4	3.4	2.2
6. I would like to do more work with graphing calculators.	3.5	3.7	3.8
7. I feel my marks in mathematics would improve if we were able to use graphing calculators for more chapters of the course.	3.4	3.3	3.4
8. After finishing the units of mathematics 11 in which we used the graphing calculator, I feel that using these calculators:			
a) helped me improve my marks	3.3	3.1	3.2
b) made it easier for me to understand the concepts of mathematics studied in this chapter	4.0	3.7	4.0
c) made mathematics more enjoyable.	3.7	3.7	3.8
9. I found it difficult to learn how to operate the graphing calculator in order to do the mathematics.	NA	1.8	1.8

prominent replies given by each of the three classes are summarized on the following pages.

1. Written Responses - Overhead Projector Class (n=29)

In answer to the question of what they liked most about having the computer and overhead projector in the classroom, the students gave many different responses, of which two were common to many of the students. Their most common reply was that this technology made it easier and faster to do the graphs and to analyze them (n=18). Their comments included, "It was more accurate in very little time" and, "It was clearer, not so tedious." The other common reply was that using this equipment made it easier to understand and made answering questions related to graphs easier to do (n=11). They wrote comments such as, "I could see the problem", and "the computer gave me a concrete picture that made the problem easier to understand."

With respect to what the students liked the least about using the overhead projector and the computer, the biggest single complaint was that they were not involved enough which resulted in those students being bored (n=6). They stated that, "It wasn't individual. We watched and copied down the answer." Another major problem in the students' minds was the software we used. Some complained that the scales were hard to read and that the teacher had to read out the numbers (n=4), while others indicated dissatisfaction with the accuracy of the computer graph, specifically mentioning that you could not be sure if a particular point was on the graph (n=3). A final common criticism about this mode of technology was that "...the computer made getting the graph look so simple, but I couldn't get the 'why' " (n=4). They commented that they became too reliant on the computer and might have learned better by hand. They suggested that repetition of graphing by hand may have made them "more familiar" with the graphs. "I felt that by using the computer we were taking a shortcut and that people might better comprehend the problem by graphing themselves" was one student's reaction.

In reply to what they would change, the favorite response was that they would like to be able to use them individually (n=9), while several said they would not change anything (n=8). A few students (n=3) suggested that if individual access was impossible, then it would be a good idea to allow the students to take turns coming up to the front of the class and using the single computer, as this would make the class more interesting and would give students not familiar with computers a new perspective.

2. <u>Written Responses - Computer Lab Class (n=28)</u>

In answer to the question of what they liked the most about using the computer lab, the students' prime response was that it made learning easier (n=13). They commented, "I could visualize", and "it was easier to learn because I could do more questions in class." Another widespread comment (n=12) linked to the previous one was that the computer allowed them to obtain and see the graphs much faster and easier with much less work, which took away the boredom of drawing graphs and saved frustration. The students wrote that they liked the fact that they "didn't have to do the graphs from scratch" and "this (the computer) simplified the work and left more time for solving problems." Other shared comments about what they liked the most about being in the computer lab included that it made learning more enjoyable (n=4), that they found the change in environment made the class more interesting (n=4), that they liked working with a partner (n=3), and that they could check their answers and therefore know when they were right (n=2).

On the negative side, they responded that what they liked least about using the computers individually was that sometimes the software was confusing (n=8). They cited examples such as needing to know what type of equation they were dealing with before being able to graph it, and the software showing different forms of the same equation.

Another common source of dissatisfaction was not being able to use the computer for the chapter test (n=6). Several students (n=5) also indicated that they thought that because they were using the computer to do the graphing, they were being denied the opportunity of doing the graphs themselves, and hence their learning was suffering. These students wrote comments such as, "It didn't allow me to do repetitious work which I think helps me learn"; "It did everything for me and I didn't learn"; "It is easier for me to see if I had done it myself"; and "Because I didn't do the graphing I lost interest and therefore did not learn as well."

In response to the question of what they would change about using the computers in the lab, the only common response was that a better software program is needed (n=6).

3. Written Responses - Graphics Calculator Class (n=24)

The class using the graphics calculators indicated that the feature they liked most about using this type of calculator was that it was faster and easier to get the graphs (n=12). They wrote comments such as "...less painful than making out a table and plotting the graph myself"; "...because it was faster the teacher had more time to explain"; and "...experimenting on graph variations was less painful than hand plotting because I could manipulate the graph faster." Almost as many students (n=9) commented that these calculators made it easier to understand what they were doing, with comments such as, "it helped me see what different shapes went with different equations." Several students (n=7) stated that they liked the picture that the calculator gave them that showed them exactly what the graph looked like, because, as one student explained, "I could see what effect changing a number or a bracket had on the graph." Other comments included stating that they made mathematics a bit more interesting (n=5), indicating that it provided a variety in the learning process (n=3), and stating that they were useful as a check for hand generated graphs they had done (n=3).

The most common reaction to the question of what they liked least about the graphics calculators was that they could not use them at home (n=5). They also complained (n=5) that they could not use the calculators on the chapter tests, with one student stating, "I didn't understand why we were using them all the time in class if we couldn't use them on tests." Another student echoed this view as he wrote, "You become dependent on a calculator, then on a test it isn't there, and this is frustrating. It was foolish to spend so much time graphing with calculators when we could have been doing paper and pencil like on the test. It made the test harder."

In response to the question of what they would change, half of the students (n=12) stated they would change nothing, while some (n=5) mentioned that they would like to have had more instructions on all functions so that they could understand the calculator fully. A few students (n=3) expressed the view that it they should be used a lot less, that in fact the extensive use of the calculators was boring.

Questionnaire - Part III: Interviews

The two students interviewed from each of the three classes were asked a series of questions that were intended to explore further the reactions of the students to using graphing devices as indicated by their responses to the two sections of the written questionnaire. The actual texts of the interviews are too long to include in this paper, however some of the central themes and significant comments made are included here. Some of the remarks made by students in the taped interviews will be combined with the information from the written questionnaires to support arguments and conclusions presented in Chapter Five. The reports of the interviews are presented one mode of technology at a time, with the comments of the two students from the Overhead Projector class first, followed by those of the two from the Computer Lab class and concluded by the thoughts of the pair from the Graphics Calculator group.

The first student interviewed was Cheryl, a Grade 12 student who was ranked number ten in the Overhead Projector class with an average of 79% at the time of the interview. Cheryl's comments were particularly interesting because she had taken the Mathematics 11 course the previous year (with a different teacher) and had passed it but was repeating it in order to upgrade her mark. The other student interviewed from the Overhead Projector class was Tracey, a Grade 11 student whose 63% average placed her twentieth in the class. The first question I wanted to ask each student was how using their form of technology had made learning easier, as this had been one of the common comments on the written questionnaires. Cheryl thought it was easier because it was more accurate and it was quicker, while Tracey liked the idea that you could see a picture of what was being explained to you.

Both students were emphatic in stating that they wished they could have learned more about how to operate the computer so that they could have gone to the computer room and worked on their own, although neither thought that many students would volunteer their free time to go to the computer room to work on mathematics. When asked if having a single computer in the room was better than no computer at all, both agreed it was, with Cheryl commenting that last year (in her class without the computer) it seemed to take so long to get the graphs. Tracey added one is better than none, but it would be a good idea to take one class and teach people how to use the software so that those who wished to could go to the computer room on their own. The two girls agreed that although a single computer was better than none, it would have been more enjoyable if the students could have worked at their own computer. To explain why, Cheryl admitted, "To understand it better, because when you were doing it in front of the class not many people were paying attention and if we did it ourselves we would understand how to do it and why we did it this way." When she was asked why people were not paying attention, she continued, "because we were just waiting for you to put it up. If we were doing it ourselves we would see how to do it, why we did it this way." When I asked Tracey if she also found this waiting time boring, as Cheryl indicated, she stated "No I didn't think so because we were all sitting there waiting to see what would happen and to see if what we had figured out was right. We were sitting in groups and if two of you thought it would do a different thing then you all kind of sat there in anticipation and then it went up - it was kind of neat." Tracey did echo Cheryl's comments about wanting to be able to manipulate the device herself, and added, "...and then you can experiment and you can say, 'OK I'm going to try this, or maybe I can try this and I can see what happens when I try this and this and you can take it a little further.'"

When asked the question about whether they were concerned about not having the computer for their unit exam, both agreed that by the time the unit was completed and they were preparing for the test, it was not a problem, although Cheryl did admit to being concerned at the beginning of the unit.

One of the questions that I was particularly interested in was whether the students liked the idea of drawing their own conclusions from their observations. The two girls agreed totally that they preferred to figure things out themselves, or as Tracey put it, "...when we had to come up with our own formulas you remember that formula so it's already memorized but then when you just tell it to us it's there and we try to memorize but it's not like we knew how we got to it...you remember it more when you do it yourself because you know it, because you did it."

A question that was especially pertinent to this class was whether, given a new type of equation to graph, they would prefer to graph it by hand and use the computer to check their graph, or let the computer do it to begin with. Cheryl was not really sure, that maybe it was a bit of both, while Tracey was firm in stating that she preferred the computer to do it first because she felt that she makes mistakes when plotting new graphs and she wanted to see the correct one right away, then she could try to reason out why it was where it was.

About the question of the effect the computer might have had on their marks, they both stated that they thought their marks were higher because of the computer. Cheryl compared it to last year and speculated that because she could see the graphs this year, graphs she knew were correct, she was able to learn better. Tracey stated that her marks improved because the style of learning we used with the computer gave her the opportunity to learn many of the concepts on her own, a learning style she is convinced helps her to learn more.

The students selected for the taped interviews from the Computer Lab class by the random number process again represented both Grade 11 and Grade 12 students. Anthony was a Grade 11 member of the Computer Lab class and ranked eighth in the class with an average of 76%. Kristen, a Grade 12 student who had taken Introductory Mathematics 11 in her grade 11 year, was also in the Computer Lab class, and ranked twentieth with an average of 61%. The first question both students were asked was how using the computer made learning easier, but both had trouble with the question although both did indicate that being able to look at the pictures (graphs) made it easier to recognize the graphs. In reply to the question of whether they experimented with the computer at any time, they gave diverse answers. Kristen replied that, "I made pictures, usually with the circles. I didn't do anything mathematical." Anthony commented that, "You did your question that was in the book on the computer, and then you said, 'What if you did this?' and you put in crazy numbers into the questions you had been doing." Both students stated that they did not come into the computer room other than during class time, but neither felt that we spent too many periods using the computers.

Some students suggested in the written questionnaires that it bothered them to be getting behind the "regular" classes, but it did not bother these two students. Kristen's reply was a simple, "It's not my fault," and Anthony commented, "If you do well on tests you don't mind." Another common concern among the students in the class was their perception that not being able to use the computers for the unit exam would hurt them on the test, but these two students disagreed with their classmates. They suggested that it was a concern early in the unit, but soon they realized that the kind of questions they were being asked could be answered without the computer, and, as Anthony put it, "...you saw it on the computer so many times it was just like, well, you knew it looked like this or looked like that."

One of the questions that I wanted to get more student input about was whether the students liked to try to form their own conclusions with respect to the mathematical material being learned. Kristen replied, "It would depend on what it was I guess. In some things I would figure if out myself anyway, and the teacher would just confirm what I thought, or else they would say what I thought was wrong. I think most people try to figure out anything themselves first." Anthony answered, "I like trying to figure things out for myself, because if someone tells you something, yeah okay, but if you actually find something out for yourself then it sticks with you."

Another idea that came out of the questionnaires was that because they were using computers, they were not learning as much. Kristen's opinion was that she probably did not, but said, "...I didn't really care why it happened. That's not really important, I don't think." She indicated that she just wanted to get through the exam. And when she was asked if the computers were restricting her learning, she replied, "What more would I have wanted to learn?" When I suggested that some students indicated that they wanted to know 'why', she responded, "But all I needed to know was what."

Both students thought that using the computer had improved their mark, Anthony because using the computer gave him so many more graphs to look at, and Kristen because the computer gave her correct results instantly.

The two students selected randomly from the Graphics Calculator class presented an interesting contrast. Myles, a Grade 12 student who had taken Introductory Mathematics 11 in his Grade 11 year, ranked twenty-seventh in the class with an average of 43%, while Jenni was a Grade 11 student whose 89% average ranked her number one. As with the other interviews, the first question these students were asked was how they thought using the graphics calculators made learning easier. Jenni had difficulty expressing herself on this topic, and could only suggest that it was easier to get the graphs and when you did get them you knew they were accurate, and finally agreed with my suggestion that perhaps "learning easier" really meant "graphing faster." For Myles, learning easier simply meant it was easier to get the graph, he really did not think he was learning much of anything from the calculator. "For me personally using the calculator made it easier to get the graph but I don't think I learned it," was his comment. I then asked him what he meant by "learned it," and he answered, "I don't think I understood it...sometimes if you work it out on paper you can understand it better than if you use your calculator. If you screw up on your calculator sometimes you don't know if you pressed the wrong button or whatever, but if you screw up on paper you can always look back at the steps if you write things down and see where you are going. I just found it easier to do things on paper." My next question to Myles was whether or not he thought it was worth using the graphics calculators. He responded, "For me personally I think it would have been better for me if I had learned to graph by hand and then used the calculator after I understood everything. You know how in elementary school the teacher would not let you use the calculator until you had learned how to add, well this is the same thing." Myles admitted that he did not spend any time experimenting with the calculator, while Jenni indicated she played with the various graphing functions and some of the other keys, and would have liked ten minutes at the beginning of some classes to keep "trying different buttons."

In response to the suggestion that perhaps six weeks of using the graphics calculators was too long and therefore boring, Jenni was positive in saying no, while Myles thought the opposite. He stated that because he was not interested in working with the calculators, sometimes in class he would do the graphing by hand when everyone else was using a calculator, and if he was given the option, he would opt to do the graphs by hand.

When asked a question that was raised in each of the three classes, namely whether they were bothered by the idea that they would not be able to use their graphing device on the chapter test, Jenni admitted that it did concern her when we started the unit, but as the lessons passed by she realized that the calculators had helped her learn what she needed to know to answer the questions, and that she did not need the calculator for them anymore. Myles, on the other hand, complained, "Yeah, that bothered me, I personally couldn't really see the point, if we were learning how to use the calculator but we couldn't use the calculator on the test."

The next question, "Do you like to draw your own conclusions, or would you rather have the teacher tell you what is going to happen?" resulted in the following exchange with Myles. "I prefer to be told." "Why?" "Because that is the way I've always been taught." "Just by someone telling you?" "Yeah, this is how you do it, and you learn this...there aren't too many classes where you just experiment in, and even math is pretty structured." "Do you think, going back to elementary school, that there should be a change? Do you think it would be better if all through school you be given more chances to get your conclusions?" "Yeah, I think it would have been, from the start, I mean like me personally I've gone through 12 years of learning the same way and you can't make me change, right?" Jenni's reply to the same question was, "A bit of both I guess, as long as what you are trying to find isn't too long so that if you think you are on the wrong path you don't get frustrated."

The observations of the students' and the teacher's behaviors and their thoughts as they progressed through the lessons form the basis of this chapter, and, along with the results of the questionnaires, suggest many interesting questions. In the next chapter these questions will be examined, and, where possible, implications and conclusions that result from these examinations will be offered.

CHAPTER FIVE

IMPLICATIONS AND CONCLUSIONS

In this chapter I examine the effect technology had on student achievement and draw some conclusions about the students' reactions to using technology. I explore the effect the use of technology had on the technologically inexperienced teacher and compare the three modes with respect to determining which of them is "best". I further present the changes I made to the lessons, and the changes I would make next time based on the results of the implementation of those lessons, and suggest some implications those changes offer for planning and instruction. In addition I outline the effects of the use of technology on both students and teacher. Finally, I examine the data from the study in order to present some implications for the technologically inexperienced teacher and to offer some final conclusions that respond to the original questions posed in Chapter One of the study.

Effect of Technology on Student Achievement

One of the questions raised in Chapter One was whether the three modes would have different effects on student achievement. In order to answer this question, scores on tests written during the study (posttests) were compared to tests written prior to the study (pretests), and an analysis of covariance was done using pretest scores as the covariate. A pretest score for each student in a class was obtained by calculating the mean for that student for six chapter tests they had written in Mathematics 11 prior to the beginning of the study. The posttest score for each student was the mean of two tests written during the study. All tests were standard classroom chapter achievement tests and were all created by the same teacher. The graphing devices were not used for any of the tests.

The analysis of covariance results showed a correlation between posttest and pretest scores of 0.74 for the Overhead Projector class, 0.78 for the Computer Lab class, and 0.76 for the Graphics Calculator class. The analysis also showed no statistically significant differences ($p > 0.05$) in posttest scores, leading to the conclusion that the mode of technology used had no differential effect on achievement. It should be emphasized that the statistics were done on scores from paper and pencil tests, that the tests did not measure what students had learned about using the devices nor did they measure students' abilities to use those devices to solve problems related to the general topics.

Students' Reactions to Using Technology

One of the major purposes of this study was to obtain students' reactions to learning mathematics with the aid of various forms of technology. Their reactions, as noted from the Likert scale and written questionnaires, from the taped interviews with selected students, and from my daily written observations, reveal information that leads to some interesting and useful conclusions. The students were unable to agree totally on any issue; there always seemed to be opposing views, although often a large percentage would lean in one direction with only a few dissenting. Often the dissenting views were the most interesting because they revealed a totally unexpected opinion, such as the case of the student who thought graphing calculators were a waste of time and who preferred to do all his graphing in a traditional manner. This was the same student who did not like the investigation approach to learning because it required him to think for himself, and he felt that since no one had asked him to think in his previous 12 years of school, it was unreasonable to ask him to start now.

With respect to the section of the questionnaire that employed a Likert scale to give students the opportunity to rate their responses on a scale from 1 (strongly disagree) to 5 (strongly agree) to several questions, a difference of half a point was deemed to be an educationally significant difference. The discussion that follows treats the three classes as individual populations rather than samples.

Technology as an Aid in Learning

The aspect all classes indicated they liked most about working with computers or graphing calculators was that these devices made learning easier. Students were asked to rate their feelings before and after working with technology about whether the devices they had used had made mathematics easier to understand. All three classes indicated positive expectations, and met or exceeded those expectations. The expectations held by the Computer Lab class, although slightly positive, were lower than those of the other two classes (OP class: before $\bar{x}$ = 3.8, after $\bar{x}$ = 4.0; Lab class: before $\bar{x}$ = 3.3, after: $\bar{x}$ = 3.7; Calc class: before $\bar{x}$ = 4.1, after$\bar{x}$ = 4.0).

The lower ratings given by the Lab class may be attributable to their attitude toward computers. In response to the question of whether they felt threatened by computers before the unit began, the students in the Computer class who had little or no experience with computers (8 out of 29) agreed that they did feel threatened ($\bar{x}$ = 3.4), while students who were inexperienced with their mode of technology in the other two classes did not share this concern about their devices (OP: $\bar{x}$ = 2.4; Calc: $\bar{x}$ = 2.2).

The written comments made by the students provided further insight as to why the devices they used made mathematics easier to understand. The majority of students in all three classes indicated that learning was easier because the devices provided them with accurate graphs they felt they could trust, graphs that provided a concrete picture they could relate to. One student commented that the calculator gave her "a visual picture so that I could know exactly what the graph looked like" and continued, "I could see what effect changing a number or a bracket had on the graph." Another student from the Overhead

Projector class suggested, "The computer gave me a concrete picture that made the problem easier to understand." These reactions support the view in the literature that students can benefit from multiple presentations of problem situations (see e.g. Demana & Waits, 1990; Ruthven, 1992).

A second major reason students gave for thinking that the devices made learning easier was the speed and ease with which they were able to obtain graphs. The majority of students indicated they liked being able to obtain graphs so quickly for the obvious reason that it was much less work. Others stated that the speed factor allowed them to obtain more graphs to analyze which helped them see the rule being developed, and that because it was faster the teacher had more time to explain. Some students also liked the idea that they could use their devices as checks for their graphs, consequently they did not have to consult the teacher to confirm their graph. As one student wrote, "You could check your answers, therefore you knew when you were right." Added another, "It shows you clearly what your answer should be in case you have doubts or questions."

The questionnaire results showed that the majority of students were convinced that learning was easier with the graphing devices, but when I probed further with the taped interviews into exactly what was being learned, students had difficulty answering the question. The information they gave in the interviews indicated that the devices certainly helped the students to learn what the rules of transformations were, but they did not help them to understand the reasoning behind the rules. The latter apparently still needs to be explained by the teacher for most of the students.

Not all students, however, agreed that technology made learning easier. A few commented that using the devices actually made learning more difficult because they were not doing much graphing in the traditional "table of values" way, a method they believed aided learning because of the repetition. According to one 'C' student from the Computer

Lab class, "It didn't allow me to do repetitious work which I think helps me learn. It did everything for me and I didn't learn." Other studies have shown similar reactions (see e.g. Ruthven 1992). Students such as this one seemed more concerned with how to obtain the graphs than what to do with them once they were graphed, and when they spoke about learning they were not referring to the concepts that I was trying to teach.

One of the disadvantages of the computers as an aid to learning from the students' vantage point was the software. Students complained that it was confusing when the equations could be given in several different forms, and they were not used to these alternate forms. Others did not like the fact that on one of the programs you were unable to determine whether a particular point was on the graph. These views were expressed by about 25 percent of the students in each of the computer classes.

Effect on Student Marks

While the students were enthusiastic in their support of the use of the graphing devices as an aid to making mathematics easier to understand, they were not as convinced that their use would help to improve their marks on the chapter test. The three classes indicated a relatively neutral view, before and after the units of work with the computers and graphing calculators, toward the question of whether they thought technology could improve their marks (OP: before $\bar{x}$ = 3.1, after $\bar{x}$ = 3.3; Lab: before $\bar{x}$ = 3.2, after $\bar{x}$ = 3.1; Calc: before $\bar{x}$ = 3.3, after $\bar{x}$ = 3.2). Five of the six students who gave taped interviews indicated that they thought the devices might have affected their marks slightly. With respect to the question of whether technology would improve their overall mathematics mark if it was used for other chapters of the course, the students again showed only mild agreement (OP: $\bar{x}$ = 3.4; Lab: $\bar{x}$ = 3.3; Calc: $\bar{x}$ = 3.4). Consequently a conclusion would appear to be that this experience with technology has given the students a feeling that technology will either not have an impact on their grades or will improve them only slightly.

On a related issue of the use of the graphing devices on tests, several students wrote comments that supported what they had been saying in class, that they could not understand why we were spending so much time working with the computers and calculators in class if they could not be used on tests. A typical comment from this group was, "I don't understand why we were using them all the time in class if we couldn't use them on tests." These students had apparently not understood the reason for using the devices. Approximately 25 percent of the students in the Computer Lab and the TI-81 classes expressed this view, so it would appear as though I did not adequately explain my rationale for using technology to teach this chapter to these students. In the taped interviews five of the six students indicated that they, too, had been concerned when we started the chapter, but as the work progressed they realized that the devices were being used to learn how to do something, and that when that something was learned they would no longer need the technological tool, consequently they were not worried about not having the devices for the test. The students' misunderstanding of why the devices were used was obviously a problem and all students in future will need to have a clear concept of the purpose for using technology.

Technology and Discovery Learning

When I started to consider teaching with the various forms of technology, the idea of discovery learning was not part of my plan. But as I explored the idea further, I began to suspect that using investigations could be a natural companion for technology. The literature is almost unanimous in supporting this type of methodology (see e.g. Kelly, 1993). I was interested, therefore, to see what the students' reactions would be to this combination.

There were few comments in the written questionnaires about the topic of discovery learning, leading me to conclude that style of learning may not be a crucial issue to most students. The few comments that were offered illustrated that among those who have an interest or an opinion on this topic, opinions were divided. In the Computer Lab class, for example, comments ranged from, "I liked the concept of trial and error for getting our own conclusions," to "I would prefer to use classroom step-by-step work because it is more effective." In the taped interviews, one student stated, "I think most students would rather be told", while another commented, "I think most people try to figure out anything themselves first." Obviously the discovery approach is ideal for some, and less appealing for others.

Two students in the Overhead Projector class indicated that they liked the idea of the whole class discussing the same question together (after they had time in their groups to do the investigation), and another commented, "it got the entire class more involved." Conversely two other students from the same class stated that they thought all they were doing was copying and they found it boring (these students participated minimally in their respective groups). Comments in the Graphics Calculator class showed some support for the investigation approach, with comments such as, "I could find things out for myself instead of taking notes," and, "It enabled us to investigate functions painlessly." In a taped interview, a student from this class answered the question of whether she prefers to be told or to discover by stating, "A bit of both I guess, as long as what you are trying to find isn't too long so that if you think you are on the wrong path you don't get frustrated."

The information provided in the questionnaires combined with that from the taped interviews leads to a not-too-surprising conclusion regarding discovery learning, with or without technology, and that is that like many other methods, it appeals to some students and not to others. For that reason I believe it should be used as one of several teaching strategies for part of any mathematics course.

<u>Enjoyment of Learning</u>

All classes indicated that they thought, before using their respective devices, that the technology would make mathematics more enjoyable, although there was a difference between the expectations of the Overhead Projector class ($\bar{x}$ = 3.5) and the Graphics Calculator class ($\bar{x}$ = 4.0) with the Computer Lab class in between ($\bar{x}$ = 3.7). At the end of the study all groups still believed that working with the graphing devices had added to the enjoyment of the course, but there was no longer any significant differences among the classes (OP: $\bar{x}$ = 3.7; Lab: $\bar{x}$ = 3.7; Calc: $\bar{x}$ = 3.8). It is interesting to note that all groups expected the work to be more enjoyable, and all found it to be so.

In looking for the factors that added to the enjoyment of the course, several seem to be common to all classes. They all stated that the speed and ease with which they were able to obtain graphs made the classes more enjoyable because it took away the tediousness of graphing. Many students felt, as I did, that the change brought about by the use of technology lent variety to the classes and made the mathematics a little more interesting and exciting. Similarly the change in environment for the students in the lab made the classes more interesting for them.

Several students commented that using technology gave them a different view of mathematics, and one even commented, "It didn't help me learn, it confused me. But it was fun." Another student who indicated that she worried because her class got behind the other classes concluded with, "...but it made math more fun."

The only negative comments general to all classes concerned the amount of time spent using the devices. In each of the three classes two or three students stated that either fewer classes should be spent using the devices, or that their use within a class should be more selective, that continued use became repetitive. On the other hand, it should be noted here that many more students in each class disagreed with their classmates and felt that the devices were not overused.

The issue of boredom surfaced in the questionnaires from the Graphics Calculator and Overhead Projector classes. In the class using the TI-81's, three students found that extensive use of the calculators was boring, while in the class using the overhead six students found copying graphs from the overhead to be uninteresting. One 'A' student in the latter class noted that since the use of the overhead tended to become monotonous perhaps it would be a good idea to not use it as much, but she did add that the class should continue to use it some because it did make the mathematics easier. The majority of the students in these two classes, however, did not find the classes boring and no one in the Computer Lab class mentioned the term 'boring'.

In general, the information from this study indicates that the majority of students liked using their respective forms of technology for a variety of reasons. Their positive reactions to technology suggest to teachers of mathematics, and to other teachers, that teachers should attempt to incorporate the use of technology into their lessons.

Students' Attitudes Towards Working With Technology

The results of the Likert scale questionnaire indicated that students generally agreed with the statement that they would like to continue to do more individual work with either the computers or the graphics calculators (OP: $\bar{x}$ = 3.5; Lab: $\bar{x}$ = 3.7; Calc: $\bar{x}$ = 3.8), results that indicates they have a positive attitude toward technology. When responding to the statement that they think the equipment they used should be used more in the teaching and learning of mathematics, they again showed a positive feeling by agreeing solidly with the idea (OP: $\bar{x}$ = 4.1; Lab: $\bar{x}$ = 4.0; Calc: $\bar{x}$ = 4.1). But the positive feeling was not unanimous. For example, a C- student from the Graphics Calculator class argued, "The school should spend its money on more important equipment" (but there was no suggestion as to what that equipment might be). In general, however, the reaction was positive. The written comments they made indicate a variety of reasons for their position, among them the sense that it makes learning easier and more enjoyable, as noted previously, and also that many of them felt computers and calculators are the future, and they expressed a belief that they should be working with the tools that will be dictating much of what they do in their later life. In the words of one student, "Keep using computers because they are the future."

The Effect on the Teacher of Using Technology

As the students and I worked through the lessons with the aid of the graphing devices, the experiences we encountered had an impact on my thoughts and attitudes towards using technology as a teaching tool. This section of the chapter examines the impact from several vantage points, from the new thinking that I was forced to undergo to the effect the process of discovery learning had on me, and finally to the broader issue of the effect technology had on my teaching in general.

New Thinking Required

Using various forms of technology in the classroom changed questions related to the use of these devices from the theoretical to the practical. I was forced to examine questions whose answers had immediate impact on what I was doing with my classes. Many were related to the larger issue of deciding exactly what I thought the purpose of technology as a teaching tool was, and specifically how and for what topics a graphing device would be beneficial. I found it necessary to formulate a workable philosophy with regard to the question of technology and teaching, and in so doing realized that there is no single answer that is best for everyone, but that teachers need to decide for themselves, based on their philosophy of education, how and why they would implement technology such as graphing devices.

As my classes and I worked through the lessons, I realized that I was restricting the use of the graphing devices to a single purpose, which was to obtain graphs faster and more accurately than had been possible in the past when we had to obtain number pairs in order to plot the graphs. But there was no creativity in the way the graphing devices were being used. I began to think that I should be finding other types of questions or topics related to the curriculum, topics for which the new technology could be a major factor in solving problems that previously were beyond our calculating capabilities. In general I began to think beyond the chapter I was doing, beyond technology only as graphing devices, to a larger picture in which technology could be used in other areas, such as statistics. In the latter stages of the technology-related lessons I began to formulate an idea that there are three lines of development as to where to employ technology: using the devices as a means of doing some previously done tasks much more quickly and accurately; solving more complex questions with technology that could not previously be solved for a given topic; and exploring new topics related to the curriculum (or not related to the curriculum depending upon the teacher's philosophy and the time available to teach the course) that could not be explored without the use of some form of technology. This approach is similar to suggestions made in the literature (see e.g. NCTM Curriculum and Evaluation Standards 1989).

Effect on Teacher's Attitude

A sense of personal excitement grew during the lesson planning stages, with the result that I was quite excited about the first lesson with each class, as one might expect.

Using a new form of technology also gave me a sense of "differentness" about those lessons that certainly had a positive effect on my enthusiasm.

As the novelty of working with something different wore off, other feelings took its place. There was the sense of victory for me and for the students when they were able to use their particular graphing devices to solve some aspect of a problem they were working on, whether it was simply a student changing a scale correctly with her computer as in Lesson Lab 3, or a student creating his own "what-if" question and exploring it with the entire class as in Lesson OP 23. In the latter situation the student inquired as to what might happen if the + sign was replaced with a - sign in the equation of a circle. The class discussed the problem, then observed the quickly obtained graph on the overhead. It gave me a very positive feeling to have a student suggest a "what-if" question and to be able to answer it so quickly and clearly.

The ability of the graphing devices to generate graphs quickly and accurately was generally a source of satisfaction for me. For example in Lesson OP 6 I used the computer and overhead for only ten minutes to correct the eight basic graphs whose shapes the students were learning, and when we finished I felt very pleased about how the computer had helped in developing the lesson. Similar situations occurred in many other lessons.

Using the graphing calculators and computers also created some fun for the students and for myself. From my perspective, it was gratifying to hear the students say "Oh" or "Yeah" in the Overhead Projector class as correct graphs that verified their conjectures were shown on the screen (for example Lesson OP 9). This use of the overhead and computer created a sense of excitement among the students that resulted in an atmosphere of fun and anticipation for all of us. If, however, in the Overhead Projector class the computer was not used sparingly, the feeling of excitement and fun was replaced by one of boredom and tediousness for me and for the students (for example Lesson OP4).

In general any class in which the students were enjoying the use of their devices and were being productive with them created an atmosphere that gave me a feeling of satisfaction and pleasure, and was most rewarding. The graphing devices we used had the potential to create an atmosphere in the classroom that made the job of teaching a more enjoyable one. Teachers should be aware, however, that careful planning of how the devices are to be used seems crucial to ensuring that enjoyment does not regress to boredom.

Frustrations in Teaching With Technology

While there were many positives about using the graphing devices in the classroom, there were also some frustrations. I was frustrated in the first few lessons with each class because it took the students considerably longer than I had anticipated to create graphs for the introductory "real-world" functions I had given them. The difficulty they had in obtaining a complete graph when the domain and range were something other than the standard default screen turned what I thought was going to be a two-period topic into a five-period topic. This is an example of the need to teach new skills to students, as was illustrated in the literature (see e.g. Dick, 1992; Hector, 1992) if a technology approach is to be used.

Another early problem concerned the issue of how to assign homework when students did not have the graphing devices readily available to them outside of the classroom. I found myself in the situation of wanting to assign certain questions, but being reluctant to because the students did not have easy access to the devices. As the lessons progressed solutions to this problem became apparent, but initially it was a frustration.

In the Graphing Calculator class and the Computer Lab class I was further frustrated by a feeling of lack of control over the class because I could not see all of the students' screens all of the time and I wondered what they were doing and how they were doing it. But the reality is that this is probably not any different than when students are following the standard pattern of working in their notebooks, it is just that when the students are using the graphing devices their graphs seemed to be more visible as I walked around the class, so I was more conscious of wondering what they were thinking. Another factor adding to my feeling of lack of control was that because the students' graphs were easier for me to see, and because the students were able to obtain more graphs in a period than before, it seemed as though I could see more incorrect graphs than I could get to in order to give help.

In the Computer Lab class there was a problem unique to that class, and that was that I found it difficult initially to generate good discussions with the class. The long, narrow configuration of the classroom, coupled with the fact that in the first couple of weeks the students could not resist the temptation to work with their computer rather than to contribute to a class discussion, made it more difficult. I found my inability to generate discussions similar to the ones the other two classes were engaging in to be frustrating.

Effect of Discovery Learning

One of the advantages of the graphing devices was to obtain graphs of functions more quickly and accurately than before, consequently it seemed logical to combine their use with student investigations, in other words with a form of discovery learning. This strategy placed an emphasis on the students developing their own conclusions, a process that proved time consuming and frustrating for many of them. Having the students form their own conclusions gave me a feeling of satisfaction, but it was also the source of frustration for two reasons. First, several students in each class had difficulty with the idea of having to actually consider some information and use it in order to create their own conclusions. They would rather have been given a rule and then asked to memorize it. This situation was not a direct result of using technology, but it was indirectly related because the graphing devices facilitated the use of discovery learning. The students' reluctance and imagined inability to form their own conclusions frustrated me. Secondly, the amount of time it took to cover the content of the chapter using the investigation approach was also a source of frustration. I felt compelled to cover the curriculum for Mathematics 11 as prescribed in the Curriculum Guide, but was discovering that the process of discovery learning was taking more periods than were available for that chapter. In each period the students appeared to be on task, but the sum of the content covered over many lessons did not seem to be enough. I began to question the number of periods each topic was taking, and I felt frustrated because I believed in the idea of students learning through investigations, but was feeling pressured because I felt I would not be able to complete the curriculum if I continued the lessons as I had planned them. As a result I felt compelled to impose time limits on some of the investigations (for example Lesson Calc 11), and to do more explaining of the topic before they got into their investigations (for example Lesson OP 13), changes that were contrary to my philosophy of teaching.

After three weeks of having the students do investigations with the graphing devices I stopped to reflect upon the process, and while I was not sure if the students were learning more or less because of this approach, I was convinced that I was enjoying the classes more in spite of the frustrations. Having students actively engaged in groups and discussing their conclusions, in short doing mathematics instead of just memorizing facts, was consistent with my philosophy on teaching mathematics and gave me a sense of both excitement and satisfaction. However after six weeks of investigations I found that I was tired of the repetitiveness of either the approach or the content (transformations) or both, and was impatient for a change. This experience forced me to think about the value of discovery learning, with or without technology, and led me to the conclusion that discovery learning has many advantages, but as with so many other aspects of education, it should not be used exclusively, but rather in conjunction with a variety of other techniques and approaches.

One of the side effects of having students follow the investigation approach was that I was forced to prepare the investigation sheets, and in so doing I found that I had to be very definite about the exact purpose of that lesson and what I hoped the students would get out of it. This forced me to re-examine each of the topics and as a result I felt I improved my understanding of them.

Effect on Teaching in General

The entire project of investigating the use of technology in the mathematics classroom had a major impact on my overall approach to teaching. It was not just the use of the modern technological devices, but the entire scenario of having to make observation notes of my lessons and the students' and my reactions to those lessons, and having to investigate each topic thoroughly in order to prepare the investigation sheets for the students that forced me to re-evaluate what and how I was teaching. I began to question some of the methods I had used in the past. For example, in each of the classes several students were asking, "Why does the curve shift the way it does?" instead of just memorizing the rule as I recall we did in previous years. I feel technology may have fostered an atmosphere in which students were asking more questions, and their specific content questions led to more general methodology questions for me.

In addition, the classroom research portion of the thesis forced me to examine my philosophy of mathematics education, and to question whether I was following that philosophy. As I continued to ponder the question, I realized that I was not involving the students enough to satisfy my goals, that I was not giving the students enough opportunity to create their own mathematics within the bounds of their experiences and capabilities, consequently I altered some of my methods in all of my courses, not just the courses that involved the new technology.

Comparison of Modes

As indicated in Chapter One, the question of which mode is "best" was to be examined from several points of view. Factors such as the ease of use, the accessibility of each mode, the effectiveness as a teaching and learning tool, the ability to help students meet the lesson objectives, the effect on teacher and student interest, and the effect on planning the lessons have been taken into account.

Ease of Use of the Modes

Students in the two classes that were actively involved in working individually with either the computers or the graphics calculators reported no difficulty in learning how to operate their respective modes. The graphics calculators, however, proved to be more awkward and time consuming to use than the computers when dealing with the section on transformation of relations because of the algebraic manipulations required to arrange the equation in the form "$y =$" before the equation could be entered into the calculator.

From my perspective as a teacher trying to work with these devices, I found that all of them took time to learn, but that there was no advantage to any mode in terms of ease of use.

Accessibility of the Technology

Although the computer and the graphics calculator seem to be relatively equal with regard to learning to operate them, there is a difference with respect to their accessibility. In my situation there was only one overhead projection pad in the school, consequently it had to be shared among any teachers wishing to use it. Fortunately only two other teachers expressed such a desire (business and computing classes), but even with such a low demand I found it very inconvenient to be constantly going back and forth between floors to retrieve the pad. A similar problem existed with regard to the computer that was used with the overhead, as it had to be transported from the Mathematics Office into my classroom on an hourly basis. The collection of these materials was annoying to the extent that one might try to do without them rather than go through the inconvenience of collecting them. Consequently I found that a computer, overhead projector and overhead projection pad need to be permanent fixtures in a classroom in order for them to provide maximum benefit. Only if they are readily available for short and sometimes spontaneous occasions during a lesson can they achieve their potential.

The graphics calculators, on the other hand, are extremely portable and can be transferred from one location to another quite quickly with a minimum of inconvenience, a factor which makes them much more attractive than the computer/overhead combination.

The computer lab also presented an accessibility problem, as this school had only one lab, making it impossible to schedule more than one mathematics class into it on a regular basis. Even trying to program a class into the lab for a week or two was difficult. I was able to program one of my classes into the lab at the time the timetable for that year was constructed, but this could not be done for more than one mathematics class, which meant that this mode could not be used on a regular basis for many mathematics classes in this school. Other schools with more extensive computer facilities might not have this problem.

Given the restrictions in the particular school used in this study, there was a definite advantage to using the graphics calculators in terms of ease of obtaining access to the graphing devices. For any given school, the advantage in that school would obviously be to the mode which had easiest access. Unfortunately none of the modes were readily available to students outside of their regular mathematics class, therefore none of the devices were able to satisfy the issue of continual access.

As an Aid to Teaching and Learning - The Ability to Demonstrate a Concept

The results of the student questionnaires indicated that all three classes rated the effectiveness of their respective modes equally as an aid to learning, although about 25 percent of the computer class did complain about inadequacies with the software they were using.

One slight general advantage the computer held over the calculator was its larger screen, which resulted in clearer, easier to read graphs. The smaller screen was a minor disadvantage for topics requiring the comparison of two graphs when those graphs almost coincided, for example $y = x^3$ and $y = x^3 + 2$.

In the early lessons in the chapter, where students were learning the different shapes of graphs that accompany different types of functions, all three modes were able to demonstrate the correct graphs clearly, although the students in the computer lab were restricted in discovering some of the shapes for themselves because of a limitation in the software being used. With the Zap-a-Graph program that was available on network, they had to know previously, for example, that $y = 2^x$ was called an exponential function, a fact not known to most students.

The translation of functions was ably demonstrated by all three modes, although the computer did have an advantage in that the larger screen made it easier for students to see certain shifts, but that problem could be partially overcome by selecting examples of functions to use on the graphics calculator that were not close to coinciding. The concepts of stretching and compressing functions were clearly demonstrated by all three modes, but the graphics calculator was slightly superior because for this topic it was advantageous to find coordinates of points on the transformed curves, and the calculator was faster than Master-Grapher and more accurate than Zap-a-Graph.

The topic of determining the correct order of transformations for relations was easier to follow using the computer because the Zap-a-Graph program allowed the transformations to be done one at a time, in any order, which permitted the students to see very quickly the consequences of changing the order. For this particular topic there was a clear advantage to using the computer over the graphics calculator.

My conclusions as a teacher were that each mode had its advantages and disadvantages, but that all three modes aided in instruction and were equally able to demonstrate the concepts because they all provided visual aids for the students.

As an Aid to Meeting Lesson Objectives

I had three objectives for this chapter: (1) students should gain knowledge of the content of the chapter and be able to apply that knowledge to answer questions related to that content, (2) students should be actively involved in obtaining their own data in order to derive their own conclusions, in other words I wanted them to be doing mathematics, not just observing, and (3) students should be discussing the mathematics they were involved with, either in their pairs or groups, or with the entire class.

With respect to the content objective, the data analyzed and reported on previously shows that there was no advantage to any of the three modes as measured by the standard classroom tests written by the students.

The Graphics Calculator and Computer Lab classes, however, were considerably more involved than the Overhead class with regard to the objective of "doing" mathematics, especially with regard to the topics related to the transformation of functions. In the former classes the students were able to input their own data and manipulate the devices. The Overhead Projector class worked in groups and gave me data to enter into the computer, but each student could not input his/her own data and react to it. The students who had individual access to a device were able to ask themselves "what if" and "why" questions and then explore the answers to those questions on their own, which many of them did. They were able to "do" more mathematics of their own making than were the students who were only able to observe what was happening on a large screen at the front of the room. The students in the Overhead Projector class did ask many of the same questions that students in the other classes asked, but they could not explore the answers themselves. In that class we would examine the questions together and although they could suggest input for the single computer, they did not have the opportunity to experiment with their own ideas.

When working on the topic of maximum and minimum problems, again the students in the Overhead Projector class were not as involved as students in the other two classes, but for this topic the students using the graphics calculators found it more difficult to arrive at a complete graph on their screen than did the students in the Lab, and as a result the students in the computer lab were able to do more problems in much less time. Consequently for this topic the computer lab appeared better than the graphics calculators which in turn seemed better than the computer/overhead combination with respect to allowing the students the opportunity to "do" mathematics.

In order to have the students become more active participants in the learning process, many of the lessons were structured on a discovery learning approach. On the topics concerning the transformation of functions, the students in the Graphics Calculator and Computer Lab classes were able to proceed through the investigation sheets at their own speed, and had time, within reason, to arrive at their own conjectures and verify them by looking at the pertinent graphs on their devices. In the Overhead Projector class the students did not have the same time freedom, because often many students had reached some conclusions and were ready to see the correct graph on the screen in order to check those conclusions before the remaining students had formed their conclusions. As a result the computer and overhead projector combination did not allow the students to become involved in discovering the rules related to transformation of functions to the same extent that the students in the other two classes did.

Another major topic in the chapter was the transformation of relations, as opposed to the transformation of functions. For this topic all three modes had disadvantages with respect to their applicability for discovery learning. The graphics calculators were not as "user friendly" as the computers with respect to entering the equations, and they were of minimal help when trying to discover any rules about the order of the transformations. The computer software used, on the other hand, did allow easy entry of equations, and did permit the students to enter individual transformations in order to examine the effect of the order of the transformations, but it required some instruction first with respect to interpreting the stretching and compressing factors, which negated some of the objectives of discovery learning. The disadvantages of using the computer/overhead combination for this topic were the same as for the other topics.

One could conclude, then, that with respect to the objective of "doing" mathematics, both the computer lab and the graphics calculators were successful for the topic of transformation of functions, that they were both also successful with respect to the topic of maximum/minimum word problems, although the calculators were more time consuming, and that for the final major topic of the unit, transformation of relations, the Computer Lab with the Zap-a-Graph software was the best, although the graphics calculators still allowed for some student interaction. For all topics the single computer combined with the overhead projector was the least successful.

With respect to the objective of having the students discussing mathematics, I found the computer lab to be the most frustrating with regard to discussions with the entire class. Research indicates other teachers have experienced the same problem (see e.g. Heid, Mattras, & Sheets, 1990). Partly because of the elongated configuration of the room, but primarily because the students always seemed eager to work at their computers, it was very difficult to maintain their attention for any discussion involving the entire class. On the other hand, within their pairs there was a great deal of discussion, so mathematics was being discussed, but I felt uncomfortable because I was not sure exactly what they were discussing and was not able to suggest changes in direction for groups whose discussions might be headed in inappropriate directions.

In the Graphics Calculator class, there was an encouraging amount of discussion within the groups, and within the entire class. One of the advantages of the calculators was that I could delay handing them out at the beginning of a class which made it easier to maintain their attention during a discussion than with the Computer Lab class.

There were also some excellent discussions in the Overhead Projector class, but they did not occur as frequently as with the Calculator class. The best discussions in the Overhead Projector class occurred when the students were asked to guess about the effect a particular transformation would have on a graph, and various ideas were suggested by the students before the graph was shown on the overhead. While this procedure could result in some good discussions, it could also result in boredom if used too often or for too long a period of time.

The degree to which any class got involved in discussions was also inversely proportional to the amount of information I gave them about a topic. If I was trying to increase the pace of a lesson by giving them more information and allowing them less time to discover concepts, then they asked fewer questions and volunteered fewer ideas. This situation happened more with the Overhead Projector class than with the other two.

All classes asked "what if" questions, and these questions often led to interesting discussions, but in the Graphics Calculator and Computer Lab classes these questions were often debated within the small groups or pairs, and only surfaced for class deliberations if the groups could not answer the question and consequently asked me, or if I overheard the question as I walked about the class. In the Overhead Projector class the "what if" questions were usually directed to me and I would use the single computer to help show the answer. The result of this situation was that in the former classes the questions were not always asked by all groups, consequently not all students were aware of the question and hence were not involved in the discussions unless these topics were elevated to class discussions, whereas in the latter class the majority of "what if" questions were brought to my attention and I could pose the question to the entire class. The advantage for the Graphics Calculator and Lab classes was that if students in the group did have a question, they were able to use their particular graphing device to experiment and play with the topic on their own, which often gave them a great sense of satisfaction.

In considering all of the above factors, my sense is that all three classes achieved the goal of discussing mathematics, but that the Graphics Calculator class was the most successful with respect to having students engaged in and benefitting from such discussions.

Effect on Interest

From the students' perspective, all modes contributed equally to the enjoyment of the class according to the results of the Likert scale questionnaire, although the open-ended portion of the questionnaire did offer some further insights. According to the written comments on the questionnaires, some students in the Overhead Projector class (6 out of

25) were bored with that particular mode, while fewer students in the Graphics Calculator class (3 out of 28) suggested they were bored, and none of the students in the Computer Lab class mentioned being bored. Over one-third of the students in the Overhead Projector class did comment that they would like to have been able to work on the computer individually.

My observations of the students during the lessons led me to the conclusion that while all modes can provide a feeling of excitement and interest in the class, the single computer and overhead was definitely the most limited in the degree and the frequency to which excitement could be generated and maintained. With this equipment some excitement can be promoted among the students, but it needs to be very carefully orchestrated by the teacher with respect to how the technology is used and how often and for what length of time. In Lesson OP 4, for example, the students were very interested during the early portion of the class as the computer was used to verify their conjectures as to the shapes of some graphs, but after 35 minutes of the same activity there was no sense of excitement in them or me. In the classes using individual devices there was a more spontaneous and lasting enthusiasm as students individually encountered graphs that surprised and challenged them.

After several weeks of use, the ability of the devices to generate interest remained almost equivalent to the level they had achieved in the first week. The students in the Computer Lab and Graphics Calculator classes were still enthused after several weeks of working with their respective devices, for example in Lesson Lab 23 they were "oohing and aahing" over their graphs when shifting circles, and in Lesson Calc 27 the students were having fun learning how to graph circles with their calculators. The students in the Overhead Projector class could still show some excitement for short periods of time, as evidenced in Lesson OP 20 when students were exclaiming "that's what I got" in triumphant tones as correct graphs for their homework were shown on the overhead, but the enthusiasm did not last as long in this class as it did in the other two classes.

With respect to how the various modes affected my interest in the lesson, I found it more interesting to work in the classes in which the students were working with individual devices, as I was able to walk about the class fielding questions and listening in on group discussions. The fact that the students in these classes seemed to be enjoying what they were doing also helped me enjoy the classes more. In the Overhead Projector class I did not enjoy sitting at the keyboard inputting data while the class just sat and waited for the results, unless it was for only one or two questions.

In terms of interest, then, it would appear that even though all classes reported that the devices added to their enjoyment of the lessons and all indicated that they would like to continue using technology, the students who had individual access, or at least access in pairs, to graphing tools sustained an interest in the activities of the lessons longer than the students in the Overhead Projector class.

Planning the Lessons

The biggest challenge to planning the lessons was to decide which topics to use technology for and how to use that technology, and those problems applied to all three modes equally. It took slightly longer to plan the lessons that involved the graphics calculators, but that was not a significant disadvantage.

The "Best" Mode

To determine which mode is "best" is a difficult task because there are so many different factors involved, as revealed in the previous discussions. After considering the various points of view, it would seem that all modes are useful for providing valuable visual aids to the Mathematics 11 content under discussion, but that there were definite advantages to any mode that could allow students to work individually on a graphing tool. These advantages included being more interested in and involved with the lessons, and being able to explore ideas on their own. But perhaps the most important aspect of the question of which mode is best is that even though one might be better than another for a given topic or in a given school, the evidence strongly suggests that any of the three modes of technology tried in this study are superior to not using any technology at all. This particular view was not expressed in any of the literature reviewed.

Changes to the Lessons

The changes referred to in this section fall into two categories, changes I made to my original lesson plans and implemented as the lessons progressed, and changes I would make the next time I was to teach a particular topic. The majority of the changes are applicable to all three of the modes of technology used; when a change was applicable to only one or two of the modes, it is so noted. Not all of the lesson topics are addressed here because not all topics required changes. The significant changes are presented in the order that the lesson topics were taught.

As stated previously I am not an expert in the use of the graphing devices used in this project, but in working with them for seven weeks with three classes I did gain some experience with them. This experience provides me with enough background to be able to expand beyond the first purpose of this section, which is to indicate what elements of the lessons did not work as well as planned in order that other teachers can avoid repeating those features, to the second, and perhaps more important purpose, which is to share the implications from the changes and to offer some suggestions for planning and teaching based on those implications.

Topic 1: Defining a Quadratic Function

The first topic in the unit, a topic that included reviewing domain and range, took four classes rather than the two that had been planned because of the difficulty the students had producing graphs for real-world problems. The students experienced considerable frustration trying to decide on a realistic domain and range and trying to produce a complete graph on their graphing devices. These frustrations hindered progress toward the eventual goal of the lesson which was to have students discover that not all functions are linear. As a result of observing their difficulties, I would make several changes to the first few lessons.

It was obvious to me that the students were restricted in their concept of domain and range by the questions they had answered in the previous chapter, questions that asked for the domain and range for abstract functions. Consequently the first change would be to do some examples of real-world problems with the class before they attempted any on their own so that they could appreciate that the selection of a domain and therefore a range can be somewhat flexible.

Next the real-world examples on their introductory investigation sheet should be changed so that the domain and range of the first few questions would show some different types of graphs without the necessity of changing the default domain and range of the graphing device. The implication with regard to the preparation of lessons is to make up teaching examples that are easily managed by the graphing devices. If the examples chosen, such as the ones used in Topic One in this study, are too unmanageable then the objective of the lesson can become lost in the manipulations of the calculator or computer. One can allow subsequent questions to become more involved, but if the initial examples are beyond the expertise of the students with the graphing device at that time, then needless confusion can result.

Another change in the lessons would be to show students an efficient way of discovering suitable domains and ranges for problems with domains and ranges considerably different from the default screen, a method devised by my students as they fought with the same problems again in Lesson Ten. This approach required students to first speculate quickly as to possible domain and range, then continue to change those limits on the graphing device by a trial and error method until a complete graph is achieved. This method may not be educationally sound, but it must be remembered that the primary objective of the lesson was to observe that graphs of different types of functions may have different shapes, and that obtaining the the graphs was secondary. The same problem with regard to finding domain and range was encountered again in Topic Ten, but as we worked through another set of real-world problems, the last change mentioned in relation to Topic One was developed so that by the end of lesson ten the problem no longer existed for most students.

Towards the end of Topic One, which was after three or four lessons, students expressed a disappointment that after all their work in learning how to get a graph for a real-world problem, they were asked only one question about that function - to determine maximum and minimum values. As a result, some questions need to be structured for this section that require students to make more interpretations from their graphs. This particular change should also be applied to Topic Ten.

With regard to specific changes about Topic One for the various graphing devices, the students using the graphing calculators were inclined to write seven and eight decimal answers for problems, indicating that they were automatically writing down all the decimals that the calculator showed them, and were not taking the time to think whether their answers were realistic. The concept of degree of accuracy of answers obviously needs to be clarified for any class using graphing calculators.

For Topic One the classes using the computers had an early problem with the Zap-a-Graph software because it required students to know the names of given functions, for example that equations of the form $y = x^3$ are cubics, but at this time the students did not know that and in fact that was one on the things they were to discover in this unit. As a result we switched to the program Master-Grapher for the remainder of Topic One, and I would use it entirely for Topics One and Two next time because it suits the objectives of the lessons better. The implication from my experience is that one program may not be adequate for teaching all topics, and that perhaps one should be prepared to alternate with a different software, assuming one is available, if the one being used is not proving satisfactory. If no other software is available, then one should not hesitate to abandon the original plan and try something else. Even though the plan may have looked good in the planning stages, it does not have to be strictly adhered to if it is not working.

With respect to the Overhead Projector class, several changes in how the individual periods concerning Topic One were conducted should be implemented. The key issue is to keep the students involved while information is being fed into the computer, specifically the questions in which the students were trying to find the domain and range. All students should write down their prediction as to a reasonable domain and range for a problem in order to get them to commit themselves to the problem, then individuals could be selected to enter their ideas into the computer, and the rest of the class could then offer suggestions as to how the domain and range shown should be modified, if a change was required. This plan was actually tried in the latter periods of Topic One and in Topic Ten and the students' reaction to it was positive. Some of the graphs that appeared, or did not appear, led to interesting discussions. But there is one note of caution about this approach, and that is that if it is done too many times in a period it tends to become boring.

Topic 2: Other Types of Functions

In Topic Two, students were to investigate other types of functions beyond the few they examined in Topic One, and each group was instructed to draw the graph of one real-world problem that had a graph different from those they had already studied. Students in both the Calculator class and the Computer Lab class were instructed to obtain their graphs by using the graphing device at their disposal. The fact that I instructed the students to use their graphing tools is an excellent example of how the graphing devices drove the early lessons. I would change those instructions next time to encourage, but not require, students to use the graphing devices in order for students to gain an appreciation that the graphing devices are simply another method to be used in solving problems.

Topic 3: Graphing $y = f(x) + q$

The students using the graphics calculators to investigate Topic Three had difficulty seeing on their calculator screens the difference between $y = x^3$ and $y = x^3 + 4$, which was the objective of the lesson. Consequently, when preparing examples for lessons dealing with the transformation of functions, examples should be chosen that clearly illustrate the concept. Not all functions are suitable. The graph of the cubic function, for example, tends to appear somewhat confusing to the students when considering vertical transformations, compressions or expansions.

In the Overhead Projector class I changed the teaching technique as I was progressing through the examples because the students were not exhibiting much interest in determining the shifts in the graphs. Instead of just having them copy down the resulting graphs, I asked them to guess where they thought the next graph would appear on the screen, and this resulted in considerable interest and interaction. This particular strategy has application to many of the lessons taught with the overhead projector and computer.

A further refinement to many of the lessons involving the overhead projector and a single computer is to involve students in entering data into the computer. During one of the lessons with this class I had to leave the room for a short time and asked a student to continue putting data into the computer, which she cheerfully did. When I returned I continued to let her operate the computer and found that she was much more engrossed in the lesson than before, and although it did not seem to affect the rest of the class in any way that I could see, it did have an effect on the student operator. Therefore it would seem logical in future lessons to let students have turns at the keyboard just to stimulate their interest.

As we progressed through the topics, I began to realize that the majority of the questions in the text book were not designed for students equipped with graphing devices. As a result, starting with Topic Four, I began to assign some questions obtained from other sources, questions that were designed for students with technology at their disposal. The next time though this unit I would be aware that many of the question sets in the text need to be scrutinized carefully with a view to deleting some questions and adding others to take advantage of the graphing devices. There are many textbooks now available that have been written for students who have access to graphing devices, and their question sets can be a valuable resource.

Topic 5: Graphing $y = af(x)$

In Topic Five, students continued to investigate the transformation of functions, and as we corrected homework the method of correcting the homework began to change, and a pattern emerged that I used in subsequent lessons and that I would use next time starting with the first lesson. My original plan had been to use the graphing devices for everything. I discovered, however, that solutions to some of the questions were easier to explain by sketching a graph on the board. As a result I would change my approach to correcting homework questions to include drawing some graphs on the board, which is actually parallel to what students are asked to do, namely sketch the graphs in their books without using a graphing device.

The majority of the topics in this unit required students to do independent investigations, a process that took more time in all three classes than I had anticipated. As a result, starting with the work in Topic Five, I began to put limits on the time the students were given to arrive at their conclusions, and I began to verbalize more. The result was that the material was covered slightly faster, but the slower students were not getting the opportunity they were earlier to create their own mathematics. All teachers who use the discovery learning approach will have to decide for themselves how much time they can devote to a certain topic, and that may dictate how much time the students are given to arrive at their own conclusions.

As the Graphics Calculator class explored Topic Five, the topic in which they were determining the differences in the graphs of $y = f(x)$ and $y = af(x)$, they had difficulty seeing the effect of 'a' because the coordinates on their screen contained so many decimals that a clear pattern was not easily visible. To partially combat this problem the investigation sheet should indicate specific integral values for x, values that will appear on the screen as integers. For example if the functions being investigated are $y = 2^{-x}$ and $y = -2^{x}$ then for the default range an x value of 2 or -2 will result in coordinates on the screen that are easy to work with, whereas x values of 1 or -1 result in coordinates that are very difficult to use. In general, examples of functions should to be constructed that will display integral values for one or both coordinates for some points on the graph in question so that students can see the relationship between coordinates without becoming buried in an avalanche of decimals. It should be noted that the TI-81 will show integral coordinates for only a few x values, and those values vary depending upon the scale being used.

Another change that would have to be made to this particular investigation sheet, as pointed out to me by the students as they worked through it, is to require students to also graph the "basic" function, in this case $y = 2^x$, so that they would have a graph to compare their new ones to. This step had been part of previous investigation sheets, but had been omitted from this one. In general students need to see the "base" curve in order to be able to compare their new graph to some original graph.

Topic 7: Graphing $y = af(x-p)+q$

In their explorations of Topic Seven, in which students were to examine the effect of doing the transformations in different orders, the groups in the Graphics Calculator class came up with several different correct orders, so I had each group with a correct order put their order on the board, and we compared and discussed them. I had not planned to do this, but I certainly would next time as it proved to be a very useful strategy. This scheme would work with any classes as the graphing devices were not used to obtain the graphs, only to check their accuracy.

Topic 10: Maximum - Minimum Word Problems

While the students in the Computer Lab class were working on Topic 10, maximum and minimum problems, a different way of showing a chain of reasoning leading towards the solution of a problem was accidentally provided by a student. She was forced to use a different computer that day, and unknowingly chose the computer that was connected to the overhead projection device, and as a result all of her steps toward a solution could be seen if the projection device was turned on. Consequently an alternate way of having students see other students' work, on a selective basis, would be to have various students take turns at the overhead-linked computer and to monitor their work, showing the class what that student was doing when it would be beneficial to the class and non-threatening to the student.

Topic 13: Graphing $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = r^2$

The objective of Topic Thirteen was to have the students expanding and compressing circles, and while all went smoothly with the computers, a problem arose with the graphics calculators. The class using the calculators was to graph the functions given on their investigation sheet, then select some points from the original graph and from the transformed graph and compare the coordinates in order to reach some conclusion. Again most of the coordinates that showed on the screen had seven or eight decimals, which obscured the relationships between the numbers. My original instructions to the class were to use the Box function on their calculator to get better approximations of the coordinates, but that also proved unworkable. We then concluded that the best approach would be to round off the coordinates shown for each point to the nearest integer or to one decimal, and to work with those values. That is the plan I might follow the next time, but it is a rather awkward one. For this particular topic the calculators were not efficient, in fact the computer was superior in terms of illustrating the concept.

Topic 14: Graphing $\left(\frac{x-p}{a}\right)^2 + \left(\frac{y-q}{b}\right)^2 = r^2$

The final topic in the unit for which graphing devices were used, Topic Fourteen, asked students to determine a correct order of transformations when combining stretching, compressing and translating to the graph of a circle. For each class I changed my original plan to a shorter one, primarily because the process of graphing with or without the graphing devices had become repetitious and boring to me and my students. The procedures that I changed to are similar to the ones that evolved during the lessons on Topic Seven, and are the ones that I would follow if I was to teach this unit again.

For the Graphics Calculator class I had originally planned to ask each group to manually draw the graph of each function given using as many different orders as they could, then to use the calculator to see which of their orders resulted in the correct graph. Instead, I had each group try only one order, check to see if that order yielded the correct graph, and if it did to list their order on the board. We then compared the lists in order to draw the conclusions. Unlike the students working with the computers, the calculator students were forced to get their graphs manually because the calculator does not allow the operator to try different orders.

With the Overhead Projector class the original plan called on each group of students to manually graph several different orders, then as a class we would use the computer to verify their graphs. The revised plan asked each group to manually try only one order, then a group was chosen randomly and their order was entered into the computer, and that graph verified. If their graph was a correct one, then any other groups who had arrived at the same graph via a different order of transformations put their order on the board and the ensuing discussion led to final conclusions re order.

The Computer Lab class was originally requested to use the computer to generate graphs using all possible orders of transformations. Instead each pair of students was asked to graph the function using one order, to check that order to determine whether that order was correct, and as with the graphics calculator group, write their order on the board if it was a correct one. Again, a class discussion using the lists on the board was used to arrive at some final conclusions. An alternate plan for this class, depending upon the time available, would be for them to continue trying different orders to see how many could give the correct graph.

In spite of which piece of equipment a class was using, all graphs had to be verified in some way to determine if they were correct. With the computer classes, the graphs were checked by selecting a point from the transformed graph and substituting its coordinates into the equation to see if they satisfied the equation. With the graphics calculators, the graphs that appeared on the screen were automatically the correct ones, barring an input error, so a student's manually derived graph could be checked by using the calculator. But for that group also, it would be faster to select a number pair from their manually generated graph and check it in the equation, as did the computer groups, so that is a procedure I would make sure they were aware of next time.

Regardless of how carefully I planned the lessons, I occasionally found that as I actually taught the lessons some changes to the original plans occurred. Sometimes I accidentally discovered a better way, while other changes were dictated my students' reactions. Occasionally the students were the ones who suggested some ideas that were better than the ones that had been planned. The implication is that no matter how much time and energy one expends in planning a lesson, one might have to actually experience teaching it in order to evaluate what will work and what will not.

Many of the things that happened during the teaching of the lessons have a significance not only for that lesson, but also suggest implications for any lessons involving the use of technology, especially graphing devices. The presentation of these implications in the previous pages should provide any teacher who is planning to use technology to teach mathematics with some useful guidelines for planning and teaching.

Implications For the Technologically Inexperienced Teacher

The comments the students made via the questionnaires and during the lessons, the changes I made to the lessons, and my experiences in teaching the lessons as reflected in my observations of and thoughts about those lessons provided me with material from which I am able to answer some of the questions raised in Chapters One and Three that have implications for the technologically inexperienced teacher. The previous section offered some implications for instruction that were related to the specific content of the Mathematics 11 chapter covered in the study; this section offers some implications that are related to technology and the teaching of mathematics in general.

Confidence and Credibility

One question raised in Chapter Three referred to my feeling of unease about using an unfamiliar graphing device in front of what can be a very demanding audience. I was concerned that the students might know more about the devices than I did, or that I might not be able to answer all of their questions about how to use them. After seven weeks with three different classes I can state quite emphatically that these were not problems at all. Very few of my students knew more about the devices than the little I knew, and those that did served as resource people and helped me and the other students. I was able to answer most questions about the operation of the devices, and those that I could not were either answered by a student, or by the cooperative computer teacher, or else we all agreed we did not know how to do something and sometimes that spurred a student on to solve the problem for all of us. I was also concerned that I did not know all of the "tricks" or "short-cuts" of a particular graphing device, and while that did cause some problems with the preparation of teaching examples for the lessons, it was not an insurmountable problem. My experience has led me to believe that a technologically inexperienced teacher need not feel intimidated or threatened by the technology or by the students who may be more familiar with it than he or she is.

The Awe of Technology

For all three devices used in the study, there was an obvious transition during the lessons from the devices being the focal point and driving the lessons to the devices being viewed as just another teaching tool and being used as a teaching aid. The excitement of using the graphing calculators or the computers was such a dominant factor in the first lesson with each class that I forgot the objective of the lesson and concentrated almost solely of the operation of the devices. By the second or third lesson with each class I began to realize that technology rather than the teacher was dictating was happening during the lesson, and I began to examine the problem.

During the second and third weeks I gradually wrestled control of the lessons away from the devices and began to use them as aids rather than as an entity in themselves. For example in Lesson OP 7 I used the computer for only a few minutes in order to produce three graphs, a use of the computer that made me feel that I was not forcing the use of the computer, but rather that I was using it as an aid to help make a point.

After four lessons with the Graphics Calculator class I realized I had been too restricted by the idea that all work must be done with a calculator and that I needed to free my thinking and change my approach. One change occurred during Lesson Calc 7 when I had students put some of their graphs on the board for correcting, because I was no longer assuming that the students' calculator-generated answers were automatically correct and understood by them. I was starting to feel more comfortable with the graphing calculator and its use, and felt that I was now using them only when it was advantageous to do so. The feeling of awe was dissipating.

The transition from technology being a driving force to being an aid in the Computer Lab class seemed complete by Lesson Lab 11 when we did not use the computers for the first 25 minutes of the class. I no longer felt obligated to use the computers just because they were there. Lesson Lab 12 gave further evidence to the transition as I used the board to explain some transitions rather than using the computers because with the board I was able to make a point more clearly than with the computer.

The net result of this transition is to suggest to technologically inexperienced teachers at the outset that they should view the graphing devices as one more method of instruction, a method to be used when advantageous to the overall objective of that lesson. The devices should not be used simply because they are there, but should be used because they are helping to meet a specific goal. They are an aid to instruction, not an end in themselves.

Planning Time

The question of the time required to construct the actual lesson plans was one that did concern me in the original planning stages. I did find that the lessons took longer to plan than I had anticipated because I had to devote a certain amount of time to learning how to operate the TI-81 and the Zap-a-Graph software. Although learning how to operate the technology was an enjoyable experience, there were some frustrations in planning the lessons. Most frustrating was the time it took to try, not always successfully, to find examples on the computer but especially on the graphics calculator that would neatly and effectively demonstrate a particular concept of a certain lesson. Also, because the text book does not contain questions that are specifically designed for students with graphing devices, additional questions had to be found. One should be aware then, that trying a mode of technology is a major time commitment with respect to planning and preparing, consequently time should be taken before starting to teach the lessons to learn how to use the devices .

Teaching Time

The use of basic scientific calculators has made it possible for students to save computational time and spend the time saved solving different types of problems, ones that required them to think beyond the level required in performing basic calculations. Similarly with graphics calculators and computers, students now have the technology to save time previously spent manually graphing functions and spend that time elsewhere. That time could be spent conducting investigations that will allow them to collect data with which to draw their own conclusions, but there may be an overall cost in time. The investigations I outlined for my students took them longer than I had anticipated. My brief experience with teaching lessons combining technology and an investigation approach implies that technology certainly makes some previous tasks much less time consuming, for example sketching graphs, but that it also opens the door for a different type of lesson that in fact could take more time to execute than lessons for the similar topic without the use of a graphing device. When trying to predict how many periods a given topic that makes use of some form of graphing device will take to teach, one should not make the assumption that using graphing devices will automatically result in less time to cover the material. While that may be the case, the opposite may also be true as was illustrated in Topic One in this study. The actual time required to teach a topic will depend upon how the devices are to be used. If, for example, the type of activities that are structured for the students include discovery learning, then one may have to plan on additional classroom time for that topic.

Technology and the B.C. Curriculum Guide - Where and When to Use Technology

Another major implication for planning is raised in the controversial question of for which topics and to what extent a graphing device should be used in teaching mathematics. The B.C. Mathematics Curriculum Guide does not adequately address the issue, and does not reflect the potential that technology has to offer. Teachers planning a unit using technology are faced with the dilemma of what traditional pencil and paper work should be replaced by technology and what should be retained; what topics should be added or deleted; and how they could best make use of the technology at their disposal. An example of deciding what content to leave in and what to leave out occurred in Topic Ten with respect to maximum and minimum word problems. With graphing devices the topic can be extended to include problems for which the equations are not in standard form, which is not one of the I.L.O.'s for Mathematics 11. Even though it takes a little more class time, and is not required, the use of technology makes it very easy to solve some interesting questions that students otherwise would miss. The potential of technology to extend mathematics is so great that it does not make sense to ignore it.

The question of whether we are being restricted by an outdated curriculum in British Columbia is a very real one, but one that average classroom teachers may feel is beyond their level of expertise and their capacity for time involvement. Consequently a very real implication for about-to-be-involved teachers is that they will have to make personal decisions as to what types of questions to use the devices for, what topics to use them for, how to use them, and whether they should be used on tests. There are presently few or no provincial guidelines to assist a teacher through these questions. But even before these questions can be considered, teachers need to seriously consider the larger questions of why they want to use the devices and what purpose they are intended to serve. And once these questions have been answered, teachers will then have to decide how they are going to marry their philosophy about the use of technology with the B.C. Curriculum Guide.

Technology as a Teaching Tool

In exploring the topic of transformations of circles, a topic I was not totally comfortable with, I found that the computer very quickly gave me graphs for my speculations about the order of transformations and helped clarify the topic for me. The exercise I went through to teach myself about this topic convinced me that students could also learn certain rules the same way I had, by following a carefully structured investigation. On the other hand, while the graphing calculators and the computers certainly helped the students to see the effect changing certain constants in an equation had on the graph of that function and to derive conclusions about the rules for transformations, they did not seem to help the students understand the "why" behind the transformations, and I found that I still needed to supply the students with explanations about "why". The various forms of technology were successful in showing students the patterns of what was happening, and helped them see the rules, but apparently could not help them understand the reasoning behind the patterns. The implication for teachers is that even if they are using technology with their students they should still be prepared to explain the reasoning behind whatever mathematical principle they are teaching.

Technology and Teaching Strategies

A major implication for the planning of all lessons involving technology is to remember that the technological devices being used are intended as aids in the teaching and learning process and are not an end in themselves. I fell into the trap of thinking the graphing devices were some sort of magical toy to be revered and was trying to use them for too many aspects of my early lessons. Teachers need to be critical of the application of their particular mode of technology, and must remain open about when they should be used. Teachers should not be reluctant to use them in place of some long-standing traditional method if beneficial, but should also be prepared not to use them if a more traditional method is better. The degree to which a teacher "lets go" of traditional methods in order to use the new technology is a decision each teacher will have to make when planning the lessons. "Control" over the traditional lesson is another issue that, according to the literature (see e.g. Ruthven, 1992), is a problem to be faced by any teacher attempting to teach with technology.

Lessons requiring some form of technology require the same basic principles and concepts of teaching that produce quality lessons not involving the use of technology. For example, in several of the lessons referred to in the previous section "Changes to the Lessons" I mentioned changes I made to the lessons while the lessons were in progress, changes such as asking students for their conjectures as to the position of a graph before showing the graph on the screen (for example OP 7). Changes such as this one are independent of technology and simply reflect good teaching techniques. When designing lessons involving the use of technology, one should not become so infatuated with the devices that one abandons the principles of good planning and teaching. The devices should not drive the lessons, rather they should be used as an aid in instruction.

Another example of how ignoring a common teaching principle can cause a problem can be found in the lessons (for example Lesson OP 1) in which I failed to work through all of the examples and questions involving graphing devices before assigning them to the students, with the result that unnecessary confusion was created for the students. Only by following the basic rule of working through examples and questions before using them in the classroom can the teacher discover the problems hidden in a question before the students do, and therefore be a little more prepared for potential questions, especially the questions that relate to the use of the graphing device. In addition, when teaching a lesson one should not hesitate to change a strategy if that strategy does not seem to be working, or if a better idea suddenly comes to mind.

The general implications with respect to teaching strategies illustrated by these examples are to avoid overusing a particular technological tool, and to remember to adhere to sound teaching principles whether or not technology is part of the lesson.

Homework

The question of what homework to assign was one that bothered me in the lesson planning stages because I suspected that most students would not have access to their particular graphing device at home. The answer to this question is linked to an earlier point raised in this section with respect to not letting the technology drive the lessons. For example, in the early lessons in this study I did not assign graphing questions for homework because I felt that most students would be unable to use a graphing device for them. However, as the lessons progressed I realized that students could, if motivated, use their non-class time, such as after school, to work with the graphing devices. Or, if they chose not to follow that route, they could still do most homework questions in the traditional way.

The implication for making up assignments is not to be restricted by the fact that students may not have graphing devices at home. In some assignments there were questions that could be done only with a graphing device; in those instances I pointed out those questions to the students in class and had them do those first, so that they could be completed with the devices before the period ended. Many questions are easier and faster to do with a graphing device, but they can also be done without it. When designing an assignment for the students, therefore, one should decide what type of question to assign, allow in-class time for questions requiring a graphing device, then make it abundantly clear to the students that any homework assigned is to be done. If they wish to use a graphing device to aid them in their work then it is their responsibility to obtain the use of one. The expectation is that the questions are to be done with or without the aid of technology. Not having the particular device at home is not an excuse for not doing the assignment.

Test Design

Just as teachers must plan the lessons, so they must plan the tests. A difficult but fundamental question that arises is whether to use the graphing devices for the tests, and there is no general solution to this problem that covers the three different modes of technology used in this study. The answer to the question will depend upon the philosophical leanings of the teacher; at this time there is no prescriptive answer. The teacher must consider factors such as the Mathematics 12 Provincial Exam which does not allow graphics calculators, and the ILO's for Mathematics 11 which also do not account for the use of these graphing devices. Classes using the graphics calculators can be asked any questions the teacher desires, but for the classes using computers, there may be some restrictions.

Students in the Overhead Projector class, for example, are unable to enter their own data, which restricts the type of question they can be asked and the thinking required to answer others. One way I discovered that this mode could be used for tests was for the teacher to enter the data, display the resulting graph, and ask students questions related to that graph. This particular scheme can be unfair for students who have difficulty reading the screen from their desks (and the student questionnaires indicated that such was the case for several students in the class). Another possibility is for the teacher to make a printout of the graphs in question and put those graphs on the test paper. In either case, the students are unable to input their own data and to create their own graphs.

In the Computer Lab class the students were in pairs, consequently I gave tests that did not require the use of the computers because I wanted a mark that was strictly a reflection of an individual's performance. I did, however, give quizzes and allow them to work with their partner in finding a solution and gave each student in the pair the group mark. This particular method of giving quizzes has a time implication for the teacher because several different quiz questions need to be constructed so that students can not obtain answers to their questions from the next computer screen.

If a teacher wished to use the computers in the lab for tests, then the student pairs would have to have a common mark for each pair for the test. If this scheme was unpalatable, then the class could be divided in half for testing purposes and students could be scheduled into the computer lab one-half of the class per period so that each student could operate his or her own computer. But this type of arrangement could create problems with the administration and other teachers. If the computer room was such that students had their own computers, then perhaps tests could be given that would make use of the computers, but it would probably require more than one form of the test, and while this is certainly possible their creation would be time consuming.

With a class using graphics calculators it is very easy to give questions involving the use of the calculators. The question then becomes, as it does with the computer class if each student has his or her own computer, what type of question to put on the tests. One feasible plan would be to divide the test into two distinct components. The first would involve questions that do not require the use of graphing devices and are related directly to the ILO's of the course, and the second would require the use of graphing calculators to do types of questions similar to those done in class that did require calculators.

For all modes of technology used in this study, the types of tests that can be designed and the various kinds of technology-related questions that can be asked will vary depending upon the mode of technology being employed. Further, within a given mode the types of questions that one wishes to ask will vary depending upon the teachers' interpretations of the purpose of technology as it relates to mathematics instruction. The issue of testing a technology-based curriculum is another issue needing more research, as indicated in the literature (see e.g. Heid, Matras, & Sheets, 1990).

Technology and Apathy

One of my concerns when planning lessons involving the overhead projector and a single computer was whether students would become bored if they were restricted to watching the graphs appear on the screen and were not involved in the operation of the computer. The students' questionnaires revealed that some students in the Overhead Projector class did find this use of technology monotonous. In addition my observation notes show that I too was occasionally bored when I was entering all the data. The implication here is obvious. If only a single computer is available for one class, apathy can result and this problem should be kept in mind when lessons are being planned. Some techniques can be built into the lessons to minimize the problem. Strategies that worked for me include restricting the use of the computer to short time intervals, allowing various students to take turns inputting the data, and requiring students to predict what the next graph in a series of transformations might look like before it appears on the screen. Another suggestion, and one that came directly from some students, would be to design an entire period for instructing the class in the operation of the software so that students could then do some independent work on their own time. But students did indicate that, even if it was boring sometimes, it was much better to use a single computer than none at all.

After seven weeks of working with the various forms of technology, it became apparent to me that while misuse of the overhead/computer combination certainly produced boredom in that class, it seems to be true that prolonged use or too much use of any device can result in a certain amount of boredom. Several students in the Graphics Calculator class also commented that they became bored, although the majority felt the opposite, consequently a suggestion would be to vary methods of instruction. Even if only one mode of technology is available, one should not use it every day or at least should vary the way it is used. A central theme returns: use technology as an aid, not as an entity unto itself.

Conclusions

The purpose of this study was to investigate the use in the classroom of three different forms of technology for teaching graphing of functions and relations. The graphing devices were to be used in the teaching of a section of Mathematics 11 in order to answer some questions regarding the use of these tools by a technologically inexperienced teacher; in particular, whether these forms of technology could be used successfully and whether such use was desirable. The evidence presented in this paper suggests strongly that such teachers should not only be able to use these devices successfully, but that it is also very desirable, from many points of view, for that methodology to be employed. The study further suggests that teachers should not hesitate to involve themselves and their students with technology, regardless of the level of expertise of the teacher with technology.

The conclusions of this study confirm much of what is written in the literature about technology and mathematics education. In addition, they provide insight into questions that are not specifically addressed in the literature. The majority of the research that was consulted for this study was written by "experts" rather than by "typical" classroom teachers, and in general they have written about the effects on learning of a particular mode of technology. This study is written from the point of view of a classroom teacher who had no bias or previous experience with the technologies used, who was simply interested in the every-day practical issues surrounding the use of the technologies, and consequently describes what it is like to use those technologies from a different, and perhaps more practical, point of view.

The response of the students to using some form of graphing tool in the classroom showed that the majority of them preferred learning using the devices, and further suggested that technology should be used more in the teaching of mathematics, preferably with equipment that allowed individual use. Their view with regard to all students having individual access to a graphing device coincides completely with the direction envisioned by many mathematics educators, who believe having continual access to a device is essential to maximizing its potential (see e.g. Hill, 1993; Ruthven, 1992).

The question of which of the three modes of technology proved to be the "best" is a complex one. As the section on Comparison of the Modes suggests, there are many factors to consider, but if one assumes that all modes are equally accessible, then for most topics in the chapters of Mathematics 11 covered in this study the advantage is decidedly to the mode that allows students to have regular individual access to a device. In this study that implies that the Graphing Calculator and the Computer Lab are the most favorable modes. In general, any mode which allows the student individual interaction with the device is superior to a non-interactive mode. In a comparison between the two interactive tools, the software that is available is a major factor in deciding which tool should be used. Given software that demonstrates what you want it to, the computers have a slight advantage over the graphics calculators. But if the software is not totally satisfactory, then the advantage is with the calculators with their small size and resulting portability. Since some topics are more suited to one mode than another, it may also be preferable to use more than one mode to teach a given unit. On the basis of this study, however, I would suggest that any of the three modes is preferable to not using any technology at all. Therefore if only one mode is available, it should be used.

If the three modes are not equally accessible, and if "accessible" is defined as "having continual access to," then because their cost is significantly lower, the graphics calculators would be the most desireable mode.

In addition to providing answers to the questions posed in Chapters One and Three, the classroom research portions of this study produced some unexpected benefits and raised some unanticipated issues. One of those benefits was the unexpected effect the procedure had on my own teaching. Taking notes and critically looking at my lessons forced me to examine how and what I was teaching and in general to re-examine my philosophy with respect to mathematics education and to evaluate whether I was following my philosophy. I had to determine specific answers to the questions of what I thought the purpose of mathematics education was and what role I felt technology had in helping achieve that purpose. As a result of examining those questions I decided to modify some of my objectives and methods, not just for the course involved in the study, but for all my courses. I believe that any teachers who decide to undertake the challenge of teaching with technology will find themselves faced with the same exercise of self-evaluation of philosophy, and will benefit from it.

In addition to forcing an examination of my own teaching, this study also encouraged me to critically examine the British Columbia Secondary Mathematics Curriculum, and led me to conclude that it is out of date and in definite need of modification. The curriculum does not reflect the availability of various forms of new technology. The Curriculum Guide mentions technology, but does not indicate which sections lend themselves to the new devices, or how those devices could be used to help meet the current course objectives. The course objectives also need to be examined to see which ones may not be relevant to our current level of technology, and what new objectives might be included. These problems are a concern not only for mathematics educators in

British Columbia, but for educators anywhere the new technology is available (see e.g. Burrill, 1992; Dick, 1992; Hill, 1993).

The entire philosophy of mathematics education needs to be debated in light of the power of the new technology. Do we continue with a curriculum that relies heavily on memorizing, or do we develop a curriculum that encourages and requires exploring and conjecturing? The answer would seem to be that we change, but changes can not be expected to happen quickly, consequently what is needed immediately are some guidelines for using technology with the current course objectives, both for teaching and testing. The problem of philosophical direction will take longer, but the Ministry of Education in British Columbia should be initiating a process to investigate the issue now.

If technology is to be used to its full potential in British Columbia secondary schools, then the Ministry of Education and the British Columbia Association of Mathematics Teachers need to assume leadership roles in providing immediate resources for teachers. Among those resources should be information about which topics are suitable for the various forms of technology and how the devices could be used. This information needs to be made available to all mathematics teachers in the province through some type of regular district workshops. As several educators have noted (see e.g. Bright et al., 1992; Ruthven, 1992) teacher-in-service is an essential ingredient in successfully introducing a new program. A one-workshop-per-year approach is not good enough. What is needed is a series of workshops, perhaps one per month or one per major curriculum strand, that would concentrate on what the teachers in that district need to know, whether it is the basics of how to operate the tools or where in the curriculum to use them.

Technology is here, it is available, and it has demonstrated, in this study at least, to be a desirable asset for both students and teachers. The next step is to provide direction, resources, and training to the teachers.

APPENDIX A

LETTER GRADE DISTRIBUTION FOR ALL MATHEMATICS CLASSES

HANDSWORTH SECONDARY SCHOOL

TERM ONE, DECEMBER, 1991

LETTER GRADE DISTRIBUTION FOR ALL MATHEMATICS CLASSES

HANDSWORTH SECONDARY SCHOOL

TERM ONE, DECEMBER 1991

Course	Percentage of Students Receiving A Given Letter Grade							Total Number of Students
	A	B	C+	C	C-	D	E	
Math 8A	0	50	0	13	38	0	0	8
Math 8	31	34	12	11	10	2	1	177
Math 9A	8	31	31	23	8	0	0	13
Math 9	16	26	13	14	14	12	3	184
Math 10A	13	40	7	13	20	7	0	15
Math 10	11	25	16	21	19	3	4	189
Math 11A	5	21	21	21	16	5	5	18
Math 11	14	30	16	17	15	4	3	187
Into Math 11	5	16	26	21	16	11	5	38
Math 12	0	22	25	33	17	2	2	60
Math 12 E	26	48	16	9	2	0	0	58

APPENDIX B

SAMPLE LESSON PLAN

SAMPLE LESSON PLAN

UNIT 6 LESSON 4

TOPIC: GRAPHING $y = f(x - p)$

A. OBJECTIVE

To be able to sketch, without the aid of a graphing devices, the graph of $y = f(x - p)$ by translating $y = f(x)$, where $y = f(x)$may be one of the basic eight functions studied in this chapter or any other function whose graph is given.

B. HOW THE GRAPHING DEVICE IS TO BE USED

Computer Lab and Graphics Calculator Classes: Students will work through a sheet of questions that will ask them to use their graphing devices to quickly obtain the graphs of several functions. These graphs will be sketched into their notebooks.

Overhead Class: Students will have the same sheet of questions as the other two classes, but the teacher and students will obtain the graphs together as the teacher or a student will generate the graphs using the computer and will display them on the large screen using the overhead projection device.

C. LESSON PLAN

1 Students, in their groups, will work through the investigation sheet, then discuss and record their conclusions.

2. Group conclusions will then be discussed by the entire class with various groups taking turns leading the discussion.

3. Some of the questions in the text book will be discussed by the entire class:

   -page 203 #3; page 245 #1, #2(b).

4. A discussion will be held regarding how to use the axis of symmetry as technique in graphing.

D. ASSIGNMENT

1. Page 203 #1c, 4, 5ac (using a graphing device for checking only)
2. Page 245 #2a, 3, 4ab, 6, 7 (#8 as a scholasship question. Students will need to use a graphing device to obtain the graph of $f(x) = \frac{1}{x^2+1}$, then use it just for checking.)
3. Some questions from a supplementary sheet. (Questions taken from *Precalculus Mathematics - A Graphing Approach* - Demana and Waits)
4. Scholarship Question - Page 204 "Investigate"

MATHEMATICS 11 CHAPTER 6 STUDENT INVESTIGATION SHEET

GRAPHING $y = f(x - p)$

A. QUESTION

1. How does the graph of $y = (x-3)^2$ difer from $y = x^2$ or $y = x^2 - 3$?
2. How does the graph of $y = f(x-3)$ differ from $y = f(x)$, where $f(x)$ is any of the eight functions studied in this unit?
3. In general, how does the graph of $y = f(x-p)$ differ from $y = f(x)$?

B. INVESTIGATION

1. Using your graphing device, graph each of the following on a standard screen, then copy the results into your notebook.

a) $y = x^2$	b) $y = (x-2)^2$	c) $y = (x+3)^2$
a) $y = \frac{1}{x}$	b) $y = \frac{1}{x+3}$	c) $y = \frac{1}{x-4}$
a) $y = \sqrt{x}$	b) $y = \sqrt{x-3}$	c) $y = \sqrt{x+2}$

C. CONCLUSIONS

Discuss your graphs in your group and arrive at a conclusion with regard to:

Changing x to $(x-p)$ in a function, i.e. changing $y = f(x)$ to $y = f(x-p)$ results in the following change to the graph of $y = f(x)$:

____

D. ASSIGNMENT (Use graphing devices only for checking your solutions.)

1. Page 203 #1c,4,5ac
2. Page 245 #2a,3,4ab,6,7
3. Scholarship - page 245 #8 - use graphing device to obtain the graph of

$$f(x) = \frac{1}{x^2+1}$$

4. Supplementary Sheet - questions to be announced

APPENDIX C

INTERVIEW QUESTIONS USED FOR TAPED INTERVIEWS
WITH STUDENTS IN THE OVERHEAD PROJECTOR CLASS

INTERVIEW QUESTIONS USED FOR TAPED INTERVIEWS WITH STUDENTS IN THE OVERHEAD PROJECTOR CLASS

The following introductory comments and questions were used as a basis for conducting taped interviews with two students in the Overhead Projector Class.

INTRODUCTORY COMMENTS

Using the computer and overhead projector was an experimental program - we were the only class at Handsworth to do it, so I am interested in student views on the program. The questionnaires that all students in the class filled in gave many interesting insights, and I would like to get your reaction to some of the points raised by the students. By the way, you were selected randomly from the class list and are being interviewed along with one other student from your class as a representative of the class and the views of the class. But the opinions you give are your own, and I would hope you would be frank. Tell me exactly what you think.

1. A majority of the students thought that using the computer made learning easier. In what ways do you think it made learning easier?
2. Would you like to have been taught more about how to work the computer so that you could go to the computer room on your own and either do homework or experiment with the program? Do you think many students would give up their unstructured time to work on their own in the computer lab?
3. A common comment was to provide students an opportunity for more hands on time with the computer. But if it is not possible to book all classes into the computer lab, should we continue to use a single computer and an overhead as we did this year, or should we drop the idea of using the computer entirely?

4. Do you think using the computer individually would have increased your enjoyment of the unit? Would it have increased your learning? Are there any other reasons why you would like to have been able to work individually at a computer?

5. Did we spend too many periods using the overhead and the computer? Did it tend to become tedious and monotonous? Was it boring (more than normal) to sit and watch and not be able to do the questions yourself?

6. Sometimes taking a different approach to learning can take more classes than a traditional method might, and consequently that class may fall behind other classes time-wise. Is this a concern to you? Do you worry if other classes are farther ahead in the text than your class?

7. We used a computer to do some of our work in the graphing unit, but obviously we could not use it for the exam. Was this ever a concern for you?

8. One of the purposes of using the computer was to quickly and accurately obtain graphs that could be used in order for students to draw their own conclusions about something. Do you like the idea of having to draw your own conclusions, or would you rather have the teacher explain what we are doing and tell you the rules?

9. One of the prime objectives of the chapter on graphing was to see how changing numbers in an equation resulted in a shift in position and /or shape of the original graph. We used the computer so that we could very quickly get the correct graphs, but several students stated that they would rather have done the graphing by hand because they felt slow repetition helps you learn better. What do you think?

10. In general, do you think repetition helps you learn?

11. Suppose you had an equation to graph (a type not done before). Would you prefer to graph it by hand and then use the computer to check your graph, or would you

prefer to have the computer do the graph and then you learn the short cut for doing it yourself from the computer graph?

12. On a line of thought similar to a previous question, do you think that because we used a computer to do the graphing that you did not learn as much about graphs? What is it that you are learning or not learning?

13. Some students expressed the view that using the computer is fast but that it is really just a short-cut that in fact really restricts learning. That is, the computer doesn't tell you "why" something is happening, it just helps you do it. What do you think about that?

14. Did you ever wonder "why" as the computer did things to a graph on the screen? Or are most students more interested in the final result and in memorizing some rule than in knowing the "why" behind something?

15. What effect, if any, do you think the computer had on your mark?

16. When you are asked to draw a graph from an equation, what do you think are the benefits of doing it with a computer compared to pencil and paper? The benefits of using pencil and paper compared to a computer?

17. In what ways is the computer and the overhead an asset (if they are) in:
    a) learning what you need for the exam?
    b) understanding what you are learning?

18. My idea in using the computer and the overhead was to have you use them to discover the rules of transformation. Then we were to use the rules to draw graphs with paper and pencil. Did I make this idea clear?

19. In summary, what do you think are the major advantages and disadvantages of using the computer with the overhead projector?

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