PREFERENCES RECOVERY IN ADDRESS MODELS OF PRODUCT DIFFERENTIATION

by

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Preference Recovery in Address Models of Product Differentiation

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ABSTRACT

Product differentiation is a feature of most modern markets. The economics of product differentiation has relied on two major approaches: the representative consumer approach and the address approach. It is argued in the literature that address models are more appropriate for studying most real cases of product differentiation. Yet little empirical work has been done in this framework, due primarily to the absence of preferences recovery techniques for address models.

The purpose of this thesis is to begin the development and implementation of preferences recovery techniques for address models. In address models, goods are described by points in a continuous space of attributes or characteristics. Consumer preferences are defined over all potential products and each consumer has a most preferred product known as his or her ideal address in the product-attributes space. Aggregate consumer preferences for diversity are captured by a preferences density function in some space of utility parameters. Preferences recovery involves the estimation of the preferences density function, given aggregate data on product attributes, prices, and quantities sold.

The bulk of the thesis is on recovering preferences in the space of lotteries. Lotteries are chosen because we need "products" that can be easily created in the laboratory to generate sufficient data. Given a parametric form for preferences from theories of choice under uncertainty, we create a parameter space that describes individual preferences for lotteries. Aggregate preferences are

represented by a probability density function in this parameter space. This density function is estimated using data generated from experiments and the proposed technique. A test based on the recovered preference density function is constructed to test if a particular theory of choice under uncertainty adequately explains the choices people make.

Using this approach, we test the expected utility (EU) theory and three generalized expected utility (GEU) theories. The results show that none of the GEU models is an improvement over the EU model in explaining the data, and that all models must be rejected as adequate models of choice under uncertainty.

As an additional application, we also demonstrate the preferences recovery in a standard address model of product differentiation and apply it to a real case of product differentiation in the context of BC ferry services.

DEDICATION

To my mother Song Xoulan whose wish was

to be able to read my letters,

with much love to

Limin Liu

and

Jesse Liu

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter One

INTRODUCTION

Traditional economic theory has been based on the assumption that firms produce a single homogeneous product---one product for each industry. Today, virtually all firms produce capital goods, consumers' goods, or services over a range of differentiated products. Over the past two decades, economists have learned to model the demand for differentiated products and the competition among firms producing differentiated goods. These developments have created a better understanding for a number of issues in international trade, industrial organization and the economics of growth.

The economics of product differentiation has relied on two major approaches: the address approach and the representative consumer approach, or non-address approach. The representative consumer approach follows Chamberlin's monopolistic competition model in which goods are simply goods, and in which any pair of goods is viewed by the consumers as having the same degree of substitution (Chamberlin's symmetric assumption in demand). In contrast, address models of product differentiation follow Hotelling's (1929) seminal article by assuming that products have meaningful descriptions, or addresses, in some product-attributes space; and that consumers have well-defined "locations" in this space. Thus, in this world, the consumer's degree of substitution between any pair of goods is not identical, and the competition among firms is localized. In their 1989 survey, Eaton and

Lipsey argue that address models seem to be more appropriate for studying most real cases of product differentiation because they are more consistent with the observed facts.[1]

In address models, goods are described by points in a continuous space of attributes or characteristics. Such models assume that (1) individual consumers have preferences defined over the space of product attributes, (2) the preferences of consumers are diverse, (3) it is possible to produce any product in the attribute space, and (4) there are significant costs of developing any product in the attribute space. In such models an array of differentiated products emerge as profit-seeking firms vie for the patronage of diverse consumers. Product development costs limit the number of products that are produced in equilibrium with the consequence that firms can exercise market power. In addition, there can be too much or too little differentiation in free-entry equilibrium, and the divergence from the optimum can be significant. This view of product differentiation raises a number of difficult policy issues. See Archibald, Eaton and Lipsey (1986) for a full discussion.

We see a key deficiency in the existing literature. While there has been some notable empirical work in the non-address branch, there

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[1] Anderson, de Palma, and Thisse (1989) argue that there is no necessary distinction between these approaches when the dimentionality of the space in which products are differentiated is large relative to the number of products, and goods are exogenously located in a symmetrical pattern in this space. Although interesting, this does not remove the distinction since the two approaches are not necessarily equivalent when the number and location of goods are endogenous.

has been little on preference estimation in the address branch.[2] In most potential applications, a major difficulty is a preference recovery problem: given aggregate data on product attributes, prices, and quantities sold, how does one go about recovering the diverse preferences that generated the data? Without knowledge of the underlying preferences, one cannot offer convincing, empirically based answers to such important questions as: Is there too much or too little product differentiation? What new product niches are likely to be profitable? What role should public policy play in markets for differentiated products? In short, we have as yet no empirical foundation which can be used either to test the theory or to calibrate it for purposes of public policy.

It is the objective of this thesis to begin the development and implementation of preference recovery techniques for address models of product differentiation. The ultimate purpose is to use these techniques to determine empirically the usefulness of the address approach to product differentiation. In particular, do real consumers

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[2] For example, and Harris (1984) used a general equilibrium analysis to calibrate a Chamberlinian model of product differentiation embedded in an open-economy model. There is also much empirical work in the discrete-choice, random preferences models arising from the early work of McFadden (e.g., McFadden, 1974). These include Train (1986), Feenstra and Levinsohn (1989) and Berry (1992). Anderson, de Palma, and Thisse (1993) contain an excellent exposition on how these models fit into the literature on product differentiation and under what conditions these models can be synthesized from an econometrics point of view. Recently, Burton (1992) adopts some nonparametric smoothing techniques to estimate an expenditure density function in the address framework of product differentiation. Finally, preferences recovery methods also have considerable appeal in marketing research (see,e.g., Kamakura and Srivastava, 1986).

see real products as points in some common attribute space? Can we recover those preferences from observed behavior and then use these preferences to predict further behavior?

To develop these preference recovery techniques and test their usefulness, we need "products" that can be created and manipulated at will in the laboratory. For this purpose, we choose lotteries. Our lotteries can be represented by a probability distribution (p_1, p_2, p_3) over a set of outcomes, (x_1, x_2, x_3). Given a parametric form for preferences from theories of choice under uncertainty, we create a parameter space that describes individual preferences over these lotteries. Aggregate preferences are represented by a probability density distribution in this parameter space. Preferences recovery refers to estimating such a density function, using choices people make in classroom experiments. To illustrate the usefulness of the technique, we construct a new test, based on the recovered preferences, to test theories of choice under uncertainty. Consequently, the bulk of the thesis is on recovering preferences in the space of lotteries and testing theories of choice under uncertainty. But to show that the preference recovery is a much more generalized issue than demonstrated in the case of lotteries, we also include an additional example, in which a standard address model of demand for differentiated products is estimated using the same methodology and applied to a real case of product differentiation.

The rest of the thesis is organized as follows. Chapter two presents a review of literature on theories of choice under uncertainty. While including a brief overview of the theoretical development, the survey focuses on the empirical studies of this branch of the literature.

In Chapter three, we present the experimental data that is used to recover preferences in the subsequent chapters. It also includes a description of the experimental design and a brief analysis of the data using existing methodologies in the literature.

The preferences recovery technique is developed in Chapter four for the expected utility model. A test is constructed based on the recovered preferences to determine if the theory adequately explains our experimental data.

Chapter five presents three generalized expected utility models as alternative models for the demand for lotteries. These models are estimated and tested using the same data and the same testing methodology.

Finally, as an additional application, Chapter six develops the preference recovery techniques in a standard address model of product differentiation and applies them to a real case of product differentiation in the context of BC ferry services. Conclusions and extensions of the thesis are also provided in this chapter.

Chapter Two

LITERATURE REVIEW

ON THEORIES OF CHOICE UNDER UNCERTAINTY

Over the past five decades, expected utility (EU) theory has dominated the theory of choice under uncertainty. However, cumulative empirical evidence in the literature has shown that people's actual choice behavior under uncertainty is systematically inconsistent with the predictions of the EU theory (For example, see Allais, 1953, 1979; MacCrimmon, 1968; Kahneman and Tversky, 1979). The "Allais paradox" (Allais, 1953) was the first example of the limited descriptive ability of the EU model.

The inadequacy of the EU model in explaining experimental data has led to theoretical efforts to propose alternative theories of choice under uncertainty. Since most alternative models are considered generalizations of the EU theory (e.g. Karmarkar, 1979, and Machina, 1982), they are classified as generalized expected utility (GEU) theories. The GEU models were designed to accommodate EU violations. Since they all include the EU model as a special case, they have more descriptive power than the basic EU model. The question is: How much better are these GEU models in explaining the data generated from laboratory experiments? Several recent empirical studies including Battalio, Kagel, and Jiranyakul (1990), Camerer (1989), Chew and Waller (1986), and Marshall, Richard, and Zarkin (1992) have been conducted to test alternative models of choice under uncertainty. The results are rather disappointing: No single theory could explain all the data collected from these studies.

This chapter reviews both the theoretical development of theories of choice under uncertainty and empirical studies of them.[3] Section 2.1 provides a historical overview of theories of choice under uncertainty. The intention is to show how the economics of uncertainty has gone from one of the most settled branches of economics to one of the most unsettled over the past decade. Section 2.2 presents the expected utility paradigm developed by Von Neumann and Morgenstern (1944) and violations associated with it. Section 2.3 outlines and examines several generalized expected utility models. Section 2.4 briefly surveys some empirical studies on testing theories of choice under uncertainty. The survey focuses on the common approach used in this branch of literature and major results found in these studies. The last section, Section 2.5, discusses problems associated with existing empirical studies, and how the current study contributes to this line of literature.

2.1 A HISTORICAL OVERVIEW

From a historical point of view, theories of choice under uncertainty can be traced back to the 17th century when modern probability was developed. Early theories of games of chance assumed that the attractiveness of a gamble with payoffs, x_1,. ..., x_n, and

[3] For a more thorough survey of literature, see Schoamaker (1989), Machina (1983a, 1983b, 1987, 1989) and Camerer (1989).

associated probabilities $p_1, p_2, \ldots, p_n$ was given by the mathematical expectations of monetary gains or losses, i.e. $\bar{x} = \sum_{i=1}^{n} p_i x_i$. The *St. Petersburg Paradox* revealed the inadequacy of this principle: Suppose someone presents you a game that involves tossing a fair coin until it comes up heads, and offers to pay you $1 if it happens on the first toss, $2 if it happens on the 2nd toss, $4 if it takes three tosses, ..., $\$2^{(n-1)}$ if it takes n tosses to land a head. How much would you be willing to pay to play this game? According to the principle of mathematical expectations, the expected value of this game is

$$\bar{x} = (1/2) \times 1 + (1/2)^2 \times 2 + \ldots + (1/2)^n \times 2^{n-1} + \ldots$$

$$= (1/2) + (1/2) + \ldots + (1/2) + \ldots$$

$$= \infty$$

However, the actual amount that people are willing to pay is finite, often less than $10. This is the St. Petersburg paradox.

To explain why people would pay only a small amount for a game of infinite mathematical expectation, Bernoulli proposed that people maximized expected utility $EU = \sum_{i=1}^{n} p_i u(x_i)$ rather than expected monetary value. The utility function $u(x_i)$ he proposed was logarithmic, exhibiting diminishing marginal utility of wealth. It can be shown that the expected utility of the coin tossing game given such a utility function is indeed finite, which was the key to resolving the St. Petersburg paradox. However, Bernoulli did not address the issue of how to measure utility, nor why his expectation principle would be rational.

It was not until John Von Neumann and Oskar Morgenstern (1944) that expected utility maximization was formally proved to be a rational decision criterion.[4] Using five quite reasonable postulates, they showed the existence of a utility index, u(.), such that the expected utility of a risky prospect, $EU=\sum_{=1}^{n} p_i u(x_i)$, represents the individual's preference ordering over risky prospects, $(p_1, \ldots p_n; x_1, \ldots, x_n)$. This is the famous expected utility theory that has played a leading role in theories of choice under uncertainty to date. Given its normative appeal and simplicity, the EU theory has been used in many applications in the economics of uncertainty since the second world war.

While most researchers at first accepted VNM's theory, Allais (1953) questioned the independence axiom, which is one of the crucial axioms in EU. By devising counter examples, he showed that the EU theory is not compatible with the preference for lotteries in the neighborhood of certainty. This has become widely recognized as the "Allais Paradox".

The Allais paradox involves the following two questions:

1) Do you prefer situation A to situation B?

Situation A:

— certainty of receiving $1 million

Situation B:

---

[4] Though axiomatic expected utility theory had been developed earlier by Ramsey (1931), the account of it given in the 'Theory of Games and Economic Behavior' by Von Neumann and Morgenstern is what made it "catch on".

— a 10% chance of winning $5 million

an 89% chance of winning $1 million

and a 1% chance of winning nothing

(2) Do you prefer situation C to situation D?

Situation C:

— an 11% chance of winning $1 million

and 89% chance of winning nothing

Situation D:

— a 10% chance of winning $5 million

and a 90% chance of winning nothing

It can be shown that, according to the EU theory, an answer of "A" to the first question implies an answer of "C" to the second question, and a choice of "B" in the first question implies a choice of "D" in the second question. However, after analyzing the answers, Allais found that 53 percent of subjects chose "A" in the first question and "D" in the second question, which is clearly inconsistent with EU predictions.

Just as the St. Petersburg paradox led Daniel Bernoulli to replace the principle of maximization of the mathematical expectation of monetary values by the principle of maximization of expected utilities, the Allais paradox has led researchers to reconsider the expected utility theory.

Over the last decade, many researchers have developed generalized expected utility theories in attempt to resolve the Allais paradox. Unfortunately, unlike the case of the St. Petersburg paradox, the Allais paradox has not yet been resolved satisfactorily.[5]

2.2 *THE EXPECTED UTILITY MODEL AND ALLAIS PARADOX*

Consider the following lottery with three final outcomes: $(p_1, p_2, p_3; x_1, x_2, x_3)$, where $\sum_{i=1}^{3} p_i = 1$ and $x_1 > x_2 > x_3$, (x_1 is preferred to x_2 which is preferred to x_3). This lottery would yield outcome x_i with probability p_i. Given fixed outcomes, such a lottery can be represented by a point in the Marschak-Machina triangle { (p_1, p_3); $p_1 \geq 0$, $p_3 \geq 0$ and $p_1 + p_3 \leq 1$ } as in Figure 2.1.[6] According to the expected utility theory, the expected utility of consuming such a lottery is given by

$$EU = P_1 \; u(x_1) + P_2 \; u(x_2) + P_3 \; u(x_3), \qquad (2.1)$$

where u(.) denotes the Von-Neumann Morgenstern utility index. The assumption $x_1 > x_2 > x_3$ implies that $u(x_1) > u(x_2) > u(x_3)$. Given the utility index, EU has the property of linearity in probabilities. Graphically, the linearity property of the EU model can be illustrated in terms of

[5] As will be discussed in section 2.3, no single alternative theory could explain all the data generated from experiments conducted in existing empirical studies.

[6] Following the existing literature, we restrict our discussions to the three-event lotteries. The Marschak-Machina triangle adopted by Marschak (1950) and popularized by Machina in the 1980's is a very convenient graphical representation of such a lottery.

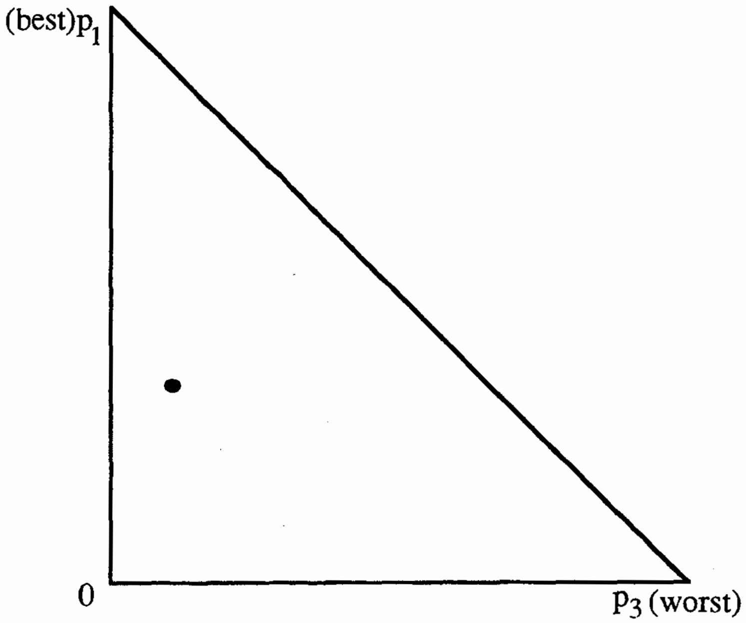


Fig. 2.1: The Marschak-Machina Triangle

indifference curves in the Marschak-Machina triangle. An indifference curve of the EU model is a set of probabilities (p_1, p_3) with the same expected utility $\bar{u}$:

$$\bar{u} = p_1 u(x_1) + (1-p_1-p_3)\, u(x_2) + p_3 u(x_3) \qquad (2.2)$$

Rewriting equation (2.2) in slope-intercept form,

$$p_1 = \frac{\bar{u} - u(x_2)}{u(x_1)-u(x_2)} + \frac{u(x_2)-u(x_3)}{u(x_1)-u(x_2)}\, p_3. \qquad (2.3)$$

The indifference curve is a straight line of slope $[u(x_2)-u(x_3)]/[u(x_1)-u(x_2)]$. Given the utility index, the slope is constant. Thus indifference curves are parallel straight lines with more preferred indifference curves lying to the northwest as in Figure 2.2a.

2.2.1 Allais Paradox and the "Fanning-Out" Effect

The *Allais paradox* is restated here for the purpose of illustrating the fanning-out effect. This problem involves choosing one lottery from each of the following pairs:

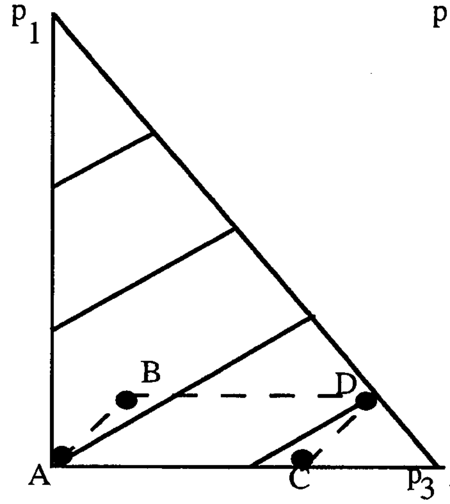
$$\begin{cases} A : (0,1,0;\ x_1,\ x_2,\ x_3) \\ B : (0.1,\ 0.89,\ 0.01;\ x_1, x_2, x_3) \end{cases}$$

$$\begin{cases} C : (0.0,\ 0.11,\ 0.89;\ x_1, x_2, x_3) \\ D : (0.1\ ,\ 0,\ 0.9;\ x_1, x_2, x_3) \end{cases}$$

where $\{x_1, x_2, x_3\}=\{\$5m, \$1m.\$0\}$. These four lotteries form a parallelogram represented by the broken lines in the (p_1, p_3) triangle, as in Figures 2.2a and 2.2b. The parallel straight lines in Figure 2.2a are EU indifference curves. If these indifference curves are flatter than the broken lines connecting lotteries A and B, or C and D, EU implies a choice of B in the first pair and D in the second pair; similarly if EU indifference curves are steeper than the broken lines, the choice would be A in the first pair and C in the second pair. However, many researchers including Allais (1953), Morrison (1967), Slovic and Tversky (1974), and Kahneman and Tversky (1979) have found that the modal if not majority of subjects have chosen A in the first pair and D in the second. According to Machina (1987), this suggests that indifference curves are not parallel but rather fan out as in Figure 2.2b.

2.2.2 Violations of the EU Theory

The Allais paradox exemplifies a class of similar violations of expected utility theory. The two most well-known violations are the common consequence effect and the common ratio effect. The Allais paradox is a common consequence violation.



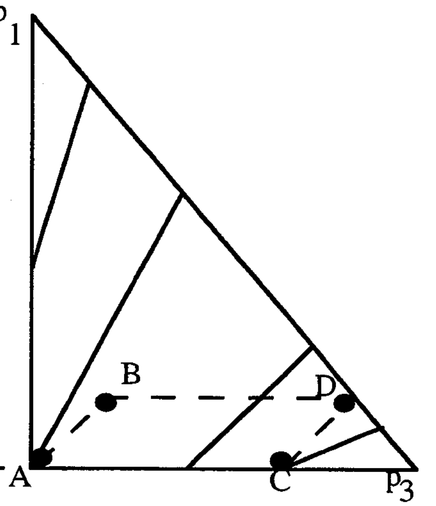


Fig.2.2a: EU Indifference Curves and Allais Paradox

Fig.2.2b: Fanning-out Indifference Curves and Allais Paradox

The common consequence effect can be demonstrated in Allais' experimental problem, rewritten using the compound lottery structures shown in Figure 2.3. Here each branch represents a sublottery. According to the expected utility theory, A is preferred to B if and only if

$$u(\$1M) > 0.89\ u(\$1M) + 0.1\ u(\$5M) + 0.01\ u(0)$$

$$\text{or} \quad 0.11\ u(\$1M) > 0.1\ u(\$5M) + 0.01\ u(0).$$

This also implies that C is preferred to D. However, as mentioned in the previous section, researchers have found a tendency for subjects to choose A in the first pair and D in the second pair. The difference between the first pair (A, B) and the second pair (C, D) is that the sublotteries in the lower branches of the first pair have a "common consequence" of $1 m, and the sublotteries in the lower branches of the second pair have a "common consequence" of $0. EU implies that these common consequences would be "irrelevant" in choosing between A and B in the first pair and C and D in the second pair. However researchers such as Kahneman & Tversky (1979), MacCrimmon(1968) and MacCrimmon and Larsson (1979), and many others, have found a tendency for subjects to choose A in the first pair and D in the second pair. Given that the sublotteries of the upper branch are the same in both pairs, such a swing in preference from more risky to less risky sublotteries in one branch of a compound lottery as the sublottery in the other branch

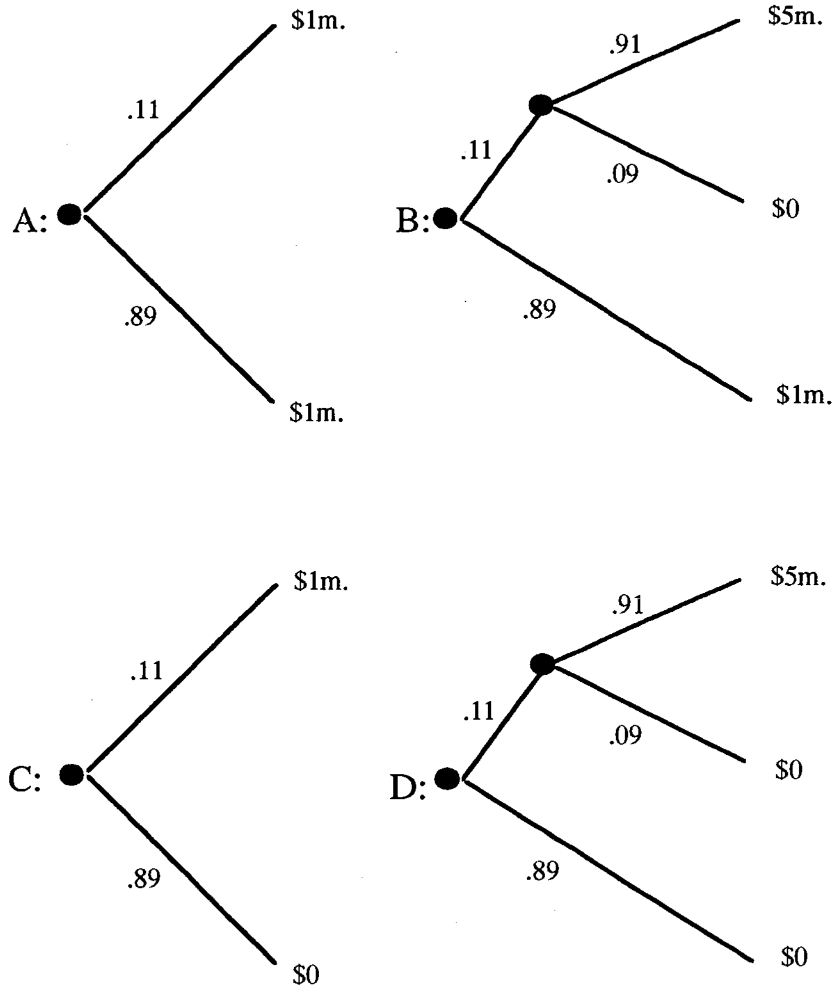


Fig. 2.3: The Common Consequence Effect

improves (in the sense of stochastic dominance) is generally known as the "common consequence effect". Intuitively speaking, as we move from the lower-right corner of the Marschak - Machina triangle to the upper left corner, people prefer not to bear further risk in the worst event, and prefer the less risky lottery.

Another class of systematic violation is called the "common ratio" effect. It can be illustrated in Figure 2.4. In Figure 2.4, $p>q$, $0 < x < y$ and $0 < r < 1$. The term *common ratio* derives from the equality of prob(x)/prob(y) in the first pair and in the second pair, which is p/q. Given the expected utility hypothesis, a rational individual should choose either L_1 in the first pair and L_3 in the second pair, or L_2 in the first pair and L_4 in the second pair. However researchers have found from experiments that the modal response is inconsistent with this EU prediction. The following is an example initially proposed by Allais (1953) and later used by Kahneman and Tversky (1979) to demonstrate the common ratio effect. In this example, y=$4000, x=$3000, p=1.0, q=0.8, and r=0.25, as shown in the parenthesis of Figure 2.4. The "common ratio" here is 1.0/0.8=1.25.

Pair 1: Choose between

L_1 (0,1,0; $400, $3000, 0)

and

L_2 (0.8, 0, 0.2; $4000, $3000, 0)

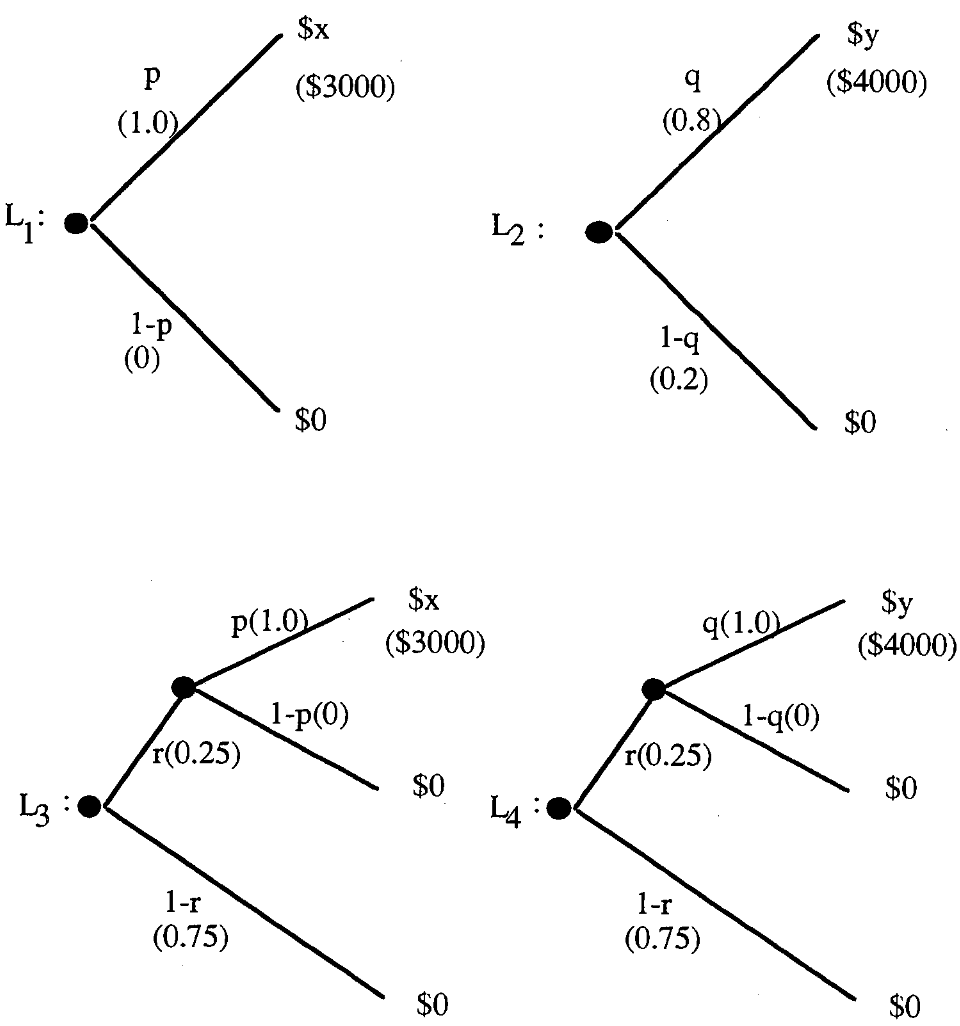


Fig.2.4: The Common Ratio Effect

Pair 2: Choose between

L_3 (0, 0.25, 0.75; \$4000, \$3000, 0)

and

L_4 (0.2, 0, 0.8; \$4000, \$3000, 0)

Kahneman and Tversky presented the gamble pairs to 95 respondents and found that that 80% of the subjects preferred L_1 in the first pair and only 65% of the subjects preferred L_3 in the second pair. Given that EU predicts either a choice of L_1 and L_3 or a choice of L_2 and L_4, the results show the common ratio effect. It can also be shown, as in Figure 2.5, this effect is consistent with fanning-out indifference curves.

In summary, a wide range of experimental violations of the EU theory have been observed. Most of them, if not all, can be interpreted by the fanning-out hypothesis.[7] Thus this hypothesis has been considered an important key in developing a generalized expected utility framework to explain violations of the EU model.

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[7] A summary of the literature is given by Machina (1982). Attention here was confined to experimental findings that have had an important impact on the development of generalized theories of choice under uncertainty.

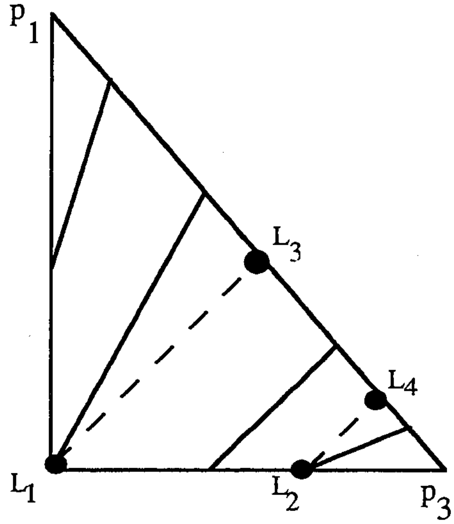


Fig.2.5: The Common Ratio Effect

and Fanning-out Effect

2.3. THE GENERALIZED EXPECTED UTILITY MODELS

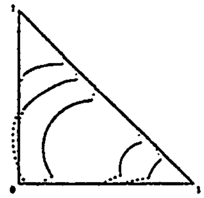
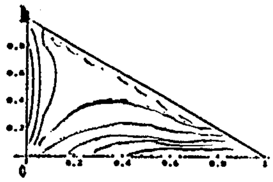
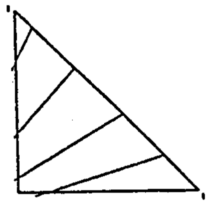
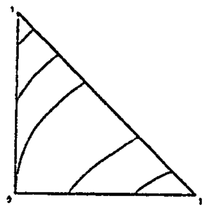
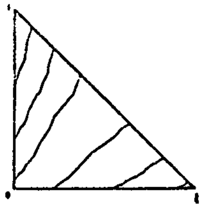
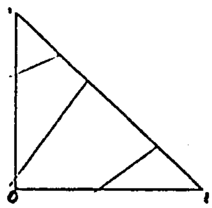
The growing body of empirical evidence against the EU hypothesis has motivated researchers to develop alternative models. Some examples of these Generalized Expected Utility (GEU) models, the researchers who have developed them, and theoretical predictions for indifference curve patterns for the three-event scenario are listed in Table 2.1.[8] Many are flexible enough to rationalize some behavior observed in experiments and inconsistent with EU theory while maintaining such basic properties as stochastic dominance, risk aversion and transitivity.

A common feature of these alternative models, except for prospect theory, is that the functional forms of the individual preference functions are more general than the EU functional form. This occurs because EU is a special case of these alternative models.

For each of the forms listed in Table 2.1, u(.) represents the utility function and w(.) stands for a probability weighting function. The other functional term, $\tau(x_i)$, in the weighted utility model, is also a weighting function that depends on final outcomes or the utility index. A superb overview and exposition of these alternative models may

[8] For thorough surveys of the GEU models, see Machina (1987) and Camerer (1989).

Table 2.1: Examples of the Alternative Models to EU

Prospect Theory $\sum_{i=1}^{n} w(p_i)\ u(x_i)$	Kahneman & Tversky (1979)	
Subjected Weighted Utility $\sum_{i=1}^{n} w(p_i)\ u(x_i) / \sum_{i=1}^{n} w(p_i)$	Karmarkar (1978,1979)	
Weighted Utility $\sum_{i=1}^{n} p_i \tau(x_i)\ u(x_i) / \sum_{i=1}^{n} p_i \tau(x_i)$	Chew & Maccrimmon (1979)	
Rank-dependent Utility $\sum_{i=1}^{n} u(x_i) \left[f(\sum_{j=1}^{i} p_j) - f(\sum_{j=1}^{i-1} p_j) \right]$	Quiggin (1982)	
The Fanning Out Hypothesis	Machina (1982)	
Implicit Expected Utility $\sum_{i=1}^{n} p_i\ u(x_i, u^*)$	Chew (1985) Dekel (1986)	

be found in Camerer (1989). In chapter five, we provide detailed descriptions of the theories of interest here. What follows is a brief description of each theory listed in this Table.

The prospect theory of Kahneman and Tversky (1979) is the only alternative model that does not generalize EU. It differs from EU in the following ways: first, all outcomes in the prospect theory are framed as changes from a reference point; second, prospects (i.e., lotteries) are edited to make them simpler to evaluate (e.g. outcomes and probabilities are rounded off or lumped together); and third, the expected utility over an edited prospect is given by a weighted probability formula as presented in Table 2.1. Kahneman and Tversky suggest that the weight function, $w(p)$, is increasing in p, subadditive ($w(p)+w(1-p)<1$), and discontinuous at the endpoints 0 and 1. They also hypothesize that the utility function $u(x)$ is asymmetrical for gains ($x>0$) and losses ($x<0$). Specifically, $u(x)$ is concave for gains and convex for losses. This theory is difficult to test because it has many more degrees of freedom, especially in the editing stage, than any other theory.

The subjective weighted utility theory was proposed by Karmarkar, 1978 and 1979. According to this model, the expected utility for a risky prospect $(p_1, p_2, p_3; x_2, x_2, x_3)$, as given in Table 2.1, depends on a weighting function $w(p_i)$, where $w(p_i)=p_i^{\alpha}/(p_i^{\alpha}+(1-p_i)^{\alpha}$, and α is an additional parameter regarded by Karmarkar as a measure of information processing performance. Low values of α ($0<\alpha<1$) underweight the objective probability p_i, high values ($\alpha>1$) overweight p_i, and when $\alpha=1$, this model reduces to the EU model. This model will be further explained in chapter five.

Weighted utility theory was developed by Chew and MacCrimmon, 1979 (see also Chew, 1983). As can be seen from Table 2.1, the weighting function, $p_i\tau(x_i)/\sum p_i\tau(x_i)$ is somewhat novel in the sense that it combines both probabilities and utilities. Depending on the choice of $\tau(x_i)$, the indifference curves of the weighted utility model can either fan-out (this corresponds to the light hypothesis of Chew and MacCrimmon), as in Table 2.1, or fan-in (the heavy hypothesis). Although the axioms suggest no obvious psychological interpretation to the weighting function, the weights seem to modify probabilities, possibly reflecting mental distortions or misperceptions, to a degree that depends on outcomes x_i.

Quiggin (1982,1985) was the first to consider a rank-dependent utility theory (called anticipated utility). This theory uses a nonlinear probability transformation function that depends on the order or rank of the outcomes. Certain specifications of the weighting function could generate nonlinear fanning-out indifference curves as in Table 2.1. As proved by Quiggin, this theory has strong axiomatic foundations. It has been used in some important applications (Quiggin, 1992). This theory is also further explained in chapter five.

In the fanning-out hypothesis, Machina uses the notion of (first-order) stochastic dominance. For three-outcome gambles, lottery A:(p_1, p_2, p_3; x_1, x_2, x_3) stochastically dominates B:(q_1, q_2, q_3; x_1, x_2, x_3) if $p_3<q_3$, and $p_1>q_1$. Graphically, point A stochastically dominates B if A lies to the northwest of B in the Marschak-Machina triangle. However, Machina did not propose specific preference functions, rather he hypothesized that the local utility functions of stochastically dominant gambles will exhibit more risk aversion (by the Arrow-Pratt measure) than local utility functions of stochastically dominated gambles. This hypothesis predicts that indifference curves, usually nonlinear, will be steeper for gambles to the northwest of the Marschak-Machina triangle.

Finally, the Implicit Expected Utility function (Dekel, 1986) generalizes the EU model by replacing u(x) by u(x,u*), where u* is the expected utility, i.e.,

$$u^* = p_1 u(x_1,u^*) + p_2 u(x_2,u^*) + p_3 u(x_3,u^*).$$

Indifference curves of Implicit EU are straight lines, but their slopes vary because u(x,u*) varies with u*. Thus this model describes a person who uses a different utility function, perhaps reflecting different degrees of risk aversion along each indifference curve.

2.4 TESTING BETWEEN ALTERNATIVE MODELS

Experiments have identified a number of well-known violations of the expected utility theory, thus giving rise to alternative models of choice under uncertainty. These alternative models, with quite different views of the behavioral processes underlying choices under uncertainty, are often able to explain some violations of EU predictions. The questions is: Are theoretical predictions of these alternative models consistent with people's actual choice behavior?

Several recent empirical studies (Chew and Waller, 1986; Camerer, 1989; Battalio, Kagel and Jiranyakul, 1990; and Marshall, Richard and Zarkin, 1992) have attempted to answer this question by designing new experiments or new empirical methods. Of these empirical studies, all but one use experimental evidence to test between the alternative models.[9]

The general tenor of conclusions from these empirical studies confirm violations of the EU predictions, but no single theory has emerged as a satisfactory alternative. In what follows, we provide only a brief description of experiments and testing methods of each study and their major results.

Chew and Waller (1986) employed a four-pair lottery structure, called the HILO lottery structure, to test weighted utility theory. The HILO lottery structure involves choices over four pairs of lotteries. Figure 2.6 shows a typical HILO structure in which gamble pairs (A_i, B_i), i=1,2,3,4 are plotted on the Marschak-Machina triangle. As can be seen from the triangle, these four gamble pairs form three parallel straight lines labelled 1, 2, 3, and 4. Since EU indifference curves of an individual are also straight lines, the EU theory predicts that the individual's choice over these four lottery pairs is either $A_1A_2A_3A_4$ if the slope of EU indifference curves is greater than the slope of line

[9] Using seat-belt-usage data, Marshall, Richard and Zarkin (1992) test Machina's fanning out hypothesis and the "light" hypothesis of Chew and Waller (1986).

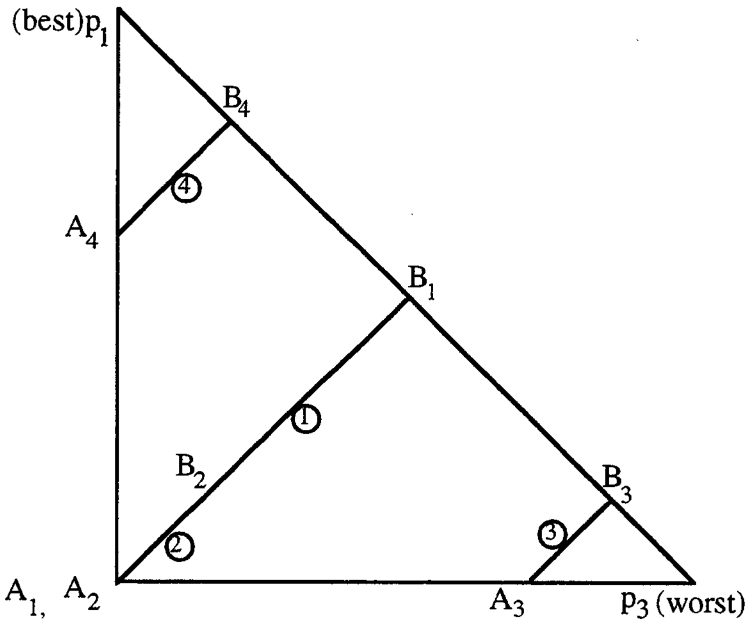


Fig. 2.6: HILO Lottery Structure

segments connecting lotteries A_i and B_i (i=1,2,3,4), or $B_1B_2B_3B_4$ otherwise. Note that pair 2 and pair 3 form an Allais type of problem. For this problem, EU predicts a choice of either A_2A_3 or B_2B_3. To elaborate, Allais' lottery structure is used to test whether individuals behave consistently over two pairs of lotteries, and theHILO lottery structure tests the consistency over four pairs. Thereforethe HILO lottery structure permits stronger empirical tests than those in the Allais lottery structure.[10] With every HILO lottery structure, there are 16 possible choice patterns. Based on observed frequencies of choice patterns implied by a particular theory, one can then test if the theory predicts the subjects' choices better than a chance prediction model. Using two HILO lottery structures, Chew and Waller tested the expected utility hypothesis (the "neutral" hypothesis), the fanning-out hypothesis (the "light" hypothesis), and the fanning-in hypothesis (the "heavy" hypothesis) of weighted utility theory. The results indicate that the "light" hypothesis with linear fanning-out indifference curves is supported by their data, that is, it predicts significantly better than a pure chance prediction model.

Camerer (1989) also conducted an experimental test of several generalized expected utility theories using an analysis of indifference curves drawn in the Marschak-Machina triangle. The theories evaluated were weighted utility theory, implicit expected utility theory, the fanning-out hypothesis, rank-dependent expected utility.

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[10] The HILO lottery structure is discussed in more detail in Chapter three.

Using responses from 14 gamble pairs with each one involving more risky and less risky gambles of the Allais type, Camerer depicted an approximate indifference curve pattern based on percentages of subjects who chose the less risky gambles over the more risky ones in a Marschak-Machina diagram. He compared these approximate indifference curves with theoretical predictions of each theory, and concluded that (Camerer, P82)

> "No theory can explain all the data, but prospect theory and the hypothesis that indifference curves fan out can explain most of them."

Just when the fanning-out hypothesis appeared to be the solution to Allais paradox, Battalio, Kagel and Jiranyakul (BKJ, 1990) found evidence of fanning-in rather than fanning-out. Battalio et al designed four series of binary choice questions of the Allais type, involving both losses and gains. Each question required subjects to indicate which of two gambles they preferred. Based on the frequencies of choice patterns generated from the subjects, and theoretical predictions of several GEU models including Rank-dependent expected utility theory (RDEU), Prospect theory and Machina's generalized expected utility model, they concluded that no single model consistently explains choices. Among the more important inconsistencies, they identified conditions generating systematic fanning-in instead of fanning-out of indifference curves in the Marschak-Machina triangle.

Like other experimental economics, the experimentally based study of theories of choice under uncertainty is open to criticism regarding the validity of the method and the generalization of the results. In the light of this criticism, Marshall, Richard and Zarkin (MRZ) (1992), for the first time, used non-experimental data (seat-belt-usage data) to construct posterior probabilities of specific types of EU violations. Adopting a Bayesian framework, MRZ estimated the basic conditional probabilities characterizing the commuter-safety lotteries and assigned posterior probabilities to the violation of the independence axiom, to Machina's fanning-out hypothesis, and to the "light" hypothesis of Chew and Waller. The results show that similar to other experimental studies, the nonexperimental data also exhibit systematic departures from the EU model and that while the nonexperimental evidence is not inconsistent with Machina's fanning-out hypothesis, it is inconsistent with the "light" hypothesis of Chew and Waller.

In summary, among the four recent empirical studies that test between alternative models of choice under uncertainty, the Chew and Waller study supports the "light" hypothesis of weighted utility theory, but MRZ found non-experimental data inconsistent with the "light" hypothesis. Both Camerer and MRZ found evidence supporting Machina's fanning-out hypothesis, yet BKJ found conditions generating systematic fanning-in of indifference curves instead of fanning-out. Prospect theory was supported by Camerer's study, but not by BKJ. To conclude this section, we borrow the statements of BKJ (P.46) as follows:

"Our overall conclusion is that none of the alternatives to expected utility theory considered here consistently organize the data, so we have a long way to go before having a complete descriptive model of choice under uncertainty."

Other independent studies (Harless, 1987; Starmer and Sugden, 1987a, 1987b) also seem to share this view.

2.5 A CRITIQUE ON EXISTING EMPIRICAL METHODS

More than two decades after Allais first challenged expected utility theory by using experimental evidence, a number of alternative models were proposed to improve the descriptive ability over the EU model. Yet existing empirical studies have not found one single theory that could explain all data. This raises questions about testing methodology and the validity of the empirical techniques used in the literature to test theories of choice under uncertainty.

As mentioned in the previous section, a common method used to test the adequacy of such models typically, the Allais type, is to compare the frequency of each choice pattern generated from the experiments with the theoretical predictions of each model. If the modal response is inconsistent with the theory, then the theory is considered to be inadequate. Typically, effort is devoted to create a new generalized expected utility model to explain the modal response.

To see what is involved in this approach, let us take one experiment from BKJ (1990) for example. Table 2.2 reproduces Table 8 for experiment set 1.1 in their paper (p.43). As shown in Figure 2.7,

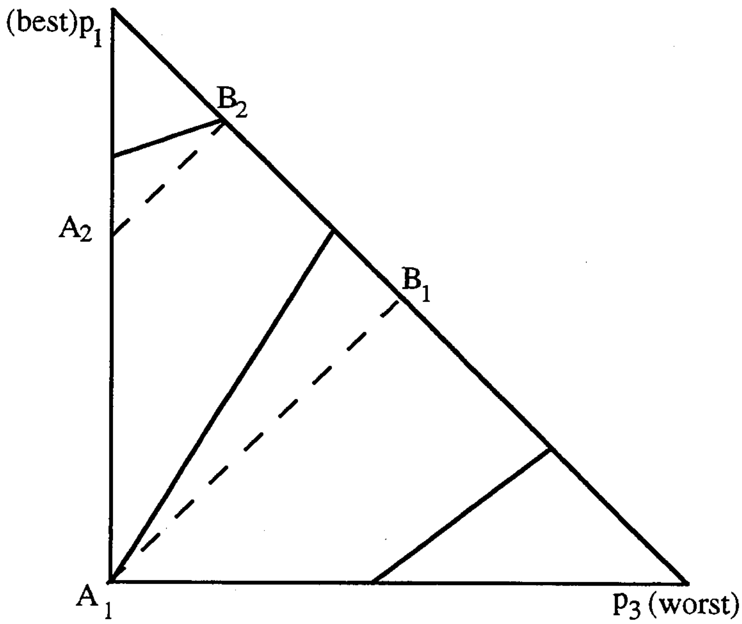

Fig. 2.7: An Experiment from BKJ

given this experiment, there are four possible choice patterns that could be generated from a sample population. Column 1 of Table 2.2 lists these choice patterns. The hypothesis that predicts each choice pattern and choice frequencies are reported in columns 2 and 3 respectively.

Table 2.2 shows that 44.5% (i.e., 16.7%+27.8%) of the subjects made choices consistent with EU theory, 11.1% of the responses is consistent with fanning-out hypothesis and 44.4% of the choices is consistent with fanning-in hypothesis. It is suggested from this result that since expected utility theory organizes less than half the data (44.5%), it is considered inadequate. Moreover, with fanning-out comprising only 20% and fanning-in 80% of the deviations from expected utility theory, the validity of fanning-out falls dramatically in this data set. In contrast, the fanning-in hypothesis may be a better alternative model with EU as a special case.

As another example, let us focus on a more complicated version of this approach adopted by Chew and Waller (1986). The following set of lotteries is picked from their study.[11]

A_1: (0,1,0; \$100,\$40,\$0) B_1: (0.5,0,0.5; \$100,\$40,\$0)

A_2: (0,1,0; \$100,\$40,\$0) B_2: (0.05,0.9,0.05; \$100,\$40,\$0)

A_3: (0,0.1,0.9; \$100,\$40,\$0) B_3: (0.05,0,0.95; \$100,\$40,\$0)

A_4: (0.9,0.1,0; \$100,\$40,\$0) B_4: (0.95,0,0.05; \$100,\$40,\$0)

[11] This set of lotteries corresponds to experiment 1: context 1a of Chew and Waller, 1986.

Table 2.2: Illustration of an existing test

lottery set 1.1:

A_1:(0,1,0; \$27,\$18,0) vs B_1:(0.72,0,0.28; \$27,\$18,0)

A_2:(0.74,0.2,0.06; \$27,\$18,0) vs B_2:(0.90,0,0; \$27,\$18,0)

Possible Choice Patterns	Patterns Consistent with	Choice Frequencies
A_1A_2	EU	6(16.7%)
B_1B_2	EU	10(27.8%)
B_1A_2	Fanning-out	4(11.1%)
A_1B_2	Fanning-in	16(44.4%)

These lotteries are also shown in Figure 2.8. From this figure, EU with parallel indifference curves predicts choice patterns: $A_1A_2A_3A_4$ and $B_1B_2B_3B_4$; the fanning-out hypothesis predicts additional choice patterns: $A_1A_2B_3A_4$ and $B_1B_2B_3A_4$; and the fanning-in hypothesis predicts additional choice patterns $A_1A_2A_3B_4$ and $B_1B_2A_3B_4$. Table 2.3 reproduces their results generated from 56 subjects. Column 1 contains all possible choice patterns, column 2 lists the suitable hypotheses and column 3 reports the observed frequencies. To test weighted utility theory, Chew and Waller used the observed choice frequencies to determine whether the EU hypothesis, the fanning-out, or fanning-in hypotheses predicted the subjects' choice pattern better than a chance prediction model. In particular, referring to Table 2.3, two of the 16 choice patterns are consistent with the EU hypothesis, therefore, for this hypothesis to predict better than a chance prediction model,the relative frequency of correct predictions would have to be significantly greater than the chance hit rate of 1/8, or 12.5%. Furthermore, since 4 of the 16 patterns are consistent with the fanning-out (or fanning-in) hypothesis, for these hypotheses to predict better than a chance prediction model, the relative frequencies of correct predictions would have to be significantly greater than the chance hit rate of 1/4, or 25%. As shown in Table 2.3, 23% (i.e., 7% + 16%) of responses are consistent with EU; 32% (7% + 16% + 4% + 5%) are consistent with fanning-in; and 53% (7% + 16% + 5% + 25%) of choices is consistent with fanning-out. Therefore from these numbers, the EU and the fanning-out hypothesis predicted significantly better than chance.

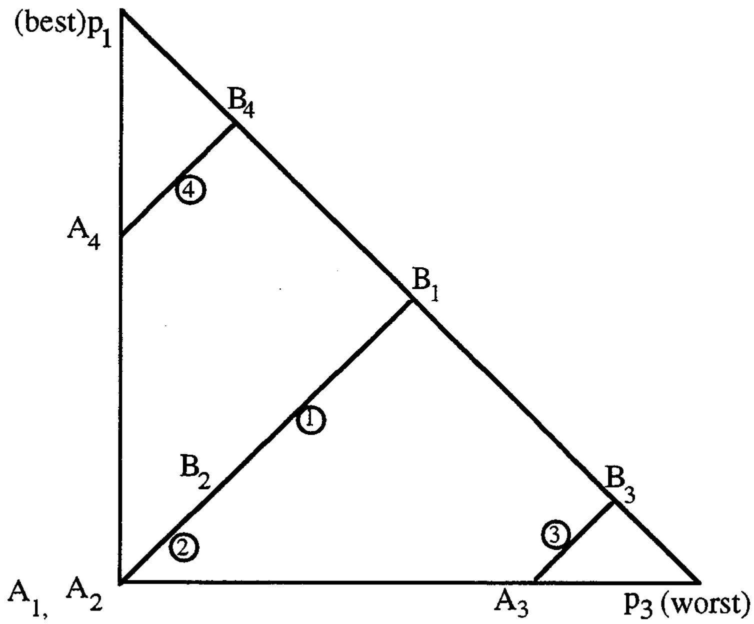

Fig.2.8: An Experiment from Chew & Waller

Table 2.3: Possible Choice Patterns, Implications of Weighted Utility and Observed Frequencies

Possible Choice Patterns	Weighted Utility Choice	Observed Frequencies
$A_1A_2A_3A_4$	EU,FO,FI	4(7%)
$A_1A_2A_3B_4$	FI	2(4%)
$A_1A_2B_3A_4$	FO	3(5%)
$A_1A_2B_3B_4$	No	0(0%)
$A_1B_2A_3A_4$	No	2(4%)
$A_1B_2A_3B_4$	No	3(5%)
$A_1B_2B_3A_4$	No	11(20%)
$A_1B_2B_3B_4$	No	2(4%)
$B_1A_2A_3A_4$	No	0(0%)
$B_1A_2A_3B_4$	No	0(0%)
$B_1A_2B_3A_4$	No	0(0%)
$B_1A_2B_3B_4$	No	0(0%)
$B_1B_2A_3A_4$	No	3(5%)
$B_1B_2A_3B_4$	FI	3(5%)
$B_1B_2B_3A_4$	FO	14(25%)
$B_1B_2B_3B_4$	EU,FO,FI	9(16%)

***EU, FO, FI** indicate that the choice pattern is consistent with Expected Utility theory, the Fanning-out and Fanning-in hypotheses, respectively.

Moreover, by comparing the three hypotheses in terms of predictive ability (i.e.,number of choices consistent with each hypothesis), Chew and Waller concluded that the fanning-out hypothesis performs the best in explaining their data.

A couple of problems are apparent from such approaches. First, results from these studies (in fact from all studies) clearly showed variations of choice patterns from a sample population, but the existing tests focus on only the modal response. The criterion that determines whether a particular theory is appropriate seems to depend on whether the modal response is consistent with the theory. This clearly ignores the possibility that different people make different choices due to taste variations. Surely, from each data set, there are always choices inconsistent with all theories. Hence, as a matter of logic, all theory should be rejected by such choices. Therefore, any attempt to find a theory that explains all choices is doomed to failure. Secondly, these tests are rather *ad hoc* and unsystematic, since no systematic statistical test was constructed to test the adequacy of theories of choice under uncertainty at the aggregate level.

In the light of these criticisms, a new approach is developed here to testing theories of choice under uncertainty. This approach is based on one important point, that is, to understand the data, one needs heterogeneity of preferences. In particular, it is assumed that individuals have diverse tastes, and that there exists a probability density function which describes the diverse tastes across individuals. Given data generated from laboratory experiments on choices over gamble pairs, the density function is estimated through maximum likelihood estimation techniques. A likelihood ratio test based on the recovered density function is constructed to evaluate a particular theory of choice. The next chapter describes the data. Chapter four explains the new empirical approach.

Chapter Three

THE EXPERIMENTAL DATA

Empirical studies on testing theories of choice under uncertainty found to date have been based on experimental evidence, with the exception of Marshall, Richard and Zarkin (1992). In general, there is an inherent trade-off between experimental and nonexperimental data. Laboratory experiments offer a high degree of control over the sampling environment, but the validity of the approach and the generalization to "real-world" phenomena is perhaps questionable. On the other hand, nonexperimental data is more convincing, but sampling controls are typically poor. Given that the primary purpose of this study is to develop some empirical techniques to calibrate models of choice under uncertainty, sampling control is important. Thus experimental data is employed in this study. Section 3.1 explains the current experimental design and procedure. Section 3.2 presents the experimental results. A brief analysis of the data using existing methodologies in literature is provided in section 3.3. Section 3.4 concludes this chapter.

3.1 Experiments

In this study, three experiments were conducted on three separate groups of subjects at three different times. Two of the experiments were used for preliminary studies. The other one was used to generate data to estimate preferences and test theories of choice under uncertainty. This section explains the experiments: lotteries, subjects and experimental design and procedure.

3.1.1 Lotteries

The lotteries were generated from the Marschak-Machina triangle. Each lottery involves three levels of payoffs: a coffee mug, a pen and nothing. The coffee mug, which cost $5.95, was a good quality mug with a landscape of Simon Fraser University (S.F.U.). The pen, with a price of $2.15, was a fine pen specially made with an S.F.U. logo on it.[12] Lotteries involving these prizes can be represented by different points on the Marschak-Machina triangle. Mugs, pens and nothing were used as prizes to avoid the possibility of local risk-neutrality results. According to the literature, such results usually arise in a choice between small money gambles when subjects make choices based on expected values of the lotteries rather than expected utilities. If students were given dollar prizes, and thought that there is a correct choice in each situation, they might be tempted to choose the lottery offering the highest expected payoff. Though preliminary experiments did not significantly show such results, we chose to use non-monetary prizes: a mug, a pen, and nothing as a precaution.

3.1.2 Subjects

Subjects were undergraduate economics students at Simon Fraser

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[12] The monetary values of these prizes were not known to the subjects at the time of experiments.

University. These students were either taking a principles of economics course or an intermediate economics course. Most of them were not familiar with the decision theory, and they had not been exposed to this type of experiment before. Some subjects were given a Crunchie chocolate bar for participating in the experiments. Some were given a chance, on a random selection basis, to actually play the lottery they picked from an experiment. A poll showed 99% of the subjects from one class claimed to have given serious responses in these experiments.

3.1.3 Experimental Design and Procedure

The experiments were conducted in two stages: a preliminary stage and a final stage. The purpose of the preliminary experiments was to gain experience in designing a more efficient and more careful experiment, that is, to generate more accurate responses for our study, and to use the data to establish appropriate empirical techniques to calibrate theories of choice under uncertainty. In this stage, we designed six sets of lotteries involving both monetary payoffs ($5, $2, $0) and non-monetary payoffs. Each set contains three lotteries generated from the Marschak-Machina triangle. The monetary payoffs were used primarily to examine the local risk-neutrality results as discussed in the literature (e.g., Quiggin, 1992). The experimental results from five different undergraduate economics classes, showed no significant difference between using money and non-money payoffs. The preliminary study also showed that the initial experimental design was limited in a number of ways: first, there was not sufficient data generated for estimation and testing; second, it was difficult to make any direct comparisons between our experiments and others since other empirical studies in the literature all used binary choices data, and we used choices from three lotteries; finally, the design was not systematic in the sense that the lotteries were generated from the Marschak-Machina triangle in a somewhat arbitrary fashion.

In the light of these preliminary studies, we designed another set of experiments to generated the data for estimating and testing the models of choice under uncertainty. In this experiment, only non-monetary payoffs were used. Subjects were 284 undergraduate economics students, who were taking a principles of economics course. They were asked to respond to 13 binary choice situations. The binary choices are described in Table 3.1 in which column 1 lists the pair numbers; column 2 will be explained later. For each pair, columns 3 and 4 describe lotteries A and B respectively, where p_1, p_2, and p_3 are the probabilities of winning a coffee mug, a pen and nothing. For example, pair 13 involves a choice between a 100% chance of winning a coffee mug (lottery A) and a 100% chance of winning a pen (lottery B). This lottery pair was designed to divide the sample into two parts: one in which subjects prefer the mug to the pen and the other one contains subjects who prefer the pen to the mug. Though both may be used to recover preferences, they should be used as separate experiments, since the assumption that $u(x_1) > u(x_2) > u(x_3)$ is necessary to maintain the graphical interpretation of EU indifference curves. The results from this show that 250 out of 284 subjects preferred the coffee mug to the

Table 3.1. Lottery Pairs Presented to Subjects

Pair No.	Situation	Lottery A* (p_1, p_2, p_3)	Lottery B (p_1, p_2, p_3)
1	1-O	(0, 1, 0)	(0.5, 0, 0.5)
2	1-I	(0, 1, 0)	(0.1, 0.8, 0.1)
3	1-L	(0, 0.2, 0.8)	(0.1, 0, 0.9)
4	1-H	(0.8, 0.2, 0)	(0.9, 0, 0.1)
5	2-O	(0, 1, 0)	(0.8, 0, 0.2)
6	2-I	(0, 1, 0)	(0.2, 0.75, 0.05)
7	2-L	(0, 0.25, 0.75)	(0.2, 0, 0.8)
8	2-H	(0.75, 0.25, 0	(0.95, 0, 0.05)
9	3-O	(0, 1, 0)	(0.2, 0, 0.8)
10	3-I	(0, 1, 0)	(0.05, 0.75, 0.2)
11	3-L	(0, 0.25, 0.7)	(0.05, 0, 0.95)
12	3-H	(0.75, 0.25, 0)	(0.8, 0, 0.2)
13		(1, 0, 0)	(0, 1, 0)

*Prizes: x_1= a coffee mug, x_2= a pen, and x_3= nothing pen. We will use only this sample because of the larger sample size.[13]

The other 12 pairs of lotteries were designed according to Chew and Waller's HILO lottery structure. As discussed in Section 2.4 of Chapter 2, the HILO lottery structure is a straightforward generalization of the Allais lottery structure. The structure is specified by two probabilities, α and β, and three outcomes, x_H, x_I, x_L, where $x_H > x_I > x_L$ (H-high outcome, I- intermediate outcome, L-low outcomes). These parameters are combined into four binary choice situations (referred to as the O, I, L, and H situations). In the O-situation, A_o offers a 1.00 chance of winning x_I, while B_o offers a β chance of winning x_L and $(1-\beta)$ chance of winning x_H. In the other situations, A_o (i=I,L,H) is obtained by constructing a lottery with an α chance of yielding A_o and a $1-\alpha$ chance of yielding the i outcome (i=I, L, H), as shown in Table 3.2.[14] B_o (i=I,L,H) is obtained by constructing a lottery with an α chance of yielding B_o and a $1-\alpha$ chance of yielding the i outcome (i=I,L,H).

In the current experiment, x_H = a coffee mug, x_I = a pen, x_L = nothing. As indicated in column 2 of Table 3.1, Lottery pairs 1-4 in Table 3.1 form HILO structure 1 in which α=0.2, β=0.5; pairs 5-8 form

13 Given that the purpose here is to recover prefe r ences over lottery pairs, not the final outcomes of lotteries, the current treatment of the sample population should not cause any sample selection bias.

14 The description of the HILO lottery structure is taken from Chew and Waller (1986). Table 3.2 is basically the same as Table 1 in their paper.

Table 3.2: The HILO Lottery Structure

situation	Lottery A	Lottery B
O	x_I	$\beta x_H + (1-\beta) x_L$
I	$\alpha A_o + (1-\alpha) x_I$	$\alpha B_o + (1-\alpha) x_I$
L	$\alpha A_o + (1-\alpha) x_L$	$\alpha B_o + (1-\alpha) x_L$
H	$\alpha A_o + (1-\alpha) x_H$	$\alpha B_o + (1-\alpha) x_H$

HILO structure 2 in which $\alpha=0.25$, $\beta=0.8$; and HILO structure 3 with $\alpha=0.25$, $\beta=0.2$ includes lottery pairs 9-12. These structures are plotted in Figures 3.1, 3.2, and 3.3 respectively.

The figures show that each structure has a different slope for the lines connecting lottery A_i and B_i: 1 for structure 1, 4 for structure 2 and 0.25 for structure 3. The numbers that appear above or below the line segments in each figure represent the lottery pair number corresponding to data given in Table 3.1.

As also illustrated by these figures, the 12 lottery pairs cover all corners of the triangle space. The objective is to use lotteries from different regions of the Marschak-Machina triangle to calibrate models of choice under uncertainty.

The experiment proceeded as follows: First, the experimenters explained to students what the experiment was all about. At the same time, sample coffee mugs and pens were circulated among students to familiarize them with the prizes. Second, a response sheet with simple instructions, reproduced in the appendix to this chapter, was handed out and explained to each student. The students were asked to read the instructions first and then wait for the experimenter to explain the lotteries. Third, using an overhead projector, the experimenter presented each pair of lotteries on a separate transparency using the diagram shown (for pair 1) in Figure 3.4. Lotteries A and B in each pair are represented by two rectangular areas of unit 1. Each rectangular area was divided into three colored areas, with the red area measuring the probability of winning a mug, the yellow area

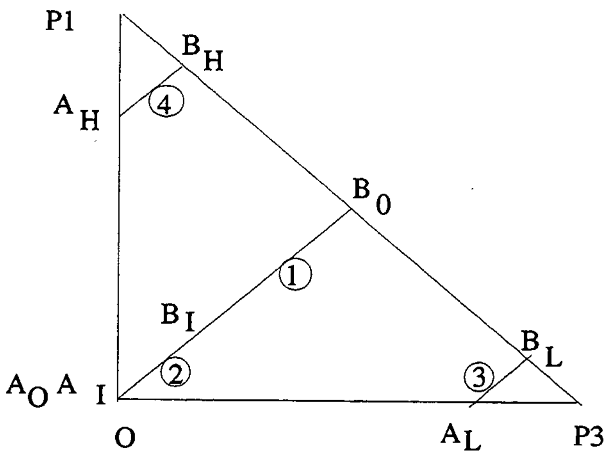


Fig 3.1: Lottery Structure 1

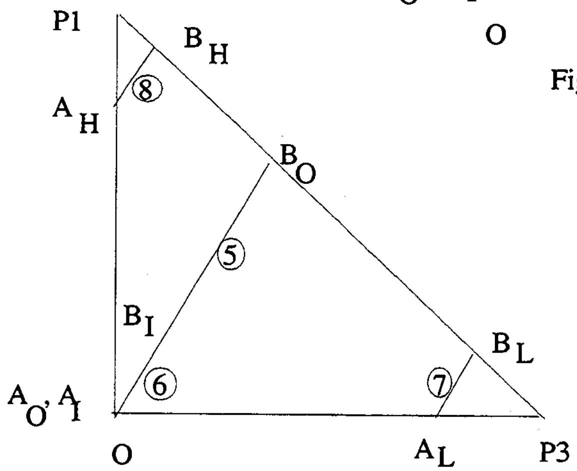


Fig 3.2: Lottery Structure 2

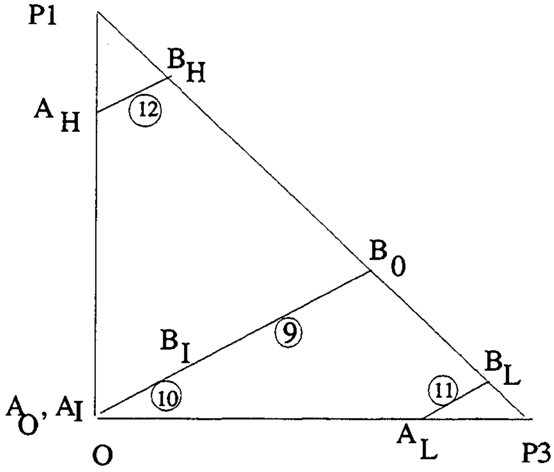
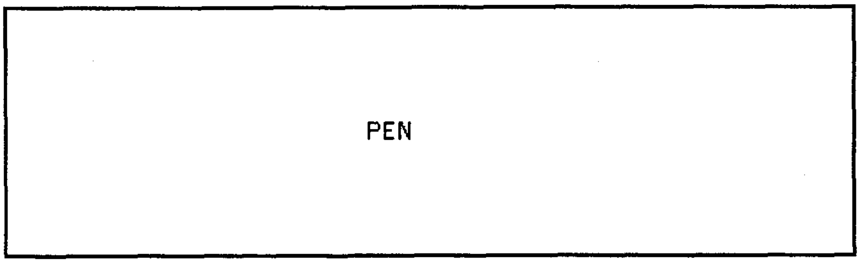


Fig 3.3: Lottery Structure 3

A: (0,1,0; mug, pen, nothing)



B: (0.5,0,0.5; mug, pen, nothing)

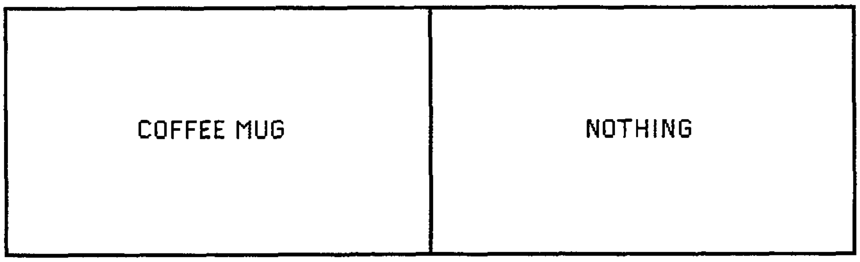


Fig. 3.4: Gamble pair 1 as presented to subjects

measuring the probability of winning a pen, and the blue area measuring the probability of winning nothing. In this example, lottery A with probability (0,1,0) is represented by the rectangular area entirely colored by yellow, and indicates a 100% chance winning a pen. Lottery B with probability (0.5,0,0.5) is represented by the rectangular area colored half in red and half in blue. It indicates a 50% chance of winning a mug and a 50% chance of winning nothing. When presenting each pair, the experimenter also verbally explained the lotteries. The students were asked to make a choice by circling either A or B on the response sheet after each pair was presented.[15] Finally, after completing all 13 pairs, the experimenters collected response sheets and rewarded each student with a crunchie chocolate bar. The experiment took approximately 30 minutes.

3.2 RESULTS

The results are reported in two parts. First, descriptive data is presented regarding the subjects' choices. Second, these choices are analyzed using the previous empirical methods adopted in the literature.

Table 3.3 reports the frequencies of A_i and B_i choices in each

[15] Following majority of the researchers, indifference curves between two lotteries was not allowed in this experiment.

Table 3.3: Frequencies of choices

choices	structure 1	structure 2	structure 3
A_O	175(70%)	94(38%)	232(93%)
B_O	75(30%)	156(62%)	18(7%)
	250(100%)	250(100%)	250(100%)
A_I	74(30%)	28(11%)	153(61%)
B_I	176(70%)	222(89%)	97(39%)
	250(100%)	250(100%)	250(100%)
A_L	140(56%)	57(23%)	212(85%)
B_L	110(44%)	193(77%)	38(15%)
	250(100%)	250(100%)	250(100%)
A_H	214(86%)	231(92%)	229(92%)
B_H	36(14%)	19(8%)	21(8%)

of the three HILO structures. From this table, we can see that the results from the 12 pairs of lotteries differ from one structure to another. In lottery structure 1, a tendency to prefer A_i alternative over B_i alternative (i=1,2,3,4) was evident, except in the I-situation. For lottery structure 2, the tendency was to prefer Bi over A_i (i=5,6,7,8), except in the H-situation. Finally in structure 3, the majority of subjects chose A_i over B_i in all H-I-L-O situations (i=9,10,11,12). Notice that the slopes of the segments connecting lottery pairs A_i and B_i are different between HILO structures (1 for structure 1, 4 for structure 2 and 1/4 for structure 3) as shown in Figures 3.1, 3.2, and 3.3 respectively. Therefore these disparate results may simply reflect these slope differences, as will be seen in the following analysis.

3.3 DATA ANALYSIS

The data is first analyzed using the existing empirical methods in the literature. The purpose here is to obtain some prior information on whether the EU theory is consistent with our data and if not, what theory could serve as a better alternative. Table 3.4 reports the observed frequencies for each HILO structure. The modal response was ABAA in structure 1(pairs 1-4), BBBA in structure 2 (pairs 5-8) and AAAA in structure 3 (pairs 9-12). The expected utility theory predicts either AAAA or BBBB in all three structures. But the percentages of the

Table 3.4: Possible Choice Patterns and Observed Frequencies of the HILO structures

Possible choice parttens	Alternative Hypothesis*	Structure 1 pairs 1-4	Structure 2 pairs 5-8	Structure 3 pairs 9-12
1 AAAA	ALL	49(19.6%)	21(8.4%)	133(53.2%)
2 BAAA	NO	2(0.8%)	0(0%)	2(0.8%)
3 ABAA	NFO	56(22.4%)	20(8%)	61(24.4%)
4 BBAA	NO	21(8.4%)	15(6%)	2(0.8%)
5 AABA	LFO	12(4.8%)	5(2%)	12(4.8%)
6 BABA	NFO	6(2.4%)	3(1.2%)	1(O.4%)
7 ABBA	NFO	41(16.4%)	47(18.8%)	12(4.8%)
8 BBBA	LFO	27(10.8%)	120(48%)	6(2.4%)
9 AAAB	FIN	4(1.6%)	0(0%)	5(2%)
10 BAAB	NO	0(0%)	0(0%)	0(0%)
11 ABAB	NO	6(2.4%)	1(0.4%)	9(3.6%)
12 BBAB	FIN	2(0.8%)	2(0.8%)	1(0.4%)
13 AABB	NO	1(0.4%)	0(0%)	0(0%)
14 BABB	NO	0(0%)	0(0%)	1(0.4%)
15 ABBB	NO	6(2.4%)	1(0.4%)	0(0%)
16 BBBB	ALL	17(6.8%)	15(6%)	5(2%)

ALL indicates all hypotheses listed in this table including EU.
NO means that no existing hypothesis could explain the choice.
NFO indicates non-linear fanning-out indifference curves.
LFO indicates linear fanning-out indifference curves.
FIN stands for fanning-in indifference curves.

subjects who made these choices are only 26.4% (19.6% + 6.8%) in structure 1, 14.4% (8.4% + 6%) in structure 2, and 55.2% (53.2% + 2%) in structure 3. Hence the EU model does not explain almost 74% of subjects in structure 1, 86% in structure 2 and 45% in structure 3. Also in Table 3.4, column 2 lists alternative hypothesis of indifference curve that may be used to explain the choices shown in column 1. For example, choices AAAA and BBBB, accounted for 26.4% (19.6% + 6.8%) may be explained by all theories including EU. Figures 3.5 and 3.6 show these choices with EU parallel indifference curves in the Marschak-Machina triangle. Choices ABAA and ABBA, which accounts for 38.8% of total population, may be explained by non-linear fanning-out hypothesis for structure 1 as in Figures 3.7 and 3.8.

To summarize the results from Table 3.4, we make the following observations: (1) No single hypothesis can organize all the data; (2) up to 14.4% of choices could not be explained by any theory listed; (3) the fanning-out hypothesis, including both linear fanning-out (LFO) and non-linear fanning-out (NFO), seem to explain a large portion of the data set. The frequencies of the choices implied by this hypothesis (i.e., choices 1,3,5,6,7,8,16 from Table 3.4) were summing up to 83.2% for structure 1, 92.4% for structure 2 and 92% for structure 3. These results suggest that the fanning-out hypothesis may be an attractive hypothesis for our data. It reflects certain behavioral regularities: sure gains are much more attractive than uncertain gains with equal expected value and small chance of a zero payoff.

To further explore the choice patterns, we also adopt an

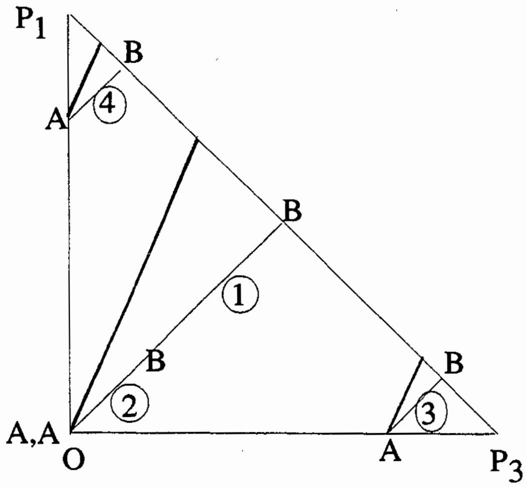


Fig. 3.5: Choice of AAAA with EU Indifference Curves

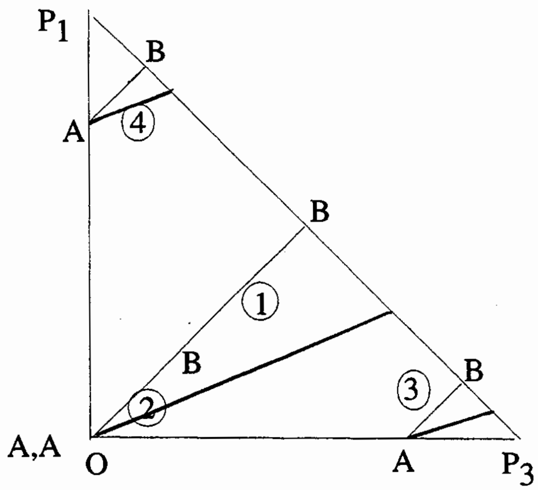


Fig. 3.6: Choice of BBBB with EU Indifference Curves

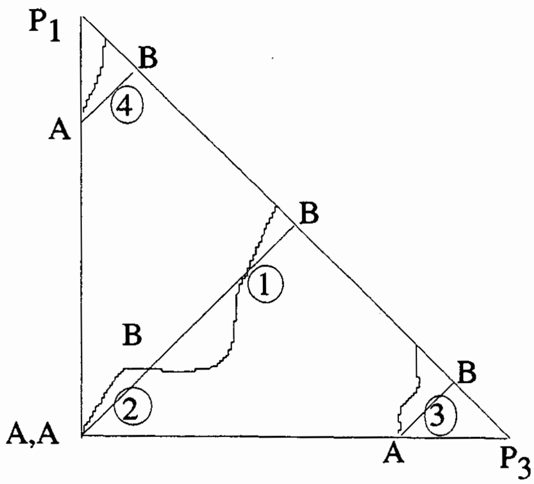


Fig. 3.7: Choice of ABAA with NFO Indifference Curves

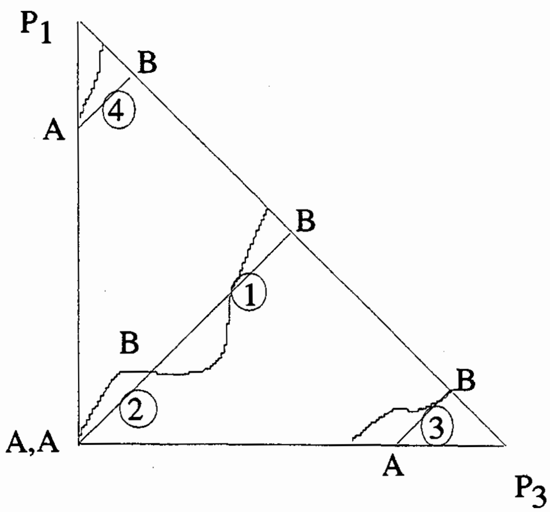


Fig. 3.8: Choice of ABBA with NFO Indifference Curves

empirical method used by Camerer (1989). In his study, Camerer proposed two ways of analyzing the data: between-subjects and within-subjects. Between-subjects tests look at patterns of averaged choices; within-subjects tests look at the averaged patterns of choices. Camerer used between-subject analyses to suggest conclusions that were verified by within-subjects analyses. This approach is another version of *the representative consumer* approach, because between-subject measurements of average behavior provide a picture of how such a hypothetical representative agent might act. Figure 3.9 shows the analysis for HILO structure 1 of our data set. In this figure, the thin lines connect the two lotteries A and B in each pair. The thick line represents the fraction of subjects who chose lottery A in the pair (the fraction is written next to the thick line). For instance, the thin line labelled 4 connects (0.8, 0.2, 0) and (0.9, 0, 0.1), the two lotteries in pair 4, 85.5% of the subjects chose A over B. The slope of the thick line is a linear function of this fraction. If all subjects chose lottery A, the thick line will be perfectly vertical; if all chose lottery B, it will be horizontal. If half chose A and half chose B, the thick line will have a slope of one (it will superimpose on the thin line connecting A and B). Therefore, according to Camerer, the thick lines are analogous to indifference curves even though they have no formal meaning. The EU model predicts that these lines will be parallel over the space of the triangle. But our results show that these lines are becoming steeper as we move from the lower-right corner to the upper-left corner of the triangle. In addition, the results from pairs 1 and 2 indicate that the

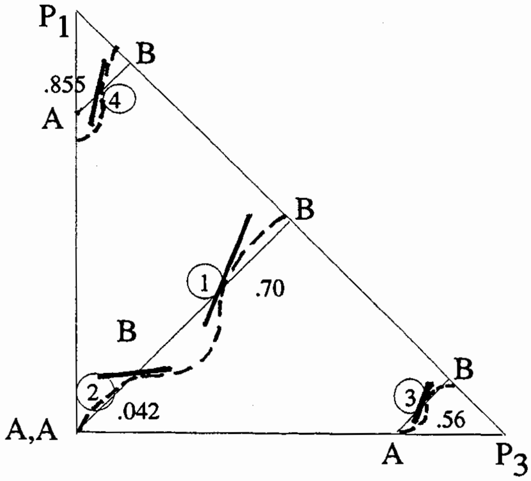


Fig. 3.9: Observed "Indifference Curve" Pattern

indifference curves may not even be straight lines inside the triangle. Its slope gets bigger when we move from the origin to the edge of the triangle. The broken line in Figure 3.9 depicts an approximate pattern of the "indifference curves". It shows that the "indifference curves" indeed fan out as we move from the lower-right corner to the upper-left corner of the Marschak-Machina triangle. Furthermore, these indifference curves may be nonlinear in the neighborhood of the origin inside the triangle. This provides a good starting point for us to search for alternative models of choice under uncertainty, which will be explained in Chapter 5.

3.4 CONCLUDING REMARKS

This chapter so far has analyzed choices generated from classroom experiments using existing methods. It was found, like many other studies, that violation of the EU model was evident in this data set, and that the fanning-out hypothesis does explain a big part of the data set. However, the data also exhibits many other different choice patterns. If we continue to pursue the analysis of a representative consumer, any theory with homogeneous preferences will be directly rejected by our data, because the data shows that there are choice patterns that are inconsistent with any theory. This is the key deficiency we see with existing methods. Furthermore, as we have also seen, the previous analysis is rather unsystematic in that no statistical inferences were made about the hypothesis. To use all information to construct a systematic test of theories of choice under uncertainty, we develop in the following chapter a new approach in which heterogeneity of preferences among individuals is assumed.

Appendix To Chapter Three

RESPONSE SHEET PROVIDED TO STUDENTS

I. Introduction

This is an experiment about lotteries. You are under no obligation to participate. The result will be used for a research project. Your cooperation would be greatly appreciated. It is not a test of whether you can pick the 'best' lottery. Which lotteries you prefer is a matter of personal taste. Please make sure that your choices are not affected by others by working silently.

II. Choices

Row	Lottery	
1	A	B
2	A	B
3	A	B
4	A	B
5	A	B
6	A	B
7	A	B
8	A	B
9	A	B
10	A	B
11	A	B
12	A	B
13	A	B

Chapter Four

PREFERENCE RECOVERY FOR THE EXPECTED UTILITY MODEL

Models of preference evaluation and demand analysis have traditionally been based on data obtained by direct observation of individual choice behavior. Such models often involve specifying a utility structure for a representative consumer. By using the appropriate statistical technique, the utility functions can be inferred. This type of approach falls into the category of either hedonic price analysis (see, for example, Rosen (1974) and Brown and Rosen (1982)) or the more conventional revealed preference analysis. Our approach differs from these approaches by assuming that individual preferences are diverse and that there exists a preference parameter space in which "consumers" are located with different "addresses" or preference parameters. Aggregate preferences for a group of individuals are described by a probability density function in the space of preference parameters. In this thesis, the notion of preferences recovery refers to the estimation of such a density function.

This chapter develops the empirical techniques used for the preference recovery for the EU model. It also constructs a test on individual choice behavior under the EU model. Section 4.1 describes the preferences recovery techniques for the EU model.Section 4.2 reports a Monte-Carlo study of the estimates. In section 4.3, the experimental data presented in Chapter 3 is regrouped for the purpose of estimating and testing the EU model. Section 4.4 provides estimations of the EU model using the preference recovery technique described above. A test, based on the recovered preference density function, is constructed to test if the EU model adequately explains the data generated from laboratory experiments. The last section summarizes the results.

4.1 PREFERENCE RECOVERY

Recall equation (2.1) in Chapter 2, the expected utility of an individual choosing lottery $(p_1, p_2, p_3; x_1, x_2, x_3)$ under the EU model is

$$EU = p_1\, u(x_1) + p_2\, u(x_2) + p_3\, u(x_3) \qquad (4.1)$$

It is assumed that $u(x_1)>u(x_2)>u(x_3)$. Without loss of generality, we assume that $u(x_1)=1$, $u(x_2)=v$, $u(x_3)=0$. The consumer's expected utility of choosing the lottery then becomes

$$EU = p_1 + p_2\, v, \qquad (4.2)$$

$$0 \le v \le 1$$

In the case of monetary payoffs, v is the certainty equivalence parameter. Higher values of v imply that the consumer is more risk averse.[16] However, when non-monetary payoffs are used, this interpretation of the v parameter is not appropriate. In this case, we shall consider v only as a diversity parameter that distinguishes one individual from another. Since the choice of any one consumer, given a set of lotteries, depends on that consumer's value of v, each consumer's preferences can be completely represented by a single value of v. Given a sample population, aggregate preferences of consumers can then be represented by a probability density function f(v). The question addressed is how to estimate f(v) using choices generated from laboratory experiments.

Different approaches may be used to estimate such a density function. Examples are nonparametric smoothing techniques and maximum likelihood estimation techniques. We shall choose the latter in this thesis because it is a well known and widely accepted technique. To use the maximum likelihood approach, we need to specify a parametric form for the aggregate preference density function $f(v;\lambda)$, where λ is a set of unknown parameters to be estimated. The statistical problem presented is to estimate λ using choices from experiments.

To see what is involved in the estimation procedure, consider the following set of binary choice lotteries:

$$A_1: (0,1,0;\ x_1,\ x_2,\ x_3) \quad \text{vs.} \quad B_1: (0.5,0,0.5;\ x_1,\ x_2,\ x_3)$$

---

[16] Machina (1982) pointed out that v/(1-v) can be used to measure risk aversion of a consumer much like the Arrow-Pratt measure. The larger is the v value, the more risk averse is the individual (larger v means steeper EU indifference curves).

A_2: $(0,1,0;\ x_1, x_2, x_3)$ vs. B_2: $(0.8,0,0.2;\ x_1, x_2, x_3)$

A_3: $(0,1,0;\ x_1, x_2, x_3)$ vs. B_3: $(0.2,0,0.8;\ x_1, x_2, x_3)$

According to equation 4.1, the expected utility of an individual choosing each of these lotteries is given by

$$EU(A_1) = v, \qquad EU(B_1) = 0.5$$

$$EU(A_2) = v, \qquad EU(B_2) = 0.8$$

$$EU(A_3) = v, \qquad EU(B_3) = 0.2$$

Note that given Equation (4.2), the expected utility of choosing any lottery with probability data (p_1, p_2, p_3) is a linear function of v. We shall call these *EU lines*. Figure 4.1a shows the EU lines of all six lotteries listed above. The 45° degree line labelled $EU(A_1)$, $EU(A_2)$, $EU(A_3)$ represents expected utilities from lotteries A_1, A_2, and A_3 for all possible values of v. The other three horizontal lines labelled $EU(B_1)$, $EU(B_2)$, $EU(B_3)$ represent the expected utilities for lotteries B_1, B_2, and B_3 respectively. Given any pair of lotteries, EU implies that the individual will choose the lottery with the higher expected utility calculated by Equation 4.1. Consider lottery pair 1 for instance; the EU lines for A_1 and B_1 intersect at v=0.5 as in Figure 4.1a. Under the EU model, any individual whose v value is less than 0.5 would choose B_1 over A_1; otherwise, B_1 will be chosen over A_1. Similarly, lottery pair 2 divides the v space into two parts (0, 0.8) and (0.8, 1), and pair 3 splits the v space into two parts (0, 0.2) and

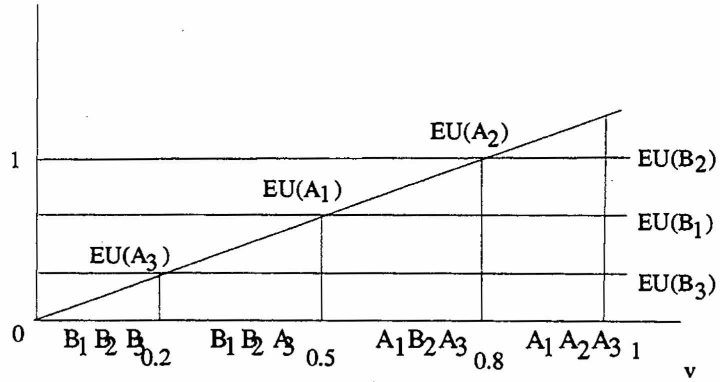


Fig. 4.1a: An Illustration of EU Choices

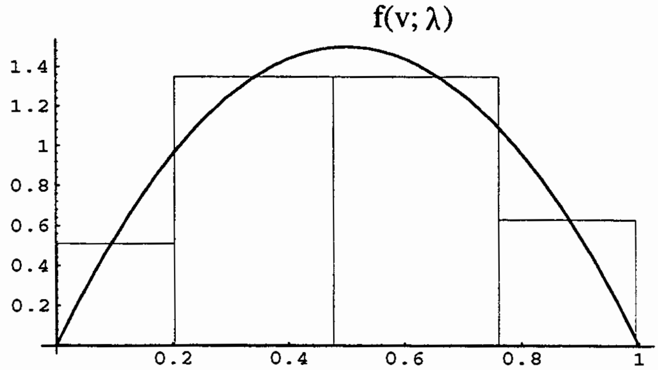


Fig. 4.1b: Histogram and Distribution of v

(0.2, 1). Therefore, the three lottery pairs divide the v space into four intervals: (0, 0.2), (0.2, 0.5), (0.5, 0.8), and (0.8, 1). Under the expected utility theory, an individual who has a v value in (0, 0.2) should choose B_1 in the first pair, B_2 in the second pair and B_3 in the third pair. Hence a choice pattern $B_1B_2B_3$ result for the three binary choice lotteries. Similarly, an individual with v value inside (0.2, 0.5) should choose $B_1B_2A_3$; an individual whose v value lies between 0.5 and 0.8 should choose $A_1B_2A_3$; finally, interval (0.8, 1) contains all the individuals who should choose $A_1A_2A_3$.[17]

In summary, given the above three pairs of lotteries, if the underlying theory (EU) is true, there will be four possible choice patterns generated from a sample population: $B_1B_2B_3$, $B_1B_2A_3$, $A_1B_2A_3$, and $A_1A_2A_3$, each with a corresponding subset of v. For convenience, we denote these choice patterns by 1,2,3,4. Given the number of subjects who choose each of these choice patterns in a sample population, a histogram based on the percentage of individuals choosing each choice pattern can be constructed as in Figure 4.1b. The probability density function $f(v;\lambda)$ drawn over the histogram is the density function to be estimated.

If we know $f(v;\lambda)$, the probability of an individual choosing pattern 1, 2, 3, 4 can be derived from Figure 4.1b as follows:

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[17] Any individual who lies at the boundary of these intervals would be indifferent between two lotteries. Since the experimental data discussed in Chapter 3 does not include individuals who expressed indifference, this case is dismissed for simplicity.

$$G(1) = \int_{0}^{0.2} f(v;\lambda)\,dv$$

$$G(2) = \int_{0.2}^{0.5} f(v;\lambda)\,dv$$

$$G(3) = \int_{0.5}^{0.8} f(v;\lambda)\,dv \qquad (4.3)$$

$$G(4) = \int_{0.8}^{1.0} f(v;\lambda)\,dv$$

G(j) (j=1,2,3,4) represents the probability that the j^{th} choice pattern is chosen. Let Q(j) denote the number of subjects who chose choice pattern j. Then the likelihood function for this data is proportionate to

$$L(\lambda) = \prod_{j=1}^{4} G(j)^{Q(j)} \qquad (4.4)$$

To get the maximum likelihood estimates of λ, we must maximize L(λ) with respect to λ.

The above estimation procedure assumes that individuals in choosing between lottery pairs strictly follow the expected utility theory. Under this circumstance, only four choice patterns from the three lottery pairs described above are possible. However, in our experimental data, more than four choice patterns are generated. In fact, for any set of three lottery pairs, there were eight possible choice patterns: $B_1B_2B_3$, $B_1B_2A_3$, $A_1B_2A_3$, $A_1A_2A_3$, $B_1A_2A_3$, $B_1A_2B_3$, $A_1B_2B_3$, $A_1A_2B_3$, which are also indexed by 1, 2, 3, 4, 5, 6, 7, 8 for convenience. Therefore given the data, without some elaboration of the model that permits other choices, one must immediately reject the model. We elaborate by introducing a trembling hand in the execution of intended choices. In particular, we make the following assumption: Individuals in making choices over a lottery pair have *trembling hands* and sometimes pick the lottery with the lower expected utility.[18] That is, people simply make mistakes in picking the "correct" lottery. Let θ be the mistake parameter representing the probability of an individual choosing the less-preferred lottery. Given the intention to choose lottery A in preference to lottery B, the individual actually chooses A with probability $1-\theta$, and B with probability θ. To elaborate further, suppose that an individual presented with lottery pair A and B has a higher expected utility for lottery A. Without a trembling hand, the probability of choosing A is 1.0 and the probability of choosing B is 0 under the EU model. In contrast, with a trembling hand, the probability of choosing A and B are $(1-\theta)$ and θ, respectively.

Given the probability of an individual falling in the jth

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[18] An alternative elaboration is to introduce an added error term to (4.2), which leads to the random utility model or a class of probabilistic choice models in econometrics literature. However this error term is often interpreted as "omissions" (e.g., unobservables and model misspecification) by researchers (See, for instance, Anderson, de Palma and Thisse, 1992). In addition, such models with systematic taste variations across individuals (i.e., different v values) are not identified with aggregate data.

choice interval: G(j), j=1,2,3,4, and the mistake parameter θ, the probability of any individual choosing each of the eight possible choice patterns is constructed as follows:

$$R(1)=(1-\theta)^3G(1)+\theta(1-\theta)^2G(2)+\theta^2(1-\theta)G(3)+\theta^3G(4) \tag{4.5}$$
$$R(2)=\theta(1-\theta)^2G(1)+(1-\theta)^3G(2)+\theta(1-\theta)^2G(3)+\theta^2(1-\theta)G(4)$$
$$R(3)=(1-\theta)\theta^2G(1)+\theta(1-\theta)^2G(2)+(1-\theta)^3G(3)+(1-\theta)^2\theta G(4)$$
$$R(4)=\theta^3G(1)+\theta^2(1-\theta)G(2)+\theta(1-\theta)^2G(3)+(1-\theta)^3G(4)$$
$$R(5)=\theta^2(1-\theta)G(1)+\theta(1-\theta)^2G(2)+\theta^2(1-\theta)G(3)+\theta(1-\theta)^2G(4)$$
$$R(6)=\theta(1-\theta)^2G(1)+\theta^2(1-\theta)G(2)+\theta^3G(3)+\theta^2(1-\theta)G(4)$$
$$R(7)=\theta(1-\theta)^2G(1)+\theta^2(1-\theta)G(2)+\theta^2(1-\theta)G(3)+\theta^2(1-\theta)G(4)$$
$$R(8)=\theta^2(1-\theta)G(1)+\theta^3G(2)+\theta^2(1-\theta)G(3)+\theta(1-\theta)^2G(4)$$

To show exactly how these R(j)'s (j=1, ..., 8) were obtained, let's take R(1) as an example: R(1) represents the probability of any individual choosing choice pattern 1, or $B_1B_2B_3$. From Figure 4.1a and Equations (4.3), the probabilities of an individual falling into each of the four choice intervals are given by G(1), G(2), G(3), and G(4). If the subject falls in the first interval, her best choice pattern would be $B_1B_2B_3$, but with mistake θ, she will execute this choice pattern with probability $(1-\theta)^3$. Therefore, the probability of an individual falling into interval 1 and choosing choice pattern 1 is $(1-\theta)^3$ G(1); if the individual falls into the second interval, her best choice pattern would be $B_1B_2A_3$. For her to choose $B_1B_2B_3$, she has to make no mistakes in pairs 1 and 2 and one mistake in pair 3. Hence the probability of an individual falling into the second interval and choosing choice pattern 1 is $(1-\theta)^2\theta G(2)$. Similarly, the probabilities of an individual falling into intervals 3, 4 and choosing choice pattern 1 are $\theta^2(1-\theta)G(3)$ and $\theta^3 G(4)$ respectively. Summing up all the probabilities, we obtain R(1) as shown in Equation (4.5). Figure 4.2 summarizes the construction of R(1).

Let Q(j) be the number of subjects choosing choice pattern j, j=1,...,8. Then the likelihood function for generating Q(j) is proportionate to

$$L(\lambda,\theta) = \prod_{j=1}^{8} R(j)^{Q(j)} \qquad (4.6)$$

A set of maximum likelihood estimates of λ and θ are obtained by maximizing L(λ,θ) with respect to λ and θ.[19]

The maximum likelihood estimation technique described above requires a choice of function f(v;λ). What we need in a density function is: (1) a function with domain $\{v|v\in(0.1)\}$, and (2) flexibility. Since the beta distribution satisfies both, we choose the beta distributions to represent the preference density functions.

The flexibility of the beta distribution is illustrated in

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[19] A computer program for the estimation written in FORTRAN by the author is available upon request.

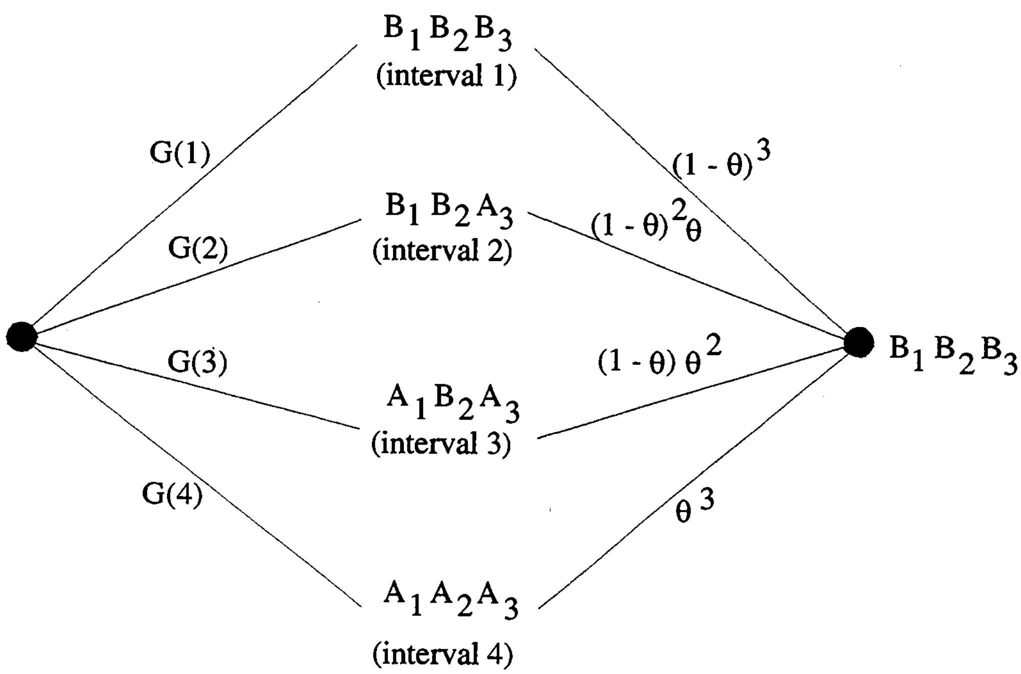

Fig. 4.2: Constructing Probability R(1)

Figure 4.3 in which the probability density function exhibits a large degree of variability to the extent that distribution has a single mode. Such flexibility is needed to recover preferences given little knowledge about the distribution of v. However, it should be noted that such flexibility is also limited. For example, the beta distribution function does not include multi-mode distributions.

The beta probability density function (PDF) is defined as

$$f(v;\ \lambda_1,\ \lambda_2) = \frac{v^{\lambda_1-1}(1-v)^{\lambda_2-1}}{\beta(\lambda_1,\lambda_2)}, \tag{4.7}$$

where $\beta(\lambda_1,\lambda_2)$ is the beta function, and

$$\beta(\lambda_1,\ \lambda_2) = \int_0^1 v^{\lambda_1-1}(1-v)^{\lambda_2-1}dv$$

Parameters λ_1 and λ_2 are the parameters to be estimated ($\lambda_1>0$, $\lambda_2>0$). Putting $f(v;\lambda_1,\lambda_2)$ into Equation (4.3), we obtained the estimates of λ_1 and λ_2 using numerical methods to maximize Equation (4.6).

The mean and variance of v are given by the following Equations.

$$\text{Mean} = \lambda_1/(\lambda_1+\lambda_2)$$
$$\text{Variance} = \lambda_1\lambda_2/(\lambda_1+\lambda_2)^2(\lambda_1+\lambda_2+1)$$

Function	λ_1	λ_2
I	2	4
II	4	2
III	1	2
IV	2	1
V	1	1
VI	0.5	0.5

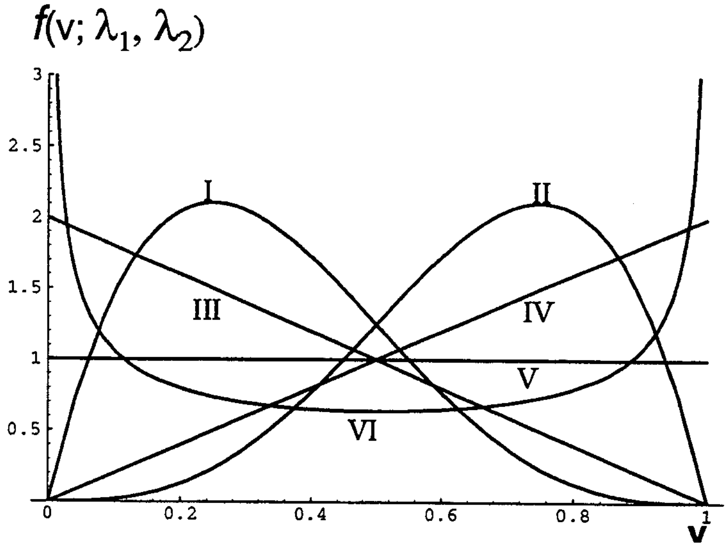

Fig. 4.3: The Beta Distribution Functions

4.2 Monte Carlo Studies

To judge the quality of the maximum likelihood estimates for our model, we conducted a series of Monte Carlo studies in the following way: (1) Choose a set of parameter values for λ_1, λ_2, θ, then the beta distribution $f(v;\lambda_1,\lambda_2)$ is given. (2) Draw a group of consumers, say 250,[20] from this distribution function. Each consumer then has a specific v value. (3) Given probabilities on a set of binary choice lotteries, make choices for each consumer according to the expected utility theory. (3) Allow for mistakes (with θ) and enumerate the choices to get aggregate data. (4) Use the aggregate data and the maximum likelihood approach to estimate the parameters; (5) Repeat the above procedure 600 times to generate a sampling distribution. Sampling properties of these estimates are examined to assess the usefulness of the estimation technique. For the chosen parameter values, (λ_1=2.0, λ_2=5.0, θ=0.05) and sample size (600), the estimated mean, variance and mean square error are reported as follows:

$$MEAN(\lambda_1)=2.0286,\ MEAN(\lambda_2)=5.0988,\ MEAN(\theta)=0.0498$$

$$VAR(\lambda_1)=0.1188,\ VAR(\lambda_2)=0.8588,\ VAR(\theta)=0.0002$$

$$MSE(\lambda_1)=0.1196,\ MSE(\lambda_2)=0.8686,\ MSE(\theta)=0.0002$$

Based on these results, a standard test on unbiasedness of each

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[20] This number corresponds to the number of subjects who participated in our experiment and preferred a coffee mug to a pen.

parameter estimate was performed. Test statistic in this case is $(MEAN-TRUE)/(VAR/600)^{1/2}$. The calculations are 2.0327, 2.6138 and 0.3466 for λ_1, λ_2 and θ respectively. The null hypothesis that parameter estimate is unbiased is weakly rejected for both λ_1 and λ_2, but accepted for θ at the 5% level of significance. It is accepted, however, in all cases at the 1% significant level. These can be taken as evidence that our estimates are reasonably good for the sample size chosen. Nonetheless, one should keep in mind the small sample bias when judging the results.[21]

4.3 Data regrouping

The HILO lottery structure discussed in Chapter 3 is a powerful tool for testing violations of the expected utility theory, however, each structure produces only two choice patterns (i.e.AAAA or BBBB) under the EU model. This raises a technical difficulty of estimating the aggregate preferences of the subjects. Referring to Equation (4.4), with two choice patterns, only two data points can be used in forming the likelihood function, which is insufficient for estimating the three parameters: λ_1, λ_2 and θ. To generate more variations in choice patterns for estimation purposes, we regroup the data into four sets, based on the following rationale: (1) The preliminary study on the data from Chapter 3 shows that the choices are not perfectly consistent with

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[21] Monte Carlo study shows that it is a small sample bias, since when increasing sample size from 250 to 2000 the bias disappears.

any theory. Furthermore, choice patterns seem to be sensitive to lotteries generated from different parts of the Marschak-Machina triangle. Hence it would be interesting to use data from one portion to calibrate the model in other portions of the triangle; (2) More than one data set was needed for testing.

The four data sets summarized in Table 4.1 are also graphically shown in Figure 4.4. They are labelled I, II, III and IV. For example, data set I, consisting of pairs 1, 5, 9, is presented by the circle in the middle of the triangle.

Figure 4.4 also reveals our experimental design, which defines the precise hypotheses to be tested. The purpose now is to develop a new approach to test whether a particular theory of choice works uniformly on the Marschak-Machina triangle, or whether individuals make consistent choices from one part of the triangle to another under a particular theory.

4.4 TESTING THE EU MODEL

Our Monte Carlo study shows that the maximum likelihood technique produces reliable estimates for "artificial consumers" who follow the expected utility theory. The question arises: Do real consumers make decisions under uncertainty in accordance with the expected utility theory? More generally, how would we know that a particular model is adequate in explaining the experimental data? To

Table 4.1: The Data Sets used for Preference Recovery

No. of data set	Lottery Pairs Involved
I	1, 5, 9
II	2, 6, 10
III	3, 7, 11
IV	4, 8, 12

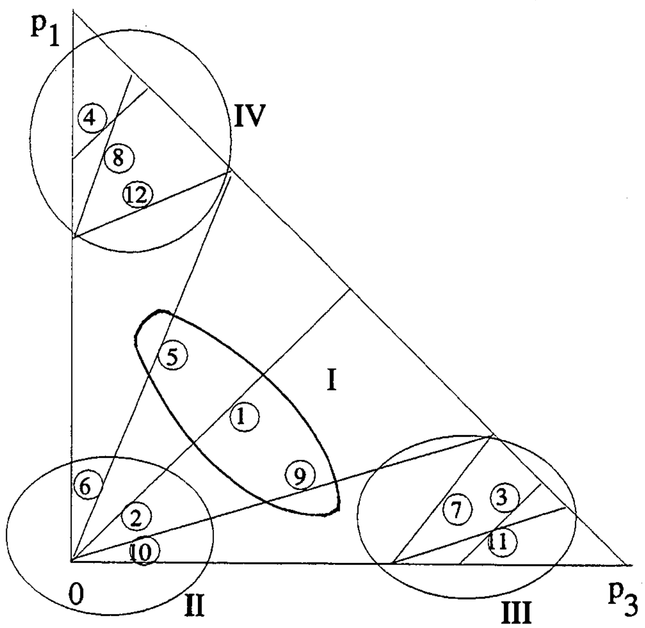

Fig.4.4: The Data Sets for Preference Recovery

be able to answer these questions, we construct a test to determine if subjects consistently follow the expected utility theory. More specifically, we use each data set to estimate the preferences density function and the mistake parameter. Since all data sets were generated from the same population, if the underlying theory is true, the parameters associated with the preferences function (i.e., λ_1 and λ_2) estimated from one set of data should not be significantly different from another assuming that the subjects were consistent in making these choices (the rationality assumption). Based on this argument, a likelihood ratio (LR) test is constructed to test the hypothesis that these parameters are statistically the same. Should the null hypothesis not be rejected, we would conclude that the choices generated under the expected utility theory were from the same preferences density function. This may indicate the validity of the theory. Conversely, should the null hypothesis be rejected, the adequacy of the theory in explaining these choices is questionable.

The LR test statistic is defined as $LR=-2\ln\tau$, where $\tau=RL/UL$, RL represents the constrained maximum likelihood value and UL stands for the unconstrained maximum likelihood value. Given the two parameters to be tested, there are two restrictions to be imposed for a test between any two data sets. Thus, the LR test statistic follows a χ^2 distribution with 2 degrees of freedom. The critical value of $\chi^2(2)$ at the 5% significance level is 5.99. Notice that we do not have a standard regression equation in which restrictions can be explicitly imposed in our model. Consequently, calculations of UL and RL are somewhat different from the conventional method. In particular, lnUL is the sum of the two maximum log-likelihood values from estimating two sets of parameters using two data sets; the RL is obtained by pooling the two sets of data through restricting λ_1, λ_2, that is, to assume the parameters λ_1 and λ_2 are the same in two data sets.[22]

Notice that our central hypothesis is that the v values of the same population in two different experiments are the same. However, if the hypothesis that the two sets v values are drawn from the same density function is rejected, then this central hypothesis will also be rejected. This indicates that individuals under the expected utility model make inconsistent choices. Therefore the validity of the underlying theory is questionable. Given this discussion, we conclude that though the constructed test statistic is biased in testing our central hypothesis, it is biased in the correct direction if the null is rejected. However, if the null hypothesis is accepted, it is not clear that we should immediately accept the theory. In this sense, the proposed LR test can be considered a test of "false" models not the "true" model.

Table 4.2 presents the parameter estimates using all data sets. The test results are reported in Table 4.3. Table 4.2 shows that the estimates are very different from one set of data to another. In all

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[22] A Monte-Carlo study was conducted to test the validity of this test. For 2000 simulation runs, the null hypothesis was rejected at the 5% level of significance for 117 times, that is, 5.85% of the total number. This indicates that the test is valid. We also conducted a Monte-Carlo study on the power of the test. The results show that the power increases as we move further away from the null.

Table 4.2: Parameter Estimates for the EU Model

Estimates using data set I	Estimates using data set II	Estimates using data set III	Estimates using data set IV
$\lambda_1 = 1.5602$	$\lambda_1 = 0.7042$	$\lambda_1 = 1.3039$	$\lambda_1 = 0.0217$
$\lambda_2 = 0.8184$	$\lambda_2 = 1.2888$	$\lambda_2 = 1.0865$	$\lambda_2 = 0.0010$
$\theta = 0.0138$	$\theta = 0.0334$	$\theta = 0.0236$	$\theta = 0.0676$

Table 4.3: LR tests for the EU Model

data sets	calculated LR	hypothesis
I & II	116.29	rejected
I & III	18.90	rejected
I & IV	196.24	rejected
II & III	50.14	rejected
II & IV	396.36	rejected
III & IV	280.30	rejected

*Critical value of $\chi^2(2)$ = 5.99 at the 5% significance level.

cases, as shown in Table 4.3, the null hypothesis that the parametersare the same is rejected. We suggest three possible interpretations: First, the beta distribution density function used for maximum likelihood estimation may be too inflexible (e.g, it is a unimodal distribution) to approximate the real density function of the population on the preference parameter v space; second, the expected utility model is inadequate in explaining individual decisions under risk.

To focus on the first possibility, we used the experimental data to generate a histogram or frequency distribution for the subjects in the v space. We then plotted both the histogram and the estimated beta distribution on one diagram. In general, the recovered beta distributions fit the histograms very well. The histograms shows that all data sets gave rise to unimodal distributions. Figure 4.5 shows examples of the histograms and the recovered beta density functions for the subjects.[23] Since we have also shown in the previous section that the maximum likelihood estimates in the simulation study are approximately unbiased, we conclude that the assumption of a beta density function is not responsible for failure of the test.

We therefore conclude that the EU model fails to explain our data. This is consistent with the results found in all other studies that people's actual choice behavior is inconsistent with what expected

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[23] These histograms are constructed in terms of v parameter space only. Parameter θ is being set at 0.

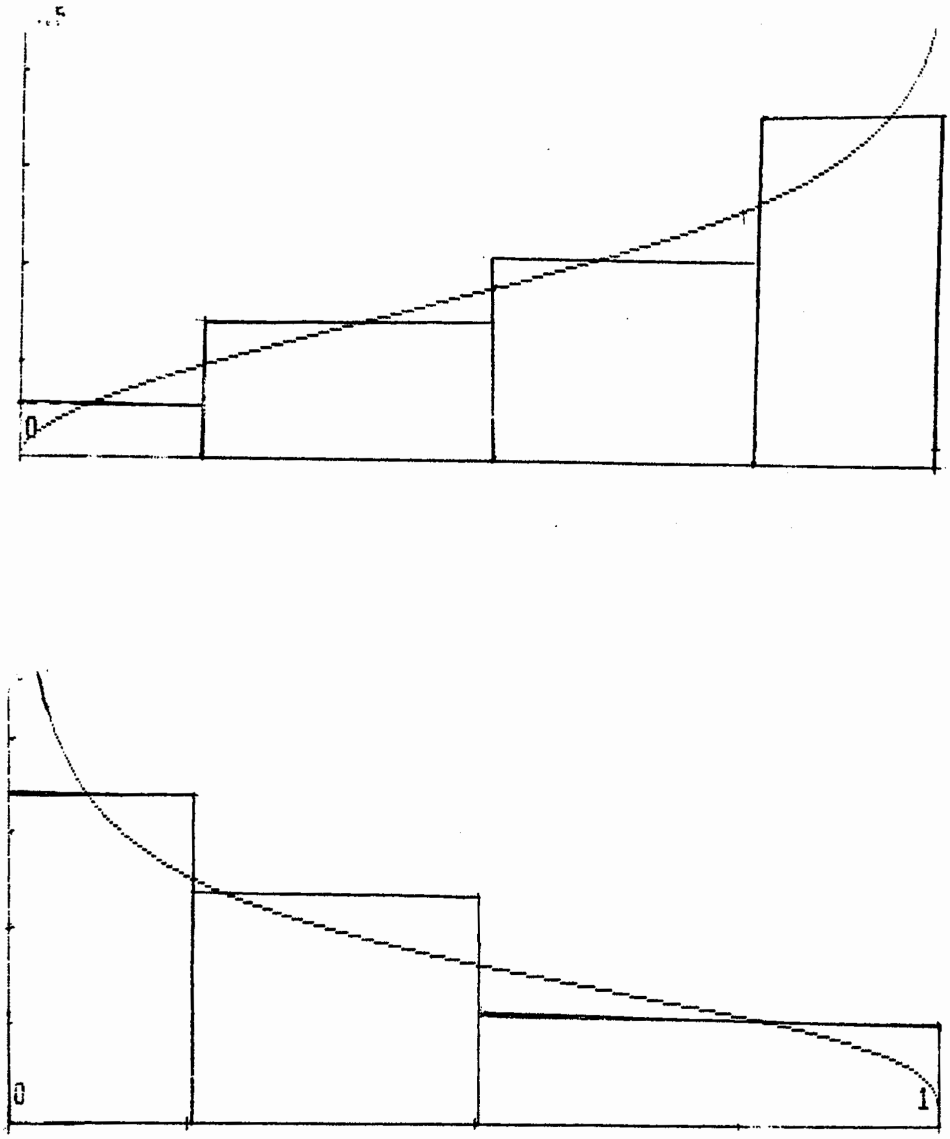

Fig. 4.5: Histograms and the Recovered Beta Density Function

utility theory predicts. The task now is to apply this new testing technique to alternative models and see if there exists an adequate theory to explain the experimental data. The next chapter explains three alternative models and testing results.

4.5 SUMMARY

This chapter presented the preferences recovery techniques for the EU model. A likelihood ratio test based on recovered preference density function was provided to test the inadequacy of EU in explaining the choices people make in laboratory experiments. The results show that EU fails to predict all our experimental data, even when we allow for randomness in the model. We then conclude that it is not an appropriate model for the data.

It should be noted that although the empirical estimation procedure was presented in terms of a particular example, its generalization is straightforward. As will be seen in the next chapter, it can be easily applied to other choice models.

Chapter Five

THE ALTERNATIVE MODELS

As with the other overwhelming evidence from laboratory experiments reporting that the expected utility model is a poor descriptor of empirically observed decision making behavior, this study found further evidence demonstrating the inadequacy of EU in explaining choices. The question is: Does there exist an alternative theory that could adequately explain the data? The purpose of this chapter is to search for such a model. We have seen from the review of literature in Chapter 2 that, over the past decade, many attempts have been made to extend the EU model in various ways to improve on the descriptive ability of the EU model. Roughly speaking, there are two classes of extensions among the alternative models. One class contains the subjectively weighted utility (SWU) models where the criterion for decision making is a weighted sum of the utility index, and the weights are some transformations of probabilities (Karmarkar, 1978; Quiggin, 1979,). Another class replaces the utility index with some function of probabilities and final outcomes (Coombs & Huang, 1970; Chew and Dekel, 1979; Machina, 1982). All these models are similar in spirit in that they are more generalized forms of expected utility and describe decision behavior in terms of maximization of the criterion. It is not the purpose of this study to create a new generalized utility theory, or to justify the existing theories. It is, rather, to search for an "adequate" theory that explains our experimental data. Two criteria were used in the searching process: First, given the preference recovery techniques presented in the previous chapter, the new models should have manageable functional forms, at least to the extent that preference can be readily parameterized; secondly, they should be able to generate fanning-out indifference curves, as the fanning-out hypothesis appears to be the most attractive hypothesis.

Based on these criteria, we focused the search on the class of subjectively weighted utility models (SWU). Such models assume that the individual first transforms the known set of objective probabilities $\{p_i\}$ of a risky prospect into corresponding "subjective probabilities" $\{w(p_i)\}$ (or "decision weights"), and then maximizes the value of $\sum w(p_i)u(x_i)$. Notice that in this class of models, the decision weight of a particular outcome x_i does not depend on the outcome, it depends only on the probability p_i. This means that the parameterization of utilities is identical to that of the EU model, as will be seen below. The theories (and authors) considered in this study are the subjectively weighted utility (SWU) theory (Karmarkar); the weighted utility theory (Chew and MacCrimmon); and the rank-dependent expected utility theory (Quiggin).

Section 5.1 presents Karmarkar's SWU model. Preference recovery and tests of this model are conducted in section 5.2. A model in which indifference curves are linear and fan out from the lower right corner to the upper left corner of the Marschak-Machina triangle, namely the linear fanning-out model (LFO), is created in section 5.3. As will be seen, this model happens to be a special case of the *weighted utility theory* developed by Chew and MacCrimmon (1979). This section also reports the test results for this model. Section 5.4 introduces *the rank-dependent utility* theory proposed by Quiggin (1982). This theory is based on a function, f(P), that satisfies the following general conditions: f(0)=0, f(1/2)=1/2, and f(1)=1. Following this theory, we construct a quadratic functional form for f(P) in which the above conditions are satisfied. This model is called the quadratic rank-dependent (QRD) utility model. Test results are also provided in this section. Section 5.5 presents a test of relative explanatory power for all the alternative models. Finally, section 5.6 makes a concluding remark.

5.1 KARMARKAR'S SWU MODEL

Karmarkar (1978) proposed a subjectively weighted utility (SWU) model as a descriptive extension of the EU model. According to this model, the utility for a lottery $(p_1, p_2, p_3; x_1, x_2, x_3)$ is defined by

$$SEU = \sum_{i=1}^{3} w(p_i)\ u(x_i) \ / \sum_{i=1}^{3} w(p_i) \qquad (5.1)$$

where

$$w(p_i)=p_i^{\alpha} \ / \ [p_i^{\alpha} + (1-p_i)^{\alpha}] \qquad (5.2)$$

In this model, prizes are mapped into utilities in the usual manner: $x \longrightarrow u(x)$, and probabilities are transformed into subjective weights: $p_i \longrightarrow w(p_i)$ as defined above. α is an additional parameter that may be regarded as a measure of probability distortion. When α=1, the SWU model reduces to the EU model. The mapping for various α values is sketched in Fig. 5.1. For $\alpha \neq 1$ the mapping has three fixed points: 0, 1/2, and 1. Thus equiprobability, certainty, and impossibility are not affected by the mapping. When $0<\alpha<1$, $w(p_i)>p_i$ for $p_i<1/2$, and $w(p_i)<p_i$ for $p_i>1/2$. This is known in the literature (e.g., Dale, 1959; Kahneman and Tversky, 1979) as subjects overestimating low probabilities ($p_i<1/2$) and underestimating high ones. Symmetrically, when $\alpha>1$, $w(p_i) < p_i$ for $p_i< 1/2$, and $w(p_i) > p_i$ for $p_i> 1/2$. This is the case of overestimating high probabilities and underestimating low ones. Figure 5.2 shows a contour plot of the indifference map of an individual with $v=0.2$ and $\alpha=0.5$.

It is argued by Karmarkar (1979) that this model can be used to explain the *Allais paradox* or fanning-out effect. Unfortunately, as proved by Quiggin (1982), this model violates the stochastic dominance property of the EU model. When such a property is imposed on Karmarkar's SWU model, the model reduces to EU. Lottery A is said to stochastically dominate lottery B if the expected utility from A is larger than that from B for all monotonically increasing utility index u(x). Given the nonlinear probability transformation of the SWU model, Quiggin argued that under certain conditions, a stochastically dominant lottery may generate a lower expected utility.[24] Nonetheless we shall use our preference recovery techniques and the LR test, to see if this model could adequately explain our data.

[24] For a thorough proof, see Quiggin (1982).

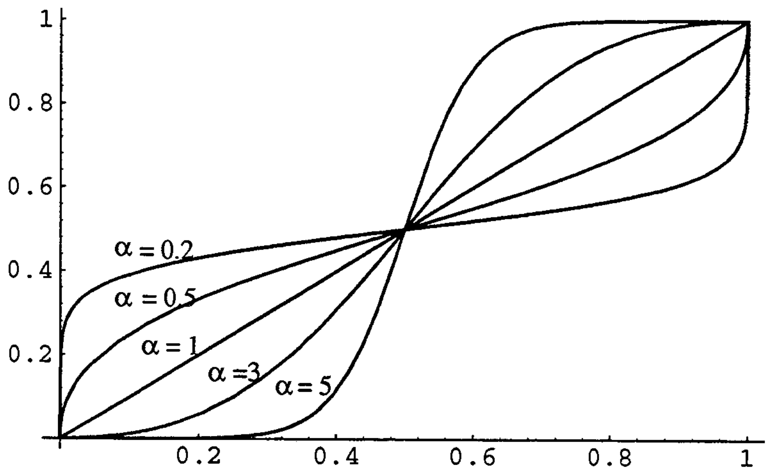

Fig. 5.1: Transformation of Probabilities

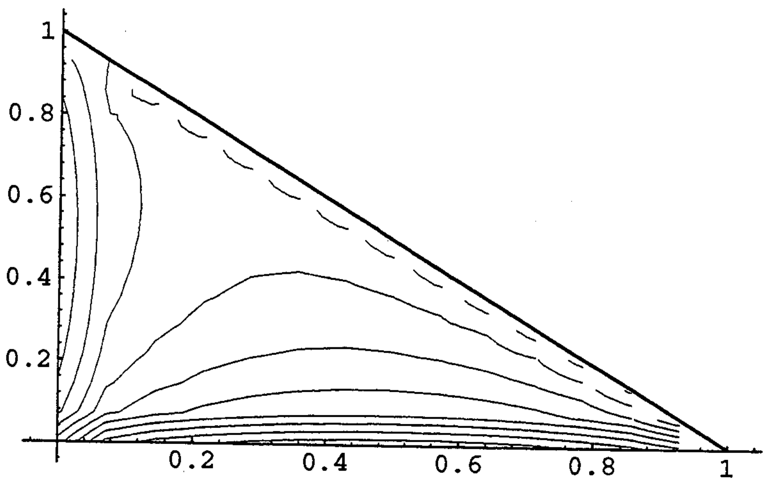

Fig. 5.2: Indifference Curves of the SWU Model

5.2 PREFERENCE RECOVERY AND TEST OF THE MODEL

From equation (5.1), an individual's preference over lottery (p_1, p_2, p_3; x_1, x_2, x_3) is completely described by the utility index: $u(x_1)=1$, $u(x_2)=v$ and $u(x_3)=0$ and the additional parameter α. Given the value of v derived from a beta distribution and the value of α, the individual's subjectively expected utility (SEU) over this lottery can be calculated by equation (5.1). With two lotteries, he or she picks the one that gives the higher SEU. As in the EU model, we continue to assume that there is a probability of θ that the individual picks the "wrong" lottery (i.e., the one with the lower SEU). Thus, preference recovery for the alternative model involves estimating four parameters: the two parameters from beta distribution function λ_1 and λ_2, the mistake parameter θ, and the probability distortion parameter α. For simplicity, both the mistake parameter and the distortion parameter are assumed to be the same across individuals. Hence the diversity of individual consumers is captured solely by the parameter v.[25]

To estimate the parameters, we construct likelihood functions of the data sets in exactly the same way as under the EU model, after transforming the objective probabilities into subjective weights. In particular, given three pairs of lotteries, a typical set of data

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[25] One could assume that the additional parameter for each alternative model is different from individual to individual. In this case, the subjects are considered to be drawn from a joint density function $f(v,z;\ \lambda)$, where z represents the additional parameter in each model, λ is a set of unknown parameters to be estimated.

presented in Chapter 4, and with each lottery represented by (p_1, p_2, p_3; x_1, x_2, x_3), there are eight choice patterns generated from a sample population. Let Q(j) be the number of subjects choosing choice pattern j, j=1,......,8. Given a beta distribution density function $f(v;\lambda_1, \lambda_2)$ and parameters θ and α, the probability of generating each of the eight choice patterns, $R_j(\lambda_1,\lambda_2,\theta,\alpha)$, is calculated. The likelihood function of generating the data is given by

$$L(\lambda_1,\lambda_2,\theta,\alpha) = \prod_{j=1}^{8} (R(\lambda_1,\lambda_2,\theta,\alpha))^{Q(j)} \qquad (5.3)$$

The estimates are obtained by maximizing $L(\lambda_1,\lambda_2,\theta,\alpha)$.[26]

Using all four data sets: I, II, III, IV (for convenience, Fig. 4.4 with data sets circled on the Marschak-Machina triangle is shown as Fig. 5.3 in this chapter), we estimated the preference density functions and performed the LR test for the SWU model. The test statistic has 3 degrees of freedom instead of 2 due to the additional parameter α. The critical value of $\chi^2(3)$ at the 5% significance level is 7.81. The test once again, was used to test the hypothesis that the parameters λ_1, λ_2 and α estimated using one set of data are the same as those estimated using another data set.

[26] The computer programs for these alternative models written in FORTRAN are also available from the author.

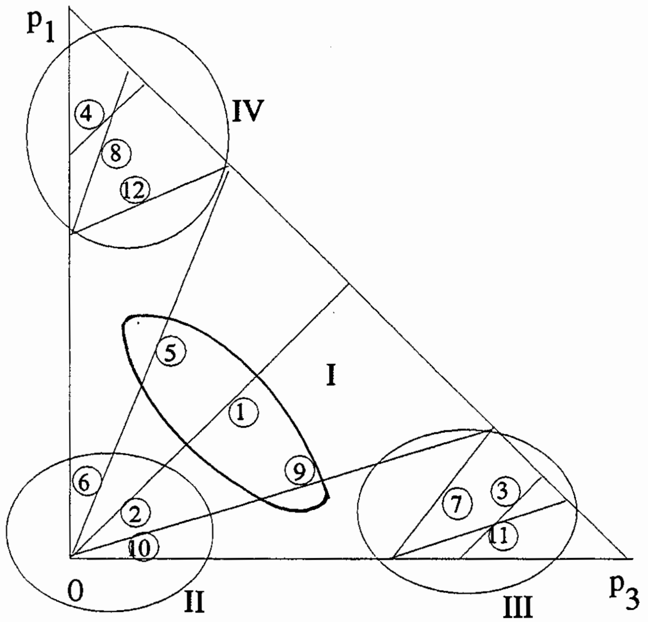


Fig. 5.3: Data Sets I, II, III, and IV

Parameter estimates are reported in Table 5.1 and test results are provided in Table 5.2. Table 5.1 shows that the parameter estimates for λ_1, λ_2, and α vary significantly from one set of data to another. Estimates for the mistake parameter θ, however, remain small and relatively stable. It is interesting to note that the estimate of α is 1 in data set IV and nearly 1 in data set III. This means that the SWU model does not improve on EU in these regions since the model reduces to EU when α=1. The estimate from data set II (α=0.42) indicates that on average the subjects overestimate low probabilities and underestimate high ones in the neighborhood of certainty (the area around the origin and inside the triangle) — a case commonly reported in the literature. But the estimate from set I (α=4.37) shows the opposite. It is not clear what causes these disparate results. Table 5.2 shows that the null hypothesis is rejected in all cases except for data sets I and III. In the context of five rejections, the one acceptance could be interpreted as a type II error. This argument is supported by comparing the estimates from data set I (λ_1= 0.27, λ_2= 0.11, θ=0.01, α=4.37) and those from data set III (λ_1= 1.30, λ_2= 1.09, θ=0.02, α=1.01). Since they appear different, the hypothesis that these parameters are the same should not be accepted even with visual inspection.

Table 5.1: Parameter Estimates for the SWU Model

Estimates using data set I	Estimates using data set II	Estimates using data set III	Estimates using data set IV
$\lambda_1 = 0.2674$	$\lambda_1 = 5.7835$	$\lambda_1 = 1.2934$	$\lambda_1 = 0.0217$
$\lambda_2 = 0.1106$	$\lambda_2 = 7.6521$	$\lambda_2 = 1.0865$	$\lambda_2 = 0.0010$
$\theta = 0.0139$	$\theta = 0.0332$	$\theta = 0.0236$	$\theta = 0.0676$
$\alpha = 4.3667$	$\alpha = 0.4177$	$\alpha = 1.0064$	$\alpha = 1.0000$

Table 5.2: LR tests for the SWU Model

Data sets	Calculated LR	Hypothesis
I & II	117.20	rejected
I & III	3.46	accepted
I & IV	197.08	rejected
II & III	31.76	rejected
II & IV	398.52	rejected
III & IV	281.26	rejected

Critical value of $\chi^2(3) = 7.81$ at the 5% significance level.

5.3 THE LINEAR FANNING-OUT MODEL(LFO)

Inspired by Machina's fanning-out hypothesis, we build an alternative model that generates linear fanning-out indifference curves, namely the Linear Fanning-out (LFO) model. The utility of a risky prospect $(p_1, p_2, p_3; x_1, x_2, x_3)$ under this model is given by

$$SEU = \sum_i w_i(p_1, p_2, p_3)\ u(x_i) \qquad (5.4)$$

where

$$w_1(p_1, p_2, p_3) = \frac{p_1}{1+\beta p_3}$$

$$w_2(p_1, p_2, p_3) = \frac{p_2}{1+\beta p_3} \qquad (5.5)$$

$$w_3(p_1, p_2, p_3) = \frac{(1+\beta)p_3}{1+\beta p_3}$$

β is an additional parameter that determines the decision weights. This parameter will be further explained below.

To show that this model has linear fanning-out indifference curves in the (p_1, p_3) space,

$$\text{let } \bar{u} = \left(\frac{p_1}{1+\beta p_3}\right) u(x_1) + \left(\frac{p_2}{1+\beta p_3}\right) u(x_2) + \left(\frac{(1+\beta)p_3}{1+\beta p_3}\right) u(x_3)$$

Recalling the following normalization $u(x_1)=1$, $u(x_2)=v$, $u(x_3)=0$,

$$\bar{u} = (\frac{p_1}{1+\beta p_3}) + (\frac{1-p_1-p_3}{1+\beta p_3})\ v$$

Solving for p_1, we have

$$p_1 = (\frac{v + \beta\bar{u}}{1 - v})\ p_3 + (\frac{\bar{u} - v}{1 - v}) \qquad (5.6)$$

Equation (5.6) clearly shows that the slope of the indifference curve increases with $\bar{u}$. It also increases with β. The larger the β value, the more rapidly indifference curves fan out. Under the condition that $\beta=0$, the linear fanning-out model collapses to the EU model. Hence β is called a "rapidity" parameter that measures the degree of fanning-out. Figure 5.4 shows the indifference curves for the LFO model when $\beta=5$.

Notice that the specification of the expected utility function given in equation (5.4) is also a special case of the *weighted utility* theory proposed by Chew and MacCrimmon (1979). In their theory, the expected utility for a lottery is expressed by

$$SEU = \frac{p_1 w(x_1) u(x_1) + p_2 w(x_2) u(x_2) + p_3 w(x_3) u(x_3)}{p_1 w(x_1) + p_2 w(x_2) + p_3 w(x_3)} \qquad (5.7)$$

where w(x) is the weighting function of the final outcome x. The theory suggests no intuitive interpretation to the weighting function. However, the weights seem to reflect misperceptions of objective probabilities. When $w(x_1)=1$, $w(x_2)=1$ and $w(x_3)=1+\beta$, the weighted

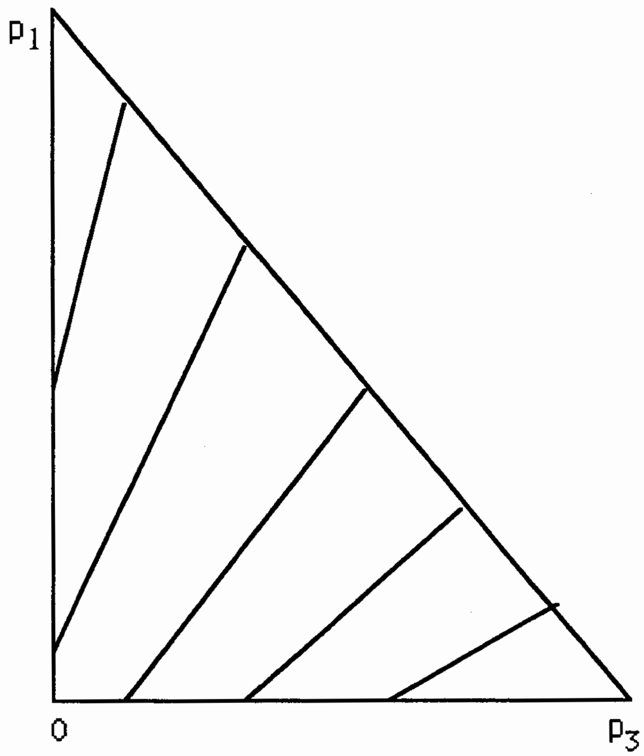


Fig. 5.4: Indifference Curves of the LFO Model

utility model becomes the LFO. Notice that from equation (5.7), these decision weights depend upon both the probabilities and outcomes that determine the utilities, while in the class of the subjectively weighted utility models decision weights (i.e., w(p)) depend on only probabilities, p.

Preference recovery for the LFO model involves estimating λ_1, λ_2, θ, and β. Using the same method and procedure described for the SWU model, we estimate these parameters and test the adequacy of the LFO model in explaining our data. The results are shown in Tables 5.3 and 5.4. Table 5.3 shows that the estimated λ_1 and λ_2 for the LFO model vary significantly from one data set to another, and that both the mistake parameter and the additional parameter β are estimated to be very small. A small β indicates a small degree of fanning-out indifference curves. As presented in Table 5.4, the null hypothesis is rejected for all data sets for the LFO model. The LR values are almost the same as those calculated for the EU model for data sets I & II and I and III. But The LR values for the other four data sets are much smaller than those calculated for the EU model, indicating some improvements. However, they are not small enough to accept the null hypothesis. Therefore the LFO model is also not an appropriate model for explaining the data.

Table 5.3: Parameter Estimates for the LFO Model

Estimates using data set I	Estimates using data set II	Estimates using data set III	Estimates using data set IV
$\lambda_1 = 1.9978$ $\lambda_2 = 2.1124$ $\theta = 0.0139$ $\beta = 0.0382$	$\lambda_1 = 0.7932$ $\lambda_2 = 1.7749$ $\theta = 0.0332$ $\beta = 0.0029$	$\lambda_1 = 1.3504$ $\lambda_2 = 1.1950$ $\theta = 0.0236$ $\beta = 0.0026$	$\lambda_1 = 0.0218$ $\lambda_2 = 0.0010$ $\theta = 0.0676$ $\beta = 0.0000$

Table 5.4: LR tests for the LFO Model

Data sets	Calculated LR	Hypothesis
I & II	117.66	rejected
I & III	19.60	rejected
I & IV	56.12	rejected
II & III	28.08	rejected
II & IV	250.66	rejected
III & IV	54.84	rejected

Critical value of $\chi^2(3) = 7.81$.

5.4 THE QUADRATIC RANK-DEPENDENT UTILITY MODEL (QRD)

In most of the subjectively weighted utility models such as the one created by Karmarkar in section 5.1, the decision weight $w(p_i)$ of outcome x_i depends on only the probability p_i. This means that outcomes with the same probability must have the same decision weight. Quiggin (1992) proposed a rank-dependent utility theory (initially called the anticipated utility theory) in which the probability transformation of p_i not only depends on p_i but also depends on probabilities of other outcomes. In particular, the transformation function $w_i(p)$ depends on all p_j for $j \le i$. According to this theory, the expected utility of a risky prospect $(p_1, p_2, p_3; x_1, x_2, x_3)$ is given by

$$SEU = \sum_{i=1}^{3} w_i(p)\, u(x_i) \qquad (5.8)$$

where,

$$\begin{aligned} p &= (p_1, p_2, p_3) \\ w_1(p) &= f(p_1) \qquad (5.9) \\ w_2(p) &= f(p_1+p_2) - f(p_1) \\ w_3(p) &= 1 - w_1(p) - w_2(p) \end{aligned}$$

$f(p)$ is a transformation function of lotteries with only two outcomes. It is used here to form the weights $w_i(p)$ for lotteries involving three outcomes. Quiggin did not suggest a specific form for $f(p)$ in his paper, but he did discuss the general properties of this function. In particular, $f(p)$ determines the pattern of probability distortion.

$f(p)>p$ for $p<1/2$ and $f(p)<p$ for $p>1/2$ imply that subjects overestimate small probabilities and underestimate high ones. More strictly, this case requires that $f(p)$ is concave on [0, 1/2] and convex on [1/2, 1], and that $f(0)=0$, $f(1/2)=1/2$, $f(1)=1$. This echoes to Karmarkar's decision weights function with $0<\alpha<1$.

According to this general description of f(p), we create a quadratic function as follows[27]

$$f(p) = \begin{cases} \gamma p - 2(\gamma-1)p^2 & \text{for} \quad 0 < p \leq 1/2 \\ (\gamma-1)+(4-3\gamma)p+2(\gamma-1)p^2 & \text{for} \quad 1/2 \leq p \leq 1 \end{cases}$$

and satisfying the restrictions,

$$f(0)=0,\ f(1/2)=1/2,\ f(1)=1$$

The parameter γ measures the distortion of objective probabilities. When $\gamma>1$, f(p) is concave on [0, 1/2] and convex on [1/2, 1], When $0<\gamma<1$, f(p) is convex on [0, 1/2] and concave on [1/2, 1]. When $\gamma=1$, there is no distortion.

Putting f(p) into equation (5.9), we obtain a version of the rank-dependent utility model, namely the quadratic utility function (QRD). Given the definition of f(p), EU is a special case of QRD when

[27] Later on, we found from a recent book (1992) Quiggin wrote that Camerer and HO (1991) employed and estimated the following functional form: $f(p) = p^{\gamma}(p^{\gamma}+(1-p)^{\gamma})^{1/2}$.

$\gamma=1$. As illustrated in Figure 5.5, the indifference curves of the QRD model for $\gamma=1.5$ are nonlinear and fan out from the lower right corner to the upper left corner of the triangle.

Table 5.5 shows the estimates for all four data sets. Once again, the estimates for λ_1 and λ_2 differ significantly from one set of data to another. The mistake parameter remains the same as in all other models. The estimated γ, however, shows that on average subjects underestimate small probabilities (or overestimate high ones) in region I ($\gamma=0.2$) and region IV ($\gamma=0.41$), and overestimate small probabilities (or underestimate high ones) in region II ($\gamma=3.01$) and region III ($\gamma=1.12$). The non-linear fanning-out is thus observed in regions II and III.

Table 5.6 reports the test results of this model. Once again, we see rejections of the hypothesis for all data sets. The difference between this set of results and those found for the EU model is the case for data sets I & III, and II and III. The LR values for the QRD model are smaller than those for the EU model. One final interesting observation from Tables 5.1, 5.3, 5.5 and 4.1, is that the estimated θ is approximately the same across all models using the same data set. Since θ measures the probability of choosing the less-preferred lottery under each model, the result indicates that the probability of an individual making a mistake is quite consistent across all theories.

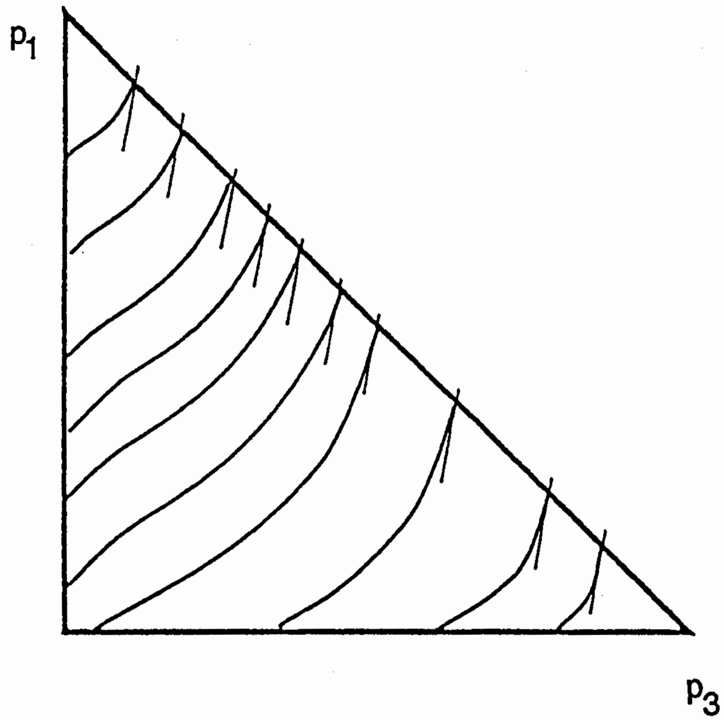


Fig. 5.5: Indifference Curves of the QRD Model

Table 5.5: Parameter Estimates for the QRD Model

Estimates using data set I	Estimates using data set II	Estimates using data set III	Estimates using data set IV
$\lambda_1 = 0.8640$ $\lambda_2 = 0.4017$ $\theta = 0.0138$ $\gamma = 0.2000$	$\lambda_1 = 0.9815$ $\lambda_2 = 1.7018$ $\theta = 0.0333$ $\gamma = 3.0108$	$\lambda_1 = 1.3278$ $\lambda_2 = 1.0641$ $\theta = 0.0236$ $\gamma = 1.1263$	$\lambda_1 = 0.0246$ $\lambda_2 = 0.0011$ $\theta = 0.0675$ $\gamma = 1.0261$

Table 5.6: LR tests for the QRD Model

Data sets	Calculated LR	Hypothesis
I & II	115.56	rejected
I & III	11.82	rejected
I & IV	185.84	rejected
II & III	30.08	rejected
II & IV	356.12	rejected
III & IV	125.26	rejected

Critical value of $\chi^2(3) = 7.81$.

5.5 A TEST OF MODEL PERFORMANCE

As discussed in the previous sections, all the alternative models include EU as a special case. In particular, when $\alpha=1$, SWU becomes EU; when $\beta=0$, LFO converts into EU; and when $\gamma=1$, the QRD models collapses to the EU model. To test if these alternative models are improvements on EU, one can perform a LR test of these restrictions. For example, the LR test of hypothesis, Ho:$\alpha=1$, for the SWU model is LR=-2(lnRL-lnUL), which asymptotically has a $\kappa^2(1)$ distribution. lnRL is the maximum log-likelihood value under the null hypothesis, lnUL is the maximum log-likelihood value under the alternative model. Acceptance of this hypothesis would, of course, imply that the alternative model does not add more explanatory power to the EU model, empirically.

Table 5.7 presents the LR tests of relative explanatory power of the alternative models for all data sets. The numbers in parentheses are the calculated LR values. The critical value of $\kappa^2(1)$ at the 5% significance level is 3.84. The results show that the null hypothesis under each model is accepted for all data sets. Therefore, these alternative models are not improvements on EU for our data.

Table 5.7: Testing Performance of the Alternative Models

Data set	Null hypothesis under		
	SWU Ho: α=1	LFO Ho: β=0	QRD Ho: γ=1
I	accepted (0.82)	accepted (1.36)	accepted (0.32)
II	accepted (0.64)	accepted (0.60)	accepted (0.18)
III	accepted (0.00)	accepted (0.04)	accepted (0.00)
IV	accepted (0.00)	accepted (0.00)	accepted (0.00)

5.6 CONCLUDING REMARKS

This chapter has presented three alternative functional forms: one from existing literature and the other two created by the author. inadequacy tests were performed on the alternative models using the same data sets. The results show that no theory can explain all the data sets. Furthermore, a test of relative performance of these alternative models as compared to the EU model shows the alternative models are not significant improvements on the EU model. Therefore we conclude that the three alternative models, each with an additional parameter, do not add much explanatory power, and that until we find a model that passes all adequacy tests, the expected utility theory, characterized by its simplicity and normative appeal of its axioms, retains its leading role in theories of choice under uncertainty.

Chapter Six

ANOTHER ILLUSTRATION

The previous chapters presented a new approach to testing theories of choice under uncertainty. In the approach it is assumed that individuals under a particular theory of choice have diverse preferences described by points in a parameter v-space, $v \in [0,1]$. Aggregate preferences of a sample population are represented by a probability distribution function of v. To study the aggregate choices people make over a set of differentiated products (e.g., lotteries), one needs to estimate such a probability density function. This way of describing the heterogeneous preferences across individuals is the spirit of all address models of product differentiation.

To extend the analysis, we demonstrate, in this chapter, the preference recovery techniques in a simple but standard address model of product differentiation. We also apply the techniques to a real case of product differentiation. Section 6.1 introduces the address model in which the consumer's preference takes a known parametric form. It also outlines a general procedure for estimating the aggregate preference density function. Section 6.2 illustrates an application to a real case of product differentiation in the context of BC ferry services. Section 6.3 reports the estimation and out-of-sample testing results. The last section discusses the potential limitations of the model and provides directions for further research.

6.1 THE MODEL

Consider a market for M differentiated products which are completely described by the quantities of K characteristics or attributes embodied in them. Let the M variants be W_1, W_2, ... W_M, where $W_j=(w_{j1}, w_{j2}, \ldots, w_{jK})$, j=1,...,M. The characteristics space is denoted $\mathbb{R}^K$, then $W_j \in \mathbb{R}^K$. Product j is offered for sale at price $p_j \geq 0$.

Suppose there is a finite number of consumers, N, each of whom is a potential customer in the market under discussion. It is assumed that each consumer buys at most one unit of one variant of the M products. The preference for variant j by an individual i is given by[28]

$$I_{ij} = v_i - \sum_{k=1}^{K} c_k (u_{ik}-w_{jk})^2 - p_j \qquad (6.1)$$

w_{jk}= amount of attribute k (k=1,...,K) possessed by variant j (j=1,...,M).

u_{ik}= consumer i's most preferred attribute k regarding to his or her ideal brand since, when all prices are equal $w_j= u_i$ maximizes net utility given in Equation 6.1.

c_k= a positive constant, measuring the marginal disutility from not buying the ideal brand with respect to attribute k (In the geographical context, this disutility

[28] This particular specification of the utility function has been used in several models of product differentiation (see, e.g., Eaton and Wooders, 1985.).

corresponds to the transportation cost).

p_j = price associated with variant j, j=1,...,M.

v_i = consumer i's reservation price for the most-preferred good, since the consumer will not buy the most-preferred good if p_j exceeds v_i, for all j=1,...,M.

Given the value of v and the most-preferred brand u, the consumer chooses among the n variants of products in the following way: (i) if maximal net (of price) utility calculated by Equation (6.1) is non-negative, the consumer buys 1 unit of the product that offers maximal net utility; (ii) if maximal net utility is negative, the consumer buys nothing.

Suppose that all consumer's preferences are known to be in the v and u parameter space $\mathbb{R}^{K+1}$ and that the population of consumers is distributed over the parameter space according to distribution function, f(u,v). The issue is how to estimate this density function given aggregate data on prices, product descriptions and quantities sold.

With M variants of products, the **(u,v)** parameter space can be partitioned into **m**+1 sets:

$$S_j = \{(u,v) \in \mathbb{R}^{K+1};\ I_j \geq I_i \text{ and } I_j > 0,\ i=1,\ldots,M\},\ j=1,\ldots,M$$
$$S_{m+1} = \{(u,v) \in \mathbb{R}^{K+1};\ I_j < 0, \text{ for all } j=1,\ldots,M\}, \qquad (6.2)$$

where S_j is defined as the market space of the variant j (j=1,...,M) and S_{M+1} is the set that no good is purchased.

These sets are illustrated in Figure 6.1 for some arbitrary prices and product descriptions in the M=3 and K=1 case. Given the number of consumers who have purchased one unit of variant j, the data provides what amounts to a histogram associated with the unknown distribution function. The statistical problem is then to recover such a distribution function from the histogram. Following the previous chapters, we adopt the maximum likelihood approach to estimate $f(u,v)$.

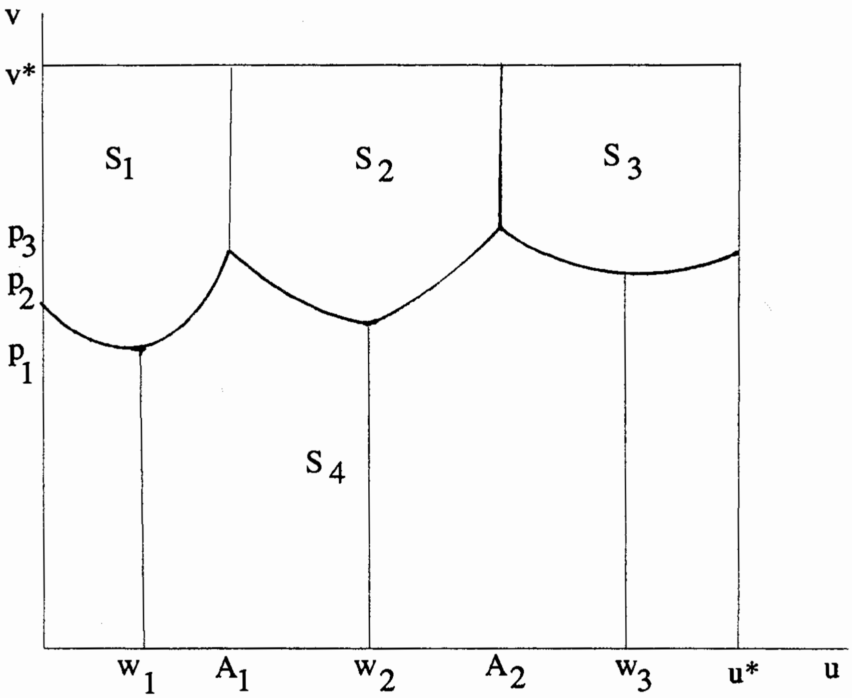
In order to apply the maximum likelihood estimation technique, we need to specify a functional form for $f(u,v;\lambda)$, where λ is a set of unknown parameters to be estimated associated with the distribution function. For instance, if (u,v) is multi-variant normally and independently distributed in $\mathbb{R}^{K+1}$ with $f(u,v)=MVN(\beta,\Omega)$, the unknown parameters to be estimated are the mean vector $\beta=(\bar{u}_1,\ldots,\ \bar{u}_K\ \bar{v})$ and the variance matrix Ω defined as

$$\Omega=\begin{bmatrix} \sigma_1^2 & & & 0 \\ & \ddots & & \\ & & \sigma_K^2 & \\ 0 & & & \sigma_v^2 \end{bmatrix} \qquad (6.3)$$

Thus, $\lambda = (\bar{u}_1,\ldots,\bar{u}_K,\ \bar{v},\ \sigma_1,\ \ldots,\ \sigma_K,\ \sigma_v)$.

Now, let $Q(j)$ be the number of units of the jth product purchased by a sample population. The likelihood function for such a sample population is expressed by

$$L(\lambda) = \prod_{j=0}^{M} R_j(\lambda)^{Q(j)}, \qquad (6.4)$$



$S_1 = \{(u,v),\ u \in (0,A_1),\ \text{and}\ v-c(u-w_1)^2 \geq p_1\}$

$S_2 = \{(u,v),\ u \in (A_1,A_2),\ \text{and}\ v-c(u-w_2)^2 \geq p_2\}$

$S_3 = \{(u,v),\ u \in (A_2,A),\ \text{and}\ v-c(u-w_3)^2 \geq p_3\}$

$S_4 = \{(u,v),\ u \in (0,\bar{u}),\ \text{and}\ v-c(u-w_i)^2 < p_i,\ i=1,\ldots,4\}$

Fig.6.1: The Market Space of Each Product

where $R_j(\lambda)$ represents the probability that good j is purchased for j=1,...,M, and $R_{M+1}(\lambda)$ is the probability of not buying any product. Thus,

$$R_j(\lambda) = \iint_{s_j} f(u,v)\,du\,dv \qquad (6.5)$$

s_j is the market space of product j given in (6.2).[29]

The maximum likelihood estimates of λ is obtained by maximizing L(λ). Therefore f(u,v) is recovered.

6.2 An Application to BC Ferries

For many years, local residents have petitioned the Government for improved service. In an effort to meet the needs of the Powell River community, the Government agreed to expand the sailing schedule on a trial basis during the 1992 summer period (June 26th to September 8th). Prior to the implementation of the new schedule, there were four trips per day from Powell River on the Sunshine coast of British Columbia to Comox on Vancouver Island. The expanded summer schedule added one additional trip for a total of five sailings per day.

In this particular application, we focus our analysis on this particular route because it allows us to conduct out-of-sample testing of the model. Specifically, we will use 1991 data to estimate the

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[29] Note that in most potential applications, u and v are bounded. Thus f(u,v) is a truncated distribution.

density function and predict the demand for 1992 with the new service. Model performance can be then evaluated by comparing the predicted data and the actual data. In addition, it is our understanding that the market for this route is largely composed of local residents. This is important since we assume a unimodal distribution function to represent aggregate preferences. Distinct market segments (e.g., local residents and tourists) may lead to a multimodal distribution.

In this application, assume that the only characteristic relevant to the consumer's choice is the time of departure, w. This implies that consumers do not choose the day of the week and/or the day of the month, or other services. Thus the attribute space has only one dimension (K=1). Given daily data on price and departure time, price per trip and assuming that aggregate preferences density function is a multiplication of two univariate normal distribution functions as

$$f(u,v:\lambda) = \frac{1}{2\pi\sigma_u\sigma_v} \exp\{ - \frac{(u - \bar{u})^2}{2\sigma_u^2} - \frac{(v - \bar{v})^2}{2\sigma_v^2} \}, \qquad (6.6)$$

the demand for different sailings on a particular day can be estimated using the proposed model via estimating $\lambda = (\bar{u}, \sigma_u, \sigma_v)$. Notice that the mean of reservation price for the most preferred sailing, $\bar{v}$ and the variance σ_v cannot be jointly identified given aggregate data. Consequently, we must impose restrictions on one of them. Since we have no a priori information on the variance of the reservation price but we do know that the mean of the reservation price $\bar{v}$ should be higher than the actual price, we choose to fix the mean. The next three sub-sections report data, estimation and out-of-sample testing results.

6.2.1 Data

The data obtained from B.C. Ferry Corporation consists of daily vehicle volumes departing from Powell River to Comox for each departure time and each day of the month in August 1991 and 1992. Since the daily average size is rather small and since we are interested in out-of-sample predictions with respect to some representative period of time not so much with respect to a particular day of the month, we have increased the sample size by aggregating the data in such a way that vehicle volumes of each sailing for all Mondays, Tuesdays etc. of August 1991 were summed up (see Table 6.1). The same aggregation method has been adopted for August 1992. Thus the empirical estimation uses the aggregate data for August 1991 as reported in Table 6.1 but not the daily data.[30] Finally, the price per sailing is flat at a rate of $20 per vehicle.

6.2.2 Monte-Carlo Study

Given that the only characteristic is time of departure (w), and the price is fixed at $20, the preference density function given in (6.6) can be estimated using the aggregate data from Mondays to Sundays

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[30] This aggregation is consistent with our assumption that consumers choose the time of departure only. Holiday is excluded in the aggregation.

Table 6.1: Aggregate Vehicle Volumes By Sailing

August 1991

An Aggregate Day	Time of Departure (w_j)			
	7:30	11:15	15:00	19:15
Monday	238	195	151	74
Tuesday	291	262	242	79
Wednesday	288	307	236	72
Thursday	358	379	326	97
Friday	437	443	391	149
Saturday	482	262	181	91
Sunday	285	295	235	97

of August 1991. To do so, we need to choose a value for $\bar{v}$, the mean reservation price. Given the actual price at $20 per trip, We fixed $\bar{v}$ arbitrarily at $25. The parameter c is also fixed at 1 for simplicity. We also need the potential population for this service since our model includes a no-set, that is, the number of people who were active in the market but did not take any sailing. Based on the data summarized in Table 6.1, we fixed the population at 1500 vehicles per aggregate day.[31]

The maximum likelihood estimates of λ are obtained using an algorithm that contains a well-known and widely available (IMSL routines) Quasi-Newton nonlinear optimization routine. Monte-Carlo studies were conducted to examine the quality of the estimates. In particular, we chose a set of parameters (e.g., $\bar{u}$ = 10.0, $\bar{v}$ = 25.0, σ_u = 5.0, σ_v=10.0) as "true" parameters in f(u,v). We then used a random number generator to draw 1500 pairs of (u.v) from this distribution.[32] Putting each pair of (u.v) into (6.1) and using 1991 prices as well as departure times, we generate 1500 utility-maximizing choices, which are aggregated to obtain the simulated vehicle volumes per sailing. Using this data and our estimation program, we obtain one set of parameter estimates. By repeating the process for 100 times, we generated 100 sets of parameter estimates. Table 6.2 reports the sampling mean and

[31] Note that the assumptions about the mean of reservation price, the marginal utility parameter c and the potential population do affect the distribution of v but not u. Monte-Carlo studies show that these numbers are within the range that the probability predictions of each sailing are insensitive to choices of these values.

[32] This number is chosen so as to correspond to the potential population for each aggregate day.

Table 6.2: Simulation Results

Parameter	True value	Estimates	
		Mean	Variance
$\bar{u}$	10.0	10.0613	0.0686
σ_u	5.0	4.9760	0.0617
σ_v	10.0	11.2686	1.3742

variance of these estimates. The hypothesis of equality between the mean and the "true" values was tested using a standard normal test. The results show that the null hypothesis is accepted for parameter σ_u, but weakly rejected for $\bar{u}$ and σ_v at the 5% significance level. Given the sample size, we consider the estimates as being acceptable.

The aggregate preferences density function is estimated using 1991 data given in Table 6.1. Table 6.3 presents the estimation results. It suggests the following: Mondays and Saturdays are obviously different from the other aggregate days of the week. In particular, they both have a much flatter distribution of u than the other days. In comparison, Tuesdays, wednesdays, Thursdays and Sundays look very similar with a mean of the most preferred departure time ($\bar{u}$) around mid-day and a relatively small variances.

6.2.3 Out-Of-Sample Testing

Now we have the underlying aggregate preferences for each aggregate day of the week, we can then use this to project the corresponding vehicle volumes for 1992. In doing so we assume of course that consumer's preferences are stable over time. Table 6.4 reports the predicted and actual vehicle volumes for each aggregate day of the week in August 1992.[33]

The predictive ability of the proposed model is examined by

[33] The potential population of each aggregate day for 1992 is also set at 1500.

Table 6.3: Parameter Estimates Using 1991 Data

Days	$\bar{u}$	σ_u	σ_v
Mondays	7.17	9.52	8.09
Tuesdays	12.82	4.90	11.85
Wednesdays	12.91	4.44	10.74
Thursdays	12.87	4.65	3.29
Fridays	13.46	4.64	12.29
Saturdays	5.57	9.03	2.25
Sundays	12.95	4.98	8.56

Table 6.4

Actual and Predicted Vehicle Volumes for August 1992

Aggregate Day		Time of Departure				
		6:00	9:00	12:00	15:30	19:15
Mondays	actual	135	214	213	190	87
	predicted	258	168	152	124	91
Tuesdays	actual	136	228	218	245	75
	predicted	173	214	253	197	91
Wednesdays	actual	164	250	252	215	73
	predicted	159	229	280	206	79
Thursdays	actual	155	249	253	260	82
	predicted	166	243	291	219	88
Fridays	actual	168	305	322	314	98
	predicted	186	247	325	270	130
Saturdays	actual	294	317	258	207	88
	predicted	491	240	203	151	105
Sundays	actual	179	288	280	309	94
	predicted	174	222	265	212	102

running simple OLS regressions of predicted volumes against actual volumes. the regression results are reported in Table 6.5. The results show the the slope coefficient ranging from .732 to1.175 is insignificantly different from 1 and the intercept is insignificantly different from 0 for the different days of the week with the exception of Fridays. When pooling all data from Mondays to Sundays, the prediction regression (the bottom row of Table 6.5) almost coincides with the perfect prediction line, that is, the line with slope coefficient of 1 and a constant term of zero as illustrated in Fig.6.2. These results indicate that overall the proposed model predicts surprisingly well, particularly considering the fact that these predictions are based on estimates with only four effective data points.

Hence, without comparing with other models, we make the following conjecture: the performance of the proposed model in terms of predictive power can be attributed to the fact that it accounts for some heterogeneity in preferences among consumers, which is the spirit of all address models.

6.3 CONCLUSIONS AND EXTENSIONS

This chapter has presented a simple address model and its application to a real case of product differentiation. This model has some attractive features for researchers interested in the positioning of new products. It starts with the intuitively appealing assumption that consumers' preferences are heterogeneous and can be represented by

Table 6.5

Regression of Predicted Volumes against Actual
August 1992

Days	R-Square	Slope	Intercept
Mondays	0.8804	1.101 (0.496) *	-25.478 (-0.409)
Tuesdays	0.9686	0.900 (1.236)	25.018 (1.029)
Wednesdays	0.9898	1.031 (0.005)	-7.735 (-0.510)
Thursdays	0.8668	0.921 (0.436)	15.759 (0.416)
Fridays	0.9526	0.732 (3.275)	53.675 (2.788)
Saturdays	0.4531	1.011 (0.021)	-2.900 (-0.020)
Sundays	0.5604	1.175 (0.358)	-43.837 (-0.338)
Pooled Sample	0.8198	0.978 (0.306)	5.200 (0.266)

* Numbers in parenthesis are the t-values. The null hypothesis is Ho: slope=1, intercept=0. The critical value at the 5% level of significance is 2.13.

① The perfect prediction line

② The model prediction line

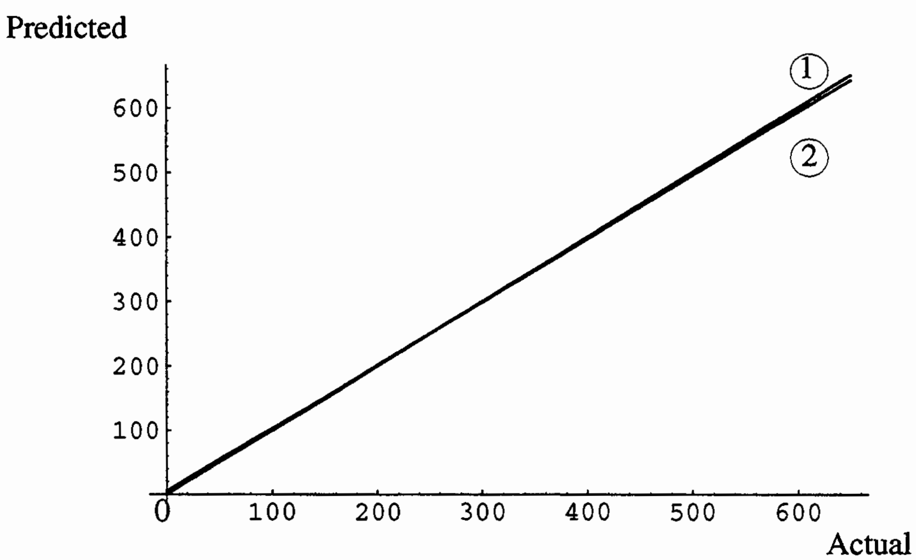

Fig. 6.2: The Regression Line of Predicted Volumes against Actual Volumes

a probability distribution function to be estimated. More importantly, it has a distinct advantage of constructing and estimating a preference density function which can be used with any new set of prices and products. Thus the introduction of new goods on the implication of different price structure can easily be investigated.

The preference recovery technique presented in this chapter, in fact in the entire thesis, is not without limitations. First, the maximum likelihood approach employed assumes a unimodal distribution of aggregate preferences in (u,v) parameter space. Should data give rise to a multimodal distribution of preference (for instance, the population is composed of distinct groups), the proposed empirical estimation procedure will not be appropriate. Secondly, as a limitation to all other discrete choice models, data is a major source of constraint. As a result, the estimation procedure inevitably involves some *ad hoc* restrictions.

The thesis can be extended along the following lines: first, to compare our method of preferences recovery with other appropriate discrete choice models with a given data set; second, to develop other preference recovery techniques, such as nonparametric smoothing techniques to estimate address models of product differentiation; finally, to relax some of the assumptions made in this thesis to test the robustness of our results.

The main purpose of thesis was to demonstrate the feasibility of preference recovery in address models of product differentiation. While the preference recovery techniques were illustrated within particular applications, they should be considered general techniques that can be applied to other cases of product differentiation.

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