TIPS: SOME COMMENTS ABOUT THE TERM STRUCTURE OF REAL YIELDS

By

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ABSTRACT

Inflation indexed securities comprise a new and developing market that provides purchasing power certainty for a bondholder and similarly constant real or inflation adjusted cost of finance for a borrower. This study documents on the characteristics of US Treasury Inflation Indexed Securities (TIPS) since their inception in 1997. Correlations of TIPS' returns with other asset classes, TIPS' real and effective nominal duration as well as the term structure of nominal yields, real yields and expected inflation are estimated until December 2004.

Particular attention is devoted to unusual patterns in real yields of the 2002 July TIPS. Effects of imperfect indexation, expected inflation and timing of the inflation uplift on an inflation indexed bond can create differences between quoted real yields and real true yields when a bond approaches maturity. Finally, this study reports that the relation between real yields with expected inflation has changed through time.

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I. INTRODUCTION

As their name suggests, inflation indexed securities are designed to help protect borrowers and investors alike from changes in the general level of prices in the real economy. They comprise a new and developing market that provides purchasing power certainty for a bondholder and similarly constant real or inflation adjusted cost of finance for a borrower. No other asset can so directly preserve the future purchasing power of savers. To further understand these new instruments, this study documents on the characteristics of US Treasury Inflation Indexed Securities since their inception in 1997.

Pricing an instrument issued for the first time is a big challenge. Investors and issuers face the difficulty to price it with no comparable instrument trading in the secondary market for reference. Coupon payments, liquidity premiums, indexation lags, limited amount of issues and tax treatment add further complexity to the yield estimates. However, increased liquidity and new issues throughout the last few years provide us with enough daily trading data to draw some conclusions.

The purpose of this study is to determine some empirical facts that could help better understand these financial instruments, especially, in relation to their closest peers, the nominal bonds. Based on the methodology used by Roll (2004), the analysis will cover the period that starts with the first auction of TIPS in January 1997 and finishes in December 2004. Correlations of TIPS' returns with other asset classes, TIPS' real and effective nominal duration as well as the term structure of nominal yields, real yields and expected inflation will be estimated.

Most of the results of this study confirm those obtained by Roll (2004) with the exception of the relation between real yields with expected inflation. The results found by extending Roll's (2004) dataset seem to contradict Roll's (1996) tax conjecture of TIPS. Additionally, particular attention is dedicated to unusual patterns in real yields of a maturing TIPS not covered in Roll (2004). Issues such as imperfect indexation, expected inflation and timing of monthly inflation may deviate quoted real yields from real true yields when a bond approaches maturity.

How TIPS work

An indexed bond is one which cash flows are linked to fluctuations in a specific price index with the aim of providing investors with means to protect the real value of their savings. In this case, TIPS are indexed every month to the CPI-U[1] with a lag of 3 months. TIPS fall under the category of capital indexed bonds, which coupons and principal rise with inflation. These instruments have been eligible for stripping since the outset of the market.

TIPS come with an embedded option often called "deflation floor". Such a feature guarantees the investor will receive at least the face value at maturity. However, during the life of the bond, the accredited principal can decline below par, declining the coupon payment below the original stated coupon amount.

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[1] Consumer Price Index for all Urban Consumers.

In the secondary market, TIPS' prices are stated as percentage of par. As a result, the settlement amount is the price multiplied by an accrual factor established officially by the treasury every day of the month for each outstanding TIPS. Accrued interest is calculated using the same accrual factor.

Historical Overview

In May 1996 the Treasury announces its intention to "issue securities that provide protection against inflation as a multiyear experiment" in order to reduce the cost of government borrowing (by saving the premium risk and completing the market) and to provide a means to observe market expectations of inflation" (Department of the (US) Treasury (1996a)).

The US treasury issued the first tranche of TIPS in January 1997, then followed by 5-year, 10-year and 30-year indexed bonds. Frequency had been low and amount issued had been in the range of 15-30 billion a year until 2003. However, due to a combination of big fiscal deficits and the recent commitment to expand and deepen the TIPS program to make a viable market to bigger investors, the US government issued approximately 70 billion of dollars in TIPS in 2004, raising to 266 billions the amount of TIPS outstanding to date (Bureau of the public debt website: www.publicdebt.utreas.gov/opd/opds022005.htm). In the present, with 17 issues outstanding and a daily trading volume of around 3.5 billions in the US, index linked securities are becoming a well established asset class, providing for the needs of larger investors such as central banks.

The Study

The study will consist of two main chapters and a conclusion. In the first chapter of this paper, returns, yields and durations of nominal and inflation indexed bonds will be estimated and described. In the second segment of the study, slope and curvature of real and nominal yield curves are estimated to describe the term structure of nominal yields, real yields and expected inflation. In the end of the paper, the conclusions from this study will be summarized in four main points.

II. TIPS CHARACTERISTICS

In this section of the study, estimates of mean, variances and correlations of returns are calculated. Real yields, real durations, empirical nominal durations (for TIPS) and factor sensitivities of TIPS to the nominal term structure are estimated and then described.

Data

A detail of the sample used in this study is described in Table 1. Daily trading data for twelve TIPS, five constant maturity bonds and three equity indexes is employed in this study. Sample periods will vary from one individual asset to another. All TIPS except the July 2002, which has already matured, will be covered since its inception until December 2004. Constant maturity bonds' data will cover from January 1997 until December 2004 except for 30-year constant maturity bonds' data which was discontinued in February 2002. Equities indexes will cover from January 1997 until December 2003.

TIPS' yields, real durations, prices and accrued interests data were kindly provided by Barclays. Constant maturity bonds and equity indexes were downloaded from the US Treasury website (www.utreas.com) and the Center for Research in Security Prices respectively.

Table 1 Data Set

Coupon	Asset	Issued	Maturity	Sample Period	
				Begin	End
A. TIPS					
3.625	Jul 2002	July-97	July-02	16-Jul-97	15-Jul-02
3.375	Jan 2007	January-97	January-07	22-Jan-97	31-Dec-04
3.625	Jan 2008	January-98	January-08	15-Jan-98	31-Dec-04
3.875	Jan 2009	January-99	January-09	15-Jan-99	31-Dec-04
4.250	Jan 2010	January-00	January-10	18-Jan-00	31-Dec-04
3.500	Jan 2011	January-01	January-11	17-Jan-01	31-Dec-04
3.375	Jan 2012	January-02	January-12	15-Jan-02	31-Dec-04
3.000	Jul 2012	July-02	July-12	15-Jul-02	31-Dec-04
1.875	Jul 2013	July-03	July-13	15-Jul-03	31-Dec-04
3.625	Apr 2028	April-98	April-28	15-Apr-98	31-Dec-04
3.875	Apr 2029	April-99	April-29	16-Apr-99	31-Dec-04
3.375	Apr 2032	October-01	April-32	16-Oct-01	31-Dec-04
B. U.S. Treasury Constant Maturity Nominal Bonds					
	3-Month			22-Jan-97	31-Dec-04
	1-Year			23-Jan-97	31-Dec-04
	5-Year			24-Jan-97	31-Dec-04
	10-Year			25-Jan-97	31-Dec-04
	30-Year			26-Jan-97	15-Feb-02
C. Equity Indexes					
	VWRETD*			22-Jan-97	31-Dec-03
	SPRTRN*			22-Jan-97	31-Dec-03
	EWRETD*			22-Jan-97	31-Dec-03

Notes: In panel C, "vwretd" is the CRSP NYSE + Amex + Nasdaq value-weighted index with dividends reinvested, "ewretd" is the CRSP equal-weighted index with dividends reinvested, and "sprtrn" is the S&P 500 Index with dividends included.

Mean and Variances

Table 2 shows the yearly average returns[2] and variances for each TIPS, constant maturity bonds and equity indexes for the sample periods detailed in Table 1. Returns for TIPS were calculated using prices and accrued interests provided by Barclays. Returns of constant

[2]Daily returns are annualized on the base of 252 trading days a year.

maturity bonds were calculated as $R_t = \frac{Y_{t-j}}{252} + D_{t-j}(Y_{t-j} - Y_t)$ where t is time, j is the time between t and the previous trading day, Y is nominal yield and D is duration.

As expected, it can be clearly seen that volatility increases with duration for TIPS as well as for constant maturity bonds. This is not the case for returns, in which bonds issued between 2000 and 2002 have outperformed the longer maturity issue of July 2013. Apparently the first ones have better captured the significant drops in yields from 2000 to the beginning of 2003 (See Figure 1 on page 8).

Overall, TIPS have considerable lower volatilities than nominal bonds. However, volatility in the most recent issues seems to be higher due to the mentioned fall in real yields between 2000 and the beginning of 2003.

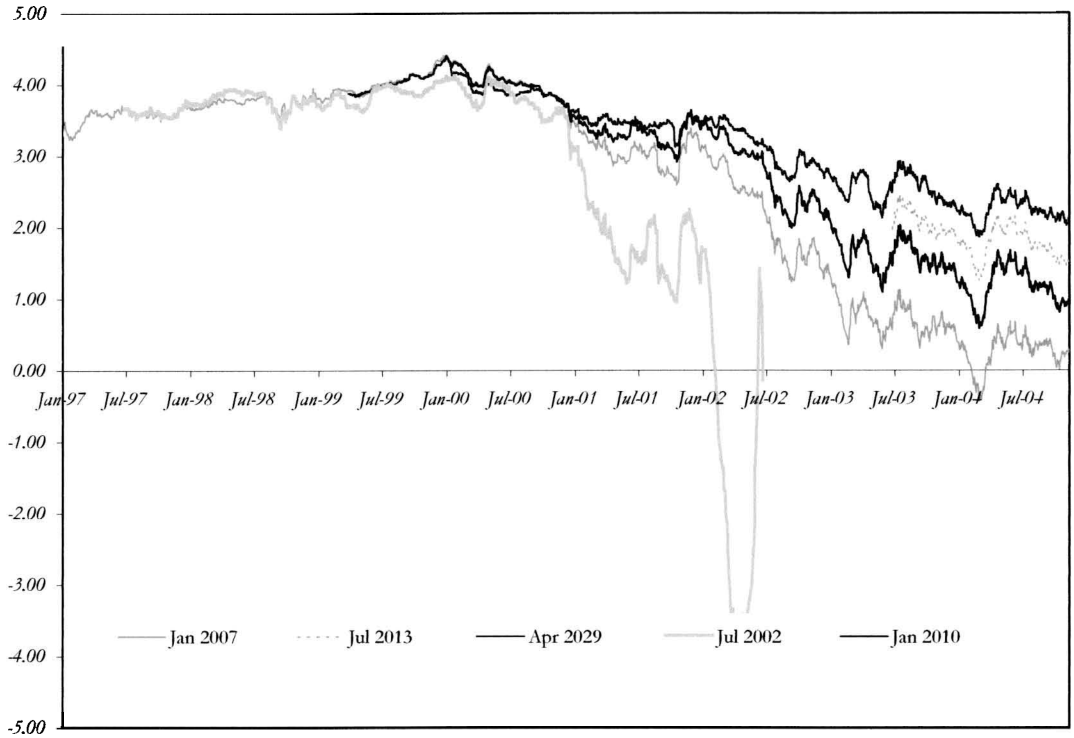
Table 2 Annualized Returns and Standard Deviations

Coupon	Asset	Mean	Standard Deviations	Observations
		A. TIPS		
3.625	Jul 2002	6.11%	1.09%	1256
3.375	Jan 2007	6.48%	2.67%	1999
3.625	Jan 2008	7.43%	3.02%	1751
3.875	Jan 2009	8.28%	3.42%	1500
4.250	Jan 2010	10.04%	4.03%	1247
3.500	Jan 2011	9.30%	4.92%	995
3.375	Jan 2012	10.28%	5.67%	745
3.000	Jul 2012	9.79%	6.32%	620
1.875	Jul 2013	7.17%	7.16%	370
3.625	Apr 2028	10.67%	7.56%	1688
3.875	Apr 2029	12.31%	8.07%	1437
3.375	Apr 2032	15.52%	11.19%	805
		B. U.S. Treasury Constant Maturity Nominal Bonds		
	3-Month	3.70%	0.23%	1987
	1-Year	4.24%	0.75%	1987
	5-Year	6.22%	4.60%	1987
	10-Year	7.15%	7.53%	1987
	30-Year	9.04%	11.04%	1269
		C. Equity Indexes		
	VWRETD	6.92%	20.33%	1734
	SPRTRN	5.33%	20.79%	1734
	EWRETD	26.73%	14.94%	1734

Yields

Figure 1 depicts the evolutions of real yields for five of the TIPS listed in Table 1. It can be observed that yields raised from 3.449% in the auction of the January 2007 TIPS in January 1997 to 4.338% in the auction of the January 2010 TIPS in January 2000. Since then, yields have gone down significantly. The auction yield of the July 2013 TIPS was 1.96% in July 2003. In December 2004, all yield of bonds maturing before 2011 had yields below 1%, ranging between 98 and 43 basis points.

Figure 1 Real Yields of TIPS from January 1997 to September 2003



Data Source: Barclays Capital.

Correlation Matrix

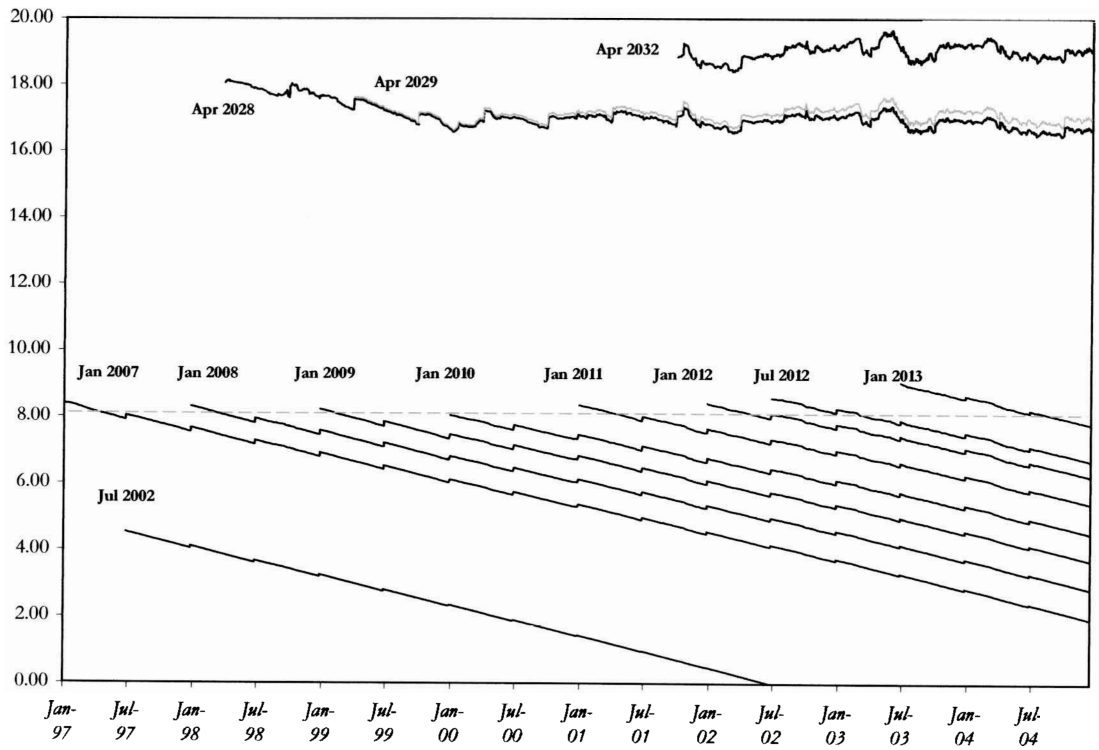
Table 3 (See page 11) provides a detail of the correlations between the returns of each individual asset listed in Table 1. TIPS are strongly correlated with each other except for the July 2002 issue which only has high correlations with TIPS of adjacent maturities. In relation to constant maturity bond, TIPS have the higher correlations with five and ten year constant maturity bonds. Except for the July 2002 issue, all TIPS hold correlations of .65 and higher with five and ten year constant maturity bonds

Both TIPS and constant maturity bonds exhibit negative correlation with stocks. This relation has been true for the sample covered in our analysis, but it hasn't been the case for longer periods. Another interesting result is that the correlations, in absolute terms, of stocks with TIPS are higher than those of stocks with constant maturity bonds.

Real Duration

Figure 2 below illustrates the evolution of real durations for the TIPS in our sample (Data calculated by Barclays Capital). Except for those of bonds with longer maturities (2028, 2029 and 2032), all real durations seem to decline smoothly through time with a small jump on each coupon date. The January 2010 issue has the lowest initial duration since real yields were at a peak when this issue was auctioned.

Figure 2 Real Durations of TIPS from January 1997 to December 2004



Data Source: Barclays Capital.

Table 3 Correlations of Daily Returns

	TIPS												Treasuries					Equities		
	Jul 2002	Jan 2007	Jan 2008	Jan 2009	Jan 2010	Jan 2011	Jan 2012	Jul 2012	Jul 2013	Apr 2028	Apr 2029	Apr 2032	3 Month	1 Year	5 Year	10 Year	30 Year	vwretd	sprtrn	ewretd
Jul 2002		0.76	0.72	0.62	0.59	0.54	0.23	N/A	N/A	0.48	0.38	0.24	0.30	0.47	0.43	0.40	0.35	-0.07	-0.05	-0.09
Jan 2007			0.98	0.97	0.95	0.94	0.94	0.94	0.94	0.77	0.77	0.80	0.20	0.55	0.67	0.65	0.46	-0.16	-0.15	-0.17
Jan 2008				0.99	0.98	0.97	0.96	0.96	0.96	0.81	0.82	0.84	0.21	0.58	0.73	0.71	0.50	-0.22	-0.22	-0.22
Jan 2009					0.99	0.99	0.98	0.97	0.97	0.85	0.85	0.86	0.20	0.57	0.76	0.74	0.51	-0.25	-0.25	-0.25
Jan 2010						1.00	0.99	0.99	0.98	0.87	0.87	0.88	0.22	0.59	0.80	0.80	0.59	-0.27	-0.28	-0.27
Jan 2011							1.00	0.99	0.99	0.89	0.89	0.90	0.27	0.60	0.82	0.83	0.64	-0.31	-0.31	-0.31
Jan 2012								1.00	1.00	0.92	0.92	0.92	0.32	0.65	0.85	0.86	0.76	-0.38	-0.38	-0.38
Jul 2012									1.00	0.93	0.93	0.93	0.32	0.65	0.86	0.88	N/A	-0.40	-0.39	-0.40
Jul 2013										0.95	0.95	0.94	0.23	0.65	0.86	0.87	N/A	N/A	N/A	N/A
Apr 2028											1.00	1.00	0.10	0.42	0.65	0.68	0.56	-0.18	-0.18	-0.17
Apr 2029												1.00	0.12	0.44	0.67	0.71	0.60	-0.20	-0.21	-0.20
Apr 2032													0.26	0.53	0.75	0.80	0.83	-0.29	-0.28	-0.30
3-Month														0.55	0.29	0.24	0.20	-0.10	-0.09	-0.11
1-Year															0.78	0.70	0.56	-0.22	-0.20	-0.24
5-Year																0.95	0.82	-0.25	-0.24	-0.28
10-Year																	0.91	-0.22	-0.21	-0.25
30-Year																		-0.02	-0.01	-0.06
vwretd																			0.98	0.85
sprtrn																				0.79

Empirical Durations and Factor Sensitivities

Duration is a measure of price sensitivity. It measures the percentage change of bond's price for a change in yield. It is very important to the reader to distinguish between real duration and nominal duration since real duration is a price sensitivity measure for a change in real yield while duration measures price sensitivity for a change in nominal yield, a combination of the change of real yield and the change in expected inflation[3].

To put TIPS and Nominal bonds in a comparable position, it is necessary to estimate empirical duration for TIPS. Two methodologies[4] are utilized to get this approximation. The first one regress TIPS' returns in the average yields of 5 and 10 year constant maturity bonds[5]. The regression is described by Equation (1)

$$R_{j,t} = \alpha_j + \beta_j \Delta Y_t + \varepsilon_{j,t} \quad (1)$$

where t is time measured in trading days, R stand for daily return for the *jth* issue and ΔY stands for the change in the average nominal yield of 5 and 10 year constant maturity bond. Since the percentage change in a price is a proxy for return, the absolute value of β_t is an estimation of the nominal duration of a particular TIPS issue. Table 4A shows these empirical durations for each individual TIPS. These estimates represent the empirical average duration of each TIPS' issue since inception until 2004. Except for the 2028 and 2029 TIPS, empirical durations increase monotonically with maturity.

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[3] Assuming nominal bonds don't pay inflation risk premium.
[4] Both methodologies are taken from Roll (2004).

The second estimate of nominal duration for TIPS and other sensitivity measures can be obtained by replicating the Litterman and Scheinckman (1991) model. This model is particularly useful for estimating the sensitivity of TIPS' returns to changes in the term structure of yields. The model allows to assess how TIPS' returns change when the shape of the term structure of nominal yields changes. The model must be run in three separate steps described below. A more technical description of the model is found in Appendix 1.

Step 1: Calculate the term structure of nominal yields for every trading day though a cross sectional estimation that regress nominal yields on durations. Equation (2) describes the regression. The three beta estimates of the model will be equivalent to the overall level, slope and curvature of the term structure. Figure 3 shows a graphical example of this estimate.

(2) $$Y_t = Level_t + Slope_t g(D_{1t}, D_{2t},.., D_{kt}) + Curvature_t g(D_{1t}, D_{2t},.., D_{kt})$$

where Y is the nominal yield, t stand for a particular day, Dit is the duration of a an particular bond on date t

Step 2: The results from Step 1 are then used to calculate the daily changes in the level, slope and curvature of the term structure between trading days. Equation (3.1), Equation (3.2) and (3.3) describes these calculations. Note that all inputs are the coefficients estimated in Step 1.

(3.1) $$Shift_t = Level_t - Level_{t-1}$$

(3.2) $$Tilt_t = Slope_t - Slope_{t-1}$$

(3.3) $$Flex_t = Curvature_t - Curvature_{t-1}$$

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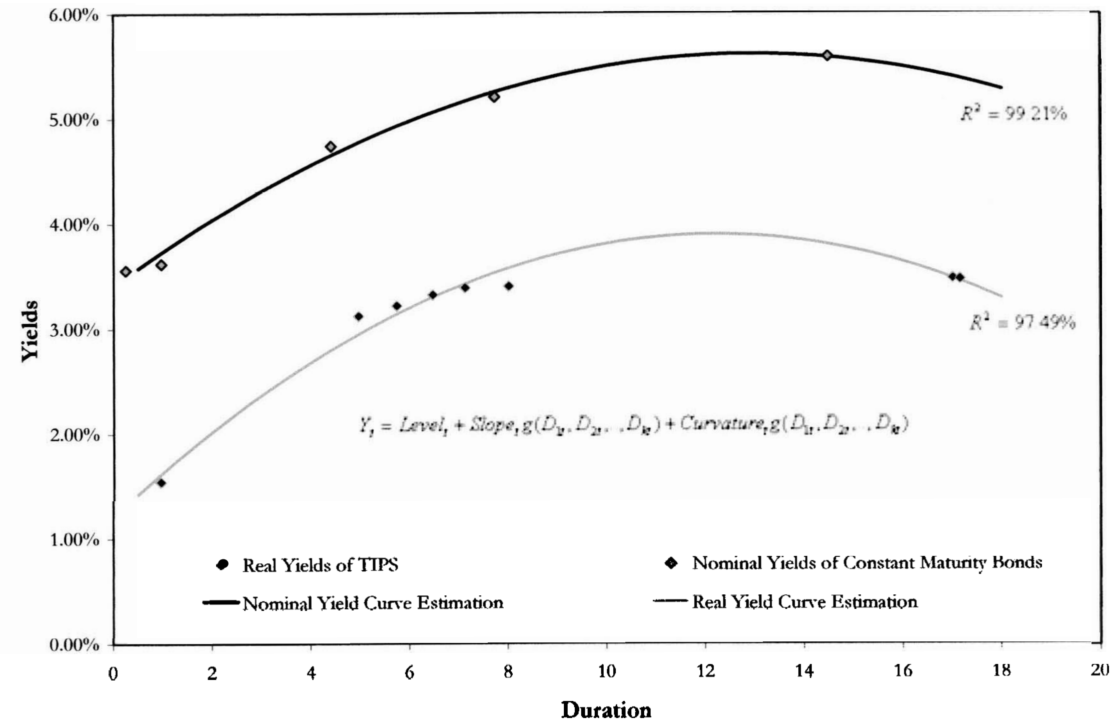
[5] 5 and 10 year constant maturity bonds have been used for this estimation since they are the nominal bonds which returns have the higher correlation with TIPS returns.

Figure 3 Estimation of the Real and Nominal Term Structure for July 25th, 2001 by regressing Equation (2)



<u>Step 3:</u> Regress TIPS' returns on the daily estimations of the shift, tilt and flex of the term structure obtained in Step 2. Equation (4) describes the equation that regress the returns of each individual TIPS issue on changes in the level, slope and curvature of the term structure. β_1, β_2 and β_3 are the sensitivities of returns to changes in the level, slope and curvature of the term structure, respectively.

(4) $$R_{t,j} = \alpha_j + \beta_{1,j} Shift_t + \beta_{2,j} Tilt_t + \beta_{3,j} Flex_t + \varepsilon_{t,j}$$

The results of this three-step estimate are presented in Table 4B. In general, the findings seem to be consistent with the results obtained in the first estimation. Sensitivities to changes in the overall level of the term structure (β_1) seem to be similar in magnitude to the empirical durations in estimated in Table 4A[6]. All sensitivities to changes in the overall level of the term structure (β_1) increase monotonically with maturity except for that of the 2028 TIPS.

The advantage of the three-step model is that it provides insights regarding the sensitivity of TIPS to changes in the level, slope and curvature of the nominal yield curve. Even with a year and a half of extra daily information, the results obtained closely resemble to those obtained by Rolls (2004). The longer-term TIPS are significantly and negatively related to the tilt factor (β_2<0), whereas the shortest-term TIPS is significantly positively related to Tilt(β_2>0). Thus, increases in the slope of the nominal yield curve decreases returns of long TIPS and increases the return of the shortest TIPS. Finally, increases in the curvature of the nominal yield curve seem to increase returns of all but one TIPS. The return of the shortest TIPS seems to be neutral to changes in the curvature of the nominal yield curve.

Overall, the explanatory power range was between 20% and 77% depending of the issue. Goodness mean and median are 57%. Adding new data to Roll (2004) model increased the explanatory power of the model for 8 of the 12 tips within the range of 1% to 6%. The explanatory power for the two 2012 TIPS remained unchanged while it decreased for the 2013 and 2032 TIPS in 8% and 2.8% respectively.

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[6] This indicates that 5 and 10 year constant maturity bonds used by Roll (2004) are appropriate for the estimations performed in Table 4A.

Table 4 TIPS Empirical Durations and Factor Sensitivities

Table 4A – Empirical Durations

$$R_{j,t} = \alpha_j + \beta_j \Delta Y_{j,t} + \varepsilon_{j,t}$$

	N	α_j	β_j	R^2
Jul 2002	1249	0.0002	-0.4807	0.17449
t-stat		12.827	-16.235	
Jan 2007	1987	0.0002	-1.8175	0.44525
t-stat		8.0126	-39.915	
Jan 2008	1740	0.0003	-2.1731	0.52323
t-stat		8.4141	-43.674	
Jan 2009	1491	0.0003	-2.5162	0.56494
t-stat		8.2848	-43.972	
Jan 2010	1239	0.0003	-3.1051	0.64395
t-stat		7.2607	-47.299	
Jan 2011	988	0.0003	-3.7661	0.68643
t-stat		5.6214	-46.459	
Jan 2012	741	0.0004	-4.5850	0.74562
t-stat		5.3569	-46.541	
Jul 2012	616	0.0003	-5.0875	0.77032
t-stat		4.5314	-45.379	
Jul 2013	366	0.0004	-5.9888	0.75881
t-stat		3.4031	-33.84	
Apr 2028	1678	0.0004	-4.9472	0.4396
t-stat		4.1708	-36.259	
Apr 2029	1428	0.0004	-5.4004	0.47281
t-stat		4.3672	-35.762	
Apr 2032	801	0.0006	-7.9191	0.60567
t-stat		3.5977	-35.032	

Table 4B - Factor Sensitivities

$$R_{t,j} = \alpha_j + \beta_{1,j} Shift_t + \beta_{2,j} Tilt_t + \beta_{3,j} Flex_t + \varepsilon_{t,j}$$

	α_j	$\beta_{1,j}$	$\beta_{2,j}$	$\beta_{3,j}$	R^2
Jul 2002	0.0002	-0.6194	0.5068	0.1541	0.20405
t-stat	12.477	-13.865	6.2948	1.4538	
Jan 2007	0.0002	-2.1509	-0.3505	0.9459	0.41677
t-stat	7.5326	-27.594	-2.5303	4.9415	
Jan 2008	0.0003	-2.6046	-0.7502	1.4659	0.4931
t-stat	7.847	-29.965	-5.007	7.0074	
Jan 2009	0.0003	-2.9879	-1.2541	2.0182	0.53003
t-stat	7.654	-28.046	-7.1876	8.0231	
Jan 2010	0.0003	-3.7495	-1.7755	2.8927	0.6089
t-stat	6.5922	-29.845	-9.1682	9.9259	
Jan 2011	0.0003	-4.7239	-2.5801	4.5803	0.6527
t-stat	4.6762	-26.198	-10.226	10.741	
Jan 2012	0.0003	-5.3713	-4.7515	7.0290	0.71056
t-stat	4.7374	-17.83	-11.108	12.081	
Jul 2012	0.0003	-5.4656	-4.1053	3.9428	0.77453
t-stat	4.5142	-17.032	-8.8039	5.5177	
Jul 2013	0.0004	-6.2852	-4.8535	4.2397	0.76246
t-stat	3.2179	-12.136	-6.4544	3.7639	
Apr 2028	0.0004	-5.9402	-6.8950	7.3341	0.49686
t-stat	4.318	-27.121	-18.351	13.962	
Apr 2029	0.0004	-6.5357	-7.5756	8.5799	0.53395
t-stat	4.5214	-25.77	-18.281	14.276	
Apr 2032	0.0005	-9.6150	-11.2970	15.4090	0.65741
t-stat	3.25	-16.631	-14.282	13.477	

III. TERM STRUCTURE OF REAL YIELDS AND EXPECTED INFLATION

This section elaborates on the interaction among the term structures of nominal yields, real yields and expected inflation. Additionally, it discusses about factors and circumstances in which estimations of real yields may deviate from true real yields.

Real Yields and Expected Inflation

Yields available at a specific moment represent those demanded by the marginal investor, and as such, they reveal to some extent the investor's expectations of future nominal yields. In the same manner, the existence of a complete and efficient debt market provides the ex ante real yield faced by borrowers and investors who want to avoid exposure inflation. If at least some investor participates in both markets, it follows that the difference between nominal and real yield must to some extent reveal participants' expectations of inflation.

The Model

The same Litterman and Scheinckman (1991) model is also used to estimate the level, slope and curvature of the real yield curve for every trading day since January 1999[7]. The same regression described in Step 1 in page 13 is applied to real yields and real durations. Figure 3 in page 14 provides an example of this estimation for July 25th, 2001. Equation (5) describes the regression with more detail.

(5) $$Y_t = Level_t + Slope_t g(D_{1t}, D_{2t}, ..., D_{kt}) + Curvature_t g(D_{1t}, D_{2t}, ..., D_{kt})$$

where Y is the Reall Yield, t stand for a particular day, Dit is the real duration of a an particular bond on date t

Following the logic in the previous section of this chapter, the term structure of real yields can be combined with the term structure of nominal yields estimated in the previous chapter[8] (Step 1 in page 13) in order to derive inflation expectations[9]. Equation (6.1), Equation (6.2) and Equation (6.3) describe the calculations.

(6.1) $$\textit{Level of Expected Inflation}_t = \textit{Nominal Level}_t - \textit{Real Level}_t$$

(6.2) $$\textit{Slope of Expected Inflation}_t = \textit{Nominal Slope}_t - \textit{Real Slope}_t$$

(6.3) $$\textit{Curvature of Expected Inflation}_t = \textit{Nominal Curvature}_t - \textit{Real Curvature}_t$$

Figure 4, Figure 5, Figure 6 plot the levels, slopes and curvatures values of the term structure of real and nominal yields. Figure 4 shows that the levels of nominal and real yields have been going down until mid 2003. Only nominal yields have started to rise since then while real yields have kept going down. It can also be noticed that expected inflation declined from 1999 until mid 2003 and has started to rise since then as. Figure 7 provides the net level of expected inflation.

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[7] January 1990 was chosen because there is enough cross sectional data (5 TIPS outstanding) to apply the model described in Appendix 1.

[8] The level, slope and curvature of the nominal term structure was calculated in the previous chapter in order to estimate the sensitivities of TIPS to shift, tilt and flex in the nominal term structure.

[9] Inflation risk premium, convexity, deflation floor option, effects of indexation lag and tax effects are assumed to be zero.

Figure 4 Term-Structure Levels for Nominal and Real Yields from January 1999 to December 2004

Figure 5 describes the slope of the real and nominal term structure. In general, the slope of the term structure of real yields tends to be smaller than that of nominal yields. This difference reflects the increasing inflation premium due to uncertainty. The slope of the real term structure started to increase significantly since Jan 2001 and made a huge hike from January 2002 to July 2002 when the real yield in the front end of the term structure sank precipitously (see July TIPS in Figure 1, page 9). When Roll (2004) calculated the term structure of real yields he excluded the July 2002 TIPS from the calculation because they exhibited very unusual low and/or negative yields when it approached maturity. On the contrary, I decided to display those yields in my estimates to later on explore some possible explanations to the huge decrease in the estimated real yields. It seems that imperfect indexation, timing of the inflation uplift on capital and/or other factors could have significantly affected the estimated real yields when the July 2002 TIPS approached maturity.

Figure 5 Term-Structure Slopes for Nominal and Real Yields from January 1999 to December 2004

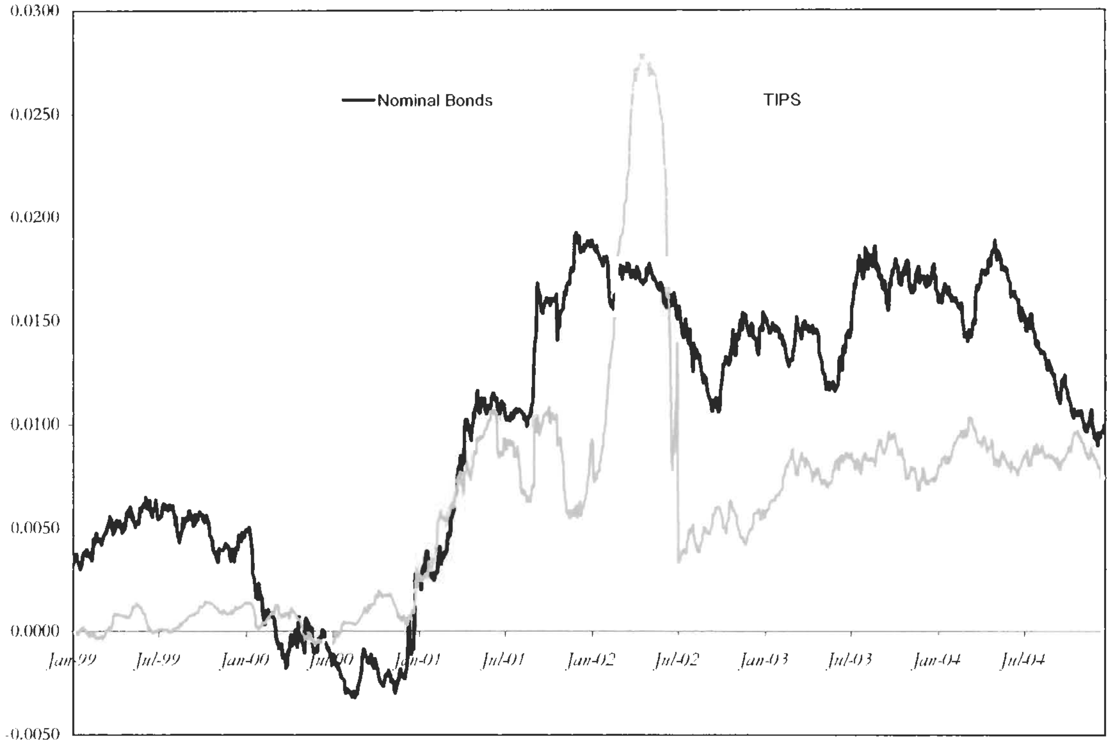

Figure 6 reports on the curvature of the term structure of real and nominal yields. As remarked by Roll (2004), the curvature of real yields has increased in the last few years. Since curvature of term structure is associated to volatility, in the increased curvature in the real term structure is consistent with the increased volatility in TIPS' returns noticed by Roll (2004). As noted in Figure 5 before, the precipitous plummeting of real yields in the short end of the term structure added high volatility/curvature to real yield term structure momentarily between January 2002 and July 2002.

Figure 6 Term-Structure Curvatures for Nominal and Real Yields from January 1999 to December 2004

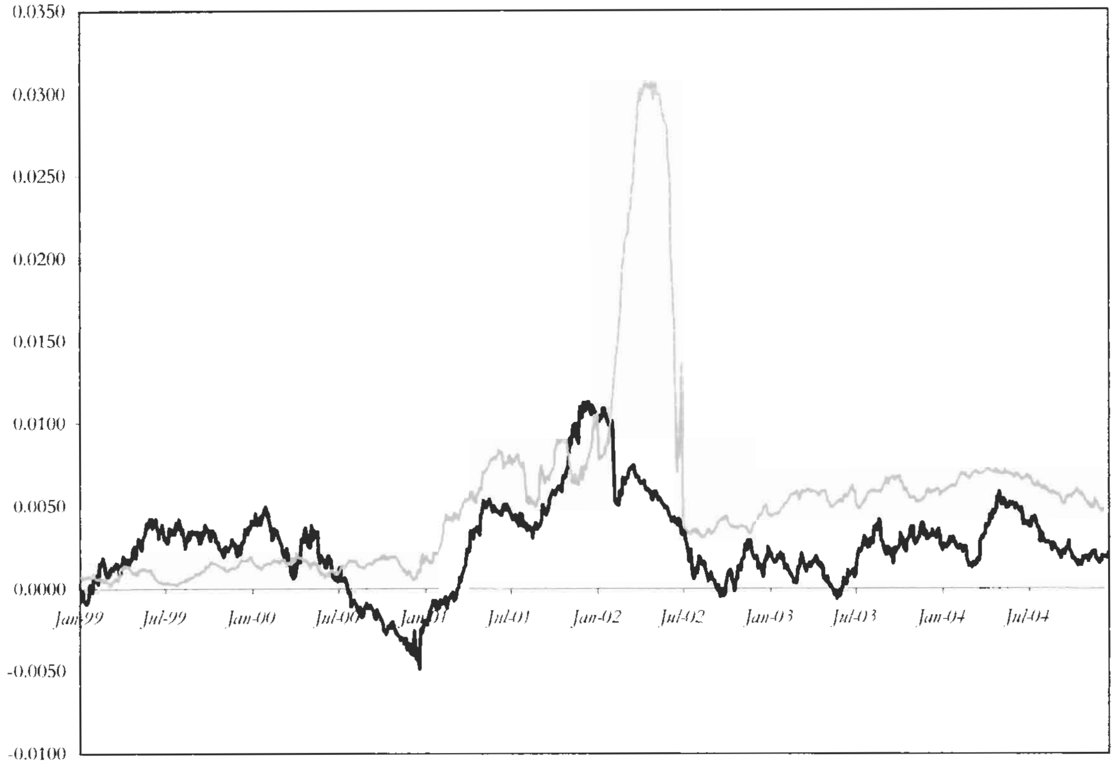

Figure 7 describes the level, slope and curvature of the term structure of expected inflation. For this estimation, the July 2002 TIPS was given a weight of cero from January 2001 to July 2002. I have taken into account that quoted real yields of the July 2002 issue might not be an appropriate measure of true real yield for reasons that will be discussed later in this chapter. Figure 7 shows frequent shifts in expected inflation. This characteristic is consistent with the result obtained by Sack (2000). He observes that all market reaction to macroeconomic events is reflected through changes in nominal interest rates, resulting in estimates of inflation expectations far more volatile than either survey forecasts or the CPI itself. This finding warns us to be cautions about high frequency changes in inflation expectations estimates. Table 5 support this position by showing that volatility of changes in expected inflation is higher than that of changes in nominal and real yields. Furthermore, Table 6 reveals that changes in nominal yields are highly correlated with changes in expected inflations.

Figure 7 Term-Structure of Anticipated Inflation from January 1999 to December 2004

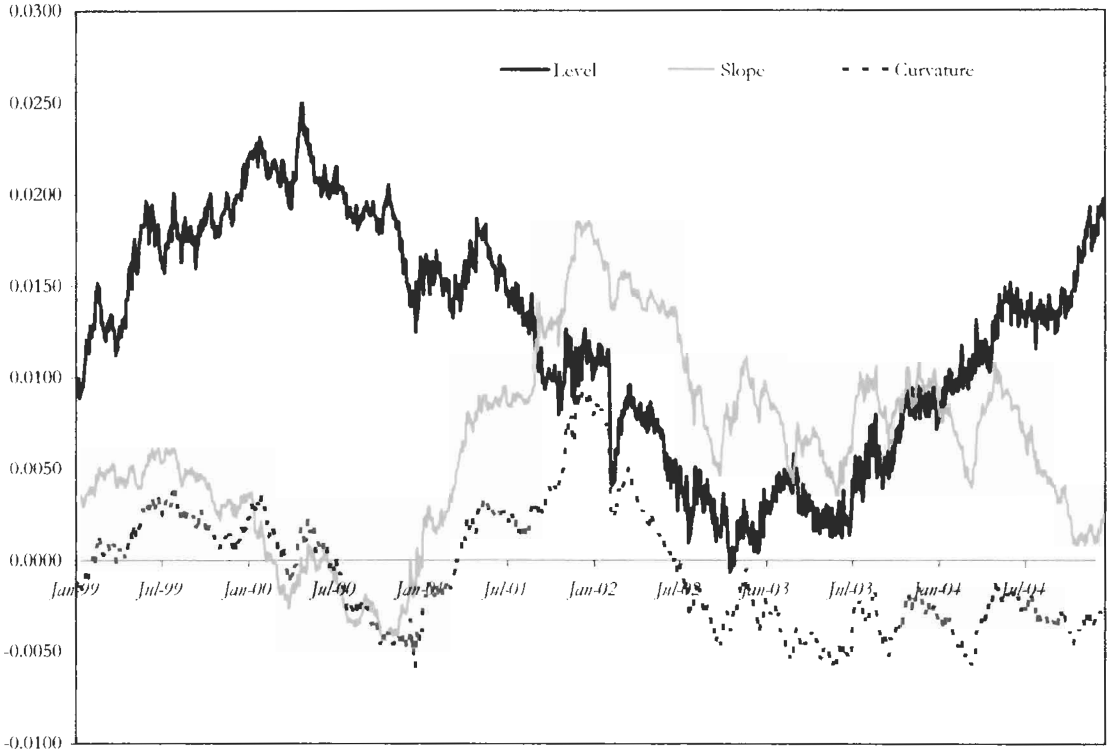

Table 5 Mean and Standard Deviation of changes in Level (Shift), Slope (Tilt) and Curvature (Flex) of the Term Structure of Nominal Yields, Real Yields and Expected Inflation.

	Nominal Bonds				TIPS			Inflation	
	Shift	Tilt	Flex	Shift	Tilt	Flex	Shift	Tilt	Flex
Mean (%)	-.0008	.0004	.0002	-.0015	.0005	.0003	.0007	-.0001	-.0001
Volatility (%)	.0593	.0298	.0281	.0337	.0155	.0103	.0682	.0330	.0297

Table 6 reports on the correlations among changes in the term structure of real yields, nominal yields and expected inflation. It shows that changes in the overall level of the nominal yield curve are practically uncorrelated with changes in the overall level of the real yield curve. Another interesting result is the negative correlation of -.499 between changes in the overall level of expected inflation and changes in the overall level of real yields

Table 6 also shows that changes in the level of the term structure of nominal yields are positively related to changes in slope and curvature in the nominal term structure. The term structure of expected inflation follows similar patterns as the nominal term structure; however, correlation coefficients are much lower. On the contrary, positive changes in the level of term structure of real yields are associated with decreases in the curvature and slope in the real term structure.

Table 6 Correlations of changes in the Term Structure of Real Yields, Nominal Yields and Expected Inflation

		Nominal Yields			Real Yields			Expected Inflation		
		Shift	Tilt	Flex	Shift	Tilt	Flex	Shift	Tilt	Flex
Nominal Yields	Shift		0.564	0.793	0.010	-0.041	-0.043	0.861	0.379	0.502
	Tilt			0.674	0.013	0.005	-0.004	0.482	0.610	0.401
	Flex				0.021	-0.039	-0.041	0.677	0.444	0.623
Real Yields	Shift					-0.375	-0.180	-0.499	0.305	0.154
	Tilt						0.916	0.155	-0.789	-0.740
	Flex							0.055	-0.728	-0.807
Expected Inflation	Shift								0.174	0.357
	Tilt									0.832
	Flex									

Other term structure considerations

As noticed in the previous section, quoted real yields of the July 2002 issue might not be an appropriate measure of true real yield. Based on the unusual patterns in real yields of the 2002 July TIPS, this section of the study will be dedicated to explore arguments why estimated real yields may deviate significantly from true real yields.

Imperfect Indexation

Imperfect indexation is caused by the indexation lag characteristic of all indexed debt. This makes an index-linked bond a combination of two instruments, a perfectly indexed real bond and a nominal bond. The shorter the time to maturity, the bigger the nominal component of the bond will be. So, when a bond is close to maturity, the quoted real yield will figure will depend on the assumptions about inflation made by investors[10] when the bond approaches maturity. The higher assumed inflation rate, the lower the true real yields will be. Figure 8 shows that the real yields of the July 2002 TIPS move in the opposite direction of the level of expected inflation when the bond approaches maturity. However, this explanation can only account for very small discrepancies between estimated and true real yields and is not a valid clarification for the huge changes in yields the July 2002 TIPS experienced during its last four months.

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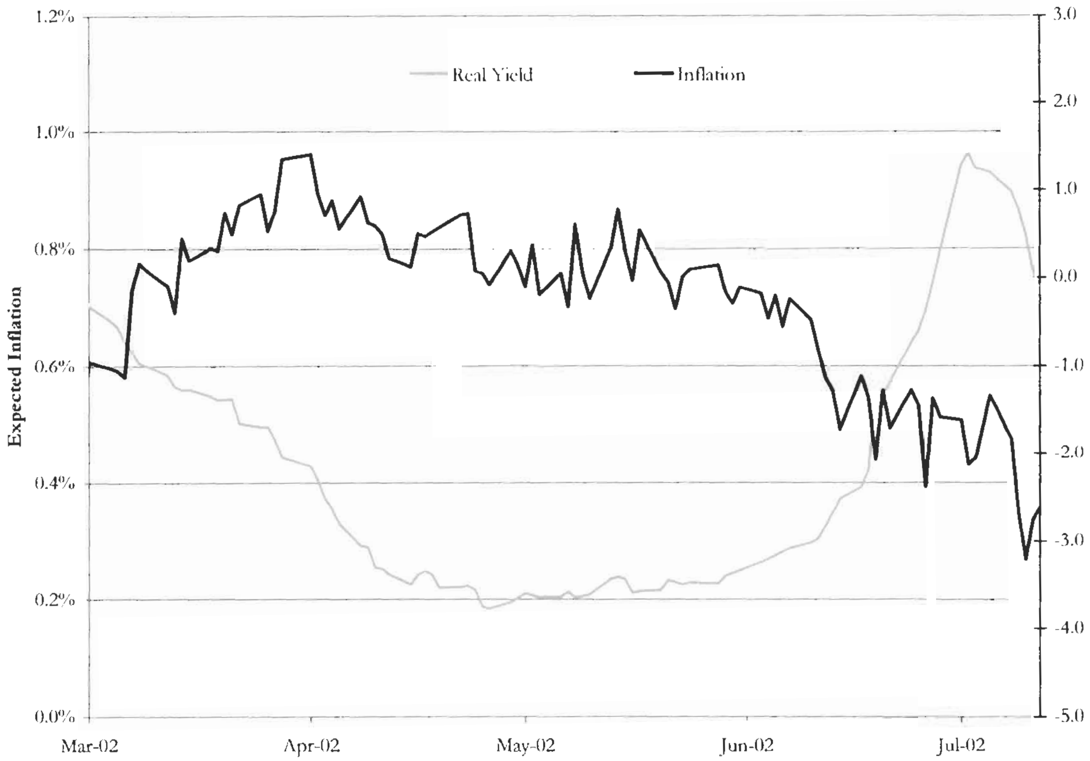
[10] Refer to Appendix 2 for a more technical description of this feature.

Figure 8 Real Yields of the July 2002 TIPS and Inflation Expectations from March 2005 to July 2002



Timing of the Inflation Uplift.

A more reasonable explanation of the huge movements in the real yields of the July 2002 TIPS is related to the seasonality of the values of the CPI-U used to calculate the inflation uplift of TIPS. Looking at the changes in the CPI-U applied to the face value of TIPS (See Table 7) for April, May, June and July 2002 give us some insights of why yields went down so precipitously for the July 2002 issue. The CPI-U for the last four months before July 2002 matured increased by 1.74% at a time when the cumulative increase of the CPI-U for the last 12 months until July 2002 was 1.63%. This means that whoever bought the July 2002 TIPS during the last four months before maturity was willing to pay a higher price (or lower real yield) in order to profit from the huge final inflation uplift.

Table 7 **Monthly TIPS' inflation uplift from January 1997 to December 2004.**

	1997	1998	1999	2000	2001	2002	2003	2004	Average uplift by month of the year
Jan	0.32%	0.25%	0.24%	0.18%	0.17%	-0.34%	0.17%	-0.11%	*0.11%*
Feb	0.19%	-0.06%	0.00%	0.06%	0.06%	-0.17%	0.00%	-0.27%	*-0.02%*
Mar	0.00%	-0.12%	-0.06%	0.00%	-0.06%	-0.39%	-0.22%	-0.11%	*-0.12%*
Apr	0.32%	0.19%	0.24%	0.30%	0.63%	0.23%	0.44%	0.49%	*0.35%*
May	0.31%	0.19%	0.12%	0.59%	0.40%	0.40%	0.77%	0.54%	*0.41%*
Jun	0.25%	0.19%	0.30%	0.82%	0.23%	0.56%	0.60%	0.64%	*0.45%*
Jul	0.12%	0.18%	0.73%	0.06%	0.40%	0.56%	-0.22%	0.32%	*0.27%*
Aug	-0.06%	0.18%	0.00%	0.12%	0.45%	0.00%	-0.16%	0.59%	*0.14%*
Sep	0.12%	0.12%	0.00%	0.52%	0.17%	0.06%	0.11%	0.32%	*0.18%*
Oct	0.12%	0.12%	0.30%	0.23%	-0.28%	0.11%	0.11%	-0.16%	*0.07%*
Nov	0.19%	0.12%	0.24%	0.00%	0.00%	0.33%	0.38%	0.05%	*0.16%*
Dec	0.25%	0.12%	0.48%	0.52%	0.45%	0.17%	0.33%	0.21%	*0.32%*
Total	**2.13%**	**1.48%**	**2.60%**	**3.40%**	**2.62%**	**1.51%**	**2.30%**	**2.51%**	**2.32%**

Figure 9 **Daily Real Yields and Returns for the July 2002 TIPS from January 2002 until July 2002**

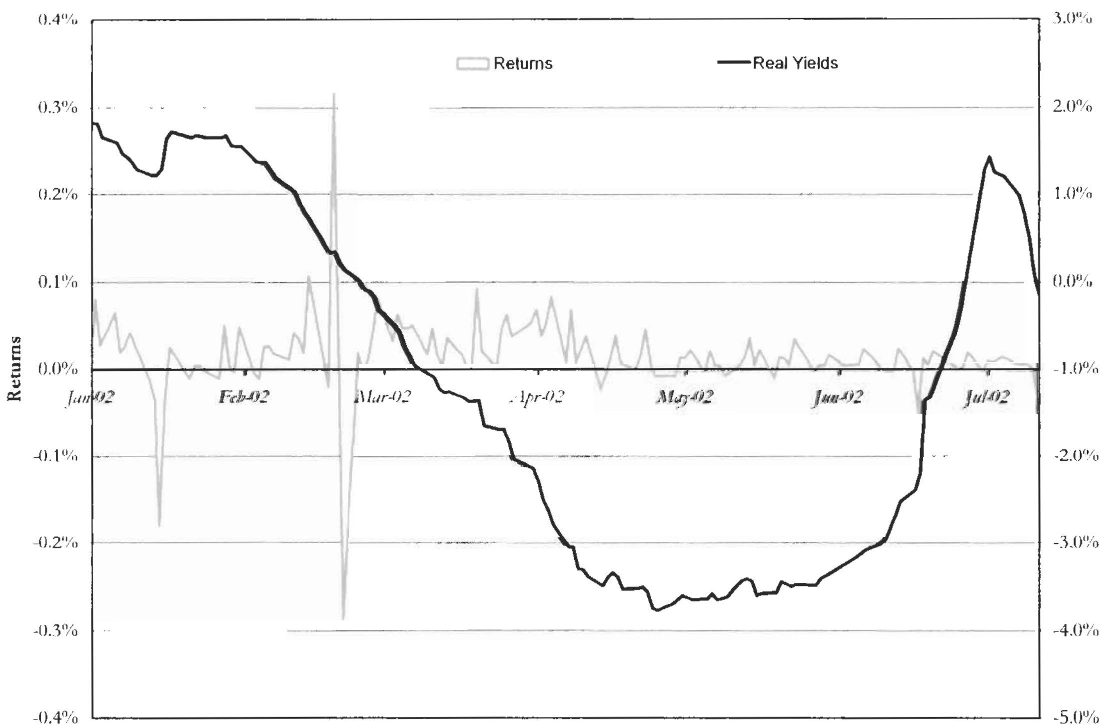


Figure 9 plots the returns and real yields of the July 2002 TIPS. Although such low and negative yields seem to be excessive, they seem to have done a good job to keep daily returns in a reasonable range of less than one tenth of a basis point.

Expected Inflation and TIPS Yields – Taxation of the inflation accrual.

Roll (2004) explores links between anticipated inflation and TIPS yields. Following his reasoning in Roll (1996), he argues that TIPS' yields might be coupled with inflation because their tax treatment stipulates full taxation of the real yield and the inflation accrual. Therefore, when inflation goes up (down), pre-tax real yields adjust upward (downward) in order to maintain an after-tax real yield. Figure 4 in page 19 shows that during the period analyzed by Roll (2004), from 1999 until mid 2003, yields and expected inflation went down consistently. However, it hasn't been the case since then. Since mid 2003, expected inflation has been steadily rising but pre-tax real yields have been going down, contradicting the tax conjecture argument about real yields.

Table 8 displays the results of regressing changes in real yields on changes in the term structure of expected inflation. Following the same methodology used to calculate Table 4, two set of regressions were done. The first one regress changes in real yields on changes in the overall level of expected inflation as shown in Equation (7).

$$(7) \qquad \Delta Y_{j,t} = \alpha_j + \beta_{j,t} \Delta \, \textit{Level of Expected Inflation}_t + \varepsilon_{j,t}$$

where t is time measured in trading days, ΔY_j stand for the change in real yield for the jth issue. The negative coefficients in Table 8A suggest that changes in real yields for all TIPS seem to have a significant negative relation with changes in the overall level of expected inflation.

The second estimates regresses changes in real yields of the *j*th asset on changes in the level (shift), slope (tilt) and curvature (flex) of the term structure of expected inflation. Equation (8) illustrates this regression.

$$(8) \qquad \Delta Y_{t,j} = \alpha_j + \beta_{1,j} \Delta \textit{ Level of Expected Inflation}_t + \beta_{2,j} \Delta \textit{ Slope of Expected Inflation}_t + \beta_{3,j} \Delta \textit{ Curvature of Expected Inflation}_t + \varepsilon_{t,j}$$

Changes in the level, slope and curvature of expected inflation are calculated by combining Equations (6.1), Equations (6.2) and Equations (6.3) as shown in Equations (9.1), Equations (9.2) and Equations (9.3)

$$(9.1) \qquad \Delta \textit{ Level of Expected Inflation}_t = \textit{Level of Expected Inflation}_t - \textit{Level of Expected Inflation}_{t-1}$$

$$(9.2) \qquad \Delta \textit{ Slope of Expected Inflation}_t = \textit{Slope of Expected Inflation}_t \textit{ Slope of Expected Inflation}_{t-1}$$

$$(9.3) \qquad \Delta \textit{ Curvature of Expected Inflation}_t = \textit{Curvature of Expected Inflation}_t - \textit{Curvature of Expected Inflation}_{t-1}$$

The results of the regression described in Equation (8) are portrayed in Table 8B and show that changes in real yields are positively related to changes in the slope of the term structure of expected inflation, which partially favours the tax conjecture of real yields by relating real yields to long term inflation expectations.

Table 8 Sensitivity of Changes in TIPS' Real Yields to changes in the Term Structure of Inflation

TABLE 8A – Sensitivity to Expected Inflation

$$\Delta Y_{j,t} = \alpha_j + \beta_{j,t} \Delta \text{ Level of Expected Inflation}_t + \varepsilon_{j,t}$$

TABLE 8B- Sensitivity to the Term Structure of Expected Inflation

$$\Delta Y_{t,j} = \alpha_j + \beta_{1,j} \Delta \text{ Level of Expected Inflation } t + \beta_{2,j} \Delta \text{ Slope of Expected Inflation } t + \beta_{3,j} \Delta \text{ Curvature of Expected Inflation } t + \varepsilon_{t,j}$$

	N	α_j	β_j	R^2	α_j	$\beta_{1,j}$	$\beta_{2,j}$t	$\beta_{3,j}$	R^2
Jul 2002	872	-0.00004	-0.06706	0.27%	-0.00005	-0.30425	0.13514	0.64452	5.42%
t-stat		-1.63680	-1.53970		-1.80180	-5.22930	1.23720	4.64310	
Jan 2007	1490	-0.00002	-0.29046	19.90%	-0.00002	-0.53577	0.39809	0.67123	51.92%
t-stat		-2.02950	-19.22800		-2.32250	-35.38300	12.41900	15.52900	
Jan 2008	1490	-0.00002	-0.29499	21.41%	-0.00002	-0.52758	0.39325	0.62728	52.03%
t-stat		-1.95740	-20.13100		-2.21660	-35.61700	12.54000	14.83500	
Jan 2009	1490	-0.00002	-0.28694	22.08%	-0.00002	-0.50351	0.37773	0.57740	51.49%
t-stat		-1.91110	-20.53600		-2.14210	-35.29800	12.50900	14.18000	
Jan 2010	1239	-0.00003	-0.30497	24.45%	-0.00003	-0.51993	0.37124	0.59750	54.36%
t-stat		-2.64880	-20.00100		-3.15600	-34.14100	11.74800	13.71200	
Jan 2011	988	-0.00002	-0.32171	26.35%	-0.00002	-0.60116	0.42923	0.82413	65.07%
t-stat		-1.81090	-18.77500		-2.55810	-38.37300	13.08000	17.24100	
Jan 2012	741	-0.00002	-0.35020	29.85%	0.00001	-0.67567	0.72358	0.71204	78.14%
t-stat		-1.54690	-17.72300		0.63401	-45.51400	20.59000	14.89700	
Jul 2012	616	-0.00002	-0.41512	36.33%	0.00000	-0.67093	0.75360	0.59534	78.60%
t-stat		-0.94139	-18.70400		0.47361	-41.49900	18.69100	9.55090	
Jul 2013	366	0.00000	-0.46083	44.12%	0.00002	-0.68440	0.65780	0.64394	78.60%
t-stat		0.23093	-16.92900		1.68620	-33.55400	11.58600	7.46610	
Apr 2028	1490	-0.00001	-0.20150	21.95%	-0.00001	-0.34647	0.06019	0.49789	40.91%
t-stat		-1.52270	-20.45500		-1.52250	-31.24200	2.56360	15.72700	
Apr 2029	1428	-0.00001	-0.20456	22.43%	-0.00001	-0.34828	0.06107	0.49835	41.30%
t-stat		-1.79580	-20.29800		-1.76110	-30.79000	2.54710	15.36700	
Apr 2032	801	-0.00002	-0.25045	27.34%	-0.00001	-0.48958	0.21601	0.74320	61.05%
t-stat		-1.49320	-17.32800		-0.66920	-34.32700	6.70190	16.90200	

IV. CONCLUSION

I replicated and extended Roll's (2004) model and found that most of their results still hold when adding a year and a half of extra information. Nevertheless, some contradictions with previous findings and additional insights are found through this study. On the whole, the results of this paper can be summarized in four points:

(1) As Roll (2004) shows, TIPS exhibit low volatility and low correlations with equity. By improving certainty of returns in an investment portfolio, adding TIPS will certainly lower the risk of an investment portfolio.

(2) Due to it complexity, pricing of TIPS have proved to be particularly difficult. It is still unknown whether real yields in the first years TIPS were issued have been a good estimate of true real yields because liquidity premiums, lack of acceptance and small market size. However, increased market size, frequent issuing, a strong commitment of the US Treasury to expand the TIPS program and the development of a CPI derivatives market in the last few years has improved pricing of TIPS and real yield estimates.

(3) Issues such as imperfect indexation due to the indexation lag characteristic of TIPS and timing of the inflation uplift may deviate real yields estimates from true real yields when TIPS are close to maturity. Timing of the inflation uplift can significantly affect estimates of real yields. The July 2002 TIPS' real yields prove this argument. From May 2002 to July 2002

the quoted real yields of this TIPS moved by more than 400 basis points. Still, its daily returns fluctuated by less than a tenth of a basis point.

(4) The results obtained by extending Roll's (2004) data set from mid 2003 until December 2004 have weakened the results obtained by Roll (2004) who found a positive relation between the level of real yields to the level of expected inflation. Roll (1996) argued that expected inflation and TIPS yields could be coupled due to the tax treatment of the inflation uplift. Therefore, in order to keep taxable demand constant, investors should demand higher real yields when expected inflation rises. Since mid 2003, real yields have been going down while expected inflation has been rising, suggesting a negative relation between real yields and expected inflation and casting doubts on Roll's (1996) conjecture.

APPENDIX 1

The methodology is divided in three steps:

STEP1: Perform a Cross Sectional Regression

$$Y_{j,t} = Level_t + Slope_t X_{L,j,t} + Curvature_t X_{Q,j,t}$$

where t is time measured in trading days. Y_j is the yield of the jth asset.

$$X_{L,j,t} = a_t + b_t D_{j,t} \text{ and } X_{Q,j,t} = -(3X_{L,j,t}^2 - 1)/2$$

$$b_t = \frac{2}{\max(D_t) - \min(D_t)} \text{ and } a_t = 1 - b_t \max(D_t)$$

D is duration. Min(D_t) and max(D_t) stand for the duration of the minimum and maximum duration assets of the set of assets used in the cross sectional regression for each trading day.

STEP 2: Calculate changes in Level, Slope and Curvature of the Term Structure

$$Shift_t = Level_t - Level_{t-1}$$
$$Tilt_t = Slope_t - Slope_{t-1}$$
$$Flex_t = Curvature_t - Curvature_{t-1}$$

STEP 3: Regress returns of TIPS on changes in Level, Slope and Curvature.

$$R_{t,j} = \alpha_j + \beta_{1,j} Shift_t + \beta_{2,j} Tilt_t + \beta_{3,j} Flex_t + \varepsilon_{t,j}$$

where R_j stands for the returns of the jth TIPS.

APPENDIX 2

Equation (1) shows that, for a perfectly indexed bond, no assumptions about inflations need to be done since the inflation cancels out.

$$(1) \qquad \Pr = \sum_{j=1}^{n} \frac{Cr\prod_{i=1}^{j}(1+\pi_i)}{(1+r_j)^j \prod_{i=1}^{j}(1+\pi_i)} + \frac{Rr\prod_{i=1}^{n}(1+\pi_i)}{(1+r_n)^n \prod_{i=1}^{n}(1+\pi_i)} = \sum_{j=1}^{n} \frac{Cr}{(1+r_j)^j} + \frac{Rr}{(1+r_n)^n}$$

where π_i is the inflation rate between date i and date i-1 and r_j is the spot real interest rate to date j.

On the contrary, for an actual inflation-indexed bond with an indexation lag, the inflation of the last period is not perfectly indexed and some assumptions regarding the inflation in the last period must be done in order to price an inflation indexed bond. Equation (2) shows this relation. For simplicity, it is assumed that to the indexation lag the coupon period is equal to the coupon period.

$$(2) \qquad \Pr = \sum_{j=1}^{n} \frac{Cr\prod_{i=o}^{j-1}(1+\pi_i)}{(1+r_j)^j \prod_{i=1}^{j}(1+\pi_i)} + \frac{Rr\prod_{i=o}^{n-1}(1+\pi_i)}{(1+r_n)^n \prod_{i=1}^{n}(1+\pi_i)} = \sum_{j=1}^{n} \frac{Cr(1+\pi_0)}{(1+r_j)^j(1+\pi_j)} + \frac{Rr(1+\pi_o)}{(1+r_n)^n(1+\pi_n)}$$

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