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IMPLICATIONS OF COMPUTERS FOR MATHEMATICS EDUCATION
IN TRINIDAD AND TOBAGO

by

Gail Yasmin Tagallie
B.A., York University, 1984

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
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ii

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The Implications of Computers for Mathematics Education in Trinidad and Tobago

ABSTRACT

In this thesis we investigate the effect of computers on the secondary school mathematics curriculum with particular emphasis on the curriculum of Trinidad and Tobago.

In the first chapter we describe, in detail, the structure of the education system in Trinidad and Tobago together with the mathematics curriculum of the secondary schools. The curriculum is not given for each grade level in the high schools. Rather it is described as the requirements for the Basic and General Proficiency examinations, administered by the Caribbean Examination Council and the Advanced Level Examination, administered by Cambridge University, London.

In the next chapter, a description of muMath, a symbolic computer-algebra system whose algebraic and analytical operations are beyond the capabilities of programming languages such as BASIC and Fortran, is given. The capabilities of muMath and the HP 28C handheld computer are observed in order to determine the effect of the computers on the teaching of mathematical topics.

Although many computer algebra systems and courseware containing modules for teaching mathematics are available, only a few are mentioned in the paper, but the mass availability and smooth implementation of the computers into the classroom are assumed. Having looked at the capabilities of the computer algebra systems and the algebra taught in the high schools in Trinidad and Tobago, a proposal is made for changes in the algebraic topics so that the curriculum takes advantage of the available technology, and also reflects the technological changes in the environment.

ACKNOWLEDGEMENTS

TABLE OF CONTENTS

CHAPTER

LIST OF TABLES

LIST OF FIGURES

ix

CHAPTER ONE

IMPLICATIONS OF COMPUTERS FOR MATHEMATICS
EDUCATION IN TRINIDAD AND TOBAGO

Introduction

The computer revolution in North America will affect education in the third world countries if only because of the migration of large numbers of international students to North American Universities. Any major changes in the high school curriculum will affect the entrance requirements of the universities, and this will put pressure on other countries to adopt some of the changes made in the North-American high schools. The lesser-developed countries might not be able to keep up with this revolution. In many cases because of the lack of funds to implement computers in the schools, the lack of teachers committed to introducing a more appropriate curriculum in the schools, and deterrents to change in the examining procedures of all students at all levels a large gap will develop between education in the developed countries and that of the lesser developed countries.

The computer is here to stay, and will continue to affect the solving process and analysis of mathematical

problems. Therefore we must search for ways to lessen the inequities which might develop because of the computer revolution. The steadily decreasing cost of microcomputers suggests that in the next five years the cost of the microcomputer will be only a small fraction of the cost of educating a child, and the courseware for these computers in mathematics can be purchased at a cost much less than the cost of the textbooks. Consequently the lack of funds would be less problematic in implementing computers into the high schools of lesser developed countries. Thus with the availability of the microcomputer, we suggest that a new and appropriate mathematics curriculum be introduced which takes into consideration the special advantages they offer.

Rationale

Heid (1983), Ralston (1985), Wilf (1982), Tucker (1987), and others were all excited by the entrance of computer algebra systems, but their projected date of the arrival of the handheld computer came far sooner than they had estimated. Ralston said, in 1985, that handheld symbolic mathematical systems would be available within a decade. However, the handheld HP 28C computer was made available to the public in January, 1987. Just as mass

production has made the calculator available at a very low cost, it is expected that soon the handheld computer algebra system will be available at a cost less than that of a textbook. The National Council of Teachers of Mathematics (NCTM, 1985) has advocated the use of calculators in schools at all grade levels in classwork, homework and evaluation. Similarly computers and relevent courseware should be used in schools as an integral part of the mathematics curriculum. They can be used not only in the calculator mode, but also as a programming and problem-solving tool surpassing all the reasons offered by the NCTM for the introduction of the calculator into the classroom.

Other educators are predicting the area of applied mathematics will change rapidly. Engelman (1970) says that it may be possible that the wrong skills are being taught in the schools,

> if the engineer will have all future integrations performed by a machine, why bother to teach a bag of tricks that he will immediately forget. Why not teach him how to use the machine as a partner in solving his problem (p. 103).

Therefore the curriculum should reflect the changing needs of the physicist, engineer and other applied mathematicians.

Ralston (1985), on the same subject, says that,

> Calculus is no longer the obvious starting point
> for the study of applied mathematics itself or
> for the study of those disciplines where
> mathematics is heavily applied. ... Thus in the
> future discrete mathematics will play a role
> equal to that of calculus in the first two
> undergraduate years of mathematics (p. 494).

While Coxford (1985) gives an opposing view,

> In order to study computer science at the
> university level, the study of calculus is
> required. Therefore changes in the algebra
> curriculum must leave in place the goal of
> algebra as preparation for calculus (p. 53).

However all agree that changes must be made to the existing curriculum, and in this thesis, suggestions are made for changes in the algebra curriculum at the high school level.

Assumptions

Many studies, too numerous to quote, support the introduction of computers and computer courseware into the schools as a teaching aid. These studies have shown how slow learners, low ability students, and above-average students have all benefited from the use of computers. Therefore a further study to show the advantages of implementing computers was not undertaken, and the advantages are acknowledged herein. Computers are

versatile, taking on the role of tutor, chalkboard, problem-solver and calculator; and teaching modules and courseware packages are becoming available faster than the publication of textbooks. There are many teaching modules and courseware packages which are appropriate for use by students, and many which are not appropriate, but only the muMath courseware and the handheld HP 28C computer have been considered in some detail in this paper.

Educators (e.g., Carpenter et al., 1980; Fey et al., 1984) have noted that there must be some changes in the high school curriculum. Other mathematics educators have stated that the introduction of computers into the classroom makes available free time (cf., Kulik, Kulik, & Cohen, 1980; O'Shea & Self, 1983). Under the assumption that the limitations of implementing computers into the schools can be overcome, recommendations have been made concerning what should be taught at the high school level, and what should be omitted from the algebra curriculum in secondary schools.

Purpose

The evaluation of muMath on the Apple IIe computer, and a brief summary of the capabilities of the HP 28C

handheld computer were given to show the power of computer algebra systems. These systems weaken the arguments for studying many topics in mathematics such as formal integration of complex expressions. Before 1968, no one had found an algorithm for integration, but in that year Robert H. Risch developed such an algorithm at the System Development Corporation in California. Solutions for integration problems which were previously obtained by paper and pencil manipulation can now be obtained on handheld computers. The availability of symbolic mathematics systems will change what is taught in secondary schools, and how it is taught. In this thesis the following questions will be addressed:

(1) Which topics will remain in, and which topics will be omitted from the mathematics curriculum, or covered in less detail?

(2) What will be done with extra time?

The following six criteria will be used to select topics to be included in the new curriculum:

1. The student should do all those things that can be done more efficiently by hand with paper and pencil, and those that cannot be done more efficiently by hand with the computer.

2.   It is more important for the student to learn the underlying concepts leaving the time-consuming activities to the computer.   The availability of calculators, handheld computers and printers, and microcomputers makes it less important to painstakingly carry out long division, addition of fractions, factorization of polynomials, but more important to be able to check the reasonableness of the result.

3.   The student should be able to perform simple arithmetic and algebraic operations.   This is to ensure that the student not only does not lose any manipulative skills in mathematics, but also does not spend too much time on mechanical operations.   The time can better be used in gaining a better understanding of mathematical concepts, and in solving problems which occur in realistic situations.

4.   The computer frees us from the drudgery of long and complex calculations, therefore the student should use the free time to explore variations of the problems, and the way these variations affect the answers.

5.   The topics, where possible, should be those necessary to exist in society and problem solving should be relevant to 'real life' situations.   The computer can simulate the

problem-solving environment, therefore, problems assigned should provide realistic experiences in the area of mathematics being taught.

6. The particular topic should be required or helpful to the student at the University level; that is, it should aid in the study of calculus and other applied and pure mathematics courses.

These questions will be applied mainly to topics in algebra not only because it is strongly emphasized in the high schools, but also because algebra provides the tools with which most problems are solved in the real world, and without algebra it would be difficult to proceed to higher mathematics. Algebra is not simply a generalization of topics in arithmetic, but also covers topics such as sets, relations and functions, problem-solving, and trigonometry; hence these topics will be considered for inclusion into the curriculum.

Procedure

In the second chapter, a description is given of the structure of the secondary schools in Trinidad along with the examinations written at the fifth (the Basic Proficiency and General Proficiency), and seventh year

(Advanced level) after entering high school. The specific objectives of the mathematics curriculum are given in the appendices along with copies of the examinations. The Caribbean Examination Council (C.X.C.) administers the Basic Profiency and General Profiency examinations, and publishes a report on the work of students in mathematics. The report includes statistics on the performance by the students on the examinations, and the areas of weakness indicated by the students' solutions to the problems. The 1984, 1985 and 1986 reports are also included in the appendices.

A description of muMath and its capabilities are given in chapter three, along with examples to emphasize its ability to handle symbols and to perform exact rational arithmetic. An evaluation of the muMath courseware was also done to determine the appropriateness of implementing muMath in the high school mathematics programme. muMath was chosen because it is the best known computer algebra system available, it is a reduction of the very powerful MACSYMA, and because the courseware is available for approximately $50.00 on the computer, the Apple IIe, most often found in high schools. MuMath is easy to use, sequential, and very powerful. The handheld

HP 28C computer is briefly described, but in comparison muMath has a larger memory, and can perform simplifications of algebraic expressions.

In the fourth chapter, we will consider whether the topics in algebra taught at the secondary school in Trinidad are needed for work in mathematics at a higher level, for the understanding of basic concepts in algebra, or whether the topic is required by the student to exist in society. The criteria listed above will be used to judge which of the topics in algebra should remain in the high school curriculum. First, the basic concepts, considered to be necessary to understand higher concepts in algebra and for the generalization of arithmetical procedures, are given. Next, some of the topics which aid in developing skills necessary to exist in society and for work at a higher level of mathematics are described.

MATHEMATICS AT THE SECONDARY SCHOOL LEVEL

IN TRINIDAD AND TOBAGO

The Republic of Trinidad and Tobago, as well as many other Commonwealth countries, adopted the British education system. In most cases this system remained in operation because it allowed those countries where there were too few schools to accommodate every student to weed out the weaker students at every level of the educational hierarchy. Also, all examinations in this system are administered from England so these Commonwealth countries were not burdened with this task, and they still had a worldwide acceptable standard of education.

In the Republic of Trinidad and Tobago there are sufficient schools to accommodate every child from the age of five to eleven years. However, from the beginning of Secondary School there are not enough spaces for every student so the places available are filled by the students who achieve the best grades. In 1979, in an effort to grant each student at least eleven years of free education and to eliminate the Common Entrance Examination (which the students write at the age of eleven years to obtain a place in the Secondary schools), more schools were built. These schools were built in the American fashion of Junior

and Senior Secondary schools, and the education system showed some signs of Americanization. However, because there still were not enough schools at the Secondary level the Common Entrance Examination was retained. In the 1970's, with the British education system still in place and American-like schools in existence, the decision was made to gradually discard the British and American education systems, and to set up the Caribbean Examination Council (C.X.C.) to administer a Caribbean education system. The first examination set and administered by the Caribbean Examination Council was in 1979. The goal to replace the British education system has not been fully realized; hence the education system which will be described in the following pages is a mixture of all three education systems existing at once.

After five years of Elementary school education the student at age 11 writes the Common Entrance Examination also known as the 11 plus examination. Approximately 28 000 students write the examination, but not all of them obtain a place in the secondary schools because there are about 20 000 places available at the secondary school level, and the top 20 000 students obtain these places regardless of grades. In 1986, 27 992 students wrote the

Common Entrance Examination and 20 230 students gained places in the Secondary Schools. At present, students who do not succeed either stay for another two years at the same Elementary school, or those who can afford to pay, go to private schools, and eventually write the University of London General Certificate of Examination at the Ordinary Level and probably the Advanced Level Examinations. An individual can register and write the University of London Examinations or the Caribbean Examination Council examinations privately, but cannot write examinations set by Cambridge University .

It is virtually impossible to determine how many private schools there are because they are run by private individuals as a business enterprise. They remain open depending on the ability of the individual to run a successful business. It is anticipated that there will be places for all students in the Secondary Schools, and the Common Entrance Examinations will be eliminated some time in the future.

Students who succeed at the Common Entrance Examination are placed in three types of schools. The students with the best marks are placed in the best school

or the first choice of the three schools requested on the

application form. The three types of schools are:

(1) a 7-year school

(2) a 5-year school

(3) a junior secondary school (3 years).

There are 24 7-year schools, and these schools are better

known as the Assisted Secondary Schools because they are

partially controlled by different religious organizations.

The 5-year schools, also known as the Government Secondary

and Composite schools, are not as prestigous as the 7-year

schools but the students here, as in the 7-year schools,

do not have to write any exams until their fifth year.

Should they obtain the necessary qualifications at the end

of the fifth year and wish to continue their education

they must be accepted by one of the 7-year schools.

Students in the 7-year schools who are successful on the

Caribbean Examination Council General examinations are

automatically allowed to sit the G.C.E. Advanced Level

examinations in the seventh year. There are 29 Government

Secondary and Composite Schools which were built and are

maintained solely by the government. There are 25 Junior

Secondary Schools also called the 3-year schools. After

three years in a Junior Secondary School the students

write another examination which determines whether they should be placed in a 7-year school, a 5-year school, or a Senior Comprehensive school. There are 24 Senior Comprehensive schools, also known as the 2-year schools, and these focus on trade skills and other skills which prepare the students for non-academic careers. These students would most likely write the Basic Proficiency Examination in subjects such as Woodwork, Fashion and Fabrics, Food and Nutrition which will be described later. The better students go on to the 7-year and 5-year schools. The students in the 7-year and 5-year schools, and those students transferred from the Junior Secondary schools are put into three groups at the end of the third year depending on the skills they have displayed in prior years. The three groups are Science, General, and Arts.

The students in the Science group are allowed to take courses in subjects such as Chemistry, Physics and "Additional Mathematics". This opportunity is not granted to students in the General and Arts groups. This implies that only students in the Science group will be able to go beyond the fifth year level in mathematics.

In the fifth year the students in the 5-year schools, the 7-year schools and the Senior Comprehensive

schools write the Caribbean Examination Council (C.X.C.) examinations, either the Basic Proficiency or General Proficiency set by the Caribbean Examination Council, or the "Additional" mathematics examination set by Cambridge University. Only students in the Science group may write the General Certificate of Examination (G.C.E.) in "Additional" mathematics which is set and marked by Cambridge University. "Additional" mathematics is an enriched form of the General Proficiency mathematics course. Up to June 1986, students who wrote only the General Proficiency Examination did not qualify to write the Advanced Level Mathematics Examination set by Cambridge University. However, the new Caribbean Examination Council has revised the curriculum, and additional topics have been included in the General Proficiency Syllabus to meet the needs of those who will be:

(a) functioning in the technological world; such as, agriculturalists, engineers, scientist, economists,

(b) proceeding to study mathematics at an advanced level,

(c) engaged in the business and commercial world.

In June, 1987, the new General Proficiency examination replaced the "Additional" mathematics

examination set by Cambridge University and London University.   Table 2.1 gives the results of the "Additional" mathematics examinations which were set by Cambridge University before they were replaced by the revised General Proficiency examination.   As mentioned previously, in most cases only the students successful in the "Additional" mathematics examination (and students who perform very well on the General Proficiency examination) have the opportunity to study for a further two years in mathematics in order to write the Advanced level examination.   Therefore, from the results in Table 2.1, we can say that approximately 750 students write the Advanced level examination.

Table 2.1

Results of the "Additional" Mathematics Examination

No. of Students	Year			
	1983	1984	1985	1986
Registered	1 715	1 603	1 593	1 030
Passed	649	686	703	714

Those students who write the Basic Proficiency Examination do not require a higher level of mathematics in their careers, and a pass at this level completes the requirements for mathematics at the high school level. The students writing the Basic Proficiency Examination come from the Arts group. The following subjects are offered on the Basic Proficiency Examination: Arts, Arts and Craft, Bookkeeping, Caribbean History, Clothing and Textiles, Craft, English A, Food and Nutrition, French, General Electricity, Geography, Home Management, Integrated Science 1, Mathematics, Metals, Office Procedures, Principles of Business, Social Studies,

Spanish, Technical Drawing, Typewriting and Woods. In this group every student writes an examination in Mathematics at Basic Proficiency level. In 1985, 5 571 students wrote the mathematics examination at this level and 4 571 obtained at least a passing grade.

The 27 subjects offered on the C.X.C. General Proficiency examination include those subjects offered at the Basic Proficiency Level. The additional topics are Agricultural Science, Biology, Chemistry, English B, Integrated Science 2, Mathematics, Physics, Principles of Accounts, and Shorthand. The students writing the General Proficiency Examination come mainly from the General group and are likely to go into vocations (secretarial work) or professions not requiring mathematics beyond the secondary school level.

Basic Proficiency Examination

The contents of the mathematics curriculum are geared towards teaching the student how to perform well in examinations. In the fourth and fifth years the students spend their time only on topics they will be tested on in the examination; the fifth year is mainly spent completing the study of topics on the examination, and practicing on

previous examinations. Before the June examination period the students write a "mock" examination under similar conditons as those of the "real" examination.

The mathematics examination in Basic Proficiency consists of two papers. On paper 1 you are allowed 1h 30min in which to complete 60 multiple-choice questions on the following:

Topic	Number of Items
Sets	15
Relations and Graphs	6
Computation	6
Number Theory	6
Measurement	6
Consumer Arithmetic	9
Statistics	6
Algebra	9
Geometry	9

On paper 11 you are allowed 2h 30min in which to complete the questions. The questions require essays or short answers, and the students have to answer all questions in

this section. The percentage of the total marks allocated
are as follows:

Topic	Mark
Relations and functions	15
Computation	10
Measurement	10
Consumer Arithmetic	20
Statistics	15
Algebra	15
Geometry	15

The specific objectives for each topic on papers 1
and 11 are described in Appendix A along with examples,
and a copy of the 1985 Basic Proficiency examination is
included in Appendix D. Table 2.2 shows the number of
students who registered for the Basic Proficiency
examination, and the number of students who actually wrote
the examinations together with the grades they obtained.

Table 2.2

Results of the Basic Proficiency Examination

Year	No. Ent.	No. Writ.	Grades				
			1	11	111	1V	V
1981	10 670	9 203	52	1 260	1 389	3 860	2 642
1984	9 414	8 109	229	876	981	4 115	1 908
1985	8 475	5 571	145	626	818	2 404	1 578
1986	5 689	5 242	158	657	1 021	2 174	1 232

Year	No. Ent.	No. Writ.	Percentages				
			1	11	111	1V	V
1981	10 670	9 203	0.57	13.69	15.09	41.94	28.71
1984	9 414	8 109	2.82	10.80	12.09	50.74	23.52
1985	8 475	5 571	2.60	11.24	14.68	43.15	28.33
1986	5 689	5 242	3.01	12.53	19.47	41.47	23.50

General Proficiency Examination

In 1987, the General Proficiency mathematics examination replaced the G.C.E "Additional" mathematics examination at the end of the fifth year of secondary school. Students are responsible for all material on the Basic Proficiency Syllabus, the compulsory section of the General Proficiency Syllabus and three questions from the Optional section of the General Proficiency syllabus. The questions are based on relations and functions, trigonometry, and vectors and matrices, and one question must be chosen from each category.

(1) Relations and functions

The specific objectives for this topic are:

(a) Sketch the graphs of the functions of the form

$y = ax^{-n}$ where $n = 1, 2,$ or 3 for a given domains,

(b) Draw the graphs of the sine, cosine and tangent function between -360 and 360 degrees,

(c) Find maximum and minimum values of quadratic functions by the method of completing the square,

(d) Apply the idea of gradient of an area under a curve to problems in the physical, biological and social sciences,

(e) Solve simple quadratic inequalities,

(f) Use linear programming techniques to solve a system of two linear equations.

(2) Trigonometry

The specific objectives for this topic are:

(a) Recognise and use the trigonometrical ratios of special angles e.g. 30, 45, and 60 degrees,

(b) Prove simple trigonometric identities containing sine, cosine, and tangent,

(c) Solve practical problems involving heights and distances in easy three dimensional situations,

(d) Use vector arithmetic to solve problems which occur in the physical world,

(e) Calculate the distance between points on the earth as a sphere, measured along parallels of latitude or meridians of longitude,

(f) Determine the latitude and longitude of a point on the earth's surface given its distance from another point whose position is known,

(3) Vectors and Matrices

(a) Perform transformations such as enlargements, rotations, reflections, shears and stretches on a 2 by 2 matrix,

(b) Give the transformation equivalent to two linear transformations of a 2 by 2 matix in the plane where the origin remains fixed,

(c) Find the determinant of a 2 by 2 matrix,

(d) Identify a singular matrix,

(e) Find the inverse of a non-singular matrix.

The specific objectives of the topics in the General Proficiency syllabus which have not been included in the Basic (core) syllabus are described in Appendix B. Table 2.4 shows the number of students who registered for the General Proficiency examination, and the number of students who actually wrote the examinations together with the grades they obtained. In Table 2.4 the grades assigned are I, II, III, IV, and V. Grade I represents a distinction, grade II is a credit, grade III is a passing mark, and grades IV and V are failures in mathematics at the fifth year level. Table 2.3 shows the total number of students who passed either the Basic or General Proficiency examinations at the fifth year level.

Table 2.3

Accumulated Results of the C.X.C. Examinations

	Basic and General Proficiency Examinations			
	1981	1984	1985	1986
Number Entered	18 453	22 695	22 480	19 399
Number Writing	16 524	19 808	17 255	17 932
Number Passed	8 063	8 351	8 478	8 246

Table 2.4

Results of the General Proficiency Examination

Year	No.Reg.	No.Writ	Grades				
			1	11	111	1V	V
1981	7 783	7 321	678	1 664	3 020	745	1 214
1984	13 281	11 699	922	2 407	2 936	3 852	1 582
1985	14 005	11 684	841	2 838	3 210	3 785	1 010
1986	13 710	12 690	776	2 596	3 038	5 353	927

Year	No.Reg.	No.Writ	Percentages				
			1	11	111	1V	V
1981	7 783	7 321	9.26	22.73	41.25	10.18	16.58
1984	13 281	11 699	7.88	20.57	25.09	32.92	13.52
1985	14 005	11 684	7.20	24.29	27.47	32.40	8.64
1986	13 710	12 690	6.11	20.45	23.94	42.18	7.30

Advanced Level Examination

The Basic Proficiency and the General Proficiency examinations are written at the fifth year. The next step is study at the Advanced level, but it is important to note that the decisions made at the third year when the students are placed in the science, general or arts categories are now important because students from the arts core cannot take mathematics at Advanced Level. Prior to the June 1987 examination, a student continues to study mathematics only if he has obtained a pass on the General Proficiency examination and the "Additonal" mathematics examination. Since the General Proficiency examination has been revised, a pass on the General Proficiency examination is required for further study in mathematics. At this stage, students who are successful in the General Proficiency examination, that is, those who obtained a pass in three out of eight subjects including mathematics, have the option to study three subjects plus the compulsory subject General Paper for two years. The 3 subjects must all be in one category; for example,

(a) History, Geography, and English Literature,

    (b) History, Geography, and Economics,

(c) History, English Literature and Economics,

(d) Physics, Chemistry and Mathematics,

(e) Chemistry, Zoology and Mathematics,

(f) Zoology, Botany and Mathematics etc..

Of course these choices are controlled by other factors such as the availability of suitable teachers in the schools, and the facilities available in the schools such as laboratory equipment. The Advanced level mathematics examination can be written in several forms where the emphasis changes for each examination.

The Mathematics Advanced Level examination is set by Cambridge University. The mathematics Advanced level syllabi B and Further B were withdrawn in 1986. Syllabi A, B and Pure mathematics will overlap with Syllabi C and Further C in 1987, and syllabi A, B and Pure mathematics will be removed in 1988. From 1988 the students will write only the Syllabus C and Further Syllabus C Advanced level examinations. Tables 2.5 and 2.6 show the number of students writing the Advanced level examinations, and the grades obtained by the students who wrote the examinations. The grades range from the highest grade A to a failure F.

Table 2.5

Number of students taking the Advanced Level

Examination

Examination	Year			
	1982	1984	1985	1986
Syllabus A	178	146	149	90
Syllabus B	478	655	678	—
Syllabus C	—	—	—	623
Pure Mathematics	10	3	4	—
Further Syllabus A	27	28	31	24
Further Syllabus B	19	18	20	—
Further Syllabus C	—	—	—	20
Total	712	850	882	757

Table 2.6

Distribution of Grades obtained on the

Advanced Level Examination

	Grades							
Year	A	B	C	D	E	O	F	Total
1982	66	89	83	74	129	171	100	712
1984	104	114	78	75	180	177	122	850
1985	109	156	84	91	174	153	115	882
1986	124	131	43	82	139	118	120	757
Total	403	490	288	322	622	619	457	3201

Total = Total number of students.

O = Pass at a level of the General Proficiency Examination.

The Advanced level examination consists of two three-hour papers each worth 50% of the total marks. The Advanced level mathematics examination Syllabus C consists of four papers. Every student must write Paper 1 which consists of several brief questions, and seven longer questions. On paper 1 students must answer all the brief questions and four of the seven longer questions. During the two years the student has spent studying for the Advanced level examination only two topics would be taught, and students will write 7 out of 10 examination questions on these two topics. Students must write seven questions on two of the three papers; Papers 2, 3 and 4 contain five questions each on Particle Mechanics, Probability and Statistics, and Pure Mathematics.

The Advanced level mathematics examination Further Syllabus C consists of four three-hour papers from which the student does Paper 1, and another one of Papers 2, 3, and 4. Paper 1 contains 12 questions on algebra, algebraic structure, analysis, complex numbers, matrices and linear spaces, and vectors; the student is expected to complete seven of the 12 questions. Papers 2 and 3 consist of ten questions each on mechanics and statistics respectively. Paper 4 consists 15 questions with five

questions each on mechanics, statistics and numerical analysis. On Papers 2, 3, and 4, the student is expected to complete seven questions. The specific objectives of Syllabi C and Further C are given in Appendix C, and the topics which are common to both syllabi C and Further C are the following:

1. Addition, subtraction, multiplication, division and factoring of polynomials;

2. Given a quadratic equation in one variable:

    (a) solve the equation,

    (b) find maxima and minima by completing the square,

    (c) sketch the graph;

3. Graphing of polynomials and simple rational functions;

4. Graph a function and its inverse;

5. Finding the sum of finite series;

6. Differentiation and integration of sums, differences, product and quotient functions;

7. Use differentiation in finding gradients, maxima, minima, rates of change and curve sketching;

8. Use simple logarithmic and exponential functions;

9. Use simple techniques of integration, including
    integration by substitution and by parts;

10. Sketch the graph of simple trigonometric functions;

11. Prove simple trigonometric identities;

12. Given a matrix, find the transpose matrix, the inverse matrix, and the determinant;

13. Use the properties of complex numbers; such as sum, product, and quotient of two complex numbers, and De Moivre's theorem.
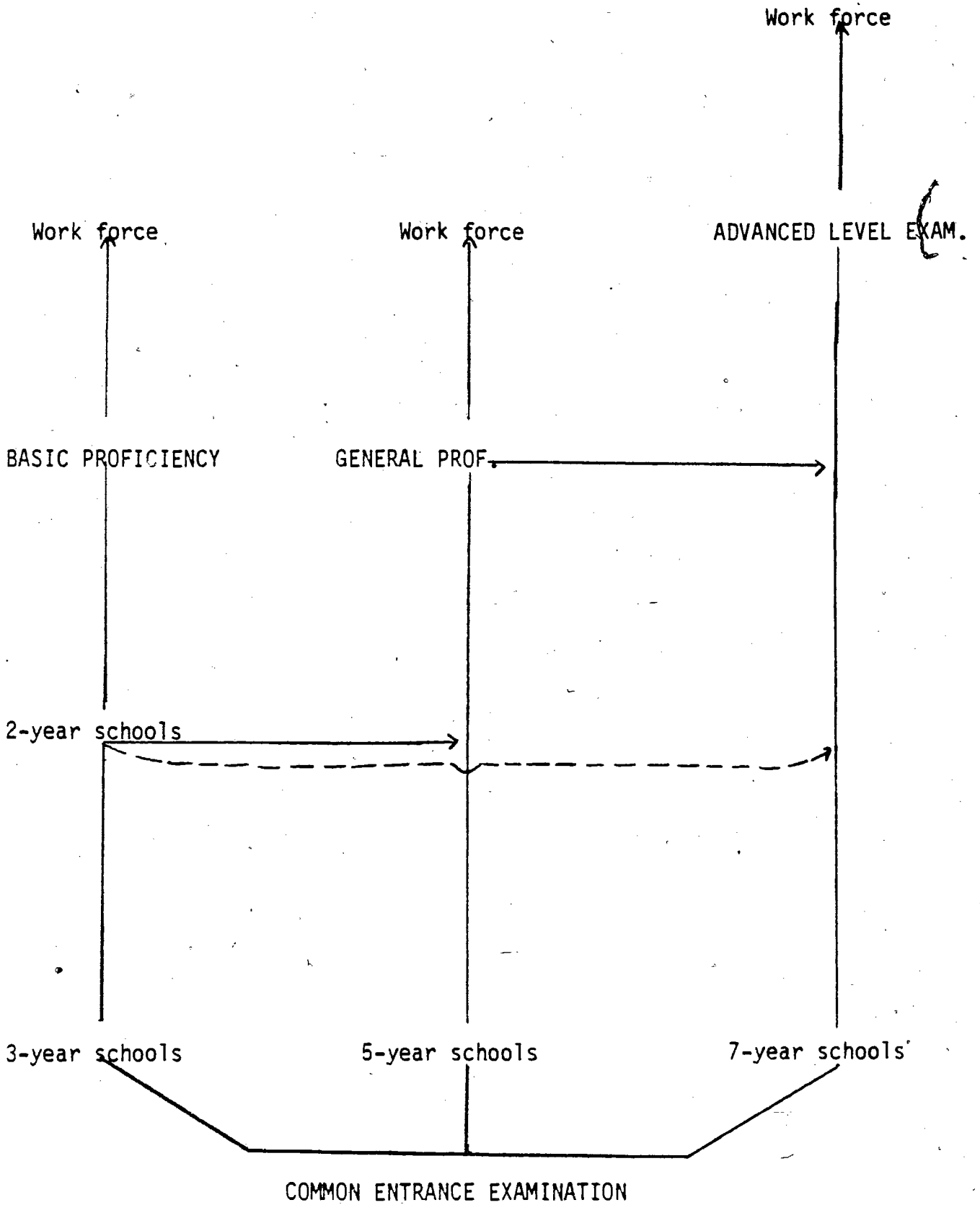
Figure 2.1 Structure of Trinidad and Tobago Education System.

CHAPTER THREE

COMPUTER COURSEWARE

Computer algebra systems exist with the capacity to manipulate algebraic symbols and to perform algebraic operations at speeds never before thought possible. The best known computer algebra systems are MACSYMA, developed at the Massachusetts Institute of Technology; REDUCE, developed at Stanford University, the University of Utah and Rand Corporation; SCRATCHPAD, developed by the International Business Machines Corporation; SMP, developed at the California Institute of Technology; and ALTRAN, developed at Bell laboratories. However, the best known computer algebra system available for the microcomputer is a reduction of MACSYMA known as muMath developed at the Soft Warehouse in Honolulu. MuMath is written in a programming language called muSimp designed by David Stoutemyer and Albert Rich, and is available for the Apple 11, Radio Shack, IBM and TRS-80 microcomputers. MuMath is described by Pavelle, Rothstein and Fitch (1981), Steen (1981), and Wilf (1982) as the most sophisticated and widely available computer-algebra system available. Its algebraic and analytical operations are beyond the capabilities of programming languages such as

BASIC and Fortran. Because of the ability of muMath to manipulate abstract symbolic algebraic expressions, muMath was chosen as the courseware most likely to be implemented in the classroom. MuMath performs as a mathematician, working with symbols and leaving calculations to the final step. Steen says " They do algebra, and calculus, and linear algebra; indeed, they do virtually everything taught in the first two years of university mathematics." (p. 250). These computer algebra systems will change the skills required both in the 'real world' and in institutes of higher learning, and eventually the school curriculum. Because computer algebra systems have proved to be very useful to mathematicians, scientists and even students, the capabilities of muMath will be discussed in the following sections to determine its suitability for the classroom.

An Overview of MuMath

Using 'muMath on the Apple 11 computer is as convenient and interactive as using a pocket calculator. Expressions are transformed and simplified when they are entered, and exact rational arithmetic and exact infinite arithmetic up to 611 digits are performed. For example,

$$1/3 + 1/6 = 1/2 \qquad 12 \ (1/2) = 2 \times 3 \ 1/2$$

$$100! = 93326215443944152681699238885626670049071$$
$$59682643816214685929638952175999932299156$$
$$08941463976156518286253697920827223758251$$
$$1852109168640000000000000000000000000000.$$

Hence the use of muMath in this calculator mode eliminates
the need to write and enter long programs. For example,
expressions can be expanded over a common denominator;
complicated trigonometric expressions can be written in
terms of the six basic trigonometric identities; linear
systems and quadratic equations can be solved; the n roots
of an nth degree polynomial can be found; matrix inverses,
matrix product and matrix dot product can be performed
when the entries are non-numeric; and the calculus
features allow one to perform symbolic integration
(definite and indefinite), find derivatives, limits and
summation of sequences. The user is free to experiment by
entering and manipulating formulae and can immediately see
results. muMath employs the mathematical symbols that the
computer does for the mathematical operations, for example

* for X

" for exponents

/ for division.
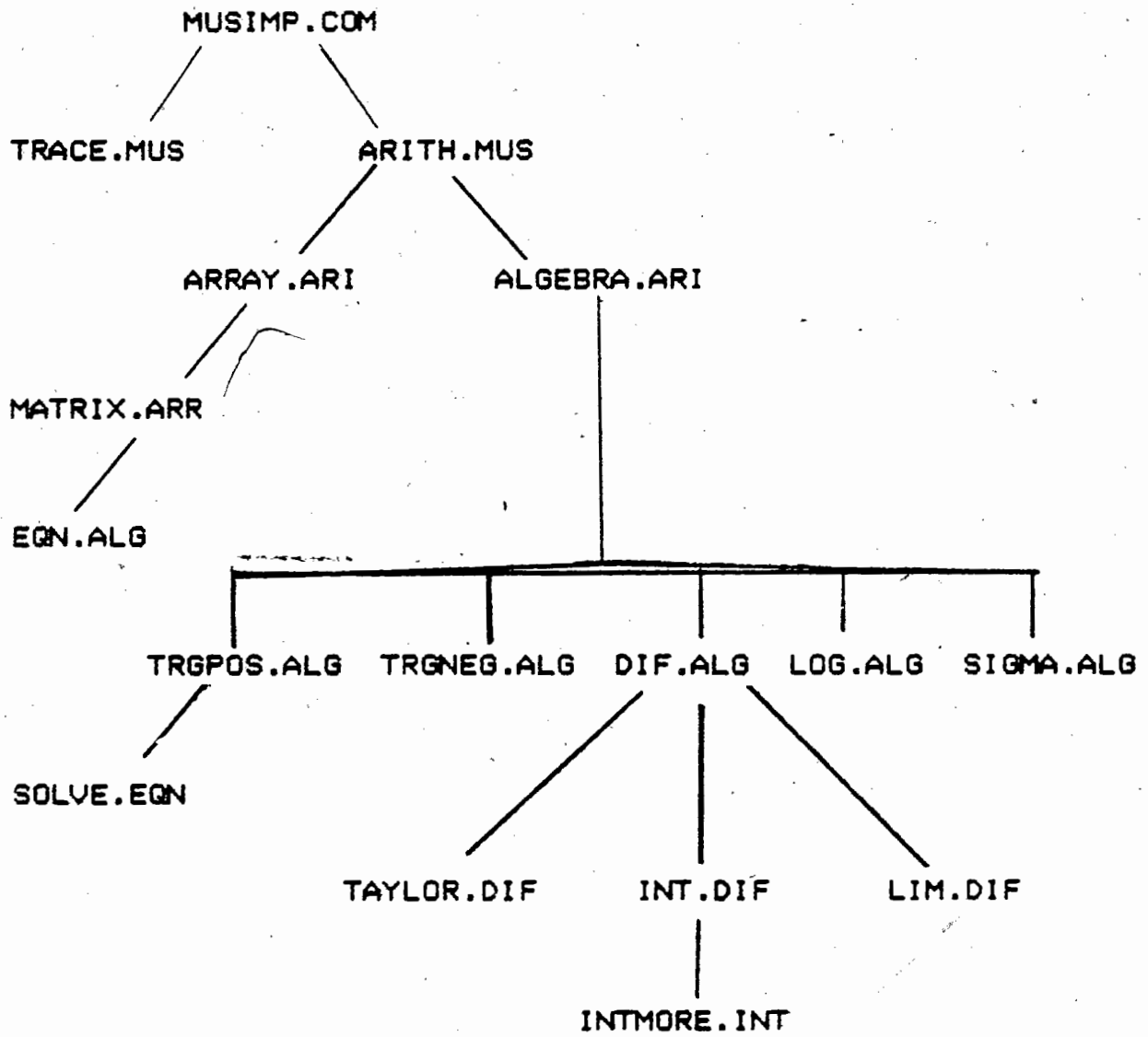
Figure 3.1 Hierarchical Structure of muMath.

In the paragraphs below, the contents of the different files will be described and the operations allowed in each file will be given.

In musimp.com the symbols: *, / , - , + , () are used for multiplication, division, subtraction, addition, and bracketing. This file does not solve n!, y^x, and other more complicated computations.

In muSimp as many files as you need can be entered into the running system (according to the mathematical hierarchy). If one is using the file Eqn.Algebra and needs to find the root of an equation, then the file Solve.Eqn can be added which gives a solution to the problem.

Arith.Mus performs rational arithmetical operations such as addition, subtraction, division, multiplication, factorials and powers. Here you can work in different bases, from two to thirty-six, by simply using RADIX (<base>) where the letters A, B, C, D, ... denote the digits 10, 11, 12, 13, ... You can perform modulo arithmetic. We can find greatest common denominator, lowest common multiple, absolute values, lengths of n!, y^x, and minimum integer. If solutions are preferred in

the decimal format rather than the rational, fractional form you use 'POINT: 10$' for ten decimal places.

Some common notations are introduced:

#E - the base of the natural logarithm 2.71828

#I - the square root of -1

#PI - the ratio of the circumference of the circle to the diameter.

Some examples of the calculations which can be performed using this file are:

$$\#E(\#I \times \#PI) = -1$$

$$4444 = 2050773823560610053645205609172376035486179836520607547294916966189360296$$

LENGTH (4444) = 73

LENGTH (100!) = 158

GCD (23 678, 549 999) = 1

GCD (54 386 742, 2 159 830 521) = 3   (Calculation takes less than 1 second.)

ABS (-15) = 15

MIN (7, -2) = -2 .

Algebra.Ari performs easy algebraic simplications using arithmetic operations. The following commands can be used:

NUMNUM- controls the distribution of factors in the numerator of an expression over a sum in the numerator

$$a * (b + c) \quad \langle ---- \rangle \quad a*b + a*c$$

DENDEN- controls the distribution of factors in the denominator over a sum in the denominator

$$1/a * 1/(b + c) \quad \langle ---- \rangle \quad 1/ (a*b + a*c)$$

DENNUM controls the distribution of factors in the denominator of an expression over a sum in the numerator

$$1/a * (b + c) \quad \langle ---- \rangle \quad b/a + c/a$$

BASEXP controls the distribution of the BASE of an expression over the exponent

$$a^{\wedge}(b + c) \quad \langle ---- \rangle \quad a^{\wedge}b * a^{\wedge}c$$

EXPBAS controls the distribution of the exponent of an expression over the BASE

$$(a*b)^{\wedge} c \quad \langle ---- \rangle \quad a^{\wedge}c * b^{\wedge}c$$

PWREXPD controls whether or not integer powers of sums are expanded in numerators and/or denominators

ZEROEXPT controls the use of the following identity that is valid for all a not equal to 0.

$$a^{\wedge}o \quad -----\rangle \quad 1$$

ZEROBASE controls the use of the following identity which is only valid for positive a

$$0^{\wedge}a \quad -----\rangle \quad 0$$

For example, EXPD expands the numerator and denominator, and then evaluates and expression,

$$EXPD \ (5x)2 = 25x$$

and FCTR semi-factors the numerator and denominator,

FCTR [(x2 + 2.x + 1)/(1 + x)] = [1 + x(2 + x)]/(1 + x).

Eqn.Alg allows equations to be solved step-wise (you have to give the next step manually). Note: you must distinguish between = and ==, where in the first case the equation is put in a simpler form and in the second it solves an equation for a given variable. An example of this procedure is: ? EQN: 6*x^2 + 3*x == 12*x;

    @ 3*x + 6*x^2 == 12*x

    ? EQN/x;

    @ 3 + 6*x == 12

    ? EQN +(-3);

    @ 6*x == 9

    ? EQN/6;

    @ x == 3/2.

Solve.Eqn finds the roots of a wide variety of equations by solving for x doing all steps automatically, for example, SOLVE (x6 + 2.x4 == -x2, x) generates the values x = -#I, x = #I, and x = 0, and SOLVE (x21 + 1, x) generates 21 values for x:

$$x = -\#E^{\wedge} (40 * \#I * \#PI / 21),$$

$$x = -\#E^{\wedge} (38 * \#I * \#PI / 21),$$

$$x = -\#E^{\wedge} (12 * \#I * \#PI / 7),$$

$$x = -\#E^{\wedge} (34 * \#I * \#PI / 21),$$

$$x = -\#E^{\wedge} (32 * \#I * \#PI / 21),$$

$$x = -\#E^{\wedge} (10 * \#I * \#PI / 7),$$

$$x = -\#E^{\wedge} (4 * \#I * \#PI / 3),$$

$$x = -\#E^{\wedge} (26 * \#I * \#PI / 21),$$

$$x = -\#E^{\wedge} (8 * \#I * \#PI / 7),$$

$$x = -\#E^{\wedge} (22 * \#I * \#PI / 21),$$

$$x = -\#E^{\wedge} (20 * \#I * \#PI / 21),$$

$$x = -\#E^{\wedge} (6 * \#I * \#PI / 7),$$

$$x = -\#E^{\wedge} (16 * \#I * \#PI / 21),$$

$$x = -\#E^{\wedge} (2 * \#I * \#PI / 3),$$

$$x = -\#E^{\wedge} (4 * \#I * \#PI / 7),$$

$$x = -\#E^{\wedge} (10 * \#I * \#PI / 21),$$

$$x = -\#E^{\wedge} (8 * \#I * \#PI / 21),$$

$$x = -\#E^{\wedge} (2 * \#I * \#PI / 7),$$

$$x = -\#E^{\wedge} (4 * \#I * \#PI / 21),$$

$$x = -\#E^{\wedge} (2 * \#I * \#PI / 21),$$

$$x = -1.$$

Array.Ari performs simple arithmetic operations on arrays. The following notations are used:

( ) column vector also (- <CTRL-L>

) - <CTRL-O>

[ ] row vector also [- <CTRL-K>

] - <SHIFT M>

([a,b,c], [d,e,f],[g,h,i]) an array with 3 rows and 3 columns. Given the matrix A = ([1, 3], [6, 5]), then you can easily find A^{-1} = ([-5/13, 3/13],

[6/13, -1/13]), Det (A) = -13, and A^2 = ([19, 18],

[36, 43]).

Matrix.Arr performs such matrix operations as matrix transpose, multiplication, power and inverse. One can also find the determinant of a matrix as well as the dot product.

Dot Product: array1 . array2

Determinant: Det (array)

Identity matrix: IDMAT (<positive integer>)

Transpose: array <CTRL-L>

Powers: array <CTRL-R>

Inverse: array ^ -1

If a matrix is a square, nonsingular matrix, then

B/A is equivalent to A * (B^{-1}).

Some examples of matrix operations are:

det ([1 1 1], [a b c], [a^2 b^2 c^2])

$$= [a - c + (b - a)(c^2 - a^2)/(b^2 - a^2)]$$

$$[b^2 - c^2]$$

$$= (b - a)(c - a)(c - b). \quad [simplified]$$

$$A = ([x \quad 0 \quad 0 \quad 0], [1 \quad x \quad 0 \quad 0], [0 \quad 1 \quad x \quad 0],$$

$$[0 \quad 0 \quad 1 \quad x])$$

$$A^{-1} = ([1/x \quad 0 \quad 0 \quad 0], [-1/x^2 \quad 1/x \quad 0 \quad 0],$$

$$[1/x^3 \quad -1/x^2 \quad 1/x \quad 0],$$

$$[-1/x^4 \quad 1/x^3 \quad -1/x^2 \quad 1/x])$$

$$A^{11} = ([x^{11} \quad 0 \quad 0 \quad 0], [11.x^{10} \quad x^{11} \quad 0 \quad 0],$$

$$[55.x^9 \quad 11.x^{10} \quad x^{11} \quad 0],$$

$$[165.x^8 \quad 55.x^9 \quad 11.x^{10} \quad x^{11}]).$$

Logarithm.Alg performs log simplication such as

log (a.b) = log a + log b

log (a/b) = log a - log b

$log \ a^n = n.log \ a$

Trapos.Alg performs the simplication of positive trigonometric expressions by reducing complicated trigonometric functions to expressions containing only sines and cosines and replacing trigonometric functions raised to an integer power greater than one to linear expressions. The control variables are:

TRGEXPD: 2$ (or any multiple of 2) converts an expression to one containing only sines and cosines,

$tan(A).cos(A) + 1/csc(A) = 2.sin(A)$

TRGSQ: 1$ converts an expression containing powers of sundry trigonometric functions to one containing cosines and vice versa,

$2 (cos (A))2 + (1/csc (A))2 = 1 + (cos (A))2$

TRGSQ: 3$ replaces an expression containing products of sines and cosines by one containing linear combinations of sines and cosines of angle sums,

$sin4(A) = 3/8 - 1/2.cos(2A) + 1/8.cos(4A)$

$cos5(A) = 5/8.cos(A) + 5/16.cos(3A) + 1/16.cos(5A)$

TRGEXPD: 5$ (or any multiple of 5) replaces integer powers of trigonometric functions by sines and cosines, and replaces powers and products of sines by linear combinations of cosines, or vice-versa,

$cos 2(A).sin(A) = sin(A) /4 + sin(3A) /4$

Trgneg.Alg simplifies trigonometric arguments. The control variables, described for Trgpos.Alg, perform the operations in the replacement of trigonometric functions by multiple angles and angle sums. Angles are assumed to be measured in radians and inverse sine, for example, is denoted by ASIN.

A = ([cos(x) sin(x) o], [-sin(x) cos(x) 0], [0 0 1])

ASIN(A) = ([cos(x) -sin(x) 0], [sin(x) cos(x) 0],

[0 0 1]).

Dif.Alg performs the first partial derivative of an expression with respect to a variable given the command DIF (expression, variable).   Higher-order   partial derivatives can be obtained. If the differentiation rule is not known to the system, you obtain a value of zero, but you can add new differentiation rules for new functions or operators. Some examples of the capacity of Dif.Alg are:

DIF (LN (3x2 + ax) = (6x + a)/(3x2 + ax))

DIF (#E^x.tan(x)/x, x)

= -#E^x.tan(x)/x^2 + #E^x.sec^2(x)/x + #E^x.tan(x)/x.

Int.Dif performs definite symbolic integration given the command INT (expression, variable).

INT (2.x/(x^3 + 1), x) =

[log (x^2 - x + 1) - log (Ix + 1I)] / 3

+ atan (2x - 1)/3$^{1/2}$

INT (x.sin^2(x), x) = -cos^2(x)/2.

Intmore.Int is an extension of Int.Dif which performs indefinite integration and enables one to perform more

complicated differentiation. Additional integration rules
can be added. Some examples are:

DEFINT $(x2, x, 0, 1) = 8/3$

DEFINT $(3a.x2, x, 0, 1) = a$

    Taylor.Dif performs Taylor series expansion given the
command TAYLOR (expression, variable, expression point,
degree[positive]).

TAYLOR $(\#Esin(x), x, 0, 4) = 1 + x + x2/2 - x4/8$.

    Lim.Dif or limits of functions performs the one-sided
limit of a mathematical expression as one of its variables
approaches a value following the command

    LIM (expression, variable, point, TRUE).

(1) Limit is a function which gives the limit of the
expression as its variable approaches the point. The
optional TRUE asks for the limit from the left. If FALSE,
or not requested, the limit is obtained from the right.

(2) Limit can be PINF or MINF denoting plus infinity or
minus infinity respectively.

    ? LIM (sin(x), x, MINF·)

    ? The appearance of the question mark tells us that
the one-sided limit does not exist.

LIM $(1/x, x, 0)$ = PINF

LIM (sin (x), x, PINF) = ?   (limit does not exist)

LIM (ln x, x, 0) = MINF

LIM ((x2 + x)/(2.x2 - x), x, PINF) = 1/2.

If the file Intmore.Int has already been loaded, you can

perform the following integrations:

DEFINT (1/x2, x, 0, 1) = PINF

DEFINT (1/x2, x, 1, PINF) = 1.

Sigma.Alg performs closed-form summations and

products, such as SIGMA (Uj, j, m, n) = Um + Um+1 +...+Un

PRODUCT (Uj, j, m, n) = Um *...* Un.

This file requires Algebra.Ari and Lim.Dif.

∫ PROD (1- xn)17, n, 1, 2) = (1 - X)17.(1 - X2)17.

The Capabilities of muMath

An overall picture of the muMath program has been

given and we will now address some questions concerning

how and whether this program could be used in the

secondary schools.

Ease of Use

The calculator mode is easy to use and the basic

operational procedure in my opinion can be taught in less

than 10 minutes at the beginning of each topic being

introduced.   The student will have to learn symbols

different from those used with paper and pencil, but these

can be introduced as you proceed through the mathematical hierarchy. Numerical calculations are speedily done on the computer; for example, Table 3.1 shows the time taken for the various calculations.

TABLE 3.1

Time Required for Various Operations

Problems	Time Taken in Seconds
30!	0.4
50!	1.5
m.x - x = .5	7
(x-2)/(x+4) + (x-1)/(X-2) = 15/7	36
x^25 + 1 = 0	56
4x^2 - 8x + 1 = 0	14.2

The major drawback in using this program for arithmetic processes is the time it takes to load each musimp file unto the desktop; Table 3.2 gives the time it takes to load each file into the desktop:

Table 3.2

Time Taken to Load File

FILE	TIME
ARITH.MUS	5m 56s
ARRAY.ARI	2m 12s
MATRIX.ARR	2m 46s
ALGEBRA.ARI	4m 30s
EQN.ALG	40s
SOLVE.EQN	2m 4s
TRGPOS.ALG	1m 53s
TRGNEG.ALG	2m 7s
DIF.ALG	1m 49s
LOG.ALG	1m 5s
SIGMA.ALG	2m 16s
TAYLOR.DIF	21s
INT.DIF	3m 45s
LIM.DIF	5m 12s
INTMORE.INT	5m 41s

As an instructional tool muMath is efficient because it allows both student and teacher to start from the basics, and move upwards in the files so that the student at a specific time will have at his or her disposal only the knowledge he or she has learnt up to that point. Later on in the course, he or she could solve all complex problems on the computer and understand what has been done at each step in the solution. Also the step-by-step format of the Eqn.Alg file allows the student to practice as much as one likes while learning how to solve algebraic equations using equal addition properties, factoring or substitution methods. Having mastered these techniques, one could, at a later time, find roots of polynomials by using simple commands.

Limitations

One cannot obtain the factors of a polynomial equation, but the roots of an equation can be obtained. The lack of factors may have been deliberate since there are supporters, such as Usiskin (1980), who recommend that factoring be deleted because factoring is only important when studying the remainder theorem and the factor theorem. Usiskin says that "factoring does not work for the vast majority of trinomials ...So we must conclude

that factoring is not a strong technique for solving a quadratic that might come up in later experiences." (p. 413).

MuMath cannot perform integration by parts except by manually instructing the computer in each step of the procedure.

The programming mode is not as easy to use as is the calculator mode, and as such it is not very convenient for teaching Geometry. This programming might be beyond the skills of a normal high school student; so if another program with all the skills of muMath described in the overview of muMath and additional plotting capabilities becomes available it would be preferable. With the improvement of programs to include graphics, the lure of a visual aid for the teaching of mathematics will make such a program more acceptable than muMath.

Another drawback is the absence of visual or graphic information. MuMath cannot provide graphs of functions. As this has always been a valuable tool in transmitting mathematical ideas to students this will be a drawback for the implementation of this program in the high schools.

The usefulness of muMath

I believe that all advocates for using calculators in the classroom will have no objections to using muMath, for it is a more sophisticated tool which is as easy to operate as the calculator and is many times more efficient. Although it does not allow simulation, it is interactive and programmable and this together with the calculator mode would make it an asset in teaching mathematics. This program, if used, seems certain to make available free time through the elimination of many manual drill problems. Concepts may be taught with the aid of simple problems, and more difficult problems can be solved by induction and the use of the computer.

I based my description of muMath on hands-on work on an Apple II microcomputer. I read through the documentation with no other outside assistance. My impression is that muMath has a great deal to offer students from Grades 7 through 12 as the material is fairly easy to master in the calculator mode and has some useful operators like differentiation and integration. Its calculations are quickly and efficiently completed as described above; that is, a pause of 1/2 to 2 seconds for easy problems which increases to 30 to 60 seconds for more

difficult problems. The speed and accuracy of muMath allow one to perform more difficult calculations and to consider the properties of a larger number of more difficult problems. MuMath has as its limitations, the lack of computer graphics, which is essential in Geometry, and no plotter for obtaining graphs of a function.

The virtue of muMath is that it allows the student to go through every step in solving equations before using simple commands to obtain a solution. It is also a computer algebra system. The advantages of this type of program over a numerical one are that it leaves all calculations to the final stage to prevent rounding off errors, and when asked to solve the quadratic equation, $4x^2 - 8x + 1=0$, instead of responding with 0.133975 and 1.866025 it gives $-2^1/2$ and $2^$ 1/2 (values better than the previous approximate calculations). Also for those who do not wish to learn how to program, it is not difficult to use.

High school students would be most likely to use muMath because it allows the user to add on files in a hierarchy. For example, one can learn arithmetic from the file Arith.Mus and then add on the file Algebra.Ari to learn to practice the manipulation of variables. While

you are using Algebra.Ari, the Arith.Mus file is still available simulating your memory of the operations you have learnt previously. Also the presence of a programming mode allows both teachers and students to use this type of instruction if it is deemed to be most effective. This, however, can be left up to the teacher because it is not generally accepted that teaching programming yields higher student achievement than simply teaching with the use of a computer. Bork (1987) writes that " Learning to program is already a rapidly increasing activity in our schools. Unfortunately, where it happens at this level it is often a disaster, harming more than helping the pupil." (p. 13). However later on in the book he suggests that grade 9 or between the ages of 14 to 16 is about the right age to begin the study of programming. Hence such decisions can be left up to the institutions or the classroom teachers.

An Evaluation of MuMath

Chambers and Sprecher (1983) have given a checklist
for the evaluation of courseware. I have used these
criteria to evaluate muMath's capabilities.

Content Issues

(1) Instructor-controlled Parameters. Can the courseware
be adjusted through the use of optional parameters to
modify problem difficulty, time allowance, problem type,
use of sound, and other variables?

Yes, the courseware can be adjusted using the
programming mode to introduce different problems such as
time-distance-rate. Also the student can spend as much
time as needed on a level which is difficult for himself
and use the Computer-Assisted Instruction (CAI) as a drill
and practice tool.

(2) Accuracy. Is the instructional content accurate,
consistent and complete?

Yes, because the files are set up in a hierarchy, the
student proceeds from one level of difficulty to another.
Moreover, each stage has all the information about lower
levels of mathematics. This completeness and consistency
allows the student the opportunity to learn the underlying

concepts. Those ideas which must be memorized, and are retained for only short periods of time, are retrievable on the computer.

(3) <u>Environment.</u> Can the courseware be used by teams as well as by individuals? Is it appropriate in unsupervised settings?

The courseware can be used by teams as well as individuals so the teacher can teach on a few screens. It is simple enough to use in any unsupervised setting as the program can be taught as described in the muMath file.

(4) <u>Pedagogy.</u> Is the content presented in a manner consistent with the style and approach? Is the content made unique for each student?

The content is not made unique for each student, but it is presented in a consistent manner. Since each child spends as long as he or she wishes on one section, the slower learners are not at a disadvantage.

(5) <u>Answer Judging.</u> Can misspellings be accepted? Can answers be anticipated? Are meaningful prompts and responses provided?

Misspellings cannot be accepted and answers are not anticipated, but meaningful prompts and responses are provided especially in the computer algebra system.

(6) <u>Branching.</u> Is the sequence of the presentation of materials determined by the answers provided by the student or is it fixed?

No it is not fixed.

(7) <u>Learning Theory.</u> Are the uses of learning theory concepts appropriate (i.e. positive reinforcement, fixed and variable reinforcement, feedback, etc).

Learning theory concepts are not used to reinforce learning. However, muMath allows one to proceed at his or her own pace and to master a section to his or her own satisfaction.

(8) <u>Time Allowance.</u> Is the anticipated amount of time required to complete the lesson appropriate?

This courseware has not been incorporated into the school mathematics curriculum and has not been used as an integral part of the course either as material or as a learning aid. Therefore there has been no attempt to set time limits for appropriate lessons.

(9) <u>Progress Reporting.</u> Does the courseware provide a summary that is useful in assisting the student? Does the summary itself represent positive reinforcement?

The courseware does allow one to read the contents of
the files but the summary does not provide positive
reinforcement for any learning activity.

(10) Professionalism. Is the courseware free of
typographical error, other careless mistakes?

There are no noticeable errors or careless mistakes in
the courseware.

Technical Issues

(1) User Friendliness. Do the instructions and the
operational aspects of the courseware represent a problem
for the student? Are responses and informational needs of
the student adequately anticipated?

The instructions and the operational aspects of the
courseware are easy to teach and will not be difficult for
the student to learn, but this depends to a large extent
on the teacher's ability to instruct the student
especially in the programming mode.

(2) Error Trapping. Do unanticipated responses cause the
courseware to halt or result in a strange system message?
Are there dead ends?

Yes, there are system messages if you type incorrect
mathematical sentences, but we have made allowances for

explaining about new symbols so these should rarely occur if one is careful.

(3) <u>Speed of Execution.</u> Is the amount of time required to load the program and evaluate responses appropriate? Are distractors used when pauses are encountered?

There are no distractors used, but the amount of time required to load the files is inappropriate. Table 2.3 gives the time taken to load the files.

(4) <u>Appearance.</u> Are the font and spacing easily read? Is there effective use made of blank spaces? Is there overreliance on text? Is effective use made of colour, graphics and sound?

There almost no reliance on the text and the muMath package does not have graphics.

Student-Oriented Review

(1) <u>Student Control.</u> Can the student control the pace of the lesson? Are the lesson parameters to be controlled by the student appropriate? Can the student easily obtain help or terminate the lesson?

Since muMath is to be part of the course material, the pace of the lessons is controlled by the teacher.

(2) <u>Freedom from Technology.</u> To what extent is the courseware independent of knowledge about microcomputers?

The courseware is almost completely independent of knowledge about microcomputers.

(3) <u>Motor Skills.</u> To what extent is the courseware dependent on eye-hand coordination, reaction time, and other physical ability?

The courseware is independent of these motor skills.

(4) <u>Motivational Value.</u> Does the courseware actively involve the students or must they be encouraged to complete objectives?

The muMath program is just a part of the curriculum and does not actively perform any of the motivational activities.

Other Symbolic Manipulators

At the International Commission on Mathematical Instruction in Strasbourg in 1985, Howson and Kahane (Churchhouse, et al., 1986) made the following comments concerning symbolic mathematical systems:

> The best known such system for large computers
> is Macsyma and the the best known for
> microcomputers is muMath. These systems do
> <u>symbolically</u> the standard processes of
> secondary school and college algebra and of
> calculus. Thus, they differentiate, they

> integrate (definite and indefinite), they do
> polynomial algebra, they do infinite precision
> rational arithmetic, they solve linear systems
> and quadratic equations - all symbolically
> although they will provide normal numerical
> answers, too, when the user wishes. ...The
> most important point to note for this document
> is that these systems are rapidly becoming more
> powerful and will soon be available on handheld
> computers. (p. 35).

No sooner said than done, in January of 1987, Hewlett Packard announced the HP 28C, a handheld computer. The HP 28C is well described by Tucker (1987). HP 28C is a normal size hand calculator which plots functions on a small dot matrix screen with its own small printer which can be operated by remote control from a distance of about 50 cm. It not only plots graphs but also does matrix operations, equation solving, numerical integration, and symbolic manipulation. It is different from the standard calculators in its use of the Reverse Polish Notation (RPN) or stack logic style of calculation. You key your input before a command, also known as an argument, and you can key in an expression in the algebraic form it appears in any textbook. The data can be entered in 10 different styles: fixed values, real numbers, binary, strings; arbitrary sequences of characters, vectors, matrices, list, names, and mathematical expressions or equations.

The stack, the amount of input, can be as large or small as required. The computer is menu driven, that is, its keys are all-purpose function keys such as logarithms, exponentials, statistics, complex arithmetic, plotting, modulo arithmetic, random number generator etc.. For example, the TRIG key produces a menu of the trigonometric functions. There are 20 menus with a varying number selections; for example, "UNITS" key gives 120 built in conversion units, and you can also program HP 28C for other conversions. Tucker says that a student should feel comfortable using this calculator after an hour or two, and after using HP 28C it is indeed possible. You do not need to understand all the mathematical concepts to use a certain topic, because the HP 28C acts an electronic tutor telling you what operations you can perform and how to perform these operations. For example, if you want to plot the graph of

$$y = (x^2 + 6x + 1) / (x^2 + 1)$$ you use the menu key "PLOT". The choices displayed at the bottom of the screen are STEQ; stores the equation, RCEQ; recalls the current equation, PMIN; sets the lower-left plot coordinates, PMAX; sets the the upper-right plot coordinates, INDEP; selects the plot independent variable, DRAW; sketchs the

graph, PPAR; recalls the plot parameters list, RES; sets

the plot, AXES; sets the intersection of the axes, CENTR;

sets centre of plot display, *W; adjusts the width of a

plot, and *H; adjusts the height of a plot to allow you

perform all these operations by pressing the key. You can

perform more than 374 operations, and because HP 28C is

fully programmable you can store your own operations.

CHAPTER FOUR

A NEW CURRICULUM

The Implications of using MuMath or any
other Symbolic Mathematical System as an
Instructional Tool

In the future an individual who wants to perform a
mathematical task will not search for the relevant
textbook, but will turn instead to a computer-based
information network for a method, or solution to his
problem. Large computer networks, and the availability of
high-powered, low-cost computers are changing the
mathematical needs of the individual. These needs are
reflected in the curriculum, and we require alterations to
the high school curriculum to satisfy those needs. In the
same way as the launch of the Sputnik started the new
mathematics movement of the 1960's (the new mathematics
was not introduced into Trinidadian high schools until
1970), the availability of the microcomputer will bring
into existence a new and more appropriate curriculum to
satisfy the needs of a computer-oriented society. Some of
those needs consist of meeting the entrance requirements
of the universities, developing in students a logical
reasoning approach to solving problems, and teaching
problem-solving skills which are appropriate to real world

situations. We will expect a curriculum which also
satisfies the needs of students in lesser developed
countries such as Trinidad. The computer can perform as a
chalkboard, drill and practice tool, tutor, and in other
ways such as in CAI (Computer-Assisted Instruction) and
CAL (Computer-Assisted Learning). We can find or write
programmes such as Logo, computers such as HP 28C, and
courseware such as TK Solver (cf., Williams; 1982) which
can perform in some of the ways listed previously, and can
perform in a less time consuming manner than the
traditional paper and pencil methods. Many mathematics
educators have stated that the introduction of computers
into the classroom makes available free time (cf., Kulik,
Kulik, & Cohen 1980; O'Shea & Self 1983). The ability of
the computer, to perform in the manner described
previously, is acknowledged; however, the aim of this
thesis is not to discuss all available courseware and
their capabilities, or to quote the numerous studies
showing the successes and in some cases failures in the
implementation of the microcomputers in the teaching of
mathematics. The aim here is to make recommendations on
what should be taught at the high school level, what
should not be taught, and what should be omitted from the

existing curriculum assuming that the problems of implementation can be overcome.

In the following section we will look at a new algebra curriculum. This curriculum; however, is not independent of the entire mathematics curriculum in the high schools. Therefore we will comment, where necessary, on other topics which should not be overlooked such as statistics which is becoming more and more important in problem solving and in interpreting graphs and data. At each step we will keep in mind the words of Atiyah (1986), " Mathematics is really an art - it is the art of avoiding brute-force calculation by developing concepts and techniques which enable one to travel more lightly." (p. 43).

In particular the following questions will be addressed:

(1) Which topics will remain in, and which topics will be omitted from the mathematics curriculum, or covered in less detail?

(2) What will be done with extra time?

The following six criteria will be used to select topics to be included in the new curriculum:

1. The student should do all those things that can be done more efficiently by hand with paper and pencil, and

those that cannot be done more efficiently by hand with the computer.

2.   It is more important for the student to learn the underlying concepts leaving the time-consuming activities to the computer.   The availability of calculators, handheld computers and printers, and microcomputers makes it less important to painstakingly carry out long division, addition of fractions, factorization of polynomials, but important to be able to check the reasonableness of the result.

3.   The student should be able to perform simple arithmetic and algebraic operations.   This is to ensure that the student does not lose any manipulative skills in mathematics, but also does not spend too much time on mechanical operations.   The time can better be used in gaining a better understanding of mathematical concepts, and in solving problems which occur in realistic situations.

4.   The computer frees us from the drudgery of long and complex calculations, the student should use the free time to explore variations of the problems, and the way these variations affect the answers.

5.   The topics where possible should be those necessary to exist in society, and problem solving should be relevant

to 'real life' situations. The computer can simulate problem solving environments; therefore, problems assigned should provide realistic experiences in the area of mathematics being taught.

6. The particular topic should be required for or helpful in future work in mathematics ; that is, it should be useful to the student at the University level in the study of calculus, or applied mathematics courses.

Elementary Algebraic Concepts

The basic concepts in algebra that are defined here are those which generalize the arithmetical procedures such as place value system; addition; subtraction; multiplication; division; prime factorization; positive and negative integer or fractional exponents; and simple factoring using the distributive law; for example, ax + bx = (a + b)x.   These are required in most algebraic procedures, and the ability to manipulate equations using these operations is necessary for higher algebra, and for solving most problems which occur in real world situations.   The ability to express a word problem as a relationship between variables and then to solve the function obtained is the underlying concept in problem-solving.   It is becoming increasingly less important to solve equations because of the numerous

computer algebra systems available which can solve equations, but it is becoming more important to be able to define the relationship between the variables. However it is still important to solve linear equations and obtain a solution set either by graphing or algebraic manipulation. The following topics are considered essential using our notion of basic concepts:

1. $a/b + c/d = (ad + bc) / bd$ and other addition, subtraction, division, and multiplication algorithms should be taught since these operations are basic to all arithmetical procedures. Also the commutative, associative and distributive laws, addition of zero, multiplication by one, and other properties used in these algorithms should be re-emphasized. It is important for the student to realize that in problems in which these operations were used, a, b, c, and d first represented the whole numbers, then fractions, decimals, rational and irrational numbers; hence they represent any real number. The computer algebra systems can work with integers, rational and irrational, algebraic symbols, and complex numbers. It is becoming increasingly less important to be able to compute long division, multiply complicated expressions, and to add fractions either mentally or on paper. All these problems can be more efficiently done on

the computer, hence it is more important to understand the steps being taken in the computation so as to check the correctness of the results.

The Caribbean Examination Council (C.X.C.) (1984) reported that students, on the Basic Proficiency examination, showed weakness in simplifying algebraic expressions such as $6a/5c + 4b/3d$. This indicates that the student did not grasp the concept of adding fractions, by first obtaining equivalent fractions with the same denominators and then adding the numerators.

$6a/5c + 4b/3d = 6a.3d/5c.3d + 4b.5c/3d.5c$

$= (18ad + 20bc)/15cd$

Learning the concept instead of solving, with paper and pencil, increasingly more difficult but similar problems which can be done on the computer seems more appropriate.

2. Some topics taught at the lower levels in disjointed parts should be brought together and re-emphasized, such as ratios, fractions and decimals which are all equivalent forms. The student should be able to convert to and from decimals, fractions, percentages and ratios.

The C.X.C. (1986) reports that on question 6 (Appendix D) most students attempted to find 2% and 3% of the gross income, but only about 20% gained more than half the possible marks. The graphic capabilities of the computer

allow one to easily show the relationship between ratios, fractions and decimals by simulating activities such as dividing a cake and sharing it among friends.

3. Measurement problems such as finding perimeter, area, and volume are very common in arithmetic, while finding the maximimum area and volume, given certain constraints, is common in algebra word problems. In most cases the type of figures used are limited because it is time-consuming to find the area of figures which are not square, rectangle, triangle, parallelogram, circle or any combination of these. These constraints do not exist on computer courseware such as the modules developed by Bork (1987). Here students can find the area of a blob on the computer screen by filling in squares where the size of the square can be adjusted so that the total area of the squares approximate very closely the area of the blob. If the student has learnt how to calculate the area of simple figures, it is no longer necessary to do many problems dealing with finding the area of a similar figure. The knowledge of simple measurement formulae; for example the area of rectangle, should be applied when finding the area of more realistic shapes such as that of a pond or lake. In this way, the student learns not only how to find areas, but also the difference between perimeter, area,

and volume, and the skills of approximation. Thus the problem of approximating the area of a circle with that of an polygon is a simple extrapolation of the ideas described before.

The C.X.C. (1984) reported that 85% attempted the question on finding the area of the given figure which required division into a rectangle and triangle, but only 5% gave good responses. The C.X.C. (1986) reported that finding the area of rectangles and squares, calculating the length of a side of a square given its area, and calculating the area of non-squares were topics in which the students were ill-prepared. The modules developed by Bork should give the student a better understanding of measurement in its simulation of realistic activities requiring measurement.

4. Algebraic manipulations- properties like exponents should be taught: (i) $a^n \times a^m = a^{(n+m)}$

(ii) $(a^n)^m = a^{(n \times m)}$

(iii) $a^{(1/n)} =$ the nth root of a

(iv) $a^{(-n)} = 1/(a^n)$.

Exponents, roots and factorization are all covered in arithmetic, but the student should be aware that the rules hold for any real number and any variable x. These three topics are used in finding the square root of a number

from its factorization. Any number can be written as a product of primes; if a prime appears more than once we can rewrite the factorization as a product of different primes to some power. If the exponents of its factors are even, then you simply use the properties of exponents to determine the square root. Suppose the prime factorization of M = A x A x A x A x B x B x C x C x D x D x D x D x D x D can be rewritten as

$$M = A4 \times B2 \times C2 \times D6 = (A2 \times B \times C \times D3)2$$

Therefore the square root of M is A2 x B x C x D3.

Algebraic manipulation develops logical and precise reasoning skills, and wherever necessary, exercises to obtain this skill should be performed. Such an exercise is recognizing patterns in when given a sequence; for example, given

$$1 + 3 = 4$$

$$1 + 3 + 5 = 9$$

$$1 + 3 + 5 + 7 = 16$$

find 1 + 3 + 5 + 7 + 9 + 11 + 13. These exercises not only develop some deductive and reasoning skills, but also tie together geometry and algebra.

Another form of algebraic manipulation is summation. Although it is an arithmetical procedure, when it is used in describing, with a set of variables, the pattern of a given sequence it becomes an algebra problem. Bork (1987)

says that logical reasoning skills must be taught because studies show that they do not develop naturally. Summation is also very important in statistics.

The C.X.C. reported that on the 1984 General Proficiency examination almost all the students attempted to solve a problem designed to test these skills, but only 10% scored half of the marks assigned. The students were also unable to work with negative fractional exponents.

5.   Factoring and solving easy equations where the distributive law is used should be taught, but factoring of quadratics and higher degree polynomials should not. The quadratic equation should be used to solve polynomials of degree two.

$$x = -b/2a + (b2 - 4ac)1/2$$

$$\text{or } x = -b/2a - (b2 - 4ac)1/2$$

This formula is important as it is used all through the mathematics programme at the university level.

The C.X.C. (1985) reported on the General Proficiency examination that about 20% of the students tried to solve the word problem involving a quadratic and a linear equation. 30% gave satisfactory responses; the others were unable to express the relationships algebraically even when given the symbols, and the students who obtained the correct equation could not

factor the expression. The examples below illustrate the type of problems done badly:

(a) If $\dfrac{8a - 1}{4} = 5$, solve for a.

(b) Simplify $6x - 2(x - 2) = ?$

(c) If $a = -3$ and $b = -5$, find $a - b$.

6. In order to solve equations, the computer will ask for the function so the student should know the difference between an equation and a function. The computer, when given a function, gives a table of pairs, (x,y), of values. Therefore it is not important for the student to manually complete a table of (x,y) values. It is, however, important for the the student to be able to give the roots, intercepts of the equation, and the performance of the graph for increasing and decreasing values of x. With this information the student can sketch the graph, or determine the correctness of the graph done on a computer. The student should not spend much time sketching numerous graphs with paper and pencil, but use the time developing the skill of describing what they should look like, and what are its minimum and maximum points, roots, domains and ranges and many other properties. The time used in sketching graphs could then be used in solving word problems.

The C.X.C. (1985) reported that on the General Proficiency examination one of the questions tested the student's ability to sketch quadratic graphs and make inferences from it; two-thirds of the students attempted this question, but only one-quarter of those who attempted the question gave satisfactory responses. The main difficulties were in solving the equation and in reading the graph. The C.X.C. (1986) also reported that many students were unaware of the characteristics, such as maximums and minimums, of the graph of a quadratic function, and the relationships between the axis of the parabola and the roots of $f(x) = 0$.

The use of the computer for the calculations imposes on the student the need to understand the relationships and characteristics inherent in the problems rather than obtaining values with no meaning.

7. Graphing of linear equations and inequalities.

Since the computer graphic system will be used in most cases to produce graphs later on, the student only needs to spend a small amount of time on paper and pencil graphing in order to understand the Cartesian co-ordinate system.

General Algebraic Topics

To determine whether the topics should remain in the curriculum or how much emphasis should be placed on the topic, the following questions will be asked: Why is this taught? Is the topic useful in the future? Can we solve real world problems with the information? Will the information be required only by a selected number of students? The topics which appear to be necessary for higher algebra or calculus, for solving problems which occur in the 'real world', and for developing in students skills such as deductive and logical reasoning ability will be listed in the following pages.

1. <u>Bases</u>

Computers, other than computer algebra systems, use binary operations. Therefore to understand the operations of the computer, the student should be able to work in different bases, to convert to and from base ten and other bases with some competency. The computer readily changes from one base to another as described earlier. For example, to convert 497 from base ten to base six:

$$497 = (82 \times 6) + 5$$

$$82 = (13 \times 6) + 4$$

$$13 = (2 \times 6) + 1$$

$$2 = (0 \times 6) + 2$$

$$497 = 2145_{six}.$$

Similarly we can describe an algorithm for converting from any base back to base 10. First obtain the digits of the number, a1, a2, a3,... a(N+1) and the base z. If the number has N+1 digits, then evaluate the polynomial

$$a1.z^N + a2.z^{(N-1)} + \ldots\ldots\ldots + aN.z + a(N+1).$$

This topic is inadequately covered in arithmetic, but it is considered important for the binary notation system. An important consequent of teaching bases is that it gives the student a good grasp of the place value numeration system which is not understand when counting is learnt by rote.

The C.X.C. (1986) reported that in the Basic Proficiency examination students showed a lack of understanding of binary operations, and in converting from base five to base ten; in the General Proficiency examination students found it difficult to write a number in expanded form in the base eight numeration system.

2. Greatest Common Divisor

Euclid's Algorithm for finding the greatest common divisor of two integers is an important topic because it forms the basis of factoring and division of polynomials.

$$a = (b \times q1) + r1$$

$$b = (r1 \times q2) + r2$$

$$r1 = (r2 \times q3) + r3$$

$$r2 = (r3 \times q4) + r4$$

This procedure continues until the remainder r is zero, and this procedure will stop because there exists a decreasing sequence of remainders r1, r2, etc.. Then we use the property that if an integer k divides M and k also divides N, then k divides (M+N).

3. Polynomials

This topic is considered to be important since it plays a very important role in higher mathematics.

(a) Algebraic sum of polynomials

Let $A(x)$ and $B(x)$ be polynomials of degree n and m respectively, where m and n are whole numbers. The next step, starting with the highest power, add coefficients of x with the same powers. All the computer algebra systems perform this operation, therefore it is important only to understand the steps in the procedure, and to practice with polynomials of degree at most four.

The computer algebra systems available perform the addition of polynomials when simplifying an expression, or when finding the roots of an equation. It is not important to perform many such exercises, but the student

should understand, and be able to describe the steps in the addition algorithm used in the courseware. In many cases the student does not understand what steps should be performed, especially when the coefficients are rational. Since factoring of polynomials of degree higher than two is not recommended, exercises in manipulating polynomials should not contain a degree higher than three or four. The use of the computer for these exercises, and shifting the emphasis to understanding the steps' encourages the development of precision and logic in thinking.

(b) Multiplication of polynomials

Multiplication of polynomials is an extension of multiplication using the distributive law. It is only necessary to perform multiplications such as,

$a(x + y) = ax + ay$, $x(ax + b) = ax^2 + bx$, and

$$(ax + b)(cx + d) = acx^2 + bcx + adx + bd$$
$$= acx^2 + (bc + ad)x + bd.$$

The product of polynomials $A(x)$ and $B(x)$ is another polynomial, say $C(x)$, of degree $n+m$. A special case of multiplication of polynomials is expanding $(x + y)^n$. The coefficients of $(x+y)^n$ can be obtained from Pascal's Triangle.

Since
$$\binom{n}{m} = \frac{n!}{m! \ (n-m)!}$$

then
```
                    1

                1       1

            1       2       1

        1       3       3       1

    1       4       6       4       1
```

For example, $(x + y)^2 = x^2 + 2xy + y^2$. This method of multiplying polynomials is used in many computer programmes, and should therefore be introduced to the students. In most cases in problem solving, the student does not require a knowledge of solving equations of degree higher than two, so it is inappropriate to spend valuable class time manipulating polynomials of higher degree.

(d)   Division of polynomials

The C.X.C. reported that on the General Proficiency examination students were unable to identify which of $a^2 + b^2$, $a^2 - b^2$, $(a - b)^2$, or $2a + b^2$ contained the factor $(a + b)$. The ability to divide simple polynomials, a quadratic equation divided by a linear, is necessary since the Advanced level examinations

require the student to sketch the graphs of rational
functions. If the students cannot obtain the graphs on a
handheld computer, a knowledge of division is needed to
find asymptotes in the graphs. However since such
computers do exist, it becomes less important as the
computer becomes more available for the student to perform
this operation. Therefore division of polynomials should
only be taught as as an extension of the Euclidean
algorithm method, for finding the greatest common divisor;
a procedure sometimes used by the computer to carry out
this operation.

$A(x) / B(x) = C(x)$ with remainder $R(x)$

$A(x) = B(x).C(X) + R(x)$

if the degree of $A(x) = m$, and degree of $B(x) = n$, then
degree of $C(x) = m-n$, and degree of $R(x) <$ or $= n-1$. For
example, if $A(x) = x^6 + x^4 + x$

$B(x) = x^2 + x$

then $C(x) = ax^4 + bx^3 + cx^2 + dx + e$

and $R(x) = fx + g$

(e) Greatest common factor of two polynomials
By extending the idea of division one step further, we can
find the greatest common factor of two polynomials $A(x)$
and $B(x)$.

$$A(x) = B(x).Q(x) + R(x)$$

$$B(x) = R(x).Q1(x) + R1(x)$$

$$R1(x) = R1(x).Q2(x) + R2(x)$$

$$\vdots \qquad\qquad\qquad \vdots$$

$$\vdots \qquad\qquad\qquad \vdots$$

degree of R2(x) < degree of R1(x) < degree of R(x), hence the algorithm stops when R(x) = 0.

(f) Finding the value of a polynomial at a point x = z.

One method of finding the value of a polynomial $A(x)$ at a point x = z is by using Horner's algorithm. This question can also be rephrased as a division problem: divide $A(x)$ by (x - z). The first step in Horner's algorithm is to write the polynomial in descending degrees of x, placing a zero wherever some power of x is missing. Now write the coefficients of the polynomial on the first row a, b, c, etc.. In the first place of the third row write the same value which appears in the first place of the first row, that is a = A. In the second place of the second row write the product of A and z, and in the second place of the third row write the sum of A.z + b = B. Now place B.z in the third place of the second row, and B.z + c = C in the third place of the third row. You will finally obtain a polynomial with coefficients A, B, C,...

and degree one less than the degree of the polynomial $A(x)$. For example, $A(x) = x^3 + 4x^2 - 3x + 7$ at $x = 2$

$$
\begin{array}{r|rrrr}
 & 1 & 4 & -3 & 7 \\
2 & & 2 & 12 & 18 \\
\hline
 & 1 & 6 & 9 & 25 \\
\end{array}
$$

Therefore $C(x) = x2 + 6x + 9$ and $R(x) = 25$. When $R(x) = 0$, the value $x = a$ is a root of the equation.

Formulating such a problem helps the student to develop the ability to set-up flowcharts, a set of logical and sequential steps, to approximate answers, and to understand the steps taken by the computer to produce a solution. In most cases the ability to find the roots of an equation and to sketch the equation is necessary only for doing well on an examination; an understanding of the steps is more appropriate to current technology.

Word Problems and Problem Solving

4. Algebra is essential in solving word problems, but a common argument used by students to explain their boredom in mathematics is that word problems such as age and mixture are not realistic, and are of little use in assisting them to cope in society. The computer allows realistic simulation of word problems; for example, TRIP,

described by Gould and Finzer (1981), is a computer system for animating time-rate-distance problems. TRIP was designed for students who can solve algebraic equations, but have difficulty setting-up the equation. VISICALC, an electronic spreadsheet for solving problems involving price, sales, cost, revenue and profit, and TK Solver, an equation solver, both designed by Software Arts are also available.

Another example is one described by Kelman (1983) which is available on the microDYNAMO courseware:

Rate = K (temperature of coffee - temperature of room) is the formula for determining how long it would take for a cup of coffee to cool to some drinkable temperature. Bork and Trowbridge (1987) also describe learning modules, using socratic dialogue written as programs in Pascal on the Terak 8510 microcomputers, which aid in problem solving. Not only is it important to be able to formulate and solve realistic word problems, it is also important to understand formulae in a problem solving context. Kelman says, "It forces the student to consider what effects different parts of the problem have on each other, and it allows them to consider those problems; which it makes no sense to talk about". (p. 83). For example, in finding the square root of a number, it would make no sense to

talk about square roots of negative numbers, but one could talk about cube roots of negative numbers. The microcomputer makes available a new kind of problem in an appropriate problem-solving environment. The ability of computer algebra systems to allow students to express a question in symbolic form, and for the computer to calculate the answer in such a form, enables the student to ask "what if" questions better than if a specific solution was obtained. It is more important to search for variations, exceptions and standard behaviours than to obtain an answer.

The following question appeared on the General Proficiency 1986 examination:

> The graph provided shows the fall in
> temperature of a body after it was put
> into a special container to cool.
> (a) Using the graph, estimate
> (i)   the temperature of the body four minutes
> after it was put into the container
> (ii) the average rate of cooling of the body,
> in degrees per minute, for the first twelve
> minutes, after being put into the container
> (iii) the rate of cooling of the body when
> its temperature was 64 degrees.
> (b) Assuming that after 20 minutes, the
> temperature of the body continues to fall at
> a constant rate, estimate the temperature of the
> body 1 hour after being put into the container.

The C.X.C. reported that 96.2% of the students attempted this question intended to test their ability to read and

interpret information presented graphically. Only 10% of the answers were rated satisfactory or better.

The C.X.C. (1986) reported that on both the Basic and Proficiency examinations students were unable to find the cost price given the selling price and percentage profit, and they had little knowledge of the terms 'gross income' and 'taxable income' in simple problems.

5.   Functions, Relations and Plotting of Functions

All the above concepts will be utilized in solving equations and the plotting of functions. An algebraic equation $f(x) = 0$ of degree n has n roots. Now suppose $x = a1$ is a root of $f(x)=0$, then

$$f(x) = (x - a1).f1(x) \qquad \text{degree of } f1(x) = n-1$$

$$f1(x) = (x - a2).f2(x) \qquad \text{degree of } f2(x) = n-2.$$

$$\vdots$$

$$\vdots$$

$$fN(x) = (x - a1).(x - a2).(x - a3) \ldots (x - aN).A$$

where a1, a2, a3, etc. are real or complex numbers, and A is constant.

The student should be able to recognize a functional relationship in a word problem, and to express the relationship as a mathematical formula. Also, because of the importance of statistics, the student should be able to give a suitable function when given a plot of some

experimental data. There are many function plotting programs available. Where it would take at least a few class sessions to discuss the differences in the graphs:

$$y = f(x) \qquad\qquad y = f(-x \qquad\quad y = 2\ f(x)$$

$$y = f(2x) \qquad\qquad y = f(x - 2) \qquad y = f(x) - 2$$

$$y = f(x + 3) \qquad\quad y = f(x) + 3 \qquad y = 2\ f(x - 2) + 3,$$

it takes only a couple of minutes to display these graphs on the screen. Hence the notion of functions, equations and plotting graphs should be taught in the high schools, but the student should not spend 5 to 6 weeks of class time learning how to set up the function, graph it and then describe its maximum, minimum, roots, etc.. The student not only learns algebra, but also some geometry; such as shifting, expanding, shrinking and other transformations. With the computer, in the case of quadratics, knowing the quadratic formula and applying it to obtain the roots, and being able to describe the function, and finally checking your image of the graph with one on a computer screen would free up lots of time to work more applications and to ask more questions about the dynamics of the function. Also the word problems should be more varied than the usual projectile and profit and loss problems. Fey et al. (1985) say of the computer in the classroom,

> ... It begins with real-world situations and
> promises the best motivation: learning
> something obviously useful. It sidesteps the
> almost unsolvable question of 'How much
> technique is enough?' by allowing students to
> progress as far as their needs and interests
> permit. It stresses informal easy-to-remember,
> and powerful successive approximation and
> graphic methods that should do much to build
> the intuition our students so often miss in
> our haste to teach manipulative rules for the
> many algorithms required by formal methods. (p.10)

The C.X.C. (1984, 1985, and 1986) reported that students showed weakness when using functional notation on both the Basic and General Proficiency examinations. The advanced level examination requires that the student should be able, when given a quadratic equation in one variable, to solve the equation, to find maxima and minima by completing the square, and to sketch the graph. All these operations can be performed by the computer, and the students should be able to perform these operations, but rather than focusing on performing these operations the time should be spent applying the knowledge to solving more 'real world' problems.

6. Matrices and Solving Linear Equations

The Advanced level mathematics examination requires that the student use Guassian elimination when solving a system of linear equation. It is an elegant method, and one which the computer uses in solving a system of linear

equations. Basic matrix arithmetic, such as addition, subtraction, scalar multiplication, and multiplication of matrices is used in solving linear equations.

(a) Addition of Matrices

Suppose A and M are two m x n matrices, then A + B is also an m x n matrix with each of its entries being the sum of the corresponding entries in A and B.

(b) Subtraction

Similarly, A - B is an m x n matrix with each entry being the difference of the corresponding entries in A and B.

(c) Scalar Multiplication

If c is a constant and A an m x n matrix, then cA is an m x n matrix where each entry in A is multiplied by c.

(d) Multiplication

Given two matrices A, an m x n matrix, and B, an n x s matrix, then A x B is an m x s matrix. For example,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 1 \times 2 + 2 \times 1 + 3 \times 0 & 1 \times 3 + 2 \times 2 + 3 \times 1 \\ 3 \times 2 + 2 \times 1 + 1 \times 0 & 3 \times 3 + 2 \times 2 + 1 \times 1 \end{bmatrix}$$

The student should be able to solve a system of linear equations in at most four variables. Solving of a system of linear equations is important in many

application problems, but there many calculators available which make hours of paper and pencil manipulation unnecessary. One needs only to know how to solve a system of linear equations in two unknowns to understand the many steps taken by the calculator to obtain an answer. Let us take three equations in three unknowns

x, y, z.

(1) $ax + by + cz - d = 0$

(2) $ex + fy + gz - h = 0$

(3) $ix + jy + kz - l = 0$

For example (1) $3x + y + 2z = 13$

(2) $x + y - 8z = -1$

(3) $-x + 2y + 5z = 13$

(1) - (2) = (4): $2x + 10z = 14$

(2) - (3) = (5): $3x - 21z = -15$

3 x (4): $6x + 30z = 42$

2 x (5): $\underline{6x - 42z = -30}$

$72z = 72$

$z = 1$

Substitute z = 1 into equation (4): $2x + 10 = 14$

$2x = 4$

$x = 2$

Substitute $x = 2$ and $z = 1$ into equation (1): $6 + y + 2 = 13$ to obtain $y = 5$. (The HP 28C handheld computer gives a solution for this system of linear equations in 10 seconds.)

Matrix operations are important for the study of higher mathematics, but the ability to solve a system of linear equations is necessary to solving 'real world' problems; therefore both topics should remain in the curriculum. However, finding inverses and determinants of matrices seems inappropriate with the available technolgy.

7. Trigonometry

Since this topic is usually introduced in the high schools (in Algebra 11 and 12 or the last two years of high school), and incorporates geometry, measurement, and functions and relations; it is an essential area of mathematics which should not be omitted. Trigonometry is required not only at University level, but for any applied mathematics course since it is useful even to the carpenter. Trigonometric concepts evolve from both arithmetic and geometry. For example, given a circle which has 360 degrees and any sector or fraction of the circle you can find the angle which the arc subtends or given a right angle triangle you can introduce the six basic trigonometric functions: sine, cosine, tangent,

secant, cosecant, and cotangent. Now that we see that trigonometry is an important link in the chain of basic mathematical concepts, we have to decide what are the basic concepts in trigonometry and what can be now done on the computer. The HP 28C displays the graphs of trigonometric functions in less than 30 seconds. Many calculators operate in both degrees and radians, and to distinguish between the two one must understand the relationship (in muMath the angles are assumed to be measured in radians).

(a). Relationship between degrees and radians.

A radian is that angle A subtended at the centre O of a circle by an arc of length equal to the radius.

2.#PI radians = 360 degrees

1 radian = 180 degrees / #PI = 57.295779513..

The student is required to be able to prove trigonometric identities and to simplify trigonometric equations because these skills are required in calculus to sketch the graph of trigonometric functions; he or she is also required to set-up an expression in a form which makes it easy to differentiate and integrate. The ability of the computer algebra systems to prove trigonometric identities, to simplify trigonometric expressions, and to differentiate and integrate complicated trigonometric

functions make it less important to perform these operations with paper and pencil. Not only is it time consuming, but it does not take advantage of the available technology. The time can be better spent in understanding that the derivative is the slope of the tangent line to the graph, and can be used to find maximum and minimum points.

8. Logarithms, Logarithmic and Exponential Functions

It is important to tie together the relationship between exponents and logarithms so that it becomes an intuitive and natural step in algebra. This topic is required in higher algebra in order to solve problems in population growth and radioactive decay. Therfore it is necessary for high school students to understand the basic operations. Graphs of both logarithmic and exponential functions are easily obtained on computers, and problem solving courseware for these type of problems will be available in the future. Therefore only the basic operations should be taught if necessary. If $y = x^a$,, then $\log_x = a$.

So one would expect the properties of logarithms to be similar to the properties of exponents:

(i) $\log xy = \log x + \log y$

(ii) $\log x/y = \log x - \log y$

(iii) $\log (x^a) = a (\log x)$

9. <u>Differentiation and Integration</u>

Although differentiation and integration are not done at the high school level in North America, it is a requirement of the Advanced level syllabus. Engelman (1970) describes another symbolic computation system, MATHLAB, as follows: "The program [has] genuine knowledge; if asked 'How will you differentiate this?" (p. 103). The program will respond by performing the requested differentiation, pausing at each step to explain what it is doing. MuMath can verify trigonometric identies, calculate limits, and perform indefinite integration for which partial fractions are commonly used. Therefore it is becoming more important to be able to perform any other than simple differentiation and integration.

Summary

We have described those topics we believe to be important and which should be taught to take advantage of the available technology. However, it is not expected that the computer will take the place of all paper and pencil tasks, but that the student should obtain the manipulative skills necessary to understand mathematical concepts, and be able to determine the reasonableness of

the answer obtained on the computers. Developing in students a better understanding of the mathematical concepts is more important than doing many exercises or learning by rote. Also the topics should not be introduced in pieces at different class levels, but in whole and integrated parts; each topic tied to the other so that there is a natural and intuitive approach to the next topic.

With the computers available, only that amount of calculation which is necessary to obtain some manipulative skill will be done with paper and pencil. At the same time the basic concepts will be introduced to the student so that the student will relate the problems to all problems of that type, and computer displays and graphs will be used to aid in understanding the topic.

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APPENDIX A

Objectives of the Basic Proficiency

Examination

Objectives of the Basic Proficiency Examination

The Basic Proficiency examination consists of questions on nine topics: sets, relations and graphs, computation, number theory, measurement, consumer arithmetic, statistics, algebra and geometry. A copy of the curriculum could not be obtained, and the specific objectives, of each of the nine topics given below, were determined from copies of the actual examinations from different years.

The specific objectives (S.O.) for Sets are:

1. Describe a set, and give examples of well-defined sets,

> Example: The set of numbers greater than -3 and less than +5 can be symbolically represented by
> (A) (n: n < -3 & n > 5)
> (B) (n: -3 < n < +5)
> (C) (n: n > -3 & +5 < n)
> (D) (n: -3 > n > +5).

2. List the members of a set from a given description;

3. Use set notation to describe a set and its subsets;

4. Identify equivalent and equal, and define the difference between the two;

5. Describe the differences between finite and infinite sets using one-to-correspondence;

6.   Calculate the number of subsets of a set of n

elements;

7.   Find the complement, union and intersection of given

sets,

   Example: On square paper where 0 is the origin
   and A is the point (5,0), show by careful drawing
   and shading the following sets:
         Q = (E: 0E< or = 3 units) and
         R = (F: AF< or = 4 units)
   (i) Clearly indicate, by shading, the region which
   represents Q intersection R.
   (ii) List five points (with integral coordinates)
   which are members of Q intersection R.

8.   Construct and use Venn diagrams to show subsets,

complements, intersection and union of sets, and

solve problems involving not more than three sets.

   Example: In a class of 35 students 25 do
   Integrated Science, 20 do Agriculture and 6 do
   neither Integrated Science nor Agriculture.
   (i) Let x represent the number of students who
   do both Integrated Science and Agriculture.
   Draw a Venn diagram to illustrate the data
   given by putting in the appropriate regions
   the number of students taking only one, both,
         or   neither   of   the   two   subjects.

   (ii) Form a suitable equation in x and solve
   it to find the number of students who do both
   Integrated Science and Agriculture.

   The specific objectives for Relations and Graphs

are:

1.   Recognize a relation;

   Example: A man had $100.   He went to a meatshop,
   a bookstore and a drugstore.   He spent three
   times as much money at meatshop as he did at the

drugstore. He spent $12 less at the bookstore
than at the drugstore . He then had $37 left.
(a) Using $x to represent the amount he spent
at the drugstore, express in algebraic terms
   (i) the amount he spent at the meatshop
   (ii) the amount he spent at the bookstore
(b) Obtain an equation for the total amount
of money spent and hence calculate the amount
he spent at the drugstore.

2. Give the set of ordered pairs which satisfy a

   relation;

   Example: Complete the table for the function
   $y = x^2 + 2x - 2$

x =	-4	-3	-2	-1	0	1	2	
y =	6	1		-3				

3. Use arrow diagrams to show relations;

4. Describe a function as a 1 to 1 mapping;

5. Use set notation to describe linear functions;

6. Use the functional notations and draw graphs to show

   variations; e.g. $f:x \rightarrow x^2$, or $f(x) = x^2$ and

   $y = 2x^2$;

   Example: Jean has twice as many 10c coins as
   25c coins in her purse. She has less than $2.00
   in coins. Let t be the number of 10c coins and
   q the number of 25c coins.
   (i) Write down two algebraic statements to
   represent the relations (a) between the number
   of each type of coin; and (b) the total value
   of each type of coin and the total amount of
   money Jean has.
   (ii) On a sheet of squared paper draw draw
   graphs to represent these two relations.
   (iii) List all the possible pairs of numbers of
   10c and 25c coins which Jean could have in her
   purse.

7. Draw a bar chart, pie chart, histogram, frequency

   polygon, and the rectangular Cartesian graph of a set

   of data;

   Example: One month Mr.Ragbeer spent some of his
   income on food (F), housing (H), and
   entertainment (E). He saved the rest (S). The
   chart illustrates how he used his month's income
   of $720.00. About how much did he save?
                 (A) $300.00
                 (B) $240.00
                 (C) $90.00
                 (D) $45.00.
8. Interpret data represented in pictorial form;

9. Draw and use graphs of linear and quadratic

   functions;

10. Find the gradient of a line as the ratio of the

    vertical rise to the horizontal shift;

    Example:
    (a) Using a scale of 1cm to represnt 1 unit
    on each axis, plot on graph paper the points
    P(2, -1) and Q(-2, 5).
    (b) Calculate the gradient of PQ.
    (c) Determine the point where PQ meets the y-axis.
    (d) Write down the equation of PQ in the form
                  $y = mx + c$.
    (e) Hence or otherwise determine the solution of
                  $3x - 2y = 4$
                  $x - y = 1$
    (f) Shade the region of y greater than or equal
    to (x - 1) for x greater than or equal to zero.
11. Solve problems dealing with exponential growth such

    as population growth, multiplication of bacteria, and

    compound interest;

12. Read and interpret graphs of exponential functions;

12. Read and interpret graphs of exponential functions;

13. Use graphs to find the solution set of a system of linear equations;

14. Find the zeroes of quadratic equations;

15. Draw graphs to represent inequalities in one or two variables, and use them to find the solution sets of problems;

The specific objectives of <u>Computation</u>.

1. Perform any of the four basis operations with rational

numbers;

Example: Which of the following is the best approximation of (575 x 29) / (4.95 x 2.78)?
(A) 1600
(B) 1200
(C) 1000
(D) 800

2. Convert from one set of units to another, given a conversion scale (e.g. convert local currencies to those currencies most commonly encountered in the Caribbean);

3. Solve problems involving fractions, decimals, percentages, ratios and rates, and be able to convert fractions to decimals, or percentages, ratios and

rates

and vice versa;

Example: Calculate the following:
(i) $19.97×17
(ii) 10% of $524.00
(iii) 125 x 0.063 x 8 x 199

4. Estimate the result of a computation and contruct a

range in which the exact value must lie;

5. Approximate a value to a given number of significant

figures and express any decimal to a given number of

decimal places;

6. Write any rational number in the standard form

(scientific number);

The specific objectives for Measurement.

1. Calculate the perimeter of a polygon and the

circumference of a circle, and their combinations

given the necessary measurements;

2. Calculate the length of an arc of a circle subtended

angles which are factors of 360 degrees;

3. Calculate the areas of rectangles, triangles,

parallelograms, trapeziums, circles and their

combinations;

Example: Construct parallelogram ABCD such that
AB = 6.5cm, BC = 4cm, and the angle DAB = 36
degrees. By making the necessary measurement
calculate the area of the parallelogram giving
your answer to the appropriate degree of accuracy.

4. Calculate or estimate the areas of irregularly shaped figures;

5. Calculate the areas of sectors of circles;

6. Calculate the surface areas and volumes of simple right prisms and pyramids;

7. Calculate the surface area and volume of a sphere;

8. Use correctly the S.I. units of measure for area, volume, mass, temperature, and time;

9. Estimate the margin of error for a given measurement;

Example: A sportsmaster timed one of his athletes over the 100 metres dash. The athlete took 11.5 seconds by his watch to cover the distance. It is believed that the actual time taken is in the range (11.5 + or - 0.1) seconds and that the length of the track in the range (100 + or - 1) metre. Use table or slide rule, to calculate the average speed, correct to 3 significant figures, of the runner, if
(i) the track was actually 99m long and he took 11.6s,
(ii) the track was 101m long and the athlete took 11.4s,
(iii) express the range within which the average speed of the runner lies in (a+ or - b)m/s.

10. Make suitable measurememts on maps, or scale drawing, and use them to determine distances and area;

11. Solve word problems involving measurements;

The specific objectives for Consumer Arithmetic.

1. Calculate profit and loss as a percentage;

2. Calculate discount and sales tax when these are given
    as a percentage;

3. Calculate marked price when cost price and percentage
    of profit, loss, or discount are given;

4. Calculate payments by installments as in simple cases
    of mortgages etc.,

    Example:
    (i) A refrigerator can be bought on
    hire-purchase by making a deposit of $480
    and 15 monthly instalments of $80 each.
    Calculate the hire-purchase cost of the
    refrigerator.
    (ii) The actual marked price of the
    refrigerator is $1 400. This includes a
    sales tax of 12%.
    Calculate the sale price of the refrigerator
    if no sales is included.

5. Calculate tax, rates and bills from instructions;

6. Calculate simple interest, depreciation, and compound
    for not more than three periods;

7. Solve   problems   involving   measures   and   money
    (includingexchange rates);

    Example: The Rates of Exchange at a bank
    are as as follows: EC$1.00 = BD$0.75
                  and US$1.00 = BD$1.98
    (i) A traveller changed EC$1 600 to
    Barbados currency. Calculate the amount
    received.
    (ii) Of the amount she received she spent
    BDS$210 and exchanged the remainder for US
    currency. Calculate the amount in US
    currency she received for this exchange.

8. Calculate returns on different types of investments;

Example: $5 000 was put in a fixed deposit
account on 1 January, 1984 for one year.
Calculate the total amount received at the
end of the period, if the rate of interest
was 12.5% per annum.

9. Solve word problems involving simple interest,

compound interest, and depreciation.

.Example: A man borrows $5000 from a bank
at 4.5 % compound interest for 5 years. He
repays the loan and interest in 60 equal
monthly installments. If the bank calculates
the total interest due in the following manner:

Interest on $5000. for 1 year

+

Interest on $4000. for 2 year

+

etc.

+

Interest on $1000. for 5 years;
then use the table to determine:
(i) The total interest due on the loan at the
end of 5 years;
(ii) The amount the man repays each month, correct
to the nearest cent.

The specific objectives for <u>statistics</u>.

1. Construct a simple frequency table from a given set

of data;

2. Given class size, determine class interval,

boundaries and class limits from a given set of data;

3. Construct a group frequency table from a given set of

data;

4. Draw and use pie charts, bar charts, line graphs,

histograms and frequency polygons;

5.˙ Determine mean, median, and mode for a given set of
data;

6. Determine when it is most appropriate to use one of
the measures of central tendency as the average for
a set of data;

7. Determine the range, interquartile, and semi-quartile
range for a set of data;

8. Use diagrams and tables to represent the outcomes of
ideal experiments;

9. Determine relative frequency and probabilities of
simple events;

Example: A table of random numbers is opened
and a 4-digit numeral is chosen. What is the
probability that the number is exactly divisible
by 5?   (A) 0.1
          (B) 0.2
          (C) 0.4
          (D) 0.5
10. Predict the expected value of a given set of
outcomes;

The specific objectives for Algebra.

1. Use symbols to represent numbers, operations,
variables   and   relations   (comutativity,
associativity);

2. Perform the four basis operations with algebraic
expressions;

3. Substitute numerals for algebraic symbols in simple algebraic expressions;

4. Translate verbal phrases into algebraic symbols and vice versa, and solve word problems;

   Example: One side of a rectangle is 5cm long and another side is x cm long. If the area of the rectangle is greater than 54 cm squared find the set of values of x and the least integral value for x.

6. Find the solution set of linear equations and inequalities in one unknown;

7. Apply the distributive law to insert or remove brackets in algebraic expressions;

8. Simplify fractions of the form aw/cy + bx/dz where a, b, c, and d are integers and w, x, y, and z can be integers or variables;

9. Use the laws of indices to manipulate expressions with integral indices;

10. Solve simultaneous linear equations algebraically;

    Example: Solve the simultaneous equations
    $$x + 3y = 5$$
    $$2x + y = 5$$

11. Use linear equations and inequalities to solve word problems;

12. Use symbols to represent binary operations, and perform simple computations with them;

The specific objectives for <u>Geometry</u>

1. Use protractor and ruler to draw and measure angles and line segments;

2. Use instruments (not necessarily restricted to ruler and compass) to construct polygons and circles;

3. Use the properties of rays, perpendiculars, and angles to draw accurate geometrical figures;

4. Specify translations in a plane as vectors, written as column matrices, and recognise them when specified;

5. Specify what are the relations between an object and its image in a plane when it is rotated about a point in that plane;

> Example: Triangle LMN with coordinates (1,2), (2,-1), (3,3) respectively is rotated through 90 degrees in an anticlockwise direction about the origin. The image of triangle LMN is triangle L'M'N'. What are the coordinates of L', M', and N'?

6. State what are the relations between an object and its image in a plane when reflected in a line in that plane;

> Example: Triangle PQR is rotated through 90 degrees about P in an anticlockwise direction. The transformation maps P onto P', Q onto Q', R onto R'. Which of the following statements are true?
> (A) Q=Q' (B) RQ perpendicular to R'Q'
> (C) Q'P' is parallel to QP   (D) RP perp.to R'Q".

7. State what are the relations between an object and
    its image in a plane under a given enlargement in
    that plane;

8. Identify a transformation in those listed in 4 to 7
    above given an object and its image;

    Example: T is a transformation which maps
    triangle LMN onto triangle L'M'N', such that
    L(-4,2) -> L'(-2,4); M(-2,5) -> M'(-5<2);
    and N(-2,2) -> N'(-2,2).
        (A) rotation through +90 degrees about N
        (B) reflection in the line y=x
        (C) rotation through -90 degrees about N
        (D) reflection in the line y = -x.
9. Identify simple plane figures possessing translation,
    bilateral and rotational symmetry;

    Example: If PQRS is any rectangle such that PQ
    is not equal to QR then PQRS has two of the
    following properties:
        I.   Rotational symmetry of order 2
        II.   Rotational symmetry of order greater than 2
        III. No axes of bilateral symmetry
        IV.   Two axes of bilateral symmetry
        V.   Four axes of bilateral symmetry
    Which are the properties: (A) II and V
    (B) I and III   (C) I and IV   (D) II and IV.
10. Recognise and apply:

    (a) the properties of polygons and circles

    (b) the properties of congruent triangles

    (c) the concept of similarity to geometric figures;

11. Use the properties of the faces, edges and vertices
    of   simple   solids   to   draw   two   dimensional
    representations of these solids;

12. Use Pythagoras theorem to solve simple problems (no formal proofs required);

13. Detemine the sine, cosine and tangent of acute angles in a right-angled triangle;

14. Use simple trigonometrical ratios to solve problems based on measurements in the physical world, e.g. heights of buildings, angles of elavation and depression, and bearings (Tables of formulae will be provided);

APPENDIX B

<u>Objectives of the General Proficiency</u>

<u>Examination</u>

Objectives of the General Proficiency Examination

The specific objectives for <u>Sets</u>.

1. Apply the results $n(A \cup B) = n(A) + n(B) - (A \wedge B)$

   in the solution of simple problems;

2. Solve problems arising from the intersection of not

   more than three sets;

3. Use Venn diagrams to represent word problems from

   which   conclusions can be made;

    Example: In President's Secondary School the
     students study many subjects including physics,
  mathematics, history, literature.   If a   student
studies physics then he studies mathematics.
 A student studies history if and only if that
  student does not study physics. Every one who
  studies history also studies literature.
  Let U = (students and President's Secondary School)
  H = (students who study history)
  L = (students who study literature)
  M = (students who study mathematics)
  P = (students who study physics)
  (Two possible Venn diagrams are given)
  Using the information given above and the
  diagrams given, briefly explain your decision
 for each of the five statements:
   I. All physics students study mathematics;
  II. Some history students study mathematics;
 III. Some literature students study history;
    IV. Some students study neither physics nor
history;
   V. All literature students study history.

The specific objectives for <u>Relations and Functions</u>

1. Recognise simple functions which have inverses;

2. Recognise the differences between functions defined by the same formula for different domains;

3. Interpret and use of the functional notation, e.g.f(x), g(x);

   Example: Given that f, g are functions such that
   f: x -> x^2 + 1,g: x -> 3x - 2, find
   (a) gf(1)   (b) fg(1)   (c) ff(-2).

4. Use graphs of a given function to determine:

   (a) the elements of the domain which have a given image   or vice versa,

   (b) the interval of a domain for which the elements the range may be positive or negative,

   (c) the interval of a domain for which the elements of the range may be greater than or less than a given value,

   (d) the roots of a given function,

   (e) the maximum and minimum values of the function over a given interval of the domain;

   Example: The graph provided shows the fall in
   temperature of a body after it was put into a
   special container to cool.
   (a) Using the graph, estimate
   (i) the temperature of the body four minutes
   after it was put into the container
   (ii) the average rate of cooling of the body,
   in degrees per minute, for the first twelve
   minutes, after being put into the container
   (iii) the rate of cooling of the body when
   its temperature was 64 degrees.
   (b) Assuming that after 20 minutes, the

temperature of the body continues to fall at a
constant rate, estimate the temperature of the
body 1 hour after being put into the container.

5. Use graphs of functions to solve simple problems;

Example: A farmer wishes to buy some goats and
cows. He has pasture for only 50 animals. Goats
cost $300 each and cows $600 each. He can expect
to make a profit of $250 on each goat and $350 on
each cow.
(a) If he has $24 000 to spend and he buys g
goats and c cows, obtain TWO inequalities
connecting g and c.
(b) Illustrate these inequalities on the same
graph.
(c) Determine how many of each type of animal
he should buy to obtain a maximum profit.

6. Find rate of change by determining the gradient of a
linear function;

7. Recognise the relation between a certain angle and
the gradient of a curve;

Example: The speed v km per hour at which a
train is travelling after t hours is given by
the equation
$$v = 5t + 6.$$
(i) Calculate the speed when t = 0.
After 2 hours the train reaches a speed of w km
per hour and maintains that speed for 1 hour.
(ii) Calculate the value of w.
The train accelerates again and its speed is
now given by the equation
$$v = 5t + 1, \text{ when } t > \text{ or } = 3.$$
(iii) Draw the speed/time graph of the journey
for t between 0 and 8 using 2 cm to represent
1 hour along the horizontal axis and 2 cm to
represent 5 km pr hour along the vertical axis.
(iv) Use your graph to calculate the distance
travelled after 8 hours.

8. Estimate the value of the gradient of a curve by
constructing a tangent to the curve at a given point;

9. Estimate the area under graphs by "counting squares" or by the trapezium rule;

The specific objectives for <u>Geometry</u>.

1. State the relations between an object and its image:

(a) as a result of a composition of transformations involving reflection in two parallel lines,

(b) under reflection in two intersecting lines not necessarily at right angles,

(c) under the transformation of a glide reflection,

(d) when it is sheared,

(e) under the transformation of stretching, including one way and two-way stretching;

Example: What is the image of the square
O(0,0),A(0,1), B(1,1), C(1,0) under the
transformation
        (1   -1
T =
        1   1) ?
A second transformation U maps (1,0) onto (4,3),
(0,0) onto (0,0), and multiplies all lengths by
a factor of 5. Write down the matrix of U.
   What is the image of OABC under the composite
transformation UT, (that is, T followed by U).

2. Locate the image of a set of points when the transformation mentioned in (1) are performed;

3. Perform successive transformations combining any two of enlargements, translations, rotations, reflections, shears and stretches;

4. Find the equation of a line given the coordinates of two points, the gradient and the coordinates of one point, or the lengths of the intercepts;

   Example: M(3) and M(8) represent reflections in
   the lines $y = 3$ and $y = 8$ respectively. If P has
   coordinates (r,s) and point $Q = M(8).(M(3)(P))$,
   then what
   (i) is the length of PQ
   (ii) are the coordinates of Q.

5. Make conjectures about the properties of and the relations between geometrical objects and prove that they are true or false;

   Example: Which of the following is never true?
   (A) a reflection followed by a reflection is
   equivalent to a rotation,
   (B) a reflection followed by a reflection is
   equivalent to a translation,
   (C) as translation followed by a rotation is
   equivalent to a rotation,
   (D) a reflection followed by a translation is
   equivalent to a translation.

6. Convert angles in degrees to radians and vice versa;

7. Calculate the length of an arc of a circle;

8. Calculate the area of a sector of a circle;

   Example: A chord PQ of a circle of radius 10 cm
   is 3.16 cm from O the centre of the circle.
   Calculate   (a) the angle in radians subtended by
   the arc PQ at O.
   (b) the length, in cm, of the arc PQ.
   (c) the area, in cm^2, of the minor sector OPQ.

The specific objectives for _Algebra_.

1. Factorize expressions of degree no higher than two;

2. Recognise the difference between those algebraic statements which represent equations and those that represent identities;

3. Solve quadratic equations by factorization, completing the square or by using the quadratic formula;

   Example: (a) Express $9x2 - 30x + 21$ in the form $(ax - b)2 - c$, where a, b, and c are integers.

4. Solve word problems involving quadratic equations;

   Example: A BWIA Tri-star jet travels 80 Km/h faster than a 747 jet liner. The Tri-star takes one hour less than the 747 jet to travel a journey of 6 280 km. Denoting the speed of the 747 jet by x km/h:
   (a) Write down in terms of x expressions for the time taken by (i) the 747 jet liner (ii) the Tri-star jet.

5. Solve a system of equations;

   Example: Find the coordinates of the points of intersection of the curve $y = 10 - x - 2x2$ and the line $y = 7 - 2x$.
   For what value of x is the expression $10 - x - 2x2$ a maximum? What is the maximum value

6. Prove simple theorems in algebra, geometry, and number theory using algebraic techniques;

The specific objectives for <u>Statistics</u>.

1. Use the midpoint of the class interval to estimate the mean of data presented in group frequency tables;

2. Calculate the mean of data presented in group frequency tables;

   Example: The scores of 8 students of 6A on a
   quiz were: 32, 57, 58, 74, 74, 98, 99, 100.
   Calculate: (a) the mean score.
   (b) the standard deviation, giving
   your answer correct to one decimal
   place.
   (c) 8 other students in a parallel
   form 6D completed the same quiz.
   The mean of their scores was 74 and
   the standard deviation was 2.88.
   Comment on the performance of the
   two forms.

3. Construct a cumulative frequency table for a given set of data;

4. Determine from the cumulative frequency table the proportion and/or percentage of the sample above or below a given value;

5. Draw and use a cumulative frequency curve;

6. Calculate the standard deviation of a set data;

7. Select a random sample and make reasonable estimates about some of the characteristics of the population;

8. Analyse statistical data, commenting on the average, the spread and the shape of the frequency distribution;

9. Calculate simple probabilities.

    Example: Six men and four women, including a
    man and his wife apply for a job at a firm.
    (a) Calculate the probanility that, if two
    applicants are selected at random, at least
    one of them is a man.
    (b) If the man and his wife are selected,
    in how many ways can two othr persons also
    be selected from the remaining applicants.
    (c) Calculate the probability that, if four
    applicants are selected at rándom, the man
    and his wife are selected.

The specific objectives for <u>Trigonometry</u>.

1. Determine the circular functions between 0 and 360

    degrees as the components of the position vector on

    the 'unit circle;

2. Determine the trigonometric ratios for angle A;

3. Use the sine and cosine formulae in the solution of

    triangles;

    Example: A vertical pole TF is standing on
    level ground. P and Q are two points 18 m
    apart on the same edge of a straight road
    running from east to west past the pole.
    The bearing of F, the foot of the pole,
    from P and Q is 20 and 50 degrees
    respectively. The angle of elevation of T
    from P is 15 degrees.
    (i) Draw a sketch of triangle FPQ
    indicating on your diagram the bearings of
    F from P and Q.
    (ii) Calculate the distance PF
    (iii) Calculate the height of the pole TF.

4. Calculate the area of a triangle given two sides and

    the included angle by means of the formula

Area of ^ABC = 1/2 ab sin C;

5. Prove simple trigonometric identities.

Example: Prove that
cos x + sin x. tan x = 1/cos x for x between
0 and 90 degrees.

The specific objectives for <u>Vectors and Matrices</u>

1. Add vectors by the triangle or parallelogram laws;

2. Show that a scalar may be distributed over addition

of vectors;

3. Associate a position vector with a given point

P(a,b);

4. Perform pre- and post- multiplication of matrices;

5. Show that matrix multiplication is not commutative;

APPENDIX C

Objectives of the Advanced Level

Examination

The Advanced level mathematics examination Syllabus C consists of four papers. Every student must write Paper 1 which consists of several brief questions, and seven longer questions. On paper 1 students must answer all the brief questions and four of the seven longer questions. Students must write seven questions on two of the three papers. Papers 2, 3 and 4 contain five questions each on Particle Mechanics, Probability and Statistics, and Pure Mathematics.

The Specific Objectives for Paper 1.

1. Addition, subtraction, multiplication and division of polynomials;

2. Find the roots of polynomials using the Factor theorem;

3. Sketch the graphs of simple rational algebraic functions;

4. Given a quadratic equation in one variable:

    (a) solve the equation,

    (b) find maxima and minima by completing the square,

    (c) sketch graphs;

5. Sketch simple curves such as $y = k(x^n)$;

6. Transform a graph of the form $y = f(x)$ to graph of the form $y = a.f(x)$, $y = f(x) + a$, $y = f(x-a)$, $y = f(ax)$;

7. Use the six trigonometric functions and their periodic properties and symmetries;

8. Simplify and solve trigonometric equations using formulae such as $\sin(A + B)$, $\sin^2 A + \cos^2 a = 1$, $1 + \tan^2 = \sec^2 A$ and $r.\cos(A+b)$;

9. Sketch the graph of simple trigonometric equations;

10. Calculate the approximations $\sin x \sim x$, $\tan x \sim x$, and $\cos x \sim 1 - 0.5 x^2$;

11. Find the angle between a line and a plane, between two planes, and between two skewed lines;

12. Find the sum of arithmetic and geometric progressions;

13. Use binomial expansion to find $(1+x)^n$;

14. Vectors in two or three dimensions, multiplication by scalars;

15. Given a function determine whether it is one-to-one and if it is find its inverse;

16. Graph a function and its inverse;

17. Use simple logarithmic and exponential functions;

18. Define the derivative using limit;

19. State the derivatives of standard functions;

20. Use differentiation in finding gradients, maxima and--
    minima, rates of change, and in curve sketching;

21. Use simple techniques of integration, including
    integration by substitution and by parts;

22. Evaluate definite integrals with fixed limits;

23. Find the area under a curve as a limit of a sum of
    areas of rectangles.

24. Solve simple problems on permutation and combination;

25. Use the method of induction in summing finite series;

26. Given the matrix A find the transpose matrix, the
    inverse matrix and the determinant;

27. Expansion of functions in the power series;

28. Integrate using the trapezium rule.

The specific objectives for <u>Particle Mechanics</u>.

1. Newton's laws of motion and motion in a straight line
   stated as functions of time, velocity or distance;

2. Problems dealing with a smooth pulley;

3. Hooke's law of elasticity;

4. Paths of projectile;

5. Uniform circular motion.

The specific objectives for <u>Probability and Statistics</u>

1. Basic probabilty laws such as

    P (A U B) = P (A) + P (B) - P (A /\ B);

2. Use of E (aX + b) = aE(X) + b

    Var (X) = E(X^2) - [E(X)]^2

    Var (aX + b) = a^2 Var(X)

    and uniform, binomial and Poisson distributions;

3. Knowledge of sampling methods and distributions, estimation of population parameters, and calculation of means;

4. Knowledge of hypothesis testing and significance of sample means.

The specific objectives for <u>Pure Mathematics</u>.

1. Find the area of a sector;

2. Conversion from cartesian to polar coordinates;

3. Knowledge of complex numbers:

    (i) algebraic and trigonometric forms,

    (ii) sum, product and quotient of two complex numbers,

    (iii) De Moivre's theorem without proof,

    (iv) use of e^i0 = cos 0 + i sin 0;

4. Curve sketching for rational functions;

5. Finding the roots of an equation by simple graphical or numerical methods;

6. Solving first and second order differential equations;

7. Vectors and vector arithmetic;

The Advanced level mathematics examination Further Syllabus C consists of four three-hour papers from which the student does Paper 1 and chooses one of Papers 2, 3, and 4.

Paper 1 contains 12 questions on algebra, algebraic structure, analysis, complex numbers, matrices and linear spaces, and vectors. Papers 2 and 3 consist of ten questions each on mechanics and statistics respectively. Paper 4 consists 15 questions with five questions each on mechanics, statistics and numerical analysis. On Papers 2, 3, and 4, the student does seven questions.

The specific objectives of <u>Algebra</u>.

1. Sketch the graph of polynomials and simple rational functions;

2. Find the sum of simple finite series.

The specific objectives of <u>Algebraic Structure</u>.

1. Define closure under associativity, commutativity and distributivity;

2. Use of equivalence classes and equivalence relations;

3. Define and use the properties of groups;

4. Describe an isomorphism between groups.

The specific objectives of Analysis.

1. Sketch the graph of the six hyperbolic functions and their inverses;

2. Find the derivative of hyperbolic functions and inverse hyperbolic functions;

3. Perform explicit and implicit differentiation, and integration;

4. Use integration to find mean values, centroids, lengths of curves, and areas under the curve;

The specific objectives of Complex Numbers.

1. Prove de Moivre's theorem for a positive integral exponent;

2. Use the properties of complex numbers.

The specific objectives of Matrices and Linear Spaces.

1. Know the properties of a linear space such as linear dependence and independence,

2. Find the determinant of a 3 x 3 matrix;

3. Find the inverse of a 3 x 3 matrix;

4. Find the eigenvalues and eigenvectors of 2 x 2 or 3 x 3 matrices.

The specific objectives of Vectors.

1. Find scalar and vector products;

2. Define the properties of vectors.

PAPER 2

The specific objectives of Mechanics.

1. Find the point of equilibrium of a rigid body under a coplanar set of forces;

2. Reduce a system of forces to a single force through a given point;

3. Find the centre of mass of a body;

4. Calculate relative velocity;

5. Calculate acceleration in circular motion;

6. Calculate harmonic motion of a simple pendulum;

7. Calculate kinetic energy.

PAPER 3

The specific objectives of Statistics.

1. Know the shapes and properties of simple distributions;

2. Use t-distribution and Chi-squared distribution;

-3. Draw scatter diagrams;

4. Calculate correlation coefficient, regression coefficient, and confidence limits;

PAPER 4

Paper 4 consists of three options: mechanics, statistics and numerical analysis. The specific objectives of mechanics and statistics are the same as in papers 2 and 3 respectively.

The specific objectives of <u>Numerical Analysis</u>.

1. Use of Taylor's and Maclaurin's series;

2. Approximate functions by polynomials, and calculate the margin of error in such approximations;

3. Solve equations of the form $f(x) = 0$;

4. Solve a system of linear equations by Guassian elimination;

5. Perform numerical integration using the trapezium rule or Simpson's rule;

6. Perform step-by-step differnetiation;

7. Fit simple curves using the method of least squares.

APPENDIX D

Copy of Basic Proficiency Mathematics

Examination
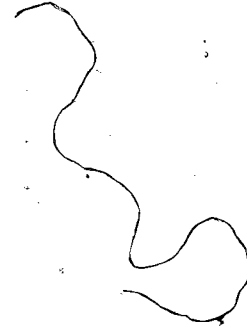
1.   Without using tables, calculate the exact value of

$$\frac{2.8 \ (4 \ - \ 2.95)}{7 \ \times \ 0.14}$$

(5 marks)

2.   (a)   Express in its simplest form

$$\frac{4x \ + \ 1}{4} \ - \ \frac{x \ - \ 2}{6}$$

(5 marks)

(b)   Given that $m * n$ denotes $m \ + \ 3n$
Evaluate $3 * (1 * 2)$

(3 marks)

3.   (a)   A car left Mandeville at 16:00 hrs and arrived in Kingston at 17:30 hrs. It travelled at an average speed of 60 kilometres per hour.   Calculate the distance from Kingston to Mandeville.

(b)   The car then left Kingston at 18:20 hrs and arrived in Montego Bay at 22:05 hrs.   If Montego Bay is 210 kilometres from Kingston, calculate the average speed of the car on the journey from Kingston to Montego Bay.   (6 marks)

4.   A man had $100. He went to a meatshop, a bookshop and a drugstore. He spent three times as much money at the meatshop as he did at the drugstore.   He spent $12 less at the bookstore than at the drugstore. He then had $37 left.

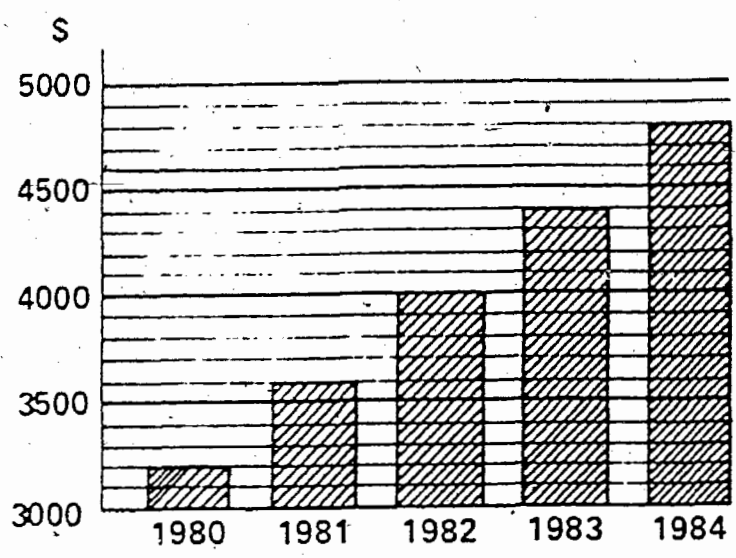(a)   Using $x to represent the amount he spent at the drugstore, express in algebraic terms

(i).   the amount he spent at the meatshop

(ii)   the amount he spent at the bookstore

(b)   Obtain an equation for the total amount of money spent and hence calculate the amount he spent at the drugstore.

(7 marks)

GO ON TO THE NEXT PAGE

5.

(a)   The bar-chart above shows the amount of money invested by a company over a five-year period.

   (i)   Write down the amounts invested in 1980 and 1983.

   (ii)   Calculate the mean amount invested per year over the 5 year period.

   (iii)   Estimate the amount invested in 1985, assuming the trend shown in the graph continues. Give a reason for your answer.

   (iv)   Calculate the angle which would represent the amount invested in 1983 if the information illustrated in the bar-chart above is to be represented on a pie chart.

                                   (11 marks)

(b)   A box contains 10 similar balls, 4 of which are yellow. The first ball taken out at random is yellow. It is not replaced. Calculate the probability that a second ball taken out at random is yellow.   (3 marks)

GO ON TO THE NEXT PAGE

- 4 -

6. (a)   S5 000 was put in a fixed deposit account on 1 January, 1984 for one year. Calculate the total amount received at the end of the period, if the rate of interest was 12.5% per annum.

    (b)   S5 000 was also put in a fixed deposit account at a different bank on 1 January, 1984 for 6 months. The rate of interest was 12.5% per annum. On 1 July, 1984 the total amount received was reinvested for a further 6 months at 12.0% per annum. Calculate the final amount received at the end of the year.

    (c)   State whether (a) or (b) was the better investment, giving a reason for your choice.   (11 marks)

7. (a)   Calculate each of the following to 2 significant figures:

        (i)   The volume of a cylindrical tin of height 20 cm and diameter 28 cm.

        (ii)   The number of litres the tin can hold given that 1 litre = 1 000 cm^3.

        (iii) The volume of a spherical ball of radius 4.2 cm.

    (b)   Calculate the least number of these balls that can be put in the tin so as to cause the water to overflow, if the tin contains 9 litres of water.   (12 marks)

GO ON TO THE NEXT PAGE

0582/F 85

8.   (a)   (i)   A refrigerator can be bought on hire-purchase by making a deposit of $480 and 15 monthly instalments of $80 each. Calculate the hire-purchase cost of the refrigerator.

(ii)   The actual marked price of the refrigerator is $1 400. This includes a sales tax of 12%. Calculate the sale price of the refrigerator if no sales tax is included.

(6 marks)

(b)   The Rates of Exchange at a bank are as follows:

$$EC\$1.00 = BDS\$0.75$$
$$\text{and} \quad US\$1.00 = BDS\$1.98$$

(i)   A traveller changed EC$1 600 to Barbados currency. Calculate the amount received.

(ii)   Of the amount she received she spent BDS$210 and exchanged the remainder for US currency. Calculate the amount in US currency she received for this exchange.

(Assume that the buying rate and selling rate for BDS$1.00 are the same.)
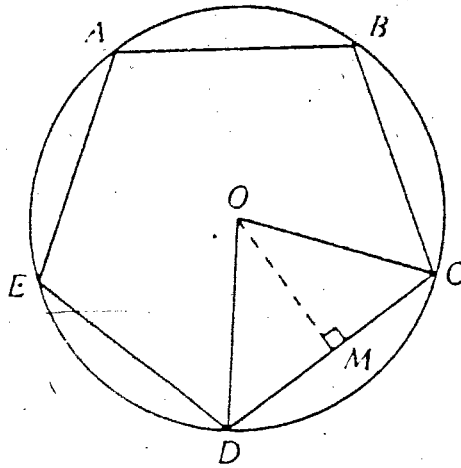
(6 marks)

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9. (a) Using a scale of 1 cm to represent 1 unit on each axis, plot on graph paper the points $P(2, -1)$ and $Q(-2, 5)$.

(b) Calculate the gradient of PQ.

(c) Determine the point where PQ meets the y-axis.

(d) Write down the equation of PQ in the form

$$y = mx + c$$

(e) Hence or otherwise determine the solution of

$$3x + 2y = 4$$
$$x - y = 1$$

(f) Shade the region $y \geqslant x - 1$ for $x \geqslant 0$.

(15 marks)

GO ON TO THE NEXT PAGE

10.

$ABCDE$ is a regular pentagon inscribed in a circle centre O, radius 12 cm, as shown in the diagram above. M is the mid-point of DC.

(a) Calculate the angle DOC (in degrees).

(b) Calculate DM.

(c) Hence, find the perimeter of the pentagon.

(10 marks)

END OF TEST

FORM TP 8418

CARIBBEAN EXAMINATIONS COUNCIL

SECONDARY EDUCATION CERTIFICATE
EXAMINATION

MATHEMATICS

Paper 2 – General Proficiency

Candidate's number . . . .

Answer Sheet for Question 7 (i)

Score	Frequency	Cumulative
1 – 10	1	1
11 – 20	3	
21 – 30	4	
31 – 40	16	
41 – 50	25	
51 – 60	27	
61 – 70	10	
71 – 80	8	
81 – 90	4	
91 – 100	2	

Write answer to 7 (iii) below.

APPENDIX E

Copy of General Proficiency Mathematics

Examination

SECTION I

Answer ALL the questions in this section.

1. (i) Evaluate, without using tables

$$\frac{10.02 \times 0.14}{0.7 \times 50.1}$$

(3 marks)

(ii) Evaluate, using tables

$$\sqrt{0.749}$$

(2 marks)

(iii) A household used 60 cubic metres of water for the first half of 1983. In 1983, water rates for domestic users for a half year were as follows:

$1.05 per cubic metre for the first 50 m^3

$1.25 per cubic metre for amounts in excess of 50 m^3

5% discount on bills paid within two weeks of billing.

Calculate the amount the household paid for the half year, assuming the bill was paid within the two-week period.

(5 marks)

2. (i) A rectangular wooden beam of length 5 metres has a cross section 20 cm by 15 cm. The wood has a density of 600 kg per m^3.

(a) Calculate the volume of the beam in cubic metres.

(b) Express the answer for (a) in standard form.

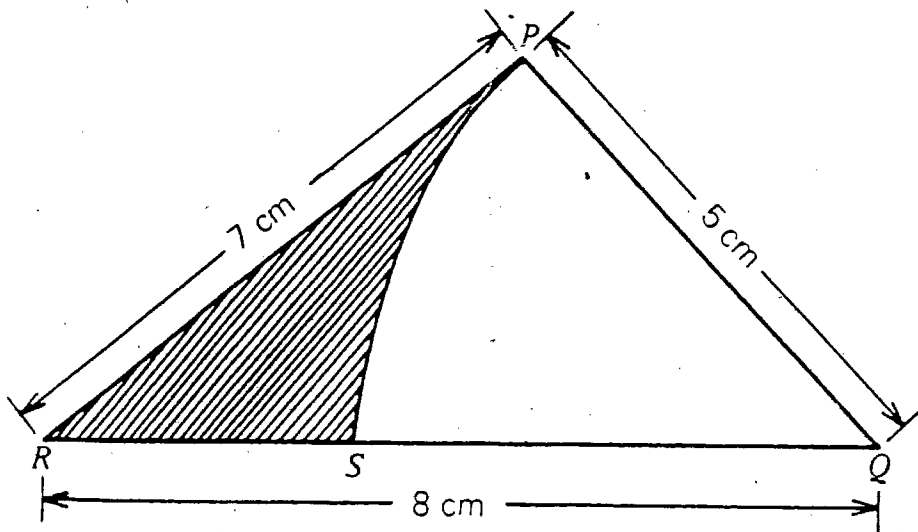(c) Calculate the mass of the beam in kilograms.

(6 marks)

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(ii) U = {natural numbers}
P = {factors of 12}
Q = {factors of 6}
R = {multiples of 12}

Draw a Venn diagram to represent these sets and show on the diagram in the appropriate regions the members of P, Q, R and $P \cap Q'$.

(5 marks)

3.

In the figure above (*not drawn to scale*) PS is an arc of a circle of radius 5 cm and Q is the centre of the circle. $RQ = 8$ cm, $PR = 7$ cm.

Calculate

(a)   the size of angle Q

(b)   the area of the shaded portion bounded by the arc PS and the line segments PR and RS. (use $\pi = 3.14$)

(11 marks)

GO ON TO THE NEXT PAGE

4. The points $A(0, 9)$ and $B(0, 4)$ are mapped by a rotation with centre C on to the points $A'(8, 7)$ and $B'(4, 4)$.

   (i) Using a scale of 2 cm to 1 unit on both axes, plot the points A, B, A' and B'.

   (ii) State

      (a) What is the relation of A and A' to C

      (b) the size of the angle BMC, where M is the midpoint of BB'.

   (iii) By suitable constructions find the coordinates of C. Measure and state the size of the angle of rotation to the nearest degree.

   (10 marks)

5. (i) Copy and complete the table for the function

   $$y = x^2 + 2x - 2$$

x	-3	-2	-1	0	1	2
y	1		-3		1	

   (ii) Using a scale of 2 cm to represent a unit on both axes, draw on graph paper the graph of the function for $-3 \leqslant x \leqslant 2$

   (iii) Draw the line $y = x + 3$ on the same graph and write down the coordinates where $y = x + 3$ cuts the curve.

   (iv) Hence, solve the equation $x^2 + 2x - 2 = x + 3$.

   (13 marks)

GO ON TO THE NEXT PAGE

6.   (i)   Simplify the expression

$$4a^{-\frac{1}{2}}\left(a^{\frac{5}{2}} - a^{-\frac{3}{2}}\right)$$

State your answer using positive indices.

                   (3 marks)

(ii)   Given $m = \dfrac{\sqrt{1 - n^2}}{n}$

express n in terms of m.

                   (5 marks)

(iii)   Let $U = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ and $V = \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}$

       (a)   Write down U^{-1} and V^{-1}

       (b)   Calculate UV

       (c)   Determine which of the following matrices $U^{-1} V^{-1}$ or $V^{-1} U^{-1}$ is equal to $(UV)^{-1}$

                   (6 marks)

0592/F 84

7.   [An answer sheet is provided for this question.]

The table below shows the distribution of scores obtained by 100 candidates in an examination.

Score	Frequency	Cumulative
1 — 10	1	1
11 — 20	3	
21 — 30	4	
31 — 40	16	
41 — 50	25	
51 — 60	27	
61 — 70	10	
71 — 80	8	
81 — 90	4	
91 — 100	2	

(i)   Complete the column for the cumulative frequency on the answer sheet provided.

(ii)   Using 1 cm to represent 10 units on both axes draw the cumulative frequency curve on graph-paper.

(iii)   From the curve estimate

   (a)   the median

   (b)   the probability that a student chosen at random obtained a score less than or equal to 35.

(12 marks)

GO ON TO THE NEXT PAGE

.SECTION II

Answer any THREE questions from this section.

Relations and Functions

8. An object moves from rest and travels for 70 seconds. The speed changes uniformly over each given time interval shown in the table below. The speed shown is the value **at the end of the time interval.**

Time Interval seconds	0 — 20	20 — 30	30 — 50	50 — 70
speed $m\ s^{-1}$	6	5	6	0

(i) Using a suitable scale, draw the speed-time graph on graph paper.

(ii) Calculate the acceleration during the last twenty seconds.

(iii) Calculate the total distance travelled in the seventy seconds.

(9 marks)

GO ON TO THE NEXT PAGE

Algebra

9. A sliding door is to be put in a wall of area $81 \, m^2$. The area of the door must not be greater than $\frac{1}{3}$ of the total wall space. The width of the door is 6 m more than the height.

Calculate the greatest possible height of the door.

(9 marks)

Geometry

10. In this question assume that the earth is a sphere with circumference 40 000 km at the equator.

(i) Given that P (0^oN, 75^oW) and Q (0^oN, x^oW) are points on the earth with Q 1 800 km west of P, calculate x.

(ii) Calculate

(a) the shortest distance between A (30^oN, 42^oW) and B (24^oS, 42^oW)

(b) the distance between R (45^oN, 50^oW) and S (45^oN, 22^oE) measured along the parallel of latitude in an eastward direction from R.

(9 marks)

Trigonometry

11.
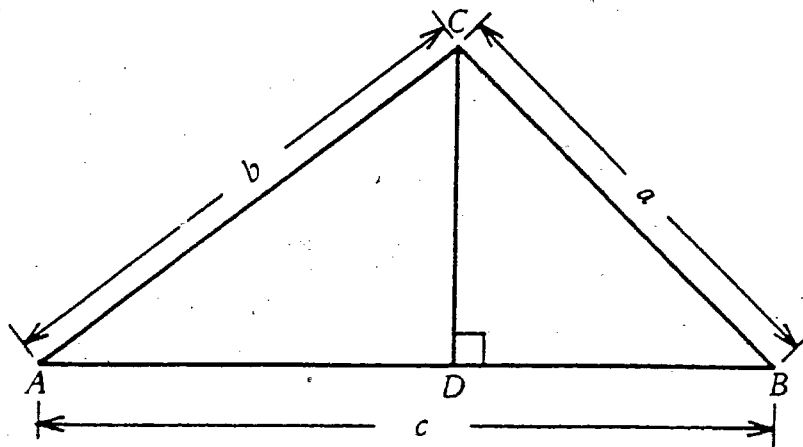
(i) In triangle ABC above, angle C is obtuse and CD is perpendicular to AB. Prove that $\dfrac{a}{\operatorname{Sin} A} = \dfrac{b}{\operatorname{Sin} B}$.

(ii) Prove that for any acute angle A

$$\frac{1}{\tan A} + \tan A = \frac{1}{\operatorname{Sin} A \ \operatorname{Cos} A}$$

(9 marks)

GO ON TO THE NEXT PAGE

Vectors and Matrices

12. The image of the point $P(x, y)$ under the transformation

$T = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ is $P'(x', y')$.

(i) Express x', y' in terms of x and y.

(ii) Calculate the coordinates of the images of the points $A(0, 1)$, $B(2, 1)$, $C(2, 4)$ and $D(0, 4)$ under T.

(iii) Plot on graph paper the figures $ABCD$ and $A'B'C'D'$.

(iv) Describe the transformation geometrically.

(9 marks)

GO ON TO THE NEXT PAGE

Statistics

13.   The table below shows the frequency distribution of the marks
      obtained by students of Forms A and B on a mathematics test.

MARKS	FORM A Frequency	FORM B Frequency
0 — 9	1	4
10 — 19	9	8
20 — 29	24	20
30 — 39	8	6
40 — 49	2	4
50 — 59	0	2
Estimated Mean		24.6
Estimated Standard deviation		12.1

(i)   Calculate an estimate of the mean and standard deviation of
      the marks of students in Form A.

(ii) **COMPARE** the means and standard deviations of the two
      marks of the two forms and **COMMENT** on these statistics.

                                                              (9 marks)

Probability

14.   (i)   In how many ways can a committee of 5 be chosen from
            5 men and 7 women?

      (ii) Calculate the probability that if a committee of 5 is chosen
            there are 2 men and 3 women on this committee.

                                                              (9 marks)

                                        GO ON TO THE NEXT PAGE

Reasoning and Logic

15. (i) (a) Prove that the statement $\sim (p \lor q)$ is logically equivalent to the statement $\sim p \land \sim q$.

(b) Given p : She is a St. Lucian,

and q : She is a mathematician

write a verbal statement which corresponds to

$$\sim (p \lor q).$$

(ii) Write the converse of the following implication

If $n = 7$ then $n^2 = 49$

If n is an integer, state whether or not the converse is true. Justify your answer.

(9 marks)

16. (i) It is given that

$$1 + 3 = 4 = 2^2$$
$$1 + 3 + 5 = 9 = 3^2$$
$$1 + 3 + 5 + 7 = 16 = 4^2$$
$$1 + 3 + 5 + \ldots + 17 + 19 = 100 = 10^2$$

(a) How many terms are there in the sequence

$(1 + 3 + 5 + \ldots + 97 + 99)$?

(b) Write down the sum of $1 + 3 + 5 + \ldots + 97 + 99$.

GO ON TO THE NEXT PAGE