

APPLICATIONS OF NON-NESTED HYPOTHESIS TESTS TO ESTIMATION OF THE  
CANADIAN PHILLIPS CURVE

by

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Applications of non-nested hypotheses  
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## ABSTRACT

The purpose of this paper is to examine three characteristics of the Canadian Phillips curve: its functional form, the inflation-expectations mechanism, and the measure of excess supply or demand in the labour market. This is done by using a variety of non-nested hypothesis tests. The results show that an ARIMA model is the most suitable inflation forecasting mechanism, that the unemployment rate is the most suitable measure of excess labour supply, and that the semi-log is the most suitable functional form.

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## CHAPTER I

### INTRODUCTION

The purpose of this paper is to estimate the Canadian Phillips curve and its functional form using non-nested hypothesis testing techniques. It is divided into five chapters. In chapter I, an introduction is given. In chapter II, the idea of non-nested hypothesis testing is introduced together with a brief discussion of all the different tests employed in our application. Problems associated with the use of these different testing techniques are also presented in this chapter.

In chapter III, different models based mainly on the models of Riddell (1981) for the Canadian Phillips curve are estimated. Various proxies for measuring excess demand/supply in the labour market, such as the vacancy rate, unemployment rate, and help wanted index are tested. In addition, different mechanisms thought to generate the expected rate of inflation are examined along with all these different proxies for excess demand/supply pressure in the labour market. Several techniques for generating the expected rate of inflation are used. These include: 1) the Box Jenkins ARIMA model, 2) the Classical Linear Regression model and 3) the Leading Indicator combined with a transfer function technique. Non-nested hypothesis tests are used to determine the relevant variables for use in chapter four.

Chapter IV tests for functional form using non-nested hypothesis tests. Several functional forms including the linear,

the quadratic, the semi-log and double-log forms are estimated for the model selected in chapter III. Non-nested hypothesis tests are then applied to discover the most appropriate functional form for the Canadian Phillips curve. Chapter V gives a summary of the results of the tests performed and some suggestions for extensions of this paper.



## NON-NESTED HYPOTHESIS TESTS

Economists are often faced with the problem of choosing between alternative model specifications and they are inclined to make their decision on the basis of their own empirical estimations, since theory seldom suggests the appropriate variables nor the correct functional form. As a result, many relations emerge when people try to capture the economic effects by using proxies that they claim to be suitable, and fit the functional form that they think to be appropriate or gives the best fit. Divergent results are very common when models are evaluated solely on the basis of their own performance regardless of whether they can predict the consequences of competing models. Some examples that can be cited are the determination of the correct functional form for the money demand equation for Canada and the determination of the correct model for the consumption function (Davidson and Mackinnon(1982)).

### 2.1 WHAT IS NON-NESTED HYPOTHESIS TESTING?

Economic researchers are familiar with nested hypotheses and some nested hypothesis tests, such as the Chow test, are popular. In a regression model,  $H_0$  is said to be nested within an alternative model,  $H_1$ , if  $H_1$  can be reduced to  $H_0$  by imposing one or more restrictions on its parameters. The Cobb-Douglas

production function, for example, can be shown to be nested within the C.E.S. production function which is reduced to the Cobb-Douglas function by simply restricting the elasticity of substitution to unity.

Non-nested hypothesis testing, on the other hand, is different. Suppose we want to consider two competing linear regression models,

$$(1) \quad H_0: Y = X\theta_0 + U, \quad U \sim N(0, \sigma_U^2)$$

$$(2) \quad H_1: Y = Z\theta_1 + V, \quad V \sim N(0, \sigma_V^2)$$

where  $Y$  is a vector of observations on the dependent variable,  $X$  and  $Z$  are matrices of observations on the independent variables and are assumed to be fixed in repeated samples, and  $\theta_0$  and  $\theta_1$  are vectors of parameters to be estimated. These two models,  $H_0$  and  $H_1$ , are said to be non-nested if  $H_0$  is not nested in  $H_1$  and  $H_1$  is not nested in  $H_0$  (i.e. it is impossible to reduce  $H_0$  to  $H_1$  or  $H_1$  to  $H_0$  by imposing any linear restrictions on the parameters of the models). When we assume for the above models that  $X$  does not lie in the space spanned by the columns of  $Z$  and vice versa, we are actually making these two hypotheses non-nested.

In using non-nested hypothesis tests, people often jointly consider other model selection criteria. In fact, non-nested hypothesis tests are not one of the discrimination criteria such as goodness of fit, instead they are just tests of model specification, just like tests for autocorrelation and heteroscedasticity. The major difference between non-nested

tests and other classical procedures is that non-nested hypothesis tests require the existence of a non-nested alternative model. They are different from the other discrimination criteria in that in applying discrimination methods, one model is chosen ultimately, while in applying non-nested hypothesis tests, it is possible to reject (or accept) both models under consideration. This special feature is due to the fact that the validity of one model is tested based on the evidence suggested by the alternative model, an idea similar to the encompassing principle. The acceptance or rejection of the model tested does not imply the rejection or the acceptance of the other model. The roles are reversed and both models are accepted or rejected individually, based on the evidence provided by the other alternative.

Before we proceed with the discussion of the different tests adopted in this paper, it is worthwhile to give a brief history of the development of non-nested hypothesis testing.

The idea of testing separate families of regressions was first proposed by D.R. Cox, while the attempt to develop tests for separate normal regressions in econometrics was led by Pesaran (1974) and Deaton (1978) whose test statistics are merely strict application of Cox's centered log likelihood ratio (CLR) criterion (Cox (1961, 1962)). The only difference between Pesaran's and Cox's test statistics is that Pesaran's test statistic is based on the logarithms of two variance estimates while Cox's statistic is based on the simple difference between

the two variances estimates. These two test criteria have the same asymptotic distribution and therefore the same asymptotic variance.

The development continued with a second feature, the method of artificial nesting (AN) of likelihoods of regression equations. It was proposed as a distinct alternative to the CLR model (Davidson and Mackinnon (1981,1982); Fisher and McAleer (1981)). The AN approach stems from the work of Hoel(1947) and Williams and Klot(1953) and was advocated by Atkinson (1970). Following that, a third feature in this literature is noted. In the case of two linear regressions, both the CLR and AN based tests are found to have surpassed the corresponding F test arising from the composite or comprehensive regression using both sets of regressors.<sup>1</sup>Hence, people turned their direction to the investigation of the small sample properties of these test statistics.

Following this short history, we can see that in general there are two major principles of non-nested hypothesis testing; the Modified Likelihood Ratio (MLR) principle of Cox and the principle of Artificial Nesting (AN). All the tests used in this paper are applications of one or the other of these principles. For simplicity, all models considered in this paper are assumed to be linear at least in the parameters. Thus, all non-linear non-nested hypothesis tests are omitted.

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<sup>1</sup> Some well argued defence of the classical F test is presented by Mizon and Richard (1982).

## 2.2 DESCRIPTION OF NON-NESTED HYPOTHESIS TESTS

With the restriction to linear models, altogether eight non-nested hypothesis tests are available for use. All these tests are discussed below.

### *1. The F test*

The F test arises from the so-called comprehensive or composite regression using both sets of regressors. With the F test a composite model is formed from the two hypotheses,  $H_0$ ,  $H_1$ , (described in (1) and (2)) as

$$(3) H_2 : Y = X\theta_0 + Z\theta_1 + U$$

where  $Z'$  is formed by deleting all the variables that  $Z$  has in common with  $X$ . An F test on the coefficient vector for  $Z'$  is performed. If this coefficient vector is found to be statistically insignificant from the zero vector, we accept  $H_0$ , otherwise, we reject it. It may be noted that if the matrix  $Z'$  contains only one column, the F test gives exactly the same result as the J test described below. When  $Z'$  has more than one column, the two approaches yield different results. Since the F test involves as many degrees of freedom as there are columns in  $Z'$ , while the J test involves only one degree of freedom, this indicates that the J test may be more powerful than the F-test. However, Pesaran (1981) has shown that these two test statistics, after suitable transformation, are the same asymptotically.<sup>2</sup>

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<sup>2</sup>Some other specification and discussion of the relationship

## 2. The Cox test

The basic principle on which the Cox test is based is that the validity of the null hypothesis  $H_0$  may be tested by examining whether it is capable of predicting the performance of the alternative hypothesis  $H_1$ : the actual performance of the alternative hypothesis is compared to the performance expected if the null hypothesis were in fact true. If the difference tests significantly different from zero, the null hypothesis is rejected; otherwise it is accepted. In the original Cox test, "performance" was defined as the ratio of the maximized likelihood of  $H_0$  to the maximized likelihood of  $H_1$ . The Cox test statistic  $N_0$  under  $H_0$  is given as

$N_0 = T_0 / \sqrt{V(\hat{T}_0)}$  where  $N_0$  is asymptotically distributed as  $N(0,1)$  under  $H_0$ .<sup>3</sup> The numerator of this statistic,  $T_0$ , is calculated as

$$T_0 = (n/2) \log(\hat{\sigma}_0^2 / \hat{\sigma}_{10}^2)$$

and the denominator is calculated as

$$V(\hat{T}_0) = (\hat{\sigma}_0^2 / (\hat{\sigma}_{10}^2)^2) * (SSR_{0,10})$$

where  $n$  is the total number of observations and

$$\hat{\sigma}_{10}^2 = \hat{\sigma}_0^2 + \hat{\sigma}_a^2$$

and  $\hat{\sigma}_a^2$  is the estimated error variance from the auxiliary regression of the fitted value of  $y$  from  $H_0$  on the model under  $H_1$ .  $\hat{\sigma}_0^2$  is the estimated error variance of the regression on (1).

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<sup>2</sup>(cont'd) between the  $N_0$  statistic and the F test statistic using confidence contours are considered in A.D. Hall .

<sup>3</sup>The derivation of  $N_0$ ,  $T_0$  and  $V_0$  are all given in Cox ( 1961, 1962)

$SSR_{0,10}$  is the sum of square errors from regressing the residuals, obtained by regressing the fitted values of (1) on  $Z$  in (2), on  $X$  in (1). Cox has proved that  $T_0$  will be distributed asymptotically as  $N(0, V_0)$  under  $H_0$  as long as  $\hat{V}_0$  is a consistent estimator of  $V_0$ .

More recently, other variants of Cox tests have been developed. One such test is the one adopted by Atkinson(1970) who replaces  $\hat{\Theta}_1$  by  $\hat{\Theta}_{10}$ , the consistent estimator of  $\Theta_{10}$ , the asymptotic expectation of  $\hat{\Theta}_1$  under  $H_0$ . It is claimed that such replacement would provide stronger evidence against  $H_1$ , should  $H_0$  be rejected by  $H_1$ . This new statistic is calculated as

$$NA_0 = TA_0 / \sqrt{V(\hat{T}_0)}$$

the numerator is given as

$$TA_0 = TL_0 + (1/2\hat{\sigma}_{10}^2) (SSR_{10} - SSR_1)$$

$SSR_{10}$  is the sum of the squares of the differences between the dependent variable under  $H_0$  and  $Z\hat{\Theta}_{10}$  which is the fitted values of  $X\hat{\Theta}_0$  on  $Z$ , and  $SSR_1$  is the sum of the squared errors of the regression on (2).

A linearized version of the Cox test (NL) which is appropriate for our application is also presented here. This linearized version is derived by Fisher and McAleer(1981), who take the upper bound Taylor Series expansion of  $T_0$ . After linearization, the statistic becomes

$$NL_0 = TL_0 / \sqrt{V(\hat{T}_0)}$$

$$TL_0 = (n/2) ((\hat{\sigma}_1^2 / \hat{\sigma}_{10}^2) - 1)$$

Similar to the  $TA_0$  statistic,  $TL_0$  is also distributed

asymptotically as  $N(0, V_0)$  under  $H_0$  and has the same variance as  $T_0$  and  $TA_0$ . The  $NL_0$  statistic also asymptotically equivalent to  $N_0$  and  $NA_0$ . To summarize the relationship among these statistics, Fisher and McAleer have proved that the following equality always holds.

$$NA_0 \geq NL_0 \geq N_0$$

### 3. *The J test and the JA test*

The J test of Russell Davidson and James, G. Mackinnon (1980), combines (1) and (2) to form one artificial nesting model

$$(3) \quad H_2: Y = (1-a)X\theta_0 + aZ\hat{\theta}_1 + U$$

where  $\hat{\theta}_1$  denotes the OLS estimates of  $\theta_1$  from (2). The test statistic is then the t-statistic on  $a$ , which will be distributed as  $N(0,1)$  if  $H_0$  is true. A proof is given in Davidson and Mackinnon (1980). The statistical significance of the t-statistic on  $a$  will denote the rejection of  $H_0$ . Both the JA and the exponential weighting procedure<sup>4</sup> are variants of the J test, but only the JA test will be discussed here. A major difference between the J and JA test lies in the estimation of  $\theta_1$ . In the former case,  $\hat{\theta}_1$  is the OLS estimate from (2); in the latter case,  $\hat{\theta}_1$  is replaced by another consistent estimate of  $\theta_1$ . This estimate is obtained by regressing the fitted values of (1) on  $Z$  and is denoted by  $\theta_1^*$ . Thus, the compound model becomes

$$(4) \quad H_3 : Y = (1-a)X\theta_0 + aZ\theta_1^* + U$$

---

<sup>4</sup>A detailed explanation of the exponential weighting procedure is available in Davidson and Mackinnon (1980).



Again the statistical significance of  $a$  would signify the acceptance or rejection of  $H_0$ . Although the two tests appear to be almost identical, there are situations where one is better than the other. James G. Mackinnon (1983) suggests that with a linear regression model with nonstochastic regressors and normally distributed errors, the JA test provides an exact non-nested hypothesis test. Russell Davidson, on the other hand, claims that the power of the JA test will be much less than the ordinary J test when neither  $H_0$  nor  $H_1$  is true. In the case when  $H_1$  is true, the JA and J test (of  $H_0$ ) seem to be equally powerful.

#### 4. The C-test, PE test and BM test

Similar to the J test and JA test, the C-test (Davidson and Mackinnon (1981a)) also hinges on the statistical significance of the parameter  $a$ . Unlike the J test where  $\theta_0$  and  $a$  are estimated jointly, the parameters  $\theta_0$  and  $\theta_1$  are all estimated before the estimation of  $a$ . Thus, the compound model formed is

$$(5) \quad H_a: Y - X\hat{\theta}_0 = a(Z\hat{\theta}_1 - X\hat{\theta}_0) + U$$

here the statistical significance of  $a$  also signifies the acceptance or rejection of the hypothesis under test. This t-statistic, however, does not share the same asymptotic characteristics as the other test statistics discussed. It is not distributed as  $N(0,1)$  even asymptotically, as proved by Davidson and Mackinnon (1980). Instead it is found to be distributed asymptotically normal, but with variance less than one. Mackinnon (1980) suggests that it is feasible to correct

this test and obtain a valid estimate of the variance of  $a$ . However, the necessarily complicated way of achieving this would make this attempt not worthwhile. Thus, the corrected test is not used in our application and the simple C-test is maintained.

The PE and BM test are discussed here for testing functional form in the last part of the paper. The previous tests are only suitable for cases when the competing models share the same dependent variable and linearity is assumed. Non-nested models that involve different transformations of the dependent variable would have to be tested by some other tests. The two tests that we are going to use are the PE test of Mackinnon (1983) and BM test of Bera and McAleer (1982).

The PE test is an elementary generation of the P test<sup>5</sup>. To make it more relevant for our discussion, we set up two hypotheses;

$$(6) \quad H_0': y_t = \sum_1 b_i x_{it} + \epsilon_t$$

$$(7) \quad H_1': \ln y_t = c_0 + \sum_1 c_i \ln x_{it} + u_t$$

where  $y_t$  is the observed value of the dependent variable at  $t$ ,  $x_{it}$  is the value of the  $i^{\text{th}}$  explanatory variable at  $t$ , and  $\ln$  is the natural logarithm. As with the P test, the PE test of  $H_0$  and  $H_1$  is a test of  $a = 0$  in an artificial regression. Under  $H_0$ , the artificial regression is expressed as

$$(8) \quad H_3' : y_t - \tilde{y}_t = b_0 + \sum_1 b_i x_{it} + a (\hat{\ln} y_t - \ln(\tilde{y}_t)) + u_t$$

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<sup>5</sup> P-test is one of the tests employed for testing non-linear non-nested regression models. Its derivation is available in most of papers discussing non-linear non-nested hypothesis testing techniques, G.R. Fisher and M. McAleer, for example.

and under  $H_1$ , it is represented as

$$(9) H_4': \ln y_t - \hat{\ln y}_t = c_0 + \sum_i c_i \ln x_{it} + a(\tilde{y}_t - \exp(\hat{\ln y}_t)) + v_t$$

where  $\tilde{y}_t$  and  $\hat{\ln y}_t$  are the predicted values of the dependent variables under  $H_0$  and  $H_1$ , respectively. The PE statistic is asymptotically distributed as  $N(0,1)$  under  $H_0$  ( $H_1$ ).

Bera and McAleer (1982) develop a similar test based on artificial regressions. Their tests of  $H_0$  and  $H_1$  are the tests of whether  $\lambda$  is significantly different from zero in the artificial regression models

$$(10) H_5': y_t = b_0 + \sum_i b_i x_{it} + \lambda \tilde{\eta}_{1t} + u_t$$

$$(11) H_6': \ln y_t = c_0 + \sum_i c_i x_{it} + \lambda \hat{\eta}_{0t} + v_t$$

where  $\tilde{\eta}_{1t}$  stands for the OLS residual obtained by regressing  $\ln(\tilde{y}_t)$  on the explanatory variables under  $H_1$ , while  $\hat{\eta}_{0t}$  is the OLS residual from regressing  $\exp(\hat{\ln y}_t)$  on the explanatory variables under  $H_0$ . This BM statistic is distributed as  $N(0,1)$  in large samples under the tested hypothesis.

### 2.3 PROBLEMS WITH THE USE OF NON-NESTED HYPOTHESIS TESTS

Since the nature of the non-nested hypothesis testing techniques allow the acceptance (rejection) of both models, problems arise when the results of the statistics indicate the rejection of both models. Actually, when none of the hypotheses under test is true, the properties of these tests are not known. Underspecification of the alternative model might affect the consistency of the test of the null hypothesis. Some cases have

been investigated and the test found to be inconsistent. For more detailed discussion concerning this, see McAleer, Fisher and Volker(1982).

There are also some other problems that are related specifically to some tests. Monte Carlo studies have indicated that there is a high tendency for the Cox test to over-reject the null hypothesis when it is true. Interpretation of the relationship among the Cox statistics also contributes some difficulties in application. Owing to the fact that the test statistics of the non-nested hypothesis techniques are usually justified on large-sample approximations, difficulty arises because there usually exist many corresponding test statistics which are also asymptotically equivalent. In practical application, we may have calculated statistics which are different numerically and yet they have the same asymptotic distribution, as in the case of the N, NA and NL statistics. It is proved by G.R. Fisher and M. McAleer that if  $H_1$  is fitting much better(worse) than it ought, relying solely on N(NA) will more likely lead to rejection of  $H_0$  than would otherwise be the case. The linearised Cox-statistic, NL, is therefore more conservative with respect to rejecting the model under test than is N(NA), when the alternative is fitting better(worse) than might be expected. For this reason, there is always room for conflict in the inferences drawn from tests. To avoid such problems, all three tests are performed herein, although they are about the same asymptotically. In some of the non-nested

cases examined in this paper, it turns out that difference in the numerical values of asymptotically equivalent tests serves to guide the interpretation underlying the rejection of the hypothesis under test. Problems associated with the F test are listed in McAleer(1982) who notes that the F test involves no optimal use of the information concerning the other rival models and it evaluates a model on the basis of its own performance only, so that the better fitting model would most likely be chosen even if it cannot predict the performance of the rival model statistically.<sup>6</sup> As a result, there is a high probability of accepting a model which might not be the true specification.

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<sup>6</sup> Since the major idea of non-nested hypothesis testing is to predict the performance of other models under the assumption that one's model is the true one, such a test seem to ignore the principle of non-nested hypothesis testing.

CHOOSING PROXY VARIABLES

This chapter explains the development of the proxies for the expected rate of inflation and uses non-nested hypothesis tests to choose among them. Simultaneously a proxy for excess supply in the labour market is tested.

3.1 Development of the inflation-expectation proxies

Three proxies, using different forecasting generation mechanisms are developed. These proxies measure expected inflation during an entire year, but are developed using monthly data.

*A. A Moving Box Jenkins ARIMA Model*

The rationale for adopting an autoregressive-integrated-moving-average (ARIMA) model is discussed in Riddell (1982). The ARIMA model employed takes the form:

$$(1-B)^d(1-B^S)^D(1-\phi_1B - \phi_2B^2 - \dots - \phi_pB^p) \\ (1-\Phi_1B^S - \Phi_2B^{2S} - \dots - \Phi_pB^{pS})CPI_t \\ = (1-\theta_1B - \theta_2B^2 - \dots - \theta_qB^q) \\ (1-\Theta_1B^S - \Theta_2B^{2S} - \dots - \Theta_QB^{QS})\epsilon_t,$$

where  $CPI_t$  is the level of the consumer price index in period  $t$ ,  $B$  is the backward shift operator defined by  $B^n x_t = x_{t-n}$ ,  $\epsilon_t$  is a white noise term with variance  $\sigma^2$  and  $(\phi_i, \Phi_i, \theta_i, \Theta_i)$  are

parameters to be estimated. Conventionally, an ARIMA model is defined specifically as  $(p,d,q)(P,D,Q)_s$  where the integers  $p,q$  denote the orders for the nonseasonal AR and MA parts respectively,  $P,Q$  are orders for the seasonal parts, and  $d, D$  represent the nonseasonal and seasonal differencing done to render the series stationary.  $S$  refers to the lags of the series taken.

In his paper (1982), Riddell has assumed that the ARIMA model is constant over time. Once the model is developed, a moving sample (add one year, drop one year) strategy is used to reestimate the parameters while keeping the form of the model constant. Our first proxy for expected rate of inflation relaxes the Riddell assumption that the ARIMA model remains constant, allowing the ARIMA model as well as the parameter estimates to change from year to year. This improves upon Riddell's specification by avoiding one possible cause of systematic error on the part of those forming the inflation expectations. Thus, an updating of both the model and the parameters is done each year to reflect the fact that people learn from their mistakes and also to capture the structural changes over time. Each yearly update is then employed to produce twelve monthly inflation forecasts.

Using data from 1963-2 to 1980-1 and a moving sample of ten years(120 observations) for each estimation period, we estimated nine ARIMA models, which are presented below.

	<u>Sample period</u>	<u>Model estimated</u>
(1)	1963-2 to 1973-1	$(1, 1, 0)(0, 0, 2)_{1,2}$
(2)	1964-2 to 1974-1	$(1, 1, 0)(0, 0, 2)_{1,2}$
(3)	1965-2 to 1975-1	$(3, 1, 0)(2, 0, 1)_{1,2}$
(4)	1966-2 to 1976-1	$(3, 1, 0)(0, 0, 1)_{1,2}$
(5)	1967-2 to 1977-1	$(3, 1, 0)(0, 0, 1)_{1,2}$
(6)	1968-2 to 1978-1	$(4, 1, 3)(0, 0, 1)_{1,2}$
(7)	1969-2 to 1979-1	$(4, 1, 0)(3, 0, 1)_{1,2}$
(8)	1970-2 to 1980-1	$(4, 1, 0)(3, 0, 1)_{1,2}$
(9)	1971-2 to 1981-1	$(3, 1, 0)(3, 0, 2)_{1,2}$

The estimated parameters for the models are listed in Table 1. All of the estimated parameters are statistically significantly different from zero except for estimates of  $\phi_3$  and  $\theta_2$  for the third model. They are included because there are statistically significant spikes at lags two and three of the PACF (partial autocorrelation function histogram) and ACF (autocorrelation function histogram) of the CPI, and dropping these two parameters would make the residuals of the model no longer white noise. Since the estimates of the two parameter are very close to being significant, these two parameters are retained.





*B. A Moving Leading Indicator Combined With A Transfer Function.*

This technique is employed because many institutions use leading indicators in forming expectations. Since twelve month forecasts are needed, a twelve month leading indicator for the CPI was constructed. The method used is that employed by Holmes (1986) in the construction of leading indicators for industrial employment in B.C.

All together, fourteen components series which are expected to lead the consumer price index were selected. They include

B1 Average weekly hours of hourly rated wage earners in Canada.

B2. Total government securities and loans outstanding.

B3. Canadian money supply.

B4. Statistics Canada's leading indicator for the Canadian economy.

B5. Toronto stock exchange index.

B6. US leading indicator.

B7. Chartered bank prime business loan rate.

B8. US foreign exchange rate in Canadian dollars.

B9. Average weekly wages of hourly rated wage earners in Canada.

B10. Total exports to the EEC.

B11. Total exports to Western Europe.

B12. Total exports to the United Kingdom.

B13. Total exports to Japan.

B14. Total exports to USA.

The component series are first massaged by smoothing, deseasonalizing, inverting and standardizing for variation. The series CPI is then regressed on each component series with a lag of twelve months to get the  $R^2$  of each component series. The  $R^2$  of the component series are summed to form the base of the weighting procedure. The weights of each component series are then calculated as the ratio of its own  $R^2$  and the sum of the  $R^2$ s. The component series are thus combined according to the weights assigned to form the first leading indicator for the period of 1963-1 to 1973-1. Altogether nine leading indicators are constructed. The  $R^2$  and weights of each component series for each leading indicator are given in Table 2 and Table 3 respectively.

Table 2: The R-squares of the Component Series for the Leading Indicators.

C.S.	P1	P2	P3	P4	P5	P6	P7	P8	P9
B1	.049	.564	.279	.006	.1864	.118	.074	.045	.032
B2	.444	.393	.329	.101	.150	.152	.096	.074	.181
B3	.374	.405	.628	.223	.105	.005	.121	.333	.300
B4	.096	.266	.714	.293	.032	.008	.005	.161	.025
B5	.006	.100	.425	.000	.033	.038	.067	.238	.002
B6	.009	.165	.551	.031	.096	.029	.003	.020	.010
B7	.141	.427	.018	.172	.162	.330	.212	.118	.057
B8	.223	.333	.046	.010	.217	.150	.170	.250	.367
B9	.492	.268	.293	.341	.397	.432	.425	.271	.115
B10	.000	.023	.106	.451	.383	.347	.205	.137	.160
B11	.035	.014	.148	.308	.415	.546	.399	.174	.166
B12	.000	.001	.092	.444	.249	.080	.013	.039	.089
B13	.400	.035	.117	.186	.349	.480	.345	.177	.093
B14	.345	.074	.247	.619	.494	.465	.342	.200	.270

P1 is the estimation period from 1963 2 to 1973 1  
P2 is the estimation period from 1964 2 to 1974 1  
P3 is the estimation period from 1965 2 to 1975 1  
P4 is the estimation period from 1966 2 to 1976 1  
P5 is the estimation period from 1967 2 to 1977 1  
P6 is the estimation period from 1968 2 to 1978 1  
P7 is the estimation period from 1969 2 to 1979 1  
P8 is the estimation period from 1970 2 to 1980 1  
P9 is the estimation period from 1971 2 to 1981 1  
C.S. stands for component series.

Table 3: The Weights of the Component Series for the Leading Indicators.

C.S.	P1	P2	P3	P4	P5	P6	P7	P8	P9
B1	.019	.184	.075	.002	.057	.037	.045	.002	.017
B2	.170	.128	.082	.032	.046	.048	.038	.033	.100
B3	.143	.132	.156	.070	.032	.002	.048	.149	.161
B4	.037	.087	.178	.092	.010	.003	.002	.072	.013
B5	.002	.033	.106	.000	.010	.012	.027	.106	.001
B6	.003	.054	.137	.009	.029	.009	.001	.009	.005
B7	.054	.139	.004	.054	.050	.104	.084	.053	.031
B8	.085	.108	.011	.003	.066	.047	.068	.112	.197
B9	.188	.087	.073	.107	.122	.136	.169	.121	.062
B10	.000	.007	.026	.142	.117	.109	.082	.061	.086
B11	.013	.005	.037	.097	.127	.172	.159	.078	.089
B12	.000	.000	.023	.139	.076	.025	.005	.017	.048
B13	.153	.011	.029	.058	.107	.151	.137	.079	.053
B14	.132	.024	.061	.194	.152	.146	.136	.089	.145

P1 is the estimation period from 1963 2 to 1973 1  
P2 is the estimation period from 1964 2 to 1974 1  
P3 is the estimation period from 1965 2 to 1975 1  
P4 is the estimation period from 1966 2 to 1976 1  
P5 is the estimation period from 1967 2 to 1977 1  
P6 is the estimation period from 1968 2 to 1978 1  
P7 is the estimation period from 1969 2 to 1979 1  
P8 is the estimation period from 1970 2 to 1980 1  
P9 is the estimation period from 1971 2 to 1981 1  
C.S. stands for component series.

To generate forecasts, a transfer function model is used where the CPI serves as the output variable and the leading indicator (LI) with a lag of twelve months serves as the input variable. With nine leading indicators, nine transfer functions are estimated together with nine ARIMA models fitted to the noise components. The transfer function we estimated takes the form :

$$(1-B)CPI = \beta LI_{t-12} + \{\theta(B)\Theta(B)\} \{\phi(B)\Phi(B)\}^{-1} \epsilon_t$$

where

$$\phi(B) = (1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p)$$

$$\Phi(B) = (1 - \Phi_1 B^s - \Phi_2 B^{2s} - \dots - \Phi_p B^{ps})$$

$$\theta(B) = (1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q)$$

$$\Theta(B) = (1 - \Theta_1 B^s - \Theta_2 B^{2s} - \dots - \Theta_Q B^{Qs})$$

The ARIMA models fitted to the noise component of the transfer functions are shown below.

<u>Sample</u>	<u>Model estimated</u>
63 2 to 73 1	(0,0,1)(0,0,2) <sub>12</sub>
64 2 to 74 1	(2,0,1)(0,0,2) <sub>12</sub>
65 2 to 75 1	(3,0,1)(0,0,2) <sub>12</sub>
66 2 to 76 1	(3,0,1)(0,0,2) <sub>12</sub>
67 2 to 77 1	(3,0,1)(0,0,2) <sub>12</sub>
68 2 to 78 1	(4,0,2)(0,0,1) <sub>12</sub>
69 2 to 79 1	(4,0,0)(3,0,1) <sub>12</sub>
70 2 to 80 1	(3,0,1)(0,0,1) <sub>12</sub>
71 2 to 81 1	(3,0,1)(0,0,2) <sub>12</sub>

The estimation results of all the models are shown in Table 5. Each of these nine models is used to generate twelve month inflation forecasts.

Table 4: Estimated Results for the Leading Indicators

Sample	$\beta$	$\phi_2$	$\phi_3$	$\phi_4$	$\theta_1$	$\theta_2$	$\phi_3$	$\theta_1$	$\theta_2$
63 2-73 1	.0014 (5.12)				-.25 (-2.61)			-.366 (-3.50)	-.292 (-2.70)
64 2-74 1	.0018 (4.82)	.236 (2.39)			-.230 (-2.34)			-.355 (-3.49)	-.398 (-3.86)
65 2-75 1	.0022 (4.34)	.277 (2.68)	.210 (1.96)		-.231 (-2.27)			-.459 (-4.09)	-.279 (-1.99)
66 2-76 1	.0023 (4.91)		.262 (2.46)		-.199 (-2.01)			-.459 (-4.16)	-.438 (-3.69)
67 2-77 1	.0025 (4.97)		.335 (3.34)		-.239 (-2.39)			-.317 (-2.92)	-.312 (-2.94)
68 2-78 1	.0028 (4.99)		.233 (2.32)	.248 (2.53)	-.373 (-3.75)	-.202 (-1.94)		-.256 (-2.46)	
69 2-79 1	.0025 (2.63)			.438 (4.14)			.492 (4.04)	-.414 (-3.34)	
70 2-80 1	.0043 (9.19)		.187 (1.94)		-.186 (-1.93)			-.394 (-4.05)	
71 2-81 1	.0045 (6.59)		.299 (3.13)		-.220 (-2.29)			-.445 (-4.71)	-.306 (-3.23)

The figures within brackets are the t-statistics.

The estimates of the parameter  $\beta$  for the leading indicators are not only statistically significant different from zero and bear the expected sign, but are also found to increase overtime. The ARIMA models fitted to the residuals of the transfer function are different from those estimated in the previous section. The major difference between the UBJ ARIMA forecasting technique and the leading indicator lies in the amount of information employed in making their forecasts. When comparing Table 1 with Table 4, the presence of the leading indicator appears to be able to capture most of the cyclical variation of the CPI, rendering the ARIMA models of the noise term more stable than those without the leading indicator.

### *C. A Moving Regression Model*

This technique is adopted because regression forecasts (mainly by economists) have been popular during the past twenty years. A simple model for the CPI is constructed for forecasting purposes. The model chosen takes the form :

$$CPI_{t+12} = b_0 + b_1 MS1_t + b_2 SCLI_t + b_3 YIELD_t + b_4 PETROL_t + \epsilon_t$$

where MS1 is the Canadian money supply, SCLI is Statistics Canada's leading indicator, YIELD is the business loan rate and PETROL is the price index for petroleum and natural gas. The series MS1 is used to measure the monetary effect on CPI, while SCLI is a proxy for expected economic activity, and the last two variables are proxies for cost push effects on the price level. The twelve month lag is required because this forecasting equation is to be used for forecasting CPI twelve months ahead.



The whole sample period is split into nine moving sample periods as was done for techniques A and B above. This model was estimated for each of these sample periods, correcting for the apparent presence of second-order autocorrelation.<sup>1</sup> This produces nine sets of coefficient estimates, reported in Table 6. Most of the coefficient estimates have the expected signs and are significantly different from zero.

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<sup>1</sup> The second-order autocorrelation are incorporated in doing the forecast for CPI.

Table 5: Estimated Results of the Forecasting Regression Models

SAMPLE	CONST	MS1	SCLI	YIELD	PETROL	DW	R <sup>2</sup>
63 2-73 1	18.3 (16.79)	.003 (29.74)	-.025 (-3.02)	0.40 (9.47)	-.025 (-2.07)	.53	.994
64 2-74 1	18.67 (17.36)	.003 (29.57)	-.024 (-3.08)	.386 (9.66)	(-2.47)	.54	.995
65 2-75 1	15.16 (34.83)	.0025 (40.64)	-.015 (-2.17)	.45 (14.23)	.0187 (2.67)	.56	.997
66 2-76 1	15.79 (39.28)	.0026 (33.13)	-0.036 (-4.71)	0.372 (8.72)	.036 (6.77)	.58	.997
67 2-77 1	15.22 (27.37)	.0025 (20.09)	-.014 (-1.44)	.53 (8.86)	.19 (2.18)	.47	.995
68 2-78 1	14.64 (24.03)	.0021 (14.77)	.0062 (.55)	.387 (5.97)	.05 (5.64)	.36	.995
69 2-79 1	15.48 (22.26)	.0017 (12.51)	.023 (2.06)	.1423 (2.47)	.083 (11.12)	.38	.996
70 2-80 1	15.58 (21.38)	.0015 (10.63)	.032 (2.91)	.0898 (1.51)	.095 (12.74)	.41	.996
71 2-81 1	15.19 (19.81)	.0011 (7.97)	.054 (4.70)	.136 (2.20)	.116 (16.49)	.44	.997

The figure within brackets are the t-statistics.

Twelve month out-of-sample forecasts are generated from each of the three models fitted to each of our nine estimation periods. This results in 108 forecasts of expected price levels from each of the univariate ARIMA, the transfer function and the regression model (techniques A, B and C respectively). Since we have the series on actual inflation, the difference between actual inflation and the expected rate of inflation is calculated and summed to calculate the bias of each of the techniques. The values calculated are 1.19, 1.13 and 1.08 for techniques A, B and C respectively. For further comparison, the forecast variance of each technique is computed. The values obtained for these three methods, represented in order, are 22.937, 15.927 and 23.944. These results indicate that the most accurate forecasts are obtained from the transfer function model since its mean squared forecast error is 17.2 as compared to 24.3 for the univariate ARIMA model and 25.1 for the regression model.

### 3.2 Estimation of the Phillips curve

With the expected rate of inflation series generated in the previous sections, the models for the Phillips curve can be estimated. This is the model derived by Riddell (1982). A full rationale is provided by Riddell and is therefore not repeated here. The model that Riddell estimated takes the form:

$$W_t = b_0 + b_1 AIB1_t + b_2 AIB2_t + b_3 AIB3_t + b_4 PE_t + b_5 CATCH_t + b_6 U_t - b_7 DEM_t - b_8 UIC_t + e_t$$

where  $W_t$  is the percentage change of wage rate at time  $t$  and is defined as  $(W_t - W_{t-1})/W_{t-1}$ ,  $AIB_{it}$  are the dummy variables used to measure the effect of the anti-inflation program in guideline year  $i$ ,  $PE_t$  is the expected rate of inflation over a one year horizon, and is defined as  $(CPI_t - CPI_{t-1})/CPI_{t-1}$ ,  $CATCH_t$  is a measure of catch-up pressure due to previously unanticipated inflation,  $U_t$  is the unemployment rate at time  $t$ ,  $DEM$  is a demographic variable which measures the contribution of exogenous changes in the composition of the labour force and  $UIC_t$  is a proxy for the changes caused by the UI act amendments.

Data for the variables  $W_t$ ,  $DEM$ ,  $U$  and the dummies for  $AIB$  are provided by Riddell  $PE$  and  $CATCH$  are constructed in the previous sections of this chapter. Our model follows closely Riddell's model discussed above, except that the variable for  $UIC$  is replaced by a new variable  $DISQ^2$  which is the total number of people disqualified for UI benefits. Further, the dummy variables are summed together to form one dummy variable  $DUMM$ . Thus, the model we estimate is

$$W_t = b_0 + b_1 DUMM_t + b_2 PE_{it} + b_3 CATCH_{it} - b_4 DEM_t + b_5 U_t - b_6 DISQ_t + e_t,$$

where  $PE_{it}$  is the expected rate of inflation generated by forecasting technique  $i$  at time  $t$ ,  $CATCH_{it}$  is the difference between the actual inflation and the expected rate of inflation generated by forecasting technique  $i$  for the past year.

Apart from using different proxies for the expected rate of inflation, the vacancy ratio which measures demand side pressure

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<sup>2</sup> $DISQ$  is obtained from Statistics Canada (73-001)

in the labour market is introduced as an alternate proxy for the unemployment rate. It is denoted as  $HWI_t$ . Altogether , six competing models are estimated by OLS: one for each combination of expected inflation measure and labour market pressure measure. A summary is shown below, and the results of estimation are given in Table 6.

<u>Model</u>	<u>Expected inflation proxy</u>	<u>labour market proxy</u>
1	generated by regression	unemployment rate
2	generated by regression	help wanted index
3	generated by ARIMA	unemployment rate
4	generated by ARIMA	help wanted index
5	generated by leading indicator	unemployment rate
6	generated by leading indicator	help wanted index

The  $R^2$  values for the models are all very similar. All of the coefficients bear the expected signs except that of HWI. Since this coefficient is not significantly different from zero in any equation and multicollinearity appears to be severe, this shortcoming is ignored. The presence of autocorrelated errors is indicated by the DW statistic, violating the assumptions of non-nested hypothesis tests. However, Mackinnon et al(1983) proved that the test statistics remain valid asymptotically. Moreover, the level of autocorrelation is comparable among all equations. Therefore, this problem is ignored and the non-nested hypothesis tests are undertaken.

With six competing models, we can form fifteen pairs of models for the tests. Since the roles of the models are interchanged, we have thirty pairs of models undergoing the tests. The results are presented in Table 7.

Table 6: Estimated Results of the Competing Models for the Phillips curve

VARIABLES	M1	M2	M3	M4	M5	M6
CONST 1	30.63 (2.31)	19.48 (1.91)	22.73 (2.63)	21.71 (2.25)	22.71 (2.61)	21.53 (2.21)
URT	-.599 (-1.039)		-.524 (-.93)		-.560 (-.99)	
HWI		-.0023 (-.06)		-.0026 (-.072)		-.002 (-.06)
DEM	-.00076 (-.932)	-.0011 (-1.37)	-.001 (-1.27)	-.0013 (-1.69)	-.001 (-1.23)	-.001 (-1.67)
Dumm	-3.33 (-3.83)	-3.87 (-4.31)	-3.49 (-4.12)	-3.98 (-4.59)	-3.43 (-4.04)	-3.94 (-4.52)
DISQ	.00005 (4.481)	.00005 (5.42)	.00004 (4.41)	.00005 (5.31)	.00004 (4.43)	.00005 (5.25)
PE1	1.61 (1.624)	1.74 (1.74)				
CATCH1	1.434 (1.553)	1.585 (1.7)				
PE2			1.85 (1.98)	2.00 (2.12)		
CATCH2			1.39 (1.54)	1.53 (1.67)		
PE3					1.97 (1.97)	2.017 (2.11)
CATCH3					1.40 (1.54)	1.54 (1.67)
DW	.607	.63	.63	.65	.63	.65
R <sup>2</sup>	.63	.62	.64	.63	.64	.64

The figures within brackets are the t-statistics.

Table 7: Estimated Results of Non-Nested Hypothesis Tests.

MODELS	J	JA	C	N	NL	NA
M1 vs M2	0.81	-.809	.038	3.82*	8.84*	13.27*
M1 vs M3	1.77	1.77	1.82	-1.3	-2.94*	-2.82*
M1 vs M4	1.79	.029	1.63	-.586	-1.32	-.959
M1 vs M5	1.64	1.64	1.98*	-11.87*	-14.97*	-26.46*
M1 vs M6	1.63	1.58	1.69	-.934	-9.79*	-20.93*
M2 vs M1	1.313	-1.31	1.07	-.29	-.663	-.68
M2 vs M3	2.15*	1.02	2.126*	-1.98*	-4.43*	-2.10*
M2 vs M4	1.82	1.82	1.89	-1.67	-3.76*	-3.09*
M2 vs M5	2.021*	1.51	1.65	-11.65*	-23.38*	-34.49*
M2 vs M6	1.635	1.64	1.46	-1.55	-1.84	-3.46*
M3 vs M1	.25	.18	.058	-.38	1.0	1.01
M3 vs M2	.179	-.455	.061	.442	1.04	1.04
M3 vs M4	.75	-.751	.046	.176	.405	-.146
M3 vs M5	-.434	-.434	-.403	.219	.516	.506
M3 vs M6	-.342	-.84	-.127	.046	1.38	1.067



M4 VS M1	.098	1.63	.53	.274	.64	1.21
M4 VS M2	1.03	.098	.027	.408	.96	.96
M4 VS M3	1.19	-1.189	.954	-3.04*	-6.96*	-17.64*
M4 VS M5	.858	-.512	.705	-.512	-.086	-1.194
M4 VS M6	-.553	-.552	-.497	.263	.609	.63
M5 VS M1	.332	.261	.087	1.76	4.15*	4.25*
M5 VS M2	.261	-.48	.092	1.48	3.49*	3.89*
M5 VS M3	.849	.85	.858	-.45	-1.02	-3.83*
M5 VS M4	.941	-.015	.403	.01	.02	7.75*
M5 VS M6	.775	-.775	.038	.43	1.00	1.42
M6 VS M1	.187	.251	.056	1.04	2.43*	4.1
M6 VS M2	1.12	.187	.612	1.27	2.96*	3.01*
M6 VS M3	1.50	.757	1.28	-1.35	-3.07*	-4.5*
M6 VS M4	.977	.976	.972	-.51	-1.16	-4.36*
M6 VS M5	1.251	1.145	1.017	-12.47*	-28.45*	-10.28*

Note : \* signifies the rejection of the model under test

Of the six models tested, the proxy for the expected rate of inflation generated by the ARIMA model stands out as the most appropriate. This is seen by examining the results reported in Table 7. The first section of Table 7, consisting of the first five lines, reports the tests of model 1, many of which indicate that model 1 should be rejected. Similarly, section 2 of Table 7 indicate that model 2 should be rejected. Section 3 contains no asterisks and this suggests that model 3 should be accepted. Section 4 contains only three rejections, and so comes close to matching model 3's performance. Both model 3 and model 4 involve the ARIMA forecasting mechanism, differing only in the choice of labour market measure. Since, as noted earlier, the unemployment rate measure seemed superior to the help wanted index in that its coefficient more frequently had the expected sign, the choice of model 3 is further supported. Section 5 and 6 of Table 7 show that models 5 and 6 are rejected only by the NL and NA tests. These rejection plus the the fact that the J and JA test statistics here have values greater than those of section 3 leads also to the choice of model 3. We thus, conclude that model 3 is the "best"<sup>3</sup> model of the six and that in the estimation of the Canadian Phillips curve, the unemployment rate is a better choice than the help wanted index.

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<sup>3</sup>Best model refers to the one that has passed all the tests performed.

## CHAPTER IV

### CHOOSING THE FUNCTIONAL FORM

The purpose of the chapter is to investigate several functional forms of the model chosen in the previous chapter. By way of comparison, we will also investigate whether it is always legitimate to choose the functional form that produces the best fit. Since theory suggests that the short run Phillips curve should be downward sloping and convex to the origin, all the functional forms attempted are based on this. These functional forms include the quadratic form and its variants (without the cross product terms), two semi-log forms, the double-log form and the linear form. These models are described as follows.

Model	Descriptions
(1)	$W_t$ is a linear function of the explanatory variables.
(2)	$W_t$ is a linear function of the inverses of the explanatory variables.
(3)	$W_t$ is a linear function of the squares of the explanatory variables.
(4)	$W_t$ is a linear function of the inverses of the square of the explanatory variables.
(5)	$W_t$ is a linear function of the natural logs of the explanatory variables.
(6)	The natural log of $W_t$ is a linear function of the explanatory variables.

(7) The natural log of  $W_t$  is a linear function of the natural logs of the explanatory variables.

The estimation results are shown in Table 8. The apparent existence of autocorrelated errors is ignored for the reasons given earlier.

Table 8: Estimated Results of the Functional Forms.

	CONST	URT	DEM	DUMM	DISQ	RATE2	cat2	DW	R <sup>2</sup>
WT	-162.53 (-1.7)	-.52 (-.93)	-.001 (-1.27)	-3.49 (-4.12)	4E-5 4.41	182.26 (1.98)	139.12 (1.54)	.63	.640
WT	CONST 45.15 (1.68)	IURT -4.83 (-.019)	IDEM 217728 (2.13)	DUMM -4.04 (-4.77)	IDISQ -140862 (-4.76)	IRATE2 -48.87 (-1.82)	ICAT2 -6E-6 (-.42)	DW .59	R <sup>2</sup> .630
WT	CONST 19.26 (.523)	SURT -.088 (-2.19)	SDEM 6E+8 (-1.69)	DUMM -3.46 (-4.04)	SDISQ 1.7E+10 (3.15)*	SRATE2 3.591 (.097)	SCAT2 789.54 (.54)	DW .58	R <sup>2</sup> .621
WT	CONST 26.03 (1.85)	ISURT -63.0 (-.73)	ISDEM 1.76E+9 (3.48)	DUMM -4.35 (-5.06)	ISDISQ -2.27E+9 (-4.28)	ISRATE2 -25.84 (-1.83)	ISCAT2 -2.49E-9 (-.28)	DW .59	R <sup>2</sup> .621
WT	CONST 84.23 (1.04)	LURT -.27 (-.069)	LDEM -11.69 (-1.36)	DUMM -3.61 (-4.32)	LDISQ 3.30 (5.15)	LRATE2 200.71 (2.16)	LCAT2 154.03 (1.72)	DW .64	R <sup>2</sup> .680
LWT	CONST -11.48 (-1.57)	URT -.11 (-2.54)	DEM 2.5E-5 (.41)	DUMM -.21 (-3.24)	DISQ 4.0E-6 (5.01)	RATE2 14.10 (1.97)	CAT2 10.44 (1.51)	DW .60	R <sup>2</sup> .680
LWT	CONST -2.20 (-.35)	LURT -.50 (-1.67)	LDEM .26 (.39)	DUMM -.22 (-3.54)	LDISQ .29 (5.8)	LRATE2 15.41 (2.14)	LCAT2 11.64 (1.67)	DW .61	R <sup>2</sup> .683

The figures within brackets are t-statistic

1. 'I' stands for the inverse of the variables.
2. 'S' stands for the square of the variables.
3. To avoid having negative values in the observations of the variables, the inflation variable RATE2 is redefined as  $CPI_t/CPI_{t-1}$ .

The non-nested hypothesis tests are then performed. In addition to the tests used in chapter III, two other tests, the PE and the BM test, are used. Results are given in Table 9.

Table 9: Estimated Results of Non-Nested Hypothesis Tests.

MODELS	J	JA	C	N	NL	NA	BM	PE
M1 vs M2	1.48	1.09	1.21	-.67	-1.49	-1.04		
M1 vs M3	1.53	.62	1.2	-.61	-1.36	-.74		
M1 vs M4	2.44*	1.46	1.72	-.84	-1.89	-1.64		
M1 vs M5	.866	-1.83	-.366	.607	1.414	1.88		
M1 vs M6							2.17*	-2.17*
M1 vs M7							2.50*	-2.77*
M2 vs M1	2.15*	1.09	2.00*	-.96	-2.09*	-1.24		
M2 vs M3	-1.09	-1.4	-.88	.57	1.33	1.52		
M2 vs M4	2.45*	1.49	2.17*	-1.31	-2.82*	-1.63		
M2 vs M5	1.80	.939	1.37	-1.44	-1.53	-1.06		
M2 vs M6							4.11*	-2.65*
M2 vs M7							4.44*	-3.66*

M3 VS M1	2.31*	.854	2.14*	-1.06	-2.32*	-1.05
M3 VS M2	2.52*	1.91	2.31*	-1.97*	-4.40*	-2.20*
M3 VS M4	2.37*	.404	2.00*	-.97	-2.12*	-.58
M3 VS M5	2.97*	1.47	2.72*	-1.61	-3.64*	-1.77
M3 VS M6						2.253*
M3 VS M7						-2.09*
						2.68*
						-2.08*
M4 VS M1	2.41*	2.08*	2.13*	-1.17	-2.98*	-2.27*
M4 VS M2	3.06*	1.82	2.81*	-1.73	-3.66*	-2.22*
M4 VS M3	3.32*	2.2*	2.95*	-1.73	-3.67*	-2.45*
M4 VS M5	2.91*	2.41*	2.26*	-.573*	-1.29	-1.06
M4 VS M6						4.95*
M4 VS M7						-3.38*
						5.16*
						-3.97*
M5 VS M1	-.03	-.45	.3	.085	.02	.57
M5 VS M2	-.19	-.30	.95	-.07	-.16	1.15
M5 VS M3	.069	-.47	.36	.09	.20	.48
M5 VS M4	.239	-.502	-.03	-.484	.55	.87
M5 VS M6						1.97*
M5 VS M7						-.74
						2.24*
						-1.17





Table 9 is interpreted in a fashion similar to Table 7. Models 1 through 5 which all have  $W_t$  rather than the natural log of  $W_t$  on the left hand side (LHS) are almost unanimously rejected by the PE and BM tests when tested against models 6 and 7 which have the natural log of  $W_t$  on the LHS. This suggests that the final choice must be made between model 6 and 7. This is not an easy choice. Model 6 is not rejected by any test against any model, whereas model 7 is rejected by the BM test for model 3. On the other hand, when models 6 and 7 are tested against each other, although there is never a rejection, the test statistic values for testing model 6 against model 7 are uniformly larger in magnitude than those for testing model 7 against model 6. This suggests that perhaps model 7 should be preferred to model 6. The closeness of their  $R^2$  reflects this dilemma of choice.

As a further investigation, the Box-Cox transformation test is done on these models (model 6 and model 7). To perform the test, we first hypothesize that the functional form of model 6 is the true functional form. Setting  $\lambda = 1$  would leave the model unchanged while setting  $\lambda = 0$  would transform all the dependent variables of model 6 into the natural log form. With model 6, we first set  $\lambda = 1$  and then set  $\lambda$  unrestricted. The log of the likelihood of these two models under two different restriction on the values of  $\lambda$  are estimated. Two times the difference between these two likelihood function estimates would produce a likelihood ratio (LR) statistic which is distributed as a

chi-square distribution with the degrees of freedom equal to the number of restrictions we build into the estimation. Although we do not restrict the value of  $\lambda$ , we have set the  $\lambda$  value the same for all the independent variables, the degree of freedom is thus one. The same procedure is repeated for model 7. The LR statistics for model 6 and model 7 are 1.13 and 4.54 respectively. When compared to the critical value of 3.84 (at the 5% significance level), the LR statistic of model 6 falls into the acceptance region while that of model 7 falls into the rejection region. Incorporating the fact that model 6 is not rejected when tested against any other model, we can conclude safely that model 6 is a better model than model 7.

Since we have determined the variables before we tested the functional form, it is legitimate to wonder whether the variables determined in chapter III would still be the most suitable variables for this functional form. Thus, the J test, JA test and the C test are repeated with the semi log form. The results remain unchanged. The selection of model 6 as the most appropriate functional form has led us to conclude that it is not always legitimate to choose the model that has the best fit in the presence of other competing models.

## CHAPTER V

### CONCLUSION

In spite of suggestions that the help wanted index is a better candidate for the estimation of the Phillips curve, the results of chapter III indicate that the unemployment rate is a better proxy, providing empirical support for the traditional Phillips curve relationship between the percentage change in wage rate and the unemployment rate.

The result that the forecasts generated by the ARIMA model offer a "better"<sup>1</sup> series for the expected rate of inflation is quite surprising. This is because the leading indicator forecasting method incorporates more information and, as noted earlier, is a better forecasting mechanism (on the basis of mean squared forecast errors calculated in chapter III.) One reason for this result may be the fact that the leading indicator is both costly and time consuming to construct and update. Its high marginal cost may be higher than its marginal benefit, making it uneconomical to employ. Note that this implies a non-Muthian definition of rational expectations.<sup>2</sup> Although the regression model has high  $R^2$  and violates no assumptions of the CLR model after correction, its poor performance supports the view that

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<sup>1</sup>Better does not refer to forecast accuracy but the suitability of the proxy for the expected inflation rate in the Phillips curve specification.

<sup>2</sup> The Muthian definition of rational expectation is that people would use all the information available to them to make their forecasts.

regression gives poor forecasts despite their lack of bias.

Although non-nested hypothesis testing is a good way to test one model against another model, it allows us to accept both models at the same time. When we cannot make a decision over the choice of the models as is the case for model 6 and model 7 of chapter 4, we have to rely on some other tests in order to determine the final model if we really want to pick one out of the two.

Of all the non-nested hypothesis tests employed in this paper, the performance of the J-test is the best of the six. This is because it has comparatively higher discriminating power over the other tests. In the testing of the functional form of the model, only the J test is able to reject both models at the same time when these two models are tested against each other. The one that displays least discriminative power is the N test. When all the other tests reject the models under test, the N test still fails to reject.

#### Further suggestions and remarks

In the testing of the functional form, only a few functional forms are examined. We have not exhausted all the possibilities. Therefore, there may exist some other functional form that can perform better. Besides, all functional forms that we tried are transformations done either to the dependent variable or all of the independent variables with the exception of the dummy

variable. There is a possibility that some of the independent variables take the form of a natural log, some the linear form and some the quadratic form. Different combinations may produce different results. Furthermore, the models tested in this paper are either linear or log linear. Thus, an attempt to apply non-nested hypothesis testing to non-linear models is recommended.

In this paper, the variables for the Canadian Phillips curve were determined first, under the assumption of a linear form, and then the different functional forms are tested assuming that the variables determined previously are the most appropriate ones. A better way to do this is to test the functional form and the variables simultaneously to avoid the potential bias arising from having to decide which one to test first.

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