THE SAMPLING DISTRIBUTION PROPERTIES OF THE BAYESIAN ESTIMATOR IN THE CASE OF AUTOCORRELATED ERRORS: A MONTE CARLO STUDY

by

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ABSTRACT

The purpose of this thesis is to examine the sampling distribution properties of the Bayesian estimator in the context of models with autocorrelated errors. Bayesian estimators have received limited attention in studies regarding autocorrelated error models due to ideological controversy and computational complexities.

The autoregressive [positive AR(1)] and the Moving Average [positive MA(1)] models are used in Monte Carlo experiments to compare the risk (mean square error) of the Pure Bayesian estimator with those of the Ordinary Least Square estimator, the Durbin estimator, the Maximum Likelihood estimator, the related 'autocorrelation' pretest estimators of the Durbin and the Maximum Likelihood estimators, and the Bayesian pretest estimator. Twenty design matrices are employed, including data from related Monte Carlo studies and a variety of real-world economic time series data.

The Pure Bayesian estimator performs better than all the other estimators if the disturbance terms are generated by the AR(1) process and the autocorrelation parameter (ρ) is greater than approximately 0.10. However, when the disturbance term follows the MA(1) process, the Pure Bayesian estimator only dominates for values of the autocorrelation parameter (δ) greater than approximately 0.50. The Ordinary Least Squares estimator dominates for lower values of the autocorrelation parameters ρ and δ.

DEDICATION

DEDICATED TO MUM, DAD, AND AKU

WHO "SLAVED" TO EDUCATE THEIR SON.

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CHAPTER 1

INTRODUCTION TO THE STUDY

1.1 INTRODUCTION

In a general linear statistical model where the disturbance terms corresponding to different observations are correlated, the Ordinary Least Squares estimator (OLS) is no longer BLUE, and thus an alternate estimator may be desired. The Generalized Least Squares estimator (GLS) is BLUE in this context but since the autocorrelation parameter is unknown a priori, it is estimated through various methods resulting in the Estimated Generalized Least Squares estimator (EGLS) which, unfortunately, does not dominate the OLS estimator over the entire range of the autocorrelation parameter. However the EGLS estimator is more efficient, in terms of variance, than the OLS estimator for higher values of the autocorrelation parameter [see Griliches and Rao(1969) and Magee et al(1987)].

Alternatively, the researcher can use traditional hypothesis testing procedures to examine if the classical assumption of uncorrelated error terms is violated or not. Depending on the outcome of this test, the researcher chooses between the OLS estimator and the EGLS estimator. This estimator, which is a dichotomous choice of estimators, is referred to as the 'Autocorrelation' Preliminary Test (pretest) estimator. This estimator has been shown to be inadmissable [see Cohen(1965) and

Zaman(1984)] because of its discontinuity property. An estimator is admissable if it is not dominated, over the entire space of the parameter, by another estimator. The inadmissability of the autocorrelation pretest estimator refers to continuous loss functions. The Stein estimator can be viewed as a way of avoiding the discontinuity feature of pretest estimators. It is weighted average of a restricted estimator and an unrestricted estimator with the weights given as a continuous function of the magnitude of the test statistic used in testing the restrictions. The weighting used by this estimator is a form of shrinkage procedure, where the unrestricted estimator is shrunk toward the restricted estimator. This procedure enables the Stein estimator to combine the two estimators in a continuous fashion instead of choosing between them, thus the Stein estimator is a form of 'smoothing' the pretest estimator. Following this Stein principle, it is a premise of this thesis that 'smoothing' pretest estimators for autocorrelated error models will result in a marked improvement in their sampling distribution properties.

The above mentioned estimators have been given considerable exposure in the existing literature while another group of estimators, the Bayesian alternatives, has received limited attention. This thesis examines the sampling properties of the Bayesian alternatives and compares the mean square error of the Bayesian alternatives to the traditional estimators used in the econometric literature in the context of autocorrelated error models.

The Bayesian estimators, though non-linear, are a form of 'smoothing' the traditional pretest estimators, thus they are a continuous function of the data. The Pure Bayesian estimator is a weighted average of an infinite number of Generalised Least Square estimators, with the weights determined by the marginal posterior density of the autocorrelation parameter. Thus the pure Bayesian estimator can be viewed as the ultimate form of 'smoothing' among all 'continuous' estimators of an autocorrelated error model. Bayesian pretest estimators are also a continuous function of the data. We structure the testing procedure such that the null hypothesis is a composite rather than a point hypothesis. This structure of the null hypothesis enables us to avoid introducing an informative prior in defining the weights used in the formulation of the Bayesian pretest estimator. The weights used are based on the posterior odds ratio of the competing hypotheses.

Under general conditions, Bayesian estimators are admissable, consistent and minimize average risk [Zellner(1971)]; this is supported by several small sample experiments [see Swamy and Rapportport(1975) and Thornber(1974)] in which Bayesian estimators have performed very well.

A Monte Carlo study is undertaken to compare the risk (mean square error) of the conventional estimators with their Bayesian competitors, for a model with autocorrelated error terms. Twenty different design matrices (data sets) comprising of economic time series data and artificial data are used to avoid the problem of generalisation from a single or few data sets.

The general conclusion from the study is that the pure Bayesian estimator is uniformly superior to all the other estimators examined over the entire range of the autocorrelation parameter for the AR(1) model except for OLS where the range is $\rho>0.10$. For the MA(1) model, the pure Bayesian estimator is the best among all the estimators only for large values of the autocorrelation parameter ($\delta\geq0.50$), but does not perform well for smaller values. OLS performs well for values of the autocorrelation parameter close to zero for both models, but was the worst among all the estimators for higher values of the autocorrelation parameter (≥0.50) for both models.

1.2 CONVENTIONAL PRE-TEST ESTIMATORS

Econometric studies often incorporate *a priori* information exogenous to the model being dealt with. This information could be some knowledge suggested by economic theory or from previous research about the parameters being estimated. Usually the validity of this extraneous information is tested, using conventional hypothesis testing procedures, and on the basis of the test, this information is either included in the estimation procedure or discarded. The estimators generated from this procedure are referred to as pretest estimators. These estimators usually have very complicated sampling distributions.

A review of the existing literature reveals that such testing procedures are applied in a variety of circumstances.

Griffiths and Beasley(1984) and Fomby and Guilkey(1978) used this procedure to test for the presence of autocorrelated errors and to choose between the Ordinary Least Square estimator and some version of the Estimated Generalized Least Square estimator depending on the outcome of the test.

Wallace(1977) and Gourieroux and Trognon(1984) investigated on the basis of 'F' and Hausman test respectively, whether to include or exclude a subset of explanatory variables in a linear model. This defines the specification pretest estimator.

Fomby et al (1984) looked at the choice of the polynomial degree in an Almon lag scheme by hypothesis testing.

These references are by no means exhaustive, but they are a few cases where pretesting has played a significant role in studies.

The sampling distributions of pretest estimators are very difficult to analyse, because they depend on a number of things: (1) The test used in the pretesting procedure (2) the level of significance chosen for the test (3) The design matrix used (Griffiths and Beasley (1984)) (4) The nature of the hypothesis being tested and (5) The methods used in estimating some parameter values (Judge and Bock(1978))

Uncertainty about inclusion or exclusion of a variable or a choice of functional form are usually incorporated into the model as a form of general linear restriction or hypothesis and traditional testing procedures are used for choice purposes. Thus the conventional pretest estimator is formulated on the basis of a restricted estimator and an unrestricted estimator. An example of a general case of pretest estimator is given in Judge et al(1985), Judge and Bock(1978) and Kennedy(1985). The specific pretest estimator we are interested in is the autocorrelation pretest estimator as per Fomby and Guilkey(1978) and Judge and Bock(1978) . Given the model

$$Y_t = X_t\beta + e_t \quad \text{.............................} 1.1$$

$$e_t=\rho e_{t-1}+u_t \quad \text{..........................} 1.2$$

the null hypothesis is of the form H_o: $\rho=0$ and the alternative hypothesis is H_o: $\rho \neq 0$. A Durbin-Watson(DW) test is used to test for the compatibility of the sample information and the null hypothesis. If the null is accepted, then the Ordinary Least Square estimator β^{OLS} is used as an estimate of β, on the other hand rejecting the null implies the use of some version of the Estimated Generalised Least Square estimator β^{EGLS}. Thus the estimator that results is based on a test of significance. Specifically,

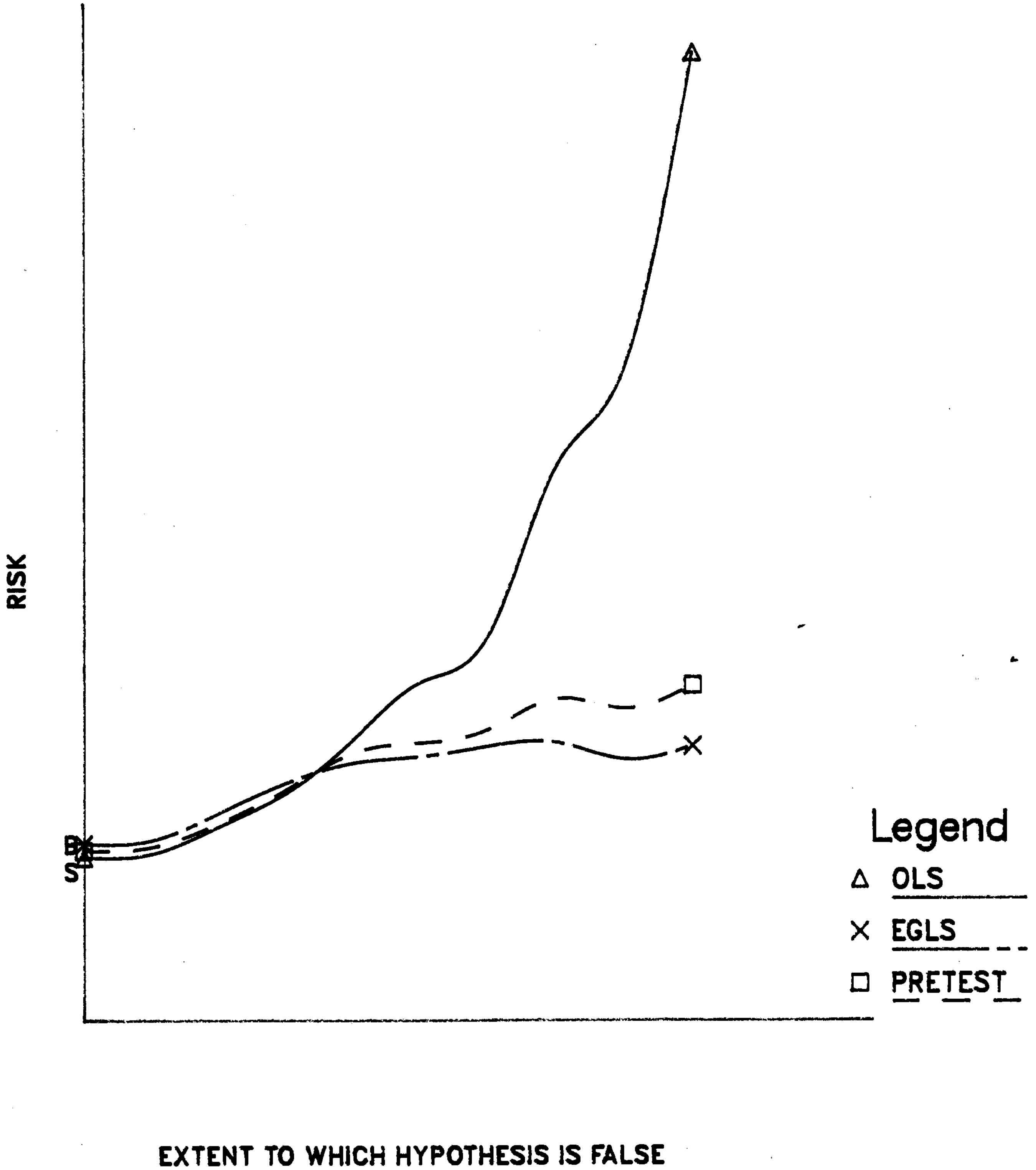
$$\beta^{PT} = I_{(A)(DW)}\beta^{OLS} + I_{(R)(DW)}\beta^{EGLS} \qquad \text{.............1.3}$$

where $I_{(...)(DW)}$ are indicator functions, which take on the values $I_{(A)(DW)}=1$, $I_{(R)(DW)}=0$ if DW (the test statistic) falls in the acceptance region and $I_{(A)(DW)}=0$, $I_{(R)(DW)}=1$ if DW falls in the rejection region. The acceptance and rejection regions in the DW test is described fully in the next chapter.

Thus the conventional pretest estimator is a dichotomous choice between OLS and EGLS estimators. It is interesting to note that in the case of the autocorrelation pretest estimator, the OLS is the restricted estimator and the EGLS estimator is the unrestricted estimator as opposed to the reverse in the general cases discussed in the literature.

The theoretical performance of the OLS, EGLS and the autocorrelation pretest estimator measured in terms of mean square error (MSE) is depicted in Figure 1.1. The comparison in the general case is fully explained in Judge et al(1985 pp 74-76) and Kennedy(1985 pp 160-161). The vertical axis measures the risk associated with the use of a given estimator, while the horizontal measures the extent to which the null hypothesis is violated. Irrespective of the 'truth' or 'falsity' of the extraneous information, i.e, whether the errors are autocorrelated or not the OLS, EGLS and the autocorrelation pretest estimators are all unbiased asymptotically, though EGLS and the pretest estimator are biased in small samples, thus discussion of the performance is based on the mean square error criterion.

FIGURE 1.1
RISK OF OLS,EGLS AND PRETEST ESTIMATOR



When the null hypothesis is true (at the origin), OLS is BLUE and thus has the least variance among all these estimators, but as the hypothesis becomes increasingly implausible, its variance increases. This behaviour could be attributed to the fact OLS loses its BLUE properties for large values of the autocorrelation parameter, thus its relative variance increases in relation to other estimators. Thus as we move further away from the origin, the risk of OLS rises as OLS consistently gets worse. Its risk can be shown as an upward sloping line starting at S.

When the null hypothesis is true, EGLS is worse than OLS , because the latter incorporates the fact that $\rho=0$. Thus it has a risk above that of OLS at the origin. As the null hypothesis becomes less true, because EGLS allows ρ to be non-zero, it attains a lower variance than OLS after a point. Its risk function is a slightly upward sloping line starting at B. The risk of EGLS intersect that of OLS at a value of ρ to the left of which OLS outperforms EGLS. This value of ρ is given as $|\rho|\leq 0.3$ in Griliches and Rao(1969), Spitzer(1979), Magee et al (1987).

The pretest estimator, which is a dichotomous choice between OLS and EGLS depending on the outcome of the test, has a mean square error between that of OLS and EGLS if the null hypothesis is true, because it is a combination of both the OLS and the EGLS estimators. If the null hypothesis is far from being plausible, the pretest estimator correctly identifies the falsity of the hypothesis and has a mean square error close to that of EGLS (shown by the risk of the pretest approaching that of EGLS as we move further from the origin). The pretest estimator performs quite well for 'very true' and 'very false' hypothesis, however in between these extremes it performs poorly. This behaviour is explained by the fact that, for a given power of test (say 60%), the pretest estimator, in repeated samples, does not always accept or reject the hypothesis. It incorrectly accepts the hypothesis 40% of the time and correctly rejects it 60% of the time. When it incorrectly accepts the hypothesis to be valid, the estimates produced are unbiased but inefficient, but when it correctly rejects the hypothesis, the estimates produced are biased and have a lower variance. The combination of these two estimates produce a pretest estimator with a higher mean square error. Consequently, between the two extremes, the pretest estimator has a mean square error above both the OLS and EGLS. The risk of the autocorrelation pretest estimator intersects that of OLS at a value of ρ approximately equal to 0.3 according to results from a Monte Carlo study done by this author. The above described behaviour of the OLS, EGLS and autocorrelation pretest estimator is confirmed by a Monte Carlo study undertaken by this author, a result consistent with the literature cited earlier. This performance of the pretest estimator stems from its dichotomous nature; that is, it is a dichotomous weighted average of the two estimators and a discontinous function of the data.

Zaman(1984) argued that "under yet unknown but probably quite general regularity conditions, discontinous functions of the data are inadmissible decision rule" Cohen(1965) also showed that because of the implicit discontinuity of the weights used in the pretest estimator, under square error loss measure, this estimator is inadmissable, thus showing that under the above criterion this estimator can be beaten.

One other problem with the conventional pretest estimator is the arbitrary choice of significance level.

Consequently an estimator which is a continous function of the data should be more appropriate than the conventional pretest estimator. Stein-like estimators and Bayesian estimators are examples of such estimators.

1.3 STEIN-LIKE ESTIMATOR

Using the orthonormal linear model, Scolve, Morris and Radhakrishnan(1972) showed that under square error loss criterion, Stein-like estimators dominate the OLS estimator over the entire parameter space. Even though most of the studies that deal with Stein-like estimators are concerned with special case models, the results suggest that over the entire parameter space, there exist some 'simple' estimators which are uniformly superior to the conventional least square estimator.

Charles Stein(1955) found that if the number of parameters is greater than two, then it is possible to improve upon the performance of the conventional (least square) estimator under square error loss measure.

In the traditional context, Stein-like estimators are formulated in terms of restricted and unrestricted estimators. The Stein estimator is a weighted average of the restricted and unrestricted estimators, where the weights depend on the magnitude of the 'F' statistic used to test the restrictions. Stein-like estimators use a shrinkage procedure, where the unrestricted estimator is shrunk toward the restricted estimator, and the shrinkage factor is determined by the 'F' statistic. In the process these estimators combine the restricted and unrestricted estimators in a continuous fashion instead of choosing between them like the pretest estimator. If the number of parameters to be estimated is greater than two, then these estimators generally have a risk everywhere below that of the conventional least square estimator.

Given the restrictions

$R\beta = r = 0$

the Stein estimator is generally specified as

$$\beta^{S} = (1 - a/u)\beta^{UR} + (a/u)\ \beta^{R} \qquad \text{1.4a}$$

where 'a' is a scalar given by (K-2)(T-K)/(K(T-K+2)), 'u' is the test statistic which has an 'F' distribution with K and (T-K) degrees of freedom and β^{UR} is the unrestricted estimator.

Consider the possibility of formulating a Stein-like estimator for the autocorrelation case where the shrinkage procedure is such that the EGLS estimator is shrunk toward the OLS estimator. In this process, 'autocorrelation' Stein estimators combine the OLS and the EGLS estimators instead of choosing between them,with the shrinkage factor depending on the magnitude on the test statistic used to test the hypothesis of autocorrelated errors . Such Stein-like estimators in the context of autocorrelated error model (an invention of this author), referred to as 'autocorrelation' Stein estimators, could be structured as

$$\beta^{as} = a(DW_c)\beta^{EGLS} + 1 - a(DW_c)\beta^{OLS} \qquad \text{1.4b}$$

(note: this formulation pertains to the case where $\rho > 0$ so that DW is expected to be smaller as the autocorrelation becomes more severe)

where

β^{as} is the 'autocorrelation 'Stein estimator

a is a scalar which depends on the design matrix

DW_c is the calculated test statistic

β^{EGLS} and β^{OLS} are the EGLS and the OLS estimators respectively

$a(DW_c)$ determines the shrinkage factor, i.e. by how much the OLS estimator has to be shrunk toward the EGLS estimator. This factor can sometimes shrink the OLS past the EGLS estimator; to prevent this from happening, the factor could be truncated after a point, and another Stein-like estimator, the 'autocorrelation' Stein postive-rule estimator is obtained. Following the parallel with the traditional Stein estimator, this estimator may be superior on the basis of mean square error to the original 'autocorrelation' Stein estimator.

To avoid a(DW) shrinking past 0 or 1, a will have to be approximately 1/2 as DW approaches 2, so that larger weights (close to 1) are placed on OLS and smaller weights(close to 0), is placed on EGLS. The opposite will be true for the case where DW approaches 0. The shrinkage procedure employed by Stein-like estimators is a form of 'smoothing' the conventional pretest estimator, thus they are a continous function of the data used. After this study had been completed, it was suggested that and extremely 'simple' way of 'smoothing' the traditional pretest estimator is to weight EGLS by $\hat{\rho}$ and OLS by $(1-\hat{\rho})$ to obtain a

Stein-like variant. Although the dramatic simplicity of this method suggest that it will be worthwile investigating, time constraints made this impossible.

The sampling distributions of traditional Stein-like estimators are discussed in detail by several authors, e.g. Judge and Bock(1978, pp 169-206), Judge et al(1985, pp 82-89) . Stein-like estimators have not been formally developed for the autocorrelated error model, thus their sampling performance may or may not differ from that of the traditional Stein estimator. However the 'smoothed' weights used by the 'autocorrelation' Stein estimators make them admissable and more attractive than the autocorrelation pretest estimator.

Despite the advantages of the Stein-rule estimators, they have a few flaws as discussed in Kennedy(1985,pp 165)

(1) their small sample distribution (properties) are unknown.

(2) in small samples, they cannot be used for hypothesis testing or creating confidence intervals because of their unknown sampling properties.

(3) They can only dominate OLS everywhere if and only if there are more than two independent variables in the statistical model.

(4) They assume the error terms to be normally distributed

(5) They are non-linear.

(6) Much of the risk gains occur when the shrinkage vector is correct.

1.4 BAYESIAN ESTIMATORS AND THE BAYESIAN PRE-TEST ESTIMATOR

Bayesian pretest estimators use as their weighting system the posterior probabilities associated with a given hypothesis. This estimator employs the notion of posterior odds-ratio, which is the ratio of the posterior probabilities on each hypothesis. The Bayesian pretest estimator is formulated as

$$\beta^{baypt} = P(H_o|Y).\beta^{OLS} + P(H_1|Y).\beta^{EGLS} \quad \ldots\ldots 1.5a$$

$$= P(H_o|Y).\beta^{OLS} + [1-P(H_o|Y)].\beta^{EGLS} \quad \ldots\ldots 1.5b$$

Bayesian and Stein-rule estimators are continuous functions of the data, unlike the traditional pretest estimator.

The Bayesian pretest estimator can be viewed as another 'smoothed' version of the traditional pretest estimator, or alternatively the traditional pretest estimator could be viewed as a special case of the Bayesian pretest estimator where the posterior probabilities associated with each hypothesis assume values of 0 or 1. The posterior probabilities used as weights are obtained by integrating the posterior density function of the autocorrelation coefficient within the range stipulated by the null hypothesis. To obtain the marginal posterior density of the autocorrelation coefficient, we combine the likelihood function $l(\beta,\rho,\sigma|Y)$ with a prior density $g(\beta,\rho,\sigma)$ to obtain a joint posterior density $g(\beta,\rho,\sigma|Y)$. (The prior density could be non-informative (ignorance prior) or informative. Note that all the prior densities used in this thesis are non informative).

Next we integrate out β and σ to obtain $g(\rho|Y)$. This marginal posterior of ρ is then integrated numerically within the range of the null hypothesis to obtain $P(H_o|Y)$. This determines the weight used in our Bayesian pretest estimator.

In this thesis, our null hypothesis is composite; i.e. it specifies a range of values of the autocorrelation parameter (rather than a single value) for which OLS outperform the EGLS estimator.Thus

H_o: $|\rho|<0.3$.

The choice of this null hypothesis makes the autocorrelation pretest estimator a special case in this context. The nature of this composite hypothesis allows us to overcome the dichotomous choice of estimators, since the probability associated with each hypothesis will not always jump from 0 to 1, but will be a continuous function of the data used. Thus our Bayesian pretest estimator can be formulated as

$$\beta^{baypt}=P(|\rho|<0.3)\beta^{OLS} + (1 - P(|\rho|<0.3))\beta^{EGLS} \quad \text{.........1.5c}$$

The pure Bayesian estimator is a counterpart of the EGLS estimator much as the Bayesian pretest estimator is a counterpart of the conventional pretest estimator. The pure Bayesian estimator is a generalization of the Bayesian pretest estimator. It is a weighted average of different Generalized Least Square estimators over the entire range of the autocorrelation coefficient, with the weights given by the marginal posterior density of the autocorrelation coefficient. Specifically

$$\beta^{*}=\int \beta^{GLS} g(\rho|Y)\, d\rho \qquad 1.6$$

The pure Bayesian estimator could be viewed as the ultimate form of 'smoothing', because it weights an infinite number of GLS estimators instead of just two estimators. A detailed discussion of this estimator is presented in chapter 2.

1.5 PURPOSE OF STUDY

The motivation for this thesis can be traced to a study by Adjibolosoo(1987). He considered a smoothed version of the traditional pretest estimator under a heteroskedastic model. The smoothed pretest estimator is a weighted average of two estimators (the OLS and the EGLS estimators) with the weighting function being a continuous function of the data rather than a discontinuous one. He concluded that the smoothed version generally had a lower mean squared error than the traditional pretest estimator, and is an attractive alternative to the conventional estimators in a model with heteroskedasticity.

Duplicating his experiment for the autocorrelated model was the initial objective of this thesis, but preliminary Monte Carlo experiments suggested that the Bayesian estimator may be uniformly superior, on the basis of MSE, to all the conventional estimators (OLS, EGLS, traditional pretest) and the smoothed pretest estimator in the context of the autocorrelated error model. The Bayesian estimator was for some cases (design matrices) even better than the GLS estimator. The GLS estimator is BLUE in the autocorrelated error model, but the Bayesian estimator is a non-linear estimator, biased in small samples, and this, coupled with the possibility of sampling errors (600 replications is used for this study), makes it conceivable that such a result could be obtained.

Our attention was then directed to the question of why the Bayesian estimator is hardly ever used in econometric analysis of such models. Despite its computational complexities, the Bayesian estimator in this context is quite costless in terms of computation given the present advanced computer packages.

Various authors have provided "simple" formulas for the computation of the Bayesian estimator under autocorrelated errors and for different cases, some of which are discussed in the next chapter. Judge et al(1985) provided a Bayesian estimator for the model with an intercept term and a stationary AR(1) process as well as the MA(1) process. Zellner and Tiao (1964) suggested a formula for a model without a constant term and a non-stationary AR(1) process, and Richard(1975) considered higher order AR processes and different prior densities.

In this thesis we will consider both the AR(1) and the MA(1) models. We will compare the sampling distributions of the Bayesian estimator to the estimators generally used in models of such nature for different design matrices including cases where the matrices contain an intercept term .

We will try and find out how the Bayesian estimator performs in relation to these other estimators on the basis of mean square error (MSE) criterion, and also try to rationalize why the Bayesian estimator is seldom used in econometric analysis of models with autocorrelated error terms.

The Stein estimator discussed in section 1.3 achieves its attractive properties through its 'smoothing' process, by using weights that are continuous function of the relevant test statistic . Its shrinking procedure is a form of 'smoothing' as opposed to a dichotomous choice of estimators. However, the Stein estimator, not unlike the traditional pretest and the Bayesian pretest estimators, considers only two estimators, the OLS and the EGLS; thus the pure Bayesian estimators ability to weight several GLS estimators over all values of the autocorrelation coefficient could be viewed as a sort of ultimate form of 'smoothing'. Because of this it is our prior belief that the pure Bayesian estimator will have equally attractive features and will generally have a lower MSE than its competitors. Thus the Bayesian estimator is likely to be superior to these conventional estimators and thus be an attractive alternative in a model with autocorrelated errors.

1.6 DESIGN OF STUDY

Monte Carlo experiments will be used to compare the sampling properties of various estimators in an autocorrelated error model. We will be concerned with the stationary AR(1) and the invertible MA(1) models, and performance of estimators will be judged on the mean square error (MSE) of the various estimators relative to that of the GLS estimator. Thus a ratio of a given estimators MSE to that of GLS that is less than one should be perceived as a better performance by the given estimator relative to the GLS estimator. For each of the models mentioned above, the following estimators, described in sections 2.3 and 2.7 below, will be examined

(1) Pure Bayesian Estimator (BE)

(2) Bayesian Pretest Estimator (BPE)

(3) Generalized Least Squares Estimator (GLS)

(4) Traditional Pretest Estimator (PTE)

(5) Ordinary Least Squares Estimator (OLS)

(6) Maximum Likelihood Estimator (MLE)

(7) Durbin Estimator (DE) (for the AR(1) model) and the MacDonald and MacKinnon(1985) EGLS (for the MA(1) model).

For each experiment the performance of nine estimators (2 traditional pretest with different EGLS components and 2 Bayesian pretest with different EGLS components, in addition to the other 5 above) will be examined.

For each model, twenty design matrices will be used with sample sizes ranging from 10 to 65. All the design matrices are described in detail in chapter 3. The design matrices consist of artificial data generated through the IMSL Fortran Library, data obtained from various studies regarding performance of estimators in an autocorrelated error model, and economic time series data obtained from Simon Fraser University data Library. 600 replications are used to determine the performance of each estimator.

For the MA(1) model, a preliminary Monte Carlo study will be undertaken to obtain a range of the autocorrelation coefficient (δ) within which the OLS estimator outperforms the EGLS estimator. For the AR(1) process, this range is given as $-0.3 \leq \rho \leq 0.3$ from studies such as Griliches and Rao(1969), Spitzer(1979) and Magee,Ullah and Srivastava (1987).

All Monte Carlo experiments are done using Fortran 77 language, and the Fortran programs used will consist of subroutines written by this author and routines called from the IMSL and NAG Fortran Library.

A review of the existing literature on performance of estimators in serial correlated error models is presented in Chapter 2. Chapter 3 consists of the model, estimation techniques used and a detailed discussion of the Monte Carlo experiment undertaken. Chapter 4 contains the results of the experiments and Chapter 5 is conclusions and recommendations based on our results.

CHAPTER 2

ESTIMATORS IN THE AUTOCORRELATED ERROR MODELS

(A REVIEW OF THE LITERATURE)

2.1 INTRODUCTION

In this thesis we will be concerned with the first order autoregressive process AR(1), and the first order moving average process MA(1).

2.2 THE AR(1) MODEL

The AR(1) model is given by

$Y_t = X_t\beta + e_t$ 2.1a

$e_t=\rho e_{t-1}+u_t$ 2.1b

where Y_t is a (Tx1) observations on the dependent variable, X_t is a (TxK) design matrix and e_t is a (Tx1) disturbance vector with $E(e)=0$ and $V(e)=\sigma^2\Omega$ and u is a random variable with mean zero and a constant variance σ^2 . ρ is the autocorrelation coefficient and for a stationary process $|\rho|$ is less than 1

2.3 ESTIMATION UNDER THE AR(1) MODEL

2.3.1 GENERALIZED LEAST SQUARES ESTIMATOR

When the true value of ρ is known, this estimator is the best linear unbiased estimator(BLUE). The estimation procedure requires multiplying equation 2.1a by ρ and lagging one period. Subtracting the resulting equation from 2.1a gives

$$Y_t - \rho Y_{t-1} = (X_t - \rho X_{t-1})\beta + (e_t - \rho\, e_{t-1}) \quad \ldots\ldots\ldots\ldots\ldots 2.1c$$

or

$$Y^* = X^*\beta + e^* \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.2$$

Ordinary Least square estimation procedure is used on 2.1c to obtain estimates of β. In terms of matrices, we will have to find a transformation matrix P such that $P'P=\Omega^{-1}$. Thus the generalized least square estimator will be

$$\hat{\beta} = (X'P'PX)^{-1}(X'P'PY) \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.3a$$

$$=(X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.3b$$

$$=(X^{*\prime}X^*)^{-1}X^{*\prime}Y^* \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.3c$$

However, if ρ is unknown, the we will have to estimate $\hat{\rho}$,by any one of the methods described later in this chapter, and use this estimate in our transformation procedure. The resulting estimator is referred to as an Estimated Generalized Least Squares estimator (EGLS).

VARIOUS ESTIMATED GENERALIZED LEAST SQUARES ESTIMATORS.

There are various forms of this estimator, but we shall discuss only the ones used in this study. The choice of these estimators has been based on the support they gained from earlier studies.

2.3.2 DURBIN (1960) ESTIMATOR: (DE)

From equation 2.1c

$$Y_t = \rho Y_{t-1} + (X_t - \rho X_{t-1})\beta + (e_t - \rho\, e_{t-1}) \quad \ldots\ldots\ldots\ldots\ldots 2.4$$

The Durbin(1960) estimator uses a two-step procedure. First Y_t is regressed on Y_{t-1}, X_t, and X_{t-1}, and the coefficient of the lagged values of Y is taken as an estimate of ρ. Even though this estimate of ρ is biased, it is consistent and efficient. Second, this estimate of ρ is used to transform the original variables as in equation 2.1c, and an OLS regression is done on the transformed variables to obtain estimates for β. This estimation procedure stops after the second stage and thus is not an iterative procedure. Notice that this estimator uses a (T-1)xT transformation matrix made up of 1 on its main diagonal and $-\rho$ as its leading (left) off-diagonal. A modified version of this estimator which has been shown to be more efficient than the original version uses a TxT transformation matrix P1, with the first row consisting of $\sqrt{1-\rho^2}$ as its first element and 0 for the others. This modified version is employed in this thesis.

2.3.3 MAXIMUM LIKELIHOOD ESTIMATOR (MLE)

If the disturbance error term e_t can be assumed to have a specific distribution (normal distribution in this case), then we can employ the technique of maximizing the log-likelihood function with respect to β to obtain the Maximum Likelihood Estimator (MLE). In the case of normally distributed error terms, this procedure provides estimates that are asymptotically equivalent to most EGLS estimators. Specific algorithms for maximizing the log-likelihood function have been suggested by several authors, Dent(1974) and Beach and Mackinnon (1978) are the most popular ones. The latter is a modification of the Cochrane-Orcutt (C-O)(1949) procedure that allows for the inclusion of the first observation. A summary of the algorithm is:

1)Starting with a choice of $\hat{\rho}=0$, calculate the C-O estimator of β including the first observation.(i.e. use P1 instead of P as the transformation matrix)

2)Substitute this value of $\hat{\beta}$ into a log likelihood function which contains the Jacobian of the transformation matrix P1. The concentrated log likelihood function is given as

$$L(\beta,\rho)= \text{const. } 1/2 \log(1-\rho^2)-T/2 \log((1-\rho^2)(y_1 - X_1\beta)^2 + \sum_{t=2}^{T}(y_t - \rho y_{t-1} - X_t\beta + \rho X_{t-1} \beta)^2 \qquad \text{2.5}$$

and maximize this function with respect to ρ, β held fixed.

3)Substitute this estimate of $\hat{\rho}$ into P1 and repeat steps 1 and 2 until sucessive values of $\hat{\rho}$ are close together. The corresponding $\hat{\beta}$ is the MLE of β. Beach and Mackinnon developed a computationally efficient way of implementing this procedure. The solution to (1) is 2.3a using the P1 transformation matrix, and they provided the solution to (2) by differentiating the log likelihood function with respect to ρ and setting it to zero, i.e.

$$\partial L/\partial \rho = \rho^3 + a\rho^2 + b\rho + c = 0 \quad \text{......2.6}$$

where

$$a = -(T-2)\Sigma A_t A_{t-1} \,/\, [(T-1)(\Sigma A_{t-1}^2 - A_1^2)] \quad \text{......2.7}$$

$$b = [(T-1)A_1^2 - T\Sigma A_{t-1}^2 - \Sigma A_t^2] \,/\, [(T-1)(\Sigma A_{t-1}^2 - A_1^2)] \quad \text{......2.8}$$

$$c = T\Sigma A_t A_{t-1} \,/\, [(T-1)(\Sigma A_{t-1}^2 - A_1^2)] \quad \text{......2.9}$$

and

$A_t = y_t - X_t\beta$ for a given β

Equation 2.6 can be solved by the standard two step procedure for solving a cubic equation.

a) θ between zero and π radians

b) calculate according to

$$\theta = \cos^{-1}((q\sqrt{27}) / (2p\sqrt{-p})) \quad \text{......2.10}$$

where

$$q = c - ab/3 + 2a^3/27 \quad \text{......2.11}$$

$$p = b - a^2/3 \quad \text{......2.13}$$

Then the desired root of 2.6 is

$$\hat{\rho} = -2\sqrt{-p/3}\cos(\theta/3 + \pi/3) - a/3 \quad \text{......2.14}$$

This equation will be used to obtain the value of $\hat{\rho}$ in our iterative procedure.

2.3.4 THE BAYESIAN ESTIMATOR (BE)

A detailed discussion of the Bayesian estimation procedure under AR(1) errors is given by Zellner and Tiao(1964), Richard(1975) and Judge et al(1985) to mention a few. There are minor differences between these approaches, because they apply to different cases as mentioned in section 1.5. Despite these differences, the basic technique of estimation in the Bayesian framework is applied in all cases. Judge et al's(1985) approach is presented below.

Following the discussion of Bayesian analysis in section 1.4 above, we would have to obtain a marginal posterior distribution of the parameters of interest by combining a prior distribution with the likelihood function and integrating out the parameters of no interest. The likelihood function of the model under the assumption of normality is given as

$$l(\beta,\rho,\sigma|Y)=f(y_1).f(y_2|y_1).......f(y_T|y_{T-1})............2.15a$$

where

$$f(y_1)=(2\pi)^{1/2}\sqrt{1-\rho^2/\sigma^2}\exp[-1-\rho^2/2\sigma^2(y_1-X_1\beta)^2]........2.15b$$

and

$$f(y_t|y_{t-1}=(2\pi\sigma^2)^{1/2}\exp[-1/2\sigma^2(y_t-\rho y_{t-1}-X_t\beta+\rho X_{t-1}\beta)^2]..2.15c$$

thus

$$l(\beta,\rho,\sigma)= \sqrt{1-\rho^2}/(2\pi\sigma^2)^{T-2} \exp\{-1/2\sigma^2[(y_1\sqrt{1-\rho^2}-X_1 \beta\sqrt{1-\rho^2})^2 + \sum_{t=2}^{T}(y_t - \rho y_{t-1} - X_t\beta + \rho X_{t-1}\beta)^2]\} \quad \ldots\ldots\ldots 2.15d$$

Combining this likelihood function with an uninformative prior density given by

$$g(\beta,\rho,\sigma) \;\alpha\; (\sqrt{1-\rho^2})^{1/2}\sigma^{-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots 2.16$$

yields the joint posterior density function

$$g(\beta,\rho,\sigma) \;\alpha\; \sigma^{-T+1} \exp[-(Y^* - X^*\beta)'(Y^* - X^*\beta)/2\sigma^2] \quad \ldots 2.17$$

where Y^* and X^* are as defined in equation 2.2. The prior density given in 2.16 follows Fomby and Guilkey(1978) and it is obtained by assuming that β, σ and ρ are distributed independently *a priori* and applying Jeffreys' rule to each parameter separately. β and $\log\sigma$ are uniformly distributed over the ranges $-\infty<\beta<\infty$ and $0<\sigma<\infty$ respectively. ρ has a beta density function with parameters (1/2, 1/2) over the range $|\rho|<1$. The prior for ρ is given as $\pi^{-1}(1-\rho^2)^{1/2}$. Combining the three separate priors gives 2.16. On intergrating out σ in 2.17, Judge et al(1985) obtained a bivariate posterior density for β and ρ as

$$g(\beta,\rho|Y) \;\alpha\; (RSS)^{-T/2} [1 + (\beta-\hat{\beta})' X^{*\prime} X^* (\beta-\hat{\beta})/RSS]^{-T/2} \quad ..2.18$$

where

$$RSS = (Y^* - X^*\beta)'(Y^* - X^*\beta)$$

and $\hat{\beta}$ is as defined in equation 2.3. Both RSS and β depend on ρ.

For a given ρ, the density $g(\beta|\rho,Y)$ is a multivariate 't' distribution with mean $\hat{\beta}$, therefore if ρ is known the Bayesian estimator is the GLS estimator, but if ρ is unknown, then we need to obtain the marginal posterior distribution for β (the parameter of interest here) $g(\beta|Y)$, by integrating out ρ in 2.18. However; this integration procedure requires the use of numerical methods

$$E(\beta|Y)=\int\int\beta\ g(\beta\rho|Y)d\beta d\rho$$

$$= \int\int\beta\ g(\beta|\rho Y)\ g(\rho|y)\ d\beta d\rho$$

$$=\int\hat{\beta}\ g(\rho|Y)d\rho \qquad \text{2.19}$$

This implies that the Bayesian estimator for β (the mean of the marginal posterior distribution of β) is a weighted average of GLS estimators with the weights given by the marginal posterior density for ρ. The density $g(\rho|Y)$ is given in Judge et al(1985) as

$$g(\rho|Y)\ \alpha\ (RSS)^{-(T-K)/2}\ |X^{*\prime}\ X^{*}|^{1/2}\ d\rho \qquad \text{2.20}$$

thus we numerically integrate 2.20 to obtain the normalising constant, then numerically integrate 2.19 to obtain the Bayesian estimator of β (BE). If one is interested in the kth element of β, β_k then

$$E(\beta_k\ |Y)=\int_{-1}^{1}\hat{\beta}_k\ (RSS)^{-(T-K)/2}\ |X^{*\prime}\ X^{*}|^{1/2}\ /\ \int_{-1}^{1}(RSS)^{-(T-K)/2}$$

$$|X^{*\prime} X^{*}|^{1/2} \, d\rho \quad \ldots\ldots 2.21$$

The Bayesian estimator in an AR(1) model is the weighted average of all the GLS estimators over the entire range of the autocorrelation coefficient ρ with the weights being the heights of marginal posterior density function of ρ.

2.4 TESTING FOR AUTOREGRESSIVE (1) ERRORS

2.4.1 DURBIN-WATSON (DW) TEST

This is the most popular test for first order autoregressive errors, it is based on the residuals of an ordinary least square regression on equation 2.1a and the statistic is given as:

$$d = \Sigma(\hat{e}_t - \hat{e}_{t-1})^2 / \Sigma\hat{e}^2_{t-1} \quad \ldots\ldots 2.22$$

For hypothesis testing of no autocorrelation i.e $H_0 : \rho = 0$ against the alternative $H_1 : \rho > 0$, the DW test rejects the null hypothesis if $d < d^*$ where d^* is the critical value for a specified significance level. A major drawback to this test is that the critical value depends on the design matrix used, thus the exact distribution of the d statistic is not known. Durbin and Watson(1950,1951,1971) overcame this problem by finding two limiting distributions of d, d_L the lower distribution and d_U the upper distribution. They used these two distributions to tabulate critical values for d_L^* and d_U^* for a given T, K and level of significance. Thus in a bounds test of no autocorrelation against postive autocorrelation, the decision rule is as follows; the null is accepted if d< d_L^* and accepted if d> d_U^*. The test is inconclusive if $d_L^* < d < d_U^*$. This inconclusive region is a major problem for this test, but exact critical values can be found by using the White(1978) Shazam procedure. Koerts and Abrahamse(1969) used a Fortran programming technique to obtain exact critical values for the DW test. Also Judge et al(1985) have suggested some approximations of the critical values which can be computed at low costs.

2.4.2 OTHER TESTS

The Berenblut and Webb (BW)(1973) likelihood ratio (g_1) test was used by Judge and Bock(1978) in conjuction with the DW test. The King(1982) $S(\rho_1)$ test was used by King and Giles(1984) in addition to the BW and DW test. The following tests are seldom used due to their computational complexities and cost. Theil and Nagar(1961), Malinvaud(1970), Henshaw(1966) and Koerts and Abrahamse(1968,1969).

2.5 THE MA(1) MODEL

A detailed survey of the MA(1) model is given in Aigner(1971), Naylor,Seaks and Wichern(1972),Pesaran(1971) and Nicholls,Pagan and Terell(1975).

The model is given by

$$Y_t = X_t\beta + e_t \qquad \text{.......... 2.23a}$$

$$e_t=\delta u_{t-1}+u_t \qquad \text{.......... 2.23b}$$

where e_t is a normal random variable with mean zero and variance σ^2 and $|\delta|<1$, i.e. we assume the process is invertible.

2.6 <u>ESTIMATION UNDER MA(1) ERRORS</u>

2.6.1 GENERALIZED LEAST SQUARES (GLS) ESTIMATOR

This method is similar to that of the AR(1) model except for the transformation matrix. If δ is known then the GLS estimator is given as:

$$\hat{\beta} = (X'\Omega^{-1}X)^{-1}(X'\Omega^{-1}Y) \qquad \text{.......... 2.24}$$

where Ω is a TxT diagonal matrix, with the elements on the principal diagonal being $1+\delta^2$, on the two leading off-diagonals δ and all other elements zero.

However, if δ is unknown, then a transformation matrix Q such that $Q'Q=\Omega^{-1}$ is required. This transformation matrix is not unique [Shaman(1973), Pesaran(1973), Balestra(1980)]. In this thesis we use the inverse of the Ω matrix described above for our GLS estimate to avoid using an inappropriate transformation matrix.

2.6.2 ESTIMATED GENERALIZED LEAST SQUARES

MacDonald and MacKinnon(1985) suggested a simple way of estimating the GLS estimator. Their method is to combine equations 2.23a and 2.23b and transform the variables as follows:

define $\xi = u_o$

then

$$Y_1 = X_1\beta + \hat{\delta}\xi + u_1$$

$$Y_2 - \hat{\delta}Y_1 = X_2\beta + \hat{\delta}u_1 + u_2 - \hat{\delta}X_1\beta - \hat{\delta}^2\xi - \hat{\delta}u_1$$

$$= (X_2 - \hat{\delta}X_1)\beta - \hat{\delta}^2\xi + u_2$$

and so on for observations 3 to T. The equation systems given above can be written as

$$Y^* = X^*\beta + Z^*\xi + u \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.25$$

where

$$X^*_t = X_t - \hat{\delta}X^*_{t-1} \qquad X^*_o = 0$$

$$Y^*_t = Y^t - \hat{\delta}Y^*_{t-1} \qquad Y^*_o = 0$$

$$Z^*_t = \hat{\delta}Z^*_{t-1} \qquad Z^*_o = 1$$

OLS regression is then performed on 2.25 to obtain EGLS estimates. It should be noted that all the transformed variables and the error term u have T+1 observations where the first observations are artificial. Since ξ is and estimate of $\hat{u}_o$, OLS regression on 2.25 will still yield (T-K) degrees of freedom.

For the EGLS estimator, the $\hat{\delta}$ suggested by these authors is the method of moments estimator of δ described in the next section below. These authors found this EGLS estimator to be less efficient than the Maximum Likelihood estimator, and sometimes less efficient than the OLS estimator. We used this procedure because of its simplicity.

2.6.3 MAXIMUM LIKELIHOOD ESTIMATOR

Under the assumption that the error terms are normally distributed, this method maximizes the log-likelihood function with respect to β, holding δ constant. This method is similar to the AR(1) method except for the likelihood function. In the MA(1) model, the likelihood function is:

$$L(\beta,\delta,\sigma^2) = -T/2\ln\sigma^2 - 1/2\ln a_n - 1/2\sigma^2(Y^*-X^*\beta)'(Y^*-X^*\beta) \quad ...2.26$$

where a_n is given as $[1-\delta^{2T+2} / 1-\delta^2]$.

Several authors have provided different algorithms for this method (*inter alia* Hannan(1969,1970), Box Jenkins(1970), Osborn(1976) Pagan and Nicholls(1976), Pesaran(1973), Balestra(1980),Godolophin and Gooijer(1982) and Sargan and Bhargawa(1983), MacDonald and MacKinnon(1985)) A specific algorithm for this method suggested by Balestra(1980) consists of the following:

1) Choose a value of δ in the interval -1 to 1 and transform the variables using Q (as above) to obtain Y^* X^* and run an ordinary least square regression of Y^* on X^*. Notice that this is equivalent to using the inverse of the Ω matrix to obtain GLS estimates.

2) Compute the quantity $B=(a_n)^{1/n} SS^*$ where a_n is as defined as earlier and SS^* is the sum of square residuals obtained from the regression in (1) above.

3) Repeat the operation for a number of δ's from -1 to 1 evenly spaced out

4) Choose the δ, and the corresponding β that gives the minimum value of B.

2.6.4 THE BAYESIAN ESTIMATOR (BE)

This procedure is similar to that of the AR(1) model except for the posterior density functions. The main ingredient, the marginal posterior of δ, is given in Judge et al(1985) as:

$$g(\delta|Y) = [1-\delta^{2T+2} / 1-\delta^2]^{-1/2} (RSS)^{-(T-K)/2} |X^{*\prime} X^*|^{1/2} \quad \ldots 2.27$$

2.7 ESTIMATING THE MOVING AVERAGE PARAMETER (δ)

Different estimators have been suggested in the literature but we discuss only the ones used in this thesis.

2.7.1 THE METHOD OF MOMENTS ESTIMATOR

The method of moments estimator of $\hat{\delta}$ provides a consistent but asymptotically inefficient estimator based on the sample autocorrelation coefficient derived from ordinary least square residuals on 2.1a.

The correlation between e_t and e_{t-1} is given as:

$\rho_1 = \delta/(1-\delta^2)$

and a consistent estimator of this correlation coefficient is

$r_1 = \Sigma\hat{e}_t\hat{e}_{t-1} / \Sigma\hat{e}^2_{t-1}$

Inverting the log likelihood function above gives:

$$\hat{\delta} = 1 \qquad -1 \le r_1 \le -0.50$$

$$= (1-(1-4r_1^2)^{1/2}) / 2r_1 \qquad -0.50 < r_1 < 0.50 \quad \ldots\ldots\ldots\ldots 2.28$$

$$= -1 \qquad 0.50 \le r_1 \le 1$$

Because r_1 is a consistent estimator of ρ_1, $\hat{\delta}$ is a consistent estimator of δ, though inefficient. If r_1 significantly exceeds $|0.50|$, then the MA(1) process is suspect and cannot plausibly be employed.

2.7.2 OTHER ESTIMATORS

The Durbin(1959) estimator is biased and inefficient in small samples but asymptotically as efficient as the MLE of $\hat{\delta}$. Anderson(1971) and McClave(1973) suggested two different methods for correcting the small sample bias of the Durbin(1959) estimator.

Another method of estimating δ, is the maximum likelihood method described above.

2.8 TEST FOR MA(1) ERRORS

2.8.1 KING (1983) TEST

This test is similar to the King(1982) test for the AR(1) process; replacing ρ_1 by δ_1. This test is shown to be Most Powerful Invariant(MPI) in the neighbourhood of $\delta=\delta_1$ for all design matrices. His test statistic is:

$$S(\delta_1) = \hat{e}^{*\prime}\hat{e}^{*} / \hat{e}'\hat{e} \ldots\ldots\ldots\ldots 2.29$$

where

$\hat{e}^*$ is the transformed generalized least square estimated residuals obtained under the assumption that $\delta=\delta_1$ and $\hat{e}$ is the ordinary least square residuals. The null hypothesis of no autocorrelation is rejected if $S(\delta_1)$ is less than the critical value $S^*(\delta_1)$. Its critical values can be calculated through methods similar to those used for the DW statistics. King (1983) has tabulated values for the 5% significance level, for both the lower and upper distributions of S(0.5) and S(-0.5). In an empirical comparison King(1983) found S(0.50) to be more powerful than the DW test against positive MA(1) errors when $\delta \geq 0.30$ (δ_1)

2.8.2 OTHER TESTS

Blattberg(1973) and Smith(1976) both found the DW test to have good power against MA(1)errors. The Lagrange Multiplier Test can also be used to test MA(1) errors

2.9 SAMPLING PERFORMANCE OF ESTIMATORS IN AN AUTOCORRELATED MODEL (REVIEW OF THE LITERATURE)

The performance of various estimators in the context of autocorrelated errors have received considerable attention in the literature. Rao and and Griliches(1969) compared the Prais-Winsten(PW) estimator with OLS in the AR(1) model and observed that for values of ρ close to zero, $|\rho|<0.3$, OLS performed better than PW, but for higher values, PW is recommended. Several authors have duplicated this study with different design matrices, but the general theme running through all of them is that they considered only two estimators, OLS and some version of the EGLS estimator. A few of these studies are Kadiyala(1968), Maeshiro(1976), Harvey and McAvinchey(1978), Spitzer(1979), Kramer(1980), Taylor(1981) and Magee, Ullah and Srivastava(1987). These authors differ as to what range of values of ρ, OLS is prefered to EGLS, but they all agree that for ρ closer to zero, OLS should be used but some version of EGLS is recommended as ρ gets larger.

The number of estimators considered were extended by various authors mentioned below, but none considered all the estimators we deal with in this thesis.

Fomby and Guilkey(1978) examined the usual selection of level of significance (α) in the Durbin-Watson test and examined the implications of this procedure by comparing the resulting pretest estimator to the Bayesian alternative on the criterion of mean squared error. Noting that pretest estimators have complicated probability distributions irrespective of the true value of the of the autocorrelation coefficient (ρ), they contended that the choice of α=0.01 or 0.05 might weight the pretest too heavily toward the OLS estimator, thus a more suitable α would have to be higher than what is conventionally used. Their model was along the lines of Griliches and Rao(1969). Given by

$Y_t = X_t\beta + e_t$ 2.36a

$e_t=\rho e_{t-1}+u_t$........................2.36b

$X_t=\lambda X_{t-1}+V_t$............2.36c

where V_t is normally distributed with mean 0 and variance 1 and $-1<\lambda<1$. They used the Prais-Winsten (1954) esimator using the Durbin(1960) estimate of $\hat{\rho}$ as their EGLS estimator. This estimator is referred to as a modified Durbin estimator. For the Bayesian estimator, they assumed an ignorance prior, and used Zellner and Tiao's(1964) solution to the autocorrelated problem. The Bayesian estimator was thus

$$\int_\beta \int_\rho \beta P(\beta\ ,\rho|Y)\partial\rho\partial\beta / \int_\beta \int_\rho P(\beta\ ,\rho|Y)\partial\rho\partial\beta$$

Thus the Bayesian approach of continuously weighting the information about ρ makes it more 'attractive' than the usual handling of the disturbance parameter by preliminary testing, because there is no need in the Bayesian case to specify a level of significance. They considered three sample sizes (15,30,45) , three values of λ (0,0.4,0.8) and eleven values of ρ. The values of λ were used to generate nine design matrices. One thousand replications and thirteen significance levels (0,0.01,0.05,0.1(0.1),1.0) were used. The critical values used for the test were computed using Koerts and Abrahmse(1969) method which gives exact critical values for the DW test. To examine if the usual choice of 0.01 or 0.05 actually minimises the MSE of each estimator they calculated the level of significance (optimal α's) which minimized the mean squared error (MSE) for each design matrix and value of ρ,and concluded that the usual choices are only suitable for quite small values of ρ. Since a priori ρ is unknown, they suggested that an average of all optimum significance levels, for all design matrices and values of ρ would be a more appropriate α level. They found this α level to be approximately 0.5.

Their observations were (1) that the OLS estimator outperformed, on the basis of MSE, both the pretest and the Bayesian estimator for ρ close to zero, but the range of better performance was smaller in the Bayesian case ($|\rho|\leq 0.1$) compared to the pretest estimator ($|\rho|\leq 0.3$). (2) For any fixed α used, the Bayesian estimator was better than the pretest estimator, but for a choice of α=0.5, the loss from using the pretest instead of the Bayesian estimator was not as substantial as the loss incurred for smaller values of α.

One limitation of this study is that it compares the conventional pretest estimator to the pure Bayesian estimator instead of its counterpart the Bayesian pretest. However, initial Monte Carlo studies that we did suggest that the Bayesian pretest is on the average better than the traditional pretest estimator on the basis of mean square error if the probability that the null hypothesis is true is less than 0.4. Fomby and Guilkey failed to compare the Bayesian estimator to the EGLS estimator or the GLS estimator and the choice of a random design matrix may have greatly affected the results. They considered only one EGLS estimator

Judge and Bock(1978) differed from Fomby and Guilkey(1978) in that they considered three different EGLS estimators. (The Durbin(1960) estimator, Cochrane-Orcutt(1949) two-stage estimator, and Prais-Winsten(1954) estimator which uses the Cochrane-Orcutt estimate of $\hat{\rho}$) but did not investigate the performance of the Bayesian estimator in their comparison. They used two pretests (the Durbin-Watson(DW), and the Berenblutt-Webb(BW) tests). Their Monte Carlo study involved twenty-five observations and two hundred replications, with ρ varied by tenths from 0.0 to 0.9. In contrast to Fomby and Guilkey(1978) they did not study a wide range of α's. They chose only three α's 0.01, 0.025, 0.05. For critical levels for the two tests, they relied on the Durbin upper distribution. They did not state how their design matrix was obtained. The criterion used to measure estimator performance was squared error loss. They discovered from their experiments that, in the Durbin-Watson pretest risks, for small values of ρ the three pretest estimators were inferior to OLS, but significantly better than OLS for higher values. Of the three pretest estimators, the Durbin pretest was the best and the Cochrane-Orcutt the worst. For the BW pretest risks, the results were similar to the DW risks, but since the former test has been shown to be more powerful than the latter , it was not surprising that the BW statistic provided slightly lower error loss for the Durbin and Prais-Winsten estimators for low values of ρ. They noted that the relative gains of one test procedure over the other were quite trivial by comparing the Durbin pretest estimator risks under the DW and BW statistics. They concluded, and quite correctly, that the risk gain are highly dependent on the choice of estimator. Their choice was the Durbin estimator.

This study was an improvement on the earlier study, but it was also limited in a number of ways. First, their choice of α might have affected the risks obtained in the study, since it may have put more weight on the OLS estimator in the generation of the pretest estimators. Second, even though they compared the pure EGLS estimators to each other, they did not compare these EGLS estimators with their corresponding pretest estimators. Third, like the earlier studies, they did not compare these estimators to the Bayesian alternatives.

In a discussion of Fomby and Guilkey(1978), Griffith and Dao (1980) demonstrated that there is a Bayesian counterpart to the class of sampling theory pretest estimators used in Fomby and Guilkey(1978). They pointed out that the Bayesian estimator used in FG(1978) was a counterpart of the EGLS estimator; they thus suggested the Bayesian pretest as an appropriate counterpart to the traditional pretest estimator. They pointed out that even though this class of Bayesian estimator does not require the setting of an arbitrary significance level, it does require the setting of some prior odds. Using a model similar to Fomby and Guilkey(1978), and a Monte Carlo experiment, they compared the mean square error of the Bayesian pretest estimator to the pure Bayesian estimator. They set a prior odds for their Bayesian pretest estimator based on a point null hypothesis of H_o: ρ=0. They concluded that for a large autocorrelation coefficient , the pure Bayesian estimator was better than the Bayesian pretest estimator, but the opposite was true for ρ close to zero. They noted that if the proabability associated with the null hypothesis of no autocorrelation is 0.4 or 0.5, the maximum possible loss of efficiency for using the Bayesian pretest instead of the pure Bayesian is quite small for small samples of 15 and 30. Placing a high prior weight through prior odds or significance level on the alternative hypothesis of autocorrelated errors results in a gain in efficiency from using the Bayesian pretest instead of the conventional pretest estimator in small samples.

Like the others, this experiment failed to consider other estimators. If the pure Bayesian estimator is a counterpart to the EGLS estimator then it will be most appropriate to compare these two estimators. Its biggest drawback is that they did not specify a rule for setting the prior odds. Their Bayesian pretest differs from ours in terms of the nature of the null hypothesis used. Secondly we establish a very objective rule for determining the posterior odds ratios, compared to the cavalier manner in which they treated the determination of the prior odds.

King and Giles(1984) extended the concept to examining the consequences of autocorrelation pretesting on hypothesis testing and prediction. They had four different design matrices with three to six regressors, two of which were eigenvectors corresponding to some K values of a tri-diagonal matrix and the other two were economic time series data on Australia. Unlike Judge and Bock(1978), they used exact critical values for their tests, obtained by employing Koerts and Abrahmse(1969) FQUAD fortran rountine. They considered two EGLS estimators (Prais-Winsten using Cochrane-Orcutt estimate of $\hat{\rho}$, and the Maximum Likelihood estimator using Beach and MacKinnon(1978) algorithm), four pretests (DW, BW, and two King(1982) $S(\rho)$ tests) They used S(0.5) and S(0.75) on the basis of King(1982) conclusion that these tests were more powerful than the DW and BW test respectively. Two significance levels (α=0.05, and 0.5) and two sample sizes (20,60) were used, and comparison of the pretest , OLS and the pure EGLS estimators was on the basis of risks (quadratic loss). They found out that the design matrix had a large impact on the results, Overall, however, they noted that there was not much difference between the risk of the pure EGLS estimators and their corresponding pretest estimators. The pretest estimators had smaller risk than their pure EGLS counterparts for ρ=0, and slightly larger risk for larger ρ. The pretest estimators were better than the OLS overall, but in the proximity of zero OLS did better than the pretest estimators. At α=0.5 and for moderate to high values of ρ the pretest estimators obtained were better than their counterparts at α=0.05, but the opposite was true for values of ρ close to zero. When the sample size used is large, these differences disappeared. Increasing α from 0.05 to 0.5 typically increased the risk of the pretest estimators when ρ was zero or small, but reduced it for larger values of ρ. Considering hypothesis testing after pretesting, they used a simple 't' test to estimate the difference between the nominal size and the actual size of the test, and found out that, with a few exceptions, the sizes of the 't' test based on OLS, EGLS, and the pretest estimators were larger than their nominal sizes. The α=0.05 pretest size at ρ=0 was closer to the nominal size than the α=0.5 pretest. For prediction after pretesting, they found out that there was not much difference between the mean squared error of prediction of any EGLS predictor and its corresponding α= 0.5 pretest predictor. For larger values of ρ, the pretest predictors were accurate especially for n=60. They favoured the MLE over the Prais-Winsten estimator, because the MLE was more efficient for low values of ρ. Choice of the autocorrelation pretest made little or no difference on the risks of the estimators. This study also ignored the Bayesian alternatives.

Griffiths and Beesley(1984) examined, by the use of Monte Carlo study, the relative efficiency (in terms of mean squared errors) of a number of estimators including pretest estimators when the errors are generated by either autoregressive (AR(1)), or a moving average (MA(1)) process. Their contribution was the incorporation of the MA(1) process which hitherto had been ignored. In addition to the traditional pretest estimator they considered a 'new' pretest estimator based on choosing between the three estimators OLS, EGLS(AR) and EGLS(MA). EGLS(AR) and

EGLS(MA) are the Maximum Likelihood estimators depending on the error process. Their pretest strategy was based on the Durbin-Watson test using the lower and upper 5% critical values for the test. They used two design matrices one of which was made up of psuedo-random numbers drawn from a normal distribution with a mean of 0 and a variance of 0.625. The other was a trended variable. Two sample sizes (20,50) and five hundred replications were used. ρ was chosen over the span -0.98 to 0.98 without constant variation. Their 'new' estimator was based on chosing between OLS, EGLS(AR) and EGLS(MA) depending on the outcome of a testing procedure to identify whether the error process was AR(1) or MA(1). They used both the DW test and the asymptotic test statistic $\sqrt{T}r$, where r is the Cochrane-Orcutt first stage estimate of the autocorrelation coefficient ρ. Thus the new estimator was of the form

$$\beta^{**}=\beta^{OLS} \text{ if } d^{*}\leq d\leq d^{**} \text{ and } |\sqrt{T}r|<1.645$$

$$=\beta^{O} \text{ if } d<d^{*} \text{ and } d\geq d^{**} \text{ or } |\sqrt{T}r|\geq 1.645$$

where

$$\beta^{O}=\beta_{AR} \text{ if } L(\tilde{\rho}) \geq L^{*}(\delta)$$

$$=\beta_{MA} \text{ if } L(\tilde{\rho}) < L^{*}(\delta)$$

$L^{*}(\delta)$ is the concentrated log likelihood for the MA(1) error model evaluated at the maximum likelihood estimate $\tilde{\delta}$ and $L(\tilde{\rho})$ is the same for the AR(1) model evaluated at the ML estimate $\tilde{\rho}$. Their conclusions were similar to the other studies with a few variations. For large values of $|\rho|$, they found that all estimators led to a considerable gain in efficiency relative to OLS. When $|\rho|$ is low, the 2 pretest estimators were better than the pure EGLS(AR), and EGLS(MA) estimators, but were worse than the AR estimator but not as bad as the incorrectly chosen MA estimator when $|\rho|$ is high. Consistent with the other studies, they found that considerable gains could be achieved by allowing for autocorrelation (either through AR or MA model or a pretest estimator) relative to ignoring autocorrelation and using OLS. For large values of δ, the correctly specified MA estimator was the best, but for small values of δ, the incorrectly specified AR estimator was preferable. From a relative efficiency standpoint, they suggested that it may be preferable to replace the OLS-AR pretest with the OLS-AR-MA pretest estimator because gains achieved when the underlying model is MA seems to outweigh losses incurred when the underlying model is AR.

Even though this study made a meaningful contribution to the literature, it had some flaws . Their choice of significance level was too low (0.05), and one does not know how the inconclusive region in the DW distribution of d was dealt with since they used inexact critical values. Also they excluded the Bayesian estimators in their study.

As a group, these studies examined, on the basis of MSE, the performance of the traditional pretest estimator, OLS and pure EGLS estimator. They almost invariably used the DW test as their test statistic, even though some considered other relevant tests in addition to this. Generally they used the conventional choice of 0.01 and 0.05 as their significance level. With regards to their findings, they concluded, generally, that, when the autocorrelation coefficient was in the proximity of zero, OLS outperforms both the parent EGLS and its corresponding pretest estimators, but as ρ diverges from zero, the MSE of the pretest and its parent EGLS estimator are substantially lower than that of OLS.

They noted that there was not much difference between the performance of the pretest estimator and its parent EGLS estimator for larger values of ρ. They agreed that the choice of the design matrix, the estimation procedure of the autocorrelation coefficient (ρ), the significance level used in the testing procedure, the test statistic used and the choice of the parent EGLS estimator all have substantial impact on the results of the experiments. Because they all differed on the models, tests and estimation techniques used, they did not give a recommendation that is appropriate for all situations though some suggested their methods for problems of the nature they investigated.

Other studies regarding the estimators in the autocorrelated error model are beyond the scope of this thesis; e.g. Giles and Beattie(1984) looked at a form of autocorrelation pretesting for a model which includes a lagged value of the dependent variable as a regressor. Durbin(1957), Guilkey(1974,1975), Godfrey(9176), Mantz(1978), Smith(1978), Harvey and Philips(1980,1981), Breusch and Godfrey(1981), and Owen(1981) all examined the consequences of testing for serial correlation in the disturbances of a simultaneous equation on the resulting estimators.

In this thesis, we will consider, in addition to many of these estimators, the Bayesian alternatives. We will consider 20 different design matrices with sample sizes ranging from 10 to 65. The choice of 20 design matrices is to eliminate the problem of generalising from a limited sampling experiment. Two significance levels (0.05, 0.50) will be used in the testing procedure, the former to reflect common practice and the latter to reflect the conclusions of Fomby and Guilkey(1978).

CHAPTER 3

THE MODEL, ESTIMATORS AND THE STRUCTURE OF THE MONTE CARLO EXPERIMENTS

3.1 INTRODUCTION

In this chapter we discuss the theoretical models, estimators and a detailed structure of the Monte Carlo experiments for both models. The design of the experiments is along the lines of Judge and Bock(1978), King and Giles(1984) and Griffiths and Beesley(1984)

3.2 THE THEORETICAL MODEL

We consider the linear statistical model

$Y_t = \beta_o + \beta_1 X_t + e_t$ 3.1

where e_t is generated by two stationary autocorrelation processes

a) $e_t = \rho e_{t-1} + u_t$............3.2

and

b) $e_t = \delta u_{t-1} + u_t$........3.3

where

$E(u_t)=0$, $V(u_t)=\sigma_u^2$, $0 \leq \rho < 1$ and $0 \leq \delta < 1$

The first of these two processes is the stationary AR(1) process and the latter is the stationary MA(1) process.

Because of the difficulties of conceiving of economic processes that generate negative AR(1) or negative MA(1) disturbances, we will consider only positive autocorrelated errors, following Judge and Bock(1978) and King and Giles(1984)

3.3 THE DESIGN MATRICES (X's)

To avoid generalisation from a single or few sampling experiments, we used 20 design matrices with sample sizes ranging from 10 to 65 for both models. All the design matrices chosen are (Tx2) dimension, with the first column being ones and the second column given by the following: (The sample sizes are in parentheses)

1) Magee, Ullah and Srivastava's (1987) X_2 variable (T=10)
2) Zellner and Tiao's (1964) rescaled investment expenditure of the U.S (T=15)
3) Beach and Mackinnon's (1978) and Griffith and Beasley's (1984) trended X variable given by $X_t = \exp(0.04t) + W_t$ where $W_t \sim N(0,0.009)$. This variable will be generated using the IMSL Fortran Library (T=30)
4) Australian Rate of Inflation (quarter to quarter percentage change in CPI (weighted average of six state capitals)) 1974-1 to 1978-4 (T=20). Taken from Clement, K.W and Taylor, J.C(1987) 'The Pattern of Financial Holding in Australia' in "Specification Analysis in the Linear Model", (1987) [ed. King and Giles]
5) Griliches and Rao's (1969) and Fomby and Guilkey's (1978) design matrix given by $X_t = \lambda X_{t-1} + V_t$. Where λ=0.4 and $V_t \sim N(0,1)$. (T=35)

The next eight vectors are taken from the data sets presented in Maddala(1988 pg 147-157)

6) Unemployment rates in the United Kingdom 1920-1938 (T=19)

7) 3-month Treasury bills interest rates in the U.S. (monthly data) January 1980 to September 1983 (T=45)

8) Food Production per capita in the U.S 1922-1941 (T=20)

9) Ratio of Disposable Income to Cost of Living Index in the U.S 1922-1941 (T=20)

10) Money Stock, M1 (currency+demand deposits+travellers cheque and other chequable deposits) for the U.S 1959-1983 (T=25)

11) Debt of Domestic Nonfinancial sector (monthly average) for the U.S December 1959-1983 (T=25)

12) Quarterly estimated real interest rate for Canada 1955-1 to 1978-2 (T=50)

13) Quarterly data on Housing Starts in Canada 1955-1 to 1978-2 (T=50)

The following 7 design matrices were taken from IMF AND IFS tapes obtained from the SFU Data Library.

14) Percent Per-annum Change in Money Supply (Canada) Quarterly data 1957-4 to 1973-4 (T=65)

15) Direct Investments in Canada in millions of dollars. Quarterly data 1970-1 1984-4 (T=60)

16) Pecent Per-annum Change in Money (Australia) Quarterly data 1957-4 to 1973-4 (T=65)

17)Percent Per-annum Changes in Consumer Price (Australia) Quarterly data 1961-4 to 1977-4(T=65)

18) Gross Fixed Capital Formation (Australia) Quarterly data 1961-4 to 1977-4 (T=65)

19) Gross National Expenditure (Canada) Quarterly data 1961-1 to 1975-4 (T=60)

20) Percent Per-annum Changes in Consumer Prices (Canada) Quarterly data 1957-4 to 1974-4 (T=65).

3.4 ESTIMATORS USED

3.4.1 THE AR(1) MODEL

The sampling performances of the following estimators were examined for the AR(1) model

1) Ordinary Least Squares $[\beta^{OLS} = (X'X)^{-1}X'Y]$

2) A modified version of the Durbin Estimator (β^{EGLS}) which uses the Prais-Winsten (1954) transformation rather than the Cochrane-Orcutt transformation, by transforming the first observation on all variables by $\sqrt{1-\hat{\rho}^2}$, where $\hat{\rho}$ is the Durbin estimator of ρ. The transformation matrix used is the TxT matrix described earlier. This estimator is included because of the support it gained from Rao and Griliches(1969), Fomby and Guilkey(1978) and Judge and Bock(1978). Using a TxT transformation matrix improves the efficiency of the Durbin estimator.(see the above studies)

3) Maximum Likelihood Estimator (β^{MLE}) using the Beach and Mackinnon algorithm. This estimator's usual asymptotic features justifies its inclusion.

4) Two autocorrelation pretest estimators (β^{PT} which uses (2) as its EGLS component and β^{MLPT} which uses (3) as its EGLS component). These pretest estimators are based on a test of significance at both 0.05 and 0.50 levels. Thus we used 4 autocorrelation pretest estimators based on significance levels and EGLS components. [$\beta^{PT}(\alpha=0.05)$, $\beta^{PT}(\alpha=0.50)$, $\beta^{MLPT}(\alpha=0.05)$, and $\beta^{MLPT}(\alpha=0.50)$].

5) Two Bayesian pretest estimators (β^{BAYPT} with (2) as its EGLS component and β^{MLBPT} with (3) as its EGLS component)

6) The Pure Bayesian estimator (β^{BAY})

7) The Generalised Least Square estimator (β^{GLS})

Thus we used eleven estimators overall in the AR(1) model.

3.4.2 THE MA(1) MODEL

For the MA(1) model the following estimators were examined

1) Ordinary Least Squares [β^{OLS}].

2) MacDonald and MacKinnon's(1985) Estimated Generalised Least Squares Estimator (β^{EGLS}) which uses the method of moments estimator of $\hat{\delta}$ coupled with equation 2.34. This estimator was used because of its simplicity.

3) Maximum Likelihood Estimator (β^{MLE}) using the Balestra(1980) scanning procedure.

4) Two autocorrelation pretest estimators (β^{PT} which uses (2) as its EGLS component and β^{MLPT} which uses (3) as its EGLS component). These pretest estimators are based on a test of significance at 0.05 .

5) Two Bayesian pretest estimators (β^{BAYPT} with (2) as its EGLS component and β^{MLBPT} with (3) as its EGLS component)

6) The Pure Bayesian estimator (β^{BAY})

7) The Generalised Least Square estimator (β^{GLS}) calculated using the inverse of the Ω matrice described in section 2.8

Thus for this model, the performance of nine estimators were examined overall

3.5 THE STRUCTURE OF THE MONTE CARLO EXPERIMENTS

3.5.1 THE AR(1) MODEL

Using the AR(1) model given by 3.1 and 3.2, the u_t were generated as pseudo-random numbers, drawn from a normal distribution with a mean of 0 and a variance of 0.0036 through the IMSLD Fortran Library with a Dseed of 123457.D0. β_o and β_1 were both chosen to be equal to one. It is important to note that the values assigned to the β vector and σ_u^2 have no influence on the results (Breusch(1980)) and were thus chosen arbitrarily.

For simplicity, only one explanatory variable was used throughout all the experiments. Even though this choice is restrictive, it is in line with many other studies. [Nicholls and Pagan(1977), Fomby and Guilkey(1978) and Griliches and Rao(1969)]. After generating the u_t's, the e_t were created by the first order autoregressive scheme given by equation 3.2 with ρ varied by tenths from 0 to 0.9. The first observation e_1 was obtained as $\sqrt{(1/(1-\rho^2))}.u_1$.

Computationally, it was possible that $\hat{\rho}$ be greater than 1 for large values of ρ, thus we set the upper limit of $\hat{\rho}$ to be 0.99999 to be able to apply the Prais-Winsten(1954) transformation, following Beach and Mackinnon(1978).

Next, β_0, β_1, X_t, and e_t were combined in the linear fashion given by equation 3.1 to obtain the Y_t values. The X's were held fixed in repeated samples for all experiments.

OLS estimates of the β's in 3.1 were used to generate the residuals necessary for computing the test statistic. Significance levels of 0.05 and 0.50 were used for the autocorrelation pretests, the former level reflecting traditional practices whilst the latter reflects Fomby and Guilkey's(1978) arguments for adopting larger significance levels. For the autocorrelation pretests, a one-sided Durbin-Watson(DW) test was used with H_0: $\rho=0$ against H_1: $\rho>0$, because as mentioned above, we are only interested in positive autocorrelation processes. For the conventional 0.05

significance level, we used the DW upper bound distribution based on the suggestion by Judge et al(1985) that it is better to use the upper critical bound rather than regard the inconclusive region as an indication of no autocorrelation. For the 0.50 significance level, we used exact critical values computed using the Shazam computer package (White (1978)). If H_0 was rejected, the EGLS estimator was used as the pretest estimator and if it was accepted, then the pretest estimator was OLS.

The choice of the Durbin Watson test instead of the other test discussed in chapter 2 was based on its simplicity, frequent use in studies of this nature and Dent's(1973) argument that compared to DW test, the likelihood ratio test for AR(1) disturbances has extremely poor power against positive autocorrelation when the sample size is small, and even for large samples, the DW test has very real power advantage. Judge and Bock(1978) also showed that the difference between the DW test and the other alternatives was trivial when the upper bound distribution is used to avoid the inconclusive region.

Computer programmes, written by this author, using Fortran 77 language and making use of some routines from the IMSLD and NAGD Fortran Libraries were employed to compute the various estimators following the formulas presented in chapter 2. The most common routines used from the Libraries were F01CDF, F01AAF, F01CKF, F01CEF,VMULFM. All experiments are done using the Double Prescision Fortran measurement unit.

To avoid the dichotomous nature of the autocorrelation pretest estimator, we used a composite null hypothesis to determine the weights for the Bayesian pretest estimator. Thus for the Bayesian pretest estimator $H_0 : \rho<0.3$ and $H_1:\rho>0.3$. We integrated numerically the posterior density of ρ over the range -0.3 to 0.3 to obtain the weight for the OLS estimator, and $(1 - P(\rho<|0.3|)$ was used as the weight on the EGLS estimator. The posterior density of ρ used for the Bayesian alternative is taken from Judge et al (1983, pg 292) rather than that given by Zellner and Tiao(1969) because our models contain an intercept term and it was more appropriate to use the former since that density is derived for models with constant terms.

To calculate the Pure Bayesian estimator, we first numerically integrated the marginal density function of ρ, $g(\rho|Y)$, over the range -0.999 to 0.999 to obtain the normalising constant, k. Next, with intervals of 0.04995, we calculated GLS estimates for values of ρ within the range -0.999 to 0.999 and then weighted these GLS estimator by the marginal density of ρ, $kg(\rho|Y)$ to obtain the heights of the marginal density of β for a given value of ρ. For purposes of employing the technique of numerical intergration, we used the formula for finding the area of a trapezoid to obtain the area for a given interval under the posterior distribution function of β and then sum these areas over all forty intervals. This technique of numerical intergration is known as the Trapezoidal method. It should be noted that the choice of the intervals and the corresponding number of GLS estimators was based on the author's observation that there was not much difference in the Pure Bayesian estimator when forty or higher intervals was used.

600 replications were used for all experiments though the number of replication is not crucial in the experiment (Lovell (1983)).

The criterion used to measure the performance of each estimator was the mean square error of the slope coefficient rather than the risk (addition of Mse's of both the intercept and slope) in line with Griffiths and Beasley(1984). The mean square error criterion was also chosen due to the fact that some of the estimators being examined are biased in small samples as discussed in chapter 2.

3.5.2 THE MA(1) MODEL

Given equations 3.1 and 3.3, the choice of the u_t's and the β vector were similar to that of the AR(1) model, since these values do not generally influence the results. Similarly, only one independent variable was used in this experiment for simplicity.

(T+1) observations on u_t were generated with mean 0 and a variance of 0.0036, so as to create T observations on e_t following the first order moving average process given by equation 3.3. To ensure that the process was invertible, we restricted our moving average parameter to be less than 1, and only positive values of this parameter were examined. Thus δ was varied by tenths from 0 to 0.9 and $\hat{\delta}$ was restricted to have a maximum of 0.999999.

The X's were held fixed in repeated samples, and the Y values were obtained by combining the β vector, e_t, and X_t in a linear fashion given by equation 3.1.

We used the King(1983) S(0.50) test as our test statistic because of King's argument that, for positive MA(1) errors, this test is more powerful than the Durbin-Watson test. δ=0.50 was chosen without reference to the data, and irrespective of the true value of δ, S(0.50) was calculated by taking the ratio of the generalised least squares residuals to the ordinary least squares residuals of equation 3.1, with the generalised least squares residuals calculated under the assumption that δ=0.50 and $\sigma^2\Omega(0.50)$ is the covariance matrix of the error term e.

The 0.05 significance level is used for the autocorrelation pretest. King(1983) has tabulated critical values for S(0.50) at the 5% level. The upper bound distribution is used, rather than regard the inconclusive region as a sign of no autocorrelation. Only the 5% level is used because this author has no knowledge of any existing algorithm for calculating the exact distribution for this test.

A one-sided (positive) test is used with H_o: $\delta=0$ and $H_1:\delta>0$, and H_o is rejected for small values of S(0.50). If H_o is rejected, the MacDonald and MacKinnon(1985) EGLS or Balestra(1980) Maximum Likelihood estimator is used as the pretest estimator, and if accepted, OLS is used.

To avoid the dichotomous choice of estimators present in the autocorrelation pretest estimator, a Monte-Carlo experiment was undertaken to determine the nature of the composite hypothesis for the Bayesian pretest estimator. This experiment revealed that, for values of the moving average parameter (δ) less than approximately 0.50, the OLS estimator has a risk less than that of the MacDonald and MacKinnon(1985) EGLS estimator for all (20) the design matrices used.

Thus for the Bayesian pretest estimator, our null and alternative hypotheses were H_o: $\delta<0.50$ and H_1: $\delta>0.50$ respectively. We numerically integrated the posterior density of δ from -0.50 to 0.50 to obtain the weight for the OLS estimator and $1-P(\delta<|0.50|)$ was the weight on the EGLS estimator. For this model, the Judge et al(1985) posterior density function for δ is used to calculate the pure Bayesian estimator and the Bayesian pretest estimator due to the fact that it is consistent with the model used in this study.

To calculate the Pure Bayesian estimator, we numerically integrated the marginal density of δ, $g(\delta|Y)$, within the range -0.999 to 0.999 and obtained the normalising constant, k. Next, we calculated forty GLS estimates at intervals of 0.04995 within the range -0.999 to 0.999, and weighted these GLS estimates by the posterior density of δ, $kg(\delta|Y)$ to obtain the heights of the marginal density of β for a given value of ρ. For the numerical intergration technique, we used the Trapezoidal method described above under the AR(1) model. The choice of forty GLS estimators is based on the observation, by this author, that there was not much difference in the Pure Bayesian estimator when forty or more GLS estimators are used.

The number of replications used to determine the performances of the various estimators and the criterion of measuring the performances used for this model is similar to those of the AR(1) case.

Similarly, Fortran programmes, written by this author, using subroutines from the IMSLD and NAGD Fortran Libraries were used to compute all the estimators following the various formulas given in chapter 2. The Double Prescision unit was used for this model as well.

The detailed computer programme for both models is presented in the Appendix.

The next chapter discusses the findings of the experiments for both models and Tables and Figures are presented to clarify these findings.

CHAPTER 4

PERFORMANCES OF VARIOUS ESTIMATORS

4.1 INTRODUCTION

In this chapter we discuss the performances of the various estimators in both the AR(1) and MA(1) models. The criterion for measuring performances is the mean square error of the slope coefficient following earlier studies such as Griliches and Rao(1969) and Griffiths and Beesley(1984).

For the AR(1) model, eleven estimators (including GLS) were examined while nine estimators (including GLS) were examined for the MA(1) model. Tables and figures 4.1 - 4.20 represent the mean square error values and analytical mean square error functions for the AR(1) model, and 4.21 - 4.40 represent the same for the MA(1) model.

An attempt to plot the mean square errors of all the estimators involved for each model proved to be messy and one cannot decipher the performance of each estimator from the others as shown in Figure 4.1A. Thus only a selection of the results of the experiments is plotted. The estimators depicted in the figures have been selected based on the average increase in their mean square error relative to that of the GLS estimator over the entire range of the autocorrelation parameter [ρ for AR(1) and δ for the MA(1) model]. This selection is representative of the various patterns that emerged in the results.

The estimators [apart from the pure Bayesian estimator (because it is the estimator of interest in this study) and the OLS (because its mean square error values rescaled the plots)] were categorized into groups based on the broad classification of estimators to which they belong. The categorization for the AR(1) model was:

1) the Durbin Estimator (β^{EGLS}) and the Maximum Likelihood Estimator (β^{MLE}) under the broad class of EGLS estimators

2) Traditional autocorrelation pretest estimators at 5% significance level; (β^{PT} which uses β^{EGLS} as its EGLS component and β^{MLPT} which uses β^{MLE} as its EGLS component) under the conventional pretest estimators group.

3) The two estimators in (2) above at the 50% significance level

4) The Bayesian pretest estimators β^{BAYPT} with β^{EGLS} as its EGLS component and β^{MLBPT} with β^{MLE} and its EGLS component under the Bayesian pretest estimators category.

For the MA(1) we had three groups:

1) MacDonald and MacKinnon(1985) estimator (β^{EGLS}) and the Balestra(1980) Maximum Likelihood estimator (β^{MLE})

2) Same as (2) in the AR(1) model

3) Same as (4) in the AR(1) model

For the AR(1) model, β^{MLE}, β^{MLBPT}, $\beta^{MLPT}(0.05)$ and $\beta^{MLPT}(0.50)$ had the least average increase in mean square error in their respective groups, whilst for the MA(1) model β^{MLE}, β^{MLBPT}, $\beta^{MLPT}(0.05)$ had the least increase. For the sake of clarity, these estimators in addition to the pure Bayesian estimator were plotted for each model, however the discussion of the results is based on the full study.

The criterion for choosing estimators to be plotted (average increase in mean square error) is calculated as the percentage increase in a given estimators mean square error over that of the GLS estimator averaged over the entire range of the autocorrelation parameter space.

The 'cases' refered to in the tables and figures represent the different design matrices enumerated in chapter three, thus 'Case 1' refers to design matrix 1, 'Case 2' design matrix 2 and so on.

4.2 DISSCUSSION OF THE RESULTS

4.2.1 THE AR(1) MODEL

Tables 4.1 - 4.20 contain the ratio of the mean square error of all the relevant estimators to that of the GLS estimator for all the design matrices used. The main observation from this experiments is 1) The pure Bayesian estimator has a mean square error that is uniformly below that of all the other estimators (except for OLS when $\rho \leq 0.10$ for some cases) over the entire range of ρ and for all the design matrices. This result is what we a priori had hoped for, because the pure Bayesian's ability to weight several GLS estimators with the posterior density function of ρ is an ultimate form of 'smoothing'.

Other observations from these results are:

2) For all design matrices, all the estimators exhibit the same general features, that is, their relative mean square error increases over the entire range of the values of ρ considered in this study.

3) In terms of average increase in mean square error over the entire range of ρ, there seems to be a clear ranking of the estimators. In all cases the pure Bayesian estimator gave the least average increase in mean square error and generally β^{MLBPT}, β^{BAYPT}, β^{MLE}, $\beta^{MLPT}(0.50)$, β^{EGLS}, $\beta^{PT}(0.50)$, $\beta^{MLPT}(0.05)$, $\beta^{PT}(0.05)$ and β^{OLS} followed in that order, with β^{OLS} having the highest average increase in mean square error over GLS.

4) When ρ is in the proximity of zero, OLS performs very well because of its BLUE properties, and is better than all the other estimators except for a few cases (1,9,11,14,17,19) where it is outperformed by the pure Bayesian estimator. It is interesting to note that for these exceptional cases mentioned above, the pure Bayesian estimator happens to be better than the GLS estimator. This observation is unusual since GLS is BLUE in an autocorrelated error model. However, the pure Bayesian estimator is a non-linear estimator and biased in small samples; and with the possibility of sampling error in this experiment(we used 600 replications), this result is not impossible.

5) When ρ is approximately less than or equal to 0.30, OLS generally outperforms β^{EGLS}, β^{MLE} and the autocorrelation pretest estimators confirming the findings of earlier studies such as Griliches and Rao, Judge and Bock(1978), Magee et al(1987), but as ρ increases, OLS loses its BLUE properties and consistently gets worse, thus its mean square error increases more rapidly than that of its competitors and becomes higher than the mean square errors of the other estimators. The mean square error of OLS generally crosses that of β^{EGLS}, β^{MLE} and the autocorrelation pretest estimators at a value of ρ approximately equal to 0.30 and β^{BAY} (apart from the cases mentioned above) at ρ approximately equal to 0.10. This result is consistent with Fomby and Guilkey(1978). The mean square error of OLS was generally lower than that of β^{MLBPT} and β^{BAYPT} at ρ approximately equal to 0.20

6) The behaviour of the autocorrelation pretest estimators in relation to their EGLS components was not unexpected. For $\rho=0$, the 5% autocorrelation pretest estimators correctly accepts the null hypothesis 95% of the time, thus more weight is placed on the OLS estimator than the EGLS estimator, and the risk of the pretest estimator is weighted towards that of the former, which at this value of ρ is the lowest among all the estimators. Thus the mean square error of the pretest is lower than that of its parent EGLS estimator for lower values of ρ, but as ρ increases, OLS gets consistently worse, and the weighting system by this pretest estimator causes its mean square error to be inflated and thus higher than the mean square error of its parent EGLS. The mean square error of the pretest estimator crosses that of its parent EGLS estimator at approximately $\rho=0.40$. The above argument holds for the 50% pretest estimator also, but because the weight placed on the EGLS and OLS are practically the same, (This pretest estimator correctly accepts the null hypothesis 50% of the time for $\rho=0$ and thus chooses OLS 50% of the time) its mean square error for higher values of ρ are not as high as the 5% pretest estimator and thus not markedly higher than that of its parent EGLS estimator. For some cases, there is not much difference between the mean square error of the pre-test estimator and its EGLS component for large values of ρ. This result confirms the findings of earlier studies such as Judge and Bock(1978) and King and Giles(1984). Specifically, pretesting has smaller risk than its parent EGLS estimator for $\rho=0$ and slightly larger risk for larger values of ρ. Secondly,

for $\rho>0.3$ pretesting is better than OLS.

7) The two Bayesian pretest estimators perform quite well (generally next to the pure Bayesian) for large values of ρ ($\rho\geq0.60$), and for most cases outperformed OLS, β^{EGLS}, β^{MLE}, and the two 5% autocorrelation pretest estimators for these values of ρ. It is interesting to note that this behaviour did not exist for all the design matrices, and for the exceptional cases, the differences between the mean square error of the estimators mentioned above (except for OLS) was very small.

8) Pretesting at the 50% level markedly improved the performance of the autocorrelation pretest, reducing the loss in efficiency of the 5% autocorrelation pretest by about 14.6%. This confirms the findings of Fomby and Guilkey(1978) about the advantage of adopting higher significance level than the traditional 0.05 level.

9) Except for the pure Bayesian estimator, no other estimator generally dominates the other estimators over the entire range of ρ; i.e. no estimator is uniformly superior to the other competitors over the whole range of ρ, though some estimator such as OLS, and the autocorrelation pretest perform quite well for low values of ρ, while the Bayesian pretest estimators, β^{EGLS}, and β^{MLE} perform well for higher values of ρ. Generally none of these estimators dominated each other entirely.

10) On the average (over all 20 design matrices) the pure Bayesian estimator's mean square error is approximately 94% of the mean square error of its closest competitior β^{MLBPT} for

$\rho \leq 0.4$ and 85% for $\rho \geq 0.50$, in relation to its worst rival (β^{OLS}), its mean squared error is 96% for lower values and 58% for higher values.

11) The choice of significance levels does not affect the relative ordering of the mean square errors, nor does it significantly change the curvature of the mean square error function of the pretest estimator, but a choice of higher significance level does decrease the absolute magnitude of the mean square error of the autocorrelation pretest estimator. Whether or not there is a significant reduction in mean square error through pretesting depends on the choice of estimators, but from this experiment, there is a significant reduction in the mean square error, if the maximum likelihood estimator is used as the EGLS component instead of the Durbin estimator.

12) The sample size of the design matrices did not make much difference in the ordinal ranking of the estimators, though it made a difference in the absolute magnitude of the mean square errors.

13) Based on the results reported in (1) above, some researchers will be drawn to the Bayesian estimator, but tradition suggest that these researchers will employ a pretest estimator based on some testing procedure to choose between OLS and the pure Bayesian estimator. Thus we investigated the performance of such an estimator based on a DW 5% test. Only 8 design matrices were used for this experiment. We observed that this 'new' pretest estimator had a mean square error function everywhere above that

of the pure Bayesian estimator though it was better than OLS for $\rho>0$. This pretest estimator was also uniformly superior to the pretest estimator that chooses between OLS and EGLS as well as the pretest that chooses between OLS and MLE.

14) However it is clear that if attention is confined to the Durbin-Watson test as in this experiment, then the recommended strategy for choice of estimators in a model with an autocorrelated error term is to use OLS if any available prior information suggest that ρ is small (say less than 0.1), otherwise use the pure Bayesian estimator. Using OLS within the range of ρ mentioned above leads to very minimal loss in efficiency and less computation cost. Another strategy is to use the pure Bayesian estimator anytime a model has an autoregressive error term, without reference to prior information, because the loss of efficiency using this estimator when ρ is small is insignificant, but substantial gain can be achieved for higher values of ρ.

On the basis of these results, the pure Bayesian estimator has proven to be a very attractive alternative to EGLS and related pretest estimators for estimating the β vector in a model with autoregressive error term.

Table 4.1

RELATIVE MSE: AR(1) (CASE 1)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.054	1.049	0.965	1.026
0.10	1.002	1.057	1.051	0.972	1.027
0.20	1.016	1.070	1.055	0.979	1.049
0.30	1.076	1.073	1.068	0.985	1.052
0.40	1.125	1.110	1.076	0.991	1.089
0.50	1.187	1.119	1.103	1.001	1.148
0.60	1.201	1.132	1.126	1.008	1.157
0.70	1.330	1.164	1.131	1.011	1.166
0.80	1.460	1.178	1.148	1.013	1.172
0.90	1.665	1.191	1.180	1.043	1.223

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.023	1.013	1.029	1.030	1.005
0.10	1.025	1.021	1.037	1.036	1.020
0.20	1.034	1.041	1.048	1.039	1.026
0.30	1.045	1.061	1.072	1.040	1.077
0.40	1.063	1.116	1.093	1.093	1.090
0.50	1.095	1.127	1.129	1.123	1.129
0.60	1.110	1.148	1.140	1.139	1.136
0.70	1.141	1.174	1.154	1.170	1.142
0.80	1.169	1.188	1.163	1.182	1.158
0.90	1.200	1.211	1.194	1.200	1.188

FIGURE 4.1A AR(1) (CASE 1)
RELATIVE MSE FUNCTIONS

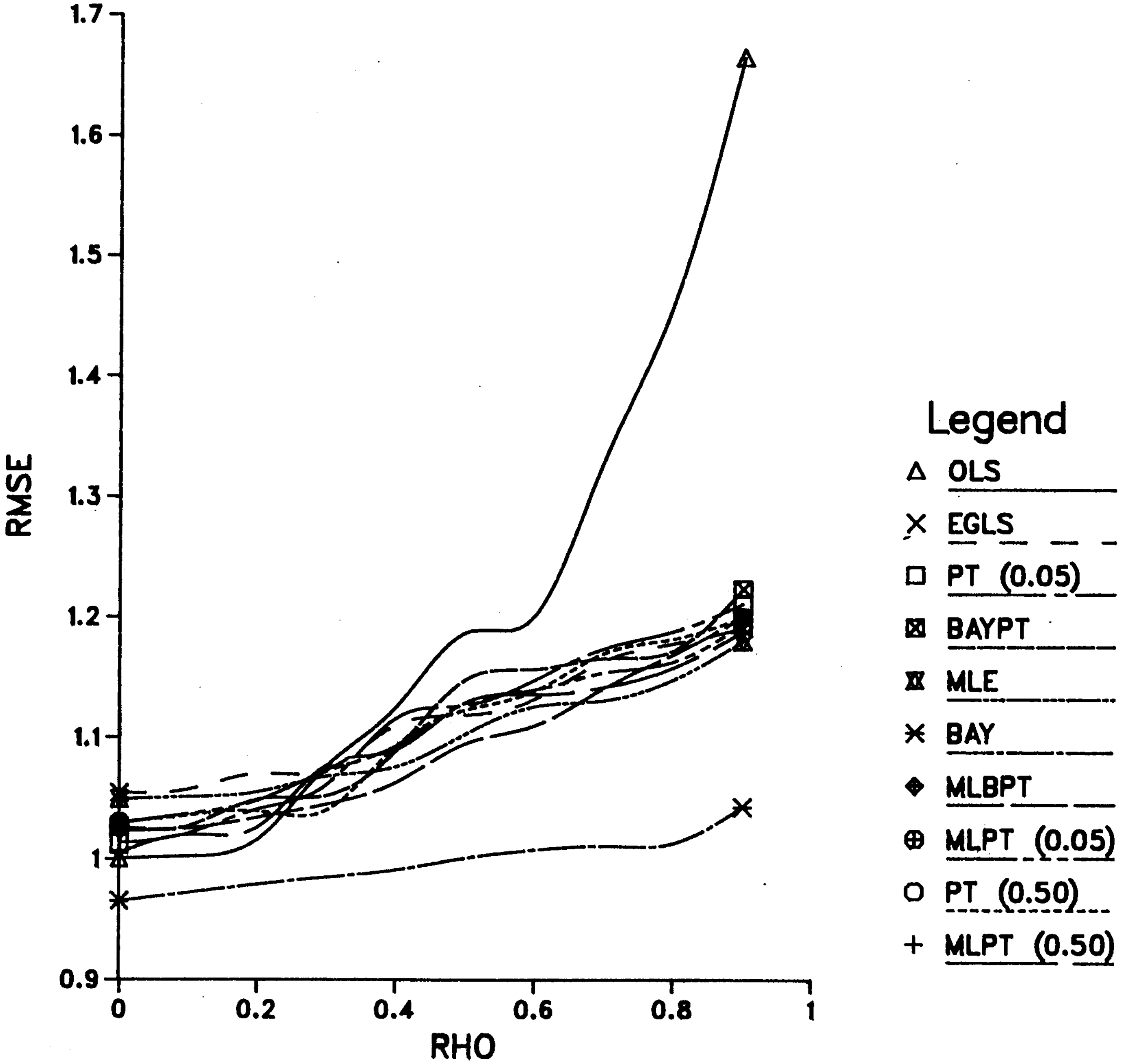


FIGURE 4.1B AR(1) (CASE 1)
RELATIVE MSE FUNCTIONS

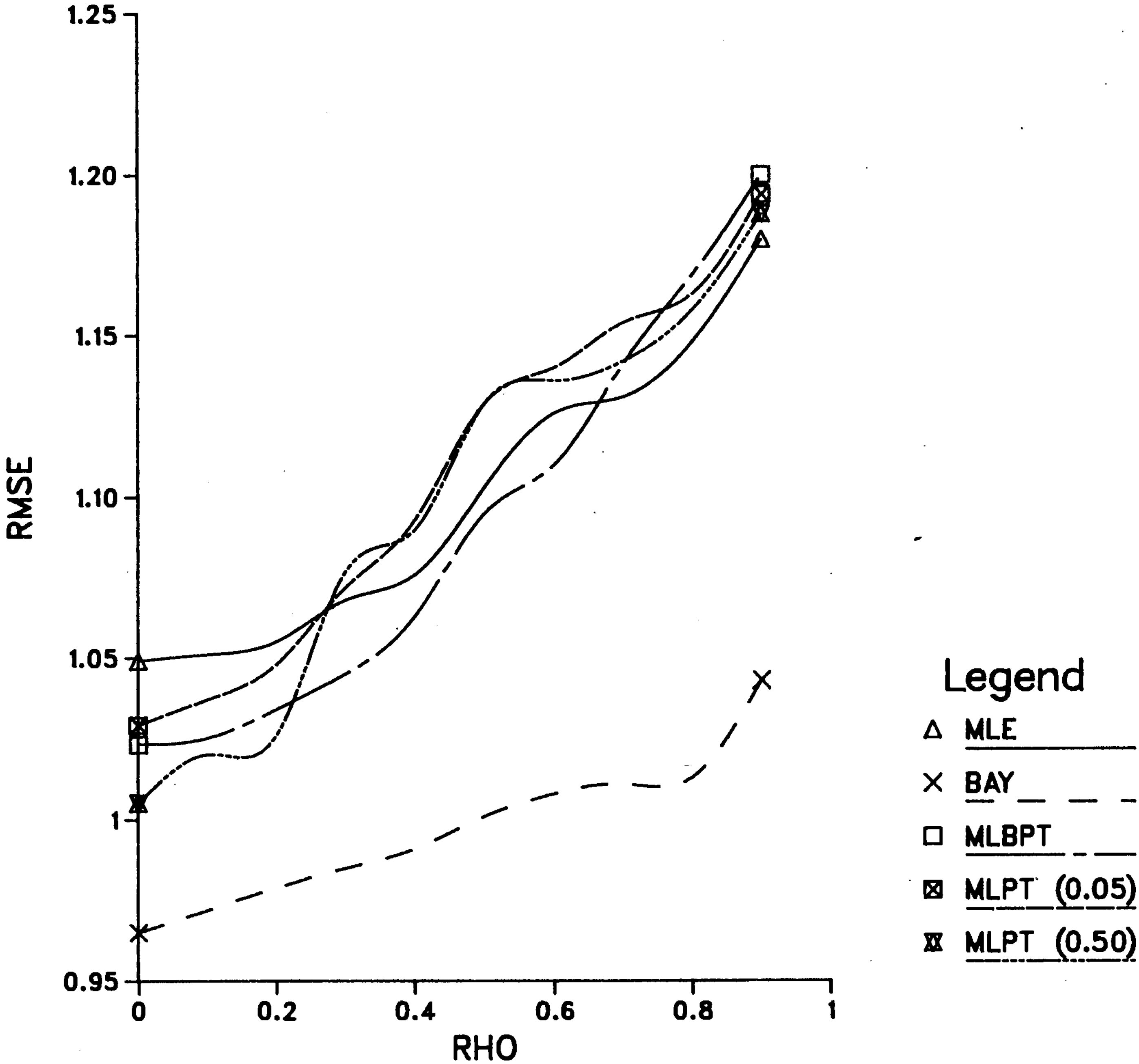


Table 4.2

RELATIVE MSE: AR(1) (CASE 2)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.077	1.106	1.003	1.020
0.10	1.001	1.063	1.117	1.005	1.017
0.20	1.072	1.137	1.214	1.010	1.074
0.30	1.138	1.194	1.287	1.014	1.141
0.40	1.421	1.297	1.342	1.039	1.263
0.50	1.820	1.366	1.361	1.042	1.343
0.60	2.034	1.467	1.455	1.047	1.424
0.70	2.881	1.484	1.463	1.053	1.445
0.80	3.729	1.502	1.582	1.078	1.509
0.90	6.056	1.712	1.669	1.185	1.731

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.050	1.028	1.031	1.023	1.044
0.10	1.078	1.022	1.041	1.021	1.048
0.20	1.112	1.086	1.104	1.061	1.119
0.30	1.192	1.156	1.171	1.137	1.186
0.40	1.283	1.360	1.361	1.253	1.280
0.50	1.312	1.435	1.414	1.371	1.365
0.60	1.403	1.574	1.563	1.463	1.468
0.70	1.464	1.724	1.663	1.509	1.494
0.80	1.645	1.830	1.779	1.579	1.618
0.90	1.742	2.260	2.157	1.748	1.670

FIGURE 4.2 AR(1) (CASE 2)
RELATIVE MSE FUNCTIONS

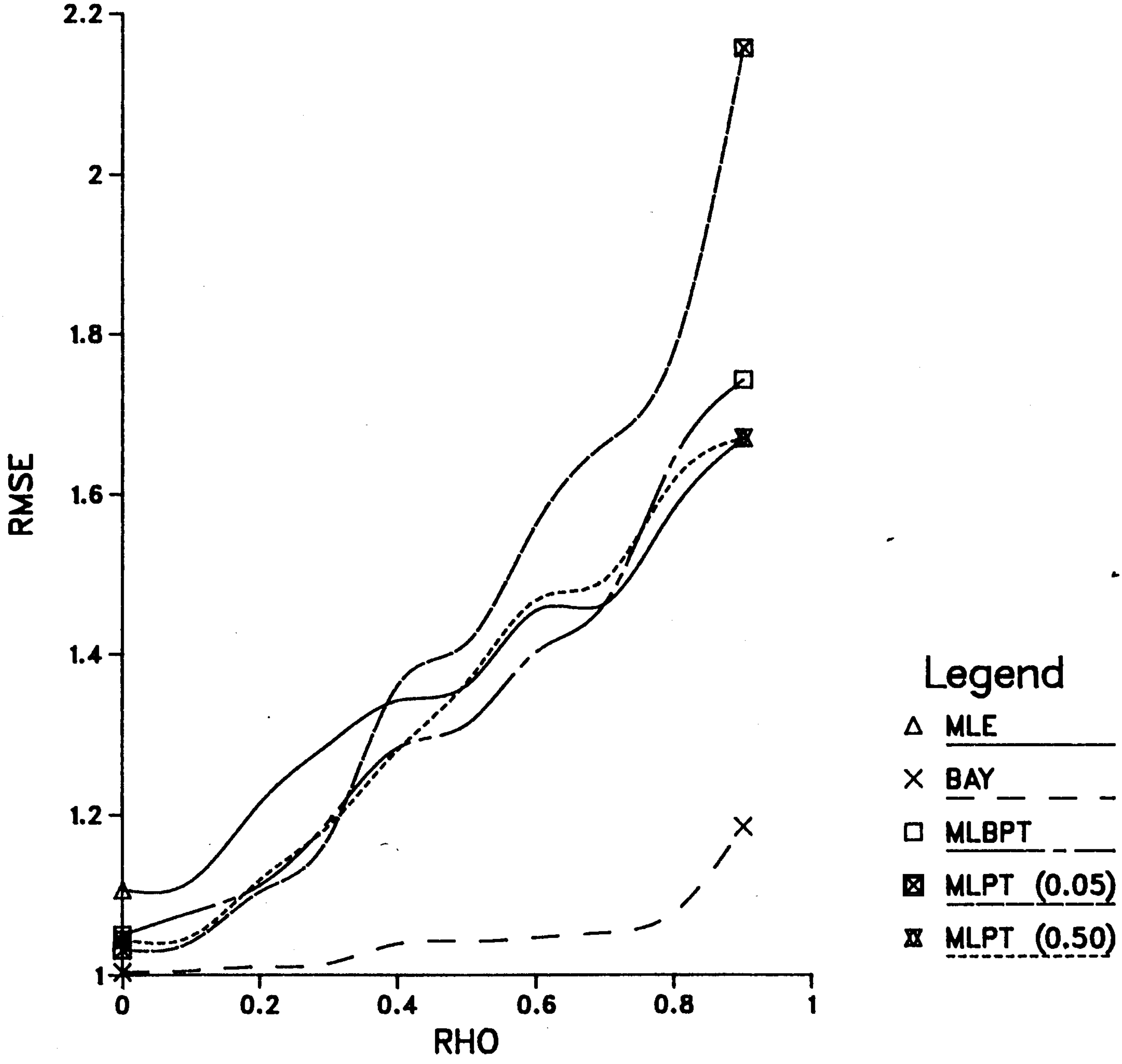


Table 4.3

RELATIVE MSE: AR(1) (CASE 3)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.039	1.026	1.005	1.013
0.10	1.007	1.060	1.041	1.009	1.011
0.20	1.025	1.062	1.068	1.011	1.037
0.30	1.029	1.082	1.073	1.013	1.048
0.40	1.101	1.088	1.087	1.015	1.069
0.50	1.223	1.120	1.136	1.024	1.089
0.60	1.353	1.124	1.152	1.039	1.105
0.70	1.454	1.196	1.218	1.043	1.190
0.80	1.485	1.202	1.238	1.055	1.196
0.90	1.906	1.447	1.452	1.120	1.443

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.030	1.001	1.010	1.022	1.004
0.10	1.049	1.012	1.019	1.049	1.017
0.20	1.069	1.049	1.056	1.050	1.038
0.30	1.070	1.082	1.067	1.065	1.042
0.40	1.078	1.102	1.090	1.076	1.077
0.50	1.121	1.141	1.147	1.130	1.145
0.60	1.135	1.155	1.165	1.135	1.163
0.70	1.211	1.239	1.244	1.201	1.224
0.80	1.220	1.246	1.251	1.212	1.249
0.90	1.449	1.513	1.480	1.449	1.453

FIGURE 4.3 AR(1) (CASE 3)
RELATIVE MSE FUNCTIONS

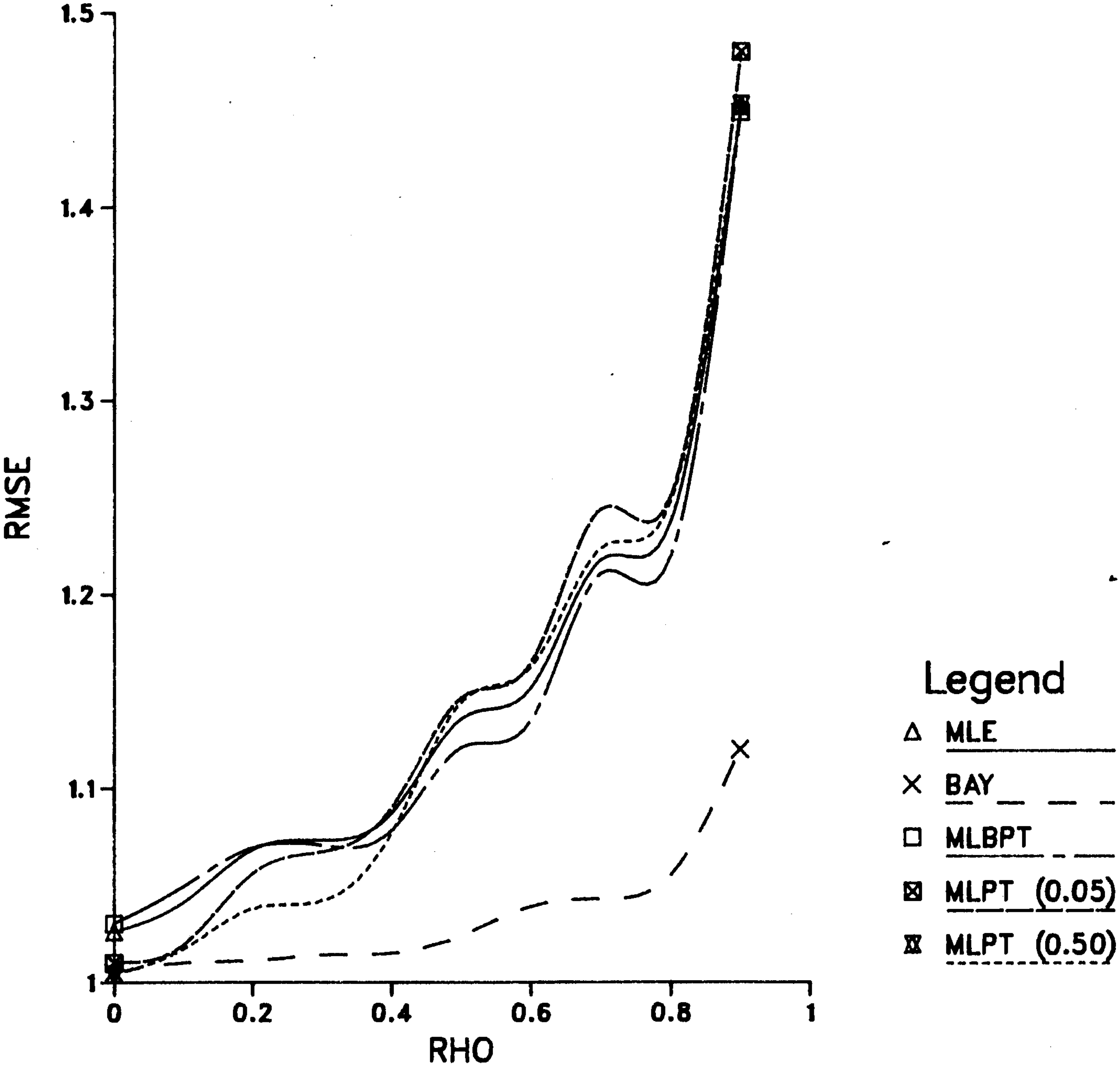


Table 4.4

RELATIVE MSE: AR(1) (CASE 4)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.041	1.053	1.002	1.020
0.10	1.002	1.046	1.068	1.010	1.024
0.20	1.008	1.059	1.081	1.015	1.034
0.30	1.030	1.071	1.085	1.028	1.038
0.40	1.084	1.100	1.090	1.037	1.077
0.50	1.261	1.106	1.096	1.053	1.091
0.60	1.278	1.153	1.110	1.068	1.135
0.70	1.688	1.145	1.113	1.077	1.144
0.80	1.959	1.168	1.174	1.085	1.157
0.90	2.575	1.174	1.200	1.098	1.178

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.045	1.012	1.014	1.010	1.014
0.10	1.066	1.025	1.068	1.032	1.023
0.20	1.070	1.051	1.072	1.044	1.036
0.30	1.088	1.059	1.082	1.053	1.044
0.40	1.098	1.082	1.094	1.076	1.054
0.50	1.100	1.129	1.122	1.114	1.100
0.60	1.102	1.158	1.130	1.160	1.117
0.70	1.105	1.160	1.149	1.164	1.128
0.80	1.155	1.201	1.201	1.175	1.176
0.90	1.167	1.209	1.233	1.187	1.205

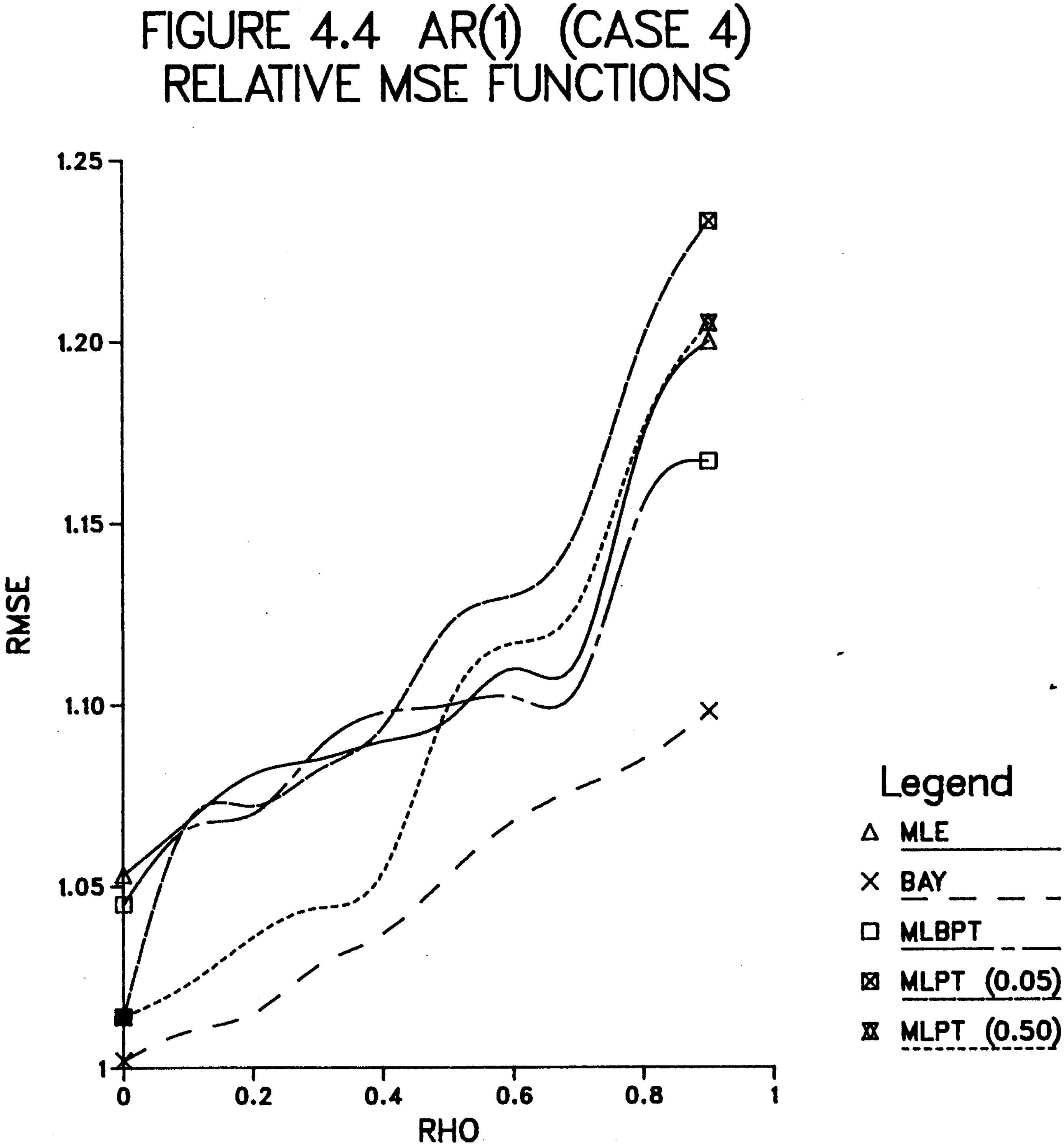


FIGURE 4.4 AR(1) (CASE 4)
RELATIVE MSE FUNCTIONS
RMSE
1.25
1.20
1.15
1.10
1.05
1
RHO
0
0.2
0.4
0.6
0.8
1
Legend
MLE
BAY
MLBPT
MLPT (0.05)
MLPT (0.50)

Table 4.5

RELATIVE MSE: AR(1) (CASE 5)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.127	1.167	1.022	1.073
0.10	1.023	1.136	1.253	1.056	1.090
0.20	1.067	1.138	1.265	1.061	1.103
0.30	1.149	1.154	1.311	1.074	1.121
0.40	1.269	1.156	1.346	1.083	1.145
0.50	1.485	1.168	1.388	1.104	1.157
0.60	1.665	1.194	1.457	1.110	1.160
0.70	1.977	1.210	1.556	1.117	1.226
0.80	1.986	1.218	1.689	1.129	1.228
0.90	2.821	1.247	1.731	1.224	1.234

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.188	1.057	1.067	1.030	1.029
0.10	1.299	1.060	1.177	1.035	1.141
0.20	1.308	1.087	1.299	1.056	1.158
0.30	1.317	1.125	1.363	1.085	1.197
0.40	1.324	1.136	1.386	1.180	1.310
0.50	1.341	1.210	1.432	1.193	1.413
0.60	1.411	1.212	1.500	1.200	1.519
0.70	1.500	1.298	1.613	1.265	1.594
0.80	1.550	1.317	1.755	1.309	1.762
0.90	1.635	1.318	1.899	1.315	1.871

FIGURE 4.5 AR(1) (CASE 5) RELATIVE MSE FUNCTIONS

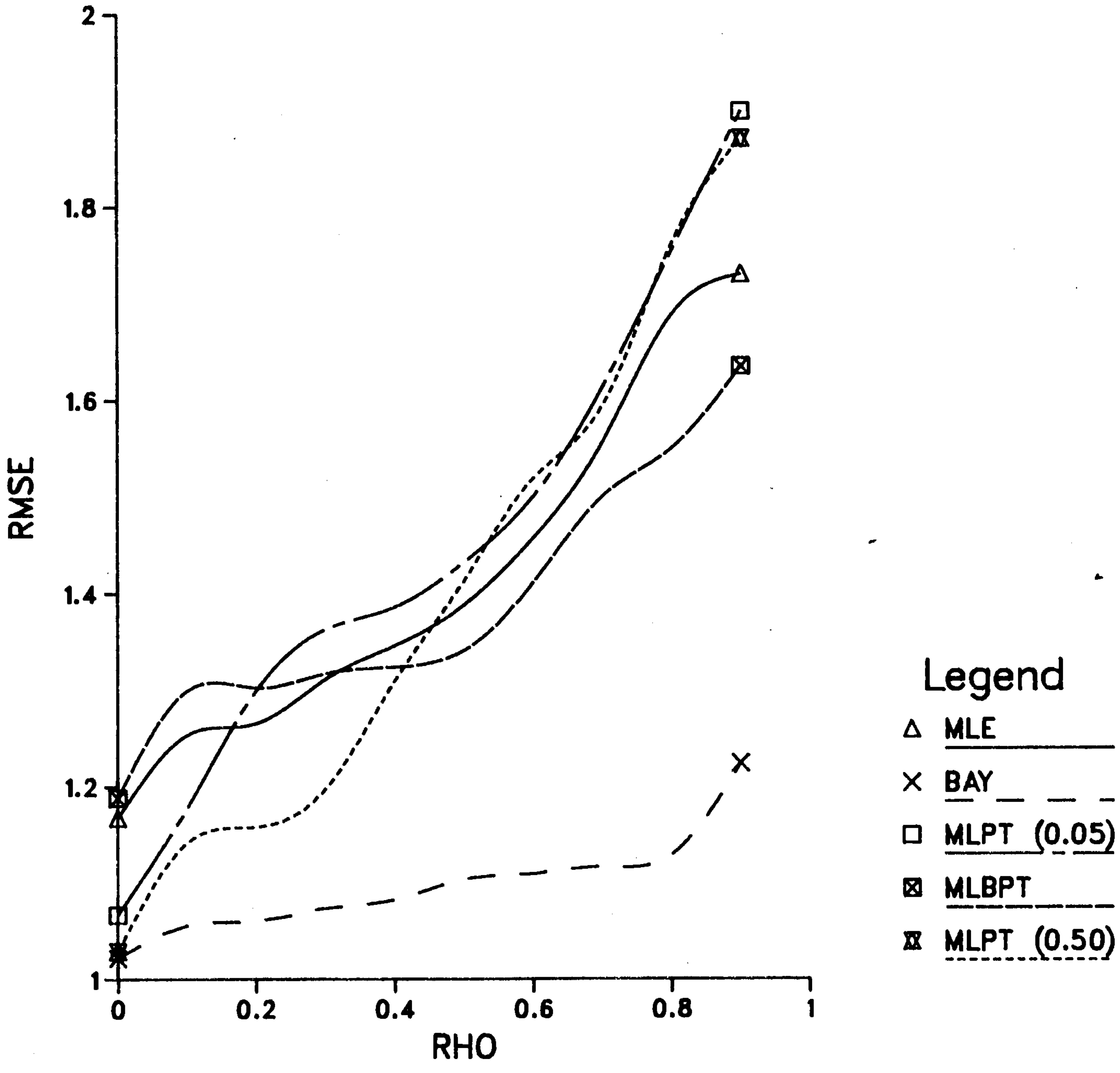


Table 4.6

RELATIVE MSE: AR(1) (CASE 6)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.063	1.084	1.002	1.018
0.10	1.017	1.081	1.090	1.005	1.027
0.20	1.043	1.116	1.118	1.031	1.057
0.30	1.147	1.182	1.213	1.036	1.127
0.40	1.294	1.208	1.217	1.051	1.159
0.50	1.566	1.286	1.311	1.060	1.270
0.60	1.683	1.344	1.324	1.100	1.305
0.70	2.539	1.478	1.370	1.126	1.479
0.80	3.150	1.506	1.444	1.167	1.505
0.90	4.466	1.533	1.651	1.171	1.525

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.026	1.013	1.008	1.016	1.030
0.10	1.032	1.050	1.059	1.017	1.044
0.20	1.059	1.066	1.070	1.080	1.047
0.30	1.137	1.155	1.213	1.133	1.156
0.40	1.163	1.191	1.325	1.164	1.265
0.50	1.232	1.335	1.345	1.290	1.309
0.60	1.288	1.370	1.475	1.356	1.377
0.70	1.350	1.618	1.477	1.517	1.418
0.80	1.398	1.630	1.578	1.539	1.475
0.90	1.556	1.633	1.766	1.556	1.673

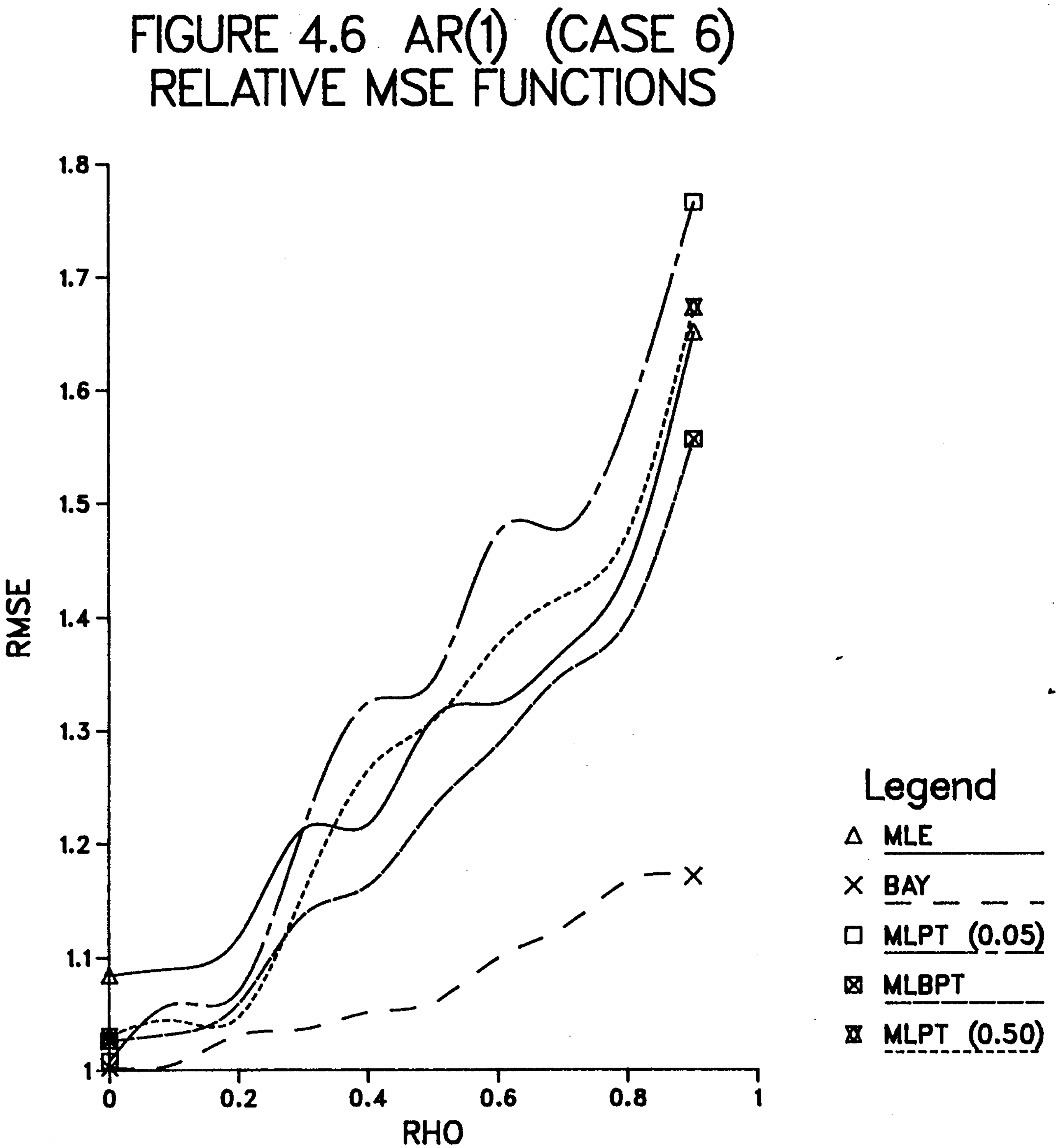


FIGURE 4.6 AR(1) (CASE 6)
RELATIVE MSE FUNCTIONS
RMSE
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0
0.2
0.4
0.6
0.8
1
RHO
Legend
MLE
BAY
MLPT (0.05)
MLBPT
MLPT (0.50)

Table 4.7

RELATIVE MSE: AR(1) (CASE 7)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.009	1.003	1.000	1.004
0.10	1.006	1.021	1.004	1.002	1.014
0.20	1.011	1.036	1.017	1.003	1.032
0.30	1.040	1.043	1.042	1.009	1.041
0.40	1.058	1.079	1.107	1.016	1.069
0.50	1.134	1.087	1.121	1.025	1.074
0.60	1.175	1.133	1.149	1.039	1.120
0.70	1.390	1.212	1.211	1.064	1.211
0.80	1.673	1.449	1.433	1.099	1.448
0.90	2.153	1.673	1.604	1.133	1.672

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.003	1.003	1.001	1.012	1.006
0.10	1.007	1.005	1.003	1.013	1.008
0.20	1.028	1.006	1.009	1.030	1.009
0.30	1.043	1.052	1.046	1.033	1.030
0.40	1.089	1.097	1.120	1.078	1.085
0.50	1.101	1.098	1.134	1.094	1.133
0.60	1.146	1.153	1.165	1.139	1.158
0.70	1.238	1.246	1.259	1.220	1.239
0.80	1.430	1.497	1.436	1.447	1.429
0.90	1.602	1.727	1.638	1.670	1.600

FIGURE 4.7 AR(1) (CASE 7)
RELATIVE MSE FUNCTIONS

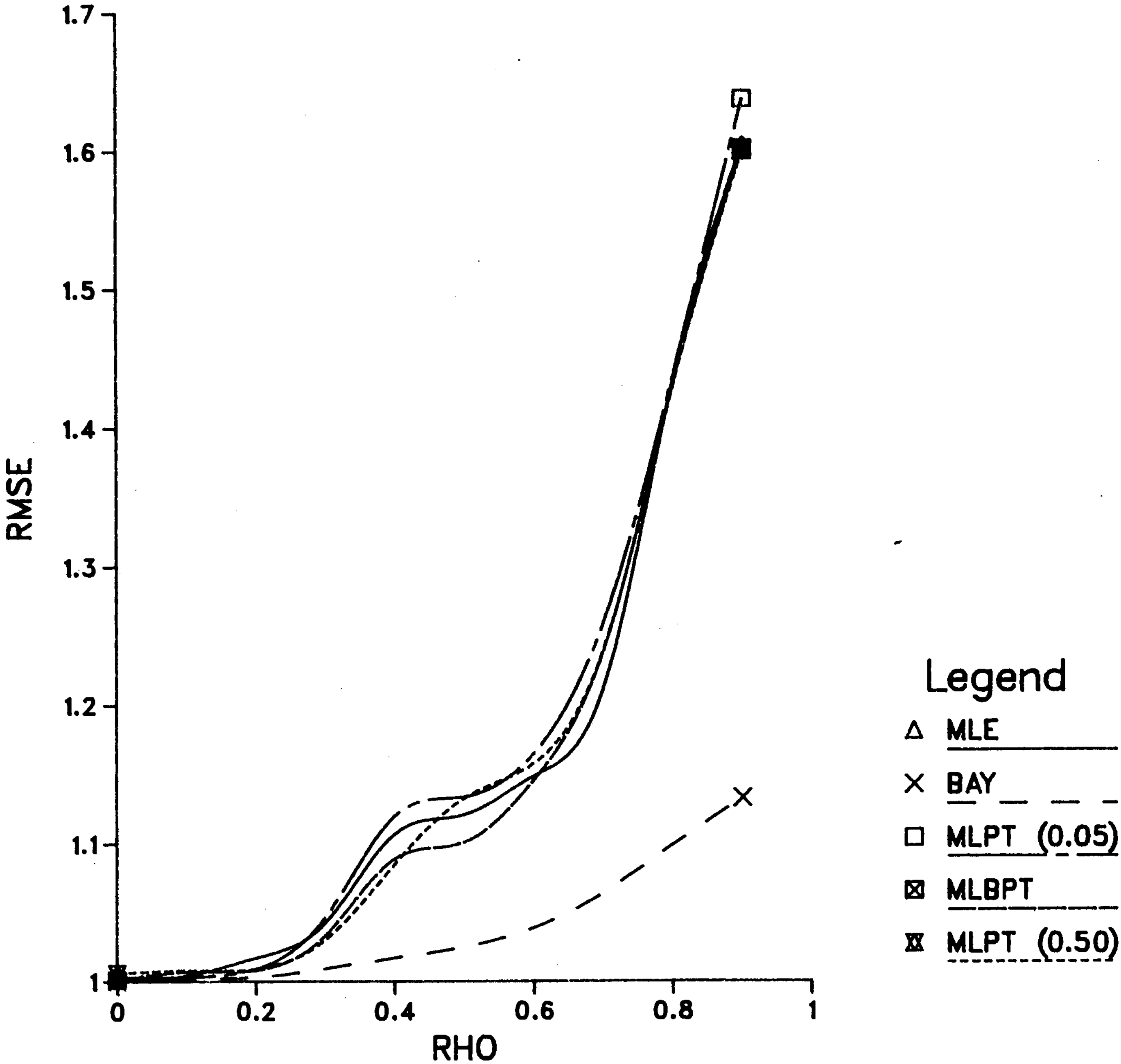


Table 4.8

RELATIVE MSE: AR(1) (CASE 8)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.051	1.110	1.003	1.017
0.10	1.003	1.066	1.189	1.006	1.019
0.20	1.038	1.107	1.200	1.008	1.043
0.30	1.094	1.141	1.226	1.020	1.052
0.40	1.276	1.147	1.243	1.029	1.077
0.50	1.636	1.264	1.397	1.121	1.238
0.60	1.942	1.334	1.468	1.149	1.312
0.70	2.749	1.515	1.772	1.271	1.503
0.80	3.528	1.730	1.861	1.484	1.724
0.90	5.538	1.944	2.266	1.699	1.939

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.055	1.025	1.034	1.038	1.071
0.10	1.091	1.032	1.035	1.050	1.090
0.20	1.110	1.038	1.139	1.093	1.150
0.30	1.130	1.138	1.147	1.110	1.154
0.40	1.200	1.243	1.266	1.125	1.169
0.50	1.341	1.450	1.469	1.251	1.400
0.60	1.416	1.593	1.612	1.356	1.495
0.70	1.794	2.038	2.050	1.556	1.794
0.80	1.936	2.465	2.423	1.791	1.873
0.90	2.542	3.583	3.512	2.150	2.457

FIGURE 4.8 AR(1) (CASE 8)
RELATIVE MSE FUNCTIONS

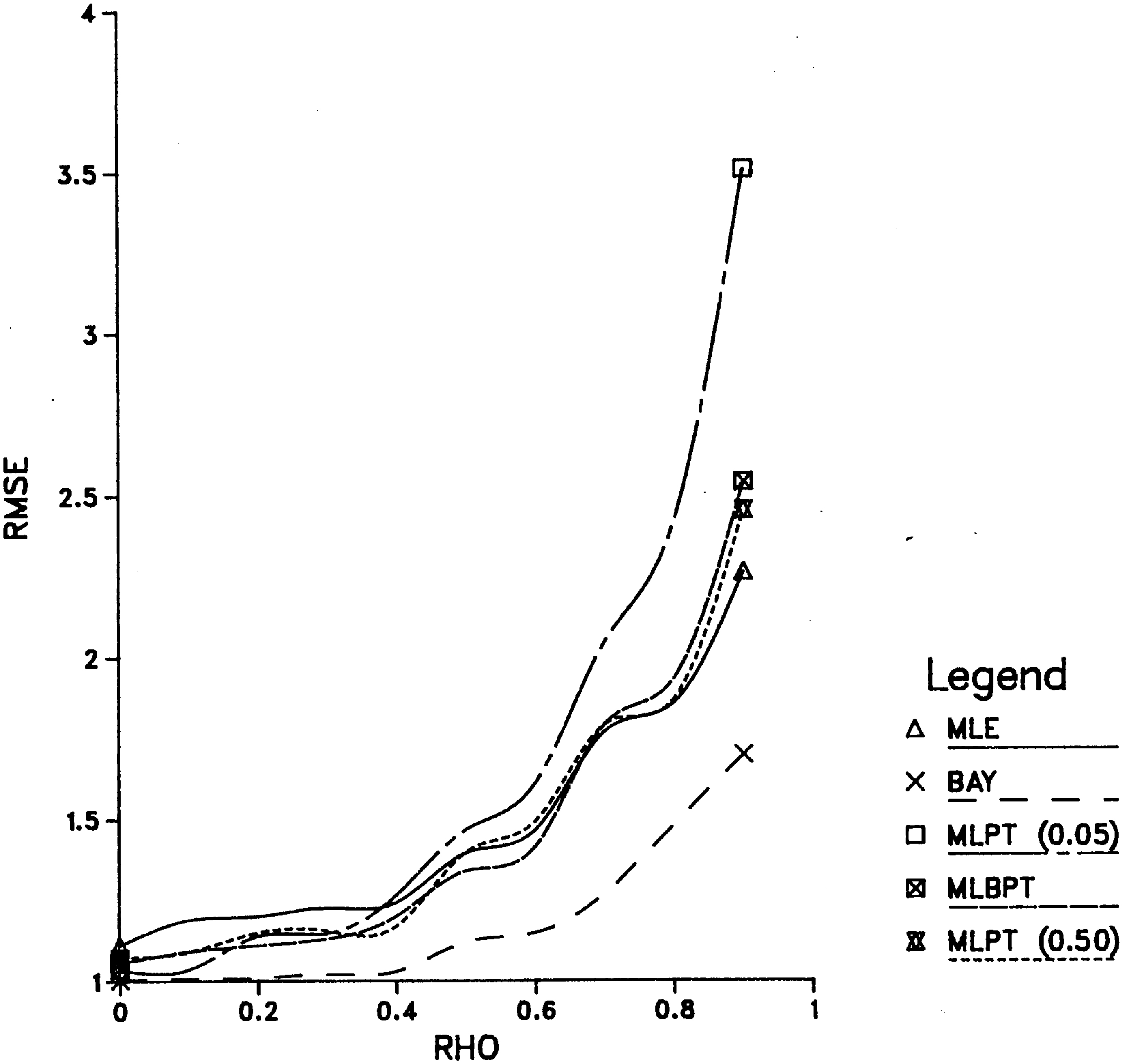


Table 4.9

RELATIVE MSE: AR(1) (CASE 9)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.037	1.046	0.931	1.010
0.10	1.006	1.038	1.051	0.946	1.012
0.20	1.018	1.053	1.056	0.966	1.025
0.30	1.062	1.081	1.081	0.972	1.055
0.40	1.157	1.123	1.100	1.005	1.103
0.50	1.338	1.133	1.125	1.019	1.119
0.60	1.345	1.152	1.211	1.100	1.143
0.70	1.618	1.154	1.287	1.107	1.149
0.80	2.015	1.193	1.351	1.110	1.191
0.90	2.486	1.252	1.399	1.118	1.246

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.013	1.017	1.025	1.020	1.026
0.10	1.018	1.018	1.045	1.023	1.032
0.20	1.020	1.030	1.050	1.024	1.037
0.30	1.046	1.061	1.061	1.063	1.062
0.40	1.077	1.123	1.119	1.127	1.080
0.50	1.109	1.147	1.128	1.139	1.152
0.60	1.133	1.153	1.227	1.154	1.237
0.70	1.201	1.165	1.312	1.160	1.294
0.80	1.276	1.225	1.383	1.200	1.360
0.90	1.311	1.259	1.431	1.246	1.401

FIGURE 4.9 AR(1) (CASE 9)
RELATIVE MSE FUNCTIONS

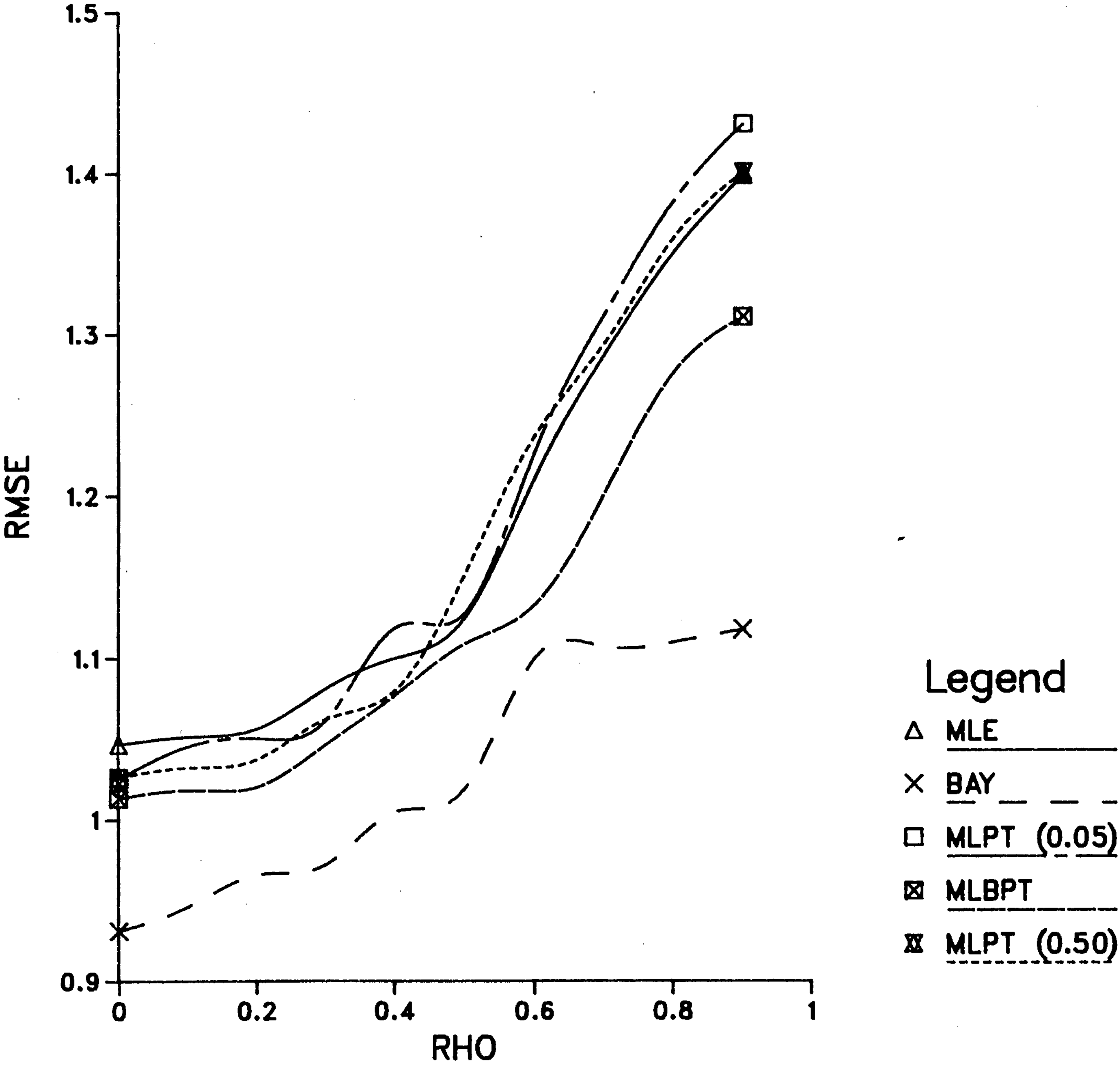


Table 4.10

RELATIVE MSE: AR(1) (CASE 10)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.008	1.006	1.001	1.009
0.10	1.005	1.034	1.024	1.009	1.038
0.20	1.008	1.047	1.042	1.016	1.046
0.30	1.013	1.048	1.052	1.021	1.055
0.40	1.046	1.052	1.053	1.029	1.059
0.50	1.094	1.056	1.088	1.035	1.065
0.60	1.105	1.073	1.125	1.042	1.068
0.70	1.196	1.084	1.165	1.057	1.071
0.80	1.271	1.100	1.191	1.069	1.073
0.90	1.344	1.130	1.221	1.084	1.128

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.006	1.001	1.000	1.001	1.000
0.10	1.028	1.026	1.018	1.029	1.014
0.20	1.046	1.031	1.031	1.042	1.016
0.30	1.059	1.046	1.037	1.043	1.029
0.40	1.064	1.059	1.060	1.044	1.038
0.50	1.079	1.060	1.099	1.045	1.048
0.60	1.097	1.090	1.161	1.078	1.136
0.70	1.145	1.111	1.191	1.095	1.174
0.80	1.190	1.138	1.242	1.109	1.203
0.90	1.220	1.150	1.285	1.135	1.249

FIGURE 4.10 AR(1) (CASE 10)
RELATIVE MSE FUNCTIONS

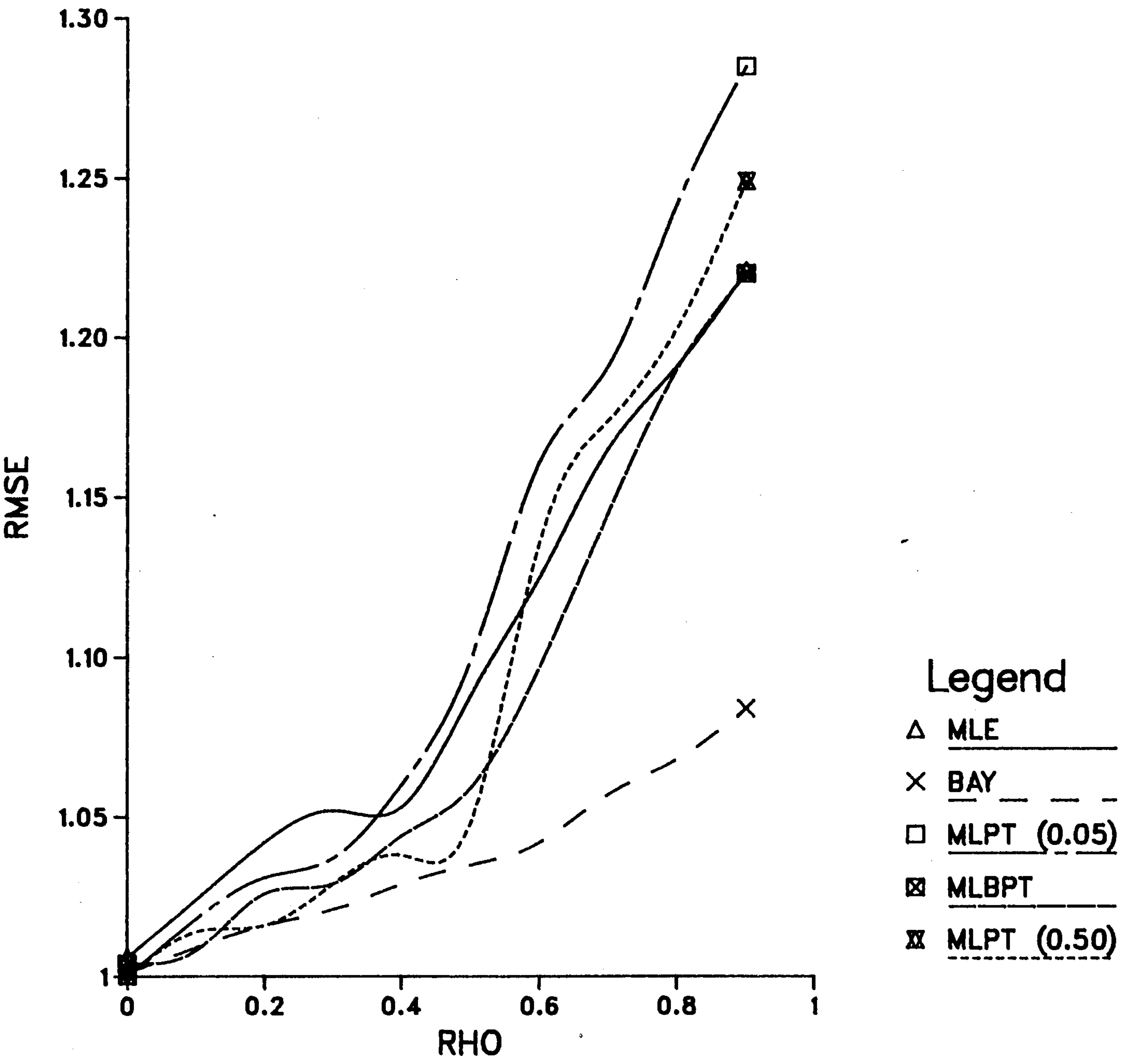


Table 4.11

RELATIVE MSE: AR(1) (CASE 11)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.004	1.010	0.952	1.006
0.10	1.005	1.029	1.025	0.968	1.011
0.20	1.010	1.036	1.029	0.969	1.021
0.30	1.016	1.059	1.050	0.973	1.038
0.40	1.059	1.063	1.052	0.980	1.056
0.50	1.104	1.073	1.065	0.985	1.064
0.60	1.126	1.099	1.066	0.988	1.099
0.70	1.244	1.134	1.093	0.995	1.135
0.80	1.313	1.176	1.105	1.001	1.176
0.90	1.409	1.211	1.158	1.044	1.211

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.010	1.006	1.001	1.000	1.002
0.10	1.019	1.012	1.016	1.018	1.014
0.20	1.024	1.019	1.021	1.026	1.018
0.30	1.031	1.041	1.036	1.040	1.033
0.40	1.043	1.058	1.055	1.064	1.051
0.50	1.055	1.067	1.071	1.065	1.068
0.60	1.059	1.100	1.074	1.090	1.075
0.70	1.091	1.140	1.098	1.129	1.099
0.80	1.104	1.182	1.114	1.176	1.106
0.90	1.159	1.228	1.169	1.210	1.158

FIGURE 4.11 AR(1) (CASE 11)
RELATIVE MSE FUNCTIONS

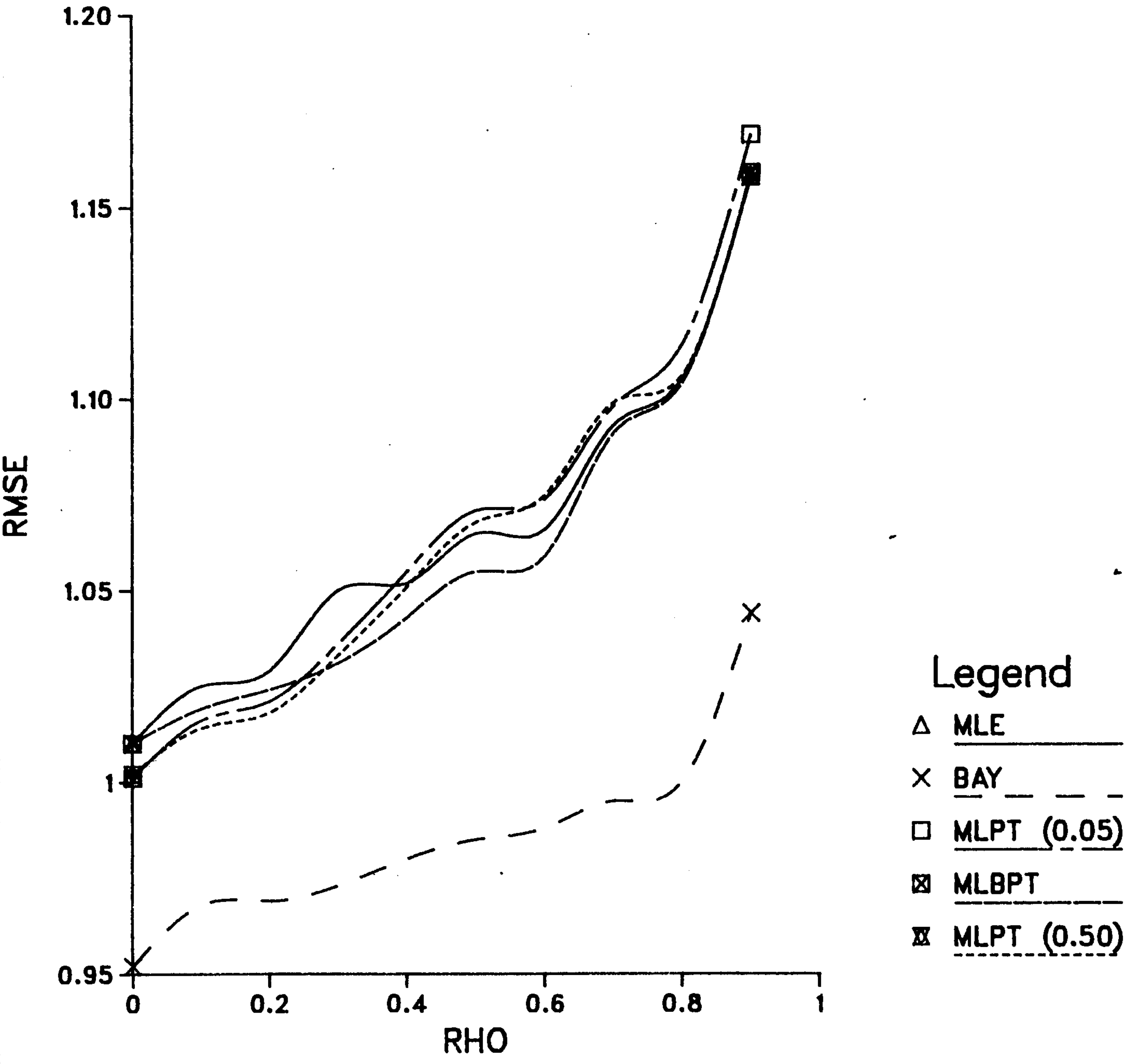


Table 4.12

RELATIVE MSE: AR(1) (CASE 12)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.007	1.015	1.001	1.001
0.10	1.001	1.033	1.040	1.003	1.015
0.20	1.021	1.040	1.046	1.005	1.022
0.30	1.047	1.060	1.058	1.007	1.043
0.40	1.081	1.061	1.076	1.015	1.049
0.50	1.126	1.074	1.105	1.023	1.053
0.60	1.259	1.126	1.122	1.033	1.129
0.70	1.478	1.238	1.173	1.034	1.234
0.80	1.558	1.241	1.209	1.056	1.236
0.90	2.004	1.268	1.212	1.073	1.265

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.016	1.006	1.008	1.006	1.005
0.10	1.048	1.025	1.028	1.030	1.037
0.20	1.053	1.027	1.060	1.031	1.042
0.30	1.065	1.064	1.063	1.051	1.049
0.40	1.075	1.074	1.083	1.057	1.067
0.50	1.083	1.082	1.111	1.058	1.089
0.60	1.112	1.161	1.139	1.135	1.130
0.70	1.168	1.253	1.189	1.237	1.193
0.80	1.202	1.260	1.220	1.238	1.223
0.90	1.207	1.288	1.228	1.267	1.230

FIGURE 4.12 AR(1) (CASE 12)
RELATIVE MSE FUNCTIONS

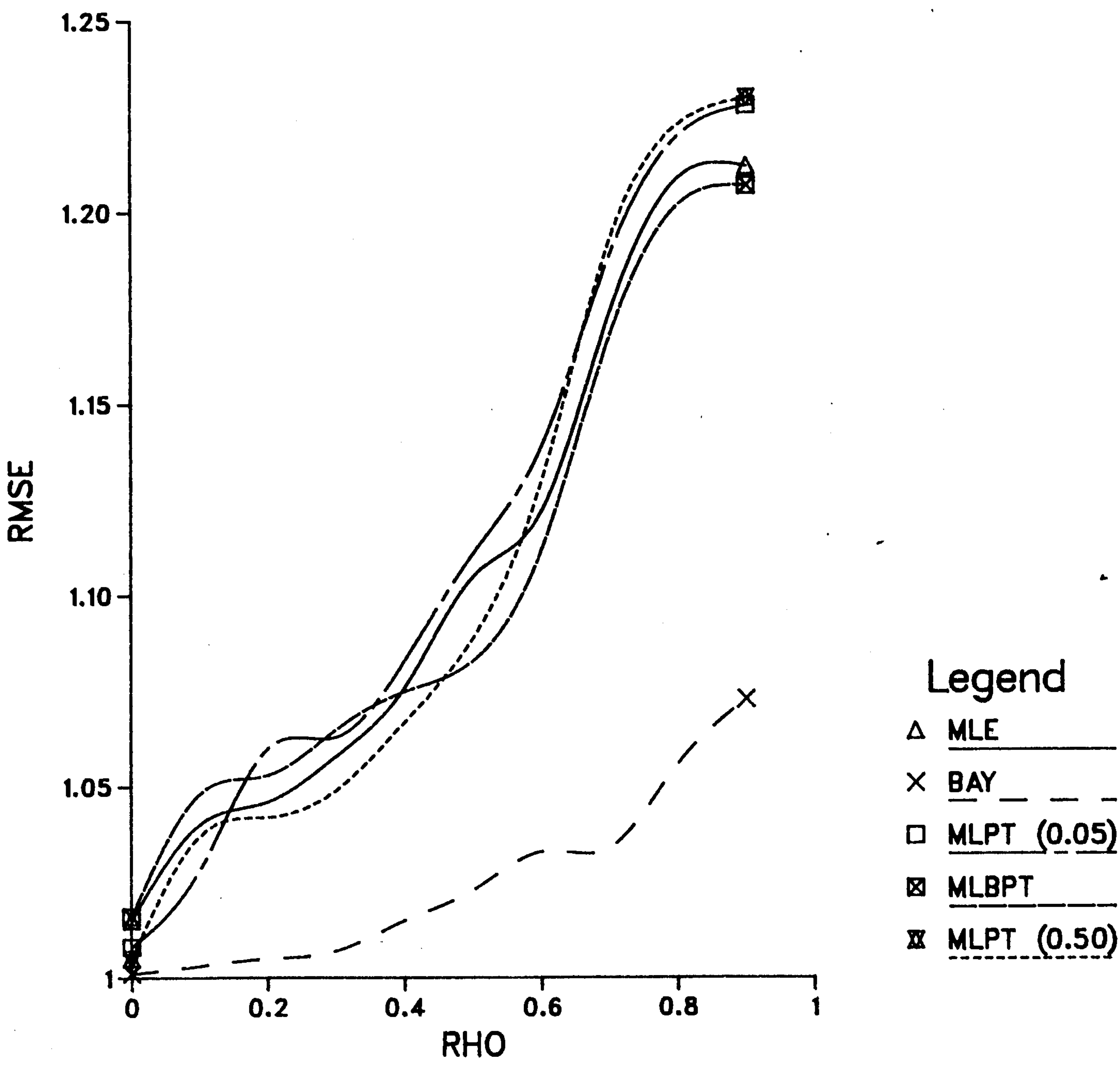


Table 4.13

RELATIVE MSE: AR(1) (CASE 13)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.054	1.070	1.004	1.012
0.10	1.012	1.068	1.079	1.010	1.020
0.20	1.034	1.078	1.100	1.015	1.037
0.30	1.078	1.084	1.109	1.019	1.051
0.40	1.186	1.104	1.110	1.025	1.078
0.50	1.268	1.123	1.131	1.037	1.095
0.60	1.467	1.131	1.134	1.053	1.114
0.70	1.811	1.244	1.221	1.056	1.233
0.80	2.008	1.260	1.269	1.066	1.251
0.90	2.672	1.288	1.273	1.068	1.293

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.022	1.045	1.057	1.037	1.052
0.10	1.024	1.049	1.065	1.050	1.064
0.20	1.047	1.067	1.072	1.059	1.080
0.30	1.058	1.075	1.079	1.063	1.083
0.40	1.078	1.145	1.139	1.094	1.138
0.50	1.100	1.162	1.158	1.108	1.146
0.60	1.108	1.183	1.185	1.123	1.154
0.70	1.220	1.340	1.304	1.245	1.230
0.80	1.275	1.372	1.362	1.249	1.270
0.90	1.289	1.388	1.399	1.284	1.279

FIGURE 4.13 AR(1) (CASE 13)
RELATIVE MSE FUNCTIONS

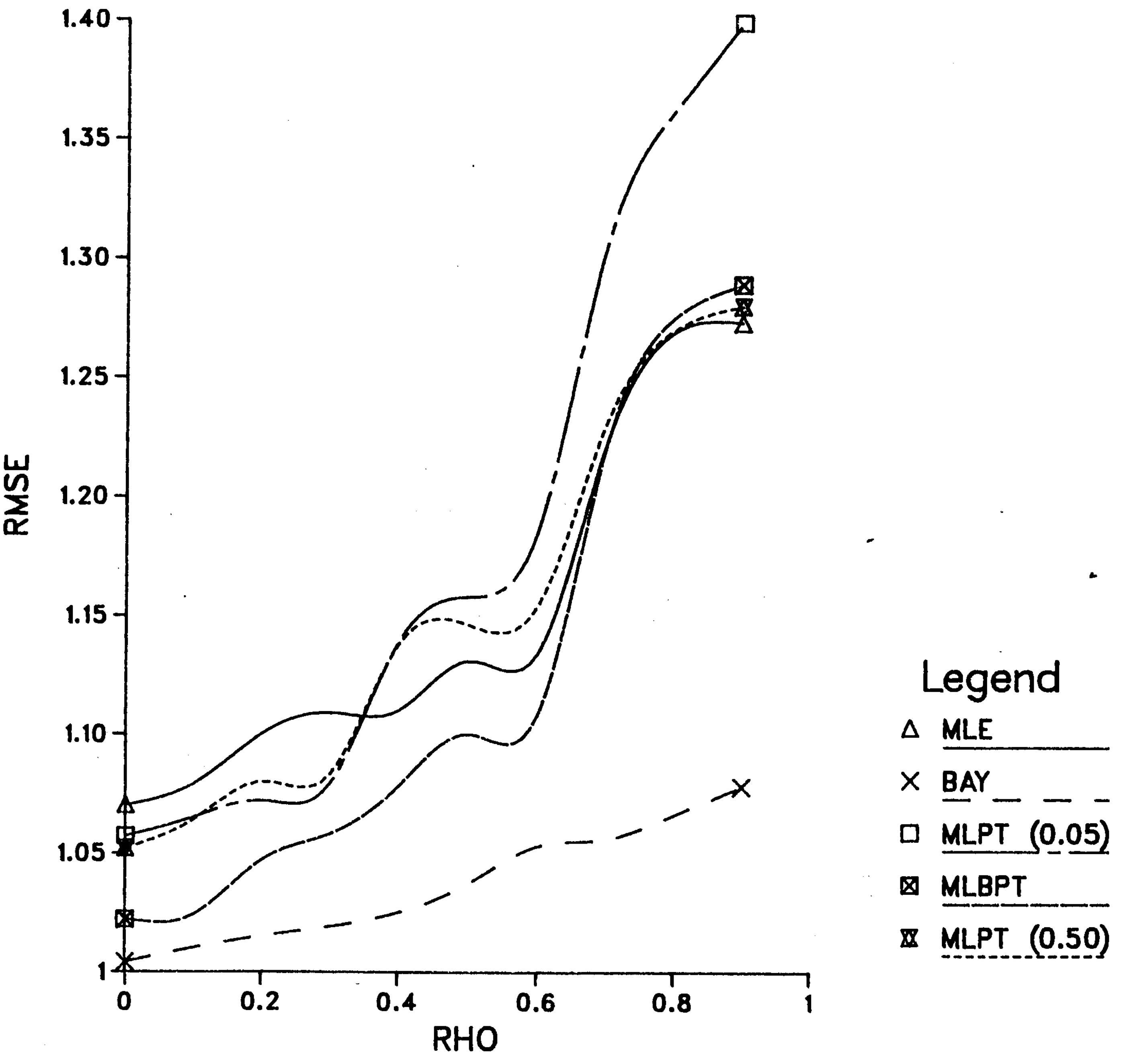


Table 4.14

RELATIVE MSE (CEse 14)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.004	1.001	0.989	1.001
0.10	1.005	1.010	1.014	0.997	1.007
0.20	1.010	1.021	1.020	0.999	1.015
0.30	1.016	1.028	1.027	1.001	1.025
0.40	1.032	1.034	1.034	1.003	1.026
0.50	1.088	1.053	1.064	1.007	1.053
0.60	1.127	1.054	1.071	1.014	1.054
0.70	1.285	1.055	1.089	1.023	1.058
0.80	1.480	1.075	1.099	1.024	1.077
0.90	1.592	1.143	1.146	1.036	1.146

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.002	1.001	1.000	1.012	1.016
0.10	1.016	1.010	1.015	1.016	1.024
0.20	1.033	1.011	1.017	1.031	1.032
0.30	1.036	1.023	1.025	1.033	1.034
0.40	1.041	1.029	1.031	1.039	1.041
0.50	1.059	1.056	1.071	1.090	1.073
0.60	1.062	1.069	1.095	1.100	1.074
0.70	1.073	1.073	1.100	1.104	1.108
0.80	1.080	1.079	1.105	1.106	1.116
0.90	1.103	1.145	1.168	1.148	1.157

FIGURE 4.14 AR(1) (CASE 14)
RELATIVE MSE FUNCTIONS

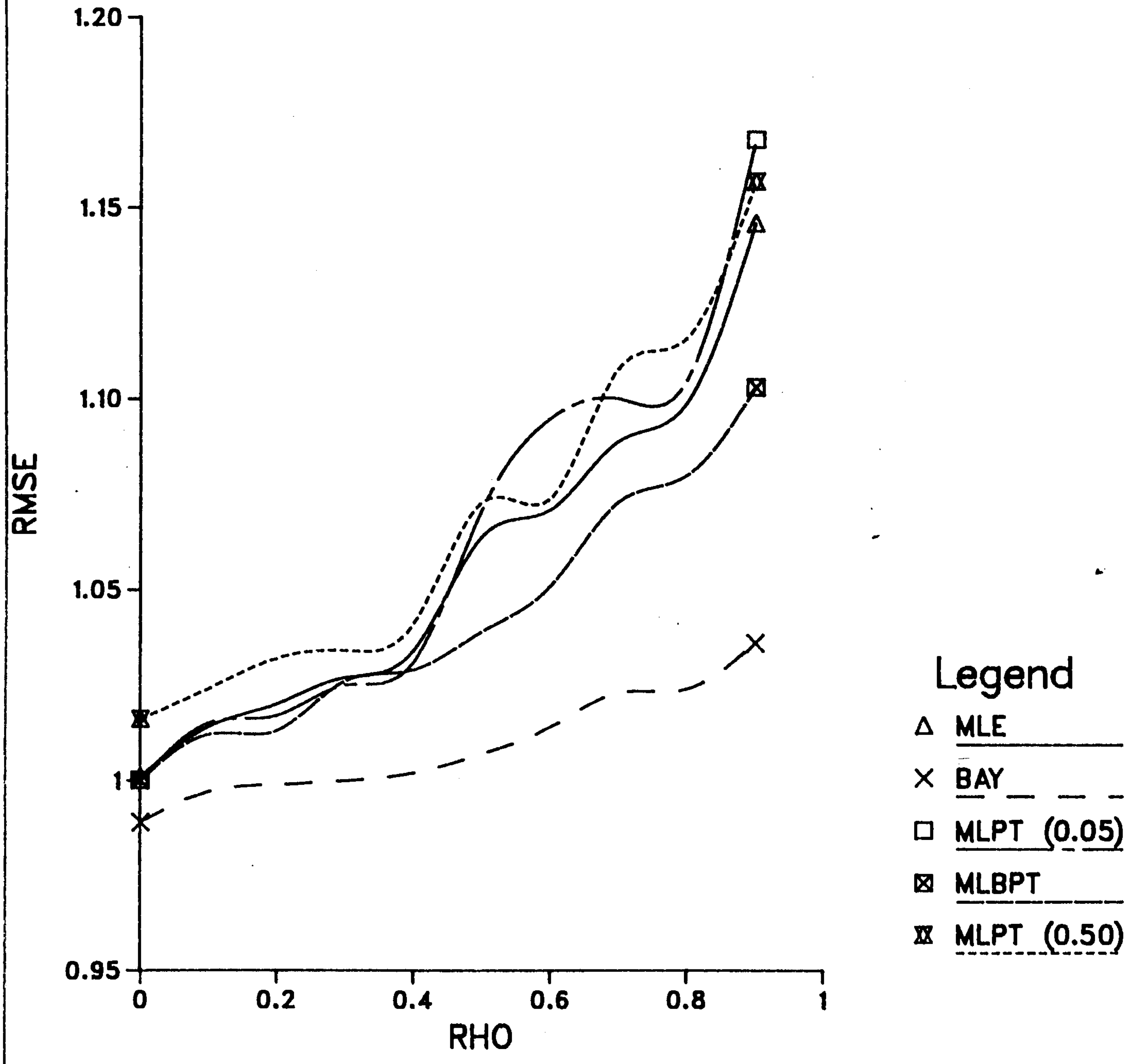


Table 4.15

RELATIVE MSE: AR(1) (CASE 15)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.071	1.113	1.010	1.022
0.10	1.024	1.078	1.114	1.017	1.036
0.20	1.106	1.145	1.185	1.020	1.090
0.30	1.110	1.169	1.241	1.028	1.127
0.40	1.303	1.218	1.254	1.041	1.197
0.50	1.558	1.279	1.289	1.055	1.270
0.60	1.744	1.294	1.309	1.064	1.278
0.70	2.419	1.384	1.335	1.087	1.398
0.80	3.132	1.523	1.398	1.166	1.547
0.90	5.184	1.669	1.487	1.295	1.726

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.041	1.019	1.027	1.033	1.060
0.10	1.050	1.054	1.068	1.058	1.093
0.20	1.106	1.115	1.129	1.070	1.099
0.30	1.172	1.137	1.165	1.120	1.176
0.40	1.216	1.281	1.287	1.180	1.203
0.50	1.258	1.361	1.346	1.254	1.300
0.60	1.371	1.372	1.376	1.266	1.376
0.70	1.399	1.617	1.585	1.378	1.380
0.80	1.420	1.816	1.698	1.507	1.412
0.90	1.511	2.120	1.800	1.649	1.504

FIGURE 4.15 AR(1) (CASE 15) RELATIVE MSE FUNCTIONS

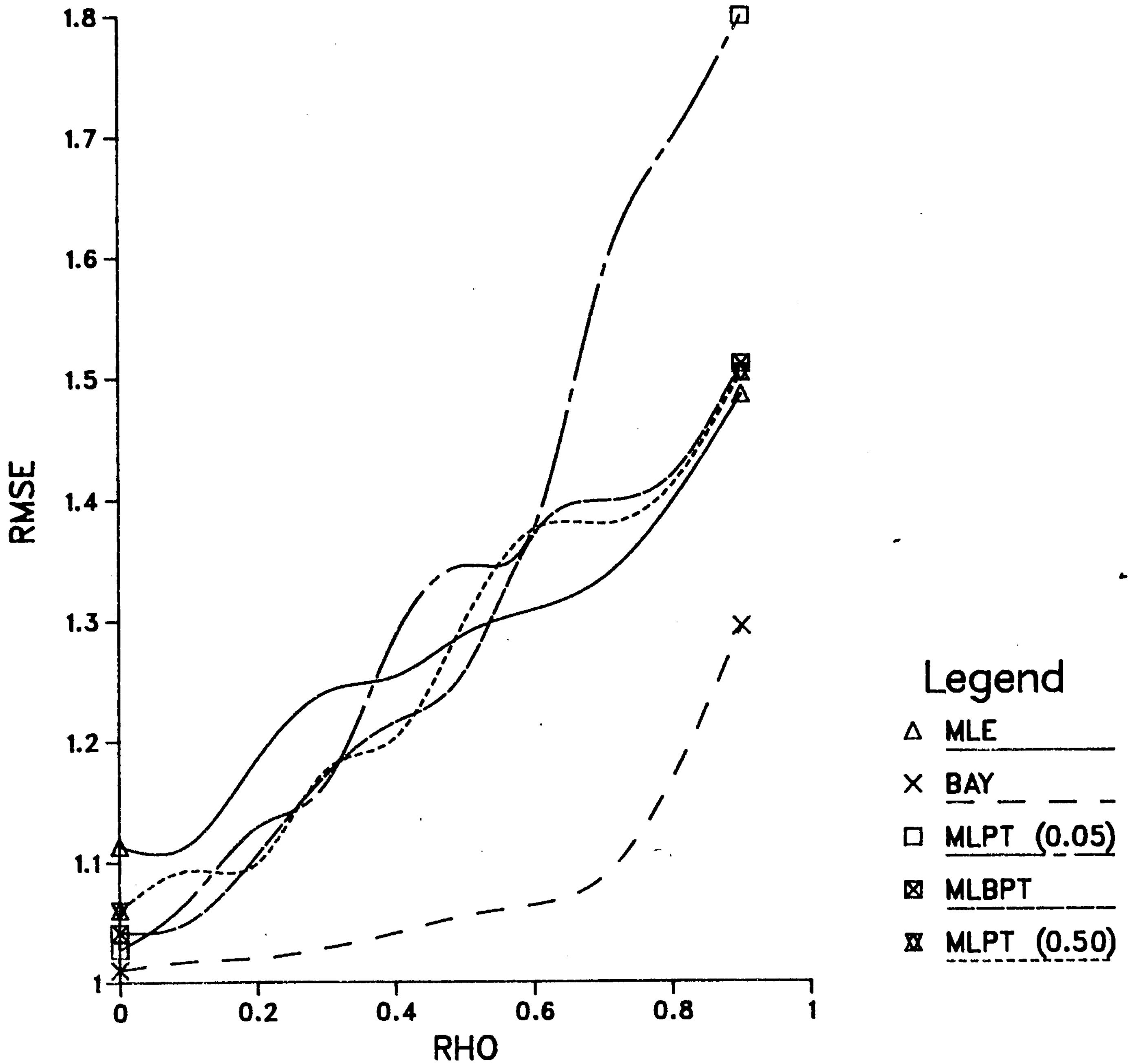


Table 4.16

RELATIVE MSE: AR(1) (CASE 16)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.005	1.006	1.003	1.001
0.10	1.002	1.011	1.009	1.006	1.004
0.20	1.003	1.015	1.014	1.007	1.008
0.30	1.012	1.016	1.029	1.008	1.016
0.40	1.047	1.031	1.035	1.014	1.022
0.50	1.050	1.033	1.041	1.020	1.036
0.60	1.092	1.044	1.042	1.022	1.042
0.70	1.126	1.064	1.054	1.024	1.064
0.80	1.176	1.069	1.077	1.025	1.068
0.90	1.252	1.114	1.083	1.032	1.116

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.005	1.001	1.004	1.000	1.002
0.10	1.009	1.003	1.008	1.013	1.007
0.20	1.014	1.006	1.019	1.038	1.010
0.30	1.020	1.015	1.032	1.039	1.033
0.40	1.028	1.016	1.043	1.089	1.046
0.50	1.030	1.035	1.061	1.116	1.058
0.60	1.034	1.038	1.068	1.143	1.067
0.70	1.041	1.061	1.071	1.187	1.075
0.80	1.049	1.070	1.091	1.193	1.089
0.90	1.161	1.113	1.100	1.215	1.100

FIGURE 4.16 AR(1) (CASE 16)
RELATIVE MSE FUNCTIONS

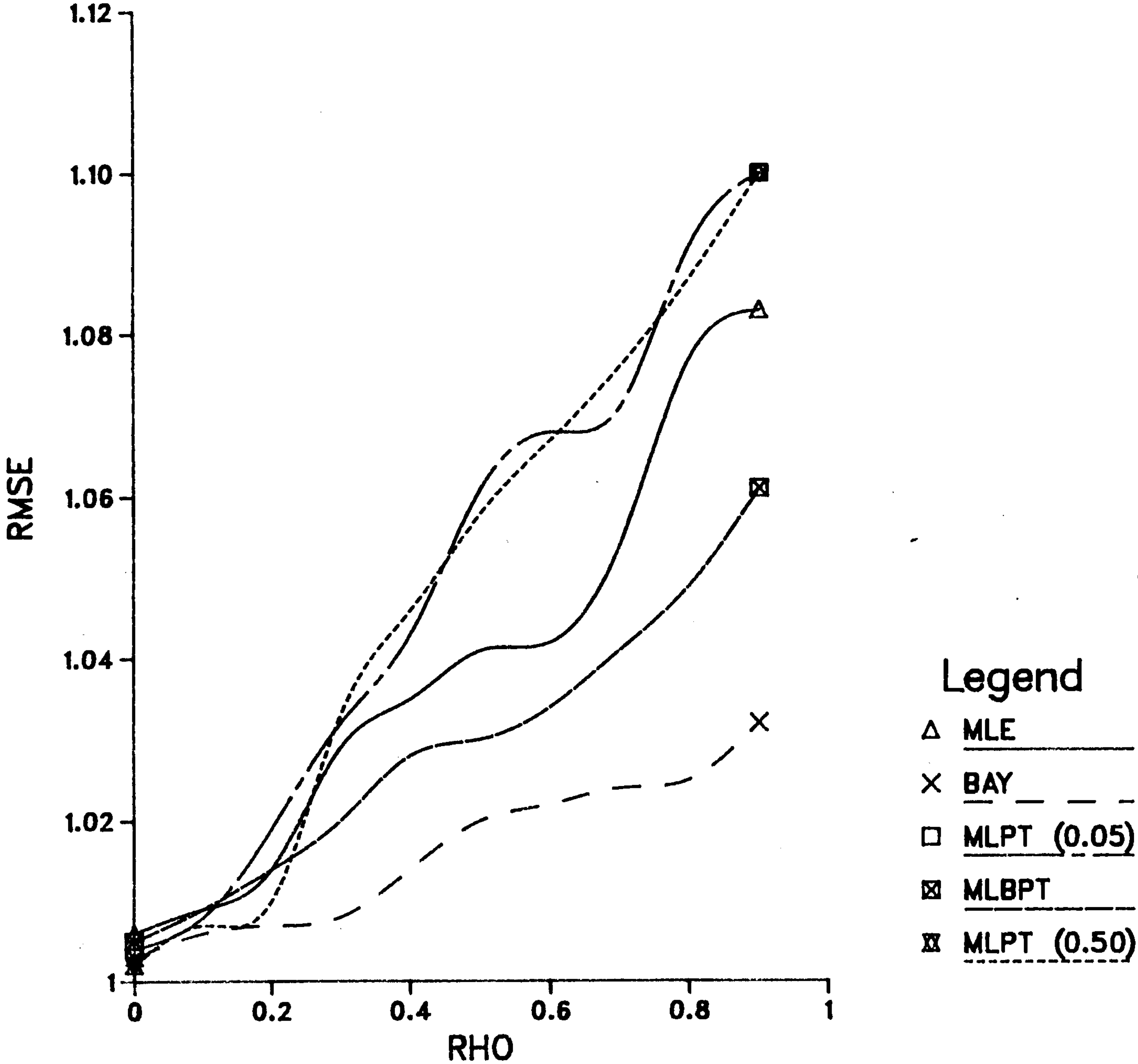


Table 4.17

RELATIVE MSE: AR(1) (CASE 17)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.006	1.004	0.940	1.007
0.10	1.001	1.022	1.022	0.951	1.010
0.20	1.008	1.026	1.036	0.961	1.014
0.30	1.024	1.034	1.043	0.970	1.016
0.40	1.057	1.082	1.062	0.972	1.056
0.50	1.163	1.112	1.114	0.992	1.097
0.60	1.123	1.114	1.138	0.995	1.105
0.70	1.469	1.160	1.181	0.998	1.157
0.80	1.627	1.243	1.239	1.022	1.243
0.90	2.245	1.353	1.343	1.088	1.352

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.001	1.011	1.009	1.001	1.003
0.10	1.010	1.019	1.014	1.011	1.012
0.20	1.022	1.022	1.030	1.019	1.029
0.30	1.024	1.034	1.050	1.026	1.038
0.40	1.043	1.062	1.075	1.075	1.056
0.50	1.106	1.113	1.127	1.108	1.111
0.60	1.112	1.130	1.146	1.110	1.144
0.70	1.182	1.193	1.210	1.160	1.186
0.80	1.236	1.270	1.254	1.243	1.238
0.90	1.343	1.396	1.365	1.352	1.344

FIGURE 4.17 AR(1) (CASE 17)
RELATIVE MSE FUNCTIONS

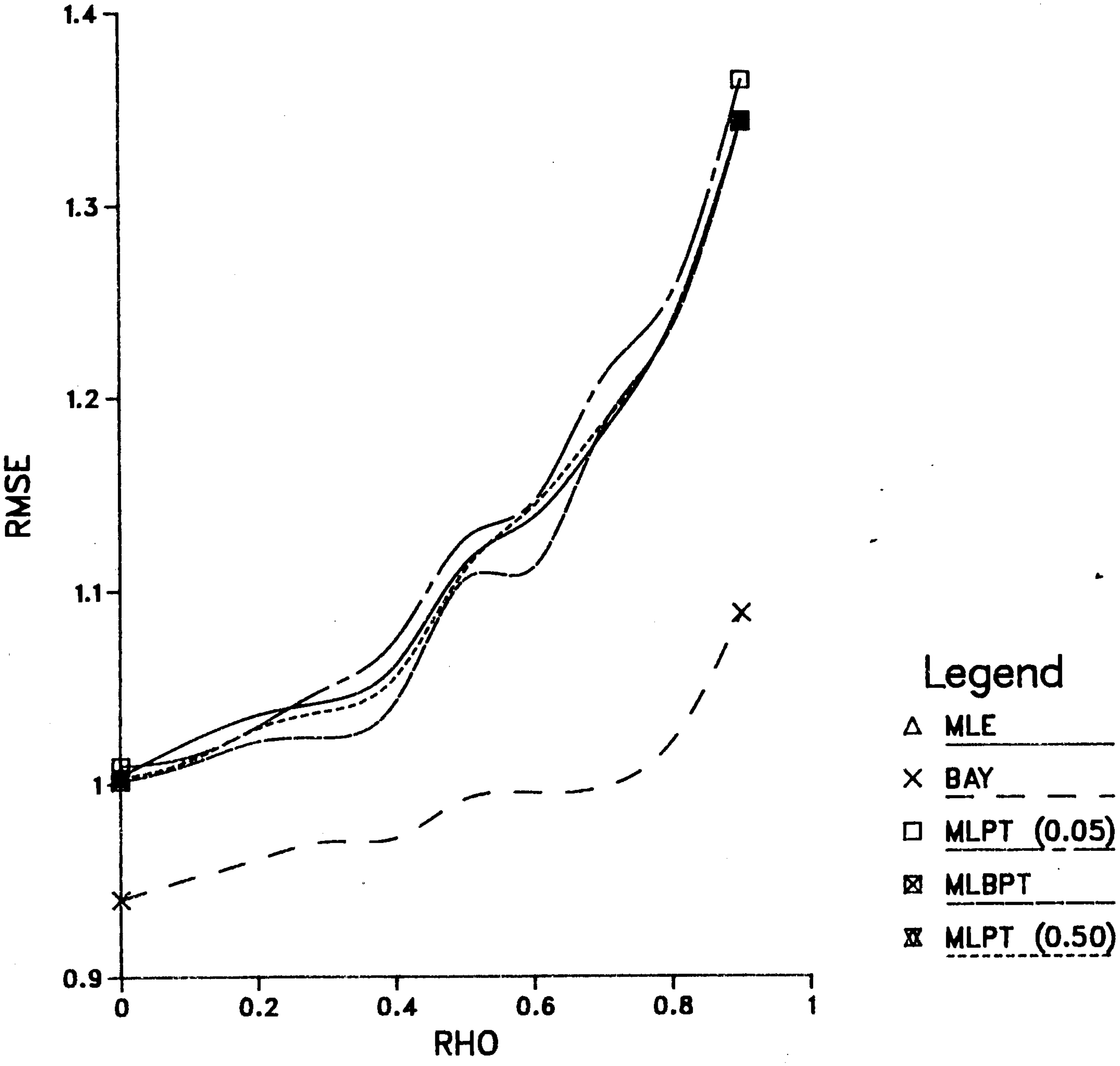


Table 4.18

Mse Of Various Estimators renative to Mse of (CASE 18)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.047	1.029	1.008	1.018
0.10	1.005	1.065	1.071	1.011	1.038
0.20	1.038	1.103	1.097	1.016	1.056
0.30	1.039	1.114	1.098	1.022	1.071
0.40	1.107	1.131	1.147	1.041	1.080
0.50	1.327	1.212	1.188	1.054	1.188
0.60	1.339	1.231	1.220	1.061	1.202
0.70	1.755	1.306	1.266	1.066	1.287
0.80	1.971	1.314	1.286	1.069	1.295
0.90	2.785	1.466	1.335	1.081	1.457

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.010	1.023	1.016	1.027	1.006
0.10	1.042	1.057	1.058	1.048	1.053
0.20	1.052	1.081	1.078	1.071	1.077
0.30	1.061	1.103	1.090	1.078	1.096
0.40	1.093	1.110	1.111	1.103	1.115
0.50	1.168	1.254	1.213	1.197	1.190
0.60	1.193	1.259	1.242	1.218	1.242
0.70	1.276	1.365	1.302	1.299	1.288
0.80	1.291	1.372	1.314	1.310	1.310
0.90	1.429	1.522	1.374	1.465	1.353

FIGURE 4.18 AR(1) (CASE 18)
RELATIVE MSE FUNCTIONS

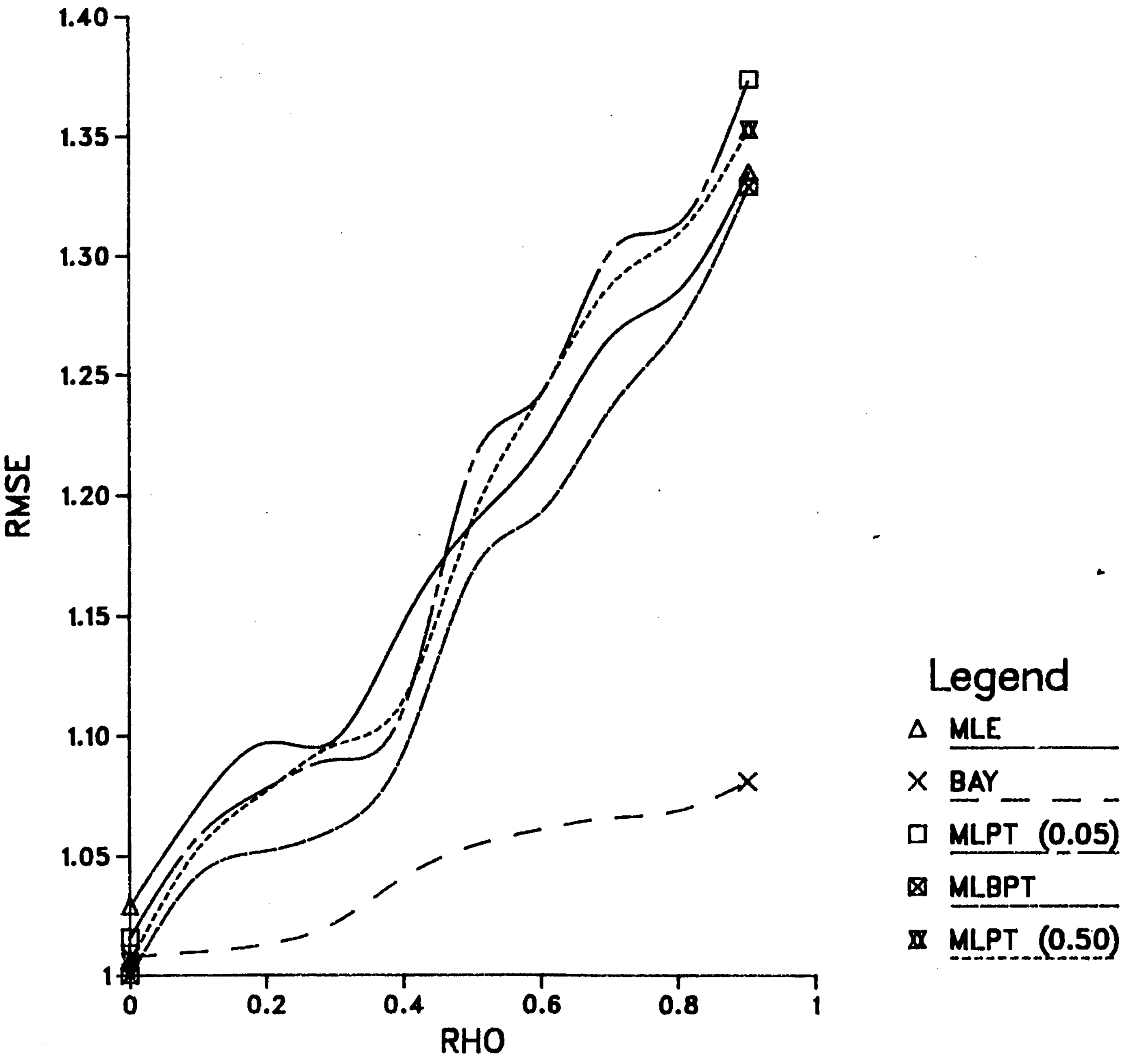


Table 4.19

RELATIVE MSE: AR(1) (CASE 19)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.017	1.012	0.937	1.003
0.10	1.007	1.023	1.025	0.945	1.005
0.20	1.013	1.045	1.026	0.954	1.022
0.30	1.014	1.052	1.032	0.965	1.027
0.40	1.059	1.068	1.037	0.977	1.040
0.50	1.066	1.054	1.064	0.986	1.055
0.60	1.099	1.084	1.067	0.993	1.073
0.70	1.234	1.089	1.093	1.000	1.085
0.80	1.308	1.120	1.125	1.020	1.116
0.90	1.394	1.178	1.182	1.029	1.174

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.006	1.001	1.003	1.006	1.001
0.10	1.008	1.014	1.013	1.012	1.011
0.20	1.016	1.032	1.021	1.038	1.022
0.30	1.018	1.038	1.025	1.040	1.024
0.40	1.020	1.061	1.029	1.052	1.030
0.50	1.052	1.080	1.068	1.061	1.057
0.60	1.057	1.089	1.073	1.077	1.060
0.70	1.089	1.103	1.100	1.086	1.100
0.80	1.122	1.129	1.131	1.120	1.126
0.90	1.178	1.191	1.189	1.178	1.182

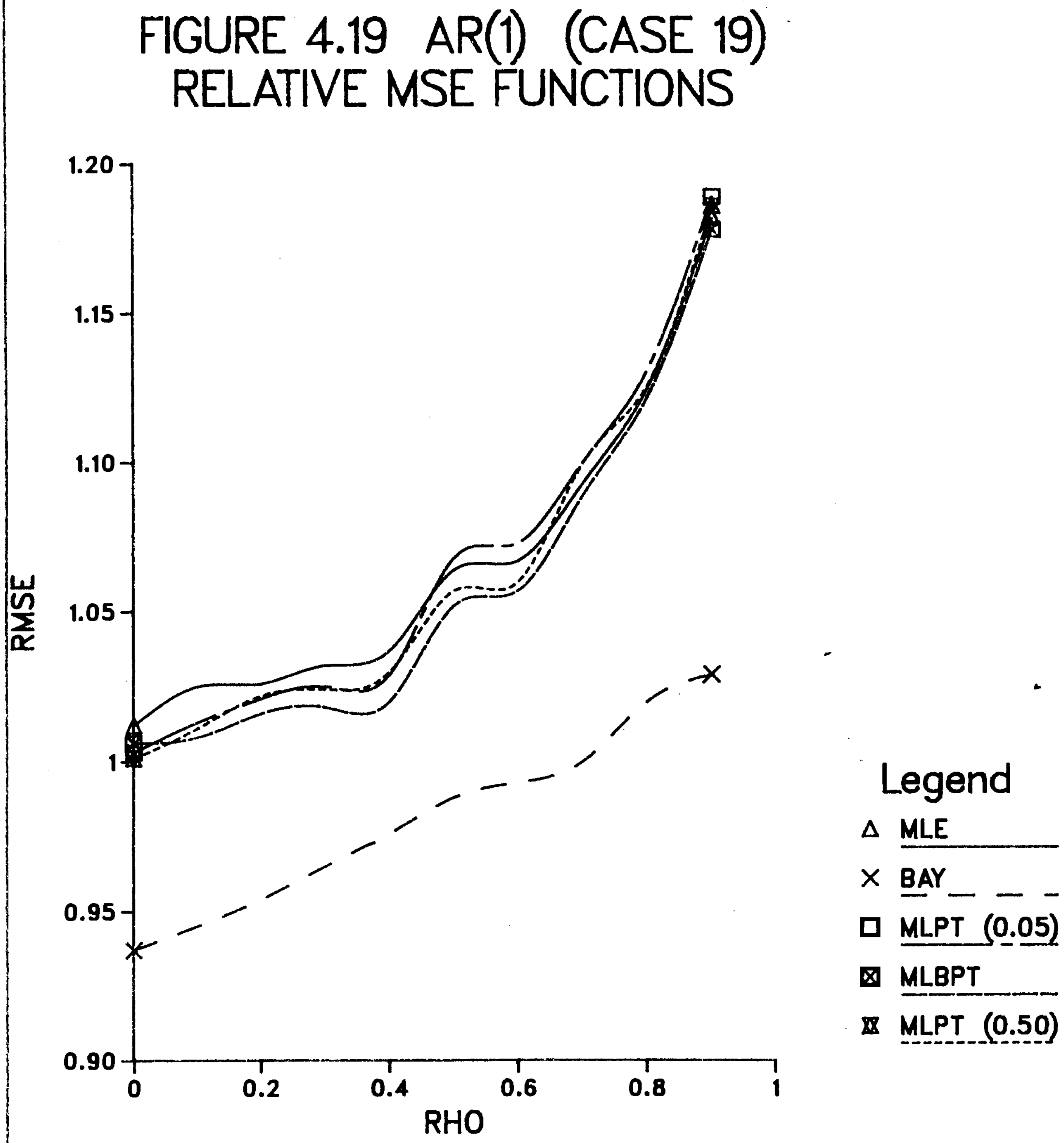


FIGURE 4.19 AR(1) (CASE 19)
RELATIVE MSE FUNCTIONS
RMSE
1.20
1.15
1.10
1.05
1
0.95
0.90
0
0.2
0.4
0.6
0.8
1
RHO
Legend
MLE
BAY
MLPT (0.05)
MLBPT
MLPT (0.50)

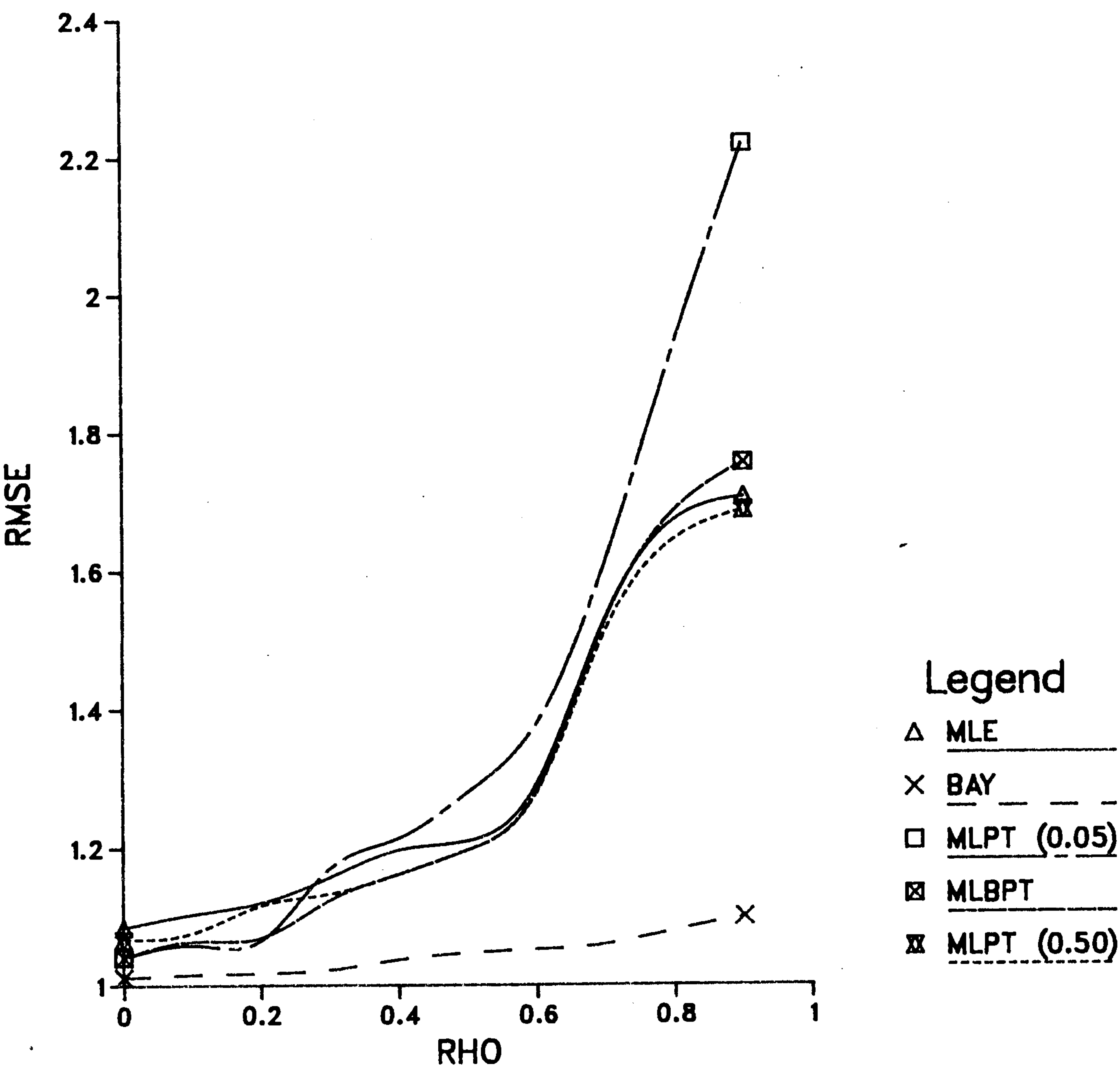
Table 4.20

RELATIVE MSE: AR(1) (CASE 20)

True ρ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.059	1.084	1.011	1.027
0.10	1.013	1.067	1.103	1.015	1.034
0.20	1.049	1.071	1.120	1.017	1.042
0.30	1.144	1.122	1.157	1.022	1.101
0.40	1.247	1.151	1.197	1.037	1.128
0.50	1.442	1.189	1.210	1.047	1.186
0.60	1.593	1.243	1.292	1.052	1.239
0.70	2.227	1.492	1.532	1.059	1.493
0.80	3.084	1.711	1.674	1.079	1.726
0.90	4.424	1.904	1.707	1.100	1.922

True ρ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)	β^{PT} (α=0.50)	β^{MLPT} (α=0.50)
0.00	1.041	1.028	1.040	1.047	1.066
0.10	1.064	1.035	1.058	1.053	1.076
0.20	1.069	1.044	1.065	1.058	1.116
0.30	1.124	1.152	1.170	1.109	1.133
0.40	1.162	1.158	1.214	1.125	1.162
0.50	1.197	1.283	1.280	1.181	1.197
0.60	1.282	1.362	1.379	1.233	1.280
0.70	1.529	1.644	1.614	1.487	1.514
0.80	1.687	2.029	1.927	1.710	1.646
0.90	1.756	2.454	2.220	1.920	1.688

FIGURE 4.20 AR(1) (CASE 20)
RELATIVE MSE FUNCTIONS



4.2.2 THE MA(1)

Table and Figures 4.21 -4.40 represent the findings for this model. Except for a few deviations, the results for the MA(1) model are similar to that of the AR(1) model. For the MA(1) model the main observation is :

1) The pure Bayesian estimator did not perform as well as its competitors for $\delta \leq 0.40$. In fact it is the second (to β^{EGLS}) worst estimator among all the competing estimators within this range of δ, but as δ increases, the increase in the mean square error of the pure Bayesian estimator is not as rapid as that of its rivals, thus leading to a mean square error that is generally below that of the other estimator for $\delta \geq 0.40$.

From this model we also observed the following:

2) Apart from case 3 (where β^{EGLS} and β^{BAY} have U-shaped mean square error functions), the mean square error of all estimators increase continuously for increasing values of δ.

3) The range of values of δ for which OLS performs very well is higher than in the AR(1) model. For the MA(1) model, OLS generally outperformed all the estimators for $\delta \leq 0.40$, but for higher values of δ ($\delta \geq 0.50$), it is outperformed by its competitors.

4) β^{EGLS} is generally the worst among all the estimators for low values of δ ($\delta \leq 0.40$). This behaviour could be attributed to the method of estimating the autocorrelation parameter δ. The method

of moments estimator of δ is used in the estimation of δ, the method could have produced poor estimates of δ, thereby leading to a β^{EGLS} with higher mean square error than would ordinarily have been the case.

5) The behaviour of the autocorrelation pretest estimator in relation to their parent EGLS estimators is the same as in the AR(1) case. For $\delta=0$ the 5% autocorrelation pretest estimator correctly accepts the null hypothesis 95% of the time and puts more weight on OLS, which has the least MSE among all the estimators at this value of δ. Thus the MSE of the pretest estimator is lower than its parent EGLS but it is above that of OLS for smaller values of δ. As δ increases, OLS gets consistently worse, thus the MSE of the pretest estimator exceeds that of its parent EGLS, but lies below that of OLS.

6) Similar to the AR(1) model, no estimator dominates the others over the entire range of δ, and thus there did not emerge a clear ordinal ranking of the estimators, though OLS, the autocorrelation pretest estimators and the Bayesian pretest estimators perform well for values of $\delta \leq 0.50$ while the pure Bayesian, β^{MLE} and β^{EGLS} did better for higher values of δ

7) On the average, the pure Bayesian estimator's mean square error is about 103% of the mean square error of OLS for $\delta \leq 0.40$, and about 70% for higher values of δ.

8) When the MSE is averaged over the entire δ space, the pure Bayesian estimator has the least average MSE, followed by β^{MLBPT}, β^{BAYPT}, β^{MLE}, β^{EGLS}, β^{PT}, and β^{OLS} in that order.

9) The size of the design matrix does not play a significant role in the relative ranking of the the estimators, However, for larger sample sizes (T=65), the range of better performance of OLS over its competitors is smaller, (approximately $\delta \leq 0.2$, while for smaller sample sizes the range was $\delta \leq 0.4$.

10) Averaging the MSE over all 20 design matrices for a given value of δ shows that the Pure Bayesian estimator is only worse than the competition for values of $\delta \leq 0.2$. For higher values of δ the Pure Bayesian estimator is markedly superior to its competitors.

11) For this model, the choice of estimators will depend on one's prior knowledge of the value of δ. If $\delta \leq 0.50$, we recommend the use of OLS since the gains in efficiency are quite substantial, while for higher values of δ the pure Bayesian is the obvious choice.

Table 4.21 (MA(1))

RELATIVE MSE (CASE 1)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.077	1.002	1.009	1.038
0.10	1.004	1.079	1.008	1.012	1.045
0.20	1.015	1.085	1.016	1.016	1.060
0.30	1.081	1.089	1.034	1.029	1.072
0.40	1.131	1.098	1.057	1.038	1.084
0.50	1.202	1.127	1.093	1.056	1.099
0.60	1.287	1.153	1.121	1.069	1.111
0.70	1.414	1.200	1.170	1.091	1.142
0.80	1.501	1.288	1.218	1.100	1.150
0.90	1.727	1.416	1.400	1.169	1.280

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.001	1.038	1.006
0.10	1.006	1.043	1.009
0.20	1.010	1.066	1.013
0.30	1.021	1.071	1.029
0.40	1.036	1.099	1.061
0.50	1.067	1.145	1.112
0.60	1.098	1.187	1.141
0.70	1.128	1.220	1.180
0.80	1.152	1.311	1.200
0.90	1.278	1.417	1.398

FIGURE 4.21 MA(1) (CASE 1)
RELATIVE MSE FUNCTIONS

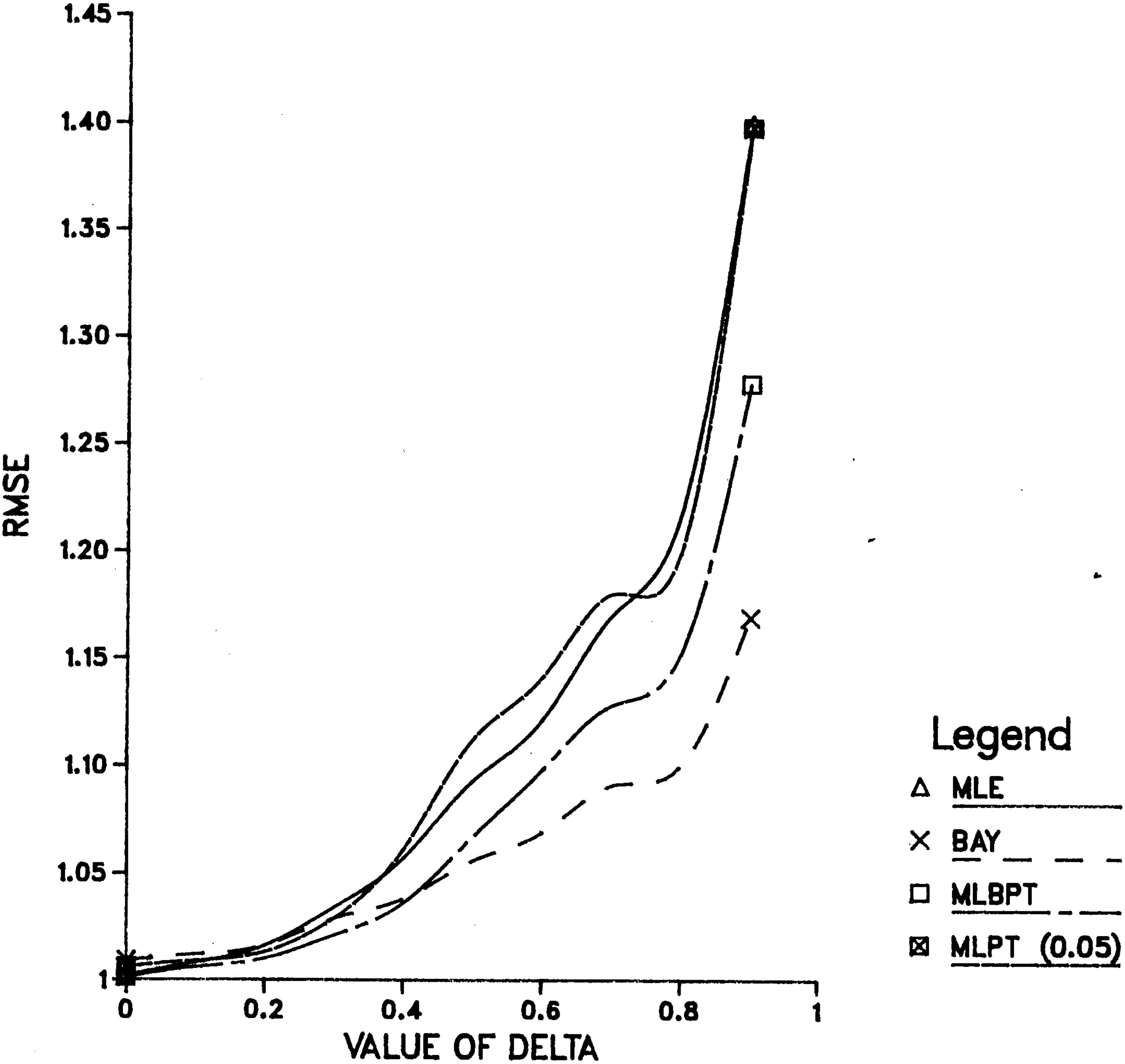


Table 4.22 (MA(1))

RELATIVE MSE (CASE 2)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.335	1.202	1.368	1.387
0.10	1.032	1.421	1.327	1.377	1.402
0.20	1.073	1.439	1.393	1.392	1.427
0.30	1.240	1.484	1.434	1.415	1.449
0.40	1.437	1.491	1.450	1.431	1.473
0.50	1.757	1.553	1.506	1.485	1.511
0.60	2.112	1.657	1.621	1.522	1.595
0.70	3.083	1.822	1.781	1.593	1.707
0.80	4.850	2.136	1.916	1.693	1.903
0.90	8.588	2.913	2.830	1.915	2.328

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.247	1.166	1.091
0.10	1.363	1.275	1.151
0.20	1.417	1.311	1.203
0.30	1.476	1.526	1.378
0.40	1.484	1.691	1.470
0.50	1.491	1.730	1.562
0.60	1.578	1.768	1.688
0.70	1.708	1.985	1.908
0.80	1.871	2.462	2.224
0.90	2.209	3.318	3.212

FIGURE 4.22 MA(1) (CASE 2) RELATIVE MSE FUNCTIONS

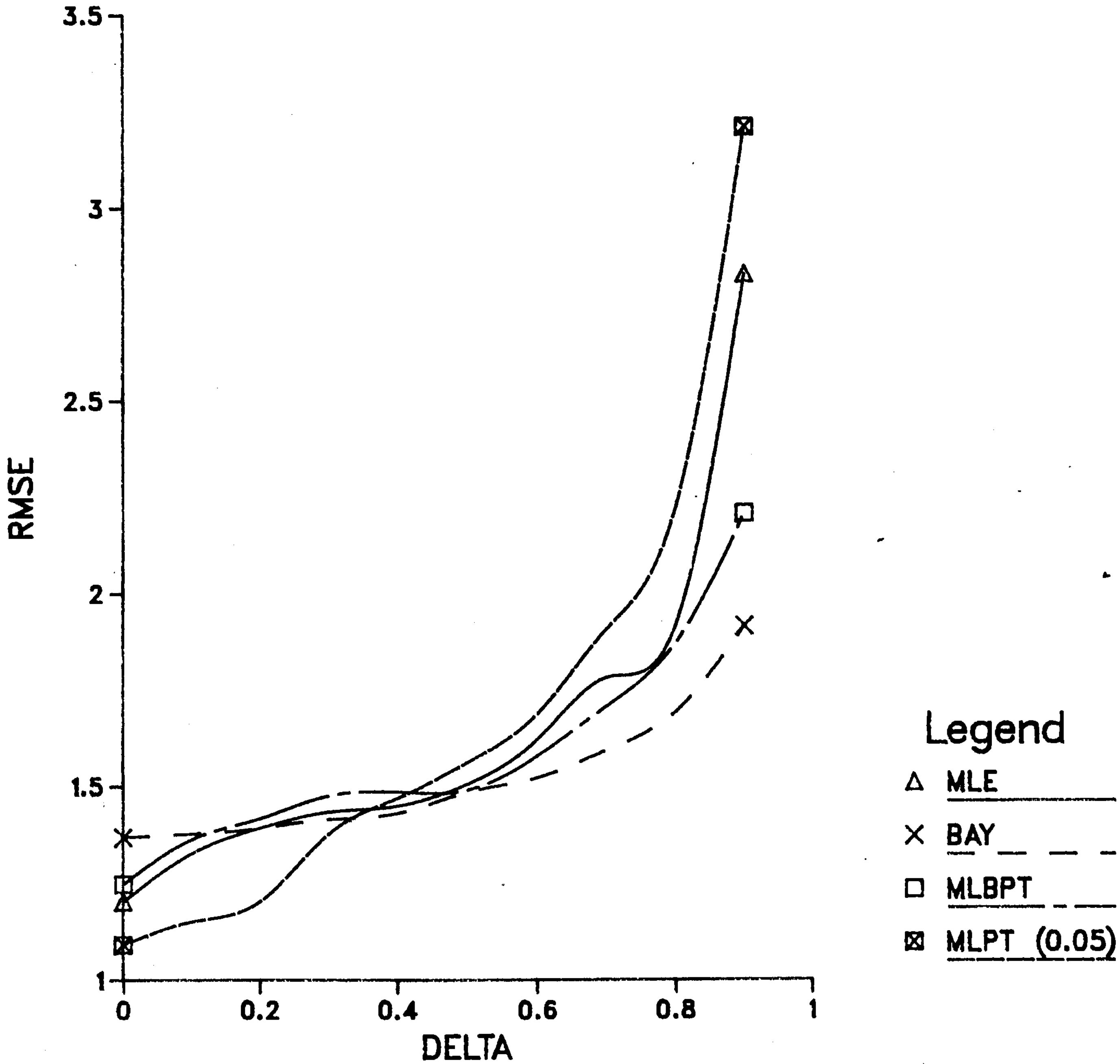


Table 4.23 (MA(1))

RELATIVE MSE (CASE 3)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	2.586	1.021	1.328	1.427
0.10	1.010	2.013	1.046	1.123	1.305
0.20	1.023	1.731	1.073	1.061	1.277
0.30	1.057	1.350	1.131	1.040	1.211
0.40	1.182	1.200	1.168	1.132	1.190
0.50	1.241	1.210	1.192	1.163	1.186
0.60	1.533	1.316	1.221	1.177	1.192
0.70	1.699	1.391	1.328	1.293	1.339
0.80	1.871	1.511	1.462	1.382	1.446
0.90	2.021	1.548	1.606	1.418	1.600

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.026	1.012	1.002
0.10	1.054	1.064	1.014
0.20	1.084	1.073	1.042
0.30	1.091	1.096	1.077
0.40	1.106	1.178	1.089
0.50	1.156	1.200	1.133
0.60	1.189	1.327	1.271
0.70	1.381	1.418	1.400
0.80	1.407	1.494	1.411
0.90	1.638	1.772	1.668

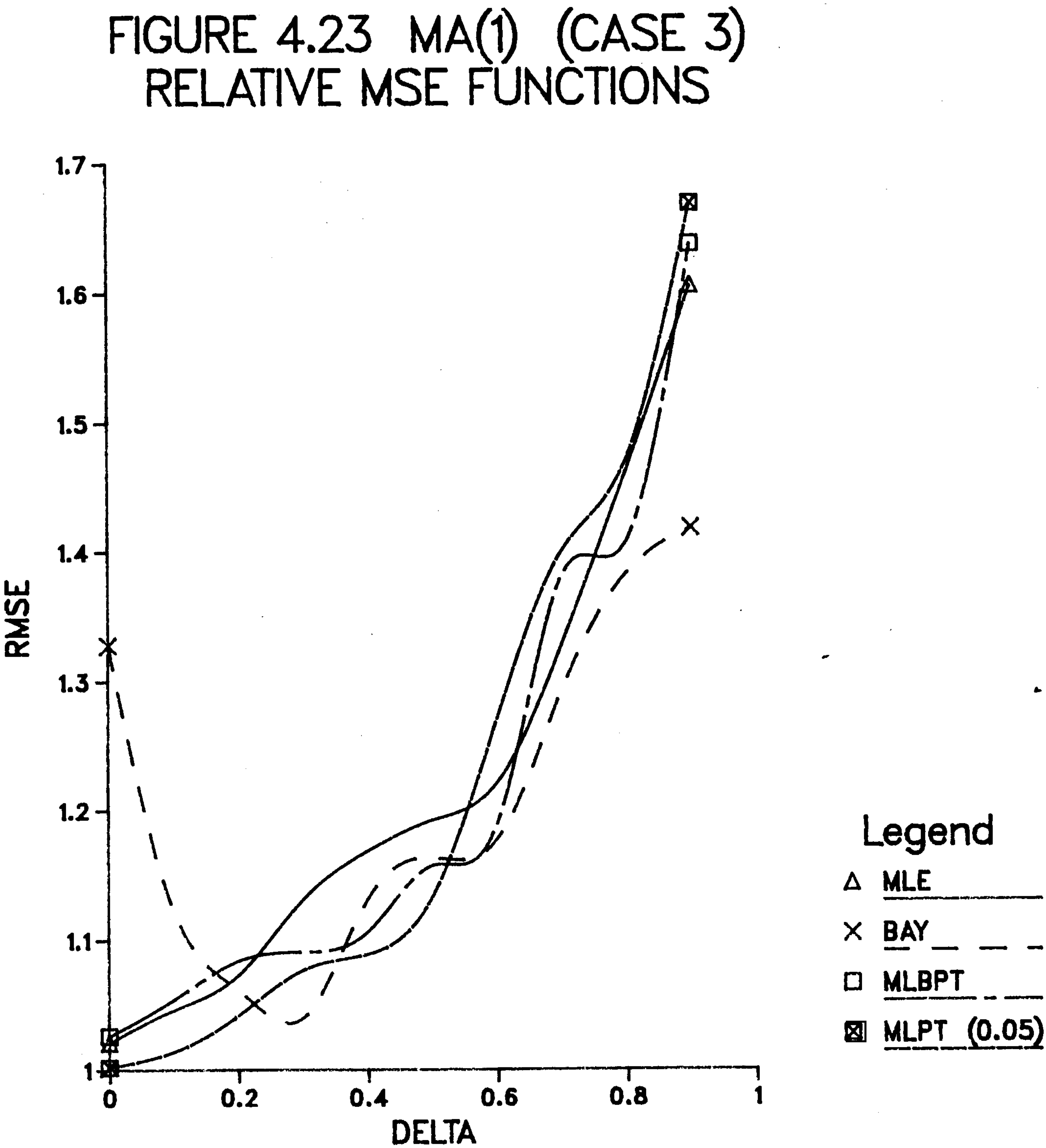


FIGURE 4.23 MA(1) (CASE 3)
RELATIVE MSE FUNCTIONS
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
RMSE
0
0.2
0.4
0.6
0.8
1
DELTA
Legend
MLE
BAY
MLBPT
MLPT (0.05)

Table 4.24 (MA(1))

RELATIVE MSE (CASE 4)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.050	1.009	1.007	1.030
0.10	1.004	1.053	1.026	1.014	1.035
0.20	1.020	1.065	1.069	1.031	1.042
0.30	1.070	1.078	1.073	1.048	1.058
0.40	1.095	1.097	1.099	1.065	1.076
0.50	1.183	1.136	1.123	1.082	1.094
0.60	1.262	1.164	1.158	1.091	1.162
0.70	1.580	1.212	1.206	1.102	1.195
0.80	1.775	1.386	1.388	1.160	1.211
0.90	2.017	1.529	1.604	1.247	1.394

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.013	1.019	1.032
0.10	1.028	1.026	1.055
0.20	1.079	1.043	1.059
0.30	1.087	1.071	1.084
0.40	1.090	1.099	1.096
0.50	1.096	1.148	1.139
0.60	1.113	1.170	1.173
0.70	1.178	1.315	1.298
0.80	1.217	1.403	1.418
0.90	1.322	1.667	1.703

FIGURE 4.24 MA(1) (CASE 4)
RELATIVE MSE FUNCTIONS

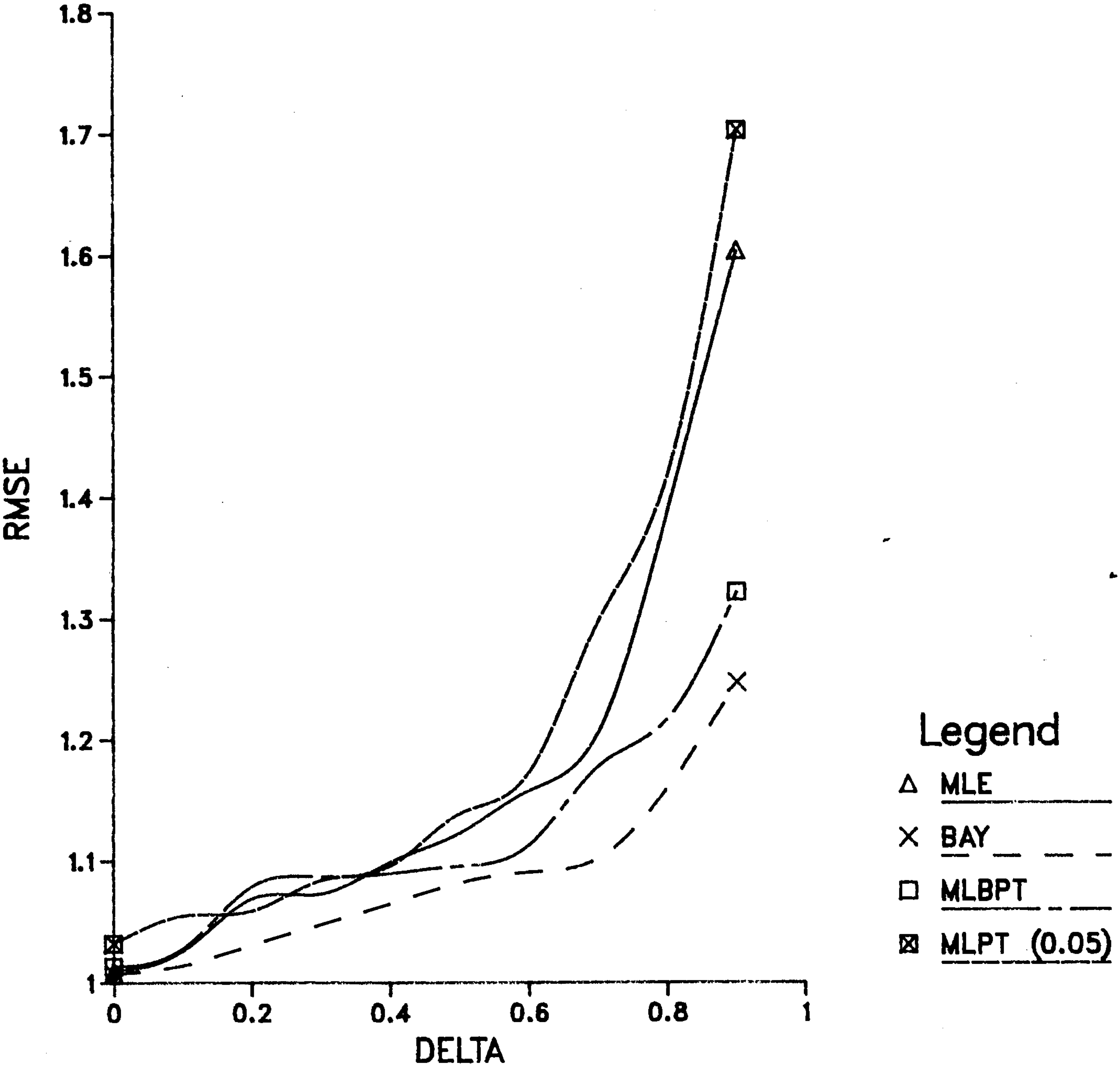


Table 4.25 (MA(1))

RELATIVE MSE (CASE 5)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.186	1.215	1.115	1.142
0.10	1.072	1.195	1.272	1.153	1.175
0.20	1.147	1.208	1.293	1.168	1.192
0.30	1.294	1.290	1.317	1.178	1.253
0.40	1.526	1.473	1.580	1.279	1.396
0.50	1.711	1.662	1.702	1.420	1.566
0.60	1.929	1.819	1.899	1.626	1.856
0.70	2.214	2.003	2.001	1.871	2.018
0.80	2.863	2.128	2.100	1.920	2.127
0.90	3.158	2.747	2.639	2.122	2.681

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.168	1.133	1.140
0.10	1.190	1.160	1.189
0.20	1.200	1.188	1.220
0.30	1.287	1.240	1.340
0.40	1.410	1.372	1.620
0.50	1.705	1.620	1.735
0.60	1.919	1.910	1.956
0.70	2.017	2.068	2.057
0.80	2.120	2.577	2.404
0.90	2.629	3.012	2.874

FIGURE 4.25 MA(1) (CASE 5)
RELATIVE MSE FUNCTIONS

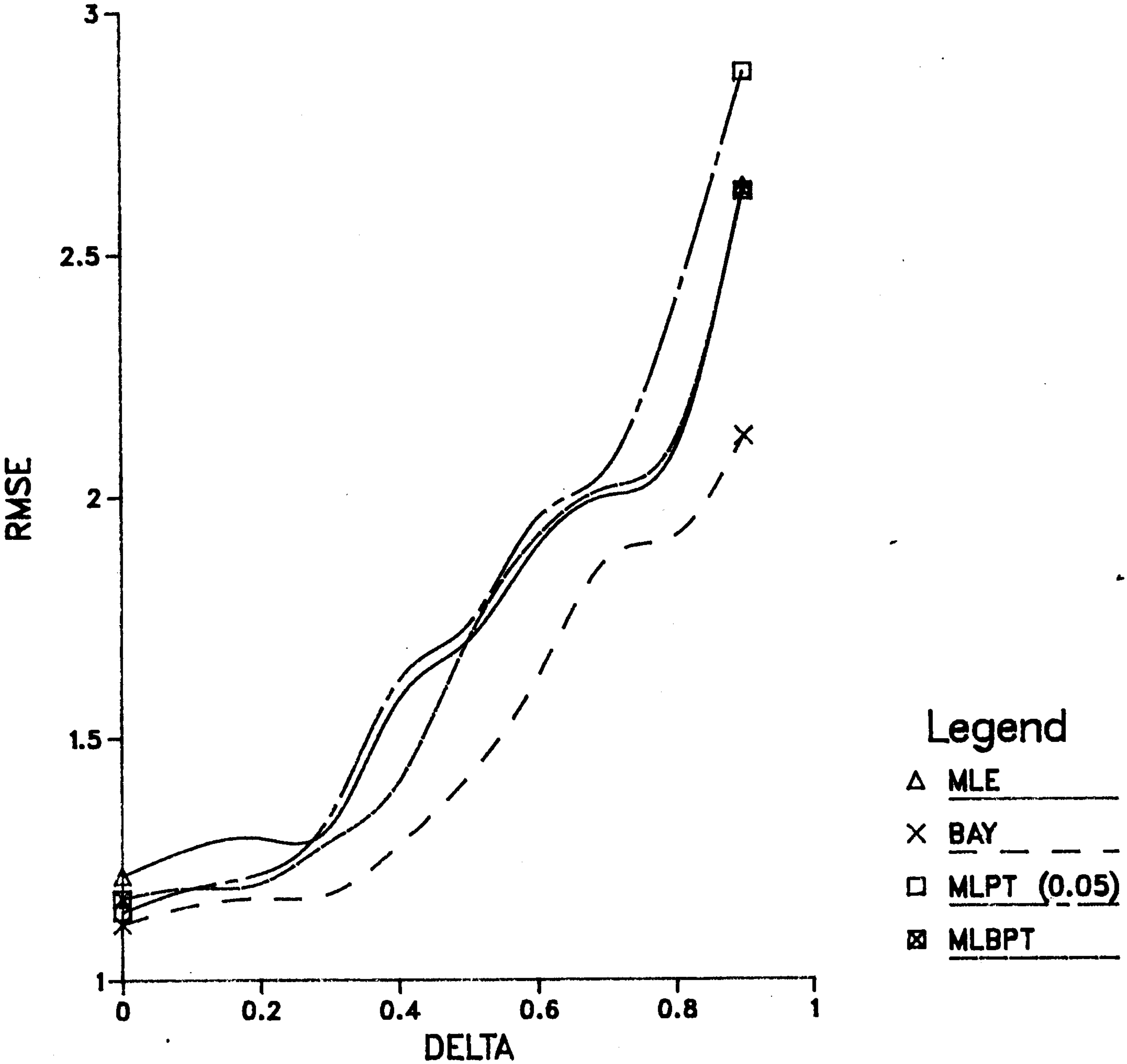


Table 4.26 (MA(1))

RELATIVE MSE (CASE 6)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.069	1.075	1.025	1.036
0.10	1.016	1.086	1.091	1.046	1.052
0.20	1.035	1.150	1.121	1.058	1.149
0.30	1.112	1.203	1.236	1.087	1.195
0.40	1.246	1.278	1.273	1.100	1.205
0.50	1.395	1.306	1.294	1.113	1.218
0.60	1.801	1.352	1.366	1.158	1.360
0.70	2.019	1.521	1.462	1.208	1.511
0.80	2.262	1.614	1.669	1.375	1.723
0.90	3.287	1.854	1.841	1.482	1.830

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.079	1.042	1.017
0.10	1.104	1.070	1.029
0.20	1.130	1.100	1.088
0.30	1.186	1.191	1.129
0.40	1.198	1.262	1.198
0.50	1.212	1.325	1.255
0.60	1.373	1.463	1.470
0.70	1.469	1.734	1.751
0.80	1.750	1.823	1.892
0.90	1.826	2.007	2.012

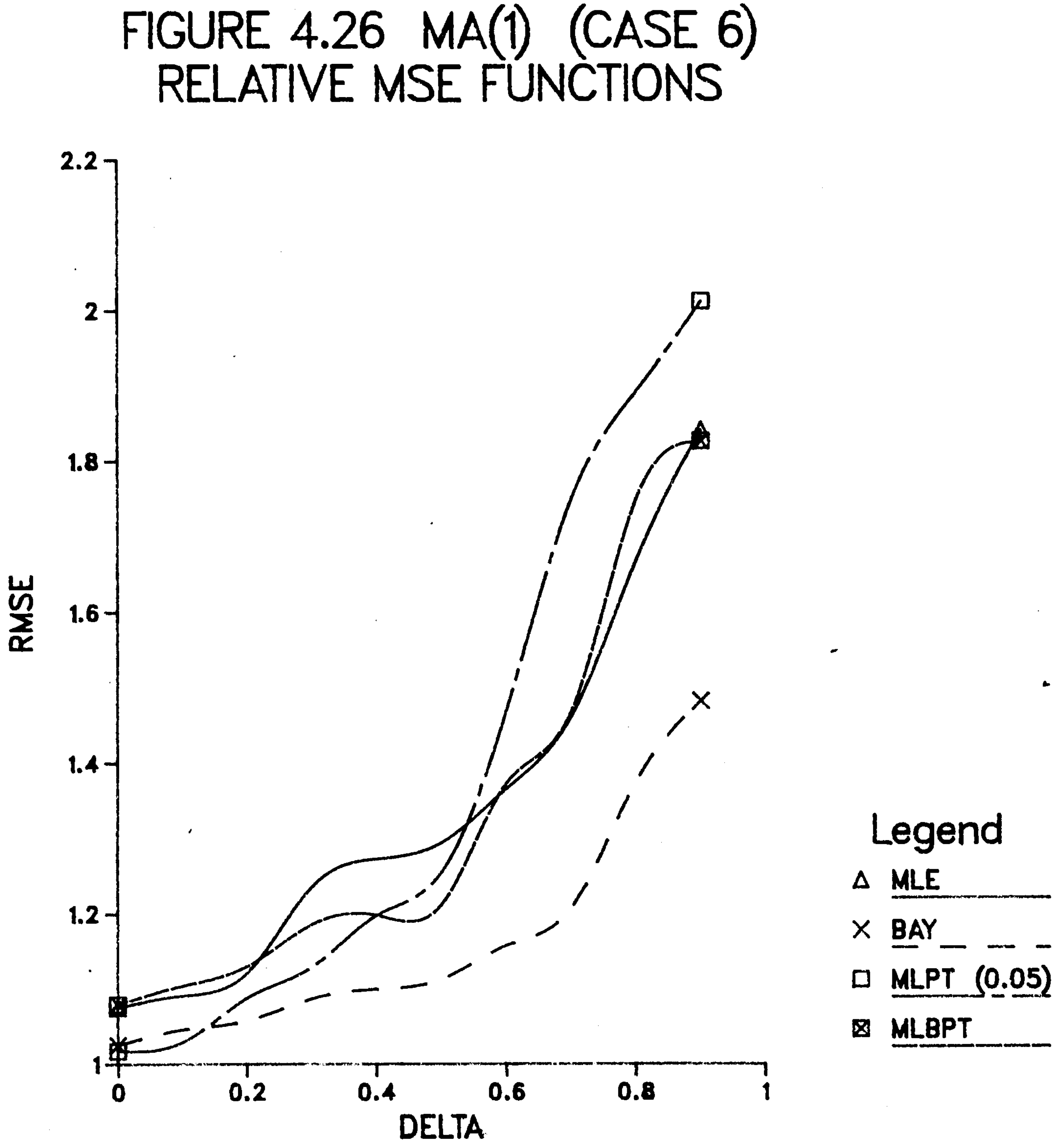


FIGURE 4.26 MA(1) (CASE 6)
RELATIVE MSE FUNCTIONS
RMSE
2.2
2
1.8
1.6
1.4
1.2
1
0
0.2
0.4
0.6
0.8
1
DELTA
Legend
MLE
BAY
MLPT (0.05)
MLBPT

Table 4.27 (MA(1))

RELATIVE MSE (CASE 7)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.048	1.012	1.036	1.033
0.10	1.008	1.056	1.037	1.042	1.049
0.20	1.029	1.071	1.052	1.059	1.063
0.30	1.066	1.089	1.088	1.067	1.075
0.40	1.093	1.103	1.113	1.080	1.099
0.50	1.153	1.126	1.135	1.095	1.135
0.60	1.221	1.150	1.148	1.103	1.153
0.70	1.477	1.203	1.200	1.187	1.231
0.80	1.765	1.431	1.440	1.287	1.452
0.90	2.181	1.701	1.683	1.306	1.721

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.009	1.022	1.010
0.10	1.026	1.031	1.024
0.20	1.049	1.065	1.033
0.30	1.075	1.073	1.071
0.40	1.108	1.096	1.100
0.50	1.139	1.146	1.149
0.60	1.152	1.183	1.167
0.70	1.228	1.311	1.270
0.80	1.465	1.600	1.559
0.90	1.689	1.921	1.835

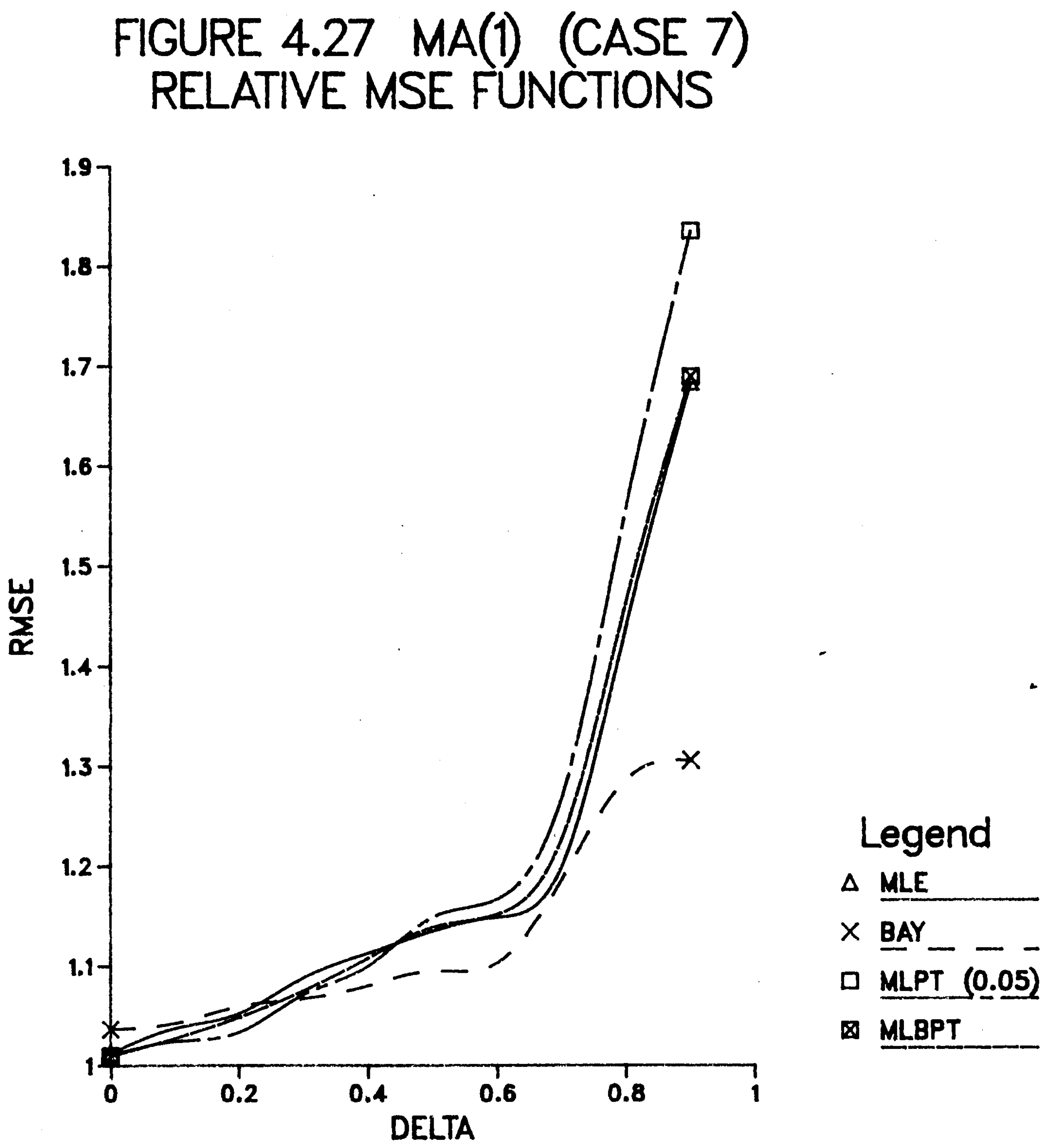


FIGURE 4.27 MA(1) (CASE 7)
RELATIVE MSE FUNCTIONS
RMSE
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0
0.2
0.4
0.6
0.8
1
DELTA
Legend
MLE
BAY
MLPT (0.05)
MLBPT

Table 4.28 (MA(1))

RELATIVE MSE (CASE 8)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.089	1.035	1.065	1.059
0.10	1.024	1.105	1.070	1.090	1.083
0.20	1.057	1.138	1.100	1.113	1.126
0.30	1.122	1.174	1.211	1.161	1.193
0.40	1.251	1.205	1.236	1.183	1.209
0.50	1.500	1.297	1.315	1.201	1.330
0.60	1.863	1.411	1.426	1.288	1.398
0.70	2.186	1.676	1.650	1.329	1.680
0.80	2.938	1.870	1.783	1.521	1.930
0.90	3.353	1.964	1.878	1.671	2.085

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.025	1.033	1.016
0.10	1.072	1.070	1.064
0.20	1.136	1.133	1.106
0.30	1.202	1.173	1.195
0.40	1.230	1.216	1.228
0.50	1.322	1.337	1.401
0.60	1.408	1.501	1.510
0.70	1.692	1.791	1.730
0.80	1.814	2.025	1.900
0.90	2.004	2.268	2.156

FIGURE 4.28 MA(1) (CASE 8)
RELATIVE MSE FUNCTIONS

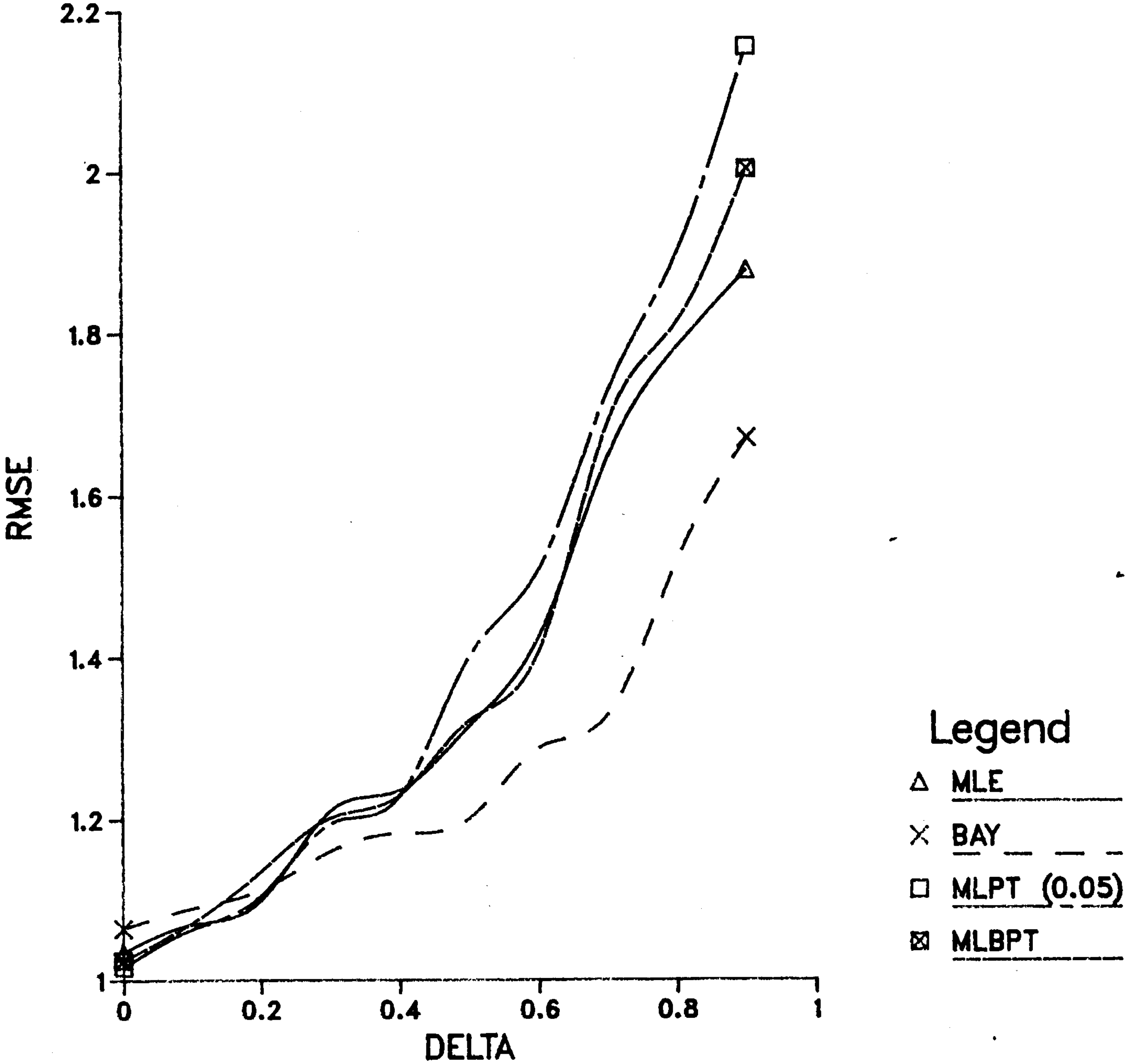


Table 4.29 (MA(1))

RELATIVE MSE (CASE 9)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.067	1.045	1.050	1.032
0.10	1.015	1.073	1.059	1.055	1.048
0.20	1.031	1.088	1.082	1.085	1.075
0.30	1.086	1.107	1.110	1.096	1.111
0.40	1.141	1.125	1.130	1.103	1.130
0.50	1.416	1.180	1.175	1.120	1.182
0.60	1.538	1.219	1.200	1.155	1.230
0.70	1.790	1.474	1.520	1.268	1.490
0.80	2.004	1.619	1.618	1.317	1.621
0.90	2.323	1.700	1.713	1.423	1.708

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.022	1.049	1.020
0.10	1.038	1.057	1.031
0.20	1.073	1.079	1.070
0.30	1.115	1.100	1.098
0.40	1.139	1.127	1.135
0.50	1.180	1.232	1.225
0.60	1.220	1.352	1.303
0.70	1.553	1.565	1.583
0.80	1.620	1.752	1.699
0.90	1.723	2.029	1.822

FIGURE 4.29 MA(1) (CASE 9)
RELATIVE MSE FUNCTIONS

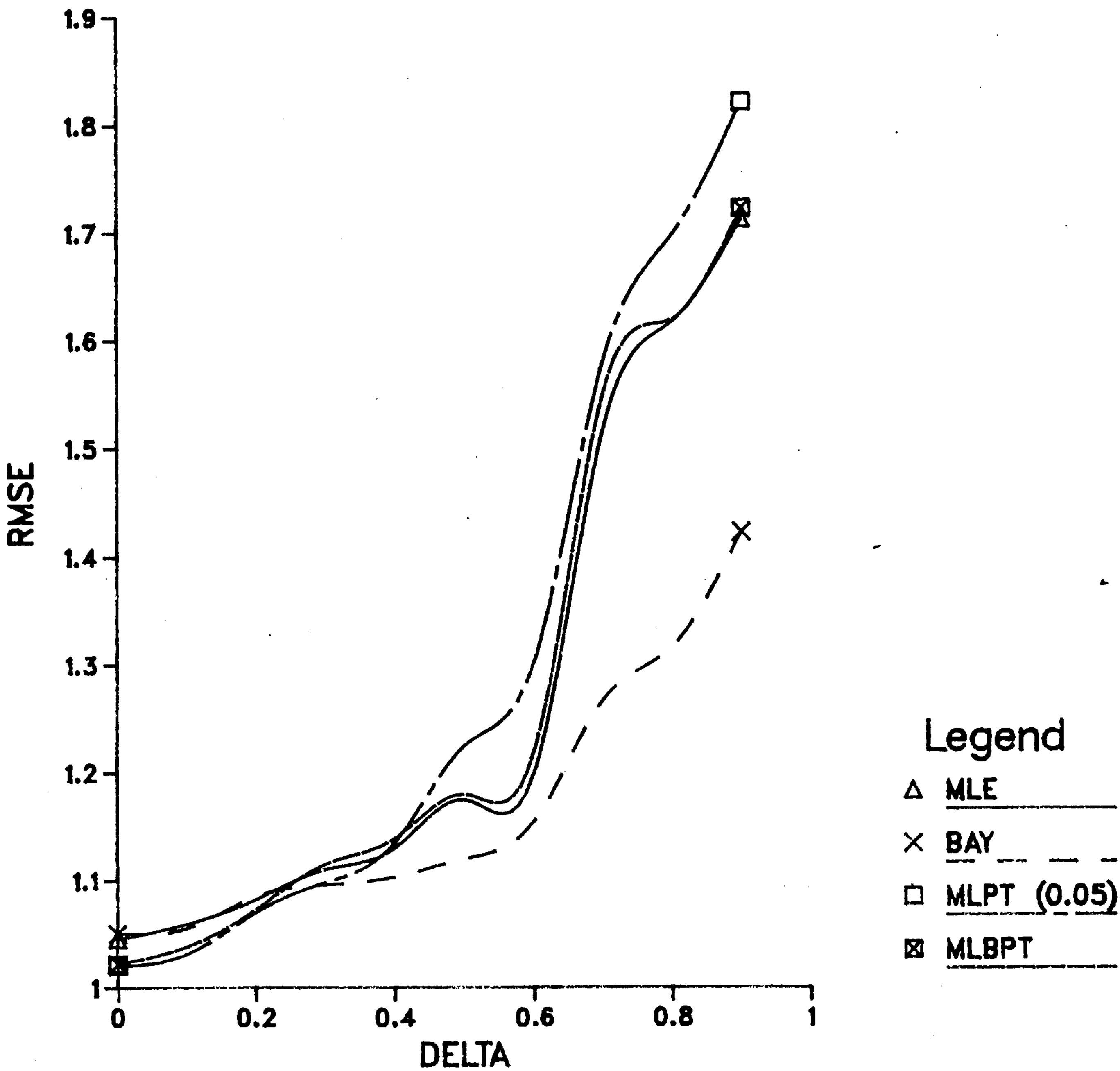


Table 4.30 (MA(1))

RELATIVE MSE (CASE 10)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.028	1.009	1.021	1.003
0.10	1.007	1.033	1.020	1.028	1.024
0.20	1.036	1.049	1.042	1.045	1.039
0.30	1.055	1.058	1.060	1.051	1.047
0.40	1.125	1.081	1.089	1.069	1.080
0.50	1.184	1.135	1.138	1.086	1.126
0.60	1.255	1.172	1.165	1.100	1.168
0.70	1.460	1.241	1.238	1.138	1.215
0.80	1.714	1.399	1.401	1.203	1.386
0.90	1.930	1.623	1.618	1.372	1.598

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.005	1.006	1.005
0.10	1.013	1.020	1.012
0.20	1.037	1.038	1.027
0.30	1.049	1.056	1.057
0.40	1.081	1.095	1.098
0.50	1.129	1.158	1.159
0.60	1.150	1.211	1.189
0.70	1.200	1.306	1.297
0.80	1.387	1.456	1.458
0.90	1.601	1.750	1.679

FIGURE 4.30 MA(1) (CASE 10) RELATIVE MSE FUNCTIONS

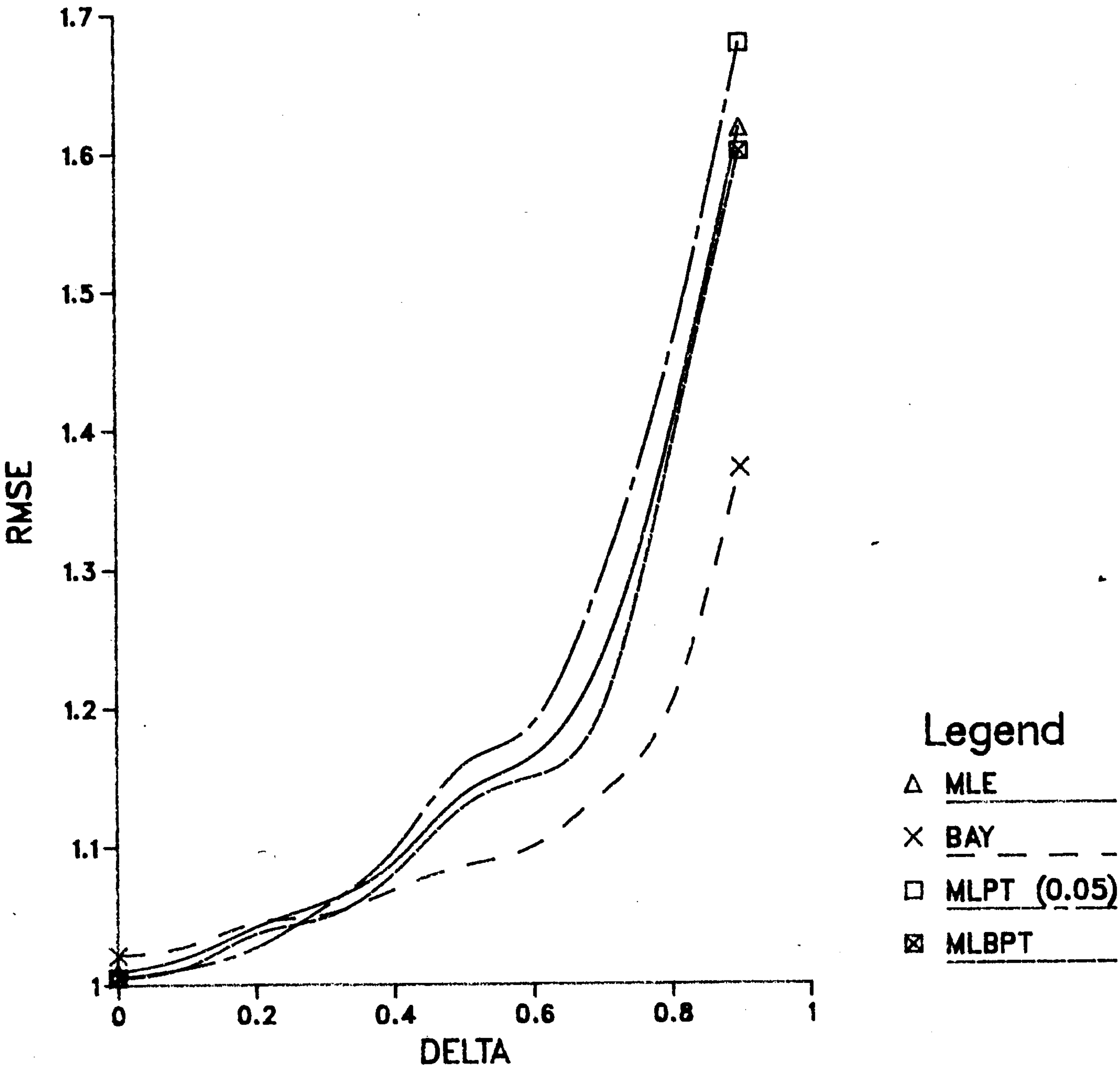


Table 4.31 (MA(1))

RELATIVE MSE (CASE 11)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.054	1.043	1.050	1.042
0.10	1.004	1.061	1.052	1.063	1.058
0.20	1.017	1.070	1.068	1.072	1.062
0.30	1.038	1.083	1.086	1.081	1.076
0.40	1.070	1.099	1.103	1.090	1.086
0.50	1.115	1.120	1.123	1.103	1.116
0.60	1.148	1.150	1.149	1.114	1.130
0.70	1.273	1.203	1.216	1.139	1.163
0.80	1.466	1.289	1.300	1.215	1.274
0.90	1.580	1.362	1.360	1.249	1.352

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.040	1.027	1.016
0.10	1.053	1.044	1.032
0.20	1.059	1.051	1.064
0.30	1.079	1.075	1.082
0.40	1.093	1.096	1.099
0.50	1.119	1.125	1.132
0.60	1.127	1.160	1.158
0.70	1.169	1.237	1.242
0.80	1.276	1.326	1.338
0.90	1.349	1.422	1.410

FIGURE 4.31 MA(1) (CASE 11)
RELATIVE MSE FUNCTIONS

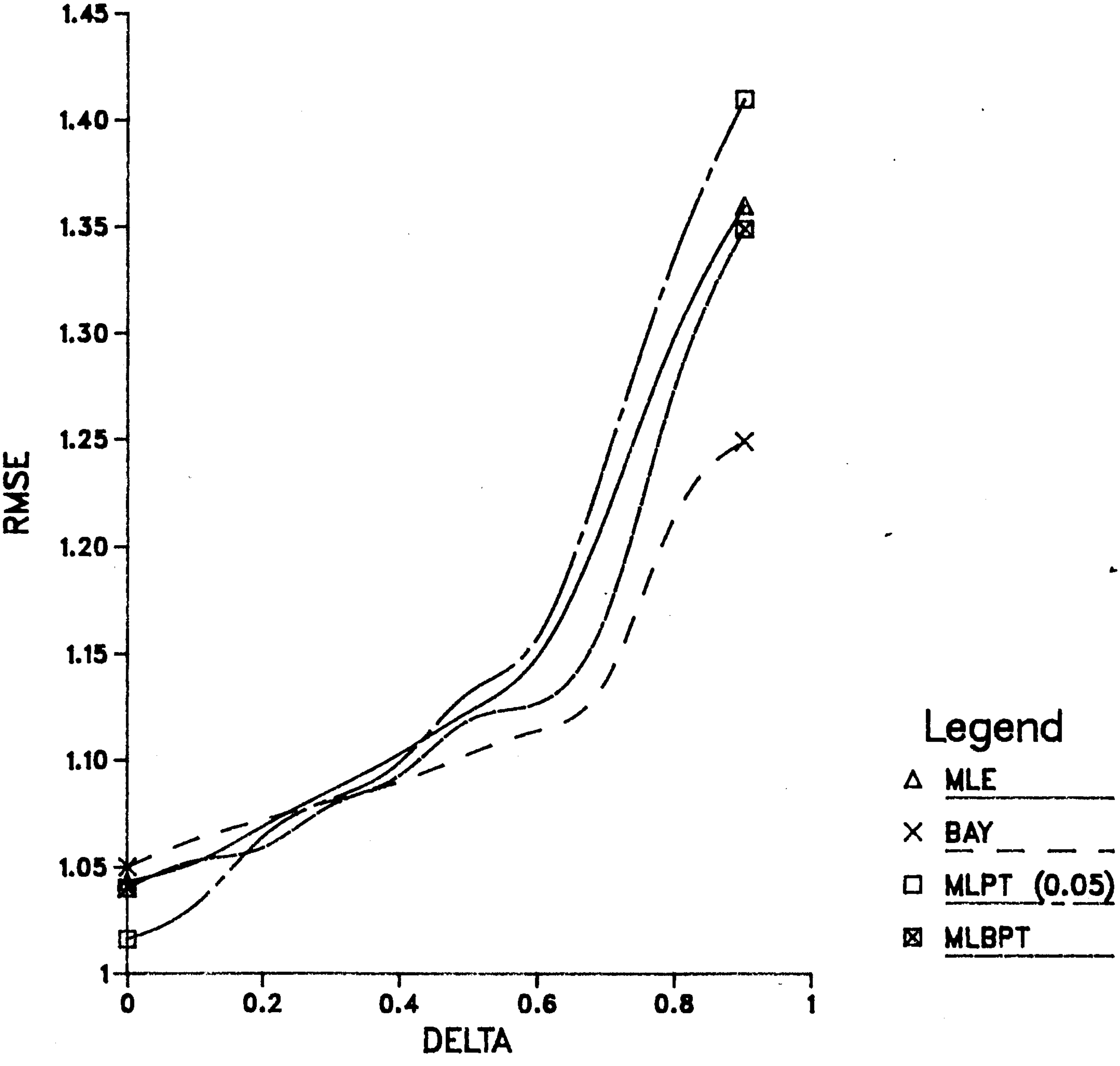


Table 4.32 (MA(1))

RELATIVE MSE (CASE 12)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.068	1.033	1.058	1.040
0.10	1.028	1.079	1.048	1.071	1.060
0.20	1.052	1.084	1.077	1.082	1.081
0.30	1.084	1.092	1.090	1.092	1.087
0.40	1.118	1.121	1.119	1.115	1.116
0.50	1.320	1.271	1.268	1.184	1.228
0.60	1.427	1.311	1.308	1.212	1.263
0.70	1.572	1.403	1.415	1.276	1.343
0.80	1.854	1.640	1.629	1.355	1.498
0.90	2.021	1.776	1.773	1.528	1.654

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.036	1.038	1.021
0.10	1.055	1.051	1.035
0.20	1.080	1.067	1.055
0.30	1.089	1.088	1.085
0.40	1.113	1.110	1.112
0.50	1.228	1.291	1.289
0.60	1.258	1.351	1.348
0.70	1.350	1.465	1.470
0.80	1.497	1.699	1.682
0.90	1.653	1.820	1.818

FIGURE 4.32 MA(1) (CASE 12)
RELATIVE MSE FUNCTIONS

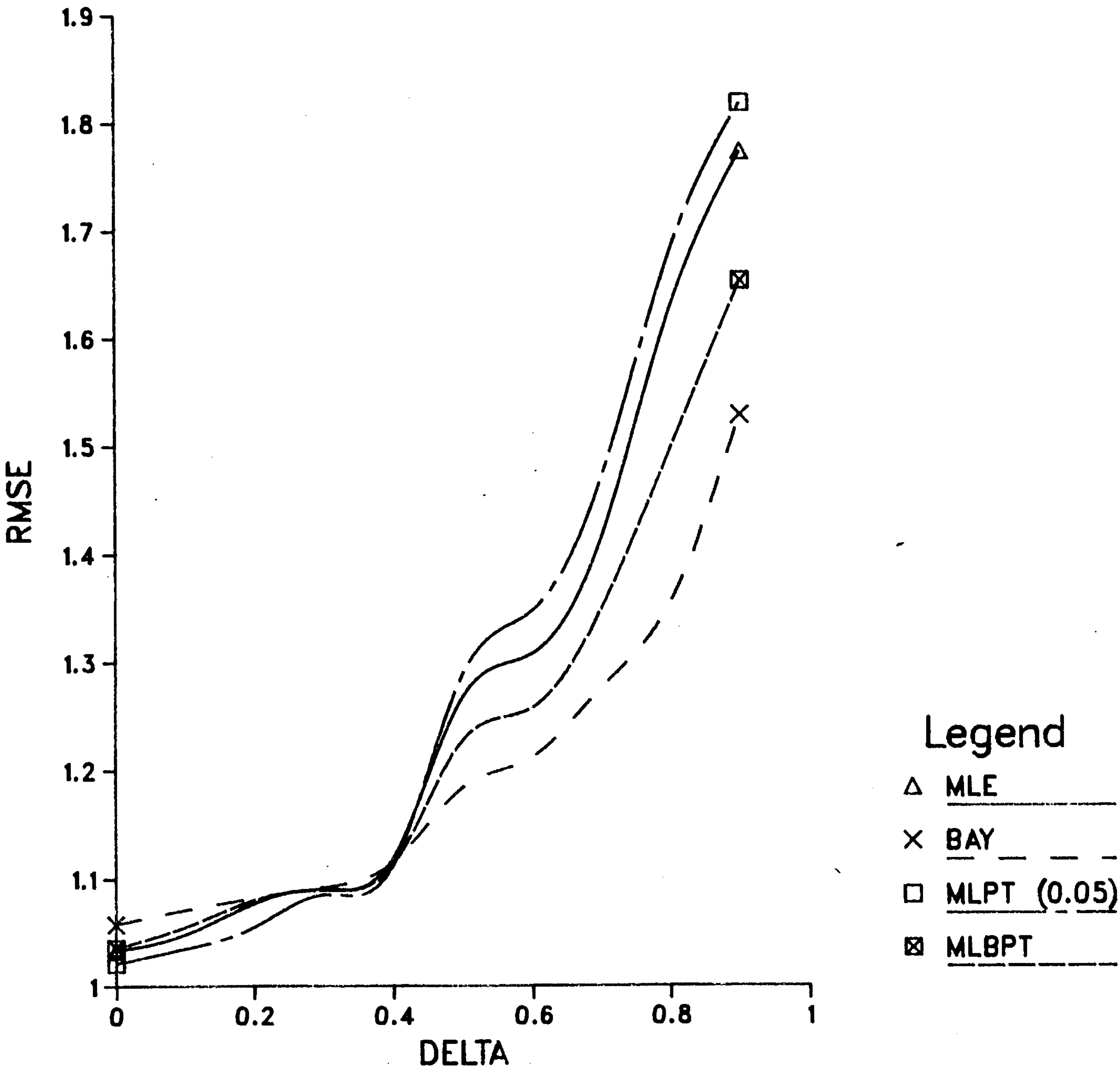


Table 4.33 (MA(1))

RELATIVE MSE (CASE 13)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.257	1.053	1.100	1.097
0.10	1.011	1.260	1.099	1.110	1.103
0.20	1.106	1.277	1.115	1.163	1.179
0.30	1.238	1.301	1.215	1.200	1.235
0.40	1.370	1.361	1.312	1.270	1.299
0.50	1.488	1.407	1.397	1.303	1.362
0.60	1.611	1.480	1.450	1.381	1.431
0.70	1.831	1.563	1.561	1.434	1.550
0.80	2.110	1.681	1.678	1.489	1.630
0.90	2.800	1.811	1.813	1.605	1.787

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.015	1.137	1.003
0.10	1.023	1.167	1.068
0.20	1.133	1.200	1.107
0.30	1.199	1.340	1.251
0.40	1.298	1.401	1.332
0.50	1.340	1.437	1.400
0.60	1.410	1.512	1.490
0.70	1.545	1.684	1.599
0.80	1.610	1.837	1.809
0.90	1.789	2.001	1.934

FIGURE 4.33 MA(1) (CASE 13)
RELATIVE MSE FUNCTIONS

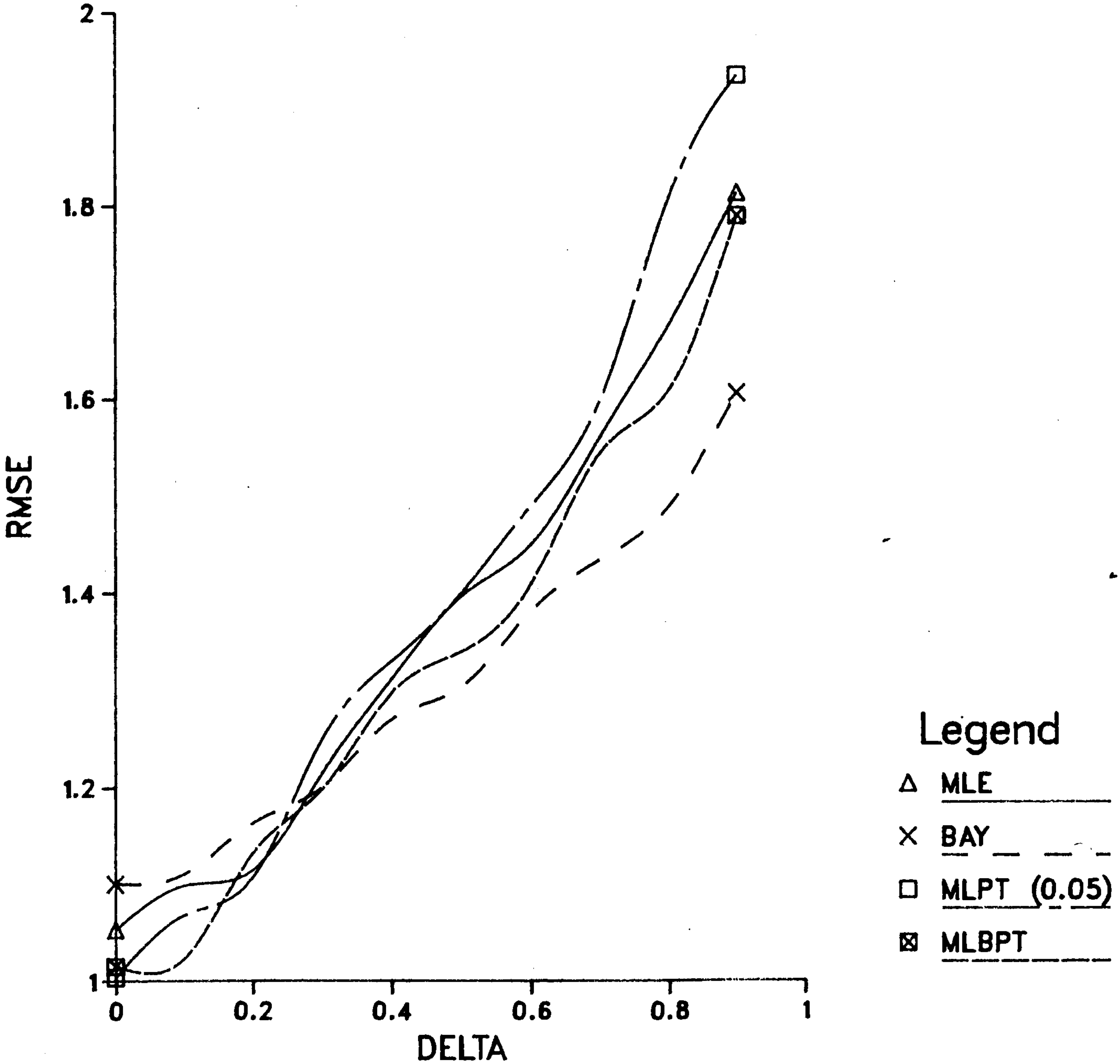


Table 4.34 (MA(1))

RELATIVE MSE (CASE 14)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.024	1.031	1.030	1.020
0.10	1.015	1.037	1.043	1.039	1.031
0.20	1.038	1.046	1.053	1.047	1.048
0.30	1.075	1.052	1.067	1.050	1.055
0.40	1.120	1.087	1.103	1.081	1.090
0.50	1.286	1.136	1.147	1.103	1.125
0.60	1.326	1.245	1.230	1.136	1.256
0.70	1.664	1.380	1.368	1.203	1.395
0.80	2.117	1.574	1.560	1.319	1.592
0.90	2.352	1.831	1.717	1.509	1.793

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.025	1.008	1.013
0.10	1.035	1.026	1.034
0.20	1.049	1.033	1.041
0.30	1.058	1.041	1.046
0.40	1.099	1.071	1.086
0.50	1.127	1.138	1.143
0.60	1.240	1.270	1.259
0.70	1.389	1.420	1.418
0.80	1.589	1.797	1.685
0.90	1.735	1.915	1.823

FIGURE 4.34 MA(1) (CASE 14) RELATIVE MSE FUNCTIONS

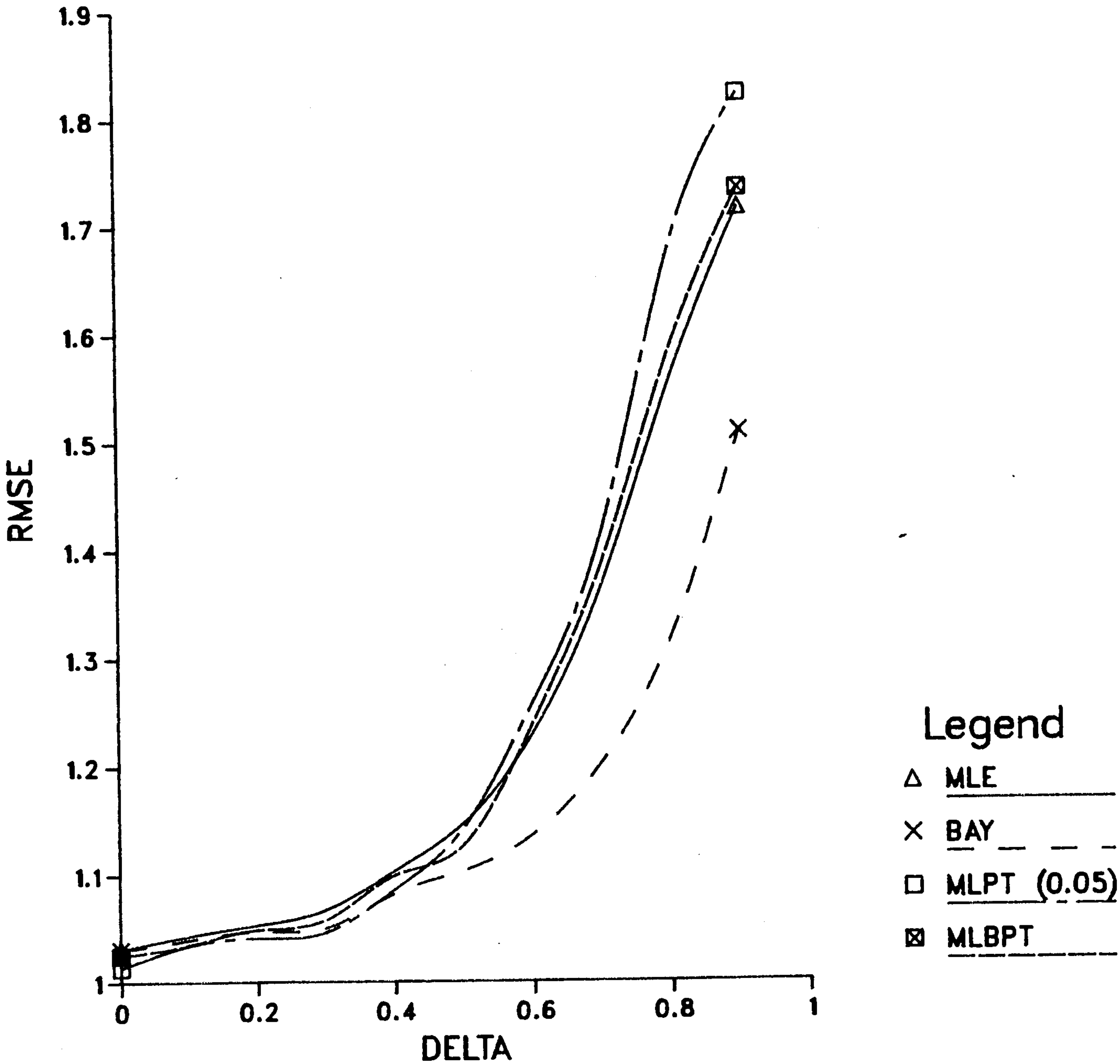


Table 4.35 (MA(1))

RELATIVE MSE (CASE 15)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.294	1.115	1.300	1.153
0.10	1.063	1.319	1.152	1.334	1.204
0.20	1.110	1.378	1.168	1.370	1.368
0.30	1.267	1.400	1.220	1.384	1.393
0.40	1.392	1.420	1.355	1.401	1.406
0.50	1.521	1.468	1.436	1.433	1.450
0.60	1.625	1.529	1.511	1.497	1.515
0.70	2.932	1.700	1.673	1.538	1.596
0.80	2.543	1.912	1.803	1.683	1.788
0.90	3.476	2.008	1.909	1.750	1.889

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.030	1.185	1.021
0.10	1.044	1.283	1.097
0.20	1.100	1.386	1.144
0.30	1.211	1.419	1.195
0.40	1.377	1.518	1.330
0.50	1.399	1.530	1.452
0.60	1.500	1.597	1.548
0.70	1.594	1.801	1.715
0.80	1.773	1.948	1.929
0.90	1.881	2.139	2.006

FIGURE 4.35 MA(1) (CASE 15)
RELATIVE MSE FUNCTIONS

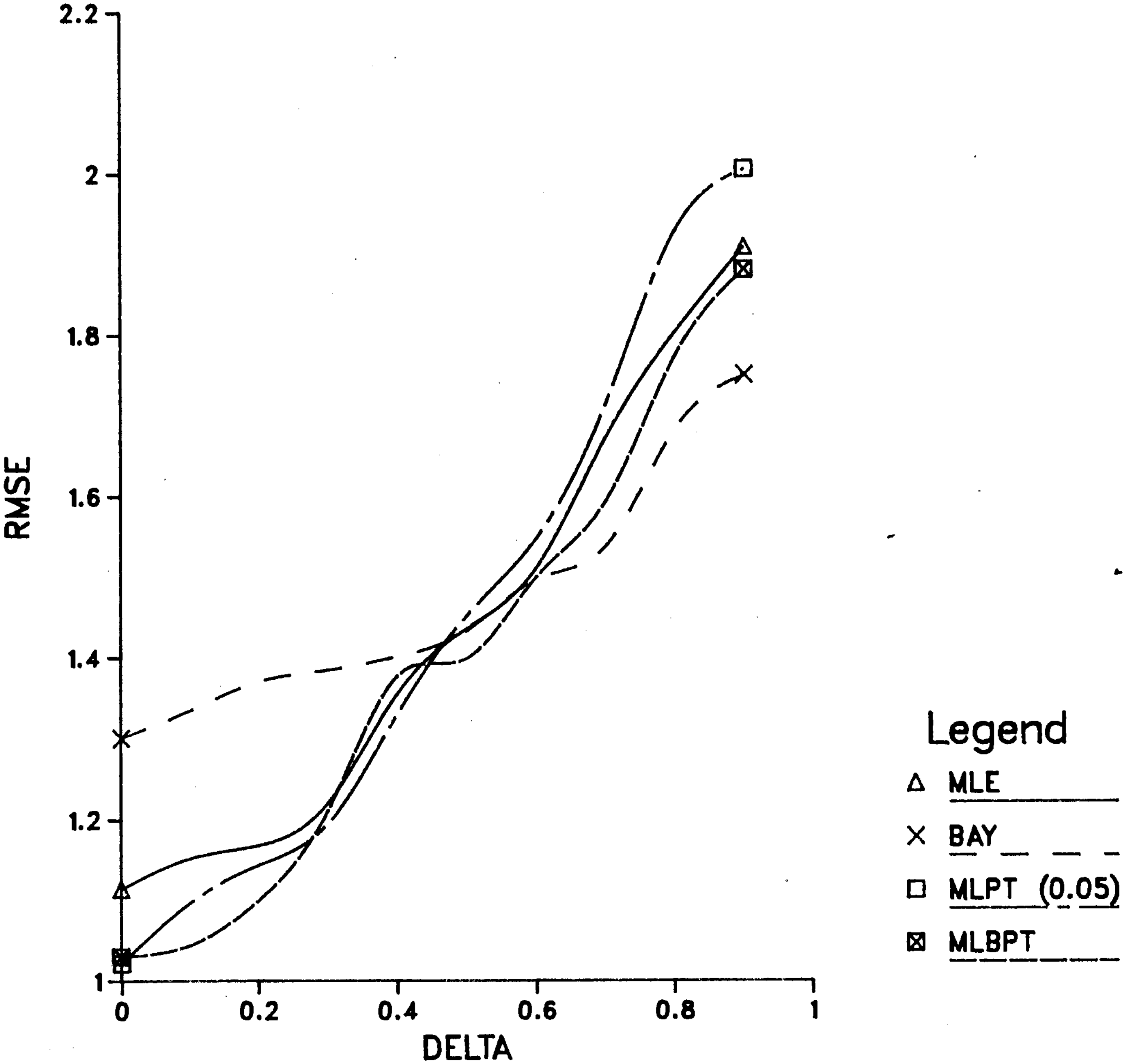


Table 4.36 (MA(1))

RELATIVE MSE (CASE 16)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.079	1.017	1.005	1.007
0.10	1.005	1.060	1.023	1.012	1.019
0.20	1.033	1.064	1.037	1.021	1.029
0.30	1.052	1.073	1.049	1.030	1.041
0.40	1.092	1.095	1.076	1.043	1.055
0.50	1.121	1.118	1.103	1.058	1.072
0.60	1.334	1.176	1.152	1.074	1.100
0.70	1.586	1.244	1.236	1.099	1.173
0.80	1.707	1.399	1.382	1.148	1.291
0.90	1.900	1.570	1.573	1.221	1.453

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.013	1.067	1.011
0.10	1.020	1.063	1.015
0.20	1.032	1.065	1.026
0.30	1.044	1.070	1.038
0.40	1.057	1.089	1.075
0.50	1.070	1.120	1.112
0.60	1.113	1.217	1.179
0.70	1.165	1.350	1.266
0.80	1.298	1.484	1.458
0.90	1.447	1.608	1.580

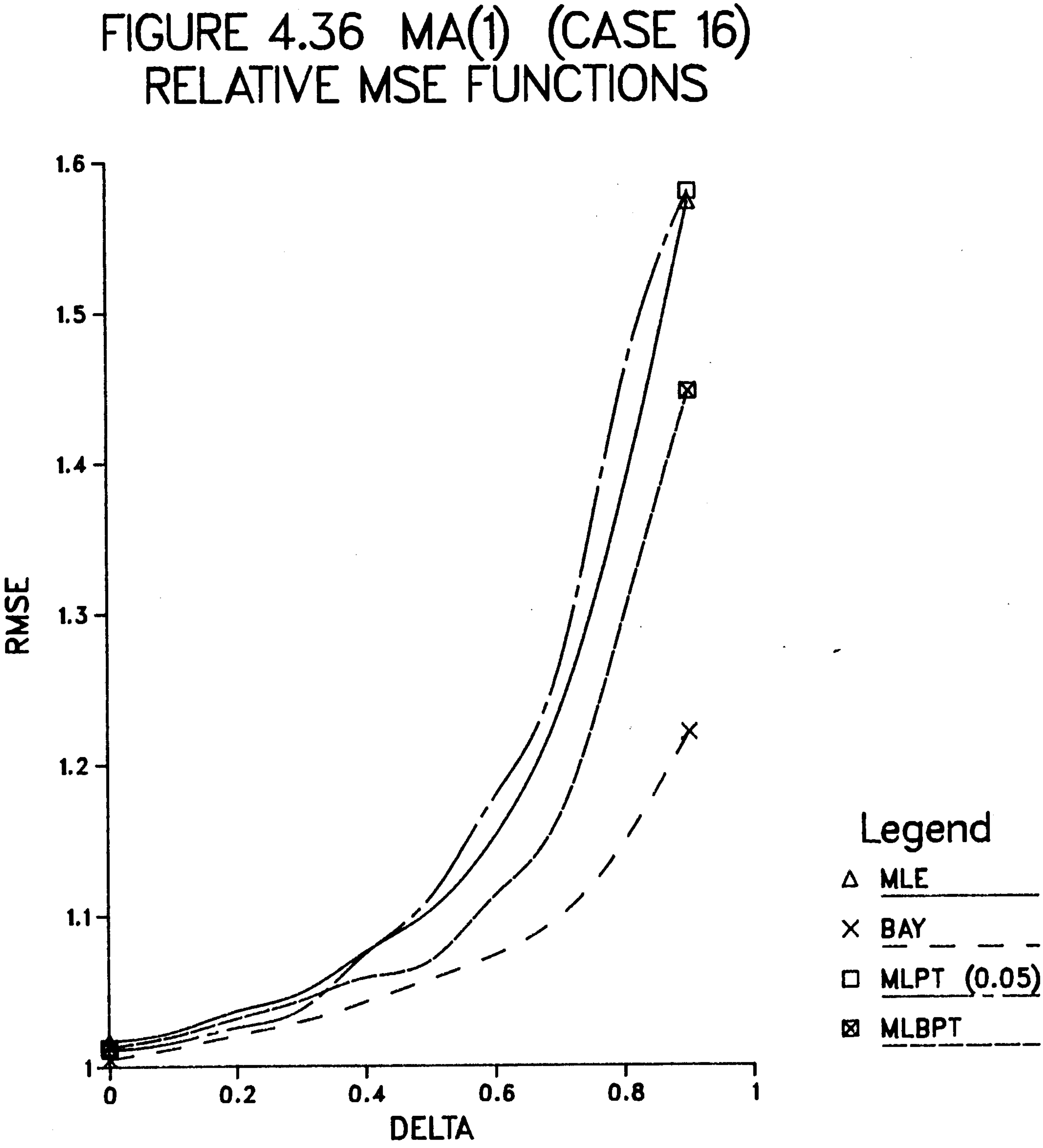


FIGURE 4.36 MA(1) (CASE 16)
RELATIVE MSE FUNCTIONS
RMSE
1.6
1.5
1.4
1.3
1.2
1.1
1
0
0.2
0.4
0.6
0.8
1
DELTA
Legend
MLE
BAY
MLPT (0.05)
MLBPT

Table 4.37 (MA(1))

RELATIVE MSE (CASE 17)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.076	1.016	1.008	1.013
0.10	1.012	1.081	1.029	1.015	1.020
0.20	1.039	1.088	1.035	1.021	1.036
0.30	1.055	1.093	1.050	1.033	1.052
0.40	1.071	1.102	1.069	1.040	1.063
0.50	1.155	1.120	1.115	1.058	1.089
0.60	1.331	1.187	1.153	1.080	1.126
0.70	1.507	1.219	1.200	1.101	1.198
0.80	1.598	1.277	1.275	1.128	1.229
0.90	2.149	1.380	1.382	1.160	1.300

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.007	1.054	1.008
0.10	1.010	1.071	1.017
0.20	1.024	1.082	1.028
0.30	1.044	1.087	1.046
0.40	1.051	1.099	1.069
0.50	1.076	1.125	1.122
0.60	1.120	1.232	1.173
0.70	1.174	1.302	1.222
0.80	1.230	1.373	1.289
0.90	1.301	1.511	1.473

FIGURE 4.37 MA(1) (CASE 17)
RELATIVE MSE FUNCTIONS

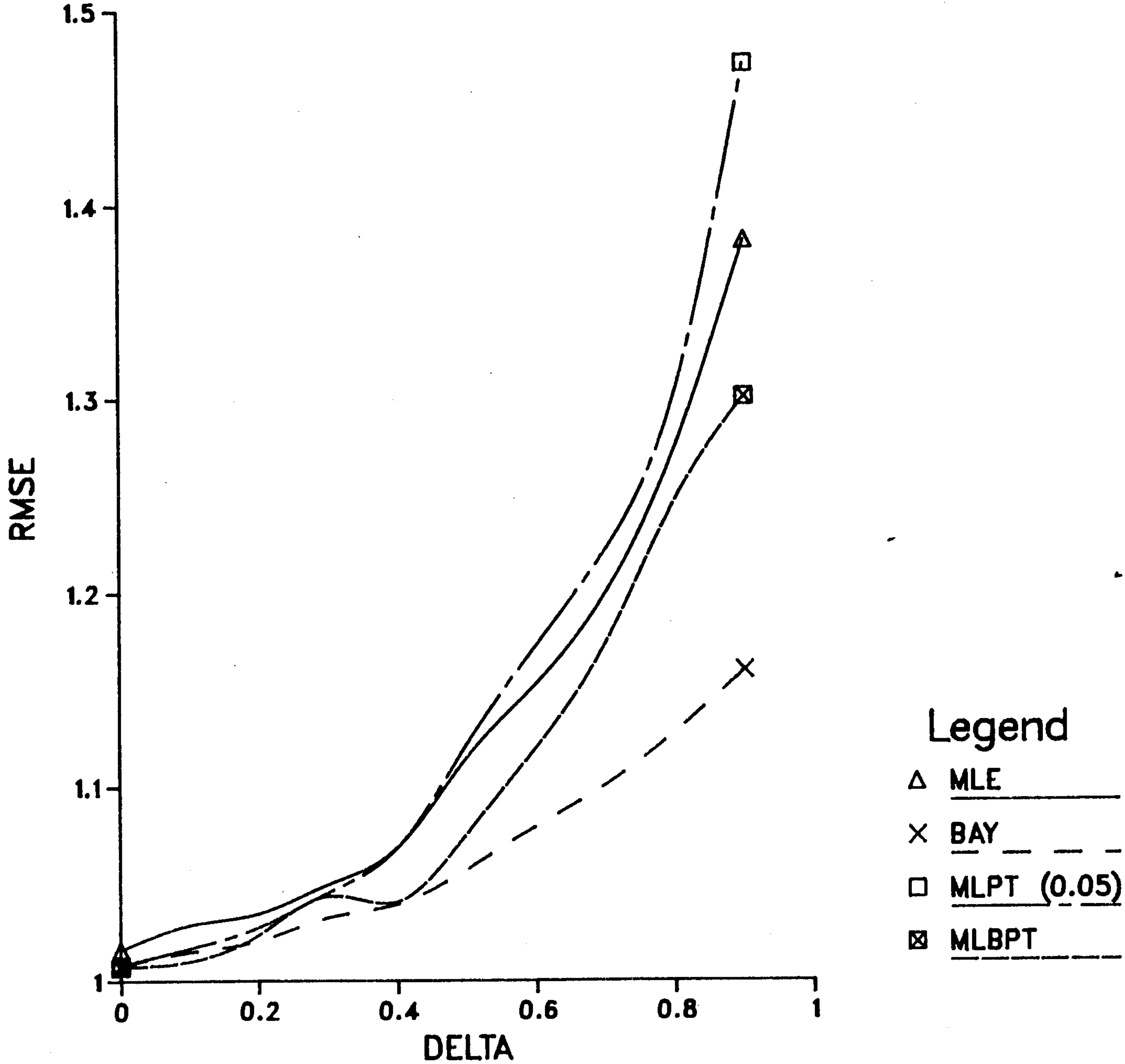


Table 4.38 (MA(1))

Mse Of Various Estimators renative to Mse of (CASE 18)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.082	1.022	1.035	1.036
0.10	1.009	1.085	1.037	1.040	1.043
0.20	1.045	1.092	1.077	1.056	1.059
0.30	1.081	1.104	1.099	1.063	1.074
0.40	1.136	1.120	1.131	1.078	1.093
0.50	1.285	1.187	1.168	1.092	1.121
0.60	1.414	1.221	1.216	1.108	1.188
0.70	1.608	1.358	1.371	1.163	1.341
0.80	1.880	1.514	1.426	1.261	1.463
0.90	2.155	1.694	1.702	1.316	1.501

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.025	1.063	1.058
0.10	1.041	1.070	1.062
0.20	1.062	1.084	1.085
0.30	1.078	1.102	1.115
0.40	1.095	1.130	1.137
0.50	1.127	1.193	1.189
0.60	1.196	1.271	1.248
0.70	1.315	1.373	1.394
0.80	1.363	1.639	1.551
0.90	1.509	1.776	1.816

FIGURE 4.38 MA(1) (CASE 18)
RELATIVE MSE FUNCTIONS

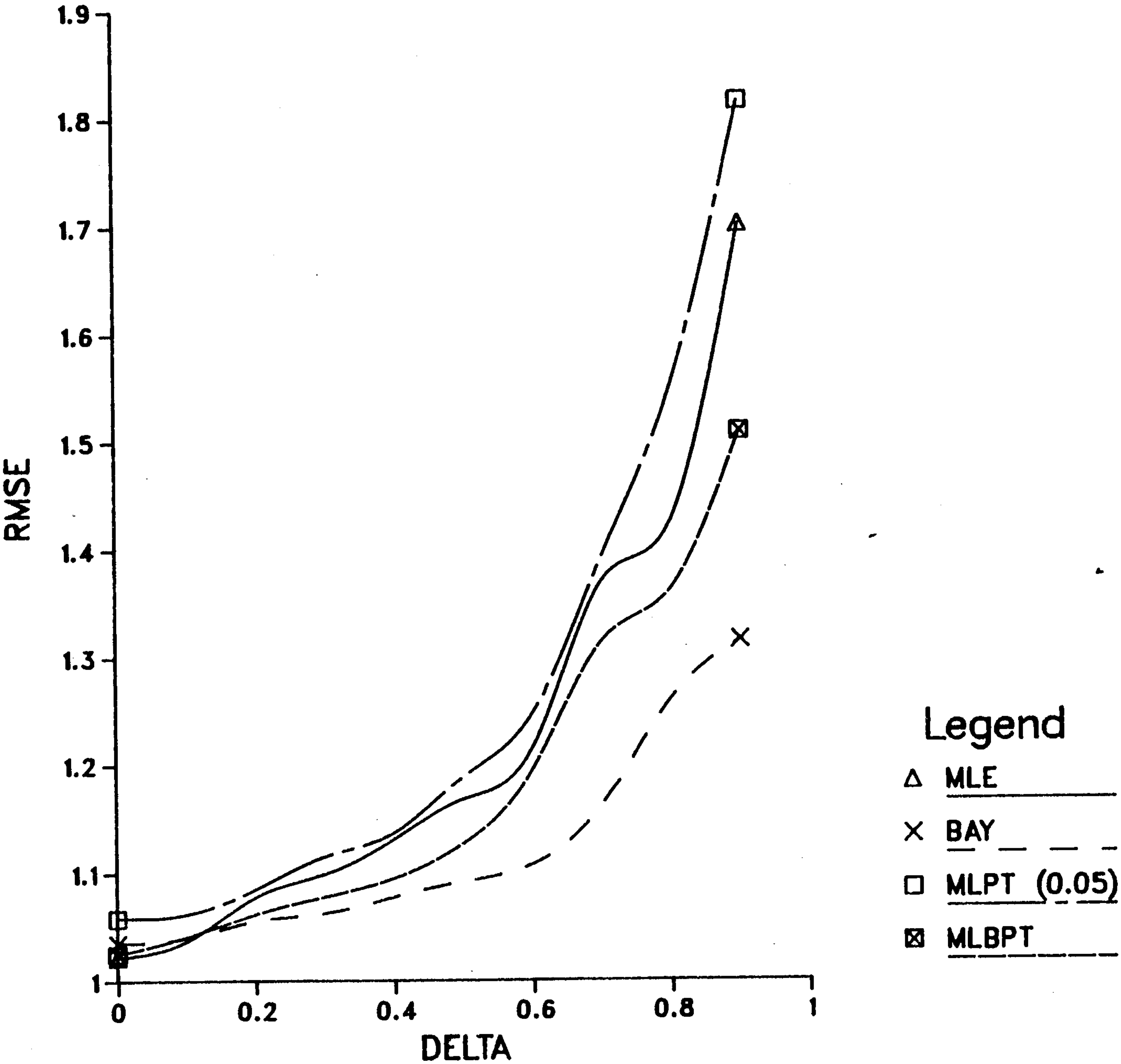


Table 4.39 (MA(1))

RELATIVE MSE (CASE 19)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.022	1.020	1.002	1.006
0.10	1.009	1.028	1.025	1.007	1.010
0.20	1.027	1.036	1.029	1.011	1.021
0.30	1.049	1.051	1.047	1.018	1.030
0.40	1.067	1.063	1.060	1.029	1.043
0.50	1.095	1.089	1.090	1.037	1.055
0.60	1.120	1.096	1.100	1.049	1.063
0.70	1.253	1.128	1.130	1.067	1.082
0.80	1.370	1.166	1.160	1.072	1.102
0.90	1.420	1.200	1.215	1.093	1.168

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.009	1.013	1.015
0.10	1.016	1.020	1.021
0.20	1.019	1.033	1.028
0.30	1.025	1.045	1.040
0.40	1.041	1.058	1.059
0.50	1.053	1.091	1.093
0.60	1.066	1.113	1.118
0.70	1.091	1.148	1.141
0.80	1.109	1.201	1.183
0.90	1.164	1.223	1.236

FIGURE 4.39 MA(1) (CASE 19)
RELATIVE MSE FUNCTIONS

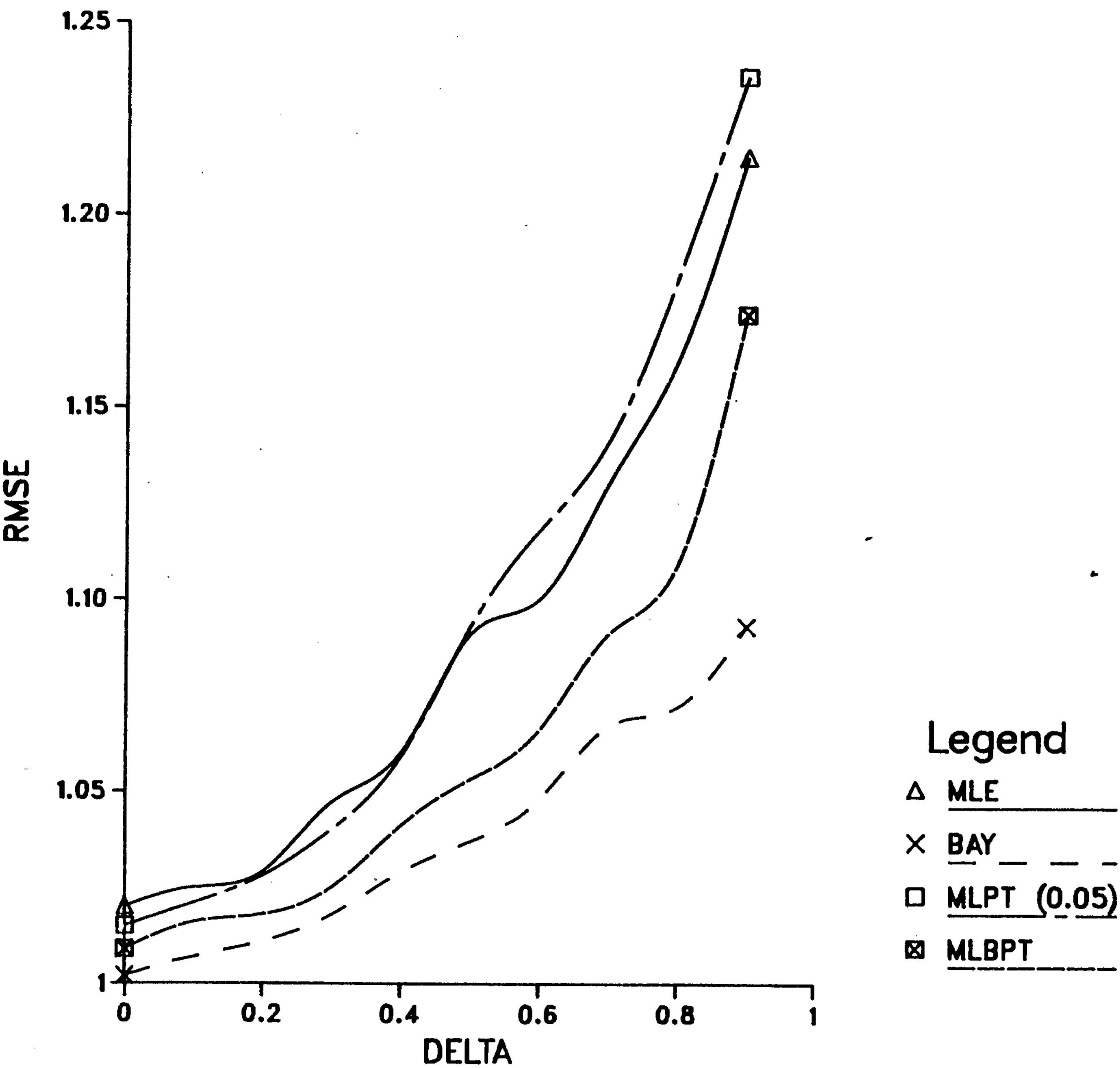


Table 4.40 (MA(1))

Mse Of Various Estimators relative to Mse of (CASE 20)

True δ	β^{OLS}	β^{EGLS}	β^{MLE}	β^{BAY}	β^{BAYPT}
0.00	1.000	1.508	1.101	1.213	1.420
0.10	1.091	1.611	1.207	1.300	1.515
0.20	1.255	1.680	1.313	1.366	1.609
0.30	1.389	1.711	1.519	1.374	1.661
0.40	1.569	1.793	1.680	1.436	1.701
0.50	1.845	1.816	1.766	1.538	1.721
0.60	1.902	1.533	1.799	1.587	1.755
0.70	2.144	1.952	1.851	1.643	1.803
0.80	2.308	2.070	1.947	1.669	1.886
0.90	2.914	2.230	2.083	1.802	2.005

True δ	β^{MLBPT}	β^{PT} (α=0.05)	β^{MLPT} (α=0.05)
0.00	1.029	1.072	1.021
0.10	1.099	1.312	1.035
0.20	1.165	1.384	1.198
0.30	1.299	1.541	1.434
0.40	1.584	1.656	1.603
0.50	1.633	1.785	1.720
0.60	1.709	1.879	1.800
0.70	1.795	1.976	1.856
0.80	1.827	2.106	1.983
0.90	1.942	2.254	2.182

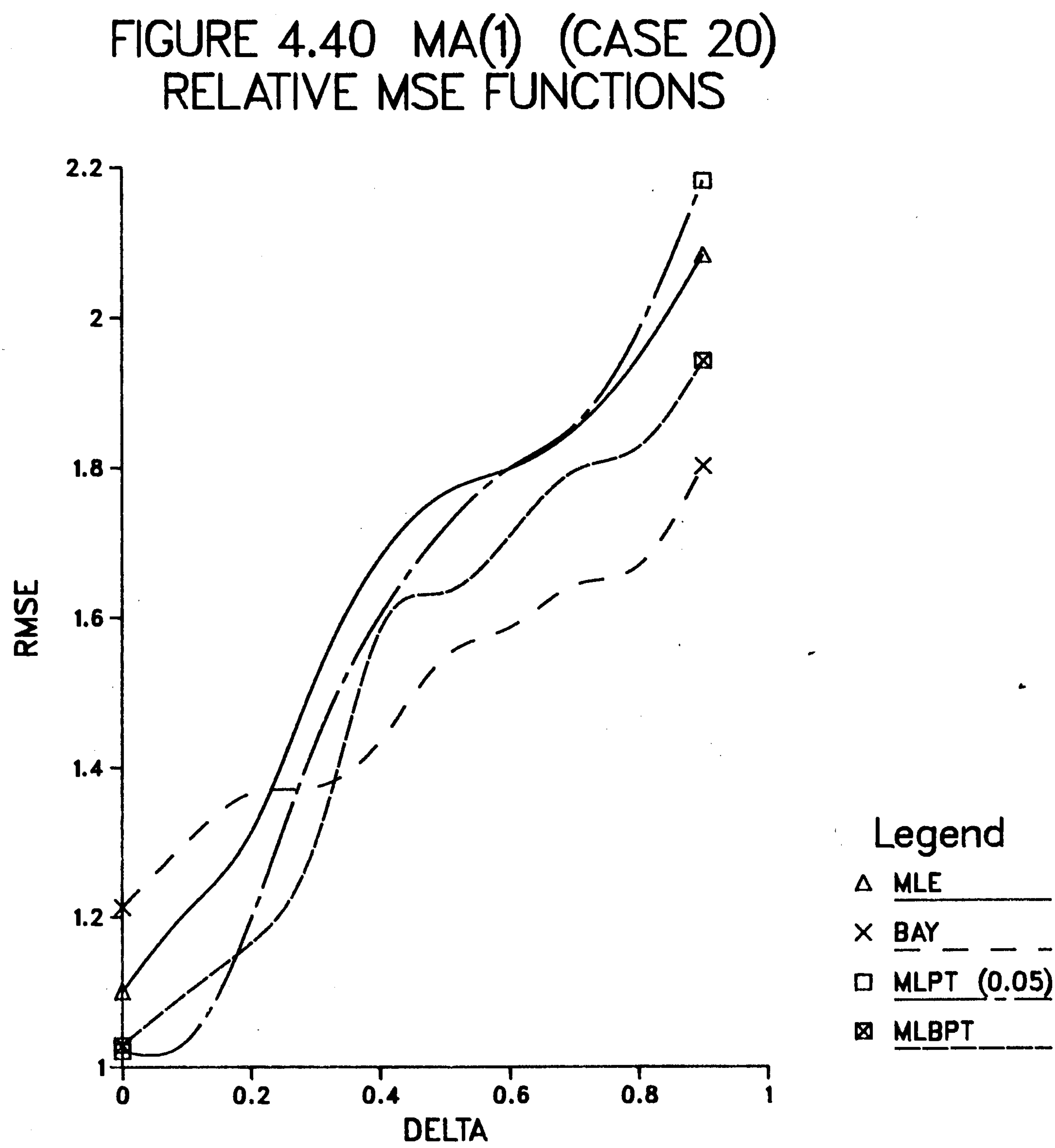


FIGURE 4.40 MA(1) (CASE 20)
RELATIVE MSE FUNCTIONS
RMSE
2.2
2
1.8
1.6
1.4
1.2
1
0
0.2
0.4
0.6
0.8
1
DELTA
Legend
MLE
BAY
MLPT (0.05)
MLBPT

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 CONCLUSION

This study, although limited in certain ways, has provided us with the conclusion that we cannot ignore the Bayesian alternative in our estimation of the β vector in a model with autocorrelated errors. For smaller values of the autocorrelation parameter,namely $\rho \leq 0.1$, OLS maintains its attractive properties but for higher values the Bayesian alternatives, especially the pure Bayesian estimator, leads to considerable gains in efficiency. For the AR(1) model, the pure Bayesian estimator performs very well and dominates all the other estimators examined over the entire range of ρ, but for the MA(1) model, it exhibited this behaviour generally only for values of $\delta \geq 0.50$.

The computational complexities of the Bayesian alternatives have been 'simplified' by the adoption of an ignorance prior leading to 'simple' formulae for this class of estimators. This author found that the computational cost, in terms of computing time (CPU), was generally five times higher for one estimate of Bayesian estimators than for one estimate of the rest of the estimators examined. It should however be noted that this time was in terms of milliseconds.

However, this drawback of the computation of the Bayesian alternative should not warrant its neglect in the econometric literature on autocorrelated error models.

5.2 RECOMMENDED TOPICS FOR FUTURE STUDY AND RESEARCH

Several areas for future research are suggested by this study.

1) This experiment could be extended to examine higher forms of autocorrelation.

2) There is an obvious need to improve on current estimates in the MA(1) model.

3) The use of the sum of mean square errors of several parameter estimates as the criterion, rather than the mean square error of a single parameter estimate.

4) Developing an 'autocorrelation' Stein estimator formally, and comparing its sampling properties with the pure Bayesian estimator.

5) Consideration of negative autocorrelated errors.

6) The performance of the Bayesian estimator in a model with two or more explanatory variables.

7) Examination of the robustness of the Bayesian estimator in a model with non-normally distributed error terms.

9) Lastly, the suggestion of using $\hat{\rho}$ as weights on EGLS and $(1-\rho$ ^) as weights on OLS could be investigated.

REFERENCES

Abrahamse, A.P and Louter, A.S(1971) "On a New Test for Autocorrelation in Least Squares Regression"<u>Biometrika</u> vol 58 pp 53-60.

Adjibolosoo, S. B-S.(1988) "Smoothing the Pretest Estimator: A Monte Carlo Study for Heteroscedasticity.<u>Unpublished Phd. Dissertation.</u> Simon Fraser University, B.C.

Aigner, D.T and Judge, G.G(1977) "Application of Pre-Test and Stein Estimation to Economic Data" <u>Econometrica,</u> vol 45 pp 1279-1288.

Balestra, P(1980) "A Note on the Exact Transformation associated with First-Order Moving Average Pocess" <u>Joural of Ecoometrics</u> vol 14 pp 381-394.

Beach, C.M and MacKinnon, J.G(1978) "A Maximum Likelihood Procedure for Regressions with Autocorrelated Errors"<u>Econometica</u> vol 46 pp 51-58.

Berenblutt, I.I and Webb, G.I(1974) "A New Test for Autocorrelated Errors in the Linear Regression Model"<u>Journal of the Royal Statistical Society,</u> Series B, vol 35 pp 33-50.

Bock, M.E; Yancey, T.A and Judge, G.G(1972) "The Statistical Consequences of Preliminary Test Estimators in Regression"<u>Journal of American Statistical Association</u> vol 68 pp 109-116.

Breusch, T.S(1980) "Useful Invariance Results for Generalized Regression Models" <u>Journal of Econometrics</u> vol 13 pp 327-340.

Cochrane, D and Orcutt, G. H(1949) "Application of Least Squaes Regressions to Relationships Containing Autocorrelated Error Terms." <u>Journal of American Statistical Association,</u> vol 44 pp 32-61

Copas, J. B.(1962) "Monte-Carlo Results for Estimators in a Stable Markov Time-Series" <u>Journal of the Royal Statistical Society;</u> Series A, No 1, pp 110-116.

Cryer, J.D ad Ledolter, J(1981) "Small Sample Properties of the Maximum Likelihood Estimator in the First-Order Moving Average Model" <u>Biometrika</u> vol 68, No 3, pp 691-694.

Davidson, J.E.H(1981) "Problems with the Estimation of Moving Average Processes" <u>Journal of Ecoometrics</u> vol 16 pp 295-310.

Durbin, J(1959) "Efficient Estimation of Parameters in Moving Average Models" Biometrika, vol 46 pp 306-316.

Durbin, J and Watson, G.S.(1950) "Testing for Serial Correlation in Least Squares Regression I" Biometrika vol 37 pp 409-428.

__________ (1951) "Testing for Serial Correlation in Least Squares Regression II" Biometrika vol 38 pp 159-177.

__________ (1971) "Testing for Serial Correlation in Least Squares Regression III" Biometrika vol 58 pp 1-19.

Effron, B and Morris, C(1973) "Stein's Estimation Rule ad its Competitors - An Empirical Bayes Approach" Journal of American Statistical Association. vol 68 pp 117-310.

Effron, B and Morris, C(1977) "Stein's Paradox in Statistics" Scientific American, vol 236 pp 119-127.

Fomby, T.B and Guilkey, D.K(1978) "On Choosing the Optimal Level of Significace fo the Durbin-Watson Test and the Bayesian Alternative" Journal of Econometrics vol 8 pp 203-213.

Folmer, H(1986) "Autocorrelation Pretesting in Linear Models with AR(1) Errors" in Lecture Notes on Mathematical Economics On Model Uncertainty and its Statistical Implications. Proceedings of a Workshop held in Groningen, The Netherlands. Dijkstra, T. K.(ed) pp 39-54.

Fuller, W. A(1976) "Introduction to Statistical Time-Series" Wiley and Sons, New York.

Goldberger, A.C(1964) "Econometric Theroy" John Wiley New York.

Griffiths, W.E and Beesley, P.A.A.(1984) "The Small Sample Properties of Some Preliminary Test Estimators in a Linear Model with Autocorrelated Errors" Journal of Econometrics, vol 25 pp 49-61

Griffiths, W.E and Dao, D(1980) "A note on a Bayesian Estimator in an Autocorrelated Error Model" Journal of Econometrics vol 12 pp 389-392.

Griliches, Z and Rao, P(1969) "Small Sample Properties of Several Two-Stage Regression Methods in the Context of Autocorrelated Errors," Journal of the American Statistical Association, vol 64 pp 253-272.

Hannan, E. and Terrel, R(1968) "Testing for Serial Correlation after Least Square Regression" Econometrica, vol 36 pp 133-150

Judge, G.G and Bock, M.E(1978) "The Statistical Implications of Pre-Test and Stein-Rule Estimators in Econometrics" North Holland, Amsterdam.

Judge, G.G., Griffiths, W.E., Hill, R.C., Lutkepohl, H and Lee, T.C(1985) "Introduction to the Theory and Practice of Econometrics" 2nd Edition. Wiley, New York.

Kadiyala, K.R.(1968) "A Transformation Used to Circumvent the Problem of Autocorrelation" Econometrica vol 36, pp 93-93.

Kennedy, P.E(1985) "A Guide to Econometrics" 2nd Edition, Basil Blackwell Ltd.

King, M.L(1983) "Testing for Moving Average Regression Disturbances" Australian Journal of Statistics, vol 25 pp 23-34.

__________(1987) "Testing for Autocorrelation in Linear Regression Models: A Survey" Specification Analysis in the Linear Model (in honour of Donald Orcutt): Ed. King, M.L and Giles, D.E. Routledge and Kegan, London, pp 19-73.

King, M.L and Giles, D.E.A(1984) "Autocorrelation Pre-Testing in the Linear Model" Journal of Econometrics, vol 25 pp 35-48

Kmenta, J(1971) "Elements of Econometrics" MacMillian, New York.

Lovell, M.C(1983) "Data Mining" Review of Economics and Statistics, vol 65 pp 1-12.

MacDonald, G.M and MacKinnon, J.G(1985) "Convenient Methods for Estimation of Linear Regression Models with MA(1) Errors", Canadian Journal of Economics, vol 18 No1 pp 107-116

Maddala, G.S(1988) "Introduction to Econometrics" MacMillian, London.

Maeshiro, A(1979) "On the Retention of the First Observations in Serial Correlation Adjustment of Regression Models", International Economic Review, vol 20, pp259-265.

Magee,L., Ullah, A. and Srivastava, U.K.(1987) "Efficiency of Estimators in the Regression Model with First-Order Autoregressive Errors" Specification Analysis in the Linear Model (in honour of Donald Orcutt) Ed. King,M and Giles, D.E.A. Routledge and Kegan, London pp 81-98.

Nakamura, A and Nakamura, M(1978) "On the Impact of the Test of Serial Correlation upon the Test of Significance for the Regression Coefficient" Journal of Econometrics, vol 7 pp 199-210.

Nicholls, D.F., Pagan, A.R and Terrel, R.D(1975) "The Estimation and Use of Models with Moving Average Distrubance Terms" International Economic Review, vol 16 pp 113-131.

Pagan, A.R.(1974) "A Generalized Approach to the Treatment of Autocorrelation" Australian Economic Papers, Series B, vol 13 pp 267-280.

Pagan,A.R and Nicholls, D.F(1976) "Exact Maximum Likelihood Estimation of Regression Models with Finite Order Moving Average Error", Review of Economic Studies, vol 43 pp 383-387.

Park, C.Y and Heikes, R.G.(1983) "A Note on Balestra's (1980) Approximate Estimator for the First Order Moving Average Process" Journal of Econometrics, vol 21 pp 387-388.

Pesaran, M.H(1973) "Exact Maximum Likelihood Estimation of a Regression Equation with a First Order Moving Average Error" Review of Economic Studies, vol 40 pp529-536.

Richard, J.F.(1977) "Bayesian Analysis of the Regression Model when the Disturbances are Generated by an Autoregressive Process" in New Developments in the Application of Bayesian Methods; Ed. Aykac, A and Brumar, C. North Holland Amsterdam. pp 185-209.

Rothenberg, T.J(1985) "The Bayesian Approach and Alternatives in Econometrics II" in Studies in Bayesian Econometrics and Statistics (in honour of Leonard Savage). Ed. Fienberg, S.C and Zellner,A. North Holland, Amsterdam. pp 55-67.

Savin, N.E. and White, K.J(1977) "The Durbin-Watson Test for Serial Correlation with Extreme Sample Sizes and Many Regressors" Econometrica, vol 45 pp 1989-1996.

Sawa, T.(1969) "The Exact Sampling Distribution of Ordinary Least Squares and Two-Stage Least Squares Estimators" Journal of American Statistical Association, vol 64 pp 923-937.

Sclove, S.L., Morris, C. and Radhakrishnan(1972) "Non-Optimality of Preliminary Test Estimators for the Multinormal Mean" Annals of Mathematical Statistics, vol 43 pp 1481 1490.

Tiao, G.C and Zellner, A(1964) "Bayes Theroem an the use of Prior Knowledge in Regression Analysis" Biometrika, vol 51 pp 219-230.

White, K.J(1978) "A General Computer Program for Econometric Models - SHAZAM" Econometrica, vol 46 pp 239-240.

White, K.J., Haun, S.A. and Horsman, N.G(1987) "Shazam: The Econometrics Computer Program" _User's Reference Manual_ Version 6.0.

Zaman, A(1984) "Avoiding Model Selection by the use of Shrinkage Techniques" _Journal of Econometrics,_ vol 25 pp 73-85.

Zellner, A(1971) 'An Introduction to Bayesian Inference in Econometrics" Wiley, New York.

__________(1975) "The Bayesian Approach and Alternatives in Econometrics I" in _Studies in Bayesian Econometrics and Statistics (in honour of Lenord Savage)_ Ed. Fienberg, S.C and Zellner, A. North Holland, Amsterdam. pp 39-54

Zellner, A and Tiao, G.C.(1964) "Bayesian Analysis of the Regression Model with Autocorrelated Errors" _Journal of American Statistical Association,_ vol 59 pp 763-778

APPENDIX A

```
C
C***THE MAIN PROGRAMME FOR THE AR(1) MODEL**********
C
 REAL*8 X(30),X1(30,2),X2(30,2),P,Y1(30,1),Y(30),LS(2,1),
*OLS(600,2),PW(2,1),EGLS(600,2),WLS(2,1),GLS(600,2),RHO,
*DW,PTE(2,1),PRET(600,2),Q,BAYS1,BAYS2,BAYES1(600,2),
*ALPHA,BAYPT(2,1),PTBAYE(600,2),PE,BM(2,1),MLE(600,2),MLES,
*MOLS,MOLS1,MEGL,MEGL1,MGLS,MGLS1,MBAY,MBPTE(2,1),M1,M2,
*MBAY1,MPRET,MPRET1,MOL,MOL1,MEL,MEL1,MRT,MRT1,MST,MST1,
*MBA,MBA1,DWA,DWV,AVA,VAA,MBPRET(600,2),PTMLB(600,2),
*MLES1,MBPE,MBPE1,PTM,PTM1,MLBPT(2,1),
*ME,ME1,MLB,MLB1,BML,BML1
      INTEGER IER,IFAIL
      DOUBLE PRECISION DSEED
      COMMON DSEED
      IFAIL=0
      DSEED=123457.D0
C
      CALL XGEN(X,X1,X2)
C
      P=-0.100
C********MAIN OUTER LOOP*******************
C
      DO 1 K=1,10
      P=P+0.100
      WRITE(7,*)'P IS',P
C**************MAIN INNER LOOP***************
```

```
      DO 2 L=1,600
      CALL YGEN(X,Y,Y1,P)
C
      CALL OLSES(X1,X2,Y1,LS)
      OLS(L,1)=LS(1,1)
      OLS(L,2)=LS(2,1)
C
      CALL ERRORS(X1,Y1,LS,DW)
      CALL RHOCAL(X,Y,RHO)
C
      CALL FINDB(RHO,X1,Y1,PW)
      EGLS(L,1)=PW(1,1)
      EGLS(L,2)=PW(2,1)
C
      CALL FINDB(P,X1,Y1,WLS)
      GLS(L,1)=WLS(1,1)
      GLS(L,2)=WLS(2,1)
C
      CALL CMLE(PE,X1,Y1,BM)
      MLE(L,1)=BM(1,1)
      MLE(L,2)=BM(2,1)
C
C*****5% AUTOCORRELATION PRETEST ESTIMATOR*******
C
      IF (DW.LE.1.62000)THEN
      DO 3 I=1,2
      PTE(I,1)=PW(I,1)
```

```
      MBPTE(I,1)=BM(I,1)
3     CONTINUE
      ELSE
      DO 5 I=1,2
      PTE(I,1)=LS(I,1)
      MBPTE(I,1)=LS(I,1)
5     CONTINUE
      END IF
      PRET(L,1)=PTE(1,1)
      PRET(L,2)=PTE(2,1)
      MBPRET(L,1)=MBPTE(1,1)
      MBPRET(L,2)=MBPTE(2,1)
C
      CALL CONS(X1,Y1,Q)
      CALL BAYS(X1,Y1,Q,BAYS1,BAYS2)
      BAYES1(L,1)=BAYS1
      BAYES1(L,2)=BAYS2

C
      CALL PROBAB(X1,Y1,Q,ALPHA)
C
      DO 6 I=1,2
      BAYPT(I,1)=ALPHA*LS(I,1)+((1-ALPHA)*PW(I,1))
      MLBPT(I,1)=ALPHA*LS(I,1)+((1-ALPHA)*BM(I,1))
6     CONTINUE
      PTBAYE(L,1)=BAYPT(1,1)
      PTBAYE(L,2)=BAYPT(2,1)
```

```
      PTMLB(L,1)=MLBPT(1,1)
      PTMLB(L,2)=MLBPT(2,1)
2     CONTINUE
C***********END OF MAIN INNER LOOP*************
C
C************MSE'S****************************
C
       CALL MSERRS(OLS,600,2,MOLS,MOLS1)
       CALL MSERRS(EGLS,600,2,MEGL,MEGL1)
       CALL MSERRS(GLS,600,2,MGLS,MGLS1)
       CALL MSERRS(BAYES1,600,2,MST,MST1)
       CALL MSERRS(PTBAYE,600,2,MBAY,MBAY1)
       CALL MSERRS(PRET,600,2,MPRET,MPRET1)
       CALL MSERRS(MLE,600,2,MLES,MLES1)
       CALL MSERRS(MBPRET,600,2,MBPE,MBPE1)
       CALL MSERRS(PTMLB,600,2,PTM,PTM1)
C
       WRITE(7,*)'BEACH AND MACKINNON DESIGN MATRIX'
       WRITE(7,*)'FOR OLS'
       CALL RETMSE(MOLS,MOLS1,MGLS,MGLS1,MOL,MOL1)
       WRITE(7,*)'FOR EGLS'
       CALL RETMSE(MEGL,MEGL1,MGLS,MGLS1,MEL,MEL1)
       WRITE(7,*)'FOR OLD PRETEST'
       CALL RETMSE(MPRET,MPRET1,MGLS,MGLS1,MRT,MRT1)
       WRITE(7,*)'FOR PURE BAYESIAN'
       CALL RETMSE(MST,MST1,MGLS,MGLS1,M1,M2)
       WRITE(7,*)'FOR BAYESIAN PRETEST'
```

```
      CALL RETMSE(MBAY,MBAY1,MGLS,MGLS1,MBA,MBA1)
      WRITE(7,*)'FOR MAXIMUM LIKELIHOOD ESTIMATOR'
      CALL RETMSE(MLES,MLES1,MGLS,MGLS1,ME,ME1)
      WRITE(7,*)'FOR MAXIMUM LIKELIHOOD PRETEST'
      CALL RETMSE(MBPE,MBPE1,MGLS,MGLS1,MLB,MLB1)
      WRITE(7,*)'FOR MAXIMUM LIKELIHOOD BAYESIAN PRETEST'
      CALL RETMSE(PTM,PTM1,MGLS,MGLS1,BML,BML1)
C
1     CONTINUE
      STOP
      END
C***END OF MAIN PROGRAM*******************
C GENERATING THE DESIGN MATRIX X1.
C X1 IS A TX2 MATRIX WITH A
C COLUMN OF ONES AND A SECOND COLUMN
C OF DATA REPRESENTING THE VARIOUS CASES
C (BEACH AND MACKINNON(1978) DESIGN MATRIX
C  IN THIS CASE)****************************
C
      SUBROUTINE XGEN(X,X1,X2)
      REAL*8 X(30),X1(30,2),X2(30,2),S,M,W(30)
      REAL R(30)
      DOUBLE PRECISION DSEED
      COMMON DSEED
      S=0.03
      CALL GGNML(DSEED,30,R)
      DO 100 I=1,30
```

```
      W(I)=R(I)*S
      X(I)=EXP(0.04*I)+W(I)
100   CONTINUE
      DO 600 K7=1,30
      X1(K7,1)=1.000
      X1(K7,2)=X(K7)
      X2(K7,1)=1.000
      X2(K7,2)=X(K7)
600   CONTINUE
      RETURN
      END
C
C  GENERATING THE Y MATRIX Y1, BY
C  FIRST OBTAINING AN AUTOCORRELATED
C  ERROR TERM  E THROUGH THE
C  THE AUTOREGRESSIVE PROCESS Et=PEt-1 + Ut
C  WHERE Ut IS NORMALLY DISTRIBUTED
C  WITH A MEAN 0 AND VARIANCE 0.0036**********
C
      SUBROUTINE YGEN(X,Y,Y1,P)
      REAL*8 X(30),Y(30),Y1(30,1)
      REAL*8 E(30),U(30),P,S,M,B0,B1
      REAL R(30)
      DOUBLE PRECISION DSEED
      COMMON DSEED
      B0=1.00
      B1=1.00
```

```
      M=0.06
      CALL GGNML (DSEED,30,R)
      DO 301 I=1,30
      U(I)=R(I)*M
301   CONTINUE
      E(1)=U(1)/((1-(P*P))**0.5)
      DO 302 K1=2,30
      E(K1)=(P*E(K1-1))+U(K1)
302   CONTINUE
      DO 500 I=1,30
      Y(I) =B0+(B1*X(I))+E(I)
500   CONTINUE
      DO 700 K7=1,30
      Y1(K7,1)=Y(K7)
700   CONTINUE
      RETURN
      END
C
C  CALCULATING OLS ESTIMATES
C
      SUBROUTINE OLSES(X1,X2,Y1,OLS)
      REAL*8 X1(30,2),X2(30,2),Y1(30,1),OLS(2,1)
      REAL*8 XX(2,2),XXI(2,2),XY(2,1),WKSP(2)
      INTEGER IER,IFAIL
      IFAIL=0
      CALL VMULFM(X1,X2,30,2,2,30,30,XX,2,IER)
      CALL F01AAF(XX,2,2,XXI,2,WKSP,IFAIL)
```

```
      CALL VMULFM(X1,Y1,30,2,1,30,30,XY,2,IER)
      CALL F01CKF(OLS,XXI,XY,2,1,2,Z,1,1,IFAIL)
      RETURN
      END
C
C     ESTIMATING THE ERROR TERMS AND
C     USING THESE TO ESTIMATE THE
C     DURBIN-WATSON STATISTIC
C
      SUBROUTINE ERRORS(X1,Y1,OLS,DW)
      REAL*8 X1(30,2),XB(30,1),OLS(2,1)
      REAL*8 Y1(30,1),EHAT(30,1)
      REAL*8 EHAT1(30),DW
      INTEGER IFAIL
      IFAIL=0
      SUME3=0.D0
      SUME4=0.D0
C
      CALL F01CKF(XB,X1,OLS,30,1,2,Z,1,1,IFAIL)
      CALL F01CEF(EHAT,Y1,XB,30,1,IFAIL)
      DO 68 I=1,30
      EHAT1(I)=EHAT(I,1)
68    CONTINUE
      DO 400 K3=2,30
      SUME3=SUME3+(EHAT1(K3)-EHAT1(K3-1))**2
400   CONTINUE
      DO 407 K4=1,30
```

```
      SUME4=SUME4+EHAT1(K4)**2
407   CONTINUE
      DW=SUME3/SUME4
C
      RETURN
      END
C
C****ESTIMATING RHO BY THE DURBIN METHOD**********
C
      SUBROUTINE RHOCAL(X,Y,RHO)
      REAL*8 Y(30),X(30),Y2(29),X3(29),X5(29,4)
      REAL*8 X6(29,4),XX(4,4),XXI(4,4)
      REAL*8 XY(4,1),TOE(4,1),RHO,WKSP(4)
      INTEGER IFAIL,IER
      IFAIL=0
C
      DO 1 I=2,30
      Y2(I)=Y(I-1)
      X3(I)=X(I-1)
      X5(I,1)=1.000
      X5(I,2)=Y2(I)
      X5(I,3)=X3(I)
      X5(I,4)=X(I)
      Y3(I,1)=Y(I)
1     CONTINUE
      DO 2 KI=1,29
      DO 3 KJ=1,4
```

```
      X6(KI,KJ)=X5(KI,KJ)
3     CONTINUE
2     CONTINUE
      CALL VMULFM(X6,X5,29,4,4,29,29,XX,4,IER)
      CALL F01AAF(XX,4,4,XXI,4,WKSP,IFAIL)
      CALL VMULFM(X6,Y3,29,4,1,29,29,XY,4,IER)
      CALL F01CKF(TOE,XXI,XY,4,1,4,Z,1,1,IFAIL)
C
      RHO=TOE(2,1)
C
      IF (RHO.GE.1.D00)THEN
      RHO=0.99999
      ELSE
      IF (RHO.LE.(-1.D00))THEN
      RHO=-0.99999
      ELSE
      RHO=RHO
      END IF
      END IF
      RETURN
      END
C
C*******USING THE PRAIS-WINSTEIN(1954) TRANSFORMATION MATRIX TO
C OBTAIN THE OMEGA INVERSE MATRIX******************************
C
      SUBROUTINE OME(P,OM11,OMEINV)
      REAL*8 OM11(30,30),OMEINV(30,30),WKSP(30)
```

```
      REAL*8 OM12(30,30),P
      INTEGER IFAIL,IER
      IFAIL=0
      DO 100 I=1,30
       DO 300 J=1,30
          OM11(I,J)=0.00
300    CONTINUE
100    CONTINUE
       OM11(1,1)=DSQRT(1-(P*P))
       DO 390 I=2,30
       OM11(I,I)=1.00
       OM11(I,I-1)=-P
390    CONTINUE
      DO 400 I=1,30
      DO 500 J=1,30
        OM12(I,J)=OM11(I,J)
500   CONTINUE
400   CONTINUE
      CALL VMULFM(OM11,OM12,30,30,30,30,30,OMEINV,30,IER)
      RETURN
      END
C
C**CALCULATING EGLS  ESTIMATES
C**BY THE DURBIN METHOD******
C***THIS IS EQUIVALENT TO USING
C***THE FINDB ROUTINE WHEN THE ESTIMATE
C***OF P IS USED.
```

```
C
      SUBROUTINE PRWIN(X,Y,RHO,EGLS)
      REAL*8 Y(30),X(30),Y2(30),X2(30)
      REAL*8 X3(30),Y3(30,1),X4(30,2),RHO
      REAL*8 X5(30,2),X1O(2,2),XOXI(2,2)
      REAL*8 XOY(2,1),EGLS(2,1),WKSP(2)
      INTEGER IFAIL,IER
      IFAIL=0
      Y2(1)=(DSQRT(1-(RHO*RHO)))*Y(1)
      X2(1)=(DSQRT(1-(RHO*RHO)))*X(1)
      X3(1)=DSQRT(1-(RHO*RHO))
      DO 1 I=2,30
      Y2(I)=Y(I)-(RHO*Y(I-1))
      X2(I)=X(I)-(RHO*X(I-1))
      X3(I)=1-RHO
1     CONTINUE
      DO 2 I=1,30
      Y3(I,1)=Y2(I)
      X4(I,1)=X3(I)
      X4(I,2)=X2(I)
      X5(I,1)=X3(I)
      X5(I,2)=X2(I)
2     CONTINUE
      CALL VMULFM(X5,X4,30,2,2,30,30,X1O,2,IER)
      CALL F01AAF(X1O,2,2,XOXI,2,WKSP,IFAIL)
      CALL VMULFM(X4,Y3,30,2,1,30,30,XOY,2,IER)
      CALL F01CKF(EGLS,XOXI,XOY,2,1,2,Z,1,1,IFAIL)
```

```
      RETURN
      END
C
C******CALCULATING THE GLS ESTIMATE
C******USING THE OMEGA INVERSE MATRIX****
C
      SUBROUTINE FINDB(P,X1,Y1,WLS)
      REAL*8 Y1(30,1),OMEINV(30,30),WOX(2,2)
      REAL*8 WO(2,25),WOXI(2,2)
      REAL*8 WOY(2,1),WLS(2,1),TRMTX(30,30)
      REAL*8 WKSP(2),P,X1(30,2)
      INTEGER IFAIL,IER
      IFAIL=0
      CALL OME(P,TRMTX,OMEINV)
      CALL VMULFM(X1,OMEINV,30,2,30,30,30,WO,2,IER)
      CALL F01CKF(WOX,WO,X1,2,2,30,Z,1,1,IFAIL)
      CALL F01AAF(WOX,2,2,WOXI,2,WKSP,IFAIL)
      CALL F01CKF(WOY,WO,Y1,2,1,30,Z,1,1,IFAIL)
      CALL F01CKF(WLS,WOXI,WOY,2,1,2,Z,1,1,IFAIL)
      RETURN
      END
C
C********USING BEACH AND MACKINNON
C********ALGORITHM TO FIND MAXIMUM
C   LIKELIHOOD ESTIMATOR.************
C
       SUBROUTINE FINDP(X1,Y1,SGLS,PE)
```

```
      REAL*8 X1(30,2),Y1(30,1),SGLS(2,1),A,B,C,D
      REAL*8 XB(30,1),E1(30,1),PA,Q,THETA,PE,E(30),SUM3
      INTEGER T,IFAIL
      T=30
      IFAIL=0
      SUM=0.00
      SUM1=0.00
      SUM2=0.00
      CALL F01CKF(XB,X1,SGLS,30,1,2,Z,1,1,IFAIL)
      CALL F01CEF(E1,Y1,XB,30,1,IFAIL)
      DO 3 I=1,30
        E(I)=E1(I,1)
3     CONTINUE
      SUM3=E(1)*E(1)
      DO 1 I=2,30
        SUM=SUM+E(I)*E(I-1)
        SUM1=SUM1+E(I-1)*E(I-1)
        SUM2=SUM2+E(I)*E(I)
1     CONTINUE
      D=(T-1)*( SUM1- SUM3 )
      A= -(T-2)*SUM/D
      B=(((T-1)*SUM3)-(T*SUM1)-(SUM2))/D
      C=T*SUM/D
      PA=B-((A*A)/3)
      Q=(C-((B*A)/3)) + ((2*(A**3))/27)
      THETA=ARCOS((Q*SQRT(27.00))/(2*PA*SQRT(-PA)))
      PE =  -2*SQRT(-PA/3)*COS((THETA/3)+(3.14/3)) - (A/3)
```

```
      RETURN
      END
C***********MLE CALCULATION*************************
       SUBROUTINE CMLE(PE,X1,Y1,MLE)
      REAL*8 Y1(30,1),X1(30,2),SGLS(2,1),SAVEP
      REAL*8 A,B,C,D,PA,Q,THETA,MLE(2,1),PE
      INTEGER T,IFAIL,COUNT
      PE=0.00
      COUNT=0
      EPS=0.00001
100   CONTINUE
      COUNT=COUNT+1
      SAVEP=PE
       CALL FINDB(SAVEP,X1,Y1,SGLS)
       CALL FINDP(X1,Y1,SGLS,PE)
       IF ((ABS(PE-SAVEP).GT.EPS) .AND. (COUNT.LE.30)) GO TO 100
 299   IF(COUNT.GT.30) GO TO 500
500       DO 4 I=1,2
             MLE(I,1)=SGLS(I,1)
 4        CONTINUE
      RETURN
      END
C
C*****CALCULATING THE NORMALISING CONSTANT
C*****FOR THE DENSITY FUNCTION
C OF THE AUTOCORRELATION PARAMETER*******
C
```

```
SUBROUTINE CONS(X1,Y1,Q)
REAL*8 X1(30,2),Y1(30,1),Q,H,H1,H2,RANGE,PO,P1
REAL*8 OM1(30,30),OR1(30,30),OM2(30,30),
REAL*8 XT(2,30),XR(2,30),XTT(2,2),XRR(2,2)
REAL*8 XRRI(2,2),XTY(2,1),XRY(2,1)
REAL*8 XRY(2,1),B1(2,1),B2(2,1),YST(30,1)
REAL*8 YRT(30,1),XST(30,2),XRT(30,2)
REAL*8 XSTB(30,1),XRTB(30,1)
REAL*8 SS1(30,1),SS2(30,1),RR1(30,1)
REAL*8 RR2(30,1),XST1(30,2)
REAL*8 XRT1(30,2),RSS1,XPX(2,2)
REAL*8 XRX(2,2),RSS2,DET1,DET2
REAL*8 WKSP(2)
INTEGER IFAIL,IER
IFAIL=0
T=30
K=2
RANGE=0.04995
H=0.D0
PO=-0.999
DO 32 K0=1,40
P1=PO+RANGE
CALL FINDB(PO,X1,Y1,B1)
CALL OME(PO,OM1,OM2)
CALL F01CKF(YST,OM1,Y1,30,1,30,Z,1,1,IFAIL)
CALL F01CKF(XST,OM1,X1,30,2,30,Z,1,1,IFAIL)
CALL F01CKF(XSTB,XST,B1,30,1,2,Z,1,1,IFAIL)
```

```
      CALL F01CEF(SS1,YST,XSTB,30,1,IFAIL)
      DO 1 I=1,30
      SS2(I,1)=SS1(I,1)
      XST1(I,1)=XST(I,1)
      XST1(I,2)=XST(I,2)
1     CONTINUE
      CALL VMULFM(SS1,SS2,30,1,1,30,30,RSS1,1,IER)
      CALL VMULFM(XST1,XST,30,2,2,30,30,XPX,2,IER)
      DET1=XPX(1,1)*XPX(2,2) - XPX(1,2)*XPX(2,1)
      CALL FINDB(P1,X1,Y1,B2)
      CALL OME(P1,OR1,OR2)
      CALL F01CKF(YRT,OR1,Y1,30,1,30,Z,1,1,IFAIL)
      CALL F01CKF(XRT,OR1,X1,30,2,30,Z,1,1,IFAIL)
      CALL F01CKF(XRTB,XRT,B2,30,1,2,Z,1,1,IFAIL)
      CALL F01CEF(RR1,YRT,XRTB,30,1,IFAIL)
      DO 2 I=1,30
      RR2(I,1)=RR1(I,1)
      XRT1(I,1)=XRT(I,1)
      XRT1(I,2)=XRT(I,2)
2     CONTINUE
      CALL VMULFM(RR1,RR2,30,1,1,30,30,RSS2,1,IER)
      CALL VMULFM(XRT1,XRT,30,2,2,30,30,XRX,2,IER)
      DET2=XRX(1,1)*XRX(2,2) - XRX(1,2)*XRX(2,1)
      PO=P1
      H1=(RSS1**((-(T-K)/2)))*(DET1**(-1/2))
      H2=(RSS2**((-(T-K)/2)))*(DET2**(-1/2))
32    H=H + 0.5*RANGE*(H1+H2)
```

```
      Q=1.00/H
      RETURN
      END
C
C
C
C CALCULATING THE PURE BAYESIAN ESTIMATOR
C WHICH IS A THE MEAN OF THE DENSITY OF B
C
      SUBROUTINE BAYS(X1,Y1,Q,BAYES1,BAYES2)
      REAL*8 H3,H4,BAYES2,TL1,TL2,GL(2,1),GL1
      REAL*8 GL2,TL(2,1),H5,H6,BAYES1,X(30),Y(30)
      REAL*8 X1(30,2),Y1(30,1),Q,H,H1,H2,RANGE,PO,P1
      REAL*8 OM1(30,30),OR1(30,30),OM2(30,30)
      REAL*8 XT(2,30),XR(2,30),XTT(2,2),XRR(2,2)
      REAL*8 XRRI(2,2),XTY(2,1),XRY(2,1)
      REAL*8 B1(2,1),B2(2,1),YST(30,1)
      REAL*8 YRT(30,1),XST(30,2),XRT(30,2)
      REAL*8 XSTB(30,1),XRTB(30,1)
      REAL*8 SS1(30,1),SS2(30,1),RR1(30,1)
      REAL*8 RR1(30,1),RR2(30,1),XST1(30,2)
      REAL*8 XRT1(30,2),RSS1,XPX(2,2)
      REAL*8 XRX(2,2),RSS2,DET1,DET2
      REAL*8 WKSP(2)
      INTEGER IFAIL,IER
      IFAIL=0
      T=30
```

```
      K=2
      RANGE=0.04995
      BAYES1=0.00
      BAYES2=0.00
      PO=-0.999
      DO 32 K0=1,40
      P1=PO+RANGE
      CALL FINDB(PO,X1,Y1,GL)
      GL1=GL(1,1)
      GL2=GL(2,1)
      CALL FINDB(P1,X1,Y1,TL)
      TL1=TL(1,1)
      TL2=TL(2,1)
      CALL OME(PO,OM1,OM2)
      CALL F01CKF(YST,OM1,Y1,30,1,30,Z,1,1,IFAIL)
      CALL F01CKF(XST,OM1,X1,30,2,30,Z,1,1,IFAIL)
      CALL F01CKF(XSTB,XST,GL,30,1,2,Z,1,1,IFAIL)
      CALL F01CEF(SS1,YST,XSTB,30,1,IFAIL)
      DO 1 I=1,30
      SS2(I,1)=SS1(I,1)
      XST1(I,1)=XST(I,1)
      XST1(I,2)=XST(I,2)
1     CONTINUE
      CALL VMULFM(SS1,SS2,30,1,1,30,30,RSS1,1,IER)
      CALL VMULFM(XST1,XST,30,2,2,30,30,XPX,2,IER)
      DET1=XPX(1,1)*XPX(2,2) - XPX(1,2)*XPX(2,1)
      CALL OME(P1,OR1,OR2)
```

```
      CALL F01CKF(YRT,OR1,Y1,30,1,30,Z,1,1,IFAIL)
      CALL F01CKF(XRT,OR1,X1,30,2,30,Z,1,1,IFAIL)
      CALL F01CKF(XRTB,XRT,TL,30,1,2,Z,1,1,IFAIL)
      CALL F01CEF(RR1,YRT,XRTB,30,1,IFAIL)
      DO 2 I=1,30
      RR2(I,1)=RR1(I,1)
      XRT1(I,1)=XRT(I,1)
      XRT1(I,2)=XRT(I,2)
2     CONTINUE
      CALL VMULFM(RR1,RR2,30,1,1,30,30,RSS2,1,IER)
      CALL VMULFM(XRT1,XRT,30,2,2,30,30,XRX,2,IER)
      DET2=XRX(1,1)*XRX(2,2) - XRX(1,2)*XRX(2,1)
      PO=P1
      H3=GL1*Q*((RSS1**((-(T-K)/2)))*(DET1**(-1/2)))
      H4=TL1*Q*((RSS2**((-(T-K)/2)))*(DET2**(-1/2)))
      H5=GL2*Q*((RSS1**((-(T-K)/2)))*(DET1**(-1/2)))
      H6=TL2*Q*((RSS2**((-(T-K)/2))*(DET2**(-1/2)))
      BAYES1 = BAYES1 + 0.5*RANGE*(H3+H4)
      BAYES2 = BAYES2 + 0.5*RANGE*(H5+H6)
 32   CONTINUE
      RETURN
      END
C
C CALCULATING THE WEIGHTS,TO OPERATIONALISE
C THE BAYESIAN ESTIMATOR.  i.e.
C THE PROBABILITY THAT OLS OUTPERFORMS EGLS..
C
```

```
SUBROUTINE PROBAB(X1,Y1,Q,ALPHA)
REAL*8 X1(30,2),Y1(30,1),Q,H,H1,H2,RANGE,PO,P1
REAL*8 OM1(30,30),OR1(30,30),OM2(30,30)
REAL*8 XT(2,30),XR(2,30),XTT(2,2),XRR(2,2)
REAL*8 XRRI(2,2),XTY(2,1),XRY(2,1),B1(2,1)
REAL*8 B2(2,1),YST(30,1)
REAL*8 YRT(30,1),XST(30,2),XRT(30,2)
REAL*8 XRT(30,2),XSTB(30,1),XRTB(30,1)
REAL*8 SS1(30,1),SS2(30,1),RR1(30,1)
REAL*8 RR2(30,1),XST1(30,2)
REAL*8 XRT1(30,2),RSS1,XPX(2,2)
REAL*8 XRX(2,2),RSS2,DET1,DET2
REAL*8 WKSP(2),ALPHA
INTEGER IFAIL,IER
IFAIL=0
RANGE=0.025
ALPHA=0.00
PO=-0.300
DO 32 K0=1,24
P1=PO+RANGE
CALL FINDB(PO,X1,Y1,B1)
CALL OME(PO,OM1,OM2)
CALL F01CKF(YST,OM1,Y1,30,1,30,Z,1,1,IFAIL)
CALL F01CKF(XST,OM1,X1,30,2,30,Z,1,1,IFAIL)
CALL F01CKF(XSTB,XST,B1,30,1,2,Z,1,1,IFAIL)
CALL F01CEF(SS1,YST,XSTB,30,1,IFAIL)
DO 1 I=1,30
```

```
      SS2(I,1)=SS1(I,1)
      XST1(I,1)=XST(I,1)
      XST1(I,2)=XST(I,2)
1     CONTINUE
      CALL VMULFM(SS1,SS2,30,1,1,30,30,RSS1,1,IER)
      CALL VMULFM(XST1,XST,30,2,2,30,30,XPX,2,IER)
      DET1=XPX(1,1)*XPX(2,2) - XPX(1,2)*XPX(2,1)
      CALL FINDB(P1,X1,Y1,B2)
      CALL OME(P1,OR1,OR2)
      CALL F01CKF(YRT,OR1,Y1,30,1,30,Z,1,1,IFAIL)
      CALL F01CKF(XRT,OR1,X1,30,2,30,Z,1,1,IFAIL)
      CALL F01CKF(XRTB,XRT,B2,30,1,2,Z,1,1,IFAIL)
      CALL F01CEF(RR1,YRT,XRTB,30,1,IFAIL)
      DO 2 I=1,30
      RR2(I,1)=RR1(I,1)
      XRT1(I,1)=XRT(I,1)
      XRT1(I,2)=XRT(I,2)
2     CONTINUE
      CALL VMULFM(RR1,RR2,30,1,1,30,30,RSS2,1,IER)
      CALL VMULFM(XRT1,XRT,30,2,2,30,30,XRX,2,IER)
      DET2=XRX(1,1)*XRX(2,2) - XRX(1,2)*XRX(2,1)
      PO=P1
      H1=Q*(RSS1**((-(T-K)/2)))*(DET1**(-1/2))
      H2=Q*(RSS2**((-(T-K)/2)))*(DET2**(-1/2))
32    ALPHA=ALPHA + 0.5*RANGE*(H1+H2)
      RETURN
      END
```

```
C
C*******MEAN SQUARE ERRORS****************
      SUBROUTINE MSERRS(BETA,N,M,MSE0,MSE1)
      REAL*8 BETA(N,M),MSE0,MSE1
      INTEGER N,M
      SUM0=0.D0
      SUM1=0.D0
      DO 1 I=1,N
      SUM0=SUM0+(BETA(I,1)-1.00)**2
      SUM1=SUM1+(BETA(I,2)-1.00)**2
1     CONTINUE
      MSE0=SUM0/N
      MSE1=SUM1/N
      RETURN
      END
C
C     RELATIVE MEAN SQUARE ERRORS
      SUBROUTINE RETMSE(BETA,PHIB,CHI,DELTA,RT1,RT2)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 BETA,PHIB,CHI,DELTA,RT1,RT2
      RT1=BETA/CHI
      RT2=PHIB/DELTA
      WRITE(7,*)'RELATIVE MEAN SQUARE ERRORS ARE'
      WRITE(7,2)RT1,RT2
 2    FORMAT(3X,2E30.8)
      RETURN
      END
```

$ENDFILE

```
C
C**THE MAIN PROGRAMME  FOR  MA(1)********
C
      REAL*8 X(65),X1(65,2),X2(65,2)
      REAL*8 D,Y1(65,1),Y(65),LS(2,1),
     *OLS(600),MM(2,1),EGLS(600),
     *WLS(2,1),GLS(600),DEST,
     *ALPHA,PTE(2,1),PRET(600),Q,BAYES1(600),
     *KING,BAYPT(2,1),PTBAYE(600),
     *BA(2,1),MLE(600),MLES,
     *MOLS(2,1),MOLS1,MEGL1,MGLS,
     *MGLS1,MBAY,MBPTE(2,1),M1,M2,
     *MBAY1,MPRET,MPRET1,MOL,
     *MOL,MOL1,MEL,MEL1,MRT,MRT1,MST,MST1,
     *MBA,MBA1,MBPRET(600),
     *PTMLB(600),PEM(600),PTM(600),
     *MLES1,MBPE1,PTM1,MLBPT(2,1),MEGL(2,1),
     *ME,ME1,MLB,MLB1,BML,BML1,D2
      REAL*8  TW(2,1),ETGL(600)
      INTEGER IER,IFAIL
      DOUBLE PRECISION DSEED
      COMMON DSEED
      IFAIL=0
      DSEED=123457.D0
C
      CALL XGEN(X,X1,X2)
C
```

```
      D=-0.100
C***MAIN OUTER LOOP******************
C
      DO 1 K=1,10
      D=D+0.100
      WRITE(7,*)'D IS',D
C******MAIN INNER LOOP***************
      DO 2 L=1,600
      CALL YGEN(X,Y,Y1,D)
C
      CALL OLSES(X1,X2,Y1,LS)
      OLS(L)=LS(2,1)
C
      CALL ERRORS(X1,Y1,LS,DEST)
C
      D2=0.50
      CALL TEST(D2,X1,X2,Y1,KING)
C
C
      CALL FINDB(D,X1,Y1,WLS)
      GLS(L)=WLS(2,1)


C
      CALL MACMAK(DEST,X,Y,MM)
      EGLS(L)=MM(2,1)
C
      CALL CMLE(X1,Y1,QMIN,BA)
```

```
      MLE(L)=BA(2,1)
C
C*****5% AUTOCORRELATION PRETEST************
      IF (KING.LE.1.061000)THEN
      DO 3 I=1,2
      PTE(I,1)=MM(I,1)
      MBPTE(I,1)=BA(I,1)
3     CONTINUE
      ELSE
      DO 5 I=1,2
      PTE(I,1)=LS(I,1)
      MBPTE(I,1)=LS(I,1)
5     CONTINUE
      END IF
      PRET(L)=PTE(2,1)
      MBPRET(L)=MBPTE(2,1)
C
C
C
      CALL CONS(X1,Y1,Q)
      CALL BAYS(X1,Y1,Q,BAYS1)
      BAYES1(L)=BAYS1
C
C
      CALL PROBAB(X1,Y1,Q,ALPHA)
C
      DO 6 I=1,2
```

```
      BAYPT(I,1)=ALPHA*LS(I,1)+((1-ALPHA)*MM(I,1))
      MLBPT(I,1)=ALPHA*LS(I,1)+((1-ALPHA)*BA(I,1))
6     CONTINUE
      PTBAYE(L)=BAYPT(2,1)
      PTMLB(L)=MLBPT(2,1)
2     CONTINUE
C
C      MSE'S
       CALL MSERRS(OLS,600,MOLS1)
       CALL MSERRS(EGLS,600,MEGL1)
       CALL MSERRS(GLS,600,MGLS1)
       CALL MSERRS(BAYES1,600,MST1)
       CALL MSERRS(PTBAYE,600,MBAY1)
       CALL MSERRS(PRET,600,MPRET1)
       CALL MSERRS(MLE,600,MLES1)
       CALL MSERRS(MBPRET,600,MBPE1)
       CALL MSERRS(PTMLB,600,PTM1)
C
       WRITE(7,*)'DEBT OF NON FINANCIAL SECTOR'
       WRITE(7,*)'FOR OLS'
       CALL RETMSE(MOLS1,MGLS1,MOL1)
       WRITE(7,*)'FOR EGLS'
       CALL RETMSE(MEGL1,MGLS1,MEL1)
       WRITE(7,*)'FOR OLD PRETEST'
       CALL RETMSE(MPRET1,MGLS1,MRT1)
       WRITE(7,*)'FOR PURE BAYESIAN'
       CALL RETMSE(MST1,MGLS1,M2)
```

```
       WRITE(7,*)'FOR BAYESIAN PRETEST'
       CALL RETMSE(MBAY1,MGLS1,MBA1)
       WRITE(7,*)'FOR MAXIMUM LIKELIHOOD ESTIMATOR'
       CALL RETMSE(MLES1,MGLS1,ME1)
       WRITE(7,*)'FOR MAXIMUM LIKELIHOOD PRETEST'
       CALL RETMSE(MBPE1,MGLS1,MLB1)
       WRITE(7,*)'FOR MAXIMUM LIKELIHOOD BAYESIAN PRETEST'
       CALL RETMSE(PTM1,MGLS1,BML1)
C
1     CONTINUE
      STOP
      END
C
C*****READING THE DESIGN MATRIX X1
C*****FROM AN INPUT FILE.*********
C
      SUBROUTINE XGEN(X,X1,X2)
      REAL*8 X(65),X1(65,2),X2(65,2)
      DOUBLE PRECISION DSEED
      COMMON DSEED
      DO 650 I=1,65
      READ(5,34)X(I)
34    FORMAT(F6.3)
650   CONTINUE
      DO 600 K7=1,65
      X1(K7,1)=1.000
      X1(K7,2)=X(K7)
```

```
      X2(K7,1)=1.000
      X2(K7,2)=X(K7)
600   CONTINUE
      RETURN
      END
C
C**** GENERATING THE Y MATRIX Y1,
C**** BY FIRST OBTAINING AN  AUTOCORRELATED
C   ERROR TERM  E, THROUGH THE
C   THE MOVING AVERAGE PROCESS
C  Et=DUt-1 + Ut, WHERE Ut IS NORMALLY
C  NORMALLY DISTRIBUTED WITH A MEAN 0 AND VARIANCE
C  0.0036.************************************8
C
      SUBROUTINE YGEN(X,Y,Y1,D)
      REAL*8 X(65),Y(65),Y1(65,1),E(65)
      REAL*8 U(65),D,S,M,B0,B1
      REAL R(16)
      DOUBLE PRECISION DSEED
      COMMON DSEED
      B0=1.00
      B1=1.00
      M=0.06
      CALL GGNML (DSEED,16,R)
      DO 301 I=1,65
      U(I)=R(I)*M
301   CONTINUE
```

```
      DO 302 K1=2,16
      E(K1)=(D*U(K1-1))+U(K1)
302   CONTINUE
C
C*******THE Y'S****************************
C
      DO 500 I=1,65
      Y(I) =B0+(B1*X(I))+E(I)
500   CONTINUE
      DO 700 K7=1,65
      Y1(K7,1)=Y(K7)
700   CONTINUE
      RETURN
      END
C********************************************
C***CALCULATING OLS ESTIMATES***************
C
      SUBROUTINE OLSES(X1,X2,Y1,OLS)
      REAL*8 X1(65,2),X2(65,2),Y1(65,1),OLS(2,1)
      REAL*8 XX(2,2),XXI(2,2),XY(2,1),WKSP(2)
      INTEGER IER,IFAIL
      IFAIL=0
      CALL VMULFM(X1,X2,65,2,2,65,65,XX,2,IER)
      CALL F01AAF(XX,2,2,XXI,2,WKSP,IFAIL)
      CALL VMULFM(X1,Y1,65,2,1,65,65,XY,2,IER)
      CALL F01CKF(OLS,XXI,XY,2,1,2,Z,1,1,IFAIL)
      RETURN
```

```
      END
C
C ***********************************
C***CALCULATING THE TEST STATISTIC
C***VIA KING'S TEST S(0.50).
C
      SUBROUTINE TEST(D2,X1,X2,Y1,KING)
      REAL*8 X1(65,2),X2(65,2),Y1(65,1)
      REAL*8 KING,B(2,1),TOP(1,1)
      REAL*8 MEGA(65,65),XB(65,1)
      REAL*8 RS(65,1),RS1(1,65),OS(2,1)
      REAL*8 XBB(65,1),RSS(65,1),RSS1(65,1)
      REAL*8 BOT(1,1),D2,MEINV(65,65)
      INTEGER IFAIL,IER
      IFAIL=0
      CALL FINDB(D2,X1,Y1,B)
      CALL OME(D2,MEGA,MEINV)
      CALL F01CKF(XB,X1,B,65,1,2,Z,1,1,IFAIL)
      CALL F01CEF(RS,Y1,XB,65,1,IFAIL)
      CALL VMULFM(RS,MEINV,65,1,65,65,65,RS1,1,IER)
      CALL F01CKF(TOP,RS1,RS,1,1,65,Z,1,1,IFAIL)
      CALL OLSES(X1,X2,Y1,OS)
      CALL F01CKF(XBB,X1,OS,65,1,2,Z,1,1,IFAIL)
      CALL F01CEF(RSS,Y1,XBB,65,1,IFAIL)
      DO 34 K=1,65
      RSS1(K,1)=RSS(K,1)
 34   CONTINUE
```

```
      CALL VMULFM(RSS,RSS1,65,1,1,65,65,BOT,1,IER)
      KING=TOP(1,1)/BOT(1,1)
      RETURN
      END
C
C**OBTAINING THE OMEGA INVERSE MATRIX USING
C   THE OMEGA MATRIX*******************
C
      SUBROUTINE OME(D,OMEGA,OMEINV)
      REAL*8 OMEGA(65,65),OMEINV(65,65),WKSP(65)
      REAL*8 D,OM13(65,65)
      INTEGER IFAIL
      IFAIL=0
      DO 100 I=1,65
       DO 650 J=1,65
           OMEGA(I,J)=0.00
650    CONTINUE
100   CONTINUE
       OMEGA(1,1)=1+(D**2)
       OMEGA(1,2)=D
       DO 300 I=2,65
        OMEGA(I,I)=1+(D**2)
        OMEGA(I,I+1)=D
        OMEGA(I,I-1)=D
300    CONTINUE
      DO 450 I=1,65
      DO 451 J=1,65
```

```
      OM13(I,J)=OMEGA(I,J)
451   CONTINUE
450   CONTINUE
      CALL F01AAF(OM13,65,65,OMEINV,65,WKSP,IFAIL)
      RETURN
      END
C
C*******USING THE METHOD OF MOMENT
C******TO ESTIMATE THE AUTOCORRELATION
C****** PARAMETER DELTA **************
C
      SUBROUTINE ERRORS(X1,Y1,LS,DEST)
      REAL*8 X1(65,2),Y1(65,1),LS(2,1)
      REAL*8 DEST,XB(65,1),EHAT(65,1)
      REAL*8 E(65),R1,SUM,SUM1,SUM2,SUM3,T
      INTEGER IFAIL
      IFAIL=0
      SUM=0.00
      SUM1=0.00
      CALL F01CKF(XB,X1,LS,2,1,65,Z,1,1,IFAIL)
      CALL F01CEF(EHAT,Y1,XB,65,1,IFAIL)
      DO 1 KI=1,65
      E(KI)=EHAT(KI,1)
1     CONTINUE
      DO 2 I=2,65
      SUM=SUM+E(I)*E(I-1)
      SUM1=SUM1+E(I)*E(I)
```

```
2     CONTINUE
      R1= SUM/SUM1
      IF (R1.LE.-0.500.AND.R1.GE.-1.000)THEN
      DEST=-1.00
      ELSE
      IF (R1.GE.0.500.AND.R1.LE.1.000)THEN
      DEST=1.00
      ELSE
      DEST=(1 - (DSQRT(1-(4*(R1*R1))))) /(2*R1)
      END IF
      END IF
      RETURN
      END
C
C*****CALCULTING EGLS VIA
C*****MACDONALD AND MACKINNON(1985) METHOD**
C
      SUBROUTINE MACMAK(DEST,X,Y,MM)
      REAL*8 X(65),X3(16),Y3(16),Z(16)
      REAL*8 Y(16),XY(3,1),X5(16,3)
      REAL*8 X4(16,3), X5(16,3),Y4(16,1)
      REAL*8 MM(3,1),XX(3,3),XXI(3,3)
      REAL*8 WKSP(3)
      INTEGER IFAIL,IER
      Y3(1)=0.00
      X3(1)=0.00
      Z(1)=1
```

```
      DO 1 I=2,16
      X3(I)=X(I-1)-DEST*X3(I-1)
      Y3(I)=Y(I-1)-DEST*Y3(I-1)
      Z(I)=DEST*Z(I-1)
1     CONTINUE
      DO 2 I=1,16
      X4(I,1)=1.000
      X4(I,2)=X3(I)
      X4(I,3)=Z(I)
      Y4(I,1)=Y3(I)
2     CONTINUE
      DO 3 I=1,16
      DO 4 J=1,3
      X5(I,J)=X4(I,J)
4     CONTINUE
3     CONTINUE
      CALL VMULFM(X4,X5,16,3,3,16,16,XX,3,IER)
      CALL F01AAF(XX,3,3,XXI,3,WKSP,IFAIL)
      CALL VMULFM(X4,Y4,16,3,1,16,16,XY,3,IER)
      CALL F01CKF(MM,XXI,XY,3,1,3,Z,1,1,IFAIL)
      RETURN
      END
C
C**CALCULATING  GLS ESTIMATES
C******USING THE OMEGA INVERSE MATRIX***
C
      SUBROUTINE FINDB(D,X1,Y1,WLS)
```

```
      REAL*8 Y1(65,1),OMEINV(65,65)
      REAL*8 WOX(2,2),WO(2,65),WOXI(2,2)
      REAL*8 WOY(2,1),WLS(2,1),TRMTX(65,65)
      REAL*8 X1(65,2),WKSP(2)
      INTEGER IFAIL,IER
      IFAIL=0
      CALL OME(D,TRMTX,OMEINV)
      CALL VMULFM(X1,OMEINV,65,2,65,65,65,WO,2,IER)
      CALL F01CKF(WOX,WO,X1,2,2,65,Z,1,1,IFAIL)
      CALL F01AAF(WOX,2,2,WOXI,2,WKSP,IFAIL)
      CALL F01CKF(WOY,WO,Y1,2,1,65,Z,1,1,IFAIL)
      CALL F01CKF(WLS,WOXI,WOY,2,1,2,Z,1,1,IFAIL)
      RETURN
      END
C
C*****MLE CALCULATION*****************************
       SUBROUTINE CMLE(X1,Y1,QMIN,MLE)
      REAL*8 Y1(65,1),X1(65,2),SGLS(2,1)
      REAL*8 X1(65,2),SGLS(2,1),AN,TOBI
      REAL*8 XB(65,1),OMEGA(65,65)
      REAL*8 OMEINV(65,65),DELTA
      REAL*8 RS1(1,65),RS(65,1)
      REAL*8 SS,Q,QMIN,CVAL,C,MLE(2,1)
      INTEGER IFAIL,IER
      IFAIL=0
      T=65
      DELTA=-0.200
```

```
      DO 100 I=1,24
        DELTA=DELTA+0.050
        CALL FINDB(DELTA,X1,Y1,SGLS)
        CALL F01CKF(XB,X1,SGLS,65,1,2,Z,1,1,IFAIL)
        CALL F01CEF(RS,Y1,XB,65,1,IFAIL)
        CALL OME(DELTA,OMEGA,OMEINV)
        CALL VMULFM(RS,OMEINV,65,1,65,65,65,RS1,1,IER)
        CALL F01CKF(SS,RS1,RS,1,1,65,Z,1,1,IFAIL)
      AN=(1-(DELTA**((2*T)+2)))/(1-(DELTA*DELTA))
      TOBI=AN**(1.0/65.0)
       Q=TOBI*SS
       IF (I.EQ.1)THEN
          QMIN=Q
          CVAL=DELTA
         DO 44 K0=1,2
          MLE(K0,1)=SGLS(K0,1)
44       CONTINUE
       ELSE
        IF(Q.LT.QMIN)THEN
          QMIN=Q
          CVAL=DELTA
         DO 392 KI=1,2
           MLE(KI,1)=SGLS(KI,1)
392      CONTINUE
        END IF
        END IF
100    CONTINUE
```

```
      RETURN
      END
C
C**CALCULATING THE NORMALISING CONSTANT
C** FOR THE DENSITY FUNCTION OF DELTA
C
        SUBROUTINE CONS(X1,Y1,Q)
       REAL*8 P0,RANGE,RANGE1,SUM,P1,X1(65,2)
       REAL*8 X1(65,2),Y1(65,1),XB(65,1)
       REAL*8 E(65,1),GL(2,1),EP(1,65)
       REAL*8 RSS,XO(2,65),XPX(2,2)
       REAL*8 TL(2,1),E1(65,1),E2(1,65)
       REAL*8 DET1,CON,DENS1,BX(65,1)
       REAL*8 CON2,DENS2,DET2,Q,GRH(65,65)
       REAL*8 RSS1,XT(2,65),XX(2,2)
       REAL*8 RRH(65,65),Q1(65,65),R1(65,65)
       INTEGER IFAIL,IER
       IFAIL=0
       T=65
       K=2
       P0=-0.999
       RANGE=0.04995
       SUM=0.D0
       DO 1 I=1,40
       P1=P0+RANGE
       CALL FINDB(P0,X1,Y1,GL)
       CALL F01CKF(XB,X1,GL,65,1,2,Z,1,1,IFAIL)
```

```
      CALL F01CEF(E,Y1,XB,65,1,IFAIL)
      CALL OME(P0,Q1,RRH)
      CALL VMULFM(E,RRH,65,1,65,65,65,EP,1,IER)
      CALL F01CKF(RSS,EP,E,1,1,65,Z,1,1,IFAIL)
      CALL VMULFM(X1,RRH,65,2,65,65,65,XO,2,IER)
      CALL F01CKF(XPX,XO,X1,2,2,65,Z,1,1,IFAIL)
      DET1=(XPX(1,1)*XPX(2,2))-(XPX(1,2)*XPX(2,1))
      CON=((1-(ABS(P0)**((2*T)+2)))/(1-(P0**2)))**(-1/2)
      DENS1=CON*(RSS**(-((T-K)/2)))*(DET1**(-1/2))
      CALL FINDB(P1,X1,Y1,TL)
      CALL F01CKF(BX,X1,TL,65,1,2,Z,1,1,IFAIL)
      CALL F01CEF(E1,Y1,BX,65,1,IFAIL)
      CALL OME(P1,R1,GRH)
      CALL VMULFM(E1,GRH,65,1,65,65,65,E2,1,IER)
      CALL F01CKF(RSS1,E2,E1,1,1,65,Z,1,1,IFAIL)
      CALL VMULFM(X1,GRH,65,2,65,65,65,XT,2,IER)
      CALL F01CKF(XX,XT,X1,2,2,65,Z,1,1,IFAIL)
      DET2=(XX(1,1)*XX(2,2))-(XX(1,2)*XX(2,1))
      CON2=((1-(ABS(P1)**((2*T)+2)))/(1-(P1**2)))**(-1/2)
      DENS2=CON*(RSS1**(-((T-K)/2)))*(DET2**(-1/2))
      P0=P1
      SUM=SUM+0.5*RANGE*(DENS1+DENS2)
1     CONTINUE
      Q=1/SUM
      RETURN
      END
C
```

```
C
C**CALCULATING THE BAYESIAN ESTIMATOR ****
C
      SUBROUTINE BAYS(X1,Y1,Q,BAYES)
      REAL*8 X1(65,2),Y1(65,1),XB(65,1)
      REAL*8 P0,RANGE,RANGE1,SUM,P1
      REAL*8 RSS,XO(2,65),XPX(2,2)
      REAL*8 E(65,1),GL(2,1),EP(1,65)
      REAL*8 DET1,CON,DENS1,BX(65,1)
      REAL*8 TL(2,1),E1(65,1),E2(1,65)
      REAL*8 CON2,DENS2,DET2,Q,GRH(65,65)
      REAL*8 RSS1,XT(2,65),XX(2,2)
      REAL*8 IT1,TT1,BAYES,RRH(65,65)
      REAL*8 Q1(65,65),R1(65,65)
      INTEGER IFAIL,IER
      IFAIL=0
      T=65
      K=2
      P0=-0.999
      RANGE=0.04995
      SUM=0.D0
C
C*******NUMERICAL INTEGRATION******************************
C
      DO 1 I=1,40
      P1=P0+RANGE
      CALL FINDB(P0,X1,Y1,GL)
```

```
      IT1=GL(2,1)
      CALL F01CKF(XB,X1,GL,65,1,2,Z,1,1,IFAIL)
      CALL F01CEF(E,Y1,XB,65,1,IFAIL)
      CALL OME(P0,Q1,RRH)
      CALL VMULFM(E,RRH,65,1,65,65,65,EP,1,IER)
      CALL F01CKF(RSS,EP,E,1,1,65,Z,1,1,IFAIL)
      CALL VMULFM(X1,RRH,65,2,65,65,65,XO,2,IER)
      CALL F01CKF(XPX,XO,X1,2,2,65,Z,1,1,IFAIL)
      DET1=(XPX(1,1)*XPX(2,2))-(XPX(1,2)*XPX(2,1))
      CON=Q*((1-(ABS(P0)**((2*T)+2)))/(1-(P0**2)))**(-1/2)
      DENS1=IT1*CON*(RSS**(-((T-K)/2)))*(DET1**(-1/2))
      CALL FINDB(P1,X1,Y1,TL)
      TT1=TL(2,1)
      CALL F01CKF(BX,X1,TL,65,1,2,Z,1,1,IFAIL)
      CALL F01CEF(E1,Y1,BX,65,1,IFAIL)
      CALL OME(P1,R1,GRH)
      CALL VMULFM(E1,GRH,65,1,65,65,65,E2,1,IER)
      CALL F01CKF(RSS1,E2,E1,1,1,65,Z,1,1,IFAIL)
      CALL VMULFM(X1,GRH,65,2,65,65,65,XT,2,IER)
      CALL F01CKF(XX,XT,X1,2,2,65,Z,1,1,IFAIL)
      DET2=(XX(1,1)*XX(2,2))-(XX(1,2)*XX(2,1))
      CON2=Q*((1-(ABS(P1)**((2*T)+2)))/(1-(P1**2)))**(-1/2)
      DENS2=TT1*CON2*(RSS1**(-((T-K)/2)))*(DET2**(-1/2))
      P0=P1
      SUM=SUM+0.5*RANGE*(DENS1+DENS2)
1     CONTINUE
      BAYES=SUM
```

```
      RETURN
      END
C
C
C******CALCULATING THE PROBABILITY THAT OLS OUTPERFORMS
C****** EGLS
C
      SUBROUTINE PROBAB(X1,Y1,Q,ALPHA)
      REAL*8 X1(65,2),Y1(65,1),XB(65,1)
      REAL*8 P0,RANGE,RANGE1,SUM,P1
      REAL*8 E(65,1),GL(2,1),EP(1,65)
      REAL*8 RSS,XO(2,65),XPX(2,2)
      REAL*8 DET1,CON,DENS1,BX(65,1)
      REAL*8 TL(2,1),E1(65,1),E2(1,65)
      REAL*8 CON2,DENS2,DET2,Q,GRH(65,65)
      REAL*8 RSS1,XT(2,65),XX(2,2)
      REAL*8 ALPHA,RRH(65,65),Q1(65,65),R1(65,65)
      INTEGER IFAIL,IER
      IFAIL=0
      T=65
      K=2
      P0=-0.500
      RANGE=0.050
      SUM=0.D0
      DO 1 I=1,20
      P1=P0+RANGE
      CALL FINDB(P0,X1,Y1,GL)
```

```
      CALL F01CKF(XB,X1,GL,65,1,2,Z,1,1,IFAIL)
      CALL F01CEF(E,Y1,XB,65,1,IFAIL)
      CALL OME(P0,Q1,RRH)
      CALL VMULFM(E,RRH,65,1,65,65,65,EP,1,IER)
      CALL F01CKF(RSS,EP,E,1,1,65,Z,1,1,IFAIL)
      CALL VMULFM(X1,RRH,65,2,65,65,65,XO,2,IER)
      CALL F01CKF(XPX,XO,X1,2,2,65,Z,1,1,IFAIL)
      DET1=(XPX(1,1)*XPX(2,2))-(XPX(1,2)*XPX(2,1))
      CON=Q*((1-(ABS(P0)**((2*T)+2)))/(1-(P0**2)))**(-1/2)
      DENS1=CON*(RSS**(-((T-K)/2)))*(DET1**(-1/2))
      CALL FINDB(P1,X1,Y1,TL)
      CALL F01CKF(BX,X1,TL,65,1,2,Z,1,1,IFAIL)
      CALL F01CEF(E1,Y1,BX,65,1,IFAIL)
      CALL OME(P1,R1,GRH)
      CALL VMULFM(E1,GRH,65,1,65,65,65,E2,1,IER)
      CALL F01CKF(RSS1,E2,E1,1,1,65,Z,1,1,IFAIL)
      CALL VMULFM(X1,GRH,65,2,65,65,65,XT,2,IER)
      CALL F01CKF(XX,XT,X1,2,2,65,Z,1,1,IFAIL)
      DET2=(XX(1,1)*XX(2,2))-(XX(1,2)*XX(2,1))
      CON2=Q*((1-(ABS(P1)**((2*T)+2)))/(1-(P1**2)))**(-1/2)
      DENS2=CON2*(RSS1**(-((T-K)/2)))*(DET2**(-1/2))
      P0=P1
      SUM=SUM+0.5*RANGE*(DENS1+DENS2)
1     CONTINUE
      ALPHA=SUM
      RETURN
      END
```

```
C
C*****MEAN SQUARE ERRORS *************************
C
      SUBROUTINE MSERRS(BETA,N,MSE1)
      REAL*8 BETA(N),MSE1
      INTEGER N
      SUM1=0.D0
      DO 1 I=1,N
      SUM1=SUM1+(BETA(I)-1.00)**2
1     CONTINUE
      MSE1=SUM1/N
      RETURN
      END
C
C     RELATIVE MEAN SQUARE ERRORS
      SUBROUTINE RETMSE(BETA,CHI,RT1)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 BETA,CHI,RT1
      RT1=BETA/CHI
      WRITE(7,*)'RELATIVE MEAN SQUARE ERRORS ARE'
      WRITE(7,2)RT1
 2    FORMAT(3X,2E30.8)
      RETURN
      END
$ENDFILE
```