

STUDENT EXPERIENCES IN A FIRST SEMESTER
UNIVERSITY CALCULUS COURSE : A STUDY USING
ETHNOGRAPHIC METHODS

by

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B.A., Yale University, 1973

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Student Experiences in a First Semester University Calculus Course:

A Study using Ethnographic Methods

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ABSTRACT

The purpose of this study was to gain and accurately depict an understanding of students' experiences in a first year calculus course. The study was motivated by my personal interest in calculus teaching and by discussions within the calculus reform movement over the past 14 years.

Ethnographic methods were used in order to produce a naturalistic description of the student experiences.

The fieldwork was conducted in a first semester calculus class at Simon Fraser University during the Fall Semester, 1991. It was an evening class held in the downtown Vancouver campus. As a participant-observer, I attended all classes and tutorial sessions, completed and submitted assigned work, and wrote both midterm tests and the final examination.

Data acquisition consisted primarily of fieldwork and interviews with the instructor and six of the students.

Data analysis included the transcription of audio tapes of the interviews; the coding and collating of this material by researcher specified topics using computer software; and a concurrent review of my field notes and the audio tapes of the lectures. The written accounts of the analyzed data were refined by weekly discussions with my thesis advisor.

The detailed record of my experiences and those of my classmates throughout the course was produced to give a sense of the challenges confronting the students.

Several conclusions arise from this study. The students perceive this course as an obstacle they must overcome before being permitted to take the courses they want. The course is not well coordinated with the high school curriculum; it assumes skills, knowledge and mathematical sophistication that are seldom taught in the secondary schools. The course tries to accomplish too many things; mathematical rigour, computational techniques, fundamental concepts, problem solving, and the use of scientific calculators are blended together into a bewildering *mélange*. There is too much material for the time allotted; many topics are dealt with superficially. The pace of the course allows no latitude for student illness, fatigue or personal upsets.

In summary, this study raises issues not yet discussed in the calculus reform debate and puts a personal face on many issues already presented.

This work is dedicated to my mother,
Barbara Jean Thomas Newell.

(September 21, 1926 - August 3, 1970)

She wanted me to care about people.

I hope that she would have been pleased with this study.

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My wife, Gail, and two sons, Jeremy and Robert, have dealt gracefully with a husband and father who, for the last three years, has been frequently absent or distracted. My love and thanks to them.

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CHAPTER I
INTRODUCTION

Rationale for the Study

The teaching and learning of calculus are subjects that are currently drawing a considerable amount of attention in the mathematics and mathematics education communities. As a high school teacher who teaches calculus to some students and prepares others to study the subject at university, I have a natural interest in the discussion and research associated with calculus pedagogy and curriculum.

The current large-scale interest in calculus reform issues began in the early 1980's when a number of mathematicians and computer scientists began to dispute the traditional pre-eminence of calculus in the college mathematics curriculum (Ralston & Young, 1983). A number of mathematicians rose to the defense of calculus, but concluded that there was need for change (Douglas, 1986).

By the mid 1980's, calculus reform issues had become mainstream in the mathematics and mathematics education communities. In 1986, a national colloquium with over 600 participants was held in Washington, D.C. (Steen, 1988).

Reporting on the colloquium, Peterson (1987) identified three premises of the reform movement, "calculus is big, important - and in trouble" (p. 317). Hundreds of thousands of students take calculus courses each year taught by tens of thousands of instructors. For many students, admission to their chosen programs of study comes only after they have successfully completed a calculus course. The calculus course is often the only university level mathematics course taken by the most educated segment of the population. Yet current calculus classes were under attack for a myriad of perceived inadequacies such as over-large textbooks, poor teaching, low standards, and irrelevant, conceptually deficient course content. Students were not having success with the course. A substantial portion of the students failed or withdrew each year.

Half a decade later, Ferrini-Mundy and Graham (1991) commented that "substantial discussion is underway in the mathematics education and mathematics communities concerning calculus pedagogy and curriculum and there is considerable momentum for a national reform of the college calculus curriculum" (p.627). The debate continues even today.

The calculus reform movement, spurred on by National Science Foundation funding, has resulted in many and diverse projects (Tucker, 1990). These projects have centered primarily on curriculum development and, generally, have not

been based on results from mathematics education research. It is a "top down" approach to curriculum development; the instructor decides what to improve and how to implement these improvements.

Ferrini-Mundy and Graham (1991) argue that "existing research on student learning in calculus has potential for strong impact on the development of calculus curriculum" (p. 629). Revitalization of calculus instruction may not be a matter of changing course content, but of paying close attention to how students learn.

One epistemological orientation of research on student learning that seems to be potentially productive is constructivism. Ferrini-Mundy and Graham (1991) comment on this.

A predominant theme in this area of research is that the learning of mathematics is a constructive process. This process occurs in the classroom context, with the learner attempting to make sense out of information by evaluating, connecting, and organizing it relative to prior experiences and existing knowledge structure. This line of research has challenged traditional assumptions about learning, such as the view of the student as the passive recipient of knowledge, or the notion that a misconception can be eradicated by the "perfect explanation" or by simply reorganizing the content. (p. 629)

My own experiences as a teacher have told me that the voices of the students must be heeded carefully if there is to be effective revision of the learning situation.

Reform of calculus curriculum and instruction is necessary for a variety of reasons. After 12 years of debate and classroom innovation, however, it is still not clear exactly what changes will be of greatest benefit to the student. A thorough understanding of the experiences of the calculus student is a prerequisite to developing useful reform initiatives.

The Nature and Purpose of this Study

The purpose of this study was to gain as thorough an understanding as possible of the experiences of students in a first year calculus course and to convey this understanding to those who might be able to use these understandings in implementing reform measures.

To accomplish these goals, I used a qualitative research design and adapted many of my research methods from ethnography. Stake and Easley (1978) summarize the goals of such a study by writing, "Seeing rather than measuring was the activity of this project" (p. C:1). The results of the study would be descriptive and interpretive, rather than normative.

Eisenhart (1988) distinguishes between the questions traditionally posed in mathematics education research and those posed by educational anthropologists. Mathematics education researchers ask, "How can mathematics teaching and

learning be improved?" and the ethnographers ask, "Why is mathematics teaching and learning occurring in this way in this setting?" (p. 100). My contention is that these are not separate questions, but that answering the "why" question should happen prior to answering the "how" question.

The purpose of doing interpretivist research, according to Eisenhart (1988) "is to provide information that will allow the investigator to "make sense" of the world from the perspective of participants" (p. 103). Of the methods used in ethnographic research, participant observation offers the best opportunity to accomplish this. Such fieldwork is holistic and contextualized; the student is not an abstracted, isolated variable but is part of a social environment. Smith (1987) says of such studies that "the researcher must personally become situated in the subject's natural setting and study, firsthand and over a prolonged time, the object of interest and the various contextual features that influence it" (p. 175).

The study was not ethnography in the sense of educational anthropology, lacking as it does any grounding in any cultural theory. It was, however, qualitative and interpretivist and did make use of the methods traditionally used by ethnographers. My understanding of what a student undergoes while working through a calculus course has been

considerably enhanced and it is my hope that the account I give will communicate much of this knowledge.

An Overview of the Study

In August, 1991 I began a brief pilot study in which I attended four lectures, observed three sessions in the Calculus Lab and had informal discussions with the instructor and some students. During this period I was most concerned with developing my fieldwork skills.

In September, 1991 I enrolled as a student in a first semester calculus course at Simon Fraser University. By the second session the instructor and my classmates were aware of my role as a researcher.

Because I continued to teach full time, the course that was available for me to attend was an evening course held at the Harbour Centre campus in downtown Vancouver. The class consisted of about 20 students, many of whom had been away from formal schooling for several years. Several of the students were recent high school graduates.

I participated completely in the course, attending all lectures and tutorial sessions, completing assigned problems and writing both midterm tests and the final examination.

I established reasonably close contact with a number of students in various ways. Because I was a calculus expert, some students sought me out for help with problems. Others

talked with me because I brought graphics capable, programmable calculators to class and demonstrated how they could be used to work on calculus problems. A small number of students caught rides with me home at the end of the evening. One student, in particular, became a key informant during these carpooling sessions. Still other students would come to me and spontaneously offer their opinions about various aspects of the course. The instructor was at all times supportive and approachable with respect to my research.

After the course was completed, I conducted and audio taped interviews with six of my classmates and the instructor.

The analysis and reporting of the data occurred over an long period of time. I transcribed each of the interviews. Subsequently, the seven interviews were collated by theme using computer software known as HyperRESEARCH (Hesse-Biber, Dupuis & Kinder, 1991). The themes were chosen by reading through the transcripts and in discussion with my thesis advisor. The material in the interviews, what was said by the participants, was written as an account centered around the themes.

My field notes were used in conjunction with audio tapes of the lectures to write a chronological account of my experiences and those of my classmates as we proceeded

throughout the course. Within the account, I have included numerous reflections that occurred to me both in the field and later as I reviewed the material. As the account was being written, I was meeting with my advisor on a weekly basis and discussing the material with him. This continuing dialogue was a significant influence in refining the account.

Several conclusions were drawn from the study. The course that I took was superior to the majority of traditional calculus courses described in the calculus reform literature. The instructor was exemplary in his organization and his care for the students. Despite these advantages, there were a number of conspicuous drawbacks to this course. The course was intended as a filter; it was a hurdle that had to be cleared before the student could take courses that he or she wanted. There was inadequate articulation between the content and expectations in this course and what was covered in high school. The course was set up so that students worked independently, in isolation rather than cooperatively. Too many aspects of mathematics were covered in the one course: techniques, concepts, applications, problem solving and rigorous use of language and symbols. Too much material was covered in the time available, forcing students into a survival "what is the minimum I need to know" mode.

The study added to the calculus reform effort discussions in several ways. Although the students voiced dissatisfaction with many aspects of the course, they did not suggest radical changes. The impetus for reform would not likely come from the students. There was no time for students to recover from illness, fatigue or upheavals in their personal lives in this course. Nor was there sufficient time for them to properly assimilate the concepts they were meant to learn. There was a reluctance among the students to work together. These factors require further study.

Of some significance to me was the impact that this style of research had on me personally. The data were not objective measurements to be statistically manipulated; they were events that I lived and shared with my classmates. Consequently, I feel I have a much deeper understanding of the student experiences and greater empathy with them.

Structure of the Thesis

Chapter II is a review of the literature relevant to this study. The literature of the calculus reform movement is cited, with particular attention being paid to those issues which helped me to form a framework for my initial observation during the fieldwork. This is followed by a summary of current research perspectives in the teaching and

learning of mathematics that led me to a qualitative research design. The literature was also reviewed to discover any prior studies closely related to this one.

Chapter III is a description of how the research was conducted. Sources supporting and explaining the methods chosen are cited.

Chapter IV is in two parts. The first part is a chronological account of my experiences as a participant observer. The second part is material from interviews with six students and the instructor. This material is arranged by theme.

Chapter V discusses the limitations of the study. This is followed by a number of conclusions reached through this study. I make several observations and recommendations with respect to these conclusions. Finally, the impact that this study has had on me is briefly discussed.

CHAPTER II

LITERATURE REVIEW

Selection of Literature for Review

The literature reviewed for this study has a diversity that is a consequence of the interaction between the research problem and the research methodology. In this chapter I review the literature that impacted on the formulation and evolution of the research problem and the literature that lead to the choice of the research methodology used in this project. In this section I categorize the literature with respect to the use that is made of its information in this study.

The first category is literature on the calculus reform movement. Material reviewed consists of articles, conference proceedings and committee reports on a decade long debate on issues concerning the college calculus curriculum. This literature is used to support the significance of the research problem. It also generated the original set of issues and questions that focused my initial fieldwork observations.

A second category which informed my original conceptualization of the research problem is current research perspectives on teaching and learning mathematics. Chief among these perspectives are a constructivist approach to

understanding student learning and an awareness of how the social and cultural contexts of a student impacts on his or her learning.

A third category involves the use of ethnographic methods in educational research. This category divides naturally into literature about the appropriateness and role of ethnography in educational research and on how to conduct such research. The material from this category is used to justify the choice and implementation of research methodology. It also served to provide structure for my actual research activities.

A fourth category in this review consists of previous studies involving qualitative research methods in science or mathematics classrooms. Reviewing the literature in this category indicated that I was not duplicating the work of any current major study. The review also contributed to the list of issues used in the analysis of the data.

Calculus Reform Movement

A Brief History of the Movement

In the late 1950's and early 1960's, the study of calculus in North America shifted from being a sophomore course taken by students majoring in engineering, mathematics, or the physical sciences to a freshman course

taken by dramatically increasing numbers of students with increasingly diverse mathematical backgrounds and aspirations. (Ralston,1985; Young,1988). For the next twenty to twenty-five years the curriculum in the first two years of undergraduate mathematics stabilized into a sequence of calculus courses supplemented by a linear algebra course. There was a remarkable consistency in content and presentation of college calculus throughout North American post-secondary institutions.

There have been critics and reformers of this calculus curriculum before the 1980's, notably John Kemeny of Dartmouth College. (Kemeny,1983). The current impetus for curriculum reform, however, began in the early 1980's with papers by Anthony Ralston of the State University of New York at Buffalo. Ralston argued that computer technology and computer science made much of the content and spirit of the traditional calculus curriculum irrelevant and inappropriate for a significant portion of the undergraduates currently taking college level mathematics courses. (Ralston, 1985). Ralston and like minded colleagues argued that calculus should be supplanted as the first undergraduate mathematics course by a course based on topics from discrete mathematics.

In the summer of 1982 a Conference/Workshop was held at Williams College in Williamstown, Maryland to discuss the need and feasibility of striking a balance between the

calculus sequence and discrete mathematics. A significant part of the proceedings was spent challenging the pre-eminence of calculus in undergraduate mathematics (Ralston & Young, 1983).

The Williamstown Conference and its follow-up activities did not result in a curriculum revolution where significant numbers of colleges and universities began to offer well developed, highly subscribed courses in discrete mathematics at the freshman level (Maurer, 1985). Indeed, surveys suggested that the primacy of calculus became more firmly entrenched in the course offerings of mathematics departments (Anderson & Loftsgaarden, 1988).

The criticisms leveled against the existing curriculum did make certain mathematicians reflect on the state of calculus instruction. One of these mathematicians, Ronald Douglas of the State University of New York at Stony Brook, concluded that "despite the changes that had taken place in mathematics, the Williamstown conferees were wrong: calculus is as important as ever." (Douglas, 1986, p. iv). He did believe, however, that there was a need to revitalize the presentation of the subject. Through the efforts of Professor Douglas, a Conference/Workshop to develop alternative curriculum and teaching methods for calculus at the college level was held in January, 1986 at Tulane University in New Orleans. The proceedings of this conference were published

in the influential document *Toward a Lean and Lively Calculus* (Douglas, 1986).

Following this conference, the Mathematical Association of America convened a Committee on Calculus Reform. Other organizations became involved in calculus reform including the National Academies of Science and Engineering and the National Science Foundation. A large colloquium with over six hundred participants was held at the end of October, 1987 in Washington, D.C. The colloquium was entitled *Calculus for a New Century*. The proceedings of the conference and other calculus reform material were published in *Calculus for a New Century : A Pump, Not a Filter* (Steen, 1988).

In 1990, the CUPM Subcommittee on Calculus Reform and the First Two Years (CRAFTY) reported on a number of these projects in *Priming the Calculus Pump: Innovations and Resources* (Tucker, 1990). Each project reviewed in this report attempted to implement innovations which directly addressed concerns about the role and teaching of calculus that had been raised in the previous decade of debate.

Following is a more detailed review of the four primary documents resulting from this decade of reform activity: the reports of the Williamstown conference in 1982, the Tulane conference in 1986, the National Colloquium in 1987, and the 1990 report on the implementation of reform.

The Williamstown Conference

In 1981 an article by Anthony Ralston heralded a decade of ongoing debate over the nature and content of the first two years of the college mathematics curriculum (Ralston, 1985). The concerns raised by Ralston lead to a conference/workshop held at Williams College in Williamstown, Maryland from June 28 to July 1, 1982. The proceedings of these meetings are published in *The Future of College Mathematics* (Ralston & Young, 1983).

Ralston (1985) argues as follows:

Within the next decade, the first two years of the college mathematics curriculum will gradually shift from the traditional calculus-linear algebra sequence to one that has balance between the traditional material and discrete mathematics. Such changes will take place almost entirely because of the impact of computer technology and computer science both on what seems important in mathematics itself and on disciplines in which mathematics is applied. (p.29)

Computer science has caused mathematicians, scientists and humanists to reconsider what is important and possible in their disciplines. Computers allow new approaches to old subjects (including pedagogy). In Ralston's opinion, discrete mathematics has become at least as important as the traditional mathematics of continuity (Ralston & Young, 1983, p. 6).

The Williamstown conference began with this theme and continued with papers and discussion with arguments for and against the restructuring of the college mathematics

curriculum. Both pragmatic and idealistic concerns were raised. The issues established here continued to be significant themes as general curriculum reform debate shifted back to the reform of the calculus curriculum.

Young (Ralston & Young, 1983) raises the issue of who was taking college level mathematics and for what purposes. One conclusion he reaches is "that vast numbers of people are studying mathematics who are not going into traditional fields that use mathematics. The process is more or less out of control" (p. 25). The homogeneity of academic goals and mathematical preparation of earlier generations of first year college mathematics students has disappeared.

Wilf (Ralston & Young, 1983) discusses with remarkable prescience the potential for change in the classroom implicit in current and soon to exist technology. He discusses symbolic manipulation programs (SMPs). Not only does he consider the impact SMPs will have on curriculum content, but he predicts the existence and capabilities of calculators with SMPs. Wilf's paper began a continuing discussion about what can be done and what should be done with technology in the mathematics classroom.

Papers by Lochhead, Greber, Norman, Zions, Scherlis and Shaw (Ralston & Young, 1983) bring forward the issue of how college mathematics are often service courses. That is, mathematics courses are taken by students so that they will

have the necessary techniques and knowledge to pursue studies in other academic disciplines. The question is what are the mathematical needs of students in varying fields of study. The "client" disciplines discussed were the physical sciences, engineering, social sciences, business and management, and computer science.

Steen (1983) submitted a provocative paper on developing mathematical maturity. The ideas in this article recur frequently throughout the calculus reform debate.

Steen cites the abilities to "to glean the essential structure from a complex situation" and to "to create new ideas by effective use of old ones" as indicators of mathematical maturity. His criteria for mathematical maturity include the abilities: 1. to use and interpret mathematical notation, 2. to express real world problems in mathematical form, 3. to perceive patterns and apply principles of symmetry, 4. to estimate and solve problems by perturbing the data, 5. to generalize, 6. to detect and avoid poor reasoning, 7. to see and use relationships among various parts of mathematics, and 8. to read and understand mathematical writing (Steen, 1983, pp. 99-102).

Steen (1983) links the study of calculus to the development of mathematical maturity when he writes:

Realism compels us to recognize that certain types of study reinforce better than others the development of mathematical maturity.

There are some compelling reasons for the widespread belief (among both mathematicians and scientists) that calculus develops mathematical maturity better than most other subjects. (p. 102)

After explaining why the study of calculus is so well suited to promoting mathematical maturity, Steen laments, "Calculus in the classroom, however, is not the same as calculus in the philosophy book" (p. 103). Calculus as it is taught today, in Steen's opinion, stifles mathematical maturity rather than encouraging it. Steen's position presaged the one adopted by later curriculum reformers: calculus should retain its central role in the freshman mathematics curriculum, but it must change so that the students have a much deeper understanding of the subject.

Kemeny (1983) talks about the impact of computers on teaching practices. He words his message strongly, stating, "the form of the first two years of math teaching and the use of computers in the classroom cannot be separated" (p. 205). To illustrate the arbitrary nature of some of the traditional approach to calculus, Kemeny offers as an example the anticipated correct answer to the problem of evaluating

$\int_0^{13} e^x dx$. The exact, closed form answer is $e^{13} - 1$. Kemeny

challenges the reader to approximate this answer to one significant figure. Kemeny points out that the integral itself is an exact closed form. There is little difference between evaluating the integral using the numerical integration capabilities of a computer or calculator or with evaluating $e^{13} - 1$ with such a device. Kemeny (1983) comments on his example:

I do not mean that [the example] as a joke - I mean it as a deep remark about mathematics, one that I overlooked for a large number of years. Now the question is, if you have two forms that are exact, why is one preferable to the other one? And if you think about that particular example, I think it's very easy to reconstruct the reason. We did not have computers but had tables of e^x . Even today [1982] there may be some advantage; you may be able to use a pocket calculator instead of a computer on $e^{13} - 1$. But the numerical integration takes less than a second on a computer. (p. 204)

In summary, advances in technology, especially in computing devices, and the advent of the Information Age resulted in challenges to the relevance of much of the mathematics taught in the first two years of university. Participants at the Williamstown Conference believe that an appropriate revision of the curriculum would involve replacing calculus with topics from discrete mathematics. In their discussions they set out a number of issues which later had to be addressed by calculus revisionists. Chief among these issues are the changing academic demographics of

freshmen, the perceived needs of client disciplines, the possible changes that new and future technology might bring to what is taught and how it is taught, and overcoming the inherent difficulties in implementing any curriculum revision on a large scale.

The Tulane Conference

At a conference held two years after Williamstown, Maurer (1985) notes that little had happened in the establishment of discrete mathematics early in the undergraduate curriculum. Indeed, speaking about schools who had received funding to implement discrete mathematics courses, he says, "They have no problem in introducing a discrete course (just lots of work to produce text materials), but their commitment to reducing the calculus seems to be eroding" (p. 259). The debate had swung from replacing calculus to improving calculus. Many of the issues, however, remained the same.

The man who organized the Tulane Conference, Professor Ronald Douglas, was reacting to the criticisms of the calculus centered curriculum raised by the Williamstown conferees. He reviews their arguments and concludes that they were wrong, "Calculus is as important as ever."

But Douglas (1986) adds to this:

There is an overwhelming case for calculus remaining as the core of the undergraduate mathematics curriculum. But, along with all of this, I realized just how much the introductory course had changed, how much it had eroded in the nearly three decades since I had studied it. Although I could advance many reasons for this, I decided that I had little interest in focusing on the causes or in continuing to participate in the "calculus versus finite mathematics" debate. Rather, I felt compelled to try to improve the way calculus is taught at American colleges and universities." (p. iv).

The conference/ workshop was held in January, 1986 at Tulane University in New Orleans, Louisiana and involved twenty-five participants. The three workshops that reported back to the conference dealt with three aspects of curriculum implementation: course content, teaching methods and implementation of the new curriculum.

The Content Workshop developed, among others, a syllabus for a Calculus I course intended for a general audience. The design of this syllabus took into account two of the issues already raised: the diverse nature of the freshman student population and the fact that, for many students, Calculus I would be the last mathematics course taken. This syllabus had a number of features that are being implemented today in a number of calculus reform projects.

The committee reports on its Calculus I syllabus:

Noteworthy features include emphasis on functions that are represented graphically (such as a curve on an oscilloscope or the Dow-Jones average) or numerically (from a table of data or a "black box" calculator), extensive use of hand calculators with "solve" and "integrate" keys, reduction in precalculus material and de-emphasis on limits and continuity, and a reorganization that allows treatment of all the elementary functions (including trigonometric, exponential, and logarithmic) from the first week on. (Douglas, 1986, p. vii)

In discussing the goals of the first year calculus course, the committee listed a number of objectives which echoed Steen's criteria for mathematical maturity. But significantly, the committee added as an objective that "students must be made aware that calculus is not taught for its own sake" (Douglas, 1986, p. ix). The past and present relevance of calculus to the way we see and deal with the physical world must be made clear to the student. To accomplish this, the committee included "honesty days" in the syllabus where time is incorporated into the course structure where the importance and relevance of the central concepts of calculus are made explicit.

One part of the conference was a workshop on teaching methods. After establishing the need for revision in the teaching methods employed in calculus courses and listing the goals for the course, the workshop committee begins a discussion of "What it Takes to Do the Job Right" (Douglas, 1986, p. xvi). Here the committee brings forward the idea of

the calculus course as being, in some sense, a language course (mathematics as the language of science). One of the goals for a calculus course is to develop precision in written and oral work. Meeting this goal requires substantial change in the tasks assigned students.

Another matter on which the committee strongly agreed was that mathematics learning at the freshman stage required extensive feedback on the work being done. The committee, however, qualifies its recommendations by stating pragmatically, "Unfortunately, many mathematics departments do not have the resources to teach the kinds of calculus courses described" (Douglas, 1986, p.xvii).

The teaching methods committee makes a comment towards the end of the report which seems particularly pertinent to my research project. They say:

Finally, we need to know more about what students learn in their mathematics classes. ... More detailed research on students' mathematics learning would be helpful, both to tell us about current difficulties in instruction and to suggest ways that might help us to improve. (Douglas, 1986, p. xx)

In his opening remarks at the conference, Douglas (1986) talks about three problems of calculus instruction. He says, "At most places, teaching calculus is viewed at best as an unwelcome chore by both young and old faculty...Few, if any, undergraduate students are inspired to major in mathematics

mathematics as a result of their calculus course" (p.18). Neither the instructor nor the student are finding the course enjoyable. Douglas argues that this is because of the "larger and different mix of generally less well-prepared students" taking calculus today and, secondly, calculus is taught "as though it could be learned passively with little contact between student and instructor" (p.19).

Epp (1986) identifies an aspect of the calculus course, often identified as the "rigorousness", as being a barrier to student understanding and enjoyment. Epp was responsible for developing a transition course in mathematical reasoning from calculus to higher-level mathematics classes. She says:

We had found that students were entering our higher-level classes woefully unable to construct the most simple proofs. I came to realize that many of my students' difficulties were much more profound than I had anticipated. Quite simply, my students and I spoke different languages. (p. 41)

Later, Epp (1986) notes that the calculus itself, with its many definitions, theorems, applications, notation, abstraction and logical complexity can be a formidable challenge to students lacking sufficient mathematical maturity (p. 46).

There is an "intellectual gulf" between calculus students and their instructors. Epp (1986) comments tellingly on two different responses to this gap.

Professors react to the gulf between themselves and their students in different ways. One reaction is to ignore it, to state the definitions and prove the theorems as if the students would understand them and were mature enough to be able to derive simple consequences (such as problem solutions) on their own....Another response, widely adopted today, is to expose the basic concepts of calculus at a moderately high level emphasizing intuition, but focus primarily on skills, and only test students on their ability to perform certain mechanical computations in response to certain verbal cues. In this approach the vast majority of students indulge their professors by listening quietly to their often inspired and beautifully intuitive explanations and the professors repay their students' courtesy by making their explanations brief, spending lots of time demonstrating procedures to solve rote problems, and never asking students to do anything on an exam that requires genuine knowledge of concepts. (pp. 47-48)

In this paragraph, Epp spells out one of the key conundrums facing the calculus reformers. Should calculus be taught at the level of student preparedness? In which case, perhaps only mechanical skills and "mimicry math" are possible. Or should standards be maintained and "survival of the fittest" be the rule of the calculus classroom? Epp maintains, "To a much greater extent than is currently the case, there is a need to respond to students' lack of sophistication, not by giving up but by helping them" (p. 50).

After offering a number of specific suggestions for modifying the current calculus curriculum and instruction,

Epp concludes that the implementation of her suggestions would make it impossible to cover all the standard topics. She echoes the conference theme, "If it comes to a choice, will we settle for superficial knowledge of a lot or deeper understanding of less? Perhaps less is more" (p. 58).

Peter Lax (1986a) emphasizes the applied nature of calculus. He speaks of the relevance of calculus to current areas of research in applied mathematics (Lax, 1986a, pp. 1-2) and how an emphasis on computing and applications can be used to enhance the teaching of calculus (Lax, 1986b, pp. 69-72). Calculus should be taught by those actively using it in their research. Numerical methods should have a more prominent role in the first course than the traditional "magic in calculus" techniques such as a number of methods of integration. The calculator and computer should be used to motivate and involve the student in the calculus classroom.

Peter Renz (1986a) addresses aspects of one of the key elements in the reform debate: the calculus text. Both the adherents of discrete mathematics and the calculus reformers agreed that the successful implementation of a revised curriculum hinged on the existence of "high quality texts that define the courses, that convince faculty of the merits of such courses, and that make it possible to teach the courses with predictable success using diverse faculty"

(Garland & Kreider, 1983,p. 209). Douglas (1986), speaking about existing calculus texts, says:

Everyone is dissatisfied with the current crop of calculus textbooks. Yet, if an author writes and manages to get published a textbook which is a little different, most colleges and universities will refuse to use it. How can we break out of this cycle? (p.13)

Renz speaks of the forces in academia which lead to uniformity in large courses such as calculus. Calculus curriculum is often determined by committees, which tend towards conservatism. As a service course for upper-level science and mathematics courses, freshman calculus must meet the perceived needs of many groups. The client faculties and courses which use calculus expect a "uniform and dependable product".

The textbook as a source of current dissatisfaction with calculus instruction and the textbook as a means of implementing curriculum revision are themes which continue throughout the calculus reform debate.

The last half of Toward a Lean and Lively Calculus is devoted to discussion papers. At this stage of the curriculum reform movement, many of the issues have been well established and already introduced into the debate. My review of this material focuses on identifying particularly lucid explanations and illustrative examples.

Renz (1986b) maintains that "it seems almost certain that tomorrow's calculus students will be less facile with

algebra and trigonometry than were the students for whom the traditional calculus course was designed" (p. 107). Paired with this lessening of skill in symbol manipulation is change in the students' attitudes. Renz (1986b) says:

My students today are less familiar with these basic algebraic and trigonometric formulas than were my previous students. Furthermore, today's students also seem less concerned that they be shown a coherent derivation of the results of calculus. ... They correctly infer that calculus itself must be correct because it seems to have worked well enough for generations of mathematicians, scientists, and engineers, and they see their problem as simply one of mastering the formulas of the subject and how to use them. (pp. 105-106)

Renz suggests that what is needed are "compelling ways to make the basic results of calculus directly obvious to the students" (p. 106).

He continues by distinguishing between slow calculus courses, a careful and rigorous introduction to the mathematical foundations of calculus, and a quick calculus which gives the students necessary techniques and concepts for studying engineering and the physical sciences. Renz argues that there is not enough time in an introductory calculus course to cover the methods and applications demanded by the client departments and to give a rigorous basis for the techniques covered. Each course, slow and quick, are appropriate for certain student populations, but in general, Renz (1986b) maintains that "every effort should be made to find less complex and subtle assumptions to

underlie the methods of calculus taught in the introductory course" (p. 109).

Stephen Rodi (1986) views the problems with calculus instruction as resulting from benign neglect of undergraduate teaching by mathematicians, rather than inappropriate textbooks. Rodi is of the opinion that quality of teaching is not a priority in many university mathematics departments.

Rodi (1986) comments on what the role of applications in introductory calculus courses should be. He says:

A real danger when research mathematicians teach calculus to freshman and sophomores is forward projection: the tendency for the mathematician to look ahead to the next generalization (the class of Riemann integrable functions as a precursor of Lebesgue integration) rather than look back to the kind of concrete problems which gave rise to calculus in the first place and which are necessary for student understanding. Insistence on dealing with practical applications purposely woven into the course can offset that bias. (p. 123)

Rodi concludes his paper by cautioning against over zealous pruning of the calculus course. A more compact calculus course would end up as "a sort of highlights of calculus" which would not foster real intellectual growth in the student. In particular, he warns against relegating all computational techniques to calculators and computers.

Rodi (1986) argues:

In the rush to pre-packaged programs, we need to be careful not to toss out the baby with the bath water. In devoting two or three classes to methods of integration, one is doing far more than developing a mechanical skill. The real, perduring learning here is the analysis of patterns, the recognition of the structure of integrands, the ability to choose the proper tool (be it substitution or partial fractions) to attack the problem. (p. 127)

Small and Hosack (1986) write about the potential of computer algebra systems (called symbolic manipulation programs elsewhere) in changing the teaching and learning of calculus. The authors maintain that routine algorithmic manipulations are emphasized in the traditional mathematics curriculum at the expense of conceptual understanding. Computer algebra systems allows for several improvements in this situation. Students using such programs would have more time to concentrate on problem solving processes. They could work on a greater variety and quantity of questions. The graphing capabilities of such systems would allow the graph to be used in the analysis of a function, rather than having the graph generated through the analysis of the function. Use of computer capabilities allows the student to develop an experimental attitude towards open ended problems. The amalgam of the possibilities listed would change student perceptions about what is important in mathematics and what it is to do mathematics.

Many students are unsuccessful in a calculus course because of their lack of proficiency in algebraic skills. In the opinion of the authors:

[Computer Algebra Systems] can offer these students the opportunity to comprehend and work with the concepts of calculus at a meaningful level. Furthermore, their work with the calculus may provide the necessary motivation for them to remedy their algebraic deficiencies. (Small & Hosack, 1986, pp. 151-152)

Stevenson (1986) writes about what the physical science "clients" might want in a revised calculus curriculum. He notes that "physics has a significant difference from mathematics in that it is an empirical science and cannot stray too far afield from observations" (p. 182). The mathematics needed by the physics student is the type most likely to be espoused by the applied mathematician. Stevenson (1986) speaks in favour of "the presence of the geometrical interpretation of calculus with frequent reference to applications" (p. 183). As well as being able to handle "sophisticated mathematical analysis and abstract thought", the physical scientist must have a well developed geometrical and intuitive feeling for mathematics.

Stevenson offers several suggestions for making the calculus a better course for science students. The faculty responsible for teaching introductory calculus should be extended beyond the mathematics departments to "applied

mathematicians" who may be employed in industry or come from other university faculties such as engineering and physics. Calculus must be presented through both a pure mathematics approach and applications. Stevenson recognizes that "a danger does exist in stressing applications and intuitive understanding to the exclusion of the beauty of mathematical logic" (p. 191). Calculus content should be determined by mathematics faculty only after "honest dialogue with other disciplines including the physical sciences" (p. 192). Also, textbook material must be modified. Stevenson argues for modular textbook and enrichment material "written by faculty with more interest in teaching than in research" (p.192).

Ash, Ash, and VanValkenburg (1986) speak about calculus for engineers and how ineffective introductory calculus courses are in teaching engineering students how to use mathematics. They identify as a primary culprit the emphasis on formalism. Students, they maintain, "simply will not learn the formalism" found in most textbooks; "the better students will succeed in reading around the abstractions, so that the textbook at least becomes useful as a source of examples" (p. 227).

Steen (1986) outlines many of the primary issues of the calculus reform debate in a list of questions. Among the issues are the idea that perhaps too many students are studying calculus; students should have prerequisite skills

before studying calculus; calculus is used, perhaps not always appropriately, as a filter for professions; students are not really taught the essential ideas of calculus, but are offered cookbook courses which over-emphasize closed-form formulas. The calculus curriculum must react to the impact of computers on what is appropriate in mathematics, what topics are appropriate for calculus and what activities are appropriate for the students. What role does the textbook have in the teaching and learning of calculus? Should calculus be a course to train the mind; convey cultural and/or scientific literacy; or to prepare students for client disciplines. The teaching of calculus can be regarded as a laboratory course or a foreign language course. Who should teach the calculus and from where do these qualified instructors come? What will the calculus be like in the year 2000?

At the end of five days, the twenty-five participants in the Tulane Conference had established the parameters of the calculus reform debate. Momentum for reform increased dramatically, resulting in a national colloquium with over six hundred participants in October of 1987.

The Calculus for a New Century Colloquium

The National Colloquium on Calculus for a New Century was held at the National Academy of Sciences in Washington, D.C. on October 28-29, 1987. Over six hundred professionals

with a stake in calculus instruction participated. The calculus reform movement was very much on the main stage of North American educational issues.

Peterson (1987) reported on the conference:

Calculus is big, important - and in trouble. This was one of the messages that came out of a recent conference at the National Academy of Sciences in Washington, D.C. on the future of calculus education. The meeting attracted more than 600 mathematicians, educators and other professionals worried about the state of calculus teaching. The large attendance reflected a growing feeling that something ought to be done to reform the way calculus is taught. (p.317)

The colloquium did not of itself raise new issues in the calculus reform debate. Many of the arguments from the Tulane conference were echoed and refined. Supporting statistics were provided in some instances. But the essential parameters of the debate had been established earlier.

Following is a brief review of those aspects of the conference that were most relevant to my study.

Tucker (1988) spoke about the calculus class as a "captive audience" from which the mathematicians and scientists of tomorrow can be identified and cultivated. Tucker projected, however, that without improvements in calculus instruction the teaching of calculus would move away from college mathematics departments to secondary school classes and to client disciplines. Special mention was made

of The Calculus Tutoring Book published by the Institute for Electrical and Electronic Engineering, a calculus textbook written by engineers for engineering students.

A substantial part of the colloquium was devoted to the views of client disciplines. Morawetz (1988) set the tone for this discussion by suggesting the slogan, "Everyone who can learn calculus *should* learn calculus" (p.18). Representatives from engineering, biology, management sciences, the physical sciences and physics itself all agreed on the importance of calculus in their disciplines, but also argued unanimously for the need for change.

Starr (1988), a historian, espouses the view that calculus is part of the core learning that every educated person should have. In response to the question "Why study calculus?", he replies to a student, "If Newton could invent it in his sophomore year, [you] can study it in [yours]" (p. 35). Starr suggests, "Calculus has to do with thinking, with concepts, with the core of a liberal education" (p. 35).

The Calculus for Physical Science discussion group debated the role of intuition, calculation and proof in the first semester of calculus. Epsilon-delta proofs were almost universally targeted for elimination. A reduction in derivative calculations and an increase in work on qualitative examples and numerical methods was encouraged. The role of proof, either formal or intuitive, became a point

of debate and no consensus was reached. Technology, the group decided, was here to stay. It should be used but should not completely replace pencil and paper exercises. Retraining teaching faculty to use technology was a concern.

The Calculus for Engineering Students group was particularly concerned with the diversity of incoming students. Pre-testing and placement programs were thought to be reasonable and effective measures. Pre-calculus courses were preferred over splitting into "hard" and "soft" calculus courses.

Groups for the Life Sciences, for Business and Social Science Students and for Computing Science spent much of their time discussing what type of mathematics was appropriate and necessary for their disciplines. The feeling was that maybe calculus was not the mathematics prerequisite needed for their courses. One group noted that they were "with few exceptions, composed of college and high school mathematics faculty. As "providers" rather than "users" of calculus courses, the group sensed a need to learn from social scientists what, in fact, their students require in their mathematics courses" (Brito & Goldberg, 1988, pp. 69-70).

The size of calculus classes and who teaches them were issues that had been discussed from the beginning of the reform debate. One discussion group spent their time

examining the role of teaching assistants in a "lean and lively" calculus. Another group addressed the issues involved in helping students from minority groups to become more successful in their studies. A third group pointed out that there was no provision at the colloquium for discussion of research and development on the teaching and learning of calculus.

Lathrop (1988) adds to the topic of what calculus should be taught to engineering students by discussing two variations of what he calls the "chicken and the egg problem". Variation one is that engineering and applied physics are best taught using calculus. But, Lathrop maintains, calculus is best learned by students working through examples in subject areas which require a prior understanding of the calculus. A second timing problem occurs when first year calculus students are subjected to an emphasis on proving theorems and studying pathological functions. These topics are not needed immediately by engineering students. Lathrop suggests a much greater degree of coordination between the mathematics and physical science courses taken by the students.

Two articles, "Calculus Reform and Women Undergraduates" (Hughes, 1988) and "Calculus Success for All Students" (Malcolm & Treisman, 1988) add to the general reform discussion in that underlying causes for failure and

strategies for success that are uncovered for minority groups tend to be applicable to the student population at large. There is a shift in these accounts from a Darwinian "survival of the fittest" classroom to a learning environment which promotes success.

Anderson and Loftsgaarden (1988) write about the "typical" calculus courses which are taught at large universities. He bases his descriptions on data on a 1985 survey on undergraduate programs in mathematics conducted by the M.A.A. The data shows that "calculus is overwhelmingly the dominant mathematics course taken by undergraduate students in universities" (p. 159) The typical student learning experience is described as follows:

Student learning procedures consist primarily of working textbook problems - usually several or many of the same sort - following model procedures given in the textbook or by a teacher. Thus students learn various pencil-and-paper algorithms for producing answers to special types of problems. They customarily read the text only to find procedures for working such problems.

Student dependence on memorized procedures to produce answers follows a similar pattern of learning in pre-calculus mathematics. In calculus, however, it requires much wider and more readily recalled background information.

The intellectual achievement for most students in learning calculus is, nevertheless, considerable. They have had to learn about new concepts much more rapidly than in earlier courses, and they have had to develop command of a wider and more diverse "bag of tricks". . . . Unfortunately, many of the tricks in the calculus bag appear irrelevant in an age where the computer and the calculator are rapidly replacing paper and pencil as tools of the trade. (pp.159-160)

The 1985 survey indicated that considerably less than 10% of the calculus courses offered at university made any use whatsoever of computers.

Anderson and Loftsgaarden identify a number of factors which might militate against curriculum reform. Among these factors two proved to be of great significance to my research. First, students are taking the course primarily to gain access to the subjects they really wish to study. Second, changing the calculus course from a procedures course to a concept course will make it more challenging for many students and will generate student resistance.

Implementing Calculus Reform

Following the "Calculus for a New Century" symposium significant amounts of "seed money" were made available through the National Science Foundation to facilitate new calculus projects. In September, 1989 the Calculus Reform and the First Two Years subcommittee of the M.A.A.'s Committee on Undergraduate Programs in Mathematics selected ten projects on which to report.

These reports, abstracts of about seventy other projects and references pertinent to calculus reform issues were published in *Priming the Calculus Pump: Innovations and Resources* (Tucker, 1990). Following is a review of three of the ten project reports.

The project at Clemson University followed a traditional calculus syllabus using a standard, widely adopted text. The innovation in the Clemson program was the regular, frequent use of a "super-calculator", the HP-28S by the students. The primary concern of the project leaders is "in increasing student interest, involvement, comprehension and retention of the subject matter" (LaTorre et al., 1990, p.12).

In the single-variable calculus course, little was done to "reform the curriculum". The course features extensive evaluation and feedback (daily quizzes, five hour tests and a final examination). The calculators are used throughout the classes and without restriction on the tests. The project leaders report that, "The classes are lively and the students are involved" (LaTorre et al., 1990, p.14).

The reform project at Dartmouth College (Baumgartner & Shemanske, 1990) does not involve radical changes in course content or presentation. What is innovative in the Dartmouth approach to calculus is the use of computers.

Students in this project are expected to gain some facility as computer programmers, using the computer as a problem solving tool.

The project instructors discuss two features critical to their approach to the use of computers.

The first is that the computing which we do is an integral part of the course. We do no demonstration just to do something on the computer. ... The second is the ability to respond to the student questions on the fly. The students see that we pick up the computer as they would a calculator, write a quick and dirty program, and get some insight into a problem. Hopefully, they are convinced to do the same. (Baumgartner & Shemanske, 1990, p.37)

The project leaders have attempted to measure student attitudes towards the use of programming in calculus courses. Student responses varied from thinking that the computing helped their understanding of concepts greatly to those who complained bitterly about having to program in a mathematics class.

Interestingly, there is anecdotal evidence that students who have taken a traditional calculus course in high school find this new approach more work than those without any calculus background.

The project at Purdue University is particularly concerned with pedagogical reform. The project report authors state:

The centerpiece of our project philosophy is the development of an emerging theory of how students learn mathematics. According to this theory, students need to construct their own understanding of each mathematical concept. Hence, we believe that the primary role of teaching is not to lecture, explain or otherwise attempt to "transfer" mathematical knowledge, but to create situations for students that will foster their making the necessary mental constructions. (Schwingendorf & Dubinsky, 1990, p. 176).

The project developers worked closely with client departments, talking with faculty and reviewing their textbooks, in order to determine what calculus was actually used in these disciplines. Experimental sections were established for two student populations: those in engineering, mathematics and physical science majors and those taking management, social science and life science courses. Rather than complain about the lack of homogeneity in their student populations, the project developers claimed, "Variations in student attitudes are helping us to synthesize a robust approach to teaching and learning calculus" (Schwingendorf & Dubinsky, 1990, p.177).

The classroom focuses on a "Socratic lecture" approach in which small groups work on problems and bring their thoughts back to the group as a whole. Students are responsible for a number of "pencil and paper" homework assignments. There are class tests and a final examination. The tests and examinations are open-ended with respect to time and the questions involve many that are non-routine, some of which are of the essay type. Teaching assistants are present not only in the computer laboratory, but also in the classes. The project report authors comment:

The situation often becomes one of students and TA's working together to figure something out. We feel that this is an important part of the overall atmosphere of our course. (Schwingendorf & Dubinsky, 1990, p. 183).

The calculus reform projects reviewed in *Priming the Calculus Pump* display a wide variety of responses to the issues raised throughout the decade. Technology impacts on both pedagogy and curriculum content. In some venues programming is deemed to be an essential element of the learning process (Baumgartner & Shemanske, 1990; Brown, Porta and Uhl, 1990); at other campuses computers are simply tools for accomplishing specific tasks (Callahan et al, 1990; Höft & James, 1990; Ostebee & Zorn, 1990). For some, computers were available only at specified times in specified locations and used only for specified tasks (Schwingendorf & Dubinsky, 1990). For others, computers were present in all aspects of the course (Brown, Porta and Uhl, 1990). The technology employed ranged from hand-held super-calculators through to networked work stations employing very sophisticated software. The intent of integrating technology into the calculus course ranged from allowing computer demonstrations in the lecture theatre through laboratory projects which allow students to explore and discover mathematics to a complete redefinition of the role and nature of the textbook.

Content reform ranges from supplementing the traditional curriculum with new materials such as projects and laboratories through "lean and lively" curricula which dropped topics in order to gain the time necessary to teach conceptual understanding to a complete rescheduling and

reworking of the standard topics. Content reform was decided upon sometimes individually, sometimes at the departmental level, and, in some cases, after extensive consultation with client departments.

Classroom procedures in some projects remain relatively unaltered. Elsewhere, group work and student exploration overtakes the "transmission of knowledge" format. Student evaluation now includes significant writing in submitted and revised laboratory or project reports. Challenging tasks are set where students may use a variety of resources. Expectations from the instructors are higher.

Content and pedagogy changes are supported by off-the-shelf textbooks, locally developed assignments and exposition, and software extensions. Future plans for many of the projects include the development of appropriate text materials.

Many of the projects deal with specific student populations: high school students who had done some work in calculus; students in engineering, mathematics and the physical sciences; students in management, social and life sciences. Minority groups and women are not specifically targetted for any project.

Student reactions to the experimental courses vary greatly. Some resent the increases in work, degree of difficulty, and in time commitments. Others value the

perceived increase in understanding, the opportunity for personal involvement in the work, and the non-standard approaches to instruction and learning.

All of the projects are by their nature more labour and resource intensive than traditional calculus courses. Reform projects will have to be intensively and critically assessed before they can be successfully sold to the mathematics community on the basis of their merits.

Current Research Perspectives on Teaching and Learning Mathematics

In the latter half of the 1970's there began a shift in attitudes in the mathematics and science education research communities as to what constituted appropriate research problems and methods. Prior to this shift, mathematics education researchers had followed the lead of other educational researchers by using the statistical model for their studies. The statistical model was adapted from experimental, quantitative studies done in biology and psychology (Sowder, 1989, p. 13). There has been a general sense of disappointment with the results of these studies which have been characterized as "high-rigour, low significance" research whose results lack relevance to the classroom teacher. (Howson & Wilson, 1986, p. 85; Sowder, 1989, p. 17).

The epistemology underlying these investigations was essentially positivism: variables were isolated and manipulated systematically to demonstrate and/or verify an immutable, objective causality among elements in an atomized learning scheme. This choice of research model was dictated more by a desire to remain within the tradition of the educational research community than by a commitment of individual researchers to specific conceptual frameworks.

As researchers focused more on the theoretical constructs which directed their investigations, the nature of the research changed. Sowder (1989) describes the motivation for this shift in research paradigms.

These conceptual frameworks have originated with European researchers and educators, who have a greater tendency than Americans to develop elaborate theoretical discussions that emphasize interpretive understanding. Mathematics educators have been attracted to more interpretive research approaches, in which investigators do not attempt to stand apart from the educational encounter so as to make an objective assessment of what is happening. Rather, they enter the classroom and try to participate in that encounter so that they can capture and share the participants' understanding of what they are teaching and learning. Along with the view of research as interpretive understanding has come a greater interest in the use of qualitative research methodologies borrowed from anthropology and sociology. Much of this interest arises as a reaction against the behaviourist program of scientific inquiry that avoids interpreting overt behaviour in terms of internal cognitive processes. Much of it also comes from disillusionment with the generalizations produced by the behaviorist program and their lack of relevance to the work of mathematics teachers. When researchers participate in the educational encounter, they enter into a dialogue with teachers and students that can begin the process of making their concepts and findings available to practitioners.

Qualitative research methods that do not depend on esoteric statistical manipulations of data appeal to researchers today and also do not seem so intimidating to teachers. (pp. 16-17)

Silver (1990), commenting on the relationship between educational research and actual educational practices, speaks of two perspectives that have gained recent adherents among mathematics education researchers:

Another theoretical perspective that has permeated the mathematics education community is a very general form of constructivism in which it is acknowledged that students actively and personally construct their own knowledge rather than making mental copies of knowledge possessed and transmitted by teachers or textbooks. (p. 7)

Later, he adds:

In recent years another research perspective has emerged that may be useful to practitioners - namely, apprenticeship. The apprenticeship notion emerges from the literature of anthropology. The apprenticeship view also suggests that a major purpose of school mathematics is to develop in students the habits of thinking and the points of view of professionals in the field; that is, the goal is learning to think mathematically. (p. 8)

Mathematics research has opened itself to the possibilities of qualitative, interpretive studies. The learner is less the recipient of knowledge than an apprentice who moves from novice to expert through the personal construction of concepts. A third element in recent research is the contextualization of learning, the social and cultural influences on teaching and learning.

D'Ambrosio (1985) speaks of ethnomathematics as a subject that "lies on the borderline between the history of

mathematics and cultural anthropology" (p.44). He distinguishes between mathematics taught in school and that which is actually practised outside of school.

Mathematics is adapted and given a place as "scholarly practical" mathematics; i.e., the mathematics which is taught and learned in schools. In contrast to this we will call *ethnomathematics* the mathematics which is practised among identifiable cultural groups. (D'Ambrosio, 1985, p. 45)

Later, D'Ambrosio (1985) adds a comment of particular significance to this study.

We may go even further in this concept of ethnomathematics to include much of the mathematics which is currently practised by engineers, mainly calculus, which does not respond to the concept of rigor and formalism developed in academic courses of calculus. (p.45)

Large-scale, naturalistic studies of science education were conducted as early as the mid-1970's. *The Case Studies in Science Education* by Stake, Easley and associates (1978) is a notable example. The authors did, however, spend a great deal of time explaining and defending their research methods.

Smith (1982) and Rist (1982) wrote articles on the appropriate use of qualitative research in the sciences. By 1987, qualitative research had become mainstream in educational research. Smith (1987) writes:

This paper constitutes a slight departure from editorial policy for AERJ [American Education Research Journal]. Far from contributing to general knowledge through empirical analysis, this paper is meant to serve a self-referent and practical purpose. It is meant to signify to the discipline that manuscripts based on qualitative research are being welcomed by AERJ editors. (p. 173)

Although qualitative research activities have gained general acceptance in the mathematics education research community, there is some question of how much research of this nature has actually been undertaken. Silver (1990) qualifies his comments on the extent to which constructivism has taken hold by saying, "many would agree that a deep appreciation of the constructivist approach has yet to take hold in the larger mathematics education community" (p. 7). Eisenhart (1988) comments specifically about ethnographic research.

During the past 10 years, there has been considerable discussion in the educational research community about the value of ethnographic research. Although the discussion has increasingly cast ethnography in a favorable light, there remain clear differences in the research activities of ethnographers and educational researchers. Relatively few educational researchers have actually undertaken ethnographic research. (p. 99)

Further on, she comments specifically on mathematics education research.

Numerous mathematics education researchers (I am thinking particularly of constructivists, of those interested in what teachers or students are thinking and actually doing in classrooms, and of those interested in the social context of mathematics education) are posing questions for which ethnographic research is appropriate. However, these researchers tend to use case studies, in-depth interviews or in-classroom observations without doing what most educational anthropologists would call ethnographic research. (Eisenhart, 1988, p. 99)

Ferrini-Mundy and Graham (1991), writing specifically on research on calculus learning, substantiate Eisenhart's thesis. They claim that "most studies in calculus have been

large-scale, quantitative attempts to model how attitudinal and experiential variables affect overall performance" and that most of these studies "have had minimal influence on understanding or performance" (p. 629). Constructivism and qualitative research are referred to favorably. "The use of qualitative techniques allows the researcher to generate a more viable model of the student's learning process than the more traditional quantitative methods" (p. 629). The qualitative research techniques listed, however, are the clinical interview, the teaching experiment, and the analysis of student errors. These techniques do not yield a "holistic depiction of uncontrived group interaction over a period of time, faithfully representing participant views and meanings" (Goetz & LeCompte, 1984, p. 51) that one could obtain using ethnographic methods.

The ethnography of a calculus class is a reasonable project, but it had yet to be done.

A Search for Prior Related Studies

The literature suggested that such qualitative studies as had been done on the learning of calculus had focused on individuals in isolation rather than on learners in the context of the micro-culture of university students. Over the course of this study, I undertook several literature

searches to verify that no research similar to mine had already been undertaken.

In November, 1990 I performed an ERIC search through the computing services at Simon Fraser University using the single descriptor Calculus. None of the articles retrieved in this search reported on a naturalistic description of student experiences. The ERIC search was updated manually in August, 1992 with similar results.

A review of Dissertation Abstracts International from 1980 to 1991 was conducted in August, 1992. The descriptors used in this search were Educ-Calculus-Ethnography-Math. Several dissertations had indeed been published reporting research using ethnographic methods to study college and university classes. (Contreras, 1988; Cooper, 1979; Dodge, 1983; Jorde, 1985; Kim, 1990; McLaughlin, 1986; Nuernberger, 1983). None of these studies dealt with calculus instruction and in none of the studies did the researcher become involved in long term participant observation.

The most recent literature search was a review of mathematics education research reported in 1992 (Suydam & Brosnan, 1993). Again, there were no studies reported that intersected to any significant extent with my study.

CHAPTER III

RESEARCH METHODS

Introduction

In this chapter I discuss the rationale for choosing to use ethnographic methods in this study. Then there is a review of the type of data accumulated in my research and the methods use to obtain them. The procedures for analyzing the data are discussed next. In the last section of this chapter, I discuss the reasons for representing the analyzed data in the manner I have in Chapter IV.

Selection of a Research Method

My choice of a research problem for this project was influenced by my feelings for the significance and beauty of the subject of calculus, my perception and concern that students beginning the study of calculus at the university were not given experiences which would encourage them to continue their studies and deepen their understanding, and by my changing ideas of what mathematics learning could and should be.

My review of the literature shows that much has been written about what is perceived to be wrong with university calculus courses and that many prescriptive measures for improving the situation have been suggested. The voice of

the student, however, is one which is not being heard in the current calculus curriculum reform debate. There is general discussion of lack of preparedness on the part of the students, not meeting the needs of the students, the dissatisfaction of the students. But I find little evidence that there has been much consultation with the students themselves. The success of any curriculum reform, however well intentioned and reasoned, will depend on the students. This study would begin articulating the student perspective. With this knowledge, the curriculum reformer would have a better sense of what should and could happen in the calculus class.

At roughly the same time as the "Crisis in Calculus" debate was occurring, there was a shift in the accepted paradigms of mathematics education research. Mathematics education research had been hitherto dominated by methodologies described as quantitative, positivistic and formalistic and which parallels work done in biology and psychology. More recent studies have employed the methods of anthropologists. Such methods have been called naturalistic, descriptive and interpretive. I have come to share the distrust of many in the possibility of understanding a complex cultural setting through the isolation, manipulation and quantification of prespecified elements of the situation. Because a culture is made of people, understanding is only as

complete as one's understanding of the views and meanings of the members of the culture. I intend that this study will be interpretivist research. About this, Eisenhart (1988) says:

The purpose of doing interpretivist research, then, is to provide information that will allow the investigator to "make sense" of the world from the perspective of participants; that is, the researcher must learn how to behave appropriately in that world and how to make that world understandable to outsiders, especially in the research community.
(p.103)

New theories and perspectives on learning mathematics were also evolving contemporaneously with the use of anthropological research methods. The view of the mathematics learner as the recipient of objective truth was being supplanted by the ideas of constructivism and ethnomathematics. My choice to use ethnographic methods in this study is based on these recently adopted perspectives on the nature of knowing and learning. The student is not a receptacle of objective knowledge, but does construct meaning from the activities associated with the classroom/lecture hall. The student does not learn in isolation. There are social interactions within the classroom and outside which profoundly affect the student's construction of knowledge. Also, there is evidence that the student's conception of what is occurring throughout a course differs greatly from that of the instructor (Sowder, 1989, p. 26). The goal of mathematics education, implicit or explicit, is to facilitate

the entry of students into the mathematical community at some level; to help the students become mathematicians.

Before becoming prescriptive, I believe it is important to understand. Eisenhart (1988) contrasts the traditional motives of mathematics education research : "How can mathematics teaching and learning be improved?", with those of educational anthropology: "Why is mathematics teaching and learning occurring in this way in this setting?" (p. 100). My contention is that these are not separate questions, but that the "why" question is prior to the "how" question.

An ethnographic study of a calculus class would "provide rich, descriptive data about the contexts, activities, and beliefs of participants" (Goetz & LeCompte, 1984, p. 17). This data would be of use to researchers and teachers, such as myself, in determining how to improve and what to improve in the calculus classroom.

Data Collection

Eisenhart (1988) identifies four methods of data collection used by ethnographers: participant observation; ethnographic interviews; search for artifacts, and; researcher introspection. I have attempted to use all four methods in this study.

Participant Observation: Preliminary Considerations

The literature on ethnographic research methods generally agrees that researcher involvement in the field (the culture under study) as a participant observer is the primary means of data acquisition. Becker (1958) describes the work of the participant observer in the following way.

The participant observer gathers data by participating in the daily life of the group or organization he studies. He watches the people he is studying to see what situations they ordinarily meet and how they behave in them. He enters into conversation with some or all of the participants in these situations and discovers their interpretations of the events he has observed. (p. 652)

Eisenhart (1988) cautions about the inherent tension between participation and observation.

Participant observation is the ethnographer's major technique for being both involved and detached from the topic of study. Participant observation is a kind of schizophrenic activity in which, on one hand, the researcher tries to learn to be a member of the group by becoming a part of it and, on the other hand, tries to look on the scene as an outsider in order to gain a perspective not ordinarily held by someone who is a participant only. (p. 105)

I discovered during the fieldwork portion of the study that the researcher's location on the observation-participation continuum is not static, but evolves over the course of the study.

Another aspect of being a participant observer to which the ethnographic fieldworker must pay particular attention is the role of the researcher at any given time and situation. Goetz and LeCompte (1984, pp.93-106) detail the issues that must be addressed. Among the multiplicity of roles I assumed in this study I was a graduate student completing research for my thesis; a secondary school instructor who was teaching material identical to that in the course under observation; a "fellow student" in the course completing assignments and taking tests; a resource for students who wanted help with their work in the course; a personal confidant for several of the students, and; a "gadfly" encouraging students to explore the capabilities of advanced handheld calculators in learning the material in this course. It was essential for me throughout the study to try to identify which role(s) I was accommodating when I made a particular observation.

The role in which I found myself in a given situation also impacted on the role of the participant(s) with whom I was dealing. "Ethnography is one of the few modes of scientific study that admit the subjective perception and biases of both participants and researcher into the research frame" (Goetz & LeCompte, 1984, p.95). The interactive and subjective nature of participant observation became abundantly clear to me as the study progressed. Goetz and LeCompte (1984, pp. 99-101) talk about the need for an

ethnographer to develop the facility to move from one set of roles to another, as is appropriate. The authors speak of boundary spanning as the ability to participate actively in the various cultures that necessarily impinge on the research. In this study, boundary spanning skills were particularly significant because of the multiplicity of backgrounds the participants brought with them.

Exactly what data was I supposed to obtain as a Participant observer? Goetz and LeCompte (1984) described my situation quite accurately.

Researchers unfamiliar with ethnography often express dismay at the prospect of attempting to record everything happening in a social situation, cultural scene, or institution group. Likewise, novice ethnographers express frustration at their inability to "get it all down." (p. 111)

Goetz and LeCompte follow this statement by admitting the goal of "getting it all down" is unrealizable and by providing a framework which I could use in focusing my observations. Intrinsic to the nature of ethnographic research and its purposes is that some of the areas of observational focus that I began with would change as I learned from my fieldwork experience. This was indeed the case. Some observational themes remained constant in my fieldnotes throughout the course. Other themes disappeared and were replaced by questions I had not considered prior to the study.

Participant Observation : Pilot Project

By the end of July 1991, I had defined a research problem which was of great personal interest to myself and fit in well with current trends and issues in mathematics education research. I was convinced that the use of ethnographic methods in my study offered me the greatest opportunity for accomplishing the goals I had for my research. Through my work in two courses in the Faculty of Education at Simon Fraser University, Foundations of Mathematics Education (Education 846) and Qualitative Methods in Educational Research (Education 867), I had developed a background in the theoretical whys and wherefores of ethnographic research. I had, however, no practical experience whatsoever in undertaking a venture of this sort. With this in mind, I determined that I would "practice" fieldwork by observing several sessions of a calculus class then in session.

With the help of my thesis advisor, I secured the permission of the Mathematics Department to act as an observer in this class. This practice fieldwork occurred on four separate days. (See Appendix A for details.) It allowed me to experience the complexity and richness of the observation process and gave me some initial insights into the challenges of balancing roles as a participant observer.

At the same time, I acquired data that I would use to compare with and contrast with fieldwork on my actual project in the fall.

A more detailed account of my experiences during this pilot study is given in Chapter IV. Following is a brief review of my activities during those two weeks.

In the first fieldwork session, I attended a lecture and made field notes. In this instance, I was purely an observer and had yet to introduce myself to the instructor or students.

Even without direct interaction with students and instructor, I found the task of meaningful observation to be daunting. I found that my observations slid from physical layout of the lecture theatre to overt behaviour of the participants (instructor and students) to appearance and body language of participants to lecture content to teaching style. Two aspects of this study became apparent. First, I would continuously need to use "boundary spanning" skills in this study. I teach calculus at a high school and my perception of what was occurring in a calculus class would in all cases be affected by that particular researcher role. Secondly, the lecture format of instruction does not lend itself to a great deal of participant interaction. If I was to make sense of the calculus student's experiences, I would have to be assiduous in my observations and aggressive in

establishing relationships with the participants. The second consideration impacted on the roles I sought to establish for myself in the primary project.

After the first lecture, I introduced myself to the instructor and we had a brief discussion. Although the texts on ethnographic methods stressed the importance of taking notes during or immediately after such a conversation, I failed to do so. Consequently, I retained none of the data from this conversation except for two of the topics discussed.

In the second lecture I attended, I continued to work on improving my skills in taking field notes. I experimented with simple coding schemes for recording notes in realtime.

After this lecture, I spoke again with the instructor. This time I made a brief set of notes after the discussion. I also went to the Calculus Lab to observe and to reflect.

In the third lecture, I experimented with the use of an audio tape recorder. This use of technology allowed me to record more data than before, but it also took up a considerable amount of my attention. Taking care of the equipment after the lecture also prevented me from talking with the instructor. The use of technology in the field alters the role and capabilities of the researcher. My experience here made real for me the truth of Wolcott's dictum that the ethnographer is first and foremost "the

research instrument" in the field (Wolcott, 1980). Goetz and LeCompte (1984) expand on this point.

Ethnographers are data collectors, or "sensitive observers, storytellers, and writers"; like human videotape recorders, the record as faithfully as possible the phenomena they see. They may use a variety of aids to assist them, including still and movie cameras, and audio and video recorders. However, unlike mechanical recording devices, which only record what they are aimed at without abstraction or analysis, ethnographers are able to go beyond a "point and shoot; no focusing necessary" approach to recording and to raise finely tuned questions, test out hunches, and move deeper into analysis of issues. (pp.101-102)

After the third lecture, I again went to observe in the Calculus Laboratory. This session in the Calculus Lab was particularly significant because a student initiated a conversation with me. I had been identified as an "expert" and my help was sought. In this way contact with other participants was established.

During the last lecture, I again experimented with taping the lecture. I accomplished little in this session save that I became more comfortable with the audio recorder.

Armed now with at least a modicum of experience in field observation, I felt somewhat more confident in beginning the study proper.

Participant Observation : Math 151

By the end of August 1991, a first semester calculus course had been chosen, in consultation with my senior advisor, and the approval of the department and instructor secured. I became a participant observer in the role of a student in this course. The selection of this course was based on the following criteria. 1) The time and location of the course made it possible for me to attend all classes and tutorials. 2) The course is a "mainstream" calculus course and is hopefully representative of the courses that are the focus of the calculus reform debate. 3) My senior advisor has taught this course before and would be able to use his experience in evaluating the validity of my reporting. 4) The course instructor had indicated a willingness to participate in this study.

Because I continued to work full time as a high school teacher throughout this project, there were constraints on which course I could use as a setting for my study. This particular course was chosen because the location and scheduling made it accessible to the researcher, rather than for criteria based on research design.

The Math 151 class chosen was atypical of Math 151 classes at the university in several respects. In this class there were about 20 students. Classes at the Burnaby Mountain campus would have several hundred students. Although

some students in the study group had just graduated from high school, the majority of the students had been away from high school mathematics for at least several years. All six students whom I interviewed were "mature" students, that is, they were not matriculating into university directly from high school. This section of Math 151 offered a one hour tutorial session with a teaching assistant each week. In contrast, students in sections taught on the main campus had the opportunity to attend a Calculus Lab where they could work together and ask teaching assistants questions during any of the many hours it was open each week.

A chronology of my activities as a participant observer in this class is listed in Appendix A. Details of what I observed, experienced and thought during my time in the field are covered in Chapter IV. Following is a brief description of my activities as a participant observer data gatherer and of the roles that I assumed in the study.

Prior to the beginning of the course, I met briefly with the instructor and discussed what I hoped to accomplish and how I intended to do so. I acquired a course outline (Appendix B), determined the times and locations of the lectures and tutorials and purchased the textbook for the course.

The course was in the evenings (Tuesday and Thursday) at the Harbour Centre Campus of Simon Fraser University. I

arranged to stay late at the school where I was teaching and to drive directly from work to the Harbour Centre Campus.

The first evening I arrived early. After locating the lecture room, I spent time mapping the room and observing the gathering of students. My field notes for the first lecture show that the "focusing and bounding" of my data collection was dictated by my background as a classroom teacher, the "crisis in calculus" issues I had encountered in my review of the literature, any student background information I could glean from overheard conversations and any overt behaviour that attracted my attention. The students were not aware of my role as researcher. There was no interaction of myself with the students this first evening; I was an observer.

At the end of the second lecture, the instructor introduced me to the class. I spoke about my project and handed out the Informed Consent by Subjects to Participate in a Research Project I had devised in compliance with the university's Research Ethics Review policies. (See Appendix C.) At this point I was identified as a graduate student doing research and as a calculus "expert". This identification prompted a conversation with a student after class and began my active, rather than passive, involvement in the social matrix of the class.

On the Sunday of the following weekend, I began another aspect of my participant role in the study. I completed the

assignment given to the class on Thursday. There were several reasons for doing this work. Although I was unlikely to find the problems difficult, I would have a better sense of the challenges faced by fellow students. I would also have some sense of the time pressure and pacing requirements that the students felt. Another important reason for doing the assignments was that it was my role as an expert that brought many of the students forward to speak to me initially. By giving them helpful advice on their work, the students were being, in some sense, "repaid" for the confidences and insights they shared with me. One unexpected effect of doing the assignments was that I became part of the evaluation scheme of the course (Appendix D). This made me more sensitive to the concerns that students had about the perceived peculiarities and inequities of the scheme.

The next lecture I arrived ten minutes early and discussed the signing and returning of the informed consent forms. I also became involved in a discussion of the solution of one of the assigned problems. It was difficult for me not to revert to the role of teacher in this context. I was only partially successful in maintaining an observer's detachment.

The Tuesday lectures were scheduled for two hours. The instructor typically allowed a ten to fifteen minute break

after the first hour. Conversation during these between participants and with me became an important source of data.

In the second hour of lecture on September 10, I began an informal intervention in the class which continued throughout the study. I introduced a student to some of the capabilities of an advanced handheld calculator (TI-81) and contrasted it with the instructor's use of a basic scientific calculator. Discussing the advantages and capabilities of the calculators initiated contact with students with whom I had yet to interact. Definitely, I expanded my database considerably through this action and role (advanced technology in the classroom advocate). On the other hand, I interfered with the "natural path" of certain students through the course and, in some sense, muddied the data I obtained from them. At that time, and currently, I rationalized my action as a shift towards participation in the participation-observation continuum. Certainly, I was to become a significant actor in the social context of the class.

Because all of the lectures were audio taped by the university and these tapes were available to the students, I did not try to do any taping myself. I did make use of a mini-tape recorder for taping my own reflections and observations as I drove home after lectures and tutorials. I used the recorder less frequently as the course progressed.

This paralleled my increased involvement in the social context of the class.

The first tutorial session was held on September 12. It was here that the tension between my roles as teacher and as a researcher came into greatest tension. There were two tutorial sessions each Thursday. One before the lecture; one after. The majority of students attended the early session. Here I had some success as a detached observer, but on occasion I felt compelled to get involved in the discussion. The second session had as few as two and as many as five students attending. In that session, I intervened more frequently and directly. Indeed, on September 12, the teaching assistant and the students miscued on where they were to meet for the second session and I spent forty-five minutes with two students in the role of teaching assistant. (It was an opportunity to interact with other students I had yet to meet.)

At the end of the evening I gave one of a students a ride back to her neighborhood. This carpooling was an opportunity to interact with the students that I had not anticipated. Up to three students availed themselves of rides home with me. Their conversation was an additional, rich source of data.

The first two weeks established a pattern of Participation, observation and reflection that was maintained

more or less throughout the study. My role as a calculus "expert" established a continuing relation with two students. My advocacy of programmable, graphics capable calculators lead to ongoing interaction with four students. Driving students home after class allowed me to gather data from three other students. One student had an ongoing interest in my research; another student saw my research as a possible forum for expressing concerns that he had. My teacher persona caused several students to seek me out on various occasions for help with their problems. I did not get to know each student in the class, but I did make contact with a significant number of them. Because many of them interacted with me in one of several roles I took in this study, I encountered a variety of perspectives.

One last role that I had throughout the participant observer phase of the study that needs to be emphasized is that of a finite human being. My field notes indicate periods of extreme fatigue and sometimes even boredom. I wrote the first midterm while taking medication for a severe cold. The demands of my teaching job were often at odds with being as thorough in my research as I thought I should be. Curiously enough, as I conversed with my fellow students, both during the course and in interviews afterward, I discovered that difficulties I was trying to overcome in doing the research paralleled challenges they faced in trying

to successfully complete the course. My limitations as a human being became a source of empathy and understanding. Being a participant observer is far more than gathering data; it is an opportunity to learn about oneself as well as others.

Ethnographic Interviews

Eisenhart (1988) describes the role of interviews in ethnographic research.

Interviews are the ethnographer's principal means of learning about participants' subjective views; thus, ethnographic interviews are usually open-ended, cover a wide range of topics, and take some time to complete. Interviews are also helpful to inform the researcher about activities beyond his or her immediate experience, such as relevant historical events or events occurring in other places. (p. 105)

She continues by describing the different types of interview available to the researcher.

These interviews take various forms: from the very informal interview, much like having a conversation with someone (except that one must try to remember the conversation so that it can be written down later); to long audiotaped sessions focused on a particular topic; to highly structured interviews in which the researcher begins with open-ended questions, then uses answers from the original open-ended questions to structure more focused questions, and then is able to convert responses into numerical form. (Eisenhart, 1988, pp. 105-106).

It was clear to me from the beginning of this project that I would need to interview a number of the participants

in some depth if I were to have any understanding of their experiences. What was considerably less clear to me was how I was going to structure and conduct these interviews. Once again, the role of the ethnographer as the "essential research instrument" and my inexperience in the appropriate techniques seemed to be an overwhelming obstacle to the success of this venture.

The text material for interviewing was less helpful than the associated material on participant observations. Goetz and LeCompte (1984) summarized the situation.

Researchers seeking guidance for interview construction find available an overwhelming array of instructions, suggestions, protocol frames, and prescriptions. Within this massive literature, contradictions abound. For each proscription on format or question structure pronounced by one researcher, other investigators suggest alternative uses for the same technique. Many of the prescriptive-proscriptive guidelines reflect differing world views, philosophical assumptions, and values held by social scientists. They also represent protocols used for differing purposes and research questions, compatible with varying theoretical frameworks and research models, and applicable to disparate research settings, participants and situations. Consequently, researchers are best served by seeking and following guidelines for interview construction that are consistent with the goals and designs of particular research projects. (p. 124).

The excellent advice at the end of the above paragraph still left me in a quandary. I brought to the study issues from my own experiences as a teacher and from my reading of

the literature on calculus reform. I believed that these issues were relevant to understanding the student's experiences. I was very interested in the students' perceptions on such matters. On the other hand, I hoped that this study would be generative in nature, rather than an exercise in testing hypotheses. That is, I hoped to be surprised by what I discovered about "participant views and meanings" in this context. The first consideration called for a structured, formal interview with a specified set of questions for each interviewed participant; the second situation called for a more informal interview with open-ended questions meant to stimulate the participant into expressing his/her own views.

I was fortunate to have the opportunity to listen to an experienced ethnographer (Haig-Brown, 1991) talk about interviewing mindsets and techniques in the summer before the study. Following her advice, I prepared as extensive a list as possible of questions exploring the issues I thought were relevant, but in the interview itself looked for opportunities for the participant to follow paths of his or her own choice.

Originally, it was my intention to select three key informants and interview them at three times during the period of the study. As it turned out, the length of time it took me to develop relationships with individual students and

the time constraints we all found ourselves under meant that I was not successful in arranging taped interviews while the course was in session. Certainly, throughout the participant observer fieldwork that was undertaken, many of the conversations I had with individuals could be construed as informal ethnographic interviews. Much of my field data is based on notes taken from and about these discussions.

After the course ended, I did arrange to tape formal, loosely structured interviews with seven participants. The interviews ranged in length from 45 minutes to over an hour and a half. Despite my relative inexperience as an interviewer, the participants managed to impart what to me were often unexpected perspectives on what they felt had occurred during the calculus course.

I began each interview with a standard set of questions and then let the conversation evolve as it would. (Questions used are listed in Appendix C.) The interviews were held in differing locales and under different conditions. The first interview occurred at 7:00 a.m. at the participant's office. One interview happened at the participant's apartment. Another was held at my house in the evening. Three interviews were done over the telephone. In one of the interviews done over the telephone, there was a follow-up discussion (untaped) at the participant's apartment ten weeks later. The last taped interview was with the instructor nine

weeks after the conclusion of the course. It was held in his office.

Interviewing participants after the conclusion of the course may not have been the optimal manner of gathering the intended data. On the other hand, the data produced by the interviews, in its consistencies and inconsistencies with my reconstruction as participant observer of what happened in course, greatly added to my understanding of the student experience.

Artifacts

The course left a number of traces after its conclusion which add to the overall description of the event. I have collected a number of them. Chief among these are the textbook; the assigned problem sets, midterm tests and final examination (Appendix D); a course outline (Appendix B); and the audio tapes of the lectures. Serendipitously, I was also able to acquire, with the instructor's approval, copies of the course and instructor evaluation forms completed by the students after the last lecture. (Blank form in Appendix E.) I also have a spreadsheet display of student marks prior to their writing the final examination. Five months after the conclusion of the course, I was given the final examinations written by the students.

I generated some written artifacts myself when doing the problem sets and writing the tests and examination. Material which I would have liked to have collected, but did not, includes student notebooks and assignments. I also did not attempt to acquire a grade distribution for the class.

Some of these artifacts set up a tension between objectivity and subjectivity in the study. Is student success in this course to be measured by final grade and ranking in the class or by his or her perception of what has been accomplished? For the purposes of this study, I maintain that the latter is more relevant.

Researcher Introspection

Eisenhart (1988) writes about researcher introspection.

The fourth method [of data collection] is probably most unusual, at least to those familiar with the positivist model of doing research. This method of data collection is *researcher introspection*. This method involves the researcher herself or himself reflecting on the research activities and context. The ethnographer regularly records the kinds of things that are happening to her or to him in the research situation. In this manner, the ethnographer tries to account for sources of emergent interpretations, insights, feelings, and the reactive affects that occur as the work proceeds. (p. 106).

This aspect of ethnographic research was legitimized for me during a class in Qualitative Methods in Education Research (Smith, 1991). A guest instructor in the course, Professor Stephen Smith, introduced me to the idea of hermeneutic phenomenological research (van Manen, 1984; van Manen, 1989). The reflective and subjective nature of a substantial part of my data did not necessarily mean that it was invalid, but rather that I had to be certain to identify the context and the role that I was assuming when recording an observation.

Researcher introspection in this study has been particularly significant in that the research process and how it has affected me as an individual has become increasingly important as the project unfolded. Ethnographic research, at least for the novice, leads to a kind of meta-research in

which the researcher himself or herself becomes his or her own object of study.

Analysis of Data

A study which employs ethnographic methods generates a vast amount of data. Goetz and LeCompte (1984) comment on the effect this has on researchers.

Many researchers find their initial confrontation with a mountain of undigested data - drawers full of field notes, notebooks full of interviews, and boxes of protocols, photographs, instruments, and other memorabilia - to be so depressing that they are reluctant to memorialize the experience in print. (p. 166)

About actual data analysis procedures they have this to say.

The process of data analysis in ethnography has been treated as art rather than science. Some experienced ethnographers reject systematizing procedures for analyzing qualitative data because such procedures might rigidify the process, resulting in a loss of the intuitive and creative qualities of ethnography. Given the dearth of literature on analysis, neophyte ethnographers find little guidance for their efforts. Nevertheless, ethnographers do analyze their data and do use formal, systematic, and logical procedures to generate constructs and establish relationships among them. (pp. 166-167)

In my review of the literature on doing ethnography, I found too much guidance, rather than too little. There is a tension among those doing qualitative research as to how systematic the data analysis procedures should be (Marshall, 1984; Miles & Huberman, 1984a; Miles & Huberman, 1984b; Tuthill & Ashton, 1984). I tried to identify and employ the features

of ethnographic data analysis that appeared consistently throughout the wide spectrum of analysis strategies described in the literature.

Eisenhart (1988) describes what I take to be the basic precepts of data analysis in a study employing ethnographic analysis.

The purpose of these [data analysis] procedures is to identify meanings held by participants and researchers and to organize the meanings so they make sense internally (to the actors) and externally (to others)

Basically, ethnographic analysis consists of text-based procedures for assuring that the views of participant and researcher remain distinct and that all aspects of the material are taken into account. Generally, the procedures involve defining "meaningful units" of the material (meaningful to the participant or researcher) and comparing units to other units.

Data collection and analysis proceed together throughout the period of the study. The collection of new material and subsequent analysis may raise new research questions or lead to insights that become incorporated into, or sometimes radically redirect, the study itself as well as later data collection.
(p. 107)

I came to see my role in this study as a filter and an interpreter of the student experiences. The data needed to be aggregated into categories or themes. The choice of these themes and their boundaries varied throughout the study.

Goetz and LeCompte (1984) speak of sequential selection as a process in which the objects of interest to the researcher in the study emerge as the study progresses (p. 175). Above all, it was the process of resolving the data into a written report, mediated through weekly discussions with my thesis

advisor, that drew me to the themes and conclusions presented in this study.

Upon completion of the data collection stage of the study I had three primary accumulations of data: my field notes, the audio tapes of the lectures, and the audio tapes of my interviews with six of the students and the instructor. Although I had multiple sets of data, only the lecture tapes and my field notes were acquired concurrently. The interviews took place after the course had finished and the structure and content of these conversations were influenced by my fieldwork experiences. Consequently, I cannot claim that the three sources of data provided triangulation. Analysis of the separate data sets did, however, lead me to convergent sets of conclusions.

I began with the interviews by personally producing verbatim transcripts. Although this process was very time consuming, it allowed me to present this material on a weekly basis to my thesis advisor. We would discuss what each participant had said in the light of calculus reform issues and our own teaching experiences.

After all the interviews had been transcribed and stored in word processor files, I produced a list of 17 issues or themes. I chose them after I had read through the transcripts again. The criteria for choosing the themes were first, the strength of the responses from the interviewed participants

on each topic and second, the relevance of the issues to what I had found during my fieldwork. A number of the themes were pre-determined by the structure of the interview. Each interviewed subject was encouraged to talk about certain aspects of their experiences. Other topics emerged from the interviews as the subjects extended the conversation beyond my prepared list of questions.

Using HyperRESEARCH, a computer program designed for the analysis of qualitative data (Hesse-Biber, Dupuis & Kinder, 1991), I coded each of the interviews according to the list of topics and then generated reports which brought together for each topic what each of the interviewed subjects had to say. These reports became the basis for the written account found in Chapter V.

In consultation with my thesis advisor, several of the topics were brought together to reduce the list of themes from 17 to the 11 used as section titles in Chapter V. Material for the section on Student Backgrounds was selected under the codes academic goals, student preparation and background. Material coded under lectures, textbooks and instructor enthusiasm was used for the section entitled Lectures and Textbooks. The section on Health and Fatigue used material coded under both campus accessibility and health and fatigue. The codes course content and pace of course were used for the material in the section The Quantity

of Material and the Pace at which It is Covered in the Course. The codes for the other sections were identical to the title of the section.

I analyzed my field notes in a manner similar to the one used with the interviews, except that I did not make use of computer software. Over an extended period of time I worked through the lectures in chronological order, creating a written account. I would listen to the university generated audio tapes while reviewing my notes. The tapes often picked up conversation outside of the formal lecture material, especially before and after lectures and during the ten minute breaks taken during Tuesday classes. Sometimes this material was new to me, sometimes it reaffirmed the observations I had made in my notes, and, at times, it evoked memories of events I had not recorded. To some extent, while writing up the account of the lectures, I relived my participant-observation experiences. There were no tapes of the tutorial sessions so that I was dependent on my notes and some reflections I had taped on a mini-tape recorder.

As with the interview material, I would present segments of my account to my advisor and we would discuss what was happening in the particular part of the course on which I had written. In this way, my analysis and interpretation was subject to the refining processes of prolonged reflection and discussion.

One consequence of this approach was the length of time between when the study was conducted and when the data was analyzed and the results written up. The primary study was conducted in the autumn of 1991 and the final interview was completed in February, 1992. It took until the end of 1993 to complete the written account of the study.

The length of time spent analyzing and displaying the data proved advantageous in my evolving understanding of what occurred. The impressions I held at the end of 1991 were not as fully developed as the ones I presented at the end of 1993. Introspection and reflective writing both require a considerable investment in time. With respect to this study, the two years of "living with the data" has had a positive impact.

Representation of the Data

Van Maanen (1988) defines ethnography as "a written representation of a culture (or selected aspects of culture)" (p. 1). He then goes on to describe three types of written accounts of ethnographic studies. The realist tale is a direct, matter-of-fact portrait of a culture. The confessional tale focuses on the thoughts and experiences of the fieldworker. The impressionist tale is a personalized account of fleeting moments of fieldwork cast in dramatic form.

Van Maanen recommends that the neophyte ethnographer begin with the realist tale, although enjoining the researcher not to mistake the reporting of detail for objectivity. I have attempted to follow his advice, although there are aspects of the confessional tale sprinkled throughout.

One consequence of how I wrote the account is its length. My purpose was to involve the reader as fully as possible in the experience of the calculus class. Not only did I want the account to corroborate the conclusions that I drew from the data, I intended that readers would generate their own questions and thoughts.

Summary

The research methodology was chosen in order to produce a descriptive, naturalistic depiction of the experiences of participants in a first semester course in university calculus. The methods of ethnographers were chosen because they appeared most likely to result in the most intimate understanding of the students' experiences and observations. Participant-observation and interviews were the primary methods of data collection. Data analysis involved a prolonged, intensive review of the data as it was transformed into text and later into a written account. Computer software was used in the coding and collating of the

interview data. Frequent discussions with my thesis advisor about the material as it was processed helped to refine the analysis. The data was represented by a lengthy, chronological account of the participant-observation stage of the study followed by the responses of the interviewed subjects collated by theme.

CHAPTER IV

STUDENT EXPERIENCES IN A FIRST SEMESTER UNIVERSITY CALCULUS COURSE

Introduction

In this chapter I present a written chronologically ordered account of my fieldwork experiences in both the pilot and actual studies. These are followed by responses made by several students and the instructor during taped interviews after the completion of the course. The responses are organized under several themes.

The Pilot Study

During the summer of 1991 I began participant observation in a Math 151 course (the first semester of the calculus sequence) on the main campus at Simon Fraser University. With the help of my thesis advisor, I secured the permission of the Mathematics Department to act as an observer in this class. This practice fieldwork occurred on four separate days. (See Appendix A for details.) It allowed me to experience the complexity and richness of the observation process and gave me some initial insights into the challenges of balancing roles as a participant observer.

At the same time, I acquired data that I would use to compare and contrast with fieldwork on my actual project in the fall.

My primary objective in this pilot study was to gain experience as an ethnographic fieldworker. Secondary objectives were to identify and clarify issues that would be of particular interest in the full study.

The fieldwork consisted of attending four 55 minute lectures, two informal half hour discussions with the instructor, and three to three and a half hours of observation and discussion with students at the calculus laboratory. I audio taped two of the lectures.

Following procedures I had gleaned from textbooks on ethnographic fieldwork, I began by recording as much detail as possible. My earliest notes contained references to student behaviour such as eating cafeteria food before and during the lecture, student attire, attendance and time of arrival, note taking techniques, student conversation, and conversation between student and instructor. There was, however, a passive quality to much of this activity. My field notes from the lectures were dominated by the course material and its presentation by the instructor. Originally, I was concerned that this pre-occupation with the teaching rather than the learning indicated that my observations were inadequate. Upon reflection, I have come to believe that my

notes and their focus accurately depict the passive role assumed by the students in this context.

In the first fieldwork session, I attended a lecture and made field notes. In this instance, I was purely an observer and had yet to introduce myself to the instructor or students. No one appeared to take note of my presence. The attempt to make meaningful observations in a specific context showed me how difficult it was to apply the general frameworks of the ethnographic methods described in textbooks to the specific situation I was studying.

I found that my observations slid from physical layout of the lecture theatre to overt behaviour of the participants (instructor and students) to appearance and body language of participants to lecture content to teaching style. Two aspects of this study became apparent. First, I would continuously need to use "boundary spanning" skills in this study. That is, I would need skills "in communicating within and across cultural groups" (Goetz & LeCompte, 1984. p. 99). I teach calculus at a high school and my perception of what was occurring in a calculus class would in all cases be affected by that particular researcher role. Secondly, the lecture format of instruction does not lend itself to a great deal of participant interaction. If I was to make sense of the calculus student's experiences, I would have to be assiduous in my observations and aggressive in establishing

relationships with the participants. The second consideration impacted on the roles I sought to establish for myself in the primary project.

A key aspect of my experience in the pilot study was my growing awareness of the many perspectives I would have to juggle as a participant observer. The first lecture I attended was just after the second "midterm" test. The first 12 to 15 minutes were dominated by the minutiae of student evaluation and grading. I found myself, as a teacher, responding negatively to this use of instructional time.

The class was scheduled to begin at 8:30 a.m. A late arrival focused my attention on the time in this first attempt at observation. I noted that it was 8:42 before the day's lecture began. The topic was Exponential Growth and Decay and was referenced by the corresponding section in the text. By this time there were 19 students in attendance.

It was now that my responsibilities as an observer began to seem overwhelming. The lecturer and his activities were conspicuous and dominating; the actions of the students were often subtle and difficult to detect. It was easy to focus on the lecture and lose sight of the class. The content of the material presented; the delivery style of the lecturer; the quirks of the lecturer in this role; and my own preconceptions of what should be happening created a set of fieldnotes that skipped quickly from one perspective to

another. All the while, I was working to bring the observation process under control. Thus I found myself adding notes about notetaking.

The content and delivery of the lecture was significant in the messages I perceived being sent to the students (although I would not know until much later in my research what messages were being received). The lecture began with some verbally presented examples of exponential growth and decay. The differential equation $\frac{dy}{dt} = ky$ was written on the overhead which caused some notetaking activity. The instructor offered that "one solution is e^{kt} ", in fact, the only solutions are Ce^{kt} " and then proceeded to outline a uniqueness proof. I reflected that the existence and uniqueness of solutions to differential equations is quite a sophisticated introduction to this topic.

But then the instructor wrote a standard problem on the overhead and the solution was offered with the comments "Here is a trick"; "You'll see why" and "I did this on a calculator". The change from at least mentioning aspects of solving differential equations to working through template problems was jarring. Not only that, I came face to face with the teacher-observer tension. I knew a better way of teaching how to solve that type of problem.

Although I was only halfway through the first lecture I was observing, I began to feel the boredom and the potential

for inattention of which the ethnography texts had warned. I was consciously and conscientiously looking for student behaviour that I could record for future interpretation. There was little that I could detect.

There was a flurry of activity at the end of the lecture as students departed or sought out the instructor. After some students had spoken with the instructor, I introduced myself to the instructor and we had a brief discussion. I did not make notes on this discussion, although the material I had covered on ethnographic methods had stressed the importance of this follow-up procedure. Consequently, except for two brief descriptors about the topics of discussion, the data from this conversation was lost.

This was the first time that I tried to explain my project to someone who was in a group I was observing. I was very pleased at the cooperation I encountered, although I am not certain that I had made my project clear to the instructor. After this, a number of students, the instructor and I headed in a convoy down the hall to outside the calculus laboratory.

In the next lecture session I continued to work on improving my field note taking skills, but often found myself recording notes on the lecture material instead of participant behaviour and interaction. I experimented with simple coding schemes for recording notes in realtime.

This second lecture was significant to my research in two respects. At the beginning of the lecture, the instructor introduced me to the students in attendance and attempted to explain my purpose in being there. I interpreted the student reaction as being polite, non-committal acceptance. What the introduction did set up, however, was a pattern for this and the later study. For good or ill, I did not seek out conversation with students unless they approached me first. I was very much concerned that my reticence in "imposing myself" on others would prove to be a crippling handicap to the research. As it turned out, after I was introduced to the class, there were students who would seek me out and initiate conversation (although to a limited degree in this preliminary work).

A second important feature of my study during that second lecture was the impact that my reading of the calculus reform literature had on my attitudes and observations.

The topic this day was l'Hôpital's Rule. I found myself incredulous that the course had moved from exponential growth and decay to limits of indeterminate forms in just one lecture, but the students did not seem particularly upset. The criticism that the traditional calculus curriculum was overcrowded and that the pace of instruction was necessarily too swift for true learning seemed particularly relevant.

Throughout the lecture I was struck by the instructor's use of phrases such as "there are some tricks"; "standard mathematical tricks" and "tricks don't always exist". The calculus was being represented as a collection of tricks. This was the "mimicry math" to which the reformers argued that calculus had been reduced. In my notes I began jotting down questions I wished the students would ask. I had to examine closely to what extent, if any, I could claim impartiality and objectivity in my observations.

After the lecture, I had another discussion with the instructor and this time, after the discussion, recorded some notes (albeit, very brief). I spoke with the him about his teaching background. This was my first informal interview. Although I had a theme for this interview, the particular questions were generated on the spot in response to the dialogue.

During the half hour that I spoke with the instructor about his teaching experiences, he was quite forthcoming and helpful. This was his third semester of teaching. He was at the university with a post-doctoral research grant and taught for the financial remuneration. He felt that he was slotted into teaching first year calculus because "the senior faculty were not interested". This particular class that he was teaching this semester was a good class because it was quiet. In this first teaching assignment, he taught a Math

110 (business pre-calculus) course with two hundred students, of whom, he estimated, about fifty were rowdy. His solution to dealing with the Math 110 class was to "push hard and cover lots of math".

The instructor had no background in pedagogy and had received no suggestions or help from departmental faculty with respect to teaching. He drew on his own experiences and memories to develop a teaching style. When he was an undergraduate studying calculus, he stopped attending lectures, attended tutorials only to pick up on the professor's insights into the material, and established the recipe READ TEXT, WORK HARD, DO WELL. The instructor deliberately choose to use informal language and humour in his lectures.

As a closing question, I asked if any of the students had asked him about the solution to problem 5 on the second midterm. (This was the problem for which every student received credit because none of them solved it.) The instructor replied that no one had. He seemed neither surprised nor upset.

The material that I gleaned from this talk was consistent with what I had read on the "Who teaches the calculus?" issue: junior faculty with little teaching experience and minimal support from the department. I took

this image with me into the full study where it was strongly challenged by the instructor's background for that class.

After this discussion and reflection, I sat and observed activity in the Calculus Lab. This observation session was in response to the perceived need to observe students outside of the lecture theatre.

For the next two lectures, I experimented with the use of an audio tape recorder to record the lecture and student conversation. The result was an audio tape in which the lecturer can be heard and some student comments can be heard. This use of recording technology was both liberating and confining.

It was liberating in the sense that I no longer strained to record all conversation; the tapes recorded aspects of the situation that I missed in real time observation. For example, in reviewing the tapes, I was struck by how much quiet time there was during the lecture. There was also a higher incidence of students initiating discussion with the instructor than my written notes suggested.

Using the tape recorder was confining in that the attention I paid to ensuring that the technology was operating correctly was attention taken away from the immediate situation. Setting up and monitoring the device distracted me from observing and the act of taping established me as a researcher and acted as a barrier to my

role as participant. In my field notes I indicated that I missed an opportunity to speak with the instructor after class because I was packing away the microphone and recorder.

This was another example of how the use of technology can change the field observation situation. These sessions brought home forcibly Wolcott's dictum that the ethnographer first and foremost is the "research instrument" during fieldwork. (Wolcott, 1980).

After this lecture, I went again to observe in the Calculus Laboratory. The session in the Calculus Lab was particularly significant because a student initiated a conversation with me. This conversation foreshadowed the manner in which many of relationships with students in the primary project would begin. The student had identified me as an "expert" in calculus and sought my help with course material. Once a discussion had begun, there was opportunity to explore the student's background, attitudes and opinions. Also, other students became willing to talk with me. It is significant that it was the student who initiated the conversation and not I. My unwillingness to "invade other people's space" was an important factor in how the study progressed.

In attending lectures I discovered that many interesting facets of the learning situation could be uncovered by inference. In this course, the tests and examination were

set by the instructor, but the marking was done by a teaching assistant. At one point the instructor commented about the teaching assistant, "He really takes it to heart when you make silly mistakes. He gets really sad." Previous to this, at the instigation of persistent student questioning about the nature of the final examination, the lecturer had spent some time trying to explain the general nature of the "three harder questions" (out of 11) that he would set for the final examination.

The fourth lecture I attended was the penultimate one in the course. Attendance was high for this lecture (about 20 out of the 32 who had written the second midterm); the students sat further forward in the lecture theatre and I noticed some new faces. There was noticeable tension among the students. At 8:40 one student got up and left. The lecture went overtime in an attempt to "finish the course" (one topic was deleted at the instructor's discretion) and to compensate for the time spent discussing the final examination.

At the end of the lecture, one student immediately approached the instructor and another queued up. Perhaps because of this line-up, one of the students approached me and asked me a question about the material. In discussion with him I found that he had taken a pre-calculus course a number of years ago and he confirmed my suspicion that he did

not remember anything about the equations of circles (material relevant to the last topic presented in class). As a participant-observer I was elated in that I had made some initial contact, albeit limited and superficial, with a student. Significantly, it was the student who approached me, rather than my seeking him out. Also, the student sought me out for help with the course material. These were two continuing patterns in my career as a fieldworker.

The calculus laboratory (known as the CLAW) was a second venue for my fieldwork. The CLAW was open during set hours with one or two teaching assistants in attendance. Solution keys for assignments; grade lists; envelopes for submitting and picking up problem sets were at the edges of the room. The teaching assistant was at a desk at the centre. Tables which could accommodate four to six students each covered the remainder of the "lab".

The environment here was in stark contrast to the lecture theatre where the bright light of the overhead and the instructor's voice focused everyone's attention front and centre. Here it was quite natural to make eye contact with someone else. Groups worked together on problem sets from various courses. Student interaction was very natural in this setting whereas it was forced in the lecture hall.

In the CLAW, students seemed quite comfortable about coming over to where I was and asking me questions. I began

to think of this participant-observation as a barter situation : my expertise in calculus for your attitudes and beliefs. One pitfall in this is that I am a classroom teacher and only incidentally a educational researcher. The truth of this is clear in my field notes which continued in one session in the CLAW for several pages of observational detail and then change abruptly for several more pages of worked examples used in teaching my research subjects.

The field notes show that I was, even at this early stage, pushing an issue which was of importance to me (and the calculus curriculum reformers in general), but appeared to be of little significance to those I was observing. This issue was the use of technology, particularly calculators, in the teaching and learning of calculus. At one stage I note that there are "maybe three calculators in sight; none being used". Five minutes later the first observed use of a calculator is duly recorded.

Although the use of calculators, particularly those which have graphing capabilities and are programmable, was originally something of a non-issue for the participants in my study, my enthusiasm for them and interest in their use actually made them an issue. This participant-observer was not a neutral element in this research.

The field notes for the last lecture were qualitatively different from my first set of notes. Because I was taping

the lecture, there were references to the Tape Counter number at particular points. There was more introspection in the latter notes, particularly with respect to the pervasive "I can teach this better" feelings that surfaced in my teacher mode. There was more material on student behaviour and responses; although student - instructor interactions continued to dominate the observed social behaviours. In these notes I was beginning to differentiate between students; giving them different codes (although I still did not know the name of a single student).

I completed my field observations for this preliminary study with an hour at the CLAW. Attendance was low and my notes did not add anything to my previous observations.

At the end of the pilot study, I had a much better sense of what the data collection stage of my research would involve. I was also made acutely aware of some of the struggles and paradoxes I would have to face as a participant-observer. I was still far more the observer than the participant at this time. But the willingness of the other participants to accept me and work with me gave me confidence. I believed that, in a full scale study, there would be sufficient interaction with the students to give me some insight into their experiences.

Participant Observation

August to September 12, 1991

By the end of August, 1991 I had selected a course for my primary study. It was a Fall Semester Math 151 course being taught on Tuesday and Thursday evenings at the Harbour Centre Campus. Approval of the Department and of the Instructor for my participation in the classes had been secured.

To begin with, as did many other students, I went to the bookstore on the main campus about one week before classes began. It was very crowded and busy in the aisles where students searched for their texts. When I located the text, I found several students puzzling over whether they were to purchase the first or second edition. (The first edition was to be used by students taking the second course in the calculus sequence, Math 152. The second edition was for the upcoming semester of Math 151.) The second edition was shrinkwrapped and more expensive than the first edition. There was some discussion as to whether the savings in cost was worth the gamble that the first edition would be of little use in the Math 151 course.

I obtained a copy of the shrinkwrapped second edition, and charged the \$70 price on my VISA. I wondered how some of

the students, working with considerably fewer financial resources than I had, felt about the price.

At home, I removed the wrapping and, using the course outline (Appendix D), browsed through the book. The text was massive, over 1100 pages and weighing more than 2 kilograms. It was profusely illustrated using several ink colours. It seemed that no topic or approach was left untried. The book embodied much of the discussion in the calculus reform movement about what was wrong with texts for a first course in calculus. Yet, in comparison with the texts I used in my own classroom, it seemed to be a very good calculus textbook.

Matching the text against the course outline, I found that Math 151 covered only about 300 pages in the text. The assumption was that most students would proceed from Math 151 to Math 152 where the same text would be used and perhaps another 300 to 400 pages would be covered. Later in the study I found out that a significant number of students will finish their study of mathematics with the Math 151 course. Also, those that do continue on to Math 152 would have to do so with some swiftness. Otherwise they might get caught with an edition change such as the one that the Math 151/152 was currently undergoing. My net feeling, before going to the first class and before talking about it with my classmates, was "too much text".

The first lecture was scheduled for 7:30 p.m. on Tuesday, September 3, 1991, in Room 1415 at the Harbour Centre Campus.

After finishing my workday, I stayed late at the school where I taught, caught a meal nearby and then drove downtown. It wasn't until later that I found my schedule was similar to that of several of my classmates. Arriving at Harbour Centre, I found parking under the building. After a frustrating experience with a balky ticket dispenser, I made my way into the building and began to search for Room 1415. My anxiety level was quite high by this time. I wondered if the same was true for my classmates, for whatever reasons.

I found the room just after 7:00 p.m. There were two or three students were already in the room. The social mood was that of an elevator, with averted glances, silence and hushed whispers.

The room itself, and the building in general, struck me as being luxurious. Room 1415 consisted of three raised, semi-circular tiers of continuous desks with comfortable, upholstered chairs for the students. In the front was a lectern and a cart with an overhead projector. The front wall was a chalkboard over which a wide overhead screen could be lowered. There were doors to either side of the chalkboard. The room was carpeted and the colours were deep blue and a shade of purple. This was a considerable step up

from the classrooms and lecture halls I had found at the Burnaby Mountain Campus.

About 7:20 there was some initial interaction between the students as one asked to look at the textbook of another. (Not all students had acquired a text before the first class!) I listened to some discussion about the relative preparedness that students felt they had for this course and why they were taking the course.

At 7:25, the instructor arrived, saying, "Good evening. This is Math 151 - Calculus. Does anyone know how to lower the overhead screen?" With prompting from the students, the screen was successfully lowered and the overhead projector turned on. The instructor introduced himself, read out the class list, cited the course description and the text. The text, identified by its author's name, Stewart, was said by the instructor to be the "bible for this course".

At 7:30 there were twenty students including myself. Several students arrived late over the next ten minutes while the instructor went over some procedural details and gave an overview of the course. He stressed the value of this course, but warned that the pace would be rapid. He reminded students of the prerequisite of at least a B standing in Algebra 12. As a high school teacher, I was a little bemused by this. The Algebra 12 course had been replaced a year and a half ago by a course entitled Mathematics 12. Fulfilling

this prerequisite was even more problematical for some of the students, I later learned, because they had been away from school for a considerable length of time and their last mathematics course was not necessarily taken in a B.C. high school.

The instructor went on at some length about the need to have sufficient background in arithmetic and algebra. He mentioned the Math 100 pre-calculus course and asked how many students had taken it. The course would be rigorous, he said, and the students would need to spend a minimum of two hours outside of class studying and working on problems for every hour of class time. It would be very difficult to make up for missed classes in this course. I looked around to see how my classmates were reacting to this doomsaying. If this discussion was designed to make weaker students reconsider their enrolment in the course, I don't know how effective it was. Most students were taking the course because it was a prerequisite for other courses they wished to study and they felt they had little choice in taking or not taking this course.

The evaluation scheme was laid out in great detail. The dates of the two midterms, Thursday, October 10 and Thursday, November 14 were set as was the amount that these one-hour tests would contribute to the student's overall grade. There were to be about nine assignments to be submitted to the

Teaching Assistant for grading. At the end, on December 3, there would be a comprehensive, three-hour examination .

The instructor provided a scale matching percent grades with letter grades. At this time, he stated a principle which was repeated and refuted on several occasions throughout my research, "A student's grade is almost directly proportional to the time and effort spent."

After establishing the 2:1 ratio of time spent outside the classroom to time in class, the instructor became quite specific about how this out-of-class time could be spent. Each lecture, he would assign sections of the text to be read. Students should attempt every odd-numbered problem in the problem set following each section. (The textbook had an answer key, but not a solution key, for odd numbered problems. The instructor assigned even numbered problems to be handed in.) Even at this stage, the instructor added a note of pragmatism. Aware that many or most of the students would not work through every odd-numbered question, he offered some strategies on selecting the problems that should be attempted. In order to get in the minimum of six hours of work each week, he exhorted the students not to procrastinate and to keep a log of their work to help keep them "on task".

Assignments were to be submitted to the Teaching Assistant, who would be responsible for marking them. After

the assignments were marked, solution keys would be available in binders in the library at Harbour Centre.

This was suggestive of how complex the teaching of calculus at university could become. The instructor chooses problems from a textbook that is prescribed by a departmental committee. Students work on these problems and submit them to a teaching assistant, who marks them on the basis of solution keys prepared by the publisher of the textbook. (The solution keys, as it turned out, were cut and paste photocopies of a Solution Key manual set for the textbook.)

Twenty minutes into the session, the instructor brought up the matter of calculators. He told students that they would need a basic scientific calculator; one costing about twenty to twenty-five dollars. "Get one and use it; one with an a^x or y^x key." Of the twenty students in class that evening, I observed four using calculators.

At about 8 p.m. the instructor seemed ready to begin the course. He asked the students to please read and study section 4 and 5 and in Chapter 1 to study, really study, 1.1 and 1.2. It was suggested that students do problems whose numbers were multiples of five on pages 34 and 40; odd numbered problems were recommended for sections 1.1 and 1.2. Those who had not had a chance to see the text were already confused. Sections 4 and 5 were in a part of the book called Review and Preview. Chapter 1 did not begin until page 56.

Some students at this point actually raised questions about what they were supposed to be reading. One student asked, "All by next Thursday?" Others asked more general questions about the type of review that would be most effective.

In the ensuing discussion it became apparent that many of the students had not done any mathematics for over three years. The instructor was concerned about this and cautioned the students who had been away from mathematics that they would have to do extra review. The instructor suggested that they acquire a Math 100 textbook. He also asked if anyone was taking computer science this semester. He warned about how time consuming such courses could be.

Again, the instructor tried to bring the conversation back to the course material. Shortly after 8:00 he began an instructional pattern which he maintained throughout the course. He read from previously prepared notes while he copied them onto an overhead, occasionally breaking to initiate or respond to student initiated student-instructor interaction.

The course material began with finding the slope of a line tangent to the graph of a function. Shortly into his presentation, the instructor broke off for a brief review of functions. He solicited choral responses from the classes and got some tentative answers. During this, the instructor said, "When you do a review, you will see..."

The lecture on the "tangent as the limit of secants" continued, but with some of the students becoming increasingly comfortable with interjecting comments or questions, albeit they seemed to be struggling with the concepts of limits. One student, grappling with the process, spoke hesitantly about the length of the secants approaching zero. The instructor moved in with, "not length, but slope; $\frac{\text{rise}}{\text{run}}$." He continues, " I think you'll agree that

$$m_{PQ} = \frac{\text{rise}}{\text{run}} = \frac{x^3-1}{x-1} ."$$

There was a faint response from the class that suggested that maybe the waters were getting deep rather quickly. I was concerned by the quick transition from the graphical picture to what must seem to many of the students to be a notationally dense representation.

But the instructor continued with a roster of values for x and $\frac{x^3-1}{x-1}$, with x taking values such as 0.99 and 1.01. It was at this stage that I observed some students using their calculators. I also noted the meticulous notetaking of the student beside me, who, I later learned, had earned a score of three on the Advanced Placement Calculus examination in high school the previous year.

The instructor maintained, with the concurrence of the class, that he had given a convincing argument that the limit of the slope is three as x approaches one. He then wrote the

point-slope form of the equation of the tangent line on the overhead.

By then it was 8:40 and the instructor declared a brief recess. These breaks during the two hour class sessions on Tuesdays were to provide some of the most significant interactions in the class.

On this first night, a number of students came forward to discuss with the instructor how they fit into the system. Several of the students already had calculus in their background. Some had done introductory work in high school; at least one had completed an Advanced Placement course. One had taken calculus five years ago in Dawson Creek and was wondering whether or not to enroll in Math 152 this semester. Others were concerned about how long they had been away from any formal study of mathematics. Typical of these was a woman who had graduated in 1983 and had no background in trigonometry. The instructor tried to advise each of the students as they came forward.

The instructor convened class again just before 9 p.m. I observed that one student, an earnest young man in a three piece suit and with a calculator, had purchased the textbook, still in shrinkwrap, during the break.

The lesson continued. "The same result can be obtained algebraically.

$$\lim_{x \rightarrow 1} [{}_{\mathbb{M}\mathbb{P}\mathbb{Q}}] = \lim_{x \rightarrow 1} \frac{x^3-1}{x-1} = \lim_{x \rightarrow 1} x^2 + x + 1 = 1 + 1 + 1 = 3."$$

Sensing uncertainty about this from the class, the instructor digressed for a while about the factoring of $a^3 - b^3$. One student in the front row attempted this factoring, but faltered. This factoring provoked nervous conversation among three students in the back tier. This was my initial observation of a theme which was later to be named (by the students) as the "Just Algebra" theme. It was the assumed background, secondary school mathematics, that was delaying the progress of the class at this point; not the calculus.

From here the instructor picked up the pace and went immediately to the generalization: for any function f at a point $(a, f(a))$ the slope of the tangent line is

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \text{ which can be written as } \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$$

After this the instructor said, "Let's do a little application. No, we're running out of time. Let's pack it in."

From the students' perspective, the theme for that first night was "can I keep up?". One student, speaking to me after the lecture, said the professor was trying to inspire fear, but at least he cared enough about his students to do

that sort of thing. The instructor conveyed the sense that the students should be able to deal successfully with a text and gain the material that they need through reading. I wondered if the instructor was displaying a "my hands are tied, what can I do?" syndrome. The curriculum is crowded and the backgrounds of the students are diverse. The students must meet the demands of the syllabus; the syllabus will not be brought down to the level of the students.

Tutorials were scheduled for Thursdays each week; one session before and one after the one hour lecture at 7:30. There was no tutorial the first week. I arrived outside of Lecture Hall 1700 at 7:25 and found both the instructor and students waiting outside. The room was in use and we were waiting our turn. The five minutes were spent with the instructor discussing problems with the students.

At 7:30 the previous class left the hall and we entered. The layout of this room was in marked contrast to the Tuesday evening classroom. This was a true lecture theatre with eleven rows of nine seats each. The seats had flipup "arm desks" for taking notes. At front there was a lectern, projection screen and two overhead projectors.

Tuesday evening we had sat in a three tier amphitheatre in comfortable chairs with ample space for notetaking, referencing the textbook or using calculators. On Thursday we were in very much a standard lecture situation. And

although there were only twenty odd students for nearly one hundred seats, the amount of space available to the individual was severely constrained.

I felt that the attitude and behaviour of the students was affected by the physical environment. There was considerably less interaction between instructor and students during the Thursday classes than on Tuesday. The front row on Thursdays was left unoccupied. Positive student activity, as I observed it, occurred in rows two and three. Those of us in rows four, five and six were relatively inactive.

Nine problems from the textbook were assigned to be handed in during tutorial sessions next week. The instructor quickly recapitulated Tuesday's lesson and then moved on to the concept of instantaneous velocity. Throughout there was frequent reference to an assumed student background in physics. The instructor drew with great deliberateness the parallels between the slope of secants/slope of tangents relationship and that of the average velocities/instantaneous velocities.

There was some student/instructor interaction during the lecture. A student in the second row caught the instructor in an error. The instructor concurred with her correction. But also, some possible interactions did not happen. A raised hand by a student in the third row went unnoticed by the instructor.

When the limit of quotients definition of instantaneous velocity was given, one student interjected that the textbook made use of negative times. The instructor responded that this had something to do with the technicality of one-sided limits and he preferred not to get into it at this time.

The lecture continued although the student's body language indicated that she still had concerns. I noted to myself this instance of a student who had read the text prior to the lecture, had thought about the material and was actively wrestling with concepts during class. But I also noted that many of my classmates were copying their hearts out and that at least two textbooks were open so that they could follow the lecture.

Since the next example in the lecture came directly from the textbook, the open books did not seem to be such a bad strategy. The example required finding the velocity of a particle at specified times given the position-time function. The function was a degree three polynomial and the velocities were calculated using $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$. The quotient limit was identified by the instructor as the "fundamental equation that we would be manipulating throughout this course; the basis of all the calculations we will be doing."

The instructor invoked the binomial theorem (no longer part of the high school curriculum, I noted with chagrin!),

briefly reviewed it and continued the example with the phrase "all we have to do now is simplify this expression and that's where some elementary algebra comes in." Also, the instructor shifted smoothly from calculating the limits for specific values of a to calculating the limit using a as a variable. After evaluating the limit the instructor finally identified it as the derivative of the position function.

A girl in the third row who had graduated from high school the previous spring was obviously concerned by what she perceived to be a tortuous and circuitous route to the solution. "Why not just use the derivative rules that we learned last year?"

The instructor replied that, yes, there were some quick formulas for finding derivatives, but that we were not at that point yet. "But I can guarantee you that on the first midterm and final examination you will have to find them this way."

The student who had raised the questions about negative time asked what had happened to the $\lim_{h \rightarrow 0}$ notation in the final answer. When the instructor replied that it was no longer necessary after the limit had been evaluated, she commented, "Essentially we have let $h=0$." The instructor replied, "We haven't let $h=0$, we have found the limit." The student was referred back to the table of values from the previous lecture.

In the last few minutes of the lecture, the instructor introduced the concept of a limit of a function. The importance of limits in the work already done was pointed out as was their central role in the further development of both branches of calculus. "It is important to understand limits and to be able to evaluate them."

The instructor defined L as the limit of a function f as x approaches a if $f(x)$ gets as close to L as we like by taking x close enough to a . With diagrams the instructor sought to clarify this definition. He qualified this work by stating, "This is kind of a big mouthful. Even though that's still a naive definition, we should look at some graphs here to see what's going on. It is easy to interpret the limit of a function geometrically."

At this point, the instructor observed that the class had officially ended. He joked with the students about continuing, "I told you it would be exciting", but elected to end the lesson there.

The instructor then asked me if I wanted to introduce myself and the research project to the class. I identified myself as a high school teacher and graduate student, outlined my project and handed out the informed consent forms (Appendix B). I did this with a fair amount of apprehension. I had no sense of what the general reaction would be, although I felt it was fairly positive after I had spoken.

Certainly, I was buoyed by the support that the instructor offered for the research and his assurance that information given to me would not come back to him.

As the students left, one asked if there would be a tutorial that evening and when the assignment was due. The instructor replied that there would be no tutorials until next Thursday, which was the due date for the assignments. I was mildly surprised when he said that he did not yet know the name of his teaching assistant. This foreshadowed an ongoing concern I had about the efficacy of the tutorial system in the context of this course.

The two hour-class session on Tuesday, September 10 was notable in that I met with students before the lecture and had some signed consent forms returned. Also, one of the students engaged me in an animated discussion about one of the assigned problems.

In lecture the geometric development of the limit concept was continued from last Thursday. Numerous diagrams were drawn on the overhead and most of the twenty students were engaged in intensive notetaking. Little student-instructor interaction occurred during the first part of the lecture, although a significant part of the lecture was delivered in a "question posed-question answered" style. The instructor answered the questions he was asking.

Upon completing a sequence of examples, the instructor referenced a theorem from the textbook. What I found interesting about this was that he made explicit mention of the "if and only if" nature of the theorem and spent some time explaining the biconditional. He spoke as if the students should be aware of such relationships in logic, but at the same time he made the effort to explain the situation in some detail. This was an instance of the instructor attempting to bridge the gap between the background knowledge the students was supposed to have and the actual experience of the students.

I was surprised to see how many textbooks were brought to class this time. At least six were open during the lecture. The student next to me had highlighted portions of the text. The instructor commented that although it was a heavy text to carry around, it was a good idea to bring it. He said that he would be making reference to material in the text, because the amount of material in these sections made it impractical to write it all down. This was just following the statement of a specific example problem from the textbook.

While working through this example, the mood and energy of the class changed dramatically as the instructor repeatedly invited choral responses to his questions. This particular example finished on a curious note when the

instructor, after establishing with the class that the limit did not exist, introduced the symbol \exists . This prompted the first unsolicited student response of the evening, "Can we write D.N.E. ?"

Again I noted that the layout of the room seemed to have a direct impact on the activity of the students. The continuous desks allowed both open textbooks and notetaking. The students in the first two rows, although not that much closer to the instructor than in the lecture hall, responded to a perceived proximity. Certainly, there was more instructor-student interaction than I had observed on Thursday (after the first half hour of notetaking, that is).

After completing this first example, the instructor went on to a second example from the text: $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}}$. During this example, I made one of my first significant breaks from observer role to intervening participant by showing the teaching assistant (attending the lecture and sitting next to me) how the problem might be done using the graphics capabilities of a TI-81 calculator. This was done while the instructor was explaining to the class that the graph of $y = (1+x)^{\frac{1}{x}}$ would give some idea of what the limit might be, but graphing such a relation was currently beyond the students' capabilities.

What the instructor did do was to borrow a calculator from a student (he had misplaced his) and after some adjusting to the device's idiosyncrasies (a source of shared amusement in the class) proceeded to determine the limit by what he termed "brute force". While using available calculating power, the instructor cautioned against the unsceptical use of the brute force method and at the same time foreshadowed the role that this limit would have in exponential and logarithmic functions. The reformer in me found himself much heartened by this direct use and advocacy of technology in the calculus classroom, albeit at a very primitive level.

A brief preamble to the next section, properties of limits, preceded a ten minute recess. The instructor spoke of moving from a naive concept of limits to a definition that could be used mathematically. The instructor began with, "Using the ϵ/δ definition of a limit from section 1.4, the eleven properties on pages 70 and 71 can be proved as theorems. Now, we're not going to prove them. The book proves one of them; a few more are done in the appendix here. But, I'm not going to look at the proofs of these things here. At this point of the game, I just want you to become familiar with the results, so you can use them in any situation that might require them." This situation further added to the

confusion I felt about what the role of proof was in this course.

These properties, in the context of this course, were to be used in algebraic manipulations allowing the evaluation of limits. The instructor spoke about how the "breaking up of a complex problem in simpler problems" is a process frequently followed in mathematics and the limit properties allowed us to reduce a complex limit expression into a number of simpler limits which could be evaluated. I came to identify asides such as this as "meta-comments" that spoke more about the nature of doing mathematics than relating directly to the course material.

During the ten minute recess there was a flurry of conversation among the students, primarily on the assigned problems. The instructor remained and answered student questions, although few had to do with mathematics per se. In particular, one student noted a discrepancy between what was covered in class and the sections assigned for reading. The reading assignments and some of the problems were in some instances ahead of the lectures. Another situation dealt with the difficulties facing a student who had acquired a first edition of the text. Yet another student was concerned that she might miss the last available bus if the lecture went as late as 9:30. One student spoke about a velocity problem. Several of the students who could not get the attention of

the instructor elected to talk with the teaching assistant. It struck me that for many students this ten minutes might be the part of the evening where their involvement is greatest.

After the break, the instructor offered a naive definition of continuity of a function at a point. The purpose of this was to explain that $\lim_{x \rightarrow a} f(x) = f(a)$ when f is continuous at a . I was bemused by the circularity of this approach, while at the same time granting the usefulness of the idea, "if f is continuous at a , let $x = a$ and evaluate $f(a)$ to evaluate $\lim_{x \rightarrow a} f(x)$ ". As the instructor tried to go on to the limit properties, one student questioned him about continuity. The instructor responded by talking about classes of familiar continuous functions and giving examples of discontinuous functions. The language used for discontinuities involved "holes" and "rips". As the digression progressed and the instructor measured the confidence of student response, he commented, "How about cotangent? You guys are running through your minds, " 'I better look up these trig functions, these graphs.' Not a bad idea." The class responded good naturedly with laughter. There was an easiness in the class at this point.

Returning finally to the algebraic manipulation of limits, the instructor identified this strategy for evaluating limits as, "This is the way you would usually work

it out. You would evaluate the limit by algebraically simplifying the function."

The use of hand calculators in evaluating limits, the "brute force" method, was relegated to the strategy of last resort.

An example from the text, $\lim_{w \rightarrow -2} \sqrt[3]{\frac{4w+3w^2}{3w+10}}$, served as a springboard for a "review digression" on polynomial and rational functions and their continuity. Lack of student response to instructor questions suggested that this review was well warranted. It took a fair amount of prompting on the instructor's part to get even one or two students to suggest that the function in the example might be continuous at $w = -2$.

Working on this one example changed the mood of the class. The variable in question was w , not x ; the limit was equal to the value that w was approaching; the question of the continuity of a rational function was disguised by the composition of this function with the cube root function. Even the arithmetical calculation of the expression with $w = -2$ caused one student to borrow a calculator from his neighbour. Student involvement faded dramatically after this example.

The instructor invited students to turn in the text to where the limit properties/laws/theorems were listed (with

red outline borders). He spoke again of the ambiguity inherent in the situation of accepting the truth of these properties without proof, suggesting that the students could prove them when they covered the next section, or would prove them if they took a course in analysis. While reading the properties from the text, the instructor carefully identified the hypotheses and conditions under which they held, but actual examples were not offered. The class was asked to "translate" the fourth listed property into "English", but by this time there was little energy left and no response was forthcoming. When the properties were used to work through the previous example, student interest and response strengthened.

Another example from the text was used:

$\lim_{x \rightarrow -2} \left(\frac{x^2}{x+2} \right) + \left(\frac{2x}{x+2} \right)$. This struck me an excellent example with which to work. The obvious initial move in evaluating the limit would be to find the limit of each of the addends. But neither of these limits is finite. So the next move was to add the expressions to form a single rational expression. Then factor the numerator and cancel the $x+2$ factors top and bottom to determine that the limit did exist and was -2 . Two important concepts came through in this example: sometimes algebraic "simplification" was done through combining rather than breaking up expressions and two functions without limits at a given point could be

algebraically combined into a function with a limit at that point. The instructor carefully, but too quickly, brought these aspects of the solution to the class' attention.

Just after rushing through this example the instructor said, "Gosh, I don't know. I sure seem to go slow...Do I seem to be going too slow or too fast for you?" The students replied that he was going too fast; the instructor stated that he was going too slow to get through the assigned material. Confirming the class' opinion on the pace, a student interjected a question about the previous, "completed" example.

Parenthetically, before beginning another example, the instructor told the class to ignore the assigned reading and questions in section 1.4. Section 1.4 dealt with the precise definition of limits upon which the proofs of the properties in section 1.3 could be built.

The third example in this sequence was announced as number 39 on page such and such. A student pointed out it was actually number 41. The instructor had found himself caught in a small way by the shift from a first to a second edition. The example, $\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x} - 1}$, was done by multiplying the numerator and denominator by the conjugate of the denominator, "a technique that usually works in this situation." The use of a calculator to determine the limit was again listed only as a last resort. Furthermore, the

instructor promised that the limit evaluation questions on the tests and examinations would be such that the student would have to use algebraic manipulations to evaluate them. The tension between student understanding of underlying concepts and student ability to perform mechanical manipulations was very apparent in this example. There was so little time to cover the material in the course, yet the instructor was obligated to work carefully through this rather limited piece of algebra. He taught it well, but to what purpose, I wondered.

At 9:20 the instructor assigned the statement of three theorems for student reading. He discussed each briefly and concluded the lecture almost as if he were talking to himself, "I'm going to assume that I'm finished now. Big deal; 1.3. I've still got 1.4 and I'm getting further behind every night. We'll just have to cope."

The class broke up into little groups and, as usual, several students approached the instructor with questions. Students also spoke with me about problems for some time after the class ended. One student had graduated from high school last spring and had been taught by a teacher I had known fairly well a number of years ago. This student quickly adopted me as a tutor. In exchange, she spoke with me in detail about her experiences here and in high school.

This was my first opportunity to establish a personal contact with my classmates in this field work.

The first tutorial session was held at 6:30 on Thursday, September 12. The teaching assistant (hereafter referred to as the TA) was a graduate student just beginning a M.Sc. program in mathematics. He had been assigned to this course by the Department of Mathematics and Statistics.

Seventeen students (including myself) attended this session. It was held in a room with tables arranged in a rectangle. Two doors were on a wall opposite a wall consisting primarily of full length windows overlooking downtown Vancouver. Chalkboards covered the wall at the front of the room (placing the doors near the front and back). The first sets of assigned problems were passed from student to student to the TA, who was at the front of the room.

After the problem sets had been collected, the TA solicited questions from the students. It became clear that he had not had an opportunity to review the material on which the students were basing their questions. Nor did he seem to have a sense of the mathematical maturity of this group. Student frustration with the situation became evident on several occasions. Comments were made about the text not being worth much, the excessive number of unanswered

questions a student had, and the cruelty of the choice of some of the problems assigned.

Throughout the hour, most of the questions were asked by just three students. Two students sitting in one corner seemed to be working collaboratively with each other and another student did bring to the class' attention the existence of a Review of Algebra as an appendix in the text.

I found myself in a quandary. I had been a teaching assistant some fifteen years earlier, so that I had quite a bit of sympathy for the situation in which this TA found himself. On the other hand, I felt that I had answers for the questions the students were asking and I was strongly tempted to take control of this session and "do the job better". Because I still had grave concerns about the role of the researcher in a study such as this, I tried to restrain myself. Sharp glances from some of the students indicated that I was stepping too far out of the participant observer role in my desire to help my classmates.

The session ended about ten minutes before the beginning of the lecture. I and several others carried a certain sense of frustration into the lecture hall. Adding to my anxiety, I encountered a student who had attended the school where I taught. We had a discussion about why I was in this course and what he was currently doing. Due to our conversation, we both entered the lecture hall after the class had begun.

The lecture tapes indicate that the instructor had been busy clarifying the assignments with a number of students. Curiously, the instructor also acted as a departmental messenger by forwarding messages to three students.

The class began with the instructor assigning sections to be read, odd numbered problems to be done, and the second problem set. A student asked if it was possible to have copies of the textbook placed on reserve in the libraries. Already, the university bookstores had run out of copies. A discussion about the availability of the texts in the library ensued.

This sort of activity raised the recurring question of the efficient use of time. Undoubtedly, these were real problems faced by the students and the instructor showed his care for the students by addressing them. On the other hand, time was at a premium in this course and I wondered if this was the best use of class time. (I was honest enough to realize that a great deal of the time in the classes I taught was also taken up by similar activities.)

The instructor told us that he was going to cover some of the content in sections 1.4 and 1.5 tonight and that it would be up to us to use the text to pick up any extra material. "Make sure you do problems. That's the key," he admonished. I was puzzled that he chose to cover section 1.4 after telling us on Tuesday that we were not going to.

Section 1.4 covered material that was critical in defining some of the major concerns I had about the course. It involved the mathematically rigorous ϵ/δ definition of the limit of a function and was contrasted with the "naive" geometrical concept that had been dealt with earlier. The degree of student acceptance of the precise definition provided a strong index of the distance between the intended curriculum and the achieved curriculum in the "intended-implemented-achieved" curriculum sequence.

The instructor carefully built the definition by reviewing with the class the diagram used previously to illustrate a point discontinuity. Although I was familiar with the definition of a limit of a function and I was impressed with the instructor's presentation of the topic, I found myself amazed at the information density of the instructor's verbal description of the definition. "I can think of these points, putting round brackets here, as being the interval, the open interval as a matter of fact, from $L-\epsilon$ to $L+\epsilon$. Now if $f(x)$ is in this interval here, it looks like it is close to L , then all I have to do is go down to a . Produce an interval down here around a and make sure that every number chosen inside this interval is close to L . In other words, I mean inside this interval here. And this argument has to be valid for any positive number ϵ . So this interval can be very tiny here. We say that L is the limit

if there is a corresponding interval around a such that whenever x is in that interval that will force $f(x)$ close to L , somewhere up inside this interval."

As the instructor moved from the verbal description and diagrams to the formal notation of the ϵ/δ definition ($f(x) \in (L+\epsilon, L-\epsilon)$), a student's question about the use of \in to represent "is a member of" made me wonder about the obstacles facing some of my classmates. On the other hand, I was impressed with the way in which the instructor contrasted using the definition (geometrically with diagrams) to establish that a limit does exist in one instance and in a second case, the limit does not exist.

The instructor quoted the definition as stated in the textbook, but took time to explain the ramifications of each part of the definition. For example, in the phrase $0 < |x - a| < \delta$, the " $0 <$ " indicates that $x \neq a$. I wondered if I was overreacting to the subtleties that I perceived in this definition. The level of mathematical maturity inherent in this material did not seem consistent with the rest of the course material.

The instructor made the meta-comment, "Generally speaking, when you are dealing with limits, there are two problems involved. The first one is to come up with a reasonable candidate for the limit. And then secondly, you can use the definition to test whether you actually have

found the limit or not." Reflecting back on the previous lecture, I found myself increasingly confused about the purpose being served by the formal definition of function limits in this course. Had not the limit properties of the previous section been introduced to verify conjectured limits?

The instructor chose an example from the text, prove $\lim_{x \rightarrow -1} (3-4x) = 7$. Using properties of absolute value with which the students may or may not have been familiar, the instructor argued that $|(3 - 4x) - 7| < \epsilon$ whenever

$|x - (-1)| < \frac{\epsilon}{4}$. The instructor then asked the class, "What would a good guess for δ be?" He waited in silence for a significant amount of time. When no response was forthcoming, he began to explain the "obvious" in detail. At the same time, he was emphasizing that the work he had done in establishing the value for δ did not in itself constitute a proof. It would be necessary to rewrite the argument "in reverse", beginning with, "choose $\delta = \frac{\epsilon}{4}$ ".

The instructor was undoubtedly sensitive to how difficult the class was finding this topic. However, after working through this example, which many clearly did not understand, he added, "The problem with this method, of course, is that I've chosen a trivial example where it's easy

to produce this expression here. Most of the time it's not. I don't want to mislead you. It's only for relatively simple cases that you can actually use the definition directly to prove that the limit exists. What will happen in reality, you will prove a series of theorems from this definition and those theorems are what we call the limit laws."

"I know this is quite a technical little proof here. Take this as an introduction to it and you want to be able to do some very simple cases like this. If you want to study this kind of material, usually you go through a calculus course and then you go back and you pick up these finer points. And what you would do is Math 141 (sic), a one semester course in Real Analysis, which tends to look at all the nitty-gritty parts of this calculus course which we tend to gloss over." The body language of my classmates suggested that very few, if any, were encouraged by the work that evening to pursue their studies of this material.

The instructor moved on to the topic of continuity with about ten minutes left in the class. The students immediately became more attentive and involved. There was student response to instructor questions and students who had stopped taking notes resumed notetaking. For me this was yet another indication that the previous topic was inappropriate for this group.

The definition of continuity and some properties of continuous functions were quickly covered and the instructor formally ended the class commenting, "I'm sorry to rush you, but I've got to pick up the pace." After class, as usual, a number of students remained to discuss problems with the instructor.

A tutorial session was scheduled for after the class. Only two students besides myself stayed. The instructor tried to locate the TA, but was unsuccessful.

I used this opportunity to talk with the two students, Thomas and Catherine, about their reactions to the lecture. Neither of them was comfortable with their understanding of the ϵ/δ definition. Catherine felt that proof in a mathematics classroom had a capital "P" on it. She associated it with proof in geometry for which "there was only one way to do it". Both students had come to this lecture without having read the relevant sections in the text, so that they did not know what the instructor would be teaching.

Catherine said that she lacked the perspective to decide what is important and what should be taught. If it was part of the course she would accept it. Yet, I found, to some extent, students do decide what is important in their acceptance or rejection of material. On Tuesday, the class accepted the material on limit properties and their use.

Later, on Thursday, they responded positively to the definition of continuity. The material on the formal definition of limits was, however, rejected. The body language was very negative ("sitting back") and student involvement, both verbally and through notetaking, was minimal.

The tension within me that had been building throughout the evening, beginning with the first tutorial session and continuing through the lecture, resolved itself when I completely abandoned the researcher role for a while and spent time teaching. I tried to illustrate and justify the material covered this evening by developing and contrasting

$\lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right)$ and $\lim_{x \rightarrow 0} \frac{\sin x}{x}$. Both of them seemed receptive of my teaching efforts.

By the end of the "tutorial hour" it was past 9:30 p.m., dark and rainy. Thomas had transportation home, but Catherine would have had to take the bus. I offered her a ride to her neighbourhood. Because of this opportunity, I found myself with a key informant.

During the drive to her neighbourhood, Catherine spoke to me about her background, her current course of studies and her learning situation (including the fact that on Thursdays, the calculus lecture and tutorial were her sixth and seventh hour of classes!).

By the end of the first two weeks of classes I felt that a number of aspects of my research had begun to come together. There were patterns and trends, some expected, some unanticipated, that I thought I would follow throughout the term.

There were the matters of tutorials, student transportation, the physical layout of the classrooms and what materials the student would bring to and use in lectures. What background mathematics was assumed and what background did the students actually have? How much time could be spent out of class on this course? How was this time spent? As it turned out, not all of these issues were pursued to any significant extent later on in my study.

The tension between the various roles I was playing was quite evident. At one moment I would be an observer, watching and recording student reactions and behaviours. Next I would be a participant, taking lecture notes and submitting problem sets. At other times, I was a researcher distributing and collecting informed consent forms and explaining my project to others. Many times I would be a classroom teacher with strong opinions on how a topic should be taught. At other times, my reading of literature from the calculus reformers would lead me to question what should be taught and again, how it should be taught.

I had established some identity within the class. Students, the instructor and, to a lesser extent, the teaching assistant knew something of me as an individual. I was beginning to interact with the members of the class in a number of ways and as I did so, I would find my focus of interest shifting as they offered varying perspectives on what was important to them and why.

September 17 to October 10, 1991

By Tuesday, September 17, many of the other students and I had reached something of a "comfort zone" in this course. Arriving five minutes before the lecture, I met with several students who asked me how my research was progressing.

Entering into the comfortable layout of Room 1430, I encountered a convivial atmosphere. There was frequent laughter and the instructor and students were bantering with each other. This was in marked contrast with the mood of last Thursday.

The instructor gave us sections to read for next class. Significantly, because "as you know, this course is crunched for time", the instructor announced that section 2.3, Rates of Change in the Natural and Social Sciences, would not be covered in lecture. Students were to read the material and attempt "every second odd problem". This and the omission of section 3.9, Applications to Economics, in the course syllabus suggested that students would have to look elsewhere to see how the calculus was used in other fields.

Assignments collected and procedural details attended to, the instructor began with section 2.1 and the formal definition of the derivative. He identified the definition

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \text{as the most important one in the}$$

course. It would occur on both midterms and finals. Students should "know it well enough to tell your grandchildren".

Furthermore, as if the instructor was aware of my concerns when he covered this material earlier, he spoke to the class about distinguishing the roles of x and h in this expression.

The first example dealt with how to find the derived function of $f(x) = \frac{2}{x+3}$. The instructor asked the class how they would evaluate the expression $\lim_{h \rightarrow 0} \frac{\frac{2}{(x+h)-3} - \frac{2}{x+3}}{h}$. There was some student response, but in general, there was no clear sense of what to do with it. The instructor commented that this limit would be impossible to evaluate directly and that, one way or another, we were led back to algebraic manipulation. It occurred to me that much of the algebra necessary in the early stages of this course dealt with simplifying rational expressions.

The student next to me, a girl named Ronnie who had adopted me as a teaching assistant, showed me her (correct) derivation of $f'(x)$. This little interchange between two students was typical of what happened fairly frequently in Tuesday evening classes. They had little or no effect on the flow of the lecture.

After completing this example, the instructor made the first of several digressions on the historical development of the calculus. He had travelled to England last summer and had visited Cambridge. There he had seen some of Newton's original manuscripts on fluxions. Later in this lecture, he

would comment on the different notation used for derivatives and the impact of Leibniz on this. There was little notetaking during these asides, but a number of students remained attentive. I made a note to myself to check with my classmates as how they felt about these digressions.

Another mathematical background impediment (rational exponents) cropped up when the instructor next established the derivative of the function $f(x) = \frac{1}{\sqrt{x}}$. Students were confused when he rewrote $\frac{-1}{x(2\sqrt{x})}$ as $\frac{-1}{2} x^{-\frac{3}{2}}$. The instructor sensed this and asked the class the value of $4^{\frac{3}{2}}$. After some hesitation, one student volunteered 6 as an answer and another responded with 8.

During the break, the instructor was besieged by several students concerned about how to approach an assigned problem: providing an ϵ/δ proof that $\lim_{x \rightarrow a} c = c$. I was unable to observe this interaction because a number of my classmates had approached me for advice on the same problem. The instructor suggested that this would be a good problem for the TA to work through in the tutorial sessions.

The timing of the assignments and the lecture material was interesting. Student attention was focused on problems they had been working on recently; the lectures were on

material that was, in most instances, "in the future" for the students' attention.

During the ten minute recesses on Tuesday evenings the instructor was essentially giving ten minute tutorials to students. This evening he called for closure to the questions by saying, "I've got to get going guys. We've got to just buzz along." He added that he would be glad to answer questions after class.

After break, the instructor's preamble established the importance of derivatives when working with rates of change and the need for being able to calculate derived functions more readily than the limit definition would allow. Hence, we would establish differentiation formulas, where "differentiation is the process of finding derivatives". The language of mathematics must be extremely frustrating for some students.

In developing the power formula, the instructor maintained that "if you mention the power rule or formula to anyone who has had a bit of calculus, they will know exactly what you mean." Mentioning that he was going to "mix up his notation, so that we would get used to it", the instructor proceeded to prove $\frac{d}{dx} (x^n) = nx^{n-1}$, $n \in \mathbf{Z}^+$. One student asked why n was restricted to positive integers and the instructor replied that the proof he was giving would be for positive integers, but that the formula could be extended for

other powers of x . He then asked if any in the class were familiar with this rule. Some were.

The proof he outlined was based on the Binomial Theorem, on which he commented, "which I hope you are all familiar with." The text offered both this proof and one based on the factorization of $x^n - a^n$. The instructor had some students guess coefficients for the expansion of $(x + h)^n$. To my surprise, he also mentioned the relation of Pascal's Arithmetic Triangle with Binomial coefficients.

I felt that a great deal of time and effort was being spent developing this particular formula. Furthermore, background mathematics was being invoked that was not relevant to any other part of the course. In fact, I questioned the value of "proving" these rules in this course. How did my classmates feel about this? Would they not accept the validity of the rules on the authority of the instructor and the text? What was at issue here: the development of the rules or the ability to use them?

Sixteen students attended the first tutorial session on September 19. The TA began with the proof that $\lim_{x \rightarrow a} c = c$. He concluded a general discussion of ϵ/δ proofs with the comment, "That's the problem with mathematics sometimes." Students responded to this with laughter.

This evening the doors to the classroom were left open, students were engaged in conversation in the corners of the

room and some street noise was coming in through the windows. I found myself distracted.

One student asked the TA to work through an odd-numbered problem from the text, thus suggesting that at least some of the people were attempting to do problems other than those that were to be submitted. In working through problems, the TA sometimes displayed lack of familiarity with the nomenclature of the text and lectures. It seemed that the students were taking the initiative in communicating with him, but the TA was responding positively to questions.

One question, however, disrupted the effectiveness of the remainder of the session. The TA began the solution in a straightforward manner, but soon found himself in difficulties from which he had trouble extricating himself. During this solution, the attention of the students wavered. When the TA made a simple error in differentiating a function, none of the students corrected him.

The class ended at 7:20 and we sought out the lecture hall, only to find that its use had been pre-empted for the evening. As a group we tracked down our classroom for the night, which was an upscale theatre larger than the usual lecture hall.

The pre-lecture discussions this evening were not so much on specific problems as on the general feelings students were having about the course and the assignments. One

student commented, "I've never been this far behind this early in a course." The instructor replied, "Let me tell you, this course is packed. You have to make sure from the very beginning that you put a lot of time into this course." He expanded on how full he felt the syllabus was and how little time there was to cover it all. The section on applications would not be covered in lecture, not because he felt it was unimportant, but because he was unable to cover everything in detail.

The instructor also pointed out to the class that the venue for the second tutorial session had been changed without his knowledge. The absence of the TA last Thursday was a consequence of not knowing where he was. I and several others had determined the location of this tutorial session this week by checking the notice board in the lobby. Coupled with the room change for the lecture this evening, this event suggested that just finding the correct classroom could sometimes be a hurdle. Several students did arrive late for the lecture this evening because of the room shift and the lecture itself was briefly interrupted by someone entering the lecture hall, glancing at the notes on the overhead and hurriedly leaving (to the amusement of the class).

The instructor reviewed the work begun last lecture and eventually reached a point where further differentiation rules would be proved. The instructor asked the class to

confirm where in the list of rules he had left off. After determining this from a student, he went back to his notes and continued rule #4, which he stated, but did not prove.

With this set of differentiation formulae, the instructor claimed that the class had sufficient information to find the derivatives of polynomial functions, "which play an important role in mathematics". Several examples were worked through. Formal derivatives of polynomial functions resulted; there was no reference to the graphs of the derived functions. Of the three representations of a function: numerical, graphical and symbolic, the symbolic representation was certainly receiving the most attention.

The instructor did pose a question to the class to see if they were grasping the concept. He asked about the relationship of the degree of a polynomial function to that of its derivative. The silence of the student response was conspicuous and disconcerting. "A good examination question," quipped the instructor. I wondered what was being learned. The instructor persisted with this question until at least one student (Catherine) had some sense of the underlying concept.

The product rule was the next differentiation formula. Fully ten minutes were spent proving this rule. The proof followed that in the text and used the limit definition of a derivative. In fact, one student responded to the

instructor's questions by referring to the text presentation. Most of the class spent their time copying the proof from the overhead notes.

The "trick" to the proof was the shift from

$$\lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} \quad \text{to}$$

$$\lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x+h)g(x) + f(x+h)g(x) - f(x)g(x)}{h} . \quad \text{The}$$

instructor pointed out that, "We are in the luxurious position of knowing what the answer is." The correct "end result" of this limit pulls us in the right direction. The instructor gave full credit to the genius of Newton and Leibniz in developing this formula. "Remember neither Newton nor Leibniz had the luxury of knowing what the answer is."

These comments went a fair distance in helping the students realize the difference between understanding a proof and creating a proof.

After developing the product rule, as an application, the derivative of $f(x) = \left(\frac{2}{x+3}\right) (x^3 - x + 4)$ was sought. My immediate reaction to this was that the derivative of $\left(\frac{2}{x+3}\right)$ was not yet "fair game" for my classmates and I was annoyed by what I felt was an inappropriate example. I was chagrined when the instructor confessed that we did not yet

have the rules to find the derivative of $(\frac{2}{x+3})$ (rewritten as $2(x+3)^{-1}$), but, in fact, this derivative was one of the examples that had been worked out earlier using the limit definition.

Catherine, who I had now identified as one of the most capable students in this class, asked the instructor if the power law could be extended to $n = -1$ for this instance. The instructor agreed that it could in this case, but ended up trying to explain the distinction between the x^n and $[f(x)]^n$ with some foreshadowing of the chain rule. This discussion, as far as I could see, involved only Catherine and the instructor.

The Quotient Rule was stated without proof and an example was completed. Moving quickly, the instructor moved on to the section on the derivatives of trigonometric functions. Although the matter had been brought up and discussed before, one student (Ronnie) again asked why section 2.3 was being omitted. The instructor said that this section was nice, dealt with applications of section 2.2, but did not offer any new theory. Was this a criterion for inclusion or exclusion of material in the course?

In order to find the derivative of the sine and cosine functions, the instructor identified the need to evaluate two

limits beforehand: $\lim_{h \rightarrow 0} \frac{\sin h}{h}$ and $\lim_{h \rightarrow 0} \frac{\cos h - 1}{h}$. The

algebraic proofs were to be found in the text. The instructor suggested that under the time constraints we faced that "brute force" evaluation of the limits was appropriate. Even this fallback to calculators required a brief review of radian measure for trigonometric functions. The limit of

$\lim_{h \rightarrow 0} \frac{\sin h}{h}$ was found to be 1 using calculators, but then,

for some reason, trigonometric identities were used to

establish that $\lim_{h \rightarrow 0} \frac{\cos h - 1}{h} = 0$. Considering the

hesitation of the class in responding to questions about trigonometry, the extra time spent might well have been warranted, if only to encourage students to review their knowledge of trigonometry.

The instructor ended the lecture about five minutes late and seven of us went off to find the room assigned for the second tutorial session. Unlike the room where the first session was held, this room had no windows and only a single door. It was quite small relative to the venue of the 6:30 tutorial.

The TA handed back assignments and explained the marking scheme. He failed to announce that solutions were available in the Harbour Centre Library. One of the students was visibly upset with the results of his assignment and crumpled

his papers. Other students, notably Catherine, were displaying the effects of fatigue.

Examples that had been brought up by students in the first session were now put forward by the TA, indicating that he was using his experiences earlier in the evening to improve the tutorial sessions at the end. While he was presenting his solutions to some problems, several of the students were working, individually or in pairs, on their homework assignments.

When the students had their turn in presenting problems for solution, I observed that some of the odd numbered questions that were done generated discussion that led students beyond the boundaries of the course expectations. Notation, especially the use of \forall and \exists , was a problem for some.

The students and TA engaged in guessing what material would show up on the midterm. This sort of discussion would take up a significant amount of time in the tutorial sessions throughout the term. It was assumed by the students that ϵ/δ proofs would be on the test.

Two concerns arose that evening that I had not considered as significant before my study: health and fatigue. The cold that the instructor had last week had worsened so that tonight's lecture was often punctuated by coughing. The instructor himself indicated towards the end

that he was "running out of steam" and that he was losing his voice somewhat. Extreme fatigue was exemplified by Catherine in the last tutorial session. She had participated strongly during the lecture, but had no energy left at all for the tutorial. I observed these factors impacting on the behaviour and performance of nearly everybody in this class, including myself, before the end of term. It occurred to me that there was no provision for being ill or tired in such a course.

By the lecture on Tuesday, September 24, my research had progressed to the stage where I was spending much of my time in conversation with my classmates. I was shifting from a detached observer frame of reference in which I typically was describing the class as a whole to that of participant who was dealing with individuals.

One consequence of this evolution in my research is that my field notes became more concerned with specific persons and their interactions with me rather than the instructor-student interactions I had been detailing earlier. The audio tapes of the lectures, especially the portions recording just before and after class, helped to continue monitoring some of the student-instructor conversation while I was otherwise engaged.

Prior to the class that evening, I spent time talking with Catherine. We spent some time discussing the spacing of

the lectures. On Thursday, the material covered on Tuesday was relatively fresh in our minds. But, during the first hour of Tuesday lectures it was often an effort to remember what had taken place the previous Thursday. Catherine also spoke about the frustration and anxiety that she felt because she sensed she was falling behind in this course. Our conversation drifted to what might be on the midterm test and Catherine's delight in "stumping" a professor with a difficult question earlier in the day.

While I was talking with Catherine, other students were bringing questions forward to the instructor. One student sought help with the evaluation of a limit of a trigonometric function. In the ensuing discussion, it turned out that the student did not realize that the function was continuous everywhere and need only be evaluated at the point in question ($x = \frac{\pi}{3}$ in this case). After the instructor had established this with the student, he tried to have her work through the evaluation. "What's $\sin \frac{\pi}{3}$?", he asked. Silence and an admission, "I don't know. Point...?" followed from the student. The instructor supplied the value of $\frac{\sqrt{3}}{2}$. The spectre of deficiencies in background math seemed ever present.

The instructor spoke briefly with one student who was older than me. This student was a professional engineer who was taking the course for "the fun of it". They spoke about how the calculus would "come back to the engineer" as he worked through the course and about the mathematics that he did in his profession.

Before beginning the lecture, the instructor was reminded to attach the microphone to his jacket so that the lesson would be recorded. He asked if anyone had accessed the tapes yet. No one, including myself, had. The assumption was that the tapes were available in the Harbour Centre Library.

He also reminded the class that solution keys for the assignments were also available in the library. Showing admirable flexibility, the instructor, responding to student feedback, changed the due date for the assignments from Thursdays to Tuesdays. This was to allow students to consult with the TA about the assigned problems before they were due.

Almost as if he had listened to the discussion I had had with Catherine about the spacing of lectures and the difficulty of remembering where we were on Tuesday lectures, the instructor began with several minutes of review of where we had been and where we going in the course. Today's topic was to find the derivatives of the trigonometric functions.

The instructor assigned (re-assigned) every second odd problem in sections 2.4, 2.5, 2.6. As these problems were not collected, there was no student accountability for their completion, I noted to myself that I should ask my classmates how many of these questions they attempted. Finally new material was begun with the problem of evaluating $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x \sin x}$. My immediate response to this example was, "Why bother?", but I soon found intrigued because I did not know what the limit was. (Nor did I feel that using l'Hôpital's Rule would be fair.) This was an aspect of mathematics I had been overlooking recently. Sometimes you do mathematics because it is fun; a puzzle to be solved. This sentiment was echoed by Catherine in a conversation I had with her later.

The instructor argued that the function in question was discontinuous at $x = 0$ and asked for student suggestions on how to algebraically manipulate the function until it could be separated into recognizable functions. The class was unable to provide any suggestions, let alone the "correct" one of multiplying the function by $\frac{1 + \cos x}{1 + \cos x}$. While multiplying, the instructor added the meta-comment, "Using the [instructor's name] rule, never multiply the denominators out. It's usually not productive." The unintentional pun notwithstanding, this comment struck me as a clear example of the apprenticeship role of mathematics students.

At the end of the example, the instructor suggested, "If you have any doubts about what I'm doing here. I don't think you do. You probably believe me. But you could revert back to the brute force method." A student spoke out, "If you got stuck on an exam, could you use a calculator?" The instructor replied that he would try to phrase the questions so that calculators would only be used as a last resort.

Twenty to twenty-five minutes into the lecture, the derivatives of the trigonometric functions came up again. More meta-comments from the instructor made me hope that my classmates were paying attention. He spoke of memorizing the derivatives of various functions; "loading your pockets with tools that you can bring out in the appropriate situation". He also noted that the derivative of a trigonometric function is also a trigonometric function, "keeping it in the family".

The development of $\frac{d}{dx} \sin x$ involved the expansion of $\sin(x+h)$. When the instructor asked the class where to proceed from $\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$ there was a lengthy silence. Finally, the engineer volunteered that expanding $\sin(x+h)$ might work. The instructor asked the students for the expansion. There was some laughter when he added, "Without looking it up in the book," because two or three students quickly went to the appendices of the text. Without the text, no one was able to give the appropriate identity.

After completing the development of the derivative of the sine function, the instructor invoked another mathematical commonplace by saying, "Similarly, $\frac{d}{dx} \cos x = -\sin x$." This was after he had pointed out that the derivative with respect to x of $\cos x$ is not the "obvious" function, $\sin x$. The instructor expanded on the mathematician's use of similarly, "I say *similarly*, in fact you can duplicate this proof. It would be exactly the same steps you would go through, except you would have the $\cos(x+h)$ here and, as this gentleman has indicated, there is probably a formula for $\cos(x+h)$."

After exhorting the class to look up the basic trigonometric identities, the instructor allowed that we would not be making much use of them in this course. They would, however, be used extensively in Math 152 for integration.

More time was spent on developing the derivative of the tangent function. Time was spent discussing how one would approach the problem initially. The instructor, rather like a magician producing a rabbit from his hat, invoked a trigonometric identity, $\tan x = \frac{\sin x}{\cos x}$ and then applied the quotient rule. "Just use a little trig identity first of all. ...Tangent, we all know, is equal to sine x over cosine x . Now why is this good? Number one, I know how to

differentiate both sine x by itself and cosine x by itself, I just gave them to you. And I also know how to do a quotient by the quotient rule. Thing solved! That's all there is to it."

Again and again, observing the development of these familiar topics, I was struck by how much "doing mathematics" was being explicated by the instructor as he sought to cover the course material. Unfortunately, the established differentiation formulae, $\frac{d}{dx} \tan x = \sec^2 x$, required some simplification. As always, the students were not told why $\sec^2 x$ was the preferred "simplified" form of the derivative. Nor did any of them seem inclined to question this "best" form of the derived function.

The instructor did point out the mnemonic for memorizing derivatives of trigonometric functions: the derivative of a trigonometric function can be found taking the opposite of the derivative of the cofunction and replacing each trigonometric function in the derivative of the cofunction by their cofunctions. For example, since $\frac{d}{dx} \tan x = \sec^2 x$, then $\frac{d}{dx} \cot x = -\csc^2 x$. In developing the derivative of $\sec x$ ($\frac{d}{dx} \sec x = (\tan x) (\sec x)$), the instructor emphasized the inertia of mathematical convention when he said, "Almost

every book writes them this way . I'm going to commute them around. This is $(\sec x)(\tan x)$."

After invoking the mathematical use of term "similarly" once again to develop the derivative of the cosecant function, the instructor spent further time explicating the patterns in the table of differentiation formulae for the trigonometric functions. "There is a summary which you must put to memory."

The lecture continued with some worked examples from the textbook involving formal differentiation. The first problem was to find the derivative with respect to x of $y = (x)(\sin x)(\cos x)$. It was worked out in detail and the "final answer" reached by using a double-angle identity. The second problem was to find the equation of the line tangent to the curve

$y = 2 \sin x$ at the point $(\frac{\pi}{6}, 1)$. In the first case, students

did not know which identity to use. In the second problem, the instructor sought dialogue with the class. But the

students were very tentative about evaluating $\sin \frac{\pi}{6}$. The

instructor sketched a graph of the curve. Asking for the value of $\sin x$ at $x = \pi$, he had no correct responses. The instructor went back to the unit circle in an attempt to help the class. He was well aware of the difficulties they were

experiencing. He concluded his mini-review of circular functions with the comment, "I'm only doing this the one time here in a bit of detail, just to remind you of what is going on here." I noted how difficult it was for a good teacher to ignore gaps in the knowledge base of his or her students.

During the break I spoke with a student who had graduated from high school in 1981. He allowed that algebraic manipulation was catching him out as he did odd numbered problems from the text. But the trigonometry was even more problematical. The trigonometry done in high school was "done once and put away...at least with algebra you had a chance to use it for three or four years in high school."

The remainder of the break I spent working through a homework problem brought to me by another student. Many of the students spent the recess talking with each other.

Class reconvened at 8:40 with the instructor saying, "It always takes longer than I think it's going to take. You know when I said to study 2.6? Scrap it. We'll have to do that next day." This brought muted cheers from the class. I made a note to myself that these people, students and instructor alike, were really working hard at this course.

Section 2.5 dealt with the Chain Rule and differentiating the composition of functions. The instructor began a straightforward development of the rule, but my

reaction at the time was that the presentation was too fast and too abstract. Rather than work with a "concrete" example, the instructor chose to talk about general functions f and g . I observed that only about 30% to 50% of the class were taking notes at this time, in contrast to a substantial majority who were taking notes during the first hour.

There was a great deal of technical vocabulary in this part of the lecture. The instructor explained what is meant when a function is said to be differentiable and talked about the non-commutativity of the composition operation. Time was spent discussing which notation is most appropriate for representing the chain rule.

The instructor continued the lecture with minimal interaction with the class. He said that he wanted to outline a proof that is "not absolutely precise" but is "better than the one given in the text". His comment that "there are still flaws in mine, but probably none of you will find them," brought up some student laughter.

The proof involved the use of several dummy variables such as u for $g(x)$ and k for $g(x+h) - g(x)$. To argue that $k \rightarrow 0$ as $h \rightarrow 0$, the instructor used the continuity of g which was based on the hypothesis that g was differentiable. By 9:05 what student attention remained was fading quickly. One student beside me was merely rocking back and forth in his chair. To his credit, the instructor persevered in

soliciting student response. But it seemed a difficult task. As an exception to the general non-response to this proof, one student, Catherine found it to be cute.

The instructor sought to connect the topic with economics and the use of the term *marginal* in economics. When he questioned the class he found that very few of the students had ever studied economics.

Having run out of acetate, the instructor moved to the chalkboard and worked through examples: find

$\frac{d}{dx} (x^2-1)^3$ and $\frac{d}{dx} (x^2-x)^{\frac{1}{2}}$. There was some positive body language (nodding of heads, sitting forward) as the lecture moved to actual examples. There was a marked contrast in attentiveness and body language of the students when they worked through these examples to the chill that pervaded the class when the proof of the chain rule was being developed.

The instructor concluded the lecture with a meta-comment that "our answers" might differ from the ones in the text because of the amount of simplification that was done.

The evening had been very full. There was development of derivatives of trigonometric functions, an exhortation to memorize some formulae, a display of formal differentiation techniques and a proof of the chain rule as well as various hints and suggestions as to how to actually "do math". It

seemed to me that many agendas were being covered more or less simultaneously.

I arrived ten minutes late for the 6:30 tutorial on September 26. About a dozen people were there when I arrived. The TA was working through some limit problems and demonstrating a number of mechanical "tricks". He seemed much more comfortable with the material and text this week than last.

Several of the student questions dealt with background material. There was a question about the value of $\sin 3\pi$ and whether $\sec^2 x$ represented $\sec(x^2)$ or $(\sec x)^2$. The latter question echoed a comment made by Catherine last Thursday about the occasional difficulty in recognizing the structure of functions arising from the composition of other functions.

One question represented a recurring theme in tutorials. Asked to evaluate the one-sided limit $\lim_{x \rightarrow 4^-} \sqrt{16-x^2}$ some of us invoked the continuity of the semicircle and simply wrote the answer (with a sketch of the graph). The TA awarded full marks only if a sequence of limit rules were used. How were we to respond on midterms and the examination? The TA replied, "On assignments, go back in the textbook and use the methods of worked examples." Once again, the text was cited as the final arbiter of correctness.

The 7:30 lecture was in the theatre. One student seated herself in the first row, twelve sat in the next three rows and the rest of the class (five of us) sat in rows five and six. I noted that one of the first students who had spoken with me, the girl who had adopted me as a teaching assistant, was no longer in the class. I had no sense of why she dropped the course. She seemed to be working hard, was asking good questions and had not seemed particularly oppressed by any aspect of the course.

The lecture began with implicit differentiation. The material was clear and well received, although, as usual for Thursday lectures, there was less student-instructor interaction than on Tuesdays. Meta-comments were sprinkled liberally throughout this topic's development. Suggestions on when to use implicit differentiation, what level of simplification for the final result is appropriate and when it was possible to check our results by solving "explicitly for y ". There was also a digression on conic sections and quadratic relations when working on the problem of finding the equation of line normal to the curve $9x^2 + 4y^2 = 72$ at the point $(2,3)$.

The example was worked through in painstaking detail. Immediately upon its completion, the instructor said, "Let's move on to the next section." It was 8:07 and some thirteen minutes remained in the lecture. Section 2.7 dealt with

Higher Derivatives. During the preamble to the section, the instructor said with good humour, "Think there is any value in calculating fourth, fifth, sixth, seventh derivatives? Nod your heads. There is some value to it." But the applications of these higher derivatives would be found in later courses.

The second derivative was motivated by defining acceleration as the second derivative of the position-time function. The instructor did comment "on the kind of a little bit of strange notation" of $\frac{d^2y}{dx^2}$. Later in the lecture, he attempted to justify this notation by running

through a possible history of how it evolved from $\frac{d(\frac{dy}{dx})}{dx}$.

His confession that he was making up the story brought forth laughter from the class.

Catherine interjected with a question about higher order derivatives of non-functional relations. The instructor took a moment to reply to her. This was an interesting instance of class time being devoted to an individual, as I suspect her question had not occurred to other students.

The class concluded with the instructor telling us that, "You've got enough to get in there now and read. Study those two sections and do the odd problems." This struck me as an interesting insight into the instructor's concept of the role his lectures played in the course.

Five of us continued to the second tutorial session. The TA handed back the marked assignments to the correct students, indicating that he had already identified the participants in the smaller, second tutorial. Building on his experiences from the earlier session, the TA initiated discussions with us based on the questions and issues brought up in the first hour. One student, Thomas, asked the TA to go through, step by step, the trigonometry involved in one solution. Another student, commenting that the TA had done this to us last time, lamented the TA's use of a limit rule that was "not on the list". The rule used turned out to be a special case of one that was listed in the text.

Both students and the TA raised issues about what was to be on the upcoming midterm. Thomas and Catherine asked for help in finding general solutions to trigonometric equations and Thomas had additional questions about the differentiation of absolute value functions (and points where the functions are not differentiable). By the end of the hour the TA had responded to questions arising from a wide range of mathematical topics.

Prior to the lecture on October 1, I had an opportunity to talk with several students. Catherine had attended the calculus lab at the Burnaby campus and found it very useful. David, a student who was working night shifts, spoke to me about his time management concerns. He was getting the

correct solutions for the assigned problems, but he felt it was taking him too long. One student I spoke with was auditing the course. He attended lectures, but did not do the assignments. Nor did he intend to write the tests.

The instructor began once again with a five to ten minute review of the last lecture and then began to work through position-velocity-acceleration linear motion

problems. The position-time function was $s(t) = \frac{t}{\sqrt{9+t^2}}$.

Differentiating to find the velocity and acceleration functions involved rational exponents, formal differentiation and rational expressions. In the midst of general silence, one student raised a question about the algebraic simplifications. While I had serious doubts about how much algebraic manipulation was appropriate in this instance, my classmates seemed to feel it was part and parcel of the course material and tried to master it. On the other hand,

when the instructor asked for the value of $25^{\frac{3}{2}}$ there was no response from the class and he had to supply the answer.

Concerning simplification of derivatives, the instructor offered the following meta-comment, "If you haven't already discovered it, if you have to take more than one derivative [second and third order derivatives] it is much easier to simplify the function and differentiate then it is to take an

unsimplified function, differentiate it and then simplify it at the end."

Completing the problem, the instructor asked the class if there was a term in physics for the rate of change of acceleration with respect to time. In my role as a student I responded with the correct term, jerk. This interaction with the instructor exemplified the comfortable relationship which existed between us.

Before the lecture, Catherine spoke to me about working on application problems. She felt it was necessary to "reduce the problems to just numbers". Trying to remain cognizant of the real world situation underlying the mathematical model just complicated the problems for her. While working through this example, the instructor had taken what I considered a rather cavalier approach to identifying units and interpreting algebraic results in terms of the model. For example, the acceleration function $a(t)$ was negative for all $t > 0$. There was no mention of what this signified in terms of the particle's motion. I found this approach to applications, both by student and instructor, to be worrisome, especially in light of the calculus reform movement's condemnation of "plug and chug" calculus.

The next section, 2.8, concerned related rates problems. In his preamble, the instructor stressed that using the chain rule was central to the solution of such problems. He also

emphasized that "everywhere you see the word rate in mathematics...it's a derivative".

The instructor gave a four step format for solving related rates problems. He suggested that his format would be somewhat easier to use than the more elaborate scheme outlined in the text. The choice of solution format was left to the student. Some time was spent listing the four steps on the overhead and then explaining each step in detail.

Finally, an example from the text was cited. The instructor suggested that if we had our textbooks with us, we should look up the problem. Every single one of us had the text with us that evening! The instructor read and expanded on the problem, asking us questions to get the class involved.

After developing a diagram and determining which rates were already given in the problem, the instructor turned his attention to what I considered the crux of related rates problems : establishing a differentiable relation between the the functions whose derivatives were given as the "related rates". Typically, this involved using background mathematics. That is, mathematics learned in high school. In this example, even with hints and guidance from the instructor, the students were unable to identify that similar triangles were involved. Student responses suggested that

they wanted to use mathematics more sophisticated than elementary geometry.

Developing the solution to this problem finished the first hour of lecture. Just after declaring a ten minute recess, the instructor advised us of the problem in the text we would be working on next if we wished to preview it. As usual, the instructor spent the recess answering student questions.

The second related rates problem involved an airplane. There was a question about whether the constant speed of the airplane was airspeed or ground speed. When it came time to "relate the rates" in this problem, the instructor asked the class, "What do you do? What is your crutch for finding the relationship, usually?" Faced with silence, he referred us back to the diagram, which was supposed to "give great insight into solution to the problem". Using the diagram, several students made suggestions for establishing the appropriate relationships. No one thought to use the Law of Cosines, so the instructor had to point it out. Even after establishing the relationship to be differentiated using the Law of Cosines, there were several questions from the class suggesting that not everyone was comfortable with the Law.

Pushing ahead, the instructor had his relationship on the overhead. When he said that implicit differentiation was the next step there was nervous laughter from the class, to

which he replied, "Sometimes it's amusing. But I don't want to lose you. Do you understand where I'm going to?" One student responded to this by saying that there would be one question like this for the (entire) exam. The instructor countered by guaranteeing that there would be one question of this type on the examination.

After completing the solution to this problem, the instructor added, "I think you can appreciate that some of these problems can be challenging." He classified the first problem as being easy and the second of being of medium difficulty, with some harder problems to be found in the exercises. When one student suggested an easier approach to the second problem, the instructor agreed with him, but strongly recommended that we follow the problem solving pattern that he had given us or the one in the text.

My field notes of the lecture include my own solutions to the two problems. I was unable to resist the temptation of the related rates problems, so that instead of recording what the instructor was presenting, I indulged myself in some pleasant problem solving. Another instance of my teacher persona dominating the researcher for the moment.

At 9:10, with ten minutes remaining, the instructor introduced the next topic, differentials and linear approximations. In passing, he referred to "one of the most important branches of mathematics", numerical analysis and

the role of approximation in the field. There was enough time to establish a diagram and define the differentials of x and y .

As the class dispersed, the instructor spoke to several students, "From the questions you ask me, it's not the calculus that is slowing you down. It's the algebra. You should do many, many questions."

The TA began the 6:30 tutorial session on October 3 with, "Are you ready for the midterm?" The students responded with laughter. The TA talked about what he thought would be on the midterm, stressing the importance of being able to use the chain rule. During this discussion one student took it upon himself to close the door to the classroom, as there was considerable incidental noise from the corridors. Thirteen students, including myself were in attendance.

The TA had already set up a problem on the chalkboard to illustrate the use of the chain rule. During the development of the solution, I noted that tonight the students seemed particularly anxious to be actively involved. Several were unable to restrain themselves from offering comments and suggestions. Curiously, during a second example involving implicit differentiation, the TA solicited student comments, but was confronted with a tired, quiet group of individuals instead.

At the completion of the second example, find $\frac{dy}{dx}$ for the relation $\sqrt{x+y} + \sqrt{xy} = 6$, the TA stopped with, "The rest is just algebra. Would you like me to finish?" One student replied, "Yes please,", while another student muttered, "Just algebra!" as a mild expletive. Most of the students conscientiously copied from the chalkboard the algebraic steps supplied by the TA. I wrote "death by algebraic manipulation" in my field notes.

As the TA worked through a second implicit differentiation problem, this time involving the relation $xy = \cot(xy)$ I noted with approval how the TA was highlighting the significant aspects of the the solution.

Following this, one student asked to TA to explain not how to do a problem, but what the problem was asking. Two things occurred to me here. The problem being discussed was an odd numbered item at the end of the problem set for section 2.4. Being an odd numbered problem, it was one that the instructor suggested the students work on. But this particular problem seemed to me to go beyond the expectations of the course. Secondly, the solution to the problem involved recognizing the theorem that if $f(x) = g(x)$ then $f'(x) = g'(x)$ for differentiable functions. The TA pointed this out as an intuitively obvious principle.

I wondered if working on problems such as this might put my classmates in jeopardy not only in terms of time management, but also self-confidence.

After the tutorial I spoke with one student in the first of several conversations we were to have. His name was Michael and he was working full time as well as taking courses (such as this one) for a M.Sc. program in Business Management. Michael was physically identifiable to me in the class because he was always in business attire with a brief case. He invariably sat in the first row during lectures and worked very hard throughout both lectures and tutorials.

Prior to the lecture, the instructor and TA spent some time discussing the transfer of assignments and information between each other. There was even some discussion of using the university E-mail service, except that the university was in the process of converting to a UNIX operating system with which the instructor was not yet familiar. The question of instructor/TA communication was something I had not considered in detail prior to this study.

The lecture began with a review of the instructor's definition of the differentials, dx and dy . The instructor emphasized that dy was a function of two independent variables, x and dx . I wondered if this was too subtle for the majority of my classmates. There was relief on my part when several minutes later he offered an alternate

description of dy in terms of the locally linear nature of continuous functions. A second concern I had was the tendency of the course to introduce an abstract, generalized situation first and to work through "concrete" examples afterwards.

The instructor offered several meta-comments about the nature of differentials. " dy could be millions. It could be very, very small. It could be positive; it could be negative. I remember when I was going through I used to always think that dx and Δx had to represent little tiny positive increments...But in fact they could be any positive or negative number, they could be large or they could be small. Very often we are interested in contexts where they are small numbers."

After working through a couple of examples, the instructor arrived at a point where he commented, "This is where differentials used to be of real significance. I must admit, this application here in the limited sense that I'm going to give it here is of no value whatsoever. The reason that it's of no value is because you now have hand calculators which can evaluate functions, almost any function, in an absolutely trivial manner by pushing a few buttons." Echoes of the calculus reform debate! The instructor did mention that this work laid the foundations for numerical methods that are still of great value.

The instructor chose to work through an approximation of $\sqrt[4]{80}$. During this development, he made constant reference to the text's notation and treatment of the topic and he sought to make his notes consistent with the textbook. Again, rather than work through the specifics of this situation and generalizing to the linearization of a function, the expression $f(x) \approx f(x_1) + f'(x_1)(x-x_1)$ was developed and $\sqrt[4]{80}$ was approximated using $f(x) = x^{\frac{1}{4}}$ and $x_1 = 81$.

Also again, evaluating rational exponents proved an arithmetic barrier for the class. Although a student was able to correctly evaluate $\sqrt[4]{81}$ there was silence when the instructor asked for the value of $81^{\frac{3}{4}}$. "What is 81 to the $\frac{3}{4}$ power? It's trivial. What is it? [silence] It's not trivial. You have to think about it."

After comparing the linear approximation with the value given by his calculator the instructor foreshadowed later work on Taylor series as a method of improving the approximation.

At 8:10 the instructor began lecturing on how differentials could be used in error analysis. After a preamble about measurement errors, the instructor directed us towards a problem in the text. At this point, after

assigning the odd-numbered problems in the section, he advised us that next week's midterm would cover to the end of this section.

The problem in the book dealt with the maximum error that would occur in the area of a circle when the maximum error in the value of the radius was known. Even though time was running short in the class, the instructor did take time to point out that the rate of change of the area of a circle with respect to the radius is numerically equal to the circumference.

The lecture concluded with the calculation of the maximum errors as percentages. Although it was clear from the computed values, it was not pointed out that the percent error in area was twice that of the radius. I wondered how many of my classmates were taking laboratory science courses and, if they were, did they recognize this as one of the rules for calculating experimental errors.

After the lecture I spent more time speaking with Michael. He was interested in the relative merits of two types of super-calculators I had brought to class.

There was another aspect to this evening which indicated how much I had swung to the participant side of the participant-observer pendulum in my studies. Catherine, whom I had been giving rides home in the evening and who I regarded as a "key informant", did not attend either the

lecture or tutorial that evening. I had seen her leave the campus just as I arrived at 6:30. I would later learn that an upheaval in her personal life had caused her to miss classes that evening. Catherine, aware that I would be concerned about her absence, had left a message for me with another student. There was also a request from her that I make notes on the lecture she would miss.

The second tutorial session began with David asking about the upcoming midterm. Thomas and David were talking about acquiring previous tests set by the instructor. The TA began to work through the examples he had used in the earlier session. He was working hard to reduce student anxiety. David commented at least twice that he was "way behind everyone else", although he had no real evidence of this and had, in fact, one of the better sets of marks for the class.

With respect to a problem from an earlier problem set whose marking some of us had disputed, the TA announced that he would add marks if we brought our assignment to him. Also, he stayed with us beyond the scheduled one hour as he tried to deal with student concerns.

The next Tuesday, October 8, I arrived early so that I could visit the library at Harbour Centre in order to locate the lecture tapes and solution keys for the assignments. The solution keys were in binders available for one-hour loans. They were photocopies of a cut and paste from a solution

manual for the text, rather hurriedly done. The tapes were available in an upstairs alcove. Tape recorders with earphones were available in study carrels nearby.

Just before the lecture the instructor spoke with the technician responsible for the classroom's overhead equipment and for audio taping the lectures. The instructor and students joked with each other briefly. There were also questions about problems and about whether or not there would be a tutorial session just before the midterm on Thursday. The TA brought in the marked assignment sets and the instructor returned them to students by calling out their names. When a number of students did not claim their papers the instructor asked in jest whether they were watching the political debate that evening or the Blue Jays game. Also, the instructor collected this week's assignments from us. These were time consuming processes.

With a "let's get going," exhortation the instructor brought us into the lecture. One student asked if it was possible to go over linearization before proceeding. The reply was no, although he relented with a brief description and offered to talk with her individually later on. "No, I'm sorry I can't go on any more. Not that I've done a great deal on it, but there's so much in here I don't have any more time."

The instructor reminded us about the midterm on Thursday and verbally listed the general topics we should be expected to encounter. In response to a student's question about whether we would be responsible for trigonometric identities, the instructor commented, " Trig identities are grade 12. I assume you know them. I'm not going to test them specifically. ... Let me make a point, since so many of you have asked me. I know that many of you have been out of school for some time and haven't done much in the way of trig for a long time. ... I will assume that you know it come the final exam."

He then proceeded into a discussion of Newton's Method (for approximating zeros of functions). There was a lengthy digression about solving equations leading into general solutions of polynomials functions and the work of Galois. On this occasion, the student beside me, David, seemed impatient with this presentation, although he had received other digressions favorably.

When the instructor began using the vocabulary of iterative approximations and recursions, I wondered as to how many of my classmates had any sort of background with these concepts and terms. The seed for the iterations was labelled an "educated guess". My response was that here was a real application for a graphics capable calculator.

Throughout the development of Newton's Method, the instructor frequently extolled the virtues of the method and the genius of Newton. "This is a method which really works in solving equations for which we haven't any formulas. This is really nice."

The instructor spoke about the limit of the sequence of values generated by the recursion formula as "almost always" being the zero of the function (or roots of the equation in his nomenclature). If the sequence failed to converge to the zero, then we should refer to alternate methods from numerical analysis.

Following this, the instructor began a long digression into the solution of polynomial equations, referring to the Fundamental Theorem of Algebra, complex conjugate roots and repeated roots. He sought student responses to questions throughout this.

To illustrate the use of Newton's Method, the instructor sought to find the real root of $x^3 + x^2 - 1 = 0$. The instructor "located" the root between 0 and 1 by using Bolzano's Principle. He recommended as a better alternative finding the point of intersection of the curve $y = x^3$ and $y = 1 - x^2$. Using 0.7 as an initial guess, he began iterating using his basic scientific calculator. His comments, "As a matter of fact, if you have a programmable hand calculator where you can program two functions into it, the function and

its derivative, you can save yourself a lot of time. That's the way you would want to do it in a real situation, but it won't pay off on a final exam or next midterm. Because I will ask you to show the formula and this kind of work here," caused me shake my head. Without any programming, but simply keying in the initial value and the recursion formula using ANSWER as the variable, I was able to complete the problem before he had finished writing the recursion formula down.

When the instructor began iterating, he kept only three digits of the seven given by his calculator for the next iteration. He described briefly why he felt justified in doing this.

A second example, solving $2\cos x = 2 - x$, required using graphs to identify the number of roots and good initial guesses for the iterations. What a great opportunity for the use of technology! Upon completing this example the instructor commented, "So you can see it's a very powerful tool. Does it really matter - suppose this was an engineering equation that you had to solve - if you have the real solution or seven or eight decimal places? Of course it doesn't. Now this little method is so popular, so easy to work with, you can write your own little computer program. ... I've got my own little program, it's shareware, it graphs the function for you and almost instantly gives eight or ten iterations." His enthusiasm was palpable. "Wonderful method

here. Take a ten minute break. Any questions before the break? This is all I'm going to say about it."

To my surprise, during the break, the instructor was not besieged with questions about what was on the midterm next Thursday. Instead, a student's query lead to a brief lecture on complex numbers and where one would encounter them in a standard undergraduate mathematics program.

There was also some discussion about the provincial political debate and the Toronto Blue Jays baseball game being televised that evening. Even my research was discussed.

The instructor began the second hour with yet another declaration of how nice Newton's Method was, this piece of calculus used in solving "algebraic problems". He answered several questions about solutions of equations.

Chapter 3 was introduced as "an application of the derivative...to use the derivative to help us sketch the graph of the function". Reasons for sketching the graphs included finding the relative and absolute extrema. Examples from economics were mentioned.

Section 3.1 dealt with critical values of a function. Briefly the graphical representation of non-differentiable points was mentioned. The instructor drew a picture of a graph which included the possible cases. We were asked to "try to copy this picture as close to mine as you can".

The graph of a continuous function on a closed interval was developed. Catherine asked what would happen if it were an open interval. The instructor replied that this was a problem he would deal with later. Later he spent some time distinguishing between extrema occurring at endpoints and at points interior to the function domain.

Using the constant function as an illustrative example, the instructor pointed out that absolute maxima and minima need not be unique. The instructor introduced and explained the symbol \forall when writing the definition of an absolute maximum on a specified domain.

A fair amount of time was spent looking at limiting and special cases of the definition. If the domain was half-open and the non-included endpoint would have been a maximum if it had been in the domain, then no absolute maximum exists for that function on that domain. The instructor took time to explain why the maximum would not exist. By questioning, he led the class in creating a function (discontinuous) which failed to have absolute extrema. Using these counterexamples, the instructor led the class to the conditions on the function f in the Extreme Value Theorem :If f is continuous on a closed interval, then f has an absolute maximum and an absolute minimum. The failure of either condition on f does necessarily mean the absolute extrema do not exist, but their existence is not guaranteed.

A significant number of aspects of mathematical proof were touched upon in this development, although none were fully developed. The class was being exposed to the elements of a mathematical proof without establishing a firm background in the process, language and notation.

Late in the evening though it was, the instructor quipped that the Extreme Value theorem was extremely important, as its name would indicate. "Not going to prove it. Roughly speaking, that's about a fourth year level proof." But care was taken to establish the statement of the theorem with some rigour.

We were reminded once again of the midterm and that the TA would be available the hour before. "Probably it would be an hour well spent with him."

After the lecture, the instructor remembered his promise to review linearization with one of the students and spent time with her. A number of students stayed to discuss problems with each other.

I arrived late for the tutorial session on October 10. There was considerable tension in the air. The numbers attending this session were low and some of those who had come, left early. Others who remained appeared nervous and confused.

The midterm was written in the lecture theatre. I found the seats with the arms that flipped over for a writing

surface to be quite uncomfortable and constricting. It was difficult to place the question sheet, examination booklet and calculator in a manner conducive to working efficiently.

The test itself consisted of seven items on two sides of a single sheet of paper. (See Appendix C.) The mark value of each question was indicated in the left hand margin. There was a total of forty marks available. The test was well laid out and quite straightforward. As my classmates confirmed afterwards, there were no surprises on this paper.

At the end of the hour when the examination booklets were collected there was a sense of relief in the room. For the majority of students, this test was not as daunting as they feared it might be. Many students remained behind after the hour to talk with the instructor and each other about the marking key and their performance on the test.

As I drove Catherine home that evening, she expressed her frustration with the one question (#7) that had caused her difficulties. Her comment that "the instructor got me" illustrated the confrontational "student versus the instructor" attitude which I encountered several times during the study. Catherine believed that she was not doing as well in this course as in the others she was taking, but, in fact, she had established herself as the most successful student in this class. Students at this stage of the term had a

difficult time answering the question, "How well am I doing?" unless they were doing very poorly.

Speaking with my classmates at this point in the term, I found that no one was experiencing a great deal of distress with respect to the course yet. Six of the allotted thirteen weeks had passed and we had covered two out of the scheduled four and a half chapters in the text. A look at the course outline suggested to me that the pace and complexity of the material were both about to increase dramatically.

October 15 to October 31, 1991

Arriving early on October 15, I spent a half hour talking with Michael. Following our discussion last week, Michael had decided to purchase an HP48S calculator and was just beginning to learn how to use it. He thought that the midterm was straightforward, but too long.

Michael had serious concerns about the availability of tutorial assistance in this course. He suggested that two hours a week would be more appropriate than one. He advised me that the Calculus Lab at the Burnaby campus was "out of bounds" for students taking this course at Harbour Centre, although several other students had already availed themselves of that facility. Michael spoke about how difficult it was to see the instructor during his office hours on Burnaby Mountain (Michael worked during the day), but he commended the instructor on his willingness to stay after lecture and talk with students. Michael also complimented the instructor on his teaching style. He felt that the instructor's habit of reading his notes while copying them onto the overhead projector slowed the rate of delivery to a manageable pace for the students.

By 7:25 the class and instructor had gathered in the classroom. The instructor initiated a discussion about how the students felt about their performance on the midterm. "How many of you found it quite difficult?" No one replied

that they had, although there was consensus that the last problem was difficult. The instructor explained the problem and announced that the solutions to the midterm would be available in the library.

It was interesting, in light of complaints from several students that there wasn't sufficient time to complete the midterm, to hear the instructor talk about the second midterm and the final examination. The second midterm would be similar in length to the first, so that those who experienced time management in the first could expect similar pressures during the second test. The final examination was designed to be the length of about two midterms. There would be three hours, however, rather than two allowed to write that paper. The final examination would be a different, less harrowing experience for those needing more time to work.

After this, the instructor reviewed in some detail the work begun last Tuesday on absolute extrema and graphs of functions. This led to the concept of relative (local) extrema. One student asked at this point why endpoints were disallowed as relative extrema in the definition given in the textbook (and later in this lecture). The instructor replied that it was a matter of definition and that the definition of local extrema might vary from text to text in this particular. "If it were productive to do so, we could always alter the definition slightly to include them [the

endpoints]. But it doesn't really make any difference because we are always going to examine the endpoints to see if we have absolute maximums or minimums." The instructor's reply illustrated the pragmatic flexibility associated with many definitions in mathematics.

Further insights into the nature of mathematical definitions were offered when the instructor spoke of $f(x_1)$ being a relative maximum if it was greater than $f(x)$ for "any nearby x " where the term "nearby" was made clear by reference to the illustrative diagram, but was not made precise. The instructor informed us that the use of the strict inequality, $f(x_1) > f(x)$ rather than $f(x_1) \geq f(x)$ was a matter of the textbook chosen. A little later a student used the weaker condition to pose the question as to whether relative extrema are achieved on an interval in which the function is constant. At this stage, how much precision is possible, appropriate and/or worthwhile?

After working through these definitions, the instructor led the class through questions to associating the local extrema on the diagram with "slopes of zero". The instructor used the diagram and insistent questioning to lead us to the ideas that a "zero derivative" does not guarantee the existence of a local extremum and that local extrema can be found where the function is non-differentiable. I found myself caught in the quandary the instructor was constantly

facing. I considered this to be good teaching, with students involved and apparently learning. But I wondered if the course schedule allowed so much time to be spent on this one topic.

After giving us the definition of a critical point, the instructor was asked whether or not an endpoint in the domain of definition could be considered a critical point (endpoints would be non-differentiable since only one-sided derivatives would exist for them in the closed interval). "Yes. Technically it does, if the domain is closed. That's true.", the instructor allowed, his tone suggesting that he might wish that his students were less zealous in seeking limiting cases for the definitions.

After considering definitions for more than half an hour, we moved to an example from the text: finding the critical numbers for $f(x) = x^3 - 3x + 1$. A point was made in the solution that, since f was a polynomial function, there were no real values of x for which $f'(x)$ did not exist. Therefore, the values 1 and -1 were the only critical numbers for f .

A second example, $g(t) = 5t^{\frac{2}{3}} + t^{\frac{5}{3}}$, did involve values of t where $g'(t)$ does not exist. Here the instructor asked if any of us knew what the graph looked like. Later, after identifying $t=-2$ as a value for which $g'(t)$ was undefined, he asked us if that alone guaranteed that a local extremum

existed there. Sketching a counter-example to show that it did not, the instructor quipped, "Sneaky, eh. You've got to admit that was pretty sneaky," provoking general amusement.

After outlining a strategy for finding absolute extrema of functions on closed intervals, the instructor worked through the example of finding the absolute maximum and minimum of $f(x) = x^2 + x - 3$ on $[-2,2]$. Yet another topic from high school mathematics came to the fore as the instructor tried to get my classmates to answer questions about the structure of the parabola. Another brief review of "prerequisite math" ensued. The instructor was explicit in his sensitivity to the plight of those who had been away from school mathematics for a time. "So all that the differential calculus does in this case is verify what we all knew, right. Cross my fingers. What I'm trying to do when I go through this is, I know that many of you have been out of school for a long time, to throw as much stuff out at you to jar your memory as much as I possibly can."

At the break, the instructor admitted that he had forgotten to set an assignment this week. No one seemed upset. Michael was showing his new calculator to a number of interested students. I spoke with Catherine about her progress in finding a new place to live. Thomas and another student discussed a problem from the previous assignment.

The instructor called us back, commenting that the pace of the course would not allow him to do as many examples as he would like. Onwards we sped to Rolle's Theorem and the Mean Value Theorem. While the statement of Rolle's Theorem was being explained to us, we were taught to distinguish between smooth and continuous curves and introduced to the notation \exists .

The instructor concluded the statement of the theorem with the words "no proof". A student interjected, "You mean there is no proof?" This caused considerable laughter and the instructor's rejoinder that a proof did indeed exist. We were given a "convincing picture" in lieu of a proof.

Several meta-comments on the nature of existence proofs were offered. We were required to know the statement of theorems, but were not held responsible for the proofs, not even for the "convincing diagrams".

Working through an example illustrating the application of Rolle's Theorem, the instructor found himself unable to satisfy the condition $f(a)=f(b)$. Checking his notes he readily confessed to us that he had been caught out by the shift from the first to second edition of the text. The notes he had prepared included an example from the first edition which was meant to illustrate the use of the Mean Value Theorem. He recovered very nicely by closing the example with, "Therefore, Rolle's Theorem does not apply."

There was some good natured laughter from the students and instructor, before we moved onto a more appropriate example.

The last part of the lecture was on the Mean Value Theorem, which the instructor identified as the Mean Value Theorem for derivatives rather than for integration. He stressed the link of this theorem with Rolle's Theorem, pointing that dropping one of the hypotheses of Rolle's Theorem alters the conclusion and that Rolle's Theorem was a special case of the Mean Value Theorem. Again, however, no proof was to be supplied. Diagrams were used to explain the theorem and to intuitively justify its validity.

More meta-comments on existence theorems were offered. The point guaranteed by the theorem was not necessarily unique and the theorem did not provide a process for finding the point(s). The absolute value function was used to show that if the differentiability hypothesis of the Mean Value Theorem does not hold, then the conclusion may not follow. The instructor emphasized the "may" in "may not follow". A significant amount of information about mathematical proofs was being presented to us incidentally.

The last example of the evening was not taken from the text. When the instructor gave us the function,

$f(x) = \sin(2x)$ on the interval $[0, \frac{2\pi}{3}]$. His aside, "I know,

trigonometry again.", provoked some laughter. We were

prompted to graph the function using the terms *amplitude* and *period*. The instructor used this as an opportunity, as he did frequently, to review secondary school mathematics. Carefully, he verified each hypothesis of the Mean Value Theorem before jumping in and determining the value guaranteed by the conclusion.

The sketch of the graph was used to estimate the value and predict its uniqueness. A calculator was used to solve

the relevant equation, $2 \cos(2x) = \frac{\sin\frac{4\pi}{3} - \sin(0)}{\frac{2\pi}{3} - 0}$. I was

impressed by the solution presented in this example. The use of a graph and the numerical "messiness" of the work and result made it seem less contrived than most other examples of this type that I had encountered.

The amount and variety of mathematics that we were exposed to in a two-hour period was daunting. We began with informal, intuitive definitions backed up by reference to diagrams. Precise definitions involving symbols for existential and universal quantifiers followed. Straightforward applications of differentiation were next, liberally laced with a review of high school mathematics. Detailed statements of two theorems were given and examined in detail, with some emphasis on the nature of conditional statements. A final example made use of diagrams, the

statement of a theorem, differentiation techniques, high school trigonometry and the use of a calculator.

On October 17, I was working late at school and consequently missed the first tutorial session. By the time I reached the lecture theatre the instructor had already begun with a preamble about curve sketching.

Perhaps it reflected my own fatigue, but there seemed to be considerably less energy in the class this evening than there had been on Tuesday. I did notice that the instructor was now identifying some students by their first names when he spoke with them.

After defining increasing and decreasing functions on intervals, the instructor defined monotonicity for functions. David asked whether or not a horizontal line would graph a monotonic function. The instructor replied with the "technicality" of non-decreasing and non-increasing functions, "It's kind of a backward approach to things. Just a little technicality and some books will take that approach. I don't think this one does."

A differentiation "test" for monotonicity of functions was stated and then proved using the Mean Value Theorem.
 $(\forall x \in (a,b)) (f'(x) < 0) \Rightarrow (f \text{ is decreasing on } [a,b])$.
Beginning the proof, the instructor said, "Now what do I have to show. If you are going to take a trip, you want to know where you are going. ... Where do we want to go here? I

want to show that f is decreasing." In effect, he was not only offering a proof to a corollary of the Mean Value Theorem, he was also instructing us in the construction of proofs in general.

The instructor asked questions throughout his presentation, trying to elicit student response. Such response as he found was weak and muddled. I found the proof to be charming, but I was sanguine about how much my classmates understood. The proof was an instance of a use of the Mean Value Theorem, which had been advertised as a "very important theorem". The students were not held responsible for the proof on tests or assignments. I had doubts as to whether the proof added to my classmates' understanding of the first derivative test. But proof, after all, is an essential component of mathematical endeavor. What role did it play in this course, for this class?

The lecture shifted from theory to application with the instructor using curve sketching techniques to develop the graph of $y = 2x^3 - 3x^2 + 2$. Over fifteen minutes of class time was spent developing and analyzing the graph of this simple polynomial function. A TI-81 gave a very reasonable sketch of the graph in less than a minute and, with the use of the NDERIV function, allowed a fairly detailed analysis of the behaviour of the graph with respect to first and second derivatives. If there were areas of the curriculum that

could be revised to allow more time for assimilation and learning, surely this topic would be a prime candidate.

After the lecture, students came forward to pick up their midterms from the TA. The instructor and I were somewhat nonplussed to find that my paper was missing. (It turned up the next week.)

There was no discussion yet of the overall results of the class on the test. Two of the students attending the second tutorial session had scored over thirty marks out of forty. One had scored in the twenties and was visibly frustrated with his results. The TA discussed solutions to some problems.

Students asked what would be done with the marks by the instructor. The TA replied that there would be "no scaling" and spoke about the "relative weight of the midterms" in the evaluation scheme. Speaking with Catherine, the TA confirmed that many students had not had sufficient time for this paper, about "one third of the students ran out of time". The lowest score was 18, but the TA was unable to say whether or not that was a failing mark.

By 9:00 p.m. the conversation about the midterm drifted to an end and the TA asked us what we wanted to talk about. He gave us a sophisticated overview of Newton's method from the perspective of numerical analysis. In his discussion about the stability of the method, the TA indicated that he

had serious reservations about the utility of the method in "real" applications. His attitude towards the algorithm was in marked contrast to the instructor's enthusiasm for it. What message was being conveyed to the students?

After the tutorial, I spoke briefly with David about the midterm. It was interesting to hear him speak of the "one week of math abstinence" that he took after preparing and writing the test.

The period before the lecture was filled with considerable energy and conversation. One person was concerned that my results on the midterm would "unbalance the curve". The instructor assured him that I was just an observer and my marks would not be considered with the rest of the class.

The instructor announced the mean and standard deviation for the midterm and used this to launch into a brief lesson on these statistics and distributions. If the students understood what he was saying, they now had a better sense of their performance on the midterm relative to the rest of their classmates.

One of the students (Charles, a professional engineer taking the course for pleasure) asked if it was possible to have the assignments in advance. He would be out of town next week and would have to miss more classes in the future. The instructor admitted that, because this was the first time

that this edition of the text was being used, he was making up the assignments on a weekly basis as we went along. The instructor did promise to have the sixth set of problems chosen by next Thursday.

As the lecture began, I noted that one student (Elizabeth) was using a microcassette recorder to tape the lecture. I found this interesting when I contrasted to the lack of use of the tapes in the library which were prepared by the university. Elizabeth must feel it was advantageous to have her own copy. I also wondered what the quality of her recording would be. During the summer, I had had limited success in audio taping lectures.

The lectures continued with a detailed description of the use of derivatives in sketching and analyzing the graphs of functions. The presentation of the material was lucid and straightforward. I noted that the instructor did not ask us questions as often as he had on other nights.

I was tired and slightly ill. Also, I had already decided that far too much time was being spent on "pencil and paper" curve sketching. This was one topic that could not resist the assault of technology. The curve sketching unit in even a traditional calculus course would undergo dramatic revision in the near future. Consequently, I found that I was not as involved a participant observer that evening as usual.

While the instructor was working through the graph of $y = x^4 - 8x^2 + 7$, I had my TI-81 calculator graph $Y1 = X^4 - 8X^2 + 7$ and $NDERIV(Y1, .01)$ on the same axes. The potential for using a graphics capable calculator for establishing the relation between the graph of a function and its derivative seemed so obvious and so great that I wrote emphatically in my notes, "Why are we doing this????"

The instructor offered some meta-comments which I thought were relevant to my concerns. "If you are trying to graph a function using the ideas associated with the derivative, you try to glean together as much information as possible as you can regarding local maximums, local minimums, absolute maximums, absolute minimums, intervals where it's increasing, intervals where it's decreasing, critical points, all this sort of stuff. You try to paste this together." It might be that the ability to sketch a graph by hand was no longer as useful as it had been, but the process of bringing together a variety of information to solve a problem would exemplify aspects of mathematical process.

As an instance of how difficult it is to schedule sequentially the presentation of information in a multi-faceted topic such as curve sketching, the instructor completed an example taken from section 3.3 of the text. Upon completion of the example, he led the class (with significant student response) into an analysis of the

behaviour of the graph outside of the interval specified in the problem. "Completing the graph" involved discussing vertical and slant asymptotes and function behaviour "at infinity". This material was covered in sections 3.5 and 3.6 of the text. The problems assigned for this week did not even go as far as these sections. Surely some of my classmates were asking themselves, "What am I supposed to know and when am I supposed to know it?"

Before graphing $y = x^4 - 8x^2 + 7$, the instructor asked us to predict the "number of bumps" the graph would have. His digression led to the conclusion the greatest number of local extrema a degree N polynomial function can have is $N-1$. I waited for the other shoe to drop, but the class was not asked why this was so, nor was this result linked to the degree of the derivative.

A great deal of technical language was passed to us during the lectures. Sometimes it was pointed out to us explicitly, as when x^4 was identified as the dominant term of the polynomial. On other occasions, the terminology was incidental. The instructor, explaining what was meant by "near a critical number", spoke of isolated singularities.

Locating the zeros of the function sent us back to high school mathematics which may or may not have been part of the curriculum for my classmates. "We are looking for the four roots of that equation. [$x^4 - 8x^2 + 7 = 0$] Generally that is

difficult to do. There are formulas for it. And we also know another way of doing it, don't we?" Silence preceded the suggestion that Newton's Method could be used. "But in this case, it is easy to solve the equation. Why is it easy to solve? Anybody know?" The instructor failed to get the response from us that he was looking for, so he identified the equation as a biquadratic. After solving the equation, with the comment, "This is a standard trick that they teach you in high school. Some of you have been out for a long time, so you'll just have to remember this," the instructor asked us, "How many have seen that before?" Silence, followed by nervous laughter resulted. The instructor mentioned that it was in the Math 100 course and then he asked me if it was in the high school course. A calculator was used for the first time to determine the approximate value of $\sqrt{7}$.

During the break I spent time with David discussing his recent purchase of a TI-81 calculator and how he was using it in his coursework. The instructor spoke with students about the scheduling of the final examination for the course and which edition of the text would be used for Math 152 next semester.

One student asked the instructor about the statistics course, Math 270. The recommendation of the instructor was to complete Math 152 before taking Math 270. This was not

because Math 270 required knowledge of integral calculus, but because success in Math 152 indicated that the student had sufficient mathematical maturity to take the statistics course. The calculus was being used as a filter not only for client disciplines, but for other mathematics courses!

In the instructor's words, "Math 152 is a corequisite for Math 270. There is not a great dependence [in Math 270] on 152 at all. But, by making sure that students are taking or have taken that course, it ensures at least a minimal level of mathematical maturity. Nice way of putting it, eh."

Students responded to this with good humour.

The second half of the evening was spent on concavity. A preamble was followed by definitions, with diagrams. Concavity was related to the second derivative of the function. Later the second derivative test for local extrema and points of inflection were introduced.

Catherine asked about the situation where the second derivative failed to exist. After answering her, the instructor added, "In this book they don't put much emphasis on that case. But we're going to look mainly at the situation where the second derivative is zero. That's good enough for our purposes here." It sounded as if the text was setting limits on what we were covering in the course. Was it indeed the "bible for this course" as the instructor had advised us at the beginning?

The first tutorial session on October 24 began with a discussion of the midterm results. To my surprise, the TA asked us if the instructor was going to "curve the results". Students replied that he would not. A minimum passing grade was identified as being around 20 to 22 marks.

The TA borrowed that week's assignment from me to see where we were working. He asked us if we were working with monotonic functions yet. One student commented about the problems, "It seems so easy when the instructor does them, but once I begin the problems..."

A curious dialogue arose when a student asked the TA to work through a specific problem from the text. The TA replied that the answer was in the back of the book. Two students replied that they did not get the answer found in the key. When the TA worked through it and yet another student said they didn't get that result, the TA replied that it was "just algebra".

The function in question, $f(x) = x\sqrt{6-x}$, led to a discussion as to whether or not the point $(6,0)$ should be considered a local minimum. It certainly looked like one on the graph. The students referred back to the lecture where the definitions of local and absolute extrema were given. Endpoints, according to the text definitions, were not to be considered as local extrema (because the endpoint could not be contained in any open interval of the domain of

definition). The general conclusions reached in the discussion were to ignore the text definitions and beg the question of endpoints as local extrema.

A discussion of concavity was disrupted by bagpipes being played in the hallway, to the general amusement of the students. I noted that many of the students seemed to have fallen behind in their work. Catherine had mentioned on Tuesday that she had done no work in this course since the midterm. Midterm tests in other subjects were taking up her time and energy.

Even the TA was somewhat uncertain as to where we were in the curriculum. He had prepared material on section 3.5, which had been covered neither in lectures nor in this week's assigned problems.

This evening, for the first time, it was necessary to clear the lecture theatre of students from the previous class. They were milling around discussing the midterm they had just written for that course.

I initiated a discussion about post midterm "letdown" with several students. One spoke about his sense of relief after writing the test; another spoke of the pressure that the other courses were putting on them. I was approached by one student with a mathematics problem and spoke with two others about their "super calculators".

The instructor had remembered his promise to Charles, the engineer, and set the problems for assignment 6. After copying the assignment onto the overhead, the instructor reviewed Tuesday's work and asked us where he had ended. After determining where he was in his notes, we continued with curve sketching.

Working with the example $f(x) = x^{\frac{2}{3}}(x^2 - 8)$, the instructor calculated $f'(x) = (\frac{2}{3}x^{-\frac{1}{3}})(x^2 - 8) + x^{\frac{2}{3}}(2x)$. "Now I'll let you go through...I'll let you do the algebra there. When you clean this up it does turn out to be fairly nice." The algebraic simplification was necessary for finding critical numbers and very useful in calculating $f''(x)$, but it is a task many of my classmates would find difficult. There was not enough time to go through the high school algebraic manipulations, but working through the simplification was something from which many in class could have benefitted.

Again, when finding the second derivative, the instructor expedited the solution by saying, "You have to differentiate the first derivative. I'm not going to do that. Let me just give you the result. I'll go "dot,dot,dot equals". When you clean this up, please confirm this, this is just algebra, this is what you are going to end up with."

At 8:12 we began another section, 3.5 Limits at Infinity. By this time I knew I was very tired and not prepared to think about a new topic. And I suspected many of my classmates were equally fatigued. But there was no time to spare. Notwithstanding this time pressure, the instructor managed, through posing a sequence of questions, to convince us that no polynomial functions had horizontal asymptotes. I was impressed by his question, "How many horizontal asymptotes can the graph of a function have?" This was the sort of task to involve the minds of students at the beginning of a topic.

Moments later a student interrupted to ask about the existence of infinity. The instructor's drawn out "well" was greeted with laughter. The description of infinity which he offered was quite informal.

The instructor finished his lecture that evening with a quiet lament, "Time goes so fast."

My use and enthusiasm for graphing calculators was well known by then. When the instructor began one of the graphing examples during the lecture a student called out, "Craig, don't cheat on this."

I spoke with a student who was auditing the course. He was simply attending the lectures; not doing assignments or taking tests. We agreed that the instructor was a particularly good teacher and the student said he planned to

enroll in as many courses taught by the instructor as he could.

In the second tutorial session, I was speaking with David about students in this class helping each other. He spoke about the "I'm all right, Jack" attitude of students and commented that, "First year students eat their own young." Competition, rather than collaboration, seemed to be the order of the day.

That evening, I gave rides home to both Catherine and Elizabeth. One of them had been at the Arts Club Theatre the evening before and had been witness to a stabbing. The violence had upset her sufficiently that she was finding it difficult to focus on her work.

I had been spending one afternoon each weekend doing the assigned problems. Assignment 5 was the first of three which involved curve sketching techniques. There were also problems involving the use of Newton's Method. I found that the assignment took two to three hours to complete. I was familiar with the material and I was using a calculator with reasonable graphing, programming, and iterative capabilities. In my field notes I asked, "How will other students fare on this assignment without similar supporting technology?" The answer was that somewhere between assignments four and five, the going in this course became substantially tougher for a number of students.

I arrived early on October 29 and spent the time checking the examination schedule posted in the main foyer. Our examination was to be written from 7 to 10 p.m. on December 3 in Room 1530. I went to look for this room. To my relief, I found that it contained chairs and tables with adequate work space rather than the cramped seats in the lecture theatre where we wrote our midterms.

The lecture began with a consideration of the behaviour of functions as $x \rightarrow \pm\infty$. Once again, after a brief review of the project at hand - curve sketching, the instructor asked a student precisely where we had finished off last class. The impression I gathered from this was that the instructor's notes for the course were a continuous whole and that he endeavored to push as far ahead through them on any given evening as time would allow.

When the instructor used a rational function to illustrate the concept of horizontal asymptotes, Catherine signalled to me conspiratorially that the instructor used a technique that the TA had previewed the previous Thursday. There was a sense of being one step ahead of the lecture, which didn't occur that frequently for my classmates.

This evening I began to note a change in the instructor's teaching style that was to become more pronounced as we rushed to the end of the course. He asking far fewer questions in his lecture and the questions he did

ask seemed to be rhetorical. The timing and nature of the questions did not seem to invite response from the students. Nor was the instructor as persistent as he had been earlier in drawing answers from the class. The tapes of the lectures indicate a shift to a style of lecture suited for a distant audience with whom no interaction was anticipated.

At the end of the second example, I observed that David had used a TI-81 calculator to confirm the sketch derived by instructor.

There was some discussion after the instructor had worked through his notes on this section. Catherine asked a question which the instructor took time to answer. And then the instructor commented, "There is one observation we should make using the first three examples. I claim that I can look at the rational functions and tell you whether the limits [as $x \rightarrow \pm\infty$] exist without doing any work at all. And, if the limit exists, I can tell you what it is. Now that would be a nice skill. There are enough exercises in here, that is what you should be aiming to do." The instructor was asking us to search for patterns and generalize specific results. A very worthwhile mathematical task!

After a brief dialogue with us on these results, it was, "All right, let's go on to the next section here. Study section 3.6 and do the odd problems."

During the preamble to this section, the instructor sought to distinguish between infinite limits and limits at infinity. In passing, he mentioned, with respect to the previous section, "You can see how easy this horizontal asymptote business is. All you have to do is take your calculator out and plug a high value for x in and see what you come close to, then plug a little bigger value in. This is the brute force method of evaluating limits." This struck me as a very legitimate way of beginning an analysis of functional behaviour far away from the origin. It was, however, a method that had not been explicitly brought to the attention of the class prior to this time, and by then the section had been officially completed.

The lecture continued until the break. Just before the recess, the instructor commented on the example he had just worked through, "In the next section I'm going to put all of this together and say "graph this function". Try and paste all of this information together and try to get a graph. You can either do that or get a slick little program, slip it into your micro, type in the formula for the function and, zap, it's there. But you can't do it on an exam. No."

By this time in the course, I had had a chance to speak with a number of my classmates on a variety of issues. There was still, however, a certain insecurity on my part about the

degree of acceptance I was having and the depth of my observations.

This evening I made it a point to talk with two students who had caught my interest, Brian and Philip. They were young men who had graduated from high school last year and always travelled and sat together. That evening I found that they came from Vancouver Island. In fact, they left the lecture at 8 p.m. that evening in order to catch a ferry to the Island. We spoke about assignment 5. I had found this assignment, with its emphasis on graphing to be quite lengthy. This pair had worked together on it using a graphics capable calculator and had still found the problems to be very difficult. They confessed that they had spent five hours in the Math Lab using a graphics capable calculator and that they still had not completed all of the graphing exercises. One of the students expressed interest in the capabilities of the TI-81 calculator which I brought with me.

I also spoke with David and Catherine about using calculators for Newton's Method. Catherine had a basic scientific model and bemoaned that fact that she frequently misplaced parentheses. David said that one of the most valuable features of the TI-81 was to see a formula in its entirety and to edit it if necessary.

The instructor spent the break talking with students about statistics courses.

David was having good success with his calculator. He had encountered the difference in the way the calculator graphed $y = x^{\frac{2}{3}}$ and $y = (\sqrt[3]{x})^2$ and had resolved the problem himself.

At 8:45 we reconvened and began an example meant to bring together all the previous skills developed in the chapter: sketch the graph of $f(x) = \frac{x+1}{x^2+x-2}$. We were led methodically through all the steps, with minimal involvement until we began consideration of vertical asymptotes. As the instructor prompted us through the traditional convoluted analysis of $f(x)$ as $x \rightarrow -2^+$ and $x \rightarrow -2^-$, it occurred to me that this was an excellent place to use the SOLVER functions available on several models of calculators or a small function evaluation program on the less advanced units. My classmates, who were wrestling with whether $f(x)$ was positive or negative at $x = -1.9$, would probably have appreciated such features.

When we investigated the existence of vertical asymptotes one student, probably suffering from information overload, interjected that we had already determined that there were no horizontal asymptotes. The instructor corrected him. There were no horizontal tangents for this graph. Not everyone was comfortable with the language of mathematics.

After spending over 30 minutes developing the graph, the instructor admired his work, "And there's your graph of this rational function. This is quite typical of what graphs of rational functions look like. Very interesting. Much more interesting than silly little polynomials."

Speaking about concavity, the instructor said, "Other things I could look at are concavity. I do not recommend it for rational functions, unless I really force you to do it, because it takes the second derivative and can get really quite messy." Here was another great opportunity to use off the shelf technology to help us understand these basic concepts.

In concluding the lecture, the instructor said, "I'll show you what I'm going to do next day, if you want to play with it." and gave us the function $f(x) = \frac{x^{\frac{2}{3}}}{x - 1}$.

Only ten students, including myself, attended the first tutorial session on Halloween. The TA was building on questions asked last week and material that he had prepared for last time, but for which we were not yet ready. During this class, I overheard one student exclaim, "I'll be glad when this course is over."

A substantial amount of time was spent by the TA detailing topics that he felt would be on the next midterm. It was recommended that we learn to like Newton's method.

One student replied that she must be dyslexic, because she never got the same answer twice.

Before lecture, the instructor spent time talking with a student about a linguistics course she was taking. Joking with the class, the instructor quipped, "I'm going to give you a treat. Well, it could be a trick or a treat." This was followed by, "If I gave you a formula for a function and asked you to graph it, which, of course, I am going to do, what would you do?"

We began to analyze the function he had given us last class. The development of this example, aside from a pair of corrections offered by Catherine and a digression about finding the cube roots of negative numbers on a hand held calculator, was straightforward, but rapid.

As the instructor progressed through the example, the pace of his presentation began to pick up. He began asking a number of rhetorical questions, which he quickly answered himself. There was student response to some of the questions, but overall there was a sense of rushing through the work. The instructor did reply, albeit briefly, to questions from the student.

This one example took another half hour to develop. One hour of lecture time had been spent on working through two curve sketching problems.

At 8:10 the instructor gave us a preamble to section 3.8, word problems involving maximum and minimum values of functions. While the instructor discussed these problems in a general context, I found myself jotting down in my notes that I was tired, fatigued, and uncomfortable. (Lower back problems were exacerbated by the lecture theatre seats.) At that time, this research instrument was not operating at full capacity. I had to believe that my classmates also experienced periods of inattention.

At 8:20 the instructor began the demonstration that the largest rectangle of fixed perimeter is a square. To my astonishment, the students did not seem familiar with these results. I too was guilty of assuming what was common knowledge. There was not enough time left to complete the problem, so the instructor told us we would continue with it next class.

In the later tutorial session, Thomas and the TA broke off to discuss a problem while David, Catherine and I worked together on implementing Newton's Method on a calculator. As a group we worked through an problem that had been suggested by a student in the first session. Towards the end, David and the TA spoke about what a first calculus course should cover. The TA thought that the best techniques should be presented first and then the history of the subject explored.

David had been reading the textbook margin notes and wanted to know "who this Fermat guy is".

At the end of the session, the TA inadvertently caused an outburst of hilarity when he asked if any of us were math majors.

November 5 to December 3, 1991

Before the November 5 lecture, I met with Brian and Philip on the escalator. I also spoke with another student (whose name I still did not know) about the examination schedule. There was some concern on her part about how late the examination was in the evening and the fact that it was being held at Harbour Centre. It occurred to me then that this was not the safest part of the city at night.

When I arrived at the classroom, Harry and Charles were discussing dams in the Columbia River system and hydroelectric power requirements. There was some degree of social interaction in the class.

Catherine and another student sought me out for advice on some problems from the assignment. Thomas arrived without notepaper, but obtained some quickly from myself and another student.

The instructor opened the lecture by asking us how we identify max/min problems and encouraging us to draw a diagram, regardless of what type of word problem we are solving. After discussing problem solving in general, we worked through an example from the text.

The presentation was notable for the lack of student interaction and for the instructor's use of a calculator to move from exact values to decimal approximations. The calculations were done in class and took some time. Some

students worked through the calculations using their calculators. There was some difficulty in arriving at consensus on the resulting approximations.

At 8:00 p.m., after telling us that we would see a max/min problem on the upcoming midterm, the instructor digressed, saying, "What other things? Let's speculate. What might we find on the final examination?" He solicited suggestions from the class and added to them.

"I want you to study section 3.10. Notice that we are leaving out section 3.9 on application of max/min theory to economic problems. Those are the sort of problems you take in Math 154 or 155 [Math 157]." Section 3.10 dealt with antiderivatives.

At this time, the instructor pointed out that there would be problems in sections 6.5, 6.8, 6.9 and 9.2 that we could not do without further work on integration. When one student suggested that we would identify these problems by not understanding them, the instructor replied, to the universal amusement of the class, "Well, you won't know how to do them. But that's not good criteria. We all know that that's not good criteria to use."

The instructor worked hard at establishing images of reversing the derivative process. He strongly suggested that it was often much more difficult to find an antiderivative than it was to differentiate. There was also a lot of

forward referencing to the Math 152 course. In fact, there was so much talk of next semester's work, one student wanted to know if we were responsible for this section in this course.

The instructor made what was for me a telling comment, "I'm not so sure that if I were designing this course, I would have left this section in. The integration is usually kept separate." The instructor was responsible for delivery, not design of the curriculum. He did take the opportunity, however, to talk about branches of calculus. This included describing differential equations as the language of physics and engineering. The instructor briefly spoke with engineer (Charles) about his background and knowledge of differential equations.

A considerable amount of effort was spent on the non-uniqueness of the indefinite integral. The Mean Value Theorem was invoked by the instructor to show that antiderivatives differed by at most a constant. "You can easily show using the Mean Value Theorem, if a function has at least one antiderivative then all of its antiderivatives consist of that one plus any arbitrary constant you might want to add and there are no other ones at all."

There were allusions to the role of proof in the study of calculus, but there did not seem to be any clear sense of what was to be accomplished by these references. We were not

held responsible for duplicating or creating proofs. Nor, as the term progressed and the rate at which we covered material increased, were we passively exposed to proofs as we had been earlier. The instructor sought to continue our exposure to proofs by referring us to the text. For our notes, he wrote and said, "It is easy to show (and the book does this actually) using Theorem 3.15 (and I want you to look at that) and Theorem 3.46. The proof of this theorem involves the Mean Value Theorem. So it does all boil down to the Mean Value Theorem. Remember, why did we study the Mean Value Theorem? That's the theorem which is really at the core of all the theoretical calculus results."

The instructor's presentation on antiderivatives was in contrast to the rapidity of his development of the min/max problems. He left a bit more time for student responses to his questions. How did he decide which topics to rush through and those on which to spend more time?

During the recess, the instructor advised us that there would be no problem assignment for this week as the second midterm was approaching. He also spent time suggesting study strategies.

My conversations during break included talking to David and the reading he was doing to learn more about the historical antecedents of calculus. The margin notes in the textbook about Fermat had caught his interest. Catherine,

discussing an assigned problem, wanted to know something about the relation of complex and real numbers. She also spoke about having studied integration in high school and her concern about the role and importance of the constants of integration.

Other students chose not to talk with their classmates. One sat back and read a Tom Robbins' novel. Another, who I encountered in the hallway, avoided eye contact with me. There was ample opportunity for a student to remain isolated in this class, despite the small number of students and the congenial atmosphere the instructor strove to establish. The lecture tapes, however, do capture many students engaged in animated conversations throughout the break. There was the opportunity for social interaction, but no forces pushing the student towards that sort of involvement.

After break, as the instructor was working hard to establish the antiderivative of x^n , the \int symbol was introduced as a stretched S, suggesting summation. In passing, the instructor mentioned that "summing certain quantities" leads to antidifferentiation. How much foreshadowing in a course such as this is wise?

While the instructor was lecturing, there was an occasional beep from where Michael sat. He was working with his HP48S calculator, probably attempting to get it to do a formal integration. The beeps occurred when he entered

information the calculator considered inadequate or insufficient. I noted to myself that I should tell him how to deactivate that particular feature.

The instructor took his material on antiderivatives well beyond that covered in the textbook. The textbook was content with a very small list of antiderivatives which included, without explanation, the restriction $n \neq -1$ when antidifferentiating x^n . "Hey, what happens then? We should just look at this on the side. What am I talking about here? If n were equal to -1 , I'd be trying this problem here, $\int \frac{1}{x} dx$. Well you know, don't you? That's the logarithm function!" The class erupted in laughter at the assumption that we recognized that particular representation of the logarithm function. The instructor wanted to show us the definition of the logarithm function, but realized he would have to distinguish between definite and indefinite integrals. Nonetheless, he continued with the definition.

Thinking back to my own teaching experience, I was struck by the thought that often the instructor needs a certain self-discipline in presenting introductory concepts. The instructor had stated that, if he were designing the course, he might not include this section on antidifferentiation. On the other hand, his presentation led into much more material than the textbook, the "bible for this course", required of us.

By 9:00 p.m. the students had lost track of where we were. The instructor sought to re-engage our attention with a series of questions, "What is the antiderivative of...?" There was little response.

There was some nice meta-commentary on how antidifferentiation was intrinsically more difficult than differentiation. "And it's not an easy problem [antidifferentiating the cosecant function] to do. It's very simple when you know the trick. Which I'm not going to show you." The ensuing laughter brought home the fact that, tonight, we were laughing at our lack of knowledge.

Properties of antiderivatives and further examples followed. Throughout, there was constant reference to the constants of integration. They seemed to take on an importance all out of proportion to their role in this introduction to the topic.

The instructor identified two of the properties he had listed as the linearity properties of antidifferentiation. He mentioned that "if you ever take linear algebra" we would encounter linear operators represented by matrices. Antidifferentiation was an example of a linear operator. This was a foreshadowing of later mathematics courses that I had not anticipated.

With fifteen to twenty minutes left, the instructor asked us to read a brief review of exponential functions in

section 6.1. A student blurted out, "We're in chapter 6 now!" After the laughter had subsided, the instructor noted that although we were indeed in chapter six, we would not cover material in the intervening chapters. This student was clearly not using the syllabus to keep track of our progress through the course. He, and I suspect many others, were along for the ride; content to find out where we were going as soon as the instructor told us.

David commented to me that he had heard the word "review" before in this course and he had serious doubts about how much of it would be nothing more than review.

The instructor developed the graph of $y = 2^x$. He got us to give him function values for $x = 0, 1, 2, 3$, but we faltered when he asked for the value of 2^{-3} . One student offered -8 as an answer. This would be a hard "review" topic for some.

What was particularly nice about developing the graphs of exponential functions prior to being able to differentiate the functions was that the curve sketching techniques could be reversed. Rather than the derivatives giving us information about the graphs, the graphs yielded insight into the nature of the derivatives. The monotonicity of the graphs of $y = a^x$ told us that $y' > 0$ for $a > 1$ and $y' < 0$ for $0 < a < 1$. When $f(x) = a^x$ was identified as a monotonic

function, Catherine quickly quipped that they were monotonous functions.

I arrived for the first tutorial session on November 7 to find Susan talking with the TA and several students discussing their travels in Europe. When the assignments were returned, I found that perhaps I was becoming too involved in my student role. The TA had marked one of my solutions according to the solutions manual he had. It was clear to me that the manual had misinterpreted the question and that my solution was correct. The TA offered to adjust my marks. Although I had lost perspective (I was tired and ill at the time), I had reason to believe that my classmates had experienced similar feelings of frustration when they had had their assignments and tests returned. The major difference was that I had the expertise to argue my case.

There were only eight people in attendance that evening, although there was a test scheduled for next week. My notes for this hour were sketchy. At one point I wrote that I had fallen asleep. I would guess that many of the students were as tired as I was.

Charles asked the TA to work through a problem in detail. Michael and the TA spoke about the use of the quadratic formula. When Michael tried to show a neighbor some work he had done on the HP48S calculator, the neighbor

exclaimed, "Get away from me!", in mock horror. I found no coherence in the events of that hour.

As the instructor was setting up for the lecture, he asked us, plaintively, "Does anyone ever listen to these tapes? Besides Craig?" Students responded by telling him how difficult it was for some of them to access the library where the tapes were. Another student spoke about using the tapes she made herself. The instructor was interested in how she used these recordings and he spoke of the potential of videotaping a lecture.

The number e was defined as $\lim_{x \rightarrow \infty} (1 + \frac{1}{x})^x$ and the instructor tried valiantly to have us identify it as an irrational number. This definition was in contrast to the textbook's definition of e as the number such that

$\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$. The instructor was embellishing the material in the "bible for the course".

After talking about $f(x) = e^x$ being the most important of the exponential functions, the instructor added, "Now a good question that should be going through your mind at this time is 'Why is this funny number e so important?' There is a very good reason for it, which we shall see in a little while." From this point, the development of the topic followed the textbook's development much more closely.

The development of e^x as the exponential function whose tangent line at $x = 0$ had a slope of 1 was new to me and I found myself being caught up in the material. I was looking forward to using this development in my own classroom. The participant observer role had been momentarily eroded by my interest in the subject matter. The fact that the instructor was using a calculator to find numerical estimates of derivatives of 2^x and 3^x at $x = 0$ added to my enthusiasm for this approach.

The instructor allowed that, "I haven't been rigorous here, but I have given you some kind of intuitive argument." His intuitive arguments seemed to be quite appropriate for this audience.

Limit definitions of derivatives were used to demonstrate that the derived function of e^x was e^x itself. One student comment to the effect that e^x was a power function indicated that the instructor was still trying to maintain interactive contact with the class. The instructor stopped his lecture and tried to make very sure that we understood that e^x was not a power function.

Eight o'clock and Brian and Philip left to catch a ferry to the island. The instructor quickly wished them a good weekend. Their departure brought our numbers down to fifteen. I noted that two students who had told me they were only auditing the course were not in attendance.

Upon establishing that $\frac{d}{dx} e^x = e^x$ (and marking it with a "happy face" on the overhead) , the instructor said, "Now isn't that beautiful. You've got to smile when you see that." His excitement continued, "Think about it. The derivative of this function is the function itself." He added, "This is a nice, nice property. Well worth emphasizing." I shared his enthusiasm and found myself hoping that some of my classmates might share this with us.

The instructor's background in statistics came to the fore again as he digressed about the Γ function. My lack of familiarity with this function gave me some sense of how my classmates might feel about this and other digressions. Interesting, but I don't understand entirely what you are trying to say.

After the beauty of working with the derivative of e^x , the examples were a letdown. They were simply a matter of formal differentiation. No interpretation or application of the results was required of us.

Closing off the lecture, the instructor, while gathering his notes, called out, "Michael, have I dazzled you?" Michael always sat in the front row and was conspicuous in his attentiveness. The instructor took opportunities to call students by their first names if he knew them. This was another indication of how he sought to transcend the simple information delivery system model of a lecturer.

Catherine advised me that she had heard that the TA was ill and had cancelled the second tutorial. Frankly, I was relieved as I felt I had run completely out of energy. What I did do with the time was to follow Catherine over to the computer lab and look at the code for a program she was developing for her computing science course.

Assignment #7 was given to us on November 12. As usual, it consisted of fifteen problems and was due the following Tuesday. It was also, as it turned out the last set of problems to be collected in this course. What I found particularly significant about this assignment was that it spanned eight different sections in two different chapters. There was at most two questions per section. Five had been the greatest number of sections covered in previous assignments. The course had now become a flat out sprint to the end in an attempt to cover all the sections listed in the syllabus. The phrase "superficial treatment" came to mind.

I arrived at 7:15 to find the instructor telling students that the calculus in this course was mostly technique and manipulation. David mentioned that he now used with ease techniques that he had found difficult earlier in the term. Charles was talking about how one of the problems in section 6.2 required the use of the natural logarithm function. I noted to myself that at least one student is still trying to work through the odd numbered problems.

Michael was concerned about what he termed algebra problems. He spoke about $x \cdot x = 2x$ and $\csc x$ as the inverse of $\sin x$. When he asked why we wrote $\csc x$ as $\text{Arcsin } x$, I realized the extent of some of the difficulties he must be facing in this course.

The lecture began with the instructor advising us that we should already be familiar with the material in sections 6.3 (Inverse Functions) and 6.4 (Logarithmic Functions) from our high school mathematics. Also, we were advised to not work on any problem that involved a definite integral. The instructor described what a definite integral looked like so that we would recognize them when we encountered them.

"Study 6.3 and also do the odd problems." We were still supposed to attempt a large number of problems in each section even when we were covering three or more sections in a evening.

After laying out some of the basic ideas of inverse functions, the instructor told us, "I'm not laying out in detail the whole theory of inverse functions. I assume that you've got it. This is essentially a reminder of what is going on with inverse functions." This attitude was not consistent with instructor's earlier efforts to remind us of what we were supposed to know.

The remainder of his development of inverse functions was seasoned with comments such as "I'm sure you will remember. I hope you will remember."

After reviewing the theory of inverse functions, we looked at $f(x) = \frac{1 + 3x}{5 - 2x}$ and its inverse. The instructor took this example from the text, but, unlike the text, he specified a domain of definition in order to ensure the existence of an inverse function. He began by sketching the graph of $y = f(x)$. Thomas registered disgust when David, sitting behind him, used his TI-81 calculator to quickly obtain a graph.

The section concluded with a theorem relating the derivative of a function with the derivative of its inverse. The instructor told us, "I'm not going to prove anything. I'm going to give you a geometric argument." The instructor made a free hand sketch on the overhead acetate. At the end, he described the theorem as "kind of an intriguing little result". It occurred to me that student input and observable reactions had been negligible for most of the forty-five minutes that had been spent on this topic.

Next up was "a little quickie review" of logarithms and properties of logarithms. "Someone give me one property of logarithms." The silence following the instructor's request was finally broken when he prompted us, "logarithm of axb , where a and b are positive numbers." Unfortunately, the

student who dared to reply gave as an answer "log a times log b". Charles, the engineer, who with the instructor and myself, was a member of the sliderule generation, noted that logarithms "allowed adders to be multipliers".

During the break, I found myself in the role of a teaching assistant. Brian and Philip sought me out and asked if they could borrow my notes for the last half hour of Thursday's lecture. I had no notes, so I redeveloped the results for them. David asked me about a counterexample he had developed for the derivatives of inverse functions result that we had just covered. It turned out that his graph of $y = f^{-1}(x)$ was incorrect. Catherine wanted me to confirm a hypothesis that she had about the derivative of an inverse of a relation.

After the recess, we went through a traditional development of the logarithm functions as inverses of exponential functions. On several occasions, the instructor asked questions, seeking student involvement. The tapes and my notes both indicate that only one student, Catherine, was responding at this time.

When the instructor introduced the notation for natural logarithms, \ln rather than \log_e , he asked us what the name was for the \log_{10} function, prompting us with "It begins with a c." There was no answer from the class. The instructor's response, "Common logarithms. You've heard of that? You're

scaring me folks," brought some laughter from the class. I was empathizing with the instructor.

With respect to the equations $e^{\ln x} = x$ and $\ln e^x = x$, the instructor asked, "How could you convince yourself that the first equation was basically true?" He answered himself, "Couldn't you take your calculator out, chose a positive number, say 7.1. And then put in 7.1, or any positive number that you want." He went on to explain where one might find the e^x and \ln functions on a calculator and to distinguish between the \ln and \log keys. The irony in using e^x and \ln functions on a calculator was that you generally had to key INV LN to access e^x . The calculator was explicit in the inverse relationship of the two functions.

The instructor continued to encourage us to use the calculator for investigating the \ln function. What is $\ln 1$? First we were referred to the graph, then he suggested we use our calculators. "Don't hesitate to play around. Learn this stuff using your hand calculator."

The derivation of $\frac{d}{dx} \ln x = \frac{1}{x}$ was followed by some examples of formal differentiation. Find the derivative of $h(y) = \ln (y^3 \sin y)$. Why?

At the end of this example, the instructor told us, "Do not simplify. I don't have time to waste on simplification here." He referred us to his instructions not to waste time on algebraic simplifications that he had put on the first

midterm and added, "I assume that everyone can do it." His comment, "Craig, why are you snickering?", brought forth laughter from students and instructor alike.

By 9:05 p.m. we were ready to look at the derivatives of $y = a^x$ and $y = \log_a x$ for $a \neq e$. While trying to establish the relationship $y = a^x \Rightarrow y = e^{x(\ln a)}$, the instructor noted that he had failed to cover the property $\ln a^x = x \ln a$ earlier this evening. With the lack of familiarity with properties of logarithms that the class seemed to have, I wondered how much of the instructor's demonstration they were absorbing. The instructor's delivery was straight lecture. When he sought student responses, only Catherine, if anybody, would reply.

Completing his proof, the instructor enthused, "That's pretty slick, isn't it? You think about it and when you've got time to study it at home. It's pretty slick. Nice and simple." He was offering us meta-comments on esthetics in mathematics. Some results are prettier than others.

The last example of the evening was an example of logarithmic differentiation. This technique was to be covered in the last ten minutes of a very full evening. Taking the logarithm of each side of an equation, using implicit differentiation, and then substituting for y in the final expression struck me as a sequence of techniques that

deserved more than ten minutes at a time when student attention couldn't be all that focused.

While closing up, the instructor, speaking to himself, said, "This course has a lot of material in it, a lot of material."

I arrived late at the first tutorial session on November 14. There were only ten students, including myself, in attendance. All four of us who attended the second tutorial session were here at the first session this evening. The TA went over the problems in assignment #6 and those brought up by students. On occasion, other students and I would intercede to help bring the TA on track when he was solving a problem. As with last week's session, there seemed to be a definite lack of structure and direction to the hour.

We went to the lecture theatre to write the midterm. At this time I felt exhausted and was taking medication for a severe cold. I sat with the midterm and just started working away at the questions, with no sense of time management or test writing strategy. Speaking with my classmates, I found that some of them had had similar experiences.

After the midterm, I spent some time with the instructor talking with him about the purpose and pace of the course. I was too tired to make notes and not sufficiently focused to remember much of what was said. This particular research instrument was rather defective at this point in time.

The lecture on November 19 began with an exchange of papers. We submitted our last problem set and picked up our midterms. The instructor had written a note on the overhead about research grants targetted for women. He was encouraging the female members of the class to consider applying.

The instructor informed us that the average score on the midterm was 26/40. He went on to reiterate his intent for the final examination: two hours worth of material and a three hour time period in which to write it. David mentioned to me that he had obtained a copy of an examination that the instructor had set for this course two years ago and it covered everything.

The lecture began with a review of graphs of logarithmic functions. Lack of response to early questions from the instructor suggested that the students were still not completely comfortable with these functions. The instructor exhorted us to become more familiar with the graphs, but he did not slow down to review them with us. In fact, he was speaking more rapidly than we were used to him doing.

The next topic was to show that $\frac{d}{dx} \log_a x = \frac{1}{x} \frac{1}{\ln a}$. When defining $\log_a x$, the instructor spoke of there being several different ways of defining such a function. The "most beautiful way" was a definition involving calculus and

would be covered next term in Math 152. We would content ourselves with a more prosaic definition viewing the logarithm function as an inverse of an exponential function.

In this definition the instructor introduced the notation *iff* for *if and only if*. Was it intended that the students should learn the language of propositional logic and quantifiers in this course? And if not, where were they supposed to learn it?

In the middle of his development, Catherine raised a question about the labelling of a diagram. After a moment the instructor agreed he had made a mistake, thanked her correction, and complimented her on her astuteness. Was Catherine the only student willing to interject a comment? Or was she the only student paying sufficiently close attention to pick up on these details?

After defining $\log_a x$, the instructor commented once again, "That's the kind of introduction to logarithms you get in grade 11 or 12. It's not nearly as nice, in my opinion, as the calculus definition." Interestingly, the instructor's rate of delivery had by this time slowed to its usual speed. He carefully developed the change of base formula and pointed out how it could be used with a calculator to evaluate something like $\log_7 5$.

The instructor then said, "Then you say, of course, why are we doing this? This isn't calculus." He answered, "The

reason is, every function you run across in a calculus course, you don't want to leave it without finding its derivative." This was a succinct statement of one of the goals of the course.

More meta-commentary on proofs were offered by the instructor. "Here's a little proof. How do you prove anything when you have an equation? The first thing you do is start with one side and see if you can just grind out the other. We've taken this approach many, many times throughout this course." If doing proofs were something that students were to learn in this course, this didn't seem like a bad introduction. But were we supposed to learn how to do proofs in Math 151?

When, inevitably, we turned to examples of formal differentiation, the instructor gave us $y = x^2 \log_{10}(x)$ to investigate. He commented, "Who knows what the graph of this thing looks like? Nobody. I don't know what it is. I doubt if you do." I reached for my graphics-capable calculator. Ignoring the potential of existing technology, even in the context of a traditional calculus curriculum, did not strike me as a reasonable attitude. The instructor seemed caught between the world of the sliderule and that of the current breed of handhelds. He encouraged us to use calculators, but fell short of realizing their potential. In my notes, I cited in the margins the many opportunities for using even a

single graphics capable calculator with an overhead projection unit.

We moved on to Section 6.7, omitting 6.6 (the logarithm defined as definite integral), and after being cautioned once again to avoid problems involving material we have not covered. An interesting task: recognizing problems that you cannot do because you lack the necessary background information. After advising of the type of problem for which we would not be responsible, the instructor seemed unable to resist pointing out that the antiderivative of $\frac{1}{x}$ was $\ln x + C$. Furthermore, the antiderivative of e^x was also e^x plus a constant of integration.

Section 6.7 dealt with exponential growth and decay problems. The instructor described the problems in this section as "real" problems that would occur in all of the sciences. In contrast, the related rates problems were "stuck in this section because they were a little application of the chain rule, but in reality these problems do not arise naturally very often."

When he asked us to study the section and do the odd numbered problems, the instructor pointed out that he was not giving any more assignments to be handed in, so we should be certain to "grind out a lot of the odd numbered problems".

The instructor carefully worked through a simple population growth model, asking us questions and trying to

establish a dialogue. I wondered if he consciously sped through topics that he considered less important and slowed down to a more interactive teaching style when working through something he felt to be more significant. We found ourselves talking about half-lives of radioactive isotopes and applications where these ideas would be used. The instructor was selling this topic. The time commitment was considerable, so I had to assume that this was intentional.

If discussing radioactive isotopes brought limited response, the instructor's example of compound interest and money growing in our bank accounts caught the attention of my classmates. There was laughter at the thought of applying exponential growth principles to our savings.

When identifying the basic assumption of exponential growth, that the rate of change of the population is proportional to the population, the instructor introduced the symbol \propto and asked if it was still taught in high school.

Shifting to the differential equation, $\frac{dy}{dt} = ky$, the instructor digressed into a long meta-commentary on the nature of solutions for different types of equations. Numbers were the solutions for quadratic equations, but functions would be the solutions for differential equations. He also spoke about the study of differential equations as an ongoing part of current mathematical research and added, "You can't do physics without differential equations."

We took a ten minute break during which I spoke with students about problems on the midterm. I gave my home and school telephone numbers to several of them who indicated they might want to get in touch with me between classes. My role as a pseudo teaching assistant was becoming more prominent as the term came to an end. The instructor was also engaged in a dialogue with several students about which problems they found most difficult on the test.

There was also a public dialogue between the instructor and myself concerning a young pianist he had seen on television the previous evening. It turned out that the musician was a student of mine. Not all conversation centered on the course.

The lecture continued with notes about differential equations and, in particular, interpreting the differential equation associated with exponential growth and decay. The solution to the differential equation was given as fact, not developed. Then we proceeded to work through a problem. The concept of a differential equation had been introduced, but none of techniques.

After the problem was solved, the instructor spoke about modifying the differential equations in the growth models to make them more realistic.

At 9:00 p.m. we started section 6.8 on Inverse Trigonometric functions. After a preamble about inverse

functions, the instructor told us that the project at hand was to look at one of the trigonometric functions and its inverse. "Remember when I looked at the derivative of the sine function, I went through a fair amount of work to get that. And after I had that, all the rest were similar and it was very easy to get their derivatives. Same basic idea here, don't you agree?" It seemed a fair comment, but my classmates laughed at it nonetheless.

The good humour continued as the instructor asked us whether or not the sine function had an inverse. Determining that it was not monotonic, it was decided that there was no inverse function and hence no need to look for its derivative. "So we just scrap this function and go on. Seems like a good idea." It was late, yet the instructor had succeeded in getting the class' attention focused for his presentation on principal domains. Good teaching under difficult conditions.

By the end of the lecture the instructor had established the graph of the Arcsin function and found its derivative using implicit differentiation, trigonometric identities and clever substitution. He offered us advice as he led us through this derivation, "Always look back to see where you were going." There were serious attempts at mathematics enculturation throughout this course. He also established

inverses for the other trigonometric functions, but did not have time to develop their derivatives.

I arrived ten minutes late for the first tutorial session on November 21. I was the tenth and last student to arrive. The TA had just finished reviewing the midterm and discussing the results. Then, he and several students engaged in a discussion of where they were in the course and how we were expected to finish in the four hours of lecture remaining. There were six and half sections left on the syllabus.

A question about inverse functions lead the TA into using the term *well-defined*. He and another student continued to discuss the concept at length. I wondered to myself what was being accomplished.

Moving on to a discussion of logarithmic functions, I gained a stronger sense of some of the difficulties the students were having with the material. One student asked if the properties for the \ln functions were the same as the properties for the \log functions.

The TA offered some hints on how to deal with the many formulas that we were recently encountering and had been told to commit to memory. My notes for this session conclude with an emphatic "course pacing - sprint to the exams".

The lecture continued with finding the derivative of the Arccos function. After that we were told that the

derivatives of the other inverse trigonometric functions were in the textbook and their derivations were similar to that of the Arccos function. Time was at a premium. Delivery of information was rapid. The instructor frequently identified topics for which we were responsible and ones that we need not commit to memory.

The instructor pointed out as he wrote out a list of differentiation formulas that there also was a corresponding list of antiderivatives.

We were asked to evaluate $\tan(\cos^{-1}(0.5))$. The instructor suggested that we use our calculators first and then evaluate from first principles. Some of the students had to be lead through the key stroke sequence to use their calculators, including setting the angle measure to RADIAN measure. As the instructor went through the traditional evaluation, I realized that most of my classmates did not have the necessary background to move from $\cos^{-1}(0.5) = \frac{\pi}{3}$ to $\tan(\frac{\pi}{3}) = \sqrt{3}$. Calculus instructors, myself included, tended to use their high school mathematics as a prerequisite background, rather than the high school mathematics of their students.

Exercises in formal differentiation followed this example. They were increasingly messy expressions culminating in an attempt to differentiate

$y = \sqrt[4]{\sin^{-1}(\sqrt{x^2 + 2x})}$. My enthusiasm for this problem was muted. The instructor did point out as a "flaw in the notation" that the $^{-1}$ in \sin^{-1} was not an exponent. This comment happened in midst of a very fast presentation of the solution.

At 8:00 p.m., with less than half an hour left, we began section 6.9 on Hyperbolic Trigonometry. The instructor mentioned that the functions in this section were useful in some fields of engineering. Brief mention of the conic sections was made and an analogy between circular functions and hyperbolic functions was alluded to. But the functions were defined as linear combinations of e^x and e^{-x} . The identity $\cosh^2(x) - \sinh^2(x) = 1$ was shown, using the exponential definitions, and the similarity to the corresponding trigonometric identity pointed out.

The derivatives of \sinh and \cosh were developed. The other hyperbolic functions were to be understood through analogy with their circular function counterparts. The inverses of the hyperbolic trigonometric functions were mentioned. Our study of this class of functions was now complete.

David, Thomas, Catherine and myself went to the second tutorial session. The hour devolved into a loose discussion on a number of topics. I spoke about extraneous roots in equations involving logarithms and the role of the absolute

value function in $\ln |x|$. Thomas and Catherine talked about the examination schedules. We all discussed who was taking Math 152 and why. The time spent was useful for my research, but I wondered how effective it was for my classmates.

Before the lecture on November 26 the instructor fielded many questions about what he would put on the final examination. His most frequent reply was, "I might." With respect to proof, he said, "When you say proof, really what choice is there? There really isn't a big variety."

The lecture proper began with the instructor assigning section 6.10 on l'Hôpital's Rule and Indeterminate Forms for study and doing odd numbered problems. In his preamble, the instructor reviewed the material on hyperbolic functions that he had covered in the previous lecture. Speaking about the inverses of the hyperbolic trigonometric functions, the instructor told us, "I asked you to read it. It was in the section. But you will not be tested on it, so you do not have to memorize anything to do with the derivatives of the inverse hyperbolic trig functions. But I hope you've read it and looked at it and maybe done a few exercises. You want to be aware of them, so that if anyone mentions them in another course you can go back to your calculus book and look them up." This was the most direct statement I had encountered of the rationale for covering this topic in this course.

l'Hôpital's Rule was introduced as a "slick rule for finding certain limits". Indeterminate forms were discussed. "In finding limits of indeterminate forms, l'Hôpital's Rule can be often used. I'm not going to prove the theorem, but I'm just going to use it and illustrate it. Don't have time to prove it." Plug and chug calculus because there wasn't enough time.

The variety of indeterminate forms must have caused some students to wonder for which limits the rule was appropriate. Considerable amount of time was spent identifying such limits.

When working through the example limits, I thought perhaps that we might confirm the limit we found through applying l'Hôpital's Rule by using our calculators. At the end of the first example, Michael asked if we were allowed to use this rule for limit problems encountered earlier. The instructor replied, "Yes, you sure can."

Examples involving indeterminate forms $\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty \times 0$, and $\infty - \infty$ were covered. This treatment of l'Hôpital's Rule allowed for mechanical manipulation, but no understanding of underlying concepts was offered. I was amazed at the algebraic contortions that were required to bring a limit to the desired form.

Limits as $x \rightarrow \infty$ caused the instructor to digress about encountering the term *googolplex* in a movie on television the

other day ("Back to the Future III"). Several students identified the movie for him. This was the most significant student-instructor interaction of the evening thus far.

At the end of the fifth example, after tortuously making our way to a limit, the instructor asked us, "Now, if you didn't believe me, what would you do to check it? To gain confidence." The reply he was seeking was not forthcoming. "Why not take your hand calculator and put in a large negative value for x?", he eventually supplied himself.

We were not done with indeterminate forms. Forms involving 0^0 , ∞^∞ and 1^∞ were discussed and the technique of taking the limit of the logarithm of the function in question was introduced. The instructor quoted a Limit Property Theorem from Chapter 1 while working through an example. It reminded me of referring back to an obscure passage from the Old Testament. The actual example, however, was quite nice.

Showing that $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^{bx} = e^{ab}$ would lend itself nicely to the development of the continuous compound interest formula.

During the break, Brian and Philip wanted to talk with me about the NDERIV function on the TI-81 calculator they had just acquired. I found that David, who joined our conversation, was more expert than I with this calculator. He was very helpful to the other students and certainly very enthusiastic about the potential of the device.

Returning from break, we learned that we were going to cover sections 9.1 and 9.2 in the remaining forty-five minutes. The concepts and amount of material that were to be covered and presumably assimilated by us in the time available were beginning to take on surreal dimensions. My own personal belief that the topic of parametric equations was a rich and significant source of concepts and applications caused me, by 8:55, to note about myself, "hunched shoulders, visceral reaction to presentation".

There was nothing wrong with the presentation except that the time available made it seem incomplete and inadequate for any understanding of the underlying concepts. A passing reference to the frequent occurrence of parametric equations in physics was made.

Derivatives of parametric equations followed next. The first derivative was acceptable, but the instructor, after conjecturing about the second derivative, asked us how to disprove a conjecture. He used a counterexample to show that

$$\frac{d^2y}{dx^2} \neq \frac{\frac{d^2y}{dt^2}}{\frac{d^2x}{dt^2}},$$

but did not follow up with the correct parametric

expression for a second derivative.

Moving on to an example, the instructor said, " $x = \cos(t)$ and $y = \cos(2t)$ and immediately your heart palpitates and you say, 'Oh, oh. We're getting into trig identities.' And you're right. By this time I assume that you are masters of

trigonometric identities." While the instructor developed the sketch of the curve, Philip was using the parametric graphing capabilities of his new calculator and proudly showing off the results to David. It was almost as if a technological fifth column was forming in the back rows of the classroom.

The development of this example was notable for the number of questions about trigonometric identities that the instructor asked and to which my classmates were unable to respond. As usual, they were facing double jeopardy: rushing through new material and foraging through their memories for long forgotten mathematics.

There was time for one example from section 9.2 and then the penultimate lecture in this course was done.

On November 28 I arrived late for the first tutorial session and spent time at the beginning talking with Brian about the zoom function for graphs on his TI-81. A sheet listing student marks on assignments and midterms was circulated.

There was a general discussion of what would be on the examination and one student asked specifically about word problems. The TA worked through some applications of l'Hôpital's Rule.

At the lecture that night course evaluation forms were handed out to the students. Afterwards the TA distributed

copies of the mark sheets. The instructor advised us to check the sheets for accuracy and bring any corrections we might have to the final examination.

At 8:00 p.m., with half an hour of instructional time left, the instructor began section 9.4, Polar Coordinates. We were told that we would not be responsible for the material in section 9.6 Conic Sections, but that we should read this material before beginning Math 152.

The treatment of polar coordinates was very similar to that of parametric equations. A great deal of material was covered in a short period of time. There was very little student involvement in the class. Even simple questions from the instructor were left unanswered.

The last example of the course was to find where there is a horizontal tangent line for the curve $r^2 = \theta$. $\frac{dy}{dx}$ was found using parametric equations with θ as a parameter.

Solving for θ so that $\frac{dy}{dx} = 0$ lead to solving $2\theta + \tan \theta = 0$.

This was done using Newton's Method for approximating roots. What a nice little problem showing how various topics from this course could dovetail! It was unfortunate that there wasn't enough time to consider the solution in detail.

Rather than have a second tutorial session that evening, the instructor, the TA, Thomas, Catherine and I went to dinner in Gastown. It was an opportunity for me to hear the

opinions and learn about the background of some of my informants. It was also a pleasant conclusion to a long and arduous term.

The following Tuesday we wrote the final examination for the course. The instructor prefaced his instructions to us by saying that we should not use the graphing capabilities or the programming capabilities or any other capabilities of which he was unaware of our calculators during the examination. To my knowledge, my classmates were scrupulous in their compliance with this restriction.

As the instructor had promised, the examination was about twice the length of the midterm. With three hours to write the paper, the pace seemed almost leisurely. The chairs and tables allowed us to arrange our work more conveniently. David, unfortunately, had hurt his back at work and was unable to remain seated for the full three hours. He left after two hours without any explanation to the instructor. Catherine, as usual, was the first to finish. The rest of us took advantage of the extra time.

At the end of three hours, 10:00 p.m. on a December night, we handed in our papers and the course was finished.

Interviews

Introduction

After the course was completed, I interviewed six of my classmates and the instructor. These interviews occurred in the period between December 13, 1991 and February 14, 1992. One interview took place in the student's workplace early in the morning. Another was conducted in the researcher's home. Three interviews were done over the telephone. One interview was in the student's apartment. The instructor, the last to be interviewed, spoke with me in his office in February.

Details of how the interviews were conducted and the resulting material analyzed are included in Chapter II. The account here is meant to tell, in their own words, the experiences and attitudes of the participants in this course.

Student Backgrounds

The students I interviewed ranged in age from their early twenties to middle fifties. Two of them were among the most successful students in the course, one was in the middle of the grade distribution, two were at the lower end of the distribution and one did not complete the course. One thing they had in common was the fact that they had all been away from high school for some time.

Catherine was a twenty-two year old who had taken this course in her second semester at Simon Fraser University. She was beginning a major in cognitive science. Prior to coming to SFU she had studied Theatre at the University of Saskatchewan for three years.

When in high school in Saskatchewan, Catherine had been enrolled in an accelerated mathematics program. Consequently, she had studied calculus in grade 12 although she had limited success with it. While studying for a theatre degree, she had taken a course which she referred to as "bunny math". "I did quite well in it, but it was a rehash of algebra and trig, basically. And it's been a while since I've done that."

Studying mathematics in high school was not a positive experience for Catherine. She said, "I had lousy teachers... and in high school there was a certain stigma attached..." She went on to talk about one classmate who worked hard at mathematics and physics. "Nice guy, really nice guy. But I didn't want to be like him."

When asked if she had taken any physics before this course, Catherine replied that she had and added, "Actually we learned a lot more about math in physics that we did in math for a while."

David was twenty-eight years old. At the time he was taking the Math 151 course, David was also working night

shifts full time. David was returning to school after an eight year absence. He had attempted two courses several years earlier, but health problems had caused him to withdraw.

In high school, David had been quite successful in mathematics and the sciences. He had graduated with a B.C. provincial scholarship. His first experience of university studies was not positive. As David said:

Suddenly in grade 12, I realized I was being shipped off to U.B.C. sciences and I wasn't sure I wanted to go there. I had no idea what I wanted to do or why I wanted to do it. But I went off to school and got truly atrocious grades, thereby giving me society's permission to go off and explore on my own.

David had transferred to Simon Fraser University at this time. "Consequently the grades I got eight years ago are pretty grim and I have to fight them to get my G.P.A. up now that I have some goals."

David's academic goals included earning a degree as a business major. His experience in the Math 151 course was so positive that he said, "In fact, I enjoyed that class so much that it made me consider possibilities that I hadn't really considered before. Possibly in math or other..." But his first goal was that initial degree in business.

Elizabeth was thirty-three years old and had originally come from Ontario. In high school she took no science courses and mathematics courses only up to grade 11. While

she was living with relatives in Germany, she developed an interest in physics.

When Elizabeth came to Vancouver in 1984, she took grade 11 and 12 mathematics and science courses at the King Edward campus of Vancouver Community College. She comments on this experience:

So when I got to Vancouver in '84, the first thing I did, a priority, was to get back to my education, fix it up. I went to the King Edward campus of V.C.C. and took all the science courses; chemistry, biology, physics, and math - grade 11 and 12 levels. And actually I did very well; that was a good period of time. I felt I learned a lot; I was a straight A student.

Elizabeth spent some time as a pharmacy technical assistant. "I did take a break from my studies at King Charles because things were getting pretty bad as far as keeping myself alive and making a living. Money was always a terrible problem." She worked as a teaching assistant at the King Edward campus and at St. Paul's Hospital.

After completing a physics 12 course at Vancouver Community College, Elizabeth enrolled in two philosophy and one linguistics course at Simon Fraser in the summer term, 1990.

She signed up for five courses in the fall of 1990, including Math 151. It was a disastrous term. She described it:

So, I just got knocked completely flat on my back in the first couple of weeks. I found that I was completely out of control and really behind by even the second week. It was a whole new setup; so much being thrown at me.

Elizabeth dropped the math and two science courses early in the semester.

During the winter term, 1991, she took Math 100 and got an A in the course. In the fall term, she was enrolled in Math 151, a physics course and a course on Euclidean geometry. Ultimately, she failed to complete any of these.

Elizabeth did not feel that she had been adequately prepared for university work in either high school or at community college. She commented:

And having no study habits from high school was a disaster. I must admit I never studied in high school. I didn't know what it really was. When I was at King Ed, it was my own determination and interest and energy that got me through. I studied a lot. I studied very hard at King Ed and put in a lot of time into everything I did there. But I don't know if I'm studying efficiently. I had enough time to do in my own style whatever was necessary to do well in my courses. But I get a feeling that that is not efficient enough for the pace and amount of material to be covered at university.

Elizabeth felt that the university was, in some ways, less accommodating to students who had been away from high school for a long time. She added:

Another thing about university. I really got a strong high schoolish feeling in a lot of ways when I went to SFU...and it made me feel very uncomfortable, I just didn't like it. It was a combination of many things. Partly, it was being surrounded by a lot of people just from high school. You could tell that a lot of instructors had geared the way they set up the course and the way they were explaining things to people, talking to people. It was geared very specifically for people right out of high school, that age group, the background. It just added to the negative aspect of my experience.

Susan was a young woman who had previously taken courses at university and was interested in studying actuarial sciences. She had already earned credit for a calculus course.

I already had credit for [Math] 154 or 157 or something like that and I thought I would be upgrading. Instead my grade goes down three letter grades of so. So I thought, "What went wrong?" In that class, I was happy to be passing.

Her poor results in Math 151 had left her unsure of her future plans. Although she intended to enroll at SFU the next semester, she planned to "go to...math where you work more individually". When I asked her if she planned to pursue actuarial science at another institution, she replied, "I guess and I guess not. Hopefully, I'm going to take other areas of study and see how it goes, but I don't know what to think after..."

Susan felt confident about the adequacy of her preparation for this course. When asked if she felt comfortable with the high school mathematics that was required for the course, she replied:

Actually I did, yes. I didn't have any difficulty in that area. I know that some parts that have left me or maybe I've never learned it and I think about it quite often. There were many areas, for example, in grade 12 where maybe I had learned it the wrong way. Now it takes ten times [the effort] to unlearn it...

Later, she added:

I found you needed your algebra. There were some parts that you obviously needed more than others. ...Some things yes, some things no. I remember he [the instructor] went through one example and I thought, "Gee, I've never covered that before."

Charles was a professional engineer in his mid-fifties. He had taken the calculus as an engineering student back in 1958. Charles was a self-confessed "course junky". He said:

I started a couple of years ago. I took a basic programming course at SFU. And I thought I would be kind of an oddball in the class. But it was really fun to find that the kids didn't care if I was twenty or fifty. They really don't seem to care, which is kind of fun. So I'm back at it.

His interest in the calculus had been kindled when his son had taken it several years earlier.

Charles had missed a number of classes during the term because his work had required him to travel. Consequently, although he did not know his grade at the time of the interview, he did not believe that he had done very well in Math 151 that term. His future plans included auditing Math 151 and then taking Math 152. "So I expect to go back and take 152. That should be one of the more interesting parts of it [the calculus] anyways, from the mathematical point of view."

Charles noted the difficulty of returning to the study of mathematics after a long absence. He commented:

Because a lot of the problems I found were gobs of rust; not just in remembering the algebra, but in getting used to sitting down in a concentrated fashion and whistling off a bunch of calculus problems.

With respect to the algebraic skills required in the course, he said:

Well certainly the algebra presented some difficulties, because it had been such a long time since I'd done it. On the other hand, I think it's fair for a professor in a course to assume that a person who takes it has the necessary prerequisites. I don't think that you can build into the course a large amount of review time.

It had been Charles's intention to take Math 100, the precalculus course, prior to enrolling in Math 151. He found that Math 100 was not offered in the evenings at the Harbour Centre Campus, which made it virtually impossible to take the course. This was a strange situation, because students taking courses at the downtown campus were likely to have been away from school for a long time and would need Math 100 as a "refresher" course. In fact, Charles recommended:

I think that the precalculus course, even if you have the necessary prerequisites, for those students who have been away for maybe more than five years, should be a strongly suggested requirement [for math 151].

He added, "It would be a lot easier to walk in there and sit down, because you would be used to working that way, first of all. And secondly, you would have freshened up on a lot of background."

Thomas was a pre-engineering student "paying his dues in the sciences". He had quite recently come to Canada from

England with the specific intention of earning an engineering degree.

Thomas was in his early thirties and had been working in an engineering firm. His decision to try for an engineering degree came, "because I had gotten to the stage where I couldn't get much further in the engineering firm I was with, not without a degree."

In Britain, Thomas had completed his O-level examinations in mathematics and the sciences. He tried one year of A-level mathematics, but decided to leave school for two years. Upon returning he had studied calculus aimed at technical students. Thomas noted that this course was "not as in depth as the stuff I've just been doing. Just basic." He did quite well in that course, but twelve years had passed since he had taken it.

His route to an engineering degree required that he take both Math 152 and a Linear Algebra course in the semester following Math 151. He had taken a physics course during the Fall 1991 term and would be taking another one in the next semester.

Thomas felt that, given the opportunity, he would have taken Math 100 before the Math 151. In terms of background mathematics, he found that the trigonometry was causing him greater difficulties than the algebra. He used college textbooks and the appendices in the course text for review

purposes. When I asked whether or not he was comfortable with logarithms and exponential functions when they came along, Thomas responded:

Well, I'm never really comfortable with anything new. But then again, logarithms are not really that new, or not to me. But it was basically the fact that I had forgotten it all, so I had to relearn it.

I prompted him with the comment, "So it came back?" Thomas replied:

It didn't come back. I had to relearn it. But when you start saying things about when you multiply logs, you actually add, I remember that. Exponentials...I don't think I covered it much before. So that's basically all new.

Later, I asked Thomas if he was comfortable with this background mathematics by the end of the course. He answered:

No, I wouldn't say I was feeling comfortable. I struggled...But I guess I would have been foolish to think that it would be otherwise. But the fact that I got through it [Math 151] has given me a lot of positive feelings.

Why Did You Take This Course?

The course evaluation forms filled out by the students indicated that the great majority of them had taken Math 151 because it was a prerequisite for courses or majors that they wished to take. The interviews added some detail as to why individuals were enrolled in the course.

Catherine set the tone by responding to the question "Why did you take Math 151?" with:

This particular math course? Well, to be brutally honest, it was a prerequisite. It was one which was not included in my program and originally I resented having to take it, because it doesn't count for my program [cognitive sciences].

Math 151 was not required in Catherine's intended major, but it was a prerequisite for mandatory courses.

Thomas said he was taking the course because he had to. "Because I'm doing this engineering program, I'm locked into this math. It's a bit of a shame", was his comment.

Susan was taking the course because of her interest in actuarial science. But she expressed her view with the comment "Why are people taking this course? There is not one soul who is taking this course just out of interest. We know that for sure."

At the other end of the spectrum were the responses of David and Charles. David said that he was required to take one calculus course for his business major, but that Math 157 would have sufficed. He chose to take Math 151 because, as he said:

It seemed to me from the description of the curriculum, 151 had pretty much what the business course had and more. And I wanted to leave the options open, if I were to decide to get a second degree in management science or computing science.

Charles decided to take Math 151 because, "I've always liked math and when I took it first time around I didn't spend as much time on it as I should have," and "My son took it a couple of years ago and I thought it was kind of interesting."

When I spoke with the instructor about the number of students taking Math 151 because it was a prerequisite, he commented:

It's used...other people use it as a filtering process. Basically they figure, if a student can get through 151,152 at Simon Fraser, they are reasonably intelligent students. It's a filtering process. If the discipline, the other discipline, has any kind of mathematical thinking at all, then they want them to come through here because they filter out the people who can't think mathematically. Business does that.

When I asked him if he felt comfortable about Math 151 being something of a litmus test, he replied:

I don't know if I feel comfortable about it, but my feeling is that it certainly is. You do well in it, get a high B or an A, and you should be able to do well...get a bachelor's degree in the sciences. But if you got a C, my feeling is that maybe the sciences is just not your strong point. Maybe you should try and look at some other area.

Student Attitudes Towards Mathematics

The participants expressed a wide variety of attitudes towards mathematics in general and the mathematics in this course in particular.

Catherine, the most successful student in the course began with various negative feelings.

At the beginning of the semester when the instructor spoke about how we should buy the algebra text, math 100, for which I had credit, and review it, I thought, "Oh god! I don't have time for this class!"

When I asked her if the time required by the course, not the difficulty of the material, was her primary concern, she replied, "Yeah, it was the time. Not that I didn't have it. I just didn't want to spend it that way."

I asked her about her feelings towards mathematics at the end of the course. She replied:

A lot better. In high school and when I was doing previous stuff, it was much more concerned with doing things in the right order of steps, getting the right answer. I couldn't believe the freedom in this. I really couldn't. All of a sudden, you didn't have to do all of the steps in exactly the same...intuition was allowed more in math, which I throve on.

As an example of one topic from which she derived great satisfaction, Catherine cited her success with related rates problems.

I liked those: related rates. Just before the final exam I went over what the textbook had to say and I figured out how to do it...which wasn't the way the professor had taught it. And all of a sudden I was thrilled with them. I wanted them. I went back and did all of them that I could, which was really interesting. Because I hadn't been able to do them at all. And secondly, because most of the class couldn't do them.

This exuberance was in stark contrast to Susan's plaint:

It would be so great if we had just one system in math, integrated, unitarian. Just one way of approaching everything. I think maybe that way we wouldn't have difficulties.

Thomas also felt that there was one appropriate solution for problems. When I asked him about this, he said:

One simple way of doing it. I guess if you try other ways, they would end up being far more problematical. So knowing... I don't know how you would know unless you practiced and practiced and practiced...and knew what to do.

David was very positive about the benefits of studying mathematics. When I asked why he chose to take a mathematics course as his first course after a prolonged absence from formal studies, he replied:

Well, I just find that it improved the way I think. The only thing comparable is a programming course in computer science. I find that it stretches me more than the typical absorb and regurgitate course. Some of the art classes struck me very much as being that way. There is no stretch. It's just a question of putting in a bit of time, doing it and proving that you have put in the time. There is more of a challenge in math in that I am going into uncharted territory. It's not a question of just absorbing different material. There are concepts and ways of visualizing things that I have not encountered before.

Elizabeth also spoke about the differences she perceived between mathematics and science courses and those in the humanities. She said:

I found that with humanity courses, with the English, philosophy or even the linguistics in the summer, with those types of courses you could get lost for a little while, or something would happen, and you could recover. But when it came to a science or math course, I felt

that, unless you were with it all the time, every day or week, as soon as something happened and you lose grip or time, it was impossible to recover.

Charles maintained that one should look at mathematics as being fun. The difficulty with the material in this course was not with the "logic of the mathematics", but with "the memorizing necessary to remember the detail of it". Later he commented:

If you have a reasonable grasp of that sort of thing, you can get by in calculus and survive with a relatively minimal amount of work, but you won't be much good at anything when you finish. But you can sort of grasp the concepts. Because the concepts in general are not all that difficult. It's to remember all that stuff.

Thomas commented on this, when I asked him how difficult he found this course relative to others.

They have all been difficult. Chemistry hasn't been so difficult because it isn't so much a concept-grasping thing. More of something that you actually learn as opposed to physics, which is more conceptual. And certainly so is math. So I guess it is the conceptual stuff I have more difficulty with.

Thomas offered another observation on mathematics courses. I listed several topics we had covered rapidly at the end of the course and asked him if he had any sense as to why those topics had been taught. He replied, "No, not really, to be quite honest. I guess there's got to be a reason." When I asked if he felt it was unusual to be taught things and not know why, he laughed and continued, "That's not unusual at all, Craig. No, that's basically how education has been going, I guess." Later, he added:

I've always been the sort of person who wants to know why something happens. But I've been told, 'Don't worry about that. You'll find out later on.' Which means that you are sitting there, your mind is whizzing around thinking, "Why is this happening?" and they are telling you just to soak up this information like a sponge. But for me, I can't really do that without knowing why. I need to link it together.

Asked if he had opportunity to establish these links in this course, Thomas answered, "There wasn't much time, really."

The instructor responded at length to questions about the intent of the Math 151 course and what he hoped that the students would accomplish. Math 151 is "a rigorous first year calculus course. The most rigorous of those that are generally given at this level here. Aimed specifically at the best science students." It was intended as "an introduction to the concepts, calculus concepts...the concepts of tangent lines, and derivatives and their applications. And some of the techniques for finding derivatives and so on."

In replying to the question as to what he would hope the students would carry away from this course, the instructor added to the rather bare description given above. He said:

Well, I guess that you always hope that you give them a bit of an ability to analyze problems and to think a little bit mathematically. You know, we sometimes talk about the mathematical level of maturity. Sometimes we give for a course a prerequisite for another course where in the second course none of the material in the first course is ever referred to specifically. But you

want people in that course whose ability to think, analyze problems, would be at that level of calculus course, if you know what I mean. So hopefully they would be able to go out and apply just the thinking process...how to analyze a scientific or mathematical problem and apply it in another situation. That would be ideal.

Later in the interview, speaking about the concept of mathematical maturity, the instructor added:

For many people that is the essence of the course. For those that are interested in mathematics, of course, it's just the foundations and they build on that mathematically or in physics, or in some of the other areas there. But, for many people, probably more than 50%, it's reaching a higher level of mathematical maturity. Some of the ideas...they are thinking about ideas. It is kind of a linear thinking fashion, taking one step after the other. Hope they can apply those ideas to totally non-mathematical areas.

What is Calculus?

To some extent, I brought my own agenda to the interviews. Two questions I asked brought responses that I had not anticipated. I asked each interview participant what she or he thought the calculus was about and I asked them what they perceived to be the role of proof and rigour in this course.

Catherine, when I asked her what she thought the calculus was about, commented on the question itself:

It's hard for me, because basically it [the calculus] is a great big fun puzzle-solving. Largely, that would be why I would do it if I were to continue, because it's a bunch of puzzles to be solved.

She did identify as a central concept "rates of change...keeping track of the way things are changing". Catherine could not see any immediate use for what she had learned this term, although she admitted, "I don't know if it turns up in computing; I don't know if it turns up in statistics; I don't know. I'll find out." Catherine thought she might take Math 152, not because she might learn something that she could use in her studies, but because she had enjoyed this course and thought she might enjoy the sequel.

Thomas responded to the "what is calculus?" question with, "I guess it's related to graphing capabilities...how things are derived from certain other information and what you can do with that information. You've put me on a spot."

When I asked him how word problems fit into his view of calculus, Thomas did not have a ready answer. "Yeah, I...[long silence]. If I knew how to do them, I know they were taught."

David responded to my question with, "What do I think the calculus is about? I'm not sure what you mean by that question." After some prompting, he elaborated:

I sort of feel like there is a huge gap in my knowledge. Like I know that I've only taken half of the lowest level. And so I can't claim to have any appreciation for any of the rest that is yet to come. But, if I had to try to explain what I was doing for just the first part...it's a means of determining anything that is in a state of change or fluctuation at a particular instant. I don't know, I feel like I've been handed an incredibly powerful tool, but not the instructions on how to operate it. It slices, it dices, it mixes, whatever.

Talking further about his frustration with the image he had been left with in this course and what he hoped would happen in Math 152, David said:

When I was talking to [the instructor] right before the final started, he said that I would definitely have a different outlook on things halfway through the second term when they started to tie the two together. I'm looking forward to that. I'm a little frustrated now in the sense that, "Okay, I've got all this that I can do now. Give me more." I'd like more time to twist it and see what I could do with it, but also, "Don't stop now. I've only got half of what I need."

When I asked Charles, who had studied calculus a number of years ago, whether his concept of what the calculus was about had changed after taking Math 151, he replied, "Well, not really. I had a pretty good feel for what the calculus

should be able to do." Earlier in the interview, Charles had described the importance of the subject.

I really think that calculus is the basis for most of mathematics and it is used in virtually every other related science and in a lot of non-science courses in some way or other. It's a basis for a lot of things.

Towards the end of the interview, we spoke with each other about Math 152. After I extolled the beauty and underlying genius of the Fundamental Theorem, Charles added:

But from an application point of view, the integration has at least as much application as differentiation, for practical applications. So it's interesting from that point of view, too.

What Role does Proof and Rigour Play in this Course?

Susan had little to say about the role proof played in the course except to say that she found it difficult. She commented, "Like proofs, I obviously find difficult. Epsilon-delta proof. Just trying to put it in abstract numbers...I just don't know."

David had considerably more to say about the role of proof and rigour in this course. When I asked him if he had expected more of the test and examination items to deal directly with proof, he responded:

Yes, I was actually very surprised. I had anticipated a lot more with respect to the rigour, as you refer to it. I find a lot of math professors tend to a "if you can do a proof you can do anything" kind of thing. And that's true if you have a full appreciation of exactly what that proof represents. But in many cases I find people memorize it and it could be hieroglyphics as far as they are concerned. They've memorized exactly what to do and put it down. I think it would have been a good idea maybe to have one or two of the simpler ones in there. Not just for the purpose of giving us more exposure to them, because if we go any distance in math, we are going to have to become very familiar with that. And we got literally almost no exposure to it whatsoever."

About his own abilities to do proofs by the end of the course, David commented:

Well, I can sit and look at all kinds of proof and go nod, nod, nod. But will I be able to duplicate it on my own? Do I understand it thoroughly enough to be able to duplicate it? One or two of the very simplest ones, now that the course is complete, I might be able to stumble my way through. But some of the more complex ones, no. I would have to spend a lot more time. If I truly understand the proof, then they are right that I do understand the concept very thoroughly. But I think, by the nature of the course, it is too easy just to

memorize something without really appreciating what it represents.

When asked if the proofs the instructor offered during lectures helped his understanding of the material, David replied:

Again, it varied from topic to topic. Some of the proofs that were used, in many cases, struck me that the reasoning was very intuitive. And I thought that it was an exercise in, okay, he has proven it, therefore it's true kind of thing. But I was quite willing to accept the point or whatever beforehand. On one or two occasions, I went into the textbook and said, "Okay, let's go through this proof slowly, step by step, and see how they got from point A to point B." And it did help. If there were any difficulties, it was usually a case of not appreciating the notation they were using.

Later, talking about the need to accurately state theorems and definitions in this course, David said:

In particular, when we were learning the concept of Rolle's Theorem and the Mean Value Theorem, he started to explain the concepts at the beginning. I went, "Oh yes, that's very intuitive. That's great." And I'm just breezing along until I caught myself making a couple of errors. And I went back and, Ah!, one of the prerequisites wasn't met. And I had been going down the wrong alley, all for naught. When, if I had looked closely at the beginning, it would have been apparent that it didn't meet the requirements.

The final examination did contain an epsilon-delta proof to confirm a limit. Charles didn't do that particular problem. He stated his feelings about it.

What ticked me off, personally, it had nothing to do with whether it should be on the exam or not, is that I remember looking at that. Both his explanation and the one in the text were difficult, I found, to apply to a problem and decide how to present it in a way that the problem requested. I finally figured it out. I was pleased with myself. But when it came to the exam, I couldn't do it.

With respect to stating theorems and definitions,

Charles said:

Personally, I have a problem with quoting definitions. I guess maybe I'm lazy when it comes to memorizing. I don't like memorizing definitions and that was what was required, to some extent. Either that or you had to have a really good feel for the subject, so that you could sort of define it in some fashion. But I don't think it was required that you quote the definition exactly as he stated it in class. But you had to show a good understanding of the subject. And proofs probably do a better job of that than definitions, in a way. Because, to do a proof, you have to understand. It's hard to memorize a proof, even specific proofs. It's difficult to memorize. You have to have some idea of what the concept is. At least, I think most of us do. To be able to sit down and do a proof, you have to remember the general approach.

Catherine had taken a course in formal logic and was intending to take another. Despite this background, when I asked her how she felt about proofs in this course, she replied:

Well, I don't quite understand what the difference between doing a proof and doing a question is. I mean, isn't a proof just a complicated question? Like show this..or derive it.

Catherine added:

I have a real mental block against proofs. Because in geometry in high school, you had to show these individual steps and you had to justify them. Again, proving in logic courses, I never know when I've done enough.

Thomas made the following comments about the role of proof in this course:

I guess it's like spelling in English. It is the formalised part of the learning process. I'm guessing. I really don't know. But a lot of people really don't

understand anything until it's been proved to them, or they refuse to believe it.

In particular, the proofs offered in lecture made Thomas "believe it was true". However, with respect to proofs in the textbook, he said, "No, I wouldn't say it was the first thing I attacked when I opened the book. It was the first thing I tended to steer clear of. I skipped basically." When I commented that he might have accepted the theorems without proof, he replied, "Basically, I'm not in a position to say otherwise." With respect to students not demanding proofs for every theorem in lecture, Thomas observed, "I guess we are all like sheep, aren't we? Being led."

Lectures and Textbooks

There were three ways that the course material was transmitted to the students : through the lectures, through the textbook and, in some instances, through audio tapes of the lectures.

Catherine found that note taking during lectures was an effective use of her time. With respect to this course, she commented:

That's one of the things I meant about the structure of math and science courses. They plan the notes so that they can be copied and understood. Not like having to take free-form notes during a lecture. Yes, I do find that because of that you see something and then you remember. You see your notes; you see exactly what was on the overhead in front of you, remember what the instructor said, maybe just an intonation in his voice which gives you a clue. Also, I personally try to elaborate notes, even what I'm thinking about.

Catherine would review her lecture notes while working on the assigned problems. "Actually, I'd probably attack the problems first and see how much of the concept I had actually grabbed. Go back, review the lecture notes." Catherine did not use tapes of the lectures because she was frustrated with the linear nature of accessing information.

And if I'm looking for one particular thing, I'll have to find it in the notes and then go through the tapes and then find out where it was in the lecture. But the amount of time involved...it seemed really ineffective to me.

When asked about how she used the textbook, Catherine said:

Originally I didn't. I read through their review and preview about what exactly functions are. That gradually came back to me. I found the textbook a bit difficult to read sometimes. But as the course went on, I started going through the textbook more and more. When studying for the exams, because you don't have access to the lectures and stuff, I would go through what the textbook would say and sometimes it was easier.

Catherine described how she read the text.

I usually wouldn't do the examples. I would just see how they did it. Again, the way the textbook is laid out, I don't know how fast some people read, but I skim very quickly. I get it, but I skim, and by the time I finish reading the question they've assigned me I've seen half the answer they've worked out, which I naturally find quite annoying. I wish they would put them all at the end of the section or something.

Elizabeth was one student who made a practice of taping the lectures. She mentioned her reasons for doing this.

I also seem to be the sort of person who picks up very little in class. I only really learn when I get home and I read the notes and textbook and I start working on things and asking questions and thinking about things. Yet I notice, over and over again, how little I pick up from lectures.

When I noted her taping of the lectures, she added:

I started doing this in philosophy and I noticed how much more I would pick up at home while I was listening to the tapes. I was appalled at how much I would miss in the classroom.

Elizabeth made her own tapes and did not avail herself of the ones made by the university, because, as she said, "I've always found it more comfortable to study at home. Also, I

found that the university hours, especially downtown, are very limited."

Elizabeth also made extensive use of the textbook. She said, "I would read the textbook very carefully and thoroughly. I would think about every word they said. I found that that took up a lot of time." When I asked if the text made sense to her, she replied:

Yes. I wouldn't read on until it made sense to me. Sometimes that meant looking back and reviewing. Or, I would try to follow every example they gave me until I thoroughly understood it. A few times, when I couldn't get it on my own, I'd mark the example and take it to the learning centre.

Susan thought the lectures were very useful. She commented:

There was a really good approach to explaining what was happening. I found it really excellent. Nothing wrong with that. But it would have been really nice if, let's say for example, you got one of those harder questions on your assignment, and you had, maybe not a lecture but something like that beforehand.

Her thoughts on the textbook were less positive.

And then there was the book. I tried to do the examples in the book and there was at least something to show how to get to the next step. But I found, quite often, that you were just left on your own.

With respect to the problems in the text, Susan commented, "I found it very frustrating that in the back of the book there were only answers. I mean, sometimes I couldn't work my way back." Susan was one of the few students with whom I spoke who used the solutions to the assigned problems that were put in the library, although she felt they would have

been of more use to her as examples for working through similar problems.

Charles was also complimentary about the quality of the lectures. He said:

The lectures were fine, I thought. One of the better classes I've taken in terms of the quality of the presentation, the clarity and the orderly fashion in which it was presented. Because even math, which should be logical and orderly, could be presented in quite a shambled manner.

Charles missed a number of classes, but found a student "who had notes I could follow" and borrowed them. The textbook, he found, "filled in some of the things that one covered in the same manner. It covered things differently than the class did. So when you are doing problems, it's a good source of information too." Charles did not make use of the solution keys or lecture tapes because, as he said, "All of those things take time and time was the biggest problem in the course."

David concurred with his classmates about the quality of the lectures. "The lectures struck me as very clearcut and straightforward. He spelled everything out very clearly." David's use of the textbook changed throughout the term.

I tried to read the textbook in the beginning. But when time constraints started to kick in, I found that it was easier, more time efficient, just to have him lecture and then when I read the textbook after the fact, understanding comes very, very quickly. It may not be as thorough as if I had solved everything myself by pre-reading it, but the time involved is a fraction of what it would have been if I had gone through the textbook by myself.

David described his reading pattern.

I think that pretty much I would start at the beginning of the section and if there was anything that I found particularly easy, I would skim over it, looking for any highlights, or anything out of the ordinary. The definitions, because [the instructor] put so much emphasis on them, I paid a lot more attention to them than I might ordinarily have.

David appraised the textbook saying:

The textbook, compared to others I have seen, I thought was actually pretty good. I've been passed down a couple of textbooks from my parents and their time. I look at these things and think, "Good grief. It's a wonder that you got through." So I am appreciative of the fact that the textbook is much more straightforward than many.

Thomas felt that the lectures were good, but that they weren't long enough. He also felt that the instructor was saying more than he was writing down. Thomas commented that he wished he had taped the lectures. When I asked if he had used the university tapes, he said he hadn't, but couldn't remember exactly why not.

Thomas felt that he had to learn how to read the textbook before using it. When I asked him what he meant by this, he replied:

Well, it was basically all the symbolisation of it all. Also the way some of it is worded. It's basically just a different language. I can imagine how some people who don't speak English as a first language...

When asked how he learned to read the textbook, Thomas commented, "I just gradually learned it, I guess. Just read it and tried to understand what they were trying to say." Thomas would sometimes read through a section from the

beginning, but more often he would "tend to go to the problem sets and then go back".

I asked the instructor about his lectures and he offered several observations. He replied:

What I try to do, unless I'm designing a course, is to take the course outline as given to me and try and present it in the best way I can. Trying to get the notation of the book. If my notation is a little different from the book, I'll try to adapt to the notation of the book which does vary a little bit.

The instructor would read his prepared notes as he copied them onto the overhead. When I remarked that one student was quite pleased with this, because it slowed the information delivery rate to a point where he could copy most of the notes, the instructor was somewhat surprised. He replied, "You know why I do that? I'll tell you why. My writing is so poor. That's my reason for doing it. This is student feedback from long ago." The instructor described yet another characteristic of his lecture style, "Typically, in that kind of course, 151, the quick question and the very small pause and then my answer to it. In that course, that's typical."

When the students were asked to describe the quality of the instruction in this course, they were universally positive.

Catherine spoke of the lectures, "Relative to at least one of the courses I took this term it was indescribably better. But the layout of the lectures and the coverage of

the material was very clear." When I mentioned the instructor's occasional digression about aspects of mathematics, Catherine said, that although it didn't add to her understanding of the subject, "It doesn't bother me, it's more fun," and, "I think it relaxed people. I think that's an important thing."

Elizabeth praised the lectures.

I found that the instructor's style was one of the best I have come across. I picked up more in his lectures than most others I'd been in. He was one of the best, if not the best, lecturer I'd had. That was a big help. That was why I continued going right to the end.

David quite enjoyed the instructor's digressions. As he said, "I liked it actually, because I'm not accustomed to instructors being enthusiastic about anything. I liked that. I'd not encountered it before."

Both Charles and Thomas concurred with David's opinion.

Charles commented:

I find that sort of thing interesting. It's interesting to hear that sort of observation or comment. I think it was a nice break. I think it's good for someone to show his interest in the subject, and to some extent, that the person who is teaching it is excited by and enjoys their subject. That carries through to the class. There is a limit to it. It's hard to know the balance. But he didn't do a lot of it. He did just enough of it to sort of let you know that there were other things besides the pure mechanical calculations of differentiating and that sort of thing.

Thomas said, "I felt they were great. I really enjoyed that part. There's a lot of background information that comes out."

The Tutorial Sessions

Questions about the tutorial sessions and their effectiveness provoked quite a bit of response in the interviews.

The instructor, when asked if there was any aspect of the structure of the course that would benefit the students, answered:

The one thing I hope would help them would be the extra hour that they have at the tutorial, where they were able to go in and ask questions. You may realize that there is so much in the course here that it is difficult to stop and and spend ten minutes answering a question. It's just too time consuming. So in that respect, in class there really isn't much in way of feedback. You have to get it outside of class. One of the places would be the tutorial. I would hope they would get some there.

The instructor described the formal relationship between himself and the teaching assistant. The TA's are unionized and their work in a course is determined by a formula set in a contract. The instructor has a set number of hours that he can ask his teaching assistant to work. He can, to some extent, apportion the use of this time. In Math 151, the TA was responsible for marking the assignments, preparing solution keys (cut and paste from a solutions manual) for the library, marking the midterms and examinations (using marking keys provided by the instructor) and, of course, for the tutorial sessions.

When asked to compare the tutorial session setup with the Math Labs conducted at the Burnaby campus, the instructor allowed that:

I think there is a preference, the students that have mentioned it to me, for the lab up here [Burnaby campus]. And I think it's a matter of time. Because those labs are used so heavily, there is so much demand that they are open almost all of the time. Down there [Harbour Centre campus], and I can sympathize with that, there is one hour a week. And if you're not quite prepared to ask a question, you're out of luck!

Catherine attended the second tutorial session which was attended by at most three other students. For her the tutorial sessions were a positive experience. She commented:

All the courses I took at the University of Saskatchewan had no tutorials. So this last term, it was like a bonus we were getting. I found them quite helpful. The problems I had with it sometimes were mine. I hadn't done the reading or caught up. I didn't know what he was talking about. Although, because he [the TA] was dealing with people who had done the problems over the weekend and encountered difficulties, he was able to address them before I got to them.

I asked Catherine if she had attended the Math Lab at any point during the term. She replied, "I did once, even though we weren't supposed to, because there was a problem and I couldn't for the life of me come up with an answer to it." Her opinion of the labs was, "They're great! I think it's a great idea that this thing exists. They were friendly and helpful."

Susan, except for one week when she attended both sessions, went to the first tutorial session. This was also the first course she had taken which offered a tutorial.

Susan asked questions and found that "when other people asked questions it would help". But somehow the tutorials did not clarify the material as much as she expected. She commented:

I know he [the TA] understands what he is doing and everything is perfect there in the tutorial. And well, he says you just can't do that. It would have helped if I had known that before I had done the assignment. A lot of things such as "where did you come up with that one".... I found in tutorial that it felt as if I was on a totally different tangent quite often....Everyone has their own method. But sometimes it really irritates me when people do things in way other than I've ever seen it before. I found there were some solutions where I would go, "Why would you do that?"

David was one student who regularly attended both tutorial sessions and made use of the Math Lab on two occasions. As such he had the widest perspective of those I interviewed. About the Math Lab, David offered, "I encountered a couple of TA's up there that struck me as very good." With respect to the tutorial sessions, he added:

The problem with the tutorial, in the first section, it was almost like having a second lecture, doing examples. Which helped. But any learning I got out of the tutorials, for the most part came out of the second one, where you could ask a specific question, or ask a question on a particular concept, as opposed to just doing examples.

Concerning the TA, David offered the opinion:

I felt that [the TA] was really lost at the beginning. But I don't think he was accustomed to being a TA. But I'm quite sure he never had to instruct a class that much lower in math than he was. And I think it took him a while to adjust to the fact that he had to go right back to the very basics. His expression "Just Algebra", you know.

Thomas also attended the second session. He did not use the Math Lab. His Physics course that term had had an open lab, but he felt that he spent much of his time waiting for a tutor to become available. About the math tutorials, Thomas offered the following comments, "I guess I could have used a few more tutorials. I learned a good deal there. I'm not so sure other people did. But they have to learn how to use the guy too." When I noted that Thomas attended a session with a small number of students, he responded, "Which meant he [the TA] was always asked to do questions which he's already [worked through]. So that was good. A bit late in the evening maybe."

Of those I interviewed, Charles was the most critical of the tutorials and the TA. He said:

The tutorial was all right in the sense that the TA was certainly willing and prepared to talk about anything he could to help the kids. There wasn't a problem there. On the other hand, his ability to present a problem solution in an orderly fashion wasn't very good. I suppose that is lack of experience. I guess it's partly the way you think, too. When you jump steps in doing a solution in a tutorial, you're not really helping people. I found it confusing.

Charles was not altogether negative about the tutorials.

There were times you could pick up something. Sometimes it would be quite clear, because you would know how to do something except that you would be missing one little step or looking at it the wrong way or something. There was some value in it, but...

Later he added:

The quality of the tutorials, it's not what it should have been. And I don't really like to pick on...I

suspect this TA was probably typical of TA's. I don't know. They are good math students, therefore they are TA's. That's no recommendation for anything. Skill in a subject doesn't mean you are a good teacher. In fact, sometimes they are exclusive of each other.

Charles had not attended the Math Lab, but felt that it was a good concept. He suggested that two hours of Math Lab before the lectures at the Harbour Centre campus would have been useful.

Health and Fatigue

The instructors and students are human. Several factors such as illness, fatigue and time spent travelling to and from classes affected attitudes and performance adversely.

Elizabeth was taking two courses at the Burnaby campus as well as Math 151 at Harbour Centre. For her, the amount of time spent travelling was a "big, big problem". Contrasting her experiences at Simon Fraser with her studies at the King Edward campus of Vancouver Community College, Elizabeth said:

I found I never had a chance or time to settle down and get into a really deep concentration as I had before when I was studying at King Ed. It was such a great experience, it was so satisfying. I really learned because I was able to get into these states, studying, where I was really concentrating, really getting down into detail, and I really enjoyed it. It was a satisfying, wonderful experience. And I found I could never get into that state when I was at SFU. And one of the problems was that I never found the time because I was constantly running around trying to get to classes, waiting for buses.

Susan explained that she never sought out the instructor or TA on the Burnaby campus because "going up there would have taken me over an hour and a half".

Catherine also took courses on both campuses. During the latter half of the semester I would drive her back to Richmond where she was living. She would frequently lament the time wasted in commuting to the university.

Two of the interviewed students spoke of chronic health problems they had and the impact it had on their studies.

David had ulcer colitis. He described his situation in the following comments.

I started to attend school a couple of years ago and I discovered I was trying to play hero. I was trying to do too many things. Then I was diagnosed as having ulcer colitis, which meant that I had to manage my body just a little bit better as well. It isn't a problem if I use a little discipline. If I don't, it lets me know if I'm misbehaving. ...But all I need is a few nights of four hours of sleep, which isn't that hard when I'm working graveyard, and it starts to object to that kind of treatment. Then it get into a vicious circle where getting the necessary sleep and maintaining the energy levels necessary to get over it becomes a problem.

At the end of the course, David not only put in some long nights of studying, he also hurt his back at work. Consequently, he was physically unable to sit through the final examination for the allotted three hours. As he said:

I did myself in with my back and then that, combined with all the studying I had done, hindered my sleep....I recognize for my next semester that first and foremost, I have to be in good enough shape to even write the exam. I was so demoralized when I walked out on that exam. I hadn't finished it. I walked out early. I would never do that normally.

Elizabeth had a recurring stomach ailment. It was one of the key elements in her decision not to complete this course this semester. She spoke of the effect of her poor health on her attitude.

I ended up often staying up almost all night trying to get assignments done. I started losing sleep and I developed this ulcer. I was really in bad shape. It came to a point where I couldn't even force myself to go on. And losing time and dealing with other distractions...once again, when you reach a certain point, when you lose a certain amount of time in these types of course, you can't recover. I just felt I was losing grip.

Talking about other courses she had begun this term, Elizabeth said:

The physics went better than all the others. I enjoyed it and I did well until the end. It was the last one I let go. And I just realized...that I wasn't going to make it through. I was like, "I surrender." Which was a big blow. I had high hopes and was planning this for a long time. Once you lose the health and energy, I was spending all my time trying to keep up. I wasn't sleeping, wasn't eating properly, long bus rides and so on and so forth. And then this stomach problem. That was the end.

The instructor gave classes on a couple of occasions even though it was evident that he was ill. When I asked him what provisions the university makes if he is unable to attend a lecture, he commented:

There will be no provisions at all. For example, if I'm sick for a week, there would be no replacement, I don't think. And the responsibility would be on me to go faster and skim a little bit and try and finish the course.

He mentioned an operation he had postponed until the end of the semester and continued:

I'll tell you something else, too. And I'm sure you are aware of it. In this job here, and in other professional jobs, people are not absent unless there is really something wrong with them. ...In this kind of job, unless I'm sick, I'm not away. And that's true, I think, of everybody here. But when they are, I don't know what provisions...If I were in a traffic accident and in the hospital for several weeks, I think they would do something. But I suspect for a week or so, nothing would be done.

The instructor was aware that many of his students were tired and sometimes ill. "The tiredness? Yes, of course they are tired. Some of them have come off of an eight hour job!" But the working assumption was that the students would soldier on through, although he admitted, "It's hard; I know."

Personal Interaction

I was interested in the amount and type of personal interaction that would evolve among the principals in this course over the semester. Some of the people had turned out to be quite gregarious and there was often conversation before, during and after classes between students, students and the instructor and students and the TA. In a number of interviews, I asked students whether they had had opportunity to work with other students, and, if they had, whether or not this cooperation had proved useful.

Catherine felt that her success in this course and the active role she took in lectures set a barrier between herself and her classmates. She mentioned:

I'm still getting some nasty comments from people sometimes. If people ask me how I'm doing in a class, I have to say, "Sorry, but I'm not having a problem with it." They become sullen.

When I asked her if she had worked with other people on the assigned problems, she responded, "Only a few times. I had a class just before math. No, and I don't work well that way anyway."

Elizabeth felt it was worthwhile to work with someone.

This was another thing I was beginning to realize from earlier term. It's very valuable to buddy up with someone. I noticed the students who were much younger than I, who came straight out of high school with a small group they knew, worked together. Even though I felt that to really learn I had to do it by myself. But once you've covered that ground, or if you run into difficulty and want to discuss it, it's good to work with another person.

She had gotten telephone numbers from two of her classmates, but did not contact them as often as she thought she might have because she felt she was not keeping up. Elizabeth also worked on the assigned problems with a tutor.

Susan did not work with other people in this course. Nor was she convinced of the benefits of working with others. She mentioned that she talked with Catherine every so often, but, as she said:

I found that she certainly knew what she was doing, but it was just too abstract. Sometimes I find it distressful when it's even more abstract than the way

I'm thinking. Actually, I find quite often that people don't understand where the problem is....Where does that part come in, where someone can really teach you and understand where your problem is?

Charles and Thomas both felt it would be advantageous to work with others, but that this course did not lend itself to people finding others with which to work. Charles said:

Part of it is because it's like going into first year university. When you take classes part-time, it's like going into first year university into the first class, because you don't know anybody. You seldom see people you've seen before. And because you're working, especially if you're working, you can only go for the class. So you tend not to establish relationships that you can sit down and work. Whereas, in university, people in the same class, first thing you know you're sitting in the library together. You're doing this or that. You have some chance to build up relationships and work with people. More difficult, I think, if you are taking classes part-time. But yes, I'm sure it would help to be able to sit down and do your assignments with someone.

Thomas also commented that it was the first semester of his studies at the university and that there was "a lot of people and you don't know a soul". The possibility of working together with others was one of the opportunities he hoped to explore in the next semester.

Technology : Calculators and Computers

The issue of technology, the use of calculators and computers in this course, was one that I brought to the study. I spoke with each of the interview subjects about their use of technology and attitude towards it.

Catherine used a simple Sharp 540D calculator for both the assignments and tests. She brought it to class, although when the instructor asked students to use their calculators, she felt that, "I'm usually not fast enough on them to do things as he asked, because I get all the steps convoluted. I can't transfer from hearing it to doing, that quickly." When I asked how she felt about some of her classmates having more advanced calculators, she replied:

It didn't bother me, it really didn't. Partly because I was doing better than they were. There was a certain sense of satisfaction, actually. No, the only advantage was time and I was never pressed for time.

Catherine felt it would be nice to acquire a super calculator "just for fun". "I thought it would be fun to play with one, but I didn't feel any need for one." For Catherine, technology wasn't a significant factor in her success. She commented:

I don't know what it was like for people that had them. It may have been a great advantage. And I don't know what it would be like if I had one. I obviously don't need them.

Elizabeth also used only a basic scientific calculator. When I asked what she thought of the programmable, graphics capable models, she said:

The only one I saw was yours that one day. I actually didn't even know that they had calculators that could do that. But I thought it was fascinating and I thought it was a really good feature, to be able to see the graph.

When I asked why she didn't obtain one that semester, she replied:

I really have to make an effort to break into this new technology. Even though after I've learned to use it, no problem, it's very useful. But it takes a lot of...whatever, to break through that block to learn to use it.

Susan was another student who was not aware of graphics capable calculators.

I found out quite late, in December at the end of the term almost. Actually, I remember our prof saying please don't use them during the exam and thinking, "What's the use of having them if you can't use them?" But actually, it would be quite helpful to learn quite a bit from it.

Susan used a TI34, "the one with all of the colourful buttons". Not only did she use it for calculations, she used it to remind herself of things such as where "sine curves, cosine curves start. Small things like that."

Charles used a programmable calculator that he had owned for a dozen years. His primary use of the unit's programming capabilities was to program Newton's Method for finding roots. When we spoke about the instructor restricting the use of some calculator functions on the midterms and examination, Charles commented:

The graphing capabilities would be one that I could see you wouldn't allow, because you try to get people to work with the concepts of what's involved. But something as mechanical as Newton's Method, I can't understand why you would be concerned.

With respect to the instructor's use of a calculator in lectures, Charles commented, "That was fine. I think the

point he was making is that a lot of these things can be done with a calculator now."

When I asked Thomas how he felt about his classmates having calculators with capabilities that his did not, he replied:

I felt that I was behind in the race. The problem is, I lost my calculator about two nights before my second chemistry midterm. So I had to buy a calculator. This is for chemistry, so I wanted one with all the constants on it. So it was \$50. So the problem is to convince my wife that I should buy another calculator for \$140.

I asked him if he used his scientific calculator throughout the course. Thomas responded, "No, not very often. Not as often as I would the graphics one. I would have not lost a lot of marks if I had that. I know that for a fact."

Thomas had taken a UNIX course and a short course in using C programming language in England. He went into these courses "not knowing a damn thing about computers". After these courses he admitted, "I'm not quite so scared about them." He was scheduled to take a course in FORTRAN programming next semester, but he had yet to use computers in his other courses.

Four students in the class obtained graphics capable calculators during the term. Michael bought an HP48SX and Brian, Philip and David acquired TI81's. David was one of the students I interviewed and he spoke at length about his use of and attitude towards his super calculator.

David was comfortable with using technology in coursework well before he obtained his TI81. In high school he had used a TI58 calculator to do linear regressions on data from his physics laboratories. And he was using the spreadsheet on his microcomputer at home. He spoke about his use of spreadsheets.

Well, for Newton's method, I found the concept, once you got it, to be quite simple. This works. But I found I was getting bogged down just with the repetitive motion of entering data, operating my thumb there. Basically, I've got a computer at home with a spreadsheet. I plugged it in there and just let it crank it through. I could see then just exactly how quickly it works and how effectively and I tinker with it a little bit. If my guess is this far off, how long will it take. Good grief, nine steps. Or, if it is this close, two steps and I've got eight digits. So I experimented a little bit that way and got a greater appreciation, I thought, for exactly how well it worked and some of its limitations too.

I asked him what had motivated him to purchase a TI81.

David said:

I was having a hard time visualizing something like trigonometry and what was happening. And I needed to picture in my mind what things looked like. I've got some graphics software on this thing, but it is very limited. I spent more time interpreting how the software worked than what the curve actually did. When it spits out this answer, what does it really mean? I found I was making it much more difficult that it had to be. When I got to see the graph of a curve, it was like, "That's it. That's not so much." When I got to see graphs repeatedly, I got a feel for exactly what their behaviour was. Up to this point, purchasing the TI81, graphing a function was a very time consuming, laborious process. And I would try to avoid it, if at all possible, for some of the uglier curves. I found I was learning to solve problems by rote. Do this, do that, without having any real understanding. By virtue of having that calculator, I could do problems much more quickly and with greater understanding. So much so that

when it came to the second midterm involving the graphs, when it came to procedures, the ten little steps or whatever, it was a breeze. I didn't even use my calculator once, I don't think. It [the function to be graphed] had a third power in it and I knew there was only so many things this could look like. So it helped me to visualize beforehand what was possible and what wasn't. The big advantage the calculator gave me was that I could absorb concepts more quickly and more thoroughly.

David not only used his calculator for graphs, he programmed it with Newton's method, used graphs to estimate limits and used the NDERIV function to plot a function and its first and second derivative functions on the same axes.

I also spoke at some length with the instructor about his use of calculators. With respect to this, he said:

Yes, they're great. I go back prior to the introduction of hand calculators period. My reaction when they came in was that they were neat. They could do things that you just didn't see being done very quickly. Matter of fact, the theory was there, but you avoided a lot that was just too time consuming. You had to be careful that you didn't get bogged down too much with a time consuming type of problem. You could end up wasting time. So when they came in, I adapted the course slightly to adjust for these calculators and I'm sure I haven't adjusted for the graphics calculator yet. But if I teach it again, I'm sure that I'm going to have to sit down and cope with it. It will help the students, but it really won't mean that I have to change things very much in terms of examinations and what not. What they will be able to do, I would foresee, is that if I have a graphing problem on it, I can test that they know the concepts, because I can demand that they show the derivative, set it equal to zero and so on. But where it would be an advantage, and certainly in a real situation you would want to do this, is in checking your results. You just punch the function in and graph it. But finding the maximums and minimums, the points of inflection and so on, they may be a little approximate, I can test the techniques to see that they know how to do that. It will have a mild impact on the way I construct tests and what not, but not a significant one.

On the other hand, from the student's point of view, I think it's great. If you can go through all that work and then confirm that your results are correct, that helps learning, I think.

When I mentioned that possibility of using overhead display units for graphing calculators in his lectures, the instructor replied that he already made extensive use of computers in teaching statistics. The instructor distinguished between teaching statistics and calculus:

When you get into multiple regression, you just can't do the problem by hand. And in calculus, I can say, "Do this graphing by hand." Sure, it takes longer, but you can go through the steps to actually construct a graph and you can do it in a reasonable amount of time. You can't do that with a multiple regression problem. That's why you just punch in the commands into a computer, project it overhead and it's right there.

The instructor did allow that using graphing capabilities of calculators would allow him to spend less time on the topic and free up time for other topics in the course. He commented, with a small laugh:

Probably, maybe the graphing could be left out to the extent that I did it with the advent of the technology here. You know, maybe you could get yourself a graphing calculator.

Mathematical Prerequisites for the Course

Both the students and the instructor were concerned about the preparation the students had in mathematics prior to taking this course. The problems that they perceived seemed exacerbated by the fact that many of them had been away from formal education for a number of years.

Catherine felt that "there has to be some time limit on the algebra prerequisite" for Math 151. Charles stated this even more strongly.

I think that the precalculus course, even if you have the necessary prerequisites, for those people that have been away for maybe more than five years or something like that, should be a strongly suggested requirement.

Paul indicated that if he had known how difficult he was going to find the Math 151 course, he would have taken the Math 100 course.

Elizabeth had taken the Math 100 course and had been quite successful in it, although she was unable to complete the Math 151 course during this term. In her first semester at Simon Fraser, she had gone to the academic counselling services. On the basis of her transcript for Vancouver Community College, she was advised to register for all the first year science courses. As Elizabeth said, "I just got knocked completely flat on my back in the first couple of weeks." She went to see another counsellor when she enrolled at the university the next term. "The counsellor I went to see just shook his head when I told him the advice I had been given." He suggested a much lighter work load for her. Elizabeth took Math 100 in the winter term of 1991 and earned an A. "Again, in the winter of 1991, I felt I could do math after all. Things were okay again."

The instructor spoke about two specific concerns he had about the students mathematical preparation in this class.

He was concerned about those who had been away from the formal study of mathematics for a long time and he had some observations about students who had studied some of the preliminary concepts of calculus in high school.

With regards to the first issue, he said:

Some of them have been out of school for four or five years, or maybe even more. There was the engineer, for example, who was obviously an intelligent, scientific kind of person there. But he had been away from mathematics for a long, long time. So that probably works against them. Sometimes they may want to take a brush up course. Most of the time, realistically, they don't want to waste their time doing it, algebra 12 or something like that.

He added, concerning the classes he taught at the Burnaby Mountain campus:

I don't find that same problem up here. Most of the students up here are very recently from high school and they haven't had a chance to forget their previous math too much.

On the second issue, the teaching of an introduction to calculus in the high schools, the instructor commented:

One of the things that does cross my mind about the group there, and I've had the situation arise several times in the past here, is that there were two students there. They went through and I believe they had taken the calculus in high school, which generally indicates that they are the better students there. But they did not do well. And

I've had that happen in the past, where people have come in from high school. There may be lots that come in from high school and do well, but this stuck in my mind. These two, and other ones in the past, although they

have had it [high school calculus], have not done well. And there's something of interest there, I think.

When I told the instructor that the provincial Mathematics 12 course had a mandatory unit on introductory calculus, he commented:

I'll tell you one thing that comes into my mind though. Not with these students here, but ones that I've had in similar situations, is that they know how to differentiate polynomials. And they think they can differentiate. I mean, a polynomial is the simplest, most trivial thing you can differentiate. They know the power rule, so they know how to differentiate. And they may be good at that, you know. I would agree with the approach you've mentioned [teaching college level calculus to well-prepared high school students]. I've even said that myself before. Look, if it's going to be in high school - it always was an optional, locally developed course - fine and dandy. But I would agree that, if they are going to do that, make it rigorous. Because they are just kidding somebody, if they think differentiating polynomials or finding max-mins of a polynomial and graphing it, if that's indicative of your abilities in calculus. It's a much bigger story than that. And so, if they are going to do it - do it. Do it full force.

We spoke further about the articulation between the high school mathematics curriculum and the university mathematics courses. Both of us found ourselves, as mathematics teachers, assuming a knowledge base for our students that at times they just did not have. The instructor said:

I was amazed. There are a few things there that I would have...but I'm not sure whether they have never seen it or they had forgotten about it. There were certainly ideas that I mentioned there, that I felt that they should know this material or at least be acquainted with it. It's like someone should be able to say in engineering, "You should, maybe not know, but be somewhat familiar with the hyperbolic trig functions." We talked about them for half an hour and at least

you've heard of them. You know they are exponential, you know their derivatives are somewhat like the derivatives of trig functions. You may not know the exact formula, but you should...But they - I just drew blank faces or else they just didn't want to put up their hands, or something. A few cases were like that. I'm not sure what the real reason was.

The Quantity of Material and the Pace at which It is Covered
in the Course

The pace of the course and the amount the material to be covered were topics that resulted in a lot of feedback from both the instructor and the students.

I listed some of the topics that we had covered in the last two weeks of lectures to Catherine and asked her what she thought about the treatment given them. With respect to the hyperbolic trigonometric functions, she commented, "I have no idea what they look like." She added:

It was very much akin to what we did in high school. Learn how. Which is fine for me. I memorize quickly. I can learn the derivatives. But, from what I gather, that's not exactly the point. So why? They weren't on the exam and I don't feel we did anything with them. Or were they on the exam? [The function \cosh occurred in one part of a formal differentiation item.]

Catherine felt quite comfortable with the material on parametric and polar representation of equations of curves. The only topic she had any hesitancy about was linearization and that was because it was covered in a lecture she had missed.

Catherine had two comments to make about the pacing of the course. With respect to the unit on curve sketching, she offered:

Like I said, the graphing. I don't know if it's that important or not. It just went on forever. However, I could probably graph any polynomial or rational function thrown at me. Possibly some other ones. I don't know.

About the pacing of the course at various times throughout the term, she mentioned:

I don't know about the rest of the class. The impression I got from some of them, and this was back when I was actually doing the six hours or whatever each week, was that it seemed to be going really quickly at the beginning. One of the people I talked to and myself, we hadn't done any math in so long that we were scrambling to keep up. Then things seemed to relax a bit. Then the lectures seemed to speed up at the end of the course. But I didn't need to scramble any more, so it didn't seem fast to me. I know a lot of people I talked to said they couldn't believe how fast things were going. They were even writing some of it off. But it didn't feel fast, just not detailed.

David was frustrated with the varying pace throughout the term, although he understood the rationale for it somewhat. He said:

I am frustrated at the timeframe they allow, the opportunity to truly expand. You're required to get the gist of the very basic concepts. And as long as you can demonstrate you have a basic understanding of the highlights of the course, you're through. But you never really get a chance to explore and inter-relate some of these concepts and see where you can go with them. The timeframe does not permit this. This was one of my disappointments with the course. That final push in the last two weeks was crazy. We started the course covering a section every lecture, lecture and a half....For the first two or three weeks, it [pace of the course] was quite adequate I thought, because being out of school as long as I have been, it took a while to

dust off the cobwebs. But if I had been fresh out of high school, I might have found it a little slow. I don't know. I can't judge any longer with respect to that. But I thought it should have been accelerated a little bit more if he [the instructor] wanted to cover the entire course. Because, covering three sections a lecture towards the end, it was just, "Okay, I have to be able to do the simplest of questions. One of everything. Exploring it will have to come at a later date."

Susan spoke about the pacing in similar terms.

The first month, September, went pretty slow compared to the amount of material. And I found in the last two or three weeks we were thrown so much work that I found that I sat down for a couple of hours just to memorize hyperbolic formulas. For which there wasn't even a question on the exam. So I thought, "Gee, what's the relevance of this one." We covered so many things and I thought, "Oh." The last bit was really rushed, but up to the first midterm everything was pretty smooth. And even the second midterm wasn't too bad. But afterwards! I think it was Newton's method or right after that, that everything accelerated. It was really rushed.

Thomas, when I asked about pacing, said, "It wasn't a uniform pace. It got very hairy towards the end, covering chapters in half a two-hour lecture." Thomas suggested that if he were to alter the course he would "try to allocated the same amount of time to each subject or maybe make the course a bit longer". He did feel that the pace was much slower in the beginning, to the distress of the instructor, but as Thomas said, "I'm not sure how he could have done it any other way. People were definitely having problems."

Charles, the engineer, spoke about the fact that sections labelled applications were omitted, "Well, a fun part of mathematics is the applications, or it should be.

Still, I can see why he did it [omitted the sections]. There just isn't time for it."

Charles did comment on some of the topics that were included in the course. He said:

I'm not sure, for the intent of this course, that hyperbolic functions and some of those things really added that much to it. I know that hyperbolic functions have a fairly specific engineering application, and a few other applications, because in one of my departments we use them all the time. But I'm not sure that in the context of the calculus course, that some of the things that we did in the last two or three weeks were essential. My own inclination would be that the course would be better if you left out the last two or three weeks and stretched the first part out and got people more comfortable with it. Because some of that stuff is, at the pace it went, you didn't learn it anyway. It didn't work. There were chapters you just whistled through.

Charles believed that the concepts of calculus, properly presented, are not that difficult for most people to learn.

He said:

None of that stuff should be that difficult. Most of the concepts in calculus, now and then when you get into rigorous proofs it is difficult, but if you are talking about the concepts, especially if you have someone who can explain it as well as [the instructor] did, are really not difficult. If you use the right language and present it correctly, the concepts aren't difficult. And I think most people with a reasonable math background, those that survived algebra in high school with a B or something like that, shouldn't have any trouble with it. Because it's not, I don't think, any more difficult than algebra or geometry or analysis. But it is set up now so that the pace is ridiculous.

For emphasis, in another part of the interview, Charles added, "It seems strange to me that the course is set up so

that you either have to be a workaholic or a real whiz at math to survive it, because it goes so fast."

The instructor was unequivocal in his feelings about the amount of material in this course and the time allotted for covering this material. He said on this subject:

Another thing that I feel about in that particular class, 151, you know my feelings, is that it is so jammed with material. My feeling is that there should be more time for it. The only reason there is not more time is, I think, at Simon Fraser, bureaucratic. I don't think there is any educational foundation for not having another hour with the course. Let things slow down a little bit and sink in a bit more. That's my feeling....It has to do with the registrar and the issuing of units. I think that's what it is.

The instructor maintained that it was not university courses in general, but this course in particular in which the "the quantity of material is too much". This had an impact on his teaching style. He commented:

I just feel continuously under pressure. I've got to do a section per hour. And some of the sections are bigger. I find it difficult to do in a way that I feel like I'm doing a good job. I know other people don't have any problem getting through the material. But I have to look back and say that it has got to be somewhat of a superficial approach just to get through all the material in that time.

Later he added:

There's no doubt about it. This is the most jammed course... This course here [Math 151] is not my preference for teaching. Not that I don't like the material. I do like the material. It's fun to teach it. But, for example, when I taught it in the colleges I got four hours to teach it in. I feel very comfortable with that. But, matter of fact, I believe this was a four hour course originally when it was set up. Then they set up the lab sections. And they wouldn't give it

fours units of credit. I guess other departments complained; it's an introductory course. Why is it that they get four units in math and only three here? There is that kind of problem. And so the hours were lined up. They were willing to give a one hour tutorial, but not add one hour to the lecture. Yet the material in the transfer courses in the colleges is done in four hours and in some colleges in five hours of lectures. Which means that there is lots more time to do examples and get a rapport, a dialogue going. But with three hours, I can't feel justified in letting that time go. I don't think I'd get the material done.

CHAPTER V

SUMMARY AND CONCLUSIONS

Introduction

In this chapter I summarize the most significant aspects of this study and show how these aspects relate to the literature reviewed for this project, specifically the literature from the calculus reform movement. This is followed by a set of conclusions I have drawn from this work and recommendations for improving those situations I believe need to be corrected. Following this, I discuss the relevance of this study to calculus reform effort. Finally, I write about the impact this study has had on me as a classroom teacher.

Summary

Chapter IV contains an extensive representation of the data that I gathered throughout the study, filtered as it is through the biases and interests of the research instrument, myself. In offering as complete a description of my experiences as was feasible, I intended that each reader of the account of the field observations and the interviews would find something or some event which relates directly with their own classroom experiences.

The collection, analysis and representation of data during this study have led to an ongoing stream of thoughts

and conclusions about the first-year university calculus experience. Following is a discussion of those aspects of the Math 151 course described in this study which I consider to be the most significant.

Positive features of the course

The students in this particular section of Math 151 enjoyed a teaching situation far superior to that of the majority of students taking the equivalent course in many North American post-secondary institutions. This comparison is made on the basis on information gathered for a special survey in 1987 (Anderson & Loftsgaarden, 1988).

Both the lecturer and the teaching assistant were fluent in English. The lecturer was regarded by the students and the researcher as an exceptionally good instructor, so much so that some students said they would take future courses taught by him because they were taught by him. One student continued to attend lectures after she had officially dropped out of the course because of the exceptional quality of his presentation.

The excellence of the lectures in this class became even more apparent when they were contrasted with the presentations of the inexperienced instructor encountered during the pilot project. It is also in contrast to the

benign neglect of calculus teaching by mathematicians referred to in the literature (Rodi, 1986).

The textbook was, in my opinion, one of the best that I have used. It contained a wealth of information as well as a wide variety of problems to be worked through. Appendices provided appropriate and necessary review material for the students who needed it. The text was profusely illustrated with figures and diagrams.

The current style of calculus textbook has been identified as a serious impediment to the quality of calculus instruction (Douglas, 1986; Garland & Kreider, 1983; Renz, 1986a). The textbook used in this course had many of the features decried in the literature, yet none of the participants in this study had any significant concerns about the textbook. Several students, in fact, mentioned that they found the text to be informative and useful.

The evaluation scheme provided relatively frequent feedback to the students on their progress. Weekly assignments were collected, marked in detail and returned promptly the next week. The two midterm tests and the examinations were generally acclaimed to be fair and "as advertised" by the instructor. Frequent feedback has been identified by some (LaTorre et al., 1990) as an essential feature for student success.

In the tests and examination the students were required to demonstrate mastery of far more skills than the mechanical symbol manipulations associated with the "plug and chug" calculus to which the calculus reform literature disparagingly refers so frequently. (See Appendix D.)

The small class size allowed for the possibility of students making contact with each other and perhaps helping each other.

The instructor advocated the use of basic scientific calculators and did not actively discourage the use of more advanced calculators and microcomputers. Although this class did not employ calculators and computers to the extent advocated by many in the calculus reform literature (Tucker, 1990), it was not confined to strictly pencil-and-paper calculations at the far end of the technological spectrum.

The classrooms were modern and well-equipped. The instructor had use of overhead projectors for all of his lectures. The lecture hall for the Tuesday classes was especially comfortable and conducive to notetaking.

The lectures were taped and the tapes were made available in the library at the Harbour Centre campus, as well as equipment for listening to them. Solution keys for the assigned problems were also available in the library.

Despite all the advantages that the students in this class had, it was not a very positive or successful experience for many of them.

Math 151 as a Filter

The image of the calculus course as a barrier to students dominates much of calculus reform literature. Steen (1988) describes the situation:

Nearly one million students study calculus each year in the United States, yet fewer than 25% of these students survive to enter the science and engineering pipeline. Calculus is the critical filter in this pipeline, blocking access to professional careers for the vast majority of those who enroll. (p. xi)

The students in this study would concur. The predominant reason given by them for taking Math 151 was that it was a prerequisite for their majors or other courses they required for their majors. Very few, if any, were able to identify the relevance of the material in this course to their majors. Those taking physics allowed that the calculus was used in physics, but that the required mathematics was taught within the physics classroom. Moreover, the techniques taught or assumed to be known in the physics classroom were generally taught considerably in advance of the techniques taught in Math 151.

The sections in the text dealing directly with applications of calculus to problems in client disciplines

were omitted from the course. The word problems in the assignments and on the tests were typical contrived "math problems" rather than real world applications. When the instructor did bring up in lecture an application of the derivative, such as its use in marginal quantities in economics, the students did not react with either enthusiasm or recognition.

One student spoke of "paying his dues" by taking this course. "Because I'm doing this engineering program, I'm locked into this math. It's a bit of a shame," he added. Another commented, "Why are people taking this course? There is not one soul who is taking this course just out of interest. We know that for sure."

Although a couple of students took this course because they wanted to, most felt they had no choice. Furthermore, there was little or no attempt to demonstrate the relevance of the subject matter or the skills learned in this course to their fields of interest.

There was little in this course that would, from the perspective of the calculus reform movement, prepare students adequately for further study in client disciplines (Ash, Ash & VanValkenberg, 1986; Stevenson, 1986).

The instructor described this course as being used by other disciplines as a filtering device. He said, "If a student can get through 151,152 at Simon Fraser University,

they are reasonably intelligent students. If the discipline, the other discipline, has any kind of mathematical thinking at all, then they want them to come through here because they filter out the people who can't think mathematically." The instructor's comments were reminiscent of the discussion by Steen (1986) on using the calculus course to develop mathematical maturity.

A typical student experience with Math 151 is that they are compelled to take the course; they have no sense of why this course is relevant to the work in the fields in which they are truly interested; and it seems as if they are being made to jump through a very high hoop in order to prove that they are worthy of studying subjects they wish.

Course Articulation with High School Mathematics

Success in this course often seemed predicated on a student's familiarity and competence with high school mathematics. The phrase "just algebra" was invoked on many occasions to indicate that the remainder of a particular solution was merely an exercise in applying secondary school mathematics techniques. For example, after differentiating a particular function, the instructor said, "Now I'll let you go through...I'll let you do the algebra there. When you clean this up, it does turn out to be fairly nice." For the second derivative of the same function, he added, "When you

clean this up, please confirm this, this is just algebra, this is what you are going to end up with."

For some students the difficulties they experienced were with trigonometry; for others it was with logarithms. There was uncertainty as to what constituted new material and old material that had to be remembered or learned anew. One student commented on his background in logarithms and exponential functions:

Well, I'm never really comfortable with anything new. But then again, logarithms are not really new, or not to me. But it was basically the fact that I had forgotten it all, so I had to relearn it...It didn't come back. I had to relearn it. But when you start saying things about when you multiply logs, you actually add, I remember that. Exponentials? I don't think I covered it much before. So that's basically all new.

One topic explicitly designated as review by the instructor was exponential functions. This was a topic that, when the instructor was creating a roster of ordered pairs for the graph of $y = 2^x$, one student offered " $2^{-3} = -8$ ". Exponents seemed to be problematical for many members of the class.

There were some pieces of mathematics that the instructor assumed would be in the students' backgrounds, but might not have been. Examples of this were techniques for simplifying radical expressions, the values of trigonometric functions for special angles such as $\frac{\pi}{6}$, solving biquadratic

equations through substitution, and evaluation of expressions with rational exponents.

Several students I interviewed after the course was over suggested that taking the pre-calculus Math 100 course would be advantageous. A couple suggested that it should be made a prerequisite to Math 151, especially if one had been away from school for a long time. It was not only a matter of having the information readily at hand. There was a need to practice its use. As one student noted, it was necessary to remove the "gobs of rust" from one's mind in order to succeed in Math 151. He commented:

A lot of the problems I found were the gobs of rust; not just in remembering the algebra, but in getting used to sitting down in a concentrated fashion and whistling off a bunch of calculus problems....Well, certainly the algebra presented some difficulties, because it had been such a long time since I'd done it. On the other hand, I think it's fair for a professor in a course to assume that a person who takes it has the necessary prerequisites. I don't think you can build into the course a large amount of review time.

The questions asked by the instructor and his comments during lecture indicate that he was clearly aware of how important it was for the students to be able to deal with the considerable quantity of "review material" that they were assumed to have mastered prior to this course. With respect to inverse functions, he said, "I'm not laying this out in detail: the whole story of inverse functions. I assume that

you've got it. This is essentially a reminder of what is going on with inverse functions." Throughout the remainder of his review he repeatedly commented, "I'm sure you will remember. I hope you will remember." He was also cognizant of the fact that many students had been away from the formal study of mathematics for quite a while. But there was only so much he could do in the context of the set curriculum and the time available.

The difficulties that the students and instructor faced with regards to prerequisite knowledge and skills are consistent with the concerns voiced in the literature. Renz (1986b) is very clear about this issue when he says "it seems almost certain that tomorrow's calculus students will be less facile with algebra and trigonometry than were the students for whom the traditional calculus course was designed" (p.107).

Personal Interaction

During the pilot study, I was concerned with how so much of my observations dealt with the behaviour of the lecturer, rather than the students. This preponderance of data about the instructor's activities continued throughout the full study. My conclusion is that this accurately reflects the imbalance in activity between the teacher and the learner in this situation.

During class time, the students were effectively passive and isolated. The instructor for this class did, on many occasions and particularly in the earlier part of the term, seek student-instructor interaction by asking questions throughout his lecture. This did not encourage student-student interaction.

Of the students interviewed, those who attended the later tutorial session which had only three or four students found it to be a much more positive experience than the earlier tutorial session, which was attended by ten to seventeen students. Those students who attended the Math Lab on the Burnaby Mountain campus were even more enthusiastic about the situation. A prominent factor in how the students rated these sessions was the amount of individual attention they received.

Although the students valued personal contact with the experts, whether it was the instructor, teaching assistant or the graduate students on duty at the Math Lab, there were barriers to gaining access to them. One barrier was the matter of transportation and time of availability. For many, it was a substantial commitment in time to go to the main campus during the day. Another barrier, especially during the lecture and tutorial sessions, was the number of other students seeking access to the experts' time.

This was a small class of approximately twenty students. During the breaks and immediately before and after the lectures a number of the students had shown themselves to be naturally gregarious. The course was sufficiently challenging that students would have benefitted from working with others in the class.

Some students attempted to establish connections with other students, but these connections failed for a variety of reasons. One student got in touch with another to discuss the assignments, but, as she said, "I found that she certainly knew what she was doing, but it was just too abstract. Sometimes I find it distressful when it's even more abstract than the way I'm thinking." Another student had obtained the telephone numbers of two of her classmates, but she failed to contact them because she felt she had fallen too far behind in the course and her concerns would not be their concerns.

Other students felt that this particular situation just did not lend itself to establishing work groups among students. One said:

Part of it is because it's like going into first year university. When you take classes part-time, it's like going into first year university into the first class, because you don't know anybody. You seldom see people that you have seen before. And because you are working, especially if you are working, you can only go for the class. So you tend not to establish relationships that you can sit down and work. Whereas, in university, people in the same class, first thing you know you're

sitting in the library together. You are doing this or that. You have some chance to build up relationships and work with people. More difficult, I think, if you are taking classes part-time.

Others ascribed the lack of working partnerships in this class to the competitive nature of the students. A very successful student felt some of her classmates resented her for her success and active participation in the class. This acted as a barrier to developing relationships with other students. Another student commented that, "It is in the nature of the first year student to eat their own young."

Pedagogical issues did not dominate the early part of calculus reform debate, but some of the more recent reform projects have centered on these issues (Tucker, 1990). In particular, cooperative interaction among students, instructors and teaching assistants have produced good initial results in student learning and attitudes (Schwingendorf & Dubinsky, 1990).

The Wide Variety of Mathematical Skills Required of the Student in the Course

On several occasions in my fieldnotes, I observed with some astonishment, the diversity of the material being presented to the students. Often, several widely divergent aspects of mathematics would be alluded to in a single two-hour lecture. For example, during the class on October 15,

the lecturer covered the concepts and definitions of local extrema and critical points; differentiation techniques were used to identify critical points; absolute extrema were defined and contrasted with relative extrema; the statements of Rolle's and the Mean Value Theorems were given, with the incidental introduction of existential quantifiers; some commentary on the nature of existence proofs was offered; and, when the Mean Value Theorem was applied to a trigonometric function, differentiation, the solution of a trigonometric equation, and the use of a scientific calculator to evaluate an expression involving trigonometric functions using radian measure all came into play.

Throughout the course "plug and chug" techniques of formal differentiation required a certain facility with algebraic manipulation. These techniques constituted a significant portion of the tasks on which students were evaluated.

Students were expected to understand the precise statements of definitions and theorems. Associated with these statements was the *ad hoc* introduction of mathematical notation. For example, existential and universal quantifiers were introduced in the ϵ - δ definition of limits. Notation was significant in the variety of ways of expressing derivatives, functions and intervals of reals. A prime example of a statement involving a profusion of notation was

$(\forall x \in (a,b)) (f'(x) < 0) \Rightarrow (f \text{ is decreasing on } [a,b])$. Open intervals, closed intervals, universal quantifiers, material implication and set membership all contained in one simple application of the derived function. Distinctions (sometimes subtle) were made between open intervals and closed intervals, one-sided limits and limits, differentiability and continuity, increasing/decreasing and strictly increasing/decreasing.

The students were introduced to a substantial amount of new vocabulary. For example, they were required to distinguish between polar and parametric representations of a curve; critical points and local extrema; local and absolute extrema; concavity and monotonicity; antiderivatives and integration.

The role of proofs in the course was never made fully clear. Proofs of moderate rigour were offered one time and "convincing arguments" supported by diagrams were offered at another time. One question on the final examination, an ϵ - δ verification of a limit, was the only time students were held responsible for creating proofs. The instructor spoke about aspects of proof theory: the structure of "if-then" statements, converses, and the nature of existence proofs while explaining the Mean Value Theorem and on tactics for creating proofs. The instructor's meta-comments such as, "Here's a little proof. How do you prove anything when you

have an equation?", offered an apprenticeship for the would be mathematics student. Such material was not officially part of the course content, however.

The student was expected to be able to move comfortably from pencil and paper computations to using scientific calculators for iterative procedures (as in Newton's Method). Students were expected to be able to evaluate trigonometric functions in radian mode and to evaluate expressions with rational exponents using handheld calculators. At the same time they might be required to simplify an algebraic expression using pencil and paper or compute a formal derivative.

Among the applications of calculus for which the students were responsible were related-rates word problems; exponential growth and decay problems; curve-sketching; physics problems (position-velocity-acceleration of a particle moving in one dimension); and some numerical analysis (linear approximations using differentials and Newton's Method for finding zeros). Notable for their omission from the course were applications from client disciplines such as economics, the social and the life sciences. Sections in the text specifically targetting these types of applications were omitted. Calculus was presented as a tool for solving mathematical problems.

In a single section of the text or, equivalently, in a single hour of lecture, the students could be exposed to a technique hitherto unknown to them which was then applied to a mathematical problem. This technique could be backed up with theory laden with notation, vocabulary and precise use of language. Such a topic was the use of differentials to create linear approximations of functions. Most students, in this age of readily accessible scientific calculators, would not immediately see the value of linear approximations. Neither would many of them understand the instructor's explanation that dy was a function of two independent variables, x and dy .

With so many themes being expounded in one course, it is understandable that the students would focus their energies on what they understood to be the examinable aspects of the material. Just what the students were to be accountable for in this course was a topic of discussion that took up considerable time and energy, both on the part of the instructor and the students.

The calculus reform literature has been dominated by curriculum issues (Ferrini-Mundy & Graham, 1991), so that it is not surprising that much of what I encountered in this study has been discussed in general terms in prior literature.

Epp (1986) speaks tellingly about the challenges faced by students as they encounter mathematical rigour for the first time. She poses as a question whether calculus be taught only at the level at which most students would succeed or should it be taught at a level where standards of mathematical rigour are maintained. Her answer is to adopt a middle ground and to develop the requisite mathematical situation in the students.

While Epp focuses on mathematical rigour, Lax (1986a) speaks of centering the calculus course on applications and numerical methods. Renz (1986b) would split the calculus into a careful and rigorous introduction to the mathematical foundations of calculus and a course which emphasizes techniques and concepts.

With the lack of consensus among the curriculum reformers as the proper focus of a calculus course, it is little wonder that the course tries to accomplish too much.

Quantity of Material Covered

Another image that pervades the calculus reform debate is that of the "lean and lively" calculus course (Douglas, 1986). Whatever the beliefs of the participants in this study about the liveliness of the course, no one would argue that it was lean.

There was consensus in the opinions of the students, instructor and researcher that too much material was to be covered in the time available. One student stated:

I am frustrated at the timeframe they allow. You never really get a chance to explore and inter-relate some of these concepts and see where you can go with them. The timeframe did not permit this. This was one of my disappointments with the course.

Another said about the pacing of the course, "It wasn't a uniform pace. It got very hairy towards the end, covering chapters in half a two-hour lecture." Yet another student commented, "It seems strange to me that the course is set up so that you either have to be a workaholic or a real whiz at math to survive it, because it goes so fast." The instructor concurred, "Another thing I feel about in that particular class, 151, is that it is so jammed with material. My feeling is that there should be more time for it."

The topics at the end of chapter 6 (hyperbolic functions and l'Hôpital's rule) and in chapter 9 (parametric and polar representation of curves) were dealt with hastily and superficially. These topics were covered after the second midterm and showed up on the final examination in three questions. Some students, when interviewed, could not even remember if some of the topics had been examined. Few, if any, students felt they had any significant grasp of the

concepts in the topics. The most successful student in the class commented on the hyperbolic trigonometric functions:

I have no idea what they look like. It was very much akin to what we did in high school. Learn how. Which is fine for me. I memorize quickly. I can learn the derivatives. But, from what I gather, that's not exactly the point. So why?

The second most successful student observed:

Because, covering three sections a lecture towards the end, it was just, 'Okay, I have to be able to do the simplest of questions. One of everything. Exploring it will have to come later.'

The pacing of the course was uneven. The instructor began slowly to bring students "on board" with respect to the assumed prerequisite background in mathematics and to facilitate the students' solid grasp on the basic concepts.

The presentation of the material began to seem more rushed "sometime around Newton's method" in the opinion of some students. There was universal agreement that the instructor sprinted through the material in the latter part of chapter 6 and in chapter 9. This rush at the end evoked a sense of panic in some students and a cynical "what is the minimum I need to know" attitude among others.

The amount of material in the course left no time for recovery if events caused a student (or the instructor) to fall behind. Illness, domestic unrest and job-related

absence and fatigue all had an impact on various members of the class. The pace of the course was such that those who fell behind felt they had to make extraordinary efforts to catch up or at least close the gap. Students finding themselves behind in this course faced double-jeopardy: catching-up and keeping-up. Concept assimilation time was at a minimum, as was exploratory investigation time.

In the opinion of at least one student, this relentless pace was characteristic of mathematics and science courses, but not university courses in general. She said:

I found that with humanity courses, with the English, philosophy or even the linguistics in the summer, with those types of courses you could get lost for a little while, or something would happen, and you could recover. But when it came to a science or math course, I felt that, unless you were with it all the time, every day or week, as soon as something happened and you lose grip or time, it was impossible to recover.

All of these factors added to the image of this course as something to be survived, rather than as a forum for learning about one of the most significant intellectual achievements of Western culture. Any improvements that might be made in the learning situation in this course would require time. This time can only be obtained through jettisoning some material from the syllabus.

Conclusions and Recommendations

Although the goals of this study were descriptive rather than prescriptive, my experiences have led me to a number of conclusions and recommendations that I believe would be of benefit to the calculus student.

The purpose of the course is not clearly articulated and communicated to the students. Currently, it is understood by most of the participants that the purpose of the university calculus course is to weed out the mathematically less able students from oversubscribed majors. The student perception of the intended role of this course is that calculus is meant to be a barrier they must pass. If there are other purposes to be served, then they are not clear to the participants in the course, especially the students. The first year of university calculus has an image problem with students.

Those responsible for the design of the calculus course should develop a clear succinct statement defining the goals of the course with respect to the students' educational development. Communicating this statement to the students in a manner which they can understand should be a priority task at the beginning of the course. The stated goals of the course must be consistent with the actual purposes of the course.

Not enough attention is paid to the demands this course makes on the mathematical maturity of the students. This

refers to more than just the background skills and knowledge of school mathematics which are explicitly stated prerequisites. Precise mathematical statements, use of mathematical symbols and vocabulary, elements of proof theory are introduced incidentally and frequently throughout the course, but are not part of a typical high school curriculum.

Where and when were the students supposed to have acquired a background in these aspects of mathematical endeavor? To what extent is it a goal of this course that the students should be taught this material? If this is one of the goals of this course, then to what extent in the evaluation scheme are the students held responsible for the material? Perhaps the formal aspects of mathematical language and reasoning should be de-emphasized in this course and made the central topics of another introductory mathematics course.

At whatever stage and level the student is exposed to mathematical rigour, it is not appropriate to assume that the students will pick it up as they go along. Some students who would otherwise do very well in such courses will be unsuccessful because they need time to construct the new knowledge. Other students will appear to be successful in assimilating these concepts, but will have established poorly constructed ideas which may cause them even more difficulty later on in their education.

The course as it is currently presented to the student has a strong "survival of the fittest" quality to it. The potential for student success in this course should be facilitated rather than having the challenging aspects exacerbated . The "sink or swim" syndrome should be reduced. I believe this can be accomplished through the active encouragement of peer interaction and cooperation; the establishment of greater opportunity for student-instructor interaction, with the roles taken by each being more equal; and by expanding the role of the math lab in student learning while decreasing the use of traditional tutorial sessions.

Following the opinions proffered in the calculus reform movement, I would argue that the first calculus course is extremely important in establishing attitudes of potential mathematicians and scientists. Because of its impact on students who will take upper-level math and science courses, this course should have proportionately greater resources, especially with respect to instructors, teaching assistants and instructional time, devoted to it.

There is too much material to be covered in this course if it is to be anything other than an academic filter. The students voiced their frustration with or their cynical acceptance of the fact that much of the material remained obscure to them, even after they had, by the evidence of their grades, successfully completed the course. There

should be sufficient time for the better students in this course to feel they have gained mastery of the central concepts. The survey of functions, such as the hyperbolic trigonometric functions, and various representation of curves, such as polar and parametric representations, should either be moved forward to the courses where they would be more fully developed or they should be fully integrated into this course. If they are to have an expanded role in this course, some other topics should be deleted, delayed or de-emphasized.

Several of the preceding recommendations could be successfully implemented through a judicious use of inexpensive, readily accessible technology. Innovative calculus curricula is being developed and taught in the context of extremely sophisticated technology (Tucker, 1990), but it is possible to make significant changes using hand-held calculators that are readily available to high school students. Calculators which are capable of graphing functions in both rectangular and polar forms, are programmable, and can calculate numerical derivatives allow the student to explore concepts numerically and graphically. The study of calculus could easily become one of informal concepts which become tools for problem solving. Formal symbol manipulation could be de-emphasized and precise

language and exact reasoning could be better learned in the context of an analysis course.

The capabilities of current calculators could transform the calculus classroom from a traditional, only moderately successful "transmission of knowledge" environment to one where the student explores and constructs his or her understanding of the central concepts, consequently gaining ownership of this knowledge. Much of what would be lost in switching to such technology would be a facility with pencil and paper techniques whose relevancy must be suspect in any case.

The Relevance of this Study to the Calculus Reform Effort

Throughout much of my fieldwork I emphasized the participant role at the expense of the observer role. I had, however, a perspective on the situation that none of the other participants had. I was particularly sensitive to those situations and conditions which might facilitate learning or militate against it. Because of the reading I had done, I also had a familiarity with a number of issues on calculus instruction and curriculum that were unlikely to occur spontaneously to my classmates.

I shared many experiences with my classmates and, as I confirmed in conversation with them, we often had similar reactions to situations. I had a different perspective than

they did, however, and a different reason for being there. Consequently, I arrived at a number of conclusions that would not occur to someone who was strictly a student in this course.

One of the strongest conclusions I came to hold was that the impetus for change in the calculus classroom would not come from the students. Although the students were forthcoming in their dissatisfaction with many aspects of their experiences throughout the course, none of them was committed to radical change. In several instances, the students accepted the blame for their lack of success in this course. They believed that they had not worked hard enough or with sufficient efficiency to succeed. None of them felt they had the expertise to prune the syllabus although most felt that there was too much material. Significantly, the most common suggestion was to increase the amount of instructional time rather than decrease the course content.

Students can inform us of those factors in a calculus course that might benefit from reform, but the actual initiatives will have to come from the educators.

This study brought out features of this course which do not receive much attention in the calculus reform literature, but have a significant impact on the student.

This course has extremely little recovery and assimilation time built into it. Illness, fatigue and

dislocations in a student's personal life can have a deleterious effect on his or her success which has no connection with the student's ability or work habits. Even without the "outside interferences" that life might be expected to bring, a student faces the daunting task of constructing a working knowledge of some very subtle concepts on very nearly the first exposure to the ideas. This course, as it is currently structured, is not consistent with the constructivist model of learning.

A third observation from the study which has no antecedents in literature I reviewed is the reluctance of the students to work together. The students had a stated preference for the Calculus Lab rather than the tutorial sessions and it was at the Lab that they had greater opportunity to work together. There was also some discussion of the factors which might impede the spontaneous formation of work groups. But for all this, I would have anticipated, considering the anxiety that many of the students felt throughout the latter stages of the course, that they would have been driven to seek help from each other. This is an issue that needs further study in the context of improving the learning situation in the calculus classroom.

This study has much to say to both the calculus curriculum designer and to the calculus classroom instructor.

The calculus curriculum designer can be convinced of the need for reform by reading the study. Those who are already involved in the reform effort can use the results of the study to refine and focus the changes they hope to effect. There are issues raised in the study which have not appeared prominently in the literature. Addressing these issues will also help in the design and implementation of beneficial changes.

The classroom instructor can use the results of this study to gain a better understanding of the learning situation in the classroom. Instructional practices can be modified to facilitate learning even within the traditional curriculum. Furthermore, the classroom instructor may find the results and methods of this study beneficial in establishing a more effective relationship with the students in his or her classes in a manner similar to that described in the following section.

Impact of the Study on the Researcher

The motivation for the study and the choice of research methods came from two years of study in which most of my ideas about learning and teaching and what constitutes valid educational research underwent significant changes.

The concept of each learner as an individual who brings to the classroom a complete life that extends well beyond the

current curriculum makes me approach my own classrooms with more diffidence and, I hope, greater sensitivity than before.

The idea of mathematics as a culture and the role of the teacher as one who enculturates students has been very useful in helping me recognize and understand some of the very peculiar things that occur in a classroom. Having listened as a student to the flow of technical jargon and the specialized use of ordinary language which occurs in a mathematics class, I am more conscious of my use of language in teaching and what different meanings I might be conveying if I am not careful.

Reading the calculus reform literature has crystallized and articulated many of the aspects of mathematics teaching that made me uncomfortable, although I was not aware of why they did so. The accounts of innovative teaching methods and curricula have led to dramatic changes in my teaching style and materials. The excitement and enthusiasm apparent in this literature have given me the energy to explore and experiment in my own workplace.

The methods of ethnography, with their emphasis on observation of interacting individuals in a cultural context, has given me a new way of looking and trying to understand my students and co-workers. The concepts of triangulation and naturalistic description have expanded my ideas of what can be done with assessment and evaluation.

Above all, my time "in the trenches" with my fellow Math 151 students has given me much greater empathy for students everywhere and the challenges they face.

APPENDIX A

CHRONOLOGY OF FIELDWORK

Pilot Project		
WHEN	WHERE	ACTIVITY
July 19, 1991; 8:30 a.m.	AQ3150 (SFU)	lecture observations
July 19, 1991; 9:30 a.m.	K9505 (SFU) (Calculus Lab.)	discussion with instructor
July 24, 1991; 8:30 a.m.	AQ3150 (SFU)	lecture observations
July 24, 1991; 9:30 a.m.	AQ corridor	discussion with instructor (30 minutes)
July 24, 1991; 10:00 a.m.	K9505 (SFU) (Calculus Lab.)	observation
July 31, 1991; 8:30 a.m.	AQ3150 (SFU)	lecture observations; audio taping
July 31, 1991; 9:30 a.m.	K9505 (SFU) (Calculus Lab.)	observation and discussion with students (2 hours)
Aug. 2, 1991; 8:30 a.m.	AQ3150 (SFU)	lecture observations; audio taping
Aug. 2, 1991; 9:40 a.m.	K9505 (SFU) (Calculus Lab.)	observation (45 minutes)

Project

WHEN	WHERE	ACTIVITY
August	SFU Bookstore	purchase text
Sept. 3, 1991; 7:00 p.m.	1415 HC	preliminary observations; observe lecture (2 hours)
Sept. 5, 1991; 7:30 p.m.	1700 HC	observe lecture (1 hour)
Sept. 10, 1991; 7:20 p.m.	1415 HC	hand out informed consent forms; observe lecture (2 hours)
Sept. 12, 1991; 6:30 p.m.	1510 HC	observe/participate in tutorial (1 hour)
Sept. 12, 1991; 7:30 p.m.	1700 HC	observe lecture (1 hour)
Sept. 12, 1991; 8:30 p.m.	1700 HC	tutorial session with 2 students, but no teaching assistant
Sept. 17, 1991; 7:30 p.m.	1415 HC	observe lecture (2 hours)
Sept. 19, 1991; 6:30 p.m.	1510 HC	observe/participate in tutorial (1 hour)
Sept. 19, 1991; 7:30 p.m.	1900 HC	observe lecture (1 hour)
Sept. 19, 1991; 8:30 p.m.	1515 HC	observe/participate in tutorial (1 hour)
Sept. 24, 1991; 7:30 p.m.	1415 HC	observe lecture (2 hours)
Sept. 26, 1991; 6:30 p.m.	1510 HC	observe/participate in tutorial (1 hour)
Sept. 26, 1991; 7:30 p.m.	1700 HC	observe lecture (1 hour)
Sept. 26, 1991; 8:30 p.m.	1515 HC	observe/participate in tutorial (1 hour)

Oct. 1, 1991; 7:30 p.m.	1415 HC	observe lecture (2 hours)
Oct. 3, 1991; 6:30 p.m.	1510 HC	observe/participate in tutorial (1 hour)
Oct. 3, 1991; 7:30 p.m.	1700 HC	observe lecture (1 hour)
Oct. 3, 1991; 8:30 p.m.	1515 HC	observe/participate in tutorial (1 hour)
Oct. 8, 1991; 7:00 p.m.	HC Library	locate lecture tapes and assignment solution keys
Oct. 8, 1991; 7:30 p.m.	1415 HC	observe lecture (2 hours)
Oct. 10, 1991; 7:00 p.m.	1510 HC	observe/participate in tutorial (0.5 hour; arrived late)
Oct. 10, 1991; 7:30 p.m.	1700 HC	wrote midterm I
Oct. 15, 1991; 6:40 p.m.	corridors HC	talk with student (40 minutes)
Oct. 15, 1991; 7:30 p.m.	1415 HC	observe lecture (2 hours)
Oct. 17, 1991; 7:30 p.m.	1700 HC	observe lecture (1 hour)
Oct. 17, 1991; 8:30 p.m.	1700 HC & ?	observe/participate in tutorial (1 hour)
Oct. 22, 1991; 7:30 p.m.	1415 HC	observe lecture (2 hours)
Oct. 24, 1991; 6:30 p.m.	1510 HC	observe/participate in tutorial (1 hour)
Oct. 24, 1991; 7:30 p.m.	1700 HC	observe lecture (1 hour)
Oct. 24, 1991; 8:30 p.m.	1700 HC	observe/participate in tutorial (1 hour)

Oct. 29, 1991; 7:00 p.m.	HC	check exam schedule; look at examination room
Oct. 29, 1991; 7:30 p.m.	1415 HC	observe lecture (2 hours)
Oct. 31, 1991; 6:30 p.m.	1510 HC	observe/participate in tutorial (1 hour)
Oct. 31, 1991; 7:30 p.m.	1700 HC	observe lecture (1 hour)
Oct. 31, 1991; 8:30 p.m.	1515 HC	observe/participate in tutorial (1 hour)
Nov. 5, 1991; 7:30 p.m.	1415 HC	observe lecture (2 hours)
Nov. 7, 1991; 6:30 p.m.	1510 HC	observe/participate in tutorial (1 hour)
Nov. 7, 1991; 7:30 p.m.	1700 HC	observe lecture (1 hour)
Nov. 12, 1991; 7:30 p.m.	1415 HC	observe lecture (2 hours)
Nov. 14, 1991; 6:30 p.m.	1510 HC	observe/participate in tutorial (1 hour)
Nov. 14, 1991; 7:30 p.m.	1700 HC	wrote midterm II
Nov. 14, 1991; 8:30 p.m.	1700 HC	discussssion with instructor
Nov. 19, 1991; 7:30 p.m.	1415 HC	observe lecture (2 hours)
Nov. 21, 1991; 6:40 p.m.	1510 HC	observe/participate in tutorial (50 minutes; arrive late)
Nov. 21, 1991; 7:30 p.m.	1700 HC	observe lecture (1 hour)
Nov. 21, 1991; 8:30 p.m.	1515 HC	observe/participate in tutorial (1 hour)

Nov. 26, 1991; 7:30 p.m.	1415 HC	observe lecture (2 hours)
Nov. 28, 1991; 6:50 p.m.	1510 HC	observe/participate in tutorial (40 minutes; arrive late)
Nov. 28, 1991; 7:30 p.m.	1700 HC	observe lecture (1 hour)
Nov. 28, 1991; 9:00 p.m.	Gastown	dinner with instructor, teaching assistant and two students
Dec. 3, 1991; 7:00 p.m.	1530 HC	wrote final examination

APPENDIX B
COURSE OUTLINE

MATHEMATICS 151-3 CALCULUS I

Fall 1991

Prerequisite: B.C. High School Mathematics 12 (or equivalent) with a grade of at least B or MATH 100 (not MATH 110). Students with credit for either MATH 154 or 157 may not take MATH 151 for further credit.

Textbook: *Calculus*, by James Stewart. Brooks/Cole Publishers, 1991 (second edition).

Review and Preview

- | | | | |
|---|----------------------------|---|--|
| 4 | Functions and Their Graphs | 6 | Types of Functions: Shifting and Scaling |
| 5 | Combinations of Functions | 7 | A Preview of Calculus |

Chapter 1 - Limits and Rates of Change

- | | | | |
|-----|---|-----|---|
| 1.1 | The Tangent and Velocity Problems | 1.4 | The Precise Definition of a Limit |
| 1.2 | The Limit of a Function | 1.5 | Continuity |
| 1.3 | Calculating Limits using the Limit Laws | 1.6 | Tangents, Velocities, and Other Rates of Change |

Chapter 2 - Derivatives

- | | | | |
|-----|--|------|---|
| 2.1 | Derivatives | 2.6 | Implicit Differentiation |
| 2.2 | Differentiation Formulas | 2.7 | Higher Derivatives |
| 2.3 | Rates of Change in the Natural and Social Sciences | 2.8 | Related Rates |
| 2.4 | Derivatives of Trigonometric Functions | 2.9 | Differentials and Linear Approximations |
| 2.5 | The Chain Rule | 2.10 | Newton's Method |

Chapter 3 - The Mean Value Theorem and Curve Sketching

- | | | | |
|-----|---|------|--------------------------------------|
| 3.1 | Maximum and Minimum Values | 3.6 | Infinite Limits; Vertical Asymptotes |
| 3.2 | The Mean Value Theorem | 3.7 | Curve Sketching |
| 3.3 | Monotonic Functions and the First Derivative Test | 3.8 | Applied Maximum and Minimum Problems |
| 3.4 | Concavity and Points of Inflection | 3.10 | Antiderivatives |
| 3.5 | Limits at Infinity; Horizontal Asymptotes | | |

Chapter 6 - Inverse Functions; Exponential, Logarithmic, and Inverse Trigonometric Functions

- | | | | |
|-----|---------------------------------------|------|---|
| 6.1 | Exponential Functions | 6.7 | Exponential Growth and Decay |
| 6.2 | Derivatives of Exponential Functions* | 6.8 | Inverse Trigonometric Functions* |
| 6.3 | Inverse Functions | 6.9 | Hyperbolic Functions* |
| 6.4 | Logarithmic Functions | 6.10 | Indeterminate Forms and L'Hospital's Rule |
| 6.5 | Derivatives of Logarithmic Functions* | | |

Chapter 9 - Parametric Equations and Polar Coordinates

- | | | | |
|-----|--|-----|-------------------|
| 9.1 | Curves Defined by Parametric Equations | 9.4 | Polar Coordinates |
| 9.2 | Tangents and Areas* | 9.6 | Conic Sections |

* The material on integration in Sections 6.2, 6.5, 6.8, and 6.9 and the material on areas in Section 9.2 is omitted in MATH 151 and will be covered in MATH 152.

APPENDIX C

Informed Consent by Subject to Participate in a Research
Project Form

and

Excerpts from request for approval of research design from
University Research Ethics Review Committee

Simon Fraser University
INFORMED CONSENT BY SUBJECTS TO PARTICIPATE IN A RESEARCH PROJECT

Note: The University and those conducting this project subscribe to the ethical conduct of research and to the protection at all times of the interests, comfort, and safety of subjects. This form and the information it contains are given to you for your own protection and full understanding of the procedures, risks and benefits involved. Your signature on this form will signify that you have had an adequate opportunity to consider the information in this document, and that you voluntarily agree to participate in the project.

Research Procedures Involving Subjects :

The researcher (Craig Newell, M.Sc.(Educ.) candidate, Faculty of Education, Simon Fraser University) proposes to do an ethnographic study of the Calculus 151 class in which you are currently enrolled/instructing.

The researcher will act as a participant observer in the role of a student throughout the course and will solicit information from participants in the course in some or all of the following ways:

- 1) Observation of subject behaviour and discourse in contexts associated with the course (e.g.; in class, in tutorial sessions, immediately before and after class).
- 2) Direct discussions between the subject and researcher.
- 3) Formal interviews by the researcher of the subject. These interviews will be recorded on audio tape.
- 4) Inspection of subject artifacts (e.g.; notebooks, homework assignments, tests, and examinations).

The researcher, for his part, will guarantee the confidentiality of information obtained and the right of any subject to withdraw participation, in part or in full, at any time.

Having been asked by Craig Newell of the Education Faculty of Simon Fraser University to participate in a research project, I have read the above paragraphs. I understand the procedures to be used in this project and the personal risks to me in taking part.

I understand that I may withdraw my participation, in part or in full, at any time.

I understand that I may register any complaint I might have about the conduct of the research with the researcher or with the Director of Graduate Programs, Faculty of Education, Simon Fraser University.

I agree to participate by allowing the researcher to obtain information from me in the ways described above during the period from September 3, 1991 through December 31, 1991.

NAME (Please print): _____

ADDRESS: _____

SIGNATURE: _____ WITNESS: _____ DATE: _____

Once signed, a copy of this consent form will be returned to you.

Request for Ethical Approval of Research.
Page 2, Item 5

5. (b) An INFORMED CONSENT BY SUBJECTS TO PARTICIPATE IN A RESEARCH PROJECT form has been devised. A copy of this form will be given to each potential subject in this project (instructor, students, teaching assistant(s)). Signed forms will be kept by the researcher, copies of the signed forms will be returned to the subjects.

A copy of this form is appended to this request.

The reporting of the study results will not use the actual names of the subjects, but will use code aliases. The audio tapes of interviews will not be released to third parties without the written consent of the interviewed subjects.

Materials accumulated in the study will remain in the secure possession of the researcher. Materials will not be released to third parties without the written consent of the subject(s) involved.

5. (a) Formal interviews will be held with subjects on up to twelve occasions. The purpose of the interviews is to gather data which will "reveal how participants conceive of their worlds and how they explain these conceptions". [Goetz, 1984 #21], p. 126 The interviews will be nonstandardized in that a different sequence and selection of questions will be employed in each instance depending on the responses of the interviewed participant.

The initial questions arise from issues in the current calculus reform debate and from learning theories that have been the focus of discussion recently in mathematics education. Subsequent questions develop from participant responses.

Examples of proposed initial questions follow:

Demographic questions:

Age, level of schooling, level of formal mathematics education and success. Academic and career ambition. Why are you enrolled in this particular course?

Lectures, assignments and evaluation methods:

What aspects of this class help you to learn the material? What aspects do not help, or actually hinder your learning?

About the assignments: are they the right length? are the right type of problems assigned?

How do you go about doing the assignments? Do you ever work with other students? Do you seek help from the instructor, teaching assistant or someone else? What do you do if you don't understand how to do a problem?

Do you use calculators or computers when you do your assignments? Why or why not? How do you use them?

How do you use the textbook in this course? What do think of the textbook; does working with it help you learn?

What do you do during lectures that help you learn?

What type of notetaking do you do? Do you want to ask questions during the lecture? Do you ask questions?

How important a factor do find the evaluation procedures in this course to be? Do the tests, examinations, and graded homeworks help you learn? How apprehensive do the procedures make you feel? Why?

Understanding and attitudes:

Why are you taking a calculus course? Is this course meeting your purposes? What more would you like to get from taking this course? What unexpected things are happening to you as you take this course?

How do you feel you are doing in this course? Why do you feel that way?

What do you think the calculus is all about? Is it important to understand the subject? Why or why not? How do you think you will use what you have learned in this course after it is completed?

Do you understand the language used in the textbook and by the instructor? What do you find about the use of language which makes it difficult? What do you think is meant by [recently covered theorem or definition]? What sorts of things do you do to help you understand the material in this course?

What do you think a mathematician does when he or she is doing mathematics? Do you think we are doing mathematics in this course? Why, or why not? What sort of mathematics will you do when you finish this course?

APPENDIX D

EVALUATION SCHEME AND INSTRUMENTS

Evaluation of student performance in this course was based on seven assigned problem sets, two one hour midterm tests, and a three hour final examination.

The problem sets were equally weighted. Performance on these sets constituted 20% of the student's overall grade.

Midterm tests were equally weighted. Performance on these tests constituted 40% of the student's overall grade.

The final examination was cumulative over the course and constituted 40% of the student's overall grade.

No scaling was applied to the percent grade earned by the student. Percent grades were translated to letter grades using a scale given to the students in the first lecture.

Problem Sets

All of the assigned problems were taken from the textbook for the course:

Calculus , by James Stewart, Brooks/Cole publishers, 1991
(second edition)

Assignment # and due date	page	problems	topic
#1; Sept. 12, 1991	60	2, 4, 8	
	68	2, 6, 12	
	102	4, 10, 6	
#2; Sept. 1, 1991	77	10, 32, 34, 48	
	85	4, 14	
	94	6, 16, 38	
	115	4, 6, 12, 16, 26, 42, 44	
#3; Sept. 26, 1991	124	24, 48, 52, 58, 66	
	134	4, 10, 18, 20, 22	
	141	4, 18, 32, 40, 46	
#4; Oct. 8, 1991	148	42, 68, 74	
	154	16, 30, 36	
	159	24, 42, 51	
	163	18, 26, 30	
	169	18, 30, 42	
#5; Oct. 29, 1991	172	8, 18, 20	
	185	12, 40, 58	
	190	4, 14, 28	
	195	4, 18, 40	
	201	10, 26, 33	
#6; Nov. 5, 1991	211	6, 8, 20, 34, 36, 38, 40, 42	
	222	12, 20, 32, 42, 46, 50, 52	
#7; Nov. 19, 1991	229	14, 40	
	235	30, 46	
	246	38, 48	
	343	34	
	348	8, 30	
	356	24, 32	
	361	54, 78	
	371	34, 62	

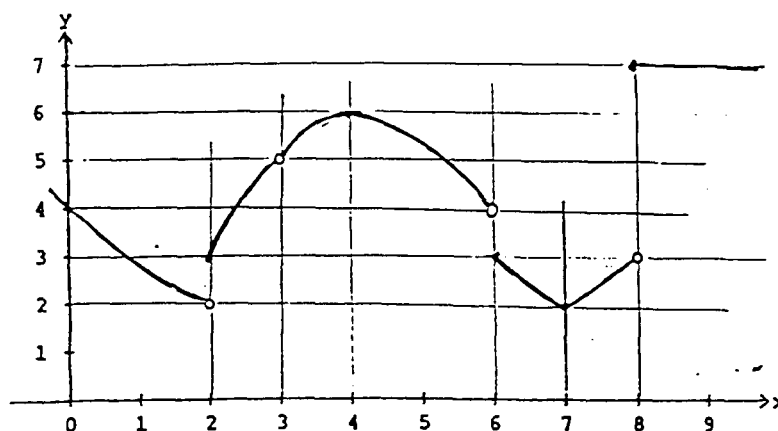
1. Define each of the terms below:

[2] a) L is the limit of the function f as x approaches a .

[2] b) $f'(x)$

[2] c) f is continuous at a .

2. Use the graph of f below to answer each question.



[1,1] a) $\lim_{x \rightarrow 2^+} f(x) =$

b) $\lim_{x \rightarrow 2} f(x) =$

[1,1] c) $\lim_{x \rightarrow 6^-} f(x) =$

d) $\lim_{h \rightarrow 0} \left\{ \frac{f(4+h) - f(4)}{h} \right\} =$

[1] e) For those values of x in the interval $[1,9]$, where is f discontinuous?

[1] f) For those values of x in the interval $[1,9]$, where is f not differentiable?

OVER==>

3. Find the value of each limit below:

[2,2] a) $\lim_{x \rightarrow -3} \left\{ \frac{x^2 + x - 6}{x^2 + 5x + 6} \right\}$ b) $\lim_{x \rightarrow 1} \left\{ \frac{\sin(x - 1)}{x^2 + x - 2} \right\}$

4. Find the derivative, dy/dx , of each function given below.
(DO NOT SIMPLIFY algebraically.)

[2] a) $y = (8x - 2)^{3/2}(\tan(x))$

[2] b) $y = x^2/(x^3 - 1)$

[2] c) $y = \sqrt[3]{\sec(2 - 3x^2)}$

[3] d) $x + y = \cos(x - y)$

[4] 5. a) Use the definition of the derivative of f given in question 1(b) to compute the derivative, $f'(x)$, for

$$f(x) = 1/\sqrt{x + 1}$$

[2] b) Find the equation of the line tangent to the graph of f at the point $(3, 1/2)$.

[4] 6. If each edge of a cube is increasing at the constant rate of 3 centimeters per second, how fast is the volume increasing when x , the length of an edge, is 10 centimeters long?

[5] 7. The radius of a spherical ball is computed by measuring the volume of the sphere (by finding how much water it displaces). The volume is found to be 40 cubic centimeters, with a percentage error of 1%. Compute the corresponding percentage error in the radius (due to the error in measuring the volume).

$$(V = (4/3)\pi r^3)$$

- [2] 1. a) State the Mean Value Theorem.
 [2] b) State the Extreme Value Theorem.
 [2] c) Define a point of inflection.
 [2] d) Define a critical number.

2. Find

- [2] a) $f'(x)$ for $f(x) = 2e^{-x}\sin(3x)$.
 [2] b) an antiderivative of $g(x) = (1/x\sqrt{x}) + e^{-x}$.
 [3] c) $\lim_{x \rightarrow 0} \left\{ \frac{e^{2x} - 1}{x} \right\}$

3. For the polynomial $f: f(t) = t^3 + t^2 - 4t + 7$ find

- [3] a) the interval(s) where f is decreasing.
 [3] b) the interval(s) where f is concave up.
 [3] c) the number of real roots of the equation $f(t) = 0$.

4. Consider the equation $e^x = \sin(x)$.

- [3] a) How many positive roots does this equation have? negative roots?
 [6] b) Use Newton's Method to approximate the value of the largest negative root of this equation by letting x_0 be the integer which is closest to the actual root. (Find x_2 .)

- [7] 5. An open box with a square base is to have a volume of 2000 cubic centimeters. What should the dimensions of the box be if the amount of material used is to be a minimum?

NOTE: No graphing functions on any calculator may be used.

- [2] 1. a) Give the delta-epsilon definition of $\lim_{x \rightarrow a} \{f(x)\}$.
- [2] b) Define what is meant by F is an antiderivative of f on the interval I .
- [2] c) Define what it means for a function to be monotonic on the interval I .
- [2] d) State Rolle's Theorem.

2. Find

[2] a) $\lim_{x \rightarrow -\infty} \left\{ \frac{2x^2 + 5x^2 - 7}{3x^2 - 8x + 12} \right\}$

[2] b) $\lim_{x \rightarrow -1^-} \left\{ \frac{x^2 - x + 1}{x^2 + 1} \right\}$

[2] c) $\lim_{x \rightarrow 0} \left\{ \frac{e^x - x - 1}{x^2} \right\}$

[2] d) $\lim_{x \rightarrow 0} \left\{ \frac{x^2 - \sin^2(x)}{\sin^2(x)} \right\}$

3. Find dy/dx for each function below. (DO NOT SIMPLIFY.)

[2] a) $y = \ln(|-4|)$

[2] b) $y = [\sin(e^{-3})]^2$

[2] c) $y = \cosh(x)/[\sec(x) - 3]$

[2] d) $y = 3^x \tan^{-1}(x/2)$

[2] e) $y = e^{\cos(x)}$

4. Compute the derivative of $f: f(x) = x - 1/x$ by using the limit definition of $f'(x)$.
[6]
5. For $x \tan(y) = y$ find
[2] a) dy/dx
[2] b) d^2y/dx^2 (Do not simplify.)
6. Suppose that the velocity at time t of a particle in linear motion is given by the formula $v(t) = t^2 + \sin(t)$.
[2] a) Find the acceleration of the particle at $t = 2$.
[3] b) If the position of the particle at $t = 2$ is 3, $S(2) = 3$, determine the position of the particle at time t .
7. a) Find a formula for the inverse of the function
[3] $f: f(x) = \sqrt{1+x}$.
[2] b) State the domain and range of f^{-1} .
[2] c) Sketch a rough graph of f^{-1} .
8. Suppose a small quantity of radon gas, which has a half-life of 3.8 days, is accidentally released into the air in a laboratory. If the resulting radiation level is 50% above the "safe" level, how long should the laboratory remain vacated?
[6]
9. Let $f(x) = Ax^2 + Bx + C$, where A , B , and C are constants with $A \neq 0$. Show that for any interval $[a,b]$, the number c guaranteed by the Mean Value Theorem is the midpoint of $[a,b]$.
[5]
10. A baseball diamond is a square with sides 90 feet long. Suppose a baseball player is advancing from second to third base at the rate of 24 feet per second, and an umpire is standing on home plate. Let θ be the angle between the third baseline and the line of sight from the umpire to the runner. How fast is θ changing when the runner is 30 feet from third base?
[7]

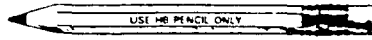
11. Sketch the graph of the function $f(x) = \frac{x + 1}{x^2 - 4}$
- [8] Label all significant aspects of the graph except points of inflection and SHOW ALL WORK.
12. Find the x-coordinates of the points on the parabola $y = x^2 + 2x$ that are closest to the point $(-1, 0)$.
- [7]
13. The curve described by the parametric curves below is called a cycloid and is a periodic function.
- $$x = 2[t - \sin(t)]$$
- $$y = 2[1 - \cos(t)]$$
- [3] a) Sketch that portion of the cycloid for $0 \leq t \leq 2\pi$.
- [3] b) Find the equation of the line tangent to the graph of the cycloid at the point where $t = \pi/2$.
14. a) How many positive roots does the equation $(x - 2)^2 = \ln(x)$ have?
- [2]
- b) Use Newton's method to approximate the largest positive root of the equation in part (a). Use the integer nearest to this root for x_0 . Show all calculations required to find x_2 .
- [6]
15. Prove that $\lim_{x \rightarrow -5} (4 - 3x/5) = 7$ using the delta-epsilon definition of limit.
- [5]

APPENDIX E
COURSE AND INSTRUCTOR EVALUATION FORM

COURSE AND INSTRUCTOR EVALUATION

FILL IN THE CIRCLES

ERASE CHANGES COMPLETELY



DO NOT USE INK
OR FELT PENS

BACKGROUND

Please answer the following questions to the best of your ability.

The results are carefully considered in decisions regarding course revisions, and promotion and tenure of faculty members.

① ② ③ ④ ⑤ Wrong
 ① ② ③ ④ ⑤ Wrong
 ① ② ③ ④ ⑤ Wrong
SAMPLE MARK
 ① ② ③ ● ⑤ Right

1. What is your cumulative grade point average?

- 3.5 or over ●
 3.0 to 3.49 ②
 2.5 to 2.99 ③
 2.0 to 2.49 ④
 below 2.0 ⑤

2. Why did you take this course?

Choose the single most important reason.

- ① It was compulsory
 ② I am interested in the subject
 ③ No alternative course available
 ④ It looked like an easy credit
 ⑤ Other reasons

GENERAL

3. How often did you attend the lectures/seminars?
 4. The course prerequisites were
 5. The overall level of difficulty for the course was
 6. The amount of work required for the course was
 7. How valuable was the course content?
 8. The course text or supplementary material was
 9. I would rate this course as
- always ① ② ③ ④ ⑤ hardly ever
 essential ① ② ③ ④ ⑤ not essential
 too easy ① ② ③ ④ ⑤ too difficult
 too little ① ② ③ ④ ⑤ too much
 very ① ② ③ ④ ⑤ not very
 relevant ① ② ③ ④ ⑤ irrelevant
 A B C D F

COURSE GRADING

10. The assignments and lecture/seminar material were
 11. The exams and assignments were on the whole
 12. The marking scheme was on the whole
- well related ① ② ③ ④ ⑤ unrelated
 fair ① ② ③ ④ ⑤ unfair
 fair ① ② ③ ④ ⑤ unfair

INSTRUCTOR AND LECTURES/SEMINARS

13. How informative were the lectures/seminars?
 14. The instructor's organization and preparation were
 15. The instructor's ability to communicate material was
 16. The instructor's interest in the course content appeared to be
 17. The instructor's feedback on my work was
 18. Questions during class were
 19. Was the instructor reasonably accessible for extra help?
 20. Was the instructor responsive to suggestions or complaints?
 21. Overall, the instructor's attitude towards students was
 22. I would rate the instructor's teaching ability as
- informative ① ② ③ ④ ⑤ uninformative
 excellent ① ② ③ ④ ⑤ poor
 excellent ① ② ③ ④ ⑤ poor
 high ① ② ③ ④ ⑤ low
 adequate ① ② ③ ④ ⑤ inadequate
 encouraged ① ② ③ ④ ⑤ discouraged
 available ① ② ③ ④ ⑤ never available
 very ① ② ③ ④ ⑤ not at all
 excellent ① ② ③ ④ ⑤ poor
 A B C D F

Course:	Semester:	Instructor's Name:
GENERAL COMMENTS		
1. What do you consider to be the strongest and weakest features of the instructor, as a teacher?		
2. What do you consider to be the strongest and weakest features of the course?		
3. Any other comments or suggestions?		
Please do not write outside the enclosed area. Use a regular sheet of paper for additional comments.		

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