TEACHING AND ASSESSING MATHEMATICS 12:
A DEPARTURE FROM TRADITION

by

Leo P. Boissy

B.Sc., University of British Columbia, 1976

THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

in the Faculty

of

Education

© Leo P. Boissy 1994

SIMON FRASER UNIVERSITY

March 1994

APPROVAL

Name: Leo Paul Boissy

Degree: Master of Science

Title of Thesis: Teaching and Assessing Mathematics 12:
                                   A Departure from Tradition

Examining Committee:

Chair: Geoffrey Madoc-Jones

_____

                                   Thomas J. O'Shea
                                   Senior Supervisor

_____

                                   Harvey Gerber
                                   Associate Professor
                                   Mathematics

_____

                                   James Sherrill
                                   Professor
                                   Mathematics Education
                                   Faculty of Education
                                   University British Columbia
                                   External Examiner

                                   Date Approved _MAR - 28, 1994_

ii

PARTIAL COPYRIGHT LICENSE

I hereby grant to Simon Fraser University the right to lend

my thesis, project or extended essay (the title of which is shown below)

to users of the Simon Fraser University Library, and to make partial or

single copies only for such users or in response to a request from the

library of any other university, or other educational institution, on

its own behalf or for one of its users. I further agree that permission

for multiple copying of this work for scholarly purposes may be granted

by me or the Dean of Graduate Studies. It is understood that copying

or publication of this work for financial gain shall not be allowed

without my written permission.

Title of Thesis/Project/Extended Essay

Teaching and Assessing Mathematics 12: A Departure from Tradition

Author:

(Signature)

Leo Paul   Boissy

(Name)

March 30, 1994

(Date)

Abstract

This study investigated the impact of using alternative teaching and assessment strategies in a Mathematics 12 class. The intent was to explore variations on traditional methods of presenting this course. The purpose was to create a more meaningful and constructive learning experience through innovative strategies that promote greater student participation and cooperation. At the same time concerns about final examination performance were to be addressed by way of focussed preparation sessions.

The method of inquiry reflected the basic principles of research in action although the research was not collaborative in nature. Data were collected through observations recorded in a teacher journal and by way of feedback obtained from students throughout the year and in year-end interviews. Student achievement results on classroom assessment instruments and the provincial final examination were collected and analysed. The types of outcomes were triangulated with the sources of information in order to cross-validate the data.

The results of the study are presented in terms of the activities undertaken during the year. These activities include classroom explorations and group work,

projects, cooperative tests, and examination preparation
sessions.

The conclusions of the study are related to each
type of activity or strategy. The classroom atmosphere
was positively enhanced by group seating arrangements and
by explorations that involved students sharing their
understandings. The projects assigned were a valuable
experience primarily because they allowed students to
take ownership of their learning, in one case allowing
them to pursue topics outside of the curriculum.
Students did not develop the mutual interdependence that
was hoped for through a cooperative testing strategy.
Rather this strategy contributed to an inflation of
grades that resulted in a significant difference between
classroom achievement and the score on the final
provincial examination. Finally, the examination
sessions had no discernible positive effect on final
examination results.

In summary it appears that the eclectic nature of
the study hindered determining clear connections between
an individual activity or strategy and any particular
result. Future research should focus on some aspects of
this study so that stronger conclusions might be drawn as
to the efficacy of the innovations.

Dedication

This study is dedicated to all teachers who take the risk
of effecting change in their classrooms.

Acknowledgements

I would like to thank Dr. Tom O'Shea and Dr. Harvey
Gerber who helped me with their advice and encouragement.
I also wish to express my appreciation to the students in
my class who showed great respect for my ability to
present an alternative experience and who encouraged me
along the way. Finally I especially thank my wife,
Vicki, and my son, Jordan, who allowed and encouraged me
to take time from our family life to complete this study.

Table of Contents

List of Tables

x

List of Figures

CHAPTER 1

INTRODUCTION

Context of the Problem

The stultifying effects of external constraints,
particularly large-scale, standardized testing, on the
teaching and learning environment of classrooms has been
well recognized. As Berlak (1985) stated,

> Teachers are spending more time teaching for tests.
> Where standardized tests dominate the curriculum,
> they dictate the content and reduce the method of
> instruction to a set of routines, and teachers
> become mere functionaries in a bureaucratic system.
> (p. 17)

If teachers of mathematics are going to succeed in
establishing a new kind of environment, one more in tune
with that envisaged by the *Professional Standards for
Teaching Mathematics* [the *Standards*] (NCTM, 1991), where
teachers extend their role beyond that of "dispenser of
knowledge and confirmer of right answers" (p. 4), then
alternative teaching and assessment strategies must be
investigated.

As many senior secondary school teachers in British
Columbia will attest, the pressure to prepare Grade 12
students for year-end provincial examinations is
considerable. Currently in British Columbia these final
examinations constitute forty percent of a student's

final mark in any course that includes such an examination as part of its overall assessment scheme. Students, and some teachers, see a successful examination as the crowning achievement of twelve years of mathematical study. Students also believe that this examination will dramatically influence their potential acceptance into post-secondary institutions.

The overall impact of these concerns, real or perceived, has been to influence the teaching and assessment strategies of teachers in such a way that students were well prepared to succeed on one particular assessment instrument. Technique has been emphasized as it is the primary means of succeeding on many questions on the examination. Extra time that might be used to delve deeper into a given topic or to explore and develop basic relationships or significant connections was often deemed a waste by students or an unrealistic extravagance by teachers. As a result, much of Mathematics 12 is seen as mere exercises in preparation for the examination, with a concommitant loss in the quality of teaching and learning of mathematics.

Many mathematics teachers I have encountered agree that a strategy that provided for exploration, projects, reports, presentations, displays, and so on would be a wonderful and useful thing. Yet these same teachers have

been quick to lament that they did not have the time to be so adventurous and, after all, they had to get their students ready for the provincial examination.

The practice of cooperative learning or group work has been similarly received. The competitive nature of most classroom assessment techniques and the final examination has not lent itself to this kind of interdependent approach. Teachers may also have feared that experimentation with alternative strategies risked leaving them open to criticism should their students perform poorly on the final examination, regardless of the benefits of such learning.

It would appear then that a dichotomy exists. On the one hand teachers recognize the value of explorative strategies and a cooperative learning environment. On the other hand they feel so compelled to prepare students for the final examination that they are unable or unwilling to break away from a traditional, didactic approach.

Some teachers became entrapped in the belief that this traditional, almost stoic, approach constituted the only appropriate model of teaching mathematics. Forces of change were quickly overwhelmed by the power of the status quo. As Ernest (1989) pointed out, this resulted "in innovations being assimilated to teachers' existing

3

models of teaching" (p. 23). Lerman, (cited in Ernest, 1989), noted that even where innovative strategies have been required under curriculum reform, projects and investigations were treated didactically.

How then is it possible to satisfy student concerns about preparedness for the final examination yet still implement innovative strategies of teaching and assessment that emphasize interdependence and understanding? If time is set aside regularly within the course for examination preparation, can student apprehension about eventual examination performance be satisfactorily addressed? If so, is it possible to teach the remainder of the time in a manner that not only provides students with the necessary skills but also provides for significant investigation and development of concepts and applications that will allow students to place their acquired skills in a context of abstract and instrumental understanding?

Background to the Study

In recent years much discussion has taken place about change in the teaching and assessing of mathematics. The Cockroft report (1982), and the new National Curriculum in Britain (Secondary Examinations Council, 1985), the *Standards* (NCTM, 1991) and various

4

curriculum reform projects in the United States, and the new mathematics curriculum in British Columbia (B.C. Ministry of Education, 1988) have all contributed to new perspectives on the teaching and learning of mathematics. All of these efforts have called for change. An emphasis on problem-solving, applications, and higher level skills is required. However, as these reports and revisions note, change will only come about through teachers taking on new roles as innovators and facilitators.

I have long felt dissatisfied with my own approaches to teaching mathematics. Too much effort has been expended attending to external pressures such as cross-grade examinations, an objective-specific syllabus, textbook-driven course outlines, and provincially mandated examinations. I appreciated the nature of the changes that were being heralded and I have sought to amend my own style in light of these calls.

Purpose of the Study

The purpose of this study was to break away from the traditional, didactic model of teaching. My specific objectives were to explore different strategies and classroom organizations that allowed for more student involvement and more conceptual development. I was to be

5

both the prime participant in the study and an observer of the activities.

This challenge to change was reflected in the methods of the study. I designed opportunities for students to take part in investigations, especially using the technology of computers and calculators, where the purpose was to promote ownership of the problem at hand. By involving students in their learning it was hoped that they would take the knowledge to heart better and more lastingly.

I required that students create projects on curricular topics, or on topics beyond the required syllabus. In this way, collectively or individually, they might develop a broader view of the topic at hand and the subject of mathematics itself.

Working together is an important skill that all students should develop. By using group work and assessment that provides for cooperation I hoped to promote a positive environment conducive to constructive interdependence. Through this mechanism I proposed to break down the fears and anxieties traditionally associated with mathematics.

Throughout the term of the study I sought to alleviate student anxieties about their final examination by providing regular preparation sessions focussed on old

papers and test-taking skills. If this could be
accomplished it was hoped that students would willingly
embrace studies not directly tied to the prescribed
curriculum and value them as positive mathematical
learning experiences.

Significance of the Study

Despite all kinds of external and self-imposed
constraints it is possible to conceive of a mathematics
class where students work together with the teacher to
achieve skills and understanding through activities that
include explorations, investigations, and projects. In
such a class students would develop a positive attitude
towards mathematics and become stimulated to further
their studies in the discipline or at least resist the
math-phobia that so many students develop from their
experiences in school.

The strategies that might be used to create such a
learning environment had to be developed within the
context of the present system of assessment that includes
final examinations. If a teaching strategy that allowed
for adequate preparation for the final examination were
utilized then student and teacher anxiety might cease to
be a hindrance to changes in teaching. Removing this one
hurdle might provide impetus for a critical review of

7

other particular strategies thus leading to an overall revision of the model of teaching.

If it could be further shown that innovative approaches could be used in tandem with a regular, systematic examination preparation scheme then the possibility of a complete revamping of the teaching model existed. The changes that so many seek might be attainable through assimilation of these innovative strategies to the teacher's own model of teaching.

It is important for the classroom teacher to recognize that research of this type is being undertaken by other teachers who recognize the day to day implications of efforts to change. While this study may or may not attain all its expectations it will serve as an example of how teachers can begin to experiment with change.

Limitations of the Study

As a participant observer I am placed in a delicate situation. It is very difficult to ignore personal biases and interpretations when making observations. In addition, observations made while teaching are often superficial and fleeting. In attempting to later recall circumstances that arose in the class it is likely that positive or negative connotations were applied depending

on my personal view of how well or poorly the lesson went. Video-taping lessons ameliorated this situation but was not convenient or possible at all times.

The findings of this study are likely to be site-specific. The student population of the school and class under study differs culturally from the average class in British Columbia in a significant way. While the actions of the teacher could be reproduced in any other class the response of the students might differ markedly. In any event, limiting the study to a single class and a single teacher affects the possibilities of generalizing the results to any other particular situation.

The very fact that the study forms one component of a graduate degree program implies that I am likely more conscious of the value of and need for change than the average teacher. Despite the professional frustration of other teachers there may not be sufficient motivation to assimilate the findings of this study to their own model of teaching. The traditional model is strongly ingrained in many mathematics teachers and the value of this study to others will be determined by their willingness to make changes to that traditional model.

Some mention must be made of the duration of the study. While I made a strong effort to maintain focus throughout the year it was difficult at times to maintain

consistency of effort and observation. This does not detract from the value of the study but it does require that I concentrate on those periods where significant activities and observations were taking place. This limitation of the study will be considered again in the final discussion on results and conclusions.

Structure of the Thesis

There are five chapters in this thesis. Chapter 1 is an introduction to the study which puts it into context and summarizes its objectives, significance, and limitations.

Chapter 2 provides a review of the literature. This section first summarizes the characteristics of traditional pedagogy. The impact of large-scale testing is then discussed with a particular emphasis on the notion of salience. Next, a review of the calls for change within the mathematics education community is included in order to put the study in perspective. Finally, the role of the teacher as researcher is discussed.

Chapter 3 describes the subjects of the study and provides a rationale for the method of inquiry. It also includes a description of the manner in which data were collected and triangulated.

10

Chapter 4 presents the findings of the study and an interpretation of them. A discussion is included on the quality and breadth of activities used in the study. The overall performance of the students is considered as well as an analysis of year-end interviews and the final examination results. The triangulation of the data is considered in a discussion on the validity of the study. Lastly, the influence on my teaching style and strategies is discussed.

Chapter 5 presents a discussion of the implications of the study. Consideration is given to possibilities of replication and improvement. Concluding remarks provide a final perspective on the limitations of the study.

CHAPTER 2

LITERATURE REVIEW

This chapter has four sections. The first section
describes the use and intent of the word "traditional" as
it is found in the literature. The second summarizes how
various authors have identified the impact of large-scale
testing. The third presents a review of the calls for
change within the mathematics education community.
Finally, the views of some authors regarding the role
teachers have in educational research are included.

The Traditional Approach

Authors, particularly those who seek to promote
change in mathematics education, have cited examples of
pedagogic approaches that may be labelled traditional in
the sense that they represent the prevailing strategies
of the time or reflect strategies that have persisted
over many years. It has been noted (NCTM, 1970) that the
reliance on textbooks during the eighteenth and
nineteenth centuries resulted in the "rule-example-
practice" teaching approach, which consisted of stating a
rule, giving an example, and setting exercises to be
followed (or, in geometry, giving proofs to be
memorized). The authors further described how the
evolution of mathematics education has been reflected, in

12

part, by cyclic efforts to change from this style of
teaching to a more inductive one. In this century the
eras of Faculty Psychology, Connectionism, and the "Back-
to-the-Basics" movement have served to solidify the
traditional rule-example-practice approach. Indeed, as
Robitaille and Dirks (1982) noted, while Thorndike's
Connectionism has lost credibility among educational
theorists "it must be recognized that many teacher- or
commercially-produced games and puzzle worksheets are
based upon a Thorndikean view of learning" (p. 10).

More recently, authors have continued to cite the
prevalence of the rule-example-practice approach, or some
similar didactic strategy such as "lecture-test" as
evidence for necessary changes in the way mathematics is
taught. In a report on mathematics education in Grades
4, 7, and 10 in British Columbia, Robitaille, Schroeder,
and Nicol (1992) found that typical mathematics classes
are "highly traditional in nature with time allocated to
review of homework, teacher lecture, and individual
seatwork" (p. 15). McKnight, Travers, and Dossey (1985),
in summarizing the *Second International Mathematics Study*
(SIMS), reported that 60% of teacher time spent in
twelfth-grade classes in the United States was used for
review, administrative duties, or supervision of
students. Students responded that the majority of class-

13

time was spent listening to teacher presentations, doing seatwork, or taking tests. It was further reported that 94% of teachers used the standard text as a primary resource. The next most highly rated (12%) resources were workbooks and supplemental texts. These results clearly demonstrate the narrow range of instructional resources traditionally used in most mathematics classrooms, especially at the senior levels.

In a background study for the Science Council of Canada, Beltzner et al. (1976) summarized the state of affairs in mathematics education.

> Too often, mathematics in high schools is treated as a 'spectator sport' rather than as a creative intellectual activity. Too often it is presented as a self-contained subject, self-motivated and divorced from other fields of human endeavour. Too often the students' satisfaction is derived, not from the joy of intellectual insight, but from getting 'marks'. Teachers and students alike are moved to question the relevance and the purpose of much of what is taught. (p. 74)

While some may argue that progress has been made since that time it is likely, given the findings of Robitaille et al. (1992) mentioned above, that most present-day classrooms still reflect these concerns.

The U.S. National Research Council (1989), in *Everybody Counts*, its *Report to the Nation on the Future of Mathematics Education*, highlighted the fact that most mathematics teachers teach the way that they themselves

14

were taught. It is evident that if such a legacy were regularly left by past teachers then the perspectives that have formed our own tradition have not changed much from those of the rule-example-practice approach.

Everybody Counts further reported that lecturing and listening is the most prevalent yet the least effective mode for mathematics learning. The implications of maintaining this tradition are significant.

> This "broadcast" metaphor for learning leads students to expect that mathematics is about right answers rather than about clear, creative thinking. In the early grades, arithmetic becomes the stalking horse for this authoritarian model of learning, sowing seeds of expectation that dominate students' attitudes all the way through college. (p. 57)

Brophy (1986) referred to the "traditional whole-class instruction/recitation/seatwork method" (p. 325) and pointed out that the efficiency of this approach often hindered the adoption of group instruction.

While many authors have used the term traditional without definition, and it is likely that teachers have their own perception of what traditional means, the evidence available indicates that a traditional pedagogy in mathematics classrooms is generally regarded to be a didactic and supervisory one. Innovation must consequently be viewed in terms of how one has moved beyond the limited dimensions of such a model.

15

The Impact of Large-Scale Testing

The term 'large-scale testing' is taken from Wideen
et al. (1991) and is used to describe testing activities
that are not under the control of the classroom teacher.
Such activities vary in intent and format but are usually
norm-referenced and made up mostly of multiple-choice
type questions. Wideen et al. classify these tests into
three major forms: syllabus-based achievement tests such
as the British Columbia Ministry of Education final
examinations, standardized achievement tests such as the
Scholastic Aptitude Test (SAT) and the Canadian Test of
Basic Skills, and program assessment instruments such as
the B.C. Assessment examinations and the program
assessment conducted by The International Association for
the Evaluation of Educational Achievement (IEA).

Wideen et al. (1991) compare characteristics of
these three types of tests and teacher-made tests. One
characteristic is the level of salience that each type of
test has for students. This is defined as an indicator
of how seriously students view the consequences of their
performance on the test. Indeed, parties other than
students could find a test salient in the sense that the
results of the test might impact significantly on a
program, school or district. Thus teachers,

administrators, and school officials at all levels would be sensitive to various large-scale tests depending on the salience of a particular test to a particular party.

The syllabus-based achievement test, being a summative assessment of achievement, has high salience for students. Other types of large-scale tests were deemed to have low salience since they are primarily diagnostic instruments. However, this analysis is situation-specific. I would anticipate that a similar study of students in the United States would indicate a high salience for the SAT as this test plays a significant role in the determination of a student's post-secondary future.

The high degree of salience of syllabus-based tests was an important consideration for my study. However, concerns related to large-scale testing transcend the boundaries of format. It is clear that whatever the mode of testing a high degree of salience will compound the general concerns about large-scale testing. Therefore I have chosen not to distinguish between types of large-scale testing in presenting this review of the literature. Various articles may have focussed on one or more of the specific types of test but I believe the concerns to be broadly based. As such, reference made to syllabus-based testing, standardized testing, or program

17

assessment should be read as reflecting on all these forms of large-scale testing.

Many authors have argued against standardized testing. Examples include Berlak (1985), Haney & Madaus (1989), Kirsner (1990), Leinwand (1985), McLean (1990), Madaus (1985), Robitaille & Dirks (1982), Shepard (1989) and Smith (1991). Criticism has focused on the argument that such large-scale testing mitigates against creative thinking, in-depth study, and the development of more complex analytic skills.

The number of examinees involved in large-scale testing necessitates a computer scoring system and hence at least a majority of the questions on large-scale tests are of the multiple-choice type. Shepard (1989) pointed out that although multiple-choice questions can elicit important conceptual distinctions "such items do not measure a respondent's ability to organize relevant information and present a coherent argument" (p. 5). Fundamental skills are more easily tested in this fashion and consequently more instructional energy is applied in this area.

While the B.C. provincial final examinations do contain a mix of multiple-choice and written-response questions the multiple-choice component makes up almost three-quarters of the student's mark on the examination.

18

In terms of the saliency factor described previously it would appear reasonable to suggest that greater emphasis is placed on a successful performance on the multiple-choice section of the examination. A corresponding emphasis on such material in the classroom is likely.

Chambers (1990) described how the influence of large-scale testing has impeded the efforts to reform mathematics teaching: "current district policies that place great emphasis on standardized test results are obstructing mathematics reform because the tests disproportionately emphasize lower-order skills" (p. 551). Robitaille and Dirks (1982) added that

> While examination syllabi are sometimes changed to reinforce content innovations, it would appear that the existence of such examinations impedes curricular change at least as often as it reinforces it. (p. 11)

Berlak (1985), Shepard (1989) and Wideen et al. (1991) suggested that one of the underlying purposes of large-scale testing is to provide a mechanism for policymakers to make teachers and districts accountable for student achievement and to force teachers to address skills that are deemed essential. When tests have such serious consequences teachers will often teach to the test. However as Shepard pointed out,

19

...teaching to the test cheapens instruction and undermines the authenticity of scores as measures of what children really know, because tests are imperfect proxies even for the knowledge domains nominally covered by the tests; and they also omit important learning goals beyond the boundaries of the test domain. (p. 5)

Shepard further described the 'dumbing down' of instruction where teachers teach the precise content of the tests instead of underlying concepts and teach this content in the same format as the test rather than as it might be used in the real world.

Research on standardized testing and science teaching in British Columbia by Wideen et al. (1991) reported many concerns about large-scale testing that are typically heard from mathematics teachers as well. Wideen et al. wrote that

Some teachers felt that government exams had reduced the opportunities for spontaneity and depth, others complained that because of time constraints imposed by the exams, they were not able to conduct as many labs [read enquiries for mathematics - author's note] as they would like. Many of these teachers felt that a subtle change had taken place toward a more content oriented delivery which discouraged both student and teacher activity. (p.59)

Later the report stated that "Even teachers who were ambivalent about the exam agreed that since its reintroduction teachers had been forced to 'concentrate on objectives, to concentrate on facts'" (p.60).

20

In spite of the concerns mathematics teachers in
B.C. may have about provincial final examinations it is a
political reality that they exist and teachers must seek
to accommodate them in their teaching strategies.
However, efforts to change the way we teach are clearly
hindered by the influence of these examinations. It is
my goal in this study to explore the possibilities of
making changes within the structure and assessment
provisions of the Mathematics 12 course as it exists.

The Call for Change

Recent studies have emphasized the need for a new
approach to the teaching of mathematics. The report of
the National Research Council in the United States,
Everybody Counts, pointed out that "...the least
effective mode for mathematics learning is the one that
prevails in most ... classrooms: lecturing and listening"
(National Research Council, 1989, p. 57). The report
suggested that the teacher's role must be modified and
expanded to facilitate and encourage classroom activities
that engage students in their own learning. In Britain,
the new National Curriculum (Secondary Examinations
Council, 1985) in mathematics endorsed innovation through
greater focus on problem-solving, applications, higher
level skills and use of microchip technologies. As

Ernest (1989) noted, "...their enactment requires major changes of the teacher, who must adopt new approaches to teaching, including a new role as facilitator instead of instructor" (p. 14).

Recent changes in the British Columbia Mathematics Curriculum (B.C. Ministry of Education, 1988) reflected many of these same hopes and expectations of change. The rationale and goals of the new curriculum emphasized that the needs of the student must be recognized and reflected in the level and quality of mathematics offered. The positive attitude generated by success at the appropriate level of mathematics "serves to reinforce the confidence necessary to ask questions, take risks and accept greater challenge" (p. viii).

The National Council of Teachers of Mathematics (NCTM, 1991), in the *Standards*, emphasized that the kind of teaching envisioned by the *Standards* differs significantly from what many teachers experienced as students and conflicts with current patterns of teaching and learning. Rather than the teacher acting as a "dispenser of knowledge and confirmer of right answers" (NCTM, 1991, p. 4), this new kind of teacher must step back and allow students to argue, experiment, invent, justify alternatives and be wrong.

22

Leinwand (1992) cited numerous impediments to change; collegial and professional isolation; lack of confidence and fear of change; fear of failure; lack of support; and insufficient time. He further suggested that teachers can overcome these obstacles by working together to alleviate fears of isolation and building a network of support within their school and beyond. He also stated that, inevitably, teachers must be prepared to take risks and experiment with new teaching strategies and assessment methods.

Cooney, Grouws, & Jones, (cited in Sowder, 1989), have suggested that research is needed that considers alternative practices: "research that relies on 'what is' limits all of mathematics instruction to current school practice, rather than allowing consideration of radically different instruction" (p.41). Sowder also reiterated the frustration of participants in the National Council of Teachers of Mathematics Research Agenda Project "who felt that many of the efforts to upgrade teaching and learning are doomed unless there is a change in assessment procedures" (p. 38). Since these assessment procedures were likely either large-scale tests of some kind, or classroom tests highly influenced by those large-scale tests, this call for change is

inevitably directed toward both the classroom teacher and district officials.

Kirsner (1990) compared what is and what might be and thereby summarized a major objective of this study:

> ...active mathematics learning requires classroom teaching that is very different from the prevalent mode of broadcasting mathematics as a body of rules and procedures. The Standards call for students to examine, transform, apply, prove, communicate, and solve mathematical problems and concepts. For this to occur, teachers must assume the more difficult role of managing classrooms where students are actively engaged in making presentations, conducting experiments, working in groups, and participating in discussions, as well as working individually. (p. 557)

The calls for change are numerous, forceful, and specific. In response, teachers can feel overwhelmed by the breadth of change necessary and expected. Efforts to alter the traditions that have hindered these changes must come. The impatience that some feel must also be tempered by the realities of the classroom, the variety of needs and abilities of the student population, and the willingness and preparedness of teachers to effect these changes. A popular phrase in the environmental movement is "think globally, act locally". Perhaps this is the approach that needs to be adopted in the movement to foment change in mathematics education. This study represents such a local response.

The Teacher as Researcher

Lerman (1990) summarized a variety of stimuli for the "teacher as researcher" movement. Top-down curriculum reform emphasized the need for teacher education through in-service courses. Teachers who recognized this need instigated investigations of changes required and subsequent curriculum implementation ramifications. In addition, the desire to improve their own practice, individually or collectively, led teachers to undertake small-scale research and report their findings in local and national publications.

Lerman adds that the traditional research model of the outsider coming into the classroom has led to a disaffection with this model on the part of teachers. Teachers complained that outsiders' investigations were often irrelevant, that outsiders recommended actions but had no responsibility for implementing them, and that the accuracy of the findings was limited by the sporadic nature of school visits. The tone of the research was often deemed to be condescending to teachers, and the reports were often written in technical jargon and published in obscure journals.

Another stimulus was the characterization of teacher research as potentially emancipatory. Stenhouse was one of the first to develop the notion of teacher as

25

researcher. Wiener wrote that Stenhouse saw that teacher research "could liberate teachers and pupils from a system of education which denied individual dignity and was predicated upon external authority and control" (Wiener, cited in Lerman, 1990, p. 26).

Lerman (1990) also reported that Schön's notion of the "reflective practitioner" has been a major influence in the teacher as researcher movement. Schön (1987) described the dilemma between the prevailing idea of professional knowledge based on technical rationality and the awareness of "indeterminate zones of practice" (p. 3) that lie beyond this rationality. Schön argued that the unique problems of practice cannot be solved by technical problem-solving means. Their solution requires critical reflection by the practitioner in order to restructure the problem, experiment with possible solutions, test new understandings and explore new phenomena.

The field of action research, first developed by Lewin in 1944, is also concerned with modifying patterns of organizational behaviour through a cycle of planning, observation, action, and reflection. Action research has also been described (Watkins, 1991) as a form of problem-solving in context with a desirable change effected through intervention experiments. Another definition presented by Watkins described action research as having

the goal of making an action more effective. In this light she noted that Schön's "reflection-in action" constituted action research.

Action research, with its requirement for research in context provides a mechanism through which teachers can undertake their own research. Various authors (Brown, 1981; Hustler, Cassidy & Cuff, 1986; Kemmis & McTaggart, 1988) have focussed on action research in education and action research has taken its place as a recognized method in some texts on research methods (e.g. Cohen & Manion, 1989).

CHAPTER THREE

METHODS

This chapter begins with a description of the subjects of the study. Included next is a discussion on the rationale for utilizing action research methods and an explanation of how the study fits into the spectrum of action research. The third section describes the manner in which the data were collected and organizes the data into three categories. The chapter ends by discussing the methods of evaluating and triangulating the data.

The Subjects

This study involved a Grade 12 mathematics class in a secondary school set in the inner city of Vancouver. The school is relatively small with an enrolment of approximately 1000 students from Grades 8 to 12. The majority of its population is made up of Asian students, many of whom display difficulties in their use of the English language, and many of whom come from socio-economically depressed situations. A survey conducted for the 1990 school accreditation report showed that only 120 out of 1008 students counted English as their first language.

The same accreditation report indicates that about 15 per cent of the graduating class go on to university or college. The total enrolment in Mathematics 12 during the year of this study was 70. The total number of students who wrote final examinations was 148.

The students in the class were not selected in any special manner, being enrolled in the class through the normal timetabling procedures. Two other Mathematics 12 classes in the school were taught by another teacher. The original class numbered 24 students and although slight fluctuations occurred throughout the year this number remained more or less constant.

All students who took part in this study were given a letter describing the objectives of the research (see Appendix A) and were required to return a form indicating they had their parents' permission to participate. Only one student's parents denied such permission. That student transferred to another Mathematics 12 class.

In many ways this study was about me more than about a mathematics class or a curriculum. I was seeking to alter my own attitudes about how to teach mathematics as well as the methods I used to teach it. If such changes could be undertaken comfortably and successfully then both my classes and I would benefit.

I have taught Grade 12 mathematics classes for 12 years although this year represented only the second year that the new Mathematics 12 curriculum was in place. My Bachelor's degree is in Mathematics and I feel confident with the subject matter of the course. However, I have long been frustrated by my inability to break away from a traditional, lecture-test type of teaching. I have also felt strongly the anxieties related to the influence of the final examination in this course. I am well aware of the impact of a student's success or failure on this test and have not wanted to put any student at a disadvantage in this regard. At the same time I have very definite opinions about what mathematics should be for students. I want students to enjoy their mathematics class and begin to see how much of mathematics is interrelated in interesting ways. I also want them to feel that their learning experiences in mathematics have been valuable whether or not they were relevant to the real world.

Rationale

The methods I have utilized in this study reflect a dilemma among research theorists regarding rigour and relevance. As Argyris and Schön (1989) stated, "if social scientists tilt toward the rigour of normal science that currently dominates ... they risk becoming

30

irrelevant to practitioners' demands for usable knowledge" (p. 612). The challenge then is to adopt standards of appropriate rigour without sacrificing relevance.

Another dilemma existed regarding the pure objectivity of the scientific research paradigm and the passivity of many qualitative methods. My objectives required a more responsive method of inquiry where feedback from observations could be used to design cr modify subsequent interventions. The necessity of intervention precluded a case study approach while my own desire to ensure relevance, at least to myself, suggested that a quantitative study was inappropriate. Given these parameters within which I wished to pursue this inquiry I determined that some form of action research was the most appropriate strategy.

Action research, as defined in Kemmis and McTaggart (1988), is

> a form of *collective* [original italics] self-
> reflective enquiry undertaken by participants in
> social situations in order to improve the
> rationality and justice of their own social or
> educational practices, as well as their
> understanding of these practices and the situations
> in which these practices are carried out.
> (p. 5)

Brown (1981) described action research as

a structural approach to innovation which allows
participants to examine their own practice and
suggests directions for improvement, by establishing
a recurring cycle of planning, action, observation,
and reflection. (p. 61)

Action research provided the most effective model of
research for my purposes although I recognized a problem
with the use of this strategy. Kemmis & McTaggart (1988)
stated very clearly that action research must be
collaborative in its implementation. They argued that
action research is a vehicle for social change and as
such must be interpersonal in its application to a
situation. In this study no formal collaboration with
other teachers nor with outside co-researchers took
place. At times I discussed my efforts with the other
Mathematics 12 teacher and received feedback but there
was no specific strategy in place to collaborate on an
ongoing and collective basis. This lack of collaboration
posed a significant problem in attempting to describe
this inquiry as pure action research.

Despite this dilemma, action research presented a
usable and effective means of undertaking this inquiry.
The problem of collaboration may only be one of role
identification. When the teacher becomes the researcher
the collaboration becomes internal and personal. There
is no outsider who is doing the research, who needs to
collaborate with a teacher that is the focus of a study.

Indeed, a form of action research, participatory action research, emphasizes the participant-observer role.

Aspects of participatory action research are present in Schön's (1987) reflective practitioner model and in the Informed Reflection approach of Case (1990). Both of these models include a dynamic, responsive framework that provides for solving specific problems in specific situations. Schön wrote that the use in practice of applied science and technique is bounded by the arts of problem framing, implementation, and improvisation. Case described a cyclic framework of establishing a question, gathering information about possible strategies, analyzing the results of their implementation, and revising the plan after reflection on its efficacy. The authors also implied that an individual or collaborative approach may be used although the collaborative strategy is recommended.

Hustler, Cassidy, and Cuff (1986) indicated that action research can be effectively undertaken by individual teachers:

> What follows is as close as we would wish to come to a definition of action research where teachers are the practitioners. They subject themselves and their practice to critical scrutiny; they attempt to relate ideas to empirical observations; they attempt to make this process explicit to themselves and others through the written word. Their prime concern is to improve their own practice in a particular situation from the standpoint of their

33

own concern or worry. For them action research seems to be a practical way forward given their concern in that situation. They use and/or design aspects of their action as teachers to find out more about effective teaching and, in our view, they do so rigourously. (p. 3)

While it is clear that the methods used in this study do not fully constitute action research as defined by some, the methods do lie well within the broad spectrum of participatory, reflective approaches that are recognized under the umbrella of research in action. The lack of collaboration may influence the validity of this study as a potential model for social or institutional change. However, such a flaw does not necessarily detract from the value of this study as a motivator of personal change.

Data Collection

This section is structured into categories related to observations, records, and informants. The type of data collected for each category is described in terms of its source and in terms of influences such as the physical and learning environment of the classroom, the variety of activities undertaken, and the opportunities for students to respond to the study.

<u>Observations</u>

<u>The journal</u>

The primary vehicle for collecting data in this study was a daily journal that I kept throughout the year. The journal documented the activities of each lesson, my reflections on how the lesson went, some thoughts on teaching various topics or using certain teaching methods, observations of the responses of students to the lessons and activities, suggestions for improving activities, and reflections on where the study was going as a whole.

The journal entries varied from day to day in quality and content. It was difficult to maintain a consistency of observation throughout the year. At times I made the entry in the class while the students worked at some activity. Most often I recorded the observations for a particular class at the end of the class or at the next convenient opportunity, which may not have been for some hours depending upon the schedule of the day.

I found it difficult to be totally objective about the entries. My own perceptions about how a particular class went coloured the tone of the journal entry for that day. However, the factual information in the journal was as accurate as I could make it and reflected

honestly the intent and implementation of various
interventions.

The environment

The manner and atmosphere in which students took
part in activities and performed tasks was an important
consideration in this study. The data collected should
be considered in the light of the structure of the
particular activity and the nature of the students'
participation.

The class was taught in what is known as a portable
classroom separated from the main building of the school.
The classroom was equipped with tables and chairs instead
of the standard student desks. This allowed for tasks to
be undertaken on a collective basis throughout the year
rather than solely on specific occasions. In effect, the
students always had the choice of working alone or
collaborating with their neighbours. The tables were
usually arranged so that groups of five or six students
were formed although this formation was altered to suit
special activities or for testing purposes. The students
generally stayed in the same informal groups throughout
the year but were often put into specific groups for
certain tasks.

Given that the majority of students were in Grade 12, most of them knew each other quite well from past years. However, some students were new to the school and some were not as well known to the class despite having been in the school for a year or more. I began the year with some cooperative activities designed to get the students to learn more about each other and make them comfortable working with the others in the class. I also made sure to regularly re-assign students to groups for activities so that all of the students had a chance to work with different people frequently.

When I selected the groups for specific activities I utilized the most recent ranking of the class in terms of their achievement to date. I sought to achieve a balance of students who were at the upper, middle, and lower levels of achievement. I also made subjective choices to place outgoing students with more introverted ones and to mix sexes as much as possible.

The goal of providing such an environment was to facilitate discussion and collaboration so that students could share their understandings and work together to solve problems and assimilate new concepts. Interaction was encouraged and expected.

Records

The records of student achievement in class and on the final examination reflected the success of the study in terms of individual student expectations and in terms of the institutional accountability of the final examination system. I considered the achievement of the students within the class in the light of my own expectations. I also considered the relative achievement of the class on the final examination compared to the other Mathematics 12 classes in the school, the district, and the province.

These records cannot be considered in isolation. I believed that the outcomes of assessment had to be influenced by the manner in which material was presented, the classroom attitude towards learning mathematics, the manner of assessment, and the preparation for the final examination. I sought to address these issues through the variety of activities that the class undertook.

Activities

Four basic types of tasks that I engaged the students in could be considered innovative in terms of my past approach to teaching Mathematics 12. These were exploratory investigations, projects, cooperative tests, and examination preparation sessions. These activities

38

reflected the essence of the changes that I was attempting to make in the implementation of the Mathematics 12 curriculum, although the examination preparation sessions were innovative only in that they were intended to open up time for more exploratory and investigative activities in the remaining class-time.

Exploratory investigations. The explorations varied from pencil and paper investigation of new concepts such as tangent lines in calculus to the use of computerized spreadsheets to investigate sequences and series. The class was also able to make use of a class set of graphics calculators to explore the effects of various transformations on the graphs of quadratic and trigonometric relations. For example, after a preliminary discussion on the fundamental period and amplitude of the function y=sinx, I assigned the task of graphing other forms such as y=5sinx, y=sinx +3, y=sin2x, etc. so that students might recognize how these new forms effected changes to the original graph. While the task in itself was not new to my repertoire, the use of the calculators made the activity more efficient and effective. This opened up time to discuss further implications of the transformations such as the possibility of choosing suitable parameters for the

equation $y=a \sin b(x+c)+d$ to model any periodic behaviour.
One problem in the Annual Problem Set (Ministry of
Education, 1991) related to tidal factors affecting the
passage of a barge through a canal. While working with
this problem I was able to refer back to the discussion
on creating a trigonometric equation that fit a given
periodic behaviour such as tides.

The students also used the graphics calculators to
explore basic and transformed graphs of exponential and
logarithmic functions. The inverse nature of these
functions was more easily examined through this
electronic medium. In addition they used the TRACE
function on the calculators to examine points of
intersection of functions or points of significance such
as intercepts.

One important aspect of the explorations was that
students had the opportunity to investigate a given topic
prior to a formal presentation or subsequent to some
initial direction that provided a focus to the
exploration. It was intended that the explorations would
stimulate interest, promote conceptual understanding, and
motivate the acquisition of the basic skills needed for
further study. Some part of the exploration often
appeared in a modified form on a subsequent test.
Students were not advised that this would occur as I did

not wish for the eventual test question to act as a
motivating force for attending to the exploration.
However, I did want to assess in some form the
understanding that students may have garnered from the
activity.

Projects. The class was assigned four projects, two
in first term, and one in each of the other two terms.
The first project (see Appendix B) related to the
application of exponential and logarithmic functions.
Students were asked to choose an application topic from
those studied in class or from personal research in the
library or elsewhere. Examples were acid rain,
earthquakes, light penetration, and population growth.
They were required to present a graphical representation
of their application and then to convert the equation for
the application into logarithmic form. They were also
required to discuss the limitations of their model and
their representative formulae. My intent was to
illustrate the inverse nature of the exponential and
logarithmic functions and to highlight the tenuous
aspects of mathematical models, especially at the
extremes of the domain.

The second project (see Appendix B) was essentially
a comprehensive algebraic and graphical review of a

chosen conic section. It was intended more as a consolidation exercise than as an exploration as we had already worked with conics for some time. The choice of conic was left to each student although in recognition of the extra work involved I awarded bonus marks for choosing an ellipse or an hyperbola. I also encouraged students to use another textbook for extra information. I wanted them to realize that there were many resources beyond their own standard text and that these other resources often provided a different view on the topic at hand.

The third project of the year (see Appendix B) was intended to promote appreciation of the breadth of topics in mathematics and to emphasize the possibility of students pursuing study of a topic on their own. A long list of mathematics topics was presented that included broad headings such as statistics or sets as well as specific topics such as powers of binomials or proof by induction. The project was designed as an individual research effort with particular emphasis on the need to find resources beyond their textbook. I assisted in this regard by making my own library of mathematics books available for loan. I was looking for students to summarize information found elsewhere in a cohesive and comprehensible manner. I did not expect students to

suddenly become expert on topics such as spherical
trigonometry or tessellations of the plane. I did hope,
however, that exposure to some of these topics might
stimulate interest at some later date in pursuing further
study in mathematics.

The final project (see Appendix B) consisted of
creating a final examination for Mathematics 12 similar
in structure to the actual examination that they were to
face. Each student or group was required to develop a
set of multiple-choice questions complete with
distractors and an answer key. The questions had to
cover the curriculum and the distractors were assessed in
terms of their value as true distractors rather than
simply as incorrect answers. The project also required
students to create two full-solution questions, one of
which had to be a geometric proof. A detailed solution
key, including a marking scheme, was expected for each
question.

The purpose of this project was to provide a
mechanism for a comprehensive review of the curriculum
and help students to develop an understanding of the way
in which multiple-choice questions were structured. It
was hoped that the familiarity that students might have
already developed with the past final examinations used
during the in-class examination preparation sessions

would have helped them design their own examination and enhanced their ability to deal effectively with this style of examination. The students also became more conscious of the significance of distractors and the manner in which such distractors could be designed to exploit common errors. In this way students could also develop their own strategies for quickly determining whether a distractor presented a potential correct answer or not.

Cooperative tests. The cooperative activities in which students engaged gave them the opportunity to share their understandings. However, these activities did not form a significant part of classroom assessment. As I observed and reflected upon the quality and style of cooperation it became clear to me that I should find some way of testing students in a cooperative manner, so that there was an element of mutual interdependence in the actual assessment instrument.

I decided to modify a traditional instrument, the classroom test, so that it contained a cooperative component. I did this by creating a two-part test where the first section was made available before the test to predetermined groups of three students of varying ability levels as determined by their relative ranking to date.

I wrote the group members' names on the blackboard at the beginning of class and then gave the groups ten minutes to consult on the first part of the test. Each group was given a single page which simply listed the questions that would appear on the first part. The single page was used to encourage consultation. At the end of the ten minutes the page was retrieved from each group. No notes were allowed to be taken. They would answer these questions on their own test papers once they had been separated and the complete test had been distributed.

The intention was to provide a last-minute cooperative preview of the questions so that all students would have an opportunity to clarify and recall any final understandings or skills. It was not intended that students attempt to solve these questions beforehand. Rather, I explained that I wanted them to discuss how to solve the questions and ensure that all students in the group had a good understanding of how to proceed.

The second part of the test comprised a further set of questions that all students were required to answer. Students saw these questions only once they had been separated and the tests handed out. Thus their achievement on this part was individually determined. Copies of these tests are included in Appendix C.

I advised the students that this system would be in effect, explaining my perception of the value of such an approach. I also explained the scheme I would use for scoring the test. I knew from past experience in other grades that students were suspicious of cooperative assessment. Comments had been made to me that the good students did all the work and the poorer students still got the marks. I therefore chose to use the average group achievement on the first part of the test as only one factor in the overall score. For example, if the first part (the cooperative preview section) of the test were valued at 20 marks and the second part (the individual section) at 20 marks, I would compute the final score by taking each student's mark out of 40 (from individual performance on the cooperative and the individual parts) and then including an extra mark out of 20 that represented the average score of that student's group on the first part of the test. Thus the final score would be out of a total of 60 with two-thirds of that mark weighted on the student's personal achievement on the complete test and one-third weighted on the average group performance on the cooperative part. The one-third component was, I hoped, sufficiently significant to ensure that real cooperation would take place in the preview groups.

I used this method of testing for much of the year although from time to time I reverted to individual testing. In addition, I revised the procedure in light of information gained from previous tests, including student feedback on the fairness and effectiveness of the process and comments from other teachers regarding the validity of such a scheme. I altered the preview so that questions like the first part were available for discussion but the actual questions were not presented. I also revised the design of the tests so that the cooperative component provided approximately twenty-five percent of the final score. This ensured that the individual student was never severely penalized for an extremely poor performance by others in their group.

Examination preparation sessions. The final type of activity that formed the basis of the study were the examination preparation sessions. These sessions were intended to be both an opportunity for review and a way in which to open up time in the remaining classes for deeper study of the topic at hand. One of the major objectives of this study was to determine if such a system were feasible and worthwhile.

An aspect of this objective concerned the anxiety that students felt about the provincial final

examination. I sought to provide a preparation session that assured the class that the needs of the examination were being addressed. I wanted them not only to be exposed to the type and style of question but also to the strategies they could employ in order to be more successful on the multiple-choice component of the examination. I hoped that the class would then be amenable to the study of problems and concepts in the remaining classes that did not relate directly to the content of the examination.

I organized the sessions on a group basis originally although this structure changed as the year progressed. The class was divided into groups, each of which was assigned a major topic area from the curriculum guide. Within each group students first worked to classify the questions on past examinations, identifying those that pertained to their topic. Working together they studied from their texts to assimilate the basic notions of each topic and applied them to the questions. I also provided the groups with topic study guides that I had created to use for this purpose (see Appendix D). I assisted when necessary but I explained that the effort was to be primarily their own. I further explained that I would be asking them to share their expertise with the other students at a later date. I emphasized this sharing and

sought to ensure that students consulted with each other within their groups as well. I hoped that the sessions would become one grand cooperative effort with me offering guidance from time to time on content and strategy. The preparation sessions were designed to initiate students into the fundamentals of each topic. In the course of the year we would eventually study each of these topics as a class to consolidate and extend the fundamentals.

The Annual Problem Set (see Appendix D) also provided opportunities for examination preparation. These problems are supplied each year by the Ministry of Education and are to constitute approximately ten percent of the course. In addition, one or more of these ten problems would appear on the final examination in some altered form. The problems are challenging and include multiple skills and concepts within each question. They provide an excellent opportunity for the application of knowledge gained not only from Mathematics 12 but also from various topics studied previously. Some of our preparation sessions were used to focus on these problems with particular attention paid to connections with the current topic of study. Students worked cooperatively on these problems or we used a specific problem for a class discussion. At times I assigned a problem as a homework

exercise to be submitted for marks. I also included
bonus questions on tests that were related to a recently
studied problem from the set.

Informants

In addition to my own reflections that I recorded in
the journal I was interested in the response of students
to the various tasks and to the study in general.
Periodically I gave the class an opportunity to give
their impressions of how well or poorly an activity had
gone.

One of the ways in which I garnered student feedback
was to use an approach suggested by Cross and Angelo
(1988). A "One-Minute Paper" handout (see Appendix E)
asked students to briefly answer two questions; "What was
the most useful/meaningful thing you learned during this
session?" and "What question(s) remains uppermost in your
mind as we end this session?". As the title suggests,
students were given only a minute or so to answer these
questions. I used this paper in a variety of situations
but primarily after introductory or review lessons.

From time to time I asked students to write down
their impressions of how a given activity, say the
cooperative tests, may have been effective or not. These
written responses helped me to plan and revise when

specific suggestions as to flaws or possible improvements were made.

At the end of the year I selected eight students on a random basis, stratified according to achievement ranking. I interviewed these students individually, asking each student a series of questions (see Appendix E) pertaining to the study as a whole. I encouraged students to answer freely and indicated that negative comments were as useful as positive ones. I audio-taped their responses and I have included transcriptions in Appendix E. I have also included a representative sample, including my interpretation of their comments in the next chapter.

Analysis of the Data

This section describes how the data were used in assessing the study. Two key elements are presented. First the concept of triangulation is described and a particular method of triangulation used in this study is identified. Secondly, the value of the various data to the study is discussed. This second part focusses on how the data collection methods reflected the objectives of the research.

Triangulation

Triangulation takes its name from the surveying and navigational technique of pinpointing a single site by using two or more locational markers. In research methodology this translates into using multiple sources of data or multiple methods in order to glean a more accurate interpretation of a situation. There are a variety of ways in which a research study can be triangulated. Cohen and Manion (1989) cited six types of triangulation: time triangulation, space triangulation, theoretical triangulation, investigator triangulation, methodological triangulation, and combined levels of triangulation.

Of these methods of triangulation, this study uses a combined approach in order to develop a more holistic view of the events studied. The effort to offer an alternative implementation of the Mathematics 12 curriculum is explored and evaluated from the perspectives of observations made, responses from student informants, and the achievement records of the students.

Evaluation of the Data

The collection of the data reflected the levels of analysis anticipated. My primary objective was to effect changes in my teaching style in a manner that provided

for satisfaction and success. It was therefore important that I document my own reflections on the innovations undertaken. I considered the success of these activities in terms of my satisfaction that learning was taking place. Further, I looked for satisfactory student achievement on the assessment instruments that I constructed. Success also had to be viewed in terms of achievement on the final examination as this reflected the institutional reality of the course. Lastly, I was interested in the satisfaction of students. It was important to me that they felt that they were learning, understanding, and achieving.

The journal recorded my own observations and feelings about how successful various innovations and interventions had been. The journal also documented the responses of the students as individuals and as a class to these activities. The achievement records of the students gave insight into the relative assessment of students in class as compared to their final examination mark. The examination mark gave a broader comparison of the achievement of the class relative to the other Grade 12 students and to the rest of the province. Finally, the One-Minute Papers and the year-end interviews provided individual responses from students as to the

effectiveness of the study and the attitude changes they might have undergone.

Data Triangulation

I sought to cross-validate data related to attitude, success, and satisfaction against their sources; journal observations, informant responses, and the student achievement records. The matrix below (Figure 3.1) illustrates which sources of data reflected which types of outcome.

Triangulation of the data within types required some measure of cross-validation across sources. For example, student success was indicated by observations of satisfactory performance in class activities, especially in terms of the effectiveness of a particular activity to engender understanding of a particular concept. It was expected that this success would be reflected in achievement on class tests, the final examination, and, where appropriate, the project assignments. Finally, responses from informants provided an indication of success from the perspective of the students themselves.

At the same time, a single source of data allowed for cross-validation of the data by type of outcome. For example, data from informants provided information as to

TYPES OF OUTCOME

		Student Attitude	Student Success	Teacher Satisfaction
	Observations	journal	journal	journal
		activity participation	effectiveness of activity	
S O U R C E S	Records		final exam results	exam results compared to others
			class tests	class tests
		projects	projects	projects
	Informants	interviews	indications of success	objectives attained
		One-Minute Papers		
		comments		

Figure 3.1: Matrix illustrating triangulation
of data sources and types.

whether students maintained a positive attitude about
their learning of mathematics and whether or not they
felt themselves to be successful in the course. This
information also indicated to me whether I had satisfied
my objectives of teaching the course in an interesting,
innovative, and effective manner.

CHAPTER FOUR

RESULTS

This chapter provides details of processes undertaken as part of the study and describes key outcomes by way of sample journal observations, responses from student informants, and the achievement records of the class.

Processes and Outcomes

The Journal

As I began recording observations in the journal I had certain objectives in mind. It was necessary to briefly describe the particular lesson or activity so as to provide background for further observations and for later review. I also wanted to record personal reflections about how the lesson had gone, how the approach compared with what had been done before, how it compared with the traditional approach, and how effective it had been in terms of student learning. In addition I wanted to document student responses through observations of activity, participation, interest level, and comments. An entry from the second class is included here to provide an example.

Furniture changed from desks to tables.
Focus Trios have been formed (2 quartets).
Focus is on functions - a topic supposedly
studied in Math 11. One recorder is used per

56

group to encourage discussion. Some students are looking into their Math 12 text. This may be a good way to encourage advance reading of material (esp. new material). All groups seem to be working well with various degrees of discussion. The recording by students is done in a variety of forms. Some accent the visual memories - graphs, vertical line test, special functions like $y=x^2$, x^3, $|x|$, $\sqrt{x}$, etc.. Some are writing phrases or key words that seemed important - domain, range, exponential, cubic etc.. The single sheet/single recorder idea works well to promote actual group work rather than individual study. Some students also recalled transformations of functions.

I have borrowed three containers from the science lab; an ordinary beaker, a tapered glass and an Erhlenmeyer flask. I handed out a sheet of graph paper with three graphs plotted on it. The graphs showed height of liquid in each container as a function of volume. The different shapes of the containers will give different shaped graphs as the containers are filled. I slowly filled each container with water in front of the class and asked the students to consider how the depth and volume were changing as the container filled. I then asked them to decide which of the three graphs represented each container.

This activity would make a good experiment. Student groups could be given different shaped containers, fill them by fixed increment in depth or volume, recording the ordered pairs and then graphing the results. I chose not to do this as the subject of functions is a brief review unit in Mathematics 12. I believe that the extra time committed to such an experiment would be better utilized in Mathematics 11. To take the extra time at this early point on an item of review may in fact disillusion the students as to what I will be doing in the rest of the course. I will save the time needed for experiments for later topics.

I had handed out the graph sheet to each student. I gave no specific instructions as to whether they should or could work together on deciding which graph matched which container.

I was pleased to note that many of them chose
to work together and much discussion took place
regarding the choices. Certainly having the
students at tables instead of desks is an
advantage.

Once the choices were agreed upon by the
class we discussed the significance of domain
and range as related to this situation. The
choice of axes for the two quantities, distance
(from the table top) and volume, is arbitrary
and in that sense domain and range are as well.
I will make clear next class the distinction
between dependent and independent variables and
how this determines the domain and range and
thus the choice of axes.

At first the entries included many details about the

topic, what was occurring from moment to moment, and what

I intended to do next or what I had neglected to do.

When the early entries were reviewed it was found that

the observations were cumbersome and tedious. However,

later attempts to focus observations only upon selective

aspects of the class resulted in a loss of perspective,

thereby reducing the effective value of the entry. The

inclusion of seemingly unimportant detail helped to give

a better overall sense of what was happening in the

class.

I sought to remain objective about observations.

However, it soon became evident that the kinds of

objectives that I had set caused the journal to become

more of a personal diary and entries thereby took on a

more subjective flavour. An excerpt from the seventh

class illustrates this.

The next activity was a focus on the review of quadratic functions. I did not use focus trios for this. The students are now seated in groups as a natural result of the furniture arrangement. I wrote four examples of quadratic equations on the board and asked the class to work together to solve them. The class set to work quickly and were successful in completing the questions in a very short time. As we went over the examples together I recognized that the class had a good recollection of how to deal with quadratic equations. I explored how to generate quadratic equations and how such solutions were represented graphically. This kind of link between the visual and the abstract is very important to me. I assigned homework exercises that provided a variety of situations to try to come to terms with where the class is at for next period.

I find the use of visuals to be very helpful in making connections between ideas in mathematics. Often, especially in a technique-oriented curriculum, visual connections are not made. The school has a program called the English as a Second Language (ESL) Project. The primary responsibility of the teachers involved is to help subject teachers develop what are called 'key visuals' for a given topic. The theory is that language difficulties impeding understanding can be overcome by a more visual approach. It has occurred to me that this approach would also be effective in mathematics. In a sense, if one agrees that mathematics constitutes a language of understanding, then using key visuals for a topic would serve to overcome the difficulties associated with learning this language . Thus graphs, mappings, flowcharts, tables, etc. may be more effective in transmitting specific and general understandings related to a given topic. This is an avenue I would like to pursue in this study and also in my other mathematics classes.

There were days when things did not go well, usually through a lack of adequate planning. These days were disappointing as I felt that I should have been pursuing innovative approaches at all times. Although this expectation was unrealistic I was still frustrated by the tendency to fall back on more traditional methods of covering the curriculum. An excerpt from early October reflects some of these concerns.

> The remainder of the class I used specific homework questions to reinforce, clarify, investigate and expand upon ideas about logarithms. I felt that what I did was effective but I was disappointed upon reflection that I was taking the responsibility for learning away from the students. Questions revolve very much around skills and techniques. I would like to inculcate some desire to understand the concept and the significance of logarithms into these students. Perhaps this can be accomplished through the study of applications. Hopefully the project assigned today will help address this concern.

> At this point, in reflection on what has taken place so far this year, I must say that I am pleased with what is occurring in my classroom. I feel that the students are involved, interested, and committed to getting the most they can out of the experience. There are a few very quiet introverted students but these do not appear dissatisfied as much as uninvolved. I would like to see these students play their full role in the overall success of the class.

> While I am pleased I am not altogether satisfied with the process as it is unfolding. I have never been one to fully plan any unit as I enjoy the side-trips into other areas that may or may not be applicable to the topic under study, but serve to broaden student understanding of the depth of what constitutes

mathematics. This habit, unfortunately, often leaves me rushing along to cover required material. For this study I sense that I need to pay closer attention to where I want to go and how well I got there. It is difficult to reflect on how effective a plan has been implemented if there is not a clear understanding of desired outcomes. This is one lesson that I am learning. It appears to me sometimes that this study is really about me and my teaching and how I can effectively implement strategies that have always seemed desirable to me. In some ways I feel that I am concocting some grand soup that in the end should be both delicious and nutritious. I often doubt the value of such a process in terms of offering any valuable advice or strategy to another teacher. Only I know what ingredients are going in to the soup.

This opportunity to reflect on what was happening was the most valuable aspect of the journal. Not all the comments reflected frustration. At times I summarized where I felt I was in terms of my objectives and where I would like to be. Here is an excerpt from the last class of the first term.

This class also marks the end of term one. Reflecting on the year so far I feel that I have made some changes that are beneficial to the class and to me professionally. I like the two part testing idea although it certainly needs refining. I like the cooperative work although given the tables in the class I find that some form of cooperation is going on all the time. In some ways I am not formalizing the cooperative learning as I feel I should. I need to work on making all students in a group feel accountable.

I feel that I am still very conservative in my approaches. I still want to discuss and review every notion with the class. This must be self-defeating if I am trying to promote

critical and independent thinking. Is that something I am trying to do? I intended this study to be an alternative implementation of the Mathematics 12 curriculum so that I could undertake novel approaches to the curriculum while also preparing the students for the examination. I seem to be drifting into a slightly modified but still very didactic approach. Certainly, I have used projects and will continue to do so but there is so much more I should be trying. On the other hand, what can reasonably be expected given the fullness of the curriculum? Perhaps only two or three innovative activities per term is as good as could be expected.

The journal observations proved to be a good source of evidence reflecting my own satisfaction with my teaching. The journal also documented the level of participation and the effectiveness of certain activities. These last observations reflected, to some extent, the attitude that students had toward the activities and the level of success attained.

The subjectivity of these observations moderated their accuracy and potential value in terms of assessing the study. However, the journal was only one element within a structure of data triangulation. Further evidence was needed from other sources.

Student Response

Students responded to the study in a variety of ways. Comments were solicited in class on an informal and irregular basis. I also had some students approach

62

me from time to time with personal observations that they wished to share. In addition I used the One-Minute Papers described in Chapter Three to garner feedback from the students after a particular activity. Lastly I interviewed a selection of students at the end of the year and audio-taped the interviews.

It should be noted that most of these students are not native English speakers and as such there were many grammatical mistakes in their comments. I have chosen to correct significant errors in grammar in order to make the interviews more clear.

In-class comments

Comments made by students, whether they were verbal or written, were generally positive. Most students appeared genuinely interested in the study as it progressed and were supportive of my attempts at innovation. At the same time they were willing to let me know when they thought that an activity was confusing or unfair.

I believe that some of the cooperation came about as a result of my openness with the class regarding the objectives I had in mind. I did not feel that the validity of the study was compromised by this sharing of my goals with the class. Indeed, I believe that such

openness constitutes an innovation in terms of developing a learning environment which has sharing of ideas and mutual understanding as a component. This reflects, in some ways, the kind of learning environment envisioned in *The Standards* (NCTM, 1991)

Not all the comments received from students were positive. Some chose to vent their frustration over a learning activity that did not suit them well. For example, after an investigation of the behaviour of the graphs of the logarithmic and exponential functions under certain algebraic transformations, I included such questions on the unit test. One student, who was a recent immigrant to Canada and who evidently had little exposure to this type of analysis, noted his frustration at not doing well on these questions.

> I don't like the grouping tests. I hate the graphing. I have the lowest test mark I ever got. I think I'll drop this class. I never had a 'C' in math class before.

In fact the student did not drop the course and his concerns about his mark were ill-founded. He made a successful adjustment to the teaching and finished the course with a B letter grade.

Negative comments were often of this type, that is, where an individual student had a personal reaction to an individual activity or event, in some cases based on performance. Positive comments, on the other hand, were

more broad, reflecting a general satisfaction with an activity or approach. Here is an example from comments solicited after a cooperative test.

> The method I think really helps. This is because I think working and discussing the test questions before the test helps people who doesn't understand the math problems itself to do better for the first part. I think this method should continue.

One student commented positively on the testing approach even though the individual's performance was adversely affected by the average group achievement:

> Under this type of test, I think my mark is affected by the average, but anyway I can learn how to teach others.

The One-Minute Papers

These papers were based on the work of Cross and Angelo (1988). The intention was to provide a vehicle to obtain student feedback regarding an activity or lesson immediately after its conclusion. Two questions were asked, "What was the most useful/meaningful thing you learned during this session?" and "What question(s) remain uppermost in your mind as we end this session?".

This method of gathering student response proved useful in many ways. I was able to quickly and conveniently identify positive and effective aspects of my teaching and those areas where improvement or further discussion was warranted. An example was a lesson in calculus on graphing functions by using the first and

second derivatives. We had previously dealt with velocity and acceleration. Students generally responded that they had a good sense of how the second derivative could help them determine if a point was a maximum, minimum or a point of inflection. Many also noted, however, that they were still unclear as to how the second derivative represented acceleration in a space-time graph. This information invited me to present a further discussion on acceleration, tying it to the second derivative, in the next lesson.

In another case I had presented the first of three activities on tangent lines that was intended to explore what a tangent to a curve is. Students responded that they had a better idea of what a tangent line was but that they still lacked a clear definition for such a thing. I had intentionally planned the activity this way so that the students would be frustrated by this uncertainty and thereby be motivated to learn the proper definition of a tangent that is so important to calculus. In this sense the papers provided feedback to me that the activity was working as I had planned it rather than giving me information about something that I expected to be clear in the students' minds. In the next lesson I was able to exploit this interest to present an effective

lesson on the tangent as the limiting line of a sequence of secant lines.

I used this activity extensively during the period where I took the class into the computer laboratory to explore the graphical aspects of sequences and series. I had previously taught lessons on the basic ideas of sequences and series and we had learned the formulas. I was very excited to give my class the opportunity to utilize computer technology to further the study of a topic in Math 12. Many classes and lessons used graphics calculators but using computers in a Grade 12 course was unusual.

Much of the work done during these periods was exploratory. Students first needed rudimentary training in how to operate an electronic spreadsheet. I then led them step by step with specific examples until they were comfortable enough to continue on their own. At each point where we changed focus I returned to a guided example to start them off. The class seemed thoroughly engrossed in the activity and appeared to be enjoying the experience. We also used the charting facility of the spreadsheet to graph the growth of arithmetic and geometric sequences and series.

The comments received by way of the One-Minute Papers suggested that the students appreciated the visual

nature of the exercise, especially the graphs. However, most students indicated some confusion about how the spreadsheet work related to the formulas studied earlier. The major concern appeared to be how to manipulate the formulas better. In this case the One-Minute Papers clarified the students' perspective on a lesson that I had interpreted in a quite different way. Upon our return to the classroom I took more time to reconsider how the work done in the computer laboratory related to the formulas we had studied previously.

The usefulness of these papers was tempered by the general inability of students to respond to the second question with clear, focussed answers to which I could react. Many times I received blank responses or comments suggesting that the students had no question outstanding in their mind. This would be an unfortunate affair if it were true. More likely the student was unable to put a concern into his or her own language that would make clear the particular problem. Certainly students for whom English was a second language would have suffered in this way. One clear example of this last problem came from a response after an exploration of tangents. In answer to the second question on the One-Minute Paper the student responded "stall (sic) don't know the exploring."

<u>The year-end interviews</u>

The year-end interviews provided an opportunity for me to assess the views of students on the study as a whole. I was interested in the attitudes and expectations that students had about the class and about mathematics in general. Eight students were randomly selected, stratified according to achievement. Transcripts of interviews are included in Appendix E.

Included here is the transcript of one student's interview and some comments on her responses. I chose the transcript of this particular student because she was best able to articulate her feelings about the course and the study.

(Researcher)   Why are you taking Math 12?

(Student)   Well, I'm interested in math and my goal is to become a teacher, as you know, and I think that it's best that I get background on every possible subject I can get.

(R)   So you're taking lots of different subjects?

(S)   Yeah. Very different, very different.

(R)   What kind of class did you expect when you started the year?

(S)   You mean knowing that this would be a little different?

(R)   No, just when you first signed up for Math 12.

(S)   I expected the usual,you know, where a teacher does a little more, gives you the notes and then

69

assigns you the homework, and then if you have any problems, the next day you go and ask, and that's what I expected, because the past four years have been like that so I'm used to it.

(R)   What have you found different about this year's class compared to other math classes you have had?

(S)   I think, the main difference is that you make us participate, whereas other teachers, they kind of, um, I'm not saying that other teachers are bad or anything, right?, well anyway, certain teachers would expect you to know certain things and they wouldn't take the time to review or anything like that. I mean, they kind of sometimes expect too much or they might ... they might just ... I don't know, maybe it's their figure is rather authoritarian, whereas you project a friendly image. And also the way you visualize things helps, whereas if you just have a board and you write numbers down it doesn't help the students because it doesn't help them to see the situations and why a certain concept or theory, or whatever, is important. So, I think that's the difference.

(R)   What activities do you remember from this year?

(S)   Getting, ... I remember the first week, or first bit anyways, we got together with different people and we talked about the different topics that we were supposed to study, but we really didn't go on with that, but just getting different people each time which gives you a chance to know other people and as well to show actually how much you know because you have to actually teach the people around you or they'll teach you.

(R)   You mean in the exam prep sessions?

(S)   Yeah. Exam preps. And those things like the tangent one, remember?, the worksheet, you gave us. The worksheet before we actually really went into it and I think that helped too because it shows you what you don't know and what you do know.

(R)   Anything else?

(S)   The projects. The projects are good too because math is not just limited to that classroom. I mean there's a lot of things that you don't know and so I think those projects helped ... for individual research and everything else.

(R)   You sort of already started to answer this
one. What activities did you enjoy or find useful?

(S)   Well, the projects, that's definite
because I think it more less forced me to kind of look
for information by myself and especially individual
projects. And, for example, the last one that we just
did, the provincial exam, that was good cause it also
forced me to review and stuff like that so, in a sense
even though it's forcing it, it does help a lot because
we're not going to do anything just sitting there without
force. I mean we do have to have some, pressure, honest
I think, for our grade or for our class specifically.

(R)   Anything other than projects?

(S)   The way you teach, like visualizing stuff.
Most people, I guess, learn better when they visualize,
I'm not sure that's true, but for me anyways, for me it
is. For some people too.

(R)   What activities did you not enjoy or find
not useful?

(S)   Remember when we first started we went
into groups and we said we'll teach each other this and
we didn't really go on with it, so I thought that was a
bit (inaudible).

(R)   Was it the fact that we didn't go on with
it? Or was it the thing itself?

(S)   Yeah, I think it was that we didn't go on
with it. No, I think the thing itself is important, and
quite important in fact, but maybe it's just because of
time limit, you know, other factors that mmm (inaudible),
that wasn't really dealt with the way it should have
been, it sort of faded.

(R)   Anything else?

(S)   No.

(R)   What activities would you have liked to
have seen in the class?

(S)   What I would have liked to have seen?

(R)   If you were controlling the class and you
could organize some activity what would you like to see?

71

(S)   Almost the same thing I guess. More
participation, but that really comes up in the students.
And we could have more group work. The only problem with
group work is that if you get new groups it's really
difficult to, ... ,if you don't know that person it's
really difficult to talk to them. That's the only
problem with group work, but, there should be more group
work 'cause I like times together.

(R)   If you could change something about the
Mathematics 12 course what would you change?

(S)   Well, add more, ... something like word
problems, something to do with real life, because
sometimes it's really difficult to relate to something
like logs or calculus when you first deal with it. You
don't know what's going on and you feel that it doesn't
...' You ask yourself 'Why do you have to learn this?'
because I'm not gonna remember this, or know this, or
need this ten years later. You tend to think that,
especially when you don't understand it. So I think
maybe if the text itself dealt with problems that were
more realistic. I dunno, use examples that show that it
can be applied to life, or something. Not so much the
course, but the way the book was written.

(R)   So, the same kind of things but just
applied more, is that what you're saying?

(S)   Yeah, yeah.

(R)   How do you feel final examinations affect
the way you learn or study?

(S)   Well, they force you to learn because you
go 'Oh I have to take this test', but I guess sometimes
people get caught up with that concept and then they
forget that learning is just learning, I mean, education
is forever. You don't just learn in school. And so,
people always have it in the back of their mind, final
exams, final exams, so they forget what learning is
really about. It's not about getting ready for final
exams, it's about learning, right?. So I think that's
the only problem with final exams.

(R)   Based on your work, how successful do you
feel that you have been in this course?

(S)   I think I tried, I think I tried. I guess
I tried harder than usual 'cause, other years I didn't

72

bother with it too much. Maybe it was the provincial, or something that really pushed me to work harder. I don't know, I think I did OK.

(R)   In terms of how you've achieved, how do you think that reflects your success? With the kinds of marks you've been getting, the results you've been getting?

(S)   Well, I guess the most important thing is what you learn, not so much the mark you get. Obviously if you have an A you're going to be happy and stuff. I think the most important thing is actually what you get out of the course, not what you get from the course.

(R)   Are you saying that it's nice to get an A but you still feel sometimes that you could ... didn't do as well as you could have or understand as well as you could have?

(S)   Yeah, yeah, yeah.

(R)   How well do you think you have understood the ideas in the course? The ideas of what a log is, the ideas of tangent and limit?

(S)   I think I've got the principles but, again, I think the most important thing is to show students how it can apply to life or else they really have no idea what's going on. I mean, sure, you can get the formula, but that doesn't tell them anything. You have to really tell them what is going on and how you can use it in life, because you can give me the quadratic formula and it doesn't mean anything to me even if I know how to use it. When you know how to apply to it I guess that's as much as ... (fades to silence).

(R)   How well do you think that you have learned the skills required for the course? Like the basic skills of manipulating expressions, working with logarithmic equations, working with trig and conic sections. How about those skills?

(S)   I think I did OK. I just kept practicing and practicing and practicing until I got it.

(R)   Practice makes perfect.

(S)   Right. And then you have to keep practicing.

73

(R)   How successful do you believe you will be
on the final exam?

(S)   I don't think that successful 'cause I
have other courses too. The tension right now! It's not
so much the course itself but other exterior factors. I
have other courses. I have six provincials to write so
it does have an effect on, not only math, but also the
other courses I'm taking like Literature, Geology,
everything.   Everything will be affected by one another
so I don't think I'll do that well. You've gotta try.

(R)   Yeah, okay, I understand what you're
saying. If you could try to focus on the math and think
about what you've seen. You've seen a lot of old exams
and you have an idea of what might be there and you have
an idea of what you've learned and how well you've done
through the year. Could you, just sort of get some idea
of whether you think you would continue to do as well on
the final, or do worse on the final, or better on the
final than you have through the year?

(S)   Maybe not as well as through the year.
Because it's like a one-shot deal.   I mean you do well,
you do well. You don't do well, you don't do well. I
mean you could have a bad day and you don't do well,
right?. So, I don't expect to do really well.

(R)   What mathematics course(s) might you be
taking at college or university?

(S)   I don't know what's out there.

(R)   Well, in terms of ... Are you going to be
doing math at all? Are you going to math as part of
science? Or, you going to do math because you will see
yourself just doing more math?

(S)   Maybe a little a bit of math. I'm not
going to concentrate on math. I'm going to concentrate
probably in English because that's where my principal
interest is.

(R)   That's what you'd like to teach one day,
is that right?

(S)   Yeah, right.

(R)   Do you have any further comments?

(S)   Well maybe for the final exams you could give out more exams. In my opinion, like, for Mrs. XXX, right?, we have English Lit and she gives us a lot, A LOT, A LOT of previous exams. And even if we don't know the material she wants us to do it and you get marks for it, but it's not so much the marks, but the fact that it makes you, and forces you to actually look up the things and deal with those questions. And I guess, as a result, you do get really acquainted with these kind of exams, whereas, I think, other courses, especially, oh, I don't know, other courses they aren't given as much. It's like just now and then, a little bit. If you gave it out periodically, maybe? Not so much, ... well it's good review because ... obviously the first exam you write, like for example, you give it two months after September or something, and the students wouldn't know as much, obviously, 'cause they've only covered a certain amount of topics, but that's not the important thing. They'd know what to expect. I guess preparation is the key to success. I mean, if you're prepared for it, I don't think there's any problem.

This student was one of the best students in the class and attained an A letter grade throughout the year. She showed interest during classes and was adept at problem solving although, like many, she was not a vocal student. She took care with her solutions and with her project work. In this interview she displayed a pragmatism typical of many students although she seemed more capable of recognizing aspects of the system that might influence her success.

Her comments about what she originally expected from the class reflected the traditional rule-example-practice approach that exists in so many mathematics classrooms. This expectation was repeated in some similar form by other students interviewed. I had recognized that this

75

approach was common and it was one of the motivating factors in my desire for change.

The subsequent comments about the perceived differences in the class were satisfying to hear. Other students also commented that the teaching approach used in the class was a positive aspect of the year. The cooperative sessions were generally popular and were deemed to be successful for the most part. The interchange of ideas, the mutual reinforcement of skills, and the informal atmosphere of these activities were appreciated. One student (see Appendix E, Interview 7) commented "The best one [activity] is getting into groups and learning a real hard section. That's the most fun I've had. Each person working hard and the rest combining it together."

The comments made in this interview regarding my visual approach to teaching mathematics were most satisfying. They served to reinforce my own reflections in the journal about using a visual approach to teach mathematical concepts. Other students commented on my style of teaching. One student noted that I tended to make notes and create examples "from my head" rather than just using the textbook. This comment suggested to me that slavish adherence to the examples and style of a textbook is one troubling aspect of the traditional model

of teaching mathematics. I had not set any particular objective in this study to consciously deviate from the text but this comment made me realize that I tended to do this a lot of the time.

The activities most commonly recalled by all the interviewees were those that this student remembered best. Those were the cooperative sessions, usually exam preparation sessions, and the projects. The satisfaction that students received from working together was remarkable. Aside from direct comments in class and in the interviews, it was clear from observations in class that this was a popular aspect of the study.

I was surprised and disappointed to note that no student interviewed recalled the computer laboratory activities on sequences and series [other than one student who I finally asked outright about this activity (see Appendix E, Interview 3)]. At the time I was enthusiastic about how well this activity had gone. I felt that student response was good and I received generally positive responses on the One-Minute Papers collected at the time. I was concerned about this as it suggested that I was misinterpreting the situation. I went back to the One-Minute Papers and found that the comments reflected positively on the exercise in terms of visualizing how sequences and series behaved. However,

77

there was an undercurrent of confusion in many comments in terms of being able to do the exercises in the text, that is, manipulating the formulae. Some comments specifically asked for notes on the topic and examples of questions.

In general, student responses supported my observations in the journal about student attitude and success. The indications were that students enjoyed the variety of activities presented and were willing and active participants. Their own indications of success were coloured by perceptions of their abilities but students did not indicate that the strategies utilized adversely affected their achievement to any marked degree The data collected from student informants by way of the comments, the One-Minute Papers and the interviews proved valuable in providing me with information about how well or how poorly I had attained my original objectives.

Student Achievement

Classroom tests

A more objective measure of student success was to be found in records of achievement on in-class tests. There were nine major tests through the year, six of which were done on the combined (individual/cooperative) basis described in Chapter Three and three others that

were done by the students as individuals. The tests are included in Appendix C.

The tests are listed and coded below. The Class% represents the percentage mark that I sent on to the Ministry of Education to form 60% of the student's final mark. This final class mark was generated from test scores, project marks, performance on the Annual Problem Set activities, and homework. The Prov% is the students percentage score on the final examination and represents the remaining 40% of their final mark. Table 4.1 presents the percentage mean, median and standard deviation for each test. Table 4.2 displays the range of percentage scores and the lower and upper quartiles. Note that the access to bonus marks on some tests resulted in scores over 100% (e.g. T2).

```
T1:     Functions (combined)
T2:     Exponents and Logarithms (combined)
T3:     Analytic Geometry (combined)
T4:     Systems of Quadratic Equations (combined)
T5:     Graphs of Trigonometric Functions (individual)
T6:     Trigonometric Identities (combined)
T7:     Sequences and Series (combined)
T8:     Calculus (individual)
T9:     Polynomials (individual)
Class%:    class mark (60% of final mark)
Prov%:     final exam mark (40% of final mark)
```

Table 4.1   Test statistics (%).

Test	Mean	Median	St. Dev.
T1	74.7	78.2	12.2
T2	75.0	79.7	18.8
T3	62.1	57.6	10.7
T4	72.6	69.6	13.7
T5	57.9	62.5	26.2
T6	57.7	53.3	18.0
T7	64.3	66.3	22.8
T8	63.0	62.1	20.4
T9	76.5	75.0	17.5
Class%	70.6	70.0	14.0
Prov%	55.8	50.0	18.0

Table 4.2   Range and Upper and Lower Quartiles (%)

Test	Min.	Max.	Q1	Q2
T1	52.1	96.9	64.9	82.7
T2	39.8	107.0	59.4	89.8
T3	50.0	85.9	54.7	67.5
T4	52.2	95.7	62.0	83.2
T5	12.5	104.2	32.3	69.8
T6	22.2	93.3	45.0	71.1
T7	14.0	107.0	48.3	77.3
T8	31.8	97.0	46.2	73.5
T9	47.9	100.0	62.5	97.9
Class%	45.0	99.0	60.0	80.5
Prov%	35.0	99.0	44.0	69.5

It would appear from the first few test results that the lower ability students benefitted from the cooperative strategy. The means were quite high and the range indicated that most students achieved a mark of better that 50%. On the second test, where the minimum was 39.8%, it was clear from the lower quartile mark (59.4%), the mean (75.0%), and the standard deviation (18.8%) that any low scores were anomalous. Once again it appears that lower end scores may have been inflated.

Such behaviour continued through the third and fourth tests.

On the fifth test, the first one done on an individual basis, the mean dropped significantly, the standard deviation was high (26.2%) and the minimum score was very low (12.5%). The lower quartile reading (32.3%) indicated a good number of low scores on this test. The upper quartile also dropped, possibly indicating a greater degree of difficulty on this test, although the top student obtained more than 100%. The wide range of scores and generally low performance was a remarkable change from performance to that date.

Tests scores made some recovery over the next four tests, the first two of which were done cooperatively, although the minimum scores and the lower quartile values indicate that performance at the lower end of the achievement scale had dropped off. The last test presents another anomaly as the marks rebound to the levels of the first tests despite this being an individual effort. This test was on the topic of polynomial functions, a topic that I have found to be well done by most students. It should also be noted that this test took place just before the start of the final examination schedule and students may have been studying more seriously than usual.

It is extremely difficult to draw strong conclusions from such a limited set of data. Many factors would have influenced student performance on these tests, including the novelty of writing a test cooperatively. However, at least initially, my own feeling about the process was that it did improve the scores of lower achieving students. The top students paid a small cost in terms of a slight drop in their percentage. The process was designed to allow students at the lower end of achievement to make gains, so I was satisfied to note that the strategy was successful. I was still concerned that students were being recognized fairly for their ability and performance. Small deviations in test scores due to the cooperative strategy were less important to me than ensuring that any overall ranking that might exist naturally in the class should remain intact. I wanted lower achievers to feel successful but I did not want brighter students to feel that they were working to their own detriment.

Shown below in Table 4.3 is the correlation matrix for the nine tests (T1 - T9), the class mark (Cl), and the final examination score (Pr).

82

Table 4.3 Correlation matrix for tests,
class mark, and provincial examination score.

	T1	T2	T3	T4	T5	T6	T7	T8	T9	Cl
T2	0.65									
T3	0.58	0.51								
T4	0.70	0.85	0.68							
T5	0.76	0.71	0.67	0.73						
T6	0.47	0.70	0.72	0.69	0.56					
T7	0.63	0.70	0.68	0.79	0.81	0.70				
T8	0.56	0.78	0.70	0.72	0.77	0.70	0.73			
T9	0.42	0.59	0.37	0.52	0.61	0.36	0.46	0.54		
Cl	0.77	0.86	0.75	0.89	0.90	0.77	0.88	0.89	0.61	
Pr	0.60	0.76	0.80	0.81	0.79	0.88	0.78	0.81	0.51	0.89

Using standard regression analysis it was determined
that correlations of at least 0.60 were statistically
valid ($p < 0.01$). The high correlation between the class
mark and the score on the provincial examination (0.89)
is noteworthy. This indicated that any relative ranking
among students was maintained through the final
examination notwithstanding any inflation of marks. In
fact, most of the correlations were good, also indicating
that this relative ranking was maintained across a
variety of tests and across cooperative and individual
testing methods. In particular, despite the evident drop
in performance, the correlation between T5 (the first
individual test) and previous cooperative tests was
significant.

The final test on polynomials, T9, again appears to
be an anomaly. The correlations between this test and
others were not significant other than for the test on

83

graphing trigonometric functions (T5) and for the final class mark.

Examination preparation and results

A key consideration in this study was whether I could adequately prepare students for the provincial final examination through regular examination preparation sessions in class. These sessions would ostensibly allow for students to learn the basic skills of each topic cooperatively and open up class time to be used in the pursuit of more conceptual understanding. My intent was to use one class approximately every five lessons for this purpose. In the final analysis of the journal I found that examination preparation or work on the Annual Problem Set that I used as a form of examination preparation constituted 20% of class time. It is arguable whether I had achieved my objective of one class in five since the recommended time to be spent on the Annual Problem Set alone was 10%. In addition the work done on the problem set was of a different type than that done in preparation for the examination itself. The examination preparation work entailed practice on old questions, recognition of common errors, multiple-choice strategies, and presentation of solutions. The work on the Annual Problem Set involved patterns, exploration, case analysis, and counting skills.

84

Student interest in group examination preparation was high at the start as evidenced in the year-end interviews. The original plan was to have students work together in groups to learn the fundamentals of a particular topic, each group having been assigned a different topic. I had prepared topic study guides for each topic so that the group would have some direction for their work (see Appendix D). Once the groups had acquired a degree of understanding of their topics the groups were to be rearranged so that information and understanding could be exchanged.

My early perception of this strategy was that it was not working as planned. Students seemed lost without specific guidance and some groups had topics that were so foreign to them (e.g. calculus, logarithms, polynomial functions) that they had trouble understanding the basic concepts. I would circulate, trying to help groups get started but I soon felt that I was in fact giving mini-lessons to the groups. I felt that if this continued I would find that a lot of time was wasted instead of saved. For these reasons I altered the plan to one where the whole class, still in groups, would treat one topic at a time or practice questions from old exams together. In this way I could watch for trouble spots and then address them briefly in front of the whole class. I was

85

more satisfied with this strategy. However, according to evidence given in the year-end interviews, some students enjoyed the original approach and were in fact disappointed that I did not continue with it. I may have acted too hastily in modifying the strategy. The non-traditional nature of this approach was exactly what I was attempting to explore and it seemed that I did not give it a proper chance to succeed. It struck me later that this fear of a new approach failing or failing to succeed quickly enough presents a great hurdle to teachers attempting to effect radical change to the classroom.

Results on the final examination were discouraging. Almost every student obtained a lower, and in some cases markedly lower, percentage than they received as their final class mark. The correlation between the drop in mark and the class mark was 0.23, a statistically unreliable value. Nonetheless it can be seen from Table 4.4 that the drop in percentage from the class mark to the exam mark appeared to be more pronounced as the class mark decreased. This appeared to reinforce the suggestion that class marks for weaker students were inflated, possibly due to the cooperative component factored into their test scores.

Table 4.4: Comparison of School and Examination
marks for the subject class.

Class mark	Exam Mark	Difference
99	99	0
93	88	-5
90	79	-11
84	74	-10
83	65	-18
78	76	-2
76	53	-23
74	56	-18
72	45	-27
70	62	-8
70	50	-20
70	45	-25
66	50	-16
65	43	-22
65	42	-23
60	45	-15
60	35	-25
58	37	-21
54	45	-9
51	37	-14
45	45	0

Without a proper control group to compare results
with it is difficult to isolate the strategies used in
this study as reasons for the lower performance on the
examination. Many factors, including study habits and
self-confidence might have influenced the results.
However, the discrepancies between class marks and exam
marks for this class and other Mathematics 12 classes in
the school can be compared. Figure 4.1 illustrates data
provided each year by the Ministry of Education regarding
distribution of letter grades. This information dates
back to the introduction of the Mathematics 12 course.
It appears that the discrepancy between the average class

87

mark and the average exam mark was greater for the subject class than for the other Mathematics 12 classes in the school although insufficient data are available to conduct a statistical test of the differences.

The pattern of difference between the school mark and the exam mark over the three years since the introduction of the Mathematics 12 course suggests that the subject class results differ from the examination mark more than usual. Once again it is difficult to attribute any difference directly to this study, although regression analysis indicates that the difference in the means of the class and exam scores in the subject class is significant ($p < 0.01$).

While most students achieved a passing mark in the course it must be noted that many in the class achieved poorly on the examination. The examination preparation sessions did not appear to have any positive effect on final examination scores.

Figure 4.1 - Mark Comparison 1990-92

(1992 *Subject Class **Others)
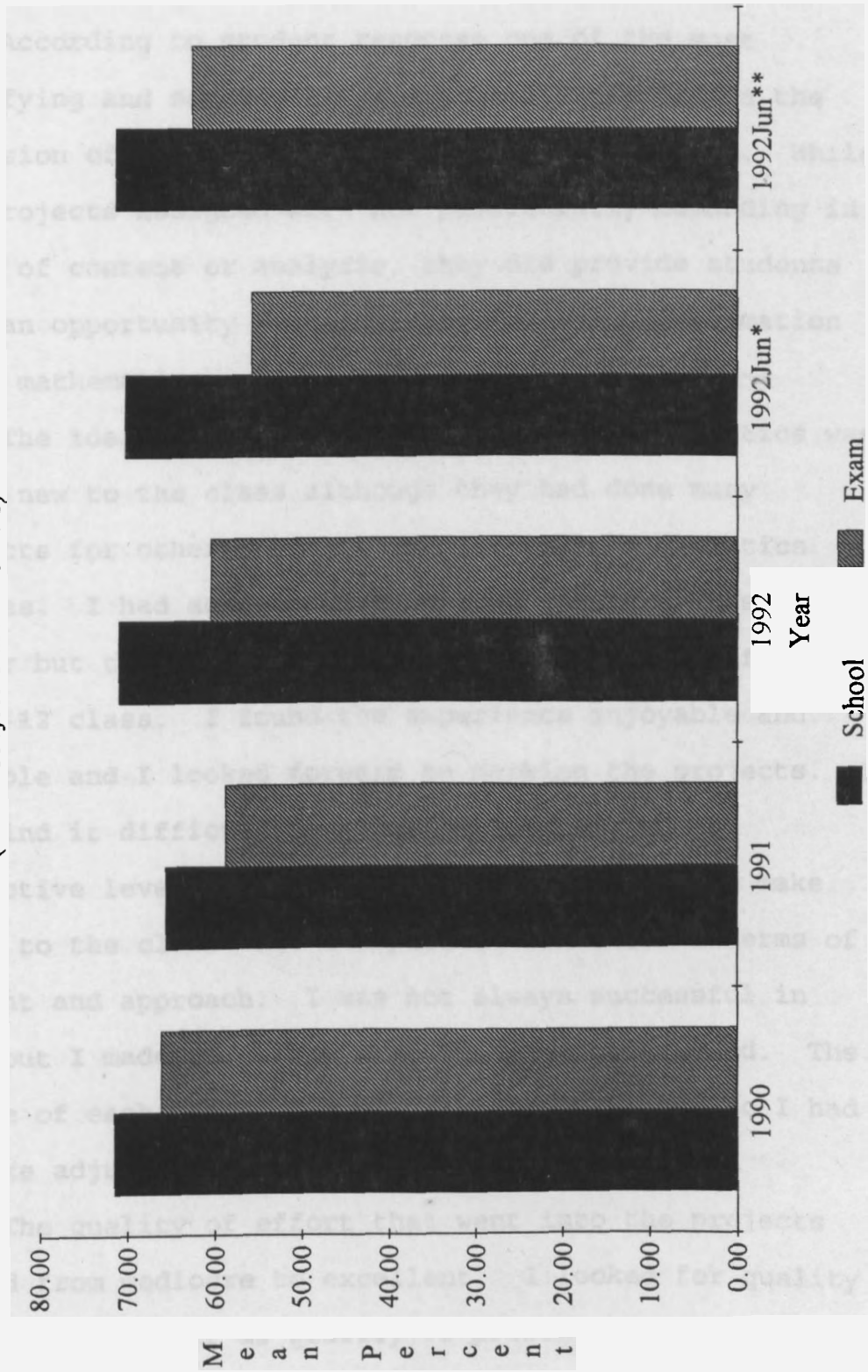
Project work

According to student response one of the most satisfying and successful facets of this study was the inclusion of a number of projects into the course. While the projects assigned were not particularly demanding in terms of content or analysis, they did provide students with an opportunity to organize and present information about mathematics.

The idea of doing projects in senior mathematics was quite new to the class although they had done many projects for other subjects and in junior mathematics courses. I had assigned projects to junior grades in my career but this was the first time that I did so for a grade 12 class. I found the experience enjoyable and valuable and I looked forward to marking the projects. I did find it difficult to adjust my marking to a subjective level for these assignments. I had to make clear to the class what I expected from them in terms of content and approach. I was not always successful in this but I made adjustments as the year progressed. The nature of each subsequent project was different so I had to make adjustment to this as well.

The quality of effort that went into the projects varied from mediocre to excellent. I looked for quality of content as well as quality of presentation. I had

90

invited students to be as creative as possible, perhaps presenting their project in the form of a video or a class lesson. No student took me up on these offers but a number of posters were created. Generally students prepared a written report.

The most interesting project assigned was the one where students were to investigate a topic from mathematics that was outside the normal Mathematics 12 curriculum. Comments received later in class indicated that students enjoyed this project because it allowed them to learn things for themselves. Given the potential depth of some of the topics I had to guide students as to how far to go with a given topic and I offered my own mathematics library for the use of the class. Other than last-minute efforts that left much to be desired, the projects were well done. The average mark on the first two projects were approximately 76% and 97% respectively. These projects were little more than expanded homework assignments and I realized that I had made them far too easy to complete. The second term project was the exploration of a topic outside the curriculum. The average mark on this one was approximately 69%. The final project was the creation of a sample examination and the average mark here was approximately 68%. The

increased complexity of the projects and the experience I had gained allowed me to make adjustments in my marking.

These last two projects were well received by the class for different reasons. As described above the second term project gave students a chance to present what they had learned on their own. The sample examination challenged students to present a relatively succinct summary of questions that pertained directly to the subject curriculum. They also had to think through potential errors to create reasonable distractors for the multiple-choice section. The project also provided a valuable mechanism for review prior to the actual examination.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

This study represented an attempt to modify the
traditional methods of teaching the Mathematics 12
curriculum. The intent was to include a variety of
activities that would be innovative and effective in
promoting understanding about mathematics at this level.
It was further intended to assess students in ways
different from those traditionally used. Lastly, in
order to address student anxiety about the final
provincial examination and to help provide the extra time
necessary for the innovative activities, the study
provided for regular examination preparation sessions
that would initiate students to the style and content of
the examination and provide a vehicle for teaching basic
skills for various topics. The first part of this
chapter outlines some conclusions as to the methods used
and the effectiveness of the various activities and
strategies. The second part of the chapter discusses
limitations, makes recommendations for improvement and
further research, and summarizes the personal impact of
the study.

Conclusions

Classroom Environment

The use of tables and arrangements promoted group communication and served to create an energetic and positive classroom environment. Students seemed to enjoy coming to this class and enjoyed the opportunity to exchange ideas when working together in small groups. These results support research in cooperative learning that indicate that classroom environment is positively enhanced by seating arrangements and by group learning strategies.

The seating arrangement in rows of desks characteristic of many mathematics classrooms reflects an adherence to a traditional model passed on from our own classroom experience in school. This arrangement serves supervisory needs but does not encourage student interaction. One need only look to the humanities or science departments in many schools to note there that tables have long been recognized as arrangements that provide for and encourage discussion. The kind of environment envisioned in the *Standards* (NCTM, 1991) requires that mathematics teachers establish arrangements that allow students to experiment together and discuss their understandings. Using tables and arranging students in groups for cooperative exercises is a

relatively simple adjustment, yet many classrooms retain the traditional desks in rows. This must change if we are to effect changes in the way that students learn mathematics.

Exploratory Investigations

Constructivists argue that students must have the opportunity to experiment with mathematical ideas in order to develop their own understandings of what is being studied. In practice, the depth of such investigations is limited by time, resources, and the commitment of the teacher to this type of pedagogy. In this study I introduced opportunities for students to explore topics through the use of graphing calculators, computers, and pencil and paper activities.

While student attitude towards these activities was positive, I felt frustrated by the lack of time to properly implement in-depth explorations. The pressure of the prescribed curriculum, despite the examination preparation sessions, left me concerned about the amount of work I had yet to cover. However, I felt strongly that the introduction of these explorations would lead ultimately to a better understanding of the topic at hand, even if the techniques related to the topic had to be dealt with separately and in a didactic manner.

Exploratory investigations are another way in which mathematics teachers can effect the changes for which there has been a call. The Annual Problem Set reflects a structured effort to provide opportunities for such exploration. These problems could be used very effectively to expand upon topics to whatever level a teacher or student chooses. A difficulty arises when the teacher tries to accomplish these activities effectively and still accommodate the provincial curriculum. To some extent the demands of the curriculum and the opportunities provided by the Annual Problem Set are working at cross purposes. The end result may very well be that recognized by Lerman in Ernest (1989); that even where innovative strategies have been required under curriculum reform, projects and investigations were treated didactically.

Projects

According to year-end interviews and journal observations, this aspect of the study was well received by students. They appreciated an opportunity to present material in an interesting and different manner, in effect putting the personal touch on their learning.

One of the projects allowed them to work together if they chose and the communication of ideas necessary in

preparing the project generally appeared to be valuable. While the quality of work varied on all projects, it was evident that these tasks were undertaken in a constructive and positive spirit, with some students going to great efforts.

It was not possible to determine the effect of these projects on achievement in the course so no conclusion is made in this regard. However, the positive feedback received and my personal observations of the value of these assignments lead me to conclude that the inclusion of projects in the Grade 12 curriculum is a positive innovation.

As mentioned earlier, projects could be developed that exploited the Annual Problem Set. These projects would then reflect a component of the prescribed curriculum, in that the Annual Problem Set carries a ten percent weighting in the final mark for the course. It may be desirable to alter the focus of the Annual Problem Set and develop instead a collection of well thought out projects that teachers can use in their classes.

I found a great deal of interest generated by a project that required students to investigate and report on a topic quite foreign to their mathematical experience. Comments received when these projects were completed and in the year-end interviews indicated that

students found it interesting and stimulating to study something on their own. I would speculate that the ownership implied in any personal or small group project adds some measure of significance to the learning experience. If true, this should be exploited and the role of projects in the curriculum should be expanded.

I found it difficult to devise projects that allowed students to truly explore a topic and build their own understanding of it. More often the assignments provided an opportunity for students to reflect upon what they had already studied. Part of the problem was my own inexperience in creating such projects. I was confused as to what actually constituted a project or report or investigation or exploration or whatever I chose to call such a thing. In the end, one project became simply a glorified homework assignment that proved unsatisfactory to me in terms of what it offered to students and what they were able to offer me as a product.

It may be appropriate for mathematics educators, through professional bodies, publications, and conferences, to educate each other about how to design and assess a project so that learning can take place along with the reporting of knowledge. Further research in this area might provide some interesting information

as to what elements are required in order to devise an effective project.

Cooperative Tests

The inclusion of a cooperative testing strategy proved not to be as useful as expected. The cooperative component of my chapter tests was intended to promote mutual interdependence where brighter students shared their understanding with others in their group as part of an in-class test. The result was that brighter students simply did the questions on the first (i.e. the cooperative) part of the test as fast as possible so that the less bright students could memorize how to do the questions on the actual paper. This was important to the brighter students as the cooperative component formed a portion of their mark. The overall effect was to inflate the scores of the weaker students. Later alterations to the testing process lessened this impact but did not effect the shared accountability that I intended. In the end I returned to tests written by individual students, if only for comparison of achievement.

Despite the apparent inflation of lower-end scores, the relative ranking of students in the class was generally reflected in the test results and the

99

correlation between the class final mark and the provincial examination score was high.

Examination Preparation

The examination preparation sessions were successful in their original format and seemed to be well appreciated by students in that form, according to feedback received from students in year-end interviews. The opportunity to work together in small groups and the sharing of collective expertise was generally deemed to be a positive experience that promoted shared learning.

However, because of my own concerns about their practicality and effectiveness, I altered the format of these sessions to one where the whole class was working on a set of particular questions suggested by me. This format was not as well received and may have served to dampen enthusiasm for the examination preparation sessions themselves. Ultimately these sessions as implemented did not appear to have had any positive impact on the final examination results.

Limitations

The eclectic nature of this study affected the quality of observations since so many different kinds of

data were being collected. In any given lesson factors such as classroom arrangement, teaching strategy, group activity, use of technology, student feedback, student achievement, and teacher satisfaction were all to be given attention and recorded in some way or another. This made for superficial data collection at times and made difficult a clear analysis of how a particular objective of a lesson had succeeded or been received. A narrower focus would have allowed me to concentrate on a few specific innovations. For example, if I had chosen to include only group work and cooperative testing strategies I could have better determined the effectiveness of these teaching and assessment strategies over the year. In this study, however, these innovations got lost among all the other changes that I was attempting to make.

The length of the study presented difficulties in maintaining enthusiasm and rigour. The quality of effort and observation would vary from time to time depending on my own state of mind. I became frustrated when I felt that I was not doing enough or excited when some activity worked well. This would colour my view of how the study was progressing as a whole. In retrospect I should have persisted with efforts to video the activities so as to have an objective record to review later. I might also

have attempted the innovations on a smaller scale, such as with a particular topic or unit, so that the process might not have become so overwhelming.

The small sample size and make-up of the subject class limit the possibility of extrapolating the results of this study to other groups. The vagaries of time-tabling in a secondary school can easily create classes with differences in attitude, motivation, and ability that would influence the outcome of this study.

In this case, many of my students were not able to communicate effectively in the English language. Thus student feedback was less clear and useful as it might be in a class of students who were comfortable in their communication skills. In addition some of my students were new immigrants to Canada and were more familiar with systems of education where there is little or no communication with the teacher or with other students on understanding mathematics. This study would have presented tremendous hurdles for these students to overcome in order to effectively share their insights with me and with other students. The innovations may have proved difficult to adjust to for any student, much less for someone who would have had to first pass them through cultural and linguistic filters.

Recommendations

While some of the results were disappointing, I believe that further research, focussed on one or two of the objectives of this study could more clearly determine their effectiveness. Specifically, the value of project work should be explored further. Projects that integrate two or three topics from the Mathematics 12 curriculum could be devised. The Annual Problem Set might provide ideas each year for the development of a long-term assignment that could include concepts from pure mathematics as well as applications to the real world. A study on methods of implementing these projects cooperatively and assessing them fairly would be valuable.

Further refinement of the cooperative testing strategy may result in a better approach to this activity. I believe that the principle of mutual interdependence is sound although some methodology must be found to ensure that students are not led into believing that their performance relative to some standard is better than it actually is. Such a testing strategy takes more time and effort to prepare and implement. Teachers need to be convinced of the value of that effort.

Finally it is recommended that further study be undertaken into self-study of topics. Such study might be assisted by a topic guide such as I prepared or simply by using the teacher as a consultant for students who are having problems. No formal class lecture would be involved. This may help send a message to students about how important it is to think for oneself. Certainly their post-secondary success will be at least partially determined by this ability.

Personal Impact

I have not continued with some of these innovations in my classes since the year of the study. The cooperative testing strategy proved difficult to continue with considering all the other demands on my time. I have continued however ensuring that border-line students have a fair chance of passing my tests. I feel that it is their right to enroll in the course despite weaknesses that may prevent them from achieving any grade higher than a pass. In conversation with other Mathematics 12 teachers in other schools I understand that students who are not achieving at a suitable level (e.g. a C or C+ letter grade) are strongly counselled to withdraw from the course. I would prefer to include other more complex and involved questions on a test that would ensure that

104

only those who are able and prepared would achieve the higher letter grades.

I have left examination preparation primarily to the end of the course. My observation and some of the comments of the participants in this study is that most students begin to prepare for the final examination only as that examination comes closer to hand. This may be a sad affair but it is certainly not an unusual one. In fact late preparation provides for convenient review of all topics, whereas year-long preparation still requires such a review thus taking more time from the course.

I am also less concerned about differences between my class mark and the mark a student obtains on the provincial examination. I feel that students are responsible to me for 60% of their mark and as such should be concerned with achieving the objectives that I set. I certainly try to ensure that the intended learning outcomes are all addressed so that the stronger students will be prepared for all possible questions and will be able to achieve as well as they might. I am less concerned with differences that arise between lower-end scores and my class mark. I feel that I may have helped a student with a different learning style achieve the minimum mark needed for advancement into a post-secondary program that they wish to pursue.

The whole issue of class by class and school by school comparisons is a volatile one. Within many schools efforts are made to ensure some fair degree of comparative standing across classes through subjective adjustments to individual teacher marks or grade-wide testing. Similarly, the score attained by students on the provincial examination is moderated by political considerations about how many students should pass or fail or obtain an A letter grade. Under new freedom of information legislation this pressure to conform to the examination score and account for differences will increase. Teaching to the examination will become the thing to do to avoid nasty questions about why class scores are different from examination scores. This is not an encouraging development.

I have continued assigning projects to my classes and have worked to refine them. I am seeking to include notions of pure mathematics coupled with applications. I want to develop projects that build a mathematical model of a given process, although the level of mathematics required may be beyond the ability of the average student. I continue to count project work as a component of my class mark.

I have also continued to include explorations of various concepts in my lessons. I have sought to improve

106

an investigation of tangents in calculus and have continued to use graphics calculators to assist in the development of graphic concepts and techniques. I also include a investigation of eccentricity of conic sections that is based on a presentation I made in one of my graduate classes. I refined the computer laboratory activities on series and sequences but due to increased use of the laboratory I have been unable to schedule adequate time for these activities.

In hindsight, I had felt anxious about the extra time that the innovations seemed to be taking. This anxiety interfered with the potential success of these activities. I found myself slipping back into the rule-example-practice mode from time to time. No doubt many attempts at innovation in mathematics education are hindered by the force of in-grained traditional strategies. I firmly believe that the courage to continue in the face of various pressures is an important element of research aimed at effecting changes to traditional methods. I sincerely hope that teachers will continue to persist in making changes despite these pressures.

Appendix A

Letter of permission

As partial fulfillment of a Master's degree in Mathematics Education at Simon Fraser University I will be undertaking a research study involving my Mathematics 12 class during the 1991-92 school year. The goal of the study is to investigate how non-traditional strategies can be used in the teaching and assessment of Math 12. The intent is to involve students in their own learning and allow them to delve deeper into specific content areas. As part of the study students will also prepare rigourously for the provincial examination through regular preparation sessions in class. Students will also be taught specific test-taking skills as part of this preparation.

The teaching and assessment methods are well-accepted alternative ones. They will include projects, presentations, reports, experiments, and enquiries. Work will be undertaken individually and in cooperative groups. I will be assisting students wherever necessary and hope to help them develop a more creative approach to mathematics and problem-solving. As part of the study a record will be kept of the activities of the class throughout the year. This will be done primarily through a journal that I will maintain. At times I may wish to record some activities on audiotape or videotape for the record or for observation later. No student will be identified in any record.

A student may withdraw from the study at any time. However, as the study requires the involvement of the whole class, any student withdrawing will be required to select another Mathematics 12 class and make any necessary timetable changes.

Please indicate below your consent to pearticipation in this study. Tear off the consent form and return it to me at the school by September 9th. Please call me at the school (255-9371) if you have any concerns or wish further information. Thank you for your cooperation.

Date: _____

I hereby consent to participate in the study described for Mr. Boissy's Mathematics 12 class.

_____   _____
Student Name   Signature

I hereby give my consent for my child to participate in the study described for Mr. Boissy's Mathematics 12 class.

_____   _____
Parent/Guardian Name   Signature

Appendix B

Projects

Mathematics 12

Exponential & Logarithmic Functions

Project   Due Wednesday October 16.   Value = 15 marks

1. Choose <u>one</u> of the following topics from references in your text. (pp 262-263, 292-293, 309-319).   You may wish to research other references in the library or elsewhere.

Compound Interest   Population Growth   Light Penetration
Acid Rain   Earthquakes   Nuclear Fallout

(2) 2. Creat a table of values for an exponential formula that illustrates your topic of study.   You may <u>not</u> use the same formula from the text but you may modify one found in the text.   Use an appropriate domain and range.

(4) 3. Carefully plot the graph of your equation from step 2.   Label your graph clearly.

(2) 4. <u>Use your graph</u> to estimate at least two ordered pairs that fit your formula.   Remember units!

(2) 5. Convert your formula into <u>logarithmic</u> form.

(2) 6. Using the formula from part 5, choose four values in its domain and compute the corresponding range values.

(3) 7. Discuss the <u>limitations</u> of your formulas. (i.e. Do they always work? What numbers don't work? What if the numbers were negative?, very large?, very small? etc.)

Mathematics 12

Conic Sections Project

Value: 25 marks   **Due: Thursday November 14**

   1.   Choose one type of conic section for your project.

      - Parabola   - Circle   - *Ellipse
         - *Hyperbola   - *Rectangular Hyperbola

      * - Bonus marks (3) will be given for submitting
          a successful project based on these conics.

(2) 2.   Describe your conic section as a locus of points
          that satisfy some condition(s). Include a sketch
          to illustrate the condition(s).
          BONUS: (2 marks)   Give a second description of
                        your conic.

(2) 3.   Give the equation of your conic in Standard Form
          with centre (or vertex) at (0,0) and again for the
          centre (or vertex) at (-3,7).

(2) 4.   Give the equation for your conic with centre at
          (-3,7) in General Form.

(4) 5.   Explain the significance of the coefficient of the
          highest powered x term in the equation from 4.
          That is, what effect does it have on the conic if
          this number were changed to a larger value?, a
          smaller one?, a negative one?

(10) 6.   Draw a neat and accurate graph of your shifted
          conic (i.e. with centre (-3,7)) on the squared
          graph paper supplied. Include any asymptotes.
          Plot six points of the conic section. If you
          use more than six points you will be penalized.

(5) 7.   Use algebra (showing all steps) to show how you
          would convert your General Form equation (with
          centre (-3,7)) back into Standard Form.

      Write your report up and staple the pages together
with a title page. You may wish to use the old Algebra
12 textbook (by Dolciani) for research. Some copies
are available in the classroom. Please sign them out
from me and return them as soon as you can so others
may use them.

Mathematics 12

Term 2 Project

** This project must be done on an individual basis **

Select one of the topics listed on the other side of this page and inform me of your choice. You may choose another topic of your own as long as I have discussed the choice with you in advance. Do your own research on the topic and prepare a presentation of the information you have found.

Your presentation may be in any form (or combination of forms) that you choose provided that it presents your topic in a substantive, cohesive, and understandable manner. Examples are; a written report, a series of posters, physical models, a chapter from an imaginary textbook, a computer or calculator presentation, an oral presentation, a lesson taught to the class, a video production, or any other method approved by me in advance. You may choose to present your topic in terms of its history or its applications. Your presentation need not be completely mathematical but must include some mathematics. Use your imagination.

The only assistance I will provide is to help with research resources. I have many books that you may borrow. The library is also a good source, especially the downtown branch. You may wish to use the university libraries but you will have to make copies of any useful material unless you have a friend or family member who can take books out for you.

The essence of this project is independent research. This skill is an important one to learn for your future at university or college. Some of these topics can be very complex so choose carefully. Ask me about the topic if you are not sure what it involves.

Mark Breakdown

10 marks - Quality of presentation (i.e. quality of writing, artwork, organization of information, oral work, etc.)

15 marks - Accuracy and Clarity (i.e. is your information presented clearly and correctly.)

20 marks - Completeness (i.e. is enough information presented to give the reader/observer an understanding of your topic.)

5 marks - "WOW" Factor (i.e. how much effort and imagination went into your work.)

113

Topics

Powers of Binomials [i.e. $(a+b)^n$]
Networks
e (The Base of the Natural Logarithms)
Complex Numbers
Proof by Induction
Symmetry
Fibonacci Numbers
Divisibility Rules
Triangles and Concurrent Lines
Tessellations of the Plane
Inverse Geometry
Number Bases
Transfinite Numbers
Linear Programming
Equations and Graphs in 3-D
Pythagorean Mathematics
Modular Arithmetic
Sets
Fractals
Vectors in the Plane
Matrices
Prime Numbers
Combinations
Permutations
Fermat's Last Theorem
Symbolic Logic
Differential Calculus
Integral Calculus
Zeno's Paradoxes
Statistics
Negative Numbers
Irrational Numbers
Probability
Spherical Trigonometry
The Normal (Bell) Curve
Focus and Directrix
Roots of Unity [i.e. nth roots of 1]
Non-Euclidean Geometry
Projective Geometry
Transformation Matrices

Mathematics 12

Term 3 Project

Due: Wednesday June 3   Marks: 40

The Mathematics 12 curriculum has eight major topic areas;

TRIGONOMETRY
QUADRATIC RELATIONS
EXPONENTIAL AND LOGARITHMIC FUNCTIONS
POLYNOMIALS
SEQUENCES AND SERIES
INTRODUCTION TO CALCULUS
GEOMETRY (PROOFS)
PROBLEM SOLVING

Your project is to create a final exam with questions based on these topics. For each of the first six topics you will create _five_ multiple-choice questions, each one having four possible answers (A,B,C,D) where only one is correct. The distractors should be designed so that they are not obviously wrong. Try to foresee errors that a student might make and then design your distractors on these erroneous results. These 30 questions will constitute Part 1 of the exam and will be worth 30 marks.

Part 2 will be made up of two problems; one a geometric proof and one of your choice. The problems must be of appropriate difficulty and they will be worth 5 marks each.

Marks will be awarded on accuracy and quality of distractors and problems. You must also submit a key for your exam. The multiple-choice key need not contain any working but the problems must have a full solution key with part marks indicated.

You may work individually or in groups of three but not in pairs. If you choose to work as a group you must notify Mr. Boissy before you start. In such cases all three members of the group will receive the mark awarded to the project.

You may use old exams and your text as guides but you must create your own questions. Ask Mr. Boissy for any further information.

NOTE: If you want to have fun, make your exam for a fictional country or province. You could even create a cover with special instructions etc.

Appendix C

Classroom Tests

Mathematics 12

Functions Test
(Group Study Version)

Show work wherever questions require it. Part marks are available for partial solutions.   The value of each question is shown in parentheses.

(1) 1.   Give the Domain and Range for the relation shown below.
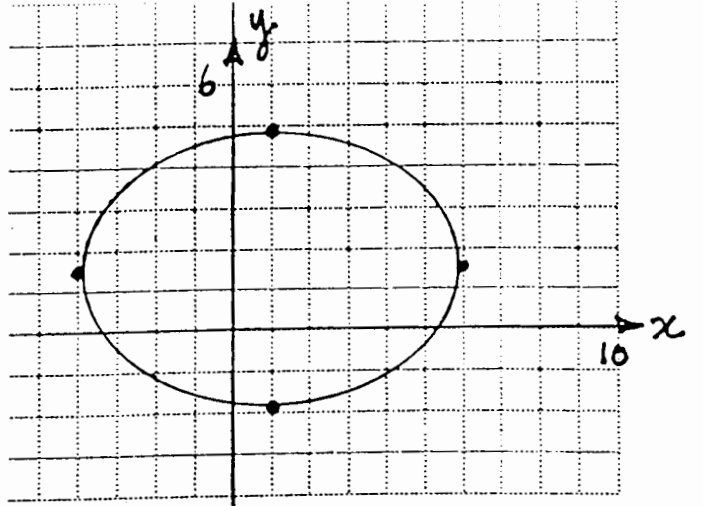
Domain:_____   Range:_____

2.   Give the Domain (D) and the Range (R) for each function.

(2) a) $f(x) = 1/x$ a) D=   R=
(2) b) $g(x) = \sqrt{x+5}$ b) D=   R=
(2) c) $h(x) = |x-3|$ c) D=   R=

3.   Find the indicated value of the function.

(1) a) $f(x) = 2x^3 - 3x + 4$ $f(-2) =$ _____

(1) b) $k(t) = 5(2^{-t})$ $k(4) =$ _____

(1) c) $g(y) = 2 \cdot |y^2 - 8| + 2y$ $g(0.8) =$ _____

(2) 4.   If $g(x) = 2/(3+5x)$ then write $g(2/x)$ in simplified form.
                                         4._____

5.   Find the inverse function $f^{-1}(x)$ in each case.

(2) a) $f(x) = 7x+6$ a)_____

(2) b) $f(x) = 3\sqrt{2x-1}$ b)_____

Mathematics 12

Functions Test

Your Name:_____   Date:_____

Group Members:_____

Show work wherever questions require it. Part marks are available for partial solutions.   The value of each question is shown in parentheses.

(1) 1. Give the Domain and Range for the relation shown below.

Domain:_____   Range:_____

2. Give the Domain (D) and the Range (R) for each function.

(2) a) $f(x) = 1/x$ a) D=   R=

(2) b) $g(x) = \sqrt{x+5}$ b) D=   R=

(2) c) $h(x) = |x-3|$ c) D=   R=

3. Find the indicated value of the function.

(1) a) $f(x) = 2x^3 - 3x + 4$ $f(-2)=$ _____

(1) b) $k(t) = 5(2^{-t})$ $k(4) =$ _____

118

(1) c) $g(y) = 2 \cdot \left| y^2 - 8 \right| + 2y$ $g(0.8) =$ _____

(2) 4.   If $g(x) = 2/(3+5x)$ then write $g(2/x)$ in simplified form.

                                                        4._____

    5.   Find the inverse function $f^{-1}(x)$ in each case.

(2) a) $f(x) = 7x+6$ a)_____

(2) b) $f(x) = 3\sqrt{2x-1}$ b)_____

Functions Test 2

Name:_____   Date:_____

Show work where questions require it. Part marks are available for partial solutions.   The value of each question is shown in parentheses.

(2) 1.   Give an example (in equation form) of a function with Domain equal to x: x ∈ R, x ≥ 0 and Range equal to y: y ∈ R, y ≥ 5   .

1._____

(2) 2.   If $f(x) = -3/(x-1)$ find $f(-2/3)$ in simplified form.

2._____

(2) 3.   If $g(x) = (x+2)/(2-x)$ find $g(x/2)$ in simplified form.

3._____

(4) 4.   If $f(x) = x^2-4$ find the equation of its inverse. Is its inverse a function?   If so find $f^{-1}(5)$.   If not, how would you restrict the Domain of $f(x)$ so that $f^{-1}(x)$ is a function?

5. Solve each quadratic equation.

(2) a) $x^2 - 3 = 7$

a)_____

(2) . b) $3x^2 - x - 10 = 0$

b)_____

(1) c) $2x^2 - 5x + 4 = 0$

c)_____

121

Exponents and Logarithms Preview

1. Simplify;

(1) a) $(n^{-3/5})(n^{7/8})$ a)_____

(1) b) $(x^{1/3})/(x^{-2/5})$ b)_____

(1) c) $(a^{2x+3})^2/(a^{x-1})$ c)_____

(2) 2. On the axes shown below sketch the graph of $y = 5^x$ with a <u>dotted</u> line and the graph of $y = 2^x$ with a <u>solid</u> line.

(2) 3. On the axes below sketch the graph of $y = (2/3)^x$ with a <u>dotted</u> line and the graph of $y = (3/2)^x$ with a solid line.

(1) 4. Give the range of $y = 3^x + 7$

                                  4._____

(2) 5. If $f(x) = a^x$ passes through the point (-3, 0.125) what is the value of a?

6. Evaluate each logarithm;

(1) a) log 100000   a)_____

(1) b) $\log_3 81$ b)_____

(1) c) $\log_2(4)$ c)_____

7. Rewrite each expression as a single logarithm;

(1) a) 7log5   a)_____

(1) b) log2 + log3 - log4   b)_____

(1) c) 3logx - (1/2)logy   c)_____

8. Solve for x;

(1) a) $\log_3 x = -2$ a)_____

(1) b) $\log_x 5 = 0.5$ b)_____

(1) c) $\log_{1/3}(x) = -3$ c)_____

(2) 9. On the axes below sketch the graph of $y = \log x$ with a <u>dotted</u> line and the graph of $y = \log_3 x$ with a <u>solid</u> line.

Exponents and Logarithms   Date:_____ Blk:____
Paper One

Part One __/28 Part Two __/22

Show work where it is required. Part marks may be available for partial solutions. The value of each question is shown in parentheses.

.   1.   Simplify;

(1)   a) $(n^{-3/5})(n^{7/8})$ a)_____

(1)   b) $(x^{1/3})/(x^{-2/5})$ b)_____

(1)   c) $(a^{2x+3})^2/(a^{x-1})$ c)_____

(2)   2.   On the axes shown below sketch the graph of $y = 5^x$ with a <u>dotted</u> line and the graph of $y = 2^x$ with a <u>solid</u> line.
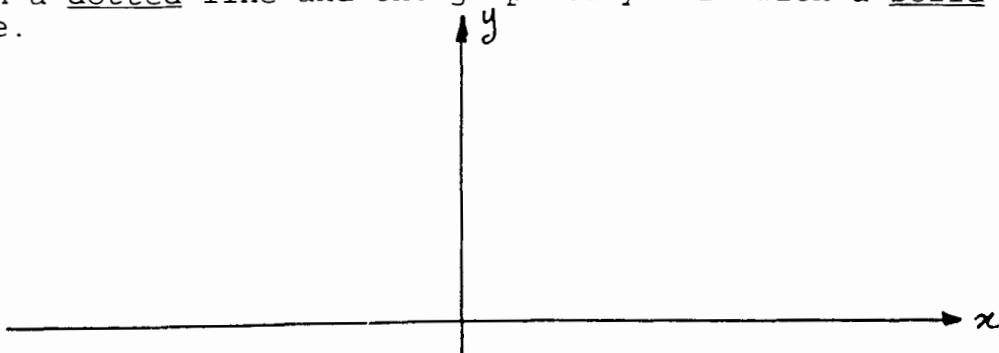
(2)   3.   On the axes below sketch the graph of $y = (2/3)^x$ with a <u>dotted</u> line and the graph of $y = (3/2)^x$ with a solid line.
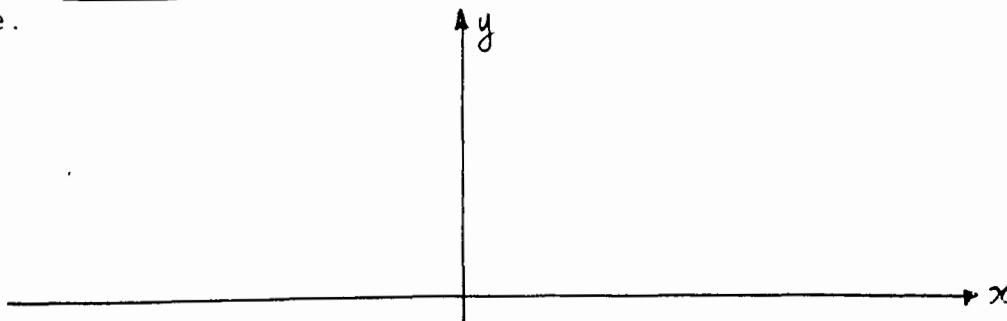
(1)   4.   Give the range of $y = 3^x + 7$ 4._____

(2)   5.   If $f(x) = a^x$ passes through the point $(-3, 0.125)$ what is the value of a?

5._____

6.   Evaluate each logarithm;

(1)   a)   log 100000   a)_____

(1)   b) $\log_3 81$ b)_____

(1)   c) $\log_2(4)$ c)_____

(1)   d) $\log_{1/2}(32)$ d)_____

(1)   e) $\log_3 /3$ e)_____

7.   Rewrite each expression as a single logarithm;

(1)   a)   7log5   a)_____

(1)   b)   log2 + log3 - log4   b)_____

(1)   c)   3logx - (1/2)logy   c)_____

(2)   d)   (1/3)[2logt + log3]   d)_____

8.   Solve for x;

(1)   a) $\log_3 x = -2$ a)_____

(1)   b) $\log_x 5 = 0.5$ b)_____

(1)   c) $\log_{1/3}(x) = -3$ c)_____

(3)   d)   log(x-10) + log(x+5) = 2   d)_____

(2)   9.   On the axes below sketch the graph of y = logx with a <u>dotted</u> line and the graph of y = $\log_3 x$ with a <u>solid</u> line.

Exponents and Logarithms
            Paper Two

        Show work where it is required. Part marks may be
available for partial solutions. The value of each question
is shown in parentheses.

        1.   Solve for x. Give any decimal answers to 3 s.f..

(2)   a) $3^x = 50$ a)_____

(2)   b) $x^{2/3} = 49$ b)_____

(2)   c) $5(7^x) = 12$ c)_____

        2.

(3)   a)   On the axes below sketch the graph of
            $y = \log_3(x-5) - 3$, showing the asymptotes clearly.
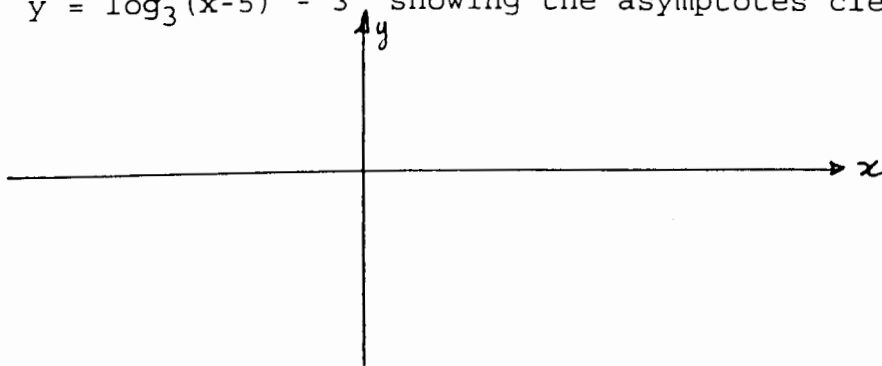
(2)   b)   Give the equations of the two asymptotes.

                                    b)_____

(1)   c)   Give the x-intercept.   c)_____

        3.   At 9:00 a.m. Mr. Boissy tells 3 students that he
              has won the lottery. These students spread the
              news so that for every 5 minutes that pass the
              number of students who have heard the news
              doubles.

(2)   a)   If 3 students know the information at time t = 0
           write down a formula for N(t) where N(t) is the
           number of students who know the information after
           t minutes. (HINT: Write out a few steps in the
           spread of the news and look for a pattern.)

.
                                        a)_____

(2)   b)   How many students know the news after 30 minutes
           have passed?

                                        b)_____

(3)   c)   What <u>time of day</u> is it when at least 1500 students
           know?

                                        c)_____

(1)   d)   Explain why this model of information spread may
           not reflect what happens in real life.

(2)   4.   Lake Muckoko has a pH reading of 4.3 while Lake
           Peeyoo has a pH reading of 2.7. A lower reading
           indicates greater acidity. How many times more
           acidic is Lake Peeyoo than Lake Muckoko? Answer
           to 3 s.f..

                                        4._____

Part 1 __/29   Part 2 __/14   Group __/10   Total __/53

Show work where it is required.   Part marks may be awarded for partial solutions.   The value of each question is shown in parentheses.

1.   Consider the points A(-3,7) and B(8,14).

(1) a)   Find the length of AB to 3 sig. figs.   a)_____
(1) b)   Find the midpoint M of the segment.   b)_____
(1) c)   If B is the midpoint of AC find the   c)_____
          coordinates of C.

2.   On the attached graph paper sketch the following graphs. Use separate axes for each graph.   Be sure to label the graph clearly and identify any special features such as centres, vertices, asymptotes, major and minor axes, and transverse and conjugate axes.   You should think first about where you wish to place the origin **before** you draw your graph.

2)   a) $(x-1)^2 + (y+3)^2 = 16$

(3) b) $(x-3)^2 + 4y^2 = 36$

(2) c) $xy = 20$

(4) d) $4(x+2)^2 + 25(y-6)^2 = 100$

(3) 3.   Find the equation of the parabola with   3._____
          vertex (2,-5) that also passes through (4,7)
(3) 4.   Write $x^2 + y^2 - 10x - 6y + 18 = 0$ in
          Standard Form. Show all algebra steps.

5.   The arch of a bridge is in the shape of a semi-ellipse. The base of the arch is 80m wide and the highest point of the arch is 30m above the base.

(3) a)   Find the equation of the ellipse that   a)_____
          forms the arch.

(3) b)   Find the height of the arch at a point 10m   b)_____
          from the centre of the arch.

(3) 6.   Write $-x^2 + 4y^2 - 4x - 32y + 56 = 0$ in   6._____
          Standard Form. Show all algebra steps.

Analytic Geometry Test Date: _____ Blk: _____
Part 1

Part 1 __/29 Part 2 __/14 Group __/10 Total __/53

Show work where it is required. Part marks may be awarded for partial solutions. The value of each question is shown in parentheses.

1. Consider the points A(-3,7) and B(8,14).

(1) a) Find the length of AB to 3 sig. figs.   a)_____

(1) b) Find the midpoint M of the segment.   b)_____

(1) c) If B is the midpoint of AC find the   c)_____
coordinates of C.

2. On the attached graph paper sketch the following graphs. Use separate axes for each graph. Be sure to label the graph clearly and identify any special features such as centres, vertices, asymptotes, major and minor axes, and transverse and conjugate axes. You should think first about where you wish to place the origin before you draw your graph.

(2) a) $(x-1)^2 + (y+3)^2 = 16$

(3) b) $(x-3)^2 + 4y^2 = 36$

(2) c) $xy = 20$

(4) d) $4(x+2)^2 + 25(y-6)^2 = 100$

(3) 3. Find the equation of the parabola with   3._____
vertex (2,-5) that also passes through (4,7)

(3) 4. Write $x^2 + y^2 - 10x - 6y + 18 = 0$ in
        Standard Form. Show all algebra steps.

     5. The arch of a bridge is in the shape of a semi-ellipse.
         The base of the arch is 80m wide and the highest point
         of the arch is 30m above the base.

(3) a) Find the equation of the ellipse that   a)_____
         forms the arch.

(3) b) Find the height of the arch at a point 10m   b)_____
         from the centre of the arch.

(3) 6. Write $-x^2 + 4y^2 - 4x - 32y + 56 = 0$ in   6._____
         Standard Form. Show all algebra steps.

Analytic Geometry Test
Part 2

Show work where it is required.   Part marks may be awarded for partial solutions.   The value of each question is shown in parentheses

(2) ·1.   Give the equations of the asymptotes for   1._____

$$6x^2 - 36y^2 = 216$$

(2)   2.   Sketch the graph of $y = \sqrt{16 - x^2}$ on the axes below.
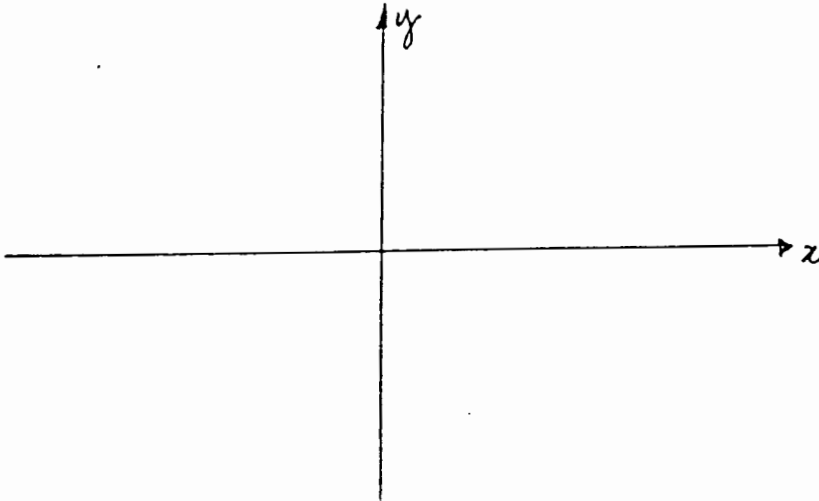
(4)   3.   A soccer ball was kicked down the field by the goalkeeper. Its path is a parabola. The maximum height attained by the ball was 30m and it landed 60m from where the goalkeeper had kicked it. Find the horizontal distance the ball was from the goalkeeper when it was 20m in the air.

3._____

(2) 4. Solve for x. $(x^2 - 9x + 19)^{(x^2 + 3x - 2)} = -1$

4._____

(4) 5. Consider the graph of $y = \log_3 x$ and the point $T(0,0)$.
R and S are points on the x-axis so that TR=4 and TS=30.
P and Q lie on the graph such that PR and QS are parallel
to the y-axis. Find the area of the quadrilateral RPQS.
Answer to 1 sig. fig.

5._____

Mathematics 12   Name:_____

    Systems of Quadratics   Date:_____ Blk:_

        Part 1   (Preview)

Totals: Part 1 __/27 Part 2 __/10 Total __/46

        Show work where it is required.   Part marks may be
awarded for partial solutions.   The value of each question
is shown in parentheses.   Leave radical answers in
simplified form.

        1.   How many solutions does each equation have? Do not
solve.

(2)   a) $x^2 + y^2 = 16$ a)_____
           $y = x - 4$

(2)   b) $x^2 - y^2 = 36$ b)_____
           $2x - y = 8$

(2)   c) $4x^2 + 9y^2 = 36$ c)_____
           $x = y^2 + 3$

        2.   Solve each system of equations.   Answer in ordered
pairs.

(3)   a) $y = 2x^2 - 4x + 5$ a)_____
           $3x - y = -2$

(4)   b) $r^2 + t^2 = 8$ b)_____
           $rt = 4$

(4)   c) $x^2 + 25y^2 = 100$ c)_____

           $x^2 - y^2 = 22$

(4)   3.   Graph the solution to this system of inequalities.

           $x^2 + y^2 > 36$

           $10x^2 + 64y^2 \le 640$

        4.   Graph the solutions of these inequalities on a
number line and write them as a set.

(3)   a) $|3x - 2| < 10$ a)_____

(3)   b) $|x - 7| \ge 8$ b)_____

Systems of Quadratics   Date:_____ Blk:_

Part 1

Totals: Part 1 __/27 Part 2 __/10 Total __/46

Show work where it is required. Part marks may be awarded for partial solutions. The value of each question is shown in parentheses. Leave radical answers in simplified form.

1.   How many solutions does each equation have? Do not solve.

(2) a) $x^2 + y^2 = 16$ a)_____
          $y = x - 4$

(2) b) $x^2 - y^2 = 36$ b)_____
          $2x - y = 8$

(2) c) $4x^2 + 9y^2 = 36$ c)_____
          $x = y^2 + 3$

2.   Solve each system of equations. Answer in ordered pairs.

(3) a) $y = 2x^2 - 4x + 5$ a)_____
          $3x - y = -2$

(4) b) $r^2 + t^2 = 8$ b) _____
           $rt = 4$

.

(4) c) $x^2 + 25y^2 = 100$ c) _____
           $x^2 - y^2 = 22$

(4) 3.   Graph the solution to this system of inequalities.
           $x^2 + y^2 > 36$
           $10x^2 + 64y^2 \leq 640$

     4.   Graph the solutions of these inequalities on a
number line and write them as a set.

(3) a) $|3x - 2| < 10$ a) _____

(3) b) $|x - 7| \geq 8$ b) _____

Mathematics 12   Name:_____

Systems of Quadratics

Part 2

(2) 1.   Write the following set as an inequality involving absolute value.

$$\{ x : -5 < x < 11 \}$$

.

1._____

(4) 2.   A rectangular piece of property has an area of $2400m^2$ and the length of its diagonal is $20\sqrt{13}$ m. Find the dimensions of the property. Include a diagram in your solution.

2._____

(4) 3.   A circle has equation $x^2 + y^2 = 16$. A parabola has equation $y = x^2 + k$ where $k \in R$. Find the values of k such that the parabola is tangent to the circle. (HINT: How many ways can this occur? How many values of y are there in each case?)

3._____

Name:_____

Trig Identities and Equations   Date:_____ Blk:_____

Preview

1.   Write each expression as a single trigonometric function:

(1)   a) $\sec\theta\sin\theta$

(1)   b) $\csc 2\beta - 1$

(1)   c) $1 - \sin 2\alpha$

(1)   d) $2\sin 48°\cos 48°$

(1)   e) $\sin 2\theta - \cos 2\theta$

(1)   f) $\cos 12°\cos 15° - \sin 12°\sin 15°$

2.   Proofs (3)

3.   If $\sin A = 3/5$, $A \in I$ and $\cos B = 4/5$, $B \in IV$, then find $\cos(A+B)$ as a reduced fraction.

4.   Solve for x, $0 \le x < 2\pi$, to 2 d.p.

$2\sin 2x - \sin x - 1 = 0$

136

Mathematics 12   Name:

Trig Identities and Equations   Date: Blk:

Part 1

Part 1 __/25 Part 2 __/10 Total __/45

Show work where necessary. Part marks may be awarded for partial solutions. The value of each question is indicated in parentheses.

1.   Write each expression as a single trigonometric function:

(1)   a) $\tan\theta\cos\theta$ a)_____

(1)   b) $\sec2\beta - 1$ b)_____

(1)   c) $1 - \sin^2\theta$ c)_____

(1)   d) $2\sin\pi\cos\pi$ d)_____

(1)   e) $\cos^223° - \sin^223°$ e)_____

(1)   f) $\sin42°\cos37° - \cos42°\sin37°$ f)_____

(1)   g) $\dfrac{\tan\left(\dfrac{\pi}{2}\right) + \tan\left(\dfrac{\pi}{3}\right)}{1 - \tan\left(\dfrac{\pi}{2}\right)\tan\left(\dfrac{\pi}{3}\right)}$ g)_____

137

(1)　h)　　$\sin(\pi/6)\cos(\pi/6)$　　　　　　　　　　　　h)_____

2.　　Prove;

(2)　a)　　$\sin(\theta - \pi/2) = -\cos\theta$

.

(2)　b)　　$\csc^2 A(1 - \sin^2 A) = \cot^2 A$

(3)　c)　　$\cot\beta + \tan\beta = 2\csc 2\beta$

(4)   3.   If $\sin\theta = -7/25$, $\theta \in III$ and $\cos\beta = 24/25$, $\beta \in IV$,
            then find $\sin(\theta - \beta)$ as a reduced fraction.

.

      4.   Solve for θ, $0 \le \theta < 2\pi$, to 2 d.p.

(2)   a) $5\cos\theta = 2$ a)_____

(4)   b) $3\cos2\theta - 5\sin\theta + 2 = 0$ b)_____

139

Part 2.

(10)   1.   Solve showing all work. Include a graphical representation of the solution.

A semi-elliptical shaped bridge, (20m wide and 12m high,supported 10 m above
an inlet bottom as shown in the diagram), spans a small passageway from an inner
harbour to the open sea. The depth of water (d, in metres) in this passageway may
be expressed as a periodic function of the time of day (t, in hours), according to
the equation;

$$d = 2.5\cos\frac{2\pi t}{12} + 9$$

A captain wishes to tow a barge 16m wide and 12m high under this bridge. One-
third of the barge is submerged. If high tide occurs at noon, between what times
in the next 12 hours can the captain take the barge to sea without doing structural
damage to either the barge or the bridge?

BARGE
16 m
12 m
WATER
d
12 m
10 m
20 m

140

Mathematics 12   Name:_____

Sequences and Series   Date:_____Blk:_

Part 1 __/30   Part 2 __/13   Total __/43
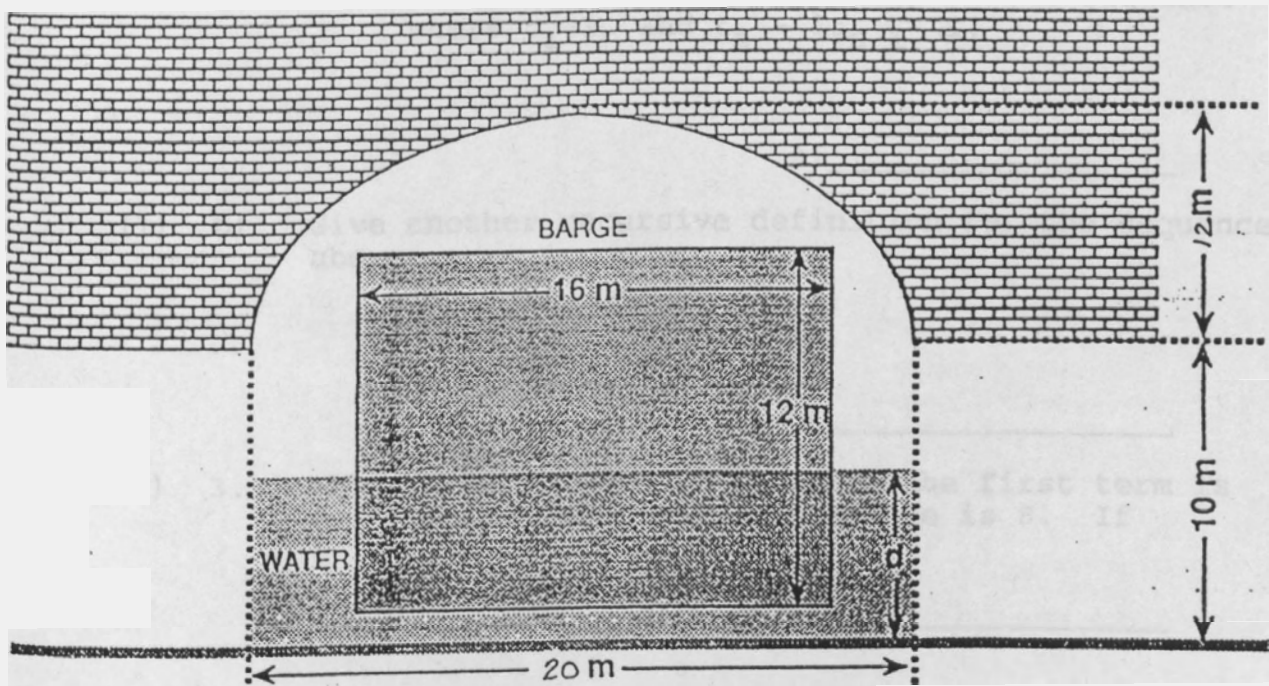
   Show work where required. Part marks may be awarded
for partial solutions. The value of each question is shown
in parentheses.

Part 1:

   1.   Find the next term in each sequence.

(1)   a)   18, -5, -28, _____

(1)   b)   0.3, -1.2, 4.8, _____

(1)   c)   4, 6, 10, _____

(1)   2. a)   Write out the first 5 terms of the sequence
               where $t_1=1$, and $t_n = 3t_{n-1}- 1$.

                        a)_____

(1)   b)   Give another recursive definition for the sequence
           above.

                        b)_____

(2)   3.   a)   In an arithmetic sequence the first term is
                   15 and the common difference is 8. If
                   $t_n = 143$, find n.

                        a)_____

(1)   b)   Find S_n for the sequence in part a).

                        b)_____

(2) 4. a) In an arithmetic series $S_{40} = -1860$. If the
            first term is 12 find the common difference.

                                    a)_____

(1) b) Find the sum of the first 14 terms of the series
    .   in a).

                                    b)_____

(2) 5. a) In a geometric sequence the first term is 6
            and an nth term is 12.4416. Find n if the
            common ratio is 1.2.

                                    a)_____

(1) b) Find the sum of the first 3 terms in a).

                                    b)_____

(3) 6. In an arithmetic sequence the fifth term is 40 and
         the ninth term is 76. Find the third term.

                                    6._____

(3)　7.　In a geometric sequence the fifth term is -648 and the third term is -72.　Find the first term.

7.＿＿＿＿＿＿＿＿＿＿＿

(2)　8.　a)　Place three arithmetic means between 7 and 28.

a)＿＿＿＿＿＿＿＿＿＿＿

(2)　b)　Place two geometric means between 18 and 1152.

b)＿＿＿＿＿＿＿＿＿＿＿

(2)　9.　Find the sum of 12 + 3 + 0.75 + 0.1875 + ...

9.＿＿＿＿＿＿＿＿＿＿＿

(2)　10.　a) Find $\sum_{k=1}^{6}(3k-4)$

a)＿＿＿＿＿＿＿＿＿＿＿

(2)　b)　Find $\sum_{n=1}^{5}(2)(-3)^{n}$

b)＿＿＿＿＿＿＿＿＿＿＿

Part 2:

   1.   Match the graphs with the descriptions; (1 mark each)

   a) an arithmetic sequence with $d<0$, $a>0$ a)_____

   b) a geometric sequence with $r>1$, $a>0$ b)_____

   c) an arithmetic series with $a>0$, $d>0$ c)_____

   d) an arithmetic series with $a>0$, $d<0$ d)_____

   e) a geometric series with $0<r<1$, $a>0$ e)_____

   f) a geometric sequence with $a<0$, $-1<r<0$ f)_____

1.

2.

3.

4.

5.

6.
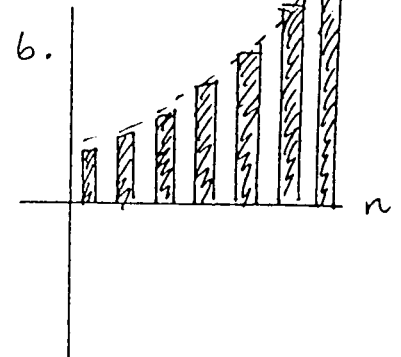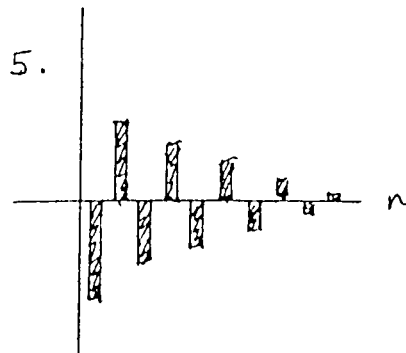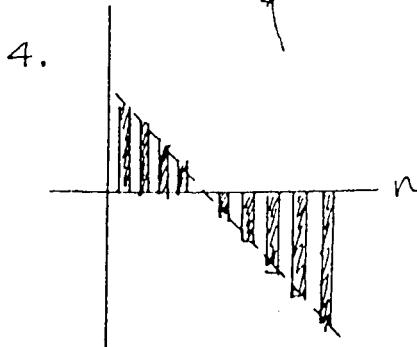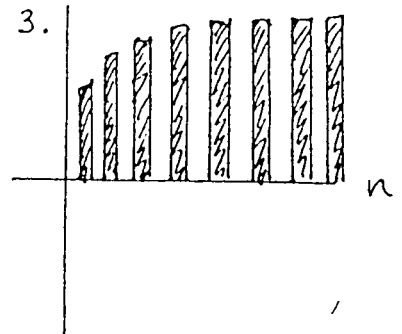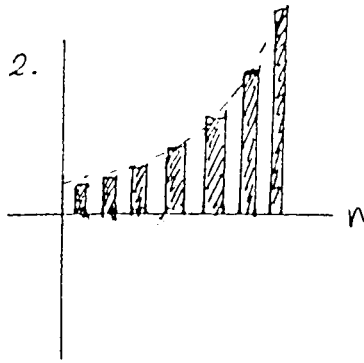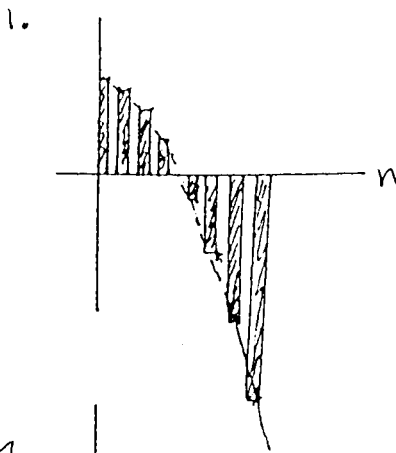
(3)   2.   In an arithmetic series $S_8 = 76$ and $S_{12} = 108$. Find the fifth term.

2._____

144

(4) 3. A ball is dropped from a height of 2m to the
         floor. On each bounce it reaches 60% of the
         height from which it fell. Calculate the total
         distance the ball travels before coming to rest.

                                3._____

BONUS QUESTIONS

Show work.

(3) 1. If 6, x, y, 27 is a sequence such that the first 3
         terms form an arithmetic sequence and the last 3
         terms form a geometric sequence, find x and y.

                                1._____

(2) 2. Find $\sum\limits_{k=1}^{12} (-2k+7) + \sum\limits_{n=1}^{\infty} 5(0.75)^{n-1}$

                                2._____

(1) 3. Find the next 2 terms of this sequence. (HINT: The
         rule is not mathematical!)

         1, 11, 21, 1211, 111221, _____

145

Mathematics 12   Name:_____

       Calculus   Date:_____Bkl:_

                                   Total: ___/33

       Show work where required.   Part marks may be awarded
for partial solutions.   The value of each question is shown
in parentheses.   Answer to the requested accuracy.

Part 1:

(2)   1.   In your own words describe what is meant by the
            slope of a tangent line to a curve at a point P.

(2)   2.   In your own words describe what it means for a
            sequence of numbers to have a limit L.

(1)   3. Find: $\lim\limits_{n\to\infty} \dfrac{3n^2-6n+4}{5n^2+7n-2}$ 3._____

(2)   4. Find: $\lim\limits_{x\to 2} \dfrac{x^3 - 8}{x - 2}$ 4._____

      5.   Find each derivative;

(1)   a) $f(x) = 5$ $f'(x) =$_____

146

(1) b) $f(x) = x^{17}$ $f'(x) =$ _____

(1) c) $y = -5x^3 - 7x^2 + 6x + 11$ $y' =$ _____

(1) d) $y = 3/x$ $dy/dx =$ _____

(2) e) $f(x) = \sqrt[3]{x} + \sqrt{x}$ $f'(x) =$ _____

(2) f) $y = (x-4)^2$ $y' =$ _____

6. An object moves along the x-axis so that its position at time t seconds is given by $x = t^3 - 5t^2 + 8t$.

(2) a) At what times is the object moving to the left? (HINT: What kind of number would the velocity be?)

                                        a) _____

(2) b) At what times is the object stopped?

                                        b) _____

(2) c) Find the velocity and acceleration at t = 4 sec.

                                c) _____

147

Part 2:

(1) 1. Write out <u>Newton's Quotient</u> for finding derivatives.

(2) 2. Use Newton's Quotient to differentiate $f(x) = x^2$. Show all steps.

(5) 3. Sketch the graph of $f(x) = x^3 - 4x$. Show all roots and coordinates of critical points.

(4) 4. The product of two positive numbers is 12. Find the smallest possible value for their sum.

4._____

Mathematics 12   Name: _____

Polynomials Functions Test   Date: _____ Blk: _____

                                Total: ___ / 24

Show work where it is required. Part marks may be awarded for partial solutions. The value of each question is shown in parentheses.

(2) 1.   If $P(x) = x^4 + 2x^3 + x^2 + 3$ find $P(-3)$.
           Show how you got your answer.

(1) 2.   What is the remainder when $5x^3 - 3x^2 + 5x - 4$
           is divided by $x-2$?

(2) 3.   If $x+3$ is one factor of $2x^3 + 3x^2 - 32x - 15$
           then factor this polynomial completely.

(1) 4.   List all the possible integral roots of
           $P(x) = x^3 - 7x^2 + 2x - 12$.

(3) 5.   Find all the zeroes of $x^3 + 4x^2 - 4x - 16$.
           Show all work.

(2) 6. List all the possible rational roots of
         $P(x) = 3x^3 + 4x^2 - 2x + 8$.

(3) 7. If $P(x) = 5x^3 - 2x^2 + mx - 4$ and $P(3) = 134$
         then find m.

(3) 8. Find all the roots of $3x^3 - 8x^2 - 9x - 2 = 0$

(3) 9. Solve for x if $x^3 + 2x^2 - 5x - 6 < 0$

(4) 10. On the accompanying graph paper sketch the graph
         of $P(x) = 2x^3 + 3x^2 - 17x + 12$. Show all the
         roots and enough points to determine the basic
         shape of the curve. Label the axes and indicate
         the scale.

Appendix D

1. Annual Problem Set

2. Examination Preparation Guides

1991-92 Annual Problem Set
Ministry of Education, Examinations Branch
Victoria, BC.

Problem 1:

Determine all values of x for which $(x^2 - 5x + 5)^{x^2 - 2x - 48} = 1$

Problem 2:

Pascal's Triangle

```
                    1
                1       1
            1       2       1
        1       3       3       1
    1       4       6       4       1
1       5      10      10       5       1
1   6      15      20      15       6       1
```

1. Write the next row of the triangle.

2. Find an arithmetic sequence within the triangle.

3. a) Find the sum of each of the first 6 rows.
    b) Give an expression for the sum of the nth row.
    c) Find the sum of all the terms in the first 20 rows.

4. Find the powers of 11 (i.e. 11^0, 11^1, 11^2, ...) in the triangle. Show how 11^5 can be determined from the triangle.

Problem 3:

If 3 of the 12 numbers around a clockface are randomly selected to form the vertices of a triangle, what is the probability that the triangle formed will be an obtuse triangle?

Problem 4:

The function $y = 2^x$ is graphed on a coordinate plane. Points R and S are on the y-axis. Point T has coordinates (0,0); RT = 20 and ST = 5. Points P and Q are on the function such that RQ and SP are parallel to the x-axis. Find the area of the quadrilateral PQRS.

Problem 5:

A semi-elliptical shaped bridge, (24m wide and 8m high, supported 10m above an inlet bottom as shown in the diagram), spans a small passageway from an inner harbour to the open sea. The depth of water (d, in metres) in this passageway may be expressed as a periodic function of the time of day (t, in hours), according to the equation

$$d = 2.5\cos\frac{2\pi t}{12} + 6$$

A captain wishes to tow a barge 16m wide and 12m high under the bridge. One-third of the barge is submerged. If high tide occurs at noon, between what times in the next 12 h can the captain take the barge to sea without doing structural damage to either the barge or the bridge?

153
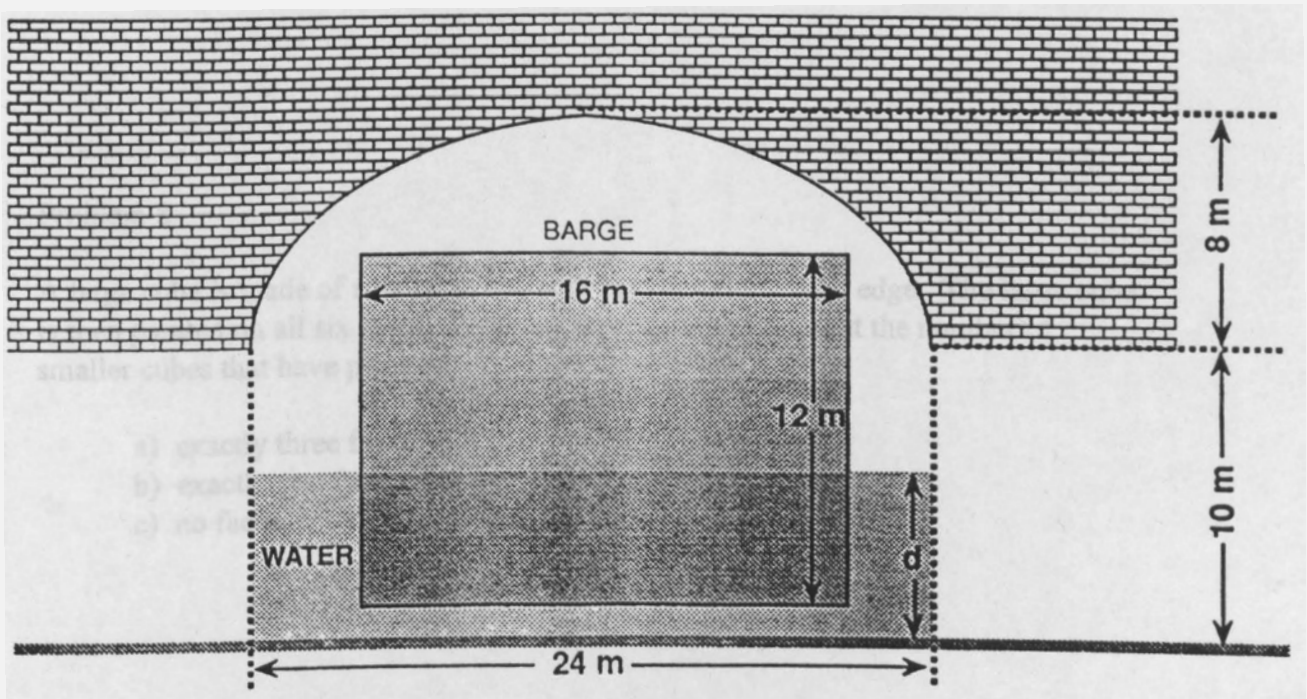
Problem 6:

How far apart are the centres of two circles with radii 5cm and 20cm if their common tangents intersect at 60°?

Problem 7:

Quarter circles are drawn in a square as shown. Determine the area of the shaded regions.

(a)
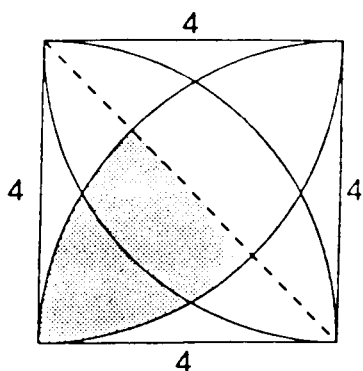
(b)
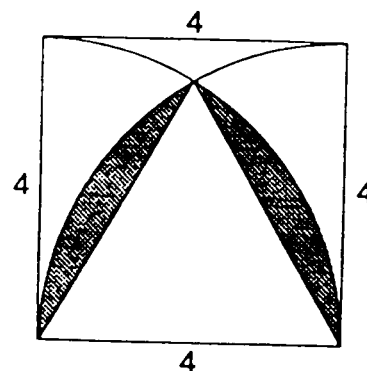
Problem 8:

A large cube is made of smaller cubes with N cubes along each edge. The large cube is then painted on all six faces. Write an expression to represent the number of smaller cubes that have paint on

     a) exactly three faces
     b) exactly two faces
     c) no faces

Problem 9:

A circle is inscribed in a square of side S. Then a square is inscribed in the circle. The process continues indefinitely.

   a) Find an expression for the sum of the circumferences of the first 20 circles.
   b) Find an expression for the sum of all the circumferences.
   c) Show that the ratio of the sum of the circumferences of all circles to the circumference of the largest circle is $(2 + \sqrt{2}) : 1$

Problem 10:

The line $x - y = -7$ intersects the curve $y = x^2 + 1$ at points A and B. Tangents which are drawn to the curve at points A and B intersect at point C. Determine the area of $\triangle ABC$.

Mathematics 12

Examination Preparation 1

Topic Study Guide

Topic: Polynomial Functions

Questions:

1. How do we divide a polynomial by another polynomial? Can you do this at least two different ways?

2. How does the remainder of the division relate back to the original polynomial and the divisor (x-a)?

3. If the remainder is zero what does this tell you about the divisor and the original polynomial?

4. How can we find a divisor (x-a) to "get started" on dividing the polynomial up into linear factors?

5. What steps do we follow to solve the polynomial equation P(x)=0 by factoring?

6. How do we write a polynomial equation P(x)=0 if we already know the roots?

7. What information do we consider when we want to graph y=P(x)?

8. How do we tell how many real roots P(x)=0 may have?

9. How do we identify possible integral or rational roots of P(x)=0?

Mathematics 12

Examination Preparation 2

Topic Study Guide

Topic: Sequences and Series

Questions:

1.   What is a sequence?

2.   How do we distinguish between an arithmetic sequence
      and a geometric one? Are these the only type of
      sequences?

3.   For an arithmetic sequence, what information will
      enable us to find;

      a)   the fifteenth term?
      b)   the common difference?
      c)   the term number?
      d)   the first term?

4.   If we know the seventh term and the twelfth term of
      an arithmetic sequence how would we determine any
      other term?

5.   For a geometric sequence, what information will
      enable us to find;

      a)   the seventh term?
      b)   the common ratio?
      c)   the term number?
      d)   the first term?

6.   If we know the fourth term and the sixth term of a
      geometric sequence how would we determine any other
      term?

7.   How would we determine if a,b,c,... is an
      arithmetic sequence, a geometric one, or neither?

8.   What is meant by a recursive definition of a
      sequence?

9. Write a recursive definition for;

   a) 2, 10, 18, 26,...

   b) 1/3, 1/6, 1/12, 1/24,...

10. What is a _series_?

11. What formulas help us find the sum of the first n terms of a geometric series? An arithmetic series?

12. What is an _infinite series_? What can this series sum to?

13. How is a repeating decimal like an infinite series? What is its common ratio? What is its sum?

14. When does the sum of an infinite series give a finite answer?

15. What is _Sigma Notation_? How is it used?

16. When we convert Sigma Notation into _expanded form_ what part of the Sigma Notation expression will give us the common difference or common ratio?

17. How do we convert from expanded form into Sigma Notation?

Mathematics 12

Examination Preparation 3

Topic Study Guide

Topic: Exponents & Logarithms

Questions:

1. Describe the three graphs; $y = 2^x$, $y = 10^x$, $y = (1/2)^x$

2. What is the <u>Range</u> of $y = b^x$? <u>Domain</u>?

3. How do we write $y = b^x$ in <u>logarithmic form</u>?

4. What do we mean by a <u>Common Logarithm</u>?

5. How can we rewrite $y = \log_b x$ in <u>exponential form</u>?

6. $10^{\log x} = ?$

7. $\log_b b^x = ?$

8. What are the rules for using logs?

9. How can we take the log of a radical?

10. Describe the three graphs; $y = \log_2 x$, $y = \log x$, $y = \log_{(1/2)} x$

11. What is the domain of $y = \log_b x$? Range? How do these compare with those of $y = b^x$?

12. Show that $y = b^x$ and $y = \log_b x$ are <u>inverse functions</u>.

13. Can we use $y = \log_b x$ for $b \leq 0$? Why or why not?

14. How can we rewrite $\log_3 5$ in terms of common logs?

15. If $b^x = c$, solve for x.

16. If $x^b = c$, solve for x.

17. Given $P = P_0(r)^n$, rewrite the formula in logarithmic form.

18. Describe each graph: $y = 2^{x-3}$, $y = 2^{x+1}$, $y = 2^x - 3$, $y = 2^x + 1$, $y = \log_2(x-2)$, $y = \log_2(x+5)$, $y = \log_2 x + 5$, $y = \log_2 x - 2$

Mathematics 12

Examination Preparation 4

Topic Study Guide

Topic: Introduction to Calculus

Questions:

1.   Explain the difference between a <u>secant</u> line and a <u>tangent</u> line to a curve at a point P.

2.   Is it possible for a tangent line to cross a curve at the point of tangency? Explain.

3.   Is it possible for a tangent line to touch a curve more than once? If so, give an example.

4.   What do we mean when we say that a sequence of numbers has a <u>limit</u>?

5.   Does the sequence $a_n = (2n^2 + 3n)/(5n^2 + 4)$ have a limit as n tends to infinity? If so, determine it in two different ways, one algebraic and one intuitive.

6.   Explain the concept of limit as it applies to functions.

7.   What do we mean when we say that a function is <u>continuous</u>?

8.   If the numerator and the denominator of a function both approach zero, how can we find the limit of the function?

9.   For a rational function $P(x)/Q(x)$, what is the significance of a vertical asymptote? A horizontal one?

10.   Write down the <u>derivative</u> of f(x) as the limit of the <u>Newton quotient</u>. Apply this definition to find the derivative of $f(x)=5x^2$.

11.   Give four different forms of notation for the derivative of $y=f(x)$.

12.   Explain how we use the General Power Rule to differentiate polynomials.

13.   Write down the Product, Quotient, and Reciprocal rules of differentiation and give an example of each one.

14.   If x(t) represents the position of an object at time = t, then how do we determine its Velocity? Speed? Acceleration?

15. Explain what is meant by an <u>increasing</u> function. A <u>decreasing</u> one?

16. Using the derivative of f(x), how can we tell when the graph of f(x) is increasing? Decreasing?

17. What do we mean by a <u>critical point</u> of f(x)?

18. Explain the difference between <u>local</u> and <u>global</u> maximum or minimum values of f(x).

19. How do we determine if a critical point is maximal or minimal?

20. Why is it important to know the domain of any function used in an extreme value problem?

Examination Preparation 5

Topic Study Guide

Topic: Trigonometry

Questions:

1.   What do we mean when we say that a function is periodic?
      What is the period of such a function?

2.   Define one radian.

3.   Show how we would convert an angle from degrees to
      radians. Radians to degrees?

4.   Convert every angle multiple of 30° (from 0° to 360°)
      into radian measure.

5.   How can we determine the length of an arc of a circle?

6.   If P(x,y) has position angle θ then give the primary
      trigonometric function values of θ in terms of x and y
      and r.

7.   If sinθ = k has a root = a then what is another root?

8.   If cosθ = k has a root = b then what is another root?

9.   If tanθ = k has a root = c then what is another root?

10.   Define the reciprocal trigonometric functions of θ in
      terms of x,y, and r where θ is the position angle of
      P(x,y).

11.   What is the relation between the cosine of an angle and
      its sine?

12.   How are the sine and the cosine of complementary angles
      related?

13.   Carefully sketch the graphs of y = sin(x), y = cos(x),
      and y = tan(x) for $-2\pi \le x \le 2\pi$. Give the period and
      amplitude for each graph.

14.   In the equation y = asinb(x-c) + d clearly describe the
      effect of each value a,b,c,d on the graph of y = sin(x).

15.   What is an identity?

16.   Using $x^2 + y^2 = r^2$, derive the Pythagorean identities.

17. Use the graph of $y = \tan\theta$ to explain why $\tan(-\theta) = -\tan\theta$.

18. Explain why $\cos(\pi - \theta) = -\cos\theta$.

19. What is a <u>co-function</u>? How are co-functions related?

20. Using $\sin(A-B) = \cos(\pi/2-(A-B))$ derive the identity for $\sin(A-B)$.

21. Using $\cos(A+B) = \cos(A-(-B))$ derive the identity for $\cos(A+B)$.

22. Derive the identity for $\cos2\theta$ and show why it can be represented in three different ways.

23. When proving identities why can't we do the same thing to both sides of the equation in order to transform it?

24. Explain the difference between <u>general</u> solutions and <u>particular</u> solutions to a trigonometric equation.

Mathematics 12

Examination Preparation 6

Topic Study Guide

Topic: Quadratic Relations

Questions:

1.   How do we find the <u>distance</u> between two points in the coordinate plane?

2.   How do we find the coordinates of the <u>midpoint</u> of a segment in the coordinate plane?

3.   What is a <u>locus</u>? How might we find the equation of a locus?

4.   What is a <u>conic section</u>? Describe how each conic section is formed from a cone.

5.   Why is an <u>hyperbola</u> with the equation $x^2 - y^2 = a^2$ or $y^2 - x^2 = a^2$ called <u>rectangular</u>?

6.   Describe the three <u>types</u> of conic sections. What are the distinguishing factors of their equations and their graphs?

7.   Write down the equations of each conic section in <u>Standard Form</u> where the centre is at $(0,0)$.

8.   Repeat #7 but with centre at (h,k).

9.   How do we convert an equation from Standard Form into <u>General Form</u>?

10.   How do we convert from General Form into Standard Form?

11.   When the equation of a conic section is in General Form how can we determine what type of conic section it is?

Appendix E

Student Response

1. One-Minute Papers

2. Year-end Interview Questions

3. Year-end Interviews 1-7
(see Chapter 4 for Interview 8)

The One-Minute Paper

Please answer each question in 1 or 2 sentences:

1) **What was the most useful/meaningful thing you learned during this session?**

----

2) **What question(s) remain uppermost in your mind as we end this session?**

Mathematics 12

Research Project 1991-2

Year-end Interview Questions

1. Why are you taking Math 12?

2. What kind of class did you expect when you
    started the year?

3. What have you found different about this year's
    class compared to other math classes you have had?

4. What activities do you remember from this year?

5. What activities did you enjoy or find useful?

6. What activities did you not enjoy or find not
    useful?

7. What activities would you have liked to have seen in
    the class? Why?

8. If you could change something about the
    Mathematics 12 course what would you change? Why?

9. How do you feel final examinations affect the way
    you learn or study?

10. Based on your work, how successful do you feel that
    you have been in this course?

11. How well do you think you have understood the ideas
    in this course?

12. How well do you think that you have learned the
    skills required for the course?

13. How successful do you believe you will be on the
    final exam?

14. What mathematics course(s) might you be taking at
    college or university? Why? or Why not?

15. Do you have any further comments?

APPENDIX E

YEAR-END INTERVIEWS

INTERVIEW 1

(Researcher)   Why are you taking Math 12?

(Student)   Because I want to get credit for it for university.

(R)   Do you need it for a certain program at university?

(S)   For science.

(R)   What kind of class did you expect when you started the year? How did you expect the class to be taught?

(S)   Ordinary.

(R)   Ordinary, what does that mean? How do you understand that?

(S)   That means that every teacher teaches the same way but with different styles. Different techniques to teach the subject. I think your class was different

(R)   Before you knew me what did you think would happen in each class? How did you think each class would be?

(S)   I expected the class to be the same as my last year's class.

(R)   And can you tell me what that was like? What would happen in a class last year? How would the teacher teach?

(S)   The teacher taught me very slowly. I think your method is much faster so I don't catch everything.

(R)   What have you found different about this year's class compared to other math classes you have had?

(S)   This year you used the past exams and worked in groups. Last year we did work as individuals.

(R)	What activities do you remember from this year? What kinds of things that we did in class can you remember?

(S)	We did our projects. We did exams. We had our tests with the group mark.

(R)	What activities did you enjoy or find useful?

(S)	The projects.

(R)	Anything else?

(S)	The group work.

(R)	What activities did you not enjoy or find not useful?

(S)	Nothing.

(R)	What activities would you have liked to have seen in the class? Is there something you wish we had done?

(S)	No.

(R)	If you could change something about the Mathematics 12 course what would you change? That's not the teacher, the way the teacher teaches, but what's in the course, the topics in the course or the order in which they're taught.

(S)	I think we should learn more about calculus.

(R)	More calculus? Why do you say that?

(S)	Because it's useful. And the graphs. And geometry.

(R)	Why would you like to see more geometry?

(S)	Because I don't know much about geometry.

(R)	So you'd like to learn more.

(R)	How do you feel final examinations affect the way you learn or study? When you know there is a government exam at the end of the year, how does that

affect the way you try to learn things or the way you study?

(S)   I try harder.

(R)   What kinds of changes would you make if you knew there was that exam coming? How would you do things differently if you knew there was an exam? Suppose there was no exam. What would you do?

(S)   No exams? I wouldn't do my homework anymore.

(R)   Based on your work, how successful do you feel that you have been in this course?

(S)   So so.

(R)   So so. Do you want to say any more?

(S)   No.

(R)   How well do you think you have understood the ideas in this course?

(S)   About 80 percent.

(R)   About 80 percent of it? So the idea of what is a tangent, what is an exponential function, what is a logarithmic function ...?

(S)   No, I don't understand much about logarithms, especially the graphs.

(R)   Yeah, you had a little trouble with that. Anything else you can think of? Any ideas that you didn't understand well or understood well?

(S)   The polynomial graphing.

(R)   Was that understood well or not understood?

(S)   It gave me trouble.

(R)   So you'd like to see some more polynomial graphs?

(S)   Yes.

(R)   How well do you think that you have learned the skills required for the course? Like actually solving equations, determining the graphs, finding the terms of sequences, things like that. Applying all of those things, proving trigonometric proofs.

(S)   I didn't really understand everything you were teaching.

(R)   How successful do you believe you will be on the final exam?

(S)   Not good. I tried last year's exam and only got 60 percent.

(R)   What mathematics course(s) might you be taking at college or university?

(S)   Calculus and Algebra.

(R)   So would you take only the courses that you are required to take for Science or would you take other math courses because you wanted to.

(S)   I think I might take a course in statistics.

(R)   Do you have any further comments?

(S)   I think you have been a really good teacher. I think your method is quite good but sometimes I couldn't understand you.

(Researcher)   Why are you taking Math 12?

(Student)   I'm taking Math 12 to go to university.

(R)   And what kind of thing do you expect to do at university?

(S)   I'm going into Commerce and they require that you have Math 12.

(R)   What kind of class did you expect when you started the year?

(S)   A normal, regular math class.

(R)   What does that mean, a normal, regular math class?

(S)   You go to class, the teacher gives you notes, you do your homework, and then you do math.

(R)   What have you found different about this year's class compared to other math classes you have had?

(S)   We had three major projects to do. The last one, where we had to make up an exam, you had to really know what to do to figure it out and so I found it useful.

(R)   What activities do you remember from this year?

(S)   Activities? Just the projects, I think. And studying for the tests.

(R)   Those were your activities, right?. Anything else in the classes that you remember?

(S)   Getting into groups and showing each other what you know to help each other with math that you don't know.

(R)   What activities did you enjoy or find useful?

(S)   I found the group that you were in useful. When one top student, one middle student, and one low student were together you could help each other.

(R)   So the mix of the groups? The way they were mixed?

(R)   What activities did you not enjoy or find not useful?

(S)   The computer lab. (The sequences and series work on computer spreadsheets.)

(R)   Can you say why?

(S)   I don't know. I find it useful if I can do it myself on paper instead of on the computer. The background, I understand it better too because I don't have to key it into the computer.

(R)   Anything else?

(S)   No.

(R)   What activities would you have liked to have seen in the class?

(S)   I think we did most of what I'd like.

(R)   Can you think of anything different?

(S)   No.

(R)   If you could change something about the Mathematics 12 course what would you change?

(S)   The curriculum?

(R)   Yes, the curriculum.

(S)   Can I change calculus?

(R)   Well, would you like to see more calculus, or less, or no calculus?

(S)   No calculus. I don't plan on making any use of it. I don't know why they teach it.. You use so many variables and it doesn't show us anything.

(R)   I guess at this point it's not important to you.

(S)   It probably will be at some point in my life, I know that for sure.

(R)   How do you feel final examinations affect the way you learn or study?

(S)   I think that it helps to show the family who you are. I don't think it affects it too much because I think it helps your mark go higher.

(R)   So it can actually help your mark?

(S)   Yeah. I didn't find them (the past exams) very tough. I know Chemistry is, but it's not like the Chem.

(R)   Based on your work, how successful do you feel that you have been in this course?

(S)   I can do better than I have now. In the beginning I did a lot more work. I thought I could handle more courses but now I realize that I can't. I put more time into something else when I needed it. In the beginning I got an A at 86 (percent) and I thought, good, I'll probably stay there for a while. Then I started working on other stuff like Chemistry and History and I'd forget about Math and it dropped.

(R)   So you had a problem balancing your workload, is that what you're saying?

(S)   Yeah.

(R)   How well do you think you have understood the ideas in this course?

(S)   I think I understand quite a bit. But when you give an assignment the problem might have, might be more advanced, have more in it. You're supposed to use what you know and learn more out of the question. I can't do that very well.

(R)   How well do you think that you have learned the skills required for the course?

(S)   Yeah, pretty well.

(R)   How successful do you believe you will be on the final exam?

(S)   I think I'm going to do well, moderate, average to what I've been doing as of now.

(R)   To about the same level, is that what you're saying?

(S)   Yeah. I've been trying to study my math harder, especially in logs.

(R)   What mathematics course(s) might you be taking at college or university?   Why? or Why not?

(S)   I'll have to take Calculus 100 and another one for statistics.

(R)   And why did you choose those courses?

(S)   I'm not sure, but that's what the course requires, for when you go into Commerce.

(R)   Do you have any further comments?

(S)   It was fun. I enjoyed it, except for some tests. Mostly I enjoyed going to class. I have these mood swings.

(R)   Mood swings? So some days were good and some were not so good?

(S)   Yeah. I suppose they were good.

(Researcher)   Why are you taking Math 12?

(Student)   It's to go to university, for science courses.

(R)   What kind of class did you expect when you started the year?

(S)   It was going to be a lot harder than Math 11 and there'd be calculus.

(R)   What kind of environment, what kind of class did you expect?

(S)   I expected it to be different.

(R)   Now, you had me as your Math 11 teacher. Did you expect things to be sort of like last year?

(S)   No.

(R)   Can you tell me what you did expect?

(S)   A lot harder tests. Maybe you would just give us the examples and just teach it to us.

(R)   What have you found different about this year's class compared to other math classes you have had?

(S)   It's fun. It's easier to understand.

(R)   What activities do you remember from this year?

(S)   Projects. Group activities. That's it.

(R)   What activities did you enjoy or find useful?

(S)   The projects were useful.

(R)   What did you like about those?

(S)   I learned about other things that I never knew about math. Doing the old exams was pretty good.

(R)   What activities did you not enjoy or find not useful?

(S)   Not really.

(R)   What did you think of the computer lab
business?

(S)   It's a lot of fun. It's really good.

(R)   Did it help you understand?

(S)   The program, it gives you a visual view of
the equations a lot faster.

(R)   What activities would you have liked to
have seen in the class?

(S)   I don't know.

(R)   If you could change something about the
Mathematics 12 course what would you change?

(S)   Maybe the order, the way things appear in
the text.

(R)   Can you think of something in particular?

(S)   You know how things go together. Maybe
calculus should go last and maybe we should learn
something first, I'm not sure.

(R)   What sort of thing should you learn first?

(S)   Trigonometry? Did we learn trigonometry
first? Yeah, I think we did. I don't know, I'm not sure
how the order is. But I'd probably like it if it was
ordered differently.

(R)   Alright. Then somehow things didn't fit
in the order you saw them?

(S)   Yeah.

(R)   How do you feel final examinations affect
the way you learn or study?

(S)   Well I have to know all the material for
the exam.

(R)   How does that affect the way you study?

(S)   I have to do a lot more questions. Those
ones like word questions which are a lot harder to
interpret.
(R)   Based on your work, how successful do you
feel that you have been in this course?

(S)   Not very well.

(R)   You feel you could have done better?

(S)   I could have, if I'd been able to
concentrate.

(R)   How well do you think you have understood
the ideas in this course?

(S)   Most of the ideas, like the limits in
calculus, I understood that, but some of the things about
graphing I just can't get. And the thing about symmetry,
the coordinates, I find that hard to understand.

(R)   Anything else?

(S)   Trigonometry. I understood that quite
well.

(R)   How well do you think that you have
learned the skills required for the course?

(S)   I know most of the equations except I
haven't practiced enough to know it really well. I know
how to do it but I can't get it right out of my head.

(R)   How successful do you believe you will be
on the final exam?

(S)   I have to do a lot more questions to
actually do well, so I don't know. I haven't been doing
too many questions so I need to prepare.

(R)   What mathematics course(s) might you be
taking at college or university?

(S)   Calculus.

(R)   Why would you take calculus?

(S)   I guess it's a really interesting type of
math.

(R)   What do you intend to pursue?

(S)   Computer programming.

(R)   So do you think mathematics will be useful
to you in that regard?

(S)   In calculations and things like that.

(R)   Do you have any further comments?

(S)   No, not really.

INTERVIEW 4

(Researcher)   Why are you taking Math 12?

(Student)   Because it's required to go to university.

(R)   What are you going on to take?

(S)   Science.

(R)   What kind of class did you expect when you started the year?

(S)   I expected it to be very hard with a lot of homework.

(R)   What about the way things are taught?

(S)   It's really good, the way you're teaching but you're not giving enough homework. We're so lazy. Looking up all your notes we thought we knew everything so we didn't have to do any homework.

(R)   But what I'm trying to get at is that before me, the first day, even last June registering for Math 12 what was in your mind about Math 12? What did you think Math 12 would be like?

(S)   Tough. Really tough.

(R)   And what about the way it was taught? How each class would be, things like that?

(S)   I thought I was going to get Mr. XXX again so I didn't really expect anything different.

(R)   What have you found different about this year's class compared to other math classes you have had?

(S)   Well, Mr. XXX, he follows the textbook from step to step. You, it's like it's from your head, you've learned all this stuff before and you give it to us from your head. You refer to the book a bit but it's not exactly from the book.

(R)   Anything else? Think of all the math classes you've ever had and then think of this one. What kind of differences have you noticed?

(S)   Not that much, not so many differences.

(R)   What activities do you remember from this year?

(S)   Group work for the prep for final examination. Group work, that's about all I remember.

(R)   Anything else?

(S)   I don't remember.

(R)   What activities did you enjoy or find useful?

(S)   Preparing for the examination, working together with partners.

(R)   What activities did you not enjoy or find not useful?

(S)   I don't think so.

(R)   What activities would you have liked to have seen in the class?

(S)   I don't know.

(R)   Take some time if you like.

(S)   A study group for the next test, or something.

(R)   Kind of a review session?

(S)   Yeah. A review session.

(R)   If you could change something about the Mathematics 12 course what would you change? Why?

(S)   More geometry. Because I think it's really hard. To me it is. I don't really know that much geometry.

(R)   Why do you think it's important to have more geometry?

(S)   For the final examination, for the provincial, there are some proofs you have to do. I don't really know how to do them because I don't know how to follow the steps.

(R)   How do you feel final examinations affect
the way you learn or study?

(S)   It makes you review everything you've
learned from the past. If you don't do well you can drop
dramatically.

(R)   How does it affect the way you learn? You
are sitting in class, if you are aware of the fact that
you have a final exam, how do you try to learn things?

(S)   Facts. I just try to learn really quick.

(R)   Based on your work, how successful do you
feel that you have been in this course?

(S)   I don't think I'm doing that well because
I'm really lazy. I didn't study that much.

(R)   Alright, but the work you did do, the test
results. How well do you think you did?

(S)   The test results? Not very good.

(R)   How well do you think you have understood
the ideas in this course?

(S)   I understood them when you wrote them in
notes. The notes were really good because I could read
them and study from them. It really helped. I
understood them, then the next day I'd go to write it (a
test) and, Holy! I know all this stuff but it's just ...
well, practice.

(R)   How well do you think that you have
learned the skills required for the course?

(S)   Practice, that's the only thing that can
really help.

(R)   So you feel that you've not learned the
skills as well as you could, is that right?

(S)   Yes.

(R)   How successful do you believe you will be
on the final exam?

(S)   I can do it, it's not really that hard,
because you showed us some tricks and that really helped.

I hope I do well for the geometry proof. I don't know if I can do it.

(R)   What mathematics course(s) might you be taking at college or university?   Why? or Why not?

(S)   Probably calculus.   Science needs calculus.

(R)   You're going into science, right.   Will you take any other math courses?

(S)   I don't know what there is.

(R)   Do you have any further comments?

(S)   This year was pretty awful for me, but just study hard and do your homework.

(R)   When you say awful, do you mean awful in mathematics or awful as a school year?

(S)   Awful in mathematics.

(R)   And why do you say that?

(S)   Because I don't really spend that much time studying or reviewing.

(R)   So you find that its a study problem. Anything else?

(S)   No, that's about it.

(Researcher)   Why are you taking Math 12?

(Student)   I just enjoy taking math and it seems easy to me although I'm not doing well.

(R)   Is there another reason you're taking Math 12?

(S)   Basically I just want to go into Science.

(R)   What kind of class did you expect when you started the year?

(S)   An environment that was hard-working and the teachers had lots of work for Grade 12s.

(R)   In terms of how the teaching were to take place, what did you expect?

(S)   It was okay how you were teaching, but I wanted you to take more examples from the book and show us rather than going off and doing a lot of stuff that was hard to understand.

(R)   And what about before it was me? What about when you first signed up for Math 12, what did you have in mind?

(S)   Well I thought there was going to be detailed examples. I know that you went into detail but you went off a lot. You went through other stuff that I didn't really understand about. You really got me off track then.

(R)   What have you found different about this year's class compared to other math classes you have had?

(S)   They're about the same.

(R)   What do you mean by the same? What is the same?

(S)   The only difference is that, before, the teacher collected homework. It made you work more. Without this you don't work as hard. They basically keep to the examples in the book, but you go off in more detail into other subjects, they go more into the examples.

(R)   What activities do you remember from this year?

(S)   Calculus. That was one I remember well, I don't know why but I did.

(R)   What about activities that we did in class?

(S)   Oh, in class. The one where we were like teachers, where we taught each other. That was good.

(R)   Anything else? Like a specific thing that we worked on in class?

(S)   All the exams, those were good.

(R)   You mean where you got together and looked at the exams?

(S)   Where we got to share each other's knowledge.

(R)   What activities did you enjoy or find useful?

(S)   Well, working together, that was (inaudible). I thought that was pretty useful too because if you don't know something and everybody else helps each other, that's good.

(R)   What activities did you not enjoy or find not useful?

(S)   When you went to talk about other stuff. When you talked about the theories. That was pretty hard to take. I lost sight of what we were supposed to be doing there.

(R)   What activities would you have liked to have seen in the class?

(S)   (Inaudible). I don't think so.

(R)   If you could change something about the Mathematics 12 course what would you change?

(S)   Actually I'd put geometry back into the curriculum so you'd study a little more of that section.

(R)   Okay, why would you want to see more geometry?

(S)   Because that section is kind of hard, I mean it's review but you should take it up because in grade 12 provincials you still have to do proofs.

(R)   So you're concerned about how you would do it on the provincial, is that what you're saying?

(S)   Yes.

(R)   How do you feel final examinations affect the way you learn or study?

(S)   Well, its okay, but I don't think it should be worth that much because you're talking a whole year that's worth 60 (percent) and a two-hour test is worth 40. It's just not as close a ratio.

(R)   But what about in terms of day-to-day in class, if you're aware that you are going to have to write a final exam, how is that affecting the way you are trying to learn or the way you do your studies?

(S)   It doesn't really affect me. I just prepare for the next test. I don't really think about the provincials.

(R)   Based on your work, how successful do you feel that you have been in this course?

(S)   Not too successful because I haven't been doing my homework and it creates extra (inaudible).

(R)   How well do you think you have understood the ideas in this course?

(S)   I think I know the basics, how to do it. I don't know how to do the real complicated problems.

(R)   Are you saying that you're comfortable with the basics but the complicated things are unclear to you?

(S)   Yeah, yeah.

(R)   How well do you think that you have learned the skills required for the course? How to do the equations, how to do the graphs?

(S)   I think that's the part where I feel most
comfortable because we did a lot of that in previous
years so it helps.

(R)   How successful do you believe you will be
on the final exam?

(S)   That's the hardest part, I think, to try
to get a B. A little bit would just bring me up.

(R)   What mathematics course(s) might you be
taking at college or university?

(S)   I think I'm going to take calculus.

(R)   Would you take that because you choose it
or because its part of the science program?

(S)   Because its part of the science program.

(R)   Would you take any other math courses?

(S)   I'm not sure yet, but I'm pretty sure that
I'll take a lot of math.

(R)   What's your direction? Do you have any
idea?

(S)   Probably engineering.

(R)   Do you have any further comments?
Anything at all, about math, about the class, the
teaching, anything.

(S)   Well, your teaching is good, right, but I
think you're too nice and it's not working. I don't
know, a person like me, I mean other students would not
take advantage of it but I don't work hard when the
teacher's not strict with me. A strict teacher will make
me work hard, but when the teacher is not strict I start
slacking off.

(R)   Anything else? Any other comments?

(S)   No, that's it.

(Researcher)   Why are you taking Math 12?

(Student)   Because that's an academic course that I need to go no to further studies at college or university.

(R)   What kind of class did you expect when you started the year?

(S)   Just basically from textbooks, and tests. But nothing that has to do with projects and provincial exams.

(R)   What have you found different about this year's class compared to other math classes you have had?

(S)   Definitely the projects that you gave us. I think its a just more fun, storytelling, you know.

(R)   What activities do you remember from this year?

(S)   Projects again. Some experiments that you showed us, with the balls, for example. The graphing, the curves. Tests.

(R)   You mean those tests where you worked together?

(S)   Yes.

(R)   What activities did you enjoy or find useful?

(S)   I thought the projects were kind of useful, especially the ones you do individually, because you research the project, the topic, you learn more about it, dig deeper into that topic, experience the topic.

(R)   What activities did you not enjoy or find not useful?

(S)   Nothing.

(R)   What activities would you have liked to have seen in the class?

(S)   Field trips, sort of. Out of class.

(R)   If you could change something about the Mathematics 12 course what would you change?

(S)   (Long hesitation) No.

(R)   How do you feel final examinations affect the way you learn or study?

(S)   Makes me kind of nervous about things.

(R)   Do you think of the final exam through the year when you have something to study?

(S)   Just at the end of the year, when its close. Two or three weeks before.

(R)   Since we were looking at final exams right from the very start were you thinking more of the final exams?

(S)   If you look at the timespan from the beginning of the year to the end of the year, I was kind of relaxed before the end.

(R)   Based on your work, how successful do you feel that you have been in this course?

(S)   I do not think I've done enough work in the course. I never really touched homework. I think that will really affect my marks at the conclusion.

(R)   In terms of how successful you've been?

(S)   Not successful.

(R)   How well do you think you have understood the ideas in this course?

(S)   The ideas?

(R)   Well, for example, what an exponential function is, what a logarithmic relationship is, what a trigonometric proof is, what calculus is about, what differentiation is about.

(S)   I quite understand except calculus and trigonometry.

(R)   How well do you think that you have learned the skills required for the course?

(S)   Not too much. I haven't really much
because of the homework I haven't done.

(R)   So you're saying that the homework, the
practice wasn't done and therefore you didn't learn so
well?

(S)   The practice, yeah.

(R)   How successful do you believe you will be
on the final exam?

(S)   Hopefully successful if I review. I do
not think I'll do quite successful because of the
homework again, which I haven't done. I hope that I'll
do well.

(R)   What mathematics course(s) might you be
taking at college or university?   Why? or Why not?

(S)   I'm not planning to go to college or
university next year. I haven't decided yet if I'll go.

(R)   Do you have any further comments?

(S)   No, nothing. I enjoyed Math 12 this year.

(Researcher)   Why are you taking Math 12?

(Student)   For my best interests. I feel that I'm going into sports medicine and without math I can't go in.

(R)   So is it a requirement?

(S)   Yes, it's a requirement.

(R)   What kind of class did you expect when you started the year?

(S)   Some hard, ... I didn't expect, ... the beginning part was mostly okay but later on it was not as hard as I expected. It's newer, newer stuff. But it's not as hard as I expected.

(R)   What did you expect, the way the course would be taught, the way the teacher would be?

(S)   Mostly, the teacher would be teaching a lesson for about two weeks, then they give you tests, you go home and learn, you don't do anything new. Same old stuff.

(R)   What have you found different about this year's class compared to other math classes you have had?

(S)   You get a variety of things. At the same time you learn the same stuff as the normal classes. You go over exams the same time, you know a lot more stuff than just straight out of the book, you know something else besides just one book. Actually I learned a lot this year than any other years that they teach because a variety of things that are happening.

(R)   What <u>activities</u> do you remember from this year?

(S)   The best one is getting into groups and learning a real hard section. That's the most fun I've had. Each person working hard and the rest combining it together.

(R)   Do you remember any other activities?

(S)   The other activity is at the beginning of the year you assigned us our parts to study from the

book. And then all of us have to tell the others what
part to study and together we combine them. And another
is our project. I never had a project in my life and
this is the first time I've had a project and it's fun.
You learn more things and discover new books about math,
not just one book that they give you.

(R)   What activities did you enjoy or find
useful?

(S)   I find useful, is the projects. Because
the projects give you a sense that there's not one book,
one math book alone. There's lots of variety of things
and you can get many information from other books that
will help you in the future.

(R)   Is that the project where you had to use
other books and study other topics?

(S)   Yes, uh-huh.

(R)   What activities did you not enjoy or find
not useful?

(S)   Actually I liked them all it's just that
the only hard thing is not about the activity, the only
hard thing about it is the way the course is running.
It's a bit, the way they teach it it's a bit too fast.
But if they could slow it down then the course would have
been great because they could introduce the new stuff.
But the teaching it went a bit fast. But otherwise it's
good. I like it.

(R)   Anything else that you can remember?
Something that we did that you didn't think was very
useful?

(S)   Not that I can remember, no. All of them
was mostly useful. The great idea I like about it, when
it comes to tests, we get into partners. The only bad
thing about it is that the kids will remember the answer
and he will write it down. The best thing, what I
suggest you do is just to have the same thing but on the
second part that you have to write, change everything,
not the same questions, but that have to do with the
section though.

(R)   What activities would you have liked to
have seen in the class? Why?

(S)   I don't know. I like the idea of group work. If I were to suggest you should get some students, ... to put the strong ones with the weak ones, which you did at the beginning. You should try to have some kind of a tutor, not like a tutor, but everytime you have any section that you're teaching you should have a group work where you put all the smart people with the weak persons and then ask them if there are any problems and walk around and see what's happening.

(R)   Some kind of peer tutoring?

(S)   Yes.

(R)   If you could change something about the Mathematics 12 course what would you change? Not the way it was taught but the content or the order in which things were done.

(S)   I like the order how it was done for the content but I don't like the way they start off too late. They should start off with calculus, cause calculus is the hardest thing to do for kids. You should start from the hardest part down to the weakest part because the hardest part is usually at the beginning where people are willing to learn and they like to learn something hard first. You give them something easy at the beginning, later on in the year they want to enjoy themselves and the hard part is harder to get in their minds.

(R)   How do you feel final examinations affect the way you learn or study? If you know you're going to have a provincial, how does it affect the way you try to learn things?

(S)   It affects me a lot because it teaches me a variety of things, such as ways that you can study and get ready for it too. Let's say you study straight, just from following topic by topic, and now you need some extra work besides those topics, what if the question asks you something besides those work? It has to do with math but it's something else. The you wouldn't know.

(R)   Based on your work, how successful do you feel that you have been in this course?

(S)   At the beginning I was more into it but later on my interest had kind of disappeared on me. I tried to pull it back but it's hard without understanding the concepts and the big view of this and that. My start, good at the beginning of the year but later on it

went slowly and slowly and a hard section came and I didn't want to learn anymore. That's what I mean by teaching the hard parts first then you start off with the easy stuff.

(R)   So overall how successful do you think you were?

(S)   Not very successful. Not as successful as I wanted it, but we'll see.

(R)   How well do you think you have understood the ideas in this course?

(S)   Pretty well but not, ... applying it, I can't. I understand the surface of it, but understand deeply?, I'm not too sure about that.

(R)   How well do you think that you have learned the skills required for the course?

(S)   The algebra part was easy but the graphing part was a lot of trouble. I had a lot of trouble with that graphing. Understanding it? So,so. Not too strong, not too weak at it. It's all up to how you interpret it and also you have to understand what the question asks you.

(R)   How successful do you believe you will be on the final exam?

(S)   Right now, when we run through the 1992 exam, it doesn't turn out too well because I don't remember mostly what's happening at the beginning. Mostly I remember what I just learned. And right now I'm trying to get back on track at the beginning, going over the questions on the exams.

(R)   So what do you think will happen on the provincial?

(S)   50-50, I would say. Above 50 (percent), I don't know.

(R)   What mathematics course(s) might you be taking at college or university?

(S)   I'm not going to college because I didn't want to go to college. I want to get straight into SFU and I find that what happens is that I don't have the high GPA, enough to get into SFU.

(R)   And if you went into that program (sports medicine) what course might you have to take?

(S)   I would start off with general math, then I'm gonna transfer into calculus.

(R)   Do you have any further comments?

(S)   The year, the class was awesome. The way it was taught, the way it was running, students were involved, projects went on. It's the first time I ever had interest in a lot of math for a long time, because projects were happening, it was really exciting to do.

(R)   Anything else?

(S)   That's about it.

References

Argyis, C. & Schon, D. (1989). Participatory action research and action science compared. <u>American Behavioral Scientist</u>, <u>32</u>(5), 612-623.

Beltzner, K., Coleman, .A., & Edwards, G.D. (1976). <u>Science Council of Canada background study No. 37: Mathematical sciences in Canada</u>. Ottawa, ON: Science Council of Canada.

Berlak, H. (1985). Testing in a democracy. <u>Educational Leadership</u>, <u>43</u>(2), 16-17.

British Columbia Ministry of Education. (1988). <u>Mathematics curriculum guide 7 - 12</u>. Victoria: Author.

British Columbia Ministry of Education. (1991). <u>The Annual Problem Set</u>. Victoria: Author.

Brophy, J. (1986). Teaching and learning mathematics: Where research should be going. <u>Journal for Research in Mathematics Education</u>, <u>17</u>(5), 323-346.

Brown, L. (1981). <u>Action research: The teacher as learner</u>. Victoria, Australia: Education Department of Victoria.

Case, R. (1990). <u>Implementation through informed reflection: A study guide for teachers</u>. Burnaby: Simon Fraser University

Chambers, D.L. (1990). School mathematics reform: Making the vision a reality. <u>School Science and Mathematics,</u> <u>90</u>(6), 549-545.

Cockroft, W.H. (1982). <u>Mathematics counts</u>. London, HMSO

Cohen, L. & Manion, L. (1989). <u>Research methods in education</u>. (3rd ed.). London: Routledge.

Cooney, T.J., Grouws, D.A., & Jones, D. (1988). An agenda for research on teaching mathematics. In D.A. Grouws & T.J. Cooney (Eds.), <u>Research agenda in mathematics education: Perspectives on research on effective mathematics teaching.</u> (pp.253-261). Reston, VA: National Council of Teachers of Mathematics, and Hillsdale, NJ: Erlbaum.

Cross, K.P., & Angelo, .A. (1988). <u>Classroom assessment techniques: A handbook for faculty</u>. Ann Arbor, MI: University of Michigan, National Center for Research on the Improvement of Postsecondary Teaching and Learning.

Ernest, P. (1989). The knowledge, beliefs and attitudes of the mathematics teacher: A model. <u>Journal of Education for Teaching</u>, <u>15</u>(3), 13-33.

Haney, W., & Madaus, G. (1989). Searching for alternatives to standardized tests: Whys, whats, and whithers. <u>Phi Delta Kappan</u>, <u>70</u>(9), 683-687.

Hustler, D., Cassidy, A. & Cuff, E.C. (Eds.). (1986). <u>Action research in classrooms and schools</u>. London: Allen & Unwin.

Kemmis, S. & McTaggart, R. (Eds.). (1988). <u>The action research planner</u>. Victoria, Australia: Deakin University Press.

Kirsner, S.A. (1990). The NCTM Curriculum and Evaluation Standards for School Mathematics: A welcome vision. <u>School Science and Mathematics</u>, <u>90</u>(6), 554-561.

Leinwand, S. (1985). Coping with the pressures of standardized math tests. <u>Learning</u>, <u>13</u>(6), 26-27 and 30.

Leinwand, S. (1992). Sharing, supporting, risk taking: First steps to instructional reform. <u>Mathematics Teacher</u>, <u>85</u>(6), 466-470.

Lerman, S. (1990). The role of research in the practice of mathematics education. <u>For the Learning of Mathematics</u>, <u>10</u>(2), 25-28.

Madaus, G.F. (1985). Test scores as administrative mechanism in educational policy. <u>Phi Delta Kappan</u>, <u>66</u>(9), 611-619.

McLean, L. (1990). Let's call a halt to pointless testing. <u>Education Canada</u>, <u>30</u>(3), 10-13.

McKnight, C.C., Travers, K.J., & Dossey, J.A. (1985). Twelfth-grade mathematics in U.S. high schools: a report from the Second International Mathematics Study. <u>Mathematics Teacher</u>, <u>78</u>(4), 292-300. Reston, VA: NCTM.

National Council of Teachers of Mathematics (1970). A history of mathematics education in the United States and Canada (37th yearbook). Washington, DC: Author.

National Council of Teachers of Mathematics. (1991). Commission on teaching standards for school mathematics: Professional standards for teaching mathematics. Reston, VA: Author.

National Research Council. (1989). Everybody counts: A report to the nation on the future of mathematics education. Washington, DC: National Academy Press.

Robitaille, D.F. & Dirks, M. (1982). Models for the mathematics curriculum. For the Learning of Mathematics, 2(3), 3-21.

Robitaille, D.F., Schroeder, T.C., & Nicol, C.C. (1992). Mathematics '90: A status report on school mathematics in British Columbia: a provincial report. Victoria, BC: British Columbia Ministry of Education.

Secondary Examinations Council (1985). The General Certificate of Secondary Education: National criteria for mathematics. Secondary Examinations Council, Department of Education and Science. London, HMSO.

Schön, D.A. (1987). Educating the reflective practitioner. San Francisco, CA: Jossey-Bass.

Shepard, L.A. (1989). Why we need better assessments. Educational Leadership, 46(7), 4-9.

Smith, M.L. (1991) Put to the test; The effects of external testing on teachers. Educational Researcher, 20(5), 8-11.

Sowder, J.T. (1989). Research agenda for mathematics education: Setting a research agenda (Vol. 5). Reston, VA: National Council of Teachers of Mathematics.

Watkins, K. (1991, April). Validity in action research. Paper presented at the meeting of the American Educational Research Association, Austin, TX.

Wideen, M. F., O'Shea, T., Pye, I., Lanzinger, I., Pye, K., Sherwood, A., & Ivany, G. (1991, June). Impact of large-scale testing on the instructional activity of science teachers. Paper presented at the Canadian Society for Studies in Education, Kingston, ON.