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# A Dynamic Growth Model of Debt Accumulation with Stochastic Investment

by

Duane W. Rockerbie

B.B.A. (Hons.), Simon Fraser University, 1984
 M.A., Simon Fraser University, 1986

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# THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

in the Department

of

Economics

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A DYNAMUC MODEL OF DEBT ACCUMULATION WITH

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### Abstract

This paper develops a dynamic model of an open economy whose objective is to maximize total utility from consumption while facing a stochastic capital accumulation. constraint. Implicit solutions for the control variables, the domestic capital stock and the stock of foreign debt, are obtained. When facing uncertainty, the optimal policy is to smooth the time path of real consumption by using one of the controls according to its implicit solutions. Necessary conditions for rapid debt accumulation are 1: that the variance of the unanticipated portion of new debt flows increases relative to the variance of the unanticipated portion of new domestic savings, and 2: the correlation between the continuous time shocks to both domestic savings and new foreign debt flows must be positive. Estimates of investment, savings, and new debt flow functions suggest that condition two is not satisfied, on the contrary, the correlation is negative for 11 out of 15 countries sampled. The predictions of the model under this circumstance are confirmed by the empirical evidence; despite a relatively higher residual variance, the countries sampled appeared to switch to foreign debt as a means to maintain consumption levels. It is quite possible that unanticipated negative shocks to domestic savings forced this behavior, despite accumulating a relatively risky asset. This could explain the rapid accumulation of debt over the last 20 years, despite high uncertainty in international capital markets. Unit-root tests on 16 South American developing economies do lend justification to the use of real shocks to investment in partly determining the time path of real gross national product.

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# Chapter 1: A Summary of the History of Sovereign Borrowing by Lesser Developed Countries

### 1.1 Introduction

Due to the recent events of the late 1970's and the "debt crisis" of 1982, an explosion of academic papers has emerged giving policy prescriptions to alleviate the relatively high levels of foreign debt held by many lesser developed countries (LDCs). The bulk of these papers do not resort to economic models to arrive at their policy recommendations. Rather they contain a mixture of historical precedent and genuine panic and fear. The knee-jerk reaction of private and institutional lenders to this crisis has generally been both to raise lending rates and to reduce the availability of credit to troubled LDCs, in an effort to cut their losses. It remains a question whether the appropriate response of lenders to the spectre of major loan defaults is to impose stringent credit conditions upon borrowers, or to gamble fresh loans with the hope of pulling the borrower out of difficulty.

This paper develops a dynamic model of an open economy whose objective is to maximize total utility from consumption while facing a stochastic capital accumulation constraint. Implicit solutions for the control variables, the domestic capital stock and the stock of foreign debt, are obtained. When facing uncertainty, the optimal policy is to smooth the time path of real consumption by using one of the controls according to its implicit solutions. Necessary conditions for rapid debt accumulation are 1: that the variance of the unanticipated portion of new debt flows increases relative to the variance of the unanticipated portion of new domestic savings, and 2: the correlation between the continuous time shocks to both domestic savings and new foreign debt flows must be positive. Estimates of investment, savings, and new debt flow functions suggest that condition two is not satisfied, on the contrary, the correlation is negative for 11 out of 15

countries sampled. The predictions of the model under this circumstance are confirmed by the empirical evidence; despite a relatively higher residual variance, the countries sampled appeared to switch to foreign debt as a means to maintain consumption levels. It is quite possible that unanticipated negative shocks to domestic savings forced this behavior, despite accumulating a relatively risky asset. This could explain the rapid accumulation of debt over the last 20 years, despite high uncertainty in international capital markets. Unit-root tests on 16 South American developing economies do lend justification to the use of real shocks to investment in partly determining the time path of real GNP.

# 1.2 Early History

The purpose of this section is to introduce the reader to the historical patterns of sovereign borrowing since the 1820's up to the late 1960's. Generally most borrowing took the form of large bond issues with moderate maturities and generous rates of interest. In the most comprehensive treatment of historic sovereign borrowing, Lindert and Morton (1989) calculated the average rates of return on the bond issues of the ten largest sovereign borrowers<sup>1</sup>. Over the period 1850-1983 the aggregate average bond premium over U.S. long-term Treasury bonds measured 1.81%. Coinciding with the development of international banking and world capital markets, the average bond premium fell over the sample period: 1850-1914: 2.36% ; 1915-1945: 1.75% ; 1946-1983: 1.17%. Despite several major defaults, Lindert and Morton conclude that lending to sovereign governments was quite profitable.

Figure 1.1 illustrates the unstable behavior of real borrowing volume in U.S. dollars<sup>2</sup>. Ignoring the spike of 1895, caused by heavy Russian borrowing, the largest

<sup>&</sup>lt;sup>1</sup> The ten countries used were Australia, Argentina, Brazil, Canada, Chile, Egypt, Japan, Mexico, Russia, and Turkey.

<sup>&</sup>lt;sup>2</sup> Note that this time-series reflects not only U.S. dollar lending, but lending in many different currencies converted to U.S. dollars at the appropriate exchange rate.

#### Chapter 1: LDC Debt Overview

lending periods were 1906-1914 and after 1970<sup>3</sup>. During the years of the Second World War up to the late 1960's, gross lending to the group of ten countries was not sufficient to offset debt service payments, causing net lending to appear negative. Lindert and Morton identified waves of high borrowing usually followed by the partial or complete defaults of heavily indebted borrowers. The first noted wave of high borrowing was during the 1820's, after which most Latin American governments defaulted. The end of the lending wave of the 1880's featured relatively few defaults. with the most notable being Argentina's partial suspension of debt-service payments, and Colombia's constant threat of complete default. Brazil, Mexico, and Russia defaulted following the 1910-14 lending wave. The cases of Mexico and Russia are different in that default followed quickly after revolution. The period of negative net lending after 1930 followed a wave of euphoric lending during the opulent 1920's. This period of credit restraint carried with it major defaults by most Latin American countries, Eastern Europe, Turkey, and China. During this period the chief means of financing international lending switched from bond issues to concessionary refunding by both governments and the I.M.F.

Statistics reflecting aggregate debt during the early years of sovereign borrowing are not available in great detail. The earliest available data goes back to 1913. Just before World War I, the largest creditor nations were the United Kingdom, France, and Germany, while the largest debtor areas were Europe, Latin America, the United States, and Asia. Table 1.1 gives a breakdown of the dollar amount of lending and percentages. The methods and objectives of the major creditors differed. In Britain, private enterprise subject to government regulation was the driving force behind international lending. In France and Germany, government was the largest source of funds for lending, largely because sovereign lending was seen as a method to achieve national objectives. Most British lending was channeled into portfolio investments (railway bonds, government

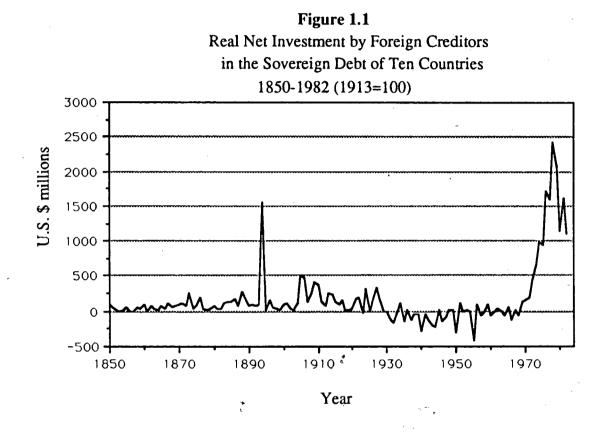
<sup>&</sup>lt;sup>3</sup> The post-1970 data is slightly exaggerated due to the inclusion of Eurodollar lending and World Bank disbursements.

and municipal securities, and bank and industrial bonds) with sovereign guarantees. By providing capital to developing countries of recent settlement, British foreign lending helped develop local economies and stimulate international trade. The bulk of French lending went to finance armaments in Russia and Latin America, while Germany provided funds to allies in order to strengthen their military power and generate a surplus in the German trade account. After World War I, France and Germany suffered great losses on these ill-advised loans.

In between the two great wars, the United States emerged as the world's largest provider of sovereign lending. During the roaring twenties, large inflows of European capital established New York as a major financial center. By 1921 the United States had far surpassed the level of lending of Britain. Unfortunately the United States was an inexperienced lender (both sovereign and private creditors). This fact, as well as the Great Depression of 1929-33, resulted in a staggering percentage of U.S. loans turning sour. Of the \$4,457 million of outstanding bonds held by the U.S. in 1945, \$2,041 million or 45.8% were in default. These were distributed geographically as follows: 86.8% of European bonds, 60.1% of Latin American bonds, 56.1% of Far Eastern bonds, and 0.3% of Canadian bonds were in default<sup>4</sup>.

From 1945 to 1950, the United States remained the world's largest creditor to developing nations with total lending and grants of some \$9,300 million. During this period foreign nations required massive capital inflows in order to rebuild war-torn economies. Unfortunately net lending to these countries was negative as seen in Figure 1.1. This rather prudent level of lending was a response to the large losses incurred before the Second World War. U.S. foreign aid slowed somewhat during the 1951-60 period to \$6,500 million, largely due to the establishment of international foreign aid agencies such as the International Monetary Fund. By 1961 official sources of lending

<sup>&</sup>lt;sup>4</sup> Angelini, Eng, and Lees (1979), p. 4.



Source: Lindert, Peter H. and Morton, Peter J. "How Sovereign Debt Has Worked." Working Paper Series No. 45: University of California-Davis, 1987.

reached 66.6% of total developed country lending. The 1960's were characterized by the tremendous growth of multinational corporations and commercial banks, the increased importance of Western Europe and Japan as providers of international credit, and the establishment and initial growth of the Euromoney and Eurobond markets. Looking ahead into the 1970's, Euromarkets quickly became established as the main alternative to the I.M.F. and the World Bank.as a source of funds for sovereign lending.

Unlike the situation at present, the lender's response to borrower default sometimes went beyond debt renegotiation. In fact, the short history of international sovereign lending is sprinkled with several interesting episodes of "gunboat diplomacy". In response to Egypt's partial default on outstanding bond issues in 1879, the British and French governments arranged for the replacement of the Khedive Ismail with his son, Tewfik. In effect, Britain and France took over control of Egypt's fiscal revenues and managed these revenues to serve the interests of European creditors. Egypt never regained her national sovereignty until after the Second World War, and only after paying a heavy debt-service load with an average interest premium of 2.92% per annum.

Throughout the early 19th century, Mexico floated large bond issues and defaulted regularly. The crisis peaked in 1859-61 when the governments and military forces of Britain, France, and Spain attempted to seize control of the customs duties previously used to pay private creditors. The situation did not improve until 1885-86, after the overthrow of two government regimes during the interval. Mexico benefited from revived lending until the Revolution of 1911.

Several other instances of diplomatic and military pressure are worthy of note. Venezuela capitulated to military pressure in 1902; the Dominican Republic suffered an invasion in 1905 by the U.S. Marines in order to seize control of customs revenues; Nicaragua suffered the same fate in 1911-12; and Mexico, Turkey, and the Soviet Union

## Chapter 1: LDC Debt Overview

	Gross Cre	ediţs		Gross Del	ots
	<u>(\$1,000 m)</u>	(%)		(\$1,000 m)	(%)
United Kingdom	, 18.0	40.9	Europe	12.0	27.3
France	9.0	20.4	Latin America	8.5	19.3
Germany	5.8	13.2	United States	6.8	15.5
Belgium, Netherlands,			Canada	3.7	8.4
Switzerland	5.5	12.5	Asia	6.0	13.6
United States	3.5	8.0	Africa •	4.7	10.7
Other Countries	<u>2,2</u>	<u>5.0</u>	Oceania	<u>2.3</u>	<u>5.2</u>
Totăl	44.0	100.0	Р	44.0	100.0

Source: A. Angelini, M. Eng, and F.A. Lees. International Lending, Risk, and the Euromarkets (John Wiley & Sons: New York), 1979.

# Table 1.1

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# Main Creditor and Debtor Countries, 1913

were denied fresh loans after their defaults before and during World War I.

Despite attempts to bring pressure upon the government of Czarist Russia between 1888 and 1914, private creditors were quite powerless to prevent the complete loss of their lent capital in 1917. During this period of large bond issues, the Russian government used the borrowed capital to build railways which serviced mainly French troop movements and armaments suppliers, all in private consultation with the French government. The Allied governments accepted these services as a form of repayment, unknown to private creditors. A portion of the borrowed capital also went towards insuring that future bond issues could be accommodated quickly. This activity took the form of bribing the foreign financial press to give favorable accounts of Russia's financial position, usually on the eve of a new bond issue. Another portion of .new borrowing was strategically placed in foreign private and central banks, whose governments might have attempted to impose trade restrictions, or some other punitive measures on Russia. By moving these large accounts in and out of banks, pressure could be brought ùpon foreign governments to reconsider their actions. Essentially no European country could afford not to lend to Russia.

Lindert and Morton (1987) note a striking consistency between countries with histories of payments problems and those countries with current payments problems. The authors suggest four possible reasons for this pattern. Certain countries, by nature of economic endowments and geography, are always more susceptible than others to external shocks which trigger debt crises. Secondly, the case of Mexico suggests that political doctrine is transferred from one policy regime to the next. Thus selfish policies such as the inflation tax, which promote debt difficulties, are not abandoned without considerable external pressure. Thirdly, governments which suffer no penalties as a result of past defaults do not fear debt difficulties in the future. Lastly, higher interest premiums and shorter maturities, which are responses to impending debt crises, could

#### Chapter 1: LDC Debt Overview

actually accelerate the onset of the crisis.

Whatever the cause of persistent debt problems in many LDCs, it is clear that these countries suffered little, if any, punishment for their irresponsible domestic policies. The "gunboat" measures already discussed were only short-term, and were usually followed by a large supply of fresh credits. The exception is the 1930-1968 period of low or negative net lending. However this period is deceiving since all LDC borrowers were credit constrained, not just the risky ones. During the sweeping popularity of trade protectionism after 1934, bilateral trade agreements favored defaulting countries as often as not.

### 1.3 The Debt Crisis Since 1970

The decade following 1969 witnessed the largest growth of LDC indebtedness in the short history of foreign lending markets (see Figure 1.1). Over the period of 1973-1984, the average annual growth in the nominal debt of the ten largest debtor countries averaged 22%, with the total debt of these countries ballooning from \$49 billion (U.S.) in 1973 to \$405 billion (U.S.) by 1984. Table 1.2 provides a breakdown of the debt explosion for the ten largest borrowers over this period. Most of this accumulation of debt took place in the mid-1970's with a gradual reduction in growth up to 1984. The late 1970's and early 1980's experienced rapidly rising real interest rates thus placing a heavy burden on LDC's to make debt service payments. In fact, by 1981 interest payments on the debt of both highly indebted countries and major borrowers<sup>5</sup> exceeded repayments of principal. This trend is seen clearly in Figures 1.2 and 1.3.

<sup>&</sup>lt;sup>5</sup> These are categories used by the World Bank. Highly indebted countries as of 1987 were Argentina, Bolivia, Brazil, Chile, Colombia, Costa Rica, Ivory Coast, Ecuador, Jamaica, Mexico, Morocco, Nigeria, Peru, Philippines, Uruguay, Venezuela, and Yugoslavia. Major borrowers were Argentina, Brazil, Chile, Egypt, India, Indonesia, Israel, Korea, Malaysia, Mexico, Turkey, and Venezuela.

Coinciding with the rapid development of Eurodollar markets, total lending to LDCs was increasingly made up of lending by private banks. In 1961, lending from official sources, such as the World Bank, the I.M.F., and the Ex-im bank, comprised some 66.6% of total lending. By 1976, this percentage had fallen to 44.9% with private banks making up the difference<sup>6</sup>. A large percentage of private loans charge a rate of interest which is LIBOR plus a country-specific risk premium. Of total loans, only 7% were floating rate loans in 1973. However by 1982 this figure rose to 37% for all borrowers, 42% for the major debtors, and in excess of 75% for Latin American debtors<sup>7</sup>.

The most commonly cited indicator of mounting debt difficulties is the debt-export ratio. Changes in this ratio should reflect the ability of the borrowing country to service its debt through export earnings. Figure 1.4 depicts the debt-export ratio for both highly indebted borrowers and major borrowers over the period 1970-1985. Generally the ratio showed little movement until 1980, when the ratio increased dramatically to near 200% for both categories of borrower. Despite the rapid accumulation of debt in the midseventies, the ratio did not respond until the 1980's when export earnings fell heavily. In most cases the fall in export earnings was caused by events beyond the control of the borrower, such as the general fall in world aggregate demand due to the global recession of 1981-82. However in some cases the cause can be attributed to overvalued exchange rates coupled with highly inflationary domestic policies. More will be said on this issue shortly.

A further revealing indicator of the debt difficulties encountered by developing countries over the period is the grant element of lending. The grant equivalent of a loan is its commitment value, less the discounted present value of its contractual debt service; conventionally, future service payments are discounted at 10 percent. The grant element

<sup>7</sup> Heffernan (1986), p. 13.

<sup>&</sup>lt;sup>6</sup> Angelini, Eng, and Lees (1979), p. 6.

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of a loan is the grant equivalent expressed as a percentage of the amount committed. It is used as a measure of the overall cost of borrowing. Loans with an original grant element of 25 percent and above are defined as concessional. Figure 1.5 illustrates the grant element of lending for both categories of borrowers over the period 1970-1985. The grant element is below 25% for the whole sample period and is negative for the early 1980's. Lending after 1969 occurred only at "hard" terms, that is, at relatively high borrowing rates and short maturities. Thus it may be surmised that lenders were partly to blame for bringing on losses due to default and rescheduling.

Unlike the pre-World War II period, the modern period of sovereign borrowing has not experienced any episodes of "gunboat diplomacy" to recover losses incurred by default. Outright default is a rarity today, even though many LDCs appear swamped by the service payments necessary for old debt. The modern period has brought with it debt rescheduling as the principal method to handle crises. In this way, problem borrowers may continuously roll-over old debt into new debt, thus avoiding any possibility of being denied future access to credit markets, as happened frequently during the historical period. Debt rescheduling does not come without heavy costs. Lending rates on new debt packages typically exceed LIBOR by a higher percentage than the old debt as well "as being of a shorter maturity. These high costs do not seem to deter problem debtors from using debt rescheduling as a method of delaying payment of old debt to the future, and obtaining a fresh inflow of new funds. Table 1.3 provides a short history of debt rescheduling during the modern period of lending 1975 - 1983. In both numbers and dollar amounts the number of official and private rescheduling has shown a fairly consistent upward trend to 1983.

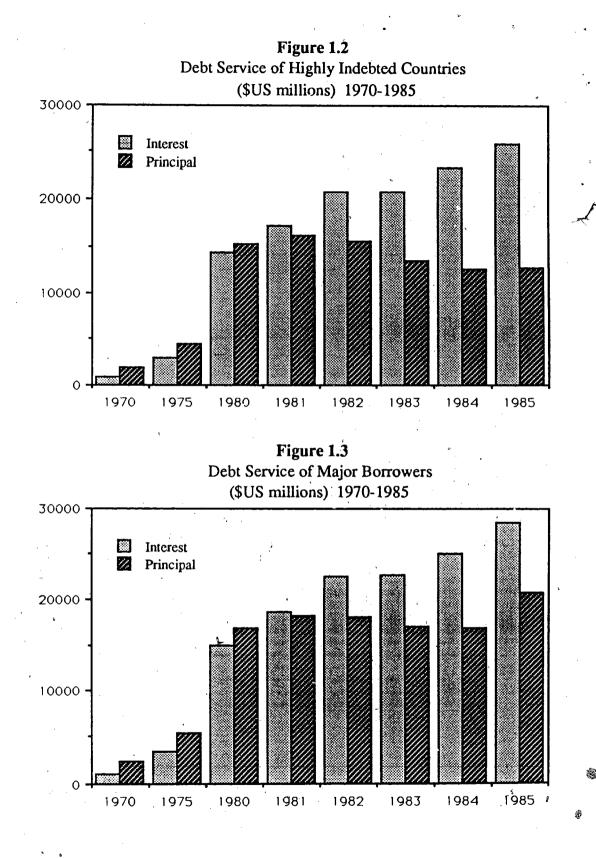
Total External Debt of the Ten Largest Borrowers, 1973-1984											
	(US \$ billions)						ł	Avg. Yr.			
Country	1973	1975	<u> 1977</u>	<u> 1978</u>	<u> 1979</u>	<u>1980</u>	<u>1981</u>	1982	1983	<u>1984</u>	<u>%                                    </u>
Mexico	8.6	16.9	27.1	33.6	40.8	53.8	67	84.6	90	90	26.0
Brazil	13.8	23.3	35.2	48.4	57.4	66.1	75.7	82.2	93	100	21.0
Argentina	6.4	7.9	9.7	12.5	19.0	27.2	35.7	38.0	40	42	20.0
Chile	3.2	5.4	5.6	7.9	9.5	11.4	15.6	17.2	18	19	18.9
Venezuela	4.6	5.7	12.3	16.3	23.7	27.5	29.3	31.3	31	32	21.0
Peru	1.4	4.0	5.7	6.4	7.9	9.2	10.0	11.2	13	.14	25.0
Indonesia	5.7	8.9	12.8	14.5	14.9	17.0	18.0	21.9	26	30	16.4
Philippines 🖌	1.9	3.8	7.1	9.3	11.2	13.9	17.3	20.7	26	32	30.0
Egypt	2.2	5.9	10.0	12.9	15.4	17.8	20.3	21.8	24	26	27.0
Nigeria	1.2	<u>1.1</u>	<u>0.9</u>	<u>3.3</u>	<u>5.2</u>	<u>7.1</u>	<u>9.9</u>	<u>11.2</u>	<u>16</u>	<u>20</u>	<u>30.0</u>
Total	49	83	126	165	205	251	299	346	377	405	22.0
% change		69.2	52.5	30.6	24.2	22.4	19.0	15.8	8.9	7.4	

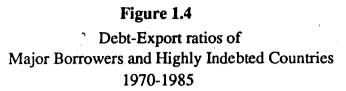
### Table 1.2

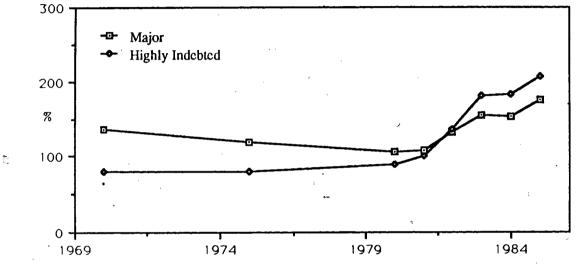
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Total External Debt of the Ten Largest Borrowers, 1973-1984

Source: Webster, Thomas J. <u>Analyzing Country Risk: Estimating the Probability of External Debt</u> <u>Repudiation in the Post-Oil-Embargo Decade</u> Unpublished Ph.D Dissertation, City University of New York, 1985.







Source: World Debt Tables, Washington: World Bank, 1986.

#### Chapter 1: LDC Debt Overview

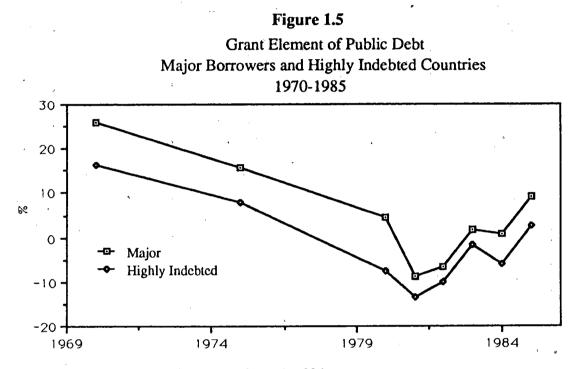
The latest major rescheduling occurred in 1987 when Brazil suspended debt service payments for most of the year. This tactic resulted in a rescheduling package of approximately \$2 billion.

Several hypotheses have been put forth in the literature for the causes of the historically high levels of indebtedness of developing countries who can ill afford it. On the supply side, the I.M.F. (1983) has identified three main causes:

1) The average real interest rate on LDC external debt over the period 1961-1970 was 4.1%<sup>8</sup>. This rate dropped to an astounding -0.8% over the 1971-1980 period. By 1981-1982, real interest rates had begun to rise dramatically as inflation declined but nominal interest rates remained high. At this time LIBOR rose to an average of 14.8% resulting in an average real interest rate in 1982 of 11.0%. The I.M.F. estimated that more than one-third of the rise in LDC debt during the late seventies was due to an increase in net interest payments, the excess of interest payments on external debt over interest received on reserves and other financial assets.

2) A sizeable deterioration in the terms of trade for most developing country debtors resulted in reduced export revenues to finance domestic investment and consumption expenditures. Already heavy debtors had to borrow even more funds in the late seventies to make up the difference. Between 1979-1982, the terms of trade for all LDCs fell by 13%, while falling by 20% for oil-importing LDCs. This rapid deterioration was largely due to the worldwide recession brought on by oil price shocks and tight monetary policies of the developed nations. The I.M.F. estimates that this effect was responsible for another one-third of the increase in LDC indebtedness.

Cline (1984).



Source: World Debt Tables, Washington: World Bank, 1986.

Incidence of Rescheduling (1975-1983)								
	Official debt	rescheduling <sup>9</sup>	Bank debt rescheduling <sup>10</sup>					
Year	Number	U.S.\$ millions	Number	U.S.\$ millions				
1975	1	230						
1976	1	270		х. — 🗣				
1977	3	249						
1978	3	1,783	2	449				
1979	4	2,987 ``	2	2,847				
1980	3	3,072	5	1,,243				
1981	° 7	1,079	6	1,65611				
1982	6	529	6	4,867				
1983	9	4,219	20	59,28812				

### Table 1.3

Incidence of Rescheduling (1975-1983)

Source: D. McFadden, R. Eckaus, G. Feder, V. Hajivassiliou, and S. O'Connell. "Is There Life After Debt? An Econometric Analysis of the Creditworthiness of Developing Countries." in <u>International Debt</u> and the Developing Countries edited by G.W. Smith and J.T. Cuddington, Washington:World Bank, 1985.

<sup>9</sup> Seven countries have rescheduled debt more than once.

<sup>&</sup>lt;sup>10</sup> Signed or agreed in principle, excluding two nonmembers of the IMF.

<sup>&</sup>lt;sup>11</sup> Excluding two Polish reschedulings with a combined total of \$4.6 billion.

<sup>&</sup>lt;sup>12</sup> Excluding a Polish rescheduling for \$1.0 billion.

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3) Increases in oil prices, particularly in oil-importing LDCs, contributed the remaining one-third of the increase in indebtedness. Between 1973-1984, the spot price of Saudi Arabian crude rose from \$2.70 per barrel to \$30 per barrel, an astounding increase even when discounted for inflation. On the other hand, large OPEC current account surpluses provided ample funds for LDCs to finance these oil price shocks. The period of the 1970's witnessed a massive movement of funds to OPEC producers, which then were recycled into petrodollars and placed in Eurocurrency banks. In 1973, OPEC ran a combined current account surplus totalling \$7.7 billion, while non-oil producing LDCs ran a combined current account deficit of \$6.2 billion<sup>13</sup>. One year later, the OPEC current account surplus expanded to \$59.5 billion, while the non-oil producing LDCs fell to a deficit of \$23.3 billion. OECD banks receiving the large gush of new deposits enthusiastically used the funds to invest in domestic and foreign real estate, government securities, precious metals, and loans to LDCs. The rapid increase in international liquidity served to postpone the LDC debt crisis until the early 1980's.

Dornbusch (1985) takes a different approach to explaining the causes of the debt crisis in the 1980's than the traditional reasons already discussed. By analyzing the separate components of external debt, Dornbusch explains how domestic policies designed to maintain an overvalued exchange rate could have contributed to high levels of debt. Formally, the increase in net external liabilities is:

Increase in net external liabilities = Private investment - Private saving + Budget deficit

In this simple relationship, external debt can the result of increases in private investment, decreases in private saving, or increases in the budget deficit. Each cause is analyzed in turn.

<sup>&</sup>lt;sup>13</sup> Webster (1985).

#### Chapter 1: LDC Debt Overview

An overvalued fixed exchange rate will lead to expectations of future exchange rate depreciations if investors are forward looking. Investment is affected through two channels. An anticipated depreciation implies capital gains on imported goods. In this case firms would purchase imported goods before the anticipated depreciation and hold them to collect capital gains. This behavior is dampened by three effects: the increasing marginal cost of carrying inventories, the uncertainty about future prices, and the opportunity cost of tieing investment funds into inventory (the nominal rate of interest). If the anticipated depreciation is not reflected in higher nominal interest rates and the variance of price changes is small, inventory investment should rise.

If fixed investment has a high import content, changes in the real exchange rate will affect the user cost of capital. If the real exchange rate is overvalued, fixed investment will be high to the reduced user cost of capital. Once the anticipated depreciation occurs, fixed investment will fall. Thus overvaluation will lead to both high inventory and fixed investment, which through through the net foreign liabilities equation, will raise debt demand *ceteris paribus*.

Multiperiod saving is a function of the present value of all future anticipated wealth (present value of after-tax labor income plus initial assets) and the real rate of interest. In this case, the real interest rate is the world nominal rate less the domestic rate of inflation. Anticipated increases in future after-tax income will cause dissaving in the current period in order to smooth savings and consumption behavior. With high intertemporal substitutability of savings and consumption, savings will fall in periods of low real interest rates and rise in periods of high real interest rates. In addition, an overvalued exchange rate reduces the price of consumer durables relative to the real rate of interest. Thus a transitory overvalued exchange rate will lead to dissaving. Through the net foreign liabilities equation, reduced savings will increase the demand for debt *ceteris paribus*.

In order to generate a mechanism between the exchange rate and the budget deficit, the assumption of Ricardian equivalence must be invoked. Any increases in the current budget deficit lead to consumer expectations of higher future taxes to finance the higher deficit. When these higher future taxes are discounted, consumers will increase current savings by the correct amount. However in the international debt situation, the link between higher future taxes and reduced current savings is not perfect since higher foreign borrowing and eventual debt default are always an available alternative. Following a policy of supporting an overvalued exchange rate will raise the current flow of debt service payments. To the extent that these higher payments are financed by additional foreign borrowing, net foreign liabilities will increase. However this effect will be dampened by consumer anticipations of higher future tax collections causing an increase in current savings. It is the possibility of debtor default without cost which causes a net increase in net foreign liabilities.

What has been the response of major lenders to the gloomy debt situation of the 1980's? Bennett and Zimmerman (1988) provide some revealing statistics for U.S. banks. Outstanding loans to LDCs fell from a total of \$152.6 billion in 1981 to \$133.6 billion at the end of 1986. Moreover only a small percentage of new lending represents fresh loans. The bulk of recent lending reflects rollovers of already existing loans or rescheduling. As measured by secondary loan markets, U.S. banks suffered large declines in the market value of their outstanding LDC debt. However it is only recently that a portion of the capital loss was written down in the form of loan loss reserves<sup>14</sup>. To counter the effects of the decline of the market values of their portfolios, U.S. banks resorted to accumulating capital through retained earnings, sales of assets, and sales of new equity and subordinated debt, as well as curtailing asset growth overall. As a result, LDC loan exposure relative to book capital fell from a peak of 243 percent in 1982 to 115 percent in 1986. However as a percentage of total lending to book capital, LDC lending has claimed a rising share since 1982. Perplexingly the decline in bank exposure

<sup>&</sup>lt;sup>14</sup> Over \$19 billion was added to loan loss reserves during 1987.

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has been most dramatic for those LDCs not experiencing debt problems. Exposure to the "Baker Fifteen"<sup>15</sup> rose as a percentage of U.S. banks' international loans outstanding from 25.9% to 31.3% between 1981 and 1986. Thus while total lending to LDCs has declined in response to the debt difficulties of the early 1980's, the share of U.S. bank exposure to the most problematic LDCs has actually increased.

### 1.4 Summary

While this chapter has suggested that the current international debt situation is not unprecedented in the short history of international capital markets, it does suggest that the magnitude of accumulated LDC debt is much higher than ever seen before in real terms. The characteristics of these markets changed considerably after the 1960's. Forms of debt switched from almost complete domination by bond issues (of moderate to long maturity and relatively low lending rates) to official and private loans (of short maturities and relatively high lending rates). The explosion of international lending in the 1970's followed a long period (1930-1968) of net negative lending caused by the overly cautious reaction of private and sovereign lenders to the defaults of the early twentieth century (particularly those of Russia and Mexico which followed revolutions), Bond premiums (over the U.S. long term Treasury bill rate) fell gradually over the pre-1970 period indicating a softening of credit terms. In the post-1969 period, risk premiums over LIBOR were generally comparable to bond premiums in the historic period, although the lending rates on short term loans were much higher than the lending rates on bonds. Perhaps coinciding with the similar risk premia is the fact that problem debtors of the historic period tend to be the problem debtors of today.

With the establishment of fixed exchange rates in 1944 under the Bretton Woods

<sup>&</sup>lt;sup>15</sup> The "Baker Fifteen" being the fifteen major LDC debtors identified in Treasury Secretary James Baker's 1985 rescheduling plan. These countries include Argentina, Brazil, Mexico, and Venezuela, among others.

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agreement, the role of the I.M.F. assumed increasing importance over the historic period. The I.M.F. had the power to approve or order changes in par values if the member country's balance of payments problem was deemed to be a 'fundamental disequilibrium'. If the problems were not of a fundamental nature, members were granted access to credit facilities. Generally the I.M.F. served its role very well until the 1970's. Between 1978 and 1980, as many LDCs experienced severe balance of payments difficulties, the average share of fund credit as a percentage of external finance for LDCs was only 2%, rising to 7% by 1981. With the rapid development of international capital markets and large supplies of petrodollars available, the I.M.F. has been reduced to a lender of last resort role. Since the adoption of flexible exchange rates, the ability of the I.M.F. to fulfill its role has been greatly eroded.

The generally accepted reasons for the rapid increase in the stocks of real debt of developing countries during the late 1970's and early 1980's, were high real interest rates, unexpected declines in the terms of trade, oil price shocks, and the maintenance of overvalued nominal exchange rates. All of these explanations, except perhaps for the last, can be characterized as unexpected real shocks. These real shocks placed a high burden on the ability of the borrowing country to generate sufficient income internally to finance debt service payments and maintain a current standard of living. To achieve these objectives, more debt must be taken on until such time that developing economies are healthy enough to generate enough savings at home. Debt can be used as a method to transfer future consumption to today in order to smoothen consumption expenditures<sup>16</sup>. This ability to smoothen consumption does not mean that the funds from new debt are merely used to purchase non-durable consumption goods, rather the new debt can be invested in durable capital equipment in order to raise output. With a constant marginal propensity to consume, consumption is maintained at previous levels. This transmission mechanism will be explained further in Chapter 3.

<sup>&</sup>lt;sup>16</sup> Likewise it can be reduced in order to transfer current consumption to the future in the case of an unexpected shock which increases the ability to produce output.

### **Chapter 2: Review of Relevant Literature**

The growth of the debt capacity and country risk literature has paralleled the growth of developmental assistance programs by major organizations, such as the World Bank and the I.M.F., to lesser developed countries since the late 1950's. Typically these studies focused on a two-stage process: 1) the determination of the "absorptive capacity" of the underdeveloped economy<sup>17</sup>; 2) an analysis of the conditions under which external finance can successfully stimulate economic growth and avoid debtor default. Since the mid 1970's the theoretical emphasis shifted from the first stage to the second, particularly focusing on optimal loan contracts to preclude debtor default. A large empirical literature emerged as well, which studied leading indicators which may give lenders information as to the risk inherent in offering a loan contract to a particular borrower. This chapter will not treat theoretical and empirical approaches separately since many empirical techniques are derived from theoretical modelling. The chapter will show that none of the models surveyed adequately explains the rapid debt accumulation of the 1970's.

The empirical analyses of debt capacity and country risk can be divided into four broad modelling techniques: discriminant models, principal components models, logistic models, and disequilibrium models<sup>18</sup>. Each method answers a different question thus the results are not comparable across models. A summary discussion of the problems inherent in these models will be provided in order to avoid unnecessary repetition.

<sup>&</sup>lt;sup>17</sup>The "absorptive capacity" is the level of investment beyond which investment can no longer be raised due to a scarcity of factors of production (Gulhati (1967)). "Absorptive capacity" is found where the marginal rate of return on investment is equal to the appropriate discount rate.

<sup>&</sup>lt;sup>18</sup> A fifth method used by Fisk and Rimlinger (1979) uses a nonparametric approach to test whether certain key economic variables are close to their values for previous cases of debt rescheduling. This approach does not test for the significance of risk indicators.

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### 2.1 Discriminant Models

Frank and Cline (1971) used discriminant analysis to analyze debt-rescheduling behavior of 26 countries over the period 1960 to 1968. The purpose of discriminant analysis is to identify independent variables which can be used to group countries into one of two categories: rescheduling and non-rescheduling. The technique is similar to a multiple regression on a binary dependent variable. Frank and Cline chose 8 independent variables and found that only three were statistically significant at the 5% confidence level: the debt service ratio<sup>19</sup>, the imports/reserves ratio, and the amortization/debt ratio<sup>20</sup>. Unfortunately the sample period contained only 13 debt reschedulings out of 145 observations leading one to question the ability of the model to statistically discern between the two groups of countries (10 of the 13 rescheduling cases were predicted), particularly when obtaining out of sample predictions.

Using a set of 20 independent variables Grinols (1976) performed a discriminant analysis for 64 countries over the sample period 1961 to 1974. Five significant independent variables were found: the debt service/reserves ratio, the disbursed debt/debt service ratio, the debt service/imports ratio, the total debt/GDP ratio<sup>21</sup>, and the total debt/exports ratio. The errors in predicting debt reschedulings were almost 50% lower than those obtained by Frank and Cline, although the two results are not directly comparable due to different samples.

Sargen (1977) incorporated a monetary approach to debt rescheduling by including as independent variables the rate of inflation and the rate of monetary growth. An increase in the price of a representative domestic non-traded good lowers the relative price of imports, hence increasing the demand for foreign capital through a worsening

<sup>&</sup>lt;sup>19</sup> The debt-service ratio is the ratio of debt repayments, including both interest and principal, over the flow of exports for a given period ot time.

<sup>20</sup> The amortization/debt ratio is the ratio of debt service payments on principal only over total debt.

<sup>&</sup>lt;sup>21</sup> Total debt, as distinguished from debt outstanding, includes debt which is undisbursed.

trade balance. Six independent variables were used over the sample period 1960 to 1975. Sargen found the rate of inflation, the rate of monetary growth, the debt service ratio, the growth rate of exports, and deviations from purchasing power parity to be statistically significant. Interestingly high inflation countries, such as those in Latin America, dominated the rescheduling group.

Despite the fact that discriminant analysis is a fairly complicated statistical procedure, it is not without it's shortcomings<sup>22</sup>. Firstly the independent variables are introduced into the discriminant function without any theoretical model to justify their inclusion (this is a problem common to all techniques). In this case the coefficient estimates are extremely sensitive to adding or dropping independent variables. Secondly despite the reporting of significance tests by the authors, these tests cannot be performed because the error distributions are not normal, making any attempt to rank the importance of the independent variables erroneous<sup>23</sup>. Thirdly the binary nature of the dependent variable leads to an unrealistic division between rescheduling and non-rescheduling countries. Debt problems are costly to renegotiate and occur only with a lag after the problem arises. Indeed some countries have consistently poor indicators but rarely reschedule their debt. Countries do not suddenly move from non-reschedulers to reschedulers (Feder and Just (1977)). Since all of the four methods to analyze country risk suffer somewhat from this problem, any further discussion will be delayed to the end of this chapter.

### 2.2 Principal Components Models

In principal components analysis a set of indicators are extracted from the sample 22 Eisenbeis (1977) provides an excellent summary of these shortcomings in much greater detail than given here.

<sup>23</sup> However discriminant analysis has asymptotic properties which make it superior to other techniques, such as logistic models. See Judge et al. (1985) page 768.

data which are linear combinations of the original independent variables<sup>24</sup>. The indicators are chosen so as to maximize the dependent variable variation explained by each. This type of analysis is especially useful when a large number of independent variables must be considered. Dhonte (1975) aggregated ten independent variables into two principal components using a sample of 13 rescheduling countries over the period 1959 to 1971. Only two-thirds of the rescheduling cases were predicted by the model. An unfortunate feature of principal components analysis is that the aggregated indicators lack any clear meaning. The variables are aggregated purely on the basis of best fit though there is no clear reason why certain variables should be combined with others. Dhonte attempted to correlate various explanatory variables with the two principal components, although these explanatory variables seem unrelated to the meaning attached to the principal components. In addition the analysis lacks explanatory power due to the small sample size, although out of sample tests were not performed.

### 2.3 Logistic Models

Since Feder and Just (1977a) the logistic model has been the most popular type of model to analyze debt capacity and country risk. Like discriminant models and principal components models, the logistic model identifies significant explanatory variables with respect to a binary dependent variable reflecting rescheduling countries and non-rescheduling countries. In addition the logistic model allows for the estimation of the probability of rescheduling for any country in the sample as well as the ability to conduct standard hypothesis tests<sup>25</sup>. Formally the logistic model assumes that the probability of rescheduling is related to a vector of independent indicators by the relationship:

<sup>&</sup>lt;sup>24</sup> A good discussion is contained in Judge et al. (1985), chpt. 22.5.

<sup>&</sup>lt;sup>25</sup> The coefficient estimates are asymptotically normal and approach normality for large sample sizes (similarly so does the t distribution). See Pindyck and Rubinfeld (1981) p. 287-301.

$$P(X) = \frac{\exp(b'X)}{1 + \exp(b'X)}$$

where b is a vector of fixed coefficients. The left-hand side cannot be observed thus a random variable with value Y=1 for rescheduling and Y=0 for non-rescheduling is substituted to obtain:

(2.3.1)

(2.3.2)

$$P(Y = 0 | X = X_i) = \frac{1}{1 + \exp(b | X_i)},$$

$$P(Y=1 | X = X_{i}) = \frac{\exp(b'X_{i})}{1 + \exp(b'X_{i})}$$
(2.3.3)

where i = 1, 2, ... n. Forming the likelihood function one obtains:

$$L = \frac{\prod_{i=1}^{n} \exp(b'X_{i}Y_{i})}{\prod_{i=1}^{n} (1 + \exp(b'X_{i}))}$$
(2.3.4)

By maximizing L with respect to the vector of coefficients b, coefficient estimates can be obtained using an iterative procedure which are consistent, efficient, and asymptotically unbiased<sup>26</sup>. The rescheduling probabilities can be obtained by substituting the estimate of the vector b into (2.3.1).

Feder and Just (1977a) used a logistic model to estimate the probability of debt

<sup>&</sup>lt;sup>26</sup> Many econometric programs will perform this maximization procedure with minimal computational cost. See for instance White and Horsman (1986) p. 105-108 for a description of the procedure using SHAZAM. An alternative procedure is to transform the estimating equation.

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rescheduling for a sample of 41 countries over the period 1965 to 1972. Of seven initial indicators examined, five were found to be statistically significant: the debt-service ratio, the imports/reserves ratio, per capita GDP, the capital inflows/ debt service ratio, and a five year moving average of export growth rates. The final estimating equation was found to be quite sensitive to model specification, particularly when the amortization/febt ratio was excluded. This could be because the causal relationship between this ratio and debt rescheduling is not clear. The likelihood ratio index (analogous to  $R^2$  in an OLS regression) scored above 0.90 for each of four regressions indicating a good fit. Despite the fact that the sample of 238 observations contained only 21 reschedulings, out of sample tests for an auxiliary set of borrowing countries performed quite well (only 4.9% of reschedulings not predicted).

Mayo and Barrett (1977) used a logistic procedure identical to that used by Feder and Just, however using a much larger sample. The data included 571 observations on 48 countries for the sample period 1960 to 1975 (2.5 times the sample size used by Feder and Just (1977a) and 4 times the number in Frank and Cline (1971)). The model included six indicators which were all found to be statistically significant<sup>27</sup>. However despite the large sample size the likelihood ratio index measured only 0.63, indicating that the model could be improved by respecification of the indicators. Unfortunately the choice of the six indicators was made from a larger set of indicators with the choice being predicated on the correctness of sign, consistency of the sign over a number of estimations, and overall significance. It would be just as interesting to the reader to learn what indicators suffered from these problems for future research. The model correctly predicted a rescheduling for 76% of the in-sample cases, however out of sample tests were not performed.

Saini and Bates (1978) tested the predictive power of a logistic model versus a

 $<sup>^{27}</sup>$  To avoid unnecessary clutter, a taxonomy of indicators used by all studies will be provided at the end of the chapter.

discriminant model and found neither model to be superior empirically 28. Their sample contained 298 observations across 25 countries for the period 1960 to 1977. Unlike previous studies the dependent variable included formal reschedulings and balance of payments support loans, which are defined as foreign loans which prevent reschedulings or payments arrears from occurring. Out of a set of 11 initial indicators, four were found to be consistently significant: the inflation rate, the rate of money supply growth, the adjusted cumulative current account balance/exports ratio, and the rate of growth of reserves. Interestingly the debt service ratio was never found to be statistically significant<sup>29</sup>.

The latest study to use a logistic model to predict debt rescheduling problems is that of Cline (1984). Cline's sample is even larger than that used by Mayo and Barrett: 670 observations over 60 countries covering the period 1968 to 1982 (only 22 cases of rescheduling). A major advantage of Cline's model over previous models is the specification of a credit market through demand and supply functions. After solving for a reduced form estimating equation, the empirical results reveal that 7 of the original 11 indicators are statistically significant: the debt service ratio, the reserves/imports ratio, the amortization rate (when included jointly with the debt service ratio), the current account deficit/export ratio, per capita GDP growth, the net debt/export ratio (with debt service ratio excluded), and the quantity of global borrowing. The ability of the model to predict reschedulings was quite adequate as only 9.1% of the in-sample debt reschedulings were unpredicted. Cline further disaggregated the sample by focusing on large debtor countries. This reduced the sample to 31 countries including several precariously near default in the early 1980's: Bolivia, Brazil, Chile, and Mexico. The results of the reduced sample model were consistent with the large sample model except

<sup>&</sup>lt;sup>28</sup> This section is summarized from Saini and Bates (1984).

<sup>&</sup>lt;sup>29</sup> The irrelevance of insolvency and illiquidity measures will be discussed later in the chapter. Saini and Bates (1984) note that previous studies adjusted the debt service ratio to account for decreases in debt service payments during and following reschedulings. This arbitrary overvaluation of the ratio may have led to it's previous statistical significance.

for the increased error rate of 21.4%, indicating that these countries respond to different variables than those of the larger sample.

While the objective of the already mentioned logistic models is to predict debt reschedulings for borrowing countries, a second strand of literature has used logistic models to analyze the terms under which loan contracts are made by using risk premiums as dependent variables. Given the difficulty of defining and measuring debt reschedulings (not enough observations), this second approach may be more appropriate. After all a default or rescheduling is the result of a violation of the terms of the loan contract between the creditor and debtor. Default and rescheduling only come to realization after a set of decisions are made by the two parties, not automatically after the probability of such an event reaches some critical level. The risk premium reflects an equality between loan demand and supply where both lenders and borrowers make the market. Rather than asking the question, "When will a certain country default?", we should realize that default is the violation of a set of contracts and instead ask "How will country risk be reflected in the terms of loan contracts?"<sup>30</sup>. Further discussion of this distinction will be delayed until the end of the chapter.

Feder and Just (1977b) is the first attempt to model the relationship between the risk premium charged on foreign loans and indicators of borrower default. The problem is to maximize the lender's utility from future debt service payments less the utility lost from possible default with respect to changes in the interest rate charged on loans<sup>31</sup>:

$$\max_{L} U = [1 - P(X)] U \{r \ \emptyset L\} + P(X) \int_{h}^{1} U \{-hL\} \Omega(h) dh \qquad (2.3.5)$$

<sup>&</sup>lt;sup>30</sup> This analogy is similar to Cheung's (1983) analysis of the definition of a firm.

<sup>&</sup>lt;sup>31</sup> Feder and Just (1980) maximizes the same objective utility function with respect to the size of the loan being considered due to the assumption of a perfectly competitive loan market. However the final estimating equation is equivalent to their earlier paper.

where P(X) is the probability of default conditional on the vector of risk indicators X, r is the interest rate charged on the loan,  $\emptyset$  is a discount factor, L is the size of the loan demanded where L = L(r), h is a random variable representing the loss rate under default, and  $\emptyset(h)$  is the subjective probability of achieving a given loss rate. After rearranging the first order condition one obtains, for a given loss rate:

$$r = \frac{\eta}{\eta - 1} \cdot \frac{P(X)}{1 - P(X)} \cdot \frac{1}{\phi} \cdot \frac{U'(-\overline{h}L)}{U'(r\phi L)}$$
(2.3.6)

The first term on the RHS is a ratio of the demand elasticity for loans. This term represents the borrower's bargaining position which depends on alternative sources of supply and the overall demand of the borrower. The impact of risk indicators on the probability of default is included in the second term, P(X)/(1-P(X)). The last term, U'(-hL)/U'(rØL), represents the extra risk premium which is charged due to risk aversion on the part of the lender (with U"<0).

Feder and Just make several simplifying assumptions to allow for econometric estimation. It is concluded that if the volume of the loan under consideration is small relative to the initial wealth of the lending bank, the last term may be negligible, thus it is excluded. The second term, P(X)/(1-P(X)), can be replaced with a linear vector of  $\beta$ coefficients and a linear vector of risk indicators if the probability of default is distributed logistically. Taking logs of the estimation equation thus far:

$$\ln r = b_0^* + \sum_{i=1}^k b_i x_i - \ln \emptyset + \ln \left\{ \frac{\eta}{\eta - 1} \right\}$$
(2.3.7)

The discount term  $\emptyset$  depends on both the loan duration, which is observable, and the perceived average cost of capital (r\*), which is not observable. Feder and Just

postulate that r\* can be expected to be the average of LIBOR rates for the previous 12 months preceding the loan, but to simplify the model they assume that this relationship is non-stochastic and thus constrain the coefficient to be -1. The last term is not known and cannot be easily observed unless the demand elasticity for loans is estimated. Since the demand elasticity will be specific for each borrowing country and may change over time, two dummy variables are introduced into the equation to account for shifts in the intercept due to these factors. For empirical purposes:

$$\ln\left[\frac{\eta_{ij}}{(\eta_{ij}-1)}\right] = \mu + u_i + v_i + w_{ij}$$
(2.3.8)

where u is a country specific error term and v is a time specific error term. Substituting (2.3.8) into (2.3.7) the regression coefficients for the risk indicators can be obtained using the method of error components outlined in Judge et al. (1985, chpt. 13.4). Six risk indicators were found to be statistically significant and of correct sign: the loan duration, the modified debt service ratio, an export fluctuations index, the imports/GNP ratio, the imports/reserves ratio, and projected GDP growth. All six indicators carry the correct sign when default risk is perceived only by the lender. However it appears that the model is incapable of generating anything other than the expected signs. After all the borrower's demand for foreign lending is only a function of the interest rate currently charged on loans implying that if the model is projected into interest rate-loans space, the demand curve is downward sloping and does not shift. Any change in the perceived risk indicators shifts the supply curve, tracing out the demand curve, but does not shift demand. It is quite likely that demand is sensitive to a portion of the risk indicators which shift supply, which if included in the reduced form estimating equation could result in different signs<sup>32</sup>. It may be much more informative to respecify the

<sup>&</sup>lt;sup>32</sup> The reduced form coefficient signs would depend on the magnitudes and signs of the responsiveness of demand and supply ato each respective risk indicator.

borrower's demand function to include exogenous shift parameters and then use a twostage estimating process to identify the responsiveness of demand and supply separately to differing risk indicators.

A second technique to derive a very similar estimating equation is utilized by Edwards (1984). The equilibrium condition for a risk-neutral lender is given by:

 $(1-p)[1+(i^*+s)] = (1+i^*)$  (2.3.9)

where p is the probability of default, i\* is the exogenously given risk free rate of interest, and s is a risk premium. Solving for the risk premium one obtains:

s = [p/(1-p)]k(2.3.10)

where  $k = (1 + i^*)$ . The expected probability of default p is a function of a set of risk indicators and is assumed to possess the same logistic distribution as given in (2.3.1). Substituting (2.3.1) into (2.3.10) a log-linear equation is obtained:

$$\ln s = \ln k + \sum_{i=1}^{n} b_{i} x_{i}$$
(2.3.11)

Replacing ln k with two error terms reflecting country-specific and time-specific effects <sup>33</sup> and inserting a random disturbance term, an estimating equation is obtained which is similar to that used by Feder and Just (1977b):

$$\ln s_{\pi} = g_{n} + a_{i} + \sum_{i=1}^{m} b_{i} x_{m} + e_{\pi}$$
(2.3.12)

<sup>&</sup>lt;sup>33</sup> Edwards (1984) and Feder and Just (1977b) assume that these effects are random. In a subsequent paper Edwards (1986) assumes that these terms are fixed.

where subscripts n and t represent country and time specific effects respectively. Edwards estimated (2.3.12) using observations on 727 public and publicly guaranteed loans granted to 19 LDC's over the period 1976 to 1980. Generally the results were disappointing with only 3 of 14 risk indicators being statistically significant: the debt/GNP ratio, the reserves/GNP ratio, and the debt service ratio. Essentially equation (2.3.10) represents the supply curve of loans made by lenders. Edwards posits that the aid supply function becomes vertical when the probability of default is one. However rational lenders will always impose a credit ceiling before this point is reached, otherwise any hope of collecting debt service on previous lending is lost. For this reason equation (2.3.12) may be mis-specified.

In a follow-up study Edwards (1986) enlarged the sample to include observations for 26 LDCs over the same sample period. Despite the larger sample size the statistical results were even more disappointing than the first study: only 2 significant risk indicators were found, the debt/GNP ratio, and the gross investment/GNP ratio, and several indicators possessed incorrect signs. Edwards also regressed (2.3.12) on bond spreads for the same sample period on the argument that since bond spreads are determined by a large number of market traders, they should reflect risk much more accurately than a spread determined by bargaining between a loan lender and borrower. Curiously the same independent variables are used for the bond market as for the loan market. It may be that LDCs supply bonds for different reasons than foreigners supply loans. Since many LDCs must typically raise the interest rate on these bonds in order to sell the issue completely, the interest rate is also a function of the quantity of bonds issued to be sold, however this simple fact is ignored. Is equation (2.3.10) a supply function for bonds or a demand function? It would appear that the risk premium on LDC bonds is determined in international capital markets which are continuously trading, implying that equation (2.3.10) is a demand function for LDC bonds. Econometric estimates would have to provide opposite signs from those expected to be

conclusive on this matter. The results indicate that bond traders are very poor risk evaluators as the estimating equation fit quite poorly. Only 3 of 12 risk indicators were statistically significant: the debt/GNP ratio, the gross investment/GNP ratio, and bond maturity. Several of the indicators had the wrong sign.

<sup>2</sup> The model used by Edwards suffers the same weakness as the more complicated model developed by Feder and Just. The estimating equation is a reduced form which has not correctly incorporated demand behavior by borrowers, thus the coefficient signs and their magnitudes are not reliable. Changes in indicators of default risk can effect the behavior of both lenders and borrowers, yet only the demand for loans is identified (actually over-identified) by the inclusion of exogenous risk indicators in the supply function. The supply curve is left unidentified. A useful extension of these two risk premium models would be to use a two-stage estimation process to identify both the demand and supply functions<sup>34</sup>.

### 2.4 Disequilibrium Models

Disequilibrium models of debt capacity and country risk incorporate the possibility of credit rationing to borrowing countries on the part of lenders. Credit rationing occurs when, at some interest rate, the demand for loans exceeds any forthcoming supply. Stiglitz and Weiss (1981) argue that there may an interest rate ceiling beyond which the lender will not enter into any loan contracts due to a rapidly increasing expected loss from default<sup>35</sup>. In this case the quantity of loans extended will be determined by supply alone, not by the intersection of demand and supply. Hence a reduced form model of loan demand and supply, such as used by Feder and Just (1977b), would be an

<sup>&</sup>lt;sup>34</sup> Unfortunately a two-stage procedure may be difficult to implement in the context of a pooled timeseries cross-section model.

<sup>&</sup>lt;sup>35</sup> For an interest rate beyond the ceiling rate, borrowers will take on riskier projects with the borrowed funds as measured by the variance of returns, in order to meet higher debt service payments. This in effect raises the size of financial loss for a given probability of default.

inappropriate estimating technique. With disequilibrium models it becomes necessary to identify separate loan demand and supply functions when foreign credit is both constrained and unconstrained.

Eaton and Gersovitz (1980) develop a two-regime model, credit rationing and noncredit rationing, utilizing the following equation system:

$D_{\rm ii}=\beta_1 X_{\rm i}{}^1+u_{\rm 1i}$		(2.4.1)
$D_{2i} = \beta_2 X_i^{1} + u_{2i}$	- <b>*</b>	(2.4.2)
$D_{i} = \min(D_{1i}, D_{2i})$	ă.	(2.4.3)
$R_i = R_{1i} = \gamma_1 X_i^{21} + v_{1i}$ if $D_{2i} \ge D_{1i}$		(2.4.4)
$= R_{21} = \gamma_2 X_i^{22} + v_{2i} \text{ if } D_{2i} < D_{1i}$	. , <i>n</i> o	(2.4.5)

Here  $D'_{1i}$ ,  $D_{2i}$ , and  $D_i$  denote, respectively, the demand for debt, the credit ceiling, and the actual debt of country i.  $R_{1i}$  is the demand for reserves when the credit constraint is not binding, while  $R_{2i}$  is reserve demand in the constrained case with  $R_i$  the actual holding of reserves. The risk indicators are contained in the  $X_i$  vector and are the ratio of imports to GNP, country population, GNP per capita, the growth rate of GNP, an index of the variability of exports, and per capita public debt<sup>36</sup>.

Although equations (2.4.1), (2.4.2), (2.4.4), and (2.4.5) are estimated simultaneously, the technique can be easiest described if split into a two-stage process. First the equations for debt demand and the credit ceiling are estimated simultaneously to yield coefficient results. Secondly the reserve demand equations are estimated where (2.4.5) contains the credit ceiling as an exogenous variable. Eaton and Gersovitz simultaneously estimate the system of five equations using an optimization program to

<sup>&</sup>lt;sup>36</sup> These risk indicators are taken from the demand for reserves literature. For a good recent survey see Bahmani-Oskooee (1985).

maximize a log-likelihood function. The novel feature of this approach is that one need not know if country i is actually credit constrained or not just by observing the levels of debt and reserves for a given year. The log-likelihood function allows for calculation of the probability that country i is credit constrained in a given year. The pooled timeseries cross-section model was estimated using observations for 45 LDCs over two years, 1970 and 1974.

The demand for debt was found to be a function of the import/GNP ratio, population, and per capita public debt. The credit ceiling was a function of the index of export variability, per capita GNP, and population. The reserve demand equations, (2.4.4) and (2.4.5), provide curious results. The unconstrained demand for reserves was a function of export variability, per capita GNP, population, and per capita public debt. Evidently larger countries with higher levels of debt rely on debt more and foreign exchange reserves less to finance consumption and investment.

One criticism of the Eaton and Gersovitz model is the lack of incorporating the interest rate charged on debt as an exogenous variable in equations (2.4.1) and (2.4.2). Eaton and Gersovitz argue that if all lenders are risk-neutral, then the same interest rate will be charged to each borrower, resulting in a zero coefficient in the pooled estimation. However it is well noted by Feder and Just (1977b) and Edwards (1984, 1986) that all borrowers are not charged the same rate of interest on borrowing. In interest rate-loan space, a zero coefficient on the interest rate would result in completely inelastic demand and supply. The question then asked by Eaton and Gersovitz is the demand curve to the right of the supply curve, or vice-versa? This question underlies the specifications of equations (2.4.1) to (2.4.3) with the amount contracted being the minimum of demand or supply. In a situation of excess supply, why would a borrower demand less than he can obtain for no additional risk premium? It is hard to imagine such a loan contract being agreed upon given that the lender and borrower enter complex negotiation procedures. Stiglitz and Weiss (1981) point out that loan supply will be upward sloping up to some

critical interest ceiling where the curve then becomes vertical. It would appear that much information is being thrown out when the supply curve is constrained to be vertical at every rate of interest. Inclusion of this information could potentially alter the signs and magnitudes of the risk indicator coefficients.

McFadden et al. (1985) estimate a model very similar to Eaton and Gersovitz with the exception of using a binary dependent variable which represents three separate regimes: excess supply of loans, moderate excess demand, and large excess demand. The resulting likelihood function is quite complicated and will not be discussed here. Demand and supply equations are specified and estimated, then by assuming a logistic distribution the slope coefficients can be used to estimate probabilities of rescheduling for several developing countries. Estimates were obtained using observations from 93 countries over the period 1971 to 1982. Significant risk indicators included: the debt service ratio, real GNP per capita, imports/GNP ratio, total debt/exports ratio, an indicator for IMF support, and an indicator for significant arrears.

Finally this section will close with a discussion noting the results of Burton and Inoue (1985). Burton and Inoue take a simple approach to the problem of modelling debt capacity and country risk by regressing the interest rates charged on new loans to LDCs on the opportunity cost of the funds (LIBOR), the quantity of each loan, and a set of risk indicators. By assuming that demand for debt is constant, the authors hope to estimate a supply curve for the lender. The model was estimated using observations for 58 LDCs over the period 1972 to 1977. Of 11 independent variables 7 were found to be statistically significant: the LIBOR rate, the loan amount, maturity, the rate of inflation, reserves/imports ratio, debt/exports ratio, country's use of it's IMF quota/total available IMF quota, and an index of political instability. By not including dummy variables to reflect changes in the intercept or slope coefficients over time and across countries these results may be unreliable. Even if these assumptions are made, proper

estimation requires the use of a GLS technique to take into account the likelihood of autocorrelated disturbances over time and heteroskedasticity between countries (see Judge et al. (1985) p. 518 for a brief discussion).

2.5 Summary

Many problems specific to each of the 14 studies have already been discussed in the chapter. The purpose of this section is to summarize the difficulties in estimating the debt capacity and country risk relationship. Generally the discussion will progress through two broad categories: the choice of a dependent variable, and the choice of a set of independent variables.

A number of dependent variables have been used in the literature to relate to country risk indicators: a binary event variable reflecting default or in some cases reschedulings or significant arrears; a risk premium charged on LDC borrowing; the loan quantity. The choice of dependent variable depends in large part on the question one wishes to ask. Binary dependent variables attempt to capture an event which may be predicted by the independent variables. Thus when considering default, a country has either defaulted or has not without any consideration for lengthy rescheduling negotiations or other types of bargaining. The usual problem with this classification is that there are not enough observed defaults to form a reliable test. In addition the use of a default-no default classification ignores the many instances of developing countries reaching repayment crises without declaring default. A default occurs only when the lender formally declares that the borrower has violated a certain condition of the loan contract. Therefore default is the result of a set of decisions made by the lender and the borrower, not just the mechanical realization of some outcome<sup>37</sup>. A failure to make current payments does not imply that future payments will not be forthcoming. For these reasons it may not be useful to use the default dependent variable to model

<sup>37</sup> Eaton, Gersovitz, and Stiglitz (1985).

country risk. As an example, the correlation coefficients between the probabilities of default calculated in Edwards (1984) and the probabilities of repayment problems calculated in McFadden et al. (1985), for the same years (1976-1980) are shown in Table 2.1. For two of the six countries, Argentina and Mexico, the correlation is not significantly different from zero.

In a demand-supply model of LDC borrowing it would seem logical that country risk would manifest itself in the price of foreign loans<sup>38</sup>. In fact Cline (1984) points out that loan negotiations typically involve an agreement as to the interest rate to be charged before the actual amount of the loan is agreed upon. Measuring the risk premium charged on foreign loans is not an easy task. The usual procedure is to subtract from the interest rate actually charged a risk free rate (LIBOR) and use the difference as the risk premium. However Eaton and Gersovitz (1980) point out that competitive lenders will impose a credit ceiling to preclude default with probability in excess of that accounted for in the risk premium on debt<sup>39</sup>. Interest spreads can reflect many other factors such as the higher costs of originating loans in certain countries and tax treatments of interest income to foreigners. These costs are usually assumed to be small relative to the size of the loan. Inclusion of these loan conditions would be including endogenous variables because the magnitudes of these variables are determined by the interest spread.

In addition, increasing the rate of interest may lower the lender's expected return, both because the best risks decide not to apply and because the higher interest charges induce borrowers to undertake riskier projects. Banks may find it profitable to charge an interest rate below that justified by the level of risk, resulting in credit rationing. On the borrower's side, the developing country may not borrow the optimal level of funds

<sup>&</sup>lt;sup>38</sup> Given that the demand and supply curves intersect at a point where the supply curve has finite slope (is not vertical) and demand is not completely elastic.

<sup>&</sup>lt;sup>39</sup> Eaton and Gersovitz (1980), p. 8.

# Table 2.1

# Correlations Between Default Probabilities

# and Repayment Problem Probabilities (1976 - 1980)

Argentina	ň	1		-0.22341
Brazil		)	• .	0.80883
Korea	4		J	0.6477
Mexico		•	1	0.19527
<b>S</b> Phillipines		, .		0.35065
Venezuela				0.95824

Note: Default probabilities obtained from Edwards (1984). Repayment problems probabilities obtained from McFadden et. al. (1985).

associated with the risk premium. The marginal return on the invested funds is typicallyset above the marginal cost of borrowing in a steady state because the government faces a constraint in raising tax revenue to service the debt.

Eaton and Gersovitz (1980) use the loan amount as the dependent variable in their credit rationing model of loan demand and supply. In Eaton, Gersovitz, and Stiglitz (1985) this practice is defended by noting that the relevant question to ask in the country risk literature is, "When will a country with certain characteristics owing a certain amount of debt under certain contractual arrangements pay or receive funds from creditors with certain characteristics?<sup>40</sup>" One could then theoretically make an estimate of the net present value of the original loan as well as additional lending. However it is possible to show that any future loans will not make a profitable expected return on their own as part of the return on additional lending is the possibility of salvaging past lending.

To estimate separate demand and supply functions properly, the loan quantity is used as the dependent variable when the rate of interest is included on the RHS's. As already mentioned, the rate of interest was not included, thus the model was constrained to use only vertical demand and supply functions, with these functions shifting due to changes in the risk indicators. In this way the loan quantity was constrained by force to be the only variable of interest as if all adjustment to risk took the form of a Marshallian quantity adjustment. It is quite likely that a large portion of this adjustment also took the form of a change in the rate of interest, which when left out, would likely give unreliable parameter estimates.

One must ask the question does a country which finds itself credit constrained resort to debt rescheduling or even default? Whether a country is credit constrained or 40 Eaton, Gersovitz, and Stiglitz (1985), p. 507.

not is not an interesting question of itself, but is only interesting when incorporated into a model which allows for other variables which are affected by country risk. It would be interesting to allow for interest rate changes before credit rationing becomes effective. A simple test reveals that the presence of credit rationing does not imply a high probability of repayments problems. The correlation coefficient between the probability of credit rationing (Eaton and Gersovitz, 1980) and the probability of repayments problems (McFadden et al., 1985) is only 0.10762 for seven LDCs in 1974. Credit rationing may be of interest in itself, but may not be related to country risk.

The major difficulty in selecting risk indicators to use as independent variables is the lack of any theory to guide the choice<sup>41</sup>. Generally the choice of independent variables has relied on the notions of illiquidity and insolvency. However illiquidity (inability to convert positive net worth into a means of payment) does not lead to default. As long as the value of the net worth is clear, creditors should be willing to supply funds, albeit at a higher interest rate. Illiquidity is usually caused ex post by the withdrawar of credit. This may imply that models which analyze default as a decision made by the lender are consistent with historical evidence. Insolvency (debt exceeds net worth) is neither necessary nor sufficient for the declaration of default. The debt of a country is usually less that the value of its assets. However since sovereign loans are made by the governments of countries, repayment is not constrained by the net worth of the country, but by the amount of net worth which can be appropriated by the government. Eaton, Gersovitz, and Stiglitz (1985) point out that it is unlikely that LDC governments are anywhere near making the maximum feasible debt service<sup>42</sup>. Despite these comments, Table 2.2 indicates that proxy measures of liquidity and solvency have

<sup>&</sup>lt;sup>41</sup> During the 1960's several papers addressed this problem, but only in the format of a discussion without any theoretical model. See for example Avramovic (1958), McKinnon (1964), Gulhati (1967), and Avramovic et al. (1968).

<sup>&</sup>lt;sup>42</sup> However for a debt situation to be sustainable it is necessary that the tax base expand quickly enough to allow the government enough revenues to service the existing debt. The expansion of the tax base is determined by private savings-investment behavior.

performed adequately in previous statistical studies.

Any independent variables chosen are subject to a simultaneity problem. Since borrowers can take actions so as to increase the likelihood that they will meet their service payments, they can increase the willingness of lenders to lend. Lenders can take actions which increase the likelihood of further loans, causing borrowers to be more willing to borrow and meet service payments. In this way the dependent and independent variables share an uncertain causal relationship. The usual solution has been to lag the independent variables, however for small sample sizes this can result in a substantial loss of degrees of freedom.

Measures that incorporate the level of existing debt have generally worked well for a simple reason. The fact that a country's debt usually sells at a discount relative to its contracted value<sup>43</sup> creates problems for new lenders. Because property rights between existing and new loans are poorly defined, new loans may be forced to share an expected loss. Unless existing creditors can be made subordinate to new creditors, the behavior of new creditors is shaped mainly by the market valuation of existing debt. Unfortunately capital markets which trade in foreign debt have not been in existence long enough to provide ample data.

Many results in the literature include endogenous variables as exogenous variables, such as GNP growth, investment, etc. The terms of trade may be truly exogenous as well as climate shocks when agriculture is important (Eaton, Gersovitz, and Stiglitz (1985)), although debt has tended to increase most when the terms of trade would seem to be high. One possible independent variable is whether debtors are prepared to adopt an IMF program or not. This may serve to distinguish between countries which do and

<sup>&</sup>lt;sup>43</sup> For instance, the contracted foreign debt of Peru was selling on public capital markets for only 6% of it's contracted value in October of 1987. Brazilian loans traded at 38% and Mexico's at 47%. Even relatively safe Colomibian debt sold at 73% of it's contracted value. See <u>The Economist</u>, October 24, 1987.

Table 2.2

A Summary of Risk Indicators

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Risk Indicator	Studies (statistical significance)
1. Debt service/exports	1,4,5,7,10,11,13
2. Debt/GNP	3,11,14
3. Total debt/exports	3,6,12,13,15
4. Net debt/exports	10
5. Net debt/amortization	-
6. Amortization rate	10
7. Amortization/debt	1,4,5,15
8. Reserves/imports	1,4,5,6,10,12,15
9. Reserves/GNP	11,14
10. Income per capita	5,9,13
11. Growth rate of income	4,5,9,10
12. Imports/GNP	4,6,9,13,14
13. Index of export variability	4,9,15
14. Export growth	5,7,15
15. Investment/GNP	6,11,14
16. Savings rate	
17. Inflation rate	6,7,8,12
18. Maturity	4,12,14
19. Loan value	12
20. Debt per capita	9
21. Inflation erosion of debt	
22. Current account deficit	10
23. Current account/GNP	11
24. Current account/exports	8
25. Population	9

# Table 2.2 (cont.d)

# A Summary of Risk Indicators

26. Government spending/GNP	r
27. Reserves variability	
28. Devaluation rate	ŵ. L
29. Real effective exchange rate	
30. Use of quota/IMF allotted quota	12 -
31. Index of political instability	12
32. Capital inflows/debt service	<b>5</b> :
33. Debt service/reserves	3
34. Disbursed debt/debt service	3
35. Debt service/imports	3
36. Monetary growth	7,8
37. Deviations from PPP	7
38. Reserves growth rate	<sup>*</sup> 8
39. Indicator for IMF support	13
40. Indicator for arrears	13

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Notes: 1-Frank and Cline (1971), 2-Dhonte (1975), 3-Grinols (1976), 4-Feder and Just (1977a), 5-Feder and Just (1977b), 6-Mayo and Barrett (1977), 7-Sargen (1977), 8-Saini and Bates (1978), 9-Eaton and Gersovitz (1980), 10-Cline (1984), 11-Edwards (1984), 12-Burton and Inoue (1985), 13-Mcfadden et al. (1985), 14-Edwards (1986), 15-Webster (1985). Dhonte's risk indicators have been excluded since they are principal components. do not anticipate servicing their debt. It is evident that debt problems frequently have their source in overambitious government expenditure plans, particularly if these plans are purely consumptive in nature (Takagi (1981)). This argues for the inclusion of some measure of fiscal responsibility in the set of independent variables.

Existing studies only include long-run country characteristics such as the variability of exports. As is evident from the economic events of the 1970's, variables which represent transitory shocks will likely play an important role in determining country risk. For instance McFadden et al. (1985) show that OPEC oil price shocks played a considerable role in the payments crises experienced by Brazil in 1982, Peru, and the Phillipines in 1983.

The role of capital flight in influencing sovereign risk has largely been ignored due to the large inaccuracies in it's measurement<sup>44</sup>. Recent literature has improved the measurement of capital flight to the point where it might be incorporated in an econometric model<sup>45</sup>. However the volume of capital flight may be highly correlated with other macro-economic variables causing inefficient significance tests. Capital flight raises the probability of debt problems through several mechanisms: reduced liquidity and higher interest rates; depreciated domestic currency under flexible exchange rates or depleted reserves under fixed rates; reduction in resources to finance investment and a reduction in the growth of income; reduced government tax revenues

<sup>44</sup> The term "capital flight" is generally associated with short-term speculative outflows or outflows resulting from political or economic uncertainty which yield no return to the country. Dietz (1986) notes that roughly half of the Latin American regions total debt of \$180 billion is held outside the region.

<sup>45</sup> Khan and Haque (1987) measure capital flight by first estimating gross capital outflows, the total stock of external claims, by subtracting from changes in external debt the current account deficit and changes in international reserves held by the central bank and net foreign assets of domestic banks. Second, "normal" capital outflows are estimated by the level of interest income earned by nationals from overseas investments and reported in the balance of payments (using foreign market interest rates to determine the volume of capital outflows). The difference between this measure and the total stock of external claims may be considered an indicator of capital flight.

for debt service; and a resulting increased need to borrow from abroad<sup>46</sup>. It is uncertain whether the inclusion of the consequences of capital flight, or the volume of capital flight itself, should be included as independent variables. In addition it is uncertain whether capital flight is truly exogenous.

Eaton (1987) developed a rational expectations model in which the expectation of increased tax liabilities for domestic residents created by the potential flight of private debt can lead to many different equilibria. One solution is no capital flight and optimal resource allocation. Another solution is 100% capital flight (a move by one borrower that increases the likelihood of default raises tax liabilities to other borrowers, causing all borrowers to flee) and almost certain default.

Efforts to model the borrower-lender relationship in the context of international contracts have been surprisingly few<sup>47</sup>. Beginning with Domar (1950), most of the literature has concentrated on defining conditions for debtor insolvency or illiquidity. The condition for insolvency is usually taken as export growth rate < interest rate. In this case debt will grow faster than exports and, in a steady state, the critical rate of interest will exceed exporf growth by an amount depending on the lenders' desired debt/export ratio for the borrower<sup>48</sup>. Illiquidity is a situation where the borrower does not possess the necessary funds to service the debt at the present time, but will possess the necessary funds in the future. Even though the borrower is solvent, he must default. This will never occur if capital markets have the foresight to perceive the future wealth of the debtor as being adequate to finance future payments. The borrower will find it necessary to arrange short-term "bridge loans" which lenders will be willing to provide. Illiquidity is usually caused ex post by the withdrawal of credit. Insolvency (debt

<sup>&</sup>lt;sup>46</sup> Khan and Haque (1987), page 5.

<sup>&</sup>lt;sup>47</sup> Eaton and Taylor (1986) provides a good review.

<sup>&</sup>lt;sup>48</sup> Eaton and Taylor (1986), p. 218. Domar and others who use this condition are ignoring no-default terminal debt conditions.

exceeds net worth) is neither necessary nor sufficient for the declaration of default. The debt of a country is usually less than the value of its assets. However since sovereign loans are made by the governments of countries, repayment is not constrained by the net worth of the country, but by the amount of net worth which can be appropriated by the government. Eaton, Gersovitz, and Stiglitz (1985) point out that it is unlikely that LDC governments are anywhere near making the maximum feasible debt service<sup>49</sup>.

It is evident from the papers surveyed in this chapter that the bulk of the literature analyzing the historically high levels of debt held by LDCs in the last 20 years concentrate on the static decision process of the lender. Using a variety of methods ranging from risk indicator regressions to game theoretic contracting models, the problem has been to derive the optimal supply curve of debt based on expected profit maximization. The lender estimates a subjective probability of default based upon a vector of relevant risk indicators specific to each borrower and computes a set of expected profit maximizing interest rate - loan quantity contracts. In order to generate the large build-up of debt observed in the 1970's, some exogenous variables must have shifted the supply curve of debt outwards, given an assumed demand curve. It is generally accepted that the large surplus of petrodollars over the decade did cause such a shift<sup>50</sup>. The literature surveyed here has probably taken the lenders decision as far as one could want to take it in comparative static maximization. However since the decision to loan today leads to uncertain income streams in the future, a dynamic maximization process is essential.

The demand for debt is implicitly thought of as a residual demand which may fall wherever it happens to in interest rate - loans space. Debt demand is simply the

<sup>&</sup>lt;sup>49</sup> However for a debt situation to be sustainable it is necessary that the tax base expand quickly enough to allow the government enough revenues to service the existing debt. The expansion of the tax base is determined by private savings-investment behavior, which is influenced by the level of debt. Hence the debt policy of the sovereign borrower partly determines the ability to service the debt over time.

<sup>&</sup>lt;sup>50</sup> Of course a completely elastic world supply of debt makes discussion of outward shifts superfluous. Apparently there is little agreement on this issue.

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difference between domestic expenditures and output. Thus the problem becomes one of determining the optimal level of expenditures (see Cooper and Sachs (1985) for example). The ramifications of taking on more debt do not appear in the first order conditions for expenditures, thus the debt decision is completely exogenous to the decision process. This seems unrealistic in countries possessing high debt-GNP ratios who are straddled with heavy debt service payments already from the existing stock of debt. It would seem the optimal solution to be taking on ever higher amounts of debt to maximize expenditures regardless of the burden on the economy. If the decision to borrow was endogenous to the decision to spend, the resulting solutions would be much more meaningful in the context of heavily indebted countries.

If profitable opportunities did not exist in LDC lending markets during the 1970's, it is certain that the large new supply of funds would have found investment opportunities elsewhere. In a nutshell, if the demand were not there the new supply would not be forthcoming, hence the neglected demand for debt is just as important in explaining the debt build-up over the period as its supply. The question is why LDCs scooped up the large new supply of funds, knowing well that the terms of the contracted debt, interest charges and maturities, were certainly not soft. The answer cannot be found in the models already surveyed here.

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### Chapter 3: A Dynamic Model of Debt Accumulation

### 3.1 Purpose

As an attempt to correct some of the deficiencies in the theoretical models covered in Chapter 2, this chapter will develop a dynamic model of the LDC debtor's decision to accumulate foreign debt over time. The decision to borrow is not a residual demand, but rather its implications are objectively considered in domestic consumptionsavings decisions. With the ability to adjust the savings rate and the stock of foreign debt to utility optimizing levels, it is determined that the demand for foreign debt is sensitive to the amount of relative uncertainty in holding it. With both risky domestic and foreign capital<sup>51</sup> available for domestic capital becomes relatively more risky to hold, and 2) the stochastic shocks to domestic and foreign capital, only that its risk increases faster than the risk attributable to foreign capital. Stronger positive correlation between the two shocks increases the stock of debt which is optimal to hold. In this way, large debt-GNP ratios can be an optimal response to an uncertain environment, rather than an irresponsible policy of living for the present.

The model will not address issues concerning conditions for loan default. Default is a possible outcome of the model only if credit is severely restricted by lenders. In most cases the borrower will choose to default if there is a gain to him from doing so net of the costs of default (such as exclusion from future credit). Default is simply a matter of unwillingness to pay. Only a few authors have considered this possibility. In Eaton and Gersovitz (1981) the debtor weighs the burden imposed by continuing debt service payments with the welfare loss due to exclusion from future credit. Knowing that the borrower faces this decision when the loan matures, the lender builds this into his

<sup>51</sup> Foreign capital and debt are identical in the presentation to follow.

decision of how much to lend. Several papers have followed this theme and developed complicated game-theoretic models of optimal loan contracts<sup>52</sup>.

The supply curve of funds determined by optimal behavior on the part of the lender is assumed exogenous. Several of the papers mentioned in the preceding chapter have explored the problem of loan supply under default risk.

The model in Section 2 will utilize a modified continuous time Brock-Merman (1972) growth model to determine the optimal accumulation of foreign debt in an infinite horizon framework. The set-up of the model is similar to that used by Bardhan (1967), but focuses on borrowing behavior in a world of stochastic capital stocks. The domestic economy is purely a production economy to keep the model simple. A discrete two-period, non-stochastic production model can be found in Cooper.and Sachs (1985). An analysis of a pure exchange economy focusing on the trade balance, with both traded and non-traded goods, can be found in Dornbusch (1983).

# 3.2 The Open Economy Model

The LDC economy is characterized by one sector which produces output Y. Only one tradeable good is produced which can be thought of as a composite commodity. With only one produced good, attention is focused on the intertemporal decision to borrow and lend in international capital markets. Output is produced by two factors, labor (L) and perfectly malleable capital (K), with the general production function  $Y_t =$ 

<sup>&</sup>lt;sup>52</sup> A representaive paper in this area is Bulow and Rogoff (1989). Kohler (1986) uses a one-period game where the borrower has the option to pay off the loan, totally default, or reschedule the old loan at new terms, when the initial loan becomes due. Since the payoff to both parties is small with total default, both the borrower and lender will prefer to reschedule if the borrower is unable or unwilling to repay. Threats to foreclose will be ignored by the borrower, since he knows it would not be in the test interests of the lender. The Nash equilibrium is rescheduling when it is necessary, with no defaults.

(3.2.1)

 $F[K_t, L_t]$ . Gross domestic product which can be allocated to consumption or investment is given by

$$Y_t = F[K_t, L_t] - i_t K_t^{\dagger}$$

where  $K_t^f$  is the total stock of debt held in the form of foreign capital at time t. All domestic production is either reinvested at home, used to pay debt service, or consumed <sup>53</sup>. Imperfections in the world capital market mean that the marginal cost of borrowing is increasing in the amount borrowed <sup>54</sup>.

 $i_t'(Kf_t) > 0, \ i_t''(Kf_t) > 0$ 

Assuming a perfect world capital market would not change the results to follow. The demand for debt would then be a residual demand occurring when domestic expenditures (consumption and investment) exceed domestic production<sup>55</sup>. With an infinitely elastic supply curve of debt, the borrower could obtain all it wanted at the going world interest rate. The decision to take on additional debt would not appear in the marginal condition for deciding how much to consume or invest. This seems unrealistic given that LDCs are charged different interest rates based on-the riskiness of large loan losses. The decision to save (and thus consume) must be a simultaneous decision with how much to borrow.

Denoting consumption by  $C_t$  and gross investment by  $I_t$ , equilibrium in the output market is described by

<sup>55</sup> Blanchard (1982) uses such a structure.

 $<sup>^{53}</sup>$  A recurring criticism of the Solow growth model is the lack of any consideration of investor expectations in deciding what amount to invest (or savers what amount to save). For a summary of this literature see Jones (1975) chapter 4.

<sup>&</sup>lt;sup>54</sup> This may represent the belief by many economists that higher levels of foreign debt are accompained by higher borrowing costs. Empirical research into this area is indecisive (see Edwards (1984), (1986)).

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$$Y_t = C_t + I_t$$

# (3.2.2)

Borrowing takes the form of physical units of foreign capital which can be imported at zero transportation cost. Interest payments are made with units of output. Depreciation of domestic capital is a constant proportion,  $\mu$ , of the total capital stock. Foreign capital bears no depreciation meaning that a depreciation rate is not part of the interest rate charged on debt. In order to differentiate between domestic and foreign capital in production<sup>56</sup>, an installation cost will be necessary for both of the form

$$C(l_t) = a\dot{K}^d + b\dot{K}^f^2$$

where the coefficients a and b are not necessarily equal <sup>57</sup>. It could be argued that new foreign capital equipment may be more costly to install due to necessary training or the importing of technicians to do the installation. Total investment is given by

$$I_t = sY(K, L) - \mu K^d - aK^d - bK^f$$

### (3.2.3)

Assuming that the labor stock grows exponentially at the known and constant rate n we have

$$L_t = L_o e^{nt}$$

### (3.2.4)

<sup>56</sup> And thus avoid an infinite elasticity of substitution as a possibility.

<sup>57</sup> The installation cost method is one way to differentiate foreign and domestic capital. Another popular technique first utilized by Bardhan (1967) is to assume a disutility of debt function  $D(k^{f})$  where  $D^{\circ}(k^{f}) < 0$  and  $D^{\circ}(k^{f}) < 0$ . The advantage of using the installation cost method is that it also addresses the issue of whether it is realistic to assume that the bounds of investment are given by  $-\infty \le I_{t} \le \infty$ . See Takayama (1985) p. 685 - 719 for a survey of installation cost functions in neoclassical investment problems.

Obviously total capital is composed of domestic capital,  $K^{d}_{t}$ , and foreign capital,  $K^{f}_{t}$ .

$$K_t = Kd_t + Kf_t \tag{3.2.5}$$

So far we have six unknowns,  $I_t$ ,  $C_t$ ,  $L_t$ ,  $K^d_t$ ,  $K^f_t$ , and  $D_t$ , but only five equations (3.2.1)-(3.2.5). One more equation is necessary to close the model. Consumption behavior is given by a simple consumption function

$$C_t = (1 - s_t) Y_t$$
 (3.2.6)

Here  $s_t$  is a parameter,  $0 < s_t < 1$ , the average propensity to save<sup>58</sup>. Full employment of capital and labor is implied by the six equation system. The trade balance has a simple interpretation in this highly stylized model. The trade balance in each period equals GDP less consumption and investment, hence a trade surplus implies net lending and a deficit net borrowing. More formally we have

$$TB_{t} = TB_{0} + \int_{t=1}^{t} \beta TB_{t} dt = (1+i) D_{0} + \int_{t=1}^{t} \beta (1+i) D_{t} dt$$

where TB<sub>0</sub> is the initial trade balance,  $D_0$  is the initial net debt position, and  $\beta$  is a discount factor.

Some comments are in order about the production function in equation (3.2.1). We assume that F exhibits constant returns to scale with diminishing returns to each factor. The following Inada conditions characterize the production function.

<sup>&</sup>lt;sup>58</sup> The parameter  $s_1$  will be used as a control variable in the model to follow. It also represents the marginal propensity to save.

 $F (aK_t, aL_t) = aF (K_t, L_t)$   $F_L (K_t, L_t) > 0, F_K (K_t, L_t) > 0$   $F_{\underline{II}}(K_t, L_t) < 0, F_{KK} (K_t, L_t) < 0$   $F_L(0) = \infty, F_K(\infty) = 0$   $F(0) = 0, F(\infty) = \infty$ 

The preparation of the model for dynamic optimization now proceeds along lines similar to those followed by Solow. With constant returns to scale on F, rewrite (3.2.1) as

$$Y_{t} = L_{t}F\left(\frac{K_{t}}{L_{t}}, 1\right) = L_{t}f(k_{p})$$
  
where  $k_{t} = \frac{K_{t}}{L_{t}}$ 

or 
$$y_t = f(k_t)$$
 where  $y_t = \frac{Y_t}{L_t}$ 

Lower case letters denote per capita values. For feasible production, we must assume  $L_t >> 0$ . We can simplify the equation system by combining (3.2.1), (3.2.2), (3.2.3), and (3.2.5).

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$$Y_t = C_t + (K^d_t + \mu K^d_t)$$

Dividing both sides by  $L_t$  we obtain

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or 
$$\frac{\dot{K}_{l}^{d}}{L_{l}} = y_{l} - c_{l} - \mu k_{l}^{d}$$

(3.2.6)

where  $y_t = Y_t/L_t$  and  $c_t = C_t/L_t$ . By differentiating  $k_t = K_t/L_t$  totally we get

$$\dot{k}_t = \frac{\dot{K}_t}{L_t} \frac{\dot{L}_t}{L_t} k_t$$

From equation (3.2.4),  $\dot{L}_t/L_t = n$ , we obtain

$$\frac{\dot{K}_t}{L_t} = \dot{k}_t + nk_t$$

Combining this result with equation (3.2.6) we get

$$k_t^d = y_t - c_t - (\mu + n)k_t^d$$
(3.2.7)

If  $k^d_t \ge 0$ ,  $c_t \ge 0$ , the time path  $(k^d_t, c_t)$  is called a *feasible* growth path which is a constraint on growth solely determined by the initial capital stock and the average propensity to save. Equation (3.2.7) simply states that the change in the domestic capital-labor ratio is equal to the new flow of domestic capital created by additional savings per capita, less the amount of domestic capital per capita which is used to replace worn out equipment and keep additional labor fully employed.

Foreign capital is used to bridge the gap between desired investment and domestic savings. To see this add the new inflow of foreign capital to both sides of the income equation  $(3.2.2)^{59}$ .

<sup>&</sup>lt;sup>59</sup> Foreign investment is not included in measures of domestic GNP.

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 $C_{i}+I_{i}+K_{i}^{f}=Y_{i}+K_{i}^{f}$ 

The accumulation equation (3.2.7) becomes

 $\dot{k}_{t} = y_{t} - c_{t} - (\mu + n)k_{t}^{d} + nk_{t}^{f} + k_{t}^{f}$ 

3.3 A Stochastic Accumulation Process

So far the model contains no uncertain elements which may force a default on interest payments on borrowed funds. Uncertainty is modeled in the evolution of the foreign capital stock as

 $dK^{f} = \widetilde{K}^{f} dt + \sigma K^{f} dz$ 

### (3.2.8)

where  $\tilde{K}^{t}$  is the mean drift of the foreign capital stock over time, and dz is the increment of a stochastic process z that obeys a Wiener process with mean zero and where  $\sigma$  is the diffusion coefficient <sup>60</sup>. Briefly a Wiener process has the following characteristics. For any time increments  $0 < t_1 < t_2 < ... < t_n$  the random variables  $z_1, z_2 - z_1, z_3 - z_2, ..., z_n - z_{n-1}$  are independent and are normally distributed with mean zero and variance  $t_j - t_{j-1}$ . If desired the Wiener process can be expressed as a white noise process by substituting  $dz = \xi dt$ where  $\xi$  is a white noise process. The diffusion coefficient  $\sigma$  is a measure of the dispersion of the disturbance term around its mean. For a discussion of the diffusion coefficient see Arnold (1974), chapter 2.

As a more general case, a second stochastic accumulation equation will also be <sup>60</sup> The diffusion coefficient is the instantaneous standard deviation of a Wiener process. considered where both foreign and domestic capital are subject to random shocks.

$$dK = \left(sY + \widetilde{K}\right)dt + \sigma_{I}K^{d}dz_{1} + \sigma_{Z}K^{f}dz_{2} \qquad (3.2.8')$$

To compute the stochastic neoclassical differential equation of growth for the economy under (3.8), we make use of Ito's lemma. We are given that

$$dK = (sY(K, L) - \mu K^{d} + \widetilde{K}^{f} - C(I)) dt + \sigma K^{f} dz$$
$$dL = nLdt$$

Letting k = K/L, Ito's lemma is stated as

$$dk = \frac{\partial k}{\partial t} dt + \frac{\partial k}{\partial L} dL + \frac{\partial k}{\partial K} dK + \frac{1}{2} \left[ \frac{\partial^2 k}{\partial K^2} (dK)^2 + 2 \frac{\partial^2 k}{\partial K \partial L} (dK) (dL) + \frac{\partial^2 k}{\partial L^2} (dL)^2 \right]$$
$$= -\frac{K}{L^2} (nLdt) + \frac{1}{L} \left[ \left[ (sY - \mu K^d + \tilde{K}^f - aK^d - bK^f) dt + \sigma K^f dz \right] dt + \sigma K^f dz \right]$$
$$+ \frac{1}{2} \left[ -\frac{2}{L} \left( \left[ (sY - \mu K^d + \tilde{K}^f - aK^d - bK^f) dt + \sigma K^f dz \right] nLdt + \frac{2K}{L^3} (nLdt)^2 \right] \right]$$
$$= \left( sy - (\mu + n) k^d - nk^f + \tilde{k}^f - ak^d - bk^f \right) dt + \sigma k^f dz \qquad (3.2.9)$$

The effect of stochastic shocks to the evolution of the foreign capital stock can best be explained as a type of stochastic depreciation rate, where the rate could be either positive or negative. Capital may simply evaporate, or alternatively, a better production technique could make it appear that the capital stock has increased. The analogue of

 $\lim_{n \to \infty} k_{\rho}^{f} \delta t = 0$ 

(3.2.9) for the case where both types of capital are uncertain is given by

$$dk = \left(sy - (\mu + n)k^{d} - nk^{f} + \tilde{k}^{f} - ak^{f} - bk^{d}\right)dt + \sigma_{k}^{d}dz_{1} + \sigma_{k}^{f}dz_{2} \qquad (3.2.9')$$

# 3.4 The Borrower's Problem and a Solution

To determine the optimal *attainable* growth path for the case where only foreign capital is uncertain, it is necessary to maximize the objective integral

$$V(t, k^{d}) = max E \int_{0}^{\infty} e^{-\delta t} u [(1 - s)y_{t}] dt$$
  
subject to  $dk_{t} = \left[ s_{t}y_{t} - (\mu + n)k_{t}^{d} + nk_{t}^{f} + k_{t}^{f} - ak^{d} - bk^{f} \right] dt + \sigma k_{t}^{f} dz$   
 $k_{0} = k(0)$  (3.2.10)

The objective is to maximize the utility derived from current consumption by controlling the policy variables  $s_t$  and  $k^f_t$ , subject to a stochastic capital constraint. The economy can be described at any time t by the state variable  $k^d_t$ . The transversality condition for this infinite horizon problem is given by

which insures that the borrower does not take on debt indefinitely in order to finance debt service.

To solve stochastic optimal control problems begin with the basic equation

$$-J_{t}(t,k^{d}) = \max_{\substack{s,k' \\ s,k'}} \left( u(t,k,s) + J_{k}(t,k)g(t,k,s) + \frac{1}{2}\sigma^{2}J_{kk}(t,k) \right)$$
(3.2.11)

where the J function is the maximized integral in equation (3.2.10) to follow without constraint. Equation (3.2.11) emerges from a lengthy derivation starting with equation (3.2.10). See Appendix A for the proof. All stochastic optimal control problems utilize the basic equation (3.2.11) which is called the Hamilton-Jacobi-Bellman equation.

Properties of the utility function include u' > 0 and u'' determined by the degree of risk aversion<sup>61</sup>. Multiplying V(t,  $k^d_t$ ) by  $e^{\delta t}$  yields a time autonomous problem. Following Kamien and Schwartz (1981), substitution of (3.2.10) into (3.2.11) yields in general form

$$-J_{t}(k^{d}) = \max_{\substack{s,k^{f}}} \left( u(t, k, s) + J_{k}(k) g(t, k, s) + \frac{1}{2} \sigma^{2} k^{f^{2}} J_{kk}(k) \right)$$
(3.2.12)

The borrowing country is going to maximize the discounted value of consumption at each point in time up to infinity, subject to a stochastic constraint on the ability of the country to supply new domestic capital for production. This is the standard Ramsey problem with debt. The domestic capital-labor ratio  $k^d_t$  is the state variable which is influenced by two control variables, foreign debt  $k^f_t$  and the average propensity to save  $s_t$ . Equation (3.2.12) can be made more specific by inserting the utility function and the domestic capital constraint to give

<sup>&</sup>lt;sup>61</sup> In order to avoid  $s_t = 1$  we could assume a utility function such as  $u = \ln c_t$  that in the limit as  $c_t \longrightarrow 0$ ,  $u_t = -\infty$ .

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$$-J_{t} = \max_{\substack{s \neq f_{t} \\ s \neq f_{t}}} \left( u(c) + J_{k} \left[ s y - (\mu + n)k_{t}^{d} - nk_{t}^{f} + k_{t}^{f} - ak^{d} - bk^{f} \right] + \frac{1}{2} \sigma^{2} k^{f} J_{kk} \right)$$
(3.2.13)

Performing the necessary maximization gives as first order conditions (dropping time subscripts)

$$- u_{c}(f(k) - i(kf)kf) y + J_{k}(f(k) - i(kf)kf) y = 0$$

$$u_{c}(1-s)(f(k) - i'(kf)kf - i) + J_{k}(s(f(k) - i'(kf)kf - i) - n) + J_{kk}kf\sigma^{2} = 0$$
(2')

From (1') it can be deduced that  $u_c = J_k$ , or the shadow price of domestic capital equals the marginal utility derived from consumption. Denote the implicit solutions for the average propensity to save and the foreign capital stock as  $s^*(k^d, \sigma^2)$  and  $k^{f*}(k^d, \sigma^2)$ . Substitution of these solutions into 1' gives:

$$-u_{C}[(1-s^{*})(f(k^{*}) - i(k^{f^{*}})k^{f^{*}})] = J_{k}$$

Differentiating with respect to  $k^d$  yields an expression for  $J_{kk}$ .

$$-u_{cc}\left[(1-s^{*})\left(f_{k}^{d}+(f_{k}^{f}-i_{k}^{f}k^{f}*-i)\frac{dk^{*}f}{dk^{d}}\right)-\frac{ds^{*}}{dk^{d}}y\right]=J_{kk}$$

Substituting the expressions for  $J_k$  and  $J_{kk}$  into (2') and rearranging gives

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$$f_{2} - i' k^{f_{*}} - i - n + r \left[ \left( f_{1} + \frac{dk^{f_{*}}}{dk^{d}} (f_{2} - i' k^{f_{*}} - i) \right) (1 - s^{*}) - \frac{ds^{*}}{dk^{d}} y \right] k^{f_{*}} \sigma^{2} = 0$$

The new term  $r = -u_{CC}/u_C$  is the Arrow-Pratt measure of absolute risk aversion. The parameter r takes on positive, zero, or negative values as the representative individual in the borrowing country is risk loving, risk neutral, or risk averse with respect to consumption. We will assume that income is valued more in bad states than in good states implying risk aversion. Note that if the representative individual is risk neutral, the risk term disappears and we achieve a similar result for foreign borrowing if risk did not exist<sup>62</sup>. Risk neutrality yields the results that negate the effects of risk, hence to continue the discussion the borrower will be assumed risk averse (r < 0). Assuming the utility function  $u = \ln c$  we can rewrite the condition as

$$f_{2} \cdot i^{*} k^{f_{*}} - i - n - \left[ \left( f_{1} + \frac{dk^{f_{*}}}{dk^{d}} (f_{2} - i^{*} k^{f_{*}} - i) \right) \frac{1}{y} - \frac{ds^{*}}{dk^{d}} \frac{1}{(1 - s^{*})} \right] k^{f_{*}} \sigma^{2} = 0$$

or in elasticities form

$$k^{f_{*}} = \frac{f_{2} \cdot i \cdot n}{i' + \begin{bmatrix} \gamma & c \\ \eta_{k} d^{*} + \eta_{k} d^{*} \end{bmatrix} k^{d} \sigma^{2}}$$

(3.2.14)

Equation (3.2.14) is the solution for  $k^{f}$  and describes the optimum marginal condition for borrowing risky foreign capital. The terms  $\eta^{y*}$  and  $\eta^{c*}$  are the optimized

 $<sup>^{62}</sup>$  Oddly most theoretical papers on optimal borrowing or lending behavior assume risk neutrality which would seem to eliminate any discussion of how risk on borrowed funds affects optimal behavior.

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elasticities of per capita income and the average propensity to consume with respect to the domestic capital labor ratio. A necessary assumption to insure that  $k^{f*} \ge 0$  is that  $f_2$ - i - n > 0. In a certainty model where  $\sigma^2 = 0$ , we have the standard factor demand solution  $k^{f*} = (f_2 - i - n)/i'$ . When  $\sigma^2 > 0$ , the optimal stock of foreign capital may rise, fall, or remain unchanged depending upon the sign of  $(\eta^y + \eta^c)$ . This condition makes The sum of these two elasticities is the elasticity of per capita some sense. consumption with respect to the domestic capital stock. With foreign capital suddenly risky to hold, the producer may wish to substitute towards riskless domestic capital, even if this means paying a higher price for it. However the extent to which this substitution occurs will depend upon whether this action raises the objective of maximizing per capita consumption. If acquiring more domestic capital raises per capita consumption,  $(\eta^y + \eta^c) > 0$ , then demand for foreign capital will be reduced. Will this always be the case? Not always because while  $\eta^y > 0$  with any normal production function,  $\eta^c$  is probably negative at the steady state solution. To see this, consider the constrained solution for  $s^*$  from (3.2.8).

$$s^{*} = \frac{(\mu + n) k^{a}}{f(\kappa) - i (k^{f_{*}}) k^{f_{*}}}$$

Assuming a Cobb-Douglas production function  $f(k) = k^{\alpha}$  and evaluating the derivative ds\*/dk<sup>d</sup> gives

$$\frac{ds^{*}}{dk^{d}} = \frac{\left(k^{\alpha} - ik^{f_{*}}\right)(\mu + n) - \left(\alpha k^{\alpha-1}(1 + \frac{dk^{f_{*}}}{dk^{d}}) - i\frac{dk^{f_{*}}}{dk^{d}}k^{f_{*}} - i\frac{dk^{f_{*}}}{dk^{d}}\right)(\mu + n)k^{d}}{\left(k^{\alpha} - ik^{f_{*}}\right)^{2}}$$

Recognizing that  $dc^*/dk^d = -ds^*/dk^d$ , the elasticity of the average propensity to consume with respect to the domestic capital-labor ratio is

$$\eta^{c} = \frac{-k^{d} \left[ \left( k^{\alpha} - ik^{f_{*}} \right) (\mu + n) - \left( \alpha k^{\alpha - 1} + \frac{dk^{f_{*}}}{dk^{d}} (\alpha k^{\alpha - 1} - i - i^{*} k^{f_{*}}) \right) (\mu + n) k^{d} \right]}{(1 - s^{*}) \left( k^{\alpha} - ik^{f_{*}} \right)^{3}}$$
(3.2.15)

The sign of  $\eta^c$  clearly depends upon the sign of the bracketed term in (3.2.15). Eliminating ( $\mu$  + n), the first term in the bracket is simply the factor payments made to domestic capital and labor. The product  $\alpha k^{\alpha-1} k^d$  is the factor payment made to domestic capital, thus we are left with the return to labor after performing the subtraction. The last term is the increase in net output when  $k^f$  increases due to the increase in  $k^{d63}$ . This amount multiplied by  $k^d$  is probably of second order smallness making the elasticity negative.

With  $\eta^{y*}$  positive and  $\eta^{c*}$  negative, the term  $\eta^{y*} + \eta^{c*}$ , or the elasticity of per capita consumption with respect to  $k^d$ , is ambiguous. The following proposition suggests an intuitive answer.

**Proposition:** Evaluated at the steady state solution where the control variables take on the expected maximized values  $s = s^*(k^d)$  and  $k^f = k^{f*}$ , the elasticity of per capita consumption with respect to  $k^d$  is always positive.

*Proof:* The proof should be intuitively obvious. The sum  $\eta^{y*} + \eta^{c*}$  gives the elasticity of per capita consumption,  $(dc/dk^d)(k^d/c)$ . Reconsider the first order condition (1'), u'(c) = J<sub>k</sub>. Remember that J is the maximized objective function when  $s = s^*(k^d)$ 

<sup>&</sup>lt;sup>63</sup> The portion of the marginal product schedule attributable to  $k^{f}$  shifts outward when  $k^{d}$  is increased.

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and  $k^{f} = k^{f*}$ . Its derivative with respect to  $k^{d}$ ,  $J_{k}$ , is always positive since u'(c) > 0. By increasing  $k^{d}$  by an infinitely small unit the J function increases, thus  $(dc/dk^{d})(k^{d}/c) > 0$ .

From equation (3.2.14), it is clear that the case of uncertain foreign capital lowers its demand relative to the certainty case. This effect will be magnified the larger is the domestic capital-labor ratio  $k^d$  and the variance of the unanticipated component of foreign investment  $\sigma^2$ .

The solution for the case where both domestic and foreign capital are uncertain is only slightly more complicated. It can be shown that the Hamilton-Jacobi-Bellman equation becomes

$$-J_{t} = \max_{s,k_{t}^{f}} \left( u(c) + J_{k} \left[ sy - (\mu + n)k_{t}^{d} - nk_{t}^{f} + k_{t}^{f} - ak^{d} - bk^{f} \right] + \frac{1}{2} J_{kk} \left[ \sigma_{1}^{2}k^{d} + \sigma_{2}^{2}k^{f} + \sigma_{1}k^{d}\sigma_{2}k^{f}\rho_{12} \right] \right)$$

where  $\rho_{12}$  is the correlation between the shocks  $dz_1$  and  $dz_2$ . Maximizing with respect to s and  $k^f$  gives the first order conditions

$$-u_{c}(f(k) - i(kf)kf) + J_{k}(f(k) - i(kf)kf) = 0$$

$$u_{c}(1-s)(f(k) - i'(kf)kf - i) + J_{k}(s(f'(k) - i'(kf)kf - i) - n) + J_{kk}k^{f}\sigma_{2}^{2} + \frac{J_{kk}}{2}\sigma_{1}\sigma_{2}k^{d}\rho_{12} = 0$$

Substituting the implicit solutions for  $J_k$  and  $J_{kk}$  as before, and solving for  $k^{f*}$  gives

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$$k^{f}_{*} = \frac{f_{2} \cdot i - n + \begin{bmatrix} y & c \\ \eta_{k}^{d*} + \eta_{k}^{d*} \end{bmatrix} \frac{\sigma_{1} \sigma_{2}}{2} k^{d} \rho_{12}}{i' + \begin{bmatrix} y & c \\ \eta_{k}^{d*} + \eta_{k}^{d*} \end{bmatrix} k^{d} \sigma_{2}^{2}}$$

(3.2.15)

The change in  $k^{f*}$  in moving from a certain environment to an uncertain one is not unambiguous, as in the previous case. The behavior of  $k^{f*}$  now depends upon the domestic capital-labor ratio, the variances of the unanticipated components of both domestic and foreign capital, and the correlation coefficient between the shocks dz<sub>1</sub> and dz<sub>2</sub>. Consider three possible situations: 1) only foreign capital is uncertain, 2) only domestic capital is uncertain, 3) both foreign and domestic capital are uncertain. In the first case the solution for  $k^{f*}$  reduces to (3.2.14). The demand for foreign capital is reduced relative to the certainty case. In the second case, (3.2.15) reduces to the certainty solution  $k^{f*} = (f_2 - i - n)/i$  because  $\sigma_2 = 0$ . The only interesting, and perhaps realistic case is where both  $\sigma_1 > 0$  and  $\sigma_2 > 0$ . The analysis will be restricted to assuming  $\rho_{12} > 0$ . The direction of change for  $k^{f*}$  now depends upon the relative changes in  $\sigma_1$  and  $\sigma_2$ . The comparative static results are<sup>64</sup>

$$\frac{dk^{f}}{d\sigma_{1}}\Big|_{\sigma_{2}>0} = \frac{\left[\eta^{y} + \eta^{2}\right]\frac{\sigma_{2}}{2}k^{d}\rho_{12}}{i' + \left[\eta^{y} + \eta^{2}\right]k^{d}\sigma_{2}^{2}} > 0$$

<sup>&</sup>lt;sup>64</sup> Strictly speaking it is not proper to consider comparative static results on the solution for  $k^{f_*}$ . The solution would require total differentiation since all the terms in (3.2.15) are functions of the two variances, excepting for n. Because we are moving from complete certainty to an uncertain environment with only an infinitely small increase in both variances, it is assumed that the changes in the other terms are of second-order smallness.

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$$\frac{dk^{f}}{d\sigma_{2}}\Big|_{\sigma_{1}>0} = \frac{\left(i'+\eta^{c^{*}}k^{d}\sigma_{2}^{2}\left(\eta^{c^{*}}\frac{\sigma_{1}}{2}k^{d}\rho_{12}\right) - \left(2\eta^{c^{*}}k^{d}\sigma_{2}^{2}\right)\left(f_{2}-i-n+\eta^{c^{*}}k^{d}\frac{\sigma_{1}\sigma_{2}}{2}\rho_{12}\right)}{\left(i'+\eta^{c^{*}}k^{d}\sigma_{2}^{2}\right)^{2}}$$

where it is now understood that  $\eta^{c^*}$  is the elasticity of per capita consumption with respect to the domestic capital-labor ratio. If  $\rho_{12} > 0$ , then it is unambiguously true that an infinitely small increase in the variance of the unanticipated component of domestic capital raises the demand for foreign capital, holding  $\sigma_2$  and everything else constant. Domestic capital becomes relatively more risky to hold thus there is a substitution effect towards foreign capital<sup>65</sup>. The effect on the demand for domestic capital of an increase in the riskiness of holding foreign capital is not so unambiguous. The numerator of  $dk^{f*}/d\sigma_2$  can be reduced to

 $\left[\frac{i'\sigma_{1}-\eta^{c^{*}}\sigma_{1}\sigma_{2}k^{d}}{2}\right]\rho_{12}-2\sigma_{2}(f_{2}-i-n)$ 

which can be assumed negative with only minor discomfort. Thus when foreign capital becomes more risky to hold, the optimal policy is to reduce its demand.

It is important to note that both comparative static results depend upon the relative variances of the two types of capital *and* the correlation between the two types of shocks. Uncertainty occurs in the model in two ways. First, a change in the relative variances makes one type of capital relatively more risky to hold and thus reduces its demand. Second, the stronger the positive correlation between the two shocks, the

<sup>&</sup>lt;sup>65</sup> To derive the net effect of an increase in the riskiness of domestic capital on the demand for foreign capital we would have to allow all variables to change by taking the total derivative.

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stronger become the comparative static results. This is because a strong correlation gives one the ability to anticipate the direction of a shock to one type of capital, given a shock to the other, increasing the confidence of making an optimal decision. Perfect positive correlation reduces the problem to only encountering uncertainty in the relative variances. Zero correlation reduces the problem to the case where domestic capital is not uncertain and equation (3.2.14). If there is no ability to anticipate the direction of a shock to one type of capital, given an observed shock to the other, we only care about expected values and not variances. Consider Figure 3.1. The expected value of the steady state solution is given by  $k^*$  where  $s = s^*$  and  $k^f = k^{f*}$ . An increase in the variance of the shock term to k<sup>d</sup> makes the s\*y\* curve more variable, but does not change its expected value<sup>66</sup>. Since we do not change our demand for domestic capital. there is no change in the demand for foreign capital as (3.2.14) predicts. However if the correlation between shocks is positive, a shock to foreign capital changes the expected value of k<sup>d</sup>, which then changes the demand for k<sup>d</sup> and k<sup>f</sup> in (3.2.15). An increase in  $\sigma_1$ then increases the probability of a large shock to k<sup>d</sup> in the same direction as the shock to  $k^{f}$ . The controls s\* and  $k^{f}$ \* will adjust to move the s\*y\* curve to some new position, perhaps s'y'.

<sup>66</sup> There must be a unique distribution for both  $k^d$  which is time-independent and independent of the initial value of  $k^d$  towards which the stochastic proces for each tends. It can be shown that the stationary distribution for  $k^d$  is given by

$$\pi(k^{d}) = \frac{m}{\frac{2}{\sigma_{1}k^{d}}} exp\left[2\int \frac{k^{d}}{\frac{s^{*}(z)y^{*}(z) - (\mu + n)z}{\sigma_{1}z}} dz\right]$$

where z(t) is a standard random, variable with zero mean and unit variance. The constant m is chosen such that  $\int_{0}^{\infty} \pi(z) dz = 1$ . It can also be shown that this distribution is Liapunov stable. See Malliaris and Brock (1981), p. 104-106.

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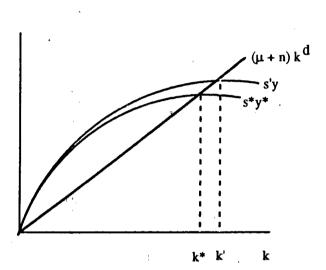


Figure 3.1

The Steady State Solution

## 3.5 A Rationale for Rapid Debt Accumulation

The purpose of the stochastic dynamic programming model developed thus far is to provide a possible scenario for the rapid build-up of LDC debt during the 1970's. If this were to occur in the context of the model, it would have to be an optimizing response to capital uncertainty. During the 1970's, capital uncertainty took the form of shocks to the prices of complementary inputs in production, most notably, the price of crude oil. It has already been noted in Chapter 1 that oil price shocks are one of three principle reasons believed by the I.M.F. to cause the "debt crisis" of the early 1980's. The "supply shock" literature<sup>67</sup> demonstrates that unexpected shocks to the prices of energy inputs shift the production function causing unexpected changes in the productivities of complementary inputs<sup>68</sup>. Essentially the stocks of capital and labor contain a stochastic element which is positive correlated with the shock to the energy input. In the continuous time framework utilized here, the rate of accumulation of the stock of capital. contains a stochastic element, rather than the stock itself. While almost any unanticipated exogenous shock can be lumped into the stochastic term of the accumulation equation (technology, human capital, natural disasters), the energy price induced supply shock was dominant during the 1970's.

Behavior similar to the extremely rapid debt accumulation of the 1970's can be obtained from the theoretical model in only one scenario. Both domestic and foreign capital contain stochastic elements which are not necessarily identical. The variance of the unanticipated component of domestic investment must rise substantially relative to its foreign capital counterpart. The borrower-producer will then substitute towards the relatively less risky foreign capital, which is analogous to rapidly accumulating debt. A

<sup>&</sup>lt;sup>67</sup> Any recent macro-economic text should provide an examination of the effects of supply shocks. One good exposition is Kennedy (1984), p. 290-302.

<sup>&</sup>lt;sup>68</sup> The borrowing LDC is assumed to be a net energy importer which means an unexpected positive shock to energy input prices reduces the productivity of complementary factors. A net oil exporting country would experience the opposite effect.

#### Chapter 3: LDC Credit Model

second requirement is a positive correlation between the shocks to domestic and foreign capital, which does not seem unlikely to be satisfied. The stronger the correlation, the more pronounced the substitution towards foreign capital. The borrower-producer maximizes his utility from consumption per capita by minimizing the riskiness of output fluctuations due to capital uncertainty.

This result is analogous to the consumption-smoothing and consumptionaugmenting behavior discussed in Frenkel and Razin (1987). In this case the same behavior has been found to exist in an infinite time model as in the two-period model used by Frenkel and Razin. The consumption-smoothing effect occurs when the variance of domestic investment rises relatively to the variance of foreign investment. Here the optimal decision is to reduce the domestic capital stock and increase the foreign capital stock in order to minimize unexpected fluctuations in income and consumption<sup>69</sup>. The consumption-augmenting effect is more subtle but potentially more powerful. Remembering that at the implicit solution for s\* in Figure 3.1, the derivative ds\*/dk<sup>d</sup> > 0 implies that as relatively riskier k<sup>d</sup> is reduced from the consumption-smoothing effect, the optimal control s\* is also reduced. This effect releases a higher percentage of income to be utilized towards valued consumption. In this way a falling savings rate is not a precursor to taking on additional debt, as several authors theorize in Chapter 2, rather it is an optimal response to taking on additional debt.

#### 3.6 Extensions

An interesting extension to the model developed in this chapter would be to build a similar model in discrete time. While algebraically more difficult, discrete time offers the ability to generate borrowing waves, such as those observed by Lindert and Morton

<sup>&</sup>lt;sup>69</sup> This can be accomplished by allowing equipment to wear out or exporting capital abroad (lending).

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(1987), by including some propagation mechanism conditioning some type of partial response to unanticipated investment shocks. For instance Kydland and Prescott (1982) use a time-to-build constraint where capital does not become productive until one period after investments are made. Greenwood, Hercowitz, and Huffman (1988) also utilize a time-to-build constraint, but also include a variable rate of capital utilization. Long and Plosser (1983) assume that current values of the technological shock, and thus production, are not observed until the next period. Any one of these propagation mechanisms could be applied to a dynamic discrete time borrowing model, which could then be calibrated and compared to observed covariances in business cycles with large borrowers.

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Chapter 4: Empirical Testing

## **Chapter 4: Empirical Testing**

4.1 Purpose

The model developed in Chapter 3 is essentially a stochastic growth model with international debt. Although the model is simpler in structure than dynamic models without international debt<sup>70</sup>, it still yields unambiguous predictions.

The critical theoretical prediction is that the borrower's optimal response to a higher level of risk in demanding either domestic or foreign capital will be to increase the relatively less risky type while reducing the other in order to buffer the economy from large drops in consumption, holding the marginal cost of borrowing constant. This is the *consumption*<sub>7</sub>*smoothing* effect. A secondary prediction is that during the period of rapid debt build-up the average propensity to save should be falling, the *consumption-augmenting* effect. The remainder of the chapter will concentrate on testing these predictions using a sample of South American LDCs<sup>71</sup>.

The model in Chapter 3 assumes that investment with foreign capital and foreign debt are equivalent. This is not the case in the real world. There is no reason why all new foreign debt must be converted into purchases of foreign capital and not used for consumption, foreign currency reserves, etc. Nevertheless the usual rule of thought is that debt is taken on to make up the difference between national savings and gross investment. It is true that foreign savings is necessary to finance any current account deficit, but this only captures a small portion of total borrowing. For this reason a separate demand and supply model for new debt is specified based on the dynamic

<sup>&</sup>lt;sup>70</sup> See for example Kydland and Prescott (1981), Long and Plosser (1983), and Greenwood, Hercowitz, and Huffman (1988).

 $<sup>7^{1}</sup>$  The theoretical predictions could just as well be applied to developed economies and tested there. However the rather spectacular build-up of debt in LDCs makes their study more relevant and interesting:

## model of Chapter 3.

The key behavioral assertion from Chapter 3 is that investment and savings decisions are made taking into account conditions in world credit markets. The taking on of additional foreign debt is not simply a residual process after optimal savings and investment decisions have been made. Investment and savings functions will be estimated simultaneously using Zellner's seemingly unrelated regressions technique.

The ratio of the residual variances for new debt and savings will be analyzed against debt/GNP ratios for 15 South American LDCs to provide a direct test of the dynamic model from Chapter 3.

In Section 4.2, investment and savings functions shall be specified and estimated in Section 4.3 for 15 South American LDCs. In Section 4.4, a reduced form new debt equation will be derived, and estimated in Section 4.5 Section 4.6 will analyze the variance ratios for the 15 LDCs versus the their respective debt-GNP ratios. Section 4.7 will conduct unit root tests to determine the importance of the real shock component of investment to the determination of the paths of real GNP. Section 4.8 provides a brief summary and conclusions.

## 4.2 Derivation of the Investment and Savings Functions

Obviously the results may depend critically on how one models anticipated investment and savings. If the econometric functions do not capture variables used by individuals to form expectations of investment and savings, the variances of the unanticipated components will be biased. This problem is almost unavoidable in any econometric equation containing expectations terms, however the problem is particularly acute in dealing with LDCs due to the lack of available data<sup>72</sup>.

<sup>72</sup> Blejer and Khan (1984) note the difficulty in estimating neoclassical investment functions for

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## 4.2.1 Investment Function

The most standard investment model in practical use is the flexible accelerator. The flexible accelerator model has the desired capital stock K\* proportional to real output y

To derive the net investment function, express the relationship in first differences and divide by y to obtain

$$I^*/y = \sigma g$$

where g is the growth rate of real income. The partial adjustment mechanism is standard and is as follows.

$$I_t - I_{t-1} = \lambda (I^* - I_{t-1})$$
 or  
 $I_t = \lambda I^* + (1 - \lambda) I_{t-1}$ 

where  $0 \le \lambda \le 1$  is the speed of adjustment. Substituting for I\* gives

$$\left(\frac{I}{y}\right)_{t} = \lambda \sigma g_{t} + (1 - \lambda) \left(\frac{I}{y}\right)_{t-1}$$

At this point a linear set of n exogenous variables is inserted to explain the behavior of the multiplier  $\sigma$ .

developing countries due to lack of available data on capital stocks and their rates of return,

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$$\sigma = f(X_1, X_2, ..., X_R)$$

The exogenous variables chosen include a riskless real interest rate, a risk-inclusive real rate of interest, the real exchange rate, the terms of trade, and the debt-GNP ratio. The use of these variables is justified below. Since user costs of capital are generally unavailable for developing countries, a proxy used was the 6 month LIBOR rate minus the anticipated U.S. rate of inflation. Obviously the observations will be the same across countries reflecting the assumption of perfect capital mobility.

The risk-inclusive real rate of interest reflects the price of a substitute for financing investment through additional savings. Annual average Euroloan nominal lending rates were obtained for each country over the sample period 1965-84. As the bulk of lending to developing countries is in U.S. dollar denomination, it was necessary to specify an information set for modelling the expected rate of U.S. inflation. This expected rate was then subtracted from the nominal Euroloan rate to arrive at a real rate of interest on international capital. The equation modelling the expected rate of U.S. inflation followed that used by Fair (1984)

$$\pi_{i} = a + \sum_{i=1}^{n} b \pi_{i-1} + \sum_{i=1}^{n} c w_{i-1} + \sum_{i=1}^{n} d m p_{i-1} + \sum_{i=1}^{n} e z z_{i-1} + v_{i-1}$$

where  $\pi_t$  is the rate of inflation measured using the GDP deflator at time t,  $w_t$  is time t wage inflation,  $mp_t$  is time t percentage changes in import prices, and  $zz_t$  is a time t demand pressure variable. The variable  $zz_t$  is the gap between potential and actual GDP as a percentage of potential GDP. Potential GDP was determined by interpolating peak to peak movements in GDP over the sample period 1948-86. The estimated equation is given below with t-values in parentheses.

 $\pi_t = -.035721 + .43228\pi_{t-1} - 1.1992\pi_{t-2} + .54919\pi_{t-3} - .28289\pi_{t-4} - .14507w_{t-1}$ (-.79206) (1.3005) (-3.3476) (1.2729)(-.66385)(-.28462) $+2.5166w_{t-2} - .71352w_{t-3} - .14652w_{t-4} + .33713mp_{t-1} + .20488mp_{t-2} - .2048mp_{t-2} - .2048mp_{t-2} - .2048mp_{t-2} - .20488mp_$ (-1.0568)(-.24237) (2.0670)(1.1328)(4.0896)  $.23227 \text{mp}_{t-3} + .03649 \text{mp}_{t-4} - .78500zz_{t-1} + 1.3192zz_{t-2} - .57668zz_{t-3}$ (-3.0927)(2.8948)(-1.2887)(-1.3756)(.21604)

+ .019936zz<sub>t-4</sub> (.0674)

Adjusted  $R^2 = 0.7555$  DW = 2.2836 F = 7.568

Since the nominal lending rate includes a spread above LIBOR for country specific risk, the risk-inclusive real rate of interest is not the same across the sample of countries. If different borrowers face different risk spreads, it does not make sense to assume that capital owners will demand a riskless rate of return on lent capital, whether they be domestic or foreign owners of capital. Figure 4.2 provides a one standard deviation error bar for each year from 1978-84 for the mean risk spread above LIBOR for 52 countries<sup>73</sup>. If the entire distribution of spreads is considered, the error bars are much larger. A table of all risk spreads for each country is contained in an appendix. Generally most previous papers do not incorporate risk premia in their real interest rates either because the nature of the paper necessitated using the risk spread for each country, which is only available in quantity starting in 1978, or risk is simply ignored (Fry (1989)). In this case both risky and riskless rates have been included.

The real exchange rate is measured as the ratio domestic prices to U.S. prices

<sup>&</sup>lt;sup>73</sup> The annual spread for each of the 52 countries is a weighted average where the weights are the magnitudes, in dollar terms, of each individual sovereign loan. These figures were compiled from various issues of Euromoney. An ANOVA test yielded a significant F-statistic of 6.437 at the 1% level of significance, indicating that the mean risk spreads are not equal throughout the sample.

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multiplied by the nominal exchange rate. When rebased to some base year, the real exchange rate measures the extent to which the nominal exchange rate is over or undervalued<sup>74</sup>. In the first chapter it was noted that an overvalued nominal exchange rate could result in expectations on the part of capital users of a depreciation in the nominal exchange rate. This would be reflected by a real exchange rate of greater than 100. To achieve capital gains, capital equipment could be imported before the anticipated depreciation and then sold or loaned after. This high inflow of new capital just might spur investment on the part of capital users.

Persson and Svensson (1985) show that an unanticipated permanent improvement in the terms of trade (export prices/import prices) may raise the domestic investmentincome ratio by raising the rate of return to capital<sup>75</sup>.

The debt/GNP ratio was included based on the solution for the model developed in Chapter 3. There it was found that the implicit optimal solution for the average propensity to save, s\*, was a function of both the domestic capital stock and the foreign capital stock.

$$dk_t^{d*} = \left[s^{*}(k_t^{d}, k_t^{f})y_t - (\mu + n)k_t^{d}\right]dt + \sigma k \beta z$$

Depreciation of the capital stock was not accounted for in the investment equations because adequate estimates of depreciation were unavailable for the entire sample period<sup>76</sup>. One technique to overcome this problem is to assume straight-line depreciation and model this as  $D_t = a + bt$  where  $D_t$  is depreciation at time t and the

<sup>&</sup>lt;sup>74</sup> Nominal exchange rates were obtained from IFS. Price levels were GNP deflators obtained from IFS.

<sup>&</sup>lt;sup>75</sup> The terms of trade index was obtained from various issues of the annual publication Statistical Abstract of Latin America (SALA).

<sup>&</sup>lt;sup>76</sup> Depreciation estimates are surprisingly available for some LDCs for recent years in the United Nations Yearbook for Latin America in the national accounts sections.

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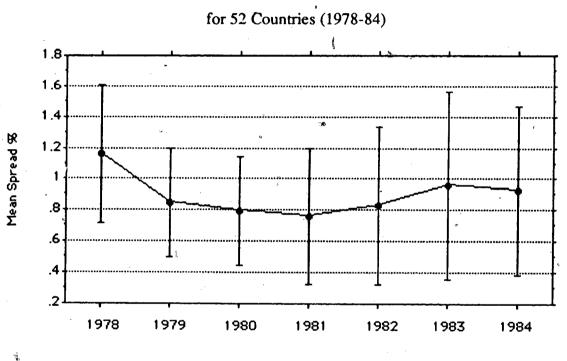


Figure 4.1

One Standard Deviation Error Bars for Mean Risk Spreads

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Note: See Appendix C for calculations. All figures taken from various issues of Euromoney.

variable t is a time trend<sup>77</sup>. By substituting this equation into the investment function, the depreciation term falls out and thus need not be measured. This is equivalent to regressing the dependent variable, investment/income, on a time trend which, for reasons already discussed at length, is an inadvisable method.

The final estimating equation for the investment function is,

 $I/Y = f(g, r, i, e, TT, D/Y, (I/Y)_{t-1})$ 

where  $r_t = riskless$  real interest rate,  $i_t = risky$  real interest rate on debt,  $e_t = real$  exchange rate index,  $TT_t = terms$  of trade index,  $D/y_t = debt$ -GNP ratio, and  $v_t$  is a disturbance term ~ N(0,  $\sigma^2_v$ ).

## 4.2.2 Savings Function

Savings is estimated using the extended life-cycle model employed by Fry and Mason (1981). In this model young income earning individuals save in order to finance consumption when they are older non-earning individuals. Each household consumes all of its resources over its lifetime (no bequests). Defining consumption C(a) and earnings Y(a) for a household of age a as a fraction of lifetime earnings, we can say that the level of household consumption L will satisfy  $L = \int C(a) da = 1$  and household earnings will satisfy  $\int Y(a) da = 1$ .

The behavior of the aggregate savings rate can be through an aggregate consumption function of the form

$$1 - s = c = \int V(a) C(a) da$$

<sup>&</sup>lt;sup>77</sup> See Fair (1984) p. 174-5.

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where V(a) is the ratio of lifetime resources of all households aged a to GNP. At the steady state solution V(a) is time autonomous and is given by V(a) = V(0) e<sup>-ga</sup> where g is the instantaneous growth rate of real GNP. Note that if g = 0, then V(a) = 1 and the savings rate is zero. Given positive growth in real GNP, the lifetime resources V(a) of young savers will exceed that of older dissavers and the savings rate will be positive, given that savings is concentrated among young households. If savings is concentrated among older households, an increase in g might lower the savings rate. Commonly it is thought that the age where mean lifetime earnings is reached ( $\mu_y$ ) is less than the age where mean consumption is reached ( $\mu_c$ ), thus the GNP growth effect should raise the savings rate.

Substituting the expression for V(a) into the expression for consumption gives

$$c = V(0) \int e^{-ga} C(a) \, da$$

Fry and Mason (1982) show that the expression for consumption can be worked out to be the rather unsurprising result

$$c = \frac{\int e^{-ga}C(a)}{\int e^{-ga}Y(a)}$$

and can be approximated by  $\ln c = \ln L + (\mu_y - \mu_c)g$ . The savings function is thus

$$ln\left[\frac{1}{1-s}\right] = -ln L + (\mu_c - \mu_y)g$$

For moderate savings rates, Fry and Mason (1982) note that  $\ln [1/(1-s)] \approx s$ . Household consumption L can be approximated by a log-linear function of a set of independent variables x,  $L = e^{-\alpha x}$ . Substituting into the savings function gives the linear savings function

## $s = \alpha x + (\mu_c - \mu_v) g$

As was the case with the investment function, the savings function will exhibit a lagged adjustment to changes in exogenous variables and thus a lagged savings rate will be included. The set of exogenous variables x will include a real rate of return on savings r, a risk inclusive real interest rate on foreign debt i, the rate of change in income attributable to terms of trade changes TTY, the debt-GNP ratio D, and the rate of inflation.

The real rate of return on savings is the same world real interest rate used in the investment equations. Again the risk inclusive real interest rate on foreign debt, i, is included in order to reflect the interactive decision process between savings and additional foreign debt.

Savings decisions are not only sensitive to the real growth rate of output g, but in an open economy they will also be sensitive to change in real income due to changes in terms of trade. Real income is defined as  $y + X(P_x/P_m - 1)$  where X is exports and  $P_x/P_m$  is the terms of trade. The terms of trade variable TTY captures the change in real income due to changes in the terms of trade: TTY =  $[\Delta(P_x/P_m)/(P_x/P_m)_{t-1}](X_{t-1}/y_{t-1})$ . An increase in g or TTY should raise the savings rate.

The debt-GNP ratio could be a preliminary indicator of imminent government action to reduce the debt overhang on the economy. Policies such as future devaluations,

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future tax increases, etc. all figure into this variable. The effect of the debt-GNP ratio on the savings rate is ambiguous because, while future evaluations will reduce real income thereby reducing the savings rate, a higher level of current borrowing may necessitate future tax increases causing a higher savings rate today (Ricardian equivalence).

Stockman (1981) using a discrete infinite time model showed that a positive inflation rate reduced the optimal capital stock for producing firms, resulting in a corresponding fall in savings. On the other hand, MacKinnon (1989) contradicts this result in a three period model by showing that a positive inflation rate is a tax on corporate dividends which makes it optimal to increase retained earnings.

As was the case with the investment function, the savings rate is likely to exhibit lagged adjustment towards the optimal savings rate, thus a lagged dependent variable is included. The life-cycle savings function to be estimated for each of 15 countries is

 $s = h(r, i, TTY, D/Y, \pi, g, s_{t-1})$ 

## 4.3 Estimates of the Investment and Savings Functions

The equations were regressed simultaneously for each of 15 countries over the period 1965-84 using annual observations. Admittedly the degrees of freedom are not large for this estimation. In order to take advantage of cross equation correlation of the residuals, Zellner's seemingly unrelated regressions method was used<sup>78</sup>. Statistical results appear in Table 4.1 below<sup>79</sup>.

<sup>&</sup>lt;sup>78</sup> All statistics and estimations were performed using the SHAZAM econometric package.

<sup>&</sup>lt;sup>79</sup> A time trend variable was included in the investment function to try to account for depreciation of the capital stock, however this variable was dropped as it did not add any explanatory power to any of the regression equations. In addition a simple flexible accelerator with expectations was tried, but the resulting fits were very poor. The DLAG option on SHAZAM was used in order to yield more efficient estimates when a lagged dependent variable is present.

## Table 4.1

# Results of Estimated Investment Equations for 15 Countries

1965-84

5

Country	g	r	i	e	TT	D/Y	(I/Y) <sub>t-1</sub>	π	R <sup>2</sup>	h	-
Bolivia	.229		.035						.655	45	-
	(.655)	`(1.02)	(.063)	(.107)	(339)	(064)	(3.33)*	(089)		1	
Brazil	.098	.392	- .001	.003	192	-11.00	.266	0352	.894	.91	
			(.033)								• .
Chile	.414	.352	· .575	944	051	.83	.07	.001	.572	4.44	
	(3.24)*	(1.15)	(1.30)	(785)	(-1.52)	(.100)	(.320)	(.070)			
Colombia	a .249	016	057	· .692	036	-22.39	.066	.048	.143	28	
	(1.04)	(127)	(235)	(1.28)	(-1.43)	(-1.08)	(.224)	(.414)			
Costa Rie	ca .363	008	1.40	209	012	-1.67	129	.161	.878	-1.43	
	(2.102)	(077)	(4.26)*	(551)	(267)	(076)	(820)	(3.76)*	,		
Dominica	an .335	.004	758	-10.76	.002	3.77	.327	.118	.954	-1.02	
	(7.02)*	(.035)	(-5.28)*	(-1.67)	(.108)	(1.20)	(3.62)*	(1.64)			
Ecuador	001	.216	668	399	.049	-1.52	.626	127	.767	-2.02	
	(071)	(1.11)	-(-1.23)	(768)	(1.37)	(513)	(3.17)*	(949)			
Guatema	la .208	002	341	8.56	.014	18	.503	.242	.665	-3.01	
	<b>(.791)</b>	(008)	(639)	(.744)	(.217)	(007)	(2.86)*	(1.49)	٦	r.	
Honduras	s .584	623	893	-6.07	044	7.02	.551	201	.645	.41	
	(3.38)*	(-2.54)*	(-1.87)	(347)	(668)	(1.37)	<sup>,</sup> (3.54)*	(749)			

. 1965-84											
Country	g	r	i	e	· <b>TT</b>	D/Y	(I/Y) <sub>t-1</sub>	π	R <sup>2</sup>	h	· · · · · · · · · · · · · · · · · · ·
Mexico	.542 (2.43)*							021 (251)	.821	3.49	
Panama								1.061 (2.78)*	.85	-1.62	
Paraguay		(.668)						.033 (.552)	.967	-1.48	
Peru	.013 (.084)	475 (-2.39)*	.839 (2.30)*		032 (-1.17)			039 (837)	.759	87	
Uruguay							.667 (3.15)*	01 (720)	.712	2.33	
Venezuel				10.34 (3.08)*					.882	62 ·	

## Table 4.1 (cont.d)

Note: An asterisk denotes significance at 95% confidence. Bracketed terms are t-ratios. h is Durbin's h test for serial correlation when a lagged dependent variable is present. It is assumed to be distributed as a standard normal.

In 10 out of 15 cases the adjustment speed coefficient was significant at the 5% level. Remember that the adjustment speed is one minus the coefficient on the lagged investment-income ratio. Those countries with high adjustment speeds (Chile, Costa Rica, Colombia, Mexico, Panama) tend to be those who are active in international capital markets and are at a relatively higher stage of development. Relatively poor countries at a low stage of development tend to have very low adjustment speeds (Paraguay, Peru, Venezuela). Most countries fall somewhere in between these two extremes.

The growth rate of real GNP was statistically significant in only 5 out of 15 cases. In the 10 insignificant cases, 8 had relatively small adjustment speed coefficients ( $\lambda$ 's). Because the coefficient associated with the growth rate of real GNP is actually  $\lambda\sigma$ , small values for  $\lambda$  reduce the magnitude of the regression coefficient, in most cases causing the coefficient to become statistically insignificant. Given small values for  $\lambda$  which are close to zero, little can be said about the magnitude of the accelerator term  $\sigma$ . Of those 5 cases which were statistically significant, all but Venezuela possess estimated accelerator terms less than one, meaning that the desired capital stock is less than the flow of real GNP each year. This result might seem odd in the instance of a developed country such as the United States, however it is not so odd in the instance of a developed country with labor intensive industry and a small domestic capital stock.

The signs on the remaining coefficients generally came out as expected. The real interest rate was statistically significant in only 4 of 15 cases at the 5% level, but always carried a negative sign. This result coincides with Fry (1988) where, in a pooled regression with a similar group of countries, the real interest rate was statistically insignificant<sup>80</sup>. It would appear that investment demand is interest rate inelastic in Latin American developing countries, perhaps explaining their recent repayments

<sup>&</sup>lt;sup>80</sup> Although Fry used the LIBOR rate minus the U.S. inflation rate, not taking into account countryspecific risk.

## Chapter 4: Empirical Testing

problems. The real exchange rate was statistically significant in only two cases, those of Paraguay and Venezuela. Apparently the expectation of a looming exchange rate depreciation does not alter investment behavior in these countries. Paraguay and Venezuela are two of three countries in the sample which at one time or another followed an exchange rate fixed to the U.S. dollar. Since currency devaluations under fixed exchange rates are usually much more dramatic and not as frequent as depreciations under flexible exchange rates, expectations may be much more important in determining investment behavior.

The terms of trade variable was not statistically significant in any case. In all cases the coefficient was of a very small order of magnitude indicating that the terms of trade is not an important determinant of investment behavior. The debt-income ratio was significant in only one case (Venezuela), but always carried the anticipated negative sign. This result was particularly disappointing since the theoretical model postulated that domestic investment decisions were not made without regard to conditions in international credit markets. The rate of inflation explained the investment-GNP ratio in only three cases, but in one case was of the wrong sign. It would appear that perhaps investment can find ways to shield itself against the inflation tax used by many of the countries in the sample. Overall the investment equations provided less than satisfactory fits, due to omitted variables and the small sample size.

The results for the savings functions are moderately successful. These appear in Table 4.2 below. The most consistent explanatory variable in the estimated savings function was the lagged savings rate (6 out of 15 cases). Generally when statistically significant, this variable indicated slow adjustment of the savings rate to its optimal value (Bolivia, Costa Rica, Ecuador, Guatemala, Paraguay, and Venezuela). This variable does not categorize the sample countries into stages of economic development as clearly as the lagged investment rate in the estimated investment functions. The nine countries which exhibited very rapid adjustment of the savings rate are at widely

1965-84										
Country	g	r	i	TTY	D/Y	(S/Y) <sub>t-1</sub>	π	R <sup>2</sup>	h	
Bolivia	.065 (.096)	.983 (1.97)	.473 (.487)		-30.89 (-2.06)			.723	.42	
Brazil	.13 (1.43)	.444 (1.99)	538 (-1.32)		-40.23 (-1.67)			.693	59	
Chile	.353 (3.26)*	.256 (.736)`	327 (748)	.609 (1.76)	.127 (.021)	.312 (1.14)	.008 (.964)	.689	09	
Colombia	.186 (1.02)	.205 (1.56)	407 (-2.05)		.007 (.001)	088 (443)	.056 (.541)	.462	- 26	
Costa Rica	.105 (.539)	.319 (2.12)	.542 (2.05)	.305 (2.47)*	30.89 (2.53)*	.621 (2.85)*		.758	-4.70	.'
Dominican	.227 (2.71)*	.221 (.957)		1	1.123 (.184)			.781	-1.66	
Ecuador	03 (250)	.326 (1.56)	136 (306)			.431 (2.43)*		.774	21	
Guatemala	.443 (3.61)*	,38 (4.31)*	444 (-1.89)		11.30 (.963)			.95		
Honduras	.271 (1.83)	.184 (.875)	.227 (.577)		-11.07 (-1.84)		03 (140)	.723	1.45	
Mexico	142 (1.51)	.013 (.286)	£		242 (0636)		.033 (1.13)	.86	-1.37	

# Table 4.2

Results of Estimated Savings Equations for 15 Countries

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## Table 4.2 (cont.d)

## Results of Estimated Savings Equations for 15 Countries

## 1965-84

Country	g	r	i	TTY	D/Y	(S/Y) <sub>t-1</sub>	π	R <sup>2</sup>	h
									·
Panama	109	.971	.991	.058	-10.78	.078	1.16	.636	02
	(684)	(3.43)*	(4.61)*	(.431)	(-3.74)*	(.421)	(3.41)*	•	
Paraguay	.097	.108	10	077	-3.99	.543	.169	.56	2.44
خ	(.318)	(.365)	(156)	(212)	(233)	(2.24)*	(1.13)		r
Peru	.271	.055	1.21	.114	2.7	.359	007	.66	30
	(1.18)	(.249)	(1.34)	(.582)	(.173)	(.969)	(083)		
Uruguay	.483	.104	944	.346	-1.12	.186	022	.662	2.03
	(2.42)*	(.529)	(-2.32)*	(1.96)	(265)	(.973)	(-1.14)		
Venezuela	.187	.02 <b>2</b>	522	061	.059	.733	.377	.915	.05
	(.943)	(.083)	(-2.79)*	(631)	(1.33)	(7.46)*	(3.86)*		

Note: An asterisk denotes significance at 95% confidence. Bracketed terms are t-ratios. h is Durbin's h test for serial correlation when a lagged dependent variable is present. It is distributed as a standard normal.

different stages of development (eg. Brazil vs. the Dominican Republic).

For all countries the savings rate is almost completely insensitive to both the real rate of interest and the risky real rate of borrowing. This could be for several reasons. First, the rate of return on savings used in the estimations may not be an accurate measure of the true after-tax rate of return. Second, capital market linkages with the rest of the world may not be perfect; there may be artificial constraints on the ability to freely move capital in and out of these countries. Third, it has already been noted in Chapter 1 that LDCs do not generally invest until the marginal efficiency of investment (MEI) just equals the real rate of return on savings due to the constraint the government faces in raising future tax revenues to meet debt-service payments on some portion of the new investment, if not completely domestically generated. With the MEI above the real rate of return on savings, the short-side rule dictates that the resulting quantity of savings will be determined by investment (the demand for savings). The effective supply curve of savings becomes vertical at the going quantity of investment and savings. Further increases in the real rate of return on savings will not increase the supply of savings since it is not in demand.

The savings rate is sensitive to the domestic inflation rate in four of the fifteen countries sampled, although its sign is unexpectedly positive in each case (albeit small in magnitude). The notion of Ricardian Equivalence suggests that individuals may anticipate that the monetary authority cannot increase the inflation rate indefinitely to collect an inflation tax. At some point in the not too distant future, the government will have to switch to a fiscal tax as the rate of inflation becomes intolerable. Forward thinking individuals may increase current savings to finance the anticipated future tax increases. More realistically there could be a strong wealth effect associated with high current rates of inflation. Income tax is collected based on the individuals money income for the previous year, but the tax payments are in current units of money. With a

high inflation rate, the actual taxes collected are small relative to current income, smaller than anticipated in the previous year when the income was earned. This is the same as an unexpected increase in after-tax wealth which may stimulate domestic savings.

## 4.4 Specification of the New Debt Market

From the model developed in Chapter 3 we derived the foreign capital (debt) dynamic demand equation

$$k^{f_{*}} = \frac{f_{2} - i - n + \begin{bmatrix} y & c \\ \eta_{k} d^{*} + \eta_{k} d^{*} \end{bmatrix} \frac{\sigma_{1} \sigma_{2}}{2} k^{d} \rho_{12}}{i' + \begin{bmatrix} y & c \\ \eta_{k} d^{*} + \eta_{k} d^{*} \end{bmatrix} k^{d} \sigma^{2}}$$
(3.2.15)

or more generally as  $k^{f*} = f(k^{d*}, \sigma^2_1, \sigma^2_2, i, n, \eta_k^{C})$ . Taking first differences debt demand can be analyzed as the flow of new debt:  $\Delta k^{f*} = f(\Delta k^{d*}, \sigma^2_1, \sigma^2_2, i, n, \eta_k^{C})$ . We also know the composition of the elasticity of consumption with respect to the domestic capital-labor ratio

$$\eta_k^C = \eta_k^y + \eta_k^c d$$

The first term on the RHS, the elasticity of per capita output with respect to the domestic capital-labor ratio, is a function of the technology possessed by producers and can be assumed constant. The second elasticity, the elasticity of the consumption rate with respect to the domestic capital-labor ratio, is behavioral since it involves the optimal behavior of the control variable s<sup>\*</sup>. From Chapter 3 we know that the optimal savings rate is a function of the state variable and the exogenous variables:  $s^* = s^*(k^d, s^d)$ 

 $\mu$ , n, r). We can assume that the domestic capital stock is a constant multiple of the level of real output,  $k^d = \sigma y$ .

Besides the independent variables discussed so far, other exogenous variables should be included to reflect uses for new debt not covered in the dynamic model of Chapter 3. Developing countries often borrow in Euromarkets and from institutional lenders to replenish exhausted foreign currency reserves. These reserves are used to stabilize exchange rates and facilitate international transactions. Thus the level of real foreign currency reserves measured in U.S. dollars (fc) may influence the decision to take on additional debt.

Due to the likely presence of an upward-sloping supply curve of loanable funds (reflecting default risk rising with level of debt) it is likely that borrowers may not acquire the optimal amount of new funds in the current year. Thus a lagged new debt/GNP ratio will be included reflecting a partial adjustment mechanism.

After all substitutions are made, the debt demand function is

$$\Delta k^{f*} = f(\Delta k^{d*}, y; \sigma^2_1, \sigma^2_2, i, n, r, fc, \Delta k^{f}_{t-1})$$

Imposing homogeneity of degree zero allows us to write the demand for new debt

$$\frac{\Delta k^{f_*}}{y} = f\left[\frac{\Delta k^{d_*}}{y}, 1, \left(\frac{\Delta k^{f_*}}{y}\right)_{t-1}; i, \dot{r}, \dot{n}, \dot{fc}\right]$$

as

Chapter 4: Empirical Testing

where the effects of  $\sigma_1^2$  and  $\sigma_2^2$  are now embedded in the error term.

As already noted in Chapter 2, the supply side of the international debt market has already been modelled in different ways quite exhaustively. The presentation here will be greatly simplified from previous papers. Generally we have

$$\frac{\Delta k^f}{y} = \phi [i, r, EURO]$$

The quantity supplied of loans is an increasing function of the country specific risk-inclusive real lending rate<sup>81</sup>, a decreasing function of the opportunity cost of lending, and an increasing function of the stock of eurocurrencies to lending banks. If this supply curve were to be estimated, a vector of risk indicators could be substituted for the interest rate i and could be any of the variables summarized in Table 2.2.

The reduced form is found by first inverting the debt demand and supply functions and then equating. This gives the reduced form equation

$$\frac{\Delta k^{f_*}}{y} = \psi \left[ \frac{\Delta k^{d_*}}{y}, \stackrel{?}{r}, \stackrel{+}{n}, E \stackrel{+}{URO}, \stackrel{+}{fc}, \frac{\Delta k^{f_*}}{y} \right]$$

which can be estimated in linear form using OLS. Its residual variance can then be compared to the residual variance of the estimated savings function to test for portfolio behavior across countries.

<sup>81</sup> As discussed at length in Chapter 2, this relationship may become negative at high lending rates due to an unacceptable risk of default.

# 4.5 OLS Estimates of the Reduced Form Debt Equation

OLS estimates of the reduced form debt equations for all 15 countries appear below in Table 4.3. Generally the results are no better than can be expected for a difference equation. Due to the reduced form nature of the estimated equations, little can be said about the interpretation of the coefficients. The fits showed considerable variation, from a high adjusted  $R^2$  of 0.6117 for Mexico to a low of -0.2681 for Colombia with a fairly even distribution in between. The lagged dependent variable was significant in only three cases, Bolivia, Panama, and Uruguay, indicating less than immediate adjustment to the desired flow of new debt. Rather perversely many of the adjustment speed coefficients were negative although insignificant. It would appear that even heavily borrowing LDCs are not constrained from achieving their desired level of new borrowing.

The signs generally came out as expected except for the sign of the lagged foreign currency variable, which frequently was negative, although insignificant. This indicates that an unexpected decrease in foreign currency reserves in the current year resulted in an increase in new debt inflows in the current year.

OLS Estimates of the Reduced Form Debt Equations

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Country	Δ(D/Y) <sub>t-1</sub>	FC	(I/Y)	• <b>N</b>	EURO	Ŕ	R <sup>2</sup>	DW
Bolivia	562	001	.005	041	.001	006	.185	2.25
	(-2.24)*	(-1.39)	(.571)	(704)	(1.83)	(-1.00)	• •	
Brazil `	.259	-1x10-5	001	004	.` .0001	001	.284	2.14
	(.658)	(300)	(435)	(385)	(1.33)	(254)		
Chile		-5x10 <sup>-5</sup>	005	.150	.0004	.004	091	1.82
	(.997)	(065)	(- 658)	(1.14)	(.819)	(.543)		
Colombia	.068	-4x10 <sup>-5</sup>	.0004	.0001	.0001	001	268	2.06
	(.182)	· (601)	(.081)	(.015)	(.960)	(702)		
Costa Rica	.11	.0001	006	002	.0001	.001	.168	1.70
	(.362)	(.424)	(-1.87)	(364)	(.548)	. (.810)	°	-
Dominican	.091	.0002	006	.047	.0002	.001	.283	2.25
	(.260)	(.297)	(750)	(1.29)	(:828)	(.244)		.*
Ecuador	.001	.0002	008	041	.0001	006	.477	2.26
A	(.003)	(1.45)	(-1.67)	(-1.01)	(.408)	(-1.82)		2
Guatemala	.56	-6x10 <sup>-4</sup>	.003	006	3x10-4	0003	.587	1.82
	(1.53)	(-3.67)*	(3.52)*	(-1.52)	(1.11)	(535)		
Honduras	.066	.0004	003	.001	.0002	.003	.412	2.03
	(.205)	(1.66)	(-1.19)	(.304)	(.231)	(1.61)		*

# Table 4.3 (cont.d)

OLS Estimates of the Reduced Form Debt Equations

,		<b>"</b>	r (19	05-04)				
Country	Δ(D/Y) <sub>t-1</sub>	. FC <sub>t-1</sub>	. (I/Y)	N	EURO	R	R <sup>2</sup>	DW
Mexico	.375	0004	003	.008	.0004	004	.612	2.29
	(.996)	(-3.94)*	(296)	(.360)	(2.91)*	(-1.03)		
Panama	.541	.003	007	.045	001	004	.369	1.29
	(4.03)*	(3.21)*	(-2.15)	(1.89)	(-2.68)	(-1.41)		
Paraguay *	325	001	002	016	.0002	003	.247	2.06
	(988)	(445)	(590)	(-1.06)	(1.24)	(-1.90)	-	
Peru	108	0002	005	.006	.001	004 .	.275	2.17
	(393)	(-2.59)*	(790)	(.160)	(2.67)*	(-1.20)		
Uruguay	.442	002	082	.093	.001	023	.496	1.73
- ·	(2.37)*	(-2.47)*	(-3.17)*	(1.26)	(1.34)	(-2.26)*		
Venezuela	362	.0002	003	.156	0002	011	.04	2.17
, _	(-1.22)	(.601)	(339)	(.499)	(412)	(-1.26)		

· (1965-84)

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#### Chapter 4: Empirical Testing

## 4.6 The Portfolio Risk Effect

Having estimated savings and debt functions for the 15 countries in the sample, we can now take the ratios of their respective standard errors and compare these across countries to the ratios of new debt to savings. The model predicts that as the ratio of the standard errors of savings to new debt increases we should observe an increase in the new debt to savings ratio. If we plot these ratios across countries an upward sloping relationship should be observed. Table 4.4 below computes these ratios and Figure 4.2 provides a scatter plot of the results.

It can be seen from Figure 4.2 that the relationship between the ratio of the savings to new debt standard errors and the new debt - savings ratio is negative, the opposite of what was expected (bands are 95% confidence limits). This perverse result was quite robust to different specifications of the investment, savings, and new debt functions. The result certainly contradicts the prediction of the dynamic model in Chapter 3, that is, that LDCs who are maximizing real consumption will practice consumption smoothing and consumption augmenting by substituting to the relatively less risky type of capital. However this result does not necessarily refute the appropriateness of the model as the sample correlations between the residuals of the savings and new debt functions were negative in 11 out of 15 cases. A negative correlation implies that savings and new debt were not being influenced by the same type of shock, even though the two markets are linked behaviorally in the model. An unexpected negative shock to savings will increase the use of foreign debt in order to maintain current consumption even though the variance of the unanticipated portion of new foreign debt may be larger. The empirical results do not refute the predictions of the model, but do provide a result which was not anticipated.

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## Table 4.4

## Summary of the Ratios of the Standard Errors of Savings to New Debt and the New Debt to Savings Ratios

Country	Ratio of Standard Errors (S/D)	Ratio of New Debt to Savings
Bolivia	5.7739 / .1008 = 57.28	.0477 / .198 = 0.2409
Brazil	1.822 / .0232 = 78.53	.0202 / .193 = 0.1047
Chile	2.722 / .1112 = 24.48	.0319 / .112 = 0.2848
Colombia	1.206 / .0209 = 57.70	.0118 / .164 = 0.0720
Costa Rica	1.1738 / .0175 = 67.07	.0118 / .108 = 0.1093
) Dominican Republic	1.8478 / .0423 = 43.68	.0303 / .131 = 0.2313
Ecuador	2.158 / .0598 =36.09	.0456 / .159 = 0.2870
Guatemala	0.88999 / .0081 = 109.88	.0091 / .132 = 0.0690
Honduras	1.8346 / .0207 = 88.63	.0377 / .118 = 0.3195
Mexico	0.56805 / .0352 = 16.14	.0255 / .198 = 0.1288
Panama	2.1617 / .0378 = 57.19	.0525 / .216 = 0.2431
Paraguay	2.6703 / .0285 =93.69	.0222 / .138 = 0.1610
Peru	2.3197 / .0604 =38.41	.0344 / .167 = 0.2060
Uruguay	1.9962 / .1703 = 11.72	.0230 / .101 = 0.2277
Venezuela	2.0597 / .1831 =11.25	.1411 / .321 = 0.4400

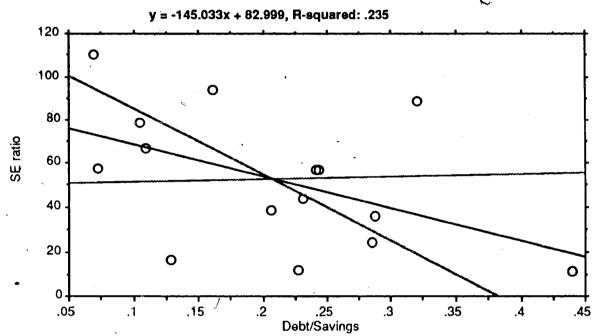


Figure 4.2

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## 4.7 Unit Roots and the Variance of Long Differences

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The stochastic growth model developed in the previous chapter is driven by real shocks to the economy. There real shocks took the form of an uncertain component of both domestic and foreign investment. This method of introducing uncertainty into dynamic model without money has become standard in the literature (Kydland and Prescott (1982), Long and Plosser (1983)). The purpose of this section is to point out statistically the importance of real shocks in determining the time path of real income, thereby giving some justification to the inclusion of the real investment shocks which drive the model of Chapter 3. Unit-root tests are performed on 15 developing countries in order to measure the importance of real shocks in determining the short-run path of real GNP. The results should not be taken as definitive evidence since these tests have been refuted by many studies<sup>82</sup>.

A time series can be decomposed into a growth or trend component, a cyclical component, a seasonal component, and a component reflecting random noise. The seasonal component can be modelled adequately using seasonal indices and is not usually a consideration in the measurement of aggregate consumption or income. The trend component is assumed to reflect real factors such as capital accumulation, population growth, and technological change. The cyclical component, on the other hand, is assumed to be transitory with nominal factors, such as monetary policy, as the prime determinants. Usually long term movements of the series are attributed to the trend component, while short term temporary fluctuations are attributed to the cyclical component. Unfortunately cycles need not only be generated by stationary movements about a long term trend<sup>83</sup>. A random walk is also a stationary path which can exhibit

<sup>&</sup>lt;sup>82</sup> See Cochrane (1988) and Rappoport and Reichlin (1989).

<sup>&</sup>lt;sup>83</sup> Stationarity can be decomposed into strong and weak stationarity. If P(.) defines the joint density function of a series x, strong stationarity implies that  $P(x_1, ..., x_T) = P(x_2, ..., x_{T+1})$  and so on. Weak stationarity implies the series x has finite second moments and the mean value of the series and its correlation function are time invariant.

cyclical movement, but does not follow a deterministic trend. Obtaining the cyclical component of a time series<sup>84</sup> can be accomplished by regressing on a time trend in the first instance (trend stationarity), or first differencing the series in the second instance (random walk). Using the wrong detrending technique can seriously impare the estimate of the variance of the series from trend<sup>85</sup>.

The basic argument of this literature is that the measurement of the variance of a time series from its trend value (if it has any trend) is used to capture the cyclical component of the series. The magnitude of this variance is very sensitive to the type of detrending procedure used, therefore a statistical method of determining whether a time series is best described by a trend stationary process or a random walk process or a mixture of both is desirable.

The unit root test, popularized by Nelson and Plosser (1982), can test whether a series is trend stationary or non-stationary (random walk), but cannot detect if the series is a combination of the two. Cochrane's (1988) variance of long differences test can detect combinations and extreme cases, however it is biased. The unit root will be used here.

To perform a unit root test one runs the regression

$$y_t = \mu + \rho y_{t-1} + \gamma t + v_t$$
 (4.7.1)

where again  $y_t$  is the natural log of the series and  $v_t$  is a disturbance term  $\sim N(0, \sigma_v^2)$ . To test for a random walk series one would want to test the joint hypothesis  $\rho = 1$ ,  $\gamma = 0$ . Unfortunately under this null hypothesis the usual t-ratios are not t-distributed, hence a joint test cannot be performed using the usual F test. Dickey and Fuller (1981) provide

<sup>&</sup>lt;sup>84</sup> We shall follow the standard convention and ignore seasonal variation.

<sup>&</sup>lt;sup>85</sup> See Appendix B for a more rigorous discussion.

tabulations for the distribution of the t-ratio for  $\rho$  for testing the null hypothesis  $\rho = 1$ and the F -distribution for the joint tests ( $\mu$ ,  $\rho$ ,  $\gamma$ ) = (0, 1, 0) and ( $\mu$ ,  $\rho$ ,  $\gamma$ ) = (1, 1, 0). Through the use of a Monte Carlo study, Nelson and Plosser determine that the likelihood of incorrectly rejecting  $\rho = 1$  when it is true is quite high. In addition the tratio for the hypothesis  $\gamma = 0$  is biased towards rejection, indicating a trend stationary series.

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The test proposed by Cochrane (1988) allows for the possibility that the time series is composed of both a trend stationary component and a random walk component. The test is utter simplicity. If  $y_t$  is a pure random walk then the variance,  $k\sigma_e^2$ , of its k-differences is increasing in the difference k. If  $y_t$  is a trend stationary series then the variance of its k-differences approaches a constant,  $2\sigma_y^2$  (in the case of k = 1, this reduces to  $2\sigma_e^2$  as already shown). If  $(1/k) \sigma_e^2$  is plotted against its k differences, the random walk model would show a horizontal line at the variance  $\sigma_e^2$ . If the actual relationship is trend stationary, the plot should decline eventually approaching zero. If the time series is actually a combination of a trend stationary component and a random walk component, the plot should decline initially and settle down to  $\sigma_e^2$ .

Results of the unit-root tests on real GNP of 15 developing countries are given in Table 4.5 below. The null hypothesis under  $\varphi_1$  is a random walk process generating real GNP, while the null hypothesis under  $\varphi_2$  is a random walk with drift. An asterisk indicates a significant F statistic at 95% confidence using critical values obtained from Dickey and Fuller (1981). The random walk was rejected in 6 out of 16 cases, however the random walk with drift was never rejected. These results suggest the importance of real shocks in determining the path of real GNP. If real shocks dominate nominal shocks to real GNP, the series will never return to a deterministic trend, as is suggested here. The results indicate that first differences of real GNP should be used rather than

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Čountry	R <sup>2</sup>	ρ	φ <sub>1</sub>	φ <sub>2</sub>				
Argentina: 1912-84	.9877	.91711	4.19	2.13				
Bolivia: 1945-84	.9931	.92438	1.68	2.26				
Brazil: 1920-84	.9964	.98997	5.29*	4.40				
Chile: 1940-84	.9746	.58826	5.75*	4.66				
Colombia: 1925-84	.9994	.99627	4.57	4.81				
Costa Rica: 1945-84	.9859	.83239	5.97*	2.47				
Dominican: 1945-84	.9950	.95746	6.47*	2.90				
Ecuador: 1940-84	.9927	.97137	4.23	2.88				
Guatemala: 1945-84	.9969	.91928	3.08	3.37	'n			
Honduras: 1925-84	.9954	.97024	4.34	3.09				
Mexico: 1920-84	.9946	.99151	4.78	4.10				
Panama: 1945-84	.9934	.90074	5.37*	4.06				
Paraguay: 1940-84	.9941	.97317	2.64	2.31				
Peru: 1945-84	.9909	.77605	5.38*	2.66				
Uruguay: 1935-84	.9751	.73053	3.99	4.49				
Venezuela: 1936-84	.9975	.91957	3.02	2.62				

Table 4.5Results of Unit-Root Tests on Real GNP for16 LDCs

Notes: 1)  $\mathbb{R}^2$  is adjusted for degrees of freedom.

2)  $\phi_1$  and  $\phi_2$  are the random walk and random walk with drift alternative hypotheses respectively.

3) • indicates significant at the .05 level.

regressing on a time trend.

### **Chapter 5: Summary and Conclusions**

The purpose of this paper was to develop a dynamic model of LDC borrowing behavior which could rationalize the unprecedented build-up of debt (in real terms) in the last 20 years or so as a result of optimizing behavior. This required a risk averse borrower borrowing foreign and domestic capital subjected to stochastic productivity shocks. The lender's behavior was given by an exogenous upward sloping supply schedule (Chapter 2 notes several treatments of a lender's decision). Unlike previous dynamic and non-dynamic models of debt behavior, the decision to take on additional debt is endogenous to the decision to consume (and thus save) giving different results than a model treating debt demand as a purely residual demand which can always be satisfied. In this sense, the large accumulation of real debt during the 1970's and 1980's by troubled LDCs could have been the result of an optimizing process to smoothen consumption over time by transferring future income to today using sovereign debt. The model suggests that LDCs were not irresponsible borrowers, a common explanation. The model also suggests that to avoid outright default of existing debt, lenders should extend further credit as prudently necessary during periods of high uncertainty in borrowing countries.

The empirical results lend support to the predictions of the theoretical model, but are not the results anticipated. Rather than explaining the rapid accumulation of foreign debt during the last 20 years as the result of new foreign debt being relatively less risky than domestic savings, the results suggest that it is likely that unexpected negative shocks to domestic savings forced LDCs to borrow internationally in order to maintain domestic consumption, regardless of which type of asset is relatively more risky. This could help explain why troubled LDCs continued to borrow funds when conditions in

international capital markets became particularly turbulent and unpredictable.

The unit-root tests on real GNP over a longer sample period suggest the importance of real shocks in determining the time path of real GNP, and thus perhaps its. components. For all 15 countries, real GNP does not tend to return to a constant trend line following a shock, suggesting that real shocks (such as those modelled in Chapter 3) dominated nominal shocks for the sample periods. Nelson and Plosser (1982) see this result as justification for the use of dynamic stochastic models utilizing real productivity shocks, although this conclusion may be too strong.

Since the statistical results hinge critically on the econometric treatment of expectations of future variables which affect investment, savings, and new debt, one can easily criticize the adaptive approach to expectations used in Chapter 4. However the merit of this approach is its small demand on data requirements. A much more complex method is the Generalized Method of Moments (GMM) estimation technique first espoused by McCallum (1976) in a relatively simple way, and later expanded by Hansen and Sargent (1980) and others in a relatively complicated process. Basically the method involves invoking a rational expectation on each of the exogenous variables of the form "  $E_{t-1}X_t = X_t + v_t$  where  $v_t$  is  $\sim N(0, \sigma^2 v)$ . Instrumental variables are formed by regressing each exogenous variable on all the other exogenous variables in the system<sup>86</sup> and then estimating the investment function using only actual values for the endogenous variables and the instruments. This technique involves extensive data requirements and is subject to large error if the actual variables used in forming the rational expectation of each exogenous variable are not the same as the variables used in estimation. Fair (1980, p. 20-22) notes that many time series data cannot distinguish between a model invoking rational expectations using the GMM technique and an adaptive expectation model<sup>87</sup>.

<sup>&</sup>lt;sup>86</sup> In this way the investment function is thought of as only one equation in a system of equations. Sims (1980) advocates the use of vector autoregressions where each variable is regressed on its own lagged values and the lagged values of other variables. Again the data requirements are prohibitive for LDCs.

<sup>&</sup>lt;sup>87</sup> The difference can be detected in the behavior of the error term, however this difference is very subtle.

This problem is compounded by the usual ad hoc way in which exogenous variables are chosen. If the theoretical model suggests the inclusion of certain exogenous variables then the GMM technique can be a very useful way to handle expectations. However in practice Fair notes that 'many models are not explicit about this, and so "extra" modelling or theorizing is needed at this point' <sup>88</sup>.

### Appendix A

The derivation of equation (3.2.9) is not difficult and is necessary to understand the stochastic optimal control method. Consider the following stochastic optimal control problem

$$J(t_0, x_0) = \max_{u} E\left[\int_{0}^{T} f(t, x, u) dt\right]$$
  
s.t.  $dx = g(t, x, u) dt + \sigma x dz$ 

where x is the state variable, u is the control variable,  $\sigma$  is the diffusion coefficient, g is the constraint function, and f is the function to be maximized. The f function is to be maximized from time to to T. Begin by breaking up the integral in (1).

(1)

$$J(t_0, x_0) = \underset{u}{\text{MAXE}} \left[ \int_{t_0}^{t_0+\Delta t} f(t, x, u) dt + \int_{t_0+\Delta t}^{T} f(t, x, u) dt \right]$$
(2)

where  $\Delta t$  is taken to be very small and positive. Equation (2) can be rewritten as

$$J(t_0, x_0) = \max_{u} E\left[\int_{0}^{t_0+\Delta t} f(t, x, u)dt + J(t_0+\Delta t, x_0+\Delta x)\right]$$
(3)

The bracketed term is the return from t<sub>0</sub> to t<sub>0</sub>+ $\Delta$ t plus the maximized return from continuing from t<sub>0</sub>+ $\Delta$ t onwards. Both terms are affected by the control u(t).

Equation (3) can be approximated by changing the first integral to the height of the curve at the lower limit of integration times the width of the interval,  $f(t_0,x_0,u) \Delta t$ . Since

 $\Delta t$  is very small we assume the control is constant over this integral. The approximation gives

$$J(t,x) \approx \underset{u}{\text{MAX E}} \left[ f(t,x,u)dt + J(t+\Delta t, x+\Delta x) \right]$$
(4)

Using Taylor's theorem expand the RHS of (4) around (t,x).

$$J(t+\Delta t, x+\Delta x) = J(t,x) + J_t(t,x)\Delta t + J_x(t,x)\Delta x + 1/2 J_{xx}(\bullet)(\Delta x)^2 + 1/2 J_{tt}(\bullet)(\Delta t)^2 + h.o.t.$$
(5)

The Wiener process is characterized by the following multiplication .89

		<u> dz dt </u>
		dz   dt   0
3	•	<u>dt   0   0  </u>

Simplifying equation (5) from the multiplication table and recognizing that

$$\Delta x = g\Delta t + \sigma x \Delta z$$
  
$$(\Delta x)^2 = g^2 (\Delta t)^2 + \sigma^2 x^2 (\Delta z)^2 + 2g \sigma \Delta t \Delta x = \sigma^2 x^2 \Delta t$$

and substituting into (4) gives

$$J(t, x) = \underset{u}{\text{MAXE}} \left[ f\Delta t + J + J_{t}\Delta t + J_{x}g\Delta t + J_{x}\sigma\Delta z + \frac{1}{2}J_{xx}x^{2}\sigma^{2}\Delta t + \text{h.o.t.} \right] (6)$$

Take the expectation of (6), subtract J from each side, and divide by  $\Delta t$  to give

 $-J_{t} = \underset{u}{\text{MAXE}} \left[ f + J_{xg} + \frac{1}{2} J_{xx} x^{2} \sigma^{2} \right]$ 

This is the so-called Hamilton-Jacobi equation.

## Appendix B

Suppose we have a time series  $y_t$  whose true form is trend stationary, such as 90

$$y_t = \alpha + \beta t + e_t \tag{4.1}$$

where  $\alpha$  and  $\beta$  are fixed coefficients and  $e_t$  is a random disturbance which is  $\sim N(0, \sigma_e^2)$ . The long term forecast of y is the mean  $\alpha + \beta t$ , thus past and present disturbance do not alter the future expectation of y. If the stationary trend process described in (4.1) accurately describes the series, the measure we are looking for is the variance of the deviations from trend,  $\sigma^2$ . If we naively take first differences of  $y_t$  we get

$$y_{t} - y_{t-1} = (\alpha + \beta t + e_{t}) - (\alpha + \beta(t-1) + e_{t-1}) = \beta + (e_{t} - e_{t-1})$$
(4.2)

The variance of the disturbance term in (4.2) is given by  $2\sigma_e^2$ , an overestimate of the variance of the series about trend<sup>91</sup>. Alternatively consider the case where the true process describing  $y_t$  is given by a random walk with drift, such as

 $y_t = \mu + y_{t-1} + u_t$  (4.3)

where  $u_t$  is a random disturbance which is  $\sim N(0, \sigma_u^2)$ . To note the difference between a random walk series and a trend stationary series write (4.3) k periods forward.

$$y_{t+k} = y_t + k\mu + u_{t+1} + u_{t+2} + \dots + u_{t+k}$$

<sup>91</sup> Assuming that  $\sigma^2$  is constant over time and  $cov(e_t, e_{t-1}) = 0$ .

<sup>&</sup>lt;sup>90</sup> Since most time series exhibit an increasing mean and variance with the age of the series, all variables are in natural log form.

Disturbances to a random walk process are permanent in the sense that past disturbances affect future expectations of y. For instance if  $u_{t+1} = 1$ , the expectation of the series increases by one unit into the future. If the true behavior of the series is represented by (4.3), regressing  $y_t$  on a linear time trend yields an erroneous estimate of the variance of the series about trend. The variance of the random walk series is given by  $k\sigma_u^2$ . Performing the regression gives

$$\beta = \frac{\sum_{j=0}^{t} (t - \bar{t}) (y_t - \bar{y})}{\sum_{j=0}^{t} (t - \bar{t})^2} = \frac{\sum_{j=0}^{t} (t - \bar{t}) u_t}{\sum_{j=0}^{t} (t - \bar{t})^2} = \frac{\sum_{j=0}^{t} (t - \bar{t}) u_t}{\sum_{j=0}^{t} (t - \bar{t})^2}$$

As the random walk process continues over time we can take the expected value of  $\beta$  by evaluating the limit

$$\lim_{t \to \infty} \beta = 0$$

since the denominator of the expression for  $\beta$  increases without limit in t. The corresponding term for the intercept  $\alpha$  is given by

$$\alpha = \overline{y} - \beta \overline{t} = \overline{y}$$

The variance of the disturbance term in the regression equation can be found by finding the variance of the forecasting error if the predicted value of y is made using the trend regression equation.

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$$\sum_{j=1}^{t} (y_{j} - \widehat{y}_{j})^{2} = \sum_{j=1}^{t} (y_{j} - \overline{y})^{2} = \sigma_{y}^{2}$$

If a linear time trend is fitted to a random walk process, the variance of the disturbance term is given by  $\sigma_y^2$ , even though the true variance of the deviations from trend is  $k\sigma_u^2$ . The former variance is bounded, while the latter variance increases without bound as the series progresses through time. The following table summarizes the results.

#### Table 4.1

Possible Estimates of the Variance of the Series From Trend

Detrending Procedure	Trend Stationary	Random Walk
Linear Time Trend	σ <sup>2</sup> <sub>e</sub>	$\sigma_y^2$
First Differencing	$2 \sigma_e^2$	kσ <sup>2</sup> u

True Description

# Appendix C

Risk Spreads over LIBOR (%)

Country	1978	1979	1980	1981	1982	1983	1984
,							
Algeria	1.5006	1.13	.894	.56	.56	.59	.874
Argentina	1.5003	.765	.637	.8	1.17	1.75	1.625
Australia	.75	.625	.612	.47	.65	.59	.13
Austria	•	•	•	.38	.38	.38	.375
Belgium	•	.438	.439	.45	.45	.47	.5
Bolivia	1.4279	1.375	•	2.22	•	•	•
Brazil	1.7	.9	.861	2.05	2.12	2.11	1.713
Canada	.3732	.75	.553	.44	.34	•	•
Chile	1.8635	.873	.875	.7	1.33	2.125	2.0
China	1.3064	.5	.666	.93	•	•	•
Colombia	1.0	.841	.733	.66	.6	1.41	1.625
CostaRica	1.0736	.913	.932	•	•	•	•
Cyprus	1.0	1.0	.25	.73	.69	•	.375
Denmark	.7758	.57	.51	.47	.41	.58	.5
Ecuador	1.1964	.936	.75	.7	.33	٠	•
Finland	.875	.625	.375	.33	.41	.34	٠
France	.6648	.449	.461	.38	.47	.47	.5
Gabon	2.25	1.821	1.5	•	2.0	•	الاست
Greece	.6643	.528	.614	.47	.5	.65	.625
Hungary	.727	.606	.75	.63	1.22	•	1.098
Iceland	1.25	.625	.5	.53	.46	.61	.375
India	1.569	.67	.53	.38	.48	.51	•

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				•		
1978	1979	<u> 1980</u>	1981	1982	1983	<u>    1984</u>
1.375	.679	.625	.51	.38	.49	.707
.75	.625	.555	.43	.43	.57	•
1.1042	.644	.599	.56	.6	.59	٠
1.7705	1.635	1.534	1.42	1.49	1.0	1.0
2.875	•	2.12	2.5	1.5	•	٠
1.25	.718	.61	.64	.52	.72	.75
.7813	.563	.5	.39	.38	.5	.487
1.625	1.646	•	•	1.5	•	1.5
1.75	.756	.653	.62	1.07	2.25	1.5
1.0856	.963	1.0	1.06	1.03	1.26	.875
.625	.379	.375	.32	.26	.25	•
•	•	1.625	1:25	1.47	٠	•
1.0	1.028	1.067	.88	.99	.83	.875
.625	.568	.375	.54	.79	.76	.185
2.25	1.996	1.659	.88	.71	.88	.875
1.4708	.878	1.25	1.25	1.41	2.0	٠
1.375	.75	٠	.58	.58	.72	•
•	1.398	1.419	1.05	1.0	2.25	1.75
1.896	.815	.89	.83	.76	•	٠
1.375	1.046	1.5	•	٠	•	•
1.0377	.882	.75	.55	.51	.73	.875
1.0	.761	.642	.72	٠	٠	٠
•	٠	.965	.76	.79	•	.625
.875	.783	.698	.54	.48	.69	.529
.625	.479	.48	.44	.41	.49	.9265
	1.375 .75 1.1042 1.7705 2.875 1.25 .7813 1.625 1.75 1.0856 .625 1.0 .625 2.25 1.4708 1.375 1.896 1.375 1.0377 1.0 .875	1.375 $.679$ $.75$ $.625$ $1.1042$ $.644$ $1.7705$ $1.635$ $2.875$ $.$ $1.25$ $.718$ $.7813$ $.563$ $1.625$ $1.646$ $1.75$ $.756$ $1.0856$ $.963$ $.625$ $.379$ $.$ $.$ $1.0$ $1.028$ $.625$ $.568$ $2.25$ $1.996$ $1.4708$ $.878$ $1.375$ $.75$ $1.398$ $.815$ $1.375$ $1.046$ $1.0377$ $.882$ $1.0$ $.761$ $.875$ $.783$	1.375.679.625.75.625.5551.1042.644.5991.77051.6351.5342.875•2.121.25.718.61.7813.563.51.6251.646•1.75.756.6531.0856.9631.0.625.379.375••1.6251.01.0281.067.625.568.3752.251.9961.6591.4708.8781.251.375.75••1.3981.4191.896.815.891.3751.0461.51.0377.882.751.0.761.642••.965.875.783.698	$1.375$ $.679$ $.625$ $.51$ $.75$ $.625$ $.555$ $.43$ $1.1042$ $.644$ $.599$ $.56$ $1.7705$ $1.635$ $1.534$ $1.42$ $2.875$ $2.12$ $2.5$ $1.25$ $.718$ $.61$ $.64$ $.7813$ $.563$ $.5$ $.39$ $1.625$ $1.646$ $\cdot$ $\cdot$ $1.75$ $.756$ $.653$ $.62$ $1.0856$ $.963$ $1.0$ $1.06$ $.625$ $.379$ $.375$ $.32$ $\cdot$ $1.625$ $1.25$ $1.0$ $1.028$ $1.067$ $.88$ $.625$ $.568$ $.375$ $.10$ $1.028$ $1.067$ $.88$ $.625$ $.568$ $.375$ $.54$ $2.25$ $1.996$ $1.659$ $.88$ $1.4708$ $.878$ $1.25$ $1.375$ $.75$ $.58$ $.1398$ $1.419$ $1.05$ $1.896$ $.815$ $.89$ $.83$ $1.375$ $1.046$ $1.5$ $.55$ $1.0$ $.761$ $.642$ $.72$ $.0377$ $.882$ $.75$ $.55$ $1.0$ $.761$ $.698$ $.54$	$1.375$ $.679$ $.625$ $.51$ $.38$ $.75$ $.625$ $.555$ $.43$ $.43$ $1.1042$ $.644$ $.599$ $.56$ $.6$ $1.7705$ $1.635$ $1.534$ $1.42$ $1.49$ $2.875$ $\cdot$ $2.12$ $2.5$ $1.5$ $1.25$ $.718$ $.61$ $.64$ $.52$ $.7813$ $.563$ $.5$ $.39$ $.38$ $1.625$ $1.646$ $\cdot$ $1.5$ $1.75$ $.756$ $.653$ $.62$ $1.07$ $1.0856$ $.963$ $1.0$ $1.06$ $1.03$ $.625$ $.379$ $.375$ $.32$ $.26$ $\cdot$ $1.625$ $1.25$ $1.47$ $1.0$ $1.028$ $1.067$ $.88$ $.99$ $.625$ $.568$ $.375$ $.54$ $.79$ $2.25$ $1.996$ $1.659$ $.88$ $.71$ $1.4708$ $.878$ $1.25$ $1.25$ $1.41$ $1.375$ $.75$ $.58$ $.58$ $\cdot$ $1.398$ $1.419$ $1.05$ $1.0$ $1.896$ $.815$ $.89$ $.83$ $.76$ $1.375$ $1.046$ $1.5$ $\cdot$ $\cdot$ $1.0377$ $.882$ $.75$ $.55$ $.51$ $1.0$ $.761$ $.642$ $.72$ $\cdot$ $\cdot$ $.965$ $.76$ $.79$ $.875$ $.783$ $.698$ $.54$ $.48$	1.375 $.679$ $.625$ $.51$ $.38$ $.49$ $.75$ $.625$ $.555$ $.43$ $.43$ $.57$ $1.1042$ $.644$ $.599$ $.56$ $.6$ $.59$ $1.7705$ $1.635$ $1.534$ $1.42$ $1.49$ $1.0$ $2.875$ $.2.12$ $2.5$ $1.5$ $$ $1.25$ $.718$ $.61$ $.64$ $.52$ $.72$ $.7813$ $.563$ $.5$ $.39$ $.38$ $.5$ $1.625$ $1.646$ $$ $1.5$ $$ $1.75$ $.756$ $.653$ $.62$ $1.07$ $2.25$ $1.0856$ $.963$ $1.0$ $1.06$ $1.03$ $1.26$ $.625$ $.379$ $.375$ $.32$ $.26$ $.25$ $.0856$ $.963$ $1.0$ $1.06$ $1.03$ $1.26$ $.625$ $.379$ $.375$ $.32$ $.26$ $.25$ $.025$ $.379$ $.375$ $.32$ $.26$ $.25$ $.025$ $.568$ $.375$ $.54$ $.79$ $.76$ $2.25$ $1.996$ $1.659$ $.88$ $.71$ $.88$ $1.4708$ $.878$ $1.25$ $1.25$ $1.41$ $2.0$ $1.375$ $.75$ $.58$ $.58$ $.72$ $$ $1.398$ $1.419$ $1.05$ $1.0$ $2.25$ $1.896$ $.815$ $.89$ $.83$ $.76$ $$ $1.0377$ $.882$ $.75$ $.55$ $.51$ $.73$ $1.0$ $.761$ $.$

Country ·	1978	1979	1980	1981	1982	1983	1984
				L			
Thailand	.9265	.727	.804	.59	.42	.38	.231
Trin./Tob.	1.06	\$	.625	.75	.75	1	1.5
Tunisia	.9762	.831	.621	.5	•	1.45	.875
U.K.	.625	5	.718	.31	٠	•	•
Uruguay	1.25	.984	1	.93	.87	•	•
Venezuela	.7324	.673	.712	.57	.83	1.85	1.875
Yugoslavia	1.7256	.899	1.125	1.22	1.33	•	1.625

Notes: All percentages are annual weighted averages with weights given by the magnitude of the individual loan transactions in addition to the number of years to maturity of each loan. The formula used (which is now used in Euromoney) is

weighted spread (%) =  $\frac{\sum_{0}^{n} \left[ \text{amount } \times \text{ spread}(\%) \times \text{maturity(years)} \right]}{\sum_{0}^{n} \left[ \text{amount } \times \text{maturity(years)} \right]}$ 

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