NATURAL GAS LIQUIDS PLANT OPTIMIZATION

An Application of Linear Programming

by

Sandro Papile

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Name: Sandro Papile

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Examining Committee:

Dr. Eng Choo

Senior Supervisor

Dr Art Warburton

Associate Professor

Date Approved: August 3, 1989

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ABSTRACT

A Linear Programming (LP) model was used to optimize weekly production scheduling at Westcoast's Taylor Natural Gas Liquids (NGL) Recovery Plant. The profit function was derived and it was shown that, because of its particular structure, profit is maximized when fluctuations between daily productions are minimized. An LP model was developed to calculate the smoothest production schedule; its solution was obtained with both simple heuristic and computer analysis. Twelve weeks of actual operating data were used to validate the model. An indication was given on how post-optimality analysis of the LP results could further help in optimizing plant operation.

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1. INTRODUCTION

1.1 General Objectives of the Study

Westcoast Energy Inc. operates a natural gas liquid (NGL) recovery plant at Taylor, British Columbia. The plant processes most of the natural gas produced in the area, extracting propane, butane and condensates.

The facility generated $19.5 million in revenue during 1987 but, as pointed out in the 1987 Annual Report of the Company, "the earnings for this project have been disappointing to date because of the low prices prevailing in the propane market".

In fact propane is increasingly difficult to sell, and plant operation is presently adjusted to limit the amount of propane produced to the amount expected to be sold. Because of thermodynamic process constrains, this translates into a limitation on the production of butane and condensates, which are the products that on a unit basis generate the highest revenue and that still enjoy a strong market. The question is asked, therefore, on whether it would not be more profitable to run the plant at maximum throughput (which would give the maximum possible quantities of butane and condensates) rather then limiting throughput for all products:

Presently, there is no unequivocal answer to this question; one of the reasons is the fact that the only model available for analysis is the accounting system. By its same nature, that system can only deliver cost and revenue data after the production decisions have been made; in fact there is a considerable time lag between the production run and the availability of cost and revenue data. Furthermore, this system does not allow us to directly tie results to the operating variables, conditions, and constraints; therefore, it does not assist the operators to optimize plant production.

In general, there is nothing in place to allow people to discuss about the expected profit of alternative situations before a production run is set up; and, except for historical data, there is very little that ties together production costs and revenues.

To date optimization of plant operation has been mostly looked at from a process point of view, and no major attempt has been made to compare the economics of alternative production scenarios. It will be scope of this study to produce a model that will allow the operator to determine best plant operation on the basis of both process and economic information available at the time a production run is scheduled.

1.2 Expected Benefits

A model that describes the profit as a function of both the economic and production variables will allow better and more informed decisions on how to set-up production schedules. In general, a model can be used to explore many alternative scenarios, compare them, and to answer the following crucial questions:

^ what are the optimal operating conditions for maximum profit?

^ are there conditions where more profit than at present could be made?

^ where should we put more effort to increase profit, and by how much can we expect that profit to increase?

^ what are the pitfalls, i.e. what are the operating conditions which generate losses and not profit?

The use of a model is likely to reduce the inefficiency of operator's decisions, and to give a better understanding of the relationship between the variables. A model, however simple, will give an overall picture of the system, with the relative importance of the various factors, and will provide a reference framework for predicting the results of a change or combination of changes in the variables without any actual trial.

A model, if realistic, will certainly be useful in many aspects, other than just production scheduling. Since it will provide a common reference framework, it will help making optimal integrated decisions, as opposed to simple maximization or minimization of some particular parameter. E.g. it will allow the test of the effect of a change in product price, as well as the impact of revisions to the terms of the feedstock supply contract, and the effect of additional capital expenditures.

From the monetary point of view there is a definite incentive to improve plant operation. This is an obvious statement for any business, but it assumes a particular significance in the case of the NGL plant where sales are in the order of $1.5 million per month, margins are more and more reduced by the volatile propane market, and earnings are disappointing shareholders. There is no doubt that the effort required to produce a model will be paid back in a very short time, if such a model is correct and usable.

2. THE NGL PLANT

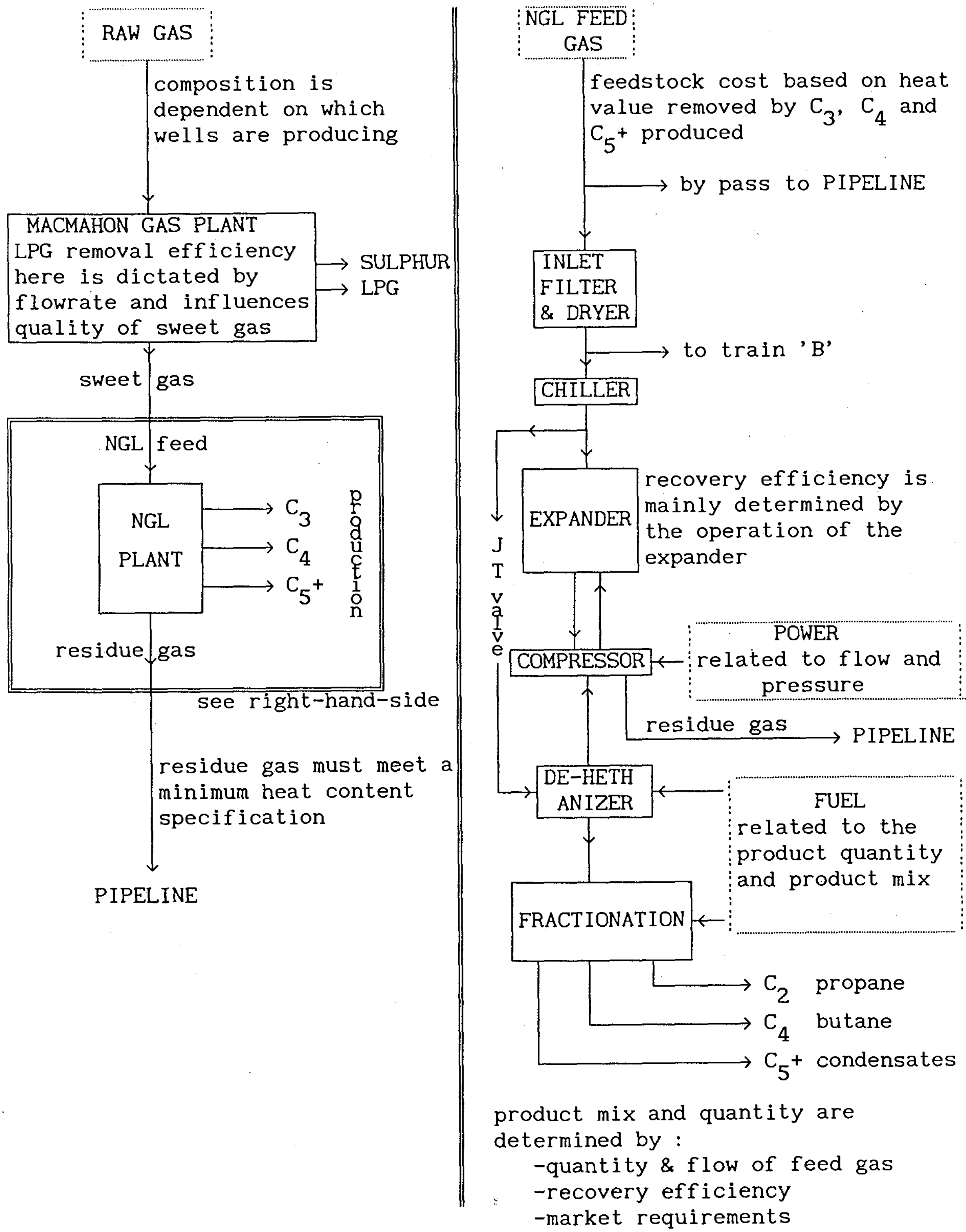
2.1 Plant Description

The natural gas gathered in the Ft. St. John area contains liquid hydrocarbons (NGL's) in sufficient volume to make their recovery profitable. In the process employed at Taylor, the gas is cooled to temperatures lower than -100 degree F, to condense out the liquids; these are then fed to a series of distillation columns which separate the various fractions. Methane and ethane are recompressed and returned to the sales gas pipeline; propane, butane, and condensates are separated and sent to pressurized refrigerated storage vessels to await transportation to market.

In this paragraph we shall give an overview of the plant, so that the links between the economic factors and the operating parameters, subject of next chapter, can be understood. Figure 2.1, which is a simplified diagram of the plant, shall be used as guidance.

Since propane, butane, and condensates are extracted, not manufactured, from the natural gas their maximum production is limited to their availability in the natural gas feed. The proportions of three products in the feed is

Figure 2.1 Taylor Natural Gas Liquid Plant Conceptual Schematic



a given, and it varies (within predictable limits) independently of plant operations; seasonal variations are typical, but fluctuations also happen on day-to-day basis, mostly due to the fact that different wells produce gas with different compositions (see table 2.1 for typical compositions of feed gas and products).

Feed quantity can be varied at wish within the minimum and maximum plant turndown. Since plant is made of two production lines which can be operated either independently or in parallel, overall flow can vary from a minimum of about 150 millions of standard cubic feet per day (MMSCFD) to a maximum of about 450 MMSCFD.

Recovery efficiency for propane can be adjusted by the operator with wide ranges (about 65% minimum, to a maximum of 95%); for butane and condensate it stays constant at about 100%. Different recovery efficiencies are associated with different costs; i.e. for the same production quantities costs are different depending on whether a high flow - low efficiency, or a low flow - high efficiency combination is used. While there are ways of going behyond these limits, they are technologically inefficient, and shall not be considered.

Removing propane, butane, and condensates from the feed gas reduces the heat value of the latter, by an amount proportional to the total amount of propane, butane, and condensates extracted; this difference in heat value between the feed and residue gas is the base for the cost of the feedstock. Restated, cost of the feedstock is directly proportional to the amounts of propane, butane, and condensate produced, through a physical characteristic of the products, which is their heat value.

Electric power is mainly used to run the compressors that return methane and ethane to the pipeline. There is one of such compressors for each production line. In first approximation, recompression power is a linear function of the feed flow. (Appendix A includes a regression analysis performed with actual plant data, which confirms this assumption is acceptable in the whole range of feed gas flow).

Natural gas supplies the heat needed for various processes, and it is paid in proportion to its consumption.

There is storage available, which provides a buffer between plant production and sales requirement; this is used both to make operation smoother (since it is always more desirable, from the technological point of view, to operate at steady state than to fluctuate), and to take care of break-downs, equipment maintenance requirements, and other events, that may reduce the plant availability, either in a planned or unplanned way. Within the weekly planning horizon, this means that some storage has to be available to meet unexpected plant shut-downs, or unexpected reduction of feed availailability. This problem will be discussed later; it suffices for now to note that it could be approached in different ways, such as increasing the production during the first days of the week (but this may prove disruptive, therefore more costly in the long run), or by taking the chance (based on past break-down experience) and not planning for it at all.

The products are shipped with both tank-cars and trucks; propane is mostly shipped via rail.

As mentioned, the operator has no control on the quality (i.e. composition) of the incoming flow; this does not mean that the concentration of the products in the feed is totally unpredictable, but it is certainly not possible to know it exactly before it gets into the process and is analyzed. Furthermore, the concentrations can theoretically vary continuosly within a pretty wide range, and life would become very complicated if some simplification was not possible. The approach that will be taken here is the same currently used at the plant, i.e. to standardize the feed quality in three categories, each characterized by a standard composition: we shall call them Lean, Design, and Rich Feeds. This approach is not as arbitrary as it may look, because the three categories referred to are build on the basis of past experience, and technical knowledge of the upstream process that affects the inlet gas composition; in fact these categories represent quite distinct feeds, easily recognizable from the chromatographic analyses.

TABLE 2.1- Typical Plant Feed and Product Compositions

(% by volume)

Component	Feed	Propane	Butane	Condensates
Nitrogen	0.45			
Methane	90.48			
Ethane	6.44	4.30		
Propane	1.91	95.28	0.62	
i-Butane	0.24	0.39	39.87	0.02
n-Butane	0.34	0.03	58.41	1.01
i-Pentane	0.05		0.94	32.99
n-Pentane Exhane	0.04		0.15	28.97
& Heaviers	0.05			37.02

2.2 Current Production Scheduling System

Every monday morning a firm order for the shipment of propane, butane, and condensates is posted by the Marketing Dpt., to be filled by the plant before the end of the week. Based on this, and on the estimated inlet composition, the operators must decide on the flow amount and recovery level to operate the plant. Other variables considered are: the inventory level in hand, and the amount of products to keep as ending inventory for the week. Forecast of future weekly orders are available from the annual and quarterly budgets, periodically adjusted on the basis of marketing information.

3. COSTS AND REVENUES

3.1 Introduction

In the following paragraph we shall review the relationship between some of the variables the operator has control on, and the cost and revenues associated with plant operation. We shall first define the process variables, then review their costs; lastly, we will briefly look at the revenues, and their implications.

To meet the production target, the operator shall adjust a number of physical variables (like pressures, temperature, flows, etc.). There are hundreds of these variables; for the purpose of this study, they will be aggregated in two macroscopic factors that can easily be related to plant production, and profit. They are: natural gas feed flow, or gas throughput, which is the amount of natural gas processed at the plant; and propane recovery efficiency, that is the overall measure of how much of the propane contained in the feed is actually extracted.

From the point of view of these variables, when the operator is given a certain target production, he can choose to operate at maximum throughput and to limit the recovery efficiency, to run at maximum efficiency and reduced throughput, or to work at any intermediate condition between these extremes.

In addition to marketing requirements, there may be other situations, plant specific, that may force the operator to work at one set of conditions instead of another. Storage level is one of the major constraints. As a consequence of operator's decision (i.e. his choice of flow and propane recovery level), and of the particular condition of the feed, the plant will use a certain amount of feed, will produce a certain product mix (revenue generating), and will require certain amounts of energy (in the form of electric power, and fuel gas).

We shall review and explain the relationship between these variables and the cost or revenues associated with them.

3.2 Process Variables

Natural gas feed flow is the amount of gas that is taken from the pipeline (see fig. 2.1) to be processed, and from which to extract propane, butane, and condensates. It is an independent variable that the operator can control, and it can be adjusted at virtually any value between 150 and 450 MMSCFD.

Feed composition is predictable, but it cannot be controlled; it is a characteristic of the gas available, and is subject to daily and seasonal variations. Composition is continuously monitored, so propane, butane, and condensate content of the feed is known, and the operator can do some adjustments to operation during a run.

Propane recovery efficiency is the ratio between the amount of propane recovered, and the amount contained in the feed. It is the macroscopic measurable effect of a certain combination of pressures, temperatures, and flows at various points in the plant; it is a convenient reference dimension that captures the effect of numerous variables. It is continuosly monitored but its control can only be done indirectly by adjusting the combination of several operating parameters. It is possible to obtain the same efficiency with different sets of operating parameters, but operator's experience, and plant instrumentation will lead to the choice of the most efficient combination.

As indicated before, recovery efficiencies for the other two products (butane and condensates) is constant (100%) because of the thermodynamic of the process.

Electric power is mostly used to run the two compressors which return the residual methane and ethane to the pipeline. Each one of those compressors may use up to 8500 KW; the actual requirement depends on gas flow, pressure, temperature, and composition on one hand, and machine efficiency on the other. On average, power requirement is a linear function of gas flow, within the practical range of plant conditions. Appendix A.2 contains the regression of actual plant data.

Fuel gas has a variety of uses, ranging from heat generation (i.e. boilers) to process functions. Its usage depends on a number of factors, and is mostly related to production rate (see Appendix). It can be seen from operating records that fuel consumption is small, and its cost is a minor component of the total cost (table 3.1).

Products are sold by volume (cubic meters) and have to meet certain composition specifications. The plant may occasionally produce off-spec products which are temporarily stored, then re-processed. There are certainly costs associated with these re-runs, but the quantities involved are definitely minimal in a well run, new plant like the Taylor facilities, and in first approximation it is safe to disregard those costs.

3.3 Costs

We shall now review each of the items that make-up the plant operating cost, and explain the structure of each one of them; we shall refer to table 3.1 which is a typical cost break-down for a plant similar to the Taylor NGL.

Operating Personnel. Unless the plant is shut-down for maintenance, the number of people required for its operation does not change with level of production. This cost is available from the annual budget.

Property Taxes. They are assessed periodically and remain fixed over the budget period.

Insurance. Premiums are fixed over the budget period, even though they are subject to periodic re-assessment and negociations.

Depreciation. It is presently calculated on straight-line, 30 year basis, and its value can be taken from the accounting records.

Maintenance. This cost is budgeted for a year and is not strictly a fixed cost, since the equipment wear and tear is somehow dependent on the level of usage. However, an average maintenance costs may be used to cover the whole range of plant operation without affecting the materiality of the results.

Feed Gas. Feedstock cost is calculated by adding a "demand charge" to a "commodity charge". The former is a fixed cost, to be paid whether or not the gas is used; the latter is a usage fee directly proportional to the amount of heat content removed from the natural gas feed by the extraction process; this is the heat value lost by the natural gas due to the removal of propane, butane, and condensate. The heat content of the unit of product is a physical constant, therefore this cost can simply be obtained by multiplying the measured (or projected) quantity of each product by the respective constant. Actual composition may vary slightly among production runs, but with insignificant effect on the constant.

Power. The B.C. Hydro contract is structured in a fashion similar to the feed gas shrinkage contract in that there are a demand, and an energy charge. The demand charge is the greater of:

- actual registered demand for the billing month;
- contractual minimum demand;

- 75% of the highest during the previous 11 billing months.

After reviewing the power bills for the past twelve months, it was concluded that, in first approximation, demand charge is a fixed cost. However, it must be recognized that it is structured to give an incentive to smooth operation of the plant and to avoid too high peaks (e.g. run the plant for two periods at half capacity instead of for one at full load).

Energy charge is a straight consumption charge, i.e. it is proportional to the number of kilowatthour (kWh) consumed during the billing month.

As mentioned earlier, power can be correlated with gas flow through the plant, therefore its cost can be calculated as a function of the gas flow.

Fuel. Fuel is taken from the pipeline and is paid on a usage basis; it is metered and paid under the same contract that covers the feed gas, and there is no demand charge.

Marketing Fee. It is a fixed fee that is paid annually to the Company that takes care of the marketing of the products; it is available from that contract.

Operating Fee. It is a contractual fee added to the plant costs to cover head office, and other service cost. It is a fixed percent of production and power costs.

3.4 Revenues

Revenues are generated by the sales of the three products. Current market prices for these products are about $\$60/m^3$, $\$80/m^3$, and $\$100/m^3$, respectively, at plant gate. These values will be used to derive the profit function, which is at the basis of the optimization model.

TABLE 3.1 - Operating Cost Breakdown for a Typical NGL Plant

a - For plant at full load:

Fixed Costs	20%
Variable Costs	80%

b - For plant at minimum capacity:

Fixed Costs	40%
Variable Costs	60%

c - Fixed Costs:

Operating & Maintenance	25.0%
Property Taxes	4.5%
Insurance	.5%
Depreciation	15.0%
General & Administration	25.0%
Shrinkage (demand charge)	25.0%

d - Variable Costs:

Power (energy charge)	15.0%
Shrinkage (commodity charge)	67.0%
Fuel	3.0%
Operating and Marketing Fees	15.0%

4. MODELLING

4.1 Problem Statement

Before we describe the approach to modelling, we shall briefly review the major items that characterize the problem. Schematically they are the following:

- Every Monday, at 8 AM, a weekly order comes in;
- Due to market demand, only propane needs to be monitored; the other two products (butane and condensate) are always produced at maximum limit;
- Projections of future orders are available (for the quarter, the year, and for the following few weeks);
- Given a weekly order, a daily schedule for the next seven days is to be determined;
- There is always the possibility of down-times due to unforeseeable circumstances, and they have to be taken in some account; typically, production tends to be higher in the first few days of the week;
- There is storage available which can be used as cushion to meet targeted order quantities;
- The weekly order quantity may be adjusted if there

is a significant economic incentive;

- Daily production level must be smoothed;
- There are two product lines, which allow a throughput between 150 and 450 MMSCFD; propane recovery efficiency can be varied between 60% and 96%; butane and condensates are recovered at 100% all the times

We shall answer the following general questions:

1. For a given daily requirement, what is the best operating setting at minimum cost, and how does it affect the other relevant factors?

2. Given a weekly requirement, build a model to determine daily production schedule to minimize cost.

Additional questions are of the type: how many weeks in advance should be considered; how to safeguard against breakdowns; how to smooth power requirements; what is the sensitivity of the various factors such as weekly order, forecasted demand, etc.

4.2 The Model

We shall answer those questions in two stages:

* First we will determine the daily production schedule that best fits the constrains;

* Then we will determine how to meet that daily schedule, while maximizing the profit.

In this work, the first stage problem was solved with an LP model; the second one in an analytical way. More specifically, the LP model allows us to determine the optimum daily production schedule on the basis of technological parameters (i.e. shipping requirements, inventory levels, plant capacity, projected future demands, etc.); once the daily schedule is determined, rules are given to determine which combination of flow and recovery efficiency to use to maximize profit.

The profit function was developed analytically from the information available on plant cost and revenues, and on the physical and technological relationship between feed and products.

The solution of the LP model was approached with both numerical and simple heuristic methods; a sensitivity analysis was done on the constrains; and actual data from the 1987 operating year were used to validate the model.

The following paragraphs will first explore the development of the profit function, its limitations, and the consequences of its particular structure on the LP formulation. The LP formulation and solution, with objective function, constraints, and both numerical and simple heuristic solutions will be presented later. Lastly, the sensitivity of the model will be reviewed, together with a discussion on extreme-case situations, and ways to approach them.

5. THE PROFIT FUNCTION

5.1 The Function and Its Analysis

When the cost items and revenues listed in chapter 3 are analyzed, it becomes clear that each of them can be expressed as a function of two variables: F, the natural gas feed flow to NGL plant; and Q_P, the amount of propane to be produced (the amounts of butane and condensate produced are proportional to the flow). Therefore, profit G can be written as:

$$G = A * Q_P + B * F + C \qquad [1]$$

where A, B, C are constants.

This equation recognizes that profit is proportional to the amount of propane produced (first term), to that of butane and condensate produced (second term), and that there are fixed costs to be covered (i.e. C, which is a negative term). Coefficients A, B, C can be regressed from past data, or calculated from the actual costs and revenues (see following §5.2).

In addition, if we base our discussion on a given feed composition, the amount of propane Q_P produced can be expressed as

$$Q_P = k * E * F \qquad [2]$$

where E is the propane recovery efficiency, and k is a proportionality constant which depends on propane concentration in the feed.

Both F and E are constrained by plant design between two extreme values, i.e.

$$F_{min} < F < F_{max}, \qquad [3]$$

and

$$E_{min} < E < E_{max} \qquad [4]$$

therefore, it is also

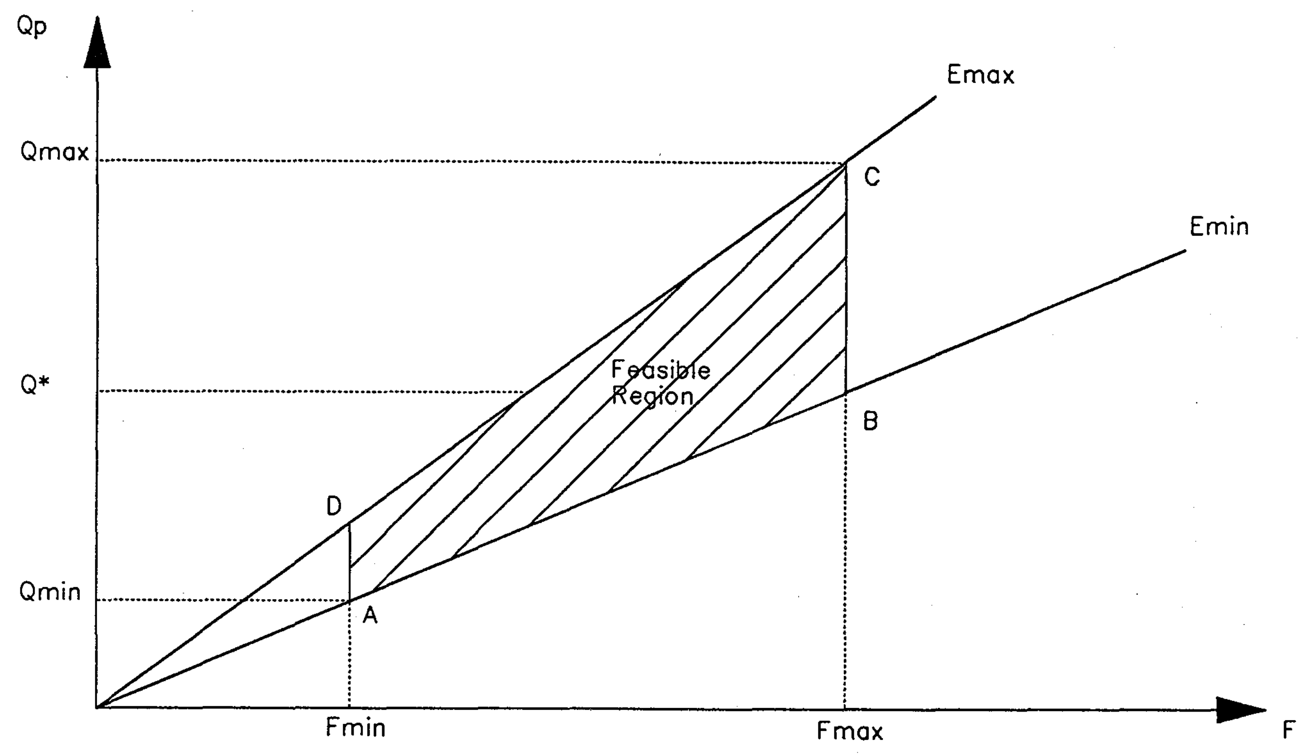
$$Q_{min} < Q_P < Qmax \qquad [5]$$

where $Q_{min} = k * E_{min} * F_{min}$

and $Q_{max} = k * E_{max} * F_{max}$

Equations [3], [4], and [5] define the boundaries of the region of feasible operation, which is represented as the shadowed area in figure 5.1.

Fig. 5.1 – FEASIBLE PROPANE PRODUCTION REGION



$Q = k * E * F \qquad E_{min} < E < E_{max} \qquad F_{min} < F < F_{max}$

Since the amount of propane Q_p to be produced is given, equation [1] says that, within the feasibility region, we achieve maximum profit when we operate at maximum flow. By looking at figure 5.1 we see that this optimal relationship between Q_p, F, and E is given by the line segments AB and BC; i.e. to produce between Q_{min} and Q_{max} we are better off operating along those segments than in any other point of the feasibility region. We shall call ABC the operating line, and it is on that line that we must operate at all times to maximize profit.

If we concentrate our attention on the points located on the operating line, i.e. if we limit our discussion to the points of maximum profit, we observe the following two situations:

1) When $Q_{min} < Q_p < Q^*$

we have $Q_p = k * E_{min} * F$

with $F_{min} < F < F_{max}$

therefore: $F = Q_p / (k*E_{min})$

and $G = [A+B/(k*E_{min})] * Q_p + C$ [6]

2) When $Q^* < Q_p < Q_{max}$

we have $F = F_{max}$

therefore: $G = A * Q_P + B * F_{max} + C$ [7]

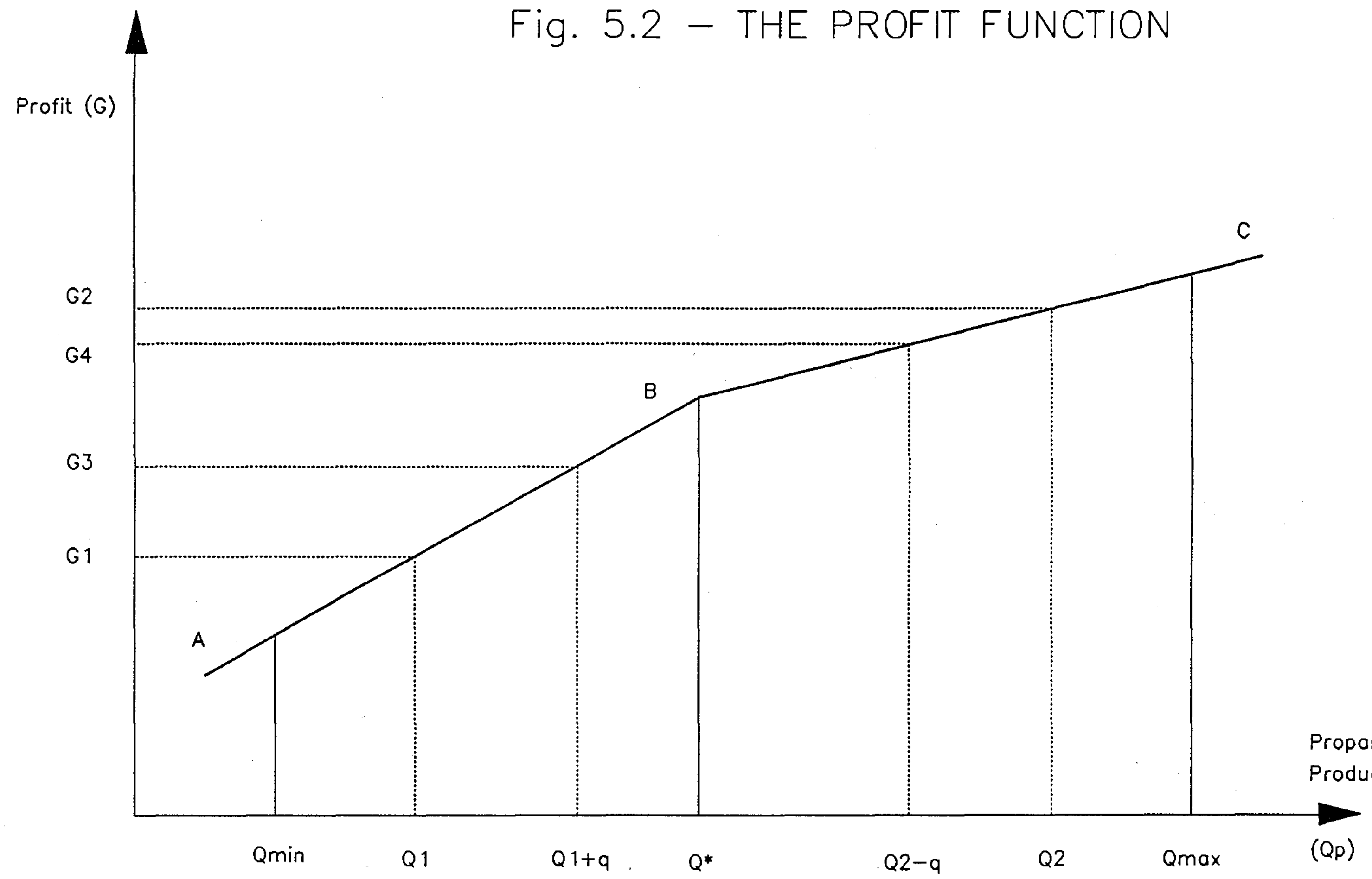
Since B, k, and E are all positive, it must be that

$$A < A + [B/(k*E_{min})]$$

which means that the slope of the function $G=G(Q_P)$ is greater at $Q_P < Q^*$ than at $Q_P > Q^*$, as shown in figure 5.2. In other words, along the operating line G is an increasing piece-wise linear concave function of Q_P. Concavity of the profit function implies that marginal profit is higher at $Q_P<Q^*$ than at $Q_P>Q^*$, and this conclusion has a very important consequence for our optimization model: it allows us to say that we shall try to avoid $Q_P>Q^*$, and that to do so we must smooth production levels. We can see this better if we consider the following.

Let us refer to figure 5.2 and let us assume that we have to produce an amount Q_P. We can do that operating one day at Q_1 and another day at Q_2, hence generating a profit G_1+G_2, but we could also produce Q_1+q one day, and Q_2-q the other day generating a profit G_3+G_4. It is evident that $G_3+G_4>G_1+G_2$ because the slope of AB is greather than the slope of BC. Thus, if we minimize deviations in daily production levels we also maximize profit. Recalling what

Fig. 5.2 – THE PROFIT FUNCTION



we said in §3.3 about the way electric power is priced, we see that by smoothing daily production we also minimize power cost.

To determine a production schedule that minimizes daily fluctuations we shall use an LP model.

5.2 Comparison with Actual Runs

<u>Note</u>: Due to the confidential nature of cost and revenue data, all units of measurement have been omitted from the formulations, as well as from the all numerical examples.

Data from 1987 production were collected to validate the formulation presented in the previous section. Coefficients A, B, and C in equation [1] were calculated from the data on current contracts and average prices for 1987. The estimated profit function is:

$$G = 61*E*F + 13.5*F - 15778 \qquad [8]$$

where:

$$E = .65 \quad \text{for} \quad 150 < F < 450, \text{ and}$$
$$.65 < E < .95 \quad \text{for} \quad F = 450$$

It is interesting to note that to avoid a loss situation (i.e. G<0) the above equation requires F to be always greather than 297. Recalling that each production line can only process feed flows up to 225 MMSCFD, the practical meaning of [8] is that the plant operates at a profit only when propane sales are high enough to need the operation of both production units.

Using [8], the operating curves were determined for three typical feed compositions, generally called Rich (figure 5.3), Design (figure 5.4), and Lean (figure 5.5). Once daily production level has been established (with the LP model), these curves shall be used to determine the optimum flow-recovery efficiency combination for the available feed composition.

Fig. 5.3 – PROPANE PRODUCTION

For Maximum Profit – Rich Feed

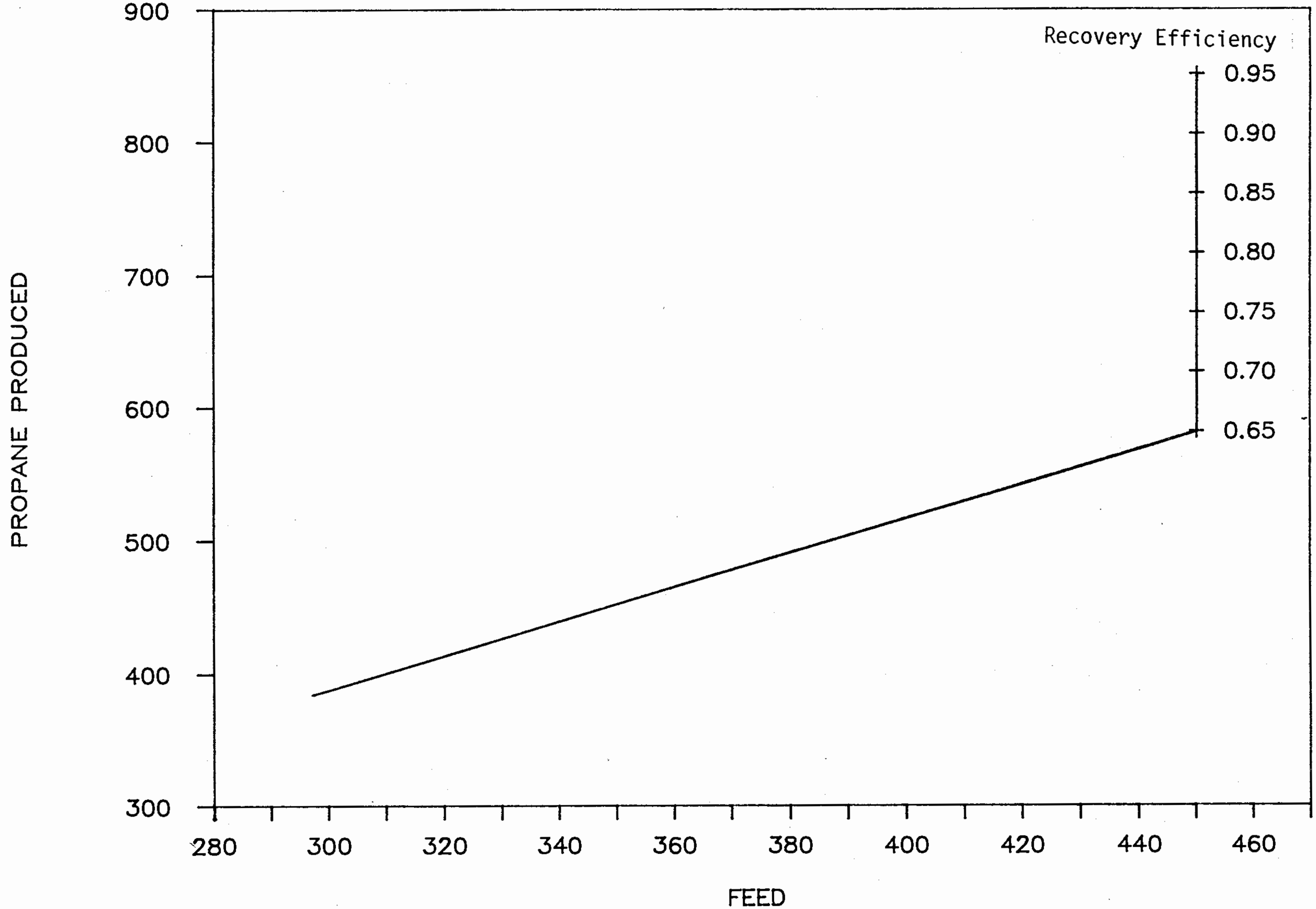


Fig. 5.4 — PROPANE PRODUCTION

For Maximum Profit — Design Feed

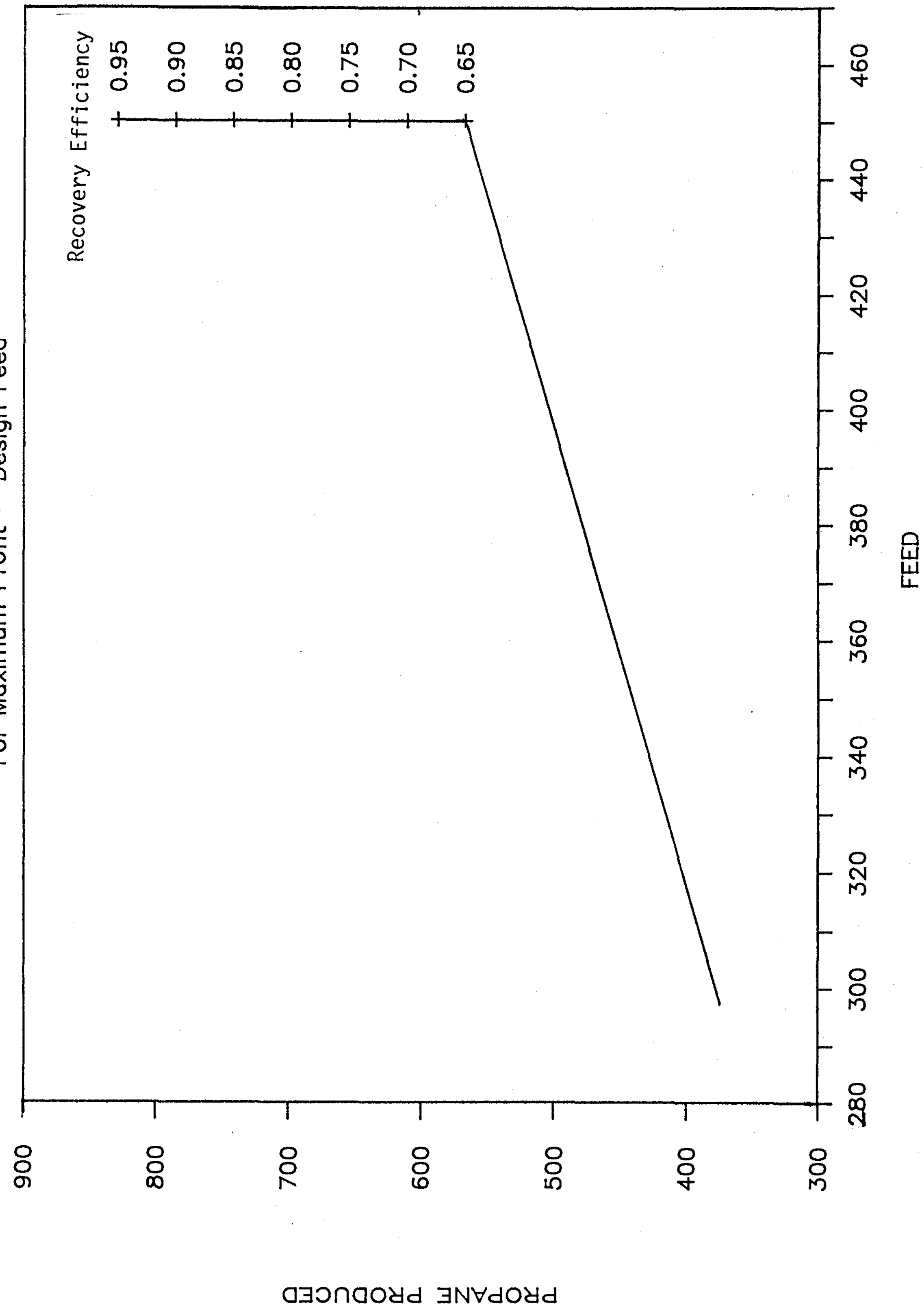
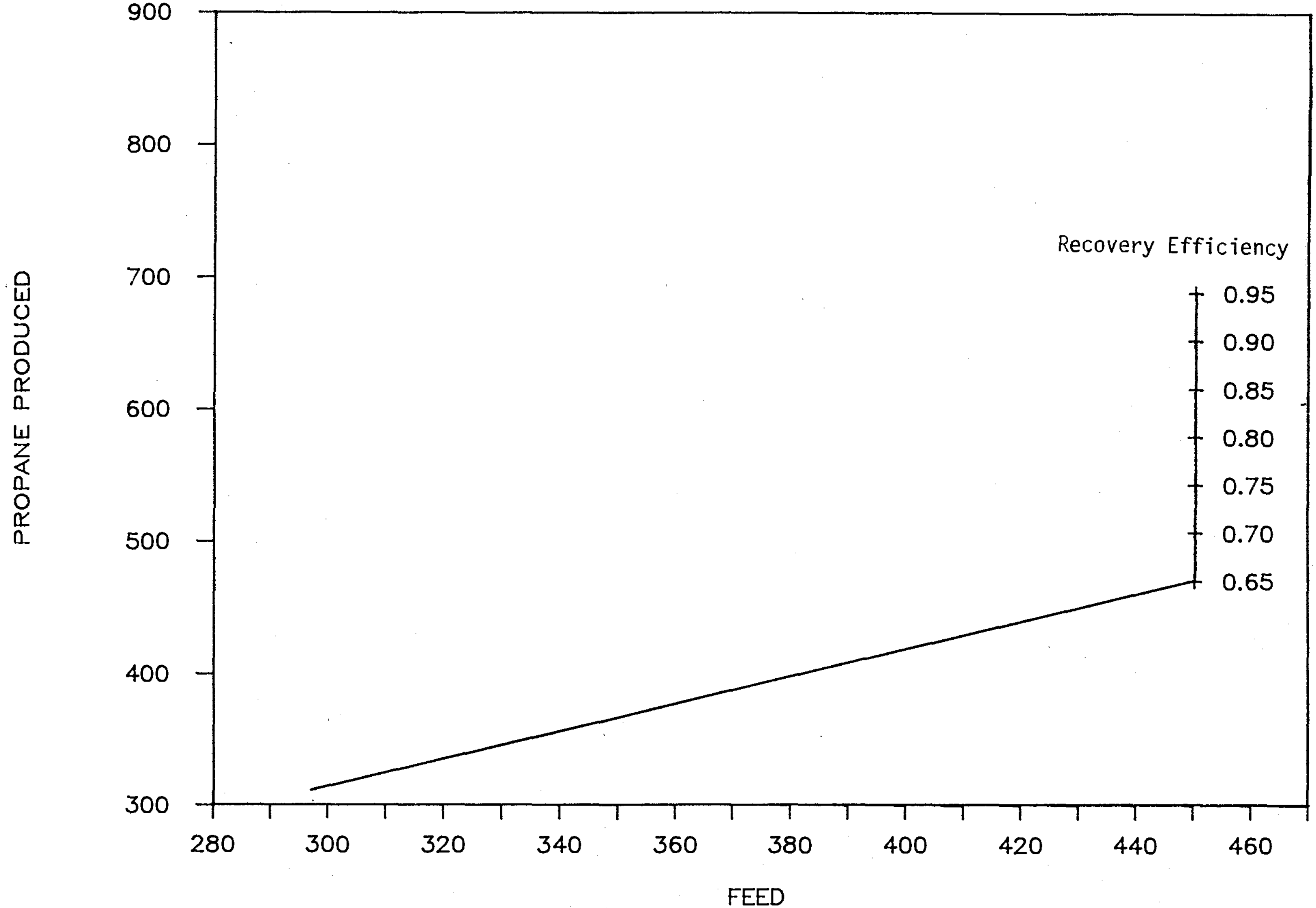


Fig. 5.5 — PROPANE PRODUCTION

For Maximum Profit — Lean Feed



6. THE LP MODEL

6.1 LP Formulation

To develop the LP model, we need to define an objective function, and a set of constraints. Based on the characteristics of the profit function observed in §5.1), our objective is to smooth the daily propane production levels, for any given target production quantity over a period of time, subject to the following workplace constraints:

1- Sales requirement for the period must be met, i.e. the sum of the daily shipments must equal the sales target;

2- The maximum daily production cannot exceed maximum plant capacity, for each given feed composition;

3- The maximum daily shipment cannot exceed the capability of the loading bay. Since both tank-cars and trucks are used, differences in the two systems have to be taken into account.

4- Production, shipment, opening and closing inventories have to be balanced;

5- Storage capacity available cannot be exceeded.

A representation of the model is given in figure 6.1; the constraints can be written as follows:

$$\text{SUM } (\text{SHIP})_i = \text{SALES}$$
$$(\text{PROP})_i < A$$
$$(\text{SHIP})_i < B_i$$
$$(\text{CLINP})_{i-1} + (\text{PROP})_i = (\text{SHIP})_i + (\text{CLINP})_i$$
$$(\text{CLINP})_i < C$$

Where:

i = 1, 2, 3, ..., n = number of periods considered;

SALES = target propane sales for the period;

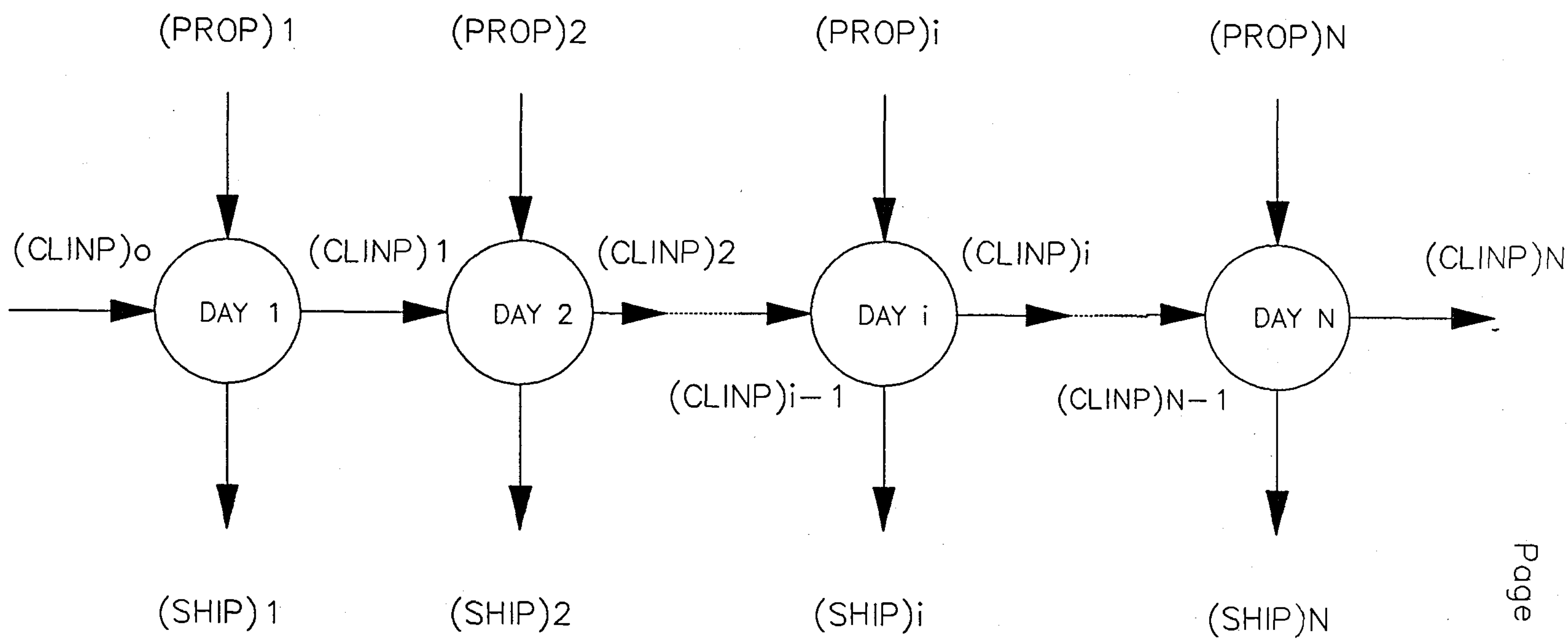
$(\text{SHIP})_i$ = shipment of product on day i;

$(\text{PROP})_i$ = propane production on day i;

$(\text{CLINP})_i$ = closing propane inventory on day i = = opening propane inventory on day $i+1$;

A = maximum daily propane production of the plant; this value is set by plant design and feed composition, and may be assumed the same for all days of the week (about 830 units);

Fig. 6.1 – SCHEMATIC REPRESENTATION OF THE LP MODEL



B_i = maximum propane shipment capacity for each day (about 1300 units during week days, and 100 units during weekends); tank cars, which carry most of the propane, are not loaded during weekends;

C = storage capacity available on site (about 1000 units).

6.2 Opening and Closing Inventory

Of the variables in the LP model, opening and closing inventory levels require some detailed discussion, because they link one period to the next, and influence future decisions.

Inventory is a necessity for any operation, but it is generally recognized as an unavoidable expense to be minimized. In the case of Taylor NGL plant, costs associated with on site storage are minimal compared to other production costs and storage operating costs vary very little with the amount of propane in the tanks. On the other hand it may be useful to maintain a certain inventory level, because it helps smoothing operation in the next period. Therefore we cannot disregard the needs for the optimization of the next period when looking at the current one.

To do this, we shall approach optimization in two steps: first, given the sales forecast for next period, we shall determine the optimum opening inventory for that period, which is also the closing inventory for the current period; then, since we know the inventory in hand, we can calculate daily production and shipping rates for the current period.

The overall procedure to optimize plant operation can be summarized in the following steps:

1- Based on the next period sales forecast, determine the optimum (or desirable) closing inventory for the current period;

2- Based on inventory in hand, and desirable closing inventory for this period, determine the daily production schedule;

3- Once the daily schedule is established, the operating line (Figures 5.3, 5.4, 5.5) is used to choose the combination of flow and recovery efficiency that maximizes profit.

When the constraints, expressed in general terms in §6.1, are examined in detail, we observe that the physical limitations of the plant tend to somewhat simplify our problem. Specifically, daily shipments have an upper limit which is the same for all weekdays, and another (lower) for the week-ends. This is due to the fact that tank cars are only loaded during weekdays, and that loading rate is limited by design.

In fact, only a minimum amount of loading is possible during weekends (about 100 units per day) compared with weekdays (about 1300 units). Given that daily propane production has a maximum at about 830 units (constrained by plant design and feed composition) we see that we can ship more than the daily production in the first five days of the week, while most of the weekend production goes to storage.

Maximum storage is limited to the physical dimensions of the tanks at the plant, and it would be impractical (and very expensive) to either add storage, or to provide storage offsite.

6.3 Heuristic Solution

Due to the special structure of the LP model, its solution can be obtained by simple heuristics. Specifically, let us assume that (by looking at next period sales forecast) we have defined that this week closing inventory must be CLINP7 = 600, and that we have in hand an inventory CLINP0 = 754. Knowing that this week sales is SALES = 4200, what is the optimum (i.e. the smoothest) production schedule?

We know that maximum shipment is the following for each day of the week (starting Monday): 1300, 1300, 1300, 1300, 1300, 100, 100; we also know that maximum inventory is 1000. Smoothest operation, obviously, is the one that requires the same production every day; in our case that can be written in the following way:

$$(\text{CLINP7} - \text{CLINP0} + \text{SALES})/7=$$
$$= (600 - 754 + 4200)/7 = 4046/7 = 578$$

This, clearly, is not feasible, because if we produced 578 on both day 6 and 7, given that we can only ship 100 on each of those days, we would end up with a volume in storage of at least:

$$\text{CLINP7} = 2 * 578 - 2 * 100 = 956$$

which is in excess of what we want. We shall then approach the problem differently.

Best schedule (in terms of smoothing) for weekends is to start with empty inventory, i.e. with CLINP5 = 0, and operate in the same fashion both on day 6 and 7. This allows to produce more on those constrained days, and to bring their production level closer to that of the less constrained ones. Then:

$$PROP6 = PROP7 = CLINP7 / 2 + SHIP6 =$$
$$= 600 / 2 + 100 = 400$$

The amount to be produced during the remaining five days can now be calculated, given that the optimum is "equal production each day". Therefore:

$$(SALES - PROP6 - PROP7) / 5 = 3400 / 5 = 680$$

We now have two questions to answer: is the solution actually feasible (i.e. does it fit all the constraints); and, can the solution be improved.

That the proposed solution is feasible is shown in the following table 6.1:

Table 6.1 - Heuristic Solution of LP

Day (i)	PROPi	CLINPi	SHIPi
0		754	
1	680	134	1300
2	680	0	814
3	680	0	680
4	680	0	680
5	680	0	680
6	400	300	100
7	400	600	100

Can this solution be improved, i.e. is there a production level between 578 and 680 for the weekdays, and between 578 ands 400 for the weekends, that is feasible? The answer is no, because we cannot produce any more during the weekend; if we did, the closing inventory level would be too high.

Let us now describe the general procedure for determining the solution for a general problem, subject to our particular set of constraints.

Step 1: Establish Feasibility. To do this, sales and production must be less than both the following values:

a) Max weekly sales =

= CLINP0 + MAX WEEKDAY PRODUCTION + WEEKEND SHIPMENT =

= CLINP0 + 832 * 5 + 200

b) Max weekly production =

= 832 * 5 + [CLINP7 - CLINP0 + 200]

Step 2: Establish Weekend Production. This shall be done assuming:

CLINP5 = 0; SHIP6 = SHIP7 = 100;

PROP6 = PROP7 = CLINP7 / 2 + SHIP6

Step 3: Establish Weekdays Production. This shall be done assuming:

PROP1=PROP2=PROP3=PROP4=PROP5, and CLINP5=0

Step 4: Check if Solution Can Be Improved. This shall be done by relaxing any of the constraints, keeping in mind that absolute optimum (i.e. the one with no constraints) is "same production level every day of the week".

The following are two example based on the above sequence.

Example 1

Requirements: SALES=5000; CLINP0=200; CLINP7=800

Solution: MAX SALES = 200 + 832 * 5 + 200 =
= 4560 < 5000

Therefore, no feasible solution exists.

Example 2

Requirements: SALES=4200; CLINP0=450; CLINP7=500

Solution: MAX SALES = 450 + 832 * 5 + 200 =
= 4810 > 4200 OK

MAX PRODUCTION = 832 * 5 + 200 + 500 - 450 =
= 4410

PRODUCTION REQUIRED = 4200 + 500 - 450 =
= 4250 < 4410 OK

Therefore, a feasible solution exists, and we could try the following one:

Weekends: PROP6=PROP7= 500/2 + 100 = 350

Weekdays: (4250 - 350*2) / 5 = 710

If we accepted this solution the situation would be that shown in Table 6.2.

Table 6.2 - Heuristic Solution of LP - Example 2

Day (i)	PROPi	CLINPi	SHIPi
0		450	
1	710	0	1160
2	710	0	710
3	710	0	710
4	710	0	710
5	710	0	710
6	350	250	100
7	350	500	100

This solution could be improved by relaxing constraint CLINP7. Since maximum closing inventory is limited by storage tank volume (1000 max), we may add 250 to each of day 6 and 7 production and inventory, obtaining:

PROP6 = PROP7 = 600

and reducing weekday production to

$$(710*5 - 250*2) / 5 = 610$$

Clearly this is a better solution for this period, if the requirement for opening inventory of next period can be relaxed.

7. THE LP SOLUTION

For the actual computer runs two LP's were used: one (called "NEXT") to determine this period best closing inventory, on the base of next period requirements; the other (called "THIS") to determine the current period production schedule. The basic difference between the two is in the way opening and closing inventories are constrained. For the actual formulation, the period chosen was the current week, since that is the way orders are taken at the plant. Obviously, the equations can be easily adapted to periods of different lenght. The meaning of each equation is explained here, and they are all shown in tables 7.1 and 7.2 (note that it is always i=1,2,...,7 except where otherwise indicated).

Objective Function PRODELTA:

Min U - L

To impose that production levels be as even as possible, the objective function was set-up to minimize the difference between maximum and minimum daily production during the period under consideration. U and L are the upper and lower limits, respectively, of the daily productions during the period.

Constraint WEEKSALES:

$SUM(SHIP_i) - TOTALSALES = 0$

It establishes that total weekly demand (TOTALSALES) has to be satisfied by the total daily shipments (SHIPi);

Constraints LOWLIMITi:

$L - PROP_i =< 0$

They calculate the smallest among the daily productions (PROPi) and assign that value to the dummy variable L;

Constraints HIGHLIMi:

$PROP_i - U =< 0$

They calculate the largest among the daily productions (PROPi) and assign that value to the dummy variable U;

Constraint MAXDAYPRD:

$U =< 832$

This imposes an upper limit to the daily production. In the way it is written it implies that maximum production is the same every day; it can be expanded and rewritten for every day, if different limits have to be imposed;

Constraints MAXSHIPi:

$SHIP_i$ =< 1300, i=1,2,...,5

$SHIP_i$ =< 100, i=6,7

These impose limitations on the maximum daily shipments;

Constraints INVBALi:

$CLINP_{i-1} + PROP_i - SHIP_i - CLINP_i = 0$

These are the inventory balance equations, that equate production (PROPi), inventory on hand (CLINPi), and shipment (SHIPi). The opening balance of the first day of the week, which is the same as the closing balance of the previous week, is called CLINP0;

Constraints MAXSTGEi:

$CLINP_i$ =< 1000

Maximum storage available is the volume of the storage spheres, and is imposed with these equations;

Constraint REQUIREDSHIP:

TOTALSALES = weekly requirement

This constraint sets the sales amount for next week;

(From this point on the equation for the two LP models, NEXT and THIS, are different).

NEXT - Constraint LOOP:

CLINP0 - CLINP7 = 0

NEXT is used to take into account next week production to calculate the optimum closing inventory for this week. To do that, as explained in §6.2, this constraint imposes that opening and closing inventory for next week are the same.

THIS - Constraint CLOSINV7:

CLINP7 = as determined from "NEXT"

Since the (desirable) closing inventory for this week is now known, this equation imposes its value;

THIS - Constraint CLINP0FIX:

CLINP0 = as known at scheduling time

This equation sets the opening inventory for this week, as actually existent at the plant.

Table 7.1 - Equations for Model NEXT

MIN	$U - L$
S.T.	$SUM(SHIP_i) - TOTALSALES = 0 \quad (i=1,2,\ldots,7)$
	$L - PROP_i =< 0$
	$PROP_i - U =< 0$
	$U =< 832$
	$SHIP_i =< 1300$
	$CLINP_{i-1} + PROP_i - SHIP_i - CLINP_i = 0$
	$CLINP0 - CLINP7 = 0$
	$CLINP_i =< 1000$
	TOTALSALES = weekly requirement

Table 7.2 - Equations for Model THIS

MIN	$U - L$
S.T.	$SUM(SHIP_i) - TOTALSALES = 0 \quad (i=1,2,\ldots,7)$
	$L - PROP_i =< 0$
	$PROP_i - U =< 0$
	$U =< 832$
	$SHIP_i =< 1300$
	$CLINP_{i-1} + PROP_i - SHIP_i - CLINP_i = 0$
	$CLINP_i =< 1000$
	CLINP7 = as determined from "NEXT"
	CLINP0 = as known at scheduling time
	TOTALSALES = weekly requirement

8. COMPARISON WITH ACTUAL OPERATION

8.1 Model Planning vs. Actual

The LP's presented above were tested against a period of twelve weeks for which production data were available. For each week the model NEXT was used first, and its results applied to the model THIS; details of the results are included in Appendix. Figures 8.1 and 8.2 show the difference between the actual production and the ones that would have been targeted for if the model had been used: clearly, the model produces a much smoother operation. The model clearly gives superior results even taking into account that it gives target volumes, which will actually vary because of fluctuations in the operating parameters and, mostly, in the composition of the feed. The weekly fluctuations were zero (i.e. same production level every day) for six weeks; more importantly, the target production level changed only ten times during the twelve weeks, as opposed to about fourty actual major changes (without taking into account the minor daily variation that could never be eliminated anyway).

Figure 8.1 TAYLOR NGL PLANT DAILY PRODUCTION

Comparison Between Actuals & Model NEXT

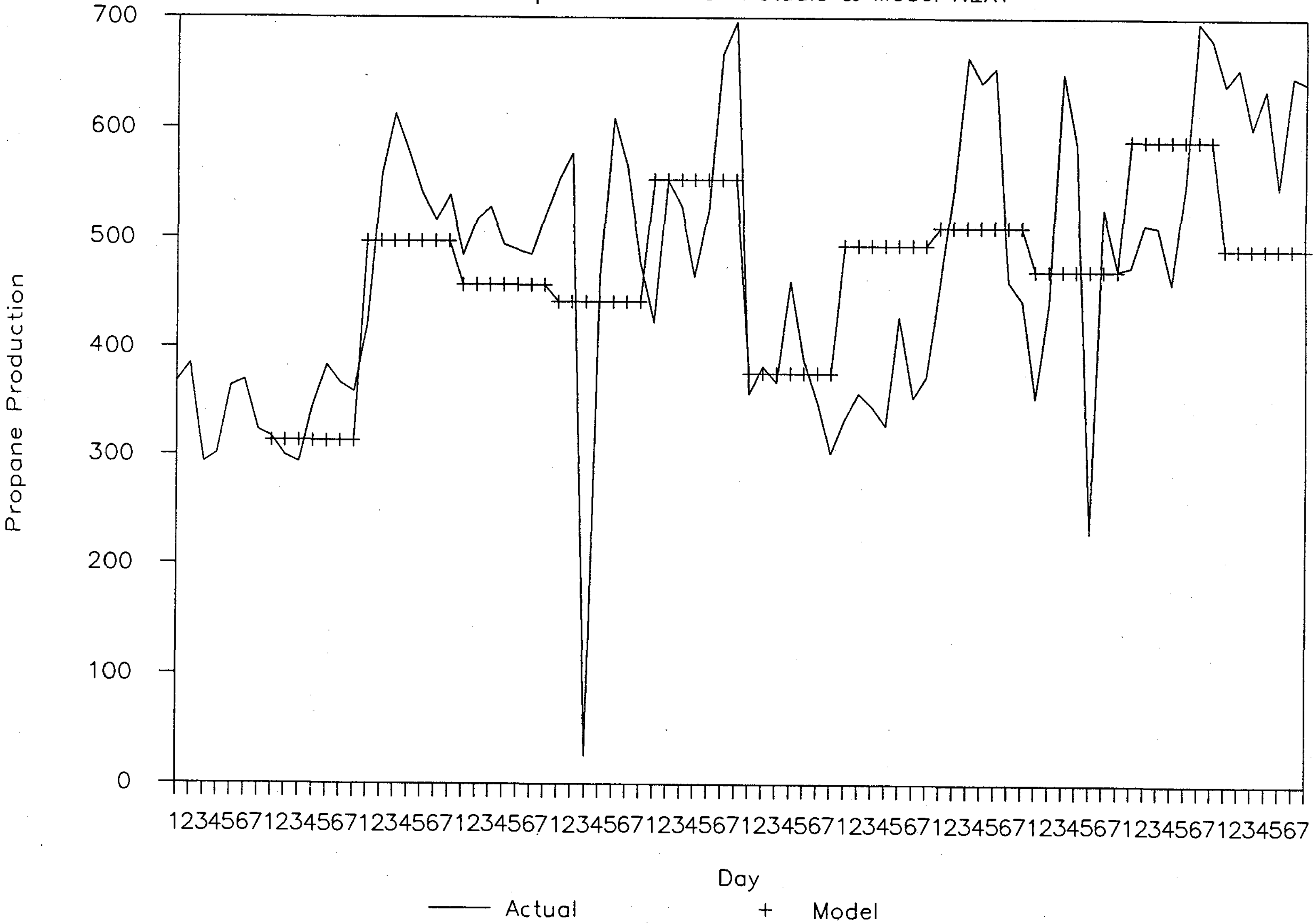
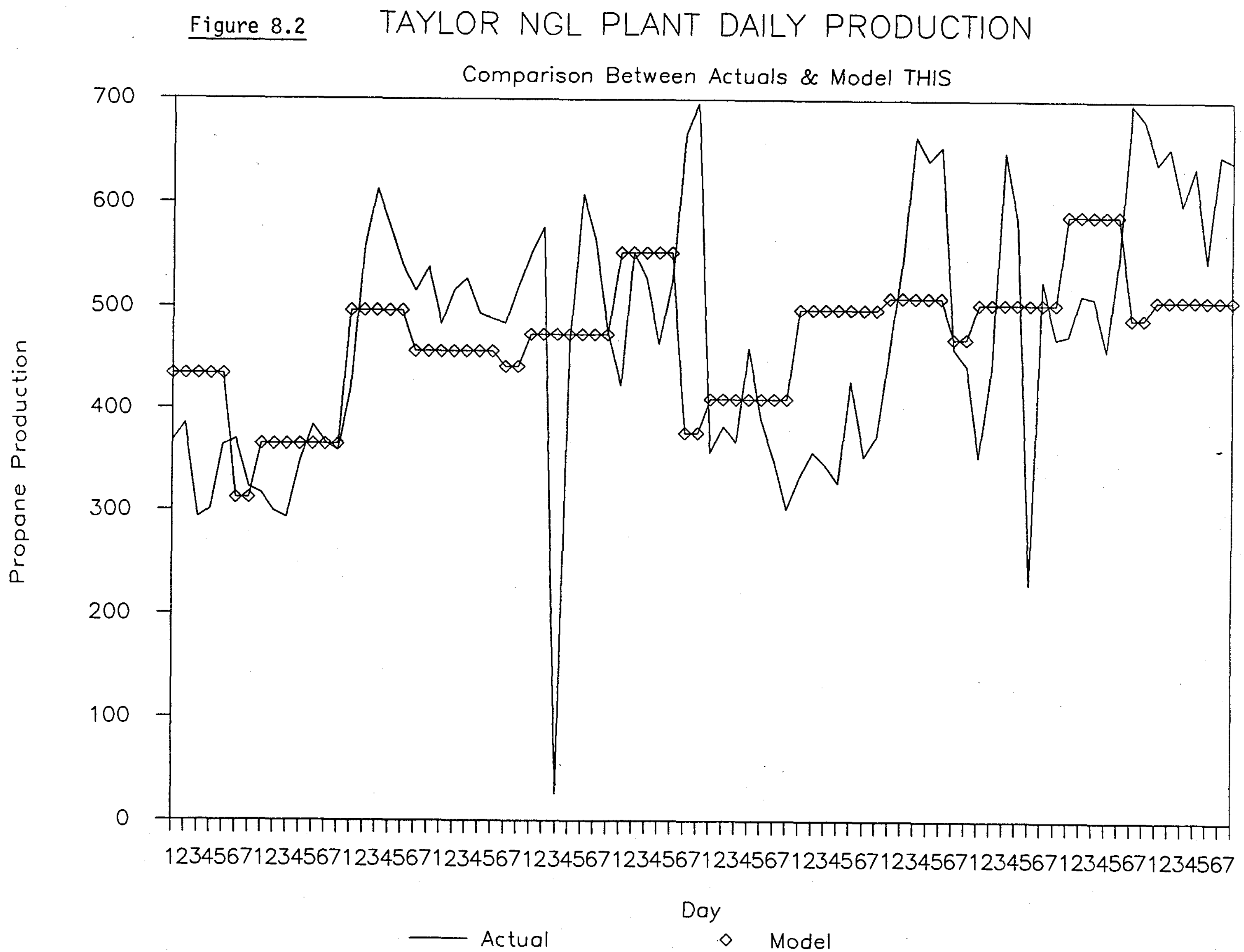


Figure 8.2


TAYLOR NGL PLANT DAILY PRODUCTION
Comparison Between Actuals & Model THIS
700
600
500
400
300
200
100
0
Propane Production
12345671234567123456712345671234567123456712345671234567123456712345671234567123456712345671234567
Day
Actual
Model

8.2 Sensitivity Analysis

One of the features of the LP model is that it allows to perform sensitivity analysis on the variables; in our case at least three variables should be scrutinized, namely the shipments during week-end (SHIP6 and SHIP7), and the weekly demand (SALES). Because of the particular formulation of our LP, we have to caution that a post-optimality analysis can only indicate the direction to move to, and cannot quantify the profit gain or loss; this would require a separate analysis.

Following the test on the twelve weeks, a demonstrative sensitivity (post-optimality) analysis was done on week 11. A look at the detailed results shows that week-end shipments are active constraints. The shadow price analysis quantifies this by saying that it would be possible to "smooth" more by shipping more during week-ends (Rows MAXSHIP6 and MAXSHIP7); it would be necessary able to ship up to about 241 units in one of the two days. In practice this may not be difficult to do: since it would require a few additional truck loads, it may be relatively easy to schedule them if enough notice is given.

We also see that a reduction in weekly sales (Row REQUIREDSHIP) would produce a smoother schedule. While it is obvious that we do not gain by selling less, this information may be used to discusss the weekly requirements with the Marketing Department, and explore with them the possibility of shifting some of the shipments to a later week.

In the same fashion, allowing the weekly closing inventory to increase appears to improve load levelling. This has to be balanced against next week forecast (where the constraining value was calculated from), and may not be obvious. Again, this is part of the knowledge that becomes available with this type of analysis, and that is certainly useful to make more informed decisions.

While some of the variables are under the operator's control, hence they can be acted on (or discussed) immediately, others require a different type of analysis, on a longer time horizon, and action by other corporate functions. For example, if we were simulating next year operation on the basis of a sales forecast, and we discovered that a larger storage capacity would allow us to operate the plant more evenly, we could use this result to evaluate the appropriate size of the storage, and see how advanageous it could be to smooth plant operation in that fashion.

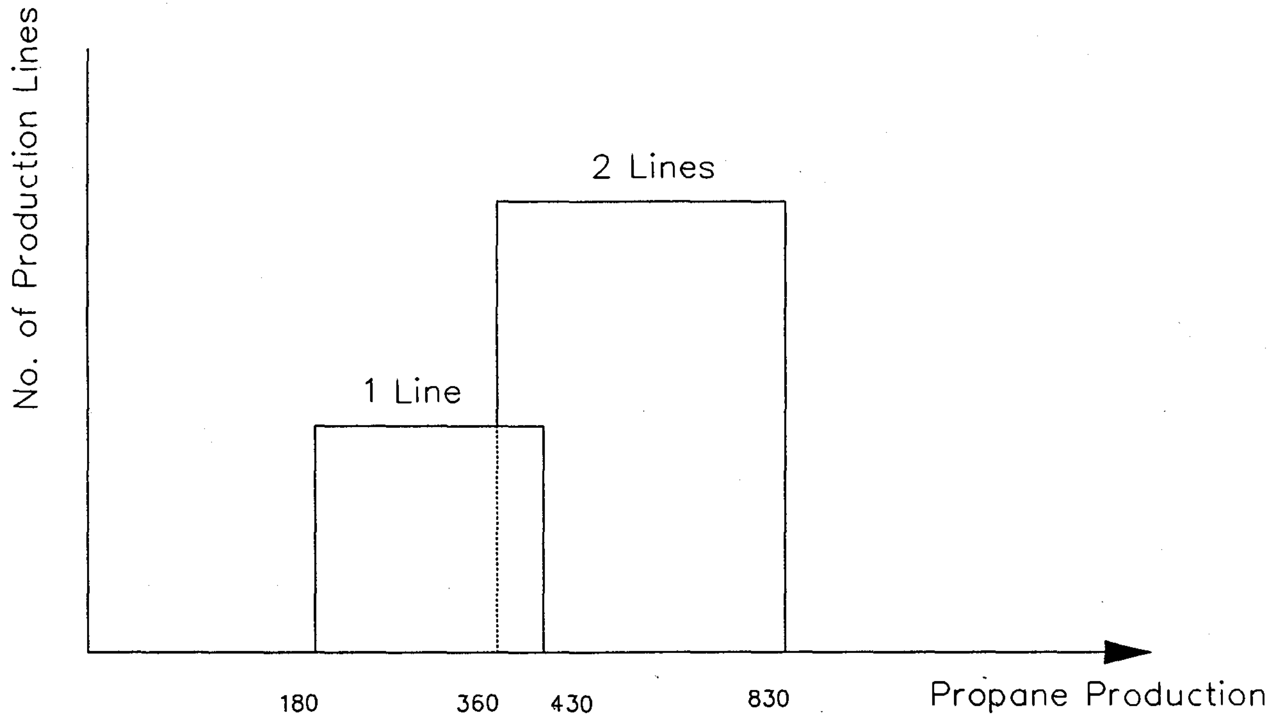
8.3 Special Cases

Lastly, we shall briefly discuss two subjects that may interfere or limit the applicability of the LP based scheduling. We refer to the potential need to handle contingencies (like equipment break-downs), and to the case when just one of the two production lines may be sufficient to satisfy demand.

Let us start with the problem of the two parallel lines, and of how to decide whether to use one or two lines to make a required production. Figure 8.3 shows the choice available at each propane production level; we see that between 180 and 360 units, one line is the only choice; between 360 and 430 units it is possible to operate either one or two lines; over 430 units two lines must be operated. So there are legitimate reasons to check whether one or two lines should be used for producing between 360 and 430 units.

One way of looking at the problem is to say that, as shown in section 5.2, there is a minimum flow corresponding to zero profit, and that flow is above the maximum capacity of one line. Therefore, if there are not enough orders to require more than that capacity, it is not profitable to fill them. Whether this means a total plant shutdown is

Fig. 8.3 — CAPABILITY OF EACH PRODUCTION LINE



still to be seen, and decided case by case. From a more general point of view, we can say that the Marketing Department is responsible to look after the sales, to insure enough contracts are in place to take care of the plant capacity.

For what concerns contingencies that may arise during the week, while the production plan is in progress, current practice is either to have a minimum storage, or to make proportionally more production at the beginning of the week; the choice being dictated by contingent factors (e.g. inventory levels) or by operators perception of the situation (e.g. type and number of problems experienced in the recent past). Following the same approach, and given that the future cannot be predicted very accurately, we can disregard contingencies for what concerns the LP model, and leave adjustements to the operator. The sensitivity analysis provided by the LP model is going to supplement the operating experience (which cannot be modelled), and the operator is going to decide what to do with daily production. We believe this is a better approach than manipulating the constrains (e.g. by restricting some of the daily outputs to imply less-than-maximum capacity) because it provides the operator with the appropriate information, but leaves the decision to his judgement.

9. CONCLUSIONS

It was demonstrated that an LP model could be used to determine optimum propane daily production levels to be used to plan the weekly production. The procedure was tested against twelve weeks of actual plant data, and the results have shown that significant improvements in plant operation can be achieved by using the model. The next logical step is to actually use the model in the plant.

This will be attempted soon, and is planned to take place in the following steps:

- Plant management will be presented with the model, its basis, its logic, so that any controversial aspect can be discussed and clarified.

- The senior operators will be involved, the objectives and the practical aspects of the model will be discussed with them, and a starting date will be decided.

- For about four weeks, the model will be used to determine target production levels, without actually using the results for production scheduling. This allows for "de-bugging", any adjustment to the local conditions, as well as familiarization of the operators with the use of the program.

- Actual (i.e. operator set schedules) shall be compared with model predictions, to confirm that these are feasible and acceptable. Post optimality analyses shall be performed to determine influence of the various factors.

- If the previous stage confirms the practicality, and the applicability of the model, and once operator acceptance and confidence has been gained, the model will be used to actually plan weekly production on a steady basis. When appropriate, post optimality analyses shall be run, and results discussed.

- After sufficient data have been collected, it may be that the model has to be adjusted or modified. Post optimality analyses shall be performed on the trends, to determine the actual value of the bottlenecks.

As mentioned in the introduction, this is the first attempt to model plant production at Taylor with a matematical tool like LP programming; as any first, it probably is a rough approximation of the reality, and refinements will be required. However, we hope that the first step is a good one, capable of giving a small but useful hand to increase profitability of Taylor Plant.

APPENDIX A.1 - PRODUCTION DATA

The following tables compile 120 days of production data collected between June 1987 and February 1988, used for the statistical analysis of Appendix A.2.

Each row contains data from one day production; variables have the following meaning (units of measurement have purposely been omitted due to the confidentiality of the information):

PROPANE	Propane produced
BUTANE	Butane produced
COND	Condensate produced
FEED	Feed gas to the plant
C3REC	Propane recovery efficiency
PWR	Total recompression power
FGNET	Net fuel gas consumption

PRODUCTION DATA

	PROPANE	BUTANE	COND	FEED	C3REC	PWR	FGNET
1	355	167	59	199	0.80	9913	0.59
2	361	153	58	198	0.80	9957	0.62
3	364	171	65	201	0.83	10046	0.59
4	370	158	64	200	0.83	10137	0.62
5	372	168	87	200	0.86	10093	0.66
6	387	163	65	200	0.87	10159	0.71
7	413	174	67	213	0.89	10137	0.72
8	593	204	111	268	0.85	13355	0.74
9	582	228	132	343	0.86	17038	1.16
10	648	252	99	344	0.89	16860	1.41
11	686	272	125	360	0.89	17320	1.47
12	714	290	115	361	0.89	17143	1.49
13	701	294	117	361	0.89	17277	1.00
14	701	293	111	362	0.90	17408	1.09
15	691	272	91	362	0.90	17210	1.36
16	679	262	85	361	0.90	17144	0.85
17	700	269	94	360	0.89	17121	1.03
18	709	276	115	360	0.90	17431	0.92
19	727	295	124	359	0.90	17586	0.98
20	709	285	133	358	0.89	17364	0.90
21	760	308	121	361	0.88	17453	1.16
22	710	302	108	361	0.89	17474	1.06

23	706	301	110	360	0.89	17276	0.90
24	731	302	112	359	0.89	17165	0.96
25	700	299	113	358	0.89	17209	0.99
26	692	286	97	351	0.88	17010	1.05
27	725	298	113	359	0.90	17055	1.25
28	686	284	94	360	0.87	16882	1.16
29	676	255	75	313	0.88	15400	0.90
30	564	230	79	347	0.86	16391	1.13
31	680	294	101	356	0.88	16944	1.20
32	286	52	16	191	0.92	8815	0.69
33	468	141	29	270	0.93	13489	0.73
34	540	162	35	283	0.96	14243	0.77
35	503	146	34	283	0.96	14286	0.76
36	481	144	32	268	0.95	13108	0.69
37	574	145	35	299	0.96	13819	0.83
38	494	144	37	291	0.96	13735	0.71
39	471	130	39	270	0.96	13622	0.67
40	303	88	35	245	0.89	12471	0.64
41	520	161	57	307	0.95	14576	0.92
42	438	117	41	264	0.96	13443	0.89
43	444	122	39	266	0.95	13376	0.72
44	391	75	23	264	0.95	13064	0.80
45	425	97	35	310	0.95	14333	0.83
46	460	95	35	305	0.95	14199	0.74
47	378	82	29	222	0.93	10727	0.62
48	331	62	18	266	0.93	12871	0.58

49	346	64	21	266	0.95	13264	0.63
50	299	51	19	219	0.93	10409	0.55
51	265	41	15	178	0.89	7988	0.47
52	298	51	18	201	0.92	8947	0.54
53	327	76	24	238	0.94	11809	0.33
54	367	109	21	255	0.96	13186	0.28
55	404	96	20	258	0.95	13365	0.09
56	424	116	22	223	0.94	10693	0.35
57	337	97	20	196	0.92	8925	0.67
58	245	41	16	181	0.93	8526	0.55
59	322	78	22	251	0.89	12044	0.59
60	400	106	28	175	0.96	8908	0.47
61	86	35	12	178	0.74	10056	0.46
62	385	107	32	209	0.93	9882	1.35
63	293	82	29	172	0.93	9213	1.47
64	301	103	68	199	0.94	9213	1.12
65	364	110	41	275	0.90	15214	1.19
66	370	103	33	213	0.93	10286	1.10
67	323	79	28	201	0.93	9476	1.03
68	317	69	25	196	0.93	9321	1.33
69	299	51	25	203	0.92	9609	0.93
70	293	62	25	200	0.93	9432	0.97
71	347	92	31	205	0.93	9671	1.11
72	384	133	41	202	0.93	9649	1.19
73	367	124	38	203	0.93	9693	1.25
74	359	97	31	207	0.92	9803	1.28

75	420	121	36	286	0.91	14012	1.35
76	558	191	52	359	0.91	17010	1.35
77	613	216	74	385	0.89	17165	1.36
78	577	207	68	357	0.89	15894	1.29
79	540	165	54	319	0.92	15252	1.42
80	516	162	53	329	0.92	15319	1.43
81	539	162	56	311	0.92	15211	1.31
82	484	158	50	298	0.91	14941	1.33
83	517	166	55	314	0.93	15274	1.28
84	528	158	52	322	0.93	15452	1.24
85	494	140	50	310	0.92	15074	1.27
86	489	134	49	306	0.92	15012	1.29
87	485	135	46	303	0.92	15057	1.28
88	520	147	48	318	0.92	15518	1.28
89	553	174	54	333	0.94	15697	1.32
90	577	183	57	256	0.93	12306	1.20
91	26	8	3	60	0.43	3729	0.36
92	720	324	199	347	0.87	16900	1.40
93	729	329	112	364	0.90	17166	1.50
94	706	296	110	363	0.89	17100	1.10
95	651	250	70	365	0.89	17126	1.00
96	674	259	83	366	0.87	17255	0.90
97	669	278	101	363	0.90	17255	0.90
98	699	282	105	365	0.89	17386	1.10
99	700	279	101	365	0.90	17166	0.90
100	652	267	68	366	0.89	17122	1.00

101	667	254	77	366	0.89	17277	0.80
102	658	256	74	362	0.88	17298	1.10
103	579	215	69	328	0.87	16394	0.90
104	571	215	50	321	0.88	15952	0.80
105	562	191	59	321	0.88	16041	1.40
106	627	233	68	356	0.89	17144	0.80
107	683	276	82	365	0.88	17541	1.45
108	638	245	77	315	0.88	15015	1.02
109	397	148	35	425	0.85	10590	0.62
110	469	194	51	318	0.87	15421	0.68
111	651	239	72	365	0.84	17254	0.91
112	621	236	53	366	0.87	16988	1.47
113	696	266	78	364	0.89	17254	0.90
114	675	280	83	361	0.88	16988	0.99
115	670	259	83	365	0.87	17143	0.72
116	672	287	86	366	0.87	17386	0.80
117	655	264	81	366	0.87	17364	0.71
118	659	251	75	366	0.87	17076	0.82
119	632	264	77	387	0.86	16811	0.77
120	534	252	49	388	0.86	15098	0.72

APPENDIX A.2 - RECOMPRESSION POWER REGRESSION ANALYSIS

The following pages contain the regression analysis done on recompression power (PWR) using data from Appendix A.1, and FEED (Plant feed gas flow) as the independent variable. The results show a very high correlation between the two variables, as well as a high significance of R^2. Figure A.1 is a plot of the data, and of the regression line. The estimated equation is:

$$PWR = 1471 + 42.84 * FEED$$

t = 32.27 (99.99%)

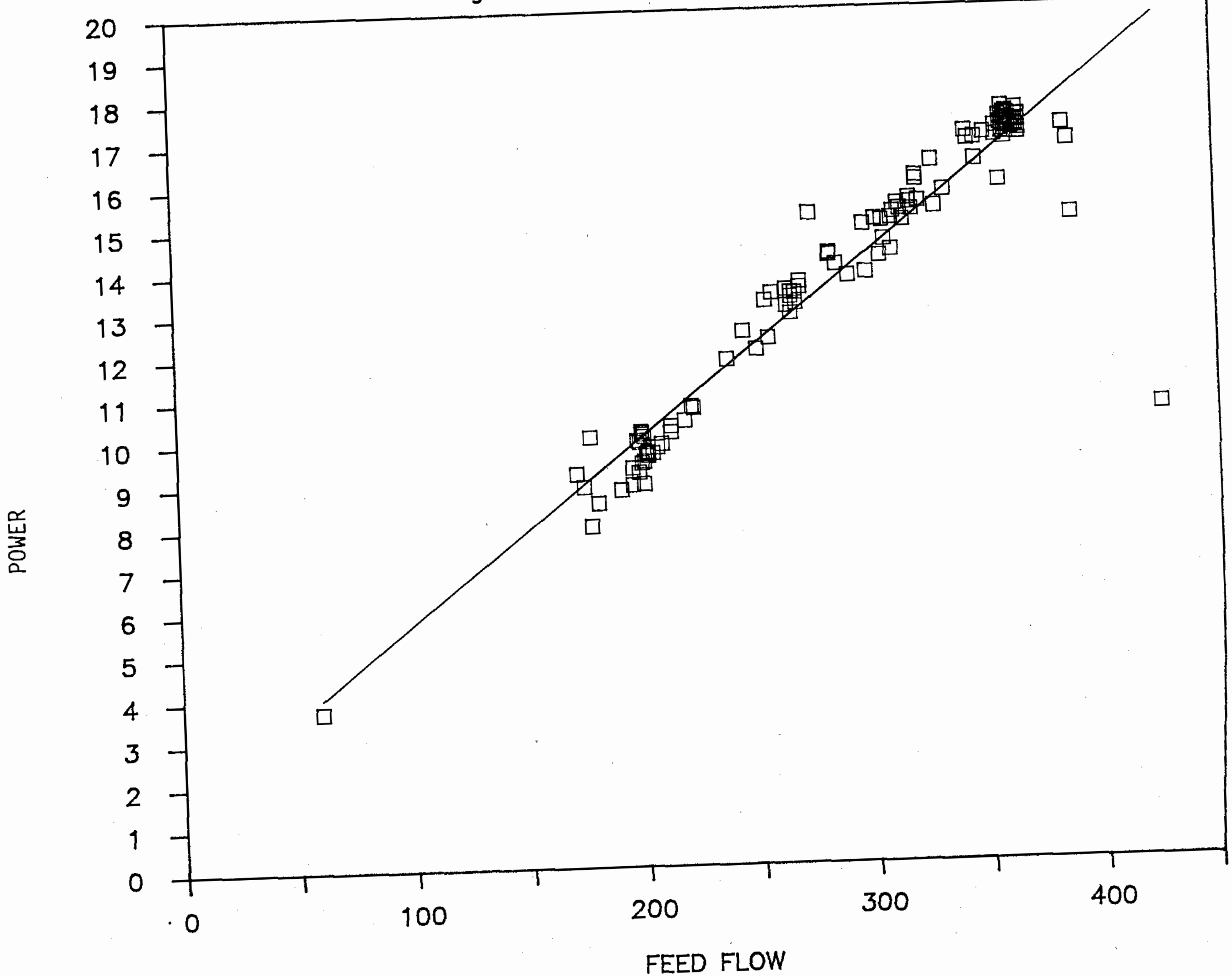
R^2 = .8982

F = 1041 (99.99%)

FIGURE A.1

TAYLOR NGL PLANT — 1987,88

Regression Analysis — Power vs. Feed



**** Multiple Regression Report ****

Dependent Variable: PWR

Independent Variable	Parameter Estimate	Stndized Estimate	Standard Error	t-value (b=0)	Prob. Level	Simple R-Sqr
Intercept	1471.016	0.0000	404.3427	3.64	0.0004	
FEED	42.84189	0.9477	1.327724	32.27	0.0000	0.8982

**** Analysis of Variance Report ****

Dependent Variable: PWR

Source	df	Sums of Squares (Sequential)	Mean Square	F-Ratio	Prob. Level
Constant	1	2.408761E+10	2.408761E+10		
Model	1	1.081511E+09	1.081511E+09	1041.17	0.000
Error	118	1.225719E+08	1038745		
Total	119	1.204083E+09	1.011834E+07		

Root Mean Square Error	1019.189
Mean of Dependent Variable	14167.92
Coefficient of Variation	7.193633E-02
R Squared	0.8982
Adjusted R Squared	0.8973

APPENDIX A.3 - LP MODELS "NEXT1" AND "THIS1"

*** NEXT1 ***

```
Constraint    1 'WEEKSALES     '
                1.000000SHIP1          +        1.000000SHIP2
   +            1.000000SHIP3          +        1.000000SHIP4
   +            1.000000SHIP5          +        1.000000SHIP6
   +            1.000000SHIP7          +       -1.000000TOTALSALES
                                       ==       0.0
Constraint    2 'LOWLIMIT1     '
                1.000000L              +       -1.000000PROP1
                                       <=       0.0
Constraint    3 'LOWLIMIT2     '
                1.000000L              +       -1.000000PROP2
                                       <=       0.0
Constraint    4 'LOWLIMIT3     '
                1.000000L              +       -1.000000PROP3
                                       <=       0.0
Constraint    5 'LOWLIMIT4     '
                1.000000L              +       -1.000000PROP4
                                       <=       0.0
Constraint    6 'LOWLIMIT5     '
                1.000000L              +       -1.000000PROP5
                                       <=       0.0
Constraint    7 'LOWLIMIT6     '
                1.000000L              +       -1.000000PROP6
                                       <=       0.0
Constraint    8 'LOWLIMIT7     '
                1.000000L              +       -1.000000PROP7
                                       <=       0.0
Constraint    9 'HIGHLIM1      '
               -1.000000U              +        1.000000PROP1
                                       <=       0.0
Constraint   10 'HIGHLIM2      '
               -1.000000U              +        1.000000PROP2
                                       <=       0.0
Constraint   11 'HIGHLIM3      '
               -1.000000U              +        1.000000PROP3
                                       <=       0.0
Constraint   12 'HIGHLIM4      '
               -1.000000U              +        1.000000PROP4
                                       <=       0.0
Constraint   13 'HIGHLIM5      '
               -1.000000U              +        1.000000PROP5
                                       <=       0.0
Constraint   14 'HIGHLIM6      '
               -1.000000U              +        1.000000PROP6
                                       <=       0.0
Constraint   15 'HIGHLIM7      '
               -1.000000U              +        1.000000PROP7
                                       <=       0.0
```

```
Constraint  16 'MAXDAYPROD   '  '
              1.000000U                +
                                       <=     832.000000
Constraint  17 'MAXSHIP1     '
              1.000000SHIP1            +
                                       <=    1300.000000
Constraint  18 'MAXSHIP2     '
              1.000000SHIP2            +
                                       <=    1300.000000
Constraint  19 'MAXSHIP3     '
              1.000000SHIP3            +
                                       <=    1300.000000
Constraint  20 'MAXSHIP4     '
              1.000000SHIP4            +
                                       <=    1300.000000
Constraint  21 'MAXSHIP5     '
              1.000000SHIP5            +
                                       <=    1300.000000
Constraint  22 'MAXSHIP6     '
              1.000000SHIP6            +
                                       <=     100.000000
Constraint  23 'MAXSHIP7     '
              1.000000SHIP7            +
                                       <=     100.000000
Constraint  24 'INVBAL1      '
             -1.000000SHIP1            +        1.000000PROP1
  +          -1.000000CLINP1           +        1.000000CLINP0
                                       ==      0.0
Constraint  25 'INVBAL2      '
             -1.000000SHIP2            +        1.000000CLINP1
  +           1.000000PROP2            +       -1.000000CLINP2
                                       ==      0.0
Constraint  26 'INVBAL3      '
             -1.000000SHIP3            +        1.000000PROP3
  +           1.000000CLINP2           +       -1.000000CLINP3
                                       ==      0.0
Constraint  27 'INVBAL4      '
             -1.000000SHIP4            +        1.000000PROP4
  +           1.000000CLINP3           +       -1.000000CLINP4
                                       ==      0.0
Constraint  28 'INVBAL5      '
             -1.000000SHIP5            +        1.000000PROP5
  +           1.000000CLINP4           +       -1.000000CLINP5
                                       ==      0.0
Constraint  29 'INVBAL6      '
             -1.000000SHIP6            +        1.000000PROP6
  +           1.000000CLINP5           +       -1.000000CLINP6
                                       ==      0.0
Constraint  30 'INVBAL7      '
             -1.000000SHIP7            +        1.000000PROP7
  +           1.000000CLINP6           +       -1.000000CLINP7
                                       ==      0.0
```

```
 Constraint  31 'MAXSTGE1     '
               1.000000CLINP1        +
                                     <=     1000.000000
 Constraint  32 'MAXSTGE2     '
               1.000000CLINP2        +
                                     <=     1000.000000
 Constraint  33 'MAXSTGE3     '
               1.000000CLINP3        +
                                     <=     1000.000000
 Constraint  34 'MAXSTGE4     '
               1.000000CLINP4        +
                                     <=     1000.000000
 Constraint  35 'MAXSTGE5     '
               1.000000CLINP5        +
                                     <=     1000.000000
 Constraint  36 'MAXSTGE6     '
               1.000000CLINP6        +
                                     <=     1000.000000
 Constraint  37 'MASTGE7      '
               1.000000CLINP7        +
                                     <=     1000.000000
 Constraint  38 'LOOP         '
               1.000000CLINP0        +         -1.000000CLINP7
                                     ==        0.0
 Constraint  39 'REQUIREDSHIP'
               1.000000TOTALSALES    +
                                     ==     2186.000000
 Objective function (PRODNDELTA  ) to be MINimised
               1.000000U             +         -1.000000L
=========
*** THIS1 ***

 Constraint   1 'WEEKSALES    '
               1.000000SHIP1         +          1.000000SHIP2
   +           1.000000SHIP3         +          1.000000SHIP4
   +           1.000000SHIP5         +          1.000000SHIP6
   +           1.000000SHIP7         +         -1.000000TOTALSALES
                                     ==        0.0
 Constraint   2 'LOWLIMIT1    '
               1.000000L             +         -1.000000PROP1
                                     <=        0.0
 Constraint   3 'LOWLIMIT2    '
               1.000000L             +         -1.000000PROP2
                                     <=        0.0
 Constraint   4 'LOWLIMIT3    '
               1.000000L             +         -1.000000PROP3
                                     <=        0.0
 Constraint   5 'LOWLIMIT4    '
               1.000000L             +         -1.000000PROP4
                                     <=        0.0
 Constraint   6 'LOWLIMIT5    '
               1.000000L             +         -1.000000PROP5
```

```
                                               <=        0.0
Constraint    7 'LOWLIMIT6      '
               1.000000L                       +         -1.000000PROP6
                                               <=        0.0
Constraint    8 'LOWLIMIT7      '
               1.000000L                       +         -1.000000PROP7
                                               <=        0.0
Constraint    9 'HIGHLIM1       '
              -1.000000U                       +          1.000000PROP1
                                               <=        0.0
Constraint   10 'HIGHLIM2       '
              -1.000000U                       +          1.000000PROP2
                                               <=        0.0
Constraint   11 'HIGHLIM3       '
              -1.000000U                       +          1.000000PROP3
                                               <=        0.0
Constraint   12 'HIGHLIM4       '
              -1.000000U                       +          1.000000PROP4
                                               <=        0.0
Constraint   13 'HIGHLIM5       '
              -1.000000U                       +          1.000000PROP5
                                               <=        0.0
Constraint   14 'HIGHLIM6       '
              -1.000000U                       +          1.000000PROP6
                                               <=        0.0
Constraint   15 'HIGHLIM7       '
              -1.000000U                       +          1.000000PROP7
                                               <=        0.0
Constraint   16 'MAXDAYPROD     '
               1.000000U                       +
                                               <=      832.000000
Constraint   17 'MAXSHIP1       '
               1.000000SHIP1                   +
                                               <=     1300.000000
Constraint   18 'MAXSHIP2       '
               1.000000SHIP2                   +
                                               <=     1300.000000
Constraint   19 'MAXSHIP3       '
               1.000000SHIP3                   +
                                               <=     1300.000000
Constraint   20 'MAXSHIP4       '
               1.000000SHIP4                   +
                                               <=     1300.000000
Constraint   21 'MAXSHIP5       '
               1.000000SHIP5                   +
                                               <=     1300.000000
Constraint   22 'MAXSHIP6       '
               1.000000SHIP6                   +
                                               <=      100.000000
Constraint   23 'MAXSHIP7       '
               1.000000SHIP7                   +
                                               <=      100.000000
```

```
Constraint  24 'INVBAL1      '
               -1.000000SHIP1          +        1.000000PROP1
  +            -1.000000CLINP1         +        1.000000CLINP0
                                       ==     0.0
Constraint  25 'INVBAL2      '
               -1.000000SHIP2          +        1.000000CLINP1
  +             1.000000PROP2          +       -1.000000CLINP2
                                       ==     0.0
Constraint  26 'INVBAL3      '
               -1.000000SHIP3          +        1.000000PROP3
  +             1.000000CLINP2         +       -1.000000CLINP3
                                       ==     0.0
Constraint  27 'INVBAL4      '
               -1.000000SHIP4          +        1.000000PROP4
  +             1.000000CLINP3         +       -1.000000CLINP4
                                       ==     0.0
Constraint  28 'INVBAL5      '
               -1.000000SHIP5          +        1.000000PROP5
  +             1.000000CLINP4         +       -1.000000CLINP5
                                       ==     0.0
Constraint  29 'INVBAL6      '
               -1.000000SHIP6          +        1.000000PROP6
  +             1.000000CLINP5         +       -1.000000CLINP6
                                       ==     0.0
Constraint  30 'INVBAL7      '
               -1.000000SHIP7          +        1.000000PROP7
  +             1.000000CLINP6         +       -1.000000CLINP7
                                       ==     0.0
Constraint  31 'MAXSTGE1     '
                1.000000CLINP1         +
                                       <=   1000.000000
Constraint  32 'MAXSTGE2     '
                1.000000CLINP2         +
                                       <=   1000.000000
Constraint  33 'MAXSTGE3     '
                1.000000CLINP3         +
                                       <=   1000.000000
Constraint  34 'MAXSTGE4     '
                1.000000CLINP4         +
                                       <=   1000.000000
Constraint  35 'MAXSTGE5     '
                1.000000CLINP5         +
                                       <=   1000.000000
Constraint  36 'MAXSTGE6     '
                1.000000CLINP6         +
                                       <=   1000.000000
Constraint  37 'MASTGE7      '
                1.000000CLINP7         +
                                       <=   1000.000000
Constraint  38 'REQUIREDSHIP'
                1.000000TOTALSALES     +
                                       ==   3371.000000
```

```
Constraint  39 'closinv7s   '  '
             1.000000CLINP7       +
                                  ==      424.571000
Constraint  40 'CLINP0FIX   '
             1.000000CLINP0       +
                                  ==     1000.000000
Objective function (PRODNDELTA  ) to be MINimised
             1.000000U            +        -1.000000L
```

APPENDIX A.4 - SUMMARY OF RESULTS FROM MODEL "NEXT" FOR TWELVE WEEKS

Each period is calculated imposing CLINP0=CLINP7; the calculated $(CLINP0)_i$ is then used as $(CLINP7)_{i-1}$ for the "$THIS_i$" runs.

WEEK	DAY	SALES	CLINPi	PROP	SHIP	CLINP0i
	0		1000	=	=	
1	1	3371				
	2					
	3					
	4					
	5					
	6					
	7		424.571			
2	1	2186		312.860	736.857	424.571
	2			312.860	312.860	
	3			312.860	312.860	
	4			312.860	312.860	
	5			312.860	312.860	
	6		212.860	312.860	100.000	
	7		424.571	312.860	100.000	
3	1	3476		496.571	1289.710	793.143

3	2			496.571	496.571	
	3			496.571	496.571	
	4			496.571	496.571	
	5			496.571	496.571	
	6		396.571	496.571	100.000	
	7		793.143	496.571	100.000	
4	1	3200		457.143	1171.430	714.286
	2			457.143	457.143	
	3			457.143	457.143	
	4			457.143	457.143	
	5			457.143	457.143	
	6		357.143	457.143	100.000	
	7		714.286	457.143	100.000	
5	1	3092		441.714	1125.140	683.429
	2			441.714	441.714	
	3			441.714	441.714	
	4			441.714	441.714	
	5			441.714	441.714	
	6		341.714	441.714	100.000	
	7		683.429	441.714	100.000	
6	1	3873	159.857	553.286	1300.000	906.571
	2			553.286	713.143	
	3			553.286	553.286	
	4			553.286	553.286	
	5			553.286	553.286	
	6		453.286	553.286	100.000	

6	7		906.571	553.286	100.000	
7	1	2636		376.571	929.614	553.143
	2			376.571	376.571	
	3			376.571	376.571	
	4			376.571	376.571	
	5			376.571	376.571	
	6		276.571	376.571	100.000	
	7		553.143	376.571	100.000	
8	1	3453		493.286	1279.860	786.571
	2			493.286	493.286	
	3			493.286	493.286	
	4			493.286	493.286	
	5			493.286	493.286	
	6		393.286	493.286	100.000	
	7		786.571	493.286	100.000	
9	1	3573	31.286	510.429	1300.000	820.857
	2			510.429	541.714	
	3			510.429	510.429	
	4			510.429	510.429	
	5			510.429	510.429	
	6		410.429	510.429	100.000	
	7		820.857	510.429	100.000	
10	1	3292		470.286		740.571
	2			470.286		
	3			470.286		
	4			470.286		

10	5			470.286	
	6			470.286	
	7		740.571	470.286	
11	1	4122		588.857	977.714
	2			588.857	
	3			588.857	
	4			588.857	
	5			588.857	
	6		488.857	588.857	
	7		977.714	588.857	
12	1	3433		490.429	780.857
	2			490.429	
	3			490.429	
	4			490.429	
	5			490.429	
	6			490.429	
	7		780.857	490.429	

APPENDIX A.5 - SUMMARY OF RESULTS FROM MODEL "THIS"

WEEK	DAY	SALES	CLINP	PROP	SHIP	U-L
	0	=	1000	=	=	=
1	1	3371	865.800	434.200	568.400	121.915
	2		0.000	434.200	1300.000	
	3		0.000	434.200	434.200	
	4		0.000	434.200	434.200	
	5		0.000	434.200	434.200	
	6		212.285	312.285	100.000	
	7		424.571	312.285	100.000	
2	1	2186	0.000	364.939	789.510	0
	2		0.000	364.939	364.939	
	3		0.000	364.939	364.939	
	4		0.000	364.939	0.000	
	5		63.265	364.939	301.674	
	6		428.204	364.939	0.000	
	7		793.143	364.939	0.000	
3	1	3476	0.000	496.571	1289.710	39.43
	2		0.000	496.571	496.571	
	3		0.000	496.571	496.571	
	4		0.000	496.571	496.571	
	5		0.000	496.571	496.571	

	6		357.143	457.143	100.000	
3	7		714.286	457.143	100.000	
4	1	3200	0.000	457.143	1171.430	15.43
	2		0.000	457.143	457.143	
	3		0.000	457.143	0.000	
	4		0.000	457.143	0.000	
	5		0.000	457.143	0.000	
	6		341.714	441.714	100.000	
	7		683.429	441.714	100.000	
5	1	3092	0.000	473.592	1157.020	0
	2		0.000	473.592	473.592	
	3		0.000	473.592	473.592	
	4		0.000	473.592	473.592	
	5		0.000	473.592	473.592	
	6		432.979	473.592	40.612	
	7		906.571	473.592	0.000	
6	1	3873	159.857	553.286	1300.000	176.714
	2		0.000	553.286	713.143	
	3		0.000	553.286	553.286	
	4		0.000	553.286	553.286	
	5		0.000	553.286	553.286	
	6		276.571	376.571	100.000	
	7		553.143	376.571	100.000	
7	1	2636	0.000	409.918	963.061	0
	2		0.000	409.918	409.918	
	3		0.000	409.918	409.918	

	4		0.000	409.918	409.918	
	5		0.000	409.918	409.918	
7	6		376.653	409.918	33.266	
	7		786.571	409.918	0.000	
8	1	3453	0.000	498.184	1284.750	0
	2		0.000	498.184	498.184	
	3		0.000	498.184	498.184	
	4		0.000	498.184	498.184	
	5		0.000	498.184	498.184	
	6		398.184	498.184	100.000	
	7		820.857	498.184	75.510	
9	1	3573	31.286	510.429	1300.000	40.14
	2		0.000	510.429	541.714	
	3		0.000	510.429	510.429	
	4		0.000	510.429	510.429	
	5		0.000	510.429	510.429	
	6		370.285	470.285	100.000	
	7		740.571	470.285	100.000	
10	1	3292	0.000	504.163	1244.730	0
	2		0.000	504.163	504.163	
	3		0.000	504.163	504.163	
	4		0.000	504.163	504.163	
	5		0.000	504.163	504.163	
	6		0.000	504.163	30.613	
	7		473.551	504.163	0.000	
11	1	4122	977.714	588.857	855.428	98.43

	2		0.000'	588.857	1300.000	
	3		0.000	588.857	588.857	
	4		0.000	588.857	588.857	
11	5		0.000	588.857	588.857	
	6		390.428	490.428	100.000	
	7		780.857	490.428	100.000	
12	1	3433	0.000	508.265	1289.120	0
	2		0.000	508.265	508.265	
	3		0.000	508.265	508.265	
	4		0.000	508.265	508.265	
	5		0.000	508.265	508.265	
	6		408.265	508.265	100.000	
	7		905.714	508.265	10.817	
13	1	3870	=	=	=	

APPENDIX A.6 - COMPARISON BETWEEN ACTUALS AND MODEL RESULTS

WK	DAY	SALES	ACTUAL DAILY	ACTUAL WKLY	DELTA SALES	INV 1000	NEXT	THIS
1	1	3371	368	2404	-967	33		434.200
	2		385					434.200
	3		293					434.200
	4		301					434.200
	5		364					434.200
	6		370					312.285
	7		323					312.285
2	1	2186	317	2366	180	213	312.860	364.939
	2		299				312.860	364.939
	3		293				312.860	364.939
	4		347				312.860	364.939
	5		384				312.860	364.939
	6		367				312.860	364.939
	7		359				312.860	364.939
3	1	3476	420	3763	287	500	496.571	496.571
	2		558				496.571	496.571
	3		613				496.571	496.571
	4		577				496.571	496.571
	5		540				496.571	496.571
	6		516				496.571	457.143
	7		539				496.571	457.143
4	1	3200	484	3517	317	817	457.143	457.143

4	2		517				457.143	457.143
	3		528				457.143	457.143
	4		494				457.143	457.143
	5		489				457.143	457.143
	6		485				457.143	441.714
	7		520				457.143	441.714
5	1	3092	553	3272	180	997	441.714	473.592
	2		577				441.714	473.592
	3		26				441.714	473.592
	4		465				441.714	473.592
	5		609				441.714	473.592
	6	*	565				441.714	473.592
	7		477				441.714	473.592
6	1	3873	423	3857	-16	981	553.286	553.286
	2		553				553.286	553.286
	3		529				553.286	553.286
	4		464				553.286	553.286
	5		525				553.286	553.286
	6		667				553.286	376.571
	7		696				553.286	376.571
7	1	2636	357	2610	-26	955	376.571	409.918
	2	*	383				376.571	409.918
	3	*	368				376.571	409.918
	4	*	461				376.571	409.918
	5	*	389				376.571	409.918
	6	*	350				376.571	409.918

7	7	*	302				376.571	409.918
8	1	3453	334	2522	-931	24	493.286	498.184
	2	*	358				493.286	498.184
	3	*	346				493.286	498.184
	4	*	328				493.286	498.184
	5	*	428				493.286	498.184
	6	*	354				493.286	498.184
	7	*	374				493.286	498.184
9	1	3573	460	3871	298	322	510.429	510.429
	2	*	545				510.429	510.429
	3	*	665				510.429	510.429
	4	*	642				510.429	510.429
	5	*	655				510.429	510.429
	6	*	461				510.429	470.285
	7	*	443				510.429	470.285
10	1	3292	354	3259	-33	289	470.286	504.163
	2		442				470.286	504.163
	3		650				470.286	504.163
	4		586				470.286	504.163
	5		229				470.286	504.163
	6		527				470.286	504.163
	7		471				470.286	504.163
11	1	4122	474	3882	-240	49	588.857	588.857
	2		513				588.857	588.857
	3		511				588.857	588.857
	4		459				588.857	588.857

11	5		548				588.857	588.857
	6		696				588.857	490.428
	7	*	681				588.857	490.428
12	1	3433	640	4367	934	983	490.429	508.265
	2	*	655				490.429	508.265
	3	*	600				490.429	508.265
	4	*	636				490.429	508.265
	5	*	546				490.429	508.265
	6	*	648				490.429	508.265
	7	*	642				490.429	508.265
13	1	3870	620	3884	14	997		
	2		675					
	3		549					
	4		390					
	5		374					
	6		623					
	7		653				*	*

APPENDIX A.7 - POST-OPTIMALITY ANALYSIS FOR CASE "THIS11"

MINIMUM solution basis

x(3)	'SHIP1 '	=	855.428
x(26)	'slk:LOWLIMIT'	=	98.4287
x(27)	'slk:LOWLIMIT'	=	98.4287
x(28)	'slk:LOWLIMIT'	=	98.4287
x(29)	'slk:LOWLIMIT'	=	98.4287
x(17)	'PROP7 '	=	490.428
x(2)	'L '	=	490.428
x(16)	'PROP6 '	=	490.428
x(10)	'PROP1 '	=	588.857
x(4)	'SHIP2 '	=	1300.00
x(5)	'SHIP3 '	=	588.857
x(6)	'SHIP4 '	=	588.857
x(30)	'slk:LOWLIMIT'	=	98.4287
x(38)	'slk:HIGHLIM6'	=	98.4287
x(39)	'slk:HIGHLIM7'	=	98.4287
x(1)	'U '	=	588.857
x(12)	'PROP2 '	=	588.857
x(13)	'PROP3 '	=	588.857
x(43)	'slk:MAXSHIP3'	=	711.143
x(44)	'slk:MAXSHIP4'	=	711.143

x(45)	'slk:MAXSHIP5'	=	711.143
x(8)	'SHIP6 '	=	100.000
x(9)	'SHIP7 '	=	100.000
x(18)	'CLINP0 '	=	977.714
x(11)	'CLINP1 '	=	711.143
x(40)	'slk:MAXDAYPR'	=	243.143
x(14)	'PROP4 '	=	588.857
x(41)	'slk:MAXSHIP1'	=	444.572
x(7)	'SHIP5 '	=	588.857
x(23)	'CLINP6 '	=	390.428
x(48)	'slk:MAXSTGE1'	=	288.857
x(49)	'slk:MAXSTGE2'	=	1000.00
x(50)	'slk:MAXSTGE3'	=	1000.00
x(51)	'slk:MAXSTGE4'	=	1000.00
x(52)	'slk:MAXSTGE5'	=	1000.00
x(53)	'slk:MAXSTGE6'	=	609.571
x(54)	'slk:MASTGE7 '	=	219.143
x(15)	'PROP5 '	=	588.857
x(24)	'CLINP7 '	=	780.857
x(25)	'TOTALSALES '	=	4122.00

MINIMUM value of the function 'PRODNDELTA ' = 98.4287

=== SHADOW PRICE ANALYSIS ===

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= CONSTRAINING ROWS =

NUMBER	ROW	SHADOW PRICE	LOWER LIMIT		UPPER LIMIT	
1	WEEKSALES	0.200000	-492.143	UNITS	1111.43	UNITS
7	LOWLIMIT6	-.500000	-196.857	UNITS	196.857	UNITS
8	LOWLIMIT7	-.500000	-196.857	UNITS	196.857	UNITS
9	HIGHLIM1	-.200000	-123.036	UNITS	492.143	UNITS
10	HIGHLIM2	-.200000	-123.036	UNITS	492.143	UNITS
11	HIGHLIM3	-.200000	-123.036	UNITS	492.143	UNITS
12	HIGHLIM4	-.200000	-123.036	UNITS	492.143	UNITS
13	HIGHLIM5	-.200000	-123.036	UNITS	492.144	UNITS
22	MAXSHIP6	-.700000	0.0	UNITS	240.612	UNITS
23	MAXSHIP7	-.700000	0.0	UNITS	240.612	UNITS
24	INVBAL1	0.200000	-492.143	UNITS	1215.71	UNITS
25	INVBAL2	0.200000	-492.143	UNITS	361.071	UNITS
26	INVBAL3	0.200000	-492.143	UNITS	736.071	UNITS
27	INVBAL4	0.200000	-492.143	UNITS	736.071	UNITS
28	INVBAL5	0.200000	-492.143	UNITS	736.071	UNITS
29	INVBAL6	-.500000	-980.857	UNITS	196.857	UNITS
30	INVBAL7	-.500000	-780.857	UNITS	196.857	UNITS
38	REQUIREDSHIP	0.200000	3629.86	UNITS	5233.43	UNITS
39	closinv7s	-.500000	0.170530E-12	UNITS	977.714	UNITS
40	CLINPOFIX	-.200000	0.0	UNITS	1469.86	UNITS

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= NON-CONSTRAINING ROWS =

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NUMBER	ROW	SHADOW PRICE	LOWER LIMIT		UPPER LIMIT	
2	LOWLIMIT1	-.0	-98.4287	UNITS	0.100000E+08	UNITS
3	LOWLIMIT2	-.0	-98.4287	UNITS	0.100000E+08	UNITS
4	LOWLIMIT3	-.0	-98.4287	UNITS	0.100000E+08	UNITS
5	LOWLIMIT4	-.0	-98.4287	UNITS	0.100000E+08	UNITS
6	LOWLIMIT5	-.0	-98.4287	UNITS	0.100000E+08	UNITS
14	HIGHLIM6	-.0	-98.4287	UNITS	0.100000E+08	UNITS
15	HIGHLIM7	-.0	-98.4287	UNITS	0.100000E+08	UNITS
16	MAXDAYPROD	-.0	588.857	UNITS	0.100008E+08	UNITS
17	MAXSHIP1	-.0	855.428	UNITS	0.100013E+08	UNITS
18	MAXSHIP2	-.0	855.428	UNITS	1588.86	UNITS
19	MAXSHIP3	-.0	588.857	UNITS	0.100013E+08	UNITS
20	MAXSHIP4	-.0	588.857	UNITS	0.100013E+08	UNITS
21	MAXSHIP5	-.0	588.857	UNITS	0.100013E+08	UNITS
31	MAXSTGE1	-.0	711.143	UNITS	0.100010E+08	UNITS
32	MAXSTGE2	-.0	0.0	UNITS	0.100010E+08	UNITS
33	MAXSTGE3	-.0	0.0	UNITS	0.100010E+08	UNITS
34	MAXSTGE4	-.0	0.0	UNITS	0.100010E+08	UNITS
35	MAXSTGE5	-.0	0.170530E-12	UNITS	0.100010E+08	UNITS
36	MAXSTGE6	-.0	390.429	UNITS	0.100010E+08	UNITS
37	MASTGE7	-.0	780.857	UNITS	0.100010E+08	UNITS

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=== COST ANALYSIS ===

=== FOR NON-BASIS VARIABLES ===

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NON-BASIC	VARIABLE	OBJECTIVE CHANGE (VALUE)	VALID UP TO	
x(19)	'CLINP2 '	-0.0	288.857	units
x(20)	'CLINP3 '	-0.0	588.857	units
x(21)	'CLINP4 '	-0.0	588.857	units
x(22)	'CLINP5 '	-0.700000	736.071	units

=== COST RANGING ON BASIC VARIABLES ===

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= EXPLICIT VARIABLES =

BASIS VARIABLE		BASIS PRICE	UPPER PRICE	LOWER PRICE
x(3)	'SHIP1 '	0.0	-0.0	-0.0
x(17)	'PROP7 '	0.0	1.00000	-1.00000
x(2)	'L '	-1.00000	-0.0	-0.100000E+08
x(16)	'PROP6 '	0.0	1.00000	-1.00000
x(10)	'PROP1 '	0.0	0.250000	-1.00000
x(4)	'SHIP2 '	0.0	-0.0	-0.100000E+08
x(5)	'SHIP3 '	0.0	-0.0	-0.0
x(6)	'SHIP4 '	0.0	-0.0	-0.0
x(1)	'U '	1.00000	0.100000E+08	0.291434E-15
x(12)	'PROP2 '	0.0	0.250000	-1.00000
x(13)	'PROP3 '	0.0	0.250000	-1.00000
x(8)	'SHIP6 '	0.0	0.700000	-0.100000E+08
x(9)	'SHIP7 '	0.0	0.700000	-0.100000E+08
x(18)	'CLINP0 '	0.0	0.100000E+08	-0.100000E+08
x(11)	'CLINP1 '	0.0	-0.0	-0.0
x(14)	'PROP4 '	0.0	0.250000	-1.00000
x(7)	'SHIP5 '	0.0	0.250000	-0.0
x(23)	'CLINP6 '	0.0	1.00000	-1.00000
x(15)	'PROP5 '	0.0	0.250000	-1.00000
x(24)	'CLINP7 '	0.0	0.100000E+08	-0.100000E+08
x(25)	'TOTALSALE'	0.0	0.100000E+08	-0.100000E+08

= AUGMENTED VARIABLES =

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BASIS VARIABLE		BASIS PRICE	UPPER PRICE	LOWER PRICE
x(26)	'slk:LOWLI'	0.0	0.250000	-1.00000
x(27)	'slk:LOWLI'	0.0	0.250000	-1.00000
x(28)	'slk:LOWLI'	0.0	0.250000	-1.00000
x(29)	'slk:LOWLI'	0.0	0.250000	-1.00000
x(30)	'slk:LOWLI'	0.0	0.250000	-1.00000
x(38)	'slk:HIGHL'	0.0	1.00000	-1.00000
x(39)	'slk:HIGHL'	0.0	1.00000	-1.00000
x(43)	'slk:MAXSH'	0.0	-0.0	-0.0
x(44)	'slk:MAXSH'	0.0	-0.0	-0.0
x(45)	'slk:MAXSH'	0.0	-0.0	-0.250000
x(40)	'slk:MAXDA'	0.0	1.00000	-0.100000E+08
x(41)	'slk:MAXSH'	0.0	-0.0	-0.0
x(48)	'slk:MAXST'	0.0	-0.0	-0.0
x(49)	'slk:MAXST'	0.0	-0.0	-0.100000E+08
x(50)	'slk:MAXST'	0.0	-0.0	-0.100000E+08
x(51)	'slk:MAXST'	0.0	-0.0	-0.100000E+08
x(52)	'slk:MAXST'	0.0	0.700000	-0.100000E+08
x(53)	'slk:MAXST'	0.0	1.00000	-1.00000
x(54)	'slk:MASTG'	0.0	0.100000E+08	-0.100000E+08

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=== QUANTITY RANGING ON BASIC VARIABLES ===

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= EXPLICIT VARIABLES =

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BASIS VARIABLE	BASIS QUANTITY	OBJECTIVE CHANGE (VALUE)	LOWER LIMIT (UNITS)	OBJECTIVE CHANGE (VALUE)	UPPER LIMIT (UNITS)
x(3)	855.43	0.0	566.57	-0.0	1300.0
x(17)	490.43	1.0000	392.00	1.0000	588.86
x(2)	490.43	1.0000	392.00	0.10000E+08	490.43
x(16)	490.43	1.0000	392.00	1.0000	588.86
x(10)	588.86	0.25000	490.43	1.0000	613.46
x(4)	1300.0	0.0	855.43	0.10000E+08	1300.0
x(5)	588.86	0.0	0.0	-0.0	877.71
x(6)	588.86	0.0	0.0	-0.0	1177.7
x(1)	588.86	0.10000E+08	588.86	1.0000	613.46
x(12)	588.86	0.25000	490.43	1.0000	613.46
x(13)	588.86	0.25000	490.43	1.0000	613.46
x(8)	100.00	0.70000	0.0	0.10000E+08	100.00
x(9)	100.00	0.70000	0.0	0.10000E+08	100.00
x(18)	977.71	0.10000E+08	977.71	0.10000E+08	977.71

x(11)	711.14	0.0	266.57	-0.0	1000.0
x(14)	588.86	0.25000	490.43	1.0000	613.46
x(7)	588.86	0.25000	490.43	-0.0	1177.7
x(23)	390.43	1.0000	292.00	1.0000	488.86
x(15)	588.86	0.25000	490.43	1.0000	613.46
x(24)	780.86	0.10000E+08	780.86	0.10000E+08	780.86
x(25)	4122.0	0.10000E+08	4122.0	0.10000E+08	4122.0

= AUGMENTED VARIABLES =

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BASIS VARIABLE	BASIS QUANTITY	OBJECTIVE CHANGE (VALUE)	LOWER LIMIT (UNITS)	OBJECTIVE CHANGE (VALUE)	UPPER LIMIT (UNITS)
x(26)	98.429	0.25000	0.0	1.0000	613.68
x(27)	98.429	0.25000	0.0	1.0000	613.68
x(28)	98.429	0.25000	0.0	1.0000	613.68
x(29)	98.429	0.25000	0.0	1.0000	613.68
x(30)	98.429	0.25000	0.0	1.0000	123.04
x(38)	98.429	1.0000	0.0	1.0000	613.68
x(39)	98.429	1.0000	0.0	1.0000	613.68
x(43)	711.14	0.0	422.29	-0.0	1300.0
x(44)	711.14	0.0	122.29	-0.0	1300.0
x(45)	711.14	0.0	122.29	0.25000	809.57
x(40)	243.14	1.0000	218.54	0.10000E+08	243.14
x(41)	444.57	0.0	0.0	-0.0	733.43
x(48)	288.86	0.0	0.0	-0.0	733.43
x(49)	1000.0	0.0	711.14	0.10000E+08	1000.0
x(50)	1000.0	0.0	411.14	0.10000E+08	1000.0
x(51)	1000.0	0.0	411.14	0.10000E+08	1000.0
x(52)	1000.0	0.70000	263.93	0.10000E+08	1000.0
x(53)	609.57	1.0000	511.14	1.0000	708.00
x(54)	219.14	0.10000E+08	219.14	0.10000E+08	219.14