FORECASTING CANADIAN MONEY SUPPLY GROWTH USING UNIVARIATE ARIMA AND MULTIVARIATE VECTOR ARMA MODELS

by

Sam H.K. Tang

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APPROVAL

Name: Sam H.K. Tang

Degree: M.A. (Economics)

Title of Project: Forecasting Canadian Money Supply Growth Using Univariate ARIMA and Multivariate Vector ARMA Models

Examining Committee:

Chairman: C. Reed

Richard Holmes
Professor
Senior Supervisor

John Chant
Professor

Drew Parker
Assistant Professor
Business Administration
External Examiner

Date Approved: 11 August 1989

ABSTRACT

The money supply growth rate is a vital financial variable in any economy. Given its importance, a relevant question is whether or not money supply growth rate can be adequately predicted by professionals outside the Bank of Canada. The objective of this paper is to deal with this question in the Canadian context. We examine how univariate ARIMA and multivariate vector ARMA models can be applied to forecasting the Canadian money supply growth rate. An attempt is made to assess the relative efficacy of our two models' predictive performances. In addition, the roles of the unemployment rate, inflation rate, nominal interest rate, and exchange rate in the determination of the money supply by the Bank of Canada are examined in application of the vector ARMA model.

In constructing our univariate ARIMA and multivariate vector ARMA models, an iterative three-stage procedure which includes model identification, model estimation and diagnostic checks on model adequacy, is used. Univariate ARIMA models rely only upon a series' past statistical characteristics to infer its future movements. Because only one time series is involved in univariate ARIMA models, the problem of finding explanatory variables in conventional regression analysis can be avoided. Multivariate vector ARMA models, which can be viewed as a set of reduced form equations associated with a simultaneous system of structural linear equations, allow whatever causal relationships that exists among the variables under study to emerge from the data; no *a priori* directional or causal constraints are imposed on the relationships. Hence, the problem involving choice of appropriate lag structures in conventional regression analysis can be avoided.

The results show that our chosen multivariate vector ARMA model generates more accurate long term forecasts than the univariate ARIMA models. Also, the results from the vector ARMA model reflect high correlation of the money supply growth rate with the nominal interest rate, exchange rate and inflation rate. The unemployment rate, however, is not significantly correlated with the money supply growth rate. This result suggests that the Bank of Canada had not paid much attention to unemployment in its management of the money supply during the period of analysis. Instead, the concern of the Bank has been to manage the money supply to control inflation.

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DEDICATION

To my parents and Alfee

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CHAPTER I

INTRODUCTION

The growth rate of the money supply plays an important role in an economy. Mismanagement of the money supply can destabilize the economy in the short-term and may cause long lasting problems. We will examine, from a theoretical perspective, how the money supply influences the economy in this introductory section.

There is a widely accepted belief among economists today that monetary expansion has, at most, minimal long-term effects on real economic variables such as the growth rate of real output and the unemployment rate.[1] In other words, money is basically neutral in influencing real output and the unemployment rate in the long-term.[2] Real economic variables are determined by a country's capacity to produce and its competitiveness in the international market which, in turn, depends ultimately on such factors as technological progress,

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[1] Two foremost proponents of this view are Irving Fisher and Milton Friedman. See Irving Fisher, *Stabilizing the Dollar* (New York: Macmillan, 1920), pp.10-11 and 29; Milton Friedman, "Money: The Quantity Theory," in *The International Encyclopedia of the Social Sciences*, Vol. X, 1968, pp.432-447. Also see general discussions by Peter Howitt, *Monetary Policy in Transition- A Study of the Bank of Canada Policy, 1982-85* (Scarborough, Ontario: C.D. Howe Institute Publication, 1986), p.13; R. Dornbusch, S. Fischer and G. Sparks, *Macroeconomics*, 2nd ed. (Toronto: McGraw-Hill Ryerson, 1985), pp.428-432.

[2] Neutrality of Money refers to the *level* of money stock having no long term real effects whereas Superneutrality of money refers to the *rate of growth* of the money stock having no long-term real effects.

natural resource endowments, and a country's international terms of trade.

The growth rate of money supply has a long-term effect on the inflation rate. Most economists' views differ only on the degree in which the money supply influences the inflation rate. In Howitt's (1986) studies of the correlation between monetary expansion and inflation in many different countries and many historical circumstances, he concludes:

> "...whenever a country has experienced severe inflation over several years, it also has had a rate of monetary expansion significantly in excess of the rate of growth of real output, on average, for several years."[3]

The long-term relationship between the money supply and inflation can be explained by the "Quantity Theory of Money"[4] which asserts that an excess supply of money is the most important cause of inflation. In Friedman's (1968) view, a country's demand for real balances, the amount of nominal money demanded divided by the price level, grows approximately at the same rate as its growth rate of real output. Thus, a growth rate of the money stock in excess of the growth rate of real output would lead to excess money supply and eventually a higher inflation rate.

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[3] Peter Howitt (1986), p.12.

[4] One may want to distinguish the Modern Quantity Theory of Money from the Classical Quantity Theory of Money. The Classical Quantity Theory of Money asserts that the price level is proportional to the stock of money whereas the Modern Quantity Theory of Money, proposed by Friedman and others, argued that the money stock is the single most important variable producing inflation. See footnote 1.

However, this conclusion follows only if the demand for real balances has a stable relationship with its major determinants — real output and the nominal rate of interest.[5] If the banking technology changes in a way that allows people to demand fewer real balances while real output is constant, the central bank may not be able to avoid inflation even if it sets the growth rate of the money supply in line with the growth rate of real output. The message from the above discussion is clear: the growth rate of the money supply has a long-term effect on inflation and the extent of this effect can be predicted if the demand for real balances is stable.

Although no real economic variables are affected by monetary policy in the long-term, monetary expansions or contractions can have short-term effects on real output and the unemployment rate. Suppose that the economy is at a full-employment level[6] of output and that the policy of the Bank of Canada is to reduce the inflation rate by using a contractionary monetary policy. The nominal money stock decreases and real balances fall at every price level. People tend to hold fewer short-term liquid

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[5] See Martin F. J. Prachowny, *Money in the Macroeconomy*, (Cambridge: Cambridge University Press, 1985), pp.125-153. Also an excellent summary of the evidence is contained in David E. W. Laidler, *The Demand for Money, 3rd ed. (New York: Harper* & *Row), 1985.*

[6] The concept of full-employment is problematic. In the 1960's it was believed that full-employment corresponds to a measured rate of unemployment of 4 percent of the labour force. Changes in the labour force toward younger and female workers who frequently change jobs increased the full-employment rate of unemployment to 6 percent in the 1970's. See Rudiger Dornbusch, Stanley Fisher, and Gordon Sparks (1985), p.10.

assets as they try to make up their cash shortages. Interest rates rise as a consequence and to the extent that expectations of inflation are unlikely to change in the short-term, real interest rates rise as well. The level of real interest rates represents the expected real cost of borrowing and the higher is this expected cost, the slower the growth of aggregate spending. As the growth of aggregate spending declines, firms start to cut back on production. They may cut back on hiring and even lay off workers. Thus, in the short-term, monetary contractions will lead to a decrease in real output and an increase in the unemployment rate. The price level also falls as result of the decrease in the labour costs as production and employment fall. In the long-term, firms will take advantage of the lower costs of production and expand output. Output and employment will return to their original full-employment levels along with a lower price level.

However, if wages are not flexible, the economy may not return to its full employment equilibrium. Wage earners may doubt the credibility of the Bank's intention to fight inflation, perceiving the anti-inflation policy as designed to lower their real wages. Under this circumstance, they will continue to bargain for higher nominal wages based on current wages. Thus, the economy will not return to its full employment level and output will remain at less than it could be at full employment.[7]

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[7] One of the central arguments between the monetarists and non-monetarists is the flexibility of wages or prices in

The short-term effects of monetary policy are strengthened through international capital movements and trade. For instance, a policy of monetary restraint which raises domestic interest rates above those of other countries, drives up the value of domestic currency by causing foreign capital in-flows, and causes domestic borrowers to seek financing outside the country. Under a flexible exchange rate policy, the domestic currency will appreciate. Exports decline and imports increase. Once again, with this worsening of the balance of trade, real output and employment growth will slow down.

In this introductory section, we have seen, from the theoretical perspective, the long-term and short-term effects of the money supply on four major economic variables. Traditionally, the Bank of Canada has been actively intervening in financial and foreign exchange markets out of fear that large fluctuations in interest rates and exchange rates could destabilize the economy.[8] A clear example of these interventions, as will be seen in the next chapter, is the fixed

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[7] (cont'd) general. In monetarists' view, prices may not be completely flexible in the short-term, but there is no doubt that prices will become more flexible the longer the adjustment period. The non-monetarists stance is that prices are relatively inflexible downwards. The time required for price adjustment to push the economy back to the full employment output level will be intolerably long. Thus, active counter cyclical policies are needed to shorten the process. On the other hand, monetarists assert that even prices are not completely flexible and private planning spending is not completely stable, so that active counter cyclical policies will do more harm than good to the economy. A clear summary of this argument is given by Robert J. Gordon, *Macroeconomics*, 2nd ed.(Boston: Little, Brown and Company, 1981), pp.364-369.

[8] Peter Howitt (1986), p18.

exchange rate policy during the 1960's and, even continuous heavy intervention in the foreign exchange market under the supposedly flexible exchange rate policy after 1970. Furthermore, the Bank of Canada has been preoccupied with maintaining "orderly" financial markets during much of its history by pegging the normal interest rates within a certain range. In order to offset fluctuations in the interest rates beyond the acceptable range, the Bank has been "leaning against the wind" — increasing monetary expansion when the interest rates are too high and decreasing monetary expansion when the interest rates are too low.[9]

The structure of this paper is as follows. Chapter II presents a brief history of Canadian monetary policy since the 1940's. Chapter III looks at the the model building technique which underlies univariate ARIMA and Multivariate vector ARMA models. Chapter IV presents empirical results from estimations of the chosen models. Finally, conclusions and summaries will be made in Chapter V.

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[9] It is clear that the Bank of Canada can not simultaneously set separate goals for the growth rate of the money supply and the nominal rates of interest. The level of interest rates are determined by the growth rate of the money supply. The Bank of Canada can have a choice of either pegging the interest rates at a certain level by adjusting the growth rate of the money supply or fixing the growth rate of the money supply by letting the interest rates to be residually determined, as it had been the case under the target growth rates of M1 policy during 1975 to 1982. See a detailed discussion of monetary control techniques in Canada from 1975 to 1982 contained in Prachowny (1985), pp.194-200.

CHAPTER II

A BRIEF SUMMARY OF CANADIAN MONETARY POLICY SINCE 1940

In the early post-war years, much of the Bank of Canada's attention was directed to keeping bond prices from collapsing, as they had after the First World War. In addition, because of the recent memory of the Great Depression in the 1930's, the government was willing to pursue expansionary fiscal and monetary policies to avoid economic downturns. Indeed, there was a general fear that recession could be forthcoming once the production of arms had been reduced.[1]

After a brief period of recession in 1949, the economy started to expand rapidly due to the increase in demand for natural resources in the United States. The outbreak of the Korean War in June 1950 provided further stimulus. Inflationary pressure was mounting. The Bank of Canada recognized that pegging interest rates and anti-inflationary monetary policies were clearly inconsistent with each other. Rigidly pegging interest rates was abandoned early in 1948. In 1950, the bank rate was raised for the first time in the post-war era, at the same time as the floating exchange rate was implemented.

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[1] Our discussion in this chapter heavily relied on two sources. First, Gordon Sparks, who presents a concise overview of the postwar conduct of monetary policy. See "The Theory and Practice of Monetary Policy in Canada" in John Sargent, research coordinator, *Fiscal and Monetary Policy*, Collected Research Studies of the Royal Commission on the Economic Union and Development Prospects for Canada no.21 (Toronto: University of Toronto Press, 1986) pp.119-149. Second, Peter Howitt provides a thorough analysis of the more recent events from 1982 to 1986. See Peter Howitt (1986), pp.91 to 107.

The Bank pursued a restrictive monetary policy throughout much of the 1950's. After a brief period of recession in the early 1950's, the economy was booming again in the mid-1950's. The unemployment rate declined substantially as the growth of GNP rose sharply. This was largely due to an increase in world demand for natural resources. Canada's attractive investment opportunities also stimulated a large influx of foreign capital which appreciated the Canadian dollar substantially. The Canadian dollar remained well above 100 U.S. cents. During 1955 and 1956, the Bank adopted a restrictive monetary policy, and interest rates were allowed to rise accordingly.

The booming economy reversed itself by 1957. The unemployment rate rose by almost three percent. However, the Bank continued to adopt a restrictive monetary policy in the face of a weakening economy. The Bank was more concerned with inflation and the balance of payments deficit than with high unemployment. Unfortunately, the restrictive monetary policy only increased the balance of payments deficit and generated an even higher unemployment rate.

The 1960's was characterized by a move to a less restrictive monetary policy and a fixed exchange rate regime. The exchange rate was fixed at 92.5 U.S. cents in May 1962. The immediate effect of the fixed exchange rate policy was a continuous increase in official foreign currency reserves which reflected an undervaluation of the Canadian dollar. The fixed exchange rate policy also led to a subsequent attempt by the United States to improve its balance of payments by restricting capital outflows to Canada.

The inflationary pressures were building up in the United States in the latter half of 1960's. They were largely due to expenditures relating to the Vietnam War. Canada was not able to stop the inflationary spill-over effects. Consequently, the increase in Canada's merchandise trade surplus put more upward pressure on the Canadian dollar. The money supply, as measured by M1, grew nearly ten percent in 1967; this was twice as much as the growth rate of M1 during the 1961-1964 period. Clearly, the Bank of Canada's ability to resist inflationary pressures from abroad was greatly limited by maintaining a fixed exchange rate policy.

The fixed exchange rate policy was finally abandoned in 1970. The growth rate of M1 was at about 2 percent per annum in 1970, the lowest level since the 1960's. The freeing of the fixed exchange rate made many analysts believe that as the Bank of Canada did not need to worry about the exchange rate, it would now commit itself to fight inflation. However, the Bank continued its heavy intervention in the foreign exchange market to resist a large appreciation of the Canadian dollar. In addition, the economic downturn in 1970 raised unemployment rates, and the Bank responded by sharply raising the growth rate of M1 to about 13 percent in 1971. In 1973 and 1974, the rate of growth of world commodity prices increased rapidly because of the large increases in the price of oil. The Canadian consumer price index rose by more than ten percent.

In retrospect, it is clear that the Bank did not use proper monetary policy on many occasions in the 1960's and 1970's. First, the fixed exchange rate policy of the late 1960's restrained it from resisting inflation. Even in the 1970's when the exchange rate was freed, the Bank continued to resist the effect of a large appreciation of the Canadian dollar on the competitiveness of the export and import-competing industries. Second, it over-reacted to the economic downturn in 1970 when it sharply increased the growth rate of M1. As was seen earlier, in the discussion of theory, the short-term effect of monetary expansion can result in a depreciation of the Canadian dollar and an increased levels of real output and employment. However, in the long-term, monetary expansion will intensify inflationary pressures. Thus, by the time of the OPEC oil price rise in 1973 and 1974, inflation was already deeply entrenched in Canada.

The Bank of Canada finally realized fully the adverse effects of monetary expansion on inflation and adopted a policy of target growth rates for narrowly defined money, M1, in 1975. This policy of *gradualism* was supposed to check inflation by restricting the growth rate of M1. The growth rate of M1 was initially set at 10 to 15 percent in late 1975 and successively lowered to 4 to 5 percent in 1980. In the course of maintaining the monetary targets, the Bank chose to use the nominal interest rate as a policy instrument. In other words, it was manipulating the demand for money on a daily basis through open market operations, such as the purchase of treasury bills, so that the annual monetary targets could be met.

Gradualism was finally abandoned in November 1982 when the Bank of Canada came under heavy attack from the economists.[2] First, *gradualism* did not effectively control inflation. Inflation rose to about 14 percent in 1981. Many argued that *gradualism* had worked too slowly to have any significant effect on reducing inflation. Second, some people contend that the effects of aggregate supply shocks caused by the oil price rises in 1973-74 and 1978-79 could not be absorbed by monetary restriction alone. They argued that the domestic price level and the unemployment rate must increase to reflect the higher price of inputs. Monetary restriction would only cause even higher unemployment in the non-resource sectors. Third, many argued that the Bank's choice of M1 as a target was insufficient to cure inflation. Banking innovations in the early 1980's had made M1 inaccurate and imprecise as a measure of the money supply aggregate. As Howitt (1986) emphasized, "the main flaw of Gradualism was its concentration on a narrow concept of the money supply, M1, rather than on a broader definition of money."[3]

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[2] One of the major opponents of the money target growth rate policy is Courchene. See T. Courchene, *Money, Inflation, and the Bank of Canada*, Vol. 11(Montreal: C.D. Howe Institue, 1981), pp.157-183.

[3] Peter Howitt (1986), p.4.

The Bank of Canada abandoned a policy of gradually restraining the money supply for a more restrictive policy in 1981. The growth rate of M1 was set at a rate lower than the previously announced target rate (M1 almost had no growth at all in 1981). It was perceived that, with inflation at intolerably high levels, reducing it would be costly, but the price would be worth it. The effect of such policies were to result in the recession of 1981-1982. Inflation came down to about 5-6 percent in 1982. Since 1982, monetary contraction continued to intensify and many blamed this highly restrictive policy to be responsible for the slow recovery of the economy from the 1981 recession.[4]

From 1985, we have seen a reversal of the restrictive monetary policy. Monetary expansion revived the economy without inflationary consequences because the Canadian economy was still operating at less than full employment. The unemployment rate in the fourth quarter of 1985 was 10.2 percent which was still higher than the estimated 6 to 7 percent of the full-employment rate of unemployment.[5] In the face of under utilization of capacity and resources, the Bank began to switch its attention to eliminating the recessionary gap. The Bank's governor stated as early as 1983:

> "The appropriate objective of monetary policy in present circumstances is a rate of monetary expansion sufficient to accommodate increasing utilization of our economic

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[4] Gordon Sparks criticized the Bank for putting too much weight on the inflationary consequences of a depreciation of the Canadian dollar when high unemployment rates in Canada were persisting. See Gordon R. Sparks (1986), p.147.

[5] See footnote 6 in Chapter 1.

> resources in a context of increasing price stability. That is what the Bank of Canada has been trying to achieve, and in my judgement the rate of monetary expansion this year has been consistent with that objective. The Bank will continue to pursue the same objective."[6]

In concluding this Chapter, one should note that monetary policy switched its goals on many occasions in the 1970's and early 1980's. The policy goal in the late 1970's was to contain inflation by gradually reducing the money supply growth rate. The interest rate was used as a policy instrument to keep the money supply growth rate of M1 on target in the short-term. From 1979 to 1982, the exchange rate was a major consideration in monetary policy decisions. The increasing volatility of U.S. interest rates and the perceived inflationary consequences of a depreication both contributed to the Bank's commitment to stabilize the exchange rate. It was not until early 1985 that the Bank slowly gave way to a less restrictive monetary policy, thereby closing up the recessionary gap that had crippled the Canadian economy since 1981.

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[6] Bouey, Notes for remarks, November 29, 1983, p.20.

CHAPTER III

TWO MODELS OF THE MONEY SUPPLY PROCESS

Univariate ARIMA Models

In this section, we will examine briefly how to construct an appropriate univariate ARIMA model to forecast Canada's money growth rate series, M1. At the outset, it is important to note that univariate ARIMA models attempt to capture the inherent correlation between observations within a single time series.[1] Thus, no other time series are required to be used as explanatory or input variables in univariate ARIMA models. Univariate ARIMA models are series-specific models. A univariate ARIMA model selected as an appropriate representation of a time series based on one estimation period may not be applicable to other time periods of the same time series. Also, caution must be taken before accepting the long-term forecasts generated by univariate ARIMA models. This is because univariate ARIMA models are incapable of capturing structural changes, which influence the movement of the time series in question, from other sources.

The well known Box-Jenkins three step iterative method is followed in choosing an appropriate model from a whole family of univariate ARIMA models. The identification stage begins with a comparison between the estimated autocorrelation function

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[1] The standard reference for univariate ARIMA models is Box and Jenkins (1970), and Pankratz (1983). Also, Makridakis, Wheelwright and Mcgee (1983) provides a systematic discussion of time series models.

diagram (ACF), estimated partial autocorrelation function diagram (PACF), generated from the time series in question, and the various theoretical ACF and PACF generated from pure autoregressive (AR), or pure moving average (MA) or mixed autoregressive moving average (ARIMA) series. One chooses ARIMA models which best resemble the pattern of autocorrelation in the series under study. It should be noted that due to exogenous effects the estimated ACF and PACF do not always exhibit well identifiable patterns. Thus, great care must be taken at the identification stage and the chosen models should be subject to diagnostic checking after estimation. The general multiplicative form of the univariate ARIMA model is:

(1) $$\Gamma(B)\ \Phi(B)\ (1-B^{s})^{D}\ (1-B)^{d}\tilde{Z}_{t} = \Delta(B)\ \Theta(B)a_{t}$$

Where,

- $\Gamma(B)$, $\Phi(B)$, $\Delta(B)$, and $\Theta(B)$ are polynomial functions of the backward shift operator B (*eg*, $(1-BZ_t) = Z_t - Z_{t-1}$) and denote seasonal AR, non-seasonal AR, seasonal MA and non-seasonal MA respectively.
- $(1-B^{s})^{D}$ is the seasonal differencing operator where D denotes the order of seasonal differencing and s denotes the time span of the seasonal cycle.
- $(1-B)^{d}$ is the non-seasonal differencing operator of d order of differencing.
- $\tilde{Z}_t$ is the time series under study and a_t is the white noise error term.

Furthermore,

$$\Gamma(B) = (1 - \gamma_1 B^s - \gamma_2 B^{2s} - \ldots - \gamma_p B^{ps})$$

$$\Phi(B) = (1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p)$$

$$\Delta(B) = (1 - \delta_1 B^s - \delta_2 B^{2s} - \ldots - \delta_q B^{qs})$$

$$\Theta(B) = (1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q)$$

Any univariate ARIMA models can be fully specified with the following complex notation: ARIMA $(p,d,q)(P,D,Q)_s$, where P and p denote the order of the seasonal and non-seasonal AR component. Q and q denote the order of the seasonal and non-seasonal MA component. The task is to specify the (p,d,q) and the $(P,D,Q)_s$ in the identification stage.

One preliminary, but important, step must be taken in the identification stage. This is to check whether or not the time series under study meets the condition of stationarity, since univariate ARIMA models are only applicable to stationary series. In Pankratz's (1983) words, "A stationary time series has a mean, variance and autocorrelation function that are essentially constant through time."[2] If a time series does not have a stationary mean, seasonal and/or non-seasonal differencing are required to induce stationarity of its mean. Also, if a time series has a variance which changes through time, transformations of the original time series should be

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[2] Alan Pankratz (1983), p.11.

performed to induce stationarity of its variance.[3] The condition of stationarity simplifies the theory underlying univariate ARIMA models and enables us to obtain useful estimates of parameters from a small set of observations. Furthermore, neither the form nor the parameters of an univariate ARIMA model for a <u>stationary</u> series will change over the entire estimation time.

After identifying one or a few appropriate univariate ARIMA models in the identification stage, precise estimates of parameters for the chosen models are derived. The most commonly used is the Non-Linear Square (NLS) estimation technique, known as "Marquardt's compromise", which is generally applicable to all univariate ARIMA models. Briefly, this estimation process, given some initial 'guess' values of coefficients, produces new coefficients which generate small sum of squared residuals (SSR) and these SSR are usually close to the minimum SSR.[4]

The diagnostic checking stage, determines whether or not the chosen models are statistically adequate. First, the model is checked for stationarity and invertibility. The stationarity requirement applies only to the AR component of the model and requires the estimated AR coefficients to satisfy some inequality conditions.[5] The invertibility requirement ensures

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[3] Stationary condition is $|\phi_1| < 1$ for AR(1), $|\phi_2| < 1$, $\phi_2 + \phi_1 < 1$ & $\phi_2 - \phi_1 < 1$ for AR(2).

[4] See Pankratz (1983), pp.192-200.

[5] See footnote 2 in this Chapter.

that smaller weights are assigned to observations which are generated farther back in time. This requirement only applies to the MA component of the model and requires that estimated MA coefficients satisfy some inequality conditions. Second, t-values greater than 2 must be ensured for coefficients in the model. Third, it is necessary to ascertain whether or not the assumption of independence of the white noise residuals has been met. If a model is an adequate representation of a time series, it should capture all the correlation in the series, and the white noise residuals should be independent of each other. Thus, any significant autocorrelation shown in the estimated white noise residuals ($\hat{a}_t$), at the ACF, indicates model inadequacy and suggests how the model should be modified.

The final stage is to obtain forecasts of the time series under study. It is common practice to compare the forecasting performance of chosen models using various statistics such as the mean absolute percentage error (MAPE), mean absolute error (MAE), root-mean square percentage error (RMSPE), and root-mean square error (RMSE). Forecasting procedures are to be discussed, in greater detail, later in this paper.

Vector ARMA Analysis

The choice of employing vector ARMA models in our study of money growth rate series is suggested by a lack of any fully developed theories regarding how the Bank of Canada reacts to the changing market conditions in the management of the money supply. Without solid theoretical argument concerning the structural form of the Bank's reaction function, the conventional econometric (*i.e.* simultaneous equations) approach must involve exceedingly arbitrary specifications. A money growth equation studied by Barro[6](1977) for the United States has two lagged values of money growth — a measure of federal government expenditures relative to normal, and a lagged unemployment rate. The specifications of the lag structure in this equation are quite arbitrary since they have no *a priori* justification. Furthermore, a lagged value of the inflation rate and the interest rate are discarded because of insignificant coefficients of these lagged terms. In sum, the problems of assigning lags and specification of explanatory variables render the use of conventional econometric models inappropriate for this study.

A vector ARMA model can be viewed as a set of reduced form equations associated with a simultaneous system of linear structural equations.[7] One major difference between vector ARMA and conventional econometric models is that only lagged relationships are included in vector ARMA models whereas econometric models can include contemporaneous as well as lagged relationships. Furthermore, vector ARMA models do not impose any

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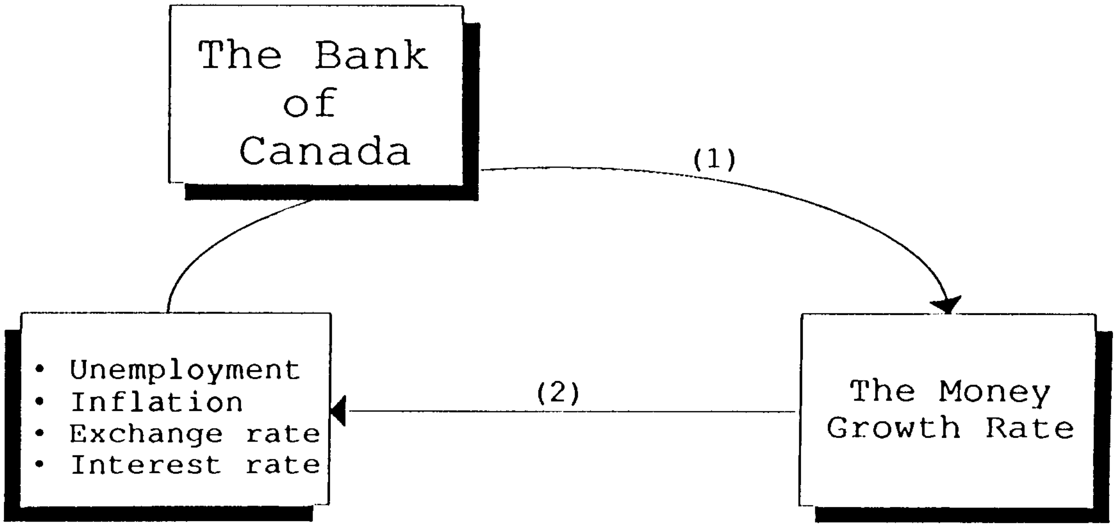
[6] $DM_t = \alpha_0 + \alpha_1 DM_{t-1} + \alpha_2 DM_{t-2} + \alpha_3 FEDV_t + \alpha_4 UN_{t-1}$

[7] The model building methodology of vector ARMA models is discussed in detail in two recent articles. See Lou-Mu Liu (1986), and Gwilym Jenkins and Athar S. Alavi (1981).

a priori constraints on the direction of causal relationships among variables and thus permits the underlying causal relationships to emerge from the data. In this study of the money growth rate, four important variables, based on observations of the Bank's past monetary policy are chosen. These variables are the unemployment rate, the inflation rate, the exchange rate and the interest rate. During the postwar period, the Bank of Canada has actively intervened in financial and foreign exchange markets to manipulate exchange rates and interest rates. Also, as fighting inflation and unemployment are two important, and repeatedly stated policy goals, it is of interest to examine the possible causal relationship that exists among these variables.

The following diagram (Figure I) illustrates a basic causal relationship linking the various variables.

Figure I



The first arrow represents the reaction function of the Bank which attempts to control or affect the specified macroeconomic variables by adjusting the growth rate of money supply. The second arrow represents the feedback effects of the change of the money growth rate on various specified variables.

However, it should be noted, that the above diagram only indicates potential causal relationships. Each input variable may *neither* influence the money growth rate *nor* be influenced by it. The purpose of this study is to empirically establish the exact nature of the link between the money growth rate and the four specified variables.

Specification of Vector ARMA Models

A general representation of a multiplicative seasonal vector ARMA models is;

(2) $\underline{\Gamma}(B)\ \underline{\Phi}(B^{s})\ \underline{\tilde{Z}}_t = \underline{\Delta}(B)\ \underline{\Theta}(B^{s})\ \underline{a}_t$

Where,

$$\underline{\Gamma}(B) = \underline{I} - \underline{\gamma}_1 B - \dots - \underline{\gamma}_p B^{p},$$
$$\underline{\Phi}(B) = \underline{I} - \underline{\phi}_1 B^{s} - \dots - \underline{\phi}_p B^{ps},$$
$$\underline{\Delta}(B) = \underline{I} - \underline{\delta}_1 B - \dots - \underline{\delta}_q B^{q},$$
$$\underline{\Theta}(B) = \underline{I} - \underline{\theta}_1 B^{s} - \dots - \underline{\theta}_q B^{qs}.$$

- $\underline{\Gamma}(B)$ and $\underline{\Delta}(B)$ are non-seasonal matrix polynomials in B where the $\underline{\gamma}$'s and $\underline{\delta}$'s are K * K matrices.
- $\underline{\Phi}(B)$ and $\underline{\Theta}(B)$ are seasonal matrix polynomials in B where

$\underset{\sim}{\phi}$'s and $\underset{\sim}{\theta}$'s are K * K matrices with seasonality s.

- * $\tilde{\underline{Z}}_t$ is a K * 1 dimensional vector series that may have been derived from an original series by transformation and differencing.
- * $\underline{a}_t$ is a sequence of K * 1 random shock vectors identically and independently distributed as a normal distribution with zero mean and covariance matrix $\underline{\Sigma}$.

The stationarity and invertibility conditions applicable to univariate ARIMA models are also required for vector ARMA models. As seen earlier, the stationarity condition ensures that the mean, variance and covariance remain essentially constant over time and the invertibility condition allows any vector ARMA model to expressed as a convergent pure vector AR model.

In our search for an appropriate vector ARMA model, an iterative approach discussed by, Tiao and Box (1981), is employed. This iterative approach is similar to the well known Box-Jenkins iterative approach used for univariate ARIMA models. In the identification stage, statistics calculated from the original data are used to tentatively select a subclass of appropriate vector ARIMA models. Three statistics are of importance here: the sample cross correlation matrices, $\tilde{\underline{\rho}}(l)$, of the original data, the estimates of the partial autoregression matrices obtained by fitting successive autoregressive models of increasing order, and the sample cross correlation matrices of the residual from each fitted autregressive model. The basic idea in the identification stage is that through examining various statistics calculated from the original data, those vector ARMA models which best resemble the pattern of correlation amongst the series are identified.

After identifying some candidate vector ARMA models in the first stage, exact estimates of the parameters for each candidate vector ARMA model are derived. Then, all the estimated vector ARMA models are subjected to diagnostic checks. Inadequate or misspecified models are dropped or modified. Valuable statistics in the diagnostic stage are the residual cross correlation matrices (CCM). These residual cross correlation matrices should present a random white noise process if a vector ARMA model is correctly specified.

After a vector ARMA model passes the diagnostic checking stage, it is subjected to a final estimation which imposes constraints on those insignificant parameters generated by the initial estimation. Then, the final *restricted* model is used to generate forecasts.

CHAPTER IV

EMPIRICAL RESULTS

This section presents the empirical results generated from the univariate ARIMA and multivariate vector ARMA models. Summaries of these results are presented in TABLE 2 through TABLE 6. All estimations are based on monthly observations starting January 1973 and ending January 1989.

Univariate ARIMA Models

The original money growth rate series[1] was found to be non-stationary (*i.e.* it has a changing mean) based on the slow decay pattern shown in the ACF. The series was then differenced and its ACF and PACF were examined. From the correlation pattern shown in the ACF and PACF of the differenced series, three ARIMA models were tentatively selected for further investigation.

Eq.(1.1) $(1-B^{12})dM1_t = (1-\delta_1 B^{12})\, a_t$

Eq.(1.2) $(1-B^{12})dM1_t = (1-\theta_1 B^9 - \theta_2 B^{13})\,(1-\delta_1 B^{12})\, a_t$

Eq.(1.3) $(1-B^{12})dM1_t = (1-\theta_1 B^9)(1-\delta_1 B^{12})\, a_t$

These three models are specific forms of Eq.(1). Note that these three models consist of MA components only. This reflects

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[1] The money supply growth rate, DM1, is the year to year monthly percentage change of M1. M1 is defined as currency outside the banks and demand deposits (Source: CANSIM B2033).

that the money growth rate series is highly correlated at lag 12 and is best modelled as an MA component. The above models were estimated by an iterative non-linear estimation technique. The results of the estimation are presented in TABLE 2. All coefficients have t-values above or close to 2, indicating statistical significance at the 5 percent level. Furthermore, the stationarity and invertibility conditions are satisfied. The residual autocorrelation functions for the above models were examined. The residual autocorrelation functions for Eq(1.2) and Eq.(1.3) do not contain significant spikes in any of their lags, indicating residuals of these two models are independent white noise terms. However, the residual autocorrelation function for Eq(1.1) contains two slightly significant spikes at lag 9 and 13 (t-values are 2.57 and -2.5 respectively). This indicates that the model has not yet captured some correlations in the series at these two lags.

Before deciding which is the best ARIMA model among the three, consideration should be given to the condition of parsimony and, what could be the most important of all, the forecasting performance of these models. The condition of parsimony requires that a low order ARIMA model is preferred to a higher order model if two models are the same in all other aspects. Parsimony suggests MODEL(1.1) should be chosen because it contains the lowest MA order (first seasonal MA order). Thus, it will be chosen as the best model for this money growth series if it can provide more accurate forecasts than MODEL(1.2) and

(1.3). Next the forecasting procedures are considered and the forecasting performances of the chosen models are compared.

Forecasting Procedures

In evaluating the forecasting performances of these models, a series of out-of-sample forecasts were generated for each of the chosen models. These out-of-sample forecasts are *ex post* forecasts because only a subset of the existing data are used for estimation. The accuracy of *ex post* forecasts can be checked by comparing them with the actual values.

In the study of the money growth rate series, there are 193 monthly observations starting January 1973 and ending January 1989. The 12 step-ahead (12 months) forecasts are started in February 1985, based on the initial 145 observations starting January 1973 and ending January 1985. The 12 step-ahead forecasts are out-of-sample forecasts because the actual values of these periods are not included in the estimation of the model which generates these forecasts. The estimation period is expanded successively by one month ahead, and 12 step-ahead forecasts were generated in each new estimation period. In the end, 37 series of 12 step-ahead forecasts are generated.

There are several statistics by which to evaluate the forecasting accuracy of our models. Two of the most popular statistics are the Mean Absolute Error (MAE) and Root-Mean Squared Error (RMSE). Also, a useful statistic related to the

RMSE and applied to *ex post* forecasts is Theil's inequality coefficient (U).[2] In practice, RMSE is more widely used and preferred to MAE since it is more sensitive to large errors than MAE. Theil's inequality coefficient (U) falls within the range 0 to 1. If U is 0, forecasts are equal to the actual values for all time periods, and there is a perfect fit. If U is 1, the forecasting performance of a model is inadequate.

Forecasting results for Univariate ARIMA models

The 37 series of 12 step-ahead forecasts allow evaluation of the forecasting performances of the three chosen univariate ARIMA models for different step-ahead forecasts. MAE, RMSE and U for the three univariate ARIMA models are presented in TABLE 3 through TABLE 5. All statistics increase in size as the number of step-ahead forecasts increases. This indicates that forecasts for the longer time horizon are less accurate than those immediately following the estimation period. This is intuitively sensible as forecasts further in time are based on preceding forecasts which are, obviously, less accurate than the actual values.[3] Thus, it is expected that all of the statistics summarizing the forecasting performances of the models increase as the number of step-ahead forecasts increase, and these results confirm this expectation.

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[2] Robert S. Pindyck & Daniel L. Rubinfeld *Econometric Models and Economic Forecasts*, (McGraw-Hill: New York, 1981), p.364.

[3] Forecasts based on preceding forecasts rather than observed values are called "bootstrap" forecasts.

The MODEL(1.1) has consistently smaller MAE, RMSE and U for most of the step-ahead forecasts. Allowing the conclusion that MODEL(1.1) is the best of the three tentatively chosen models since it provides more accurate forecasts, as well as being the most parsimonious model, despite the fact that it has two slightly significant spikes in the residual ACF.

The MODEL(1.1) can be expressed in difference equation form:

Eq.(1.1.1) $$dM1_t = dM1_{t-12} - \delta_1 a_{t-12} - a_t$$

This equation is equivalent to Eq.(1.1) which is expressed in backshift operator form.

Vector ARMA Model

Chapter II identifies the Bank of Canada's four major monetary policy goals pursued in the past. These four goals are a stable exchange rate and interest rate, and lower inflation and unemployment levels.[4] Clearly, lower inflation and unemployment cannot be achieved simultaneously in the short-run. Monetary expansions will lead to a reduction in unemployment and increase of output in the short-run, but the price level would also rise as a consequence. As stated earlier, the Bank has been

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[4] The unemployment rate is the monthly average unemployment rate, based on the total labour force, which includes military personnel (Source: CANSIM D767289). The inflation rate is the year to year monthly percentage change of the Consumer Price Index (Source: CANSIM D484000). The exchange rate is the Canadian dollar index against group of ten (G-10) currencies (1970=100) (Source: CANSIM B3418). The interest rate is the monthly average Treasury-bill rate (Source: CANSIM B14007).

overwhelmingly intervening in the foreign exchange and money markets for the reasons of maintaining an "orderly" financial market and fighting inflation. Thus it is of interest to incorporate all known information in the vector ARMA model analysis.

Five time series, notably the money growth rate, unemployment rate, inflation rate, exchange rate and interest rate are used in the vector ARMA model analysis. All series contain 193 monthly observations starting January 1973 and ending January 1989. Results of the estimation for the chosen vector ARMA model and various statistics summarizing the forecasting performance of this model are presented in page 32 and TABLE 6.

Tentative Specifications

In the initial identification stage, the cross correlation matrix and partial autoregression matrices were generated from the original data[5] The cross correlation matrix of the original data contains persistently large correlations in all its lags, indicating an autoregressive model is required. The order of an AR model can be specified by examining the chi-square (χ^2) statistic after each successive higher order AR model is fitted to the data. The χ^2 statistic did not indicate any significant

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[5] It has been shown (Liu, 1986; Tiao & Box {1981}) that differencing of several non-stationary series jointly can lead to the problem of over-differencing and the resulting model may be non-invertible. Thus, in general, it is not advisable to difference several time series simultaneously even though they are non-stationary.

improvements in the goodness of fit after an AR(2) model was fitted to the data. Thus, an AR(1) or at most an AR(2), model should be adequate. Furthermore, the residual cross correlation matrix (RCCM) of an AR(1) model was inspected and found that large residual correlations were present in its short lags, (Lag 1 and 2), indicating a mixed VARMA model is appropriate. Following the quideline of parsimony, a mixed VARMA(1,2) model was tentatively selected.

Estimation and Diagnostic Checking

An ARMA(1,2) model was fitted to the data using an iterative conditional likelihood method.[6] The estimation results are presented in TABLE 1. The RCCM of the ARMA(1,2) was inspected again. No significant residual correlations were present in its short lags (Lag 1 and 2) and only a few significant residual correlations occurred in the further lags. The VARMA (2,2) model also showed similar results, but by the criterion of parsimony, a VARMA(1,2) is preferred to a higher order VARMA(2,2) model. Hence, it was concluded that an VARMA(1,2) model is an adequate representation of the data.

A VARMA(1,2) model can be expressed as:

Eq.(2.1) $$(\underline{I} - \underline{\delta}_1 B)\ \underline{Z}_t = \underline{C} + (\underline{I} - \underline{\delta}_1 B - \underline{\gamma}_2 B^2)\ \underline{a}_t$$

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[6] This estimation technique is discussed in detail by Lui (1986) and because of the complexity of mathematics involved, it will not be described in any detail in this paper.

Eq.(2.1) is a specific form of Eq.(2). In the initial estimation, no constraints were imposed upon the parameters in the model. However, those statistically insignificant parameters (t-values below 2.00) generated from the initial estimation should be constrained to zero, and the restricted ARMA(1,2) model must be re-estimated. The final restricted ARMA(1,2) model expressed in the usual difference equation form appears on the following page.

Table 1

Estimation Results of a Restricted Vector ARMA(1,2) Model.

Eq.(3.1)
$$\begin{array}{lllllll} \text{DM1}_t & = 8.443 & + 0.795\text{DM1}_{t-1} & + 0.322\text{Inflat}_{t-1} & - 0.049\text{Exch}_{t-1} & - 0.429\text{Int}_{t-1} & + a_{1,t} \\ & (4.878) & (24.090) & (4.128) & (3.063) & (6.703) & \end{array}$$

Eq.(3.2)
$$\begin{array}{lllllll} \text{Unemp}_t & = 1.010 & - 0.030\text{DM1}_{t-1} & + 0.905\text{Unemp}_{t-1} & + 0.136a_{2,t-1} & + 0.484a_{2,t-2} & + a_{2,t} \\ & (3.447) & (-2.308) & (31.207) & (-2.092) & (-7.563) & \end{array}$$

Eq.(3.3)
$$\begin{array}{llllll} \text{Inflat}_t & = 0.707 & - 0.060\text{Unemp}_{t-1} & + 0.971\text{Inflat}_{t-1} & + 0.093a_{3,t-1} & + a_{3,t} \\ & (4.337) & (-4.615) & (88.273) & (-2.385) & \end{array}$$

Eq.(3.4)
$$\begin{array}{llllllll} \text{Exch}_t & = -1664 & + 0.081\text{DM1}_{t-1} & + 1.0\text{Exch}_{t-1} & + 0.142\text{Int}_{t-1} & 0.430a_{4,t-1} & - 0.165a_{4,t-1} & + a_{4,t} \\ & (-1.359) & (3.24) & (90.909) & (2.958) & (3.44) & (3.438) & \end{array}$$

Eq.(3.5)
$$\begin{array}{lllllll} \text{Int}_t & = 1.539 & + 0.047\text{DM1}_{t-1} & - 0.017\text{Exch}_{t-1} & + 0.942\text{Int}_{t-1} & + 0.327a_{5,t-1} & + a_{5,t} \\ & (1.789) & (2.938) & (-2.125) & (29.438) & (-4.809) & \end{array}$$

* Where;

DM1, Inflat, Exch, Unemp and Int

denote the money growth rate, inflation rate, exchange rate, unemployment rate and interest rate respectively.

Forecasting results for the vector ARMA model

The forecasting procedures ,described earlier are used to evaluate the forecasting performance of the chosen vector ARMA model. The forecasting procedures consist of generating 12 step-ahead forecasts of the money supply growth rate from each estimation period. The initial estimation period consists of 145 observations starting January 1973 ending January 1985. Using Eq. (3.1), 12 step-ahead forecasts based on the initial estimation period can then be calculated. The initial estimation period is, then, expanded one month (step) ahead, and another 12 step-ahead forecasts are calculated based on the expanded estimation period which includes 146 observations. This procedure is to be repeated successively until the last step-ahead forecast reaches January 1989 (the last available observation). In the end, 37 series of 12 step-ahead forecasts are generated.

The MAE, RMSE and U statistics which are used to summarize the 37 series of 12 step-ahead forecasts are presented in TABLE 6. Since the RMSE is more sensitive to large errors than MAE, it will be given more attention in the result section. In comparsion of the RMSE of univariate ARIMA models and the chosen vector ARMA model, one major finding is that the vector ARMA model consistently generates smaller RMSE than those of the univariate ARIMA models for the *last* 6 step-ahead forecasts, while univariate ARIMA models performed somewhat better than the chosen vector ARMA model for the *first* 6 step-ahead forecasts.

The conclusion based on present forecasting results is that the vector ARMA model is more suitable for long-term forecasts (6 step-ahead and onward) than the univariate ARIMA models. This finding is consistent with the general structure of vector ARMA models which captures the dynamic relationships amongst several relevant variables and uses these established relationships to make forecasts. The main point here is that long-term variations of a time series under study would likely be caused by the movements of some relevant variables. If the relationships between the time series under study and its relevant variables can be established in advance, as vector ARMA models attempt to do, then the long-term variations of the time series under study can be predicted accurately.

Interpretation of the vector ARMA model results

The results represent a slight departure from the conventional belief of the ways in which the Bank of Canada has been managing the money supply. Eq.(3.1) indicates that the money growth rate in current time periods is heavily influenced by last period's money supply growth rate. This is not at all surprising since any particular monetary policy is likely to be carried out for more than a month at a time. It is, thus reasonable to expect a high correlation between two consecutive periods of the money supply growth rate. It is also interesting to have found that unemployment does not influence the current money supply growth rate at all. Despite the general belief that one of the policy goals of the Bank is to maintain a high output and employment level, Eq.(3) indicates the Bank did not pay much attention to the employment level in its monthly management of the money supply growth rate. The Bank had been preoccupied with fighting inflation most of the 1970's and early 1980's. Since the policy of *gradualism* failed to lower inflation in the late 1970's, highly restrictive monetary policy was in place in 1981 to lower inflation; a recession was seen as the price Canadians had to pay to achieve this goal.

Another highly significant influence of the current money supply growth rate is the last period's interest rate. Eq.(3.1) shows that there is an inverse correlation between the last period's interest rate and current money supply growth rate, implying the Bank's intention to minimize fluctuations in the interest rate on a monthly basis. This inverse correlation is consistent with the Bank's policy of maintaining an "orderly" financial market. Also, the result is consistent with the controlling mechanism through which the Bank controlled the money supply during the target money growth rate policy period (1975-1981). That is, the Bank had been manipulating the demand for money by keeping the interest rate within a certain range so that the target money growth rate could be achieved.[7] For example, the predetermined range of interest rate which was consistent with the target range for the growth rate of M1 for a particular year was, perhaps, in the range of 8 to 10 percent.

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[7] See Prachowny (1985) p.196

The Bank would have reason to believe the money supply is growing too quickly if the interest rate falls below the lower bound. The Bank, then, would buy fewer treasury bills or cut back its purchase of government bonds in open market operations.

The current money supply growth rate is also highly correlated with the last period's inflation rate and exchange rate. First, the positive correlation between the last period's inflation rate and current money supply growth rate signifies that monetary policy tended to be procyclical of the general business cycle in the immediate term. One obvious reason is that the Bank has very little knowledge about the last month's inflation rate since inflation data are published only once every three months.[8] Furthermore, a procyclical monetary policy coincides with the Bank's commitment to fight inflation when the economy is already in a downturn. Second, Eq.(3.1) indicates that the last period's exchange rate is inversely correlated with the current money supply growth rate. That is, any depreciation (appreciation) of the Canadian dollar in the last period leads to a decrease (increase) in the current money supply growth rate.[9] This result is consistent with our prior belief of the ways in which the Bank had been managing the exchange rate. The Bank's governor stated in 1983,

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[8] Prachowny (1985) p.198.

[9] Note that our definition of exchange rate is the amount of Canadian dollar one must give up to exchange for a unit of foreign currency. Thus, an increase in the exchange rate indicates a depreciation of the Canadian dollar.

> "any major depreciation of the Canadian dollar at this time would be strongly inflationary in an environment where confidence that inflation is under control is still so fragile." [10]

If the monetary policy is designed to stabilize short-term fluctuations in the exchange rate, it must be used in a way to eliminate any excess supply or demand for the Canadian dollar that had brought the exchange rate out of some predetermined range in the first place. Eq.(3.1) implies that Bank of Canada had been actively attempting to dampen the short-term fluctuations in the exchange rate by buying up more Canadian dollars in the foreign exchange market, thereby decreasing the money growth rate, when the exchange rate is higher (depreciation) than the Bank wants it to be.

Eq.(3.2) through Eq.(3.5) specify the feedback effects the money supply growth rate has on the input variables. All of the input variables are highly correlated with their corresponding terms lagged one period ago. Furthermore, the results indicate that the last period's money supply growth rate asserts significant influence on the unemployment rate, exchange rate and interest rate of the current period. In Eq.(3.2), the negative correlation between the last period's money supply growth rate and the current unemployment rate reinforces the earlier theoretical discussion about the short-term impacts of the money supply growth rate on the output and unemployment

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[10] Gerald K. Bouey, Notes for remarks to the Investment Dealers Association of Canada, Toronto, November 20, 1983; reprinted in Bank of Canada Review, December 1983, p.23.

level. That is, monetary restriction leads to a lower output and higher unemployment in the short-term only.

CONCLUSION

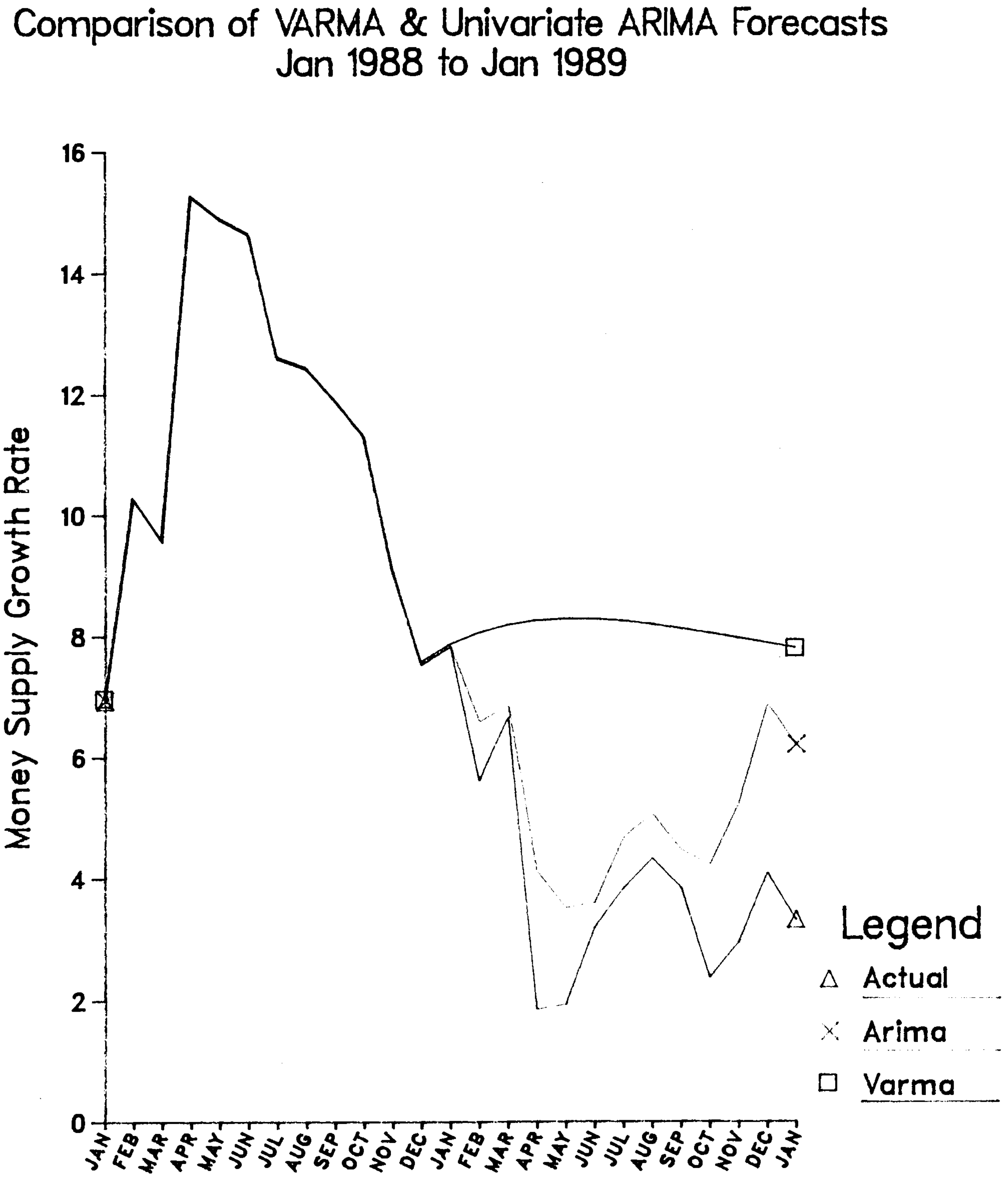
The analysis supports the view that the Canadian money supply growth rate is significantly influenced by the interest rate, exchange rate and inflation rate during the period studied. The absence of the unemployment rate in the money supply growth rate Eq.(3.1) reflects the Bank's preoccupation with the policy goal of lowering inflation during the period of analysis. The results also reflect that the Bank's practice of using the exchange rate and interest rate as policy instruments to control inflation.

The present results lend support to monetarists views on the role in which the money supply growth rate plays in the economy. More specifically, the money supply growth rate has only short-term effects on the real economic variables such as unemployment, as indicated by Eq.(3.2). On the other hand, the long-term causation relationship between the money supply growth rate and the inflation rate argued by the monetarists cannot be established from the present analysis.

Long-term forecasts generated by the vector ARMA model are more accurate than those generated by univariate ARIMA models. This result is consistent with the general structure of vector ARMA models which captures the relationships among several relevant variables and uses these established relationships to make forecasts.

The present analysis does not make any attempt to examine the applicability of our vector ARMA model for a different period of the money supply growth rate. It is important to know how the emphasis of the Bank of Canada change over time. This task will be left for future investigations.

Figure 2



Source: Cansim b2033

TABLE 2

Univariate ARIMA Model Results

	Moving Average Coefficients		
	Non-Seasonal MA	Non-Seasonal MA	Seasonal MA
Lags	9	13	12
Eq. 1.1	– –	– –	0.7728
			(16.32)
Eq. 1.2	-0.1506	0.1412	0.7704
	(-2.10)	(1.96)	(16.11)
Eq. 1.3	-0.1767	– –	.07650
	(-2.48)		(15.76)

Note: t values in parentheses.

TABLE 3

Out-of-Sample Forecasts Comparison for Univariate ARIMA Models

Mean Absolute Error			
Step-Ahead Forecast	Eq. 1.1	MAE Eq. 1.2	Eq.1.3
1	1.13183	1.16368	1.19921
2	1.4222	1.46467	1.56516
3	1.99777	2.01784	2.15729
4	2.29022	2.33444	2.58931
5	2.62145	2.67884	2.97608
6	3.01027	3.0368	3.35935
7	3.26025	3.29677	3.66812
8	3.45217	3.47932	3.86272
9	3.48849	3.53424	3.94135
10	3.49235	3.52473	3.93857
11	3.45335	3.44532	3.85229
12	3.3604	3.3514	3.76535

TABLE 4

Out-of-Sample Forecasts Comparison for
Univariate ARIMA Models

Root Mean Squared Error

Step-Ahead Forecast	Eq. 1.1	RMSE Eq. 1.2	Eq.1.3
1	1.37861	1.37576	1.43095
2	1.76041	1.8009	1.90478
3	2.35107	2.37448	2.53347
4	2.70126	2.74598	2.26187
5	3.07149	3.11702	3.38622
6	3.46457	3.51233	3.81969
7	3.73662	3.8211	4.1729
8	3.97778	4.07066	4.44077
9	4.13904	4.24789	4.63026
10	4.28472	4.38563	4.77345
11	4.38491	4.46492	4.86554
12	4.50537	4.57052	4.95655

TABLE 5

Out-of-Sample Forecasts Comparison for
Univariate ARIMA Models

	Theil's Inequality Coefficient (U)		
Step-Ahead Forecast	Eq. 1.1	Eq. 1.2	Eq.1.3
1	0.088056	0.087607	0.099099
2	0.112409	0.114308	0.120566
3	0.151509	0.151616	0.161117
4	0.175884	0.17691	0.189786
5	0.20254	0.203144	0.219021
6	0.230895	0.231356	0.249049
7	0.253075	0.255433	0.275454
8	0.27484	0.277527	0.298063
9	0.292358	0.295877	0.316381
10	0.309542	0.313952	0.334076
11	0.32268	0.327385	0.347487
12	0.340549	0.345895	0.363949

TABLE 6

Out-of-Sample Forecasts Statistics for Vector ARMA Model

Step-Ahead Forecast	Mean Absolute Error (MAE)	Root-Mean Squared Error (RMSE)	Theil's Statistics (U)
1	1.14759	1.5970	0.102065
2	1.55505	2.00985	0.127941
3	2.08083	2.66058	0.169239
4	2.38776	3.05946	0.194483
5	2.73236	3.35768	0.21343
6	2.93311	3.61475	0.229952
7	2.98055	3.66541	0.233784
8	3.11935	3.79089	0.243234
9	3.28774	3.89449	0.251927
10	3.3560	3.97103	0.25915
11	3.46124	3.99697	0.263536
12	3.54456	4.05005	0.270226

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