USING COOPERATIVE LEARNING TO PREPARE STUDENTS FOR MATHEMATICS CONTESTS

by

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ABSTRACT

Improvement in teaching practice begins with the individual teacher. In this study the researcher attempted to improve his teaching by experimenting with cooperative learning followed by reflection on the process. Thirty-five students from five schools in the Burnaby School District #41 were gathered for six sessions to work together to review for The Canadian Mathematics Competition sponsored by The University of Waterloo. The subjects were chosen by their mathematics teachers with the stipulation that the students had taken part in The Canadian Mathematics Competition the previous year.

Data were collected from individual scores on the contests, from a pretest and posttest survey of attitudes, from videotapes and notes, and from journal entries. Evaluation of the researcher's progress was continual and attempted to follow the model of action research. The improvement on individual test scores was significant ($F=5.62$, $df=1,52$, $p=.02$), with mean scores rising from 78.9 to 96.4 (17.5 difference) for students participating in the study compared to an increase of 82.6 to 89.8 (7.2 difference) for non-participants within the district. Gains in attitude, as measured on 15 items on a "mathematics as a process" scale were generally positive. Student images on the videotapes and their responses in the journals constitute a powerful appeal to use cooperative techniques in mathematics problem solving.

Tell me,
I Forget

Show me,
I Remember

Involve me,
I Understand

Chinese Proverb

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I would like to thank the students for their sincere efforts to immerse themselves in the activities presented to them and to provide reflective comments on their experience to me.

TABLE OF CONTENTS

LIST OF TABLES

PAGE

LIST OF FIGURES

PAGE

CHAPTER 1
INTRODUCTION

PURPOSE OF THE STUDY

This study has, as its general objective, to create a setting for an educational model based on "stimulation of learning." An attempt was made to create a social setting in which interaction with other people was a norm, where discussing the process of mathematics had value as high as or greater than results (written answers), and where learning to do mathematics was intrinsically motivating. This is in alignment with Sowder's (1989) goals for reform:

> Emphasis must shift from drill in paper-and-pencil computations to experience in the use of conceptual, analytical, and problem-solving techniques of mathematics. This shift will require a fundamental restructuring of the educational environment from the current "transmission of knowledge" model into one based on "stimulation of learning." The transition will involve fundamental changes in the content, modes of instruction, teacher education, and methods of assessing student progress. (p. 7)

To this end, I chose to use the review of mathematics contests to facilitate the shift from teacher dominance in a classroom setting to students cooperating with each other to solve problems. I felt that it was not necessary to perform a great deal of formal teaching, and that the students would be able to fall readily into the role of working together in preparation for the mathematics competition.

The study examines student's perceptions as they participate in the project and identifies relevant activities and procedures which will enhance student learning. For each session, I planned activities, implemented them , and reflected on observations made by myself and other teachers involved in the project. This process was repeated as a cycle for each of the six sessions.

OVERVIEW OF THE STUDY

Thirty-five Grade 10 students from five of the six secondary high schools in Burnaby met at Schou Education Center at approximately bi-weekly intervals to engage in mathematics activities and competitions. The Schou Education Center is an elementary school that has been converted to the district resource center and, with assembly and conference rooms, there is little left to remind one that it was once a school. The primary purpose of the meetings was to make mathematics a social and enjoyable activity. It was emphasized to the students that cooperative learning was the main focus and that the competition was for motivation. The competitive nature of the meetings was to develop a teamwork approach in the students from each school, and they were told to practice old mathematics exams on their own as well as participating at these meetings. The mathematics activities were centered on problem solving and topics not normally studied in class. For this reason I chose to use mathematics competitions as the vehicle to promote cooperative learning.

MATHEMATICS CONTESTS IN B.C.

Mathematics competitions are available for all students in British Columbia from Grade 7 to Grade 12. The Canadian Mathematics Competition is organized by the University of Waterloo and is the source for the Gauss contests for Grade 7 and Grade 8, Pascal for Grade 9, Cayley for Grade 10, Fermat for Grade 11, and Euclid for Grade 12. The Mathematical Association of America produces the American High School Mathematics Examination (AHSME) for Grade 12.

Students write Pascal, Cayley, Fermat, and AHSME contests in the last week of February, the Euclid in mid-April, and the Gauss at the end of May. The tests and answer key for the Gauss are provided to the school districts, which are then responsible for administering, marking, and compiling the results. The Pascal, Cayley, and Fermat tests must be administered by the schools at the specified time and the tests are sent to Waterloo where they are marked and the results compiled. The Euclid tests are marked at the University of British Columbia (UBC); the AHSME is marked at Simon Fraser University (SFU) and next year it will be marked by the schools and the results compiled at SFU. All are multiple-choice tests except for the Euclid.

The results of these tests generally are not used for students' marks. This situation provides teachers with a non-threatening source of enrichment materials. However, it is different for the Grade 12 students, since UBC reviews the Euclid scores and SFU examines the AHSME scores in determining university acceptance in certain programs and for scholarship applications.

ORGANIZATION OF THE ACTIVITIES

The students were drawn from volunteers at each school who had written the Pascal contest (1990) and who intended to write the Cayley contest in 1991. They did not have to be the top students, although very weak students might be intimidated in participating in the group activities. One assumption made was that the students that volunteered for this project already possessed a high level of motivation, and that this would enhance the project's success because group effectiveness would not be hampered by dysfunctional individuals that could exist in a normal classroom. It was expected that the students would get together at their individual schools to prepare for the competitions. It was up to the sponsor teachers at each school to select and prepare the cohort.

The meetings took place at Schou Educational Center . Each school was responsible for the transportation of the students, but the supervision was undertaken by myself (researcher) and Ivan Johnson (consultant and district mathematics coordinator). The decision to use Schou rather than a school was based on the desire to create a new environment in order that the "culture" created would not reflect preconceived notions of "school." I felt that a setting away from a school might be more conducive to students changing their behaviour from competitiveness to cooperation with mathematics tests. Teacher participation was invited, but the recommendation by department heads at a district meeting that the program be a "pullout" program for enrichment precluded this option.

The students met six times, spaced at two week intervals in order that the students would have an opportunity to meet on their own to review materials and develop their own strategies and for the researcher to reflect on each session and plan for the next. All sessions incorporated Cayley type questions in review format in the first few minutes of a meeting. The program of each session remained flexible to accommodate student's wishes and the "learning on the spot" of the researchers. One hypothesis of this study is that cooperative learning will improve achievement on the Cayley Contest. The more interesting part of the study is the exploration of the effect of this process on the students and on the researchers, and the possibility of gaining new insights from a "stimulation model of learning."

LIMITATIONS OF THE STUDY

A major limitation of the study occurs if a comparison of contest scores is to be made. The subjects were exposed to twelve hours of treatment, whereas there was no control over other groups that are compared, and it is likely that they did not benefit from as much intervention. There is also the Hawthorne effect to consider. Each student was made aware that I was a student from SFU, and that I was conducting a study using cooperative learning techniques to prepare for mathematics contests when they signed the release form to participate in this project. At the conclusion of the meetings, some of the students approached me and wished me "good luck" on the project.

Another limitation is that the subjects were not chosen in a random manner. The study began with twenty-two males and thirteen females; twenty-four orientals and eleven caucasians.

STRUCTURE OF THE THESIS

Themes

The research was centered around four themes. The evidence for each of these themes was drawn from a number of sources.

1. The first theme centered on whether the intervention had an effect on students' scores on mathematics contests. To address this, the students' scores from their Pascal contest (Grade 9) were compared with their scores from the Cayley contest (Grade 10), and with scores obtained by Burnaby students who did not take part in the study.
2. The second examines change in the attitudes of the students. The survey "Mathematics as a Process" (Robitaille, O'Shea, & Dirks, 1982 pp 149-151) was used as the attitude questionnaire applied at the beginning and at the end of the program and the two results were compared. A scale was used to measure how the students view the nature of mathematics as a discipline. To be positive on this scale is to view mathematics as a field where speculation and

heuristics are important rather than just rules; where the discipline is changing rather than stagnant; and where there is opportunity for creative endeavor. The students also maintained a journal, and this was analyzed to detect any changes in attitude as they occurred.

3. A third theme was to observe the enculturation process of a collection of "bright" mathematics students in a new environment (different from their school or their classroom) as they worked on different material (more focussed on non-algorithmic problem solving and also different in content) with a new process (cooperative learning). Interpretation of notes and video tapes attempted to document any changes.

4. A fourth theme was to examine the effect on the teacher participants of attempting to implement an innovation in educational process. This view of research was suggested by Richardson (1990):

> Clandinin (1986) and Clandinin and Connelly (1986) suggested, through a case study, that teachers' personal narratives or constructions of their personal biographies interact with particular situations to help teachers acquire practical knowledge (p.13).

An examination of personal reflections combined with collaborative discussions with other participant observers would provide interpretive research data.

Chapter Organization

This study has been organized into five chapters. Chapter I introduces the study and describes the connection between mathematics contests and cooperative learning in this project, sets out the limitations, and outlines the remainder of the thesis.

In Chapter II, the literature is reviewed. Proposed curriculum changes and mathematical educational research relevant to this study are outlined. A description of action research and the application of triangulation is provided.

Chapter III contains a description of the study, beginning with the conception of the idea for the project. The methodology and the six research cycles are described.

Chapter IV is a presentation of the results of the contests, attitudinal survey, and observations collected from students.

Chapter V is a collection of my thoughts as I analyzed the events in which I participated on this project. The nature of action research is cyclic, so some reflections are also presented as part of the cycle in Chapters III & IV as well.

CHAPTER II
REVIEW OF THE LITERATURE

CURRICULUM CHANGES

I became interested in this research topic as a result of momentarily stepping back in my "traditional classroom" and observing the activities that the students seemed most interested in. They would pay attention to the lesson and work on the assignments from a sense of responsibility and politeness, but most seemed interested in the social milieu of the class and of school in general. I saw that my pride in being a "good" teacher resulted from how effective I was in motivating students to "work" on their assigned tasks in spite of the distractions that seemed more interesting to the students.

Is "teaching" molding the students only by engaging them in activities that we think are efficient and appropriate for the mastery of content? We consider cognitive development through the theories of Piaget and Skemp, but we don't concede the necessity of addressing social development for the students. As Cusick (1973) noted:

> More and more, as I continued in the school, I saw that the students' most active and alive moments, and indeed the great majority of their school time, was spent not with teachers and subject-matter affairs, but in their own small-group interactions which they carried on simultaneously with their class work (p.58)

After considering such issues, I felt the use of group work as an instructional technique in school might address the need of students to socialize while also providing for learning. As I learned more about cooperative learning strategies, my desire to try the strategies was tempered by a lack of confidence and fear of the experiment having a negative impact on the classes I was currently teaching. The idea of trying cooperative learning as a way of helping students participating in the mathematics contests satisfied several needs. It would provide me with a group of intrinsically motivated students with whom I could concentrate on practicing strategies and techniques. I would not have to deal with classroom management problems such as wide ranges in ability, motivation, and discipline. The students would be provided with coaching for the contests and enrichment. Improvement in test scores would provide an indicator of how I was doing. Finally, what may be important to many teachers as they embark on a project of this nature, if I was failing, it was fairly easy to "pull the pin."

While I was contemplating the process with which I would involve students in their learning, I had a discussion with an SFU classmate who was an elementary teacher. She informed me that my goals were already being implemented in the elementary schools. I decided to visit Forest Grove Elementary school in Burnaby to observe the activities there. What I observed was an environment where there were very few classrooms with desks in rows; the rooms were organized around centers or themes of learning, with the obvious presence of manipulatives. Education was functioning under the "stimulation of learning" model that Sowder (1989) had described. This visit impressed upon me the need for secondary teachers to make the necessary changes recommended by the *Year 2000* (1990) document, otherwise we would be doing a disservice to the students leaving an environment such as they had experienced at Forest Grove Elementary to come to a secondary school atmosphere where, for example, I had been willing to perpetuate a "transmission of knowledge" model of learning.

In preparing students for the future, teachers have to examine what they are currently doing and what the needs of the students for the future are going to be. Present methods of lecturing and individual assignments, emphasis on classroom control over students' interactions, and structured whole class activities are geared to prepare students for an industrial society where punctuality, conformity and the ability to perform algorithmic tasks are valued. The mission statement from the British Columbia Ministry of Education, *Year 2000: A Framework For Learning* (1990) attempts to bridge the concepts of curriculum as cognitive development and as one of social adaptation.

> The purpose of the British Columbia school system is to enable learners to develop their individual potential and to acquire the knowledge, skills, and attitudes needed to contribute to a healthy society and a prosperous and sustainable economy (p. 2).

The Year 2000 Document suggests social development as one of the guiding principles. "Social interaction provides opportunities to examine one's knowledge and beliefs, and contributes to the motivation to learn (p.4)." It also recommends that educational programs focus on developing skills and attitudes that result from working with other students (p.5).

It was also at this time that I began reading the National Council of Teachers of Mathematics' (NCTM) Curriculum and Evaluation Standards for School Mathematics (1989) and found in it reinforcement for my idea of having students interact with each other. The statement "Our premise is that *what* a student learns depends to a great degree on *how* he or she has learned it" (p.5) combined with goal #4 for students:

> *Learning to communicate mathematically.* The development of a student's power to use mathematics involves learning the signs, symbols, and terms of mathematics. This is best accomplished in problem situations in which students have an opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural. As students communicate their ideas, they learn to clarify, refine, and consolidate their thinking. (p.6)

made me feel that this project had some merit.

EDUCATIONAL RESEARCH

The concepts embodied in the theories of constructivism, mathematical enculturation, cooperative learning, and problem solving are interwoven. Rather than attempt to unravel and classify how these ideas relate to the research program, I will address the issues related to the study under these main classifications.

Constructivism

Constructivist learning theory suggests that people collect, discover or build their knowledge through the course of involvement in an activity. Sowder (1989) suggests three variations of this model:

1. The first perspective centers on the notion of "doing" mathematics. "To know" from this perspective means that students "do" mathematics by abstracting, inventing, proving, and applying. Students then construct their mathematical knowledge from these purposeful activities.

2. The second constructivist position involves "cognitive modeling," the psychological procedures for constructing representations of mathematical knowledge as cognitive procedures and schemata.

3. The third constructivist view of knowledge regards knowledge as the product of a social process. This view was represented by Lave, Smith, and Butler (1989), Schoenfeld (1989), and Kieran (1988), in the context of mathematics teaching. Their position is that a lot of what students "know" comes from their social and cultural experiences, and not just from planned instruction in schools. (p. 22)

The third view presented by Sowder (1989) suggests a reason for proceeding with the project to create more social experiences for students in their mathematical activities and to attempt to document the effectiveness of this change.

Mathematical Enculturation

Mathematical enculturation as an educational approach suggests that teachers should teach the values of mathematical culture because values are embedded in the mathematics that children should learn. A description that Bishop gives of mathematical culture is "the activities of counting, locating, measuring, designing, playing and explaining have, individually and in interaction, been instrumental in developing the complex symbolizations and conceptualizations of Mathematics, as we know the internationalised discipline today (p 82)." "Formal mathematical enculturation has as its goal the induction of children into the symbolisations, conceptualisations and value of Mathematical culture." (Bishop, 1988, p.89) The values which Bishop wishes to stress are stated as follows: "I therefore will present a Mathematics curriculum structure which allows rationalism to be stressed more than objectism, where progress can be emphasized more than control and where openness can be more significant than mystery" (p.95). This is in contrast to the public notion that in school mathematics, following rules is more important than reasoning, conforming to other's expectations takes precedence to personal growth, and where it is acceptaple for children to perceive that mathematics is mysterious. In a cooperating atmosphere where students are encouraged to express themselves, they will be able to interact socially to construct their hierarchy of values. This research program was not able to address all of the ideas of the enculturation process, but attempted to incorporate such ideas as: removing the teacher from the position of an authoritative mathematics figure, promoting the opportunity for student explanations, group work, activities as a major focus of students' work, and students working on projects (Bishop, 1988, pp. 95-111). By creating a cohort of students from different schools at a location removed from their familiar schools, it was hoped that a new cultural experience could be created which was more conducive for the enculturation process.

Cooperative Learning

The advocacy for group learning is presented in the Curriculum and Evaluation Standards for School Mathematics by the National Council of Teachers of Mathematics (1989):

> Teachers foster communication in mathematics by asking questions, of posing problem situations that actively engage students. Small-group work, large-group discussions, and presentation of individual and group reports-both written and oral-provide an environment in which students can practice and refine their growing ability to communicate mathematical thought processes and strategies. Small groups provide a forum for asking questions, discussing ideas, making mistakes, learning to listen to others' ideas, offering constructive criticism, and summarizing discoveries in writing. Whole-class discussions enable students to pool and evaluate ideas; they provide opportunities for recording data, sharing solution strategies, summarizing collected data, inventing notations, hypothesizing, and constructing simple arguments (p. 79).

The activities planned in the program of research were designed with cooperative learning strategies as a guiding principle. Various structures as proposed by Spencer Kagan and the student team learning that Robert Slavin developed were used. The current status of research on cooperative learning and student achievement is summed up by Slavin (1990):

> There is wide agreement among reviewers of the cooperative learning literature that cooperative methods can and usually do have a positive effect on student achievement. Further, there is almost as strong a consensus that the achievement effects are not seen for all forms of cooperative learning but depend on two essential features, at least at the elementary and secondary level. One of these features is group goals, or positive interdependence; the cooperative groups must work together to earn recognition, grades, rewards, and other indicators of group success. Simply asking students to work together is not enough. The second essential feature is individual accountability: the group's success must depend on the individual learning of all group members (p.52).

In the present study, the use of Slavin's model of student team building in cooperative learning with competition against other school teams was designed to bring the students towards positive interdependence and the final writing of the 1991 Cayley Test to stress individual accountability. A controversy about cooperative learning is its applicability to all grade levels:

> One issue is whether cooperative learning is effective at all grade levels. Newmann and Thompson (1987) question whether cooperative learning is effective in senior high school (grades 10-12). There is ample evidence that these methods are instructionally effective in grades 2-9, but relatively few studies examine grades 10-12. More research is needed in this area (Slavin, 1990, p. 53).

The students in this study were in Grade 10 and their results on the Cayley test provided a small sample for this research. Providing a social setting may improve student's motivation to participate in mathematics. The students' contributions may be partial and not entirely correct, yet they will see themselves as engaging in interpretation. "The public setting also lends social status and validation to what may best be called the disposition to meaning construction activities" (Resnick, 1988, p.40). This program of research also examined the affective results of using cooperative learning, including changes in motivation, self-esteem, willingness to attempt non-algorithmic problems, attitudes towards mathematics, and ability to relate to others. Appendix A contains a summary of the advantages of using cooperative learning strategies for students. Appendix B contains an analysis of five models of cooperative learning based on work by the Johnsons, Slavin, Kagan, Sharan, and Bellanca & Fogarty (Bellanca and Fogarty, 1991, p. 243).

Problem Solving

Greater emphasis is being placed on problem solving throughout the high school mathematics curriculum. The Math 12 students in British Columbia must complete a "Problem Set" this year, with more such questions being placed on the provincial exam than in previous years. The questions on the Cayley tests approach mathematics from the problem-solving perspective. These questions demand more concentration and intellectual activity than textbook assignments from the students. Placed in a cooperative learning setting the researcher, as observer, can study the student's cognition processes. As Shavelson, Webb, Stasz, & McArthur (1988) state:

> It is an excellent setting to study student's problem-solving processes because it avoids directions to "think aloud": students freely verbalize their thoughts to one another. Having just solved the problem themselves, some students are in an excellent position to understand what their peers don't know--perhaps better than teachers and tutors (p.221).

The opportunity to share problem solving activities with peers may assist students to develop the necessary skills for themselves. The conditions are described by Resnick (1988):

> Socially shared problem solving, then, apparently sets up several conditions that may be important in developing problem-solving skill. One function of the social setting is that it provides occasions for modeling effective thinking strategies. ..... This process opens to inspection mental activities that are normally hidden. Observing others, the student can become aware of mental

> processes that might otherwise remain entirely implicit. Something about performing in social settings seems to be crucial to acquiring problem solving habits and skills. "Thinking aloud" in a social setting makes it possible for others to critique and shape a person's performance, something that cannot be done effectively when only the results, but not the process, of a thought are visible (pp.39-40).

Finally, the types of problem solving activities that the students use on the Cayley test questions places greater demands on higher level cognitive thinking than teacher-prepared tests based on curriculum content. The need to place emphasis on problem solving is described in the NCTM Curriculum and Evaluation Standards Document (1989):

> Henry Pollak (1987) a noted industrial mathematician, recently summarized the mathematical expectations for new employees in industry:
> *The ability to set up problems with the appropriate operations
> *Knowledge of a variety of techniques to approach and work on problems
> *Understanding of the underlying mathematical features of a problem
> *The ability to work with others on problems (p.4).

If students are to meet these expectations, teachers should make the effort to become skilled at providing these experiences for them.

ACTION RESEARCH

Two major ideas influenced the direction that this research project followed. The first is the impact of research on learning. The change that must be made in teaching is described by Steen (1988):

> I hardly need to point out that most post-secondary mathematics-- indeed, most mathematics at any level -- is taught by lecture, with homework exercises for practice and examinations for enforcement. Lecturing and examining may be the easiest way to

> teach mathematics, but they are by no means the most effective. ..... Effective teaching for today's students requires a more diverse repertoire of approaches, including in addition to lectures, homework, and examinations, new opportunities for group work, for extensive writing, for oral practice for exploration and experimenting, for modeling projects, and for computer activities.

The second influence is the direction that educational research should be moving. For research to be useful and meaningful to teachers, a more practical approach is recommended. d'Entremont (1988) reports "Research would be a means by which teachers themselves are able to reflect on their practice, modify their procedures and improve their teaching. There must be a link between research and the everyday world. Action research may be that missing link." (p. 48)

I had not been conscientious in keeping up with developments in research on teaching and learning since I completed my education program at university 17 years ago. There always seemed such a gap between the theory and the practice that my colleagues and I were more committed to developing lessons and new content, rather than thinking of the process of learning.

The school district where I am employed has introduced a professional growth plan to be adopted by teachers beginning in 1990. The introduction to the plan (Burnaby School District #41, 1990) states:

> The Burnaby School Board and the Burnaby Teacher's Association are committed to a professional growth program which promotes self-analysis, encourages a collaborative approach to professional development, places greater responsibility on individual teachers for their own growth. .....Through a combination of research, experimentation and reflection, each teacher will take an active role in a process of growth and development which will

ultimately enhance the quality of instruction in the classroom. (p.5)

This concept of teachers as researchers is not in alignment with the past practice that research is done by "outside" specialists, and that the dissemination of research results is an effective way of improving educational practice. The proposal in the professional growth plan describes the process of action research, and the question in my mind is, how do I relate this to my previous conception of educational research? A formal definition of educational research (Verma & Beard, 1981) describes the process as:

> an organized and deliberate effort to collect new information or utilize existing information for a specific and new purpose. It is directed towards seeking answers to worthwhile, fairly important and fundamental questions through the application of sound and acceptable methods. (p. 18)

I compared this with d'Entremont's definition (1988, p. 53):

> Research should be involved in helping teachers to improve the learning experience of the children in their classes; it should be helping us to better understand our practices in order to move confidently into improved ways of working.

My conclusion was that I was prepared to accept d'Entremont's definition and to use it as a guide to improve my practice.

The Kemmis and McTaggert (1988) model of action research describes the process as a group and its members undertaking to:

1. develop a plan of critically informed action to improve what is already happening.

2. act to implement the plan.

3. observe the effects of the critically informed action in the context in which it occurs

4. reflect on these effects as a basis for further planning, subsequent critically informed action and so on, through a succession of cycles. (p.11)

A model proposed by Case (1990) has four stages which he calls a framework for informed reflection:

1. **Focus.** Identify possible questions or concerns you might have about your plan, select those that are both realistic to consider and likely to have a pay-off, given the resources available to you.

2. **Investigate.** Formulate a plan to address your questions by asking three questions: " What information do I need?" "Who can provide that information?" and "How can I best obtain the information from that source?" Then carry out the information-gathering plan.

3. **Analyze.** Organize and summarize the information collected into appropriate formats and interpret what the results tell you about your teaching.

4. **Initiate.** On the basis of your analysis, evaluate the courses of action open to you and, if appropriate, initiate those actions which you believe will further enhance your teaching. (p.4)

I chose to follow a model of action research drawn from Kemmis and McTaggert, with a modification from their insistence on collaboration as an essential feature of their program. Although I would have preferred to maintain this feature, the three hours of collaboration with others on this project was minimal. The feature which I liked from the Kemmis and McTaggert model was the use of the research cycles, and I feel that this feature enabled me to learn a great deal from this project.

Gurney (1989) provides further justification for following this line of research:

> The argument for teachers adopting the dual role of teacher-researcher has been made most persuasively by Stenhouse (1975, 1979, 1981a, 1981b), Ruddick (1985), Elliot & Adelman (1973a, 1973b), Nixon (1981) Hopkins (1985) and Whitehead (1983a, 1983b, 1986). Ruddick & Hopkins (1985) rest their argument for research as a basis for teaching on two principles: first, that teacher research is linked to the strengthening of teacher judgement and consequently to the self-directed improvement of practice; second, that the most important focus for research is the curriculum in that it is the medium through which knowledge is communicated in schools. (pp. 15-16)

I do not intend to provide in comprehensive detail the theory of reflective practice, but Sparks-Langer and Colton (1991) describe the breadth of this form of research:

> Many terms and concepts are joined together in this view of reflection: case studies of the tacit wisdom that guides practice (Shulman 1987), the inclusion of craft knowledge in teacher assessment practices, (Leinhardt 1990), the legitimacy of viewing teaching as art (Eisner 1982, Kagan 1988), defining teaching as improvisational performance (Yinger 1987), teacher action research (Cochran-Smith and Lytle 1990), and the appearance of qualitative studies using narrative inquiry (Connelly and Clandinin 1990). The common thread through all these is the emphasis on the validity of teacher's judgements drawn from their own experiences. This view is sympathetic with Schon's notion of "giving reason" because it is the teachers themselves whose voices comprise the story. (p.42)

In my search of the literature I found it interesting that a particular journal, Educational Leadership, chose "Cooperative Learning" (47 (4)) and "The Reflective Educator" (48 (6)) as themes for two recent issues. This coincidence reinforced my thinking on the relevance of my project.

TRIANGULATION

I have attempted to combine quantitative and qualitative techniques in this project. Triangulation in the form of using different methodology in data collection has been used to support the validity of the research findings. Forward (1989) describes the process of triangulation:

> Clearly such an approach is directed towards classroom research but it can be easily extended to apply to a wider area of educational research. Denzin (1978) has identified four main triangulation types: data triangulation using a variety of data sources; investigator triangulation using different researchers or evaluators; theory triangulation using different perspectives to interpret data; and methodological triangulation using multiple methods to study a single event or programme. (p. 35)

The triangle formed by quantitative results and the two qualitative results of student and teacher reflections is the means by which I am triangulating the data. The quantitative results consist of the Cayley scores and the survey "Mathematics as a process" described in Robitaille, O'Shea, & Dirks (1982) pretest and postest scores. Qualitative data comes from reflections by students in their journals and interviews. Video-taped images provide data about students which the researcher can interpret. The researcher also creates qualitative information from notes and analysis of students' work.

One problem which I experienced was connecting my prior conception of educational research, the literature, and my experience as a researcher in this project. I read five theses to familiarize myself with what I thought the Faculty of Education at SFU expected. Coursework, discussions with colleagues, Education Research Information Clearinghouse (Eric) searches, and review of educational journals provided exposure to literature reviewed for this thesis. The problem I encountered when I began writing the thesis was transcribing the qualitative information from the students and from my reflections to a thesis format. It was very difficult to place in print the reactions, feelings, and involvement that were part of the research.

CHAPTER III

DESCRIPTION OF THE STUDY

THE STORY

The idea for this project was seeded in the spring of 1988 when I was given a choice of co-sponsoring a field trip to Cariboo College in Kamloops with 18 top mathematics students accompanied by nine teachers from all of the secondary schools in Burnaby, or sponsoring a rugby tour to Kelowna with 18 of the toughest boys from my high school. I chose the former.

What struck me was the quick manner in which the shy, achievement-oriented students socialized. Their common interest was mathematics, and a good deal of their communication was mathematical, often in a fun kind of way. During the course of the two day field trip, I saw students who usually worked independently in a quiet way in class in animated conversation about a mathematics topic with somebody from a different school whom they had just met. Some comments that struck me were:

"Hey! There are other wierdos like me out there"

"It's not weird, this is reality."

The innovation that I conceived of at that time was the creation of an atmosphere of cooperation (teamwork) of students working together mathematically with the stimulus of competition with students from other schools. The framework for this enterprise would be a mathematics competition based on the students' grade level. The district has a program for extracurricular sports competition, so it seemed natural to have mathematics competitions. If the students could get together for mathematical activities, they should also benefit. This might motivate some students to prepare for mathematics contests more than by taking old tests and reviewing them individually.

Another experience which reinforced my conception of the value of this approach occurred when I participated in an "Algebra 12 Retreat" to Camp Elphinstone in June 1989. The activities there focussed on cooperative learning to review the government examination. This social activity was more fun than reviewing individually, and it also provided a structure in which the students were to work. The students indicated that this was a positive experience for them, and my observation was that they worked very hard on the mathematics activities.

I first attempted to implement the mathematics contests as a vehicle to experiment with cooperative learning in September,1989. I wrote up a proposal (Appendix C) for the Burnaby District Mathematics Department Head meeting, and it was favourably received. It was suggested that I should make further contact with the schools to implement the meetings. Subsequent phone calls to department heads showed them to be supportive, but there were difficulties with time and personnel resources. Typical responses were:

"We are working on the Pascal and Cayley math contests now, wait until they are out of the way,then we'll start with the Grade 8's."

"I can't find someone to sponsor the Grade 8's"

"My teachers think it's a good idea but are afraid to commit themselves because they think that they will get themselves into something that they don't know very much about (i.e. cooperative learning) or that they will get involved in a lot of extra work."

"We have decided to prepare for the contest on our own."

I found my interest and commitment waning. There was enough to do without creating more work that didn't seem to lead anywhere, with the only apparent reward of trying an idea in the hopes of creating enrichment for a relatively small number of students and making learning more meaningful for them. I made one more attempt at the project by setting a meeting date and phoning the schools to see if they could make the meeting. With a commitment from only three out of six schools I was very disappointed and decided to cancel the project.

I reflected on my failure to implement the program and I felt a personal rejection by others in my commitment to the value of creating motivating and activity based problem solving for students. I decided I might have had more success through a personal telephone appeal to acquaintances teaching at the other schools instead of following bureaucratic channels via the district department heads. I certainly would have had a friendly audience in order to sell my impression of the quality of the innovation, and the participation would have been collegial from the beginning. Perhaps the quality of the innovation was not clear to the participants. In hindsight, I think that the problems were due more to a lack of teacher time to participate, difficulty in communicating with other schools and inertia in the system, than to lack of interest in the project. In order to induce more involvement on the teachers' part, I presented a very loose proposal of the student meetings. The first meeting was arranged with students participating in an activity that would allow teachers freedom to discuss the project and to plan future sessions with others. This process would have allowed the other teachers to build ownership in this project. I had not heard of action research, and the activities that I was pursuing were very individualistic. Teaching in my experience is a very personal activity and I had not thought in a clear manner to create a collaborative venture.

A month passed and the pressure was on to start preparing my Grade 8 students for the Gauss Contest. I began by doing what I had always done. I passed out old exams to my more motivated students and asked them to start reviewing for the test that they would be writing in one and a half months. They asked me why they should do this extra work, and I launched into my practiced speech on how this work was enrichment and developed more creative thinking than the regular mathematics that we do. They asked me if the tests counted, and I told them "No", but that it gave them an opportunity to see how they stack up against other students in the country, and that it gave them yearly practice in contests until Grade 12, when the contests do count in terms of university entrance and scholarships. They seemed to accept this explanation. I structured my classes so that this enrichment was not seen as extra work. I encouraged students to work in pairs and posted solutions for them to check. I monitored the students' progress by giving a practice test and found that they did not do very well. I felt that they worked on the contest materials as if it was different work, but that it was still just an assignment.

It was at this point that I thought of having an inter-class competition within our school. For the second cycle of this action research I visited the other Mathematics 8 classes, and with their teacher's permission, challenged the students to a competition in my room at lunch time. At these visits, I explained the format of the competition to the students. Each class was to send a team of four students who would have to work together cooperatively to provide a solution. Questions would be given one at a time to all the teams. Scoring would be 2 points to the first team to complete a question correctly, minus 1 point for an incorrect solution. The rest of the teams had 3 minutes to complete the question after the first successful team. They would receive 1 point if they completed successfully. I also invited the teachers to participate.

I felt that this cycle of the innovation was successful. The competitions in my room were high energy, intensive and fun. I liked the effect of the penalty because it forced consensus upon the group. In one instance, in the heat of competition one of the brighter students in the group "forcibly persuaded" his reluctant peers to accept his solution which later turned out to be incorrect. This was an important lesson for the group, and my observations indicate important growth took place. In trying to remember what happened, I wish that I had made more observations and kept notes; even interviewed some of the students. I do remember that students who were not participating volunteered to assist by keeping score, timing, and handing out materials. One of the teachers teaching the Grade 8's came and helped me for one of the sessions and the other two came and observed part of a lunch time. Their comments were that they liked what they saw, noticed that the students were very involved and that they thought that it was a good idea. The students participating returned to their classes with enthusiasm and a commitment to be better prepared for the next encounter. I noticed within my own classes that the students made an effort to classify questions and to complete as many different types as possible. They also seemed more committed to completing the old tests. We had four sessions before we ran out of time and the students had to write the contest. The comments to me by the school mathematics department head and invigilator of the Gauss Contest, was that he was impressed by the number of students who volunteered to write the contest, and by the seriousness with which they participated. Our school came first in the district on the Gauss Contest.

The third cycle of this action research was in partial fulfillment of the requirements for the master's program at SFU. This time I chose to work with Grade 10 students, for several reasons. The first reason was that we had the previous year's scores from the Pascal Contest results, and these data could be used in a quantitative manner to compare with the results of the Cayley Contest that the students wrote this year. Perhaps these results could be used as a baseline to compare the descriptive results of the qualitative data. The second reason was that the students are more mature and critical than the Grade 8's that I had worked with previously, although it seems that Grade 8's are more keen and that they can be readily motivated without as much effort as the older students. The process of having the students maintain a journal made me think that the more mature Grade 10's would be more reflective with their entries than the younger students.

I presented a proposal (Appendix D) at the first district mathematics department head meeting. The meeting was productive in that the department heads agreed to have their schools participate in the competitions. They agreed, in principle, with the first proposal, but they felt that I would have better success implementing the project if the students were pulled out of their classes and met at 1:00 p.m. I consented to go along with this plan, but indicated that I had wanted more teacher involvement which would have been possible with after school sessions, not so much for assistance to run the program but I thought that other teachers might become interested in this project. They would also provide a different perspective for reflection and feedback, and might develop a substantial degree of ownership. I asked the district mathematics coordinator if it was possible to get release time for the teachers, thus providing a reward for them with the use of substitutes, but the answer was no. I was hoping that the district would give financial support because this would be one way of demonstrating cooperative learning in mathematics to all of the schools in the district. The department heads indicated that if a teacher was interested, they would try to arrange internal coverage. My experience with this is that teachers are always reluctant to ask their colleagues for this service. Ivan agreed to provide support for this project and he made a commitment to be at all the sessions. A quote from Kemmis and McTaggart (1988) is appropriate here "Negotiation and compromise may be necessary - but compromises must also be seen in their strategic context. Modest gains may do for the time being" (p. 12).

It was at this time that I became aware of the literature on action research from my SFU Education 830 course on Implementing School Programs. It became clear to me that I was engaged in action research in that I was following a cycle of 1) planning, 2) acting & observing and 3) reflecting on a thematic concern. What I was doing diverged from action research in that I was not working in a strong collaborative network of colleagues. This point is made in Kemmis & McTaggart's (1988):

> Action research is not individualistic. To lapse into individualism is to destroy the critical dynamic of the group and to risk falling victim to the fallacious liberal notion that all educational practices and values which they purport to realize are equally defensible. (p. 15)

When I began the program at SFU my professional identity was very individualistic. Driscoll and Lord (1990) attribute this common phenomena to the evolution of the one-room schoolhouse from the 19th century:

> ....the combined effect conveyed the impression that teaching was a highly transient profession and one where teamwork was unlikely to develop. Hence the egg-crate metaphor: with the turnover as high as it was-and was expected to be-it was convenient to design the typical school as if it were a collection of one-room schoolhouses. Teachers were separated from one another, and collegiality was not encouraged. Thus a basic component of a healthy profession was stifled from the start.(p. 239)

The egg-crate metaphor was D. Lortie's metaphor, and Fullan (1982) expands on the reasons why teachers find it difficult to collaborate:

> Partly because of the physical isolation and partly because of norms of not sharing, observing, and discussing each other's work, teachers do not develop a common technical culture. (p.108)

In retrospect, this is my justification for not beginning this project in a collaborative manner.

THE STUDENTS

If I had been entirely responsible for the selection of students for the study, I would have chosen the top four students from each school. I did not know the students, and this would probably have been the most convenient method to ensure acquiring "motivated" students for the study. Instead, I chose to delegate the task to the sponsor teachers. The main motivation was to involve the teachers, hoping that they would become interested in the project and would take a more active role. They also knew the students fairly well, and might know which students would be interested in this activity. The only stipulations for selecting students were that they had written the Pascal in 1990 and were planning to write the Cayley in 1991. None of the schools sent only their top four students. Many sent students with average scores in the Pascal. Thirteen of twenty-eight subjects had scores below 50 % on the Pascal. This creates an interesting situation in terms of the results of the study. If the goal was to have Burnaby Schools place well with their team scores, the top students should have been selected for treatment. If the goal was to measure the change in scores, then average students might have a chance of gaining more on the contest than students already scoring fairly high.

Some schools sent all males, while others made an effort to send a balance of males and females. Twenty-two males and thirteen females; twenty-four orientals and eleven caucasians began the study.

The students were of high-ability, and there were no problems with respect to discipline, although three students were not well motivated during the sessions. Two of these students interacted immaturely and were frequently off task, distracting their table-mates and did not make productive use of the available time. One student did not participate effectively with his group, and was often observed reading a science fiction paperback.

COLLABORATION

The literature on action research stresses that collaboration is essential to the success of this method of improving teaching practice. I identify the lack of collaboration as one major weakness of this project. There were many efforts made to involve other teachers, but it was very difficult to coordinate their time with the meetings which took place at Schou. An exception was the constructive collaboration with Ivan Johnson, Coordinator of Schou Education Center. Ivan is more experienced with cooperative learning techniques and has presented workshops at conferences and for various school districts. His experience helped me avoid some errors, such as lecturing too long instead of maximizing the time that students would be interacting with each other. After each session we would review the activity and identify the strengths and weaknesses.

Cynthia Garton, enrichment teacher for the gifted and talented students of Burnaby, was able to attend two sessions and provided feedback on her observations.

Tom O'Shea, Senior Supervisor from SFU, was able to attend the December 6, 1990 session, and his assistance with the videotaping of the activities provided a different perspective to the sessions. His comment on the time spent on student presentations of problem solutions provided me with a different idea to try to maximize student involvement.

Collaboration is necessary for providing a different point of view, new ideas, support and an effective means for reflection. In a study of implementing change Wideen, Carlman, and Strachan, (1987) state:

> At all stages of the process it appeared important for teachers to engage in a process of reflection. Those who were able to reflect realistically upon their teaching and to learn from that reflection, appeared to have a great advantage in making improvements to their classroom practice. (p. 250)

The usefulness of being able to work with other teachers on this project became clearer to me as time progressed. I was simultaneously involved in planning an outdoor math camp with five other teachers, utilizing active and cooperative learning as the themes. The experience of being a participant in cooperative learning and practicing it with peers made me realize how important it is to experience the process to really appreciate the difference between implementing a theory and being a part of it. My experience with these people also involved "processing" (cooperative learning terminology) the sessions, which to me is the same as "reflecting" or "analyzing" (action research terminology). It is difficult to admit that the bulk of my educational training has been administered to me via the "transmission model" and that I am attempting to implement a more active and reflective model of education without having been exposed to the experience myself. This brief exposure (approximately 30 hours) to collaboration on the math camp project was very rewarding. I feel that more collaboration with others on this thesis project would have enhanced this project commensurably. It is one recommendation that I would suggest to SFU with the next cohort of mathematics teachers enrolled on the master's program that they work together on a project to be implemented for students.

THE SETTING

Schou Educational Center is an elementary school that has been converted to the district resource center for Burnaby. It is an old building, but the rooms have been updated to provide office spaces and assembly or conference rooms. We worked in the Assembly Room, with decorative cedar panelling, no windows, video projection equipment and table seating. I felt that it was important that the students felt empowered to participate in the decision-making process and that meetings at schools would have been more restrictive on the students' initiative to become more involved. The importance of considering the setting for this project is described by Bishop (1988):

> So the learner's role is to construct ideas, and the social (and physical) environment's initial role is to allow ideas to be constructed . Furthermore the environment must promote negotiation, while the learner must respond to that kind of feedback and become more involved. These are the essential features of learning in general and of the enculturation relationship in particular. (p.127)

By departing from a regular school atmosphere, it was hoped that the setting would have a positive effect on the attitudes of the students. The physical arrangement was changed every meeting and the groups sat in different parts of the room.

A proposed agenda was presented to the students at the beginning of each session, with a break for juice and cookies occurring mid-way through the meeting. I had anticipated the breaks to be the high point at these sessions for the students. At the beginning, they were fairly important. During the first breaks, students took the opportunity to become acquainted with people from other schools, and this helped relax the atmosphere. As the sessions progressed, the break was taken as just one of the transitions of the program, and quite often the students would continue working for up to five minutes before they went for the refreshments and then continued right on with their task. The discussions during the break most often centered on the problem the group was working on at the time. Mumbling through a cookie was a norm in these conversations.

In general, I would describe the setting as "serious", with activities structured solely by the myself in the first two sessions, with a transition to a more relaxed and informal framework with student input to the activities and their timing. My observation, although it is most likely not without bias, is that the students became more involved in the problems that they were working on and also with communicating with each other when the need arose. I felt that because they were allowed to decide when to cooperate that they felt more responsible for their learning.

USING THE VIDEO CAMERA

Recording the events for research was a new experience for me. In hindsight, I would recommend that anyone wishing to use this tool for educational research would benefit from training prior to any attempts at implementation. With the exception of the technical use of the videocamera, my only experience in the use of videos in education was a seminar hosted by David Pimm (SFU 1990) which exposed me to the difficulty of actually interpreting what was happening in a pre-recorded event.

The equipment that I used was a Magnavox (model 1005) camcorder and for editing I used two Panasonic (model NV-8500) videocassette recorders with individual colour monitors inter-linked by a Panasonic (model NV-500) editing controller. A very useful feature of the editing controller is the use of a counter that displayed real elapsed time. This allowed me to "log" the tapes, and in the case of the first two sessions when the camera was mounted on a tripod and was running continually, the elapsed time furnished data on the progress of events of each session. Another advantage of mounting the camera on a tripod and setting it running in a corner of the room with the zoom on wide angle is that the students soon forget about it, and candid shots of events in about one-half of the room are recorded. One disadvantage of this procedure is that the quality of the picture is not very good, the audio picked up is virtually useless, and reviewing the tape requires great concentration and frequent rewinding to view the salient details. It is a tedious task to review a video that has been recording constantly whether there has been any action or not.

Tom O'Shea visited the December 6,1990 session, and volunteered to operate the camera. This brought to my attention that there are three benefits to having a camera operator. The first is that the picture is focussed on the event being studied, and detail that is not apparent in unmanned videos can be captured. The second is that the camera follows the action, recording significant events, and making the review of the tape much more interesting. The third is that the camera operator brings his or her own perspective and values to the fore, that is, by watching what the camera swings to, you realize what the camera operator is interested in, and this is not necessarily what you would have followed.

One use of videos that could be useful is the analysis of interaction between people. A typical example is the Flanders interaction analysis chart (FIAC) which categorizes verbal interaction every 3 seconds. I think that the videos would be less intrusive than an observer stationed within a group performing this analysis. I modified the procedure for categorization every ten seconds and noted that as the sessions progressed there was movement away from my domination to more activities with student group work. I did not apply this technique extensively for two reasons. I had abandoned the study of problem solving by students and I felt that my video tapes did not track any particular group long enough to record significant changes.

On the basis of insights I gained in this project, I would recommend that teachers use video tapes of their classrooms, no matter what their degree of sophistication in the use of this technology is. Just the simple act of watching video recordings of events experienced in the classroom generally increases the quality of the reflection that one engages in on their practice. I feel that I have been able to grasp deeper insights of the process of learning and to make more detailed comments to support my interpretation by being able to review these tapes.

FEEDBACK FROM THE JOURNALS

The students were asked to maintain a personal journal. In the beginning, they were asked to respond to specific questions, but as they became more conditioned to writing about their observations, they were asked to make general comments. I began with specific questions, hoping to make it easier for them to engage in the process of reflective thinking and to provide direction in the kind of information that I was interested in. In hindsight, I wonder if I had structured the process too much at this early stage, and removed the opportunity for more creative and individual responses. As I began to make the questions more open-ended, the quality of the responses did not meet my expectations. Consequently, the journals did not provide me with responses which incorporated reflections of which I thought the students were capable. Another factor which may have contributed to the sterility of the journals is that the students did not develop a sense of ownership of them. I collected the books at the end of each session to process the information. What I unwittingly did was deprive the students of the opportunity to reflect more privately and leisurely on any issues which they may have been dealing with. I have seen some applications of journal writing in which students include doodling and sketches. This is one approach which I would like to try in order to personalize the journal, which may result in student's providing much richer insight to both themselves and the reader.

I had anticipated three results from using the journals. First the students would have a diary of their feelings and attitudes as the consequence of the activities that they were engaged in, and any changes in attitude would also be recorded. Secondly, I hoped that by thinking of their learning process, the students would become more responsible for it. Finally, the journals would provide a means for the students to communicate personally with me.

Surbeck, Han, and Moyer (1991) provide a structure for assessing responses to journals based on the following categories:

Reaction:
1. Positive Feeling
2. Negative Feeling
3. Report
4. Personal Concern
5. Issues

Elaboration:
1. Concrete Elaboration
2. Comparative Elaboration
3. Generalized Elaboration

Contemplation:
1. The Personal Focus
2. The Professional Focus
3. The Social Ethical Focus (p. 25-27)

If the complete sequence (reaction-elaboration-contemplation) is used, the writer progresses from "personal and concrete reactions to a more sensitive social and ethical perspective" (p.27).

THE RESEARCH CYCLES

The intent here is not to report the events play-by-play at each of the meetings with the students. The agendas of each session, the video logs, and the activity sheets assigned to the students provide a guide to the events. What follows is a description of what I think were important events and the action taken to improve practice.

On October 25 I met with Ivan Johnson to finalize the plans for this Cayley Review. I had met with Ivan twice before and most of the organizational planning had been completed. Bookings for rooms, equipment, refreshments and nametags had been made. Ivan had a general idea as to what I wanted to achieve, and although I had an outline of the events for all sessions, we both agreed to keep the plan "loose" so that we could adapt and make changes as we felt necessary. I had decided to plan more activities than the two hours of each session, just in case the students were quicker than I had anticipated. Ivan was more experienced with cooperative learning strategies and I asked for advice on "group processing" of the cooperative activity. This is in direct reference to Johnson and Johnson's (1990) model in which the group takes time to discuss how well the group performed on the cooperative task. We agreed that with the limited amount of time that we had, that we would try to focus more on the mathematics than on this aspect of cooperative learning.

November 5, 1990

The first session began very formally. The students completed the attitude scale "Mathematics as a process" reported in Robitaille, O'Shea, & Dirks (1982). When this was completed, I stated my goals, asked the students what their goals were, described the history of mathematics contests and how they can affect their personal lives. I described cooperative learning versus competition, and highlighted the benefits of each strategy. I described non-algorithmic problems and how group work may allow them to be solved more pleasurably. According to the video log, this introduction took twenty-five minutes to accomplish. A review of the body language of the students on the video and a conversation with Ivan Johnson confirmed that this activity was too "teacher focussed" and too long for an introduction to what I was trying to propose. The mind is willing, but old habits are hard to break.

As an exercise, I asked the students to think about whether they would like to work cooperatively with another student or by themselves on the task of completing ten questions from the 1990 Cayley test. Each student was asked to sit at a table with a student from a different school, beside each other if they wanted to work cooperatively, or opposite each other if they were competitive. After this exercise they were to rejoin their school based group to cooperatively submit one answer sheet for the ten questions. A pan of the room with the video camera indicated that all of the students desired to work cooperatively, but most were not able to overcome the difficulty of working cooperatively with a complete stranger. When the students re-formed their school based groups, they worked quite well together, with heads together and often more than one of them out of their chair.

I had intended to capture both audio and video/audio recordings at all of the sessions. When I placed the microphone of the audio tape recorder at the center of the table at which a group was working very cooperatively, they immediately "clammed-up." After this occurred several times I decided to dispense with the idea of obtaining audio recordings of the individual group sessions. I had originally planned to study the process of students working together in problem solving, but decided that I couldn't handle both machines by myself.

The student groups from Burnaby South did not attend this session due to an administrative mixup. Support from other staff was not as great as I had hoped for. The only sponsor teacher that was able to attend was Carol Quinn from Cariboo Hill.

I was disappointed after this session. I had anticipated that the students would be very keen to interact with each other, but it was discouraging to observe that the students interacted with others for only about thirty minutes of the two hours that they were there. I felt that a major effort was going to be required to improve the situation. The first plan was to involve the students more, and I proposed to have them work in pairs. Keep it simple and focus on interaction.

November 23, 1990

At the beginning of this session I was still really interested in studying the process of group problem solving. I had given the students within each group instructions at the first session to use one color of pen and that the group would solve the problem on one sheet of paper to hand in. The attempt to study the problem solving process was not successful, the failure being the result of several factors. The major factor was probably due to myself being too busy directing the events of the session and monitoring student groups, leaving little time for manning the video camera and keeping my own notes. Another problem was that the students and I were not well acquainted and we had not developed a rapport. I think that it required some knowledge of what I expected them to do plus a little more trust before they would have supplied me with the data I was attempting to collect. After this session I abandoned my attempt to study the process of problem solving.

We began the meeting with student presentations of the problems they had worked on at the last session. It was a disappointing start. Some of the groups had not attempted to complete the problems even though they knew that they might be called upon to go to the front of the group and present a solution. The presentations were slow, but I still felt committed to using student presentations for several reasons. One example was the case where a student (Bruce V.) presented a textbook solution to a standard problem, followed by a volunteer (Bruce C.) who presented a different solution to the same problem that was much more elegant. I think that it is important for students to see different patterns of thought, and it also gives students a sense of pride when they are able to show their creativity. I also felt that presentations were a method to ensure individual accountability and group interdependence. The individual is on the spot when he or she has to present a solution, and the group is dependent on the individual because he or she is the source of "points." To a certain degree, the recognition of the group's competence and status among all of the schools is also dependent on the individual's performance in this competitive structure. The method of choosing presenters was by random selection of ballots coded to the student's name tags. Many students didn't like this system, but were not able to suggest functional alternatives.

I felt that we had made some success this session. The students were engaging each other more effectively and the atmosphere seemed to be more relaxed and friendly. The students seemed to recognize that we didn't have any classroom rules and I felt that their behaviour was more adult. Upon reflection, I would compare the maturity of the socialization to that of a meeting of teachers which I attended where they were engaged in the group task of creating examination questions. Humour was noted in several parts of the video.

December 6, 1990

I was struggling with the use of student presentations, they were slow and not as efficient as I had hoped that they would be. The students lacked experience in presenting to a large group and they were not well prepared. I lectured for five minutes at the beginning of this session justifying the use of this format in our meetings, hoping that there would be improvement.

In reviewing this video I realized the advantage of having another teacher be the camera operator. Tom's eye followed activities and details that I would not have or he did it in a manner that was different than mine. He recorded a group arriving late, and the exact time of their arrival. He followed the explanation of a problem within a group. He captured a group working very well cooperatively as well as a student completely off task, reading a pocketbook. When I saw that, I reviewed the other videos searching for other evidence of inefficiencies. I realized that the students were not on task as well as I had assumed, even with this group of "high ability students." Following this session, I decided that videos taken by a camera operator are much superior to a stationary tripod camera operating with wide open zoom and no personal input. In the next session, I operated the camera myself but found that I was much busier with other activities, and I taped less of the sessions than I had hoped.

What I also found interesting about Tom's camera work is that he recorded a great deal of footage of a particular group that I thought was working very well cooperatively and were developing their communication skills with regards to mathematical problem solving. This group happened to be younger by a year and, eventually, placed ninth in

Canada on the Pascal contest.

It was at this time that I considered the groups from Cariboo and Central to be working very well together, and making a serious attempt to build a cooperative team. The students from Alpha were concerned about missing class time and each group was attending alternate sessions, destroying any continuity of progress.

In their journals, the students had indicated a desire to work and socialize more with the students from the other schools. I implemented the jigsaw technique (Johnson & Johnson, 1990) as a method of creating new groups that were formed on the basis of students from different schools. Each member of the school based group of 4 was assigned a number from 1 to 4. I instructed the 1's to go to the flip chart at the left front of the room, the 2's to the right front, the 3's to the back right, and the 4's to the back left. Since there were only four flip charts at Schou Education Center, I decided to work with four large groups in order to provide the observer with the opportunity to follow the group discussion. This decision resulted in the jigsaw groups being too large. Each group had eight students in it, which in hindsight, was a mistake. Another observation based on review of the videotape was that the boys seemed to dominate the discussions.

After this session, I began to think of the structure of the competition and felt that with these students, it was not necessary to create such an artificial situation of extrinsic rewards with the accumulation of team points. It appeared to me that the time used for keeping score could be better utilized by students interacting with each other. I reduced the emphasis on the team scores and the students' motivation seemed sustained by their intrinsic interest. Kagan (1990) states:

> If teachers structured things so that there always was an extrinsic reward for learning or cooperating, or so that it was always adaptive to cooperate, they would rob students of experience in settings which student's interests' and needs provide the sole basis of cooperating and learning, and experience in settings in which cooperation is not the most adaptive response. (p.4:13)

I felt particularly disappointed with this session. I had high hopes of impressing Tom since this was the only meeting that he was going to attend. Tom suggested that the time spent on student presentations to the entire group may not be making optimal use of the available time. Student presentations were important for the presenter, but not necessarily for the receiver, especially if she or he already knows the solution. I began to think of situations where I could have a student present to a smaller group. This process would give the students much needed practice in this type of activity. An activity which I planned to utilize more would be to have the students work in pairs and negotiate solutions acceptable to each other and then present their solution to the other two members of their group. This would increase the number of presentations being made by the students.

I had participated in a cooperative group session that Ivan had organized for Grade 7 students, and these younger children seemed to be able to work much more comfortably with each other than the Grade

10 students with whom I was working with. When I expressed my concern with Ivan, he pointed out the students work with each other cooperatively much more often in elementary schools, and also that many of these students were working in groups in which they had already developed cooperative skills.

Although these comments eased my sense of lack of accomplishment initially, they raised more serious concerns for me. If elementary students are becoming proficient with cooperative learning techniques now, what will happen to them when they reach the high schools, where many teachers are still teaching under the "transmission model" of educating children. It was at this time that I visited Forest Grove elementary school and observed quite a different process of learning. The discrepancy between what is happening in elementary schools and high schools became apparent to me when I observed the cooperative skills that the eleven and twelve year olds possess and those that the fifteen and sixteen year olds don't. This was further motivation for me to acquire more skill and experience in a style of learning that my students would already have experienced when they arrive in my classroom in a few years.

February 7,1991

The weather conditions necessitated a change in schedule to the end of January, then school exams caused a further delay. I feel that the two month interval did not have a good effect on the project. The students had lost focus on the problems that we were working on and were slow to reach the level of cooperation at which they were working prior to the Christmas holidays. Students from Burnaby South did not attend because the school was having a "career day."

The student presentations of problems was slow and not particularly inspiring. The students were not well prepared. This activity was aborted and I presented a brief lesson on using graphing calculators, specifically the Casio fx7000. Obtaining a set of these calculators required some effort, first making several phone calls and then two trips. The set belongs to the British Columbia Association of Mathematics Teachers (BCAMT), and can be borrowed by any member, although this is not widely publicized. I had two reasons for using this activity. The first was that the calculators provide a manipulative device that the students can actively engage with. By working in groups, they could help each other develop their skills in using this technology. It provided a focus for communicating, and in most groups each member was able to be a contributor.

To describe the process generally, the students "played" with the calculator individually, exploring the keypad functions on their own. When they began the assignment, each person brought her or his experience to the application, and the group was able to effectively use the calculator. The second reason for introducing this innovation is that I feel that it provides a powerful means by which students can bridge the theory between algebra equations and visual representations of them. This is a recommendation made in the NCTM Curriculum and Evaluation

Standards (1989): "The integration of ideas from algebra and geometry is particularly strong, with graphical representation playing an important connecting role..." (p.125) Although the students were aware that they could use calculators on the 1991 Cayley, they assumed that the graphing calculator would not be allowed, and some questioned (in their journals) the validity of using them in our sessions. In general, most felt that it was a useful activity and also "a lot of fun."

I felt that I was developing in the students a sense of control in the direction that the sessions were taking. I asked them for feedback in their journals, and when time permitted discussed the appropriate application of the activities with them, in large group as well as small group and individually. Since they knew that it was a new experience for me, and that I was trying to improve my practice (stated to them several times) as well as providing an opportunity for them to improve their skills, they were very forthcoming with ideas, suggestions and criticism. I observed, in reviewing the videotapes, two cases of student input to the group decision-making process as we were working as a large group.

At the end of this session I had mixed emotions regarding this project. I felt pretty good in achieving some success in creating an environment in which students felt that they had some control of their learning. I felt depressed at the very partial success in completing tasks on Cayley contest activities. One such task that had not been well done was the group preparation of a mock Cayley that the group would administer to a different team. This was to be handed in to me this week in order that I could review it and advise them of any necessary changes or errors. Only two tests were handed in. At this point I did not feel that there was going to be any effect on their test scores based on the amount of review that they had completed.

A change in practice was the time of the meetings. The last two sessions were planned to occur in the mornings in response to student's comments that they always missed the same afternoon classes.

February 14,1991

Valentine's day coincided with celebrations for Chinese New Year, and in alignment with requests by students in their journals to become more friendly with the people from other schools, time was taken to perform an ice-breaking exercise. The task was for students in their jigsaw groups (not based on their school) to learn the name of a new face, and one significant fact about that person in order to share it with the class. This activity worked very well, and I wished that I had done this at the first meeting. The students enjoyed the experience, and it gave them the opportunity to speak informally before the entire group. The activity created a much more positive social atmosphere in which the students worked.

I was anticipating interesting work from the students at this session. I felt the task of preparing a mock Cayley exam while working cooperatively as a group should highlight many social skills and creative talent. The tests that the students had created varied from very good to two groups that failed to complete the task. Burnaby Central put together a professional looking "fake Cayley" which they had used an IBM computer to assemble and it was printed out complete with graphic diagrams. School teams exchanged tests, which they completed cooperatively within their own groups. When completed, the originators of the tests marked, with discussion, the solutions and returned them. It should be noted that only one copy of the test was available and I think that this does have an effect on the way the test is written and marked.

After allowing the writers time to go over their test results, the school groups were jigsawed to form groups of four (two students from each school). They took each test and performed two tasks. The first was to ensure that the group understood the questions and the solutions, or if there were any errors in the test, to rewrite the questions. The second task was to analyze the test for authenticity in representing Cayley questions, and quality (i.e. creative or interesting to write). In thinking about the events, there are two routines that I would change. When the students were jigsawed and discussing the tests, I overheard and observed many interactions between the students that was rich in information regarding how they thought and felt about what they were doing. I think I would attempt to have at least two cameras recording this activity. The second change would be to give the students a clearer idea of what my expectations were regarding the post-briefing session. I think that I would have them write an evaluation of each test.

When I interviewed some of the students, I discovered that it was difficult for them to prepare the tests collegially. When forced to work on the project at a computer terminal, as the group from Central did, collaboration was facilitated by the need to use technology. However, one group that prepared the test resorted to assigning sections of the test to individuals to prepare it. I was disappointed that some groups did not feel comfortable in creating their own questions, but resorted to reviewing old Cayleys and selecting questions from various tests.

In terms of action research, it would have been ideal to repeat this process, at least for me. It was a very interesting activity and I had many ideas that I would have tried which would benefit my experience.

I felt happy with the quality of this session. The activities were interesting to the students, and, especially for those that had prepared the tests, a very good way to extend their experience in writing tests. I liked the results of the students processing an exam after it was written, and incorporated this activity into the final session.

February 21,1991

The final session was guaranteed to be well attended. Promises of pizza and time for social activities created anticipation for the students. The first activity was to write the posttest of the "Mathematics as a process" attitude scale from the Second International Mathematics Study (SIMS) survey (Robitaille, O'Shea, & Dirks (1982)). Before I handed out the Fake Cayley that I had prepared, I asked them to consider the strategy that they would like to follow when writing it. When the test was handed out, they wrote individually for approximately thirty minutes. Most wanted to work individually to understand the question before working with others. Groups started to work cooperatively after this initial period of silence. The structure of the test affected interaction. The first section is relatively easy and did not require collaboration. The last part of the hour the groups worked very cooperatively, in pairs or in groups of four as they deemed necessary or if they were all stuck on the same problem. As they began to realize that time was running out, they worked cooperatively within their small groups but conscious of the fact that they were competing against other schools. They assigned tasks to each other in order to produce one answer sheet with some degree of consensus.

During their break I marked the tests and returned it to them. They were then assigned jigsaw groups to find solutions to all questions. Students, selected by the peers in their jigsaw groups, were then asked to present solutions to the more difficult questions. The session ended with journal entries and then a social hour (lunch time) with pizza and juice.

The highlight of this session for me consisted of the three groups of students who came to me at the end and thanked me personally for hosting the activity, telling me that they enjoyed it, and also that they felt they had benefitted from the experience.

THE INTERVIEWS

I interviewed five students at the end of the sessions to gather more in-depth data about how the students felt about the events at Schou

Education Center. The interviews were conducted at the student's school during her or his mathematics class, lunch time, or after school. They varied in duration from 15 to 40 minutes. I found that the girls were more willing to volunteer their ideas, and interviews conducted during class time seemed to progress at a more relaxed pace.

I was not experienced with conducting interviews, so this was another learning experience. It was difficult to know how much to "lead" the student into a question. As I conducted the interviews, I went through a cyclic process of not providing sufficient information to the student in order that she or he knew what I was asking, or of prejudicing the student's response by giving too much of my point of view. An example is the question "did cooperative learning assist you in any way with respect to social skills?" When no explanation was given, the response was "no", but when I explained the question in detail, the students responded "yes" and I was not sure that I received an unbiased answer.

I did not know the students well, and the dialogue at the beginning of the interviews was hesitant, as we built some trust. At the conclusion of approximately thirty minutes, I felt that I had gotten to know them better than some students that I had in a class for a whole year. They seemed to consider each question carefully, and often the time before a response would be three to five seconds. The reflective nature of their comments in the interviews provided a dimension to the student-teacher relationship that is difficult to establish in a classroom setting. As I gained experience with this technique, I found myself extending its application beyond probing their thinking to sustaining and extending their thinking.

The plan to conduct the interviews at the conclusion of the treatment was made on the assumption that much information could be collected at this time. This was appropriate to two questions that arose from the results of the project. The first was the anomaly in the response to Questions #4 & 8 in the SIMS survey described on page 68, the second was the discrepancy in performance on the Cayley contest by the students from Alpha Secondary School described on page 78. However, doing the interviews at the end resulted in the loss of the opportunity to incorporate the interviews into a cycle of learning on my part for improving my technique with this skill.

CHAPTER IV
RESULTS

CAYLEY CONTEST RESULTS

Before analyzing the Cayley contest results, it should be pointed out that the contest committee has made a change, starting with the 1991 contests, in philosophy (Isenegger, personal communication, 1990) which has resulted in significantly higher scores this year than in the previous year. The change has been "a conscious effort to make the first two parts of the contests easier so that students will feel better about their performance." The 1990 Pascal average score for 28 880 students was 53.30 while the 1991 Cayley score was 73.25 for 19 869 students. While these figures suggest a change of 19.95 in average scores, the greatest change occurred in the lower range of scores. To have a standing in the 99 percentile of the official competitors, a student had to score higher than 104.00 in 1990 on the Pascal, while he or she had to score higher than 115.75 in 1991 on the Cayley. This suggests a change of 11.75 in average scores at the higher 99 percentile level. An uncontrolled factor is the loss of 8011 official competitors, or 28 % of the sample. My guess is that many of these students scored low on the Pascal in 1990 and were discouraged to write the contest in 1991, and they would have reduced the Canadian average score on the 1991 Cayley. For the students participating with cooperative techniques at Schou Education Center, their average 1990 Pascal score placed them at the 85 percentile, as did their 1991 Cayley average score. From this perspective, the results indicate a null gain.

A comparison of Burnaby students that wrote the Cayley contest indicate gains in scores for the students participating in the study. A difference in scores shown in Table 1 of 17.5 for students in the project compares well with a change of 7.2 for Burnaby students not participating in the study. The F test results were $F=5.62$, $df=1,52$, $p=0.02$. Appendix E contains the Pascal and Cayley scores for each student participating at Schou Education Center and also the difference between these two scores. Appendix F contains the Pascal and Cayley scores for each student not participating at Schou Education Center. It may be suggested that since this project was taking place in Burnaby, sponsor teachers at the schools might not have prepared all of the students for the contest as they had in previous years. For this reason, the scores for a cohort of students from St. George's School, a private school outside the district for which scores were readily available, was analyzed. Appendix G contains these scores. A difference of 10.8 is very close to the 11.75 change in scores for students in the 99 percentile of official competitors in Canada.

Thirteen subjects in this study ($n=28$) obtained Pascal (1990) scores less than 50% and this group achieved an average 28.1 gain on the Cayley (1991) score. The fifteen higher scoring students showed an average gain of 9.5, indicating a ceiling effect for higher ability students.

TABLE I

Summary of Cayley Contest Results

Group	1990 Pascal	1991 Cayley	Difference
Burnaby n=28	78.9	96.3	17.5
Males n=19	82.3	97.2	14.9
Females n=9	73.8	93.9	20.1
Orientals n=19	77.1	96.2	19.1
Caucasian n=9	82.7	96.8	14.1
Burnaby Non-participants n=26	82.6	89.8	7.2
Canada (all competitors)	53.3 n=28880	73.3 n=19869	20.0
St. George's n=20	86.1	96.9	10.8

NOTE: Individual Cayley Scores increase from 0 to 150 in 0.25 point increments. The results of Table I are rounded to one decimal place.

In Table 1 it is interesting to note that the the difference in the mean scores of the four different groups (male, female, oriental, and caucasian) in 1990 was 8.9 while this difference in 1991 was 3.3. An interpretation I would like to suggest is that the cooperation amongst the members of the four groups provided this leveling effect on the scores between the groups.

Another source of comparison on the relative success of this project are the standings that the Burnaby schools made in the Canadian Team Honour Roll for the top fifty teams. In 1990, one school (Burnaby North) placed (ninth) on the Honour Roll for the Cayley Contest. In 1991, three schools (Burnaby Central, 12th; Burnaby North, 30th; and Burnaby South, 40th) placed on the Cayley Contest Honour Roll. This data is contained in Appendix H. Three of the five schools that participated placed in the top fifty of 1334 schools in this competition. In order to compare the relative improvement with respect to the particular grouping of individuals, the 1990 Pascal Honour Roll consisted of two Burnaby schools (Alpha, 28th; and Burnaby North, 40th). It is interesting to note that Alpha did not place in 1991 and fell from honour roll status. I attribute this situation to the fact that the school was having in-school term examinations at the time of the contest Three students from Alpha involved in this project did not write the Cayley, and three students that did write had gains in 1991 of -18.75, -6.50 and -11.00. The two students that achieved positive results had gains of 15.50 and 12.25. Alpha students also did not appear to be as committed as much as the other schools to reviewing for this competition. They expressed concern that they were missing too much class time at school and on average each Alpha student attended three of six sessions.

A comparison of the relative position of the Burnaby schools on the Provincial Zone (9) Cayley results in Appendix I shows the mean of the school rank on the 1990 Pascal results of 7.0, compared with the 1991 Cayley results of 5.4 indicates improvement for the Burnaby schools.

A problem with the two previous comparisons of Honour Roll teams is that they are composed of the top three student scores from each school, and this selection resulted in some teams that were not composed entirely of students participating in the study (Central, one; North, none; South, all). (See Appendix J.)

An unexpected development resulted from a parent becoming aware of the project through other students at school. She persuaded me to accept another group of students into the cohort. Because it was a late entry, I was not aware that the students were one year younger than the rest and that they would be writing the Pascal contest in 1991, not the Cayley. This group seemed to work very well cooperatively, and it was the group that Tom O'Shea focussed the camera on a lot of the time when he filmed the December 6,1990 session. The four students that participated had the top scores from Cariboo Secondary, and the school team placed 13th on the Canadian Honour Roll, of 1416 schools. This is one of the interesting aspects that the flexibility of action research bestows on a project of this nature.

ATTITUDINAL RESULTS

A survey "mathematics as process" was applied as pretest to the students at the beginning of the first session and as posttest at the conclusion of the last session. The scale presented to the students suggests that "to be positive is to view mathematics as a field where speculation and heuristics are important rather than just rules; where the discipline is changing rather than fixed; and where there is opportunity for creative endeavor by imaginative persons" (Robitaille, O'Shea, & Dirks, 1982, p. 149).

Items marked with an asterik are negatively worded, and are re-coded. The results showed a mean positive gain of 0.10 in student's attitudes towards "mathematics as a process" with a pretest mean of 3.64 compared to a posttest mean of 3.74. A 0.10 gain on a four point spread represents a 2.5% gain. Relatively larger gains occurred on items 11 and 13, with students' views of "mathematics is a set of rules" changing to a more flexible approach.

The survey and item results are presented in Figure 1.

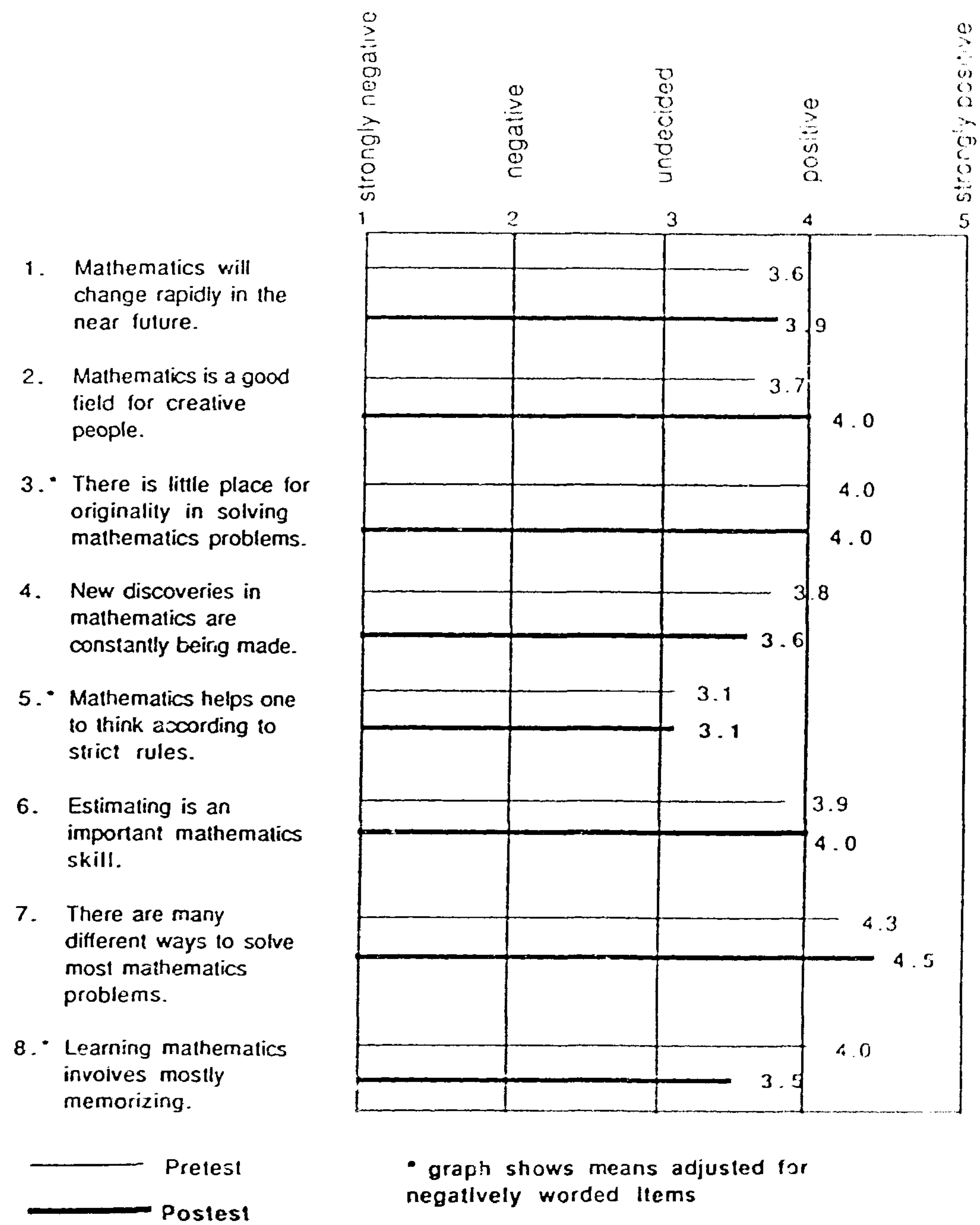


FIGURE 1. Item means on the "Mathematics as Process" Attitude Scale

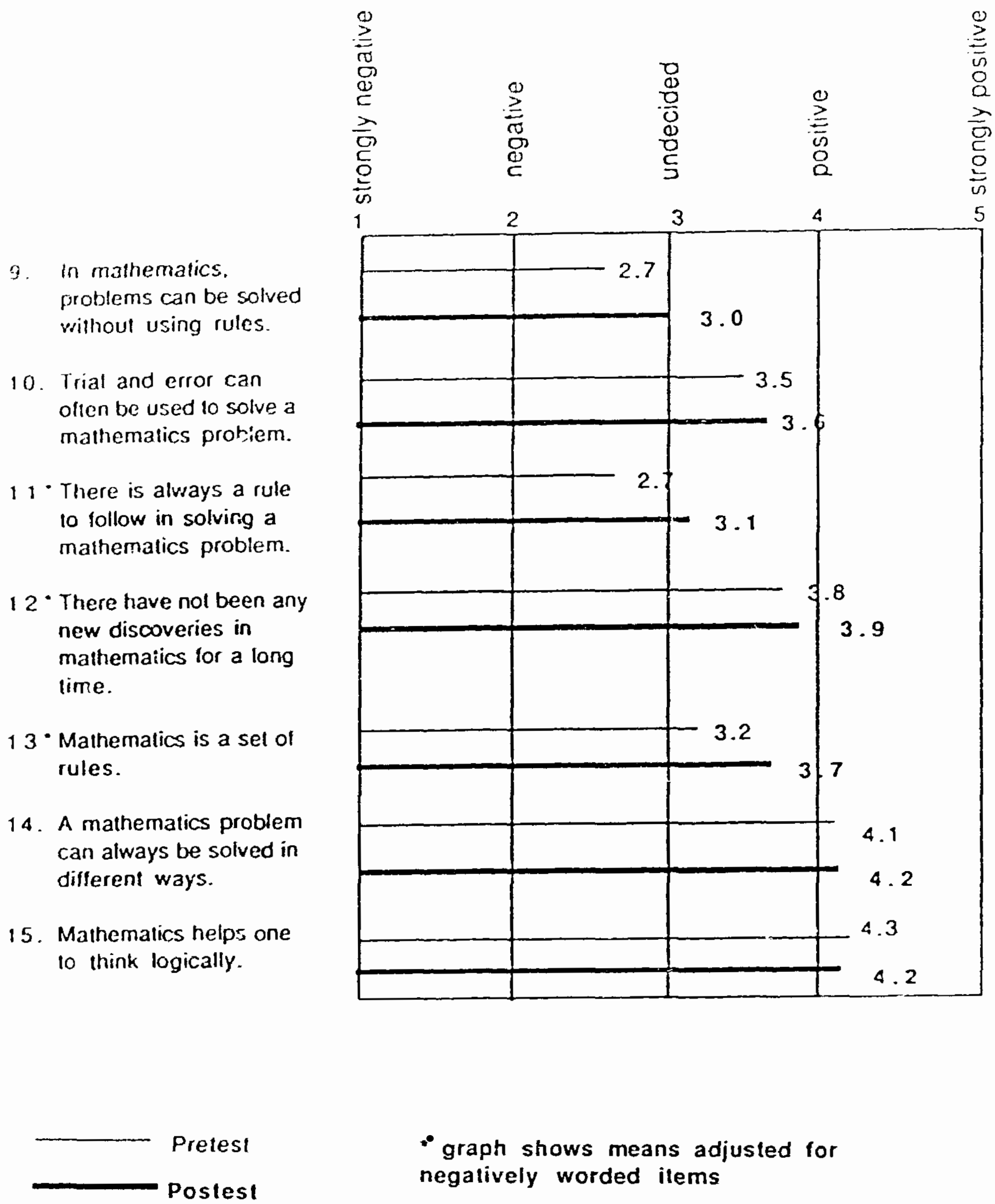


FIGURE 1. Item means on the "Mathematics as Process" Attitude Scale

Items 3, 5, 6,14, and 15 showed little change but were generally positive; the remaining items indicated desirable changes in attitude, except for items 4 and 8. Two of the students that I interviewed suggested that these results may be due to the topics (quadratic formula and trigonometry) being studied in class during this two month interval, and the survey results on these two items were influenced more by the memorizing that they had to complete in class than by the events at Schou.

RESULTS FROM VIDEOTAPES

The Panasonic videocassette recorders had a counter that indicated real elapsed time and this feature was very helpful. I reviewed the tapes and created a log of the activities. By reading the record of events I was able to refresh my memory of the activities as well as relate this with the amount of time spent. The record (Appendix K) can be reviewed quickly to provide cues to the researcher's memory as he or she attempts to recall the events.

It is difficult to use the written language to describe the results captured visually by the video camera. There were moments of frustration, boredom, and time off task as well as shots displaying positive group interdependence, active learning, enthusiasm and enjoyment recorded at the sessions. The tapes were selective and did not register all activities. The viewer must place his or her own interpretation onto these events. My interpretation is that the students benefitted from these activities.

RESULTS FROM JOURNAL ENTRIES

The summary of student journal entries recorded in Appendix L portrays some of the reactions that the students had with using cooperative learning strategies while reviewing for the Cayley contest. For example, reactions to the jigsawing event provided me with feedback for planning the following sessions. Comments such as "We should have some more "jigsaw" groups (4 responses)." "It's nice to know different people, today's session has been fun" prompted me to use more jigsaw activities. Another reaction worth noting is the 18 positives of 29 responses to the question; "Do you think that you will make more of an effort to work with other people on math questions?"

A student's elaboration "by letting the student explain their answers it makes things easier to understand" indicated to me that I accomplished some of my goals, such as to have the students become more reflective about their learning.

Remarks in the journals that give an indication that the students are really understanding the process in which they are engaging is apparent in journal comments that show contemplation; "I think that this project should be made available to more students at other academic levels....set up for C & D students [students with C & D marks] where some added exposure to math may be useful to them."

Most students indicated that they enjoyed the experience at Schou. They also expected the experience to help them to write the Cayley test in terms of new techniques (learned from peers), improved attitude and confidence, and by the extra practice on the questions.

Their critical comments helped me to plan the sessions and also involved them in the process of being responsible for their learning. Their positive comments sustained me when I was feeling discouraged with our progress.

The students were not given specific instructions in the use of journal writing. I was inexperienced with their use, and clumsily attempted to guide them with a structure of questions. This is an area in which further research would benefit the improvement of educational practice. I think that the research should examine the results of various approaches to journal writing, with consideration for different practices for different age groups. When I read with a reflective attitude the students' journals about events which I had experienced as well, the approach gave me a sense of being a participant-observer.

The changes in student's self-esteem were varied. Some students felt better because they realized that there were many more people who were similar to themselves, that is, they were not as weird as they thought they were because they liked math. Some students did not feel as important as they had because they realized that they were not as unique as past experience had led them to believe. Allan (1991) describes this phenomena in her review:

> Programs designed for gifted students have trivial effects on self-esteem (Kulik 1985). Why are these results counter to prevailing expectations? Kulik (personal communication) raises an interesting point on the relative importance of effects of labeling

> versus the effects of daily classroom experience. He suggests that the labeling (by placement of a student into a low-medium-high group) may have some transitory impact on self-esteem but that impact may be quickly overshadowed by the effect of the comparison that the student makes between himself or herself and others each day in the classroom. (p. 64)

RESULTS FROM INTERVIEWS

Most of the interviews were taped, and the tapes were transcribed. This allowed me to mentally reconstruct the interview and to reflect on the interaction between myself and the student. I was able to inspect certain events by looking at the transcript and listening to the emotion of the student on the tape. In several instances, I turned the tape off during the interview, and I noticed that the students were more willing to describe in detail events which they felt affected their performance. Issues such as their relationship to their teacher at school would not be discussed with the tape recorder on. I conducted one interview without the tape recorder, and the tone was quite different than the interviews with a tape recorder. The transcription of the tapes was time consuming. I think that in a school setting, I would use interviews with students, but I doubt that I would tape record and transcribe them.

CHAPTER V
REFLECTIONS

COOPERATION

My original intention in developing a program to use the mathematics contests together with cooperative learning strategies was to provide a more motivating structure for the students to prepare for the contests. I also felt that it would be a "safe" way to become skilled in cooperative learning strategies, because it would not interfere with regular classroom activities. As I became more involved with the project, I began to see more of the values in the innovation, and more support for the rationale of using cooperative learning. For me this would make teaching more meaningful for the students, and it would solve several problems if done properly. Talking would be a requirement in the class, not an undesirable intrusion. Students would get immediate feedback from their peers, they wouldn't have to wait for me to provide the social mathematical interaction. As I became more aware of what I was doing, through my reflections on the activities with the students and through the university course work, I began to realize the need to develop a more collaborative approach to the research. This realization did not hit me suddenly, but was rather a slow process. At first it was an intuitive reaching out, as in the proposal in Appendix A, where I arranged the first meeting in a manner that I hoped would attract other teachers to come and participate in the planning. I suppose that I was trying to create the mutual-adaptation model for change (Fullan, 1982, p.31). Then it was an appeal for teacher release time at the department head meeting at the presentation of my third proposal. Now that I have read the literature on action research regarding collaboration I have a better understanding of the need for collaboration at the early stages of implementing an innovation. Concurrent with the implementation of this project, my experience in working cooperatively with five other mathematics teachers to plan the activities at the 1991 BCAMT Math Camp was very important for my knowledge of cooperative learning. One of my later regrets was that the university did not promote collaboration of students to work on a thesis. I had made many friends in the master's program at SFU with whom I exchanged ideas about my thesis, it was too bad that I could not have done this in greater depth.

The theme that I had tried to develop for the students by using cooperative learning is also a theme that is parallel to what I think teachers should be doing, that is, collaborating. Aoki mentions in his address to the members of the Canadian Association for Curriculum Studies (1986):

> A situated curriculum is a curriculum-as-lived. It is curriculum in the presence of people and their meanings. It is an experienced curriculum. I like to call it the first order curriculum world.(p.4)

I think that Aoki is referring to the meaning of curriculum as content as well as the experiences that students will share. By working actively with each other, students will create more experiences. If teachers work collaboratively on curriculum, their shared meanings will more likely create an "experienced curriculum." I think this project has helped me to understand his comment now, where I didn't when I first read it. It is very difficult for a teacher who has experienced education as the "transmission of knowledge" and who has been trained to use this method to change to an educational model of "stimulation of learning."

I had three reasons for believing in the success of this project. The first was the quality of the innovation. I quote Neil Davidson (1990):

> Systematic and frequent use of small-group procedures has a profound positive impact upon the classroom climate; the classroom becomes a community of learners, actively working together in small groups to enhance each person's mathematical knowledge, proficiency, and enjoyment. Frequent use of small groups also has an enlivening and invigorating impact on the professional lives of mathematics teachers. (p.1)

If these gains could be demonstrated, most teachers would be willing to give cooperative learning a try; if not in their classrooms, then maybe as a coach on a mathematics contest team where there is less to lose if failure is encountered. The second reason was the enthusiasm of the students. If they have fun, find that they are getting better academic results and are meeting new friends, they will want to continue. The third reason was that I think that a teacher-initiated innovation with advocacy from a central administrator has a legitimizing effect. In Fullan's (1982) words: "there is a strong body of evidence which indicates that fellow teachers are often the preferred and most influential source of ideas. (p.46)"

PROBLEMS

I did not always feel confident, my feelings about the project changed with each session. After the second meeting with the students, I became concerned that the results of the project would not meet my expectations. The lack of personal contact and support from other teachers appeared to be the dominant factor. The students were not completing tasks which I had expected them to do at their schools. My contact and influence on them was minimal compared to what their teachers could have provided. Other teachers are required to support the structure of the competitions and also to reflect on the process of implementation.

Another problem was the conflict between goals of the innovation. It was difficult for students to incorporate cooperative learning strategies with the concept of mathematics contests. During the interviews at the end of the sessions, one student confided that he did not like to help other students because he felt that his own (high) score would not look so spectacular because he had helped raise other students' scores.

I was trying to implement cooperative learning for students through mathematics contests, with assistance from teachers. I feel that part of my initial failure to convince teachers to get involved was the result of not being able to clearly convey to them my own goals and that these goals might be similar to theirs if they had been able to examine them more closely. How could I describe to other teachers an activity that would be interesting for them without personal contact. "It is essential that the nature and purpose of the change are explicit. Implementation often fails because the intentions of the curriculum are not clearly stated or understood" (Taylor & Werner, 1989, p. 15). Perhaps the format of my proposal was too complex and ambitious. Implementation at the school level would be a lot simpler. It might have been easier to get commitment from fellow staff members and communication should not have been as difficult if this project was at the school level. The problems, commitment and communication, became major obstacles when approached at the district level. I had considered using only one school for the selection of subjects in the study, but chose to involve all of the schools in the district. One reason for this decision was that more teachers would be made aware of the study and I felt that I had a better chance of involving teachers that were really interested in the project.

Finally, the issue of the average age of the mathematics teachers (old) in Burnaby and over-worked teachers might have been the reasons for lack of commitment. Teachers may have seen this innovation as creating more work for them in the manner that physical education teachers are expected to coach sports teams, so there would be parallel expectations of mathematics teachers to coach mathematics teams.

DISCOVERIES

When I compared the Burnaby scores with St. George's School, I realized that there may be differences in gains made by girls compared with boys. St. George's School is an all-boy school. The results in table

1 show the girls to have improved more than the boys. The difference of 5.2 between the gains in average scores indicates that one might examine more closely the achievement gains by boys and girls in using cooperative group work in problem solving. Phelps and Damon (1989, p.640) suggest "girl's learning seems to be particularly affected by peer feedback. Dweck and Bush (1976) found that girls showed significantly greater tendencies to improve their intellectual performance when given constructive feedback than did boys." Unfortunately, I do not think that the results in this study are very accurate since the subjects were not randomly chosen. It is interesting to note that when female teachers made the selection process, there was equal representation of the sexes, whereas when male teachers made the selection, only one of the teachers ensured equal representation. The girls in this study had a much lower average score in 1990 than the boys did, and the ceiling effect on scores would limit the boys' gain in 1991 more than the girls. Further study in achievement between boys and girls when cooperative learning has been used would be of interest. This might be a learning style which would help maintain a more equal balance in the sex ratio of males and females in mathematics classes.

The consideration of differences in results based on sex led me to look at differences based on race. From my observations that the orientals were generally more quiet and more competitive, my guess was that the caucasians would perform better on the contest because they were basically more verbal and cooperative. An explanation for the orientals doing better by 4.9 on the average scores (see Table 1) is that this group had much more to gain by learning to work cooperatively. My experience is that culture has more to do with student achievement than race. Most of the oriental students in this study were first or second generation immigrants. Again this is a speculation, and a more rigorous study needs to be carried out to substantiate any claims. There may also be a confounding effect of the oriental girls' scores on the differences in scores.

A thought occurred to me that the videos could be used to assist the affective development of the students. It struck me that teenagers often expend a great deal of time in front of a mirror creating a "static" image of themselves. If they could observe themselves in "action" with other peers, I think that this would assist them in developing an appreciation of their image in a dynamic manner as they view themselves interacting in a social situation. This is a method of "empowering" the individual to be responsible for his or her learning, and might be a useful tool to improve cooperative learning in groups.

An important discovery from the interviews was the reason why Alpha did so poorly on the Cayley contest. I discovered that the students at Alpha Secondary school had written school-based term exams at the time scheduled for the Cayley contest. Some students did not write the Cayley due to conflicts in the exam schedule, while others wrote it immediately before or after school exams. "For one thing, when we were doing our Cayley was during exams (school). There's a lot of *stress*, so I didn't feel that, umm, comfortable writing it during that Cayley." And another comment "Our school came first in Burnaby in the Pascal last year, and I think we were overconfident."

Listening to students describe how they felt about their participation in the project gave me feedback that is richer than any response in a questionnaire or numerical improvement in test scores. One student's description of her feelings reinforces my conjecture that this was a good project. Her response to the question "How did you feel about your results on the Cayley?" was "I was *really* proud of it. Only one or two marks off the *top!* Besides the fact that I did a *lot* better this year than last year."

Based on the results of this study, one could not say definitively that cooperative learning strategies help improve student's scores on the Cayley contest. The improvement that the students in this project made could be attributed to the increased time (14 hours), the increased attention (Hawthorne effect), the change in philosophy of the contests made by Waterloo, and to school based activities. However, I think that it would be safe to conclude that if a teacher's goal is to improve scores on contests, having the students work cooperatively is a fun and motivating way that provides rich educational social experiences for the participants, with gains in academic results. The converse statement--if a teacher's goal is for students to work cooperatively together in a setting rich with educational social experiences, then having them work on review for mathematics contests will provide good results--is also true.

PERSONAL PROFESSIONAL GROWTH

I feel that I have helped the students in many ways. If not improving cooperating skills, then providing an awareness of what cooperation means. If not improving test scores, then at least providing an appreciation for how they feel about writing tests and what they might do to improve their scores. If not improving their attitude about mathematics, then providing experiences to understand the formation of their attitudes. It is very rewarding that students said in interviews that they have a good feeling about writing mathematics contests and that they feel that they have a good idea of the types of activities that they would like to engage in to prepare for future contests . Some students have told me that they feel that they have a personal commitment to try to improve in future contests.

Was this action research? Had I reflected critically and altered practice in alignment with new experiences? The concept of professional development or improvement is closely tied to action research. This project was very important to me with respect to improvement in my teaching practice. I learned about cooperative learning techniques, about reflective practice with action research, and a new way of relating to the students that makes them feel that they are also responsible for their learning. I have begun acquiring technical skills which I believe practicing teachers should utilize. Video taping, use of journals, interview techniques and collaborating with other people are some of these skills.

A question remains. Is this educational research? I think that there were two studies written up in this thesis. One is the use of cooperation by students for the review of the Cayley Contest. The other study is of a traditional teacher who is attempting to "break out" of that position in order to improve practice. The second study may be of more interest for those teachers attempting to do the same thing.

I think that by combining quantitative and qualitative research methods, one is able to reach a different kind of conclusion that is of interest to teachers. The question "How can I teach students how to write mathematics tests in a manner that would be interesting and fun for them?" can be answered more creatively than two separate questions; a quantitative study "How can I improve test scores?" and a qualitative study "How can I make mathematics more fun?"

In conclusion, this project has been appealing to me because it has given me the opportunity to "live my educational values" by addressing the issues that Fullan (1982, p.21 &116) raises with the statement "Individual, interpersonal and social attitudes and skills appropriate for a democratic society do not receive the equal attention that Dewey so clearly argued they should and that the rhetoric of the formal goal statements of schools and governments implies. " I have made a move to creating more situations for students to be active and to use discussion and collaboration. I am more aware of the power of allowing students to feel that they are responsible for their own learning and can exert some influence. I began reflecting on the possible changes that I could make to my teaching several years ago. Perhaps with the insights I have gained, I may see the process of change bear fruit to the seed planted back then.

APPENDIX A
Rationale for using cooperative learning:

Rationale for using cooperative learning:

(Davidson, 1990, p. 4)

1. Small groups provide a social support mechanism for the learning of mathematics. Students have a chance to exchange ideas, to ask questions freely, to explain to one another, to clarify ideas and concepts, to help one another understand the ideas in a meaningful way, and to express feelings about their learning. This is part of the social dimension of learning mathematics.
2. Small-group learning offers opportunities for success for all students in mathematics (and in general). Students within groups are not competing one against another to solve problems. The group interaction is designed to help all members learn the concepts and problem-solving strategies.
3. Mathematics problems are ideally suited for group discussion because they have solutions that can be objectively demonstrated. Students can persuade one another by the logic of their arguments.
4. Mathematics problems can often be solved by several different approaches. Students in groups can discuss the merits of different proposed solutions and perhaps learn several strategies for solving the same problem.
5. Student in groups can help one another master basic facts and necessary computational procedures in the context of games, puzzles, or the discussion of meaningful problems.
6. The field of mathematics is filled with exciting and challenging ideas that merit discussion. One learns by talking, listening, explaining, and thinking with others, as well as by oneself. Buck (1962, p.563) puts it this way:

> *Let me remind you that student-student interactions are also important in learning, and that at the professional level, much mathematical research springs from discussions between mathematicians. Moreover, a test of understanding is often the ability to communicate to others; and this act itself is often the final and most crucial step in*

the learning process.

7. The role of small groups in mathematical communications is addressed in the *Curriculum and Evaluation Standards for School Mathematics* by the National Council of Teachers of Mathematics (1989):

   Teachers foster communication in mathematics by asking questions of posing problem situations that actively engage students. Small-group work, large-group discussions, and presentation of individual and group reports-both written and oral-provide an environment in which students can practice and refine their growing ability to communicate mathematical thought processes and strategies. Small groups provide a forum for asking questions, discussing ideas, making mistakes, learning to listen to others' ideas, offering constructive criticism, and summarizing discoveries in writing. Whole-class discussions enable students to pool and evaluate ideas; they provide opportunities for recording data, sharing solution strategies, summarizing collected data, inventing notations, hypothesizing, and constructing simple arguments.

8. Mathematics offers many opportunities for creative thinking, for exploring open-ended situations, for making conjectures and testing them with data, for posing intriguing problems, and for solving nonroutine problems. Students in groups can often handle challenging situations that are well beyond the capabilities of individuals at that developmental stage. Individuals attempting to explore those same situations often make little progress and experience severe and unnecessary frustration. (p 16 Neil Davidson)

APPENDIX B
Bellanca & Fogarty
Blueprints for thinking in the cooperative classroom

Bellanca& Fogarty
Blueprints for thinking in the cooperative classroom
(1991, p. 243)

THE FIVE APPROACHES

A brief look at the major approaches will help teachers clarify the pluses and minuses of each and understand the tremendous wealth of successful cooperative tools that have been developed.

Cooperative Learning: Five Models

MODEL	CREATOR	DESCRIPTION	PLUSES	MINUSES
The Conceptual Approach	Johnsons Cohen	Theories of cooperation, competition and expectation-state theory	+ creative teachers create + can easily enhance what experienced teacher already does	- time away from content - no recipes - extra planning time - not step-by-step - unskilled teachers - full commitment
Curriculum Packages Approach	Slavin	Curriculum packages that have cooperative learning structured into the materials	+ easy to train + daily + pretested strategies + instructional variety (HOT/Dir. instruction)	- no direct teaching of social skill - discourages transfer - not a lot of curriculum packages available
A Structures Approach	Kagan	A repertoire of interactive strategies	+ simplicity in structures + easy to use + builds repertoire of strategies	- cutesy - assumes transfer - if restricted to low level tasks
The Group Investigation Model	Sharan & Sharan	The ultimate classroom jigsaw	+ inquiry + social skills + creative problem solving + facilitates skills + gives depth to content	- not good for curriculum coverage - if students have poor social skills - if parents want same assignment for all
The IRI Synthesis with HOT	Bellanca Fogarty	A synthesis of the four cooperative learning approaches with higher order thinking focus	+ synthesis + creative application + transfer	- needs training - needs commitment from school & district

APPENDIX C
First proposal for using cooperative learning
Math 8 Enrichment

Sept. 1989

MATH 8 ENRICHMENT

A. Proposal: Burnaby will have six math competitions for Grade 8 students, one at each of the high schools each month until the Gauss test is written.

B. Intended Goals:

1. Promote interest in mathematics for students.
2. Provide enrichment for students.
3. Improve test scores on math contests (Gauss).
4. Students will have fun with math.
5. Students will improve their communication skills.

C. Structure:

1. Each school will host a competition.
2. A sponsor teacher at each school will design :
   - (a) Type of competition (e.g. test, problem, etc)
   - (b) Format of competition, e.g.
     - i) team approach- hand in one set of solutions
     - ii) sum of individual scores
     - iii) random selection of a team member to present the solution.
     - iv) etc.
3. A school team of five members will participate at each competition. A suggestion is to maintain a pool of students in a club and select the students for each competition.

D. **Ivan Johnson** finds and coordinates the sponsor teachers from the different schools.

Proposed Schedule for Math 8 Competitions

Dec. 7 Burnaby South

Students will write a practice Guass test. The scoring for the competition will be the sum of all of the individual scores of the team.

The sponsor teachers will meet and discuss strategies for the competitions at the other schools.

tentative schedule, to be resolved on Dec. 7 at Burnaby South;

Jan 18 Cariboo Hill

Co-operative learning.

The competition will be by random selection of students from the different teams to explain the group's solution.

Feb. 15 Alpha

Test based on problems requiring written solutions. The team will hand in one solution sheet for marking, with stress placed on correctness, conciseness, readability and elegance as desired qualities.

Mar. 15	Central	TBA
Apr. 15	Burnaby North	TBA
May 10	Moscrop	TBA

APPENDIX D
Proposal sent to teachers at the beginning of the study Math 10 Enrichment

Proposal by: Ron Woo
revised:Sept. 30,1990

MATH 10 ENRICHMENT

A. Proposal: Four Grade 10 students from each high school will meet at Schou every second week to engage in math activities or competitions.

B. Intended Goals:

1. Promote interest in mathematics for students.
2. Provide enrichment for students.
3. Improve test scores on math contests (Cayley).
4. Students will have fun with math.
5. Students will improve their communication skills.
6. Students will meet others interested in solving math problems

C. Structure: An attempt will be made to provide different types of learning experiences for the students.

1. Format of meetings:
   - i) team approach- hand in one set of solutions
   - ii) sum of individual scores
   - iii) random selection of a team member to present the solution.
   - iv) simulations
   - v) perhaps some enrichment activities?

2. A school team of four members (same people) will participate at each meeting at Schou.
3. R. Woo will provide materials and supervise the activities, sponsor teacher attendance is preferred, but not necessary.

Proposed Schedule for Math 10 Activities

Mon. scoring Nov. 5, 1990	Students will write a practice Cayley test. The for the competition will be the sum of all of the individual scores of the team.
Fri. Nov.23,1990	Co-operative learning. The students will be given problems to solve collectively in their groups, but the solution is to be explained by a student randomly selected from the group.
Thur. Dec. 6,1990	Test based on problems requiring written solutions. The team will hand in one solution sheet for marking, with stress placed on correctness, conciseness, readability and elegance as desired qualities.
Thur. Jan. 10,1991	Students will participate in a simulation activity.
Thur. Jan 24,1991	Students will prepare a mock Cayley exam and answer key and bring it to this meeting. Their test will be administered to another group. The scoring for each school will be based on the fairness and correlation of their designed test plus the acquired score on the test that they wrote.
Thur. Feb.7,1991	Each student will write a mock Cayley exam co-operatively with another student.

Meetings

Place: Schou Education Center
Time: 1:00-3:00 p.m.
Goodies: Juice and cookies (no lunch)
Transportation: arrange through your school
Bring: Calculator, pencil, & problem solving skills

APPENDIX E
Participant results on Contests for 1990 & 1991

PARTICIPANT RESULTS
ON CONTESTS FOR 1990 &1991

Burnaby participants		Pascal 1990	Cayley 1991	Differences
ALPHA1-PASCAL				
dh		120.75	102.00	-18.8
bh		94.75	110.25	15.5
jh		88.00	81.50	-6.5
ALPHA2				
cw		104.75	93.75	-11.0
cm		88.50	100.75	12.3
CARIBOO 1- GAUSS				
kb		82.00	82.25	0.3
bh		72.50	99.00	26.5
tc		64.75	106.75	42.0
rb		64.00	74.75	10.8
kj		56.25	82.00	25.8
CENTRAL-EINSTEIN				
rc		83.25	113.00	29.8
gj		97.25	124.50	27.3
bm		83.75	102.00	18.3
sm		71.50	97.25	25.8
NORTH1-NEWTON				
aw		101.00	103.75	2.8
kc		97.25	98.50	1.3
ss		85.75	106.75	21.0
ec		79.50	91.00	11.5
NORTH2-ABEL				
rs		70.50	108.25	37.8
sw		61.00	97.50	36.5
jc		61.00	76.00	15.0
eo		57.75	80.75	23.0
SOUTH1-EUCLID				
jm		95.50	114.50	19.0
jt		54.25	108.50	54.3
rs		79.50	75.25	-4.3
SOUTH2-FERMAT				
sg		79.75	113.00	33.3
cly		55.00	75.75	20.8
fs		60.25	79.25	19.0
AVERAGE		78.9	96.4	17.5

APPENDIX F
Non-participant results on Contests for 1990 & 1991

LIST OF NON-PARTICIPANT STUDENTS, Burnaby school district				
		PASCAL 1990	CAYLEY 1991	Differences
ALPHA				
sk		63.75	82.50	18.8
lk		84.50	68.50	-16.0
CENTRAL				
tc		89.75	89.25	-0.5
jn		67.25	81.25	14.0
NORTH				
sh		102.25	90.50	-11.8
vl		101.25	85.00	-16.3
wc		97.50	100.75	3.3
sb		96.75	89.50	-7.3
cr		96.75	93.25	-3.5
jm		94.25	97.00	2.8
al		92.50	92.75	0.3
ew		92.50	115.00	22.5
ec		92.50	113.50	21.0
ac		92.25	88.25	-4.0
mr		90.00	69.50	-20.5
ac		87.75	89.00	1.3
rh		83.75	96.25	12.5
bn		82.50	92.25	9.8
cn		82.25	107.00	24.8
bg		80.75	109.25	28.5
SOUTH				
cl		82.00	110.00	28.0
ss		90.00	107.50	17.5
jb		68.75	85.75	17.0
cb		46.25	71.25	25.0
ms		41.25	59.75	18.5
mg		47.50	51.25	3.8
AVERAGE		82.6	89.8	7.2

APPENDIX G
St. George's School Results on contests for 1990 & 1991

NON PARTICIPANT ST. GEORGE'S SCHOOL				
		PASCAL 1990	CAYLEY 1991	difference
ka		117.50	124.00	6.50
hc		80.50	105.50	25.00
ma		87.50	105.50	18.00
hh		97.00	104.50	7.50
as		92.75	101.75	9.00
ya		72.00	101.75	29.75
vrk		82.50	101.50	19.00
ba		119.00	101.00	-18.00
cm		98.75	100.75	2.00
hr		81.00	100.25	19.25
mj		116.25	98.00	-18.25
es		81.50	96.75	15.25
kb		85.00	96.00	11.00
k		105.25	94.25	-11.00
bc		89.75	93.25	3.50
wjt		65.00	90.75	25.75
fg		54.50	87.00	32.50
sb		60.00	86.00	26.00
ps		63.75	75.00	11.25
bj		71.50	73.50	2.00
				0.00
				0.00
Average		86.05	96.85	10.80

APPENDIX H
Canadian Honour Roll
for 1990 & 1991

Canadian Team Honour Roll / Palmarès des équipes à l'échelle nationale

	SCHOOL ECOLE	LOCATION ENDROIT	PROV PROV	SCORE NOTE
1	ZION HEIGHTS JUNIOR H.S.	NORTH YORK	ONT	413.75
2	ST. JOHN'S-RAVENSCOURT SCHOOL	WINNIPEG	MAN	388.00
3	ERIC HAMBER SEC. SCHOOL	VANCOUVER	B.C.	364.25
4	WOBURN C.I.	SCARBOROUGH	ONT	359.50
5	ST. GEORGE'S SCHOOL	VANCOUVER	B.C.	352.75
6	LISGAR C.I.	OTTAWA	ONT	344.75
7	ST. ANDREWS JR. H.S.	WILLOWDALE	ONT	344.00
8	TEMPO SCHOOL	EDMONTON	ALTA	340.50
9	KILLARNEY SEC. SCHOOL	VANCOUVER	B.C.	338.75
	ST. MICHAEL'S UNIVERSITY SCH.	VICTORIA	B.C.	338.75
11	BELL H.S.	NEPEAN	ONT	336.50
12	ACADIA JR. HIGH	WINNIPEG	MAN	335.25
13	PRINCE OF WALES SEC. SCHOOL	VANCOUVER	B.C.	334.75
14	SENTINEL SEC. SCHOOL	WEST VANCOUVER	B.C.	332.50
15	WINDFIELDS JR. H.S.	NORTH YORK	ONT	330.75
16	MARTINGROVE C.I.	ETOBICOKE	ONT	328.75
17	TORONTO FRENCH SCHOOL	TORONTO	ONT	327.25
	ST. PAUL'S CATHOLIC H.S.	NEPEAN	ONT	327.25
19	EDITH ROGERS JR. H.S.	EDMONTON	ALTA	326.75
20	PARKVIEW ELEM.-JR. HIGH SCHOOL	EDMONTON	ALTA	326.25
21	CROFTON HOUSE SCHOOL	VANCOUVER	B.C.	326.00
22	WATERLOO C.I.	WATERLOO	ONT	325.50
23	THORNLEA SECONDARY SCHOOL	THORNHILL	ONT	325.00
	UNIV. OF TORONTO SCHOOLS	TORONTO	ONT	325.00
25	ST. ROSE JR. HIGH SCHOOL	EDMONTON	ALTA	324.75
26	LONDON CENTRAL S.S.	LONDON	ONT	324.25
27	ECOLE SECONDAIRE SAINT-THOMAS	POINTE-CLAIRE	QUE	322.00
28	ECOLE JOSEPH-FRANCOIS PERRAULT	MONTREAL	QUE	321.75
	ALPHA SEC. SCHOOL	BURNABY	B.C.	321.75
30	MAILLARD JR. S.S.	COQUITLAM B.C.	B.C.	321.25
31	SEAQUAM SECONDARY SCHOOL	DELTA	B.C.	320.25
32	GRANDVIEW HEIGHTS JR. H.S.	EDMONTON	ALTA	320.00
33	WEST HILL SECONDARY SCHOOL	OWEN SOUND	ONT	319.75
34	UNIVERSITY HILL SEC. S.	VANCOUVER	B.C.	318.50
35	STELLY'S SECONDARY SCHOOL	BRENTWOOD BAY	B.C.	318.25
36	ECOLE EDUCATION INTERNATIONALE	ST-HUBERT	QUE	317.75
37	QUEEN ELIZABETH HIGH SCHOOL	CALGARY	ALTA	317.00
38	CAMBIE JR. SECONDARY SCHOOL	RICHMOND	B.C.	316.75
39	COLONEL BY S.S.	GLOUCESTER	ONT	316.50
40	BURNABY NORTH SEC. SCHOOL	BURNABY	B.C.	313.50
41	MERIVALE H.S.	NEPEAN	ONT	312.75
	ST. ROBERT CATHOLIC H.S.	GORMLEY	ONT	312.75
43	LOWER CANADA COLLEGE	MONTREAL	QUE	312.00
	CEDAR HILL JR. S.S.	VICTORIA	B.C.	312.00
45	ROYAL WEST ACADEMY	MONTREAL WEST	QUE	311.75
46	MARKHAM DISTRICT H.S.	MARKHAM	ONT	311.50
47	LAKE TRAIL JUNIOR SEC. S.	COURTENAY	B.C.	311.25
48	EARL HAIG S.S.	WILLOWDALE	ONT	310.00
49	GLENFOREST S.S.	MISSISSAUGA	ONT	309.75
	ALBERT CAMPBELL C.I.	SCARBOROUGH	ONT	309.75

Rank / Position	Scores / Notes
51 - 100	308.25 - 292.00
101 - 150	291.75 - 279.00
151 - 200	278.50 - 271.00
201 - 250	270.50 - 263.75
251 - 300	263.50 - 257.50
301 - 400	257.25 - 246.25
401 - 500	246.00 - 237.00
501 - 600	236.75 - 227.25

There were 1388 schools enrolled
1388 écoles étaient inscrites

Canadian Team Honour Roll / Palmarès des équipes à l'échelle nationale

	SCHOOL ECOLE	LOCATION ENDROIT	PROV PROV	SCORE NOTE
1	EARL HAIG S.S.	NORTH YORK	ONT	410.50
2	CAMBIE JR. SECONDARY SCHOOL	RICHMOND	B.C.	396.50
3	ERIC HAMBER SEC. SCHOOL	VANCOUVER	B.C.	391.00
4	WOBURN C.I.	SCARBOROUGH	ONT	386.00
5	R.C. PALMER JR. SEC.	RICHMOND	B.C.	385.50
6	UNIV. OF TORONTO SCHOOLS	TORONTO	ONT	383.75
7	WEST VANCOUVER SEC. SCHOOL	WEST VANCOUVER	B.C.	379.50
8	J.N. BURNETT JR. SEC.	RICHMOND	B.C.	378.25
	THE WOODLANDS S.S.	MISSISSAUGA	ONT	378.25
10	FREDERICTON HIGH SCHOOL	FREDERICTON	N.B.	376.25
11	COBEQUID EDUCATION CENTRE	TRURO	N.S.	375.00
12	BURNABY CENTRAL S. S.	BURNABY	B.C.	372.75
13	ST. JOHN'S-RAVENSCOURT SCHOOL	WINNIPEG	MAN	369.75
14	LISGAR C.I.	OTTAWA	ONT	369.00
15	OAKVILLE TRAFALGAR H.S.	OAKVILLE	ONT	366.00
16	ESQUIMALT SECONDARY SCHOOL	VICTORIA	B.C.	364.25
17	WATERLOO C.I.	WATERLOO	ONT	362.75
18	HALIFAX WEST H.S.	HALIFAX	N.S.	362.25
19	JOHN RENNIE H.S.	POINTE CLAIRE	QUE	361.50
20	ST. MICHAEL'S UNIVERSITY SCH.	VICTORIA	B.C.	360.75
21	MERIVALE H.S.	NEPEAN	ONT	357.25
	ALBERT CAMPBELL C.I.	SCARBOROUGH	ONT	357.25
23	UPPER CANADA COLLEGE	TORONTO	ONT	356.00
24	ECOLE SECONDAIRE DES SOURCES	DOLLARD DES ORMEAUX	QUE	355.75
25	VANCOUVER COLLEGE H.S.	VANCOUVER	B.C.	354.00
26	MISS EDGAR AND MISS CRAMP'S	WESTMOUNT	QUE	351.50
	ROYAL WEST ACADEMY	MONTREAL WEST	QUE	351.50
28	WIDDIFIELD S.S.	NORTH BAY	ONT	350.25
29	VINCENT MASSEY S.S.	WINDSOR	ONT	350.00
30	BURNABY NORTH SEC. SCHOOL	BURNABY	B.C.	349.25
31	MAILLARD JR. S.S.	COQUITLAM	B.C.	348.00
32	GEORGE S. HENRY ACADEMY	NORTH YORK	ONT	346.75
33	EASTVIEW S.S.	BARRIE	ONT	346.50
34	WESTMOUNT SECONDARY SCHOOL	HAMILTON	ONT	346.00
	SIR CHARLES TUPPER S.S.	VANCOUVER	B.C.	346.00
36	BIALIK H.S.	MONTREAL	QUE	345.50
37	A.Y. JACKSON S.S.	WILLOWDALE	ONT	344.75
38	ECOLE INTERNATIONAL SCHOOL OF	1208 GENEVA	ONT	344.00
39	CROFTON HOUSE SCHOOL	VANCOUVER	B.C.	343.75
40	BURNABY SOUTH SR. SEC.	BURNABY	B.C.	342.25
41	BRENTWOOD COLLEGE	MILL BAY	B.C.	342.00
	LOWER CANADA COLLEGE	MONTREAL	QUE	342.00
43	F.E. MADILL S.S.	WINGHAM	ONT	341.50
	WALTER MURRAY C.I.	SASKATOON	SASK	341.50
45	MARTINGROVE C.I.	ISLINGTON	ONT	341.25
46	SAINT JOHN H.S.	SAINT JOHN	N.B.	340.75
47	SAINT-CHARLES GARNIER	QUEBEC CITY	QUE	340.25
48	MARKHAM DISTRICT H.S.	MARKHAM	ONT	340.00
49	ERINDALE S.S.	MISSISSAUGA	ONT	339.50
50	L'AMOREAUX C.I.	AGINCOURT	ONT	339.25

Rank / Position	Scores / Notes
51 - 100	338.75 - 328.25
101 - 150	328.00 - 319.75
151 - 200	319.50 - 314.00
201 - 250	313.75 - 309.00
251 - 300	308.50 - 302.25
301 - 400	302.00 - 293.50
401 - 500	293.25 - 284.00
501 - 600	283.75 - 275.25

There were 1334 schools enrolled
1334 écoles étaient inscrites

APPENDIX I
Comparison of zone 9
Pascal 1990 & Cayley 1991 Results

1990
PASCAL CONTEST/CONCOURS PASCAL

PROVINCE OF BRITISH COLUMBIA/PROVINCE DE COLOMBIE-BRITANNIQUE
ZONE 09 TEAM LIST/LISTE DES EQUIPES DE ZONE 09

	SCHOOL ECOLE	LOCATION ENDROIT	SCORE NOTE
1	SENTINEL SEC. SCHOOL	WEST VANCOUVER	332.50
2	ALPHA SEC. SCHOOL	BURNABY	321.75
3	BURNABY NORTH SEC. SCHOOL	BURNABY	313.50
4	HILLSIDE MIDDLE SCHOOL	WEST VANCOUVER	293.75
5	SUTHERLAND SECONDARY SCHOOL	NORTH VANCOUVER	288.50
6	BALMORAL JR. SEC. SCHOOL	NORTH VANCOUVER	287.00
7	BURNABY CENTRAL S. S.	BURNABY	282.75
8	HANDSWORTH SEC. SCHOOL	NORTH VANCOUVER	274.25
9	ST. THOMAS MORE H.S.	BURNABY	268.50
10	BURNABY SOUTH SR. SEC.	BURNABY	268.00
11	WINDSOR SECONDARY SCHOOL	NORTH VANCOUVER	249.25
12	SEYCOVE SECONDARY SCHOOL	NORTH VANCOUVER	243.00
13	ARGYLE SEC. SCHOOL	NORTH VANCOUVER	241.75
14	COLLINGWOOD SCHOOL	WEST VANCOUVER	224.00
15	CARIBOO HILL S. S.	BURNABY	219.25
16	CARSON GRAHAM SEC. SCHOOL	NORTH VANCOUVER	206.75
17	MOSCROP JR. SEC. SCHOOL	BURNABY	195.25

1991
CAYLEY CONTEST/CONCOURS CAYLEY

PROVINCE OF BRITISH COLUMBIA/PROVINCE DE COLOMBIE-BRITANNIQUE
ZONE 09 TEAM LIST/LISTE DES EQUIPES DE ZONE 09

	SCHOOL ECOLE	LOCATION ENDROIT	SCORE NOTE
1	WEST VANCOUVER SEC. SCHOOL	WEST VANCOUVER	379.50
2	BURNABY CENTRAL S. S.	BURNABY	372.75
3	BURNABY NORTH SEC. SCHOOL	BURNABY	349.25
4	BURNABY SOUTH SR. SEC.	BURNABY	342.25
5	HANDSWORTH SEC. SCHOOL	NORTH VANCOUVER	333.25
6	SENTINEL SEC. SCHOOL	WEST VANCOUVER	332.00
7	CARSON GRAHAM SEC. SCHOOL	NORTH VANCOUVER	331.50
8	ALPHA SEC. SCHOOL	BURNABY	316.00
9	SUTHERLAND SECONDARY SCHOOL	NORTH VANCOUVER	314.50
10	CARIBOO HILL S. S.	BURNABY	312.50
11	SEYCOVE SECONDARY SCHOOL	NORTH VANCOUVER	310.00
12	MOSCROP JR. SEC. SCHOOL	BURNABY	296.25
13	COLLINGWOOD SCHOOL	WEST VANCOUVER	296.00
14	ARGYLE SEC. SCHOOL	NORTH VANCOUVER	284.75
15	ST. THOMAS MORE C.I.	BURNABY	281.75
16	BALMORAL JR. SEC. SCHOOL	NORTH VANCOUVER	273.50

APPENDIX J
Match up of students participating at Schou and winning school team

SCHOOL TEAMS SELECTED FOR CAYLEY STANDINGS			
ALPHA			
*	bh		110.25
	ps		103.75
*	dh		102.00
		SCORE	316.00
CARIBOO			
	fc		106.75
*	tc		106.75
*	bh		99.00
		SCORE	312.50
CENTRAL			
	hp		128.75
*	gj		124.50
	mw		119.50
		SCORE	372.75
NORTH			
	jc		120.75
	ew		115.00
	ec		113.50
		SCORE	349.25
BURNABY SOUTI			
*	jl		114.75
*	jm		114.50
*	sg		113.00
		SCORE	342.25
SCHOOL TEAM S			
CARIBOO			
*	sl		135.00
*	km		131.25
*	jb		116.25
		SCORE	382.50
*PARTICIPANT AT SCHOU			

APPENDIX K
Videotape logs

APPENDIX K

VIDEOTAPE LOG NOVEMBER 5,1990
Camera operator: mostly on tripod
Ron Woo and Ivan Johnson when handheld

TIME	DESCRIPTION
0:00	Goals, intentions
2:10	Why are you here? Journals
6:00	Explicit reason for writing Cayley. Fun when working together.
7:00	Does the Cayley count?
8:15	Implicit reason for writing
9:15	What is important?
10:00	Working in teams.
11:00	Cooperation or competition?
11:46	Cooperation
13:11	Teaching others reinforces knowledge
15:22	focus non-algorithmic problems
16:45	anxiety and "freezing" on problems
17:12	structure of meetings
19:24	evaluation
22:06	cooperative structures
25:30	start of exercise
27:00	forming pairs
28:09	working
32:00	decide on seating arrangement
36:10	multiple choice strategy
59:00	Break
1:10:05	working session, cooperation, North and Cariboo, seats in the air
1:22:14	reversion to individual work
1:35:00	Will question at next session be from Cayley?
1:38:00	end

VIDEOTAPE LOG NOVEMBER 23,1990
Camera operator: mostly on tripod
Ron Woo and Ivan Johnson when handheld

TIME	DESCRIPTION
0:00	solution of question
2:47	choose student to do question
3:18	group Euclid does #18
4:30	group Fermat does #18

5:50	Bruce H. does question
11:57	Bruce C. does question
12:48	elegant solution
15:02	group work, inter-dependence
16:00	Rene attempts question
18:45	Jim solves by elimination
21:24	calculators used on Cayley
22:30	communicate
23:18	work in pairs
24:00	North working
36:50	Central and Cariboo; South
41:17	share answers
42:00	break
42:15	bonus marks
54:00	good sharing - Central
54:35	North
55:30	ensure interdependence
1:00:25	North working as a group
1:01:05	Cariboo
1:02:30	humour
1:05:00	South
1:09:00	Rene B. reading book
1:12:30	Tanya C. joins group
1:15:15	Erika presents quietly
1:18:58	Jason B. presents
1:28:40	Shannon S. presents
1:30:30	#5 lawnmower problem
1:30:59	humour
1:33:42	good explanation
1:33:40	Gavin presents
1:39:00	journals
1:44:30	scores
1:53;30	end

VIDEOTAPE LOG DECEMBER 6,1990
Camera operator: Tom O'Shea

TIME	DESCRIPTION
0:60	North students
1:00	Math Camp Video
5:46	Math Camp "sell" Ivan
7:48	guide on use of journal
8:33	Woo- having presenter provides motivation, helps practice

	dealing with anxiety
13:25	agenda, journal comments
14:35	Jigsaw
15:80	problem demo
17:22	group Descarte not prepared
17:53	enthusiasm for competetion
18:40	S.S. L. explains series problem
26:00	good problem, pan too fast, long question
27:30	"can you see an easier way?" - take sum of series as 1st term
29:00	Gavin J absent
31:00	Sanjay presents geometry explanation
34:00	example of why student explanations in presentations are not an effective use of group time
37:00	Woo presents
40:30	students express emotion
40:50	hand for effort
41:36	jig saw activity
44:48	Jason B. then Bruce C. guide #3, others not observing, Jim T. and Ron C. not participating with group but doing own thing.
47:20	Jason attempts group involvement
49:33	"does everybody understand?"
49:50	Ron makes point
50:50	His 2nd point is right!
51:50	girls become involved
54:40	all students involved
55:00	tutor-tutee
57:40	pan of all four jigsaw groups
58:30	Woo - closure on this activity
59:30	collegial decision making "what do you think, do you want to go over explanations now?"
1:00:36	Break
1:01:22	Cariboo group
1:04:20	heads together, buns in the air
1:06:00	Woo - comment on thought processes
1:08:00	goofing off
1:08:25	tutoring
1:10:10	Alpha group
1:12:00	assignment re: calculators and Cayley
1:12:40	pan of students working on assignment
1:13:23	request for help from sponsors
1:13:45	journal entries
1:15:40	Tom voice over describing jigsaw
1:16:10	end

good quality video

camera focussed on Cariboo Grade 9 group much of the time

difficult to listen to problem solving, even though the camera was focussed on the particular group - too much background noise.

decide to take other videos with handheld mode

VIDEOTAPE LOG FEBRUARY 7, 1991
Camera operator: Ron Woo

TIME	DESCRIPTION
0:45	student presentation
1:23	Shannon reading
1:40	North presents
2:50	Mike of Central presents
5:00	calculator question
5:50	graphing calculators
7:27	Shannon working with three boys
8:08	very independent work
8:53	Alpha working
9:17	end

VIDEOTAPE LOG FEBRUARY 14, 1991
Camera operator: Ron Woo

TIME	DESCRIPTION
0:13	ice-breaking, learn another person's name and one fact about him/her, present this to the group
0:38	marking fake Cayley
1:17	Cariboo group
1:23	jigsaw marking
1:46	"I did that question, it was wrong"
2:03	group processing test
2:32	end

VIDEOTAPE LOG FEBRUARY 21, 1991
Camera operator: Ron Woo

TIME	DESCRIPTION
0:00	intro to Fake Cayley
3:18	students writing
4:20	Cariboo cooperating
5:52	9:30 am students are mostly working individually
6:44	Gavin in ecstasy
7:18	Bruce V. working individually
8:12	more discussion
10:08	North group (all girls) and Central
10:22	Alpha
10:56	jigsaw
11:10	end

APPENDIX L
Log of journal entries

LOG OF JOURNAL ENTRIES

I have attempted to summarize the student's responses in a short but readable format that captures the essence of what they are communicating. The numbers contained in brackets following comments indicate the number of students that responded in that manner.

NOVEMBER 5
JOURNAL ENTRIES

1. WHY ARE YOU HERE?

to do better on Cayley (14)
to study topics different from regular math class (5)
for new experience (cooperative learning) (6)
to benefit researcher (1)
summary "provide a more 'easy' atmosphere for everyone to work in and we will improve our problem solving skills for the Cayley Contest"

2. HOW DID YOU FEEL ABOUT WORKING DURING THE "AWAY SESSION"

difficult (7)
did not enjoy (4)
did not mind (3)
good experience (4)
enjoyed (1)
summary "I found it difficult to work with strangers"

3. WERE THE QUESTIONS TOO HARD FOR THE FIRST SESSION?

yes (11)
some (3)
no/right level (7)

4. WOULD YOU LIKE TO SEE ANYTHING DONE DIFFERENTLY?

no (5)
don't know (4)
yes (14)-more demos by teacher
more work in pairs
metaphysical calculators
no video taping
more work with home group
more time to work on problems

5. GENERAL COMMENTS.

great experience
It's fun
glad to be here
this is interesting
I thought it was well organized
less talk, more action
"I think we should get to know people in the other schools better and go over the answers to some of the tougher problems."

NOVEMBER 23
JOURNAL ENTRIES

1. SHOULD WE TIME THE PRESENTATIONS?

no (26) causes nervousness, difficult to judge how much time to allow.
yes (3) to speed things up, to encourage better preparation.

2. DO YOU HAVE A BETTER METHOD OF CHOOSING THE PRESENTER?

yes (15) group chooses own(8)
each group gets one question
choose volunteer
no (14) present system O.K.
I don't support presentations (1)

3. ARE THERE TOPICS THAT YOU WANT US TO TEACH?

yes (8) anything that would help us solve questions
Grade 11 stuff
factoring, long equations
geometry, graphing

no (10) rather work on problem solving and different approaches
do not teach methods

4. YOUR OWN COMMENTS

this session was better than the last because we did more problems and there was less talking (teaching) (4)
how do we find out the names of other students? (2)
more time for problems
"I had difficulty working with two members in my group"
"presentations uncomfortable"
"by letting the student explain their answers it makes things easier to understand"
"I enjoyed what happened today. I am looking forward to coming again."

JOURNAL ENTRIES
DEC. 6

1. HOW MANY OLD TESTS HAVE YOU DONE AS OF TODAY?

one (2)
two (8)
three (5)
four (7)
more than five (4)

2. HOW MANY HOURS HAVE YOU WORKED WITH ANOTHER PERSON ON CAYLEY REVIEW?

the responses were not accurate because some students included the time at these sessions while others did not

3. HAS THIS PROJECT HAD ANY INFLUENCE AT YOUR SCHOOL (OTHER STUDENTS THAT ARE NOT INVOLVED) ON REVIEWING FOR CAYLEY & IF SO WHAT ARE THE CHANGES?

no (22)

4. YOUR COMMENTS (REMEMBER, THESE ARE THE MOST IMPORTANT.)

"the conferencing will not help on the Cayley because that is an individual test"

"I enjoyed getting together with the schools and working out the problems"

"We should have some more "jigsaw" groups. (4) It's nice to know different people,today's session has been fun"

"I found the jigsaw quite enlightening. It was a new, fun and exciting way of learning. Try not to have too much to do though, today was well planned"

"I think that this project should be made available to more students at other academic levels....set up for C & D students where some added exposure to math may be useful to them"

JOURNALS

FEB. 7

1. HOW DID YOU FEEL ABOUT THE CALCULATORS (DEGREE OF DIFFICULTY, AMOUNT OF TIME AVAILABLE, SUITABILITY WITH RESPECT TO THE CAYLEY)?

was fun (5)
hard (3)
frustrating (2)
interesting(3)

2. ARE SOLUTIONS TO PROBLEMS COMPLETE? SHOULD WE BE TEACHING MORE TOPICS?

"I thought this was to practice for the Cayley. If so, why are we using graphing calculators which are not available to us, affordable, or legal to use since they are programmable

JOURNAL
CAYLEY REVIEW
FEB. 14

1. WHAT DID YOU THINK OF THE OTHER GROUP'S TEST?

hard (4)
well designed (7)
interesting (2)
appropriate (2)
easy (2)
good and bad (1)
too much copying (2)
not clear (5)

2. DID DISCUSSING THE TEST AFTER IMPROVE YOUR INSIGHT?

no (9)
yes (11) need more time

"through discussion people's feelings and contributions are valuable"

"fun, it was interesting to find out how many ways there are to answer the same question"

3. WHAT DID YOU THINK OF TODAY'S SESSION?

"it was a good idea to see what other people expected on the test"
"making and doing the fake Cayley test was really fun"

4. WAS THE PACING TOO SLOW OR DID YOU FEEL THAT THE EXTRA TIME FOR SOCIALIZING APPROPRIATE?

"going too fast"
"the extra time for socializing was a waste of time. Would be better used for other things"
"the extra time for socializing was a very good idea, except some of the members seemed to close themselves off"

"I think the socializing was appropriate, the atmosphere was better. It made learning math a little better.
"I think the session was the most fun of all our sessions so far; just interacting by ourselves is fun" (3)

FEB. 21
JOURNAL ENTRIES

1. DO YOU FEEL THAT THESE SESSIONS WILL HELP YOU IN WRITING THE CAYLEY?

no (1)
undecided (1)
yes (23) "the sharing of solutions helps one learn other types of solutions"
"it builds up confidence and provides more experience"
"I think it will improve my score by 10 points"

2. IF THE ANSWER TO #1 WAS YES; HOW DID THEY HELP?
(confidence, techniques, attitude, etc.)

new techniques (14)
confidence and attitude (9)
practice (7)
"these sessions will help because some of the presentations are really thorough."
"The mock Cayley

3. DID YOU ENJOY DOING PROBLEM SOLVING IN GROUPS?

no (4)
somewhat (3)
yes (22)
very much (1)
"in groups it is much easier, and there is a bigger pool of ideas to work with"

4. DO YOU THINK THAT YOU WILL MAKE MORE OF AN EFFORT TO WORK WITH OTHER PEOPLE ON

MATH QUESTIONS?

no (8) "I have nowhere to do this "
"we are not encouraged to in class"
maybe (3) "I have a busy schedule and cannot do this out of school"
yes (18) "I will definitely make more of an effort to work with others"

5. HOW IMPORTANT IS THE SOCIAL ASPECT OF A SITUATION TO THE LEARNING THAT TAKES PLACE?

not (1)
unsure (7)
very (18) it is more fun (2)
"I think the social aspect was a big bonus added to this program"
"I like a very relaxed atmosphere"

6. CAN YOU THINK OF ONE CONCEPT THAT YOU LEARNED FROM ANOTHER STUDENT THAT HAD AN INTERESTING SOCIAL LINK? IF SO WHAT WAS THE SITUATION?

this question was too difficult for them. many said yes (5) but could not describe the situation.

7. HOW WOULD YOU IMPROVE THIS PROGRAM IF IT WAS TO BE REPEATED?

more help from teacher (1)
more time for activities, too rushed (2)
mix up groups more (6)
improve transportation
create a more lively atmosphere
duration (too short 2) too long (2)
more high level math
more individual work
no student explanations

8. YOUR COMMENTS PLEASE!

"we missed a lot of school" (3)
"latter stages of program were very enjoyable, mid-program very boring (presentations), beginning O.K."
"very social atmosphere, nice people"
"The atmosphere is excellent, the test is exciting and challenging. It gave me a friendly feeling"
"good experience" (3)
"I think this was very good, things like this should be done again"
"This program has helped me with many important math skills that I am going to use in the future."

APPENDIX M
Agendas and student activities of the meetings

SIMON FRASER UNIVERSITY
FACULTY OF EDUCATION
Information and Consent form for Cooperative learning and the Cayley Contest.

Dear Student and Parents:

I am conducting a research project with twenty-four students from grade 10 in Burnaby. The purpose of this letter is to ask for your consent for ________________ to participate in this project. This project has been approved by the Math Department Heads of Burnaby and Ivan Johnson, Coordinator of Schou Education Services, Burnaby. It has also been examined and approved by Dr. Tom O'Shea, Faculty of Education, Simon Fraser University.

The purpose of this research is to examine the effects of gathering students from different schools to participate in an atmosphere of cooperation to review for the Cayley Contest. This should create the opportunity for students to communicate and participate actively with others in a manner which hopefully will enrich their mathematical understanding. The students will be asked to complete evaluation forms and to maintain a journal of their perceptions of the learning processes that they are participating in. The sessions will also be videotaped. The information collected will be confidential. The results of this project will be made available to you on request.

Your participation is voluntary and you may withdraw at any time. Thank-you for your cooperation.

Sincerely,
Ronald Woo

I agree to participate in the project described above.

Name:________________________ Signature:________________________

I agree to allow my daughter/son to participate in the project.

Name:________________________ Signature:________________________

TASKS

1. Within your home group, choose one of the following colors and use it for ***all*** of your work done here: black, blue, red, or pencil.

2. Review as many of the old Cayley exams that you can. The first 30 minutes of each meeting will be devoted to reviewing these questions.

3. With the members of your group, begin preparing a mock Cayley exam. This will be administered to one of the other schools. They will grade your group on how fair and representative it is, but you will mark their test with your answer key.

4. Please maintain a journal of your thoughts and feelings about the activities, your perceptions of individual and group performances and any changes that you felt that occurred.

AGENDA
NOV.5

SURVEY FORM

INTRODUCTION

GOALS & FORMAT OF MEETINGS

ACTIVITY 1: TEN CAYLEY QUESTIONS - AWAY AND HOME GROUPS
ONE SHEET HANDED IN PER GROUP

BREAK: JUICE & COOKIES

LESSON ON WRITING MULTIPLE CHOICE TESTS: MR. JOHNSON

ACTIVITY 2: TEN CAYLEY QUESTIONS, RANDOM SELECTION OF
PRESENTER

JOURNAL ENTRY

SURVEY 1

Name:

	strongly negative 1	negative 2	undecided 3	positive 4	strongly positive 5
1. Mathematics will change rapidly in the near future.					
2. Mathematics is a good field for creative people.					
3. There is little place for originality in solving mathematics problems.					
4. New discoveries in mathematics are constantly being made.					
5. Mathematics helps one to think according to strict rules.					
6. Estimating is an important mathematics skill.					
7. There are many different ways to solve most mathematics problems.					
8. Learning mathematics involves mostly memorizing.					
9. In mathematics, problems can be solved without using rules.					
10. Trial and error can often be used to solve a mathematics problem.					
11. There is always a rule to follow in solving a mathematics problem.					
12. There have not been any new discoveries in mathematics for a long time.					
13. Mathematics is a set of rules.					
14. A mathematics problem can always be solved in different ways.					
15. Mathematics helps one to think logically.					

FORMAT OF MEETINGS

1. The meetings will usually start with a review of actual Cayley questions or similar type questions to be solved cooperatively without notes.
2. There will be a lesson related to the Cayley or problem solving.
3. There will be a break with juice provided.
4. There will be another work session.
5. You will be requested to write your thoughts in your journal. These will be specific questions or general reactions to the process we are engaging in. You are welcome to make any comments you wish anytime.
6. Scoring for the day will be tallied and a winner for the session will be announced. Evaluation is not firmly decided at this point, but we will be giving bonus points for groups demonstrating that they are working cooperatively.

INTRODUCTION

1. Why are you here? Please make a journal entry here.
   Explicit Reason: To review for the Cayley in a "fun" way.
   Why write the Cayley? see how you stack up against others
   Does it count? no, not until grade 12, when the AHSME and the Euclid determine universty entrance and scholarships.
   Implicit Reason: To provide you with the opportunity to experience a different perspective on the way of learning, a perspective based on the belief the mathematical learning consists of students constructing mathematical concepts and procedures as in contrast to the notion that mathematics is a "received" body of knowledge in which the teacher's role is to "transmit" and the student's role is to "receive". Have you ever thought "when am I going to use this (factoring for example)?
2. Is this cooperation or competition? Both!! The stress will be on cooperative learning. The skill to work in a group is a desireable attribute in the real world. Do you agree?? Practice in school will help develop the skill, and it also helps learning. My experience is that I only really understood math well when I began teaching it. If you are helping a fellow student, he will also benefit from your explanation because it is more likely to be at his level than that of the teacher's. The competition is for fun, and thats all I need to say about it.

Name:
School:
Cayley Answer Sheet

Question	Answer
11	
16	
18	
19	
20	
21	
22	
23	
24	
25	

____
40

AGENDA
Nov. 23

1:00-1:20	REVIEW: CAYLEY 1990#19-25	(10)
1:20-1:50	ACTIVITY 1: TEN CAYLEY QUESTIONS -DO IN PAIRS, SHOW WORK, ONE SHEET HANDED IN PER PAIR. EACH MEMBER WRITE WITH DESIGNATED COLOR.	(40)
1:50-2:00	MULTIPLE CHOICE TEST STRATEGY	
2:00-2:05	BREAK: JUICE & COOKIES	
2:05-2:20	ACTIVITY 2: HOME GROUPS COLLABORATE ON THE ~~TEN~~ CAYLEY QUESTIONS, HAND IN ONE ANSWER KEY.	(200)
2:20-2:40	STUDENT PRESENTATION OF RESULTS.	(10)
2:40-2:55	JIGSAW PROBLEM	
2:55-3:00	JOURNAL ENTRY	

TOTAL SCORE=220

roles or expected behaviors

1. *Problem restater*: Tells students to get to know the problem by restating it in their own words. Students are to state the information provided and the information they seek.
2. *Elaborator*: Asks, "Does this problem remind us of any problem previously solved by the class?"
3. *Strategy suggester/seeker*: Suggests possible alternative strategies to use in solving the problem and/or asks others to do so. Asks, "What's another strategy we could use?"
4. *Approximator*: Asks, "What range of answer would be reasonable?" so that group members estimate and approximate the answer before solving it exactly.
5. *Review/Mistake Manager*: If the group missed the problem, asks, "What can we learn from this mistake?" If the group solved the problem correctly, asks, "How may our solution be improved?"
6. *Confidence builder*: Says, "We can do it!"

POLYHEDRON. A solid bounded by plane polygons called the *faces*; the intersections of three or more edges called the *vertices*. The numbers of faces, edges and vertices in polyhedra in general obey Euler's law: $f + v = e + 2$. A *simple polyhedron* is topologically equivalent to a sphere, i.e. it has genus 0. See Concave, Convex Polyhedron; Regular Polyhedron; Topological Transformation.

MARKING SCHEME FOR PRESENTATION

GETS UP IN FRONT OF GROUP	1	
IDENTIFIES GOAL	1	
IMPLEMENTATION Strategy:	complete=6 1 secondary error=5 2 secondary errors=4 conditions overlooked=3 strategy initiated=2 strategy not clear=1	
PRESENTATION	(vocalizes=1)+(eye contact=1)	
	total	

Name:
School:
Cayley Answer Sheet

Question	Answer
1	B
2	B
3	A
4	B
5	E
6	D
7	C
8	D
9	C
10	D

____
40

Name: ____________________

____

Nov. 23
Cayley Review
Set 2

School:

1 The number of positive integers that are less than 500 and that are *not* divisible by 2 or by 3 is

(A) 168 (B) 167 (C) 166 (D) 165 (E) 83

There are 499 integers to consider. Of these 249 are divisible by 2, 166 are divisible by 3, and 83 are divisible by 6, i.e. by both 2 and 3. Hence the number of integers that are not divisible by 2 or 3 is

$$499 - [249 + 166 - 83] = 167.$$

2 For the sequence $13^5 - 13, 14^5 - 14, 15^5 - 15, \ldots, n^5 - n, \ldots$, the largest number which exactly divides each of the terms of the sequence is

(A) 60 (B) 30 (C) 20 (D) 12 (E) 6

For the sequence $13^5 - 13$, $14^5 - 14$, $15^5 - 15$, ..., $n^5 - n$, ..., the largest number which exactly divides each of the terms of the sequence is

(A) 60 (B) 30 (C) 20 (D) 12 (E) 6

Solution

$$n^5 - n = (n-1)(n)(n+1)(n^2+1)$$

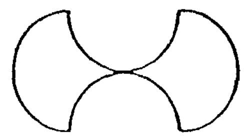
Note that $13^5 - 13 = (12)(13)(14)(170)$ which is divisible by all of the given integers.

But $14^5 - 14 = (13)(14)(15)(197)$ which is not divisible by 60, 20, or 12.

Since $(n-1)n(n+1)$ is the product of three consecutive integers, it is divisible by 6. If it is also divisible by 5 then $n^5 - n$ is divisible by 30. If $(n-1)(n)(n+1)$ is not divisible by 5, n is of one of the forms $5k \pm 2$.

Then $n^2 + 1 = (5k \pm 2)^2 + 1 = 25k^2 \pm 20k + 5$ which is divisible by 5. In either case $n^5 - n$ is divisible by 30.

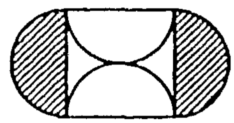
3 . The figure is constructed of four equal semicircles, two of which are tangent at their mid-points. If the diameter of each of the semicircles is one unit, then the area, in square units, of the interior of the figure is



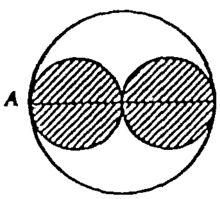
(A) 1 (B) $\pi\sqrt{2}$ (C) $\frac{\pi}{4}$

(D) π (E) $\frac{\pi}{2}$

Draw the four diameters shown, thus forming a square with side of length 1. Since the area of the two shaded semicircles outside the square is equal to the area of the two white semicircles inside the square, the given figure has an area equal to the area of the square, that is, one square unit.
The answer is A.



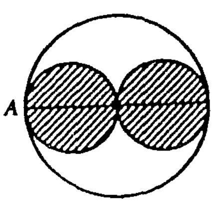
4 AB is a diameter of the large circle and passes through the centres of the two small circles. If the radius of each small circle is one unit and the radius of the large circle is two units, then the ratio of the shaded area to the unshaded area is



(A) 2 : 1 (B) 1 : 1 (C) 1 : 2

(D) 3 : 4 (E) none of these

The shaded area is $2[\pi(1)^2] = 2\pi$.
The area of the large circle is $\pi(2)^2 = 4\pi$.
The unshaded area is $4\pi - 2\pi = 2\pi$.
The ratio of the shaded area to the unshaded area is $2\pi : 2\pi = 1 : 1$.
The answer is B.



5 A rectangular house which measures 20m by 10m has an outside electrical outlet at a corner of the house. An electric mower, connected by a cord to the outlet, can reach a maximum distance of 15m. The largest area of lawn, in square metres, which can be cut is

(A) $225\pi - 50$ (B) $\frac{725\pi}{4}$ (C) 225π (D) $\frac{675\pi}{4}$ (E) 175π

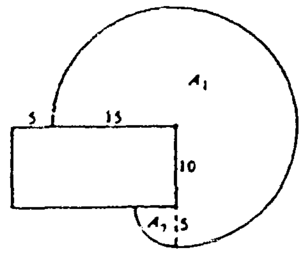
Solution

The total area that can be cut is

$$A_1 + A_2 = \frac{3}{4}(\pi \cdot 15^2) + \frac{1}{4}(\pi \cdot 5^2)$$

$$= \frac{675\pi}{4} + \frac{25\pi}{4}$$

$$= \frac{700\pi}{4} = 175\pi \text{ m}^2.$$



ANSWER: (E)

6 In the sequence 5, 16, 27, ... each term is 11 greater than the preceding term. A term in this sequence is

(A) 90 (B) 91 (C) 92 (D) 93 (E) 94

Solution 1.

The terms of the sequence are 5, 16, 27, 38, 49, 60, 71, 82, 93, 104, ...
93 is a term in the sequence.

Solution 2.

Every term in the sequence is of the form $11n - 6$, where n is a positive integer. The only answer of this form is 93 since $93 = 11(9) - 6$.

ANSWER: (D)

7 The integers greater than 1 are arranged, four in each row, in five columns as follows:

a	b	c	d	e
2	3	4	5	
	9	8	7	6
10	11	12	13	
	17	16	15	14

If the pattern is continued, 1000 will occur in column

(A) a (B) b (C) c (D) d (E) e

The integers greater than 1 are arranged, four in each row, in five columns as follows:

a	b	c	d	e
2	3	4	5	
	9	8	7	6
10	11	12	13	
	17	16	15	14

If the pattern is continued, 1000 will occur in column

(A) a (B) b (C) c (D) d (E) e

Solution

Even integers appear only in columns a, c, and e. Those in column a are of the form $8t + 2$ ($t = 0,1,2,\ldots$) ; those in column e are of the form $8t + 6$; and those in column c are of the form $4t$. Since $1000 = 4 \times 250$, it will be in column c.

8 The sum $1^4 + 2^4 + 3^4 + 4^4 + \cdots + n^4$ is given by the expression

$$\frac{6n^5 + an^4 + bn^3 - n}{30}.$$

The value of $a - b$ is

(A) −25 (B) −15 (C) −5 (D) 5 (E) 25

Let $S(n) = \dfrac{6n^5 + an^4 + bn^3 - n}{30}$

Therefore, $S(1) = \dfrac{6 + a + b - 1}{30} = 1^4$

$$a + b = 25. \quad (1)$$

Also, $S(2) = \dfrac{6(2^5) + a(2^4) + b(2^3) - 2}{30} = 1^4 + 2^4$

$$192 + 16a + 8b - 2 = 17(30)$$
$$16a + 8b = 320$$
$$2a + b = 40. \quad (2)$$

Subtracting (1) from (2) gives $a = 15$.

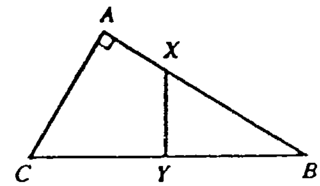
Thus $b = 10$ and $a - b = 5$.

The answer is D.

9 ABC is a right-angled triangle with $AB = 4$ and $AC = 3$. If the triangle is folded along the line XY, vertex C coincides with the vertex B. The length of XY is

(A) $\frac{8}{3}$ (B) $\frac{5}{3}$ (C) $\frac{15}{8}$

(D) $\frac{10}{3}$ (E) $\frac{5}{4}$

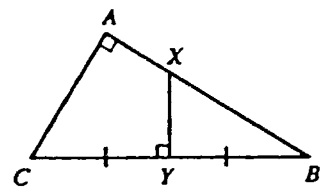


Since $AB = 4$ and $AC = 3$, $BC = 5$.
Since Y is the midpoint of CB, $BY = \frac{5}{2}$.
Since triangles XBY and CBA are similar,

$$\frac{BA}{BY} = \frac{AC}{XY}$$

$$\frac{4}{\frac{5}{2}} = \frac{3}{XY}.$$

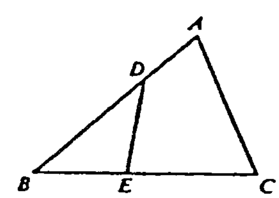
Hence, $XY = \left(\frac{5}{2}\right)\left(\frac{3}{4}\right) = \frac{15}{8}$.
The answer is C.



10 In ABC, D divides AB in the ratio $1:2$, and E divides BC in the ratio $3:4$. If the area of BDE is 6, then the area of ABC is

(A) 12 (B) $13\frac{1}{2}$ (C) 16

(D) 21 (E) $15\frac{3}{4}$



10. Join CD. Since ΔDBE and ΔDEC have a common altitude from D,

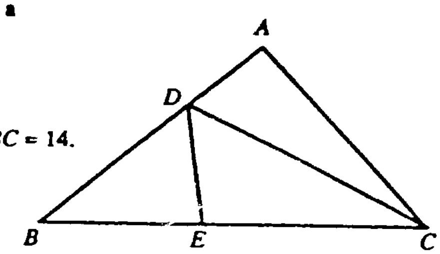
$$\frac{\Delta DBE}{\Delta DEC} = \frac{BE}{EC} = \frac{3}{4}.$$

Since $\Delta DBE = 6$, $\Delta DEC = 8$, and $\Delta DBC = 14$.

Similarly, $\frac{\Delta CAD}{\Delta CBD} = \frac{AD}{DB} = \frac{1}{2}$.

Since $\Delta DBC = 14$, then $\Delta CAD = 7$.

Therefore $\Delta ABC = \Delta DBC + \Delta CAD = 21$.



AGENDA
CAYLEY REVIEW
DEC. 6

1. Math Camp
2. Review Questions from last day- Presentations
3. Jigsaw - Group by #s, 1,2,3, 4&5

    -solve problem

    -return to home groups and teach others your problem

    -presentations
4. Multiple choice strategy

BREAK

5. This week's set of CAYLEY questions. Written solutions required, one set handed in per team. Marks- neatness 1

    - correct answer 1
    - strategy inititated 1
    - stategy completed 1

    } 4
6. Presentations
7. Assignment : Calculators may be used on the contest. How will this change the questions and the nature of the test? Find five questions on old Cayleys that will not be asked or be replaced in a different form.
8. Journal Entries.

Cayley Assignment
Group Name:______________________

1. Assignment : Calculators may be used on the contest. How will this change the questions and the nature of the test? Find five questions on old Cayleys that will not be asked or be replaced in a different form.

2. The following question might be a calculator type question. Can you solve it?

   Determine the following to three decimal places:

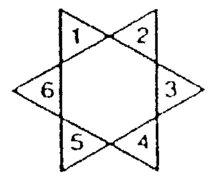
$$\cfrac{3}{1+\cfrac{3}{1+\cfrac{3}{1+\cfrac{3}{1+\cfrac{3}{1+\cfrac{3}{\cdots}}}}}}$$

3. Your Group must start the planning and construction of the mock Cayley test for the January 24 meeting. Organize the test into three sections, with five questions in each. The test is to be multiple choice, and be sure to put some planning in the choice of "distractors". Do not try to make the test too hard, but attempt to approach the level of the "regular Cayley". Try to have the test typed. (You may want to ask your teachers if you can use their "Math Type" program. The test will be out of 60 (3 sections x 5 questions x 4 marks)

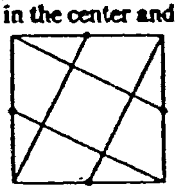
NOV 23 CAYLEY
JIGSAW SET

① **Can you use exactly six congruent line segments and arrange them so as to form six congruent triangles?**

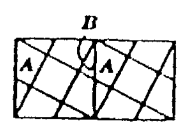
The use of the word *exactly* is important. We must use exactly six segments; however, the statement of the problem allows for more than six triangles just as long as we have six congruent triangles.



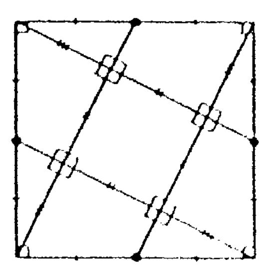
② **Given a square of area A, each vertex is joined clockwise to the midpoint of one opposite side. Describe the figure formed in the center and find its area.**



Square of area $A/5$

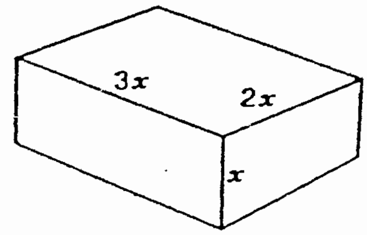


By congruent triangles we can prove that the figure is a square. Then by duplicating the figure next to itself, we can see that each triangle unites with each trapezoid to yield a square with the same dimensions as the center square. Five equal squares are formed in the large square.



(3) If the ratio of the lengths of the edges of a right-rectangular prism is 1:2:3 and the total surface area is 550 square units, find its volume.

750 cubic units



$$6x^3 = \text{Volume}$$

$$2 \cdot x \cdot 2x + 2 \cdot x \cdot 3x + 2 \cdot 2x \cdot 3x = 550$$

$$4x^2 + 6x^2 + 12x^2 = 550$$

$$22x^2 = 550$$

$$x^2 = 25$$

$$x = 5$$

(4) Find the sum of *all* proper fractions whose denominators are less than or equal to 100.

2475

$$\frac{1}{2} + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + \left(\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5}\right) + \cdots + \left(\frac{1}{100} + \frac{2}{100} + \cdots + \frac{99}{100}\right)$$

$$= \frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \frac{4}{2} + \cdots + \frac{99}{2}$$

$$= \frac{1}{2}(1 + 2 + \cdots + 99) = 2475$$

AGENDA
CAYLEY REVIEW
FEB. 7

1. PRESENTATIONS OF QUESTIONS FROM DEC. 6.

2. HAND IN: a) CAYLEY SOLUTIONS FROM DEC. 6
   b) TEST THAT YOUR GROUP HAS PREPARED
   c) ASSIGNMENT SHEET

3. USING THE GRAPHICS CALCULATOR.

4. CAYLEY SET BASED ON GRAPHING.

BREAK

5. PRESENTATIONS ON GRAPHING QUESTIONS.

6. JOURNAL ENTRIES.

7. MULTIPLE CHOICE STRATEGIES.

Equations of Lines

Multiple Choice Questions

1. If $x \Diamond y$ is defined to be $2x - 5y$, then $y \Diamond x = x$ for

   (A) $y = 1$ only (B) $x = 0$ only (C) $y = \frac{1}{5}x$ (D) $y = 3x$ (E) no value of y

2. The number of points in which the graphs of $4x - y = 0$, $3x + 2y - 9 = 0$, $x = 2$, and $y = \frac{3}{2}$ intersect is

   (A) 6 (B) 5 (C) 4 (D) 3 (E) 7

3. If $ax + 3y = 5$ and $2x + by = 3$ represent the same straight line, then $a + b$ equals

   (A) 5 (B) $\frac{77}{15}$ (C) $\frac{19}{15}$ (D) $\frac{31}{5}$ (E) $\frac{77}{10}$

4. If $\frac{x}{4} + \frac{y}{5} = \frac{19}{20}$, where x and y are positive integers, then $x + y$ is

   (A) 9 (B) 19 (C) 20 (D) 4 (E) 5

5. If a and b are the x- and y-intercepts of a line which passes through the point (2, 1), then

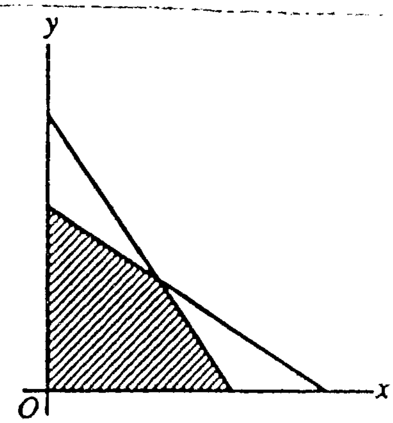
   (A) $a(b - 1) = 2b$ (B) $a = 2b$ (C) $b = 2a$
   (D) $b(a - 1) = 2a$ (E) none of these

6. The lines $x = 0$, $y = 0$, and $2x + y = 4$ form a triangle. The number of points with integral coordinates which are inside this triangle is

   (A) 1 (B) 2 (C) 3 (D) 4 (E) more than 4

7. The shaded region in the diagram is bounded by the lines $3x + 2y = 30$, $2x + 3y = 30$, $x = 0$, and $y = 0$. Its area is

(A) 30 (B) 60 (C) 84
(D) 90 (E) 150



8. A lattice point in the plane is a point whose coordinates are integers. The number of lattice points on the line $3x + 4y = 59$ which are in the first quadrant is

(A) 2 (B) 3 (C) 4 (D) 5 (E) an infinite number

9. Two perpendicular lines intersect at the point (9, 2). If the x-intercept of one line is double the x-intercept of the other, then a possible sum of these x-intercepts is

(A) $\frac{17}{2}$ (B) 10 (C) $\frac{51}{2}$ (D) $\frac{45}{2}$ (E) 5

10. The line with equation $y = 3x + 1$ is reflected in the line $y = 4$. The equation of the reflected line is

(A) $y = \frac{1}{3}x + 1$ (B) $y = -\frac{1}{3}x + 7$ (C) $y = -\frac{1}{3}x + 6$
(D) $y = -3x + 6$ (E) $y = -3x + 7$

AGENDA
CAYLEY REVIEW
FEB. 14
Happy Valentine's Day & Happy Chinese New Year

1. Review of last week's questions.

2. Write student Cayleys.

3. Break

4. Mark tests.

5. Review test with students that wrote your test.

MARKING & DISCUSSING THE TESTS

1. OBTAIN THE TEST THAT YOUR GROUP MADE UP AND MARK IT.

2. LIST THE ERRORS THAT THE WRITER'S MADE, ANALYZE THEM, AND PREPARE TO DEFEND YOUR TEST IN A DISCUSSION WITH THEM. RETURN THE TEST TO THE WRITERS.

3. PREPARE A CRITIQUE OF THE TEST THAT YOU WROTE IN PREPARATION OF A DISCUSSION WITH THE MARKERS.

4. FORM TWO GROUPS, ONES AND TWOS FORM ONE GROUP, AND THREEES AND FOURS FORM THE OTHER. DISCUSS THE TWO TESTS.

5. RETURN TO YOUR ORIGINAL GROUPS AND WRITE A SHORT REPORT OF THE MAJOR POINTS OF DISCUSSION.

6. FROM THE TWO TESTS THAT YOU WORKED ON, PICK ONE QUESTION FROM EACH SECTION WHICH YOU THINK WOULD MOST LIKELY BE ON A CAYLEY, AND LIST THESE THREE.

GROUP PAIRING

DESCARTE (Cariboo)	EINSTEIN (Central)
GAUSS (Cariboo	ABEL (North)
PASCAL (Alpha)	FERMAT (South)
NEWTON (North)	EUCLID (South)

FAKE CAYLEY CONTEST

BY BURNABY CENTRAL

Good Luck!

Part A (5 credits each)

1. The value of $\frac{(0.2)^2}{2}$ is

(A) 0.2 (B) 0.1 (C) 0.04 (D) 0.02 (E) 0.01

2. If $15x + 20 = 25$, then the value of x is

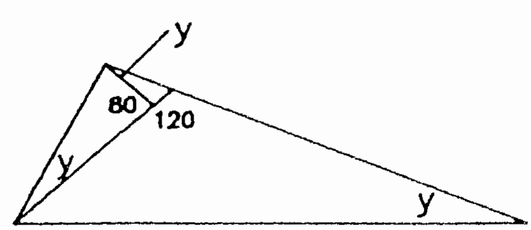
(A) -10 (B) $-\frac{1}{3}$ (C) $\frac{1}{3}$ (D) $\frac{5}{7}$ (E) 3

3. The value of $\frac{\sqrt{9+16}}{\sqrt{16}}$ is

(A) 4 (B) 3 (C) (D) $\frac{7}{4}$ (E) $\frac{5}{4}$

4. In the diagram, all given measures are in degrees. The value of y is

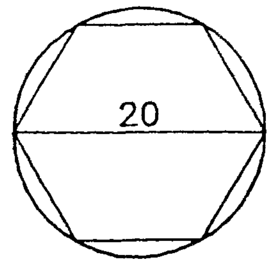
(A) 15 (B) 20 (C) 30
(D) 45 (E) 50



5. If $3x + 2 = y$, then the value of x, in terms of y, is

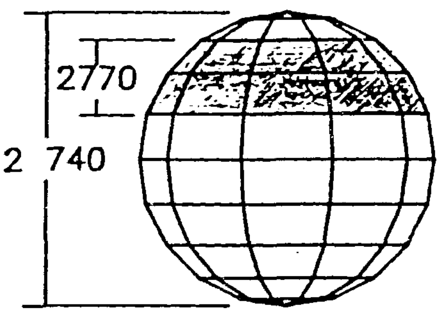
(A) $y-1$ (B) $y-2$ (C) $y-5$ (D) $\frac{y-2}{3}$ (E) $\frac{y+2}{3}$

Part B (5 credits each)



6. Find thr area of a regular hexagon inscribed in a circle with a 20 cm diameter to the nearest centimetre.

   (A) 260 (B) 300 (C) 314 (D) 410 (E) 433



7. If the diameter of the world is 12 740 km, assuming that the world is an exact sphere, and the height from the Tropic of Cancer to the Arctic Circle is 2770 km, find the surface area of the shaded region to the nearest millions of km in terms of Π.

   (A) $3.5\Pi \times 10^7$ (B) $7.1\Pi \times .10^7$ (C) $9.2\Pi \times 10^7$ (D) $1.8\Pi \times 10^7$ (E) $1.62\Pi \times 10^8$

8. A girl is flying a kite. She is holding the string 1 metre off the ground. If the string is 30° to the ground and is 1 kilometre long, how high is the kite from the ground in metres?

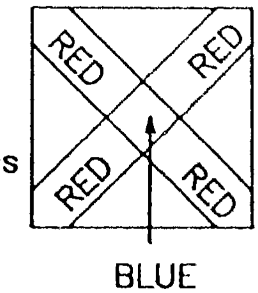
   (A) 1000 (B) 867 (C) 501 (D) 485 (E) 354

9. A forest fire is burning at a rate of 20 square metres per minute. Two fire fighters are in a canoe 5 kilometres away. If the fire fighters could continuously paddle at a rate of 30 km/h, and there is a current of 5 km/h flowing against the boat, how much of the forest will burn in the time they took to reach the scene?

   (A) 240 m^2 (B) 200 m^2 (C) 199 m^2 (D) 170 m^2 (E) 4 m^2

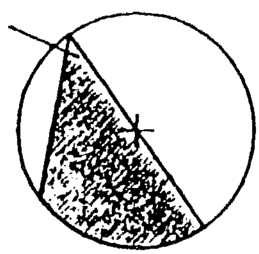
Part C (5 credits each)

11. A square flag has a red cross of uniform width with a blue square in the center on a white background as shown. (The cross is symmetric with respect to each of the diagonals of the square.) If the entire cross (both the red arms and the blue center) takes up 36% of the area of the flag, what percent of the area of the flag is blue?



(A) 0.5 (B) 1 (C) 2 (D) 3 (E) 6

12. Find the area of the shaded region subtended in a circle with a 30 mm diameter.



(A) $\frac{225}{2}$ (B) 225 (C) $\frac{225\pi}{2}+\frac{225}{4}$
(D) $\frac{225\pi}{2}$ (E) $\frac{225\pi}{4}+\frac{225}{2}$

13. One train leaves a station heading west. A second train heading east leaves the same station 2 h later and travels 15 km/h faster than the first. They are 580 km apart 6 h after the 2nd train departed. How fast in the train heading west travelling.

(A) 15 (B) 45 (C) 30 (D) 35 (E) 50

14. Five bundles of hay are weighted in pairs to form these weights; 120, 122, 124, 125, 126, 127, 128, 129, 130 What is the weight of the 3rd heaviest bundle.

(A) 63 (B) 60 (C) 66 (D) 65 (E) 61

FEB. 21
CAYLEY REVIEW
AGENDA

8:50 SURVEY FORMS

9:00 DISCUSS WITH EACH OTHER YOUR STRATEGY FOR WRITING THE FAKE CAYLEY, e.g.:
- working individually, in pairs, or groups of 4?
- work individually for 30 minutes, then in pairs, then groups of 4?

9:05 FAKE CAYLEY

10:05 BREAK & MARK TEST

10:15 JIGSAW TO ATTEMPT TO FIND ALL SOLUTIONS FOR THE TEST.

#1'S WILL DO	1-5-9-13-17-21-25
#2'S	2-6-10-14-18-22
#3'S	3-7-11-15-19-23
#4'S	4-8-12-16-20-24

11:00 CLOSURE ON TEST

11:15 JOURNAL ENTRIES

11:30 PIZZA

12:20 END OF SESSIONS

FAKE CAYLEY

/150

	ANSWER	MARK
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
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24		
25		

GROUP NAME: ______________________

SCHOOL: ______________________

INDIVIDUALS:

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THE BURNABY MATHEMATICS COMPETITION

BURNABY
SCHOOL DISTRICT 41

FAKE CAYLEY for Grade 10

Thursday, February 21, 1991
Supported by Schou Education Centre

Time: 1 hour

Instructions

1. You may talk to any one.
2. You may use any calculator, geometry sets, graph paper.
3. Be certain that you know a few multiple choice strategies. One point will be deducted from incorrect answers.
4. Diagrams are not drawn to scale.

Part A (4 credits each)

1. The value of 0.01−0.9 is:

(A) −0.91 (B) 0.89 (C) −0.89 (D) −0.091 (E) 0.91

2. $(1.1-1.2)(1.1+1.2) =$

(A) 2.65 (B) −2.65 (C) −0.23 (D) 0.23 (E) −0.01

3. If $5 \le a \le 10$ and $20 \le b \le 30$ what is the maximum value of $\frac{a}{b}$?

(A) 0.5 (B) 0.25 (C) 0.3333 (D) 0.16666 (E) none of these

4. If $x = -3$, then $x^{-x} = ?$

(A) 27 (B) −27 (C) −9 (D) 9 (E) 1/27

5. The product of the ages of three teenagers is 4590. How old is the oldest?

(A) 19 (B) 18 (C) 17 (D) 16 (E) 15

6. The probability of rolling a "9" with a pair of dice is :

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{5}$ (D) $\frac{1}{6}$ (E) $\frac{1}{9}$

continued

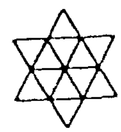
7. A four digit number reads the same whether it is read from the right or the left. It is also the same when looked at upside down. The number and its reflection in a mirror are also the same. What is the number?

(A) 3333 (B) 1010 (C) 1111 (D) 6969 (E) none

8. If the digits of a certain three digit number are added and the result is cubed, the final answer is the original number. What is the number?

(A) 125 (B) 216 (C) 343 (D) 512 (E) 729

9. How many triangles are in this figure?



(A) 12 (B) 14 (C) 18 (D) 20 (E) 22

10. Express as a common fraction: $\left[\left(\frac{1}{2}\right)^{-1}+\left(\frac{1}{3}\right)^{-1}+\left(\frac{1}{5}\right)^{-1}+\left(\frac{1}{7}\right)^{-1}\right]^{-1}$

(A) $\frac{17}{1}$ (B) $\frac{1}{17}$ (C) $\frac{175}{17}$ (D) $\frac{17}{175}$ (E) 175

Part B (5 credits each)

11. The volume of a cube with a surface area of 48 is:

(A) $24\sqrt[3]{3}$ (B) $16\sqrt{3}$ (C) $16\sqrt{2}$ (D) $192\sqrt{3}$ (E) $12\sqrt[3]{2}$

12. A bag contains 5 blue marbles, 4 white marbles, and 3 red marbles. If 3 marbles are randomly selected from the bag, what is the probability that the marbles selected will be of the same color?

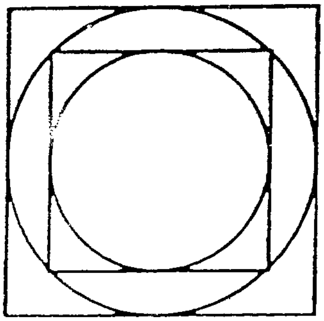
(A) 0.2500 (B) 0.1818 (C) 0.0682 (D) 0.0455 (E) 0.0156

13. Give the best approximation of: $1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cdots}}}}$

(A) 1.51 (B) 1.61 (C) 1.71 (D) 1.81 (E) 1.91

continued

14. Find the ratio of the area of the smaller square to the area of the larger square?

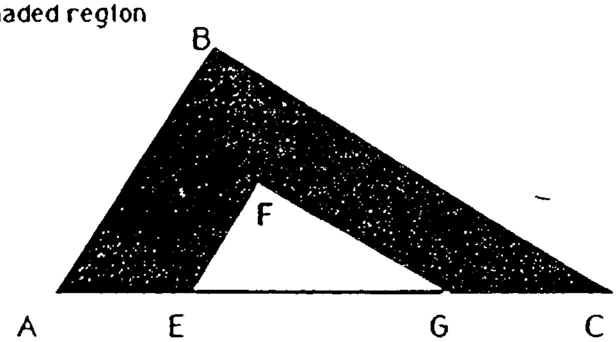


(A) 1:2 (B) 1: $\sqrt{2}$ (C) 1: $2\sqrt{2}$ (D) 1: $\sqrt{3}$ (E) 1: $3\sqrt{3}$

15. The average age of a group consisting of doctors and lawyers is 40 years. If the average age of the doctors' ages is 35, and that of the lawyers' ages is 50, find the ratio of the number of doctors to lawyers.

(A) 2:1 (B) 3:1 (C) 1:2 (D) 3:2 (E) 2:3

16. Given $\triangle$ ABC $\sim$ $\triangle$ EFG, $\angle$ F is a right angle, AB=6, AC=10, FG=3, Find the area of the shaded region



(A) 20.625 (B) 20.375 (C) 20.225 (D) 19.875 (E) 19.625

17. The smaller angle (in degrees) between the hands of a clock at 12:25 is:

(A) 132.5 (B) 137.5 (C) 150 (D) 137 (E) none of these

18. If x is positive, which of the following expressions must be less that 1?

(A) $\frac{1}{X}$ (B) $\frac{(1+X)}{X}$ (C) X^2 (D) $\frac{(1-X)}{X}$ (E) $\frac{X}{(X+1)}$

19 An equilateral triangle ABC has an area of $\sqrt{3}$ Point P is an arbitrary point in the interior of the triangle What is the sum of the distances from p to AB, AC, and BC?

(A) 1 (B) $2\sqrt{2}$ (C) $\sqrt{3}$ (D) $\sqrt{2}$ (E) $2\sqrt{3}$

continued

20. If A*B = AB+1 and A⊕B = (A+B)/2, then find 3*[(7⊕3)⊕(3*4)]

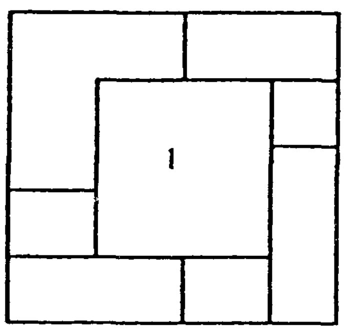
(A) 21 (B) 19 (C) 24 (D) 31 (E) 28

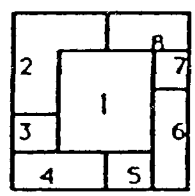
Part C (6 credits each)

21. For what value of "m" will the triangle formed by the lines
$y=-4$ $y=mx+7$ and $y=-mx+7$ be equilateral?

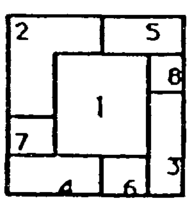
(A) $\sqrt{3}$ (B) $2\sqrt{2}$ (C) $\sqrt{2}$ (D) $3\sqrt{2}$ (E) $2\sqrt{3}$

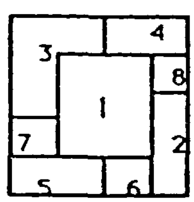
22. Eight square sheets of paper, all the same size, have been placed on a table. They overlap as shown. One sheet, marked 1 is shown completely, and the seven others are only partly exposed. Number the squares from the top layer to the bottom.

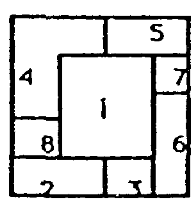


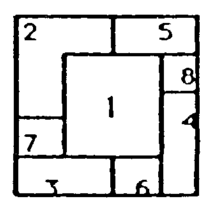


(A)









(B) (C) (D) (E)

23. Find the value of x if $$\left(\frac{4}{9}\right)^{\sqrt{x}} = \left(\frac{2}{3}\right)^{3x-\sqrt{x}}$$

(A) 169 (B) 81 (C) $\sqrt{13}$ (D) $3\sqrt{13}$ (E) $3\sqrt{3}$

24. Find the sum of all proper fractions whose denominators are less than or equal to 100.

(A) 47.25 (B) 464 (C) 1728 (D) 2475 (E) none of these

25. If $x+y=5$ and $x^2+3xy+2y^2=40$, find $x+2y$

(A) 4 (B) 8 (C) 16 (D) 20 (E) 25

INTERVIEW QUESTIONS COOPERATIVE PREPARATION FOR THE CAYLEY CONTEST

1. How did you feel about:
   a) the video camera?
   b) the Journals?
   c) score keeping?
   d) your results on the Cayley? Did they meet your expectations in relation to the time that you spent?
   e) helping others?

2. a) Was the group the right size?
   b) How did the group function?
   c) Did you or anyone in the group monitor the group's performance? (make observations or suggestions)

3. a) How much time outside of these sessions did you spend preparing for the Cayley:
      i) by yourself?
      ii) with other people (who)?
   b) How was the 'mock Cayley' prepared by your group?

4. a) Did you look at the Cayley Results? What is your reaction to them?
   b) Did you know the students on the list that were not part of the Schou meetings? If so, how did they prepare for the Cayley?
   c) Was there any program of review for the Cayley available at your school?

5. Did cooperative learning assist you in any way with:
   a) social skills
   b) self-esteem
   c) self-direction
   d) liking for math or math contests?

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