A COMPARISON OF SCIENTIFIC HEURISTIC AND TRADITIONAL METHODS OF TEACHING MATHEMATICS

by

DOMINIC ANTHONY ALVARO

B.Sc., University of British Columbia, 1961

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE

in the Faculty
of
Education

SIMON FRASER UNIVERSITY

April, 1973

APPROVAL

Name: Dominic Alvaro

Degree: Master of Science (Education)

Title of Thesis: A Comparison of Scientific Heuristic and Traditional Methods of Teaching Mathematics

Examining Committee:

Chairman: Arthur H. Elliott

John V. Trivett
Senior Supervisor

A. J. Dawson

John L. Berggren

D. Bruce Harrison
External Examiner
Associate Professor
University of Calgary, Alberta

Date Approved: April 19/73

ABSTRACT

The purpose of the study was to compare two distinct methods of teaching mathematics in a secondary school. A quasi-experimental design was utilized with an experimental group of students being taught by the researcher using controlled discovery techniques. The major focus of this method was the subordination of teaching to learning (also called the scientific heuristic approach). A control group was also taught by the researcher using more standard teaching techniques involving a teacher-centered classroom climate.

A questionnaire was constructed by the author, and administered to people of varying degrees of expertise while they viewed video-tape recordings of the lessons with the school students. Analysis of the questionnaires supplied evidence of the two distinct methods of teaching used.

The data used to compare the two methods comprised pre and post-test measurements of achievement in mathematics, attitude towards mathematics and self concept of the experimental and control groups. The data was statistically analyzed in order to test the following null hypotheses:

1. There will be no significant difference between the scores of the group of students taught by traditional methods and those taught by scientific heuristic methods, with respect to <u>achievement in mathematics</u>.

2. There will be no significant difference between the scores of the group of students taught by traditional methods and those taught by scientific heuristic methods, with respect to <u>attitude towards mathematics</u>.

3. There will be no significant difference between the scores of the group of students taught by traditional methods and those taught by scientific heuristic methods, with respect to self concept.

The results of the statistical analysis indicated a significant increase in achievement for both the experimental and control groups. The traditionally taught control group, however, scored significantly higher in achievement than did the scientific heuristically taught experimental class. The students of the scientific heuristic class showed a significant improvement in their attitude towards mathematics, a result not found in the control class.

All evidence considered, it was concluded that neither method of teaching mathematics was shown to be superior over the other.

ACKNOWLEDGEMENTS

I wish to express my sincere appreciation to all who assisted me in completing this thesis, in particular:

Professor John V. Trivett who showed unquestionable eagerness to help, especially in the number of hours he spent reading and rereading the thesis and the promptness by which it was done, for his numerous hints and suggestions and finally for his perseverance for clarity and flow which did much to improve the standard of writing of the paper;

Dr. A.J. Dawson for his moral encouragement and the prompt and sincere fashion in which he helped with the thesis, especially with the organization of its content;

Dr. J.L. Berggren for the conscientiousness with which he offered helpful criticism, particularly on the section on the nature of mathematics;

Miss Beth Carolan for her help with the video-taping of the two classes and for giving a very understanding ear during the teaching session;

Dr. Norman Robinson for the congenial and prompt way in which he helped with the statistical analysis of the data;

Professor Jack Young of the University of British Columbia for his help in administering the questionnaire;

Mr. Don Fraser, principal of Summer School 1970, North Vancouver School District, for his willing cooperation during the teaching period;

And, finally, The Board of the School District of North Vancouver whose cooperation made possible the pilot project and the experiment.

D.A.A.

TABLE OF CONTENTS

CHAPTER PAGE

CHAPTER PAGE

LIST OF TABLES

TABLE		PAGE
XI.	COMPARISON OF SCORES FOR SH-CLASS ON SELF CONCEPT	84
XII.	COMPARISON OF SCORES FOR TRAD-CLASS ON SELF CONCEPT	85
XIII.	ANALYSIS OF COVARIANCE BETWEEN SH-CLASS AND TRAD-CLASS ON FINAL SCORE OF SELF CONCEPT WITH PRETEST SCORE OF SELF CONCEPT AS COVARIATE	86
XIV.	COMPARISON OF BOYS' SCORES WITH GIRLS' SCORES OF TRAD-CLASS BEFORE THE EXPERIMENT	87
XV.	COMPARISON OF PRETEST AND POST-TEST SCORES FOR BOYS OF TRAD-CLASS	88
XVI.	COMPARISON OF BOYS' SCORES WITH GIRLS' SCORES OF TRAD-CLASS AFTER THE EXPERIMENT	89
XVII.	COMPARISON OF PRETEST AND POST-TEST SCORES FOR GIRLS OF TRAD-CLASS	89
XVIII.	COMPARISON OF SCORES FOR SH-CLASS AND TRAD-CLASS ON ACHIEVEMENT ON 'SH-TYPE' TESTS	92
XIX.	COMPARISON OF PRETEST SCORES FOR BOYS OF SH-CLASS AND TRAD-CLASS	191
XX.	COMPARISON OF POST-TEST SCORES FOR BOYS OF SH-CLASS AND TRAD-CLASS	191
XXI.	COMPARISON OF PRETEST SCORES FOR GIRLS OF SH-CLASS AND TRAD-CLASS	192
XXII.	COMPARISON OF POST-TEST SCORES FOR GIRLS OF SH-CLASS AND TRAD-CLASS	192

TABLE. PAGE

TABLE PAGE

LIST OF GRAPHS

CHAPTER I

THE NATURE AND THE PURPOSE OF THE STUDY

I. INTRODUCTION

Change is a fact of present-day life. In the last ten years we have experienced not only quantitative changes but also qualitative changes in our life styles. It is not surprising then that a great variety of people, be they psychologists, educators, teachers or students have and are advocating modifications, radically or otherwise, in the education system, in teaching strategies, in curriculum design and in the role of the teacher. Indeed, if the world continues to change at the present rate probably one of the most significant contributions education can make is to assist the adults of tomorrow in adapting and coping with change.

This report deals exclusively with possible changes in the teaching of mathematics in schools. In doing so, however, mathematics is not viewed as an isolated discipline. Mathematics is viewed as very much a part of the student's general education and as preparation of the student for tomorrow's world.

To say that change in mathematics teaching is needed may be a lot easier than to say what it is that needs changing, and perhaps more difficult is to outline how these changes should occur. However, some mathematics educators acknowledge the necessity for change and do attempt to delineate methods for this change.

The author of this study outlines a method of teaching mathematics in accordance with some of the changes these advocators indicate. Further, he compares the effects that this newly defined method of teaching had on a select group of boys and girls. Their attitude towards mathematics, their self concept, and their achievement in mathematics were all examined and so were the results obtained of teaching a comparable group of students by a traditional method, using the same criteria in both cases.

The study does, however, limit any conclusions or generalizations to a select group of people because of the size of the sample, duration of the study, content and methods used.

II. BACKGROUND AND REVIEW OF THE LITERATURE

In this section the writer reviews the literature on the need for change in education, the direction some psychologists and educators suggest this change should take, and finally how this need for change and its direction applies to the teaching of mathematics.

Carl Rogers, a prominent educator and psychologist, in his book Freedom To Learn states:

> I rely on the potentiality and wisdom of the human being - if this potential can be released - to bring about desperately needed changes in education before it is too late.[1]

---

1. Rogers, C., Freedom To Learn. Charles E. Merrill Publishing Co., Columbus, Ohio, 1969, p.viii.

It is interesting to note that along with the urgency for change in education, Rogers also implies that the wisdom and potentiality to bring about this change essential for the benefit of all, exists in human beings. This would imply that we as teachers have only to find the way to bring about change.

> ... Unless we give strong positive attention to the human interpersonal side of educational dilemma, our civilization is on its way down the drain ... Only persons, acting like persons in their relationships with their students can even begin to make a dent on this most urgent problem of modern education.[2]

Rogers is not only concerned with curriculum changes, or changes in methodology, he is concerned with the interpersonal relationships between students and teachers. In fact he is very concerned with the role of the teacher. He believes that such attributes as realness or genuineness on the part of the teacher are basic essentials. He goes on to say:

> Teaching and imparting knowledge makes sense in an unchanging environment. But if there is one truth about modern man, it is that he lives in an environment which is <u>continually changing</u>.
>
> ....
>
> The goal of education, if we are to survive, is the facilitation of change and learning.[3]

---

2. Ibid, p.125.

3. Ibid, p.104.

Rogers refers to teaching as 'facilitating learning'. He also prefers to think of students learning, rather than teachers teaching - hence his statement:

> My experience has been that I cannot teach another person how to teach.[4]

So far we have talked about change in education but this implies also a change in mathematics education.

In the remainder of this section attention is focused first on what some writers consider to be required changes in the teaching of mathematics. The modifications suggested are then placed in a broader framework, one which draws attention to the powers of children and the effects these powers have on the teaching-learning environment of the classroom. It is from this brief survey of the literature that generalizations are sought which give rise to the purpose of this study.

Dawson views mathematics as it is taught today:

> ... mathematics today lacks a mode of inquiry.[5]

---

4. Ibid, p.152.

5. Dawson, A.J., "A Model of Mathematics Instruction:. University of Alberta, unpublished doctoral dissertation, 1969, p.7.

He defines the mode of inquiry of mathematics as the patterns, methods, or procedures utilized by the creative mathematician. Dawson advocates that mathematical knowledge grows in a manner described by the philosophy of Karl Popper, namely 'Critical Fallibilism'.

> The fallibilistic position is one which characterizes the growth of knowledge as being a conjecture and refutation process ... all knowledge is tentative and subject to constant and never ending criticism.[6]

Dawson implies that the mathematics today is not taught by a discovery method, nor is it taught as knowledge which is subject to a never ending change.

How is it taught then? Polya contends that:

> Our knowledge about any subject consists of information and of know-how. If you have genuine bona fide experience of mathematical work on any level, elementary or advanced, there will be no doubt in your mind that, in mathematics, know-how is much more important than mere possession of information. Therefore, in the high school, as on any other level, we should impart, along with a certain amount of information, a certain degree of know-how to the student.
>
> ....
>
> The teacher should know what he is supposed to teach. He should show his students how to solve problems - but if he does not know, how can he show them? The teacher should recognize and encourage creative thinking - but

---

6. Ibid, p.12.

> the curriculum he went through paid insufficient attention to his mastery of the subject matter and no attention at all to his ability to reason, to his ability to solve problems, to his creative thinking. Here is, in my opinion, the worst gap in the present preparation of high school mathematics teachers.
>
> ....
>
> What is know-how in mathematics? The ability to solve problems - not merely routine problems but problems requiring some degree of independence, judgment, originality, creativity. Therefore, the first and foremost duty of the high school in teaching mathematics is to emphasize methodical work in problem solving. This is my conviction;...[7]

Speaking generally of education before becoming specific in relation to one discipline, Gattegno in his book, What We Owe Children states:

> ... A stable society uninterested in questioning tradition was served well by transmission of well-preserved statements about wisdom and truth
>
> ....
>
> But in the changing world one discovers that the ability to forget is needed as much as the capacity to retain and that there is no value in taking the time to fix in one's mind what no longer obtains. No one in such a world is prepared to pay a heavy price for what is no longer functional.[8]

What he is referring to here of course is his objection to education to-day being based on memory. He goes on to criticize the traditional approach:

---

7. Polya, G., Mathematical Discovery: On Understanding Learning and Teaching Problem Solving, two volumes. John Wiley and Sons Inc., New York, 1962. Vol. 1., p.vii-viii.

8. Gattegno, C., What We Owe Children: The Subordination of Teaching To Learning. Outerbridge & Dienstfrey, New York, 1970. pp.6-7.

> .. in this approach, knowledge is conceived as pre-existing and as coming down, through the teacher, from those gifted people who managed to produce it ... teachers are those people who take knowledge down from the shelves where it is displayed and hand it out to students who presumably need only memory in order to receive it. This process is conceived as the way the student comes to own knowledge. The key to this view-and to the whole traditional way of teaching - is the tacit belief that memory is a power of the mind.[9]

He goes on to describe traditional teaching:

> Teachers give lessons, teachers also give homework, there are also reviews, they also test, and they do not stop with one cycle of reviewing and testing ... Traditional schools have a curriculum that is based upon teachers providing children with showers of knowledge, the kind of knowledge that is not 'know-how'. A consequence of this kind of teaching is that knowledge is passed on to them rather than something they themselves own.[10]

However, Gattegno offers an alternative. His basic philosophy which he calls the 'subordination of teaching to learning', is based on the belief that education rests on the 'powers of children'.

> ... when we look at children as owning the powers they actually have, and how they function, we are overwhelmed with possibilities for education ....[11]

Gattegno lists the task of the teacher who wants to subordinate teaching to learning:

---

9. Ibid, p.4.
10. Ibid, pp.14-15.
11. Ibid, p.11.

1. To understand that his students are persons with a will and that in an individual, the will is the source of change.

2. Acknowledge the existence of a sense of truth which guides us all and is the basis of all knowing.

3. Find out how knowing becomes knowledge.

4. Consider the economy of learning. Teachers can learn by watching these continuous transformations which are the laws of the economy of life and should make them allies, rather than work counter to them.[12]

With reference to the teaching of mathematics, Gattegno argues that:

> The role of the teacher of mathematics is to recognize that a student who can speak has a large number of mental structures which can serve as the basis for awareness that will enable him to transform these structures into mathematical ones. In particular, Algebra, defined as operations upon operations, is already the endowment of all students of all ages and to work from it will make every child into a budding mathematician. In such an approach, mathematics teaching becomes the task of making students aware of themselves as the basis of reaching the dynamics of mathematical relationships and offering them the situations that involve all sorts of these relationships.
>
> ....
>
> If teachers technically know how to take advantage of all the ways of knowing present in their student, the outcome is subordination of teaching to learning, a know-how for teacher that they will come to own as all other know-hows are learned, through trial and error, practice and mastery.[13]

---

12. Ibid, pp.53-65

13. Ibid, pp.70-73.

William Purkey, who is not a mathematician, nor an educator in mathematics, but whose work is nevertheless appropriate to the learning of mathematics, presents evidence accumulated from the research done by people such as Combs, Bledsoe, Brookover, Gill, Allport, and a host of others which support the statement that self concept plays a dominant part in students' success or failure in school. One such statement is the result of conclusions drawn from a study by Gill (1969) who found that patterns of achievement in public school students are significantly related to how the students see themselves. The actual quotation as it appears in Purkey's book is as follows:

> The results of this study support the conclusion with such convincing uniformity that the importance of the self concept in the educational process seems to need more emphasis than is presently given to it.[14]

If credibility is given to such comments, the need for change becomes self-evident. One interesting observation should however be noted at this point. It would at first glance appear that although those quoted are all in favour of educational changes, they have different priorities. This is not to imply that any of their ideas are contradictory. On the contrary, their comments seem to be complementary, but let us examine other statements by such authors before summarizing. (on pp.14-16.)

---

14. Purkey, W., Self Concept and School Achievement. Prentice-Hall, Inc., Englewood Cliffs, 1970, p.18.

Purkey also says:

> Teaching methods can be adapted so that definite changes of the kind sought for will occur in the self without injury to the academic programme in the process.[15]

He then lists some of the qualities the teacher should possess:

1. The teacher must have a good attitude about himself and be a model of authenticity to his students.
2. He shows the students that he is genuinely interested in them, has confidence in them, offers guidance toward the solution of their problems, and demands an appropriate degree of competence.

Purkey further states:

> Six factors seem particularly important in creating a classroom atmosphere conducive to developing favourable self-images in students. These are:
>
> 1. Freedom: ... the student ... needs the opportunity to make meaningful decisions for himself ... must have the freedom to make mistakes ... freedom of choice ... freedom to explore and to discover for themselves.
> 2. Challenge: High academic expectation and a high degree of challenge on the part of the teachers have a positive and beneficial effect on students.
> 3. Respect: A basic feeling by the teacher for worth and dignity of students.

---

15. Ibid, p.44

4. Warmth: A warm and supportive educational atmosphere is one in which each student is made to feel that he belongs in school and that the teachers care about what happens to him.

5. Control: Not ridicule, and embarrassment, but a firmness which shows the student that the teacher cares about him.

6. Success: Provide an educational atmosphere of success, rather than failure.[16]

Rogers criticizes traditional teaching in this way:

> ... in the vast majority of our schools, at all educational levels, we are locked into a traditional and conventional approach which makes significant learning improbable if not impossible. When we put together in one scheme such elements as prescribed curriculum, similar assignments for all students, lecturing, as almost the only mode of instruction, standard tests by which all students are externally evaluated and instructor chosen grades as the measure of learning, then we can almost guarantee that meaningful learning will be at an absolute minimum.[17]

Rogers says that the elements essential for 'self-initiated, experiential learning' are not obtained from the teaching skills of the teacher, or his scholarly knowledge of his subject, or his use of audio visual aids or upon his lectures and presentations or upon an abundance of books, although all of these will help. The facilitation of significant learning rests upon certain attitudinal qualities which exist in the personal relationships between the facilitator and the learner. He claims that

16. Ibid, pp.50-55.

17. Rogers, op. cit., p.5.

these qualities are 'realness' and 'prizing'.

> Realness: .... the facilitator is a real person, being what he is, entering into a relationship with the learner without presenting a front or a façade.
>
> ....
>
> Prizing: .... it is - a non-possessive caring. It is an acceptance of this other individual as a separate person, having worth in his own right. It is a basic trust - a belief that this other person is somehow fundamentally trustworthy.[18]

Many of the ideas on how freedom in the classroom may be realized by the students have been implied above. This additional one is of particular importance in the teaching of mathematics:

> The teacher sets the stage of inquiry by posing problems, creating an environment responsive to the learner, giving assistance to the students in the investigative operations. This makes it possible for pupils to achieve autonomous discoveries and to engage in self-directed learning. They become scientists themselves, on a simple level, seeking answers to real questions, discovering for themselves the pitfalls and the joys of the scientists' research. They may not learn as many scientific 'facts' but they develop a real appreciation of science as a never-ending search, a recognition that there is no closure in any real science.[19]

Purkey's and Rogers' comments on the teacher are almost identical. Purkey uses the terms 'authenticity' and 'genuinely interested' while

---

18. Ibid, pp.106-108.

19. Ibid, p.136.

Rogers substitutes 'realness' and 'caring'.

The classroom's atmosphere described by Purkey through such terms as 'freedom' and 'challenge' ... will foster student inquiry and self discovery which Rogers says are essential for significant learning.

Jean Piaget who is considered by many psychologists, educators, and others to be the foremost contributor to the field of intellectual development, gives a good account of his educational goals in the following statement:

> The principal goal of education is to create men who are capable of doing new things, not simply or repeating what other generations have done - men who are creative, inventive, and discoverers. The second goal of education is to form minds which can be critical, can verify, and not accept everything they are offered. The great danger today is of slogans, collective opinions, ready-made trends of thoughts. We have to be able to resist individually, to criticize, to distinguish between what is proven and what is not. So we need pupils who are active, who learn early to find out by themselves, partly by their own spontaneous activity and partly through material we set up for them; who learn early to tell what is verifiable and what is simply the first idea to come to them.[20]

The classroom and the teacher described by both Purkey and Rogers would indeed make a significant contribution towards achieving Piaget's aim of education.

---

20. Duckworth, E., "Piaget Rediscovered," in Piaget Rediscovered. (R.E. Ripple and V.N. Rockcastle, editors.) Cornell University, 1964, p.5.

It was mentioned earlier that Dawson developed a model of mathematical instruction, using the works of Popper, Polya, and Lakatos. From the point of view of this model the main roles of the teacher and students could be summarized by the following statements:

The Role of the Teacher:

1. Set the initial problem and learning situation.
2. Aid the students in developing effective testing procedures.
3. Provide counter examples and suggestions designed to force the students to expand their mathematical knowledge.
4. Guide the students in mapping an unknown mathematical terrain.
5. Allow students to create, revise, and expand their own maps (a map which will never be complete in every detail).

Students' Role(s):

1. To create, analyze, revise and expand his map of the mathematical wilderness which he is exploring.
2. (Like the teacher), to adopt a rational and critical attitude towards proposed maps.
3. To play a very active role in this learning situation.
4. To learn to create the map, not to memorize and reproduce it on demand.

   i.e. develop skills and attitudes for attacking problems fallibilistically.

Obviously a central theme of the fallibilistic approach which is itself based on a conjecture and refutation process, is the process of inquiry rather than seeing the discipline as a finished product. The process of inquiry was stressed in all the other previous quotes.

Although Dawson does not dwell on the interpersonal relationship between the students and teacher his comments do imply the teacher's awareness of his students as human beings with needs as previously outlined by Purkey and by Rogers.

A comment made earlier was that although the people referred to in this section all advocate change, their priorities differed. Moreover, it was contended that their comments were at the very least complementary. Below is a summary of the main points made by Rogers, Purkey, Dawson, Polya and Gattegno.

Rogers believes that human beings have the potentiality and the wisdom to learn and adapt to change. He says that learning to adapt to change is the only thing that makes sense in a changing world. The teacher can facilitate this change by promoting inquiry, by being empathetic, by showing trust and most important, by being real.

Purkey's main concern is the idea of self concept. He believes that a student who possesses a good self concept is well equipped to learn. He also believes that a person who is experiencing success in learning will develop a better self concept. A good learning situation and one which will produce a good self concept on the part of the students is a classroom which can produce an atmosphere of: Freedom, challenge, respect, warmth, control and success. To achieve this the teacher must be above all authentic. As educational psychologists both Purkey and Rogers tend to direct their attention towards the human interpersonal characteristics, between student and teacher. These implications are just as valid in the teaching of mathematics as they are in the teaching of social studies, English or science.

Dawson deduces from the fallibilistic approach that all knowledge is tentative, and subject to constant and never-ending change. We must teach by inquiry. Dawson's theory is consistent with Rogers' ideas on constant change, and the process of inquiry.

Polya's main concern is to show students how to solve problems. This seems to imply that students should be exposed to and encouraged in the process of inquiry. Dawson is in agreement with him.

Gattegno says all human beings possess a sense of truth, which appropriately adheres to change, which is a sense of learning. It is the teacher's role to take advantage of these attributes already present within all human beings and which are favourable to learning.

Again we see the close similarity in what these writers are saying. The mathematics educator's remarks are directed to the teaching of mathematics, but the similarity present in the ideas expressed by both groups is unmistakable.

They agree on change in methods of teaching. The mathematics educators believe in teaching for understanding through the process of inquiry. The psychologists believe that meaningful learning occurs when teachers are authentic and guide students in creating an atmosphere whereby students can discover for themselves.

III. PURPOSE OF THE STUDY

The purpose of this study is to investigate a 'Scientific Heuristic approach' to the teaching of mathematics as compared to a more traditional approach, and to assess as a result, the effect on students' self concept, their attitude towards mathematics and their achievement in mathematics.

To do this students were randomly assigned to two classes. One class was taught by a scientific heuristic method, the other by a traditional method. Each class was administered achievement, attitude and self concept tests before and after the teaching period.

The data obtained from these tests were analysed using appropriate techniques in order to test specifically the following null hypotheses:

1. There will be no significant difference between the scores of the group of students taught by traditional methods and those taught by scientific heuristic methods, with respect to <u>achievement in mathematics</u>.

2. There will be no significant difference between the scores of the group of students taught by traditional methods and those taught by scientific heuristic methods, with respect to attitude towards mathematics.
3. There will be no significant difference between the scores of the group of students taught by traditional methods and those taught by scientific heuristic methods, with respect to self concept.

In many ways this is a descriptive study which attempts to define and delimit the nature and scope of scientific heuristic methods. Relatively little research has been done in this area. Consequently, there is a great need for the collection of data which will contribute to the development of a more precise description of scientific heuristic methods.

To this end students' papers, video tapes, audio tapes, personal statements and anecdotes were accumulated and some of these are presented later in this thesis.

IV. LIMITATION OF THE STUDY

General limitations of this study include: Subject content, duration of the study, size and nature of sample, descriptive nature of traditional and scientific heuristic methods.

The study was restricted to the discipline of mathematics. The teaching period which yielded the data was based on a session of four weeks.

The sample comprised forty-six students who had previously 'failed' the 'regular course' and were collected from one school district only. Furthermore, although the definitions of traditional and scientific heuristic methods are based partly on literature and partly on the investigator's own experience, it must be noted that different interpretations and/or descriptions of these methods could lead to different results.

Due to the abovementioned restriction the results of this study are subject to further investigation.

V. ORGANIZATION OF THE THESIS

The thesis is subdivided into four additional chapters: Chapter II outlines the terms and methods of teaching mathematics used in this experiment. Chapter III describes the methods by which the experimenter carried on his investigation. Chapter IV is subdivided into four main sections. The first outlines the various statistical computations and different tests used to compare the mean scores of the two classes (pretest and post-test) on attitudes towards mathematics, self concept and achievement in mathematics. The second presents evidence of the two distinct methods of teaching mathematics used in this experiment. This is followed by a diary, which contains the teacher/experimenter's personal notes, written during the teaching of the two classes. The fourth and final section comprises high school students' own written feelings on the content of the course and the methods by which they were taught. The last chapter, V, contains a summary of conclusions, delimitations of the study, generalizations, and areas suggested for further investigation.

CHAPTER II

TRADITIONAL AND SCIENTIFIC HEURISTIC APPROACHES TO TEACHING MATHEMATICS - DEFINITION OF TERMS

I. MATHEMATICS

Methods of teaching mathematics are influenced by the nature of the subject itself. A brief look into the history of mathematics shows that two most influential aspects have always been and still are firstly, mathematics is in constant growth and thus apparently in never-ending change, and secondly, there is the process by which this growth occurs.

With regard to the first aspect, it would appear that early mathematics evolved out of the engineering and administrative needs of the civilization - as, for example, was the case in Babylon and Egypt. The mathematics then had a certain 'cookbook' character and though many problems were ingeniously solved and the basic operations of arithmetic became completely routine, the concept of a proof was foreign to the mathematics. The Greeks however successively developed an axiomatic mathematics. Their intent was to show that certain statements were necessarily true, once the truth of a few basic statements was admitted. They achieved their most notable success in plane and solid geometry and trigonometry.

The origins of algebra can be traced to the Babylonians. With their positional number system and complete mastery of quadratic equations with real roots the early development of the subject owes much to them. Their treatment of algebra was verbal and non-symbolic. The Greeks generally gave algebraic propositions a geometric expression (see, for example, Book II of Euclid's Elements). Around 350 A.D. Diophantus of Alexandria working rather within a Babylonian tradition introduced symbols representing plus (+), minus (-) and equals (=) of today. He also used letters for unknown quantities in arithmetic and treated arithmetic problems analytically.

We know but little of the development of mathematics during the first fifteen hundred years after the birth of Christ. In the early part of the seventeenth century Descartes used the symbolical algebra of the sixteenth century to analytically investigate geometrical problems, giving rise to analytic geometry. This together with the growing tendency to associate Kinematical concepts with curves, provided some of the methological and conceptual tools which resulted in the invention of the infinitesimal calculus by Leibnitz (1684) and Newton. For over a century after that mathematicians occupied themselves with extending and applying the calculus.

Secondly it can be noticed that the mathematics we read now is written in well organized and logical packages so as to economize energy of thought. But it is a misconception to conclude that mathematics existed always in that form. It should be pointed out that mathematics in the making neither is, nor has been, necessarily logical. Mathematicians have often arrived at the truth purely by instinct. Jordain in his book The Nature of Mathematics puts it this way:

> In mathematics it has, I think, always happened that conceptions have been used long before they were formally introduced, and used long before this use could be logically justified or its nature clearly explained. The history of mathematics is the history of a faith whose justification has long been delayed, and perhaps is not accomplished even now.[21]

Two relatively recent comments on the nature of the mathematical process by contemporary mathematicians are stated below. Polya, expresses it in this way:

> Mathematics is regarded as a demonstrative science. Yet this is only one of its aspects. Finished mathematics presented in a finished form appears as purely demonstrative, consisting of proof only. Yet mathematics in the making resembles any other human knowledge in the making. You have to guess a mathematical theorem before you prove it; you have to guess the idea of the proof before you carry through the details. You have to try and try again. The result of the mathematician's creative work is demonstrative reasoning, a proof; but the proof is discovered by plausible reasoning, by guessing. If the learning of mathematics reflects to any degree the invention of mathematics, it must have a place for guessing, for plausible inference.[22]

Halmos identifies the process as this:

> Mathematics - this may surprise you or shock you some - is never deductive in its creation. The mathematician at work makes vague guesses, visualizes broad generalizations, and jumps to unwarranted conclusions.

---

21. Jordain, P., "The Nature of Mathematics," in The World of Mathematics. (J.R. Newman editor) Simon and Schuster, New York, 1950, Vol.1, pp.34-35.

22. Polya, G., Mathematics and Plausible Reasoning. Princeton, 1954. Vol. 1, p.vi.

> He arranges his ideas, and he becomes convinced of their truth long before he can write down a logical proof. The conviction is not likely to come early - it usually comes after many attempts, many failures, many discouragements, many false starts. It often happens that months of work result in the proof that the method of attack they were based on cannot possibly work, and the process of guessing, visualizing and conclusion - jumping begins again. A reformulation is needed - and - this may surprise you - more experimental work is needed ...[23]

The similarity in what Polya and Halmos say about mathematics and how it grows is significant. They say that mathematics as it is created or developed is based on trial and error even if eventually it is condensed into neat logical packages. Even the outstanding invention of the calculus was accomplished long before the rigorous logical basis for it - the concept of a 'limit' - was introduced. Mathematics in fact was criticized for this lack of rigor at this time. Philosophers were the main critics, not the mathematicians themselves.

Not until new generations of mathematicians (Cauchy, Abel, Weierstrass, to name a few) arose was much significant advance made in establishing the logical foundation of the subject and that took two hundred years - from the time of Newton and Leibnitz to the end of the nineteenth century.

The work of mathematicians is still continuing as is the growth of mathematics. In fact there have been more articles in mathematics

---

23. Halmos, P.R., "Mathematics as a Creative Art", American Scientist. (56,4, 1968) pp.379-382.

published in the last decade than in all previous time. However the process by which this growth occurs is still as described by Polya and Halmos.

In summary therefore, there seems a notable consistency in how mathematics was particular, then it became more general, and eventually it was justified and expressed logically.

Mathematics itself changes because of its growth, and one view of this growth process has been outlined. It is not unlikely therefore that there are implications from this for methods of teaching mathematics, old and new. The following sections under the headings 'Scientific Heuristic Methods of Teaching Mathematics' and 'Traditional Methods of Teaching Mathematics' detail such implications.

II. TRADITIONAL METHODS OF TEACHING MATHEMATICS

Under traditional methods mathematics too often appears to students as a finished body of knowledge, accumulated by creative individuals known as mathematicians and presented as such by teachers who have mastered the body of knowledge and who in turn try to teach it to their students in this finished form.

To accomplish this traditional model the teachers have evolved a certain mode of behaviour.

A traditional teacher strives to be in control of his class; he assumes the role of an authoritative figure. The student is usually expected to get permission from the teacher to speak. All students are expected to be silent when the teacher is talking, and try to understand his explanation.

The students sit in rows, in desks which face the teacher. During the lesson (usually one hour) they remain at their desks, unless they have the teacher's permission to perform such tasks as working at the boards, sharpening their pencils or leaving the room. In most cases (especially in academic subjects at the high school level) the teacher follows a specified curriculum set by the Provincial Department of Education. This implies that the teacher is responsible for the student's mastery of a given number of mathematical facts. In order to maintain his role of controlling the class he has to demonstrate to them that he is the authority of what he is trying to teach. He must be well prepared in his presentation of facts and conduct his class with poise, confidence and competence. He must also be prepared to answer any of the student's questions. The mastery of facts is helped by home assignments and review sheets. Students' achievement is in part measured by tests and also by the grade they obtain on teacher marked projects and home assignments. The teacher keeps a record of these marks and eventually presents the students with a grade.

The traditional teacher of mathematics relies mainly on the lecture method of teaching. A typical lesson may start as follows. The teacher uses the first fifteen to twenty minutes to introduce a new concept.

He uses a demonstrating device such as a large slide rule or he explains verbally or questions the students. The concept is then illustrated on the board by the teacher working out some related examples for ten minutes or so.

'Seat work' follows. This is the part of the lesson when students usually work at their desks, independently attempting to solve problems similar to the ones the teacher worked out on the board. During the seat work the conscientious teacher supervises the students' work by making sure that the students are working quietly so they don't disturb others, as well as giving individual help, or he may even help a group of students if enough of them are having difficulty with the same problem.

Finally he might even call the whole class to attention if enough of them are having difficulty and explain a particular problem to all.

Following the seat work or the question period involving the seat work, the teacher follows up with a compulsory home assignment. Such an assignment is usually designed to give the students practice, and possibly to add some problem that extends beyond what has been done in class. The home task is usually the same for all the students, although the more gifted students get their reward by finishing more quickly.

A follow-up lesson usually involves a discussion of the homework. Some traditional teachers also believe in running an occasional check to see if the students have completed their assigned work. It is not uncommon for the teacher to levy a penalty such as a detention if a student fails to complete his task and cannot present him with what he considers to be a valid reason.

The discussion of the assignment usually means that the teacher is willing to work out solutions on the board. He quite often tries to involve the students in working these problems by asking them questions so that the problem is really being solved as a collaboration between students and himself. The teacher, however, is the authority and it is up to him to decide how many questions are asked and how much time can be given a student to reply before somebody else is asked. During this time the teacher expects the whole class to pay attention to what is being done.

It is very important that in a traditional discussion the students are attentive and show interest, as this is considered to be an essential part in maintaining good morale in the classroom. A competent teacher while discussing a problem makes use of a questioning technique which is intended to motivate the slower students as well as the faster ones. As a rule, discussion of the home assignment does not take more than ten or fifteen minutes of the period, since the class must get on with the course; invariably the main objective appears to be to 'cover the course'! However, if a student is still in need of help even at the end of such a discussion a conscientious teacher may offer his services after class. Once the assignment has been discussed the teacher may vary the lesson. He may want to give a short written quiz on the previous day's work or proceed with a new lesson in a fashion similar to what has been described before.

If the quiz is given, the task of marking the test-papers and recording the corresponding grades follows.

Marking can be done in various ways. One of these is to have students exchange papers and have them mark according to the teacher's answers. If this procedure is followed the teacher usually records the marks and then has the papers returned to the owners. Depending on the students' performance on the test the teacher may be willing to explain at the board once again how some of the questions are worked. On the other hand he might feel that what the students need is more practice to master the required concept, in which case he will set a short home assignment which may involve only some of the students. If, however, in his opinion most of the students have done relatively poorly he might consider re-teaching and then follow up with more practice, and finally give another home assignment.

A second way of dealing with the quiz papers, after the quiz has been administered, is to collect the test papers, mark them after class, record the marks and bring the papers back to the students the following day. Using this procedure the teacher will save the time which would have been taken in class to mark the papers and to record the results. Furthermore, by marking these test sheets he will gain insight as to the kind of errors the students made, thus enabling him to do a better job of explaining the correct way to solve the problem.

Testing, home assignments, and review sheets are an essential part of the traditional teacher's method of teaching. Tests vary in length.

They may be five to ten minutes in duration, sometimes called 'daily quizzes'. There are 'chapter tests', approximately thirty minutes in duration. Some last for a full hour. In most cases when students get a 'long' test, they get advance notice to prepare for it. These tests result in the accumulation of marks which form a considerable portion of a student's grade and that will eventually determine whether he passes or fails the course.

'Review sheets' are also very important because they are used to diminish the students' loss of retention. These again may vary in length and also in how they are used. Some teachers prefer to use tests five minutes in duration, cumulative in nature and given as frequently as every other day.

These details of traditional approach seem quite in accord with the conclusion that historically the mathematics itself was developed in simple, prestructured forms and was thereby communicable as such directly to others. As we have previously suggested this is a misinterpretation of what really occurred but gives credence to the methods prevailing during the early years of public education.

Another essential part of the traditional methods of teaching mathematics can be made by the use of teaching aids and materials. The traditional classroom is usually well supplied with chalk boards, possibly on three of the four walls, with a bulletin board on the other wall. Further, the teacher makes use of instruments and equipment such as compasses, rulers, set squares, large slide rules, coloured chalk, graph boards, overhead projectors, opaque projector, slides, film strips, and films.

III. SCIENTIFIC HEURISTIC METHODS

'Scientific Heuristic Methods' are concerned with those of controlled discovery - 'discovery' because a great deal of the learning of each student arises from his reactions to situations, 'Scientific' because the intent always is to closely scrutinize all assumptions made with amendment as further realities are revealed.

The teacher who utilizes scientific heuristic methods sees his role as one of facilitating learning. He believes in creating an environment (of which he himself is part) whereby the students can inquire for themselves. He creates situations where free discussion takes place among the students and he plays the role of a participant and guide rather than as the authority for the discussion.

One way or another the teacher recognizes that growth in mathematics over the centuries occurred because of the way humans learn and live, and he reproduces aspects of this in his classroom methods.

The teacher is of course still an authority on the subject, knows about the logical basis, has mastered the conventional resulting forms but he minimizes telling or correcting the students. Rather he encourages them to search for their own answers by his asking appropriate questions and presenting them with suitable media.

Mathematics in the classroom is not presented in concise deductive finished packages, anymore than was achieved by man through history, but as rediscoverable facts and generalizations by the students. Testing may take place in many forms, but its main purpose is to facilitate learning, in particular as a diagnostic medium both for the teacher and the student. When using assignments the recognition of student differences are dealt with. The main philosophy in grading is to minimize teacher-assigned grades and maximize student self-evaluation.

The scientific heuristic teacher believes in the students' mastery of mathematical facts but he does not believe this to be the sole purpose of the study of mathematics. With scientific heuristic methods the acquisition of information (which has almost been the sole aim of traditional mathematics teaching) results as a by-product of the study of mathematical systems, patterns, and relationships from as much first hand experience as possible.

Students are encouraged to be responsible, but in an atmosphere of freedom, with flexible seating arrangements to allow formation of small groups, movement about the room or involvement in a variety of learning activities.

There follow elaborations on some matters already raised:

A, telling; B, an environment for discovery; C, mathematics as a finished product; D, right and wrong answers; E, use of materials; F, testing; G, seating arrangement of the students; and H, home assignments.

A. Telling: Supposedly telling is a form of communication. How effective is it? Let us analyze this concept further by considering it as a system involving the teller, the speech (what is being told by the teller) and the hearer.

In communicating ideas by telling, the teller needs to have a mastery of the art of being able to put into words exactly what he wants to say. Assuming that the teller can do just that, he still has no guarantee that the hearer will eventually annex the ideas intended by the teller. There are at least two reasons for this: One is interference with the speech, and the other is the ability on the part of the hearer to assimilate the speech into ideas. Let us discuss the former reason.

(i) Interference - there exist various kinds of 'noises' which can interfere with an intended message from being received by a hearer. Some of these are: Sound, sight, physical feelings, moral pressure, diversions and rhythms.

a) Sound - it would be very difficult for a teacher to communicate, say Pythagoras' Theorem, to a class of thirty students while two or three of the students in different parts of the room are each conversing privately to neighbouring students about some other interesting topics of their own. It would be equally difficult to talk to the students while building construction is going on outside the classroom, while the school band is practising in a nearby classroom, while the class across the hall is vigorously singing French songs in French.

b) Sight - certain physical idiosyncrasies on the part of anyone talking to students can prove distracting. These include: Juggling chalk, waving the blackboard pointer, a particularly distracting mode of dress.

c) Physical Feelings - these include: The discomfort of having to sit still and quietly at a desk for a relatively lengthy period of time especially in the heat of the late spring and summer months, or discomfort from thirst or hunger, or maybe witholding the 'call of nature'.

d) Moral Pressure - many students subject to traditional lecture methods have a feeling that they must comprehend what the teacher is telling them. This tends to be upsetting to some students and thus quite distracting.

Distractions can also stem from such pressing thoughts as the anticipation of a test the following period, or from worrying about an assignment for another course. Students can also suffer as a result of excessive classroom pressure on top of personal worries arising from home problems.

e) Diversions - sometimes the scenery outside the window under various climatic conditions, such as a snowy day or sunny spring day, can create inappropriate diversions. Similar interference can also arise from the activities of other groups in an open area, or the activities of certain inattentive students.

f) Interfering Rhythms - could include: A teacher speaking in a monotone, anyone in the class coughing repeatedly, the teacher's use of a particular phrase or word in a predictable fashion.

Clearly, there are many difficulties in communication by the method of telling. The scientific heuristic teacher is conscious of them.

However, he is also aware that the need for some telling is not eliminated. Not all facts are discoverable, and sometimes too much time would be needed if the discovery method was viewed as a fetish. Conventional labels need to be given, for they are the words which have been arbitrarily given by society and will therefore be used by other people with whom, sooner of later, students will need to work. It is normally impossible to get clues from concepts themselves as to what a concept is called, but in mathematics there is frequently a system of clues. Consider the three examples which arise during the experimental teaching in this study:

Example 1: "My hair is black and curly, I am 5'10-3/4" tall, dark and was born in Italy. In fact I could do better than that, I could give you a picture of me, and then ask you what is my first name ... I bet you could not guess what it is. I suppose you could say that it is probably not Thor or Ming or even Warwick, and you would probably be right. After all, those are not traditional Italian names. But could you pick one from names like Pasquale, Antonio, Carlo, Cesare, Roberto? I suppose you could if I looked like a Carlo, or a Roberto, or an Antonio. But do I? One sure way to obtain the name would be for me to tell you".

Clearly, in this example the name must be told. It cannot be discovered.

Example 2: "If six tens is conventionally called sixty, what is four tens called?"

Here there are clues which lead to 'forty'.

Example 3: "What is the name for a five sided polygon?"

In this last case a student might discover the name 'five sided polygon' or perhaps 5-gon, but if the technical name is needed he has to be told the conventional 'Pentagon'.

(ii) Assimilation - However, even when the effects of 'noises' are minimized as the purposeful result of a changed environment or because teachers and students have honestly discussed and accepted the types of interference which can occur in learning situations, communication is still not guaranteed. The process of assimilation has to be taken into account. It is even conceivable that this process is more essential (than the former) for successful communication.

One can possibly understand another's thought through 'high noise level' but it is impossible to grasp an idea if one's previous experience does not allow for assimilation and accommodation of the fact. Ginsburg and Opper express Piaget's theory:

> New mental structures evolve from the old ones by means of the dual process of assimilation and accommodation. Faced with novel experiences, the child seeks to assimilate them into his existing mental framework ...
>
> ....
>
> If one tries to teach a concept to a child who does not yet have available the mental structure necessary for its assimilation, then the resulting learning is superficial.[24]

---

24. Ginsburg, Herbert and Opper, Sylvia, Piaget's Theory of Intellectual Development. Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1969, pp.223-225.

Similarly, Gattegno writes:

> ... communication does not follow from expression. The second is necessary for the first, but communication happens only when the miracle of the meeting of two minds takes place.[25]

A teacher aware that a student does not understand can attribute the difficulty to laziness, defiance or inability in a vague 'he - is - dumb' way. Appreciation of the facts of assimilation on the other hand, helps a teacher realize that try as they can, expending much energy and listening attentively, there will still be examples when students cannot jump the gap from their present state to make contact with the idea being presented. A teacher with the latter knowledge may see alternative strategies or accept that the student must mature before the particular communication has any chance for success. A classroom example follows on pages 49-50.

B. An Environment for Discovery: An argument often presented against a discovery approach is stated in this fashion:

> "It took many people a long time to accumulate all the facts which a student is exposed to in a high school mathematics course so how could the students re-discover all of these facts in approximately one hundred hours (number of hours of instruction in a high school mathematics course)?"

25, Gattegno, Caleb, The Adolescent and His Will. Outerbridge and Dienstfrey, New York, 1971, p.139.

Perhaps it is not possible for the students to re-discover all these mathematical facts in the duration of a mathematics course, but the argument does not eliminate the fact that the teacher can try to set up an environment which will help the students re-discover as much as is possible in the given amount of time.

How does one set up such an environment? The scientific heuristic teacher arranges situations in which students interact amongst themselves, with teaching materials, and the teacher. He will play a role of a planner, guide and a catalyst in these interactions.

To illustrate, consider a specific example, such as the study of sets - one of the most fundamental concepts of mathematics. The teacher can make available several sets of 'attribute blocks' - pieces of wood of different shapes, colour, thickness or size. The reason for having several sets of such blocks is to encourage and facilitate the formation of small study-groups of students. More students can then manipulate the blocks at any one time. The teacher first suggests some free time for the students to familiarize themselves with the material but eventually he will suggest to the groups the activity of sorting. The only direction the teacher gives may be, "can you sort these blocks?" Such activity will, with guidance, enable the students to see that there are many ways of sorting the blocks, and present each student with the opportunity to discover for himself at least one method of sorting.

The role of the teacher varies. He might help one particular group by asking a question, help another by suggesting a related exercise, or simply ignore a group if he feels his presence is not needed. This is where he plays the role of a 'catalyst', remaining somewhat neutral yet being essential to the learning process.

Suppose the aim of the activity is to illustrate 'intersection of sets' as actually happened during lessons 14 and 15 with the scientific heuristic class. The teacher can guide the students' sorting in a way which will include such standard criteria as thickness, colour, shape and size. The class can eventually agree on using only such criteria as these. Once rules are agreed upon the teacher can suggest a 'one difference game'[26] then a 'two difference game'.

These games (which are interesting in their own right) will provide the students with a background useful in understanding intersection. Again the role of the teacher is to assist the students to play their own game, but he in no way deprives them of the experience of playing the game themselves.

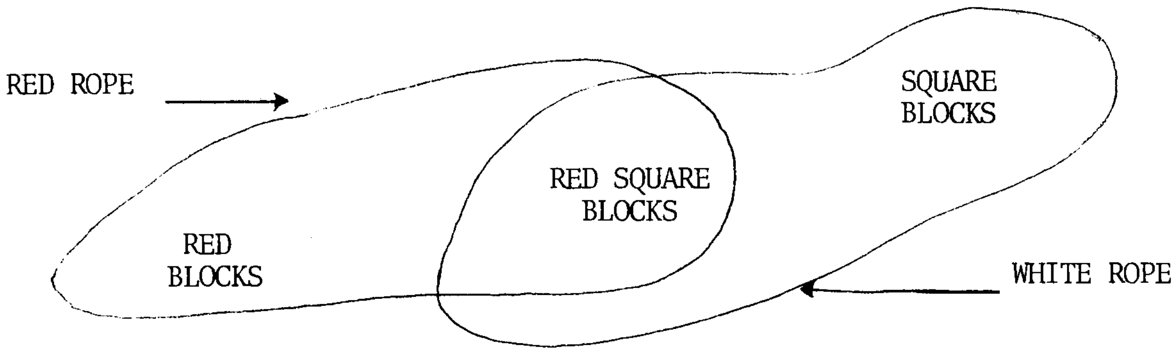
As the students indicate readiness for further learning, the teacher might suggest 'three difference games' and 'four difference games'. These are closely related to the previous.

For other groups he could introduce a slightly different problem, by

---

26. Trivett, J.V., Games for Children to Play In Learning Mathematics. Cuisenaire Co. of America, Inc., New York, September, 1973.

making available several coloured ropes, with the following remarks, "Here are two ropes, a red one and a white one. Tom, could you put the red blocks inside the loop of the red rope, and the square blocks inside the loop of the white rope?" Much discussion follows. But the students eventually realize that in order to fulfill the restriction presented by the problem the ropes must be overlapped, as in the diagram below.



These fundamentals of mathematics can be studied by students with the use of the blocks or similar media without necessarily the knowledge of conventional labels, such as 'intersection', 'union', 'disjoint' and 'complement'. Such names are the kinds of facts which even a scientific heuristic teacher tells, but perhaps after the use of non-standard names.

C. Mathematics, a Finished Product: History shows that mathematics was made by the work of many men. Sometimes they worked together; at other times they argued and most of the time they used what was passed down to them by the labour of others before them. However, to quote Halmos once again:

> Mathematics ... is never deductive in its creation. The mathematician at work makes vague guesses, visualizes broad generalizations, and jumps to unwarranted conclusions. He arranges and rearranges his ideas, and he becomes convinced of their truth long before he can write down a logical proof ....[27]

The heuristic teacher believes that this should also be the approach with the students in his classroom. Before a concept is presented as a precisely written abstraction, the students should be helped to observe it in more concrete and particular ways, then investigate it, discuss it with their fellow students and later state it in a general and consistent way.

D. Right and Wrong Answers: During a student's search for answers he may in certain instances seek approval or disapproval. In such cases, the scientific heuristic teacher believes that it is more desirable that it come from the student's peers. However, whether the approval comes from the teacher or peers, expressions such as 'I agree' or 'I disagree' are favoured over 'right' or 'wrong'. The objection is not in the words 'right'or 'wrong', but only if these words suggest an authoritarian approval or disapproval.

There is one other very important reason as to why a scientific heuristic teacher avoids a continual use of 'right answer' and 'wrong answer'.

---

27. Halmos, P.R., "Mathematics as a Creative Art". The American Scientist. (56, 4, 1968), p.579.

When a student gives a response to a question, his response could be the result of a different 'game' from what the teacher, or the other members of the class, are playing. The role of the scientific heuristic teacher here would be to try to discover whether the student is playing a different game and whether the student is being consistent in playing his own game. It could be that the student's response is just as appropriate as that of the teacher, but perhaps the student's assumption of different rules is giving him a different response to the same question. If this is the case, it becomes a matter of the student noting his use of different rules, and changing his rules if he wishes to conform and play the game which the teacher and/or the rest of the class is playing.

To illustrate, let us examine the following problem:

$$\frac{1}{3} + \frac{5}{9} = \ ?$$

Suppose the student responds with $\frac{10}{8}$ adding the 1 to the 9 and the 3 to the 5 by confusing addition with division, the scientific heuristic teacher might then ask 'Johnny' - any student - to explain how he arrived at the 'answer'. Johnny might resort to counters, pebbles or to coloured rods to explain his answer, but in doing so, is most likely of course to fail. He will find an inconsistency with the physical world.

On the other hand, an alternative might consist in posing to Johnny a further question: "What is $\frac{5}{9} + \frac{1}{3} = \ ?$"

If the student is consistent in playing his 'game' and responds with $\frac{8}{10}$ the teacher could proceed in this fashion:

> "I notice you have called 5 + 3 the same as 3 + 5, namely 8. Don't you also want this reversing principle to work with fractions?"

If Johnny answers <u>yes</u>, the teacher will confront the student with the lack of commutativity in his system. Should the answer be <u>no</u>, there is still at least one other argument which can be presented to him.

Teacher: What is $\frac{2}{3} + \frac{1}{10} = ?$

Student: $\frac{12}{4}$ (if he is still playing the same game)

At this point the teacher can ask for the standard name for $\frac{12}{4}$, which might be answered as 3. Johnny now is faced with accepting a game which yields a result such as:

$$\frac{2}{3} + \frac{1}{10} = 3$$

It is very unlikely that Johnny himself will accept this conclusion from his own reasoning. He therefore rejects his present game as not viable for adding fractions!

Returning to the previous illustration where Johnny was persuaded to apply his own game in a physical context it does not follow that this approach will always be inapplicable.

Consider another possible response:

$$\frac{1}{3} + \frac{5}{9} = \frac{6}{12}$$

Johnny and the teacher know that he has added numerators and denominators - a common 'mistake' by students at the junior secondary level. Pushed on to some interpretation Johnny might say:

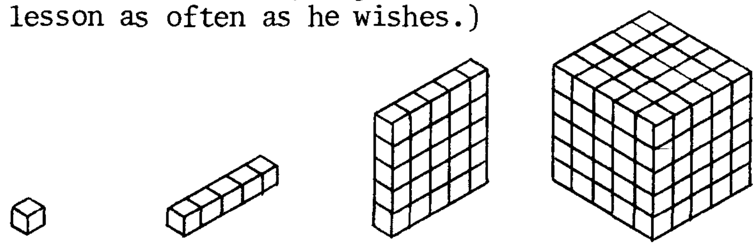
> "$\frac{1}{3}$ represents the games won per games played by the Montreal Canadiens hockey club at home during their first week of schedule. $\frac{5}{9}$ is their away record for the next three successive weeks. How would one express the club's record at the end of the first month? Clearly, $\frac{6}{12}$, would do just that."

Indeed, it is highly improbable that a student, say, in a grade eight class working on fractions would give $\frac{6}{12}$ as the 'answer' to the problem $\frac{1}{3} + \frac{5}{9} = ?$, and then claim that he is calculating the won/per game record of the Montreal Canadiens hockey club, but it is salutary for the scientific heuristic teacher to know that it _is_ a possibility.

All examples above point out that it may be a better learning situation for the teacher to suspend judgment of 'right' or 'wrong' until he finds the reason behind the student's written work. Furthermore, it becomes more meaningful for the student if the teacher helps the student see for himself whether he is 'right' or 'wrong'. Finally, the teacher must also be aware that it is not always possible for him to facilitate discovery for the student.

There is more to be said however, for there is increasing evidence from well known mathematicians and educators of today, and recent years, that approaches to 'rightness' and 'wrongness' need drastic changes.

Consider the following excerpt from the 'Journal of Research and Development in Education' by Robert B. Davis.

> <u>The Girls Who Saw Too Many Cubes.</u>
>
> This issue of what is, or is not observable, is one of the fundamental questions of mathematics. The film entitled 'The Concepts of Volume and Area' shows four girls working with Dienes MAB block (base 5). These blocks are illustrated below. (This film is available from the Madison Project, so every reader may be his own observer, and he may repeat his observations of the same lesson as often as he wishes.)
>
> 
>
>
> The task is to imagine that a 'long' is made up out of units, glued together; similarly for a 'flat' and for a 'block'. The specific problem is this: The girls are handed a box containing a large number of units, a large number of longs, a large number of flats, and a reasonably large number of blocks. Imagining the longs, flats, and blocks to be made up out of units, how many units are there altogether (counting longs, flats, and blocks) in the box? The correct answer is of the order of magnitude of several thousands.
>
> The class is divided into small groups, so the teacher leaves the four girls at their task and goes away to work with other groups. When he subsequently returns, he finds out that two girls (Pat and Debbie) have different answers. Debbie is fairly confident that she is right, and is quite articulate about what she did. Pat is more hesitant about revealing to the teacher that she does not

> agree with Debbie (a kind of fearfulness that I would diagnose as evidence that Pat has had the wrong kind of educational experiences somewhere along the way). The teacher does, however, elicit Pat's answer; as the discussion goes on, it becomes clearer that she very much believes that Debbie is wrong (yet Pat was originally going to remain silent and, presumably, go home one more child who believes that things in school never work out the way you expect them to, but that the prudent student will always 'let them have them have their way' which seems to me to constitute one of the major failures of our educational efforts). Pat cannot imagine how Debbie could possibly be right.
>
> The point at issue turns on whether a block is made out of 125 units, as Debbie argues (and stacks up flats to prove her point), or out of 150 units, as Pat argues (and points to the twenty-five units on each of the six faces, 6 x 25 = 150, so there!)[28]

It is not too difficult to discover from the above example that Pat and Debbie are playing two different '<u>games</u>'. Pat is calculating what other people call the 'surface area' of the block, while Debbie is calculating what they call the 'volume'. Once again the teacher being aware of the existence of the two different games, is not necessarily immediately able to help Pat to play Debbie's game, but at least he did not regard Pat's efforts with a discouraging remark such as "That's wrong Pat". After all, Pat was right in playing Pat's game.

Let us continue Davis' remarks because they will give us some insight as to why it might not be an easy task for the teacher to get Pat to play Debbie's game.

---

28. Davis, Robert B., Mathematics Teaching with Special Reference to Epistemological Problems, <u>Journal of Research and Development in Education</u>. (Monograph numbers 1/Fall, 1967) pp.15-16

> There is much that can be said about this lesson. The teacher mainly allows Pat and Debbie to argue back and forth between themselves. He does occasionally attempt to interject a question intended to be provocative; for example, by attempting to use precisely Pat's own argument in simpler cases (e.g., in the case of a 'long'), where the falseness of it may be clearer. But Pat never backs down.
>
> What is relevant here is that, at the end of the lesson, Pat still doesn't understand. She is, however, quite visibly puzzled, and is obviously thinking about the question.
>
> Now: Should the teacher have told Pat the answer? Let me say that I, personally, don't know what the teacher should have done. He could have introduced the distinction between a unit of area and a unit of volume. He didn't, and the reason he gives is that he felt Pat "wasn't ready".
>
> Now it is pretty clear that the teacher could have taught Pat a verbal answer, so that she would say the right thing <u>whether she knew what it meant or not</u>. (The entire traditional ninth-grade algebra course is based on a sequence of tricks to get children to write down on paper what appear to be correct answers, although the student more often than not does not know what it all means, if anything.)
>
> The teacher, feeling that Pat lacked readiness, chose not to do this. Now, many objective paper-and-pencil tests could easily reveal whether Pat thought the answer to be 125 or else 150. Are these test missing something important? The teacher believed that they are, which is why he behaved as he did.[29]

Davis, with his story makes two vital points: Firstly, a teacher must be certain of the rules by which a student bases a response before a judgment of 'right' or 'wrong' is passed. Secondly, he mentions the concept of 'readiness'. This was discussed previously as <u>assimilation</u> on pages 36-37. Ginsburg and Opper state Piaget's view again:

---

29. Ibid, pp.16-17

> ... intellectual development seems to follow an ordered sequence ... Certain things cannot be taught at any level, regardless of the method adopted. It is of course possible to accelerate some types of learning to a certain extent by use of suitable environmental stimuli. For instance, if a child of the pre-operational period is fairly close to achieving the structure of concrete operations, suitable physical experience may expedite the process, with the result that the structure may be acquired somewhat earlier than if no such experience had been presented. But presentation of the same experience to an infant would not have the same effect. The infant lacks much of the experience and mental development necessary to achieve concrete operations and would consequently not have available an appropriate mental structure into which he could fruitfully assimilate the planned experience to fit his own level of understanding. He might learn something from it, but not what the teacher had in mind.[30]

It is true that the above is aimed at the teaching of young children for whom the sum of 11 apples and 7 apples is 'concrete' but the addition of 11 and 7 is not. On the other hand a student (say at the grade eight level) who just begins to use a 'variable' will consider 11 + 7 as concrete but not 11N + 7N. Moreover, at the grade nine level (age fourteen or fifteen) a student who understands a statement such as 11N + 7N = 18N may not be ready immediately to appreciate a similar equation:

$$3(N + 5) \quad + \quad P(N + 5) \quad = \quad (3 + P) \quad (N + 5)$$

Furthermore, the teacher's verbal 'telling' may not achieve meaningful understanding for the student. However, if the student does understand 11N + 7N = 18N, he may be very close to grasping the other equation. A

---

30. Ginsburg and Opper, op. cit., pp.225-226.

'scientific heuristic' teacher would try to build the student's experience so that the 'jump' from one concept to the other may be possible. He could do this by leading the student and suggesting substitutions such as:

$11N + 7N = 18N$*

'substitute $\triangle$ for N'	....	$11\triangle + 7\triangle = 18\triangle$
'substitute $\bigcirc$ for $\triangle$'	....	$11\bigcirc + 7\bigcirc = 18\bigcirc$
'substitute for 18 an equivalent name using 11 and 7'	....	$11\bigcirc + 7\bigcirc = (11 + 7)\bigcirc$
'substitute R for $\bigcirc$	....	$11R + 7R = (11 + 7)R$
'substitute x for 11 and y for 7'	....	$xR + yR = (x+y)\ R$
'substitute $(R + 2)$ for R'	....	$x(R+2) + y(R+2) = (x+y)(R+2)$

The teacher is trying to have the student evolve his own examples by using schema he already possesses. It is the way that the student will 'really' understand the concept - discover it by himself. It may be however, that the student is not ready to discover the concept, even with such efforts, in which case the teacher must sense this and postpone the discussion. Telling the student could result in the student's memorizing the fact, which may yield only superficial learning.

The art of knowing how to encourage students to advance comparatively

---

* This example occurred in Lesson 9 of the scientific heuristic class.

on their own is relatively new in the learning of mathematics. Any teacher attempting this will experience difficulties in some topics. One such difficulty arose with this experimenter during the course of this investigation.

He wished to promote discussion amongst the students but not without their respecting their colleagues' opinions nor without their being certain of the rules by which the discussion was waged.

The problem was tackled by having the class engage in a game called 'Fiddle'.*

<u>The Fiddle Game</u> - Each student was given a sheet of paper with the following directions - "Below we have a game called 'Fiddle'! Can you discover how to play it?"

The game consisted in discovering a pattern of an operation on different sets of two number pairs - (171-174 pages of appendix for the actual sheets). Verbal directions by the teacher before he issued the 'sheets' to each student were as follows: "Please sit yourselves in a way which will help you <u>not</u> to consult with your fellow students in playing the game. The object is to discover the rules of the game individually."

---

* This took place in the scientific heuristic class, Lesson 15.

Ample time was given to the students to study their papers. When all the students agreed that they knew how to play the game the experimenter (the teacher) wrote on the board this question:

$$(8,7) \wr (5,7) \longrightarrow ?$$

Six students were asked to write their own answers on the chalk board. Of the six answers given there were four different solutions. (e.g., (40,49), (13,7), (3,7), (13,14), (13,7), (3,7).) What followed was an almost immediate uproar of discussion and argument amongst the students! Such comments as 'you are wrong', 'you must be out of your mind', 'you don't know what you are talking about' were heard. And this went on for about ten minutes until one student asked the teacher if he could change his original stipulation of the game, so that he could consult with his friend, and thus show him that he was 'right' and his friend was 'wrong'. The teacher agreed. It didn't take long for them to discover that the teacher had given them sheets with four <u>different</u> patterns. (appendix pages 171 - 174.) Four stencils had been made each for a different game, although the directions were identical and sheets physically looked the same, at least from a distance! Six copies of each stencil, twenty-four sheets of paper in all, were then 'shuffled' and given to the students.

It was evident that this method of introducing a 'discrepant event' proved to be a dramatic way of highlighting the need for suspension of judgement. Before one accuses another of being 'wrong' one should at least assure himself of playing the same game!

E. Use of Materials: A scientific heuristic teacher believes that the use of physical materials are often vital for the learning process. Ginsburg and Opper have something to say about this:

> ... a good school should encourage the child's activity, and his manipulation and exploration of objects. When the teacher tries to bypass this process by imparting knowledge in a verbal manner, the result is often superficial learning. But by promoting activity in the classroom, the teacher can exploit the child's potential for learning, and permit him to evolve an understanding for the world around him. This principle (that learning occurs through the child's activity) suggests that the teacher's major task is to provide for the child a wide variety of potentially interesting materials on which he may act. The teacher should not teach, but should encourage the child to learn by manipulating things.[31]

So does Davis:

> ... Much experience with concrete objects may aid in the learning of mathematics (and I personally believe it can help tremendously) ...[32]

Physical aids were therefore important in the scientific heuristic teaching of this experiment. In the last few years there has arisen a wide range of such material, some of them being selected for use by the students in the experimental class. Rather than adding exceptions of the detailed implications for each topic taught, some notes of reference are given:

---

31. Ginsburg and Opper, op. cit., p.221.

32. Davis, op. cit., p.14.

Topic.	Material or Method
1. Fractions and Fractional algebraic expressions.[33]	Cuisenaire Rods
2. Sets - 'difference games' intersections and union of sets.[34]	Attribute Blocks
3. Operations with signed numbers and algebraic expressions.[35]	Adaptation of 'The Battle of the Red and Blue Numbers'.
4. Solving equations (mainly: linear - one variable).[36]	'Think of a Number Equations'

There are other considerations of aspects of classroom teaching which a teacher adopting a scientific approach needs to consider, although they may well be of minor importance. Three of them became concerns during this particular experiment: Testing, seating arrangement of students, and assignments. A brief note on each follows:

F. Testing: In traditional methods of teaching mathematics tests are usually in written form, made, administered and marked by the teacher. The main purpose of the test is to help the teacher in assigning students' grades. In scientific heuristic methods, tests[37] may be in

---

33. Gattegno, C., Mathematics With Numbers in Colour. Book IV. Lamport Gilbert and Co. Ltd., Reading, England, 1963.
34. Trivett, John V., op. cit.
35. Frederique. Les Enfants et la Mathematique. Marcel Didier, Montreal, 1970.
36. Trivett, John V., Journal of the Provincial Intermediate Teachers' Association, Vol. 12, No. 1, March 1972.
37. A sample test administered to the scientific heuristic class in this experiment is included in the Appendix. Furthermore, the first ten questions of this test were administered to the traditional class and a comparison of the results as well as comments are included in the section Statistics.

written or oral form. The teacher is interested in discovering whether the students have learned and also in allowing the students to find out whether they have learned. The teacher avoids correcting students if the test is oral and minimizes marking students' written tests. The idea is to create a situation whereby the students can correct their own work. This minimizes the role of the teacher as the authoritarian and promotes inquiry, responsibility and independence within the students.

G. Seating Arrangement of Students: The most important aspect of the seating arrangement of the students is that it is flexible. They should be free to move about the room so that they can help themselves to teaching materials at the appropriate times. This does not imply irresponsible action; in fact once they have finished with the equipment they are expected to put it back. Students should be free to carry on independent study or to work in small groups, but whatever the activity they should keep in mind the well-being of their fellow students.

H. Home Assignments: Home assignments can be used to help students master certain specific skills and to pormote further study. Quite often in a classroom where teaching is traditional the same assignment is given to the whole class. This can create a boring situation for those students who have already mastered the topic(s) pertaining to the assignment. On the other hand it creates a frustrating affair for those students who are not capable of solving problems involving the assignment.

In a scientific heuristic classroom when the teacher gives an assignment he tries to take into account the individual differences of his students. He also tries to promote responsibility within the students for the completion of the assignment and finally he encourages the students to correct their own work.

There follows formal definitions of terms such as 'self concept' and 'attitude towards mathematics' which have already appeared in the section entitled Background and Review of the Literature.

The author assumes that one's conception of one's ability to do mathematics is 'closely allied' to self concept and to attitude towards mathematics. Therefore, as part of this experiment, an attempt was made to measure both self concept and attitude towards mathematics of the students.

IV. SELF CONCEPT

A vast amount of literature on self concept has been written to date. Wylie's[38] comprehensive review of research on this topic is possibly the most definitive. However, Purkey's work which has been quoted by the writer is more recent and more relevant to this study. His review of the literature on self concept by such people as Lecky (1945), Rogers (1951), Jersild (1952), and Combs and Snygg (1959) resulted in his composite

---

38. Wylie, R.C., The Self Concept: A critical survey of pertinent research literature. University of Nebraska Press, Lincoln, 1961.

definition of the self:

> The 'self' is a complex and dynamic system of beliefs which an individual holds true about himself, each belief with a corresponding value.[39]

Purkey shows relationship between self concept and achievement:

> Although the data do not provide clear-cut evidence about which comes first - a positive self concept or scholastic success, a negative self concept or scholastic failure - it does stress a strong reciprocal relationship and gives us reason to assume that enhancing the self concept is a vital influence in improving academic performance.
>
> ....
>
> Judging by the studies considered thus far, there is no question that there is a persistent relationship between the self and academic achievement. However, a great deal of caution is needed before one assumes that either the self concept determines scholastic performance or that scholastic performance shapes the self concept. It may be that the relationship between the two is caused by some factor yet to be determined. The best evidence now available suggests that it is a two-way street, that there is a continuous interaction between the self and academic achievement, and that each directly influences the other.[40]

The experimenter offers no evidence of previous research on the effect that self concept has on achievement or the effect that achievement has on self concept, although one of the aims of the study was an

---

39. Purkey, William, Self Concept and School Achievement. Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1970, p.7.

40. Purkey, op. cit., pp.23-27.

attempt to test the above hypothesis. The test used to measure self concept is a recent one, and thought to be one of the best available (see description pp.60-61).

In any case, the amount of time devoted in teaching the two classes was relatively short. This could be an essential drawback in the lack of significant effect that any different methods of teaching might have on a measure of self concept.

V. ATTITUDE TOWARDS MATHEMATICS

Lewis R. Aiken Jr. (1970) in his article Attitudes Towards Mathematics claims that there is no standard definition of the term attitude, but he goes on to say:

> attitude: in general it refers to a learned predisposition or tendency on the part of an individual to respond positively or negatively to some object, situation, concept or another person.[41]

He also claims that very little research has been done on attitudes towards mathematics prior to the last twelve years, but during the past decade the number of dissertations and published articles on this topic has increased substantially. Aiken states that he has reviewed more than

---

41. Aiken, Lewis R. Jr., Attitudes Towards Mathematics, Review of Education Research, Guildford College, 1970. Vol. 40, No. 4, p.551.

two dozen journal articles concerned with this subject, and in his paper he discusses many major topics (relevant to the study undertaken here) including descriptions of various tests that measure attitudes towards mathematics.

Aiken in the article does not define attitudes towards mathematics, hence this concept will be defined according to Krech, Crutchfield and Livson, (1969):

> An enduring system of positive or negative evaluations, emotional feelings, and action tendencies with respect to various aspects of mathematics learning.[42]

The test used to measure attitude towards mathematics for this experiment was a recent one (description of this test is on pages 64-66 and the test itself is on pages 159-163). It was designed by C.W. Montgomery,[43] who reviewed the works of Shaw and Wright, (1967); Kahn, (1969); Aiken, and Krech et al. and others.

Numerous investigations have been carried out in the past to compare the effects that organization of subject matter would have on attitude towards mathematics and the effects created by traditional curricula.

---

42. Krech, D., Crutchfield, R.S., and Livson, N., Elements of Psychology, Alfred A Knopf, New York, 1969. Second edition p.813.

43. Montgomery, C.W., "Imperial Study of the Constructive Predictive Validity of Five Types of Instruments to Measure Students' Attitudes Towards Mathematics", University of British Columbia, Unpublished Doctoral Dissertation in progress.

Aiken states:

> ... investigators who have compared SMSG (School Mathematics Study Group) and traditional curricula in elementary and junior high school (Hungerman, 1967; Osborn, 1965; Phelps, 1964; Woodall, 1967 ...) found that the mean mathematics attitude scores of students taught by SMSG curriculum was not significantly greater than (and even more negative in some reports, e.g. Osborn, 1965), the mean attitude score of students taught by the traditional curriculum ...[44]

Of course the author's experiment differed in the sense that it involved not the comparison of the effects that two different curricula have on attitude but the effect that two different methods have on attitude, self concept and achievement. Furthermore, the experiments mentioned by Aiken have been the subject of severe criticism! Aiken himself points out:

> Before one goes too far interpreting the above results however, it should be emphasized that in these investigations the available subjects were not assigned at random to the two types of curricula. The investigators merely analyzed data obtained from existing groups. In some cases the investigators attempted to assure themselves that the groups did not differ significantly in their pretest scores; in other cases the investigators used analysis of covariance in an attempt to control for initial group differences. But without random assignment of subjects to conditions, there is little control over extraneous variables.[45]

---

44. Aiken, op cit., p.582.

45. Ibid, p.582.

In the investigation undertaken by the writer the students were assigned randomly to the two groups, so this would seem to meet one of Aiken's criticisms. Another was met by having the same person teach both groups for Aiken also states:

> ... the teacher, rather than the curriculum, still appears to be the more influential variable as far as attitudes are concerned.[46]

The interaction between attitudes and behaviour has been given a great deal of attention in recent years (see Festinger et al.[47]). Perhaps not as extensively researched, but certainly not of less concern, is the effect of attitudes towards mathematics or performance. Aiken states:

> The relationship between attitudes and performance is certainly the consequence of a reciprocal influence, in that attitudes affect achievement and achievement in turn affects attitudes.[48]

It is this hypothesis which prompted the setting of the test already quoted.

---

46. Ibid, p.581.

47. Festinger, L., et al., Conflict, Decision and Dissonance. Stanford University Press, Stanford, Calif., 1964.

48. Aiken, op. cit., p.558.

CHAPTER III

THE DESIGN OF THE EXPERIMENT

I. EXPERIMENTAL DESIGN

The research design for this experiment can be called 'Pretest - Post-test Control Group Design'. It is depicted schematically by the following figure:

$$\text{Group 1:}\quad R \quad M_b \longrightarrow T_1 \longrightarrow M_a$$
$$\text{Group 2:}\quad R \quad M_b \longrightarrow T_2 \longrightarrow M_a$$

The R in the model indicates that the two groups were assigned randomly, M_b corresponds to measurement before the experiment (pretest), T_1 and T_2 refer to the two different teaching methods and M_a signifies measurement after the teaching session (post-test).

In this experiment the two groups were taught separately, meeting each week day for one and a half hours for a four week period during the summer of 1971. The class that comprised twenty-two students was taught using scientific heuristic methods; the other class, of twenty-four, was taught traditionally.

Appropriate self concept, attitude and achievement tests were administered to both classes before and after the session. Copies of all these tests are included in this report, as part of the Appendix.

The data gathered was analyzed using t-tests and analysis of covariance to test the following null hypotheses:

1. There will be no significant difference between the scores of the group of students taught by traditional methods and those taught by scientific heuristic methods, with respect to achievement in mathematics.
2. There will be no significant difference between the scores of the group of students taught by traditional methods and those taught by scientific heuristic methods, with respect to attitudes towards mathematics.
3. There will be no significant difference between the scores of the group of students taught by traditional methods and those taught by scientific heuristic methods, with respect to self concept.

A two-way analysis of variance might at first glance appear as the test to use for the analysis of the data. However a closer look at the relatively small number of subjects in each cell led the researcher to the decision that the use of t-tests would be more appropriate.

The Pilot Project

A pilot project, involving twelve students, to gain personal experience and to improve the development of scientific heuristic methods using the topics outlined on pages 29-54, was run during May and June, 1971.

This project was further used to evolve details within the course and also to finalize the course content. Finally, a selection of attitude, self concept and achievement tests were administered to the dozen students with the view of making final selections and to gain insight into adequate procedures for the main study.

II. SAMPLE

The forty-fix students utilized for the main study were drawn from those who had failed the mathematics grade nine course during the 1970-1971 school term in various schools in the North Vancouver school district, British Columbia. Each available student was assigned randomly to one of the two classes.[a]

III. DESCRIPTION OF TESTS

Self Concept Test

The self concept test was constructed by the staff of the Instructional Objectives Exchange, whose extensive research of existing literature on the subject included the works of Wylie and Purkey. The test is a direct

---

(a) One exception was in the case of two students who had to be assigned to the traditional class because of timetable difficulties.

self report ('what an individual believes about himself' as opposed to self concept - 'what an individual says about himself')[49] device of the Likert-type, comprising eighty questions on a seven point scale. This test is designed to offer subscale scores which reflect different dimensions of the learners' self concept. The four dimensions are:

1. Family - one's self-esteem yielded from family interactions.
2. Peer - one's self-esteem derived from peer relations.
3. Scholastic - one's self-esteem derived from success or failure in scholastic endeavors.
4. General - a comprehensive estimate of how the self is esteemed.

The Instructional Objectives Exchange designed this particular test especially for high school students. They also refined it so as not to produce appreciable variability in the learner's response.

Attitude Test

This is a sixty question Likert-type test with fifteen categories and four items in each category. The items are statements to which students are asked to respond on a seven-point Likert scale. The fifteen categories are:

---

49. Combs, D.W., Soper, D.C., and Courson, C.C., The Measurement of Self Concept and Self Report. University of Florida, Educational and P.M., 1963. Vol. XXIII, No. 3.

1. Class participation.
2. Job participation.
3. General like or dislike of mathematics.
4. Attitude towards teachers and parents.
5. Extra class participation, i.e., clubs, extra reading.
6. Friends, i.e., how you feel about discussing mathematics with friends.
7. Attitude towards mathematics text.
8. Memory, recall - good or poor.
9. Attitude towards different mathematics problems.
10. Amount of time spent working on mathematics problems.
11. Test, feelings towards mathematics exams.
12. Studying mathematics - how much time, and how difficult.
13. Comparison between mathematics and other subjects.
14. Problem difficulties, i.e. word problems.
15. Marks, i.e., confidence towards mathematics marks.

Montgomery computed the KR20 coefficient, $r = 0.957$, using results from 570 students. As a check on content validity the test correlates, $r = 0.880$, with the Minnesota Pupil Opinions Instrument, which was also administered to the same students.[50]

---

50. The Minnesota Pupil Opinion is a Guttman-type 94 items designed by Dr. Cyril Hoyt in an unpublished study in 1960.

Achievement Test

The achievement test (traditional type) was constructed in part with items taken from 'Mathematics 7 - 9'[51] and modified into multiple choice questions. The rest of the test comprises items designed in collaboration with Simon Fraser University professors.[52]

A concern of the researcher was to ascertain the advantage which the traditional class might enjoy as a result of sitting a test to which they were accustomed. For this reason a scientific heuristic-type test was designed and administered to both classes. The test appears in the Appendix (pp.156-158).

IV. RATIONALE FOR THE PROCEDURE OF HOW TESTS WERE ADMINISTERED

As indicated earlier, identical forms of the self concept, attitude and achievement tests were administered to both classes before and after the session.

The writer was aware of some disadvantages such as 'carry over'

---

51. A book of test items for students of mathematics grade 7 - 9, published by Instructional Objectives Exchange, P.O. Box 24095, Los Angeles, Calif., 90024.

52. These items were designed as a supplement to test certain topics common to B.C. Mathematics Curriculum.

inherent with the procedure, and also of the possibility that the comparable forms of these tests may be available or even constructed. However, the research performed by C.O. Neidt[53] and C.W. Montgomery[54] supports the contention that duplicate tests may be administered before and after a session (especially one of a duration of as long as four weeks), without the occurrence of undesirable effects such as the recall of the content of the test items from the previous administration. For instance, the self concept and attitude tests administered here consist of sixty and eighty questions respectively, each question based on a seven-point scale. It would be a difficult task indeed for any student to remember the exact responses to many of the eighty or sixty questions after four weeks.

Achievement Test as Pretest

Keeping in mind that these students had 'failed' the mathematics 9 course at least once before enrolling for the summer session, one might have had reservations in presenting them with an achievement test for it would probably appear relatively difficult for practically everybody. This might have a discouraging effect on some, or might even reinforce any negative feeling they possessed about themselves or towards mathematics. With these thoughts in mind plus the fact that legitimate comparisons would be improved by the process of randomization[55] of the two classes one could

---

53. Neidt, C.O., Changes in Attitudes During Learning. ERIC ED. 003227, 1964.

54. Montgomery, op. cit.

55. Popham, W. James, Baker, Eva L., Systematic Instruction. Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1970, p.158.

have built a strong case for administering the achievement test only at the end of the session. However, by doing that, the investigation would have been deprived of:

1. A measure of the increase in achievement which took place in either class.
2. A comparison of achievement versus self concept or attitude within an individual student.

e.g., Johnny scored poorly, on achievement, attitude and self concept before. At the end of the session his achievement rose significantly. Did his self concept and attitude also improve?

It was decided that there was much to be gained by administering achievement tests before and after the session. But so as to diminish discouraging effects on the students the investigator, when administering the test to both classes before the session, took great care to make the students aware that such tests would be used solely for the diagnosis of their mathematics background. This was done verbally and also by the directions which appeared on the front page of the examination itself. (See Appendix pages 156-164).

Use of Pseudonyms

Because of the nature and design of both the self concept and the attitude towards mathematics test, it was important that the students answer the test items truthfully. To facilitate this, caution was taken to preserve the anonymity of the participants. Details of the exact procedure used appears on the cover of the test which is included in the Appendix.

End-of-Session Administration of Achievement Test

An agreement was made by the teacher and students of the scientific heuristic class that a final test for the purpose of assigning their grades would not be administered. Thus a different instruction page was used for each class when the achievement test was administered at the end of the session. The scientific heuristic class was told that the test was not going to be used for the purpose of grading. The traditional class on the other hand was told that the test would count for half of their grade. Both classes had been advised of this final event during the session. (For the actual directions consult the Appendix for the scientific heuristic and traditional tests on pp. 170-172.)

Course Content

The contents of the course taught during the session included topics from the regularly prescribed course for Mathematics nine in the British Columbia curriculum. Such topics involved: Discussion of real numbers, sets, linear equations in one and two variables, factoring of quadratic polynomials, inequalities, and operations with rational algebraic expressions.

III. METHODS

Traditional Methods

In the traditional class the main procedure of teaching was the lecture approach - the teacher being the authority. Each lesson followed the basic format as outlined in the section - Traditional Methods of Teaching Mathematics (pp.24-30).

An example of how fractions and rational algebraic expressions were taught in the traditional class is as follows:

The students were told about the fundamental properties of fractions, i.e.:

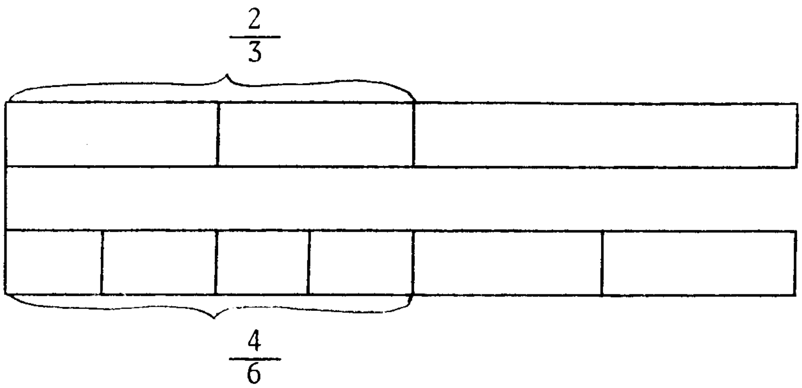
given $\frac{a}{b}$ a, b integers $b \neq 0$

then $\frac{ar}{br} = \frac{a}{b}$ r, any integer except 0.

This was illustrated with examples such as:

a) $\frac{2}{3} = \frac{4}{6}$

and diagrammatically so:



Secondly they were given a review of prime factors, such as:

Factor into Primes

a) 125

b) 345

c) 1001

The above was followed by practice on finding the lowest common multiple.

<u>Example</u>: Find the L.C.M. of:

a) 36,48

b) 72,45

c) 27,162

This led to practice on finding the L.C.M. using algebraic expressions:

Find the L.C.M. of:

a) $2n^3y^2$, 3^2y^3

b) $18n^5y^3$, $36ny^6$

c) $34x^3y^4$, $42xy^3$

Finally came the addition of algebraic expressions such as:

a) $\frac{1}{6} + \frac{3}{4}$

b) $\frac{1}{3a} + \frac{1}{6a}$

c) $\frac{1}{6n^3} + \frac{1}{15n}$

d) $\frac{5}{24n^2} + \frac{b}{18n^3}$

e) $\frac{3}{ab^3} + \frac{2}{a^3b}$

For a full traditional treatment see any standard text, as for example, Modern Algebra, Book I, Dolciani et al., or Modern Elementary Algebra, Nichols, et al.

Scientific Heuristic Methods

In contrast to the traditional method for fractions and rational algebraic expressions, manipulations with colored pairs of rods were initially used. Students were challenged, first, to find pairs of "trains" each of which gave, when measured, an example of the same rational number class. The rods are such that a large number of equivalent pairs were discovered quickly and in recording them, students obtained the conventional forms:

e.g. (3,11) (6,22) (9,33) (18,66) ...

or $\frac{3}{11} = \frac{6}{22} = \frac{9}{33} = \frac{18}{66} = \ldots$

Variations upon the same theme evoked from the students the observation that, "there is no need to use the rods once you see that a new pair belongs to the same family if it can be generated from one of the existing pairs by multiplication of the first component (the numerator) and the second component (the denominator) by the same number".

From examples such as: $\frac{5}{7} = \frac{40}{56} = \frac{5\cdot 11}{7\cdot 11} = \frac{5\cdot 7\cdot 22}{7\cdot 7\cdot 22}$

students proceeded to invent expressions of their own, like:

$$\frac{A}{B\ C} = \frac{A\cdot R\cdot N^3}{B\cdot C\cdot R\cdot N^3}$$

and

$$\frac{11A}{5A^2B^3} = \frac{22A^4B^3}{2\cdot 5\cdot A^5B^6}$$

Addition of two pairs was discovered, necessarily by having to have the second components of each pair equal, leading to:

$$\frac{5}{7} + \frac{3}{11} = \frac{(5\cdot 11) + (7\cdot 3)}{7\cdot 11}$$

and thence to expressions like:

$$\frac{a}{6n^2} + \frac{b}{4n^3} = \frac{(a\ 4n^3) + (6n^2 \cdot b)}{6n^2 \cdot 4n^3}$$

Later discussion of possible short cuts produced the traditional form:

$$\frac{a}{6n^2} + \frac{b}{4n^3} = \frac{2an + 3b}{12n^3}$$

A full description of this approach is found in Gattegno's <u>Arithmetic with Numbers in Color</u>, Books IV, V.

CHAPTER IV

THE RESULTS OF THE INVESTIGATION

I. INTRODUCTION

This chapter includes the results of the data analysis with respect to achievement in mathematics, attitude towards mathematics and self concept. It also offers evidence of two distinct methods of teaching used in the experiment. Further sections include a 'diary' (teacher's personal feelings on his teaching experience) written during the teaching session of the experiment, comments by the students of both classes on their learning experiences during the session, a summary of the results of the statistical analysis of the data and finally a comparison of mean achievement scores of both classes on a scientific heuristic-type test.

II. STATISTICAL ANALYSIS

The classes' mean scores on achievement in mathematics, attitude towards mathematics and self concept were analysed by t-tests and analysis of covariance in order to test the null hypotheses stated on page 17. The format of this section includes the statement of each null hypothesis, presentation of data and its corresponding statistical analysis, and finally the summary of the results and the conclusions.

Further analyses were, however, made with the data available to test two questions which arose after the initial findings:

1. Were the performances of boys significantly different from the performance of the girls?
2. Did the ages of students affect the outcomes?

The data in this section are presented informally without precise statements of null hypotheses. Intercorrelational analysis and t-tests were utilized as statistical means of seeking answers to these questions.

Achievement in Mathematics

Null Hypothesis 1.

There will be no significant difference between the scores of the group of students taught by traditional methods and those taught by scientific heuristic methods, with respect to achievement in mathematics.

TABLE I

COMPARISON OF PRETEST SCORES OF SH-CLASS (SCIENTIFIC HEURISTIC CLASS) OF TRAD-CLASS (TRADITIONAL CLASS) ON ACHIEVEMENT IN MATHEMATICS

MEAN SH-CLASS	MEAN TRAD-CLASS	S.D.[x] SH-CLASS	S.D. TRAD-CLASS	D.F.[y]	t
8.09	8.71	3.52	3.21	44	-0.61

t Value not significant at .05 level.

N, SH-CLASS = 22

N, TRAD-CLASS = 24

TABLE II

COMPARISON OF POST-TEST SCORES OF SH-CLASS AND POST-TEST SCORES OF TRAD-CLASS ON ACHIEVEMENT IN MATHEMATICS

MEAN SH-CLASS	MEAN TRAD-CLASS	S.D. SH-CLASS	S.D. TRAD-CLASS	D.F.	t
11.55	16.46	3.14	3.59	44	-4.81

t Value significant* at the .01 level.

N, SH-CLASS = 22

N, TRAD-CLASS = 24

Table II indicates that the measure of achievement in the TRAD-CLASS after was higher than in the SH-CLASS after (the experiment), and significant at the .01 level.

---

x Standard deviation.

y Degrees of freedom.

* Level of significance (for all the results of this paper) is given for two-tailed test.

TABLE III

COMPARISON OF PRE AND POST-TEST MEAN SCORES
FOR SH-CLASS ON ACHIEVEMENT IN MATHEMATICS

MEAN PRETEST	MEAN POST-TEST	S.D. PRETEST	S.D. POST-TEST	D.F.	t
8.09	11.50	3.52	3.14	42	-3.36

t Value significant at .01 level.

N = 22

This comparison indicates that the achievement of the SH-CLASS increased and was significant at the .01 level.

TABLE IV

COMPARISON OF PRE AND POST-TEST MEAN SCORES
FOR TRAD-CLASS ON ACHIEVEMENT IN MATHEMATICS

MEAN PRETEST	MEAN POST-TEST	S.D. PRETEST	S.D. POST-TEST	D.F.	t
8.71	16.46	3.21	3.59	46	-7.72

t Value significant at .01 level.

N = 24

The pretest - post-test comparison for the traditional class (Table IV) indicates an increase in achievement as a result of the teaching session.

Summary of Results

On achievement the data (Table I) clearly shows that there was no significant difference between the mean scores of the two classes before the experiment. It also shows that the students in both classes scored significantly better after the session (Tables III and IV) but the students in the traditional class achieved significantly higher scores than those in the scientific heuristic class (Table II)

Conclusion

Although both classes scored significantly better on achievement as a result of the teaching session, the traditional classes' scores were significantly higher and thus the Null Hypothesis 1 was rejected.

Attitude Towards Mathematics

Null Hypothesis 2.

There will be no significant difference between the scores of the group of students taught by traditional methods and those taught by scientific heuristic methods, with respect to attitude in mathematics.

TABLE V

COMPARISON OF PRETEST SCORES OF SH-CLASS AND PRETEST SCORES OF TRAD-CLASS ON ATTITUDE TOWARDS MATHEMATICS

MEAN SH-CLASS	MEAN TRAD-CLASS	S.D. SH-CLASS	S.D. TRAD-CLASS	D.F.	t
161.68	203.54	39.99	51.26	44	-3.00

t Value significant at the .01 level.

N, SH-CLASS = 22

N, TRAD-CLASS = 24

The results of Table V show that the measure of the attitude of the students in the TRAD-Class before the experiment was higher than in the SH-Class, and the difference was significant at the .01 level.

TABLE VI

COMPARISON OF PRE AND POST-TEST MEAN SCORES FOR SH-CLASS ON ATTITUDE TOWARDS MATHEMATICS

MEAN PRETEST	MEAN POST-TEST	S.D. PRETEST	S.D. POST-TEST	D.F.	t
161.68	192.00	55.00	55.00	42	-2.04

t Value significant at the .05 level.

N = 22

This pretest - post-test comparison indicates that the attitude of the students of the SH-class improved significantly.

TABLE VI

COMPARISON OF PRE AND POST-TEST MEAN SCORES FOR TRAD-CLASS ON ATTITUDE TOWARDS MATHEMATICS

MEAN PRETEST	MEAN POST-TEST	S.D. PRETEST	S.D. POST-TEST	D.F.	t
203.54	199.58	51.26	51.19	46	0.26

N = 24

This comparison indicates that the attitude towards mathematics of the TRAD-Class did not improve.

Although this experiment was performed using randomized groups, the pretest scores of the classes on attitude towards mathematics were significantly different. For this reason the analysis of covariance test was utilized in order to adjust the post-test scores, thus making possible a meaningful comparison for the final scores on attitude towards mathematics.

TABLE VIII

ANALYSIS OF COVARIANCE BETWEEN SH-CLASS AND TRAD-CLASS ON FINAL SCORE OF ATTITUDE WITH PRETEST SCORE OF ATTITUDE AS COVARIATE

GROUP	MEAN	MEAN SQUARE	D.F.	F	B
SH-CLASS	161.7 (210.7)*	1337.2	1[a] 43[b]	5.68	.856
TRAD CLASS	203.5 (182.45)*	7596.5	44		

F Value significant at .05 level.

N, SH-CLASS = 22

N, TRAD-CLASS = 24

Summary of Results

The results of Table V indicate that the pretest scores of attitude towards mathematics for the two classes were significantly different. Table VI which gives a comparison of the adjusted final scores of attitude implies that if the two classes' pretest scores had been equal then the post-test scores would have significantly different - the attitude of the SH-class would have been higher.

---

* Adjusted

[a]between

[b]within

Conclusion:

The analysis of covariance test (Table VI) indicates that the attitude towards mathematics of the students of the SH-class was significantly higher than that of the TRAD-class as a result of the teaching session. Null Hypothesis 2 was therefore rejected.

Self Concept

Null Hypothesis 3.

There will be no significant difference between the scores of the group of students taught by traditional methods and those taught by scientific heuristic methods, with respect to self concept.

TABLE IX

COMPARISON OF PRETEST SCORES OF SH-CLASS AND PRETEST SCORES OF TRAD-CLASS ON SELF CONCEPT

MEAN SH-CLASS	MEAN TRAD-CLASS	S.D. SH-CLASS	S.D. TRAD-CLASS	D.F.	t
343.23	378.79	46.70	40.49	44	-2.71

t Value significant at the .01 level.

N, SH-CLASS = 22

N, TRAD-CLASS = 24

The measure of self concept of the students of the TRAD-class before the experiment was higher than in the SH-class, and the difference was significant at the .01 level.

TABLE X

COMPARISON OF POST-TEST SCORES OF SH-CLASS AND POST-TEST SCORES OF TRAD-CLASS ON SELF CONCEPT

MEAN SH-CLASS	MEAN TRAD-CLASS	S.D. SH-CLASS	S.D. TRAD-CLASS	D.F.	t
349.86	394.54	47.69	47.76	44	-3.10

t Value significant at the .01 level

N, SH-CLASS = 22

N, TRAD-CLASS = 24

Table X indicates that the measure of self concept of the students in the TRAD-Class after the experiment was higher than in the SH-class, and significant at the .01 level.

TABLE XI

COMPARISON OF PRE AND POST-TEST MEANS SCORES FOR SH-CLASS ON SELF CONCEPT

MEAN PRETEST	MEAN POST-TEST	S.D. PRETEST	S.D. POST-TEST	D.F.	t
343.23	349.86	46.70	47.69	42	-0.46

N = 22

TABLE XII

COMPARISON OF PRE AND POST-TEST MEAN SCORES FOR

TRAD-CLASS ON SELF CONCEPT

MEAN PRETEST	MEAN POST-TEST	S.D. PRETEST	S.D. POST-TEST	D.F.	t
378.79	394.54	40.49	47.76	46	-1.21

N = 24

Summary of Results of t-Tests

The comparison of the self concept pretest scores of the SH-class with the self concept pretest scores of the TRAD-class followed by a corresponding comparison using post-test scores revealed that the measures in both comparisons differed significantly at the .01 level (Tables IX and X correspondingly). However, when a comparison of the pretest and post-test scores was made for both classes separately, it was found that there was no significant change in the scores of either class (Tables XI and XII).

TABLE XIII

ANALYSIS OF COVARIANCE BETWEEN SH-CLASS AND TRAD-CLASS ON FINAL SCORE OF SELF CONCEPT WITH PRETEST SCORE OF SELF CONCEPT AS COVARIATE

GROUP	MEAN	M.S.	D.F.	F.	B
SH-CLASS	343.2 (364.5)	1164.9	1 43	2.31	.791
TRAD-CLASS	381.1	2690.4	44		

F Value for significance at .05 is 4.07.

N, SH-CLASS = 22

N, TRAD-CLASS = 24

Conclusion

The analysis of covariance test verifies what had already been indicated by the t-tests. The self concept of one group did not increase significantly over the other as a result of the teaching session. Therefore, the null Hypothesis was accepted.

Analysis of Data by Sex

As mentioned on page 75 , additional data were analyzed. Comparisons of the mean scores on achievement, attitude and self concept by sex were made. The results appear on Tables XIV - XVII and XIX - XXVI. However, only the data dealing with the comparisons which investigate the effect that the teaching session had on the self concept of the boys and girls of the traditional class will be presented informally on the pages that follow. The other data which did not appear to produce any apparent relevance to the study are placed in the Appendix.

The results of Table XIV indicate that the self concept of the boys and the girls did not differ significantly before the experiment. Furthermore, the pretest - post-test comparison for the boys also showed no significant increase in self concept (Table XV).

TABLE XIV

COMPARISON OF BOYS' SCORES WITH GIRLS' SCORES
OF TRAD-CLASS BEFORE THE EXPERIMENT

CRITERIA	MEAN BOYS	MEAN GIRLS	S.D. BOYS	S.D. GIRLS	D.F.	t
ACHIEVEMENT	8.44	9.25	2.81	3.83	22	-0.56
ATTITUDE	204.69	201.25	41.38	66.72	22	0.15
SELF CONCEPT	374.50	378.38	44.84	28.05	22	-0.71

t = 2.07, for significance at .05 level.

N, BOYS = 16

N, GIRLS = 8

TABLE XV

COMPARISON OF PRETEST AND POST-TEST SCORES
FOR BOYS OF TRAD-CLASS

CRITERIA	MEAN PRETEST	MEAN POST-TEST	S.D. PRETEST	S.D. POST-TEST	D.F.	t
ACHIEVEMENT	8.44	16.31	2.81	3.27	30	-7.08[a]
ATTITUDE	204.69	202.06	41.38	41.97	30	0.17
SELF CONCEPT	374.50	376.69	44.84	37.86	30	-0.14

[a]significant at the .01 level.
N = 16

The results of Table XVI show that the girls' self concept increased significantly when compared with the boys'. A further test comparing girls' pretest and post-test scores on self concept should confirm whether the girls' self concept did in fact increase as a result of the teaching session. This test was run and the results appear in Table IX.

TABLE XVI

COMPARISON OF BOYS' SCORES WITH GIRLS' SCORES
OF TRAD-CLASS AFTER THE EXPERIMENT

CRITERIA	MEAN BOYS	MEAN GIRLS	S.D. BOYS	S.D. GIRLS	D.F.	t
ACHIEVEMENT	16.31	16.75	3.27	4.15	22	-0.27
ATTITUDE	202.06	194.63	41.97	69.59	22	0.32
SELF CONCEPT	376.69	430.25	37.86	45.43	22	-2.92[a]

[a] significant at the .01 level.

N, BOYS = 16

N, GIRLS = 8

TABLE XVII

COMPARISON OF PRETEST AND POST-TEST SCORES
FOR GIRLS OF TRAD-CLASS

CRITERIA	MEAN PRETEST	MEAN POST-TEST	S.D. PRETEST	S.D. POST-TEST	D.F.	t
ACHIEVEMENT	9.25	16.75	3.83	4.15	14	-3.52[a]
ATTITUDE	201.25	194.63	66.72	65.59	14	0.19
SELF CONCEPT	387.38	430.25	28.05	45.43	14	2.13

[a] significant at the .01 level.

N = 8

Table XVII indicates that the self concept of the girls in the TRAD-class did not increase significantly as a result of the teaching period. But it is interesting to note that the t-value for significance at the .05 level is 2.15.

Just as important is the fact that the girls' self concept in the TRAD-class improved significantly more than the boys' (Table XVI).

Intercorrelation Analysis

An intercorrelation Analysis was run using sex, age, self concept, attitude and achievement as the variables. The purpose of this was to enquire into the possibilities of further conclusions directly from the analysis and to gain further insight into the kinds of statistical computations it might suggest which would in turn lead to additional assertions. The analysis supplied neither new conclusions nor further investigations. The tables containing the results are included in the Appendix.

Summary of Results of the Statistical Analysis of the Data

Comparisons of the mean scores of attitude towards mathematics, self concept and achievement in mathematics for the two classes show that:

1. Both classes showed a significant increase in achievement in mathematics.

2. The traditional class showed significantly higher achievement in mathematics than the scientific heuristic class.
3. The scientific heuristic class showed a significant improvement in their attitude towards mathematics.
4. The girls in the traditional class showed an improvement in self concept, although this improvement did not reach statistical significance ($t = 2.15$ at .05 is required for significance whereas $t = 2.13$ was obtained). However, a comparison of girls versus boys (traditional class) on self concept showed that the girls improved significantly (.01 level) more than the boys.

Scientific Heuristic-Type Achievement Test

At the end of the session a 'SH-type' test was given to both classes (test is included in the Appendix, pages 141-143) to investigate the classes' performance on this type of test. The results (Table XII) show that the students of the scientific heuristic class achieved significantly higher (.01 level) than those of the traditional class when both classes were administered a 'SH-type' achievement test.

TABLE XVIII

COMPARISON SCORES FOR SH-CLASS AND TRAD-CLASS

ON ACHIEVEMENT ON 'SH-TYPE' TEST

CRITERIA	MEAN SH-CLASS	MEAN TRAD-CLASS	S.D. SH-CLASS	S.D. TRAD-CLASS	D.F.	t
ACHIEVEMENT	16.3	11.1	3.82	4.86	44	2.88

t Value significant at the .01 level.

N, SH-CLASS = 22

N, TRAD-CLASS = 24

II. VIDEO-TAPE REGARDING TWO DISTINCT METHODS OF TEACHING

To offer evidence that the two classes were exposed to two distinct methods of teaching, video-tape recordings of most lessons were taken by a competent person assisting the teacher-experimenter. These tapes are available for viewing.

The evidence is presented as part of the thesis in the form of Tables (Tables XXXIV - XLII). The method of extracting evidence from the tapes involved the construction of a questionnaire, the administration of the questionnaire to people (unrelated with the students of the class) who viewed the tapes, the scrutinizing of answered questionnaires and lastly an analysis of the results.

The Tape Viewers

The members of this group were chosen by the experimenter in a way which would approach a random sample. They comprised: Teachers, administrators, university professors, students, relatives, people who did not know the experimenter, tradesmen, labourers, people of professional occupations.

The Questionnaire

A questionnaire comprising fifty questions of the type illustrated by the example below was constructed by the experimenter. (Appendix pages 180 - 186.)

Question: Does the teacher appear to insist or expect that all the students try the exercise(s) he suggests?

yes no ?

After viewing a tape, the viewers were asked to answer each of the fifty questions by circling exactly one of three possible answers (yes, no, ?), according to their reaction prompted by the tape they saw.

The questionnaire originally devised was subjected to two successive revisions. These revisions were necessary to avoid ambiguities (in a few questions) which became apparent after successive administrations. However, seldom was a question omitted or another substituted. In nearly all cases only the wording of a question was changed. Since the questionnaire remained basically the same, the results of the three papers were utilized. Only the final questionnaire is included in the Appendix (pages 180 - 186).

Administering the Questionnaire

The administration of the questionnaire involved two procedures, the second of these being used only for the viewers named on Table XXXVIII.

The first procedure:

1. Allow the viewers to read the questionnaire (to familiarize themselves with its content) and the directions on the front page (to learn how to use it).
2. View the full (thirty-minute) tape.
3. Answer the questionnaire.

The second procedure:

1. Allow the viewers to read the questionnaire and the directions on the front page on how it would be used.
2. View half of the tape (the first fifteen minutes).
3. Answer the questionnaire.

4. Watch the second half of the tape (the last fifteen minutes).
5. Answer a duplicate form of the questionnaire.

A disadvantage of the first procedure was in expecting the viewer to recall the whole lesson (thirty minute tape) when answering the questionnaire. This could have encouraged the viewer to base his responses on impressions created by the latter part of the tape. To diminish this effect the second procedure was used.

Standard for Scoring

In order to arrive at a standard for scoring the questionnaire, a group of eight 'experts' - experienced teachers and university professors of varying methods of teaching mathematics - were asked to identify each of the fifty questions as traditional, scientific heuristic or neutral. They were used to test the investigator's identification of the questions.

For a question to be identified as an element of one of the sets mentioned, an agreement of at least six of the eight experts was necessary. In nearly all cases, the experts were unanimous with each other and with the experimenter. (Appendix Table XXXIII)

This produced the standard by which each question was labelled S (scientific heuristic), T (traditional) or N (neutral).

The identification of each question as S, T or N based on an affirmative response is shown in Table XXXIII of the Appendix pages 198 - 199.

The Scoring

Evaluating the questionnaire responses involved comparisons of answers with the standard obtained by the experts. Only those questions answered 'yes' or 'no' were considered.

The Scores

The viewers' scores appear in Tables XXXIV - XLII.

e.g.

VIEWER	TAPE	SCORE
TEACHER OF ENGLISH	VBO186 SH VBO177 TRAD	36S 2T 7S 31T

This example shows that a participant viewed one tape and judged it as showing a scientific heuristic lesson by agreeing with thirty-six of the thirty-eight criteria determined by the 'experts', and a second tape as a traditional lesson by agreeing with thirty-one of the thirty-eight.

The Graphs

To make the interpretations of the scores visually clearer, graphs of the results of all traditional tapes and all scientific heuristic tapes were constructed. Each graph shows one quadrant of a Cartesian Plane with a 'Traditional Trend' as the ordinate and the 'Scientific Heuristic Trend' as the abscissa. The horizontal and vertical lines labelled 'Maximum Score' bound a square with the axes which is divided into two congruent triangles by the 'Neutral Line'.

A participant's score of say 2S 28T would be represented as a number pair (2,28). This number pair when plotted on the graph results in a point in the triangular region which is representative of the viewer's identification of the tape as traditional. Conversely, a score of 30S 5T yields the number pair (30,5) and a point indicating a scientific heuristic lesson.

Once all the ordered pairs were plotted, separate means were calculated for all values of the ordinate (range) and values of the abscissa (domain). These means resulted in a new corresponding pair which represented an average trend of the response for all the participants shown in a particular graph. This average was diagrammatically represented by a 'ray' passing through the new point of the graph, and the origin (Graphs 1 - 18).

TABLE XXXIV

RESPONSES BASED ON THE ORIGINAL QUESTIONNAIRE

USING THE FIRST PROCEDURE

MAXIMUM SCORE = 39*

PARTICIPANT	TAPE	SCORE
1. RETIRED ELEMENTARY TEACHER	VB 0194 SH	15 S[a] 6 T[b]
2. PROFESSOR OF EDUCATION U.B.C.	VB 0194 SH	23 S 11 T
3. HOUSEWIFE	VB 0192 SH	31 S 4 T
4. WIFE	VB 0180 TRAD	7 S 30 T
	VB 0189 SH	31 S 1 T
	VB 0191 SH	38 S 0 T
5. EXECUTIVE B.C. DRIVER EDUCATION	VB 0177 TRAD	2 S 35 T
	VB 0186 SH	36 S 1 T
6. COMMERCE TEACHER HIGH SCHOOL	VB 0177 TRAD	2 S 36 T
	VB 0186 SH	34 S 2 T
7. ENGLISH TEACHER	VB 0186 SH	36 S 2 T
	VB 0177 TRAD	7 S 31 T

[a]S - in favour of scientific heuristic.

[b]T - in favour of traditional.

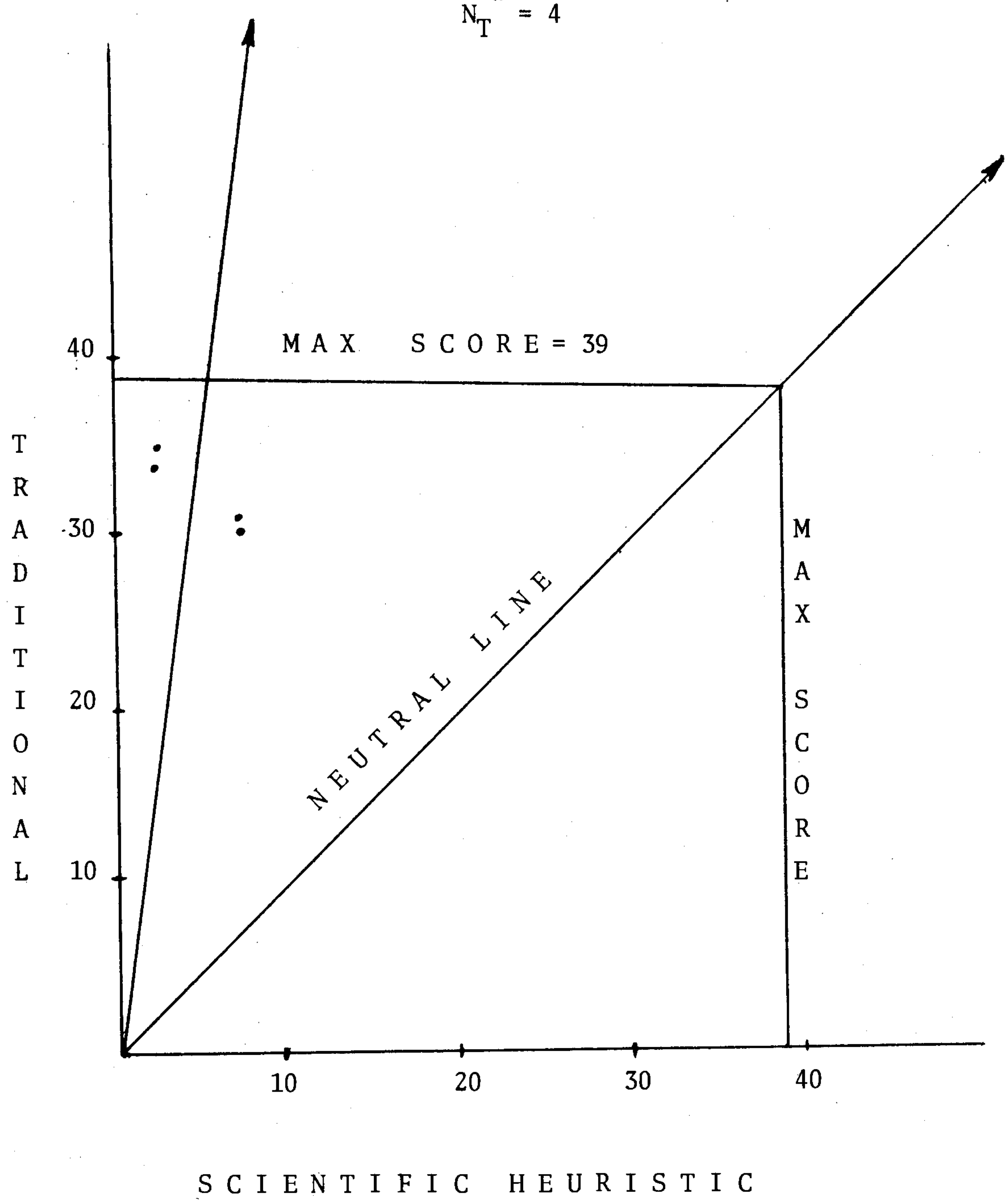
* The viewers' responses seldom totalled the maximum score. Some questions were left unanswered.

GRAPH I

RESULTS OF THE TRADITIONAL TAPES

TAKEN FROM TABLE XXIV

$N_T^* = 4$



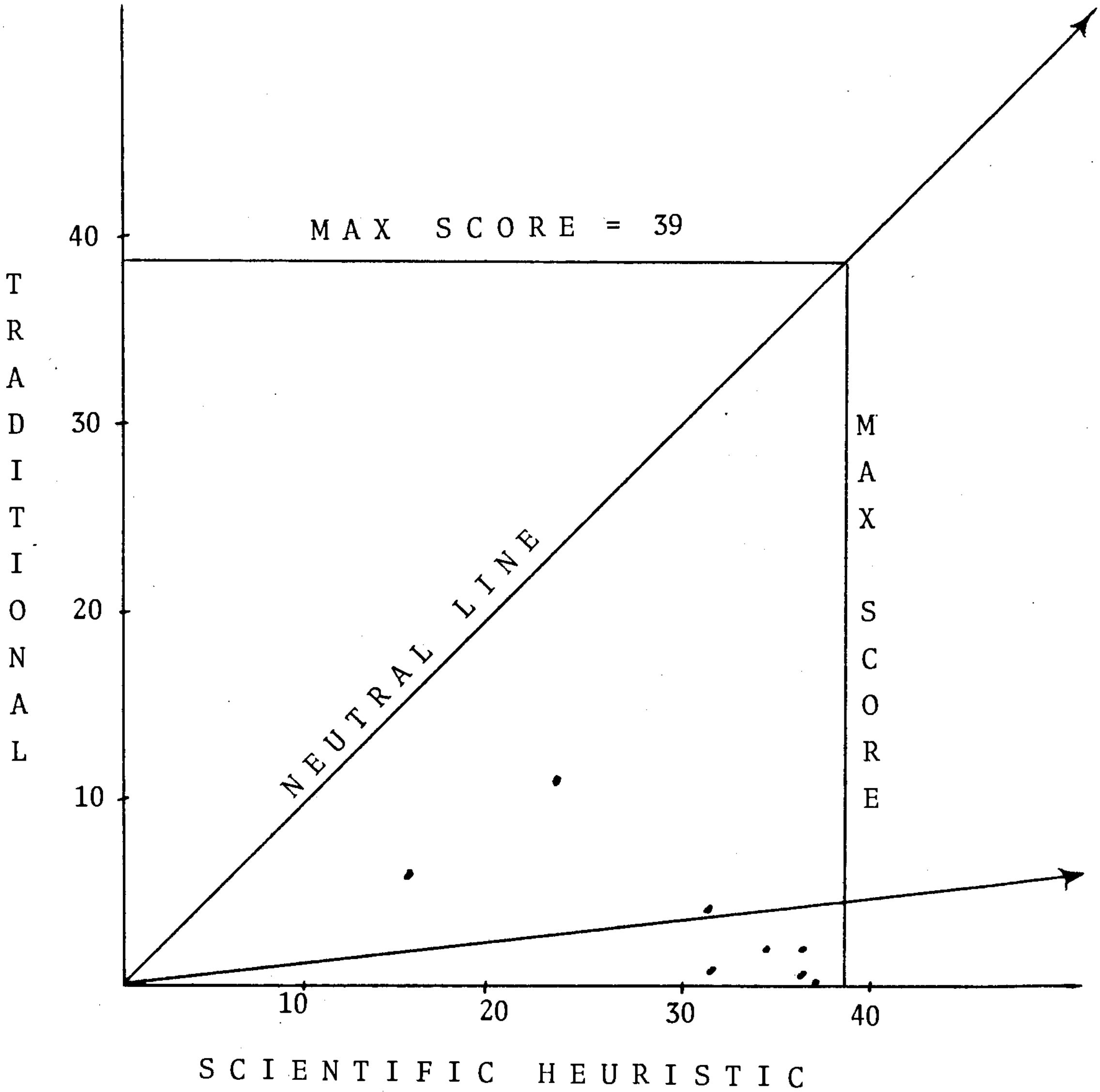
*N_T - Number of viewers of traditional tapes.

GRAPH 2

RESULTS OF THE SCIENTIFIC HEURISTIC

TAPES TAKEN FROM TABLE XXXIV

N_S* = 8



*N_S - number of viewers of scientific heuristic tapes.

Graphs 1 and 2 indicate that the participants listed on Table XXXIV identified the two different methods.

Comments from these people were helpful in the revision of the first questionnaire.

Table XXXV lists the responses to the revised questionnaire by university students enrolled in the faculty of education. These students were unknown to the experimenter (teacher in the tape) and the experimenter was unknown to the students.

TABLE XXXV

RESPONSES OF THE FIRST GROUP ON THE FIRST REVISION

OF QUESTIONNAIRE USING FIRST PROCEDURE

MAXIMUM SCORE = 42

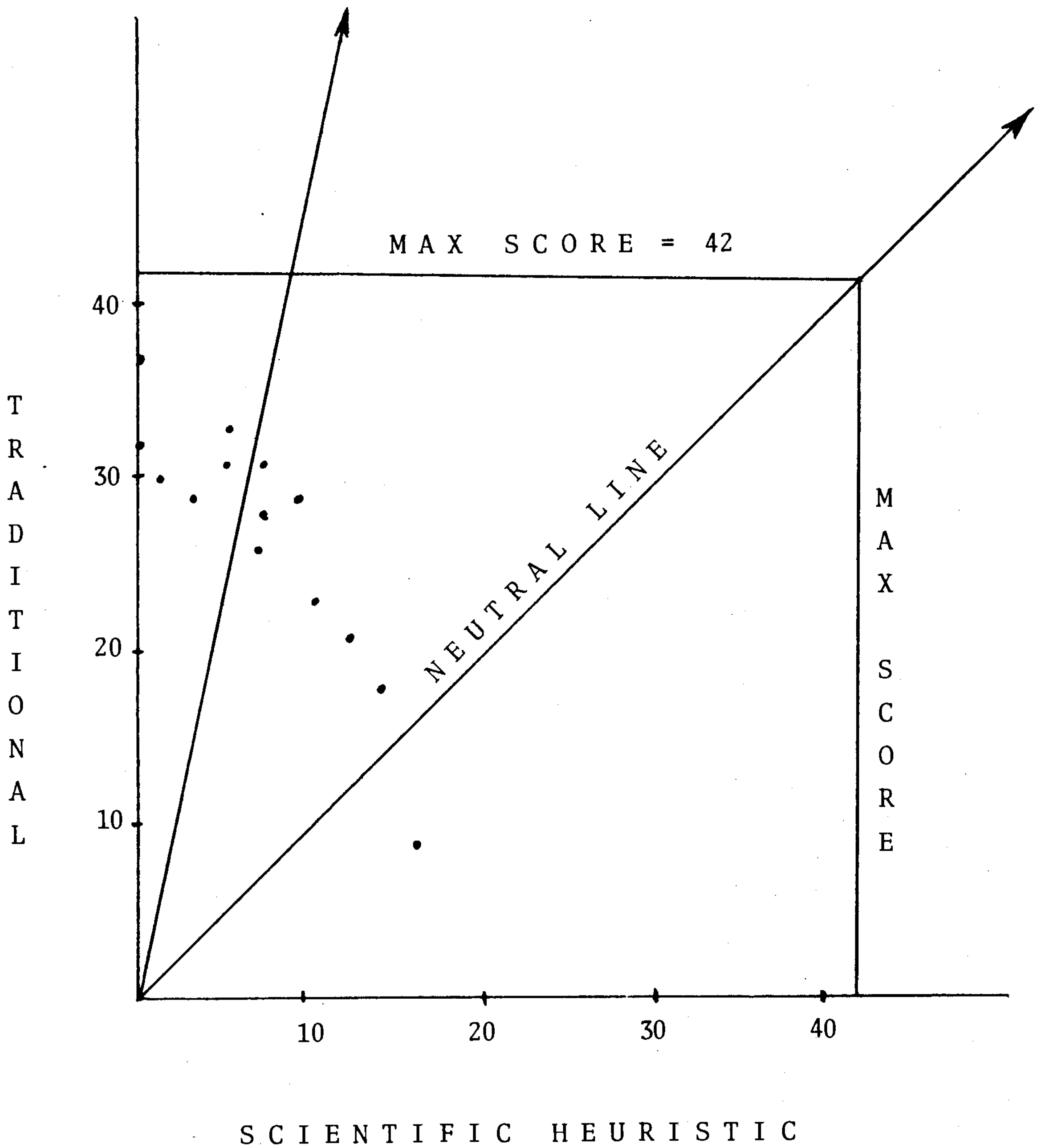
PARTICIPANTS	TAPE VB 0193 TRAD.		TAPE VB 0179 S.H.	
1. STUDENT 1	5 S	33 T	33 S	5 T
2. STUDENT 2	16 S	9 T	27 S	4 T
3. STUDENT 3	0 S	32 T	35 S	0 T
4. STUDENT 4	14 S	18 T	30 S	4 T
5. STUDENT 5	1 S	30 T	36 S	0 T
6. STUDENT 6	12 S	21 T	35 S	3 T
7. STUDENT 7	7 S	31 T	32 S	4 T
8. STUDENT 8	3 S	29 T	28 S	1 T
9. STUDENT 9	5 S	31 T	35 S	0 T
10. STUDENT 10	7 S	26 T	20 S	10 T
11. STUDENT 11	10 S	23 T	36 S	2 T
12. STUDENT 12	9 S	29 T	28 S	8 T
13. STUDENT 13	6 S	28 T	31 S	4 T
14. STUDENT 14	0 S	36 T	26 S	3 T

GRAPH 3

RESULTS OF TAPE VB 0193 (TRADITIONAL)

TAKEN FROM TABLE XXXV

$N_T = 14$

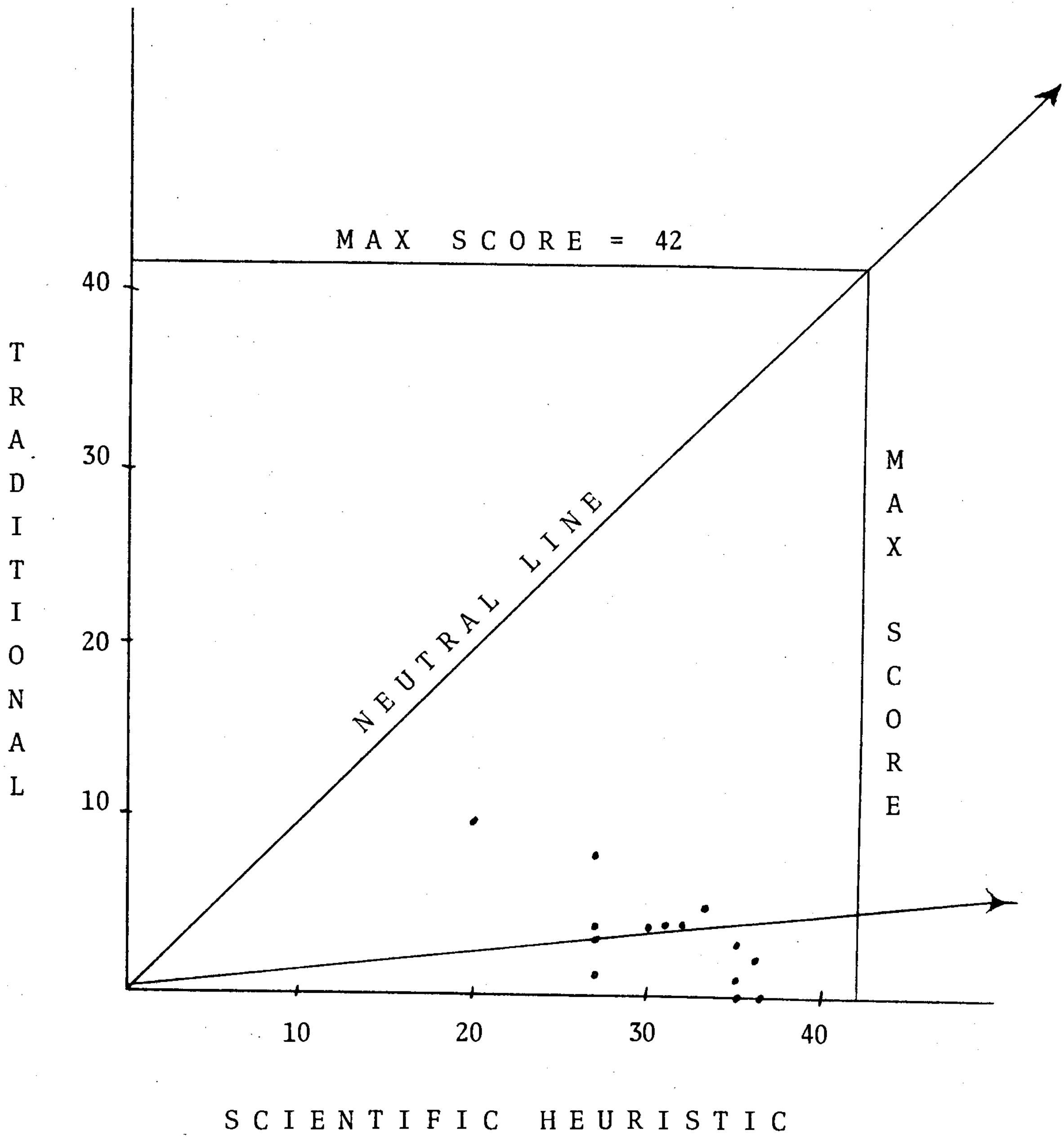


GRAPH 4

RESULTS OF TAPE VB 0179 (SCIENTIFIC HEURISTIC)

TAKEN FROM TABLE XXXV

$N_S = 14$



The university students with the exception of one, also identified the two distinct methods.

The next Table (XXXVI) shows people mostly involved in education. They too agreed with the previous groups. However, some of them offered some suggestions leading to the second and final revision. The changes consisted mainly in altering the wording of a few questions.

The final questionnaire was then administered to a different group of people (Table XXXVI) still using the first procedure. The results shown in Graph 7 and 8 coincide with previous ones.

TABLE XXXVI

RESPONSES OF SECOND GROUP TO QUESTIONNAIRE (AFTER FIRST REVISION)

USING THE FIRST PROCEDURE

MAXIMUM SCORE = 42

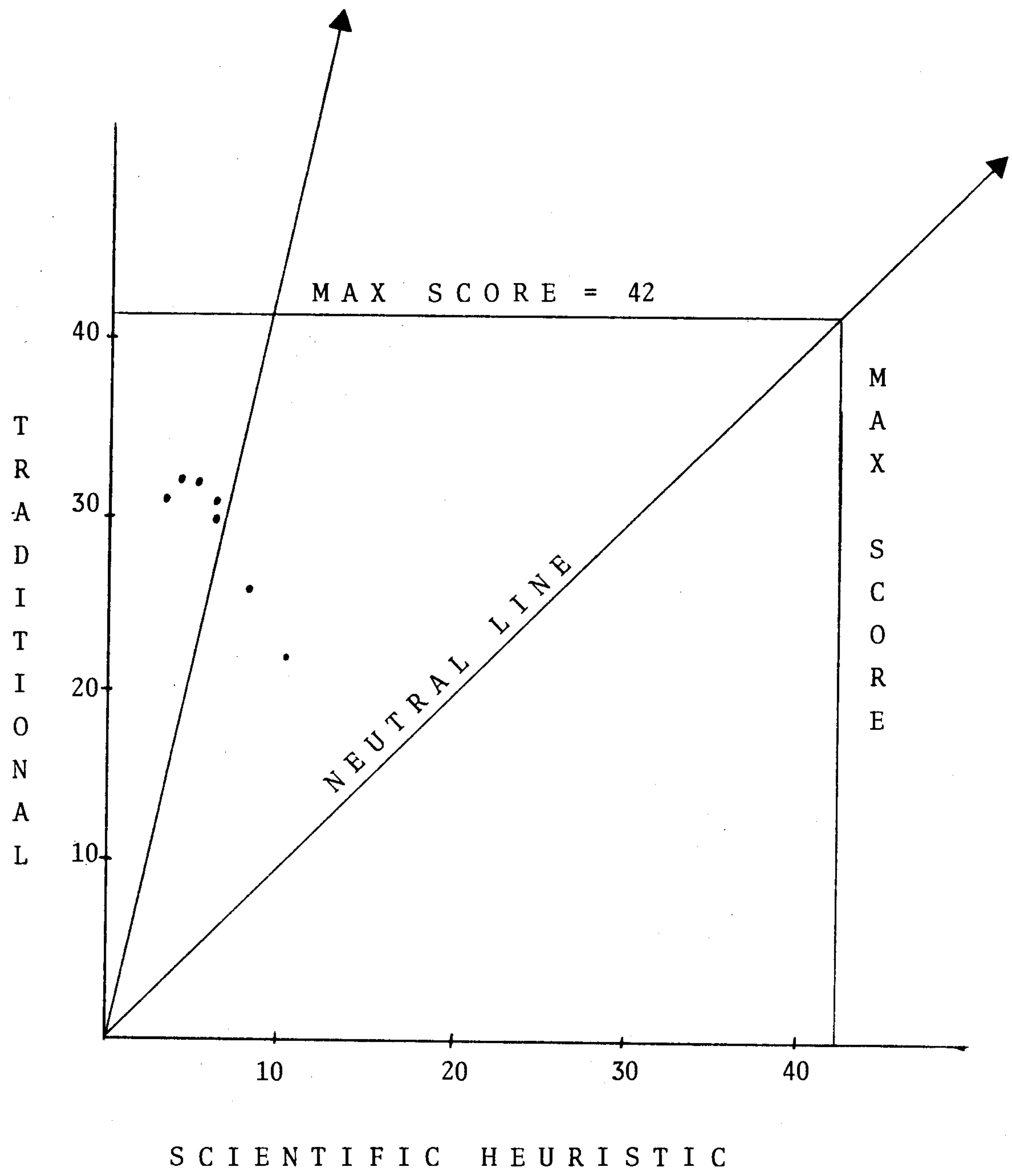
PARTICIPANT	TAPE	SCORE
PROFESSOR (EDUCATION)	VB 0179 SH	31 S 5 T
HIGH SCHOOL TEACHER (PHYSICS)	VB 0179 SH	18 S 11 T
MASTER CANDIDATE (EDUCATION)	VB 0179 SH	27 S 8 T
ELEMENTARY SCHOOL TEACHER (RET.)	VB 0179 SH	29 S 4 T
UNIVERSITY STUDENT	VB 0179 SH	27 S 3 T
HIGH SCHOOL SUPERVISOR	VB 0186 SH VB 0177 TRAD	25 S 12 T 10 S 22 T
ELEMENTARY SCHOOL SUPERVISOR	VB 0177 TRAD VB 0186 SH	3 S 31 T 30 S 6 T
HIGH SCHOOL PRINCIPAL	VB 0186 SH VB 0177 TRAD	33 S 3 T 5 S 32 T
ASSISTANT SUPERINTENDANT OF SCHOOLS	VB 0177 TRAD VB 0186 SH	6 S 31 T 28 S 6 T
HIGH SCHOOL COUNSELLOR	VB 0186 SH VB 0189 SH VB 0180 TRAD	39 S 0 T 29 S 3 T 6 S 30 T
TEACHER OF SENIOR ENGLISH (HIGH SCHOOL)	VB 0186 SH VB 0189 SH VB 0180 TRAD	36 S 2 T 30 S 4 T 4 S 32 T
LABOURER	VB 0180 TRAD VB 0189 SH	8 S 26 T 34 S 3 T
UNIVERSITY STUDENT	VB 0180 TRAD	2 S 31 T

GRAPH 5

RESULTS OF TRADITIONAL TAPES

TAKEN FROM TABLE XXXVI

$N_T = 7$



GRAPH 6

RESULTS OF SCIENTIFIC HEURISTIC TAPES

TAKEN FROM TABLE XXXVI

$N_S = 14$

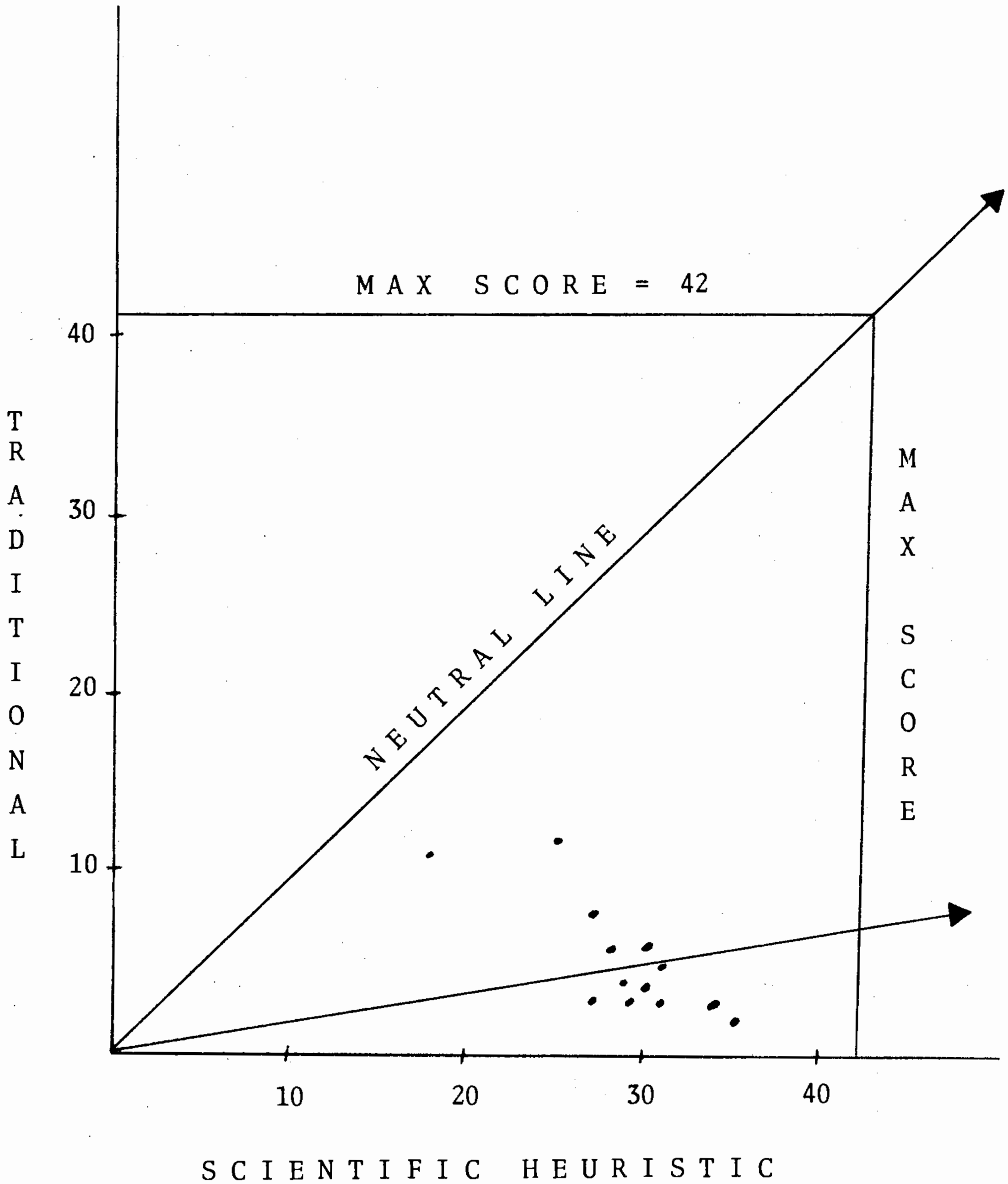


TABLE XXXVII

RESPONSES TO FINAL QUESTIONNAIRE (SECOND REVISION)

USING FIRST PROCEDURE

MAXIMUM SCORE = 44

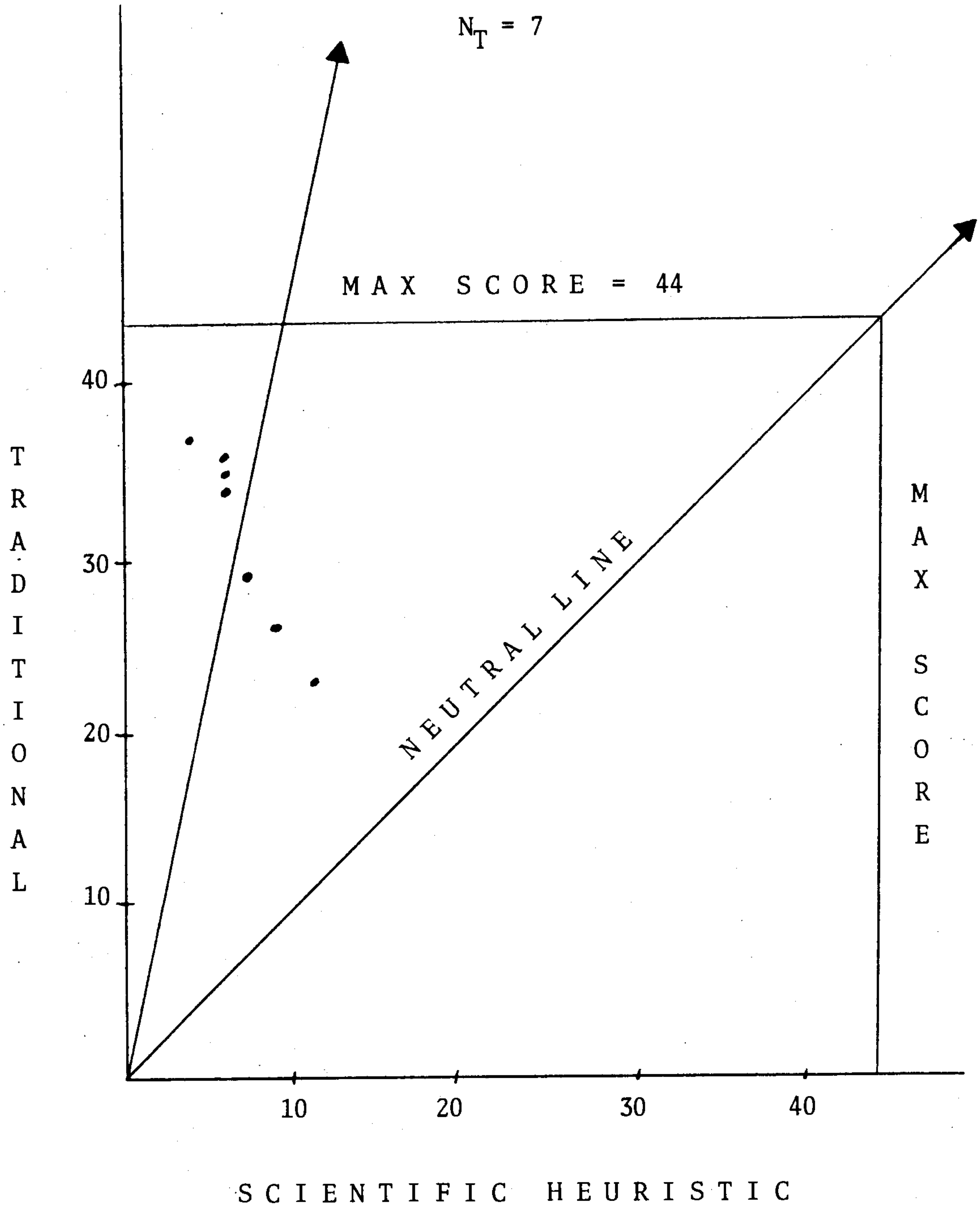
PARTICIPANT	TAPE VB 0177 TRADITIONAL		TAPE VB 0186 SCIENTIFIC HEURISTIC	
1. MATHEMATICS TEACHER	6 S	36 T	40 S	2 T
2. WIFE	7 S	29 T	39 S	2 T
3. STUDENT TEACHER (FRENCH)	4 S	27 T	28 S	8 T
4. ENGLISH TEACHER	9 S	26 T	38 S	1 T
5. ELEMENTARY SCHOOL TEACHER	11 S	23 T	32 S	1 T
6. BROTHER - UNIVERSITY STUDENT	6 S	34 T	32 S	3 T
7. BROTHER - MECHANIC	6 S	36 T	40 S	2 T

GRAPH 7

RESULTS OF TRADITIONAL TAPES

TAKEN FROM TABLE XXXVII

$N_T = 7$

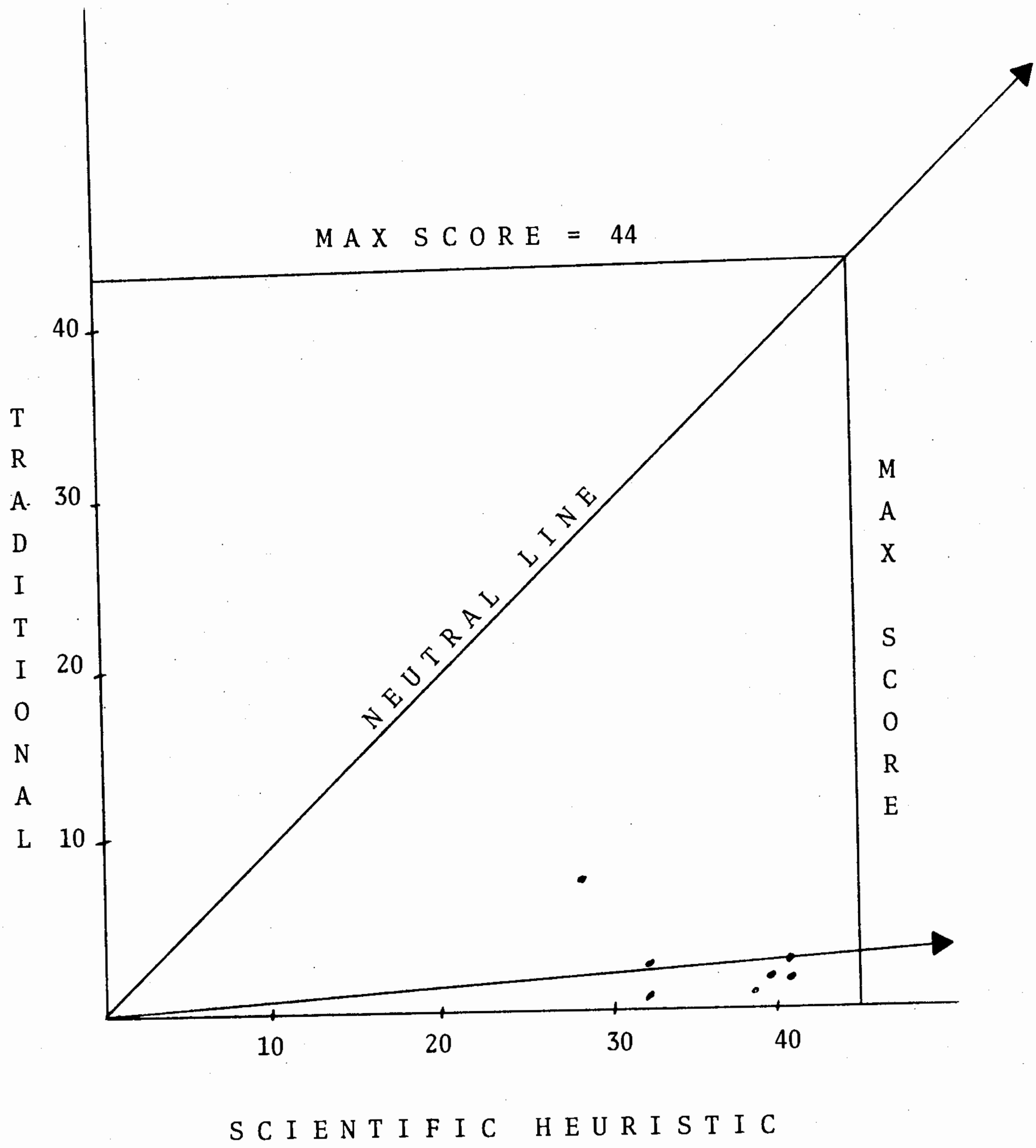


GRAPH 8

RESULTS OF SCIENTIFIC HEURISTIC TAPES

TAKEN FROM TABLE XXXVII

$N_S = 7$



As was mentioned on page 94, a second procedure was employed. Table XXXVIII identifies the viewers (group of varied occupations) and Graphs 9 and 10 indicate the results which are much the same as all previous ones.

TABLE XXXVIII

RESPONSES TO THE FINAL QUESTIONNAIRE USING THE SECOND PROCEDURE

MAXIMUM SCORE* = 44

PARTICIPANT	TAPE VB 0177 TRADITIONAL		TAPE VB 0186 SCIENTIFIC HEURISTIC	
1. SOCIAL STUDIES TEACHER	3 S	40 T	34 S	3 T
2. PROFESSIONAL JAPANESE INTERPRETER	8 S	30 T	38 S	3 T
3. BIOLOGY GRADUATE (1971)	15 S	24 T	26 S	7 T
4. CHEMISTRY GRADUATE	9 S	34 T	34 S	7 T
5. STORE CLERK	6 S	29 T	26 S	3 T
6. ECONOMICS GRADUATE (1971)	2 S	42 T	24 S	5 T
7. HOUSEWIFE	8 S	20 T	26 S	13 T
8. BARBER	8 S	28 T	26 S	8 T
9. B.C. TELEPHONE ELECTRICAL WORKER	11 S	24 T	28 S	7 T
10. UNIVERSITY ASSOCIATE	3 S	26 T	40 S	0 T

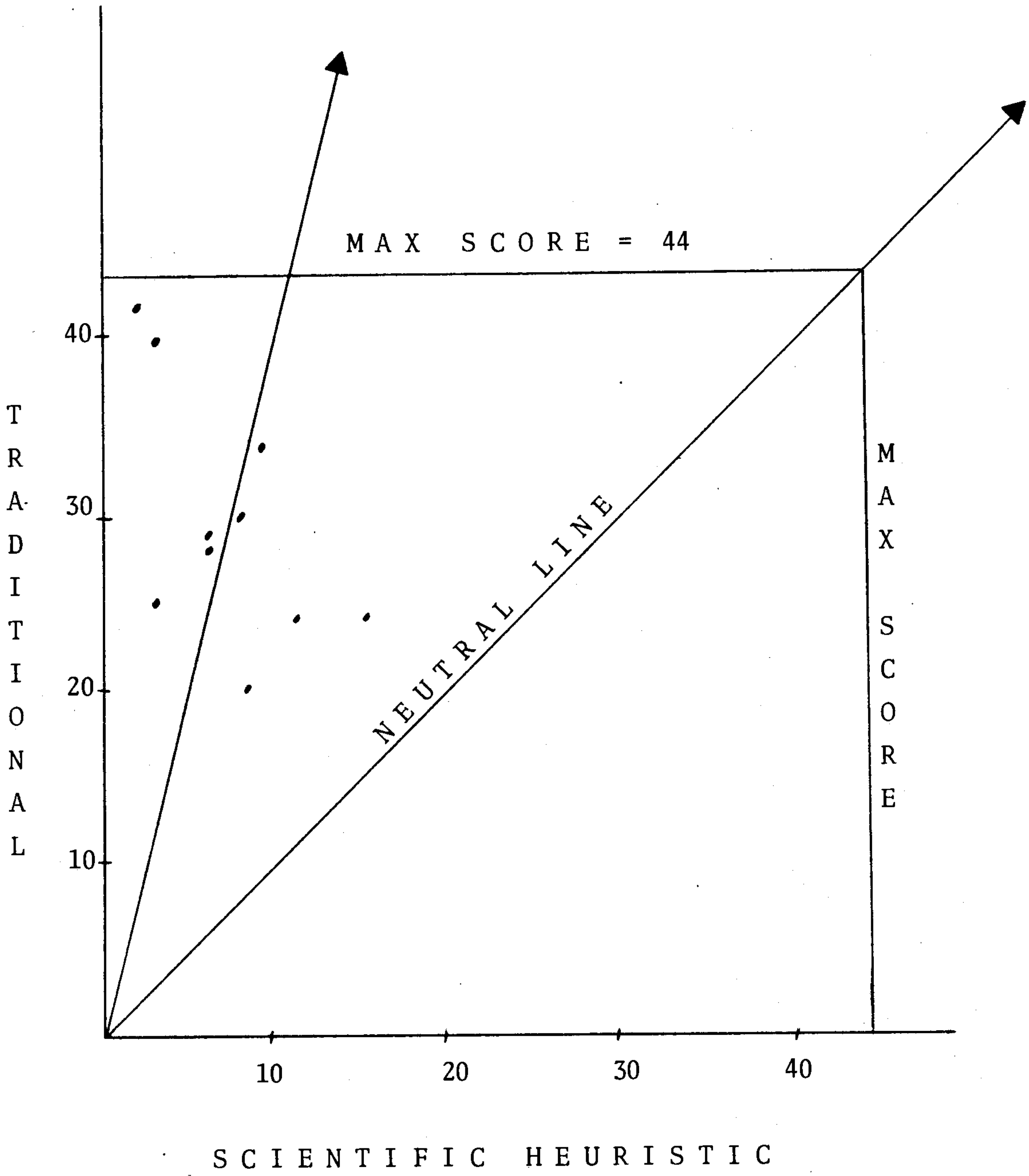
* The participants' scores represent a mean of the two fifteen-minute questionnaire results.

GRAPH 9

RESULTS OF TRADITIONAL TAPE (VB 0177)

TAKEN FROM TABLE XXXVIII

$N_T = 10$

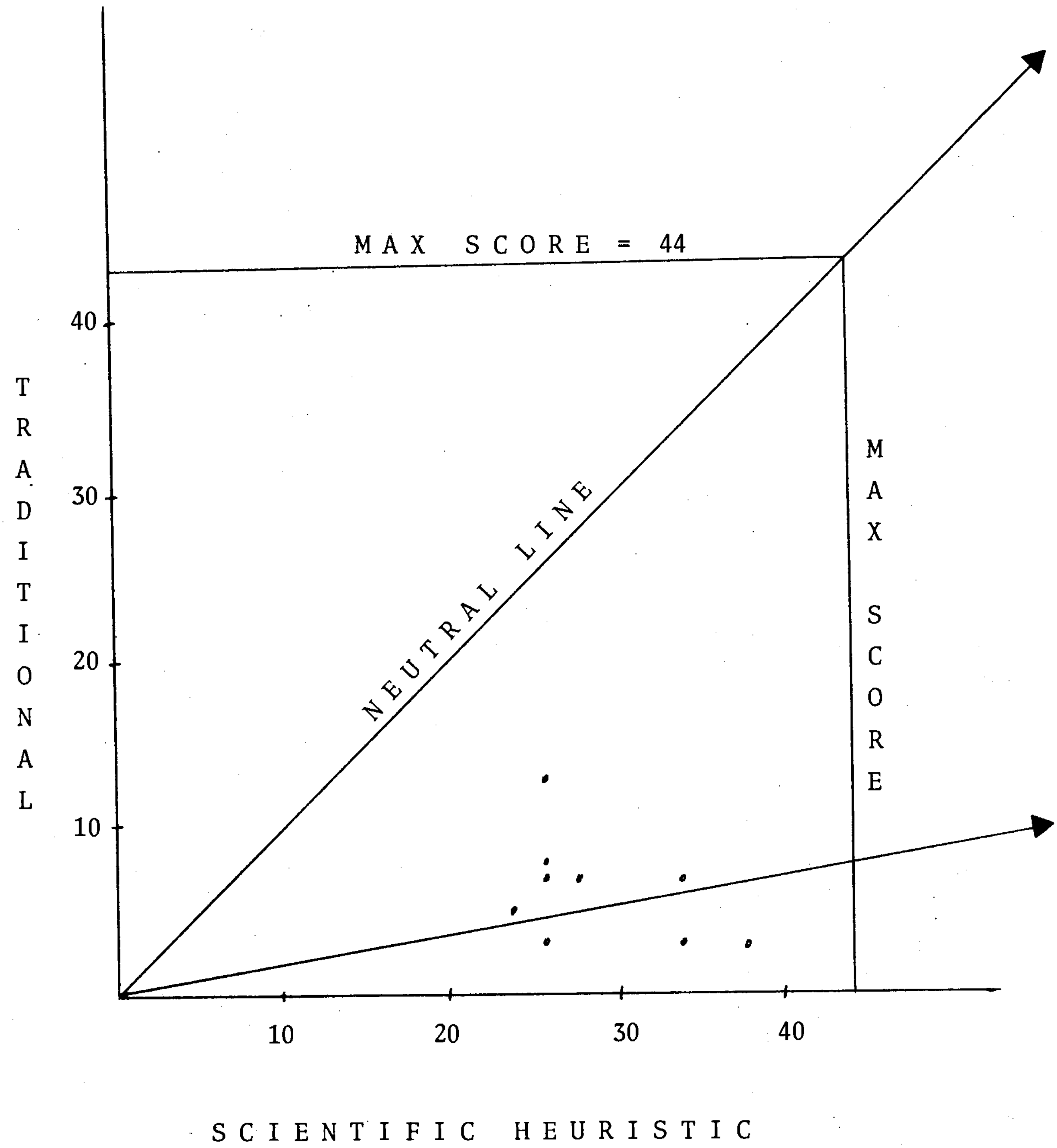


GRAPH 10

RESULTS OF SCIENTIFIC HEURISTIC TAPE (VB 0186)

TAKEN FROM TABLE XXXVIII

$N_S = 10$



No additional people besides those shown in Tables XXXIV - XXXVIII. viewed tapes. However, the responses of these viewers were analyzed further by categorizing them according to: Educators, university students, relations, and others of varied occupations. The results are shown by the Graphs 11 - 18.

TABLE XXXIX

EDUCATORS

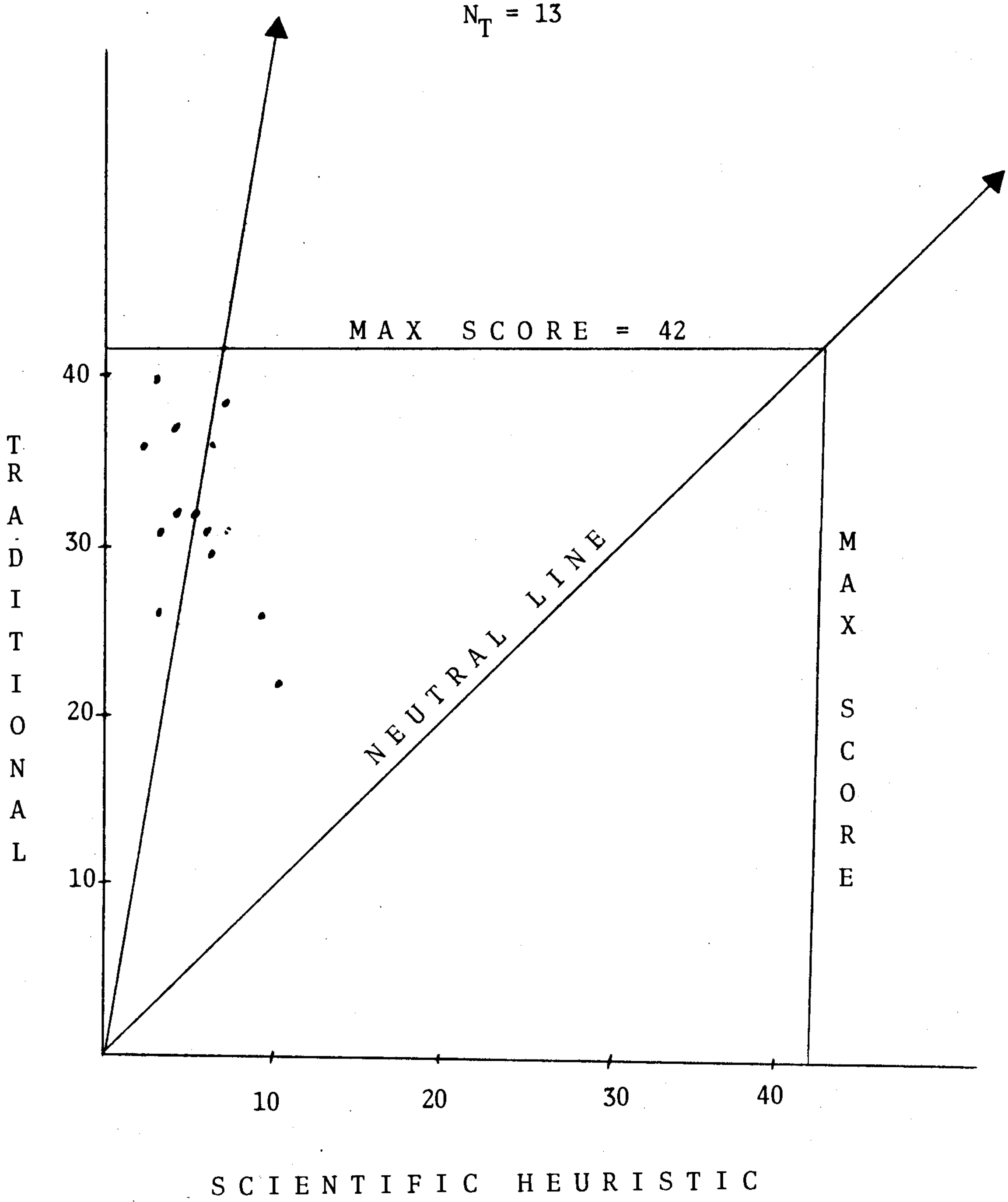
PARTICIPANT	TRAD-TAPE	SH-TAPE
RETIRED ELEMENTARY TEACHER		15 S 6 T
UNIVERSITY PROFESSOR (EDUCATION		23 S 11 T
COMMERCE HIGH SCHOOL TEACHER	2 S 36 T	34 S 2 T
ENGLISH TEACHER (SECONDARY)	7 S 31 T	36 S 2 T
UNIVERSITY PROFESSOR (EDUCATION)		31 S 5 T
PHYSICS TEACHER		18 S 11 T
ELEMENTARY TEACHER		29 S 4 T
HIGH SCHOOL SUPERVISOR	10 S 22 T	25 S 12 T
ELEMENTARY SUPERVISOR	3 S 31 T	30 S 6 T
HIGH SCHOOL PRINCIPAL	5 S 32 T	33 S 3 T
SUPERINTENDANT (ASSISTANT)	6 S 31 T	28 S 6 T
HIGH SCHOOL COUNSELLOR	6 S 30 T	39 S 0 T
ENGLISH TEACHER	4 S 32 T	30 S 4 T
MATHEMATICS TEACHER	6 S 36 T	40 S 2 T
FRENCH TEACHER	4 S 37 T	28 S 8 T
ENGLISH TEACHER	9 S 36 T	38 S 1 T
SOCIAL STUDIES TEACHER	3 S 40 T	34 S 3 T
UNIVERSITY ASSOCIATE	3 S 26 T	40 S 0 T

GRAPH 11

EDUCATORS AS VIEWERS OF TRADITIONAL TAPES

(TABLE XXXIX)

$N_T = 13$

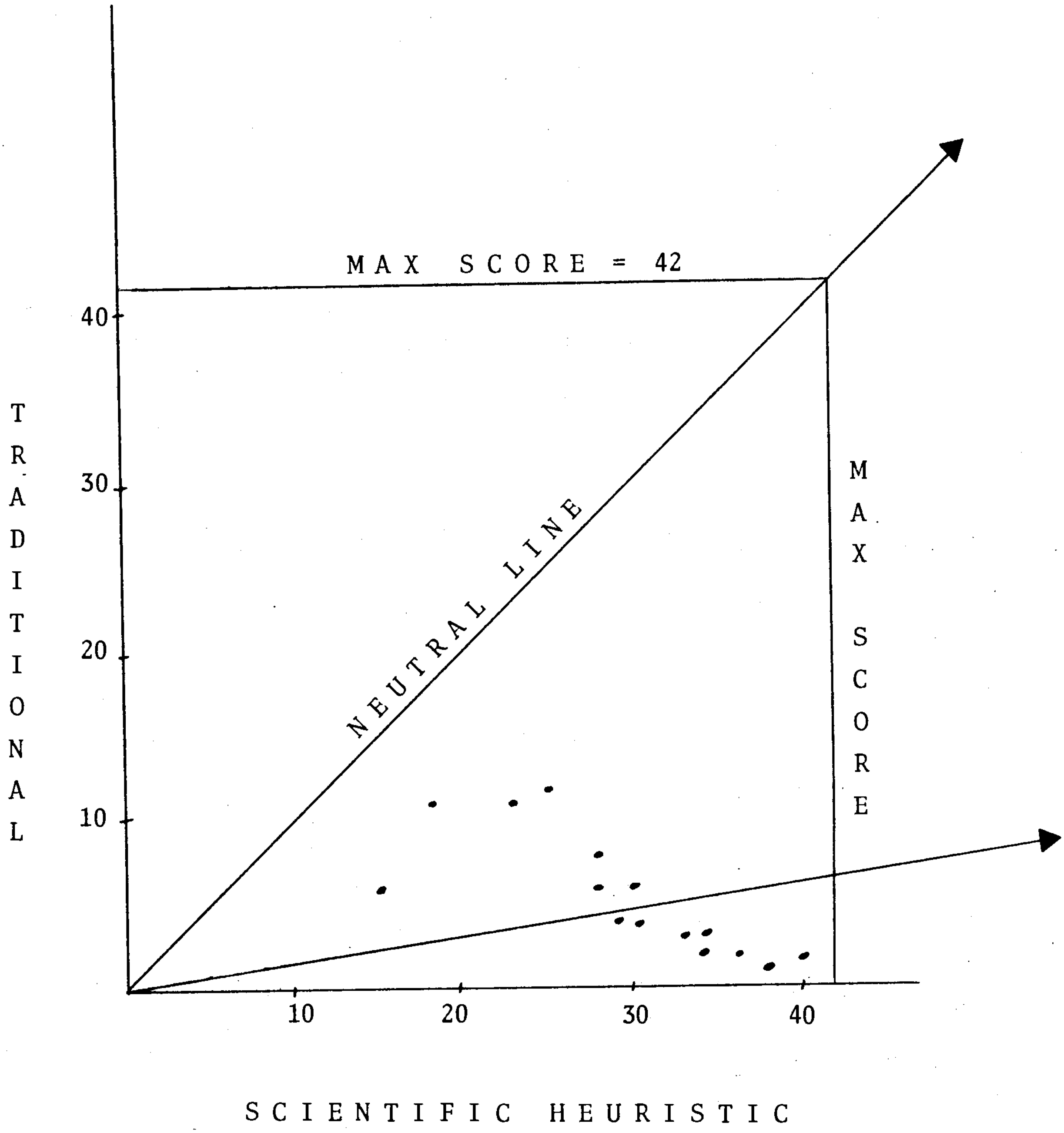


GRAPH 12

EDUCATORS AS VIEWERS OF SCIENTIFIC HEURISTIC TAPES

(TABLE XXXIX)

$N_S = 18$



Graphs 11 and 12 indicate as before that the educators identified decisively the two different methods.

TABLE XL

UNIVERSITY STUDENTS

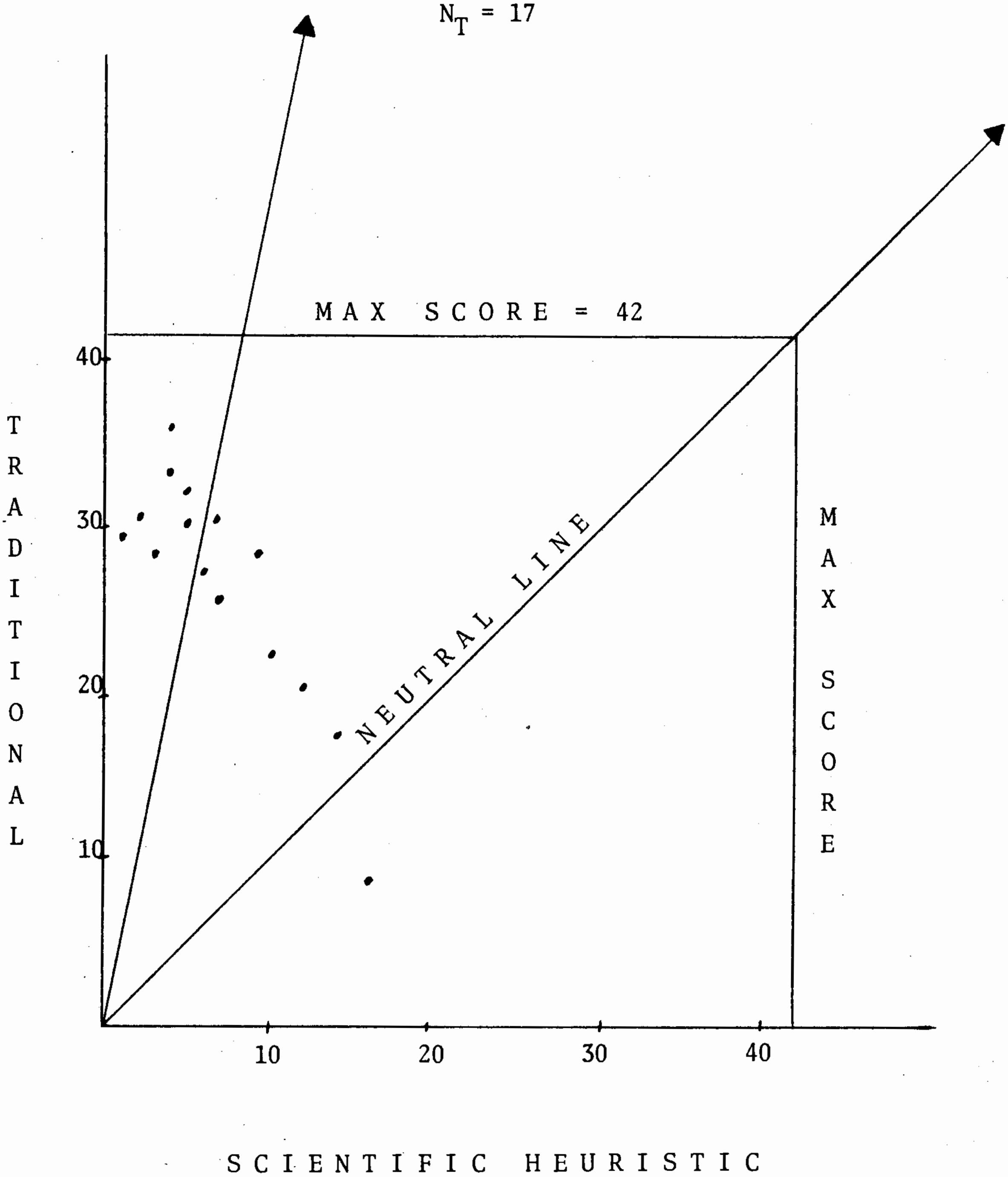
PARTICIPANTS	TRAD-TAPE		SH-TAPE	
STUDENT 1	5 S	33 T	33 S	5 T
STUDENT 2	16 S	9 T	27 S	4 T
STUDENT 3	0 S	32 T	35 S	0 T
STUDENT 4	14 S	18 T	30 S	4 T
STUDENT 5	1 S	30 T	36 S	0 T
STUDENT 6	12 S	21 T	35 S	3 T
STUDENT 7	7 S	31 T	32 S	4 T
STUDENT 8	3 S	29 T	28 S	1 T
STUDENT 9	5 S	31 T	35 S	0 T
STUDENT 10	7 S	26 T	20 S	10 T
STUDENT 11	10 S	23 T	36 S	2 T
STUDENT 12	9 S	29 T	28 S	8 T
STUDENT 13	6 S	28 T	31 S	4 T
STUDENT 14	0 S	36 T	26 S	3 T
UNIVERSITY MASTERS CANDIDATE			27 S	8 T
UNIVERSITY FRENCH STUDENT	4 S	37 T	28 S	8 T
SECOND YEAR COLLEGE STUDENT	6 S	34 T	32 S	3 T
UNIVERSITY STUDENT	2 S	31 T	27 S	3 T

GRAPH 13

UNIVERSITY STUDENTS AS VIEWERS OF TRADITIONAL TAPES

(TABLE XL)

$N_T = 17$

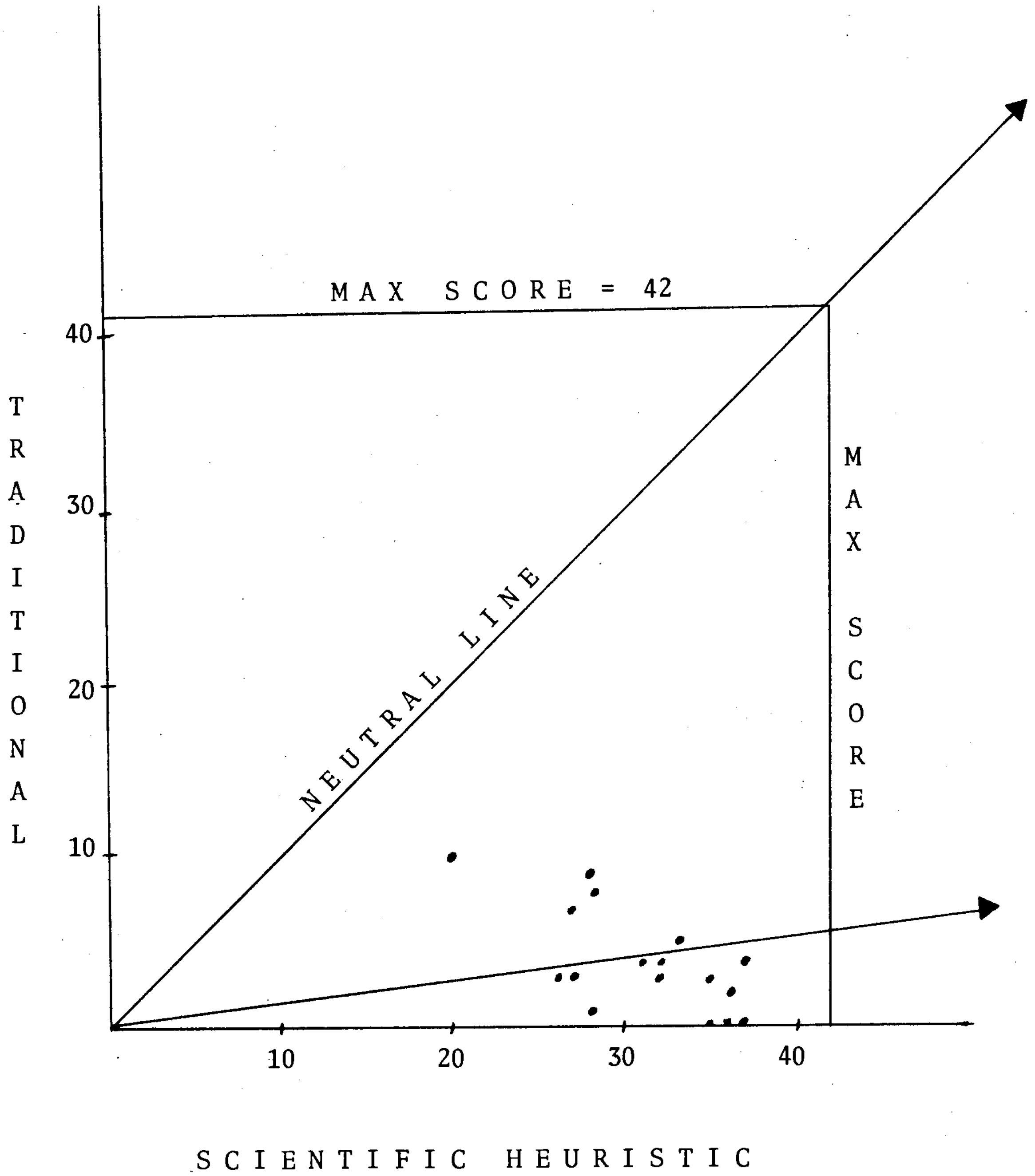


GRAPH 14

UNIVERSITY STUDENTS AS VIEWERS OF SCIENTIFIC HEURISTIC TAPES

(TABLE XL)

$N_S = 18$



The university students also identified the two distinct methods of teaching. Similar results were obtained (as indicated in Graphs 15 - 18) involving relatives and people of varied occupations as viewers.

TABLE XLI

RELATIVES

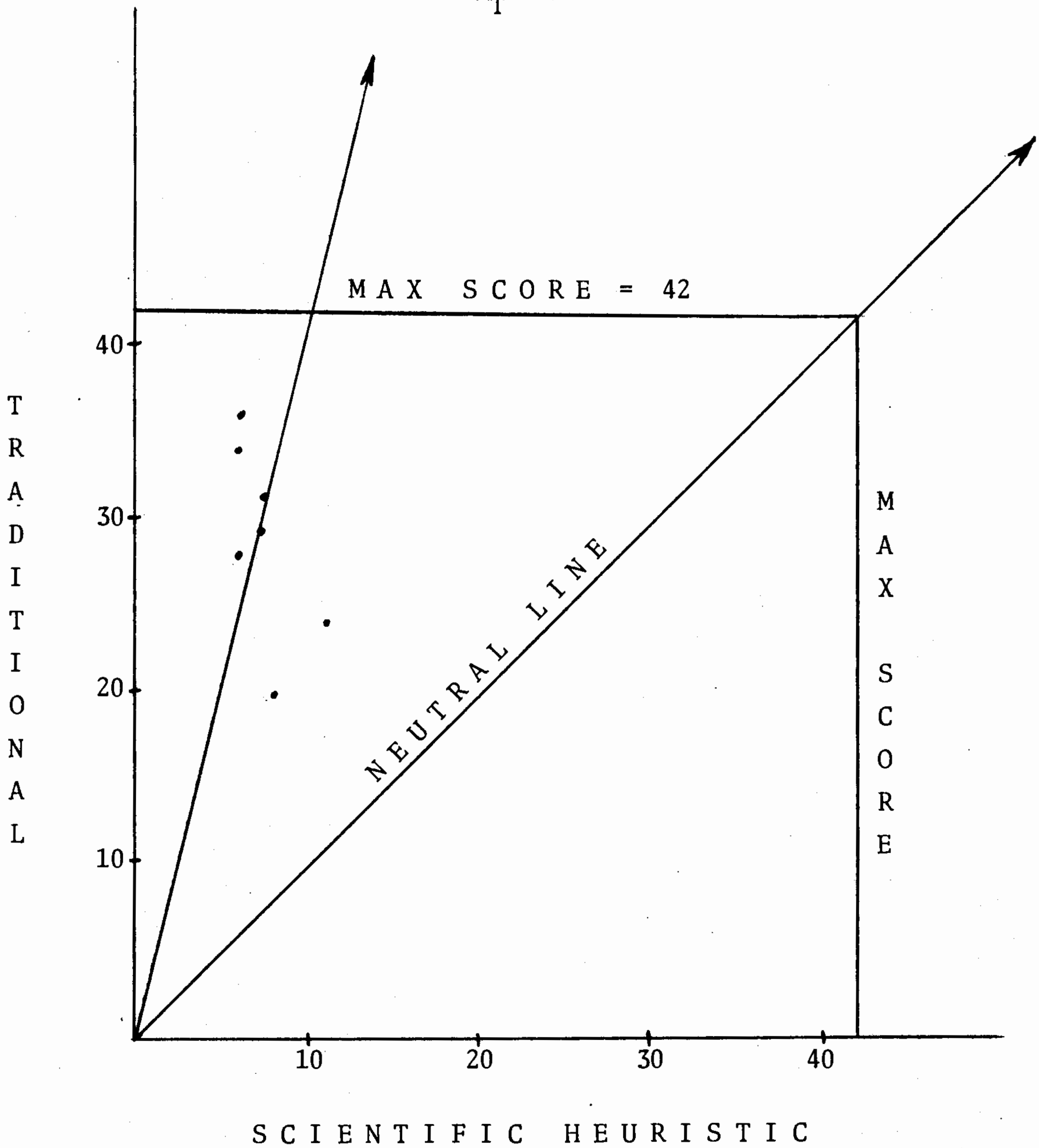
PARTICIPANTS	TRAD-TAPE		SH-TAPE	
WIFE	7 S	30 T	38 S	0 T
WIFE	7 S	29 T	31 S	1 T
BROTHER	6 S	34 T	32 S	3 T
BROTHER	6 S	36 T	40 S	2 T
COUSIN	6 S	28 T	26 S	8 T
BROTHER-IN-LAW	11 S	24 T	28 S	7 T
SISTER-IN-LAW	8 S	20 T	26 S	13 T

GRAPH 15

RELATIVES AS VIEWERS OF TRADITIONAL TAPES

(TABLE XLI)

$N_T = 7$



GRAPH 16

RELATIVES AS VIEWERS OF SCIENTIFIC HEURISTIC TAPES

(TABLE XLI)

$N_S = 7$

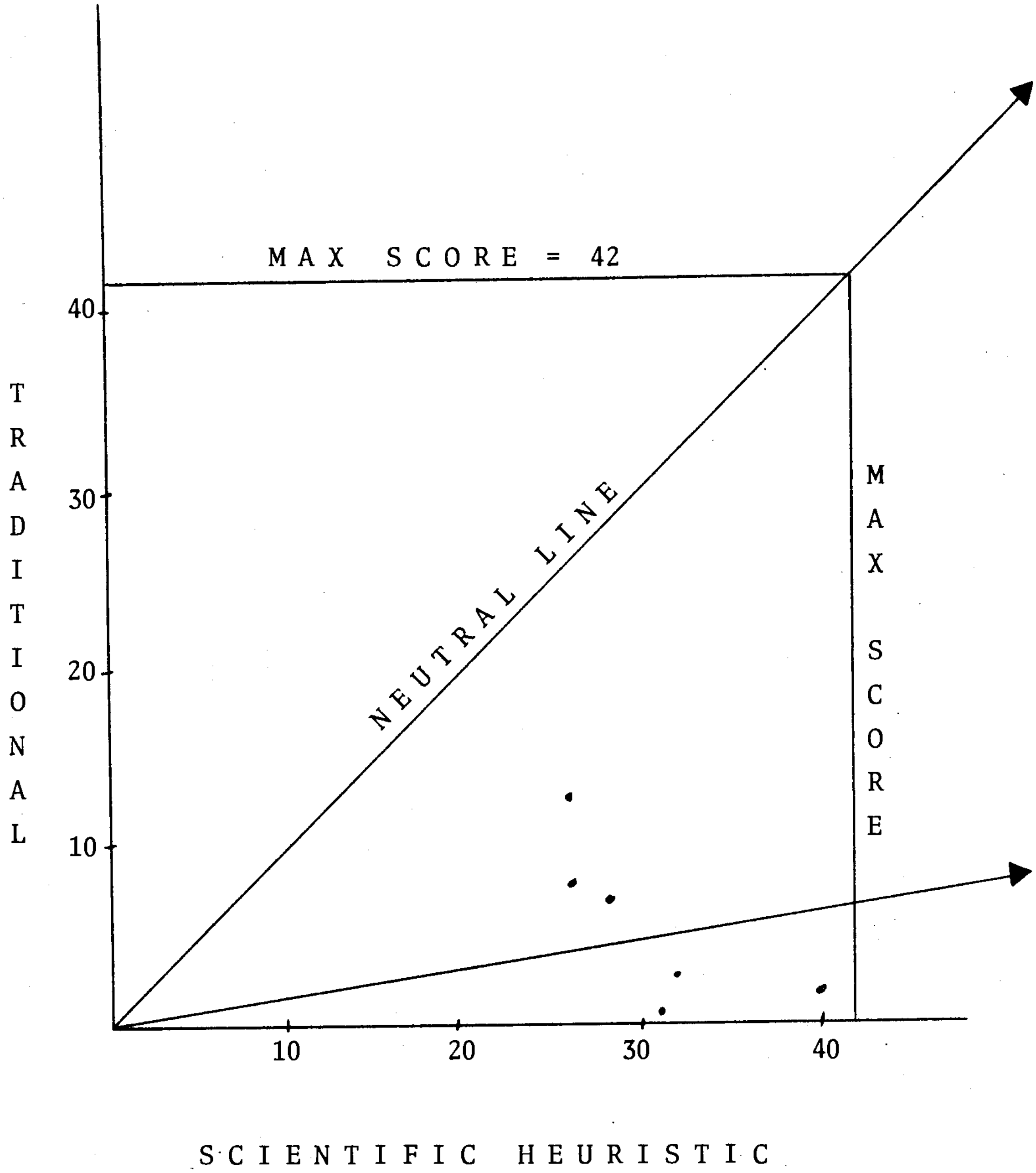


TABLE XLII

PEOPLE OF VARIED OCCUPATIONS

PARTICIPANT	TRAD-TAPE		SH-TAPE	
BUSINESSMAN	2 S	35 T	36 S	1 T
LABOURER	8 S	26 T	34 S	3 T
JAPANESE INTERPRETER	8 S	30 T	38 S	3 T
BIOLOGY GRADUATE (1971)	15 S	24 T	26 S	7 T
CHEMISTRY GRADUATE (1971)	9 S	34 T	34 S	7 T
STORE CLERK	6 S	29 T	26 S	3 T
ECONOMICS GRADUATE (1971)	2 S	42 T	24 S	5 T
BARBER	6 S	28 T	26 S	8 T

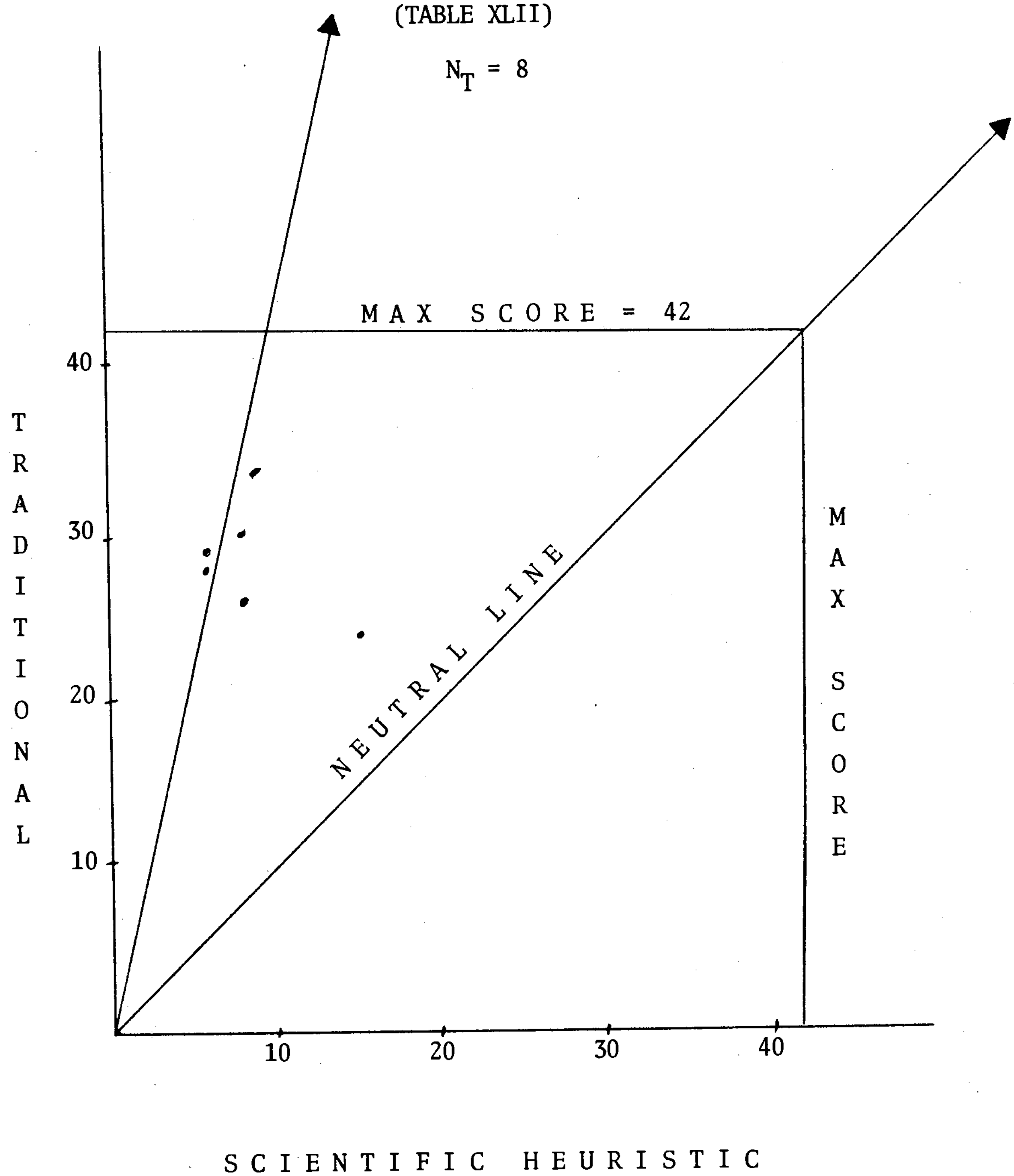
GRAPH 17

VARIED-OCCUPATION GROUP AS VIEWERS

OF TRADITIONAL TAPES

(TABLE XLII)

$N_T = 8$



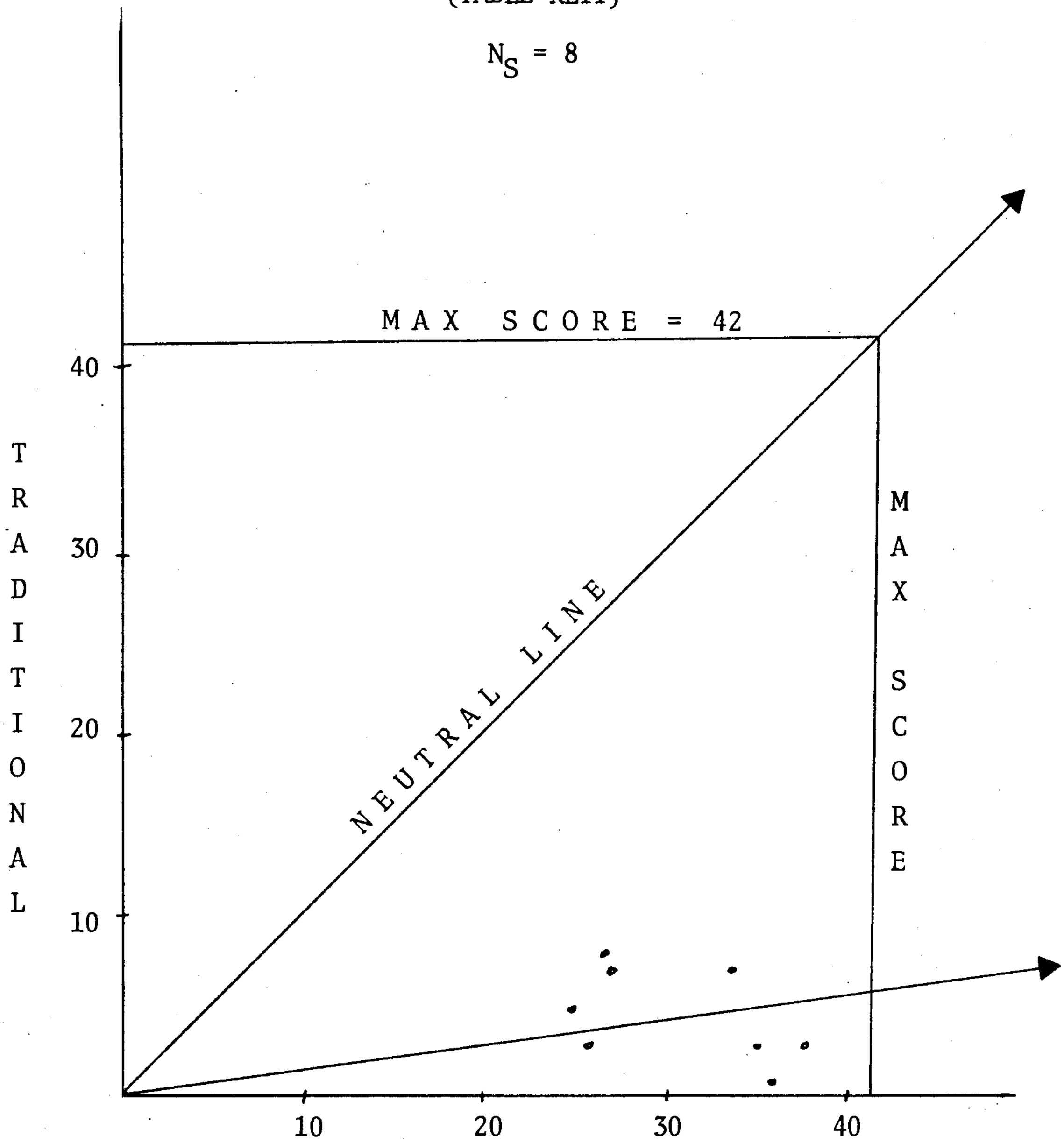
GRAPH 18

VARIED-OCCUPATION GROUP AS VIEWERS

OF SCIENTIFIC HEURISTIC TAPES

(TABLE XLII)

$N_S = 8$



SCIENTIFIC HEURISTIC

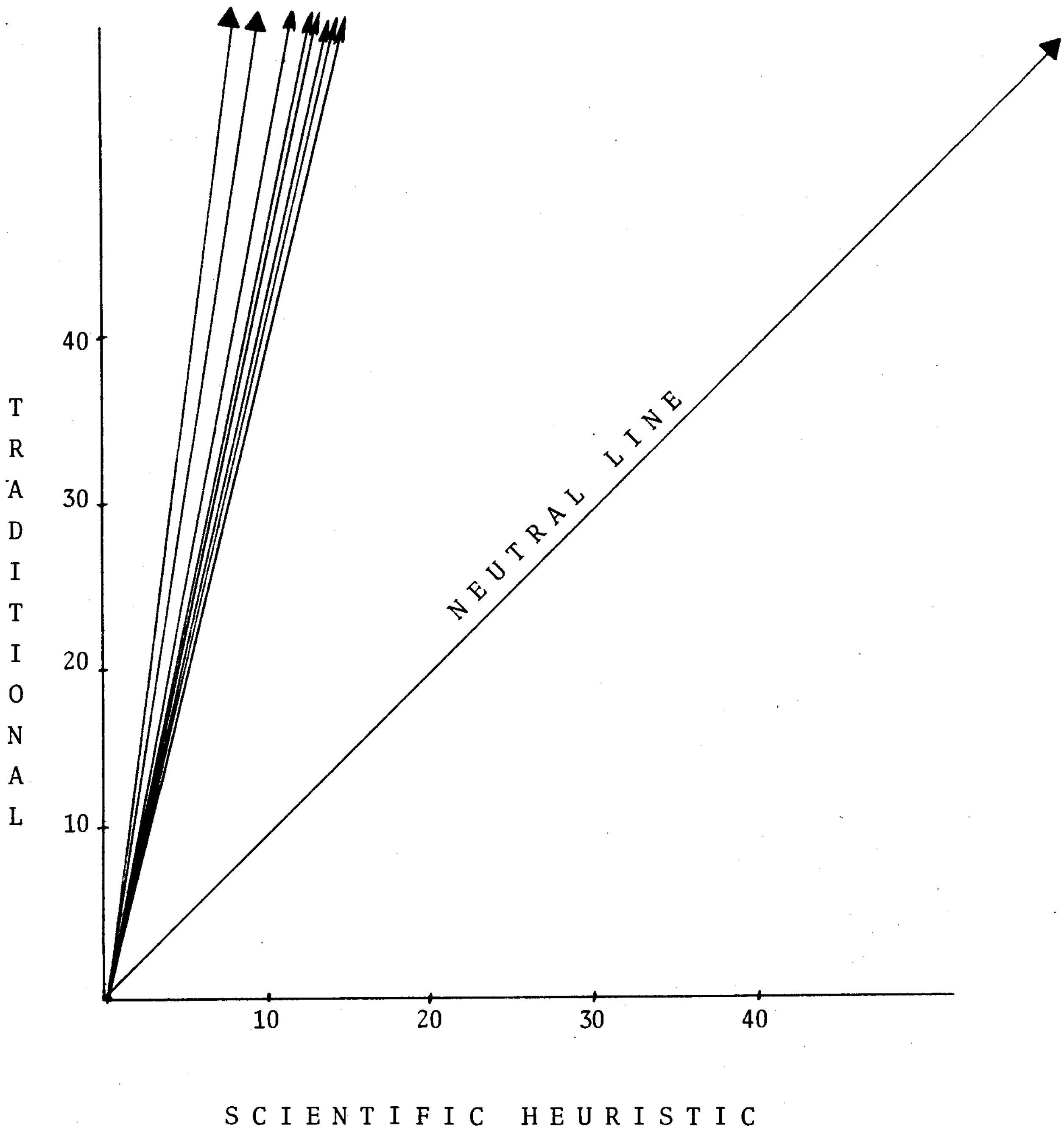
Graph 19 indicates a visual comparison of the results of all the previous traditional graphs.

Graph 20 does the same for the scientific heuristic graphs.

GRAPH 19

SUMMARY OF RESULTS OF ALL

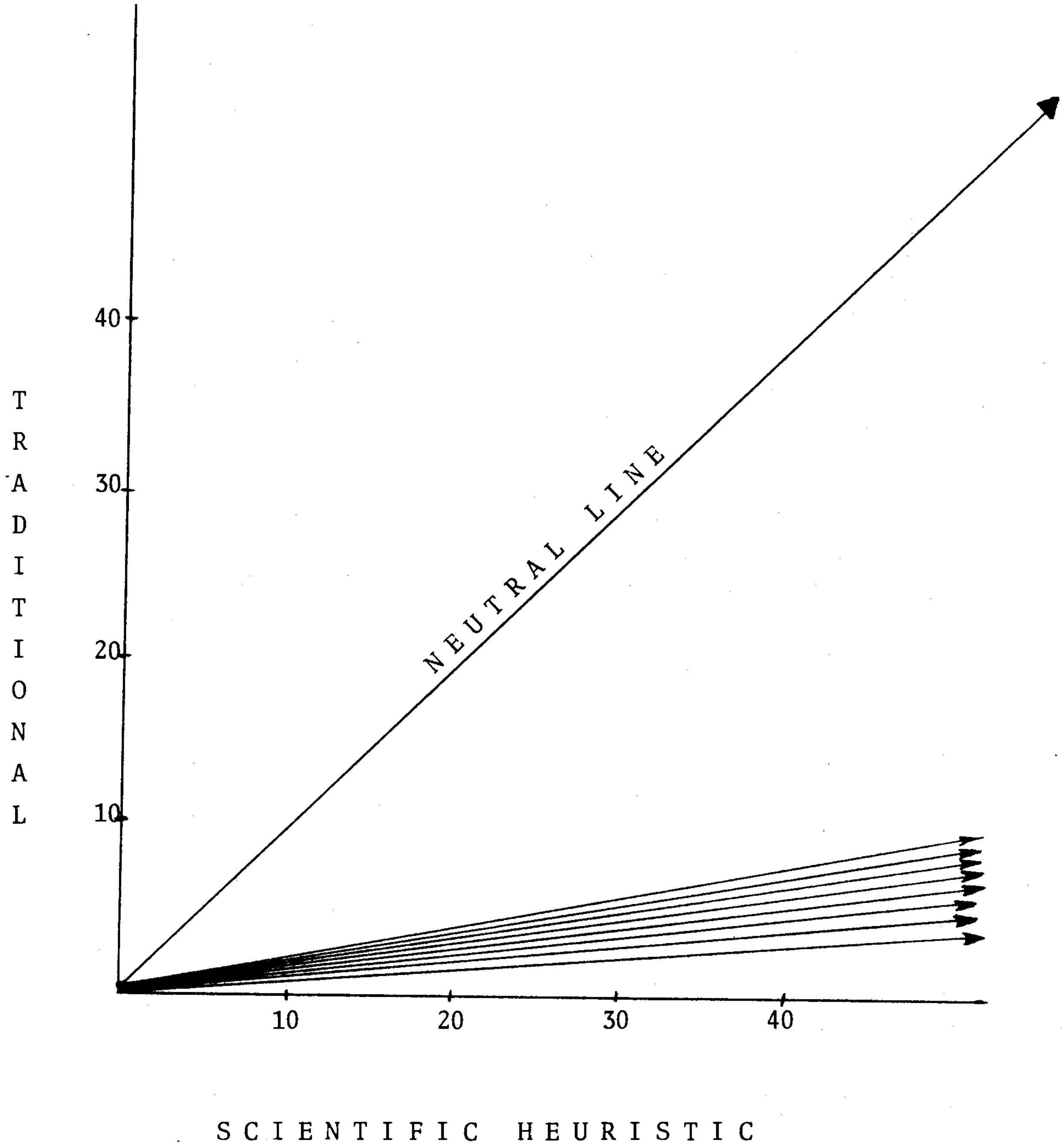
TRADITIONAL GRAPHS



GRAPH 20

SUMMARY OF RESULTS OF ALL

SCIENTIFIC HEURISTIC GRAPHS



Summarized Comments on Evidence Pertaining to the Use of Two Different Methods of Teaching Mathematics

The results presented on the preceding pages indicate with little doubt that the great majority of the people who viewed the respective tapes identified the two distinct methods of teaching.

Graphs 19 and 20 show decisively that all the groups involved in the video-tape experiment responded correspondingly to the criteria which the experts considered as peculiar to the scientific heuristic lessons, and similarly to the traditional lessons.

III. DIARY

July 13, 1971:

Today I taught the sixth of eighteen lessons (two periods used for testing purposes). I am finding the task of behaving differently in each class a little difficult. I find that occasionally I must hold myself back from being traditional in the scientific heuristic class, and from being scientific heuristic in the traditional class! However, I find it more difficult to be traditional in the traditional class. I find that telling the student about facts is very difficult for me. It makes me feel almost apologetic. On the other hand, the hard part in the scientific heuristic class is holding myself back from insisting that they do the homework I suggest. I wish sometimes that I could say "this is very important so I expect you to do it".

I adjusted quite well (I think) to the fact that some students continue working amongst themselves while I speak, even though I feel that what I have to say is important to them. So far, I find that the hardest part is when I am away from the classes. I think about what I should do the following day almost continuously. I spend a great deal of time viewing the video tapes made in class and writing accounts of what occurred during the lessons. But is seems that a lot of my energy is going into thinking about what has happened and what I will do the following day in each class. Even though the heuristic class has run very well up to now, I still seem to be more concerned about it. I find that I have to restrain myself not to push the students in doing what I think they should do.

The two classes are quite different. The scientific heuristic class:

1. Has a rug on the floor.
2. The desks are placed in no special arrangement.
3. The students sit in different places at different times.
4. They are urged to correct each other and told not to be afraid to make mistakes.
5. They get no tests for grading purposes, only sheets to diagnose difficulties and to see if any learning has taken place.
6. Students don't do the same things at the same time.
7. They are encouraged not to be afraid to <u>disagree</u> with the teacher or with their fellow students if they are not convinced about the truth of any topic.
8. Some of the students have already begun to correct each other by discussing amongst themselves.

The traditional class:

1. The room is an ordinary classroom.
2. The desks are arranged in rows.
3. Homework is compulsory.
4. Discussion among students is not encouraged and I insist on having their full attention when I am explaining.
5. Review tests are given every day.
6. The lessons proceed as follows:

   i. Discussion of assignment.

   ii. Review test is administered.

   iii. They exchange papers amongst themselves and mark the test according to my answers.

   iv. These marks are read out and recorded.

   v. The students get their papers back.

   vi. Discussion of the quiz then takes place.

   vii. Then it is time for the introduction of the new work.

   viii. In the introduction some discussion is allowed; different ideas by the students are considered.

   ix The discussion does not last long and once the concept is introduced examples are worked out by the teacher with the help of the students.

   x. Then an assignment is given on the new work as well as on previous work for review.

July 14, 1971: (after 7th lesson)

In the scientific heuristic class we spent nearly seven lessons on rational numbers and rational algebraic expressions (mostly the addition of rational algebraic expressions). I find that some of the students still don't understand as much as I would like them to. They have learned a lot though, not only about simple fractions but about some rules on exponents, and also about certain things that must have bothered them for a long time.

Here are some examples:

1. $P^4 = P \cdot P \cdot P \cdot P$
2. $\frac{P^4}{P^3} = \frac{P \cdot P \cdot P \cdot P}{P \cdot P \cdot P} = P$
3. $5 + a \neq 5a$
4. $\frac{a}{2} = \frac{1}{2} \neq \frac{1}{2a}$
5. What must be placed in the blank to complete the equation? $B^2 \times \_\_\_ = B^5$

When they were first faced with this question many thought the answer was 3. Now they all appear to agree that B^3 should go in the blank.

What bothers me among other things is the fact that (although I know, given enough time, we would do a lot), we seem to be falling behind in comparison to the traditional class. At least we appear to be working further ahead with the traditional class. This is not to imply that the students in the traditional class have learned more.

In the scientific heuristic class we seem to operate in a more natural way. The students are learning, rather than me teaching. Many of the facts they are learning aren't what I had anticipated. We discuss many different areas of mathematics, so they learn some concepts which are not supposed to be part of this course. In the traditional class I can guide students so that they learn more of what I had planned for them.

There are some students in the scientific heuristic class who don't work well by themselves. They seem more at home if they can get me to explain things to them.

Today, two of them were playing X's and O's! They had actually tried what had been suggested and seem to have satisfied themselves with what they had gotten out of the activity. I was a little bothered by it obviously; somehow I thought that they should have pursued it further. I started them working again by asking a few questions they could not answer.

In the traditional class I find it difficult at times to play the authoritarian role.

There has been the occasional student who has been coming late. I made a rule that my door would be closed to anybody who is late for class.

Today three people came late. I didn't let them in but the secretary came to plead for them about ten minutes later; I let them in, but scolded them and threatened them further.

Although I do it, I find it very difficult telling the students of the traditional class mathematical facts. Often I would rather help them to discover the facts for themselves. I feel I should at least allow discussion amongst them.

July 16, 1971. (Saturday)

My over-all impression in the traditional class is that most of the students are developing a really good attitude. They listen when I talk, listen when a fellow student talks, and wait patiently for someone who is getting special attention from the teacher. Their attitude seems to have changed in a very good way. Some of the students come to talk to me after class and joke around.

They seem to be jovial. They appear to have a feeling of satisfaction. I guess they feel they are learning. I am beginning to feel a lot closer to them. I feel that they are getting to know what I am like, and I think they like me.

July 18,

After I finished teaching, I discussed the two classes (Friday, July 16th's) with an observer who watched both classes. He said:

> The attitude of the students was good in both classes. But in my biased opinion I preferred what went on in the traditional class a little more. I was impressed by how well behaved and how patient the rest of the students were when you were explaining to a single student.

He felt that the scientific heuristic class did a deeper level of math, and if he were to use one of these methods to train a research mathematician, then he would definitely use the scientific heuristic method.

He thought that for the calibre of students one encounters at the summer school the traditional method might be a bit better. He also added in a sincere way that he was not sure about his opinions.

I find it extremely difficult to prepare for the scientific heuristic class, since I find it almost impossible to know how the students will influence the lesson. Furthermore, I am finding that it is very difficult to devise methods to enable the students to discover for themselves. I spend an unbelievable amount of energy trying to devise these ways.

I hope that a scientific heuristic teacher can become more effective in facilitating the students' learning as he gains more experience.

July 23, (Friday)

Today in the scientific heuristic class we re-discussed the home assignment. Some asked me to put my answers on the board for them. I refused, and suggested they try it on their own. They seemed to try, but whether <u>or not</u> they still understand the game we had been playing, most of them did very little. Some still seem to just sit around.

In playing with the attribute blocks, there was a considerable difference between the students in this class and the ones in the pilot project, or students I taught at night school. The summer school students don't appear to show much resourcefulness, nor much enthusiasm. They seem to work with the blocks only if I am with them. With one group I started the one-difference game, but very soon after I left they started what appeared to be useless 'fiddling'. So far, the blocks don't appear to be the success they were with the night school students or the students of the pilot project.

Finally, I was notified by three students that today would be their last day in class. Although it is not uncommon at summer school for students to try and leave the course early, I was very disappointed at their missing a whole week of the course. The thought that they might not like the teaching occurred to me, and this upset me a little.

July 26.

Today the students in the traditional class appeared quite enthusiastic. Many were very eager to go to the board and put up their solutions for the problem of the home assignments. Some did.

A university professor who came to observe both classes claimed that he saw evidence of 'cheating' going on while they were writing a test.

July 27

Even though the students in the traditional class sat in stationary desks in rows and the freedom to interact amongst them is almost negligible, I have managed to keep their interest. Most of them appear to be happy. The lesson still follows the same format!

1. Discuss home assignment
2. Quiz.
3. Mark it.
4. Record it.
5. Discuss it.
6. Introduce new work.
7. Practise with new work at desks.
8. Start home assignments.

But they seem to like it. I get the impression they think they are learning and it is almost as if they are more than willing to put up with some of the unpleasantness of the classroom such as:

1. No talking except with the teacher's permission.
2. Getting tested every day.
3. Having to pay attention to the teacher at all times.
4. No lateness or absenteeism.
5. Teacher checking the homework.

About the only thing that should keep them happy, is the fact that they seem to be acquiring some skills such as:

1. Operating with negative rational numbers.
2. Solving equations.
3. Adding and multiplying of algebraic expressions.

I lecture them on good behaviour quite often. But I also joke with them occasionally.

At this point in the scientific heuristic class, the students are quite free in the room. They can sit on the floor, talk to someone while I (the teacher) am talking. They may join the rest of the group if they wish, work independently or just remain idle.

Rather than lecture to them I try to get them to make their own decisions, correct their own work. Home assignments are occasionally self imposed which means in most cases just investigating further what they were doing in class. I present the class with sheets involving review activities but it is left as the students' responsibility to read these and work out the solutions by themselves, or as a group or in consultation with me. Mathematics is quite often presented as a game. There are many games. To play a game you must establish some rules and then be prepared to accept them and stick with them. So it is with mathematics. When we play a mathematics game we must define our rules carefully, agree upon them, be consistent with them. In this way we succeed in playing the same game as our fellow students if we wish it. I really believe that the effect this course is having on these students is terrific, but I doubt that many of them really appreciate fully what it is doing for them. They seem still to want to be told. They find it uncomfortable to be put in a position where they have to decide for themselves. Also I find that their perseverence is of short time span. It appears to me that once they understand a little they think they know it all and thus they think there is no need to pursue something you already know. But if they don't understand some concept quickly, they dislike to persevere and try to learn about it. I guess the teacher's role is to help the students to discover just enough to keep them guessing, so as to maintain their interest. They show little enthusiasm with many of the ideas that fascinated the students at night school or the ones of the pilot project.

July 28.

So far I can honestly say that I am doing things in both classes which give me satisfaction - and doing things in both classes which I dislike. I think that there are students in the scientific heuristic class which would benefit more by being members of the traditional class and vice-versa.

It almost makes me feel that possibly the best method is that which suits the individual student. At least there have been occasions when I wanted to lecture and be more authoritative (because I thought I knew what was best for them) in the scientific heuristic class. But there were times when I wanted to use the discovery approach in the traditional class so badly that I almost got sick over not doing it.

IV. COMMENTS MADE BY STUDENTS

Traditional Class

1. I really thought this was a worthwhile course. I enjoyed it very much.

2. I like the teacher. He's excellent. He really cares. He drills it into your head until you know it. I'm very glad I came. Next year even if I don't have to come I will. Thank you for everything. He was the best teacher I've ever had.

3. When I found out I failed math I felt real bad about it and was really upset about going to summer school. But now not only am I glad I came, I enjoyed every period. It was just the right length and I think I learned more in Mr. Alvaro's class than in all my years at this school. Not all teachers have the ability to put things across, but Mr. Alvaro did. He taught me all the things that I never could do in grade nine math. Thanks a lot, I enjoyed your class.

4. I liked it very much. It was fun.

5. I thought that the class this year was quite good. I enjoyed the teacher even if I did miss some classes. The progress was good. I enjoyed this year's summer school.

6. This course was well planned. We did not rush on to other things as soon as we finished another one.

7. I thought that this was a waste of time!

8. Much of the work was review for me but I did learn some things. I think we spent too long on doing equations.

9. I have learned more through this summer school class than during the regular course. I really enjoyed math this way. The little tests really help you not forget math. Mr. Alvaro is a better than average teacher and knows what he is doing.

10. Not bad. I learned more in summer school than all year.

11. I really enjoyed this summer course. I feel really as if I have learned something and I understand it well.

12. I think I got a lot out of this math course because everything was reviewed lots of times. I liked this math class!

13. I think that this mathematics course was a good one and I learned something out of it.

14. I thought the course was very good and I benefitted from it very much. It helped me to understand what I didn't at the beginning of grade nine.

15. I learned a lot in this class - I like how Mr. Alvaro doesn't mind if you ask him a lot of questions.

Scientific Heuristic Class

1. I enjoyed this math class very much. I thought the teacher was excellent, and the methods I learned more here than ever before.

2. Great.

3. I don't know if my answers coincide with my remarks but I thought that this class was the neatest class for math I have ever been in; (i) you can disagree when you want to; (ii) ask questions, etc.

4. I think that there should be more methods of teaching math in a free speech environment. And more teachers that are as easy to cope with as Mr. Alvaro.

5. I felt that this kind of a class was a lot of fun. It got a lot more people involved than any of my other math classes in the past. The one thing I thought was missing, was when we were playing the game and someone disagreed, and then after a little while we all came to an agreement, the thing I thought was missing was someone to tell us whether we agreed on the right answer or not. But otherwise I thought that the class was a lot of fun and I do believe I did learn something.

6. I found the classes very interesting. Very different from the way I had been taught. This system taught me more, because we spent more time on the problem.

7. I feel from my own personal involvement in the experiment, that it has been truly successful. I wouldn't have thought it possible to learn so much in math in the short time we've had. Discovering and learning with each other helps create interest, and a desire to learn. A genuine desire, and interest for the students shown by the teacher, creates a good learning situation. Trying out different ways to discover a problem, by using different techniques, has given me an insight into the involvement of mathematics. I have thoroughly enjoyed and benefitted from this course.

CHAPTER V

CONCLUSIONS, DELIMITATIONS AND AREA OF FURTHER INVESTIGATION

I. SUMMARY OF RESULTS

Achievement in Mathematics

The analysis of the data indicated that both the traditional and the scientific heuristic class scored significantly better on the post-test than they did on the pretest relative to mathematical achievement. The traditional class however, out-performed the scientific heuristic class on the post-test.

Two reasons for the traditional class' greater achievement might be:

1. The test yielding the comparison of achievement of the classes was based on a traditional-type test, the implication being that the students in the traditional class would enjoy an advantage over the students in the scientific heuristic class because the test was the type to which the traditional students were accustomed. In fact, when achievement was measured by a 'SH-type' test the scientific heuristic class did significantly better than the traditional class.

2. The students in the traditional class might have been more highly motivated to achieve on the test. Half of their grade was based on

the results of their performance in this test. The students in the scientific heuristic class were aware of the fact that their performance on the test would not affect their passing the course.

Attitude Towards Mathematics

The students in the scientific heuristic class showed significant improvement in their attitude towards mathematics as a result of the session. The students in the traditional class did not show significant improvement.

The pretest mean score for the scientific heuristic class was 161.68, the post-test mean score was 192.00. As a contrast the traditional class pretest mean score was 203.54, the post-test mean score was 199.58.

However, a very interesting fact is that the measure of attitude towards mathematics in the traditional class was greater both before and after the experiment when compared with that of the scientific heuristic class.

Could it be that the increase in attitude for the scientific heuristic class was a 'catch-up' factor? After all, the measure of attitude for both classes at the end of the experiment was not significantly different. Could it be that the improvement in attitude in the scientific heuristic class could have been brought about by the traditional method as well? These questions are possibilities, however, for one must remember that the students in the scientific heuristic class had been exposed to traditional methods of teaching possibly throughout their whole previous mathematical instruction, and they for some reason maintained that particular level of attitude. Nevertheless, it seems that the scientific heuristic method did bring about an increase in the measure of their attitude towards mathematics.

Self Concept

Neither class showed a significant imporvement in the measure of their self concept after the experiment. This is in disagreement with Purkey's hypothesis. However, one very important limiting factor must be noted at this juncture. The amount of teaching time used to bring about a change in the students' self concept was brief. It is one thing to expect change in one's attitude towards mathematics during such a relatively short period of time, but to expect the change in how a person thinks about himself, might be a little presumptuous and unrealistic.

In any event, it should be mentioned that in the comparison of pre-test mean scores and post-test mean scores of self concept for the girls of the traditional class, a t-value of 2.13 was obtained. A t-value of 2.15 would have yielded significance at the .05 level. Furthermore, a post-test comparison of self concept scores for the girls and boys indicated that the girls' self concept increased significantly (.01 level) more than the boys'. Why was this increase in self concept only for the girls of the traditional class? A relatively recent study by

Patricia Sexton[56] suggests that girls in a traditional class achieve more than the males and this fact combined with Purkey's hypothesis that self concept is directly proportional to achievement might explain the girls' increase in self concept.

The statistical results would indicate that the scientific heuristic method of teaching seems to improve students' attitudes towards mathematics. This being the case, could one not make the conjecture that if students' attitudes improve they should be better prepared to achieve at a future date?

The traditional class showed a higher increase in achievement. Would this not imply that an increase in achievement in turn promotes a good attitude?

To answer the last two questions, it is obvious that one would need a teaching period considerably longer than four weeks. However, this may be difficult to do. This experimenter experienced a great deal of anxiety (even with this relatively short span of time - four weeks) in teaching both classes, each by a different method.

---

56 Sexton, P.C., The Feminized Male. Random House, New York, 1969.

II. CONCLUSIOSN AND AREAS OF FURTHER INVESTIGATION

The results of this study did not indicate clear evidence that one method of teaching (scientific heuristic) is superior to the other (traditional).

From a subjective point of view, the experimenter enjoyed feelings of satisfaction, success and failure in both classes. It could be that the best method of teaching is the one which best suits a particular circumstance.

The most rewarding aspect of this experiment on the author's part was what he learned as a result of the new experiences produced by the experiment.

As was indicated in the section entitled 'Summary of Results', many questions remain unanswered.

The most significant factor in obtaining more decisive results with an experiment of this type would appear to be the amount of time devoted to the teaching session. However, as was already mentioned, this experimenter experienced appreciable anxiety in trying to teach by two different methods consecutively. It is difficult to say how successful anyone else attempting a similar study would be.

A future study which could yield more meaningful results would be one that would measure more long range effects that different methods of teaching would have on students' success in mathematics.

BIBLIOGRAPHY

BIBLIOGRAPHY

A. BOOKS

Combs, D.W., Soper, D.C. and Courson, C.C. The Measurement of Self Concept and Self Report. University of Florida, Educational and P.M., 1963.

Dienes, Z.P., and E.W. Golding, Set, Numbers and Powers, The Educational Supply Association Limited, School Materials Division, Pinnacles, Harlow, Essex, 1966.

______, The Power of Mathematics. Hutchison Educational, London, 1964.

Festinger, L., et al. Conflict, Decision and Dissonance. Stanford University Press, Stanford, California, 1964.

Fishbein, Martin. Readings in Attitude Theory and Measurement. John Wiley and Sons, Inc., New York, 1967.

Frederique. Les Enfants et la Mathematique. Marcel Didier, Montreal, 1970.

Gattegno, C. What We Owe Children: The Subordination of Teaching to Learning. Outerbridge and Dienstfrey, New York, 1970.

______, Mathematics with Numbers in Colour: Book IV. Lamport Gilbert and Co. Ltd., Reading, 1963.

______, The Adolescent and His Will. Outerbridge and Dienstfrey, New York, 1971.

Ginsburg, H. and S. Opper. Piaget's Theory of Intellectual Development: An Introduction. Prentice-Hall, Inc., Englewood Cliffs, 1969.

Instructional Objectives Exchange. Measures of Self Concept: Grades K-12. (P.O. Box 24095) Los Angeles, California 90024, 1970.

______, Mathematics 7 - 9. (P.O. Box 24095) Los Angeles, California 90024, 1970.

Krech, D., R.S. Opper and N. Livson. Elements of Psychology. Alfred A. Knopf, New York, 1969, Second Edition.

Neidt, C.O. Changes in Attitudes During Learning. ERIC ED.., 003227, 1964.

Polya, G. Mathematics and Plausible Reasoning. (Two Volumes) Princeton University Press, Princeton, 1954.

Polya, G. Mathematical Discovery. (Two Volumes): On Understanding, Learning and Teaching Problem Solving. John Wiley and Sons, Inc., 1967.

Popham, James W. Educational Statistics: Use and Interpretation. Harper and Row, Publishers, New York, 1967.

Popham, James W. and L. Eva Baker. Systematic Instruction. Prentice-Hall, Inc., Englewood Cliffs, 1970.

Purkey, William W. Self Concept and School Achievement. Prentice-Hall, Inc., Englewood Cliffs, 1970.

Rogers, C. Freedom to Learn. Charles E. Merrill Publishing Co., Columbus, Ohio, 1969.

Sexton, Patricia C. The Feminized Male. Random House, New York, 1969.

Trivett, J.V. Games for Children to Play in Learning Mathematics. Cuisenaire Company of America, New York, in press, 1973.

Wylie, R.C. The Self Concept: A Critical Survey of Pertinent Research Literature. University of Nebraska Press, Lincoln, 1961.

B. ARTICLES AND JOURNALS

Aiken, Lewis R. Jr. "Attitude Towards Mathematics", Review of Educational Research. Vol. 40, No. 4, 1970.

Davis, Robert B. "Mathematics Teaching with Special Reference to Epistemological Problems", Journal of Research and Development in Education. Monograph No. 1, Athens, Fall, 1967.

Duckworth, E. "Piaget Rediscovered", in Piaget Rediscovered. R.E. Ripple and V.N. Rockcastle, editors, Cornell University, 1964.

Halmos, P.R. "Mathematics as a Creative Art", American Scientist. 56, 4, pp. 375-389, 1968.

Jordain, P. "The Nature of Mathematics", in The World of Mathematics, J.R. Newman, editor, Simon and Schuster, New York, 1950, Vol. 1.

Trivett, John V. <u>The Official Journal of the Provincial Intermediate Teachers' Association</u>. Vol. 12, No. 1, March, 1972.

C. UNPUBLISHED SOURCES

Dawson, A.J. "A Model of Mathematics Instruction". Doctoral Dissertation, University of Alberta, 1969.

Montgomery, C.W. "Empirical Study of the Constructive Predictive Validity of 5 Types of Instruments to Measure Students' Attitudes Towards Mathematics", Doctoral Dissertation in progress, University of British Columbia, 1973.

Osborn, K.H. "A Longitudinal Study of Achievement In and Toward Mathematics of Selected Students Using School Mathematics Study Group Materials". Doctoral Dissertation, University of California, Berkeley, 1965.

Phelps, J. "A Study Comparing Attitudes Towards Mathematics of SMSG and Traditional Elementary School Students". Doctoral Dissertation, Oklahoma State University, 1963.

APPENDICES

APPENDIX A - INCLUDES TESTS, QUESTIONNAIRES, MATHEMATICAL GAME ('FIDDLE') AND TABLES XIX - XXXIII.

APPENDIX B - COMMENTS MADE BY PEOPLE WHO WATCHED THE TWO CLASSES WHILE IN SESSION AND ITEMS OF CORRESPONDENCE.

APPENDIX A

MATH 9 - SH-TEST*

1. Give three equivalent fractions to:

   a) $\frac{7}{11}$ __________________

   b) $\frac{A}{N^2}$ __________________

2. If you can discover how to play the following game, supply three examples of your own:

$$(6,25) \; \S \; (2,5) \longrightarrow (3,5)$$
$$(18,16) \; \S \; (9,8) \longrightarrow (2,2)$$
$$(17,15) \; \S \; (4,3) \longrightarrow (\tfrac{17}{4},5)$$
$$(R, \triangle) \; \S \; (P,X) \longrightarrow (\tfrac{R}{P}, \tfrac{\triangle}{X})$$

____

____

____

3. Express as a single rational number or as an integer:

   a) $(-6) - (-3)$ __________________

   b) $2(-3) - (-7)$ __________________

   c) $\frac{6 - (-8)}{5 - 7}$ __________________

   d) $\frac{(-4)(-32)(-6)}{(-8)(-12)}$ __________________

4. Consider: $3\,(a \cdot b)$

   Which of the following expressions do you think are equivalent to it?

   a) $3a \cdot 3b$ b) $9ab$ c) $3ab$ d) $a(3b)$ e) $(3a)b$

---

* Only the first of ten questions of the test were given to the trad-class. This precaution was taken so as not to subject the trad-class to any mathematics which they had not been exposed to.

5. Supply two examples which follow the same pattern as:

$$2N + 3N = (2 + 3)N$$

$$aN + 3N + 5N = (a + 3 + 5)N$$

$$KN + 6N + 3N + eN = (K + 6 + 3 + e)N$$

____

____

6. Express as a binomial:

a) $-3N - (-10) - 5N$ ____________________

b) $-(N-12) - 2(3N-8)$ ____________________

7. According to <u>Antonino</u> all the following numbers except one of them follows a certain pattern. If you think you understand Antonino's game write the exception.

$\frac{13}{12}, \frac{14}{11}, \frac{15}{10}, \frac{17}{8}, \frac{18}{7}, \ldots. \frac{21}{5}, \ldots \frac{23}{2}$ __________

8. Express as a single rational expression:

a) $\frac{3}{8N} - \frac{1}{2N}$ ____________________

b) $\frac{A}{N^3R} - \frac{2B}{NR^2}$ ____________________

c) $\frac{a}{98N^5} + \frac{3}{56N}$ ____________________

9. Can you supply the missing number?

$$10^4 \longrightarrow 10{,}000$$

$$10^3 \longrightarrow 1{,}000$$

$$10^2 \longrightarrow 100$$

$$10^1 \longrightarrow 10$$

$$10^0 \longrightarrow \boxed{}$$

$$10^{-1} \longrightarrow \frac{1}{10}$$

$$10^{-2} \longrightarrow \frac{1}{100}$$

10. If you can discover a pattern in the following, supply an example of your own:

$$\frac{2}{3} + \frac{5}{7} = \frac{2\cdot 7 + 5\cdot 3}{3\cdot 7}$$

$$\frac{2}{3} + \frac{5}{7} + \frac{11}{13} = \frac{2\cdot 7\cdot 13 + 5\cdot 3\cdot 13 + 11\cdot 3\cdot 7}{3\cdot 7\cdot 13}$$

11. Using the blocks contained in a single bag:
    a) Describe what you think would comprise the intersection of: Red Blocks, Thin Blocks, and Small Blocks.
    b) How many blocks do you think will comprise the union of yellow blocks with small blocks?

12. Make up a mathematical game, and let your teacher discover how to play it.

STUDENT ATTITUDE TOWARDS MATHEMATICS

SIMON FRASER UNIVERSITY PROFESSIONAL DEVELOPMENT CENTRE

This is part of a study in methods of teaching mathematics at the high school level. The results will be used in a research project at Simon Fraser University. The results will not be made known to anyone but the research team. You will be asked to write a PSEUDONYM on this form. A pseudonym is a fictitious name; one which you will choose for yourself. It is of the utmost importance that you will use this same name on any other form, when asked to do so. The only reason you are asked to write a PSEUDONYM on these forms is so that the research team can match and compare the results of this form with the results of other forms which you will be asked to complete. They are NOT interested in identifying the writer.

PSEUDONYM	AGE	SEX

INSTRUCTIONS:

The statements in this form describe a person's feelings or thoughts about mathematics. You are asked to indicate how each of the statements describes your feelings or thoughts about mathematics. You will indicate your answer by marking an X over one of the numbers shown on the scale for each item. Only one answer is allowed for each item, and you should choose one answer for each item.

Do not stop long to think about any one statement, because we are interested in your first impression. There are no "Right" or "Wrong" answers or choices. The answer you mark should describe JUST HOW YOU FEEL OR THINK ABOUT MATHEMATICS.

SCALE KEY:

Strongly Agree 7
Mostly Agree 6
Agree 5
Neutral 4
Disagree 3
Mostly Disagree 2
Strongly Disagree 1

STUDENT ATTITUDES TOWARDS MATHEMATICS - TEST I

Strongly Disagree	1	2	3	4	5	6	7	Strongly Agree	1.	I forget mathematics easily.
Strongly Disagree	1	2	3	4	5	6	7	Strongly Agree	2.	I am glad on the days we do not have mathematics.
Strongly Disagree	1	2	3	4	5	6	7	Strongly Agree	3.	I find my mathematics book too hard.
Strongly Disagree	1	2	3	4	5	6	7	Strongly Agree	4.	I remember most of the things I learned in mathematics.
Strongly Disagree	1	2	3	4	5	6	7	Strongly Agree	5.	I do not use mathematics during the summer vacation.
Strongly Disagree	1	2	3	4	5	6	7	Strongly Agree	6.	I do not enjoy discussing mathematics with my friends.
Strongly Disagree	1	2	3	4	5	6	7	Strongly Agree	7.	Mathematics tests are easy for me.
Strongly Disagree	1	2	3	4	5	6	7	Strongly Agree	8.	I have always liked mathematics.
Strongly Disagree	1	2	3	4	5	6	7	Strongly Agree	9.	My mathematics book is interesting.
Strongly Disagree	1	2	3	4	5	6	7	Strongly Agree	10.	I do not like being asked questions in mathematics.
Strongly Disagree	1	2	3	4	5	6	7	Strongly Agree	11.	I do not get good marks in mathematics but I do not worry about it.
Strongly Disagree	1	2	3	4	5	6	7	Strongly Agree	12.	I worry about my marks in mathematics.
Strongly Disagree	1	2	3	4	5	6	7	Strongly Agree	13.	I would like to read other books about mathematics besides the one we use in class.
Strongly Disagree	1	2	3	4	5	6	7	Strongly Agree	14.	I like my mathematics book better than most school books.
Strongly Disagree	1	2	3	4	5	6	7	Strongly Agree	15.	I wish mathematics classes were shorter.

Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	16.	I like to miss mathematics classes.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	17.	I like mathematics the best of all my school subjects.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	18.	I dislike taking tests in mathematics.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	19.	I think mathematics classes are enjoyable.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	20.	I do not enjoy studying from my mathematics book.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	21.	I often forget how to do one kind of mathematics problem after I have worked on other kinds.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	22.	It is not easy for me to begin doing my mathematics homework.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	23.	I often think "I can't do it" when a work problem seems hard.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	24.	I can usually finish my mathematics homework in class.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	25.	Mathematics is easy for me.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	26.	I do not remember much mathematics over the summer vacation.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	27.	My friends like mathematics.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	28.	I would rather get high marks in mathematics than in other subjects.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	29.	I like to answer questions in mathematics class.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	30.	Mathematics is uninteresting for me.

Strongly Disagree 1 2 3 4 5 6 7 Strongly Agree	31.	If I do not get a mathematics problem right away, I like to keep working to find the answer.
Strongly Disagree 1 2 3 4 5 6 7 Strongly Agree	32.	I feel sure of myself in mathematics.
Strongly Disagree 1 2 3 4 5 6 7 Strongly Agree	33.	I often wish I had fewer mathematics problems to do.
Strongly Disagree 1 2 3 4 5 6 7 Strongly Agree	34.	I prefer sets of mathematics problems that are all alike rather than sets having different kinds mixed.
Strongly Disagree 1 2 3 4 5 6 7 Strongly Agree	35.	I wish that mathematics tests were easier.
Strongly Disagree 1 2 3 4 5 6 7 Strongly Agree	36.	Mathematics is hard for me.
Strongly Disagree 1 2 3 4 5 6 7 Strongly Agree	37.	I am glad when it is time for mathematics class.
Strongly Disagree 1 2 3 4 5 6 7 Strongly Agree	38.	I enjoy working at hard problems in mathematics.
Strongly Disagree 1 2 3 4 5 6 7 Strongly Agree	39.	Mathematics problems take too much time.
Strongly Disagree 1 2 3 4 5 6 7 Strongly Agree	40.	Most of my friends think mathematics is dull.
Strongly Disagree 1 2 3 4 5 6 7 Strongly Agree	41.	Mathematics is one of my favourite subjects.
Strongly Disagree 1 2 3 4 5 6 7 Strongly Agree	42.	I would rather do mathematics than read books.
Strongly Disagree 1 2 3 4 5 6 7 Strongly Agree	43.	I enjoy talking to my teacher about mathematics.
Strongly Disagree 1 2 3 4 5 6 7 Strongly Agree	44.	I like to do word problems in mathematics.
Strongly Disagree 1 2 3 4 5 6 7 Strongly Agree	45.	I would take mathematics next year even if I did not have to.

Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	46.	I like the easy problems in mathematics.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	47.	I like to do extra work in mathematics when I have time.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	48.	I like to work mathematics problems with my friends.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	49.	I would rather be an author than a scientist.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	50.	I would like a job which used some mathematics.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	51.	I like taking tests in mathematics.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	52.	I would like a job which never used any mathematics.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	53.	I would like a job which used a great deal of mathematics.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	54.	I do not enjoy talking to my parents about mathematics.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	55.	Mathematics homework often takes more time than my other school subjects.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	56.	I hate mathematics.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	57.	Mathematics is not my best subject.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	58.	I would rather write a story than work mathematics problems.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	59.	I hate to start doing my mathematics homework.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	60.	I do not have to spend much time on mathematics to keep up.

STUDENT ATTITUDE TOWARDS SELF

SIMON FRASER UNIVERSITY PROFESSIONAL DEVELOPMENT CENTRE

This is part of a study in methods of teaching mathematics at the high school level. The results will be used in a research project at Simon Fraser University. The results will not be made known to anyone but the research team. You will be asked to write a PSEUDONYM on this form. A pseudonym is a fictitious name; one you will choose for yourself. It is of the utmost importance that you will use this name on any other form, when asked to do so. The only reason you are asked to write a PSEUDONYM on these forms is so that the research team can match and compare the results of this form with the results of other forms which you will be asked to complete. They are NOT interested in identifying the writer.

PSEUDONYM	AGE	SEX

INSTRUCTIONS:

The statements in this form describe a person's feelings or thoughts. You are asked to indicate how each of the statements describe your feelings or thoughts about yourself. You will indicate your answer by marking an X over one of the numbers shown on the scale for each item. Only one answer is allowed for each item, and you should choose an answer for each item.

Do not stop long to think about any one statement, because we are interested in your first impression. There are no "Right" or "Wrong" answers or choices. The answer you mark should describe JUST HOW YOU FEEL OR THINK ABOUT YOURSELF.

SCALE KEY:

Strongly Agree	7
Mostly Agree	6
Agree	5
Neutral	4
Disagree	3
Mostly Disagree	2
Strongly Disagree	1

STUDENTS ATTITUDES TOWARDS SELF

Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	1.	I like to meet new people.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	2.	I can disagree with my family.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	3.	Schoolwork is fairly easy for me.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	4.	I am satisfied to be just what I am.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	5.	I ought to get along better with people.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	6.	My family thinks I don't act as I should.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	7.	I usually like my teachers.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	8.	I am a cheerful person.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	9.	People often pick on me.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	10.	I do my share of work at home.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	11.	I often feel upset in school.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	12.	I often let other people have their way.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	13.	Most people have fewer friends than I do.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	14.	No one pays much attention to me at home.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	15.	I can get good grades if I want to.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	16.	I can be trusted.

Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	17.	I am easy to like.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	18.	There are times when I would like to leave home.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	19.	I forget most of what I learn.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	20.	I am popular with kids my own age.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	21.	I am popular with girls.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	22.	My family is glad when I do things with them.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	23.	I often volunteer in school.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	24.	I am a happy person.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	25.	I am lonely very often.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	26.	My family respect my ideas.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	27.	I am a good student.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	28.	I often do things that I'm sorry for later.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	29.	Older kids do not like me.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	30.	I behave badly at home.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	31.	I often get discouraged in school.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	32.	I wish I were younger.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	33.	I am always friendly towards other people.

Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	34.	I usually treat my family as well as I should.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	35.	My teacher makes me feel I am not good enough.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	36.	I always like being the way I am.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	37.	Most people are much better liked than I am.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	38.	I cause trouble to my family.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	39.	I am slow in finishing my school work.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	40.	I am often unhappy.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	41.	I am popular with boys.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	42.	I know what is expected of me at home.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	43.	I can give a good report in front of the class.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	44.	I am not as nice looking as most people.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	45.	I don't have many friends.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	46.	I sometimes argue with my family.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	47.	I am proud of my school work.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	48.	If I have something to say I usually say it.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	49.	I am among the last to be chosen for teams.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	50.	I feel that my family always trusts me.

Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	51.	I am a good reader.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	52.	I don't worry much.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	53.	It is hard for me to make friends.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	54.	My family would help me in any kind of trouble.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	55.	I am not doing as well in school as I would like to.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	56.	I have a lot of self control.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	57.	Friends usually follow my ideas.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	58.	My family understands me.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	59.	I find it hard to talk in front of the class.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	60.	I often feel ashamed of myself.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	61.	I wish I had more close friends.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	62.	My family often expects too much of me.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	63.	I am good in my schoolwork.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	64.	I am a good person.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	65.	Sometimes I am hard to be friendly with.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	66.	I get upset easily at home.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	67.	I like to be called on in class.

Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	68.	I wish I were a different person.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	69.	I am fun to be with.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	70.	I am an important person in my family.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	71.	My classmates think I am a good student.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	72.	I am sure of myself.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	73.	Often I don't like to be with other children.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	74.	My family and I have a lot of fun together.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	75.	I would like to drop out of school.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	76.	I can always take care of myself.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	77.	I would rather be with kids younger than me.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	78.	My family usually considers my feelings.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	79.	I can disagree with my teacher.
Strongly Disagree	1 2 3 4 5 6 7	Strongly Agree	80.	I can't be depended on.

SUMMER SCHOOL

MATHEMATICS NINE

<u>These questions are NOT for the purpose of assigning grades</u>

They will be used to discover the concepts you understand, so that we will not dwell on what you already know. This will give us time to concentrate on those areas with which you may have had some problems.

Name ______________________ Date ______________________

Teacher ______________________ Time of Class ______________

INSTRUCTIONS:

1. For each of the following questions there is exactly ONE correct answer; circle the letter of the answer you consider to be correct.
2. Do <u>NOT</u> spend too much time with any particular question. Do all the ones you think you can, then come back to the others.
3. Do all your rough work on the question sheets.

SUMMER SCHOOL

MATHEMATICS NINE

The purpose of these questions is to offer the research team another means of finding out what learning has taken place during these classes. The RESULTS WILL NOT BE USED FOR GRADING PURPOSES.

Name ____________________ Date ____________________

Time of Class ____________

INSTRUCTIONS:

1. For each of the following questions circle the letter which you consider corresponds to the most appropriate answer.
2. Do NOT spend too much time with any particular question. Do all the ones you think you can, then come back to the others.
3. Do all your rough work on the question sheets.

SUMMER SCHOOL

MATHEMATICS NINE

FINAL TEST

Name ______________________ Date ______________________

Teacher ______________________ Time of Class ______________

INSTRUCTIONS:

1. For each of the following questions circle the letter corresponding to the correct answer.
2. Do NOT spend too much time with any particular question. Do all the ones you think you can, then come back to the others.
3. Do all your rough work on the question sheets.

ACHIEVEMENT TEST

1. Which integer is equivalent to:

$$\frac{(-8)\ (-16)}{(-4)}$$

a) -8 d) -32

b) 32 e) None of these

c) 8

2. On the number line how many integers are there between:

$$-2\frac{2}{3} \text{ and } 5\frac{1}{4}$$

a) 7 d) 5

b) 6 e) None of these

c) 8

3. $3 \times 4^2 =$ __________ ?

a) 144 d) 49

b) 48 e) 72

c) 24

4. An expression equivalent to (-2) - (-3) is?

a) -6 d) 6

b) 1 e) -1

c) -5

5. If $x + y$ is equal to $x + z$, then:

a) $x = y$ d) $x = z$

b) $y = 2z$ e) z is larger than y

c) $y = z$

6. $\frac{18 - (-3)}{7 - 9}$ is equivalent to:

a) $\frac{15}{-2}$ b) $\frac{-15}{16}$ c) $-10\frac{1}{2}$

d) $\frac{21}{-16}$ e) $-1\frac{5}{16}$

7. What is the result if both numerator and denominator of $\frac{a + b}{a - b}$ $(a \neq b)$ are multiplied by $(a + b)$?

a) $\frac{2(a+b)}{a^2 - b^2}$ b) $\frac{(a + b)^2}{(a - b)^2}$

b) $\frac{a^2 + b^2}{a^2 - b^2}$ e) $\frac{a^2 - b^2}{a^2 - b^2}$

c) $\frac{(a + b)^2}{a^2 - b^2}$

8. $6(7 - x) - 2(x - 8)$ is equivalent to:

a) $58 - 8x$ d) $34 - 8x$

b) $58 - 4x$ e) $50 - 3x$

c) $34 + 1x$

9. Solve within the set of real numbers:

$$2x - 10 = 6 - 10x$$

a) $\left\{\frac{3}{4}\right\}$ d) $\left\{\frac{4}{3}\right\}$

b) $\left\{\frac{1}{2}\right\}$ e) $\{-2\}$

c) $\left\{\frac{-1}{2}\right\}$

10. The largest number which will divide evenly into <u>both</u> 24 and 18 is:

a) 72 d) 6

b) 18 e) None of these

c) 24

11. Which of the following is the Greatest Rational Number?

$$\frac{10}{16}, \frac{11}{17}, \frac{12}{18}, \frac{13}{19}$$

a) $\frac{11}{17}$

b) $\frac{12}{18}$

c) $\frac{13}{19}$

d) $\frac{10}{16}$

e) They are all equivalent since the numerator and denominator are both changing by one, in each case.

12. If $\sqrt{2}$ can be approximated by 1.414, then $\sqrt{18}$ can be approximated by:

a) 9

b) 12.726

c) 6

d) 4.242

e) None of these.

13. Give the additive inverse of: $x - y$

a) $\frac{1}{x-y}$

b) $\frac{1}{y-x}$

c) $y - x$

d) $(-1)\ (y - x)$

e) None of these.

14. If $2x^2 + bx - 6 = 0$ when $x = 3$, what is b?

a) -8

b) -4

c) 4

d) 8

e) 12

15. An expression equivalent to $(r^2\ s^3)^3$ is:

a) $r^2\ s^6$

b) $r^5 s^6$

c) $r^6 s^9$

d) rs^{15}

e) $(r^2 s^2)^3$

16. $\left(\frac{2}{5}\right)^2 \times \left(\frac{5}{3}\right)^3$ can be expressed as:

a) $\frac{20}{27}$ d) $\left(\frac{2}{3}\right)^6$

b) $\frac{4}{27}$ e) None of these.

c) $\left(\frac{2}{3}\right)^5$

17. Which of the following equations is equivalent to $2x - y = 3$?

a) $2x + 3 = y$ d) $4x - 2y = 3$

b) $y + 3 = 2x$ e) None of these.

c) $x = \frac{3 - y}{2}$

18. $7 = a^2$

a) 'a' is an odd real number.

b) 'a' is even real number.

c) 'a' is a decimal.

d) 'a' is a positive real number.

e) None of these.

19. Which of the following (is/are) not (a) prime (s):

1, 7, 13, 29, 49.

a) 29 d) 1, 49

b) 49 e) They are all primes.

c) 1, 29, 49.

20. If $3x + 4 = 11$ and $x = 6$, then $4x + 4 = 17$.

a) Commutative principle of addition.

b) Associative principle of addition.

c) Distributive principle.

d) Equation principle of addition.

e) Commutative principle of multiplication.

21. If 'A' denotes John's present age in years, what algebraic expression in 'A' will denote three times his age in two years?

a) $3A + 2$ d) $3(A + 2)$

b) $3A - 2$ e) $3(A - 2)$

c) $3A - 2$

22. An expression equivalent to $\frac{-15r^6s^8}{3r^3s^4}$ $(r \neq 0,\ s \neq 0)$ is:

a) $5r^3s^4$ d) $5r^{-3}s^{-4}$

b) $-5r^2s^2$ e) $-5r^9s^{12}$

c) $-5r^3s^4$

23. An equivalent expression for $2(a + b) - (2a - b)$ is:

a) $-a$ d) $-3b$

b) 0 e) $-a + b$

c) $3b$

24. Solve within the set of real numbers: $\frac{3}{4}x = -5 + 2x$

a) -6 d) 4

b) 6 e) -20

c) -4

25. The sum of $\frac{5}{77}$ and $\frac{3}{98}$ can be expressed as:

a) $\frac{7 + 3(11)}{7 \cdot 2 \cdot 11}$ d) $\frac{14 + 33}{7^2 \cdot 11 \cdot 2}$

b) $\frac{1 + 3}{7 \cdot 2 \cdot 11}$ e) None of these.

c) $\frac{14 + 3(11)}{7^2 \cdot 11 \cdot 2}$

26. What is the solution set of the linear equations:

$$x + y = 2,\ x - y = 2?$$

a) (1,1) b) (2,2) c) (2,0)

d) (1,2) e) (0,2)

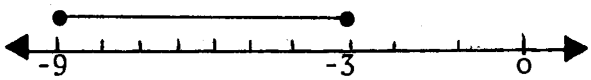
27. An equivalent expression for $\frac{5}{ab^2} + \frac{x}{ba^2}$ is:

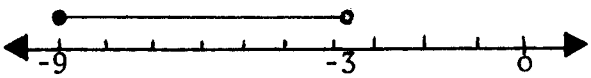
a) $\frac{5 + x}{a^2b^2}$ d) $5ba^2 + xab^2$

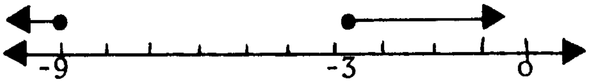
b) $\frac{5a + bx}{a^2b^2}$ e) None of these.

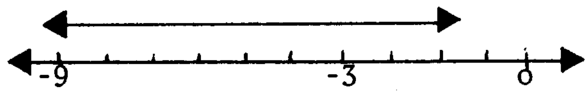
c) $\frac{5 + x}{ab}$

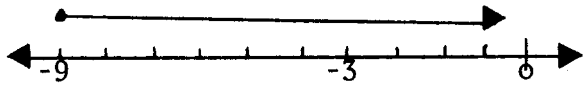
28. The graph of the solution set of: $\{x: x \geq -9\} \cap \{x: x < -3\}$

a) 

b) 

c) 

d) 

e) 

29. Represent the following statement algebraically:

"3 is subtracted from 4 times a number and the result is divided by 7."

a) $\frac{4N - 3}{7}$ d) $4N - \frac{12}{7}$

b) $\frac{4(N-3)}{7}$ e) None of these.

c) $\frac{4N}{7} - 3$

30. A single rational expression for $\frac{a}{b} - \frac{b}{c}$ is?

a) $\frac{a}{c}$ b) $\frac{-a}{c}$ c) $ac - b^2$

d) $\frac{ac - b^2}{bc}$ e) $a - b$

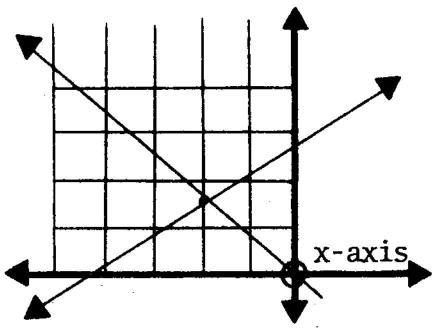
31. A number is multiplied by 5 and 4 is added to their product. The result is 32. The original number is:

a) $5\frac{3}{5}$ d) $6\frac{3}{4}$

b) $7\frac{1}{5}$ e) Problem cannot be solved without more information.

c) $2\frac{2}{5}$

32.



What is the solution set of the system of equations graphed on the left:

a) (-2, -1) b) (4,3)

c) All number pairs on both lines.

d) $\emptyset$ e) (-4,3)

33. Solve within the set of real numbers: $\frac{2h - 3}{4} - 1 = \frac{h - 5}{3}$

a) $\left\{\frac{1}{2}\right\}$ d) $\emptyset$

b) $\{-6\}$ e) None of these.

c) $\left\{\frac{13}{2}\right\}$

QUESTIONNAIRE (FINAL)

VIDEO-TAPE NO: __________

Name: ______________________________ Date: ________________________

DIRECTIONS:

1. Indicate your response by circling either YES or NO or ?.

2. The ? indicates that the tape perhaps does not show significant evidence to warrant a conclusive response such as YES or NO.

3. The YES or NO should be based on an evident trend rather than a single occurrence!

1. Does the teacher <u>appear eager</u> to <u>correct</u> students?

YES NO ?

2. Does the seating arrangement of the students suggest an atmosphere of orderliness?

YES NO ?

3. Does the teacher appear to insist or expect that all the students try the exercise(s) he suggests?

YES NO ?

4. When the teacher asks the students questions does it seem that his intent is to create inquiry within the students?

YES NO ?

5. Is the students' participation <u>MAINLY</u>: Asking the teacher questions, answering the teacher's questions, paying attention to the teacher or being involved with seat work?

YES NO ?

6. Is mathematics presented as a game in which the discovery of the rules is one of the main purposes?

YES NO ?

7. Does the teacher appear to be willing to discuss the assignment at a future date (the following day)?

YES NO ?

8. Would you say that the teacher relies on relatively conventional teaching materials such as text book, work sheets, explanation and exercises?

YES NO ?

9. Does the teacher appear anxious to keep all his students busy?

YES NO ?

10. Does it appear that the teacher follows a method of demonstrating, in talking, or on the board, then asking the class to try similar questions?

YES NO ?

11. Do the students appear free to move their seats so as to allow flexible groupings?

YES NO ?

12. Would you say that the teacher appears to the students as playing a role of <u>judge</u>, of what is <u>right</u> and <u>wrong</u>?

YES NO ?

13. Does the teacher's behaviour suggest an 'eagerness' to TELL the student(s) the CORRECT method to solve a problem or the CORRECT answer to a particular question?

YES NO ?

14. Does the teacher appear to provide opportunity so that any student may voice an alternate method to a problem?

YES NO ?

15. Does the teacher's method suggest that all the students be engaged in the same activity at the same time?

YES NO ?

16. Does the teacher appear to expect that his students ask his permission to speak, for example, raising of hands?

YES NO ?

17. Does it appear that the students regard the teacher as the authority who will TELL them about mathematics?

YES NO ?

18. Do the students appear free to discuss or argue with one another?

YES NO ?

19. Does the teacher appear to be playing the role of a guide towards the students' search for an answer?

YES NO ?

20. Do the home assignments demand a common set of 'answers' from all students?

YES NO ?

21. Does the teacher appear to stress the importance of memorizing the rules of mathematics?

YES NO ?

22. Does the teacher show that he is interested in helping the students arrive at an agreement of certain rules governing patterns in mathematics?

YES NO ?

23. Does it appear that materials are used by the students for the discovery of mathematical concepts?

YES NO ?

24. Do the students appear free to move about the room?

YES NO ?

25. Does it appear that one of the teacher's main interests is to present mathematical facts for mastery?

YES NO ?

26. Does the teacher appear to promote within the student the confidence for him to express his answer without fearing nasty repercussions by the rest of the class? (This includes the teacher.)

YES NO ?

27. Do the students engage in activities which precipitate discussion amongst them?

YES NO ?

28. Would you say that part of the teacher's plan is to give the students an opportunity to try exercises at their desks with the main purpose of mastering mathematical facts?

YES NO ?

29. Does the teacher appear to fulfill his teaching role mostly from a designated area of the classroom?

YES NO ?

30. Does the teacher create situations which allow the students to discover certain patterns in mathematics?

YES NO ?

31. Do the students appear comfortable?

YES NO ?

32. Does the teacher appear willing to be side-tracked from what he appears to want to teach?

YES NO ?

33. Does it appear that the teacher believes in using quizzes for the purpose of grading students?

YES NO ?

34. Do the students appear free to consult with one another even though the teacher is engaged in discussion with other students at the same time?

YES NO ?

35. Does it appear that mathematical facts are actually in the process of being developed by the class?

YES NO ?

36. Does the teacher's method of teaching appear to have an intent to have all the class's attention when he is talking:

YES NO ?

37. In your opinion does the teacher appear to act in an authoritarian manner, on what is being taught?

YES NO ?

38. Does it appear that students are tested on what is being taught <u>MAINLY</u> by teacher-prepared tests?

YES NO ?

39. Do the students appear hesitant to question or disagree with each other, the rest of the class, or the teacher?

YES NO ?

40. When the teacher asks a question does it seem that he anticipates rewarding the students' response with an indication implying 'correct!' or 'wrong'?

YES NO ?

41. Do the students challenge each other's answers and/or methods?

YES NO ?

42. Does the teacher appear prepared to capitalize and exploit upon student discussions?

YES NO ?

43. Does the teacher TELL the students how problems are solved, rather than LEAD THEM TO THEIR OWN SOLUTIONS?

YES NO ?

44. Does it appear to be an intent of the teacher to promote discussion among students?

YES NO ?

45. Does the teacher appear willing to help all his students?

YES NO ?

46. Does it appear that the teacher is willing to discuss the solution of a particular problem even to a single student?

YES NO ?

47. Does the teacher TELL his student(s) about mathematical facts?

YES NO ?

48. Does the teacher appear to exploit the opportunity to explore a different method of solving a problem suggested perhaps by the students?

YES NO ?

49. Does the students appear to exhibit a discernible eagerness to seek their own answer(s)?

YES NO ?

50. Does the teacher's method of teaching indicate an intent to have the students UNDERSTAND the mathematical concepts presented, rather than the memorization of rules?

YES NO ?

MATH 9 S.S.

(S.H.)

Below we have a game called 'FIDDLE'!

Can you discover how to play it?

1. $(3,5)\ \S\ (2,5) \longrightarrow (1,5)$
2. $(6,13)\ \S\ (5,13) \longrightarrow (1,13)$
3. $(7,11)\ \S\ (3,11) \longrightarrow (4,11)$
4. $(15,4)\ \S\ (17,4) \longrightarrow (-2,4)$
5. $(-16,101)\ \S\ (18,101) \longrightarrow (-34,101)$
6. $(-45,37)\ \S\ (40,37) \longrightarrow (-85,37)$
7. $(-17,53)\ \S\ (-3,53) \longrightarrow (-14,53)$
8. $(16,35)\ \S\ (a,35) \longrightarrow (16-a,35)$
9. $(N,11)\ \S\ (Y,11) \longrightarrow (N-Y,11)$
10. $(a,b)\ \S\ (c,b) \longrightarrow (a-c,b)$
11. $(\square ,X)\ \S\ (\triangle ,X) \longrightarrow (\square - \triangle ,X)$

MATH 9 S.S.

(S.H.)

Below we have a game called 'FIDDLE'!

Can you discover how to play it?

1. $(3,5) \wr (2,5) \longrightarrow (5,5)$
2. $(6,13) \wr (5,13) \longrightarrow (11,13)$
3. $(7,11) \wr (3,11) \longrightarrow (10,11)$
4. $(15,4) \wr (17,4) \longrightarrow (32,4)$
5. $(-16,101) \wr (18,101) \longrightarrow (2,101)$
6. $(-45,37) \wr (40,37) \longrightarrow (-5,37)$
7. $(-17,53) \wr (-3,53) \longrightarrow (-20,53)$
8. $(16,35) \wr (a,35) \longrightarrow (16+a,35)$
9. $(N,11) \wr (Y,11) \longrightarrow (N+Y,11)$
10. $(a,b) \wr (c,b) \longrightarrow (a+c,6)$
11. $(\square ,X) \wr (\triangle ,X) \longrightarrow (\square + \triangle ,X)$

MATH 9 S.S.

(S.H.)

Below we have a game called 'FIDDLE'!

Can you discover how to play it?

1.	(3,5)	$\wr$	(2,5)	$\longrightarrow$	(6,25)
2.	(6,13)	$\wr$	(5,13)	$\longrightarrow$	(30,169)
3.	(7,11)	$\wr$	(3,11)	$\longrightarrow$	(21,121)
4.	(15,4)	$\wr$	(17,4)	$\longrightarrow$	(15X17,16)
5.	(-16,101)	$\wr$	(18,101)	$\longrightarrow$	$(-288,101^2)$
6.	(-45,37)	$\wr$	(40,37)	$\longrightarrow$	$(-45\text{x}40,37^2)$
7.	(-17,53)	$\wr$	(-3,53)	$\longrightarrow$	$(51,53^2)$
8.	(16,35)	$\wr$	(a,35)	$\longrightarrow$	$(16a,35^2)$
9.	(N,11)	$\wr$	(Y,11)	$\longrightarrow$	$(NY,11^2)$
10.	(a,b)	$\wr$	(c,b)	$\longrightarrow$	(ac,b^2)
11.	$(\square,X)$	$\wr$	$(\triangle,X)$	$\longrightarrow$	$(\square\triangle,X^2)$

MATH 9 S.S.

(S.H.)

Below we have a game called 'FIDDLE'!

Can you discover how to play it?

1. $(3,5) \wr (2,5) \longrightarrow (5,10)$
2. $(6,13) \wr (5,13) \longrightarrow (11,26)$
3. $(7,11) \wr (3,11) \longrightarrow (10,22)$
4. $(15,4) \wr (17,4) \longrightarrow (32,8)$
5. $(-16,101) \wr (18,101) \longrightarrow (2,202)$
6. $(-45,37) \wr (40,37) \longrightarrow (-5,74)$
7. $(-17,53) \wr (-3,53) \longrightarrow (-20,106)$
8. $(16,35) \wr (a,35) \longrightarrow (16+a,70)$
9. $(N,11) \wr (Y,11) \longrightarrow (N+Y,22)$
10. $(a,b) \wr (c,b) \longrightarrow (a+c,2b)$
11. $(\square, X) \wr (\triangle, X) \longrightarrow (\square + \triangle, 2X)$

TABLE XIX

COMPARISON OF PRETEST SCORES FOR BOYS
OF SH-CLASS AND TRAD-CLASS

CRITERIA	MEAN SH-CLASS	MEAN TRAD-CLASS	S.D. SH-CLASS	S.D. TRAD-CLASS	D.F.	t
ACHIEVEMENT	8.63	8.44	3.71	2.81	30	0.16
ATTITUDE	165.63	204.69	41.03	41.38	30	-2.60[a]
SELF CONCEPT	347.63	374.50	49.66	44.84	30	-1.56*

a significant at .05 level.

* Note - t-value for significance at .05 is 2.04.

N, both classes = 16.

TABLE XX

COMPARISON OF POST-TEST SCORES FOR BOYS
OF SH-CLASS AND TRAD-CLASS

CRITERIA	MEAN SH-CLASS	MEAN TRAD-CLASS	S.D. SH-CLASS	S.D. TRAD-CLASS	D.F.	t
ACHIEVEMENT	11.75	16.31	3.47	3.27	30	-3.70[b]
ATTITUDE	191.38	202.06	52.58	41.97	30	-0.62
SELF CONCEPT	347.31	376.31	48.65	37.86	30	-1.85

b significant at the .01 level.

TABLE XXI

COMPARISON OF PRETEST SCORES FOR GIRLS
OF SH-CLASS AND TRAD-CLASS

CRITERIA	MEAN SH-CLASS	MEAN TRAD-CLASS	S.D. SH-CLASS	S.D. TRAD-CLASS	D.F.	t
ACHIEVEMENT	6.67	9.25	2.43	3.83	12	-1.34
ATTITUDE	151.17	201.25	34.98	66.72	12	-1.55*
SELF CONCEPT	331.50	387.38	35.08	28.05	12	-3.06[a]

a significant at the .01 level.
* Note - t-value for significance at .05 level is 2.18.
N, SH-CLASS = 6
N, TRAD-CLASS = 8

TABLE XXII

COMPARISON OF POST-TEST SCORES FOR GIRLS
OF SH-CLASS AND TRAD-CLASS

CRITERIA	MEAN SH-CLASS	MEAN TRAD-CLASS	S.D. SH-CLASS	S.D. TRAD-CLASS	D.F.	t
ACHIEVEMENT	11.00	16.75	1.91	4.15	12	-2.92[c]
ATTITUDE	193.67	194.63	60.94	65.59	12	-0.03
SELF CONCEPT	356.67	430.25	44.30	45.43	12	-2.81[b]

b & c significant at the .05 level.

TABLE XXIII

COMPARISON OF PRETEST AND POST-TEST SCORES
FOR BOYS OF SH-CLASS

CRITERIA	MEAN PRETEST	MEAN POST-TEST	S.D. PRETEST	S.D. POST-TEST	D.F.	t
ACHIEVEMENT	8.63	11.75	3.71	3.47	30	-2.38[a]
ATTITUDE	165.63	191.38	41.03	52.58	30	-1.50
SELF CONCEPT	347.63	347.31	49.66	48.65	30	0.02

a significant at the .05 level.

N = 16.

TABLE XXIV

COMPARISON OF PRETEST AND POST-TEST SCORES
FOR GIRLS OF SH-CLASS

CRITERIA	MEAN PRETEST	MEAN POST-TEST	S.D. PRETEST	S.D. POST-TEST	D.F.	t
ACHIEVEMENT	6.67	11.00	2.43	1.91	10	-3.14[b]
ATTITUDE	151.17	193.67	34.98	60.94	10	-1.35
SELF CONCEPT	331.50	356.67	35.08	44.30	10	-0.996

b t-value for significance at .01 is 3.17.

N, GIRLS = 6

TABLE XXV

COMPARISON OF BOYS' SCORES WITH GIRLS' SCORES
OF SH-CLASS BEFORE THE EXPERIMENT

CRITERIA	MEAN BOYS	MEAN GIRLS	S.D. BOYS	S.D. GIRLS	D.F.	t
ACHIEVEMENT	8.63	6.67	3.71	2.43	20	1.15
ATTITUDE	165.63	151.17	41.03	34.98	20	0.73
SELF CONCEPT	347.63	331.50	49.66	35.08	20	0.70

t = 2.09 for significance at .05.

N, BOYS = 16

N, GIRLS = 6

TABLE XXVI

COMPARISON OF BOYS' SCORES WITH GIRLS' SCORES
OF SH-CLASS AFTER THE EXPERIMENT

CRITERIA	MEAN BOYS	MEAN GIRLS	S.D. BOYS	S.D. GIRLS	D.F.	t
ACHIEVEMENT	11.75	11.00	3.47	1.91	20	0.478
ATTITUDE	191.38	193.67	52.58	60.94	20	-0.083
SELF CONCEPT	347.31	356.67	48.65	44.30	20	-0.392

t = 2.09 for significance at .05 level.

TABLE XXVII

INTERCORRELATION ANALYSIS USING PRETEST SCORES OF BOTH CLASSES

	SEX	AGE	ATTITUDE	SELF CONCEPT	ACHIEVE-MENT
SEX					
AGE	-.1549				
ATTITUDE	.0487	-.0497			
SELF CONCEPT	.0231	-.0002	.3685[a]		
ACHIEVEMENT	.0530	-.1343	.2294	-.1586	

[a] significant at .05 level.

N = 46

TABLE XXVIII

INTERCORRELATION ANALYSIS USING POST-TEST SCORES OF BOTH CLASSES

	SEX	AGE	ATTITUDE	SELF CONCEPT	ACHIEVE-MENT
SEX					
AGE	-.155				
ATTITUDE	-.022	.059			
SELF CONCEPT	.321[a]	.032	.188		
ACHIEVEMENT	.028	.021	.131	-.013	

[a] significant at .05 level.

N = 46.

TABLE XXIX

INTERCORRELATION ANALYSIS USING PRETEST SCORES OF SH-CLASS

	SEX	AGE	ATTITUDE	SELF CONCEPT	ACHIEVE-MENT
SEX					
AGE	.047				
ATTITUDE	-.161	-.028			
SELF CONCEPT	-.154	.251	.273		
ACHIEVEMENT	-.248	-.184	.093	-.324	

N = 22

TABLE XXX

INTERCORRELATION ANALYSIS USING PRETEST SCORES OF TRAD-CLASS

	SEX	AGE	ATTITUDE	SELF CONCEPT	ACHIEVE-MENT
SEX					
AGE	-.355				
ATTITUDE	-.032	-.127			
SELF CONCEPT	.150	-.344	.244		
ACHIEVEMENT	.119	-.093	.310	-.077	

N = 24

TABLE XXXI

INTERCORRELATION ANALYSIS USING POST-TEST SCORES OF SH-CLASS

	SEX	AGE	ATTITUDE	SELF CONCEPT	ACHIEVE-MENT
SEX					
AGE	.047				
ATTITUDE	.019	.207			
SELF CONCEPT	.087	.194	.166		
ACHIEVEMENT	-.106	.057	.081	-.513[a]	

[a] significant at the .05 level.

N = 22

TABLE XXXII

INTERCORRELATION ANALYSIS USING POST-TEST SCORES OF TRAD-CLASS

	SEX	AGE	ATTITUDE	SELF CONCEPT	ACHIEVE-MENT
SEX					
AGE	-.355				
ATTITUDE	-.069	-.109			
SELF CONCEPT	.529[a]	-.179	.183		
ACHIEVEMENT	.057	-.182	.137	-.233	

[a] significant at .01 level.

N = 24

TABLE XXXIII

THE EXPERT'S RESULTS*

IN IDENTIFYING THE QUESTIONS OF THE QUESTIONNAIRE (FINAL REVISION) AS: TRADITIONAL (T), SCIENTIFIC HEURISTIC (S) OR NEUTRAL (N)

QUESTION	EXPERTS' RESPONSE A	B	C	D	E	F	G	H	IDENTIFICATION
1	T	T	T	T	T	T	T	T	T
2	T	N	N	T	N	T	T	N	N
3	T	T	T	T	T	T	T	T	T
4	S	S	S	S	S	S	S	S	S
5	T	T	T	T	T	T	T	T	T
6	S	S	S	S	S	S	S	S	S
7	N	N	S	N	N	N	N	N	N
8	N	T	T	T	N	T	T	T	T
9	T	T	T	T	N	T	T	N	T
10	T	T	N	T	T	T	T	T	T
11	S	S	S	S	S	S	S	S	S
12	T	T	T	N	T	T	T	T	T
13	T	T	T	N	T	T	T	T	T
14	S	S	S	S	S	S	S	S	S
15	T	T	T	T	T	T	T	T	T
16	T	T	N	T	T	T	T	T	T
17	N	T	T	T	T	T	T	T	T
18	S	S	S	S	N	S	S	S	S
19	S	S	S	S	S	S	S	S	S
20	T	T	T	T	T	T	T	T	T
21	T	N	T	T	T	T	T	T	T
22	N	S	S	S	S	S	S	S	S

* The results are based on affirmative responses.

TABLE XXXIII

(CONTINUED)

QUESTION	EXPERTS' RESPONSE A	B	C	D	E	F	G	H	IDENTIFICATION
23	S	S	S	S	S	S	S	S	S
24	S	S	S	S	S	S	S	S	S
25	T	T	T	T	T	T	T	N	T
26	S	S	N	S	S	S	N	S	S
27	S	N	S	S	S	N	S	S	S
28	T	T	T	T	T	T	T	T	T
29	T	T	T	T	N	T	T	T	T
30	S	S	S	S	S	S	S	S	S
31	N	N	N	N	N	N	N	N	N
32	S	S	N	N	N	S	N	S	N
33	T	T	T	T	T	T	T	T	T
34	S	S	S	S	S	S	S	S	S
35	N	S	S	S	S	S	S	S	S
36	T	T	T	T	T	T	T	T	T
37	T	T	T	T	T	T	N	N	T
38	T	T	T	T	N	T	T	T	T
39	T	T	T	T	T	T	T	T	T
40	T	T	T	T	T	T	T	T	T
41	S	N	S	S	S	S	S	S	S
42	S	S	S	N	S	N	S	S	S
43	T	T	T	T	T	T	T	T	T
44	S	S	S	S	S	S	S	S	S
45	N	N	N	N	N	N	N	N	N
46	N	N	N	N	N	N	N	N	N
47	T	T	N	T	T	T	T	T	T
48	S	S	S	S	S	N	S	S	S
49	S	N	S	S	S	N	S	S	S
50	N	S	S	S	S	S	N	S	S

NOTE: Questions: 2, 7, 31, 32, 45 and 46 were considered neutral.

APPENDIX B

PAGE

a) COMMENTS ON THE TWO CLASSES BY PEOPLE WHO WATCHED THE CLASSES IN PERSON:

Simon Fraser University,
Burnaby 2, B.C., Canada.
July 27, 1971.

Mr. Dominic Alvaro,
Professional Development Centre,
Simon Fraser University.

Dear Dominic:

This is my response to your request for some impressions of the differences I observed between the two classes I watched you teach yesterday.

Physically, of course, the arrangement of people in the two classes was quite different. In the first the children sat in rows, at their desks, while you maintained the time-honored distance from them, standing at the front of the room lecturing - save for occasional forays along the rows to check on their work. In the second the students broke into groups which they formed and seated themselves on rugs scattered throughout the room, and there began playing the various math games. In this situation you went from group to group and, I felt, participated with each group not as an outsider but as a member. I felt that you were not an alien there, come to observe, but a participant come to play.

In the first class I noticed a fair bit of cheating on the short quiz you gave - hurried consultations with eyes looking the other way. The students were caught in a system and were going to beat it, by fair means or foul. In the second class I noticed that during the games each person seemed interested in figuring out for himself what the rules were and there was very little consultation between students, even in this situation where it was certainly permissible.

As I worked with students during both classes, my impression was that many students in the first, when they encountered difficulty, seemed content for me to lead them to "The Answer". In the second class I did not feel this passivity, this eagerness to be led.

Interestingly enough, it seemed that in the first class all the students were always busy - solving exercises, writing the quiz, copying homework assignments from the board. In the second class, while more students were engrossed, a few dropped out of the game very quickly and simply sat along the wall. They lost interest in finding your rules for the game (a one-difference game) and, there being nothing else to keep them busy, they stayed idle. (It evidently did not occur to them to make up their own game.)

You, also, behaved differently in the two classes. I have mentioned some of these differences above, but to sum it up briefly here, in the first class you were the straw boss, setting out the tasks to be done, the oracle, revealing mathematical truths, and the judge, deciding which answers were correct and which were wrong. None of these roles appeared in the second class, where you effectively combined the roles of guide and participant in an intelligent manner.

These, then, are some of the differences I observed. There is one final difference, however, and it is the most important. It lies in the image of mathematics which, I felt, you effectively conveyed to the second class. Here mathematics was presented not as a corpus of knowledge which the student must learn and understand, but as a series of patterns to be created and structures to be built. It is your realization of the truth of this statement which lends significance to the other differences noted above.

Yours very truly,

J.L. Berggren.

TO WHOM IT MAY CONCERN:

The following notes are recollections of time spent in the two classes involved in the project. I had been asked to video-tape the lessons for future reference and to that end I was in the classroom almost full-time.

My impressions of course are subjective, although quite strong, and are being written some months after the teaching took place.

I noted that the traditional class seemed to accept, as usual procedure, the way the class was run, and soon settled into routine, whereas the scientific heuristic class appeared puzzled by an apparent lack of structure and took some time to accept the fact that they were not being organized in the usual way and that they were expected to make a lot of decisions that (some said) they were not accustomed to making for themselves, such as whether or not a topic had been covered in sufficient depth, whether individuals needed homework for more practice and whether or not they would take tests to give some indication of what they had learnt.

The way in which tests were given and the students' attitude towards them was, I felt, one of the major differences between the two classes. In the traditional class, which had daily tests, with cumulative marks and which was working towards a comprehensive test at the end of the course, I noted a general feeling of apprehension and a particular anxiety about not being able to finish on time. (Through talking more extensively with one student who was experiencing great difficulty I heard expressed the feeling that tests that "count" are such a nervous strain that a student finds it hard to put together what he or she knows.)

There was a rise of tension in the room as daily marks were read out and recorded. Students had difficulty marking an answer which was not exactly the same as the one the teacher had given. A small minority of students would change the answers to some questions when they received their papers back and claim extra marks. As the final test approached, questions about what might be on the test and how important it would be to passing the course became more frequent and urgent.

In the scientific heuristic class the students were constantly re-assured that passing the course would not depend on test results. They were given the choice of taking teacher-prepared tests at various times throughout the course or finding some other way of demonstrating what they had learned. Most took the occasional tests and seemed to use them as a means of assessing their own strengths and weaknesses. In the early stages they showed concern that tests were returned unmarked and asked the teacher to indicate which answers he agreed or disagreed with. By the end of the course they seemed generally able to compare results with each other, discuss how they had achieved their results and come to an acceptable agreement. At first they seemed to find it hard to think in any other terms than "right" or "wrong" answers, on which the teacher was the only authority, but gradually came to accept that different results could mean different ways of looking at the same problem.

At the end of the course many students in both classes expressed satisfaction with and gratitude for the instruction they had received and the evident personal interest that was shown in them, even for such a relatively short period of time. I know that an extraordinary amount of time and energy was spent in the preparation for and conduct of these classes, and I think the students were fortunate to reap the benefits of such concerned, capable teaching.

E.E. Carolan.

Dear Dominic:

After some discussion of the nature of your experiment, you invited me to observe the remedial classes under your care in North Vancouver.

Considering the improvements in the general climate of the classroom teaching claimed by the new approach, I thought that the following aspects of behaviour should be carefully observed:-

a) The degree of 'involvement' of the students in the work at hand.
b) Their attitude to the teacher cum instructor.
c) The 'knowledge of mathematics' gained by the class.
d) The social attitudes operating among the students.
e) The 'democratic versus dictational' relations between teacher and class.

I noted the following differences between the groups:-

<u>ad a)</u> Members of the 'conventional' class appeared more involved in the work. Some members of the 'progressive' class took little or no part in the work, they appeared bored and somewhat hostile. Most students in the latter class took some time to become involved in the discussions.

<u>ad b)</u> Students in the conventional class were friendly towards the teacher. A few seemed upset when told that some of their results were 'wrong'. Most students in the other class were equally friendly but several displayed impatience and ill temper when the teacher was not forthcoming with 'correct' answers.

<u>ad c)</u> I could make no judgments on the progress of the two classes in mathematics because of the completely different nature of the work. The conventional class was learning or drilling certain skills, the progressive class was discussing possible solutions to problems.

<u>ad d)</u> There was some cooperation and discussion among students in both classes but naturally, far more in the progressive class, except in the case of those students who opted out.

<u>ad e)</u> In the conventional class I observed the presently accepted norms of teacher leadership. Since the teacher was successful, the students seemed to accept the leadership easily and without resentment. In the progressive class group leaders appeared spontaneously but were not accepted by all group members with good grace. Some appeals against these vocal students were made to the teacher and objections were raised when the teacher did not intervene instantly.

General Conclusions:-

The major problem encountered in all social experiments, is that of controlling all variables, so as to be able to observe the results of changing one or another variable. In this case, I suspect, previous conditioning caused some of the children in the progressive group to doubt whether they were being taught 'real mathematics'. The teacher had no control over this factor.

One is persuaded, on purely logical grounds, that the progressive 'curriculum' and method should generate finer mathematicians in the future but in the absence of sufficient case-studies the assertion cannot be made.

My observations have not lead me to believe that the progressive method produces a more desirable social climate in the classroom. It is possible that 'natural' leaders are more desirable than appointed teachers - maybe because they are less benevolent. Such paradoxes are amusing, but are they either humane or constructive?

There are several other aspects of the problem which I failed to consider before and during my visit, such as; disciplined work, informed versus uniformed guidance, etc. I hope you will be able to touch upon them in your thesis.

I wish you all the best in your future work.

Sincerely yours,

M.D. Garber.

Dear Dominic:

As you requested, here is a copy of the relevant material concerning definition of attitude, taken from Chapter 1 of my pending dissertation.

I administered the Likert-type instrument, along with the other four attitude instruments, and two standardized achievement instruments, to hundreds of students both during the Pilot Study in North Vancouver, and during the Main Study in Victoria.

For the Pilot Study, I found the reliability coefficient, as measured by the Cronbach Alpha formula, to be 0.9436 (252 students). For the Main Study, the Cronbach Alpha was 0.9351 (812 students). Inter-correlations between the Likert-type and the other four types was found to be fairly good, indicating a reasonable degree of validity for this instrument.

I have a strong suspicion that the Likert is still the best of the lot for predicting achievement, but must await the detailed statistical analysis which is still taking place, before I can be certain. It also appears that a very much more significant amount of R^2 can be picked up by using the fifteen category-scores than when using a simple global-score.

I hope the enclosed material is what you are looking for. I expect to be in Vancouver, in about a month, to complete the statistical analysis, and will give you a call. I'd like to hear a lot more about your study.

Sincerely,

Monty.

EXCERPTS FROM MY DISSERTATION:

"An Empirical Study of the Constructive and Predictive Validity of Five Types of Instruments to Measure Students' Attitudes Towards Mathematics."

... in progress.

... "A search of the literature by the writer has revealed that in a great majority of the studies on attitudes, the attitude instruments used resulted in a single global score for each subject, either explicitly or implicitly suggesting that attitude scores lie on a bi-polar unidimensional continuum. On the other hand, most of the current text books on social psychology (Krech, Crutchfield and Livson, 1969) and theories of learning (Travers, 1967), suggest that attitude has at least three dimensions - cognitive, affective and conative. Most authors further rationalized that attitude instruments traditionally measure only the affective dimension (Shaw and Wright, 1967). A recent study by McKie and Foster (in process, 1971) has indicated that two or more students can be found to have the same global score using a Semantic Differential instrument and yet when their subscores on three empirically selected factors are graphed, there often resulted quite different profiles. The implication was that partitioning of the global attitude score might result in accounting for a higher portion of the variance when achievement in mathematics was the criterion. J.F. Fedon (1958) seemed to be supporting this conjecture when he observed that there was a general feeling that various aspects of arithmetic were enjoyable and necessary, but not always meaningfully significant. Khan (1969) reported higher than usual relationships when predicting achievement from subscores on measures of ability, intelligence, attitudes, motivation and study habits. He stated that the results suggested the usefulness of subscores as compared to an overall score, and cast doubt on the assumption that attitude, motivation, and study habits can be represented unidimensionally.

II DEFINITIONS

Over the past half century, the rapidly increasing number of definitions for the concept of attitude has contributed greatly to the problem of obtaining consistent results in attitude research studies. G.W. Allport (1935), after surveying more than one hundred different definitions of attitude, concluded by defining attitude as a mental and neural state of readiness, organized through experience, exerting a directive or dynamic influence upon the individual's response to all objects and situations with which it is related. Of more importance, L.L. Thurstone (1928) contributed a unidimensional definition which has been adopted by a great number of researchers since that time. He stated that attitude was the affect for or against a psychological object. He further postulated that the affect could be located on a linear continuum with a neutral point or zone and two opposite directions, one positive and one negative. Most instruments designed to measure attitudes have proceeded from this point of view, producing a single global score which could be located on such a bipolar continuum. Because research based on this concept of attitude has not resulted in consistent significant behavioural prediction, the unidimensional view has been questioned.

More recently, attitude has been defined as a concept consisting of three dimensions - cognitive, affective and conative. In support of this concept, three items have been selected from the attitude-measuring scales constructed for the International Study of Achievement in Mathematics (Husen, 1967). The statement, "In mathematics there is always a rule to follow in solving problems", required the student to make a cognitive evaluation in order to express his agreement or disagreement with it. The statement, "I enjoy everything about school", demanded an assessment of the affective sentiment of the student agreeing or disagreeing with it. The statement, "I dislike school and will leave just as soon as possible", forced the student to indicate his behavioural intentions in order to either agree or disagree with it.

As was pointed out in the previous section, most textbooks present some version of this definition of attitude as a tri-dimensional concept. Some writers, notably Shaw and Wright (1967), reject this notion. For the purpose of this study, the modern definition seems more appropriate. The writer believes, however, that although some items in most attitude instruments do tap each of these three dimensions, the effect of these measurements is lost when a global score is produced to satisfy a uni-dimensional affective continuum concept.

As applied to the learning of mathematics, the concept of attitude towards mathematics will be defined, following Krech et al. (1969) as an enduring system of positive or negative evaluations, emotional feelings, and action tendencies with respect to various aspects of mathematics learning.

Since every definition of attitude implies that attitude is a latent variable, there cannot exist a direct way to observe or measure a student's attitude towards mathematics. As a consequence, it is necessary to accept the statement that inferences may be made about an underlying attitude by observing what a person says or does in relevant situations. (Corcoran and Gibb, 1961). Thurstone (1928) faced this problem earlier and offered a solution which has been universally adopted by researchers designing instruments to measure attitudes. He argued that agreement or disagreement by a subject with a verbal statement constituted an opinion, and that an opinion symbolized an attitude, and therefore an opinion could serve as the carrier of the symbol of the attitude of the subject.

For the operational purposes of this study, the responses which are given by a student to a statement or to a question about various aspects of mathematics will be taken as a measure of the student's attitude towards that aspect of mathematics. This position is taken with the full realization that only a small part of the latent attitude may be measured in this manner. This position, however limiting, has also been taken by most other researchers in the field of attitude measurement.

The term "attitude scale" has widely differing connotations depending on the type of attitude-measuring instrument being used. In the Semantic Differential type of instrument, each set of bi-polar adjectives, together with the seven points provided for the subject's response, is referred to as a scale. Traditionally, those scales which can be empirically shown by subsequent factor analysis to be evaluative are combined to yield a measure of the subject's attitude toward the mathematical concept being rated. The measures on several concepts are summed to yield an overall attitude score, representing the subject's attitude towards mathematics. When using a Guttman-type of instrument, the responses to several items related to a particular category of mathematics are combined to form an attitude scale. Several scales can then be combined to yield a measure of the subject's attitude toward mathematics. With a Likert-type instrument, the responses to all items are summed to produce a global measure of attitude toward mathematics, and the entire instrument is often referred to as a scale. The word scale, then appears to have a different meaning at each of three levels. In this study, the confusion will be side-stepped by avoiding the use of the word "scale".

For operational purposes, the following terminology will be used:

1. Item-score: will be taken to mean the numerical value assigned to the subject's response; to a statement on the Likert- and Q-Sort types; to a question in the Guttman type; to a set of bi-polar adjectives (a scale) on the Semantic Differential type.
2. Category-score: will be taken to mean the numerical value assigned; to a subset of related items in the Likert and Q-Sort types; to a category (or scale) in the Guttman type; to a concept in the Semantic Differential type; and to a category in the Teacher's Rating Instrument.
3. Global-score; will be taken to mean the number assigned as the overall measure of the student's attitude towards mathematics on each of the instruments."

EXTRACT FROM ELEMENTS OF PSYCHOLOGY - Second Edition
David Krech, Richard S. Crutchfield and Norman Livson
Alfred A. Knopf, New York, 1969.

UNIT 49 - SOCIAL ATTITUDES pp. 809-824.

Thurstone and Chave's Classic - "Attitude towards the Church" is reprinted and analysed. (45 Thurstone statements). Then the authors say:

... "Note how we derived a measure of your attitude toward the church. Our raw data consist of a sample of your agreements and disagreements with certain statements about it. From this pattern we inferred an underlying disposition that we call an "attitude". An attitude is not itself directly observable.

Note that every statement in the scale refers to the church. The church is the social object of the attitude. An attitude is always organized around an object; it always has a focus. The object may be anything that has psychological reality for the individual - the individual himself, a person, a group, a nation, or a political issue, a scientific theory, a commercial product.

Let us examine the content of some of the statements in the Scale. Statement 34 reads, "I think the organized church is an enemy of science and truth," and statement 44 reads, "I believe the church is a powerful agency for promoting both individual and social righteousness". In these illustrations the operative phrases are "I think" and "I believe". Many of the items in the scale are intended, clearly, to provide indicators of the individual's evaluative belief about the church.

Statement 5 reads, "When I go to church I enjoy a fine ritual service with good music." Statement 43 reads, "I like the ceremonies of my church but do not miss them much when I stay away". The responses of the individual to these two statements enables us to say something about how he feels about the church. Assent would indicate liking, dissent disliking.

Finally, if a person agrees with statement 30, (I think the country would be better off if the churches were closed and the ministers set to some useful work) we would infer that he favors a specific form of restrictive Action toward the church.

From this content analysis, we conclude that the concept of attitude refers to a complex inner disposition that consists of three components; an evaluative belief component, and an action-orientation component. We can now summarize our discussion of the concept of attitude in a formal definition. An attitude is a complex organization of evaluative beliefs, emotional feelings, and action orientations focused upon an object, predisposing the individual to respond to the object in certain ways."

... All the foregoing was taken from page 813.