

NONLINEAR MODEL OF FOREARM SUPINATION

by

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ABSTRACT

A two component muscle model is used as basis for digital computer simulation of human muscular contraction by means of an iterative process. The contractile (CC) and series elastic (SEC) components are seen as lumped components of structures which produce and transmit torque to the external environment.

The CC is defined by four parameters, torque, angular velocity, angle and activation. These parameters are described by a series of non-planar torque-angle-angular velocity surfaces stacked on top of each other, each surface being appropriate to a given level of muscular activation. The SEC is described similarly along dimensions of torque, extension, angle and activation.

Two subjects performed a large number of contractions against varying moments of inertia and at four levels of perceived exerted effort. From these, characteristics of the CC and SEC were estimated. CC characteristics were found to be highly non-linear. Both torque and angular velocity were influenced by muscle length and activation.

The determined muscle characteristics were used to predict individual contractions for both subjects. This successful prediction of muscle behaviour validates both the model and method of determining muscle parameters. The approach is not limited to forearm supination, but could be used for any other

muscle or muscle group. The efficiency and accuracy of the model allows analysis of many aspects of muscle behaviour including optimization studies.

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I. INTRODUCTION

Terry Fox was able to run more than halfway across Canada with one artificial leg. This exemplifies the high level that design of prostheses has achieved. However, if we compare his rather laborious way of running with the superhuman ability of bionic people on the TV shows, a large gap between imagination and reality becomes apparent.

Though it seems impossible to design artificial limbs stronger and better than real ones, there is still much room for improvement of current designs. As muscle provides the driving force for the natural system, modelling should start at the muscular level.

Hill (1938) first proposed a model of muscle. It consisted of a contractile component, the CC, and an undamped elastic component in series with the above, the SEC. The model is illustrated in Figure 1a. Later Hill (1949b) added a parallel elastic component to the system. This, however was only important when large loads were applied or when the muscle was stretched beyond its natural length.

Hill (1938) also found a characteristic relation between exerted force and speed of shortening of the CC, shown in Figure 1b.

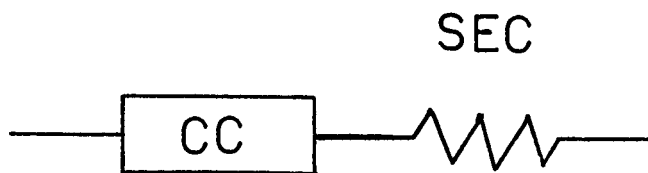
Figure 1. Hill's model (a) and equation (b).

CC is the contractile component;

SEC the series elastic component.

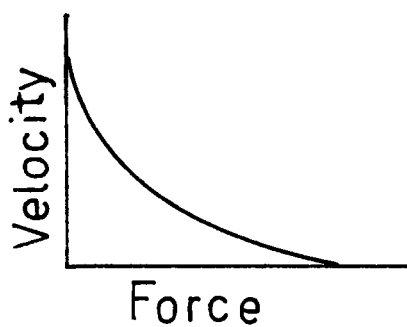
Parameters of equation are: P is force, P_0 is the isometric maximum; v is velocity; a and b are Hill's constants.

a) Hill's Model



b) Hill's Equation

$$(P+a)(v+b)=(P_0+a) b$$



The SEC was assumed to have non-linear characteristics. Hill (1949b) pointed out that knowledge of the CC force-velocity relation and SEC characteristics are sufficient to calculate the time course of a muscular contraction. This is still the goal of muscle modelling and a variety of approaches have been taken. The following will give an overview of existing literature on the topic.

II. LITERATURE REVIEW

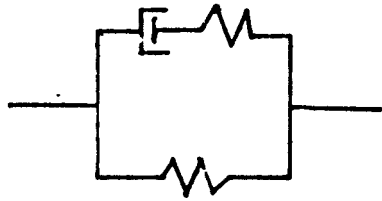
a) Linear Models

To simplify the calculations involved in determining the CC and SEC characteristics many researchers have transformed Hill's (1938) model to a linear equivalent. Lord Kelvin (William Thomson, 1824-1907) first described the dynamics of the "standard linear model" consisting of two springs and a dashpot. This model is depicted in Figure 2a. Alfrey and Doty (1945, see also Fung, 1971) further showed that this model can also be represented by the one shown in Figure 2b¹. Especially in research concerned with cardiac muscle, the model shown in Figure 2a is called a "Maxwell model" and the one presented in Figure 2b a "Voigt model" of muscle (eg. Brady, 1968). However, since the two models can be shown to be equivalent and the naming is rather arbitrary, a distinction between them seems artificial (Alfrey and Doty, 1945; Fung, 1971).

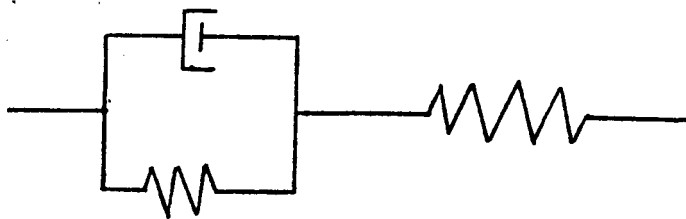
¹However the springs would have different constants.

Figure 2. Standard linear model (a), and alternative form (b)

a) Standard Linear Model



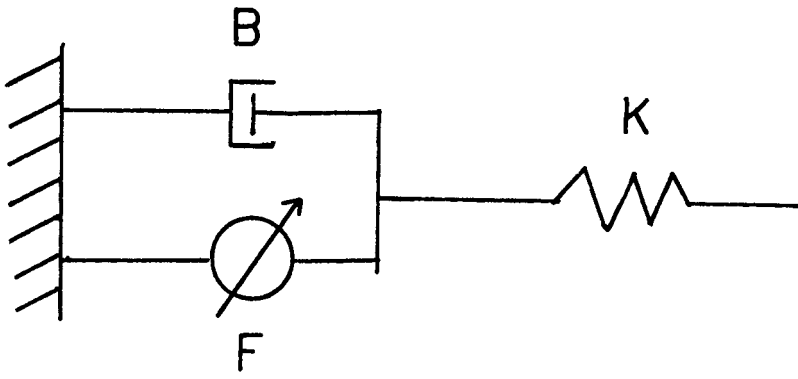
b) Alternative Form



Houk et al. (1966, see also Houk, 1963, reviewed in Milhorn, 1966, ch. 17) proposed a linear version of Hill's (1938) model similar to Lord Kelvin's standard linear model. The non-linear CC is linearized by replacement with a dashpot, and a force generator. The SEC is assumed to have linear spring characteristics. The model is depicted in Figure 3. The above is a second order linear system which can be analysed easily. With the use of Laplace transformation the frequency response can be determined and with inverse Laplace transformation the time course of contractions can also be calculated. Using cat plantaris muscles Mannard and Stein (1973) found that responses of the muscle were similar to those expected from a system described above. Bawa et al. (1976a) analysed the response of cat plantaris muscle when elastic loads were placed in series with it, and observed behaviour similar to that of a second order linear system. Crowe et al. (1980) also incorporated a linear dashpot in their model which was based on evidence from Alexander and Johnson (1965) and the results of their own experiments (Van Atteveldt and Crowe, 1980). However, they also had to include a non-linear parallel elastic element to explain their results (Crowe et al., 1980).

Figure 3. Linear muscle model

B is viscosity of dashpot, K stiffness of spring and F force produced by force generator.



Though most authors acknowledge the presence of non-linearities, their relative importance is not agreed upon. Bawa et al. (1976a) found that even though plantaris muscle of the cat can be analysed as a second order linear system, the results for soleus muscle could not be fitted by the response of a linear system. Further Bawa et al. (1976a, Fig. 6) acknowledge and show considerable variations of the rate constants describing the second order system with muscle length, stimulation rate and stiffness of the spring placed in series. Also when calculating the effective muscular stiffness from the sum of the two elastic components and comparing it with the estimate derived from the low frequency gain of the system² the two values of the same parameter are not very close, especially for small added springs (see Bawa et al., 1976b, Fig. 3) and large muscle specimen (see Bawa et al., 1976b, Table 1). For a perfect second order system equality of the two estimates would be expected. Chapman and Harrower (1977) determined the parameters of the force-velocity curve of the CC and the compliance of the SEC of rat gastrocnemius muscle under isometric conditions. Comparing a linear and a non-linear model to explain their results, these authors conclude that linear approximation is not appropriate for the experimental conditions analyzed.

²see Bawa et al. (1976b) for derivation of relevant equations.

b) Non-Linear Models

From the above it follows that for exact and complete description of muscle behaviour a non-linear model seems necessary. Several researchers have constructed such models, some building very elaborate and complicated ones. The most notable attempts at this are explained in Hatze's (1973, 1977) papers. He uses a multitude of differential equation to describe every aspect of muscle behaviour. The model claims to be based on the cross-bridge theory, but the complexity necessitates estimation of a great number of parameters and relationships. Nevertheless, Hatze successfully predicts human contractions and has used the model for optimization studies (Hatze, 1976), and to analyze stretch responses (Hatze, 1981). Due to the complexity, and the fact that the model is an electro-magnetic view of muscle rather than a mechanical one, this model will not be further discussed here. As will be shown in this thesis, a much simpler model can also successfully predict human contractions.

Julian and Moss (1976) outlined a model based on Hill's (1938) idea. Besides the CC and SEC they added an activation mechanism that regulates CC activity (see also Julian, 1969). Aspects of these three muscle components are discussed in more detail in the following sections.

c) The Contractile Component

The contractile component is the force generator of the muscle. Huxley (1957) first proposed the now widely accepted cross-bridge theory as the force generating mechanism. Since then much has been published on the details of the mechanism, including its biochemistry (Harrington, 1979; Tregear and Marston, 1979) and models incorporating cross-bridge behaviour (Julian and Moss, 1976; Huxley and Simmons, 1973). A summary of the current state of the theory, including some of the criticism of it (see also Noble and Pollack, 1977, 1978) is provided by Julian et al. (1978). T.L. Hill (1974) discusses almost every aspect of the theory in exhaustive detail.

i) CC Force-Velocity Relationship

The macroscopic aspect of the CC is very well researched. Hill (1938) observed the characteristic relationship between force and velocity and analyzed the heat liberated by a contracting muscle. The constant 'a' of Hill's equation (see Figure 1b) is then related to the heat liberated per centimeter of shortening, whereas b defines the absolute rate of energy liberation. As Hill (1965:64) states, the "characteristic equation was not invented, it emerged." Nevertheless purely mechanical data can also be used to find the parameters of the

equation. In a twitch against small loads the tension developed and the velocity reached can be measured and then fitted by Hill's equation (Hill, 1949b). In this case the parameters are more or less "invented". A high order polynomial could be used with equal accuracy to fit the curve.

Another possible approach is the analysis of force records. At any point where $dF/dt=0$ (ie. where the rate of change of force is zero), the rate of change of SEC length must also be zero (according to Hill's, 1938 model) and therefore SEC velocity is equal at both ends. From that it follows that the CC velocity must be equal to external muscle or muscle fiber velocity at this time. In this way a force-velocity curve can be constructed from a number of trials.

ii) CC Force-Length and Velocity-Length Relation

Hill (1938) stated that for the determination of the force-velocity curve the muscle should not be allowed to shorten very much since the isometric maximum, P_0 , will then change. Other authors also found considerable variation of isometric tension when changing muscle length (eg. Rack and Westbury, 1969; Edman, 1978, 1979). As is consistent with the cross-bridge theory, at very short sarcomere lengths the tension developed is low and rises to a plateau with increasing sarcomere length, falling off again once the sarcomeres are stretched too much.

Once the muscle is stretched very far the tension rises again due to extension of the parallel elastic component.

The maximal velocity of shortening also depends on the sarcomere length, but in a different manner. At very short lengths it behaves similarly to the force-length relation, but after showing a plateau maximal velocity increases sharply with further extension (Edman, 1979).

In human motion the combined action of several muscles in most movements and their varying mechanical advantage throughout a movement make analysis of length influences difficult. An apparent loss of external force could be the result of the changes in either length or mechanical advantage.

d) Elastic Components

i) The Series Elastic Component

Hill (1938, 1950) and Abbott and Mommaerts (1959) assume the series elastic component (SEC) to be passive. Other authors (eg. Huxley and Simmons, 1971, 1973) consider the SEC to be an active component. The question of whether the large extensibility of resting muscle is due to either elasticity of the non-activated CC or a non-existent SEC cannot be answered by gross biomechanics. SEC properties can, due to the mentioned extensibility of resting muscle, only be studied in activated

muscle (Hill, 1950). Jewell and Wilkie (1958) assert that the SEC characteristics are not influenced by muscle length, however this assumption has not yet been verified.

ii) The Parallel Elastic Component

Some muscle properties could be explained better by assuming the presence of an elastic component parallel to the CC. However, Hill (1950) observed that it seemed to be of importance only when muscle is stretched beyond its normal length. Mathematically the parallel elasticity can be incorporated with the SEC (eg. see Bawa et al., 1976b)³. The original distinction between the two might have originated with Hill's feeling that the SEC resides mostly in tendon and the parallel elasticity in the sarcolemma (Hill, 1950), although he never states this outright (see also Hill, 1965:350).

e) Activation

A variety of approaches have been taken in attempts to define and measure activation. Muscles are activated by nerve impulses which result in the release of calcium within the

³In the same way that electrical capacitors can be combined, mechanical compliance can also be collected to give an overall compliance of the system (Blessner, 1969:98).

muscle fibres. This calcium release then provokes the change in the configuration of the cross-bridges. Activation could thus be measured at the nervous level, chemically, or by defining a parameter containing all or some of the processes involved.

The concept of "active state" and maybe effort perception are examples of the last group. Calcium dynamics have been measured in isolated muscles and muscle fibres and incorporated into models based on the cross-bridge theory (eg. Julian, 1969). Use of the electromyogram (EMG) is an attempt to measure the nervous input to the muscle. In cats and isolated muscles measurement or setting of the stimulation rate is possible. The following section deals with each of these methods of determining the level of activation in more detail.

i) Stimulation Rate

In fibre experiments monitoring of the applied stimulation rate allows quite accurate determination of the level of activation. Generally a sigmoid relationship is observed between stimulation rate and developed force (Cooper and Eccles, 1930; Matthews, 1959; Rack and Westbury, 1969; Julian and Sollins, 1973).

Monitoring of stimulation rates is possible in human experiments by measuring nerve impulses in the nerves leading to certain muscles and by direct stimulation of these nerves. Due to the involvement of several muscles and varying recruitment

levels in most movements the first is unreliable and the second method tends to be rather painful.

ii) EMG as an Estimate of Activation

An indirect measurement of stimulation rate is the electromyogram (EMG). The EMG is a large and easily measured biological signal. Since nerves carry the excitatory stimuli to the muscle fibres, there must also be some relation between EMG and activation. However, description of this relation has proven somewhat difficult. EMG recordings show extreme fluctuations, and considerable processing has to be done to get a smooth curve suitable for evaluation. Filtering, rectifying or integration is most frequently used. Lippold (1952) observed linear relationships between integrated EMG and force, as did Bigland and Lippold (1954a) when measuring EMG during contractions of calf muscles in young adults. In another experiment the above authors (Bigland and Lippold, 1954b) recorded EMG while stimulating the abductor digiti minimi via the ulnar nerve with electrodes attached at the elbow. They could identify some individual motor units and observed a sigmoid relation between developed tension and frequency of discharge of motor units. A similar curve was also recorded by Kanosue et al. (1979).

Because of the large fluctuation of raw EMG, averaging over time or over space with the use of a number of electrodes (eg. Lam et al., 1979) usually improves the correlation between EMG

and force. However, the EMG is also influenced by a variety of other conditions such as the subject, processing, electrodes (Lam et al., 1979) and muscle length (Grieve and Pheasant, 1976).

Further integrating or filtering of EMG can cause considerable loss of information. Chapman and Calvert (1979) found that due to this need for smoothing, EMG gives a rather poor estimate of activation in muscle, especially when rapid changes in activation occur or during gross dynamic contractions (see also Calvert and Chapman, 1976). Despite some recent improvements due to novel methods of processing EMG (Hogan and Mann, 1980a, 1980b), serious doubts about its usefulness as an estimate of activation remain.

iii) Chemical Parameters of Activation

Any muscular contraction must be preceded by action potentials, which cause the release of calcium ions and after a series of other events this leads to the contraction (Stein and Wong, 1974; Julian and Sollins, 1973). Julian (1969) first added equations describing calcium dynamics to his model based on the cross-bridge theory. After the arrival of a nerve impulse, activation increased with a short time constant, which was responsible for a quick rise in free calcium. Subsequently a long time constant was responsible for the slower calcium uptake. Stein and Wong (1974) used Julian's (1969) equation for

their similar model. They also calculated the response to varying degrees of activation. They could not produce the typical sigmoid relation between degree of activation and developed tension observed by other researchers. They explained this with a change of calcium ion dynamics in the middle range of activation.

In human experiments monitoring of the calcium ion level in muscle is not possible without disturbing the movement.

iv) Active State

Hill (1938) only considered active versus inactive muscle. The arrival of the nerve impulse was supposed to set up the so-called "active state" of muscle. Hill (1949a) defined the intensity of the "active state" as the force developed at a zero CC velocity. Since even in isometric contraction the CC shortens when extending the SEC, a quick stretch of the muscle was used to compensate for this internal shortening. Then however, one must assume that the quick stretch does not influence the CC (Pringle, 1960). This potential influence is difficult to determine so that definition and measurement of "active state" have always been unclear.

Julian and Moss (1976) compared results from a model that included an activation parameter with the results of some of the classical experiments. They came to the conclusion "that classic active state concepts are not definitive concerning the time

course of the level of activation during a contraction." Then one is only left "to wonder whether the term (active state) has any exact meaning" (Hill, 1965:68).

v) Perception of Effort as Estimation of Activation

A more vague concept of activation, but possibly also an estimate of stimulation and recruitment (via efferent copy), is measurement of perceived effort. It seems quite logical to expect a close relationship between the exerted effort and the perception of it. Bell (1833, quoted in Granit, 1972) first talked about the "consciousness of exertion", and later Renqvist (1927) observed that equal forces resulted in equal perception. Though the exact origin and nature of the information underlying perception is still controversial (Cooper et al., 1979), surprisingly consistent results have been reported by many researchers.

Three experimental approaches have been used to quantify the relationship. One is the estimation of the magnitude of a produced force, another the production of a force of a specified magnitude and the third matching of a pre-given force level. A variety of scales have also been derived.

Generally a high correlation between exerted force and perceived force is reported (Borg, 1970, 1972; Cooper et al., 1979; Stevens and Mack, 1959; Eisler, 1962). The relation is mostly described as a power function with an exponent between 1

and 1.7. Stevens and Mack (1959) point out that though the value is constant for each subject, considerable variation between individuals is common. Cooper et al. (1979) analyzed isometric and dynamic contractions of quadriceps and isometric contraction of adductor pollicis. They come to the conclusion that even though there is some difficulty defining perception of effort

"a single motor performance, be it an isometric or a dynamic contraction, can be perceived with remarkable precision by both a small and large muscle group. Both 'estimation' and 'production' procedures can be used. This precise perception has been shown to be reproducible (high test-re-test correlation) and as such could serve as a practical basis for training programmes in patients, and for the analysis of muscular sensations in various job situations and everyday life activity." (Cooper et al., 1979)

vi) Activation and Maximum CC Velocity

In the sliding filament theory, the number of calcium ions released is assumed to determine the number of cross-bridges that are attached at a given time. Therefore activation should influence only the force developed, but not the velocity of contraction (Julian and Sollins, 1973). Edman (1978) reaches this conclusion from his experimental results. Other researchers, however, obtained different results. Julian and Sollins (1973) calculated the force-velocity relationship for frog muscle fibres from isometric twitches and compared the curves derived from data before, at and after the twitch force peaked. Since activation would be much lower towards the end of

the twitch than at the beginning, the results can be used to estimate the influence of activation on the F-V curve. Though the effect was more pronounced for tension, velocity also decreased considerably with decreasing activation (Julian and Sollins, 1973). Petrofsky and Phillips (1980) made similar observation using cat muscles that were stimulated via the ventral roots.

Bigland and Lippold (1954a) also compiled force-velocity curves for different levels of activation (as estimated by EMG). Isometric force decreases with decreasing activation, whereas maximal velocity (V_{max}) is shown unaltered. The curves they show however are extrapolated to this finding since maximal velocity reached in their experiments was less than half of theoretical V_{max} (Bigland and Lippold, 1954a, Figure 5).

The relation between V_{max} and activation is probably quite complex (Julian and Sollins, 1973) and difficult to conceptualize. However the effects of it can be incorporated into activation models (Julian and Sollins, 1973; Phillips and Petrofsky, 1980).

vii) Activation and Length

Rack and Westbury (1969) found an interaction between stimulus rate and muscle length. The steep part of the typical sigmoid curve relating force and activation was located at higher stimulus rates for shorter muscle lengths than for longer

ones. Taylor and Ruedel (1970) and Ruedel and Taylor (1971) presented evidence that at a very short sarcomere length the central core of muscle fibres becomes inactivated. Since this sarcomere length was well below physiological ranges encountered, application of these findings to human contraction is difficult.

Julian et al. (1978) discuss the influence of length on activation and come to the conclusion that there is no large influence, except maybe in a few specialized muscles or lengths outside the physiological limits.

III. SUMMARY OF LITERATURE REVIEW

For the design of functional prostheses an understanding of muscles is necessary since they provide the driving force for natural systems. Initially Hill (1938) proposed a muscle model consisting of a contractile component (CC) in series with an elastic component (SEC). Both components were assumed to display non-linear mechanical properties. Houk et al. (1966) linearized this model to simplify mathematical analysis. However, despite the ability of linear models to explain the behaviour of some muscles under certain conditions (eg. Mannard and Stein, 1973; Bawa et al., 1976a, 1976b), non-linearities inherent in muscle prevent general applicability of such designs (Chapman and Harrower, 1977).

For a non-linear model to be of use, properties of its components need to be described. Hill (1938) showed that a typical relationship between force and velocity of the CC exists, and this has since been considered a fundamental property of the CC. Muscle length also greatly influences developed tension (Rack and Westbury, 1969; Edman, 1978, 1979).

In muscle fibres estimates of activation of the CC can be derived from calcium concentration. However when analyzing human movements the situation becomes rather complex. Though EMG is often used as a measure of activation (Lippold, 1952; Bigland

and Lippold, 1954a, 1954b; Kanosue et al., 1979; Lam et al., 1979), necessary processing limits its applicability. Its usefulness for movement analysis can be doubted (Calvert and Chapman, 1977; Chapman and Calvert, 1979). An attractive alternative is measurement of effort perception which has been shown to correlate well with produced force (Bell, 1833; Borg, 1970, 1972; Cooper et al., 1979).

While the SEC can be described by its length-tension curve (Hill, 1950), this curve might depend somewhat on both the muscle length as well as the state of activation.

The sliding filament theory (Huxley, 1957) allows explanation of many macroscopic observation with microscopic structures, however the exact location of the series (or cross-bridge) elasticity for example is still debated (Jewell and Wilkie, 1958; Tameyasu and Sugi, 1979; Huxley and Simmons, 1971). The use of Hill's (1938) model as a conceptual one rather than as a representation of actual components avoids confusion (Pringle, 1960). Determination of the relationships between the force the CC develops and activation, length and velocity, as well as the determination of the SEC characteristics, should nevertheless allow construction of a model that closely approximates the muscle behaviour under a wide range of experimental and real conditions.

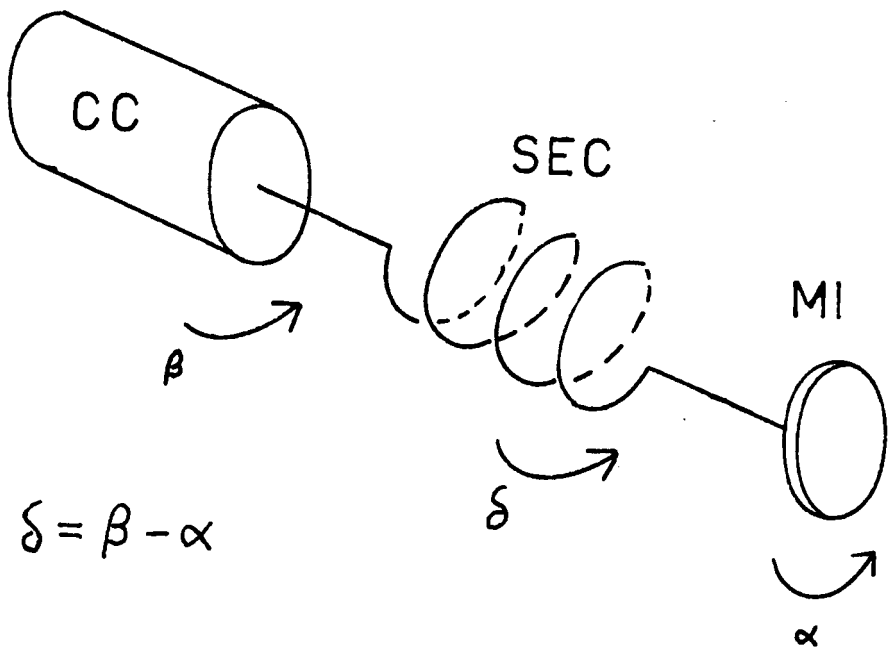
IV. THE MODEL

a) Introduction

From the above literature review it can be seen that a model based on Hill's (1938) proposal with an activation parameter added (Julian, 1969) seems appropriate to describe muscle behaviour. Such a model is depicted in Figure 4. Since joint movement is almost always rotational, the model will be rotational too.

Figure 4. Proposed Muscle Model

CC is the contractile component, SEC the series elastic component, and MI a mass with moment of inertia MI; α is limb rotation, β is CC rotation, the difference δ is SEC extension.



b) Structural Elements of the Model

As shown in Figure 4, the proposed model consists of a CC and a SEC.

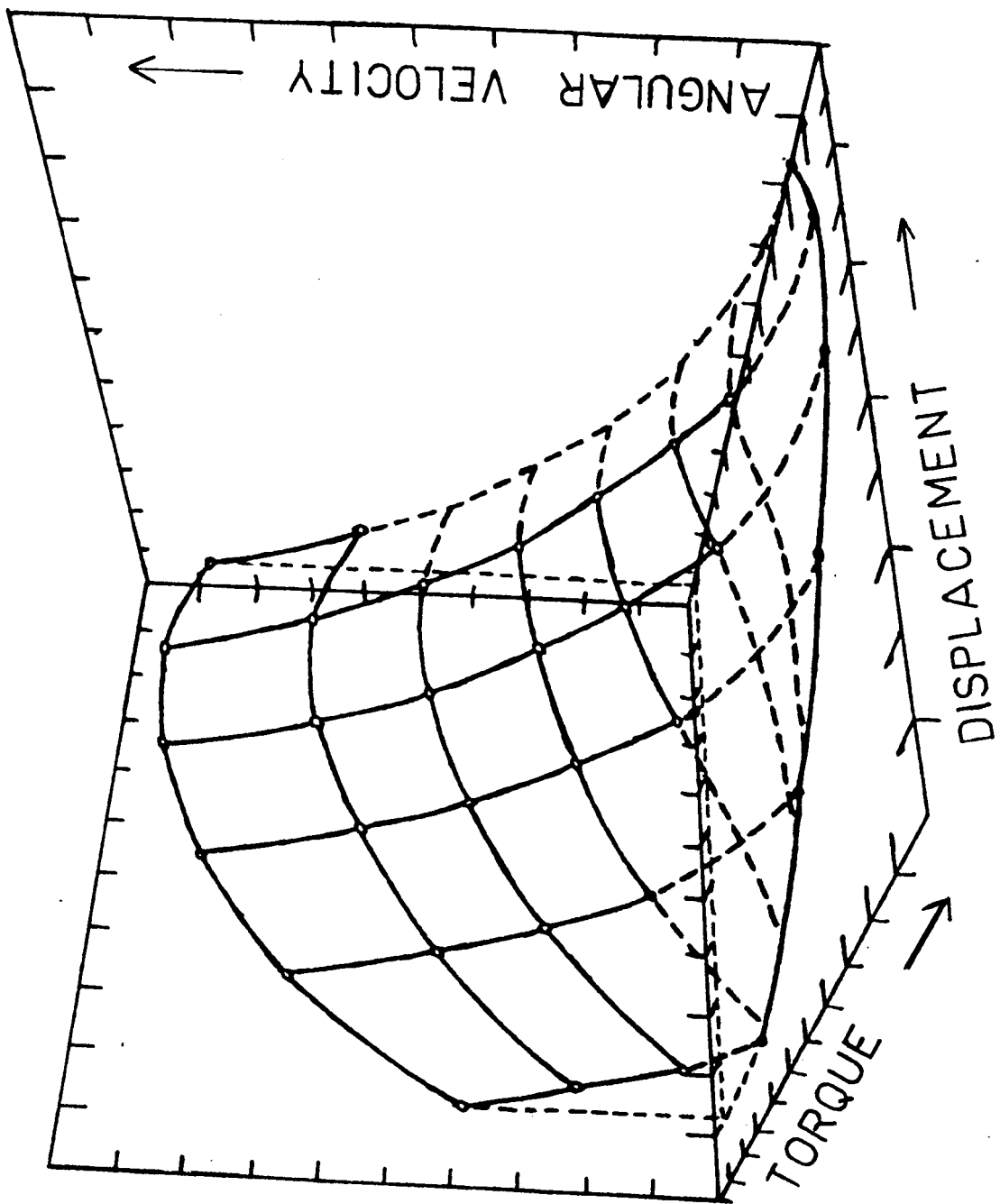
The CC is a lumped component, consisting of all torque generating structures involved in the analyzed movement. This component is not separated into individual muscles, fibres or motor units, nor is the rotational movement translated into the really linear action of the muscles themselves.

Similarly the SEC is considered a lumped elastic component, consisting of all elasticity residing in the cross-bridges, tendons and any other elastic structures. These elements are those which transmit torque from the CC to the external environment.

c) Functional Properties of the Contractile Component

Figure 5 shows a typical three-dimensional torque-angle-angular velocity surface. Since this is a rotational model, muscle length is defined by the angular displacement of the moving limb in reference to some absolute position. Velocity is the angular velocity.

Figure 5. Typical torque-angle-angular velocity surface
Redrawn from Bahler et al., 1968



In this model a fourth dimension, activation, is added. Thus the CC is described by a series of non-planar torque-angle-velocity surfaces, stacked on top of each other. Each surface is then appropriate to one level of activation, and is defined in three planes by the torque-velocity curve at optimal length, the isometric torque-angle relation, and the velocity-angle relation at a constant, small torque. Equations (1) to (6) list all required relationships:

$$\text{for } \dot{\beta}=0, A=1 \quad T=f(\alpha) \quad (1)$$

$$\text{for } \alpha=\alpha_0, \dot{\beta}=0 \quad T=f(A) \quad (2)$$

$$\text{for } \alpha=\alpha_0, A=1 \quad T=f(\dot{\beta}) \quad (3)$$

$$\text{for } T=0, A=1 \quad \dot{\beta}_{\max}=f(\alpha) \quad (4)$$

$$\text{for } T=0, \alpha=\alpha_0 \quad \dot{\beta}_{\max}=f(A) \quad (5)$$

$$\text{for } T=f(\alpha), \dot{\beta}=0 \quad A=f(\alpha) \quad (6)$$

where T is torque,

A is activation,

$\dot{\beta}$ is CC velocity, $\dot{\beta}_{\max}$ is the maximum possible,

α is angular muscle displacement, α_0 is the optimal displacement (maximal isometric torque).

d) Functional Properties of the SEC

The SEC characteristics can be defined similarly to those of the CC. Transmitted torque can depend on activation and muscle length as well as SEC extension. Equations (7) to (12)

describe the functions needed:

$$\text{for } \alpha = \alpha_0, A = 1 \quad T = f(\delta) \quad (7)$$

$$\text{for } \delta = \delta_0, A = 1 \quad T = f(\alpha) \quad (8)$$

$$\text{for } \alpha = \alpha_0, \delta = \delta_0 \quad T = f(A) \quad (9)$$

$$\text{for } A = 1, T = T_0 \quad \delta = f(\alpha) \quad (10)$$

$$\text{for } \alpha = \alpha_0, T = T_0 \quad \delta = f(A) \quad (11)$$

$$\text{for } \delta = \delta_0, T = T_0 \quad A = f(\alpha) \quad (12)$$

where $T, \alpha, \alpha_0,$ and A are the same as for the CC,

T_0 is the maximum reached over all lengths,

δ is SEC extension, δ_0 is the maximum possible,

e) Equations of the Movement

The experimental situation depicted in Figure 4 is that of an inertial load of the moment of inertia MI being rotated by the limb. According to the equations shown in (1) to (6) the torque at a given time t can be described by (13):

$$T(t) = f(A(t), \dot{\beta}(t), \alpha(t)) \quad (13)$$

α and β as shown in Figure 4.

Equation (13) can be solved for $\dot{\beta}$ once all the functions are described. From Figure 4 it also follows that the externally measured torque must be equal to that transmitted by the SEC, and is thus a function of the parameters describing the SEC in (7) to (12).

Further, the differential equation (14) also applies to the system:

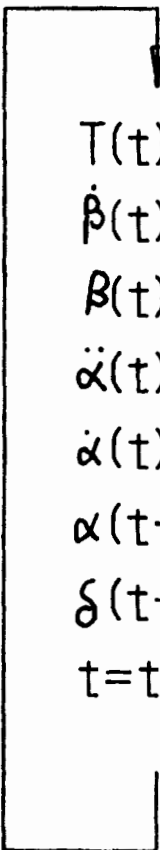
$$T(t) = M I x \ddot{\alpha}(t) \quad (14)$$

Not all equations describing the system can be solved simultaneously. The block diagram shown in Figure 6 therefore solves them in an iterative routine. The time course of the activation is known, or preset. SEC extension (δ) is initially zero and is set at the desired initial displacement. Using the known functions, the time course of the mechanical variables of the system can be calculated.

Figure 6. Iterative routine used in muscle model

T is torque, α , β , δ and MI as in Figure 4;

t is time, dt the time interval used for calculations.


$$T(t) = f(\alpha(t), \zeta(t), A(t))$$

$$\dot{\beta}(t) = f(T(t), A(t), \alpha(t))$$

$$\beta(t) = \beta(t-dt) + \dot{\beta}(t) \times dt$$

$$\ddot{\alpha}(t) = T(t) / MI$$

$$\dot{\alpha}(t) = \dot{\alpha}(t-dt) + \ddot{\alpha}(t) \times dt$$

$$\alpha(t+dt) = \alpha(t) + \dot{\alpha}(t) \times dt$$

$$\zeta(t+dt) = \beta(t) - \alpha(t+dt)$$

$$t = t + dt$$

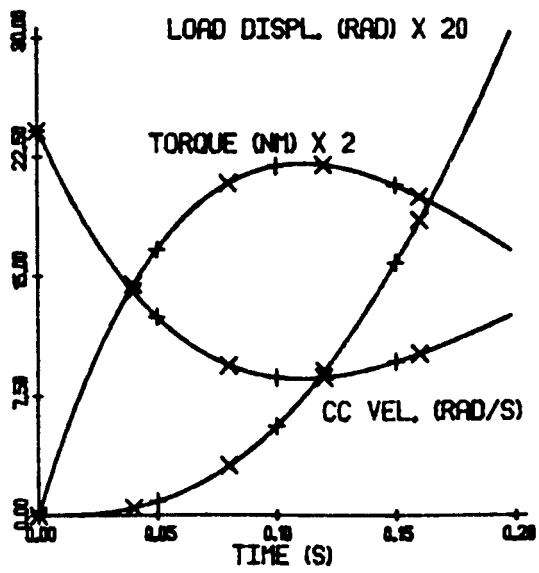
f) Verification of Computational Routine

The programme shown in Figure 6 is implemented in the computer language APL on an IBM 4341 computer. The user is asked to specify functions (1) to (12) as well as the experimental condition.

When using linear functions, the system becomes a second order-linear system and can be evaluated by solving the differential equations describing such a system. Appendix A shows these equations and their solution by using Laplace transforms. This then allows an estimation of the error introduced by the iterative routine itself. Figure 7 shows a comparison of the two solutions. Muscle parameters are those given by Houk (1963) for forearm supination. Moment of inertia is 0.1 kgm^2 . All calculated parameters show very close agreement. For the torque-time curves, the average error is 0.39%.

Since the current model does not depend on linearity of the functions, the error for a non-linear muscle is likely to be of the same magnitude.

Figure 7. Verification of computational routine
x model output, + calculated as described in Appendix A.



V. DETERMINATION OF MUSCLE PARAMETERS FOR FOREARM SUPINATION

a) Introduction

According to Wilkie (1950) the following points should be considered when analyzing human motion:

A geometrically simple joint should be chosen.
Only few muscles should be involved in the movement.
The movement should not disturb the rigid fixation of the rest of the body.
Only slight skill should be involved to allow high reproducibility. (Wilkie, 1950:249)

Forearm supination was chosen, since it does fulfil the above criteria and since the equipment to analyze this movement was available. Youm et al. (1979) described the biomechanical aspects of this movement in considerable detail. Houk (1963) presented a linear model for this movement. The model developed with the method described below will be non-linear.

According to the model of Figure 4, at all inflection points of the torque-time curve in a single contraction the external angular velocity is equal to CC velocity. Finding a large number of such inflections allows determination of the CC torque-angular velocity relationship. Reconstruction of the trials then allows determination of the SEC characteristics.

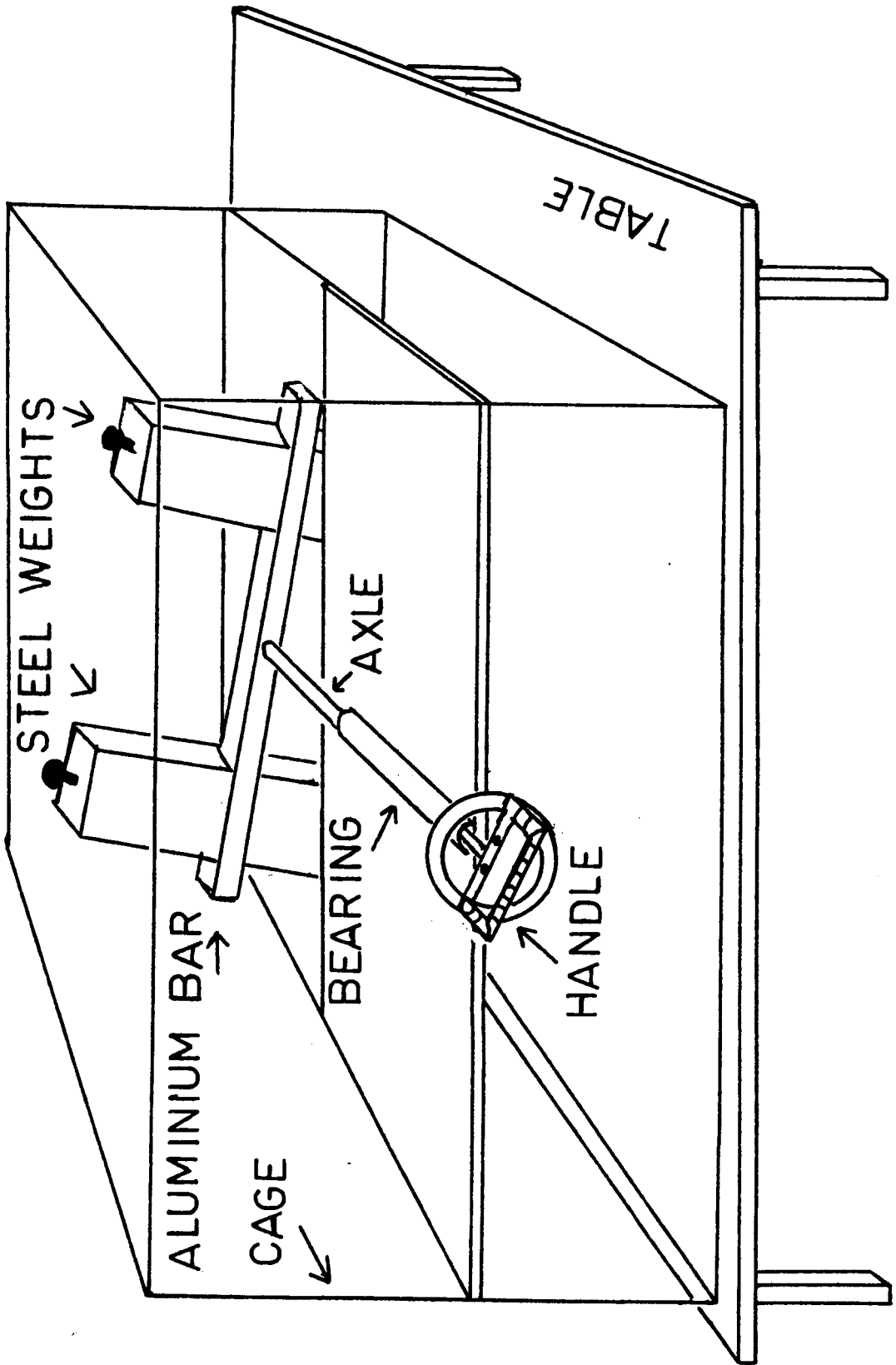
b) Materials and Methods

i) Apparatus

Four foil strain gauges (Type WA-06-250BG-120, linearity of strain 0 to 1/1000 of unstrained length)¹ were glued to a handle and formed the arms of a Wheatstone bridge circuit. The handle was connected to an aluminum rod by an axle. Balanced sliding steel masses on this rod, which was perpendicular to the axis of rotation, allowed variation of the moment of inertia of the system. The minimum was .03 kgm² and the maximum moment was .26 kgm². Figure 8 shows a sketch of the apparatus.

¹M-M, Romulus, Michigan

Figure 8. Sketch of experimental apparatus
The dots on the handle represent the strain gauges



A polarized light goniometer (POLGON)² was used to measure angular deflection of the handle. This device projected polarized light onto a stationary reference and one moving light detector. The support equipment allowed determination of the angle between the two detectors, however a 40msec time lag was introduced in the processing. A complete description of this apparatus can be found in Mitchelson (1976). Position was measured as the deviation from the horizontal (palm facing downwards when gripping the handle). Digital differentiation of the displacement trace provided velocity. The two traces, torque, and displacement were digitized at 200 Hertz and stored in a LSI11 Computer³. Starting position, effort level and moment of inertia were recorded manually.

When EMG was used, the signal was amplified, filtered (32-800 Hertz), and recorded on UV paper (TECA TE4)⁴. The width of the envelope around the record was taken as the measure of electrical activity.

ii) Experimental Procedure

The subject stood relaxed in front of the apparatus. The arm was fully adducted, the elbow rested in a U-shaped support to prevent any arm movement other than pronation/supination. The subject grasped the handle and was asked to perform a supination

²TECA, White Plains, N.Y.

³Modified version, TRACAN Electronics, Vancouver, B.C.

⁴TECA, White Plains, N.Y.

movement. Starting position was varied randomly from trial to trial. The subject was asked to exert maximal effort, three-quarter, half, and one-quarter of this. Therefore each complete series consisted of 80 trials (5 moment of inertias x 4 starting positions x 4 effort levels).

Isometric contractions were performed over the full range of movement to determine the isometric torque-angle relation, and the relation between exerted effort and produced torque. For each level of effort the maximal velocity was determined by attaching the POLGON sensor to the hand, and asking the subject to perform an unloaded contraction. For one subset of trials EMG was recorded to establish the relation between EMG and exerted effort. Subjects AC and RB completed two and five series of trials respectively.

c) Data Analysis

Data were entered into an IBM 4341 computer for further analysis with APL programs. Initially a matrix containing torque, velocity and displacement for those times when $d\dot{l}/dt=0$ (T =torque) was constructed. A piecewise linear or hyperbolic function fitted to the isometric torque-length relation described the isometric maximum for each length encountered.

A variety of procedures were tried for the fitting of the torque-angular velocity relation. Non-linear regression

utilizing BMDP⁵ or IMSL⁶ surface fitting routines was found to be too unstable for this application. The experimental results were also not fitted well by Hill's equation, as illustrated in Figure 9. The data were therefore fitted piecewise by low-order polynomials, which were then converted to one fifth order polynomial. Figure 9 illustrates the result of this fitting procedure.

This relation was then used in an iterative routine to determine SEC stiffness. According to the model in Figure 4, SEC length is the difference between CC and load displacement. CC velocity could be calculated from the fitted relationship, integration provided CC displacement and load displacement had been measured. Relating SEC length to torque allowed calculation of the stiffness of the SEC. The above procedure was repeated for every trial. A block diagram illustrating this routine is included as Figure 10. The APL programme used is included in Appendix B.

The fitting of the matrix and determination of SEC characteristics was done separately for each subject and level of activation.

⁵P series of Biomedical Computer Programs (BMDP) developed at the Health Sciences Computing Facility of the University of California at Los Angeles.

⁶International Mathematical and Statistical Library (IMSL), IMSL Inc., Houston, Texas.

Figure 9. Comparison of experimental data, and fitted curves,
using Hill's equation and procedure outlined in text.

* Observed velocities

	standard error	correlation
— Hill's curve	4.88	0.848
- - - fitted curve	0.31	0.999

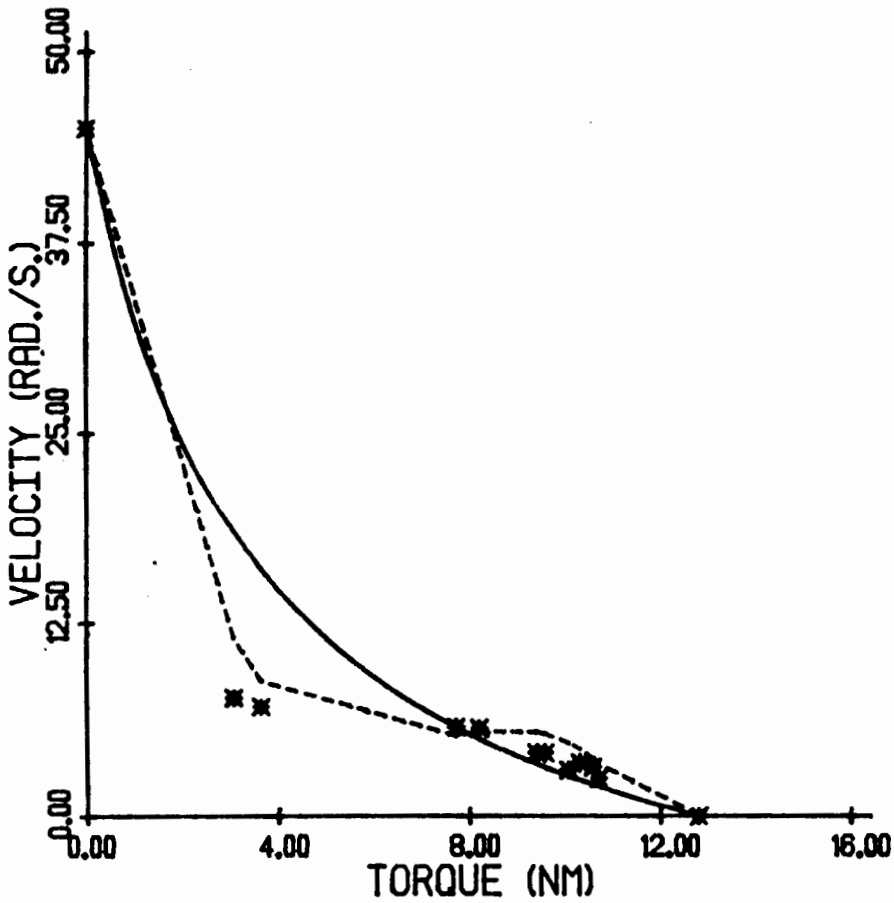
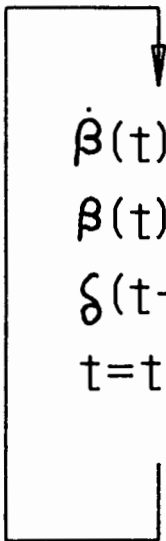


Figure 10. Block diagram of routine used to calculate SEC
extension. β is CC displacement, $\dot{\beta}$ CC velocity, α limb
displacement, t time and dt time interval used in calculations.



$$\dot{\beta}(t) = f(T(t), A(t), \alpha(t))$$

$$\beta(t) = \beta(t-dt) + \dot{\beta}(t) \times dt$$

$$\delta(t+dt) = \beta(t) - \alpha(t+dt)$$

$$t = t+dt$$

d) Results

To validate effort perception as a means of estimating activation, EMG was recorded for a series of contractions. Figure 11 shows the resulting curves in which 11a is the isometric EMG-torque relation, and 11b shows the EMG-effort and peak torque-effort relationships in dynamic and static contractions.

Figure 12 and 13 are sample records of all steps of the analysis. Here all trials are shown for one external loading condition and starting position. The torque-time and displacement-time curves are recorded, load velocity is differentiated displacement. The other curves are calculated once the CC characteristics have been determined, and are then used for SEC description.

Figure 11. Effort-EMG curves

- a) Isometric EMG/torque curves for four positions. EMG is corrected for length influences and therefore is represented as a fraction of unity. Also shown are values of EMG and torque at the four levels of effort and each position.
- b) Comparison of static and dynamic EMG-effort and torque-effort curves at one starting position.
-

a) Subject: AC

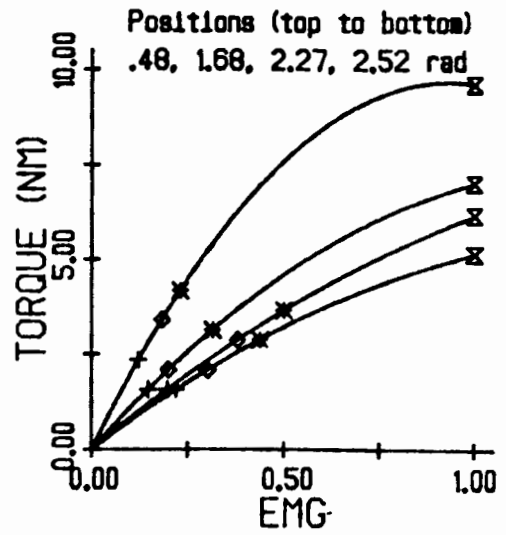
Effort levels:

⊗ 1.00

* 0.75

◇ 0.50

+ 0.25



b) Subject: AC

x static

□ dynamic

Position: 1.68 rad

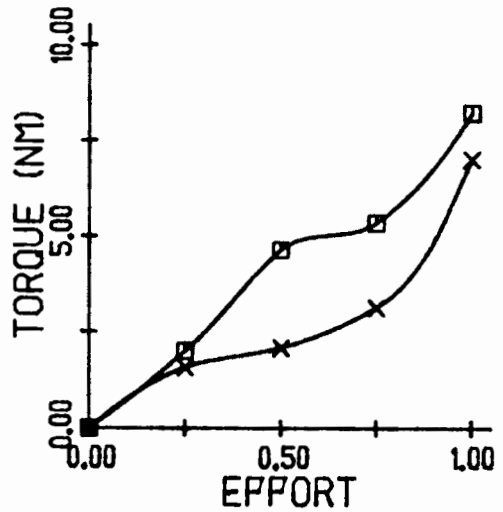
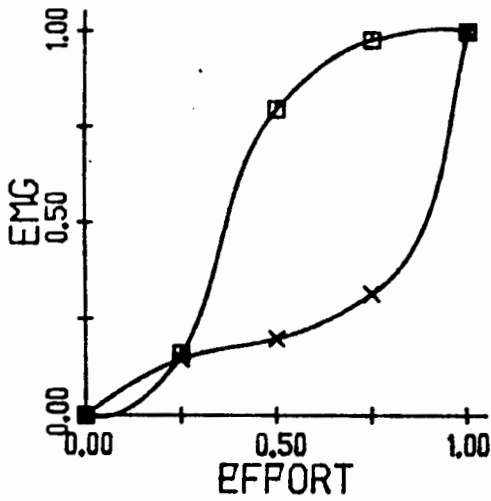
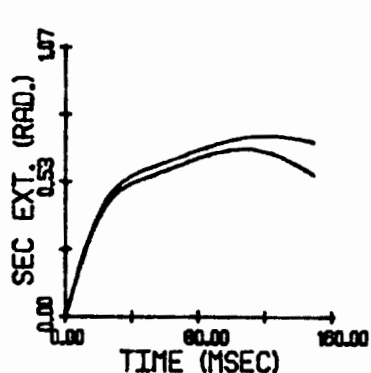
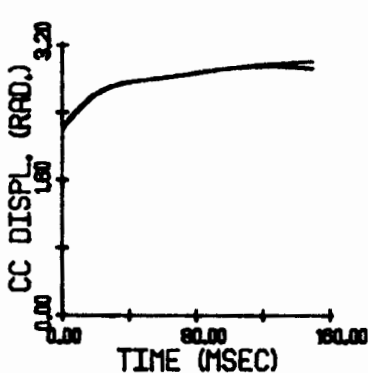
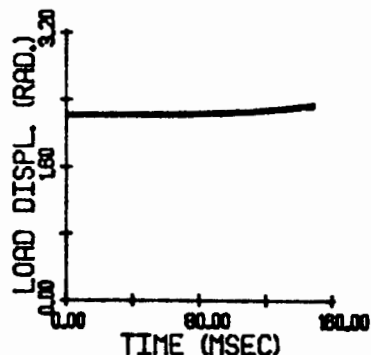
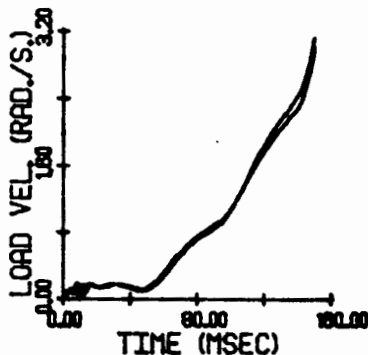
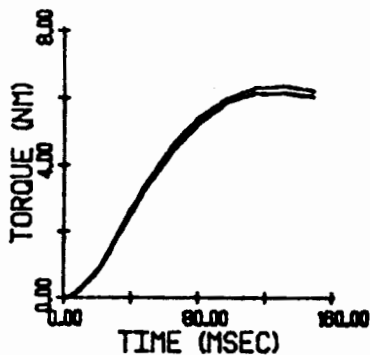


Figure 12. Sample recording and results of analysis for two
 trials of subject AC

Subject: AC
Effort: 1.0

Moment of inertia: $.21 \text{ kgm}^2$
Starting position: 2.27 rad



Subject: RB
Effort: 1.0

Moment of inertia: .03 kgm²
Starting position: 2.27 rad

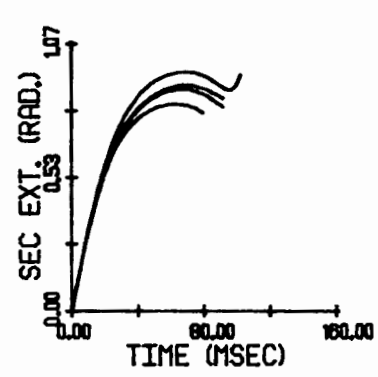
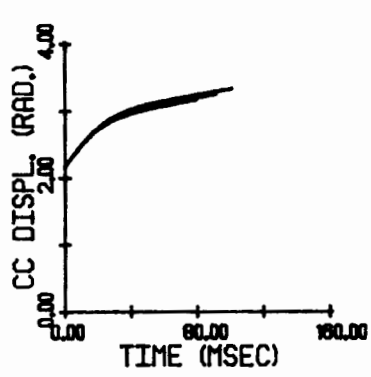
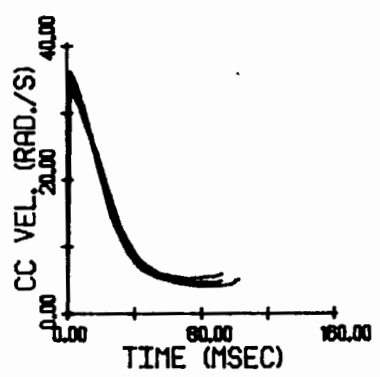
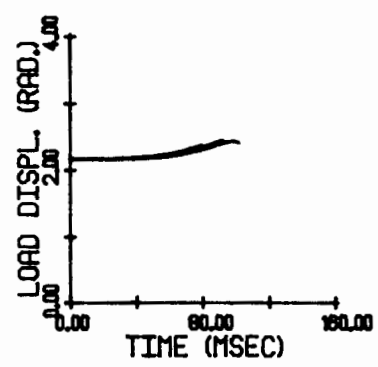
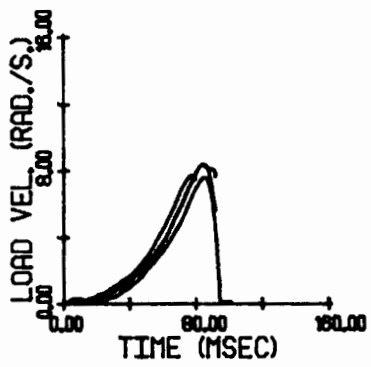
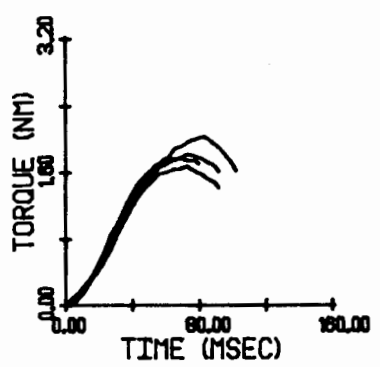


Figure 14 illustrates the wide dispersion of torques and velocities made available through the experimental technique. The peak torque, velocity at time of peak torque and displacement at time of peak torque are plotted against the 20 experimental conditions. Certain regular influences of length, external load and activation on torque and velocity are quite obvious from these results.

Figure 15 shows the determined isometric torque-angle relations for the two subjects. As can be seen in Figure 11b the dynamic torques at half and three-quarter effort were 1.8 and 1.6 times greater than static torques respectively. Thus the curves at half and three-quarter effort were multiplied by these factors. This raised them to a level which enveloped the dynamic peak torques in the same way as they did at full and one-quarter effort.

When determining the CC torque-angular velocity relation both torque and velocity have to be corrected for length influences. The influence of length on velocity is difficult to estimate. That there is such an influence nevertheless is illustrated by Figures 16 and 17. Here only the torques are corrected for length influence, and consistent variation in velocities with length is apparent. Figures 18 and 19 show the families of uncorrected torque-angular velocity relationships determined for the subjects. Note that both maximal CC velocity and isometric torque vary with limb position.

Figure 14. Average peak torque, velocity, and displacement at
time of peak torque for all conditions.

a) Subject AC, b) Subject RB

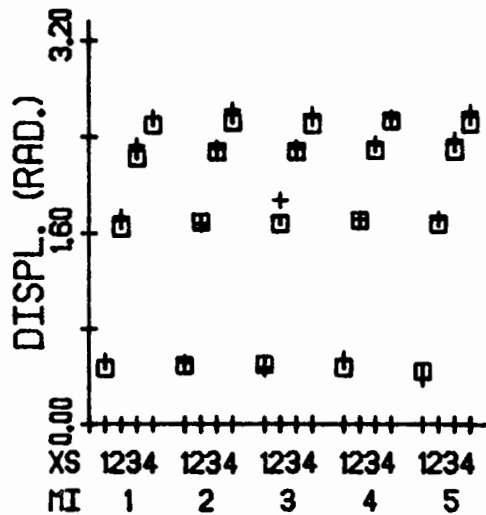
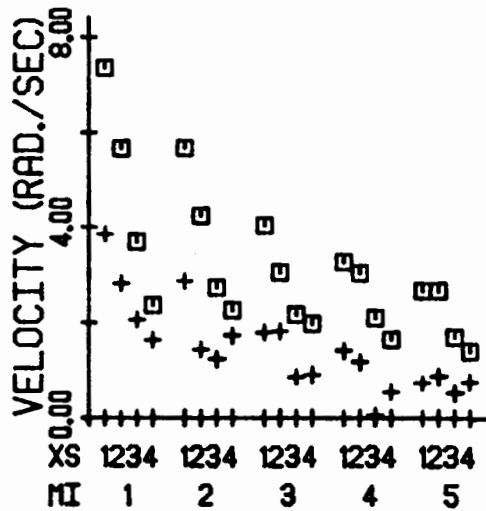
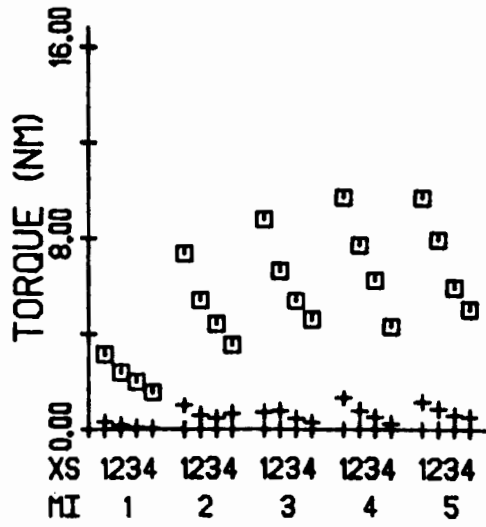
XS denotes starting position, 1 is .48 rad, 2 is 1.68 rad, 3 is
2.27 rad and 4 is 2.52 rad.

MI denotes moment of inertia, 1 is .03 kgm², 2 is .07 kgm², 3 is
.14 kgm², 4 is .21 kgm² and 5 is .26 kgm².

a) Subject: AC

□ Full effort

+ Quarter effort



b) Subject: RB

□ Full effort

+ Quarter effort

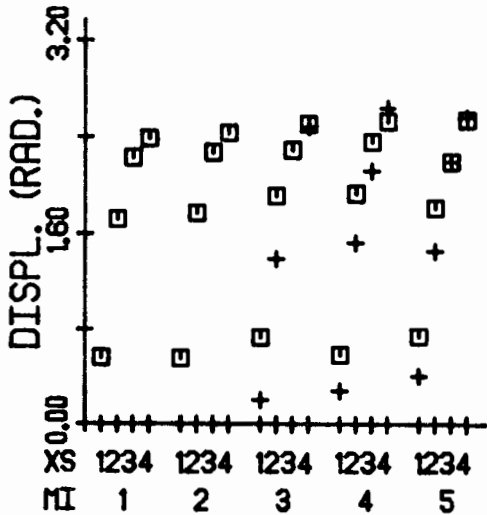
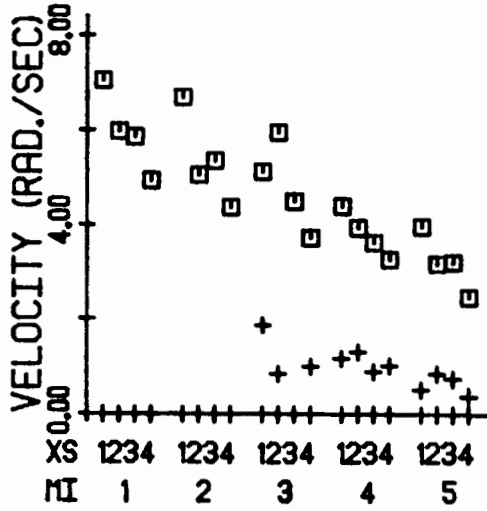
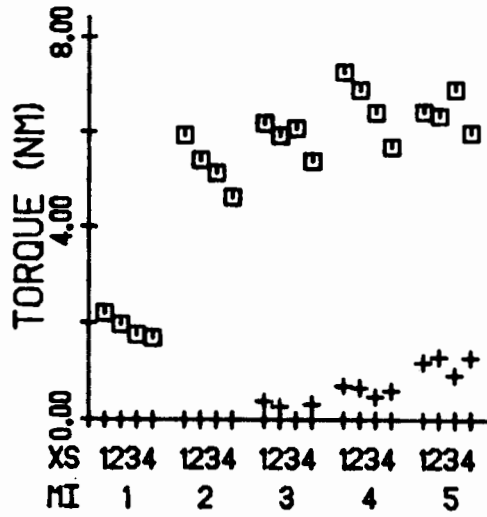
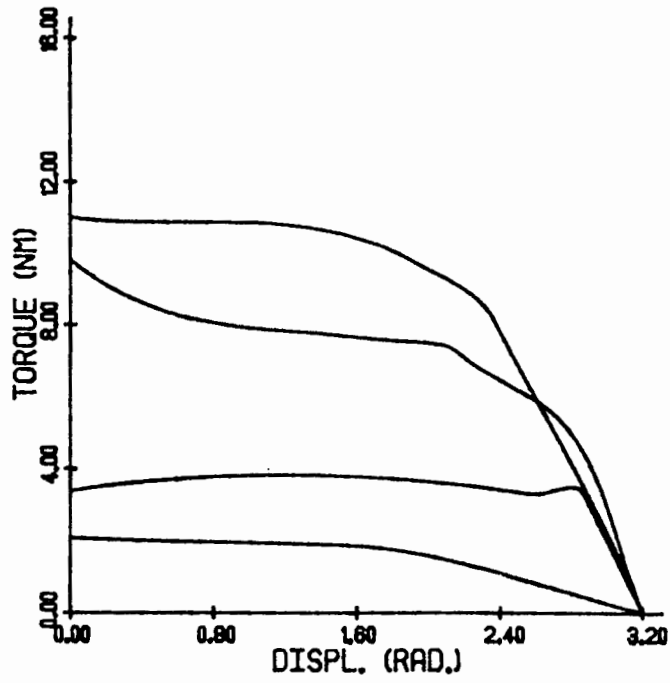


Figure 15. Families of isometric torque-displacement curves
All four effort levels are shown, from top to bottom in the
order of 1.0, 0.75, 0.5, 0.25.

a) Subject: RB



b) Subject: AC

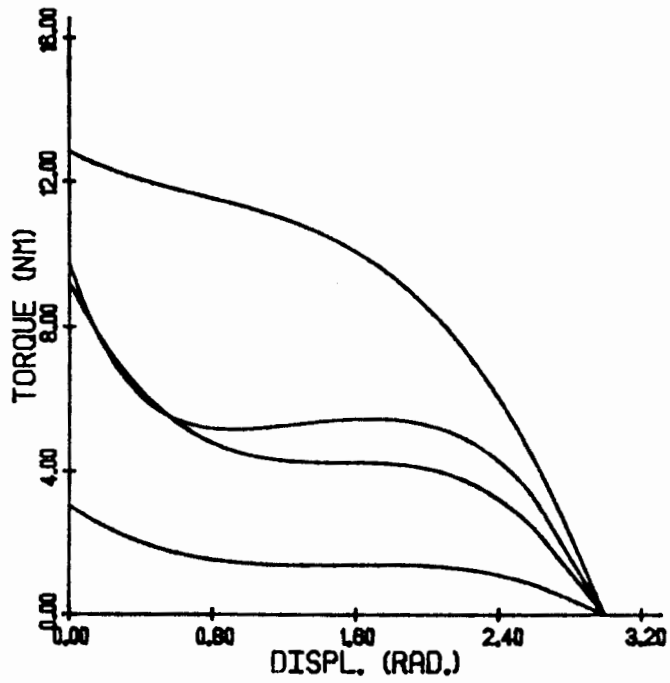


Figure 16. Velocities achieved at peak torque against peak
torque corrected for length influences

Subject AC

Subject: AC

Position:

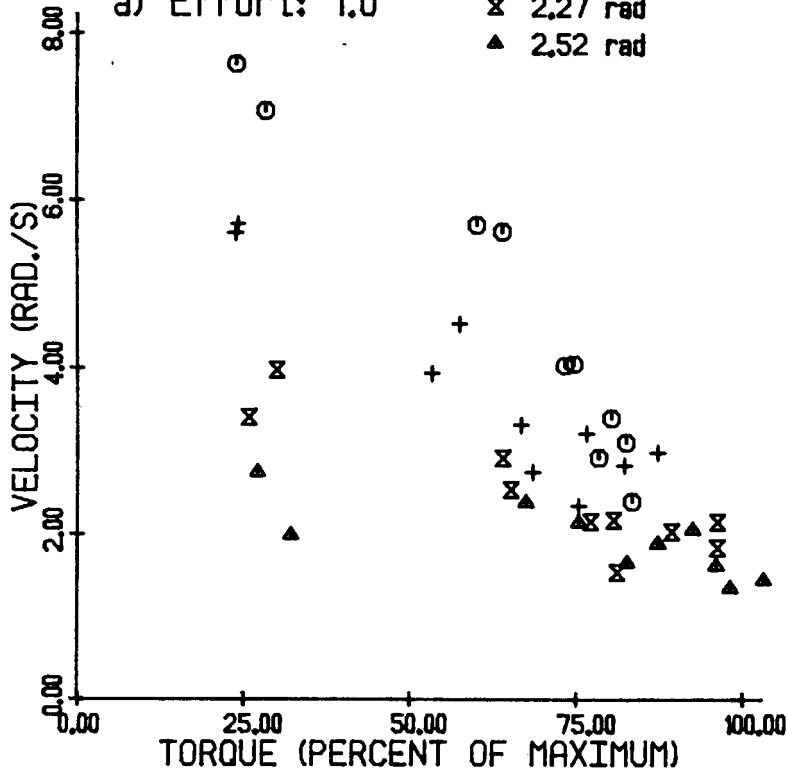
○ .48 rad

+ 1.68 rad

x 2.27 rad

▲ 2.52 rad

a) Effort: 1.0



b) Effort: 0.5

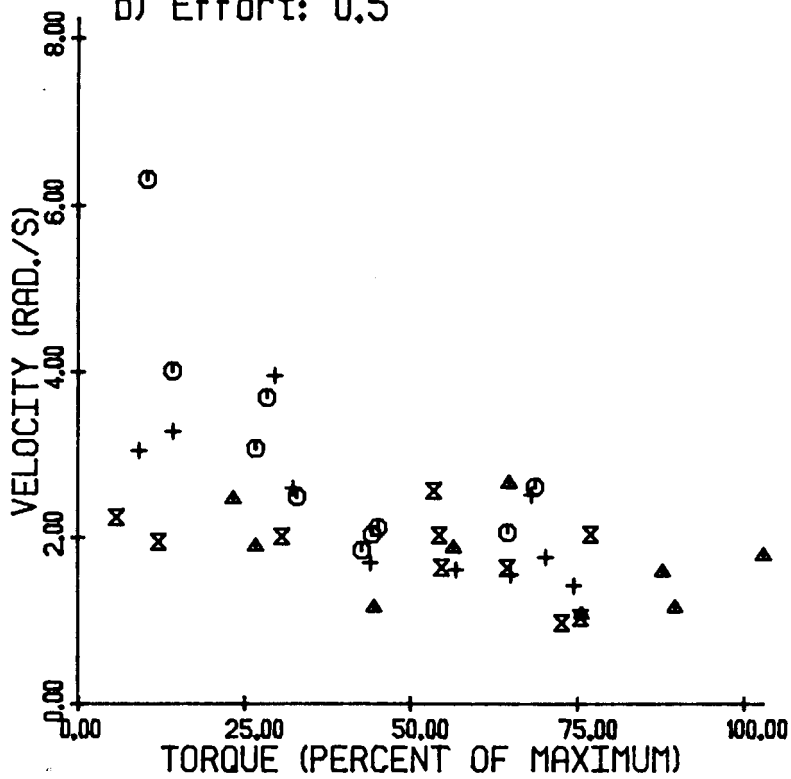


Figure 17. Velocities achieved at peak torque against peak
torque corrected for length influences

Subject RB

Subject: RB

Position:

○ .48 rad

+ 1.68 rad

× 2.27 rad

▲ 2.52 rad

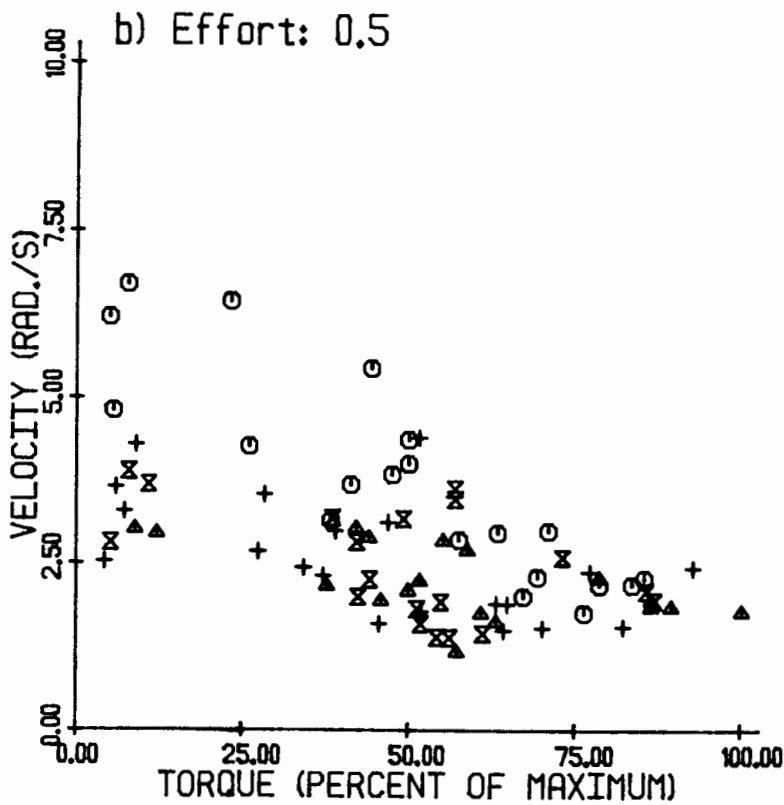
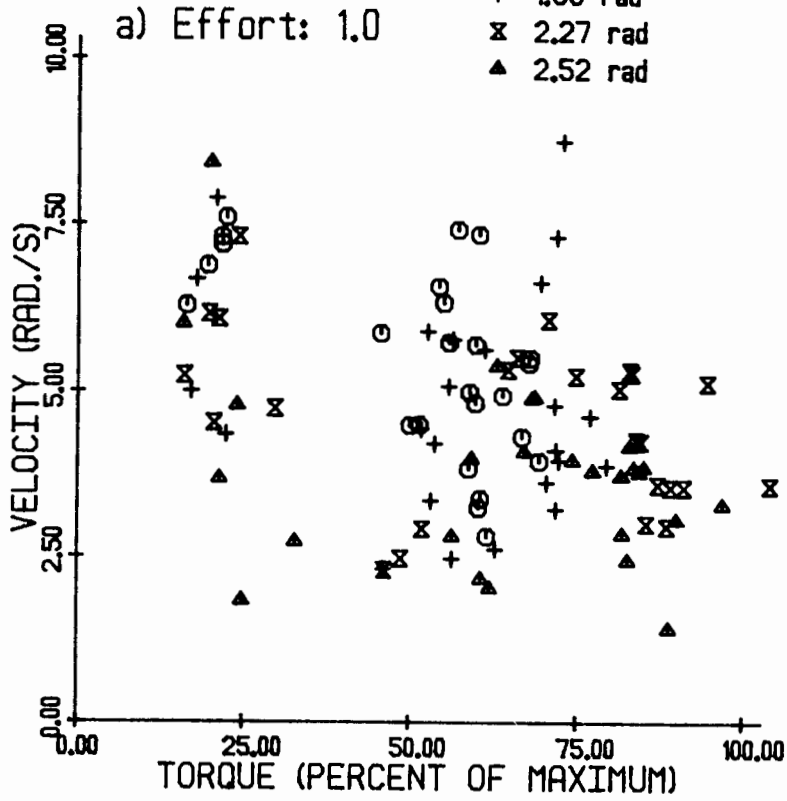


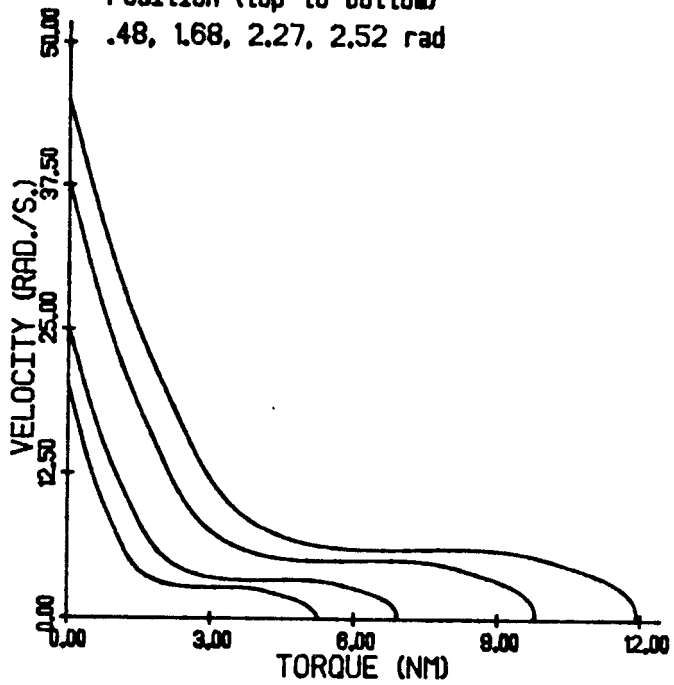
Figure 18. Families of torque-angular velocity curves of the CC
at four positions and effort levels: Subject AC

Subject: AC

Effort: 1.0

Position (top to bottom)

.48, 1.68, 2.27, 2.52 rad



Position: .48 rad

Effort (from top to bottom)

1.0, 0.75, 0.5, 0.25

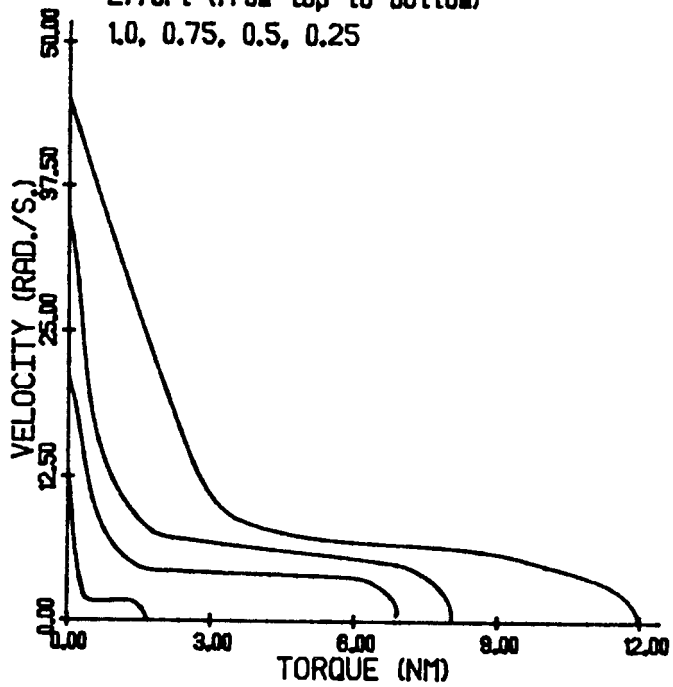


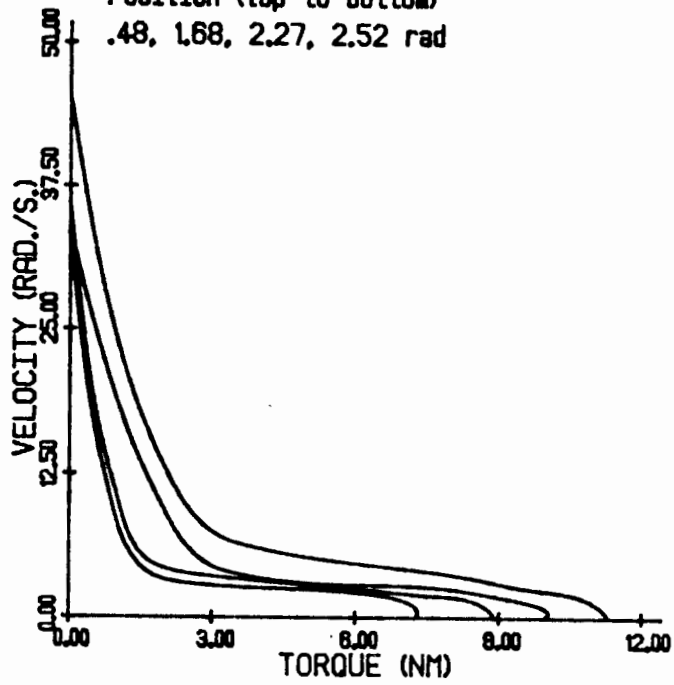
Figure 19. Families of torque-angular velocity curves of the CC
at four positions and effort levels: Subject RB

Subject: RB

Effort: 1.0

Position (top to bottom)

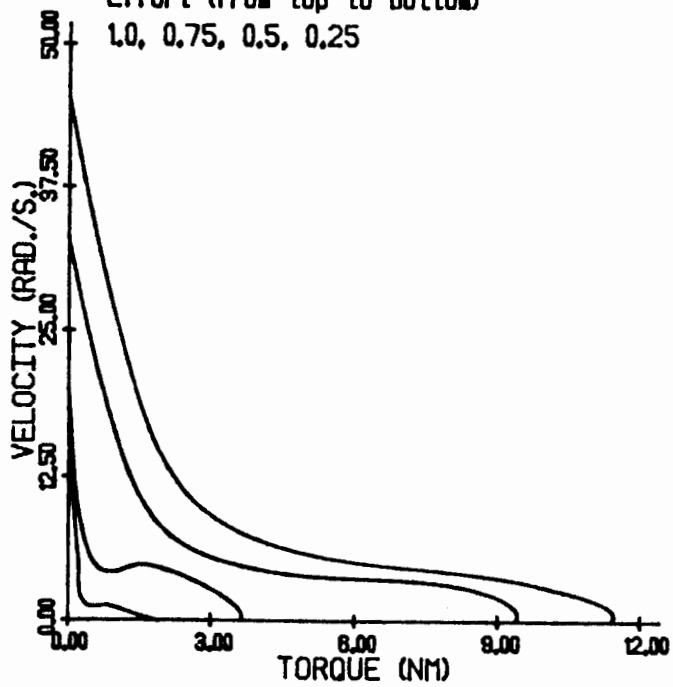
.48, 1.68, 2.27, 2.52 rad



Position: .48 rad

Effort (from top to bottom)

1.0, 0.75, 0.5, 0.25



The next step was the determination of SEC characteristics. For each trial a torque-SEC extension curve was obtained. Figure 20 shows some examples. In 20a series of trials at one position and all 5 inertias are compared. There is some irregular variation with moment of inertia in apparent SEC characteristics. Only at the lowest moment of inertia did the SEC consistently appear very compliant. Figure 20b illustrates the much clearer length influence on SEC characteristics. The very typical turnover of the curves can also be seen in Figure 20a. This will be discussed in more detail later. Figure 21 and 22 show the determined SEC characteristics for both subjects.

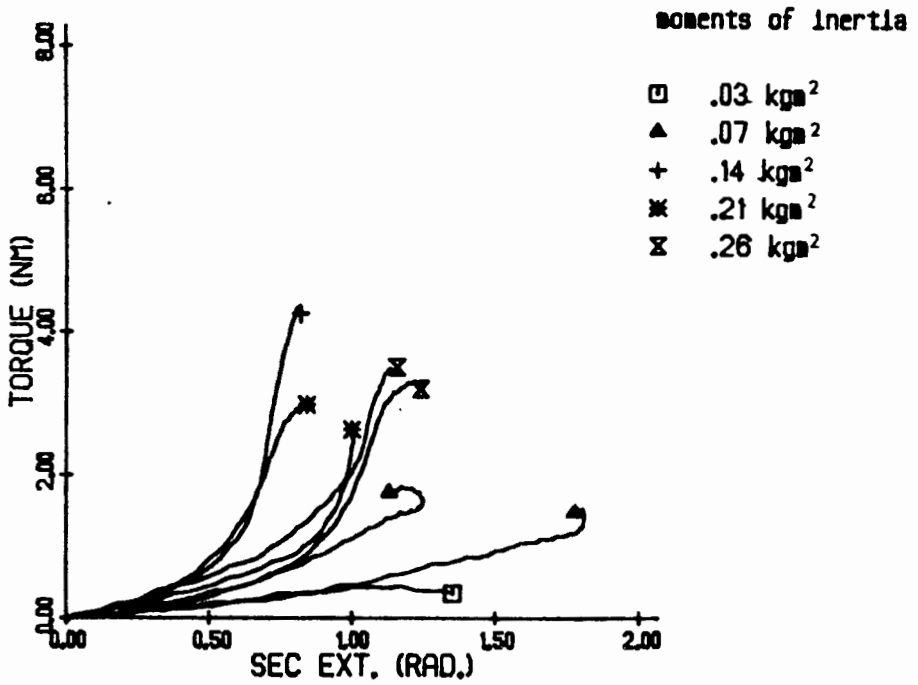
As outlined in the Model section, 12 relationships are required to completely describe the muscle model. These are collected in Figures 23 to 26.

Figure 20. Examples of individual torque-SEC extension curves as
determined by analysis.

Because of recording errors two trials of the conditions
depicted in Figure 20b had to be ignored.

Subject: AC Effort: 1.0

a) starting position: 1.68 rad



b) moment of inertia: .14 kgm²

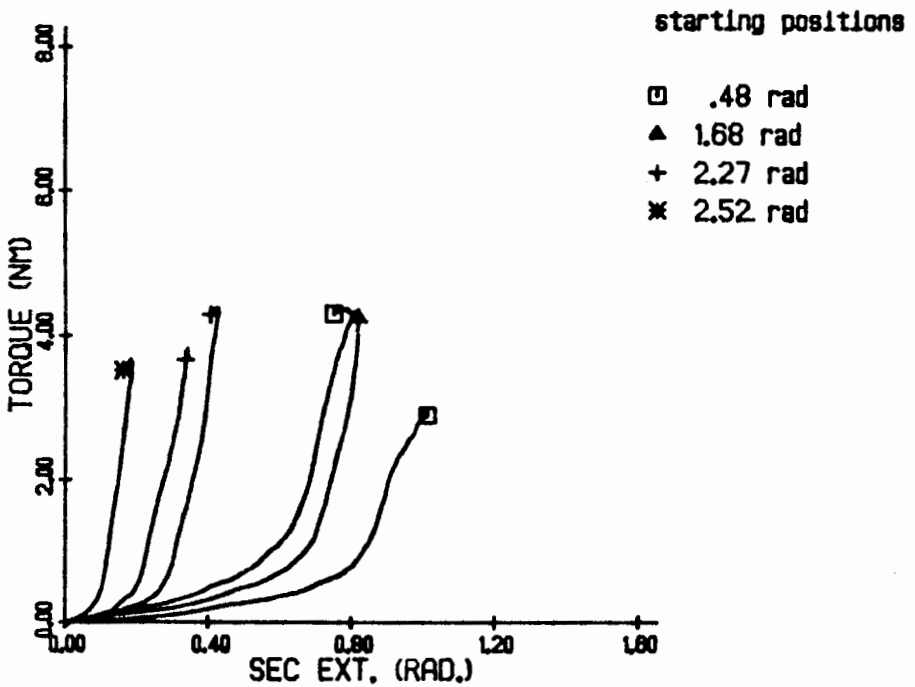


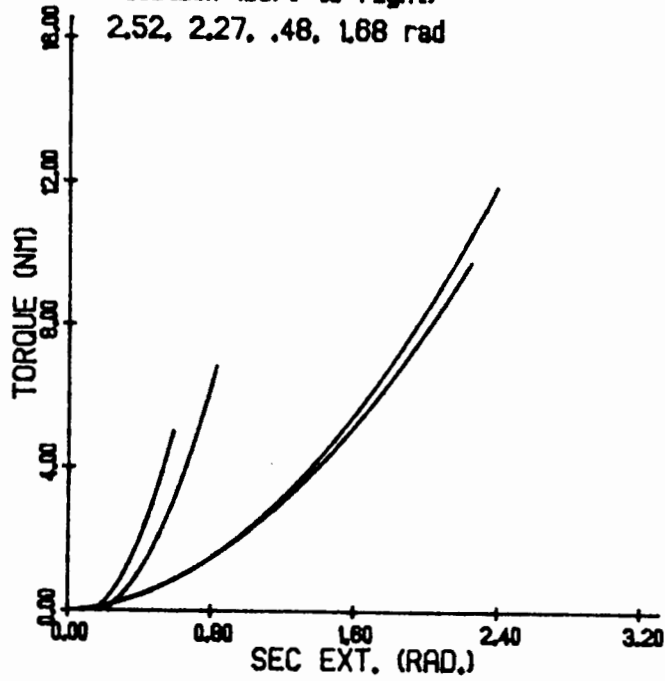
Figure 21. Families of torque-SEC extension curves for four
positions and effort levels: Subject AC

Subject: AC

Effort: 1.0

Position (left to right)

2.52, 2.27, .48, 1.68 rad



Position: .48 rad

Effort (from left to right)

1.0, 0.75, 0.25, 0.5

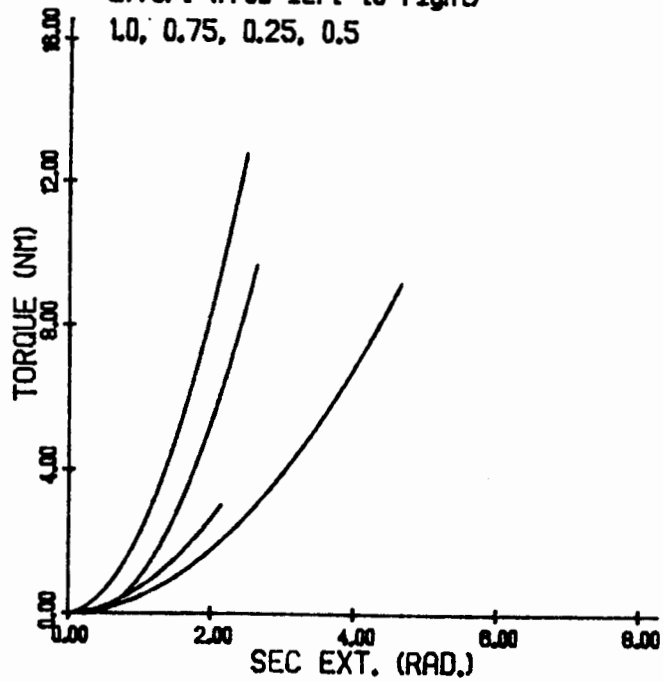


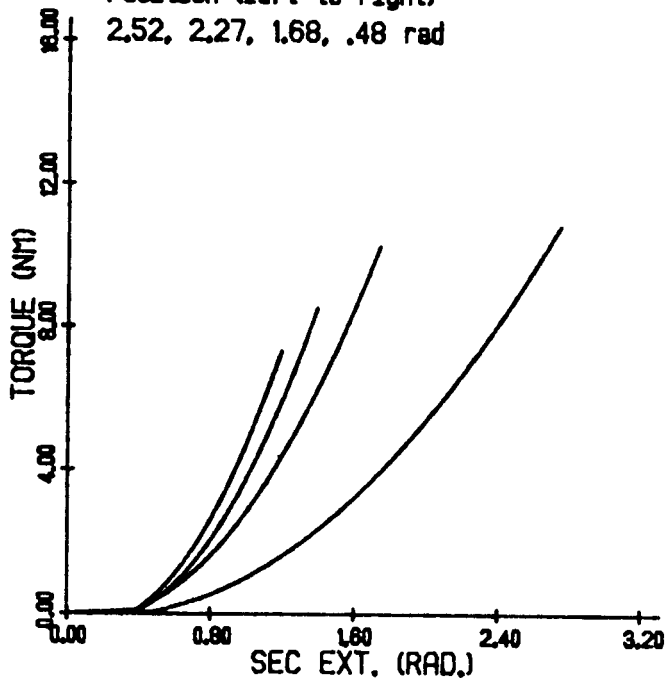
Figure 22. Families of torque-SEC extension curves for four
positions and effort levels: Subject RB

Subject: RB

Effort: 1.0

Position (left to right)

2.52, 2.27, 1.68, .48 rad



Position: .48 rad

Effort (from left to right)

0.5, 0.75, 1.0, 0.25

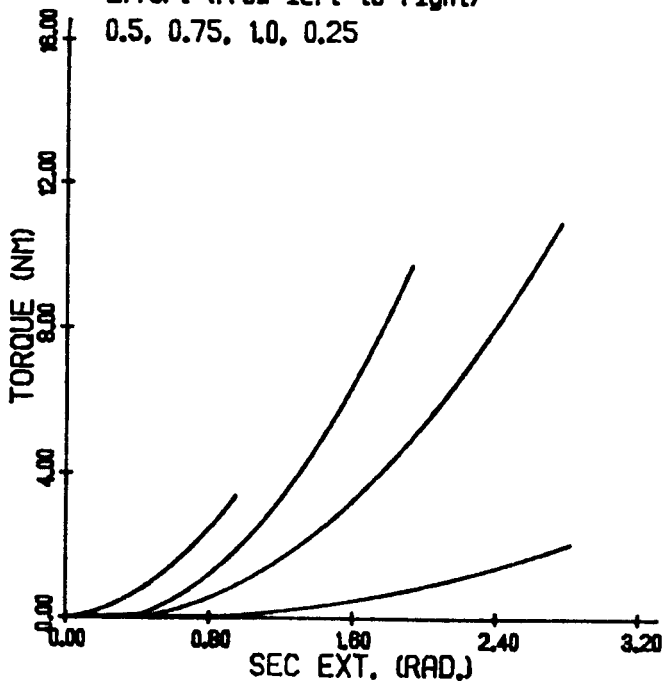


Figure 23. Complete description of CC properties of muscles
involved in forearm supination for subject AC.

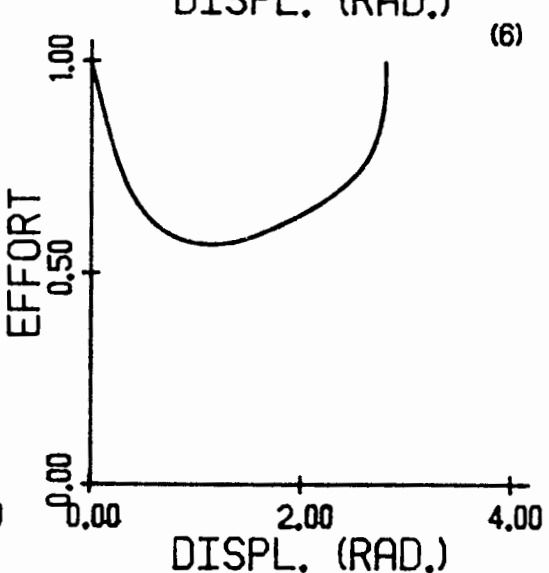
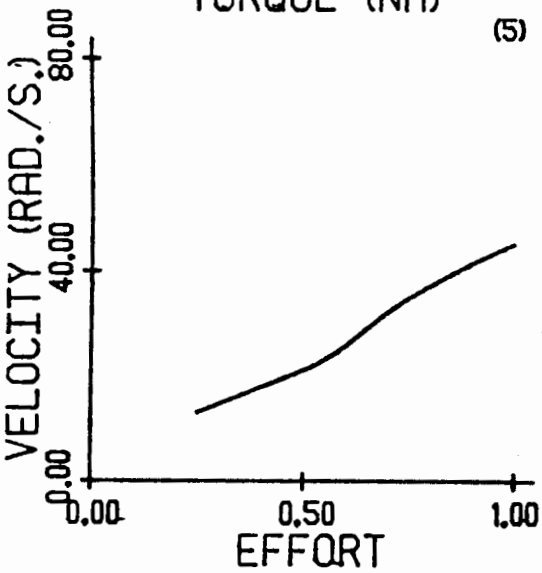
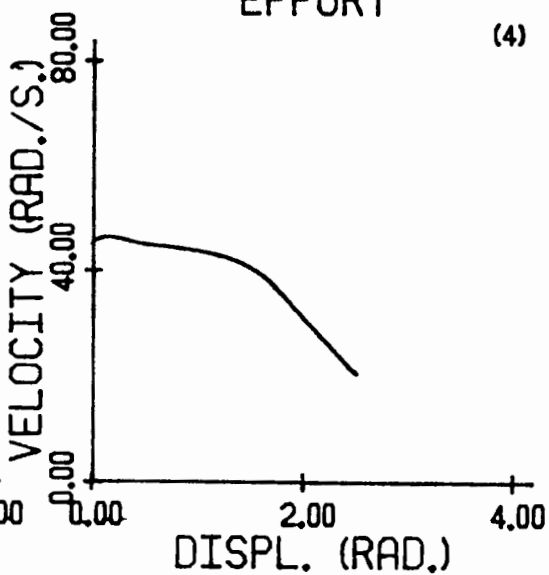
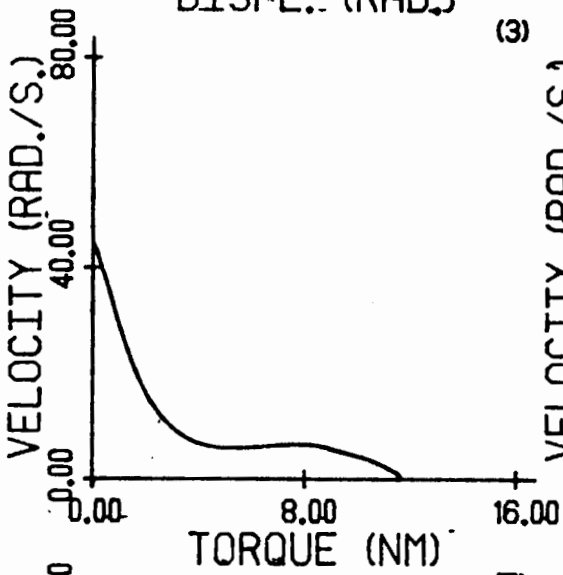
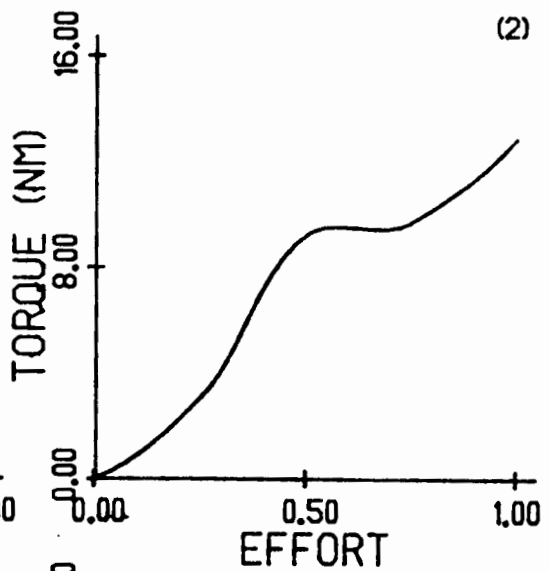
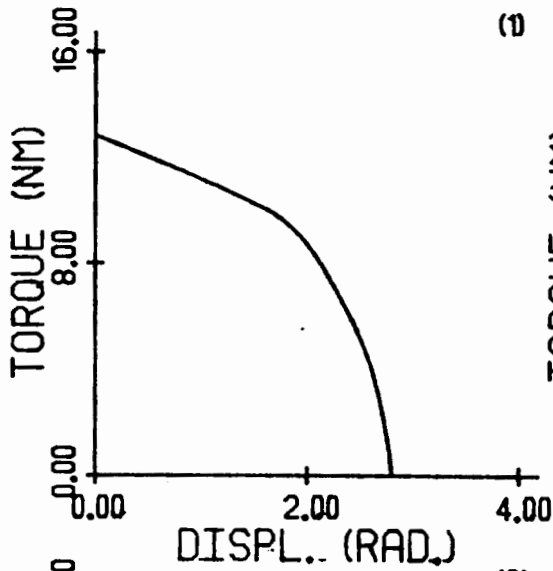


Figure 24. Complete description of SEC properties of muscles
involved in forearm supination for subject AC.

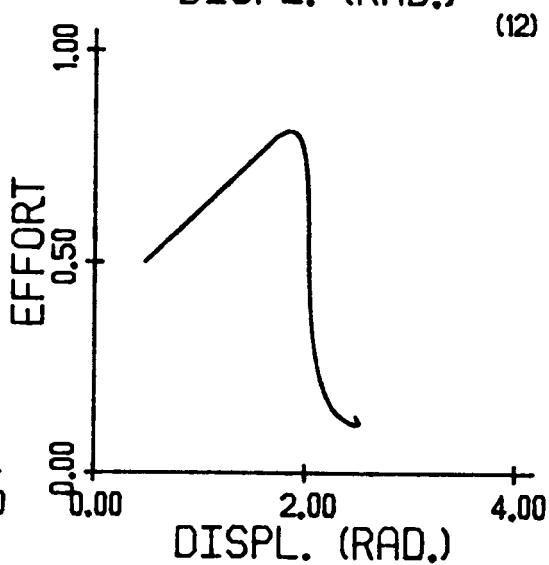
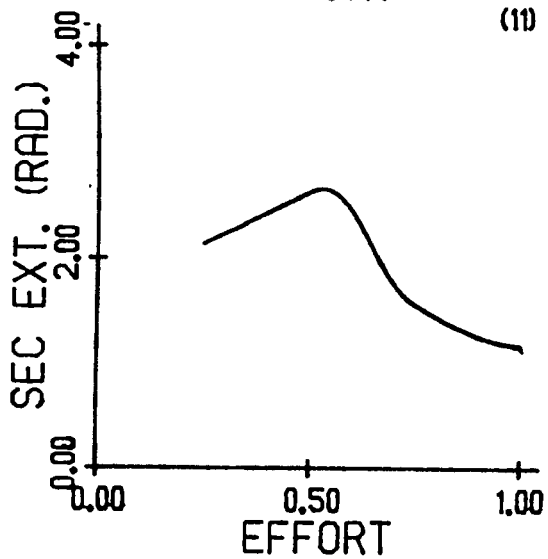
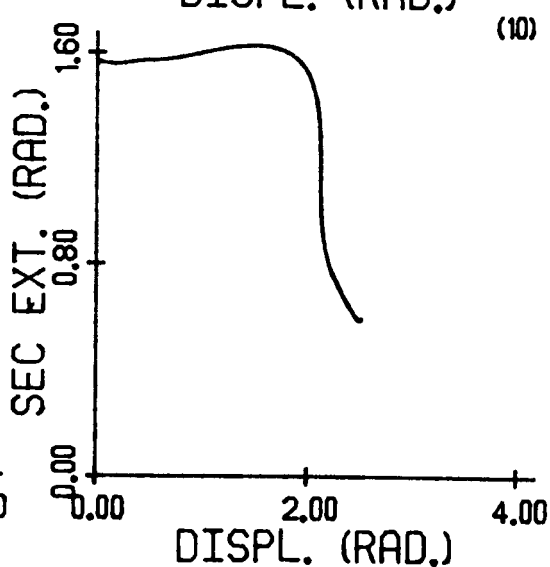
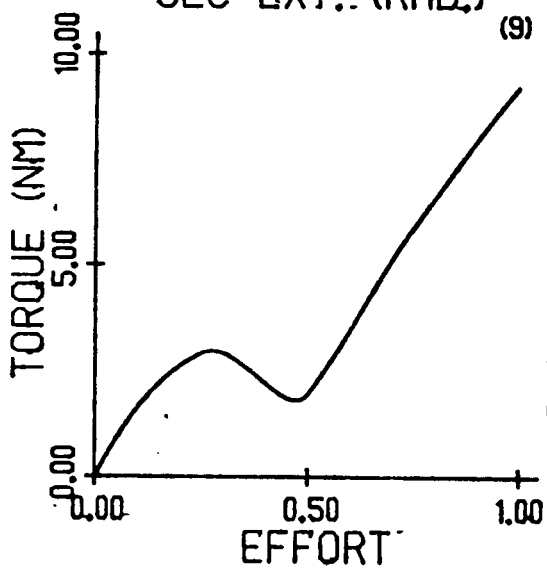
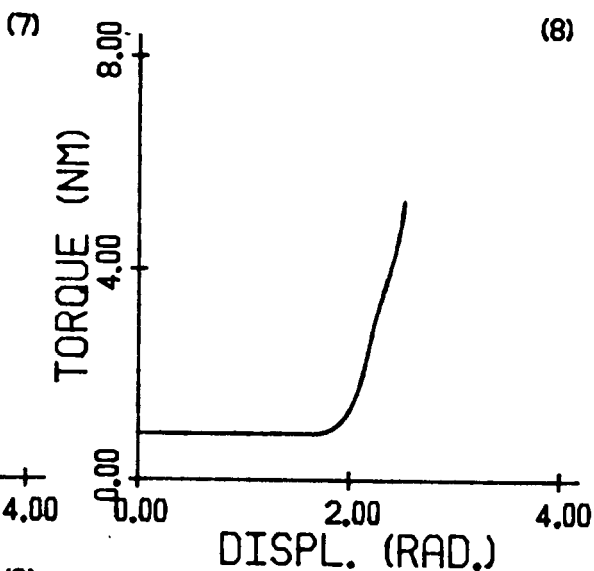
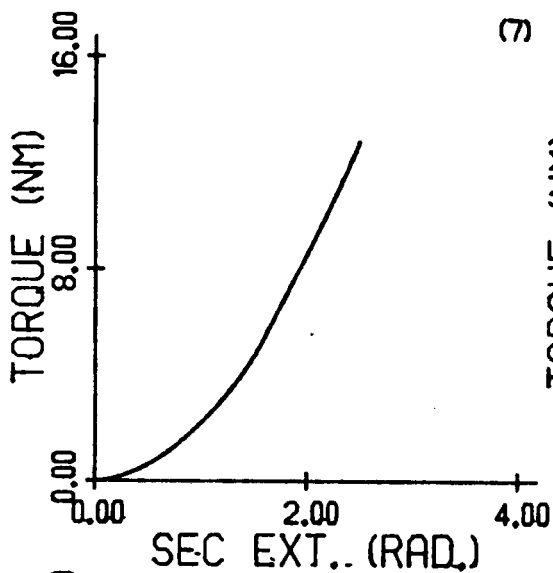


Figure 25. Complete description of CC properties of muscles
involved in forearm supination for subject RB.

Subject: RB

CC description

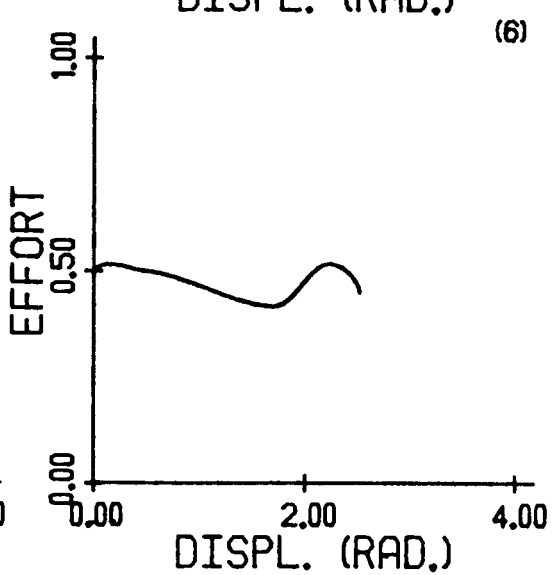
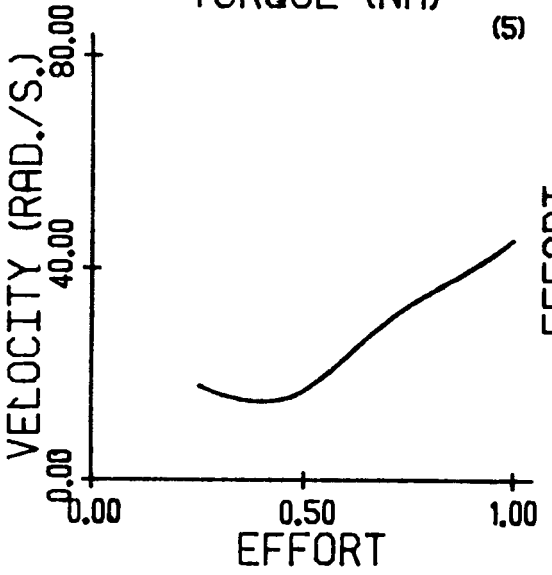
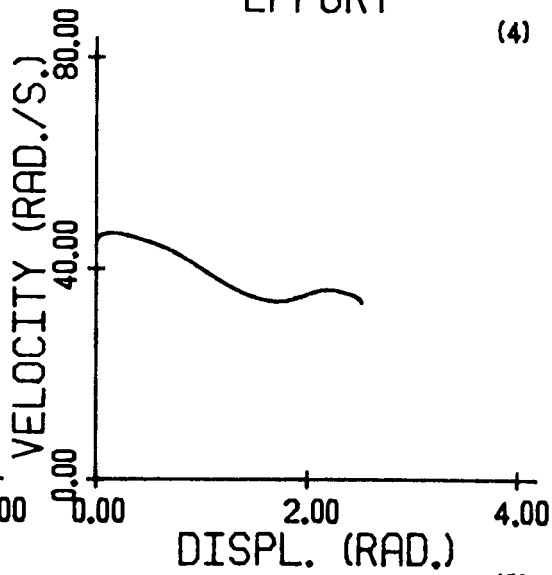
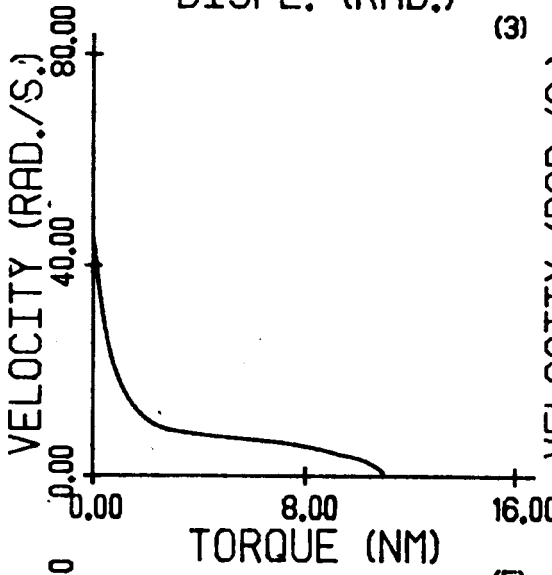
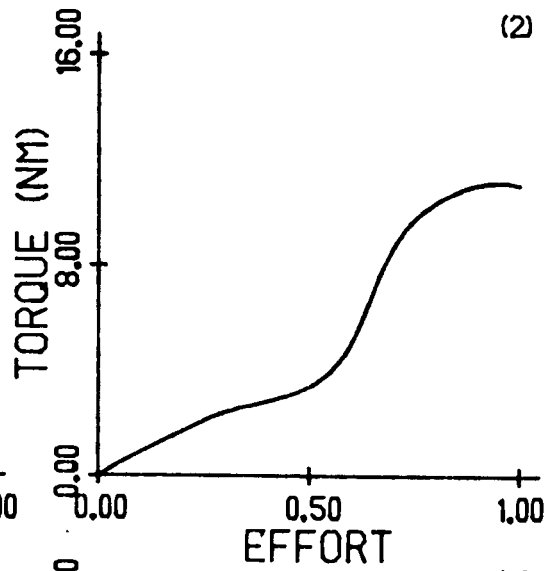
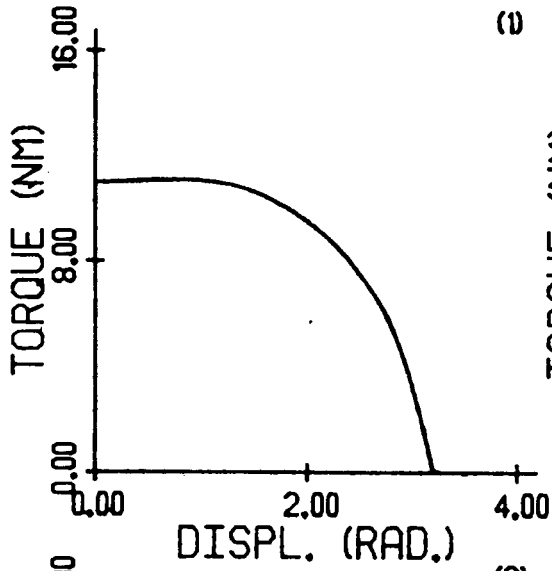
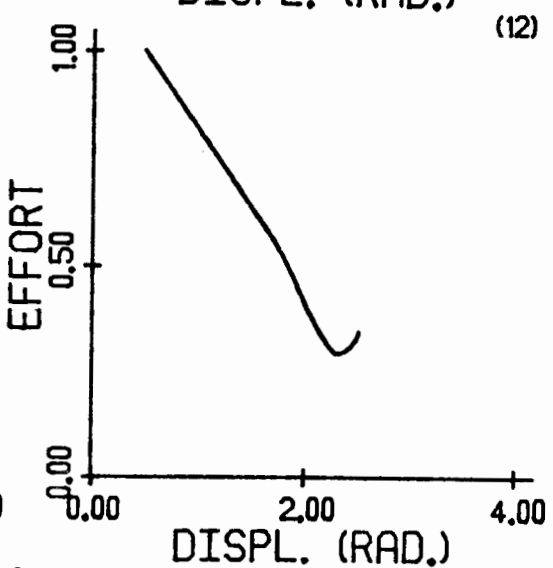
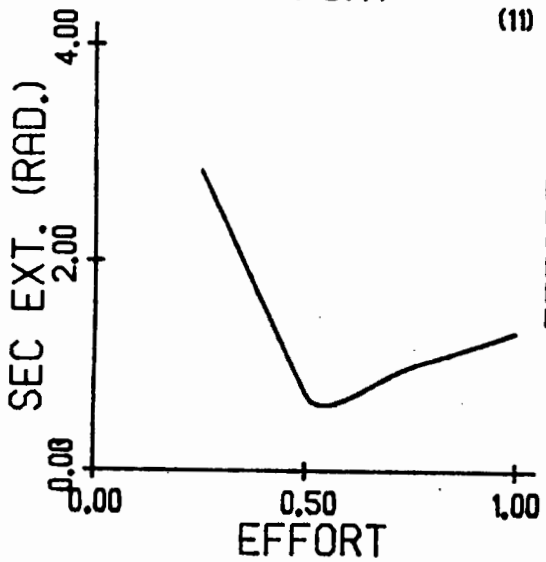
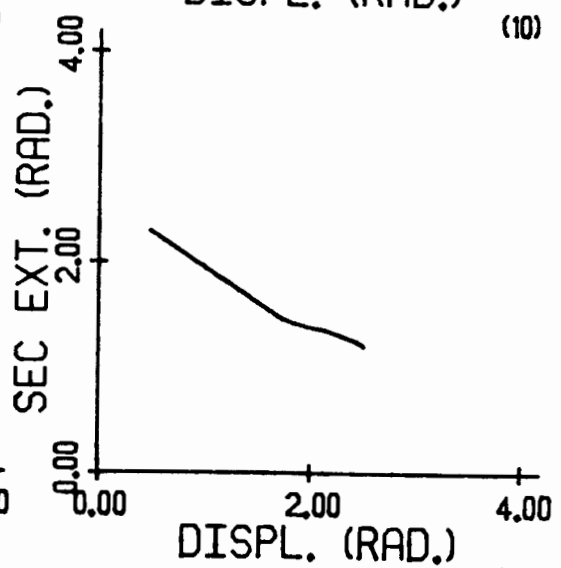
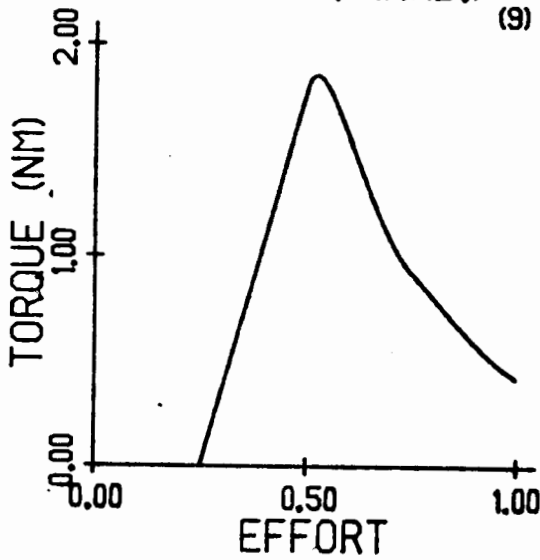
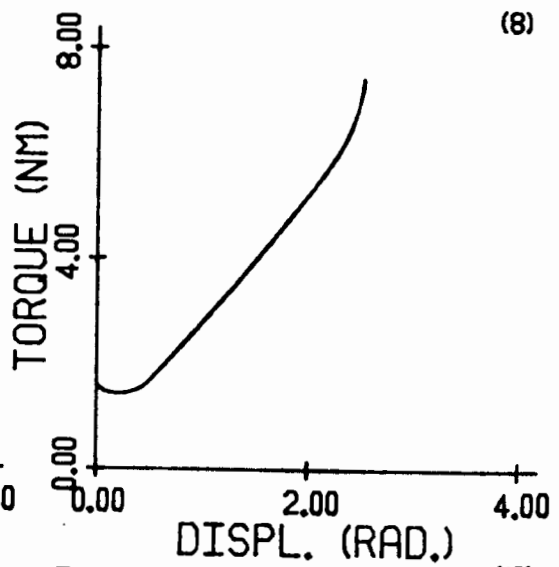
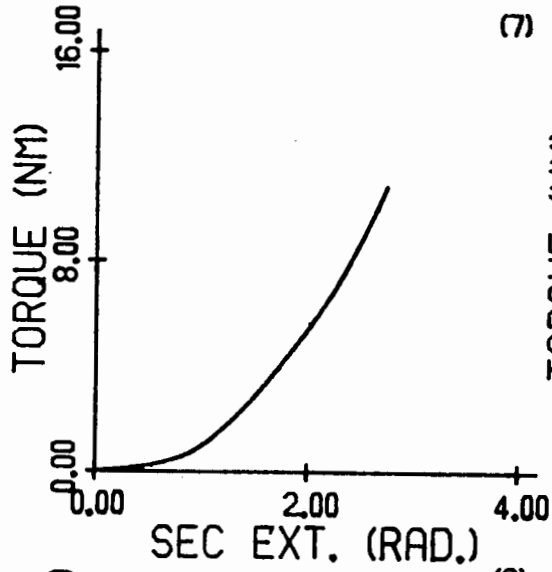


Figure 26. Complete description of SEC properties of muscles
involved in forearm supination for subject RB.



e) Discussion

i) Variation between Subjects

Due to the large amount of data collected and the extensive analysis performed only two subjects were tested. However, since one (AC) is very trained in the movement analyzed, whereas the other (RB) was more or less a novice, some interesting differences became apparent. Most obvious is the much greater regularity of repeated trials in AC, when comparing torque-time curves (Figures 12 and 13). Even for maximal effort, RB shows variation not only in the peak torque achieved, but also in the rise of torque, ie. the peak torque is shifted in time. This was very rarely observed for AC. Since this phenomenon is apparent at all levels of effort, it is likely not due to a difference in the accuracy of effort perception, but may be related to the level of training. The much greater repeatability of trials for AC was apparent throughout all experimental conditions, though variations increased for both subjects when performing at submaximal effort.

ii) Validity of Effort Perception as a Measure of Activation

Effort perception is used as an activation parameter since it seems very close to real life situations. We generally do not decide to what level of EMG we want muscle activity to rise, though this can be learned very well, but rather estimate the effort necessary to achieve our goal. The good repeatability when using effort perception that has been reported in the literature (see review) was also observed here. The fact that a constant transfer function between effort and EMG can be established validates the measure further (see Figure 11). Activation is the most difficult parameter to estimate in human contractions, and though effort perception seems to give better results than other methods, this still remains the weakest aspect of the modelling process.

The very large difference between static and dynamic effort (Figure 11b) is surprising. Cooper et al. (1979, Fig. 4) also show some difference, but not to any comparable degree. However their velocities were much lower than the ones achieved in this experiment. The reason for the difference between static and dynamic effort perception is open to speculation. EMG differs by a factor of up to 6, whereas produced torque varies only by a factor of 1.4 to 1.8. This could point to the possible attempt to achieve about the same torque level in the dynamic situation

as in the static trial at the same effort level. For this set of experiments, the only requirement of the estimation of activation is constancy from trial to trial. Other speculations about effort perception are therefore left to be substantiated by other experimenters.

It should also be noted that the peak dynamic torque reached in the dynamic contraction shown in Figure 11b is slightly above the isometric maximum. This occurrence is discussed in more detail later.

iii) Validity of Experimental Procedure

One goal of the experimental design was to achieve a wide dispersion of torques and velocities. Figures 14, 16 and 17 illustrate how successful this was. Dynamic torques recorded reached from just detectable to very close to (or even above) isometric maximum. The torques were also very evenly spread throughout this range. Nevertheless, a considerable gap between the largest angular velocity recorded in the sets of experiments, and the maximal unloaded velocity remained. This region of the torque-angular velocity relation has to be assumed linear. The two turning points seen in the torque-angular velocity curves are however within the range of experimental data.

iv) CC Characteristics

As expected, isometric torque depends considerably on muscle length, which was measured here by angular displacement. It is interesting that this influence becomes less important over a large segment of the total range at the lower effort levels (Figure 15).

That velocity is influenced by muscle length is illustrated by Figures 16 and 17. For AC the influence is very clear at full effort, but not so strong for lower effort levels. The wider spread of trials obscures the relation somewhat for RB, but it can be seen well in Figure 17b. The influence of position on velocity was determined from these results and then extrapolated to maximal velocity. The maximum reached in the unloaded contraction is set equal to maximal CC velocity; this is likely to be a conservative estimate. Figures 18 and 19 show that the torque-angular velocity relation of the CC varies fairly regularly with both position and effort. Despite the inherent non-linearity of the curves, they contain substantial linear segments.

The behaviour of the CC close to zero velocity is very different from that reported by other researchers. These curves cannot be fitted by Hill's equation (Figure 9). Pringle (1960) pointed to the discontinuity of the torque-velocity relation at zero velocity. This discontinuity seems shifted to a higher

velocity in muscles participating in forearm supination. The occurrence of dynamic torques which are larger than isometric torques shows that this is not the result of the fitting procedure or some other error in the analysis. According to accepted muscle physiology this seems impossible. However, with curves shaped like those shown in Figures 18 and 19, a very small error in the estimation of isometric torque could result in this situation.

Maximal CC velocity is taken here as the maximal velocity reached in an unloaded contraction. This estimate is likely to be too low. However, even if CC maximum velocity is double that used here, the influence on the results is comparatively small as shown in Figure 27.

Also, since activation is unlikely to rise instantaneously, neither will CC velocity. Therefore it is necessary to estimate the rise time. This is the only parameter estimated in the model, and as Figure 28 illustrates the use of either a very small value or the very high estimate of 10 msec results only in a small shift of the torque-SEC extension curve. In both Figure 27 and Figure 28 the stiffness of the SEC is unaffected over a large part of the range of its extension. During any contraction the CC operates close to the maximal velocity for a very short period of time. This is the reason why the effect induced by the rise of activation and maximal CC velocity is confined to the early stages of the contraction only. As the early stages of the

contraction are likely to involve compression of the soft tissues of the hand, the true SEC stiffness of the muscles is likely to be ill defined and errors in estimation of activation rise time and maximal CC velocity are therefore irrelevant.

Figure 27. Effect of assuming higher maximal CC velocity on the
torque-SEC extension relationship

max. CC velocity:

— 11 rad./s.

- - - 22 rad./s.

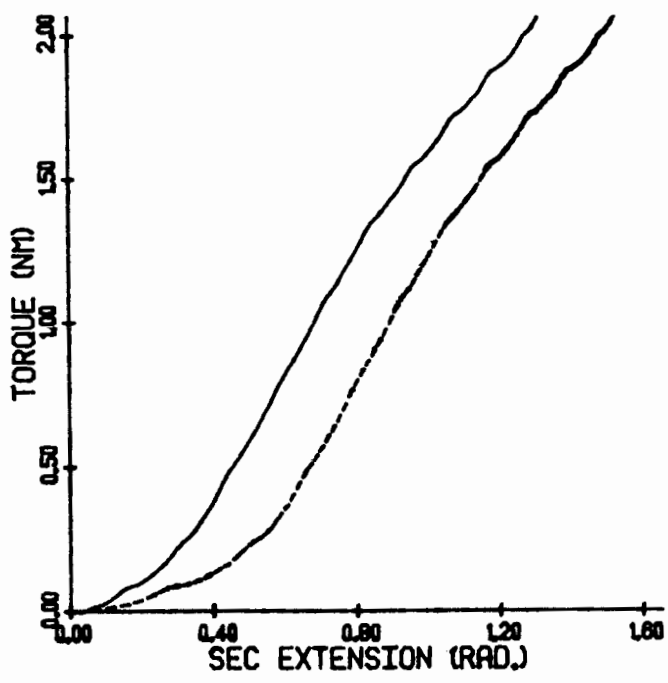
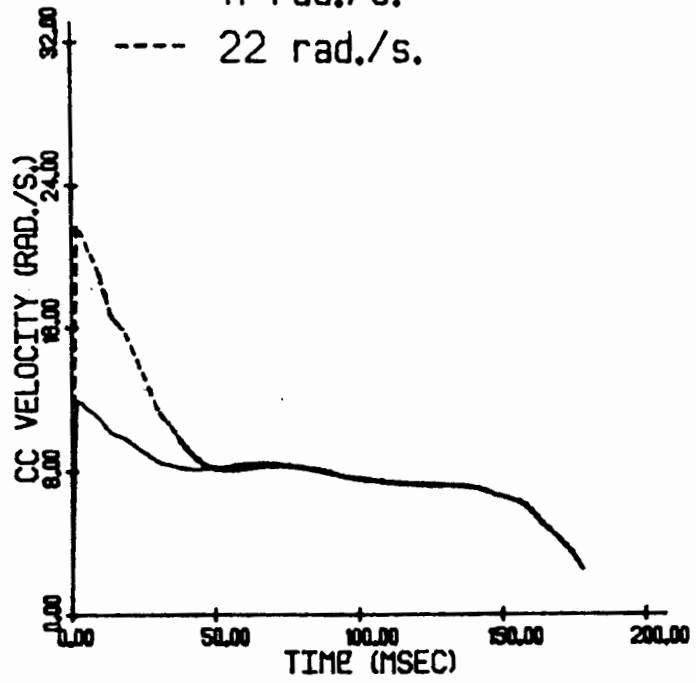
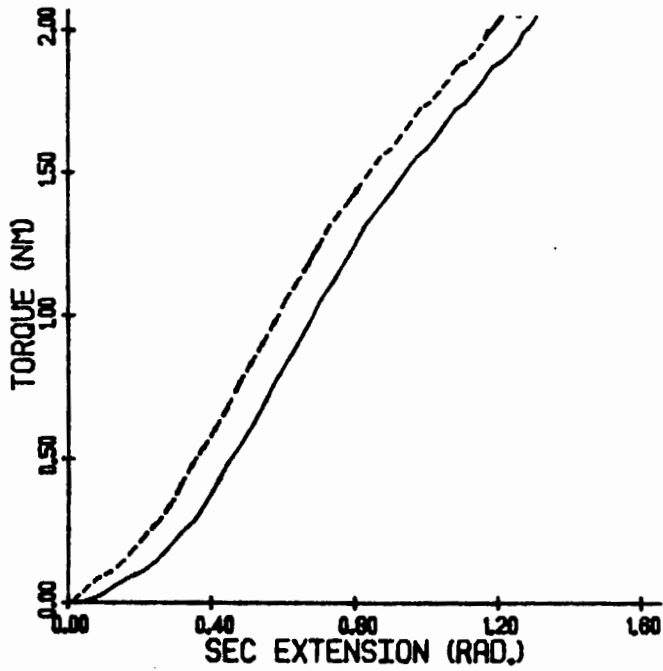
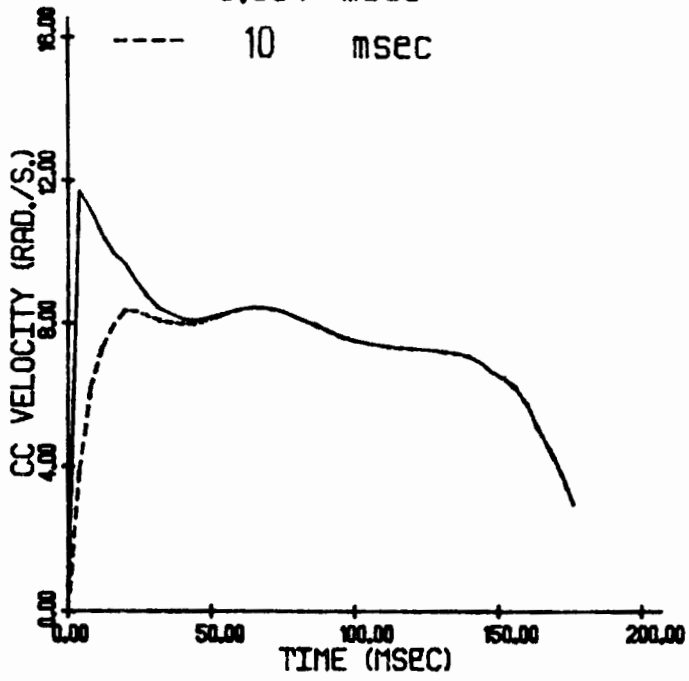


Figure 28. Effect of rate of rise of activation on the
torque-SEC extension relationship

rise time constant:

— 0.007 msec
- - - 10 msec



v) SEC Characteristics

Once the CC characteristics are known SEC parameters are calculated. Ideally these should not depend on factors other than those outlined in the model section. However, as Figure 20 illustrates, the curves for one effort level and one starting position (for which the peak torques occurred at the same displacement) are spread considerably. Contributing to this are errors in the estimation of CC characteristics, as well as some apparent changes with the moment of inertia. These are likely to result from the subject being able to see and feel what inertia is being used, and behaving differently in each situation. Particularly the manner of grasping the handle would show great influences. Though it would have been preferable to have subjects grasp the handle very firmly at the beginning of each trial, this was not possible since it triggered premature data collection. Subjects reported a feeling of the handle "running away" at the low moments of inertia. Such low torques observed in this condition are hardly sufficient to compress the soft tissues of the hand. This could explain the extremely compliant SEC characteristics observed under these conditions.

The influence of moment of inertia was neglected when fitting SEC relations, since the contribution was very irregular

and not significant (ANCOVA)⁷ for inertias other than 0.03 kgm². Contributions by effort and position were significant.

The turnover of curves visible in Figure 20 was also neglected. Explanation of this phenomenon is difficult, particularly since in the same experimental condition turnover to the left and right occurred. Some, but not all, of the turns to the left are due to the fact that the POLGON record flips over at 180 degrees. As can be seen for the trials in Figure 13, this causes a very sharp drop in load velocity, which then in turn would lead to a serious underestimation of SEC extension. This recording error did occur occasionally. A real hysteresis might be present in the SEC and show up as turns to the right in individual torque-SEC extension curves. In this situation, the torque at a certain SEC extension is lower after the peak torque has been reached than before at the same SEC extension. Since SEC velocity is positive before peak torque and negative after, the presence of a damper in parallel with the SEC would cause the hysteresis apparent in some trials. So attributing any turns to the left, and only those, to recording error, could lead to the suggestion that the SEC is not an undamped muscle component. Alternatively, both turns could be attributed to errors. Any estimation error in the CC characteristics also causes such behaviour. It is for this reason that the significance of the

⁷ANCOVA fits linear curves to data, and then compares slopes and intercepts. This is not necessarily valid for these data.

phenomenon was ignored.

The dependence of SEC characteristics on position and effort is shown clearly in Figures 21 and 22, and in relations (8) to (11) of Figures 24 and 26. From the cross-bridge theory one would expect the SEC to become stiffer as activation increases since most of the stiffness presumably resides in the cross-bridges. The experimental results for AC support this clearly. Subject RB shows a different trend. From these results then one must also conclude that the SEC is an active component, or at least consists to a considerable degree of active components.

From the SEC curves (Figures 21 and 22) and relations (8) and (10) in Figures 24 and 26 it is apparent that SEC stiffness increases with displacement. Since displacement is the deviation from the horizontal (palm facing down) muscle length of supinators actually decreases as displacement increases. Thus an increase in stiffness is observed when muscle length decreases. Haugen and Sten-Knudsen (1981) observed the opposite in frog sartorius muscle fibres. One possible reason for the discrepancy is the contribution of changing mechanical advantage to the length influences observed.

vi) Goodness of Fit

The CC characteristics generally fitted experimental results very well (see Figure 9). Correlations were always above

0.9 for AC and 0.8 for RB. This is significant for both subjects at the 1% level (Snedecor and Cochran, 1967, p.557).

Since SEC characteristics were fitted from calculated extension, any error made elsewhere would result in significant changes in SEC characteristics. For both subjects correlations were generally between 0.55 and 0.7, which is also significant at the 1% level.

However, the success of the determination of muscle parameters can ultimately only be judged by the predictive power of the model. This criterion is tested as explained in the next chapter.

VI. PREDICTION

The claim to validity for any model depends on the accuracy of prediction it allows. The model presented here attempts to describe muscle parameters in one individual, and therefore should allow prediction of contractions by that same individual. As the next figures illustrate, this has been fairly successful for both subjects. Also drawn is the response of the best fitting linear system and as can be seen its predictive power is considerably lower.

Figure 29. Predicted and observed torques in repeated
contractions under conditions shown for subject AC

Subject: AC

- observed
- * predicted
- - linear model output

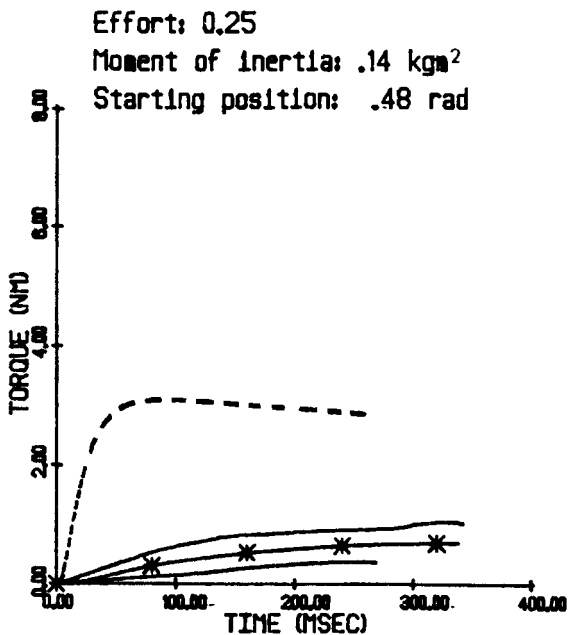
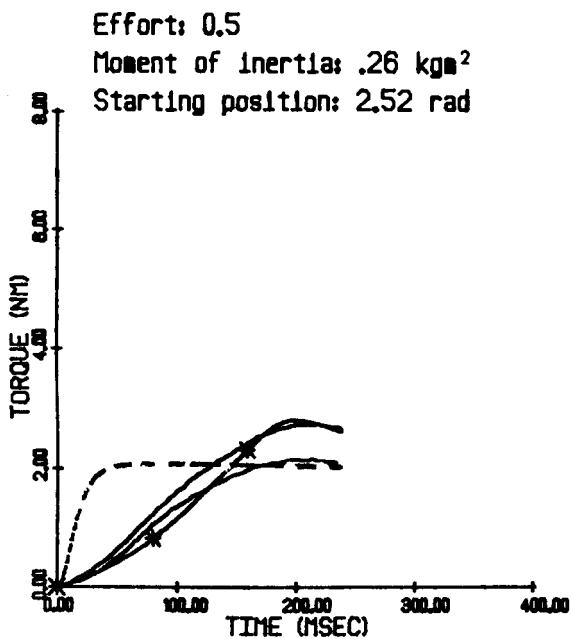
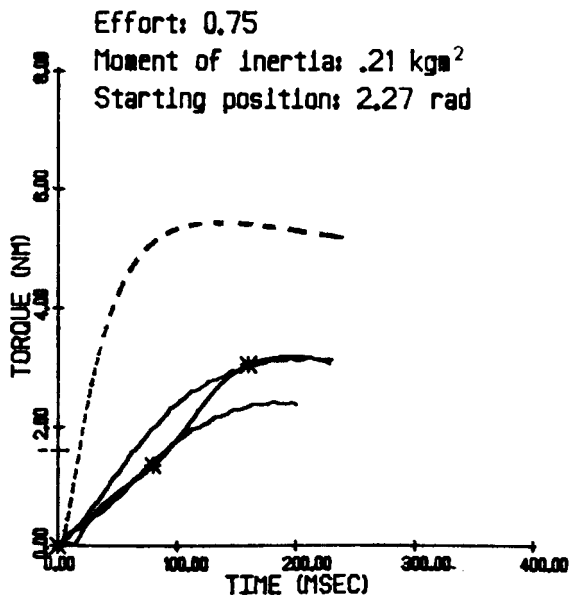
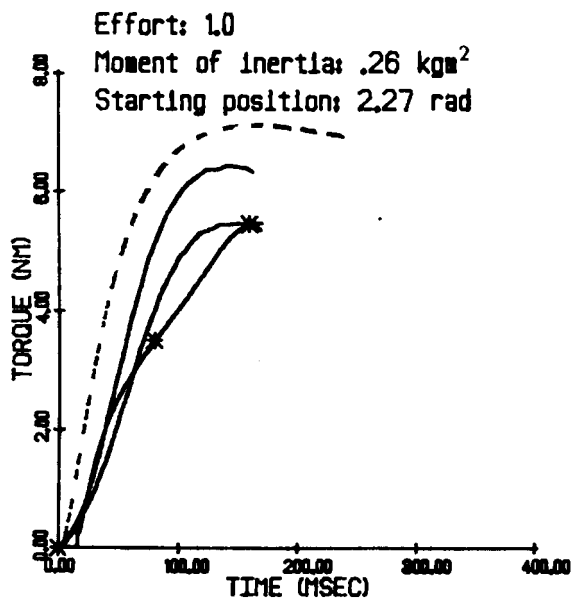
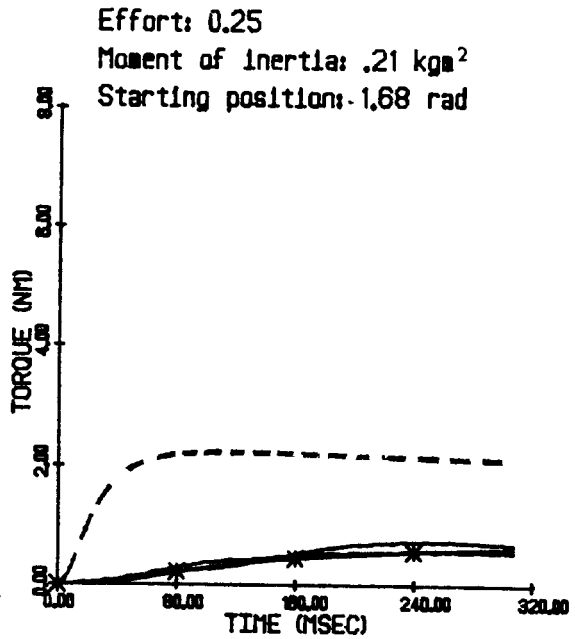
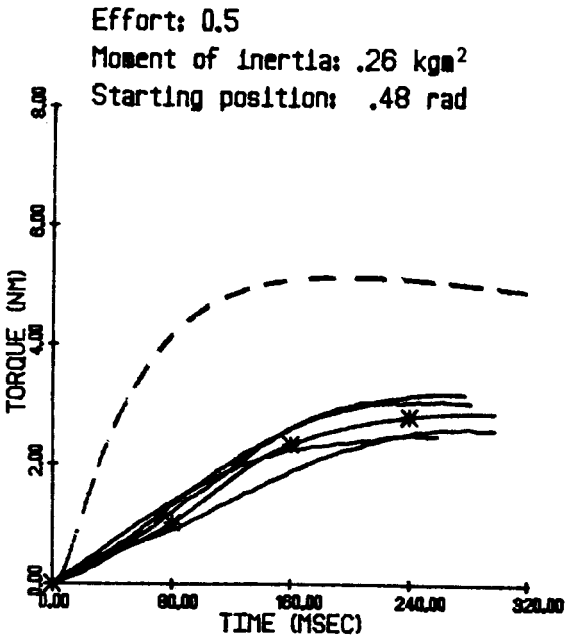
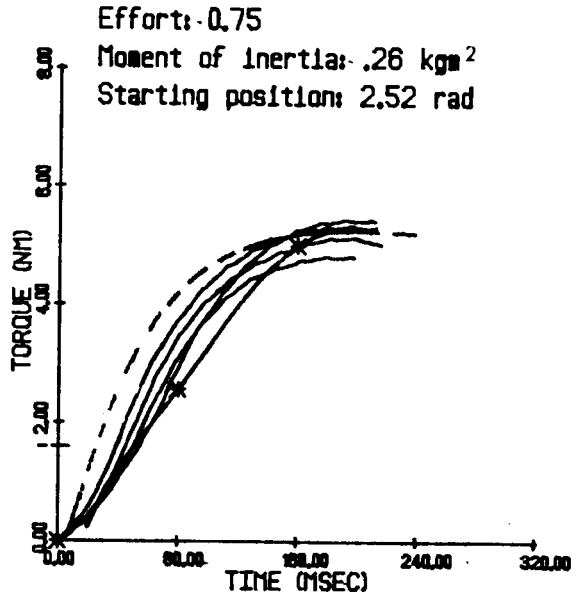
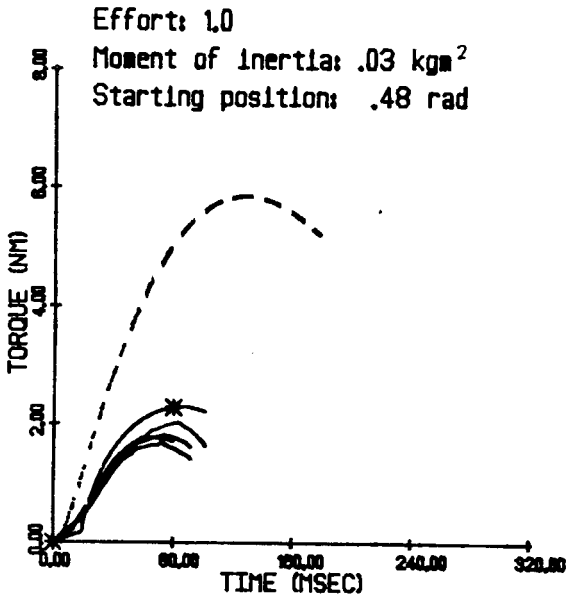


Figure 30. Predicted and observed torques in repeated
contractions under conditions shown for subject RB

Subject: RB

- observed
- * predicted
- - linear model output



Nevertheless, there are a few trials (not shown) where prediction was very poor. A number of reasons exist. All errors made anywhere in the analysis will contribute to the difference between observed and predicted torque. This includes not only description of the SEC and CC, but also the estimate of the moment of inertia.

The predominant influence, however, is probably errors in the fitting of SEC characteristics. All parameters other than torque (ie. load velocity, load displacement, CC velocity, CC displacement and SEC extension) showed very close agreement. Poor reproduction was always limited to a particular displacement, and one or two inertias. It seems likely that some error occurred in the determination of parameters under the condition in question. It is worthwhile to point out that the parameters used are exactly as determined by the analysis. No corrections were made to account for outlying trials or influences other than the ones outlined in the model. Quite obviously, the SEC torque-extension curves could easily be changed to allow very close fit of predicted versus observed torque time curves. Since this work is not an exercise in curve fitting, but the test of a new modelling approach, any such tinkering would have seemed inappropriate. When the goal is a very exact prediction of muscle output under a limited range of experimental conditions, correcting the relations such as to optimize predictive power would seem justified, and can be done

easily. When SEC parameters were fitted using only one condition, predicted torque was always very close to observed curves.

VII. CONCLUSIONS

The thesis has two separate aspects. First a new theoretical model of muscle behaviour is developed, and independently proven to be appropriate. The iterative process used introduces negligible error and allows the mechanical outcome of a variety of normal muscular contractions to be evaluated parsimoniously. The model is defined by 12 relations, all of which can be determined by the experimental method outlined here. The only parameter to be estimated is the rise time of activation, and model output is not sensitive to this over a reasonable range.

In the second part, parameters describing the single equivalent muscle involved in forearm supination are determined. The use of effort perception as a means of ascertaining constant activation was shown to be feasible for dynamic contractions. The great non-linearity of muscle parameters, and an apparent discontinuity of the torque-angular velocity relation at a small velocity other than zero are interesting findings. At lower levels of effort, both the torque-angular displacement as well as the torque-velocity relations show long linear segments with very sharp turns at the end of the ranges of displacement and torques. This could be interpreted as an attempt to make the system more linear and thus more predictable at submaximal

effort. A difference between the two subjects was noted. The highly trained one (AC) could reproduce a contraction much more accurately. This was independent of activation since the difference was also very pronounced at full effort.

The model and muscle description was tested by reproducing all recorded trials. The predictive power of the model was superior to that of a linear model. This new model has several distinct advantages over other models provided in the literature. Due to its relative simplicity the computations take very little time. A preliminary Fortran implementation of the programme shown in Figure 6 needed 5 seconds of CPU time on an IBM 4341 for the calculation of a 1 second contraction with a time step of 1 millisecond. Such a small time step is only necessary for extremely fast movements; usually a 5 millisecond step is sufficient, in which case only 1.4 seconds are required for the same calculations.

One poorly understood aspect of muscle physiology is the integration of the activation parameter. In this model activation influences on each of the other parameters can be studied independently, and their relative importance can be estimated. Due to the efficiency of the model, optimization studies in human movement become possible as well. For these a large number of contractions under varying conditions are necessary, and therefore the model has to be fairly economical. Since all mechanical parameters are determined continuously, the

instantaneous energy and power output, as well as other variables of interest, can be determined and used in these studies.

It is also hoped that this model will contribute to better understanding of muscle and movement, and in this way also facilitate the development of better prosthetic devices.

Appendix A: Second Order Linear System

In a linear system (see Figure 3) the CC is replaced by a torque generator, producing the torque T , and a dashpot with the viscosity B parallel to it. The SEC is replaced by a linear spring of stiffness K . Using the same notation as in Figure 4, the following equations apply:

$$T - B\dot{\beta} = K(\beta - \alpha) \quad (A-1)$$

$$MIx\ddot{\alpha} = K(\beta - \alpha) \quad (A-2)$$

Solving (A-2) for β , and substituting in (A-1) gives

$$T = (BxMI/K)x\ddot{\alpha} + MIx\dot{\alpha} + B\dot{\alpha} \quad (A-3)$$

Taking the Laplace transform of (A-3), and solving it for α , results in

$$L(s) = (TxK/(BxMI))x(1/(s^2(s^2 + (K/B)s + K/MI))) \quad (A-4)$$

where $L(s)$ is the transform of α .

Assuming a step input of T at time zero, the solution found by inverse Laplace transform is

$$(t) = (T/B)x(t - (2\gamma/\omega_n)) + (1/(\omega_n(1-\gamma^2)^{1/2}))x\exp(-\gamma\omega_n t)x\sin(\omega_n(1-\gamma^2)^{1/2}t - \phi)$$

(A-5)

where $\phi = 2\tan^{-1}[(1-\gamma^2)^{1/2}/(-\gamma)]$

$$\omega_n = (K/MI)$$

$$\gamma = ((KxMI)/(2xB))$$

Double differentiation of limb displacement provides acceleration, which is related to torque by (A-2). Once the

torque is known, CC velocity can be calculated using (A-1).

Appendix B: Programme Used for SEC Analysis

The following is a copy of the APL programme used to determine SEC characteristics.

```

[0]
[1]  ▽ KCAL2;MM;I;LX;TD;J;D;SUM;PN;CU;CX;THETA;Z;K;I;XX;ZXM;RES;KP
[2]  MM←I←1
[3]  SUM←0
[4]  L1:SUM←SUM+KEEPN[MM;1]
[5]  LX←(KEEPN[MM;1])↑((SUM-KEEPN[MM;1])↑KEEP[;4])
[6]  TD←(KEEPN[MM;1])↑((SUM-KEEPN[MM;1])↑KEEP[;2])
[7]  D←KEEPN[MM;2]
[8]  TUPOLY←POLIES[KEEPN[MM;4];]
[9]  J←1
[10] CX←1+LX
[11] L2:PN←LX[J]↓(Φ(FLPOLY÷MAXT[1;1]))
[12] CU←(1-(*(-KEEP[(SUM+J-KEEPN[MM;1]);1]÷TAU))×(TD[J]÷PN))↓(ΦTUPOLY)
[13] CX←CX+CU×D
[14] Z←CX-LX[J]
[15] KEEP[(SUM+J-KEEPN[MM;1]); 5 6 7 8]←(TD[J]÷Z),Z,CU,CX
[16] →((PLX)≥J+1)/L2
[17] →((KEEPN[;1])≥MM←MM+1)/L1
[18] !!
[19] !!
[20] ('MINIMUM KSEC: ',T(L/KEEP[;5]))
[21] ('MAXIMUM KSEC: ',T(T/KEEP[;5]))
[22] ('UNWEIGHTED MEAN: '),T((+/KEEP[;5])÷(KEEP[;5]))
[23] KPOLY←(1 2 3) POLYFIT KEEP[; 2 6]
[24] !!
[25] !!
[26] 'TORQUE SEC EXTENSION RELATION:'
[27] 'A THIRD ORDER POLYNOMIAL FITTED TO ALL DATA POINTS HAS THE'
[28] (T'COEFFICIENTS')
[29] 10 4 TKPOLY
[30] !!
[104]

```

Variables appearing in the programme are:

CV: instantaneous CC velocity.

CX: instantaneous CC displacement.

D: time interval between points.

FLPOLY: polynomial describing isometric torque-length relation.

J: counter, reset for each trial.

KEEP: Nx8 matrix, where N is the total number of data points for this set of trials and columns correspond to: time, torque, load velocity, load displacement, instantaneous SEC stiffness, SEC extension, CC velocity and CC displacement.

KEEPN: Nx4 matrix, where N is the number of trials in this set and columns correspond to: number of points in this trial, time interval, moment of inertia, and starting position.

LX: load displacement vector, contains values for one trial.

MAXT: absolute isometric maximum, used to scale the isometric torque-length relation to values between zero and 1.

MM: counter for the number of trials processed.

PN: isometric torque at current load displacement.

POLIES: group of polynomials, each describing the CC torque-velocity relation at a particular displacement.

SUM: counter for the total number of data points processed.

TAU: time constant of rise of activation.

TD: torque vector, contains values for one trial.

TVPOLY: CC torque-velocity polynomial appropriate for current displacement.

Z: instantaneous SEC extension.

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