AN EMPIRICAL ANALYSIS OF THE PRICING OF KNOCK-IN REVERSE EXCHANGEABLE SECURITIES

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by

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ABSTRACT

This paper studies the pricing of knock-in reverse exchangeable securities (RES). They are securities that are initially issued as straight bonds paying regular coupons. During the life of the securities, if prices of underlying stocks hit pre-determined triggers, the securities will be knocked in, which gives issuers options at maturity date to either redeem the securities in cash, or to deliver a perspecified number of shares. In this paper, the payoffs of **RES** are initially replicated with bonds and options, and then building block approach is used to value each component. It is thereby possible to estimate theoretically "fair" terms of issuance, and contrast these with effective terms. **A** significant overpricing of 5.48% is found if knock-in features are artificially removed, but if the features are added back, an underpricing of -1.98% is observed, which shows that the knock-in feature of the securities is significantly undervalued by issuers.

Keywords: structured products; reverse exchangeable securities; reverse convertible bonds; building block approach; issuance cost

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DEDICATION

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To my beloved wife Rofi:

Without your pushing, I cannot be through.

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1 INTRODUCTION

The last decade has seen a strong increase in the number of different financial derivates that are being offered on financial markets. This has greatly expanded people's investment strategies. A financial derivative, by definition, refers to a financial instrument from which payoffs are paid at maturity (European style), or paid any time before expiry date (American style). The amount and timing of those payoffs must be derived from the performance of the underlying assets. There is a wide range of possible underlying assets: they can be commodities, stocks, interest rates, or an index. The main types of derivatives include futures, forwards, options and swaps. Financial derivatives are always designed to satisfy specific requirements of markets. Weather derivatives, for example, were first introduced to the market in 1996 due to the deregulation of the power industry¹. The deregulation forced the role of the Utility to change from a monopoly to a market participant, and hence forced utility companies to manage their exposures including weather risks. Since then, the weather derivative market has been significantly expanded. In the first eight months of 2005, more than 600,000 weather contracts were traded at Chicago Mercantile Exchange with the notional value over 16 billion. 2

A large branch of financial derivatives is labeled "structured products". Examples include the Reverse Exchangeable securities issued by ABN AMRO, Equity Linked Securities issued by Citigroup and, High Income Trigger Securities issued by Morgan Stanley. These financial instruments combine contingent claim features of traditional options and the provisions for fixedincome cash flows. Generally, they have higher coupon rates than straight bonds but as a

¹ In 1996, Koch Energy (now Entergy-Koch) and Enron entered a Heat-Degree-Day (HDD) swap for the winter of 1997 in Milwaukee, WI. This HDD swap allowed Koch Energy to hedge the downsize risk to its revenues in the event of a mild winter.

 2 Source: Chicago Mercantile Exchange

consequence bear much higher risks³. Their popularity is evidenced by their statistical numbers: at the end of 2005, the outstanding notional amounts of equity linked derivatives, as reported by the Bank for International Settlement (BIS), reached USD5.057 trillion,⁴ while the world's entire GDP for the year of 2005 was just USD44.4 trillion. 5

As structured products become more and more popular, financial economists have become more and more interested in researching the efficiency of this market. Among various equity-linked securities, reverse exchangeable securities (RES) issued by major international banks such as ABN AMRO and ING Bank are widely analyzed in the academic financial literature. Burth, Kraus and Wohlwend [2001] study discount certificates and reverse convertible securities in the Swiss market. Wilkens, Erner, and Roder (2003) study the same financial instruments in the German market. Szymanowska, Ter Horst, and Veld (2005) study the Dutch RES market. One of the apparent reasons why RES are chosen as research samples is the fact that RES' payoffs can be fully represented by a linear combination of basic financial instruments: straight bonds and European put options on underlying stocks. An ABN AMRO RES issued on US market on October 5, 2005, for example, has a one-year term with a coupon rate of 9%. The security is linked to the common stock of Best Buy Co., Inc. Before its maturity date, October 5, 2006, the security pays coupons quarterly as regular bonds do, but at expiry, ABN AMRO may not pay back the principal of the RES. If the market price of Best Buy stock on October 5,2006 is greater than the stock price on the pricing date (i.e. the initial price)⁶, ABN AMRO will pay the principal of the RES; if the stock is lower than that initial price, the RES will be converted to a pre-determined number of shares of the Best Buy stock. The payoff at the expiry day will be exactly same as a put option, written to ABN AMRO, at a strike as the initial price of Best Buy

 3 For example, the coupon rate of a reverse exchangeable note issued by ABN AMRO on April 7, 2006 reached as high as 17% per year.

⁴ Source: Bank for International Settlement (BIS) – Statistics: http://www.bis.org/statistics/index.htm

⁵ Source: International Monetary Fund, World Economic Outlook Database, April 2006

⁶ Pricing dates of RES are normally five days before issuance date. Please see the attached offering prospectus of a ABN AMRO reverse exchangeable security.

stock. Since the bond and the put option can be theoretically valued, the RES—the combination of these two components—can be valued as well.

For a reverse exchangeable security, since the embedded put option is favourable to the issuer, its coupon rate must be set fairly high in order to compensate for the bond purchasers' written put option. Due to the high coupon rate, a RES appears more attractive than a straight bond, especially when market interest rates are low. However, since there is no protection for investors' principals, a RES is much riskier than a straight bond. In addition, because a RES issuer will never exercise its put option if the price of the underlying stock goes above the strike price, a RES purchaser can not earn more than the sum of the pre-determined interest payments and hislher principal. Because of this ceiling, the investor's potential of benefiting from a bullish stock market is designed away. It is well known that the holding period return of a financial instrument is made up of two parts: accrued income (such as interest and dividends), and capital gains. In the case of a RES, since the part of capital gains is taken away from investors, they require higher coupon rates for compensation. We can intuitively conclude that the most important value-driver of a RES is the performance of its underlying stock: the higher volatility of the underlying stock, the higher the coupon rate that can be expected.

The situation will be much more complicated when some "sweeteners" such as "knockin" or "knock-out" features are added to a plain vanilla RES. **A** knock-in RES initially starts as a "normal" straight bond but converts into a reverse convertible bond once the price of the underlying stock knocks at a pre-determined barrier, e.g. 80% of initial price of the stock. Normally, the period during which the knock-in feature can be activated starts at the trade date until the third day prior to expiry date of the bond. Purchasers of a knock-in RES would realize their maximum benefit if the underlying stock did not hit the triggering barrier during the entire life of the knock-in feature. Apparently, such a feature can make bond purchasers' situation better off, and we can therefore expect that the coupon rate of a knock-in RES would be lower than that of a plain vanilla RES if all else is equal.

An opposite instrument to knock-in RES is "knock-out" RES, which initially starts as a RES but converts into a "normal" straight bond if the price of the underlying stock reaches up to a pre-determined level. Similar to the knock-in feature, the "knock-out" provision increases the probability for bond purchasers that they can realize their maximum benefit—the higher interest earned from the bond. As a result, the coupon rate of a "knock-out" RES can therefore be expected to be lower than that of a plain vanilla RES if all else is equal.

In this paper, I use the building block approach, which is also the fundamental method used in financial engineering, to dissect knock-in reverse exchangeable securities. My basic valuation model is similar to Benet, Giannetti, and Pissaris's (2005), except that I add one more "block" to analyze knock-in features. In Benet, Giannetti, and Pissaris's (2005) paper, they study the fair value of plain vanilla RES. However, the knock-in reverse exchangeable securities are currently more popular in the market,⁷ and therefore such a feature is modeled in the valuation formula of this paper. Specifically, Monte Carlo simulation is applied to simulate the probability that a straight bond will convert into a RES. Although there are no samples of "knock-out" RES in my study, the model in this paper can also be applied to them. Furthermore the way simulating "knock-out" should be the same as that for "knock-in" except that the barrier for a "knock-out" is in an opposite direction.

I find that as the knock-in feature is added to a RES, its theoretical issuance cost can be significantly reduced. Before knock-in features are added to RES, the sampled securities are significantly "overpriced", which is consistent with the results of Benet, Giannetti, and Pissaris's (2005) study. But once the knock-in probability is simulated and added in the model, most of the

Between July, 2005 **and July,** 2006, **ABN AMRO issued total** 65 **reverse exchangeable securities on US market, among which 54 are knock-in** RES **and only 11 are plain vanilla** RES.

RES become "underpriced". Here the overpricing is defined as the situation that the "fair" coupon rate, which is given by the valuation model, is greater than the effective coupon rate. Because the theoretical coupon rate is expected to pay for corresponding level of risks, investors will have a "bad" investment if they are compensated by a lower effective coupon. On the contrary, the underpricing is defined as the situation that the theoretical coupon is lower than the effective coupon. Under such a situation, investors are compensated more than the "fair" level of the corresponding risks exposed. A coupon spread is defined as the difference between a "fair" rate and its corresponding effective rate. A positive coupon spread is therefore overpricing and negative is underpricing. In this paper, the "fair" coupon rate is calculated in two ways: one with a knock-in feature, the other without. When there is no knock-in feature, sampled **RES** are significantly overpriced with an average positive spread of 5.48%; when the knock-in probability is simulated and added to the model, **37** out of total 46 samples have negative coupon spreads, and the average of underpricing is -1.95%. This shows that while investors in plain vanilla reverse exchangeable securities are suffering considerable economic losses, investing in a knockin RES looks like real bargain for purchasers.

The remaining of this paper is organized as following: Section 2 describes the data that are used in this paper; Section **3** contains a detailed description of the analyzing methodology; Section 4 describes the procedure of simulating knock-in probabilities and valuing call options; Section *5* briefly explains the results of the analysis; and the conclusion is presented in Section 6.

2 DATA DESCRIPTION

The study sample consists of the entire set of 54 knock-in RES issued by ABN AMRO on the US market from July, 2005 to July, 2006. The information regarding those knock-in RES is pulled out from ABN AMRO's website: www.us.abnarnromarkets.com. It should be noticed that the knock-in RES issued during this time period were not listed on any securities exchange, and the investors are supposed to be willing to hold the securities until their maturity dates. Although affiliated institutions of ABN AMRO intend to make the market for those securities from time to time in off-exchange transactions, their secondary markets are still very inactive. 8 The issuance terms of the knock-in reverse exchangeable securities were collected from corresponding published prospectuses, and their summary descriptive characteristics are presented in Table 1 on page 25.

The original sample of 54 knock-in reverse exchangeable securities is reduced because it is not possible to find information regarding prices and trading volume for corresponding call options on all underlying stocks. However, that information is critical for the calculation of an implied volatility. The implied volatility of a financial instrument is the volatility derived from the market price of a derivative based on a theoretical pricing model such as the Black-Scholes-Merton model. Therefore, the lack of market prices makes it impossible to calculate the implied volatility. In total there are 8 securities in the sample that suffer from this problem.⁹ Consequently, there are 46 knock-in reverse exchangeable securities (85% of the original sample) of which the "fair" coupon rates are able to be calculated.

⁸Please see **ABN** AMRO RES Prospectus dated Sep. 17,2003 and Prospectus Supplement dated Sep. 18, 2003.

⁹ The 8 knock-in RES that are dropped from the original sample list include: App-2, Charles, Chev, DRH, ING, JET-1, JET-2, and Sirius-1.

The market prices of the underlying stocks and their dividend yields are retrieved from Yahoo Finance. Yahoo Finance provides several different dividend yields for each stock including: forward dividend yield, trailing dividend yield, and the 5-year average dividend yield. The difference between the forward and trailing dividend yield is the fact that the former is calculated by using forecasted dividends in the future 12 months while the latter is calculated by using the dividend paid in the last 12 months. Since I intend to simulate the stock prices in the future, the forward dividend yields are more relevant to the analysis and are thus adopted. Additionally, the dividend yields released by Yahoo Finance are discrete rates. In order to be properly applied in deriving implied volatility through the Black-Scholes-Merton model, we have to convert the discrete rates into their continuous versions.

Another important input data is the risk-free rate. Instead of Treasury rates, the LIBOR is used as the risk-free rates in this paper. The historical LIBOR on daily basis are released by British Bankers' Association (BBA).

3 METHODOLOGY

3.1 A brief introduction of building block approach

The term "building block" is originally from architecture design, and now is widely used in various sciences including finance. Many complicated financial instruments, in particular structured products, are combinations of some basic instruments such as bonds, stocks, forward contracts or options. We can therefore analyze the value of each component, and then connect each part to form the structured product. An illustrative example is a callable and putable convertible bond, which is a combination of (A) a straight bond, (B) a call option which allows the bond issuer to call the bond when market interest rates go down, while the bond's price goes up, (C) a put option giving bondholders the option to sell back the bond to the issuer when market interest rates go up while the bond price goes down , and (D) a call option granting the bondholders the right to convert the bond to a pre-determined amount of common shares. Mathematically,

callable and putable convertible bond value = straight value of bond

+ value of the call option on the stock

- value of the call option on the bond

+ value of the put option on the bond

As discussed before, the building-block approach is especially useful in handling today's more and more complicated "structured products". In this paper, the building block approach is used as the fundamental tool for analysis. A knock-in reverse exchangeable security may look complicated, especially when we are facing its over 100 pages of offering prospectus. However, by using the building block approach, we can immediately dissect such a product into three blocks: a straight bond, a European put option, and the probability of being "knocked-in".

3.2 Block 1: bond

The theoretical value of a coupon-paying bond is the sum of present values of all future cash flows. For a coupon-paying bond, the future cash flows include periodically paid coupons and the repayment of principal. The value of such a bond, P, can be expressed as:

$$
P = \sum_{t=1}^{nT} P(0, \frac{t}{n}) F \frac{c}{n} + P(0, T) F \tag{1}
$$

Here, $P(0,t)$ is the present value of zero coupon bond paying principal of \$1 at time t, which works as the discounting rates in the model; n is the number of coupon payments in one year, e.g. $n = 2$ if the coupon is paid semi-annually; F is the principal paid at time 0 to purchase the considered bond; c is the annual coupon rate of the considered bond.

In the above formula, the coupon rate and principal can be easily observed in the bond market. The choice of discount rates is somewhat difficult but very critical for the accuracy of valuation. While some derivatives traders use Treasury rates to define the payoff from a derivative, most traders prefer LIBOR rates because Treasury rates are artificially low due to a number of tax and regulatory issues.¹⁰ Another reason why LIBOR rates are adopted in this paper is because RES are issued by major banks for which the probability of default exists. Meanwhile, LIBOR rates are quoted by major international banks, and the quotes already include the credit spread of those banks

¹⁰ See Chapter 4 of Hull (2006)

3.3 Block 2: put option

The maturity payoff of the embedded put option in a RES can be expressed as:

$$
payoff = \begin{cases} 0 * 1_{\{S_T \ge K\}} \\ qS_T * 1_{\{S_T < K\}} \end{cases} \tag{2}
$$

Here, q is the conversion ratio of the considered RES; S_T is the price of underlying stock at expiry day. K is the strike price of the option. $1_{\{\omega\}}$ is a dummy variable of which the value equals to 1 when the event of ω occurs, otherwise equals to 0. To calculate the present value of the above payoff, I use the Black-Scholes (B-S) model. Under the B-S model, a put price is given by:

$$
P_0 = Ke^{-r_f T} P(0, t) N(-d2) - S_0 N(-d1)
$$
 (3)

Where $d1 = \frac{\ln(S_0/K) + (r_f + .5\sigma^2)T}{\sqrt{T}}$ and $d2 = d1 - \sigma\sqrt{T}$. N(.) denotes the cumulative $\sigma\sqrt{T}$

normal distribution function; S_0 stands for stock price at time 0; r_f is the continuously compounding risk-free rate. The same reasons as discussed in part 2.3, I use LIBOR rather than Treasury rates as my risk-free rate in the B-S model.

3.4 Block 3: the likelihood of knock-in

A knock-in RES is not a born structured financial product when it is issued, but during the period from the trade date to the third trading day prior to the maturity date, once the price of underlying stock hits the triggering level, e.g. 80% of initial stock price, the security is "knocked in"-i.e. it is converted from a pure bond into a normal reverse exchangeable security. This can be illustrated by a knock-in RES issued by ABN AMRO on April 26, 2006: the security has a term of one year with a coupon rate of 14.55%. Although it was issued on April 26, the security had a pricing date of April 21 on which the initial price of underlying stock--common stock of Apple Computer—was observed at \$67.04. The triggering level of knock-in was set up at 80% of the initial price. During the life of the security, if the stock price of Apple Computer never reaches or falls below \$53.63 (80% of the initial price, \$67.04), the security will end up as a straight bond at maturity even if Apple's price at that time is below the strike price of \$67.04. However, if the stock price does hit, at least one time, the triggering level, the security will end up as a RES: if the stock price at maturity is below \$67.04, investors will receive 14.916 shares of Apple stock for every \$1000 principal (l4.916=1000/67.04).

Apparently, the knock-in feature can significantly influence the issuance cost of a RES and therefore must be modeled into our valuation process. To achieve this, we can simulate the evolving paths of stock prices. According to prospectuses of most knock-in RES, knock-in may occur "at any time during regular business hours of the relevant exchange on any trading day during the life of the Securities".¹¹ Since stock prices in the real world evolve continuously, the shorter time intervals are used in the simulations, the more accurate the knock-in probability that can be obtained. However, because the Monte Carlo simulation has drawbacks of timeconsuming and computation-consuming, there is a trade-off between accuracy and computation. In this paper, a daily-basis time interval is chosen, in consideration of the fact that the sampled RES have only two different terms: nine months (three quarters) and one year.

To simulate evolving paths of stock prices, we have to assume the stochastic process of the stock price. In finance, a generally accepted assumption is that stock prices evolve as geometric Brownian motion. Mathematically, the process can be expressed as:

$$
dS = \mu S dt + \sigma S dz \tag{4}
$$

Where S is the underlying stock price; μ is the drift (expected return) of the stock; σ is the volatility of the stock's returns; and dz is a standard Wiener process. In practice, it is more

¹¹ Initial offering prospectus of ABN AMRO RES dated on June 2, 2006.

accurate to simulate $ln(S)$ than simulating S itself.¹² From Ito's lemma, we can derive that the process of ln(S) can be expressed as: $d \ln S = (\mu - \frac{\sigma^2}{2})dt + \sigma dz$. It further follows that:

$$
S_t = S_0 e^{(\mu - 0.5\sigma^2)t + \sigma \epsilon \sqrt{t}}
$$
 (5)

Here **E** is a random sample from a normal distribution with mean zero and standard deviation of 1.

Once the stock price at each time interval is simulated, we can easily build our third block: the likelihood that the knock-in feature can be activated during the life of the security. The probability is the portion of "knocked-in" stock price paths to the total simulated stock price paths. For example, if we simulate 5000 stock price paths and there are 2000 paths in which at least one price at any time interval hits the trigger, the probability of knock-in would be 40% (200015000).

3.5 Integration of the blocks

Once each of blocks in the structured product of RES is built up, we can integrate all components and derive our valuation model:

$$
F = prob (knockin) * [\sum_{r=1}^{nT} P(0, \frac{t}{n}) F \frac{c}{n} + P(0, T) (1_{\{s_T \geq K\}} F + 1_{\{s_T < K\}} qS_T)]
$$
\n
$$
+ [1 - prob (knockin)] * [\sum_{r=1}^{nT} P(0, \frac{t}{n}) F \frac{c}{n} + P(0, T) F]
$$
\n(6)

All notations are the same as defined above.

Equation (6) is the theoretical model in valuing RES, but is difficult to apply directly. **A** typical methodology to solve this problem is to replicate payoffs with other portfolios. Those portfolios normally consist of more primary securities such as bonds, stocks, and options. There

l2 See Chapter 17, Hull *(2006)*

already exist well established models used to valuing those primary securities. This method is widely used in literature. For example, Chen and Sears [1990] price S&P Industrial Portfolio (SPIN) in this manner, while Wasserfallen and Schenk [1996] similarly examine a cross-section of 13 'capital-protected' Swiss structured products. To derive a workable model we can replicate the expiry payoff of reverse exchangeable securities by creating synthetic portfolios. Suppose we create a portfolio, A, at time 0, with a long position of q units of underlying stock and a short position of q units of call options. Here q is the conversion ratio of the RES replicated. Such a portfolio has a cost of $-qS_0 + qC_0$. At time T, if the underlying stock price, S_T , is less than the strike price, the call will not be exercised, leaving the value of the portfolio to be qS_T ; if S_T is greater than the strike price, the value of the portfolio can be expressed as $qS_T - q(S_T - K)$. Since the put option embedded in the RES normally has the initial stock price as its strike price, K is therefore equal to S_0 , and the expiry payoff of the portfolio can be simplified as qS_0 , which is exactly the principal of RES. Mathematically, the valuation formula for a RES can be expressed as:

$$
F = \sum_{t=1}^{nT} P(0, \frac{t}{n}) F \frac{c}{n} - qC_0 + qS_0 \tag{7}
$$

 C_0 is the price of European call option at time 0 with the same strike price as the embedded put option in a RES. Similarly, we can replicate the expiry payoff of a RES by creating another portfolio, a short position of q put options on underlying stocks and a long position of qK zero coupon bonds, and thus obtain another equivalent equation:

$$
F = \sum_{t=1}^{nT} P(0, \frac{t}{n}) F \frac{c}{n} - qP_0 + P(0, T) qK^{13}
$$
 (8)

¹³ The equation (7) can also be directly derived from equation (6) through put-call parity, S-P(0,T)K=C₀-P₀.

 P_0 is the price of European put option at time 0 with the same strike price as embedded put option in RES. Although both Equation (7) and (8) can be used in pricing RES, it is much more difficult to find the price of a zero coupon bond with matched maturity and rating grade than to obtain prices of underlying stock. As a result, Equation (8) is abandoned and Equation (7) is used in this paper. To consider the feature of knock-in, we have to modify the Equation (7) to get a more general model as:

$$
F = prob(knockin) * [\sum_{t=1}^{nT} P(0, \frac{t}{n}) F \frac{c}{n} - qC_0 + qS_0]
$$

+ [1 - prob(knockin)] * [\sum_{t=1}^{nT} P(0, \frac{t}{n}) F \frac{c}{n} + P(0, T)F] (9)

For a RES without knock-in feature, the prob(knock-in) equals to 1, and the Equation (9) will collapse to the Equation (7).

3.6 Calculation of "fair'' coupon rate

As we discussed above, a RES is made up of a bond and a written put option. In order to compensate the written put, reverse exchangeable securities offer higher coupon rates than normal bonds do. Since the RES on US market are initially issued into market at par (see Appendix C: an offering prospectus of a knock-in RES), the observed coupon rates of those RES can be regarded as the equilibrium price that issuers are willing to offer and investors are willing to accept. But do those observed coupon rates sufficiently compensate risks involved in knock-in RES? To answer this question, we can assume a "fair" or theoretical coupon rate, c, in Equation (9) as an unknown variable; the theoretical values of the call, C_0 , can be calculated by the Black-Scholes-Merton model; the probability of knock-in is simulated; the stock price, S_0 , and the issuing price of the RES, F, can be observed in the market. Thus we can solve the equation to obtain the "fair" coupon rate. As we defined in Section 1, the coupon spread (CS) is the

 $\Delta \sim 10^{11}$ and $\Delta \sim 10^{11}$ and

difference between the "fair" and effective coupon rates; overpricing is observed if CS is positive while underpricing if CS is negative.

 $\Delta \phi = 10^{11}$ and $\Delta \phi$

-4 ANALYSIS PROCEDURE

4.1 Simulating block-in probability

The event that the stock price hits triggering level may happen anytime during the life of knock-in RES. However, since it is impossible to know in advance the underlying stock price at the time when the considered RES is issued, we therefore need to simulate the evolving paths of stock prices. For each path, if any simulated price reaches or falls below the triggering level, such a path is "knocked-in". The proportion of total "knocked-in" paths to the total simulated paths is exactly the probability that we are looking for.

From our stock price evolving model, $S_t = S_0 e^{(\mu - 0.5\sigma^2)t + \sigma \epsilon \sqrt{t}}$, we can intuitively expect a lower knock-in probability for Stock A than Stock B, in cases where the both stock have a same triggering levels but Stock A has a higher expected return, **p,** as well as a lower volatility, **o.** An accurate estimation of these two parameters directly determines the accuracy of our simulations. For the expected return, there are several ways that we can use for estimating: we can use a onefactor model such as CAPM (Sharpe, 1964), or multi-factor models such as the Fama-French **3** factor model (Fama and French, 1996). Alternatively, we can also use a stock's historical return as a proxy for its expected return, which is the method adopted in this paper. For volatilities, there is a large amount of literature showing that the implied volatility is superior to historical, databased estimation. However, it is almost impossible to find call or put options with time to maturity and strike price exactly matched with the put options embedded in RES. This causes a problem due to the existence of a "term structure of volatilities" (Brenner and Subrahmanyam, 1988) and the "volatility smile".¹⁴ To obtain meaningful volatilities, the same method as Ter

l4 See Chapter 16, Hull (2006)

Horst and Veld (2006) used in their study is adopted for the calculation of implied volatilities. This method uses linear interpolation to derive implied volatilities from a volatility surface. An example of calculating implied volatility is provided in Appendix B. The estimations of expected returns, implied volatilities, simulated probabilities, and the knock-in triggering levels are presented in the Panel A of Table 2 on page 27. From this panel, it appears that when all else is equal, a higher expected return leads to a lower knock-in probability. For example, App-1 and GM have the same knock-in trigger, 80%, and very close volatilities, 44.717% and 44.48% respectively, but App-1's expected return (23.63%) is much higher than GM's (3.674%). The knock-in probability for GM $(63.9%)$ is almost 20% higher than App-1's $(44.47%)$. The same reasoning is adopted for the triggering level and the volatility, and the conclusion from Panel A is that higher triggering levels and higher volatilities result to higher knock-in probabilities.

To better disclose the relationship among those variables, I run a regression between the knock-in probabilities (dependent variable) and triggering levels, expected returns, and volatilities (three independent variables). The results are presented in the Panel B of Table 2. Consistent with the Panel A, the results in Panel B disclose: (1) a strong negative relation between the expected return and the probability: every one percent increase in expected return will lead to about 0.43 percent decrease in the knock-in probability; (2) a strong positive relation between the probability and the volatility: every one percent increase in volatility will result in about 1.27 percent increase in the probability; and (3) a strong positive relation between the probability and the triggering level: one percent increase in triggering level will lead to 1.72 percent increase in the probability. All parameters are statistically significant.

4.2 Pricing option component

Using the standard Black-Scholes (B-S) model to price call or put options, we need five input variables: (1) current price of underlying stock (S_0) , (2) strike price (K) , (3) time to maturity

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(T), (4) risk-free interest rate (r), and (5) the volatility of the return on the underlying stock (σ) . The first three variables can be easily observed in the market. For the risk-free interest rate, due to the reasons mentioned in Section 3.2, LIBOR rates rather than Treasury rates are adopted in the Black-Scholes formula. As to the volatility, the same implied volatilities as calculated in the last section are used.

However, in the standard Black-Scholes (1973) model (B-S), the underlying stock is assumed to pay zero dividends during the lifetime of the option, which is not the case for many underlying stocks in our sampled RES. Consequently, the Black-Scholes-Merton (1973) model (BSM), which is for dividend-paying stocks, is adopted in this paper. The difference between these two models is that the latter one uses S_0^p replacing S_0 in B-S formula¹⁵. The estimations of various variables needed in BSM model and the theoretical call prices calculated are presented in Table 3 on page 29.

In Table 3, the initial stock price $(S₀)$, the strike price (K) , and the time to maturity (T) are pulled out from the offering prospectuses; The risk-free interest rate (r) is the LIBOR rate on the issuance date; the dividend yield (δ) is of continuous version through formula: $\delta_{continuous} = \ln(1 + \delta_{discrete})$; and the volatility (σ) is the implied volatility derived from the market prices of relevant call options.

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¹⁵ S_0^p is called a "dividend peeled" stock price, which is in fact the current stock price S0 deducted from which the present value of future dividend is deducted. Mathematically, $S_0^p = S_0 e^{-\delta T}$.

5 RESULTS

5.1 Previous empirical research

Previous empirical research mainly covers European markets. Burth, Kraus and Wohlwend (2001) study reverse convertibles and discount certificates in Switzerland. Based on a static arbitrage argument, they derive their valuation formula by replicating the cash flows of the structured products. They report an average overpricing for reverse convertibles of 3.22% and for discount certificates of 1.4%. Szymanowska, Ter Horst and Veld (2005) study reverse convertible bonds in the Dutch market. The Netherlands has a very active market in long-term call options. Therefore, Szymanowska, Ter Horst and Veld (2005) conduct a model-free analysis by directly comparing reverse convertibles to combinations of put options traded on the options exchange, and bonds. They find an average overpricing of 23%. Benet, Giannetti and Pissaris (2005) study the reverse exchangeable securities in US market. They use a similar valuation method as Burth, Kraus and Wohlwend (2001). They compare the coupon spread between "fair" coupon rates and effective rates by using different estimations of volatilities. They find that by using short-term call implied volatilities, there is an average overpricing of 5.97%; by using LEAPS implied volatilities ¹⁶, the average overpricing is 4.05%. Overall these previous empirical studies unanimously conclude that reverse convertible bonds are overpriced in markets researched.

To properly explain why the reverse convertible bonds are consistently overpriced, Szymanowska, Ter Horst and Veld (2005) design a financial experiment and find that two behavioral factors, framing and cognitive errors, partly explain the overpricing. Similarly, Benet,

¹⁶ LEAPS is short for Long-term Equity Anticipation Securities. Those securities are long-term stock or index options including call and put with expiration dates up to three years in the future.

Giannetti and Pissaris (2005) telephone interviewed traders familiar with RES and explain the overpricing with investors' naivety about this complicated financial instrument.

5.2 Findings of this paper

This paper mainly focuses on the influence of the added-up "sweetener" of the knock-in feature. To find out that the influence of such a sweetener on issuance cost of RES, two different ways are used in the calculation: the first way exactly follows the methodology as described in Section 3, and the second way is the same in most aspects except that the knock-in features are artificially removed from those securities while all other issuance characteristics are kept unchanged. The coupon spreads calculated by these two ways are presented in Table 4 on page 31.

From Table 4, we can observe that if there were no knock-in features, the sampled RES were significantly overpriced at an average positive spread of 5.48%, which is consistent with the result of Benet, Giannetti and Pissaris (2005)'s study. However, since the sampled securities have "knock-in" features, we have to calculate their "fair" coupon rate by considering such characteristic. As we discussed before, the integrations of the knock-in features can greatly favor investors. In fact, this integration drives the coupon spread from a positive 5.48% to a negative 1.95%, which means that knock-in RES are under-priced. This result is consistent, at least to some extent, with the finding of Szymanowska, Ter Horst and Veld (2005). They find that in the Dutch market, the plain vanilla contracts are more expensive than knock-in reverse convertibles.

We can better understand why the knock-in **RES** in US market are much less expensive-in fact underpriced-than their plain vanilla counterparts if we compare these two types of securities. During July 2005 to July 2006, **ABN** AMRO issued 54 knock-in **RES** and 11 plain vanilla **RES** on US market. The average effective coupon rate for knock-in **RES** is 11.03% while the average rate for plain vanilla is 9.61%, which is against the theory that knock-in

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features are favourable to investors. The effective coupon rates of plain vanilla and knock-in RES are presented in Table 5 on page **33.**

People might argue that the discrepancy as described above can be explained away because different underlying stocks have different risks and volatilities, thus driving effective coupon rates of knock-in RES higher. However, such arguments appear somewhat weak if we see the specific example of Texas Instruments. In January, April, and July of 2006, ABN AMRO respectively issued three reverse exchangeable securities all linked to the performance of Texas Instruments common stock. In these three RES, one is plain vanilla and two are knock-in. We would expect the coupon rates for knock-ins to be lower than that for plain vanilla RES but this is not the case. The coupon rate for the plain vanilla is 10% while one knock-in is 9.5% and the other is 10.5%. Similar examples include McDonald's and Motorola. Apparently, the knock-in characteristic did not make much difference when the issuer designed those financial instruments.

5.3 Some caveats

First of all, the knock-in RES issued by ABN AMRO in US market are privately placed and their market makers make the market on an off-exchange basis. As a result, the volatility of embedded put options may have a different nature from that of options traded in organized exchanges. In my analysis, I use exchange-traded options to calculate the implied volatility for the options embedded in RES. The implied volatility of exchange traded options may not properly reflect the true nature of RES, thus distorting the findings of this paper. However, even if there is some distortion, the main conclusion will remain the same. This is partly evidenced by the example of Texas Instruments as discussed in section 5.2.

Second, in the US market, the long-term call options are not actively traded. This may cause some problems when we use the market quotes of those illiquid options to calculate the implied volatility. The relationship between the implied volatility and net buy pressures is well addressed by Bollen and Whaley (2004). Fortunately, all the sample RES have terms of one year or nine months, and the options on most underlying stocks still have some relatively active market for these maturities.

 $\mathcal{L}(\mathbf{q})$, where $\mathcal{L}(\mathbf{q})$ is a set of

6 SUMMARY AND CONCLUSION

In recent years, various equity linked securities are dynamically introduced to the market. The research comparing theoretical value of those derivatives to their effective price is also becoming a hot spot in finance literature. This paper focuses on one specific equity linked security—knock-in reverse exchangeable security. This type of security initially starts as a straight bond but has relatively high coupon rate. During the life of the security, if the price of the underlying stock goes down to a trigger level, the security is "knocked-in" and become a reverse exchangeable security, which gives the issuer an option to either pay back the principal in cash or to deliver a pre-determined number of shares.

The building block approach is used in this paper to dissect the security into three components: a bond, a written put option, and the probability that the security will knock in. A no-arbitrage framework is applied in this paper to replicate the payoffs of a knock-in RES. As a result, the value of a knock-in RES can be represented by values of more basic securities such as bonds and call options. The likelihood of knock-in is simulated by using Monte Carlo simulations, and the probability obtained is, as a critical block, integrated into the valuation model of the reverse exchangeable security.

The results indicate a significant overpricing of RES if the knock-in feature is removed from a sample security while all other issuance terms remain unchanged. This artificial removal is a reasonable analysis procedure because a comparison between plain vanilla and knock-in RES shows that the issuer did not add much premium for the knock-in feature when the securities are issued, Consequently, when knock-in features are added back to those securities, their theoretical value significantly increases, which leads to lower "fair" coupon rates. A positive coupon spread of 5.45% is reported if knock-in characteristic is peeled away. The result is consistent with previous empirical studies on plain vanilla RES in the US market. But when the knock-in feature is added back, an under-pricing of 1.95% is observed. The results show that while investors in plain vanilla reverse exchangeable securities are suffering significant economic losses, the investors in knock-in RES are enjoying abnormal returns.

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APPENDICES

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Appendix A Tables

Table 1 Composition and Characteristics of knock-in RES sample

This table contains characteristics of total 54 "knock-in" RES issued by ABN AMRO from July 2005 to July 2006. Information pertaining to RES symbol, amount of principal borrowed, issuing and maturity date, coupon rate, underlying stock, conversion ratio, and "knock-in" trigger are provided below (coupons paid semi-annually or quarterly in arrears; strike price is that of the underlying stock on issuance date).

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Table 2 Panels A and B: Knock-in probability

Panel A

presents parameters needed in simulating knock-in probabilities, including initial stock price (S_o) expected return **(p),** volatility (a), and triggering level. Probabilities simulated are presented in the last column.

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Table 2 Panel B: Regression results among variables

Panel B presents the results of a regression between knock-in probability (dependent variable) and expected return, μ (independent variable 1), volatility, σ (independent variable 2), and triggering level, γ(independent variable 3)

$$
P = \alpha + \beta_1 \mu + \beta_2 \sigma + \beta_3 \gamma + \varepsilon
$$

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Table 3 Theoretical call option price

This table presents parameters needed in calculating the theoretical call price, including initial stock price (S_0) , strike price (K) , riskfree rate (r) , dividend yield (δ) , volatility (σ) , and time to maturity (T).

RES Symbol	S ₀	Κ	٢	δ	σ	T	Call price
Alcoa	32.57	32.57	5.483%	1.980%	26.156%	1	3.8434
App_Kn1	67.04	67.04	5.441%	0.000%	44.470%	0.75	11.4281
Cat	72.7	72.7	5.140%	2.078%	22.870%	1	7.5152
Ches	31.43	31.43	4.678%	0.568%	36.701%	1	5.1153
CSN	29.73	29.73	5.140%	9.531%	55.040%	0.75	4.8504
CVDRD	24.04	24.04	5.693%	1.489%	30.361%	1	3.313
Conoco	61.17	61.17	5.630%	2.176%	33.460%	1	8.8746
Corning	21.16	21.16	4.810%	0.000%	43.317%	1	4.0614
Dell	25.68	25.68	5.418%	0.000%	31.902%	1	3.891
ENSCO	53.69	53.69	5.295%	0.300%	39.432%	1	9.5313
Freeport	54.75	54.75	5.361%	2.372%	33.702%	0.75	6.7855
GE	33.93	33.93	5.630%	2.859%	16.073%	1	2.5646
GM	34.55	34.55	4.280%	3.537%	44.480%	1	5.972
Harmony	16.21	16.21	5.363%	0.000%	48.513%	0.75	2.9754
HD	33.84	33.84	5.698%	1.784%	28.179%	1	4.3261
JET-3	11.82	11.82	5.080%	0.000%	45.411%	0.75	2.0374
Lyondell	28.62	28.62	4.440%	4.018%	33.502%	1	3.7079
McDonald	32.29	32.29	4.716%	1.882%	18.745%	1	2.8024
MGM	41.14	41.14	4.735%	0.000%	26.779%	1	5.2983
Micron	14.59	14.59	4.735%	0.000%	28.508%	1	1.9749
Mittal	33.4	33.4	5.079%	1.930%	34.710%	1	4.9699
Motorola-1	21.94	21.94	5.180%	1.094%	29.993%	1	2.9929
Motorola-2	20.42	20.42	5.401%	1.094%	29.737%	1	2.787
XTO	43.06	43.06	5.161%	0.698%	32.383%	0.75	5.4363
Noble	82.6	82.6	5.418%	0.200%	41.421%	1	15.3801
UH	44.42	44.42	5.401%	0.100%	27.318%	1	5.9298
Urban	19.34	19.34	5.400%	0.000%	43.397%	1	3.7678
PD	86.94	86.94	5.441%	1.000%	44.367%	0.75	14.3921
Starbucks	35.99	35.99	5.483%	0.000%	32.900%	1	5.5999
Valero	61.5	61.5	5.506%	0.600%	34.256%	1	9.6602
Red Hat	25.93	25.93	5.630%	0.000%	41.850%	1	4.9271
Ultra	52.58	52.58	5.630%	0.000%	51.521%	1	11.8962
WAL	48.17	48.17	5.693%	1.587%	19.853%	1	4.7052
SEARS1	154.6	154.6	4.163%	0.000%	29.740%	1	21.228
SEARS2	119.84	119.84	4.851%	0.000%	39.446%	1	21.2671
TEXAS1	35.34	35.34	5.329%	0.400%	28.936%	1	4.855
TEXAS2	28.75	28.75	5.698%	0.400%	26.433%	1	3.7104
SLB1	68.51	68.51	5.329%	0.800%	34.671%	1	10.7284
SLB ₂	65.11	65.11	5.693%	0.800%	36.522%	1	10.7587

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 $\label{eq:2.1} \frac{1}{\sqrt{2}}\int_{\mathbb{R}^{2}}\left|\frac{d\mathbf{r}}{d\mathbf{r}}\right|^{2}d\mathbf{r}d\mathbf{r}$

Table 4 "Fair" coupon rates with and without knock-in features

This table presents "fair" coupon rates for knock-in RES and for RES of which the knock-in features are artificially removed. CS stands for coupon spread which is defined as the difference between the "fair" coupon rate and the effective coupon rate.

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Table 5 Comparison between knock-in and plain vanilla

This table contains the knock-in RES and plain vanilla RES issued by ABN AMRO on the US market from July 2005 to July 2006.

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Appendix B An example of calculating implied volatilities

Volatility Table

Suppose we are going to calculate the implied volatility (IV) for an option with a strike price of \$45 and time to maturity (TTM) of 0.6 year.

Step 1. Using linear interpolation horizontally:

Let $X1 = IV$ for strike price of 45 and TTM of 0.5,

 $X2 = IV$ for strike price of 45 and TTM of 0.7,

$$
\frac{|45-40|}{|X1-0.37|} = \frac{|50-45|}{|0.4-X1|} \Rightarrow X1 = 0.385
$$

$$
|45-40| \qquad |50-45|
$$

$$
\frac{|45-40|}{|X2-0.42|} = \frac{|50-45|}{|0.49 - X2|} \Rightarrow X2 = 0.455
$$

Step 2. Using linear interpolation vertically:

Let $Y = IV$ for strike price of 45 and TTM of 0.6,

$$
\frac{|Y - X1|}{|0.6 - 0.5|} = \frac{|X2 - Y|}{|0.7 - 0.6|} \Rightarrow Y = 0.42
$$

 $\label{eq:2.1} \hat{A}^{(1)} = \hat{A}^{(1)} + \hat{A}^{(2)}$

Appendix C An offering prospectus of a knock-in RES

For copyright reasons, the offering prospectus cannot be reproduced here. Please see http://www.abnamromarkets.com/pdf/US00079FQT20/US00079FQT20_EN_Prospectus.pdf for this prospectus in detail.

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