VALUE AT RISK METHODOLOGIES: DEVELOPMENT, IMPLEMENTATION AND EVALUATION

by

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Bachelor of Arts 2003

PROJECT SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ARTS

In the
Faculty
of
Business Administration

SIMON FRASER UNIVERSITY

Summer 2006

APPROVAL

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Degree: **Master of Arts**

Title of Project: **Value at Risk Methodologies: Development, Implementation and Evaluation**

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Date Approved: August 2, 2006

ABSTRACT

Value at Risk (VaR) is a useful concept in risk disclosure, especially for financial institutions. In this paper, the origin and development as well as the regulatory requirement of VaR are discussed. Furthermore, a hypothetical foreign currency forward contract is used as an example to illustrate the implementation of VaR. Back testing is conducted to test the soundness of each VaR model. Analysis in this paper shows that historical simulation and Monte Carlo simulation approaches have more advantages than the delta-normal approach based on the fact that these two approaches capture the option involved portfolio features and pass three back testing models which are used to test the soundness of the VaR models.

Keywords: Value at Risk; back testing; historical simulation; Monte Carlo simulation; delta-normal

EXECUTIVE SUMMARY

The paper gives a detailed introduction of a useful risk management technique, value at risk (VaR), which was developed in the 1990s and is widely used by regulators and financial institutions to measure the magnitude of risks in frequently traded portfolios. VaR measures the worst expected loss over a pre-set time horizon at a given confidence level. The development, implementation and evaluation of VaR methodologies are illustrated in the paper.

In the first section, the concept of VaR and the comparison between traditional valuation model and the VaR model are introduced. Three major financial institution failures occurred in the early 1990s because of the lack of efficient risk management tools. These failures are then introduced to illustrate the reason of private sectors' invention of the VaR technique. Furthermore, Basel Committee of Banking Supervision launched the Basel Accords in 1988, 1996 and 2004 to regulate the risk disclosure of financial institutions. The VaR technique is proved by the Basel Accord and is widely used by financial institutions.

The second section will provide a VaR calculation literature review which focuses on parametric approach, which is delta-normal, and non-parametric approach including historical simulation and Monte Carlo Simulation.

The Third section will replicate the paper from Linsmeier and Pearson (2000) to implement three VaR methodologies. Similar to Linsmeier and Pearson (2000) which calculates the VaR for a US dollar and British pound foreign currency forward contract using the above three methodologies, this paper calculates the VaRs of a US dollar and Canadian dollar foreign currency forward contract. The contribution of this paper comes from the calculation of more

than one period of VaRs. VaRs spread over 1899 periods are calculated by repeating the same procedure in Linsmeier and Pearson (2000) 1899 times using historical data going back to 1899 periods. More importantly, the unconditional, independent and conditional coverage back testing models are used to test the validity of the VaR models the paper is using, tests Linsmeier and Pearson (2000) do not provide. These tests are back testing techniques. Philippe (2000) describes back testing as a formal statistical framework that consists of verifying that actual losses are in line with projected losses. The test results show that historical simulation and Monte Carlo simulation in this paper provide unbiased VaR estimates for this foreign currency forward contract. The delta-normal approach in this paper fails these three tests; therefore, it is not a good method to calculate VaRs for this contract.

To my parents, thank you for your unconditional

love and continued support for so many years.

献给我的父母，谢谢你们这么多年来

无条件的爱和持续的支持！

ACKNOWLEDGEMENTS

I would like to gratefully acknowledge Professor Chris Veld for his inspiration, his enthusiasm, his patience, and his great effort to guide me through this paper. I could not make it happen in such a short time line without the help of Professor Chris Veld. I also wish to thank Professor George Blazenko and Professor Christophe Perignon for their helpful comments and suggestions.

I am grateful to all my classmates from the Financial Risk Management program, Segal School of Business, Simon Fraser University, for being good friends of me, providing supports on my programming, and creating such a fun environment in this intense learning period.

I would also like to thank my parents' continued support to bring me opportunities to explore new things in the world. To them I dedicate this paper.

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1.1 The Concept of Value at Risk

In today's highly sophisticated financial market, companies are broadly involved in widely diversified portfolios outside their core businesses. Such involvements enable companies to take advantage of a large number of financial instruments in order to achieve the goals of hedging and leveraging. Due to the complicated characteristics of financial instruments, it is difficult for both senior management and investors to understand the promising aspects of an investment portfolio and make the investment decision by knowing what the risks are within their tolerance. This is why the Value at Risk (VaR) concept was introduced to describe the market risk of a portfolio in a commonly understandable method. Hull (2006) describes VaR concept as "an attempt to provide a single number summarizing the total risk in a portfolio of financial assets". It measures the worst expected loss over a pre-set time horizon at a given confidence level. Specifically, VaR is the lowest quantile of a projected distribution of profits and losses during a specific time period. When the time horizon T and the confidence interval q% are chosen, VaR can be calculated. Then we have q percent confidence that the loss will not exceed V dollars in the next T trading days, where V is the value of VaR.

In general, VaR can be applied for the portfolios with normally distributed changes in the value of the portfolio. And it is the loss at (100-q) quantile over the next T days.

1.2 Comparison between an Asset Valuation Model and the VaR Model

Philippe (2000) demonstrates that there are several similar aspects in the methodology of the VaR risk management approach and a fundamental asset valuation model. Consider the traditional martingale valuation approach, to obtain the present value at time t, f_t, of an asset from its future value S at time T; we can take the discounted expectation from the future value F(S):

$$f_t = E[e^{-r(T-t)}F(S_T)]$$

The assumption of the martingale approach is risk neutrality which uses the risk-free rate as the discount rate. While the goal of the VaR approach is to measure the variation in value of an asset at certain trading date T:

$$\text{VaR}(q, T) = E[F_T] - Q[F_T, q]$$

Where $Q[F_T, q]$ is the quantile under the confidence level q. $E[F_T]$ is the expected value of a portfolio at time T which is assumed to be zero. Both approaches require a model for the estimation of prices. The valuation approach considers the mean, the expectation of a return distribution while the VaR approach illustrates the possible variation in the returns.

Besides the common points of these two approaches, (Philippe, 2000) mentions two differences regarding these approaches. The valuation model requires a precise estimation number to make trading decisions while the VaR approach only provides a rough idea of the magnitude of the potential losses. Another difference is that the valuation approach requires the assumption of risk neutrality while the VaR approach can be operated in actual distributions. Table 1.1 lists the similarities and differences of two approaches:

Table 1.1 Comparison of the Valuation model and the VaR model

	Derivatives Valuation	VaR
Principle	Expected discounted value	Distribution of future values
Focus	Centre of distribution	Tails of distribution
Precision	High precision needed for pricing purposes	Less precision needed, simply approximate tails
Distribution	Risk-neutral distributions and discounting	Actual, objective distributions

Source: Jorion, Philippe. Value at Risk: The Benchmark for Controlling Market Risk. Blacklick, OH, USA: McGraw-Hill Professional Book Group, 2000. p28.

1.3 The Origin of VaR

The VaR approach serves as a good indicator of how risky the investment would be. Therefore it is useful for investment decision making and supervision. In this section the original of the VaR concept is examined by introducing some failures in trading financial instruments and the regulations corresponding to those disasters.

1.3.1 Financial Risk Management Failure Cases

1.3.1.1 The Collapse of Barings's Bank

On February 26, 1995, the Barings Bank went bankrupt after 233 years since its foundation. The failure of this bank was due to a lost of $1.3 billion from derivatives traded by one single trader, the 26-year-old Nicholas Leeson. He was the head of derivatives trading at the Singapore office. He took a large long position on the Nikkei 225 index betting that the Japanese stocks would rise. Leeson continued using the 88888 account to trade beyond limits by changing computer records. Every time the index dropped, he increased his long positions. These added up to $7 billion. Futures markets require daily settlement. When the index dropped with 15% in the first two months of 1995, the Barings Bank suffered large losses from margin calls on the wrong positions. When this failure caught people's attention on better operational risk control, the abuse of computer uses, it also indicated the necessity of better internal audit system and methodology, a complete measurement of risks for trading positions (LeBaron, 2005).

1.3.1.2 The Loss of Metallgesellschaft in Hedging Positions

In December 1993, Metallgesellschaft (MG) announced a loss of approximately $1.5 billion cash flow problems in maintaining a hedging strategy operated by a subsidiary company MG Refining and Marketing Inc (MGRW). By 1993, MGRW had sold forward contracts of oil products amounting to 180 million barrels at prices over the spot rate. Customers could lock the price over a period of 10 years. The unique feature of these contracts was an option clause enabling the customers to terminate the contracts before the maturity date if the price of front-month New York Mercantile Exchange (NYMEX) futures contract exceeded the price MGRM provided to the customers. Holders could receive one-half of the difference between the futures price and the fixed prices times the total volume left to be delivered on the contract. To hedge the possible increase in price of their forward contracts, MGRM hedged their position using a "rolling hedge" of entering a series of short-term, 3-months futures contracts. In the long run, say, 10 years, the net gain from the total short-run futures contracts would converge to the gain of buying and holding a single forward contract with a maturity of 10 years.

However, this "rolling hedge" required sufficient funding for daily settlements and margin calls in case of a sudden and sharp decrease of the future contracts. The hedging went wrong when the oil cash price fell from $20 to $15 in 1993. But MGRM's parent company in Germany did not have an incentive to provide a billion dollars for the futures contracts' margin calls when MGRM themselves did not have sufficient funds to do this. The liquidation of those futures contracts immediately led to a reported loss of $1.3 billion. In fact, positive and negative cash flows would balance out over the entire life of the hedge. The parent company's limited understanding of the true risk of the strategy accounted for partial reasons of the loss.

1.3.1.3 The Loss of Orange County, California Government Fund

The orange county government fund loss is a typical case indicating the necessity of the risk management approach VaR. Bob Citron was in charge of managing a $7.5 billion portfolio of the government fund. This fund was invested by county schools, cities, special districts and the county itself. In order to raise more capital with existing fund, he used reverse repurchase agreements[1] to borrow approximately $12.5 billion for a $20 billion investment in agency notes with average maturity of about 4 years. At the time, the interest rates were falling, so short-term funding cost was lower than medium-term yields. By refinancing the funding in a short period of time, the borrower can capture the advantage of the decrease in the interest rate. The strategy went well at first.

In February 1994, the increase of interest rate led to margin calls from Wall Street brokers providing short-term financing on the paper loss of the fund. When the news was disclosed, investors tried to pull out their capital. Eventually, the fund defaulted on its collateral payments. This led to a liquidation of its collateral. In January 1995, the realized loss amounted to $1.8 billion following the liquidation of remaining securities in the fund.

Orange County's loss is a typical case of risk unawareness. Citron reported his portfolio as no risk because he planned to hold to maturity. On the other hand, the lack of regulations for recording unrealized profits and losses of a municipal investment pool could be counted as another reason for Orange County's loss.

[1] Reverse Repurchase Agreement: A purchase of securities with an agreement to resell them at a higher price at a specific future date. This is a way to borrow money and allow the securities to be held as collateral. Reverse repos occur most often in government securities, and often also in other securities that are highly valued and thus considered a good source of collateral. (WebFinance, 2005)

1.3.2 Private Sector Innovation of VaR Method

1.3.2.1 G-30 Report

The Group of Thirty (G-30), a private, non-profit organization founded in 1978 and is composed of bankers, regulators and academics from leading industrial nations to achieve a deepen understanding of international economic and financial issues by meetings twice a year. [2] Its publication in 1993, Derivatives: Practices and Principles, which came to be known as the G-30 report, is the first the use the word VaR.[3] This work provides a comprehensive study of derivatives markets with recommendations to help dealers and end-users better understand and manage the risk of derivatives activities. The recommendations included the role of boards and senior management, the implementation of independent risk management functions and the various risks that derivatives transactions entail.

The report recommends dealers and end-users who manage portfolios to mark-to-market daily and to assess portfolios with both VaR and stress testing. While the G-30 Report focuses on derivatives activities, its recommendation can be used in other traded instruments. Therefore it plays an important role on defining financial risk management in the 1990s.

1.3.2.2 J.P. Morgan's RiskMetrics

In October 1994, observing the fluctuations on the firm's earning and balance sheets, JP Morgan launched its free RiskMetrics service, intending to promote the use of Value at Risk among the firm's institutional clients. The service provided a technical document describing how to implement a VaR measure. Furthermore, it contained a system and data to estimate volatilities using an exponential moving average of volatility to capture rapid change in volatility. Their

[2] The Group of Thirty, "About the Group," 30, http://www.group30.org/about.php.
[3] Riskglossary.com, "Group of 30 Report," http://www.riskglossary.com/link/group_of_30_report.htm.

database included financial time series in bond markets, money markets, swaps, foreign exchange, commodities, spread products, equity indices and so forth.[4]

These data were in the form of time series and a covariance matrix and was updated quarterly from historical data. Trading units would update their position deltas by e-mail every day. These updates in key factors were aggregated to express the combined portfolio value as a linear polynomial of those factors. In this way, the standard deviation of the portfolio value could be calculated. Various VaR metrics were generated with the assumption that the portfolio value was normally distributed.[5]

J.P. Morgan launched this free system with a desire to increase greater transparency of market risks, to provide sophisticated risk management tools to other users who were unable to develop such system and to promote J.P. Morgan's methodology as an industry standard

1.4 Regulatory Standard with VaR

1.4.1 Securities and Exchange Commission (SEC)'s Requirement of VaR Disclosure

In January 1997, the SEC declared the requirement of disclosing quantitative information about risks of financial instruments in all fillings, effective for fiscal years after June 15, 1998. Before SEC's requirement, security analysts and accountants had a feeling that users were confused by insufficient disclosure of the potential effect of derivatives activity on corporate profits. Also there was no detail reporting guidelines providing detail disclosure. Nearly all companies claimed that they used derivatives to hedge rather than to speculate, but large losses incurred by derivatives trading led the public to think the contrary. Therefore, SEC required

[4] Bill Goffe, "JP Morgan's RiskMetrics," Academia Sinica, http://www.sinica.edu.tw/main_e.shtml.
[5] Riskglossary.com, "RiskMetrics," http://www.riskglossary.com/link/riskmetrics.htm.

registrants to disclose quantitative information of market risks. There were three alternatives that companies could use for information disclosure.

1. A tabular presentation of expected cash flows and contact terms summarized by risk category.
2. A sensitivity analysis expressing possible losses for hypothetical changes in market prices and quotes.
3. Value-at-risk measures for the current reporting period, which is to be compared to actual changes in market values.

Philippe (2000) demonstrates that the SEC rules were welcomed by users of financial statements. From these three approaches, financial industries preferred the VaR reporting approach, because unlike the sensitivity analysis, VaR reveals little information about the direction of exposure. However, on the other hand, the new rule imposed compliance costs, resulting in corporations' lack of enthusiasm to do so.

1.4.2 Basel Committee on Banking Supervision

The Basel Committee is playing an important role in standardizing bank regulations across jurisdictions. Its foundation can be traced back to 1974 when the German regulators forced the Bank Herstatt with a messy financial situation into liquidation. Responding to the Herstatt's failure, a group of ten (G-10) countries (actually eleven countries), including Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Sweden, Switzerland, the United Kingdom, the United States, and Luxemburg formed a committee under the platform of the Bank for International Settlements (BIS). The Basel Committee on Banking Supervision focuses on defining roles of regulators in cross-jurisdictional situations, ensuring that international banks or banks holding companies do not escape comprehensive supervision by a regulatory authority and promoting uniform capital requirements, so banks from different countries may compete with one another. The Bank of International Settlements stated that:

The Committee does not possess any formal supranational supervisory authority, and its conclusions do not, and were never intended to, have legal force. Rather, it formulates broad supervisory standards and guidelines and recommends statements of best practice in the expectation that individual authorities will take steps to implement them through detailed arrangements - statutory or otherwise - which are best suited to their own national systems. In this way, the Committee encourages convergence towards common approaches and common standards without attempting detailed harmonisation of member countries' supervisory techniques.

1.4.2.1 1988 Basel Accord

In 1988, the committee published its first guideline for banking supervision, the so-called Basel I. The 1988 Basel Accord was aiming at providing a minimum standard of capital requirements to form a buffer of credit risk. It ranked banks' capital into two tiers. The capital elements are illustrated in Table 1.2:

Table 1.2 1988 Basel Accord Capital Elements

	Capital Elements
Tire 1 (Core Capital)	Paid-up share capital/common stock
	Disclosed reserves
Tire 2 (Supplementary Capital)	Undisclosed reserves
	Asset revaluation reserves
	General provisions/general loan-loss reserves
	Hybrid (debt/equity) capital instruments
	Subordinated debt

Source: International Convergence of Capital Measurement and Capital Standards

Tire 1 capital or core capital should represent at least 50% of a bank's total capital. The 1988 Basel Accord defined the credit risk charge (CRC), which is the minimum capital requirement, as the product of a Cook Ratio, 8% and the total risk-weighted assets, which will be illustrated clearly in Table 1.3. The CRC formula is shown as below:

$$\text{Credit Risk Charge (CRC)} = 8\% \times (\sum_i \omega_i \times Asset_i)$$

The risk weights ω_i and their corresponding asset types were defined in Table 1.3

Table 1.3 Risk Weights and Asset Types

Weight (ω_i)	**Asset Type**
0%	Cash held, Claims on OECD governments and governments in national currency
20%	Cash to be received, Claims on OECD banks, Claims on non-OECD banks below 1 year, Claims on multilateral development banks
50%	Residential mortgage loans
100%	Claims on private sector (corporate debts and equity), Claims on non-OECD banks above 1 year, Real Estate, Plant and Equipment

Source: Jorion, Philippe. Value at Risk: The Benchmark for Controlling Market Risk. Blacklick, OH, USA: McGraw-Hill Professional Book Group, 2000. p56.

OECD countries are members of the Organization for Economic Cooperation and Development. It currently comprises of Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Korea, Luxemburg, Mexico, the Netherlands, New Zealand, Norway, Portugal, Portugal, Slovak Republic, Spain, Sweden, Switzerland, Turkey, the United Kingdom, and the United States. [6]

Although 1988 Basel Accord is a milestone in banking supervision, there are several significant drawbacks in it. First, there is no recognition of term structure effects. It assigned the same weight to a loan with 2 years maturity and a loan with 30 years maturity, where the latter is more likely to default; second, there is an inadequate differentiation of credit risk; third, there is no recognition of diversification effects. The credit risk in a diversified portfolio is much lower than a non-diversified one; fourth, there is no recognition of market risk that rose from banks' trading of derivatives.

[6] Organisation for Economic Co-operation and Development, "Member Countries," OECD, http://www.oecd.org/countrieslist/0,3025,en_33873108_33844430_1_1_1_1_1,00.html

The 1996 Basel Accord Amendment added a capital charge for market risk, Market Risk Charge (MRC) to the total risk charge. MRC is based only on a bank's trading book containing short-term financial instruments. On the other hand, the banking book contains other instruments such as loans. The Basel Committee came forth with an internal model approach in 1995 to allow banks to choose their own internal value at risk methods. The committee specified several critical elements in choosing an appropriate VaR model. Quantitative parameters for VaR include a horizon of 10 trading days, 99% confidence level and observation period of at least one year which should be updated quarterly. The formula to calculate MRC using VaR is shown below (Philippe 2000):

$$\mathrm{MRC}_t^{IMA} = Max(k \times \frac{1}{60}\sum_{i=1}^{60} VAR_{t-i}, VAR_{t-1}) + SRC_t$$

MRC contains two elements. The first is the interest rate risk in the portfolio where the long and short position in different securities can be off-set. The second one is called specific risk charge for each security regardless it is a short or a long position. K is a multiple factor determined by local regulators and has an absolute floor of 3 (Amendment to the Capital Accord to Incorporate Market Risks, 1996).

Furthermore, the 2004 Basel Accord, so called Basel II incorporated external and internal credit ratings to set up different asset type weights. It also took into account an operational risk charge (ORC) in calculating total risk charge. So the minimum risk charge now consists of three parts, credit risk charge, market risk charge and operational risk charge. Since banks can choose their internal VaR model to calculate market risks, VaR method gained a significant research merit in compliance issues.

2 VAR IMPLEMENTATION LITERATURE REVIEW

Generally, approaches to calculate VaRs can be separated into two groups, linear approximation of the portfolio risks with assumption of joint normal (or log-normal) distribution of the market parameters and historical or Monte Carlo simulation-based approach.

2.1 Linear Approximation Approach

Duffie and Pan (1997) measure VaR with linear approximation approach based on two models: a model of random changes in the prices of the underlying instruments (equity indices, interest rates, foreign exchange rates and so on) and a model for computing the sensitivity of the prices of derivatives to the underlying prices. The approach to measure VaR was defined as follows: First, build a model for simulating changes in prices and volatilities over the VaR time horizon. The model can be a parameterized statistical model, a "bootstrap" of historical returns refreshed by recent volatility estimates; Second, build a data-base of portfolio positions and estimate the size of "current" position in each instrument; third, develop a model to revaluate the derivative prices given changes in the market prices and volatility. The model can be an explicit price formula, a delta-based approximation or an analytical approximation of a pricing formula; fourth, simulate the changes in market values of the portfolio using an independently generated scenario of underlying market returns in each instrument.

Simons (1996) mentions this approach in his paper as parametric VAR, which is based on the estimate of the variance-covariance matrix of asset returns. The matrix is obtained using historical time series of asset returns to calculate their standard deviations and correlations. The assumption of a normal distribution of asset returns means that the variance-covariance matrix completely describes the distribution. He describes portfolio risk as its variance, a function of the variance of the return on each instrument in the portfolio, as well as the correlations between each pair of returns. Only if the returns in the portfolio are perfectly correlated, the variance of the portfolio equals the simple sum of the variances of the individual positions. In the case of diversification, the risk that any investment contributes to the portfolio is less than the risk of that investment alone. In other words, the risk of the portfolio is less than the sum of the risks of its parts in the diversified portfolio.

Other linear approximation approaches can be seen in Philippe (1996), Pritsker (1997), J.P. Morgan RiskMetrics (1996), Beder (1995), and Stambaugh (1996).

2.2 Historical and Monte Carlo Simulation Approach

Historical or Monte Carlo simulation-based tools can be called as non-parametric approach, which are used when the portfolio contains nonlinear instruments such as options.

Mauser and Rosen (1999) describe the computation of VaR under simulated-based approach (historical or Monte Carlo approach) as relying on a complete valuation of the portfolio return under a set of scenarios. The profit and loss of a portfolio can be calculated straightforward given a particular "base case" scenario such as representative of current market conditions. Let v_i^0 denote the current value (the base scenario) of instrument i and v_{ij}^t denote the future value at time t of the instrument in scenario j. Furthermore, Mauser and Rosen (1999) define $\Delta v_{ij} = v_i^0 - v_{ij}$ to be the loss of instrument i in scenario j. If the current position in instrument i is x_i, then the loss of the portfolio caused by scenario j is $L_j(x) = \sum_{j=1}^{N} x_i \Delta v_{ij}$. Mauser and Rosen (1999) suppose the likelihood or weight of scenario j is p_j, order the profit and loss from most positive to most negative (order the portfolio return from the biggest to smallest) and compute the cumulative scenario probability. The non-parametric 100(1- α) % VaR equals the return (loss) in scenario j that the cumulative probability first hit or exceeds α .

Other discussions about non-parametric VaR implementation can be seen in Bucay and Rosen (1999), Pritsker (1997), J.P. Morgan RiskMetrics (1996), Beder (1995), and Stambaugh (1996). Litterman (1997a, 1997b), Lucas also discuss optimization problems involving VaR in their works.

3 VAR IMPLEMENTATION

The VaR method is a technical approach trying to provide a quantitative estimation of the possible losses, the magnitude of the risk for certain financial instruments. As mentioned in the last section, there are two common approaches to estimate VaR, the nonparametric and parametric (linear approximation) approaches. The nonparametric approach estimates the VaR for an underlying financial instrument whose profit and loss distribution can be linear or non-linear. The parametric approach estimates the VaR with an assumption of a normal distribution of the underlying instrument. The nonparametric approach includes historical simulation and Monte Carlo simulation methods and delta-normal is the commonly used method of the parametric approach.

3.1 Choice of Parameters

All of these approaches involve the choice of a financial instrument with a holding period of t days and a probability of x percent that an entity's loss will not exceed a certain VaR level during the next t days. The choice of holding days could be 1-day, 5-day, 10-day or an even longer period. It is determined by an entity's holding period of an underlying asset. Financial firms who actively trade financial instruments would use a 1-day horizon, whereas investors and non-financial corporations may use longer periods. The VaR number applies to the potential loss of a certain portfolio in the next trading period, so the underlying assumption is that the portfolio should remain unchanged during this holding period. This assumption is not feasible for financial firms which trade and modify their portfolios according to the spot market condition. That is why they should recalculate their portfolio VaR at a higher frequency. A Commonly used method to convert a 1-day VaR to t-day VaR is to multiply a 1-day VaR by $\sqrt{t}$.

Note that the longer the holding period, the larger the VaR of underlying asset would be. Because the longer the holding period, the larger the profit and loss would be dispersed out from the initial day's value. A typical choice of the probability x can be 1 or 5 percent. The choice of x is determined by the risk manager and the way he or she wants to interpret the VaR to explain a potential loss. The Basel committee recommends a 1% probability and the RiskMetrics system uses 5% probability. Also note that the smaller the probability x, the larger the VaR value would be. Because VaR discloses the left tail risk of a profit and loss distribution, the smaller the probability that a profit and loss would exceed a certain level, VaR, the larger the VaR would be.

In order to compare VaR across different entities, it is necessary to carefully adjust the parameter x and t to the same level.

3.2 Choice of Important Market Factors

Linsmeier and Pearson (2000) illustrate that in order to estimate the VaR of a portfolio, it is necessary to identify feature market factors which are likely to influence the profit and loss of the portfolio. However, taking into account every factor that may affect the portfolio is also infeasible, since even for a simple instrument, possible factors include different maturity dates. In order to reduce the computation work load while remaining the accuracy of VaR, we can identify limited feature market factors of that portfolio and estimate VaR according to the price change of those factors.

Typically, market factors can be chosen by constructing a portfolio using simple instruments that can replicate the payoff of that portfolio. Therefore the risk manager can analyze those instruments or market factors independently and combine those market effects to estimate the VaR of a portfolio. Specifically, for a foreign currency forward for a US company to deliver USD10 million and receive CAD12 million in 3 months, we can replicate the payoff of that contract by a portfolio with a long position in a 91-day zero-coupon bond with a face value of CAD12million and a short position in a 91-day zero-coupon bond with a face value of USD 10 million. From the replication portfolio, it would be easy to find out the possible market factors are the spot exchange rate, the 3-month US dollar interest rate, and the 3-month Canadian interest rate. A detailed example will be provided later in the next section in order to illustrate the composition of this portfolio.

3.3 The Hypothetical Foreign Currency Forward Contract

To examine the VaR implementation details, I will start from a scenario that a three months foreign currency forward contract is held by a US company at some time in the past. On the delivery date, the US company will deliver USD10 million and receive CAD12 million. Suppose the current date is June 28, 2006, so the contract has 91 days remaining until the delivery date of September 27, 2006. At the current date, the three-month US dollar interest rate r_{USD} is 5.01 percent, the three-month Canadian dollar interest rate r_{CAD} is 4.32 percent and the spot exchange rate is 0.88928 USD/CAD. The mark-to-market value of the forward contract expressed in USD value can be obtained from a formula including two interest rates from two countries and the spot foreign exchange rate on June 28, 2006. Specifically,

USD mark-to-market value

$$= [(\text{Exchange rate in USD/CAD}) \times \frac{CAD12million}{1 + r_{CAD}(91/360)}] - \frac{USD10million}{1 + r_{USD}(91/360)}$$

$$= [(0.88928 \text{ USD/CAD}) \times \frac{CAD12million}{1 + 0.0432(91/360)}] - \frac{USD10million}{1 + 0.0501(91/360)}$$

=USD681,190.

The fact behind this calculation is that the first leg of the forward contract is equivalent to a CAD-denominated 91-day zero-coupon bond and the other leg is equivalent to a USD-denominated 91-day zero-coupon bond.

From this calculation, we could find out the truth that the value of a currency forward contract is determined by interest rates in the two counties involved and the spot exchange rate. The next day, June 29, 2006, the value of the foreign currency forward contract would be changed because of the floating interest rates and exchange rate.

3.4 Historical Simulation Approach

In this section, the implementation of the VaR methodology follows Linsmeier and Pearson (2000) to estimate VaR for a 3-month foreign currency forward contract using historical simulation approach.

Historical simulation applies the movement features of the market factors in the past to generate possible movements of those factors in the next day and obtain the possible profits and losses of the underlying portfolio. Because this method uses the historical features to image the features in the future, it does not require the distribution of the market factors to be normal. Therefore it can be applied to estimate the VaR of a portfolio that includes options content.

This approach involves using the historical actual changes dating back to N periods of the current portfolio holding period to construct a hypothetical distribution of potential future profits and losses with N observations and obtain VaR number by picking the one at the x quantile of the distribution. Specifically, we can estimate N sets of hypothetical market factors using historical trend and use them to calculate N hypothetical mark-to-market values of the portfolio on the next trading day. Then N hypothetical portfolio profits and losses can be obtained by subtracting the current mark-to-market value of the portfolio from each hypothetical mark-to-market values of the portfolio. The last step is to read off the VaR on the next trading day at certain quantile of the hypothetical profits and losses distribution of this portfolio.

Again, suppose the current day is June 28, 2006, a US company enters into a foreign currency forward contract to deliver USD10 million on the delivery date in 91 days and receive CAD12 million in exchange. The holding period is one day (t=1); the probability that the profit and loss will exceed VaR is 5%(x=5) and the 100 most current business days (N=100) would be used to obtain 100 sets of hypothetical market factors changes which can eventually be used to calculate 100 hypothetical portfolio profits and losses on the next trading day. The implementation of the historical simulation approach can be illustrated in five steps.

Step one, to identify the featured market factors and the formula to express the mark-to-market value of the forward contract in terms of the market factors. From the analysis above, the market factors are the three-month US dollar interest rate, the three-month Canadian interest rate and the spot USD/CAD exchange rate. Also, the formula in Section 3.3 with a long position in a Canadian dollar-denominated zero-coupon bond with a face value of CAD12 million and a short position in a dollar-denominated zero-coupon bond with face value of USD10 million can be applied to the calculation of mark-to-market value of the forward contract.

Step two, to collect historical data of the market factors for the last N periods. In this example, we need to obtain the three-month US dollar interest rate, the three-month Canadian interest rate and the spot USD/CAD exchange rate for the last 100 business days. In the next step, we need to construct the hypothetical values of the market values using daily changes in those historical data.

Step three, to calculate 100 daily profits and losses, we need to plug in 100 sets of hypothetical values of the market factors into the formula in Section 3.3 to calculate hypothetical mark-to-market values of the portfolio and then subtract the current (June 28) market-to-market of the portfolio from each hypothetical mark-to-market value of the portfolio. The key to complete this step is to obtain 100 sets of hypothetical values of the market factors. We should calculate 100 sets of daily percentage changes in those market factors and combine them with the current (June 28) value of the market factors to obtain 100 sets of hypothetical values of the market factors. Table 3.1 shows the detailed calculation of the first set of hypothetical values of the market factors and the mark-to-market value of the forward contract:

Table 3.1 Calculation of Hypothetical June 29th Mark-to-Market Profit/Loss on Forward Contract: Historical Simulation Method

Measure of Value		Market Factors			Mark-to-Market Value of Forward Contract (USD)
		USD Interest Rate(% per year)	CAD Interest Rate (% per year)	Exchange Rate (USD/CAD)	
1	Actual values as of close of business on 6/28	5.01	4.32	0.8893	681,382
2	Actual values as of close of 101 business days before 6/28	4.47	3.51	0.8742	
3	Actual values as of close of 100 business days before 6/28	4.47	3.54	0.8770	
4	Percentage change 101 business days before to 100 business days before	0.00	0.8547	0.3203	
5	Hypothetical future values on 6/29 calculated using rates from 6/28 and percentage changes from 101 business days before to 100 business days before	5.01	4.3204	0.889328	714,216
6	Hypothetical 6/29 mark-to-market profit/loss on forward contract				32,833

The market factors values on June 28th are used to calculate the mark-to-market value of the forward contract on June 28th which is shown on Line 1. Next, the value on June 29th would be determined. To do this, the actual change of market factors from 101 business days before June 28th (Line 2) to 100 business days before June 28th (Line 3) is deemed as the hypothetical change from June 28th to June 29th (Line 4). June 29th value can be obtained with the following formula:

Value of June 29th = Value of June 28th $\times$(1+ percentage change from 101 business days before to 100 business days before June 28th)

After a set of hypothetical June 29th market factors values are calculated, we can obtain the hypothetical mark-to-market value of the portfolio on June 29th, the actual mark-to-market value on June 28th is subtracted from that on June 29th to get the hypothetical profit and loss of June 29th (Line 6).

To obtain the second hypothetical mark-to-market of the portfolio, the market factors on June 28th and the percentage change in the market factors obtained from the 100 business days before June 28th to 99 business days before June 28th is used to get the second set of hypothetical market factors values and the corresponding market-to-market value of the portfolio is calculated. More specifically, suppose current date June 28, 2006 is time t=i, the percentage change of market factors dating back form t=i-101to t=i-100 is deemed as the percentage change from t=i to t=i+1 (June 29) and used to calculate the possible market factor values on June 29, 2006. Using the same standard, the percentage change from t=i-100 to t=i-99 is used to calculate another set of possible market factor values on June 29. The same steps are repeated until 100 sets of hypothetical market factors are obtained. Then, 100 hypothetical mark-to-market value of the portfolio can be calculated.

Step four, to order the mark-to-market profits and losses from the largest profit to the largest loss and select the 96th profit and loss from the sorted hypothetical profits and losses as the 95% 1-day VaR. Table 1.5 shows the details of the 100 hypothetical daily mark-to-market profits and losses ordered from largest profit to largest loss using historical simulation method. The bold one on the 5% quantile, -89,422 is the 95% 1-day VaR of this foreign currency forward contract. To be specific, there is 95 percent opportunity that the loss of the forward contract on the next trading day, June 29th, 2006 will not exceed USD89, 422 dollars.

Table 3.2 100 Hypothetical Daily Mark-to-Market Profits and Losses Ordered from Largest Profit: Historical Simulation

Order Number	Market Factors			Hypothetical Mark-to-Market Value of Forward Contract (USD)	Change in Mark-to-Market Value of Forward Contract (USD)
	USD Interest Rate(% per year)	CAD Interest Rate (% per year)	Exchange Rate (USD/CAD)		
1	5.020	4.372	0.902	832,600	151,400
2	5.010	4.320	0.902	829,910	148,720
3	5.010	4.310	0.899	797,140	115,950
4	5.020	4.320	0.897	773,400	92,204
5	5.032	4.343	0.897	771,700	90,507
6	4.988	4.320	0.897	768,730	87,533
7	5.010	4.300	0.896	763,080	81,886
8	5.061	4.320	0.896	761,220	80,028
9	5.021	4.342	0.896	758,660	77,468
10	5.032	4.320	0.896	758,240	77,049
91	5.071	4.320	0.884	620,720	-60,473
92	5.010	4.308	0.884	618,920	-62,279
93	5.010	4.257	0.882	600,010	-81,189
94	5.021	4.344	0.882	599,450	-81,748
95	4.969	4.341	0.882	594,850	-86,342
96	5.041	4.309	0.882	591,770	**-89,422**
97	4.988	4.285	0.881	586,710	-94,482
98	5.020	4.310	0.880	572,700	-108,490
99	5.010	4.320	0.879	555,560	-125,630
100	5.020	4.320	0.878	548,930	-132,270

3.5 Monte Carlo Simulation Approach

As another nonparametric VaR approach, Monte Carlo Simulation approach has a number of similarities to the historical simulation approach. Linsmeier and Pearson (2000) mention that the main difference is the method used to generate hypothetical market factors. Other than using percentage changes in the past to estimate possible changes of the market factor on the next day, we can choose a statistical distribution captures the possible changes in the market factors. This specific statistical distribution can be called random number generator. After the random number generator is chosen, a large number of hypothetical changes in the market factors can be generated and a large number of hypothetical profits and losses on the current portfolio can be calculated. Remaining steps in choosing VaR are the same as those in historical approach.

The detail steps to calculate VaR of the same foreign currency forward contract using Monte Carlo simulation approach can be illustrated as follows:

Step one, to identify the key market factors that would approximate the potential impacts to the mark-to-market value of the forward contract. The same market factors as those in historical approach, the three-month US dollar interest rate, the three-month Canadian interest rate and the spot USD/CAD exchange rate are chosen.

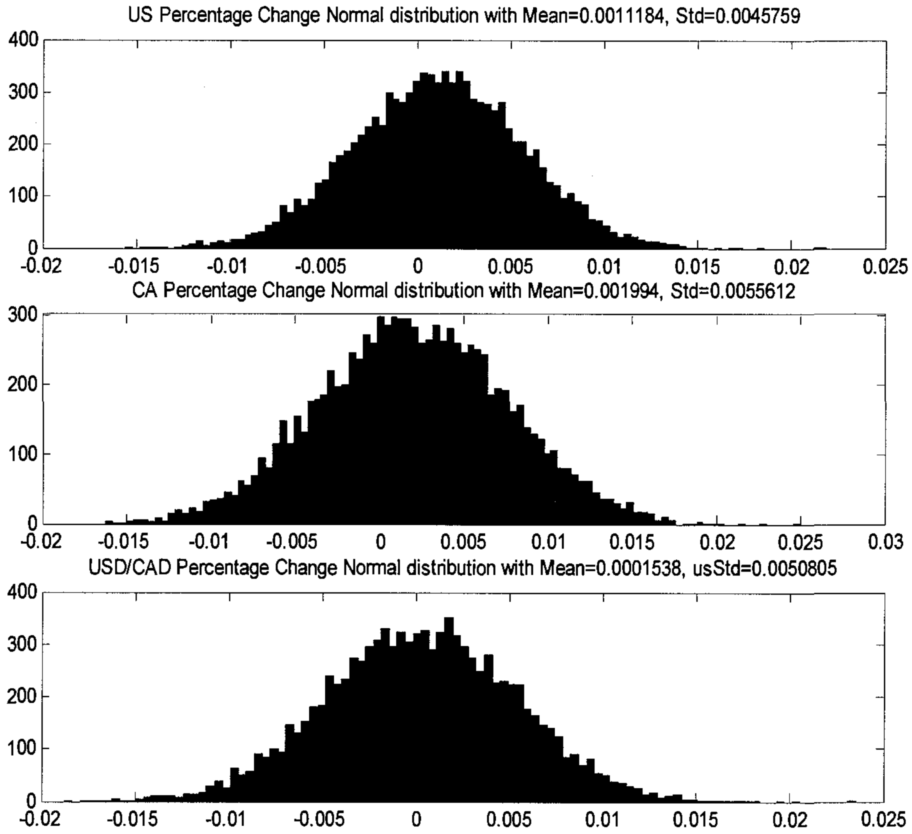
Step two, to determine a distribution for changes in the market factors. Choosing a specific distribution for the market factors is the key difference from other approaches. For either historical approach or delta normal approach, which would be demonstrated later in this paper, a risk manager can only estimate possible future values based on historical data. Linsmeier and Pearson (2000) illustrate that the distribution is free to be chosen by the risk manager designing the risk management system. Therefore, it could be any distributions that the manager thinks reasonably capture possible future changes in the market factors. This paper first estimates the mean and standard deviation of 100 sets of percentage changes of the historical market factors which are dating back 100 periods from the current date, June 28, 2006. The paper then assumes a normal distribution of future possible changes in market factors and constructs a distribution with the estimated mean and variance based on the historical data. The estimated normal distribution parameters obtained from t=i-101 to t=i-1data are listed in Table 3.3:

Table 3.3 Normal Distribution Parameters: Monte Carlo Simulation Method

		Mean	Standard Deviation	Estimation Period
1	US Interest Rate Percentage Changes	0.0011184	0.0045759	t=i-101 to t=i-1
2	Canadian Interest Rate Percentage Changes	0.001994	0.0055612	t=i-101 to t=i-1
3	USD/CAD exchange Rate Percentage Changes	0.0001538	0.0050805	t=i-101 to t=i-1

Step three, to generate 100 hypothetical daily profits and losses of the forward contract using random number generator. Three normal distributions for the above market factors can be constructed with the estimated means and standard deviations. Random numbers representing the future possible percentage changes of the market factors can be drawn from the distributions with means and standard deviations estimated above. Figure 3.1 displays the three random number generators constructed by the above normal distribution parameters. Those parameters are estimated from 100 days historical data, therefore, the new distributions capture the historical movement features of each market factor. 100 sets of hypothetical percentage changes of three market factors can be randomly drawn from these distributions to eventually calculate 100 hypothetical mark-to-market profits and losses of the forward contract.

Figure 3.1 Market Factor Value Percentage Change Random Number Generators: Monte Carlo Simulation Approach



Step four is the same as that in the historical simulation approach. The 100 hypothetical profits and losses are ordered from the largest profit to the largest loss and the 96th profit and loss from the sorted hypothetical profits and losses is the 95% 1-day VaR. Table 3.4 shows the details of the 100 hypothetical daily mark-to-market profits and losses ordered from largest profit to largest loss estimated from Monte Carlo simulation method. The bold one on the 5% quantile, -196,140 is the 95% 1-day VaR of this foreign currency forward contract. To be specific, there is a 95 percent chance that the loss of the forward contract on the next trading day, June 29th, 2006 will not exceed USD196, 140 dollars.

Table 3.4 100 Hypothetical Daily Mark-to-Market Profits and Losses Ordered from Largest Profit: Monte Carlo Simulation

Order Number	Market Factors			Hypothetical Mark-to-Market Value of Forward Contract (USD)	Change in Mark-to-Market Value of Forward Contract (USD)
	USD Interest Rate(% per year)	CAD Interest Rate (% per year)	Exchange Rate (USD/CAD)		
1	5.079	4.324	0.907	893,710	212,520
2	4.975	4.333	0.907	891,220	210,020
3	5.020	4.289	0.905	867,590	186,400
4	5.027	4.283	0.904	860,690	179,500
5	4.984	4.335	0.904	849,110	167,920
6	4.997	4.253	0.903	843,590	162,390
7	4.960	4.291	0.902	833,080	151,890
8	4.975	4.382	0.902	829,110	147,920
9	5.029	4.304	0.901	821,890	140,700
10	5.032	4.296	0.901	819,920	138,730
91	5.026	4.323	0.877	533,260	-147,940
92	5.058	4.297	0.876	530,870	-150,330
93	4.967	4.360	0.877	528,060	-153,130
94	5.057	4.351	0.875	516,500	-164,690
95	5.026	4.263	0.873	489,760	-191,440
96	4.963	4.354	0.873	485,050	**-196,140**
97	4.971	4.320	0.870	450,100	-231,090
98	5.008	4.342	0.869	439,000	-242,200
99	5.037	4.314	0.868	431,220	-249,970
100	5.019	4.337	0.867	419,630	-261,570

3.6 Delta-Normal Approach

Linsmeier and Pearson (2000) describe the delta-normal approach as a method to calculate VaRs based on the assumption that the underlying market factors have a multivariate normal distribution. The calculated profits and losses are also normal distributed. For a normal distribution with the mean of zero, possible outcomes less than or equal to 1.65 standard deviations below the mean occur only 5 percent of the time. It has an equivalent meaning of a 95% 1-day VaR. Therefore, based on the forward contract example, we can calculate VaR using following formula:

VaR=-[(Expected change in portfolio value)-1.65(standard deviation in portfolio value)]

Hull (2006) defines α_i as the amount invested in position i, σ_i as the daily volatility of the ith asset and ρ_{ij} as the coefficient of correlation between returns on asset$_i$ and asset$_j$. The variance of the portfolio denoted by σ_P^2 is given by:

$$\sigma_P^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \rho_{ij}\alpha_i\alpha_j\sigma_i\sigma_j$$

Step one, to calculate portfolio variance. Following Linsmeier and Pearson (2000), this paper defines the position value in the US interest rate as the value of the US interest rate leg of the forward contract in Section 3.3 which is -9,874,900. The US dollar demonstrated position value in the Canadian interest rate as the value in the Canadian interest rate leg which is 10,556,000, holding the spot exchange rate constant. The position value of the exchange rate can be defined as the value in the Canadian interest rate leg which is also 10,556,000, holding the Canadian interest rate constant. Linsmeier and Pearson (2000) demonstrate that from a perspective of a US company and risk mapping concerns, a position in a Canadian-denominated bond is exposed to changes in two market factors.

Using the three market factors' historical data dating back to 100 business days of current date June 28th, 2006, a three by three mentions variance-covariance matrix P is constructed and W is set to be a scalar with three position values as the elements.

W= [US interest rate position value, Canadian interest rate position value, exchange rate position value]

= [-9,874,90010,556,000 10,556,000]

To calculate the portfolio variance is equivalent to complete the following matrix calculation:

$$\sigma_P^2 = W \times P \times W'$$

Step two, to obtain portfolio standard deviation and multiply it by the critical value of 1.65 for a normal distribution under the 95% confidence interval. The VaR of the portfolio:

VaR=-[(Expected change in portfolio value)-1.65(standard deviation in portfolio value)]

=-[0-1.65(σ_P)]

The calculated VaR is USD 2,652,900, interpreting the possibility of 5% for the profit and loss of the forward contract next day to be -USD2, 652,900.

4 VAR METHODOLOGIES EVALUATION

4.1 VaR Methodologies Comparison

The qualitative comparison of the three methodologies is conducted by Linsmeier and Pearson (2000). The comparisons of the ability to capture the risks of options and option-like instruments, ease of implementation, ease of communication, and reliability of results are provided.

4.1.1 Ability to Capture Options and Option-Like Instruments

Portfolio values containing option and option-like instruments are volatile with the option exercising decisions; therefore, linear approximations are unlikely to approximate the true distributions of those option involved portfolio values. Historical and Monte Carlo simulation methodologies re-compute the value of the portfolio for each "draw" of corresponding market factors. That is the reason that these two methods incorporate the option exercising decisions and provide the correct distribution of the portfolio value. In contrast, the linear approximation in delta-normal approach does a poor job in capturing those option decisions. In the special case with 1-day holding period, delta-normal approach might work well because the portfolio value is unlikely to vary significantly from last day. In a longer holding period such as one month, a large change in portfolio value is likely to happen. Therefore, linear approximation provided by delta-normal approach is not reliable.

4.1.2 Ease of Implementation

Historical method is conceptually easy to implement, since only percentage changes using old data are used to estimate possible future market factor values. Historical simulation does not require advanced econometrics technique or finance modelling to calculate the possible change in underlying market factors. On the other hand, it highly relies on historical data, thus this technique is difficult to implement when sufficient time series data in portfolio involving market factors in less well developed capital markets are difficult to obtain.

Monte Carlo approach is difficult to implement because it requires great expertise and judgement in econometrics and finance to select the right distributions, or random number generators to capture market factors' risks.

It is easy to implement delta-normal approach using available software packages which cover prevalent market factors. Although calculating standard deviations and correlations of the market factors are straightforward, obtaining reliable market factors for all maturities in all currencies representing the portfolio can be difficult.

4.1.3 Ease of Communication

Historical simulation has a good advantage in communicating the results to senior managers because of the conceptual simplicity. Monte Carlo simulation is very difficult to explain. The choice of statistical distribution and the design of random number generator are tricky to most people. Delta-normal method is difficult to communicate with people lack of technical training to understand the mathematics of the normal distribution in calculating portfolio standard deviation and VaR too.

4.1.4 Reliability of Results

These three methods all rely on historical data. Historical simulation is the unique one which relies directly on historical data. It is risky that the last 100 days was a period with low volatility in market rates and prices, the estimated VaR is likely to underestimate the true risk of the underlying portfolio.

Monte Carlo simulation and delta-normal methods also rely on historical data to estimate distribution parameters. The problem is not as severe because by assuming distributions to estimate future possible price movement, some historical features are "erased" in the process of replacing actual historical data with data drawn from distributions such as normal distribution.

Another potential problem for these two methods is that the selected distributions may not adequately capture the true distributions of market factors which are likely to have fat tails or likely to be log-normal distributed relative to the normal distribution. In this case, it is likely to underestimate the true risk of the underlying portfolio too.

4.1.5 Realistic Assumptions

Unrealistic assumptions are likely to lead to biased estimated results. Monte Carlo simulation and delta-normal approach in this paper assume the historical data to be normally distributed. In the contrary, in reality, portfolio returns might be log-normal distributed or have fat tails. Whilst, historical simulation does not require any assumptions about the historical data; therefore, it has a great advantage to provide an unbiased estimation of VaR.

Concerns about the reliability of the methodologies can be addressed by back testing technique which is performed by collecting a series of VaR numbers and actual mark-to-market portfolio profits and losses. The criteria and detailed process to implement the testing will be illustrated in the next section.

4.2 Back Testing

Value at risk methodologies are meaningful only when they are compared to actual performance. Philippe (2000) explains back testing technique as "a formal statistical framework that consists of verifying that actual losses are in line with projected losses". In order to compare projected losses with actual losses, the history of VAR forecasts should be compared with their realized portfolio returns systematically. Back testing can also be called as reality checks. Philippe (2000) believes the failure of estimated VaR may due to faulty assumptions, wrong parameters, or inaccurate modelling. For a correct VaR model, the observations falling outside the estimated VaR should be in line with the confidence level. The number of times when actual losses are greater than estimated VaRs can be known as the number of exceptions. For instance, for 100 95% VaR observations, the number of exceptions should approximately be 5. It should be noticed that too few exceptions can be problematic because it indicates that the VaR model is set to be too conservative.

4.2.1 Setup for Back Testing

In Section 3, a 1-day VaR for June 29, 2006 is estimated by three methodologies, historical simulation, Monte Carlo simulation and delta-normal. Recall June 28, 2006 is time t=i. For each method, historical data dating back to 100 business days are analyzed by three methods to estimate the possible VaR on June 29, 2006, time t=i+1. By completing the above procedures, only one VaR number is obtained.

To test the validity of a VaR model, large quantities of VaRs are needed to be compared with the actual portfolio profits and losses. To provide enough VaR observations, this paper repeats the same procedure performed in Section 3 1899 times to obtain 1899 VaRs and then compares them with the actual portfolio profits and losses to calculate the number of exceptions. Since June 28, 2006 is the most current day and data is not available starting from June 29, 2006, this paper calculates VaRs dating back to 1899 days. In this way, three sets of VaRs starting from time t=i-1898 to time t=i are calculated by three methodologies. Detailed VaR values for these periods will be provided in the next section when back testing is conducted.

4.2.2 Back Testing: Unconditional Coverage Model

The concept failure rate is introduced by Philippe (2000). It is the portion of times that VaR is exceeded in a given sample. He defines N as the number of exceptions and T as the number of observations. N/T gives the value of failure rate. As mentioned above, a large number of exceptions indicate that the VaR model underestimates the actual risk. On the other hand, if the number of exceptions are extremely small, it overestimates the risk by setting a greater VaR, therefore the inappropriate VaR misleads the financial institution to set aside greater capital buffer which is actually not necessary most of the time.

A test is needed to know whether N is too small or too large. If we set a confidence level, say, 10%, we have confidence that the actual losses would not be greater than VaR 90% of the time. A null hypothesis that p=0. 1 can then be set up. To see whether the number of exceptions is neither too large nor too small, the confidence region is set as below to calculate acceptable exceptions. The regions are defined by a log-likelihood ratio:

$$LR_{UC} = -2\ln\left[(1-p)^{T-N} p^{N}\right] + 2\ln\left\{[1-(N/T)]^{T-N} (N/T)^{N}\right\}$$

which follows a chi-square distribution with one degree of freedom under the null hypothesis that p is the true probability. The null hypothesis can be rejected if LR>2.7.

4.2.3 Back Testing: Conditional Coverage Models

The unconditional coverage model ignores the time variation in the data. Philippe (2000) introduces another back testing model incorporating time variation to reject VaR models with exceptions clustering in a short time. Under a good VaR model, exceptions should evenly spread over time. If 10 exceptions happen in 2 weeks, it indicates that the VaR model does not capture the volatility in that market. An independent test over the deviations was developed. A deviation indicator is set to 0 if the VaR is not exceeded and to 1 otherwise. T_{ij} is the number of days in which state j occurred in one day while state i occurs in previous day and π_i as the probability of observing an exception on state i the previous day. Table 4.1 displays the computation of parameters for the conditional coverage model.

Table 4.1 Computation of Back Testing Parameters

		Conditional		Unconditional
		No Exception	**Exception**	
Current Day	**No Exception**	$T_{00} = T_0(1-\pi_0)$	$T_{10} = T_1(1-\pi_1)$	$T(1-\pi)$
	Exception	$T_{01} = T_0(\pi_0)$	$T_{11} = T_1(\pi_1)$	$T(\pi)$
Total		T_0	T_1	$T = T_0 + T_1$

Source: Jorion, Philippe. Value at Risk: The Benchmark for Controlling Market Risk. Blacklick, OH, USA: McGraw-Hill Professional Book Group, 2000. p141.

Specifically, T_0: the number of days without exceptions;

T_1: the number of days with exceptions;

π_0: the probability of observing no exceptions yesterday;

π_1: the probability of observing an exception yesterday;

π: the probability of observing an exception;

T_{00}: the number of days without an exception today conditional on observing no exception yesterday;

T_{01}: the number of days with an exception today conditional on observing no exception yesterday;

T_{10}: the number of days without exception today conditional on observing an exception yesterday;

T_{11}: the number of days with an exception today conditional on observing an exception yesterday.

The relevant independence test statistic is

$$LR_{ind} = -2\ln[(1-\pi)^{(T_{00}+T_{10})}\pi^{(T_{01}+T_{11})}] + 2\ln[(1-\pi_0)^{T_{00}}\pi_0^{T_{01}}(1-\pi_1)^{T_{10}}\pi_1^{T_{11}}]$$

The first term is the maximized likelihood under the hypothesis that exceptions are independent across days. Specifically, $\pi = \pi_0 = \pi_1 = (T_{01} + T_{11})/T$.

Conditional coverage must satisfy the following combined test statistic:

$$LR_{CC} = LR_{UC} + LR_{ind}$$

4.2.4 Back Testing: Implementation

This paper calculates 1899 sets of VaR with three methodologies and compares them with actual profits and losses. Following the above criteria, numbers of exceptions under conditional and unconditional approaches are counted. Parameters for the above three test statistics are thus obtained. LR_{UC}, LR_{ind} and LR_{cc} can be calculated from the above formulas. The critical value for chi-square distribution with 1 degree of freedom and 10% confidence level is 2.705543971. The null hypothesis that exceptions happen 5% of the time can be rejected when LR_{UC}, LR_{ind} or LR_{cc} is greater than the critical value. Based on the analysis of 1899 sets of VaRs and actual profits and losses, VaR models estimated from historical simulation and Monte Carlo simulation pass the unconditional test, independent test and the conditional coverage. But VaR model estimated from delta-normal approach fails those three tests.

The total number of exceptions is calculated from historical simulation and is shown in Table 4.2. This table contains the first and the last 10 sets of VaRs estimated by historical simulation for 1899 periods. The corresponding number of period, actual mark-to-market profits and losses and statement verifying the existence of an exception are provided. For instance, Line 4 shows the detail information at period t=i-1895, 1895 business days before the current date June 28, 2006. The actual profit and loss at that period is -77812 and the estimated VaR is -73045. Therefore, the actual loss exceeds the estimated VaR. The statement of 1 is made, indicating that an exception is observed at that period. By summing up the number of statement of 1, the total number of exceptions 104 is obtained.

T_{00} is the number of days without exceptions today conditional on no exceptions yesterday. In column T_{00}, the statement of 0 is made only when no exceptions today and yesterday are observed. For instance, in Line 4, an exception is observed today, therefore, the statement of 1 is made. In Line 5, an exception is observed yesterday, therefore, the statement of 1 is made.

T_{01} is the number of days with exceptions today conditional on no exceptions yesterday. In column T_{01}, the statement of 1 is made only when no exceptions are observed yesterday conditional on an exception is observed today. For instance, in Line 4, no exception is observed yesterday and an exception is observed today, therefore, the statement of 1 is made. In Line 5, there is an exception on yesterday and no exception today, therefore, the statement is 0 is made.

T_{10} is the number of days with exceptions yesterday and without exceptions today. For instance, in Line 5, there is and exception yesterday and no exception today, therefore, the statement 1 is made.

T_{11} is the number of days with exceptions today and yesterday. For instance, in Line 4, only an exception on today is observed, therefore, the statement of 0 is made.

Table 4.2 Computation of Back Testing Parameters: Historical Simulation

	Time Periods	Actual Mark-to-Market Profits and Losses	95% 1-day VaR	Number of Exceptions T_1	T_{00}	T_{01}	T_{10}	T_{11}
1	i-1898	-32017	-65735	0	-	-	-	-
2	i-1897	29074	-64346	0	0	0	0	0
3	i-1896	114720	-73742	0	0	0	0	0
4	i-1895	-77812	-73045	1	1	1	0	0
5	i-1894	-18450	-72874	0	1	0	1	0
6	i-1893	-5472	-72798	0	0	0	0	0
7	i-1892	13903	-72929	0	0	0	0	0
8	i-1891	2846	-72945	0	0	0	0	0
9	i-1890	4065	-73082	0	0	0	0	0
10	i-1889	-22789	-72813	0	0	0	0	0
91	i-9	-53766	-86839	0	0	0	0	0
92	i-8	-45628	-86470	0	0	0	0	0
93	i-7	-1857	-86456	0	0	0	0	0
94	i-6	60938	-86955	0	0	0	0	0
95	i-5	116810	-91108	0	0	0	0	0
96	i-4	-127960	-90015	1	1	1	0	0
97	i-3	-60896	-89473	0	1	0	1	0
98	i-2	3112	-89482	0	0	0	0	0
99	i-1	23927	-89676	0	0	0	0	0
100	i	-31376	-89422	0	0	0	0	0
Total				**104**		**100**	**100**	**4**

Note: Number of exceptions equals to 0 if VaR is not exceeded and to 1 otherwise.

Table 4.3 shows the calculation of three back testing formulas and corresponding results. As calculated above, the total number of exceptions during 1899 periods is 104. Therefore, the total number of days without exceptions T_0 is 1795 which is equal to the total number of period, 1899 minus the days with exception, 104. T_{00}, the number of days without exceptions today conditional on no exceptions yesterday is obtained by counting the number of zeros in column T_{00} at Table 4.2. T_{01} is obtained by counting the number of statement 1 from column T_{01} at

Table 4.2. The number of T_{01}=100 in this table is the same as the number 100 in Table 4.2, column T_{01}. The value T_{10} and T_{11} in this table is the same as those in Table 4.2. $\pi = T_1 / (T_0 + T_1)$ is the unconditional probability of observing an exception. $\pi_{01} = T_{01} / T_0$ is the conditional probability of observing an exception today conditional on observing an exception yesterday. $\pi_{11} = T_{11} / T_1$. The values of LR_{UC}, LR_{ind} and LR_{cc} are calculated by plugging in parameters obtained above to the formulas in Section 4.2.3. None of the values are greater than the critical value 2.705543971 with 1 degree of freedom and 10% confidence interval, therefore, the VaR model, historical simulation method survives the unconditional coverage, independence test and conditional coverage back testing. The above historical simulation method provides unbiased and independent estimation results for the VaR of the forward contract.

Table 4.3 Back Testing for Historical Simulation

Parameters	Parameter Values	Back Testing Results
T_0	1795	
T_1	104	
T_{00}	1694	
T_{01}	100	
T_{10}	100	
T_{11}	4	
π	0.05476567	
π_{01}	0.05571031	
π_{11}	0.03846154	
LR_{uc}	0.88189142	Do Not Reject
LR_{ind}	0.6258772	Do Not Reject
LR_{cc}	1.50776862	Do Not Reject

Note: Model is rejected when LR is greater than 2.705543971

The total number of exceptions calculated from Monte Carlo simulation is shown in Table 4.4. This table contains the first and the last 10 sets of VaRs estimated by Monte Carlo simulation for 1899 periods. The corresponding number of period, actual mark-to-market profits and losses and statement verifying the existence of an exception are provided. For instance, Line 4 shows the detail information at period t=i-1895, 1895 business days before the current date June 28, 2006. The actual profit and loss at that period is -77812 and the estimated VaR is -77408. Therefore, the actual loss exceeds the estimated VaR. The statement of 1 is made, indicating that an exception is observed at that period. By summing up the number of statement of 1, the total number of exceptions 96 is obtained.

In the column T_{00}, the statement of 0 is made only when no exceptions today and yesterday are observed. For instance, in Line 4, an exception is observed today, therefore, the statement of 1 is made. In Line 5, an exception is observed yesterday, therefore, the statement of 1 is made.

In the column T_{01}, the statement of 1 is made only when no exceptions are observed yesterday conditional on an exception is observed today. For instance, in Line 4, no exception is observed yesterday and an exception is observed today, therefore, the statement of 1 is made. In Line 5, there is an exception on yesterday and no exception today, therefore, the statement is 0 is made.

T_{10} is the number of days with exceptions yesterday and without exceptions today. For instance, in Line 5, there is an exception yesterday and no exception today, therefore, the statement 1 is made.

T_{11} is the number of days with exceptions today and yesterday. For instance, in Line 4, only an exception on today is observed, therefore, the statement of 0 is made.

Table 4.4 Computation of Back Testing Parameters: Monte Carlo Simulation

	Time Periods	Actual Mark-to-Market Profits and Losses	95% 1-day VaR	Number of Exceptions	T_{00}	T_{01}	T_{10}	T_{11}
1	i-1898	-32017	-55669	0	-	-	-	-
2	i-1897	29074	-62902	0	0	0	0	0
3	i-1896	114720	-73517	0	0	0	0	0
4	i-1895	-77812	-77408	1	1	1	0	0
5	i-1894	-18450	-64033	0	1	0	1	0
6	i-1893	-5472	-60396	0	0	0	0	0
7	i-1892	13903	-61084	0	0	0	0	0
8	i-1891	2846	-71273	0	0	0	0	0
9	i-1890	4065	-64707	0	0	0	0	0
10	i-1889	-22789	-68209	0	0	0	0	0
91	i-9	-53766	-86735	0	0	0	0	0
92	i-8	-45628	-94068	0	0	0	0	0
93	i-7	-1857	-80611	0	0	0	0	0
94	i-6	60938	-84500	0	0	0	0	0
95	i-5	116810	-70771	0	0	0	0	0
96	i-4	-127960	-77386	1	1	1	0	0
97	i-3	-60896	-92783	0	1	0	1	0
98	i-2	3112	-83863	0	0	0	0	0
99	i-1	23927	-88503	0	0	0	0	0
100	i	-31376	-98960	0	0	0	0	0
Total				**96**		**93**	**93**	**3**

Note: Number of exceptions equals to 0 if VaR is not exceeded and to 1 otherwise.

Table 4.5 shows the calculation of three back testing formulas and corresponding results. As calculated above, the total number of exception during 1899 periods is 96. Therefore, the total number of days without exceptions T_0 is 1803 which is equal to the total number of period, 1899 minus the days with exception, 96. T_{00}, the number of days without exceptions today conditional on no exceptions yesterday is obtained by counting the number of zeros in column T_{00} at Table 4.2. T_{01}, T_{01} and T_{11} are obtained from Table 4.4 and are 93, 93 and 3 respectively. $\pi = T_1$

$/(T_0+T_1)$, $\pi_{01}=T_{01}/T_0$ and $\pi_{11}=T_{11}/T_1$ are 0.05057956, 0.05160932 and 0.03125, respectively. The values of LR_{UC}, LR_{ind} and LR_{cc} are calculated by plugging in parameters obtained above to the formulas in Section 4.2.3. None of the values are greater than the critical value 2.705543971 with 1 degree of freedom and 10% confidence interval, therefore, the VaR model, Monte Carlo simulation method survives the unconditional coverage, independence test and conditional coverage back testing. The above Monte Carlo simulation method provides unbiased and independent estimation results for the VaR of the forward contract.

Table 4.5 Back Testing for Monte Carlo Simulation

Parameters	Parameter Values	Back Testing Results
T_0	1803	
T_1	96	
T_{00}	1709	
T_{01}	93	
T_{10}	93	
T_{11}	3	
π	0.05055292	
π_{01}	0.0515807	
π_{11}	0.03125	
LR_{uc}	0.01218005	Do Not Reject
LR_{ind}	0.89916904	Do Not Reject
LR_{cc}	0.91134909	Do Not Reject

Note: Model is rejected when LR is greater than 2.705543971

5 DISCUSSION AND CONCLUSION

Value at risk is a useful tool to provide information about potential risk level of a portfolio. Although it is based on some assumptions that are difficult to be satisfied, it quantifies the level of risk. Therefore it is becoming widely used by regulators to improve risk disclosure. This technique is also useful in providing information for risk managers to make trading decision taking into account the underlying risks of the portfolio. But it is noted that, good expertise is needed in choosing parameters and models to ensure an appropriate VaR model.

The back testing results indicate that historical simulation and Monte Carlo simulation provide unbiased VaR estimations of the hypothesis portfolio. From the qualitative analysis of model comparison in Section 4.1.5, VaR estimations from historical simulation are more reliable because unlike Monte Carlo simulation, historical simulation does not assume a specific distribution of the historical data. The result that Monte Carlo simulation passes the test might due to the limited historical data, 100 days' data used to estimate the next day's VaR and the assumption of normal distribution of historical data. The reasons may lead to a "coincidence" for Monte Carlo simulation in this paper to pass the back testing. It is also possible that the historical data are really normally distributed, so Monte Carlo simulation in this paper does provide an unbiased estimation for VaRs. Due to the limitation of assumptions in Monte Carlo simulation and delta-normal approaches, the most popular risk disclosure method used by major financial institutions is historical simulation approach.

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