HOW ACCURATE ARE VALUE-AT-RISK MODELS AT CANADIAN COMMERCIAL BANKS?

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ABSTRACT

Value-at-Risk (VaR), a measure of the dollar amount of the potential loss from adverse market moves, has become a standard benchmark for measuring financial risk. In most developed countries, commercial banks are required by regulators to compute their VaR on a daily basis. VaR estimates serve as a major determinant of the banks' capital requirement. In our study, we evaluate the accuracy of the VaR models of the six largest Canadian commercial banks. We provide evidence that the current models used by these banks for their VaR estimation are excessively conservative. An implication of this systematic overstatement is that the required capital for these banks is larger than what it should be, which is costly for these banks. We propose alternative models for computing VaR, and we show that, unlike the current VaR models used by the banks, our models are not rejected by the data.

Keywords: Finance; Commercial Banking; Risk Management; Market Risk; Value-at-Risk

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1 INTRODUCTION

Value-at-Risk (hereafter VaR), a measure of the dollar amount of potential loss, has become an essential tool for risk managers. This risk measure provides a quantitative measure of downside risk by providing maximum loss over a target horizon with a given level of confidence. VaR summarizes the effects of leverage, diversification, and probabilities of adverse market price movements in a single dollar amount. VaR disclosure improves risk management in commercial banks as it forces them to develop a systematic process of risk measurement.

VaR is widely used by financial institutions, fund managers, and nonfinancial corporations to control their exposure to market risk. Bank regulators have adopted this measure of risk as the major determinant of the capital that banks are required to maintain to cover potential losses arising from the market risks they bear.

With the implementation of Amendment to the Capital Accord to Incorporate Market Risks (Basel Committee on Banking Supervision, 1996a), a bank with significant trading activity must calculate a capital charge for market risk using either its own internal risk measurement model or a standard procedure developed by the Committee. The internal model approach requires banks to provide adequate capital against potential trading losses. The internal models will only be accepted by the regulators when a bank can demonstrate the quality of its model to the supervisor through the backtesting of its output using one year of historical data (Basel Committee on Banking Supervision, 2004). The Market Risk Amendment also requires that VaR must be computed on a daily basis with a 99^{th} percentile, one-tailed confidence interval. The banks' models must accurately capture the unique risks associated with market factors. However, there is no particular type of VaR model required by the Basel Committee. Banks will be free to use any model based, for example, on variance-covariance matrices, historical simulations, or Monte Carlo simulations. In the real world, the computing methods applied by commercial banks are often more complex. To manage market risks, major commercial banks have developed large scale VaR models.

The trading accounts at large commercial banks have grown substantially. Should analysts and investors use VaR disclosure to compute the risk of banks' trading portfolios? Jorion (2002) has investigated the relation between the trading VaR disclosed by a small sample of U.S. commercial banks, and variability of their trading revenues. The empirical results suggest that VaR disclosures are informative in that they predict the variability of trading revenues. Although VaR disclosure has been shown to be correlated with banks' trading profits and losses (hereafter P&L), the accuracy of banks' VaR models remains the most important concern for analysts and investors.

In this study, we examine the accuracy of the six Canadian banks' VaR models. We analyze daily data on historical trading P&L extracted from their annual reports, and estimate the VaR using alternative models. We evaluate the accuracy of the banks' VaR models in two ways. First, we implement an unconditional coverage test. We find that the VaR estimates tend to be excessively conservative relative to the 99 percentile of P&L. Second, we assess the performance of the banks' VaR models by comparing the banks' VaR with our own VaR estimates computed using two standard models, i.e., the GARCH-based VaR model and the historical simulation VaR model. The GARCH-based model generally provides smaller VaR and permits comparable risk coverage with less regulatory capital. Furthermore, the historical simulation models with volatility updating outperform bank VaR models as well. An important result is that, unlike the current VaR models used by the banks, our models are not rejected by the data.

The remainder of this study is organized as follows. Section 2 provides an overview of Canadian banks' VAR models, and presents the methodology used to evaluate the performance of the VaR models. We also try to understand why banks are systematically overstating their VaR estimates. Section 3 presents five alternative VaR models and compares their results with the banks' current VaR calculations. Section 4 provides a general conclusion.

2 VALUE-AT-RISK OF CANADIAN BANKS

In this study, our evaluation of Canadian banks' VaR models is based on the market risk disclosure, especially the information from the daily P&L and VaR, in their annual reports. Therefore, Canadian banks that have been publicly disclosing figures of the daily P&L and VaR are included in our sample. They are the six largest Canadian commercial banks: Bank of Montreal (BMO), Royal Bank of Canada (RBC), Toronto-Dominion Bank (TD), Canadian Imperial Bank of Commerce (CIBC), Scotiabank (also called Bank of Nova Scotia, BNS) and National Bank of Canada (NBC). These banks have been disclosing the daily P&L and VaR since 1999, enabling us to evaluate the VaR forecasts of these banks for the period from 1999 to 2005.

2.1 Regulation and Banks' VaR Models

Established in 1987, the Office of the Superintendent of Financial Institutions Canada (hereafter OSFI) supervises and regulates all banks in Canada. Since 1997, Canadian banks have been required to compute their VaR measures under the guidance of the OSFI. The general requirements on VaR approaches can be found in the Guideline that the OSFI issued in November 1997. The Guideline, an implementation of the Basel Capital Accord, allows Canadian banks to choose their own models which must be approved by the OSFI. The regulator also requires the banks to backtest their VaR models with hypothetical daily P&L.[1] Where an institution has not yet developed the ability to calculate hypothetical P&L on a daily basis, the previous day's VaR should be compared to the next day's daily P&L. According to their annual reports, Toronto-Dominion Bank and Scotiabank report actual trading revenues while other banks use hypothetical P&L.

[1] Hypothetical P&L are the profits and losses that would have occurred if the portfolio at the previous day's close were held constant for the current day, assuming no additional transactions are made.

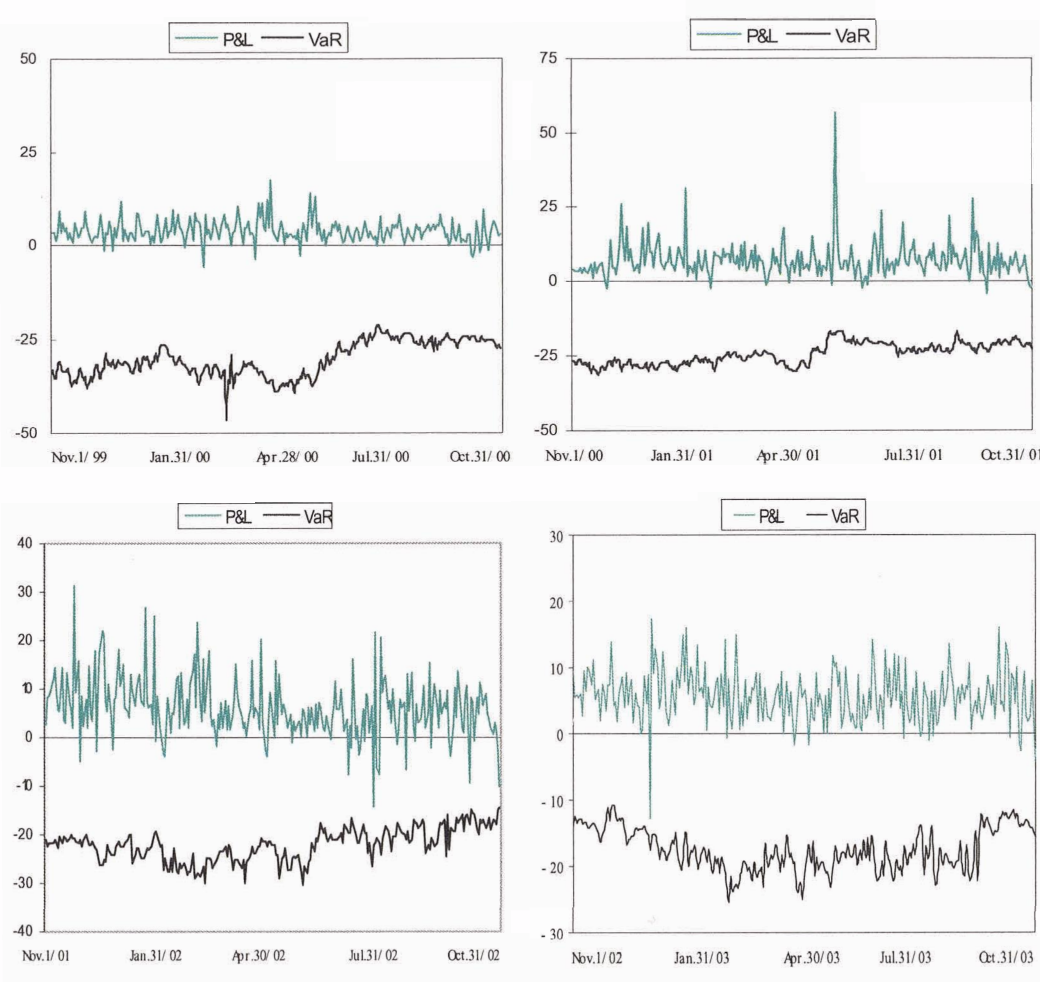
The six Canadian banks compute and disclose daily, one-day ahead VaR figures at a 99% confidence level. However, the method used by each bank to generate its VaR estimates is not clearly explained in their annual reports. Scotiabank reports using a historical simulation based on 300 days of market data; TD Bank estimates VaR by creating a distribution of potential changes in the market value of current portfolio and using the most recent 259 trading days of market price and rate changes; NBC's simulation model is based on two years of historical data. RBC uses a historical simulation of the previous 500 trading day scenarios to determine the VaR for its trading portfolio. BMO and CIBC do not provide such details.

2.2 Data Extraction

Since neither the banks included in our sample nor the regulators agree to provide us with official historical data, we extract daily P&L and VaR statistics from graphs available in banks' annual reports. We develop a Matlab-based application which allows us to extract a time-series from a graph, which is usually saved in a JPG or BMP format. Therefore, it should be mentioned that this paper is based on estimated data of Canadian banks. When comparing the graphs based on our estimated data and the original data in the annual reports, we find that our data is reliable since our estimated graphs match the actual ones very well. Figure 2.1 shows a set of graphs plotted based on estimated data. These graphs are virtually identical with the actual ones in TD Bank's annual reports, which are available on its website.

However, the limitation of this method arises when the original graphs are either inaccurate or incomplete. For example, TD Bank does not disclose actual trading revenues in its 2004 and 2005 annual reports. Therefore, we cannot backtest its VaR for this period. Similarly, we have to discard Scotiabank's 1999 data to make our analysis consistent since it does not disclose VaR and trading P&L graphs in its 2000 and 2001 annual reports. Regardless of this limitation, the number of observations is large enough to implement our backtesting procedure and the sample is large enough to construct our new historical simulation and GARCH models.

Figure 2.1 Estimated P&L and VaR of TD



Note: P&L and VaR are measured in millions of Canadian dollars.

2.3 Backtesting VaR

The regular backtesting method required by Basel Accord is to directly compare the hypothetical (or actual) P&L against the VaR and calculate the number of times when the P&L exceeds the VaR ("exceptions"). For example, over 300 trading days, the number of exceptions should be no more than three if daily VaR is set at the 99% level. However, such a simple backtest cannot tell us much about the accuracy, nor can it show the conservativeness of the VaR model. Especially for the six Canadian banks in this study, this backtest has limited power in model testing. As Table 2.1 shows, none of them experienced more than one exception. This does not mean that the models they use are accurate. We have reason to question the validity of the banks' VaR models simply by eyeballing the statistics in Table 2.1.

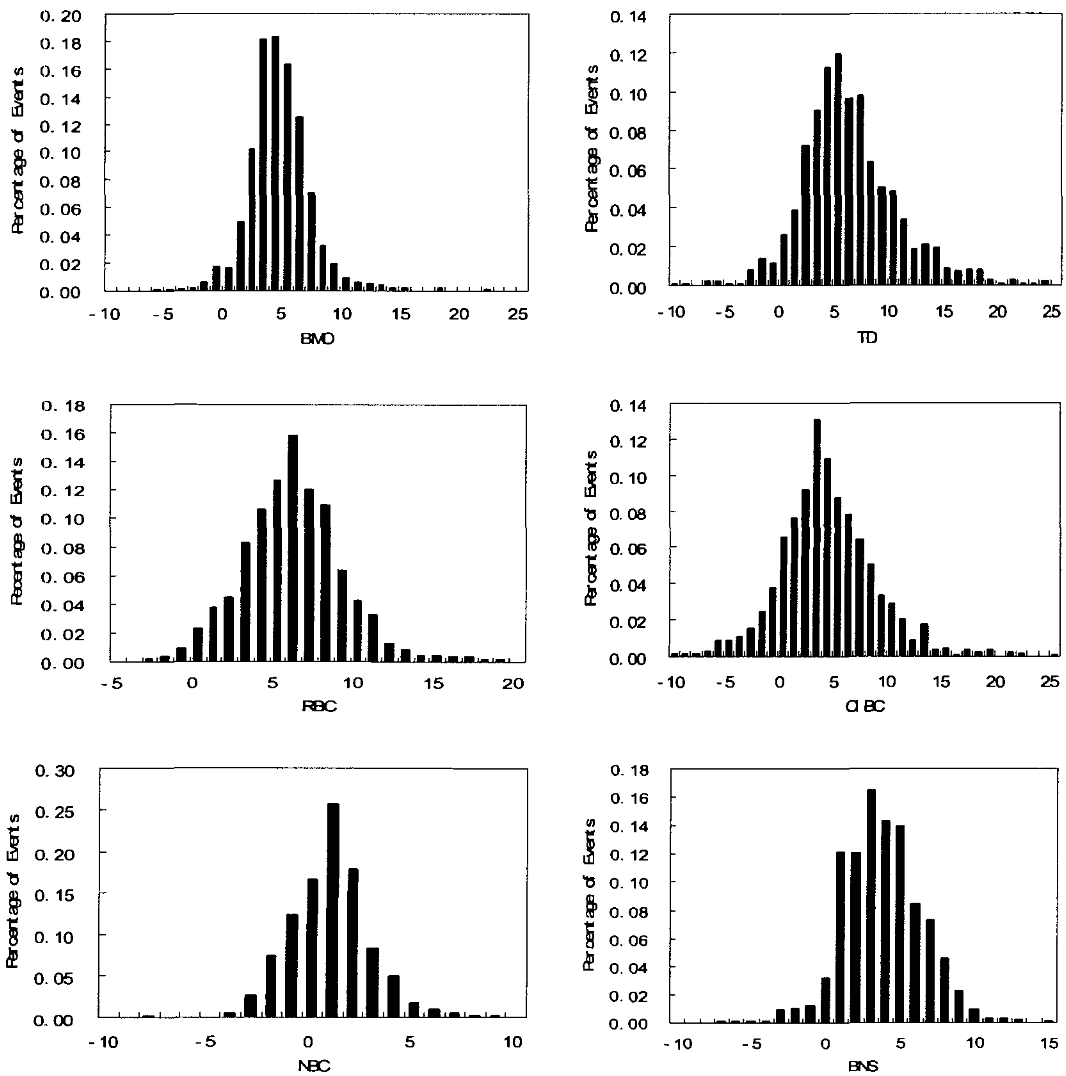
Table 2.1 Bank Daily P&L and VaR Summary Statistics

		Daily Profit and Loss					Daily VaR	
Bank	Obs	Mean	Standard Deviation	99^{th} Percentile	Excess Kurtosis	Skew-ness	Mean VaR	Exceptions
BMO	1,268	3.76	2.79	-2.07	18.13	1.90	-18.77	0
TD	1,015	5.74	5.20	-4.82	12.65	1.92	-23.61	0
RBC	1,258	5.47	3.14	-1.25	1.15	0.41	-11.47	0
CIBC	1,301	3.57	4.63	-7.11	5.09	0.90	-10.34	0
NBC	943	0.39	1.91	-3.86	1.01	0.24	-4.62	1
BNS	1,012	3.32	2.71	-3.25	1.86	0.04	-8.06	1

Note: The statistics in this table are based on our estimated daily data of the six largest Canadian commercial banks from November 1999 to October 2005. Obs denotes the number of observations. The observation horizon varies among the six banks: BMO (11/2000-10/2005), TD (11/1999-10/2003), RBC (11/2000-10/2005), CIBC (11/2000-10/2005), NBC (11/2001-10/2005), and BNS (11/2001-10/2005). P&L and VaR are measured in millions of Canadian dollars.

Positive skewness statistics in Table 2.1 suggest that the P&L in Canadian banks tend to be right-skewed. This is confirmed in Figure 2.2 where empirical distributions of P&L are plotted. The right-skewed characteristics and positive means together imply less risk. But the fact that the mean VaR is more than six times the 99^{th} percentile P&L for three banks suggests a high risk.

Figure 2.2 P&L Distributions of Six Canadian Banks



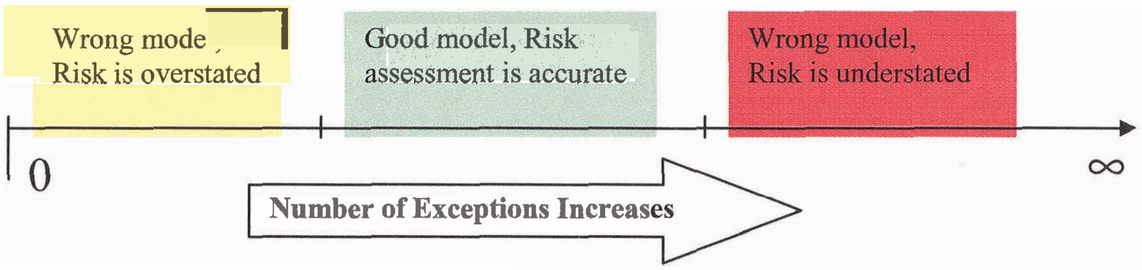
Note: The X-axis represents the level of P&L, and the Y-axis measures the percentage of events with the same P&L level. All figures are based on our estimated daily data. P&L are measured in millions of Canadian dollars.

In order to further assess the accuracy of VaR models used by Canadian banks, we implement a coverage test on bank VaR. The rules of the Basel Framework (Basel Committee on Banking Supervision, 1996b) for backtesting the internal model approach are also derived directly from such a test. The methodology of the unconditional coverage test we used was explained in detail by Jorion (1997, Pages 132-140). The purpose of this log-likelihood ratio test is to see whether the number of exceptions is acceptable. The formula used is:

$$LR_{UC} = -2\ln\{(1-p)^{T-N}p^{N}\} + 2\ln\left\{(1-\frac{N}{T})^{T-N}(\frac{N}{T})^{N}\right\} \qquad LR_{UC} \sim \chi_1^2 \qquad (1)$$

where LR_{uc} is the unconditional coverage, p is the target violation rate, N is the number of exceptions, T is the total number of observations, Chi-square critical value χ_1^2 is set at a 10% level which approximately equals 2.71. If $LR_{uc} < 2.71$, we cannot reject the null hypothesis that the model is correct, or in other words, the actual number of exceptions is equal to the expected number of exceptions. Otherwise, we conclude that the model is wrong. For each bank, we compute a non-rejection range where N makes $LR_{uc} < 2.71$. This is illustrated in Figure 2.3.

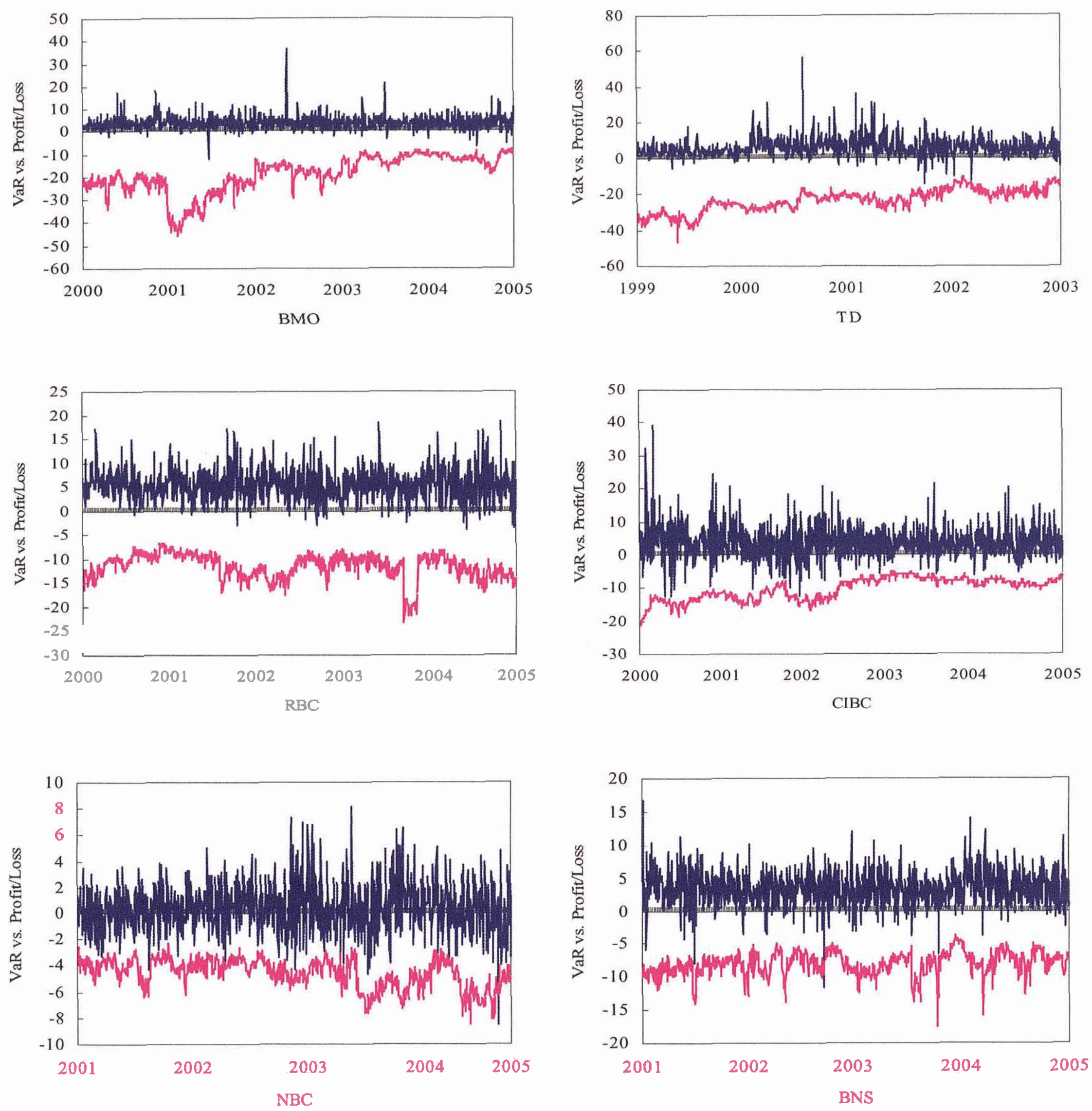
Figure 2.3 Illustration of Unconditional Coverage Test on VaR Models



Note: The number of exceptions increases from left (zero) to right (infinity).

Jorion (1997, Pages 132-140) suggests a large number of observations to make the coverage test more powerful. As Table 2.1 shows, NBC has 943 observations while the other five banks have over 1,000 observations. These samples are large enough for the coverage test. However, the coverage test has a mathematical problem when there are no exceptions. From the equation (1), we can see that the value of LR_{uc} is not defined if N equals zero (i.e., when there is no exception). In fact, Canadian banks in our study are exactly this case. There is no exception for four out of six banks. This can be seen in Figure 2.4 in which we present the VaR and P&L data for the six banks over the entire sample period.

Figure 2.4 VaR vs. P&L for Six Canadian Banks



Note: For each figure, the line on the top represents P&L while the line on the bottom is the VaR. All figures are based on our estimated data. P&L and VaR are measured in millions of Canadian dollars.

Despite this mathematical problem, we consider a zero exception case as the sign of a wrong model. This is consistent with the intuition behind the coverage test. Moreover, besides directly testing banks' VaR, we design a coverage test on discounted banks' VaR under different discount rates to see how conservative the banks' VaR are. For instance, if the bank VaR is 10 at time t while the discount rate is 50%, then the discounted VaR at time t is 5. The discount rate per se cannot exactly tell the degree of conservativeness of the banks' VaR. However, it does show us a signal of conservativeness if the discount rate is particularly high.

We first implement the coverage test on the non-adjusted banks' VaR. The results show that all of the models are wrong since none of them experienced more than one exception. Then we increase the discount rate on the banks' VaR until $LR_{uc} < 2.71$, which gives us a range of discount rates which make the model acceptable. Table 2.2 and Figure 2.5 contain more details.

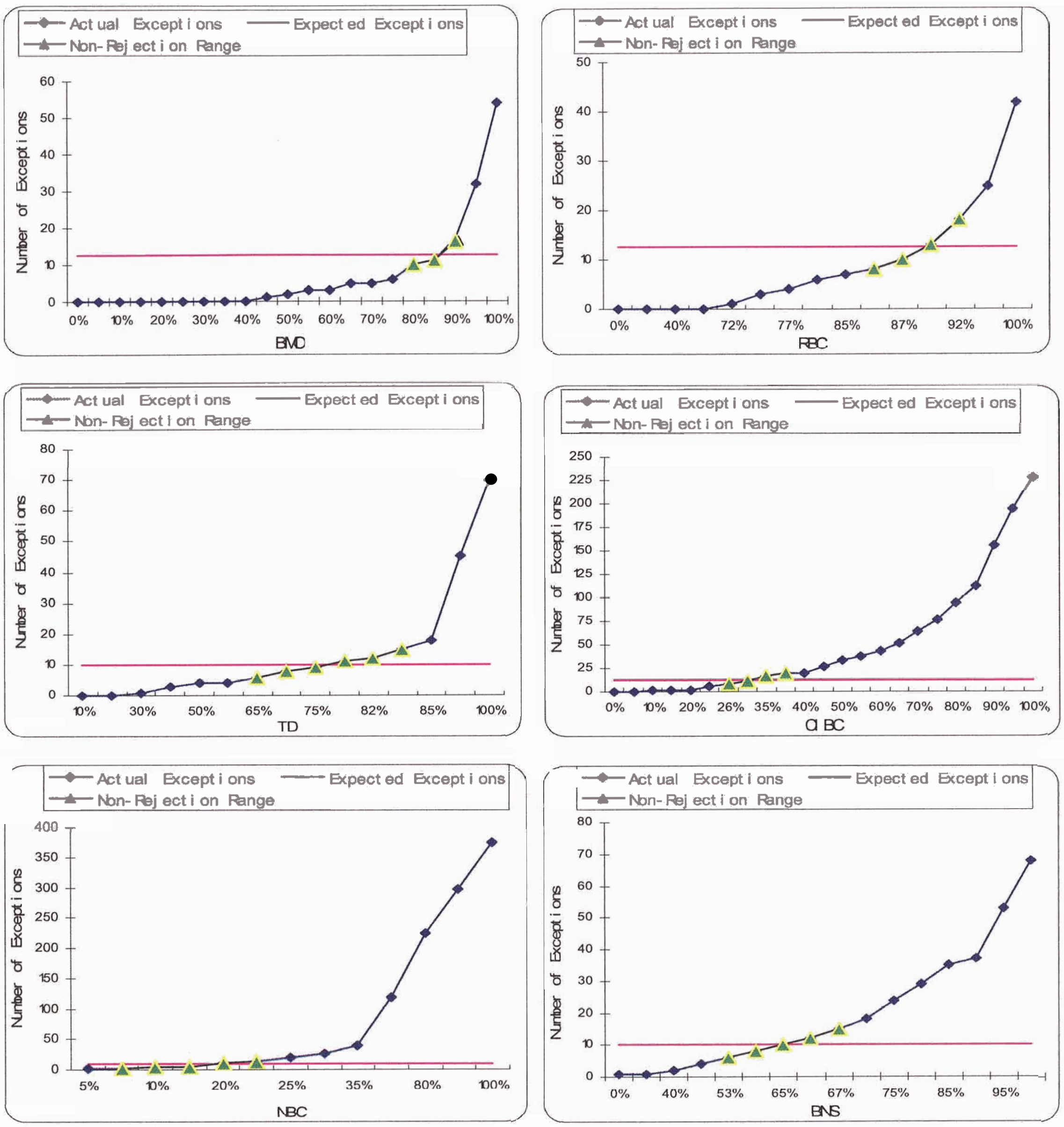
Table 2.2 Discounted VaR and Exceptions

Bank	Number of observations	Number of losses (%)	Discount rate when the first exception appears (%)	Discount rate within non-rejection range (%)	Number of exceptions within non-rejection range
BMO	1,268	28 (2.2%)	45	[80.0~90.0]	[10~17]
TD	1,015	70 (6.9%)	30	[65.0~83.5]	[6~15]
RBC	1,258	42 (3.3%)	72	[86.0~92.0]	[8~18]
CIBC	1,301	229 (17.6%)	10	[26.0~39.5]	[8~19]
NBC	943	373 (39.6%)	0	[6.0~21.5]	[3~14]
BNS	1,012	68 (6.7%)	0	[53.0~67.0]	[6~15]

Note: The statistics in the last two columns have a one-to-one relationship. For example, BMO would have 10 exceptions if its VaR was discounted by 80 percent, while it would experience 17 exceptions if its VaR was discounted by 90 percent. The non-rejection range is determined with a 99% confidence level.

From Table 2.2 and Figure 2.5, we can see that four banks need a discount of over 50 percent to reach the non-rejection range. The VaR of BMO and RBC could have been discounted by more than 80 percent without being rejected by the test for admitting too many exceptions. Such high discount rates indicate that conservativeness is a common feature in the VaR models used by Canadian banks.

Figure 2.5 Discounted Bank VaR and Exceptions.



Note: The X-axis represents the discount rate on bank VaR and the Y- axis measures the number of exceptions. All figures are based on our estimated daily VaR.

2.4 Possible Reasons for Overstating VaR

The development of the Basel rules tells us that the regulators are very concerned about the underestimation of risks and capital adequacy. From the regulator's perspective, financial institutions tend to underestimate their VaR to avoid high capital requirement. However, this is not the case for Canadian banks. Berkowitz and O'Brien (2002) observed similar behaviour using data from six large U.S. banks. We propose a series of potential reasons that explain why Canadian banks tend to overstate VaR.

Any loss that exceeds the VaR can be perceived as a risky signal by investors. To attract investors, banks have the incentive to understate the risks they are exposed to, and therefore are willing to overstate the VaR in their public reports. Another reason is related to the competition among banks. Clients of commercial banks tend to consider safety as a critical factor. Any signal of high risk in a bank may impair its relationship with clients and, therefore, harm its competitive capability. The third reason is regulation. Although, theoretically, exceptions in backtest are allowed for an effective VaR model according to the Basel requirement, banks tend to avoid taking the risk of being punished by the regulator.

Another point we should mention is that all of the six Canadian banks report their aggregate VaR, which is the summation of the VaR due to different market risk factors and subtracting the correlation estimation (the so called diversification effect). The example in Table 2.3 shows more details of the aggregate VaR. Since some approximations have to be applied when determining the VaR for each risk category and correlation between risk factors, the error in the aggregate VaR tends to be amplified. Banks which are sensitive to VaR disclosure may tend to exaggerate each component VaR or to underestimate the correlation effect. As a result, the aggregate VaR may be excessively overstated.

Table 2.3 Example of Aggregate VaR of BMO

Risk Category	Year-end	Average	High	Low
Commodity VaR	3.2	4.2	13.0	1.0
Equity VaR	3.8	4.9	7.1	2.8
Foreign Exchange VaR	0.4	0.6	2.2	0.1
Interest Rate VaR	3.8	4.4	8.9	2.5
Correlation	(5.5)	(6.6)	(10.2)	(3.7)
Aggregate VaR	5.7	7.5	14.7	4.0

Data source: BMO annual report 2005. All data are measured in millions of Canadian dollars.

3 ALTERNATIVE VAR MODELS

In order to further discuss the overstatement of the six banks' VaR, we construct five alternative VaR models. All of these alternative models are basic VaR approaches which have been discussed and applied by many others in the VaR literature. We apply the backtesting methodology on each model as above. The result shows that any of these models outperforms the banks' VaR models in terms of unconditional coverage test.

3.1 ARMA_GARCH Model

In this model, we combine ARMA (1, 1) and GARCH (1, 1). It can be illustrated as follows:

$$R_t = \mu + \phi R_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1} \tag{2}$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{3}$$

where R_t represents the P&L at time t, σ_t is the volatility at time t, ε_t is the error term of the regression at time t.

The parameters in the model are estimated on each business day with data up to that day. For each bank, we use rolling out-of-sample forecasts. All forecasted daily data is based on the previous 250 samples, which means that the out-of-sample estimates are updated daily. For example, TD Bank has 1,015 daily P&L observations. Its ARMA_GARCH result includes 765 sets of out-of-sample forecasted data. Based on the forecasted data, the VaR with 99% confidence level is calculated by using the following formula:

$$ARMA\_VaR_t = \hat{R}_t - 2.33 * \hat{\sigma}_t \quad (4)$$

where $\hat{R}_t$, obtained from equation (2), is the forecasted daily P&L at time t; $\hat{\sigma}_t$, obtained from equation (3), is the forecasted volatility at time t.

Table 3.1 ARMA_GARCH VaR Results

Bank	Number of Observations	Non-Rejection Range	Number of Exceptions	Unconditional Coverage (LR_{UC})	Accept/ Reject
BMO	1,018	[6~15]	8	0.51	Accept
TD	765	[4~12]	8	0.02	Accept
RBC	1,008	[6~15]	9	0.12	Accept
CIBC	1,051	[6~16]	6	2.3	Accept
NBC	693	[4~11]	7	0.01	Accept
BNS	762	[4~12]	14	4.34	Reject

Note: The Chi-square critical value is set at a 10% level which approximately equals 2.71. The last column denotes whether the VaR model should be accepted or rejected.

From Table 3.1, we can see that the ARMA_GARCH model performs pretty well except for the BNS case. According to the Basel requirement for backtesting, the ARMA_GARCH model should not be rejected for five banks. Figure 3.1 shows how the ARMA_GARCH VaR is compared with banks' VaR for each bank. Apparently, ARMA_GARCH VaR is much lower than banks' VaR for four out of six banks.

3.2 AR_GARCH Model

Similarly, inspired by the ARMA_GARCH model, we combine AR (2) and GARCH (1, 1) to form the AR_GARCH model. The only difference is that we replace the term ε_{t-1} by R_{t-2}. It can be illustrated as follows:

$$R_t = \mu + \phi_1 R_{t-1} + \phi_2 R_{t-2} + \varepsilon_t \quad (5)$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2. \tag{6}$$

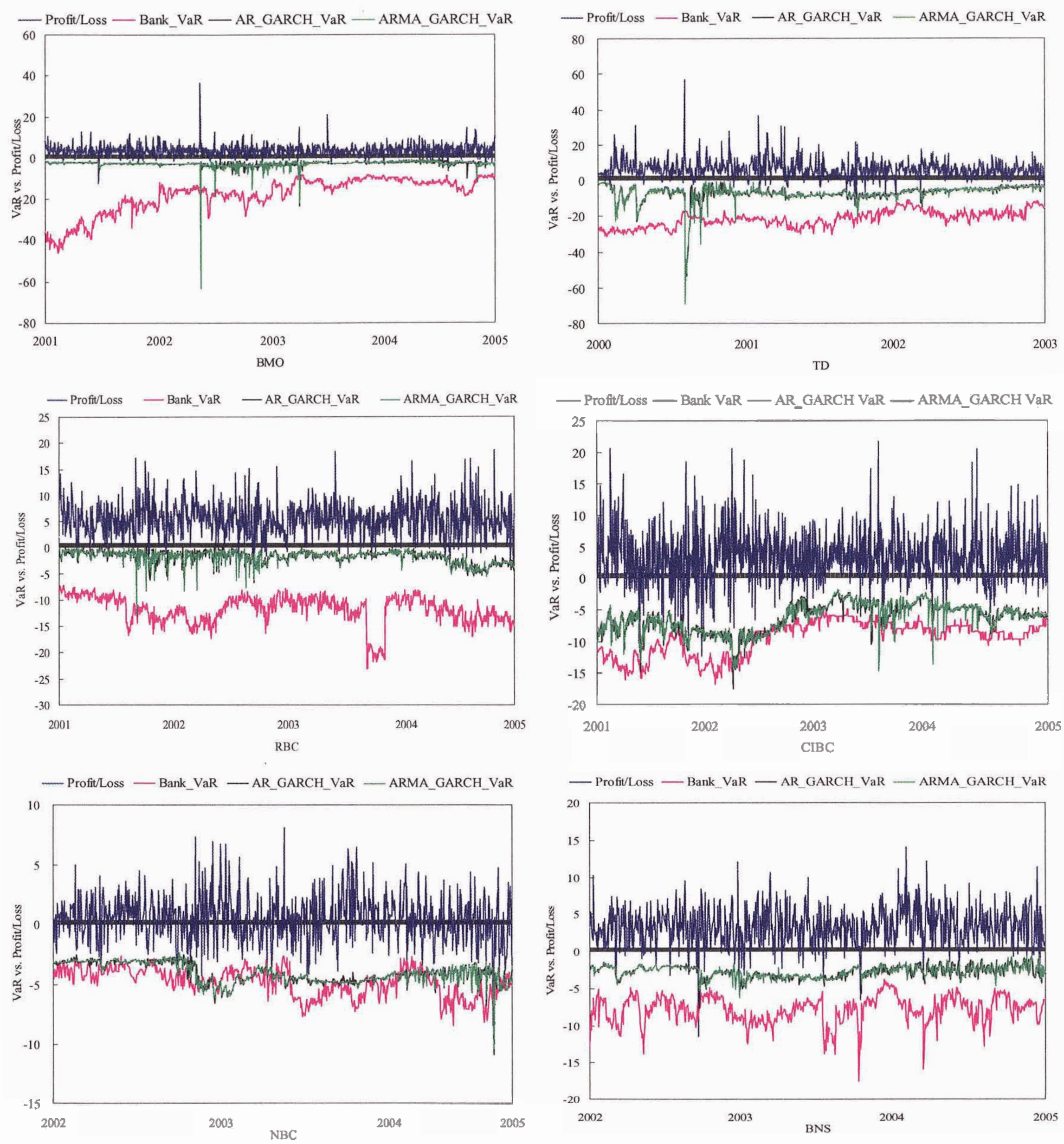
The way we forecast daily data and compute VaR is exactly the same as what we have done with the ARMA_GARCH model. Table 3.2 suggests that the AR_GARCH model shows better performance than the ARMA_GARCH model although it needs less computation. The AR_GARCH model can be applied to each bank without any rejection in terms of the unconditional coverage. Figure 3.1 provides more details of the AR_GARCH model. Note that the line representing AR_GARCH_VaR is hardly recognized from the line of ARMA_GARCH_VaR in Figure 3.1 since their values are much closer to each other for most of the time period.

Table 3.2 AR_GARCH VaR Results

Bank	Number of Observations	Non-Rejection Range	Number of Exceptions	Unconditional Coverage (LR_{UC})	Accept/ Reject
BMO	1,018	[6~15]	7	1.12	Accept
TD	765	[4~12]	8	0.02	Accept
RBC	1,008	[6~15]	9	0.12	Accept
CIBC	1,051	[6~16]	10	0.02	Accept
NBC	693	[4~11]	8	0.16	Accept
BNS	762	[4~12]	10	0.69	Accept

Note: The Chi-square critical value is set at a 10% level which approximately equals 2.71. The last column denotes whether the VaR model should be accepted or rejected.

Figure 3.1 Comparison of Banks' VaR and VaR from AR_GARCH and ARMA_GARCH Models.



Note: AR_GARCH_VaR is the VaR obtained from the AR_GARCH model while ARMA_GARCH_VaR is from the ARMA_GARCH model. All data are measured in millions of Canadian dollars.

3.3 Simplified Historical Simulation Model

Another simple method for calculating VaR is the historical simulation. This method involves using historical changes in market factors to construct a distribution of potential future portfolio P&L. Then, based on the potential distribution, VaR with a 99% confidence level is determined as the loss that is exceeded only 1% of the time. This method is discussed in detail in Linsmeier and Pearson (2000). The advantage of the historical simulation is that it requires no assumption on the distribution of P&L.

Since we do not know the details of each bank's position, a regular historical simulation model cannot be implemented. Alternatively, we use a simplified historical simulation in which we only assume that historical P&L represent today's level. In order to make the results comparable, we choose the same size of rolling sample, i.e., each out-of-sample estimate is based on 250 daily data. Under the above assumption, we sort the 250 historical daily P&L from the largest loss to the largest profit, and then the 99% VaR is just the value of the third largest loss (i.e., $250 \times (1 - 99\%) \approx 3$).

Table 3.3 Simplified Historical Simulation VaR Results

Bank	Number of Observations	Non-Rejection Range	Number of Exceptions	Unconditional Coverage (LR_{uc})	Accept/ Reject
BMO	1,018	[6~15]	14	1.30	Accept
TD	765	[4~12]	15	7.03	Reject
RBC	1,008	[6~15]	14	1.38	Accept
CIBC	1,051	[6~16]	8	0.66	Accept
NBC	693	[4~11]	13	4.29	Reject
BNS	762	[4~12]	6	0.37	Accept

Note: The Chi-square critical value is set at a 10% level which approximately equals 2.71. The last column denotes whether the VaR model should be accepted or rejected.

Table 3.3 shows that the simplified historical simulation is rejected for TD and NBC cases but accepted for all other banks. This is not surprising since we do not make any adjustment on historical data. However, the result does draw our interest as the number of exceptions falls in or close to the non-rejection range. That is why we want to discuss historical simulation VaR with volatility updating below.

3.4 Historical Simulation Model with Volatility Updating

The method known as historical simulation VaR with volatility updating was proposed by Hull and White (1998). The intuition behind this method is to incorporate volatility changes into historical data instead of using changes in market factors to construct a historical simulation. In this paper, we estimate the volatility using both the AR_GARCH model and the ARMA_GARCH model. The formula we use to generate the hypothetical P&L is:

$$\tilde{R}_t = \frac{\tilde{\sigma}_t}{\sigma_s} R_s \qquad (7)$$

where $s = t\text{-}1, t\text{-}2, \ldots, t\text{-}250$, $\tilde{R}_t$ is the hypothetical P&L at time t, R_s is the actual historical P&L at time s, $\tilde{\sigma}_t$ is the forecasted volatility at time t, σ_s is the volatility at time s.

Then we sort the generated hypothetical P&L as we do in the simplified historical simulation above and use the same way to find the VaR. Table 3.4 and Table 3.5 show the backtest results of the historical simulation models with volatility updating. Both AR_GARCH and ARMA_GARCH volatility updating models outperform the simplified historical simulation model by reducing exceptions in most cases except for CIBC. Figure 3.2 shows how the VaR obtained from the three historical simulation models in our study perform compared with the banks' VaR. Apparently, the VaR from these alternative models are much lower than the banks' VaR for four out of six banks.

Table 3.4 Results of Historical Simulation VaR with AR_GARCH Volatility Updating

Bank	Number of Observations	Non-Rejection Range	Number of Exceptions	Unconditional Coverage (LR_{UC})	Accept/ Reject
BMO	1,018	[6~15]	9	0.14	Accept
TD	765	[4~12]	5	1.05	Accept
RBC	1,008	[6~15]	9	0.12	Accept
CIBC	1,051	[6~16]	6	2.3	Accept
NBC	693	[4~11]	4	1.47	Accept
BNS	762	[4~12]	2	5.9	Reject

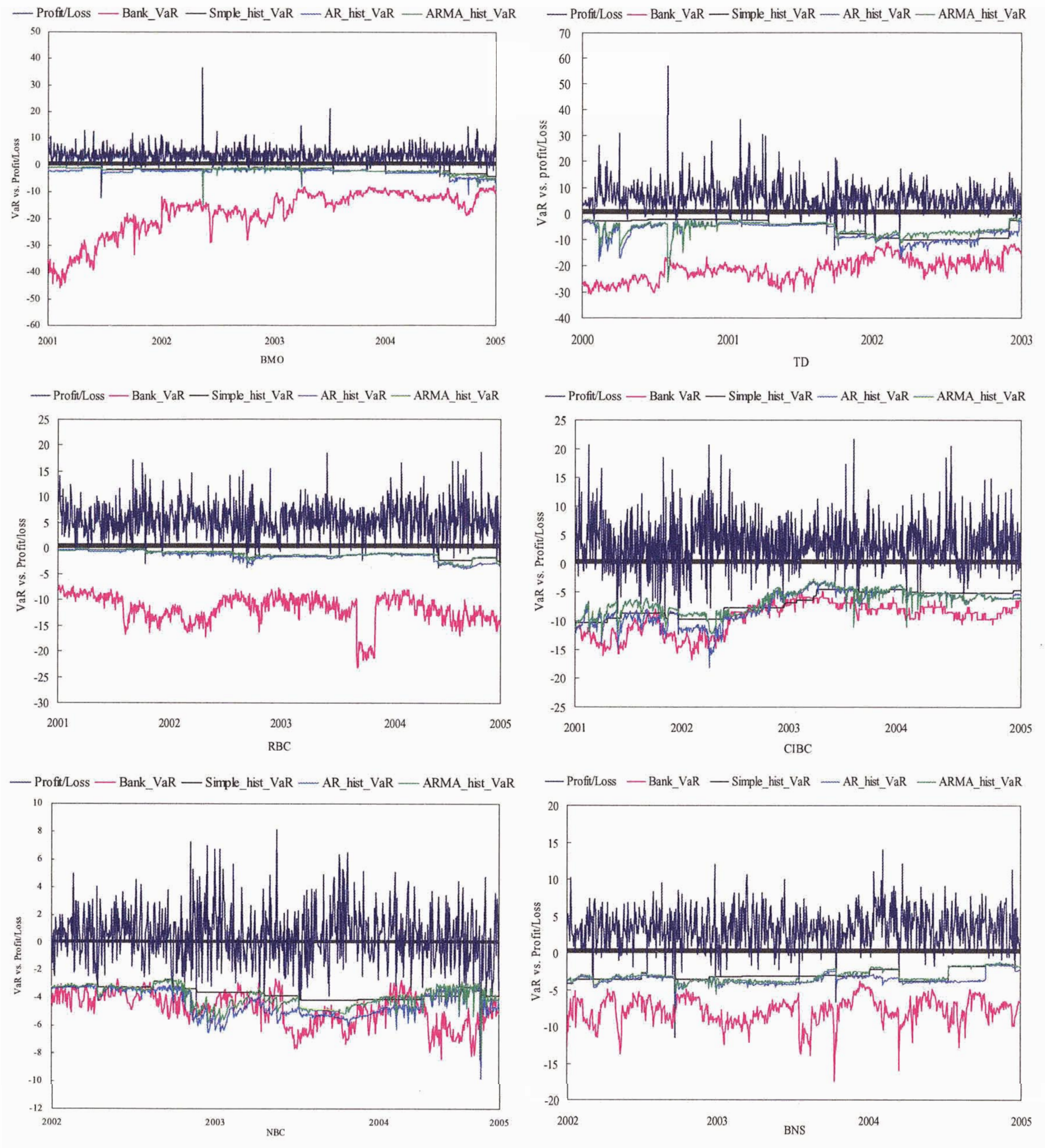
Note: The Chi-square critical value is set at a 10% level which approximately equals 2.71. The last column denotes whether the VaR model should be accepted or rejected.

Table 3.5 Results of Historical Simulation VaR with ARMA_GARCH Volatility Updating

Bank	Number of Observations	Non-Rejection Range	Number of Exceptions	Unconditional Coverage (LR_{UC})	Accept/ Reject
BMO	1,018	[6~15]	12	0.31	Accept
TD	765	[4~12]	9	0.23	Accept
RBC	1,008	[6~15]	12	0.35	Accept
CIBC	1,051	[6~16]	9	0.23	Accept
NBC	693	[4~11]	5	0.6	Accept
BNS	762	[4~12]	5	1.03	Accept

Note: The Chi-square critical value is set at a 10% level which approximately equals 2.71. The last column denotes whether the VaR model should be accepted or rejected.

Figure 3.2 Comparison of Bank VaR and VaR From Various Historical Simulation Models.



Note: Simple_hist_VaR is based on the simplified historical simulation model; AR_hist_VaR is obtained from the historical simulation model updated with AR_GARCH volatility; ARMA_hist_VaR is computed from the historical simulation model updated with ARMA_GARCH volatility. All daily data are measured in millions of Canadian dollars.

4 CONCLUSION

This study investigates the Value-at-Risk (VaR) computed and disclosed by the six largest Canadian banks. Most of these banks have been disclosing daily P&L and VaR continuously since 1999, which, therefore, provides us with a sufficiently large sample for backtesting the VaR and constructing alternative VaR models. We extract daily P&L and VaR data from financial reports. By comparing graphs of estimated and true data, we are confident that our estimated data are reliable.

We backtest the VaR models used by the six Canadian banks, and find that none of them has experienced more than one exception during the past five years. Such low violation rates lead us to question the accuracy of the VaR models from a statistical point of view. We design and implement an unconditional coverage test on discounted banks' VaR in that the traditional coverage test is not suitable for these Canadian banks. Our main conclusion is that Canadian banks are excessively conservative in forecasting their VaR. In addition, we propose five alternative VaR models. The way we construct these models is anything but creative. However, most of the alternative models outperform VaR models in Canadian banks in the sense of modelling effectiveness. This result also supports our claim that Canadian banks overstate their VaR.

According to the Basel Amendment (Basel Committee on Banking Supervision, 1996a), banks need to calculate capital charges using VaR if they measure market risks by internal models. Although there are some possible reasons for banks to overstate their VaR, it should not be ignored that a high VaR will induce a high capital charge. This means overstatement of VaR can be costly for banks. Computing and analyzing such cost would be very informative, but this calculation is beyond the scope of our study.

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