A CASE STUDY OF THE MATHEMATICAL BEHAVIOR OF A GIFTED LEARNING DISABLED SECONDARY STUDENT

by

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A Case Study of the Mathematical Behavior of a Gifted Learning Disabled Secondary Student

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Abstract

This case study explores the mathematical behavior of a gifted learning disabled secondary student for the purposes of examining his way of thinking, assessing the nature of his mathematical giftedness, and making recommendations for his future learning experiences.

The method of inquiry is qualitative in nature. For the data collection, ten audio-taped interviews were conducted. Eight of these interviews were with the subject of the case study and one each with the subject's brother and the subject's special education teacher. In the interviews with the subject, he was asked to describe what he was thinking while he attempted to solve various problems. He was also asked to write a computer program, and the mathematics involved were examined. Various samples of his work were also collected.

In order to analyze the data, the interviews were transcribed and coded. The codes were then classified and compared. This led to the formation of a model describing the subject's mathematical behavior. The model is then explained and supported with data and is followed by some recommendations for his future learning. Some of these recommendations were then implemented and the subsequent results were described.

The study concludes that the subject is a highly self-motivated learner only in situations where the curriculum or learning activities match his special interests. The study shows that he has learned quite a sophisticated mathematics curriculum of his own through the completion of personal projects on the computer. Since his special learning needs are not being met by the school system, the conclusion is made that more

flexibility in the mathematics curriculum and evaluation procedures must be implemented before the subject and others like him will reach their academic potential.

Dedication

This study is dedicated to the "Kurts" in our schools.

Acknowledgments

I would like to thank Dr. Tom O'Shea and Dr. Leone Prock for their encouragement and advice. I am also grateful to those who gave up their time to participate in this study. Specifically, my gratitude is extended to Kurt's brother, to Kurt's special education teacher and especially to Kurt himself, who contributed so much to my enjoyment of this study and to my growth as a teacher. Finally, my special thanks go out to my wife and children, who made many sacrifices in order that I could complete this study.

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Chapter 1
Introduction

Context of the Study

Research in the last ten to fifteen years has indicated that gifted learning disabled students have been among the least served groups in schools (Gunderson, Maesch, & Rees, 1987; Landrum, 1989). While programs for the gifted exist in some schools, and programs for the learning disabled exist in most schools, the gifted learning disabled (GLD) are usually placed in programs for the learning disabled in order to correct a deficiency. Thus, their strengths are virtually ignored while all the attention is focussed on their weaknesses. Furthermore, Suter and Wolf (1987) have suggested that GLD students often have their giftedness masked by their handicapping condition and that many appear to function at an average level in which neither giftedness nor learning difficulties are recognized.

Gifted learning disabled students often display a low self-esteem and tend to be highly self-critical. This seems in large measure due to the student's frustration in being unable to account for the discrepancy between superior cognitive ability and an inability to succeed at basic academic tasks (Suter & Wolf, 1987). Baum and Owen (1988) report that GLD students have a greater sense of inefficacy in school and that this most likely increases their motivation to avoid school tasks. Bandura (cited in Baum & Owen, 1988) defines self-efficacy as a person's perception of being able to organize and carry out some action. These perceptions can then have a strong influence on a person's motivation. Successful experiences increase one's self-efficacy, which in turn motivates better performance. Baum (1988), in a study involving seven elementary school GLD children, found that focussing on the strengths of these children through an enrichment program improved their self-esteem and had a possible indirect effect of improving academic achievement.

Nothing in the literature suggests that any studies have been done on GLD students specifically in the area of mathematics. The case studies reviewed by the researcher (Burnett, 1981; Erlwanger, 1975; Harel, 1990; Jordan, 1981; Landis & Maher, 1989) all deal with average to high achievers or children with learning problems in mathematics. These studies are useful because they provide information about individuals' conceptualizations, and conclusions can be drawn that might have curriculum or pedagogical implications for the classroom teacher.

Given the special characteristics of GLD students, the fact that they are under-served by the education community, and the apparent lack of study of their mathematical behavior, a case study of an individual GLD student might be a step toward understanding the special learning needs of GLD students. According to Hawkins (cited in Zehavi, Bruckheimer & Ben-Zvi, 1988):

> The really interesting problems of education are hard to study. They are too long-term and too complex for the laboratory and too diverse and non-linear for the comparative method. They require longitudinal study of individuals, with intervention a dependent variable, dependent upon close diagnostic observations. (p. 421)

Background

The initial motivation for this study was provided at the beginning of October, 1990, at the secondary school at which I was employed. A grade 10 student, whom I'll name Kurt, had just written and subsequently failed a mathematics unit test. However, the test seemed to reveal that he had considerable talent in problem solving. Even though he performed poorly on the "skills" section of his test, he solved one particular problem in a most general and abstract fashion using a technique that was totally surprising to me. I was also aware that Kurt had learning disabilities because he was in a learning assistance program at the school in grades 8 and 9. The combination of Kurt's apparent mathematical talent along with his possible disabilities captured my interest and as a result, led to the development of this study.

Influenced by Hawkin's quote above (p. 2) and the research of Erlwanger (1975), I decided to undertake a qualitative case study centered on Kurt. The methods that I chose were adapted primarily from the writings of Goetz & LeCompte (1984), Miles & Huberman (1984), and Merriam (1988). The study consisted of 8 audio-taped observation-interview sessions, over the period 15 February,1991, to 7 June, 1991, during which Kurt attempted to solve various problems and explain his thinking or reasoning. The initial sessions were meant to provide formative information for the purpose of planning subsequent interviews. Interviews were also conducted with his special needs instructor and his brother. The interviews were then self-transcribed, coded and analyzed using qualitative methods.

In September, 1991, Kurt was enrolled in my mathematics 11 course. I attempted to give him an informal enrichment program by allowing him the option of choosing alternative assignments which might interest him rather than requiring him to do all the "drill" exercises that were assigned to everyone else. These assignments were to be weighted at 50% of his overall grade. The assignments were collected and I kept journal notes on many of his comments over the course of the year.

Research Objectives

The general objective of this study was to observe and analyze a learning disabled secondary student's mathematical behavior in order to identify and understand phenomena related to his way of thinking. The initial phase focussed on determining the student's sense of self-efficacy regarding school mathematics and mathematics in general. This was followed by an attempt to examine the student's mathematical thinking, partly to assess his giftedness in mathematics and partly to draw inferences regarding his future learning experiences. This objective led to the creation of a possible model to explain his mathematical behavior. The study was meant to be exploratory in nature and not completely limited to investigating any preconceived questions.

Limitations

Being qualitative, the study must necessarily reflect my perspectives and personal biases. For example, other researchers might study the data that I collected in my initial interviews and then proceed in a completely different direction, dictated by their own personal view points and interests. Thus, it is important for the reader to have some insight into my perspectives on mathematics education in order to be able to judge how they possibly affected the way I perceived and interpreted my data.

Firstly, I do not believe that the current British Columbia high school curriculum, with its multitude of intended learning outcomes, truly reflects the nature of mathematics because, in my opinion, it leaves students with the impression that mathematics is simply a large body of "truths", skills, and techniques that are applied to solve "contrived" problems. Mathematical facts and techniques are generally memorized and it is rare for the student to experience the processes that a practicing mathematician in industry might be involved in. For example, Tuttle (1990), an insurance actuary, argues that approximate solutions to equations are often desirable and that honest disagreements about the right answer occur daily in his work. Furthermore, there is also the possibility that there are several right answers, depending on the context, and that solutions must often be "sold" to their perspective audience. That is not to say that a certain amount of factual knowledge and development of certain skills are not important. Rather, I believe they are prerequisite skills and could be de-emphasized as major outcome goals of a program.

Secondly, it is my belief that the existence of government examinations in grade 12 motivates teachers in lower grades to spend most of their time teaching the prerequisite intended learning outcomes for the next grade level. As a result, they spend considerably less time on activities that are open-ended and exploratory in nature and that would require students to learn how to effectively communicate their ideas to each other. I am therefore frustrated with having to compromise what I believe is worthwhile mathematics with what I believe is essentially meaningless mathematics for most students.

Given that I am a secondary mathematics teacher interested in improving my teaching and assuming that knowledge of how students learn and think is necessary in designing effective teaching strategies, a study such as this one should help me make some decisions in how to plan my future teaching. It may also help me to argue the case for a revised curriculum which might benefit the majority of students rather than the minority.

One limitation of this study is that the findings are unique to the individual being studied and that the conclusions cannot be generalized to other individuals. However, this does not deny the possibility that the study's findings may have implications elsewhere.

Another limitation to the study is my double role as teacher and researcher. I have developed a very good relationship with Kurt since the beginning of the study, to the point where, before classes, he freely initiates conversation about some of his philosophical and mathematical ideas. As his teacher and friend, I wanted to see him succeed in mathematics. As a researcher, I had to try to describe his behavior and evaluate his work with as little bias as possible. In evaluating his alternative assignments given in the academic year following the interviews, it was difficult to apply an objective standard because very often his understanding was good but his communication of ideas was poor. Furthermore, I tried not to put pressure on him to do any of the alternative assignments because I wanted to see if he would be intrinsically motivated by them. On the other hand, he was already offered an incentive in order to improve his mark because the assignments were to make up 50% of his final mark.

There is also the possibility that, knowing that his mathematical behavior was being studied as part of a master's thesis, Kurt may have exaggerated or left out certain information that might be of importance to the study.

Although there are accepted definitions for terms such as gifted and learning disabled, it may still be difficult to precisely characterize these students using this terminology, because there will always be discrepancies between the students' actual characteristics and those described in the literature.

Finally, there is the possibility that, being an inexperienced interviewer, I may have unintentionally influenced the direction of Kurt's thinking in any given interview. Although it was my intention not to make any judgemental or praising comments about his work, there could have been times when he sensed something from my tone of voice or body language.

Chapter Organization

This thesis is organized into five chapters. Chapter 1 is an introduction to the study which describes the context of the investigation, summarizes the research objectives, characterizes the method of inquiry used, discusses the limitations and outlines the remainder of the thesis.

Chapter 2 contains a review of the literature. Included is a summary of the characteristics of gifted learning disabled students and a description of some programs that may meet their special needs. Giftedness in mathematics is then discussed along with an outline of instructional considerations for the mathematically gifted. Finally, various case studies in mathematics education are reviewed in order to help justify the use of the case study method.

Chapter 3 describes the method of inquiry. The subject of the investigation is introduced and a rationale for performing a qualitative case study is offered. Following this is a detailed account of the data collection procedures and the subsequent analytical techniques.

Chapter 4 is a presentation of the findings and my interpretations. It begins with an account of the subject's story; his history and his character, and illustrates how Kurt compares with "typical" GLD students. Next, the nature of his mathematical giftedness is described. Finally, a model is proposed of the subject's mathematical behavior, along with its educational implications. The model is then supported with data from interviews and preliminary recommendations are made with regard to the subject's mathematics education. An account is then given of the implementation of some of the preliminary recommendations.

Chapter 5 is a discussion of the practical implications of the study as applied to Kurt and to teachers of students with similar characteristics to Kurt. Also included is an account of the significance of the study and some concluding remarks.

Chapter 2
Literature Review

Identification of the Gifted Learning Disabled

Whitmore and Maker (1985) define giftedness as " the capability or potential for exceptional achievement and contributions in a specific area of human ability" (p. 8). More specifically related to education, " the mentally gifted child can be defined simply as one with exceptional *potential* [italics added] for a) learning, b) achieving academic excellence in one or more subject areas, and c) manifesting superior mental abilities through language, problem solving, and creative production" (Whitmore, 1981, p. 107).

On the other hand, Whitmore (1981) defines handicapped children as "those whose normal learning and development are impaired by one or more specific conditions and who therefore need special education and related services in order to develop their abilities" (p. 107). The person that could be classified as gifted learning disabled would then require special education services both to help overcome or accommodate specific learning disabilities and to fully develop those talents that are superior.

While it is generally accepted that gifted learning disabled (GLD) persons exist in our schools, identification, and as a result, service for such students can often be difficult for several reasons. Firstly, the GLD person's special talent may be masked by specific learning disabilities (Suter & Wolf, 1987, Whitmore & Maker, 1985). For example, Whitmore and

Maker (1985) describe the case of Marcia, a woman with a severe reading disability, who used her superior thinking ability and memory to develop a coping mechanism to compensate for her weaknesses and allowed her to pass through all her years of schooling as a " C student". Whenever Marcia was assigned a paper related to a novel, even though she could not successfully read the novel, she gained enough information through discussions in class and with her friends, that her superior memory allowed her to write papers which received passing grades.

Secondly, stereotypic expectations for the gifted may cause the GLD student to be overlooked. Whitmore (1981) includes the following stereotypes: that the gifted child excels in all areas; that the best indicator of intellectual ability is the use of advanced, appropriate and fluent language; that gifted students are highly motivated to achieve excellence in school; and that the gifted child is mature, independent, and self-directed. Similarily, the label of learning disabled has been found to negatively influence teachers' referral recommendations for the gifted and talented. In a study of 68 public schools, Minner, Prater, Bloodworth and Walker (1986) found that teachers, even those trained in special or gifted education, were generally unwilling to consider referring or placing a handicapped gifted child into a program for gifted students.

Thirdly, GLD persons may lack the ablility or opportunity to evidence their superior mental abilities. (Whitmore, 1981; Whitmore & Maker, 1985) This may be partly due to the expectations (limitations) and style of the classroom teacher or, as is often the case (Landrum, 1989), the GLD students are placed in special programs for the learning disabled in order to correct specific deficits.

Notwithstanding the obstacles to identification discussed above, researchers have recently defined many of the common characteristics displayed by GLD students (Baum, Emerick, Herman & Dixon, 1989; Baum & Owen, 1988; Gunderson, Maesch & Willis Rees, 1987; Huntley, 1990; Landrum, 1989; Silverman, 1989; Suter & Wolf, 1987; Whitmore, 1980; Whitmore, 1981; Whitmore & Maker, 1985; Cygi &Wolf, 1981; Yewchuk & Bibby, 1986). The following is a brief summary of these characteristics:

1. While school performance is often poor, these students often have hobbies and interests that require keen motivation and creative thinking. They usually become "expert" in this area.
2. GLD students tend to be good problem solvers, have high abstract thinking ability, and are often highly creative.
3. Subscales that assess verbal reasoning abilities (comprehension and similarities) tend to yield high scores; scores on digit span, arithmetic, coding, reflecting attention, and concentration tend to be low.
4. These students have difficulty understanding the discrepancy between their poor performance and their superior thinking ability.
5. They have difficulty setting realistic goals; low goals are unacceptable while high goals seem out of reach.

6. GLD students are highly self-critical, may not take constructive criticism well and are resistant to influence.
7. They are frequently more reflective in the learning process and thus usually have difficulty with time restraints on standardized tests.
8. They may experience great frustration because they might understand higher level, abstract ideas or problems, yet may not have the skills to do simple math calculations or be able to express their ideas in writing. The result is usually very low self-esteem and lack of confidence.
9. These students are frequently highly disorganized and tend to "forget" to do their homework.
10. Their long and short term memory is often impaired.
11. They may be visual or auditory processing deficient or have visual motor integration problems.
12. They are often poor at foreign languages.
13. They often seem "spacy" and inattentive in class.
14. These students are often test phobic, achieving poor test results.

Whitmore and Maker (1985) have suggested the following early and reliable indicators of giftedness in the learning disabled:

1. Their oral language often shows a more advanced vocabulary, a more complex language structure and syntax and perhaps fluency of ideas as well.
2. They have specific deficits related to visual or auditory

memory. For example, a GLD person may have a superior memory for facts, general knowledge, concepts, principles, or events and yet show deficits related to memory for specific details such as letter or number sequences.

3. They possess superior problem solving skills and reveal exceptional analytical ability, use of information and logic, and creative manipulation of alternatives and aspects of the problem.
4. They have a keen curiosity and drive to know and often ask profound questions.

Meeting the Needs of the Gifted Learning Disabled

Since GLD students' achievement is often low, they have been regularly placed in remediation programs to improve their weaknesses. However, research has shown (Baum, 1984) that focussing on basic skills at the expense of encouraging and developing special talents can result in low self-esteem, lack of motivation, depression and stress. In order to meet the special needs of these individuals, it helps to first understand their system of motivation. Whitmore (1980), in her study of gifted underachievers, defines two psychological dimensions of a child's motivational system that are directly related to the degree to which the student is achievement motivated, apathetic toward school work, or negatively unmotivated to achieve in school. These are:

a) the child's self- esteem as a result of self-perceptions shaped by experiences that have communicated the degree to which the child is apt to be accepted, liked, competent, and successful; and b) the child's expectation of finding participation in school work rewarding, meaningful, and useful, also derived from past experiences. (Whitmore, 1980, p. 213-214)

Simply put, childrens' self-concepts and the learning opportunities (curriculum and instruction) that they are exposed to have a direct effect on their classroom behavior.

Whitmore and Maker (1985) identify several motivating factors for gifted persons with disabilities including the following:

1. a perceived possibility of success in a learning experience that is challenging. Often, repetitious drill work does not appear challenging enough and the child is apt to be motivated not to participate. On the other hand, if the experience seems far too difficult, the child may not even attempt it.
2. the fear of failure,
3. the degree of match between an individual's interests and abilities and the nature of the learning opportunities,
4. positive models or successful models that have similar disabilities,
5. a positive vision of what the individual can become, and
6. accurate self-knowledge. Motivation to achieve can be negatively affected by unrealistically high or low estimation of ability or disability.

Whitmore (1980) proposes that in order to facilitate the development of a higher self-concept, these students require a learning environment that is cooperative and free of the pressure and stress of competitiveness; in which the child's special talents are made highly visible; in which his or her weaknesses are de-emphasized, although still acknowledged privately; and that has an atmosphere of freedom to test their ideas without the threat of failure or rejection. She further recommends that GLD students be given a much more challenging curriculum as these children "tend to perceive school curriculum as irrelevant to their interests and needs, not useful to them, unchallenging and unrewarding" (Whitmore, 1980, p. 216). Such a curriculum would include student-centered programs with student choices from alternatives and would require a teacher highly skilled in methods of inquiry, problem solving and creative thinking. Finally, Whitmore (1980) suggests that

> considering the minimal conditions that must be met to meet the basic needs of gifted children, including the desirable characteristics of teachers effective with the gifted child, it is very possible that the regular classroom may, in many instances, be the most restrictive environment for the gifted child. (Whitmore, 1980, p. 404)

Using a formal program which supports the recommendations forwarded by Whitmore, Baum (1988) conducted a study in which seven elementary school GLD pupils were subjected to an enrichment program which incorporated skill development into the production of new knowledge through independent or small group investigations. The program was based on Renzulli's (1977) Enrichment Triad Model. Renzulli (1977) developed his model around the assumption that "the greatest source of student satisfaction almost always resulted from the student's freedom to pursue topics of their own choosing in a manner with which they themselves felt most comfortable" (p. 16). Thus two of Renzulli's program objectives were:

firstly, students will spend most of their time pursuing "their own interests to whatever depth and extent they so desire; and they will be allowed to pursue these interests in a manner that is consistent with their own preferred style of learning" (p. 5). Secondly,

> the primary role of the teacher in the program for the gifted and talented students will be to provide each student with assistance in (1) identifying and structuring realistic solvable problems that are consistent with the student's interests, (2) acquiring the necessary methodological resources and investigative skills that are necessary for solving these particular problems and (3) finding appropriate outlets for student products. (p. 10)

The model consists of three types of enrichment activities. Type I activities are general exploratory activities designed in order to help the students become aware of areas of study that may be of sincere interest to them. Students are not only made aware of particular fields of knowledge but also of what professionals in that field do. Activities include reading, field trips to observe professionals at work and presentations by various resource persons.

Type II activities are group training activities designed to provide students with the skills necessary to solve problems in a variety of areas. The exercises help the learner to develop the processes that better enable him or her to deal more effectively with content. Some examples of these processes include: critical thinking, problem solving, reflective thinking, divergent thinking, and many other processes related to Bloom's taxonomy.

Type III activities are individual and small group investigations of real problems in which the students become actual investigators of real problems by using appropriate methods of inquiry. "The child takes active part in formulating both the problem and the methods by which the problem will be attacked" (Renzulli, 1977, p. 30). The students are not simply reporting about other people's conclusions but are drawing their own conclusions based on their analysis of "raw data".

In the study that Baum (1988) conducted, seven GLD students from grades two and five met once a week for two and one-half hours at a resource center for the gifted. The activities that the students engaged in were appropriately based on their own interests, learning styles and academic strengths. The inquiries provided open-ended challenges requiring divergent thinking, discussion and experimentation and involved real problems based on student interests. In the initial weekly sessions Type I and Type II activities were provided in order to stimulate interest in possible future investigations and to develop the necessary communication and technological skills to compensate for poor reading and writing skills. Later sessions involved all the students in initiating a whole-group project requiring creative production. They wrote, using rhymed couplets, and illustrated, using photography, a unique children's book on unusual ways to pop a balloon.

Eventually conferences were held with each individual in order to help in defining a real problem, in determining a target audience for the study, and in deciding the appropriate form of the product. Students were required to sign contracts in which clear expectations were developed. Some of the projects eventually undertaken were: computer programs, qualitative research studies, a comparison of the attitudes of adults and children about wearing bicycle helmets and a slide-tape show of fifth grade attitudes to nuclear war which was eventually sent to politicians.

The results of the study indicated that learning behavior improved as demonstrated by increased time on task and sustained effort toward completion. The children's motivation was also found to be improved. Unexpectedly, it was also found that the "academic achievement in four out of seven children improved dramatically" (p. 229). After completion of the program, one student no longer required support services, another gained four grade levels in reading while two others began to show improvements in all subject areas.

The educational implications of the study as suggested by Baum were the following:

1. educators must focus on the gift or talent of the individual,
2. GLD students require a supportive environment which values and appreciates individual abilities,
3. students should be given strategies to compensate for their learning problems as well as direct instruction in basic skills, and
4. the students must become aware of their strengths and weaknesses and must be helped to cope with the wide discrepancy between them.

Huntley (1990) also used The Enrichment Triad model to develop what she called the Search Handicapped Outreach Program (SHOP) for the gifted learning disabled. Students were selected on the basis of having met Connecticut's criteria for learning disabled and having scored an IQ of greater than 120 on the WISC-R (Wechsler Intelligence Scale for Children-Revised). Students completed an independent project based on a topic of interest. The project included appropriate methods of inquiry, identification of an audience and development of an original product. For example, one student interested in jets wrote to a major company and requested information. From the sources sent to him, the student discovered that birds could be a hazard for jet engines. As a result, he designed a screen for the engines and returned his plan to the company.

Gifted Learning Disabled in the Mainstream

Since many, if not most, schools do not have specific programs for either the gifted learning disabled, or the gifted, if their needs are to be met at all, it must be done in the regular classroom setting. Researchers (Baum, 1984; Baum, Emerick, Herman & Dixon, 1989; Moller, 1984; Silverman, 1989; Suter & Wolf, 1987; Yewchuk & Bibby, 1986) have suggested the following alternative strategies for instruction and evaluation:

1. provide tape recording of lectures,
2. provide opportunities for experiential learning,
3. provide opportunities for choices in the modes of presentation,
4. use a multisensory approach to learning because students should be assisted in locating information in a form they are able to understand,
5. use alternative evaluation methods (e.g. oral),

6. use inductive teaching stategies, holistic methods and activities requiring synthesis,
7. use more meaningful materials, high interest activities, computers and more challenging curricula which expose the student to a broad range of topics,
8. use activities designed to circumvent problematic weaknesses and highlight abstract thinking and creative production, and
9. provide an opportunity for the child to meet mentors and role models with similar patterns of handicap, excellence, and vocational preference.

Since all students will bring their own special learning style to the classroom, these strategies would benefit not only the GLD students, but all children.

Giftedness in Mathematics

Krutetskii (1976) insists that "one should distinguish between ordinary 'school' ability for mastering mathematical information, reproducing it, and using it independently and creative mathematical ability, related to the independent creation of an original product that has social value" (p. 21). This is partly due to the fact that ordinary school testing does not uncover the true nature of a person's ability because "tests are oriented only toward a quantitative expression of the phenomenon under consideration and in no way reveal its qualitative characteristics" (Krutetskii, 1976, p. 13). In a vast study, spanning twelve years, he used both qualitative and quantitative analysis of the solution of specially designed experimental mathematical problems by students with various abilities in mathematics, primarily for the purpose of investigating

> the structure of mathematical giftedness (as a unique combination of abilities) at school age; in other words, to undertake an analytic "decomposition" of this integral property of the mind into the individual components that occupy an essential place in its structure. (Krutetskii, p. 77-78)

Krutetskii defines the ability to learn mathematics as

> individual psychological characteristics (primarily characteristics of mental activity) that answer the requirements of school mathematical activity and that influence, all other conditions being equal, success in the creative mastery of mathematics as a school subject - in particular, a relatively rapid, easy, and thorough mastery of knowledge, skills, and habits in mathematics. (p. 74-75)

Students were given a problem and asked to try to explain their thought processes. For example, Krutetskii's instruction to a pupil would proceed as follows:

> Think aloud. You do this, don't you, when you are solving a problem alone at home? Write down on paper everything that comes into your head in connection with the solution. I am interested not in your final decision, not in the time it takes , but in the process itself. Do not try to explain anything to anyone else; pretend there is no one here but yourself; do not tell about the solution but solve it. (p. 93)

Krutetskii's conclusions about the characteristics of mathematically gifted students can be summarized as follows:

1. These students have a formalized perception of mathematical material. This means that:

"capable pupils perceive the mathematical material of a problem *analytically* (they isolate different elements in its structure, assess them differently, systematize them, determine their 'hierarchy') and *synthetically* (they combine them into complexes, they seek out mathematical relationships and functional dependencies)". (p. 227-228)

2. These students have the ability to generalize mathematical objects, relations and operations. This means that they recognize specific problems as ones representative of a general class and work out a general method for solving problems of the given type. They generalize rapidly and broadly.
3. They have the ability to curtail the process of mathematical reasoning and the corresponding system of operations. The student is able to by-pass many links in the reasoning process.
4. The student demonstrates flexibility of mental processes. These students easily switch from one method of operation to another, rather than being influenced by stereotypical conventional methods of solution and can reconstruct the knowledge and skills required to adapt to new situations.
5. These students strive for clarity, simplicity and economy ("elegance") in a solution.
6. Students demonstrate reversibility of mental processes in mathematical reasoning. This means that the student can easily switch from a direct to a reverse train of thought.
7. These students have a good memory for mathematical generalizations. In other words, " the memory of a mathematically able pupil is markedly selective: the brain retains not all of the mathematical information that enters it,

but primarily that which is 'refined' of concrete data and which represents generalized and curtailed structures" (p. 300).

8. These students possess a mathematical cast of mind. This means that these students tend to pay attention to the mathematical aspects of the world and notice the mathematical relationships in it.
9. These students tend to be tireless when doing mathematics.

Other researchers (Heid, 1983; NCTM, 1987) have used Krutetskii's findings as a basis for describing the characteristics of gifted mathematics students. Sowell, Zeigler, Bergwell and Cartwright (1990) suggest that mathematically gifted students often appear bored in the regular classroom setting and that "some may perceive that long practice exercises are a waste of time because they already understand the mathematics and see no reason for the practice" (p. 147). This may, in some instances, result in underachievement. Citing the cases of well known research mathematicians such as Lobachevskii, Krutetskii (1976) argues that it is often the case that capable students in mathematics show little interest in it and do not display great success in learning the subject unless the teacher is able to "awaken" their interest in it.

Meeting the Needs of the Mathematically Gifted Student

The National Council of Teachers of Mathematics (NCTM) (1987) advocate the following sixteen essential components of any program for the mathematically gifted:

good mathematics	learning resources
sound pedagogy	integration of content
teacher competence	planning and development
higher order thinking	evaluation
applications and problem solving	student concerns
study skills and work habits	mobility
individual differences	status
encouragement of creativity	communication skills

Above all else, Renzulli, Reis and Smith (cited in NCTM, 1987) suggest that the gifted mathematics student requires something worthwhile to think about and work on, as well as "the nurturance of a creative, knowledgable and sensitive teacher" (p. 60).

Hersberger and Wheatley (1989) developed a program for gifted fifth and sixth graders which centered around the use of the computer, calculator and various problem solving techniques in solution of difficult problems. The emphasis was placed on thinking processes, including self-monitoring of thinking processes and a limited amount of time was spent on computations.

Students worked independently, in small groups or together with the whole class. Groups were determined according to student interests and learning styles, and grades were deemphasized in order to encourage students to focus on the tasks themselves. Students were mostly evaluated on the basis of group and individual projects, and few tests were given.

Hersberger and Wheatley (1989) found that the use of computer programming resulted in better problem solving. There was a constant necessity for students to develop efficient program debugging methods, which forced them to reflect on the mode of thinking and the processes that they used in solving the problems. Testing their ideas on the computer helped students become less dependent on their teacher for explanations.

Through his research into award winning programs for the mathematically gifted, Campbell (1988) found that such programs were always problem-oriented, independent study programs which had regularly scheduled classes (before school or at lunch). Students of mixed grade levels solved highly imaginative and open-ended problems, which, in some cases, were assigned for homework. There were no texts used and no exams given. In addition, teachers in these programs were given extra preparation time or reduced supervision and homeroom duties.

Stanley, Lupkowski and Assouline (1990) proposed the following considerations for mathematically gifted youth:

1. Acceleration may provide the best educational option, although this does not mean racing through a standard sequence of learning outcomes at the expense of enrichment.
2. Students should be given a curriculum of "highly satisfying" mathematics that is at the appropriate level for the individual.
3. Summer programs could be made available.

Elsewhere, these authors recommend the use of a mentor model for talented students (Lupkowski, Assouline, & Stanley, 1990). In a once weekly, 2 or 3 hour meeting with a gifted student, a mentor would provide the challenges and motivation that such a student requires. Other authors likewise advocate the use of mentors (NCTM, 1987; Yewchuk & Bibby, 1986).

Given that programs for the gifted in mathematics are not always possible, researchers (Heid, 1983; NCTM, 1987) have suggested several classroom level options. These are summarized below:

1. Be flexible about assignments. Allow students to plan some of their own investigations.
2. Select problems and activities that have a variety of approaches and solutions, that have many levels of solution (extendible problems) and that are multistrand.
3. Make additional resources available.
4. Spend time with these students in extra dialogue.
5. Provide opportunities for the students to communicate with others about their work.

Use of Case Studies in Mathematics Education

Case study methods in mathematics education are not new. Erlwanger (1975) used a case study to describe the nature of several childrens' conceptions, beliefs, emotions and views concerning mathematics and how these appear to guide their mathematical behavior. Using the observation/interview method on a sixth grader named Benny, Erlwanger found that Benny's underlying conceptions about mathematics led to the justification of procedures and answers according to his own beliefs and intuition independent of what he was taught. The nature of children's conceptions depend on the learning environment and may be quite different from an adult's view.

Erlwanger concluded that unless a child's conception is understood, his or her observable mathematical behavior cannot be explained. As a result, inappropriate learning experiences and remediation may be prescribed. Since evaluations based on tests and occasional conferencing are not enough to reveal conceptions (many analyses of these tests are based on adult inferences about the nature of the child's thinking), Erlwanger recommends the use of qualitative methods requiring close observation of the students' mathematical behavior.

Burnett (1981) and Landis and Maher (1989) have also conducted similar case studies in which mathematical behavior was monitored. Landis and Maher observed a fourth-grade pupil in a classroom setting and looked for the heuristics used, the modes of representation employed and the connections made by the student between current representations and those used in previous work. Their observations confirmed that these mathematical behaviors do indeed exist in natural classroom settings.

Burnett (1981), in a more extensive analysis of the mathematical understanding of two senior secondary school students, used videotape, audiotape, typed protocols and work samples in order to examine the following:

1. The function language served while engaged in mathematics.
2. Evidence of rules, rule-governed behavior and algorithmic thinking.

3. The degree to which the students explicitly indicated their awareness of what they were doing.

4. The students' problem solving heuristics, strengths and weaknesses.

5. The representations used in solving problems.

Burnett used the results to diagnose difficulties and make recommendations for future learning experiences. He also outlined the curriculum implications of his study.

In the area of learning disabilities, Jordan (1981) conducted a case study of an eleven year old girl for the purpose of exposing the symptoms of her mathematical misunderstanding and then diagnosing the misunderstanding.

Speer (1983) claimed that it is quite effective to use observation and interview techniques in order to collect and analyze data for the purpose of forming a profile or "cognitive style map" of the individual student. "Once a student's educational cognitive style has been determined, it is possible to structure prescriptive instruction that best meets the style requirements" (Speer, 1983, p. 38).

These studies all suggest that case studies of individuals, using qualitative methods, may be the best means of gaining an understanding of a student's mathematical behavior.

Chapter 3
Method

The Subject

The motivation to begin a case study of an individual began early in the fall of 1990. The subjest, Kurt, was familiar to me only in that he had brothers and sisters graduate from the school in previous years and that he had been involved in a learning assistance program in grades 8 and 9. In early October of his grade 10 year, he wrote a unit test on radical arithmetic and received a failing grade. On one of the most difficult items on the test, however, Kurt demonstrated, in my opinion, considerable talent. The problem was to determine side **AD** in triangle **ABD** shown below, given that **AC** = **CD** and that **AB** = **BC** = **1**. Kurt's work is illustrated below.

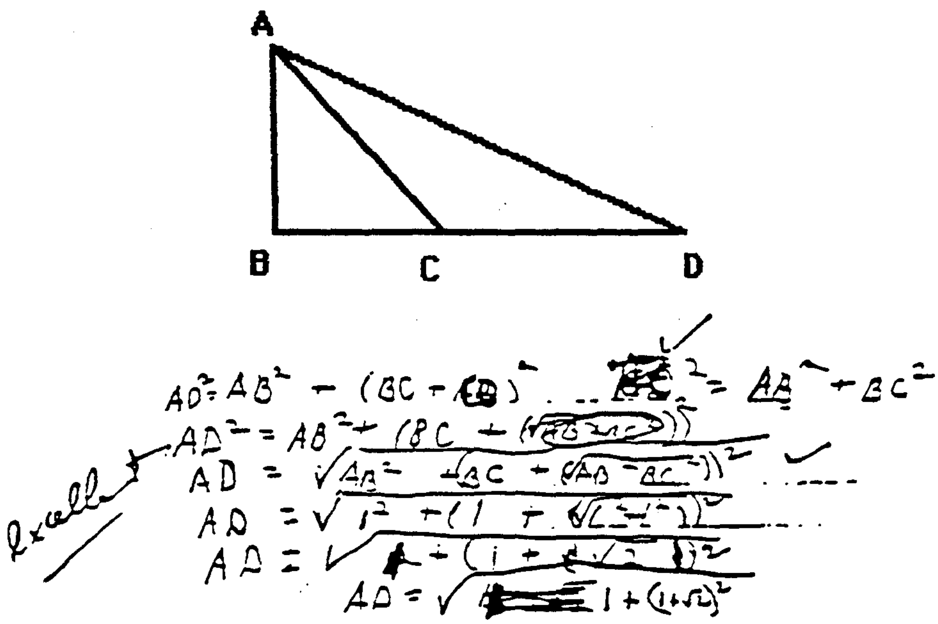


Figure 3.1 – First Indication of Talent

It seemed perplexing that a student could create a general algorithm to solve the problem in the most abstract way, substitute the numbers and then not simplify or calculate the final answer. Most capable students would probably have followed similar steps, except that they would have worked with the numbers only and not the abstract variables. What seemed even more of a contradiction was that he was unable to pass the "easy" part of the test involving radical arithmetic. The matter seemed worthy of further study.

At the start of the study, Kurt was a fifteen year old grade 10 student. He had a history of learning difficulties and was receiving extra attention from the special education teacher in the school. Psychoeducational assessments conducted by the Vancouver General Hospital in 1983 and again by the Vancouver Children's Hospital in 1989 revealed that Kurt had superior intellectual ability coupled with specific deficits that made it difficult for him to cope in an ordinary academic setting. His biggest academic difficulty was consistently producing assignments. As a result of his low productivity throughout his academic history, Kurt consistently achieved below average grades. In contrast, he became very skillful in the use of personal computers. A more detailed account of Kurt's strengths and disabilities follows in chapter 4.

Rationale

The research design that one chooses is influenced by many factors such as philosophical perspectives, personal experiences, cultural ideologies (ie. the culture of the mathematics/science educator) and the views of other researchers in one's field of study. If one tends to view the world as objective reality in which all things, including social beings, are affected by variables that can be manipulated, then one would tend to favor a quantitative research design in which hypothesized causal relationships or correlations between variables can be tested experimentally and where confidence (or lack of confidence) in the results is guaranteed statistically. In this mode of inquiry, hypothesis testing or confirmation of theory is the strongest motivation. Alternatively, if one views social reality as being affected by peoples' subjective experience and the meanings that they derive from their experiences, then it would seem that a more qualitative approach would be favored, since subjective meaning would be difficult to measure quantitatively. Merriam (1988) writes,

> in a qualitative approach to research, the paramount objective is to understand the *meaning* (author's italics) of an experience. In contrast to quantitative research, which takes apart a phenomenon to examine component parts (which become the variables of the study), qualitative research strives to understand how all the parts work together to form a whole. (p. 16)

It would be inappropriate to dismiss either type of research method without first examining carefully the theoretical framework and research questions that guide the study. Goetz and LeCompte (1984) suggest that the "primary criterion for selection, development, and implementation of a research model is whether a design allows the researcher to address effectively the research goals and questions posed" (p. 48).

Since the major purpose of the study is to investigate phenomena related to Kurt's mathematical behavior in order to decide what the implications are for his future learning experiences, the findings of the investigation will naturally be rooted in his unique personal experience. Generalization will be next to impossible. What Stake (1978) calls "naturalistic generalization" is a more realistic goal. "What becomes useful understanding is a full and thorough knowledge of the particular, recognizing it also in new and foreign contexts" (Stake, 1978, p. 6). Thus it is hoped that the study will have resonance for other educators, particularly mathematics educators.

Given these goals, a qualitative case study seems the more appropriate mode of inquiry. Merriam (1988) writes,

> investigators use a case study design in order to gain an in-depth understanding of the situation and its meaning for those involved. The interest is in process rather than outcomes, in context rather than a specific variable, in discovery rather than confirmation. (p. xii)

Furthermore, "research focused on discovery, insight, and understanding from the perspectives of those being studied offers the greatest promise of making significant contributions to the knowledge base and practice of education" (Merriam, 1988, p. 3).

Specifically in the area of mathematics, Erlwanger (1975) cites Piaget's view that since children develop their knowledge through their own activity, an idiographic research method is required "involving a close and detailed study of children through observations followed by flexible and exploratory interviews with children about phenomena arising from these observations" (Erlwanger, 1975, p. 165). Lamon (cited in Burnett, 1981) suggests:

> Work in small groups of subjects or even with one subject at a time,using qualitative methods, should be conducted for the purpose of penetrating the mental activity of the subjects and analyzing mental processes when working exclusively in mathematics. (p. 17-18)

Merriam (1988) defines a case study as an investigation of a bounded system, that is, a person, institution, social group, a program or an event and that it is characterized as being particularistic, descriptive, heuristic, and inductive. When the study involves only one person, it can be argued that the conclusions cannot be extended to the general population because the chances are slim that the individual chosen will be representative of the population. However, as argued above, generalization in the traditional sense may not be the goal. It is possible that the individual might be representative of some sub-population (such as the gifted-learning disabled), in which case the study might be applicable to the sub-population. It is the inductive nature of the inquiry that is, moreover, important. Many scientists and mathematicians make discoveries inductively, form hypotheses and then set out to "prove" their theories or theorems deductively. The published result is usually a deductive argument, but the real work of discovery remains hidden in the scribbled notes at the laboratory or office. Getzels (1973) argues that Freud used himself as the case study in order to make his discoveries and that Piaget's work derived from the observation of his own children. He writes, "let us not be too scornful of N=1; it is not the N that matters but what is done with it" (Getzels, 1973, p. 18).

Data Collection

The major form of data collection used in this study consisted of in-depth interviews (audiotaped) and observations of the student engaged in mathematics (pen and paper note taking). These sessions had as their purpose the exploration of Kurt's mathematical thinking. This involved presenting him with non-curricular mathematical activities and then observing how he proceeded. I chose or designed the activities to reflect the interests and abilities of Kurt, but at the same time, they required different types of mathematical thinking. For example, activities were chosen that might require or involve the use of conjectures, predictions, generalizations, justification, different problem solving strategies, different modes of representation, and algorithmic thinking. The activities also had to be challenging enough to expose any giftedness that Kurt might possess. Protocols of the subject's work, school reports and psychoeducational assessments were also collected. Finally, I kept a journal of my reflections in order to write my first impressions of the interviews.

The first eight interviews with Kurt were conducted over a period of 5 months from 15 February, 1991 to 7 June, 1991. It was initially intended that the interviews be scheduled at intervals of every two to three weeks at a time and place convenient to Kurt. The lengths of the sessions were to be flexible, depending on the nature of the activity and the disposition of the subject. This was intended to ensure that he did not feel he had to partake in a session out of a sense of obligation to myself as the researcher. Coincidentally, Kurt was scheduled with a "study skills block" (in place of French) with the specials needs/ resource person and this period was also my preparation block. With the full support of the resource person and Kurt himself, it became convenient to meet during this 56-minute time period.

On the first meeting, which took place in the resource person's study skills room, I asked Kurt to complete a 50-question mathematics attitude survey (see Appendix B, p. 119) adapted from surveys in The Second International Mathematics Study (Robitaille, O'Shea, and Dirks, 1982). The purpose of the survey was to get a "quantitative" measure of Kurt's attitudes and views specifically related to mathematics as a process, his personal reaction to the study of mathematics, his view of mathematics in terms of its practical value, and his attitude toward calculators and computers. Immediately after completing the survey, I conducted an interview to gain information related to Kurt's self-efficacy, interests and dislikes in mathematics. (see Appendix C, p. 124 for a sample interview transcript) Examples of some of the questions I asked are: "What sorts of things or activities would appeal to you in doing math? What do you dislike about mathematics, that's been in your experience so far? Generally speaking, at the start of a math test which your teacher gives you or if you know there is a math test coming up, how do you usually feel about your chances of success on the test? If you had your choice, what sorts of activities or topics would you like to do in learning mathematics?" The purpose of the last question was to help me decide some possible mathematical activities in which to engage Kurt later in the study. I thought that more productive mathematics might result if the subject was keenly interested in what he was doing. Although I had written out questions in advance in order to guide the interview, spontaneous questions and probes arose as the situation developed.

During the latter part of the session, I gave Kurt a non-curricular cubic equation to solve and my role became that of observer. I informed him that the purpose of the study was to reveal his mathematical understanding and that I would neither intervene to teach him anything, nor acknowledge the "correctness" or "incorrectness", nor do any remediation. Furthermore, the problems were to be extra-curricular in nature in order to investigate his natural ability. I recorded my observations of what he was doing and collected his own written work. In addition, I asked probing questions in order to attempt to reveal deeper understanding (or misunderstanding). For example: "What did you just realize when you said "no wonder"? What did you just try to do? What are you thinking? Why did you do that?"

During the second session, I had to repeat some of the questions from the first interview because the sound quality was poor. The problem was solved by subsequent use of the same machine for both recording and transcribing. I devoted the remainder of the session to exploring Kurt's ability to generalize patterns of numbers.

The third session was a problem solving session requiring Kurt to use some prior knowledge of coordinate geometry. Due to technical difficulties, the session was not audio-taped. However, I took extensive notes and immediately following the session, I made a photocopy of Kurt's work and wrote down all that I remembered directly on the protocol. What took place was later transcribed into type.

The fourth session took place, as did the remaining sessions, in the counsellor's office, where an IBM computer was available for use. The purpose of this session was to explore a computer program, previously created by Kurt and written in Turbo Pascal programming language, in order to investigate what mathematics was involved. Kurt explained various aspects of the program, and based on what was observed, I spontaneously thought of a problem for him to program. Since he was using trigonometry in his programs in order to change the direction of moving objects, and had in previous interviews shown, in my opinion, an unclear understanding of basic trigonometry, I felt that another problem that could possibly involve trigonometry might produce new insight into his understanding. I asked him to create a program in which an object (a projectile) revolves around in a circle near the bottom of the computer screen, much the same as a rock in a sling is rotated. Another object travels in a straight path across the top of the screen. The objective was for the user of the program to release the projectile as it is revolving, in order to make contact with the object moving across the screen. Subsequently, the fifth, sixth and seventh sessions were all concerned with the programming of this problem. I took notes of the mathematics used, Kurt produced diagrams at various stages when necessary and all sessions were audio-taped.

The final session with Kurt was an investigation of his ability to make, and subsequently support, conjectures in geometry.

Next, I conducted an interview with Kurt's special needs instructor, who had worked with him previously during his elementary school years. The purpose of this semi-structured interview was to determine the nature of Kurt's disabilities and learning strengths according to the professional opinion of the instructor and to triangulate with data obtained in his psychoeducational assessments.

The last formal interview that I conducted for this study was with Kurt's older brother Ken. During the course of the sessions with Kurt, Ken's name often came up as a person that Kurt consulted on computing matters, and it became obvious that they were very close. I felt that Ken might offer more insights into the way his brother thinks. The interview with Ken took place in August,1991, and was also audio-taped.

In February, 1992, after I had written an account of Kurt's early years (see Kurt's Story, p.48-60), I asked his mother and Kurt himself to read it and to make any comments that they wished. Both approved the description that I had written.

Since Kurt was a student in my class, I also had occasional opportunities to collect samples of his class work, both written and oral, that seemed unique or interesting.

In addition, I started a journal immediately following the fourth interview in order to record my impressions of what transpired and to write out any hunches or perspectives as they occurred. Moreover, anything of interest that came up in the classroom, such as explanations, were also recorded in this journal.

During July of 1991, I analyzed all the data that I had collected up to that point. As a result of the analysis, I produced a possible model explaining Kurt's mathematical behavior. I also made some preliminary recommendations regarding his future mathematics education. Since Kurt was scheduled into my mathematics 11 course in the fall of 1991, I felt it might be an opportunity to try out some of my ideas. As a result, I suggested to him in September that since his ability and interests lay in problem solving and computing, I would assign him various enrichment activities and problems that would account for 50% of his mark and the regular homework would be de-emphasized. I had intended to provide him with as many choices of problems or investigations as I could find and tried to relate the work to computing. For example, I asked him to investigate the successive perimeters of the Koch snowflake (see Figure 4.6, p. 100) and to write a computer program to print them out. Other examples of assignment choices were: to write a paper on fractals, to write a computer program to calculate the roots of quadratic equations and to program the computer to make simple tesselation designs (because he had shown an ability in drawing and art). Any background information that may have been required for completion of the assignments was either taught in class or provided by me outside of class.

The other 50% of his mark would be made up from regular classroom work and testing. He initially responded positively to the idea and was willing to give it a try. Thus, throughout the fall term and for the remainder of the year, I collected the extra work that he attempted and made journal entries of any interesting or revealing conversations between us.

Data Analysis

The data analysis used in this study is based on the ideas presented by Goetze and LeCompte (1984) and Miles and Huberman (1984). They suggest a preliminary analysis of each interview or observation immediately after it occurs. In reference to leaving all analysis to the end of data collection, Miles and Huberman (1984) argue,

> It rules out the possibility of collecting new data to fill in gaps, or to test new hypotheses that emerge during analysis; it tends to reduce the production of what might be termed "rival hypotheses" that question the field worker's routine assumptions and biases; and it makes analysis into a giant, overwhelming task that both demotivates the researcher and reduces the quality of the work produced. (p. 49)

Circumstances dictated that I could not undertake a detailed, systematic analysis until eight interviews with Kurt and one interview with the special needs instructor were completed. That is not to say, however, that I did no prior analysis. As previously indicated, any ideas or perspectives were noted in my journal. In addition, the first computer session was a result of the first interview with Kurt and the first computer session led directly to a new and unique problem for him to solve on the computer.

I transcribed all the interview data myself. This in itself was helpful because it provided an opportunity to note immediately any interesting phenomena suggested by the data. In addition, any diagrams drawn or work done by Kurt on separate pieces of paper were photocopied and pasted into the interview transcripts in their appropriate locations. Next, I scanned each interview transcript for phrases, sentences, paragraphs, equations, diagrams or expressions that indicated or described some concept or category which for the most part "derive from research questions, hypotheses, key concepts, or important themes" (Miles & Huberman, 1984, p. 56). The codes themselves were words or short phrases and were generated as I proceeded through the data, rather than having been created ahead of time (although their creation would still be influenced by the research questions). This coding scheme is analogous to what Goetz and LeCompte (1984) refer to as "analytic induction" or what Miles and Huberman (1984) describe as "essentially the more empirically 'grounded' approach advocated by Glaser (1978)" (p. 57). I also kept the codes on a separate sheet of paper, grouped according to transcript number, for easy reference and for later comparison to see which codes naturally fit together. Some examples of the codes that emerged are: preferences, dislikes, feelings, confidence, pressure, creativity, increase in understanding, strategy, use of past experience, justification, explanation, generalization, relationships, language, creation of variables, connections, organization, humor, learn by accident, modify to adapt, knowledge, seeks simpler solution, and so on.

In addition to coding, as I read the data, I took marginal notes and wrote questions down when ideas arose. Since the data were transcribed leaving a wide right margin, there was plenty of room for notes. Goetz and

LeCompte (1984) suggest that the "notes taken while scanning constitute the beginning stages of organizing, abstracting, integrating, and synthesizing, which ultimately permit investigators to tell others what they have seen" (p. 191).

Immediately after each interview was transcribed, I wrote a contact summary in a style suggested by Miles and Huberman (1984). Similarly, Goetze and Lecompte (1984) also advocate a written summary arguing that it summarizes "the major events and issues discovered in the course of investigation. Writing such a summary helps the researcher to withdraw from minute details and look for the larger picture that emerges" (p. 192). An outline of the form of such a summary follows:

Summary #__

1. Context:

2. Main Issues That Stood Out:

3. Information Obtained/Inferences Made Relating to Research Questions:

self-efficacy/interests/self-concept

mathematical behavior

mathematical giftedness

GLD

disabilities

4. Any Other Interesting Phenomena:

5. New Questions:

I indexed each entry in parts three and four of the summary with the page number of the interview transcript from which the entry was derived.

The next step in the analysis involved classifying all the codes into larger categories. The construction of each category was motivated by what Miles and Huberman (1984) call a "checklist matrix" which "organizes several components of a single, coherent variable"(p. 96), and by a "conceptually clustered matrix" which "has its columns arranged to bring together items that 'belong together'"(p. 110) Each code became a subheading under which interview data that matched that particular code was entered. (see Appendix F, p. 140)

The subheadings (codes) of two of the larger categories derive directly from the literature and helped me firstly, to compare Kurt's behavior to the characteristics of gifted learning disabled students, and secondly, to decide whether Kurt was mathematically gifted. Figure 3.2 on the following page outlines these categories which form a type of checklist.

The remaining larger categories evolved from my attempts to classify the additional codes that had emerged from the data. Each major category and the codes that belong with them are outlined in Figure 3.3 on p. 47.

As the data were grouped and sorted according to the plan above, it became easier for me to speculate on some conclusions and perspectives.

Checklist: Mathematical Giftedness	**Checklist: Characteristics of the Gifted Learning Disabled**	
Formalized Perception/ Relationships	Hobbies	Problem Solving
Generalization	Discrepancy	Subscales
Curtailment of Reasoning	Goals	Languages
Economy/Flexibility	Self-Critical	Spacey
Reconstruction of Mental Processes	Reflective	Test-Phobia
Generalized Memory	Frustration	Deficiencies
Mathematical Cast of Mind	Disorganized	Memory
Energy		
These codes are derived from Krutetskii's (1976) characteristics of the mathematically gifted found summarized on p. 21-22 of Chapter 2.	The above codes were adapted from the characteristics of GLD students summarized in the literature review, p. 11-12	

Figure 3.2 - Codes For Mathematical Giftedness and GLD

Personal Characteristics

Self - Efficacy/Self - Concept	Attitude Towards Mathematics
Comfort	Feelings Toward Mathematics
Pressure	Preferences
Confidence	Dislikes
Self - Efficacy/ Self - Concept	Importance/ Usefulness
Humor	
Self - Motivated	
High Standards	
Nervousness	
Feelings About Misunderstanding	

Problem Solving Strategies	Mathematical Processes	Factual Knowledge
Trial & Error/ Experiment	Creating Relationships	Use of Language +/-
Use of Past Experience	Generalization	Trigonometry
Use of Simpler Cases	Justification	Pythagorean Theorem
Seeks Simpler Solutions	Pattern Recognition	Absolute Value
Diagrams	Explanations	Tangent/Radius Property
Use of Systems of Equations	Conjecturing	Transformational Geometry
Help from Brother	Error Recognition	Use of Inequalities
Reads Manual	Sudden Ideas/Realizations	Own Formulae
Pen & Paper Scribble	Curtailment of Reasoning	Geometry
	Planning/Organization	
	Creativity	

Figure 3.3 - Categories and Codes for Personal Characteristics

Chapter 4
Findings and Interpretations

Most of the quotations that I have included in this chapter come from transcribed interview data that were organized into categories (Chapter 3, Figures 3.2 and 3.3, p. 46-47) according to the various characteristics of Kurt that I decided to study. Where applicable, I referenced these quotes with an interview number (Roman numeral) and the page number of the interview transcript. Where dialogue occurs between Kurt and myself, my questions are preceded with R:, and Kurt's comments are preceded with K:. Other information came from anecdotal notes in my journal and documents from Kurt's school files.

Kurt's Story

At the start of this study, Kurt was a fifteen year old grade ten student. He is the youngest member of a family with ten children. Kurt's difficulties began very early in his life when he had to undergo open heart surgery at seven months of age in order to remedy a hole in his heart. During his primary years at school, Kurt was characterized in a 1983 Vancouver General Hospital psychoeducational assessment as having poor concentration ability, letter reversals and poor articulation, and difficulty in remembering past learning, especially unstructured learning. In addition, there was an indication of a slight delay in visual-motor organization. On the other hand, he showed good reading comprehension and high average intelligence. Despite the fact that his teachers felt that he was bright, Kurt was labelled a "reluctant learner" because he was distracted easily, and had difficulty producing work. In grade 3, his teacher requested that a portable partition be set up around his desk during seatwork in order to help him focus and concentrate.

During the intermediate years, he continued to have difficulty concentrating, took a long time to copy things down, and often neglected completing assignments. His grade 5 teacher commented on one report card:

> Frankly, if it wasn't for the fact that Kurt must finish at least one assignment before going home each day, he would have no thoroughly completed assignments. It must be realized that Kurt is accomplishing one quarter of the workload expected at this time.

Occasionally, he was kept in at the lunch hour and after school for hours in order to do his work. One teacher even placed him in the hallway from time to time in the hope of getting him to focus on getting some work done. However, these measures did not motivate Kurt to be more consistently productive. Eventually, he was even suspended from school for a day so that his missing work could be completed. Even with a day to catch up on his work, Kurt found he could still not face those assignments and the negative feelings associated with them. His mother ended up having to help him with most of them so that he could return to school.

Socially, Kurt has also had difficulties. His grade 5 teacher reported that he had a difficult time making friendships.

Kurt grew up in a family that placed tremendous value on knowledge and education. By the time he reached grade 8, three of his four brothers and all five of his sisters were either attending or had graduated from university (the exception was his brother Ken, who was in grade 12 at the time) and all were very successful in many different areas of study. As a result, Kurt was always exposed to a very high level of conversation and discussion at home, as well as having had siblings who provided a model of studentship. It became very difficult for him to understand why he was not achieving at a high level in school, unlike others in his family. He felt he was a failure.

The most recent psychoeducational assessment conducted at Vancouver Children's Hospital in February, 1989, included the following list of scaled scores that Kurt earned on the WISC-R:

Table 4.1 - Kurt's WISC-R Scores

Verbal Subtests	Scaled Score	Performance Subtests	Scaled Score
Information	13	Picture Completion	15
Similarities	16	Picture Arrangement	10
Arithmetic	15	Block Design	18
Vocabulary	14	Object Assembly	14
Comprehension	13	Coding	6
Digit Span	10	Mazes	13

Scaled scores between 9 - 11 are considered to be well within the average range on this test. The assessment concluded that Kurt's scores on the

WISC-R are "indicative of an individual with overall superior cognitive skills who has a specific area of cognitive deficit which would make it frustrating and difficult to indicate his capabilities within the regular academic system".

Within the area of verbal skills, Kurt's scores were considered significantly above average with the exception of Digit Span, on which he scored in the average range. However, the assessment concluded that in comparison to his other scores, this score was significantly lower, indicating that Kurt's short term memory ability with numbers was significantly lower relative to the other verbal skills measured.

Within the area of Performance Subtests, Kurt's exceptionally high score on Block Design was an indication of well developed spatial skills. In contrast, his below average score on the Coding subtest was indicative of a significant motor output problem. This means that Kurt had a great deal of difficulty putting down on paper what he was thinking.

In addition to the WISC-R, other tests were also performed in the 1989 assessment including the Developmental Test of Visual Motor Integration (Beery), the Peabody Picture Vocabulary Test - Revised (PPVT-R, Form L), the Detroit Test of Learning Aptitude (DTLA-2), the Rey Auditory-Verbal Learning Test (REY), the Incomplete Sentences - High School Form (Rotter), and the Wide Range Achievement Test - Level 2 (WRAT-R2). The overall assessment summarized Kurt's difficulties as follows:

> It is not possible to determine whether these learning difficulties are a result of neurological deficit secondary to cardiac surgery in view of the fact that Kurt has one older

> brother who exhibited dyslexia in early grades. Regardless of the source of Kurt's difficulty, the fact remains that he has cognitive ability in the superior range coupled with significant developmental motor output, short term memory, and sequential processing problems which [unreadable] he has difficulty completing written assignments, works slowly, is disorganized, has difficulty remembering multi-step instructions, and feels frustrated with his academic performance. If the motor component is removed from tasks, Kurt can perform at an above average level.

As recently as March, 1990, Kurt's family hired an out of school psychologist from an independent assessment agency to conduct an interview with Kurt. The psychologist's report reaffirmed previous studies and characterized him as having visual-motor impairment, a very weak short-term memory and difficulty with the written word. According to the assessment, Kurt dislikes being the center of attention and does not readily seek clarification of confusing instructions even though he realizes that he must do so. The assessment continued:

> If he is not sure of the nature of an assignment, he will delay doing it, or not do it at all, rather than do something poorly. He worries about whether something is going to be good long before it is even started, let alone completed. If he thinks his effort will not meet his expectations, he puts it off entirely.

Not surprisingly, he has often been characterized by teachers as being "lazy". The assessment also concluded that as a result of all his frustrations and lack of success (academic records show below average achievement across all subject areas), Kurt's self-esteem was suffering. Furthermore, he experienced "nervousness" and recently began suffering from migraine headaches.

Finally, the above assessment cites Kurt's strengths including being a good listener, being a good problem solver, and being skillful in the use of computers.

Of the fourteen characteristics of gifted learning disabled students described in the literature (Chapter 2, p.12-13), Kurt has matched eleven of them. I have underlined each of these characteristics as they are discussed.

Kurt has become an "expert" in the use of computers. His interest in computers began in grade 5 or 6. The family purchased a computer and Kurt began observing his older brother Ken's use of the machine. Being very artistic, with a love of drawing, Kurt began to see the possibilities of expressing his talents in computer graphics. His interest in computers was eventually intensified to the point that computing became a major pastime for him. For example, during an interview in April, 1991, Kurt explained how one of his programs functioned and mentioned in passing that:

> I started this program in the summer and finished it a couple months into school and I was kind of experimenting with it all of the time. iv 12.

In the next interview he commented:

> It's kind of funny, someone asked if I knew a lot of computers and I said that I did , a lot more than a lot of people in this school, not of course including Mr. Bird, (computer science teacher) and I got the impression that they thought I was being arrogant or something or other. I went home and I thought, well, if I were being arrogant, I don't see how that could be because if I didn't know more than these people, I'd be ashamed because I've been working with these for like five years. v 20.

The assessment of Kurt by non-school psychologists indicated that he is a good problem solver. This ability will be discussed later in relation to his mathematical behavior.

Kurt's psychoeducational assessments (Table 4.1, p. 50) have yielded high scores on subscales that assess verbal reasoning abilities, whereas his scores on digit span were significantly lower and his scores on the coding subscale were very low.

Kurt has a difficult time understanding the discrepancy between his poor performance and his perception of his ability. This is illustrated by the following exchange, specifically regarding mathematics:

> R: Do you feel that you have achieved as high as your potential might allow?
>
> K: No (laughs).
>
> R: Do you have any idea why that might be the case?
>
> K: Uh, it's hard to say, uh. . . . . I think I could be better at mathematics but I don't really know why I'm not living up to the standards I've set for myself. Uh, ... it boggles me (laughs). ii 2.

Kurt has difficulty setting and meeting realistic goals. Low goals are unacceptable. For example, having to do drill exercises in mathematics are pointless to him because he already understands how to do them. On the other hand, if he thinks he cannot meet his own or other people's expectations in an assignment, he puts it off entirely and would rather accept a mark of zero. For example, the special needs instructor at the school gives the following account:

> . . . . He feels that he is a failure in . . . the routine of note taking, he's afraid that he's maybe missed out something, that he's not competent enough at note taking in class or, note taking from a book, or answering a comprehension question. He feels that, "what if I've missed a point"? So he's very anxious about that

> whole area and for the most part, he will not even attempt to answer comprehension questions because, in the past, he's felt insecure that he may not have covered all the material. He won't complete parts of things. For example, he had a project, which was basically a very simple, step by step assignment, that was all laid out for him and he was very concerned that he was supposed to have four pages of notes. He was very concerned from step one that he was not going to be able to stretch four pages of notes out of that assignment and so, he didn't bother to even attempt it, even though the material that was supposed to be included was very well laid out and was very predictable. We had done one as an example together and when he saw it only came to two pages, he did not want to be bothered having anything to do with that assignment because he had heard from classmates that they were supposed to have four pages. When I explained to him, "well, why not hand in what you have done, you've covered the assignment, don't worry about the length of the assignment", he got very angry and said, "what's the point, I have to have four pages, that's what everyone else has". His teachers coached him in the same area but he refused to have anything to do with that assignment because he didn't feel that he could have the volume that was required. He wouldn't hand in parts of the assignment to get partial marks because "that isn't perfection, that is not something I will be proud to hand in, or have anything to do with it, so I won't even begin it, won't even attempt it". ix 6-7.

Kurt seems to be more reflective in the learning process, and as a result, has had past difficulty with time constraints on tests. When he was in grade 9, teachers were asked to give him extra time to complete exams and tests. In grade 10, during a mathematics test, he spent 20 minutes trying to answer one question, completely oblivious to the fact that he still had most of his test to complete. Even though he was being allowed extra time, I had to remind him to move on. The previous example is probably more closely related to Kurt's perfectionist and one directional nature described by the special needs instructor:

> Kurt tends to be a one directional type person. He tends to have the type of concentration that he concentrates on one thing at a time. He can't sort of, you know, walk and chew gum at the same time. He has to think of one area, one focus, and tends to be a real perfectionist in that area as well. . . . . He tends to, I think, go off in one direction and forget the big picture. He tends to get very bogged down if there's something he's concerned about that takes up all his time and energy and he doesn't see that, maybe in perspective, it's not all that important. . . . . That's the kind of learner he is. Kurt kind of lives for the moment and he doesn't see a lot of things at once, he only sees one thing at a time and so that kind of awareness of time and managing that time and seeing how it connects to other things in the environment is a real disability that he has. ix 3-4

Kurt has always experienced great frustration at his inability to produce work in school despite the fact that his mind often operates at a much higher level of understanding than most of his peers. At a recent meeting with his mother, she quoted him as saying that there seems to be a "glass wall" up between himself and his teachers in that he can see what is expected of him and yet he cannot seem to "smash" that wall down in order to give back to the teachers what they expect. His brother Ken offers the following explanation as to why Kurt doesn't produce:

> I think, in general, homework is kind of harder for him than most and I had the same experience, because sometimes it's hard, you sit at a desk, and particularily for me writing an essay, and I'm trying to transfer that to what he's feeling on a broader scale. When I write an essay, a lot of times I'll just sit and stare at the page and I just can't get anything out and this goes on for a long time and so you just tend to drop it after a while. I think he's just kind of writing off homework as something he can't do because he does have problems doing it. For some things at first, it'll take him a long time to get it out. Now for math, I think it seems to me if he just did the homework, it would just come real quickly. x 6-7.

As his psychoeducational assessments have indicated, Kurt has tended to be highly disorganized and, as illustrated above, has avoided doing most of his homework assignments no matter what the subject was. The common complaint by most of his teachers, both in the past and currently (grade 11), is that he has the ability to be highly successful, but does not complete his assignments. He does not respond to a highly structured curriculum. Even his computer science teacher has given up trying to get Kurt to hand in curriculum work. On the other hand, if something captures his interest, he responds brilliantly. His English 11 teacher was so impressed by an essay Kurt wrote on the novel Lord of the Flies, that the teacher gave it to the principal to read. When asked why he had not handed in work on another novel, Kurt responded, "after reading four pages of the book, I couldn't get interested, so I didn't bother anymore". Thus, his overall achievement remained low. During a recent meeting between Kurt, the counsellor, the special needs instructor and Kurt's mother, it was suggested that his Accounting 11 class be dropped (he was failing badly) in order that school time be structured into his schedule to allow him time to complete assignments in some of his required or more important courses.

One of Kurt's major organizational difficulties is with time management. One of the initial goals of the special needs teacher working with Kurt in his grade 10 year was to develop strategies for him to focus on priorities and to plan for time management. The following is a description of some of the difficulties Kurt had with organization of his time as observed by the special education teacher:

. . . . at the beginning of the year when we first started planning for time management, we did like an hour block and blocked it all out for the evening, and how he was going to use his time. Even to extra-curricular type things, when was he going to go to the computer lab, when was he going to have dinner, etc. That was . . . he couldn't handle that emotionally. His mother changed the time of dinner by fifteen minutes and he said he couldn't use his schedule at all, it just threw the whole thing off. He had no flexibility at that stage and he was very anxious about minor changes that he saw, that were disruptive to him because of that sort of lock-step approach, you know, "if it doesn't happen the way it is predicted, I can't deal with it". He was very anxious, I mean physically he looked very anxious and . . . just the frustration that he walked around with. ix 5

Kurt seems to have a deficiency in his short term memory, as his psychoeducational assessments have concluded. He has a history of having difficulty remembering things such as running shoes for physical education class and track practice or special equipment required for certain classes. This may also be due to organizational difficulties, such as a lack of routine, rather than just memory problems.

Kurt's psychoeducational reports have also indicated a visual-motor output problem.

Another characteristic that Kurt shares with many other gifted learning disabled students is a tendency to have difficulty learning foreign languages. In Kurt's case, the school counsellors decided that since he achieved so poorly in grade 9 French, he would be given a study skills tutorial in grade 10 in place of French.

Motivation seems to be one of the biggest factors in Kurt's productivity. His special education teacher explains:

I think a key to Kurt is whether or not he thinks something is worth doing and unfortunately in life, we don't always have those

> kinds of choices. Things like the mundane things of learning how to calculate in math are necessary to learn things at a higher level. Well Kurt sometimes, I think maybe, because he learns a different way, can go beyond those steps, doesn't necessarily have to learn them as systematically as some students do, in areas that he's gifted in, such as math. So, when he has to be forced to go through things such as calculations, he doesn't see them as being valuable or relevant and so he doesn't devote the time to them. He tends to have, I think, a kind of arrogance in that respect, where he thinks he is in a position to decide what's valuable and what's not valuable and I think it's tied to success as well. ix 5-6.

In early January, 1992, I spoke with Kurt about his progress in all his subject areas. I had heard that he was doing poorly or failing in half his courses and asked him why he wasn't producing. He suggested to me that if he did all his school work all the time, he wouldn't have any "spare time" to work on his personal projects. Knowing that he wishes to attend a university in the United States after high school, I asked him how he was going to achieve the grades that would allow him entrance to such a university, if he didn't start producing in school. He could not answer. Approximately a month later, when the topic of spare time came up again, he explained that he needed the spare time to "escape" the highly structured day spent at school. He needed his spare time to express his intellectual and artistic tendencies and he did it through the use of the computer.

In summary, Kurt has been frustrated for years with the demands of a highly structured school curriculum. Despite his high intellectual ability, his school achievement has been low, with the exception of the odd assignment that captures his interest. He does not complete most assignments for a number of reasons: he is not interested and the curriculum is not intellectually stimulating enough for him; he feels he cannot meet his own or the teachers' expectations; or he just cannot get anything down on paper. The key factor in Kurt reaching his potential to produce seems to be his perception of whether or not something has intrinsic value, especially that it be intellectually stimulating and that he be able to express himself through a mode of representation that he is comfortable with, such as the computer. The school system has failed to provide for Kurt's specific needs and as a result he has failed to be successful at school.

The Nature of Kurt's Mathematical Giftedness

After having identified Kurt as someone who demonstrates many of the common characteristics associated with gifted learning disabled students, my next goal was to determine the nature of his giftedness, particularly in relation to mathematics. In Chapter 3, I outlined how codes were developed in order to sort the interview data into meaningful categories, one of which was mathematical giftedness (see Chapter 3, Figure 3.2, p. 46). The data were sorted into a checklist (Appendix F, Mathematical Giftedness, p.142) where each code represented a characteristic of mathematical giftedness. The number of data entries under each characteristic of the checklist enabled me to decide, with some confidence, which attributes also belonged to Kurt. With the exception of the last one, for each of the following characteristics which are underlined, there were three or more examples from the data. However, I have reported only the most interesting or revealing examples.

Kurt has <u>demonstrated logical thought about quantitative and spatial relationships and the ability to think in mathematical symbols. Thus he has a formalized perception of mathematics</u>. For example, in the third interview he wrote two relationships involving the sides of a triangle based on the following problem:

> A triangle whose area is 4 is bounded by the two coordinate axes and also by the line whose slope is 4 and whose y-intercept is b. If b>0, what is the value of b?

After he had substituted into the area formula, he suddenly said, "ok, yes" and wrote, side2 = 4, then scratched out the =4 and replaced it with =b. I asked him to explain what he was doing. He said, "2 unknowns, but side2 has a relationship to side1. I can figure out what side1 is in terms of side 2. It reduces to one variable and the equation is solvable". The sequence of steps he used (and which I have numbered) are illustrated in Figure 4.1 on the following page.

He was solving a system of two equations with two unknowns without having ever encountered the technique (at least not to my knowledge) in his school curriculum and before we had dealt with the topic in grade 10. It was interesting to see him use word variables instead of letters and I suspect that this is due to his frequent use of word variables in computer programming.

Step 4 in Figure 4.1 seems to be pointless. However, Kurt wrote this simple equivalence down as an aid in remembering how to manipulate proportions because he was trying to solve the equation in step 3 for side2.

He also made several errors. The first obvious error occurs in his diagram where he drew a line with a negative slope (although if the problem is solved using a right triangle only, it would not matter). Secondly, he did not labelled the diagram and perhaps as a result, became confused in his definition of variables (see step 2, Figure 4.1). Thirdly, step 9 should be $(\text{side } 1)^2 = 32$. Notwithstanding these errors (or possible

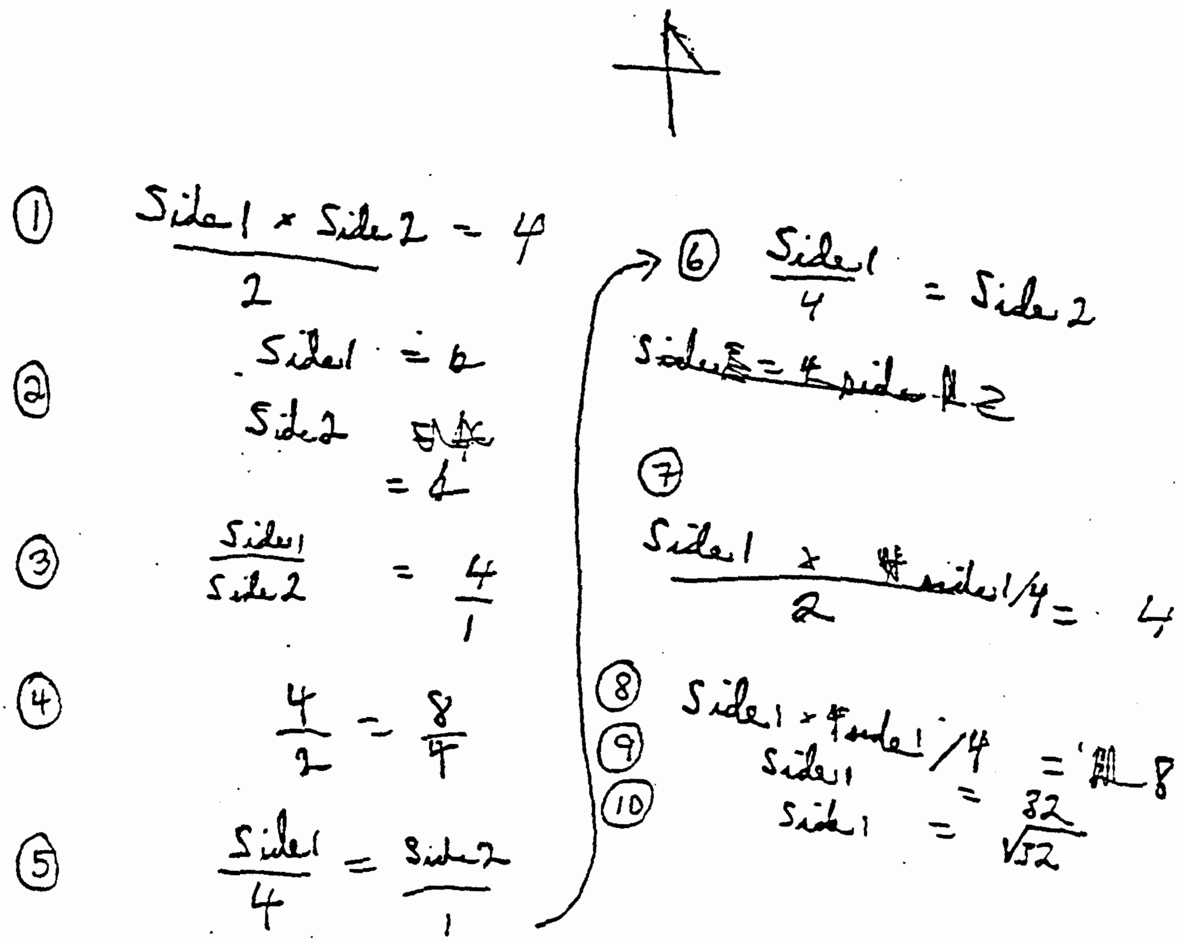


Figure 4.1 – Use of System of Equations

misconceptions), I consider Kurt's comfort in using variables and symbols and the ease with which he identifies relationships between quantities as being more important in describing his overall ability. This talent for seeing the mathematical structure in a problem will also be evident in other examples below.

Kurt has solved problems which involved rapid and broad generalization of mathematical objects, relations and operations. For example, the problem that motivated this case study (Figure 3.1, p.30) is an example of one of Heid's (1983) observations,

> It is not uncommon for a gifted student to solve a problem on its most general level, to generalize algorithms for solving whole catagories of problems of the type given, and then to neglect answering the particular question stated in the problem. (p. 223)

Kurt has the ability to curtail the process of reasoning. His work is usually very sparse, often not showing many of the operations required in reaching an answer. For example, when I taught problems with systems of equations in grade 10, I encouraged students to look for totals to suggest equations and variables. For example, if 5000 dollars was invested, partly at 9% and partly at 10.5% and the total interest earned was 483.75 dollars, the problem was to find the two amounts invested. It was suggested that students might proceed in such a way as to eventually produce the following equations:

$$x + y = 5000 \quad \text{(total amount invested)}$$

$$.105x + .09y = 483.75 \quad \text{(total interest earned)},$$

where x and y are the different amounts invested. However, when students were asked to try this very question (it had not been done as the example), within one or two minutes, Kurt asked me to come to his desk and asked if "this was ok":

$$(5000 - x)\,.09 = 483.75 - y$$

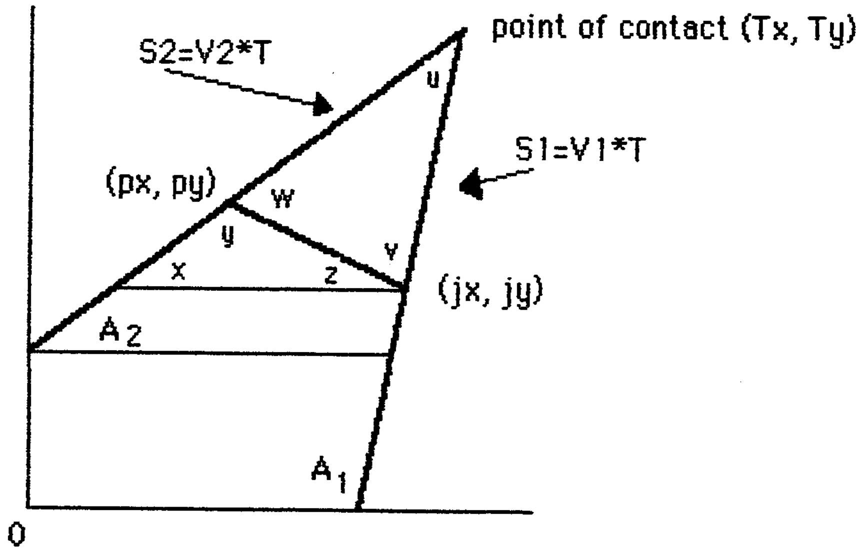
$$x(1.05) = y \quad \text{[Kurt's arithmetic error]}$$

Nothing else was written on his page. I assumed that Kurt defined (in his head) his variables such that y is the interest earned at 10.5% and x is the amount of money invested at 10.5%. Only one of these quantities (x) matches the quantities asked for in the original problem, but the other could later be calculated. I should have asked follow-up questions in order to determine if that was how he, in fact, defined his variables, and in order to determine how he would arrive at his final answer using these as his variables. However, since this took place in class, I didn't have the presence of mind, nor the time, to do this follow-up. Kurt's set of equations is what I would consider a unique system of equations, ones which had never occurred to me before I met him. Assuming that Kurt understood what he was doing (and I am very confident that he did), it is also an example of his considerable ability to describe mathematical relationships between quantities.

Another excellent example of his curtailment of reasoning in addition to his ability to write relationships is illustrated by some work which he recently (January 1992) submitted to me voluntarily. I have typed it in order to make it easier to read and so that I could make comments at various points. A copy of the original work is included in Appendix D (p. 136). After class on 2 December, Kurt mentioned that he would like to write a computer program that would involve a cannon shooting down a jet streaking across the screen (in a straight line). I was not immediately sure of the mathematics of motion that would be involved and suggested he talk to his physics and computer science teacher about it. He eventually brought up the problem again on 10 January, 1992 and showed me a discovery that he had made. What he had written on his page was the Law of Sines in trigonometry! I asked him how he figured it out and he replied that at first he experimented with ratios of the angles of a triangle to its sides and then he eventually tried some trigonometry. (It is not unusual for Kurt to experiment with trigonometry, as will become evident later in the chapter.) By the end of January he handed his work to me. I scanned it quickly and suggested that he first write a statement of the problem and define all his variables, since he handed me only a diagram and some mathematical equations related to it. Two days later he handed me the following:

> If I want to shoot down a jet at the co-ords (Tx, Ty) moving at the speed V_1 and at angle A_1, what angle (A_2) must I shoot my projectile at if I am at the co-ords (px, py) and the projectile travels at the speed of V_2 which is greater than V_1. T is the unknown time at which the collision takes place.

See Figure 4.2 on the following page for his work. Note here that in writing the formula for v (see **, Figure 4.2) Kurt bypasses several steps of algebra in using the sine law. I acknowledge the fact that he had time to work on this on his own time and might have included the missing steps somewhere else, but it is typical of him not to bother to write in steps of algebra, even when he has the time. He has also made errors. In the formula for v (** , Figure 4.2), note that V_2 and V_1 should be be switched. Also A_1 is given and A_2 was supposed to be the unknown,



$x = A_2$

$z = \mathrm{Tan}\left(\frac{py-jy}{px-jx}\right)$ [this should be arctan]

$y = 180 - (x + z)$

$w = 180 - y$

$T*V_2 = S_2$

$T*V_1 = S_1$ Therefore, $\frac{V_2}{V_1} = \frac{S_2}{S_1}$

** $v = \arcsin\left(\frac{\sin(w)}{V_2} V_1\right)$ by sine law

$A_1 = z + v$

Figure 4.2 – Kurt's Linear Projectile

whereas Kurt wrote his equations as if the opposite were true.

Kurt has also demonstrated flexibility of mental processes and strives for clarity, simplicity, economy and rationality of solutions. For example, in the fourth, fifth, and sixth interviews, Kurt was creating a program called David and Goliath in which a "rock" was launched in order to strike an object (Goliath) moving across the top of the computer screen. David's "sling" and "rock" were simulated by a line segment rotating in a circle at the bottom of the screen (see Figure 4.4, p.78). The Goliath that Kurt created moving across the top of the screen was originally square. Determining whether the projectile hit Goliath would have required at least two inequalities in order to test if the projectile was within the range of the maximum and minimum x and y coordinates of the square. Suddenly he had an insight:

> K: Oh, it would be interesting if I had a circular Goliath. (chuckles) I could use the pythagorean theorem and only make one equation.
>
> R: How's that?
>
> K: Um, well what I'd do is I'd take, I'd just put in location, the distance, your delta x I guess would be location x . . .location x minus um . . . gx and your y would be location y minus gy and once you've got that you've got your distance from Goliath and uh, when he's within a certain distance, he's going to be within a certain circle. vi13
>
> later:
>
> K: Maybe I should try that, it'll be easier than this. vi14

Kurt displayed a mathematical cast of mind; a mathematical perception of the world as shown in the following (and only) example from the data:

> R: You mentioned something to me at the end of the last session about one of the reasons you enjoy math; that you always try to see .... what was it that you said?
>
> K: How it relates to things in real life..... I, I, there are not a lot of useless math problems. There's lots of stuff you can apply mathematics to.... just, you know, it helps me think logically. They seem to be sort of intertwined, logic and mathematics. ii 3

Kurt's Mathematical Behavior

From the data presented thus far, I believe that it is easy to conclude that Kurt is both learning disabled and mathematically gifted. However, the major goal of the research was to explore and describe Kurt's mathematical behavior in order to draw inferences regarding his future learning activities. I tried to analyze the other five major categories (in addition to Characteristics of GLD Students and Mathematical Giftedness) that emerged from the data (Self-Efficacy/ Self-Concept, Attitude Towards Mathematics, Problem Solving Strategies, Mathematical Processes, and Factual Knowledge (Chapter 3, Figure 3.3, p. 47) with the objective of finding relationships or connections among them. This analysis led to the formation of the "causal network" (Miles and Huberman, 1984) shown in Figure 4.3 (p. 69). A brief overview of this model of Kurt's mathematical behavior is given below.

The prime motivating factor in Kurt's mathematical "life-world" is his interest in computing (represented by the rectangular component at the top) which began during his intermediate years (grades 5 and 6) in

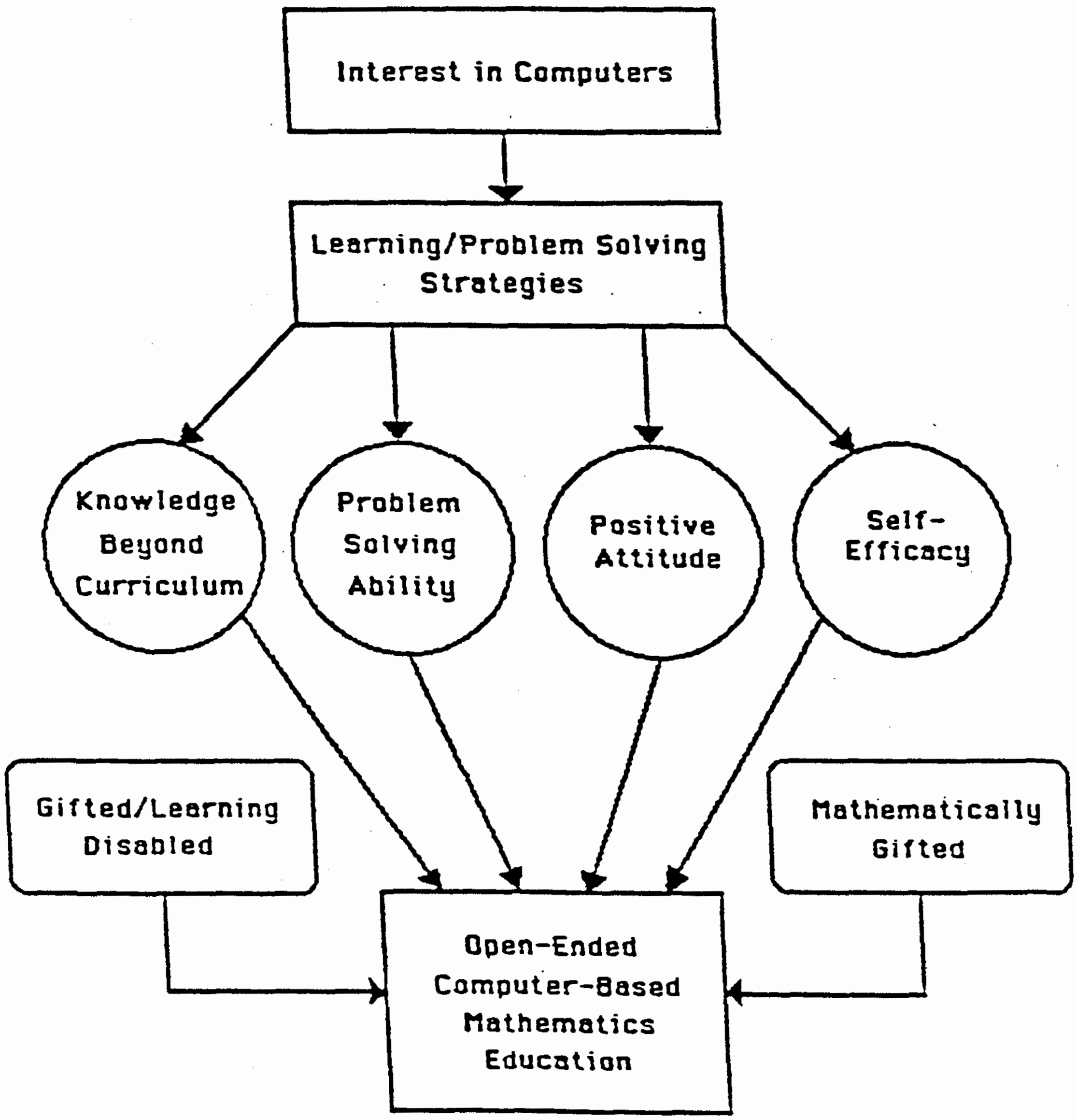


Figure 4.3 - Model of Kurt's Mathematical Behavior

elementary school. This interest has required him (and led him, as signified by the arrow) to develop learning and problem solving strategies (represented by the second rectangular component) in order to create designs and programs on the computer. The use of these strategies and his subsequent successful experiences on the computer have helped Kurt acquire (as signified by the arrows) several attributes (represented by the circular components). Firstly, he possesses mathematical knowledge, some of which is quite sophisticated and extends well beyond the required curriculum. Secondly, he has become a good problem solver. Thirdly, he has developed a very positive attitude toward mathematics, and finally, Kurt has acquired a greater sense of confidence and self-efficacy in mathematics when using a computer. These four traits, in addition to being gifted learning disabled with a particular gift in mathematics (represented by the rectangular components with "rounded" corners), lead me to believe (as signified by the arrows) that in order to be successful in school mathematics, Kurt must be offered an open-ended, computer-based mathematics education (represented by the rectangular component at the bottom) in which evaluation emphasis is placed on his special abilities . The above model (Figure 4.3) of Kurt's mathematical behavior will be explained in detail in the remainder of this chapter.

Kurt's <u>interest in computers</u> began when he was in grade five and was initially stimulated by his brother Ken, who was in high school at the time and who also spent a great deal of time on the computer. As he became more computer literate through the help of his brother, Kurt became interested in programming in order to create games and graphics designs. This in turn required the use of mathematics. For example, he explains,

> What I'm working on now is I wanted to make sort of like a little snake that would crawl along on the computer and you would rotate it and it would go in that direction and each little part of the snake that was following along would follow the snake. But, it wouldn't do exactly, it would kind of swerve because of momentum and stuff. I like doing that kind of thing. I've tried to figure out how I'm going to get that into the computer, how am I going to get it - what are the calculations I'm going to be doing to make it do that and that sort of thing. i 3-4.

In order to learn the required mathematics to write his computer programs, Kurt had to develop <u>learning and problem solving strategies.</u> These included primarily trial and error experimentation, the use of computer manuals and getting help from his brother Ken. Later on in high school he also received some help from the computer science teacher.

Kurt seemed to prefer learning things independently through the use of trial and error. This is illustrated in the following interview dialogue:

> R: If you had your choice would you prefer to work on your own to learn something or would you rather be told. For example, if you had to learn a new topic, would you prefer to look up the topic in a book or would you prefer to be shown?
>
> K: I'd really like to be able to do it on my own, but I may not necessarily get it. But, some of the ways I figured out, for example, how sine and cosine worked, the way I figured that out was my brother was making a program that drew a circle and I wanted to know how he did it. So, I looked into his program and I just looked at it and tried all sorts of little things with the parts that I thought were actually making the circle, kind of figuring out where it was. I finally figured out what were the parts, what were the main things that were making the circle,

making the x coordinates this, what y coordinates and I figured out how to do it and that was kind of fun.

R: And you did that all on your own?

K: Yeah and . . . I like doing that sort of thing.

R: Did your brother help you to figure out what the sine and cosine meant?

K: I kind of initially played around with it a bit and for the first few times I just took his program and would modify it a bit. I would change certain variables in there and I'd look and see what happened and a lot of times for a while there I was just kind of using that program to make other programs because I wasn't quite familiar with it. But after a while, after using it, I got to know it and I was comfortable and so now I could make up something from scratch. But in way he sort of introduced me to the idea because if he didn't make the program, I wouldn't have found it. i 4-5.

The following example again illustrates Kurt's use of trial and error in his learning. In the fourth interview, Kurt was explaining how part of one of his programs functioned:

R: Did Ken teach you about the sine and cosine graphs?

K: He made a little program - I asked him how to draw a circle and uh, he had it drawn point by point on the apple when I was in grade 4 or 5. I looked at it and uh . . . it took me quite a while but I figured out that what was making the thing rotate was the sine and cosine. So I experimented with that and learned how it worked.

R: And the fact that the sine never increases above 1 and stuff like that, you just figured that out by experimenting? (Kurt mumbles affirmatively) You didn't read up about it in a book at all?

K: No.

R: Ok.

K: Ok, that here I sort of have it named after the parts of a ship, portx um, that's the left side, and um . . .so . . . oh, I think um . . . oh yeah, as you'll notice with the portx and the starx and the stary um, . . . instead of adding stuff to the tilt, instead all I did was I put a minus and a plus there, to do the perpendicular thing, you know, that we're doing now.

R: With perpendicular slope?

K: Yeah.

R: So where did you learn that? Did you pick that up just from experimenting too?

K: Yeah, I picked that up in . . . grade 6 or something or other, I happened to be fiddling with a program where I wanted the thing, instead of, I wanted it to rotate 90 degrees and I wasn't sure how to do it and somehow I stumbled upon it. (chuckles)

R: From experimenting? Good for you.

K: I think it was totally by accident, because I accidently put a plus or something where I should have put a minus and I thought, oh hey, that's interesting, the thing's turning funny (chuckles). So I checked that out and (mumbles something inaudible). Anyway, this here, uh, just make sure you don't get confused. iv 10-11.

At the end of the first interview, I gave Kurt a non-curricular cubic equation to solve and he tried techniques related to the solving of linear equations. When these did not work out, he became a little frustrated:

K: Uh, I have to divide everything by 5 and divide everything by 2 and divide everything by 3 but I was only dividing one of them by 5 and one of them by 2 and one of them by 3. . . . . . (19 sec later) I can't get . . [inaudible]

R: That's ok. Suppose you were allowed to use the computer. Do you have any ideas on how to solve the equation using a computer?

K: . . . . I might do several things uh, I might try replacing

> everything with variables or repacing everything with numbers.
>
> R: When you say replacing with numbers, what do you mean?
>
> K: Like uh, I'd, well, it wouldn't be with the computer actually, it probably, I might take everything, replace a number, see – then work it out, see . . . (inaudible) . . . there's no unknown and then see what kind of answer I get and sometimes that helps me see how to solve the problem. On the computer, if I replace it with variables sometimes it helps me look at everything in relation to everything else like so I can see a little more clearly sometimes when uh, . . . what, sort of like uh, when you have a number and you want to express it in terms of another uh, when I put it with variables sometimes I can look at it and it gives me a better idea of uh, . . . what it is, uh, what it is the problem actually means. i 11.

Thus it is easy to conclude that trial and error is a major learning strategy for Kurt. His brother Ken confirms this:

> R: I was asking Kurt how he learned trigonometry and he mentioned to me that you wrote him a program that drew a circle and then that's basically how he learned. Did you formally teach him trigonometry?
>
> Ken: I think a few times I've sat down with him on different occasions as far as that program goes. On a number of different occasions that's happened with trigonometry and everything.
>
> R: So you would write a program and then sit down with him and explain how it worked?
>
> Ken: Um, ya. Sometimes he just, he would get a very bare bones explanation from me and then he'd just experiment with it himself.
>
> R: Yes, that kind of leads to my next question. How independent would you say he is in learning his math and computer skills? Do you give him a lot of detailed help or just . . . .
>
> Ken: No, I don't give him much detailed help.

R: He just fiddles around with the programs?

Ken: Ya and I assume he's doing a lot of investigations himself. Just recently he asked me this question. He wanted to find out, he had these little programs with little dots moving around on the screen, and now that we have a better computer and a better language, he'll have like a little spaceship moving around or a little man tracking another person and uh he was asking me how to, how he would get a missile to hit an object in the sky. And so I was trying to show him the way of investigation to figure out how to hit this missile and he argued with me and I said it would take the quadratic formula. He didn't know the quadratic formula and I think it would be impossible for him to find out how the missile hit the object without having the quadratic formula. So he's since been interested in having me teach him that. x 2

Kurt also makes use of manuals to learn some of the mathematics that he needs. A good example of this follows:

R: How did you first discover the absolute value function? Was it part of your .... programming ... Did you look at the programming manual and look at the different functions to see what they do?

K: Yeah, every once in a while um, when I was - like this is in BASIC too, absolute value, so every once in a while I would flip through the thing and ... it's not a big coincidence that uh, I happen to get absolute value because it's one of the first ones, so (chuckles). vii 3.

Kurt easily remembered techniques that he had utilized in the past and frequently thought about how he could use them in new situations. The following example illustrates this point:

R: If I gave you a problem, is the first thing you might think of is to how to use the computer? Is that how you would try to solve the problem?

K: Um, actually a lot of time I'll just do it on paper and pencil. But uh, ... sometimes in a way I'll figure out how to do those problems because I thought, oh hey, I did something like that on computer and I worked it out this way. i6

Over the years, in order to create his programs, Kurt has learned a great deal of quite sophisticated mathematical knowledge which was beyond the curriculum he received in school. Especially impressive is his use of trigonometry, particularly polar coordinates (although he does not know it by that name) in order to move objects in different directions on the computer screen. His explanations show that his understanding of the trigonometric functions in his programs is unclear. However, he possesses "tacit" knowledge, that is, he knows how to use polar coordinates and adapt them to new situations (such as the program created during the interviews), but cannot really explain how or why. The following dialogue shows that his understanding of trigonometry is unclear:

> R: Do you know the connection between the sine and the cosine and the circle, the properties of sine and cosine that give you the circle?
>
> K: I'm not sure. (laughs) I don't even know if I know what I'm doing but I can make it rotate and I know that, . . . I have a rough idea of how it's going to work, like if I put a number into sine and cosine, I know that if I increase the number by a certain amount it will go like that-
>
> R: Back and forth?
>
> K: Yeah and uh, that sine is sort of the same thing but a little bit different. I don't know exactly how sine is different from cosine.
>
> R: Do you mean that as you increase one you are decreasing the other, is that what you're saying?
>
> K: Well, no, I increase both of them and then . . . (laughs) I don't know, it's hard to explain. I could probably learn a little bit more about sine and cosine um, because I'm not sure exactly what the difference is between the two. I've tried looking in physics

books and tried to look around in every mathematics book I could find and I still didn't get a very good idea. Then after a while I just kind of forgot about it and just . . .[inaudible]. i 5-6.

David and Goliath

In creating his program called David and Goliath, Kurt needed to make an object spin around in a circle. He did this by spinning a line (representing the "sling"). The point at the end of the line represented the "rock" that would be released in order to hit the object (Goliath) moving across the screen (see Figure 4.4 on the following page). In order to rotate this point he had to have the computer pre-calculate 32 positions around the unit circle. (In refining his program after all the interviews were over, he changed this to 128 positions.) His explanation of what he was doing totally confused me at the time we were sitting at the computer for the interview and it was not until I had the chance to study the program myself, that I understood what he was doing. The following excerpt is his explanation on how he gets the object to rotate:

R: Sine, cosine array ,0 point, point, 0 decimal, decimal 15.

K: Ok, uh, this here, that's uh the lowest number you can put in the array (inaudible) and this is the highest number you can put in there. So it'll be 0, 1, 2, 3, 4, 5, any number from 0 to 15. [He later changes this to 0 to 31]

R: Just integers or any number in between?

K: Uh, no just integers, but uh, the number itself, you can input a real number as (inaudible) uh, because . . . uh, oh, well it's sort of like you have um, . . . you don't just have sine and cosine, you have sine 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15.

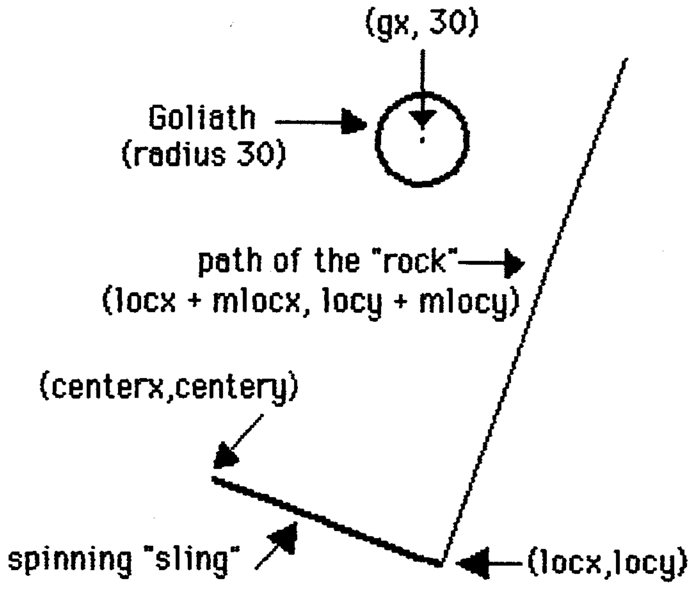


Figure 4.4 – Screen Image of "David and Goliath"

R: You mean the sine of 1?

K: No, it's just the first um, it's the first –

R: Oh, because you've got 16 positions, is that what you're–

K: Yeah, so I save the position, the first position as sine 1, the second one as 2 and so on.

R: And then down this way would be?

K: And then cosine I do the same thing for.

R: So then you've got a position like cosine1, sine3 or something like that, and the 1 and 3 you substitute numbers for, is that right? Or does that have something to do with the different positions of turning it?

K: Well, it's sort of like uh . . . (inaudible)

R: Would it be easier to write it down, what it does?

K: Um, . . . see, I put in a number, tilt, and that is going to be um, that's, that tilt can equal anything from 0 to 15 and that'll give it the . . . .um. . . that's where it will save the position . . .um . . .so , then because those are not like preset in there, it'll, it'll calculate and it will say sine, this particular sine will equal and give it the position (inaudible)

R: I'm not sure I ah . . .

K: (something inaudible) You have sine . . . . . 2 and uh, as of yet they don't mean anything.

R: So that 1 and 2 are actually variables?

K: Yeah. They're, they're individual variables uh, . . . and what it does is, at the beginning they are just equal to 0 and they're not useful until I put something into them. So, I make a calculation and for . . . for the first sine, which will be like tilt will be equal to 0 the first time uh, it puts it into here-

R: It puts 0 into here for tilt.

K: Ya, and it times it by pi divided by 16, so that'll give it a position there and so on. Uh, . . . and that way it's able to calculate one of the positions and then it puts it, it stores it in this variable. That way I can call it up without calculating it each time.

R: So it does them all ahead of time.

K: Yeah.

R: Does that give you coordinates, x and y coordinates, sort of?

K: Yeah. And so like uh, by the time I get to uh . . . v 3-4.

Later:

R: Now, what would be the idea here if you've got something moving around? How do you get that thing moving around, using your sine and cosine?

K: Uh yeah, I just add to the tilt each time and um . . . when it gets -

R: But, doesn't that just rotate the object, how do you get it actually moving across or down or something?

K: Oh, you mean when I release it?

R: No, I'm going to get to that. Does that sine[tilt] stuff move it from one place to another here?

K: Well, no that, that'll be just like a fixed position. I'll just move it from fixed position to fixed position.

R: Do you determine those fixed positions to begin with already?

K: Well yeah, they've been determined up there uh, so all I have to do is say tilt = tilt plus or minus depending on which key I press here-

R: The sine and cosine array give you 16 positions to begin with already, around the circle?

K: Well no, they don't give me 16 positions, I calculate them down here and then um, once I've got them stored, I can move from one to the other, just as long as when I pass 0 and 15, I reset it back to (inaudible).

R: Why did you go from 0 to 15? Could you make that more?

K: Uh, yeah.

R: But that's because you've got 16 positions, is that why?

K: mmm hmm.

R: So if you wanted to make 32 positions, you would have to change this too?

K: I think I was going - wait a second - mind if I just look at - something looks wrong there, cause I don't know if that should be 31. Uh, no it shouldn't be 31. . . Oh, cause I was going to change this. Ya, because 15 isn't very many positions for this. (makes a change to program line) Yeah, that's kind of funny, (chuckles) thought it looked a little wierd.

R: So that gives you 32 positions around the circle.

K: mmm hmm. v 6-7.

What he has done is create a variable called tilt, which takes on values from 0 to 31 corresponding to 32 positions around a circle. The program then calculates cosine(pi/16*tilt) and sine(pi/16*tilt) for the various values of tilt and stores them in arrays called cosine[tilt], sine[tilt] respectively. This gives the x and y coordinates of 32 positions around the unit circle. He then used polar coordinates to calculate the position of the object on the outside of the circle:

locx = cos[tilt]*radius

locy = sin[tilt]*radius

R: Any time you need to write stuff, just let me know here because I've got a paper and pen.

K: I never usually do have to write stuff. That worries me because most programmers do (laughs). My brother says it would help my organization but um . . . Ok . . .(types stuff - types in locx: cos[tilt] *radius)

R: Now what does that do? Locx? x-coordinate?

K: That'll be the location of uh, the little object flying around.

R: That's - oh, I see ok. cos[tilt] times radius, that's the . . .

K: How far out it's going to be. v 13.

It was interesting to observe that Kurt wrote virtually nothing down on paper in the three sessions that he worked on this program and then he did so only when I needed clarification. I got a sense that Kurt was very proud that he could write a program without planning it first on paper. In my opinion, it also seems to be a sign of giftedness that he can keep track of so many things in his mind.

Kurt later recognised that the coordinates above were in error because the origin on the computer screen is in the top left corner:

> R: Oh, so you changed these now, right? You added something, you went locx . . . Does the colon mean an equal to?
>
> K: Well, when you're working with certain variables, you have to put a colon in front of the equal sign.
>
> R: Centerx, that's where the center is, plus . . . .cos . . . .that put's it on the outside of the circle, is that right? Is that what this does?
>
> K: What do you mean, well yeah, it figures out where it is on the circle.
>
> R: Where it is on the circle. Ok, that's why you had to know the center first, right?
>
> K: mmm hmm, . . . otherwise it would have been determining it from here (chuckles). [He points to the top left corner of the screen.]
>
> R: Yeah, that's right because this radius here depends on where the center is, right? Is that why?
>
> K: Uh, it, it would, it would just be like if I - changing the center would mean that, if I didn't have centerx there, it would be rotating around zero. [He means the origin, top left corner of the screen]
>
> R: Ok, that's right, that's right. v 16.

He therefore used transformational geometry in order to relocate the circle somewhere in the lower middle of the screen. He assigned specific coordinates to the center of the circle and wrote the following formulae to represent the position of the "rock":

locx = centerx + cosine[tilt]*radius

locy = centery + sine[tilt]*radius

Kurt knew that in order to launch the "rock", he had to release it in a path that was perpendicular to the radius (sling) of the circle. This is evident in the following dialogue that took place when the initial planning for the program took place:

> K: I thought like, what I thought was, this would be the point, and it would be changing the slope all the time, like from there to there, and then to shoot the thing I'd just take this -
>
> R: Oh, and fire that off at it.
>
> K: I'd just uh, I wouldn't fire in that direction, but I'd take, I'd know that was our slope, right? And then I would just take something perpendicular to it.
>
> R: Oh, to throw it out.
>
> K: Yeah.
>
> R: Why perpendicular?
>
> K: Uh, well because it's not going to fire out that way [points in outward direction]. It's going to be moving, . . . because the rock is moving that way, so-
>
> R: So if you let it go here, it's going to be perpendicular to this line here [the radius-sling]?
>
> K: Yeah.
>
> R: Do you know that just from intuition or did you know that already from geometry?
>
> K: Well . . .
>
> R: You just have a feeling for it, is that right?
>
> K: Yeah (chuckles). iv 18.

In the next interview, he explained how he would actually launch the rock:

> R: Alright, when it gets to the point of release, when this thing is coming around here, what will be the mathematics of that?
>
> K: Ok, I'll take uh, the ... position.
>
> R: The position from where you're releasing it?
>
> K: mmm hmm. Uh, then I'll take, I'll take it's slope, well, I'll take the reciprocal of it, the negative reciprocal like what we were doing in math (chuckles). v 8. [He is referring to the negative reciprocal of the slope of the radius (sling).]

A few moments later:

> R: Ok, so then you take the negative reciprocal .... of the slope. . .. of that thing there? Now what? How does it actually shoot up to some ... it has to go in that direction, right?
>
> K: Yeah, then I'd times it by the speed, to make sure it goes in the right direction.
>
> R: Multiplied by what speed?
>
> K: Uh, at which it's moving from position to position. So like if it's moving at 1, it's going to change position - each time it'll be at a new position, but say it's moving at .25, each 4 times it'll be at a new position but it will appear like (inaudible)
>
> R: Ok, so at this point, in order to get it moving along at the speed that it was rotating, right, is that what you're saying you'd have to multiply .... what I'm trying to figure out is what the mathematics is, how you're going to get this thing to move from here to here.
>
> K: Ok, uh...
>
> R: From this point here to that line there.
>
> K: Well, once I've calculated all that stuff, I'll store it in a variable, and I'll just each time, I'll, uh .... I'll store it in two variables, I'll store the x and the y and then um ...

R: The x and y of this position here when you let it go, ok and then you increase - then what do you do?

K: And then I just forget all this circle stuff and I'll go to a new routine where it'll, it'll add to this position, the x and y each time and eventually . . .

R: And how do you calculate those new positions, based on what?

K: Based on (inaudible)

R: Based on the slope.

K: Yeah, just keep on adding to it and then eventually it'll find out whether or not it hits that. Um, I guess I could calculate before hand . . . you wouldn't get the fun of seeing it miss it (chuckles). v 9-11.

Therefore, in order to create the launch, he defined new coordinates which would move in a perpendicular direction to the turning radius of the rock by using the following formulae:

mlocx = -sine[tilt]*speed

mlocy = cosine[tilt]*speed,

where speed is the speed of rotation of the rock and controls how fast the rock travels toward its target. Recall that the position of the spinning rock was defined as:

locx = centerx + cosine[tilt]*radius

locy = centery + sine[tilt]*radius.

Therefore, I can see that Kurt understands that (cosx, sinx) and (-sinx, cosx) define perpendicular vectors (although I never used vector terminology with him). When I asked him to give the coordinates of a point that is a 90 degree rotation of the point (x,y), he responded with (-y,x).

In order to make the rock move along a straight path from launching, Kurt again used transformational (translations) geometry:

> R: And what moves it from one location to the next?
>
> K: That's what I'm going to do in procedure collision. I'll say uh, locx = locx + mlocx. vi 7-8.

The other formula is locy = locy + mlocy. He has these in a subroutine which keeps adding equal increments to the position of the rock and thus makes it move in a straight line.

A final example of Kurt's use of personal computer projects to increase his mathematical knowledge is his discovery of the Law of Sines referred to earlier in this chapter (p. 65).

In summary, Kurt has made use of some quite sophisticated mathematical knowledge in the area of trigonometry (such as polar coordinates) and in the area of transformational geometry required to rotate objects 90 degrees and to move objects along straight line paths. A complete copy of the program David and Goliath is included in Appendix E (p. 137). The program in the appendix is a slightly more refined version of David and Goliath than the one described in the interview text, since Kurt has since worked out some of the "bugs". However, the mathematics that he used is still the same as in the original draft of the program.

In order to test the location of an object within certain boundaries, he has made use of the Pythagorean Theorem (in the form of the distance formula) and the absolute value function, both of which he understands clearly. For example, in order to test to see if the "rock" collides with Goliath in the program discussed above, Kurt used the following inequality:

If $30 > \sqrt{(\text{locx} - \text{gx})^2 + (\text{locy} - 30)^2}$ then . . . .

Goliath is a circle with radius 30 and center (gx, 30). Thus, if the distance between the "rock" and the center of the circle is greater than 30, the "rock" will keep moving. If the distance is less than 30, collision takes place and the program outputs "you hit!".

In order to test to see if the "rock" leaves the screen, Kurt uses the following inequalities:

$|320 - locx| > 320$

$|240 - locy| > 240$,

where the screen has a width of 640 and a height of 480. As he was writing these program lines I began to probe him for his understanding of absolute value:

R: So, where do we begin?

K: Well now I'm checking on whether it leaves the screen.

R: You're checking to see if it leaves the screen. Now you said something to me a minute ago that you found another way to make your decisions?

K: Yeah, I know when we were going to make this a box, I was going to check it four times but I figured out a way of not uh, of doing that with, without doing that.

R: But didn't you change it to a circle?

K: Ya, ya, but like this screen would be similar cause it's just one big box.

R: Oh, I see.

K: So I was just going to do it the way I was going to do it over there uh ... but I figured out another way of doing it uh ..

R: To determine if it goes off the screen.

K: With only, only checking twice.

R: And how is that?

K: What it does is it - that's uh 320 is the middle of the screen along here and locationx is where the object is along here and uh, . . . it takes the center of the screen minus locationx and it changes that into an absolute value.

R: Which is what, what's an absolute value?

K: It's, it's positive.

R: Do you know what it is though? Do you know what it does for you?

K: Well uh, I don't know, I just consider it as getting rid of all the signs (chuckles).

R: Getting rid of the negatives.

K: Yeah. So no matter what it is, it changes it to a positive, so I can work with it.

R: So what is it in fact giving you then if it changes it to a positive?

K: I guess the difference between two things whether or not it's on this side or on that side. And that's what this is doing so, um if it's 320 past this point, which would be if it's greater than 320 past this point uh, then it'll know that it's off the screen on that side, um, if it's less than then it'll be off the screen on that side and either way it'll tell me that it's greater than 320.

R: Ok, so how did you figure that out, were you just thinking about it one day or?

K: Um, it just popped in my head just now.

R: How did you know to use the absolute value function in your program? Have you used it before?

K: Yeah, I've used it once or twice before.

R: To do the same sort of thing?

K: Well no, to do something (inaudible), well actually I think I have (inaudible) before. vii 1-3.

The above dialogue reveals that Kurt has a good understanding of absolute value and also shows that he was in search of the shortest, most efficient way of solving a problem.

Besides the new knowledge that Kurt has aquired through his use of the computer, he has become a skillful problem solver, often easily recognizes relationships between quantities, and is comfortable working with symbolism in mathematics. The writing of the program David and Goliath in itself shows that Kurt is a good problem solver. Other examples of his problem solving ability were already discussed earlier in the chapter in relation to his mathematical giftedness. Of course, programming a computer requires frequent definition of variables and their expression in terms of dependent variables. Kurt explained to me after our fifth session (not on tape but noted in my journal) that he thinks he is a good problem solver because of his work with computers. He went on to say that in order to create his programs, he needs to create equations and relationships.

Kurt's computing hobby has also led to a positive attitude toward mathematics as exemplified by the following exchange:

R: Suppose we take the computer out of it completely?

K: It would get a little more boring. (laughs)

R: You'd find it a lot more boring, eh?

K: Uh, yeah. Uh, I'd probably still like it though. It just uh, it just makes it a lot more interesting. I find it even easier to understand when I'm working with it on the computer. i 6.

He prefers challenging and unique problems:

K: I kind of like uh, . . . what I do on the computer a lot - I'll take an idea and I'll have it in my head but if it's all in written words and pictures and uh, . . . I'll put it into a mathematics problem.

R: Something's in words and pictures? -(Kurt interrupts)

K: Yeah in my head and then I'll, I'll, figure out how I can get the computer to do it or, . . .how I uh, . . . solving like a practical problem and putting it into numbers and algebra. i1

and later,

R: If you had your choice, what sorts of activities or topics would you like to do in learning mathematics?

K: Definitely computers, uh, . . .

R: Can we get any more specific? You don't have a huge repertoire or background in mathematics to pick from, but if I said to you, "Ok Kurt, we've got 3 months of time in the mathematics classroom and I'd like you to be involved in your choice of what we could study", what types of things would you like to do?

K: I know what I'd like to do but when it comes to saying it (laughs) . . .

R: You are finding it hard to express what you'd like to do?

K: Yes.

R: Just throw out any word, you don't have to express yourself in a full sentence, just throw out any idea.

K: Like uh, I know my favorite part when I'm working with computers, . . . like when i'm making a program or something, I'll be taking . . . . [inaudible] . . . I just love really interesting and complicated problems to do something different than what I've been doing before. I like to try new things. I like using sine and cosine a lot. i 3-4.

On the other hand he dislikes mundane things like arithmetic and must see some challenge or worth in what he's doing:

> R: Like for instance, is there anything you've done in mathematics that perhaps your teacher has given you and you think "Oh God, this is boring" or "I wish I didn't have to do this", something that just doesn't appeal to you?
>
> K: When it's stuff maybe I already know really well that I can almost get the answer to . . . [inaudible] . . . challenge.
>
> R: So you like the challenge.
>
> K: Yeah. i 2.

In the next interview:

> K: Uh, . . . Okay, uh, . . . If I could, I wouldn't deal with the Arithmetic at all. (chuckle) Well, not, not completely, of course-
>
> R: You mean calculations, and stuff like that?
>
> K: Yeah, uh, I prefer Algebra, stuff like that. ii 1.

The results of an attitude survey (see Appendix B , p.119) given to Kurt at the start of the study, as compared to the means for grade 8 and 12 students in British Columbia, are summarized in Table 4.2 on the following page. Kurt's mean scores are considerably higher than the mean scores for grades 8 and 12 students on all scales tested, but particularly so on the scale testing his attitude toward computers and calculators. These statistics reinforce the fact that he has a positive attitude towards mathematics, especially when used in combination with computers. The survey was adapted from a survey developed by Robitaille, O'Shea and Dirks (1982).

Table 4.2 - Results of Attitude Survey

Scale	Gr. 8 Mean	r *	Gr. 12 Mean	r*	Kurt's Mean
Math as a Process	3.05	.78	3.13	.72	3.6
Math and Myself	3.31	.90	3.51	.92	3.8
Math in Society	3.43	.81	3.43	.88	4.0
Calculators/ Computers	3.06	-	3.45	-	4.5

* **Hoyt's Reliability Coefficient**

Kurt is comfortable with his mathematical abilities and yet cannot articulate or understand why he has not been a high acheiver (as previously discussed in this chapter, p. 54). For example,

R: How do you feel about your abilities in Mathematics?

K: Uh. I'm not sure, uh. . . . . . . I can't be too bad because I have a lot of fun working with computers and uh, I use a lot of Algebra when I'm . . .

. . . Uh, . . . I don't know. I find it hard to look at that question from an objective point of view because I'd like to be good at Mathematics, but I don't know if I am. I, I think it's important to be good at Mathematics. ii 2.

However, his <u>sense of self-efficacy</u> seems to be dependent on his experiences on the computer. For example, during the second interview, in trying to determine an expression for the sum of the first n odd integers, Kurt was having difficulty and suddenly said "this in computers, I'd have an easy way out of this" and chuckled. Unfortunately I did not have the presence of mind to explore this statement further. Kurt's greater sense of self-efficacy in mathematics in using a computer may be due firstly to the fact that the computer gives him feedback on his errors, which he can then immediately correct and secondly, to the fact that it does not make arithmetic errors. For example, during the seventh interview, I was asking him something about the error messages:

> R: Oh you mean it sends you to the cursor? The error message? Does it send the cursor to where the mistake was?
>
> K: Yeah.
>
> R: Oh, that's good. Oh, yes I see.
>
> K: It's a great way to solve all my problems. (chuckles) vii 8.

In contrast with Kurt's sense of comfort while working on the computer is the sense of pressure he feels when doing written work or writing tests. The following sequences illustrate these points:

> R: If I gave you a problem, is the first thing you might think of is how to use the computer? Is that how you would try to solve the problem?
>
> K: Um, actually a lot of time I'll just do it on paper and pencil. But uh, . . . sometimes in a way I'll figure out how to do those problems because I thought, oh hey, I did something like that on computer and I worked it out this way. It's kind of funny, when I see it on a piece of paper it seems different from seeing it on the computer even though it's basically the same. Like . . . I, I put

my brackets here and there and I make sure that everything is arranged accordingly so that uh, like I pay attention to all the rules that I normally do when I'm, when I'm doing stuff on paper. But in a way it's different. It could be just that **I feel more comfortable on the computer, like there's not the pressure there because uh, . . . I don't know, like I use the pythagorean theorem sometimes um, to calculate distance a lot on the computer but sometimes I can still boo boo when I'm, when I'm doing it on paper whereas I never make that mistake on the computer.**

R: Right, so why is that?

K: I don't know, maybe it's just that I feel pressure.

R: When you say "boo boo" , what do you mean? The calculation?

K: Well, on the computer, first of all I'll always write out the pythagorean theorem right and it can get very complicated on the computer. But most of the time uh, sometimes the problem will be a couple of lines and I'll have a bracket here and a bracket there and that will be the end of the pythagorean theorem (laughs) and inside there will be all sorts of individual little problems that will calculate each little thing. I could probably make it more simpler I guess but, sort of like uh, say you have the length here and the width there and uh, instead what I have for the length I have such and such minus this and what not times and then I can get a lot of, I can get a lot of variables stuffed into there and I still don't make a mistake as much as if I were doing it on paper. **Maybe I just feel more comfortable. . . . and in a way it's better on the computer because if I make a mistake, I know it's easier to figure out where I went wrong because the computer will still do the program right, it will still go through the program.** But, it will uh, because it'll usually go through the program several times and like in less than a second, I get to see what's going on and I see how the numbers progress, usually on the screen with graphics and stuff. So I can see, oh I bet I know what happened, I probably put a minus here where I should have put a plus or I times it and I should have divided it or I didn't square it over here when I should have and I can kind of see what's going wrong. i 6-7.

With regard to his feeling about the chances of success on written math tests, Kurt explains:

> Uh, . . . mediocre, I don't have a lot of confidence when I'm under pressure. i 2.

So it seems clear that traditional written work and tests create pressure for Kurt, which lead to a lack of confidence and a lower sense of self-efficacy which in turn might have led to lack of success. On the other hand, in working with the computer, in addition to the enjoyment he receives, he develops a greater self-confidence which enables him to freely and creatively experiment with his ideas.

I also easily concluded that Kurt is highly self-motivated in the area of computers. Kurt confirms this in the following excerpt:

> K: Usually I never write instructions (chuckles) in my programs.
>
> R: Because you are the only one using them?
>
> K: Most of the time I write them for myself and once in a while someone in the computer room will play with it. vii4

Initial Recommendations For Kurt

The fact that Kurt is a self-motivated, independent learner in the area of mathematics and computing, together with his sophisticated level of knowledge for his age, his strong problem solving skills, his positive attitude and sense of confidence and self-efficacy in mathematics when using the computer, lead to the conclusion that Kurt's needs are not being served by the present school system. Kurt's special education teacher explains:

> I think he would benefit from a very open-ended kind of education where he can make choices about what kind of project he would like to do and how he would go about gathering information in a way that he can work with. There are a variety of ways that he could do that, whereas I don't think the traditional style um ... simply taking notes and writing reports um, using one textbook source, that kind of thing, is what motivates him. He's a very creative kind of thinker and the options aren't necessarily there for him, and haven't been there for him in most areas. ix 9.

After analyzing all the data in the summer following the interviews, I concluded that Kurt should be offered an open-ended, computer-based mathematics education. I felt that I, as a caring, hardworking teacher familiar with, and sensitive to, his special needs, could integrate such an education into the regular curriculum for Kurt. He would still need to be taught the required curriculum as preparation for provincial examinations and therefore, he would still be required to write all the same tests as other students in his class. However, my evaluation of Kurt would place much greater emphasis on a modified homework program. His daily homework could include a choice of one from several problems or investigations related to the curriculum and which, if possible, would require the use of the computer or a graphics calculator that would be made available to him. In addition, he would be required to complete some mathematics projects which would require a greater amount of time to complete, perhaps weeks or months.

Implementation of Initial Recommendations

By the end of September, 1991, Kurt (now in grade 11) and I had agreed that he would have the opportunity to do optional assignments and that the regular homework assignments would be de-emphasized. We did not discuss the number of assignments that would be completed, only that each time he handed one in, a new one would be assigned. I also asked him to make sure that he completed at least three or four questions from the class homework in order to test his understanding of the concepts covered in class. The optional assignments were to make up 50% of his final mark although I put no pressure on him regarding deadlines. My evaluation of his work was rather subjective and I tried to take an holistic approach. For example, I assigned marks to things like statement of the problem, understanding of the problem, clarity of presentation, use of symbols and variables and creation of relationships. However, I was not completely consistent, primarily because I was learning as I went along and did not always have clear in my mind what I wanted to evaluate as I assigned some work. What was important to me was that Kurt would be motivated and interested in his school mathematics and at the same time, improve his grades.

Throughout the first term (which lasted until mid November) Kurt had completed only four optional assignments. I considered this to be a rather low output, because none of the assignments was very lengthy. Many of the reasons for his low output have already been discussed earlier. Others will be discussed below. Of the four completed assignments in the first term , I will present two of the more interesting examples.

The first assignment was a set of three problems from the enrichment sections of his textbook, from which he was to choose one. He chose the following investigation and his solution follows:

If n is a perfect square, what is the next perfect square?

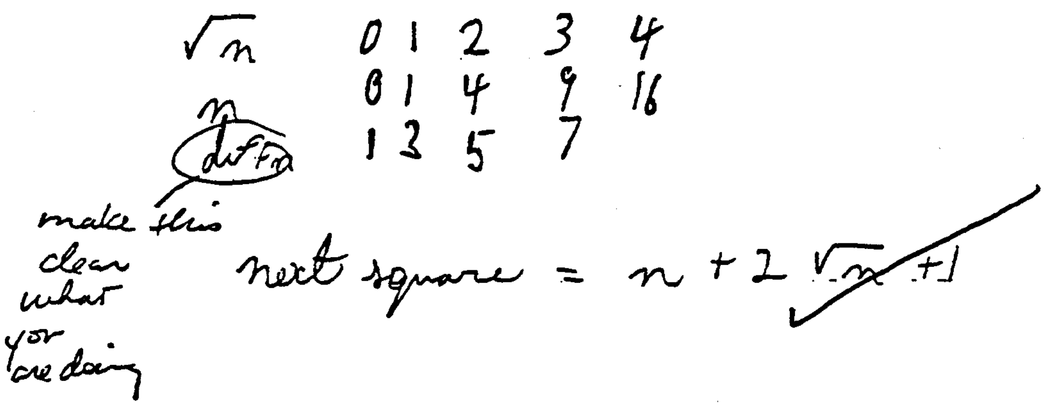


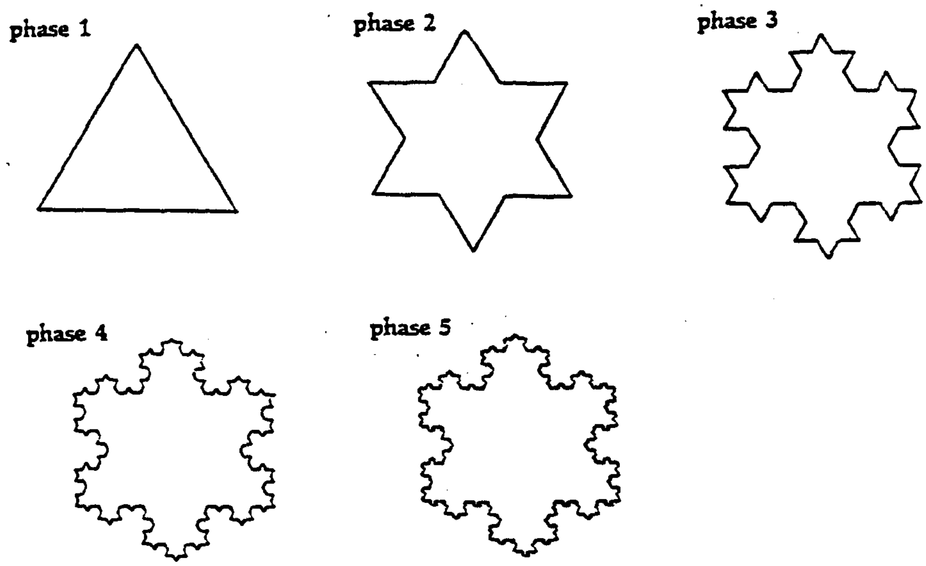
Figure 4.5 - Perfect Square Investigation

The work above is another example of Kurt's ability to see relationships and make generalizations.

On 17 October I presented him with the following problem suggested by Johnson (1991):

> The Koch "Snowflake" is made by taking an equilateral triangle and adding, at the middle of each side, an equilateral triangle that is one third the perimeter of the original triangle; and so on. (p. 63)

a) Investigate how each successive perimeter of the Koch Snowflake relates to the previous one and then write a program that will print out the perimeters of the first 45 phases using a phase 1 perimeter of 27. Based on your results, what happens to the perimeter of the Koch snowflake as you keep increasing the number of phases indefinitely?



b) In a similar way, investigate the successive areas of the Koch Snowflake and write a program to print out the areas of the first 45 phases. What happens to the area as the number of phases is increased indefinitely?

He immediately read the problem and remarked, "Oh, I'm sure I'll have fun with this one". The very next class he brought in the work shown in Figure 4.6 on the following page.

Notice that Kurt uses a general side length rather than the specific length of 9 as suggested by the problem. Furthermore, instead of just finding how each successive perimeter relates to the previous one, Kurt

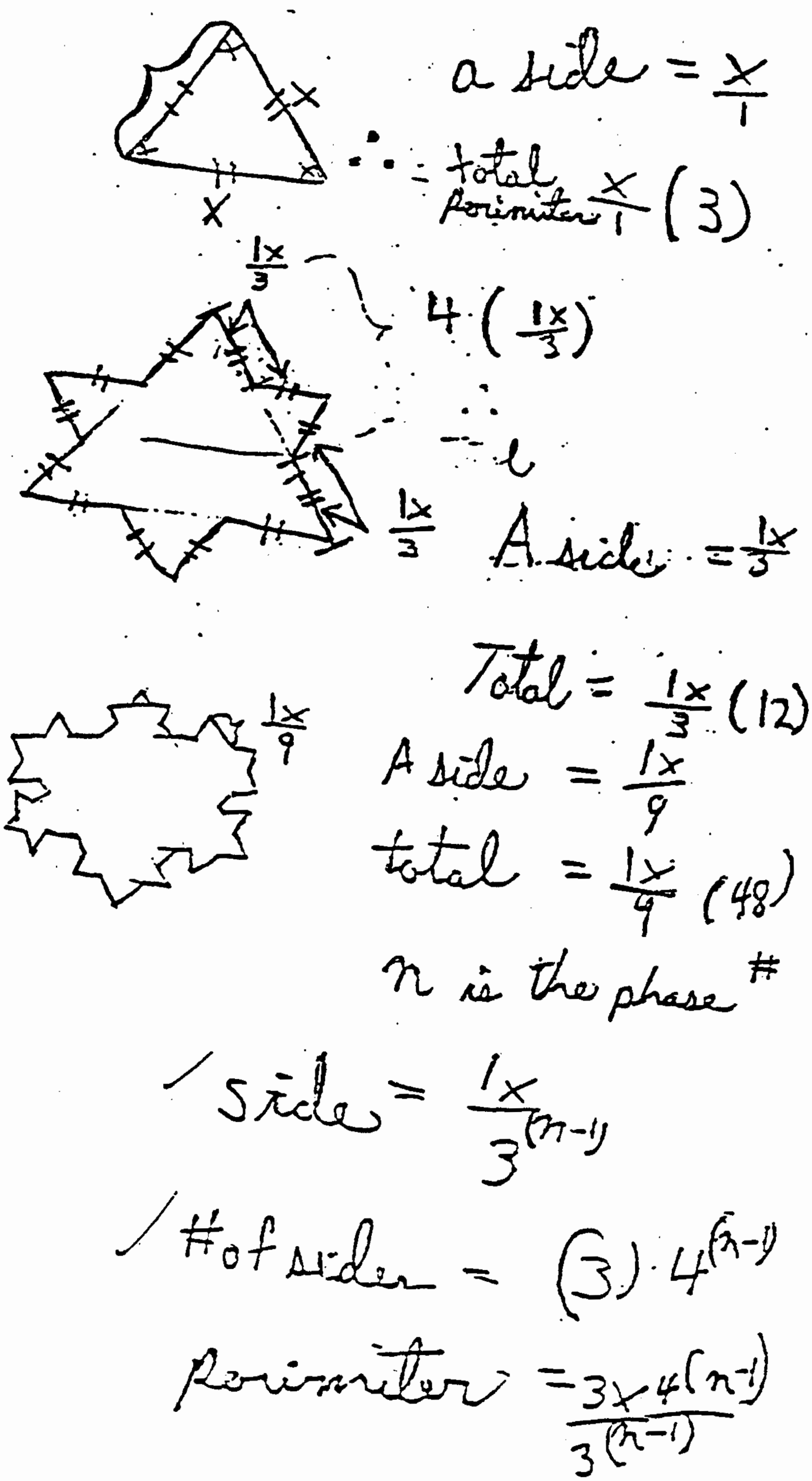


Figure 4.6 - Koch Snowflake

goes beyond that and writes the general formula for the perimeter of the n^{th} phase! To my knowledge, Kurt had not formally studied the properties of sequences and series as this topic is first introduced in the grade 12 curriculum. During the next class, Kurt began investigating similar patterns for squares instead of equilateral triangles and was trying to generalize the problem to a higher level. Thus the problem led to a more open ended investigation. Kurt's approach to this problem shows that he is more interested in the general solution rather than the specific solution.

I then asked Kurt if he had written the computer program yet and he had not. Over the next few weeks I kept asking him for the program. On 28 October, he said that he wrote the program but forgot it at home. On 4 November, he said that he could not find the program and thought he might have lost it, but that he would redo it. On 7 November, he said that he had wanted to work on it that morning, but that the staffroom door was closed. (I'm not sure what he meant, because if he wanted to see me, I am usually in my classroom.) On 11 November, I went to look for him in the computer lab, where he was producing a beautiful graphic design. I asked if he had worked on the snowflake program and he complained that a certain input statement had "escaped" him, so he abandoned the effort.

I finally decided to leave it alone and, since reporting time was near, I had to calculate his mark. His average on the four assignments he completed was 82%, combined with a class mark of 56%, giving him an average of 69% for his first term mark. He also made a good effort at attempting some questions from the majority of his daily class homework assignments.

At the start of the second term, I decided that since he was keen on graphic design, I might suggest that he do some art work involving mathematics. On 27 November, I presented him with part of a paper I had written on "Escher" type tesselations and asked him if it were possible for him to create such tesselations with the computer. On 2 December, Kurt said, "sorry, I haven't done anything on the tesselations yet". Then after class that day he presented me with his missile problem, discussed earlier in this chapter (see p. 64-66). He also mentioned in a very matter of fact manner that he had three projects on his mind simultaneously. His computer science teacher later told me, "Ya, and he seems to work on a different one every day".

The next assignment that I asked him to consider was the writing of a program that would calculate the roots of a quadratic equation. One week later he asked when I would like it completed because, in his words, "I'm busy trying to fix my car track game so that the keys don't have to be banged in order to make the car turn. Otherwise Mr. Bird [the computer science teacher] won't network it for other people to use". He never did write the quadratic equations program.

Kurt devoted the remainder the second term to his missile project and at one point he mentioned to me that he just had to show his brother Ken that he could create programs on his own without borrowing ideas from his brother. I suspect that Kurt's concern grew out of an accusation that his brother made. Kurt's brother Ken explains:

> I think one of the main thrusts in my teaching him, one of my main goals is trying to get him (which I'm very unsuccessful in getting him to do) to start a program from scatch. To use very

> structured programming, like to sit down and write the program and plan it all out beforehand. I was successful a little bit because I know he'll sketch the variables and stuff to the program and then sit down with the computer. So he's getting the idea of how to program but I've been trying to push him to do that. I was recently accusing him of just taking my programs and just reworking them but I think, I found out that that wasn't really the case, he got some ideas from my programs and he did use some of my procedures which sometimes I wrote for him, but other than that he's been on his own. x 3.

During all of second term, the only assignment Kurt handed in was the mathematics of his missile project which was described earlier in the chapter (p. 64-66). He did not respond to any other suggestions that I made including the possibility of writing a paper on fractals, for which I gave him several resources. Needless to say, I could not weight the one assignment very heavily in his second term mark. He had also completely given up on doing any class assignments, not just in mathematics but in virtually all his other subject areas as well. Despite his lack of productivity on assignments, his unit test marks actually improved. With the exception of a badly failing mark on his quadratic equations test (which he said he forgot about), he averaged 79% on the remaining three tests that he wrote in second term.

One exception to his non-compliance in completing assignments was in English class, in which he eventually handed in all outstanding assignments for second term and scored the second highest mark on the grade 11 English exam, which he completed in the required time limits. His teacher and I discussed Kurt's tendency to be so selectively productive and the teacher made an interesting remark about Kurt's attitude to Social Studies saying "Kurt described Social Studies as a memorization course and that he was not interested in memorizing facts and therefore saw no use in doing the assignments despite the fact that he was aware of his graduation requirements".

In addition to being successful in his Mathematics and English courses, Kurt seems to have also made strides in improving his social skills. His teachers have recently noticed that he seems much happier and less withdrawn than he was in previous years.

Even though the program of alternative assignments worked reasonably well for the first two months, it soon became apparent that it would not work over the long term. One reason for this may have been my inability to find enough interesting problems for him to choose from. I consulted many books on problem solving in computing and found very little work that would be at the appropriate level for Kurt or found the work to be almost too trivial, requiring the programming of a simple given formula. In addition, finding interesting problems or investigations related to the curriculum was difficult.

A second possible reason for the program's lack of success was Kurt's internal motivation. I thought that a 50% weighting on alternative assignments might be a powerful incentive to be productive . However, based on Kurt's academic history, I should have realized that marks do not motivate him. Kurt has his own personal agenda and unless it matches an externally imposed curriculum, he may not respond to that imposed curriculum.

In the following chapter I will discuss some of the theoretical and practical implications of this study.

Chapter 5
Conclusion

Practical Implications for Kurt

In my opinion, one of the most significant descriptions of Kurt's difficulties was his very own analogy with that of a "glass wall" between himself and his teachers (Chapter 4, see p. 56). He reflected that he could see what was expected of him and yet could not break through the barrier, transparent though it was. He also clearly knows what is expected of him in terms of graduation requirements and prerequisites for post secondary education and yet he will not or cannot conform to the expectations of the system. Any disability (if it is a disability) that Kurt still has is related solely to his low productivity in school related tasks. His recent improvement in his Mathematics and English classes are evidence that he has overcome earlier deficits related to written output and time restraints on tests. Furthermore, I do not believe that these improvements are a coincidence. Kurt has developed excellent relationships, both with myself and his English teacher. The English teacher and I have shown extra interest in Kurt and have placed great value on his talents and contributions, and I believe as a result, he has responded positively. At the end of Kurt's grade 10 year, after all the interviews with him had ended, his special education teacher commented:

> I know that his mother was talking to the principal from the elementary school that he attended and said how happy she was with his whole self-concept this year, that he has really changed in terms of his acceptance of himself, he's happy, he has friends,

> his attitude has changed dramatically towards himself and towards others and towards school, which doesn't mean that he's that much more successful in school, but he sees himself as being more accepted, and I think, certainly what you've done with him and the sort of, time to talk about things has really helped him. He needs that time to reflect and bounce things off people . . .ix 3

I believe that Kurt has a deep-rooted intellectual need for personal value and meaning in the educational tasks *he decides* to undertake. It is as if he must have total unique ownership of the solution to a problem or indeed often the *creation* of the problem (as in the case of his missile problem) before he sees value in completing it. He has a deep need to be creative, something which the computer and, more recently, his writing, have allowed him to be.

Therefore, if conformity is one of the goals of our educational system, then Kurt is disadvantaged because he finds it impossible to conform to the system's expectations. I am confident that there are other students with characteristics similar to Kurt who are also silently suffering throughout their school years. Renzulli and Reis (1991) argue:

> Whole group instruction, prescribed and didactic curriculum, and an emphasis on standardized achievement and minimum competence have turned our schools into dreary places that cannot begin to compete with non-school interests, extra-curricular activities, and endless hours in front of the television set. (p. 28)

However, if creativity is one of the goals of our educational system, then Kurt is willing and able to produce and must have his needs and talents attended to.

Current trends in educational reform in British Columbia, with the intended adoption of many of the recommendations of the Year 2000 document, are a step in the right direction toward meeting the needs of students like Kurt. The principles of the Intermediate Program stating that the curriculum and assessment should be learner-focussed, that the curriculum should provide students with choices, and that assessment should help students make informed choices, are especially important. In addition, the integration of special needs (including the gifted) students into the regular classroom setting has become the accepted norm. However, I believe that if Kurt's intellectual potential, at least in mathematics, is to be realized, then it must be through a program for gifted mathematics students or an alternate school program modelled after something similar to the Enrichment Triad Model (Renzulli, 1977) discussed in Chapter 2. Certainly some skill development must be built into such a program, but I have seen that Kurt can learn more sophisticated mathematical knowledge than his peers through his own investigations - his own curriculum. The current mathematics curriculum with its multitude of content-oriented intended learning outcomes is stifling to Kurt's potential because it does not allow sufficient class time to explore ideas.

A program of study might be developed that does not necessarily have the same content objectives for each person that enrolls in it, but is based on a curriculum of process objectives. For example, students could be encouraged to choose investigations that would involve one or more of the following processes: inductive investigation or argument, deductive proof, algorithmic thought and generalization, conjecture, and problem solving of various types. Many of the useful basic algebraic skills that are part of the current curriculum would probably be required in many of the investigations that students would undertake. The specific content that each person learns would be dependent on the types of investigations that the student chose.

The teacher's role in such a program would be to act as a mentor and resource person and generally as someone to discuss ideas with. Kurt would also have to be under some pressure to produce, perhaps in the form of a negotiated contract. He might also benefit from co-operative work experience, perhaps with a firm that produces software and computer games.

Such a program of study would also include some small group investigations. Certainly Kurt would benefit from working with someone that has complementary skills to his own. It would also fulfill the need to be social during the learning process in addition to encouraging verbal communication of mathematics between students.

Implications for Teaching

The study of Kurt has evoked an awareness in me that there may be others (and not just gifted students) with similar needs in mathematics classrooms . Kurt is an extreme case of someone who needs to be motivated with challenging learning activities and self-chosen topics of study. Given that it is highly unlikely that specialized programs such as the one described above would be readily available for students such as Kurt, provisions must be made to integrate these students into the regular classroom. These provisions, however should be beneficial to all students. The implications for teaching are threefold: the curriculum must be flexible, new resource materials must be readily available, and new evaluation procedures must be developed.

The current mathematics curriculum does not have enough flexibility built into it to allow teachers to deviate from the intended learning outcomes prescribed and thus provide students such as Kurt a source of stimulation. The argument might be made that a creative teacher can cover all the curriculum in a stimulating way but reality dictates that the majority of teachers will follow what the curriculum guide suggests. The current Biology 12 curriculum at least offers teachers optional topics and I see no reason why the same flexibility could not be built in to the mathematics curriculum. Bishop (1988), in offering curriculum suggestions, argues that the concepts listed in curricula should not be viewed as 'topics' as in examination syllabuses. He explains:

> They are offered as *organizing* [italics author's] concepts in the curriculum which provide the knowledge frame. They should be the foci of concern, approached through activities in rich environmental contexts, explored for their mathematical meaning, logic and connectedness, and generalized to other contexts to exemplify and validate their explanatory power. (p. 100)

Bishop further proposes that one third of the curriculum be devoted to project work and an additional one third to investigation type activities. Thus, there is plenty of flexibility in terms of the content that a student such as Kurt might choose to investigate.

As mentioned in the previous chapter, I felt that one of the limiting factors to the success of the program of alternative assignments that I gave to Kurt was the difficulty that I had in finding appropriate material, especially sources of computer projects that require the use of mathematical concepts related to the curriculum. Undoubtedly, there are interesting materials for projects and investigations scattered throughout university libraries but the fact remains that they are not easily accessible to the classroom teacher. With the great number of students that teachers are responsible for, it would be difficult for a teacher to go far out of his or her way to provide for the special needs of one or two students, and yet that is what is ideally expected. What is needed is for some researcher to collect and classify, according to topic area, all the investigations and projects currently available at the various grade levels. These could then be published and sent to the schools along with the curriculum guides. In this way, mathematics teachers would have an immediate source of interesting materials that could be given as possible choices to students such as Kurt.

Given that Kurt is not motivated by grades or marks and assuming that there are others like him, perhaps the emphasis in evaluation should shift toward a more formative type. Furthermore, if the curriculum were to become more flexible as suggested above, then standarized examinations for all students would have to be eliminated or at least altered. For example, an evaluation of Kurt might include a description of the mathematical content and processes that he has used in his projects. This might be aided by requiring him to keep a journal where he could record his reflections on any particular investigation. The report would also include areas that require improvement. Then, rather than a test mark being the measure of his ability, the formative report and the products that he creates become the measure of his achievement. After all, that is the way that the real world operates outside school.

Currently, the only options available to Kurt regarding provincial examinations in mathematics are cosmetic in nature, meaning that the actual content of the exam remains the same, but that he may be allowed extra time, or to take the exam orally, for example. However, I believe that there could be other options. If, for example, the curriculum were to become much more flexible, then the provincial examination might eliminate multiple-choice questions and instead provide a wide range of choices of problems to solve from the content areas that are in the curriculum.

Another alternative would be to waive the examination requirement altogether, and instead provide an option for gifted students such as Kurt whereby their evaluation would be based on a project that would be carried out at the end of the year over a one or two week period. Students would have several options including computer projects, research papers and investigations all requiring students to demonstrate that they have met various content and process objectives. The projects would be completed in class under the supervision of the teacher. I realize of course that the evaluation of such work would require more effort and cost more money than would the passing of computer cards through a scanner, but gifted students such as Kurt should be considered an investment for the future.

Significance of the Study

This study is significant in at least three ways. Firstly, the study reaffirms many of the characteristics of gifted learning disabled students and, in a realistic way, makes the reader aware that these students exist in our British Columbia schools. Perhaps the study may have resonance for other mathematics educators and offer possible insights into the minds of students with characteristics similar to Kurt's. Thus, the study may be a source of ideas on how to motivate such students in the regular classroom.

Secondly, the study presents a challenge to curriculum planners in mathematics. Because a good education is every person's right in this province, curricula and resources should be available that are flexible enough to suit everyone's learning style. This study has suggested that the current British Columbia mathematics curriculum is not suitable to all individuals, especially some of the more creative students.

Finally, conducting this research project has laid the ground work for me to be a much improved classroom teacher. Prior to undertaking this research, I more often than not paid lip service to the notion of enriching individual students while teaching the required curriculum. Working with Kurt has shown me that it is possible to at least offer the option of enrichment to all students. I have recently suggested optional assignments, worth a small percentage of the term grade, to all students in both my science and mathematics classes. The need to offer students a variety of choices in curriculum and assignments has never been clearer in my mind.

Concluding Remarks

The nature of gifted learning disabled children is very complex and puzzling. It seems a contradiction that people could be both gifted and learning disabled and yet, as this study reveals, the existence of these types of students is not uncommon. GLD students are intelligent, creative, and strong problem solvers and often have a keen interest in a hobby. On the other hand, visual or auditory processing difficulties and/or motor integration problems may result in them having a great deal of difficulty expressing themselves in traditional ways. Other deficits, such as in short term memory, and their usual unorganized nature, may also hinder their efforts at being successful in school. In addition, they usually show patterns of non-compliance with assignments. As a result of all their difficulties, GLD students are usually low or average achievers. The frustration felt by these students at the discrepancy between their superior thinking ability and their low achievement usually leads to a lower self-esteem.

Early identification of gifted learning disabled students is extremely important. In Kurt's case, psychoeducational assessments revealed his superior intelligence in early elementary school. Throughout his elementary school years, teachers were aware of both his high intellectual ability, and some of his difficulties. However, I suspect that most teachers felt that his potential ability would eventually be realized, despite the traditional approaches to teaching that Kurt was not responding to. Through no fault of their own, Kurt's teachers were probably not fully aware of, and therefore could not possibly understand, the nature of the GLD student. These students must not only be identified in elementary school, but must also have their special learning needs met soon after identification.

The following is a list of characteristics of a gifted learning disabled student, as portrayed through my experience with Kurt. I am not attempting to generalize these to all GLD students, as indeed no GLD student ever shows all the characteristics listed in the literature. They are meant only to make teachers aware of some of the possible characteristics that these "mystery" students might exhibit.

1. Early psychoeducational assessments indicated high intelligence coupled with specific learning deficits.

2. He was forgetful.

3. He was articulate and had a good sense of humor.

4. He was highly motivated, creative, and an "expert", in an area of interest outside the school curriculum. (ie. computers)

5. He did not readily respond to a highly structured curriculum which required much memorization of factual knowledge or completion of "drill" type exercises. Non-compliance with assignments was frequent, and as a result, achievement was usually low.

6. He was not motivated by marks, grades, or post secondary

requirements. Interesting, challenging and preferably self-discovered or self-created problems provided the motivation. Unusual, strange or "wierd" phenomena often attracted his attention.

7. He preferred to learn independently, choosing his own curriculum of study. He often worked on several personal projects at the same time, frequently jumping from one to another.

8. He clearly understood what was expected of him from his teachers and yet could not bring himself to respond to those expectations, either because he perceived the task as being impossible to perfect, or because it just may not have been interesting enough for him to "waste" time on.

9. He loved to discuss his special interests. Often this occurred before and after class.

Appendices

Appendix A - Letters of Authorization

The following is a letter of request to the parents of Kurt for the participation of their son in this study. A similar letter was also sent to Kurt. The second letter is a request for authorization to access Kurt's confidential school file.

Simon Fraser University
Faculty of Education

<u>Informed Consent by Parent for Participation of a Minor in a Research Project in Mathematics Education</u>

Dear Mr. and Mrs. ________________,

The purpose of this letter is to ask for your consent to have your son, Kurt, participate in a research project that I will soon be undertaking. This project has been examined and approved by Dr. Tom O'Shea, Faculty of Education, Simon Fraser University.

The purpose of the project is to examine your son's mathematical behavior. He will be asked to attempt to solve a variety of mathematical problems using any means which are available to him. While he is solving the problem he will be asked to try to explain what he is thinking. My role will be only to observe and record his responses, to ask probing questions and to clarify any misunderstanding that he might have in interpreting what a problem is asking. I will then attempt to analyze the information in order to determine Kurt's characteristic patterns of thought, his strengths, his weaknesses and his interests.

This study might be important for his future learning of mathematics because if a teacher knows how a student thinks, then learning activities might be structured in such a way as to best take advantage of the pupil's strengths. In addition, the project may also benefit other educators because there may be more students who learn and think in similar ways to Kurt.

The information collected and the analysis of his work will be confidential and will not negatively influence in any way Kurt's school marks or evaluation. All recorded information will be destroyed at the conclusion of the project and the final results of this study will also be available to you.

Your consent for the participation of Kurt will be very much appreciated and is completely voluntary. It is your right to withdraw Kurt from this research project at any time. Thank you very much for your cooperation.

Sincerely,

Reg Cichos

I agree to allow my son, Kurt, to participate in the research project described above.

Name____________________________ ____________________________

(please print) (signature)

Date____________________________

Simon Fraser University

Faculty of Education

Dear Mr. or Mrs. __________,

I am writing to request your permission to access and photocopy some of Kurt's school files in order to gather certain personal information regarding Kurt. Specifically, in order to properly complete my research project, it will be necessary for me to obtain his entire academic history and his psychoeducational assessments conducted by Vancouver General Hospital, Vancouver Children's Hospital and the firm of psychologists hired by yourselves to do an independent assessment of Kurt . Again, all information will be kept strictly confidential and the photocopies will be destroyed after the project is complete. Thank you for your cooperation.

Sincerely,

Reginald A. Cichos

I authorize Reginald A. Cichos to obtain the information requested above.

Name ______________________ ______________________

(please print) (signature)

Date ______________

Appendix B - Attitude Survey

The following attitude survey was adapted from Robitaille, O'Shea, and Dirks (1982). The survey questions presented here are classified according to the scales that they were designed to test. The test items were scrambled before they were administered to Kurt. Kurt's response is given in brackets beside each question. Questions with a * at the end are negatively worded.

Mathematics Attitude Survey

The following survey measures how you feel about various aspects of mathematics. On the following few pages are statements about mathematics. Read each statement carefully and decide how you feel about it. Rate each statement on a scale of 1 to 5, as follows:

5 means "I strongly agree"
4 means "I agree"
3 means "I cannot decide"
2 means "I disagree"
1 means "I strongly disagree"

Math as a Process

1. Mathematics will change rapidly in the near future. (2)

2. Mathematics is a good field for creative people. (4)

3. There is little place for originality in solving mathematics problems. *(2)

4. New discoveries in mathematics are constantly being made. (2)

5. Mathematics helps one to think according to strict rules. *(2)

6. Estimating is an important mathematics skill. (3)

7. There are many different ways to solve most mathematics problems. (4)

8. Learning mathematics involves mostly memorizing. *(1)

9. In mathematics, problems can be solved without using rules. (2)

10. Trial and error can often be used to solve a mathematics problem. (4)

11. There is always a rule to follow in solving a mathematics problem. *(3)

12. There have not been any new discoveries in mathematics for a long time. *(3)

13. Mathematics is a set of rules. *(2)

14. A mathematics problem can be solved in different ways. (5)

15. Mathematics helps one to think logically. (5)

Math and Myself

16. I really want to do well in mathematics. (5)

17. My parents really want me to do well in mathematics. (4)

18. I am looking forward to taking more mathematics. (5)

19. I feel good when I solve a mathematics problem by myself. (4)

20. I usually understand what we are talking about in math class. (4)

21. I am not so good at mathematics. *(2)

22. I like to help others with mathematics problems. (1)

23. If I had my choice I would not learn any more mathematics. *(1)

24. I feel challenged when I am given a difficult mathematics problem. (4)

25. I refuse to spend a lot of my own time doing mathematics. *(1)

26. Mathematics is harder for me than than for most persons. *(2)

27. I could never be a good mathematician. *(3)

28. No matter how hard I try I still do not do well in mathematics. *(3)

29. I will work a long time in order to understand a new idea in mathematics. (4)

30. Working with numbers makes me happy. (4)

31. It scares me to have to take mathematics. *(4)

32. I usually feel calm when doing mathematics problems. (5)

33. I think mathematics is fun. (5)

34. When I cannot figure out a problem, I feel as though I am lost in a maze and cannot find my way out. *(4)

Math in Society

35. It is important to know mathematics in order to get a good job. (5)

36. Most people do not use mathematics in their jobs. *(3)

37. I would like to work at a job that lets me use mathematics. (4)

38. Mathematics is useful in solving everyday problems. (3)

39. I can get along well in everyday life without using mathematics. *(1)

40. Most of mathematics has practical use on the job. (3)

41. Mathematics is not needed in everyday living. *(1)

42. A knowledge of mathematics is not necessary in most occupations. *(2)

Calculators and Computers

43. It is less fun to learn mathematical ideas if you use a hand-held calculator. *(1)

44. If you use a hand-held calculator you do not have to learn to compute. *(1)

45. Using a hand-held calculator can help you learn many different mathematical topics. (3)

46. Solving word problems is more fun if you use a hand-held calculator. (3)

47. Computers solve problems better than people do. *(1)

48. Using computers makes learning mathematics more mechanical and boring. *(1)

49. Everybody should learn something about computers. (5)

50. Computers do lots of good things for people. (5)

Appendix C - Sample Interview

This interview was originally transcribed with a wide right margin, in order to leave room for coding and commentary. In the interest of saving space, the wide margin has been removed.

INTERVIEW NUMBER TWO, 6 MARCH 1991

R: Kurt, what sorts of things or activities appeal to you in Mathematics?

K: Uh, . . . I think I like, uhm, word problems because they have some meaning behind them. Uh, . . . I like using problems that I can apply to things.

R: So if you had your choice, is that all you would want to do is to solve problems throughout the whole year? What else would you like to involve in your learning?

K: Uhm, . . . I like to use computers, of course. Uhm, ...

R: Maybe there are some things that you dislike. What types of things to you dislike, and maybe we can go from there.

K: Uhm, (chuckle), . . . Uh, . . . I don't really like, uh, . . . I don't know. (chuckle)

R: When does Mathematics bore you?

K: Uh, . . . Okay, uh, . . . If I could, I wouldn't deal with the Arithmetic at all. (chuckle) Well, not, not completely, of course -

R: You mean like calculations, and stuff like that?

K: Yeah, uh, I prefer Algebra, stuff like that.

R: How do you feel about Geometry?

K: Uh, it's okay, I guess. I don't, I haven't done a lot of Geometry, uh, . . . I guess except for the stuff on the computer. It's uh, I like it. It's okay.

R: How do you feel about your abilities in Mathematics?

K: Uh. I'm not sure, uh. . . . . . . I can't be too bad because I have a lot of fun working with computers and uh, I use a lot of Algebra when I'm . . . Uh, . . . I don't know. I find it hard to look at that question from an objective point of view because I'd like to be good at Mathematics, but I don't know if I am. I, I think it's important to be good at Mathematics.

R: What would be some of the reasons?

K: Uh. Well. . . I think programming will be useful and of course I use mathematics, uh, everyday sort of stuff . . . depending on what carreer I go into, uh, I know my one brother Ken is thinking about engineering. He phoned up recently and he told me that and of course that would be something you need mathematics for. . . . . . or say if you just wanted to do home renovations and things.

R: So you think it might be useful at home too?

K: Ya.

R: Do you feel that you have achieved as high as your potential might allow?

K: No (laughs).

R: Do you have any idea why that might be the case?

K: Uh, it's hard to say, uh. . . . . I think I could be better at mathematics but I don't really know why I'm not living up to the standards I've set for myself.
Uh, ... it boggles me (laughs).

R: If you had your choice what sorts of things or activities would you like to do in learning mathematics?

K: Anything to do with computers (laughs) . . . I have a lot of fun with computers. . . . . . I would like to learn more about trigonometry; it seems interesting from the little bit that I do know.

R: You mentioned something to me at the end of the last session about one of the reasons you enjoy math; that you always try to see . . . . what was it that you said?

K: How it relates to things in real life. . . . . I, I, there, there are not a lot of useless math problems. There's lots of stuff you can apply mathematics to.. . . just, you know, it helps me think logically. They seem to be sort of intertwined, logic and mathematics.

R: Ok, here are a series of little questions that involve patterns. It says what's my rule? So just read it and then I'm going to ask you if you really understand what the question is asking.

a moment later. . .

R: Do you understand what is meant by "find the nth term"?

K: You mean uh, how would you be able to calculate the next uh, number?

R: Ok, that's what you're asked to do initially is to give the next few numbers, but then suppose I asked you now, which term would this be? (point to the first term)

K: First term. (R points to the next term) Second term. Third, fourth, fifth.

R: Ok, now suppose I asked you to calculate the eighteenth term (Kurt interjects "ok") without having to go through that whole sequence, how could you calculate the eighteenth term, that's what it's asking.

K: Ok.

R: You understand?

K: Un huh.

R: Ok, so why don't you try the first one there, fill in what the next few numbers would be and see if you can come up with a formula in terms of the term that you have-the nth term.

(25 seconds)
R: Is that a multiplication symbol?

K: Uh, yeah. (chuckles) looks kind of funny doesn't it.

R: How do you know for sure that that (the formula) is correct?

K: Um, because I could take every one of these and do that and then I could keep on going what I think is the next pattern and this works for every one that I can come up with - pick any one of these that I know would fit this pattern at random and I know it would work for that.

(continues working - 2:42 later S begin chuckling)
R: What's that? What's happening between - what's happening here to get the pattern going?

K: I think it just - plus five each time.

(30 seconds later)
R: Now, how did you get the plus two here S?

K: Um, ... they uh, they weren't multiples of five but each one had five extra than the next one and each of them could, um, be minused by two and it would equal a multiple of five, so I knew that n times five plus two in each case would give me one of these.

R: Does that in fact work?

K: Yuh.

R: Does that work for the second term? . . . . . Ok, how would it work then for the second term?

K: Uh, n would be one and it would be times five-

R: For the second term n would be one? the second term n would be?

K: Oh, uh, oh yeah (surprise) uh. . . . . . wait a second, I think I got myself confused, so in each of these the first one of n is one?

R: Yeah, that's the first term. This would be the first term (points), the second term, third term, fourth term, fifth term and so on.

K: Ok, then I have to change this a bit.

R: So, you put an n-1 in there?

K: Yeah.

(continues working - 1:15 later)
R: Are you saying here add 3 to n and then divide by 2?

K: Ya. uh no. No, (laughs) not any more (laughs and makes a change).

R: What is this supposed to say now?

K: Uh, n plus 2 divided by 2.

R: And that works for all of them?

K: Yeah.

R: Ok.

(continues work - 1:55 later)

R: In these harder examples, numbers 6 through 9,10 you' re figuring out the rule in your head and then you are writing the sequence down. . . . . . So, what's the rule then for number nine?

K: n plus 2, no, n plus (chuckles) - keep on wanting to add more than I should, n plus 1 uh, . . . times . . . 8, and then, n plus 1 times 8 minus . . . no, plus 1.

R: May I ask you something Kurt before you go on to the next one? What makes you add one to n here all the time? See in all these formulas you're putting n plus something in brackets and then dividing by 2 , ok, for these ones that were a half (point to fractional examples) - here you went (n-1)*5 +2, here you went (n+1)*8+1 - Why do you put the n+1 in brackets?

K: Uh, because these are supposed to be the first term, right?

R: Right.

K: So, n equals 1?

R: Right.

K: And uh, . . . it just doesn't fit to have n equal to 1 (laughs).

R: You mean it just doesn't fit when you just have n, is that what you're saying?

K: Yeah.

R: But how do you determine the pattern like that - how do you determine that you have to add one- just by looking at the first term?

K: Uh, usually uh, after I've gone through it, I look at the relationship between these and I get the rest of the stuff and then I figure out that, well, that's kind of times 2 really and then so I figure well I guess I'll have to add one to n. Like I sort of figure out what the relationship is between these guys and then I figure out where it starts.

R: And what's the relationship between each number in number 9?

K: Uh, there uh, in number nine they're uh, times 8 plus 1.

R: How did you know to times by 8? What made you see that it was eight that you have to multiply by?

K: Well, I knew there had to be uh, they couldn't be . . . uh . . . they didn't seem to make any sense the way they were there so I tried minusing numbers and looked at what I got and if some of them weren't . . . I don't know. . . when I minused 1 it looked like it needed minusing or something or other (laughs).

R: Well, in each case here you either multip[lied or divided by something. Now what made you decide on what number to multiply by?

K: Uh, . . . What do you mean like (laughs)?

R: Ok, well see here your pattern is n times 3 and here it's n times 2 and here it's n times 3 plus 5 and here it's n minus 1 times 5 and suddenly here you are dividing by 2, here you're dividing by two, now you're back to multiplying by a number. How do you figure out that number there, that you have to multiply by?

K: Uh . . . well . . . I'm not sure exactly, uh, I think I must have minused or something, subconsciously.

R: Ok, let's go through the next one, number 10, and try to think out-loud as to how you are figuring out the pattern so that I can here what is going on in your head.

K: Ok, uh, . . . they look like they're multiples, uh, . . . and 9 and 23 don't uh, don't have uh, aren't any multiples of something but I know they must be because of the way that they are - if you subtract 23 from 9, if you subtract 37 from 23, I know I'm going to get the same number uh.

R: So what number do you get in that case?

K: I get . . . I don't know how I figured it . . . well it just looks like it's being multiplied - I kind a . . .

R: Ok, so you subtract the numbers and what would you get in this case, what is that common number that you get?

K: Uh, . . . fourteen.

R: Ok, go ahead and try to come up with the rule and the next few terms.

K: 21 is close to 23 so if I minus by 2 and if I minus by two they both come to be a multiple of 7, um, so it's whatever times 7 plus 2 ,um . . . . .

R: How come it doesn't work?

K: Oh (laughs) because, un, well I think the rule works but I messed up somewhere.

R: Show me that the rule actually works for the first three.

K: Uh, 1 times 7 equals 7 plus 2 equals 9. . . . . no it doesn't work (laughs), it doesn't work. Uh . . . ok.

R: Does it work for the next one?

K: ... if n were 1 there, it were 3 there ... and ... if it were 5 there ... ok, uh, ... n times 2 minus 1 times 7 plus 2.

R: Does that work now?

K: Works for the first 3 uh, ... ya.

R: What did each one of these questions have in common? What was similar about all of them?

K: Oh, you already said that, they're either multiplied or divided by uh, ..., and one's either multiplied or divided by somewhere along road, in here it was after a plus or something.

R: Ok but the actual patterns themselves - forget the rules for a minute - what is similar about all the patterns?

K: uh, ... you can say that uh, the first and the second number are different in the same way than the second and the third number are different.

New Problem

R: Find a rule to calculate the sum of the first n odd integers. Ok, now ask if you don't understand.

K: Oh wait, I get it now, I get it now.

R: Tell me how you know you get it?

K: Well you mean the first n odd integers, uh, you want an odd number every time,um, I'm not sure if I get it now.

R: You can write something down if you want.

K: Ok, yeah, that might help. Uh, ...

R: Why don't you write some numbers down first.

K: You add them all, right?

R: Ok, what's this number here?

K: Uh, oops, yeah . . . uh . . .

R: Right and you have to add those together, right? And that would be the first how many odd integers?

K: Four.

R: Ok, so now I'd like you to come up with a formula. For example, if I asked you what would be the sum of the first 25 odd integers, I'd like you to be able to tell me with a formula, what that sum would be, without actually adding up all the numbers.

K: That doesn't do it, it just calculates one of them - the last one (refers to his rule n*2+1).

R: What does this give you - this 2n plus 1?

K: The last number.

R: Ok, so that just gives you the number, right?

(long pause)

K: This in computers I'd have an easy way out of this (laughs).

R: What are you thinking now when you write 4 times 2 + 1.

K: I'm thinking that - how I would get that- uh, yeah and so I'm just sort of substituting and then -

R: What does this give you, 4 times 2 plus one?

K: That will give me 7.

R: No.

K: No. Uhm, ok then that helps, I know I'm doing something wrong. (laughs)

R: Well you just finished telling me a minute ago that this 2n +1 only gives you the last term, right? I want you to find the sum of all of them.

K: But I'm thinking that would help me, uh, ... (long pause)

R: Well, what is the sum up there?

K: uh, 16. . . . . .

R: What are you thinking?

K: I'm thinking that (chuckle) . . . I don't think I know how to do this because . . . I'd have to have series of numbers, an indefinite series of numbers depending on what n is.

R: No. [I probably wasn't listening because what he said is correct, he just hasn't determined how to generate the series yet] I'd like to know specifically, if I ask you to find the sum of the first 4 odd integers, that is my answer. If I ask you to find the sum of the first 6 odd integers, you add them up and give me an answer. Ok. Now what I'm asking you to do now is if I asked you to find the sum of the first 100 odd integers, can you give me a rule that will give me the answer like that? (snaps fingers)

S: (long pause) I still don't get it because . . . If I want the sum of all these integers, this only gives me the last one, and seeing n could equal anything, it could equal the next hundred integers, the next 101 uh, I'd need to do this
. . . . . . I'd, I'd need to do equation similar to this uh, for I don't know how many times, wouldn't I?

R: Well, test out some other sums.

K: From 0 to n, right?

R: No, no you're starting with 1.

K: Sorry, from 1 to n.

R: No, that's not n. n is the first n terms, this n is 4 here.

K: But it will start with one, right.

R: It will always start with one. Could you write out a representative for me of the sum that you would have to find - the sum of the first n terms? Like this is the sum of the first four terms, could you write out for me what it might look like to add the first n terms, the first n odd integers?

K: What do you mean?

R: Ok, write out the sum of the first 3 odd integers.

(he does this)
R: Ok, write out the sum of the first five odd integers.

(he does this)

R: Write out the sum of the first n odd integers.

K: Ok, uh oh,oh, ok (insight)I think I know what you mean now. (long pause)

R: Where does it have to start?

K: At 1.

R: What's a way to show that it actually starts at 1 and goes higher and higher and ends at the nth term?

pause
R: Ok, you've had no experience at writing out these things, but anyways, can you see what the answer might be, what the sum might be from the few that you've done?

S: Well, I don't know, I,I keep on seeing these,uh, a series of, of . . . those things, uh, of n times 2 minus 1 -

R: (interrupts) Those are the numbers that you are adding, right, but what about the sums?

K: And then add that to n minus . . . minus n minus 1.

R: But what about the answers, what do the answers look like?

K: uh, oh, ... ok, ok, uh, . . . that's easier.

R: That's what I want you to find, I want you to find the answers, I want you to find the sums.

K: Ok, I could have been approaching this in an easier way. Ok, ... I'm going to do that little pattern thing, uh, ...(long pause- 45 sec. - writes something down)

R: Is that your answer?

K: It fits. I guess so. Uh.

R: Circle it there so I know where you are on the page - n squared is your answer.

K: Yeah.

R: And it fits, right? That's it.

K: (laughs)

R: Maybe you misunderstood my question.

S: I, I could have, I should have thought of that because. . .

R: You were concentrating on the terms instead of on what you were supposed to concentrate on, the sum, right?

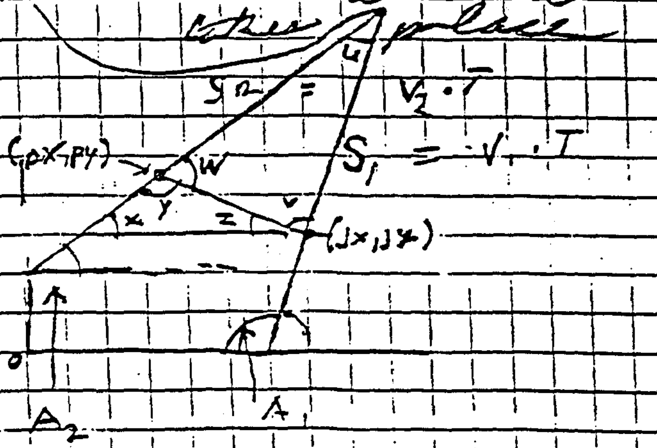
Appendix D – Kurt's Missile Problem

This is a copy of the actual work that Kurt handed in to me as an optional assignment. The transcribed version is in Chapter 4, Figure 4.2, p. 66.

Q: If I want to shoot down a jet at the co-ords (Jx, Jy) moving at the speed V_1 at an angle of A_1, what angle (A_2) must I shoot my projectile at if I am at the coord (px, py) and the projectile travels at the speed of V^2 which is greater than V^1

T is the amount unknown time at which the collision takes place

point of collision



$x = A_2$

$z = Tan\left(\frac{py - jy}{px - jx}\right)$

$y = 180 - (x+z)$

$w = 180 - y$

$T \cdot V_2 = S_2$
$T \cdot V_1 = S_1$ $\therefore$ $\frac{V_2}{V_1} = \frac{S_2}{S_1}$

$v = \arcsin\left(\frac{\sin(W)}{V_2} V_1\right)$ by sine law

$A1 = z + v$

Appendix E - David and Goliath Computer Program

```
program david_and_goliath;

uses graph,crt;

const
  Radius=40;rspeed=1;gspeed=0.25;
  SpecialKey=#0;Up=#72;Down=#80;left=#75;right=#77;Home=#71;Pgup=#73;

var
sine,cosine: array [0..127] of real;

key:char;

locx,locy,gx,speed,mlocx,mlocy,
olocx,olocy,ogx,tilt,
rlocx,rlocy,
orlocx,orlocy: real;
centerx,centery,collision,subend,launch,inc,grdriver,grmode: integer;

procedure initialize;
begin
centerx:=320;
centery:=300;
locx:=0;
locy:=0;
speed:=0;
gx:=640;
collision:=0;
subend:=0;
launch:=0;
grdriver:=DETECT;
INITGRAPH(grdriver,grmode,'');
SetGraphMode(VGAHI);
CLEARVIEWPORT;
end;

procedure MakeSinCosArray;
begin
for inc := 0 to 127 do
   begin
   tilt:=inc;
   sine[trunc(tilt)]:=sin((pi/64)*tilt);
   cosine[trunc(tilt)]:=cos((pi/64)*tilt);
   end;
end;


procedure keys;
begin
  IF KEYPRESSED THEN
    begin
    key:=READKEY;
    sound(5);
    if key = SpecialKey then
       begin
       key:= ReadKey;
       CASE key OF
```

```
                  left:
                  begin
                  speed:=speed-rspeed;
                  end;

                  right:
                  begin
                  speed:=speed+rspeed;
                  end;

                  up:
                  begin
                  launch:=1;
                  end;
               end;
            end;
         end;
      end;

procedure update;
begin
tilt:=tilt+speed;
if tilt>127 then tilt:= ((tilt/128-trunc(tilt/128))*128);
if tilt<0 then tilt:= 128+((tilt/128+trunc(tilt/128))*128);
olocx:=locx;
olocy:=locy;
ogx:=gx;
locx:=centerx+cosine[trunc(tilt)]*radius;
locy:=centery+sine[trunc(tilt)]*radius;
gx:=gx-gspeed
end;

procedure draw;
begin
setcolor(12);
line(trunc(locx),trunc(locy),centerx,centery);
setcolor(2);
circle(trunc(gx),30,30);
end;

procedure erase;
begin
setcolor(0);
line(trunc(olocx),trunc(olocy),centerx,centery);
circle(trunc(ogx),30,30);
end;

procedure launch_it;
begin
mlocx:=-sine[trunc(tilt)]*speed;
mlocy:=cosine[trunc(tilt)]*speed;
rlocx:=locx;
rlocy:=locy;
end;
```

```
procedure newupdate;
begin
tilt:=tilt+speed;
if tilt>127 then tilt:= ((tilt/128-trunc(tilt/128))*128);
if tilt<0 then tilt:= 128+((tilt/128+trunc(tilt/128))*128);
olocx:=locx;
olocy:=locy;
ogx:=gx;
gx:=gx-gspeed
end;

procedure collide;
begin
orlocx:=rlocx;
orlocy:=rlocy;
gx:=gx;
rlocx:=rlocx+mlocx;
rlocy:=rlocy+mlocy;
putpixel(trunc(rlocx),trunc(rlocy),15);
if 30>sqrt(sqr(abs(rlocx-gx))+(sqr(abs(rlocy-30))) then
   begin
   collision:=1;
   subend:=1;
   end;
end;

procedure screen;
begin
if abs(320-rlocx)>320 then
   subend:=1;
if abs(240-rlocy)>240 then
   subend:=1
end;

begin                              {main}
repeat
initialize;
makeSinCosArray;
repeat
keys;
update;
draw;
erase;
until launch=1;
launch_it;
repeat
newupdate;
collide;
draw;
erase;
screen;
until subend=1;
setcolor(4);
if collision=1 then outtext('You hit!');
delay (350);
until subend=0;
end.
```

Appendix F - Data Sorted into Categories

This appendix includes all the data that I classified according to the major categories that emerged during analysis. I further grouped the data according to whether they matched a particular code or not. Occasionally, my observations or comments were also included. Originally, I left a wide margin to leave room for additional remarks, but in the interest of saving space, I have removed the margin.

Checklist: Characteristics of GLD

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1. **Hobbies**

Computers-5 years up to 1991, V20

K: I started this program in the summer and finished it a couple months into school and I was kind of experimenting with it all of the time. iv13

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2. **Discrepancy**

I think I could be better at mathematics but I don't really know why I'm not living up to the standards I've set for myself. Uh, ... it boggles me (laughs). ii2

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3. **Goals**

If he thinks he cannot meet his own or other peoples' expectations in an assignment, he puts it off entirely and would rather accept a mark of zero.

. . . . He feels that he is a failure in . . . the routine of note taking, he's afraid that he's maybe missed out something, that he's not competent enough at note taking in class or, note taking from a book, or answering a comprehension question. He feels that, "what if I've missed a point"? So he's very anxious about that whole are and for the most part, he will not even attempt to answer comprehension questions because, in the past, he's felt insecure that he may not have covered all the material. He won't complete parts of things. For example, he had a project, which was basically a very simple, step by step assignment, that was all laid out for him and he was very concerned that he was supposed to have four pages of notes. He was very concerned from step one that he was not going to be able to stretch four pages of notes out of that assignment and so, he didn't bother to even attempt it, even though the material that was supposed to be included was very well laid out and was very predictable. We had done one as an example together and when he saw it only came to two pages, he did not want to be bothered having anything to do with that assignment because he had heard from classmates that they were supposed to have four pages. When I explained to him, "well, why not hand in what you've have done, you've covered the assignment, don't worry about the length of the assignment", he got very angry and said, "what's the point, I have to have four pages, that's what everyone else has". His teachers coached him in the same area but he refused to have anything to do with that assignment because he didn't feel that he could have the volume that was required. He wouldn't hand in parts of the assignment to get partial marks because "that isn't perfection, that is not something I will be proud to hand in, or have anything to do with it, so I won't even begin it, won't even attempt it". ix 6-7.

see interview ix, p. 3,7 for examples.

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4. **Self-critical**

no examples

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5. **Reflective**

In grade 9 year, teachers were asked to give extra time in completing tests. Spent 20 minutes on one test question for me, completely forgetting or ignoring other questions.

Kurt tends to be a one directional type person. He tends to have the type of concentration that he concentrates on one thing at a time. He can't sort of, you know, walk and chew gum at the same time. He has to think of one area, one focus, and tends to be a real perfectionist in that area as well. . . . . He tends to I think go off in one direction and forget the big picture. He tends to get very bogged down if there's something he's concerned about that takes up all his time and energy and he doesn't see that maybe in perspective, it's not all that important. . . . . ix 3

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6. **Frustration**

Frustrated at the lack of success in basic things. See interview ix.

Brother Ken: I think, in general, homework is kind of harder for him than most and I had the same experience, because sometimes it's hard, you sit at a desk, and particularily for me writing an essay, and I'm trying to transfer that to what he's feeling on a broader scale. When I write an essay, a lot of times I'll just sit and stare at the page and I just can't get anything out and this goes on for a long time and so you just tend to drop it after a while. I think he's just kind of writing off homework as something he can't do because he does have problems doing it. For some things at first, it'll take him a long time to get it out. Now for math, I think it seems to me if he just did the homework, it would just come real quickly. x 6-7.

---

7. **Disorganized**

Tends to not do homework regularly (special ed. teacher, myself, others)
Work is sloppy, sparse. (int. iii, viii)
Time management difficulties: That's the kind of, I think the kind of learner he is. Kurt kind of lives for the moment and he doesn't see a lot of things at once, he only sees one thing at a time and so that kind of awareness of time and managing that time and seeing how it connects to other things in the environment is a real disability that he has. ix 4

. . . . at the beginning of the year when we first started planning for time management, we did like an hour block and blocked it all out for the evening, and how he was going to use his time. Even to extra-curricular type things, when was he going to go to the computer lab, when was he going to have dinner, etc. That was . . . he couldn't handle that emotionally. His mother changed the time of dinner by fifteen minutes and he said he couldn't use his schedule at all, it just threw the whole thing off. He had no flexibility at that stage and he was very anxious about minor changes that he saw, that were disruptive to him because of that sort of lock-step approach, you know, "if it doesn't happen the way it is predicted, I can't deal with it". He was very anxious, I mean physically he looked very anxious and . . .

just the frustration that he walked around with. ix 5

Psychoeducational assessments-disorganized

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8. **Memory**
Psychoeducational assessments- forgetful, difficulty remembering past learning, especially unstructured, poor short term memory

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9. **Deficiencies**
Psychoeducational assessments- delay in motor output
Non-school psychologist- visual motor impairment

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10. **Problem Solving**
Plenty of examples in data - test question, computer program, coordinate geometry problem.

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11. **Subscales**
WISC - R 1989
High scores in similarities, arithmetic, vocab., comprehension, picture completion, block design, object assembly.
Low scores in digit span, coding, picture arrangement.

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12. **Expert**
Computers:
I started this program in the summer and finished it a couple months into school and I was kind of experimenting with it all of the time. iv 12.

" It's kind of funny, someone asked if I knew a lot of computers and I said that I did, a lot more than a lot of people in this school, not of course includung Mr. Bird - uh, and I got the impression that they thought I was being arrogant or something or other and I went home and I thought, well, if I were being arrogant, I don't see how that could be because if I didn't know more than these people, I'd be ashamed because I've been working with these for like five years". v20

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13. **Languages**
French was dropped from his schedule in grade 10. Poor performance and time with special ed. teacher was required.

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14. **Spacy** - no examples found

Checklist: Mathematical Giftedness

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1. **Formalized Perception/Relationships**
K: I kind of like uh, . . . what I do on the computer a lot - I'll take an idea and I'll have it in my head but if it's all in written words and pictures and uh, . . . I'll put it into a mathematics problem.

R: Something's in words and pictures? -(S interrupts)

K: Yeah in my head and then I'll, I'll figure out how I can get the computer to do it or, . . .how I uh, . . . solving like a practical problem and putting it into numbers and algebra. i1

K: I find it fun to use the computer and (inaudible) at the same time. I like taking something from words and putting into numbers and stuff and in a way it kind of increases my understanding of what I'm talking about - what I'm thinking about. i4

R: But how do you determine the pattern like that - how do you determine that you have to add one- just by looking at the first term?

K: Uh, usually uh, after I've gone through it, I look at the relationship between these and I get the rest of the stuff and then I figure out that, well, that's kind of times 2 really and then so I figure well I guess I'll have to add one to n. Like I sort of figure out what the relationship is between these guys and then I figure out where it starts. ii7

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creates variables for a problem quickly (iii), evident throughout study.

In session iii he has written two relationships involving side1 and side2 of a triangle based on given information. He suddenly says, "ok, ok, yes" and writes, side2 = 4, then scratches out the =4 and replaces it with =b. I ask him to explain what he's doing. He says, "2 unknowns, but side2 has a relationship to side1. I can figure out what side1 is in terms of side 2. It reduces to one variable and the equation is solvable". iii2.

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a problem given to his group in class was to <u>graph</u> two linear profit relationships, described verbally, and determine when one company's profits overtook the other company's. This was an introduction to linear systems. Steven wrote two, 2variable equations and began to solve the system algebraically. (notes, April 18)

In teaching problems with systems of equations, students were encouraged to look for totals to suggest equations and variables. For example, if 5000 dollars was invested, partly at 9% and partly at 10.5% and the total interest earned was 483.75 dollars, then it was suggested that they might proceed in such a way as to produce the following equations:

$$x + y = 6000$$

$$.105x + .09y = 483.75$$, where x and y are the different amounts invested. However, when students were asked to try this very question (it hadn't been done as the example), he asked me to come to his desk and asked if this was ok,

$$(5000 - x)\ .09 = 483.75 - y$$

$$x(1.05) = y$$ (arithmetic error his)

<u>Nothing else was written on his page</u>, but one can see that y is the interest earned at 10.5% and x is the amount of money invested at 10.5%. This is quite different. (notes, April 30, 1991)

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ability to think in symbols is clear in his production of the computer program David & Goliath.

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the problem in interview viii is a less sucessful example of finding relationships, but he was at least attempting it.

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2. Generalization

the example that motivated the study is an example of one of Heid's (1983) observations, "It is not uncommon for a gifted student to solve a problem on its most general level, to generalize algorithms for solving whole catagories of problems of the type given, and then to neglect answering the particular question stated in the problem" (p. 223)

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after writing the formula for the nth term of a number of arithmetic sequences:
R: What did each one of these questions have in common? What was similar about all of them?

K: Oh, you already said that, they're either multiplied or divided by uh, . . . , and one's either multiplied or divided by somewhere along road, in here it was after a plus or something.

R: Ok but the actual patterns themselves. Forget the rules for a minute, what is similar about all the patterns?

K: uh, ... you can say that uh, the first and the second number are different in the same way than the second and the third number are different. ii9

Next, I asked him to show me that the answer is in fact correct. First he thought it might be the wrong sign. Then he began talking about the slope equation. He gave the slope as - y intercept/ x intercept. I asked him why negative and he answered because the line slopes down and shows it on the diagram. (he new this from class as well, but was not shown the intercept form of slope formula) iii3

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3. **Curtailment of Reasoning**
shows very little work as indicated in the systems of equations example above.

In interview viii, where relationships are right triangle trigonometric, he first writes an acronym learned in class to remember trig ratios but, instead of for example, writing sinA=x1/L1, he will go directly to sinA*L1=x1 orsin(A+B) *L2=x2 in his work. pp10, 16

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missile problem

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4. **Economy, Flexibility**
In writing the program David & Goliath, his Goliath moving across the top of the screen was originally square. Determining if the projectile hit Goliath would have required inequalities (2 or 4)testing the range of x coordinates and y coordinates. Suddenly:
K: Oh, it would be interesting if I had a circular Goliath. (chuckles) I could use the pythagorean theorem and only make one equation.

R: How's that?

K: Um, well what I'd do is I'd take, I'd just put in location, the distance, your delta x I guess would be location x . . .location x minus um . . . gx and your y would be location y minus gy and once you've got that you've got your distance from Goliath and uh, when he's within a certain distance, he's going to be within a certain circle. vi13

later:
K: Maybe I should try that, it'll be easier than this. vi14

While working on David & Goliath, he frequently borrows stuff from another program, which he refers to here:

K: (inaudible stuff) about my car program is, it works and everything but it doesn't really need the pythagorean theorem uh . . .

R: What doesn't need it?

K: The car program. See . . . . . (types - goes back to car program) this spot here is where it uses the pythagorean theorem to um, calculate um, what the momentum of it's going to be, including all the slippage and stuff, but I think you could just use ratios, like . . . . it could probably just times a certain number by .9 and that would mean 90% of it would go in whatever direction and then the other one would be times point one, add them together, divide by 2. . . . or no . . . . . . yeah, I'd just add them together and that would (inaudible). vi16

---

in interview vii, he's determining if the projectile leaves the screen, which he originally did by using 4 inequalities:

K: So I was just going to do it the way I was going to do it over there uh . . . but I figured out another way of doing it uh . .

R: To determine if it goes off the screen.

K: With only, only checking twice.

R: And how is that?

K: What it does is it - that's uh 320 is the middle of the screen along here and locationx is where the object is along here and uh, . . . it takes the center of the screen minus locationx and it changes that into an absolute value. vii1-2.

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5. **Reconstruction of Mental Processes** no examples

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6. **Generalized Memory** no examples

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7. **Cast of Mind**
R: You mentioned something to me at the end of the last session about one of the reasons you enjoy math; that you always try to see the . . . . what was it that you said.

K: How it relates to things in real life. . . . . I, I, there are not a lot of useless math problems. There's lots of stuff you can apply mathematics to.. . . just, you know, it helps me think logically. They seem to be sort of intertwined, logic and mathematics. ii3

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8. **Energy**
In interview vii, he worked for 55 (including overtime) minutes without solving the problem completely. He never gave up. (could that be partly because I was watching him?)

He certainly never seems to get tired of computing.

Problem Solving Strategies

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1. Trial and Error/Experimenting

R: If you had your choice would you prefer to work on your own to learn something or would you rather be told. For example, if you had to learn a new topic, would you prefer to look up the topic in a book or would you prefer to be shown?

K: I'd really like to be able to do it on my own, but I may not necessarily get it. But some of the ways I figured out, for example how sine and cosine worked, the way I figured that out was my brother was making a program that drew a circle and I wanted to know how he did it. So I looked into his program and I just looked at it and tried all sorts of little things with the parts that I thought were actually making the circle, kind of figuring out where it was. I finally figured out what were the parts, what were the main things that was making the circle, making the x coordinates this, what y coordinates and I figured out how to do it and that was kind of fun.

R: And you did that all on your own?

K: Yeah and . . . I like doing that sort of thing.

R: Did your brother help you to figure out what the sine and cosine meant?

K: I kind of initially played around with it a bit and for the first few times I just took his program and would modified it a bit. I would change certain variables in there and I'd look and see what happened and a lot of times for a while there I was just kind of using that program to make other programs because I wasn't quite familiar with it. But after a while, after using it, I got to know it and I was comfortable and so now I could make up something from scratch. But in way he sort of introduced me to the idea because if he didn't make the program, I wouldn't have found it. i4-5

R: That's ok. Suppose you were allowed to use the computer. Do you have any ideas on how to solve the equation using the computer?

K: . . . . I might uh, I might do several things uh, I might try replacing everything with variables or repacing everything with numbers.

R: When you say replacing with numbers, what do you mean?

K: Like uh, I'd, well, it wouldn't be with the computer actually, it probably, I might uh, take everything, replace a number, see - then work it out, see . . . (inaudible) . . . there's no unknown and then see what kind of answer I get and sometimes that helps me see how to solve the problem. On the computer, if I replace it with variables some times it helps me look at everything in relation to everything else like so I can see a little more clearly sometimes when uh, . . . what, sort of like uh, when you have a number and you want to express it in terms of another uh, when I put it with variables sometimes I can look at it and it gives me a better idea of uh, . . . what it is, uh, what it is the problem actually means. i11

R: Did Ken teach you about the sine and cosine graphs?

K: He made a little program - I asked him how to draw a circle and uh, he had it drawn point by point on the apple when I was in grade 4 or 5. I looked at it and uh . . . it took me quite a while but I figured out that what was making the thing rotate was the sine and cosine. So I experimented with that and learned how it worked.

R: And the fact that the sine never increases above 1 and stuff like that you just figured that out by experimenting? (Kurt mumbles afirmatively) You didn't read up about it in a book at all?

K: No.

R: Ok.

K: Ok, that here I sort of have it named after the parts of a ship, portx um, that's the left side, and um . . .so . . . oh, I think um . . . oh yeah, as you'll notice with the portx and the starx and the stary um, . . . instead of adding stuff to the tilt, instead all I did was I put a minus and a plus there, to do the perpendicular thing, you know, that we're doing now.

R: With perpendicular slope?

K: Yeah.

R: So where did you learn that? Did you pick that up just from experimenting too?

K: Yeah, I picked that up in . . . grade 6 or something or other, I happened to be fiddling with a program where I wanted the thing, instead of, I wanted it to rotate 90 degrees and I wasn't sure how to do it and somehow I stumbled upon it. (chuckles)

R: From experimenting? Good for you.

K: I think it was totally by accident, because I accidently put a plus or something where I should have put a minus and I thought, oh hey, that's interesting, the things turning funny (chuckles). So I checked that out and (mumbles something inaudible). Anyway, this here, uh, just make sure you don't get confused, uh - iv10-11

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2. **Use of Past Experience**

R: If I gave you a problem, is the first thing you might think of is to how to use the computer? Is that how you would try to solve the problem?

K: Um, actually a lot of time I'll just do it on paper and pencil. But uh, . . . sometimes in a way I'll figure out how to do those problems because I thought, oh hey, I did something like that on computer and I worked it out this way. i6

(one minute later - Kurt struggling with equation)

R: Now this is a non-curricular example. I'm always going to be giving you questions that are not from the curriculum. You don't necessarily have to take approaches that, um - (Kurt interrupts)

K: I'm just trying this because I'm familiar with them.

R: Because you've learned it in school.

K: Yeah. i8

---

remembers something he did earlier in session:
R: That's what I want you to find, I want you to find the answers, I want you to find the sums.

K: Ok, I could have been approaching this in an easier way. Ok, ... I'm going to do that little pattern thing, uh, ...(long pause- 45 sec. - writes something down) ii13

At the very beginning of the programming of David & Goliath:
R: Let's just see - can you clear this and start - you'd want to work on paper first, eh? Before you start programming anything?

K: Sometimes I do a little of both um, sometimes I don't , I don't know . . . Ok, um, I'll probably keep some of this program seeing it'll help me out (chuckles while saying this). A lot of times I don't make anything up from scratch anymore. iv14

K: An equilateral triangle 60-60 . . . . can't remember what the ratio of the . . . uh, would it be ok to ask what the, (chuckles) what the . . .um . . ratio is of this side to this side, because I can't remember. Like you know how we were doing special triangles. viii6

R: Now can you give me some indication as to why you drew that line?
(lines on outside of triangle)

K: Um . . . .it, it just helps me think because um . . . like. . . graphing (also strategy for geometry)kind of um . . . . . usually when I get confused with like uh, some sort of trigonometry thing, um . . . . .what I like to do is, like when I was thinking of three dimensional trigonometry, what would help me the most was when I was, like drawing-when I was thinking of the images in my head, and moving them around and stuff, um, and how I was going to do it, I uh . . . . sort of like drew, I had the point where uh . . . .like say I had a line that I had determined using sine and cosine, I would um . . .think about it kind of like this, and this is x um, this is y and this is x and um . . . .well now this could take me a while to explain (chuckles)

R: Ok, what I'm reading - you can tell me whether I'm reading what you're saying correctly or not- you're trying to interpret the geometry on a grid.

K: mmm hmm.

R: Using coordinates, is that right?

K: mmm . . . . .ok viii9

K: (begins to chuckle) . . . .(writes out SOH CAH TOA, memory device taught in math 10 to remember trig ratios) viii10

K: Even though it doesn't look like it. (chuckles) . . . . .this makes me think of, what do you call it . . . . . .that uh IBM (inaudible) 3-dimensional graphics . . . projecting things, because this is a lot like it . . I think. . . . . (begins chuckling) . . .

R: So you're labelling everything with variables. . . ..

K: Ooooooh. . . . . . Ok, ok, I think I got something now. . . . . . . .that uh, that cliff thing, that that cliff um . . . .

R: Oh, the cliff problem in class?

K: That would be the lake there, and these would be the banks.

R: Oh, I see, like the question on your test?

K: Yeah. . . . Ok, that's going to make things easier now (chuckles) . . .now that I've drawn that 90 degrees down there . . .ok, because I knew I had to fit a 90 degree angle in there somewhere and I couldn't figure out how I was going to do it. . . . Ok, ... the problem is what does z have to do with anything . . . . .um . . . . problem is i have everything upside down here so I can barely read it . . . . . . hmm . . . I don't know if that's going to help me . . . . . L2 . . . . viii14

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3. Use of Simpler Cases

In comparing the terms of a cubic equation:

R: Why are you comparing them? In what sense are you comparing them?

K: Uh, . . . It probably doesn't help but, uh, . . . it will help me get an idea . . . maybe how I could put them in another form or something that would . . . work better. (chuckles) . . . . (55 sec later) I'm thinking of percentages for some reason. . . i9

He next writes the proportion side1/side2 = 4/1 but is not sure how to solve it, so he writes the proportion 4/2 = 8/4 in order to remember the relationship between the terms. iii2

R: So what are you trying to do now?

K: This is confusing (chuckles) . . . um . . .I'm just going to try to write an equation for . . . that um . . . that . . .for what side one (S1) is going to equal, depending on the length and the angle. . . . . Yeah ok, what's that triangle that we - that's an isosceles (mumbling something)

R: What do you mean?

K: Like say if I had an example, maybe it might help me out a bit.

R: Oh, a specific example?

K: Yeah. viii5

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4. Seeks Simpler Solutions

see economy/flexibility in Mathematical Giftedness category.

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5. Diagrams

Examples of Kurt's's use of diagrams are frequent. Anything involving geometric concepts or coordinate geometry, stimulated the production of diagrams. (see Knowledge of geometry and Strategies-use of past experience.

Diagrams in session viii were rotated frequently, perhaps in the hope of something "clicking" (field notes)

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6. **Use of Systems of Equations**
see formalized perception/relationships in Mathematical Giftedness category.

Referring to the diagram he drew (viii14), Steven wrote the following equations:
sin(A+B)*L2=X2
sin(A)*L1=X1 and S2=X2-X1 and then wanted A
R: Why are you trying to determine that angle?

K: Because I use it here.

R: You mean you're trying to find a specific measure for it?

K: No, no no no no, (chuckles) . . .um . . .because I use it here and A isn't really in the triangle, I have to figure out what A is in relation to B um . . . that way instead of just putting an A in there I can just plug in some sort of B thing .

R: Oh, I see.

K: Um . . .sort of like the same thing what I'm doing up here. I'd find X2 and X1 first and (mumbles) uh . . . .and once I've done that, this part will be easy. . . . should be anyway. viii16

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7. **Help from Brother**
see Trial and Error/Experimenting this section.

Re: car program
R: So you programmed this from scratch yourself?

K: Um, I got a little help from, my brother, um, doing this really neat little procedure up here where it checks the keys uh, . . . he told me that I could use this special procedure called "case key of"(??) and then instead of - what I would have done before is I would have had a lot of "if" statements - "if key = left" - instead it's more like one "if" statement. iv5

R: Ok, how did you find that out? Did Ken help you with that?

K: Um . . . You mean . . . Oh, the pi thing?

R: Yeah.

K: Well, uh . . . . he uh, uh . . . I guess I could have figured it out myself um, . . . he actually plugged in that particular thing in there because I forgot to put it in um . . .

R: So did he just tell you that this is the thing that rotates something?

K: Well, not this part, it was just the pi thing. Other than that I would have had uh . . .I probably just would have put 8 there uh . . . because uh . . . well I knew that you had to use pi, I just forgot to put it in there uh . . .iv8

Special ed. teacher :with the computer, he's always had that interest and always had good experiences with the computer, and with people associated with computers, like Mr. Bird and his brother particularly, who has spent so much time with him, they're very happy time that he remembers with them. ix 11

8. Reads Manuals

R: How did you first discover the absolute value function? Was it part of your .... programming ... Did you look at the programming manual and look at the different functions to see what they do?

K: Yeah, every once in a while um, when I was - like this is in BASIC too, absolute value, so every once in a while I would flip through the thing and ... it's not a big coincidence that uh, I happen to get absolute value because it's one of the first ones, so (chuckles) vii3

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9. Pen and Paper

R: So how did you first design this, did you... ?

K: Well, first I took like, uh a piece of paper and just sort of put numbers on it and kind of drew out a ... (R: interjects "grid") a grid, yeah.
R: And then you decided on those coordinates based on what you have on your - what's the maximum number of units that you have across the screen?

K: I think across it's 640 and down it's 480, I think. iv2-4

Factual Knowledge

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1. Use of Language

K: I'm thinking that (chuckle) ... I don't think I know how to do this because ... I'd have to have series of numbers, an indefinite series of numbers depending on what n is. iii1

I ask him to explain what he's doing. He says, " 2 unknowns - but side2 has a relationship to side1 and I can use the slope and side1. I can figure out what side1 is in terms of side 2. It reduces to one variable and the equation is solvable". iii2

K: They're different. They've got a, they've got the, the square and the cube there. One is cubed and the other one is squared so I can't minus them. i8

R: May I ask what you're thinking?

K: Uh, ... i'm not sure what to say (laughs). Um, these relate in a way but I can't do anything with them so um, ... I'm trying to figure out how I could sort of compare them, solve these ... you can, you can look at them and they're all sort of powers of x, 3 is a power of x to the 0 and the second one is just x to the power of one and uh, 2 is x to the power of second and this one here is two x to the power of third, so ... i9

R: How did you know to multiply by 8? What made you see that it was eight that you have to multiply by?

K: Well, I knew there had to be uh, they couldn't be ... uh ... they didn't seem to make any sense the way they were there so I tried minusing numbers and looked at what I got and if some of them weren't ..

. I don't know. . . when I minused 1 it looked like it needed minusing or something or other (laughs). ii7

K: . . . it adds your position where the car is and then it timeses the slope so that it'll be, so that the point won't be right next to the location. iv16

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2. **Trigonometry**

R: Do you know the connection between the sine and the cosine and the circle, the properties of sine and cosine that give you the circle?

K: I'm not sure. (laughs) I don't even know if I know what I'm doing but I can make it rotate and I know that, . . . I have a rough idea of how it's going to work, like if I put a number into sine and cosine, I know that if I increase the number by a certain amount it will go like that-

R: Back and forth.

K: Yeah and uh, that sine is sort of the same thing but a little bit different. I don't know exactly how sine is different from cosine.

R: Do you mean that as you increase one you are decreasing the other, is that what you're saying?

K: Well, no, I increase both of them and then . . . (laughs) I don't know, it's hard to explain. I could probably learn a little bit more about sine and cosine um, because I'm not sure exactly what the difference is between the two. I've tried looking in physics books and tried to look around in every mathematics book I could find and I still didn't get a very good idea. Then after a while I just kind of forgot about it and just . . .(inaudible) i5-6

R: Why do you use the sine and cosine?

K: Um, . . . that's just to make sure that uh, see if I weren't to use sine and cosine, . . . I don't know, uh, well, why wouldn't I? (chuckles)

R: Well, what does the sine and cosine do for you?

K: Uh, well it means all I have to do is, is like here, what it does is, there's this number called tilt and I just increase it from 0 to 15 and for each one it will go through this procedure plugging the, the . . the, plugging the, the . . thing tilt which is whatever number in between here and here -

R: Ok.

K: And uh, it'll do that 16 times.

R: And what's this pi/8?

K: Uh, that's uh . . . I think if I don't do that I'll run up, I'll get a really wacko uh, . . . like it'll increase by too much so it'll be like going around 5 times-

R: So do you understand what the pi divided by 8 is though?

K: yuh.

R: What's that?

K: Ok, well, the divided by 8 is too make sure that I don't go all 360 degrees.

R: So pi is how many degrees?

K: Uh, pi? Pi is 3.14 (nervous chuckle) Well, it's uh . . .that, just pi I think would be uh, would be 1 degree. . . . No, no, no, sorry 360 degrees.

R: Would be pi.

K: Yeah. And so I divide it by 8 to get my . . . uh, actually, I haven't, . . . you know I did this once and I forget how I did it (chuckles).

R: Well, because you've got 16 different directions, right?

K: Yeah.

R: And pi divided by 8 would only give you 8 directions.

K: Yeah, so I guess it's . . . pi would have to be 180 then.

R: That's right. iv6-8

Using polar coordinates:

R: What I'm trying to get at is how does the sine and cosine cause the car to tilt? Like what's the calculation that goes on there? You've got this little car here in a triangle, right? What makes it turn let's say, to here? (point on screen) To that direction there?

K: Oh, uh . . . well because the . . . (looks for some paper to draw on, which I give him) when the cosine uh . . probable can't draw this very well . . . goes something like uh . . . roughly like that . .

R: The cosine gragh? (Kurt draws overlapping sine and cosine graphs)

K: Yeah. The sine and cosine, so there's, at one point they're uh, midway in between, they're uh .75, they'll be .75 at 45 degrees, so uh . . . and they never go above 1 so I have like a ratio . . . of uh the direction between uh where my x will be in a certain direction and y.
iv8

Uses a variable which he calls sine[ ] cosine[ ] where [ ] = first 0 to 15, then later, 0 to 31, and represent 32 positions around a circle, each the same angle away from the previousposition. v3, 7

Uses a variable called tilt, which takes on values between 0 and 31 corresponding to the positions around circle. Then caculates sine(pi/16*tilt) and cosine(pi/16*tilt) and stores them in sine[tilt], cosine[tilt] respectively. This gives him the positions of x and y coordinates of 32 positions around the unit circle (although not explained in that fashion by him) v3,4

Uses polar coordinates to calculate the position of the object on the outside of the circle. locx = cos[tilt]*radius, locy = sin[tilt]*radius (pp13) and later recognises (pp16)this is a mistake because the origin on the computer screen is in the top left corner and therefore must make use of transformational geometry in order to relocate the circle somewhere in the lower middle of the screen.

```
locx = centerx + cos[tilt]*radius
locy = centery + sin[tilt]*radius
```

For more see summary 6

Uses right triangle trig in session

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3. **Pythagorean Theorem**

Uses it in problem that motivated the study. see protocol

Uses it in the programming of David and Goliath – see Economy/Flexibility in the Mathematical Giftedness category

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K: (inaudible) about my car program is, it works and everything but it doesn't really need the pythagorean theorem uh . . .

R: What doesn't need it?

K: The car program. See . . . . . (types – goes back to car program) this spot here is where it uses the pythagorean theorem to um, calculate um, what the momentum of it's going to be, including all the slippage and stuff, but I think you could just use ratios, like . . . . it could probably just times a certain number by .9 and that would mean 90% of it would go in whatever direction and then the other one would be times point one, add them together, divide by 2. . . . or no . . . . . . yeah, I'd just add them together and that would (inaudible). vi16

In trying to find a relationship between the line joining the midpoint of adjacent sides of a triangle and the remaining side:
K: I keep on thinking distance though. (chuckles) the pythagorean theorem, um . . . . see what I keep on thinking is I keep on seeing this little triangle in my mind, like this . . .

R: Right.

K: And I know this and . . . . I keep on figuring um . . . . where this is and I calculate that by distance but . . . it doesn't seem to . . . (chuckles)

R: So what's this? Is this a coordinate system?

K: Um . . . Yeah. viii13

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4. **Absolute Value**

K: I started this program in the summer and finished it a couple months into school and I was kind of experimenting with it all of the time. I had it cycle through the colors and I just added a bunch of stuff. Um, so that part . . . . . it takes the x distance from whwre you are from the center of the check point and the y distance um . . . . let me think, it either adds them or takes the average . . . oh, oh I know, it takes the x distance, the absolute value of where your location and minus the center of the check point, for both the x and y, and if both of them are less than a certain number, it knows you're within the square, uh, the check point. iv13

R: To determine if it goes off the screen.

K: With only, only checking twice.

R: And how is that?

K: What it does is it - that's uh 320 is the middle of the screen along here and locationx is where the object is along here and uh, . . . it takes the center of the screen minus locationx and it changes that into an absolute value.

R: Which is what, what's an absolute value?

(here seems to be evidence of Kurt looking for the shortest, most elegant method of solving a problem)

K: It's, it's positive.

R: Do you know what it is though? Do you know what it does for you?

K: Well uh, I don't know, I just consider it as getting rid of all the signs (chuckles).

R: Getting rid of the negatives.

K: Yeah. So no matter what it is, it changes it to a positive, so I can work with it.

R: So what is it in fact giving you then if it changes it to a positive?

K: I guess the difference between two things whether or not it's on this side or on that side. And that's what this is doing so, um if it's 320 past this point, which would be if it's greater than 320 past this point uh, then it'll know that it's off the screen on that side, um, if it's less than then it'll be off the screen on that side and either way it'll tell me that it's greater than 320.

R: Ok, so how did you figure that out, were you just thinking about it one day or?

K: Um, it just popped in my head just now.

R: How did you know to use the absolute value function in your program? Have you used it before?

K: Yeah, I've used it once or twice before.

R: To do the same sort of thing?

K: Well no, to do something (inaudible), well actually I think I have (inaudible) before.

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5. Tangent Perpendicular to Radius of Circle at the Point of Contact

R: You have this thing spinning around in a circle and if you want to hit it (the target) there, where are you going to release this thing? The intention is that the person who is controlling this thing is the one that presses the button to let it go on a straight trajectory to hit that thing, to intercept whatever is coming across the screen here.

K: I thought like, what I thought was, this would be the point, and it would be changing the slope all the time, like from there to there, and then to shoot the thing I'd just take this -

R: Oh, and fire that off at it.

K: I'd, I'd just uh, I wouldn't fire in that direction, but I'd take, I'd know that was our slope, right? And then I would just take something perpendicular to it.

R: Oh, to throw it out.

K: Yeah.

R: Why perpendicular?

K: Uh, well because it's not going to fire out that way (points in outward direction). It's going to be moving, . . . because the rock is moving that way, so-

R: So if you let it go here, it's going to be perpendicular to this line here (the radius-sling)?

K: Yeah.

R: Do you know that just from intuition or did you know that already from geometry?

K: Well . . .

R: You just have a feeling for it, is that right?

K: Yeah (chuckles). iv18

R: Alright, when it gets to the point of release, when this thing's coming around here, what will be the mathematics of that?

K: Ok, I'll take uh, the . . . position.

R: The position from where you're releasing it?

K: mmm hmm. Uh, then I'll take, I'll take it's slope, well, I'll, I'll take the reciprocal of it , the negative reciprocal like what we were doing in math (chuckles).

R: Right,(this "right" is not meant to judge correctness, instead it's meant to convey my understanding, although I should have chose a different word) now just a sec now. You got it, you got it at a position here. Negative reciprocal of what?

K: That, oh no, well, it won't mean anything if it's 0, so it'll just stay at 0, but what I'll do is (inaudible - seems to be showing something on paper?)

R: You mean, ok, so you're taking a slope of a line from the center of the circle.

K: Uh huh.

R: And then taking the negative reciprocal of that?

K: Yeah. And of course if it's moving that way - it wouldn't make sense going that way - so I'll times it by the speed at which you're rotating it. That way if it's negative (inaudible) v8-9

R: Times negative speed. Now why did you multiply by negative speed?

K: I multiplied by negative speed because it's going to be (still enering stuff while answering), it's not going to be the actual direction of the, of the . . . where the line is, it will be perpendicular to it. . . .although I might have to reverse and make this negative speed (inaudible)

R: So speed is what? What mathematical quantity would speed be? vi2

R: See, if you're at a point (x,y) . . .and then you simply multiply, take the negative x, see here's (-x,y) (drawing it on page) does that make it pependicular?

K: No, (-y,x) . You, you'd switch them around.

R: Is that what you've done?

K: Yeah uh, . . . I did that originally, (chuckles) but I didn't know I had done it, so when you told me, I switched it around a second time but I didn't need (inaudible)

R: So you'd switch, you'd take this value's negative and put it as the x and this value's positive and put it as the y. vi4-5

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6. **Transformational Geometry**

R: Along the horizontal.

K: Uh, a variable, and I'll say uh, let's call it x for now. x equals x plus or minus, depending on what side its moving, uh if I'm moving that way , I make it minus, that's 0, 0 over there uh, I'll say x equals x plus or minus a certain number, like whatever the speed. v6

R: Ok, so at this point, in order to get it moving along at the speed that it was rotating, right, is that what you're saying you'd have to multiply . . . . what I'm trying to figure out is what the mathematics is, how you're going to get this thing to move from here to here.

K: Ok, uh. . .

R: From this point here to that line there.

K: Well, once I've calculated all that stuff, I'll store it in a variable, and I'll just each time, I'll, uh . . . . I'll store it in two variables, I'll store the x and the y and then um . . .

R: The x and Y of this position here when you let it go, ok and then you increase - then what do you do?

K: And then I just forget all this circle stuff and I'll go to a new routine where it'll, it'll add to this position, the x and y each time and eventually . . . v10

R: And what moves it from one location to the next?

K: That's what I'm going to do in procedure collision. I'll say uh, locx = locx + mlocx.

R: Oh, ok go ahead.

K: Loc short for location. Anyway uh . . .(continues) vi7-8

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7. **Use of Inequalities**
Uses inequalities to test the locations of objects on screen. See computer programs.

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8. **Own Formulas**
In reference to the area of the triangle formed by the coordinate axes and a straight line :
I ask, "what are you thinking"? He replies, " area = y-intercept times x intercept divided by 2. iii1

Next, I asked him to show me that the answer is in fact correct. First he thought it might be the wrong sign. Then he began talking about the slope equation. He gave the slope as - y-intercept/ x intercept. I asked him why negative and he answered because the line slopes down and shows it on the diagram. (he new this from class as well, but was not shown the intercept form of slope formula) iii3

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9. **Geometry**
R: How do you feel about Geometry?

K: Uh, it's okay, I guess. I don't, I haven't done a lot of Geometry, uh, . . . I guess except for the stuff on the computer. It's uh, I like it. It's okay. ii1-2

R: Now can you give me some indication as to why you drew that line?

K: Um . . . .it, it just helps me think because um . . . like. . . graphing kind of um . . . . . usually when I get confused with like uh, some sort of trigonometry thing, um . . . . .what I like to do is, like when I was thinking of three dimensional trigonometry, what would help me the most was when I was, like drawing-when I was thinking of the images in my head, and moving them around and stuff, um, and how I was going to do it, I uh . . . . sort of like drew, I had the point where uh . . . .like say I had a line that I had determined using sine and cosine, i would um . . .think about it kind of like this, and this is x um, this is y and this is x and um . . . .well now this could take me a while to explain (chuckles)

R: Ok, what I'm reading - you can tell me whether I'm reading what you're saying correctly or not - you're trying to interpret the geometry on a grid.

S: mmm hmm.

R: Using coordinates, is that right?

S: mmm . . . . .ok viii9

Attitude Towards Mathematics

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1. **Feelings Towards Math**
R: The first question is, do you like mathematics?

K: Yes. i1

K: I think it's important to be good at mathematics. ii2

R: Suppose we take the computer out of it completely?

K: It would get a little more boring. (laughs)

R: You'd find it a lot more boring, eh?

K: Uh, yeah. Uh, I'd probably still like it though. It just uh, it just makes it a lot more interesting. I find it even easier to understand when I'm working with it on the computer. i6

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2. Preferences

R: What sorts of things or activities would appeal to you in doing math?

K: Like, uh, . . . would I like exponents or different problems?

R: Yeah, do you like solving problems, do you like just doing calculations, you know, that type of thing?

K: I kind of like uh, . . . what I do on the computer a lot - I'll take an idea and I'll have it in my head but if it's all in written words and pictures and uh, . . . I'll put it into a mathematics problem.

R: Something's in words and pictures? -(Kurt interrupts)

K: Yeah in my head and then I'll, I'll, figure out how I can get the computer to do it or, . . how I uh, . . . solving like a practical problem and putting it into numbers and algebra. i1

R: So you like the challenge.

K: Yeah. i2

R: If you had your choice, what sorts of activities or topics would you like to do in learning mathematics?

K: Definitely computers, uh, . . .

R: Can we get any more specific? You don't have a huge repertoire or background in mathematics to pick from, but if I said to you," Ok Kurt, we've got 3 months of time in the mathematics classroom and I'd like you to be involved in your choice of what we could study, what types of things would you like to do?

K: I know what I'd like to do but when it comes to saying it (laughs) . .

R: You are finding it hard to express what you'd like to do?

K: Yes.

R: Just throw out any word, you don't have express yourself in a full sentence, just throw out any idea.

K: Like uh, I know my favorite part when I'm working with computers, . . . like when I'm making a program or something, I'll be taking . . . . (inaudible stuff) . . . I just love really interesting and complicated problems to do something different than what I've been doing before. I like to try new things. I like using sine and cosine a lot.

R: When you say you like to figure out formulas to do certain things, does that mean for example, that if someone gives you a problem in computing that you want to solve with the computer and you have to find some mathematical formula to program the computer to do the job for you, is that the type of thing you mean?

K: Yeah, like uh, . . . what I'm working on now is I wanted to make sort of like a little snake that would crawl along on the computer and you would rotate it and it would go in that direction and each little part of the snake that was following along would follow the snake but it wouldn't do exactly, it would kind of swerve because of momentum and stuff - I like doing that kind of thing - I've tried to figure out how I'm going to get that into the computer, how am I going to get it - what are the calculations I'm going to be doing to make it do that and that sort of thing. i3-4

R: So basically, you'd like to involve the computer as much as you could in your learning of math, is that right?

K: I find it fun to use the computer and (inaudible) at the same time. I like taking something from words and putting into numbers and stuff and in a way it kind of increases my understanding of what I'm talking about - what I'm thinking about.

R: What sorts of things or activities appeal to you in Mathematics?

K: Uh, . . . I think I like, uhm, word problems cause they have some meaning behind them. Uh, . . . I like using problems that I can apply to things.

R: Okay, So if you had your choice, is that all you would want to do is to solve problems throughout the whole year? What else would you like to involve in your learning?

K: Uhm, . . . I like to use computers, of course. Uhm, ... ii1

R: How do you feel about your abilities in Mathematics?

K: Uh. I'm not sure, uh. . . . . . . I can't be too bad because I have a lot of fun working with computers and uh, I use a lot of Algebra when I'm . . Uh, . . . I don't know. ii2

R: If you had your choice what sorts of things or activities would you like to do in learning mathematics?

K: Anything to do with computers (laughs) . . . I have a lot of fun with computers. . . . . . I would like to learn more about trigonometry; it seems interesting from the little bit that I do know.

R: You mentioned something to me at the end of the last session about one of the reasons you enjoy math; that you always try to see . . . what was it that you said.

K: How it relates to things in real life. . . . . I, I, there, there's not a lot of useless math problems. There's lots of stuff you can apply mathematics to. . . . just, you know, it helps me think logically. They seem to be sort of intertwined, logic and mathematics. ii3

Special Ed. teacher: I think a key to Kurt is whether or not he thinks something is worth doing and unfortunately in life, we don't always have those kinds of choices. Things like the mundane things of learning how to calculate in math are necessary to learn things at a higher level. Well Kurt sometimes, I think maybe, because he learns a different way, can go beyond those steps, doesn't necessarily have to learn them as systematically as some students do, in areas that he's gifted in, such as math. And so when he has to be forced to go through things such as calculations, he doesn't see them as being valuable or relevant and so he doesn't devote the time to them. And he tends to have, I think, a kind of, arrogance in that respect, where he thinks he is in a position to decide what's valuable and what's not valuable and I think it's tied to success as well. One example is social studies. He reads very well. He has very good background knowledge. He likes history. He likes discussing it, but he does not, he says he is not a good social studies student. Social studies is a wipe-out for him. And his attitude is very poor towards it. So as a result, even though his comprehension is very good, even though he's interested in the subject area when you're discussing it, he does not respond to the structure that it's taught in. But he will not, say, sit down and study for a test. If it's something he's remembered from reading it the first time, he will recall that, but if it takes any kind of work, any review, any sitting down and disciplining himself to review things, that he doesn't see as being possible to be successful at, because he maybe had lack of success very early on, on written exams, for example, or didn't want to take the time to go and read over those notes again, um, certain things he doesn't want to invest time in. So he's really made a lot of decisions about what he likes and doesn't like at a very young age, which is very difficult because we're in the job of exposing him to many things that he may not like. And as a result that makes him disabled in the sence that . . . he, he doesn't fit the pattern of the typical student and unfortunately, school is structured to meet an average range of students that learn in a certain style, a very sort of, structured approach and Kurt doesn't learn that way. ix 5

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3. Dislikes

See special ed. teacher's comments above.

R: Like for instance, is there anything you've done in mathematics that perhaps your teacher has given you and you think "Oh God, this is boring' or 'I wish I didn't have to do this", something that just doesn't appeal to you?

K: When it's stuff maybe I already know really well that I can almost get the answer to . . . (inaudible stuff) . . . challenge. i2

R: Suppose we take the computer out of it completely?

K: It would get a little more boring. (laughs)

R: You'd find it a lot more boring, eh?

K: Uh, yeah. Uh, I'd probably still like it though. It just uh, it just makes it a lot more interesting. I find it even easier to understand when I'm working with it on the computer. i6

R: When does Mathematics bore you?

K: Uh, . . . Okay, uh, . . . If I could, I wouldn't deal with the Arithmetic at all. (chuckle) Well, not, not completely, of course -

R: You mean like calculations, and stuff like that?

K: Yeah, uh, I prefer Algebra, stuff like that. ii1

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4. **Importance/Usefulness of Mathematics**
K: I find it fun to use the computer and (inaudible) at the same time. I like taking something from words and putting into numbers and stuff and in a way it kind of increases my understanding of what I'm talking about - what I'm thinking about. i4

K: Uh, yeah. Uh, I'd probably still like it though. It just uh, it just makes it a lot more interesting. I find it even easier to understand when I'm working with it on the computer. i6

K: Like uh, I'd, well, it wouldn't be with the computer actually, it probably, I might uh, take everything, replace a number, see - then work it out, see . . . (inaudible) . . . there's no unknown and then see what kind of answer I get and sometimes that helps me see how to solve the problem. On the computer, if I replace it with variables some times it helps me look at everything in relation to everything else like so I can see a little more clearly sometimes when uh, . . . what, sort of like uh, when you have a number and you want to express it in terms of another uh, when I put it with variables sometimes I can look at it and it gives me a better idea of uh, . . . what it is, uh, what it is the problem actually means. i11

K: Uh. I'm not sure, uh. . . . . . . I can't be too bad because I have a lot of fun working with computers and uh, I use a lot of Algebra when I'm . . . Uh, . . . I don't know. I find it hard to look at that question from an objective point of view because I'd like to be good at Mathematics, but I don't know if I am. I, I think it's important to be good at Mathematics.

R: What would be some of the reasons?

K: Uh. Well. . . I think programming will be useful and of course I use mathematics, uh, everyday sort of stuff . . . depending on what carreer I go into, uh, I know my one brother Ken is thinking about engineering. He phoned up recently and he told me that and of course and he told me that and of course that would be something you need mathematics for. . . . . . or say if you just wanted to do home renovations and things.

R: So you think it might be useful at home too?

K: Ya. ii2

Self-Efficacy / Self-Concept

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1. **Comfort**
R: How do you feel about your ability in math?

K: I'm not sure uh, ... some stuff I seem to have a natural feel for and other things kind of confuse me. Most of the time I actually feel pretty comfortable. i2

K: "It's kind of funny, when I see it on a piece of paper it seems different from seeing it on the computer even though it's basically the same. Like . . . I, I put my brackets here and there and I make sure that everything is arranged accordingly so that uh, like I pay attention to all the rules that I normally do when

I'm, when I'm doing stuff on paper. But in a way it's different. It could be just that I feel more comfortable on the computer, like there's not the pressure there because uh, . . . I don't know, like I use the pythagorean theorem sometimes um, to calculate distance a lot on the computer but sometimes I can still boo boo when I'm, when I'm doing it on paper whereas I never make that mistake on the computer. i6

K: . . . "I can get a lot of variables stuffed into there and I still don't make a mistake as much as if I were doing it on paper. Maybe I just feel more comfortable. . . . and in a way it's better on the computer because if I make a mistake, I know it's easier to figure out where I went wrong because the computer will still do the program right, it will still go through the program. But, it will uh, because it'll usually go through the program several times and like in less than a second, I get to see what's going on and I see how the numbers progress, usually on the screen with graphics and stuff. So I can see, oh I bet I know what happened, I probably put a minus here where I should have put a plus or I times it and I should have divided it or I didn't square it over here when I should have and I can kind of see what's going wrong. i7

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2. **Pressure**

R: Generally speaking, at the start of a math test which your teacher gives you or you know there is a math test coming up, how do you usually feel about your chances of success on the test?

K: uh, . . . mediocre, I don't have a lot of confidence when I'm under pressure. i2

R: If I gave you a problem, is that the first thing you might think of is to how to use the computer? Is that how you would try to solve the problem?

K: Um, actually a lot of time I'll just do it on paper and pencil. But uh, . . . sometimes in a way I'll figure out how to do those problems because I thought, oh hey, I did something like that on computer and I worked it out this way. It's kind of funny, when I see it on a piece of paper it seems different from seeing it on the computer even though it's basically the same. Like . . . I, I put my brackets here and there and I make sure that everything is arranged accordingly so that uh, like I pay attention to all the rules that I normally do when I'm, when I'm doing stuff on paper. But in a way it's different. It could be just that I feel more comfortable on the computer, like there's not the pressure there because uh, . . . I don't know, like I use the pythagorean theorem sometimes um, to calculate distance a lot on the computer but sometimes I can still boo boo when I'm, when I'm doing it on paper whereas I never make that mistake on the computer.

R: Right, so why is that?

K: I don't know, maybe it's just that I feel pressure. i6-7

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3. **Confidence**

R: Generally speaking, at the start of a math test which your teacher gives you or you know there is a math test coming up, how do you usually feel about your chances of success on the test?

K: uh, . . . mediocre, I don't have a lot of confidence when I'm under pressure. i2

R: So you'd switch, you'd take this value's negative and put it as the x and this value's positive and put it as the y.

K: Yeah. But I think uh, cause it's not as much slope, it also has a direction, I might end up going, it might be spinning like this and end up going in that direction, but I'll find out soon enough.

R: When you try it.

K: Yeah. vi5

R: Oh you mean it sends you to the curser? The error message? Does it send the curser to where the mistake was?

K: Yeah.

R: Oh, that's good. Oh, yes I see.

K: It's a great way to solve all my problems. (chuckles) vii8

K: ......put....plus...um.......I don't know if I'm doing this the right way......

R: Well it doesn't matter whether you're doing it the right way or not. We're just trying to see what you're thinking here. So don't worry about it. viii11

R: I'd like to have a copy anyway.

K: I think I'll give a few grade 11 and 12's copies if they want them...Ok,...keep that and change some numbers around. (he's borrowing stuff from his car program)..... iv15

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4. Self-Efficacy / Self-Concept

R: How do you feel about your ability in math?

K: I'm not sure uh,.....some stuff I seem to have a natural feel for and other things kind of confuse me. Most of the time I actually feel pretty comfortable.

R: Do you feel that you have achieved as high as your potential might allow?

K: No.

R: Can you think of any reasons why?

K: (long pause - no response)

R: Generally speaking, at the start of a math test which your teacher gives you or you know there is a math test coming up, how do you usually feel about your chances of success on the test?

K: uh,... mediocre, I don't have a lot of confidence when I'm under pressure. i2

R: How do you feel about your abilities in Mathematics?

K: Uh. I'm not sure, uh....... I can't be too bad because I have a lot of fun working with computers and uh, I use a lot of Algebra when I'm...Uh,...I don't know. I find it hard to look at that question from an objective point of view because I'd like to be good at Mathematics, but I don't know if I am. I, I think it's important to be good at Mathematics. ii2

At the end of the session (5), after the tape recorder was turned off, I suggested to Kurt that it might be good to evaluate his performance in school more an a problem solving basis, because that what he seems to enjoy and is skillful at. He then mentioned that he thinks he is good at problem solving because of his work with computers. According to Kurt, in order to create his programs, he needs to create equations and relationships. He knows so much about computers because he's worked on them for five years. journal

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R: Do you feel that you've achieved as high as your potential might allow?

K: No (laughs).

R: Any idea why that might be the case?

K: Uh, it's hard to say, uh . . . I think I could be better at mathematics but I don't really know why I'm not living up to the standards I've set for myself. Uh, ... it boggles me (laughs). ii2

In trying to determine an expression for the sum of the first n odd integers, Kurt is having some difficulty and says "This in computers I'd have an easy way out of this (laughs). ii10

K: I can imagine the way it could be if I could type (chuckles) v13

R: You've obviously fiddled around with this stuff a lot already, eh?

K: Yeah. It's kind of funny, someone asked if I knew a lot of computers and I said that I did, a lot more than a lot of people in this school, not of course includung Mr. Bird - (laughing from R) - uh, and I got the impression that they thought I was being arrogant or something or other and I went home and I thought, well, if I were being arrogant, I don't see how that could be because if I didn't know more than these people, I'd be ashamed because I've been working with these for like five years. (laughter) v20

R: You're just writing relationships down.

K: Yeah. . . . . See if I can figure out that they're parallel, I should be able to figure out what their lengths are (chuckles) . . . . .I've never tried anything like this before.

R: Well, ... that's the whole point. I want you to try things that you've never seen before. viii13-14

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5. Humor

R: Ok, what do you dislike about mathematics, that's been in your experience so far?

K: Division by zero. (laughs) i1

K:Anyway, when you get here, around here I think it is, there's a check point and it says , ok you're done and uh, it increases your score uh, by the time left. So sometimes, I guess if you go around to slow you get negative times (chuckles). iv4

K: Well if it hit this, if it hit this, hit this, it would forget checking whether it went off the screen and um . . . . if it went off the screen then I'd just have it ask you if you want to try again.

R: Ok, well that's interesting.

K: Maybe I could have it do anything like I could insult the guy now. (chuckles) vi15

R: You're going to call that angle up at the top A.

K: Um . . . I guess that's the top from where you are (chuckles).

R: That's right. It's the side from where you are. viii3

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6. **Self-Motivated**
I don't know, it's hard to explain. I could probably learn a little bit more about sine and cosine um, because I'm not sure exactly what the difference is between the two. I've tried looking in physics books and tried to look around in every mathematics book I could find and I still didn't get a very good idea. Then after a while I just kind of forgot about it and just . . .(inaudible) i6

K: Um, yeah. Usually I never write instructions (chuckles) in my programs.

R: Because you're the only one using them?

K: Most of the time I write them for myself and once in a while someone in the computer room will play with it. . . . . (continues) vii4

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7. **High Standards**
K: Uh, I think I could be better at mathematics but I don't really know why I'm not living up to the standards I've set for myself. Uh, ... it boggles me (laughs). ii2

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8. **Nervousness**
R: Why do you use the sine and cosine?

K: Um, . . . that's just to make sure that uh, see if I weren't to use sine and cosine, . . . I don't know, uh, well, why wouldn't I? (chuckles) iv6

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9. **Feelings About Misunderstanding**
K: And then there's times when you're just sort of stuck and you can afford to do this (chuckles). . . . . Ok, there's one thing though I don't get about this and I really should look into and it's uh, this thing having to do with checking this key press. My brother put it in there and I know if I take it out it doesn't work. I don't get why not though.

and later on same page,
R: But you need it.

K: But I need it. (chuckles) . . . . It's rather annoying though because when you're programming, you like to assume you know everything in your program, what it does and when it's one thing there it gets annoying. iv23

Mathematical Processes

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1. **Creating Relationships** see Formalized Perception / Relationships in Mathematical Giftedness category

2. Generalization see Generalization in Mathematical Giftedness category

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3. Justification

R: How do you know for sure that that (the formula) is correct?

K: Um, because I could take every one of these and do that and then I could keep on going what I think is the next pattern and this works for every one that I can come up with - pick any one of these that I know would fit this pattern at random and I know it would work for that. ii4

Next, I asked him to show me that the answer is in fact correct. First he thought it might be the wrong sign. Then he began talking about the slope equation. He gave the slope as - y-intercept/ x intercept. I asked him why negative and he answered because the line slopes down and shows it on the diagram. (he new this from class as well, but was not shown the intercept form of slope formula)

He finally substitutes the solution into the area equation, but instead of using radical arithmetic (taught at the start of the year), he feels he needs a calculator to do the rest of the check. iii3

K: Ok these, yeah these are parallel.

R: Which are parallel?

K: These two lines.

R: Is there any way you can prove that?

K: Um . . . I'm thinking about it. . . . . (12 seconds later) Ok, well the side along here is double to the sides here and they have the same angle . . . so . . . the ratio of the sides is going to be the same . . . and seeing they have the same angle, they're going to have to be parallel. (SAS for similarity?) . . . uh . . . . .yeah viii2-3

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4. Pattern Recognition

He determines the general formula for the nth term of the sequence 2,7,12,17,22, . . . as (n-1)*5 + 2 (first wrote n*5 + 2)

R: Now, how did you get the plus two here Kurt?

K: Um, ... they unh, they weren't multiples of five but each one had five extra than the next one and each of them could, um, be minused by two and it would equal a multiple of five, so I knew that n times five plus two in each case would give me one of these.

R: Does that in fact work?

K: Yuh.

R: Does that work for the second term? . . . . . Ok, how would it work then for the second term?

K: Uh, n would be one and it would be times five-

R: For the second term n would be one? the second term n would be?

K: Oh, uh, oh yeah (surprise) uh. . . . . . wait a second, I think I got myself confused, so in each of these the first one of n is one?

R: Yeah, that's the first term. this would be the first term (points), the second term, third term, fourth term, fifth term and so on.

K: Ok, then I have to change this a bit.

R: So, you put an n-1 in there?

K: Yeah. ii4-5

R: Can I ask you something Kurt before you go on to the next one. What makes you add one to n here all the time? See in all these formulas you're putting n plus something in brackets and then dividing by 2 , ok, for these ones that were a half (point to fractional examples) - here you went (n-1)*5 +2, here you went (n+1)*8+1 - Why do you put the n+1 in brackets?

K: Uh, because these are supposed to be the first term, right?

R: Right.

K: So, n equals 1?

R: Right.

K: And uh, . . . it just doesn't fit to have n equal to 1 (laughs).

R: You mean it just doesn't fit when you just have n, is that what you're saying?

K: Yeah.

R: Ok.

R: But how do you determine the pattern like that - how do you determine that you have to add one- just by looking at the first term?

K: Uh, usually uh, after I've gone through it, I look at the relationship between these and I get the rest of the stuff and then I figure out that, well, that's kind of times 2 really and then so I figure well I guess I'll have to add one to n. Like I sort of figure out what the relationship is between these guys and then I figure out where it starts.

R: And what's the relationship between each number in number 9?

K: Uh, there uh, in number nine they're uh, times 8 plus 1.

R: How did you know to times by 8? What made you see that it was eight that you have to multiply by?

K: Well, I knew there had to be uh, they couldn't be . . . uh . . . they didn't seem to make any sense the way they were there so I tried minusing numbers and looked at what I got and if some of them weren't . . . I don't know. . . when I minused 1 it looked like it needed minusing or something or other (laughs).

R: Well, in each case here you either multip[lied or divided by something. Now what made you decide on what number to multiply by?

K: Uh, . . . What do you mean like (laughs)?

R: Ok, well see here your pattern is n times 3 and here it's n times 2 and here it's n times 3 plus 5 and here it's n minus 1 times 5 and suddenly here you are dividing by 2, here you're dividing by two, now you're back to multiplying by a number. How do you figure out that number there, that you have to multiply by?

K: Uh . . . well . . . I'm not sure exactly, uh, I think I must have minused or something, subconsciously -(R interrupts)

R: Ok, lets go through the next one, number 10, and try to think out-loud as to how you are figuring out the pattern so that I can here what is going on in your head.

K: Ok, uh, . . . they look like they're multiples, uh, . . . and 9 and 23 don't uh, don't have uh, aren't any multiples of something but I know they must be because of the way that they are - if you subtract 23 from 9, if you subtract 37 from 23, I know I'm going to get the same number uh.

R: So what number do you get in that case?

K: I get . . . I don't know how I figured it . . . welkl it just looks like it's being multiplied - I kind a . . .

R: Ok, so you subtract the numbers and what would you get in this case, what is that common number that you get?

K: Uh, . . . fourteen.

R: Ok, go ahead and try to come pu with the rule and the next few terms.

K: 21 is close to 23 so if I minus by 2 and if I minus by two they both come to be a multiple of 7, uhm, so it's whatever times 7 plus 2 ,uhn . . . . .

R: How come it doesn't work?

K: Oh (laughs) cause, uhn, well I think the rule works but I messed up somewhere.

R: Show me that the rule actually works fort the first three.

K: Uh, 1 times 7 equals 7 plus 2 equals 9. . . . . no it doesn't work (laughs), it doesn't work. Uh . . . ok.

R: Does it work for the next one?

K: . . . if n were 1 there, it were 3 there . . . and . . . if it were 5 there . . . ok, uh, . . . n times 2 minus 1 times 7 plus 2.

R: Does that work now?

K: Works for the first 3 uh, ... ya.

R: What did each one of these questions have in common? What was similar about all of them?

K: Oh, you already said that, they're either multiplied or divided by uh, . . . , and one's either multiplied or divided by somewhere along road, in here it was after a plus or something.

R: Ok but the actual patterns themselves, ok, forget the rules for a minute. What is similar about all the patterns?

K: uh, ... you can say that uh, the first and the second number are different in the same way than the second and the third number are different. ii6-9

In trying to find a formula for the sum of the first n odd integers, he says:
K: I'm thinking that (chuckle) . . . I don't think I know how to do this because . . . I'd have to have series of numbers, an indefinite series of numbers depending on what n is.

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5. **Explanation Using "If It Wasn't So, Then . . ."**
R: How come you use the sine and cosine?

K: Um, . . . that's just to make sure that uh, see if I weren't to use sine and cosine, . . . I don't know, uh, well, why wouldn't I? (chuckles) iv6

R: And what's this pi/8?

K: Uh, that's uh . . . I think if I don't do that I'll run up, I'll get a really wacko uh, . . . like it'll increase by too much so it'll be like going around 5 times-

R: So do you understand what the pi divided by 8 is though?

K: yuh.

R: What's that?

K: Ok, well, the divided by 8 is to make sure that I don't go all 360 degrees.

R: So pi is how many degrees?

K: Uh, pi? Pi is 3.14 (nervous chuckle) Well, it's uh . . .that, just pi I think would be uh, would be 1 degree. . . . No, no, no, sorry 360 degrees.

R: Would be pi.

K: Yeah. And so I divide it by 8 to get my . . . uh, actually, I haven't, . . . you know I did this once and I forget how I did it (chuckles).

R: Well, because you've got 16 different directions, right?

K: Yeah.

R: And pi divided by 8 would only give you 8 directions.

K: Yeah, so I guess it's . . . pi would have to be 180 then. iv7

6. Conjecturing

R: Well, what I'm doing - I want you to come up with a conjecture. Do you know what a conjecture is?
K: Um . . . .

R: If you can see if there is a connection between those two lines somehow.

K: Ok these, yeah these are parallel.

R: Which are parallel?

K: These two lines. viii2

R: Is there anything else you might be able to say about those two lines?

K: Well one's going to be a certain . . . . . . If I knew this angle and the length of this side um . . . I could figure out what the length of this side is . . .I think.

R: What would you say is the relationship is between the two lengths.

K: Uh . . . . can I name this - I'm going to call this angle A.
viii3

R: Can I ask you this? Have you thought yet. . . have you any idea or any guesses as to what the ratio of that thing would be to that? Which is the original problem.

K: Um . . . . 75?

R: You think this is three quarters of that.

K: Yeah. viii12

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7. Error Recognition

R: What's going on in your head right now when you see that there, that $x^3 - x^2 + x =$ what, -60? What are you trying to do?

K: Uh, I'm thinking how I can get rid of the exponents. (38 sec later scribbles earlier stuff out) Wait a second, no that doesn't work.

R: Why?

K: Uh, I have to divide everything by 5 and divide everything by 2 and divide everything by 3 but I was only diviving one of them by 5 and one of them by 2 and one of them by 3.
(19 sec later) I can't get . . (inaudible) i10-11

K: Oh, you know I'm going to have a funny looking circle because it's going to be on 0, right so it's just going to go (inaudible-automatic?)

R: Oh, ok.

K: So, I better change that. (types) vi18

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8. Sudden Ideas/Realizations

R: Oh I see. But this thing here, isn't gx, isn't that the thing you've got moving across the top of the screen, Goliath?

S: It'll change loction x if I do that, so I better not, so I'll have (I don't think he was listening) gx = gx + 1 and that we'll have to (inaudible mumbling). . .Oh, I missed something up here. No use having Goliath if you can't see where he is. vi8

R: Ok, so how did you figure that out, were you just thinking about it one day or?

K: Um, it just popped in my head just now.

R: How did you know to use the absolute value function in your program? Have you used it before?

K: Yeah, I've used it once or twice before.

R: To do the same sort of thing?

K: Well no, to do something (inaudible), well actually I think I have (inaudible) before. vii 2-3

R: Well if that's what you want to pursuit, draw yourself an x and y axis.

K: Well I just . . . got another idea right now.

R: Oh, ok.

K: I don't know if this is going to help any, it probably won't. (Chuckles)

R: What's the idea?

K: (mumbles)

R: Ok, you're drawing the perpendicular again.. . . . . flipping back to a previous diagram . . . . .

K: Ok um . . . . I'm going in the same direction again um viii12

R: So you're labelling everything with variables. . .. .

K: Ooooooh. . . . . . Ok, ok, I think I got something now. . . . . . . .that uh, that cliff thing, that that cliff um . . . .

R: Oh, the cliff problem in class?

K: That would be the lake there, and these would be the banks.

R: Oh, I see, like the question on your test?

K: Yeah. . . . Ok, that's going to make things easier now (chuckles) . . .now that I've drawn that 90 degrees down there . . .ok, because I knew I had to fit a 90 degree angle in there somewhere and I couldn't figure out how I was going to do it. . . . Ok, ... the problem is what does z have to do with anything . . . . um . . . . problem is i have everything upside down here so I can barely read it . . . . . . hmm . . . I don't know if that's going to help me . . . . . L2 . . . . .viii14-15

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9. **Curtailment of Reasoning** see Mathematical Giftedness category

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10. **Planning/Organization**
On a second page, he writes side/4 = side, seemingly losing track of which side is which, but he knows he has to substitute into the area equation.

Next, he writes side2 = 4 side1, and then scratches out the 2, scratches out the 1 and replaces it with the 2.
Next he substitutes (for side2) into the area equation and obtains (side * side/4)/2 = 4
Then, solves to obtain side = square root of 32. iii2

R: Let's just see - can you clear this and start - you'd want to fiddle with a piece paper first, eh? Before you start programming anything?

K: Sometimes I do a little of both um, sometimes I don't , I don't know . . . Ok, um, I'll probably keep some of this program seeing it'll help me out (chuckles while saying this). A lot of times I don't make anything up from scratch anymore. iv14

R: What is "keys". Oh, that tells you that you can use certain-

K: Um, well that'll be the subroutine where I'm going to put in . . .like it'll check for when you fire and stuff.

R: So you have to make a separate subroutine for that later. Is that what you're doing here, naming these things?

K: Yeah, I'm just kind of like, it's convenient in a way, cause then you can just do a general thing of what you're going to do. Like I know -I don't do this a lot - but I know my brothers uh, Jerry and Ken, before they do all this they'll get a paper and they draw little pictures with a, like a line and uh . . .it'll just sort of say, "do this"-

R: Flowchart.

K: Yeah, a flow chart. And it'll be very general.

R: You don't do those things?

K: Uh, I do them once in a while when the program gets very complicated, so that I don't lose . . .(chuckles)

R: Sight of what you're doing. But you don't think this is going to be that complicated.

K: Uh, no. (chuckles) . . . .(enters more stuff) iv20-21

R: Any time you need to write stuff; If you need to write anything, just let me know here because I've got a paper and pen.

K: I never usually do have to write stuff. That worries me because most programmers do (laughs). My brother says it would help my organization but um . . . Ok . . .(types stuff - types in locx: cos[tilt] *radius) v13

K: Oh, I know, gx for Goliath (chuckles)

R: gx for goliath, ok (I laugh)

K: gx, gx minus 1 (inaudible) . . . I do things in a mish-mash sort of way.

R: Yeah, you're going back and forth aren't you. v14

R: What about instructions to the person using the program, that you'd have to put in there somewhere too, eh?

K: Um, yeah. Usually I never write instructions (chuckles) in my programs.

R: Because you're the only one using them?

K: Most of the time I write them for myself and once in a while someone in the computer room will play with it. . . . . (continues) . . . .

R: What are you looking for now?

K: I was wondering whether I put my variables that aren't constants underneath this or above it.

(continues) . . . what I should have done is listed these out as I went through them.

R: Those are your variables.

K: Yeah.

R: Now you've got to go through all of it and look them up.

(continues)
K: Then after I figure out what they are I'm going to have to sort them into integers and real numbers. vii4

Tends to write all over the page without regard to order. see protocols.

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11. Creativity
R: When you say you like to figure out formulas to do certain things, does that mean for example, that if someone gives you a problem in computing that you want to solve with the computer and you have to find some mathematical formula to program the computer to do the job for you, is that the type of thing you mean?

K: Yeah, like uh, . . . what I'm working on now is I wanted to make sort of like a little snake that would crawl along on the computer and you would rotate it and it would go in that direction and each little part of the snake that was following along would follow the snake but it wouldn't do exactly, it would kind of swerve because of momentum and stuff - I like doing that kind of thing - I've tried to figure out how I'm going to get that into the computer, how am I going to get it - what are the calculations I'm going to be doing to make it do that and that sort of thing. i3-4

R: So you've drawn it out in a different shape?

K: (no response) . . . . this may be uh . . . makes me think of uh . . . . that uh . . . . .maybe if I just assume that this is perpendicular to that.

R: Ok.

K: Even though it doesn't look like it. (chuckles) . . . . .this makes me think of, what do you call it . . . . . .that uh IBM (inaudible) 3-dimensional graphics . . . projecting things, because this is a lot like it . . I think. . . . .(begins chuckling) . . . viii14

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