

ESTIMATING IMPLIED CORPORATE DEFAULT PROBABILITIES FROM BOND PRICES

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ABSTRACT

In this paper we investigate a reduced-form model for dynamically estimating the risk-neutral default probability distribution of a set of US corporations using the constantly changing information in the corporate bond market. The strength of this approach lies in its simplicity and reliance on market sentiment embedded in the bond prices, and does not rely on stale and sometimes inaccurate accounting information. This reduced-form model approach of estimating implied creditworthiness of firms has the advantage over traditional credit ratings.

Keywords: Reduced Form Model, Implied Default Probability, Credit Ratings.

*To our wives
for their support and encouragement
during our absence from home.*

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1 INTRODUCTION

In light of the recent financial debacles and the public outcry over the demise of companies such as Enron and WorldCom the credit rating agencies have come under scrutiny for failing to predict credit related events. Many finance professionals are questioning the ability of the credit rating agencies to accurately predict the default of companies or their ability to warn investors of imminent financial danger. The three biggest agencies; Moody's, S&P and Fitch, together account for the overwhelming majority of credit scoring of private and public debt all over the world. S&P alone rates \$30 trillion dollars worth of debt, which includes over 750,000 securities and over 40,000 borrowers (The Economist 2005a). The practice of credit rating has a long history but its real boom began with a Wall Street analyst named John Moody who created an easy to understand credit scale and published his first book Analysis of Railroad Investments in 1909. By 1924, Moody's ratings covered almost the entire US bond market. Numerous other companies began emerging: Poor's Publishing Company in 1916, Standards Statistics Company Inc followed in 1922 and later Fitch Publishing Company in 1924. (Poor's and Standards Statistics later merged in 1941 to form S&P) (Cantor and Packer 1994). A copy of each companies rating scale can be found in Table 1.

In the early years, the rating agencies had to build their reputation on their ability to accurately predict the default of US companies. Their information was by and large accepted by the public who in turn paid the rating agencies for such information. Any ratings given by one of the agencies was subject to public scrutiny. In addition poor assessment could result in the demise of the entire agency. The credit rating agencies were only as good as their last rating and relied on their reputation to build a successful business. Their sheer existence in today's financial

world should be a testament to their abilities but lately there has been considerable backlash among many in the finance industry regarding the ability of credit agencies to accurately rate companies.

“Corporate finance professionals are questioning the accuracy and timeliness of credit ratings and believe the Securities and Exchange Commission (SEC) must take a more active role in promoting competition among the credit rating agencies, according to a new survey released today by the Association for Financial Professionals (AFP).” (The Economist 2005b, p. 67).

In the above survey, 36% of recently upgraded companies believed their ratings were inaccurate, 48% believed the ratings not timely. Of the organizations that recently received a downgrade, 50% believed their rating to be inaccurate. While finance professionals question the validity of the credit ratings, the public at large questions how the rating agencies could continue to hold investment grade ratings (see Table 1 for investment grade) for Enron and WorldCom right up until the companies went bankrupt.

One of the arguments in defence of the rating agencies' lack of action in the Enron bankruptcy is the difficulty of their position. Enron had many contracts with debt holders that had downgrade triggers, enabling the debt holders to call for cash if Enron was downgraded below investment grade. Credit rating agencies knew that if they downgraded Enron below investment grade they would effectively force Enron into bankruptcy with all the cash calls that would have ensued. With Enron fighting to turn things around in October 2001, the credit rating agencies held an investment grade rating on the company right up until its eventual collapse in December of 2001 (Hull 2006, p. 496). Was this the right thing to do? Enron went bankrupt anyway; could the agencies have saved a lot of people a tremendous amount of money by forcing Enron's hand earlier? How could the agencies that are supposed to be objective and autonomous have found themselves in such a position? The rating agencies have denied that the “triggers” in Enron's debentures had anything to do with their reluctance to downgrade the company.

However there is no denying that rating agencies have become institutionalized into the financial structure of North America.

In this study we show that a modified reduced-form model of credit risk based on Duffie and Singleton framework that infers default probabilities from observable bond prices does a better job of predicting counterparty defaults in the short-term than rating agencies. The paper is organized as follows: Section 1 provides background to the credit rating agencies and some of the problems with using their work as an accurate predictor of default. Section 2 presents a literature review of default models (structural and reduced-form models). Section 3 provides the data and methodology used to test the validity of an alternate default prediction model to credit ratings. Section 4 will be the results of the test and Section 5 our conclusion.

2 BACKGROUND

2.1 History of Credit Rating Agencies

After the stock market crash in 1929 and the hard times of the early 1930's as the capital markets were in disarray, the Federal Reserve Banks began to use bond ratings as a way to examine the credit worthiness of their member banks. A system was devised to use the credit ratings as a way to identify the safety of the member banks portfolios. But it was not until 1931 when the first formal rule incorporating ratings was enacted. The US Treasury adopted credit ratings as a proper measure of quality of Nation Banks bond accounts. This was a big stamp of legitimacy to credit rating agencies that changed the market forever. Specifically, the Comptroller of the US Treasury ruled that bonds rated BBB (or equivalent) or higher could be carried at cost by the banks but any bonds with a lower credit rating were to be written down by some fraction (Cantor and Packer 1994). Many of the state banks followed the move. In 1936 the Comptroller issued the following ruling:

“The purchase of ‘investment securities’ in which the investment characteristics are distinctly and predominantly speculative, or ‘investment securities’ of a lower designated standard than those which are distinctly and predominantly speculative is prohibited...*

....*The terms employed herein may be found in recognized rating manuals, and where there is doubt as to the eligibility of a security for purchase, such eligibility must be supported by no less than two rating manuals.” (Partnoy, 1999 p. 688)

The ruling meant that any assets held by the banks must be rated by an agency and have an acceptable rating level. At the time of the ruling there were approximately 2000 publicly traded bonds listed of which only 1000 met the specifications laid out by the Comptroller. Prior to the ruling many banks invested in bonds below the BBB rating but following the ruling they

could no longer do so. Not only did this change the way the banks operated but it also led to changes for the credit rating agencies. Up until this point in time, bonds had been rating by the agencies after they had been brought to market. The new law created a need to have the bonds rated prior to entering the market. The Credit rating agencies had become the gatekeepers to the capital markets (Partnoy 1999). By 1970 as the demand for rating agencies increased due to an increase in default on Commercial Paper, the credit rating agencies began to charge the issuers for ratings (Cantor and Packer 1994). The agencies no longer had to rely solely on their reputation in the marketplace to earn a profit.

In 1975, the rating agencies that would be considered acceptable in reference to the previously passed laws were formally named by the Securities and Exchange Commission (SEC). The SEC referred to these agencies as Nationally Recognized Statistical Ratings Organizations (NRSRO's). S&P, Moody's and Fitch were the first and only ones to be included in the definition of NRSRO. Over the years others have been added to the list but today the total only stands at five. The first three (S&P, Moody's and Fitch) are still the most dominant.

At the time of the creation of NRSRO's the SEC also incorporated a net capital requirement for banks, specifically Rule 15c3-1 under the Securities and Exchange Act of 1934. The net capital rule required banks to hold a certain amount of capital in cash. The rule was made in order to ensure that the banks had enough cash on hand and would not declare bankruptcy as a result of defaults on some of their assets at the same time as a large amount of requests for withdrawals from customers. Having cash on hand made sure the bank could still service depositor's requests. The rule was more specific on how to calculate the amount of capital required, even specific enough as to use the rating systems of the NRSRO's as a measure of capital required. The lower the rating of bonds the bank held as assets, the higher the capital requirement.

In March of 2002 in testimony before the senate committee on government affairs concerning the role of Credit Rating Agencies in US securities markets, Isaac C. Hunt Jr., Commissioner, US Securities and Exchange Commission stated the following regarding NRSRO's:

“The term "NRSRO" was originally adopted by the Commission solely for determining capital charges on different grades of debt securities under the Commission's net capital rule, Rule 15c3-1 under the Securities Exchange Act of 1934 *(...)* The requirement that the credit rating agency be "nationally recognized" was designed to ensure that the firm's ratings were credible and that the ratings were reasonably relied upon by the marketplace.” (Federal Reserve 2002).

The rules first put forth in 1936 to only allow banks to have bonds in their portfolios that were rated above investment grade, were now coupled with the ruling of 1975 that there were differing capital charges in reference to different rating levels. The rating agencies gained increasing importance. The creation of the NRSRO took it one step further by designating only certain rating agencies as acceptable for issuing a rating on the debt.

2.2 Results of the Legislation

Following the ruling the dynamics of the rating industry had changed. The NRSRO designated rating agencies were no longer held accountable to the public they supposedly served. The SEC had granted these few companies as the sole rating publications with any importance in the market. The members of the NRSROs had a steady stream of business, not only did companies have to deal with a NRSRO first, before issuing any debt, they also had to pay the NRSROs to obtain a rating. Through the 1980's and 1990's, regulations surrounding investments made by Pension Funds, Mutual Funds and other large institutional investors ballooned; all including the designation of an NRSRO rating for debt instruments (list of regulations to 1994, Table 2). The NRSRO's rating system became ingrained in the financial markets forever. Even if a new company was to developed a new and better system for predicting default of companies

it may never have gained any recognition or acceptance. The only ratings that Banks, Pension Funds, Mutual Funds, and investors are concerned with are NRSRO ratings.

As a result of all the regulations passed regarding the allowable investments of Pension Funds, Banks, Mutual Funds and others, there is a large and identifiable credit spread between investment grade and non investment grade debt in the market. The large credit spread is created by the differences in demand; a result of the large institutional investor inability to invest below investment grade. Table 3 shows that as credit ratings decrease, investors demand higher returns from the debt issuer, which intuitively makes sense. However, once the issue receives a rating below investment grade, the spread jumps dramatically, reflecting the large decrease in demand, that is forced through the legislation mentioned above. For a one year to maturity corporate bank bond the spread jumps from 91 basis points to 605 basis points once the rating drops below investment grade (from Baa3 to Ba1) (Table 3). Obviously it is in the company's best interest to at least have their debt rated investment grade.

Criticism surrounds the NRSRO's business model, claiming that it has a built-in conflict of interest. Ratings are paid for by the issuers of bonds and other forms of tradable debt, not by investors who use them. The rating agencies insist that they can be completely independent of the firms who are paying their bills. They argue that internal firewalls bar analysts from fee discussions, and each issuer accounts for a tiny proportion of their revenues. Central to their argument is the fact that their businesses stand or fall on their reputation for independence, and they would never risk that (The Economist 2005c).

2.3 NRSRO Track Record

The NRSROs publish information on corporate defaults to show their track record on default prediction. Table 4 shows the cumulative corporate average default rates of varying credit ratings. As illustrated the higher the initial credit rating the less likely the issue eventually

defaults. While this is a testament to the abilities of the credit rating agencies to rate long term performance of corporate bond issues, there is a tremendous amount of criticism regarding the NRSRO's poor short term performance. For example the following list of recent failures:

- Enron Rated Investment grade Nov 28 2001 - Defaulted Dec 2 2001 (4 days later)
- WorldCom Investment grade April 2002 – Defaulted July 2002
- California Utilities rated Investment grade 2 weeks prior to default.
- Global Crossing rated Investment grade March 2002 – Defaulted July 2002
- AT&T Canada rated Investment grade Feb 2002 – Defaulted Sept 2002
- Parmalat rated Investment grade 45 days before bankruptcy.

Losses from Enron and WorldCom alone were in excess of \$100 Billion (Egan 2005).

With such a tremendous responsibility and with so many recent missteps by the NRSRO's what does the SEC think about the ability of the credit rating agencies? The SEC is the only governing body with any authority over the NRSROs, nevertheless to this day it still does not regulate them. In March of 2002, Isaac C. Hunt Jr., Commissioner, US Securities and Exchange Commission, found himself in front of the Senate on Government Affairs testifying concerning the Role of Credit Rating Agencies in the U.S. Securities Markets. Commissioner Hunt stated the following regarding the track record of NRSRO's:

“Over the course of its history the Commission has considered a number of issues regarding credit rating agencies. Not surprisingly, many of the instances in which either the Commission or Congress reflected on the need for regulation coincided with a large scale credit default such as the Orange County default and the default of the Washington Public Power Supply System ("WPPSS") bonds. Ten years ago the Commission seriously considered the need for oversight authority of credit rating agencies, given their increasing role in the financial and

regulatory systems. The Commission at that time did not reach a consensus on the need for regulation.” (Federal Reserve 2002).

The response was simple. The SEC still believed the NRSROs were doing an adequate job of rating debt in the financial markets. However the pressure from government on the SEC did not end.

In April of 2003 the Honorable Richard H. Baker, Chairman, Subcommittee on Capital Markets, Insurance and Capital Market Enterprises issued a letter to the SEC following up on the Subcommittee’s credit rating agency hearing in 2003. One of the questions asked was the following:

“Do you believe the NRSRO’s have continued to serve the public in light of this recent history: continuing to rate Enron “investment grade four days before bankruptcy; California utilities “A-“ two weeks before defaulting; WorldCom “investment grade” four months before defaulting on loans? We now understand that other rating agency firms, which have not received NRSRO status from the SEC staff provided investors with more timely warnings of the financial problems of those issuers.” (Federal Reserve 2003)

The inquiry presented quite a list of recent missteps by the NRSROs, not to mention the Chairman’s remark on the fact that other non-NRSRO rating issued warnings ahead of the disasters. In June of 2003 the SEC responded in defense of the NRSROs:

“For their part the rating agencies generally take the position that they rely on issuers and other sources to provide them with accurate and complete information and typically do not audit the accuracy or the integrity of the issuer information” (Federal Reserve 2003)

Knowing that the rating agencies do not audit any of the information they receive from the issuer they are rating is less than comforting. Chairman Baker further questioned the SEC about the observations that “rating triggers” in debentures can further accelerate debt obligations for issuers and that rating agencies may be reluctant to downgrade issuers because of this. If this is the case, then what purpose if any do the rating agencies serve? The response from the SEC was as follows:

“(...) To date, in its study of credit rating agencies, Commission staff has been unable to confirm allegations of rating agency reluctance to downgrade an issuer subject to a rating trigger for fear the downgrade would trigger a default....” (Federal Reserve 2003)

Through 2002 and 2003, the SEC continued to defend the NRSROs objectivity and ability to produce relevant market information.

2.4 Through-The-Cycle Methodology

The NRSROs response to criticisms regarding their slow adjustment of ratings is their efforts to use a “through-the-cycle methodology” when rating companies. The NRSROs admit that due to the link of various investments to their ratings, and soon Bank capital requirements (Basel II), there is a dichotomy between both stable and timely ratings (Altman and Rijken 2005). Individual investors value timely ratings whereas large institutional investors prefer to see stable ratings which would not force them to rebalance their portfolios frequently.

Through-the-cycle methodology for ratings consist of two parts; 1) Focus on permanent component of default risk, and 2) Prudent migration policy (Altman and Rijken 2005). According to the first part, rating agencies try to avoid short term fluctuations and only measure the permanent long term component of default risk. The attempt is to try and avoid excessive rating reversals sending inconsistent signals to the market. The second part refers to the fact that only substantial changes in the long term component of default risk will lead to a ratings change and even then the rating would only be partially adjusted (Altman and Rijken 2005). To help quantify the effects of this methodology on rating stability, rating timeliness and default protection a study was conducted in March 2005 by Edward Altman and Herbert Rijken. The study attempted to benchmark agency ratings against credit scoring models, which are assumed to serve as a proxy for investors point in time perception of credit quality of a company.

Altman and Rijken used accounting data from the period of January 1981 – July 2002 to formulate credit scores using a regression model. The credit scores were then converted into credit ratings similar to those used by S&P and compared to data from S&P's CREDITPRO database. To attempt to quantify the effect of the through-the-cycle methodology, the credit rating agencies' ratings were compared to a Long Term Default Prediction Model (LDP). LDP is a point in time model incorporating all available credit information using both temporary and permanent components of default risk. In contrast the through-the-cycle methodology only incorporates permanent components of default risk. LDP uses a 6 year time horizon just greater than the length of temporary credit cycles of 4 years.

Results of the study showed that, rating through-the-cycle delays the timing of rating migrations by 0.56 years at the downside and 0.79 years at the upside. This slow reaction of ratings adjustments is consistent with the prudent migration policy of through-the-cycle methodology. Furthermore, the accuracy of predicting one year defaults drops by 8% when using the through-the-cycle methodology (Altman and Rijken 2005). In fact, the LDP outperforms the agencies' credit ratings for predicting up to 4 years to default. On the other hand, the credit rating agencies outperform the LDP model for defaults greater than 4 years away. Suggesting that by using temporary components of default risk the LDP model is able to predict defaults within a short term time horizon and react with rating migrations faster. While the credit rating agencies focus on permanent components of default risk with the use of a through-the-cycle methodology may be a better indicator of default for long term horizons.

The results of Altman and Rijken (2005) may show that by using the through-the-cycle methodology, credit rating agencies are more useful for long term investors such as pension funds or mutual funds. Shorter term investors may need to take into consideration the lack of temporary component of default risk in the rating agencies' policies. Unfortunately many investors rely too heavily upon the ratings put forth by the NRSROs and lack a clear alternative

method to predict the default of corporate debt. Furthermore while the differences between the through-the-cycle methodology and other methodologies that focus on temporary and permanent components of default risk can be quantified, the differing methods cannot explain the lack of rating migration by the NRSROs. As shown previously, the NRSROs failed to change ratings on Enron, WorldCom, Global Crossing, AT&T Canada, Parmalat and the California Utilities just prior to these companies claiming bankruptcy. These are only examples of clear incompetence over the last 5 years; other oversights have shocked the public prior.

The lack of regulation and competition in the industry has led to an alarming amount of criticism and resulted in the NRSROs track record to appear less than first-class. The immense amount of laws and regulations passed over the last 30 years restricting investments for institutions to different levels of credit worthiness have been put forth in hopes of protecting the public from disaster. However there is an inherent conflict of interest in the way that the NRSROs operate. Bond issuers are forced by law to use the NRSROs to bring their securities to market and on top of that, the issuers must pay the NRSROs to initially rate their debt and then continue to pay the NRSROs to update that rating through time.

With all the controversy surrounding the poor performance of the NRSROs to accurately predict some of the largest corporate defaults in recent history, investors need to find alternative methods for analyzing the credit risk in their portfolios. Other default prediction models have been developed and the majority of the models can be separated into two distinct classes; Structural models and Reduced Form models. We will now investigate the merits of the two models and show that reduced form models can be used by businesses and investors alike to predict the default of their counterparties or bond assets.

3 LITERATURE REVIEW

3.1 Structural Models

Structural models are driven by fundamentals and model the evolution of firm assets, with default occurring when the value of firm assets falls below some threshold level. In this framework, debt is a contingent claim on firm assets and firm assets are usually modelled as a geometrical Brownian motion. This class of models represent the oldest approach to modelling of credit risk for valuation purposes, originating from the Black-Scholes and Merton model. This approach is suited to the study of corporate issuers, where actual firm value can be identified using the balance sheet data. The concept can with slight amendment be extended to sovereign issuers using national stock indices as proxies for the value.

Generally, under the structural models framework, the firm's equity can be viewed as an option on the underlying value of the firm assets. Therefore equity holders would only be incented to repay the maturing debt obligations in cases where the option was "in the money" with respect to the market value of the firm. This would only be the case when the market value of a firm's assets exceeded the par value of the maturing liabilities. Otherwise the equity holders would elect to let the option "expire," thus defaulting on the liabilities. The probability that this "option" is out of the money is theoretically related to the probability that the firm will default.

In the Merton model, it is assumed that the firm's capital structure consists of: debt with notion amount K , in form of zero coupon bonds with maturity T and its current value equal to $B(t, T)$ and equity with total value today equal to $S(t)$. From the fundamental balance sheet equation, the firm's total assets must equal the sum of equity and its liabilities.

Therefore, the stock price and bond price are linked via equation:

$$V(t) = S(t) + B(t, T) \quad (1)$$

From the fundamental Merton assumption, that default only occurs at maturity of the bond, the bond payoff is given by:

$$B(T, T) = \min(V(T), K) \quad (2)$$

Therefore, the firm is solvent if its asset value is greater than the redemption value of the debt otherwise, the firm is insolvent and bond holders have a first claim on its assets. This implies that shareholders are residual claimants with payoff at maturity given by:

$$S(T) = \max(V(T) - K, 0) \quad (3)$$

3.1.1 Extensions to Merton Model

The Merton model is largely viewed as the benchmark for all structural models of default risk. However, some of its assumptions present some limitations. The capital structure of a firm is assumed to have only issued zero coupon bonds with single maturity. However, in reality firms debt structures are also composed of coupon bonds with differing maturities.

Also, the evolution of the risk-free term structure is assumed to be deterministic. Shimko (1993) combines the mechanism of default with the Vasicek interest rate model hence allowing for stochastic interest rates. The results are compatible with those of deterministic case, and credit spreads are increasing function of the short rate volatility and its correlation with firm value.

Because of indenture provisions and safety covenants protecting the bondholders during the life of the bond, it is therefore unrealistic to assume that default of the bond issuers only

occurs at maturity of the bond. Black and Cox (1976) introduced the first passage time mechanism where time of default can be modelled as the first time the firm value crosses a certain boundary. Therefore, the time of default as a random variable can be given by:

$$\tau = \min[t \geq 0 | V(t) = K(t)] \quad (4)$$

Where $K(t)$ represents a stochastic boundary. Geske (1977) treats the liability claims as compound options. In this framework, Geske assumes the firm has the option to issue new equity to service debt. Longstaff and Schwartz (1995) introduce stochastic interest rates into the structural model framework to create a two-factor specification. Leland and Toft (1996) consider the impact of bankruptcy costs and taxes on the structural model. In their framework, they assume the firm issues a constant amount of debt continuously with fixed maturity and continuous coupon payments. Collin-Dufresne and Goldstein (2001) extend the Longstaff and Schwartz (1995) model by introducing a stationary leverage ratio, allowing firms to deviate from their target leverage ratio in the short run, only.

While empirical evidence is still scant, a few empirical researchers have begun to test these Merton model extensions. Lyden and Saraniti (2000) compare the Merton and the Longstaff-Schwartz (1995) models and find that both models under-predicted spreads; the assumption of stochastic interest rates did not seem to change the qualitative nature of the finding. Eom, Helwege, and Huang (2003) find evidence contradicting conventional wisdom on the bias of structural model spreads. They find structural models that depart from the Merton framework tend to over-predict spreads for the debt of firms with high volatility or high leverage. For safer bonds, these models, with the exception of Leland-Toft (1996), under-predict spreads.

3.1.2 Applications of Structural Models

Firm value models are popular among analysts adopting a bottom-up approach because of their focus on fundamentals. They are particularly useful for practitioners in the credit portfolio and credit risk management fields. The intuitive economic interpretation of the model facilitates consistent discussion regarding a variety of credit risk exposures. Corporate transaction analysis is also possible with the structural model. If an analyst wants to understand the impact on credit quality of increased borrowing, share repurchases, or the acquisition of another firm, the structural model naturally lends itself to understanding the transaction's implications. In general, the ability to diagnose the input and output of the structural model in terms of understandable economic variables (e.g. asset volatility as a proxy for business risk, the market's assessment of an enterprise's value, and the market leverage) facilitates better communication among loan originators, credit analysts, and credit risk portfolio managers.

However, firm value models are not readily applicable to special situations such as leveraged buyouts or take-overs where debt becomes riskier while equity valuations increase. Nonetheless, their basis in fundamentals makes them a good starting point for developing more complicated models that are better able to explain market behaviour.

The industry application based on firm value model is given by Expected Default Frequencies® (EDF) provided by KMV Corporation. EDF is a commercial/proprietary estimator of default, where the default boundary is inferred from the balance sheet data. This approach is simply an extension of the Merton model by recognizing the fact that neither the underlying firm value nor its volatilities are directly observable. Instead they are directly inferred from the value of equity and volatility of equity (through equity prices). This data gives a measure of what is often referred to as “distance to default”, which is then mapped to actual default frequencies via a proprietary database of corporate defaults. KMV argues that their model is a better predictor of corporate default than credit ratings. Applying EDF to characterize credit quality, Bohn (1999)

finds evidence that the term structure of credit spreads is hump-shaped or downward sloping for high yield bonds.

3.1.3 Limitations of Structural Models

Structural models have some inherent shortcomings. Firstly, modelling firm value as a diffusion process, implies that default events are predictable and consequently bond prices converge to their default value and short credit spreads tend to zero. This means that default is never a surprise because we can see it coming as the asset price falls. Unfortunately this is seldom observed in practice.

Secondly, the calibration of structural models is data sensitive, and even worse, this data is not generally available. It is difficult to estimate the asset values and volatilities from balance sheet data. To fully implement the firm value model, one would be required to take into account all the various claims on firm assets –a very difficult and perhaps unfeasible task. Also, fitting a term structure of bond prices would require a term structure of asset value volatilities, and asset values, which are not readily observable.

Thirdly, firm value models quickly become cumbersome and slow to compute as we move towards a more complicated firm debt structure that may be composed of both zero coupon bonds and coupon bonds with differing maturities. For instance, if we introduce a coupon paying bond in the debt structure then its pricing is dependent on whether the firm value is sufficient to repay the coupon interest on designated coupon payment dates. Also, if the issuer has two zero coupon bonds outstanding at a given time, the price of the longer maturity bond is conditional on whether the firm is solvent when shorter maturity bonds matures. This makes the pricing formulae very complex.

3.2 Reduced Form Models

3.2.1 Model

Recently new forms of default models have emerged under the title Reduced Form Models. This new form is in relative infancy with little empirical evidence but there has been much work done on the subject; Jarrow and Turnbull (1995), Duffie and Singleton (1996, 1999), Lando (1998), Madan and Unal (1998). Unlike structural models which model default on fundamental firm characteristics, reduced form models treat default as an exogenous event. The probability of default is explained using statistical properties based on pricing of fundamental liquid market instruments. The methodology is much closer to that of actuarial sciences than to the corporate finance methods employed with structural models (O’Kane and Schlogl 2001).

Jarrow and Turnbull (1995) provided the framework for default based on exogenous events using an analogy between default and currency devaluation. The time to default was assumed to be exponentially distributed with an intensity parameter. Duffie and Singleton (1996) proposed a formula to solve for the probability of firm default based on an exponential default probability. Lando (1998) built upon the previous models to include representation of the default intensity as a Cox diffusion process. The model incorporated dependence of default intensities with underlying state variables of the firm including default free interest rates, stock prices, and credit ratings. Previous literature by Jarrow and Turnbull (1995) and Madan and Unal (1998) assumed default intensities as independent of firm characteristics. All reduced form models are based on the assumption of the probability of default being exponential distributed. To understand the validity of this assumption we now describe the origins and basic underlying principles of the idea.

Time to default can be thought of as an independent continuous random variable. Let $F(t)$ denote the distribution of the time until default as,

$$F(t) = \Pr(\tau \leq t) \quad t \geq 0 \quad (5)$$

where $F(t)$ represents the probability that the security will default prior to time t , and τ is the time of default. Therefore the survival function would be:

$$S(t) = 1 - F(t) \quad t \geq 0 \quad (6)$$

where $S(t)$ represents the probability that the security will survive to time t .

Using actuarial notation we can construct a discrete time probability statement about a security that has survived x years as follows:

$${}_t q_x = \Pr[\tau \leq x + 1 \mid \tau > x] \quad (7)$$

where ${}_t q_x$ represents the probability that a security will default within the next year given it has survived x years. The symbol ${}_t q_x$ is called the marginal default probability, a credit curve for a security using a discrete model (with $t = 1$) can be thought of as a series of marginal default probabilities $q_0, q_1, q_2, \dots, q_n$.

Without some process for modelling the default event itself our above equations for time until default are rather useless. To remedy this, reduced form models assume that default events occur according to a Poisson counting process. The Poisson process has among its properties the following:

1. The numbers of changes in non overlapping intervals are independent for all intervals. (memoryless)
2. The probability of two or more changes in a sufficiently small interval is 0.

These relationships make sense when related to firm defaults because it is realistic to assume that the probability of default in any period is independent of all other periods and the probability of two defaults in any period is 0. For the purposes of default modelling we will just consider the first Poisson event or the first arrival of default because a firm can only default one time.

The behaviour of the Poisson process is determined by $\lambda(t)$ known as the intensity of the process, also called the arrival rate of default or hazard rate. If we consider the intensity process to be a continuous process we can rewrite our conditional probability formula to be based on a small change in time as follows:

$$P[\tau \leq t + dt | \tau > t] = \lambda(t)dt \quad (8)$$

The above equation can be interpreted as the instantaneous probability of default given survival to time t , where the probability of defaulting in the next instant is proportional to the hazard rate $\lambda(t)$ and time t . Given a Poisson process the probability of obtaining exactly n successes (i.e. a default) is described in the limit of the binomial distribution as follows:

$$P(n | N) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n}. \quad (9)$$

As N (our sample size) becomes large the Poisson process formula from above will approach a Poisson distribution, where the probability of an event (x) from a Poisson distribution is:

$$p(x) = e^{-\lambda} \frac{\lambda^x}{x!}. \quad (10)$$

We can extend this formula to give the probability that $\tau > t$, or the probability that τ (time to default) does not occur in the interval $[0, t]$ (Recall we are only concerned with this interval because we are only interested in the first default event from the Poisson counting process):

$$\begin{aligned} P(\tau > t) &= e^{-\lambda t} \frac{(\lambda t)^0}{0!} \\ &= e^{-\lambda t} \end{aligned} \quad (11)$$

where $P(\tau > t)$ is also known as $S(t)$ the Survival probability. From this we can derive the default probability $F(t)$ as $1 - S(t)$ or:

$$\begin{aligned} P(\tau < t) &= 1 - S(t) \\ &= 1 - e^{-\lambda t}. \end{aligned} \tag{12}$$

With the understanding of the Poisson process and how it relates to risky bond default we can now further examine the reduced form models put forth. Probably the most frequently used and well known method was developed by Duffie and Singleton (1999). In their model a bond can be priced in terms of a short rate r and Equivalent Martingale measure Q . The resulting equation is as follows:

$$V_0 = E_0^Q \left[\exp \left(- \int_0^T (r + \lambda L) dt \right) X \right] \tag{13}$$

where X is the value of expected cash flows and L is the fractional loss rate of X in case of default. This model attempts to discount the expected cash flows of the bond not only by the short rate r but also by a measure of the default risk. The inclusion of L in the equation takes into account that in the event of default only some percentage of the credit risky bond is lost and a portion is recovered. With this model we can use observed bond market prices to calculate the default intensity λ . The easiest method to show the calibration of this model is to assume X is a zero coupon bond. Using the observed market price of this bond V_0 we can work backwards to solve for λ if a loss rate (L) is a known or an assumed rate. We can extend the Duffie and Singleton framework to show a formula for extraction of λ for coupon bonds with an observable market price and a known loss rate (L).

$$V_0 = E_0^Q \left[c \int_0^T \exp \left(- \int_0^t (r + \lambda) dt \right) \right] + E_0^Q \left[\exp \left(- \int_0^T (r + \lambda L) dt \right) \right] \tag{14}$$

where c denotes the coupon.

3.2.2 Recovery Assumptions

Recovery is defined as what proportion of the bond is recovered in the event of default, or using the Duffie and Singleton (1999) notation, recovery would be $1 - \text{Loss Rate}$. Recovery assumptions have significant impact on the model and ultimately on the calculation of the default intensity λ . Previous work by Jarrow and Turnbull (1995) had assumed that recovery was a fraction of an equivalent default free zero coupon bond (sometimes denoted as the equivalent recovery approach). If we introduce some notation to let $B^d(0, T)$ to equal the price of a defaultable bond then:

$$B^d(0, T) = RB(0, T) + (1 - R)B_0^d(0, T). \quad (15)$$

The price of the defaultable bond is the sum of the recovery (R) on a default free bond plus (1-R) of the price under zero recovery. The problem with this approach is that it inherently sets an upper limit on the bond credit spread. Consider $y(0, T)$ to denote the continuous yield of a non-defaultable bond and $y^d(0, T)$ to be the continuous yield of defaultable bond. Equation (15) would then imply that:

$$e^{-y^d(0, T)} = Re^{-y(0, T)} \text{ so that } y^d(0, T) - y(0, T) \leq \frac{1}{T} \ln\left(\frac{1}{R}\right). \quad (16)$$

For example if we assume the equivalent recovery approach then if $R=60\%$ then for a maturity of 10 years the maximum credit spread is 510bps. This constraint can become a problem if we are modelling the senior bonds of a high yield issuer, but is not necessarily a problem unless the bonds have a long maturity (O'Kane and Schlogl, 2001).

Duffie and Singleton (1999) introduced the fractional recovery approach that was shown in Equation 1. The recovery was not dependant on a similar risk free bond as with the Jarrow and Turnbull (1995) approach but rather dependant upon the value of the security itself at any

particular time. The fractional approach solves the problem of imposing a maximum credit spread but neither the fractional approach nor the equivalent recovery approaches agree with market convention. Under market convention, in the event of default all future coupons are not collected and the holder of the bond is paid any recovery based on the face value of the instrument not market value as implied by the fractional approach. Therefore the best method to use would be to assume recovery based upon some fraction of face value of the bond. We will explain further our recovery assumptions in the data and methodology description.

3.2.3 Default Intensities Term Structure

Default intensities can be assumed to be constant through time (Jarrow and Turnbull (1995)), deterministic but changing through time or stochastic (Lando (1998)). Given that it is common for corporations to issue more than one bond and to have a variety of maturities it is therefore possible to solve for a variety of default intensities across differing maturities. If we assume the default intensity to be a piecewise deterministic function through time then we can calculate the varying default intensities using the different bond maturities. For instance a one year corporate bond will allow us to calculate the year one default intensity $\lambda_{0,1}$. Then taking a two year bond from the same issuer we can solve for the $\lambda_{0,2}$ and so on. Finally a term structure of default intensities $(\lambda_{1,2}, \lambda_{2,3}, \lambda_{3,4}, \dots, \lambda_n)$ can be calculated in the same manner as one would bootstrap an interest rate curve to build a term structure of interest rates. What we will be left with is a picture of the stepwise varying default intensity of a particular issuer through time. The detail of the term structure, i.e. the number of steps involved, is limited to the available corporate bond structure. Typically, a flat intensity is assumed for any years greater than the longest corporate bond maturity.

3.2.4 Liquidity Considerations

As the understanding and modelling capabilities of default probabilities has evolved the credit derivative market has expanded as well. One of the most widely used products is the credit default swap. This instrument has a payoff to the protection buyer usually $1-R$ ($1 - \text{recovery rate}$) upon default of an underlying instrument. The protection seller is paid a premium to offer this protection and if no default occurs will collect the premium and incur a profit. Credit Default Swaps (CDS) provide protection to direct bond issues or basket of issues and the premium paid for protection is directly related to the probability of default of the bond issue. Previously we have shown a method to solve for the default intensity using market observable corporate bond prices. Likewise we can use the same method to solve for the default intensity using market observable CDS premium.

Longstaff, Mithal, Neis (2004) identified some important differences between CDS contracts and corporate bond contracts that can have an influence in the calculated default intensities based on the two instruments. First, bond securities are in fixed supply whereas new CDS can be created an infinite amount of times, therefore there is less supply and demand pressure placed on CDS. Secondly, if an investor would like to liquidate a position in a CDS they can simply take an offsetting position in another CDS contract. On the other hand corporate bond positions cannot easily be unwound if there is little or no interest from new buyers and this sometimes makes it expensive to liquidate a corporate bond position. Finally, it is very costly to short corporate bonds whereas CDS are relatively easy to buy and sell. These differences in features allow the CDS to be much more of a liquid instrument than the corporate bond even referencing the same issuer.

Longstaff, Mithal, Neis (2004) compared the calculated default intensities based upon corporate bond prices and CDS's for the same underlying company. The CDS is a much more liquid instrument and therefore we can reason that the CDS derives its entire value based on the

default probability of the underlying company. The default intensity calculated by using a CDS is presumed to be the true default intensity. Any difference between the default intensity of the CDS and the corporate bond default intensity is due to the increase in liquidity cost that bond holders must receive a premium for. Longstaff, Mithal, Neis (2004) investigated a variety of corporate bonds and their corresponding CDS's and found that the average spread attributed to liquidity is the following: AAA/AA 47.3bps, A 68.6 bps, BBB 72.6 bps, and BB 117.2 bps based on Treasury Bonds used as the risk free rate.

4 DATA AND METHODOLOGY

To test the predictive power of the Duffie and Singleton (1999) reduced form model we use weekly bond price data from a variety of US corporations known to have defaulted or experienced financial distress in the past to calculate the default intensities (λ), leading up to bankruptcy. Using these default intensities we calculate the risk neutral probability of default each week up to the known time of actual default. Furthermore, we test the model against a corporation that has not defaulted and is still in existence at the time of writing this paper. This contrast allows us to show that the model does more than just predict default of companies in distress but that it can also accurately forecast survival probability.

In order to apply the Duffie and Singleton (1999) framework, we make a number of assumptions. Firstly, we treat callable bonds and non-callable bonds equally by not altering the price of callable bonds to reflect the inherent call option embedded in them because callable bonds are rarely ever called. Secondly, we assume continuous coupon payments, all semi-annual coupon rates will be converted to a continuous rate. This simplifies the model and allows us to develop a closed form solution for the default intensity. Thirdly, for each weekly estimated default probability we assume a constant term structure of interest rates. Lastly, any recovery rate assumptions will be based upon Moody's report "Default Rates and Recovery of Corporate Bond Issuers 1920-2003". The historical average recovery rate reported for corporate debt by Moody's was 39.5% (Moody's Investor Service 2004).

Our initial investigation of the reduce form model will be a simplistic model based upon the Duffie and Singleton (1999) framework. In this initial model we assume that there is no

recovery in the event of default and also that the default intensity λ is constant over time. We then modelled the price of a risky corporate bond as follows:

$$V(0, T) = c \int_0^T e^{-(r+\lambda)t} dt + e^{-(r+\lambda)T} \quad (17)$$

where λ is the hazard rate, c is the coupon rate, and r is the risk-free rate. Solving equation (17), we obtain a closed form solution for the market price of a risky bond $V(0, T)$ that can be used to calculate the default intensity λ :

$$V(0, T) = \frac{c}{r + \lambda} + \left[1 - \frac{c}{r + \lambda} \right] e^{-(r+\lambda)T}. \quad (18)$$

Using the shortest term bond from a corporation's outstanding bonds we solve for the default intensity. Once the default intensity is calculated we can use equation (12) to calculate the default probability incrementally over the next 6 years. We repeat this process each week leading up to a default event or up to the most recent bond prices reported in the case of no default. From this process, we obtain a series of default probabilities that are then compared to Moody's historical default probabilities based on bond ratings.

To build upon this analysis we introduce recovery rates into the model. The recovery rates are based on the study by Moody's as previously discussed. By introducing a recovery in case of default, equation (17) becomes:

$$V(0, T) = c \int_0^T e^{-(r+\lambda)t} dt + e^{-(r+\lambda)T} + R\lambda \int_0^T e^{-(r+\lambda)t} dt. \quad (19)$$

Solving equation (19) we obtain a closed form solution for the price of risky corporate bond with recovery R .

$$V(0, T) = \frac{c + R\lambda}{r + \lambda} + \left[1 - \frac{c + R\lambda}{r + \lambda} \right] e^{-(r+\lambda)T}. \quad (20)$$

Once again we use equation (12) to calculate the default probability incrementally over the next five years.

After investigating the results based upon a constant default intensity assumption we introduce a deterministic but changing default intensity term structure. For this we use a term structure of corporate bonds from the company investigated rather than just one short maturity bond. Consider a firm that has issued a series of coupon bonds, $V(0, t_1), V(0, t_2), V(0, t_3), \dots, V(0, t_n)$. For each of the n bonds we evaluate equation (20) to solve for n default intensities $\lambda(0, t_n)$. To calculate default probabilities we again use equation (12) but since the default intensity is longer assumed constant the results should be a more accurate reflection of the default probability through time.

Finally, we extend the Duffie and Singleton (1999) framework to include a liquidity premium. The value of the liquidity premium δ will be based upon the work done of Longstaff, Mithal and Neis (2004). Including the liquidity premium δ , equation (19) becomes:

$$V(0, T) = c \int_0^T e^{-(r+\lambda+\delta)t} dt + e^{-(r+\lambda+\delta)T} + R\lambda \int_0^T e^{-(r+\lambda+\delta)t} dt. \quad (21)$$

Solving equation (21), we obtain a closed form solution for the price of a risky corporate bond with recovery and liquidity parameters:

$$V(0, T) = \frac{c + R\lambda}{r + \lambda + \delta} + \left[1 - \frac{c + R\lambda}{r + \lambda + \delta} \right] e^{-(r+\lambda+\delta)T}. \quad (22)$$

Again we use a deterministic but changing default intensity based upon the term structure of issued bonds to calculate default probabilities.

Historical weekly bond data was collected from Bloomberg on four US companies: Kmart, WorldCom, and Calpine who have all defaulted in the past and Boeing that continues to

operate in good financial standing. The data for Kmart is from January, 2001 to June 2002; during this time Kmart experienced financial difficulties which ultimately led to their default on January 22, 2002. Moody's held an investment grade rating on Kmart until December of 2001 when they downgraded the company to below investment grade (Baa3 to Ba2). We have two bond issues for Kmart used to construct a term structure of default intensities, one bond maturing in 2004 and another maturing in 2006. For WorldCom we have data that begins in July 2000 and ends May 2002, our data consists of four bonds with maturity dates of 2003, 2004, 2005, and 2006. WorldCom began to experience serious financial difficulties in 2002 and eventually defaulted in July of the same year. The default of WorldCom was a surprise to many in the financial community and Moody's held an investment grading rating on WorldCom until only a month before default. For both Kmart and WorldCom, we investigate whether a reduced form model does a better job of predicting default than the Moody's ratings.

Calpine had experienced financial difficulties since December 2001 and had been rated below investment grade since that time. Our data for Calpine begins in October, 2004 and runs till June 2006 with bond maturities of 2006, 2007, 2008, 2009, 2010. During the period of our data set, on December 20, 2005 Calpine filed for Chapter 11 bankruptcy protection. Calpine is an example of a company in distress but since filing for Chapter 11 they have been restructuring and trying to become profitable again. By investigating this data we are able to observe how the reduced form model behaves when a company is already below investment grade and experiencing difficulties. Furthermore we are able to see if the reduced form model can predict the probability of Calpine surviving Chapter 11 proceedings as they struggle to restructure. Finally, we obtained data for Boeing Corporation that is in good financial standing (A rated) and has been stable for a long period of time. Our data consists of bond prices from August, 2003 to June, 2006 with bond maturities of 2006, 2007, 2008 and 2010. By investigating Boeing we are able to observe how the reduced form model behaves when a company is not experiencing any

financial difficulties and test the ability of the model to correctly predict the extremely low probability of default.

Also, to investigate the correlation between equity prices and default probabilities implied from bond prices, we obtained stock prices for all companies in our study from the CHASS database of the University of Toronto.

Besides bond and equity price data, we also collected historical interest rate data consisting of swap rates from the US Federal reserve website. The swap rate curve was used as the risk free rate rather than the treasury curve because swap rates are more widely used by practitioners when discounting cash flows in the fixed income markets (Longstaff, Mithal and Neis 2004). Weekly swap rate data was collected for 1, 2, 3, 4, 5, 7, and 10 year maturities. We used a standard linear interpolation algorithm to compute the appropriate discount factor for our bond data. The liquidity premium used was a premium to swap rates based on Longstaff, Mithal and Neis (2004). The premium for Boeing and WorldCom used was 14.2bps and 2.9bps respectively based on results in the above study for these companies. We used the average liquidity premium to swap rates for investment grade companies of 16.8bps for Kmart and 61.9bps for non-investment grade companies for Calpine. The latter two companies were not directly involved in the Longstaff, Mithal and Neis (2004) study and therefore we used the average liquidity premium calculated.

5 RESULTS AND ANALYSIS

5.1 Implied Default Probabilities vs. Recovery Assumption

Implied default probabilities for all companies are sensitive to recovery assumptions. Implied default probabilities estimated under zero recovery assumption are lower than those estimated under a positive recovery rate of 39.5% - the historical average recovery rate according to Moody's. The reason for this is that the recovery rate sets an artificial floor for the bond price, implying that if the bond price is below the historical recovery rate the bond would be in default even if this is not the case. It is possible for the bond price to be below the recovery of face value without being in default. The larger the recovery assumptions the higher the artificial floor value for the bond and the larger the probability of default as the bond approaches that artificial floor. This is synonymous with the diffusion to barrier under structural models of credit risk, where the level of barrier directly influences the chances of default. In Figure 2 and 3, we present the plots of implied default probabilities for Calpine and WorldCom respectively under the two recovery assumptions. For the case of Calpine, where bond prices fell below the historical average recovery value in late 2005, the one year probability of default went to 100% and our model correctly predicted default.

However, another key observation is that the recovery value assumption seems to affect the level, but not the shape of the implied default probability curve. Therefore, increasing the recovery rate has the effect of sliding the curve upwards on the vertical plane. Conversely, a reduction in recovery rate slides the curve downwards on the vertical plane. Hence, the default probability under one recovery value is related to the default probability under another recovery value through a simple analytical linear relationship. This outcome provides us with a great

degree of confidence that our initial recovery assumption of 39.5% does not have a large impact on our results. Differing recovery assumptions will only lead to differing levels of predicted financial distress but even a zero recovery assumption would still predict a rather large probability of default.

5.2 Implied Default Probabilities vs. Liquidity Assumption

From Figure 1, we observe that liquidity premium does not seem to influence default probabilities as evidenced by the near negligible change in implied default probabilities of Calpine Corporation when investigated under the two liquidity assumptions. However, based on theory, a changing liquidity premium would be desirable as contrasted to a constant premium as used in our paper. This is because, as the risky bond approaches maturity, one would assume that the liquidity premium increases. At this point, we not aware of any research study that provides empirical evidence for appropriate levels of liquidity premiums as a bond approaches default. The liquidity premiums used in this paper, were very small and could not produce a significant change in implied default probabilities.

5.3 Constant vs. Time-Varying Intensity

To compare the assumptions of Constant and Time-Varying default intensity we used our model that incorporates 39.5% recovery and the liquidity premiums for each respective company. Default probability was calculated for all companies out to six years for each set of weekly bond prices. The results can be seen in Figure 8 through Figure 15.

As one would expect, the series of graphs under the constant intensity model reflect the strict exponential relationship of the nearest bond default intensity and time. While the assumption of constant intensity has no effect on the probability of default in one year it can have a large impact on probabilities greater that one year. In all the graphs we see that there are subtle

differences between the two assumptions as we observe small ripples in the outer year default probabilities under the time varying intensity scenario. Some differences are much more obvious and probably the two graphs that reflect this the most are Figure 6 and Figure 9 for Calpine. We can observe that in the beginning of our sample the probability of default in the next year is the same for both scenarios. The difference can really be seen on the probability of default in 2 or 3 years where we see that it is much lower under the constant intensity assumption than under the time varying assumption. We know that the time-varying assumptions is a better representation of the true default probability for Calpine because it takes into account the default intensity for 2 or 3 years represented in the bond prices rather than just extrapolating the intensity from one short maturity bond. Therefore it is important to recognize this difference when evaluating credit risk and especially important to recognize it when pricing financial derivatives such as credit default swaps.

If we look at the time-varying intensity graphs for all the companies in our sample a noticeable pattern emerges. In all cases where the company was in financial distress the probability of default in the 5th or 6th year pushed towards 100% faster than the probability of default in one year. Once again Figure 9 for Calpine is a great representation of this as we can see that even early in our data set the probability of default in the 5th or 6th year was close to 100% while the probability of default in the 1st year was less than 20%. This is a great reflection of the underlying market interpretation of the financial standing for Calpine. The probability of default within one year may have been low but within the 5th or 6th year, most of the market participants predicted this company to default. We can also observe this phenomenon when looking at Figure 10 for WorldCom. Shortly before the company default in mid 2002, we see that the probability of default in one year is approximately 30-40% while the probability of default in the 5th or 6th year is almost 100%.

Some key differences emerge between constant and time varying intensity when looking at the graphs for Boeing in Figure 8 and Figure 12. Under the constant intensity assumption we see the default probability growing with time as it should under such an assumption but we do not see the same results under time varying intensity. Figure 8 shows that under the time varying assumption the probability of default in the 5th or 6th year is actually lower than under the constant intensity assumption. This is because under the constant assumption we have the nearest term intensity growing exponentially through time when in reality, the risk neutral longer maturity intensities are actually lower than the exponentially grown shorter term intensities. This reflects the markets perception that the probability of default for Boeing in 5 years is almost the same as the probability of default in the next year. It would be impossible to obtain this result with just a constant default intensity assumption.

The similar default probability for one year all the way to six for Boeing also supports the interpretation that the high default probabilities in the 5th and 6th years for the companies in financial distress are a true representation of that distress. Since we know that Boeing is a company that is in good financial condition we would expect to see low default probability throughout time, if this was not the case it would undermine the results of higher probabilities in the outer years for the Kmart, WorldCom and Calpine. Therefore, when evaluating the credit risk of a corporation it is important to interpret the entire term structure of default probabilities as it can provide valuable information of the likelihood of the company surviving in the future.

5.4 One-Year Implied Default Probability

5.4.1 Kmart Corporation

Figure 4 shows weekly results of the model predictions for the probability of default in one year compared to Moody's probability of default. Moody's probabilities are based on Table

6, Moody's average one year transition matrix from 1980-1999. The matrix shows the probability of moving from one rating to the next over the course of one year as well as the probability of defaulting in one year based on the current rating. Early in Figure 4 we can see that according to Moody's ratings the probability of default was less than 1% (0.2%) for Kmart which was rated investment grade (Baa3). At the same time the probability of default predicted by our model was approximately 5% and began to climb in early September 2001. Moody's changed their rating for Kmart in Dec 2001 to below investment grade but the rating change was only to Ba2 which would imply the one year default probability to be 1.54% (Table 6). In contrast our model describes more severe financial distress for Kmart and a one year probability of default to be approximately 15% at the time of downgrade and the probability of default according to our model climbs from there to almost 50% at the time of default. The jump in default probability according to Moody's is a result of the further downgrade of Kmart to Caa mid way through January of 2002, just prior to default.

When evaluating the stock price performance of Kmart in relation to the default probabilities we can see a strong negative correlation between the probability of default and the stock price. At this time it is difficult to infer whether the negative correlation is a reflection of the bond market driving the stock prices or vice versa. However we can observe that although the stock price for Kmart had been in a decline in the early part of the figure, the probability of default did not have any significant change with our model. With this observation we can presume that our model does not just respond to changing stock price performance but rather to actual financial distress. Clearly, financial distress will also have an impact on stock price performance, therefore as time passes and news of Kmart's financial distress emerges, the default probabilities and stock prices form a clearer negative correlation.

5.4.2 WorldCom Corporation

Perhaps one of the most interesting financial disasters of the past few years has been the demise of WorldCom due to the large number of investors in the financial community that were surprised with the companies bankruptcy. Moody's and other credit rating agencies were caught off guard and many carried investment grade ratings of WorldCom right up until just before its eventual default. Many believe that it was impossible to predict any default of such a large and seemingly stable company because bankruptcy was considered a consequence to poor accounting practices that the majority of the financial community knew nothing about.

Figure 5 displays the results of our model's one year default prediction compared to Moody's default prediction. Moody's carried an investment grade rating on WorldCom up until May 2002 when the Sr. Unsecured debt was lowered from Baa2 to Ba2 just below investment grade. At this time we can see the probability of default in one year based on Moody's ratings move from 0.2% to 1.54%. On the other hand our model shows the default probability of WorldCom is relatively the same as Moody's in the early part of the time sample and a slight increase in default probability in approximately February or March of 2001. Considerable changes to the one year default probabilities based on our model begin in Jan 2002, a full 5 months before Moody's downgrades WorldCom below investment grade. In fact our model already has a one year default probability of approximately 30% for WorldCom at the time that Moody's finally downgrades the company in May 2002 (WorldCom would default two months later in July).

Looking at the stock price performance of WorldCom we can see that the stock had been under considerable pressure in the latter part of 2000, dropping by more than 50% from \$49 to approximately \$15. At the same time we do not see any significant change in the one year default probability of WorldCom estimated by our model. This provides further evidence that our

reduced form model does not simply respond to stock price performance but rather responds to actual financial distress.

5.4.3 Calpine Corporation

Calpine provides us with an example of a company that is already in financial distress. The company had been downgraded by Moody's to below investment grade in December of 2001 and then further to a Caa1 in October of 2003. Moody's default probability of a bond with such a rating is 27.7% and is reflected in Figure 6. Our reduced form model actually has a lower probability of default when compared to Moody's 27.7% in the early part of the time series. As time passes we can see the probability of default in one year based on our model to be fluctuating almost in unison with the stock price. Closer to the end of 2005 we see the probability of default for our model eclipse the probability based on Moody's ratings and shoot all the way to 100% default probability a full two and a half weeks before Calpine actually files Chapter 11.

After the Chapter 11 filing our model shows the one year default probability of Calpine to be decreasing through time. This may seem counter-intuitive because Calpine has already filed Chapter 11 however our model is reflecting the fact that Calpine is in the midst of restructuring in order to remain solvent and meet its future bond payments. After the December 2005 Chapter 11 filing our model is able to incorporate current market news and any developments as Calpine negotiates with creditors. As information is made available to the public it is reflected in bond prices and therefore into our model as well. At the end of our time series Calpine is still under Chapter 11 protection and we do not know for sure whether they will emerge from it or not. However, our model represents that the probability of default is decreasing and therefore reflects the markets current intuition that Calpine has a chance to emerge from Chapter 11 and meet future payments.

The results for Calpine show that our model is much more flexible than current ratings information from Moody's and other rating agencies. Once Calpine had filed for Chapter 11 Moody's dropped their rating on the company and therefore we are unable to infer the probability of Calpine's survival from Moody's ratings. On the other hand our reduced form model reflects current market sentiment and can be used to predict the risk neutral probability of Calpine surviving.

5.4.4 Boeing Corporation

Boeing is an example of a corporation that is in extremely good financial standing, the company has held an investment rating of A from Moody's for the entire sample period we investigated. Figure 7 shows that based on Moody's A rating of Boeing the probability of default within one year is very low (0.02%). Likewise our model shows that the probability of default is very low, during our time sample it is never higher than approximately 2% with an average of less than 1% (0.44%). Furthermore we can see once again that our model does not seem to be influenced by stock price. Boeing Corporations stock price doubles over the course of our time series but this does not seem to have any impact on the default probability calculated by our model.

These results of our models default predictions for Boeing show that the model can also accurately reflect the strong financial standing of companies.

6 CONCLUSION

In this paper we analysed a reduced-form model for dynamically estimating the risk-neutral default probability distribution of a set of US corporations using the constantly changing information in the corporate bond market. From the results, it is evident that the reduced form approach does a better job of estimating corporate defaults than credit ratings especially in the short-term. Bond rating agencies follow a through the cycle methodology with their ratings transitions that does not seem to accurately predict the short term probability of default. Furthermore, our results show that implied default probabilities are not influenced by stock price performance but rather actual financial distress. This method can be applied to any firm with outstanding bonds of different maturities.

The strength of this approach lies in its simplicity and reliance on the constantly changing market sentiment embedded in the bond prices, and does not rely on stale and sometimes inaccurate accounting information. Furthermore, the term-structures can be expressed using simple analytic formulas that can be easily used for sensitivity analysis or stress testing. Therefore, the approach can be easily and cheaply included in enterprise-wide risk management systems.

Our results show that recovery assumptions can have a significant impact on the calculated default probabilities but the recovery value assumption seems to affect the level, but not the shape of the implied default probability curve. This result means that the recovery assumption can be arbitrarily set depending on preferences. We also researched the effect of a liquidity assumption on the model but found that the liquidity premiums used were not significant

enough to have any material affect on the results. Further analysis could be done on what affect time varying liquidity premiums would have on default probabilities.

The applications of this approach are wide and varied. Bank lending officers can use the approach to complement the traditional qualitative analysis hence making better judgement when processing corporate credit applications. It is also feasible to apply the approach to entire portfolios of bonds and the risk manager can present detailed information (i.e. expected credit losses) to management of the bank and investors. Also, in order to price structured products such as Collateralized Debt Obligations (CDO), financial institutions can use this approach to compute default probabilities of the corporate counterparties. Besides being greatly useful when monitoring creditworthiness of counterparties, the risk-neutral default probabilities can be used to assess the market's perception on one's own financial strength. This is perhaps useful in determination of the best timing of bond or stock issues. Lastly, this simple approach can be extended to assessing the creditworthiness of bond-issuing sovereigns.

FIGURES

Figure 1: Calpine Corporation Implied Default Probabilities and Liquidity

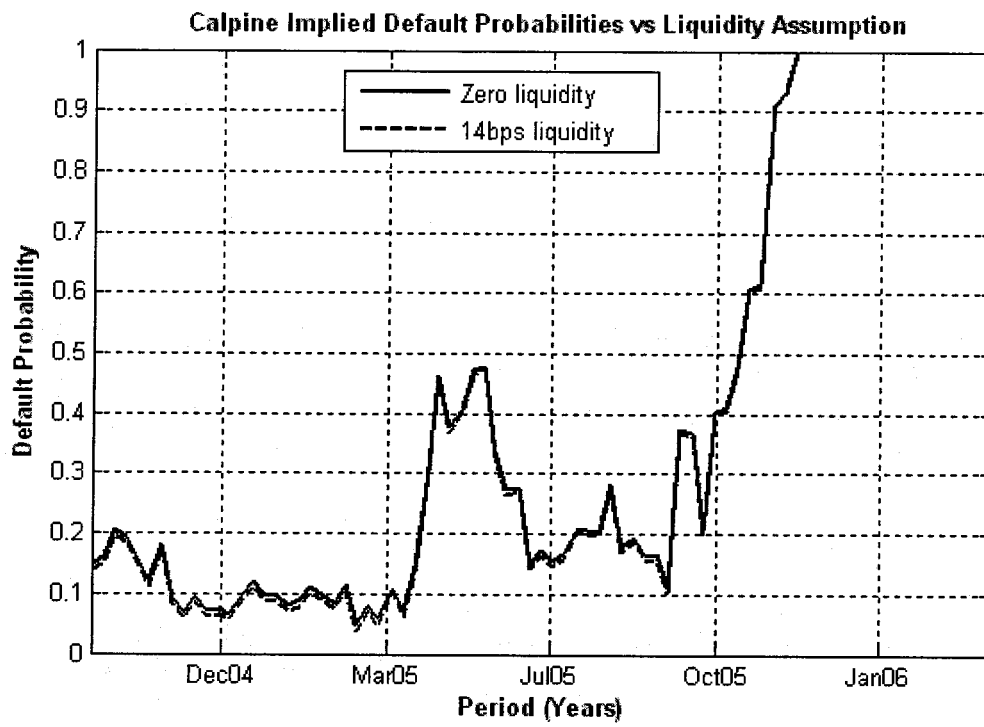


Figure 2: Calpine Corporation Implied Default Probabilities and Recovery

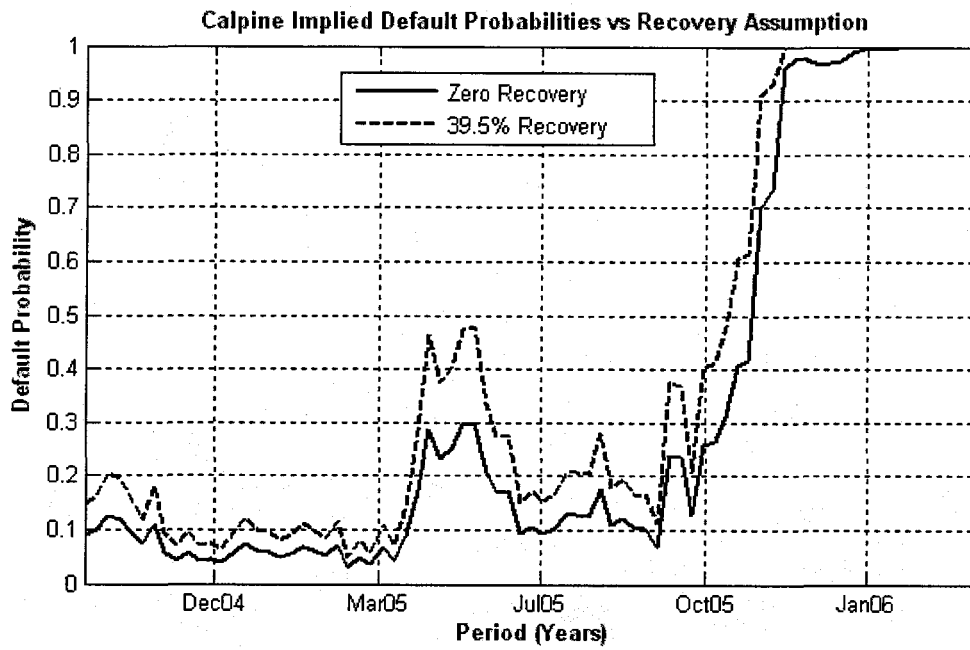


Figure 3: WorldCom Corporation Implied Default Probabilities and Recovery

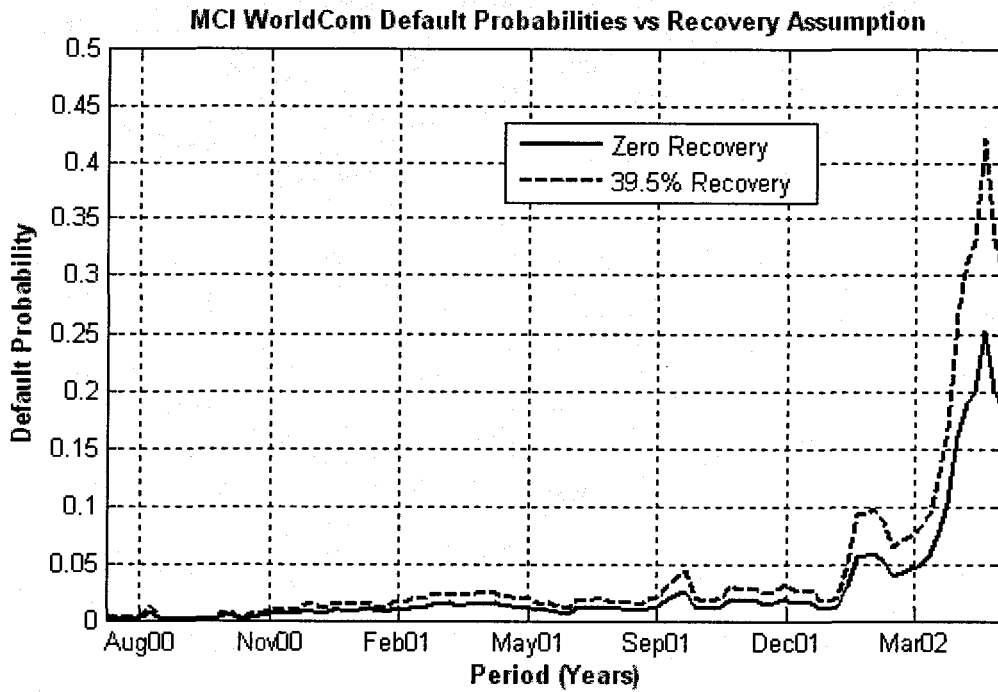


Figure 4: Kmart Corporation

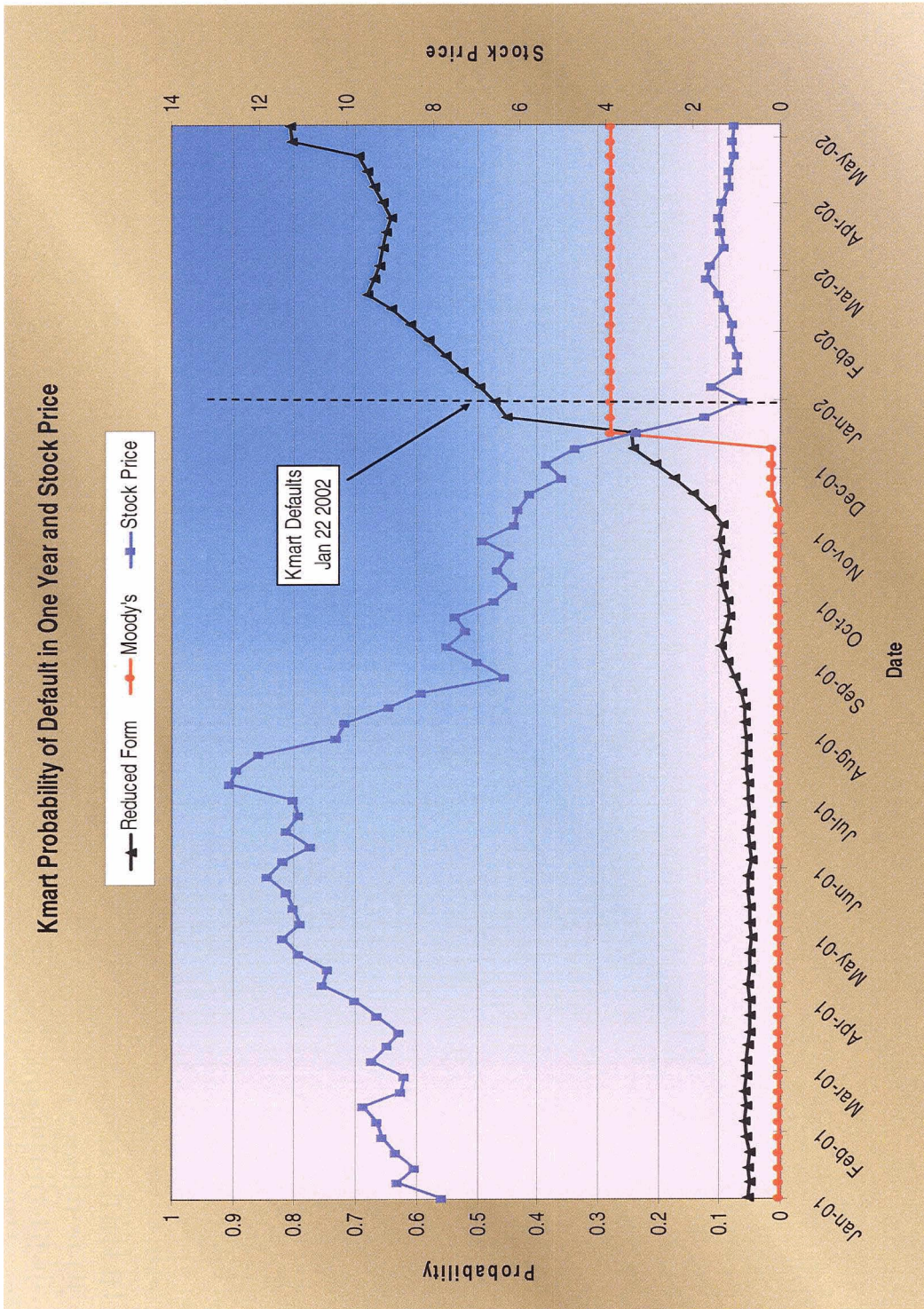


Figure 5: WorldCom Corporation

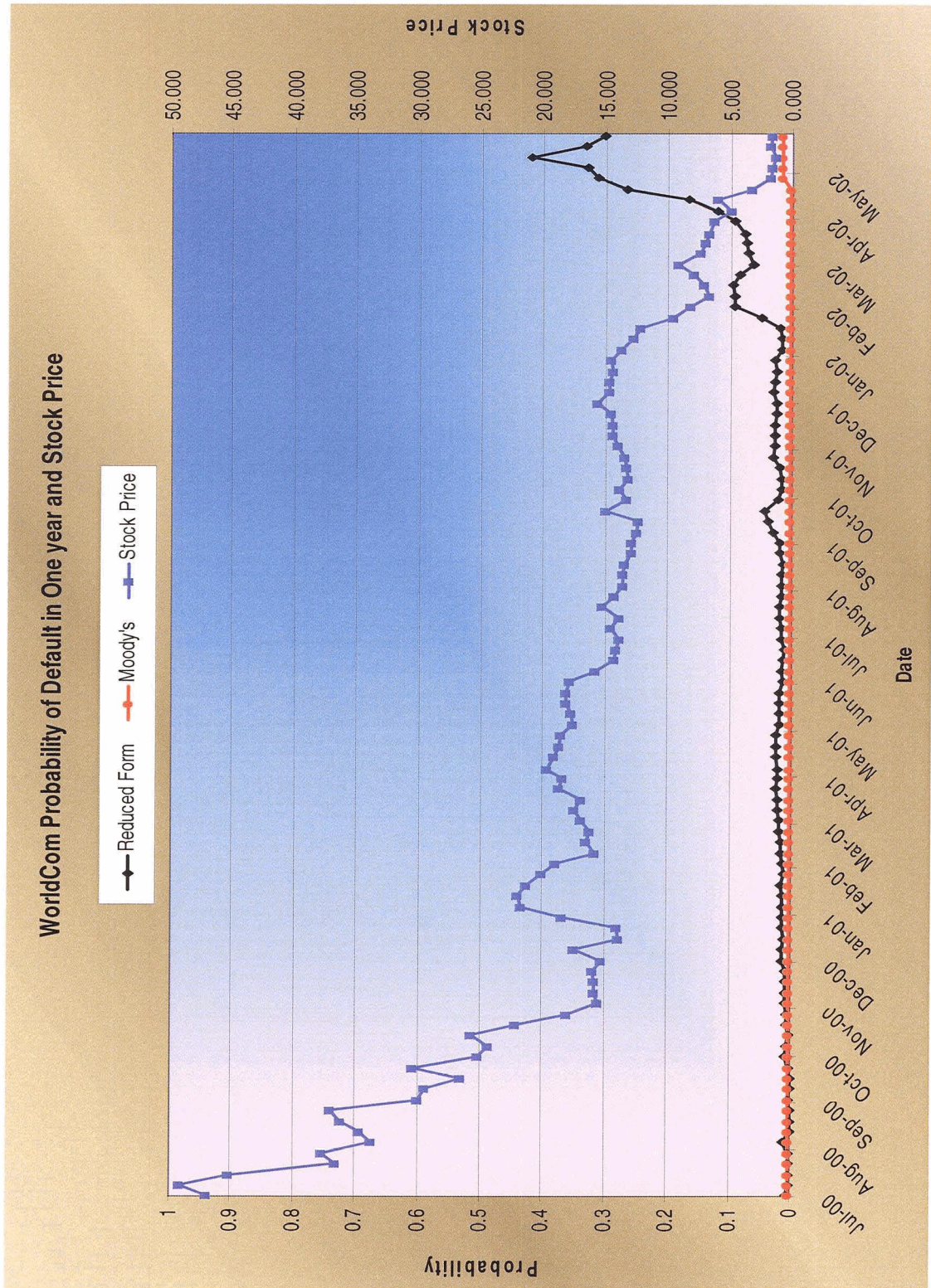


Figure 6: Calpine Corporation: Default and Equity Price

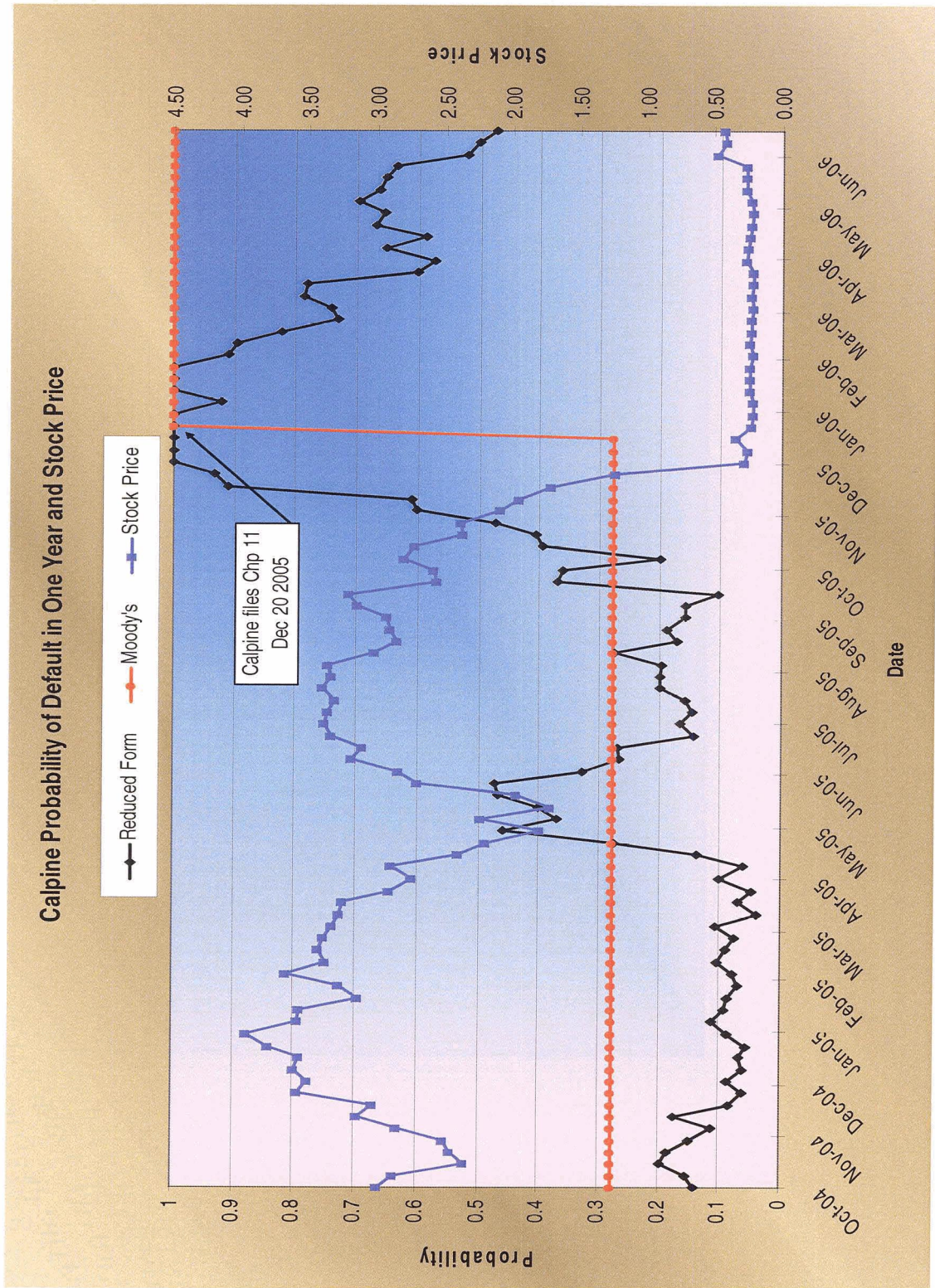


Figure 7: Boeing Corporation: Default and Equity price

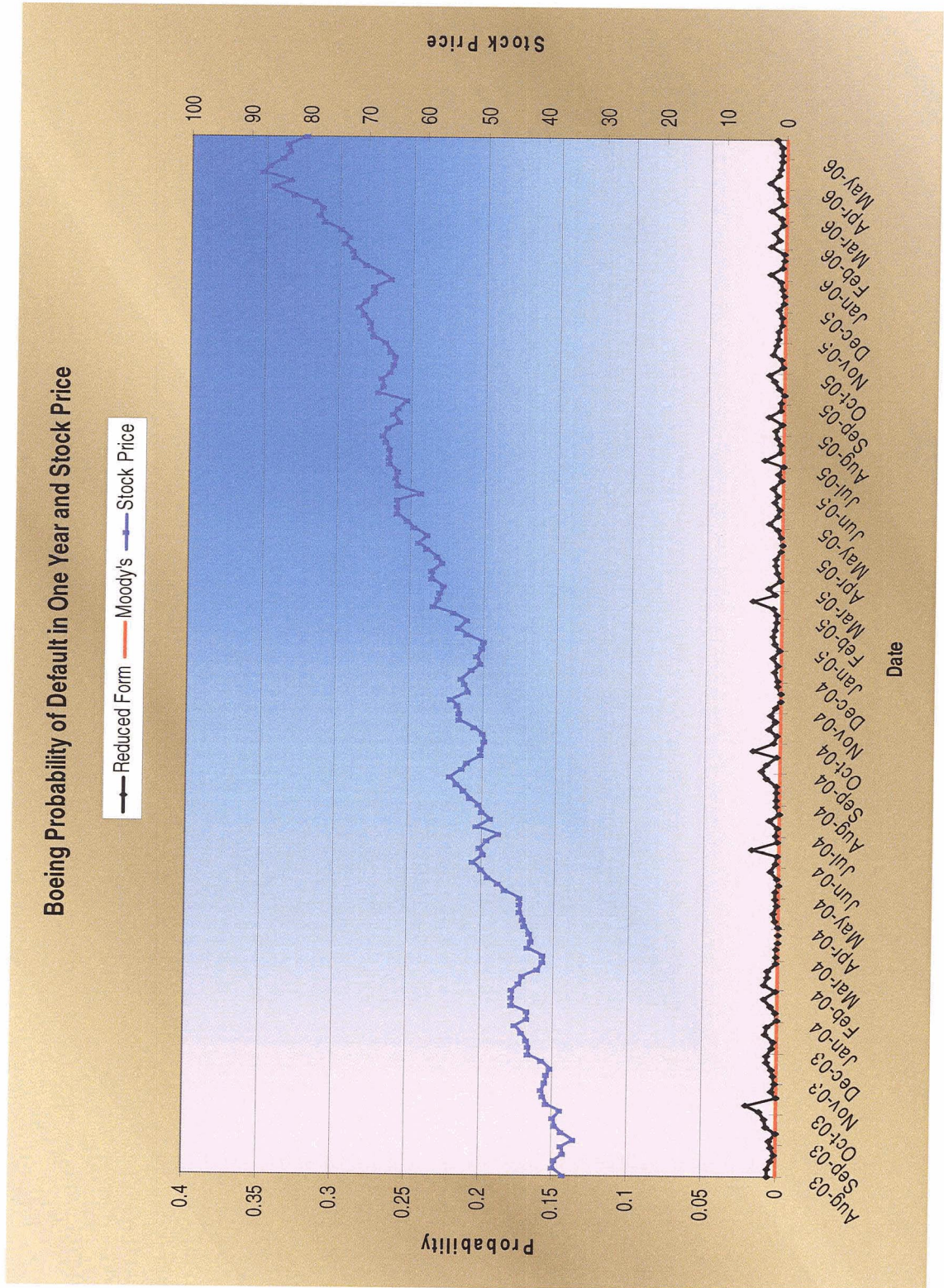


Figure 8: Boeing Varying Intensity Term Structure

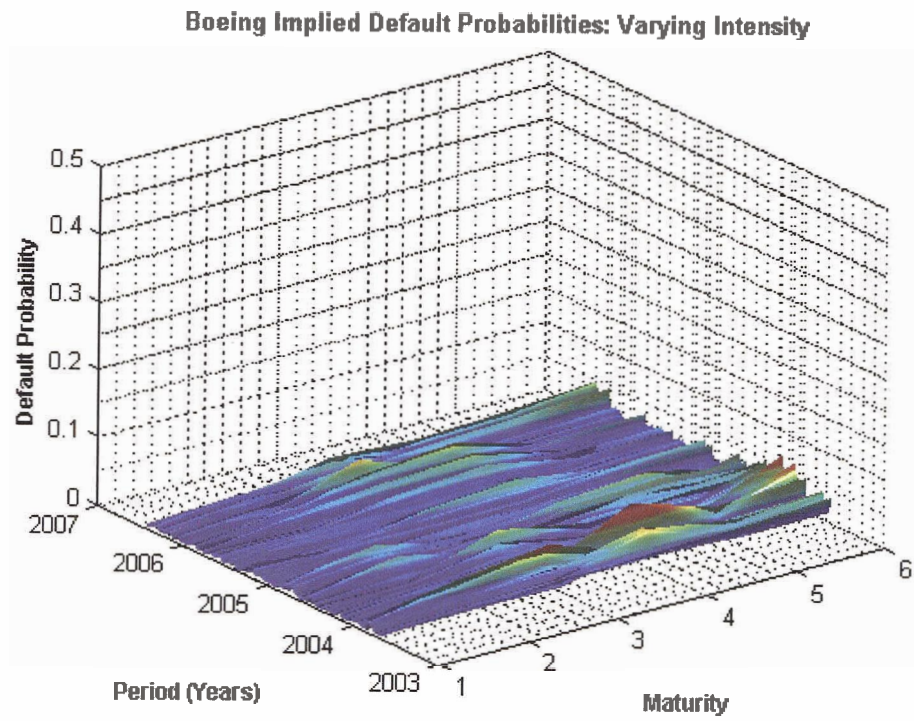


Figure 9: Calpine Varying Intensity Term Structure

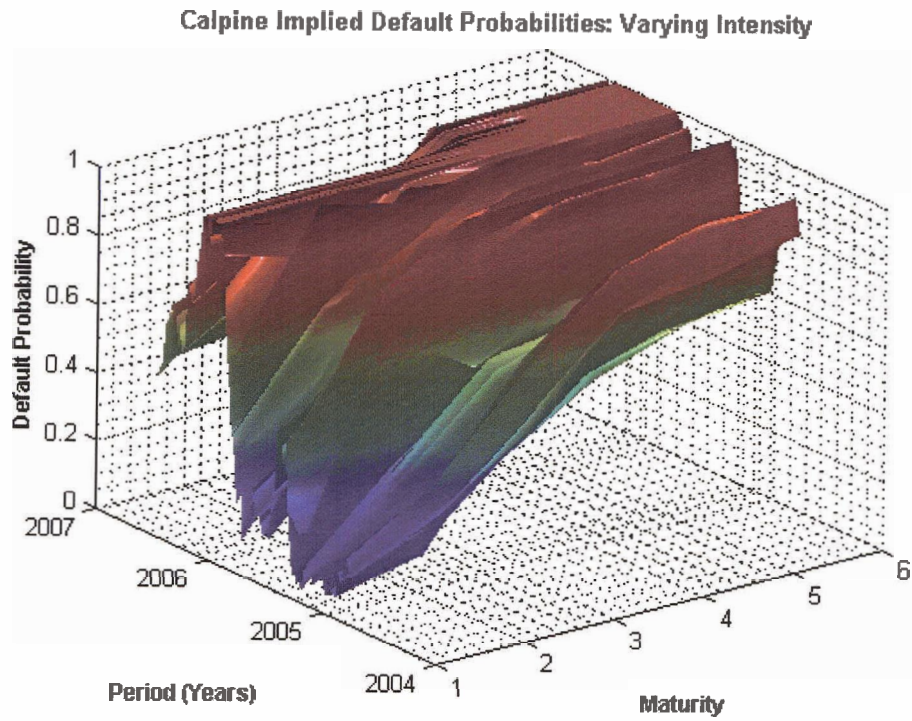


Figure 10: WorldCom Varying Intensity Term Structure

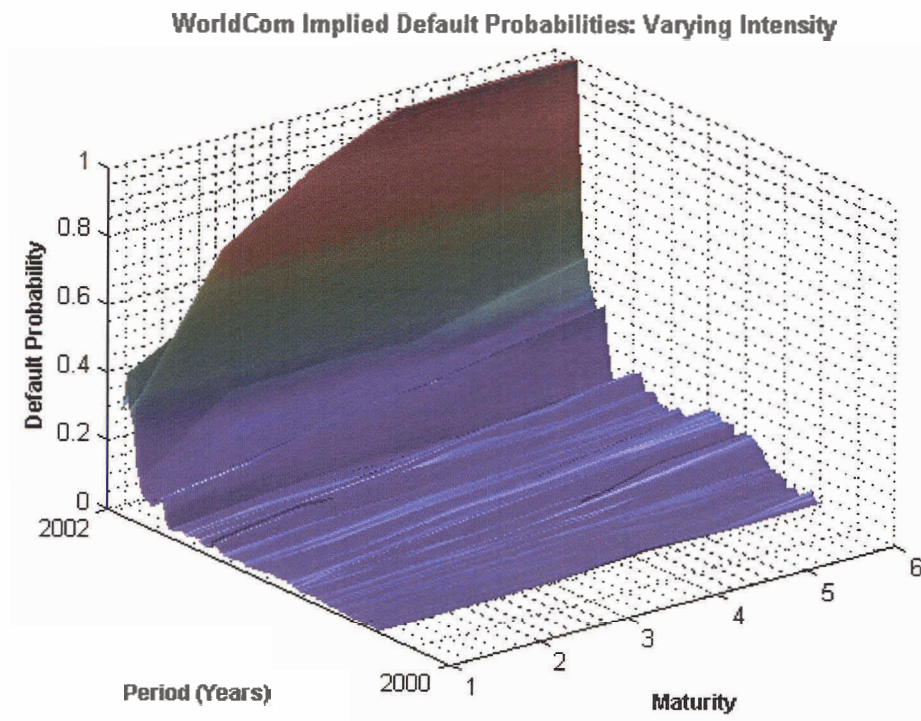


Figure 11: Kmart Varying Intensity Term Structure

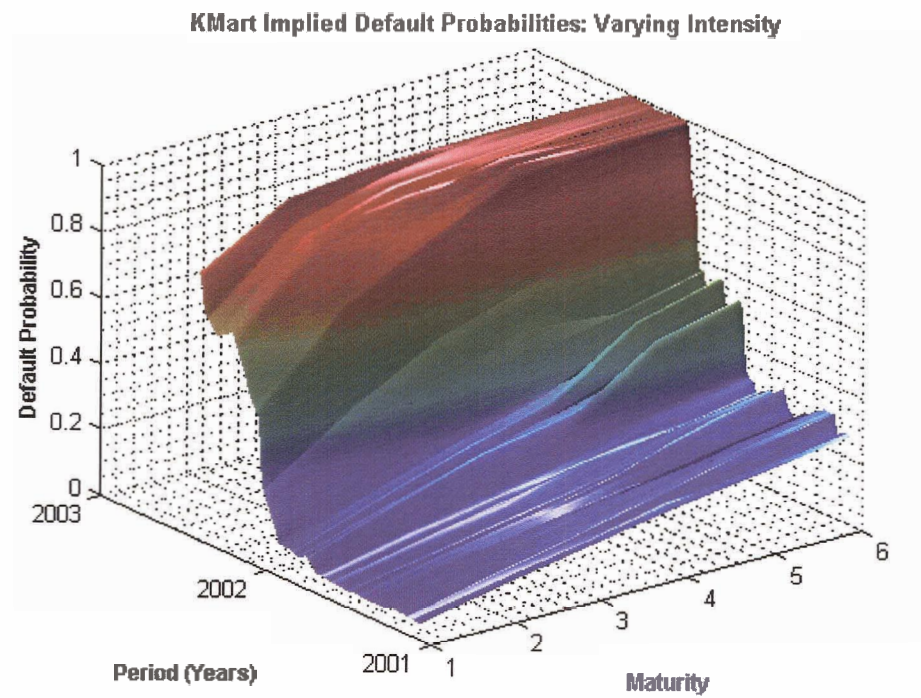


Figure 12: Boeing Constant Intensity Term-Structure Chart

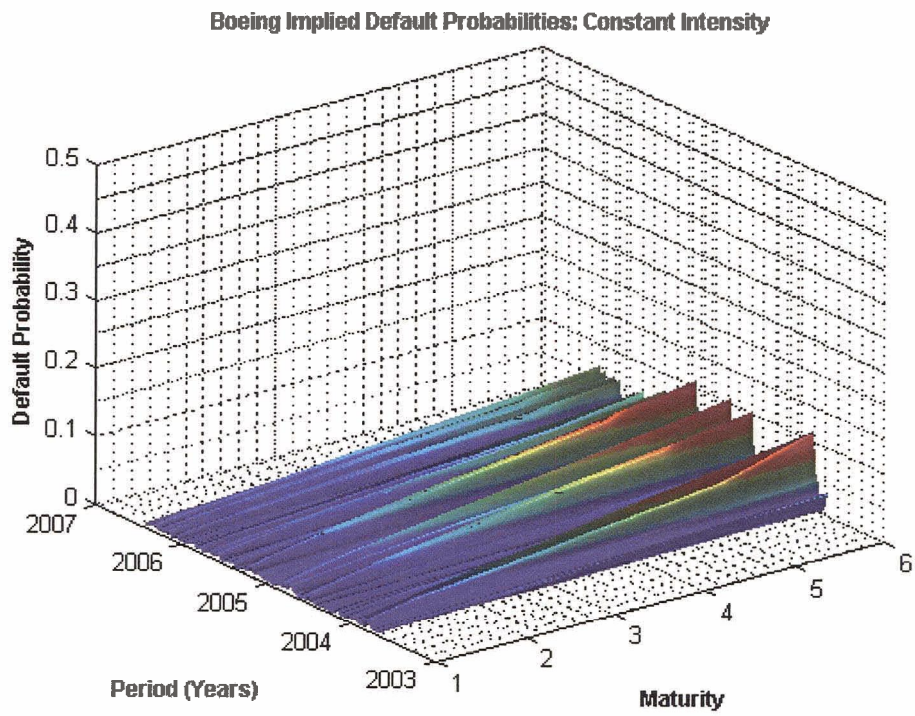


Figure 13: Calpine Constant Intensity Term-Structure Chart

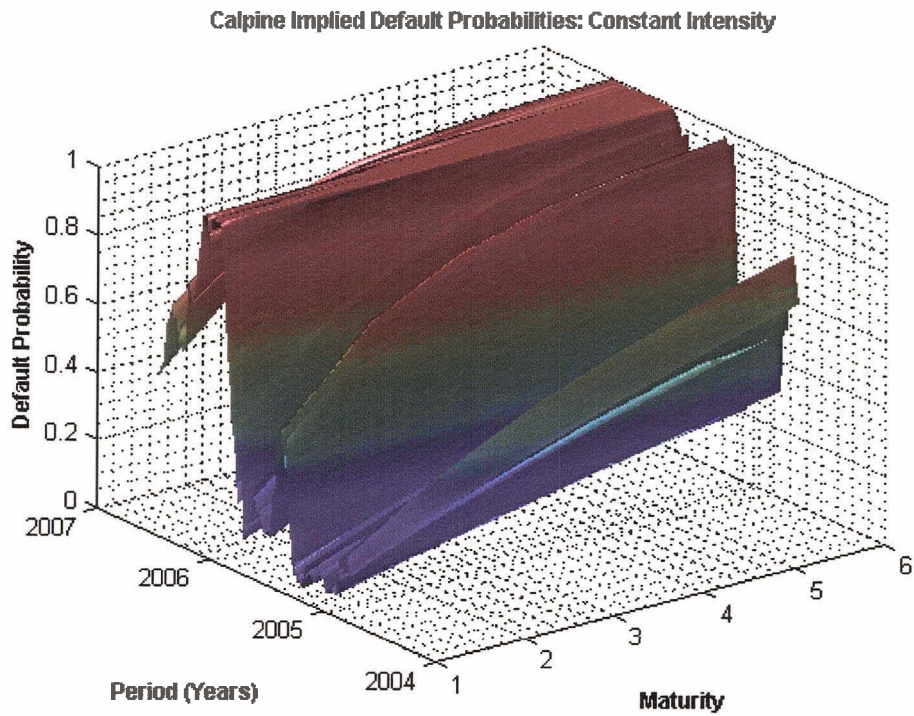


Figure 14: WorldCom Constant Intensity Term-Structure Chart

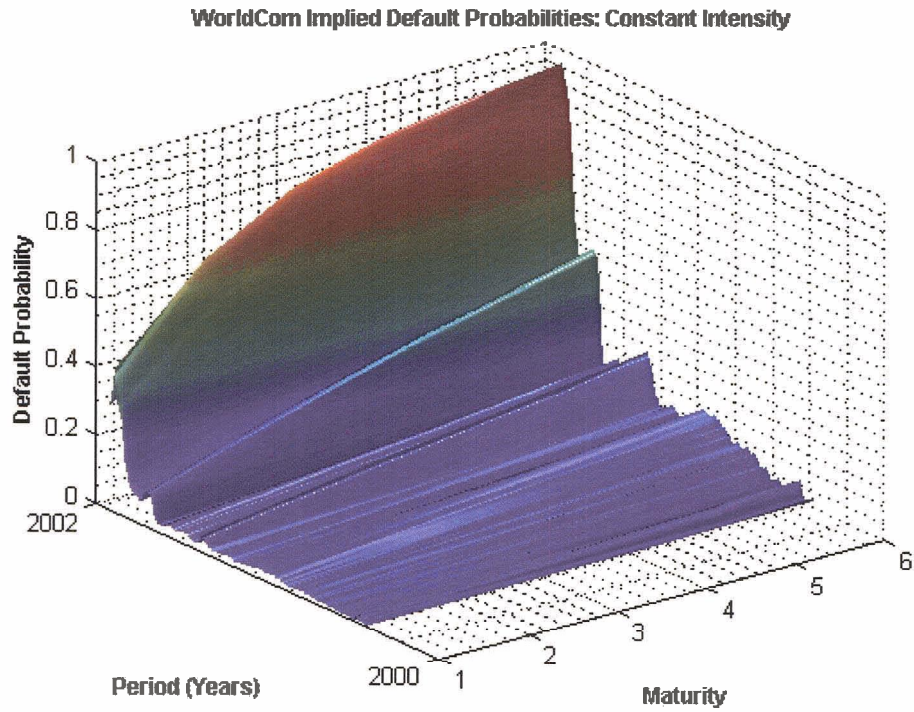
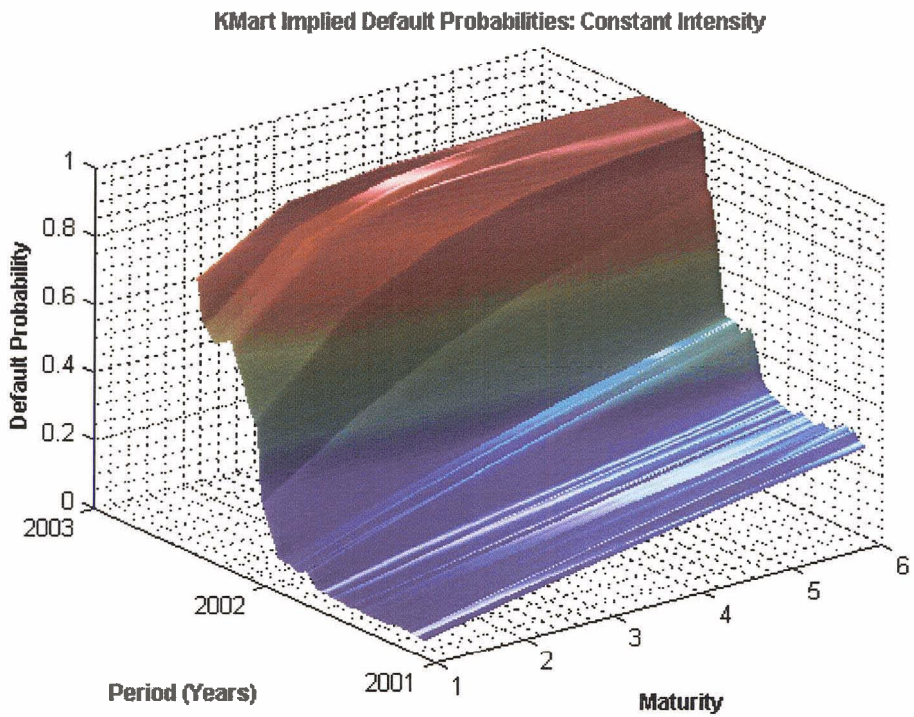


Figure 15: Kmart Constant Intensity Term-Structure Chart



TABLES

Table 1: Summary of Corporate Bonds Ratings and Symbols

Fitch	Moody's	S&P	Summary Description
Investment Grade – High Credit Worthiness			
AAA	Aaa	AAA	Gilt edge, prime, maximum safety
AA+	Aa1	AA+	High grade, high-credit quality
AA	Aa2	AA	
AA-	Aa3	AA-	
A+	A1	A+	Upper-medium grade
A	A2	A	
A-	A3	A-	
BBB+	Baa1	BBB+	Lower-medium grade
BBB	Baa2	BBB	
BBB-	Baa3	BBB-	
Speculative – Lower Credit Worthiness			
BB+	Ba1	BB+	Low-grade, speculative
BB	Ba2	BB	
BB-	Ba3	BB-	
B+	B1	B	Highly speculative
B	B2		
B-	B3		
Predominately Speculative, Substantial Risk, or in Default			
CCC+	Caa	CCC+	Substantial risk, in poor standing
CCC		CCC	
CC	Ca	CC	May be in default, very speculative
C	C	C	Extremely speculative
		C1	Income bonds—no interest being paid
DDD			Default
DD			
D		D	

Data Source: Rating Agency Websites

Table 2: Selected Uses of Rating in Regulation

Information is presented in the following order: Year Adopted; Ratings-Dependent Regulation; Minimum Rating; How Many Ratings?; Regulator/Regulation.

- 1931; Required banks to mark-to-market lower rated bonds; BBB; 2; OCC and Federal Reserve examination rules
- 1936; Prohibited banks from purchasing "speculative securities"; BBB; Unspecified; OCC, FDIC, and Federal Reserve joint statement
- 1951; Imposed higher capital requirements on insurers' lower rated bonds Various; N.A.; NAIC mandatory reserve requirements
- 1975; Imposed higher capital haircuts on broker/dealers' below-investment-grade bonds; BBB; 2; SEC amendment to Rule 15c3-1 the uniform net capital rule
- 1982; Eased disclosure requirements for investment grade bonds; BBB; 1; SEC adoption of Integrated Disclosure System (Release #6383)
- 1984; Eased issuance of nonagency mortgage-backed securities (MBSs); AA; 1; Congressional promulgation of the Secondary Mortgage Market Enhancement Act of 1984
- 1987; Permitted margin lending against MBSs and (later) foreign bonds; AA; 1; Federal Reserve Regulation T
- 1989; Allowed pension funds to invest in high-rated asset-backed securities; A; 1; Department of Labor relaxation of ERISA Restriction (PTE 89-88)
- 1989; Prohibited S&Ls from investing in below-investment-grade bonds; BBB; 1; Congressional promulgation of the Financial Institutions Recovery and Reform Act of 1989
- 1991; Required money market mutual funds to limit holdings of low-rated paper; A1[a]; 1[b]; SEC amendment to Rule 2a-7 under the Investment Company Act of 1940
- 1992; Exempted issuers of certain asset-backed securities from; BBB; 1; SEC adoption of Rule 3a-7 under the registration as a mutual fund Investment Company Act of 1940
- 1994 Proposal; Would impose varying capital charges on banks' and S&Ls' holdings of different tranches of asset-backed securities; AAA & BBB; 1; Federal Reserve, OCC, FDIC, OTS Proposed Rule on Recourse and Direct Credit Substitutes

a Highest ratings on short-term debt, generally implying an A- long-term debt rating or better.

b. If issue is rated by only one NRSRO, its rating is adequate; otherwise, two ratings are required.

Data Source: Cantor and Packer (1994) Vol. 19, Issue 2.

Table 3: Reuters Corporate Spreads for Banks for February 10, 2003 (in basis points,)

<u>Ratings</u>	<u>1 yr</u>	<u>2yr</u>	<u>3 yr</u>	<u>5 yr</u>	<u>7 yr</u>	<u>10 yr</u>	<u>30 yr</u>	
Aaa/AAA	27	36	47	61	74	88	107	
Aa1/AA+	33	49	53	61	84	99	118	
Aa2/AA	35	54	56	75	87	101	121	
Aa3/AA-	37	57	58	80	91	105	130	
A1/A+	61	73	77	95	107	123	145	
A2/A	64	76	79	97	108	125	148	
A3/A-	68	79	82	101	115	128	150	
Baa1/BBB+	81	98	104	124	153	180	202	
Baa2/BBB	84	106	112	130	158	183	207	
Baa3/BBB-	91	111	117	135	163	190	212	Investment Grade
Ba1/BB+	605	615	625	635	655	675	695	Below Investment
Ba2/BB	615	625	635	645	665	685	705	Grade
Ba3/BB-	625	635	645	655	675	695	715	
B1/B+	775	785	795	825	865	905	955	
B2/B	785	795	805	835	875	915	965	
B3/B-	795	805	815	845	885	925	975	

Data Source: www.bondsonline.com (Bond Market Association) website

Table 4: Cumulative Average Corporate Default Rates*

Rating	1 year	5 years	10 years	15 years
AAA	0.00	0.10	0.45	0.61
AA	0.01	0.30	0.85	1.35
A	0.04	0.61	1.94	2.98
BBB	0.29	2.99	6.10	8.72
BB	1.20	11.25	19.20	22.59
B	5.71	25.40	33.75	38.63
CCC/C	28.83	50.85	56.45	59.44
Investment Grade	0.11	1.20	2.71	3.92
Speculative Grade	4.91	20.22	28.25	32.42
All Rated	1.64	7.08	10.45	12.51

*By years after initial rating

Data Source: The Economist (2005a). 374(8419), p67-69.

Table 5: Moody's Average one year Transition Matrix for 1980-1999

	Aaa	Aa	A	Baa	Ba	B	Caa-C	Default
Aaa	89.30%	10.15%	0.05%	0.00%	0.03%	0.00%	0.00%	0.00%
Aa	0.96%	88.42%	10.04%	0.38%	0.16%	0.02%	0.00%	0.04%
A	0.08%	2.34%	90.17%	6.37%	0.81%	0.22%	0.00%	0.02%
Baa	0.09%	0.39%	6.42%	84.48%	6.92%	1.39%	0.12%	0.20%
Ba	0.03%	0.09%	0.50%	4.41%	84.25%	8.65%	0.52%	1.54%
B	0.01%	0.04%	0.17%	0.58%	6.37%	82.67%	2.98%	7.17%
Caa-C	0.00%	0.00%	0.00%	1.10%	3.06%	5.89%	62.17%	27.77%
Default	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Data Source: O'Kane, Schlogl (2001)

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