

# **VALUATION OF EARLY EXERCISE PREMIUM ON CURRENCY OPTIONS: THE PUT-CALL PARITY APPROACH REVISITED**

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PROJECT SUBMITTED IN PARTIAL FULFILLMENT  
OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ARTS

In the  
Faculty  
of  
Business Administration

Financial Risk Management

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SIMON FRASER UNIVERSITY



Summer 2006

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# APPROVAL

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**Degree:** Master of Arts

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## ABSTRACT

Previous studies on American options have shown that European style models do not reflect early exercise premium (EEP). This project expands on the Poitras, Veld and Zabolotnyuk (2006) paper which applies the put-call parity method to currency options data from American options traded at PHLX for EEP. We define a wider range for in-the moneyness and use a rolling volatility for the volatility parameter. We estimate the early exercise premium as a percentage of option price (REEP) for calls and puts to be 7.329%, 6.122%, respectively. We then regress the REEP against moneyness, interest differentials, and time to maturity and volatility. Our results show that REEP is strongly and positively correlated with interest rate differentials and time to maturity. The effect of moneyness is less apparent. The effect of volatility on REEPs of put options is significantly negative, which coincides with the results of Poitras, Veld and Zabolotnyuk (2006).

**Keywords:** early exercise premium, currency options, put-call parity, American options.

# **DEDICATION**

To my family and friends.

John Lee

To my family.

Mei Xue

## **ACKNOWLEDGEMENTS**

We would like to thank our senior supervisor, Dr. Chris Veld for his guidance and assistance on this project. We would also like to thank our second reader, Dr. Geoffrey Poitras.

John Lee and Mei Xue

## **STATEMENT OF CONTRIBUTION**

This project is a collaborative effort between John Lee and Mei Xue. John Lee is responsible for the introduction, literature review as well as the brief section on currency market. Sections on options premiums, put-call parity, early exercise boundaries and methodologies were written by Mei Xue. She also handles most of the data work. Results and conclusion is mostly written by John Lee.

John Lee and Mei Xue

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# 1 INTRODUCTION

The development of the international financial market in the second half of the 20th century provides multinational companies, banks and individual investors with unprecedented opportunities and challenges to speculate on the direction of foreign exchange rate as well as hedge their currency exposure. One of the tools made available to these market participants is foreign currency options. While options contracts have existed in the over-the-counter markets and at exchanges since the 1600's, the 1970's saw three major events that anticipate the advent of currency options.

The first event emanates from the global macroeconomic environment. Towards the end of World War II, a proposal was made to set currencies in parity to gold and International Monetary Fund was established to oversee the operation of the new monetary system. IMF member countries were required to intervene in order to keep fluctuation of exchange within a range of 1%. In spite of some periods of devaluations in the post-war years, the Bretton Woods agreement which had, somewhat effectively, kept exchange rates amongst member countries within some fixed ranges. Nonetheless, the Agreement became more and more difficult to maintain as waves of devaluations, spurred on by significant discrepancies in the growth rates and inflations of different countries, grew more intense in the 1960's. At the outset of the collapse of Bretton Woods in early 1970's, central banks in Japan, US and European countries were no longer willing to intervene to maintain fixed exchange rates (Taylor, 2003). This cleared the way for a floating exchange rate system, where central banks are not expected, and in practice, do not participate in foreign exchange market on a regular basis, though there remain some countries that continue to peg their currencies to a benchmark currency. This new era of floating exchange

rate has led to a predictably dramatic increase in volatility of exchange rates. Consequently, the need to hedge foreign currency risks for both multinationals and other types of firms has grown over the past two decades.

Two other events that contribute to the development of foreign currency options are more specific to the development of options trading and pricing theories. Before discussing these events however, it would be advisable to introduce a few basic concepts on options.

The two most common types of options are calls and puts. Options, unlike futures, do not have linear payoff profiles. For instance, a call option allows the holder of the instrument (someone who is long the call) the right, but not the obligation to buy the underlying asset at a previously agreed upon price known as the exercise or strike price. It follows that someone who is short the call option is obliged to sell the underlying to someone who is long the call at the exercise price regardless of what the market price may be. A put option, on the other hand, allows the holder of the instrument the right, but not the obligation to sell the underlying asset at the strike price. A short position in the put entails the obligation to buy the underlying from someone who is long the put option.

A further division exists in terms of the style of the option. European style options can only be exercised at the maturity of the contract, while American options can be exercised anytime prior to the expiry of the options. The payoff of Asian options is dependent on the average of the underlying prices during the life of the contract, while Bermudan options restrict exercise to certain dates. Other exotic options such as barrier and chooser, binary, lookback, compound and exchange options are also available in the over-the-counter markets (Hull, 2006, pp.529-549). Since American options have exactly the same features as the European options *plus* the option to exercise early, it must be the case that, *ceteris paribus*, an American option should be worth just as much if not more than its European counterpart. The discrepancy between the

two prices should be attributed to the ability to exercise prior to maturity. This difference is called the early exercise premium.

In 1973, the Chicago Board of Options Exchange (CBOE) was created, and 16 stock options began trading on that exchange in April of that year. Other exchanges such as the American, Philadelphia, Pacific and the Midwest exchanges followed suit. Since then, options with different underlying assets, including foreign currencies have been introduced at several exchanges.

The Philadelphia Stock Exchange (PHLX), one of the oldest stock exchanges in the US, made option trading on foreign currencies available in December 1982 and by 1988 currency options were trading in daily volumes as high as \$4 billion in underlying value. Currently, currency options on Canadian dollar, the Euro, Australian dollar, British Pound, Japanese Yen, and Swiss Franc are traded on PHLX. Prior to the introduction of Euro in 1999, the options on Deutsche Mark were also traded in large volume at PHLX.

Currency options are also traded at the Chicago Mercantile Exchange (CME) and the London International Financial Futures Exchange (LIFFE). The former offers options on more than a dozen foreign currencies as well as options on cross rates (exchange rate between two non-U.S. dollar currencies), while the latter provides foreign exchange options on US dollar quoted in Euro, and options on the Euro quoted in U.S. dollars. In September 1985 the CBOE began offering European-style foreign currency options, but these option contracts failed to attract significant volume, and were transferred to PHLX on August 1987.

Also in 1973, the Black-Scholes model for options was published which provides an intuitive and insightful solution to option valuation. Although this model is not without its limitations, its practical uses were immediately recognized. For instance, the assumption of constant volatility is not consistent with the empirical market results, nor do prices of underlying

assets such as individual stocks necessarily follow a lognormal distribution. Instead of pricing option using the Black-Scholes assumptions, the currency options traders became aware of the volatility smile inherent in the prices of currency options, and employ a different distribution that allows for fatter tails on both ends of the distribution (Hull, 2006). Nevertheless the status of Black-Scholes formula as a model for a base-case scenario which can be extended further to apply to pricing of a variety of options has not been lost.

An aspect of option pricing theory that does not depend on the assumptions made by Black-Scholes is the put-call parity, which establishes the relationship between the price of a call option and that of a put option, using a no-arbitrage argument, provided that the call and the put have the same underlying asset and time of maturity. It is important here to be specific about the style of the options, for put-call parity conditions are significantly different for European and American options.

This project attempts to determine the early exercise premium of both put and call options by applying the concept of put-call parity. It is an extension of a paper by Poitras, Veld and Zabolotnyuk (2006), who used daily data on currency options from PHLX to determine the level of early exercise premium for both call and put options. The organization of this project is as follows. Section II reviews the current literature on both the efficiency of foreign currency options and the early exercise premium of American options. Section III provides details on the options traded on PHLX and the CME as well as a brief description of the over-the-counter markets. Section IV discusses the key determinants in the level of early exercise premium for currency options. Section V offers an extended discussion on the concept of put-call parity. Section VI analyzes how to determine the optimal exercise for American options. Section VII offers a detail description of the methodology used in this study. The data used for this study is discussed in Section VIII. In Section IX, we present the results obtained, and provide an analysis of these results, which include a comparison of our results with those of Poitras, Veld and

Zabolotnyuk (2006). Finally section X summarizes the project. Appendices and References can be found in Sections XI and XII respectively.



## 2 LITERATURE REVIEW

After the publication of the Black-Scholes paper (1973), many studies have been carried out to extend the Black-Scholes model, which provides analytical solution to European options, to the valuation of American options. In many instances, closed form solutions can not be obtained as path dependency proves to be a formidable challenge. The possibility of early exercise renders the pricing of American options rather difficult, as the decision to exercise is contingent on the path the underlying asset that takes (Back, 2006).

Roll (1977), Geske (1979) and Whaley (1991) approximated the value of an American call option through constructing and valuing hypothetical portfolios of European options that mimic the payoff of the American options. Brennan and Schwarz (1977), Geske and Johnson (1984), Barone-Adesi and Whaley (1987) all proposed analytical approximation methods to value American options (Poitras, 2002 p.464).

Just as the timing and likelihood of exercising early for stock option is influenced by dividends, the probability of exercising a currency option is dependent on the level of foreign interest rate. More specifically, if the foreign interest rate is sufficiently above the domestic interest rate, then the call option holder may choose to exercise (i.e.: give up time value of the option in order to earn the foreign interest rate).

The theme of efficiency of currency options market has generated much interest. Amongst earliest efforts include Bodurtha and Courtadon (1986) and Shastri & Tandon (1986), who analyze the deviation from boundaries for foreign currency options. While these studies point out the inadequacies of Black-Scholes model as the tool of valuing American style options,

these violations themselves do not provide insights into what are the drivers of these deviations (Jorion and Stoughton, 1989).

Using a modified version of the Black-Scholes model, Shastri and Tandon (1986) examine the efficiency of the foreign currency options market by conducting both ex-ante and ex-post tests. The ex-ante tests examine the ability of an option trader to take advantage of the deviation from the model price. Essentially, the study by Shastri and Tandon uses a European pricing model to arrive at trading strategies for American options. Their ex-post results indicate that while there are numerous opportunities of abnormal profits, these tend to disappear if the trades are executed the day after the observation. This suggests that during the sampling period of the ex-ante period, the currency options market on the PHLX appear to be relatively efficient.

Bodurtha and Courtadon (1986) test the deviation from both the early exercise and the put-call parity boundary conditions. The key idea underpinning their study is that if these boundary conditions were violated, then the market is clearly not efficient. As in the case of Shastri and Tandon, the currency options data from PHLX is used. Using a transaction database that takes transaction costs and synchronicity into consideration, Bodurtha and Courtadon show that the currency options market is efficient. In particular, they find that while there are a considerable number of violations of put early exercise boundary, and some violations of put-call parity conditions when transaction costs are ignored. However, adjusting for non-synchronicity has the effect of significantly reducing the number of put-call parity condition violations as well as moderately lessening the incidents of early exercise boundary violations. Moreover, inclusion of transaction costs result in dramatic decrease in deviation from early exercise boundaries. Thus they reach the conclusion that put-call parity violation is particularly sensitive to synchronicity of the data, whereas that of early exercise boundary is influenced by transaction costs.

While these studies offer insights into the efficiency of currency options market, they have yet to address the issue of early exercise premium. Focusing on the probability of early exercise, Jorion and Stoughton (1989) analyze the relationship between the market value of exercise premium and the parameters that drive these values. Specifically, they develop comparative statistics by assuming that the spot exchange rate follows a diffusion process. Jorion and Stoughton assert that for any given time during the life of the option, there is a critical spot rate, that once reached, would be optimal to exercise an American call option. Similarly, another critical, optimal exercise spot rate exists for the American put option at every point in time. Jorion and Stoughton are also the first to use European style options traded briefly on the CBOE in conjunction with the American style options trading at PHLX. They determine the five parameters of interest to be the moneyness of the option measured as  $\log(S/K)$ , both domestic and foreign interest rates, volatility of the exchange rate and time to expiry.

In essence, they argue that a higher spot rate means an increased likelihood to reach the critical spot rate for the call option before maturity, while the likelihood of reaching the critical spot rate for the put option is decreased. In the same vein, increasing the exercise price will reduce the probability of the spot exchange rate reaching the critical spot rate, whereas the effect will be the opposite for the put option.

Jorion and Stoughton hypothesize that changing time to maturity and volatility, like adjusting the spot and strike price, have the same sign as the effect on the premium of the options. In the case of early exercise premium on a call option, while increasing the time to expiration will push the critical spot rate upward, this effect is more than offset by the fact that a longer-life contract has higher likelihood of reaching the critical value. The same can be said about volatility where increased dispersion of spot rate dominates the effect of a higher exercise threshold. The same argument can be put forward for the effect on the early exercise premium of the put option. Higher foreign interest rate represents a higher opportunity cost to call option holders. Thus it will

increase the likelihood of exercise; for put option holders, a higher foreign interest rate makes them wish to hang to the foreign currency in order to earn the high return, thus reducing the probability of early exercise. The reverse is true for the domestic interest rate.

In running a multiple regression to test their hypothetical model with the daily data obtained from both PHLX and CBOE, Jorion and Stoughton find that for call options, most of the parameters are significant with the signs agreeing with the hypotheses. However, the results for put options are less convincing as the t-statistics were too low for the coefficients of the parameters to be considered significantly different from zero, as can be seen from Table A. Moreover, the sign for the volatility parameter of the put option is the reverse of what would be expected. Nevertheless, the result of regressing the actual market early exercise premium against an approximated model premium shows a higher  $R^2$  than when a multiple regression was applied with a slope close to 1. This suggests that the simple valuation model captures the relationship between the determinants and the premium much better than a linear model. The corresponding  $R^2$  for put options was, however, virtually unchanged.

While European options on spot and futures contracts have an equivalent value given that no early exercise is possible, the same cannot be said about their American style counterparts (Bodurtha & Courtadon, 1995). Options on spot and those on futures differ significantly in their early exercise probabilities and values as well as the behaviour of their traders. In their 1995 paper, Bodurtha and Courtadon compare the data from of American-style options on both spot and futures currency options, employing an implicit difference method to arrive at the value of premium and early exercise boundaries. Currency options data on spot and futures exchange rate are obtained from PHLX and CME, respectively. Their study yields two interesting and important results. First, the effects of volatility and domestic interest rate on the early exercise probability differ from early exercise premiums for options on futures. That is, they find that the higher volatility will increase early exercise premium for in-the-money futures calls, but reduces the

probability of early exercise. In the same vein, raising the domestic interest rate increases the early exercise premium, yet lowers the likelihood of exercising early. They also conclude that observed early exercise behaviour in both the PHLX spot options and CME futures options markets conform to the standard pricing model for American options, since with a few exceptions, the empirical exercises occur at the theoretically prescribed boundaries.

Zivney (1991) examines the measurement of early exercise premium by using observation from prices of the CBOE's S&P 100 index options. He uses the violation of put-call parity for European style option to approximate the early exercise premium. Given the put-call parity equation

$$c - p = S - D_T - Xe^{-rT},$$

where  $c$  and  $p$  are the prices of European call and put, respectively,  $D$  is the dollar value of dividend payment,  $X$  is the exercise price, and  $T$  is the time to expiry. It is possible to determine an implied interest rate for the put-call parity to hold if the dividend payment is deterministic. Assuming the condition of no-arbitrage, deviation from European put-call parity allows for the possibility of exercising early. Taking the implied interest derived from the nearest money call and put pairing, Zivney determines the net value early exercise to be the difference between the price differential of observed American call and put and price differential of the nearest money call and put that satisfies put-call parity conditions. That is,

$$A = (C - P) - (C - P)' = (C - P) - (S - D_T - Xe^{-r_0'T}),$$

Where,  $r_0'$  is the risk-free rate that corresponds to put-call parity for nearest money pairing.

Zivney's results show that  $A$ , the early exercise premium increases with moneyness of the option and time to expiration as can be seen in Table B. A higher interest rate increases the

early exercise premium for call options on a stock index, but the opposite is true for puts. He also notes that put options have substantially higher early exercise premiums than call options and he asserts this demonstrates the failure of previous models to account for early exercise. The results he obtained are also in line with previous studies that suggest put options were undervalued relative to calls.

De Roon and Veld (1996) point out that equity index options, such as the one used by Zivney, do not correct for dividend payments, thus deviation from put-call parity may stem from the early exercise premiums of both call and put options. They instead examine put-call parity on a performance index since the dividend payments are automatically re-invested, so that underlying index behaves like non-dividend paying stock. In this case, only prices of put options will contain early exercise premium.

First De Roon and Veld determine the upper and lower bounds for value of call option. Recognizing that in a dividend-free world, the American and European call share the same upper bound, they show that the difference between the upper and lower bound of the call option can be used to represent the maximum premium on the put. The actual early exercise premium, however, is the difference actual value of call and upper bound of American call.

Using the options on the Frankfurt-based DAX, which trades on the Amsterdam Stock Exchange (ASE), De Roon and Veld show that premium for early exercise, is consistently positive. In regressing the differential of actual call price and upper American boundary against inputs of option pricing, they find that moneyness, implied volatility and domestic interest rate have the positive sign in the coefficients as predicted. However, as Table C shows the time to maturity appears to be significant.

Another analysis of early exercise premium can be found in Engström and Nordén (2000) who examine the American style equity put options traded on the Swedish exchange for options

and derivatives. Taking advantage of the fact that dividend payment occurs only once a year in Sweden and using a sampling period of seven months, they are able to solely attribute early exercise premium obtained to the put option, since call option would never be exercised early in the sampling period. By way of comparison, they also calculate a difference between theoretical values of American options based on MacMillan (1986) and Barone-Adesi and Whaley (BAW) (1987) and European options based on Black-Scholes.

Of particular interest is the fact that Engström and Nordén apply a modified control variate technique to empirical found premium to value the American put options. In essence, the regression coefficients from the estimation period are used to obtain early exercise premiums and then adding the premium to the Black-Scholes price. The accuracy of this method can be examined by checking the obtained option value with the market price of the put, as well as the theoretical values from the BAW model. Their results suggest that the value of early exercise premium is quite significant, with average premium higher for put-call parity measure than the theoretical measure outlined above. Table D shows that the premium increases with time to maturity of the option and moneyness. Nevertheless, they find that effect of interest rate and implied volatility depends on the moneyness of the options. The modified control variate technique appears to work rather well when compared with the Barone-Adesi and Whaley model of pricing American put. This is especially true when looking at the deep in the money options where the performance of control-variate technique is significantly superior.

Most recently, Poitras, Veld and Zabolotnyuk (2006) apply the method Zivney uses to derive to the early exercise premium of index option to currency options. The data they consider are from January 1992 to September 1997 on the six relatively liquid currency options traded on PHLX. Currency options are a primary candidate for early exercise because foreign risk-free interest rate is to the holder of a currency call what dividend payments are to equity option holder; both represent opportunity cost of keeping the option alive (Hull, 2006). They identify

and discard options that are very near the money since the value of  $(C-P)-(c-p)$  in order to address of problem of having confounding effect of having the early exercise attributed to both the call and put options. In addition, the options prices that do not conform to put-call parity conditions would naturally have to be eschewed from this analysis.

They hypothesize that the early exercise premium is mainly influenced by relative interest rate between the foreign and domestic currencies, and time to maturity as well as volatility. They derive the foreign interest rate from the covered interest rate parity, after using the 3-month Eurodollar as the proxy for the domestic (U.S.) risk-free interest rate. Using a multiple linear regression in which the REEP, relative early exercise premium (early exercise premium as a percentage of option value) is regressed against the parameters of interest differential, moneyness, time maturity and implied volatility, they estimate that REEP is 6.88 % for call options, and 5.71% of option value for put options.

The results for their regression agree with ex-ante hypotheses for the most part. Interest rate differential (defined in their study as subtracting domestic from foreign rate) is positive for puts and negative for calls. Time to expiration date increases the premium for both puts and calls. The effect of ratio of moneyness on early exercise premium increases is insignificant for calls, is significant for puts. However, as indicated in Table E, the coefficient for call options is not significant, but is for puts. For the volatility parameter, while volatility increases the REEP for calls, the regression implies that it does the opposite for that of put options. Interestingly, this result is quite similar to Jorion and Staughton (1989), who also find a negative, albeit insignificant, sign for effect of volatility on early exercise premium for put. Nevertheless, the results obtained by Poitras, Veld and Zabolotnyuk are relevant when European option models are used to determine the values of the American-style currency options.



**Table A: Jorion and Stoughton (1989)**  
**Regression Model of the Early Exercise Premium for Currency Options using data from PHLX (American-style) and CBOE (European-style)**

<b>Premium = <math>\alpha_0 + \alpha_1 \log(S/K) + \alpha_2 r + \alpha_3 r^* + \alpha_4 \sigma + \alpha_5 T + error</math></b>					
<b>PARAMETERS</b>	<b>HYPOTHESIZED</b>		<b>RESULTS</b> (*denotes significance at 5% level)		
	Call	Put		Call	Put
Constant Term			<i>Coefficient Estimate</i>	0.0729	0.0021
			<i>t-statistics</i>	0.76	0.03
Moneyness of the Options (log(S/K))	+	-	<i>Coefficient Estimate</i>	0.4185*	-0.0766
			<i>t-statistics</i>	3.13	-0.43
Exchange Rate Volatility, $\sigma$	+	+	<i>Coefficient Estimate</i>	0.3286*	-0.1757
			<i>t-statistics</i>	1.98	-1.02
Domestic Interest Rate, $r$	-	+	<i>Coefficient Estimate</i>	-0.0163	0.0035
			<i>t-statistics</i>	-1.28	0.43
Foreign Interest Rate, $r^*$	+	-	<i>Coefficient Estimate</i>	0.0061*	-0.0012
			<i>t-statistics</i>	2.58	-0.54
Time to Maturity, $T$	+	+	<i>Coefficient Estimate</i>	0.0631	0.0637
			<i>t-statistics</i>	1.79	1.81
			$R^2$	0.0475	0.0070

Data source: Jorion and Stoughton (1989).

**Table B: Zivney (1991)**  
**Regression Estimates of Value of Early Exercise for S&P Index Options at CBOE**

$A = a + b_1 f(S - X) + b_2 T + b_3 r_o' + error$					
PARAMETERS	HYPOTHESIZED		RESULTS (*denotes significance at 5% level)		
	Case 1 (Calls in the money) A's>0	Case 2 (Puts in the money) A's<0		Call	Put
Constant Term			Coefficient Estimate	-0.1630**	0.4304**
			t-statistics	-6.67	11.97
Moneyness of the Options (S-X)	+	+	Coefficient Estimate	0.0411**	-0.1119**
			t-statistics	31.34	38.66
Time to Maturity, T	+	-	Coefficient Estimate	0.00311**	-0.0044**
			t-statistics	11.82	-11.58
Implied Daily Interest Rate, $r_o'$	+	-	Coefficient Estimate	26.94	-57.92*
			t-statistics	1.47	-2.18
			$R^2$	0.3298	0.5118

Data source: Zivney (1991).

Note: In Zivney (1991), the conversion process from calls to puts means the parameters for puts are reverse of what is obtained in multiple regression.

**Table C: De Roon and Veld (1996)**  
**Regression Estimates of Value of Early Exercise for put options on DAX trading at ASE**

$D_t = \beta_0 + \beta_1 M_t + \beta_2 \sigma_{impl,t-1} + \beta_3 r_t + \beta_4 (T - t) + \beta_5 D_{t-1} + \varepsilon_t$			
PARAMETERS	HYPOTHESIZED	RESULTS	
		(*denotes significance at 5% level, **denotes significance at 1% level)	
	Put		Put
Constant Term		Coefficient Estimate	-0.62**
		t-statistics	-3.59
Moneyness of the Options, $S_t(X-I_t)$	+	Coefficient Estimate	0.06**
		t-statistics	5.20
Implied Standard Deviation of Call, $\sigma_{impl,t-1}$	+	Coefficient Estimate	1.04**
		t-statistics	2.97
Daily interest Rate, $r_t$	+	Coefficient Estimate	9.75**
		t-statistics	2.89
Time to Maturity, T-t	+	Coefficient Estimate	-0.08
		t-statistics	-1.06
$D_{t-1}$		Coefficient Estimate	0.66**
		t-statistics	11.27
		$R^2$	0.8887

Data source: De Roon and Veld (1996).

**Table D: Engström and Nordén (2000)**  
**Regression Estimates of Value of Early Exercise for American put options of Swedish equity options**

$\text{Early Exercise Value} = \alpha + \beta_1 \left( \frac{X}{S} \right) + \beta_2 (T - t) + \beta_3 (r) + \beta_4 \sigma + \varepsilon$				
PARAMETERS	HYPOTHESIZED	RESULTS		
			Put-Call Parity	BAW Model
Constant Term		Coefficient Estimate	-1.2477	-1.6499**
		P-Value	0.0513	0.0001
Moneyness of the Options (X/S)	+	Coefficient Estimate	8.4672**	7.0016**
		P-Value	0.0001	0.0001
Time to Maturity, T-t	+	Coefficient Estimate	2.4372**	2.3575**
		P-Value	0.0001	0.0001
Interest Rate, r	+	Coefficient Estimate	-65.638**	-42.903**
		P-Value	0.0001	0.0001
Implied Volatility, $\sigma$	+	Coefficient Estimate	-3.3201**	-5.2788**
		P-Value	0.0001	0.0001
		$R^2$	0.3611	0.4923

Data source: Engström and Nordén (2000).

Note: Engström and Nordén (2000) find that for PCP method, when dividing the options into out-of-the money, at-the-money and in-the-money groups, the out-of-the-money and at-the-money groups show positive and significant coefficients for interest rate and volatility as hypothesized, but the in-the money group continues to have significant and negative coefficients for these two parameters.

**Table E: Poitras, Veld and Zabolotnyuk (2006)**  
**Regression Estimates of Value of Early Exercise for Currency Options on PHLX**

<b>REEP = <math>\alpha + \beta_1(r - r_f) + \beta_2(T - t) + \beta_3(S / K) + \beta_4(\sigma_{call,t-1}) + \varepsilon</math></b>					
<b>PARAMETERS</b>	<b>HYPOTHESIZED</b>		<b>RESULTS</b> (*denotes significance at 5% level, **denotes significance at 1% level)		
	In-the-money Calls (S/K>1.005)	In-the-money Puts (S/K<0.995)		Calls	Puts
Constant Term			<i>Coefficient Estimate</i>	0.23	0.49*
			<i>t-statistics</i>	1.11	2.43
Interest Differentials (r-r <sub>f</sub> )	-	+	<i>Coefficient Estimate</i>	-0.90**	0.77*
			<i>t-statistics</i>	-6.33	5.74
Time to Maturity, (T-t)	+	+	<i>Coefficient Estimate</i>	0.10**	0.05**
			<i>t-statistics</i>	4.92	3.49
Moneyness of Options, (S/K)	+ (non-linear)	- (non-linear)	<i>Coefficient Estimate</i>	-0.24	-0.43*
			<i>t-statistics</i>	-1.16	-2.13
Implied Volatility, $\sigma_{call,t-1}$	+	+	<i>Coefficient Estimate</i>	0.38**	-0.43**
			<i>t-statistics</i>	2.80	-4.20
			$R^2$	0.43	0.30

Data source: Poitras, Veld and Zabolotnyuk (2006).

## **3 THE CURRENCY OPTION MARKET**

### **3.1 The Philadelphia Stock Exchange (PHLX)**

The Philadelphia Stock Exchange (PHLX), the oldest stock exchange in the U.S., was established in 1790. Although it is not a major commodity futures exchange, it was the primary currency options exchange in the United States for many years. It has the distinction of being the first exchange to offer currency options (American-style) contracts in December 1982. Moreover, PHLX also offers customized currency contracts alongside its standardized option contracts. In recent years, however, the Chicago Mercantile Exchange (CME) has overtaken PHLX in terms of volumes and open interest (Poitras, 2002).

Standardized options contracts at PHLX typically have standardized contract size, strike intervals, expiry date, price quoting as well as premium settlement. The exchange currently offers currency options on six major currencies, Australian dollar (AUD), British Pound (GBP), Canadian dollar (CAD), Euro (EUR), Japanese Yen (JPY) and Swiss Franc (CHF). Both American and European style options are available for these currencies, with the Euro currency enjoying the largest trading volume. Even within the confines of these standardized settings, there are considerable degrees of choice available to traders. For instance, PHLX offers mid-month and month-end as well as long-term expiration contracts. Currency options contracts typically are available for trading on quarterly months of March, June, September and December as well as two near-term months. The issuer and guarantor of the contract is the Options Clearing Corporation (OCC). As the expiry settlement date is typically on third Thursday or Wednesday (for contracts expiring in March, June, September and December), the last day of trading for these contracts is the preceding Friday. Appendix A shows standardized currency options offered on the PHLX as of July 2006.

In response to the success of the over the counter market, PHLX launched customized currency options contracts. These offer the traders to customize such specifics as strike price, date of expiry of up to two years, and premium can be expressed either in units of the base currency or as a percent of the underlying currency. With a few exceptions, any pairing of the currencies approved for trading (in the customized setting, this means the six currencies available in the standardized setting as well as US Dollar and Mexican Peso (MXP). Nevertheless, even customized options are inflexible in terms of contract size (fixed size for each currency) and style (exclusively European). Designed mainly for institutional traders, customized contracts have minimum opening transaction size. In addition, the customized option series are not continuously traded or quoted. But instead are distributed as text message on such quotation terminals as Reuters and Bloomberg. Appendix B shows customized currency options traded on the PHLX as of July 2006.

### **3.2 The Chicago Mercantile Exchange (CME)**

Founded in 1898 as the Chicago Butter and Egg Board (renamed as Chicago Mercantile Exchange in 1919), the CME, also known as the Merc, focused on the trading agricultural products in its early years. Nevertheless, this exchange quickly evolved to include other derivative contracts. It claims to be the world's leading exchanges for futures and options in the area of interest rates, stock indices, foreign exchange and commodities. In 1972, CME became the first exchange to offer financial derivatives by offering currency futures on seven major currencies. In fact, it is currently the leading currency futures and options on currency futures in the world in terms of volume. Prices are quoted for both pit-trading and trading on electronic platform. In the mid 1980's the CME also began offering options on major currencies, with much success. It now offers a wider of selection of foreign currencies and cross rates options than PHLX, and enjoys a much higher volume. The underlying of these options are typically the

currency futures traded at the CME. Appendix C shows the options on foreign currency options futures offered at the CME.

### **3.3 The Chicago Board of Options Exchange (CBOE)**

The Chicago Board of Options Exchange initiated trading of European style currency options in September 1985, in an attempt to take advantage of the fact that some hedgers had no assignment risks for contract that matches the maturity of the futures contract. Since these contracts were virtually identical from those trading at the PHLX with the exception of style exercise, the contracts from the two exchanges could be monitored for arbitrage opportunities. Moreover, they serve as good subjects in studying the early exercise premium, since these premiums can actually be measured directly. Unfortunately, the CBOE currency options never generated sufficient volume, and by August 1987, they were transferred to the PHLX.

### **3.4 Over-the-counter Market (OTC)**

The over-the-counter options market is not a recent phenomenon, for it grew alongside the exchange traded currency options (Taylor, 2003). The interbank currency option market is now by far the largest and most important market for currency options, with large institutions as the dominant players. The popularity of this market will likely continue as more banks and corporations seek to hedge their currency exposures with more derivatives products.

The OTC market has generally no upper limit on the size of the deal, but US \$10 million appears to be the norm for the bare minimum. The contracts are typically tailored to the needs of parties though most OTC contracts are European style. In the absence of a clearing agency, it is imperative that market participants consider the element of credit risk, keeping in mind that typically for option contracts such as call or put, credit risk arises when the option becomes in the

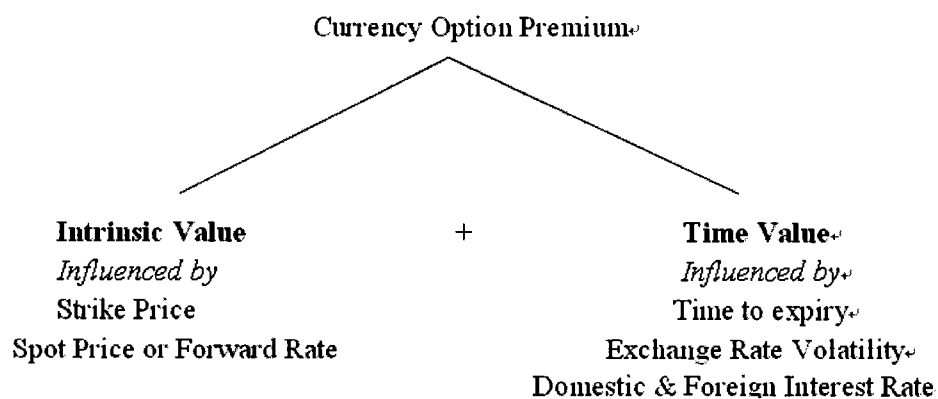


money (Klein, 1996). A comparison of exchange-traded currency options and those traded over-the-counter is provided in Appendix D.

## 4 OPTION PREMIUMS

As option holders have the right but not the obligation to exercise, all options have non-negative values. That is, the option prices (values) or premiums are always greater than or equal to zero. Option pricing theory assumes that the option premium can be split into two parts: intrinsic value and time value. Premiums that are quoted should combine an element of both components. The amount of premium depends on several factors, such as exchange rate volatility, relevant interest rates (which in the case of currency options means both domestic and foreign risk-free rate), time to expiration, strike price and option style.

**Figure 1: Components of Currency Option Premium**



*Based on 2000, Coyle, page 61*

### 4.1 Intrinsic Value

The intrinsic value can be defined as the amount the option would be worth if it were exercised immediately. In other words, it is the difference, between the strike price and the spot price. A currency call option has value at expiry by the amount of the spot price is higher than the strike price. A currency put option has value at expiry by the amount of the spot price is below

the strike price. At the time of expiry, an option will be worth its intrinsic value. Prior to the expiration date, an American style option will always be worth at least its intrinsic value, otherwise there will be riskless profit opportunities for the arbitrageur. However, a European style option can be traded at a discount to its intrinsic value right up to the expiration date because there is no possibility of exercising early and thus it is difficult for the arbitrageur to take advantage any risk-free profit opportunity before the expiration date (Coyle, 2000).

The intrinsic value for call or put options will be the maximum of zero and the difference between the spot and strike price.

$$\text{Call: } \textit{Intrinsic Value} = \text{Max}(0, S_T - K)$$

$$\text{Put: } \textit{Intrinsic Value} = \text{Max}(0, K - S_T)$$

The intrinsic value can be used as a reference to the moneyness of options. An option is in-the-money if the intrinsic value is positive and is at-the-money if the intrinsic value is zero.

**Table F: The relationship between the intrinsic value and the moneyness**

Strike Price (K) VS. Spot Price (S)		Intrinsic Value	Moneyness
Same	K = S	ZERO	At-the-money
Less Favourable	Put: K < S	NEGATIVE: the underlying exchange rate must move to the strike rate to take the intrinsic value to zero.	Out-of-the-money
	Call: K > S		
More Favourable	Put: K > S	POSITIVE: difference between strike price and spot price	In-the-Money
	Call: K < S		

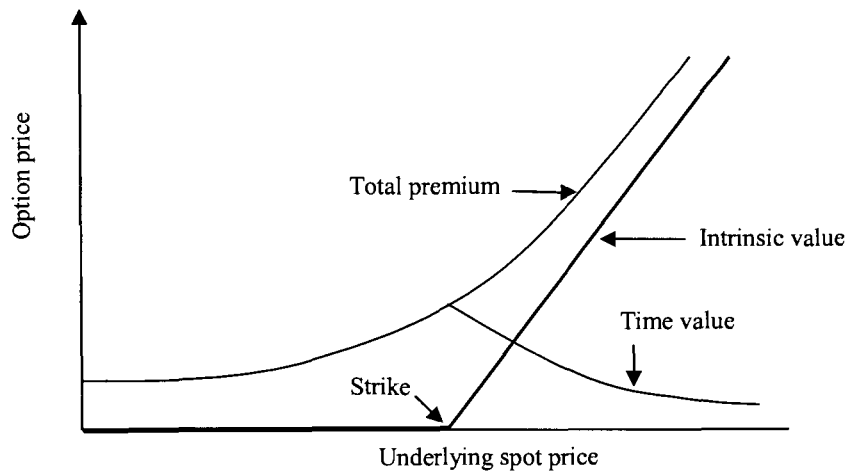
Based on 2000, Coyle, page 66

## 4.2 Time Value

The time value of an option is the difference between its premium and its intrinsic value. The time value is the more difficult to estimate than its definition may suggest. First, only if the option has some time remaining before expiry will it have any time value. The level of time value depends upon the degree of probability that the underlying market price, the exchange rate in case of currency options, could move sufficiently in the period up to the maturity to create some profit for the option holder. Therefore, the time value is sometimes referred to as a 'risk' premium, sometimes 'net' or 'extrinsic' premium (Sutton, 1990, page 30). The time value of currency options is determined by five factors: spot price, strike price, volatility of exchange rate, time to expiry, and interest rate differentials. At any time before the expiration date, the in-the-money option premium is made up of intrinsic and time value. At-the-money or out-of-the-money options have strikes at or above (call)/below (put) spot price, thus they have no intrinsic value and are made up solely of time value.

The time value of an option tends to be at its maximum when the spot price is equal to the strike price. As the deeper in-the-money an option becomes, the more its time value decreases (Sutton, 1990, page 31). Figure 2 illustrates this point. Once the option is very deep in-the-money, it loses virtually all its time value and trades only for intrinsic value. This is because the option will almost certainly be exercised and therefore the buyer will not pay, nor will the writer demand, any risk premium above intrinsic value. This suggests that it is theoretically possible for American options to have zero time value and for European options to have a negative time value.

**Figure 2: Intrinsic and time value for Call Option**



*Source: 1990, Sutton, page 31*

Even if the option is out-of-the-money, it still may have some value (time value only) (Sutton, 1990, page 31). This is because the market believes that there is a chance that the spot will move higher (call) or below (put) the strike price before the option expires, assuming there is sufficient time remaining before expiration date; indeed, the longer the maturity of the option the greater the premium. The reason is because the chances of larger price movements are greater for the longer-dated option and consequently the option is more likely to move in-the-money and be exercised. As the option approaches the maturity, time value falls to zero. While it is generally true that long dated options have more time value than short dated options, it should be noted that the effect of time value on option premiums is not linear. The premium for at-the-money options declines at an accelerating rate towards the expiration date.

### **4.3 Volatility of Exchange Rate**

The volatility is a statistical measure of the amount or percentage by which the spot price of the underlying assets is expected to fluctuate during a given time period. Volatility describes

the size of likely price variation, specifically of price variations around the trend rather than the trend itself. The greater the chances of the underlying currency moving higher or lower over the time period of the option, the higher will be the premium. Therefore, the more the underlying currency is expected to fluctuate, the greater will be the currency option premium. Interestingly, the volatility effect grows as the maturity of the option increases and falls as the option moves deeper into the money.

Volatility is a key variable in option pricing, but the value of this parameter cannot be easily obtained. While volatility cannot be observed directly, the difficulty lies in the fact that future volatility, which is unknown at present time, needs to be estimated as it is an input in the option pricing formula. Essentially there are two ways of estimating volatility. First, historical volatility, which involves calculating the standard deviation of a given series of past prices, can be considered. The historical method relies on the volatility which adequately explains past market movements to forecast how the market is likely to move in the future. For historical volatility, it is important to note that the results will vary with the length of the time period used. Choosing a long period for rolling volatility will reduce the effect of noise, while a shorter period can better reflect the recent trend. An alternate method to obtain a volatility estimate is to solve the option valuation equation backwards, taking the price of the option in the market as given and finding the volatility that would make the theoretical value equal to the market price. This volatility figure is known as the “implied volatility” for the option, since the volatility estimate is implied by the price that investors observe in the market.

One of the advantages implied volatility is that it is forward looking. Historical volatility estimates can not account for new information that is expected to change the underlying asset’s volatility in the future. Nevertheless, the volatility that is relevant for establishing the value of an option is the volatility of its underlying asset from the present until option expiration, and this may not be completely captured by the implied volatility measure. One problem with using

implied volatility is that each option produces its own figure, the volatility that makes the theoretical value equal to the current market price for that option. But a multiplicity of volatility figure is inherently contradictory. Moreover, as the implied volatility method selects the figure that reconciles the theoretical value with the market price, trying to find options in the market that are mispriced relative to their underlying asset becomes a futile exercise. A third problem, related to the second, is that the procedure inherently incorporates into the volatility estimate all sources of mispricing, such as data errors, effects of the bid-ask spread as well as temporary imbalances in supply and demand. Fourth, option model-based forecast requires a number of assumptions to produce a useful volatility estimate (Figlewski, Silber and Subrahmanyam, 1990, page 97-99).

In short, both ways of obtaining volatility estimate are valid under the proper conditions. In practice, most professional option traders pay attention to both, sometimes blending them together into a single composite figure. There is no single best approach (Figlewski et al, 1990).

#### **4.4 Interest Rate Differentials**

The effect of interest rates on the option premium is another important component of the premium for currency options. In currency options, there are two interest rates involved, the domestic interest rate and the foreign interest rate. It is the interest rate differentials, not any single interest rate that affects the premium of currency options. The premium of a currency call option will increase if the domestic interest rate rises relative to the foreign interest rate. This is because the increase in domestic rate increases the cost of borrowing the domestic currency and thus makes the currency option alternative more attractive. For put options, which is simply the equivalent of the right to borrow in the foreign currency, a decrease in domestic rate or an increase in the foreign rate will make them more attractive (Sutton, 1990, page 40-41).

The effect of interest rate differential changes on currency option premium can be summarised as follows:

1. If the spot rate remains unchanged, a rise in the domestic interest rate relative to the foreign currency interest rate, or a fall in the foreign interest rate relative to the domestic interest rate, will increase the premium for a currency call option and decrease the premium for a currency put option.
2. If the spot rate remains unchanged, a fall in the domestic interest rate relative to the foreign currency interest rate, or a rise in the foreign interest rate relative to the domestic interest rate, will decrease the premium for a currency call option and increase the premium for a currency put option.

However the relationships are reversed when early exercise premium is considered in place of straight premium.

## 4.5 Bounds for Currency Option Premium

In this part, the upper and lower bounds for option premium will be discussed. If an option price is above the upper bound or below the lower bound, then there are profitable opportunities for arbitrageurs.

### 4.5.1 Upper Bounds

An American or European call option gives the holder the right to buy a set amount of a currency. Under no conditions, can the option price be worth more than the spot rate ( $S$ ) of the deliverable foreign currency. Hence,

$$c \leq S; C \leq S$$

If these relationships were not true, an arbitrageur could easily make a riskless profit by buying the spot and selling the call option.

Similarly, for an American or European put option, since it gives the holder the right to sell a set amount of a currency at the strike price ( $K$ ), the strike price is thus the maximum value for a put option. Hence,

$$p \leq K; P \leq K .$$



If these relationships were not true, an arbitrageur could make a riskless profit by writing the option and investing the proceeds of the sale at the risk-free interest rate

#### 4.5.2 Lower Bounds for European options

In the absence of arbitrage opportunities, a European call option must satisfy:

$$c \geq Se^{-r_f(T-t)} - Ke^{-r(T-t)}$$

Appendix E shows the detail of derivation.

Because the worst that can happen to a call is that it expires worthless, its value cannot be negative. This means that  $c \geq 0$ , and hence the lower bound of a European currency call is as follow:

$$c \geq \text{Max} [0, Se^{-r_f(T-t)} - Ke^{-r(T-t)}]$$

For a European put option, in the absence of arbitrage opportunities, it should satisfy:

$$p \geq Ke^{-r(T-t)} - Se^{-r_f(T-t)}$$

Also, since a European put option cannot be worth less than zero, the lower bound of a European currency put is as following:

$$p \geq \text{Max} [0, Ke^{-r(T-t)} - Se^{-r_f(T-t)}]$$

#### 4.5.3 Lower Boundary for American Options

The value of an American option must be at least as much as that of an otherwise identical European option because American options allow the possibility of early exercise prior to the expiration date. In addition, American options can never be worth less than the immediate exercise value (DeRosa, 2000). These conditions are combined to give the following lower bounds for American calls and puts.

Calls:

$$C \geq \text{Max} [0, S - K, Se^{-r_f(T-t)} - Ke^{-r(T-t)}]$$

Puts:

$$P \geq \text{Max} [0, K - S, Ke^{-r(T-t)} - Se^{-r_f(T-t)}]$$

## **5 PUT-CALL PARITY FOR CURRENCY OPTIONS**

The put-call parity describes the relationship between the prices of European put and call options with the same exercise price and time to maturity. It shows that the value of a call with a certain strike price and exercise date can be deduced from the value of a put with the same strike price and exercise date and vice versa. An interesting feature of the put-call parity is that it is based on a particularly simple no-arbitrage argument. That is, if the actual put or call price deviates from the parity price, an arbitrage opportunity exists for investors to earn more than the risk-free rate of return without any investment. However, the put-call parity can only indicate a relative mispricing of the put with respect to the call or vice versa, rather than indicate which of the two options is mispriced. This relationship does not require any assumption about the probability distribution of the asset price in the future (Hull, 2006, Page 375-376). The put, call and the underlying asset form an interrelated complex, in which any two of the three instruments can be combined in a way to replicate the future payoff of the third instrument. The put-call parity can also be used to price a call (put) option as long as the put price (call price), the spot price of the underlying assets, and related interest rates are known. Different forms of put-call parity for European options and American options exist because American options must take account of the possibility of early exercise (Zivney, 1991).

### **5.1 Put-Call Parity for European Currency Options**

A foreign currency has the property that the holder of the currency can earn interest at  $r_f$ , the risk-free interest rate prevailing in the foreign country. According to the no-arbitrage condition, the difference between the price of a European put and call with the same strike price and expiration date should be equal to the difference between the present value of the strike price

and the present value of the deliverable quantity of foreign currency (discounted at the foreign interest rate). Thus, the European put-call parity for currency option has the following form (Hull, 2006, page 314):

$$c - p = Se^{-r_f(T-t)} - Ke^{-r(T-t)}$$

or,

$$c + Ke^{-r(T-t)} = p + Se^{-r_f(T-t)}$$

According to the well-known covered interest rate parity (CIRP), the relationship between the spot exchange rate (S) and the forward exchange rate (F) is as follows:

$$F = Se^{(r-r_f)(T-t)}$$

When interest rates vary unpredictably (as they do in the real world), forward and futures prices are in theory not the same due to marking-to-market property of the futures contract. Also, a number of other real-world factors, such as taxes and transactions costs may cause discrepancies between forward and futures prices. Nevertheless, the theoretical differences between forward and futures prices for contracts that last only a few months are sufficiently small as to be negligible in most circumstances. Some empirical studies, such as Cornell and Reinganum (1981) and Chang and Chang (1987), show that the difference between foreign exchange futures and forward prices is statistically insignificant. Since futures contracts are standardized contracts traded on an exchange whereas forward contracts are not standardized contracts traded over-the-counter, it is naturally easier to find prices for futures contracts. Thus, futures prices ( $f$ ) are used as substitutes for forward prices in this project. Moreover, futures contracts in the same expiration cycle as options than forward ones should be used if possible in order avoid the problem of non-synchronicity.

Combining the put-call parity with futures prices, the relationship can be expressed as following:

$$c + Ke^{-r(T-t)} = p + fe^{-r(T-t)}$$

or,

$$c - p = (f - K)e^{-r(T-t)}$$

## 5.2 Put-Call Parity for American Currency Options

If early exercise is possible, the above put-call parity will not hold. For American options, the put-call parity can only be given as a lower and an upper bound instead of equality.

For no-dividend paying stocks, the boundaries are (Hull, 2006, page 215):

$$S - K < C - P < S - Ke^{-r(T-t)}$$

Similarly for American currency options, the boundaries according to the put-call parity should take the form (DeRosa, 2000, page 124-126):

$$C + Ke^{-r(T-t)} - S \leq P \leq C + K - Se^{-r_f(T-t)}$$

or,

$$Se^{-r_f(T-t)} - K \leq C - P \leq S - Ke^{-r(T-t)}$$

## **6 EARLY EXERCISE OF AMERICAN CURRENCY OPTIONS**

If market efficiency is assumed as would be the case under normal market conditions, two conditions may cause an American currency option to be exercised early. One has to do with the fact the holder requires the foreign currency before the exercise date. Secondly, an option will be exercised early if the holder stands to lose money or sacrifice a profit by continuing to carry the option position. These are conditions under which it is optimal to early exercise.

### **6.1 Sufficient Conditions for Early Exercise**

A sufficient condition for the optimal early exercise rather than selling an American option is that it is trading for less than its intrinsic value, which would be the case if the market itself is inefficient. That is,

$$\begin{aligned}C &< S_T - K \\P &< K - S_T\end{aligned}$$

Under normal circumstances, the market should be efficient, and therefore the option should always be worth at least its intrinsic value; consequently it would be more rational to sell the option rather than to exercise it.

### **6.2 Necessary Conditions for Early Exercise**

#### **6.2.1 American Calls**

According to Gibson (1991), the necessary condition for early exercise depends on both the option's being in-the-money and the spread between the domestic and foreign interest rates.

For an American currency call, a necessary condition for optimal early exercise is when the present value of the interest earned on the foreign investment at the foreign interest rate is greater than the forgone interest that could be earned by investing the exercise price at the domestic rate prior to expiration.

$$S[1 - e^{-r_f(T-t)}] > K[1 - e^{-r(T-t)}]$$

On one hand, the opportunity cost of early exercise is the forgone interest that could be earned by investing the strike price at the domestic rate between the exercise date and maturity date.

$$K[1 - e^{-r(T-t)}]$$

On the other hand, the opportunity cost of not exercising early is the interest that could have been earned by investing the deliverable quantity of foreign currency at the foreign rate between the exercise date and maturity date.

$$S[1 - e^{-r_f(T-t)}]$$

The opportunity cost of carrying the option is thus the difference between the above two formulas. In other words, the optimal moment to exercise is when the opportunity cost of exercising early is less than that of not exercising early. It will be optimal to early exercise an American currency call when it is deep enough in-the-money that the time value is approaching zero and when the foreign currency is at a discount. That is, a necessary condition for early exercise of an American currency call is that the interest rate is higher in the foreign country than the home country.

However, even if the interest on the deliverable quantity of foreign currency is much greater than that on the exercise price of domestic currency, the option might still be worth more than its opportunity cost of carrying if the volatility is sufficiently high. Thus this condition is only necessary, but not sufficient.

## 6.2.2 American Puts

Optimal exercise of an American put requires that the interest earned on the deliverable quantity of foreign currency between the exercise date and the expiration date is less than that on the exercise price of domestic currency. That is,

$$S[1 - e^{-r_f(T-t)}] < K[1 - e^{-r(T-t)}]$$

For an American put, the optimum moment to early exercise is when the option is deep in-the-money and when the foreign currency is at a premium. That is, a necessary condition for early exercise of an American currency put is that the domestic interest rate is higher than the foreign interest rate.

Once more, this is a necessary, but not sufficient condition. The reason is that at sufficiently high levels of volatility, the put may become even deeper in-the-money and thus more than compensate for the opportunity cost of keeping the option alive.



## 7 METHODOLOGY

There are potentially several methods of obtaining the value of early exercise premium. First of all, if American and European options exist on the same asset, then the early exercise premium can be computed directly by subtracting the European option value from the American. Unfortunately, there are no simultaneous liquid markets for otherwise identical American and European options in most situations. Second, the early exercise premium can be calculated as the difference between the market price of the American option and the value of a European option generated by an option pricing model. However, the observed measure is a function of the particular option pricing model employed, making it susceptible to model errors. For example, the option pricing model may not exactly capture the marketplace's assessment of the impact of the dividends, changing volatility, changing interest rate or early exercise probability. Alternatively, one can measure the value of early exercise by examining deviations from European put-call parity. Put-call parity relies upon the idea of a duplicating portfolio, which consists entirely of observable market prices. Since this method does not rely on a specific option pricing model, the problem of jointly testing the pricing model can be averted. Zivney (1991) used this third method to investigate the early exercise premium of S&P 100 index options. This project adopts the same approach to estimate the unobserved early exercise premium on currency options as Zivney (1991).

Derivation from put-call parity could be due either to the early exercise premium or simply to market inefficiencies. However, Kamara and Miller (1995) find that the violations from put-call parity of European options are much less frequent and smaller than that of American options (Engström & Nordén, 2000). This suggests that the major part of the derivations for

American options could be attributed to the early exercise premium. Zivney (1991) states in any given pair of calls and puts that have identical strike prices and time to expiration, one contract will be in-the-money and the other out-of-the-money. Since the in-the-money options have higher probability of being exercised than the out-of-the-money option, the in-the-money option should have a larger value for early exercise built into its market price. In other words, the value of C-P will be relatively high if the call is in-the-money and relatively low if the put is in-the-money.

Poitras, Veld and Zabolotnyuk (2006) use put-call parity to study the early exercise premium for currency options traded on the PHLX. They find that the early exercise premiums are slightly higher for call options than for put options and that these premiums are strongly influenced by time to maturity and the interest rate differential. This project will extend their study by changing the moneyness ratio and replacing two independent variables in the multiple regression model.

First, American currency calls and puts that have the same trade date, same underlying currency, same exercise price, and same expiration date are paired together. Then, the following steps are performed to study the early exercise premium of currency options.

## **7.1 Estimation of Foreign Interest Rate**

Three-month Eurodollar interest rates are used as the domestic interest rates. The Eurodollar interest rates are then applied to the Interest Rate Parity (IRP) to determine the foreign interest rates. For this purpose, the futures prices with the same expiration cycle as the traded options are used and the formula is as following:

$$F(0, N) = \frac{1 + \frac{r(0, T)}{4}}{1 + \frac{r_f(0, T)}{4}} F(0, n)$$

Rearranging the above formula, the implied foreign interest rate can be computed as:

$$r_f(0, T) = \frac{r(0, T)F(0, n) - 4[F(0, N) - F(0, n)]}{F(0, N)}$$

Where, N-n is equal to three months. F(0, n) is the nearest futures contract while F(0, N) is the futures contract that is expired three months after the nearest contract. All these futures contracts have the same trade date as the option. r and  $r_f$  are quarterly compounded annual interest rates.

## 7.2 Consistency with the Boundaries of American Put-Call Parity

The option prices are checked for consistency with the boundaries of the American put-call parity:

$$Se^{-r_f(T-t)} - K \leq C - P \leq S - Ke^{-r(T-t)}$$

The upper and lower boundaries for call options are, respectively:

$$\begin{aligned} C &\leq P - Ke^{-r(T-t)} + S \\ C &\geq P + Se^{-r_f(T-t)} - K \end{aligned}$$

The upper and lower boundaries for put options are, respectively:

$$\begin{aligned} P &\leq C - Se^{-r_f(T-t)} + K \\ P &\geq C + Ke^{-r(T-t)} - S \end{aligned}$$

Pairs of calls and puts that have prices outside the boundaries of American put-call parity will be excluded from the study to avert inaccurate measure of the early exercise.

## 7.3 Estimation of (c-p)

In order to obtain the early exercise premium, the price difference between an otherwise identical European call and put (c-p) is required. This difference can be calculated from the European put-call parity.

$$c - p = Se^{-r_f(T-t)} - Ke^{-r(T-t)}$$

## 7.4 Valuation of Early Exercise Premium

Following Zivney (1991), the unobserved early exercise premium (EEP) can be estimated by subtracting the observed theoretical European option price differentials from the observed American option price differentials, which leads to:

$$EEP = (C - P) - (c - p) = (C - P) - (Se^{-r_f(T-t)} - Ke^{-r(T-t)})$$

American option can be considered as including the European option prices and the early exercise premium as the following:

$$\begin{aligned} C &= c + EEP_c; & P &= p + EEP_p \\ EEP &= (C - P) - (c - p) = EEP_c - EEP_p \end{aligned}$$

This estimated EEP consists of a relatively large value (in absolute terms) for the option in-the-money and a relatively small value for the option out-of-the-money. Therefore, EEP is expected to be positive for in-the-money calls and negative for in-the-money puts.

As options move deeper into the money, the early exercise premium will approach zero. Thus, when puts are very deep-in-the-money:

$$EEP = (C - P) - (c - p) = EEP_c$$

When calls are very deep-in-the-money:

$$EEP = (C - P) - (c - p) = -EEP_p$$

The early exercise premium for near-the-money options consists of the premium for both calls and puts; therefore it is impossible to attribute the EEP solely to either calls or puts in this case.

The estimated early exercise premium for the currency options is then checked for consistency with the boundaries for the early exercise premium (EEP) according to the put-call parity.

$$c - p = Se^{-r_f(T-t)} - Ke^{-r(T-t)}$$

$$Se^{-r_f(T-t)} - K \leq C - P \leq S - Ke^{-r(T-t)}$$

From the above two relationships, the upper and lower boundaries for the early exercise premium, (C-P)-(c-p), are as follows:

$$EEP < S - Se^{-r_f(T-t)}$$

$$EEP > Ke^{-r(T-t)} - K$$

In this project, the prices of the option pairs are already checked for consistency with the boundaries of American put-call parity, thus the early exercise premium for these options should also be consistent with the above boundaries.

## 7.5 Subgroups of Moneyness

Moneyness is defined as the ratio of the spot price to the exercise price(S/K). Following Poitras, Veld and Zabolotnyuk (2006), the options data are divided into two subgroups with respect to moneyness which are in-the-money puts ( $S < K$ ) and in-the-money calls ( $S > K$ ). The exact definitions are as following:

Group 1:	In-the-money put	$S/K < 0.992$
Group 2:	In-the-money call	$S/K > 1.008$

The ranges of (S/K) for the groups are arbitrary. Poitras, Veld and Zabolotnyuk (2006) define in-the-money put as  $S/K < 0.995$  and in-the-money call as  $S/K > 1.005$ . Because the ranges of (S/K) in their study are very small and close to at-the-money options, we widen the range in

order to be more certain about the source of early exercise premium and to examine the size of the early exercise premium at a different level of moneyness.

Options with a spot price to strike price ratio of between 0.992 and 1.008 are classified as near-the-money options which will not be considered in this project, because the early exercise premium for these options can be attributed to both call and put options.

## 7.6 Hypotheses Testing

Based on Poitras, Veld and Zabolotnyuk (2006), a modified multiple regression model is used to test four hypotheses about the early exercise premium. The dependent variable in this model is the relative early exercise premium (REEP). This is the early exercise premium as a percentage of the option price.

$$REEP = \alpha + \beta_1(r - r_f) + \beta_2(T - t) + \beta_3 \ln(S / K) + \beta_4(\sigma_{Hist}) + \varepsilon$$

First, the relative early exercise premium (REEP) depends on both the domestic and foreign interest rates. For calls, the REEP should increase when the foreign interest rate increases relative to domestic rate. This effectively makes the possibility to exercise early becomes more attractive, and the value of REEP is expected to increase if the difference becomes larger. The situation for the put is the reverse, since if the call is in-the-money, the put is out-of-the-money. Here the REEP should increase if the domestic interest rate is higher than the foreign interest rate. The second hypothesis is that the early exercise premium increases with time to maturity for both calls and puts. Since the holder of a longer maturity American option has all the possibilities the holder of a shorter maturity option has, plus the additional possibilities coming from the extra time to expiration, the value of REEP is expected to increase if the time to maturity increases. Early exercise premium is expected to increase with the amount the option is in-the-money. The deeper in-the-money the option is, a higher likelihood that the option will be exercised early. For calls, when the spot price is higher than the exercise price, calls are in-the-money and thus, are

more likely to be exercised. The early exercise premium should increase as the ratio of spot price to strike price ( $S/K$ ) or  $\ln(S/K)$  increases. The effect for puts is the opposite. The early exercise premium should decrease in absolute terms as the ratio of the spot price to exercise price ( $S/K$ ) or  $\ln(S/K)$  increases because puts are moving in the direction of out-of-the-money. Thus, the sign of the coefficient should be negative. However, when an option is very deep in the money, the option value will be lower than the value of exercising early, thus the early exercise premium would approach zero again. Thus non-linear regressions may be better capture the relationship between the two variables. Thus, apart from natural logarithm of money ratio, squared moneyness and squared root of moneyness are both possible candidates for the moneyness parameter. Moreover, if the option is sufficiently deep in the money that time value approaches zero, then it will be optimal exercise if  $r_f > r$  for calls, while reverse is true for puts (Poitras, 2002, page 479). This suggests that for both the call and put groups, we can further divide the sample into groups with positive and negative interest differentials, and examine whether the effect of moneyness is different from each of these subgroups.

The effect of the volatility is intuitively not as clear. A higher volatility would increase the likelihood that the stock price reaches a level low enough to trigger early exercise, but the optimal exercise boundary level for all maturity will simultaneously be raised. The actual outcome depends on which effect dominates. Jorion and Stoughton (1989) state that the early exercise premium is hypothesised to be an increasing function of the volatility of the underlying security.

In summary, the signs of the coefficients for call options and put options are expected as following:

$$REEP = \alpha + \beta_1(r - r_f) + \beta_2(T - t) + \beta_3 \ln(S / K) + \beta_4(\sigma_{Hist}) + \varepsilon$$

<b>Call:</b>	+	-	+	+	+
<b>Put:</b>	+	+	+	-	+

To test the hypothesis, regressions will be run for both in-the-money put and in-the-money call.

## 7.7 Historical Volatility

Poitras, Veld and Zabolotnyuk (2006) use the implied method to test the effect of volatility on the early exercise premium of currency option. In this project, we will use historical volatility of the underlying which is the spot exchange rate for currency options and compare our regression results with that of Poitras, Veld and Zabolotnyuk (2006).

Historical volatility of spot exchange rates is the actual volatility which is the sample standard deviation of percentage rates of return of the spot exchange rate over a period of time (DeRosa, 2000, P. 97-98). The rate of return of the spot exchange rate,  $R_t$ , is calculated as the log differences in the spot rate:

$$R_t = \ln(S_t) - \ln(S_{t-1}) = \ln\left(\frac{S_t}{S_{t-1}}\right)$$

Where,  $S_t$  and  $S_{t+1}$  are successive observations on the spot exchange rate. To measure volatility continually, the unbiased sample rolling volatilities which are estimated by entering the daily rate of return are used.

$$\sigma_{t+s} = \sqrt{\frac{\sum_{i=t}^{t+s-1} (R(i) - \bar{R})^2}{s-1}} \quad \text{for } t = 1, 2, \dots, n-s$$



Where,  $S$ , the number of observations in the sample, represents the size of rolling-period and  $\bar{R}$  is the incorporating sample mean.

To choose an appropriate value for  $s$ , we need to consider the relationship between the length of the data set and the accuracy of the historical volatility as well as issues of statistical significance. More data generally lead to more accuracy, but data that are too old may not be relevant for predicting the future volatility. For this project, we set  $s$  equal to 20. That is, daily data for the last 20 trading days are used to estimate the historical volatility. Since the put options and call options have the same underlying assets, this historical volatility will be used for both put and call regressions.

An alternative to moving average is the exponentially weighted moving average (EWMA) which assigns more weights to the more recent data. The formula is  $\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2$ , where the estimate,  $\sigma_n^2$ , is the volatility for day  $n$ , made at the end of day  $n-1$ , is computed from  $\sigma_{n-1}^2$  (an estimate that was made at closing of day  $n-2$ ), and  $u_{n-1}^2$ , which is the most recent daily percentage change (Hull, 2006, page 463-464). Following J.P. Morgan's Riskmetrics database, we use a  $\lambda$  of 0.94. We will compare the effect of using EWMA with that of using a simple moving average to see whether either one improves the results of the regression.

## 8 DATA

As in Poitras, Veld and Zabolotnyuk (2006), American style currency options data from the Philadelphia Stock Exchange (PHLX) are used in this study. The data range from January 2, 1992 to September 24, 1997, over which period the currency options on the PHLX experienced active trading and high volumes. During the period, the PHLX traded options on the Australian dollar, British Pound, Canadian Dollar, Deutsche Mark, Japanese Yen, Swiss Franc, European Currency Unit, French Franc, Deutsche Mark / Japanese Yen, British Pound/ Deutsche mark, and British Pound/ Japanese Yen. This project only uses the data from the first six currencies, as they were the most actively traded options. Data from the PHLX include trade data, currency symbol, options type, expiration month, exercise price, number of contracts, number of trades, opening and closing prices, spot price, high and low prices, and time of trade. For this study, data on the exercise price, expiration date, spot price, and the closing prices of the options from the PHLX database are used.

After the screening process, 2,389 pairs of American call and put options that have the same trade date, underlying value, expiration month, and exercise price are sorted out. Then, 1,420 options that are at-the-money are eliminated, because in this case it is not possible to attribute the EEP solely to either puts or calls. From the remaining 969 options, those that are not consistent with the boundaries of the American put-call parity (213) are deleted from the data group to avoid inaccurate measure of the early exercise premium. Finally, 314 observations are eliminated, because they have a negative EEP, which is likely to be caused by non-synchronous trading of the options. The remaining sample in this study consists of 442 observations. These

observations are classified into 2 groups: 230 in-the-money puts in group 1 and 212 in-the-money calls in group 2. Appendix F summarizes the option selection process.

Three-month Eurodollar interest rates, obtained from the US Federal Reserve Board website, are used as the domestic interest rate. Eurodollar rates are more appropriate than T-bill rates to be the proxy for the risk-free rate. Given the regulation and market structure, the domestic markets (T-bill markets) may be less efficient than the Eurodollar markets. More importantly, Covered Interest Parity does not hold for T-bill rates, which are typically 100 basis points lower than Eurodollar rates (El-Mekkaoui & Flood, 1998). The effect of this yield differential will be more pronounced for American options because it will also affect the probability of early exercise. Thus, the Eurodollar interest rates are applied to the Covered Interest Rate Parity to determine the foreign interest rates.

Futures on currencies traded on the International Money Market Division of the Chicago Mercantile Exchange (Chicago IMM) are used to calculate foreign interest rates. The differences between forward and futures prices for contracts that last only a few months in the foreign exchange markets are insignificant. In addition, futures on currencies traded on the Chicago IMM have the same expiration cycle as currency options traded on the PHLX. The futures prices for the period from January 1992 to September 1997 are obtained from the Thomson Financial Datastream database.

Unlike Poitras, Veld and Zabolotnyuk (2006), who use implied volatility from the option for the volatility parameters, we use a historical, 20-trading day rolling volatility. Spot exchange rates, which serve as the value of the underlying asset for currency options, are obtained from the online database at Pacific Exchange Rate Service website (<http://fx.sauder.ubc.ca/data.html>). A summary statistics of all the parameters are provided in Appendix G.

## 9 RESULTS

In order to value the relative early exercise premium (REEP), the data are divided into two parts: in-the-money puts ( $S/K < 0.992$ ) and in-the money calls ( $S/K > 1.008$ ). Appendix H and Appendix I show the actual prices of the American options as well as their upper and lower bounds according to put-call parity for group 1 and group 2. Observations that are not consistent with the boundary conditions are deleted from the study to ensure the early exercise premiums calculated are reliable. Appendix J and Appendix K show the calculated early exercise premium (EEP) and upper and lower bounds according to put-call parity. Since the prices of the remaining options are consistent with the bounds of American put-call parity, the early exercise premiums for these options are all consistent with boundary conditions.

As Table G indicates, the REEP for the put group is 6.122%, which is higher than the 5.71% obtained from Poitras, Veld and Zabolotnyuk (2006). For each of the currencies, the level of REEP is also higher than the corresponding REEPs in the Poitras Veld and Zabolotnyuk (2006), the lone exception being the Japanese Yen which has an estimated REEP that is actually lower in our study. REEPs vary greatly among different currencies, from a low of 2.795% for the British Pound, to a high of 8.459% for the Japanese Yen. This is not surprising given that there is significant variation in the level of foreign interest rates.

Table H shows the REEP for the call group is 7.329%, again above the corresponding figure of 6.88% from the study by Poitras, Veld and Zabolotnyuk. It is therefore not surprising that the REEP for the individual currencies in our study are also higher than those of Poitras, Veld and Zabolotnyuk (2006). The relative early exercise premium for each currency is again large in

absolute term, though it is the Deutsche Mark that exhibits the highest premium at 11.005% while the Yen has the lowest at 5.635%.

Since the definition of near-the-money range in our case,  $0.992 \leq \frac{S}{K} \leq 1.008$ , is wider than that of Poitras, Veld and Zabolotnyuk ( $0.995 \leq \frac{S}{K} \leq 1.005$ ), it is natural that we would have fewer observations for both calls and puts. However, the fact that the REEPs we obtain are higher than those of Poitras, Veld and Zabolotnyuk (2006) is unexpected. While this may be attributable to the relatively small sample size in our study, our sample in both the call and the put group may not contain enough options that are sufficiently deep in-the-money to show the effect of decline in early exercise premium.

**Table G: REEP of Group 1 (In the money Put, S/K < 0.992)**

	<b>No. of observations</b>	<b>Average premium as % of option price (REEP)</b>	<b>Average US minus foreign interest rate</b>	<b>Standard Deviation of REEP</b>
<b>Overall</b>	<b>230</b>	<b>6.122%</b>	<b>1.998</b>	<b>0.060</b>
Australian Dollar	11	4.948%	-1.610	0.044
British Pound	29	2.795%	-1.862	0.033
Canadian Dollar	21	5.288%	-0.680	0.054
Deutsche Mark	45	4.686%	1.936	0.031
Japanese Yen	87	8.459%	4.188	0.074
Swiss Franc	37	5.803%	2.544	0.055

**Table H: REEP of Group 2 (In the money Call, S/K > 1.008)**

	<b>No. of observations</b>	<b>Average premium as % of option price (REEP)</b>	<b>Average US minus foreign interest rate</b>	<b>Standard Deviation of REEP</b>
<b>Overall</b>	<b>212</b>	<b>7.329%</b>	<b>-1.913</b>	<b>0.086</b>
Australian Dollar	21	5.854%	-1.530	0.037
British Pound	61	8.075%	-3.067	0.096
Canadian Dollar	9	7.811%	-2.221	0.049
Deutsche Mark	67	11.005%	-2.602	0.097
Japanese Yen	21	5.635%	1.217	0.077
Swiss Franc	33	7.881%	-0.533	0.086

Table I and Table J display the results of running a multiple linear regression of the relative early exercise premium (REEP) on the four parameters (moneyness, time to maturity, interest rate differentials, and volatility) for both put and call groups as well for each of the six currencies within the group.

Table I shows the results of the multiple linear regression for the put group. With the exception of the volatility, all coefficients are of the expected sign for the overall put options. The REEP of puts is positively related to both the difference between the domestic (US) and the foreign interest rate and time to maturity. Moreover, both coefficients are statistically significant on the 1%-level. The results for the moneyness ( $\ln(S/K)$ ) also give the hypothesized sign; the relationship between REEP and  $S/K$  is less apparent as the coefficient is not statistically significant. However, as the hypothesized relationship between moneyness and REEP is not linear, squared moneyness and the squared root regressions are also applied to the overall put options. As Table K shows, the effect of these alternatives, however are minimal. The coefficient of volatility for put option is significantly negative, which is contrary to our hypothesis. With the exception of the significance of the moneyness parameter, all these results are consistent with the empirical study of Poitras, Veld and Zabolotnyuk (2006) since they yield a significantly positive relationship for the interest rate differential and the time to maturity, a significantly negative relationship for the moneyness, as well as a significantly negative relationship for the volatility. A further test on volatility is conducted by replacing 20-day moving average volatility with EWMA with  $\lambda$  of 0.94. However, as Appendix M indicates, the effect of this substitution has negligible impact on the results of overall regression. Table I shows that among the individual currencies, the Canadian Dollar and Japanese Yen display the same results as the overall put options. As Appendix L indicates, the correlation coefficients between each of the independent variables are all below 0.4, we are relatively confident the effect of multicollinearity is negligible.

Table J reports the results of the multiple regression for the call group. For the overall call options, the relative early exercise premium is negatively related to the difference between the domestic (US) and the foreign interest rate and positively to the time to maturity, which matches our hypothesis. Also, both coefficients are statistically significant on the 1% level. Just as in the case of puts, the relationship between the REEP and the moneyness ( $\ln(S/K)$ ) appear to be insignificant. In fact, using the squared moneyness and squared root of moneyness to the overall call options does little to improve the significance of the coefficient, as can be seen in Table L. In addition, the result also indicates that the effect of volatility on REEP of is insignificant. These results for the first and second hypotheses are consistent with the study of Poitras, Veld and Zabolotnyuk (2006) in which they find that the call REEPs are significantly influenced by the interest rate differential and time to maturity. For the third and fourth hypotheses, however, the results only differ from Poitras, Veld and Zabolotnyuk (2006) in that they find the coefficient for volatility to be positive and significant as hypothesized. Our results however, show that neither volatility nor moneyness is significant. As can be seen in Appendix L the effect of multicollinearity is also dispelled by the low correlation coefficients between the variables. As a further test on volatility parameter, EWMA is substituted for a 20-trading-day moving average, and has very little impact on the regression results. The results are shown in Appendix M. As for the results amongst the individual currencies, Australian Dollar, British Pound, Japanese Yen and Swiss Franc, all display the same results as the overall call options, as Table J indicates.

Given that options that are sufficiently deep in the money will be optimal to exercise if the interest rate differential is  $r - r_f < 0$  for calls, and  $r - r_f > 0$  for puts, we should expect that by dividing each of the call and put group into subgroups in accordance with the interest rate differentials a more apparent relationship between the moneyness and REEP can be revealed. However, Appendix N shows that the interest rate differential does not have a considerable



impact on the relationship between moneyness and REEP; the coefficients remain insignificant. This can probably be explained by the fact that there are very few options in the data that are sufficiently deep-in-the-money to satisfy the hypothesized conditions.

Even though in this study we modify the in-the-money range, adjust the S/K parameter, and select an alternative method of estimating volatility, overall, the regressions results for the REEPS for the put options in our study are very similar to those in Poitras, Veld and Zabolotnyuk (2006). Previous studies such as Engström and Nordén (2000) and Poitras, Veld and Zabolotnyuk (2006) all have negative coefficients for the volatility parameters in the case of put options. On the other hand, as Table A indicates, Jorion and Stoughton (1989) find all the parameters for puts to be insignificant. Given that like the result Poitras, Veld and Zabolotnyuk (2006), the coefficient of the volatility that we obtain for call options is insignificant at 5%, we conclude that selecting alternate proxy for volatility estimate does not improve the result of the regression.

**Table I: Regression Results for Group 1: In-the-money Put (S/K<0.992)**

$$REEP = \alpha + \beta_1(r - r_f) + \beta_2(T - t) + \beta_3 \ln(S / K) + \beta_4(\sigma_{Hist}) + \varepsilon$$

		Intercept	r-r <sub>f</sub>	T-t	LN(S/K)	Volatility	R <sup>2</sup>
<b>Overall</b> 230 Observations	<i>Coefficients</i>	0.0467**	0.8452**	0.0584**	-0.2475	-4.5008**	0.2832
	<i>t-Statistics</i>	4.8191	6.3527	4.4467	-1.0888	-2.9436	
<b>Australian Dollar</b> 11 Observations	<i>Coefficients</i>	0.0849	1.3008	-0.0376	-2.5281	-11.7299	0.4994
	<i>t-Statistics</i>	1.3409	0.6831	-0.1554	-1.3456	-1.2495	
<b>British Pound</b> 29 Observations	<i>Coefficients</i>	0.0481*	-0.8488	-0.0086	0.0205	-6.0122	0.1810
	<i>t-Statistics</i>	2.4595	-1.7343	-0.2028	0.0414	-1.5444	
<b>Canadian Dollar</b> 21 Observations	<i>Coefficients</i>	0.0897*	1.1129	0.1453	-0.0326	-16.5759	0.6063
	<i>t-Statistics</i>	2.6404	1.9867	2.1118	-0.0365	-1.6190	
<b>Deutsche Mark</b> 45 Observations	<i>Coefficients</i>	0.0801**	-0.0713	0.0152	0.6032	-5.1744*	0.2046
	<i>t-Statistics</i>	3.0909	-0.0842	1.0847	1.3263	-2.2291	
<b>Japanese Yen</b> 87 Observations	<i>Coefficients</i>	-0.0144	1.8197**	0.1061**	-0.1586	-3.5752	0.4227
	<i>t-Statistics</i>	-0.6217	5.2120	4.7785	-0.4401	-1.1527	
<b>Swiss Franc</b> 37 Observations	<i>Coefficients</i>	0.0683*	0.2030	0.1107	0.1777	-5.0808	0.1960
	<i>t-Statistics</i>	2.1463	0.3930	2.1130	0.2836	-1.4965	

Note: \*denotes significance at 5% level, \*\*denotes significance at 1% level.

**Table J: Regression Results for Group 2: In-the-money Call (S/K>1.008)**

$$REEP = \alpha + \beta_1(r - r_f) + \beta_2(T - t) + \beta_3 \ln(S / K) + \beta_4(\sigma_{Hist}) + \varepsilon$$

		Intercept	r-r <sub>f</sub>	T-t	LN(S/K)	Volatility	R <sup>2</sup>
<b>Overall</b> 212 Observations	<i>Coefficients</i>	0.0262	-1.0455**	0.1028**	-0.0100	-0.6819	0.3806
	<i>t-Statistics</i>	1.8337	-7.5527	6.1933	-0.0393	-0.3778	
<b>Australian Dollar</b> 21 Observations	<i>Coefficients</i>	0.0560	-3.0672*	0.0570*	-0.1841	-13.0510*	0.5011
	<i>t-Statistics</i>	1.7024	-2.8819	2.3966	-0.3631	-2.6104	
<b>British Pound</b> 61 Observations	<i>Coefficients</i>	0.0173	-3.0324**	0.0605*	-0.2174	-7.6447*	0.6598
	<i>t-Statistics</i>	0.7096	-10.1891	2.2253	-0.5717	-2.1571	
<b>Canadian Dollar</b> 9 Observations	<i>Coefficients</i>	-0.1102	0.6695	0.1671*	2.5411	30.9661	0.7215
	<i>t-Statistics</i>	-1.3120	0.5479	3.7055	1.1450	1.1796	
<b>Deutsche Mark</b> 67 Observations	<i>Coefficients</i>	-0.0089	-1.2031**	0.1527**	-0.0880	3.3522	0.6097
	<i>t-Statistics</i>	-0.3622	-3.5775	4.7934	-0.2111	0.9383	
<b>Japanese Yen</b> 21 Observations	<i>Coefficients</i>	0.1492**	-1.7301*	0.0201	-1.0466	-7.2538	0.4874
	<i>t-Statistics</i>	2.9954	-2.1683	0.3555	-1.7898	-0.8620	
<b>Swiss Franc</b> 33 Observations	<i>Coefficients</i>	0.0415	-0.0417	0.1922**	-0.6737	-2.6263	0.4101
	<i>t-Statistics</i>	1.6717	-0.3627	3.8668	-1.3001	-1.3414	

Note: \*denotes significance at 5% level, \*\*denotes significance at 1% level.

**Table K: Comparison of Regressions with Alternate Moneyness for Group 1**

	<b>Intercept</b>	<b>r-r<sub>f</sub></b>	<b>T-t</b>	<b>LN(S/K)</b>	<b>Volatility</b>	<b>R<sup>2</sup></b>
<i>Coefficients</i>	0.047**	0.845**	0.058**	-0.248	-4.501**	0.283
<i>t-Statistics</i>	4.819	6.353	4.447	-1.089	-2.944	
	<b>Intercept</b>	<b>r-r<sub>f</sub></b>	<b>T-t</b>	<b>(S/K)<sup>2</sup></b>	<b>Volatility</b>	<b>R<sup>2</sup></b>
<i>Coefficients</i>	0.183	0.845**	0.058**	-0.136	-4.507**	0.283
<i>t-Statistics</i>	1.519	6.354	4.441	-1.111	-2.949	
	<b>Intercept</b>	<b>r-r<sub>f</sub></b>	<b>T-t</b>	<b>(S/K)<sup>1/2</sup></b>	<b>Volatility</b>	<b>R<sup>2</sup></b>
<i>Coefficients</i>	0.554	0.845**	0.058**	-0.507	-4.502**	0.283
<i>t-Statistics</i>	1.202	6.353	4.445	-1.095	-2.945	

Note: \*denotes significance at 5% level, \*\*denotes significance at 1% level.

**Table L: Comparison of Regressions with Alternate Moneyiness for Group 2**

	<b>Intercept</b>	<b>r-r<sub>f</sub></b>	<b>T-t</b>	<b>LN(S/K)</b>	<b>Volatility</b>	<b>R<sup>2</sup></b>
<i>Coefficients</i>	0.026	-1.046**	0.103**	-0.010	-0.682	0.381
<i>t-Statistics</i>	1.834	-7.553	6.193	-0.039	-0.378	
	<b>Intercept</b>	<b>r-r<sub>f</sub></b>	<b>T-t</b>	<b>(S/K)<sup>2</sup></b>	<b>Volatility</b>	<b>R<sup>2</sup></b>
<i>Coefficients</i>	0.034	-1.046**	0.103**	-0.008	-0.680	0.381
<i>t-Statistics</i>	0.282	-7.553	6.199	-0.068	-0.377	
	<b>Intercept</b>	<b>r-r<sub>f</sub></b>	<b>T-t</b>	<b>(S/K)<sup>1/2</sup></b>	<b>Volatility</b>	<b>R<sup>2</sup></b>
<i>Coefficients</i>	0.049	-1.046**	0.103**	-0.023	-0.681	0.381
<i>t-Statistics</i>	0.099	-7.553	6.195	-0.047	-0.377	

Note: \*denotes significance at 5% level, \*\*denotes significance at 1% level.

## 10 CONCLUSION

Previous studies have shown that early exercise premiums for American-style options are substantial and should not be ignored in pricing American foreign currency options. Jorion and Stoughton (1989) examine the early exercise premium (EEP) by taking the difference in the prices of American- and European-style options and hypothesize the effects of moneyness, interest rates, time to maturity and volatility on EEP. Zivney (1991) proposes an alternate estimate of early exercise premium using deviation from European put-call parity. De Roon and Veld (1996) and Engström and Nordén (2000) demonstrate examples of isolating the source of early exercise premium to one type of options, while Bodurtha and Courtadon (1995) show that the relationship between probability of early exercise and early exercise premium is not linear.

This project extends the study of Poitras, Veld and Zabolotnyuk (2006) on early exercise premium for currency options, which utilizes the deviation from European put-call parity to determine early exercise premium, and eliminate the effect of early exercise from both calls and puts by eschewing the near-the-money options. As in Poitras, Veld and Zabolotnyuk (2006), the data for currency options on Australian Dollar, Canadian Dollar, British Pound, Deutsche Mark, Japanese Yen and Swiss Franc trading at the Philadelphia Stock Exchange from January 2, 1992 to September 24, 1997 are examined. We use the natural logarithm of moneyness, squared moneyness, as well as squared root of moneyness separately as regressors to see if they better capture the non-linearity of the EEP and moneyness. In addition, a rolling historical volatility is used for the volatility parameter to examine the effect of an alternate volatility estimate.

After excluding near-the-money options from the sample we are able to divide the data into a call group and put group, though we use a wider definition of near-moneyness in order to

reduce the effect of having two sources for early exercise premium. Like Poitras, Veld and Zabolotnyuk (2006) and Jorion and Stoughton (1989), we hypothesize that moneyness, time to maturity, interest rate differentials and volatility are all factors that influence the magnitude of the early exercise premium. Instead of using implied volatility, a 20-day moving average of historical volatility is chosen as a proxy for the volatility of the option. Alternatively, an exponentially weighted moving average for volatility is also tested as the volatility parameter.

Like Poitras, Veld and Zabolotnyuk (2006), we calculate early exercise premium as percentage of the option value. Our REEPs for call and put are 7.329% and 6.122% respectively, while are the greater the REEPs found in Poitras, Veld and Zabolotnyuk (2006), even though our samples contain fewer nearer-the money data points. This is probably because our sample data is not sufficiently deep-in-the money for the early exercise premium to decrease in a significant way that it outweighs the increase from the higher likelihood of exercise.

Running a multiple regression against the four parameters, we find that the time to maturity does lead to higher REEP, and interest differentials is negatively correlated with REEPs of calls, and positively correlated with the REEPs of puts. For the moneyness parameter, we find that using the natural logarithm of moneyness as the regressor yields the same result as Poitras, Veld and Zabolotnyuk (2006), who simply used moneyness as the variable. In both cases the coefficients for call and puts are insignificant. Further regressions were done using squared and squared root moneyness as regressors, but the effects were minimal. The results for volatility are also not straight forward; the coefficient for the put option is significant, while that for the call is not. This suggests volatility estimates relying historical data does not greatly enhance the results of the regression, though like implied volatility, they suggest that negative relationship between volatility and REEP for puts. In addition, our results show that the relationship between moneyness and REEP is not significantly influenced by interest rate differentials, as the moneyness parameter does not become significant in either subgroup for both calls and puts.

## APPENDICES

### APPENDIX A: Contracts Specifications for PHLX Standardized Currency Options Contracts

**Standardized Contracts** carry specified contract terms for features such as contract size, strike price intervals, expiration dates, price quoting and premium settlement. PHLX offers standardized options on six major currencies, with either American- or European-style exercise; maturities available range from monthly to as long as two years, with a choice of mid-month or month-end expiration.

#### **Expiration Months**

March, June, September and December + two near-term months

#### **Expiration Date/Last Trading Day**

Assuming it is a business day, otherwise the day immediately prior:  
Friday before the third Wednesday of expiring month

#### **Expiration Settlement Date**

Third Thursday of expiring month, except for March, June, September, and December expirations which are the third Wednesday.

Note: Expiration date and expiration settlement date maybe subject to change due to holidays

#### **Exercise Style**

American and European

#### **Trading Hours**

2:30 a.m. to 2:30 p.m. Philadelphia time, Monday through Friday.

#### **Issuer and Guarantor**

The Options Clearing Corporation (OCC)



	Australian Dollar	British Pound (£)	Canadian Dollar	Euro (€)	Japanese Yen (¥)	Swiss Franc
Ticker Symbols (American/European)	XAD/CAD	XBP/CBP	XCD/CCD	XEU/ECU (Even Strike) XEB/ECB (odd Strike)	XJY/CJY	XSF/CSF
Half-Point Strike Three Near-Term Months Only	XAZ/CAZ	n.a./n.a.	XCD/CCD	n.a./n.a.	XJZ/CJZ	XSZ/CSZ
Alternate Symbols <sup>1</sup>	XAY/CAY	XBX/CBX XBY/CBY	XCV/CCV	XEV/ECY	XJJ/CJJ XJV/CJV	XSX/CSX
Contract Size	50,000	31,250	50,000	62,500	6,250,000	62,500
Position & Exercise Limits	200,000	200,000	200,000	200,000	200,000	200,000
Base Currency	USD	USD	USD	USD	USD	USD
Underlying Currency	AUD	GBP	CAD	EUR	JPY	CHF
Exercise Price Intervals						
Three Nearest Months	1¢	1¢	.5¢	1¢	.005¢	.5¢
6, 9 And 12 Months	1¢	2¢	.5¢	1¢	.01¢	1¢
Premium Quotations	Cents per unit	Cents per unit	Cents per unit	Cents per unit	Hundredths of a cent per unit	Cents per unit
Minimum Premium Change	\$. (00)01 per unit = \$5.00	\$. (00)01 per unit = \$3.125	\$. (00)01 per unit = \$5.00	\$. (00)01 per unit = \$6.25	\$. (0000)01 per unit = \$6.25	\$. (00)01 per unit = \$6.25
Margin	USD	USD	USD	USD	USD	USD

Source: [www.phlx.com](http://www.phlx.com)

## APPENDIX B: Contracts Specifications for PHLX Customized Currency Options Contracts

**Customized Contracts** allow users to customize all aspects of a currency option trade including: choice of exercise price, customized expiration dates of up to two years, and premium quotation as either units of currency or percent of underlying value.

### Currency Pairs

Any two currently approved currencies, including the U.S. dollar, may be matched for trading. Either may represent the base or underlying currency. For example, USD/JPY (strike prices expressed in USD per JPY) or JPY/USD (strike prices expressed in JPY per USD). Note: AUD and MXP may be matched with the USD only (premiums for AUD contracts must be denominated in USD).

### Contract Size

Contract size for the currency pairs is determined by the underlying currency. The sizes for the underlying currency are as follows:

AUD	GBP	CAD	EUR	JPY	MXP	CHF	USD
50,000	31,250	50,000	62,500	6,250,000	250,000	62,500	50,000

### Premium

Premium may be expressed in units of the base currency or as a percent of the underlying currency. For example, the premium of a USD/EUR contract could be expressed in U.S. cents per EUR (a quote of 1.23 =  $.0123 \times 62,500 = \$768.75$ ), or as a percentage of Euros (a quote of 2.16% =  $.0216 \times 62,500 = \text{EUR } 1,350$ ).

### Exercise Price

Any price level to four characters. For instance, a USD/GBP option could have an exercise price of \$1.543 per GBP.

### Exercise Style

European-Style

### Expiration

A participant may trade a customized currency option with either a standard expiration date ("Standard-expiry Option") or with a customized date ("Custom-dated Option"). Standard-expiry Options and Custom-dated Options have distinct exercise and assignment processes.

### Exercise and Assignment

Standard-expiry Options conform to existing exercise and assignments practices for all standardized contracts.

Custom-dated Options follow a unique exercise and assignment process on expiration day as reflected below (All times are Eastern Time):

8:00 a.m. Trading ceases in expiring Custom-dated Options.

10:00 a.m. Window closes for exercise instructions in expiring Custom-dated Options. Subsequently, OCC will disseminate a preliminary indication of the percent of open interest exercised in each series. The PHLX and OCC will employ a pro-rata assignment process.

10:15 a.m. Custom-dated Options expire and final assignment notification based on a pro-rata assignment process begins.

On expiration day, no new series may be created for trading which will expire that day - trades may only occur in previously established options series.

**Position and Exercise Limits**

Position and exercise limits are 200,000 contracts (100,000 contracts for MXP).

**Minimum Transaction Size**

Opening transactions may be for any amount equals or exceeds 50 contracts. Subsequent trades in series with open interest must be for amounts which equal or exceed 50 contracts, unless the position is being reduced or closed-out.

**Price and Quote Dissemination**

Request For Quotes (RFQ), quotes, and trades are disseminated as administrative text messages over the Options Price Reporting Authority (OPRA). Currently the text messages can be received via the quotation terminals of Reuters and Bloomberg:

Reuters:	UCOM/FLXA - monitor page/summary of all activity
	[swift code]/FLXA - information on underlying currency
	FLEXOPY - list of customized option terminology
Bloomberg:	NH PHL

**Customer Margin**

Margin is subject to the same margin rules and requirements as the standardized currency option contracts. In addition, margin offsets may be allowed on options with the same underlying currency.

**Trading Hours**

2:30 a.m. to 2:30 p.m. Eastern Time, Monday through Friday.

**Issuer and Guarantor**

The Options Clearing Corporation (OCC).

Source: [www.phlx.com](http://www.phlx.com)

## APPENDIX C: Contracts Specifications for Currency Future Options on CME

Currency	Contract Size	Point Description	Contract Months	Strike Price Interval	Globex Trading Hours
Australian Dollar	100,000 AUD	\$10 per contract	Four months in the March quarterly cycle, two months not in the March cycle (serial months), plus 4 Weekly Expiration Options	\$0.005	Mon/Thurs 5:00 p.m.-7:15 a.m. & 2:00 p.m.-4:00 p.m. Sun & Hol 5:00 p.m.-7:15 a.m.
Brazilian Real	\$100,000 BRL	\$5 per Contract	Twelve consecutive contract months plus four weekly expirations	\$0.005	Mon/Thurs 5:00 p.m.-7:15 a.m. & 2:00 p.m.-4:00 p.m. Sun & Hol 5:00 p.m.-7:15 a.m.
British Pound	62,500£	\$6.25 per Contract	Four months in the March cycle and two months not in the March cycle (serial months), plus 4 Weekly Expiration Option	\$0.01	Mon/Thurs 5:00 p.m.-7:15 a.m. & 2:00 p.m.-4:00 p.m. Sun & Hol 5:00 p.m.-7:15 a.m.
British Pound [European]	Same as above	Same as above	Same as above	Same as above	Sun/Fri 5:00 p.m.-4:00 p.m. LTD 9:00 a.m.
Canadian Dollar (CAD)	100,000 CAD	\$10 Per Contract	Four months in the March cycle and two months not in the March cycle, plus 4 weekly expirations.	\$0.005	Mon/Thurs 5:00 p.m.-7:15 a.m. & 2:00 p.m.-4:00 p.m. Sun & Hol 5:00 p.m.-7:15 a.m.
Canadian Dollar (CAD) [European]	Same as above	Same as above	Same as above	Same as above	Sun/Fri 5:00 p.m.-4:00 p.m. LTD 9:00 a.m.
Czech Koruna (CZK)	4,000,000 CZK	\$4 Per Contract	Four months in the March Quarterly Cycle, Mar. Jun. Sep. and Dec. and two months not in the March cycle (serial months) plus four weekly expirations. (Not yet listed).	\$0.0001	Mon/Thurs 5:00 p.m.-7:15 a.m. & 2:00 p.m.-4:00 p.m. Sun & Hol 5:00 p.m.-7:15 a.m.
EURO FX	125,000 €	\$12.50 per Contract	Four months in the March cycle, Mar, Jun, Sep, Dec and two months not in the March cycle (serial months), plus four weekly expirations	\$0.005	Mon/Thurs 5:00 p.m.-7:15 a.m. & 2:00 p.m.-4:00 p.m. Sun & Hol 5:00 p.m.-7:15 a.m.
EURO FX [European]	Same as above	Same as above	Four option contract months in the March Quarterly Cycle (Mar, Jun, Sep, Dec), and two option contract months not in the March Quarterly Cycle, that is, serial months (Jan,	Same as above	Sun/Fri 5:00 p.m.-4:00 p.m. LTD 9:00 a.m.

Currency	Contract Size	Point Description	Contract Months	Strike Price Interval	Globex Trading Hours
			Feb, Apr, May, Jul, Aug, Oct, Nov), plus four weekly expirations (One March Quarterly, two serial months and four weeklies on GLOBEX, except two March quarterlies, two serial months, and four weeklies on GLOBEX).		
EURO/BP	125,000 €	6.25 £ per Contract	Four months in the March quarterly cycle and two months not in the March cycle (serial months), and four weekly expirations.	0.0025£/€	Mon/Thurs 5:00 p.m.-7:15 a.m. & 2:00 p.m.-4:00 p.m. Sun & Hol 5:00 p.m.-7:15 a.m.
EURO/JPY	125,000 €	1,250 ¥ per Contract	Four months in the March quarterly cycle, two months not in the March cycle (serial months), and four weekly expirations.	0.5¥/€	Mon/Thurs 5:00 p.m.-7:15 a.m. & 2:00 p.m.-4:00 p.m. Sun & Hol 5:00 p.m.-7:15 a.m.
EURO/CHF	125,000 €	12.50 CHF per Contract	Four months in the March quarterly cycle, two serial months, and four weekly expirations.	0.0025 CHF/€	Mon/Thurs 5:00 p.m.-7:15 a.m. & 2:00 p.m.-4:00 p.m. Sun & Hol 5:00 p.m.-7:15 a.m.
EURO/CZK	125,000 €	4 € per Contract	Four months in the March Quarterly Cycle, Mar. Jun. Sep. and Dec. and two months not in the March cycle (serial months) plus four weekly expirations.(not yet listed)	0.0001 €/CZK	Mon/Thurs 5:00 p.m.-7:15 a.m. & 2:00 p.m.-4:00 p.m. Sun & Hol 5:00 p.m.-7:15 a.m.
EURO/HUF	125,000 €	3€ per Contract	Four months in the March Quarterly Cycle, Mar. Jun. Sep. and Dec. and two months not in the March cycle (serial months) plus four weekly expirations. (Not yet listed).	0.00001€/HUF	Mon/Thurs 5:00 p.m.-7:15 a.m. & 2:00 p.m.-4:00 p.m. Sun & Hol 5:00 p.m.-7:15 a.m.
EURO/PLN	125,000 €	5€ per Contract	Four months in the March Quarterly Cycle, Mar. Jun. Sep. and Dec. and two months not in the March cycle (serial months) plus four weekly expirations. (Not yet listed).	0.001€/PLN	Mon/Thurs 5:00 p.m.-7:15 a.m. & 2:00 p.m.-4:00 p.m. Sun & Hol 5:00 p.m.-7:15 a.m.
Hungarian Forint (HUF)	30,000,000 HUF	\$3 per Contract	Four months in the March Quarterly Cycle, Mar. Jun. Sep. and Dec. and two months not in the March cycle (serial months) plus four weekly expirations. (Not yet listed).	\$0.00001	Mon/Thurs 5:00 p.m.-7:15 a.m. & 2:00 p.m.-4:00 p.m. Sun & Hol 5:00 p.m.-7:15 a.m.
Israeli Shekel (ILS)	1,000,000 ILS	\$10 per Contract	Four (4) months in the March quarterly cycle. Two (2) serial option months (non-March cycle	\$0.001	Mon/Thur 5:00 p.m.-4:00 p.m. Sun & Hol 5:00

Currency	Contract Size	Point Description	Contract Months	Strike Price Interval	Globex Trading Hours
			months, e.g., January, February, April, May, July, August, October and November). Four (4) weekly options listed.		p.m.-4:00 p.m. LTD
Japanese Yen	12,500,000 ¥	\$12.50 per Contract	Four months in the March cycle and two months not in the March cycle (serial months), plus four weekly expirations.	\$0.00005	Mon/Thurs 5:00 p.m.-7:15 a.m. & 2:00 p.m.-4:00 p.m. Sun & Hol 5:00 p.m.-7:15 a.m.
Japanese Yen [European]	Same as above	Same as above	Four option contract months in the March Quarterly Cycle (Mar, Jun, Sep, Dec), and two option contract months not in the March Quarterly Cycle, that is, serial months (Jan, Feb, Apr, May, Jul, Aug, Oct, Nov), plus four weekly expirations (One March Quarterly, two serial months and four weeklies on GLOBEX, except two March quarterlies, two serial months, and four weeklies on GLOBEX).	Same as above	Sun/Fri 5:00 p.m.-4:00 p.m. LTD 9:00 a.m.
Mexican Peso (MXP)	500,000 MXP	\$5 per Contract	Twelve consecutive calendar month options plus one deferred March quarterly cycle contract month. Four weekly options, with a monthly underlying future.	\$0.000625	Mon/Thurs 5:00 p.m.-7:15 a.m. & 2:00 p.m.-4:00 p.m. Sun & Hol 5:00 p.m.-7:15 a.m.
New Zealand Dollar (NZD)	100,000 NZD	\$10 per Contract	Four months in the March cycle and two months not in the March cycle (serial months), plus four weekly expirations	\$0.005	Mon/Thurs 5:00 p.m.-7:15 a.m. & 2:00 p.m.-4:00 p.m. Sun & Hol 5:00 p.m.-7:15 a.m.
Polish Zloty (PLN)	500,000 PLN	\$5 per Contract	Four months in the March Quarterly Cycle, Mar. Jun. Sep. and Dec. and two months not in the March cycle (serial months) plus four weekly expirations. (Not yet listed).	\$0.001	Mon/Thurs 5:00 p.m.-7:15 a.m. & 2:00 p.m.-4:00 p.m. Sun & Hol 5:00 p.m.-7:15 a.m.
Russian Ruble (RUB)	2,500,000 RUB	\$25 per Contract	Four months in a Quarterly Cycle, Mar, Jun, Sep, & Dec.	\$0.00025	Mon/Thurs 5:00 p.m.-7:15 a.m. & 2:00 p.m.-4:00 p.m. Sun & Hol 5:00 p.m.-7:15 a.m.
South African Rand (ZAR)	500,000 ZAR	\$5 per Contract	Twelve consecutive calendar month options plus one deferred March quarterly cycle contract month. Four weekly options, with a monthly underlying future.	0.00250	N/A

Currency	Contract Size	Point Description	Contract Months	Strike Price Interval	Globex Trading Hours
Swiss Franc (CHF)	125,000 CHF	\$12.50 per Contract	Four options in the March cycle, two months not in the March cycle (serial options), plus 4 Weekly Expiration Options.	\$0.005	Mon/Thurs 5:00 p.m.-7:15 a.m. & 2:00 p.m.-4:00 p.m. Sun & Hol 5:00 p.m.-7:15 a.m.
Swiss Franc [European]	Same as above	Same as above	Same as above	Same as above	Sun/Fri 5:00 p.m.-4:00 p.m. LTD 9:00 a.m.

*Note: All Contracts are American-style unless indicated otherwise.*

**APPENDIX D: Comparison between Options Traded on Exchange  
and The Over-The-Counter Market**

	<b>Exchange-Traded</b>	<b>Over-The-Counter</b>
<b>Contract Size</b>	Specified	Negotiable
<b>Expiration</b>	Standardized (with some flexibility)	Negotiable
<b>Exercise Style</b>	Standardized (with some flexibility)	Negotiable
<b>Strike Price</b>	Standardized (with some flexibility)	Negotiable
<b>Deal Method</b>	Open-Outcry / Electronically	Electronically (Dealer Network)
<b>Commissions</b>	Negotiable	Net Price
<b>Regulations</b>	Securities Exchange Commission, Options Clearing Corporation, National Association of Securities Dealers, etc.	Self-Regulated
<b>Risk</b>	Borne by Clearing House	Counter-Party Risk
<b>Market Participants</b>	Public customers, Corporate and Institutional users	Corporate and Institutional users



## APPENDIX E: Derivation of European Option Boundaries

Consider the following two portfolios:

*Portfolio A:* one European call option plus an amount of domestic currency equal to  $Ke^{-r(T-t)}$

*Portfolio B:* an amount of foreign currency equal of  $Se^{-r_f(T-t)}$

At the expiration date, the payoff of each portfolio would be:

	$S_T \leq K$	$S_T \geq K$
Portfolio A	$0 + K = K$	$(S_T - K) + K = S_T$
Portfolio B	$S_T$	$S_T$
Portfolio A – Portfolio B	$K - S_T \geq 0$	$(S_T - S_T) = 0$

Hence, portfolio A is always worth as much as, and can be worth more than, portfolio B at the option's maturity. It follows that in the absence of arbitrage opportunities this must also be true today. Hence,

$$c + Ke^{-r(T-t)} \geq Se^{-r_f(T-t)}$$

or,

$$c \geq Se^{-r_f(T-t)} - Ke^{-r(T-t)}$$

Since options values are always non-negative. Therefore, the lower bound for a European call is:

$$c \geq \max[0, Se^{-r_f(T-t)} - Ke^{-r(T-t)}]$$

A similar argument produces the lower bound for a European put:

$$p \geq \max[0, Ke^{-r(T-t)} - Se^{-r_f(T-t)}]$$

## APPENDIX F: Data Selection Process

1. Selecting all the options that are not at-the-money ( $S/K \geq 1.008$  or  $S/K \leq 0.992$ )

Year	Put-call pairs with the same trade date, underlying value, expiration month, and exercise price	Not near-the-money
1997	257	117
1996	419	119
1995	414	238
1994	471	195
1993	469	212
1992	359	131
<b>Total</b>	<b>2389</b>	<b>969</b>

Number of eliminated options that are near-the-money:  $2389 - 969 = 1420$ .

2. Selecting option prices which are consistent with the boundaries of the American put-call parity

Currency	Put Group ( $S/X < 0.992$ )	Call Group ( $S/X > 1.008$ )	Total
Australian Dollar	23	32	55
British Pound	61	91	152
Canadian Dollar	43	18	61
Deutsche Mark	68	117	185
Japanese Yen	102	65	167
Swiss Franc	62	74	136
<b>Total</b>	<b>359</b>	<b>397</b>	<b>756</b>

Number of eliminated options that are not consistent with bound conditions:

$$969 - 756 = 213.$$

3. Selecting option prices which have positive EEP

Currency	Put Group ( $S/X < 0.992$ )	Call Group ( $S/X > 1.008$ )	Total
Australian Dollar	11	21	32
British Pound	29	61	90
Canadian Dollar	21	9	30
Deutsche Mark	45	67	112
Japanese Yen	87	21	108
Swiss Franc	37	33	70
<b>Total</b>	<b>230</b>	<b>212</b>	<b>442</b>

Number of eliminated options which have negative EEP:  $756 - 442 = 314$ .

## APPENDIX G: Statistical Summary

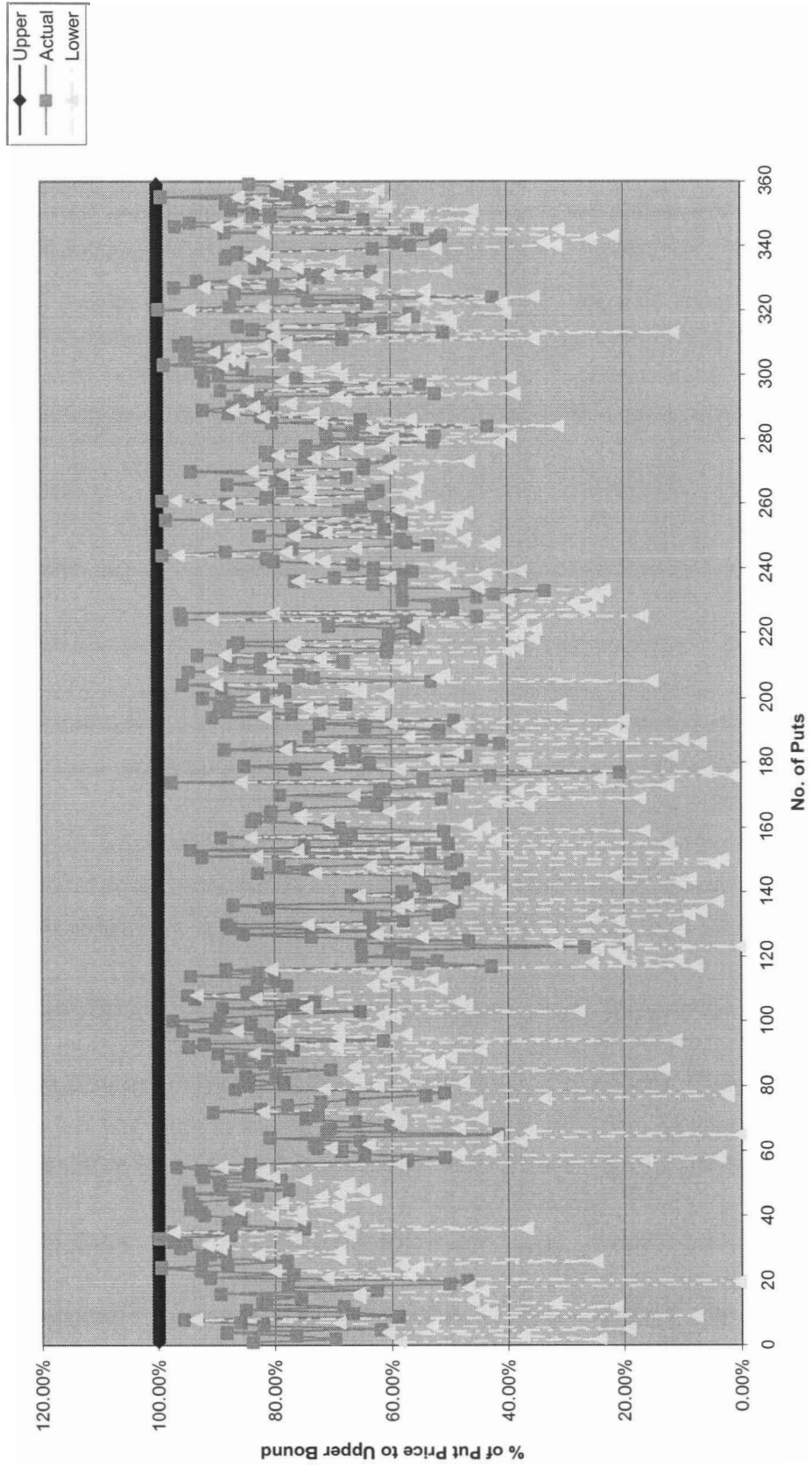
### Group 1: In-the-money Put (S/K<0.992)

	$r-r_f$	T-t	LN(S/K)	Volatility
Minimum	-0.059505489	0.008219178	-0.0945651	0.001334
Maximum	0.071019355	0.98630137	-0.0082338	0.017263
Mean	0.019980473	0.302680167	-0.0221075	0.005676
Standard Error	0.001801908	0.018128153	0.00103443	0.000155
Standard Deviation	0.02732729	0.274927056	0.01568797	0.002345
Sample Variance	0.000746781	0.075584886	0.00024611	5.5E-06

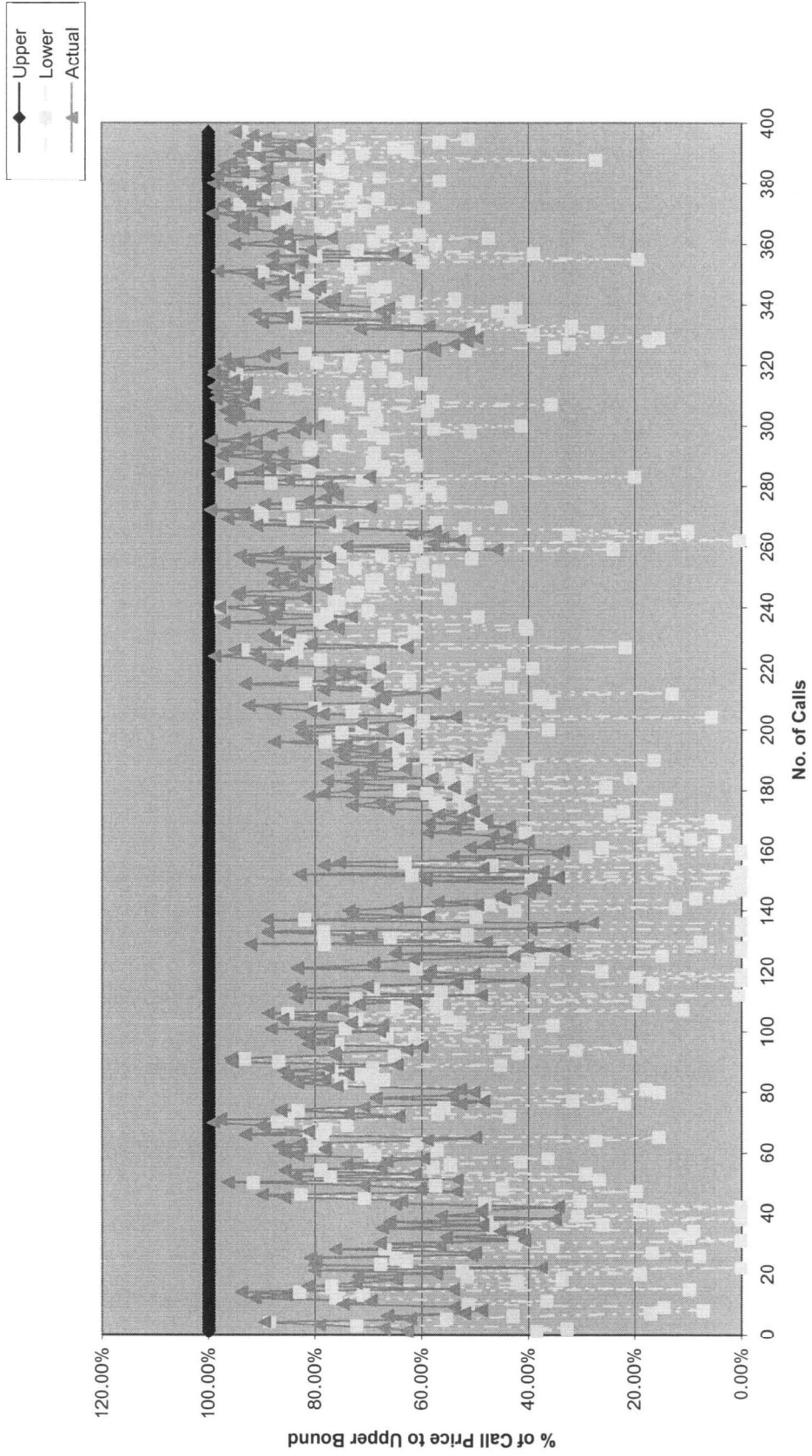
### Group 2: In-the-money Call (S/K>1.008)

	$r-r_f$	T-t	LN(S/K)	Volatility
Minimum	-0.283336879	0.019178082	0.00798332	0.001867
Maximum	0.053956117	1.005479452	0.11377501	0.017263
Mean	-0.01913203	0.360726286	0.02439203	0.006248
Standard Error	0.002418659	0.020303329	0.00129015	0.00018
Standard Deviation	0.0352162	0.295620934	0.01878492	0.002621
Sample Variance	0.001240181	0.087391736	0.00035287	6.87E-06

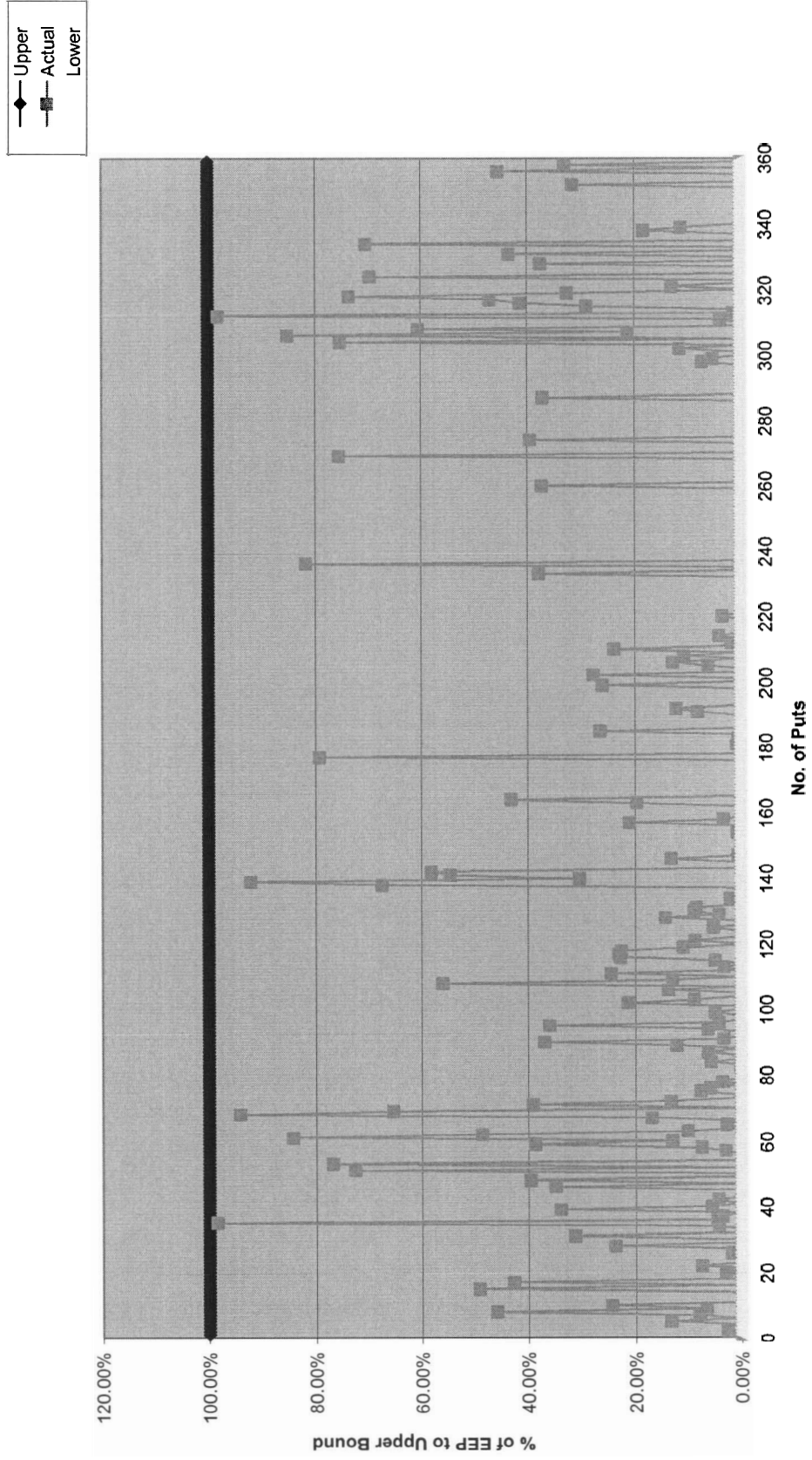
### APPENDIX H: Upper and Lower Bounds of In-The-Money Puts (Group 1)



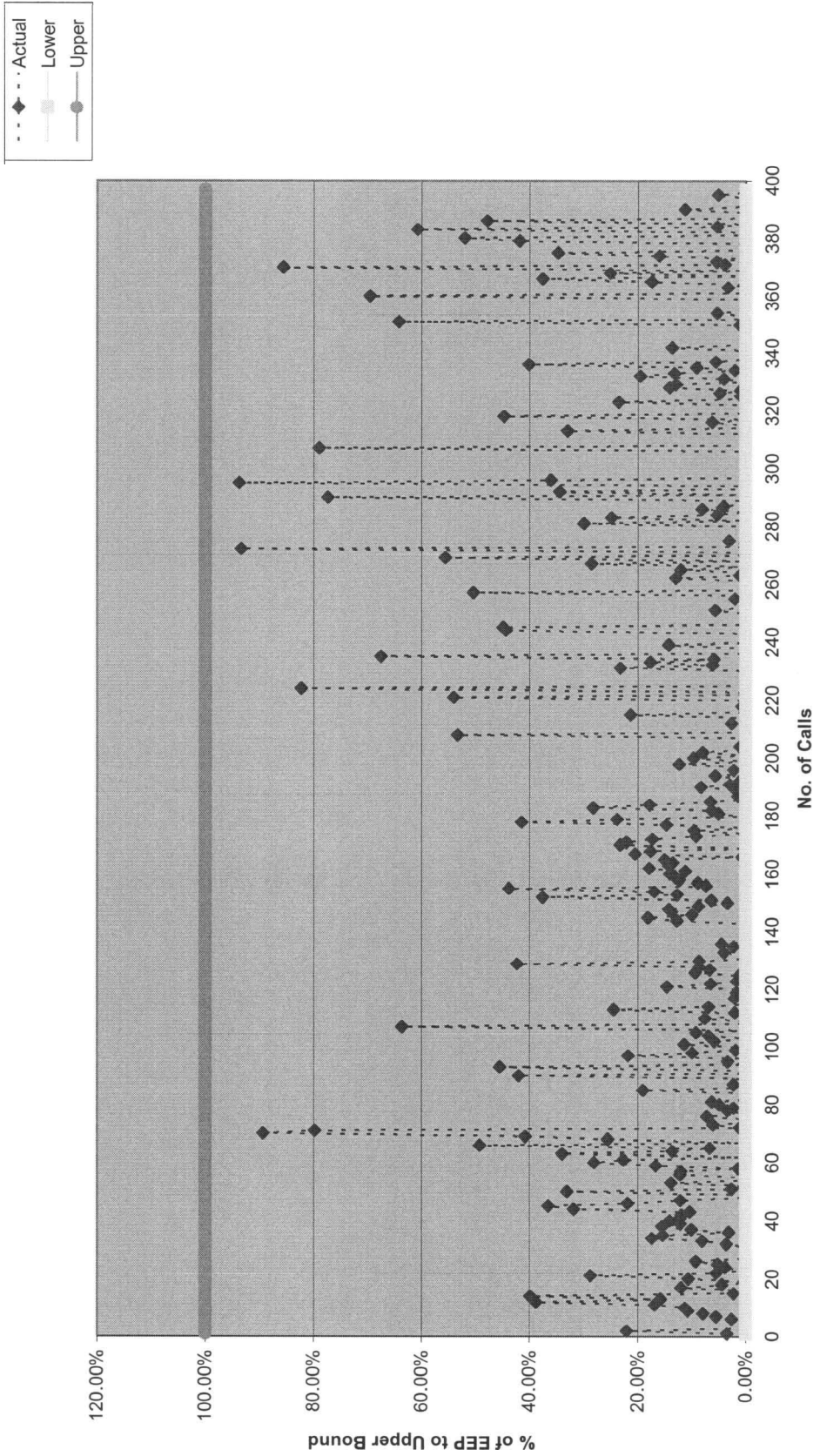
# APPENDIX I: Upper and Lower Bounds of In-The-Money Calls (Group 2)



**APPENDIX J: EEP Upper and Lower Bounds for In-The-Money Puts (Group 1)**



### APPENDIX K: EEP Upper and Lower Bounds for In-The-Money Calls (Group 2)



## APPENDIX L Correlation Coefficients of Explanatory Variables

### Correlation Coefficients of parameters for put options

$\rho$	$r-r_f$	T-t	LN(S/K)	Volatility
$r-r_f$	1.000	0.308	-0.138	0.158
T-t	0.308	1.000	-0.090	0.129
LN(S/K)	-0.138	-0.090	1.000	-0.266
Volatility	0.158	0.129	-0.266	1.000

### Correlation Coefficients of parameters for call options

$\rho$	$r-r_f$	T-t	LN(S/K)	Volatility
$r-r_f$	1.000	-0.246	-0.057	0.036
T-t	-0.246	1.000	0.128	0.034
LN(S/K)	-0.057	0.128	1.000	0.046
Volatility	0.036	0.034	0.046	1.000



## APPENDIX M: Regression with EWMA Volatility

		Intercept	r-r <sub>f</sub>	T-t	Ln(S/K)	EWMA Volatility	R <sup>2</sup>
<b>In-the-money Put</b> 230 Observations	<i>Coefficients</i>	0.045**	0.835**	0.059**	-0.254	-4.082**	0.280
	<i>t-Statistics</i>	4.678	6.270	4.462	-1.106	-2.731	
<b>In-the-money Call</b> 212 Observations	<i>Coefficients</i>	0.023	-1.047**	0.103**	-0.014	-0.064	0.380
	<i>t-Statistics</i>	1.685	-7.539	6.181	-0.055	-0.043	

Note: \*denotes significance at 5% level, \*\*denotes significance at 1% level

## APPENDIX N: Positive and Negative Interest Rate Differentials

### Group 1: In-the-money Put (S/K<0.992)

		Intercept	r-r <sub>f</sub>	T-t	Ln(S/K)	Volatility	R <sup>2</sup>
<b>(r-r<sub>f</sub>) &gt; 0</b> 164 Observations	<i>Coefficients</i>	0.011	1.828**	0.074**	-0.013	-4.787**	0.315
	<i>t-Statistics</i>	0.743	6.519	5.097	-0.048	-2.627	
<b>(r-r<sub>f</sub>) &lt; 0</b> 65 Observations	<i>Coefficients</i>	0.056**	-0.497	-0.012	-0.040	-5.938**	0.138
	<i>t-Statistics</i>	4.265	-1.290	-0.318	-0.114	-2.755	

Note: \*denotes significance at 5% level, \*\*denotes significance at 1% level

### Group 2: In-the-money Call (S/K>1.008)

		Intercept	r-r <sub>f</sub>	T-t	Ln(S/K)	Volatility	R <sup>2</sup>
<b>(r-r<sub>f</sub>) &gt; 0</b> 49 Observations	<i>Coefficients</i>	0.076**	-0.667	-0.051*	-0.289	-1.903	0.198
	<i>t-Statistics</i>	5.159	-1.827	-2.016	-0.765	-1.335	
<b>(r-r<sub>f</sub>) &lt; 0</b> 160 observations	<i>Coefficients</i>	0.011	1.424**	0.137**	0.144	-3.848	0.395
	<i>t-Statistics</i>	0.625	-7.045	6.917	0.470	-1.499	

Note: \*denotes significance at 5% level, \*\*denotes significance at 1% level

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