A THEORETICAL AND EMPIRICAL OBSERVATION OF AGGREGATE PRODUCTION IN BRITISH COLUMBIA

by

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ABSTRACT

This research investigates the applicability of the Cobb-Douglas and Constant Elasticity of Substitution production models to the British Columbia economy. Both models have been estimated by multiple regression techniques. Where non linear functional forms do not permit linear transformations a recursive technique was employed. With the exception of capital stock, the data used were obtained from the Department of Economics and Statistics in Victoria and the Dominion Bureau of Statistics in Ottawa. Capital figures on provincial basis are not available, and therefore an extention of the Vintage Model was developed from which the required capital figures were obtained.

During the period 1959 - 1969 it was found that aggregate production was more responsive to capital components than to labor. In each subperiod, and the total period, where the elasticity of substitution was empirically determined its value was consistently less than unity, with magnitude around .36. The returns to scale component for both the Cobb-Douglas and Constant Elasticity of Substitution was greater than unity suggesting increasing returns over this period. The constrained model of the Cobb-Douglas proved to be statistically inferior to its unconstrained form. Technological influences were only significant within the Constant Elasticity of Substitution model, and these were of the neutral type, suggesting no significant changes in input productivity over the period had occurred.

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CHAPTER I

NEO-CLASSICAL PRODUCTION THEORY

1.1 The Production Function

The production function is an abstract formulation expressing a relationship between a particular level of output and the inputs necessary to generate this level of output. Moreover, the relation of the production function not only suggests causality between inputs and outputs, but also inputs with inputs. The economist in utilizing the production function attempts to capture the technical relation between the outputs on the one hand, and the inputs on the other. In so doing, the economist assumes, *a priori*, that there exists a particular physical transformation process which abstracts from the particular considerations immediate to designers or engineers. As such, the production function does not direct itself to any specific transformation process, rather, it is concerned with a spectrum of possible procedures of production. In other words, although the production function is implicitly related to the physical and technical concerns of production, (which are not directly measurable), it is what one might consider a "fabrication of the marginal economists".[1]

One may rationalize, that because of its abstractions the production function is of little use, nevertheless, one may also argue it is this abstractness which renders its source of value. In a real sense, it is the abstraction inherent in the concept of a production function, that allows the economist to examine a wide spectrum of problems, e.g., growth factors and relative income shares. Thus we arrive at our macro-economic production function, *ipso facto*, incorporating the heterogeneous outputs and inputs through aggregate measures of national product and factor inputs embodied in production. The obscureness from aggregation results primarily from the definition of the particular aggregate components. As a consequence, we take as given the technical possibilities defined for the particular production function utilized. Hence, we are summarizing the various technological components which restrict economic behavior and thereby focus on the main concern of production theory - a measure of the degree to which the various inputs can be substituted for each other in the production process. Let us now move to a specific set of criteria, imposed by the Neo-Classical economists, which in turn suggest means of measuring the relative substitutability, inherent in economic analysis.

1.2 The Neo-Classical Production Function

The Neo-Classical type production function, for a given level of technology, finds expression as:

$$Q = q(x_1, x_2, \ldots\ldots\ldots\ldots, x_n) \tag{1.1}$$

where Q and x_i, $(i=1,2\ldots n)$, represent measures of output and inputs respectively. The predominance of the above production function has held its role in economic theory by virtue of its plausibility and mathematical convenience. The above structure includes the assumption of continuous first and second order partial derivatives throughout the region of definition, namely:

$$Q \geqslant 0$$

$$x_i \geqslant 0 \qquad \text{for all } i, \ (i=1,2,,,n)$$

The given level of technology constrains that the realization of Q must be single valued: if however, for given inputs the technological process leads to a multiple valued Q, then the underlying process must be misspecified. To avoid this occurrence another assumption is added; id est, the inputs are consolidated in such a fashion that a maximum level of output will result from given input combinations. In statu quo, limiting ourselves with an optimum level of Q for given input, we are then interested in those particular values of $\partial Q/\partial x_i$ such that;

$$\partial Q/\partial x_i \geqslant 0 \tag{1.2}$$

Namely, output is an increasing, or at least non decreasing function of the various x_i. It follows that relations (1.1) and (1.2) define the region of substitution as the set of efficient levels of Q (for given x_i) belonging to the production function (1.1). With this let us now direct our attention to the basic considerations of economic theory.

(a) The Marginal Rate of Substitution:

Without loss of generality let us consider the two input situation, the most common in aggregate production analysis. As such equation (1.1) now reads,

$$Q = q(x_1,x_2) \tag{1.3}$$

Considering marginal variations at a realized value of Q , belonging to equation (1.2) we have:

$$dQ = \sum_{i=1}^{n} (\partial Q/\partial x_i) \; dx_i \tag{1.4}$$

providing, of course $\partial Q/\partial x_i \geq 0$. Hence, $dQ > 0$, will occur only via the use of larger levels of one input and analogously, a decrease in the amount of an input $(dx_i < 0)$ will be made up by using greater quantities of another input.

Accordingly, if x_i is defined by x_1 and x_2 , we can demarcate a production surface, throughout which, combinations of (x_1,x_2) can be continously varied. Infinitesimal movements along the production surface will necessarily have to satisfy....

$$dQ = \sum_{i=1}^{n} (\partial Q/\partial x_i) \; dx_i = 0$$

and as such partial variations in x_1 and x_2 , with Q remaining constant, must yield:

$$dx_1/dx_2 = - \frac{\partial Q/\partial x_2}{\partial Q/\partial x_1} \tag{1.5}$$

which is negative, since dx_2 and dx_1 are of opposite sign, in the relevant region of substitution, <u>id est</u> (1.3) , (1.4). This value delimitated by $-\dfrac{\partial Q/\partial x_2}{\partial Q/\partial x_1}$ is known as the marginal rate of substitution.

(b) Elasticity of Substitution and Returns to Scale

Consider the most common case of the production function homogeneous of degree one. Let W be defined as a constant such that $W > 0$, therefore equation (1.1) can be expressed as,

$$WQ = Wq(x_1, x_2 \ldots\ldots x_n) = q(Wx_1, Wx_2 \ldots\ldots Wx_n) \qquad (1.6)$$

Such a production function is characterized by so-called constant returns to scale. Proportional changes in all x_i, $(i=1\ldots n)$, by W will result in a similar change in Q by WQ. The significance of the homogeneity property becomes apparent when a measure of the substitutability of the inputs is desired. This in turn is more commonly referred to as the elasticity of substitution. Here we are interested in the rate at which one input must be substituted for another input, in order to leave the level of Q unchanged. This relation was suggested in statement (1.5). Using the relationship developed in (a), namely that equation (1.5) declined as x_1 was substituted for x_2, we can merely expand (1.5) to attain a measure of the input-ratio resulting from a change in one of the inputs. The elasticity of substitution is generally around unity, but always positive depending on the effects induced by a change in the marginal rate of substitution as well as the resulting change in the input ratio in the opposite direction. In other words, the elasticity of substitution E_s is the negative slope of the relationship of the (x_i/x_j) , $(j \neq i)$ $(i=1....n,\ j=1....n)$, to $-\frac{\partial Q/\partial x_j}{\partial Q/\partial x_i}$, i.e.,

$$E_s = - \frac{d(x_i/x_j) \quad (\partial Q/\partial x_j/\partial Q/\partial x_i)}{d(\partial Q/\partial x_j/\partial Q/\partial x_i) \quad (x_i/x_j)} \tag{1.7}$$

The returns to scale component implicit in the technical characteristics of production, cannot be ruled out by assumption. That is, for given increases in factor inputs, does this result in greater, equal, or smaller proportional changes in Q ? The impact on output[2] , presented by the returns to scale, can be dichotomized into, external and internal sources. External relates to general industrial development and internal results from changes with the production concerned i.e., changes dependent on the resources of the industry. In the present context no specific identification to either will be given since aggregate analysis does not permit this dichotomy which is manifested in single industry studies. With the present framework established let us now turn to a specific production function of the Neo-Classical type - the Constant Elasticity of Substitution .

1.3 Constant Elasticity of Substitution: A specific form of the Neo-Classical production function.

The constant elasticity of substitution (C.E.S.) production function was pioneered by Arrow, Chenery, Munhas and Solow[3] early in the sixties. It is a specific case of equation (1.1) where:

(i) the production function must be linearly homogeneous.

(ii) the elasticity of substitution must be constant, although not necessarily unity.

In its most common form, and the one adopted here, the C.E.S. is comprised of two factor inputs, capital (K) and labor (L), and can be expressed as:

$$Q = Z\left[AK^{-a} + BL^{-a}\right]^{-\frac{b}{a}} \qquad (1.8)$$

where: Z = scale parameter depicting efficiency, which accounts for changes in Q without changes in capital or labor.[4]

A = a capital intensity parameter (a relative measure).

B = (1-A) the respective labor parameter.

b = the returns to scale.

a = a parameter functionally related to E_s.

In the previous sections certain criteria were established for a Neo-Classical production function. Therefore, let us now consider each of these criteria, for the specific C.E.S. production. The basic conditions imposed in the previous sections were:

(i) That each of the respective inputs display positive marginal products - in the present context K and L .

(ii) Each of the respective inputs, over a particular range of the production surface, should have a decreasing - yet positive marginal product, that is:

$\partial^2 Q/\partial L^2 < 0$

$\partial^2 Q/\partial K^2 < 0$

(iii) The particular returns to scale should not be specified, a priori. Rather the degree of returns should be empirically determined.

(iv) That there exists some upper boung on Q , given one input as variable while the other (s) are fixed. In the context of the C.E.S., if L is constant, and K is varied, then the limit of Q should be finite[5].

(a) Positive Marginal Products

The marginal products of labor and capital find expression, for the C.E.S., as:

$$\partial Q/\partial L = bZBL^{-a-1} \left[AK^{-a} + BL^{-a}\right]^{-\frac{b}{a}-1}$$

let $X^{1} = Bb/A^{a/b}$ then,

$$\partial Q/\partial L = X^{1} L^{-1-a} (Q)^{1+a/b} \qquad (1.9)$$

and $\partial Q/\partial K = bZAK^{-a-1} \left[AK^{-a} + BL^{-a}\right]^{-b/a-1}$

let $X^{11} = AbZ^{-a/b}$ then,

$$\partial Q/\partial K = X^{11} K^{-1-a} (Q)^{1+a/b} \qquad (1.10)$$

From the above derivatives we can readily ascertain that both (1.9) and

(1.10) are positive - providing that b, L, and K are positive[6]. Hence, we can conclude that condition (i) is satisfied for the C.E.S.

(b) Decreasing Marginal Product

In order to investigate the properties of the respective marginal products we resort to the two expressions derived in section (a). First considering the marginal product of labor, using relation (1.9) we have:

$$\partial Q/\partial L^2 = BZ^{-a}(1+a)(Q/L)^a \frac{(\partial Q/\partial L)K - Q}{L^2} \quad (1.11)$$

The marginal product of capital can be considered from equation (1.10) as:

$$\partial^2 Q/\partial L^2 = AZ^{-a}(1+a)(Q/K)^a \frac{(\partial Q/\partial L)K - Q}{K^2} \quad (1.12)$$

From the above expressions we can recognize that for positive values of A and B, and with the aid of Euler's theorem, the numerator on both the above expressions will be negative[7]. Moreover, as long as this state of nature exists then both $\partial^2 Q/\partial L^2$ and $\partial^2 Q/\partial K^2$ will be of opposite sign to $(1+a)$, which has important implications to be considered. In consequence for positive values of A and B., and providing $a > -1$, the marginal products of labor and cpaital are positive - but decreasing and thus fulfill the second Neo-Classical requirement.

(c) Returns to Scale

Although the above considerations have assumed a value of b=1, it can be seen from equation (1.8) that b was not initially specified - rather it was undetermined and its value was left to empirical estimation. As such this condition of empirical determination can be satisfied for the Neo-Classical framework.

(d) Asymptotic Properties of the C.E.S.

The existence of asymptotes implicitly imposed in the Neo-Classical framework, does not lend itself to easy examination. Although not yet considered in the context of the C.E.S., the asymptotic properties are implicitly dependent on the elasticity of substitution. Accordingly let us first derive an expression for the elasticity of substitution for the particular case at hand - the C.E.S. - and then using this derived relation identify its significance with respect to the asymptotic properties of the C.E.S.

Using relation (1.7) derived previously, and substituting (1.9) and (1.10), we can express the E_s for the C.E.S. as:

$$E_s = \frac{1}{1+a} \tag{1.13}$$

Consider the case if $a < 0$, or that $E_s > 1$, then holding capital fixed and increasing labor without bound, therefore:

$$\lim_{L \to \infty} Q = \lim_{L \to \infty} Z \left[AK^{-a} + BL^{-a} \right]^{-b/a} \Bigg|_{-1<a<0} = \infty$$

Even without using the necessary binomial expansion it is apparent that the limiting value of output, as the labor input is increased without bound, is undefined. For the other case, where $0 < a < 1$, we have,

$$\lim_{L \to \infty} Q = \lim_{L \to \infty} Z \left[AK^{-a} + BL^{-a} \right]^{-b/a} \Bigg|_{0<a<1} = ZA(-ab/1-a)K^{a}$$

These results seem somewhat tenuous at first - at least from a theoretical point. However, if we carefully examine these two results we can see that this is what we should anticipate a priori. Namely, as long as $a < 0$ for equation (1.13), we see that E_s must be greater than one. Therefore if the elasticity of substitution is greater than one, it would suggest that the inputs are so similar to each other, that output can be completely supported by one or the other, and consequently, levels of output would respond to increases in one of the inputs. In the second situation, when $a > 0$ the two inputs are less substitutable for each other - and thereby impose upper limits on Q, regardless of input level[8]. Therefore, providing $E_s < 1$ the asymptotic properties imposed by the Neo-Classical framework, are upheld. However, under conditions where $E_s > 1$, we find that the C.E.S. does not conform to this fourth requirement. It should be noted that if $E_s > 1$ then the decreasing marginal product criteria is similarly violated[9].

1.4 A Limiting Case of the C.E.S. - The Cobb-Douglas Production Function

Up until the arrival of the C.E.S., most empirical production analysis within the framework of the Neo-Classical world, employed the Cobb-Douglas (C.D.) production function. The C.D. production function has received much attention primarily because of its econometric, and mathematical properties, which rendered great usefullness to the empirical economist[10]. The major features of the C.D., pertain to the homogeneous properties, and to the elasticity of substitution between the inputs. In general, constant returns to scale are assumed, concomitant with this assumption, we find that the elasticity of substitution of the C.D., is <u>unity</u>; if the elasticity of substitution is other than unity , then the production function would be of the C.E.S. type. In spite of this, if the returns to scale are other than unity we find that the C.D. production is still applicable - it is however, a more general form of the C.D.

In the following derivation, let us assume a C.E.S., production function, exhibiting both constant returns, and a constant elasticity of substitution of unity. With this in mind let us examine the marginal rate of substitution for the C.E.S.,

$$dK/dL = \left[A/1-A\right]\left[(L^{1+a}/K^{1+a})\right] \tag{1.14}$$

Now taking the limit of dK/dL as $a \to 0$ (which is the same as $E_s \to 1$)

$$\lim_{a \to 0} dK/dL = \left[A/\ (1-A)\right]\ \left[L/K\right] \tag{1.15}$$

The above expression is the marginal rate of substitution for a particular functional form with an elasticity of substitution equal to unity; what we now wish to ascertain is whether or not this marginal rate of substitution, results from an exact differential, and/or was derived from the C.D. type production function. In order to demonstrate that (1.15) is an exact differential consider the following:

Setting $dK/dL = dQ$, we can see that:

$dQ = [\partial Q/\partial L]\, dK + [\partial Q/\partial K]\, dL$, must hold by the definition of exact differential equations. Examining $\partial Q/\partial L$, we can suggest that its expression takes the form of $(1-A)/L$ from equation (1.15). Similarly for $\partial Q/\partial K$, we can substitute from (1.15), giving A/K. Now rewriting dQ , above, in terms of the results yields:

$$dQ = \left[(1-A)/L\right] dL + \left[(A/K)\right] dK \qquad (1.16)$$

Differentiating the above expression, over K and L, separately, gives:

$Q_L = 0$ and $Q_K = 0$, so that (1.16) is an exact differential[11].
As such, let $q_K = A/K$. Integrating gives:

$$q = A \ln K + C(L)$$

where $C(L)$ is the 'constant' of integration, yet implicitly related to the labor component. Differentiating q with respect to L yields,

$$q_L = C'(L)$$

and this expression must equal $\partial Q/\partial L$. Therefore

$$C\ (L) = (1-A)/L$$

or $$C(L) = 1-A \ln L + C_1$$

where C_1 is a constant of integration.

Thus: $$Q(K,L) = A \ln K + 1-A \ln L + C_1 \qquad (1.17)$$

which is tautologous to:

$$Q(K,L) = K^{A}L^{1-A} + C_1 \tag{1.18}$$

where $C = \ln C_1$, since C_1 is only the constant of integration which can assume any value, and therefore suggests:

$$Q(K,L) = CK^{A}L^{1-A}$$

which is in fact the C.D. production function with constant returns to scale and unitary elasticity substitution. With this result it should be intuitively obvious that the properties of the C.D., with respect to Neo-Classical criteria will also follow. As such, no attention will be directed to fulfillment of the Neo-Classical criteria, rather it shall be stated that the criteria are satisfied, and if the reader wishes he can pursue this on his own.

1.5 Technology and the Production Function

In considering a production function as a relation between inputs on the one hand, and output on the other, we are implicitly assuming some underlying technology. Murray Brown[12] suggests that there are four characteristics of a production function which in turn determine an abstract technology. These characteristics of technology are returns to scale, efficiency of technology, the intensity of capital in technology, and the ease at which inputs are substituted for each other. For three of these characteristics the previous sections of this chapter have already considered, however, for the efficiency of technology no explicit consideration has yet been given.

Technological progress, the result of the above characteristics can result in two basic impacts on the level of output. The first of these is neutral change. Neutral change affects the realization of the particular level of output, without affecting the input - input relation. The changes in output resulting from neutral changes are generally the efficiency components of technology, not directly pertinent to returns of inputs. The other technological change is non-neutral change; here the alteration of the technology does affect the rate at which one input can be substituted for the other, which is generally attributed to possible changes in productivity of the inputs; e.g., education and training for labor[13].

In the previous sections, C and Z, were explicitly included as parameters. These variables are advanced only as measures of the neutral technological element. Nevertheless, other changes in the level of output can ensue from alterations in the other production parameters such as the intensity parameters. In later sections we shall turn our discussion to specific forms of econometric techniques, offering some aid in deciphering technological influences.

N O T E S

Chapter 1

1. Brown (1966), p.12

2. Marshall, A., <u>Principles of Economics</u>, MacMillian, (1922), p.266

3. Arrow, et. al. (1961)

4. The technological parameter is examined in detail in latter chapters as well as at the final section of this chapter.

5. It should be noted that the positive values of each of the marginal products is independent of b, the returns to scale parameter.

6. This particular occurrence is not always true, since for the exercise at hand the value of b has been assumed as unity. It should be noted that for values of b greater than one the marginal products may be increasing.

7. Draper and Klingman, <u>Mathematical Analysis</u>, Harper & Row, (N.Y. 1967) pp. 229-302

8. Asymptotic considerations with respect to the capital input are analogous.

9. <u>Loc. cit.</u>

10. <u>Loc. cit.</u>

11. Protter, Morrey, <u>University Calculus</u>, Addison Wesley, (1964)

12. <u>Op. cit.</u>

13. Griliches, Z., "Research Expenditures, Education and the Aggregate Agricultural Production Function", <u>American Economic Review</u>, 1964

CHAPTER 2

CAPITAL THEORY & MEASUREMENTS OF CAPITAL STOCK

2.1 Introduction

Over the last two decades capital theory and/or the theory of capital accumulation has received much attention among economists. Perhaps, the first formal presentation of capital theory can be identified as early as 1896, by Wicksell,[1] who reinterpreted and expanded the earlier consideration of Böhn Bawerk. However, Wicksell's attempt to formalize a theory of capital was not pursued by his contemporaries. Rather, capital theory as such was not to be entertained in economic theory until the field of economic development and growth analysis became vogue. This resulted from the fact that economists concerned with analyzing and developing theories of growth realized that an explicit incorporation of capital and it's determinants was of prime importance to study the factors relevant to economic growth. Consequently, over the last two decades a multitude of theoretical and empirical studies on capital and it's related components, have appeared in journals and texts of economics. Hence, the choice of what is relevant and what's not is some-what subjective, and therefore is a matter of interpretation of what is required, and most relavant for the particular problem at hand. Correspondingly, the following sections of this chapter will outline briefly, some of the more "important" studies of capital, with particular emphasis on those theories which are relevant to present analysis.

2.2 Theories of Capital: An Overview

As mentioned, the determination of a particular level of capital stock, determinants of this stock, and therefore a theory of investment are major areas of research in economics. The most outstanding theories of the demand for capital goods and accordingly their services - particularly for the present study are: the Neo-Classical Theory, Duesenberry's Model, and Solow's Vintage Model. These theories have not been received without severe criticism, and many contemporary writers argue that none of the above theories provide an adequate explanation of the determinants of capital, id est, investment behavior. This opinion has developed as a result of the fact that many objectives of businessmen are not only diometrically opposed, but more important, are not known. A classic statement of this fact was emphasized by Lutz, when he said, "it is one of the surprising things about capital theory that no agreement seems to have been reached as to what entrepreneurs should maximize".[2] With this in mind let us now turn to an explicit consideration of the above theories.

2.3 The Neo-Classical Theory of Capital Accumulation

(a) General Considerations

The basis of the Neo-Classical theory of capital accumulation is centered on the behavior of the entrepreneur, who is assumed to maximize some measure of profit. The areas of primary objection to such a criteria are : first, a considerable amount of non-empirical analysis has been devoted to the determinants of businessmens' motivation, and this has arrived at a conclusion that basic marginal considerations are largely irrelevant in the decision processes; the second objection to the Neo-Classical criteria, is that the Neo-Classics have failed to properly formulize a theory of investment complimenting the basic Neo-Classical criteria; the last and most fundamental objection, raised by Haavelmo, is that there is no a priori justification for any continuous demand for investment by an entrepreneur, Haavelmo goes on the say:

> "What we should reject is the naïve reasoning that there is a demand schedule for investment which could be derived from a classical scheme of producers' behavior in maximizing profit. The demand for investment cannot simply be derived from the demand for capital. Demand for a finite addition to the capital can lead to any rate of investment, from almost zero to the infinity, depending on the additional hypothesis we introduce regarding the speed of reaction of capital users. I think the sooner this naïve, and unfounded, theory of the "demand-for-investment schedule" is abandoned, the sooner we shall have a chance of making some real progress in constructing more powerful theories to deal with the capricious short run variations in the rate of private investment."[3]

The first objection has been severely criticized by White who after critically examining the available evidence, concludes, that as a result of the defective data (defective on non econometric standards), there does not exist any support of the "non marginalist" decision process.[4] Jorgenson rejects the second objection to the Neo-Classical framework;[5] Jorgenson argues that none of the tests reported in early literature are based on a rigorous statement of theory. Moreover, the corresponding assumptions with respect to the lags between the derived demand for capital services and actual investment are much too restrictive. Jorgenson suggests that the statement of theory, accompanying earlier studies, paid little or no attention to the role the price of capital goods, the rate of interest and various taxes, would play in the determination of the stock of capital. These components are of utmost importance in the determination of the user cost of capital, referred to by Jorgenson, which he argues should be explicitly accounted for in determination of the demand for capital goods in the Neo-Classical framework.[6]

The last objection, raised by Haavelmo is basically the result of confusion. Namely, Haavelmo confuses the concept of a demand for investment schedule of a correct Neo-Classical type, with an abstract notion of investment occurring over time but without any systematicity. The theory of investment purported by the Neo-Classical economists advance how changes in investment demand,in turn, lead to changes in capital stock over time, and can therefore, quite reasonably suggest an investment function explained by the level of, or change in, aggregate demand.

Another way of looking at the Neo-Classical theory, is that the optimim level of capital accumulation can be achieved if the firm attempts to maximize the utility associated with a consumption stream over time subject to the constraint of the production process, and an assumption pertaining to the relationship between the rate of capital accumulation and net investment. More specifically investment is not an end in itself, but rather a means of distributing consumption over time. Also, in connection to the maximizing of the utility from consumption, Jorgenson proposes that under various conditions this approach is tautologous to the maximization of the firms net wealth. Consequently, the problem then becomes a simple mathematical exercise, resulting in a solution for the demand of services from the corresponding set of capital goods.

The central feature of this theory is the dependence of the demand for capital goods on the relative factor prices. When a level of capital stock, deemed optimal, is found we can then move to a formalization of an investment theory. Thus, the demand for capital should not be mistaken as a demand for investment.[7] In addition, the short run determination of investment behavior depends on the lag effects, in responding to changes in demand for capital goods.[8] As such, with a change in the desired stock of capital some investment project will be initiated, however, before actual investment occurs, several stages have to be passed. Initiation of the contract, issuing of orders, appropriation of funds, various levels of work must be completed, and perhaps other steps, and then the actual investment occurs.

(b) Optimal Capital Accumulation: A Formal Theory

Let us assumethat the demand for capital is determined in such a way that the net worth of the firm is maximized, where net worth is defined as the present value of all future streams of net revenue of the firm. Prices, interest rates, and wages are assumed fixed, but for the moment, let us assume, that the effect of taxes on this stream of net revenue is negligible. Defining a particular realization of a single ouput by Q^*, the corresponding labor required for Q^* as L^*, and the required investment in capital stock by I^*, as well as p, w, r, their respective prices, the net revenue of the firm can be defined as:

$$R(t) = PQ^*(t) - wL^* - qI^* \tag{2.1}$$

and the present value of net worth is

$$W = \int_0^\infty e^{-rt}R(t)dt \tag{2.2}$$

First, the rate of change in the flow of capital services is proportional to the flow of net investment. Net investment, on the other hand, is total investment less replacement investment, furthermore, replacement investment is assumed to be proportional to the level of capital stock K. Therefore, the constraint can be expressed as:

$$dK = I^* - CK \tag{2.3}$$

where C is the constant of proportionality.

The second constraint is the relationship between the level of output and the level of inputs, i.e., capital and labor services, namely a production function:

$$F(Q_1 K_1 L) = 0$$

Employing the technique of Lagrangian multipliers, we can derive the following relationships.....

$$\partial Q/\partial L = W/P$$

$$\partial Q/\partial K = \frac{r(v+C) - dq}{P} \qquad (2.4)$$

.... which are the familiar marginal productivity conditions. The expression $(r(v+C) - dq)$ represents the shadow price of implicit rental of one unit of capital service per period of time,[9] also called the user cost of capital mentioned earlier. Assuming q (the capital gain or loss) as being transitory, i.e., attaching an expected value of zero to it, and labelling the user cost of capital by C, we have:

$$\partial Q/\partial K = C/P \qquad (2.5)$$

where $C = q(r+C)$ From this last expression, the demand for capital stock can be obtained.

This generalized solution can be expanded for more complex situations, where explicit recognition of the user cost, C, being affected by taxes, tax rates, tax incentives, and depreciation policys of governments, etc., can be incorporated. The first part of the above equation, i.e., the marginal productivity theory condition preserves its form C/P, only C takes new values according to new situations. For a detailed analysis, Jorgenson (1963) can be consulted[10].

2.4 Vintage Model

Consider the normal Cobb-Douglas and constant Elasticity of Substitution (C.E.S.) production functions outlined in chapter 1:

$$Q(t) = Ce^{vt} L(t)^{1-A} K(t)^{A} \tag{2.6}$$

or

$$Qt = Z (AK^{-a} + BL^{-a})^{-b/a} e^{vt} \tag{2.7}$$

The term e^{vt} denoted the pace of technical progress, that is, if L and K are kept constant, we still expect the production function to shift upward over time. This results from changes in the productivity of the factor inputs as well as technical changes not attributable to either inputs. Even if we ignore, for the moment, the econometric and estimation problems which arise in the use of above functions, we do not know, moreover, we cannot explicitly interpret the process of technical change. Many attempts have been made to identify the so-called residual e^{vt}. In these undertakings the majority of work has centered upon using data on education as an additional variable, to see if the relative magnitude of e^{vt} can be reduced. However, as Solow has correctly noted in one of his earlier articles regarding the residual:

> "It is true that the notion of time shift in the (production) function is a confession of our ignorance rather than a claim to knowledge; they (i.e., the shifts) ought to be analyzed further into such components as improvements in the skill and quality of labor force, returns to investment in research and education, improvements in techniques within industries,...etc."[11]

Both of the above equations treat technical progress as something like time and motion study — some way of organizational improvement. But the most striking assumption is that ALL capital equipment, old and new, participate equally in the technical progress. This is to say that new improvements, inventions, and innovations apply equally to old and new capital. Here is the flaw in the argument. A simple observation of most of innovations shows that they need to be implemented in the new kinds of capital equipment, i.e., technical change is embodied in the latest model of capital equipment.

Empirically speaking, in 1957, Solow used a production function of Cobb-Douglas type and came up with the results that the technical progress has contributed more than anything else to growth in output (about 85%).[12] He basically assumed a disembodied process of technical progress, one that drops from the sky. Many criticisms followed these results until Jorgenson and Grilliches[13] in 1967 formally demonstrated that technical change and improvements are almost completely embodied (non neutral technical change) in capital and labor, with disembodied technical progress accounting for only 3.3% of the growth of output (while growth of inputs together with their relative improvement, explain the rest).

This was the basis of development of vintage models, i.e., models in which capital equipments of different age are treated differently. Denoting gross investment at time v by K_V , meaning the total output of capital goods at that time period, it is also equal to $I(v)$, investment in period v . Using a simple assumption with respect to depreciation and obsolescence, namely that the capital equipments are exposed to a constant mortality rate ∂ (i.e., avarage life expectancy is $1/\partial$), then we can write:

$$K_V(t) = K_V(v)e^{-\partial(t-v)} = I(v)e^{-\partial(t-v)} \qquad (2.8)$$

The above equation gives the value of capital stock of vintage v in period t (say, capital stock of vintage 1945 in period 1970) in terms of original value of capital (i.e., in time period v) discounted to present period by the mortality rate ∂ .

From the above equation, the overall level of capital stock at a certain point in time, say t , can be found to be the summation of present value of capital stocks of all vintages, i.e.,

$$K(t) = \int_{-\infty}^{t} I(v)e^{-\partial(t-v)}\,dv = \sum_{v=-\infty}^{t} I(v)e^{-\partial(t-v)} \qquad (2.9)$$

Considering the fact that the data for capital stock is generally rare and unavailable, it is possible to use the last equation to develop a technique for estimation of (2.8) and then use a recursive system to estimate the level of capital stock at any time period.

The other striking results of vintage models is in regard to growth. Solow[14] and Phelps[15], both show that:

(a) the average life of capital stock $1/\partial$ depends only on the rate of growth and rate of depreciation and not on the investment ratio, I/Y, and

(b) the asymptotic rate of growth is independent of the investment ratio. For the present analysis the theory of the vintage model beyond this point does not contribute to the problem at hand. However, both Solow and Phelps in the previously mentioned articles do discuss in detail, further extentions if the reader desires. As such, let us now pursue the approach of the vintage model in the particular study at hand.

2.5 Estimation of the Level of Capital Stock For B.C. Using the Vintage Model.

In the previous section (2.4) we arrived at the equation:

$$K(t) = \sum_{v=-\infty}^{t} I(v)e^{-\partial(t-v)} \qquad (2.10)$$

At this time it was suggested that this equation could be used for estimation of ∂, the average mortality rate, and later for the estimation of provincial capital stock. This has been done by the following procedure:

(a) Assume that the average mortality rate of the capital stock of Canada is the same as that of British Columbia. This assumption means that average lifetime of capital stock for the nation and the province is the same.

(b) Expand equation (2.10) to obtain:

$$K(t) = I(-\infty)e^{-\partial(t-\infty)} + \ldots I(t-3)e^{-\partial(3)} \ldots + I_t \qquad (2.11)$$

or

$$K(t-1) = \ldots\ldots\ldots\ldots\ldots \quad I(t-3)e^{-\partial(2)} + I(t-1) \qquad (2.12)$$

Multiplying the equation (2.12) by $e^{-\partial}$, we get,

$$e^{-\partial}K(t-1) = I(-\infty)e^{-\partial(t-\infty)} \ldots\ldots\ldots\ldots + I(t-1)e^{-\partial} \qquad (2.13)$$

Subtracting equation (2.11) from equation (2.13), we get

$$K(t) - e^{-\partial}K(t-1) = I(t) \qquad (2.14)$$

Since the data for the capital stock of Canada is available for years preceding 1957, the above equation can be used to estimate the average mortality rate of Canadian capital stock. From equation (2.14) we can write:

$$\partial = \ln\frac{K_{t-1}}{K_t - I_t} \qquad (2.15)$$

Using the available data, for the eleven years (1947 - 1957): on capital stock estimates for Canada, an average measure of ∂, ∂^*, can be estimated. The values of ∂ generated over this period were as follows:

0.06654	0.06268	0.09067	0.08054
0.06521	0.05369	0.07017	0.08858
0.06654	0.04429	0.06443	

The average ∂^* is 0.0683, suggesting that the average life of Canadian capital stock is 1/0.0683 or approximately 15 years.

(c) Now expanding equation (2.14) for the year 1969, results in;

$$K(1969) = I(1950)e^{-19\partial} + I(1951)e^{-18\partial} + \ldots\ldots + \ldots I(1967)e^{-2\partial} + I(1968)e^{-\partial} + I(1969) \qquad (2.15)$$

Since it was found that the average life of a typical piece of capital is approximately 15 years, a reasonable approximation of K(1969) for B.C. can be obtained if we start the above series from 1949, i.e., a twenty-year period and use the values of investment in B.C. from I(1949) to I(1969).

If a set of data for investment in B.C. were available[16] (at least from 1930 to 1969), one could use the same method to approximate the level of capital stock, but unfortunately the available data ends in 1949, consequently the present analysis is forced to use a weaker approximation. (Namely 1949.) With these estimates of capital data and from the published statistics figures 1, 2, 3, and 4 were generated. In figure 1, the relationship between capital and provincial product is plotted. This graph is the most important, as all figures of capital stock are produced by the above vintage model. Figure 2, also using the derived capital figures, demonstrates the dependence of various levels of capital stock and investment over the period (1959 - 1969). The remaining graphs are included for general interest, as to the trend of investment over the period under consideration and the relative affect upon the adjusted labor force.[17] As a natural choice, with the established framework on capital and general production theory let us now move to the theoretical possibilities of estimation, employed in the aggregate analysis.

FIGURE I

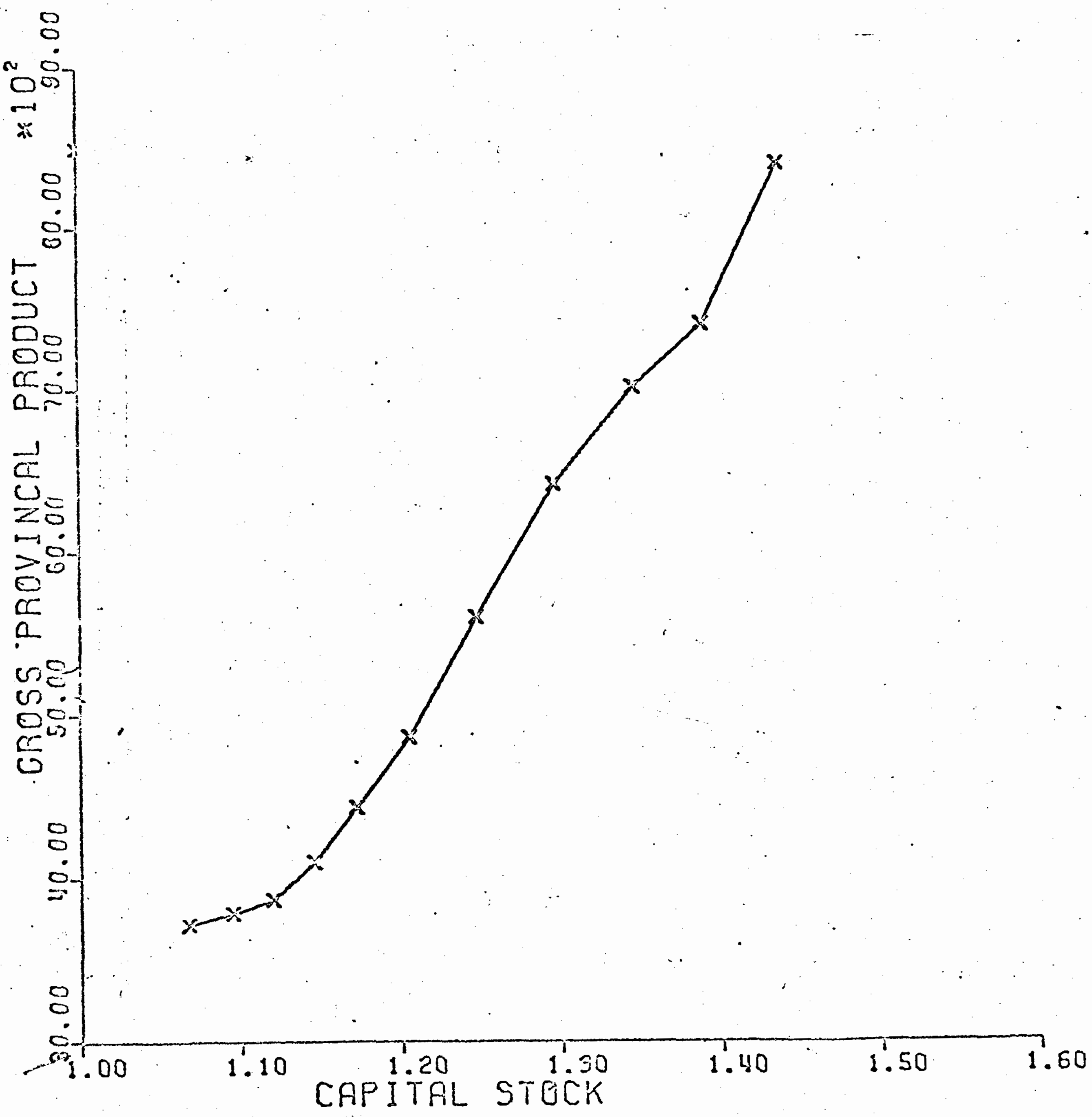

GROSS PROVINCAL PRODUCT *10²
30.00
40.00
50.00
60.00
70.00
80.00
90.00
1.00
1.10
1.20
1.30
1.40
1.50
1.60
CAPITAL STOCK

FIGURE 2

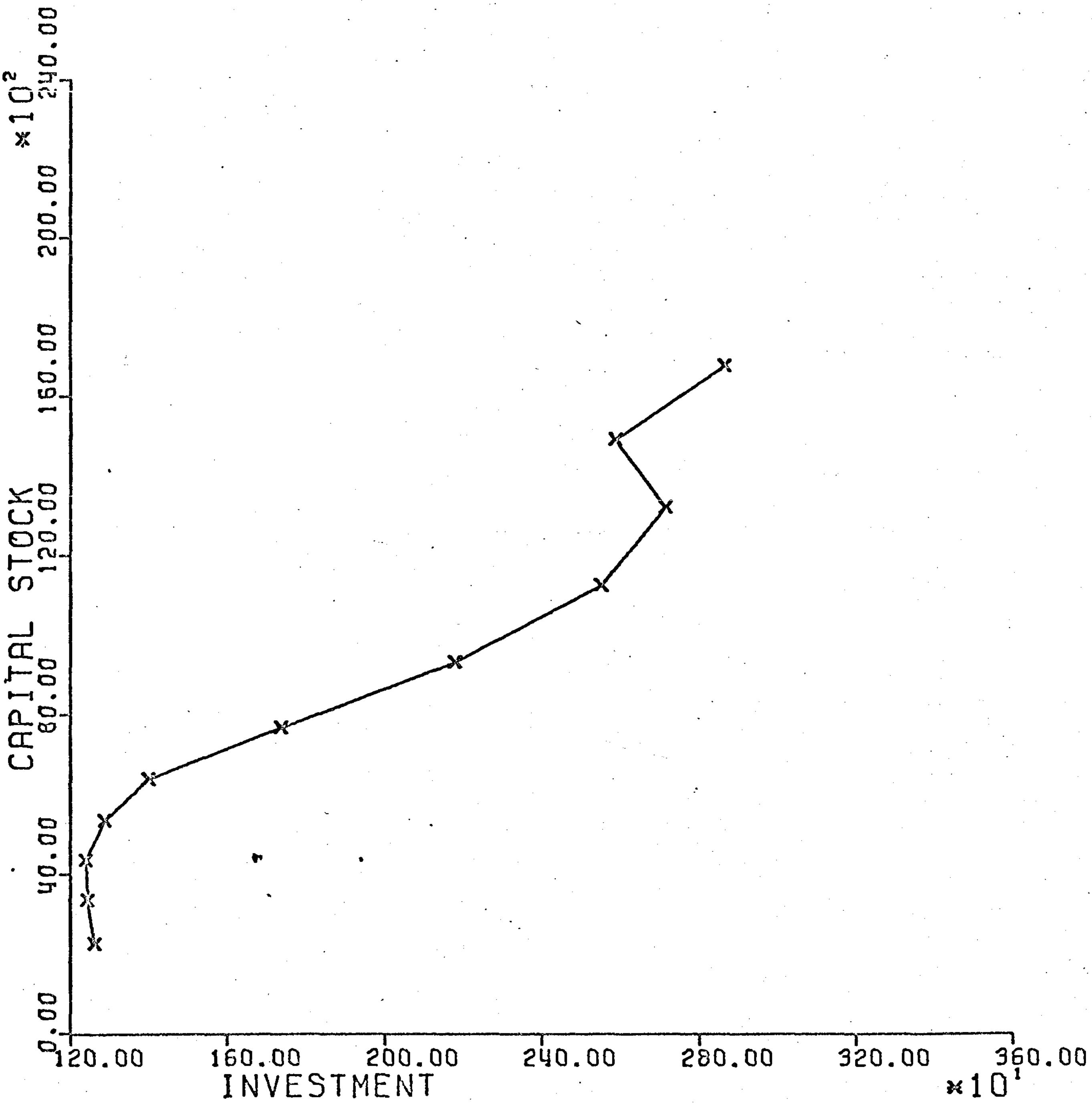

CAPITAL STOCK
×10²
0.00
40.00
80.00
120.00
160.00
200.00
240.00
120.00
160.00
200.00
240.00
280.00
320.00
360.00
INVESTMENT
×10¹

FIGURE 3

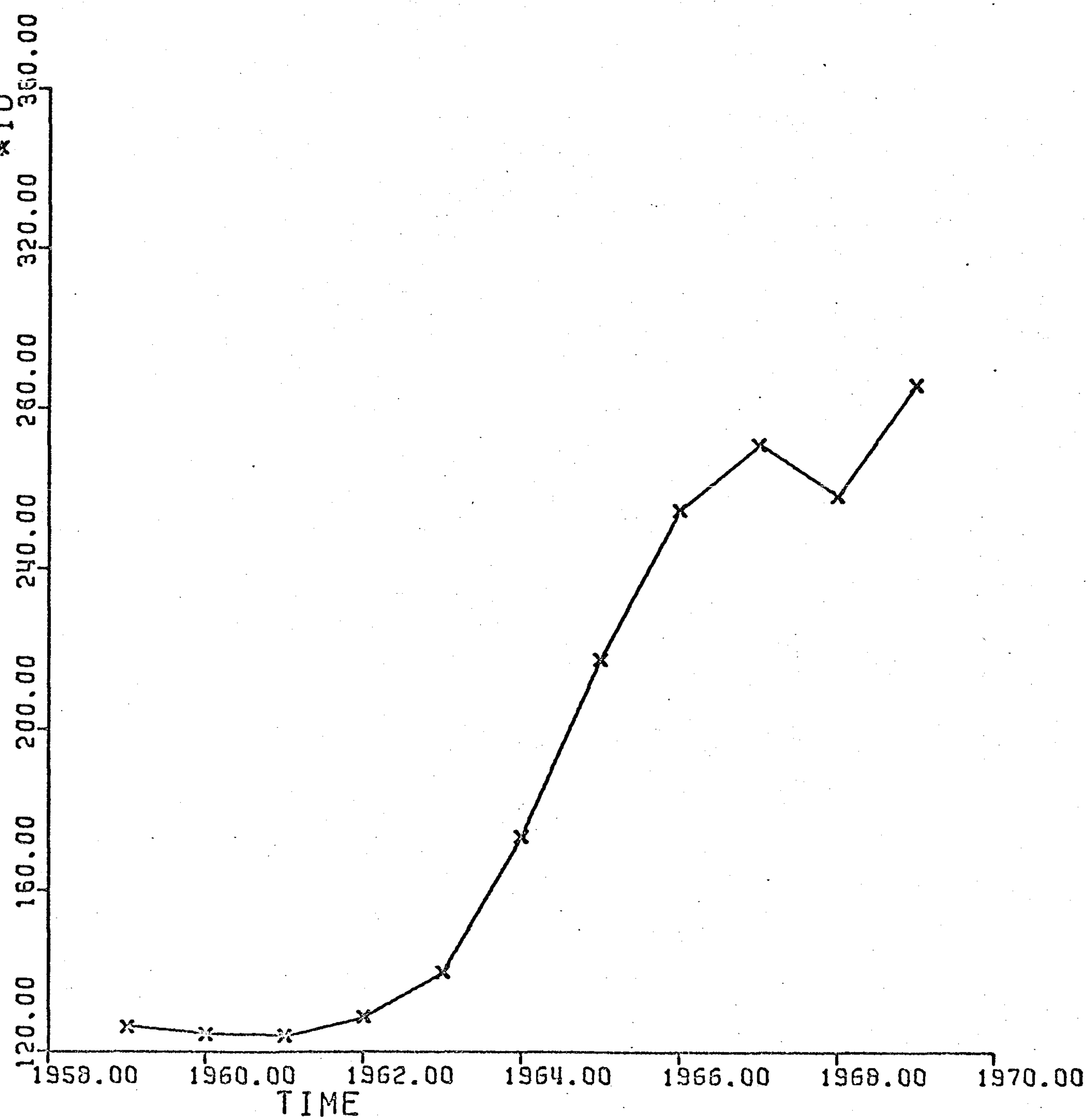
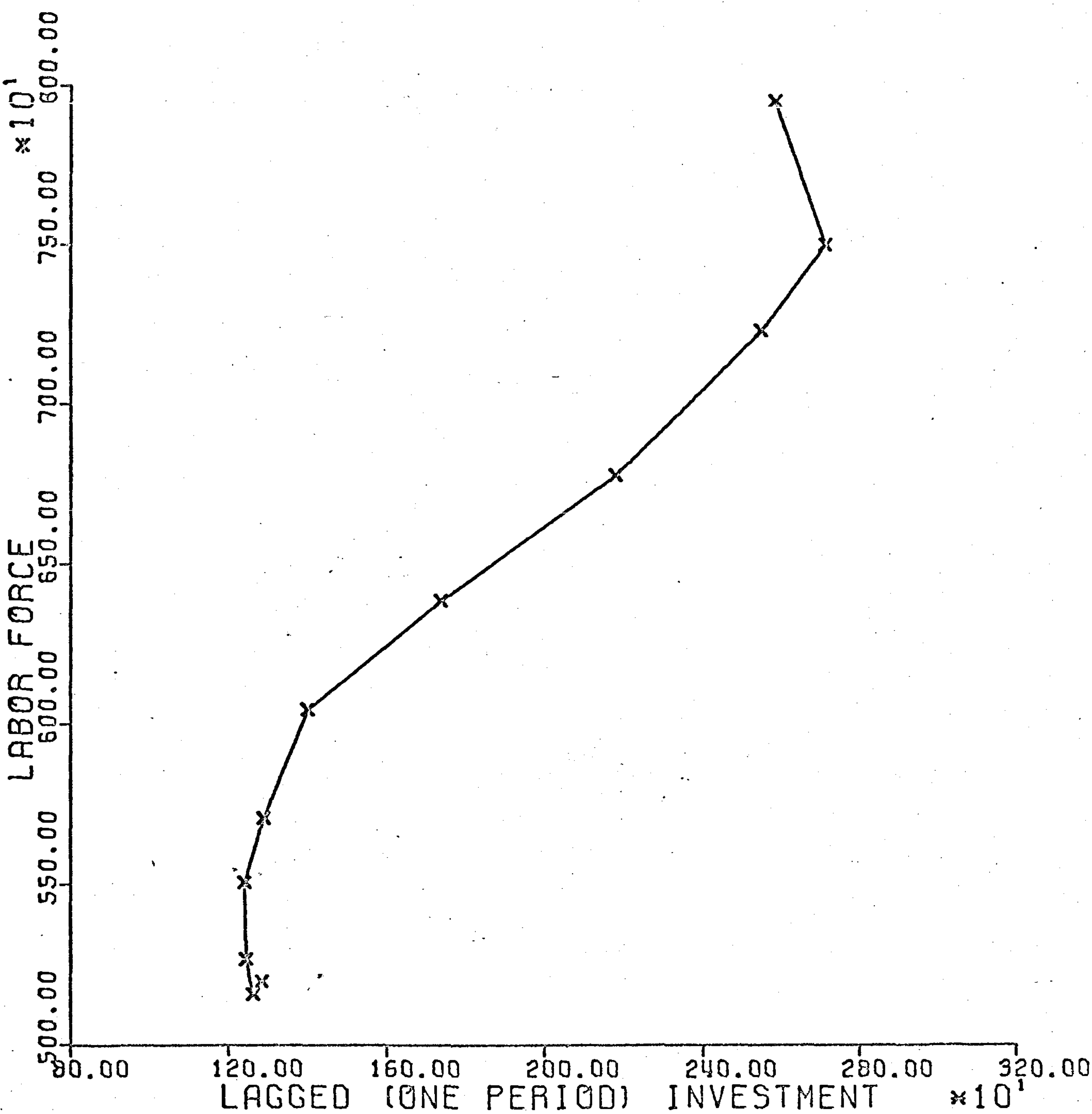

120.00
160.00
200.00
240.00
280.00
320.00
360.00
1958.00
1960.00
1962.00
1964.00
1966.00
1968.00
1970.00
TIME

FIGURE 4


LABOR FORCE ×10[1]
500.00
550.00
600.00
650.00
700.00
750.00
800.00
80.00
120.00
160.00
200.00
240.00
280.00
320.00
LAGGED (ONE PERIOD) INVESTMENT ×10[1]

N O T E S

Chapter 2

1. Wickwell, K ., (1896), p.53.

2. Lutz (1961), p.6

3. Haavelmo (1960), p.170

4. White (1956), pp.565-587, the data referred to was a result of a questionnaire of over 1,000 English firms, and later a similar study of 13,000 American firms.

5. Jorgenson (1967), pp.131-132

6. This concept of user cost was first introduced by Keynes in his General Theory........, and discussed at length in the appendix of chapter 6, of the same. Also, see Abba Leiner for some elucidating comments (1943), pp,131-132

7. Hirshleifer (1958), p.205

8. Jorgenson (1963), p.248

9. Ibid., p.249

10. Loc.cit.

11. Solow (1959), p.159

12. Solow (1957)

13. Jorgenson and Grilliches (1967), pp. 249-283

14. Loc. cit.

15. Phelps (1962), pp.548-567

16. Summary of Economic Activity in British Columbia, (1958-1969). Plus additional information with respect to investment was obtained from Mr. Knight assistant director of the Department of Economics and Statistics, Victoria, B.C.

17. See chapter 4, for explicit considerations of the fashion in which all graphs should be interpreted.

CHAPTER 3

ESTIMATION TECHNIQUES for AGGREGATE PRODUCTION FUNCTIONS

3.1 Introduction

Before one can confront theory with data there exists certain issues which must be settled. In the case of empirical production analysis, we must identify certain procedures whereby estimates of the parameters under examination can readily be secured. As such, the following chapter will focus on two important econometric problems. The first of these problems pertains to estimation of the parameters such as; the intensity parameters, the returns to scale, and the elasticity of substitution (at least in the case of the C.E.S.). The second problem to be investigated relates to the measurement of the technological components of the respective production functions under consideration. In order to do justice to these problems, the first two sections of this chapter will be devoted to the methods of estimating the parameters of the C.D. and C.E.S. production functions, respectively. The third section, will deal with the technological components in a manner which will be useful for the empirical results - at least from the prospective of interpretation.

3.2 Estimation of the Cobb-Douglas Production Function

Up to this point, all discussion has focused on the economic model, where all problems of measurement have been ignored. What we now wish to establish is a statistical or econometric model. In order to transform the economic model to an econometric model necessitates the inclusion of a latent variable u . The latent variable represents technical factors unique to a particular observation, i.e., u is unique to the i^{th} observation on Q , L , and K . Consequently, the previous economic model now becomes:

$$Q = Z K^{A} L^{B} u \tag{3.1}$$

The above model is now an econometric model, via the incorporation of u , the latent variable. The justification for u entering multiplicatively results from convenience, with respect to statistical estimation.

The latent variable is a device whereby recognition is given to the fact that we cannot expect estimates A , B and Z to exactly determine the level of Q_t , given values of K_t and L_t at time t . Rather, we would expect that other factors - random in nature - will cause the equality between Q and K , L , to be violated. However, since these influences are random over time, we expect that on the average, u should have a value of zero, suggesting that its influence on Q over time is negligible.

As suggested in the first chapter, the C.D. specification combines simplicity with certain attractive theoretical properties. That is, due to the limitation imposed by estimation techniques, both theoretical and computational, direct estimation for the above econometric model is not possible. Rather, a transformation is applied, which in turn converts this equation to a linear function - which has available a host of possible estimation procedures. The most common estimable form of the C.D. is,

$$\ln Q = \ln Z + A \ln K + B \ln L + \ln u \qquad (3.2)$$

As can be seen estimation of the above equation produces estimates of both the technological and input parameters. However, a problem which often occurs from utilization of the above equation is that $\ln K$ and $\ln L$ are highly correlated, which in turn affects the level of realibility that can be placed on the statistical estimates of A and B. In order to avoid this problem an alternative formulation of (3.2) can be utilized, which results in a value of A, while simultaneously reducing the collinear problems of $\ln K$ and $\ln L$, that is,[1]

$$\ln(Q/L) = \log Z + A \log(K/L) + u \qquad (3.3)$$

The above relationship, although alleviating the collinearity problem, offers a serious constraint; that is, estimation of A, and therefore B, where $B = 1-A$, requires constant returns to scale are explicitly incorporated in the estimation procedure.

The interpretation of $\hat{A}$ and $\hat{B}$, the statistical estimates of A and B, in the C.D. model, are merely elasticities of output, with respect to capital and labor. The sum of these elasticities, as suggested in chapter I, determine the returns to scale implied by the observations used to generate $\hat{A}$ and $\hat{B}$. With respect to the estimate of $\mathfrak{T}$, $\hat{\mathfrak{T}}$, its interpretation will be considered in a later section in conjunction with ∂ of the C.E.S.

Little more need be said of the C.D., within the context of estimation. It should be apparent why its use has been so frequent among economists in empirical research. Interpretation and estimation of each of the elasticities are readily obtained, procedures to eliminate problems of multicollinearity are also readily applicable. Both of these functions are estimated with a comparison of the parameter results in the following chapter. As such, let us now pursue a more interesting problem - estimation of the parameters of the C.E.S. production function.

3.3 Estimation of the C.E.S. Production Function

Consider the case where a direct logarithmic transformation, as was undertaken in the previous section, is applied to the C.E.S. production function:

$$\ln Q = \ln Z - b/a \ln(AK^{-a} + BL^{-a}) \qquad (3.4)$$

Referring to chapter 1, it will be remembered that the estimates which were desired were; a, b , ∂ , A , and B , for given data of Q , L , and K respectively. Nonetheless, upon close examination of the above equation, and its orginal functional form, we can readily see that direct estimation is not possible. Namely, even with the logarithmic transformation we find that $(AK^{-a} + BL^{-a})$ does not permit a direct procedure for estimation, since three unknown parameters, A , B , and a , are included in this expression. Accordingly, we must now employ additional functional specifications in order to finesse the desired parameter estimates.

Various procedures have been developed by economists, yet many of these techniques do not allow a direct estimation of the returns to scale factor, b . One possible procedure would be to estimate b , via the C.D. model, where (A+B) are not constrained, then using these estimates one could apply the returns to scale factor of the C.D., to the C.E.S. This procedure was considered as a possibility, but after contemplation it was recognized that any production function has certain unique properties, the most important of course is the determination of the interaction of the inputs. Moreover, this interaction of the inputs implicitly affects the returns to scale, since we realize that the C.D. and the C.E.S. are functionally related only under very special circumstances (see section 1.4), it should be intuitively obvious that utilization of the parameter for returns to scale of the C.D. will not - and has not - any theoretical or empirical validity. However three procedures are generally adopted, the C.E.S. with additional functions, from which estimation of the orginal parameters of the model can be accomplished. Let us now look at each of these in turn, and cosider their plausibility, from both a theoretical as well as an empiriacl stand point.

First let us impose the assumption, providing the relevant production function is in fact a C.E.S., that all business concerns contributing to the aggregate values of Q, and therefore their derived demands for capital and labor, are done in such a fashion as to minimize costs, then we can redefine our marginal relations of chapter 1, as:

$$r_{KE}/WL_t = \frac{\partial Q_t/\partial L_t}{\partial Q_t/\partial K_t} \tag{3.5}$$

We can relax the strong assumption of cost minimization of all productive concerns by incorporating our stochastic component which specifically accounts for incomplete cost minimization, thus equation (3.5) now becomes:

$$r_{Kt}/WL_t = \frac{\partial Q/\partial L_t}{\partial Q/\partial K_t} \; u_t \tag{3.6}$$

Substituting from (1.14) for (3.6) yields:

$$r_{Qt}/W_{Lt} = \frac{A}{1-A} \left(\frac{K}{L} \right)^{1+a} u_t \tag{3.7}$$

Therefore, to transform equation (3.7) into a form which can be estimated a logarithmic transformation is applied, resulting in:[2]

$$\ln(r_{Rt}/W_{Lt}) = \ln \left(\frac{1-A}{A} \right) + (1+a) \ln \left(\frac{K}{L} \right)_t + \ln u_t \tag{3.8}$$

To compute estimates of $\left(\frac{1-A}{A} \right)$ and $(1+a)$, requires only, the wage rate, price of capital, and data on their corresponding aggregates K and L. The estimates of $\left(\frac{1-A}{A} \right)$ and $(1+a)$, allow us to transform equation (3.5) so that estimates of ∂, b, and v can be easily obtained That is,

$$\ln Q = \ln Z + v_t - \frac{b}{a} \ln(AK^{-a} + 1-AL^{-a})$$

Substituting the estimates generated from equation (3.8) yields the following.....

$$\ln Q = \ln Z + vt - \frac{b}{a} \ln(\hat{A}K^{-a} + (1-A)L^{-\hat{a}}) \qquad (3.9)$$

...from which we can obtain estimates of Z , v , and b . This procedure has definite econometric appeal since the collinear relationship, mentioned earlier, between capital and labor has been avoided.[3] Consequently the degree of reliability which can be placed in these estimates - at least from a statistical point of view - is desirable.

Another procedure that can be adopted, in estimating the C.E.S. parameters, employs a direct estimation technique, on equation (3.4), without extraneous fucntional relations. Moreover, the following technique has a theoretical advantage over the previous method. That is, assumptions regarding cost minimization behavior, in addition at the assumption regarding the acceptance of the production function, explicitly incorporated can come under a host of theoretical arguments. The procedure to be outlined does not require the former of the two assumptions, rather we must only accept the existence of the production function and disregard any possible cost considerations. There do exist certain other properties however, which may prohibit its use - mainly problems of computation.

This procedure, generally referred to as the Gauss-Newton[4] method, utilizes least square techniques. The normal equations used in the estimation procedure are derived from the original function, which has been linearized by a Taylor series expansion. The use of a Taylor series technique results from intuitive "guesses" by the researcher, with respect to the parameters under consideration. The parameter values are then revised by an iteration procedure, until convergence is satisfied. It should be noted that convergence may require a large number of iterations in order for convergence, as such this may deter one from adopting it as a suitable estimation technique. In the case of our C.E.S., its first order Taylor series approximation can be written as:[5]

$$Q = Q(Z,A,a,t) = Q_1 + \frac{\partial Q}{\partial Z}(Z-Z_1) + \frac{\partial Q}{\partial A}(A-A_1) + \frac{\partial Q}{\partial a}(a-a_1) + \frac{\partial Q}{\partial s}(s-s_1) \qquad (3.10)$$

where $s = b/a$, now let $Q^* = (A_1K^{-a}\partial + (1-A)\,L^{-a}\partial)$, where the indicated 1's , represent initial values assigned, rewriting equation (3.10) with the above substitutions yield:

$$Q(Z,A,a,s) = Q_1 + Q_1^{-s_1}(Z-Z_1) + Q^{*-(1+s_1)}\, s_1 Z_1 \,(K^{-a_1}-L^{a_1})\,(A-A_1) - sZ^{-(1+s_1)}\, Q^*\,(A_1K^{-a_1}\ln K + (1-A)\,L^{-a_1}\ln L)\,(a-a_1) + ZQ^{*-s_1}\ln Q^*\,(s-s_1)$$

Now values of Q denote predictions determined, based on the linearized model, with Q_1 being an evaluation of Q , for intial values of the parameters; as such we can derive an expression, of the relationship between the predictions and actual values of Q , say Θ , as

$$\Theta = \sum_{L=1} (Q_i - \{Q_i\})^2$$

where values of $(Z-Z_1)$......$(s-s_1)$ can be determined via minimizing with respect to each s- component $(Z-Z_1)$, on etc.

It should be apparent, that values of the parameters, determined in this fashion are estimates of the orginal non linear form of (3.5) , this follows if the correction factors $(Z-Z_1)$.....etc., vanish at convergence. The basic problem of this method is that convergence may well never occur, simply because the initial guesses were "too far" from the true values. However, if one had the proper computation facilities, evaluation of this series could be undertaken without too much difficulty. In addition, the estimates derived in this fashion are "true" parameter estimates for the functional relationship (3.6) and therefore this procedure should be the adopted method of parameter estimation, but because of insufficient computational devices available, the proceeding chapter will utilize the first method.

Other procedures have been adopted by Hilhorst'[6] and Dhrymes,[7] based on Solow's earlier investigations. These procedures are similar to that outlined at first, in that all use extraneous functions, in an attempt to finesse the parameter estimates of the C.E.S. Nonetheless, it should be noted that these procedures utilizing extraneous functional relations have statistical aggregation problems, in that as more functions are utilized in estimating the parameters, we find the errors arising with each function estimated.

3.4 Measurement of Technological Factors in the C.E.S. and C.D. Production Functions

In section (1.5) , of chapter 1, we considered certain types of technological change.[8] These technological changes had distinctive affects on output, where the input relation was affected in only one of the two possible technological changes. Neutral technological change, was that type of technical change where no change in the input relation occurred, rather the volume of output change, without changes in the volume of the inputs, and as such the marginal rate of substitution was unaffected. The other change, non neutral, directly affected the productivity necessitates, under our cost minimization assumption, that the input composition will change, to more or less capital using. Measurement of either of these technological changes is one of the more difficult problems in empirical production analysis.

The C.D. production function can be estimated in either of the two forms suggested in the previous chapters, i.e.,

$$Q = \ln Z + vt + A \ln K + B \ln L \qquad \text{(i)}$$

$$Q = \ln Z + A \ln K + B \ln L \qquad \text{(ii)}$$

In case (i) , we have attempted to capture technological components by two variables, Z and vt . Changes in the parameter Z denote neutral change since the rate of substitution of capital for labor is not affected. The parameter v , a trend component, is asserted to be a measure of efficiency, but yielding only another operational means of quantifying neutral changes in technology. In order to utilize this parameter to account for structural changes in production , which can be deemed as neutral changes is to attempt to divide the estimation period. That is, rather than considering a time span of T periods, we should break the time span into sub-periods, which in total are the T periods. Hence we can estimate for k time intervals $(k < T)$, and then for K+1 to K+n , where $K+n \leq T$, so that a comparison of the parameters generated during the first k periods, and those generated during the (K+n) periods can be compared statistically. In order to provide conclusive evidence that a structural change has occurred we can incorporate the use of a variance test on each of the distributions for the latent variables, of each time span; providing the variance test does not suggest that different structures have generated the latent variables, we can suggest that no change has occurred at least for the neutral components. However if we discover that the structures are statistically insignificant, yet the parameter values of L and K , are significantly different for the two time intervals then there is reason to believe that an embodied change, such as quality changes in one or both of the inputs, has occurred. However, to distinguish whether it is capital or labor inclined can only be suggested by changes in the magnitude of the respective parameters, which has obvious problems of interpretation. For the following empirical work, estimates of both C.D. forms will be undertaken, however, due to insufficient data, the changes in the estimation span will be for $k=5$, $k+1 \ldots\ldots k+b$, so that large structural changes will not be expected a priori.

The technological components of the C.E.S. are similar to those suggested in relations (i) and (ii). The identification of the non neutral and the neutral technological changes are somewhat more obvious. That is, if we use the relation e^{ft} of output per labor unit as a function of the marginal product of labor, plus a time shift component (e^{vt}), then as long as v is significant, we can suggest that changes occurring have been of the neutral type. Another plausible approach is to estimate a trend component in equation (3.8), and then use its estimates in equation (3.9) i.e.,

$$\ln(r_{et}/W_{Lt}) = \ln\left(\frac{A}{1-A}\right) + (1+a)\ln\left(\frac{L}{K}\right) + \ln(e^{ft}) + \ln u_t \qquad (3.11)$$

where estimates of f, $\hat{f}$, are then substituted into equation (3.9) as follows.

$$\ln Q = \ln Z + vt - \frac{b}{\hat{a}} \ln\left[e^{\hat{f}t}\left\{\hat{A}K^{-\hat{e}} + 1-\hat{A})\,L^{-\hat{a}}\right\}\right] \qquad (3.12)$$

This new component reflects changes occurring because of changes in the relative factor prices over time. As such, the inclusion of the second trend component will account for obsolescence since all capital data have been derived by the vintage approach, which yields net stock figures. Consequently, since a net stock measure is employed, it is possible for us to capture measures of obsolescence which are part of the depreciation component. Now if the degree of obsolescence is negligible then we would expect measures of $\hat{f}$ to be insignificant, however for values of $\hat{f}$ which are statistically significant we can readily realize that $\hat{f}$ must account for some measure of technological change, specifically non neutral change since it would suggest that there exists an influential component of time, which does have considerable impact on the price ratio of the inputs over time. That is, since we are not regressing output on these variables, then we cannot expect a neutral component to have had any effect on the relative prices, however a non neutral change would most definitely have an influence on the relative prices. As such, when equation (3.12) is fitted, we have already explicitly accounted for non neutral changes via $\hat{f}$, and therefore the measure vt, should produce a reasonable indicator of the neutral technological change which may have occurred over the period under consideration.

The estimation procedure for the C.E.S. will also be attempted for various sub periods, since although measures of v and f may provide measures of technological components <u>over</u> time, we cannot determine the points in time where these components (if any) have become influential, i.e., at what <u>point</u>, if any, has any significant technological affect occurred, during the interval under consideration, this in turn is captured by the Z component.

In summary, we can see that the C.E.S. model seems to provide more recognizable measures of the technological components than does the C.D. model. That is, the C.E.S. can measure those technological characteristics included in the C.D. plus the obsolescence component which renders estimates of v which have been determined "free" of the non neutral components. With this framework at hand, let us now proceed to the empirical analysis of aggregate production in British Columbia within the context of the C.D. and C.E.S. models.

N O T E S

Chapter 3

1. Cramer (1968), p. 271

2. Mc Carthy (1965), p. 74

3. Klein (1968), p. 33

4. Hartley, H.O., "The Modified Gauss-Newton Method for the Fitting of Non Linear Regression Functions by Least Squares", Technometrics, (1961)

5. Draper and Klingman, Mathematical Analysis, Harper and Row, 1967

6. Leibenstein, H., "The Proportionality Controversy and the Theory of Production, Quarterly Journal of Economics, 1955

7. Dhyrmes (1968), pp. 357-363

8. The section drqws heavily on Brown (loc. cit.)

CHAPTER 4

AN EMPIRICAL OBSERVATION OF AGGREGATE PRODUCTION FOR BRITISH COLUMBIA

4.1 Introduction

Up to this point our primary concern has been focused on the theoretical aspects of production and estimation. With the theoretical framework established we can now turn to the empirical results for the B.C. economy. As such, the following chapter is orientated to give a comprehensive review of the observed results of the theoretical constructs of previous chapters. The first of the proceeding sections pertains to the decision criteria and data composition, employed in the statistical sections, with emphasis on the effects of certain estimation problems. In the remaining sections we shall examine the mathematical functions, in light of the observed data, from both statistical and economic perspectives, in addition, particular emphasis is given the elasticity and returns to scale components. The concluding section attempts to consolidate the theoretical implications of the observed results in analyzing aggregate production in British Columbia.

4.2 Decision Criteria and the Data

Various estimation runs were completed on both the C.E.S. and C.D. production models. Certain circumstances, particularly the (Q/L) function of the C.D. type, proved to be statistically insignificant but was included for purposes of comparison. The criteria which was used to evaluate the "usefulness" of the parameter estimates is as follows:

(a) In each of the functions estimated, there must be at least two parameters which are statistically significant from zero. This criteria was used in all functions, i.e., both production and extraneous functions. (see chapter 3).

(b) The standard error of each parameter estimated must be small enough to allow a minimum value of 1.860 for the students t .

(c) Depending upon (a) and (b) , measures of goodness of fit (R^2), adjusted for degrees of freedom, Durban Watson, and simple correlation coefficients of the regressors, were then employed. In the statistical tests of the underlying structures, for structural changes, a variance test on each of the respective distributions of the latent variables was used. This test employed the assumption that the distribution of the disturbance terms were normal, and therefore a ratio of their respective variances came within the framework of an F-test with the appropriate number of degrees of freedom (depending on the number of parameters estimated).

In those circumstances where multicollinearity and/or autocorrelation were present, parameter estimates were accepted providing their t statistics were greater than that specified in (a). That is, both of these problems imply that the standard errors of the parameter estimates will be larger than otherwise, yet no bias will be present, id est:

Let $Y = XB + u$

where $Y = TX1$ vector of observations on the regressand

$X = TX(1+K)$ matrix of observations on the regressors

$B = (1+k) X1$ vector of coefficients - to be estimated

$u = TX1$ vector of latent variables

Where estimates of the B vector are generated by least squares methods we know that a consistent estimator of the B vector, B^*, is

$$B^* = (X^1X)^{-1} X^1Y$$

Now if we substitute for Y, we have:

$$B^* = (X^1X)^{-1}X^1 (XB+u)$$

$$= B + (X^1X)^1 u$$

If the problem of multicollinearily is present then we can readily see that the values of $(X^1X)^{-1}$ will be large, since the collinear relationship results in a small value of $|X^1X|$, and so, as $X^1X \rightarrow 0$, i.e., the linear relationship approaches an exact form, then the elements of the adjoint of (X^1X) when divided by $|X^1X|$, as $|X^1X| \rightarrow 0$, suggests these values will be large, however as can be demonstrated, if we take the probability limit of B^*, even when multicollinearity exist, there exists no bias in B^*, that is :

$$\underset{T\rightarrow\infty}{\text{plim}}\ B^* = B + \underset{T\rightarrow\infty}{\text{plim}}\ T^{-1} (X^1X)\ u$$

$$= B + \underset{T\rightarrow 0}{\text{plim}}\ T^{-1} (X^1X)\ \underset{T\rightarrow 0}{\text{plim}}\ T^{-1}\ u$$

where $\underset{T\rightarrow\infty}{\text{plim}}\ T^{-1}\ u = 0$, by assumption, indicating that B^* is a constant estimator of B. However, if we examine the variance covariance matrix, of B^*, we find:

$$V.C._B = \text{plim } T^{-1} (B^*-B) (B^*-B)^1$$

Substituting our orginal estimate of B^* we find expression for the variance covariance matrix of B as:

$$V.C._B = (X^1X)^{-1} X^1 uu^1 X(X^1X)^{-1}$$

Since $uu^1 = \partial^2 I$, of the homoskedasticity assumption of C.L.R.M. we have by definition:

$$V.C._B = \partial^2(X^1X)^{-1}(X^1X)(X^1X)^{-1}$$

$$= \partial^2(X^1X)^{-1}$$

The above relationship depicts the influence of multicollinearity upon the C.L.R.M. Because we use the variance of the residuals as an estimate of ∂^2, we see that as $(X^1X)^{-1}$ increases as result of multicollinearity the respective variances (the diagonal elements of $\partial^2(X^1X)^{-1}$), increase and in consequence reduce the probability of the parameter estimates, B^*, testing statistically significant. The problem of autocorrelation has similar effects, no bias, but the $\partial^2 I$, relation will not hold, since there exists a covariance of the latent variables implying $\partial^2 C$, where C is a matrix with 1's down the diagonal, but non zero values on the off diagonal elements, and as above the influence of this problem effects only the variance covariance matrix of B^*, and not the point estimates. Since neither of these problems affect the point estimates, their impact is not specifically considered. Nevertheless, if a parameter estimate is "almost" statistically significant, attention is then directed to the degree of multicollinearity or autocorrelation.

A final note on the goodness of fit criteria. This component of the decision process was considered only as minimal in importance. This follows from the fact that if multicollinearity is present it is possible to generate large values of R^2 without having any parameters statistically significant from zero. In those cases where R^2 is suggested as a reasonable measure it always means an adjusted R^2, for degrees of freedom, and that parameter tests were statistically significant.

In the following empirical section, data has been obtained from both the Dominion Bureau of Statistics, and the Department of Economics and Statistics in Victoria. For all dollar values, these have been deflated with 1961 being the base period. However, due to the unavailability of provincial figures on employment, at least on an aggregate provincial level, all labor data was adjusted by the national unemployment rates. Also, all agricultural components of output and labor have been netted out of the aggregate measures of output and labor used in the regressions. In the following empirical section the data used is:

Q_2 - Gross provincial product in millions of dollars, deflated at 1961 prices, net of agricultural output.

K_t - Estimated net capital stock figures, deflated at 1961 prices.

L_t - Adjusted labor figures, in hours per week, where the values have been adjusted for employment percentages.

TABLE I :
Parameter Estimates of the Unconstrained Cobb-Douglas Production Function.

Period	lnT	A	B	A+B	v	R^2_A	$\frac{\sum u^2_{s1}}{\sum u^2_{s2}}$	$\frac{\sum_{t=2}^{N}(u_t - u_{t-1})^2}{\sum_{t=1}^{N} u_t^2}$
1959 - 1963	4.214 (1.89)*	1.071 (2.78)	.248 (3.3.)	1.319	.004 (1.12)	.9895	1.56	1.80
1963 - 1969	4.124 (1.67)	1.049 (2.34)	.281 (2.72)	1.330	.007 (1.43)	.9892		1.28
1959 - 1964	4.294 (1.83)	1.061 (2.71)	.267 (3.37)	1.328	.003 (1.04)	.9885	1.59	1.71
1964 - 1969	4.087 (1.71)	1.037 (2.41)	.284 (2.61)	1.321	.002 (1.21)	.9890		1.23
1959 - 1965	4.114 (1.79)	1.083 (2.63)	.239 (3.14)	1.322	.018 (1.34)	.9868	1.67	1.87
1965 - 1969	4.112 (1.67)	1.018 (2.31)	.291 (3.19)	1.309	.004 (1.41)	.9879		1.17
1959 - 1969	4.27 1.71	1.034 (2.63)	.267 (3.19)	1.301	.009 (1.39)	.9892		1.32

* Values in parenthesis are the t-statistics.

r_{wt} - Prime interest rate, of Canadian chartered banks.

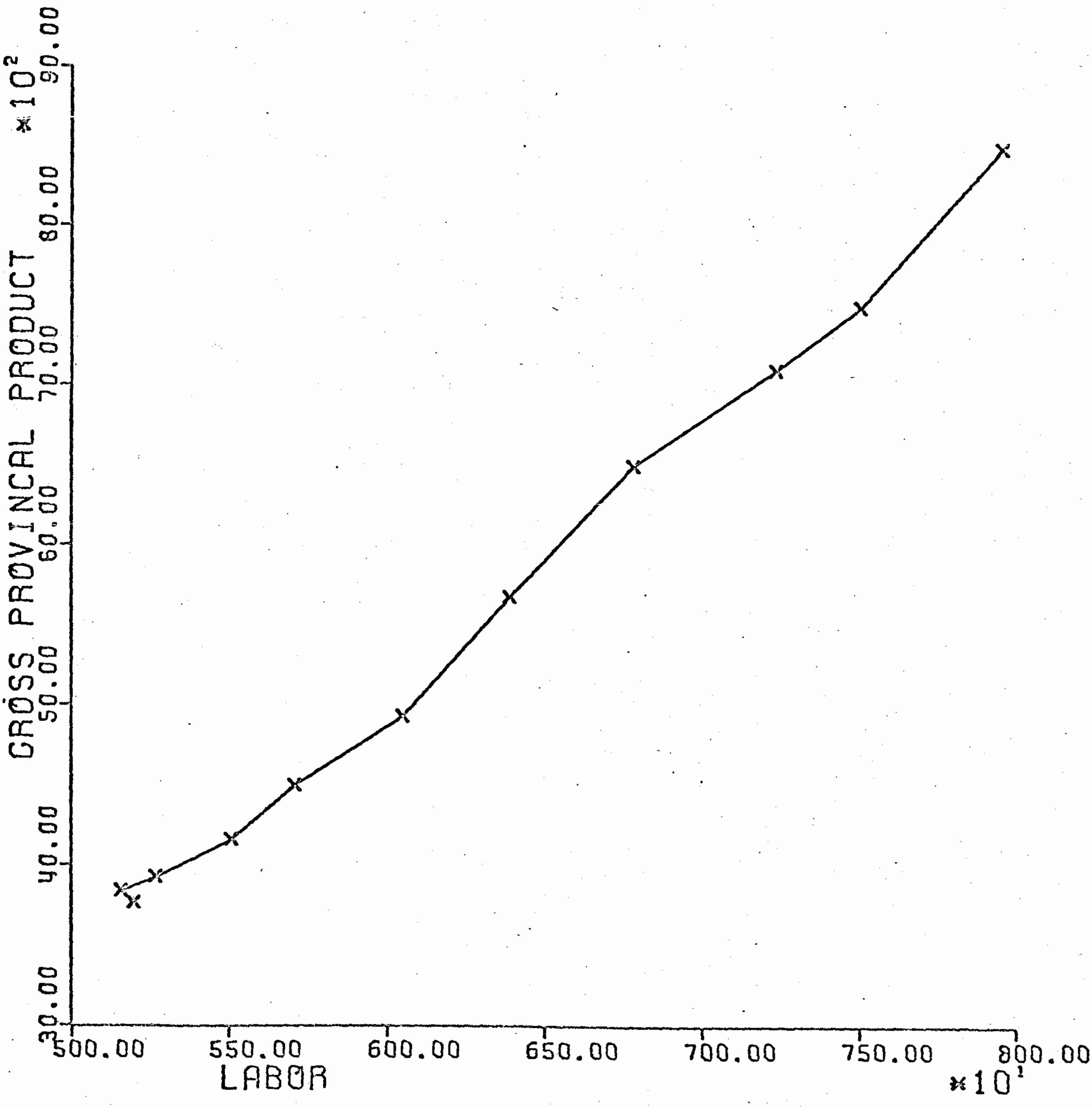
W_{Lt} - Weighted average of weekly earnings, for the adjusted labor figures.

These variables have been transformed and appear in graphs throughout the following pages. However, caution must be taken when examining the computer graphs since the procedure used to order the data vectors does so in such a fashion as to provide equal proportions, for each of the two scales. Also in the cases where aggregate figures are used, most points on the axis are raised to various powers, and should be noted for correct interpretation.

4.3 Empirical Results of the Unconstrained Cobb-Douglas

Table I, represents the outcome of the various regressions performed on the C.D. model. The main point of importance for this type of production model is that the returns to scale, implied by the data, can be easily determined. With the exception of lnT and v, all parameter estimates are significant under the established criteria. Examining the estimates of the capital and labor parameters we can see that the absolute importance of capital is predominant in all periods. Nevertheless, in carefully examing the estimates of A and B, we can see that in the later periods (1966 - 1969), the magnitude of the labor parameter appears to be larger, at least when contrasted to the early period (1959 - 1963). Futhermore, another interesting point to note is the decline in the capital parameter

FIGURE 5


GROSS PROVINCAL PRODUCT *10²
90.00
80.00
70.00
60.00
50.00
40.00
30.00
500.00
550.00
600.00
650.00
700.00
750.00
800.00
LABOR
*10¹

for later periods. Now, although these changes are not statistically significant, we should not put aside possible implication of this occurrence. That is, if we examine the estimate, (A+B) , the returns to scale component, we can see that it maintains a consistency around 1.3 , even with the parameters of labor and capital changing over time, which offers support for the homogenuity assumption of the production function.

The parameters included for the technological elements, were insignificant for all periods. From a statistical perspective, the relevance of lnT and v_s are primarily aids in fitting the data. Namely, v_t will generally aid in reducing autocorrelation in the residuals, while lnT , generally assists in reducing variation about the observed and calculated values of the regressand. From an economic point of view, these parameters are incorporated to capture technology and its influence (if any) on the particular structure under examination. The only interpretation which seems to be somewhat tenable is that neutral changes were of no significant importance, yet because we observe a change in the relative size of the labor and capital parameters we might purport that the technological change was of the non neutral type, somewhat bias to labor. If this is what has been occurring - where changes in the level of output are gradually becoming labor orientated - then we might argue that various components, e.g., education, training, etc., are being embodied in the labor unit, and therefore technological changes may well have been occurring but not of the non neutral type. Moreover, there is some evidence

TABLE II :
Estimates of a Cobb-Douglas with Constant Returns.

Period	lnZ	A	B	A+B	v	R^2A	$\frac{\sum u_{s1}^2}{\sum u_{s2}^2}$	$\frac{\sum_{t=2}^{N}(u_t - u_{t-1})^2}{\sum_{t=1}^{N} u^2}$
1959 - 1963	5.312 (2.13)*	.657 (5.31)	.343	1 1	.006 (3.14)	.9813	1.01	.64
1963 - 1969	5.417 (2.01)	.704 (4.92)	.296	1 .	.009 (2.79)	.9861		1.31
1959 - 1964	5.361 (2.11)	.678 (5.79)	.322	. .	.013 (3.04)	.9826	1.38	.94
1964 - 1969	5.214 (1.91)	.713 (5.31)	.287	. .	.014 (2.99)	.9869		1.04
1959 - 1965	5.351 (2.04)	.681 (5.64)	.319	. .	.012 (3.01)	.9810	1.21	1.01
1965 - 1969	5.283 (1.87)	.741 (5.38)	.259	. .	.011 (3.01)	.9857		1.12
1959 - 1969	5.314 (1.77)	.719 (5.21)	.281	. .	.010 (2.71)	.9881		1.41

* Values in parenthesis are the t-statistic.
B = 1-A

to support this, in that the statistical tests on the variances of the latent variables provided no evidence of structural changes.

Both multicollinearity and autocorrelation were present. Correlation of capital and labor variables was extremely high (.94) for all periods. However, as can be seen in table I, this did not precipitate insignificance in the parameter estimates. Correspondingly, it would appear that the unconstrained C.D. is a most reasonable candidate for an aggregate production model of B.C., however before concluding this let us first examine the remaining aggregate alternatives.

4.4 A Constrained Cobb-Douglas Model

Another set of regressions was performed for the same periods, where constraint $A+B=1$, was imposed on the previous C.D. model, the results appear in table II. As we can readily note, all parameter estimates have changes not only in magnitude, but also with respect to statistical significance. For the neutral technological components it is now implied that there were significant influences attributable to exogenous sources, not directly related to the inputs. This change in itself is not that striking, yet when consideration is given to the remaining estimates, we find that other changes which have occurred make these empirical results somewhat unacceptable. Parameter estimates of capital, although plausible, have been reduced in magnitude, at least with respect to the unconstrained model. The only explanation for the large change in the statistical results

TABLE III:
Estimates of the Factor Price Equation for the C.E.S.

Period	$\ln \frac{A}{1-A}$	a	1+a	$E_s = \frac{1}{1+a}$	f	R^2_A	$\frac{\sum u^2_{s1}}{\sum u^2_{s2}}$	$\frac{\sum_{t=2}^{N}(u_t - u_{t-1})^2}{\sum_{t=1}^{N} u^2}$
1959 - 1963	-.407 (7.14)	2.261 (1.94)	3.261	.361	.031 (1.66)	.912	2.14	.986
1963 - 1969	-.421 (7.32)	2.279 (1.86)	3.279	.343	.013 (1.71)	.872		.914
1959 - 1964	-.402 (7.29)	2.267 (1.96)	3.267	.330	.018 (1.43)	.908	2.19	.968
1964 - 1969	-.396 (7.31)	2.281 (1.91)	3.281	.321	.017 (1.69)	.871		.938
1959 - 1965	-.413 (7.23)	2.262 (1.89)	3.262	.359	.028 (1.61)	.901	2.41	.976
1965 - 1969	-.398 (7.27)	2.282 (1.93)	3.282	.320	.073 (1.72)	.875		.937
1959 - 1969	-.401 (7.39)	2.279 (1.88)	3.279	.343	.009 (1.31)	.887		1.014

* Values in parenthesis are the t-statistic.

is that in the determination of the capital parameter we excluded one regressor, namely labor. In doing so we have reduced the amount of variation explained, in output. Consequently this explanation has been shifted to the other regressors - the technological components. From a statistical viewpoint, when consideration is given to the changes occurring in the statistical estimates, we may suggest that this particular model is of little empirical or economic significance.

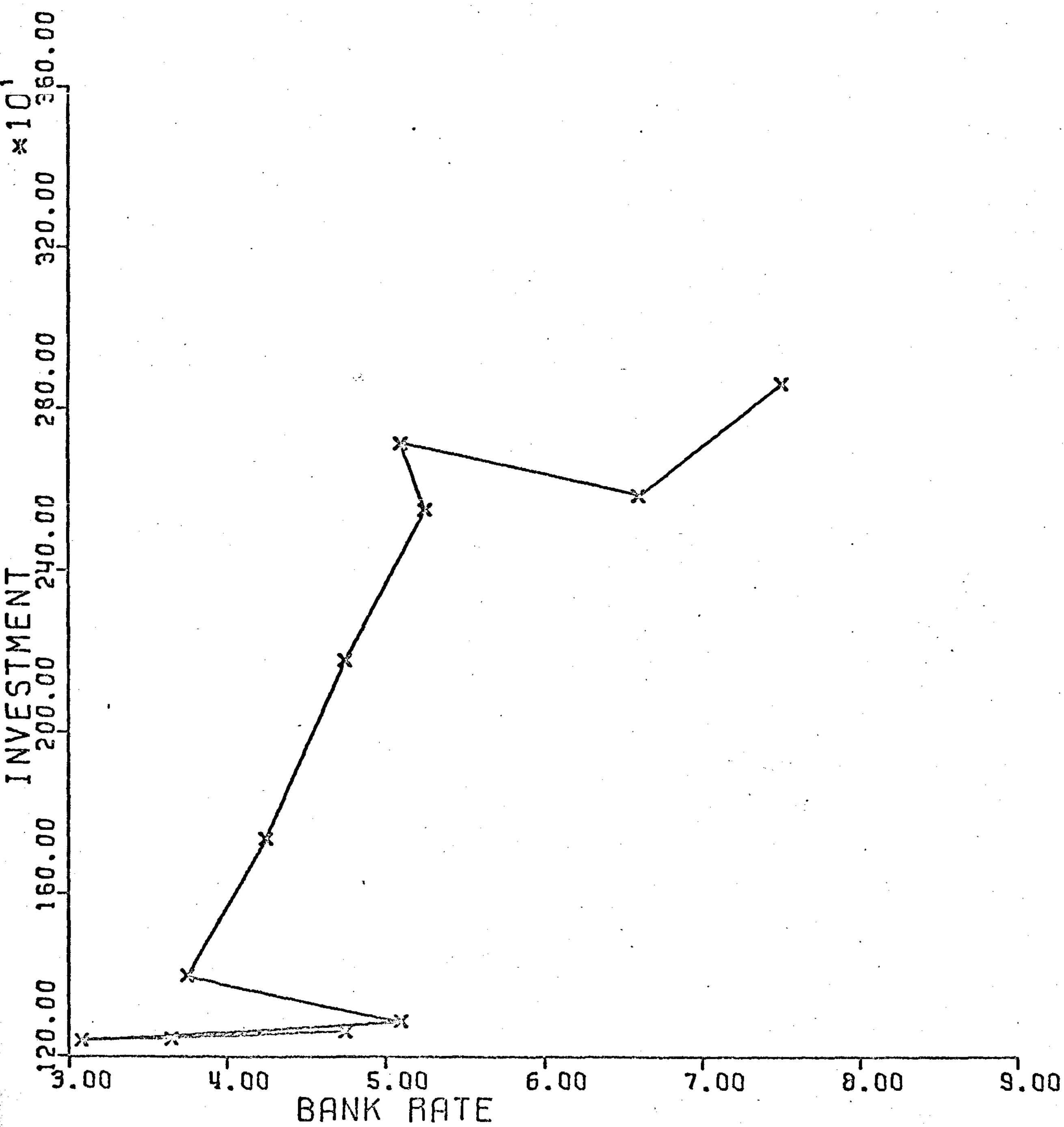
Although, this unique formulation of the C.D. was offered as a "computationally" better procedure, by alleviating problems of multicollinearity it would appear that the advantages, at least in the present context, were at the expense of relatively good estimates under the adopted criteria. Consequently, further discussion will exclude this particular formulation of the C.D. model.

4.5 The Factor Price Equation of the C.E.S. Model

In chapter 3, it was proposed that a feasible solution to estimating the parameters of the nonlinear C.E.S. model was to incorporate the use of an extraneous equation. Although other procedures were advanced at this time, the choice of the factor price relation was the result of computational limitations. Using the data outlined in section 4.2 , on capital and labor prices and the assumption of partial cost minimization equation (3.8) was estimated; the results of which are listed in table III.

As can be seen, with the exception of f , all parameter estimates

FIGURE 6



INVESTMENT
*10¹
120.00
160.00
200.00
240.00
280.00
320.00
360.00
3.00
4.00
5.00
6.00
7.00
8.00
9.00
BANK RATE

are significant at the established acceptance level. From the estimates of $\ln\frac{1-A}{A}$, in the first column, the approximate magnitude of the capital intensity parameter can be derived. For all periods the value of $\hat{A}$ is in the neighborhood of .697 (based on absolute values of $\ln\frac{1-A}{A}$), suggesting as in the C.D. model, that capital plays the major part, with respect to the two inputs, in determining the level of output. Nevertheless, as should be noted this conclusion in the present context is considerably different, than that of the previous model. That is, upon examining the estimated values for the elasticity of substitution it is observed that there occurs a consistency around .34 markedly different from the implicit unitary value inherent to the C.D. model. Measures of $f(\ln e^{ft})$, the non neutral technological parameters, appeared insignificant for all periods, similar to those suggestions of the C.D. model.[1] Likewise, as in the previous estimates, there does not appear to be any structural change occuring, for the periods under observation.

Before moving to our prime objective, vis a vis the factor price equation - the C.E.S. - it should be noted that the use of this relation is manifested in many of the important parameter estimates to which this study is devoted. That is, not only are we able to derive an estimate of the C.E.S. structure in the context of production analysis, but it is a means whereby estimation of other parameters - such as the non neutral element - can be readily obtained. Let us now turn to the derived estimate of the structural C.E.S., utilizing $\hat{A}$, $\hat{a}$, excluding $\hat{f}$, for the

TABLE IV :
Estimates of the C.E.S. Parameters.

Period	lnZ	v	$\frac{b}{a}$	b	R^2A	$\frac{\sum u^2_{s1}}{\sum u^2_{s2}}$	$\frac{\sum_{t=2}^{N}(u-u_{t-1})^2}{\sum_{t=1}^{N} u^2}$
1959 - 1963	1.874 (4.31)	.0021 (10.12)	.596 (3.14)	1.33	.9832	3.68	1.54
1963 - 1969	1.714 (4.39)	.0017 (9.71)	.611 (3.74)	1.39	.9835		1.57
1959 - 1964	1.831 (4.71)	.0026 (11.14)	.635 (3.31)	1.44	.9837	4.33	1.60
1964 - 1969	1.701 (4.81)	.0019 (10.16)	.597 (3.61)	1.37	.9839		1.64
1959 - 1965	1.819 (4.61)	.0026 (11.14)	.599 (3.38)	1.36	.9821	3.91	1.38
1965 - 1969	1.689 (4.83)	.0018 (10.93)	.616 (3.59)	1.41	.9861		1.78
1959 - 1969	1.718 (4.67)	.0024 (11.41)	.617 (3.48)	1.40	.9842		1.68

* Values in parenthesis are the t-statistic.

above mentioned reason.

4.6 The Structural C.E.S.: A Derived Estimate

Incorporating the above information, equation (3.9) was estimated, the outcome of which can be examined in table IV. The first and perhaps most notable change, with respect to the previous equations, is the level of significance for each of the technological components. The returns to scale factor, although a derived measure, yields an estimate very much consistent with the previous C.D. model. That is, in all periods there was a definite trend around 1.38, although higher, approximately in the neighborhood of the C.D. estimates.

Upon close inspection there does appear one major occurrence, not consistent with previous estimates. Namely, the variance test on the latent variables. Although not statistically significant its proximity to the acceptance level was so close, that one may suggest that there was a reasonable degree of difference in the variance of the latent variable for each period, this may have resulted in the large degree of significance with respect to the technological components. Another reason for the large values on the variance tests, may be a result of the use of the derived estimates of the factor price equation, in the sense of compounding errors of estimation.[2] In all periods, autocorrelation was present, which for previously suggested reasons, may also have had some influence on these ratios. In all, the estimates generated in the C.E.S. were good, at least in a statistical context, as for the economic interpretation, let us draw upon the previous results to establish a basis of interpretation.

4.7 Summary and Conclusions

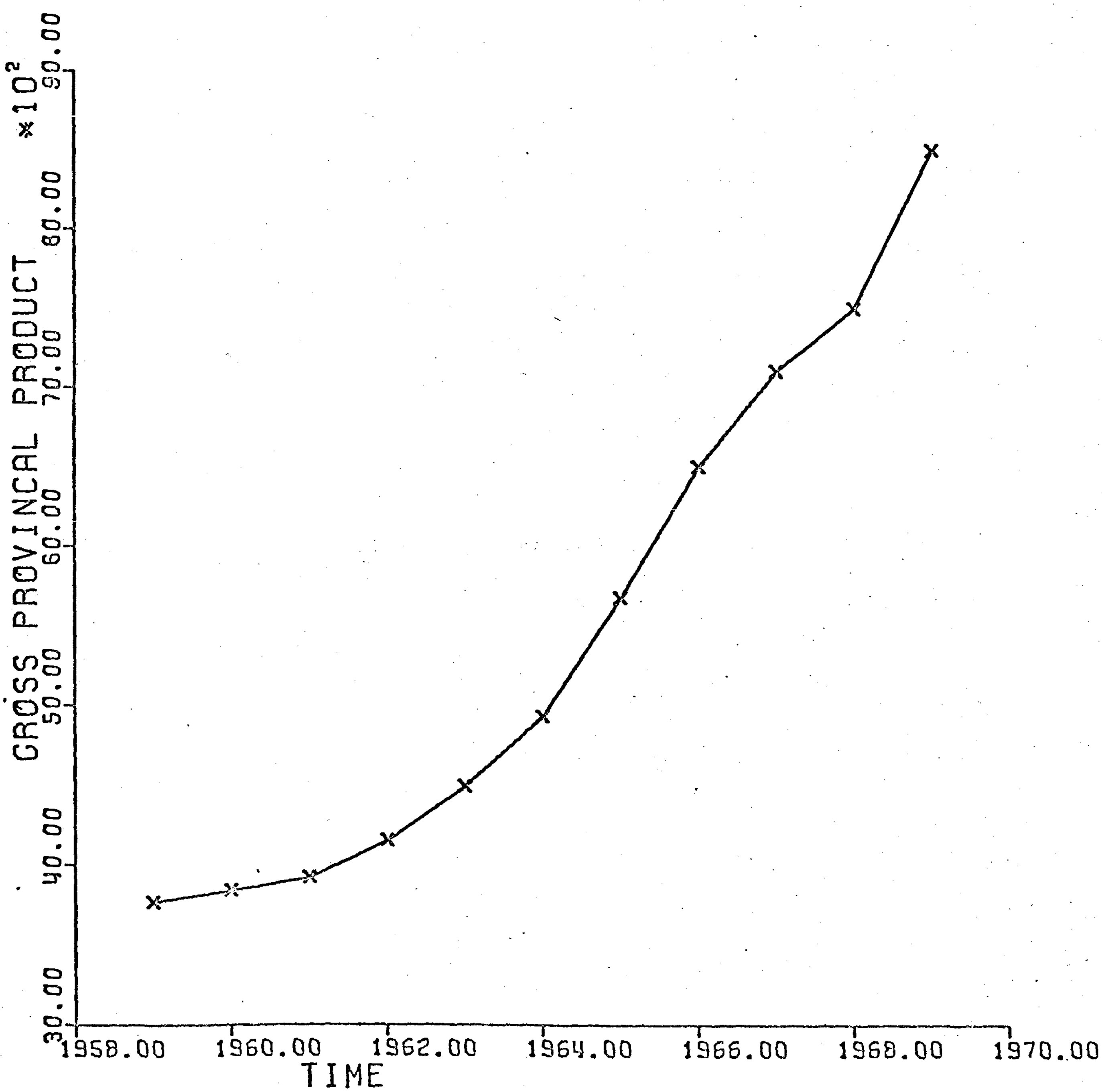
Throughout the preceding sections all discussion has been focused on relating theoretical considerations to an observed occurrence. In chapters one and two, attention was directed at the primary components of production and capital theory. Chapter one outlined the framework, of the Neo-Classical school, with respect to production. In particular, the Cobb-Douglas and Constant Elasticity models were examined in lieu of certain criteria imposed by the Neo-Classics. At this time it was suggested that one of the major aspects of production theory was to suggest how inputs could be substituted for each other concommitant with the returns to scale, suggested by empirical analysis. In chapter two, a brief outline of the Neo-Classical considerations with respect to capital was given, as well as other theories also relevant to capital accumulation. Moreover, it was also noted that at present, there existed no capital data and as a consequence the vintage model was utilized to alleviate this problem. Hence with the theory of production and capital well formulated attention was then directed to econometric procedures, whereby the previous considerations could be integrated and tested empirically. With this established, concentration was then focused on the aggregate case at hand: British Columbia production.

Here, in the present chapter the previous sections have been concerned with reporting the empirical findings, of the relevant theoretical constructs established in earlier chapters. In each observation on the particular production model, close examination was given to the parameter estimates from both a statistical and economic point of view. However, from each of these a decision of which production model seems to be the "most" applicable, theoretically and empirically, does not appear possible.

That is, in the first estimate of the Cobb-Douglas it was found that both capital and labor variables were statistically significant, under the established criteria. As for the estimates of the Constant Elasticity formulation similar results were suggested. Moreover, the relative magnitude of each of the input parameters, were also similar, in that the Cobb-Douglas model advanced that the capital input was of greater relative importance in affecting the particular value of output. Similarly, the latter case also proposed the relative predominance of capital in the production process. Another feature of striking similarity was the respective estimates of the returns to scale parameter. In both models an increasing returns value was implied by their empirical results.

It was also noted that certain differences arose, and yet the above similarities existed. Namely, in the Cobb-Douglas formulation as was suggested in chapter one, implicitly incorporates an assumption that the elasticity of substitution for the factors is unity, throughout time. On the other hand, although assumed constant, the Constant Elasticity model allows us to estimate a value of the elasticity of substitution, which need not

FIGURE 7


GROSS PROVINCAL PRODUCT *10²
90.00
80.00
70.00
60.00
50.00
40.00
30.00
1958.00
1960.00
1962.00
1964.00
1966.00
1968.00
1970.00
TIME

be unity. Nevertheless, when these two models are estimated the latter yields an estimate of the elasticity. In the case at hand it was found that empirically it was considerably less than unity (.36) for all periods. Other striking differences arose as well, the first of these was the statistical significance of the technological components of the Constant Elasticity formulation, where similar elements tested insignificant in the Cobb-Douglas function. The last discrepancy between the two models was the suggestive change in the structures in one case (almost) whereas there was definitely no change in the other, at least under the F-test adopted.

In conclusion it would seem that those empirical results which can be concluded are: that during the period under examination (1959-1969) the aggregate production was more responsive to the capital element than labor, and : during the period considered, aggregate production definitely enjoyed increasing returns to scale. These results may not appear too important, yet when these and the above results (elasticity of substitution) are compared to other empirical studies we find there exists a definite similarity. That is, Fergeson, Dhrymes, etc., all found relatively low elasticities of substitution. As for the measures on the returns to scale, these results are also consistent with other empirical production studies. In consequence, it can be purported that the present analysis has successfully served to indicate how the major components of Neo-Classical production theory occur in the present context. It also provides two models from which a forecasting procedure can be adopted which may provide a definite means whereby decisions regarding the applicability of these models could more obviously be ascertained.

NOTES

Chapter 4

1. In our previous discussion of the C.D., it was mentioned that although no statistical difference was suggested, there was a definite change in the magnitude of A and B from period to period.

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