

INFLATION, EXPECTATIONS AND
NOMINAL RATES OF INTEREST:
AN EXAMINATION OF THE
FISHER EFFECT, 1955-1978

by

G. Edward Anderson

B.A.(Hon.) Simon Fraser University, 1977

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF ARTS
in the Department
of
Economics and Commerce



G. Edward Anderson 1979

SIMON FRASER UNIVERSITY

June 1979

All rights reserved. This work may not be reproduced in whole or in part, by photocopy or other means, without permission of the author.

APPROVAL

Name: George Edward Anderson

Degree: Master of Arts

Title of Thesis: Inflation, Expectations and Canadian Nominal Rates
of Interest: An Examination of the Fisher
Effect, 1955-1978

Examining Committee:

Chairperson: K. Okuda

Dennis R. Maki
Senior Supervisor

S/T. Easton

Robert Brown
External Examiner
Dean of Arts

Date Approved: June 28, 1979

PARTIAL COPYRIGHT LICENSE

I hereby grant to Simon Fraser University the right to lend my thesis or dissertation (the title of which is shown below) to users of the Simon Fraser University Library, and to make partial or single copies only for such users or in response to a request from the library of any other university, or other educational institution, on its own behalf or for one of its users. I further agree that permission for multiple copying of this thesis for scholarly purposes may be granted by me or the Dean of Graduate Studies. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Title of Thesis/Dissertation:

Inflation, Expectations and Canadian Nominal Rates of Interest:
An Examination of the Fisher Effect, 1955-1978

Author: _____

(signature)

George Edward Anderson

(name)

79/06/28

(date)

Abstract

This paper examines the relationship between expected future rates of inflation and movements in nominal rates of interest, known as the Fisher Effect. The interest rates selected for the study were six monthly Canadian nominal bond series, spanning a period of twenty-three years, 1955-1978. The sample of bond rates was chosen so as to include two short term Treasury Bill rates, two longer term averages of government bond rates, and two private Finance Company bond rates.

For the purposes of this study, the "real" rate of interest, determined by equilibrium conditions in the capital, goods and bonds markets, was taken to be a constant over the period examined. Empirical estimates of anticipated inflation were generated via two theories, "adaptive expectations" and "rational expectations", and regressed upon the above-mentioned nominal bond rates. Also included in the regressions was a constant term, which, under the assumptions outlined, served as an estimate of the "real" component of the bond rate under consideration.

The coefficient associated with each estimate of anticipated inflation was of major interest to the study, since, in theory, its value should always fall between zero (complete rejection of the Fisher Effect hypothesis), and one (perfect compensation for expected inflation, as Fisher postulated). In

fact, values of greater than one were found on occasion, but after adjusting the model slightly to account for taxation effects, in no case was the coefficient found to be significantly different from one.

After testing the selected rates of interest under the very different assumptions of the "adaptive" and "rational" expectations hypothesis statistical evidence, obtained via the Box-Jenkins procedure, was presented as a means of showing why both theories should be expected to yield similar predictions with respect to future inflation. A demonstration was made showing how the estimated autoregressive structure underlying Canadian rates of inflation renders the theories indistinguishable under reasonable assumptions about the economy's functioning.

The overall conclusion arising from the study is that the Fisher Effect hypothesis appears valid from the sample of Canadian nominal bond rates examined.

Acknowledgment

I would like to thank Lillian Prosegger, whose patience and help in the editing and typing of this thesis contributed greatly to its completion.

Table of Contents

Approval.....	ii
Abstract.....	iii
Acknowledgment.....	v
List of Tables.	viii
A. Introduction.....	1
B. The Data.....	11
C. The Fisher Effect Under the Adaptive Expectations Approach to Expectations Formation.....	14
I. The "Single Variable" Model.....	14
I.1.1 The Optimal Time Horizon of Past Price Level Changes.....	20
I.1.2 Almon Lags and Canadian Interest Rates.....	25
I.2 Empirical Results of Testing Canadian Nominal Interest Rates Under the Modified Y and K Assumptions.....	32
I.3 Indirect Almon Lags and Nominal Interest Rates....	37
I.4 Empirical Tests of the Multivariable Adaptive Expectations Model.....	41
I.5 Summary of Part I.....	51
D. The Fisher Effect Under Rational Expectations Assumptions..	57
II.1 Introduction.....	57
II.2 The Rational Expectations Model.....	61
II.3 Empirical Estimates as to the Behavior of Monetary and Real Growth Rates Over The Period 1955 - 1978.....	71
II.4 Empirical Tests of the Fisher Effect Under Rational Expectations Assumptions: The Mussa Model.....	78

II.5 An Alternate Rational Expectations Model.....	92
II.6 A Rational Expectations Approach to Explaining the Historical Success of Adaptive Expectations.....	97
II.7 Summary of Part II.....	105
E. Conclusions.....	110
Appendix: Charts.....	117
Bibliography.....	132

List of Tables

Table I : $RN_t = f(\text{Almon } P_t, \dots P_{t-47})$	30
Table II	34
Table III : $RN_t = a + bP^*t$	38
Table IV : $RN_t = a + g_1 M_3t + \text{Almon}(P_t, \dots P_{t-n})$	46
Table V : $RN_t = a + g_1 m_3^*t + g_2 M_3^*t-1 + \text{Almon}(P_t \dots P_{t-n})$	47
Table VI : $RN_t = a + g_1 M_1^*t + g_2 M_1^*t-1 + \text{Almon}(P_t \dots P_{t-n})$	48
Table VII : RN_t Subjected to Box-Jenkins Analysis.....	54
Table VIII : Autoregressive Structures, Monetary.....	73
Table IX : Autoregressive Structures, Growth Indices.....	74
Table X : OLSQ: $CPI_t = a + b(M - \text{Growth})$	79
Table XI : CORC: $CPI_t = a + (M - \text{Growth})$	81
Table XII : OLSQ: $RN_t = a + bP^*t + et$	86
Table XIII : OLSQ: $RN_t = a + bP^*t + et$	87
Table XIV : CORC: $a + bP^*t + et$	89
Table XV : CORC: $RN_t = a + bP^*t + et$	90
Table XVI : CORC(2): $RN_t = a + bCPI_t + et$	96
Table XVII : Autoregressive Structure, C.P.I.....	100
Table XVIII : CORC: $RN_t = a + b(CPI_{HAT}) + et$	104
Table XIX : Summary of "a" Estimates.....	112
Table XX : Summary of "b" Estimates.....	113

A. Introduction

Through much of the twentieth century, the Fisher Effect has been regarded as a theoretically logical, but empirically unimportant determinant of nominal bond rates in North America. Indeed, during long periods of relative price stability, such as the 1950's and early 1960's, the consequences and predictions arising from Fisher's theory were all but forgotten by government and private sector lenders and borrowers. The experience of the 1970's, however -- specifically prolonged inflation accompanied by the expectation throughout the economy of continued price level increases -- has changed the view that the Fisher Effect may be disregarded. In fact, it has become an important consideration to anyone currently involved in forecasting or analyzing interest rate movements.

According to Fisher, the nominal interest rate associated with a bond should conceptually be divisible into two distinct components; a "real" component, which he believed would be basically stable with respect to time, and an inflation/deflation premium to compensate the holder for a rise/fall in the real value of the asset during its period to maturity.

In the case of a government bond, for example, the "real" component of the nominal interest rate would, in equilibrium,

reflect the opportunity cost of (i.e. the foregone return on) investment in the capital market, adjusting for period to maturity, risk factors, and so on. During a period of expected price stability, said return would leave the investor indifferent to buying a capital asset or bond. If the real return to the capital asset were expected to rise, however, due to an increase in the money value of its output relative to the original price of capital purchase, as in the case of an inflationary period, the investor would no longer be indifferent to the capital versus bond alternative. Equilibrium between the two markets would only be restored when return to the bond again matched the opportunity cost of investing in the capital market - in other words, if a premium were paid to the bond holder, over and above the real rate, equal to the expected price level rise during the period to maturity of the instrument.

In one sentence, then, the Fisher Effect, at the theoretical level, postulates a perfect positive relationship between expected future rates of inflation/deflation and one component of nominal interest rates.

The purpose of this paper will be to statistically examine the influence of the Fisher Effect over a recent period, 1955 - 1978, with respect to Canadian nominal rates of interest. The period 1955-1978 was selected for two reasons; first, data for this period was readily available for several bond series, but second, and more importantly, such a time span captures a broad

range of inflationary experience. Since this paper deals primarily with the inflationary expectations component of nominal bond rates, a variety of actual observed inflation rates would seem a major criterium in the selection of sample data.

Perhaps the most difficult issue encountered when undertaking a study of the Fisher Effect is the choice of method one should employ when attempting to statistically generate the inflationary expectations of bond holders. Such a decision rests upon the nature of expectations formations, and the process whereby expectations are formed is far from clear. In fact, the two leading hypotheses put forward regarding expectations formation differ dramatically in approach.

The first, which has come to be known as the "adaptive expectations hypothesis", holds that future beliefs regarding inflation result from an extrapolation process with respect to the behavior of a key economic variable, or variables. The bond holder, it is postulated, believes at a given time "t", that inflation in period "t+1" will follow some trend established in periods "t", "t-1", "t-2" and so on. If the actual rate of inflation in period "t+1" differs from that anticipated, perception of the "trend" is altered in the bond holder's mind, leading to a change in expectations for the next, future period.

In basic terms, then, a bond holder is assumed to concentrate upon information gathered from the present value and past behavior of a critical economic variable, over some time

frame, when generating inflationary expectations. Not surprisingly, such a variable commonly cited is the observed rate of inflation itself. The behavior of additional variables over time might also be of interest to a bond holder under the adaptive expectations assumptions, provided that series contained information which would normally be incorporated into the behavior of the inflation rate, but, perhaps due to the existence of lags, is not.

The second approach to expectations formation has been termed "rational expectations". This hypothesis revolves about the notion that a rational bond holder, at time period "t", will wish to use all relevant information about the future which is available, when predicting the rate of inflation in time period "t+1". If the bond holder perceives some relationship between present behavior of a variable, and future changes in the rate of inflation, it is assumed his estimate of future inflation would incorporate said knowledge. It is important to recognize that such information may or may not be transmitted directly via the past behavior of a key variable. As such, the past behavior of the inflation rate might be of remote interest to a bond holder under rational expectations, in marked contrast to the adaptive expectations theory.

The approach suggested by each hypothesis was deemed different enough that a decision was made to test the Fisher Effect under each set of assumptions separately. As a result,

this paper is divided into two parts, where similar tests are carried out. As will be seen, however, the findings differ dramatically depending upon the expectations component employed.

Before detailing the equations used to simulate each approach, one important point should be made. Throughout this paper it will be assumed, for simplicity, that the real component of nominal bond rates has remained stable over time, as Fisher hypothesized.¹ A rise or fall in nominal bond rates will therefore be assumed to be in response to changes in the second component, inflationary/deflationary expectations. This assumption allows the constant term in a regression equation to act as an estimate of the real rate of return to a given bond, as will be seen, and would not seem to distort the models to any major extent. Nonetheless, it should be recognized that constraining the real rate of return to remain constant over time could introduce a possible source of error into the

¹ There have been, in fact, good arguments made as to why the real rate of interest might be expected to fall during an inflationary period. Robert Mundell, for example, (in Monetary Theory: Inflation, Interest and Growth in the World Economy. Goodyear Publishing Company, Santa Monica, California, 1971. pp. 14-22), shows that a fall in desired holdings of real cash balances during an inflationary period will cause the real rate of interest to decline.

Also, empirical estimation of the stability property of the real rate of interest has been undertaken. See, for example, Eugene Fama, Short Term Interest Rates as Predictors of Inflation, American Economic Review, June 1975, Vol 60, #3. P. 269.

results.²

Mathematically, the Fisher Effect may be written as follows:

$$(1) \quad R_{Nt} = R_{Rt} + P^*t$$

where R_{Nt} = nominal rate of return to the bond at time "t"

R_{Rt} = real rate component associated with bond at time "t"

P^*t = expected rate of inflation at time "t"

² Specifically, such an error in measurement will result in a biased estimator "b", as follows, where "B" is the true estimate:

$$b = \frac{B (\text{var } P^*t) + \text{cov} (v, P^*t)}{\text{var} (P^*t) + \text{var} (u)}$$

where P^*t = expected rate of inflation as of time "t"
 var = variance associated with
 cov = covariance associated with
 v = measurement error associated with "RRt"
 u = measurement error associated with "P*t"

The problem comes when " $\text{cov} (v, P^*t)$ ", assumed to be negative, plays a significant factor. The assumption in this paper is that such a term is very small, but it should be noted that a source of downward bias is present.

Both expectations theories mentioned attempt to explain how values of P^*t are generated. Since the first hypothesis - adaptive expectations - posits a form of error learning, or extrapolation of perceived trends in a critical series or set of series, an equation based upon (2) will be used in the first part of this study. It is assumed that the observed rates of present and past inflation act as the key variable when inflationary trends are being generated.

$$(2) P^*t = f(Pt, Pt-1, Pt-2, \dots, Pt-n; Ot)$$

Pt = actual rate of inflation in percentage change between period "t-1" and "t"

Ot = other factors than price levels entering formation of expectations adaptively at time "t", based upon information from time series in addition to past inflation

Equation (2) states that expectations of future inflation, as of time "t", are generated via two means - extrapolation of trends derived solely from past price level changes encompassing some period "t-n", and through consideration of additional factors which should affect present inflation rates, "Ot", but do not due to lags in the economy of some known length.

Assuming a constant real rate component of nominal bond interest rates (R_{Rt}), substitution of equation (2) into (1) yields:

$$(3) \quad R_{Nt} = a + f(P_t, P_{t-1}, P_{t-2}, \dots, P_{t-n}; O_t)$$

which is the general form of the equation statistically tested when adaptive expectations assumptions were employed.

Not surprisingly, the rational expectations hypothesis takes a very different mathematical form. Rather than relying upon past and present values of key variables, rational expectations postulates the inclusion of all relevant information about the future which is available at time "t".

The rational expectations model we will employ may be written as follows:

$$P^*t = (1/v+1) \sum_{j=0}^{\infty} E_t [(M_{t+j+1} - M_{t+j}) - (Y_{t+j+1} - Y_{t+j})] * (v/v+1) \exp j$$

where P^*t = expected future rate on inflation in periods $t+1, t+2, \dots$, as of period "t"

v = Partial income elasticity with respect to interest

M = Natural log of money supply M_1

Y = Natural log of real output, measured

by real G.N.P.

Although the above expression might appear complex, it states simply that the expectation of inflation in the period "t+1" depends upon expected weighted increases in the money supply, over and above real growth in all future periods "j", which are being examined.

The paper is divided into two central Parts, with a Data section preceding, and a Conclusions section following, the main presentation of the study's findings in Parts I and II. The format followed in each of the two Parts is identical. First, a published American model of expectations formation is reviewed, and the model's major assumptions examined. Secondly, each model is tested for Canadian data. Next, an alternative model suggested by the initial study is presented and tested for Canadian data. Finally, another model, consistent with the assumptions of each Part, but differing significantly from the major model's structure, will be detailed and tested using Canadian data.

Part I of this study tests the Fisher Effect under the assumptions of Adaptive Expectations. The specific model detailed and tested is based upon a 1969 study by William Yohe and Dennis Karnosky, published in the St. Louis Federal Reserve

Bulletin.³ The model, (and as such, most of Part I) is "single variable" in nature, as defined earlier -- i.e. expectations formation is solely dependent upon the behavior of a single variable over time. Tests of a "multivariable adaptive expectations" model will also be presented in Part I.

Part II of the study is centered upon approaches to expectations formation suggested by the Rational Expectations hypothesis. The first model set out and tested in this part follows from a paper by Michael Mussa, published in the Journal of Monetary Economics, in 1975.⁴ Also included in Part II is an attempt, using Rational Expectations assumptions, to explain the success of earlier models employing the hypothesis of Adaptive Expectations. As will be shown, at least in the case of Canada, both theories are compatible to a surprising degree.

As mentioned, the last section of the paper contains a brief set of conclusions, arising from the study.

³ Yohe, W and Karnosky, D. Interest Rates and Price Level Changes, 1952-69. Federal Reserve of Bank of St. Louis Review, 1969. P. 18.

⁴ Mussa, Michael. Adaptive and Regressive Expectations in a Rational Model of the Inflationary Process. Journal of Monetary Economics, Vol. I. 1975.

B. The Data

Six series of nominal interest rates were examined over the period studied; two Treasury Bill rates, two government bond rates, and two Finance Company Paper rates. The short term interest rates used, and the periods of data availability were :

- 1) 30 day Finance Company Paper rate;
1961 - 1978, Cansim # = B 140395
- 2) 90 day Finance Company Paper rate;
1957 - 1978, Cansim # = B 14017
- ✓3) 3 month treasury bill rate;
1955 - 1978, Cansim # = B 14007
- ✓4) 6 month treasury bill rate;
1955 - 1978, Cansim # = B 14008

The long term bond rates were :

5 "Cansim numbers" are the codes associated with data series from Statistics Canada, available for access with computer programs such as "Massager 1973".

✓ 5) 1-3 year government bonds
average interest rate; 1955-1978,
Cansim # = B 14009

✓ 6) 3-5 year government bonds
average interest rate; 1955-1978,
Cansim # = B 14010

All data were in monthly form. Monthly changes in the Consumer Price Index (not seasonally adjusted), were selected as the measure of inflation, due primarily to the fact that the C.P.I. receives much publicity, and as a result is widely regarded as "the" inflation rate by most consumers. When used in regression analysis, this change in the index was calculated using the formula:

$$Pt = ((CPI - CPI(-1))/CPI(-1)) * 100$$

to represent the percentage change in inflation per month.

When rational expectations tests were run, money supply and real economic growth data were necessary. The money supply series selected were "M1" (total Canadian dollars held in demand deposit accounts plus total currency), and "M3" (M1 plus time and other deposits). Real economic growth was represented by

three indices - Real Gross National Expenditure, the Index of Industrial Production, and Real Domestic Product, all base year 1961.

For the period 1955-1974, M1 was not recorded under a Cansim number. For the purposes of this study, therefore, it was calculated by adding total dollars in Canadian currency and total Canadian dollars in demand deposits for each given month. After 1975-1, the Bank of Canada Review published M1 data directly. The additional series used, then, were :

- 6) Money supply M1; 1955 - 1978,
= total Canadian currency (Cansim # = B 2001) plus total Canadian deposits in dollars (Cansim # = B 459)
- 7) real Gross National Expenditure (G.N.E. indexed) ; 1955 - 1978, (Cansim # = D 40476)
- ✓ 8) Index of Industrial Production , 1955 - 1978, (Cansim # = D 5760)
- 9) Real Domestic Product, (indexed), 1955-1978, (Cansim # = D 100215)

C. The Fisher Effect Under the Adaptive Expectations Approach to Expectations Formation

I. The "Single Variable" Model

As mentioned in the introduction, two forms of the Adaptive Expectations process may be tested from an equation such as (3), reproduced below as (3a) :

$$(3a) \quad R_{Nt} = a + f(P_t, P_{t-1}, P_{t-2}, \dots, P_{t-n}; O_t)$$

In section I.4, where a multivariable model will be used, factors "O_t" will be explicitly recognized in regression form. Throughout the majority of the present part, however, we will test only a single variable form of inflationary expectations generation, thus ignoring "O_t".

The decision to purposely omit said information rests upon the reasoning that present and past price level changes already convey all information relevant to the estimation of future inflation rates. This is a rather strong assertion, since it implies that changes in nominal and real variables at any given time are immediately translated into changes in price levels. Nonetheless, proponents of the one variable hypothesis would

argue that adjustment may be assumed fast enough, given present communication methods, that it is correct to be concerned only with current and past price level behavior.

A different argument leading to the same conclusion might be that regardless of whether present and past inflation rates actually do incorporate all necessary information for accurately generating expectations or not, it is reasonable to assume that the average bond buyer forms expectations primarily upon past and present rates, due to the low information costs associated with such a method. A one variable approach would therefore yield good estimates of the inflation/deflation component of the nominal interest rate on bonds, at least from the demand related forces pushing nominal bond rates toward equilibrium.

In either case, proponents of the one variable hypothesis argue that expectations in period "t" (P^*t) are generated adaptively through weighted emphasis on past price level changes:

$$(4) P^*t = b_1P_t + b_2P_{t-1} + b_3P_{t-2} + b_4P_{t-3} + \dots + b_nP_{t-(n-1)}$$

where P_t = % change in price level from period "t-1" to period "t",

b_n = weight associated with period "t-(n-1)" 's past inflation rate

Recent inflation rates are believed to be assigned relatively large weights, generally declining such that:

$$(5) \quad b_i > b_{i+1} \quad (i = 1, 2, \dots, n-1)$$

It is argued that beyond some point in time "t-(n-1)" the influence of more distant price level changes is so insignificant as to be ignored; statistically, the coefficient becomes unstable to the point where "bn" cannot be shown different from zero with 95% probability.

Combining equation (4) into (1) and again assuming a constant real rate component, we can statistically estimate the equation:

$$(6) \quad R_{Nt} = a + b_1 P_t + b_2 P_{t-1} + \dots + b_n P_{t-(n-1)}$$

where again R_{Nt} = nominal rate of return to the bond at time "t"

P_t = % change in C.P.I. from period "t-1" to "t"

Two considerations become important when an equation such as (6) is being dealt with: first, the time period, "t", over which changes in the price level are to be measured (i.e. weekly, monthly, quarterly, yearly, etc.); and secondly, the length of lag to be included in the regression (how large a value of "n" would be appropriate). The latter problem appears one which the data itself can settle -- inclusion of all significant variables in conjunction with a "stable" R² (R squared) value would seem logical.⁶ As we shall see in section I.1.1, however, the significance of variables depends in this case upon the length of lag chosen, meaning this simple rule is not an operational one.

The "best" period over which to measure price changes is also non-obvious, but a recent study by William Yohe and Dennis Karnosky⁷ suggested that past monthly changes in price levels yield good explanatory power for monthly levels of U.S. nominal

⁶ By "stable" we mean that no significant increase in R² can be obtained from the inclusion of extra variables. Obviously R² will never be truly stable in such a case, since the addition of variables will always increase R² - it is the relative "jump" in R² associated with each extra variable which will be of interest.

"Significant" variables are those whose coefficient values can be shown different from zero with a statistical probability of 95%.

⁷ Yohe W, and Karnosky, D. Interest Rates and Price Level Changes, 1952-69. Federal Reserve Bank of St. Louis Review, 1969. P. 18. Hereafter, references to Yohe and Karnosky will follow from this paper.

bond rates. Since data were available for Canada in monthly form, there would seem nothing gained by aggregating the data to quarterly or yearly form; hence monthly changes in nominal bond and inflation rates were selected for this study. Periods of less than a month (i.e. weekly) were not tested due to non-availability of data.

The above mentioned study by Yohe and Karnosky also found evidence of two important characteristics with respect to length of past lag (time horizon) in formation of inflationary expectations over the period 1952 - 1969. These findings were 1) a lag of only 24 months in past inflation rates needed to be taken into account for a satisfactory explanation of expectations generation. The addition of further lagged variables of the inflation rate beyond "t-24" months led to only minor increases in R2 values; and 2) the length of lag chosen had almost no impact upon coefficient values (the "b"'s in equation (6)). Further, beyond a 24 month lag, coefficients tended to be small and insignificant. Both points lead to the important conclusion that expectations of future inflation are drawn from very recent past price changes, in contrast with earlier studies.⁸

⁸ A mean lag of approximately twenty years was found in one study by David Meiselman ("Bond Yields and the Price Level : The Gibson Paradox Regained" in Deane Carson (ed.) Banking and Monetary Studies. Homewood, Illinois, R.D. Irwin, 1963. pp. 119-122.)

This assumption was not carried into the Canadian counterpart of Y and K 's study for reasons outlined in section I.1.1 which follows. A second major facet of the Y and K study was incorporated into testing for Canada, though, and this was with respect to the method employed when constraining the "b" coefficients, as a means of avoiding multicollinearity problems, in the statistical estimation of an equation such as (6). Before the procedure, known as Almon Lagging, was adopted for Canadian data, however, properties of the weighting coefficients generated via such a technique were examined (and outlined in section I.1.2) to ensure that the coefficients were being constrained in a theoretically sensible manner.

After establishing the optimal number of lagged price level changes, "t-n-1", to be included in a regression such as (6), (section I.1.1), and that the Almon Lag procedure does indeed generate a satisfactory decay weighting system (section I.1.2), the details and results of testing the Fisher Effect for Canada, using a model similar to Y and K's, are outlined in section I.2.

8 (cont'd) Yohe and Karnosky (op. cit.) also mention two studies, the first by Milton Friedman and Anna Jacobson Schwartz (Trends in Money, Incomes and Prices, 1867-1966, unpublished manuscript, National Bureau of Economic Research, Nov. 1966. Chapter 2, pp. 110-143) which found a mean lag of some twenty-five to thirty years, and a second study done by Suraj B. Gupta (Expected Rate of Change of Prices and Rates of Interest, unpublished dissertation, University of Chicago, 1964) which found a mean lag of around sixteen years.

I.1.1. The Optimal Time Horizon of Past Price Level Changes

Before presenting the empirical results arising from a Canadian counterpart of the Y and K model, shown earlier as (6), it would seem instructive to "side-step" for the moment into a very brief discussion about one assumption made in the U.S. study regarding the number of coefficients "n" which are relevant.

For convenience, the Y and K model is reproduced below as (6a):

$$(6a) \quad R_{Nt} = a + b_1 P_t + b_2 P_{t-2} + \dots + b_{25} P_{t-24}$$

Such a diversion would seem warranted on the grounds that the impact of an expectation proxy may well vary dramatically in a one variable case depending upon the variable's length of lag considered. Yohe and Karnosky determined that a 24 month lag captured the important effects of formulation of expectations. As will be shown, such does not appear to be the case for Canada over the same period.

Unfortunately, exact duplication between studies was not achieved due to non-availability of nominal bond rates prior to 1955 (and later in some cases - see Data section, Page 11 earlier), but the non-inclusion of data in the early 1950's

would not seem to dangerously distort the comparability between monthly Canadian and American results. For reference, Y and K 's regression results are reproduced in Chart I of the Appendix.

Data for Canadian nominal bond rates were chosen so as to include both private sector and government rates, whereas Y and K dealt exclusively with private sector bond rates. Rather than selecting a representative short term and long term bond rate to correspond to Y and K's two rates, all six Canadian bond rates were run separately as left hand side variables in equation (6). Lags in percentage change of the consumer price index for 24, 36 and 47 months⁹ were regressed upon each short and long term rate. Mathematically these regressions took the form:

$$(8) \text{ RNT} = a + b_1\text{Pt} + b_2\text{Pt}-1 + \dots + b_{25}\text{Pt}-24$$

$$(9) \text{ RNT} = a + b_1\text{Pt} + b_2\text{Pt}-1 + \dots + b_{37}\text{Pt}-36$$

$$(10) \text{ RNT} = a + b_1\text{Pt} + b_2\text{Pt}-1 + \dots + b_{48}\text{Pt}-47$$

⁹ Another slight non-comparability factor between studies entered due to the computer program's inability in the Canadian study to process more than 49 R.H.S. variables. Hence, only 47 (rather than the desired 48) lagged variables plus current inflation rate plus a constant were run.

If Y and K 's contention, (that only "t-24" variables matter) is correct, the coefficients attached to all "t-(24 +i)" - where $i = 1, 2, \dots, 23$ - coefficients should be very small and/or not significant. Graphically, each set of coefficients from the "t-24", "t-36" and "t-47" monthly runs on a given interest rate appear plotted on the same graph (X axis = lag, Y axis = coefficient) in Charts III - VIII in the appendix. These charts are directly comparable to Y and K's Chart I.

Significant "b" values are shown in Chart III to VIII as a '.' and insignificant values as a '0', although the reader should be aware that due to multicollinearity these "t" test values may be unreliable.

Two important observations may be drawn from the Canadian results in Charts III - VIII with respect to varying the length of time horizons in the one variable model. First, the length of lag chosen seems to have a great effect upon the magnitudes of certain coefficients, and generally upon R2 values (although autocorrelation makes R2 and "t" statistics somewhat unreliable). For example, Chart III, and to a lesser extent IV, correspond roughly to Y and K's Chart I. Charts V to VIII, however, all indicate strong and significant coefficients from roughly t-25 to t-36 months on the 36 month lag. These are so large in the case of the 3 month treasury bill rate and 1-3 year government bond average rate that the coefficients during the

t-25 to t-36 month period are of the same magnitude to expectations as those found in the first twelve months.

The second point worth noting in comparing Canadian and U.S. studies is the difference in R2 statistics, and in the Canadian case, the small Durbin-Watson statistic. Yohe and Karnosky argue that increasing the lag from 24 to 48 months boosts the R2 from only .498 to .536 on the long term rate, and from .591 to .630 on the short term rate. Little explanatory power is gained, then, by looking back further into the past for expectations formation. The U.S. study does not mention a Durbin-Watson statistic.

With the exception of Chart III, all Canadian regressions show a marked increase in R2 values attributable to increasing the lag from 24 to 47 months (Charts IV and V show an almost doubling, for example). Even more importantly, however, in all cases the Durbin-Watson statistic is so low that severe autocorrelation is indicated, making all R2 values unreliable. It would seem likely that Y and K would also have encountered this problem, although no mention is made of it.

The conclusions arising from the Canadian - U.S. data comparison are two-fold. First, a two year lag in price level changes does not appear adequate in terms of explanatory power ("stability" of R2 as lag increases), nor able to satisfactorily generate coefficients (where "stability" of coefficients regardless of whether longer lag terms are added is

desired).¹⁰ Rather, in the case of Canadian data, a 47 month lag appears superior on these grounds.

The second conclusion concerns statistical problems encountered in the one variable lag procedure undertaken. Autocorrelation proved severe in the monthly data. Equally problematic was multicollinearity, causing very unstable coefficients to be generated from an equation such as (6). Solutions to these problems differ, of course, but it might be mentioned that first differencing the raw data failed in several instances to satisfactorily remove autocorrelation.

As a result of testing, then, equation (6) appears to be in need of modification to remove multicollinearity. The method employed for such a task by Y and K in their study was to impose a constraint upon the weights (coefficients in (6)), generated via the Almon Lag procedure. In the following section, the weighting coefficients generated by such a procedure are examined for Canadian past rates of inflation, over a period longer than "t-24" months (which is the period Y and K select) for reasons outlined above.

10 Here we refer to stability primarily in the sense of the magnitude of "b" values with respect to the lag chosen. If increasing the lag dramatically changes the weight of a given past price level change, we cannot be satisfied with arbitrarily cutting off the lagging experiment at that point. Rather, the lag should be extended until relative stability is reached.

I.1.2 Almon Lags and Canadian Interest Rates

Irving Fisher believed an arithmetically declining weighting system for past levels of price changes would be adequate with respect to generating an expectations function - i.e. a weight of 9 for period $t-1$, 8 for $t-2$ and so on, divided by the sum of the weights, to yield coefficient constraints for past price variables. The problems with such a method conceivably arise due to the approach's need for the arbitrary choosing of some time period over which to apply the weights, and, in a different vein, the uncertainty as to whether in fact arithmetically declining weights accurately capture the expectations generating process. It might be argued, for example, that exponentially decaying, or for that matter, almost any decay pattern would be "a priori" as reasonable a weighting system from theory as an arithmetic decay process.

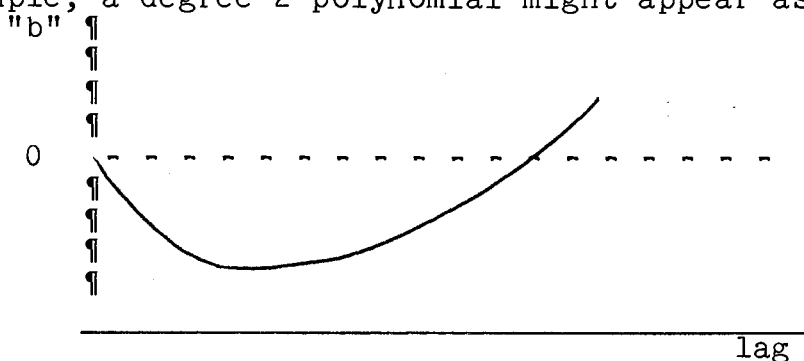
Fortunately, the Almon Lag procedure offers a solution to the latter of these problems. By selecting a weighting pattern which maximizes explanatory power vis-a-vis changes in the left hand side variable, the Almon Lag method ensures that the weights imposed are at least optimal from the standpoint of impact of independent variables upon that variable which is to be explained. The arbitrary choice decision is not eliminated, however, as the lengths of lag selected, and the degree of polynomial must be specified. The Almon Lag procedure also

raises the uncomfortable possibility that the weights selected by such criteria might not even decline generally over time.¹¹

The question of choosing the best lag period "t-n-1" was discussed for Canadian data in the last sub-section of this paper. A time horizon of 47 months was selected on the grounds of stability of coefficient values and on the relative steadiness of R2 values with such a lag. When applying the computer program,¹² however, a maximum of only 45 lagged values could be included. This restriction would not seem an important problem -- in fact the variables lost are in all cases small and insignificant.

The "correct" degree of polynomial as a postulated decay pattern is a much more difficult selection task, since theory is of little help in this matter. A degree 1 polynomial can likely be rejected on the grounds that some recognition lag exists, and

¹¹ For example, a degree 2 polynomial might appear as follows:



¹² The program alluded to is the "Massager 1973" program. In addition, a constraint of $b=0$ must be imposed as the last coefficient of any Almon Lag. Hence only 44 non-zero values were obtained.

thus the weights of decay coefficients might well rise before declining over some range. Yohe and Karnosky decided upon a degree 6 polynomial in their study; indeed a reasonable rationale could likely be found for most degrees, including the largest possible which the computer program will handle, since this is the least arbitrary (most responsive) option.

This study followed Y and K's decision as to a degree 6 polynomial, however, for two reasons. First, using the same polynomial decay pattern increased the comparability between studies, which was deemed desirable. More importantly, though, a degree 6 polynomial "fitted" the unconstrained coefficients well (see Charts III - VIII, for example) and allowed good flexibility in choices of "b" patterns.

Having selected the shape and length of decay for weighting a regression such as (11) :

$$(11) R_{nt} = a + b_1 P_t + b_2 P_{t-1} + \dots + b_5 P_{t-4}$$

we are in a position to analyze whether the Almon Lag procedure, which Y and K incorporated into their study, gives rise to theoretically sensible results when applied to Canadian data. In particular, we should be examining the results for two features.

First, the shape of decay weighting function should be markedly different for shorter versus longer term bond rates. This is so because in the case of short term bonds, most recent

price level changes are far more likely to affect a bond holder's expectations than would be the case with the holder of a longer term bond, whose returns during maturity of the bond will be subject to inflation rate changes over a greater period.

Secondly, the magnitudes of "t" scores in the longer term bond rates should be noticeably different from shorter term rates as more distant inflation rates are considered. Over a long lag there will always be some correlation between changes in distant price levels and current movements in a nominal bond rate. However, in the case of longer term bonds, the relevant past horizon should be longer than is found with shorter term bonds, as was argued above. As such, the same distant inflation rate change may have miniscule relevance to a short term bond over repeated samples, while having a distinct effect on expectations for a longer term bond.

As an example, consider a change in price level occurring 24 months in the past. To a short term bond buyer of 90 days or less, this change would likely be unimportant -- certainly in relation to changes within the past year. To a potential buyer of a five year bond, on the other hand, this change may be of substantial impact to the premium demanded as compensation for inflation over the next five years. The same movement in inflation during period "t-24" months, then, is theoretically of only trivial importance to the inflationary component of a term bond, while being an important determinant in the nominal

interest rate of a longer term bond. Statistically, such a relationship over repeated sampling should be evidenced by an insignificant coefficient "b25" for the short term bond, but a significant coefficient "b25" for the longer term bond, quite apart from the magnitude of each respective "b25" coefficient.

To repeat, then, "a priori" we should expect two features of a desirable weighting pattern. First, coefficients on short term expectations should be large with respect to recent past price level changes, decreasing rapidly. Long term expectations should be generated over a longer past horizon, and weighted more evenly. Hence, we would expect smaller coefficients over a longer period for long term bond rates. Secondly, we would expect more non-relevant correlation with short term expectations and more distant lags than would be the case for longer term expectations. Statistically, for lags of the same length, this should appear as larger "t" scores with longer term bonds than would be found with shorter term bonds, especially in the more distant past.

Table I shows the first thirty-six coefficient lag values from equation (11), for each of the six nominal interest rates selected. Due to autocorrelation, a Hildreth-Lu modification was applied after generation of coefficients, meaning the "t" scores

Table I

Rnt = f(Almon P t....P t-47); Hildreth-Lu; 1955-78

t-	30 day	90 day	mon 3	mon 6	year 1	year 3
0	.277(2.7)	.238(2.5)	.171(2.4)	.232(3.0)	.269(4.0)	.227(3.9)
1	.423(3.6)	.336(3.0)	.248(2.9)	.271(3.0)	.264(3.4)	.253(3.8)
2	.514(4.0)	.402(3.3)	.278(2.9)	.277(2.8)	.241(2.8)	.246(3.3)
3	.568(4.0)	.443(3.3)	.295(2.8)	.277(2.5)	.221(2.4)	.237(2.9)
4	.591(4.0)	.462(3.3)	.301(2.8)	.275(2.4)	.205(2.1)	.226(2.7)
5	.589(4.1)	.463(3.3)	.299(2.7)	.269(2.4)	.191(2.0)	.213(2.5)
6	.568(4.0)	.450(3.3)	.290(2.7)	.261(2.3)	.179(1.9)	.200(2.4)
7	.531(3.9)	.426(3.2)	.277(2.6)	.252(2.3)	.168(1.8)	.185(2.3)
8	.484(3.7)	.394(3.0)	.261(2.5)	.242(2.3)	.159(1.8)	.171(2.2)
9	.440(3.8)	.358(3.0)	.275(2.8)	.259(2.6)	.172(2.1)	.186(2.6)
10	.383(3.3)	.318(2.7)	.256(2.6)	.248(2.5)	.165(2.0)	.172(2.4)
11	.324(2.7)	.276(2.3)	.238(2.4)	.239(2.3)	.159(1.9)	.158(2.1)
12	.265(2.2)	.235(1.8)	.221(2.1)	.229(2.2)	.154(1.7)	.144(1.9)
13	.211(1.6)	.195(1.5)	.206(1.9)	.221(2.0)	.149(1.6)	.132(1.6)
14	.161(1.2)	.159(1.2)	.194(1.8)	.214(1.9)	.145(1.6)	.120(1.5)
15	.116(.86)	.126(.93)	.183(1.7)	.207(1.8)	.142(1.5)	.109(1.3)
16	.077(.57)	.097(.72)	.175(1.6)	.202(1.8)	.138(1.5)	.100(1.2)
17	.046(.35)	.073(.55)	.170(1.6)	.197(1.8)	.135(1.5)	.092(1.2)
18	.021(.16)	.055(.42)	.168(1.6)	.193(1.8)	.134(1.5)	.086(1.1)
19	.004(.03)	.042(.33)	.168(1.7)	.190(1.8)	.132(1.5)	.083(1.1)
20	-.007(-.1)	.033(.27)	.170(1.7)	.188(1.9)	.130(1.6)	.080(1.1)
21	-.011(-.1)	.029(.25)	.174(1.8)	.187(1.9)	.129(1.6)	.079(1.1)
22	-.010(-.1)	.030(.26)	.180(1.9)	.185(1.9)	.128(1.6)	.080(1.1)
23	-.004(-.0)	.035(.30)	.186(1.9)	.183(1.9)	.128(1.6)	.082(1.1)
24	.005(.04)	.044(.36)	.194(1.9)	.181(1.8)	.128(1.5)	.086(1.2)
25	.018(.15)	.055(.44)	.201(2.0)	.179(1.7)	.128(1.5)	.091(1.2)
26	.034(.26)	.069(.54)	.209(2.0)	.176(1.6)	.129(1.4)	.098(1.2)
27	.050(.38)	.085(.64)	.215(2.0)	.173(1.6)	.130(1.4)	.105(1.3)
28	.067(.49)	.101(.75)	.220(2.0)	.168(1.5)	.131(1.4)	.113(1.4)
29	.083(.61)	.134(.86)	.224(2.0)	.163(1.4)	.132(1.4)	.122(1.5)
30	.097(.73)	.147(.98)	.225(2.0)	.156(1.4)	.133(1.4)	.131(1.6)
31	.108(.83)	.160(1.1)	.224(2.0)	.148(1.3)	.134(1.5)	.139(1.7)
32	.166(.92)	.169(1.2)	.220(2.1)	.139(1.3)	.134(1.5)	.147(1.9)
33	.121(.99)	.175(1.3)	.214(2.1)	.128(1.2)	.134(1.6)	.154(2.0)
34	.122(1.0)	.177(1.4)	.204(2.0)	.116(1.1)	.134(1.6)	.160(2.1)
35	.117(.97)	.174(1.5)	.191(1.9)	.103(.99)	.132(1.6)	.163(2.2)
36	.109(.88)	.167(1.4)	.175(1.8)	.089(.84)	.129(1.5)	.164(2.2)
Sum=	7.95	8.06	8.72	7.67	6.35	6.16

(shown in brackets) should be reliable.¹³ As may be seen, Almon Lag generated coefficients stand up well in relation to the "a priori" conditions outlined. Short term rates, especially the 30 day and 90 day rates, do in fact show associated large "recent" coefficients which quickly drop to below the .100 level (t-17). By contrast, the 1-3 year average government bond rate is still above the .100 level after "t-36" months. The 3-5 year rate, in fact, shows significant coefficients greater than .150 at "t-36" months. The first condition, then, that shorter term rates be associated with larger early coefficients, dropping in magnitude quickly compared with longer rates, appears satisfied generally. In the case of the 3 month treasury bill rate, however, it might be noted that the coefficients do not decline as rapidly in weight as might be expected.

The second feature of Table I worth noting is that "t" scores react generally as predicted with respect to type of bond

13 We noted only that "t" scores "should" be reliable since the Durbin-Watson statistic for these runs could not be tested, due to the lack of significance tables for large numbers of variables and observations. The largest values commonly calculated are for 5 explanatory variables and 100 observations - this study used 44 explanatory variables and 200+ observations. The boundaries for k=5, n=100 case are D.W.(upper)= 1.65 and D.W.(lower)= 1.44 at the 1% certainty level. Calculated Durbin-Watson statistics in this study were: 30 day finance co. paper rate = 1.84 ; 90 day finance co. paper rate = 1.65 ; 3 month government bond rate = 1.42 ; 6 month government bond rate = 1.50 ; 1-3 year government bond rate = 1.42 ; and 3-5 year government bond rate = 1.39 . Therefore, the "t" value shown in brackets in Table I should almost certainly be accurate.

and length of lag of inflation. For shorter term rates, only recent weights are subject to small standard errors, while in the longer term rates, coefficients retain significance further into the past. Again, however, the 3 month treasury bill rate did not conform to expectations. The 1-3 year bond rate also showed surprisingly short lived significant coefficients .

To this point we have attempted to establish the appropriateness of two assumptions which are incorporated into the rest of Part I of this study - that a period "t-n-1" should be of no less than 47 months, and that the Almon Lag procedure generates estimates of "b" coefficients for equation (11) which are compatible with theory. The next three sections outline the testing, and conclusions arising from, the Fisher Effect relationship between Canadian nominal interest rates and inflationary expectations, based upon the trends deduced from past behavior of an economic series or series.

I.2 Empirical Results of Testing Canadian Nominal Interest Rates Under the Modified Y and K Assumptions

When testing Y and K 's constraint weighted model, mathematically represented earlier as equation (11), and reproduced below for convenience as (11a):

$$11a) \quad R_{Nt} = a + b_1P_t + b_2P_{t-1} + \dots \\ + b_4P_{t-4}$$

two separate runs were made in addition to testing the data for the period 1955-1978. The data were split into pre-1969 and post-1969 groups, since the period 1969-78 might be hypothesized as being subject to stronger, or at least larger, expected inflation rates than the period 1955-1968. The results of these runs are shown in Chart IX of the Appendix. Table II lists the same data after the Hildreth-Lu modification was applied to remove autocorrelation.

The most interesting facet of Table II may well be the value of "a", which is the constant variable in equation (11). Since the one variable hypothesis suggests that only price level changes affect nominal interest rates, this constant becomes an estimate of the "real rate" component of nominal bond rates for the periods shown.

Excluding the two treasury bill rates momentarily, an interesting pattern emerges when scanning the columns of Table II vertically. In all cases the values of "a" increase with the length to maturity of the bond. This finding supports the argument that the real return on a longer term investment should be greater than on a shorter term, to compensate the holder for

Table II

	1955-1968	1969-1978	1955-1978
1. 30 day	a = 2.106 (5.39) R2 = .931 P = .790 D.W. = 1.77	a = 3.62 (2.47) R2 = .913 P = .913 D.W. = 1.82	a = 3.141 (5.17) R2 = .952 P = .911 D.W. = 1.83
2. 90 day	a = 2.395 (1.80) R2 = .917 P = .957 D.W. = 1.47	a = 4.001 (2.70) R2 = .917 P = .917 D.W. = 1.79	a = 3.319 (4.78) R2 = .949 P = .929 D.W. = 1.65
3. 3 mon	a = 1.87 (2.30) R2 = .879 P = .903 D.W. = 1.67	a = 2.294 (1.37) R2 = .971 P = .976 D.W. = 1.04	a = 2.34 (3.34) R2 = .966 P = .950 D.W. = 1.42
4. 6 mon	a = 2.266 (4.21) R2 = .871 P = .831 D.W. = 1.67	a = 2.833 (1.80) R2 = .963 P = .964 D.W. = 1.25	a = 3.005 (4.89) R2 = .960 P = .935 D.W. = 1.50
5. 1-3 year	a = 2.686 (4.72) R2 = .877 P = .885 D.W. = 1.59	a = 4.793 (4.89) R2 = .924 P = .925 D.W. = 1.32	a = 3.632 (7.85) R2 = .955 P = .924 D.W. = 1.42
6. 3-5 year	a = 3.14 (5.52) R2 = .896 P = .915 D.W. = 1.46	a = 5.402 (6.93) R2 = .916 P = .910 D.W. = 1.39	a = 4.04 (9.39) R2 = .960 P = .931 D.W. = 1.39

where a = constant

R2 = explained variation from residuals

P = rho

D.W. = Durbin-Watson Statistic

the inconvenience of illiquidity. The reason treasury bills were excluded is that they are held largely by banks as reserves. By law these institutions must hold treasury bills; hence the real return on these instruments is most likely influenced by factors other than the "cash - illiquidity" choice which the public faces.

A second feature of interest in Table II comes from reading across the columns horizontally. In all cases, the value of "a" is greater in the post-1969 than pre-1969 period. This could be because, in fact, the real rate of return has increased over time, but this seems unlikely. Rather, it would seem plausible to interpret this rise to non price-explanatory variables which are missing from equation (11). In other words, if factors other than past inflation rates have become increasingly important since 1969 in determining expectations, the influence of these variables could show only in the constant term. If such were the case, the real rate need not have increased, (indeed it may have even decreased), but the one variable hypothesis can attribute "non-lagged inflation variation" only to the constant in this model. We will return to this possibility later in the paper, when considering the multivariable alternative of adaptive expectations, and in the Conclusions section, where the effects of changes in taxation are considered.

Finally, it is worth noting that Y and K found constant values similar to those in Canada in their U.S. study over the

same period. For the years 1952-1969, Y and K calculated a constant for the short term rate of 2.283, and a long term rate constant of 3.090 over an unconstrained, 48 lagged (inflation rate) regression. These values appear quite close to Canadian results in the similar period, as theory would predict.

While the Almon Lag method employed by Y and K yields useful information about the constant term in equation (11), an important consideration remains unexplored, and this is the relative impact which a change in past prices has directly upon nominal interest rates. For example, if expectations are fully captured in interest rates we would expect a coefficient value of "c = 1.00" in equation (12) :

$$(12) Rnt = a + cP^{**t}$$

where P^{**t} is the "best guess" at the expected rate of inflation at time "t". If expectations are conversely not translated into nominal interest rates at all, "c" should have a value of zero. It might be noted, as an aside, that equation (12) is simply equation (1) written in statistical estimation form.

In the next section of Part I, an equation such as (12) will be estimated. Separate runs will be made, as before, for the periods 1955-1978, 1955-1968, and 1969-1978.

I.3 Indirect Almon Lags and Nominal Interest Rates

Before an equation such as (12) could be estimated, a variable P^{**t} was needed. This "best guess" as of time "t" at the expected inflation rate should be generated, according to the one variable model, from changes in present and past price levels. Given the desirable characteristics of Almon Lag determined coefficients outlined earlier, a suitable method for obtaining an estimate of P^{**t} would seem to be via regressing present inflation rates upon past weighted rates, and using the "P hat" value (calculated inflation rate) as P^{**t} . This was, in fact, the procedure undertaken.

The results of estimating equation (12) are shown in Chart X of the Appendix. Table III shows the same results after the Hildreth-Lu modification was used, again to remove autocorrelation. The major difference between these two exhibits is the large change in "c" values - from magnitudes of greater than 5.00 in Chart X to approximately 1.50 in Table III. The magnitudes of "c" in Table III should be more reliable than in Chart X, however, since the "tracking" of residuals causing autocorrelation may lead to serious distortions in a given sample, such as appears in Chart X. Once this "tracking" has been corrected, as in Table III, the coefficient should assume a more accurate statistical estimate of the true relationship.

Table III

$$RN_t = a + bP^*t$$

(where $P_t = d_1P_t + d_2P_{t-1} \dots d_{45}P_{t-44}$)

	1955-1969	1969-1978	1955-1978
1. 30 Day	a = 4.720(5.81) b = 1.111(1.11) R2 = .917 P = .962 D.W. = 1.74	a = 6.081(6.06) b = 2.524(2.10) R2 = .909 P = .936 D.W. = 1.79	a = 5.442(6.01) b = 1.984(2.46) R2 = .947 P = .965 D.W. = 1.79
2. 90 Day	a = 4.564(4.95) b = 1.139(1.31) R2 = .898 P = .971 D.W. = 1.36	a = 6.542(6.33) b = 2.089(1.78) R2 = .912 P = .943 D.W. = 1.74	a = 5.518(5.12) b = 1.636(2.20) R2 = .941 P = .974 D.W. = 1.60
3. 3 mon T.B.	a = 4.025(6.32) b = 1.028(1.09) R2 = .863 P = .949 D.W. = 1.60	a = 7.211(.000) b = 1.049(1.54) R2 = .967 P = 1.00 D.W. = .961	a = 5.535(.000) b = .966(1.71) R2 = .955 P = 1.00 D.W. = 1.32
4. 6 mon T.B.	a = 4.225(7.79) b = 2.096(2.07) R2 = .863 P = .932 D.W. = 1.63	a = 6.628(4.36) b = 1.271(1.69) R2 = .961 P = .984 D.W. = 1.21	a = 5.393(5.27) b = 1.622(2.67) R2 = .959 P = .978 D.W. = 1.49
5. 1-3 year	a = 4.527(7.42) b = 1.082(1.43) R2 = .861 P = .959 D.W. = 1.58	a = 5.869(9.22) b = 2.371(3.25) R2 = .920 P = .942 D.W. = 1.22	a = 5.154(6.20) b = 1.800(3.42) R2 = .947 P = .977 D.W. = 1.43

(Table III con't next page)

Table III (con't)

6. 3-5	a = 4.912(4.78)	a = 6.557(11.64)	a = 5.650(.000)
year	b = 1.052(1.77)	b = 1.759(2.66)	b = 1.331(2.94)
	R2 = .872	R2 = .909	R2 = .954
	P = .985	P = .939	P = 1.00
	D.W. = 1.44	D.W. = 1.28	D.W. = 1.37

where a = constant
 b = slope coefficient
 R2 = explained variation from residuals
 P = correlation coefficient
 D.W. = Durbin-Watson Statistic

As mentioned, the value of "c" should not exceed 1.00 in the long run. Information taken from Table III, then, would seem to indicate consistent undershooting of expectations, or an inadequate expectations variable P**t. Consistent undershooting (i.e. demanding, on average, a lower inflation premium to bond holding than is actually provided) over as long a period as twenty-three years seems highly unlikely.

A solution to this problem might well be that price change variables alone are not adequate in explaining how expectations are formed. If other variables enter the generation of expectations function, the variable P**t will never be correctly estimated - it may be either too large or too small. Combined with the evidence presented in the last sub-section of Part I, that variables other than price appear missing (increased

constant values over time from the Almon regressions), such an explanation -- i.e. that the "single variable" model is inadequate for explaining generation of expectations -- appears promising. In the last section of Part I a version of the multivariable adaptive expectations model will be tested for Canada. As will be seen, however, the addition of time series variables other than price, (at least in the cases tested), does not succeed in removing the puzzling undershooting phenomenon alluded to.¹⁴

In defence of the one variable adaptive expectations model, however, it should be pointed out that that no value of "c" in Table III can be shown significantly different from 1.00 with 95% certainty. Thus, even though each coefficient but one is greater in magnitude than 1.00, it should not be forgotten that the variable P^{**t} may actually be a legitimate estimate of the expectations component of the nominal rates shown, but due to the small number of samples (18), values greater than 1.00 appeared. The mean value of "c" might, in fact be less than 1.00 - statistically it is conceivable, though improbable.

It might also be mentioned that the constant term values of

¹⁴ Examination of the "rho" values in Table II will also show the modification comes very near to first differencing, at which point the constant term loses all meaning. It may be that this factor alone is responsible for the behavior of the constant estimates after removal of autocorrelation.

Table III are large by comparison with Table II.¹⁵ This also seems most likely attributable to the poor explanatory power of the single generated variable P^{**t} , rather than to a different real rate component of nominal bond rates being indicated by comparison to Table II. The estimates are not different enough, however, to indicate that measurement error cannot logically account for the differences between the two Tables.

I.4 Empirical Tests of the Multivariable Adaptive Expectations Model

The final test of the Adaptive Expectations hypothesis which we will employ on the selected Canadian nominal bond rates is of the "multivariable", rather than "single variable" variety. The desirability of testing such a model follows from an observation noted in the preceding section -- i.e. that the "c" coefficient in equation (12), reproduced below as (12a):

$$(12a) \quad R_{Nt} = a + cP^{**t} + u_t$$

was found to have an unexpectedly high value. One explanation for this finding could be that the variable P^{**t} was

¹⁵ This "problem" will be dealt with in the Conclusions section of the paper, where a plausible solution follows after inclusion of taxation considerations presently outside the model.

significantly biased due to the non-inclusion of information which bond holders do in fact use adaptively, but which is not transmitted via present and past inflation rates.

It might be recalled that P^{**t} was statistically derived from a process described by equations (13) and (13a) below:

$$(13) \quad P_t = a + d_1 P_{t-1} + d_2 P_{t-2} + \dots + d_{45} P_{t-45} + u_t$$

$$(13a) \quad P^{**t} = P_t - u_t$$

where P_t = % change in C.P.I. from period "t-1" to "t"

P^{**t} = inflationary expectation over period to maturity of bond

u_t = estimated unexplained error as of time "t" from O.L.S. regression

d_1 = Almon weighted coefficient for variable lagged "1" period

In words, equation (13a) states that the best guess as to future inflation comes, over time, from forming a single variable estimate, P^{**t} , which minimizes the error between the predicted value of inflation, as of period "t-1", and that

observed in period "t".

The crucial assumption under the single variable hypothesis is, of course, that the error term "ut" cannot be made significantly smaller in a consistent fashion over time through the inclusion of added variables into an equation such as (13). If such a decrease were in fact possible, a better value of P^{**t} could be determined through the addition of said information -- which is precisely the postulate of the Multivariable Adaptive Expectations hypothesis. The fact that the coefficient attached to P^{**t} appeared to be surprisingly large, then, suggested that a better explanation of nominal interest rate movements might come from explicit consideration of variables in addition to past and present rates of inflation.

There would seem little argument that, in the long run, inflation is a result of excess money creation over and above increases in the real growth of an economy. The single variable adaptive expectations model assumes that such a relationship holds in the very short run as well -- i.e. that no significant lag exists between excess money creation and price level adjustment. As such, the most accurate inflationary trend is transmitted via the inflation rate itself.

If a lag in the transmission of such information were, in fact, to exist for Canada, it would seem reasonable to assume it to be a very short lag, given present communication methods, government disclosure of money creation statistics, and the

rents available to anyone outguessing the market (which would be assumed, under adaptive expectations, to be relying upon a single variable hypothesis). Perhaps the best argument could be made for a lag of one month before monetary and real growth shocks impact fully upon the inflation rate, since a one month delay exists before "accurate" monthly data is published by government for a given monthly period. Until that point, only guessing as to government actions can take place, and one might expect significant portions of the market only to react to "hard" information.

To be safer, however, it was assumed that some monetary and/or real growth information is not transmitted to price level change for a period of up two months and longer. The multivariable hypothesis test took the mathematical form, then, of equation (14) below:

$$(14) \quad R_{Nt} = a + g_1 M^*t + g_2 M^*t-1 + \dots \\ \dots + g_r M^*t-r-1 + \text{Almon } (P_{t+n})$$

where M^*t = excess money creation over real growth between periods "t-1" and "t" in percentage form

$$\text{Almon } (P_{t+n}) = d_1 P_t + d_2 P_{t-1} + \dots + d_n P_{t-n-1}$$

from equation (13)

Two series were selected for the monetary creation variable, M1 and M3, as defined in the Data section earlier, P.11. Real growth was taken to be a constant for a given month, equal to one third the percentage growth in G.N.E. over the three month reported period.

Besides using two estimates of "M*t", separate equations were run including "M*t-1", "M*t-2" etc, and then re-run excluding the furthest lagged "M*" term in the previous regression, for comparison of explained variation (R²) statistics. Both M1 and M3 runs yielded similar results, and as such, only the M3 runs are listed, in the Appendix, as Charts XI and XII. The same results are listed after the Cochrane-Orcutt procedure was employed to remove as much autocorrelation as possible, in Tables IV and V. In addition, Table VI lists the result of running "M*t" and "M*t-1" using M1 rather than M3 as a measure of monetary activity. No results are presented for regressions including lagged "M*t" terms more distant than "t-1", since the conclusions arising from such runs are no different than can be deduced from Tables IV, V, and VI.

In short, the findings presented in Tables IV, V, and VI give little if any credence to the existence of lagged information having a noticeable impact upon nominal rates of interest, via inflationary expectations. In all cases the "M*" coefficients are small (at the largest ".01"), suggesting on

Table IV

Cochrane Orcutt : $RN_t = a + g_1 M3^*t + \text{Almon (Pt, \dots Pt-n)}$
(1955-78)

$$1) \text{ Day30t} = \begin{matrix} 3.58 \\ (3.80) \end{matrix} - \begin{matrix} .002M3^*t \\ (-.724) \end{matrix} + \text{Almon (Pt, \dots Pt-n)}$$

R2 = .95 rho = .915 D.W. = 1.83

$$2) \text{ Day90t} = \begin{matrix} 3.35 \\ (5.06) \end{matrix} - \begin{matrix} .004 M3^*t \\ (-1.28) \end{matrix} + \text{Almon (Pt, \dots Pt-n)}$$

R2 = .96 rho = .918 D.W. = 1.73

$$3) \text{ Mon3t} = \begin{matrix} 2.60 \\ (3.08) \end{matrix} - \begin{matrix} .003 M3^*t \\ (-1.59) \end{matrix} + \text{Almon (Pt, \dots Pt-n)}$$

R2 = .97 rho = .957 D.W. = 1.40

$$4) \text{ Mon6t} = \begin{matrix} 2.93 \\ (1.99) \end{matrix} - \begin{matrix} .003 M3^*t \\ (-1.52) \end{matrix} + \text{Almon (Pt, \dots Pt-n)}$$

R2 = .97 rho = .974 D.W. = 1.32

$$5) \text{ Year1t} = \begin{matrix} 3.80 \\ (6.13) \end{matrix} - \begin{matrix} .006 M3^*t \\ (-2.86) \end{matrix} + \text{Almon (Pt, \dots Pt-n)}$$

R2 = .96 rho = .939 D.W. = 1.43

$$6) \text{ Year3t} = \begin{matrix} 4.27 \\ (9.39) \end{matrix} - \begin{matrix} .002 M3^*t \\ (-1.29) \end{matrix} + \text{Almon (Pt, \dots Pt-n)}$$

R2 = .97 rho = .930 D.W. = 1.43

where $M3^*t = \% \text{ increase in M3 growth above } \% \text{ real}$

growth from "t-1" to "t"

Table V

Cochrane Orcutt : $RN_t = a + g_1 M3^*t + g_2 M3^*t-1 + \text{Almon (P)}$
(1955-78)

$$1) \text{ Day30} = 3.59 - .002 M3^*t + .003 M3^*t-1 + \text{Almon (P)}$$

$$(3.81) \quad (-.711) \quad (.786)$$

$$R2 = .94 \quad \rho = .915 \quad \text{D.W.} = 1.83$$

$$2) \text{ Day90t} = 3.35 - .004 M3^*t + .002 M3^*t-1 + \text{Almon (P)}$$

$$(5.06) \quad (-1.27) \quad (.465)$$

$$R2 = .96 \quad \rho = .917 \quad \text{D.W.} = 1.73$$

$$3) \text{ Mon3t} = 2.62 - .003 M3^*t + .002 M3^*t-1 + \text{Almon (P)}$$

$$(3.12) \quad (-1.58) \quad (.664)$$

$$R2 = .97 \quad \rho = .957 \quad \text{D.W.} = 1.40$$

$$4) \text{ Mon6t} = 3.04 - .003 M3^*t + .001 M3^*t-1 + \text{Almon (P)}$$

$$(1.82) \quad (-1.49) \quad (.343)$$

$$R2 = .97 \quad \rho = .975 \quad \text{D.W.} = 1.32$$

$$5) \text{ Year1t} = 3.80 - .006 M3^*t - .000 M3^*t-1 + \text{Almon (P)}$$

$$(6.11) \quad (-2.85) \quad (-.019)$$

$$R2 = .96 \quad \rho = .939 \quad \text{D.W.} = 1.43$$

$$6) \text{ Year3t} = 4.27 - .002 M3^*t - .000 M3^*t-1 + \text{Almon (P)}$$

$$(9.38) \quad (-1.28) \quad (-.118)$$

$$R2 = .97 \quad \rho = .930 \quad \text{D.W.} = 1.43$$

where $M3^*t-1 = \% \text{ increase in } M3 \text{ above } \% \text{ increase}$
in real growth between periods
"t-2" and "t"

$$\text{Almon (P)} = d1Pt + d2Pt-1 + \dots + d46Pt-45$$

Table VI

Cochrane Orcutt : $RN_t = a + g_1 M1^*t + g_2 M1^*t-1 + \text{Almon (P)}$

$$1) \text{ Day30t} = 3.55 + .010 M1^*t - .016 M1^*t-1 + \text{Almon (P)}$$

$$\begin{matrix} (3.67) & (.673) & (-1.12) \\ R2 = .94 & \rho = .918 & D.W. = 1.83 \end{matrix}$$

$$2) \text{ Day90t} = 3.32 + .006 M1^*t - .007 M1^*t-1 + \text{Almon (P)}$$

$$\begin{matrix} (4.80) & (.457) & (.533) \\ R2 = .96 & \rho = .921 & D.W. = 1.79 \end{matrix}$$

$$3) \text{ Mon3t} = 2.62 - .005 M1^*t - .001 M1^*t-1 + \text{Almon (P)}$$

$$\begin{matrix} (2.85) & (-.529) & (-.141) \\ R2 = .97 & \rho = .960 & D.W. = 1.39 \end{matrix}$$

$$4) \text{ Mon6t} = 3.16 - .005 M1^*t - .007 M1^*t-1 + \text{Almon (P)}$$

$$\begin{matrix} (1.86) & (-.544) & (-.732) \\ R2 = .97 & \rho = .977 & D.W. = 1.36 \end{matrix}$$

$$5) \text{ Year1t} = 3.85 + .001 M1^*t - .004 M1^*t-1 + \text{Almon (P)}$$

$$\begin{matrix} (5.37) & (.137) & (-.397) \\ R2 = .96 & \rho = .949 & D.W. = 1.36 \end{matrix}$$

$$6) \text{ Year3t} = 4.28 - .000 M1^*t + .001 M1^*t-1 + \text{Almon (P)}$$

$$\begin{matrix} (8.83) & (-.006) & (.191) \\ R2 = .97 & \rho = .933 & D.W. = 1.41 \end{matrix}$$

where $M1^*t = \% \text{ increase in } M1 \text{ above } \% \text{ increase}$

in real growth between "t-1" and "t"

$$\text{Almon(P)} = d1Pt + d2Pt-1 + \dots + d46Pt-45$$

average a one percent increase in excess money creation transmits an impact of less than 1/200 of a percent directly to nominal bond rates, and even then, in a negative direction.

In only three cases out of eighteen are the coefficients of "M*t" and "M*t-1" significant, and in no case is the coefficient significant when associated with lags of more than one month. The "R2" statistic (explained variation) is increased slightly with the addition of extra explanatory variables (for example, compare Tables III and IV), but such is virtually always the case as more right hand side variables are added, and the increase is modest at best.

Perhaps most damaging to the assumptions of the multivariable hypothesis is the sign of the lagged excess money variables. They are negative, which would seem to suggest a temporary lowering of nominal rates following monetary expansion. This effect is theoretically sensible in the short run, but should not be included as part of the Fisher Effect, which deals with a situation where all markets have adjusted in real terms to a monetary shock. The small size and general insignificance of the coefficients, however, suggests that the success of a policy designed to lower interest rates significantly through monetary expansion was very modest, and short lived over the period examined, if employed for that reason.

As might be recalled, the major criticism of the single variable hypothesis was that, on average, it underpredicted the size of the inflationary premium of nominal bond rates, if anything. The finding that a multivariable model would leave the prediction unchanged, then, and in addition would suggest a very small decrease in nominal rate, does nothing to encourage the view that the multivariable adaptive expectations model -- at least in the form tested -- yields a more credible estimate of the Fisher Effect.

The evidence from Tables IV, V and VI -- especially the tiny and often insignificant coefficient associated with "M*t" -- would seem to indicate that monetary information is translated quickly, if not immediately, into the inflationary component of nominal bond rates. While such a finding is of interest to the adaptive expectations model, the real impact of such a situation (assuming it to be the case), falls on the hypothesis to be tested in Part II, "rational expectations". Indeed, the rapid incorporation of monetary information into price level changes is a critical assumption to that theory. We will return to this, and related assumptions, in Part II, but before leaving the discussion of adaptive expectations, a few general remarks might be made in closing.

I.5. Summary of Part I

In this part of the paper, aspects of the Adaptive Expectations hypothesis were tested for Canada. It was determined that a lag of 47 months in price level changes was superior to shorter time horizons (in contrast to the U.S. findings of Yohe and Karnosky) on grounds of stability of R² and coefficient magnitudes, and by examination of "t" scores, with respect to generating inflationary expectations.

Almon Lag estimated coefficients were examined to ensure that weights selected conformed generally to theory. This was important since the procedure weights the independent variables according to maximum explanatory power of the dependent variable, and as such almost any weighting pattern may emerge from the process. It was determined, however, that in this case the generated coefficients declined in a manner well suited to theory. As a result the technique was employed in constraining the impact of past inflation rates upon expectations generation.

Constant values were estimated for Canadian nominal interest rates, which under the adaptive expectations model should correspond roughly to the "real rate" component of said nominal rates. This of course assumes that real interest rates have remained constant over time, a simplifying assumption incorporated into this study. Estimates of Canadian real rates of interest were found to be close to published American real

bond rate findings.

Evidence was found that the real rate of return tends to increase with the length to maturity of the asset - which one might predict from theory, assuming that more liquid assets are preferred to less liquid assets.

Inconclusive evidence was found with respect to the impact of expectations entering nominal interest rates - in no case was an expectations coefficient value found significantly different than 1.00, or perfect adaptation, but at the same time estimates of this coefficient were consistently greater than one. The lack of estimated values less than 1.00 lead to the strong suspicion that the generated variable, P^{**t} , derived solely from past price levels changes, was less than satisfactory; as such a new expectations hypothesis, termed the multivariable adaptive expectations hypothesis, which took explicit account of information besides past and present inflation rates, was tested.

It was determined that the postulate, of information not already incorporated into past and present inflation rates, but shaping expectations of inflation, was not statistically sound. Rather, it would seem that information is rapidly transmitted from the monetary sector to price level changes, which is consistent with the single variable hypothesis.

One statistical point might also be made. Throughout Part I, the phenomenon of autocorrelation persisted when estimates of

the Fisher Effect were run. Such a problem is not unusual in time series data, but the inability of either the Hildreth-Lu (H-L) or Cochrane-Orcutt (C-O) techniques to remove said autocorrelation to a safe extent on several occasions, led to a question of why the problem was so severe. In an attempt to shed light in this area, the Box-Jenkins (univariate) technique¹⁶ was applied to residuals from regressions undertaken, as well as to the nominal rates of interest themselves.

Analysis of the residuals led to no new answers. The remaining autocorrelation after H-L or C-O was applied, was picked up by the Box-Jenkins technique, but no autoregressive structure could be identified whose constant or moving average terms were statistically significant from zero at the 95% confidence interval.

Analysis of the autoregressive structures of the nominal bond rates themselves proved interesting, however. Table VII displays the best structural autoregressive equations estimated for each bond rate.¹⁷ The important point to note from Table VII is that the error structure for all nominal bond rates except the 30 Day rate is of the moving average variety. The 30 Day nominal rate is first order autoregressive, (AR(1)), structured,

¹⁶ Box, G. and Jenkins, G. Time Series Analysis, Forecasting and Control, Holden-Day, San Francisco, 1976.

¹⁷ The procedure used was "ESTIMATE" from the Econometric Software Package program.

Table VII

RNt Subjected to Box-Jenkins Analysis

1) $\text{Day30t} = .121 + .980 \text{Day30t-1} + \text{et}$
 (1.55) (82.85)

R2 = .947 Chi squared (12) = 26.2 (10 d. of f.)
 Chi squared (24) = 46.3 (22 d. of f.)

2) $\text{Day90t} = .000 + 2 \text{Day90t-1} - \text{Day90t-2} + .989\text{et-1} + \text{et}$
 (.110) (669.4)

R2 = .950 Chi squared (12) = 27.7 (10 d. of f.)
 Chi squared (24) = 52.4 (22 d. of f.)

3) $\text{Mon3t} = .000 + 2 \text{Mon3t-1} - \text{Mon3t-2} + .776 \text{et-1} + \text{et}$
 (.079) (20.64)

R2 = .961 Chi squared (12) = 26.1 (10 d. of f.)
 Chi squared (24) = 49.5 (22 d. of f.)

4) $\text{Mon6t} = .001 + 2 \text{Mon6t-1} - \text{Mon6t-2} + .680 \text{et-1} + \text{et}$
 (.711) (21.81)

R2 = .974 Chi squared (12) = 4.9 (10 d. of f.)
 Chi squared (24) = 21.3 (22 d. of f.)

5) $\text{Year1t} = .000 + 2 \text{Year1t-1} - \text{Year1t-2} + .612 \text{et-1} + \text{et}$
 (.723) (78.53)

R2 = .965 Chi squared (12) = 15.0 (10 d. of f.)
 Chi squared (24) = 30.8 (22 d. of f.)

(Table VII con't next page)

period moving average error pattern. Such a structure implies an infinitely declining series in autoregressive format, after successive " ϵ_{t-i} " 's ($i=1,2,3,\dots$) are substituted for. We will deal in depth with such a substitution in the final section of Part II, so it will not be discussed further here, except to note that given a declining series error structure, the inability of the Hildreth-Lu and Cochrane-Orcutt procedures is not surprising. They estimate one term " r " which must approximate an infinite series of declining coefficients, and hence will never fully capture, and remove autocorrelation ideally. Again, the Tables in Part I reflect this estimating inability of the H-L and C-O techniques by a low Durbin-Watson statistic after correction has been undertaken.

At the time of writing, a multivariate Box-Jenkins computer package -- which would be useful in dealing with such an autocorrelation situation -- was not available for use. As such, data were adjusted to remove first order autocorrelation, and the results reported. One should be aware, however, that the swings in coefficient estimates before and after H-L or C-O procedures were applied, may be, in part, a reflection of the underlying autocorrelation structure itself.

D. The Fisher Effect Under Rational Expectations Assumptions

II.1 Introduction

In this part of the study, models falling under the heading of "rational expectations" will be used to statistically generate estimates of the inflationary expectations component of nominal bond rates, P^*t in equation (1). The major model of Part II will be detailed in the next section of the paper, but before becoming immersed in specifics it would seem prudent to briefly clarify the critical differences between the family of models known as rational expectations, and those which are classified under the heading of adaptive expectations. This section will also serve to outline the more important assumptions incorporated into all of the models which we will be testing in this part.

Rational Expectations models are based upon the idea that a decision maker at a given point in time t will formulate an estimate about the behavior in a variable as of period $t+1$, based upon information relevant to his future decision which is available at time t . The decision maker is "rational", then, in the sense of recognizing that the occurrence of some event during period t , will lead to a predictable outcome in period

"t+1" (subject, of course, to some probability, since the future is never perfectly certain). Ideally, a decision maker would want to collect all such possible indicators of the future, but costs associated with the collection and interpretation of data would dictate that information be collected only to the point where the marginal value of collection would equal expected marginal value of a correct prediction.

This assumption differs sharply from the adaptive expectations method of examining "trends" in past values of a variable or variables. Current information is included under these models only if already incorporated into the behavior of a key variable, and even then, its predictive power is diminished greatly when the value of the key variable is weighted by a coefficient less than one. It is interesting to note, however, that in a situation where no information about the future existed, (or where the collection and/or interpretation of data was so expensive that the cost of even the first unit of information exceeded its expected marginal value), a rational person would incorporate no current information apart from trends in the variable of interest -- in other words, rational expectations would become adaptive expectations.

In an attempt to further clarify the difference between approaches in the two theories, the following example might be offered. Suppose a baseball game is being played under cloudy skies. In the second inning rain begins to fall and the game is

halted. A decision must be made as to waiting and attempting to finish the game or calling it off completely. (Thus, as of time "t", a decision must be made regarding "t+1" -- in this case "t" being equal to perhaps one hour.) Under adaptive expectations, one would weight the fact that it had not rained in any of the previous forty-eight hours, and a decision would be arrived at using some weighting scheme for that information. Under rational expectations, one would want to know the wind conditions, the probability of clearing in the area an hour after the rain started (based on past observations) or a weather forecast, which would incorporate all such information.¹⁸

The decision reached, be it to wait or postpone the game will always have a probability of being the correct or incorrect one. It would seem reasonable to assume, however, that in the majority of cases, the decision dictated by rational expectations would be that followed by most decision makers. Such reasoning is based upon the logic that weather can be predicted fairly accurately one hour in advance based upon current information about the future.

The problem with predicting inflation during period "t+1" from period "t" -- which is the nature of the Fisher Effect -- is clearly not as straightforward as was the baseball example.

18 The weather forecast is an example of an economy of scale, with regard to collecting, analyzing and distributing current information. Such services are available in many prediction areas, including of course, inflationary forecasting.

For one thing, the period of prediction (one month, in this study) dictates a lower probability of being correct than a period ahead of only an hour, as was the earlier case. Secondly, one cannot "see" inflation coming over the horizon as easily as clouds or sunlight. Our measuring devices are somewhat more suspect in relation to future inflation than weather. Thirdly, it is arguable whether the average person understands how to "rationally" predict inflation; i.e. there is a question as to the decision makers' ability to correctly perceive the relevant information to his decision. [One could argue, for example, that in a situation where no one understands why an event occurs, that adaptive expectations might well be the method employed when forecasts of another such event occurring are made.]

As a means of dealing with these and other potential problems arising from the rational expectations approach, two major assumptions will explicitly be made about the Canadian economy when testing is undertaken. First, we will assume that information about the activity of government -- specifically in the money creation areas, is current, and reported accurately enough that decision makers can collect such information confidently. Further, we will assume the cost of collection is not prohibitive. Neither of the above assumptions seems unrealistic, but without them, the theory of rational expectations becomes suspect.

Secondly, we will assume for the purposes of this paper that enough decision makers understand the economy, that no consistent misunderstanding of signals occurs. Thus, we assume that an increase in money creation over and above real growth in the economy will be viewed as an inflationary pressure. This is similar to an assumption that markets clear instantaneously, including capital markets, or that prices are perfectly flexible. Given the evidence of Part I, (that increases in excess money creation over real growth led to a virtually insignificant effect upon bond rates outside of the effect upon current inflation rates themselves), this assumption, too, seems not to be a dangerous one.

With these overall assumptions in mind, we turn in the next section to the specific model employed where Canadian data were tested.

II.2 The Rational Expectations Model

The model tested in this section of Part II follows closely from work published by Michael Mussa¹⁹ (Journal of Monetary Economics, 1975). We begin by assuming a money demand function

¹⁹ Mussa, Michael. op cit. Adaptive and Regressive Expectations in a Rational Model of the Inflationary Process.

employed by Phillip Cagan²⁰ and others:

$$(15) \text{MDt} = \text{Yt} + \text{Pt} - v\text{P}^*t$$

- where
- MDt = log of money demanded at "t"
 - Yt = log of real economic growth as of time "t"
 - Pt = log of price level as of time "t"
 - P*t = expected rate of inflation as of time period "t"
 - v = partial income elasticity with respect to interest rates²¹

²⁰ Cagan, Phillip. The Monetary Dynamics of Hyperinflation, in M. Friedman ed. "Studies in the Quantity Theory of Money", University of Chicago Press, Chicago. 1956.

²¹ The elasticity may be derived from equation (15):

$$\frac{\text{MDt}}{\text{Pt}} = \text{Yt} e^{-v\text{P}^*t}$$

Taking natural logarithms and differentiating, and then dividing both sides by "d ln P*t"; and finally multiplying and dividing the R.H.S. by " M/P " and " P "(dropping time subscripts for convenience) :

$$v = \frac{- \widehat{(M/P)}}{\widehat{(P^*t)}} \times \frac{P^*t}{(M/P)}$$

Since inflation is the expected percent change in price levels between "t" and "t+1", and since at a given time "t" a certain information set exists, we can write:

$$(16) P^*_t = E_t [P_{t+1} - P_t \uparrow I_t]$$

where I_t = information set as of time "t"

$E_t []$ = expected value operator as
of time "t"

Assuming markets clear continuously (i.e. money supplied = money demanded), and dropping the information set symbol for convenience, substitution of (16) into (15) yields:

$$(17) M_t = Y_t + P_t - v E_t [P_{t+1} - P_t]$$

By manipulating (17), we obtain:

$$(18) P_t = 1 / (1+v) (M_t - Y_t + E_t [P_{t+1}])$$

Since an equation for period "t" exists, it follows that period "t+1" relationships between price level and right hand side variables would be generated identically:

²¹(cont'd) where " \wedge " represents "percentage change".

$$(19) P_{t+1} = 1 / 1+v (M_{t+1} - Y_{t+1} + E_{t+1} [P_{t+2}])$$

Assuming $E_t E_{t+1} = E_t$:

$$(20) E_t [P_{t+1}] = 1 / 1+v E_t [M_{t+1} - Y_{t+1} + E_t [P_{t+2}]]$$

Using the identity established in (18), one can subtract the left side of (18) from the left hand side of (20), and the right hand side of (18) from the right hand side of (20); and keeping in mind that " $E_t [M_t] = M_t$ ", and " $E_t [Y_t] = Y_t$ ", we obtain:

$$(21) E_t [P_{t+1} - P_t] = 1 / 1+v E_t [(M_{t+1} - M_t) - (Y_{t+1} - Y_t) + E_t [P_{t+2} - P_{t+1}]]$$

Updating (20) continuously (i.e. for "t+2", "t+3" and so on) and substituting into the right most component of (21), " $E_t [P_{t+1+i} - P_{t+i}]$ " ($i = 1, 2, \dots$), we can obtain an expression stretching infinite periods into the future. We assume, however, that after some point "t+r" the weighted expectation of future increases in excess money creation over and above real growth

"Et [(Mt+r+1 - Mt+r) - (Yt+r+1 - Yt+r)]" becomes insignificant. After such infinite substitution for "Et [Pt+1+i - Pt+i]" (i = 1,2,.....), Equation (21) expands to:

$$(22) \text{ Et } [Pt+1 - Pt] = 1/1+v \sum_{j=0}^{\infty} \text{ Et } [(Mt+j+1 - Mt+j) - (Yt+j+1 - Yt+j)] (v/v+1) \exp j$$

Equation (22) states that the expected percent change in inflation between periods "t" and "t+1" is determined from a weighted average of current beliefs about future increases in money created above real economic growth. The weighting term "(1/1+v) * (v/v+1) exp j" can be seen to diminish rapidly as "j" becomes large, suggesting that only near future activities in excess monetary creation play an important role in expectations formation; i.e. that "t+r" is not a distant future horizon.

To this point nothing has been said with regards to how a decision maker might arrive at an empirical estimate of "Et [Pt+1 - Pt]". Indeed, unless something is known about the process by which "Mt+1" and "Yt+1" are generated, the theory is of little practical importance. Fortunately, knowledge is available about both terms; "Mt+1" is known to be a function of the monetary authority's decision regarding how large the money supply should be at time "t+1", and "Yt+1" can be shown (and will be shown later) to follow a distinct, and to a large extent

predictable, path through time.

To follow Mussa's reasoning, an equation describing monetary growth might appear as follows:

$$(23) M_{t+1} - M_t = m_{t+1} + \Delta m_{t+1} - y_{et} + e_{t+1}$$

where m_{t+1} = long term monthly growth
decided upon by the monetary authority

Δm_{t+1} = change in long term growth
rate as of time "t+1",
assumed to be distributed
normally with mean zero
and finite variance

y_{et} = deviation of actual growth
in the money supply from
the target " $m_t + m_t$ ",
partially compensated for
in period "t+1" by a fraction "y"

e_{t+1} = random error term as of
time "t+1", distributed
normally with mean zero
and finite variance

Equation (23) has four major right hand side components and each deserves comment. First, " m_{t+1} " represents the long term growth rate decided upon by the monetary authority, and as such, could equal " m_t ". This long term target growth rate may or may not be announced, but, as will be seen, it can be inferred statistically from behavior of the money stock over time. Second, the term " Δm_{t+1} " represents a decision as of period " $t+1$ " to alter the long term growth rate of the money stock. This term is not easily measured, since it is indistinguishable from the last term " e_{t+1} " -- both of which are assumed random normal variables, with mean zero and finite variance. The remaining term " e_t " represents the error, known to the monetary authority, in achieving the desired growth rate in period " t " (" $m_t + \Delta m_t$ "). It is assumed under this model that some portion of this error, " y ", is compensated for by the monetary authority during the subsequent period " $t+1$ ". This term should also be statistically detectable, even if unannounced.

An equation for " Y_{t+1} " can similarly be postulated. Logically, such an equation would seem to fit one of two forms:

$$(24) \quad Y_{t+1} - Y_t = b_{t+1} + u_{t+1}$$

or

$$(25) \quad Y_{t+1} - Y_t = b_{t+1} + d_{ut} + u_{t+1}$$

Equation (24) characterizes the general assumed behavior of real G.N.P. growth - i.e. a random walk over time about a constant growth trend. Equation (25), on the other hand, displays cyclical behavior in percentage increases in real growth, where an error in expected growth tends to be consistently carried over into future growth rates, until such time as factors outside the equation cause a negative " u_t ", which then passes on into " Y_{t+1} " as a recessionary influence. As will be seen, monthly measures such as the Industrial Index of Production and Real Domestic Product display behavior similar to (25).

Knowing the hypothesized structure of " M_{t+1} " and " Y_{t+1} " allows a great simplification of equation (22) to be made. Assuming, as mentioned, that beyond some future period " $t+r$ ", the impact of future changes in monetary and real growth activity are inconsequential, the final term of interest in equation (22) becomes:

$$(26) \quad \frac{1}{v+1} E_t [(M_{t+r+1} - M_{t+r}) - (Y_{t+r+1} - Y_{t+r})] (v/v+1) \exp r$$

Substituting the behavior of monetary and real growth hypothesized in equations (23) and (24), into (26), leads to:

$$(27) \quad \frac{1}{v+1} E_t [(M_{t+r} + m_{t+r+1} + \Delta m_{t+r+1} - \\
\text{yet}_{t+r} + e_{t+r+1} - M_{t+r}) - (Y_{t+r} \\
+ b_{t+r+1} + d_{t+r} + u_{t+r+1} - Y_{t+r})] \\
* (v/v+1) \exp r$$

By assumption:²²

- 1) $E_t [\Delta m_{t+r+1}] = 0$
- 2) $E_t [\text{yet}_{t+r}] = y E_t [e_{t+r}] = 0$
- 3) $E_t [d_{t+r}] = d E_t [u_{t+r}] = 0$
- 4) $E_t [e_{t+r+1}] = 0$
- 5) $E_t [u_{t+r+1}] = 0$
- 6) $E_t [m_{t+r+1}] = \hat{m}_t + r E_t [\Delta m_t] \\ = \hat{m}_t$
- 7) $E_t [b_{t+r+1}] = \hat{b}_t + r E_t [\Delta b_t] \\ = \hat{b}_t$

which means the final term solves to :

$$(28) \quad \frac{1}{v+1} (\hat{m}_t - \hat{b}_t) * (v/v+1) \exp r$$

²² An additional assumption might have been added: " $E_t [b_{t+r+1}] = 0$ ", but the term b_{t+r+1} was deleted from equation (27) due to the assumed constant growth nature of G.N.E.

Applying expression (27) recursively for each of the remaining "r-1" terms, and employing similar logic that all expected future values of "mt+i", "et+i", and "ut+i", (where i = 1,2, ..."r-1") would be equal to zero at time "t", the solution to (22) appears as follows:

$$(29) \quad E_t [P_{t+1} - P_t] = \frac{1}{v+1} \sum_{j=0}^{\infty} E_t [(m_{t+1} - y_{t+1}) - b_{t+1}] (v/v+1)^j \exp j$$

which further simplifies to:

$$(30) \quad E_t [P_{t+1} - P_t] = (\hat{m}_t - (1/v+1) \hat{y}_t) - \hat{b}_t$$

Equation (30) represents the statistical equation we will employ when generating the inflationary expectations component of nominal bond rates, in the next section of this study. The reasoning underlying such an equation is straight-forward: the rational decision maker expects inflation in future to be a reflection of government's long term excess monetary growth decisions compensated by a correction factor if desired monetary growth was not equal to average growth for the present period. It should be noted that the terms " \hat{m}_t ", " \hat{b}_t ", and " \hat{y}_t " are not necessarily known values, they represent the decision makers best estimate of the true parameter, based upon experience

and/or current information.

II.3. Empirical Estimates as to the Behavior of Monetary and Real Growth Rates Over The Period 1955 - 1978

In the preceding section, equations such as (23), (24), and (25) were suggested as possible processes for explaining the movements over time in the monetary and real growth sectors of an economy. As a first step towards testing the Fisher Effect for Canada, two series of monetary aggregates (M1 and M3, standardly defined) and three indices of real growth (G.N.E., Index of Industrial Production and Real Domestic Product, all base year 1961) were subjected to the Box-Jenkins technique to determine the autoregressive structure embodied in each series.

Tables VIII and IX present the statistical results of those runs. All data were arbitrarily broken into separate sub-series representing pre-June 1, 1970 and post-June 1, 1970. It was decided that the Canadian monetary authority's decision to change from a fixed to floating dollar as of that date was significant enough to warrant separate analysis, and from examination of the coefficients on both the constant and moving average terms, such a decision appears valid. It might be noted that results for the G.N.E. index are not listed. No significant break was found in this series before and after June 1, 1970; the autoregressive structure of the index over the entire period

1955 - 1978 appeared as follows:

$$(31) \quad Y_{t+1} - Y_t = 1.834 + u_t$$

Since G.N.E. data is published in quarterly form, converting " $Y_{t+1} - Y_t$ " into monthly estimates was accomplished by assuming a constant monthly trend in growth, " \hat{b}_t ", equal to one third that of the constant quarterly growth estimate, 1.834 percent, or " $\hat{b}_t = .612$ ".

As Table VIII shows, the selected monetary aggregates behaved in a fashion very similar to that predicted by Mussa's theory. "M3" for example, exhibited a strong constant growth component over the two time series examined, as well as a one period adjustment term, where " y " = .423 and .289 for the periods 1955-1970(5) and 1970(6)-1978 respectively. The Chi-Square statistics indicate that during the previous twelve month period, almost no significant autocorrelation remains unexplained by the equation listed.

The monetary aggregate "M1", however, exhibited behavior less well predicted. For the period 1955-1970(5), a strong constant and one period lag correction component were isolated, but a two period lagged error term whose coefficient value was positive and significantly different from zero, was also found to be present. With respect to the latter M1 sub-period, 1970(6)-1978, two lagged error terms were again found to be

Table VIII

Autoregressive Structure of Monetary Aggregates

1) 1955 -- 1970(6) :

$$\%M1_t = .381 - .289 \text{ et-1} + .179 \text{ et-2} + \text{et}$$

(2.69) (-3.97) (2.46)

R2 = .109 Chi squared (12) = 101.6 (9 d. of f.)
 Chi squared (24) = 193.4 (21 d. of f.)

2) 1970(6) -- 1978 :

$$\%M1_t = .955 + .037 \text{ et-1} + .422 \text{ et-2} + \text{et}$$

(8.18) (4.04) (4.56)

R2 = .069 Chi squared (12) = 28.8 (9 d. of f.)
 Chi squared (24) = 46.6 (21 d. of f.)

3) 1955 -- 1970(6) :

$$\%M3_t = .568 - .423 \text{ et-1} + \text{et}$$

(6.69) (-6.29)

R2 = .139 Chi squared (12) = 15.4 (10 d. of f.)
 Chi squared (24) = 37.6 (22 d. of f.)

4) 1970(6) -- 1978 :

$$\%M3_t = 1.320 - .289 \text{ et-1} + \text{et}$$

(12.72) (-2.84)

R2 = .056 Chi squared (12) = 28.4 (10 d. of f.)
 Chi squared (24) = 45.6 (22 d. of f.)

Table IX

Autoregressive Structure of Growth Indices

1) 1955 - 1970(6) :

$$\%IIP_t = .551 + .228 e_{t-1} + .449 e_{t-2} + e_t$$

(8.42) (4.22) (6.88)

R² = .255 Chi squared (12) = 117.5 (9 d. of f.)
Chi squared (24) = 229.4 (21 d. of f.)

2) 1970(6) - 1978 :

$$\%IIP_t = .494 + .442 e_{t-1} + .356 e_{t-2} + e_t$$

(5.12) (4.74) (3.76)

R² = .259 Chi squared (12) = 66.7 (9 d. of f.)
Chi squared (24) = 157.2 (21 d. of f.)

3) 1955 - 1970(6) :

$$\%RDp_t = 1.065 + .982 e_{t-1} + e_t$$

(33.54) (96.36)

R² = .415 Chi squared (12) = 90.6 (10 d. of f.)
Chi squared (24) = 157.2 (22 d. of f.)

(Table IX con't next page)

Table IX (con't)

4) 1970(6) - 1978 :

$$\%RDp_t = .731 + .909 e_{t-1} + e_t$$

(10.53) (24.20)

R² = .404 Chi squared (12) = 79.0 (10 d. of f.)
Chi squared (24) = 139.1 (22 d. of f.)

where %IIP = percent change in Industrial Index of
Production between periods "t" and "t-1"
%RDP = percent change in Real Domestic Product
between periods "t" and "t-1"

present and associated with coefficients significantly different from zero. In this instance, however, both coefficients were positive, and not negative as was expected. Further, the Chi-Square test in the 1955-1970(5) series showed that large amounts of autocorrelation remained even after explicit consideration was made of the second error term.²³

The presence of positive lagged error components in a monetary aggregate raises a theoretical problem. These terms

²³ The presence of autocorrelation indicated the need of first or even second differencing before "strict" stationarity could be achieved. As will also be the case later, however, the structure " Mt+1 - Mt " was deemed desirable enough that some remaining autocorrelation could be traded off against the added benefit of having the estimate in percentage change form.

would seem to suggest a cyclical nature to money creation, where unexpected growth in the money supply led to even larger future growth, rather than a "cutting back" by the monetary authorities. Such variation might be related to the movements of Canada's floating dollar over the period 1970(6)-1978, but a satisfactory theoretical solution is difficult to formulate.

Regardless of the theoretical problems suggested by the latter behavior of the M1 series, one very important point emerges from Table VIII - each data series followed a consistent and statistically stable pattern through time, over the period examined. A rational decision maker as of time "t", then, could conceivably have arrived at an estimate as to the longterm growth component and autoregressive structure of each monetary series, leading to a further estimate regarding " $E_t [M_{t+1}]$ ", using only current information at time "t". This is not to suggest the estimates would be exact for any period, but, over time, such behavioral relationships would enable the analyst a better probability for a correct guess about a variable's value as of time "t", than any averaging process would be expected to. Hence, the behavior of monetary growth in Canada would be compatible with the rational expectations model as set out to this point in Part II of the paper.

As Table IX shows, the real growth components of the economy, over the period examined, also reacted in a predictable, moving average fashion. All series exhibited a

positive constant growth trend and at least one significant moving average coefficient term. In the case of real growth indices, the sign of the moving average term was postulated as being positive, consistent with observations of business cycles. In this matter, estimated equations for real growth conformed well with theory. The only statistical problem in Table IX is indicated by the Chi-Square statistic, indicating in all cases shown that some autocorrelation remained unaccounted for by the structure tested. However, the equations shown are compatible with the formulation " $Y_{t+1} - Y_t$ ", and as such, are in a desirable structure for testing.

As shown, therefore, two monetary aggregates, M1 and M3, and three real growth indices G.N.E., Industrial Index of Production, and Real Domestic Product, exhibit characteristics over time which are conducive to estimating excess monetary growth in period "t+1", subject to some probability of error.²⁴ A problem now presents itself in determining which of the six potential combinations of M1, M3, G.N.E., I.I.P., or R.D.P., a rational decision maker would select for estimating future excess monetary creation -- each could act as a suitable

24 The probability of a correct estimate in period "t+1" should greatly increase under such circumstances, compared with an estimate based solely upon past "trends". The past success of the adaptive expectations hypothesis in this respect would seem somewhat puzzling, therefore, but as will be shown later, the error structure of inflation rates allows comparable accuracy to be derived from either the adaptive or rational expectations approaches.

candidate when testing the Fisher Effect in a rational expectations model such as Mussa suggests. The criterion decided upon, however, was the past success rate of each predictor, vis-à-vis observed inflation in period "t+1".

II.4. Empirical Tests of the Fisher Effect Under Rational Expectations Assumptions: The Mussa Model

Before the testing of an equation similar to (12) in Part I could be undertaken, an estimate was required for the expression " $\hat{m}_t - (1/1+v) \hat{y}_t - \hat{b}_t$ ". As mentioned in the previous sub-section, two monetary aggregates were found to generate stable values of " \hat{m}_t " and " \hat{y}_t ", and three real growth series offered potential terms " \hat{b}_t " and " \hat{d}_t ".

As a preliminary measure, each of the six combinations²⁵ suggested by these findings was regressed against the actual rate of inflation one period in future, to determine which estimators performed best in predicting inflation. The regressions took the form:

$$(32) \quad \widehat{CPI}(t+1) = a + b(\widehat{M} - \widehat{Y}) + e_{t+1}$$

²⁵ The six combinations alluded to are: 1) $\widehat{M1} - \widehat{G.N.E.}$; 2) $\widehat{M1} - \widehat{I.I.P.}$; 3) $\widehat{M1} - \widehat{R.D.P.}$; 4) $\widehat{M3} - \widehat{G.N.E.}$; 5) $\widehat{M3} - \widehat{I.I.P.}$; and 6) $\widehat{M3} - \widehat{R.D.P.}$.

where \hat{M} = $\hat{M1}$ and $\hat{M3}$ respectively
 \hat{Y} = G.N.E., I.I.P. and R.D.P.

separately

ϵ_{t+1} = random error term in period "t+1",
 assumed to be distributed normally
 with mean zero and finite variance

One would assume that the estimator which best predicted inflation over time should be that chosen by rational decision makers as the most desirable indicator of future inflation.

The results of these six runs are listed in Table X, and Table XI after removal of autocorrelation. As is clear from these Tables, Equations (5) and (6) employing future estimates of M3 and M1 percentage growth minus expected growth in real domestic product, do not predict inflation well. The coefficients attached to the prediction term, in fact, do not prove statistically different from zero.

On the other hand, future estimates of percent M3 growth minus the expected real growth term generated by G.N.E. data, shown as Equation (1), performed well in relation to those tested. After autocorrelation was removed, for example, the coefficient corresponding to the prediction term in equation 1, Table XI, proved some seven times greater than the next largest

Table X

$$\text{OLSQ : } \%CPIt = a + b (\hat{M} - \hat{\text{Growth}})$$

1955-78

- | | | |
|----|--|---|
| 1) | $\%CPIt = .206 + .313 M3EXt + et$ $(6.95) \quad (6.86)$ $R2 = .142$ | $\ \ \quad M3Ex = f(\hat{M3} - \hat{Y})$ $D.W. = 1.42$ |
| 2) | $\%CPIt = .322 + .108 M1EXt + et$ $(13.84) \quad (3.84)$ $R2 = .050$ | $\ \ \quad M1EX = f(\hat{M1} - \hat{Y})$ $D.W. = 1.24$ |
| 3) | $\%CPIt = .337 + .034 M3EXt + et$ $(14.67) \quad (3.12)$ $R2 = .033$ | $\ \ \quad M3EX = f(\hat{M3} - \hat{IIP})$ $D.W. = 1.10$ |
| 4) | $\%CPIt = .342 + .028 M1EXt + et$ $(14.98) \quad (2.89)$ $R2 = .029$ | $\ \ \quad M1EX = f(\hat{M1} - \hat{IIP})$ $D.W. = 1.09$ |
| 5) | $\%CPIt = .414 + .006 M3EXt + et$ $(15.82) \quad (1.71)$ $R2 = .014$ | $\ \ \quad M3EX = f(\hat{M3} - \hat{RDP})$ $D.W. = 1.09$ |
| 6) | $\%CPIt = .412 + .006 M1EXt + et$ $(15.65) \quad (1.62)$ $R2 = .012$ | $\ \ \quad M1EX = f(\hat{M1} - \hat{RDP})$ $D.W. = 1.06$ |

where EXT = excess monetary creation predicted as of period "t-1" from autoregressive structure shown as " $f(\hat{M} - \hat{\text{Growth}})$

Table XI

Corc: $\%CPI_t = a + b (\hat{M} - \hat{\text{Growth}}) + et$

1955-78

- 1) $\%CPI_t = .279 + .147 M3EX_t + et$ ¶¶ $M3EX_t = f(\hat{M3} - \hat{Y})$
 (6.97) (2.85)
 R2 = .23 Rho = .382 D.W. = 2.07
- 2) $\%CPI_t = .339 + .006 M1EX_t + et$ ¶¶ $M1EX_t = f(\hat{M1} - \hat{Y})$
 (8.68) (.247)
 R2 = .23 Rho = .476 D.W. = 2.15
- 3) $\%CPI_t = .339 + .022 M3EX_t + et$ ¶¶ $M3EX_t = f(\hat{M3} - \hat{IIP})$
 (9.07) (2.08)
 R2 = .23 Rho = .456 D.W. = 2.14
- 4) $\%CPI_t = .340 + .015 M1EX_t + et$ ¶¶ $M1EX_t = f(\hat{M1} - \hat{IIP})$
 (8.93) (1.72)
 R2 = .23 Rho = .469 D.W. = 2.16
- 5) $\%CPI_t = .413 + .006 M3EX_t + et$ ¶¶ $M3EX_t = f(\hat{M3} - \hat{RDP})$
 (9.68) (2.07)
 R2 = .22 Rho = .455 D.W. = 2.14
- 6) $\%CPI_t = .409 + .006 M1EX_t + et$ ¶¶ $M1EX_t = f(\hat{M1} - \hat{RDP})$
 (9.34) (1.87)
 R2 = .23 Rho = .471 D.W. = 2.10

where EXt = predicted excess monetary creation
as of time "t-1"

prediction coefficient (.147 vs. .022). Table X also suggests that in Equation (2), expected percentage increases in M1 minus expected growth in G.N.E. fared relatively well in testing. In the process of removing first order autocorrelation via the Cochrane-Orcutt technique, however, a surprising drop occurred in both the magnitude and significance of the prediction term (i.e. .108 in Table X, to .006 in Table XI).

Another point is worth noting about Tables X and XI. Generally, as one scans equations (1) to (6) vertically, two trends seem evident, 1) equations employing M3 estimates outperform those incorporating M1 predicted values; and 2) expected growth in G.N.E. outperforms the Index of Industrial Production and Real Domestic Product indices as indicators of future inflation. Performance, in this instance refers to the measured size and significance of prediction coefficients, and, before first differencing, explained variation (R^2) in predicting future inflation.²⁶ It should be reasonable to assume, therefore, that a rational decision maker, seeking the best information about inflation in period "t + 1", would be

²⁶ Interestingly, relative success in prediction, attributable to the estimate of real growth component, appears inversely proportional to the sensitivity of the output measure represented by each index. Monthly R.D.P. varied more than I.I.P. (i.e. variance of % R.D.P. = 96.96; variance of % I.I.P. = 18.57) and both predicted poorly compared with a constant expected G.N.E. monthly growth trend.

most interested in the behavior of M1 and M3, over and above the expected constant growth rate in G.N.E..

For the remainder of this study, therefore, findings are not reported when based upon the assumption that a rational decision maker would choose to rely on estimates generated from I.I.P. or R.D.P. when forming inflationary expectations.

27

Having decided upon the most desirable estimates of " \hat{m}_t ", " \hat{b}_t ", and " \hat{y}_t ", based upon the criteria on inflationary prediction records over time, only one further detail required attention before testing could be undertaken -- specifically, the choosing of an estimate of "v" in Equation (30). It was decided that rather than assuming one value for this parameter, (the income elasticity of money holding during inflation), reasonable bounds for possible "v" values should be established, and tests of Equation (30) be carried out between these probable "end points".

The value of "v" should, a priori, be greater than or equal to zero, since income elasticities are not assumed negative for normal goods, including money. As such, "v = 0" was tested as the lower bound. With respect to an upper bound, the value of "v = 2" was selected, following reasoning that

27 Testing was carried out using " b_t " values generated via these "second best" growth series, however, as a check, but were found to be very erratic due to the volatility of the I.I.P. and R.D.P. components of " $E_t [P_{t+1} - P_t]$ ".

the true value of such an income elasticity would most likely fall in the inelastic, or at most, unit elasticity region.²⁸ As such, the probability of committing a Type I statistical error (i.e.) the true parameter value of "v", falling beyond the bound established as the upper end point in the sample was deemed sufficiently small, for "v = 2".

Having specified the method for estimating " $P^*t = Et [Pt+1 - Pt]$ ", an equation such as (33) was tested:²⁹

$$(33) \quad R_{Nt} = a + b_1 P^*t + e_t$$

and the results listed in Tables XII and Table XIII, where Table XII shows the results of generating " \hat{m}_t " and " \hat{y}_t " from the M1 series, and Table XIII from the M3 series.

Perhaps the most important test statistic reported in Tables XII and XIII is the Durbin-Watson statistic, which once again indicated strong autocorrelation in the regression

²⁸ Realistically, the magnitude of "v" should be very close to zero. Recalling the two components of "v" are: 1) $d(M/P) / dP^*t$ and this is multiplied by 2) $(P^*t) / (M/P)$, the product should be a small fraction if only because (2) is a small number.

²⁹ The "hat" values used in the run 1955-1978 were compiled from pre- and post-June 1, 1970 runs, since under rational expectations an assumption is made that a rational decision maker would not use the same inflationary estimation procedure under significantly different information sets -- i.e. the knowledge that currency has been allowed to float in value rather than remain pegged.

results. First order autocorrelation was removed via the Cochrane-Orcutt technique, and the modified equations presented later in the paper, but before leaving Tables XII and XIII, two points should be made.

First, with respect to the autocorrelation noted, it might be recalled that such a characteristic is believed not to lead to bias in estimated coefficients over large samples. While the present examination certainly does not constitute a large sample case, and bias may well be a problem, there should be no consistent error expected in the values of "a" and "b1" as reported. What is anticipated, however, is that the "R2" and "t" statistics are biased upwards by the autocorrelation.

This leads to an interesting possibility suggested by Tables XII and XIII. As was the case in Part I, the estimated coefficient "b1" in equation (33) should always fall between zero (no anticipated inflation compensated for by the nominal bond rate) and one (perfect compensation). The "b1" values from Tables XII and XIII are shown to be almost always significantly larger than one, but it should be noted that the "t" statistics are suspect, due to the above mentioned problem of autocorrelation. If the true "t" value were to be lower than estimated, it is possible the "b1" coefficient, in

Table XII

$$\text{OLSQ : } R_{Nt} = a + bP^*t + e_t \quad \uparrow \quad P^*t = \hat{M1} - \hat{Y}$$

RNt	V = 0	V = 1	V = 2
1. Day 30t	a = 5.59 (37.05) b = .971 (5.93) R2 = .136 D.W. = .234	a = 5.25 (33.42) b = 1.77 (8.09) R2 = .227 D.W. = .232	a = 5.11 (32.02) b = 2.10 (8.86) R2 = .260 D.W. = .180
2. Day 90t	a = 5.55 (43.19) b = 1.08 (7.04) R2 = .155 D.W. = .247	a = 5.25 (40.52) b = 1.93 (9.69) R2 = .257 D.W. = .239	a = 5.14 (39.51) b = 2.26 (10.65) R2 = .295 D.W. = .182
3. Mon 3t	a = 4.56 (37.03) b = 1.02 (6.86) R2 = .143 D.W. = .221	a = 4.30 (34.39) b = 1.81 (9.28) R2 = .235 D.W. = .204	a = 4.20 (33.39) b = 2.12 (10.12) R2 = .267 D.W. = .148
4. Mon 6t	a = 5.20 (39.83) b = .783 (5.43) R2 = .113 D.W. = .191	a = 4.95 (36.03) b = 1.40 (7.91) R2 = .184 D.W. = .179	a = 4.85 (34.57) b = 1.65 (7.81) R2 = .209 D.W. = .136
5. Year 1t	a = 5.11 (50.38) b = .990 (8.01) R2 = .186 D.W. = .290	a = 4.85 (48.57) b = 1.78 (11.41) R2 = .317 D.W. = .292	a = 4.75 (47.98) b = 2.09 (12.72) R2 = .365 D.W. = .218
6. Year 3t	a = 5.46 (55.79) b = 1.01 (8.46) R2 = .203 D.W. = .319	a = 5.18 (54.89) b = 1.84 (12.47) R2 = .356 D.W. = .342	a = 5.06 (54.85) b = 2.17 (14.15) R2 = .416 D.W. = .259

Table XIII

$$\text{OLSQ} = R_{Nt} = a + bP^*t + e_t \quad \uparrow \quad P^*t = \hat{M}_3 - \hat{Y}$$

1955 - 1978

R _{Nt}	v = 0	v = 1	v = 2
1. Day 30t	a = 5.09 (25.07) b = 1.82 (6.37) R ² = .152 D.W. = .232	a = 5.03 (30.36) b = 2.46 (9.33) R ² = .278 D.W. = .109	a = 5.01 (30.18) b = 2.51 (9.47) R ² = .284 D.W. = .096
2. Day 90t	a = 5.14 (31.61) b = 1.81 (7.38) R ² = .166 D.W. = .252	a = 5.07 (37.92) b = 2.63 (11.28) R ² = .317 D.W. = .108	a = 5.04 (37.81) b = 2.68 (11.46) R ² = .324 D.W. = .094
3. Mon 3t	a = 4.28 (27.07) b = 1.58 (6.59) R ² = .134 D.W. = .178	a = 4.18 (31.91) b = 2.42 (10.42) R ² = .278 D.W. = .070	a = 4.16 (31.81) b = 2.48 (10.64) R ² = .286 D.W. = .060
4. Mon 6t	a = 4.93 (27.60) b = 1.27 (5.00) R ² = .097 D.W. = .142	a = 4.81 (32.20) b = 1.92 (7.94) R ² = .212 D.W. = .076	a = 4.79 (32.04) b = 1.97 (8.13) R ² = .220 D.W. = .067
5. Year 1t	a = 4.81 (37.27) b = 1.59 (8.10) R ² = .189 D.W. = .257	a = 4.72 (46.20) b = 2.40 (13.24) R ² = .383 D.W. = .113	a = 4.70 (46.29) b = 2.46 (13.59) R ² = .396 D.W. = .095
6. Year 3t	a = 5.07 (42.11) b = 1.75 (9.57) R ² = .245 D.W. = .322	a = 5.01 (53.80) b = 2.52 (15.23) R ² = .451 D.W. = .117	a = 4.99 (53.99) b = 2.57 (15.62) R ² = .464 D.W. = .094

actual fact would prove not statistically significant, at the 95% certainty level, from one.

Secondly, the pattern in coefficient values of both "a" and "b1" when "v" is increased from zero to one, and from one to two is of interest. Since the "a" value consistently falls, as expected, and the "b1" value consistently rises, also as would be predicted from the relationship shown in equation (33) as "v" is increased, the "end point" method appears credible. Depending upon one's belief as to the true "v" value, therefore, an estimate may be drawn in relation to the monotonically increasing behavior of the "b1" coefficient, and decreasing behavior of the "a", or constant term, over the range " $0 \leq v \leq 2$ ".

As mentioned, removal of first order autocorrelation was undertaken for equation (33), and these results are presented in Tables XIV and XV - again where Table XIV corresponds to " \hat{m}_t " and " \hat{b}_t " estimates derived from M1, and Table XV, to the same parameters, but estimated from M3. Changes in both constant and coefficient magnitudes between the two previous Tables and the current results are most noticeable. Table XV, in fact, even displays negative "b1" coefficient values -- suggesting that expected increases in excess monetary growth leads on average to a fall in nominal bond rates. In almost all cases, the coefficients derived in Tables XIV and XV cannot be shown significantly different from zero at the 95% probability level,

Table XIV

$$\text{CORC: } R_{Nt} = a + bP^*t + e_t \quad \uparrow \quad P^*t = M1 - Y \quad \wedge \quad \wedge \quad (1955-78)$$

RNt	v = 0	v = 1	v = 2
1. Day 30t	a = 6.69 (4.70) b = .043 (1.13) R2 = .947 D.W. = 1.75 P = .977	a = 6.66 (4.77) b = .078 (1.04) R2 = .947 D.W. = 1.75 P = .977	a = 6.63 (4.82) b = .106 (.954) R2 = .947 D.W. = 1.75 P = .976
2. Day 90t	a = 6.88 (5.97) b = .047 (1.38) R2 = .952 D.W. = 1.65 P = .977	a = 6.84 (6.07) b = .089 (1.29) R2 = .952 D.W. = 1.65 P = .976	a = 6.80 (6.14) b = .123 (1.21) R2 = .952 D.W. = 1.65 P = .976
3. Mon 3t	a = 6.53 (4.19) b = .019 (.538) R2 = .970 D.W. = 1.36 P = .986	a = 6.74 (4.97) b = .013 (.245) R2 = .970 D.W. = 1.36 P = .986	a = 6.69 (5.07) b = .013 (.170) R2 = .970 D.W. = 1.36 P = .958
4. Mon 6t	a = 6.81 (3.59) b = .019 (.661) R2 = .958 D.W. = 1.46 P = .987	a = 6.78 (3.62) b = .034 (.594) R2 = .958 D.W. = 1.46 P = .987	a = 6.74 (3.68) b = .045 (.526) R2 = .958 D.W. = 1.46 P = .987
5. Year 1t	a = 6.53 (7.57) b = .003 (1.16) R2 = .963 D.W. = 1.43 P = .978	a = 6.49 (7.77) b = .050 (1.11) R2 = .963 D.W. = 1.43 P = .978	a = 6.45 (7.90) b = .078 (1.06) R2 = .963 D.W. = 1.43 P = .978
6. Year 3t	a = 7.12 (7.34) b = .010 (.468) R2 = .971 D.W. = 1.44 P = .983	a = 7.05 (7.64) b = .017 (.400) R2 = .971 D.W. = 1.44 P = .983	a = 7.02 (7.76) b = .021 (.333) R2 = .971 D.W. = 1.44 P = .983

Table XV

$$\text{CORC: } R_{Nt} = a + bP^*t + e_t \quad \uparrow \quad P^*t = \hat{M}_3 - \hat{Y}$$

RNt	v = 0	v = 1	v = 2
1. Day 30t	a = 7.30(4.19) b = -.190(-2.39) R2 = .950 D.W. = 1.75 P = .982	a = 7.32(4.17) b = -.253(-.778) R2 = .948 D.W. = 1.73 P = .982	a = 7.24(4.22) b = -.152(-.619) R2 = .948 D.W. = 1.73 P = .982
2. Day 90t	a = 7.51(5.18) b = -.136(-2.19) R2 = .954 D.W. = 1.66 P = .981	a = 7.58(5.24) b = -.312(-1.05) R2 = .954 D.W. = 1.64 P = .983	a = 7.49(5.29) b = -.210(-.936) R2 = .954 D.W. = 1.64 P = .982
3. Mon 3t	a = 7.98(4.09) b = -.065(-1.37) R2 = .971 D.W. = 1.38 P = .990	a = 7.64(4.49) b = -.146(-.633) R2 = .971 D.W. = 1.35 P = .989	a = 7.55(4.54) b = -.082(-.471) R2 = .971 D.W. = 1.35 P = .989
4. Mon 6t	a = 8.51(2.68) b = -.131(-2.15) R2 = .961 D.W. = 1.49 P = .993	a = 8.27(2.92) b = -.202(-.796) R2 = .960 D.W. = 1.46 P = .992	a = 8.16(2.94) b = -.131(-.699) R2 = .960 D.W. = 1.46 P = .992
5. Year 1t	a = 7.98(5.17) b = -.038(-.329) R2 = .971 D.W. = 1.40 P = .988	a = 7.72(5.76) b = -.216(-.977) R2 = .962 D.W. = 1.39 P = .987	a = 7.67(5.77) b = -.159(-.955) R2 = .962 D.W. = 1.36 P = .987
6. Year 3t	a = 7.97(5.60) b = -.013(-.329) R2 = .971 D.W. = 1.44 P = .989	a = 7.72(6.35) b = -.136(-.727) R2 = .971 D.W. = 1.43 P = .988	a = 7.71(6.29) b = -.084(-.600) R2 = .971 D.W. = 1.43 P = .988

and the estimated constant component of nominal bond rates rises in all cases far above any seemingly logical level (i.e. "a = 7.00" is the lowest estimate of real return from Table XV over the period 1955-1978).

It might be noted, as well, that the "rho" value chosen by the Cochrane Orcutt technique is extremely close to one in all cases. This might help to explain the questionable estimates of constant values. Even with such large "rho" values, however, the majority of regression results still exhibit Durbin Watson statistics less than the "1.65" safety margin. As such, many estimates of "R2" and "t" values in Tables XIV and XV remain suspect even after the Cochrane-Orcutt procedure was applied.

The results of testing Equation(33), then, would seem truly inconclusive. On the one hand, Mussa's model appears to generate theoretically logical results as evidenced in Tables XII and XIII. Severe autocorrelation was noted as a potential problem, however. On the other hand, Mussa's theory, at least as tested here, generated near nonsense results when the Cochrane-Orcutt technique was applied. Most disturbing, however, is the drop in "b1" values, from greater than one, to near zero (and even below zero in Table XV). The "t" statistics also undergo severe drops in value -- falls by a factor of ten or more are not uncommon in these Tables.

Clearly any decisions regarding the Fisher Effect and rational expectations, based upon the previous tests, would be

most dangerous. Fortunately, however, the assumptions of rational expectations lead to a variation on Mussa's model and allow additional testing of the relationship between inflationary expectations and Canadian bond rates.

II.5 An Alternate Rational Expectations Model

Mussa's model assumed a decision maker would consistently expect future inflation to correspond to an estimate of " $\hat{m}_t - \hat{y}_t - \hat{b}_t$ ". In this section we will expand our assumption as to rationality somewhat, and obtain a different estimate of the nominal bond rate component " P^*t ". As will be shown, the statistical results generated by such an estimator will appear consistent in many ways to the findings resulting from the estimates of Equation (12) in Part I.

The additional assumption explicitly made in this section of the study is that a rational decision maker will, over time, incorporate into his information set knowledge of past prediction successes and failures. If, for example, a decision maker knows with a 95% probability that his past estimates of inflation when generated by a specific procedure, were consistently twice as large as that actually observed, it would seem sensible to assume that his current estimate of expectation of inflation, " P^*t " would be adjusted to account for the known bias of past predictions.

It would not be difficult to ascertain such a success rate over past predictions -- indeed Tables VIII and IX employed just such a test of the effectiveness of certain predictors in estimating future inflation rates. Over time, therefore, we will assume a rational person would use two kinds of information when generating variable "P*t". First, he would incorporate knowledge as to the behavior of monetary and real growth aggregates at time "t" as Mussa argued. Secondly, however, he would temper any estimate arrived at with knowledge of any consistent (i.e. statistically significant) deviation in the prediction coefficient's value from the assumed value of "b=1", from Table XI.

The best estimate of inflation, incorporating knowledge of past success rates could be mathematically represented in two steps below:

$$(34) \quad CPI_t = a + b_2 [\hat{m}_t - (1/v+1)\hat{y}_t - \hat{b}_t] + u_t$$

$$(35) \quad \hat{CPI}_t = CPI_t - u_t$$

where [] = expected excess money
creation beyond real growth
in period "t + 1"

\hat{CPI}_t = calculated percent change
in CPI which minimizes the
sum of squared error in
O.L.S. regression.

Equation (34) generates an estimate of inflation in the manner hypothesized by Mussa. Equation (35) tempers the variable "P*t" with a fraction of some magnitude, "b2", determined via an O.L.S. regression between actual percentage changes in C.P.I. on the right hand side, and the predicted inflationary values on the left hand side.³⁰

This "modified" estimate, then, consistent with all assumptions of rationality, could be tested against movements in Canadian nominal bond rates:

$$(36) \quad R_{Nt} = a + c \hat{CPI}_t + e_t$$

Such a procedure was carried out, and the results listed in Chart XIII of the Appendix. The same results are shown in Table XVI after the removal of first order autocorrelation via the modified Hildreth-Lu procedure.

30 The "hat" values selected were taken from an O.L.S. regression after modified via H-L to remove first order autocorrelation.

Once again, the "rho" value selected to remove autocorrelation proved troublesome. The estimates of the constant real rate of return are greater than would seem reasonable, and several "t" statistics appear as ".000", due to the selection of "rho = 1.00" values.

The " c " coefficients, however, from Table XVI prove interesting. Although insignificant when "rho" is less than 1.00, they fall between values of "0.00" and "1.00" as would be expected from coefficients associated with the inflationary component of nominal bond rates. In the cases where " rho " was selected equal to one the coefficients generally support theory well, falling in an area between ".200" and "1.00", and showing significant "t" values.³¹ The variability associated with O.L.S. and differenced results throughout the study serves as a caution, however, as to the validity of assuming the true coefficient "c" is accurately represented by estimates in Table XVI. As may be seen between Chart XIII and the current Table, the magnitude and significance of " c " terms, once again depends dramatically upon whether a process to remove autocorrelation has been applied or not.

³¹ While the "t" statistics proved unreliable throughout much of Table XVI, the selection of a " rho = 1.00 " value should not cause bias in coefficient estimation (assuming, of course, that the error structure may be approximated by a first order autoregressive estimate.)

Table XVI

CORC: 1) $R_{Nt} = a_1 + b_1 \hat{CPI}_{t-1} + e_t$ ¶ $CPI_t = f(M_{1t} - Y_t)$
 2) $R_{Nt} = a_2 + b_2 \hat{CPI}_{t-3} + e_t$ ¶ $CPI_t = f(M_{3t} - Y_t)$

RNt	(1)	(2)
1. Day 30t	$a_1 = 6.30$ (5.05) $b_1 = .083$ (.469) $R^2 = .946$ $D.W. = 1.72$ $P = .977$	$a_2 = 6.55$ (4.49) $b_2 = .111$ (.581) $R^2 = .947$ $D.W. = 1.70$ $P = .981$
2. Day 90t	$a_1 = 5.97$ (4.08) $b_1 = .107$ (.731) $R^2 = .944$ $D.W. = 1.57$ $P = .984$	$a_2 = 6.80$ (.000) $b_2 = .168$ (1.08) $R^2 = .942$ $D.W. = 1.53$ $P = 1.000$ *
3. Mon 3t	$a_1 = 5.13$ (.000) $b_1 = -.0005$ (.000) $R^2 = .956$ $D.W. = 1.28$ $P = 1.000$ *	$a_2 = 5.75$ (.000) $b_2 = .050$ (.425) $R^2 = .954$ $D.W. = 1.24$ $P = 1.000$ *
4. Mon 6t	$a_1 = 6.17$ (4.17) $b_1 = .148$ (1.13) $R^2 = .957$ $D.W. = 1.46$ $P = .986$	$a_2 = 7.91$ (.000) $b_2 = .197$ (2.37) $R^2 = .952$ $D.W. = 1.40$ $P = 1.000$ *
5. Year 1t	$a_1 = 5.11$ (.000) $a_2 = .260$ (2.52) $R^2 = .948$ $D.W. = 1.34$ $P = 1.000$ *	$a_2 = 6.13$ (.000) $b_2 = .267$ (2.37) $R^2 = .941$ $D.W. = 1.26$ $P = 1.000$ *

(Table XVI con't next page)

Table XVI (con't)

6. Year 3t	a1 = 5.57 (.000)	a2 = 6.02 (.000)
	b1 = .222 (2.49)	b2 = .216 (2.27)
	R2 = .957	R2 = .955
	D.W. = 1.41	D.W. = 1.38
	P = 1.000 *	P = 1.000 *

II.6. A Rational Expectations Approach to
Explaining the Historical Success of Adaptive
Expectations

As a final exercise involving rational expectations and nominal rates of interest, an explanation will be put forward as to the historic success of the adaptive expectations approach in generating inflationary expectations proxies. As will be shown, the inflation rate has historically behaved in a manner such that the rational expectations and adaptive expectations approaches become indistinguishable statistically -- and as such, a rational person, using current information, could be expected, over large samples, to arrive at a very similar expected inflation rate in period " t + 1 " to someone using a weighted average of past rates of inflation.

To see why such might be the case, let us assume that a rational decision maker exists, and understands the general nature of the relationship between future monetary and real growth, and inflation in period "t+1". Assume, however, that the data on monetary and real growth is not current, trustworthy or regularly available to the decision maker. Instead, assume he has access only to the outcome of past monetary decisions and real growth -- past and present rates of inflation.

Since an autoregressive component is thought to exist in at least one major determinant of inflation -- specifically the money supply process, to follow Mussa's reasoning -- it might be sensible to subject the inflation rate itself to analysis for an autoregressive structure. In effect, any autoregressive features of the inflation rate should be nothing more than a "shadow" of the autoregressive structure believed to exist with respect to the process of excess money creation. A rational decision maker, therefore, could make future predictions based upon observed outcomes (i.e. inflation rates), without ever being aware of the structure of the process (or, in fact, even the process itself) giving rise to that outcome (i.e. excess money creation).

Such analysis of past Canadian inflation rates was undertaken, and the results listed in Table XVII. As before, the data was broken into pre- and post- 1970(6) subperiods, since a rational decision maker would be assumed to realize that a significant change in monetary activity (again, due to the

decision to release the "pegged" Canadian dollar) could have a noticeable impact upon inflation. As well, a single run for the period 1955-1978 was made and is shown as equation (1), Table XVII.

Perhaps the most significant result from Table XVII is the behavior of the Chi-Squared statistic between the 1955-1978 run and the 1955-1970(5) and 1970(6)-1978 runs. Equation (3) captures all significant autocorrelation, even to a lag of 36 periods. Equation (2) is not quite as successful in capturing autocorrelation, but the Chi-Squared statistic for a 12 month lag is not dangerously beyond the safety margin. Certainly by comparison to equation (1), a break at June 1, 1970 allows each sub-series to greatly improve upon the explanation of the autocorrelation structure. As a result of this improvement, the equations (2) and (3) become of prime interest with respect to inflation. Interestingly, the sign of the moving average coefficient changes from negative, as theory would predict, in equation (1), to positive in equations (2) and (3).

For simplicity we will employ equation (3), Table XVII, (whose constant term is insignificant, and will be ignored), to illustrate why the Adaptive Expectations hypothesis explains inflationary behavior well, but equations (1) or (2) could also have been used. For the present, we will also define $X_t = (P_t - P_{t-1})$; X_t equals the first difference of percentage growth in inflation. Employing these simplifications, equation (3) from

Table XVII

Autoregressive Structure of the Consumer Price Index

(1) 1955 - 1978:

$$\begin{aligned} \%CPI_t &= .345 - .360 et-1 + et \\ &\quad (12.01) \quad (6.52) \\ R^2 &= .158 \end{aligned}$$

$$\begin{aligned} \text{Chi squared (12)} &= 182.7 \quad (10 \text{ d. of f.}) \\ \text{Chi squared (24)} &= 304.8 \quad (22 \text{ d. of f.}) \end{aligned}$$

(2) 1955 - 1970(6):

$$\begin{aligned} \%CPI_t - \%CPI_{t-1} &= .001 + .983 et-1 \\ &\quad (2.17) \quad (192.9) \\ R^2 &= .387 \end{aligned}$$

$$\begin{aligned} \text{Chi squared (12)} &= 33.4 \\ \text{Chi squared (24)} &= 51.7 \end{aligned}$$

(3) 1970(6) - 1978:

$$\begin{aligned} \%CPI_t &= .00004 + \%CPI_{t-1} + .887 et-1 + et \\ &\quad (.957) \quad (17.81) \\ R^2 &= .146 \end{aligned}$$

$$\begin{aligned} \text{Chi squared (12)} &= 11.9 \\ \text{Chi squared (24)} &= 17.6 \end{aligned}$$

where $\%CPI_t$ = percent change in CPI from period "t-1"
to period "t"

and rejection bounds for hypothesis that significant
autocorrelation remains are :

$$\text{Chi squared} = 33.9 \quad (22 \text{ d. of f.})$$

Table XVII may be written:

$$(37) \quad X_t = .887e_{t-1} + e_t$$

or:

$$(38) \quad e_t = X_t - .887e_{t-1}$$

Since we have statistically estimated a stable structural equation such as (38) above, we can backdate both sides of the equation to solve for "e_{t-1}":

$$(39) \quad e_{t-1} = X_{t-1} - .887e_{t-2}$$

Substituting equation (39) into (37):

$$(40) \quad X_t = .887(X_{t-1} - .887e_{t-2}) + e_t$$

Repeated backdating allows substitutions for "e_{t-i}" (i = 1, 2, ..., n), where the influence of the "n+1 th" term is arbitrarily set equal to zero. The resulting equation derived under rational expectations assumptions, therefore, appears:

$$(41) \quad X_t = .887X_{t-1} - .787X_{t-2} + .698X_{t-3} \\ - .619X_{t-4} + \dots + (.887)^n X_{t-n} \\ + e_t$$

$$= \sum_{i=1}^n (.887)^i X_{t-i} * (-1)^{i+1}$$

Equation (41) states that rational expectations assumptions lead to a forecast as of period "t" based on a declining weighted combination of past values of inflationary increases over some past time horizon "n" periods into the past. When "Xt" are re-substituted for in terms of "Pt", Equation (41) becomes:

$$(42) \quad P_t = 1.887P_{t-1} - 1.674P_{t-2} + 1.458P_{t-3} \\ - 1.297P_{t-4} + \dots \\ + (.887)^{n-1} + (.887)^n P_{t-n} + e_t \\ = \sum_{i=1}^n (.887)^{i-1} + (.887)^i P_{t-i} * (-1)^{i+1}$$

Due to the nature of the autoregressive structure of inflation, therefore, -- technically known as "one term moving average" -- the adaptive expectations hypothesis embodies the essential characteristics of a rational expectations forecast. Although the hypotheses are dramatically different, the best inflationary guess as to period "t+1", whether beginning from an adaptive expectations framework, or the assumptions underlying rational expectations, appear grounded upon identical

procedures. It should not be surprising, therefore, that published studies in the past have obtained good statistical predictions of inflation based upon adaptive expectations techniques -- in fact the approach could fit equally well under the heading of rational expectations.

Since the weighting pattern emerging from equation (42) differed from that used under the Almon Lag procedure in Part I, one would expect a different value of "P*" to be generated than was the case earlier in the study. As such, equation (12), (section I.2), reproduced below:

$$(43) \quad R_{Nt} = a + cP^*t$$

was tested for the newly created variable "P*t".

Results of this test are listed in Chart XIV and, after removal of first order autocorrelation, in Table XVIII. It is interesting to note the similarities in both estimated constant and coefficient terms between Charts XIV and X, and Tables XVIII and III, where variables P*t were created from different weighting coefficients being applied to the same lagged price level changes.

Perhaps the only important difference between Tables XVIII and III is the observation that in the latter case, the

Table XVIII

$$\text{CORC: } R_{nt} = a + b(\text{CPIHATt}) + et$$

1955-1978

(where CPIHAT were taken from 1955 - 1970,
1970-1978 runs)

- (1) Day 30t = $\begin{matrix} 5.77 & + & 2.01 & \text{CPIHATt} & + & et \\ (5.03) & & (2.05) & & & \end{matrix}$
R2 = .95 Rho = .970 D.W. = 1.76
- (2) Day 90t = $\begin{matrix} 5.96 & + & 2.13 & \text{CPIHATt} & + & et \\ (6.15) & & (2.40) & & & \end{matrix}$
R2 = .95 Rho = .971 D.W. = 1.66
- (3) Mon 3t = $\begin{matrix} 6.09 & + & 1.41 & \text{CPIHATt} & + & et \\ (5.11) & & (2.02) & & & \end{matrix}$
R2 = .97 Rho = .985 D.W. = 1.37
- (4) Mon 6t = $\begin{matrix} 6.14 & + & 1.58 & \text{CPIHATt} & + & et \\ (3.78) & & (2.05) & & & \end{matrix}$
R2 = .96 Rho = .987 D.W. = 1.47
- (5) Year 1t = $\begin{matrix} 5.45 & + & 2.59 & \text{CPIHATt} & + & et \\ (7.82) & & (4.71) & & & \end{matrix}$
R2 = .97 Rho = .974 D.W. = 1.38
- (6) Year 3t = $\begin{matrix} 5.91 & + & 2.04 & \text{CPIHATt} & + & et \\ (9.03) & & (3.71) & & & \end{matrix}$
R2 = .97 Rho = .977 D.W. = 1.44

coefficient associated with "P*" is in no case significantly different from a value of "1.000" at 95% probability levels. One of Table XVIII's coefficients, on the other hand -- that associated with the 1-3 year average bond rate (equation 5) -- proves statistically different from "1.000".

As was the case in Part I, (section I.3), a strong suspicion lingers that, even though it is statistically possible that the true value of five of the coefficients estimated in Table XVIII equals "1.000", and sampling error accounts for the "b" greater than one recordings, the consistent appearance of values greater than "1.000" suggests that the structure of equation (43) -- or equation (12) in Part I -- overly restricts the behavior of variable "P*t".

II.7 Summary of Part II

Tests of the Fisher Effect and Canadian nominal bond rates were carried out in Part II under the assumption that Rational Expectations postulates were in effect during the period 1955-1978. Specifically, bond holders were assumed in this part to understand the process of inflation, and have access to data necessary for the generation of inflationary predictions. The data were also assumed inexpensive and accurate enough that the benefit of improved inflationary predictive power equalled the cost of data collection and interpretation at the margin.

The Mussa model for generating " P^*t " ($E_t [P_{t+1} - P_t]$), was presented, and assumptions made as to the theoretical structure underlying monetary creation and real components of inflation. Two series of monetary aggregates, M1 and M3, were analyzed using the Box-Jenkins procedure, leading to a statistical estimate of the autoregressive components of each series. Three indices for real growth -- G.N.E., I.I.P., and R.D.P., base year 1961 -- were similarly analyzed using the Box-Jenkins technique, and each was found to embody a stable autoregressive structure over time.

The selection of the most efficient monetary and real growth information, (which a rational decision maker would wish to use when estimating future inflation), was decided upon using the historical success rate of each predictor as of period " t ", and the actual rates of inflation observed in period " $t+1$ ". O.L.S. regression showed that the expected value at " t " of M1 and M3 minus one-third the constant expected quarterly growth rate in G.N.E., yielded the best statistical fit for inflation during period " $t+1$ ". Nominal bond rates were therefore tested against " $E_t [M1_{t+1} - G.N.E.t+1]$ " and " $E_t [M3_{t+1} - G.N.E.t+1]$ " as right hand side variables, again assuming the real rate of interest to be constant over the period 1955-1978.

Separate regressions were run assuming different values of " v ", the partial income elasticity with respect to interest. End points of " $v = 0$ " and " $v = 2$ " were chosen, and a run of " $v = 1$ "

reported, although the true value of "v" was deemed very close to zero.

The effectiveness of these "P*" measures was difficult to ascertain due to autocorrelation removal problems. From O.L.S. regression, the coefficients associated with "P*t" were generally significantly different from "1.000", but the "t" statistics supporting that conclusion were believed biased upwards due to autocorrelation. When the H-L or C-O techniques were used to remove first order autocorrelation, however, coefficient values dropped dramatically, and in the "Et [M3t+1 - G.N.E.t+1]" case, changed sign, becoming negative. The problem was due to a "rho" value being selected very near one in each case.

Due to the inconclusiveness of testing the Mussa model as presented, the working definition of rationality was expanded, and knowledge of past successes in predicting inflation was assumed incorporated into the information set as of time "t". The expected values arising from Mussa's model were weighted, then, by a coefficient "z", producing a prediction term " \hat{CPI} " equal to " $b_2 * Et [\hat{M} - \hat{Y}]$ ".

This new predictor for inflation was regressed against nominal bond rates and the estimates of constant and "P*t" coefficients analyzed. It was noted that those estimates corresponded closely with estimates obtained in Part I of the study. Removal of autocorrelation again proved troublesome --

"rho" values were set close or equal to one -- and as a result the constant term rose to unacceptably large estimates (on theoretical grounds) of the real rates of return. The "b" coefficients tended to fall between "0.000" and "1.000" as would be expected but the magnitudes of the coefficients were much lower in value than found in earlier parts of the study, and often proved statistically insignificant.

As a final test of the rational expectations theory, an assumption was made that a rational decision maker would chose to examine observed inflation rates, due to uncertainty about the processes underlying the creation of money and/or real growth. It was postulated that any autoregressive moving average component of inflation would merely follow from the effect of earlier, autoregressive movements in the money creation process. A stable moving average structure was estimated for past rates of inflation via Box-Jenkins.

It was shown that through substitution, the rational expectations model could be set in terms of an adaptive expectations framework, which suggested a solution to the question of why earlier adaptive expectations models predicted inflation well. The new "P*t" variable generated via the autoregressive structure -- different from "P*t" determined earlier due to a slightly different weighting scheme -- was regressed upon nominal bond rates. After removal of autocorrelation, the "c" coefficients were found to be not

significantly different from one in five of six cases, but greater than one in magnitude in all cases, as was true of the "P*t" coefficient estimated when adaptive expectations assumptions were employed. As was the case earlier in Part I, doubt was raised as to the likelihood of bond holders consistently being compensated for more inflationary losses than they would expect or demand -- the conclusion following from "c greater than 1.00" findings.

As will be argued in the Conclusions section which follows, however, the consistent overestimation of "P*t" coefficients is quite compatible with theory, but explicit account must first be taken of a bias as yet not compensated for in this paper.

E. Conclusions

Throughout the paper, estimates of the two nominal bond rate components, RR_t (the real rate of return) and P^*t (the compensation for the expected rate of inflation) were sought. Since the real rate was assumed constant, the major emphasis of the study concerned generating alternate values of P^*t , and estimating equations of the form:

$$(44) \quad R_{Nt} = a + bP^*t$$

Estimates of the real rate of return varied but when excluding results found after the H-L or C-O modification was applied and "rho" values were chosen very close to "1.00" -- since the constant term ceases to correspond to an estimate of the real rate of interest under such a situation -- the magnitudes of the various terms centered in an area between "3.00" and "4.00" percent annually. For example, eight such terms are listed in Table XIX, selected from Tables and Charts throughout the study. With the exception of Columns (3) and (4), representing the estimates drawn from running the Mussa model's " $E_t [P_{t+1} - P_t]$ " as " P^*t ", the variance of generated real rates for a given bond is not large.

Table XIX

Summary of "a" Estimates

	TABLE II	IV	XII	XIII	CHART IX	XIII	XIV
1. Day 30	3.41	3.58	5.59	5.09	3.32	3.59	3.29
2. Day 90	3.31	3.35	5.55	5.14	3.38	3.69	3.49
3. Mon 3	2.34	2.60	4.56	4.28	2.88	2.89	2.61
4. Mon 6	3.01	2.93	5.20	4.93	3.16	3.54	3.22
5. Year 1	3.63	3.80	5.11	4.81	3.76	3.52	3.45
6. Year 3	4.04	4.27	5.46	5.07	4.15	3.89	3.83

The magnitude of these real rate estimates appears unusually high, however. In past times of zero expected inflation, (and hence nominal rates equalling real rates of return), the market did not generate interest rates above three percent for bonds of similar risk or maturities to those shown. In fact, the nominal rates were generally found in the neighborhood of between two and three percent annually.

The estimated value of the "b" coefficient in Equation (44) also appeared upward biased from this study. Some very large estimates of the P*t coefficient from O.L.S. regressions can be disregarded, however, as almost certainly being biased, due to autocorrelation in the small sample case. (for example see Charts X ad XIV, where calculated "b" terms exceed "6.00" with great regularity.)

Some estimates of the "b" coefficient were also found, which, after the H-L or C-O modifications were applied seem unusually low and proved statistically not significantly different from zero (i.e. Tables XIV, XV, and XVI) and as such would not seem representative of the overall study. However, arising from both the adaptive expectations, and modified Mussa model approaches, several "b" estimates were generated with very similar magnitudes proving significantly different than zero with 95% probability. Some of these values are shown in Table XX, again compiled from Tables and Charts throughout the study.

As was alluded to in the Summary sections of Parts I and II, " $b > 1$ " values are theoretically most unlikely over a period as long as twenty three years. Coupled with the observation that real rate estimates appear unusually high as well, the most logical conclusion would seem to be that some critical bias factor has not been taken account of satisfactorally in any of the regression equations outlined.

Table XX

Summary of "b" Estimates

TABLE	III	XII	XIII	XVIII	XIIa
1. Day 30t	1.984	.971	1.82	2.01	1.77
2. Day 90t	1.636	1.08	1.81	2.13	1.93
3. Mon 3t	.966	1.02	1.58	1.41	1.81
4. Mon 6t	1.622	.783	1.27	1.58	1.40
5. Year 1t	1.800	.990	1.59	2.59	1.78
6. Year 3t	1.331	1.010	1.75	2.04	1.84

One such factor excluded from this study was the effect of taxation upon the behavior of nominal bond rates. As work by Michael Darby³² has suggested, such an effect will lead to the over estimation of the true values of "RRt" and the true "b" coefficient we shall designate as "B". As will be shown, when account is explicitly taken of taxation effects, "a" values fall

32 Darby, Michael R. The Financial and Tax Effects of Monetary Policy on Interest Rates, Economic Inquiry, June 1975 P.231-244.

well within the expected neighborhood (annual rates of about 2.5%) and "B" coefficients fall in magnitude very near a value of one.

Since taxation is applied to nominal returns from bond holding, both the real rate of return and the compensation for expected inflation are affected. The return a bond holder receives, then, can be shown as:

$$(45) \quad R_{Nt} (1 - T) = R_{Rt} + P^*t$$

$$\text{or: } (46) \quad R_{Nt} = (1 / 1-T) R_{Rt} + (1 / 1-T) P^*t$$

where T = marginal tax rate.

When estimating the coefficients "a" and "b" in this study, no account was taken of the fact that logically "a" and "b" should represent the following relationships:

$$(47) \quad a = 1 / (1-T)$$

$$\text{or:} \quad R_{Rt} = a (1-T)$$

$$(48) \quad b = 1 / (1-T) B$$

$$\text{or:} \quad B = b (1-T)$$

where B = true coefficient associated with P*t

What is being suggested, then, is that bond holders are aware of taxation, and further, that before undertaking current investment they demand compensation for said losses of income via taxation. If the real rate component is established where the market clears (i.e. supply of investment funds demand for funds, conditional upon different internal rates of discount), the real rate demanded should include compensation for tax losses. Similarly, a person demanding compensation for inflation will expect such a premium to be in effect after taxes and not before, or the compensation would not truly keep the bond holder insulated from inflation.

The magnitude of "T" in Equations (47) and (48) may be taken as being between ".20" and ".50", for Canada, depending upon the income received, and the bond holder's other income.³³ Assuming some mid point, however, of ".35" as representative of the tax faced by the "average" bond holder, Equations (47) and (48) would become, for example:

$$(49) \quad RRt = .65(a)$$

³³ These figures were calculated from the taxes payable per income bracket. Source: 1979 Canadian Tax Guide.

$$(50) \quad B = .65(b)$$

In other words, when account is taken of taxation, the estimates of "a" and "b" no longer appear inappropriate. The figures listed in Tables XIX and XX, after adjusting for taxation effects, indicate values of the real rate of return being near 2.5% annually, and in no case shown is "B" significantly different from "1.000" -- the estimates generally fall below, but very close to that number.

The major findings of this study may be summed up, therefore, into one sentence -- every indication is that bond holders do indeed understand the process of, and demand compensation for, inflation. This conclusion is not surprising -- in fact, if anything else were the case one would likely become suspicious. Nonetheless, at a time when governments continue to generate expectations of inflation by monetary actions, while at the same time publicly promising a future lowering of interest rates, the point is certainly far from an irrelevant one.

Appendix

Chart I

Yohe and Karnosky Regression Coefficients

Ordinary Least Squares: 1961 - 1969

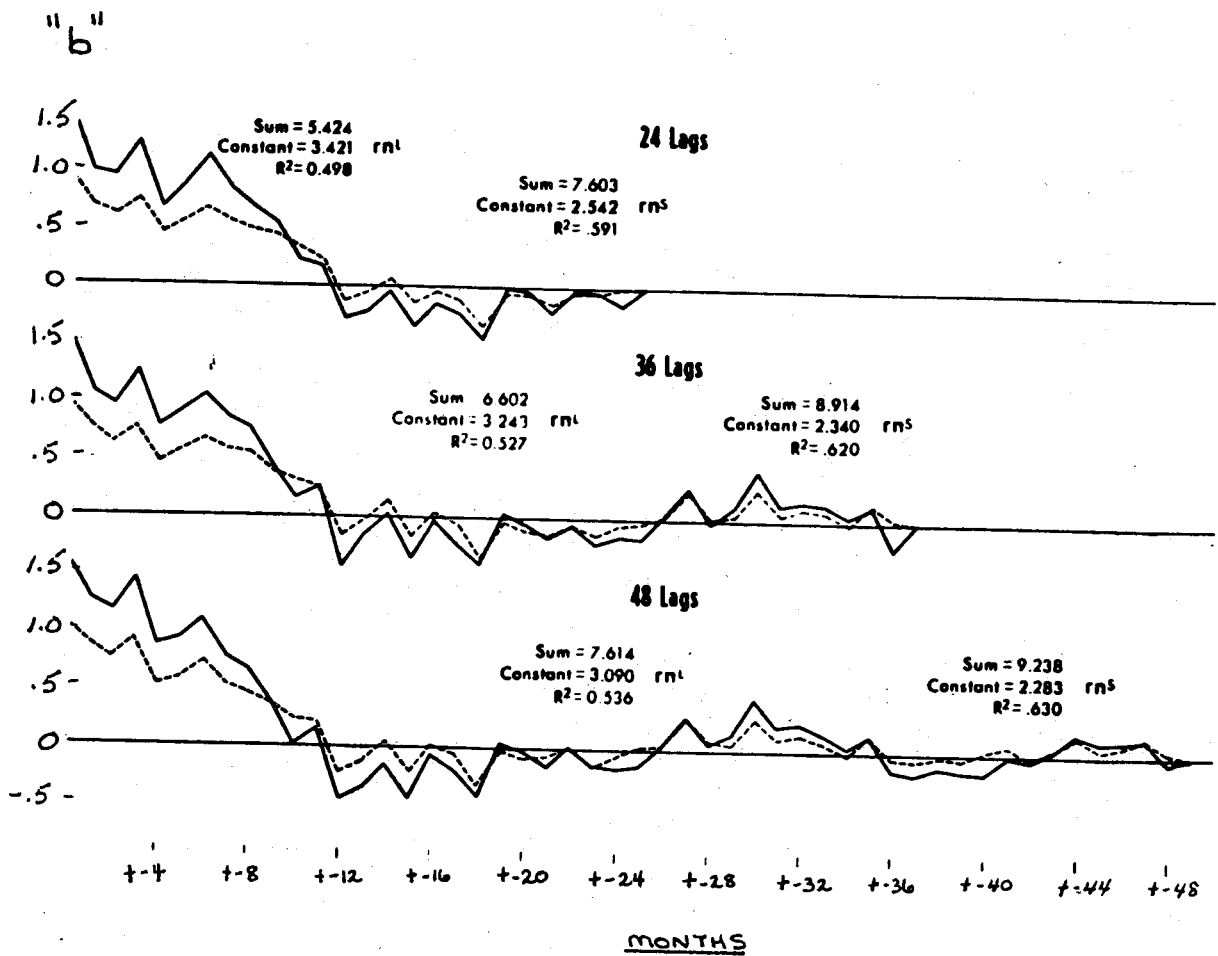


Chart II

Yohe and Karnosky Regression Coefficients

Almon Lag Coefficients: 1961 - 1969

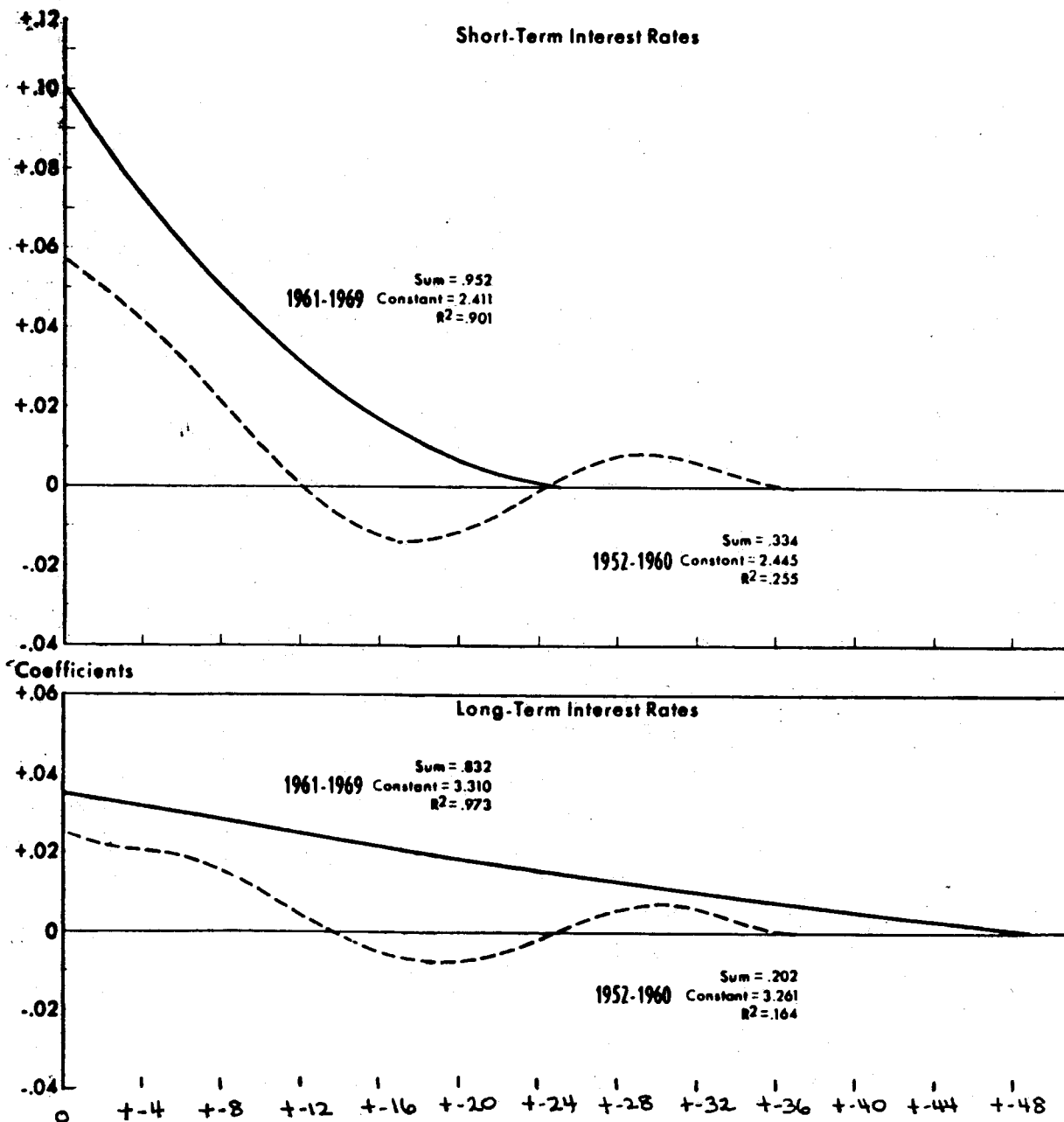
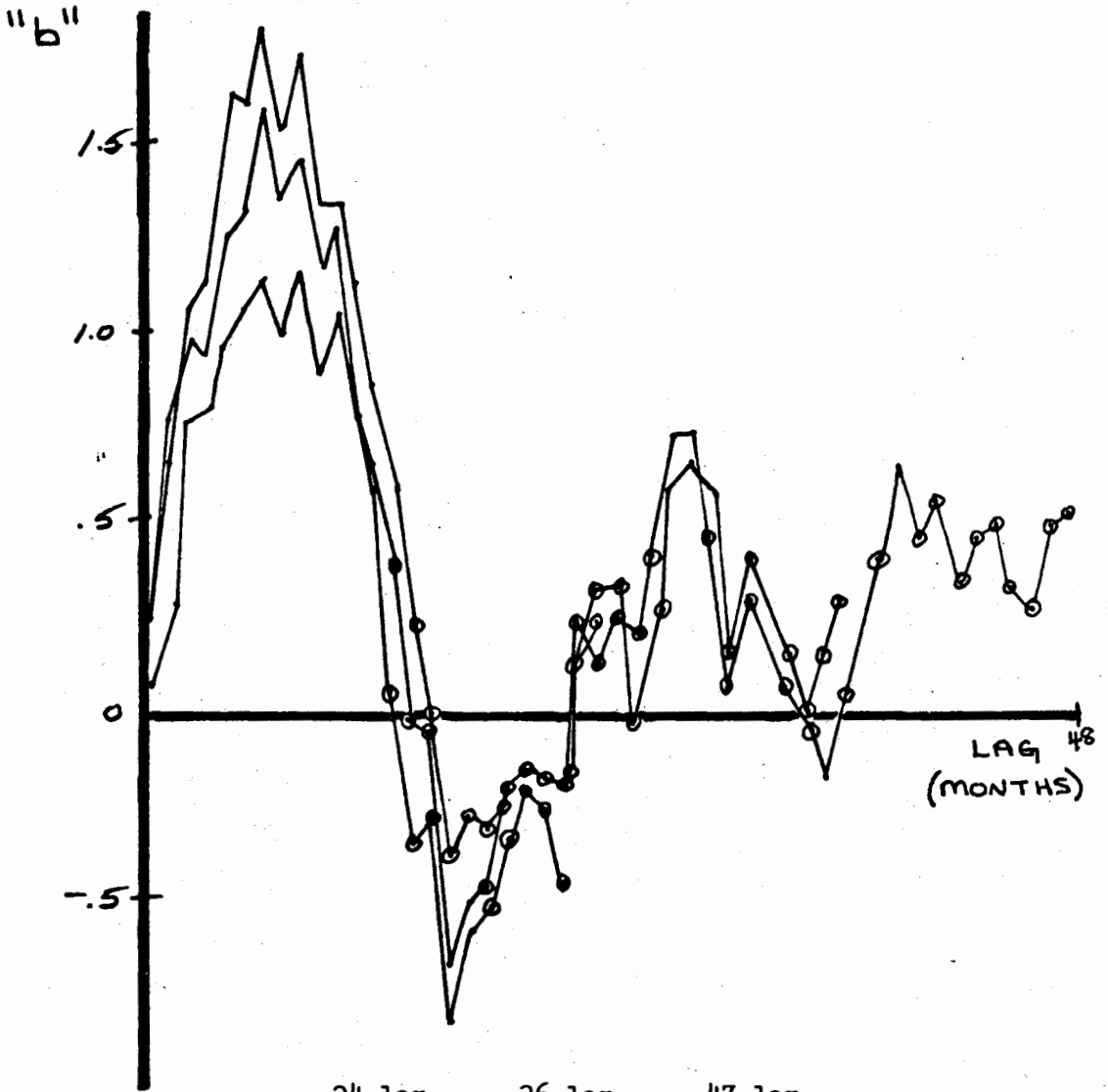


Chart III

30 Day Finance Co. Paper Rate

Olsq coefficients: 1961 - 1969

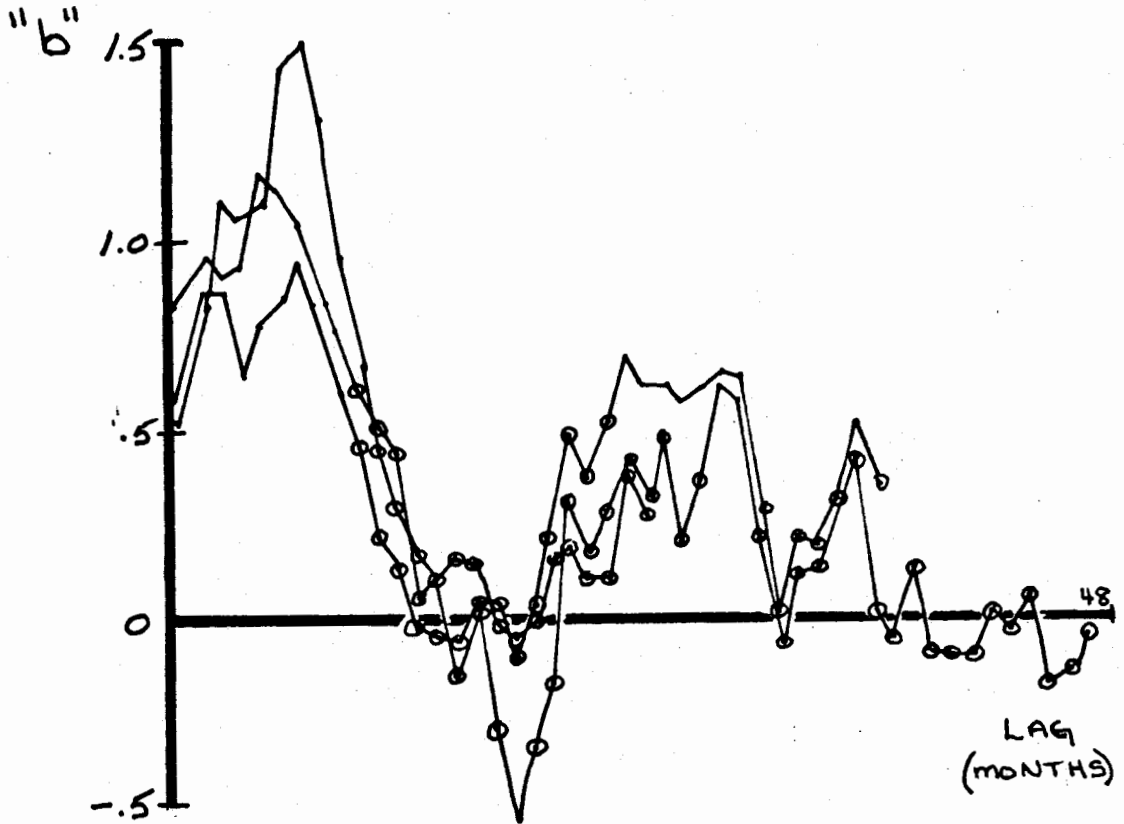


24 lag	36 lag	47 lag
R2 = .941	R2 = .958	R2 = .983
DW = .336	DW = .395	DW = .755

Chart IV

90 Day Finance Co. Paper Rate

Olsq coefficients: 1961 - 1969



24 lag	36 lag	47 lag
R2 = .592	R2 = .854	R2 = .928
DW = .161	DW = .347	DW = .371

Chart V

3 Month Treasury Bill Rate

Olsq coefficients: 1961 - 1969

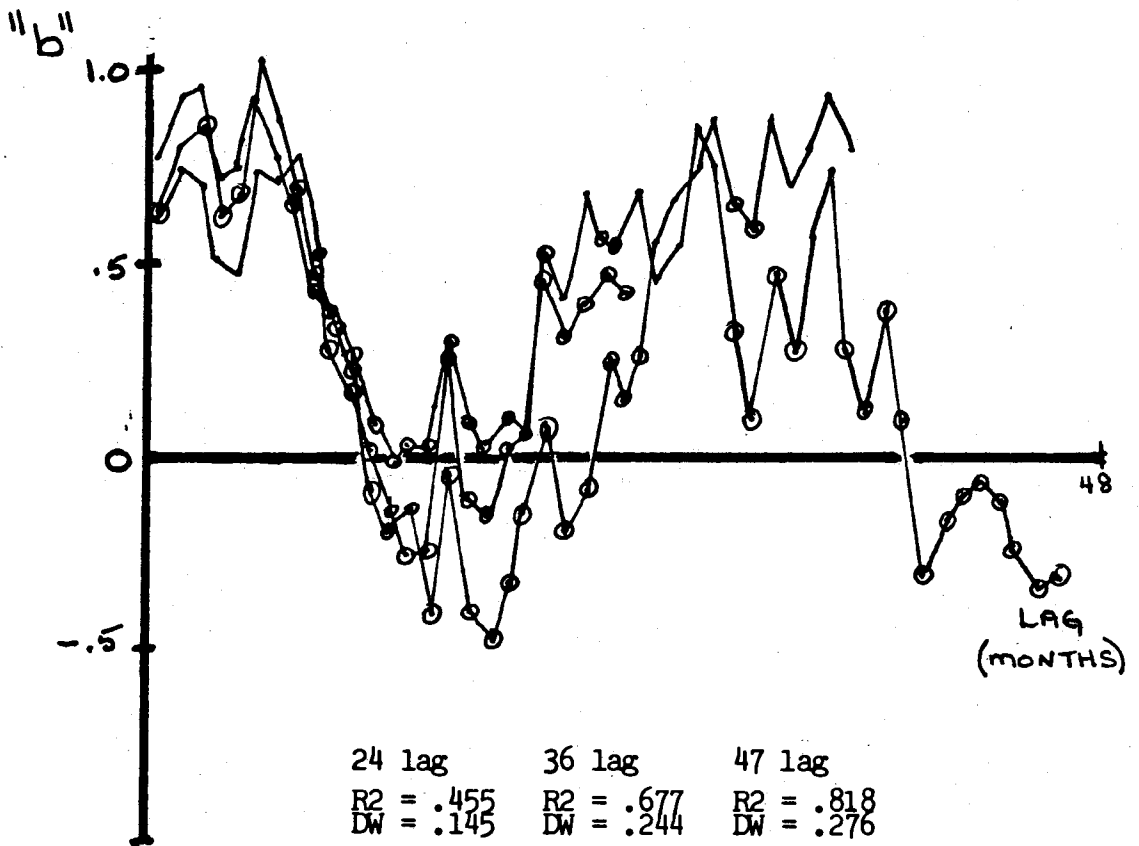


Chart VI

6 Month Treasury Bill Rate

Olsq coefficients: 1961 - 1969

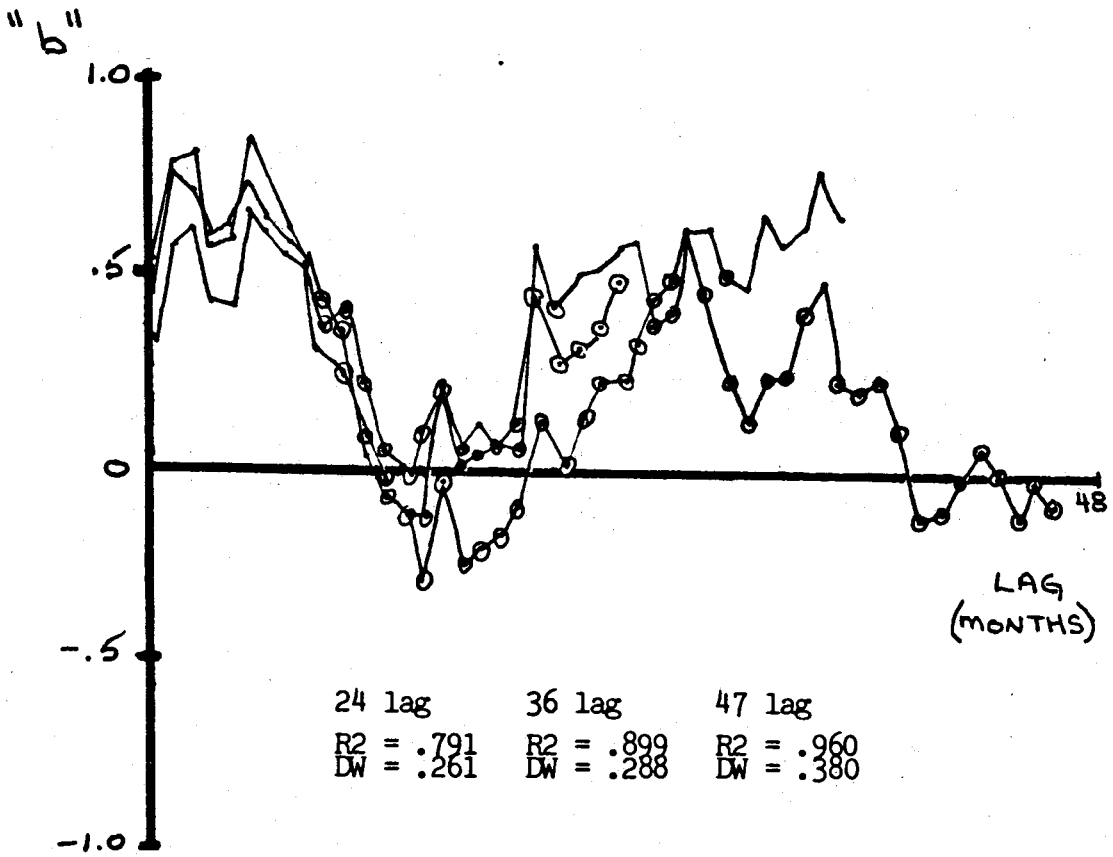


Chart VII

1 - 3 Year Average Bond Rate

Olsq coefficients: 1961 - 1969

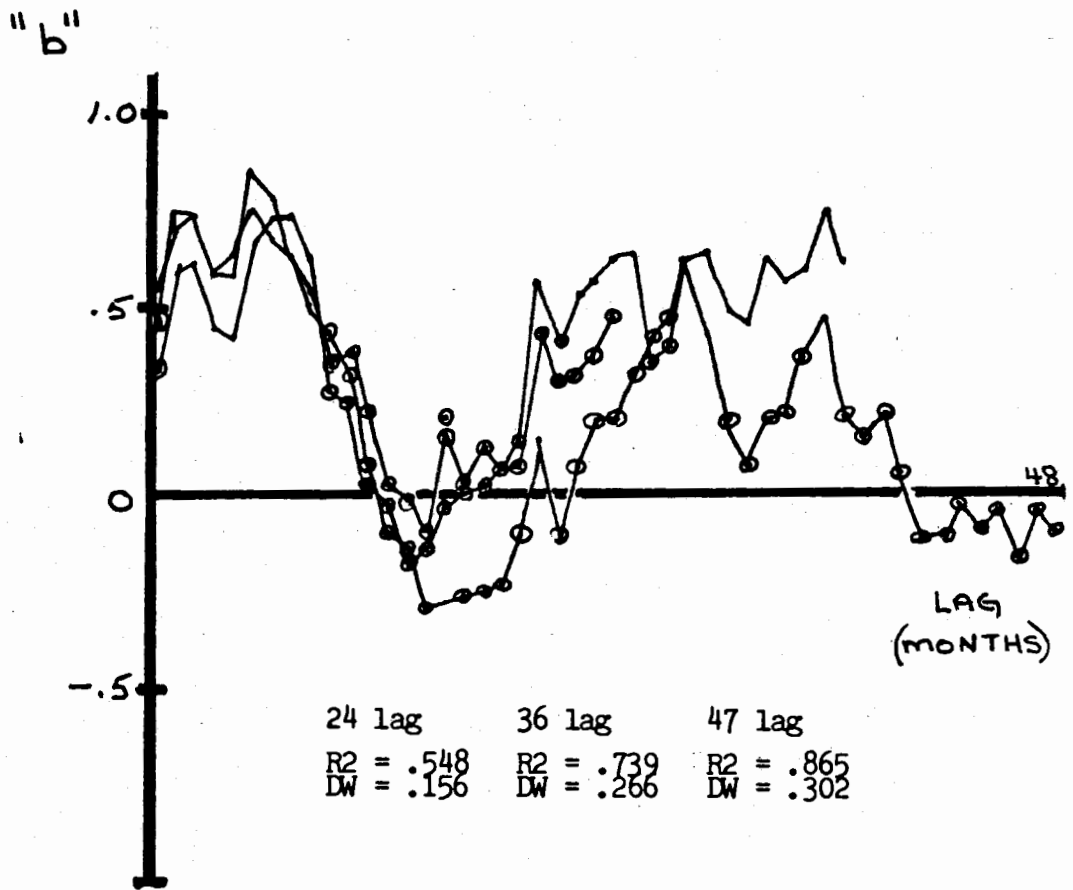
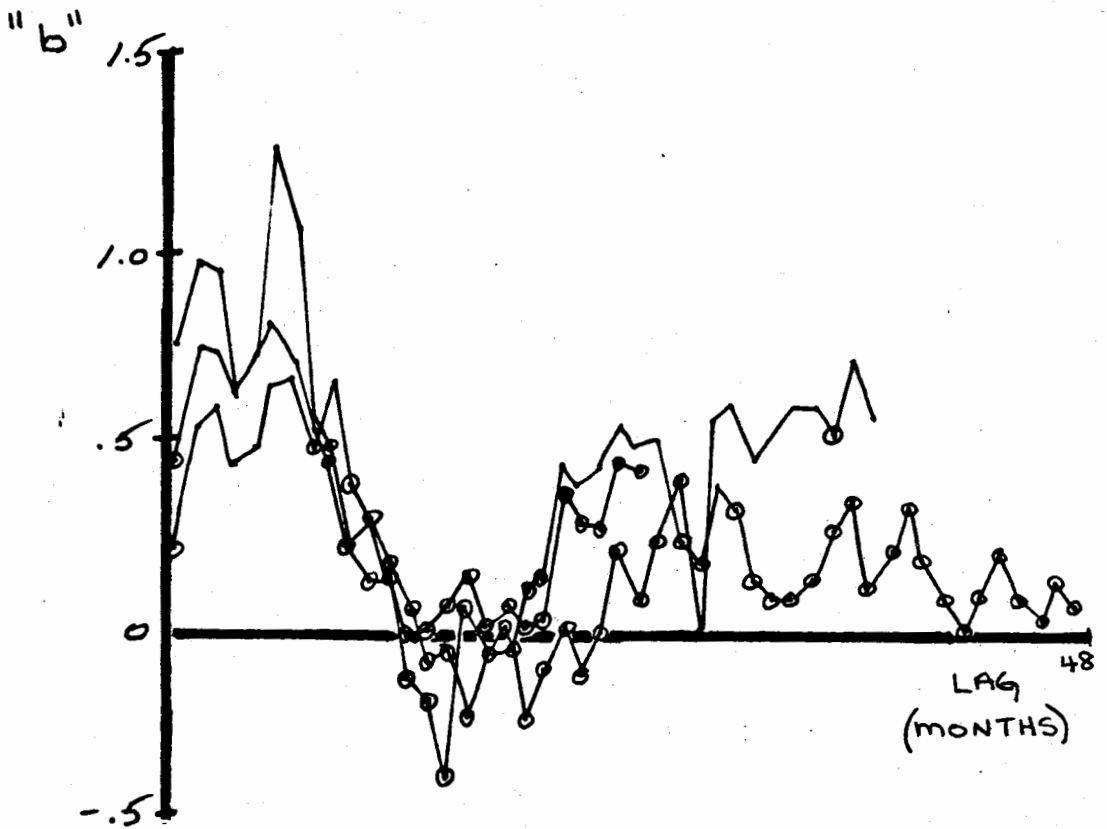


Chart VIII

3 - 5 Year Average Bond Rate

Olsq coefficients: 1961 - 1969



24 lag	36 lag	47 lag
R2 = .542	R2 = .676	R2 = .890
DW = .126	DW = .493	DW = .317

Chart IX
 OLSQ: $RNt = a + \text{Almon}(\text{Pt}, \dots, \text{Pt}-n)$

i	1955-1968*	1969-1978	1955-1978
1. 30 day	a = 2.284(16.97) R2 = .867 D.W. = .581 NOBS = 108	a = 4.196(11.40) R2 = .535 D.W. = .213 NOBS = 117	a = 3.316 (24.24) R2 = .747 D.W. = .209 NOBS = 225
2. 90 day	a = 2.480(12.77) R2 = .660 D.W. = .272 NOBS = 123	a = 4.517(12.34) R2 = .528 D.W. = .203 NOBS = 117	a = 3.676(27.59) R2 = .712 D.W. = .184 NOBS = 240
3. 3 Mon	a = 2.222(11.87) R2 = .602 D.W. = .321 NOBS = 123	a = 2.732(8.36) R2 = .620 D.W. = .079 NOBS = 117	a = 2.883(24.18) R2 = .730 D.W. = .126 NOBS = 240
4. 6 Mon	a = 2.531(14.33) R2 = .655 D.W. = .409 NOBS = 116	a = 2.947(9.34) R2 = .622 D.W. = .105 NOBS = 117	a = 3.158(26.27) R2 = .728 D.W. = .150 NOBS = 233
5. 1-3 Year	a = 2.965(22.30) R2 = .602 D.W. = .397 NOBS = 123	a = 4.695(19.67) R2 = .520 D.W. = .195 NOBS = 117	a = 3.764(40.20) R2 = .752 D.W. = .181 NOBS = 240
6. 3-5 Year	a = 3.384(30.90) R2 = .737 D.W. = .362 NOBS = 123	a = 5.346 R2 = .540 D.W. = .227 NOBS = 117	a = 4.152(50.02) R2 = .787 D.W. = .168 NOBS = 240

* 1960-68 in case of 30 day, 1956-68 for 90 day, and 1959-68 for 6 mon.

Note: NOBS = Number of Observations

Chart X

OLSQ: $RN_t = a + bP^*t + et$

i	1955-1968	1969-78	1955-1978
1. 30 day	a = 1.87(10.62) b = 12.42(16.35) R2 = .716 D.W. = .660	a = 3.770(9.91) b = 6.55(10.01) R2 = .465 D.W. = .194	a = 2.926(18.86) b = 7.958(23.03) R2 = .704 D.W. = .226
2. 90 day	a = 2.75(13.51) b = 10.07(11.15) R2 = .507 D.W. = .324	a = 4.038(10.69) b = 6.432(9.89) R2 = .459 D.W. = .188	a = 3.357(22.88) b = 7.544(22.45) R2 = .678 D.W. = .206
3. 3 Mon	a = 2.550(12.553) b = 7.796(8.67) R2 = .383 D.W. = .297	a = 3.009(8.017) b = 6.392(9.89) R2 = .459 D.W. = .083	a = 2.78(19.43) b = 6.77(20.61) R2 = .641 D.W. = .139
4. 6 Mon	a = 2.779(13.15) b = 7.684(8.29) R2 = .376 D.W. = .284	a = 3.183(8.94) b = 6.342(10.35) R2 = .482 D.W. = .103	a = 3.012(21.10) b = 6.634(20.55) R2 = .646 D.W. = .152
5. 1-3 Year	a = 3.18(21.06) b = 7.198(10.78) R2 = .490 D.W. = .370	a = 4.793(19.18) b = 4.046(9.40) R2 = .435 D.W. = .155	a = 3.641(34.31) b = 5.826(23.98) R2 = .707 D.W. = .193
6. 3-5 Year	a = 3.58(29.42) b = 6.853(12.73) R2 = .573 D.W. = .408	a = 5.519(25.417) b = 3.435(9.19) R2 = .423 D.W. = .172	a = 4.058(41.76) b = 5.671(25.48) R2 = .732 D.W. = .186

Chart XI

$$\text{PDL: } R_{Nt} = a + \underset{1955-78}{bEXt} + \text{Almon}(Pt, Pt-1, \dots, Pt-n)$$

$$(1) \text{ Day } 30t = \begin{matrix} 3.88 \\ (20.30) \end{matrix} - \begin{matrix} .014 \\ (-1.68) \end{matrix} M3Rt + \text{Almon}(Pt, Pt-1, \dots, Pt-n)$$

$$R2 = .69 \qquad D.W. = .230$$

$$(2) \text{ Day } 90t = \begin{matrix} 3.74 \\ (27.49) \end{matrix} - \begin{matrix} .018 \\ (-2.33) \end{matrix} M3Rt + \text{Almon}(Pt, Pt-1, \dots, Pt-n)$$

$$R2 = .76 \qquad D.W. = .227$$

$$(3) \text{ Mon } 3t = \begin{matrix} 2.96 \\ (24.51) \end{matrix} - \begin{matrix} .024 \\ (-3.19) \end{matrix} M3Rt + \text{Almon}(Pt, Pt-1, \dots, Pt-n)$$

$$(4) \text{ Mon } 6t = \begin{matrix} 3.20 \\ (19.77) \end{matrix} - \begin{matrix} .020 \\ (-2.67) \end{matrix} M3Rt + \text{Almon}(Pt, Pt-1, \dots, Pt-n)$$

$$R2 = .72 \qquad D.W. = .155$$

$$(5) \text{ Year } 1t = \begin{matrix} 3.80 \\ (39.48) \end{matrix} - \begin{matrix} .022 \\ (-3.68) \end{matrix} M3Rt + \text{Almon}(Pt, Pt-1, \dots, Pt-n)$$

$$R2 = .77 \qquad D.W. = .235$$

$$(6) \text{ Year } 3t = \begin{matrix} 4.18 \\ (49.42) \end{matrix} - \begin{matrix} .012 \\ (-2.25) \end{matrix} M3Rt + \text{Almon}(Pt, Pt-1, \dots, Pt-n)$$

$$R2 = .80 \qquad D.W. = .196$$

Chart XII

$$\text{PDL: } R_{nt} = a + bEX_t + cEX_{t-1} + \text{Almon}(Pt, \dots Pt-n) \\ 1955-1978$$

$$(1) \text{ Day } 30t = 3.88 - .014 M3R_t - .001 M3R_{t-1} + \text{Almon}(Pt, \dots Pt-n) \\ (20.19) \quad (-1.68) \quad (-.125)$$

$$R2 = .69 \qquad \qquad \qquad \text{D.W.} = .232$$

$$(2) \text{ Day } 90t = 3.75 - .019 M3R_t - .005 M3R_{t-1} + \text{Almon}(Pt, \dots Pt-n) \\ (27.30) \quad (-2.36) \quad (.453)$$

$$R2 = .76 \qquad \qquad \qquad \text{D.W.} = .233$$

$$(3) \text{ Mon } 3t = 2.97 - .024 M3R_t - .028 M3R_{t-1} + \text{Almon}(Pt, \dots Pt-n) \\ (24.96) \quad (-3.27) \quad (-1.25)$$

$$R2 = .75 \qquad \qquad \qquad \text{D.W.} = .211$$

$$(4) \text{ Mon } 6t = 3.22 - .021 M3R_t - .012 M3R_{t-1} + \text{Almon}(Pt, \dots Pt-n) \\ (19.82) \quad (-2.76) \quad (-1.18)$$

$$R2 = .73 \qquad \qquad \qquad \text{D.W.} = .183$$

$$(5) \text{ Year } 1t = 3.80 - .022 M3R_t - .006 M3R_{t-1} + \text{Almon}(Pt, \dots Pt-n) \\ (39.40) \quad (-3.73) \quad (-.789)$$

$$R2 = .77 \qquad \qquad \qquad \text{D.W.} = .243$$

$$(6) \text{ Year } 3t = 4.18 - .012 M3R_t - .004 M3R_{t-1} + \text{Almon}(Pt, \dots Pt-n) \\ (49.24) \quad (-2.28) \quad (-.517)$$

$$R2 = .80 \qquad \qquad \qquad \text{D.W.} = .199$$

Chart XIII

$$\begin{array}{l} \text{OLSQ: 1) } R_{Nt} = a_1 + b_1 \text{CPI}_{t} + e_t \\ \quad \quad \quad 2) R_{Nt} = a_2 + b_2 \text{CPI}_{t} + e_t \end{array} \quad \begin{array}{l} \uparrow \\ \uparrow \end{array} \quad \begin{array}{l} \text{CPI}_{t} = f(M1_t - Y_t) \\ \text{CPI}_{t} = f(M3_t - Y_t) \end{array}$$

RNt	(1)	(2)
1. Day 30t	$a_1 = 3.78 (14.13)$ $b_1 = 6.14 (9.38)$ $R^2 = .283$ $D.W. = .473$	$a_2 = 3.59 (12.77)$ $b_2 = 6.66 (9.73)$ $R^2 = .295$ $D.W. = .466$
2. Day 90t	$a_1 = 3.87 (16.92)$ $b_1 = 5.92 (10.17)$ $R^2 = .276$ $D.W. = .456$	$a_2 = 3.69 (15.52)$ $b_2 = 6.51 (10.75)$ $R^2 = .297$ $D.W. = .469$
3. Mon 3t	$a_1 = 3.05 (13.74)$ $b_1 = 5.43 (9.51)$ $R^2 = .243$ $D.W. = .380$	$a_2 = 2.89 (12.30)$ $b_2 = 5.98 (9.89)$ $R^2 = .256$ $D.W. = .378$
4. Mon 6t	$a_1 = 3.67 (15.85)$ $b_1 = 5.10 (8.95)$ $R^2 = .257$ $D.W. = .405$	$a_2 = 3.54 (14.07)$ $b_2 = 5.57 (9.04)$ $R^2 = .259$ $D.W. = .370$
5. Year 1t	$a_1 = 3.69 (20.51)$ $b_1 = 5.14 (11.12)$ $R^2 = .306$ $D.W. = .472$	$a_2 = 3.52 (18.51)$ $b_2 = 5.71 (11.68)$ $R^2 = .324$ $D.W. = .491$
6. Year 3t	$a_1 = 4.08 (23.32)$ $b_1 = 4.99 (11.10)$ $R^2 = .305$ $D.W. = .464$	$a_2 = 3.89 (21.43)$ $b_2 = 5.61 (12.01)$ $R^2 = .337$ $D.W. = .514$

Chart XIV

CORC: $Rnt = a + b(\text{CPIHATt}) + et$
1955-1978

(where CPIHAT were taken from 1955-1970,
1970-1978 runs)

- (1) Day 30t = 3.29 + 6.90 CPIHATt + et
(19.44) (19.33)
R2 = .62 D.W. = .154
- (2) Day 90t = 3.49 + 6.90 CPIHATt + et
(25.41) (21.85)
R2 =
- (3) Mon 3t = 2.61 + 6.73 CPIHATt + et
(19.80) (21.86)
R2 = .63 D.W. = .092
- (4) Mon 6t = 3.22 + 6.00 CPIHATt + et
(21.11) (18.36)
R2 = .59 D.W. = .110
- (5) Year 1t = 3.45 + 5.86 CPIHATt + et
(32.32) (23.51)
R2 = .66 D.W. = .114
- (6) Year 3t = 3.83 + 5.74 CPIHATt + et
(38.52) (24.77)
R2 = .68 D.W. = .100

Bibliography

- Barro, R.J. Rational Expectations and the Role of Monetary Policy, Journal of Monetary Economics, Vol. 2, 1976. pp 1-32.
- Box, G. and Jenkins, G. Time Series Analysis: Forecasting and Control, Holden-Day, San Francisco, 1976.
- Cagan, P. "The Monetary Dynamics of Hyperinflation" in M. Friedman ed. Studies in the Quantity Theory of Money, University of Chicago Press, Chicago, 1956.
- Darby, M. The Financial and Tax Effects of Monetary Policy on Interest Rates, Economics Inquiry, June 1975. pp 231-244.
- Fama, E. Short Term Interest Rates as Predictors of Inflation, American Economic Review, Vol. 60, NO. 3, June 1975. p 269.
- Fisher, I. The Theory of Interest, New York, 1930.
- Gibson, W. Interest Rates and Inflationary Expectations: New Evidence, American Economic Review, Vol. 58, No. 5, December 1972. p 854.
- Gibson, W. Price Expectation Effects on Interest Rates, Journal of Finance, Vol. 25, March 1970. pp 19-34.
- Lucas, R.E. Expectations and the Neutrality of Money, Journal of Economic Theory, Vol. 4, April 1972a. pp 103-124.
- Mundell, R. Monetary Theory: Inflation, Interest and Growth in the World Economy, Goodyear Publishing Co., Santa Monica, Calif., 1971.
- Mussa, M. Adaptive and Regressive Expectations in a Rational Model of the Inflationary Process, Journal of Monetary Economics, Vol. 1, 1975. pp 423-442.
- Muth, J. Rational Expectations and the Theory of Price Movements, Econometrica, Vol. 29, No. 3, July 1961. pp 315-335.
- Sargent, T. Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment, Brookings Papers on Economic Activity, Vol. 2, 1973.
- Sargent, T. and Wallace, N. Rational Expectations and the Theory of Economic Policy, Journal of Monetary Economics, Vol. 2,

1976. pp 169-183.

Sargent, T. and Wallace, N. Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule, Journal of Political Economy, March-April, 1975.

Yohe, W. and Karnosky, D. Interest Rates and Price Level Changes, Federal Reserve Bank of St. Louis Bulletin, Dec., 1969. p 18.