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EVIDENCE ON THE STATIONARITY OF SYSTEMATIC RISK

by

Simon M. Wheatley

M.A., University of Aberdeen, 1977

A PROJECT SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF ARTS
in the Department
of
Economics and Commerce

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Abstract

The Mean-Variance Capital Asset Pricing Model (CAPM) is probably the most widely-researched equilibrium pricing model for capital assets. Systematic risk (beta) plays a central role in the theory, and much attention has been focused on what constitutes the correct procedure for its estimation. Several authors have observed that beta may not remain stationary through time. They have further indicated that estimates of beta, derived ignoring the effects of non-stationarity, may be of only limited value, from the viewpoint of both the portfolio manager, and the academic researcher.

This study applies a new statistical procedure to investigate beta-non-stationarity. The analysis concentrates on a well-known empirical study of the CAPM, in which an assumption of beta-stationarity appeared explicitly. The evidence provided by this paper does not support the assumption.

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A. Introduction

The capital asset pricing model (CAPM) of Sharpe-Lintner-Mossin (XIX, XIV and XV) provides a linear relationship between the expected return on an asset, and that asset's systematic risk (beta). Although the derivation of the CAPM requires a number of fairly strict assumptions, it remains the most widely-researched equilibrium pricing model for capital assets. It has intuitive appeal, and its simplicity holds considerable promise for making it suitable for practical applications in a variety of fields.

The model, constructed within a single-period framework, has been subjected to a large number of empirical tests, necessarily constructed within a multi-period framework. No assumptions as to the stationarity of systematic risk appeared in the derivation of the CAPM. The theory does indicate, indeed, that an asset's systematic risk may alter through time as a result of changes in capital structure and the adoption of projects from a different risk class than that of current operations. However, a necessary condition for the econometric procedures employed in testing the model, has been that systematic risk should remain stationary over some period of time.

This paper examines in some detail the validity of this assumption of beta stationarity, with particular reference to Black, Jensen and Scholes' (BJS) tests of the CAPM (II). It should be emphasised that the current author is aware of criticisms of these and other tests, of a more fundamental nature, that have been made by Roll (XVII). However this paper is primarily concerned with the general implications of beta non-stationarity for empirical research, for which the BJS study provides a suitable example.

Section B gives a review of previous work. Section C discusses model specification and describes the tests used in the current paper, which are somewhat novel to the field of Finance. The empirical results appear in Section D, whilst Section E provides conclusions.

B. A Review of Previous Work

One of the first attempts to address the question of the possible non-stationarity of systematic risk was a study by Blume (III). Blume computed rank and product moment correlation coefficients between betas, estimated over two consecutive seven-year periods, for beta ranked portfolios of various sizes. He concluded that whilst naively extrapolated assessments of future risk for larger portfolios were remarkably accurate, those for individual securities were of limited value in forecasting future risk. However his study did not contain statistical tests of this proposition.

Fabozzi and Francis (VIII) investigated the stability of betas over bull and bear market conditions. Their study employed dummies to test whether the model as a whole remained stable over the two regimes, and whether if it did, the instability was due to a shift in alpha or a shift in beta. However, they found no evidence that the model differed significantly between the two market conditions.

In a later study, Fabozzi and Francis (IX) used a procedure suggested by Theil (XXI) for analysing the residuals from an OLS regression, in order to test for the stability of a model. Their results indicated that beta was non-stationary for a significant minority of the stocks studied, and that the degree of

non-stationarity was sufficient to render beta, for those particular securities, unstable from the viewpoint of a portfolio manager.

Recognizing the possible non-stationarity of systematic risk, Bar-Yosef and Brown (I) reexamined an earlier study of Fama, Fisher, Jensen and Roll (FPJR) of stock splits. They used moving regressions of length 60 months to analyse the stability of beta in the period 54 months either side of the stock split. Their findings were that the betas estimated rose up to the time of the stock split, and then tailed off somewhat. However no evidence as to the statistical significance of this observation was provided. They argued, FPJR's study overstated the benefits to the existing shareholders, since at least part of the increased return observed, prior to the split date, could be accounted for in terms of an increase in risk.

Drawing on Rubinstein's work, Brenner and Smidt (V) argued that whilst there were theoretical grounds for believing systematic risk to vary through time, the measure of absolute risk might remain fairly stationary. They defined the following relationship:

$$B_j = B_j/V_j$$

where B_j represents the absolute risk of the j th firm; and v_j denotes its value.

A comparison was made of the two models:

$$r_{jt} = \alpha_j + \beta_j \cdot r_{mt} + e_{jt} \quad (1)$$

$$r_{jt} = \alpha_j + B_j \cdot (r_{mt} / V_{jt-1}) + e_{jt} \quad (2)$$

No significant difference was found as to the fit each model provided, and when the stability of each parameter, β_j and B_j , was tested using an analysis of covariance, both appeared to exhibit significant non-stationarity.

Transition matrices were employed by both Sharpe and Cooper (XX) and Roenfeldt, Griepentrog and Pflaum (RGP) (XVI) to investigate the stability of betas between two time periods. RGP then performed Chi-square tests on the data, and on the basis of these tests concluded that no significant non-stationarity existed.

In a recent study, Kon and Lau (XIII) used a maximum likelihood estimation procedure. Whilst they placed restrictions on the number of separate regimes (they allowed for two and three separate regimes), they placed no restrictions as to the ordering of the data. They found that significant non-stationarity existed in the model, and that much of this could be attributed to non-stationarities in beta.

Since we are at least partially interested in the stability of systematic risk from a practical point of view, it is

worthwhile noting that several authors have suggested that, in a Bayesian context, OLS estimates of beta will be biased. Vasicek (XII) shows that given the prior knowledge that betas are distributed with mean one, and that their probability density function is of a bell-shaped form, OLS estimates of betas that are less (greater) than one, will be biased downwards (upwards). An intuitive explanation of this is that, given the betas are distributed in the way described, an estimate of beta less (greater) than one has a higher probability of having a negative (positive) error associated with it than a positive (negative) error, due to the shape of the density function of the population of betas. Blume (IV) in a later study adjusted his beta estimates to take account of this bias, but concluded that the bias was relatively small, and that, however, there existed real non-stationarities in the betas, and that these non-stationarities took the form of a pronounced tendency for betas to regress towards their mean (one) over time.

C. Model Specification and Statistical Procedures

If the capital market is in equilibrium at time $t - 1$, then the CAPM will be given by:

$$E(R_{jt}) = R_{ft} + \{E(R_{mt}) - R_{ft}\}\beta_{jt} \quad (3)$$

where $E(R_{jt})$ is the expected return on the j th security or portfolio in period t ; R_{ft} is the risk-free rate of interest; $E(R_{mt})$ is the expected rate of return on the market portfolio; and $\beta_{jt} = \{\text{cov}(R_{jt}, R_{mt})\} / \{\text{var}(R_{mt})\}$ is the measure of systematic risk for security or portfolio j .

The market model is given by:

$$r_{jt} = \alpha_j + \beta_{jt} \cdot r_{mt} + e_{jt} \quad (4)$$

this may alternatively be expressed:

$$r_{jt} = \alpha_j + \bar{\beta}_j \cdot r_{mt} + \{(\beta_{jt} - \bar{\beta}_j) \cdot r_{mt} + e_{jt}\} \quad (5)$$

where $r_{jt} = R_{jt} - R_{ft}$; $r_{mt} = R_{mt} - R_{ft}$; and $\bar{\beta}_j$ denotes the mean of the β_{jt} over the testing period. An OLS estimate derived from (3) will be an unbiased estimate of $\bar{\beta}_j$, but in general it will not be an unbiased estimator of the true β_{jt} in any one period.

Two approaches may be identified in investigating possible non-stationarities. One approach is to place a restriction on

the number of different values the β_{jt} may take, and then to test for a significant difference between these β_{jt} . A second approach concerns analysing either the OLS residuals or the so-called recursive residuals, in order to test for the stability of the regression model.

The tests used in this paper were developed by Brown, Durbin and Evans (VI). Their tests are aimed at giving the researcher a variety of statistical 'tools' with which to investigate the stability of a model, and these tests are all contained on a program, TIMVAR, produced by the Central Statistical Office London, England. A brief description of the tests is given below.

The Cusum Test

Given the following regression model:

$$y_t = a + b \cdot x_t + e_t \quad (6)$$

where the e_t are assumed to be normally and independently distributed errors with constant variance, the recursive residual is defined as:

$$w(r) = \{y_r - (a_{r-1} + b_{r-1} \cdot x_r)\} / \sqrt{1 + x_r'(X'_{r-1} X_{r-1})^{-1} x_r} \quad (7)$$

where $w(r)$ is the r th recursive residual; y_r is the r th observation on the dependent variable,

y ; a_{r-1} and b_{r-1} are the OLS estimates of a and b derived

from a regression involving only the first $r-1$ observations;
 x_r is the 2 by 1 vector of observations for the r th period;
and X_{r-1} is the $r-1$ by 2 matrix of observations for the first
 $r-1$ periods. Thus the recursive residual may be thought of as an
appropriately-transformed prediction error. On the null
hypothesis: no switch of regimes, the $w(r)$ will be normally and
independently distributed, with expectation zero and constant
variance. Further the Cusum is defined as:

$$W(r) = \sum_{j=1}^r w(j)/s \quad (8)$$

where s denotes the mean square residual over the entire
period. Under the null hypothesis, $W(k+1), \dots, W(T)$ will be a
sequence of approximately normal variables such that:

$$E\{W(r)\} = 0$$

$$\text{Var}\{W(r)\} = r - k$$

$$\text{Cov}\{W(r), W(s)\} = \min(r, s) - k$$

Thus the $W(r)$ will, on the null hypothesis of stationarity,
follow a Brownian motion starting at time = k ¹. The Cusum test
involves investigating whether the $W(r)$ depart significantly

¹Note that recursive regressions may be run forwards through
time or backwards through time. Thus two sets of Cusums can be
derived, looking at both sets yields additional insights as to
possible instability in the regression model.

from this process. TIMVAR produces plots of the Cusum series against time. Examples of such plots appear in figures 1 through to 4 (the results of these are discussed in greater detail in the following section). The procedure adopted for testing non-stationarity in the regression model is to draw bounds through the points $(k, \pm a \sqrt{(T-k)})$, $(T, \pm 3a \sqrt{(T-k)})$, where a will be:

- 1.143 for 1% significance
- 0.948 for 5% significance
- 0.850 for 10% significance

Plots of Cusums that cross these boundaries indicate non-stationarity in the regression model, at the given level of significance, the switch of regimes occurring at that point where the boundary is crossed. This will occur when successive recursive residuals take on the same sign, thus indicating that the model estimated using data largely from earlier periods (or later periods, if the recursive regressions are being done backwards) yields biased forecasts, suggesting a switch of regimes has taken place. It can be shown that when the Cusum series crosses one of these boundaries, the following test statistic will exceed a , for the given level of significance:

$$| \{ \sqrt{(T-k)} \cdot W(r) \} / \{ (T-3k) + 2r \} | \quad (9)$$

TIMVAR prints the maximum of these for each series of recursive regressions, and these figures appear in Tables 5 and 6.

The Cusum of Squares Test

The Cusum of squares is defined as:

$$S(r) = \frac{\sum_{j=1}^r w(j)^2}{\sum_{j=1}^T w(j)^2} \quad (10)$$

An appropriate transformation of the Cusum of squares yields (on the null hypothesis of stationarity) Pyke's modified Kolmogorov-Smirnov statistic C^+ . TIMVAR produces plots of the Cusum of Squares series. Examples of such plots appear in figure 5. The procedure adopted is to draw a pair of lines:

$$S(r) = \pm C_0 + (r-k)/(T-k)$$

with C_0 :

0.1579 for 1% significance

0.1301 for 5% significance

0.1166 for 10% significance

These figures will understate the significance of our results, because unfortunately the correct figures for our particular sample size were not available at the time of

writing. Cusum of Squares series that cross these boundaries indicate non-stationarity in the regression model, at the given level of significance. When this occurs the following statistic will exceed C_0 :

$$|S(r) - (r-k)/(T-k)| \quad (11)$$

TIMVAR prints the maximum of these values for each set of recursive regressions, and these figures appear in Tables 5 and 6.

The Homogeneity Test

The homogeneity statistic is calculated from the moving regressions, and represents a standard analysis of covariance approach. The test statistic is given by:

$$\{[SSE(R) - SSE(U)]/[kp - k]\} / \{[SSE(U)]/[T - kp]\}$$

(12)

where $SSE(R)$ represents the error sum of squares of the regression over the whole period; $SSE(U)$ is the sum of the error sums of squares of the separate regressions over each of the sub-periods; k is the number of exogenous variables (including the constant); T is the length of the whole period under

consideration; n is the length of each moving regression; and p is the number of subperiods.

On the null hypothesis: no switch of regimes, the test statistic will be F distributed with $[kp - k]$ and $[T - kp]$ degrees of freedom.

Quandt's Log-Likelihood Statistic

Quandt's log-likelihood statistic is given by:

$$Q = \ln \{L[H_0]/L[H_1]\} \quad (13)$$

that is, the log of the ratio of the likelihood function under the null hypothesis (stationarity) to the likelihood under the alternative hypothesis (two regimes). In order to locate the break point, the minimum of this statistic is taken (the length of the two sub-periods being allowed to vary). Under the assumption of normality this is equivalent to choosing that break-point (and the sub-periods associated with it) that minimises the $SSE(U)$ of the test statistic given in (12). Unfortunately the distributional properties of this estimator of the break-point are not known, thus in interpreting the results this must be borne in mind.

The above tests are all designed to test for the stability of a regression model. This paper is particularly interested in

the stability of one of the parameters of a model, beta. To test for the stationarity of beta individually, a regression model of the following form was employed:

$$r_{jt} = \alpha_j + \beta_j \cdot r_{mt} + \delta_{11} \cdot \text{DUM1} + \delta_{12} \cdot \text{DUM1} \cdot r_{mt} + \delta_{21} \cdot \text{DUM2} + \delta_{22} \cdot \text{DUM2} \cdot r_{mt} \\ + \dots + \delta_{(p-1)1} \cdot \text{DUM}(p-1) + \delta_{(p-1)2} \cdot \text{DUM}(p-1) \cdot r_{mt} + e_{jt} \quad (14)$$

where DUMk takes on the value 1 in the kth sub-period and 0 in all other sub-periods; p is the number of sub-periods; and all other terms are as previously defined. The pth sub-period provides the reference period. The choice of sub-periods was designed to match those used in the calculation of the homogeneity test statistic. The test statistic calculated from a joint test of the significance of all the dummy parameter coefficients in the above model would be identical to the homogeneity statistic.

D. Empirical Results

The empirical work in this study centres on the possible non-stationarity of the betas of the the portfolios used by BJS in their tests of the CAPM. BJS argued that the method by which their portfolios were formed assured the stationarity of their betas through time. A check of this assumption was made by BJS. The model, given by (5), was estimated over four non-overlapping sub-periods, and the regression results were tabulated. From a visual inspection (they undertook no formal tests) beta was judged to be 'fairly stationary' for the majority of the portfolios. Given this conclusion, the best possible estimate of a beta, was thought to be provided by a regression over the whole period. These estimates were then used in the second pass regressions (of the sub-periods, as well as of the full period). BJS did indicate that the alphas did not appear to remain stationary over time, and they argued that this reflected the presence of a zero-beta factor, and that the Sharpe-Lintner-Mossin CAPM might be misspecified.

The study of BJS, in common with other tests of the CAPM,

used portfolio data ², in an effort to reduce the standard error of the estimates of the betas, and thus minimise the asymptotic biases associated with the second pass regressions, in which the estimated betas appear as regressors. Since formation of these portfolios involved use of a great deal of computer time, and hence expense, it was decided to use a set of portfolios that were already available, and had been formed in a somewhat similar manner. The portfolios were those employed in a study by Grauer (XI). Instead of the ten portfolios used by BJS, our data set consisted of twenty beta-ranked portfolios.

In investigating the stationarity assumption of BJS, five portfolios were employed, portfolios 1, 5, 10, 15 and 20 (high beta to low beta). The initial OLS regression results over the full 467 month period, January 1934 through to August 1972, are given in tables 1 and 2. BJS employed an equally weighted index as their market proxy, hence we shall be primarily concerned with the results using this index, although it proves interesting to note what difference, if any, it makes in using a value weighted index.

As observed in previous empirical studies, the high beta portfolios had negative intercepts associated with them, and the low beta portfolios had positive intercepts associated with them, the theoretical value of the intercept being, under the

²If the returns on the securities forming a portfolio are not perfectly correlated, then the standard error of the estimate of the beta of that portfolio will be greatly reduced.

Sharpe-Lintner-Mossin hypothesis, zero.

Second pass regressions of the type performed by BJS, were repeated for our data using all twenty estimates of portfolio betas. That is, the following regression was run:

$$\bar{r}_j = \gamma_0 + \gamma_1 \cdot \beta_j + e_j \quad (13)$$

where \bar{r}_j is the mean excess return of the j th portfolio over the risk-free rate for the testing period. From (3), the hypothesised value of γ_0 is zero, and the hypothesised value of γ_1 is \bar{r}_m , the mean excess return of the market portfolio over the risk-free rate, for the testing period. The results are due to Grauer (XII), and appear in tables 3 and 4. The betas used for the sub-periods were estimated in those sub-periods. The similarity of these results to those obtained by BJS would appear to confirm the validity of using the portfolios formed by Grauer as proxies for the BJS portfolios.

Tables 5 and 6 give the results produced by TIMVAR. Using an equally weighted proxy for the market portfolio, two of the Cusum statistics exceeded the critical value at the 5% level, whilst all of the Cusum of Squares statistics exceeded the critical value at the 1% level. The homogeneity statistics produced additional evidence of non-stationarity, with four of these exceeding the critical value for sub-periods of length 60, and three exceeding the critical value for sub-periods of length 120.

With a value weighted proxy for the market portfolio, a far greater degree of non-stationarity seems to exist. Three of the Cusum statistics exceeded the critical value at the 5% level. Again all of the Cusum of Squares statistics exceeded the critical value at the 1% level. The homogeneity statistics all exceeded the critical value at the 1% level barring one, which was however significant at the 5% level.

Evans (VII) suggests that the Cusum of Squares test may not always yield identical results to the Cusum test. He argues that large consecutive recursive residuals of opposite sign will show up in the Cusum of Squares series, but not in the Cusum series. Further the drawing of the bounds involved in the Cusum test is an approximation, the formulae for the true bounds being mathematically intractable. The bounds used in the Cusum test will tend to understate the significance of the results. The formulation of the Cusum of Squares test also involves an approximation, and this approximation may lead to an overstatement of the significance of the results. These facts might explain the rather different results yielded by the Cusum and Cusum of Squares tests that were found in this study.

An interesting finding was that Quandt's log-likelihood ratio reached a minimum, in virtually every instance, during World War II. It is difficult to know what interpretation to put on this result. It is not clear from the data whether a fundamental shift in the structure of the market came about as a

result of the War, or whether betas shifted considerably both before and after the War, and that this finding is merely a result of the constraint imposed by the use of this statistic vis-a-vis the existence of only two regimes.

Figures 1 through to 4 show the plots of those Cusum series that indicated instability in the regression model. Similarly Figure 5 shows a sample of the plots of those Cusum of Squares series that indicated instability in the regression model. The evidence that these present does not seem to point to a switch of regimes during the War years.

To test whether the instability observed in the regression model arose through shifts in alpha or shifts in beta, a regression of the form: (12) was run. The results of these appear in Table 7. Both alpha and beta appeared to move between sub-periods. The betas of the low risk portfolios appeared to have shifted considerably through time.

Table 1

Summary of Results for the Regression

$$r_{jt} = \alpha_j + \beta_j \cdot r_{mt} + e_{jt}$$

Based on an Equally Weighted Proxy for the Market Portfolio (Fisher's Arithmetic Index)

Portfolio (j)	$\hat{\alpha}_j$	$\hat{\beta}_j$	$t(\hat{\alpha}_j)$	$t(\hat{\beta}_j)$	s.e.	R ²	D.W.
1	-0.3424	1.3411	-15.65	62.25	0.0291	0.8928	2.20
5	-0.0455	1.0466	-2.68	62.63	0.0226	0.8940	1.86
10	0.1190	0.8838	6.70	50.49	0.0237	0.8457	2.16
15	0.2501	0.7510	16.08	49.03	0.0207	0.8379	2.09
20	0.5564	0.4469	30.70	25.04	0.0241	0.5742	1.77

Table 2

Summary of Results for the Regression

$$R_{jt} = \alpha_j + \beta_j R_{mt} + e_{jt}$$

Based on Value Weighted Proxy for the Market Portfolio (contained on the CRSP tapes)

Portfolio (j)	$\hat{\alpha}_j$	$\hat{\beta}_j$	$t(\hat{\alpha}_j)$	$t(\hat{\beta}_j)$	s.e.	R^2	D.W.
1	-0.7397	1.7388	-17.22	40.93	0.0415	0.7827	2.03
5	-0.4186	1.4193	-15.42	52.88	0.0262	0.8574	2.07
10	-0.2049	1.2073	-7.81	46.51	0.0253	0.8231	2.10
15	-0.0385	1.0392	-1.79	48.82	0.0210	0.8367	2.01
20	0.3755	0.6275	15.31	25.88	0.0237	0.5902	1.81

Table 3

Summary of Results for the Regression

$$\bar{r}_j = \gamma_0 + \gamma_1 \cdot \beta_j + e_j$$

Based on an Equally Weighted Proxy for the Market Portfolio (Fisher's Arithmetic Index)

Period	$\gamma_1 \equiv \bar{r}_m$	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$t(\hat{\gamma}_0)$	$t(\hat{\gamma}_1 - \gamma_1)$	R^2	D.W.
1/34-12/71	0.0119	0.0058	0.0066	7.006	-5.952	0.745	1.95
1/34-6/43	0.0178	0.0054	0.0122	2.729	-2.582	0.621	2.11
7/43-12/52	0.0130	0.0082	0.0050	5.195	-4.804	0.318	1.74
1/53-6/62	0.0087	0.0090	0.0009	5.162	-4.479	0.014	1.87
7/62-12/71	0.0084	-0.0027	0.0115	-1.254	1.448	0.601	2.22

Table 4

Summary of Results for the Regression

$$\bar{r}_j = \gamma_0 + \gamma_1 \beta_j + e_j$$

Based on a Value Weighted Proxy for the Market Portfolio (contained on the CRSP tapes)

Period	$\gamma_1 \equiv \bar{r}_m$	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$t(\hat{\gamma}_0)$	$t(\hat{\gamma}_1 - \gamma_1)$	R^2	D.W.
1/34-12/71	0.0084	0.0047	0.0058	4.842	-3.386	0.741	1.67
1/34-6/43	0.0088	0.0040	0.0090	1.815	0.096	0.614	1.88
7/43-12/52	0.0110	0.0074	0.0047	4.307	-4.374	0.350	1.75
1/53-6/62	0.0085	0.0084	0.0015	4.460	-3.654	0.031	1.84
7/62-12/71	0.0055	-0.0039	0.0103	-1.711	4.008	0.617	2.22

Table 5

Summary of Results Produced by TIMVAR

Based on an Equally Weighted Proxy for the Market Portfolio (Fisher's Arithmetic Index)

Portfolio	Recursive Regressions		Recursive Regressions		Moving Regressions	Homogeneity Statistic	Log-likelihood
	Backwards	Forwards	Backwards	Forwards			
	Cusum	Cusum of Squares	Cusum of Squares	Cusum of Squares	Length	Length	Ratio a minimum at
1	0.3205	0.2899*	0.6635	0.2916*	60	120	3/46
5	0.4446	0.3513*	1.0474**	0.3214*	2.07**	4.70*	8/43
10	0.5518	0.4393*	0.8576#	0.4275*	2.17**	2.24	11/43
15	0.8738#	0.4124*	1.1767*	0.3762*	5.65*	6.66*	2/43
20	0.6814	0.3815*	0.8862#	0.2616*	7.64*	18.62*	2/43

* = significant at the 15 level; ** = significant at the 5% level; # = significant at the 10% level.



Table 6

Summary of Results Produced by TIMVAR

Based on a Value Weighted Proxy for the Market Portfolio (contained on the CRSP tapes)

Portfolio	Recursive Regressions		Recursive Regressions		Moving Regressions		Log-likelihood	
	Backwards	Forwards	Homogeneity Statistic	Ratio a	minimum at			
	Cusum	Cusum of Squares	Cusum of Squares	Length	Length			
				60	120			
1	0.7376	0.2622*	1.0784**	0.1517*	5.71*	13.69*	1/46	
5	0.5281	0.2097*	0.5603	0.1592*	3.56*	6.20*	7/44	
10	0.9357#	0.3352*	1.4399*	0.2716*	3.67*	8.43*	11/43	
15	0.3948	0.3426*	1.1647*	0.3071*	3.18*	4.36*	5/40	
20	0.4970	0.2408*	0.9267#	0.2218*	3.35*	2.74**	11/42	

* = significant at the 1% level; ** = significant at the 5% level; # = significant at the 10% level.

Table 7

Summary of Results for the Regression

$$r_{jt} = \alpha_j + \beta_j \cdot r_{mt} + \delta_{11} \cdot DUM1 + \delta_{12} \cdot DUM1 \cdot r_{mt} + \delta_{21} \cdot DUM2 + \delta_{22} \cdot DUM2 \cdot r_{mt} + e_{jt}$$

where DUM1 = 1 for 1/34-12/43; 0 otherwise.

and DUM2 = 1 for 1/44-12/53; 0 otherwise.

Based on an Equally Weighted Proxy for the Market Portfolio (Fisher's Arithmetic Index).

The standard error of each parameter estimate appears in brackets below each parameter estimate.

Portfolio	$\hat{\alpha}_j$	$\hat{\beta}_j$	$\hat{\delta}_{11}$	$\hat{\delta}_{12}$	$\hat{\delta}_{21}$	$\hat{\delta}_{22}$	R^2	D.W.
1	-0.32 (0.05)	1.32 (0.04)	-0.02 (0.05)	0.02 (0.05)	-0.06 (0.07)	0.05 (0.07)	0.893	2.21
5	-0.11 (0.03)	1.11 (0.03)	0.10 (0.04)	-0.11 (0.04)	-0.05 (0.06)	0.05 (0.06)	0.898	1.89
10	0.04 (1.01)	0.96 (0.04)	0.12 (0.04)	-0.11 (0.04)	0.07 (0.06)	-0.07 (0.06)	0.849	2.20
15	0.16 (0.03)	0.85 (0.03)	0.12 (0.04)	-0.12 (0.04)	0.11 (0.05)	-0.11 (0.05)	0.847	2.17
20	0.32 (0.03)	0.68 (0.03)	0.33 (0.04)	-0.33 (0.04)	0.15 (0.06)	-0.15 (0.06)	0.633	1.83

Figure 1 Cusum Series - Backwards Recursive Regressions

PORTFOLIO 15 Based on an Equally Weighted Proxy for the Market Portfolio

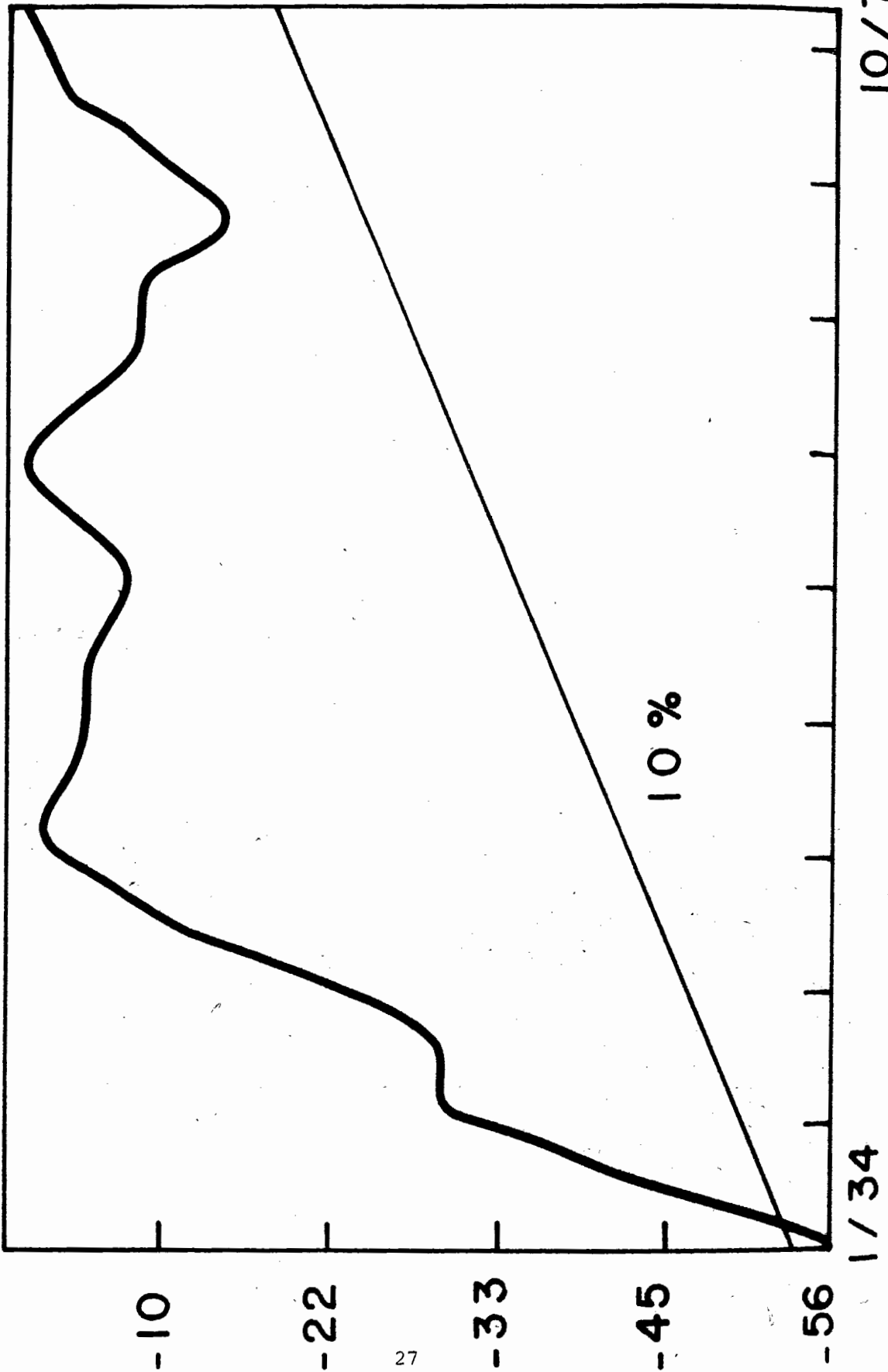


Figure 1 Cusum Series - Backwards Recursive Regressions

PORTFOLIO 15

Based on an Equally Weighted Proxy for the Market Portfolio

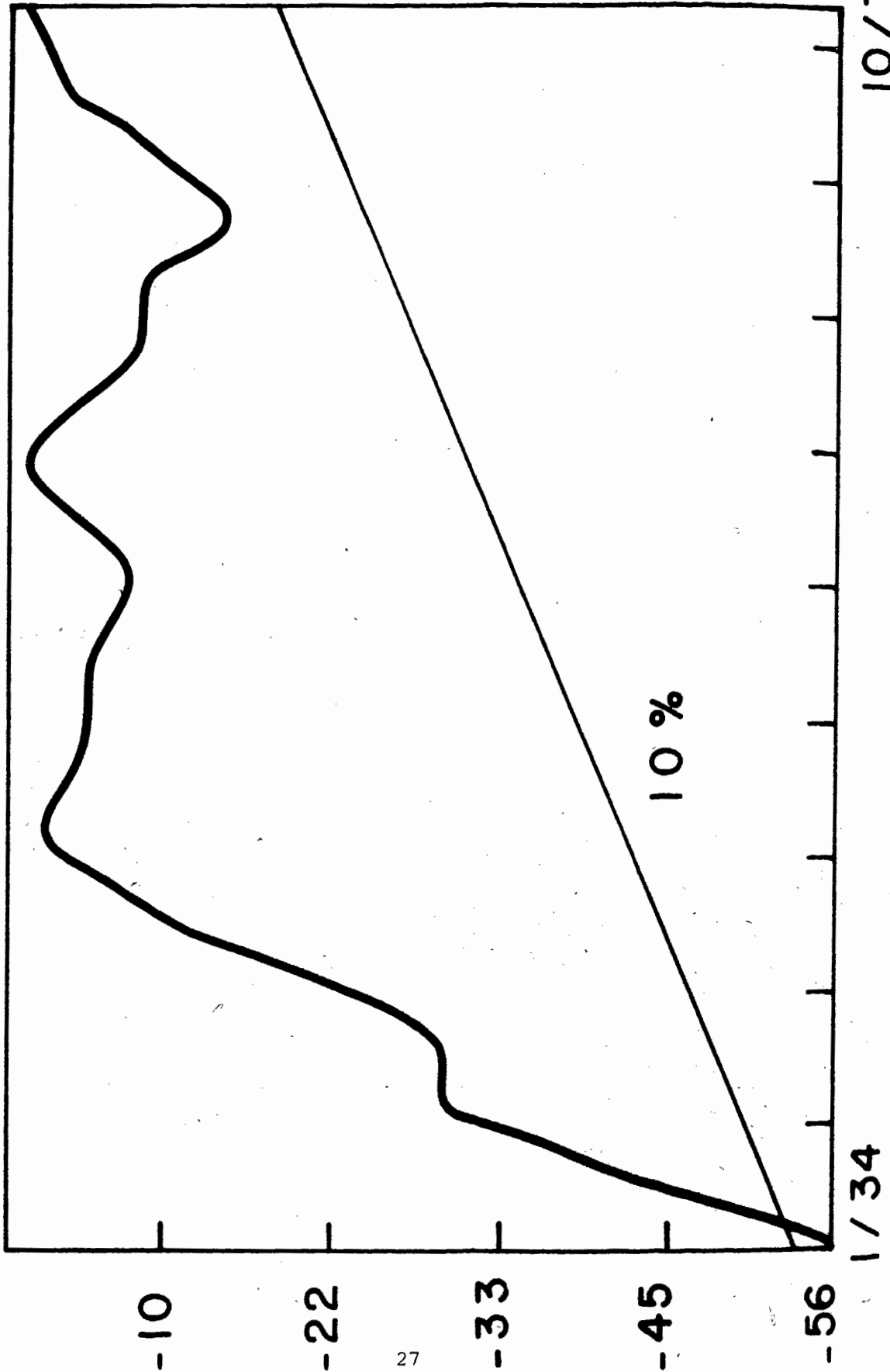
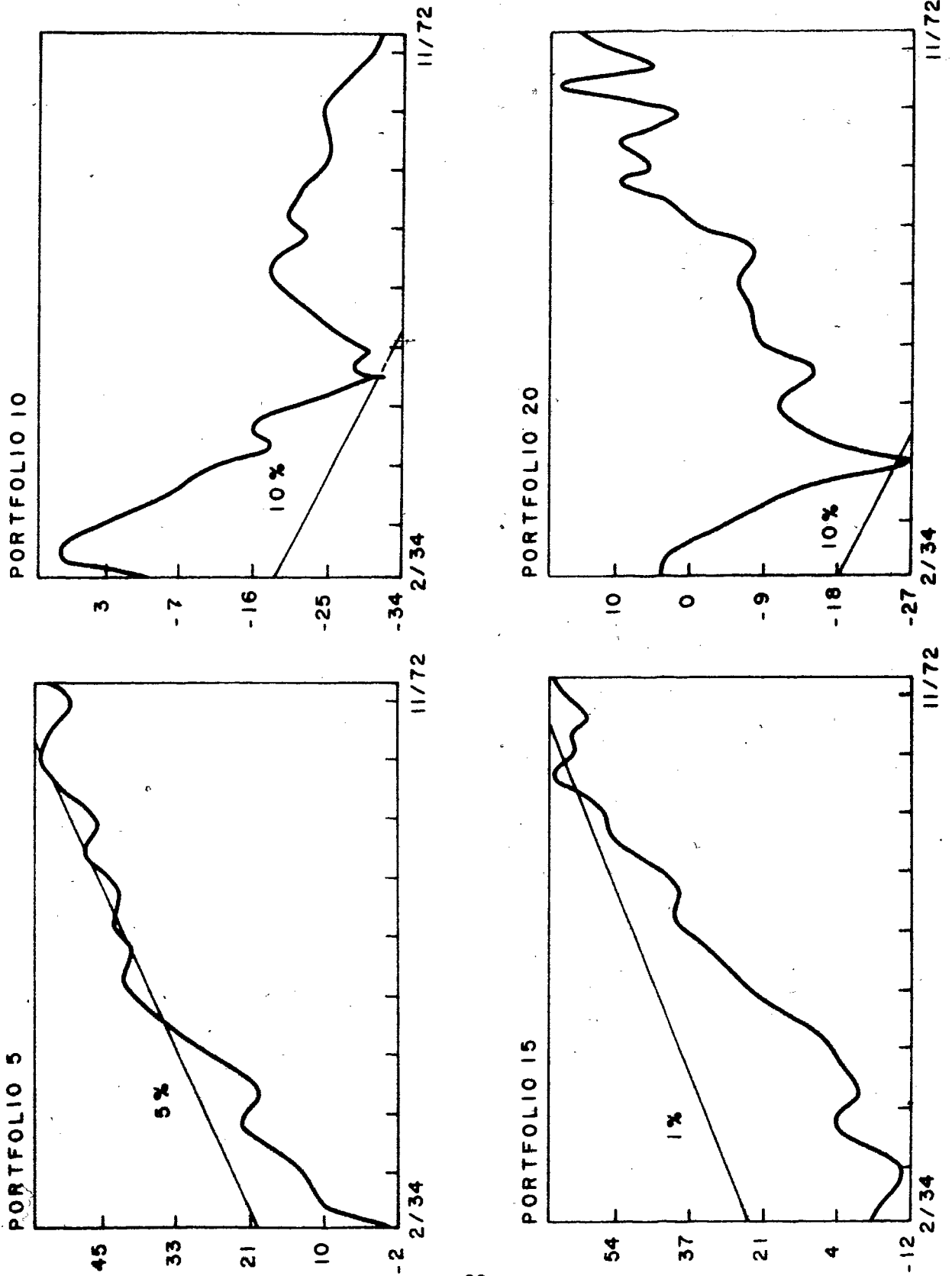


Figure 2 Cusum Series - Forwards Recursive Regressions
 Based on an Equally Weighted Proxy for the Market Portfolio



PORTFOLIO 10

Based on a Value Weighted Proxy for the Market Portfolio

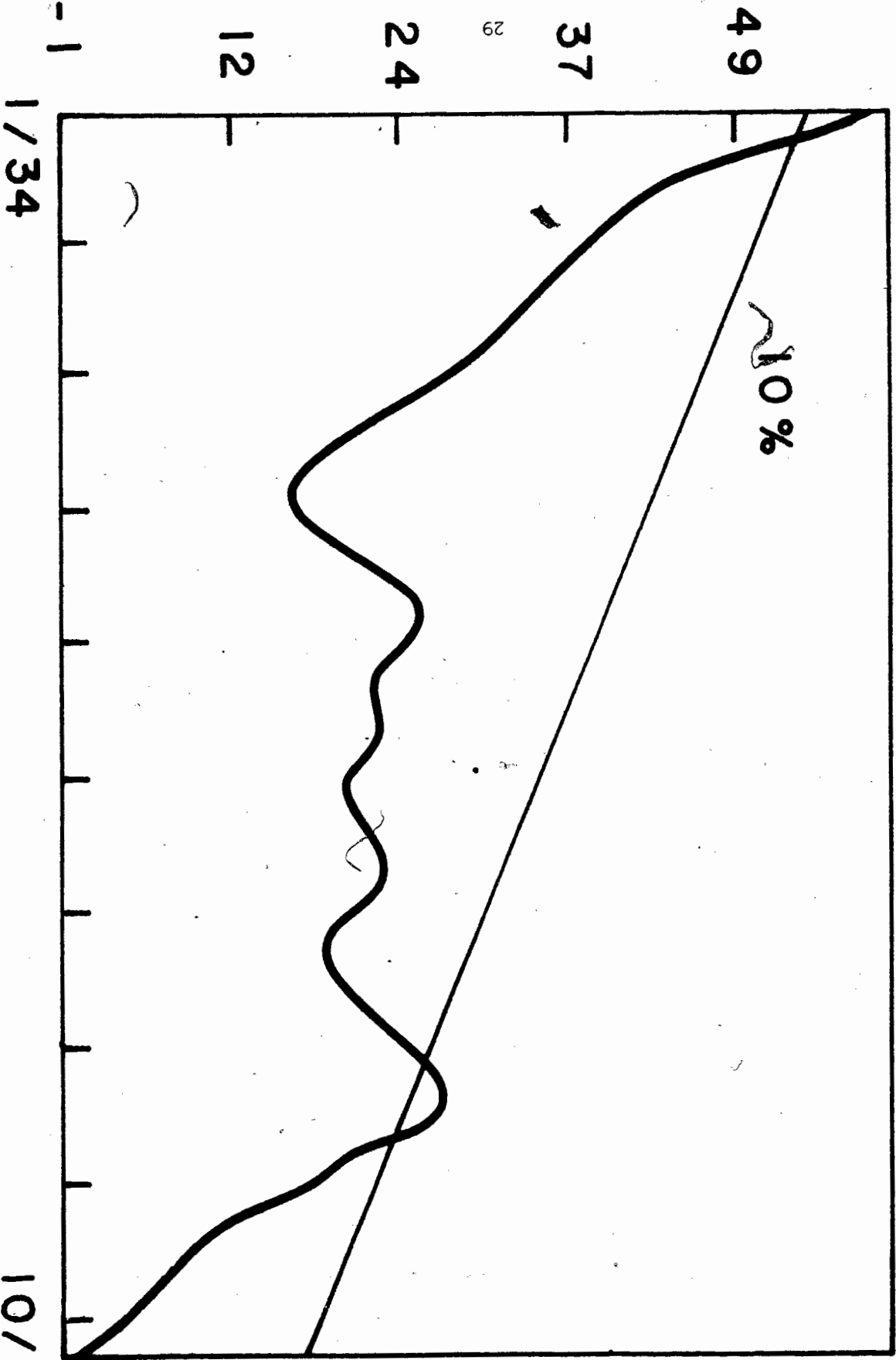


Figure 3 Cusum Series - Backwards Recursive Regressions

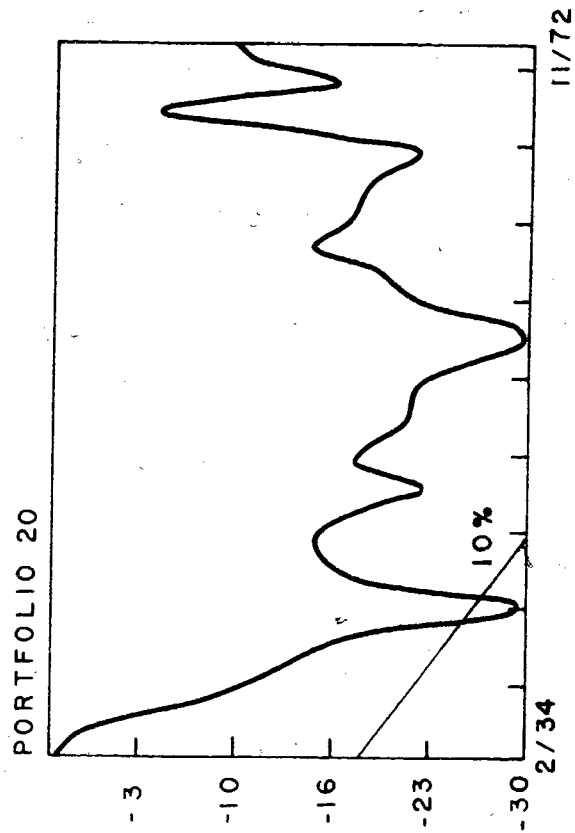
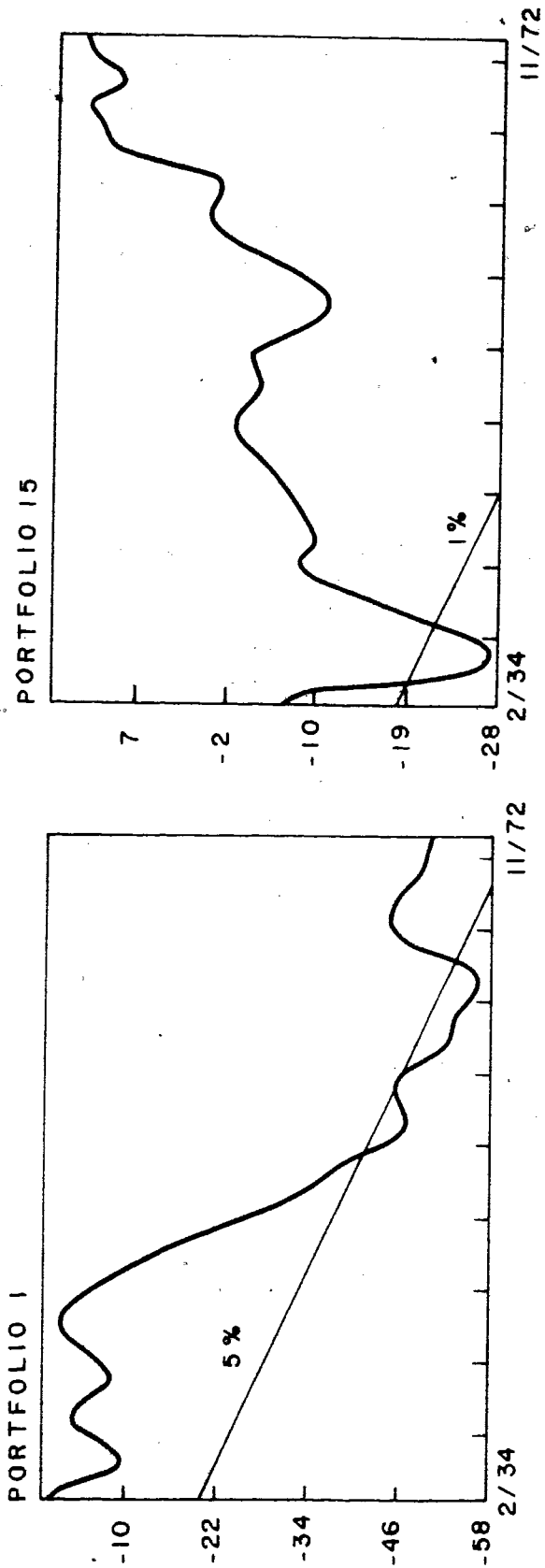


Figure 4 Cusum Series -
 Forwards Recursive Regressions
 Based on a Value Weighted Proxy
 for the Market Portfolio

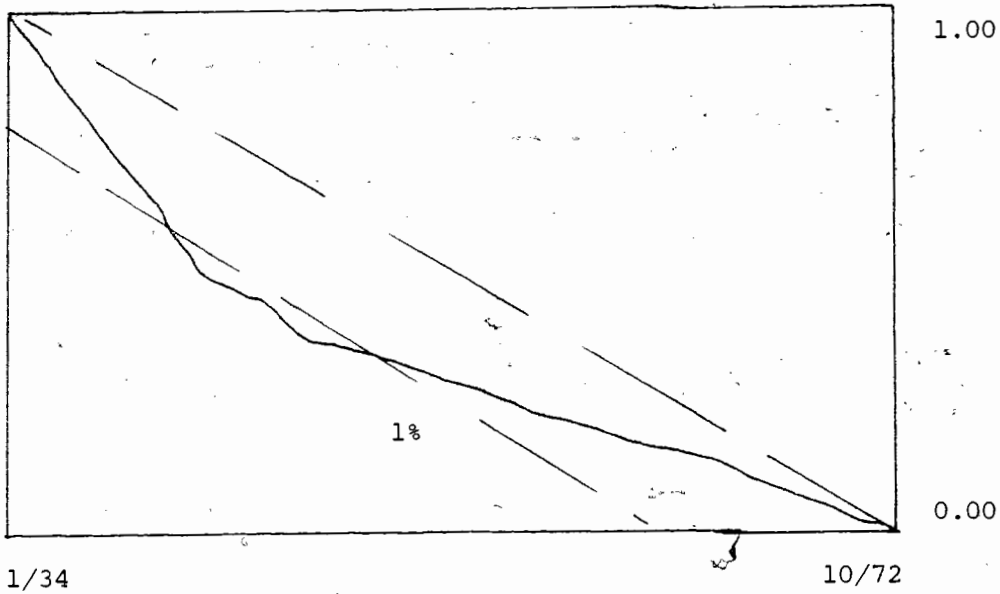
Figure 5

Plots of Cusum of Squares Series

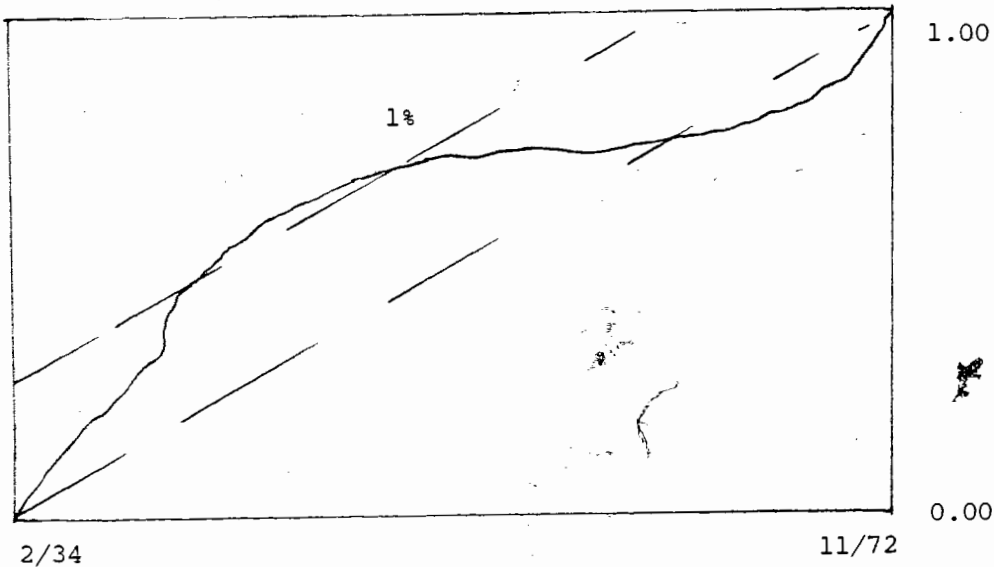
Based on an Equally Weighted Proxy for the Market Portfolio

(Fisher's Arithmetic Index)

Portfolio 15 (backwards)



Portfolio 20 (forwards)



E. Conclusions

Our results indicate that the betas of the portfolios studied do not appear to be stationary through time. The implication of this finding is that the time period over which an estimate of beta is to be used will dictate the time period over which that estimate should be calculated. The shorter the time period over which the model is judged to be stationary, the less information will one have with which to calculate an estimate of beta. This is an unfortunate fact. Thus at any point in time we may choose between an estimate of beta derived from a regression over an appropriate sub-period, that will be unbiased, but may have a large standard error, due to small sample size, or an estimate of beta derived from a regression over the whole period, that will in general be biased but may have a smaller standard error associated with it³

³From equation (5) the variance of the residual:

$$z_{jt} = \{(\beta_{jt} - \bar{\beta}_j) \cdot r_{mt} + e_{jt}\}$$

will be given by:

$$\text{var}(z_j) = \text{var}(\beta_j) \cdot \text{var}(r_m) + \text{var}(e_j)$$

where it is assumed that:

$$\text{cov}(\beta_j, r_m) = \text{cov}(\beta_j, e_j) = \text{cov}(r_m, e_j) = 0$$

Ignoring the non-stationarity of beta will in general lead to bias in the second pass regressions in tests of the CAPM. Taking into account non-stationarities in the betas will lead to estimates of the betas, that whilst unbiased, may have larger standard errors associated with them. This increment in the standard errors of the betas may increase the asymptotic bias present in the second pass regressions.

The findings of this paper do not appear to offer comfort to the empirical researcher. However it is as well that the researcher be aware of potential deficiencies in his data.

 3 (cont'd) thus the variance of an estimate of β_j derived from a regression using T observations will be given by:

$$\text{var}(\hat{\beta}_j) = \{\text{var}(\beta_j) + \text{var}(e_j)\} / \left\{ \sum_{t=1}^T (r_{mt} - \bar{r}_m)^2 \right\}$$

Although $\text{var}(\beta_j)$ may be significantly greater than zero, implying the existence of beta non-stationarity, the denominator will increase with increasing sample size, and this will tend to reduce $\text{var}(\hat{\beta}_j)$.

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