

THEORY OF THE MONOPOLY UNDER UNCERTAINTY

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B.A., California State University, 1971

A THESIS SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ARTS

in the Department

of

Economics and Commerce

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SIMON FRASER UNIVERSITY

June 1978

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ABSTRACT

In the traditional monopoly theory, it is argued that a monopoly firm produces the optimum output quantity characterized by its marginal revenue being equal to marginal cost, and a monopsony firm employs the optimum input quantity characterized by its marginal expenditure being equal to marginal revenue product of input. Traditional in these theories is the presumption that agents act as if economic environments were nonstochastic. However, recent studies of firms facing uncertainty indicate that several widely accepted results of the riskless theories must be abandoned.

This paper extends the traditional monopoly theory to the case where monopolists are interested in maximizing expected utility from profits and they are risk-averse. This paper also examines the behaviors of a monopoly firm facing demand uncertainty and monopsony firm facing cost uncertainty. In the last section of this paper, we will examine the implications of uncertainty in the case of bilateral monopoly. For the main tools of analysis, we draw upon the work of Sandmo.

This paper shows that the optimum conditions of the traditional monopoly theory do not hold under uncertainty. It will be concluded that increased demand uncertainty leads to a decline in the monopolist's output and that increased cost uncertainty leads to a decline in the monopsonist's employment of input, provided that firms are risk-averse.

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Theory of the Monopoly under Uncertainty

In the traditional monopoly theory, it is argued that a monopoly firm produces the optimum output quantity characterized by its marginal revenue being equal to marginal cost, and a monopsony firm employs the optimum input quantity characterized by its marginal expenditure being equal to marginal revenue product of input. Traditional in these theories is the presumption that agents act as if economic environments were nonstochastic. Although it was shown by Radner [18] that one can still apply traditional theorems, in the case of competitive firms, when firms have unlimited computational capacity even if uncertainty exists and agents have heterogeneous information, recent studies of firms facing uncertainty indicate that several widely accepted results of the riskless theories must be abandoned. A number of writers in recent years have extended the traditional monopoly theory to the case where monopolists are interested in maximizing expected utility from profits and are risk-averse.

The purpose of this paper is to examine the behaviours of a monopoly firm facing demand uncertainty and a monopsony firm facing cost uncertainty. In section IV of this paper, we will also examine the implications of uncertainty in the case of bilateral monopoly. For the main tools of analysis, we draw upon the work of Sandmo [20], who has examined the behaviour of the competitive firm under price uncertainty. However, Sandmo is unable to derive categorical effects of a marginal increase in uncertainty whereas this paper shows that increased demand uncertainty leads to a decline in the monopolist's output and that increased cost uncertainty leads to a decline in the monopsonist's employment of input, provided that firms are risk-averse.

I Monopoly under Uncertainty

In the traditional riskless monopoly theory, it is assumed that the demand price for the monopolist's product is a function of the output and that the monopolist knows, with certainty, the demand function relating the two variables and its cost function. The optimum quantity produced by the monopolist under certainty is then characterized by its marginal revenue being equal to its marginal cost. The monopolist considered in this paper likewise knows its demand function as well as its cost function; the difference being that the demand function now contains a random variable ϵ over which the monopolist has no control. In this section, we will examine the implications of introducing uncertainty via the random variable ϵ in the demand function and derive its comparative statics properties.

We will assume that the stochastic demand function faced by the monopolist is of the form:

$$(1) \quad P = a(\epsilon) - bZ,$$

where Z is its output and ϵ is a random variable.

Profit is given by:

$$(2) \quad \Pi = Z \cdot P(Z, \epsilon) - C(Z) - k,$$

where $C(Z)$ is its total variable costs assumed to be known with certainty and k represents fixed costs and $P(Z, \epsilon)$ is the stochastic demand price faced by the monopolist.

The monopolist maximizes the expectation of its utility from profits:

$$(3) \quad E[U(\Pi)] = \int U[Z \cdot P(Z, \epsilon) - C(Z) - k] \cdot f(P) dP,$$

where $f(P)$ represents its subjective probability density function with $E[P] = \bar{P}$.

Differentiating (3) with respect to its control variable Z yields the first and the second order conditions:

$$(4.a) \quad F = E[(R' - C') \cdot U'(\Pi)] = 0, \quad R' = MR \text{ and } C' = MC$$

$$(4.b) \quad S = E[(R' - C')^2 \cdot U''(\Pi) - (2b + C'')U'(\Pi)] < 0, \quad C'' = \frac{d(MC)}{dZ}$$

Rewriting (4.a) as $E[C' \cdot U'(\Pi)] = E[R' \cdot U'(\Pi)]$ and then subtracting $E[\bar{R}' \cdot U'(\Pi)]$ from both sides, we obtain:

$$(5) \quad (\bar{R}' - C')E[U'(\Pi)] = E[(\bar{a} - a)U'(\Pi)],$$

where $\bar{R}' = E[R']$ and $\bar{a} = E[a]$.

Proposition (I.A):

A risk-averse monopoly firm under uncertainty will produce less than it would under certainty.

In order to prove the proposition, we will rewrite the profit function (2) by adding and subtracting $Z \cdot \bar{P}$ so that:

$$(6) \quad \Pi = \bar{\Pi} + (P - \bar{P}) \cdot Z,$$

where $\bar{\Pi} = Z \cdot \bar{P} - C(z) - k$.

We will then show that the monopoly firm's optimum output solution is characterized by its \bar{R}' greater than C' so that it is to the left of the traditional optimum solution under certainty assuming that the expected demand price under uncertainty corresponds to the demand price of the certainty case. Case (i) Suppose that the stochastic demand price is under-estimated, ex ante. Then:

$$(7) \quad \begin{aligned} P > \bar{P} &\rightarrow U(\Pi) > U(\bar{\Pi}) \\ &\rightarrow U'(\Pi) < U'(\bar{\Pi}) \\ &\rightarrow (\bar{P} - P)U'(\Pi) > (\bar{P} - P)U'(\bar{\Pi}) \\ &\rightarrow E[(\bar{P} - P)U'(\Pi)] > U'(\bar{\Pi}) E[\bar{P} - P] = 0 \\ &\rightarrow E[(\bar{a} - a)U'(\Pi)] > 0 \end{aligned}$$

Therefore, LHS of (5) is also positive, and since $E[U'(\Pi)]$ is always positive, we have:

$$(8) \bar{R}' > C'.$$

Case (ii) Suppose that the stochastic demand price is over-estimated, ex ante.

Then:

$$\begin{aligned} P < \bar{P} &\rightarrow U(\Pi) < U(\bar{\Pi}) \\ &\rightarrow U'(\Pi) > U'(\bar{\Pi}) \\ &\rightarrow (\bar{P}-P)U'(\Pi) > (\bar{P}-P)U'(\bar{\Pi}) \\ &\rightarrow E[(\bar{P}-P)U'(\Pi)] > U'(\bar{\Pi})E[\bar{P}-P] = 0 \\ &\rightarrow E[(\bar{a}-a)U'(\Pi)] > 0. \end{aligned}$$

Therefore, LHS of (5) is also positive, and since $E[U'(\Pi)]$ is always positive, we have the same result that $\bar{R}' > C'$ as in (8).

Proposition (I.B):

Increased uncertainty decreases output of a monopoly firm.

We will now consider the effects of a marginal increase in uncertainty by defining the variability of the density function of the demand price in terms of mean preserving spread. This requires one multiplicative shift parameter and one additive shift parameter. Let us define:

$$(9) a' = \gamma a + \theta,$$

where γ is the multiplicative shift parameter which gives a stretching of the probability distribution around a constant mean and θ is the additive shift parameter.

Then the mean preserving spread type of shift in the density function of a' leaves the mean $E[a']$ unchanged, that is, $dE[a'] = \bar{a}d\gamma + d\theta = 0$, or:

$$(10) \frac{d\theta}{d\gamma} = -\bar{a}.$$

We can now write the profit function as:

$$(11) \Pi = Z[(\gamma a + \theta) - bZ] - C(Z) - k,$$

and the new first order condition becomes:

$$(12) F = E[(\gamma a + \theta - 2bZ - C') \cdot U'(\Pi)] = 0$$

Totally differentiating (12) with respect to γ , taking account of (10) and then evaluating at the initial point, $\gamma=1$, $\theta=0$, we obtain:

$$(13) \frac{dZ}{d\gamma} = \frac{Z}{S} E[(R' - C')(\bar{a} - a)U''(\Pi)] + \frac{1}{S} E[(\bar{a} - a) \cdot U'(\Pi)],$$

where $S = E[(R' - C')^2 U''(\Pi) - (2b + C'')U'(\Pi)]$.

This expression for the change in output due to increase in uncertainty is decomposed into two separate terms. We know the sign of the second term

$E[(\bar{a} - a)U'(\Pi)]/S$ to be negative since, from (7), $E[(\bar{a} - a)U'(\Pi)]$ is positive

and S is required to be negative. Therefore, we must prove that

$E[(R' - C')(\bar{a} - a)U''(\Pi)]$ has positive sign in order to prove our proposition (I.B).

We can rewrite $E[(R' - C')(\bar{a} - a)U''(\Pi)]$ as:

$$\begin{aligned} E[(R' - C')(\bar{a} - a)U''(\Pi)] &= E[(R' - C')(\bar{R}' - R')U''(\Pi)] \\ &= E[(R' - C')\{(\bar{R}' - C') + (C' - R')\}U''(\Pi)] \\ &= E[(R' - C')(\bar{R}' - C')U''(\Pi)] + E[-(R' - C')^2 U''(\Pi)] \\ (14) \quad &= (\bar{R}' - C')E[(R' - C')U''(\Pi)] + E[-(R' - C')^2 U''(\Pi)]. \end{aligned}$$

The second term in (14) is always positive. We must then show that the sign of $E[(R' - C')U''(\Pi)]$ is positive in the first term since $(\bar{R}' - C')$ is positive from (8).

Let a^* be the value of a such that $R' = C'$ and let Π^* be the corresponding profit level.

Case (i) $a > a^* \rightarrow \Pi > \Pi^*$

$$\begin{aligned} &\rightarrow \rho(\Pi) < \rho(\Pi^*) \\ &\rightarrow \rho(\Pi) \cdot [-(R' - C')U'(\Pi)] > \rho(\Pi^*) [-(R' - C')U'(\Pi)] \\ &\rightarrow (R' - C')U''(\Pi) > -\rho(\Pi^*) \cdot [(R' - C')U'(\Pi)] \\ &\rightarrow E[(R' - C')U''(\Pi)] > -\rho(\Pi^*) \cdot E[(R' - C')U'(\Pi)] = 0 \end{aligned}$$

where $\rho(\Pi) = \frac{-U''(\Pi)}{U'(\Pi)}$

since $E[(R'-C')U'(\Pi)] = 0$ from the FOC. Therefore,

$$(15) E[(R'-C')U''(\Pi)] > 0.$$

Case (ii) $a < a^* \rightarrow \Pi < \Pi^*$

$$\begin{aligned} &\rightarrow \rho(\Pi) > \rho(\Pi^*) \\ &\rightarrow \rho(\Pi)[-(R'-C')U'(\Pi)] > \rho(\Pi^*)[-(R'-C')U'(\Pi)] \\ &\rightarrow (R'-C')U''(\Pi) > -\rho(\Pi^*)[(R'-C')U'(\Pi)] \\ &\rightarrow E[(R'-C')U''(\Pi)] > -\rho(\Pi^*)E[(R'-C')U'(\Pi)] = 0 \end{aligned}$$

Therefore, the expression in (14) is positive, which implies that the expression in (13) is, in turn, negative and our proposition (I.B) is proved.

Proposition (I.C):

Increased fixed costs decrease output of a monopoly firm with decreasing absolute risk aversion.

To determine the effect of a small increase in fixed cost k , we totally differentiate (4.a), obtaining:

$$(16) \frac{dZ}{dk} = \frac{1}{S} E[(R'-C')U''(\Pi)] .$$

Since the sign of S is negative from (4.b) and the sign of $E[(R'-C')U''(\Pi)]$ is positive from (15), the sign of dZ/dk is negative and our proposition (I.C) is proved.

Proposition (I.D):

Increased demand increases output of a monopoly firm with decreasing absolute risk aversion.

Since the demand is stochastic, it does not make sense to speak about the effect of increased demand for the monopoly firm's output. However, it seems intuitively natural to have an upward shift of the demand schedule by an amount θ . Such an increase in demand will leave the distribution of price conditional on quantity unaffected except for a higher mathematical expectation of price.

We will suppose that the size of upward shift in demand, θ , remains invariant with respect to Z .

Let $p^* = a - bZ + \theta$ define the new demand curve resulting from the upward shift where θ is the shift parameter. The new first order condition becomes:

$$(17) F = E[(R' - C' + \theta)U'(\Pi + Z\theta)] = 0.$$

Totally differentiating (17) and evaluating at initial demand, $\theta=0$, we obtain:

$$(18) \frac{dZ}{d\theta} = -Z \cdot \frac{dZ}{dk} - \frac{1}{S} E[U'(\Pi)].$$

This expression in (18) is similar to the Slutsky equation familiar from the traditional demand analysis. It says that the change in quantity for an increase in demand can be decomposed into two separate effects, one of which is analogous to a decrease in fixed costs, and the other a pure substitution effect. The sign of the first term is positive since we know from (16) that dZ/dk has a negative sign.

The sign of the second term is also positive since $E[U'(\Pi)]$ is always positive. Therefore, the expression (18) has a positive sign. In other words, increased demand increases output of a monopoly firm with decreasing risk aversion and our proposition (I.D) is proved.

II Monopsony under Uncertainty

In the traditional riskless theory of monopsony it is assumed that the supply price for the monopsonist's input is a function of the amount of input employed and that the monopsonist knows, with certainty, the input supply function relating the two variables and its marginal revenue product function. The optimum quantity of input employed by the monopsonist under certainty is then characterized by its marginal revenue product being equal to its marginal expenditure of the input. The monopsonist considered in this paper likewise

knows its input supply function as well as its marginal revenue product schedule, the difference being that the input supply function now contains a random variable ϵ over which the monopsonist has no control. In this section we will examine the implications of introducing uncertainty via the random variable ϵ in the input supply function for the theory of monopsony and derive its comparative statics properties.

We will assume that the stochastic input supply function faced by the monopsonist is of the form:

$$(19) W = \alpha(\epsilon) + \beta Z,$$

where Z is its input and ϵ is a random variable.

Profit is given by:

$$(20) \Pi = P \cdot h(Z) - Z \cdot W(Z, \epsilon) - k,$$

where P is the fixed price of the monopsonist's output and $h(Z)$ is its production function assumed to be known with certainty and k represents the fixed costs and $W(Z, \epsilon)$ is the stochastic input supply function faced by the monopsonist.

The monopsonist maximizes the expectation of its utility from profits:

$$(21) E[U(\Pi)] = \int U[P \cdot h(Z) - Z \cdot W(Z, \epsilon) - k] g(W) dW,$$

where $g(W)$ represents its subjective probability density function with $E[W] = \bar{W}$. Differentiating (21) with respect to its control variable Z yields the first and the second order conditions:

$$(22.a) F = E[(MRP - ME) \cdot U'(\Pi)] = 0, \text{ where } MRP = P \cdot h' \text{ and } ME = W + ZW',$$

$$(22.b) S = E[(MRP - ME)^2 \cdot U''(\Pi) + (P \cdot h'' - 2\beta)U'(\Pi)] < 0.$$

Rewriting (22.a) as $E[\text{MRP} \cdot U'(\Pi)] = E[\text{ME} \cdot U'(\Pi)]$ and then subtracting both sides from $E[\overline{\text{ME}} \cdot U'(\Pi)]$, we obtain:

$$(23) \quad (\overline{\text{ME}} - \text{MRP})E[U'(\Pi)] = E[(\overline{\alpha} - \alpha) \cdot U'(\Pi)],$$

where $\overline{\text{ME}} = E[\text{ME}]$ and $\overline{\alpha} = E[\alpha]$.

Proposition (II.A):

A risk-averse monopsony firm employs less input than it would under certainty.

In order to prove this proposition, we will rewrite the profit function (20) by adding and subtracting $Z \cdot \overline{W}$ so that:

$$(24) \quad \Pi = \overline{\Pi} + Z \cdot (\overline{W} - W),$$

where $\overline{\Pi} = P \cdot h(Z) - Z \cdot \overline{W} - k$.

We will then show that the monopsony firm's optimum input solution is characterized by its $\overline{\text{ME}}$ less than MRP so that it is to the left of the traditional solution under certainty assuming that the expected supply price under uncertainty corresponds to the certainty supply price of the input Z.

Case (i) Suppose that the stochastic supply price is under-estimated, ex ante.

Then:

$$(25) \quad \begin{aligned} W > \overline{W} &\rightarrow U(\Pi) < U(\overline{\Pi}) \\ &\rightarrow U'(\Pi) > U'(\overline{\Pi}) \\ &\rightarrow (\overline{W} - W)U'(\Pi) < (\overline{W} - W)U'(\overline{\Pi}) \\ &\rightarrow E[(\overline{W} - W)U'(\Pi)] < U'(\overline{\Pi})E[\overline{W} - W] = 0 \\ &\rightarrow E[(\overline{\alpha} - \alpha)U'(\Pi)] < 0 \end{aligned}$$

Therefore, LHS of (23) is also negative and since $E[U'(\Pi)] > 0$, we obtain:

$$(26) \quad \overline{\text{ME}} < \text{MRP}.$$

Case (ii) Suppose that the stochastic supply price is over-estimated, ex ante.

Then:

$$\begin{aligned}
 W < \bar{W} &\rightarrow U(\Pi) > U(\bar{\Pi}) \\
 &\rightarrow U'(\Pi) < U'(\bar{\Pi}) \\
 &\rightarrow (\bar{W}-W)U'(\Pi) < (\bar{W}-W)U'(\bar{\Pi}) \\
 &\rightarrow E[(\bar{W}-W)U'(\Pi)] < U'(\bar{\Pi})E[\bar{W}-W] = 0 \\
 &\rightarrow E[(\bar{\alpha}-\alpha) \cdot U'(\Pi)] < 0 .
 \end{aligned}$$

Therefore LHS of (23) is also negative and since $E[U'(\Pi)] > 0$, we have the same result that $\bar{ME} < MRP$.

Proposition (II.B):

Increased uncertainty decreases employment of input by a monopsony firm.

We will now consider the effects of a marginal increase in uncertainty by defining the variability of the density function for the supply price in terms of mean preserving spread. This requires one multiplicative shift and one additive shift parameter. Let us define:

$$(27) \alpha' = \gamma\alpha + \xi ,$$

where γ is the multiplicative shift parameter which gives a stretching of the probability distribution around a constant mean and ξ is the additive shift parameter.

Then the mean preserving spread type of shift in the density function of α' leaves the mean $E[\alpha']$ unchanged, that is, $dE[\alpha'] = \bar{\alpha}d\gamma + d\xi = 0$, or

$$(28) \frac{d\xi}{d\gamma} = -\bar{\alpha} .$$

The profit function can now be written as:

$$(29) \Pi = P \cdot h(Z) - Z[(\gamma\alpha + \xi) + \beta Z] - k .$$

And the new first order condition becomes:

$$(30) F = E[(Ph' - \gamma\alpha - \xi - 2\beta Z) \cdot U'(\Pi)] = 0 .$$

Totally differentiating (30) with respect to γ , taking account of (28) and then evaluating at the initial point $\gamma=1$, $\xi=0$, we obtain:

$$(31) \frac{dZ}{d\gamma} = \frac{Z}{S} E[(MRP-ME)(\alpha-\bar{\alpha})U''(\Pi)] + \frac{1}{S} E[(\alpha-\bar{\alpha})U'(\Pi)].$$

This expression for the change in the employment of inputs due to increase in uncertainty is decomposed into two separate terms. We know the sign of the second term to be negative since $E[(\alpha-\bar{\alpha})U'(\Pi)] > 0$ from (25) and S is required to be negative. Then, we must show that the first term is also negative, that is, we must show that:

$$(32) E[(MRP-ME)(\alpha-\bar{\alpha})U''(\Pi)] > 0.$$

We can rewrite (32) as:

$$\begin{aligned} E[(MRP-ME)(\alpha-\bar{\alpha})U''(\Pi)] &= E[(MRP-ME)(ME-\bar{ME})U''(\Pi)] \\ &= E[(MRP-ME)\{(ME-MRP) + (MRP-\bar{ME})\}U''(\Pi)], \end{aligned}$$

which can be separated into two terms:

$$(33) E[-(MRP-ME)^2U''(\Pi)] + (MRP-\bar{ME})E[(MRP-ME)U''(\Pi)].$$

The first term in (33) is always positive. To show that the second term is also positive, we must show $E[(MRP-ME)U''(\Pi)] > 0$ since $(MRP-\bar{ME}) > 0$ from (28).

Let α^* be the value of α such that $ME=MRP$, and let Π^* be the corresponding profit level.

$$\begin{aligned} \text{Case (i) } \alpha > \alpha^* &\rightarrow \Pi < \Pi^* \\ &\rightarrow p(\Pi) > p(\Pi^*) \quad \text{where } p(\Pi) = \frac{-U''(\Pi)}{U'(\Pi)} \\ &\rightarrow p(\Pi)[- (MRP-ME)U'(\Pi)] > p(\Pi^*)[- (MRP-ME)U'(\Pi)] \\ &\rightarrow (MRP-ME)U''(\Pi) > -p(\Pi^*)(MRP-ME)U'(\Pi) \\ &\rightarrow E[(MRP-ME)U''(\Pi)] > -p(\Pi^*)E[(MRP-ME)U'(\Pi)] = 0 \end{aligned}$$

since $E[(MRP-ME)U'(\Pi)] = 0$ from the FOC.

Therefore,

$$(34) E[(MRP-ME)U''(\Pi)] > 0.$$

$$\begin{aligned} \text{Case (ii) } \alpha < \alpha^* &\rightarrow \Pi > \Pi^* \\ &\rightarrow p(\Pi) < p(\Pi^*) \\ &\rightarrow p(\Pi)[- (MRP-ME)U'(\Pi)] > p(\Pi^*)[- (MRP-ME)U'(\Pi)] \\ &\rightarrow (MRP-ME)U''(\Pi) > -p(\Pi^*)(MRP-ME)U'(\Pi) \\ &\rightarrow E[(MRP-ME)U''(\Pi)] > -p(\Pi^*)E[(MRP-ME)U'(\Pi)] = 0. \end{aligned}$$

Therefore, the expression in (33) is positive, which, in turn, implies that the expression in (31) is negative, and our proposition (II.B) is proved.

Proposition (II.C):

Increased fixed costs decrease employment of inputs by a monopsony firm with decreasing absolute risk aversion.

The effect of a small increase in fixed costs can be obtained by totally differentiating the FOC (22.a):

$$(35) \frac{dZ}{dk} = \frac{1}{S} E[(MRP-ME) \cdot U''(\Pi)]$$

Since the sign of $E[(MRP-ME) \cdot U''(\Pi)]$ is positive from (34) and S is negative, dZ/dk is negative and our proposition (II.C) is proved.

Proposition (II.D):

Increased costs decrease employment of inputs by a monopsony firm with decreasing absolute risk aversion.

Since the supply price of input is regarded as random, it does not make sense to speak about the change in the input costs for the monopsony. However, it seems intuitively natural to have an upward shift of supply curve for the monopsony firm's inputs by an amount, ξ . Such an increase will leave the distribution of supply price of input conditional on quantity of inputs unaffected except for a higher mathematical expectation of the supply price. We will suppose that the size of the upward shift, ξ , remains invariant with respect to quantity of inputs.

Let $W^* = \alpha + \beta \cdot Z + \xi$ define the new supply curve resulting from the shift where ξ is an additive shift parameter. The new first order condition is:

$$(36) F = E[(MRP - ME - \xi)U'(\Pi - \xi Z)] = 0$$

Totally differentiating (36) and evaluating at the initial supply condition, that is, at $\xi = 0$, we obtain:

$$(37) \frac{dZ}{d\xi} = Z \cdot \frac{dZ}{dk} + \frac{1}{S} E[U'(\Pi)] .$$

The sign of the first term is negative since $dZ/dk < 0$ from (35) and the sign of the second term is also negative since S is negative and $E[U'(\Pi)]$ is positive. In other words, increase of costs decreases employment of inputs by a monopsony firm with decreasing absolute risk aversion.

III The Case of Bilateral Monopoly under Uncertainty

Bilateral monopoly is a market situation with a single seller and a single buyer. A monopolist does not have an output supply function relating price and quantity. As is well known from the traditional analysis under certainty, he selects a point of his buyers' demand function that will maximize his profit. On the other hand, a monopsonist does not have an input demand function. He selects a point on his sellers' supply function that maximizes his profit. It is not possible for the seller to behave as a monopolist and for the buyer to behave as a monopsonist at the same time. Consequently, the normal market mechanism fails to operate and the terms of trade must be settled by bilateral bargaining.

The monopolist in this market produces product Z and operates under cost conditions given by $TVC = \psi(Z)$. He uses a single input for producing Z and buys the input in a competitive market at a fixed price, which is assumed to be known with certainty. On the other hand, the monopsonist uses the monopolist's output Z as an input, and the revenues he can obtain from using various amounts

of this input, Z are given by the function, $TRP = \phi(Z)$. The monopsonist's revenues are obtained by selling his product in a competitive market at a fixed price, which is assumed to be known with certainty.

III.A Dominant Seller under Uncertainty

The monopolist is assumed to confront the monopsonist with the MRP schedule of the monopsonist. However, the buyer's MRP is not known and appears stochastic to the monopolist. On the other hand, the monopsonist is assumed to behave like a competitor in the sense that he must offer to purchase various quantities of Z specified by the dominant monopolist.

Under these conditions, the monopolist's profit function is given by:

$$(38.a) \quad \Pi_s = Z \cdot \phi'(Z) - \psi(Z) - k,$$

where $\phi'(Z)$ is the monopsonist's MRP. And the monopolist maximizes

$$(38.b) \quad E[U(\Pi_s)] = \int U[Z \cdot \phi'(Z) - \psi(Z) - k] f(\phi') d\phi',$$

where f is the monopolist's subjective probability density function of the monopsonist's MRP, $\phi'(Z)$.

The first order condition is:

$$(39.a) \quad \frac{dE[U(\Pi_s)]}{dZ} = E[(\phi' + Z\phi'' - \psi')U'(\Pi_s)] = 0, \text{ or}$$

$$(39.b) \quad E[\psi' \cdot U'(\Pi_s)] = E[(\phi' + Z\phi'') \cdot U'(\Pi_s)]$$

and the second order condition is assumed to be satisfied.

This expression (39.b) is similar to the traditional condition for profit maximization under certainty, equating the monopolist's MR and MC. However, it was shown in (8) that expected marginal revenue is greater than marginal cost under uncertainty. In the Figure (III.A), the quantity of Z exchanged, Z^*_s , for example, is the solution to the equation (39.b) and the price per unit

of Z can be computed by evaluating $E[\phi'(Z)]$ at $Z = Z_s^*$. However, the traditional solution under certainty would have been at the point (Z_s, P_s) assuming that AR and MR schedules in the diagram correspond to mean values of the respective schedules.

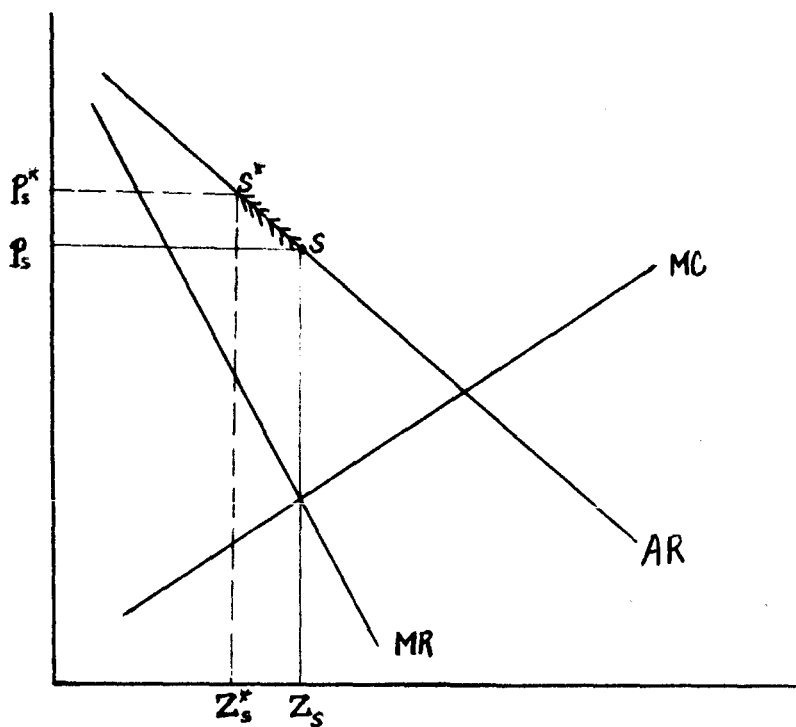


Figure (III.A)

III.B Dominant Buyer under Uncertainty

We now reverse the roles of the two traders and assume that the monopolist confronts the monopsonist with the MC schedule of the monopolist. However, the seller's MC is not known and appears stochastic to the monopsonist. On the other hand, the monopolist is assumed to behave like a competitor in the sense that he must offer to sell various quantities of Z specified by the dominant monopsonist.

Under these conditions, the monopsonist's profit function is given by:

$$(40.a) \pi_B = \phi(Z) - Z \cdot \psi'(Z) - k,$$

where $\psi'(Z)$ is the monopolist's MC schedule. And monopsonist maximizes

$$(40.b) E[U(\pi_B)] = \int U[\phi(Z) - Z \cdot \psi'(Z) - k] g(\psi') d\psi',$$

where g is the monopsonist's subjective probability density function of the monopolist's MC, $\psi'(Z)$.

The first order condition is:

$$(41.a) \frac{dE[U(\pi_B)]}{dZ} = E[(\phi' - \psi' - Z\psi'') \cdot U'(\pi_B)] = 0, \text{ or}$$

$$(41.b) E[\phi' \cdot U'(\pi_B)] = E[(\psi' + Z\psi'') U''(\pi_B)],$$

and the second order condition is assumed to be satisfied.

This expression (41.b) is similar to the traditional condition for the monopsonist's profit maximization under certainty. However, it was shown in (26) that expected marginal expenditure is less than marginal revenue product under uncertainty. In the Figure (III.B), the quantity of Z exchanged, Z_B^* , for example, is the solution to the equation (41.b) and the price paid per unit of Z can be computed by calculating $E[\psi'(Z)]$ at $Z = Z_B^*$. However, the traditional solution under certainty would have been at the point B , or Z_B and P_B assuming that the monopsonist's AE and ME schedules correspond to mean values of the respective schedules.

III.C Bilateral Monopoly Solution under Uncertainty

Under conditions of bilateral monopoly it is more reasonable to assume that both traders will try to exercise their monopoly power. We may, for example, suppose that each trader assumes his opponent to act like a competitor, while he himself will offer terms of trade in accordance with his monopolistic

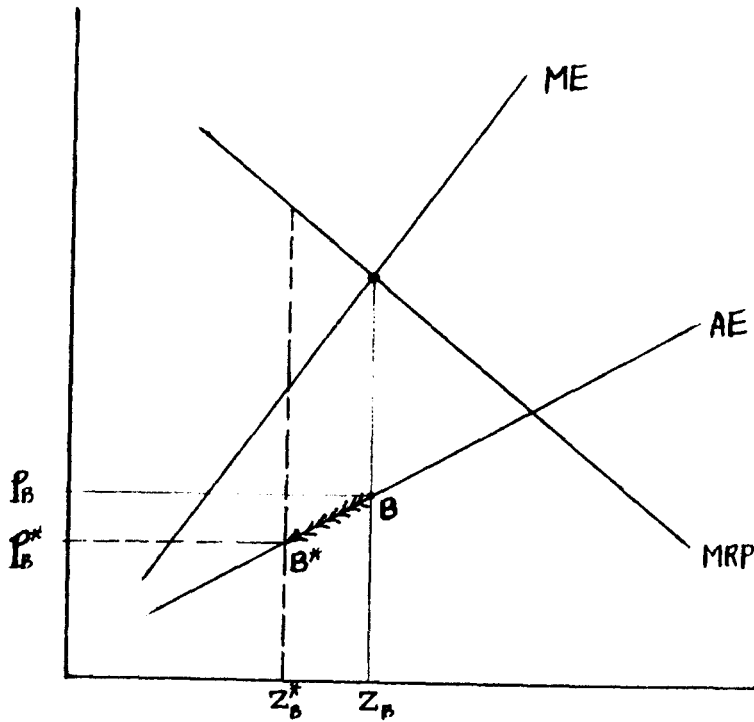


Figure (III.B)

position. In the certainty case, the monopolist would maximize profits by operating at output Z_S and price P_S , in the Figure (III.C), where his MC equals MR. The monopsonist would maximize profits by operating at output Z_B and price P_B where his ME equals MRP. The objectives are inconsistent, so the price and quantity under bilateral monopoly are said to be indeterminate. In the uncertainty case, on the other hand, the monopolist would maximize profits by operating at output Z_S^* and price P_S^* , where his expected MR is greater than MC (i.e., $\bar{R}' > C'$) from (8). The monopsonist would maximize profit by operating at output Z_B^* and price P_B^* , where his expected ME is less than MRP (i.e., $\bar{ME} < MRP$) from (26). The price and quantity under bilateral monopoly in this uncertainty case is also indeterminate.

It seems likely that the price will lie somewhere between P_B and P_S , and that output will lie somewhere between Z_S and Z_B in the certainty case.

In the uncertainty case, it seems likely that price will lie somewhere between P_B^* and P_S^* , and the output will lie somewhere between Z_S^* and Z_B^* . But we cannot make any more specific predictions. A theory based upon either profit maximization or expected utility maximization is unable to yield a more specific prediction. Other factors, such as bargaining power, negotiating skill and public opinions, are likely to play an important role in determining the nature of the final outcome. However, some of the important features of the uncertainty case are that the gap between the monopolist's asking price (i.e., demand price) and the monopsonist's offering price (i.e., supply price) has been widened, and the range of possible amounts of Z to be traded has been shifted downward due to presence of uncertainty as shown in the Figure (III.C).

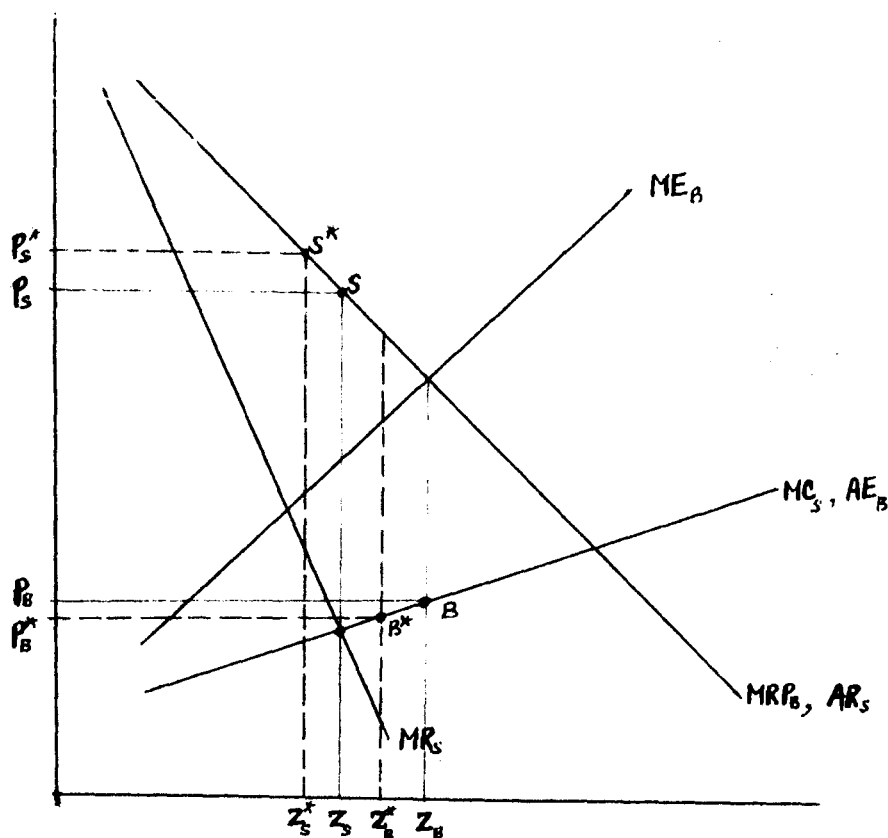


Figure (III.C)

III.D Collusion Solution

It is now assumed that the market participants will recognize their mutual interdependence and make available all the relevant information concerning the monopolist's cost schedules and the monopsonist's demand schedules so as to eliminate the uncertainty that exists between the two traders. It is further assumed that the traders maximize the utility of their joint profit and that the utility function is well defined. In order to reach a satisfactory agreement as to price and quantity, the bargaining process can be separated into two steps. First, the participants determine a quantity that maximizes the utility of their joint profit, and then determine a price that distributes the joint profit among them.

The utility function of their joint profit is:

$$(42) U[\pi_J] = U[\pi_S + \pi_B] = U[ZP - \psi(Z) + \phi(Z) - ZP] = U[\phi(Z) - \psi(Z)]$$

Maximizing (42) with respect to Z:

$$(43.a) \frac{dU[\pi_J]}{dZ} = [\phi'(Z) - \psi'(Z)] \cdot U'(\pi_J) = 0, \text{ or}$$

$$(43.b) \phi'(Z) = \psi'(Z),$$

and the second order condition is assumed to be satisfied.

The utility of their joint profit is maximized at the output level at which the seller's MC equals the buyer's MRP. As will be shown in the next section, the collusion solution is the same as the competitive solution at Z_C in the Figure (III.D).

However, the competitive price does not necessarily follow from the collusion solution. For the prescribed quantity, the seller will try to receive as high a price as possible, and the buyer will try to pay as low a price as possible. As in the case of the bilateral monopoly situation, the price in

the collusion case is also indeterminate. There are several possible outcomes for the price, and one possible situation is to assume that either trader can do no worse than his own monopolistic solution. All we can say is that the price will lie somewhere, for example, between P_B and P_S and that the quantity to be traded is Z_C in the Figure (III.D).

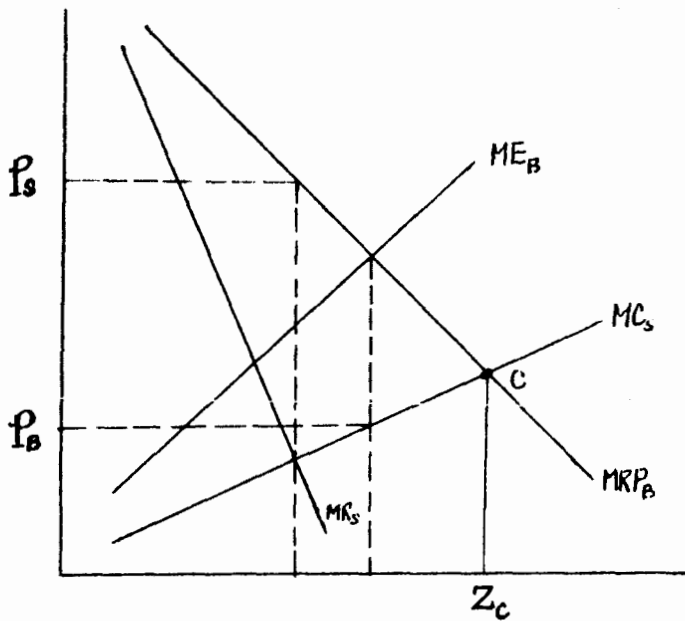


Figure (III.D).

III.E Competitive Solution and Summary

Consider now the price and quantity that would be achieved if both seller and buyer were price takers under certainty. The demand and supply functions would now be both effective and the competitive equilibrium quantity under certainty, Z_C , is determined by equating the demand function and the supply function. That is:

$$(44) P_C = \phi'(Z) = \psi'(Z).$$

This gives the same quantity as the collusion solution quantity given by (43.b). The competitive price P_C equals both the MRP of the buyer and the MC of the seller under certainty. This traditional solution may not be a likely outcome for a market characterized by bilateral monopoly and uncertainty, but it is discussed here as another useful reference point. An example of a competitive solution under certainty is shown by the point C in Figure (III.D).

Some of the results of a comparison of the dominant monopoly, dominant monopsony, bilateral monopoly, collusion and competitive solutions in Figure (III.E) may be generalized to cover all cases in which the demand function has negative slope and the supply function has positive slope. The monopoly and monopsony solution points under uncertainty as well as under certainty will always lie to the left of the intersection of the demand and the supply functions, which gives the solution point for both collusion and competitive cases. Therefore, we have the following inequalities:

$$(45.a) \quad Z_S^* < Z_S < Z_C \quad \text{and}$$

$$(45.b) \quad Z_B^* < Z_B < Z_C .$$

Figure (III.E) shows that $Z_S^* < Z_B^*$ and $Z_S < Z_B$. However, these results do not always hold. These results depend upon the slopes of the demand and supply curves as well as the extent of risk aversion by traders.

On the other hand, since the collusion and competitive equilibrium lie to the right of the monopoly equilibrium on the demand curve, and to the right of the monopsony equilibrium on the supply curve, we obtain the following inequalities:

$$(46.a) \quad P_S^* > P_S > P_C \quad \text{and}$$

$$(46.b) \quad P_B^* < P_B < P_C .$$

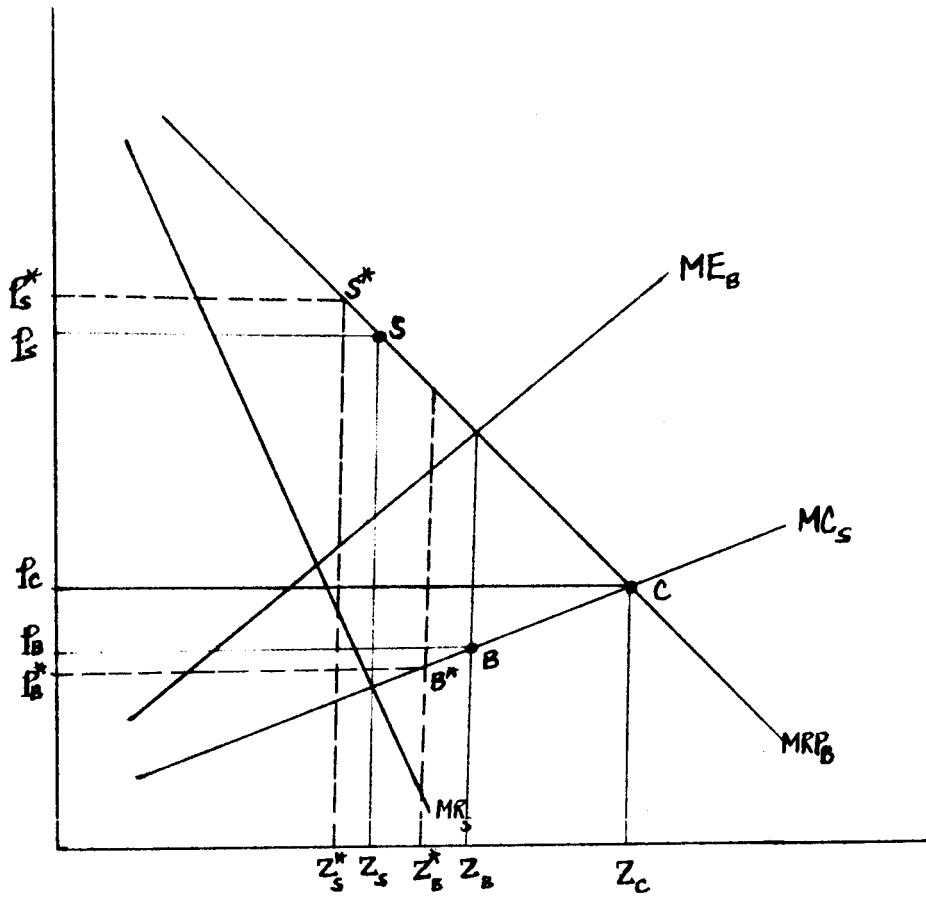


Figure (III.E)

References

- (1) Arrow, Kenneth J. Aspects of the Theory of Risk-Bearing. (Helsinki, 1965).
- (2) Baron, David P. "Price Uncertainty, Utility, and Industry Equilibrium in Pure Competition," International Economic Review. Vol. 11, No. 3, (1970), pp. 463-480.
- (3) Batra, Raveendra N. and Ullah, Aman. "Resource Allocation in a General Equilibrium Model of Production Under Uncertainty," Journal of Economic Theory. Vol. 8 (1974), pp. 50-63.
- (4) "Competitive Firm and the Theory of Input Demand under Price Uncertainty," Journal of Political Economy. Vol. 82(3), (May 1974), pp. 537-548.
- (5) Borch, Karl H. The Economics of Uncertainty. (Princeton Press, 1968).
- (6) Cyert, Richard M. and Charles L. Hedrick. "Theory of Firm: Past, Present, and Future: An Interpretation," Journal of Economic Literature. Vol. X(2), (June 1972), pp. 398-412.
- (7) Fellner, W. "Prices and Wages under Bilateral Monopoly," Quarterly Journal of Economics. Vol. 61 (1947), pp. 503-532.
- (8) Hadar, Josef and William R. Russel. "Rules for Ordering Uncertain Prospects," American Economic Review. Vol. LXI, No. 1 (March 1969), pp. 25-34.
- (9) Iwai, Katsuhito. "The Firm in Uncertain Markets and its Price, Wage and Employment Adjustments," Review of Economic Studies. Vol. XLI(2), No. 126 (1974), pp. 257-276.
- (10) Leland, Hayne E. "Theory of the Firm Facing Uncertain Demand," American Economic Review. Vol. LXII, 3 (1972), pp. 279-290.
- (11) "On the Existence of Optimal Policies under Uncertainty," Journal of Economic Theory. Vol. 5 (1972), pp. 35-44.
- (12) Mayer, Wolfgang. "The Rybczynski, Stolper-Samuelson, and Factor-Price Equalizations under Price Uncertainty," American Economic Review. Vol. 66, No. 5 (1976), pp. 797-808.
- (13) McCall, J.J. "Competitive Production for Constant Risk Utility Functions," Review of Economic Studies. Vol. XXXIV(4), No. 100, pp. 417-420.
- (14) Mills, Edwin S. "Uncertainty and Price Theory," Quarterly Journal of Economics. Vol. LXXIII (February 1961), pp. 41-62.
- (15) Nelson, Richard R. "Uncertainty, Prediction and Competitive Equilibrium," Quarterly Journal of Economics. Vol. LXXV (1961), pp. 41-62.
- (16) Oi, Walter. "The Desirability of Price Instability under Perfect Competition," Econometrica. Vol. 21, 1 (January 1961), pp. 58-64.

- (17) Pratt, John W. "Risk Aversion in the Small and in the Large," Econometrica. Vol. 32 (1964), pp. 122-136.
- (18) Radner, Roy. "Competitive Equilibrium under Uncertainty," Econometrica. Vol. 31, 1(January 1968), pp. 31-58.
- (19) Rothschild, Michael. "Increasing Risk," Journal of Economic Theory. Vol. 2 (1970), pp. 225-243.
- (20) Sandmo, Agnar. "On the Theory of the Competitive Firm under Price Uncertainty," American Economic Review. Vol. XLI, 1(1971), pp. 65-73.
- (21) Spindler, Zane A. "Endogenous Bargaining Power in Bilateral Monopoly and Bilateral Exchange," Canadian Journal of Economics. Vol. VII, No. 3, (1974), pp. 463-473.