

EDGEWORTH MODELS OF THE FIRM

by

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ABSTRACT

This is a theoretical work concerned with the behaviour of firms operating in duopolistic and oligopolistic product markets. Starting from a very simple duopoly model, assumptions are systematically relaxed, in order that we may ascertain the limiting effects of those assumptions. This method of enquiry also allows the critical examination and/or incorporation of a number of previously-suggested models of firm behaviour, at those points in the analysis where the structural and behavioural assumptions coincide.

The analysis begins with a reformulation and extension of the pure Edgeworth model of duopoly, which assumes identical products, price strategy, short term profit maximization, and the firms' failure to recognize their mutual dependence. Price instability is the general rule in this model. The requirements for price stability, and the nature of the price instability, are examined in detail. When product differentiation is allowed into the simple model, price stability becomes the general rule. The concept of product differentiation is broadened to include both asymmetry and discontinuity in aggregate consumer preferences. These latter concepts generate stable price and sales differentials, as do differences in the firms' cost structures. Price instability is shown to be feasible under limited circumstances in the differentiated-products-duopoly model.

The model is then generalized to oligopoly, first for the identical

products case, and then with product differentiation. Once again price stability is the exception rather than the rule in the identical products case, and the opposite is true in the differentiated products case. As in the preceding chapter, the demand functions of the firm are specified in detail, taking into account the features of aggregate consumer preferences, and the limited productive capacities of other firms. When prices stabilize, a pattern of price differentials will be established which depends upon the relative cost and product differentiation advantages of the competing firms.

Next, the assumption that firms fail to recognize their mutual dependence of actions is relaxed. The firm's "conjectural variation" must then be assumed, in order that the demand functions are determinate. Several possibilities are examined at this point, including the "minimax" assumption, "conscious parallelism", price-leadership, kinked demand curves, and non-price competition. When long run aspects are considered, such as the entry of new firms, the objective function is changed. Long run profit maximization is assumed, and this causes firms to contemplate the issues of "limit" pricing, takeover, and erection of barriers to entry, for example. An index of oligopoly price is constructed which describes the location of the prevailing price level, which in turn is shown to depend upon the "limit" price, the price which would induce unilateral price cutting, and the price which would cause some firms to go out of business. In conclusion, several further extensions and modifications of the model, possible empirical tests, and some policy conclusions, are suggested.

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CHAPTER 1

INTRODUCTION, AIMS, AND SCOPE

I. The Theory of the Firm

The subject matter of this dissertation may be classified, most simply, as relating to the 'theory of the firm'. This area of the discipline is the bridge between Microeconomic Theory and the field of Industrial Organization. The theory of the firm has traditionally drawn its analytical techniques from the former, and has relied upon the latter to suggest (and to substantiate) the structural and behavioural assumptions employed. The theory of the firm seeks to explain and predict the market behaviour of business firms, in terms of their competitive environment. Accordingly, it covers the spectrum of possibilities from perfect competition to pure monopoly. These extremes are well charted but seldom discovered. Monopolistic competition theory, which introduces product differentiation into an otherwise perfectly competitive framework, is also settled to the point that it can be presented quite unequivocally in standard textbook presentations. There remains the problem of oligopoly, which is by no means settled, and which is the specific topic of this study.

The theory of oligopoly is forbidding territory. The fewness of sellers causes a mutual interdependence of action which does not enter the

other major market forms. A necessary precondition for an accurate prediction of the behaviour of oligopolists is thus to correctly specify the impact the firm feels its action will have on its rivals, and the reaction of the rivals to that impact. The correct specification of the firms' objective functions are essential to this prediction. The conventional short run profit maximization assumption, however, appears less suited to oligopoly than to the other market forms. In perfect and monopolistic competition, entry of new firms cannot be prevented and the existing firms should maximize short run profits as rational long run behaviour. In monopoly, entry is blockaded, and again the firm should maximize short run profits. But in oligopoly, short run profit maximization will attract entry of new firms and the subsequent dilution of next-period profits, unless entry barriers are sufficiently high. Clearly barriers to entry must be incorporated into the analysis as an explanatory variable. So, too, must we consider other factors which may motivate oligopolists, such as the public relations impact of their actions, desire for a quiet life, meglomania, etc. In the other market forms these objectives are for the most part, either of no consequence, impossible to achieve, or come hand-in-glove with short run profit maximization. The oligopolist, however, has some room for discretionary action in these areas.

Further, the symmetry which characterizes the other market forms is usually absent from oligopoly. The firms of an oligopolistic industry are not all the same size, do not have similar cost structures, and do not necessarily have similar degrees of product differentiation around some central tendency. Perfect competitors of course, are forced to adopt the

most efficient scale of plant, and their products are identical. A monopolist has no other firm with which to compare itself, since its product is unique. Monopolistic competition, with free entry, forces firms to adopt the most efficient means of producing their particular variant of the product, and the logical extension of free entry is that product differentiation will occur in an n-dimensional continuum and is, therefore, symmetric.¹ Oligopoly, with restricted entry, allows firms to maintain price and cost differentials, and to have differing scales of plant which co-exist from period to period.

Thus the oligopoly problem is not simple and straightforward. And there is no universally accepted theory of oligopoly which has been forthcoming, despite the extensive writings in this area. Fragments of a theory do exist, however, and it is to this that we now turn.

II. The Present State of Oligopoly Theory

It is not my purpose to review the vast literature on oligopoly theory. This would be redundant; if not inferior, in view of the surveys and review articles which are available. (See particularly [137], [127, ch.5], [99], [81], [96]). In any case, those particular models and theories which bear directly on the subject matter of this thesis are examined at points in the following chapters where it is more appropriate to do so. At the present point of the analysis we shall confine ourselves to some generaliz-

1. Monopolistic competition, with significant barriers to entry, need not be symmetric, since some firms may have advantages others cannot achieve or emulate. This complication is passed over here, since it is, in effect, the same as the n-firm oligopoly case where firms do not recognize their mutual dependence, and as such is examined in chapter four.

ations regarding the present state of oligopoly theory.

Perhaps the major problem is that there is a multiplicity of different models and associated controversies. All of these may be relevant for some particular firms, although some appear to be of very limited applicability. Others, however, suffer only because they are partial theories which have not yet been extended and/or integrated into a more general model. A major purpose of this thesis is to remedy this problem of fragmented effort. First we must dispose of a couple of major areas of the literature which one might expect to be covered by this thesis, but which are not.

The first of these is the "Stability of Cournot Oligopoly" controversy which raged throughout the nineteen-sixties, and to which contributions still appear. Cournot [28] started it all in 1838. The stark simplicity of his structural and behavioural assumptions proved a blessing to the publishing aspirations of a later generation of mathematical economists. Cournot assumed two sellers with unlimited capacity to produce an identical product. Each seller would adjust his output in the expectation that his rival's output would remain constant. Prices and outputs were subsequently dynamically stable. Theocharis [153], in slightly less than two pages, touched off a re-examination of the model, by demonstrating that if there are more than three sellers, the model is dynamically unstable. Fisher [48], McManus and Quandt [92], Bishop [20], Hahn [60], and Okuguchi [109] have shown that with more realistic cost and demand assumptions, the oligopoly model is stable. This is particularly so with a continuous adjustment process. With discrete adjustments, stability is obtained with

a large number of sellers under more limiting conditions. Hadar [57] introduced product differentiation, which in turn contributes to stability. Sato and Nagatani [125] modified the behavioural assumption regarding the reaction of rival sellers. Cournot's zero "conjectural variation" assumption was replaced by one where the rivals are expected to vary their outputs in response to an initial change. The greater output adjustment expected, (that is, the larger the conjectural variation), the more likely is stability. Hosomatsu [75] introduced uncertainty regarding the demand situation and the production levels of rivals. Neudecker [105] provided an alternative proof. Tarr [151] allows firms to modify their conjectural variations through a "learning" process, as have Okuguchi [111], [112], Cyert and DeGroot [32] and Friedman [50].

And so the discussion continues, but to what avail? It remains predicated, as Bertrand [15] noted in 1883, upon the presumption that firms adjust their output levels, and allow the forces of the market to find the market clearing price. But oligopolists do not appear to be output-strategic, and moreover it is not clear that a competitive price determination process² would work efficiently amongst a few large firms. More realistically, it would seem, oligopolists set prices, and allow outputs to find their equilibrium levels. Accordingly this thesis will proceed on the presumption that firms are price-strategic, at first in the manner suggested by Bertrand, and Edgeworth [42], and later with appropriate modifications.

2. That is, that excess supply at the prevailing price would force the price down to eliminate the surplus, while excess demand would cause price to rise and eliminate the shortage.

Another area of the literature which we shall largely by-pass is the application of the "Theory of Games" to oligopoly problems. This development followed the work by Von Neumann and Morgenstern [156] and is best exemplified in Shubik [133] and Sherman [131]. This variant of the oligopoly literature treats the oligopolistic market as a forum for conflict amongst the firms. The central behavioural assumption is that firms choose their course of action in order to bestow the least advantage or greatest disadvantage, upon their rivals. In constant-sum games this "minimax" assumption allows the firm to maximize its own expected payoff. The outcome is stable dynamically if there is a "saddle-point" from which it is to no-one's advantage to depart. If no saddle-point exists, which is perhaps more likely, the game is still determinate if each player adopts a "mixed" strategy. That is, the payoffs are weighted by the probabilities that that particular strategy will be employed, and a minimax strategy will allow the player to maximize the "expected value" of the game. With more than two players, however, the formation of coalitions will allow multiple equilibria possibilities. Product differentiation widens the range of possible outcomes even further.

But game theory does not appear to be the salvation of the oligopoly problem. (For critical appraisals, see [137, p.526], [127, pp. 140-145] and [47, p. 354].) First, it assumes an extremist competitive situation. Firms are probably much less antagonistic than the minimax assumption supposes. Moreover the constant-sum game cannot admit the strategy of collusion, which is evident in some form in most oligopoly markets. Secondly, mixed strategies require complete secrecy regarding the strategy

to be played, thus maximizing the uncertainty of rivals. It is doubtful, however, that the firms could always maintain this secret until the moment of impact. Further, it is not clear that they would even want to, given the risk-aversion and preference for stability which large firms may be expected to exhibit. Thirdly, firms may not generally have sufficient information to enable them to calculate the payoffs for various strategy interactions.³ Thus it would seem that the game theoretic approach to the problem of duopoly is of fairly limited applicability. It is more useful, however, for some static decision problems within a much broader framework of analysis, such as the decision to indulge in non-price competition. Accordingly its discussion will be confined to this context in later chapters.

We come now to the "multiplicity of models" problem of oligopoly theory. The most familiar model is that of profit maximization in the short run. Firms envisage their cost and demand curves, and set price and output by equating marginal costs and revenues. Other models assert that firms do not try to maximize profits, but that sales, growth, and managerial utility are important in the firm's objective function. Another model rejects the presumption that firms wish to maximize anything, stating instead that firms merely "satisfice". Pricing for profit maximization is also rejected by another school of thought, which propounds that price is determined by a markup over costs rather than by marginalist principles.

3. It is conceded, of course, that many economic models are vulnerable to this criticism. In reality, uncertainty clouds the firm's estimation of its demand curve, ceteris paribus conditions do in fact change, and the construction of cost curves requires interpolation between calculable points, to mention a few problem areas.

Others feel that prices are set in order to inhibit the entry of new firms, and thus depend on the barriers to entry. Prices may be set individually, as the simple profit maximization model suggests, or jointly via a collusive agreement. Alternatively, some firms may act as price leaders and others as price followers. A kinked demand curve may be envisaged, inhibiting price adjustments. Or, price strategy may be secondary to advertising and other forms of non-price competition.

Few, if any, of these models can be said to be "wrong", since they are the logical extension of their premises. But since their assumptions are different, slightly in some cases, radically in others, these various models are not strictly comparable, and remain as scattered strands of a theory of oligopoly. Is a reconciliation possible? Are they each consistent with some wider theory of oligopoly? In the following chapters each of the above-mentioned models will be examined and an attempt is made to incorporate each model into the analysis.

III. Aims and Method of this Study

The aims of this study are threefold. First, it is intended to clarify certain models of firm behaviour which have been suggested in the past. Secondly, attempts are made to extend the analysis of particular models such that they may contribute more to the theory of oligopoly. And thirdly, effort is taken to synthesize a number of existing models into a more unified theory of oligopoly, in conjunction with the development of the analysis of this dissertation.

The method of analysis is the classical approach to oligopoly. We start with an oversimplified model of duopoly and progressively relax assumptions in order to broaden the scope of the analysis. The starting point is the Edgeworth model of duopoly. This simple model is often regarded as an intellectual curiosity, since its assumptions are so palpably unrealistic and its predictions are so often incorrect, in view of our observations of the real world. Yet the model serves the purpose in intermediate microeconomic textbooks, of introducing the student to the intricacies of a price-strategy competitive model where the firms are crucially interdependent. From this point, the student is led to more complex interrelationships amongst firms, more sophisticated strategies and objective functions, and eventually develops an appreciation, if not an understanding, of the complexity of the oligopoly problem. The simple duopoly model thus provides a foundation for the analysis of oligopoly.

It would seem, then, that much is to be gained by an intensive examination of the duopoly situation. By strengthening the foundations we may expect to derive stronger analysis at every higher level. Accordingly the following chapter is confined to a restatement, clarification, and extension of the simple Edgeworth duopoly model. In this form it is expected to better serve as an introduction to oligopolistic interdependence, since its subtleties are exposed and critically examined, and new directions of enquiry are pursued.

The ensuing chapters enhance the usefulness of the model still further, as the progressive relaxation of assumptions improves the model's

ability to explain and predict the structure of prices, market shares, profitabilities, and competitive strategies of actual business firms. In chapter three, differentiated products replace the identical products of the simple case. In chapter four the analysis is extended to the oligopoly case of more than two firms. Recognition of mutual dependence is allowed in chapter five, in contrast to the non-cooperative assumption of all earlier chapters. In chapter six the firms consider the long run implications of their actions, rather than being concerned simply with the short run, as in earlier chapters. Chapter seven summarizes the development of the model of the previous chapters and makes some concluding comments.

The model thus becomes progressively more sophisticated and, hence, more in accord with the real world, as the chapters progress. This methodology facilitates the incorporation of various other models of firm behaviour. Since models are only different as long as their underlying assumptions differ, and since the assumptions are progressively modified in this analysis, points arise at which existing models and the present analysis are strictly comparable. At these points the aims of this study, viz to clarify, extend, and synthesize a number of previously suggested models of firm behaviour, are pursued and, hopefully, fulfilled.

CHAPTER 2.

EDGEWORTH DUOPOLY REVISITED

I. Introduction

In 1897, Francis Y. Edgeworth [42] developed a price-strategic model of "two competing monopolists", following Bertrand's 1883 criticism [15] of the output-strategic model which Cournot [28] had suggested in 1838. If this exemplifies the pace of criticism of the duopoly model, then it is no surprise that Edgeworth's simple exposition of the model is found today in many intermediate textbooks. For the standard exposition is not without fault. It is neither very clear in its presentation, nor complete in its examination of the ramifications of the model. Nichol [106], Shubik [132], [133], [134], [135], [136] and others [98], [130] have made valuable contributions to our understanding of the simple model, and to its further development, but many questions and unexplored issues remain.

The purpose of this chapter is to clarify and extend the Edgeworth duopoly model within the context of the original assumptions. In order to facilitate these aims a new approach is adopted. The revised format of the model should promote better understanding of the "mechanics" of the model, and allow both extension and discussion of previous contributions. The major issues in which we shall be interested are as follows. Will prices and market shares be stable in the short run? If not uni-

versally, under what conditions will prices stabilize? When prices are unstable, over what range will they fluctuate, and what determines this range? If the fluctuation of prices is cyclic, how long does it take for prices to complete one full cycle, and on what does the periodicity depend? To these issues we now turn.

II. The Pure Edgeworth Model

The model assumes two firms producing identical products for sale in a compact market. Cost structures are identical, with constant average variable costs out to full capacity output, at which all costs would become infinite in the short run.¹ Market demand is static throughout the short run. A useful analogy is the demand for daily newspapers where the same people appear every day to purchase, (or not purchase, depending on price), one unit of the commodity from either of two street-corner paper-boys. The duopolists are day-to-day profit maximizers who do not recognize their mutual dependence. Consequently they have neither cooperative nor malicious intentions toward each other. Each regards the other's price of yesterday as datum, and may set a new price each morning which then prevails for at least one day. To avoid the complications of unsold articles we assume that commodities are produced on order at the

1. For the most part, the following analysis is confined to the Edgeworth structural assumption that each firm's full capacity output level is less than that amount which would satisfy the entire market demand at a price equal to minimum average variable costs. The Bertrand assumption, that each firm can satisfy the entire market, leads to considerably different conclusions, as will be noted.

time of the transaction.²

Let us call the duopolists firm A and firm B. Suppose that as we begin our observation of the behaviour of these firms, A is setting a slightly lower price than is B. In Fig. 2-I A's price is shown as P_1 , while B's price is P_m , the industry monopoly, or cartel, price. Since products are identical (and consumers are presumed to maximize utility subject to a budget constraint) A's demand curve will be horizontal and this firm will be selling full capacity output, Q_c . Firm B will be selling only to the residual demand. In terms of Fig. 2-I, B's sales will equal Q_1 , which is approximately³ equal to the portion of the total market demand which A was unable to satisfy at price P_1 . That is;

$$q_b = Q_b - q_a^* \quad (2.1)$$

where the subscripts a, b denote firms A, B; q represents the sales of the individual firm signified by the subscript; q^* is the full capacity output (sales) level for the individual firm; and Q is the total market demand at the price of the firm which is specified by the subscript.

Since the firms are profit maximizers, we would expect them to be constantly evaluating their alternative price/output opportunities. Let

2. This replaces Edgeworth's equivalent assumption that production and disposal are costless. Shubik [133, p.91] and Levitan [89] have examined the implications of inventory costs, as a modification of the Edgeworth model.

3. This is only approximate, as a small number of buyers will leave the market due to the slightly-higher price negating their consumer surplus, as explained below.

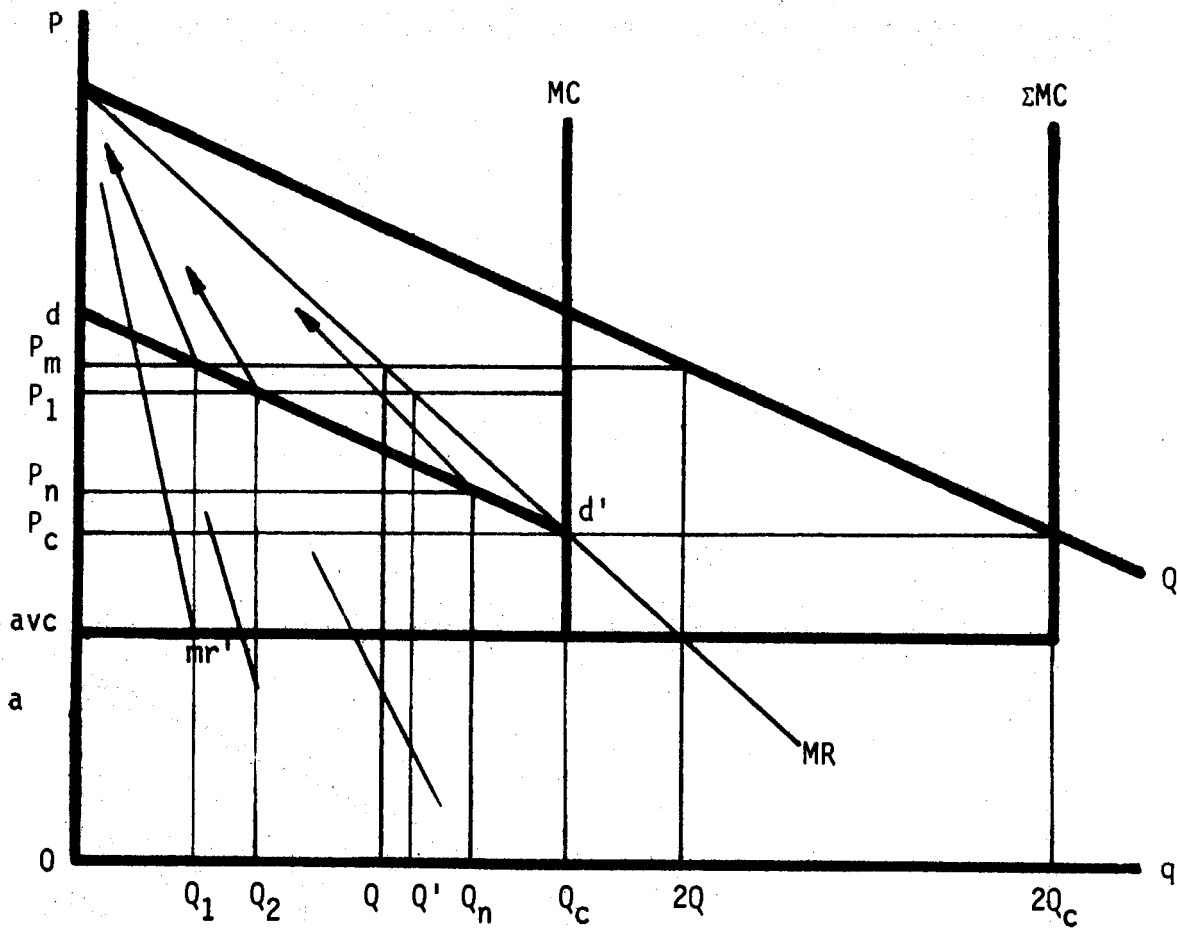


Fig. 2-I The Pure Edgeworth Duopoly Situation with Identical Cost Structures.

us consider firm B. What will his demand function look like? For prices above his present price the residual demand will decline as buyers drop out of the market due to the rising price exhausting their consumers' surplus. We may specify the residual demand at all higher prices if we make an assumption concerning which particular buyers comprise the residual demand at the price immediately above A's price. Let us assume that the arrival of the buyers, in the market place, is random with respect to the amount of consumer surplus they each expect to derive from the consumption of the commodity. Now, note that a certain proportion of buyers at A's price, that is q_a^*/Q_a , were early enough to obtain the product at A's price. Given the "random" assumption we can say that of those buyers who would have paid B's higher price, (that is, Q_b), the proportion q_a^*/Q_a were lucky enough to purchase the commodity at A's price. Subsequently $1 - q_a^*/Q_a$ of those Q_b buyers remain as the residual demand. Hence firm B's demand function for all prices higher than A's may be expressed as follows:

$$q_b = Q_b \left(1 - \frac{q_a^*}{Q_a} \right) \dots (2.2)$$

As B increases his price, the market demand, Q_b , and hence q_b , will decline as buyers progressively drop out of the market as their consumer surplus is negated.⁴ This section of the demand curve, given A's price of P_1 , thus begins at the coordinates (Q_1, P_m) in Fig. 2-I, and converges on

4. Note that when A's and B's prices are only slightly different, Q_a approaches Q_b , and equation (2.2) reduces to (2.1).

point P_0 on the price axis. For price decreases, however, the demand curve for B is characterized by discontinuities. If B lowers his price slightly, to equal A's price, the firms will share the market equally, and the relevant coordinate will be the point (Q', P_1) in Fig. 2-I. The third part of the demand curve will be horizontal, at a price slightly below the rival's price, and reflects the infinite cross-elasticity of demand between the firms' products. The fourth section will be the market demand curve below the point where it is met by the horizontal section, since the individual firm's demand curve is constrained by market demand.⁵

Because the demand function has discontinuities, firm B's profit function is not differentiable over the entire range of outputs. Instead B will need to calculate the maximum profit from each of the sections of the demand function, and compare them, choosing the price/output combination which yields the profit maximum maximorum. Consider the first section of the demand curve, given by

$$q_b = Q_b \left(1 - \frac{q_a^*}{Q_a} \right) \dots (2.2)$$

As yet we have no specification for the total market demand. Call P_b the price of firm B, P_0 the intercept of the market demand curve on the price axis, and M the reciprocal of the slope of the curve in absolute terms.

5. Strictly, the third section of the firm's demand function will be a horizontal discontinuity at a price slightly below the rival's price. The only concrete point in this section is where it is constrained by the market demand curve.

The market demand curve at B's price is then

$$Q_b = M (P_o - P_b) \dots (2.3)$$

Introducing constant variable costs per unit, k_b , and total fixed costs, F_b , we can specify B's first profit function as

$$\Pi_b = \left[(P_b - k_b) M (P_o - P_b) (1 - q_a^*/Q_a) \right] - F_b$$

$$\text{or } \Pi_b = \left[\left(-P_b^2 - k_b P_o + P_b P_o + k_b P_b \right) (1 - q_a^*/Q_a) M \right] - F_b \dots (2.4)$$

The first order condition for profit maximization is to set the derivative of (2.4), with respect to P_b , equal to zero. This in turn implies

$$P_b = \frac{P_o - k_b}{2} \dots (2.5)$$

That is, the profit maximizing price lies halfway between the point P_o (price axis intercept) and the level of marginal costs.⁶ Note that since marginal costs remain constant over a substantial range of output in this model, the profit maximizing price on any demand curve converging on point P_o will be constant over that same range. Thus B, at price P_m in Fig. 2-I, is already exercising his best option with respect to the first section of

6. The second order condition, that $-2M (1 - q_a^*/Q_a) < 0$, is satisfied, since M was introduced as an absolute value. I am indebted to John Palmer, of the University of Western Ontario, for his comments on this section.

his demand curve, since P_m was chosen from a demand curve which also emanates from the P_0 intercept.

Now consider price reductions. If B lowers his price slightly, equalling A's price, he moves to the point Q, P_1 which is half of the total market demand at that price. Profit at this point is thus

$$\Pi_b = \left[(P_b - k_b) \ M/2 \ (P_0 - P_b) \right] - F_b \quad \dots \dots (2.6)$$

But if B lowers his price slightly below that of his rival, his demand curve will be horizontal out to the market demand curve. If B's capacity limit is less than the market demand at this price, profit will be

$$\Pi_b = (P_b - k_b) \ q_b^* - F_b \quad \dots \dots (2.7)$$

Note that equation (2.7) will always give a profit at least equal to the profit of equation (2.6). Since only a slight reduction of price is necessary to go from half of the market demand, to full capacity, profit from the latter will exceed that from the former in all except the extreme cases where half the market is the same or greater than full capacity. Hence the strategy of slightly undercutting the rival's price will dominate that of matching its price, except when the firms are both at full capacity and each set the market clearing price.

In the case where the slightly-lower-priced firm's demand is constrained by the market demand at an output level less than that firm's capacity limit, profit will be

$$\Pi_b = \left[(P_b - k_b) M (P_o - P_b) \right] - F_b \quad \dots (2.8)$$

In this case the fourth section of B's demand curve is feasible, and B must be assured that this is the optimal price on this section of the demand curve. Equation (2.8) is the relevant profit function. Setting the derivative of this, with respect to P_b , equal to zero,

$$\frac{\partial \Pi_b}{\partial P_b} = MP_o - 2MP_b - Mk_b = 0$$

implies

$$P_b = \frac{P_o - k_b}{2}$$

which is, of course, the price as indicated by the first section of B's demand curve, since both sections emanate from point P_o , and average variable costs are constant. But note that this price does not lie on the fourth section of the demand curve. If B were to set this price it would sell only residual demand, since B's price would exceed A's. The closest B can get to the price determined by equation (2.8) is to set a price slightly below A's. Thus the strategy of slightly undercutting A's price dominates any strategy involving a larger price cut.⁷ And since a slight price undercut dominates matching the rival's price, B has only two choices; to stay at price P_m or to slightly undercut A's price.

7. We shall see that neither firm will ever set price higher than that price which is halfway between marginal cost and the price intercept. Thus the firm adjusting price must choose a price below this unless it intends to sell only to the residual demand.

Thus B will compare the profit from equation (2.4) with whichever of equations (2.7) or (2.8) which is relevant and depending upon whether or not the slight price undercut allows full capacity sales. In Fig. 2-I it will be seen that if B undercuts A's price, he will sell full capacity output, and this is clearly more lucrative than remaining at price P_m . Following B's price cut, A is reduced to satisfying the residual demand, which is now slightly larger, at Q_2 in Fig. 2-I. A's demand function will also have four sections. For price increases it will rise towards the price axis intercept P_0 , and for price reductions it will be a point, a horizontal section, and finally the lower part of the market demand curve. By analogy with B's case above, it will be seen that only the first and third sections on this curve are relevant. Price P_m is the profit maximizing price on the first section, for reasons explained earlier. A should compare the profitability at this price with that at the price slightly below B's price, which allows full capacity. In Fig. 2-I it is evident that the latter will be the preferred strategy.

And so it continues. The firms alternately cut prices and expand to full capacity, and are reduced, in turn, to selling only the residual demand. Since the price reductions will be only slight, the firm with the undercut price finds itself on the curve dd' , in Fig. 2-I, which Shubik [133, p.82] has called the "contingent demand" schedule. Note that it is described by equation (2.1), and may be defined as the residual demand, contingent upon the price difference being very small. It is thus a special case of equation (2.2) which is the residual demand for all prices of B higher than A's price, when the buyers arrive randomly. The

contingent demand curve must begin at point d on the price axis, since the lower priced firm is able to satisfy market demand completely when the higher price is slightly above point d . It lies parallel to the market demand curve, and it must terminate at point d' , since at this price market demand is large enough to allow both firms to sell full capacity output, and any further price cuts would simply reduce revenue.⁸

But will the price be bid down until both firms are at full capacity? This depends upon the relative profitabilities given by equations (2.4) and (2.7) or (2.8). The further the prices fall, the less profitable is full capacity output, and the more profitable it becomes to set price P_m . Consider the price P_n in Fig. 2-I, which is chosen such that the profit level with full capacity output and price P_n , is just equal to profits at the higher price P_m . Hence if firm A is setting the (relatively lower) price P_n , firm B will prefer to raise price to P_m rather than to undercut price P_n . Firm A will continue to sell full capacity output at price P_n , but will realize that he may enlarge profits by raising price to a level fractionally below B's price P_m . As long as A is the lower-priced firm it will be the first choice of all buyers. But there are fewer total buyers at price P_m , and A's raising price causes B to be placed on the contingent demand curve at price P_m . We are back where we started! Once again, B has a strong profit incentive to undercut A's price, and the price cutting will continue until price P_n , after

8. This contingent demand curve is appropriate for both firms, since their cost structures are identical. The parallelism between the contingent demand curve and the market demand curve depends upon the vertically-rising marginal costs.

which a price rise will be more attractive to the firm with the contingent demand. Thus, prices will fluctuate between the floor and ceiling prices P_n and P_m , and Shubik [135, p.268] has called this the "Edgeworth Cycle".⁹

III. The Amplitude of the Edgeworth Cycle

In this section we are concerned with the location of the price floor and ceiling, and the resultant range of price fluctuations. What factors cause this range to be greater or smaller? Under what circumstances will there be no price cycle? It is evident that an important determinant of the price instability rests with the relationship between the size of the firm's plants and the market demand. For it was the existence of excess capacity which made it profitable to undercut the rival's price, as we saw in Fig. 2-I. Similarly, when residual demand was sufficiently large, the firms were motivated to raise prices again. Price stability will require a situation in which it is neither profitable to raise, nor profitable to lower, the prevailing price level.

We may thus envisage two extreme situations in which there will be no Edgeworth cycle. The first is that where plant sizes are so small, that the ceiling price allows both firms to sell full capacity output. No incentive to cut prices exists, since this would simply reduce revenues. Price increases are likewise less profitable, since according to the reaction function for price increases, equation (2.2), and the associated profit function, equation (2.4), the joint profit maximizing (ceiling)

9. Prices do not "cycle" in the strictest sense. They move smoothly down to price P_n , but leap directly up to the ceiling price P_m .

price is optimal. A second circumstance of price stability would occur if capacities were so large relative to the market, that each firm could satisfy the entire market at a price equal to its minimum average variable costs. No incentive to lower price exists, as firms will not accept prices below average variable costs, since the latter are discretionary costs. Similarly there is nothing to be gained by any higher price, since residual demand is zero at any higher price. Hence prices would stabilize at the market clearing price.¹⁰

Between these two extremes, price instability will prevail. As soon as the joint capacity of the firms is so large that it cannot be cleared at the monopoly price, an incentive will exist to cut prices, and later to raise them again. The amplitude of the Edgeworth cycle will increase as the capacities of the firms are enlarged relative to the market demand. The ceiling price is the same, as long as average variable costs are constant at the same level, as has been argued earlier. The floor price, however, falls closer and closer to the "clearing" price, as capacities are enlarged.¹¹ This is illustrated in Fig. 2-II, where six different capacity sizes are superimposed upon an otherwise ceteris paribus situation. The price P_n is in all cases the price floor, and is that price

10. Strictly, this second circumstance of price stability does not occur within the confines of the pure Edgeworth model, since it involves the Bertrand assumption that the firm's output constraint is not binding.

11. Given a particular level of constant costs, any particular price will allow a larger profit when it applies to a larger output. This price will have to fall further before the profit from undercutting is inferior to that at P_m . But in addition, residual demand at the ceiling price is smaller when capacities are larger relative to the market. Hence profit at the ceiling price is lower, and therefore the price floor must be lower still before the profit from undercutting just equals the profit from raising price to P_m again.

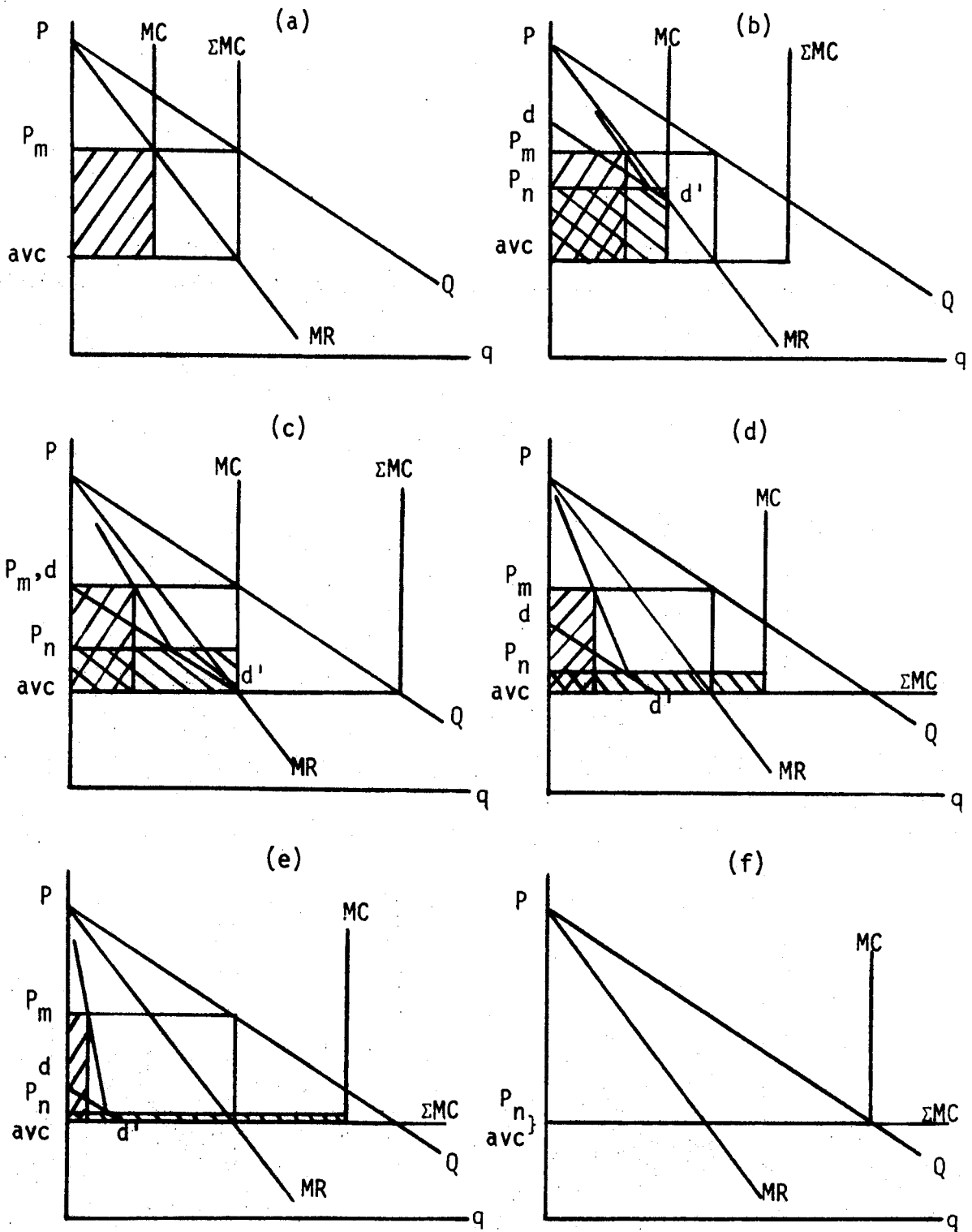


Fig. 2-II The Effect of Capacity Size upon the Price Cycle.

where the profit from a further price reduction would be inferior to the profit at the ceiling price P_m . The equivalent profit rectangles, neglecting fixed costs, are shaded oppositely in each diagram. Note that the Edgeworth cycle reaches its maximum when each firm has capacity almost sufficient to satisfy market demand at a price equal to minimum average variable costs. When each firm does have sufficient capacity to satisfy the market, as in Fig. 2-II (f), the price cycle abruptly disappears.

In the above we have held the demand situation constant and varied capacity limits. Let us now investigate the implications of differing demand structures in relation to any given cost conditions. The smaller is the slope of the market demand curve, the smaller will be the total decline in prices before both firms are at full capacity. Moreover, the smaller the slope, the greater the likelihood that the marginal revenue curve will cut a given industry marginal cost curve in its vertical section, and thus the more likely is price stability.¹²

One last variation on this theme is instructive. Suppose the firms now have differing cost structures. Let us presume that firm A is the relatively high-cost, small-capacity firm, and that the cost curves of the firms are represented by MC_a and MC_b in Fig. 2-III. Since capacities are different there will now be two contingent demand curves. A's curve

12. The extreme situation of perfect market price elasticity would ensure price stability regardless of capacity sizes. This is a trivial outcome, however. Market price elasticity subsumes elements of income elasticity and cross-elasticity (against all other products). Cross elasticity is likely to be relatively small, by definition of the duopoly, and income elasticity is not likely to be infinite, which it would need to be for perfect market price elasticity.

is labelled d'_a , and applies when B has the slightly lower price. Similarly, B's curve is d'_b . Due to the different levels of minimum average variable costs, the profit maximizing price on the firms' reaction functions for price increases will also differ. These are shown as P'_a for A, and P'_m (the industry monopoly price) for B. What will be the amplitude of the price cycle? It is clear that each firm will prefer to raise its price sometime before the price reaches the level of that firm's minimum average variable costs. Firm A, the higher-cost firm, reaches this crucial decision first, at price P_n in Fig. 2-III. The floor price is, once again, the price at which a further price reduction for either of the firms will return an inferior profit as compared to raising price against the residual demand. Firm A subsequently raises price to P'_a . But firm B may now raise price as high as P'_b yet still sell full capacity output. This reduces A's residual demand to zero. He must undercut B's price, and prices subsequently tumble towards P_n .

When cost structures differ, then, the amplitude of the cycle¹³ is determined by the higher-cost firm since this firm chooses both the floor and ceiling turning points. Given two firms with differing cost structures, the amplitude of the fluctuations will vary with the size of plants relative to the market, and inversely with the market elasticity, as outlined above. It is interesting to note that if minimum average variable costs differ, prices will not stabilize, (barring per-

13. Note that the "cycle" is now even more irregular in its path from floor to ceiling. Prices jump up, from P_n to P'_a , then jump down to P'_b , and then progress smoothly down to P_n before jumping back to P'_a again.

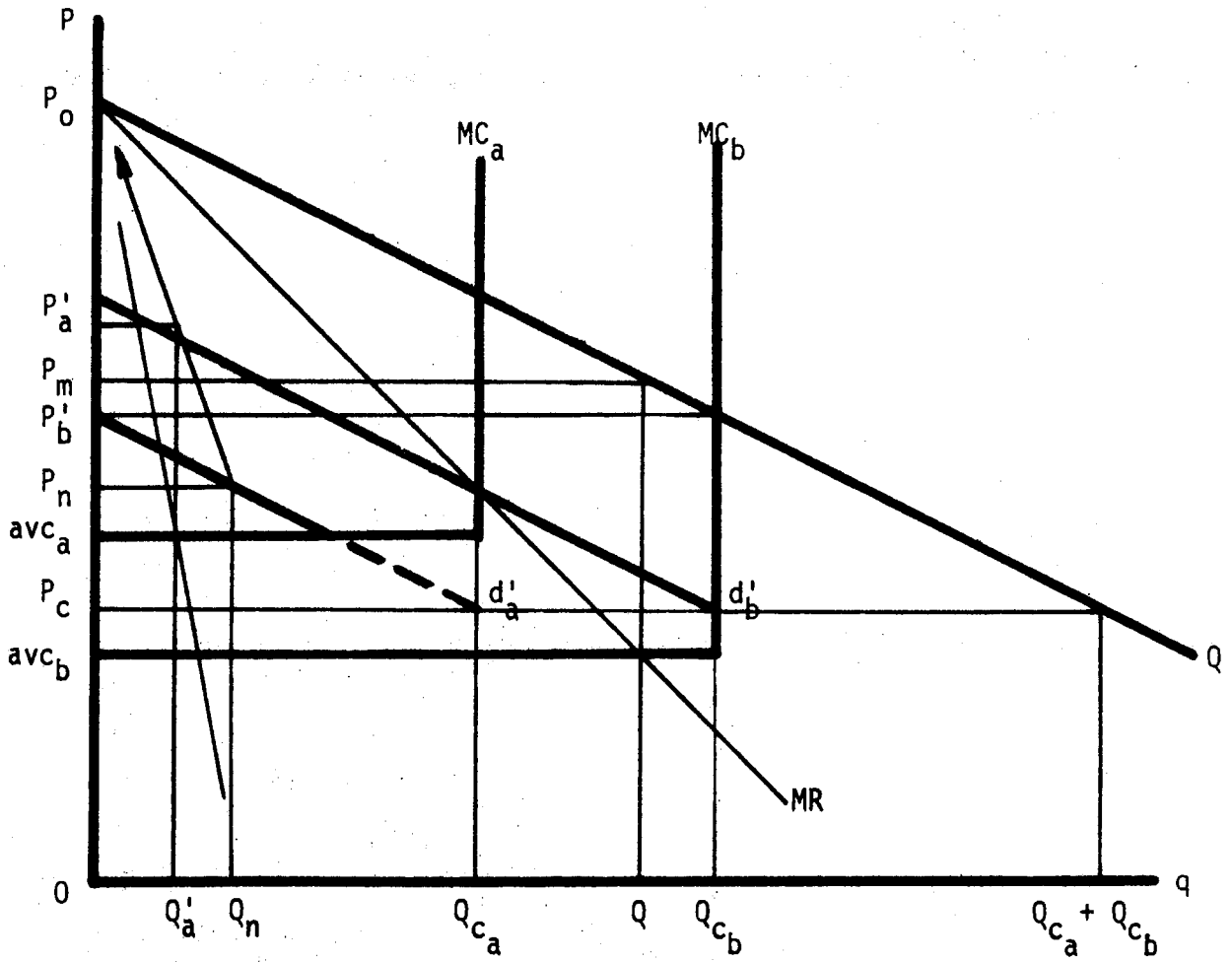


Fig. 2-III The Pure Edgeworth Duopoly Situation with Differing Cost Structures.

fectly elastic market demand), regardless of how small the firms are in relation to the market. Even when both firms are at full capacity, profit maximization requires that the higher-cost firm set a higher price than the lower-cost firm. The low-cost firm will then raise its price, and prices will subsequently cycle. At the other extreme price will stabilize, but only after the higher-cost firm is forced to cease production. The low-cost firm will set a price just below the average variable costs of his rival, and completely satisfy the market at that price.¹⁴

IV. The Frequency of the Edgeworth Cycle

In this section we shall examine the underlying determinants of the period of the price cycle, when instability prevails. That is, we are interested in how long it takes for prices to degenerate from P_m to P_n and rise back up again. What factors might cause this period to be shorter or longer?

The initial assumption that the sellers set their prices in the mornings, to prevail throughout the day, means that the period of the cycle will be the number of days it takes for prices to fall from P_m to P_n , plus one day for the price to be raised back to P_m again, in the

14. The introduction of diminishing returns (curvilinear marginal cost curves) complicates the diagrammatic presentation, but does not change the conclusions. The maximizing prices on each firm's reaction functions will no longer be constant over a wide range of outputs, but will fall as marginal costs fall, and will be higher as marginal costs rise. The floor and ceiling prices remain determined by the higher cost firm, and the conditions for price stability in the short run are unaltered, except that instead of an absolute full capacity, the firm will consider itself at full capacity when price equals marginal cost.

identical costs case, for example. Strictly, if products are identical, consumers are rational, and information is perfect, cross-elasticity of demand between the two goods approaches the infinite. Hence the price cut needed to attract all buyers is infinitesimal, and, thus, the period will be infinitely long. More realistically however, a certain minimum (finite) price differential will be required to attract the attention of all buyers. Given the size of the minimum price cut, and given that prices may be changed daily, the frequency of the price cycle depends upon the amplitude of the cycle, ceteris paribus. But why do we specify that prices are changed daily, rather than at some more frequent interval? The underlying presumption is that it takes a full day for each seller to find out his rival's price, and to act accordingly. How does one seller find out the other's price? Either he poses as a buyer, obtains the information from a buyer, or simply recognizes that his sales have dwindled to the residual demand.¹⁵ Having obtained the price information, the undercut firm must set his new price. All of this takes, presumably, one full day to accomplish.

The frequency of the price cycle thus appears to bear a strong relationship to the efficiency of information systems faced by the two firms. Firstly, the minimum price cut which serves to attract the preference of all buyers is related to the extent of the imperfections in

15. The model formulated by Stigler [145] may represent the mechanism by which a seller recognizes that the other seller has reduced his price. Stigler envisages the firm calculating the loss of its own 'old customers', the gain of other firms' 'old customers', and the gain of 'new customers' entering the market for the first time. A significant variation from the normal values of these flows would indicate that a competitor is secretly (to the other sellers) cutting its price.

the transmission of information from sellers to buyers. The more efficient is this information flow, the smaller will price cuts need to be, and the longer will be the period of the cycle, ceteris paribus. Secondly, the presumption that prices are changed daily is, in effect, a specification of the imperfections in the information flow between sellers. The more efficient is this information system, the more quickly will a retaliatory price cut be forthcoming, and the shorter will be the period of the cycle, ceteris paribus. The more efficient the former aspect of the information environment, and the less efficient the latter aspect, the longer will be the period of the cycle. In the event that these aspects of information systems are positively related, the frequency of the cycle will depend upon the balance between the two forces which tend to shorten or lengthen the period of the cycle.

V. Conclusions

Stability of prices is the exception rather than the rule in the simple Edgeworth model. Given the underlying assumptions, price stability is possible in only two circumstances. One is that where the firms capacities are sufficiently small relative to the market demand that neither firm wishes to depart from the "cartel" profit maximizing price. The second situation requires the Bertrand capacity assumption; that both firms have sufficiently large capacities that they could each satisfy the entire market demand at a price equal to minimum average variable costs. Shubik is aware of both of these equilibrium possibilities, but has dismissed the former case as trivial. [132, p.424]. Thereafter, he and his

associates seemed to indicate that the only equilibrium price will be the "efficient" price, or that price which is equal to minimum average variable costs.¹⁶ The reader is led to understand that the only meaningful case of price stability occurs as a result of invoking the Bertrand capacity assumption, or puristically, that price stability only occurs trivially in a strictly Edgeworth model. But to dismiss the small-capacities equilibrium case as trivial appears to be unreasonable in the light of the short run nature of this model. In the short run the market demand curve may shift,¹⁷ but firms' capacities must remain constant. It is quite feasible that the market demand curve could shift sufficiently far to the right, thus enabling both firms to attain full capacity sales at the "cartel" price. Price stability would then prevail throughout the short run or until demand conditions slackened.

The amplitude of the cycle was found to depend upon the ratio of firms' capacities to market demand, and upon the inelasticity of market demand. As noted above, the relationship of capacities to market demand could change in the short run, by virtue of shifts in the market demand curve. If the demand curve shifted outward, we would predict a progres-

16. Shubik defines the efficient point as follows: "... at this point any further price cut by one firm merely lowers its own profits. At the efficient point price the optimal production rates of both firms just saturates the market" [133, p.95]. Also, he has defined the efficient point solution as the price equal to marginal costs [98, p.144]. With curvilinear cost functions these two statements amount to the same thing, but with constant variable costs out to the point of full capacity, as in this exposition, the price which just allows both firms to sell full capacity will be above the constant level of marginal costs in all except the Bertrand case.

17. The demand curve may shift in the short run due to seasonal factors, or changes in consumer tastes, incomes, expectations, or prices of complementary goods.

sively smaller range of price fluctuations with eventual price stability, (if this did not already exist), and the opposite as the demand curve moves leftward. We notice in fact, that firms appear to be more willing to give discounts and lower prices when demand is relatively slack.

It should be briefly noted that the amplitude of the price cycle depends very heavily upon the assumption underlying the formation of the residual demand. In the foregoing we followed Edgeworth and assumed that buyers lucky enough to purchase at the lower price were chosen randomly with respect to their expected consumer surplus. Much of Shubik's work, however, involves the assumption that residual demand is minimized by virtue of the lower-priced firm selling to customers in descending order of their expected consumer surplus at that price. (See [132, pp.419-425], [133, pp.86-91], [130, pp.31-34]). Since the floor and ceiling prices each have a dependence upon the residual demand, it is clear that the cycle will be affected. We leave this question until chapter five, however, since it is outside the terms of reference of the simple model. The basic Edgeworth model assumes failure to recognize mutual dependence between sellers, whereas a firm choosing its customers in order to minimize the demand remaining for the other firm implies that this recognition is in fact made.

The frequency of the price cycle was seen to depend upon the minimum necessary size of the price cut, and the length of the Edgeworthian day. If the flow of information from sellers to buyers was perfect, then an infinitesimal price cut would suffice. If not, a larger, more noteworthy, price cut must be instigated. The Edgeworthian day may be quite long

when it takes a considerable time for one seller to find out the actual price of the other seller. The period of the cycle will be longer, the greater the efficiency of the flow between sellers and buyers, and the lesser the efficiency of the flow between sellers. Real world situations where buyers' information is good and sellers' information is relatively poor, may be approximated in some basic manufactures markets, such as steel and cement. The sellers' products approach homogeneity when manufactured to specification, and the buyers, having the required technical knowledge, tend to shop around for the best price. When transactions are made as a result of secret tenders, the sellers will not be certain of the successful bid price until after the contract is awarded.

The analysis of this chapter was intended to clarify and extend our understanding of the basic Edgeworth duopoly model. The method of exposition, with increased emphasis upon the mechanics of the model, is expected to have facilitated a more general appreciation of the model. As far as I am aware, this chapter contributes the following elements to the literature on the model. It makes the clear distinction between the contingent demand function of the residual ~~demand~~ demand function for all higher prices, carefully defining the latter. It explicitly discusses the factors underlying what I have called the "amplitude" of the price cycle. It provides an information-efficiency explanation of the "frequency" of the price cycle. Further, the often complex advances made by Shubik et.al have been clarified, and in some cases qualified, in the light of this analysis.

In the following chapter we begin the process of extension of the pure model by the systematic relaxation of certain restrictive assumptions that are employed in the basic model.

CHAPTER 3

PRODUCT DIFFERENTIATED DUOPOLY

I. Introduction

In this chapter the basic Edgeworth duopoly model is modified by the relaxation of the homogeneous products assumption. At the outset, it should be noted that product differentiation is a subjective matter. If the consumer has any preference for one commodity over the other, at equal shop-door prices, then the products are differentiated in his eyes. This preference may be based on real or imagined quality differences; location of the sellers and associated shopping cost and inconvenience; packaging; courtesy; expected after-sale service; and the prejudices, ignorance, apathy, etc. of the buyer. Sraffa [143] was one of the first to make the point that identical-products models were inadequate in view of the often strong preferences held by consumers. Hotelling [76] later incorporated this into his model of two firms producing cider of different degrees of sweetness. Hotelling's model was not really in the Edgeworth mould, however. Stability was attained when firms priced at average total cost, and the firms competed by changing their location on the product differentiation spectrum in the long run. Product differentiation has been introduced into the Edgeworth model by Shubik [133], [136], and his associates [130], [89]. Their treatment of the notion of product differentiation,

however, has been confined to a particular case, as we shall see.

In what follows we shall establish a general form of the demand function faced by the individual firm. It will incorporate certain extensions to the concept of product differentiation. The price adjustment process is examined, and the conditions for stability and instability are determined. Computer simulations of the adjustment process were employed, and some results are noted. Apart from differentiated products, all other assumptions are Edgeworthian. Firms are price-strategic, and attempt to maximize their immediate profit without recognizing their mutual dependence. Hence they expect their rival's price to remain fixed. Output capacities are limited, and marginal costs are constant to the point of full capacity.

II. The Firms' Demand Functions

We begin by examining the form that product differentiation may take. The division of the total sales between the two firms, at a particular pair of prices, will depend upon the structure of the aggregate preference at those prices or price differential. We shall define product differentiation first in terms of its symmetry or asymmetry, and later in terms of its continuity or discontinuity. Next we take account of the possibility that excess demand may arise as a result of the limited capacities of the firms, causing some buyers to purchase their non-preferred product.

(i) Symmetric and Asymmetric Product Differentiation

Symmetric product differentiation implies that when prices are equal, the two firms exactly share the market demand at that price. It further implies that when prices differ, the total demand will be divided between the two firms in a ratio depending upon the price differential, and regardless of which firm sets the higher or lower price. For independent price movements away from any equal prices situation then, both firms will gain or lose sales along a demand curve of the same slope. At any particular price level, price elasticity of demand will be the same for either firm. Asymmetric product differentiation implies that when prices are equal, the market divides unequally. For example, a widely-advertised product may command two-thirds of the market whenever its price, and the price of a lesser-known substitute, are equal. The individual demand curves need not have the same slope, in this case, since price elasticities may be the same, or different, at any particular price.

The demand functions faced by the duopolists, before consideration of possible discontinuity of product differentiation, or the effects of limited capacities, may be expressed as follows:

$$D_a = m_a Q_b + (P_a - P_b) \frac{dq_a}{dp_a} \dots (3.1)$$

and $D_b = m_b Q_a + (P_b - P_a) \frac{dq_b}{dp_b} \dots (3.2)$

where the subscripts a, b denote Firm A, B; D is the individual firm's demand; Q is the market demand at the price of the firm specified by the

subscript; P is the price of the individual firm specified by the subscript; dq/dp is the reciprocal of the slope of the specified firm's demand curve; and m is the proportion of the total market demand that will prefer to purchase from the specified firm when prices are equal. It is clear that in the symmetric case $m_a = m_b = 1/2$, and that the reciprocal slope terms will have the same value. If prices are equal, the latter term in the equations vanishes, and the firms share the market on the basis of the m values. A price cut by either firm would cause it to gain sales while the other firm loses sales. This is assured by anchoring the demand of one firm to some portion (m) of the total market at the other firm's price, plus or minus some part of demand which depends upon the price differential. Thus, if initially prices were equal, and firm A cuts price, it will move along its demand curve, while B's demand curve will shift back. If B now matches A's price, it will move along its demand curve, and A's demand curve will shift back. In the symmetric case the firms would once again share the market equally.

(ii) Continuous and Discontinuous Product Differentiation

Product Differentiation means that consumers do not view the competing products as being exactly the same. Some buyers will view the products as close substitutes, while others will consider them distant substitutes. As long as they are considered to be substitutes there will be a pair of prices for each consumer at which he will be indifferent between the two products. We presume that buyers preferences are such that as the price difference widens, buyers switch over to the lower-priced product smoothly and linearly. All this is implied in equations (3.1) and (3.2), and we

shall call this phenomenon "continuous" product differentiation. That is, no buyer has an absolute preference for either of the products regardless of relative prices. "Discontinuous" product differentiation, then, arises when some buyers do not view the two products as substitutes. Whatever the price of one product, some buyers will purchase the second product up to the point where its price is so high that their consumer surplus would be negative. These buyers then leave the market rather than purchase the other commodity.

To the extent that some buyers will only ever purchase the product of one firm, the demand for this product will not fall to zero until the last of these "loyal" buyers has his consumer surplus negated by the rising price-level. These buyers constitute a private market for that firm, which we shall assume depends linearly upon that firm's price. These loyal buyers will thus be the minimum sales for any given price level. Call H the intercept of this private demand curve, G the slope of this curve, and q' the minimum, or "loyal" quantity demanded. Thus, for Firm A, the "loyal" quantity demanded may be expressed as follows:

$$q'_a = (P_a - H_a) / G_a \quad \dots (3.3)$$

and similarly for firm B:

$$q'_b = (P_b - H_b) / G_b \quad \dots (3.4)$$

Incorporating this into equations (3.1) and (3.2) we have:

$$D_a = q'_a + m_a (Q_a - q'_{ba} - q'_a) + (P_a - P_b) \frac{dq_a}{dp_a}$$

and (3.5)

$$D_b = q'_b + m_b (Q_b - q'_{ab} - q'_b) + (P_b - P_a) \frac{dq_b}{dp_b}$$

. . . . (3.6)

which are subject to: $0 \leq q'_a \leq D_a \leq Q_a - q'_{ba}$

$0 \leq q'_b \leq D_b \leq Q_b - q'_{ab}$; $m_a + m_b = 1$; $0 \leq m_a \leq 1$;

$0 \leq m_b \leq 1$, and $\frac{dq}{dp} < \infty$. The notation q'_{ba} means the value of B's

minimum quantity demanded at the level of A's price. That is, P_a is substituted for P_b in equation (3.4) to find how much of the total market is not accessible to firm A at A's particular price level. Note that if product differentiation is both symmetric and discontinuous, the parameters H and G must have the same value for firms A and B. If product differentiation is continuous, q'_a and q'_b are zero for all prices of firms A and B, of course.

(iii) The Full Capacity Feedback Effect

Since we have postulated that the productive capacity of each firm is limited, a situation may arise in which the excess demand for one product is partly redirected back towards the other product. If, for example, firm B is at full capacity sales, firm A is able to raise price without B selling any more, despite the extra demand being diverted to B's product. Some of these unsatisfied demanders can be expected to go back and purchase

from A, because although they prefer A to B at that particular price differential, they prefer A to nothing at all. Thus there is an additional element which enters A's demand function whenever there is excess demand for B's product. In order to specify the size of this full capacity feedback effect, we need to make an assumption about the composition of the excess demand. In keeping with the Edgeworth model we assume that of those buyers who wished to purchase B's product, the satisfied buyers were chosen randomly with respect to their expected consumer surplus. This allows us to 'scale down' the excess demand which remains at B's price to find out how many still have non-negative consumer surplus at A's price.

The excess demand for B's product at price P_b , is equal to $D_b - q_b^*$. The higher is A's price above P_b , the smaller will be the proportion of this excess demand which still has non-negative consumer surplus. At the intercept of the market demand curve, P_o , all of the excess demand is exhausted. Thus we may envisage two rays converging upon the price axis intercept P_o ; one from the coordinates (q_b^*, P_b) and the other from (D_b, P_b) . At price P_b the horizontal distance between these rays is the amount of the excess demand, $D_b - q_b^*$, and at all higher prices the horizontal distance is the amount of the excess demand which remains willing to purchase at the higher prices. The ray joining q_b^* (at P_b) to the intercept P_o , may be described as follows

$$P_b = P_o + X (q_b^*) \dots \dots (3.7)$$

Since P_b , P_o , and q_b^* are known, we may solve for the slope term, X:

$$X = \frac{P_b - P_o}{q_b^*} \dots (3.8)$$

Similarly, the ray joining D_b (at P_b) with the intercept P_o , is expressed as

$$P_b = P_o + Y (D_b) \dots (3.9)$$

and we may solve for the slope term, Y , since D_b is known. Hence

$$Y = \frac{P_b - P_o}{D_b} \dots (3.10)$$

Now, for any higher price set by A , the feedback effect will be the difference between equations (3.9) and (3.7), evaluated at price P_a rather than at P_b . Hence

$$P_a = P_o + Xq \dots (3.11)$$

will give the value, q , on the first ray,

$$q = \frac{P_a - P_o}{X} \dots (3.12)$$

which must be subtracted from the value on the second ray. To find the latter, we know, from (3.9) that

$$P_a = P_o + YD \dots (3.13)$$

and thus the second value, D , is given by

$$D = \frac{P_a - P_o}{Y} \dots \dots (3.14)$$

Thus the full capacity feedback effect for firm A, call it E_a , is given by

$$E_a = D - q$$

Substituting from (3.14) and (3.12) we have

$$E_a = \left(\frac{P_a - P_o}{Y} \right) - \left(\frac{P_a - P_o}{X} \right)$$

Further substitution from (3.8) and (3.9) gives

$$E_a = \frac{P_a - P_o}{P_b - P_o} (D_b) - \frac{P_a - P_o}{P_b - P_o} (q_b^*)$$

or

$$E_a = \frac{P_a - P_o}{P_b - P_o} (D_b - q_b^*) \dots \dots (3.15)$$

By a similar procedure for firm B we would obtain

$$E_b = \frac{P_b - P_o}{P_a - P_o} (D_a - q_a^*) \dots \dots (3.16)$$

Equations (3.15) and (3.16) are subject to non-negativity, since the feedback effect only exists if there is excess demand for the other

firm.¹

The general form of the ceteris paribus demand functions for firms

A and B may thus be written as:

$$D_a = q'_a + m_a (Q_a - q'_{ba} - q'_a) + (P_a - P_b) \frac{dq_a}{dp_a} + E_a \quad \dots \dots (3.17)$$

and

$$D_b = q'_b + m_b (Q_b - q'_{ab} - q'_b) + (P_b - P_a) \frac{dq_b}{dp_b} + E_b \quad \dots \dots (3.18)$$

which are subject to

$$0 \leq (q'_a + E_a) \leq D_a \leq (Q_a - q'_a) \quad \text{and}$$

$$0 \leq (q'_b + E_b) \leq D_b \leq (Q_b - q'_b) \quad \text{respectively.}$$

1. It should be noted that this specification of the feedback effect involves the assumption that "loyal" buyers are satisfied first, and that the excess demand is composed entirely of "continuous" buyers. One justification for this is that "loyal" buyers may be regarded as having "subscriptions" to the product, while the "continuous" buyers go to the "news-stand". This explanation accords well with the "loyal" buyers' total commitment to one or other product. Relaxing this assumption would involve a further discounting of the excess demand. If, for example, all buyers arrive randomly, and if $q^*_b/D_b = 2/3$, then 1/3 of the "loyal" buyers will be unsatisfied. These must be subtracted from the excess demand before it is discounted by the relationship between the firms' prices, as in equations (3.15) and (3.16). Thus, under this latter assumption, the feedback effect must be smaller at all higher prices, as compared with the feedback effect under the assumption made in the text.

(iv) The Additivity Problem of Market Demand

In the preceding chapter we defined

$$Q_b = M (P_o - P_b) \quad (2.3)$$

as the market quantity demanded at the price level set by firm B. This demand curve is characterised by the intercept P_o on the price axis, and the reciprocal of the slope, M . Since products were identical, the price of firm A was immaterial to the specification of total market demand at the price level chosen by firm B. When products are differentiated, however, an additivity problem arises in the specification of market demand. (See particularly Triffin [154]). In the general case we are unable to specify a unique market quantity demanded when there is a price differential in the market. The mean price is usually unsuitable, since aggregate demand will depend upon which firm has the higher price, and how wide is the price differential. Only in the case of symmetric product differentiation will the mean of the two different prices, when substituted for P_b in equation (2.3), give the same aggregate quantity demanded as would be obtained by the simple addition of A's and B's quantity demanded at their respective prices, as calculated from equations (3.17) and (3.18). This is true because the firms' demand curves have equal slopes and at equal prices each firm has half of the market. Thus whichever firm causes the mean price to be lower by a given amount, the total sales will expand by the same amount. In the asymmetric case of product differentiation, however, either firm may lower the mean price by, say, ten percent, but the change in aggregate sales will depend upon the slope of the demand curve of

the price-cutting firm relative to that of the other firm.

In the general case, when products are differentiated, market demand may be defined only in terms of equal prices. Whenever asymmetry of product differentiation exists, the arithmetic mean price will not accurately predict the summation of the two firms' sales. Once again we shall assume that the market demand is linear with its intercept on the price axis at P_0 , and the reciprocal of its slope being M . Modifying equation (2.3) slightly, we have

$$Q = M (P_0 - P) \quad \dots (3.12)$$

where it is understood that P will be the price of both firms in the asymmetric case, and any mean price in the symmetric case. The individual firm's demand curves given by equations (3.17) and (3.18) show Q with a subscript indicating either the price of firm A or firm B. Note that this serves only to specify the starting point from which the firm's equal-prices share of the market will diverge as its price diverges from that of the other firm. The subscript to Q indicates at what level this starting point is to be evaluated.

III. The Price Adjustment Process

The ceteris paribus demand curve for the individual firm will have a number of kinks at certain price levels. An example is shown in Fig. 3-I. As firm A varies its price, given B's price P_b , quantity demanded will vary along the kinked line shown. The lower kink, labelled with

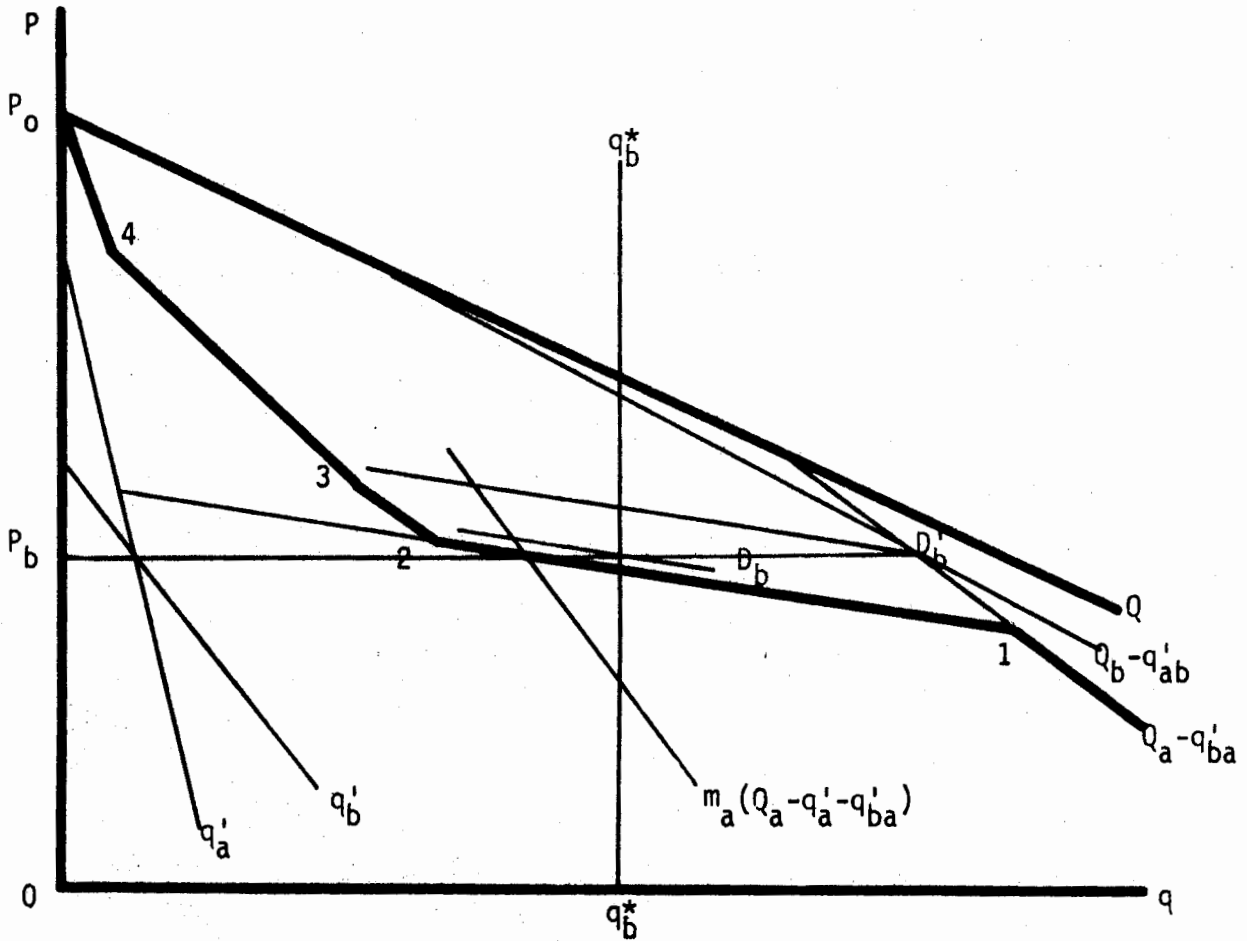


Fig. 3-I Differentiated Products Duopoly - Demand Curve for A, given B's price and capacity.

N.B. This example is one of asymmetric discontinuous product differentiation, since the "loyal" shares, shown by q'_a and q'_b , are different. However, $m_a = m_b = 1/2$, and the slopes of A's and B's demand curves are the same, for diagrammatic clarity.

the numeral 1, accords with the fact that A's demand, as he cuts price, is constrained by the accessible portion of the market demand, viz $Q_a - q'_{ba}$. (Recall that q'_{ba} is B's loyal buyers, evaluated at A's price level). For prices above this kink, A's demand would fall until at price $P_a = P_b$, A and B would share the continuous part of the market, that is $(Q_a - q'_a - q'_{ba})$, in the ratio $m_a : m_b$, which in this case is one-half each. Kink number 2 arises shortly above B's price, and is the result of the full capacity feedback effect beginning to operate, since B's demand curve will have shifted to the right, (D_b) , allowing B to sell full capacity at price P_b . At point 3 there is another kink upwards since A's rising price has alienated the last of the buyers in the continuous section of the market, and the feedback effect now adds directly to the "loyal" buyers on curve q'_a . Previous to this price level, A's demand was the sum of three elements: loyal, continuous, and feedback. Although the feedback element was increasing in magnitude, the loss of continuous buyers more than offset that increase, and the demand curve was negatively sloping. Above the price at point 3, the feedback now adds to the q'_a curve which allows demand to decrease much less for a given price increase, as compared to the continuous element. Hence the demand for A kinks upward at this point. Note that at the same price level of firm A, the excess demand for B reaches its maximum, since there are no more "continuous" buyers left to switch over to firm B. The feedback effect thus also reaches its maximum at this price level, and at higher prices of firm A this constant excess demand is discounted to the extent that A's price departs B's price and approaches the price intercept P_0 , as detailed in equation (3.15). The fourth kink arises when the last

of the "loyal" buyers leaves the market, and thereafter A's demand, up to the point P_0 , is confined solely to the feedback buyers.²

Since the individual firm's demand curve is kinked in a number of places, the marginal revenue function will be disjointed at these price (or output) levels. Thus the marginal revenue "curve" may intersect the marginal cost curve more than once, and/or it may pass through one or more of the discontinuities in the marginal revenue curve. As in the preceding chapter, the firm must compare the profit maximums from each section of the demand function, and choose the price/output combination which allows the "maximax" profit. The procedure for this is very similar to that outlined in chapter two, and hence will not be re-iterated here in algebraic form. Instead we shall concern ourselves with the conditions for stability and instability. Stability and instability depend upon these profit functions, of course, since it is the relative profitability on the different functions which causes the firm to decide whether to hold price stable, or to destabilize prices by choosing another price.

(i) Price Stability

In the Edgeworth model the firms choose the profit maximizing price level on the presumption that the price of the other firm will remain constant. If the other firm's price subsequently changes, and if this affects the first firm's sales and profits, this firm will adjust price

2. If the intercept of A's loyal demand was at P_0 , or if product differentiation were continuous, the fourth kink would not arise.

again, once more expecting the rival's price to remain fixed. The price adjustment process will thus continue until the price adjustment of one firm is so small as to have a negligible impact on the other firm's profits. If this occurs, there will be stability of prices and market shares. Let us look briefly at a case where stability does prevail.

In Fig. 3-II we depict the simple case of symmetric and continuous product differentiation. For further simplicity we have assumed identical cost structures, and that by coincidence, the firms are each setting the price P_m . At this price they are each selling output level Q , and we may imagine them both at the coordinates marked A' . Both firms face the demand curve³ shown as $AA'A''$. If we assume that firm A now independently adjusts price so as to maximize profits, he will cut his price to P_1 and move to full capacity output. This causes firm B to lose sales: in fact, he is now located at point B, facing the demand curve $BB'B''$ ⁴. Firm B's price adjustment would then be to price P_2 , where marginal revenue equals marginal costs. This causes firm A to be at point C, facing the demand curve CC'' , and he subsequently adjusts price to P_3 . We note that the adjustments have become progressively smaller, and that price and sales appear to be converging upon equilibrium values, such as P_e and Q_e ,

3. In its entirety, the demand curve would extend rightward until it meets the market demand curve, where it would kink downwards. For unilateral price increases there is no feedback effect since one firm would price itself out of the market (at price A) and the other firm would not yet be at full capacity.

4. This demand curve must have the same slope as $AA'A''$, and must pass through B' , since at that point both firms would set price P_1 and share the market equally.

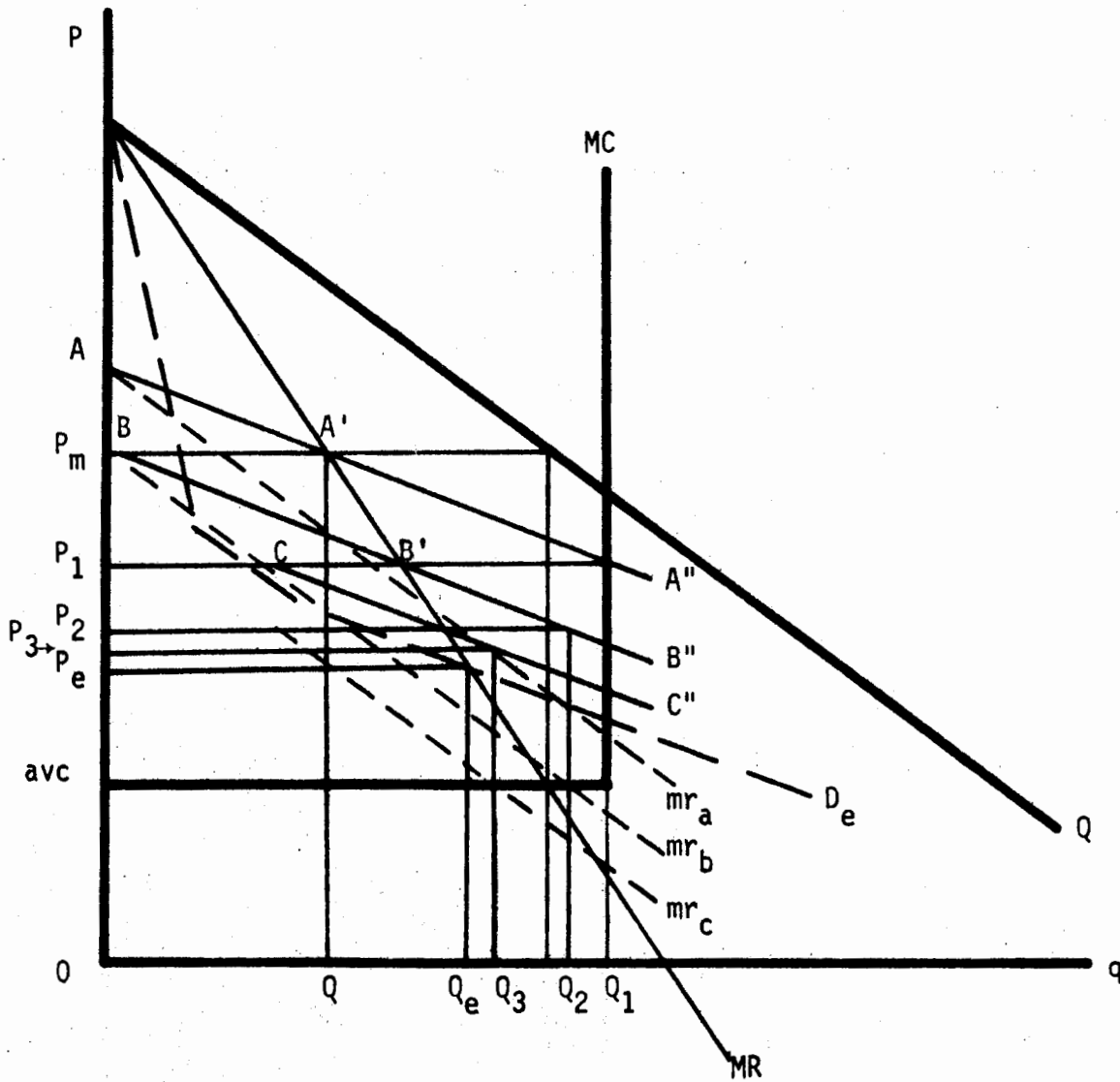


Fig. 3-II Price Stability in Symmetric and Continuous Product Differentiated Duopoly.

where both firms would set the same price and share the market equally.⁵

To confirm that the price and quantity values would tend to converge upon equilibrium values, the adjustment process was computer simulated. In order to simulate the model, parameter values were inserted to describe the characteristics of the market demand curve, slopes of the individual firm's demand curves, fixed and variable cost levels, full capacity outputs, and the product differentiation parameters. As a starting point it was presumed that the firms were setting the industry monopoly price, this allowed an initial $P_a = P_b$ situation from which A's price might depart, and also allowed us to see when the duopolists chose to act like a cartel. When A had adjusted price maximally, all the appropriate variables were re-evaluated at the new price differential, and then B adjusts his price maximally, and so on. The model was initially simulated with parameter values consistent with Fig. 3-II.⁶ After five iterations, in each of which both firms took the initiative in adjusting price to maximize profits, the sales of each firm were affected only at the second decimal place when the other firm adjusted price. After ten iterations prices and sales of the firms were equal to the eighth decimal place before and after adjustments. For all intents and purposes, we may

5. There is no profit incentive to raise price, since the feedback effect, in this case, is not large enough at any higher price, to cause a price rise to be a more profitable strategy as compared to holding the price constant at price P_e . The next-most profitable price is one close to P_1 (above the "feedback" kink in the demand curve shown as D_e , which passes through the coordinates Q_e, P_e) where the profitability is approximately six percent less than at P_e .

6. These were: market demand curve intercept, 52, slope, 0.362; firm's capacities 41 each; firm's demand curve slope reciprocals, 3.375; fixed costs, 100 each; variable costs, constant at 14 for both firms; product differentiation was symmetric and continuous. The cartel price level was 33.

regard the market as being stable at the following values:

Table 3.1 Initial Case - identical costs, symmetric and continuous product differentiation.

| | A | B |
|---------|----------|----------|
| Sales | 37.0986 | 37.0986 |
| Price | 24.9921 | 24.9921 |
| Profits | 307.7952 | 307.7952 |

Following the initial run, certain cost and demand parameters were systematically varied in order to ascertain their impact on the equilibrium prices and sales. The equilibrium price was made to occur at the cartel price by one or a combination of three changes. Firstly the full capacity limits of the firm were reduced until the industry monopoly sales level was equal to or greater than the firms' combined capacities. Secondly, the price intercept of the market demand curve was increased until the same result was obtained. Thirdly, the slope term of the market demand curve, which is involved in market price elasticity, was reduced until again the firms could sell full capacity output at the industry maximizing price.⁷

A price differential is introduced by any one of the following changes. Firstly, if variable costs are different, ceteris paribus, then the firm with the lower variable cost will tend to sell more output, possibly full

7. In each of these cases, the actual level of the cartel price was different, of course, in accordance with the changed relationship between market size and productive capacity.

capacity, by virtue of the lower price which it sets to maximize profits. Table 3.2, when compared with table 3.1, shows the result of varying only the variable cost level of one firm.

Table 3.2 Differing Variable Costs - A's now 10, B's remain at 14.

| | A | B |
|---------|----------|----------|
| Sales | 41.0 | 35.6951 |
| Prices | 23.5896 | 24.5763 |
| Profits | 457.1747 | 277.5233 |

Secondly the firm with the advantage of asymmetry of demand (the larger share when prices are equal), ceteris paribus, will set a higher price and sell the larger output. Compare table 3.3 with table 3.1.

Table 3.3 Asymmetric product differentiation:

$$m_a = .333, m_b = .667$$

| | A | B |
|---------|----------|----------|
| Sales | 33.3774 | 41.0 |
| Prices | 23.8896 | 27.0036 |
| Profits | 230.0909 | 433.1510 |

Thirdly, a price differential is introduced when the firms have different capacity output levels, ceteris paribus. The firm with the smaller capacity is able to set a higher price, yet still sell its full capacity.

Compare table 3.4 with table 3.1.

Table 3.4 Differing Capacity Levels: A's now 30,
B's remains 41.

| | A | B |
|---------|----------|----------|
| Sales | 30.0 | 39.6523 |
| Prices | 27.5441 | 25.7488 |
| Profits | 306.3258 | 365.3258 |

In the composite cases of the above three changes, the outcome depends upon the relative strengths of the factors involved. A situation could be generated, for example, where prices are equal, due to the advantage of asymmetric demand being just offset by the high cost and low capacity levels of the same firm.

The introduction of discontinuous product differentiation had no effect as long as this discontinuity was symmetric. Only when the "loyal" markets differ, is there an impact on sales and prices. The firm with the larger "loyal" market has higher sales, price, and profits compared with the other firm, because it is, in effect, operating in a larger market. Compare table 3.5 with table 3.1.

Table 3.5 Asymmetric Discontinuous Product Differentiation:
B has a "loyal" share of 10% of the total market
at all prices.

| | A | B |
|---------|----------|----------|
| Sales | 35.6712 | 38.4463 |
| Prices | 24.5692 | 25.3915 |
| Profits | 277.0190 | 337.9628 |

If we compound this asymmetry by varying the ratio $m_a : m_b$, the firm with the advantage in both aspects of product differentiation enjoys even higher sales, price, and profits. Alternately, if the firm with the larger share at equal prices has the smaller "loyal" market, then the advantage of the former is offset, in some degree, by the disadvantage of the latter. And, of course, larger or smaller price differentials can be generated by giving one or other firm relatively higher costs, or by varying the size and elasticity of total market demand and/or capacity levels.

The computer simulations of the model serve to illustrate that the introduction of product differentiation into the Edgeworth model can lead to a great variety of outcomes, especially when in conjunction with differences in other cost and demand parameters. Prices may stabilize, but the level at which they stabilize and the price differential which is established, depends upon at least a dozen variables. We have seen that equal prices do not necessarily indicate collusion, nor that the firms necessarily face identical cost and demand conditions, since prices may stabilize very close to each other due to the countervailing effects of various cost and demand conditions. On the other hand, prices may not stabilize, and to this we now turn.

(ii) Price Instability

Levitan and Shubik [89], following Shapley and Shubik [130], and Quandt [120], have shown that the introduction of product differentiation does not guarantee the achievement of price stability in the Edgeworth

model⁸. Instability of prices will occur if at some point in the descent of prices (towards the equilibrium values mentioned in the previous section) the feedback demand becomes sufficiently large that it is more profitable to raise the price than to reduce it further. A situation where this happens is depicted in Fig. 3-III. If no regard has been given to the feedback effect, prices will gravitate to P_e , since at this price marginal revenue equals marginal costs and there is no further incentive to cut price. For expositional clarity we show an isoprofit curve lying tangent at point E. Since we have assumed constant variable costs, the isoprofit curve is a rectangular hyperbola out to the capacity limit.

Now let the firms become aware of the full capacity feedback effect which accrues as one firm raises its price. If this effect is such that the demand function kinks upward to cross the original isoprofit curve, then either firm may attain a higher level of profit by raising price. In the Figure a new tangency is evident at point J, vertically above the point where the marginal revenue associated with one section of the demand function intersects the marginal cost curve. Since the profit at J exceeds that at E, each firm will be motivated to independently raise price to the higher level. Suppose A does in fact raise price to P_j . As soon as B realizes that it may raise price to some extent and still sell full capacity, it will, and A's demand curve will move leftward to some lower isoprofit curve. A continuing series of price adjustments will thus ensue as the prices are bid down towards P_e , and raised again, and so on.

8. I have commented upon this elsewhere. (See [40]). Much of this section relies heavily upon that earlier comment.

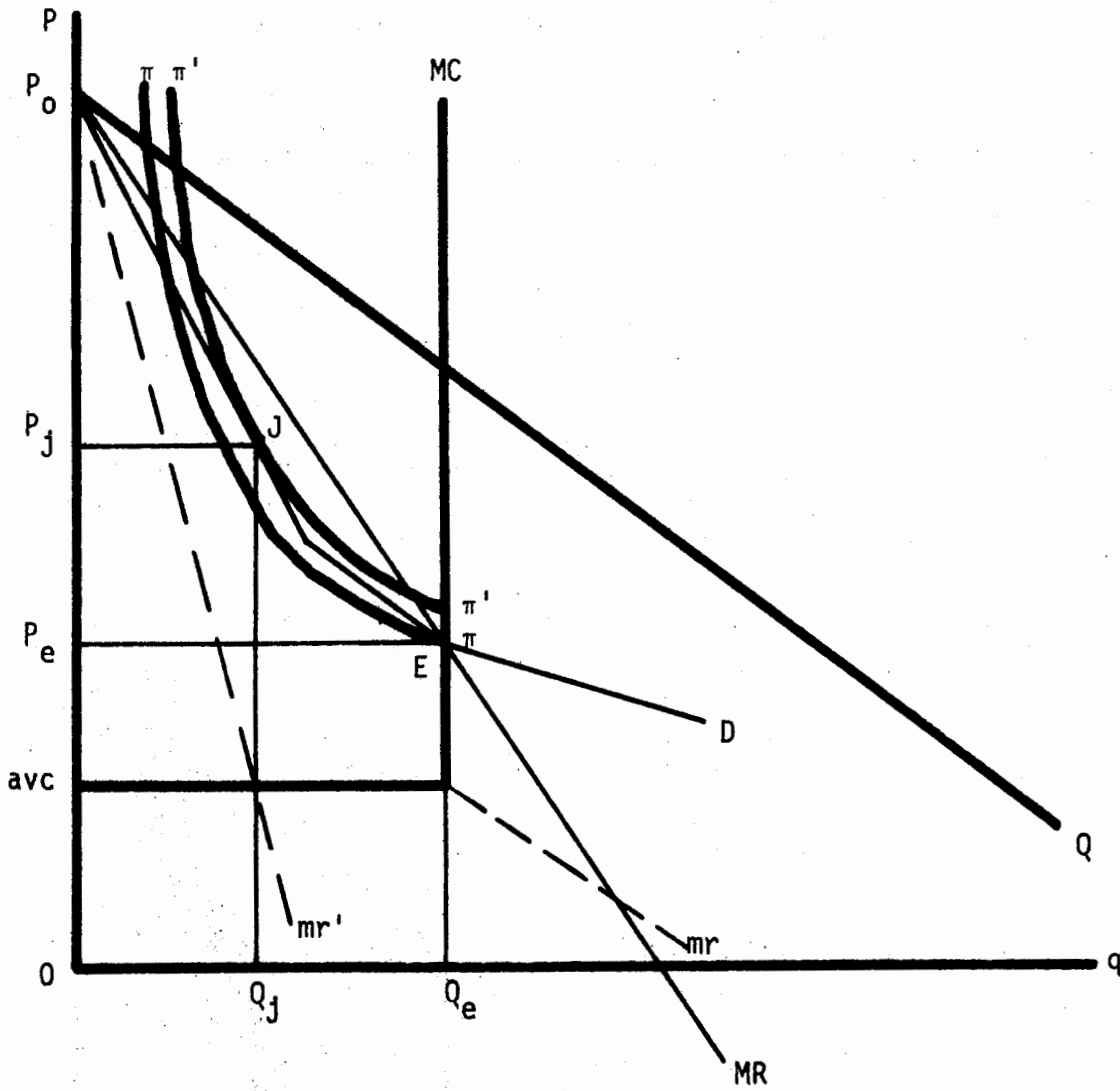


Fig. 3-III Price Instability in Differentiated Products Duopoly.

Whether one of the upper sections of the demand function becomes tangent to a higher isoprofit curve, depends upon the factors underlying the shape and position of both curves. With regard first to the demand function, the smaller is the ratio of the other firm's capacity output level to the total market demand in the vicinity of price P_e , the sooner the curve will kink. The steeper is the firm's demand curve before the kink, the closer to vertical will be the curve after the kink, since the feedback effect will more nearly offset the loss of the "continuous" buyers as price rises. Further, since at higher prices the feedback demand adds to the "loyal" demand, the larger is the latter element at any price, the steeper will be the firm's demand curve. And lastly, the steeper is the market demand curve, the more steeply rising will be the demand function at higher price levels.

The isoprofit curve will be a rectangular hyperbola out to the capacity output limit if there are constant average costs. If average costs fall and later increase, the isoprofit curve will take on a U-shape, with the sides of the U becoming more steep as first increasing and later decreasing returns become more severe. Given any isoprofit curve, it is clear that as the "continuous" section of the demand curve becomes steeper, the tangency with the isoprofit curve must occur at a smaller proportion of a firm's given capacity. The slope of the "continuous" section of the firm's demand curve depends in part upon the cross-elasticity of demand between the firms' products. The greater the degree of product differentiation the steeper will be the "continuous" section. Given any demand function, the steeper is the left-hand-side of the isoprofit curve, the

less likely is it to cross the demand function. Thus the conditions most conducive to price instability would be a slight degree of product differentiation, constant average variable costs up to full capacity output, and firms having little or no excess capacity at point E. This contrasts with our expectations of actual duopolies, of course, where products are more often substantially differentiated, and where firms are motivated, as we shall see in chapter six, to hold a significant proportion of excess capacity.

Note that the relationship of capacity size to market size is an important determinant of price stability, as it was in the identical products case. The larger are plant sizes in relation to the market demand, the higher one firm will need to raise the price before the other firm attains full capacity. Only after this point will the feedback effect begin, and its size is less likely to cause a price increase to be more profitable, ceteris paribus. This is illustrated in Fig. 3-IV where four different sizes of plant are superimposed upon the same market demand situation. In part (a) the plant size is so small that the price P_e puts both firms at full capacity output. The demand curve thus kinks immediately, for price increases above P_e . In this case the profitability at price P_j exceeds that at price P_e , and thus there will be price instability. In parts (b) and (c) plant sizes are progressively larger, the feedback effect is seen to become progressively smaller, and there is no price, in either situation, which yields a greater profit than does price P_e . In part (d) the plant size is sufficiently large that in order to allow the other firm to attain full capacity, the firm raising price

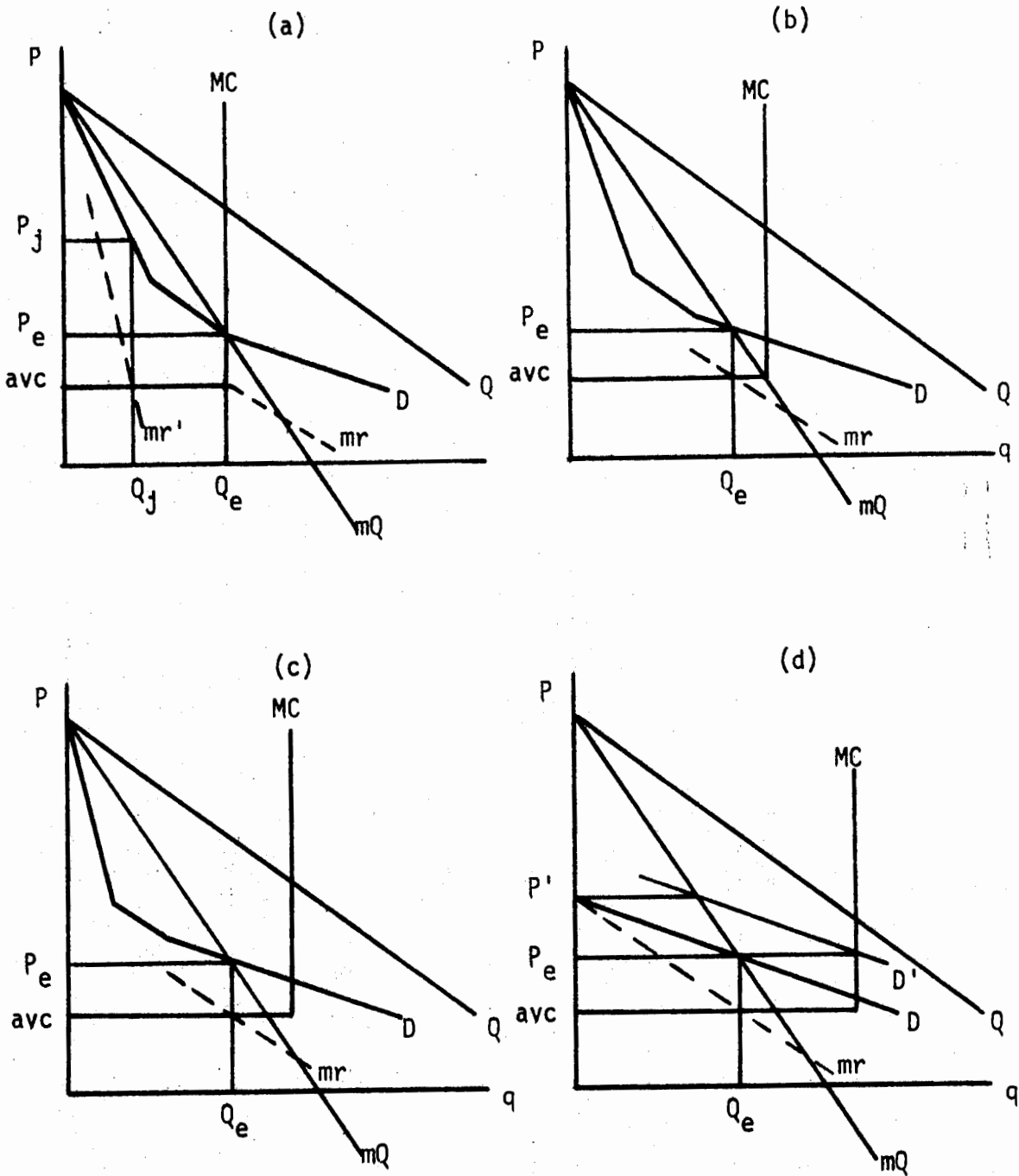


Fig. 3-IV The Stabilizing Effect of Larger Capacities in Differentiated Products Duopoly.

must set a price so high, (P'), that the last of the "continuous" buyers has already switched to, and been satisfied by, the other seller. Thus there is no feedback demand in this case. Price stability, it will be noted, was obtained somewhere between parts (a) and (b). Given the characteristics of market demand, and the cross-elasticity between the two products, price stability depends upon the relationship between size of plant and size of the market demand.

Alternatively, we might say that given a cost structure, and market demand situation, instability is less likely the greater is the degree of product differentiation. When instability exists, then, firms may remove it by further differentiating their products. If firms prefer stability to instability this would be a short run means of attaining stability, while the above-mentioned effect of larger plant sizes could be utilized in the long run.

IV. Conclusions

Contrary to the findings in the identical products case of duopoly, stability of prices is common in the differentiated products case, while instability of prices is the exception to the rule. The levels at which prices will stabilize, and the differences in the firms' prices, were seen to depend upon a variety of cost and demand parameters. Product differentiation was introduced in the twin dimensions of symmetry and continuity, with interesting implications for the subsequent prices and relative sales levels. The re-channelling of excess demand back to the higher-priced seller, which I have called the full capacity feedback

effect, may be sufficiently large that it will aid in the destabilization of the price level, but we have seen that this eventuality depends upon a somewhat unlikely combination of other cost and demand conditions.

To my knowledge, the demand functions for the individual firms have been specified with considerably more generality here, than has been done before. Product differentiation has been carefully defined in terms of the concepts of symmetry and continuity, which I have introduced. Shubik [133], [136], Shapley and Shubik [130], and Levitan and Shubik [89] have previously discussed product differentiation in the price strategy duopoly situation. Their treatment, however, appears to confuse elements of symmetry and discontinuity. As I have noted elsewhere, [39], their analysis would seem to be confined to the symmetric case, but their treatment of a discontinuous element is not done symmetrically. Moreover their discussion of what I have called discontinuous product differentiation is confined to a special case. They presume that if differentiation is discontinuous, then a constant proportion of the market demand will be loyal to a particular seller at various prices. My specification is not constrained in that manner. And lastly, the full capacity feedback effect, as specified here, is general, and does not rely on the constant proportion of discontinuous product differentiation, as did my earlier [39] respecification of the Shapley and Shubik feedback effect [130].

In the following chapter the Edgeworth duopoly model, both in its identical products form and its differentiated products variant, is extended to the oligopoly case.

CHAPTER 4

EDGEWORTH OLIGOPOLY

I. Introduction

In this chapter the Edgeworth model is modified to include more than two firms. At first, products will be assumed identical, and the analysis will expand upon chapter two. Then product differentiation is allowed, and the analysis follows upon that of chapter three. For the most part the conclusions of the previous chapters will stand, as duopoly is a special case of oligopoly: in oligopoly an individual firm contemplates a competitive strategy which will take place at the expense of all other firms, while in duopoly there is simply one other firm.

II. The Identical Products Case

(i) The firm's demand function

When there are n firms, the demand for the individual firm depends crucially upon its price ranking. Thus it is necessary to identify those firms with higher, lower, and equal prices in relation to the firm under consideration. Let us call D_i the demand for the i^{th} firm's product, where $i = 1, 2, \dots, n$; q_i^* is the full capacity output (sales) level of the i^{th} firm; and Q_i is the total market demand at the i^{th} firm's price. When we are considering the price ranking of the i^{th} firm, there will be

k lower-priced firms and h higher-priced firms, subject to $k + h \leq n - 1$ and both k and h being non-negative. It follows that there will be $n - k - h$ firms with prices equal to that of the i^{th} firm, (one of these being the i^{th} firm).

Consumers will wish to purchase from the lowest-priced firm which has not yet sold out, since the products are identical and consumers are rational. Thus the demand for the lowest-priced firm will be constrained only by the extent of market demand, and it will be the first firm to sell full capacity output, if $q_j^* \leq Q_j$, $j = 1$. The demand remaining for the next-lowest-priced firm depends not only on the margin between q_j^* and Q_j , but also on the assumption that is made regarding the composition of the first firm's customers. If they were a random selection of the potential customers at that price, then the proportion q_j^*/Q_j of the total market demand at all higher prices will already have obtained the product, and thus will be no longer in the market. The second-lowest-priced firm, if it is able to sell full capacity, will also remove a fraction (the ratio of its full capacity output to the demand which remained for it) from the market at all higher prices. The demand which remains for the i^{th} firm will thus be Q_i , the total market demand at the i^{th} firm's price, minus the fraction of Q_i which was already satisfied by the lower-priced firms. That is:

$$D_i = Q_i - \sum_{j=1}^k (q_j/Q_j) Q_i \dots (4.1)$$

subject to $0 \leq q_j \leq q_j^*$ and $q_j \leq Q_j$

If the capacities of all lower-priced firms are sufficient to satisfy the market at the i^{th} firm's price, there will be zero residual demand. If not, and if the resulting residual demand puts the i^{th} firm at less than full capacity, then all firms with prices higher than that of the i^{th} firm will have zero sales.

The above expression is valid only for the simple case where there are no other firms with prices exactly equal to the price of the i^{th} firm. If there were, then these firms would share the residual demand equally, subject to their full capacity constraints. If these full capacity constraints are not equal, then the remainder of the demand for those firms which go to full capacity will be shared equally amongst the other firms at the same price level, and if residual demand remains it will go to successively higher-priced firms until exhausted. The more general form of the i^{th} firm's demand is as follows:

$$D_i = \frac{\left(Q_i - \sum_{j=1}^k (q_j/Q_j) Q_i \right)}{n - k - h} +$$

$$\frac{1}{n - k - h - f} \left(\sum_{j=1}^f \left(\frac{\left(Q_i - \sum_{j=1}^k (q_j/Q_j) Q_i \right)}{n - k - h} \right) - q_j^* \right)$$

. . . . (4.2)

where there are $j = 1, 2, \dots, f$ equal-priced firms which go to full capacity since they are unable to satisfy their share of the residual demand ($0 \leq f \leq h - 1$).

It can be seen that if all prices and capacities are equal, all firms share the market equally, and equation (4.2) reduces to:

$$D_i = \frac{Q_i}{n} \quad \dots (4.3)$$

subject to $0 \leq D_i \leq q_i^*$

Had we assumed that the lower-priced firms satisfy the buyers in order of their expected consumer surplus, as has sometimes been done [133], [130], and thus minimized the residual demand, the demand for the i^{th} firm would be

$$D_i = Q_i - \sum_{j=1}^k q_j \quad \dots (4.4)$$

when there are no other firms at the same price level as the i^{th} firm, and, otherwise

$$D_i = \frac{\left(Q_i - \sum_{j=1}^k q_j \right)}{n - k - h} + \frac{1}{n - k - h - f} \sum_{j=1}^f \left(\frac{\left(Q_i - \sum_{j=1}^k q_j \right)}{n - k - h} - q_j^* \right) \quad \dots (4.5)$$

In this analysis we will use the 'random' assumption, since the 'minimization' assumption appears inappropriate for the following reasons. Firstly, it would seem to imply that the firms wish to, and are able to,

discriminate amongst their customers in a complex fashion, indicating that the firms recognize their mutual dependence, which in turn is not allowed under the Edgeworth assumptions. Alternatively, it might imply that buyers organize themselves before trading begins, qualifying for the lower prices on the basis of relative consumer surplus, which is an unlikely interpretation to place upon the model.

The demand function for any one firm thus depends upon the prices of all other firms, the capacity output limits of the other firms, the size of the market demand at all price levels, and the assumption underlying the composition of the residual demand. The demand function will have discontinuities and kinks in abundance as the i^{th} firm varies its price, ceteris paribus. As the i^{th} firm's price is decreased, the firm captures more and more of the market demand, as it continues to undercut rivals' prices. This is shown graphically in Appendix A to this chapter.

(ii) Price stability conditions

Price instability will be the rule, rather than the exception, in the identical products case of Edgeworth oligopoly. If a price is to be the stable price, it must be the price at which each and every firm maximizes profits. If any one firm could increase profits by independently raising or lowering its price, then the prevailing price is not the equilibrium price. We need, therefore, to discover the possible circumstances under which there is no incentive for any firm to either raise or lower price. No firm will increase profits by raising price if the residual demand at all higher prices is either zero or so small that it is better to maintain the present price rather than raise price to

maximize against the residual demand. No firm will wish to lower its price if it is already selling its full capacity output level, or if any further price cut causes price to fall below minimum average variable costs. Any sufficient combination of these circumstances should therefore provide a situation of price stability.

Let us examine a case analogous to the Bertrand case of chapter two. If any $n - 1$ firms have sufficient capacity to satisfy the market demand at a price equal to their minimum average variable costs, then the residual demand for the remaining firm, if it sets any higher price, will be zero. If all cost structures are equal all firms will be forced to set price equal to minimum AVC. No firm can sell any output at any higher price, and no firm is willing to accept a lower price, thus this price is a stable price and the i^{th} firm's demand is given by equation (4.3). By further analogy with the pure duopoly case, one other situation of price stability suggests itself. If all firms have the same cost structure, and if the industry monopoly price allows all firms to sell full capacity, then the industry monopoly price will be the equilibrium price. No firm will wish to reduce price, since it is selling its maximum output at the prevailing price. No firm will wish to raise price, since the profit maximizing price on the residual demand curve (equation 4.2) will be the same as the prevailing price. These can be shown to be the only cases of price stability. Cost structures must be identical, and capacities such that either (i) $n - 1$ firms can satisfy the total market at a price equal to minimum AVC, or (ii) the n firms cannot satisfy any more than half of the total market demand at a price equal to minimum

AVC (ie. the market marginal revenue curve cuts the aggregate marginal cost curve in the vertical section of the latter).

If the firms' capacities are such that in aggregate they lie between the two situations mentioned above, then there will be cyclical fluctuations of the price level. In Fig. 4-I there is depicted a case of four firms with equal cost structures, but whose combined capacity is greater than half the market at a price equal to minimum AVC, but is not large enough such that any $n - 1$ firms can satisfy the market demand at that price. Prices will fall no further than P_e , since all firms would be at full capacity at that price. Any one firm would then gain by raising price along the residual demand curve dd' to maximize profits at price P_m . But this in turn will induce each of the other $n - 1$ firms to simultaneously (although independently) raise price to a level below P_m , in the expectation of still selling full capacity at this higher price. Subsequently all firms are at less than full capacity, and each firm expects to increase profits by cutting price. Prices therefore cycle between the ceiling P_m (the industry monopoly price) and the floor which can be no lower than P_e .¹

To show the importance of identical cost structures for price stability, we now examine one of the capacity cases which is conducive to a stable price, but in conjunction with unequal cost structures.

1. The floor may be higher than P_e if it is more profitable to raise price to P_m rather than cut it further, at a price higher than P_e , and this depends upon the specific cost and demand structures. The larger are the firms capacities relative to the market demand, the closer will the floor price be to absolute minimum price, which is P_e in this case, or the level of constant marginal costs, in Fig. 2-II for example.

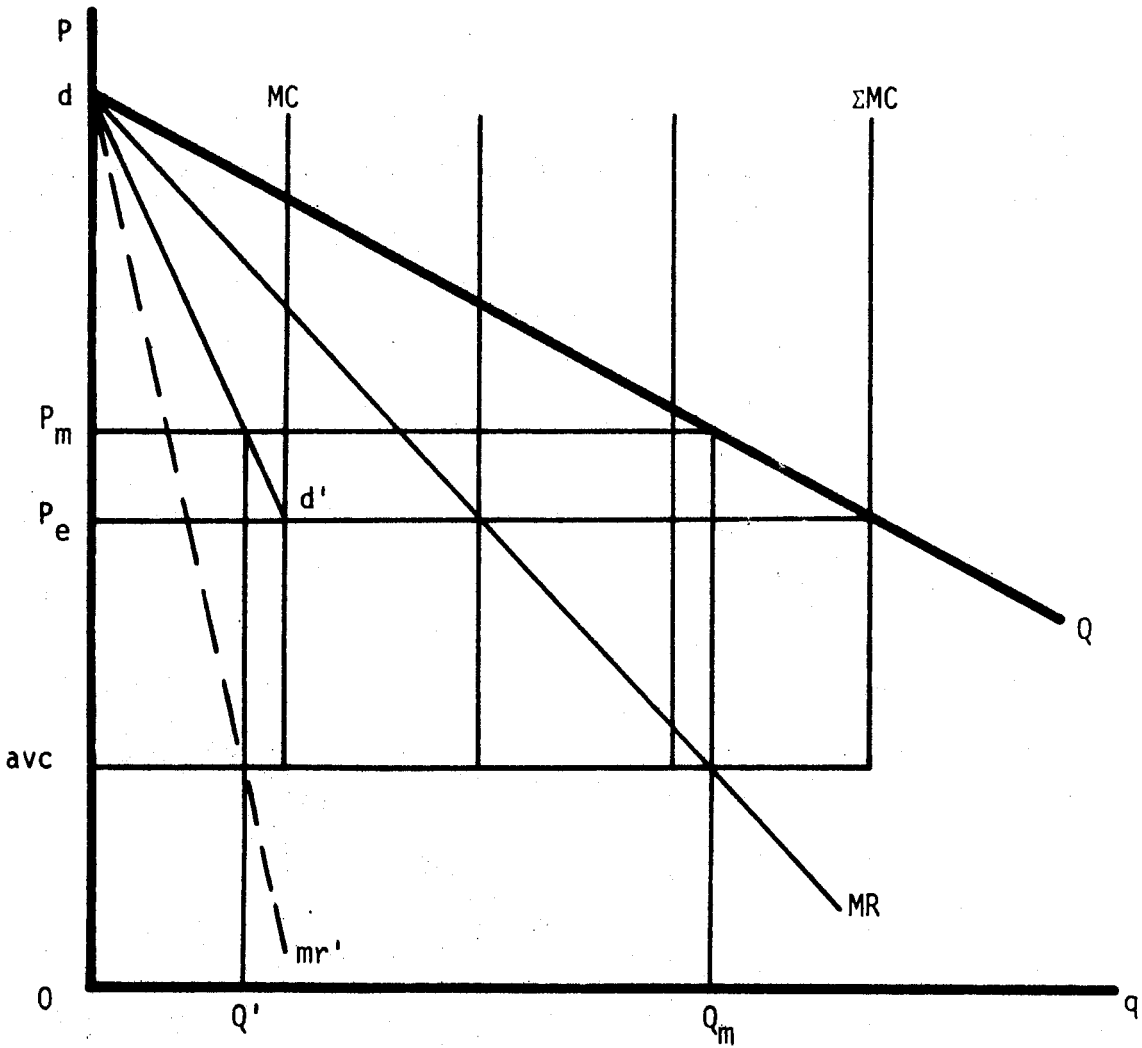


Fig. 4-I Price Instability in the Identical Products Case of Oligopoly with Identical Costs.

Suppose that $n - 1$ firms could satisfy the market demand at a price equal to the minimum AVC of the highest-cost firm within this $n - 1$ subset of firms. This situation is depicted in Fig. 4-II. It is clear that prices will be cut down below the level of the fourth firm's minimum AVC and he will be forced to close down. Yet prices will fall still further as the remaining three firms attempt to independently maximize profits. Each firm compares the profit from a further price cut with that which would result from raising the price to maximize against the residual demand. We note that the highest-cost firm, (of the remaining three), will be the first to find it more profitable to raise price, since he will be closer to his cost floor, and his capacity limit is no larger than that of any other firm. In Fig. 4-II the third firm would prefer to raise price, rather than cut it further, at the price P_0 . This is the price floor.² In raising price, the third firm will set price P' . But this firm, in failing to recognize the mutual dependence of firms does not take into account that the two lower-cost firms will raise their prices in an attempt to increase profits without losing sales, and that the fourth firm will be motivated to begin production again and set a price slightly below P' . Since ceteris paribus does not hold for any of the firms, their expectations will not be fulfilled, and this will precipitate another round of price cutting. Prices thus fluctuate cyclically between P_0 and P' .

2. If the combined capacities of the two lowest-cost firms were almost sufficient to satisfy the market at a price equal to the third firm's cost floor, then the third firm would have very little residual demand at higher prices, and would be prepared to cut prices almost to the level of his minimum AVC before the price floor would be reached.

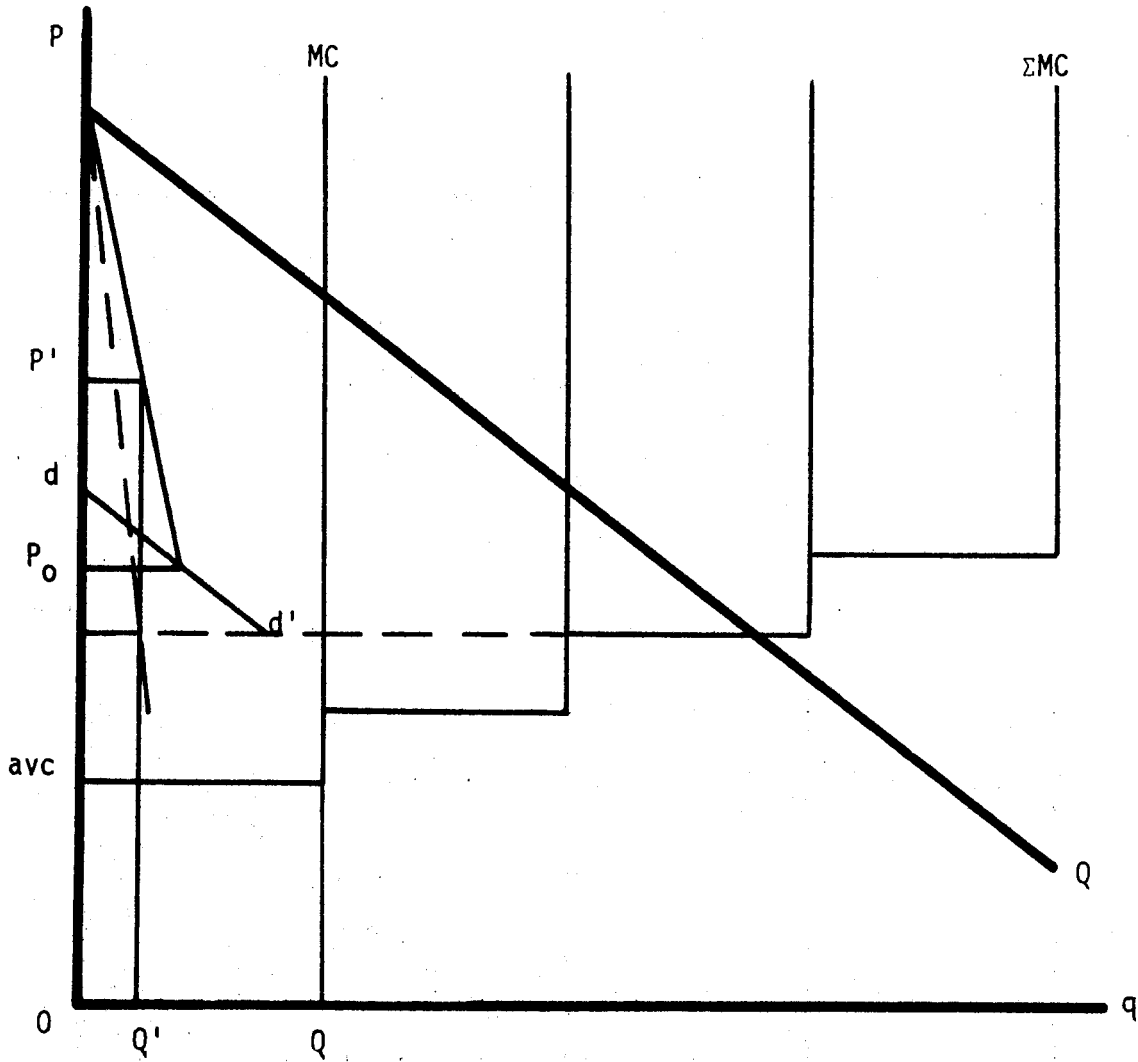


Fig. 4-II Price Instability in the Identical Products Case of Oligopoly with Differing Costs.

Curvilinear cost functions do not change the above conclusions. In Fig. 4-III is depicted the curvilinear case which is analogous to the rectilinear case of Fig. 4-I. Note that P_e will be the price floor, since each firm will sacrifice profit by setting any lower price. From this price any one of the firms will expect to gain by raising price against the residual demand, along dd' to price P_m . Prices will then be cut down to the floor price P_e , and so on. The other cases could similarly be shown with curvilinear cost structures; however this becomes quite complex graphically. Instead, we move now to an examination of the ramifications of both product differentiation and oligopoly in an Edgeworth model of firm behaviour.

III. The Differentiated Products Case

(i) Demand Functions

Continuous Product Differentiation

Under the Edgeworth assumptions one firm will adjust price on the presumption that all others will hold their own prices constant. Let us call the firm which changes price the "initiating" firm, and the remaining $n - 1$ firms the "passive" firms. The initiating firm will gain or lose sales along a negatively sloped demand curve, the slope of which will depend upon the price elasticity of demand for his particular product. The loss of sales to each of the passive firms, will be some share of the aggregate loss of sales to the passive firms, and will depend upon the cross elasticity of demand between the initiating firm's product and that particular passive firm's product. We will need to establish two demand

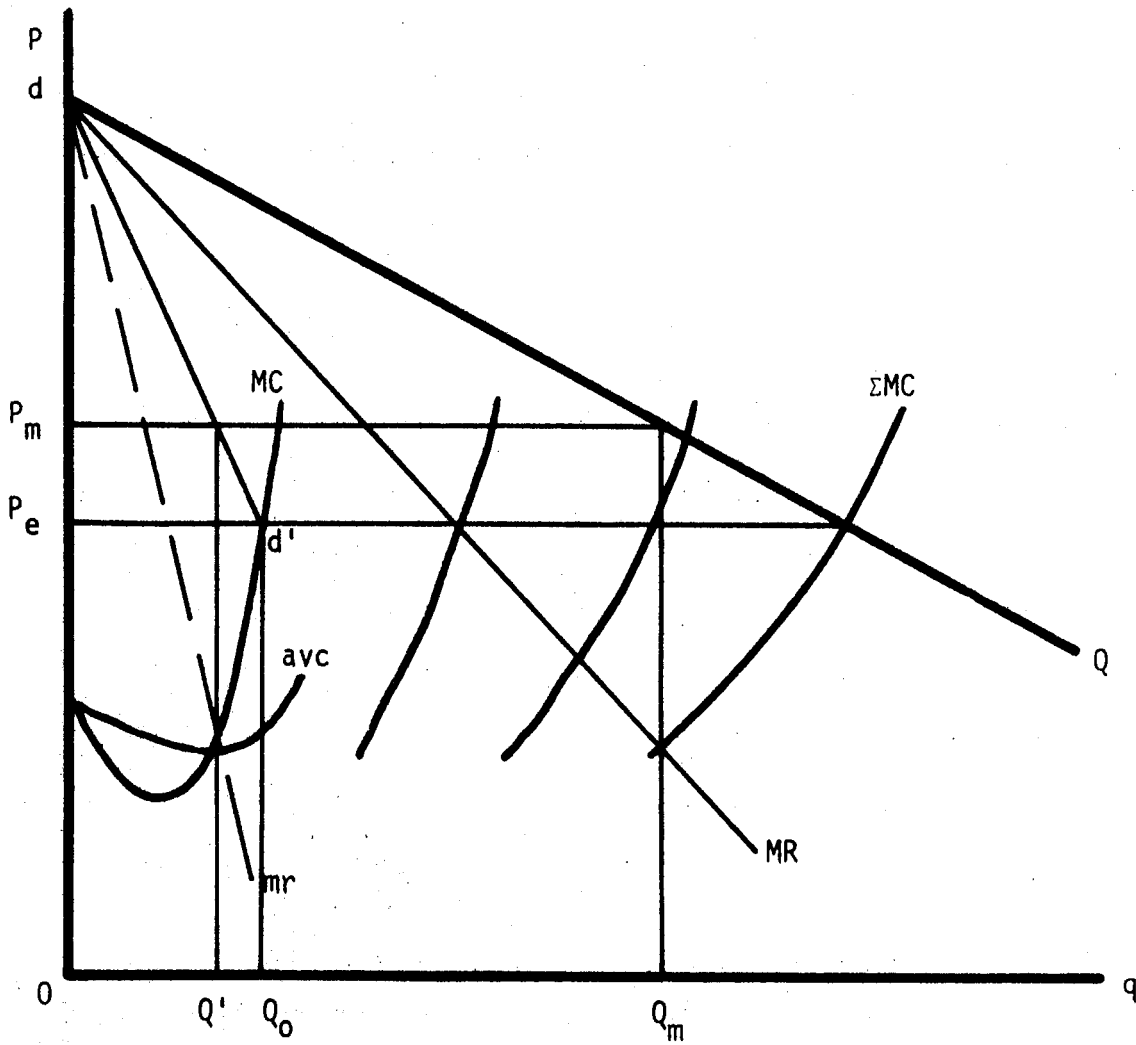


Fig. 4-III Price Instability in the Identical Products Case of Oligopoly with Curvilinear Costs.

functions; one that applies to the firm when it is the initiating firm, and one that is appropriate when it is the passive recipient of another firm's actions.

A demand function for any firm must not simply explain how the firm is expected to gain or lose sales when it, or another firm, changes price, but must explain in addition why the firm sells a particular quantity at a particular price. We therefore require a starting point. As in the preceding chapter we will assume that at equal prices each firm will have a share, m , of the total market demand when all prices are equal, which is presumed to be invariant for all levels of equal prices, or at least, for all prices within the relevant range or neighbourhood of the prevailing price. For symmetric product differentiation these shares are equal, and hence $m = 1/n$, but for the asymmetric and the general case we will specify, by the subscript, the share of the i^{th} firm as m_i , where $i = 1, 2, 3, \dots, n$. Thus, if all firms were to set the same price as the i^{th} firm, his demand would be

$$D_i = m_i Q_i \quad \dots \dots (4.6)$$

which is a generalization of equation (4.3). Since all prices are equal, $P_i = P_x$, that is, the i^{th} firm's price equals the mean price. In the symmetric case of product differentiation we can say that if the i^{th} firm sets any mean price (i.e. not necessarily an equal-prices mean price) his share will be m_i of the total demand at the mean price. That is:

$$D_i = m_i Q_x \quad \dots \dots (4.7)$$

If the i^{th} firm's price is above the mean, its demand will be less than this amount, and if its price is below the mean price, its demand will be greater. Thus, if dq_i/dp_i is the reciprocal of the slope of the i^{th} firm's ceteris paribus demand curve, then

$$D_i = m_i Q_x + (P_i - P_x) \frac{dq_i}{dp_i} \dots (4.8)$$

describes the quantity demanded of the i^{th} firm at the initial position. Suppose this firm now initiates a price change; call the new price P'_i . The firm will move along the ceteris paribus demand curve, and the new quantity demanded is given by

$$D_i = m_i Q_x + (P'_i - P_x) \frac{dq_i}{dp_i} \dots (4.9)$$

and is subject to $0 \leq D_i \leq Q_i$. Note that the mean price, P_x , in equation (4.9), is the initial mean price; that is, it includes P_i rather than P'_i . In the symmetric product differentiation case P_x will be the simple arithmetic mean of all prices. For the asymmetric case this mean price will need to be weighted, since it matters which firms have which prices above and below the arithmetic mean. The specification of the actual weights appears to be a quite complex issue, and is not attempted here.³

3. In the asymmetric case, the aggregate demand will be greater, for any particular structure of prices, if the firms with the larger dq_i/dp_i terms have the relatively lower prices, than if these firms have the relatively higher prices. The weights, which must be attached to each firm's price, would appear to vary positively with the dq_i/dp_i terms and the price of each firm, but the specific form of this relationship is not explored here.

The initiating firm's gain of sales will arise from two sources. In some part it is at the expense of the passive firms, and in the remaining part it is due to the 'entry' of new customers into the market due to price being lowered sufficiently to allow them non-negative consumer surplus at the initiating firm's price. Before we can specify the demand for each passive firm we must separate these two elements and subtract the latter from the total in order to find the aggregate loss of sales of the passive firms.

Even if the i^{th} firm does not become the lowest-priced firm by virtue of its price cut, new buyers will have entered the market due to the price cut. To see this, note that product differentiation implies that at a price of say \$10 for product A, some consumers will prefer to pay \$11 for product B, and \$12 for product C. This implies that the ratio of marginal utility to price is greatest for C, next-highest for B, and lowest for A. Slightly higher prices of B and C could produce an indifference situation, e.g.

$$\frac{MU_a}{\$10.00} = \frac{MU_b}{\$11.01} = \frac{MU_c}{\$12.05}$$

Suppose that these are the prevailing prices, and that some consumers are indifferent amongst the products, and thus choose with the aid of a chance mechanism. If the price of B were now reduced to \$11.00, some of the buyers of A and C will switch to B. In addition, some other individuals, who were not previously buying from any firm, or who were considering

buying an extra unit, now decide to purchase B. An example of an individual who would now enter the market is one whose preferences for the marginal unit are as follows:

$$\frac{MU_a}{\$9.00} = \frac{MU_b}{\$11.00} = \frac{MU_c}{\$10.00}$$

That is, the price of A would need to be less than \$9.00, and the price of C less than \$10.00, before this individual would prefer to buy one of these instead of B as his marginal unit.

The extent to which new customers enter the market when the i^{th} firm cuts price is presumably related to m_i , which is in effect an index of the firm's "popularity" or of the market consciousness regarding that firm's product. Let us make the rather plausible assumption then, that the i^{th} firm's price cut will cause m_i times the total change in market demand, (which would occur if all firms cut price to the same extent), to actually enter the market. If $m_i = 1/2$, for example, the i^{th} firm will gain half of the increase in market demand which would occur if all firms cut price to the same extent. Market demand was specified in the preceding chapter as

$$Q = M (P - P_0) \dots (3.12)$$

where P_0 is the price axis intercept, M is the reciprocal of the slope of the curve, and P is the relevant price level. We need to quantify Q

at the two prices of the i^{th} firm, and subtract one from the other.

That is,

$$\begin{aligned} Q'_i - Q_i &= \left[M (P'_i - P_o) \right] - \left[M (P_i - P_o) \right] \\ &= MP'_i - MP_o - MP_i + MP_o \\ &= M (P'_i - P_i) \end{aligned}$$

Thus the i^{th} firm's gain of "new" customers is

$$m_i \left[M (P'_i - P_i) \right]$$

and the aggregate loss of sales to the passive firms must be

$$(P'_i - P_i) \frac{dq_i}{dp_i} - m_i \left[M (P'_i - P_i) \right]$$

which simplifies to

$$(P'_i - P_i) \left(\frac{dq_i}{dp_i} - m_i M \right)$$

There is no particular reason to expect that the impact of this aggregate loss on each of the passive firms will always be absolutely equal, but it does seem a fair assumption to expect the loss of each particular passive firm to be proportional to its previous share of the

market. We therefore postulate that the share of the lost sales will be $(m_j/1 - m_i)$ for the j^{th} (passive) firm, where $j \neq i$, since i refers to the initiating firm. Thus the passive firms share in the loss of sales in the same proportion as their sales previous to the price cut of the initiating firm. For example, if four firms shared the market $1/2$, $1/4$, $1/8$ and $1/8$, and the largest firm cuts the price, then the next firm would sustain twice the loss of each of the third and fourth firms; that is, half of the total sales loss of the passive firms.

Given the price adjustment of the i^{th} firm, the quantity demanded for each of the passive firms will be

$$D_j = m_j Q_x + (P_j - P_x) \frac{dq_j}{dp_j} + \left(\frac{m_j}{1-m_i} \right) \left[(P_i' - P_i) \left(\frac{dq_i}{dp_i} - m_i M \right) \right] \dots \dots (4.10)$$

subject to $0 \leq D_j \leq Q_j$

That is, the individual passive firm will move away from his equal-prices share to the extent that his own price departs the mean, and to the extent that the initiating firm changes price, given their relative shares of the market at equal prices. The mean price, P_x is, again, the initial mean price. After the i^{th} firm's price adjustment there will be a new mean price, and any one of the n firms may contemplate a movement along a curve given by (4.9) and located by the prevailing mean price.

Discontinuous Product Differentiation

It will be recalled from the preceding chapter that product differentiation is said to be discontinuous if there are some buyers who will never buy a particular product, due to prejudice, or to the fact that the conception of the range of feasible substitutes differs amongst buyers. When there are only two firms we may simply say that those buyers who will never buy one product must buy the other, if they purchase at all. With $n > 2$ products however, there may be some buyers who will only ever consider one particular product, others who will only consider a subset of two, and so on, up to some buyers who may regard all n products as substitutes.

To the extent that some buyers will only ever purchase the i^{th} product, the demand for the i^{th} product will not fall to zero until the last of these buyers has his consumer surplus exhausted by the rising price level. These buyers, then, constitute a private market for the i^{th} firm, which we shall assume depends linearly upon the i^{th} firm's price, as we did in equation (3.3) of the preceding chapter. Hence

$$q_i' = (P_i' - H_i) / G_i \quad (4.11)$$

where H_i is the intercept, and G_i is the slope, of this private, or "loyal" demand curve. The amount q_i' is thus the minimum sales for any given price level.

We need to subtract from the market those buyers who similarly have

one-product loyalty to one of the other $n - 1$ products since these are buyers that the i^{th} firm will never sell to. That is, there will be

$$\sum_{j=1}^{n-1} \left[(P_j - H_j) / G_j \right] = \sum_{j=1}^{n-1} q'_j$$

buyers who will never be interested in the i^{th} product.

Those buyers who are interested in the i^{th} product and one or more of the j^{th} products, and who will decide which to actually purchase on the basis of relative prices and preferences, are already included in the demand function, by the slope term dq_i/dp_i , which describes the rate at which the particular firm loses or gains sales when it raises or lowers price. The slope of the i^{th} firm's demand curve will be flatter around the prevailing price, the greater is the number of consumers who consider the i^{th} product to be among their feasible subset of the n products. That is, the greater the number of substitutes that the buyers in aggregate consider in relation to the i^{th} product, the more elastic will be the i^{th} firm's demand for price changes in the vicinity of the prevailing price, ceteris paribus.

The demand curve for the initiating firm may thus be written as

$$D_i = q'_i + m_i \left[Q_x - \sum_{j=1}^{n-1} q'_j \right] + (P'_i - P_x) \frac{dq_i}{dp_i} \dots (4.12)$$

and the quantity demanded for each of the passive firms is:

$$\begin{aligned}
 D_j = & q'_j + m_j \left(Q_x - \sum_{t=1}^{n-1} q'_t \right) + (P_j - P_x) \frac{dq_j}{dp_j} \\
 & + \frac{m_j}{(1 - m_i)} \left[(P'_i - P_i) \left(\frac{dq_i}{dp_i} - m_i M \right) \right]
 \end{aligned}$$

. . . . (4.13)

where $t = 1, 2, 3, \dots, n - 1$ and $k \neq j$.

The Full Capacity Feedback Effect

If the price structure is such that one or more firms are unable to fully satisfy the demand coming their way, some part of this excess demand will flow to the other firms which are not yet constrained by their capacity limit. It will be recalled from the preceding chapter that one problem in specifying the feedback effect is to calculate how many of these excess demanders will drop out of the market rather than buy a substitute product. A second problem arises in the oligopoly context: how do we determine the way in which the feedback effect distributes itself amongst the recipient firms?

Those consumers that choose to drop out of the market after they are unable to obtain their first-choice product, presumably do so because their second-choice product, at its given price, offers them less utility for the incremental dollar's expenditure than does some other (outside) commodity. Those that stay in the market and purchase their second-choice (or next-preferred, available) product, must do so because this substitute

offers a higher ratio of marginal utility to price as compared to all 'outside' goods and services. This total full capacity feedback effect will be divided amongst the firms which are still able to meet the demand.

As the initiating firm raises its price, it will move back along the curve given by equation (4.12) until any one of the passive firms reaches full capacity sales. For further price increases the initiating firm will lose sales at a lesser rate, until a second firm attains full capacity, whereupon its sales reduction for successively higher prices is at an even lesser rate. The i^{th} firm's demand curve will thus kink at every price level which causes another passive firm to achieve full capacity sales. The quantity demanded for each of the passive firms will increase as the initiating firm raises its price. This increase will come from two sources. The first is as specified by equation (4.13). The second source of additional demand will be the feedback effect from other passive firms which have reached their full capacity limits.

The accurate specification of the feedback effect which accrues to each firm in the oligopoly model proves to be considerably more intractable than it was in the duopoly case. The major problem arises in attempting to determine how the feedback effect splits amongst the firms which are able to satisfy it. The answer to this question would involve specification of the taste and preference patterns for each consumer at all possible sets of prices. Not only must this preference information refer to the n products under examination here, but all other goods and services as well. Rather than pursue such an exercise, and in view of the limited practical usefulness of the result even if it were achieved,

a different approach has been taken here. We shall incorporate the feedback effect into the demand functions, but it will remain unspecified at this point. In the Appendix B to this chapter a specification is attempted using assumptions and proxy variables in lieu of the more complex relationships which exist. While not theoretically accurate, it may be a sufficiently close approximation to be of use for empirical testing purposes.

Once again we shall signify the feedback effect by the letter E, with a subscript to denote the particular firm. Hence the demand functions for the firm, first when it takes the role of initiating firm, and then when it is a passive firm while another firm adjusts price, become

$$D_i = q'_i + m_i \left(Q_x - \sum_{j=1}^{n-1} q'_j \right) + (P'_i - P_x) \frac{dq_i}{dp_i} + E_i$$

and (4.14)

$$D_j = q'_j + m_j \left(Q_x - \sum_{t=1}^{n-1} q'_t \right) + (P_j - P_x) \frac{dq_j}{dp_j}$$

$$+ \frac{m_j}{1 - m_i} \left[(P'_i - P_i) \left(\frac{dq_i}{dp_i} - m_i M \right) \right] + E_j$$

. . . . (4.15)

respectively. Given these demand functions, we turn now to an examination of the structure of prices that will evolve in particular market situations.

(ii) Price Levels and Differentials

At any point in time there will exist a set of n prices. This will not be an equilibrium set of prices if any one firm can expect to augment its profit by adjusting price to some other level. If this occurs, then the quantity demanded for every other firm will change, and those firms will wish subsequently to adjust their prices. Given the assumptions of the model, price stability will exist only if every firm is satisfied with the price structure: that is, if every firm's prevailing price is the one that maximizes the profits of each particular firm, given the prices of all other firms.

Since the graphical analysis for the differentiated-products-oligopoly case becomes quite difficult to follow, we shall proceed verbally and by analogy with the differentiated-products-duopoly case of the preceding chapter. Suppose all firms are initially setting the cartel price.⁴ Each firm's sales will be determined by the product differentiation parameters, m_i , and the extent of each firm's "loyal" market q_i' . Since the firms do not recognize their mutual dependence we expect each firm to envisage a rather elastic ceteris paribus demand curve, given by equation (4.14). Each firm will expect to enhance its profits by reducing price, because marginal revenue and marginal cost must intersect to the right of the present output level, unless the firm is already at full capacity. Thus each firm will independently lower its price. Finding the situation

4. Alternatively, we might postulate simply that we begin observing the firms at any set of prices, but that would prolong the exposition considerably.

to have changed relative to its expectations, each firm will then re-adjust price in pursuit of profit maximization.

Thus prices are expected to gravitate downwards from the cartel level. By analogy with the diagrammatic presentation of the preceding chapter, we expect the magnitude of the price adjustments to become progressively smaller as the firms continue to cut prices. This follows from the fact that as other firms cut prices, any particular firm's demand curve shifts to the left. The associated marginal revenue curve must cut the non-negatively sloped marginal cost curve at progressively smaller output levels. Thus the firm will make progressively smaller price adjustments in order to move from the coordinates determined by equation (4.15), where it has been placed by another firm's initiation of a price cut, to the profit maximizing price which lies on its ceteris paribus demand curve, given by equation (4.14).

Although gravitating downwards by progressively smaller adjustments, prices may not eventually stabilize. This depends upon the magnitude of the feedback effect which a firm can expect to obtain by independently raising its price. But note that the feedback effect begins only when one or more firms attain full capacity output. We have argued above, and we saw graphically in Fig. 3-II, that as price levels degenerate, the firms' outputs will become smaller, and hence the firms are likely to be at less than full capacity. Thus in raising price an individual firm may not receive any feedback effect until its price has been elevated a substantial amount. Even then, the feedback effect which is forthcoming must be distributed amongst all other firms which are not yet at full

capacity.

Thus the amount of the feedback effect which accrues to the initiating firm may be quite small, in absolute terms, at higher prices. Whether it is small enough to prevent price being raised, however, depends upon the relative profitability of raising price to maximize profits against the feedback demand, as compared with holding price constant or lowering it further. This, in turn, would depend, in part, upon the relative capacities of the firms. If plant sizes differ, the firms with smaller plants will be the ones that find it more profitable to raise price from any particular level, if any firms do, since a slightly lower price will be less remunerative when it applies to a smaller output than to a larger output. It would also depend on the firm's level of minimum average variable costs: the lower are these the further a firm will cut price rather than raise it, ceteris paribus. Next, it depends upon how the feedback demand is distributed amongst the recipient firms. If, as suggested in Appendix B to this chapter, this distribution depends in some way on the value of the product differentiation parameter m for each firm, then the firms with the larger m 's are more likely, ceteris paribus, to raise price against the feedback demand. Lastly, and as we saw in chapter three, the demand for any firm at higher prices is composed of three elements, which we have identified as loyal, continuous, and feedback. The greater are the first two of these elements, the more likely it is, ceteris paribus, that any given amount of feedback demand will cause it to be more profitable to raise price. Accordingly, we could construct a situation in which the feedback effect will destabilize the price level, due to a sufficient combination of the above factors.

When prices do stabilize, the levels at which they settle will depend upon the cost and demand situation. When cost structures are identical, and product differentiation is symmetric, the oligopolists will each set the same price. This follows by analogy with the duopoly case, as we saw in Fig. 3-II. Similarly, firms with the advantage of any asymmetry of product differentiation will set relatively higher prices and enjoy larger sales, ceteris paribus. Lower cost firms will set slightly higher prices and sell larger outputs, ceteris paribus. These latter two statements are based on the computer simulation results of chapter three. The oligopoly result can be shown to be similar. The most essential difference in the adjustment process is simply that when one firm initiates a price adjustment the remaining firms share in the consequences, as determined by equation (4.15), whereas in the duopoly case there was only one other firm which suffered the consequences. We have postulated that the price adjustment process will cease when the impact of any firm's price adjustment, upon each other firm's output level is considered negligible. Given that the passive firms in oligopoly share the impact we might expect the adjustment from the cartel to the equilibrium prices to be achieved more quickly than in the duopoly case.

(iii) Chamberlin's 'Large Group' Model

The Edgeworth model of oligopoly, with differentiated products is, in effect, very similar to E.H. Chamberlin's model of monopolistic competition. (See [25, ch.5], [86, pp.405-408] and [47, ch.10]). In the latter model all firms expect to move along a relatively elastic demand

curve for independent price adjustments, but since all firms adjust simultaneously, they move instead along a relatively inelastic demand curve and merely maintain their market share. In the Chamberlin model, the assumption is made that each firm is so small relative to the market that it expects its actions to go unnoticed. In effect, this is the same as the Edgeworth assumption that the initiating firm expects the prices of all other firms to remain constant when it adjusts its price. In the Chamberlin model there is no mutual dependence of actions, and in the Edgeworth model this mutual dependence is not recognized. Hence the firms act in the same manner, since their objectives are the same in the two models.⁵

Chamberlin dwelt upon the "symmetric" case of the large group model, by which he meant "that both demand and cost curves for the 'products' are uniform throughout the group", ([25, p.82]), and it is this case which is commonly represented in the textbooks. Equilibrium in this case is characterized by the firms selling equal quantities at identical prices, as in the parallel Edgeworth oligopoly case. Chamberlin dealt briefly and somewhat intuitively with the "diversity of conditions surrounding each producer". [25, pp.110-113]. Cost differences and product differentiation advantages will, he said, allow prices and output levels to vary amongst firms. For the most part he is concerned with adjustment to long run equilibrium, but where he considers insurmountable barriers to the entry of similar products, his analysis is similar to that of short run Edgeworth oligopoly. "Peculiarities of any individual esta-

5. This was first noted by Triffin [154, p.34].

blishment which cannot be duplicated lead to ... higher prices (and) ... larger sales". [25, p.112]. His analysis of this more general case is not very rigorous however, as he relies largely upon empirical examples to support his conclusion.

The present analysis is able to confirm Chamberlin's conclusion regarding the development of price and sales differentials in the general case. Moreover, the Edgeworth oligopoly model, by analogy, adds considerable rigour to the Chamberlin analysis. Since the behavioural assumptions of the two models are essentially similar, we could insert the cost and demand parameters for any "large group" case into equations (4.14) and (4.15) to determine the exact nature of the equilibrium structure of prices and price differentials. It has been shown here that the equilibrium price structure will arise as a result of competitive price adjustments by the firms, and its final form will depend upon the relative cost and demand advantages of the constituent firms. Although implied, this was not explicitly shown, to my knowledge, by Professor Chamberlin.

IV. Conclusions

When the Edgeworth duopoly model is generalized to include more than two firms, the results are basically unchanged. With identical products, prices tend to be unstable, except in the extreme circumstances of firm sizes either being very small or very large in relation to the market. Even then, costs must be identical to preserve price stability. With differentiated products, prices tend to be stable, except in certain cases

where the full capacity feedback effect would cause destabilization. Price differentials will exist in the general case where cost and product differentiation parameters differ, just as they did in the preceding chapter.

A major contribution of this chapter is in the specification of the demand functions faced by the oligopolistic firm. In both the identical and differentiated products cases, the demand for a particular firm is a function, not only of its own price, but also of the prices of other firms and the capacity levels of other firms, this latter element giving rise to the residual demand and feedback effect. The notions of asymmetry and discontinuity of product differentiation were introduced into the Edgeworth oligopoly model for the first time. This allowed a substantial contribution to the theory concerning price and market share differentials. The underlying determinants of the magnitude of the feedback effect were examined at some length, and in an appendix to this chapter a specification is attempted. The analysis of price and market share differentials of the asymmetric product differentiation case adds, by analogy, to our appreciation of the more general case of Chamberlin's "large group" model, which uses equivalent behavioural assumptions.

Following the appendices to this chapter we begin to explore the effects of allowing the firms to take into account the ramifications of their short run profit maximization strategies. To this point, we have postulated that the firms fail to recognize that their actions are mutually dependent, or that present actions could damage future sales levels. This remains the major shortcoming of the model. In chapter five, the firm

foresees that its actions will bring forth retaliatory adjustments from rivals, and that it may employ other strategies in an effort to expand sales. In chapter six we extend this recognition of mutual dependence to the firms' taking cognizance that the entry of new firms may be attracted by excessive short run price levels.

Appendix A to Chapter 4

Graphical Representation of the Firm's Demand Curve in
the Identical Products Case of Edgeworth Oligopoly

For simplicity, suppose there are only five firms with equal cost structures. Four of these firms have established prices, and the fifth firm wishes to know the demand for its product at various price levels. Call the firms A, B, C, D, and E, and presume that the prices of the first four are P_a , P_b , P_c , and P_d , as shown in Fig. 4A-I.

When E's price is above those of all other firms, its quantity demanded will be the residual demand at the price chosen. We determine the magnitude of the residual demand as follows. Firm A, at the lowest price, sells full capacity output, and removes the ratio q_a^*/Q_a from the market at all higher prices. Firm B has $(1 - q_a^*/Q_a) Q_b$ as its market at price P_b . He, too, is able to sell full capacity and thus removes q_b^*/Q_b from the market at all higher prices, which is shown as the difference between the lines dA and dB. Firm C similarly sells full capacity and the market at higher prices is $dQ - dC$. At price P_d , Firm D will sell less than full capacity. Hence residual demand will be zero at all higher prices for firm E.

If E sets its price equal to that of D, the demand available at that price is shared equally, and is shown by the point M. Slightly

undercutting price P_d , E will move to point N, and then slide down NO until he reaches price P_c . When both E and C set price P_c the demand for each will be shown as point R. For further price cuts E moves along ST until his price is equal to B's price. Since demand is sufficiently large that half of that demand allows B to stay at full capacity, E stays at point T for price P_b . Further price cuts send him to U, then along UV, then to W, and then along WQ when he is the lowest priced firm. Given the prices, and capacities, of the other firms, the demand curve for firm E is shown as the disjointed line $dP_d - M - NO - R - ST - UV - WQ$.

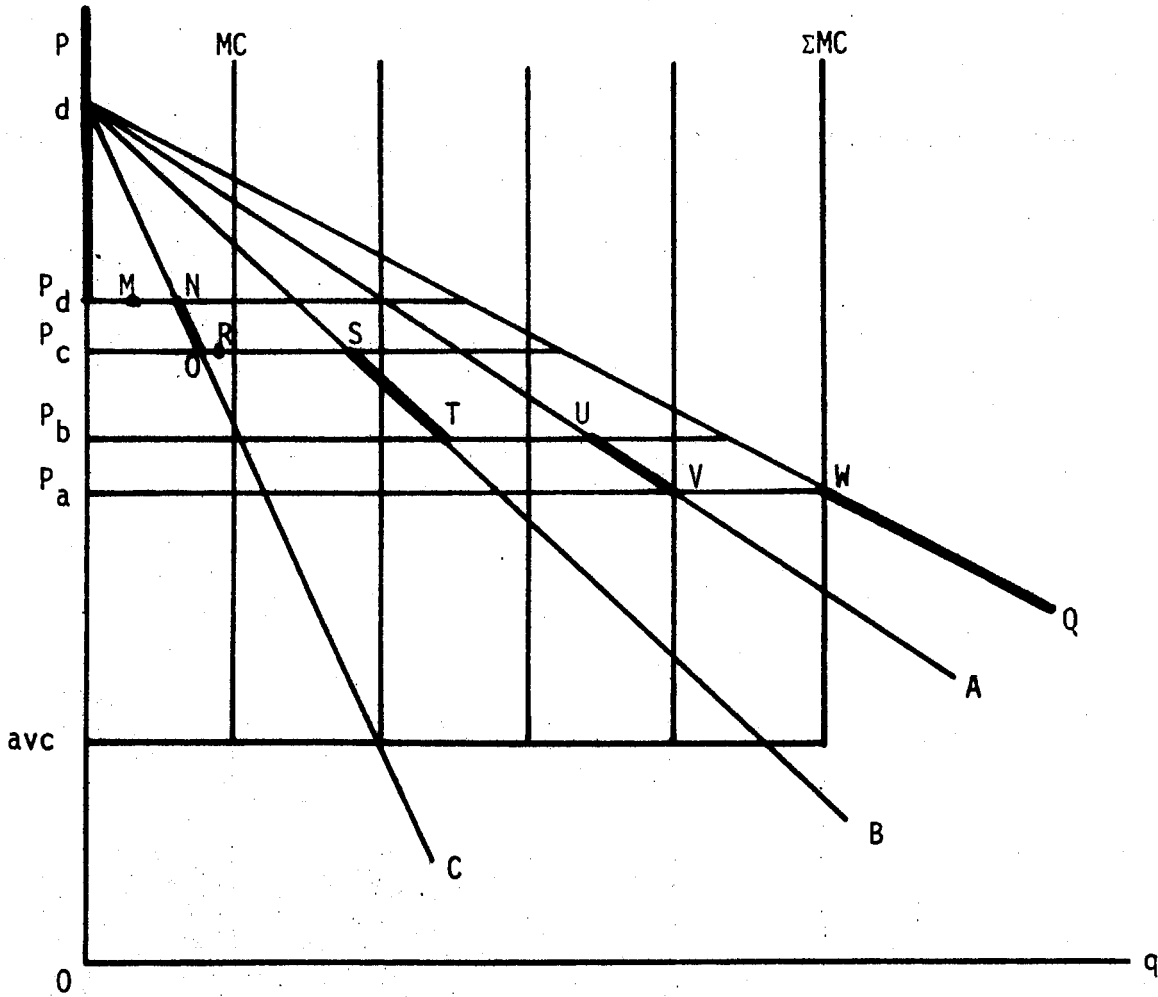


Fig. 4A-I The Demand Function for the Firm in Identical Products Oligopoly.

Appendix B to Chapter 4

An Approximate Specification of the Full Capacity
Feedback Effect for the Oligopoly Case.

The full capacity feedback effect will arise when some firms are unable to satisfy the entire demand for their product at the prevailing structure of prices. The problem is firstly to determine how much of this excess demand remains willing to purchase an alternative product at higher prices, and secondly to determine the distribution of these remaining buyers amongst the firms which are able to accommodate them.

The total amount of excess demand will be the sum of the excess demand for all those firms which attain full capacity. That is

$$\sum_{f=0}^e (D_f - q_f^*) \quad \dots \quad (4B.1)$$

where $f = 0, 1, 2, \dots, e$ represents those firms which are at full capacity sales. If the prices of the firms which are not at full capacity are all equal, we assume these firms will share in the excess demand in the same ratio that they share the market demand when prices are equal. Thus we postulate that the i^{th} firm will be the second choice (or the best available choice) of

$$\left(\frac{m_i}{1 - \sum_{f=0}^e m_f} \right) \left(\sum_{f=0}^e (D_f - q_f^*) \right) \dots \dots (4B.2)$$

buyers, where $i = 1, 2, 3, \dots, n - e$ and $i \neq f$. But not all of these buyers will actually purchase the i^{th} product, notwithstanding that it is their best choice still available, since some of the buyers will prefer to spend their limited income on an entirely different commodity. The extent to which buyers decide not to purchase their best-available choice would seem to depend on the relationship between the price of their first choice (unavailable) product, and the price of the best-available product. In order to avoid the complex specification of the preference pattern of all consumers, we shall assume that if the price of the i^{th} firm is above the mean of the e firms' prices, then we will discount the i^{th} firm's share, and if it is below this mean, we will inflate the share. This discount or inflation factor will be the ratio of the mean price of the full capacity firms to the price of the i^{th} firm. That is,

$$\left(\frac{1}{e} \sum_{f=0}^e P_f \right) / P_i = \bar{P}_e / P_i \dots \dots (4B.3)$$

One further discounting or inflating factor is necessary if the prices of the other (non-full capacity) firms differ. If the i^{th} firm's price is above the mean of these $n - e$ firms, his share of the feedback demand will be discounted, and if it is below it will be inflated, in both cases

by the ratio of the mean price of these $n - e$ firms to the price of the i^{th} firm. That is,

$$\left(\frac{1}{n - e} \sum_{g=1}^{n-e} P_g \right) / P_i = \bar{P}_g / P_i \quad \dots \quad (4B.4)$$

Given the above assumptions the full capacity feedback effect now may be specified as

$$E_i = \frac{m_i}{\left(1 - \sum_{f=0}^e m_e \right)} \left(\sum_{f=0}^e (D_f - q_f^*) \right) (\bar{P}_e / P_i) (\bar{P}_g / P_i) \quad \dots \quad (4B.5)$$

where $i = 1, 2, 3, \dots, n - e$; $f = 0, 1, 2, \dots, e$; and $i \neq f$. Similarly for any of the passive firms

$$E_j = \frac{m_j}{\left(1 - \sum_{f=0}^e m_e \right)} \left(\sum_{f=0}^e (D_f - q_f^*) \right) (\bar{P}_e / P_j) (\bar{P}_g / P_j) \quad \dots \quad (4B.6)$$

The discounting or inflating factors used in this specification were assumed to be a simple relationship between the price of the firm under consideration and the mean price of certain other firms. Alternatively, these modifying factors might be expressed as some function of the relationship between the i^{th} (or j^{th}) firm's price and the relevant mean prices.

CHAPTER 5.

THE RECOGNITION OF MUTUAL DEPENDENCE

I. Introduction

Perhaps the most unsatisfactory aspect of the Edgeworth model is the assumption that individual firms will adjust prices along a ceteris paribus demand curve. More specifically the firm which initiates a price change, does so on the expectation that the price(s) charged by its rival(s) will remain constant.¹ In conjunction with this myopia, the firms never learn from experience that the increased profitability of a price adjustment is quite short lived, since the price adjustment of any one firm necessitates retaliatory price adjustments by other firms which are similarly acting in their own self-interest. In short, the firms fail to recognize their mutual dependence.

Without recognition of mutual dependence (R.M.D.) the continuing price adjustments, that were a feature of the identical products situations in most cases, and of the differentiated products situations in some cases, may be expected to have exercised a deleterious effect upon the sum of profits of any particular firm over a period of time, and for

1. This is certainly the most criticised aspect. See, for example, [47, p.341], [86, p.388] and [127, p.134].

the following reasons. First, unless the product is produced instantaneously and at the time of the transaction, the firm may find itself incurring the cost of excess stocks, spoilage or obsolescence, if its sales fall below expectations. Secondly, there are costs of changing prices. Items must be re-ticketed, and the price change must be communicated to the potential customers. Thirdly, the instantaneous hiring and firing of variable factors, implicit in the frequent adjustments of price and quantity, may neither be possible nor costless. Fourthly, when the firm is faced with such uncertainty as to output levels and hence input requirements, it will presumably be unwilling to enter into longer term contracts and bulk orders of material, for example, and thus will be unable to take advantage of these cost-saving strategies. Similarly economies of scale from larger plant sizes may never be considered in the face of the attendant investment uncertainty. And lastly, frequent price adjustments would be expected to have an adverse impact upon the public image of the firm. To the extent that all firms engage in frequent price adjustments, the market demand parameters may be influenced, and if some firms adjust price more frequently, or by greater magnitudes than others, this would be reflected in the product differentiation parameters.

When firms do recognize their mutual dependence, a wide range of possibilities exist, their form depending upon the understanding or agreement which arises out of the recognition. At the upper extreme is the formal cartel, in which firms set price and output levels to allow joint profit maximization. At the lower extreme is the simple recognition that one's price adjustment will cause the sales and profits of rival firms

to be affected, with no understanding or agreement being made.² Within these limits is the "spontaneous coordination" discussed by Fellner [46, p.16ff] and the "conscious parallelism" involved in price leadership patterns, which is apparently very informal and completely implicit.³ Whenever there is an understanding amongst the firms, effective collusion may be generated, since the effectiveness of a collusive agreement appears to depend in large part upon the personalities of the people of the industry, the specific cost and demand conditions, and other factors.

In the following we wish to examine the impact of R.M.D. upon the models developed thus far. First we shall discuss non-collusive R.M.D., and later collusive R.M.D. Thus a definitional problem arises: at what point does the behaviour of firms become collusive? This is obviously a very complex question. For the purposes of this chapter, however, we shall exclude price leadership patterns and other conscious parallelism from the definition of collusion, since these are generally tacit, implied understandings that arise out of long experience in an industry, and persist despite the intensive observation of anti-trust legislation in the U.S. experience, for example. (See [127, esp. ch.6]). Collusion will be taken to imply the existence of a relatively formal and explicit agreement regarding price and output levels. A further subcategorization for

2. Kahn [80, p.9] has called this the "first degree of collusion". It should, more appropriately, be called the first degree of R.M.D., since collusion is usually understood to involve an agreement of some type.

3. The term "conscious parallelism" was introduced by William Hamburger [62, p.226].

expositional purposes is made between price strategic R.M.D. and non-price strategic R.M.D. In the course of the discussion, several well-known models of oligopoly will be referred to. It is taken for granted that the reader is familiar with these models, since they adorn the pages of most intermediate microeconomic textbooks. Accordingly they are not reproduced or explained in full here.

II. Non-Collusive, Price Strategic, R.M.D.

When oligopolists recognize their mutual dependence, and expect that their rivals will react to any price adjustment they make, the ceteris paribus demand curve will only be appropriate for the special case in which rivals are expected to do nothing. More generally the firms will envisage a mutatis mutandis demand curve: that is, one which will show the quantity demanded at various prices, taking into account the reactions of rivals. The expectation regarding the nature of the reactions of rivals is possibly the greatest single specification problem in oligopoly theory. At one extreme of this "conjectural variation" we might expect the rivals' reactions to be completely malicious, intending to inflict the largest possible damage upon the initiating firm. The other extreme is that rivals' reactions will be cooperative in view of some mutually rewarding strategy such as joint profit maximization. It is more likely, however, especially in the case of non-collusive R.M.D., that the conjecture of firms regarding the rivals' reactions will lie somewhere between these two extremes. Further, the reactions of rivals may differ amongst rivals at any point of time, and for any one rival at different points in time.

(i) The "Minimax" Behavioural Assumption

The "minimax" assumption of game theory is an application of the first extreme. Under this assumption the firm will choose the strategy which affords the best of all worst-possible outcomes, presuming that its rival wishes to inflict the maximum damage. Shubik [132, pp.419-425], [133, pp.86-91], has applied this assumption to the Edgeworth duopoly model. It is instructive to pursue this a little further since this assumption has important implications for the amplitude (and hence the frequency) of the price cycle. It will be recalled that in the earlier chapters we assumed that the buyers lucky enough to purchase at the lower price were chosen randomly with respect to their expected consumers' surplus. Shubik has also examined the polar case where residual demand is minimized by the choosing of customers for the lower-priced commodity in descending order of their expected consumer surplus.

If a firm expects its rival to choose its customers in this manner then its envisaged demand curve for all prices higher than the rival's price will no longer be given by the equation (2.2) which was discussed in chapter two. Instead, the contingent demand curve, equation (2.1) becomes the envisaged demand curve. This equation becomes the upper section of the firm's demand curve because the lower-priced seller is expected to have selectively satisfied all of those customers represented by the uppermost end of the market demand curve up to the point of his full capacity. Residual demand is thus equal to the lower part of the market demand curve, to the right of the last buyer which the lower-priced seller was able to satisfy.

How does this affect the price cycle? Consider Fig. 5-I. Where the price ceiling under the "minimization" assumption is found on the contingent demand curve dd' , where the associated marginal revenue curve, mr' , intersects the marginal cost curve, and is consequently the price P' . Note that this price is lower than P_m , which would be the price ceiling under the "random" assumption. The price floor for the "minimization" assumption is found when the higher-priced firm would prefer to raise price to P' rather than cut it further. From casual observation, it would appear to be quite close to the clearing price P_e in Fig. 5-I. The floor price for the "random" case must be substantially above P_e , since the profit rectangle at that price is obviously smaller than that on the appropriate reaction function (which in this case is coincident with the MR curve) at price P_m .⁴ Thus both the floor and ceiling prices may be lower under the "minimization" assumption, and certainly will not be higher. The amplitude is smaller however, as compared to the "random" case, since the ceiling price differential is greater than the floor price differential.

The reduction in the amplitude of the price cycle arouses the suspicion that the "minimized" residual demand case may be stable at times

4. The price floor for the "random" case cannot be below that for the "minimization" case, since the same or a higher price can be obtained for any given quantity in the former case, as the reaction function [equation (2.2)] emanates from the contingent demand function, (which is the appropriate reaction function when residual demand is minimized). Thus, as prices are being pressed downwards, it will be more profitable to raise the price against the random residual demand before (or no later than) it will be more profitable to raise the price against the minimized residual demand.

when the "random" case would be unstable. Consider Fig. 5-II. Suppose prices have degenerated to P_e . If residual demand has formed randomly, price will then be raised to P_m (along a reaction function coincident with MR) since this is more profitable. This price would be undercut and instability would prevail. But if residual demand had been minimized, the price would be stable at P_e , since no higher or lower price is more profitable. On the basis of the reasoning in chapter two, we can say that the more elastic is the market demand, and/or the larger the plant sizes relative to the market size, the more likely that the "minimization" assumption would lead to stable prices when the "random" assumption would not.

It is a relatively simple matter to apply the minimization of residual demand assumption to the other cases, as we saw in chapter four where the demand function of the identical products oligopolist was modified to fit that assumption. Our findings regarding the smaller amplitude of the price cycle appear, intuitively, to apply to the other cases as well. Whenever there is price instability under the random formation of residual demand (or the feedback effect) this instability would be expected to be of lesser amplitude, or perhaps disappear, if the lower-priced firm chose the "best" customers first. We note that it is somewhat unreasonable to expect a firm to be able to discriminate amongst its customers on the basis of their expected consumer surplus.⁵ Alternatively, perhaps the buyers organize themselves, in order of expected consumer surplus, in order to qualify for the lower price. Both of these interpretations

5. The firm may wish to act like this in order to force the other firm(s) into eventual bankruptcy.

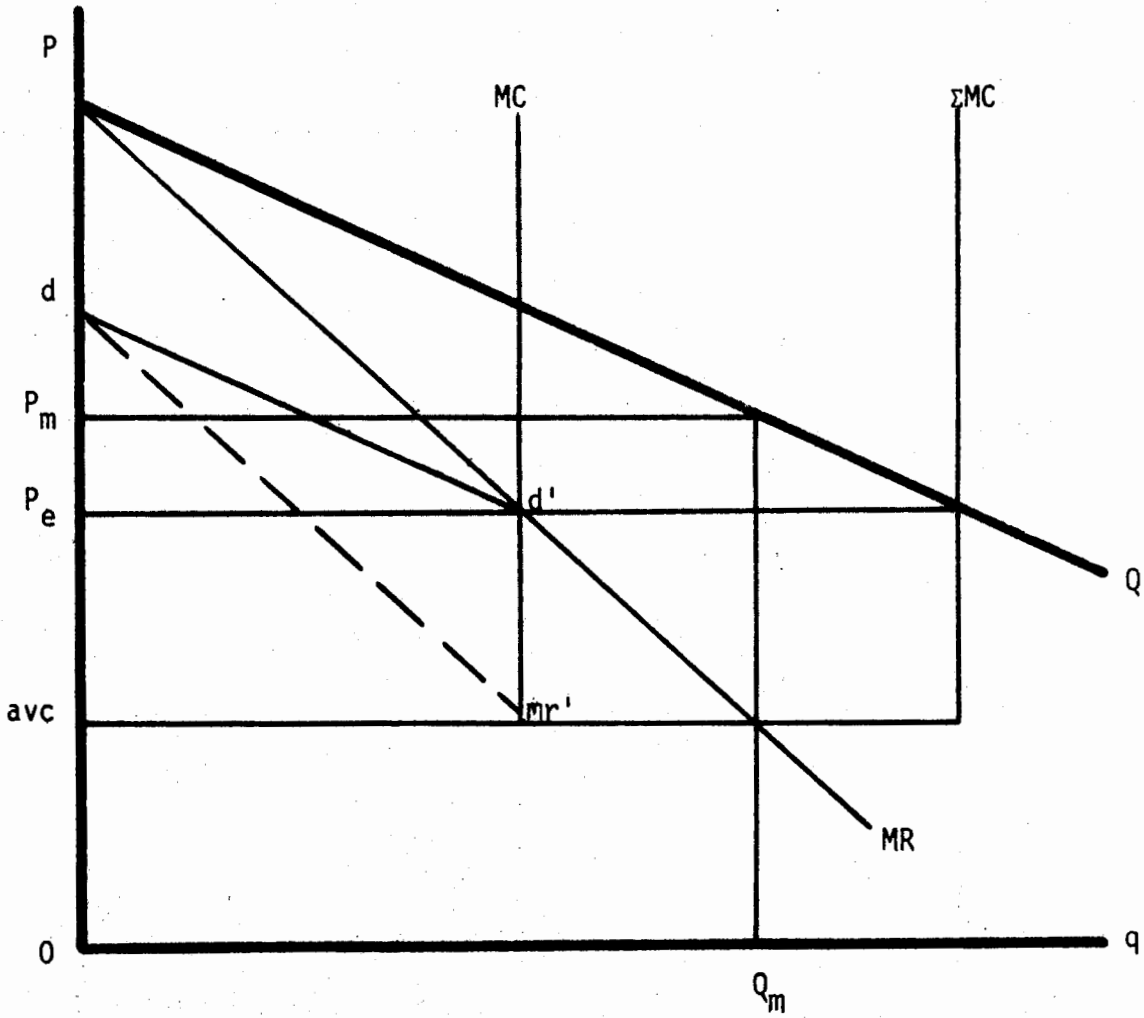


Fig. 5-II Removal of the Price Cycle due to the Minimax Assumption.

involve the well-known problems inherent in attempting to make inter-personal comparisons of utility.

We now turn to some situations more in accord with what we more commonly observe in the actual business world, where firms appear to be more likely to adopt a live-and-let-live policy, rather than to exhibit the aggressiveness which characterizes minimax behaviour.

(ii) Conscious Parallelism and Price Leadership

Given that the cost and other advantages mentioned in the introduction to this chapter may accrue to the firm when its output level is relatively stable, we shall make the initial assumption that all firms wish to maintain their output level. A firm intending to initiate a price change may expect all other firms to adjust their own prices by the same proportion. With this assumption, the mutatis mutandis demand function, for the identical products case of either duopoly or oligopoly, will be the equal-prices demand curve encountered previously, that is

$$D_i = m_i Q_i \quad \dots \quad (5.1)$$

where D_i is the demand for the i^{th} firm; $m_i = 1/n$ in the identical products and symmetric product differentiation case, and is the share of the market at equal prices for the i^{th} firm; $i = 1, 2, 3, \dots, n$; and Q_i is the total market demand at the price level of the i^{th} (and all) firms. This expression is also valid for the symmetric and continuous case of product differentiation. If each firm sets its price along this

demand curve, the outcome of the identical products case, and the symmetric-continuous product differentiation case, of both duopoly and oligopoly (with identical cost structures) will be that all firms set the industry monopoly price. This follows directly from the analysis of chapters two, three, and four, and presumes that buyers are chosen randomly.

When costs differ, and/or product differentiation is not symmetric, the profit maximizing prices along the firms' equal-prices demand curves will differ, as we have seen in the preceding chapters. Looking first at the identical products cases, (with differing costs) it would seem that the higher cost firm(s) will appreciate the futility of attempting to set price above that of the lower-cost firm. We may expect, therefore, that the lower-cost firm will be recognized as the price leader. This model is well known. (See [47, pp.360-361]). Each of the higher-cost firms will expand output to the point where its marginal cost equals the price chosen by the low cost firm, or until its sales equal $1/n^{\text{th}}$ of the total market demand at that price, whichever is the smaller. Not shown in the typical textbook treatment of this price leadership model, is the outcome if marginal costs of one or more of the price-followers equals price at an output level smaller than $1/n^{\text{th}}$ of the total market. This will cause excess demand for the product at that price, and the resultant feedback effect will cause the price-leader to raise price until this excess demand is eliminated.

When cost structures and/or product differentiation parameters differ we saw that prices would usually settle to a stable pattern of price

differentials, when the firms envisage ceteris paribus demand curves.⁶ Given R.M.D. and the assumption that any firm will expect all other firms to adjust price by the same proportion that the initiating firm does, we would expect the whole structure of prices to move upwards with the established price differentials being unchanged, and consequently all firms enjoying their same relative sales position in the industry. Perhaps not all firms would have the courage to initiate such a price adjustment, but would certainly follow a move made by another firm. A "barometric" price leader may emerge, being a firm which commands the respect of all other firms and which has strong consumer acceptance. (See [146, ch.18]). In the event that this firm's estimation of the appropriate price level is not shared by other major firms, some further small adjustments may be necessary to attain a structure of prices which is satisfactory to all firms. (See [127, pp.170-173]).

The barometric price leader will adjust price to the level which it judges as "appropriate". This may not simply mean that the price-leader seeks the price which will maximize all firms' profitabilities on their mutatis mutandis demand curves. Rather it may mean that price is adjusted to a level just below that which would induce any one firm to ignore the consequences of his action and independently cut price. We note that the incentive to make a price cut increases as the price level moves closer to the "cartel" level, since firms are more likely to have excess capacity at higher prices. The incentive to cut prices is re-

6. See particularly pp.53-56 in chapter three and pp.87-92 in chapter four.

inforced if a significant time lag is expected before rivals find out and react to the price cut. The incentive to undercut prices is further strengthened the greater is the probability that the rivals will not react at all. That is, the rivals may tolerate relatively small price cuts by an individual firm rather than precipitate a price war, or involve themselves in costly re-ticketing of articles and informing the public of the price changes. An additional factor which appears to be very important in the determination of the height of the actual price level above the non-cooperative floor level is the length of each firm's time horizon. (See [137, pp.531-532]). Firms with relatively short time horizons will be more likely to desire short term gains at the expense of more distant profits, and hence will be more likely to initiate price cuts. The adverse impact of frequent price adjustments on costs and product differentiation parameters may well be recognized by firms with a shorter time horizon, but other considerations such as impending bankruptcy, old age of the owners, or capriciousness, may prevail. The extent to which firms resist the temptation to cut prices even though such an action would be profitable over a short time horizon, may be referred to by an "index of industry discipline" which would represent the loyalty of the firms to their joint welfare.⁷ This would be very similar in concept to Fellner's "esprit de corps" in which the firm "places above individual advantage (his) good standing in the group, or the interests of the group". [46, p.43].

7. This concept is developed further in the next section of this chapter.

We might expect, therefore, that the actual level of the price structure between the non-cooperative floor and the joint maximization ceiling will vary amongst markets and over time, depending upon the "conjectural variation", capacity utilization ratios, efficiency of information systems, firms' time horizons, and the "index of industry discipline". If all firms have little excess capacity, and/or are certain that rivals will retaliate almost immediately, and/or if industry "discipline" is strong, we would expect prices to be maintained quite near to the joint maximization levels. For other structural, behavioural, information, and "discipline" situations, the firms may learn from experience that a stable level of prices can only be maintained at a level sufficiently near to the non-cooperative floor such that the advantage to be derived from undercutting is marginal.

It would seem possible that the above could be specified algebraically, although it would depend very heavily upon assumptions underlying the time horizons and the "conjectural variation" of firms, as well as upon other more-easily handled assumptions. Since the "horizon" and "conjectural variation" issues are very complex issues in reality, no simple specification would suffice. For an empirical study it would seem possible to experiment with proxy variables, the experience with which may contribute to a useful development of the theory.

(iii) The Kinked Demand Curve Put Straight

The "conjectural variation" assumption made by the price followers

in a situation of price leadership is exactly that made in the kinked demand curve (K.D.C.) model of oligopoly price behaviour. (See [61], [149], and [146, ch.18]). If firms expect all firms to follow a price cut, but that no-one will follow a price rise, which they initiate, then the firm's imagined demand curve is kinked at the prevailing price.

The K.D.C. model is unable to explain the general level or structure of the prevailing prices. Given the analysis of the preceding chapters, however, we are able to contribute to this question. With regard first to the structure of prices there are two major possibilities. Firstly, prices may have been degenerating to the non-cooperative levels, (explained in chapters three and four) when the firms eventually recognized their mutual dependence, and learned the futility of their price cutting. If they then expect that further price cuts will be matched and that price increases will be ignored by rivals, then their imagined curves are kinked at the last price which they set. Alternatively, perhaps the firms had already bid prices down to the non-cooperative floor levels, and upon reflection realized the error of not recognizing their mutual dependence. The emergence at this point of a price-leader (or cartel organization) would allow all prices to be raised by a proportion which the price-leader (or cartel) judges as appropriate.

The actual level which is chosen will depend upon the extent to which the price-leader feels prices may be raised before it becomes too tempting for any firm to undercut the chosen price level for individual gain. Our analysis allows us, upon observation of a kinked demand situation, to state how far prices have been raised above the non-cooperative floor

price, or alternatively, to what extent do prices fall short of the "cartel" price level. To see this, recall that the kink in the demand curve gives rise to a vertical discontinuity in the marginal revenue curve, and that if the firm is profit maximizing, the marginal cost at that output must have a value equal to one of the limits, or between the limits, of the discontinuity in the marginal revenue curve. (See [141], [87]). Now, note that if the marginal cost curve intersects the uppermost point of the discontinuity, then marginal cost is at that point equal to the marginal revenue associated with the ceteris paribus demand curve. We saw in preceding chapters that this is the equilibrium condition for the non-cooperative floor price. Alternatively, if the MC curve intersects the lowermost point of the discontinuity, then the firm's price is at the "cartel" level, since the MR associated with the mutatis mutandis demand curve is here equal to the marginal cost.

Refer to Fig. 5-III, overleaf. If the price for any particular firm was set at P_f , the kink arises at point H where the ceteris paribus demand curve $d_1 d'_1$ crosses the mutatis mutandis demand curve DD' . The latter curve is the "share of the market" demand curve, and may be regarded as being equal to $m_i Q_i$ in the notation of this analysis. The marginal revenue curve is $d_1 A B MR$. Note that the marginal cost curve, MC , passes through point A, and is thus equal to the marginal revenue associated with the $d_1 d'_1$ demand curve at price P_f . The firm has no incentive to either raise or lower its price independently, and thus this is the non-cooperative floor price, (if, similarly, no other firm adjusts price). Now let the firms jointly raise price at the suggestion of a price leader or

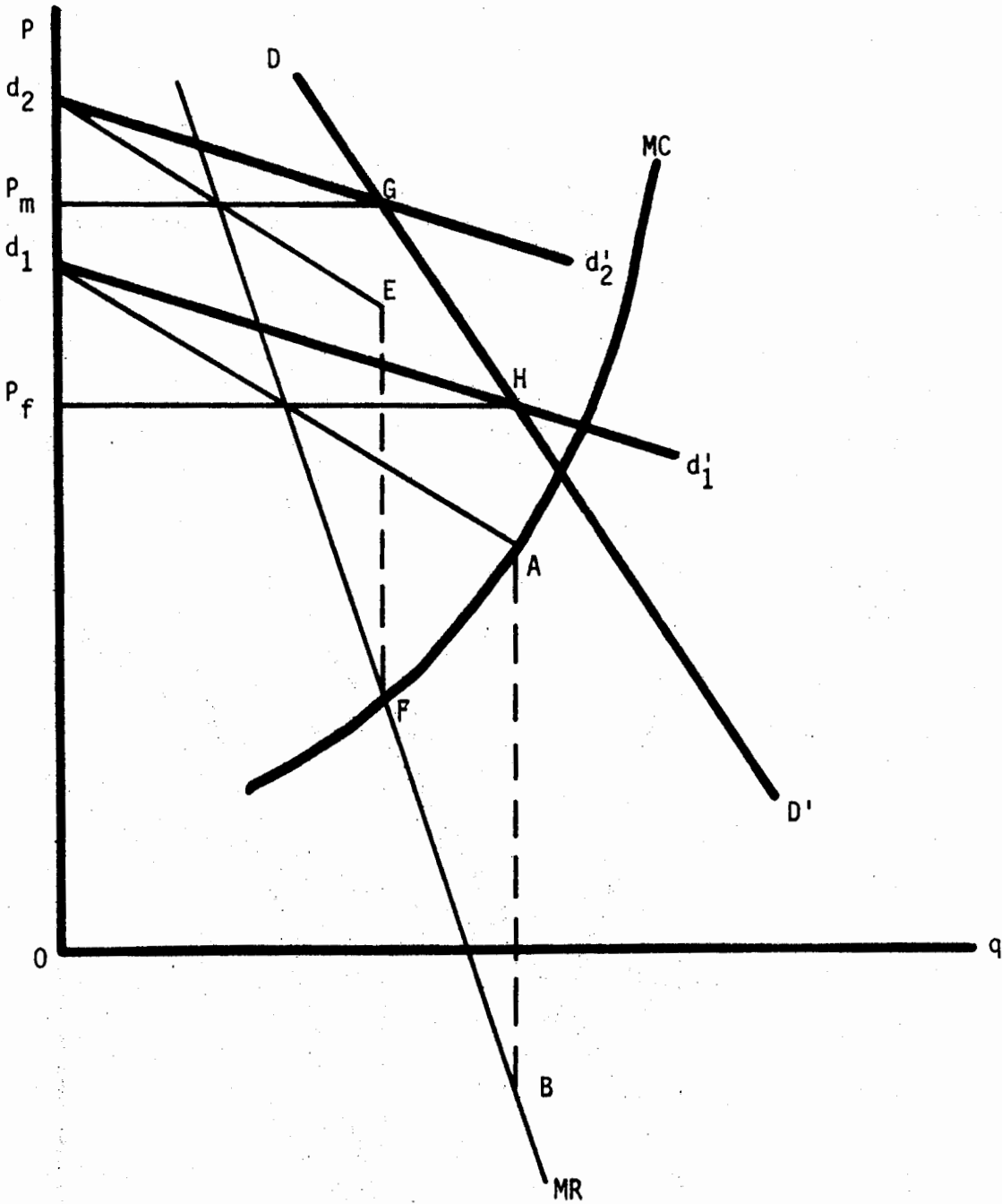


Fig. 5-III Upper and Lower Limits to the Kink in the Kinked Demand Curve.

collusive organization. This particular firm will move along the DD' curve shown. Price will be raised no higher than P_m , for at this price the marginal revenue curve associated with the mutatis mutandis demand curve intersects the marginal cost curve, at point F, and this is the "cartel" price level. Since no firm will follow a further price increase, the demand curve is kinked at point G, and the marginal revenue curve becomes $d_2 E F MR$.

Thus P_f is the floor price and P_m is the "cartel" ceiling price. To the extent that price lies above P_f , the marginal cost curve will pass through the discontinuity at a point further down the discontinuous section. It follows that if we observe a kinked demand curve situation, we could, by moving the kink up and down the mutatis mutandis demand curve, ascertain the upper and lower limits to the price which the firm may set, given the cost and demand parameters for all firms, and the market demand parameters. This, in turn allows us to say that the firm's price is X dollars, or Y percent, above the non-cooperative floor price for that firm, or below the "cartel" price for that firm. This suggests a means to quantify the "index of industry discipline" which was mentioned above. If we set the spread between the floor and cartel prices equal to unity, then the index will be the fraction corresponding to the actual price set by the firm. That is:

$$I = \frac{P_k - P_f}{P_m - P_f} \dots (5.)$$

where I is the value of the index, P_k is the actual price at the observed

kink, P_f is the non-cooperative floor price, and P_m is the "cartel" price. The index attains a maximum value of unity, when the actual price is the "cartel" price, and a minimum value of zero when the actual price is the non-cooperative floor price. Between these extremes the index will provide a linear scale of the loyalty firms feel towards the pricing decision of the price leader(s), amongst other things, as we shall see in chapter six. Note that the index may give different readings for different firms within the same industry, since cost and demand conditions may vary amongst firms. In this case we would need, for consistency, to refer to the price-leader's index, or the lowest-cost-firm's index, for example, in order to conduct inter-industry comparisons of the index.

The above explanation of the structure and level of prices in a K.D.C. situation adds substantially to the theory on this subject. To my knowledge, nobody has previously improved upon the "full-cost" rule-of-thumb explanation of the price level and structure, given by Hall and Hitch in their original paper [61]. We shall see, in an appendix to chapter six, that the "full-cost" explanation is not necessarily inconsistent with the above explanation.

Perhaps the major virtue of the K.D.C. model is that it offers an explanation of price rigidity in the face of cost and demand changes. But the distinction needs to be made between those changes which are specific to the particular firm under examination, and those which are incurred by all or most firms in the industry. We often observe firms raising their prices in response to cost and demand changes which affect all firms in the industry. For firm-specific changes the firm does

imagine a kink at the prevailing prices and will only raise or lower price if the changed cost or demand conditions cause the marginal cost curve to intersect the marginal revenue at a point other than within the vertical discontinuity of the latter. For industry-wide changes, such as a significant increase in the wage level, or a general upsurge in demand, the firm will not envisage a kink at the prevailing price, since it expects that all firms will wish to raise price. The firm might assume that all firms will follow a price rise which is sufficient to restore the firm's profit margin to its previous level. Given the followship of all other firms (or their simultaneous action) the mutatis mutandis section of the demand curve extends upward to the new price level, where it would then kink. This is not made clear in the textbooks generally.⁸

We note that when the price-leader raises price, or when all firms independently raise price, each firm's price is raised by a certain proportion rather than by a certain absolute amount, in order to maintain relative standings in the market. If any firm attempted to raise price more than proportionately, we may envisage this firm as pricing above his kink, and relative market standings will change to the detriment of that firm. If any firm raised price less than proportionately this would presumably induce a competitive response from its rivals, and we may imagine that firm as (temporarily) pricing below his kink. The demand curve for the price-leader would be of constant slope through its new price, since all firms will follow both its price increases and decreases, as long as

8. Ferguson [47, pp.346-347] and Bilas [18, pp.247-248] make no mention of it. Scherer treats the joint reaction to an industry-wide change as a "contradiction" of the K.D.C. model, [127, pp.148-149].

this firm remains the accepted price leader.

III. Collusive, Price Strategic, R.M.D.

We turn now to the collusive variant of R.M.D., which arises when all firms agree to adopt a set of objectives which presumably are expected to enhance their joint welfare. In fact it would seem that explicit collusion is only necessary when a non-collusive situation is unable to maintain the industry "discipline" required to prevent erosion of the price level. One would expect the firms to have an incentive to organize a collusive agreement whenever it is profitable for any one firm to institute a price cut, but where all firms eventually lose, relative to the situation of continuing collusive price levels. Each firm would be expected to support a structure which prevents or inhibits any other from initiating a price war.

Although the firms agree explicitly to collude as to price, it does not follow that they will choose the joint profit maximizing price. To the extent that the cartel is unable to prevent one or more firms from opting out of the agreement, and to the extent that it would be profitable to renege on the collusive agreement, the price level will need to be below the cartel price, in order that the attraction of unilateral action is lessened. Once again the exact location of the general price level will depend on factors such as the extent of un-utilized capacity, the time lag before price-shading is discovered, the probability of retaliatory price-cutting, the shortest time horizon of the member firms, and the loyalty of the firms to their joint welfare. In addition we should

note that since the cartel is a joint organization of the firms, this body may be able to exert additional pressures, such as boycotts to suppliers of the price-cutting firm, which in turn may reduce the expected profits, and hence the incidence, of price cutting. Collusion need not be as overt and explicit as in a cartel, of course. It may be implicit and informal, yet produce similar results, although the less formal and explicit is the agreement the less joint pressure the firms would be expected to be able to bring to bear upon the recalcitrant firm. Consequently we would expect the collusive price level to be lower when collusion is less formal, ceteris paribus.

The existence of a collusive agreement implies a somewhat stronger joint concern for the avoidance of price competition than does the existence of non-collusive (but parallel) action. The ability of the firms to agree upon a joint strategy is evocative of the solid understanding which they have regarding their interdependence of welfare and the advantages that each will derive as a result of price stability and output predictability. The collusive price may be raised to just below that point where a dissenting or selfish voice is expected to be heard. Given the peer-group pressure, the morale-boosting effects of a collusive agreement, and the motivation which is demonstrated by the willingness of firms to enter such an agreement, one would expect this point to be higher, ceteris paribus, for a collusive group than for an equivalent group which exhibits only conscious parallelism. In addition the personalities in the industry, and the length of time that they have co-existed, no doubt exercise some influence on the level to which prices may be

raised.⁹

A collusive agreement to maintain prices at a certain level will probably cause the firms to imagine a kink in their demand curves at the prevailing prices. Price increases would not be followed (unless instigated by the cartel leaders) and price decreases would attract defensive price retaliation, apparently signifying the breakdown of the previous price agreement. Given the kink, we can ascertain the upper and lower limits of the price level, and thus calculate the index of industry discipline represented by the actual price level.

The collusive agreement might be expected to be less satisfactory for some members of the agreement than for others. Some firms may feel severely constrained by the agreement, wanting to expand sales, but will be reluctant to break the agreement by independently adjusting price. An alternate outlet exists for their competitive tendencies, however, in the form of non-price competition, and it is to this that we now turn.

IV. Non-Price Competition

When firms recognize their mutual dependence, and learn that price competition causes output instability (which probably raises costs) and may precipitate expensive price-wars, they may be expected to look elsewhere for a means to expand their profits. Advertising, product improvement, changes in packaging, and improved customer service are some of the

9. Scherer [127, Ch.6] elaborates upon this. The U.S. Steel industry, in earlier years, exemplifies the value of social intercourse amongst rival firm leaders as a means of maintaining industry "discipline. See particularly [127, p.211, p.457].

means by which a firm may increase its sales at its prevailing price level. Non-price competition is a relatively safe means to increase sales. It is not sudden, like price-competition, either in its conception or its impact. The initiator of a non-price strategy may expect a reasonable period of increased sales to elapse before its rivals are able to respond in kind. Non-price competition is also much more subtle than price competition, and not so easy to emulate. An effective advertising campaign, or a beneficial product improvement may gain repetitive sales that rivals can never recoup. Furthermore, non-price strategies appear to be accepted as the "gentlemanly" means of competing, in large part as a more acceptable outlet for the latent aggressiveness of firms, than is price-competition. (See [127, ch.4]).

Marginalist theory suggests that firms will incur costs of additional advertising up to the point where marginal cost of this increased advertising just equals the marginal revenue which is attributable to the increased advertising effort. (See Chamberlin [25, ch.7]). Both the cost and demand curves are expected to move in an upwards direction, as the advertising expenditure mounts. Since there are likely to be diminishing returns to advertising, and increasing cost per unit of output, we might expect an optimal level of advertising to be found by marginalist principles.

This is probably only true for a monopolist, however, or for a firm which is sufficiently small relative to the market that its actions may pass unnoticed by any other firm. For oligopolists the benefit of an advertising campaign depends in large part upon what rivals are simul-

taneously doing. To some extent advertising efforts are mutually cancelling, and this will be especially so if rivals mount simultaneous campaigns. It has been hypothesized that oligopolists face a prisoners' dilemma problem with regard to advertising. (See [127, pp.335-337]). Due to the uncertainty concerning the campaigns their rivals may be planning, they are forced to proceed with elaborate campaigns of their own to avoid considerable loss of profits which would occur if their rivals initiated new campaigns and they themselves did not. By acting independently, and without sufficient communication amongst rivals, each firm expends more than the optimum: that is, the marginal cost of these efforts exceeds the marginal revenue derived.

This unnecessary loss of profits could be avoided, however, by collusion as to non-price variables as well. If firms colluded with respect to their product differentiation expenditures they would reduce the mutually cancelling element and bring about the marginal equality of costs and revenues attributable to these efforts.¹⁰ But the impact of further product differentiation is difficult to predict, and may leave some firms unsatisfied with the agreement. In addition, to close the avenue of non-price competition may well increase the pressure for an individual firm to resort to the potentially dangerous means of price-competition. Thus it is not unexpected that firms in actual business environments appear reluctant to engage in agreements regarding non-price

10. "Product differentiation expenditures" is being used to include advertising, packaging, promotion, services, etc. It is possible, of course, that some product differentiation expenditures have mutually amplifying effects when firms simultaneously indulge in them.

strategy, preferring to rely on the latter method of stimulating sales, even in cases where demand at the established price levels is considerably depressed.¹¹

Where the firm makes its pricing decision on the basis of its imagined ceteris paribus demand curve, as in earlier chapters, we would expect that firm to wish to adjust price, in most cases, after incurring product differentiation expense. This is due to the unlikelihood that cost and demand functions will react to additional expenditures in such a way that the prevailing price would remain the profit maximizing price. But, given R.M.D. and a kinked demand curve, there will be a given price level for each firm, departure from which will cause profits to decline, unless the price-leader(s) initiate the price adjustment. All firms except the price-leader(s) are therefore limited in the amount of additional product differentiation expenditure they may incur in the short run, to the point where the marginal cost curve intersects the uppermost point of the vertical discontinuity of the marginal revenue curve. Any further expenditure, even if it did cause demand to increase by more than costs, would be precluded by the firm's unwillingness to independently raise price.

In terms of the Edgeworth models developed in this study, the intent of non-price competition is to change the values of the product differentiation parameters which appear in the firm's demand functions. The firm wishes to obtain a larger share of the market at prevailing prices, and

11. A great deal of case study information on non-price competition is available in Scherer [127], particularly chapter 14.

to lose less sales if its price is undercut by rivals. In terms of equations (4.14) and (4.15), the firm's demand functions, this translates to a desire to increase the share of "continuous" buyers when prices are equal (viz. m_i); to reduce the reciprocal of the slope of the individual firm's demand curve (viz. dq_i/dp_i); and to increase the "loyal" demand by increasing the value of the intercept, H_i , and by reducing the slope term, G_i , which appear explicitly in equation (4.11) of the previous chapter. The extent to which these parameters actually change depends upon the effectiveness of the advertising and promotion expenditures, of course. To gain a net increase in sales, a firm's non-price strategy must be superior to those of its rivals, and/or it must attract new buyers into the market.¹² Conversely, and as intimated in an earlier chapter, certain policies of a firm may have an adverse impact on that firm's product differentiation parameters. Poor public relations, due to frequent price adjustments, discriminatory hiring, indiscriminatory firing, pollution, and the like, may be expected to alienate potential customers.

It should also be noted that price instability in the differentiated products cases of duopoly and oligopoly, which could conceivably arise as a result of the feedback effect, is ruled very unlikely as a result of the firm's recognition that its actions have a public relations impact. The instability which arose was dependent upon the assumption that firms take into account only the immediate effects of raising price against the

12. In which case the market demand value Q_i will be increased. There are, in fact, likely to be external benefits of an individual firm's advertising efforts. See [127, p.334].

feedback demand. If in fact consumers become annoyed or resentful of such a policy, the firm's product differentiation parameters may change over time as a consequence. The firm will expect, or eventually learn, that periodically exploiting the market by raising price against the feedback demand, causes it to lose sales in the longer term. Thus the profit maximizing response to this would be to allow prices to stabilize at the level to which they would otherwise degenerate.

No attempt will be made here to specify the impact of non-price strategies upon the demand function of the individual firm. It would appear to be a very complex matter, and is largely peripheral to the aims of this study. Some work on this area has been done, by Baumol [13] and Schuster [128] particularly, and much remains to be done.

V. Conclusion

When the firms of an industry recognize their mutual dependence, a number of modifications must be made to the predictions which arose from the models of earlier chapters. Firstly, if firms do recognize their mutual dependence but do not come to any agreement or understanding, they may expect their rival(s) to attempt to minimize the residual (or feedback) demand by a process of discrimination amongst customers. We saw that, in the identical products case of duopoly, this strategy would reduce the amplitude of the price cycle, or remove it altogether, compared with what would happen under an assumption of random choice of customers. Shubik has stressed the dependence of the cycle upon the assumptions underlying the "reconstitution" of the residual demand.

(See [132, pp.419-428], [133, pp.81-113], and [130, pp.31-34]). The above discussion is the first, to my knowledge, which explicitly outlines the nature of this dependence. It can be shown, furthermore, that the minimization of the residual or feedback demand will reduce or remove the fluctuations in prices, if such fluctuations would otherwise exist, in each of the other models discussed in the preceding chapters.

Attention was then given to the implicit and informal understandings which may arise from the firms' R.M.D. This gave rise to a consideration of price-leadership models in which we expect one or more firms to raise the general price structure to some level exceeding the non-cooperative price floor. This level would be the highest price the leader(s) expect can be charged without any firm preferring to initiate an independent price adjustment. It is believed that a contribution has been made to the literature on the kinked demand curve, as a result of the earlier analysis. By observation of the placement of the firm's marginal cost curve in relation to the discontinuity in the marginal revenue curve, we are able to say how far above the non-cooperative floor price, and how far below the "cartel" price, is the firms present price. This breakthrough allowed the construction of an "index of industry discipline" which would appear to be useful for inter-industry comparisons of collusive loyalty in the short run context.

The firms' recognition of their mutual dependence may lead them to enter into more explicit and formal agreements, the quintessence of which is the cartel. The extent to which firms may raise prices above the non-cooperative equilibrium levels will depend upon certain factors which were

enumerated, but we would expect the general level to be above that which would prevail without such a collusive agreement. Non-price competition is expected to assume considerable importance when firms recognize that their price adjustments may be easily matched. Advertising and promotion efforts provide a means of increasing sales which is neither as easy to copy, nor as likely to provoke aggressive reactions, as compared with price strategy.

For the most part, the demand functions of the firm have not been respecified to incorporate the modifications which R.M.D. requires. The "minimax" case allowed a relatively simple change in the functions as we saw. But if the firm does not know what to expect by way of retaliation from his rivals, its demand function for independent price adjustments becomes indeterminate. In order to have determinate demand functions we made a number of simple assumptions concerning rivals' reactions to price changes by an individual firm. There is no great specification problem for the constant-market-shares demand curve, or for the kinked demand curve. When non-price competition is allowed, however, specification becomes considerably more difficult. One could not simply express increased sales as a function of current (or past) product differentiations expenditures, since a wide dispersion of results may occur, depending upon what rivals are simultaneously doing. One could not even discuss sales responses to product differentiation expenditures of a single firm, with ceteris paribus, since dollars spent on product differentiation will not likely have the same qualitative impact for any two firms, or for any two different advertising campaigns.

In the following chapter the firms consider the long run implications of their actions.

CHAPTER 6

LONG RUN CONSIDERATIONS

I. Introduction

In this chapter we depart from the assumption that firms will attempt to maximize their profits in the short run. By replacing that objective function with one relating to the long run, we are able to introduce a number of new strategies into the models discussed so far. These strategies include pricing to prevent entry of new firms, efforts to bankrupt a rival, and changing the scale of plant. The issue of full-cost or markup pricing is discussed in an appendix, and is found not necessarily inconsistent with this analysis.

It was noted in the previous chapter that when firms recognize their mutual dependence they will look beyond the immediate and direct results of their pricing decision. We saw that a firm might abandon the objective of maximizing immediate profits in favour of a less aggressive strategy which is expected to garner a larger profit over a longer period of time. The time horizon envisaged by the firm may or may not exceed the 'short run' period of that firm, but since changes in plant size were not considered in the preceding discussion it was implied that firms were attempting to maximize profits over the short run. Given that

barriers to the entry of new firms are not absolute, short run profit maximization implies a similar sort of myopia to that criticized in the preceding chapter, since it infers that firms do not foresee that their profits may attract new firms, and that their future profits may subsequently be reduced.

But if the time horizons of firms are shorter than the short run period, firms will attach no significance to the entry of new firms, and short run profit maximization is rational behaviour. To proceed, we require some basis for the presumption that time horizons are generally longer than the short run period. Following the Berle and Means Study [14] a great deal of attention was given to the fact that the firms in a wide range of oligopolistic markets were generally very large in relation to the economy as well. (See Scherer [127, ch.3]). Rather than being owned and managed by a single entrepreneur, large firms are characteristically of corporate form, with the attendant feature of separation of ownership and control. The corporation is regarded, by shareholders and management alike, as an ongoing institution, a legal entity which will exist in perpetuity despite turnover of shareholders and replacement of managers. The interests of the owners and managers are served by the continued survival of the firm. Moreover, both these groups could be expected to see that it will be to their continuing benefit if the firm at least maintains its relative standing in the industry, and perhaps in the economy generally. Thus it appears reasonable to contend that the relevant time horizons which exert influence on the pricing decision in

oligopolistic markets are longer than the short run.¹ If, on the other hand, oligopolists were primarily sole proprietors (with no heirs) we might expect the converse.

Thus it would seem to be a generally untenable proposition that firms which are concerned with their long term existence and market standing, would at the same time attempt to maximize their short run profits. Pursuit of the latter may induce the entry of new firms, which in turn may jeopardize the attainment of the former. If not short run profit maximization, then what is the firm's objective? Baumol [12] has suggested sales maximization, subject to the meeting of a (minimum) profit constraint. Others have suggested the maximization of multivariate utility functions, where profits, and other objectives generate utility. [34], [159]. Growth has been suggested as the prime aim of the firm. [116], [96], [11]. 'Satisficing' rather than maximizing has been suggested in response to the claimed inadequacy of the data systems and control mechanisms of the firm. [139] A lively debate ensued over a prolonged period as to the validity of the various theories. (For a review see Machlup [95]). No solid consensus of opinion has emerged, with the advocates each claiming that their particular behavioural assumption is valid for a least some business firms.

II. The Objective Function Replaced.

A more promising contender for the firm's objective function is that

1. This is supported by the discussion concerning time horizons and the firm's objective functions in Silberston's review article. [137, pp.530-533].

firms attempt to maximize their long run profits. (See [46, p.36], [150, p.42n], [16, p.299], [2, passim], [69, passim] and [173], for example). This accords well with our reasoning that firms wish to continue their existence and at least maintain their market standing. Long run profits are necessary for the attainment of these objectives, and if the estimation of long run profitability takes into account the possibility of entry, this will avoid the conflict which arises with short run profit maximization.

Moreover, it would seem that many of the proposed alternatives to short run profit maximization are compatible with a strong concern for longer run profits, given the uncertainty that exists in the estimation of future profits. The sales maximization hypothesis, for example, may be equivalent to a long run profit maximization hypothesis, where repeat purchases are a feature of the market, where loyalty to particular firms over time is strong, or where there is a "lock-in" effect of complementary sales. (e.g. special films for special cameras and special projectors for the special film, replacement parts and accessories for consumer durables, and the like). Maximizing sales, subject to a profit minimum in the short run will ensure a continuing flow of customers in the long run, which in turn augers well for the firm's continued existence and relative market standing.

The growth of the firm certainly requires profits, since these are an important source of the necessary finance. And since growth is presumably desired over a protracted period, then long run profits will be preferred over profits which may be short-lived. Multivariate utility

functions which include the goodwill of customers and the public generally, the comforts of a quiet life, etc., may similarly amount to long run profit maximization. The firms continued existence and market standing require continuing public goodwill, and the preference of some managers for the quiet life will presumably inhibit the firm's propensity to be continually seeking short run profit maximization which in turn could induce new entry. In effect we are saying that firms do what they do in the short run because they expect that their actions now will enhance profits some time in the future. A firm which makes a charitable donation, contributes to a political campaign, installs anti-pollution equipment voluntarily, or refrains from a price rise at the suggestion of Government officials, is no doubt thinking in terms of the public relations impact of its action, which in turn will influence future profitability of the firm. Similarly, expanded sales now, at the expense of additional profits, are expected to generate repeat purchases in the future. The lower price which is set in the short run to expand sales, serves to broaden the future market for the firm's product.²

But the future is uncertain, and a firm's actions in the short run can only be based upon its expectations concerning the long run course of events. Hence if firms perceive their futures differently we would expect them to behave differently in the short run, in response to the same stimuli. In retrospect the firm might see that its expectations were poor, and may revise its future plans accordingly. But changes

2. Williamson [160] and Leland [88] show the differences which arise when the firm alternately maximizes sales, growth, or long run profits. In many cases the results do in fact converge with each other.

occur in the business world and the circumstances surrounding a decision are seldom the same at different points of time, and thus uncertainty continues to exist in varying degrees. This is a central point in the argument of the 'satisficing' school of thought regarding the firm's objectives. Firms, it is said, face considerable uncertainty as to costs and demand even in the short run, and adjust prices, promotional expenditures and other variables whenever it appears that one of their minimum objectives will not be attained. Once the sales objective is attained, for example, the managers apply their efforts to satisfying the next constraint which is not yet met, by adjusting control variables on the basis of their imperfect information systems and their imperfect expectations of the effect of these adjustments. In effect, satisficing managers appear to act by short run criteria designed to ensure the continued existence and maintained market standings of the firm they control.

Of course, the objective to maximize long run profits is not the same as to ensure continuity of existence and the maintenance of relative market standing. The maximization of long run profits is defined as maximization of the present value of the expected future stream of annual profits. Depending upon the conformation of this stream, and upon the discount rate applied, maximization of this sum may dictate heavy exploitation of the market in the first few periods, with a much reduced share of the market over subsequent periods. In fact, going out of business is quite compatible with this view of long run profit maximization, but is completely incompatible with the objective of continuity and growth. For firms with relatively long time horizons however, the discount rate

would be relatively low, and immediate profit maximization would only be called for in the case where it is impossible to prevent entry, unless rapid secular decline in market demand is envisaged. In the light of the uncertainty which exists, one might expect firms to adopt fairly cautious attitudes, to minimize the possibility of unwanted outcomes such as the dilution of their market share and reduced future profits. Further, the firm must view the expected profits of future periods with increasing skepticism the further into the future it looks. Thus at any particular point in time, the firm will be predominantly concerned with its existence over the next few time periods, and will take actions that are expected to be profitable, yet not alienate consumer goodwill or invite vigorous competition from existing or potential firms.

Thus we explicitly state the firm's objective function as follows. We assume that the firm attempts to maximize its expected long run profits, subject to its continued existence and maintained relative market standing.³ Note that the firm's conception of long run profits is subject to its vision being clouded by the uncertainty of future events. Such an objective function would be extremely difficult to specify, however, (see [1], [46, ch.5]), and it is doubtful that firms would ever attempt to do so. But an outward manifestation of this objective might well be a strategy of pricing to deter the entry of new firms, for it is the entry of new firms which poses the greatest threat to the firm's

3. If continued existence implies negative long run profits, the firm is expected to eventually exit the industry, of course.

continued existence, market standing, and profitability. We turn now to this issue of "limit" pricing, and other short run strategies consistent with the revised objective function.

III. Short Run Behaviour

In this section we shall discuss three strategies which firms may employ in the short run in an attempt to achieve the maximization of long run profits. In the first place, firms may place an upper limit on their short run price, which is low enough that entrants expect to be unable to make a profit. Secondly, established firms may attempt to force the exit, takeover, or merger of one or more existing firms, and thus expand market shares and/or experience economies of scale. And thirdly, established firms, both separately and jointly, may erect and maintain barriers to entry in the form of product differentiation expenditures.

(i) Limit Pricing

The idea of pricing to limit or deter the entry of new firms arose from the works of Kaldor [82], Andrews [2], Harrod [63], and Bain [4]. The theory solidified following later works by Hicks [69], Hahn [59], Bain [7] and Sylos-Labini [150]. An account of the development is available in Osborne [113] and a good outline of the simple model is in Scherer [127, pp.219-234]. More recent advances are those by Baron [9] and Gaskins [54]. We shall examine the two major cases, mutations of which present no significant problems.

In the first case, the firms jointly recognize, (or follow the example of the price-leader, who recognizes) the potential of entry of small-scale, relatively-high-cost firms at the periphery of the industry. If allowed to enter, these firms may prosper and grow and eventually make substantial inroads into the market share and profitability of the original firms. Preferring to avoid this, the existing firms set price just below the level of minimum average costs which the entrant firm is expected to incur. The potential entrant thus never enters, as he does not expect to earn even a normal profit. In the second case the existing firms hold no cost advantage over the potential entrant, which is expected to be a relatively large-scale firm. The existing firms are motivated to set the price at a level such that the entry of another firm would depress the price below the entrant's minimum average costs, given that the existing firms maintain their production at the pre-entry levels. The potential entrant is thus inhibited by the expectation that his entry would not be profitable.

Is limit pricing a rational, long run profit maximizing strategy? One wonders why the firms should lower the price before the point when the entrant actually enters the industry. If the existing firms have a cost advantage they could presumably lower prices after entry occurs, and drive the entrant out again, suffering reduced profits (or losses) only for the duration of the actual skirmish. Several factors weigh against this, however. First, prevention is probably better than cure. Once a firm has entered, and especially if it is backed by wealthy or conglomerate interests, it can be expected to make a sustained effort to "ride

out the storm", and may make accelerated efforts to reduce costs and/or achieve economies of scale. Secondly, the existing firms' credibilities are weakened by first charging entry-inducing prices, and then threatening to hold prices down until the entrant is ruined. The entrant may hang on for a prolonged period in the expectation that the existing firms will relent and allow the new firm to "join the crowd". In this case, the pre-existing firms may incur reduced profits, or losses, for a substantial period. Thirdly, the extent of price reduction required to deter entry is much less than that required to force the same firm to leave. The entry-detering price is just below minimum average costs, while the exit-forcing price is just below minimum average variable costs. It may well be preferable to set the former price over relatively long periods, rather than the latter over relatively short periods. And fourthly, forcing an entrant to leave the industry might be construed as illegal predatory pricing by the relevant authorities, and attract censure, cease and desist orders, or fines, to the conspiring firms. Thus setting the entry-detering price, plus making the implied threat of stiff competition if subsidized entry is threatened, may well be the long run profit maximizing strategy.

Regarding the large-scale entry model, the major problem is that successful deterring of entry depends upon the credibility of the threat made by the existing firms that they will maintain their output at pre-entry levels and allow the resultant excess demand to press prices below the level of minimum average costs. The potential entrant might reason that the existing firms will eventually accept his entry, in their own

interests, and all firms will raise the price to the new entry deterring level and thus restore profitability. The original firms wish to prevent entry and thus should make their threat explicitly and apparently in the greatest sincerity, but if their bluff is called we would expect them to take the rational course and accept the entrant into their limit pricing agreement such that they may quickly adjust price to a more profitable long run level. Thus it would seem that entry of a large-size low-cost firm could not be prevented, unless the entrant firm's financial backing was inferior to that of the existing firms. For example, if the entrant must make profits within a certain period in order to repay creditors, and the existing firms' financial reserves were more than sufficient to cover losses during this period, and both the entrant and existing firms knew this, then entry may be successfully deterred, if the threat made by the existing firms to maintain output is sufficiently credible.

There are circumstances in which the existing firms will not practice limit pricing even when their objective is to maximize long run profits. Bain [7, p.22] introduced the term "blockaded entry" to describe those cases where the limit price exceeded the short run profit maximizing price. The existing firms in this case would have no incentive to raise prices high enough to attract entry, and would maximize long run profits by maximizing successive short run profits. In this case, barriers to entry are likely to be very great, either in the form of absolute cost disadvantages, product differentiation loyalties, or inability of the market to support another firm operating at minimum optimum scale. Bain

also introduced the term "ineffectively impeded" entry, to describe the case where the limit price is so low that firms find it advantageous to attract entry until a higher limit price is appropriate. That is, the firms maximize short run profits for the first few periods and then adopt limit pricing in subsequent periods, since this strategy is expected to contribute more to long run profits (see [7, ch.1]).

A word of clarification is necessary regarding "the" limit price. First, the above analysis refers to a static market demand situation. If the entrant expects the market to expand, or to eventually "win over" a substantial portion of the market, he may enter despite the limit price, in the expectation of eventually making profits. The simple limit price theory implies that if the entrant cannot make positive profits initially, it will be deterred from entry. But firms may enter expecting to make initial losses which will later be outweighed by profits. Entry takes place, one presumes, if the present value of expected future profits exceeds the present value of expected initial losses by enough to make this a more attractive investment proposition than the investor's other opportunities. But this needs to be qualified to cover the case where profits begin too late, that is, after the entrant has been declared bankrupt and has dissolved. If creditors fore-close before the entrant "breaks-even", and the firm is unable to re-finance its loans, then the entry will be unsuccessful. Given the uncertainty involved in predicting future profits the entrant would no doubt prefer a suitable safety margin included in the "break-even" period.

Secondly, since the limit price depends upon the expectations of the existing firms with respect to the cost structure of the potential entrant firms, there is a risk element involved. If the limit price is not, in fact, low enough to prevent entry, due to prediction errors, then entry will occur anyway. If the firms are risk-aversers, they will set a lower limit price than otherwise, to reduce the consequences of errors in their predictions. The more averse to the risk of entry are the established firms, the greater margin for error they are likely to have between the limit price and expected minimum cost levels of potential entrants. (See [9, p.670]).

In terms of the model developed in the preceding chapters of this thesis, the introduction of limit pricing is relatively simple. Recognition of mutual dependence is a prerequisite, of course, and either the price leader, or the collusive body, will choose the price level on the basis of their expectations concerning the limit price. Thus we have added another criterion to the "appropriateness" of the chosen price. In chapter five the appropriate price was the highest price that would inhibit unilateral price adjustments. The added consideration is that the price is not so high that it will attract entry of new firms. The lower of these two prices will be the one chosen, since it will both deter entry and inhibit unilateral price action.

The index of industry loyalty thus takes on another dimension, when we consider that the price charged by the firm depends upon the potential of entry as well. The index, which simply describes the relationship of

the firm's actual price level to the range in which it might lie, does not explain the level of price. But the explanatory factors are now two-fold: that is, the threat of independent price cutting from within the group, and the threat of incursion by new entrants. The actual price may in fact be the firms' estimation of the "limit" price, if the threat of independent price cutting does not occur until price is somewhere above the "limit" price. But note that the index says nothing, per se, about the height of the barriers to entry. This would need to be expressed as some function of the monetary value by which price exceeds average costs. The index simply indicates the relative location of the actual price between the upper and lower price limits: viz. the complete collusion-blockaded entry price, and the zero collusion price.

(ii) Predacity, Takeover, and Merger

Since the exit of any firm would presumably expand the market share and enhance profits of all or most other firms, a more positive strategy to ensure long run profitability might be to drive rival firms into bankruptcy. This strategy will only be employed if entry is blockaded, or "effectively impeded", of course. In the latter case the firms will need to practice limit pricing to maintain the increased degree of market concentration. Given that new entrants will not nullify the effect of the exit of a firm, predatory pricing requires either that the intended victim has significantly higher costs than the other firms', or that the intended victim has significantly more restricted access to loan finance. A price war of sufficient duration would, in either of these circumstances, cause the intended victim to actually leave the industry due to his over-

whelming losses. Predatory pricing is almost necessarily a joint activity, since predacity by any single firm would usually spread its effects too thinly over all other firms. The price war and consequent losses or reduced profitability would be over in a much shorter period if they were the result of a collusive strategy on the part of a subset of firms, or if the price leader took it upon himself to lead prices downwards to eliminate one or more rivals.

When firms serve different geographical areas, or where product differentiation extends over a spectrum, price discrimination may be a sufficient form of predatory pricing. If the intended victim sells primarily in one geographical area, or has relatively few product lines, a "localized" price war might be all that is required to force that firm's exit. The "cost" of the price war to the other firms would be less in aggregate, and presumably the distribution of this "cost" amongst the other firms will be roughly proportionate to the benefits derived subsequent to the exit of one or more firms. If not, the relatively disadvantaged firms may be offered side payments or other concessions in the "implicit bargaining" process of oligopoly markets, (see [46]), to induce them to cooperate, if their cooperation is necessary.

In most advanced economies, however, predatory pricing is specifically outlawed by legislation intended to preserve competition in the interest of consumers and the public generally. But the illegality of predatory pricing need not cause it to be abandoned as a competitive strategy. Just as pollution, although illegal, is sometimes indulged in because it is more profitable to pollute and pay fines if detected,

one might expect the predatory pricing decision to be more economic than legal in its foundations. It is more likely to be indulged in, one presumes, the lower is the likelihood of detection, conviction, heavy fines and/or imprisonment of the conspiring personnel, given a situation where predacity is potentially profitable but illegal.

The potential profitability of predacity will depend upon the firms' expectations regarding future profits under the status quo, as compared with future profits during and after the price war. As well as expected future price levels, cost expectations are paramount. Elimination of rivals would expand the market shares of the remaining firms, and may allow the attainment of economies of firm size and/or of plant size. Other costs changes may be foreseen as the result of changing technology and factor prices. On the demand side, expectations must be formed regarding the growth of the market over time, and possible changes in consumer taste and preference patterns. In the light of all these considerations then, the price leader or a collusive bloc of firms will opt either to continue the status quo, or institute a predatory pricing policy.

Other policies of predacity may be followed, and, if they are more subtle, may be preferred to predatory pricing, since the risk of conviction will presumably be lower. One such policy might be to threaten boycotts of suppliers if they were to supply the intended victim (which would need to be a relatively small customer of the supplier, vis-a-vis the aggregate of the conspirators' demand). Another policy designed to destroy a rival might be to conduct advertising or rumour campaigns which

tend to denigrate the intended victim's reputation or product. Contrived bad publicity may be very difficult to nullify, given the persuasiveness of certain media forms and the consumer's inability to confirm the validity of either side of the story.

If unable to force the exit of a rival the predator firms may instead attempt to take-over the firm, and subsequently retire its plant and equipment, and/or expand sales and profitability. Mergers are similar in intent, since they are presumably undertaken to achieve economies of size, or to "rationalize" industry capacity, advertising expenditures, research and development efforts, and other duplication. (See [127, pp.103-122] and [146, ch.8]). Although profit considerations may dictate the take-over or merger of one or more firms, this avenue is only available when all parties to the transaction are willing, unless a stock market take-over is possible.⁴ Independence of spirit, stubbornness, or misanthropy, may over-ride simple profit considerations in the real world, of course. Moreover, takeovers and mergers may be subject to official permission being given by the above-mentioned custodians of effective competition. If judged contrary to the public interest, the union is prohibited, and the parties are then left to pursue some lesser degree of joint action, or become competitors again.

Takeover and merger activity may be represented, in the context of the model developed here, as an additional non-price competitive strategy,

4. If the owners of the firm which is to be taken over believe the firm is worth more than the offer, they will not be willing, of course.

to the extent that a reduced price is not necessary to effect the takeover or merger.⁵ Predacity intended either to force another firm's exit, merger or takeover, enters the present model under the question of the "appropriateness" of the price chosen by the price-leader(s). The price-leader(s), if convinced that increased concentration of the market will enhance the long run profits of themselves, or of all remaining firms, will lead prices down to a level which is considered sufficiently low to force one or more firms to comply with the wishes of the price-leader(s).

Thus there is another explanatory factor behind the index of industry discipline. Prices may be held below the cartel level, not due to the fear of unilateral price adjustments, and not due to the fear of new entry, but in pursuit of increased concentration of the market.⁶ We note that the term "industry discipline" is appropriate for the concept of the firms resisting the impulse to undercut prices for individual gain, but is less appropriate as a descriptive term for the limit or predatory price. Let us use the term "index of oligopoly price" to mean the value of the index when price is chosen in a long run context, and retain the "index of industry discipline" to refer to the degree to which firms feel they can raise price without inducing unilateral price cuts from existing competitors. Thus the index of oligopoly price could not

5. In the event that a party to the takeover or merger is initially unwilling to be a party, reduced price levels (with consequent lower profits) may help change the firm's mind.

6. In this vein, and considering that predatory pricing is often illegal, the price-leader(s) may set the price such that the peripheral firms are able to exist, but are unable to expand their scale of operation or market share, over time.

exceed the index of industry discipline, although these would be equal if the limit price was at a higher level, (and predacity was not considered feasible). The actual price will be the lowest of the limit price, the predatory price, or the highest price which the price-leader(s) dare set without inducing unilateral price action.

(iii) Product Differentiation Barriers

A third class of strategies which a firm may invoke in the short run is to strengthen its product differentiation advantages. Thus, a potential entrant will be confronted by even greater limitations upon the price it may charge for any given quantity, and upon the cost that will be incurred to sell any particular quantity. Product differentiation barriers may be erected jointly by the established firms, and/or individual firms may seek to strengthen their particular position against the incursions of a new firm. Joint action may take the form of eulogizing the virtues of long-standing experience in the industry, and the ramifications for superior service, durability of the product, trade-in values, and the like. In other words, the established firms may seek to cultivate a prejudice against the products of new firms, which in turn may be very difficult and quite expensive for the entrant firm to overcome. The expectation of this expense and difficulty may inhibit entry.

Product differentiation efforts by individual firms have already been discussed in the context of non-price competition, but when the potential of entry is recognized, some firms more than others may feel the need to erect barriers around their own particular product. Since products are differentiated, the entrant's product will be a closer

substitute for some products than for others. The entrant may have the discretion of entering the product differentiation spectrum at any point, and each established firm would no doubt prefer that the entrant steal sales from the other firms rather than itself. Thus, firms will make efforts not to be the weakest link in the chain; that is, to avoid having its product easily imitated by a new entrant.

Just as in the non-price competition framework firms might tend to "overspend" on product differentiation efforts, it is feasible that a similar prisoner's dilemma problem will exist with respect to barrier-raising expenditures. Each firm faces the possibility that if it spends less than its rivals it may be "entered against" by a new firm. To protect itself against this outcome, each firm will be motivated to spend at least as much as it expects its rivals to spend, with the probable result that some element of their expenditures are mutually-cancelling. A collusive agreement may be possible but this would be likely to encounter problems similar to those mentioned in the previous chapter. Bain [7, ch.6] has indicated that product differentiation barriers are perhaps the most important deterrents to new entry, since the other major barriers (absolute costs and scale economies) are likely to be eroded by technological progress and conglomerate entry. Limit pricing is often insufficiently prohibitive, and post-entry predacity is illegal. It is evident then that firms will be concerned with product differentiation as perhaps the most effective means of preventing entry. Any short run analysis which purports to show that firms spend too much on advertising, and other product differentiation efforts, may be missing the point of

these expenditures.

Increased product differentiation expenditures for "barriers" purposes may be treated in this model as a fixed cost, since they are intended as a precautionary tactic to protect future output levels. (Expenditures for non-price competition purposes no doubt have both a fixed and variable cost element, although it may prove extremely difficult to separate them in practice). Since all firms are inclined to undertake product differentiation expenditures due to the prisoner's dilemma problem, we would expect these expenditures to be incurred by all firms as a "built-in" element of their costs. Price determination proceeds as the model suggests. A structure of price differentials will evolve which is to the satisfaction of all firms. If there is no agreement arising out of the R.M.D., then all firms will set their non-cooperative floor prices, which depend upon the relative cost and product differentiation advantages of the particular firms. When any one firm changes the relative cost and/or product differentiation position, by virtue of its advertising efforts for example, a new structure of price differentials is expected to develop. To the extent that firms implicitly or explicitly agree to follow the price-leadership of a firm, the general price level will be above the floor level. It will be below the cartel level to the extent that unilateral price adjustments, entry of new firms, and predatory pricing are considered.

IV. Long Run Adjustment

The short run is usually defined in terms of the fixity of factor

inputs, and, strictly, as the period of time it takes to vary the input of the "most-fixed" factor. At the end of the short run the firm may change plant size, technology, or location, to any other possible combination known to it, and then continues operating in the next short run period. It is evident that the short run periods of the various firms are not likely to be coincident. Any one firm may embark on a new short run period, while other firms remain constrained to their "old" output and cost ranges. In the analysis of the preceding chapters however, we presumed that all cost functions remained constant while the price adjustment process was carried out. But with non-coincident short run periods, the period of "peace", during which no firm introduces new plant, may be quite shortlived. Thus prices may be adjusting towards a set of equilibrium values, when a new set of equilibrium values becomes appropriate. R.M.D. takes care of perhaps most of this apparent price instability, however, since prices may be adjusted more quickly and directly to the newly "appropriate" levels, and/or non-price competition may replace price adjustments, as we saw in chapter five. Note that the entry of new firms is similar in impact to the expansion of plant size by an existing firm, and the exit of a firm has the same consequences as contraction of plant size. Firms become aware that cost and/or demand conditions have changed, and may wish to change the level and/or structure of prices.

Firms will change the scale, technology, or location of plant, on the basis of actual changes in the state of technology, factor cost ratios, and market size or composition, since these will most likely cause the

present plant to be non-optimal. But expectations of future changes will also influence the decision. If growth of the market, changing tastes, and improving technology are expected, then the firm may include these factors in its calculations, attaching weights to reflect the degree of certainty and risk aversion which the firm feels. Similarly new products may be introduced on the basis of expectations regarding the trend of consumer tastes and the desirability of being "first on the scene" if the product were to gain wide consumer acceptance.

There would seem to be three fairly compelling reasons why firms will build new scales of plant which incorporate excess capacity, even when no increase or change in the composition of market demand is expected.⁷ First there is the precautionary motive. If the firm were to experience mechanical breakdown, or a labour strike, demand will be satisfied for a while out of inventories, and some customers may elect to wait for delivery. When the firm is back in production it will wish to "catch up" with demand and restore its preferred inventory levels, to guard against sales losses it would incur if a similar stoppage occurred too soon afterwards. Secondly, there is the aspirational motive. No doubt the firm aspires to expand its sales and/or market share. If it has some capacity in reserve it is able to take advantage of other firm's production stoppages and unexpected increases or shifts in demand. Thirdly, there is the defensive motive, which is directed

7. Excess capacity is defined, for these purposes, to mean the ability to produce more than the current output level at a similar level of average costs. Firms may be expected to choose plants which allow constancy, or near-constancy, of average costs over a wide range of output, since this allows flexibility of output levels without substantial changes in per unit costs.

primarily against potential entrants. If the existing firms have excess capacity, potential entrants will be less likely to enter the industry, since the existing firms will be able to satisfy expected and unexpected increases in market demand of probably considerable magnitudes. Alternatively, if entry forced prices down, the existing firms would not immediately meet their full capacity constraints, and would therefore allow no full capacity feedback effect to benefit the entrant.

In terms of the model, with firms attempting to maximize their long run profits, we would thus expect firms to take the decision to build new plant as soon as it became "worthwhile" to do so. The decision will be based on actual and expected cost and demand factors, and the resultant new plant would be expected to contain a significant proportion of 'planned' excess capacity.⁸ This excess capacity has certain implications for the price instability which arose in earlier chapters due to the limited capacities of the firms. It will be recalled that price instability occurred in the product differentiated duopoly and oligopoly models if, at some point in the downward progression of prices, it became more profitable for any firm to raise price against the feedback demand. We saw that the feedback demand is larger, at any particular price of the initiating firm, the smaller are the full capacity output levels of the other firms. This was illustrated in chapter three by Fig. 3-IV. There, it was shown that larger plant capacities relative to any given market demand will cause prices to stabilize, due to the reduced size, or non-

8. The likelihood of excess capacity persisting in long run equilibrium has been suggested, via different reasoning, by Harrod [63], Hicks [69], and Hahn [59].

existence, of the feedback demand. Thus, if firms plan to have significant excess capacity, at expected output levels, then we expect the feedback effect to be correspondingly smaller. In conjunction with the firm's recognition of the public relations impact of price instability, discussed in chapter five, this development virtually rules out cyclical price instability in the product-differentiated models.

In the identical-products models, planned excess capacity, by itself, will remove price stability only if this causes either firm in the duopoly model, or any $n - 1$ firms in the oligopoly model, to be able to supply the entire demand at a price equal to minimum average variable costs. It was illustrated in chapter two, by Fig. 2-II, that the amplitude of the price cycle increases as capacity sizes are enlarged relative to the market, until the cycle abruptly disappears when the Bertrand capacity condition is met. The existence of planned excess capacity therefore tends to enhance price instability in the identical products case. Recognition of mutual dependence, however, is expected to militate against instability, as we discussed in chapter five.

V. Conclusions

Long run profit maximization was introduced as the objective function of the firm. Rather than adopt the conventional approach that the firm maximizes the sum of the discounted future profits stream, we recognize that the firm is likely to be unable to make such calculations in its uncertain environment. Instead, the firm will pursue strategies which it believes will enhance long run profits. It will attempt to

broaden its market over time by product differentiation efforts, by building excess capacity in case it has the opportunity to expand sales, and by attempts to increase the market concentration of the industry via takeover, merger, and forced exit. On the other hand, the firm might make efforts, independently, and jointly with the other firm(s), to prevent the entry of new firms. Limit pricing, threats of retaliation, product differentiation barriers, and excess capacity are the major means of achieving this latter objective.

These longer run considerations are likely to cause the price level to be stable in those product-differentiated cases where price instability would otherwise prevail. This follows from the firms motivation to build excess output capacity in the long run with its subsequent impact upon the size of the feedback demand.⁹ In the identical products case, planned excess capacity is expected to further destabilize the price level, except where it allows the Bertrand capacity condition to prevail. As compared to the short run cases where price was stable, we expect the long run considerations to cause price to be stable at a lower level. This is true to the extent that barriers to entry are less than complete, and that firms may wish to eliminate an existing competitor. If entry is "blockaded" we recognize that price may rise as high as the "cartel" level, if there is no threat of unilateral price action from existing firms, and if there is no intention to impoverish an existing firm.

9. Stability of prices also follows if the firms agree to adopt a "limit" price, of course.

The term "index of oligopoly price" was introduced to describe the degree to which price is below the "cartel" level, and above the non-cooperative floor, when long run considerations are taken into account. The index of industry discipline refers only to the extent to which firms would raise prices before any one firm would be tempted to independently cut price, and without consideration being given to potential entry or elimination of a competitor. The value of the index of oligopoly price is obtained from equation (5.2). The actual price set will be whichever is the lowest, of the limit price, the predatory price, and the highest price which can be set without independent price cutting.

The major contributions of this chapter, to the literature of this area, are as follows. The limit pricing model was incorporated into the analysis of the preceding chapters, and may provide the upper limit to the price which firms choose. A second possible determinant of the price level is the predatory price; that which is calculated to impoverish one or more firms. The index of industry discipline, introduced in chapter five, was broadened to become the index of oligopoly price, when long run considerations are reflected in the actual price level. Three motives for the building of planned excess capacity were suggested: precautionary, aspirational, and defensive, or entry-inhibiting.

In the appendix to this chapter, the issue of full-cost or markup pricing is examined. Although this principle has been said to be contrary to the conventional theory of the firm, it is found to be quite compatible with the foregoing analysis.

APPENDIX A to CHAPTER 6.

Markup or Full-cost Pricing

The marginalist theory of the firm has been heavily criticized for its presumption that firms "know" the values of costs and revenues over a substantial range of outputs, and are thus able to maximize profits by adjusting price and/or output until the marginal equality is achieved. Empirical studies of actual pricing policies of business firms have tended to show that firms operate with considerably less information and greater uncertainty than is supposed by the marginalist analysis. Hall and Hitch [61] were perhaps the first to indicate that rather than marginalist principles, the rationale underlying pricing decisions for many firms was that the selling price is determined as a "markup" over "prime cost".

"Prime cost" includes labour and material outputs, and specifically excludes overhead costs, and thus accords fairly well with the economist's concept of average variable costs. The "markup" is a percentage of these costs which is added on, and which is intended to contribute to overhead costs and the firm's profits. A common presumption in the markup pricing literature is that firms will choose the markup to achieve only "normal" profits, and thus not attract entry of new firms. Price is then equal to the average total costs, or the opportunity cost of all factors employed per unit, and hence the term "full-cost" pricing. (See [63], [69], [59]).

It is clear however that the markup need not always be that which allows normal profit, since if entry is impeded at all, a price above average cost will still inhibit entry, and in any case the respondent firms admit to adjusting their markup percentage up and down on occasions when market conditions change significantly. (See [61, pp.19-20]).

An alternative approach to the cost base of the final price is the "standard volume" average cost which is reportedly used by some large corporations. This approach is employed when the actual volume of sales is subject to significant uncertainty, but where the price is typically constant over an extended period of time, as in the automobile market. (See [127, p.174]). To calculate S.V.A.C. the firm estimates its total costs, including overheads, at a volume of production chosen on the basis of that firm's sales in the preceding year or two. This "standard" volume may be chosen somewhat conservatively, at say, 80% of the firm's full capacity output. The markup which is then applied is intended to return a target profit rate at the "standard" volume of sales. To the extent that actual sales fall short of the "standard" volume, profits fall short of the target level, or rate. Conversely when sales exceed expectations, profits are similarly greater.

The examination of this type of pricing decision has generated considerable controversy. Machlup [93] has defended the marginalist theory with reasoned argument, while Earley [41] has added impassioned empiricism. Friedman [52, ch.1] offers the view that if a theory predicts adequately, it is valuable in spite of the unrealism of its assumptions. Alchian [1] and Stigler [146, ch.7] have propounded the principle that profits are

essential to the survival of the firm in a Darwinian business world.

Hahn [59] and Andrews [2] have implied that markup pricing is a rational policy for firms which are in market equilibrium, since they are simply maintaining the status quo.

In fact, the marginalist and markup pricing principles may be reconciled as follows. We know, from elementary price theory, that marginal revenue is related to price, and the coefficient of price elasticity of demand, as follows:

$$MR = P \left(1 - \frac{1}{e} \right)$$

Now, assuming that the firm wishes to maximize profits, we have

$$MC = P \left(1 - \frac{1}{e} \right)$$

or

$$P = MC \left(\frac{e}{e - 1} \right)$$

If costs are constant over the relevant range of output, $MC = AVC$. Hence,

$$P = AVC \left(\frac{e}{e - 1} \right)$$

Suppose $e = 5$, then $P = AVC \left(\frac{5}{4} \right)$ or

$$P = AVC + \frac{1}{4} (AVC)$$

that is, price is equal to a 25% markup on average variable costs. In choosing the size of the markup the firm may, in effect, be making a rough estimate of the coefficient of price elasticity. (See [18, p.231]).

Two main assumptions were made in the above reconciliation. The first was that the firm would choose the markup percentage to maximize profits. It has been argued in chapter six that this is probably a quite reasonable assumption if taken in the long run viewpoint, with due cognizance being given to potential entry. The second assumption, that average variable costs are constant in the neighbourhood of the firm's current output, is fairly well founded both in the short and the long run senses, according to various empirical studies. (See Sherman [121, pp.55-60] and Bain [6]). Thus the reconciliation of the markup principle and the marginalist principle appears to be on fairly solid ground when the latter is taken in its long run context. The greater part of the controversy revolved around the observation that firms did not maximize their short run profits. The controversy dissolves if the objective function of the observed firms was to maximize their long run profits.

Thus we fall back on the Friedman-like view that firms may attempt to maximize their (long run) profits, but that they do it in their own way, lacking the precise information required by the economist's finely-honed tools. Machlup [93] makes a very convincing case that firms attempt to maximize profits, but do it in terms of average or total, rather than marginal, concepts. It is noted in various parts of the literature that the margin added to costs varies with market conditions, barely contributing to overheads when demand is slack, and allowing high profits when

demand is strong. The caveat is often appended, however, that care is taken not to set a profit margin so high that new firms are attracted to enter the industry. (See [41], [93], [59], [61]). All of this would seem to be consistent with the presumption that firms, in the face of various elements of risk and uncertainty, attempt to maximize their long run profits.

Furthermore, the adoption of markup pricing policies, by the firms in an industry, facilitates their collusion or "conscious parallelism". Where cost structures are similar, the "agreement" may be quite informal or tacit, otherwise a trade association or other subset of firms may announce a "representative" average cost figure and a "fair" profit margin. Responses by businessmen to the Hall and Hitch, and Earley, surveys ([61], [41]), show a strong belief in the "fairness" of price, and the ethical position involved in departing from the "fair" price. A price which allows a high profit margin might be taken to imply an intent to "gouge" the consumer, whilst a low profit margin might be construed as an attempt to bankrupt a fellow businessman. Avoiding these extremes locks the firm into an "ethical cartel" and helps preserve the stability of prices and of market shares. [93, pp.541-543]. The full or prime cost of each firm, if costs are similar, or of the price-leader or "representative" firm if costs are dissimilar, may thus be used as the focal point for collusive or parallel pricing behaviour.¹

1. The idea of the "ethical cartel" is clearly related to the concept of "industry loyalty" which was described in chapter five. The index of industry loyalty would indicate the collusive strength of the implicit agreement involved in the ethical cartel. The more strongly do firms hold the belief that prices divergent from the (designated) fair price are unethical or immoral, the more firms will forego the opportunity to increase their individual profits by unilateral price adjustments.

Finally, markup pricing simplifies the problem of allocating overhead costs amongst different types of output when the firm produces joint products. The markup applied to each product's prime costs will reflect the degree of competition that particular product encounters in its market. Where demand is more elastic and competition more intense, markups will be lower, as compared to other products which compete in markets where collusion or "conscious parallelism" prevail. The products with higher markups contribute more to overheads and profits than do the more competitive lines. Overall, the markup policy is probably intended to return a target profit rate, which must not be so high as to attract entry of new firms. Similarly, if new entrants could produce one of the products alone, without producing the other products which are "joint" to the existing firms, then the markup on that particular product must not be so high as to attract entry.

In conclusion, it would appear that a policy of markup pricing is not inconsistent with the model which has been developed in the preceding chapters. Rather it is an alternate means of choosing price. Once we recognize that the firm has very little information regarding its demand curves, and that it must proceed by trial and error in choosing the optimum price, a rule-of-thumb method appears eminently sensible. The actual markup depends upon a number of factors, not the least of which are the threat of entry and the threat of unilateral price action, which were argued to be major determinants of the value of the index of oligopoly price developed here.

CHAPTER 7

CONCLUSION

I. What has been done

The aim of this study was, as stated in chapter one, to clarify, extend and synthesize certain existing strands of the theory of the firm. The intended result was that a theory of the firm should arise which is capable of adding to our understanding of the market behaviour of firms, price levels and structures, non-price strategies employed, and the actual sales levels of the individual firms. To achieve this objective, several issues were analysed in greater depth than had been done previously, and a number of theoretical innovations were introduced.

The analysis began with the simple Edgeworth duopoly model, and systematically departed from the more restrictive assumptions of that model. At each stage the model was subjected to close scrutiny, to determine whether or not prices would stabilize. If so, at what level, and with what price differentials, would prices settle? If not, over what range would prices vary? We found that prices are typically unstable when products are identical, and typically stable when products are differentiated, in both the duopoly and oligopoly cases. Product differentiation was defined in two senses, (those of symmetry and con-

tinuity), and was seen to play an important role in the determination of the price level and structure. Recognition of mutual dependence allowed prices to be raised above the non-cooperative floor levels, but long run considerations may prevent the firm from raising price to the "cartel" level in the short run. These results are detailed in the text of chapters two to six, and are summarized in the concluding section of each chapter. They need not be re-iterated here.

What is the explanatory power of this model when applied to a particular market situation? Can it explain why a particular firm sets a higher price, yet satisfies a disproportionately large part of market demand? Alternatively can it explain why very small firms exist on the periphery of industries, charging high prices to consumers who want a special variant of the commodity? Both of these questions could be answered on the basis of the relative cost and product differentiation advantages of the firm, as explained in chapters three and four. The general level of prices may be above the non-cooperative floor, and prices may remain fixed over extended periods of time, by the reasoning of chapters five and six. An index of oligopoly price was constructed to describe the extent to which the firm's actual price level lay above the floor price and below the "cartel" price. A high index value would suggest substantial barriers to entry, strong industry discipline, and that elimination of existing competitors was probably not intended. A low index value could indicate either insubstantial barriers to entry, or weak industry discipline, or the price-leader's intention to eliminate a rival firm. We could not say which, a priori, and would need to look

closer at the underlying factors and the past experience of the industry.

What does the model predict? It predicts that prices will not generally fluctuate cyclically, given the recognition of mutual dependence. Moreover, firms will be reluctant to raise price unless they are the recognized price-leader, or unless there is the strong expectation that all other firms will simultaneously raise prices as well. Firms will indulge in non-price competition to improve their sales at the given price level, and for the purposes of strengthening a barrier to the entry of new firms. Firms will build excess capacity in order to protect their sales level, to gain sales when the opportunity arises, and to inhibit the entry of new firms. Firms will tend to set prices which are lower than those which would maximize their short run profits.

The major contributions of this study to the existing literature would appear to concern the demand functions of the firms. Product differentiation was introduced in the more general case of asymmetry and discontinuity of aggregate consumer preferences. Parameters to describe these phenomena were inserted in the demand functions, and these helped to generate price and sales differentials amongst the firms. When the firms recognize their mutual dependence they are expected to raise prices above the non-cooperative floor levels, each by the same proportion, in order to maintain their relative positions. At the new price levels firms will envisage a kinked demand curve, unless they are the recognized price leader, or are confident that all firms will follow a price increase as well, due to a similar cost or demand increase. The intensive examination of the price determination process, in the non-cooperative framework,

allowed the conclusion that the non-cooperating floor price will be the lower limit of price in the conventional kinked demand curve model. The upper limit will be the "cartel" price, or the point where the firms marginal cost curve intersects the mutatis mutandis marginal revenue curve. The actual price can thus be represented by the "index of oligopoly price" which was introduced here. In addition to the innovations, it is expected that some of the previous models may be more easily understood by virtue of their exposition here.

II. Directions for Further Research

Much remains to be done in the theory of the firm. In this study a number of questions were left unanswered. One of these was the specification of the weights which one would need to attach to the prices of the oligopolists in the asymmetrically differentiated case to obtain a unique value for market demand at every (weighted) mean price of the firms, regardless of which firms set which prices. Another was the accurate specification of the full capacity feedback effect in the differentiated products oligopoly case, although this becomes something of an intellectual curiosity in a world of mutual dependence recognized and long run considerations, as we have seen.

The impact of advertising, and other product differentiation expenditures, upon the product differentiation parameters in the firm's demand function was also left unexplored. Similarly the factors underlying the index of industry discipline, which cause one industry to collude "better" than another, remain largely unexamined. The question of how the price-

leader estimated the limit price, the predatory price, or the price which will induce unilateral price action, also was not examined in any detail.

III. Possible Empirical Tests

It would seem that parts of the model developed here could be subjected to empirical testing. It would be instructive to confirm, or deny, a number of the assumptions that were employed in the model. Some of the questions which might be pursued are as follows. Do firms imagine ceteris paribus curves, and do the slopes of these differ amongst firms? Is the parameter m constant over the range of prices considered by the firms? Do the firms' product differentiation parameters stay constant from year to year, or do they change in response to annual model changes, for example? Do the "loyal" markets of the firm differ in size and elasticity? Do they even exist? Is the feedback or residual demand ever contemplated? Is the market demand curve adequately represented as being linear over the range of prices charged?

Do collusive organizations maintain a higher index of oligopoly price than groups of firms which are merely consciously parallel? Are prices raised by similar proportions by all firms, in response to a price-leaders action, or subsequent to a general cost increase, for example? Does the index of oligopoly price vary over time in a particular industry? Does it vary across industries, and can this be related to some proxy variables for industry discipline and the extent of entry barriers? Does the structure of prices change when a firm introduces new (low-cost) plant, or when product innovation occurs? How much excess capacity do

firms desire to have? These, and other issues, would seem deserving of empirical investigation.

IV. Policy Conclusions

The index of oligopoly price may prove to be of some value for policy in the areas of price control and illegal price agreements, for example. If the index were calculated for a particular firm, and was seen to increase substantially due to reduced costs, with output constant, a price control board might establish grounds for demanding that prices decrease at the retail level to reflect this saving. Alternatively, if the index were calculated for a group of firms, and was found to be quite high, this might provide cause for an investigation regarding the possibility of collusive price-fixing.

To calculate the index, one would need to have information regarding the slopes of the ceteris paribus and mutatis mutandis demand curves in the vicinity of the actual price/sales coordinate of the firm. In addition the firm's marginal cost curve would need to be estimated. The former requirement would seem to pose the greater problem. Estimates of the market demand, plus an estimation of the parameter m for the particular firm may allow a satisfactory representation of the mutatis mutandis demand curve. With regard to the ceteris paribus demand curve, one may be able to extrapolate from recent experiences of unilateral price adjustments in that particular industry. Alternatively it could be estimated on the basis of the number and closeness of substitutes, for example.

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Abbreviations of Journal Titles

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| A.E.R. | American Economic Review |
| C.J.E. | Canadian Journal of Economics |
| E.J. | Economic Journal |
| E.R. | Economic Record |
| I.E.R. | International Economic Review |
| J.E.T. | Journal of Economic Theory |
| J.I.E. | Journal of Industrial Economics |
| J.P.E. | Journal of Political Economics |
| O.E.P. | Oxford Economic Papers |
| Q.J.E. | Quarterly Journal of Economics |
| Q.R.E.B. | Quarterly Review of Economics and Business |
| Restuds. | Review of Economic Studies |
| S.E.J. | Southern Economic Journal |
| W.E.J. | Western Economic Journal |

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