MODEL BASED FAULT DIAGNOSIS IN COMPLEX CONTROL SYSTEMS—ROBUST AND ADAPTIVE APPROACHES

by

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Abstract

This thesis deals with model based fault diagnosis problems for several classes of systems with complexities such as uncertainties and nonlinearities. To deal with system complexities, robust and adaptive approaches are used as the main tools. To focus more on fault isolation and estimation, novel observer and output estimator based fault diagnosis schemes are proposed.

Chapters 2 to 4 employ robust approaches to deal with complexities such as nonlinearities and nonparametric uncertainties. Robust observers, that is, Unknown Input Observers (UIOs) and Sliding Mode Observers (SMOs), are designed to solve fault diagnosis problems for Lipschitz nonlinear systems and Takagi-Sugeno fuzzy system represented uncertain nonlinear systems. UIO and SMO based fault diagnosis schemes, whose main novelty lies in the fault isolation, are proposed.

Chapters 5 and 6 also use robust approaches to attack more challenging complexities such as unmatched uncertainties. A novel idea which advocates output estimator design and abandons the state observer design is proposed. Robust output estimator based fault diagnosis schemes are developed for a class of linear systems with both matched and unmatched non-parametric uncertainties. The output estimator approach is extended to a more general class of linear systems, and a high-order sliding mode differentiator based actuator fault diagnosis scheme is designed, which is the first in fault diagnosis.

Chapters 7 and 8 use adaptive approaches to cope with complexities such as parametric uncertainties. Adaptive output estimator based fault diagnosis schemes are designed for sensor and actuator fault diagnosis problems in unknown linear Multi-Input Multi-Output (MIMO) and Multi-Input Single-Output (MISO) systems. A novel idea involving integration of fault isolation design functions into controller designs is put forward in actuator fault diagnosis.

The results in this thesis demonstrate that: 1) the proposed robust observer based fault diagnosis schemes are powerful in dealing with matched uncertainties and certain types of nonlinearities; 2) the proposed robust output estimator (and output derivative estimator) based fault diagnosis schemes are powerful in counteracting unmatched non-parametric uncertainties; and 3) the adaptive output estimator approach is very promising and powerful in coping with parametric uncertainties.

The thesis concludes by discussing important open problems for future research.

Keywords: Fault diagnosis; control systems; observer; output estimator; robust; adaptive

Dedication

To my dear wife Xiaoxiu Shi, my bright and handsome son Jeffrey Chen, and my lovely and beautiful daughter Jessica Chen.

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Table of Acronyms

AFIX	Actuator Fault Isolation Index
BJFDF	Beard-Jones Fault Detection Filter
DOS	Dedicated Observer Scheme
FDI	Fault Detection and Isolation
FDIA	Fault Detection, Isolation and Accommodation
FDIE	Fault Detection, Isolation and Estimation
FDP	Fault Detection Problem
FEP	Fault Estimation Problem
FIP	Fault Isolation Problem
FITI	Fault Isolation Time Interval
GOS	General Observer Scheme
HOSMRD	High-order Sliding-mode Robust Differentiator
IORIAFIX	Input-Output Relation Induced Actuator Fault Isolation Index
LME	Linear Matrix Equation
LMI	Linear Matrix Inequality
LPF	Low Pass Filter
MIMO	Multi-Input Multi-Output
MISO	Multi-Input Single-Output
NFD	Nonlinear Fault Diagnosis
NUIO	Nonlinear Unknown Input Observer
NUIOIFIX	NUIO Induced Fault Isolation Index
OEIAFIX	Output Estimator Induced Actuator Fault Isolation Index

PDC	Parallel Distributed Compensation
SFIX	Sensor Fault Isolation Index
SISO	Single-Input Single-Output
SMO	Sliding Mode Observer
SMOIAFIX	SMO Induced Actuator Fault Isolation Index
SMOISFIX	SMO Induced Sensor Fault Isolation Index
TS	Takagi-Sugeno
UIO	Unknown Input Observer
UIOC1	Unknown Input Observer Condition One
UIOC2	Unknown Input Observer Condition Two
UIOC3	Unknown Input Observer Condition Three
UIOIAFIX	UIO Induced Actuator Fault Isolation Index

Nomenclature

A	the system state matrix
$A_{f,s}$	the filter state matrix with respect to the set s
$\underline{A}_s$	the augmented state matrix for sensor FDI with respect to the set s
B	the system input matrix
$B_{f,s}$	the filter input matrix with respect to the set s
B_s	the matrix composed by those
	columns of B corresponding to the set s
$\bar{B}_s$	the complementary matrix of B_s
$\underline{B}$	the augmented input matrix for sensor FDI
$\underline{B}_s$	the augmented matrix related to y_s for sensor FDI
b_i	the ith column of B
C	the system output matrix
C_s	the matrix composed by those
	rows of C corresponding to the set s
$\bar{C}_s$	the complementary matrix of C_s
$\underline{\bar{C}}_s$	the output matrix of the augmented system for sensor FDI
C_m^l	the number of combinations of
	getting l elements from m elements
c_i	the ith row of C
D	the system unknown input matrix
$d(t)$	the unknown input or non-parametric uncertainty vector
$\underline{D}$	the augmented unknown input matrix
E_s	one of the UIO gain matrices dependent on the set s
e_s	the state estimation error dependent on the set s

$f(x)$	a nonlinear function
$\underline{f}(z_{aug,s})$	the augmented nonlinear function of $f(x)$ for sensor FDI
$\underline{f}(\hat{z}_{aug,s})$	the estimate of $\underline{f}(z_{aug,s})$
$\bar{G}_s$	one of the UIO gain matrices dependent on the set s
g_{num}	the number of residuals below the chosen threshold
I	the identity matrix
$i_1, \cdots, i_l$	integers belonging to either S_I or S_O
K_s	one of the UIO gain matrices dependent on the set s
$\bar{K}_s$	a matrix satisfying the LMI in (2.16)
L_s	one of the UIO gain matrices dependent on the set s
l	an integer belonging to either S_I or S_O
m	the number of inputs
M_s	one of the UIO gain matrices dependent on the set s
N_s	one of the UIO gain matrices dependent on the set s
n	the dimension of the system state
n_f	the number of faults
P_s	a symmetric positive definite matrix dependent on the set s
p	the number of outputs
Q_s	a symmetric positive definite matrix dependent on the set s
q	the number of unknown inputs
R^n	the n dimensional real Euclidean space
$R^{n\times m}$	the n by m real linear matrix space
$Re(\chi)$	the real part of a complex number χ
R_s	a positive definite design matrix dependent on the set s
S	a set
2^S	a set consisting of all the subsets of S
S_F	a set of faults
S_I	a set defined as $\{1, 2, \cdots, m\}$
S_O	a set defined as $\{1, 2, \cdots, p\}$
s	a set included in 2^S
s_j	a set included in 2^S

T_{detect}	the fault detection time
t	time
u	the input vector defined as $(u_1 \ \cdots \ u_m)^T$
u_i	the ith element of the control input vector u
u_s	the column vector composed by those elements of u corresponding to the set s
$\bar{u}_s$	the complementary vector of u_s
u^H	the healthy input vector defined as $(u_1^H \ \cdots \ u_m^H)^T$
u_s^H	the column vector defined the same way as u_s
$\bar{u}_s^H$	the column vector defined the same way as $\bar{u}_s$
$u_{i_j}^{fe}$	the estimate of the i_j actuator fault
V_s	a Lyapunov function dependent on the set s
$W1_s$	a matrix introduced in the LMI in (2.16)
$W2_s$	a matrix introduced in the LMI in (2.16)
$W3_s$	a matrix introduced in the LMI in (2.16)
X_s	a design matrix in UIO dependent on the set s
X^+	the pseudo inverse of matrix X
X_{11}	a matrix introduced in the LMI in (2.16)
X_{12}	a matrix introduced in the LMI in (2.16)
x	the state vector
$\dot{x}$	rate of change of x
x_i	the ith element of the state vector x
$\hat{x}$	the estimate of x
$\hat{x}_s$	the estimate of x dependent on the set s
Y_s	a design matrix in UIO dependent on set s
$\bar{Y}_s$	a matrix satisfying the LMI in (2.16)
y	the output vector
y_i	the ith element of the output vector y
y_s	the column vector composed by those elements of y corresponding to the set s
$\underline{y}_s$	the output vector of the augmented system for sensor FDI

$z_{aug,s}$	the state of the augmented system
	for sensor FDI with respect to the set s
z_s	the state of UIO in Chapter 2.
$\hat{z}_{aug,s}$	the estimate of $z_{aug,s}$
$z_0, z_1, \cdots, z_n$	the states of an nth sliding mode robust differentiator
α_i	functions in back-stepping controller design
β_i	functions in back-stepping controller design
γ	the Lipschitz constant
Δ	the length of the fault isolation time interval
η	the filter state related to system outputs
θ	the unknown parameter vector
$\hat{\theta}$	the estimate of the unknown parameter vector θ
θ^*	the optimal parameter vector in function appoximators
θ_s	unknown system parameter vector related to the set s
$\hat{\theta}_s$	the estimate of the unknown parameter vector θ_s
λ	the filter state related to control inputs
μ	a sliding mode term
μ_s	a sliding mode term dependent on the set s
ξ	the filter state related to system outputs
ξ_s	the filter state related to y_s
ρ, ρ_{1s}, ρ_{2s}	constants in sliding mode terms related to the set s
ρ_i	constant design parameters in sliding mode terms
τ_i	functions in back-stepping controller design
υ	the filter state related to control inputs
ϕ	the empty set
χ	a complex number

Chapter 1

Introduction

A typical control system, which is shown in Fig. 1.1, consists of actuators, sensors and a process to be controlled. Actuators are used to generate the desired inputs in order to control the process to behave as expected, while sensors provide all the measurements needed for computing the desired inputs and for monitoring the system. A practical control system is designed in such a way that the desired performances can be achieved when all actuators, all sensors, and all components of a process work normally.

Unfortunately, no real control systems are free of faults. In fact, actuators, sensors, and the components of a process in any control system may be faulty. Throughout this thesis, a *fault* is defined as any change in an actuator, sensor, or process component that leads to any undesired system performances (excluding a complete breakdown of the control system, which is defined as a *failure*).

When actuator faults occur, the faulty actuators are no longer able to generate the desired control inputs. Some examples of actuator faults are damage in bearings, deficiencies in force and momentum, defects in gears, aging effects, and stuck faults. When sensor faults occur, correct measurements needed for computing control inputs

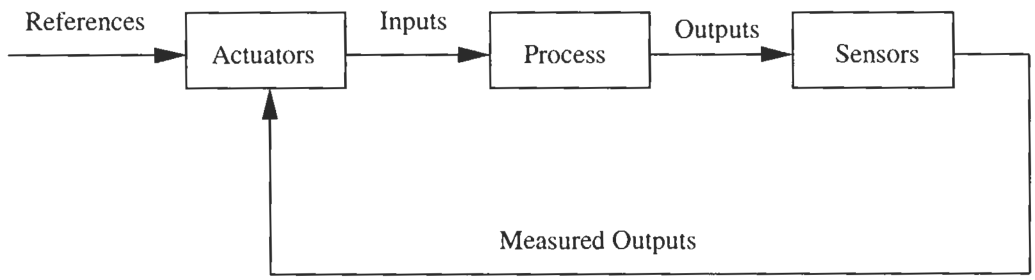

Figure 1.1: A typical control system

and for system performance monitoring can not be provided. Typical examples of sensor faults are scaling errors, drift, hysteresis, dead zone, and contact failures. When some components of the process are faulty, the original process has changed into a different process so that the controller designed for the original process is no longer able to achieve the expected system performance. Some examples of component faults are cracks, ruptures, leaks, loose parts, and abnormal system parameter variations.

Faults can lead to production deterioration or damages to machines that not only cost a vast amount of money, but can also lead to disasters that claim both property and human life. According to [1], the explosion at the Kuwait Petrochemical's Mina Alahmedi refinery in June, 2000 resulted in about 100 million dollars in damages. The paper also noted that minor accidents in the chemical industry cost billions of dollars every year. Much worse than the loss of money, aircraft accidents, due to faults in the control systems, may result in tragedies that make many families lose their loved ones. Some recent examples of such events are described in [2].

The growing demands for quality, cost efficiency, reliability, and human safety in modern control systems call for fault diagnosis. The research on fault diagnosis has attracted many people from civil and military industries as well as universities

[3, 4, 5, 6, 7, 8, 9, 10, 11], and interest in this research field is still increasing [1, 12].

Because various kinds of complexities (uncertainties and/or nonlinearities) are unavoidable in practical systems, any practical fault diagnosis should be carried out by taking complexities into consideration. Such complexities make fault diagnosis problems in control systems very challenging, and many fault diagnosis problems are still largely open. This observation, together with the great importance of fault diagnosis, motivates the research of this thesis. Robust approaches and adaptive approaches are chosen in this thesis work to deal with different kinds of complexities in control systems in order that better solutions could be provided for some inadequately solved or unsolved fault diagnosis problems.

1.1 Complexities in Control Systems

Because the fault diagnosis research is carried out for complex systems, some discussions on system complexities are presented in this section.

In control systems, uncertainties and nonlinearities are two basic types of complexities. Uncertainties can be divided into two classes: parametric uncertainties, which are characterized in terms of unknown parameters, and non-parametric uncertainties, which include modelling errors and disturbances. Nonlinearities could be classified as special nonlinearities (e.g., Lipschitz type nonlinearity and bilinear type nonlinearity), and general nonlinearities.

The presence of uncertainties and nonlinearities in control systems constitutes a major challenge to model based fault diagnosis. For example, the existence of uncertainties, even in linear systems, makes the observer design very difficult or sometimes impossible to achieve exact state estimation [13]. The difficulty encountered in observer design for nonlinear systems is well known. So far, nonlinear observer design can only be accomplished systematically for some special classes of nonlinear systems; for example, Lipschitz systems [14, 15, 16], bilinear systems [17, 18, 19, 20], linearizable systems [21, 22], and other special types of nonlinear systems [23, 24, 25]. For general nonlinear systems, no universal method for observer design is available.

To see the complexities in control systems more clearly, a classification of control systems based on nonlinearity and uncertainty is given in Figure 1.2.

1.2 The Tasks of Fault Diagnosis and Related Problems

Given a complex system, the tasks of fault diagnosis considered in this thesis are fault detection, fault isolation and fault estimation, which are defined as follows:

- **Fault detection** is to make a decision on whether or not faults have occurred in control systems;
- **Fault isolation** determines the number and the location of faults; and
- **Fault estimation** estimates the faults.

This thesis investigates the following problems that are closely related to the tasks of fault diagnosis:

- **Fault detection problems:**
  - **FDP1** Is fault detection possible?
  - **FDP2** How to detect the faults?
- **Fault isolation problems:**

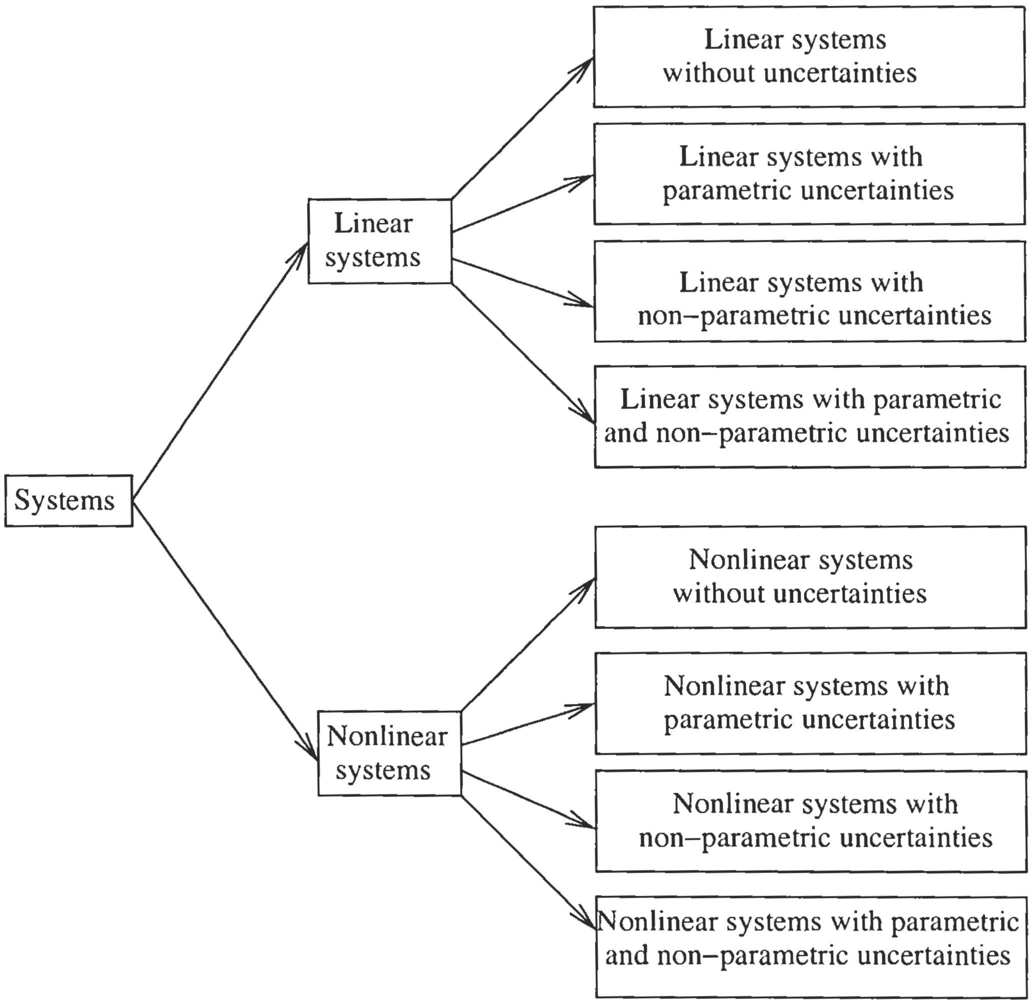

Figure 1.2: System classification based on complexities

- **FIP1** Is fault isolation possible?
- **FIP2** How many faults can be isolated simultaneously?
- **FIP3** How to design fault isolation schemes to isolate single/multiple faults?

- **Fault estimation problems:**
  - **FEP1** Is it possible to estimate the faults?
  - **FEP2** How to estimate the faults?

Because fault detection is needed in any practical control system such as a car engine, it has been studied extensively in the literature. As already shown in the literature, fault detection problems, usually easier than fault isolation and estimation problems, are solved better than fault isolation and estimation problems [1, 3, 4, 5, 6, 7, 8, 9, 10].

Fault isolation, although almost equally important as fault detection, has received much less attention. Some important results are found in [1, 3, 4, 5, 6, 7, 8, 9, 10] and the references listed therein. As noted in several recently published works [13, 26, 27, 28, 29], although solutions have been provided for fault isolation problems for some control systems such as aircraft control systems, certain fault isolation problems are not solved satisfactorily and there are still open problems for many other complex control systems.

Fault estimation used to be regarded as less important [9] than fault detection and isolation and has been studied even less. However, it is very useful for fault accommodation and fault tolerant control [11] in aircraft control systems, and is gaining more interest because some fault estimation techniques can be used directly for both fault detection and fault isolation. Some examples using the sliding mode based fault estimation techniques have been proposed in [27, 30, 31], but much work remains to be completed.

1.3 The Purpose of This Thesis

This thesis is to solve model based fault diagnosis problems defined in Section 1.2 for several classes of systems with complexities such as uncertainties and nonlinearities. In accordance with Fig. 1.2, the systems considered include nonlinear systems with matched non-parametric uncertainties (formerly called unknown inputs in the literature), linear systems with both matched and unmatched non-parametric uncertainties, and linear systems with parametric uncertainties.

The fault diagnosis problems will be solved based on observer design as well as output estimator design. The main tools used to deal with system complexities are robust and adaptive approaches. Based on the observation that fault detection is solved better than fault isolation and estimation, the research of this thesis will focus more on fault isolation and estimation problems.

1.4 Model Based Fault Diagnosis–A Literature Review

Fault diagnosis methods may be classified into two major groups: model-free methods and model based methods. The advantages and disadvantages of these methods can also be found in [9].

According to [9], model-free fault diagnosis methods include: **physical redundancy**, **special sensors installed for fault diagnosis purpose**, **limit checking**, **spectrum analysis**, and **knowledge based logic reasoning**.

Model-free fault diagnosis method can not capture the system dynamics sufficiently, and thus can not be used in fault diagnosis problems of systems with rich dynamics such as fault diagnosis of a car engine which undergoes frequent start and stop. Model based fault diagnosis makes use of both quantitative mathematical models and qualitative models. Because this thesis is devoted to deterministic quantitative mathematical model based methods, a detailed review will only be given on research in this area. For simplicity, in the rest of this thesis, the word *model* is used to stand for *a deterministic quantitative mathematical model*.

1.4.1 Model Based Fault Diagnosis Methods

It is widely accepted that model fault diagnosis consists of two stages: residual generation and decision making based on residual evaluation [32]. The residuals are generated for fault diagnosis purpose. Corresponding to different residual generation techniques, model based fault diagnosis methods that have been developed in the literature can be divided into four groups:

- parity space approach
- parameter estimation approach
- observer based approach
- direct output estimator based approach

Because neither a parity space approach nor a parameter estimation approach is used in this thesis, no review is given here for these two approaches. However, researchers interested in the parity space approaches can consult [6, 9, 33] and those interested in the parameter estimation approaches can check [4, 12].

Both observer based approach and direct output estimator based approach are employed in this thesis. A detailed review of each approach will be given in the following two subsections.

1.4.2 Observer Based Fault Diagnosis

Observer-based approach is the most extensively used method in model based fault diagnosis. Various types of observers have been proposed for fault diagnosis purposes: the Beard-Jones fault detection filter, the unknown input observer, the sliding mode observer, the adaptive observer, the H_∞ observer, and the iterative learning observer.

In this subsection, only four types of observer based fault diagnosis will be reviewed: the Beard-Jones fault detection filter based fault diagnosis, the unknown input observer based fault diagnosis, the sliding mode observer based fault diagnosis, and the adaptive observer based fault diagnosis. Readers interested in the H_∞ observer based fault diagnosis are referred to [34, 35, 36], and those interested in the iterative learning observer based fault diagnosis are referred to [37, 38, 39].

1. **Beard-Jones Fault Detection Filter (BJFDF) Based Fault Diagnosis**

   In [40], fault detection filter was first proposed to generate directional residuals for linear systems without uncertainties. The main idea of the BJFDF is that each directional residual is designed in correspondence to a particular fault or a particular group of faults. This approach was refined in a geometric framework in [41] and [42]. The design problem of BJFDF was later investigated in [43, 44, 45, 46]. Since 1990s, various extensions of the BJFDF have been conducted including the robust BJFDF design in [35, 47, 48, 49], the BJFDFs for singular perturbed systems and time delay systems in [50] and [36], and the BJFDFs for Lipschitz nonlinear systems and affine nonlinear systems in [51, 52] and [53].

2. **Unknown Input Observer (UIO) Based Fault Diagnosis**

In this thesis, the terms *unknown inputs* and *non-parametric uncertainties* will be used interchangeably. The design of observers for systems subject to unknown inputs has attracted considerable attention in the past, and many types of UIOs are now available. Reduced order linear UIOs are designed in [54, 55, 56, 57], while full order linear UIOs have been designed in [58] and [59]. UIOs for nonlinear systems were designed in [18, 19, 20, 60, 61, 62, 63, 64, 65, 66, 67], where [18, 19, 20, 60] considered bilinear systems, [61, 62, 63, 64] were devoted to Lipschitz nonlinear systems, and [65, 66, 67] attempted designs for more classes of nonlinear systems.

In order to accomplish fault diagnosis efficiently for systems with uncertainties, generating residuals that are insensitive to those uncertainties is desirable. If uncertainties are treated as unknown inputs, UIOs can be readily used for fault diagnosis. Amongst the various robust fault diagnosis schemes, the UIO based fault diagnosis scheme is one of the schemes that have been studied the most extensively (see [29, 61, 62, 66, 68, 69, 70, 71, 72, 73, 74, 75] and the related references listed therein). Many existing fault diagnosis schemes based on UIO were proposed only for linear uncertain systems [69, 70, 71, 72, 74, 75]. Developing nonlinear robust fault diagnosis schemes based on UIO has been attempted. A UIO based fault diagnosis scheme for bilinear systems was proposed in [73]; UIO based fault diagnosis schemes were designed in [29, 62, 61] for Lipschitz nonlinear systems; and nonlinear UIO based fault diagnosis schemes have been proposed in [66, 68] for a more general class of nonlinear systems that are in a suitable form or can be transformed into that structure.

The main difficulty in nonlinear UIO based fault diagnosis is the design of nonlinear UIOs, because no systematic design method is available for general uncertain nonlinear systems. Besides the design difficulty, most existing UIO based schemes assume that the fault distribution matrix is known, which is often not the case for fault isolation problems, and many of them are only devoted to fault detection or single fault isolation. Even for linear systems, the fault diagnosis problems raised in Section 1.2 have not been solved completely, and this fact motivated the research in [29], where a relatively complete solution was provided for Lipschitz nonlinear systems. As for general nonlinear systems, solving the fault diagnosis problems using UIO design is still largely open.

3. **Sliding Mode Observer (SMO) Based Fault Diagnosis**

   Because sliding mode observers (SMOs) are robust to uncertainties, they can be used in robust fault diagnosis.

   In general, the SMO based fault detection and isolation (FDI) techniques are classified into two categories. The first category uses SMOs to make the output estimation error insensitive to uncertainties, but sensitive to faults ([27, 76, 77, 78]). [76, 77, 78] only considered the fault detection problem, while a scheme in [27] focused on the fault isolation problem.

   The second category employs SMOs to reconstruct or estimate the faults [27, 30, 31, 79, 80, 81, 82]. In [30, 31, 79, 80], fault detection and isolation problems for linear systems were solved under the assumption that the fault distribution matrix is known. In [78, 81], the solutions for fault detection and isolation problems were provided for nonlinear systems under structural constraints. Again, the distribution of faults is assumed to be known, and the construction of the state transformation for nonlinear systems is not an easy task.

In [27], two schemes were proposed for a class of uncertain Lipschitz nonlinear systems to remove the need for knowing the distribution of faults. However, the assumption that all the system inputs can be reconstructed may not be possible for some systems. This assumption is removed in [82]. Since Lipschitz nonlinear systems are only a restricted type of special nonlinear systems, designing SMOs to solve fault diagnosis problems for general nonlinear systems still remains to be solved.

4. **Adaptive Observer Based Fault Diagnosis**

Although many adaptive observers have been designed for both linear [83, 84, 85] and nonlinear systems [86, 87, 88, 89, 90, 91, 92], the adaptive observer design for an unknown linear MIMO system remains unsolved because none of the existing adaptive observers are applicable.

In the fault diagnosis community, two types of adaptive observer based fault diagnosis schemes have been proposed. One type assumes that the systems (or nominal systems) are known, and faults can be properly parameterized. The works in [93, 94, 95, 96] belong to this type, where persistent excitation conditions are required. The schemes in [97, 98, 99] belong to this type too, where a compact convex region to which the unknown parameter vector θ^* belongs needs to be determined using some knowledge about the faults.

Another type deals with systems with unknown parameters and does not make assumptions on the faults. The works in [73, 100, 101, 102, 103] belong to this type. Although [100, 103] considered nonlinear systems, how to apply these adaptive schemes to unknown linear systems is not clear. The only adaptive observer fault diagnosis scheme for unknown linear systems was proposed in

[102], where a proportional-integral adaptive observer was designed for fault diagnosis for single-input single-output (SISO) linear systems. For general unknown multi-input multi-output (MIMO) linear and nonlinear systems, how to use adaptive approaches to solve the related fault diagnosis problems is still an open problem.

1.4.3 Direct Output Estimator Based Fault Diagnosis

A necessary assumption for the observer based fault diagnosis is that systems under consideration are observable or at least detectable. When the systems under study are not detectable, observer design is impossible, and thus the observer based approach cannot be used for fault diagnosis.

Another limitation of observer based fault diagnosis is that asymptotical state estimation using an observer is sometimes impossible even for linear observable systems whose unknown inputs do not satisfy certain matching conditions [13]. The unknown inputs, which do not satisfy certain matching conditions, is termed as unmatched unknown inputs in this thesis. If unmatched unknown inputs are present, observer based fault diagnosis using asymptotical state estimation might not be possible.

One well known approach that could be used for fault diagnosis of systems not detectable is the parity space approach. It was first developed for discrete-time systems in [32, 104], and was later extended to continuous systems in [105]. When the systems have parametric uncertainties, parity space approach is very hard to use if not impossible. Note that the functional observers developed in [106] are actually a generalized output estimator based on the rather complicated special coordinate basis transformation in [107]. This thesis uses different approaches to achieve output estimator design.

Because only output estimators are actually needed for fault diagnosis purpose, it is possible to abandon the idea of observer design through employing the idea of direct output estimator design. The idea of direct output estimator design for fault diagnosis was first proposed and studied systematically in [13] for a class of linear systems with unmatched unknown inputs. This idea was extended in [108], where direct estimators for both the outputs and their derivatives were designed for the purpose of fault diagnosis.

Because the existence of a direct output estimator does not necessarily require the systems under study to be detectable, direct output estimator based fault diagnosis removes the assumption needed for observer based fault diagnosis. This idea is particularly useful for adaptive fault diagnosis. Using the idea, sensor fault diagnosis problems are solved elegantly for MIMO linear systems with parametric uncertainties [109], and actuator fault diagnosis problems are solved also for multi-input-single-output (MISO) linear systems with parametric uncertainties [110]. Because the direct output estimator based approach is a novel approach developed only very recently, much work is needed for linear systems with both parametric and non-parametric uncertainties as well as for various types of uncertain nonlinear systems. Direct output estimator based fault diagnosis will gain more popularity and make more contributions to model based fault diagnosis in the future.

1.5 Thesis Contributions

The contributions of the thesis are summarized below.

1. **Fault diagnosis of nonlinear systems with matched non-parametric uncertainties–Robust observer based approach**

- For a class of Lipschitz nonlinear systems with matched non-parametric uncertainties, a novel UIO is proposed. To ease the design difficulty, a Linear Matrix Inequality (LMI) based UIO design approach is developed. By employing a bank of the proposed UIOs, a UIO based robust fault diagnosis scheme is proposed, which provides solutions for the actuator fault detection and isolation problems raised in Section 1.2. The scheme, when applied to linear systems, is also new.

- For the same class of uncertain Lipschitz nonlinear systems, an SMO based robust fault diagnosis scheme is designed in a parallel manner. Unlike the UIO based scheme, which does not solve the fault estimation problems, the SMO based approach is able to solve all the problems raised in Section 1.2 for actuator faults.

- For a class of nonlinear systems with matched non-parametric uncertainties and that can be represented by Takagi-Sugeno (TS) fuzzy models, a UIO based robust fault diagnosis scheme is constructed with the intention to extend the ideas employed in the UIO based scheme for Lipschitz nonlinear systems to more general nonlinear systems. The design of the UIO is more difficult and is formulated as an LMI problem in order to ease the design difficulty. Both the actuator fault detection and isolation problems are solved.

2. **Fault diagnosis of linear systems with both matched and unmatched non-parametric uncertainties–Robust direct output estimator based approach**

- For a class of linear systems with both matched and unmatched non-parametric uncertainties and with relative degree one, a canonical system form is first established to split the non-parametric uncertainties into matched and unmatched uncertainties. Based on the canonical system form, a robust actuator fault diagnosis scheme based on the direct output estimator design is proposed using sliding mode techniques. It provides solutions to all the problems raised in Section 1.2, and its advantage is that it can be applied to certain systems where observers can not be designed to achieve asymptotical state estimation.

- For a more general class of linear systems, which have both matched and unmatched non-parametric uncertainties, a relative degree larger than one, and are not necessarily detectable, an input-output relation is derived. By extending the idea of direct output estimation to the direct estimation of both the outputs and their derivatives and by employing the input-output relation, a robust fault diagnosis scheme based on direct estimation of outputs and their derivatives is designed for actuator fault diagnosis, which is the first scheme using robust high-order sliding-mode robust differentiators (HOSMRDs). The scheme is able to solve all the problems raised in Section 1.2 for actuator faults. Its advantage is that it can be applied to systems that are not detectable, where observer based fault diagnosis schemes are impossible to use.

3. **Fault diagnosis of linear systems with parametric uncertainties–Adaptive direct output estimator based approach**

- For a class of linear multi-input multi-output (MIMO) systems with unknown system parameters, a new fault diagnosis scheme is proposed for adaptive sensor fault detection and isolation problems. The scheme abandons the idea of designing adaptive observers to estimate all the states and employs the design of adaptive output estimators to estimate only the outputs. Firstly, an MIMO system is decomposed into a group of MISO systems and a transfer function description for each MISO system is presented. Secondly, inspired by [83, 111] and based on each transfer function as well as for each output, an output equation, suitable for output estimator design, is derived by filtering the corresponding output and all the inputs properly. Thirdly, using the derived output equations, adaptive output estimators are designed for all outputs. Finally, based on the designed output estimators, the adaptive sensor fault detection and isolation problems are solved. The proposed fault diagnosis scheme enables us to treat each output separately, and thus makes the difficult sensor fault isolation problem an easy task. It does not require the original systems to be detectable. No such scheme has been proposed even for known linear MIMO systems in the literature.

- Actuator fault diagnosis in linear systems with unknown system parameters is much harder than sensor fault diagnosis, which is why an adaptive actuator fault diagnosis scheme is designed only for unknown linear MISO systems. The original systems do not have to be detectable, and the designed scheme is even new for known linear systems. Again, the scheme abandons the idea of designing adaptive observers to estimate all the states and employs the design of an adaptive output estimator to estimate only the output. To solve the detection problem, an adaptive estimate of the output signal is constructed. By comparing it with the output of the system, any type of actuator fault can be detected. In order to solve the much more complicated fault isolation problems using an adaptive approach, only constant actuator faults are considered, which arise when the actuator output (such as a valve) is stuck at some fixed value. A novel idea which entails controller design for fault isolation is proposed. Thus, the controller in this case is not only designed to meet the control objective, but also to help with fault isolation, in case of an actuator failure. To accomplish this, assuming that there are m inputs, a group of additive functions, called *fault isolation design functions*, in $m-1$ inputs is introduced solely for fault isolation purpose. Assume that only fewer than $m-1$ faults can occur, to isolate the faults, $C_m^1 + \cdots + C_m^{m-1}$ adaptive estimates of the output are defined. Isolation is accomplished by comparing these estimates with the output of the actual system.

1.6 Thesis Outline

The remainder of this thesis is organized as follows: Chapter 2 proposes a UIO based robust fault diagnosis scheme for a class of Lipschitz nonlinear systems with matched non-parametric uncertainties, which solves the fault detection and isolation problems. In a parallel manner, Chapter 3 develops an SMO based robust fault diagnosis scheme for the same class of systems considered in Chapter 2. The scheme not only solves the fault detection and isolation problems, but also the fault estimation problems. Chapter 4 designs a UIO based robust fault diagnosis for a class of uncertain nonlinear systems, which has matched non-parametric uncertainties and can be represented by TS fuzzy models. The scheme is able to provide solutions for the fault detection and isolation problems. Robust approaches are used as tools to deal with matched non-parametric uncertainties in these three chapters, and all fault diagnosis schemes are based on observer design. Chapter 5 is concerned with systems with both matched and unmatched non-parametric uncertainties. By developing a canonical system form, which separates the matched and unmatched uncertainties explicitly, an output estimator, other than a state observer based fault diagnosis scheme, is constructed using robust approaches. The scheme provides solutions for all the problems raised in Section 1.2. In Chapter 6, the main ideas in Chapter 5 are extended to more general uncertain linear systems, where a robust actuator fault diagnosis scheme is designed based on an input-output relation and the use of robust high-order sliding-mode differentiators. The designed scheme again solves all the problems raised in Section 1.2. Adaptive approaches are used in Chapter 7, where an adaptive sensor fault diagnosis scheme is presented for linear MIMO systems with parametric uncertainties. Based on a novel idea called *controller design for fault diagnosis*, Chapter 8 proposes an adaptive actuator fault diagnosis scheme for linear MISO systems with parametric uncertainties. Finally, Chapter 9 provides conclusions and future works.

1.7 Publication Notes

All the works in this thesis have been either published or submitted for publication. The main results on fault diagnosis for a class of Lipschitz nonlinear systems with matched non-parametric uncertainties in Chapter 2 was published in [29]. The works based on the direct estimation of outputs, presented in Chapter 5, appeared in [13]. The fault diagnosis scheme based on the direct estimation of outputs and their derivatives was reported in [108]. Chapter 8 is adapted from the paper in [110], which has been published in the refereed journal *International Journal of Control.*

The work reported in Chapter 3 was submitted to a refereed journal,revised according to the reviewers' comments, and is pending for publication. The works accomplished in Chapter 4 and Chapter 7 have been accepted by a refereed conference.

Chapter 2

UIO Based Fault Diagnosis for Uncertain Lipschitz Nonlinear Systems

In this chapter, the fault diagnosis problems for a class of Lipschitz nonlinear systems with matched non-parametric uncertainties are considered using a novel UIO design.

2.1 Introduction

In the monitoring and diagnostic of complex dynamical systems that are subject to non-parametric uncertainties, robust approaches are usually employed. A robust fault diagnosis scheme is a procedure that can generate residuals that are sensitive to faults, but insensitive to uncertainties and/or unknown disturbances.

To deal with various types of unknown inputs (or non-parametric uncertainties), two strategies using robust approaches have been developed. One strategy is to remove the effect of the unknown inputs completely by designing fault diagnosis schemes that are invariant to the unknown inputs. Schemes based on the design of unknown input observers (UIOs) and sliding mode observers (SMOs) adopt this strategy. The other strategy is to attenuate the effect of the unknown inputs to a minimum level in certain sense; i.e., minimizing the H^∞ gain of the unknown inputs. Generally, this strategy will lose the invariant property to matched unknown inputs.

The UIO based robust FDI problem has been studied extensively; however, most existing UIO based fault diagnosis schemes were proposed only for linear uncertain systems [71, 75, 70, 72, 69, 74]. Built upon the reduced order UIO design proposed in [56] and for a broad class of faults that can be represented by a state space model, fault detection and estimation problems were solved successfully in [69]. Through proper state transformations, [70] was able to decompose the original system into two subsystems. A reduced order UIO was designed and used to solve component and actuator fault isolation problems. Similar to [70], [72] presented a new method to design reduced order UIOs and designed a bank of UIOs to isolate one single fault. Using a special canonical form obtained also by state transformation, a reduced order UIO was designed easily in [74], and a particular actuator fault and sensor fault isolation problem was solved successfully. A full order UIO was designed using a parametric approach for robust fault detection in [75].

The development of robust fault diagnosis schemes based on nonlinear UIO design has been attempted. A fault diagnosis scheme based on reduced-order UIO design for bilinear systems was proposed in [73]. [61] extended linear UIO design to a class of Lipschitz nonlinear systems and developed sufficient condition for the existence of the proposed UIOs using linear matrix inequalities (LMIs) and linear matrix equalities (LMEs). Then, by treating the actuator faults as unknown inputs, a bank of UIOs was designed to isolate one single actuator fault. However, finding a solution that satisfies the LMIs and LMEs is not an easy task. Assuming the fault distribution matrix is known (though often not the case for a fault isolation problem), [62] also proposed a UIO based fault diagnosis scheme that required to solve a more difficult parametric Lyapunov equation. Sensor fault diagnosis for a class of uncertain Lipschitz nonlinear systems was considered in [112], where LMI technique was used to design the observer, but disturbances were not taken into consideration. For a more general class of nonlinear systems that are in a suitable form or can be transformed into that structure, [66, 68] proposed a fault diagnosis scheme based on a bank of nonlinear UIOs.

Besides the design difficulty, most existing UIO based schemes assume that the fault distribution matrix is known, which is often not the case for fault isolation problems. Many schemes are only devoted to fault detection or single fault isolation. Moreover, if not properly designed, existing UIO based schemes will fail to isolate a single fault or even to detect faults . Therefore, even for linear systems, the fault diagnosis problems raised in Section 1.2 have not been solved completely. This observation motivated the research in this chapter, where a relatively complete solution is provided for the fault diagnosis problems of a class of uncertain Lipschitz nonlinear systems. Because uncertain linear systems can be viewed as special cases of uncertain Lipschitz nonlinear systems, the proposed fault diagnosis scheme can be applied to uncertain linear systems.

The remainder of this chapter is arranged as follows. In Section 2.2, the system is described, the problems are formulated, and then particular system structures are developed for the sake of both actuator and sensor fault diagnosis. In Section 2.3, a novel diagnostic UIO with a special property suitable for fault isolation purposes is proposed with the necessary condition and sufficient conditions for its existence. The

LMI based sufficient condition provides a systematic way to design the UIO using the LMI toolboxes. Based on a new concept, which is called *UIO Induced Actuator Fault Isolation Index (UIOIAFIX)*, Section 2.4 solves the actuator fault diagnosis problems using the novel UIO design technique. In Section 2.5, two examples illustrate the design of the proposed fault diagnosis scheme and how to test effectiveness of the scheme. One example considers Lipschitz nonlinear systems with non-parametric uncertainties while the other considers a practical example, where a linearized model of a tailless jet fighter taken from [113] is used. Conclusions and discussions are made in the last section.

2.2 Problem Formulation and Particular System Structures for Fault Diagnosis

2.2.1 Problem Formulation

The uncertain nonlinear systems considered are of the following form

$$\begin{aligned} \dot{x} &= Ax + f(x) + Bu + Dd \\ y &= Cx \end{aligned} \tag{2.1}$$

where the state vector $x = (x_1 \ \cdots \ x_n)^T \in R^n$, the output vector $y = (y_1 \ \cdots \ y_p)^T \in R^p$, and the input vector $u = (u_1 \ \cdots \ u_m)^T \in R^m$. $f(x)$ is a known vector function of x, and $d \in R^q$ is the unknown input vector which may consist of disturbances and/or other system uncertainties. A is the system state matrix in $R^{n\times n}$, B is the system input matrix in $R^{n\times m}$, C is the system output matrix in $R^{p\times n}$, and D is the system unknown input matrix in $R^{n\times q}$. For notational convenience, let $B = (b_1 \ \cdots \ b_m)$ and $C = (c_1^T \ \cdots \ c_p^T)^T$.

The following assumptions are needed:

- Assumption $A21$: A, B, C, D are known, both B and D are of full column rank, and $p \geq q$.

- Assumption $A22$: For $f(x)$, a positive constant γ exists such that

$$\|f(x) - f(\hat{x})\| \leq \gamma \|x - \hat{x}\| \tag{2.2}$$

  for all $x, \hat{x}$.

Remark 2.2.1 *$f(x)$ satisfying A22 is said to be a Lipschitz function. The inequality (2.2) is the well known Lipschitz condition. Although Lipschitz nonlinear systems are a restricted class of nonlinear systems, they still represent a broader class of systems, which include linear systems as special cases(corresponding to $f(x) = 0$). Given the fact that most fault diagnosis has been studied for linear systems, it is not so restrictive to study the fault diagnosis problems of Lipschitz nonlinear systems. Also note that some general nonlinearities can be treated as unknown inputs, therefore (2.1) actually includes a fairly broad class of uncertain systems.*

Two fault diagnosis problems are formulated as below:

- **Actuator fault detection and isolation (FDI) problems** – Assume that only actuator faults can occur, the objective is to carry out a systematic study on the fault detection and isolation problems in Section 1.2.

- **Sensor FDI problems** – Assume that only sensor faults can occur, the objective is to carry out a systematic study on the fault detection and isolation problems in Section 1.2.

2.2.2 A Particular System Structure for Actuator FDI

For notational simplicity, throughout this thesis, let ϕ denote the empty set, and 2^S denote the set consisting of all subsets of a given set S. Additionally, two sets are defined as $S_I = \{1, 2, \cdots, m\}$ and $S_O = \{1, 2, \cdots, p\}$.

To develop a particular system representation for actuator FDI, for any $s = \{i_1, \cdots, i_l\} \in 2^{S_I}$ with $1 \leq l \leq m$, denote $B_s = (b_{i_1} \ \cdots \ b_{i_l})$, and define $\bar{B}_s$ as the complementary matrix of B_s consisting of the remaining columns of B. Similarly, denote $u_s = (u_{i_1} \ \cdots \ u_{i_l})^T$ and $\bar{u}_s$ as a column vector consisting of the remaining components of u.

Now, by rewriting (2.1), a particular system structure is obtained as follows:

$$\begin{aligned} \dot{x} &= Ax + f(x) + \bar{B}_s \bar{u}_s + B_s u_s + Dd, \\ y &= Cx. \end{aligned} \tag{2.3}$$

Remark 2.2.2 *This system structure is obtained by regrouping the system inputs. It allows the designer to treat any combination of inputs as unknown inputs. By treating each of the C_m^l combinations of inputs in u_s as unknown inputs, the system structure is especially convenient for fault isolation.*

2.2.3 A Particular System Structure for Sensor FDI

Similarly, to develop a particular system structure for sensor FDI, for any $s = \{i_1, \cdots, i_l\} \in 2^{S_O}$ with $1 \leq l \leq p$ and $s \neq \phi$, denote $C_s = (c_{i_1}^T, \cdots, c_{i_l}^T)^T$, and define $\bar{C}_s$ as the complementary matrix of C_s consisting of the remaining rows of C. Also denote $y_s = (y_{i_1} \ \cdots \ y_{i_l})^T$ and $\bar{y}_s$ as a vector consisting of the remaining components of y.

As in [31], y_s is filtered as

$$\dot{\xi}_s = A_{f,s}\xi_s + B_{f,s}y_s \tag{2.4}$$

where $A_{f,s}$ is chosen to be Hurwitz, and $A_{f,s}$ ($\in R^{l\times l}$) and $B_{f,s}$ are chosen as any invertible matrices in $R^{l\times l}$.

By defining $z_{aug,s} = (x^T \; \xi_s^T)^T$, $\underline{y}_s = (\bar{y}_s^T \; \xi_s^T)^T$, and using (2.1) and (2.4), a particular system structure is obtained as

$$\begin{aligned} \dot{z}_{aug,s} &= \underline{A}_s z_{aug,s} + \underline{f}(z_{aug,s}) + \underline{B}u + \underline{B}_s y_s + \underline{D}d \\ \underline{y}_s &= \underline{\bar{C}}_s z_{aug,s} \end{aligned} \tag{2.5}$$

where

$$\begin{aligned} \underline{A}_s &= \begin{pmatrix} A & 0 \\ 0 & A_{f,s} \end{pmatrix}, \; \underline{B} = \begin{pmatrix} B \\ 0 \end{pmatrix}, \; \underline{B}_s = \begin{pmatrix} 0 \\ B_{f,s} \end{pmatrix}, \\ \underline{D} &= \begin{pmatrix} D \\ 0 \end{pmatrix}, \; \underline{\bar{C}}_s = \begin{pmatrix} \bar{C}_s & 0 \\ 0 & I \end{pmatrix}, \; \underline{f}(z_{aug,s}) = \begin{pmatrix} f(x) \\ 0 \end{pmatrix}. \end{aligned} \tag{2.6}$$

Remark 2.2.3 *This system structure is obtained by regrouping and filtering the outputs. If y_s is treated as unknown inputs, it is easy to see that (2.3) and (2.5) actually have the same system structure. This observation is sufficient to develop UIO based schemes only for actuator FDI because, with only a few slightly different matrix manipulations, sensor FDI can be solved using the same schemes. Therefore, in the rest of this chapter, only actuator FDI is considered.*

2.3 A Novel Nonlinear Diagnostic UIO

As stated in Remark 2.2.3, because considering actuator FDI problems is sufficient, a novel diagnostic nonlinear UIO is only proposed for system (2.3). In this section, sufficient conditions for its existence are presented, and an LMI based sufficient condition is derived for the purpose of UIO design.

2.3.1 A Diagnostic UIO

For any s, it is desired to design a UIO such that only the inputs in u_s, besides d, are treated as unknown inputs. In this way, the state estimation error will be insensitive to the $i_1 \cdots i_l$th actuator faults, but sensitive to any other actuator faults. Because the UIOs designed is specially for the purpose of fault diagnosis, it is called a diagnostic observer.

u^H is defined as the healthy actuator output vector; that is, when all actuators are healthy, $u^H = u$, otherwise, $u^H \neq u$. Let u_s^H and $\bar{u}_s^H$ be defined in the same way as u_s and $\bar{u}_s$, respectively. By treating u_s as an unknown input vector, a diagnostic UIO for (2.3) is introduced as follows:

$$\begin{aligned} \dot{z}_s &= N_s z_s + \bar{G}_s \bar{u}_s^H + L_s y + M_s f(\hat{x}_s) \\ \hat{x}_s &= z_s - E_s y \end{aligned} \tag{2.7}$$

where $N_s, \bar{G}_s, L_s, M_s$ are defined as

$$\begin{aligned} N_s &= M_s A - K_s C, \\ \bar{G}_s &= M_s \bar{B}_s, \\ L_s &= K_s(I + CE_s) - M_s A E_s, \\ M_s &= I + E_s C. \end{aligned} \tag{2.8}$$

By defining $e_s = \hat{x}_s - x$, the following is easy to derive:

$$\dot{e}_s = N_s e_s + M_s(f(\hat{x}_s) - f(x)) + \bar{G}_s(\bar{u}_s^H - \bar{u}_s) - M_s B_s u_s - M_s D v. \tag{2.9}$$

Clearly, all the observer gain matrices defined by (2.8) are determined by E_s and K_s. For fault diagnosis purposes, E_s and K_s should be chosen such that the observer given by (2.7) and (2.8) satisfies the following requirements:

- UIO Condition 1 (UIOC1) $M_s(D \;\; B_s) = 0$.
- UIO Condition 2 (UIOC2) $\bar{G}_s$, that is, $M_s\bar{B}_s$, is of full column rank.
- UIO Condition 3 (UIOC3) N_s is Hurwitz.

The following remark presents some discussions on the above requirements.

Remark 2.3.1 *If $UIOC2$ is not satisfied, that is, $\bar{G}_s$ is not of full column rank, then e_s is not affected by any faults such that $\bar{u}_s^H - \bar{u}_s \neq 0$ and $\bar{G}_s(\bar{u}_s^H - \bar{u}_s) = 0$. In such a case, correct fault isolation cannot be made based on a bank of UIOs. This implies that existing UIO based fault isolation schemes (none of which have such a condition) may fail if not properly designed. Existing fault detection schemes based on a UIO may encounter the same problem (that is, faults may not be detected). This is the reason why $UIOC2$ is needed to improve the performance of the proposed FDI scheme.*

The novelty of the proposed diagnostic observer is discussed in the following remark.

Remark 2.3.2 *The novelty of the diagnostic observer is that 1) all combinations of the inputs can be treated as unknown inputs; 2) $UIOC2$ is required particularly for fault diagnosis purposes; and 3) $\bar{u}_s^H$ (instead of $\bar{u}_s$ as in conventional UIO design) is also used in the observer design for purpose of fault diagnosis.*

2.3.2 Conditions for the Existence of the UIO

In this subsection, a necessary condition for the existence of the UIO given by (2.7) and (2.8) is provided first. Then, sufficient conditions are derived, and an LMI based sufficient condition is given to ease the difficulty in the UIO design.

Based on the results obtained in [30, 59, 114], proving the following necessary condition for the existence of the UIO given by (2.7) and (2.8) is straightforward.

Theorem 2.1 *If the observer given by (2.7) and (2.8) exists such that $UIOC1 \sim UIOC3$ are satisfied, the following two conditions must be met:*
1) there exist E_s and a full column rank matrix X_s such that

$$E_s C(D\ B_s\ \bar{B}_s) = (-D\ \ -B_s\ \ -\bar{B}_s + X_s); \tag{2.10}$$

2) for any complex χ with $Re(\chi) > 0$, rank $\begin{pmatrix} \chi I_n - A & D & B_s \\ C & 0 & 0 \end{pmatrix} = n + q + l$.

The uncertainties that satisfy the above necessary conditions are called matched uncertainties.

Remark 2.3.3 *Compared with conventional UIOs, the condition $UIOC2$ actually constrains the feasible solutions of E_s, which in turn shrinks the feasible set of all feasible E_s. However, for the purpose of fault isolation, this condition is necessary and has to be added. This fact will be shown more clearly later in the fault isolation problems.*

For simplicity, it is assumed rank $C(D\ \ B_s\ \ \bar{B}_s) = m + q$. Then, for any X_s of full column rank, E_s can always be given in the following form:

$$\begin{aligned} E_s &= (-D\ \ -B_s\ \ -\bar{B}_s + X_s)(C(D\ \ B_s\ \ \bar{B}_s))^+ \\ &\quad + Y_s(I - (C(D\ \ B_s\ \ \bar{B}_s))(C(D\ \ B_s\ \ \bar{B}_s))^+) \end{aligned} \tag{2.11}$$

where $X^+ = (X^T X)^{-1} X^T$ and Y_s can be chosen freely.

Remark 2.3.4 *Under the condition that the rank $C(D\ \ B_s\ \ \bar{B}_s) = m + q$, E_s always has solutions for any X_s of full column rank, which means one has the freedom to choose X_s.*

In the remainder of this subsection, sufficient conditions for the existence of the UIO given by (2.7) and (2.8) will be derived, which satisfies $UIOC1 \sim UIOC3$.

The first sufficient condition is given in Theorem 2.2.

Theorem 2.2 *Under assumptions A21 and A22 and assuming that* $\bar{u}_s^H = \bar{u}_s$, *if there exist* E_s *and* K_s *such that*

1. $M_s(D\ \ B_s) = 0$;
2. $M_s\bar{B}_s$ *is of full column rank;*
3. *there exists a symmetric positive definite matrix,* P_s, *satisfying the following matrix inequality*

$$N_s^T P_s + P_s N_s + \gamma P_s M_s M_s^T P_s + \gamma I < 0, \quad (2.12)$$

then $UIOC1$ *to* $UIOC3$ *are satisfied, and, moreover,* e_s *exponentially approaches zero, and is thus made invariant with respect to* u_s *and* d.

Proof. Because 1 is the same as $UIOC1$, 2 is the same as $UIOC2$, and 3 implies $UIOC3$, $UIOC1 \sim UIOC3$ are satisfied.

Now, it needs to show e_s approaches zero exponentially fast.

Because $UIOC1$ is true, $M_sB_s = 0$ and $M_sD = 0$. Using these and $\bar{u}_s^H = \bar{u}_s$, (2.9) becomes

$$\dot{e}_s = N_s e_s + M_s(f(\hat{x}_s) - f(x)). \quad (2.13)$$

For convenience, let $-Q_s = N_s^T P_s + P_s N_s + \gamma P_s M_s M_s^T P_s + \gamma I$. By choosing a Lyapunov function as $V_s = e_s^T P_s e_s$ and differentiating it with respect to t along (2.13), one gets

$$\begin{aligned} \dot{V}_s &= e_s^T[N_s^T P_s + P_s N_s]e_s + 2e_s^T P_s M_s[f(\hat{x}_s) - f(x)] \\ &\leq e_s^T[N_s^T P_s + P_s N_s]e_s + 2\gamma \|e_s^T P_s M_s\| \|e_s\| \\ &\leq e_s^T[N_s^T P_s + P_s N_s]e_s + \gamma \|e_s P_s M_s\|^2 + \gamma \|e_s\|^2 \\ &= -e_s^T Q_s e_s. \end{aligned} \quad (2.14)$$

Because $Q_s > 0$, (2.14) implies that e_s will exponentially approach zero as t goes to infinity. ∎

The problem remaining is how to design E_s and K_s such that all the conditions needed are met. According to Theorem 2.2, the design of these matrices involves solving a highly nonlinear matrix inequality (2.12), which is a very difficult task. To overcome the difficulty encountered in designing E_s and K_s, an LMI based sufficient condition will be derived in the remainder of this section.

For simplicity, the following notations are introduced:

$$\begin{aligned} W1_s &= -(D\; B_s\; \bar{B}_s)(C(D\; B_s\; \bar{B}_s))^+, \\ W2_s &= I - (C(D\; B_s\; \bar{B}_s))(C(D\; B_s\; \bar{B}_s))^+, \\ W3_s &= (0\; 0\; \bar{B}_s)(C(D\; B_s\; \bar{B}_s))^+. \end{aligned} \tag{2.15}$$

The LMI based sufficient condition is given in the following theorem.

Theorem 2.3 *Under assumptions A21 and A22, and assuming that $\bar{u}_s^H = \bar{u}_s$, and if there exists a solution of $P_s > 0$, $R_s > 0$, $\bar{Y}_s$ and $\bar{K}_s$ for the following LMI*

$$\begin{pmatrix} X_{11} & X_{12} \\ X_{12}^T & -I \end{pmatrix} < 0 \tag{2.16}$$

where X_{11} and X_{12} are defined as

$$\begin{aligned} X_{11} = {} & [(I + W1_sC)A]^T P_s + P_s(I + W1_sC)A + (W3_sCA)^T R_s + R_s(W3_sCA) \\ & +(W2_sCA)^T\bar{Y}_s^T + \bar{Y}_s(W2_sCA) - C^T\bar{K}_s^T - \bar{K}_sC + \gamma I, \end{aligned} \tag{2.17}$$

and

$$X_{12} = \sqrt{\gamma}[P_s(I + W1_sC) + R_sW3_sC + \bar{Y}_s(W2_sC)]. \tag{2.18}$$

Then, by letting $Y_s = P_s^{-1}\bar{Y}_s$, $K_s = P_s^{-1}\bar{K}_s$, *and* $X_s = P_s^{-1}R_s\bar{B}_s$, *all the gains defined by (2.8) can be computed easily as follows:*

$$\begin{aligned} E_s &= W1_s + P_s^{-1}R_sW3_s + P_s^{-1}\bar{Y}_sW2_s, \\ M_s &= I + E_sC, \\ N_s &= M_sA - K_sC, \\ \bar{G}_s &= M_s\bar{B}_s, \\ L_s &= K_s(I + CE_s) - M_sAE_s. \end{aligned} \tag{2.19}$$

and the observer given by (2.7) and (2.8) satisfies $UIOC1 \sim UIOC3$ *and can make* e_s *approach zero exponentially.*

Proof. Using the definitions of E_s, $W1_s$, and $W2_s$, one obtains

$$E_s = W1_s + (0\ 0\ X_s)(C(D\ B_s\ \bar{B}_s))^+ + Y_sW2_s. \tag{2.20}$$

Using $X_s = P^{-1}R_s\bar{B}_s$, the definition of $W3_s$ and $Y_s = P^{-1}\bar{Y}_s$, one derives

$$E_s = W1_s + P^{-1}R_sW3_s + P^{-1}\bar{Y}_sW2_s. \tag{2.21}$$

(2.21) shows that E_s can be computed using the first equation in (2.19). Because E_s and K_s can be computed, by definition, all the other observer gains can be computed using (2.19).

In the remaining part of this proof, it will be shown that the observer given by (2.7) and (2.8) satisfies $UIOC1 \sim UIOC3$ and can make e_s exponentially approach zero.

First, since Y_s and X_s exist, E_s exists and can be computed by (2.21), which is equivalent to (2.11). Therefore, by definition, E_s satisfies

$$E_sC(D\ \ B_s) = 0, \tag{2.22}$$

and

$$E_s C \bar{B}_s = X_s = P^{-1} R_s \bar{B}_s. \tag{2.23}$$

(2.22) and (2.23) imply both conditions 1 and 2, required by Theorem 2.2, are satisfied using the fact B is of full column rank. Second, it will be shown that condition 3 in Theorem 2.2 is also satisfied; that is, there exists $P_s > 0$ such that (2.12) is true.

Using (2.21) and the definitions of $\bar{K}_s$, $\bar{Y}_s$, N_s, M_s, X_{11}, and X_{12}, it is easy to show that

$$\begin{aligned} X_{11} &= [(I + E_s C)A]^T P + P(I + E_s C)A - C^T K_s^T P_s - P_s K_s C + \gamma I \\ &= N_s^T P_s + P_s N_s + \gamma I, \end{aligned} \tag{2.24}$$

and

$$\begin{aligned} X_{12} &= \sqrt{\gamma} P_s (I + W1_s C) + R_s W3_s C + \bar{Y}_s (W2_s C) \\ &= \sqrt{\gamma} P_s (I + E_s C) = \sqrt{\gamma} P_s M_s. \end{aligned} \tag{2.25}$$

It is well known that

$$\begin{pmatrix} X_{11} & X_{12} \\ X_{12}^T & -I \end{pmatrix} < 0$$

is equivalent to $X_{11} + X_{12} X_{12}^T < 0$. This fact, together with (2.24) and (2.25), implies (2.12).

It has been proved that all the conditions needed by Theorem 2.2 are met. Therefore, according to Theorem 2.2, $UIOC1 \sim UIOC3$ are met and e_s is made to exponentially approach zero. ∎

Remark 2.3.5 *Designing UIOs for linear systems is not an easy task., but the design of UIOs for Lipschitz nonlinear systems is even harder, and no systematic way has been proposed. The contribution here is that an LMI based sufficient condition given* *by (2.16) $\sim$ (2.18) is derived for a class of Lipschitz nonlinear systems. This LMI condition provides a systematic way to solve the difficult design problem of E_s and K_s using the efficient algorithms provided by Matlab's LMI toolbox.*

Remark 2.3.6 *Because the literature has paid little attention to $UIOC2$, no technique has been proposed to ensure $UIOC2$. In the proposed UIO design, a technique is provided for ensuring that $UIOC2$ is met automatically as long as the LMI given by (2.16) $\sim$ (2.18) has a feasible solution.*

Remark 2.3.7 *The LMI based UIO design procedure is still valid for $s = \phi$ if one uses $\bar{B}_s = B$ and $(D \quad B_s) = D$. If there exists a UIO for such a case, the uncertainties represented by d are called* **matched**. *The UIO designed for this case could be used for fault detection.*

2.4 The Solution of Actuator FDI Problems

In this section, the actuator FDI problem formulated in Section 2.2 is investigated. The solution of this problem is closely related to a new concept, which is called the *UIO Induced Actuator Fault Isolation Index (UIOIAFIX)* and is defined based on the existence conditions of a bank of UIOs.

Definition 2.4.1 *System (2.1) is said to have a UIO Induced Actuator Fault Isolation Index (UIOIAFIX) equal to l if and only if Theorem 2.1's two conditions 1) and 2) are satisfied for all the sets of the form $s = \{i_1, \cdots, i_l\} \in 2^{S_I}$ with l being the largest number that has this property.*

For simplicity, AFIX is used to stand for UIOIAFIX in the remaining part of this chapter.

Remark 2.4.1 *If there does not exist any $l > 0$ such that $AFIX = l$, then system (2.1) does not have any AFIX. For consistency, $AFIX = 0$ is used to denote this situation.*

The AFIX has the following property.

Lemma 2.4.1 *For system (2.1), one always has $0 \leq AFIX \leq min\{m, p-q\}$.*

Proof. If $AFIX = l > 0$, by definition, both conditions 1) and 2) in Theorem 2.1 are satisfied for any set of the form $s = \{i_1, \cdots, i_l\}$. Obviously, condition 2) requires $(D\ B_s)$ is of full column rank and has rank $l + q$. This fact, together with condition 1), implies that $l + q$ must be less than the number of rows of C, that is, $l + q \leq p$, which is equivalent to $l \leq p - q$. ∎

To accomplish fault isolation, two observer schemes are proposed in the literature: the *dedicated observer scheme*(DOS) [115], and the *generalized observer scheme*(GOS) [6]. Both schemes aim to properly design a group of N residuals (i.e., $r_1, \cdots, r_N$). The idea of DOS is to design the residuals such that r_i is only sensitive to the ith fault or fault group, but insensitive to all other faults or fault groups. On the contrary, the idea of GOS is to design the residuals such that r_i is sensitive to all faults or fault groups except the ith one.

Because of the form of the diagnostic observer given by (2.7) and (2.8), only GOS scheme could be designed for this thesis. For a fixed l, if, for each set $s = \{i_1, \cdots, i_l\}$, a UIO is designed, there are a total of C_m^l UIOs.

Theorem 2.4 *Under assumptions A21 and A22, and if a bank of UIOs of the form given by (2.7) and (2.8) are used for actuator fault diagnosis, then the maximum number of actuator faults that can be simultaneously isolated is equal to the Actuator Fault Isolation Index (AFIX).*

Proof. Two cases for $AFIX = 0$ and $AFIX > 0$, are considered.

For the case $AFIX = 0$, the proof needed is to show that no single actuator fault can be isolated and only fault detection is possible. In order to isolate one single fault, m residuals must be designed based on m UIOs of the form given by (2.7) and (2.8) such that each residual is insensitive to only one actuator fault but sensitive all other actuator faults. If this could be done, one would have $AFIX \geq 1$ by definition, which contradicts $AFIX = 0$. This proves that single fault isolation is impossible. Note that $AFIX = 0$ is equivalent to $s = \phi$, then, according to Remark 2.3.7, a UIO is still possible to design such that $UIOC1 \sim UIOC3$ are satisfied, which means that fault detection is possible.

Suppose $AFIX = l > 0$, by the definition of $AFIX$, for each set $s = \{i_1, \cdots, i_l\}$, a UIO given by (2.7) and (2.8) exists and can be designed such that $UIOC1 \sim UIOC3$ are satisfied. Without loss of generality, l actuators are assumed to be faulty. Because $UIOC1$ and $UIOC3$ are satisfied, it follows from (2.9) that

$$\dot{e}_s \quad = \quad N_s e_s + M_s(f(\hat{x}_s) - f(x)) + \bar{G}_s(\bar{u}_s^H - \bar{u}_s). \tag{2.26}$$

If the faulty actuator group is $u_{i_1}, \cdots, u_{i_l}$, $\bar{u}_s^H = \bar{u}_s$, which, together with (2.26), implies that $lim_{t\to\infty} e_s = 0$ according to Theorem 2.2. For any $s \neq \{i_1, \cdots, i_l\}$, then $\bar{u}_s^H - \bar{u}_s \neq 0$. This together with $UIOC2$ and (2.26) implies that e_s will not tend to zero.

Based on the above discussions, if r_s is defined as $ey_s = Ce_s$, then r_s is insensitive to any fault in the actuator group $u_{i_1}, \cdots, u_{i_l}$, but is generally sensitive to any other actuator faults. According to the idea of GOS, this implies that it is possible to isolate l faults simultaneously.

The possibility of isolating $AFIX = l > 0$ faults has been proved.

To design a GOS to isolate $l+1$ faults, any combination of $l+1$ inputs must be treated as unknown inputs. Assume that, for all sets of the form $s = \{i_1, \cdots, i_{l+1}\}$, a UIO given by (2.7) and (2.8) can be designed such that $UIOC1 \sim UIOC3$ are satisfied. Then, by the definition of $AFIX$, $AFIX = l+1$, which contradicts the fact that $AFIX = l$. This completes the proof. ∎

Actually, Theorem 2.4 has provided a solution for **FDP1**, which is shown more clearly in the following corollary.

Corollary 2.4.1 *Under the assumptions of Theorem 2.4, if* $AFIX > 0$*, actuator fault detection is always possible, and if* $AFIX = 0$*, fault detection may or may not be possible.*

Proof. For the case when $AFIX > 0$, according to Theorem 2.4, it is obvious that actuator fault detection can be achieved simply because fault isolation can be accomplished. For the case when $AFIX = 0$, if the UIO for the case when $s = \phi$ can be designed such that $UIOC1 \sim UIOC3$ are satisfied, actuator fault detection can be carried out. However, if a UIO can be designed such that $UIOC1$ and $UIOC3$ can only be satisfied under the condition that $M_s B = 0$, actuator fault detection cannot be achieved because the UIO is insensitive to any actuator faults. This completes the proof. ∎

Theorem 2.4 has also provided a solution for **FIP1** and **FIP2**, as shown in the following corollary.

Corollary 2.4.2 *Under the assumptions of Theorem 2.4, if* $AFIX = 0$*, actuator fault isolation is impossible; if* $AFIX = 1$*, only one single fault can be isolated; and if* $AFIX > 1$*, actuator fault isolation can be achieved for a single fault, two faults, and up to* $AFIX$ *faults.*

Theorem 2.5 provides theoretical support for the design of a GOS based fault isolation scheme.

Theorem 2.5 *Under assumptions A21 and A22, and assuming that all conditions in Theorem 2.2 are met and $CM_s\bar{B}_s$ is of full column rank for all s, the bank of UIOs designed can make all residuals satisfy the property of GOS; that is, each residual is only insensitive to faults in a particular actuator group while sensitive to all other faults outside the actuator group.*

Proof. For any set s, according to Theorem 2.2, e_s will exponentially approach zero for any input in u_s. Hence, $r_s(t)$ is insensitive to any fault in the actuator group corresponding to u_s. For faults outside this actuator group, they will lead to $\bar{u}_s^H - \bar{u}_s \neq 0$. This fact, together with the fact that $CM_s\bar{B}_s$ is of full column rank, implies that $r_s(t)$ is sensitive to the faults outside the actuator group corresponding to u_s. This completes the proof. ∎

Theorem 2.6 serves as a foundation for determining number of faults.

Theorem 2.6 *Under all conditions in Theorem 2.5, if the number of actuator faults is $0 < n_f \leq AFIX$, then the number of residuals (i.e.,$r_s(t)$), which are insensitive to the n_f faults, is equal to $C_{m-n_f}^{AFIX-n_f}$.*

Proof. Because there are n_f faults, the number of actuator groups defined according to u_s, which include the faulty actuators, is equal to $C_{m-n_f}^{AFIX-n_f}$. According to Theorem 2.5, all these $C_{m-n_f}^{AFIX-n_f}$ residuals are insensitive to the n_f faults, and any other residual is sensitive to some of the n_f faults. ∎

Remark 2.4.2 *If there are n_f faults and $n_f \leq AFIX$, exactly $C_{m-n_f}^{AFIX-n_f}$ residuals are insensitive to the actuator faults. Therefore, once the number of residuals insensitive to the actuator faults is determined (g_{num}), the number of faults n_f is found by solving $C_{m-n_f}^{AFIX-n_f} = g_{num}$.*

For any set $s = \{i_1, \cdots, i_{AFIX}\}$, r_s denotes the residual defined according to ey_s. The following result is useful in fault isolation.

Theorem 2.7 *Under the assumptions of Theorem 2.5, suppose only g_{num} residuals are under the threshold and $C_{m-k}^{AFIX-k} = g_{num}$ has an integer solution for k. Denote these residuals as $r_{s_1}, \cdots, r_{s_{g_{num}}}$, and define $s_j = \{i_1^j, \cdots, i_{AFIX}^j\}, 1 \leq j \leq g_{num}$, and let $S_F = \bigcap_{j=1}^{g_{num}} s_j$. Then, the number of elements in S_F is the number of faults and each element in S_F identifies a particular fault.*

Proof. According to Theorem 2.5, $C_{m-n_f}^{AFIX-n_f}$ residuals should be under the threshold if there are n_f faults. Because g_{num} residuals are under the threshold, that is, $r_{s_1}, \cdots, r_{s_{g_{num}}}$, and the equation $C_{m-k}^{AFIX-k} = g_{num}$ has an integer solution for k, faults can only occur in actuator groups corresponding to $u_{s_1}, \cdots, u_{s_{g_{num}}}$. This fact, together with the definition of S_F, supports the conclusions of the theorem. ∎

Assuming that fewer than $AFIX$ actuator faults can occur at the same time, the following actuator fault detection and isolation scheme is designed for solving the fault detection problems(**FDP1** and **FDP2**)and the fault isolation problems (**FIP1**, **FIP2**, **FIP3**):

- Step 1 Compute $AFIX$.
- Step 2 If $AFIX = 0$, no fault can be isolated and only fault detection is possible. Fault detection can be achieved using the UIO designed for the case $s = \phi$. Stop.
- Step 3 If $AFIX > 0$, both fault detection and fault isolation can be accomplished. If $AFIX = m$, then let $l = m - 1$ and go to Step 4. If $0 < AFIX < m$, then $l = AFIX$, go to Step 4.
- Step 4 Fault detection and isolation

1. For each set $s = \{i_1, \cdots, i_l\}$, design a UIO given by (2.7) and (2.8) satisfying $UIOC1 \sim UIOC3$.

2. Define residuals $r_s(t) = \|Ce_s\|^2/N_{normal}(t)$, where $N_{normal}(t)$ is chosen such that $r_s(t) \leq 1$ when only possible faulty actuators correspond to u_s; and $r_s(t) \geq 1$ otherwise.

3. The threshold is chosen as 1.

4. Fault Detection: If any of the C_m^l residuals is larger than the threshold at any time constant, faults are detected. Otherwise, no fault has been detected.

5. After faults are detected, the fault detection time is denoted as T_{detect}, and a fault isolation time interval(FITI) is chosen as $(T_{detect}, T_{detect} + \Delta)$ with Δ suitably large. Fault isolation is carried out on FITI.

6. For residuals that are below the threshold on the FITI, monitor each of them on the FITI to determine the tendency of the residuals.

7. Count the number of residuals that are below the threshold and that have no tendency to increase, and denote the number as g_{num}.

8. If $g_{num} = 0$ and if $l = m - 1$, all actuators are faulty. If $l < m - 1$, then more than l actuators are faulty and exact fault isolation cannot be achieved. Stop.

9. If $g_{num} = 1$, then $n_f = l$ and there are l actuator faults. If r_s is the only residual that is under the threshold, then the $i_1 \cdots i_l$th actuators are faulty. Fault isolation is completed, and stop.

10. If $g_{num} > 1$, then solve $C_{m-n_f}^{l-n_f} = g_{num}$ for n_f. If there is no integer solution for n_f, then the number of faults occurred can not be determined and fault

isolation cannot be accomplished. Stop. If an integer solution of n_f exists, the number of faults is equal to $n_f < l$.

11. If the number of faults $n_f < l$ is determined and $n_f < l$, $C_{m-n_f}^{l-n_f} = g_{num}$ actuator groups have residuals under the threshold. Denote these actuator groups as $u_{s_j}, 1 \leq j \leq g_{num}$ with $s_j = \{i_1^j, \cdots, i_l^j\}, 1 \leq j \leq g_{num}$.

12. Fault Isolation for $g_{num} > 1$: Let $S_F = \bigcap_{j=1}^{g_{num}} s_j$, and if $S_F = \{i_1, \cdots, i_{n_f}\}$, then the faulty actuators are the $i_1 \cdots i_{n_f}$th actuators.

Remark 2.4.3 *The novelty of this fault detection and isolation scheme lies mainly in the isolation part. No such scheme has been proposed in the literature. The scheme has several novel elements: 1) fault detection can be achieved as a byproduct of fault isolation; 2) the number of faults and the isolation of faults can be accomplished using only C_m^l UIOs; 3) the idea of combining the concepts of fault isolation time interval (FITI) and the tendency checking to carry out fault isolation is very useful. For fast fault isolation, one may want to use a small FITI, however too small a FITI may lead to a wrong fault isolation decision, which can be seen later in Example 2 for the slow incipient fault case. Obviously, there is a tradeoff between fast fault isolation and obtaining a right decision. The combination of these two concepts provides a better way to overcome the difficulty. One can use a relatively small FITI to realize fast fault isolation, while using tendency checking to reduce the wrong fault isolation rate. This idea is especially useful for slow incipient fault isolation and is also clearly shown in Example 2 for slow incipient fault isolation.*

Remark 2.4.4 *Sensor fault diagnosis can be accomplished in almost the same way as actuator fault diagnosis by defining a similar concept called the Sensor Fault Isolation Index (SFIX). Note that AFIX is generally not equal to SFIX. In fact, based on (2.5) and (2.6), it can be shown that $0 \leq SFIX \leq \frac{p-q}{2}$, which implies that the number of* *sensor faults that can be isolated is less than $\frac{p}{2}$ even for linear systems with $d = 0$. However, for linear systems with $d = 0$, the number of sensor faults that can be isolated may be equal to p [46]. Although sensor fault isolation can not be solved completely by treating it as an actuator fault isolation problem, the sensor fault isolation method in this chapter provides an alternative to existing sensor fault diagnosis methods.*

2.5 Examples and Simulation Results

In this section, two examples are provided to show how the proposed fault diagnosis scheme is designed and how to test its effectiveness. The first example shows that the proposed UIO FDI scheme works for uncertain Lipschitz nonlinear systems. The other example considers a linearized model of a tailless jet fighter taken from [113], which provides an opportunity to deal with a more practical problem and to verify that the proposed UIO based FDI scheme can indeed be used for the fault diagnosis of linear systems. For both examples, the proposed UIO FDI scheme is used to accomplish actuator FDI and simulation results are presented.

2.5.1 Example 1 with Simulation Results

The following uncertain nonlinear system is considered:

$$\begin{aligned} \dot{x} &= Ax + f(x) + Bu + Dd \\ y &= Cx \end{aligned} \tag{2.27}$$

where

$$A = \begin{pmatrix} -0.6344 & 0.0027 & 0 & 0.9871 & 0 \\ 0 & -0.0038 & 0.1540 & 0 & -0.9876 \\ 0 & -8.2125 & -0.7849 & 0 & 0.1171 \\ -0.5971 & 0 & 0 & -0.5099 & 0 \\ 0 & -0.8887 & -0.0299 & 0 & -0.0156 \end{pmatrix},$$

$$B = \begin{pmatrix} -0.0459 & -0.0395 & -0.0133 \\ -0.0047 & 0 & 0.0031 \\ 3.783 & 0 & 1.8255 \\ -2.5115 & -1.9042 & -0.9494 \\ -0.0453 & 0 & -0.2081 \end{pmatrix}, \; D = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \; C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix},$$

$f(x) = (\,0.2sin(10x_5) \quad 0.1cos(2x_4) \quad 0 \quad 0 \quad 0\,)^T$ and $d = 0.05sin(2t)$.

For this system, $AFIX = 3$, which is equal to the number of actuators. Let $l = m - 1 = 3 - 1 = 2$ and $s = \{i_1, i_2\}$. Then, based on the LMI based sufficient condition, a UIO of the form given by (2.7) and (2.8) is written as

$$\begin{aligned} \dot{z}_s &= N_s z_s + \bar{G}_s \bar{u}_s^H + L_s y + M_s f(\hat{x}_s), \\ \hat{x}_s &= z_s - E_s y. \end{aligned} \tag{2.28}$$

Because there are three sets of the form $s = \{i_1, i_2\}$, that is, $\{1,2\}$, $\{1,3\}$, and $\{2,3\}$ respectively, three UIOs can be designed. The observer gain matrices are computed easily using Matlab and its LMI toolbox.

In the simulations, $N_{normal}(t) = 0.00001$ is chosen, and r_{12}, r_{13}, and r_{23} are defined according to $\{1,2\}$, $\{1,3\}$, and $\{2,3\}$. The simulation results are plotted in Figure 2.1.

At $5s$, the first actuator has an abrupt fault, and does not produce any output, i.e., $u_1 = 0$. Based on the proposed FDI scheme, the fault diagnosis is accomplished as follows: faults are detected after 0.07 seconds because the residual r_{23} goes beyond the threshold. By choosing $FITI = (5.07s, 7.5s)$ and checking the tendency of r_{12} and r_{13}, it is found that they are well below the threshold and the tendency for them is to decrease. Therefore, it is concluded $g_{num} = 2$. By solving $C_{3-n_f}^{2-n_f} = 2$, it yields $n_f = 1$, which implies that the number of faults is one. Because r_{12} and r_{13} are under the threshold, $S_F = \{1\}$, which means the first actuator is faulty. The decision is correct in this case.

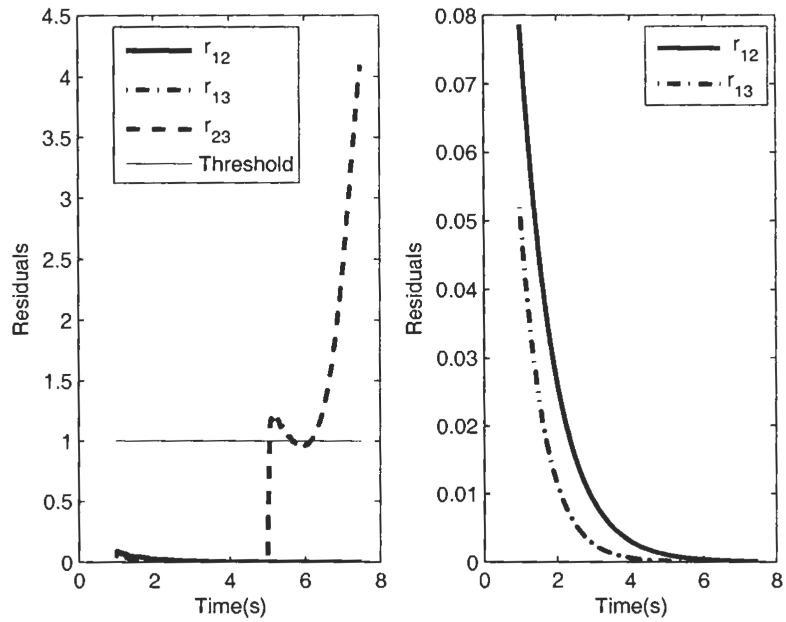

Figure 2.1: Fault isolation with tendency checking – Nonlinear case

2.5.2 Example 2 with Simulation Results

A linearized model of a tailless jet fighter taken from [113] is given as.

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{2.29}$$

where $x = (\alpha, \beta, p, q, r)^T$ with α being the angle of attack, β being the sideslip angle, and p, q, r being the roll rate, pitch rate, and yaw rate, respectively. The control u is defined as $u = (\delta_{el}, \delta_{er}, \delta_{pflap}, \delta_{amtl}, \delta_{amtr})^T$ with $\delta_{el}, \delta_{er}, \delta_{pflap}, \delta_{amtl}, \delta_{amtr}$ being the deflections of left and right elevons, pitch flap, and left and right all moving tips, respectively. The system matrices are defined as follows: A is the same as in Example

1, $C = I_5$, and

$$B = \begin{pmatrix} -0.0459 & -0.0459 & -0.0395 & -0.0133 & -0.0133 \\ -0.0047 & 0.0047 & 0 & 0.0031 & -0.0031 \\ 3.783 & -3.783 & 0 & 1.8255 & -1.8255 \\ -2.5115 & -2.5115 & -1.9042 & -0.9494 & -0.9494 \\ -0.0453 & -0.0453 & 0 & -0.2081 & 0.2081 \end{pmatrix}.$$

For this system, $AFIX = 5$, which is equal to the number of actuators. Now, let $l = m - 1 = 5 - 1 = 4$. As in Example 1, let $s = \{i_1, i_2, i_3, i_4\}$, based on the LMI based sufficient condition, a UIO given by (2.7) and (2.8) is

$$\begin{aligned} \dot{z}_s &= N_s z_s + \bar{G}_s \bar{u}_s^H + L_s y + M_s f(\hat{x}_s), \\ \hat{x}_s &= z_s - E_s y. \end{aligned} \tag{2.30}$$

For all possible sets of the form s, i.e., $\{1,2,3,4\}$, $\{1,2,3,5\}$,$\{1,2,4,5\}$, $\{1,2,4,5\}$, and $\{2,3,4,5\}$, five UIOs can be designed. Again, the observer gain matrices are computed easily using Matlab and its LMI toolbox.

In the simulations, for the sake of comparisons, two types of actuator faults are introduced. The first type is abrupt faults (here, the first three actuators became at and after $5s$, that is, $u_1 = u_2 = u_3 = 0$), the second type is slow-changing faults (e.g., $u_1 = e^{-0.05t} u_1^H; u_2 = e^{-0.05t} u_2^H; u_2 = e^{-0.05t} u_3^H$ at and after $5s$).

In this example, $N_{normal}(t) = 0.0001$ is chosen, and $r_{1234}(t)$, r_{1235}, r_{1245}, r_{1345}, and r_{2345} are computed according to the proposed FDI scheme. The simulation results for abrupt fault detection are plotted in Fig. 2.2, while those results for slow-changing fault are presented in Fig. 2.3.

Fig. 2.2 and Fig. 2.3 show: abrupt faults are detected very quickly with a detection time of $0.07s$; while slow-changing faults are detected quite slowly with a detection time of $5.71s$.

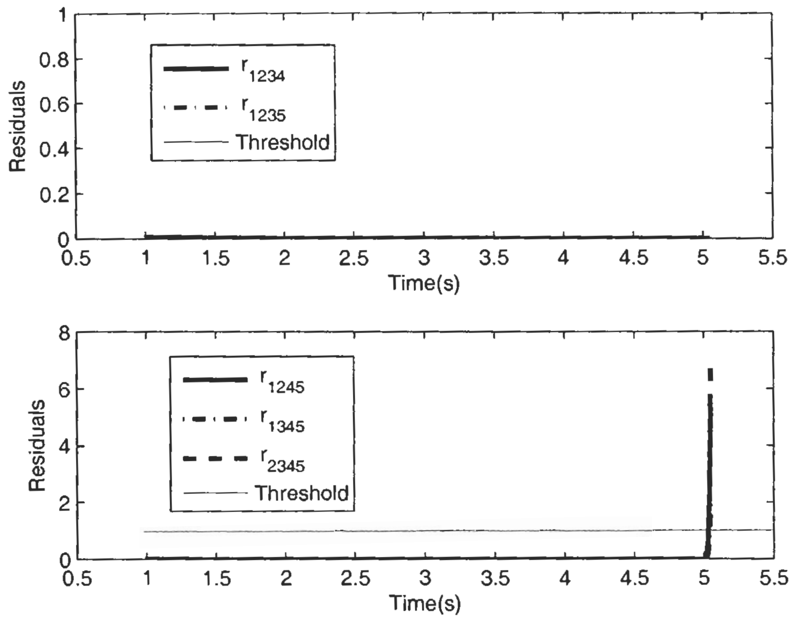

Figure 2.2: Fault detection – Abrupt fault case

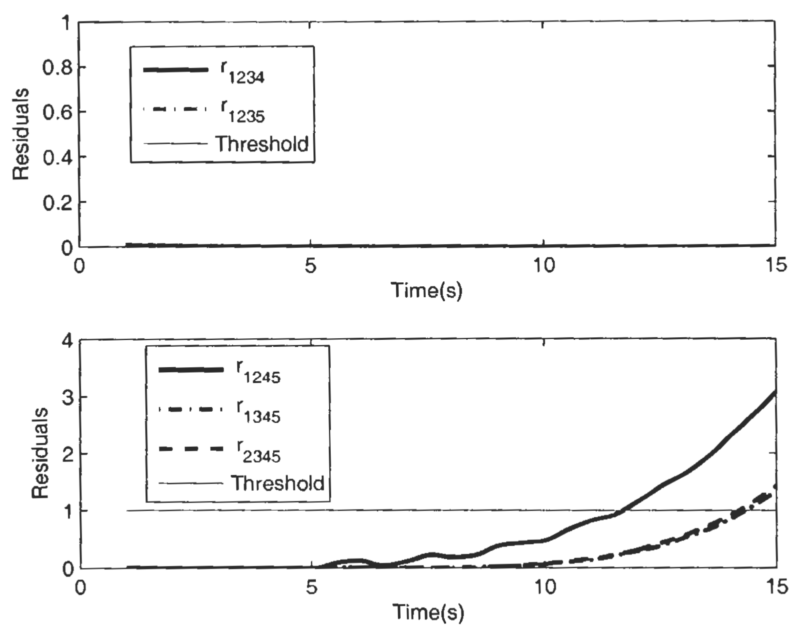

Figure 2.3: Fault detection – Slow-changing fault case

The simulation results for abrupt fault isolation are presented in Fig. 2.4. In such a case, $FITI = (5.05s, 5.5s)$ is chosen. Fig. 2.4 shows that r_{1234} and r_{1235} are so small compared with the threshold that they are almost at the same level of the time axis while r_{1245}, r_{1345}, and r_{2355} are far beyond the threshold. Therefore, according to the proposed FDI scheme, $g_{num} = 2$. By solving $C_{5-n_f}^{4-n_f} = 2$, it yields $n_f = 3$; i.e., three faults occurred. Additionally, $S = \{1, 2, 3\}$ means the first three actuators are faulty, which is a correct isolation decision. The fault isolation time is within one second.

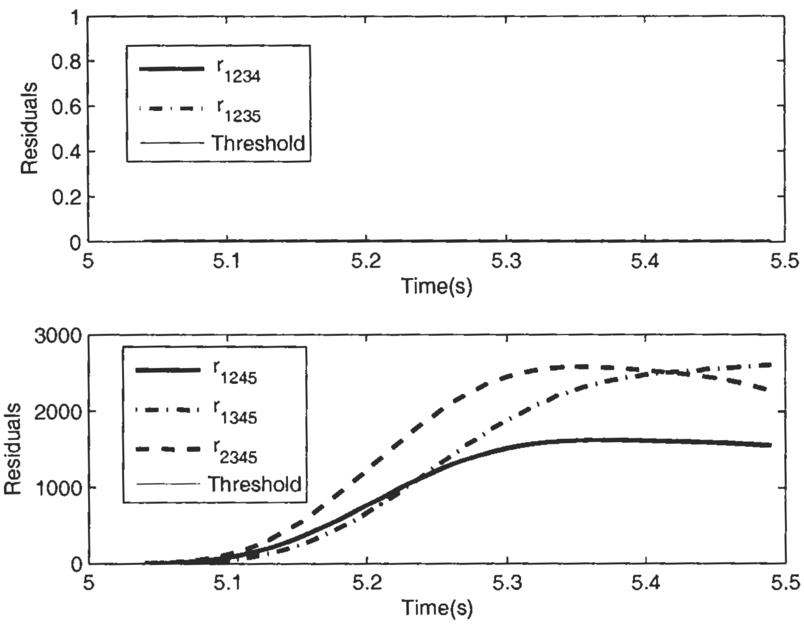

Figure 2.4: Fault isolation – Abrupt fault case

The simulation results for slow-changing fault isolation are plotted in Fig. 2.5, Fig. 2.6, and Fig. 2.7.

For the slow-changing fault case, $FITI = (11.71s, 15s)$ is chosen. Then, base on Figure 2.5 and as in the abrupt fault case, it can be concluded that the first three actuators are faulty, which is again a correct isolation decision. However, if

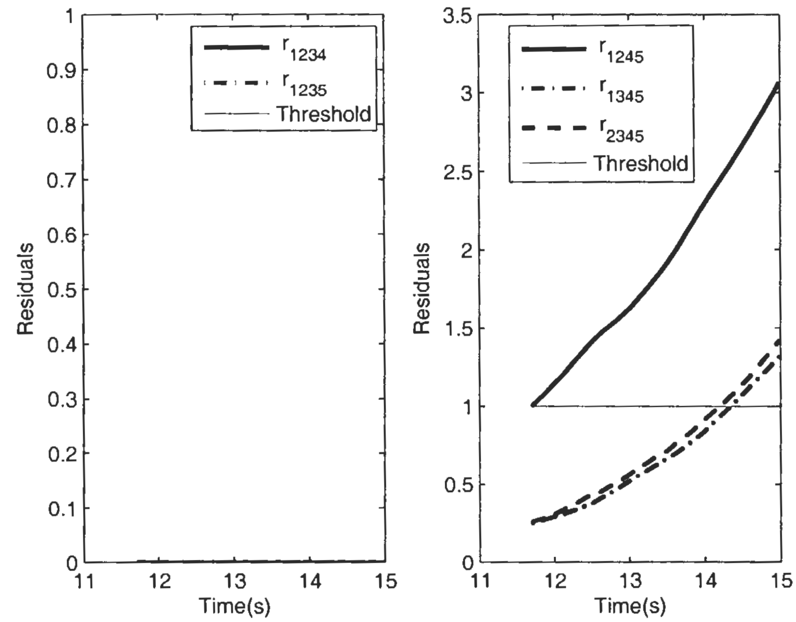

Figure 2.5: Fault isolation – Slow-changing fault case

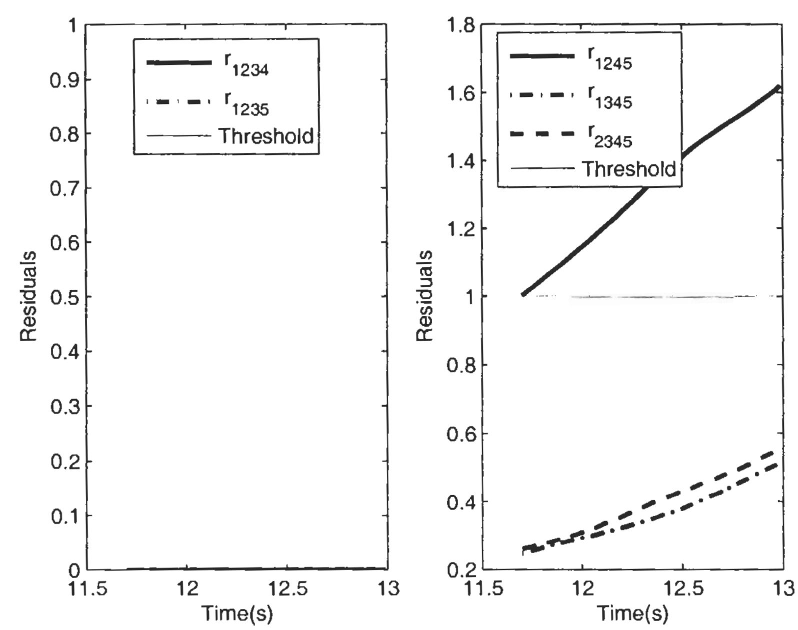

Figure 2.6: Fault isolation without tendency checking – Slow-changing fault case

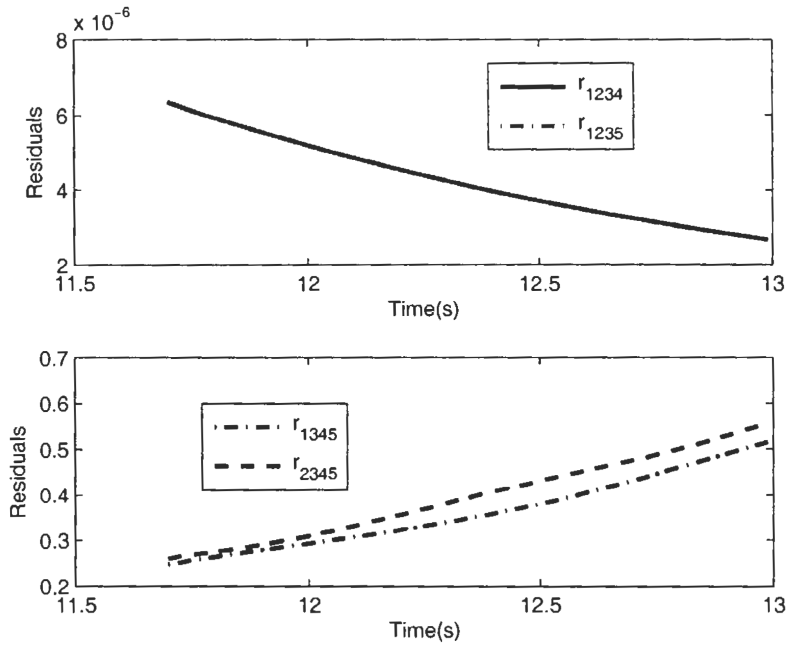

Figure 2.7: Tendency checking – Slow-changing fault case

$FITI = (11.71s, 13s)$ is chosen, Fig. 2.6 shows that r_{1234}, r_{1235}, r_{1345}, and r_{2345} are all below the threshold. If tendency checking is not completed, $g_{num} = 4$, which yields $n_f = 1$ and $S = \{3\}$. The conclusion is only one fault occurred, where the third actuator is faulty. Unfortunately, this decision is wrong. However, if tendency checking is implemented, as shown in Figure 2.7, there is a clear tendency for both r_{1234} and r_{1235} to decrease and for both r_{1345} and r_{2345} to increase. This observation implies that both r_{1345} and r_{2345} may exceed the threshold in the future and only r_{1234} and r_{1235} will stay below it. At this time, one may make a decision that only r_{1234} and r_{1235} will stay below the threshold and that $g_{num} = 2$, which will lead to a correct decision. With the help of tendency checking, fault isolation can be achieved two seconds earlier.

This example shows that the proposed UIO based FDI scheme works very well for abrupt actuator faults in terms of the time and accuracy of making decisions on fault detection and isolation. Compared with abrupt actuator faults, slow-changing faults need a longer time to be detected and isolated. The idea of combining the FITI and tendency checking works well for slow-changing faults and may help isolate faults correctly in shorter time.

2.6 Conclusions and Discussions

Using the UIO design to achieve actuator or sensor fault detection and isolation has been studied in a detailed and systematic way for a class of uncertain Lipschitz nonlinear systems.

Firstly, by regrouping the system inputs or by regrouping and filtering the system outputs, a particular system structure has been developed, which is in suitable form for UIO design.

Secondly, based on the particular system structure, a novel diagnostic UIO has been designed with a special property suitable for fault isolation purpose. Necessary and sufficient conditions for its existence were provided. The LMI based sufficient condition enables the designers to use Matlab's LMI toolbox, which makes the difficult UIO design problem much easier.

Thirdly, given that the proposed nonlinear UIOs are used for FDI, answers have been provided for the formulated actuator FDI problem, which are closely related to a new concept called the *UIO Induced Actuator Fault Isolation Index (UIOIAFIX)*. To be specific, fault detection can be performed if $UIOIAFIX \geq 0$, which solves **FDP1**; fault isolation can only be accomplished for the case where $UIOIAFIX > 0$, which provides a solution for **FIP1**; the number of faults can be isolated is equal to $UIOIAFIX$, which answers **FIP2**; **FDP2** and **FIP3** are solved through proposing a UIO based FDI scheme, which was given in detail and presented in steps.

Two examples were given to show how the proposed FDI scheme was used as well as the effects of the FDI scheme. The simulation results in the first example show the proposed FDI scheme works well in abrupt fault detection and isolation for uncertain Lipschitz nonlinear systems. The simulation results for the linearized tailless jet fighter model not only showed that the FDI scheme can detect and isolate the abrupt fault very fast, but it can also detect and isolate slow-changing faults effectively.

Because uncertain Lipschitz nonlinear systems only represent a restricted class of uncertain nonlinear systems, more research is needed for more general uncertain nonlinear systems. One extension in this direction has already been completed in this thesis and will be presented in Chapter 4.

Another restriction of the proposed fault diagnosis approach is its ability to do sensor fault isolation. As mentioned in Remark 2.4.4, even for known linear systems with $d = 0$, the number of sensor faults that can be isolated is less than $\frac{p}{2}$. This restriction is completely removed in Chapter 7 by an approach based on an adaptive output estimator for linear systems with only parametric uncertainties.

Chapter 3

SMO Based Fault Diagnosis for Uncertain Lipschitz Nonlinear Systems

In this chapter, the same class of systems considered in Chapter 2 is revisited. The research is carried out in a parallel manner to solve not only fault detection and isolation problems, but also fault estimation problems by employing SMO design.

3.1 Introduction

As mentioned in Chapter 2, besides UIOs, SMOs can also be used to deal with matched uncertainties in fault diagnosis. Since an SMO based strategy is used to accomplish the fault diagnosis tasks in this work, only a review of this class of fault diagnosis schemes is provided.

In general, the SMO based FDI techniques are classified into two categories. The first category uses SMOs to make the output estimation error insensitive to uncertainties such as disturbances or unknown nonlinearities, but sensitive to faults [27, 76, 77, 78]. The fault detection problem was only considered in [76, 77, 78], while a scheme in [27] focused on the fault isolation problem. The second category employs SMOs to reconstruct or estimate the faults [27, 30, 31, 81]. In [30] and [31], fault detection and isolation problems for uncertain linear systems were solved under the assumption that the fault distribution matrix was known.

Nonlinear SMO based FDI problem has also been the subject of studies in recent years. For a class of systems that have, or can be transformed into, a special canonical form, SMO based FDI schemes were designed in [78, 81]. The distribution of faults was assumed to be known, and the construction of the state transformation was not an easy task. In [27], a scheme was proposed to reconstruct all the inputs (both normal and faulty) for a class of uncertain Lipschitz nonlinear systems, which removed the need for knowing the distribution of faults. However, some systems may not satisfy the assumption that all the inputs can be reconstructed. This observation, along with the desire to solve fault estimation problems that are not considered in the UIO based fault diagnosis, motivated the research reported in this chapter.

The main purpose of this chapter is to present a detailed and systematic study of the design process of an SMO based fault diagnosis scheme for a class of Lipschitz uncertain nonlinear systems, which can solve fault detection, fault isolation, and fault estimation (FDIE) problems.

The remainder of the chapter is arranged as follows. In Section 3.2, fault diagnosis problems are formulated. In Section 3.3, diagnostic SMOs that are suitable for the purpose of fault isolation are presented. The SMO existence conditions are given, and the properties of the proposed SMOs are investigated. Finally, LMI based methods are derived to provide a systematic approach for the design of the proposed SMOs. Section 3.4 solves both actuator and sensor FDIE problems based on concepts called *SMO Induced Actuator Fault Isolation Index* (SMOIAFIX) and *SMO Induced Sensor Fault Isolation Index* (SMOISFIX). In Section 3.5, an example is presented to show the effect of the proposed FDIE scheme in detecting, isolating, and estimating actuator faults that change slowly or fast. Finally, conclusions and discussions are made in the last section.

3.2 Problem Formulation

The uncertain nonlinear systems of the form (2.1) are revisited under assumptions $A21$ and $A22$. For simplicity, in this and the following chapters, all notations that are not explicitly defined are defined the same way as those in Chapter 2.

The following two fault diagnosis problems are formulated:

- **Actuator FDIE problem** – Assuming that only actuator faults can occur, the objective is to carry out a systematic study on the design of diagnostic SMOs to solve all fault detection, isolation, and estimation problems.

- **Sensor FDIE problem** – Assuming that only sensor faults can occur, the objective is to carry out a systematic study on the design of diagnostic SMOs to solve all fault detection, isolation, and estimation problems.

3.3 Nonlinear Diagnostic SMOs

In this section, two diagnostic nonlinear SMOs are proposed based on modifying the well-known Walcott-Zak SMO; one is for system (2.3), and the other is for system

(2.5). Necessary conditions for the existence of the proposed observers are provided. Finally, the properties of these observers along with systematic LMI based methods for their design are investigated.

3.3.1 A Diagnostic SMO for Actuator FDIE

For any given set $s = \{i_1, \cdots, i_l\} \in 2^{S_I}$ with $1 \leq l \leq m$, it is desired to design an SMO such that, in addition to $d(t)$, the inputs in u_s are treated as uncertainties. In this way, the state estimation error will be insensitive to the actuator faults in the actuator group u_s, but sensitive to any other actuator faults outside of the group. Because the proposed SMO is specially designed for the purpose of fault diagnosis, in the remainder of this chapter, it is referred to as a diagnostic observer.

To design the required SMO for (2.3), the following assumption is needed:

- Assumption $A31$: For a given set $s = \{i_1, \cdots, i_l\} \in 2^{S_I}$ with $1 \leq l \leq m$, assume that L_s, $F1_s$, $F2_s$, a positive definite symmetric matrix P_s, and $\epsilon_s > 0$ exist such that

$$\begin{aligned} (A - L_sC)^T P_s &+ P_s(A - L_sC) + \epsilon_s I < 0, \\ 2\gamma P_s - \epsilon_s I &< 0, \\ P_sB_s = C^T F1_s^T \quad &, \quad P_sD = C^T F2_s^T. \end{aligned} \tag{3.1}$$

If Assumption $A31$ is satisfied, then an SMO can be designed as

$$\dot{\hat{x}}_s = A\hat{x}_s + L_s(y - C\hat{x}_s) + f(\hat{x}_s) + \bar{B}_s\bar{u}_s^H + B_s\mu 1_s + D\mu 2_s \tag{3.2}$$

where $\hat{x}_s$ is the estimate of x, $\mu 1_s$, and $\mu 2_s$ are defined as

$$\mu 1_s = \begin{cases} -\rho 1_s \frac{F1_s ey_s}{||F1_s ey_s||}, & ||F1_s ey_s|| \neq 0 \\ 0, & ||F1_s ey_s|| = 0 \end{cases}$$

and

$$\mu 2_s = \begin{cases} -\rho 2_s \frac{F2_s ey_s}{\|F2_s ey_s\|}, & \|F2_s ey_s\| \neq 0 \\ 0, & \|F2_s ey_s\| = 0, \end{cases}$$

with $ey_s = C\hat{x}_s - y$ and $\rho 1_s$ and $\rho 2_s$ chosen to be large enough constants.

By defining $e_s = \hat{x}_s - x$, the following is derived:

$$\begin{aligned} \dot{e}_s &= (A - L_s C)e_s + (f(\hat{x}_s) - f(x)) \\ &+ \bar{B}_s(\bar{u}_s^H - \bar{u}_s) + B_s(\mu 1_s - u_s) + D(\mu 2_s - d(t)). \end{aligned} \tag{3.3}$$

Remark 3.3.1 *Several comments about the designed SMO are listed as follows. Firstly, it is an SMO for Lipschitz uncertain nonlinear systems, which has not been designed in the literature. Secondly, the second matrix inequality in (3.1) is introduced to ensure the asymptotic stability of (3.3) and to also reduce the observer design difficulty. Other observer techniques for Lipschitz nonlinear systems may lead to a more difficult design problem; for comparison, see the observer design techniques in [52] and [112]. Thirdly, the use of $\bar{u}_s^H$ other than $\bar{u}_s$ is convenient for fault diagnosis and is different from conventional SMOs. Fourthly, while conventional SMOs can not, the SMO in this chapter can be used directly for fault isolation without any modification because of the dependence of the SMO on the set s. Finally, treating all the inputs in u_s as uncertainties removes the need to know the distribution of faults.*

3.3.2 A Diagnostic SMO for Sensor FDIE

For any given set $s = \{i_1, \cdots, i_l\} \in 2^{S_O}$ with $1 \leq l \leq p$, it is desired to design an SMO such that, in addition to $d(t)$, the outputs in y_s are treated as uncertainties. As a result, the state estimation error will be insensitive to the sensor faults in the sensor group y_s, but sensitive to any other sensor faults outside of this group.

Denote $y^H = (y_1^H \ \cdots \ y_p^H)^T$ as the healthy output vector; that is, when all sensors are healthy, one has $y^H = y$, otherwise, $y^H \neq y$. Let $y_s^H = (y_{i_1}^H \ \cdots \ y_{i_l}^H)^T$ and let $\bar{y}_s^H$ denote a vector consisting of the remaining components of y^H.

To design the required SMO for (2.5), the following assumption is needed:

- Assumption $A32$: For a given set $s = \{i_1, \cdots, i_l\} \in 2^{So}$ with $1 \leq l \leq p$, assume that L_s, $F1_s$, $F2_s$, a positive definite matrix P_s, and $\epsilon_s > 0$ exist such that

$$\begin{aligned} (\underline{A}_s - L_s\underline{\bar{C}}_s)^T P_s &+ P_s(\underline{A}_s - L_s\underline{\bar{C}}_s) + \epsilon_s I < 0, \\ 2\gamma P_s - \epsilon_s I &< 0, \\ P_s\underline{B}_s = \underline{\bar{C}}_s^T F1_s^T \quad &, \quad P_s\underline{D} = \underline{\bar{C}}_s^T F2_s^T. \end{aligned} \tag{3.4}$$

If Assumption $A32$ is satisfied, then an SMO can be designed as

$$\dot{\hat{z}}_{aug,s} = \underline{A}_s\hat{z}_{aug,s} + L_s(\underline{\bar{y}}_s - \underline{\bar{C}}_s\hat{z}_{aug,s}) + \underline{f}(\hat{z}_{aug,s}) + \underline{B}u + \underline{B}_s\mu 1_s + \underline{D}\mu 2_s \tag{3.5}$$

where $\hat{z}_{aug,s}$ is the estimate of $z_{aug,s}$, $\mu 1_s$, and $\mu 2_s$ are defined as

$$\mu 1_s = \begin{cases} -\rho 1_s \frac{F1_s ey_s}{\|F1_s ey_s\|}, & \|F1_s ey_s\| \neq 0 \\ 0, & \|F1_s ey_s\| = 0 \end{cases}$$

and

$$\mu 2_s = \begin{cases} -\rho 2_s \frac{F2_s ey_s}{\|F2_s ey_s\|}, & \|F2_s ey_s\| \neq 0 \\ 0, & \|F2_s ey_s\| = 0, \end{cases}$$

with $ey_s = \underline{\bar{C}}_s\hat{z}_{aug,s} - \underline{\bar{y}}_s$, and $\rho 1_s$ and $\rho 2_s$ are design constants.

By defining $e_s = \hat{z}_{aug,s} - z_{aug,s}$, the following is derived:

$$\begin{aligned} \dot{e}_s &= (\underline{A}_s - L_s\underline{\bar{C}}_s)e_s + (\underline{f}(\hat{z}_{aug,s}) - \underline{f}(z_{aug,s})) + L_s(\underline{\bar{y}}_s - \underline{\bar{y}}_s^H) \\ &+ \underline{B}_s(\mu 1_s - y_s) + \underline{D}(\mu 2_s - d(t)) \end{aligned} \tag{3.6}$$

where $\underline{\bar{y}}_s^H = ((\bar{y}_s^H)^T \ \xi_s^T)^T$.

3.3.3 Necessary Conditions for the Existence of SMOs

Based on either [30] or [114], necessary conditions for the existence of the two SMOs can be given in Lemma 3.3.1 and Lemma 3.3.2.

Lemma 3.3.1 *A necessary condition for the existence of the SMO given by (3.2) is that the rank $C(B_s\ D)$=rank $(B_s\ D)$, and that for any complex number λ with $Re(\lambda) \geq 0$, the following is true*

$$rank \begin{pmatrix} \lambda I_n - A & B_s & D \\ C & 0 & 0 \end{pmatrix} = n + rank(B_s\ D).$$

Lemma 3.3.2 *A necessary condition for the existence of the SMO given by (3.5) is that the rank $(\bar{C}_s D)$ =rank D, and that for any complex number λ with $Re(\lambda) \geq 0$, the following is true*

$$rank \begin{pmatrix} \lambda I_n - A & D \\ \bar{C}_s & 0 \end{pmatrix} = n + rank D.$$

Remark 3.3.2 *The necessary conditions in the two lemmas are not new results; they have already been proved or implied by [114] and [30]. The reason for their presence is to show their dependence on the set s. Their dependence on the set s leads to the introduction of new concepts called the SMO Induced Actuator Fault Isolation Index (SMOIAFIX) and the SMO Induced Sensor Fault Isolation Index (SMOISFIX), which will be defined in Section 3.4.*

As in Chapter 2, the uncertainties satisfying the above necessary conditions are called matched uncertainties.

3.3.4 Properties of the Designed SMOs

In this subsection, the properties of the proposed SMOs by (3.2) and (3.5) are investigated, and two results are given below.

Theorem 3.1 *Under assumptions A21, A22, and A31, if* $\bar{u}_s^H = \bar{u}_s$, *and* u_s *is bounded, then the SMO given by (3.2) ensures that* e_s *exponentially approaches zero and thus is invariant with respect to* u_s *and* $d(t)$.

Theorem 3.2 *Under assumptions A21, A22, and A31, if* $\underline{\bar{y}}_s = \underline{\bar{y}}_s^H$, *and* y_s *is bounded, then the SMO given by (3.5) ensures that* e_s *exponentially approaches zero and thus is invariant with respect to* y_s *and* $d(t)$.

Because the proof of Theorem 3.1 and Theorem 3.2 are similar, only the proof of Theorem 3.2 is provided. *Proof.* Because $\underline{\bar{y}}_s = \underline{\bar{y}}_s^H$, it follows from (3.6) that

$$\begin{aligned} \dot{e}_s &= (\underline{A}_s - L_s\underline{\bar{C}}_s)e_s \\ &+ (\underline{f}(\hat{z}_{aug,s}) - \underline{f}(z_{aug,s})) + \underline{B}_s(\mu 1_s - y_s) + D(\mu 2_s - d(t)). \end{aligned} \tag{3.7}$$

By choosing a Lyapunov function as $V = e_s^T P_s e_s$ and differentiating it with respect to t along (3.7),

$$\begin{aligned} \dot{V} &= e_s^T[(\underline{A}_s - L_s\underline{\bar{C}}_s)^T P_s + P_s(\underline{A}_s - L_s]e_s + 2e_s^T P_s(\underline{f}(\hat{z}_{aug,s}) - \underline{f}(z_{aug,s})) \\ &\quad +2e_s^T P_s[\underline{B}_s(\mu 1_s - y_s)] + 2e_s^T P_s D(\mu 2_s - d(t)) \\ &\leq -\epsilon_s\|e_s\|^2 + 2\gamma\|P_s\|\|e_s\|^2 + 2e_s^T P_s[\underline{B}_s(\mu 1_s - y_s)] \\ &\quad +2e_s^T P_s D(\mu 2_s - d(t)). \end{aligned} \tag{3.8}$$

By applying the two matrix equalities in $A32$ and choosing $\rho 1_s > \|y_s)\|$ and $\rho 2_s\|d(t)\|$ (this is possible because y_s and $d(t)$ are bounded),

$$\dot{V} \leq -\epsilon_s\|e_s\|^2 + 2\gamma\|P_s\|\|e_s\|^2 = -(\epsilon_s - 2\gamma\|P_s\|)\|e_s\|^2. \tag{3.9}$$

Because $2\gamma P_s - \epsilon_s I < 0$ implies that $2\gamma\|P_s\| < \epsilon_s$, it follows from (3.9) that e_s will converge to zero exponentially as t goes to infinity. ∎

Remark 3.3.3 *Under the conditions in Theorem 3.1, e_s and thus ey_s will tend to zero no matter what u_s is; that is, e_s and thus ey_s will converge to zero asymptotically even if $u_s \neq u_s^H$. Hence, e_s and ey_s are insensitive to those actuator faults occurring amongst the actuator group u_s. Similar comments can be made for the sensor faults.*

3.3.5 LMI Based SMO Synthesis

In order to design the SMO given by (3.2) or (3.5), one has to solve (3.1) or (3.4) for L_s, $F1_s$, $F2_s$, a positive definite matrix P_s, and $\epsilon_s > 0$. Because (3.1) and (3.4) consist of two matrix inequalities and two matrix equations, solving them directly is very difficult. Fortunately, both (3.1) and (3.4) can be reformulated as LMIs using the technique introduced in [27] and thus the difficult SMO synthesis problem can be carried out easily using Matlab's LMI toolbox. For simplicity, only (3.1) is formulated as LMIs because (3.4) can be dealt with in a similar manner.

Using the technique in [27], it is easy to show that the two matrix inequalities in (3.1) are equivalent to the following two matrix inequalities:

$$\begin{aligned} \bar{A}^T\bar{P}_s + \bar{P}_s\bar{A} + \bar{C}^T\bar{Y}_s^T + \bar{Y}_s\bar{C} + \epsilon_s\bar{M}_s &< 0 \\ 2\gamma\bar{P}_s - \epsilon_s\bar{M}_s &< 0 \end{aligned} \tag{3.10}$$

where $\bar{T}_s$ can be any nonsingular matrix, $\bar{A} = \bar{T}_s A \bar{T}_s^{-1}, \bar{C} = C\bar{T}_s^{-1}$, $\bar{P}_s = (\bar{T}_s^{-1})^T P_s \bar{T}_s^{-1}$, $\bar{Y}_s = -\bar{P}_s\bar{T}_s L_s$, and $\bar{M}_s = (\bar{T}_s^{-1})^T\bar{T}_s^{-1}$.

Let $G_s = (B_s\ D)$ and $F_s^T = (F1_s^T\ F2_s^T)$. According to [30], if CG_s is of full column rank, a linear change of coordinates $\bar{T}_s$ can be found such that

$$\begin{aligned} \bar{G}_s &= \bar{T}_s G_s = \begin{pmatrix} 0 \\ 0 \\ \bar{G}_s^2 \end{pmatrix} \\ \bar{C} &= C\bar{T}_s^{-1} = [0\ \ T_{o,s}] \end{aligned} \tag{3.11}$$

where the upper zero block of $\bar{G}_s$ is in $R^{(n-p)\times(l+q)}$, the lower zero block is in $R^{(p-l-q)\times(l+q)}$, $\bar{G}_s^2$ is in $R^{(l+q)\times(l+q)}$ and nonsingular, and $T_{o,s}$ is in $R^{p\times p}$ and is orthogonal. Guidelines for computing $\bar{T}_s$ can be found in [30].

If $\bar{P}_s$ is defined accordingly to $\bar{G}_s$ as

$$\bar{P}_s = \begin{pmatrix} \bar{P}_1 & [\bar{P}_{12}\ \bar{P}_{13}] \\ [\bar{P}_{12}\ \bar{P}_{13}]^T & \bar{P}_2 \end{pmatrix}, \tag{3.12}$$

according to [27], $P_s G_s = C^T F_s^T$ is equivalent to

$$\bar{P}_s = \begin{pmatrix} \bar{P}_1 & [\bar{P}_{12}\ 0] \\ [\bar{P}_{12}\ 0]^T & \bar{P}_2 \end{pmatrix}. \tag{3.13}$$

Now, by allowing $\bar{P}_s = \begin{pmatrix} \bar{P}_1 & [\bar{P}_{12}\ 0] \\ [\bar{P}_{12}\ 0]^T & \bar{P}_2 \end{pmatrix}$, (3.1) is equivalent to (3.10). Also note that (3.10) consists of two LMIs, (3.1) has been reformulated as LMIs given by (3.10). After a feasible solution for $\bar{P}_s > 0$, $\bar{Y}_s$ and $\epsilon_s > 0$ is found for (3.10), P_s,L_s, and F_s can be computed easily as

$$\begin{aligned} P_s &= \bar{T}_s^T \bar{P}_s \bar{T}_s, \\ L_s &= -\bar{T}_s^{-1} \bar{P}_s^{-1} \bar{Y}_s, \\ F_s &= (T_{o,s}^{-1} \bar{P}_2 \begin{pmatrix} 0 \\ \bar{G}_s^2 \end{pmatrix})^T. \end{aligned} \tag{3.14}$$

Remark 3.3.4 *The LMI based SMO design procedure is still valid for $s = \phi$ if $\bar{B}_s = B$ and $(D\ \ B_s) = D$. If an SMO for such a case exists, the uncertainties represented by $d(t)$ are called* **matched**. *The SMO designed for this case is useful for fault detection.*

3.4 The FDIE Strategy

In this section, solutions for the problems formulated in Section 3.2 using SMO design are provided. In order to accomplish this, new concepts called the *SMO Induced*

Actuator Fault Isolation Index (SMOIAFIX) and *SMO Induced Sensor Fault Isolation Index* (SMOISFIX) are defined based on the existence conditions of a bank of SMOs as described in Definition 3.4.1.

Definition 3.4.1 *System (2.1) is said to have an SMO Induced Actuator Fault Isolation Index (SMOIAFIX) (or an SMO Induced Sensor Fault Isolation Index (SMOISFIX)) equal to l if and only if the conditions required by Lemma 3.3.1 (or Lemma 3.3.2) are satisfied for all $s = \{i_1, \cdots, i_l\} \in 2^{S_I}$ (or $s = \{i_1, \cdots, i_l\} \in 2^{S_O}$). l is defined as the largest number that has this required property.*

For simplicity, AFIX and SFIX are used to stand for SMOIAFIX and SMOISFIX respectively in remainder of this section.

Remark 3.4.1 *The concepts of AFIX and SFIX depend on the system matrices and thus are system characteristics. The role of disturbances on the AFIX or SFIX is also clearly shown. If there is no $l > 0$ such that $AFIX = l$ or $SFIX = l$, system (2.1) does not have any AFIX or SFIX. For consistency, this situation is denoted as $AFIX = 0$ or $SFIX = 0$. For $AFIX > 0$, assume that $rank(B_s \;\; D) = rank(B_s) + rank(D)$ for any $s = \{i_1, \cdots, i_{AFIX}\} \in 2^{S_I}$. Showing that $0 \leq AFIX \leq p - q$ or $0 \leq SFIX \leq p - q$ is easy.*

Remark 3.4.2 *The motivation for introducing AFIX and SFIX is to characterize the maximum number of faults that can be isolated. No such concept has been proposed in the literature although the determination of the maximum number of faults has been discussed (more often implicitly or qualitatively). For example, refer to the results based on invariant subspaces in [42] and those based on structured residuals in [9] and [10]. The invariant subspace based results use geometric methods, while AFIX and SFIX are defined based on algebraic conditions. Those structured residual based results focus on what kind of residuals have to be constructed to isolate faults, while AFIX and SFIX are more clearly related to the system structure.*

Fault diagnosis can be implemented using the fault reconstruction technique [30, 31, 27] via a single SMO design if $AFIX = m$. Therefore, only the case for $AFIX < m$ is considered for actuator fault isolation and estimation. As argued in Chapter 2, because the elements in u_s or y_s are treated as uncertainties, only the GOS scheme can be employed based on the structure of the designed SMO. The the following of this section, answers are provided for all the problems posed in Section 3.2 by employing the designed SMO based GOS fault diagnosis scheme.

Theorem 3.3 *Under assumptions* A21, A22, *and* A31, *and assuming that only actuator faults can occur, and that the diagnostic observer given by (3.2) is used for actuator fault isolation, the maximum number of actuator faults that can simultaneously be isolated is equal to the Actuator Fault Isolation Index (AFIX).*

Theorem 3.4 *Under assumptions* A21, A22, *and* A31, *as well as assuming that only sensor faults can occur, and that the diagnostic observer given by (3.5) is used for sensor fault isolation, the maximum number of sensor faults that can simultaneously be isolated is equal to the Sensor Fault Isolation Index (SFIX).*

Proof. Only the proof of Theorem 3.3 is given because the other theorem can be proved similarly.

By definition, no single actuator fault can be isolated if $AFIX = 0$. Therefore, the theorem needs to be proved for the case when $AFIX > 0$.

By the definition of the $AFIX$, for each set of the form $s = \{i_1, \cdots, i_{AFIX}\} \in 2^{S_I}$, an SMO given by (3.2) can be designed. In total, a bank of C_m^{AFIX} SMOs can be designed for all sets of the form s. Assume that the number of actuator faults is n_f and $n_f \leq AFIX$. According to Theorem 3.1, for any u_s, which includes all the faulty actuators, e_s and ey_s tend to zero asymptotically. Hence, at least $C_{AFIX}^{n_f}$ sets are of the form s such that e_s and ey_s tend to zero asymptotically. If exactly $C_{AFIX}^{n_f}$ sets are such sets, the n_f faulty actuators can be isolated. The detailed method for achieving fault isolation will be described later in this section.

The remaining proof is that it is not possible to isolate l faults with $l > AFIX$.

To design a GOS to isolate l faults, all inputs in u_s must be able to be treated as uncertainties in the design of SMOs of the form (3.2) for all sets of the form $s = \{i_1, \cdots, i_l\} \in 2^{S_I}$. In such a case, it would lead to an *Actuator Fault Isolation Index* no less than l, which contradicts the definition of the *Actuator Fault Isolation Index* because $l > AFIX$. This completes the proof. ∎

Actually, Theorem 3.3 has provided solutions for **FDP1**, **FDP2**, **FIP1**, and **FIP2** at the same time. This point is illustrated more clearly in the following corollaries.

Corollary 3.4.1 *Under the assumptions of Theorem 3.3, if $AFIX = 0$, actuator fault isolation is impossible and fault detection is possible; if $AFIX = 1$, only one single fault can be isolated; if $AFIX = l > 1$, actuator fault isolation can be performed for one up to l faults.*

Proof. For the case that $AFIX = 0$, let $s = \phi$, according to Remark 3.3.4, if an SMO exists such that it is invariant to the unknown inputs $d(t)$, it is obvious that fault detection is possible using such an SMO. The other points of this corollary are already proved in Theorem 3.3. ∎

Corollary 3.4.2 *Under the assumptions of Theorem 3.3, if $AFIX > 0$, the maximum number of actuator faults that can be simultaneously isolated is $AFIX$.*

Remark 3.4.3 *Two corollaries can also be given for sensor FDIE in the same manner.*

For system (2.1), assume that only one type of fault can occur, and also that fewer than $AFIX$ actuator ($SFIX$ sensor) faults can occur at the same time. The goals are to design a GOS scheme using as few SMOs as possible to detect the faults, to determine the number of faults, and to isolate and estimate the faults. To accomplish these goals, a set of actuator groups characterized by u_s with $s = \{i_1, \cdots, i_{AFIX}\} \in 2^{S_I}$ are defined. For each actuator group u_s, an SMO is designed to make the output estimation error insensitive to any fault in this actuator group, but sensitive to any fault outside this actuator group.

The following theorem provides theoretical support for the possibility of designing GOS based actuator FDIE schemes.

Theorem 3.5 *Under assumptions A21, A22, and A31, as well as assuming that only actuator faults can occur, that all the conditions in Theorem 3.1 are met, and that $C\bar{B}_s$ and $(B\ D)$ are of full column rank for any set s, a bank of SMOs can be used to define residuals that satisfy the property of GOS. That is, each residual is only insensitive to faults in a particular actuator group and sensitive to all other faults outside the actuator group.*

Proof. For any set s, if all actuator faults are inside the actuator group u_s, as implied by Theorem 3.1 and pointed out in Remark 3.3.3, ey_s is insensitive to all actuator faults.

On the other hand, any actuator fault outside the actuator group u_s will result in $\bar{u}_s^H - \bar{u}_s \neq 0$, which makes $\bar{B}_s(\bar{u}_s^H - \bar{u}_s) \neq 0$. Because $(B\ D)$ is of full column rank, the nonzero term $\bar{B}_s(\bar{u}_s^H - \bar{u}_s)$ in (3.3) cannot be attenuated by $\mu 1_s$ and $\mu 2_s$. This fact, together with (3.3), implies that e_s is sensitive to any faults outside the actuator group u_s. Note again because $C\bar{B}_s$ is of full column rank, ey_s is sensitive to any faults outside the actuator group u_s.

It has been proved that for any set s, ey_s is insensitive to all actuator faults inside the actuator group u_s, but sensitive to any actuator faults outside u_s. In total, one has a bank of C_m^{AFIX} SMOs, and thus has C_m^{AFIX} output estimation errors(ey_s). Clearly, C_m^{AFIX} residuals can be defined using ey_s to satisfy the property of GOS. ∎

The situation is a bit more complicated for sensor FDIE. In this case, L_s is first partitioned according to matrix $\underline{\bar{C}}_s$ as $L_s = \begin{pmatrix} L_s^{11} & L_s^{12} \\ L_s^{21} & L_s^{22} \end{pmatrix}$. By defining $L1_s = ((L_s^{11})^T \quad (L_s^{21})^T)^T$, the following theorem provides theoretical support for the design of the GOS based sensor FDIE schemes.

Theorem 3.6 *Under assumptions A21, A22, and A32, as well as assuming that only sensor faults can occur, that all conditions in Theorem 3.2 are met, that $\bar{C}_s L_s^{11}$ is of full column rank, and that rank$(L_s^{11} \ D)$ = rank(L_s^{11})+rank$D$, a bank of SMOs can be used to define residuals that satisfy the property of GOS. That is, each residual is only insensitive to faults in a particular sensor group and sensitive to all other faults outside the sensor group.*

Proof. By definition, $L_s(\underline{\bar{y}}_s - \underline{\bar{y}}_s^H) = L1_s(\bar{y}_s - \bar{y}_s^H)$. Using this fact, (3.6) becomes

$$\begin{aligned} \dot{e}_s &= (\underline{A}_s - L_s\underline{\bar{C}}_s)e_s + (\underline{f}(z_s) - \underline{f}(z_{aug,s})) \\ &+ L1_s(\bar{y}_s - \bar{y}_s^H) + \underline{B}_s(\mu 1_s - y_s) + \underline{D}(\mu 2_s - d(t)). \end{aligned} \tag{3.15}$$

By the definition of $\underline{B}_s$ and $\underline{D}$ and employing (3.15), the term $L1_s(\bar{y}_s - \bar{y}_s^H)$ will not be attenuated by $\mu 1_s$ and $\mu 2_s$ under the assumption that rank$(L_s^{11} \ D)$ = rank(L_s^{11})+rank(D). Using this fact, the theorem can be proved using the same arguments in the proof of Theorem 3.5. ∎

The theorem below serves as a foundation for determining the number of faults.

Theorem 3.7 *Under the conditions in Theorem 3.5 (or Theorem 3.6), if the number of faults is $0 < n_f < AFIX$ (or $0 < n_f < SFIX$), then the number of residuals, which are insensitive to the n_f faults, is equal to $C_{m-n_f}^{AFIX-n_f}$ (or $C_{m-n_f}^{SFIX-n_f}$).*

Proof. Because there are n_f faults, the number of sets of the form $s = \{i_1, \cdots, i_{AFIX}\}$ (or $s = \{i_1, \cdots, i_{SFIX}\}$), which include all faulty actuators (or sensors), is equal to $C_{m-n_f}^{AFIX-n_f}$ (or $C_{m-n_f}^{SFIX-n_f}$). According to Theorem 3.5 (or Theorem 3.6), all residuals corresponding to these sets are insensitive to the n_f faults and any other residual is sensitive to the faults. This completes the proof. ∎

Remark 3.4.4 *If there are n_f faults and $n_f < AFIX$ (or $n_f < SFIX$), exactly $C_{m-n_f}^{AFIX-n_f}$ (or $C_{m-n_f}^{SFIX-n_f}$) residuals are insensitive to all faults. Based on this fact, the number of faults n_f can be found by solving $C_{m-n_f}^{AFIX-n_f} = g_{num}$ (or $C_{m-n_f}^{SFIX-n_f} = g_{num}$), where g_{num} is the number of residuals that are insensitive to all faults.*

For any set $s = \{i_1, \cdots, i_{AFIX}\}$ (or $s = \{i_1, \cdots, i_{SFIX}\}$), the residual defined according to ey_s is denoted as r_s. The following result is useful in fault isolation.

Theorem 3.8 *Under the assumptions of Theorem 3.7, suppose only g_{num} residuals are under the threshold and $C_{m-n_f}^{AFIX-k} = g_{num}$ (or $C_{m-n_f}^{SFIX-k} = g_{num}$) has an integer solution for k. Denote those residuals as $r_{s_1}, \cdots, r_{s_{gnum}}$, define $s_j = \{i_1^j, \cdots, i_{AFIX}^j\}, 1 \leq j \leq g_{num}$ (or $s_j = \{i_1^j, \cdots, i_{SFIX}^j\}, 1 \leq j \leq g_{num}$), and let $S_F = \bigcap_{j=1}^{g_{num}} s_j$. Then, the number of elements in S_F is the number of faults, and each element in S_F identifies a particular fault.*

Proof. According to Theorem 3.7, at least $C_{m-n_f}^{AFIX-n_f}$ (or $C_{m-n_f}^{SFIX-n_f}$) residuals are under the threshold if there are n_f faults. Because g_{num} residuals (i.e., $r_{s_1}, \cdots, r_{s_{gnum}}$) are under the threshold, and the equation $C_{m-n_f}^{AFIX-k} = g_{num}$ (or $C_{m-n_f}^{SFIX-k} = g_{num}$) has an integer solution for k, faults can only occur in actuator groups $u_{s_1}, \cdots, u_{s_{gnum}}$ or sensor groups $y_{s_1}, \cdots, y_{s_{gnum}}$). This fact, together with the definition of S_F, proves the conclusions of the theorem. ∎

Assume that the fault isolation has been achieved using the technique provided in Theorem 3.8 and $S_F = \{i_1, \cdots, i_{n_f}\}$ is obtained. To estimate the faults, it is further assumed that $(B_s\ D)$ (or $(L_s^{11}\ D)$) is of full column rank for any set s (otherwise faults may be mixed with $d(t)$ and may not be correctly estimated). A certain set s_0 with the smallest residual is picked up for fault estimation. Because e_{s_0} tends to zero, and $\dot{e}_{s_0}$ is assumed to tend to zero, according to (3.3) (or (3.6)) and the idea of using a low-pass filter to estimate the equivalent control, the following approach is proposed to estimate the faults:

$$u_{i_j}^{fe} = LPF(\mu 1_{s_0}(i_j)) - u_{i_j}^{H}, 1 \leq j \leq n_f \tag{3.16}$$

where $\mu 1_{s_0}(i_j)$ is an element in $\mu 1_{s_0}$ that corresponds to the index i_j, and LPF denotes a low-pass filter.

Remark 3.4.5 *The fault estimation method based on SMO design has been proposed in the literature. In [27, 30, 31], in order to estimate faults, continuous terms are used to approximate the discontinuous terms in SMOs. If one does not want to modify the SMO, an alternative method is to use a low-pass filter to achieve fault estimation, which is used in this chapter.*

Now, assuming that the number of faults that can occur are less than the $AFIX$ (or $SFIX$) at the same time, and that $AFIX < m$ for actuator FDIE, based on the results obtained, the following algorithm summarizes the proposed actuator (or sensor) FDIE strategy.

FDIE Algorithm

Step 1. Compute $AFIX$ (or $SFIX$).

Step 2. If $AFIX = 0$ (or $SFIX = 0$), no fault can be isolated. Fault detection can be achieved using the SMO for the case $s = \phi$. If $0 < AFIX$ (or $0 < SFIX$), then go to Step 3.

Step 3. Perform fault detection and isolation:

1. For each set s, design an SMO given by (3.2) (or (3.5)).
2. Define residuals $r_s(t) = \|Ce_s\|^2/N_{normal}(t)$, where $N_{normal}(t)$ is chosen such that $r_s(t) \leq 1$ when only actuators (or sensors) in u_s (or y_s) are possibly faulty, and $r_s(t) > 1$ otherwise.
3. The threshold is chosen as 1.
4. Fault Detection

   If any of the C_m^{AFIX} (or C_m^{SFIX}) residuals are larger than the threshold at any time constant, faults are detected. Otherwise, no fault has been detected.
5. After faults are detected, denote the fault detection time as T_{detect}, choose a fault isolation time interval (FITI) as $(T_{detect}, T_{detect} + \Delta)$ with Δ suitably large, during which one wishes to perform fault isolation.
6. Count the number of residuals that are below the threshold, and denote the number as g_{num}.
7. If $g_{num} = 0$, more than $AFIX$ actuators (or $SFIX$ sensors) are faulty and exact fault isolation cannot be achieved. Stop.
8. If $g_{num} = 1$, there are $AFIX$ actuator (or $SFIX$ sensor) faults. If r_s is the only residual that is under the threshold and $s = \{i_1, \cdots, i_{AFIX}\}$ (or $s = \{i_1, \cdots, i_{SFIX}\}$), then the $i_1 \cdots i_{AFIX}$th actuators (or the $i_1 \cdots i_{SFIX}$th sensors) are faulty. Fault isolation is performed. Stop.
9. If $g_{num} > 1$, solve $C_{m-k}^{AFIX-k} = g_{num}$ (or $C_{m-k}^{SFIX-k} = g_{num}$) for k. If no integer solution exists, the number of faults occurred cannot be determined and

fault isolation cannot be performed at this moment. Choose a larger Δ, and go to Step 3.6. If an integer solution k_0 exists, the number of faults is k_0.

10. After the number of faults is determined, $s_j = \{i_1^j, \cdots, i_{AFIX}^j\}, 1 \leq j \leq g_{num}$ (or $s_j = \{i_1^j, \cdots, i_{SFIX}^j\}, 1 \leq j \leq g_{num}$) is obtained and $S_F = \bigcap_{j=1}^{g_{num}} s_j$ is computed.

11. Fault Isolation for $g_{num} > 1$: Denote $S_F = \{i_1, \cdots, i_{k_0}\}$. Then, the faulty actuators (or sensors) are the $i_1 \cdots i_{k_0}$th actuators (or sensors).

Step 4. Perform fault estimation

Pick up a set s_0 which corresponds to the best residual (or the smallest), then use (3.16) to estimate the faults by letting $n_f = k_0$.

Remark 3.4.6 *The above FDIE algorithm has provided systematic and detailed methods to perform FDIE. In the algorithm, fault detection is accomplished as a byproduct of fault isolation. The algorithm can determine how many faults can be isolated, make a decision on the number of faults, isolate, and estimate the faults. The idea of determining the number of faults is interesting and enables us to perform the FDIE only using C_m^{AFIX} (or C_m^{SFIX}) SMOs, which is the smallest number of SMOs needed for a GOS scheme (or for actuator FDIE where one assumes $AFIX < m$). The idea of using fault isolation time interval (FITI) to accomplish the fault isolation is also needed for more solid isolation decisions. For fast fault isolation, a small FITI can be used, but too small of an FITI may lead to a wrong fault isolation decision. Obviously, there is a tradeoff between fast fault isolation and an accurate decision.*

Remark 3.4.7 *The FDIE scheme is based on defining residuals using the output estimation error. For the case that $AFIX < m$, it is not clear how to accomplish fault* *diagnosis through directly reconstructing the faults employing SMOs like in [30, 31] and [27] because the fault distribution matrix is not known.*

Remark 3.4.8 *Note, under the conditions required in this chapter, an unknown input observer (UIO) based FDI scheme may also be designed in a similar way. SMOs are chosen for the following reasons. Firstly, the SMO based method can provide the estimation of the faults directly while existing UIO based methods usually do not. Secondly, SMO based schemes are an alternative to UIO based schemes, and a person more familiar with SMOs but not with UIOs can design and use SMOs.*

3.5 An Illustrative Example and Simulation Results

In this section, an example is given to show the effects of the diagnostic scheme when applied to an actuator FDIE problem of uncertain Lipschitz nonlinear systems.

Given a system

$$\begin{aligned} \dot{x} &= Ax + f(x) + Bu + Dd(t) \\ y &= Cx \end{aligned} \tag{3.17}$$

where

$$A = \begin{pmatrix} -0.6344 & 0.0027 & 0 & 0.9871 & 0 \\ 0 & -0.0038 & 0.1540 & 0 & -0.9876 \\ 0 & -8.2125 & -0.7849 & 0 & 0.1171 \\ -0.5971 & 0 & 0 & -0.5099 & 0 \\ 0 & -0.8887 & -0.0299 & 0 & -0.0156 \end{pmatrix},$$

$$B = \begin{pmatrix} 0.0213 & 0 & 0 \\ 0 & 0.0356 & -0.0006 \\ 0 & -0.0004 & 0.0427 \\ -0.0001 & 0 & 0 \\ 0 & -0.0113 & 0.0018 \end{pmatrix}, \quad D = \begin{pmatrix} 0.0106 \\ 0.0116 \\ 0.0212 \\ -0.0001 \\ -0.0029 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix},$$

$$f(x) = (\, 0.05sin(x_2) \quad 0.01(x_1 + sin(x_5)) \quad 0.03(cos(x_3) + sin(x_5)) \quad 0 \quad 0\,)^T,$$

and $d(t) = sin(0.5t)cos(2t)$.

For this system, $AFIX = 2$, which is less than the number of actuators. Because $S_I = \{1, 2, 3\}$, there are in total three sets with two elements; that is, $\{1, 2\}$, $\{1, 3\}$, and $\{2, 3\}$. Using the LMI based design approach, three SMOs of the form given by (3.2) can be designed for $s = \{1, 2\}$, $s = \{1, 3\}$, and $s = \{2, 3\}$, respectively.

In all simulations, the normalization signal is chosen as $N_{normal}(t) = 0.001$, and three residuals, r_{12}, r_{13}, r_{23}, are defined according to $s = \{1, 2\}$, $s = \{1, 3\}$, and $s = \{2, 3\}$, respectively. For comparison, the scheme is tested on two types of incipient faults: one is changing slowly, and the other is changing faster (specific definition of faults are given later in this section). Simulation results are plotted in Fig. 3.1 to Fig. 3.4. The first two figures are the results for the slowly changing actuator faults, while the last two are for the faster changing actuator faults.

At $t = 5s$, the first and second actuators have faults defined as

$$u_i(t) = \begin{cases} u_i(5) + Pa(t-5)/10, & 5 < t \leq 15 \\ u_i(5) + Pa - Pa(t-15)/20, & 15 < t \leq 25 \\ u_i(5) + Pa/2, & t > 25, \end{cases}$$

where $i = 1, 2$, and Pa is a parameter which indicates the changing rate of the faults. $Pa = 2$ and $Pa = 10$ are chosen to simulate the slow and fast actuator fault scenarios.

For the slow actuator fault case, the FDIE is performed as follows. Because the residual r_{13} in Fig. 3.1 exceeds the threshold, faults are detected at $5.16s$, which is

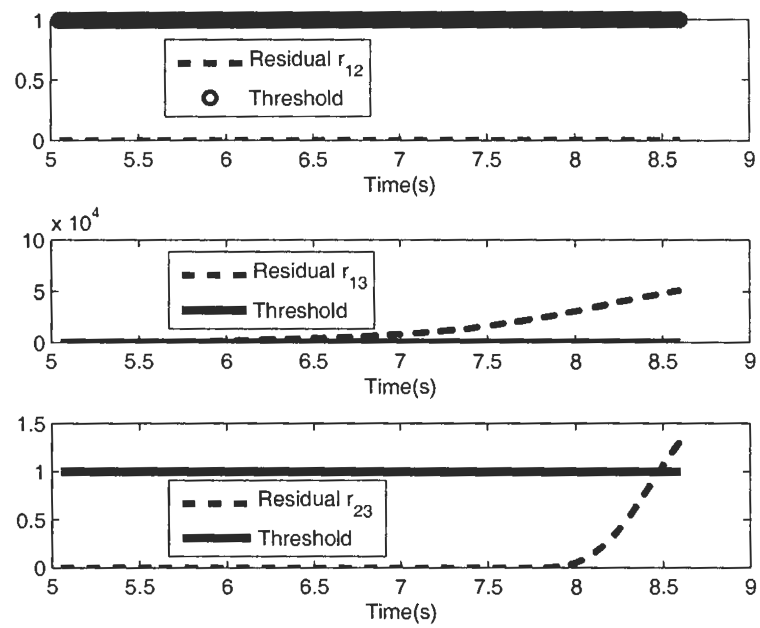

Figure 3.1: Nonlinear fault detection and isolation – Slow actuator fault case

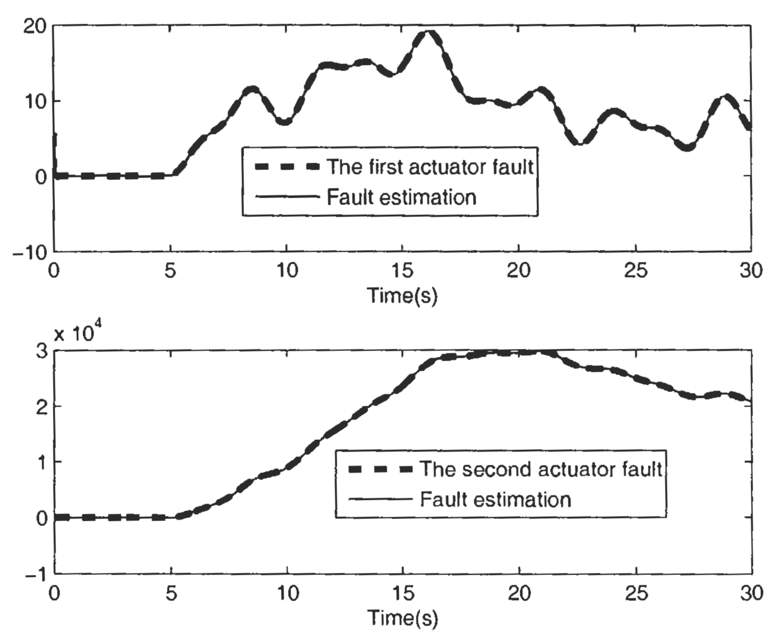

Figure 3.2: Nonlinear fault estimation – Slow actuator fault case

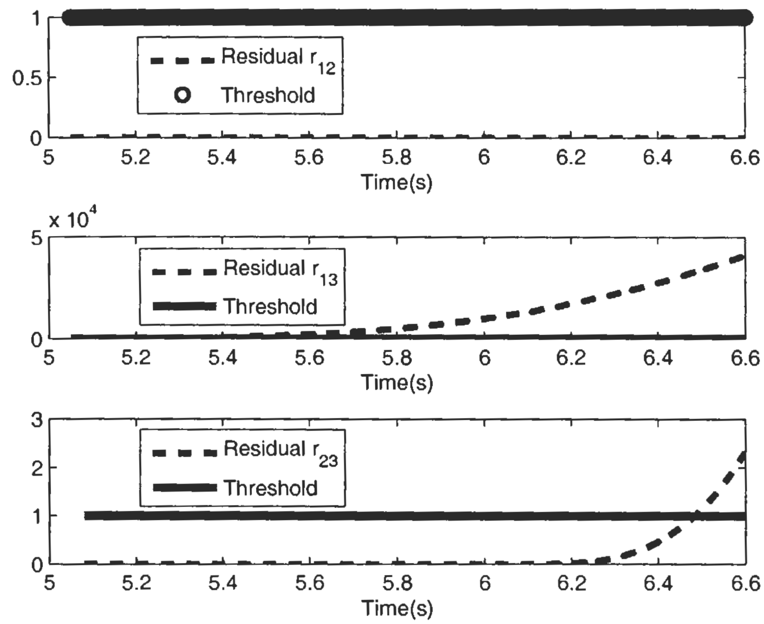

Figure 3.3: Nonlinear fault detection and isolation – Fast actuator fault case

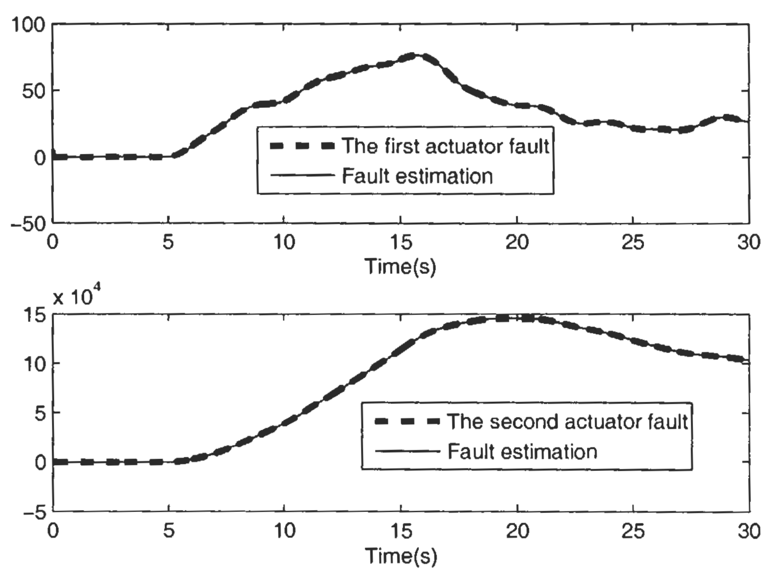

Figure 3.4: Nonlinear fault estimation – Fast actuator fault case

$0.16s$ after faults occurred. By choosing $FITI = (5.16s, 8.60s)$, as shown in Fig. 3.1, only r_{12} is well below the threshold; therefore $g_{num} = 1$. By solving $C_{3-k}^{2-k} = 1$, $k = 2$, which implies that the number of faults is two. Because r_{12} is under the threshold, $S = \{1, 2\}$, which means both the first and second actuators are faulty. The faults are isolated within four seconds after they were detected, and the decision is correct. Fig. 3.2 shows that accurate fault estimation has been achieved.

For fast actuator fault case, the FDIE is performed as follows. Because the residual r_{13} in Fig. 3.3 exceeds the threshold, faults are detected at $5.11s$, which is $0.11s$ after faults occurred. Similar to the slow fault case, by choosing $FITI = (5.11s, 6.60s)$, it follows from Figure 3.3 that two actuator faults are present and both the first and second actuators are faulty. The faults are isolated within two seconds after faults were detected, and the decision is correct. Again, as shown Fig. 3.4, the actuator fault can be estimated very accurately.

Comparing the results for both cases, it is easy to see that faster changing faults can be detected and isolated sooner.

3.6 Conclusions and Discussions

Sliding mode observer design to achieve actuator or sensor fault detection, isolation, and estimation has been studied in a detailed and systematic way for a class of uncertain Lipschitz nonlinear systems.

Based on the particular system representation developed in Chapter 2, a diagnostic SMO was designed with a special property suitable for fault isolation purposes. Necessary conditions were provided for the existence of SMOs for both actuator and sensor fault cases. To make the SMO design easier, LMI based design approaches were derived to enable designers to use commercially available software such as Matlab's

LMI toolbox to carry out the otherwise difficult SMO design.

Given that the proposed nonlinear SMOs are used for FDIE, answers were provided for all the fault diagnosis problems raised in Section 1.2 by designing FDIE schemes, which were presented in steps. An example was given to show how to use the FDIE scheme and to also show its effectiveness. The simulation results show that the proposed FDI scheme does work well in detecting, isolating, and estimating both slow and fast changing faults for uncertain Lipschitz nonlinear systems in terms of providing correct FDI decisions and accurate fault estimation.

Similar discussions as those made in Chapter 2 can be made here on the limitations of the SMO based fault diagnosis. Again, more research is needed to extend the proposed FDIE schemes to more general nonlinear systems.

Chapter 4

UIO Based Fault Diagnosis for Uncertain Nonlinear Systems Represented by TS Fuzzy Models

This chapter extends the results in Chapter 2 to more general nonlinear systems with matched non-parametric uncertainties, which can be represented by Takagi-Sugeno (TS) fuzzy systems.

4.1 Introduction

Because powerful design methods to deal with nonlinearities are lacking, existing nonlinear unknown input observers (NUIOs) are often designed under various restrictions. Since UIO design for general nonlinear systems is still largely an open problem, NUIO based nonlinear fault diagnosis remains also as an area for further research. This chapter considers the NUIO design and NUIO based nonlinear fault diagnosis problems.

Takagi-Sugeno (TS) fuzzy systems can approximate general nonlinear systems or represent many of them [116]; therefore, they are promising for solving many nonlinear observer design and control problems. In addition, TS fuzzy systems also have the advantage that they make it possible to use the rich control system theory that has been developed over many years, which is the reason why nonlinear systems represented by TS fuzzy models are chosen to be considered.

Because results related to UIO design and UIO based fault diagnosis have already been reviewed in Chapter 2, only the results related to fuzzy observer design based on TS fuzzy systems and their application to fault diagnosis will be reviewed.

Because a TS fuzzy system is a blending of local linear models, many fuzzy observers are designed by extending the observer design techniques developed for linear systems. Fuzzy observers are designed for two cases: for the case that the premise variables do not depend on unmeasured state variables [117, 118, 119, 120, 121, 122, 123, 124, 125, 126]; and the case that the premise variables depend on unmeasured state variables [124, 127, 128].

All the observers in the cited references are designed based on the so-called parallel distributed compensation (PDC) concept. The main idea of PDC is like this: a local observer is first designed for each local linear model, then the overall observer is obtained by blending all the local observers in the same or similar way as the local linear models are blended.

In [117], the separation property for linear systems was proved to be also true for TS fuzzy systems, and the stability of the fuzzy observer was provided as its byproduct. In [124], fuzzy observers were designed for both cases mentioned earlier in this introduction, where relaxed sufficient stability conditions were proved. LMI based designs were proposed for both the controller and observer design. In order to further relax the sufficient stability conditions, sufficient conditions that are less conservative were established in [118] and [125]. Fuzzy Thau-Luenberger Observers were proposed in [127] for Lipschitz nonlinear systems, and an LMI based design technique was presented. Two sliding mode observers were designed in [128] for TS fuzzy systems with affine local models. In [121], robust observers were designed for TS fuzzy systems whose local models have uncertainties. For TS fuzzy systems whose local models have both uncertainties and unknown disturbances, a robust observer with L_2 gain was designed via solving the LMIs in [126]. For fuzzy systems whose local models have unknown inputs, a sliding mode observer was proposed to estimate the states in [120].

Fuzzy observers are not only used for control purpose, but also for fault diagnosis. In [123], fuzzy observers based on TS fuzzy systems were designed for the fault diagnosis of induction motors in railway systems. To deal with the unknown inputs presented in the local models, fuzzy sliding mode observers were designed to perform fault diagnosis for the three-tank benchmark system in [119] and for a two-tank hydraulic system in [122]. The robust observer in [126] was also designed for robust fault detection.

In this chapter, several NUIO design and nonlinear fault diagnosis problems are addressed. For NUIO design, the following problems are studied:

- NUIO1 Extending linear UIO design techniques to TS fuzzy systems.

- NUIO2 Determining conditions under which the state estimation error dynamics is stable and invariant to unknown inputs.

Given that the designed NUIOs are employed to perform fault diagnosis, the fault detection and isolation problems raised in Section 1.2 are also investigated.

The literature has few results on NUIO design for nonlinear systems that can be represented by TS fuzzy systems. As for the fault diagnosis problems, no systematic study exists that addresses all of these problems. The main purpose of this chapter is to solve all the NUIO design and nonlinear fault diagnosis problems in a systematic way for nonlinear systems represented by TS fuzzy systems.

This chapter is arranged as follows. In Section 4.2, the system of interest is first described, and is then represented as a TS fuzzy system. In Section 4.3, the two NUIO design problems (i.e., $NUIO1$ and $NUIO2$) are solved. In Section 4.4, a particular TS fuzzy system structure is proposed by regrouping the system inputs for the purpose of actuator fault diagnosis. Also shown is that sensor fault diagnosis can be reformulated as actuator fault diagnosis. In Section 4.5, the nonlinear fault detection and isolation problems are investigated for actuator faults and some results are derived. In Section 4.6, the Lorenz's chaotic system with multi-inputs is chosen as an example to show the effectiveness of the designed NUIOs and nonlinear fault diagnosis schemes. Conclusions and discussions are provided in the last section.

4.2 Nonlinear Systems and its TS Fuzzy System Representation

The uncertain nonlinear systems considered are of the following form

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t), d(t)) \\ y(t) &= Cx(t) \end{aligned} \tag{4.1}$$

where $x(t) \in R^n$ is the state vector, $y(t) \in R^p$ is the output vector, $u(t) \in R^m$ is the input vector, $d(t) \in R^q$ is the unknown input vector which may consist of system uncertainties and/or disturbances, and C is a constant output matrix.

Remark 4.2.1 *For the general nonlinear output equation given by $y(t) = h(x(t))$, if $\frac{\partial h}{\partial x}$ is of full row rank, a state transformation $z = \Phi(x(t))$ with $(z_1, \cdots, z_p)^T = h$ can be found such that in the new coordinates, the output vector takes on the form of $y(t) = Cx(t)$. Therefore, the output equation $y(t) = Cx(t)$ is without loss of generality.*

Takagi-Sugeno fuzzy systems are used to represent (4.1), and the ith rule of the TS fuzzy systems is of the following form:

Plant Rule i:

IF $z_1(t)$ is M_{i1}, $\cdots$, and $z_N(t)$ is M_{iN}, THEN

$$\begin{cases} \dot{x} = A_i x(t) + B_i u(t) + D_i d(t), i = 1, 2, \cdots, r \\ y(t) = Cx(t), \end{cases}$$

where M_{ij} is a fuzzy set and r is the number of fuzzy rules. $z_j(t), 1 \leq j \leq N$ are the premise variables, which are assumed to be independent of $u(t)$. In this chapter, the linear models in the consequent parts are called local linear models.

Given $(x(t), u(t), d(t))$, the final outputs of TS fuzzy systems are inferred as follows:

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{r} h_i(z(t))\{A_i x(t) + B_i u(t) + D_i d(t)\} \\ y &= \sum_{i=1}^{r} h_i(z(t)) C x(t) = Cx(t) \end{aligned} \tag{4.2}$$

where

$$\begin{cases} \sum_{i=1}^{r} h_i(z(t)) = 1 \\ h_i(z(t)) \geq 0, i = 1, 2, \cdots, r. \end{cases}$$

In this chapter, (4.2) is called a TS fuzzy system representation of (4.1).

The following assumption is needed.

Assumption $A41$: $A_i, B_i, D_i, 1 \leq i \leq r$ and C are known.

4.3 NUIO Design and Stability Conditions

In this section, the linear UIO design technique developed in [59] is extended to design NUIOs for the following two cases:

- Case A, $z_1(t) \sim z_N(t)$ do not depend on unmeasured state variables.
- Case B, $z_1(t) \sim z_N(t)$ depend on unmeasured state variables.

NUIO problems are studied for both Case A and B, and the results are presented in Subsection 4.3.1 and Subsection 4.3.2.

4.3.1 NUIO Design and Stability Conditions–Case A

The concept of PDC is used here to design an NUIO for (4.2), and the ith observer rule is of the following form:

Observer Rule i:

IF $z_1(t)$ is $M_{i1}, \cdots,$ and $z_N(t)$ is M_{iN}, THEN

$$\dot{w}(t) = A_i w(t) + G_i u(t) + L_i y(t), i = 1, 2, \cdots, r.$$

The overall fuzzy observer is given as

$$\begin{aligned} \dot{w}(t) &= \sum_{i=1}^{r} h_i(z(t))\{N_i w(t) + G_i u(t) + L_i y(t)\} \\ \hat{x}(t) &= w(t) - Ey(t) \end{aligned} \tag{4.3}$$

where $N_i, G_i, L_i, i = 1, 2, \cdots, r$ and E will be specified later in this subsection.

By defining $e(t) = \hat{x}(t) - x(t)$, it follows from (4.2) and (4.3) that

$$\begin{aligned} \dot{e}(t) &= \sum_{i=1}^{r} h_i(z(t))N_i e(t) + \sum_{i=1}^{r} h_i(z(t))\{N_i + K_i C - (I + EC)A_i\}x \\ &+ \sum_{i=1}^{r} h_i(z(t))\{G_i - (I + EC)B_i\}u - \sum_{i=1}^{r} h_i(z(t))(I + EC)D_i d(t) \end{aligned} \tag{4.4}$$

where $K_i = L_i + N_i E$.

Theorem 4.1 provides a sufficient condition for the observer given by (4.3) to be an NUIO.

Theorem 4.1 *For the observer given by (4.3) and under assumption A41, the error dynamics given by (4.4) is globally asymptotically stable at the origin if* $K_i, i = 1, 2, \cdots, r$, E, *and a positive definite symmetric matrix* P *can be chosen such that*

$$\begin{aligned} N_i &= (I + EC)A_i - K_i C, \\ G_i &= (I + EC)B_i, \\ L_i &= K_i - N_i E, \\ ECD_i &= -D_i, i = 1, 2, \cdots, r, \end{aligned} \tag{4.5}$$

and

$$N_i^T P + PN_i < 0, i = 1, 2, \cdots, r. \tag{4.6}$$

Proof. Using (4.5), it follows from (4.4) that

$$\dot{e}(t) = \sum_{i=1}^{r} h_i(z(t)) N_i e(t) \tag{4.7}$$

Using (4.6), the stability result can be proved easily. ∎

Theorem 4.1 implies that the observer given by (4.3) is an NUIO, that is, $e(t)$ asymptotically approaches zero and is invariant with respect to the unknown inputs in $d(t)$.

Equation (4.5) implies that

$$EC(D_1 \ \cdots \ D_r) = -(D_1 \ \cdots \ D_r) \tag{4.8}$$

To ensure the existence of E, another assumption is needed.

Assumption $A42$: $rank(C(D_1 \ \cdots \ D_r)) = rank((D_1 \ \cdots \ D_r))$.

Under assumption $A42$, a nonsingular matrix T_D exists such that $(D_1 \ \cdots \ D_r)T_D = (D_{T1} \ 0)$, where D_{T1} is of full column rank. This fact, together with assumption $A42$, implies that $rank(CD_{T1}) = rank(D_{T1})$, and thus that CD_{T1} is of full column rank. Based on the fact that CD_{T1} is of full column rank, equation (4.8) requires all the possible solutions for E to have the following form:

$$E = -D(CD_{T1})^+ + Y(I - CD_{T1}(CD_{T1})^+) \tag{4.9}$$

where Y is any compatible matrix and $X^+ = (X^TX)^{-1}X^T$.

Because all possible solutions for E must have the form of (4.9), the only freedom left in E is the matrix Y; once Y is chosen, E is chosen. Also that $N_i, G_i, i = 1, 2, \cdots, r$ can be determined once $L_i, i = 1, 2, \cdots, r$ and E are chosen. Now, the design of observer gain matrices is reduced to the design of $L_i, i = 1, 2, \cdots, r$ and Y such that the sufficient conditions in Theorem 4.1 can be satisfied, which is a very difficult design problem. In order to provide an efficient design method, the sufficient conditions given by (4.5) and (4.6) are reformulated as LMIs. The result is given in Theorem 4.2.

Theorem 4.2 *For the observer given by (4.3) and under assumption* $A41$, *the error dynamics given by (4.4) is globally asymptotically stable at the origin if there exist* $\bar{K}_i, 1 \leq i \leq r$, $\bar{Y}$, *and a positive definite symmetric matrix* P *such that the following LMIs are satisfied*

$$\begin{aligned}[(I + UC)A_i]^TP + P(I + UC)A_i \ &+ \ (VCA_i)^T\bar{Y}^T + \bar{Y}(VCA_i) \\ &- \ C^T\bar{K}_i^T - \bar{K}_iC < 0, 1 \leq i \leq r \end{aligned} \tag{4.10}$$

where $U = -D(CD_{T1})^+$ *and* $V = I - CD_{T1}(CD_{T1})^+$, $K_i = P^{-1}\bar{K}_i$, $Y = P^{-1}\bar{Y}$, *and the observer gains are computed using (4.9) and (4.5).*

Proof. Clearly, using (4.9), the LMI based conditions given by (4.10) are equivalent to those conditions required in Theorem 4.1. This completes the proof. ∎

4.3.2 NUIO Design and Stability Conditions–Case B

For Case B, the observer design is also based on the concept of PDC, but becomes more involved. The ith observer rule is of the following form:

Observer Rule i:

IF $\hat{z}_1(t)$ is M_{i1}, $\cdots$, and $\hat{z}_N(t)$ is M_{iN}, THEN

$$\dot{w}(t) = A_i w(t) + G_i u(t) + L_i y(t), i = 1, 2, \cdots, r$$

Similar to Case A, the overall fuzzy observer is given as

$$\begin{aligned} \dot{w}(t) &= \sum_{i=1}^{r} h_i(\hat{z}(t))\{N_i w(t) + G_i u(t) + L_i y(t)\} \\ \hat{x}(t) &= w(t) - Ey(t) \end{aligned} \tag{4.11}$$

where $N_i, G_i, L_i, i = 1, 2, \cdots, r$ and E will be specified later in this subsection.

By defining $e(t) = \hat{x}(t) - x(t)$, it follows from (4.2) and (4.11) that

$$\begin{aligned} \dot{e}(t) &= \sum_{i=1}^{r} h_i(\hat{z}(t))N_i e(t) + \sum_{i=1}^{r} h_i(\hat{z}(t))\{N_i + K_i C - (I + EC)A_i\}x \\ &+ \sum_{i=1}^{r} h_i(\hat{z}(t))\{G_i - (I + EC)B_i\}u - \sum_{i=1}^{r} h_i(\hat{z}(t))(I + EC)D_i d(t) \\ &+ (I + EC)\Delta(t) + \sum_{i=1}^{r}(h_i(\hat{z}(t)) - h_i(z(t)))(I + EC)D_i d(t) \end{aligned} \tag{4.12}$$

where $K_i = L_i + N_i E$ and $\Delta(t) = \sum_{i=1}^{r}(h_i(\hat{z}(t)) - h_i(z(t)))\{A_i x(t) + B_i u(t)\}$.

Theorem 4.3 provides a sufficient condition for the observer given by (4.11) to be an NUIO.

Theorem 4.3 *For the observer given by (4.11) and under assumption* $A41$*, the error dynamics given by (4.12) is globally asymptotically stable at the origin if* $\|\Delta(t)\| \leq \gamma\|e(t)\|$*, and if, for any* $1 \leq i \leq r$*, there exist* N_i*,* L_i*,* G_i*,* E*, and a positive definite* *symmetric matrix P such that*

$$\begin{aligned} N_i &= (I + EC)A_i - K_iC, \\ G_i &= (I + EC)B_i, \\ L_i &= K_i - N_iE, \\ ECD_i &= -D_i, \end{aligned} \tag{4.13}$$

and

$$N_i^T P + PN_i + \gamma P(I + EC)(I + EC)^T P + \gamma I < 0, i = 1, 2, \cdots, r. \tag{4.14}$$

Proof. Using (4.13), (4.12) becomes

$$\dot{e}(t) = \sum_{i=1}^{r} h_i(\hat{z}(t))N_i e(t) + (I + EC)\Delta(t). \tag{4.15}$$

By choosing $V(t) = e^T(t)Pe(t)$ and differentiating it along (4.15), one obtains

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^{r} h_i(\hat{z}(t))e^T(t)(N_i^T P + PN_i)e(t) + 2e^T P(I + EC)\Delta(t) \\ &\leq \sum_{i=1}^{r} h_i(\hat{z}(t))e^T(t)(N_i^T P + PN_i)e(t) + 2\|e^T P(I + EC)\|\|\Delta(t)\| \\ &\leq \sum_{i=1}^{r} h_i(\hat{z}(t))e^T(t)(N_i^T P + PN_i)e(t) + \gamma(2\|e^T P(I + EC)\|\|e(t)\|) \\ &\leq \sum_{i=1}^{r} h_i(\hat{z}(t))e^T(t)(N_i^T P + PN_i \\ &+ \gamma P(I + EC)(I + EC)^T P + \gamma I)e(t). \end{aligned} \tag{4.16}$$

Clearly, using (4.14), (4.16) implies the conclusions of the theorem are true. This completes the proof. ∎

Remark 4.3.1 *The observer design for Case B is quite different from that for Case B, and moreover, a stronger condition (i.e., (4.14)) is needed to guarantee the globally* *asymptotic stability. Also, (4.14) is much harder to solve than (4.6) because less information is available to use.*

Remark 4.3.2 *Using similar arguments to those in Theorem 4.3, it can be proved that the results of the theorem still hold if* $\|P(I+EC)\Delta(t)\| \leq \gamma\|e(t)\|$ *and (4.14) is replaced with the following inequalities*

$$N_i^T P + PN_i + \gamma I < 0, i = 1, 2, \cdots, r. \tag{4.17}$$

Detailed design for this situation is not given because it is very similar to Case A. If (4.14) fails to have any feasible solution, (4.17) can be used as an alternative.

As in Case A, the following conditions in terms of LMIs are derived.

Theorem 4.4 *For the observer given by (4.11) and under assumption A41, the error dynamics given by (4.12) is globally asymptotically stable at the origin if* $\|\Delta(t)\| \leq \gamma\|e(t)\|$*, and if, for* $1 \leq i \leq r$*, there exist* $\bar{K}_i$*,* $\bar{Y}$*, and a positive definite symmetric matrix* P *such that the following LMIs are satisfied*

$$\begin{pmatrix} X_i & \sqrt{\gamma}[P(I+UC)+\bar{Y}(VC)] \\ \sqrt{\gamma}[P(I+UC)+\bar{Y}(VC)]^T & -I \end{pmatrix} < 0, 1 \leq i \leq r \tag{4.18}$$

with X_i *being defined as*

$$\begin{aligned} X_i &= ((I+UC)A_i)^T P + P(I+UC)A_i + (VCA_i)^T \bar{Y}^T + \bar{Y}(VCA_i) \\ &\quad -C^T \bar{K}_i^T - \bar{K}_i C + \gamma I, 1 \leq i \leq r, \end{aligned} \tag{4.19}$$

and if $Y = P^{-1}\bar{Y}$*,* $K_i = P^{-1}\bar{K}_i$*, and* $N_i, L_i, G_i, i = 1, 2, \cdots, r$ *and* E *are computed according to (4.13) and* $E = U + YV$*.*

Proof. Clearly, the LMIs given by (4.18) and (4.19) are equivalent to (4.14). This complete the proof. ∎

4.4 Particular TS Fuzzy System Structure for Fault Diagnosis

Similar to Chapter 2, system structures are developed for the purpose of fault diagnosis. All undefined notations in this section are the same as those defined in Chapter 2.

4.4.1 A Particular System Structure for Actuator FDI

For any $1 \leq j \leq r$ and any $s = \{i_1, \cdots, i_l\} \in 2^{S_I}$ with $1 \leq l \leq m$, denote $B_{j,s} = (b_{j,i_1} \ \cdots \ b_{j,i_l})$ and define $\bar{B}_{j,s}$ as the complementary matrix of $B_{j,s}$ consisting of the remaining columns of $B_j = (B_{j,1} \ \cdots \ B_{j,m})$.

Now, by rewriting (4.2), a particular system structure is obtained as follows:

$$\begin{aligned} \dot{x} &= \sum_{j=1}^{r} h_j(z(t))\{A_j x(t) + \bar{B}_{j;s}\bar{u}_s + B_{j;s}u_s + D_j d(t)\} \\ y &= Cx \end{aligned} \tag{4.20}$$

By defining $D_{j;s} = (B_{j;s} \ D_j)$ and $d_{aug}(t) = (u_s^T \ d^T(t))^T$, (4.20) can be rewritten as

$$\begin{aligned} \dot{x} &= \sum_{j=1}^{r} h_j(z(t))\{A_j x(t) + \bar{B}_{j;s}\bar{u}_s + D_{j;s}d_{aug}(t)\}, \\ y &= Cx. \end{aligned} \tag{4.21}$$

Remark 4.4.1 *The system structure given by (4.21) is obtained by regrouping the inputs and treating some inputs (i.e., the inputs included in u_s) as unknown inputs, which is very convenient for actuator fault diagnosis. One reason for this system structure is that (4.21) has the same form as (4.2) and thus the UIO design techniques developed in the last section can be used directly. The other reason is that s can be any set, and C_m^l UIOs can be designed for all sets of the form s.*

4.4.2 A Particular System Structure for Sensor FDI

To develop a particular system structure for sensor FDI, for any set s, y_s is filtered as

$$\dot{\xi}_s = A_{f,s}\xi_s + B_{f,s}y_s \tag{4.22}$$

where $A_{f,s}$ is chosen to be Hurwitz and $B_{f,s}$ as any invertible matrix.

By defining a new state as $z_{aug,s} = (x^T \ \xi_s^T)^T$ and using (4.2) and (4.22), a particular system structure is obtained:

$$\begin{aligned}\dot{z}_{aug,s} &= \sum_{j=1}^{r} h_j(z(t))\{\underline{A}_j z_{aug,s} + \underline{B}_j u + \underline{B}_s y_s + \underline{D}_j d(t)\} \\ \underline{y}_s &= \underline{\bar{C}}_s z_{aug,s}\end{aligned} \tag{4.23}$$

where

$$\underline{A}_j = \begin{pmatrix} A_j & 0 \\ 0 & A_{f,s} \end{pmatrix}, \underline{B}_j = \begin{pmatrix} B_j \\ 0 \end{pmatrix}, 1 \leq j \leq r, \tag{4.24}$$

$$\underline{B}_s = \begin{pmatrix} 0 \\ B_{f,s} \end{pmatrix}, \underline{D}_j = \begin{pmatrix} D_j \\ 0 \end{pmatrix}, \underline{\bar{C}}_s = \begin{pmatrix} \bar{C}_s & 0 \\ 0 & I \end{pmatrix}. \tag{4.25}$$

By defining $\underline{D}_{j,s} = (\underline{B}_s \ \underline{D}_j)$ and $d_{aug}(t) = (y_s^T \ d^T(t))^T$, (4.23) can be rewritten as

$$\begin{aligned}\dot{z}_{aug,s} &= \sum_{j=1}^{r} h_j(z(t))\{\underline{A}_j z_{aug,s} + \underline{B}_j u + \underline{D}_{j,s} d_{aug}(t)\}, \\ \underline{y}_s &= \underline{\bar{C}}_s z_{aug,s}.\end{aligned} \tag{4.26}$$

Remark 4.4.2 *The system structure given by (4.26) is developed for sensor fault diagnosis. Using the system structure to perform sensor fault diagnosis is convenient because the sensor faults can be treated as actuator faults. Also, (4.26) has the same form of (4.2) and similar to the actuator fault case, NUIOs can be designed and used to perform sensor fault diagnosis.*

4.5 Nonlinear Fault Diagnosis Based on NUIO Design for TS Fuzzy Systems

In this section, the nonlinear fault detection and isolation problems (i.e., $FDP1$, $FDP2$, $FIP1$, $FIP2$, and $FIP3$) are solved by designing NUIOs based on the particular system structures developed in the previous section. Because sensor faults can be treated as actuator faults, only actuator fault diagnosis problems are investigated.

4.5.1 Fault Detection Using One NUIO

Under the assumption that no fault occurs and that the NUIO given by (4.3) is designed such that $e(t)$ tends to zero, then, by defining $r(t) = \|\hat{y}(t) - y(t)\| = \|Ce(t)\|$, this NUIO makes $r(t)$ tend to zero as well. Based on this observation, $FDP1$ and $FDP2$ can be solved as follows:

$$\begin{cases} \textit{Faults are detected}; & if\ lim_{t\to\infty} r(t) \neq 0 \\ \textit{No fault is detected}; & otherwise \end{cases}$$

4.5.2 Actuator Fault Isolation Using a Bank of NUIOs

Actuator fault isolation based on the NUIO design technique developed earlier in this chapter is now addressed.

For any set of the form s, an NUIO that can be used for fault diagnosis for Case A has the following form:

$$\begin{aligned} \dot{w}_s(t) &= \sum_{j=1}^{r} h_j(z(t))\{N_{j,s}w(t) + G_{j,s}\bar{u}_s^H(t) + L_{j,s}y(t)\} \\ \hat{x}_s(t) &= w_s(t) - E_s y(t) \end{aligned} \tag{4.27}$$

where $\bar{u}_s^H(t)$ is the desired input vector, $\bar{u}_s^H(t) = \bar{u}_s(t)$ if no actuator related to $\bar{u}_s(t)$

is faulty, and the matrices, $N_{j,s}$,$G_{j,s}$, $L_{j,s}$, E_s, and $P_s = P_s^T > 0$, satisfy

$$\begin{aligned} E_s C D_{j,s} &= -D_{j,s}, \\ N_{j,s} &= (I + E_s C)A_j - K_{j,s}C, \\ N_{j,s}^T P_s &+ P_s N_{j,s} < 0, \\ G_{j,s} &= (I + E_s C)\bar{B}_{j,s}, \\ L_{j,s} &= K_{j,s} - N_{j,s}E_s, j = 1, 2, \cdots, r. \end{aligned} \tag{4.28}$$

The design of $N_{j,s}, G_{j,s}, L_{j,s}, E_s$ for $j = 1, 2, \cdots, r$ can be carried out by solving LMIs as shown in Section 4.3.1.

For Case B, an NUIO that can be used for fault diagnosis has the following form

$$\begin{aligned} \dot{w}_s(t) &= \sum_{j=1}^{r} h_j(\hat{z}(t))\{N_{j,s}w_s(t) + G_{j,s}\bar{u}_s^H(t) + L_{j,s}y(t)\} \\ \hat{x}_s(t) &= w_s(t) - E_s y(t) \end{aligned} \tag{4.29}$$

where the matrices, $N_{j,s}, G_{j,s}, L_{j,s}, L_{1j;s}, L_{2j;s}, j = 1, 2, \cdots, r,\ E_s$, and $P_s = P_s^T > 0$, satisfy

$$\begin{aligned} E_s C D_{j,s} &= -D_{j,s}, \\ N_{j,s} &= (I + E_s C)A_j - K_{j,s}C, \\ N_{j,s}^T P_s + P_s N_{j,s} &+ \gamma_s P_s (I + EC)(I + EC)^T P_s + \gamma_s I < 0, \\ G_{j,s} &= (I + EC)\bar{B}_{j,s}, \\ L_{j,s} &= K_{j,s} - N_{j,s}E_s, \end{aligned} \tag{4.30}$$

where $\Delta_s(t) = \sum_{j=1}^{r}(h_j(\hat{z}(t)) - h_j(z(t)))\{A_j x(t) + \bar{B}_{j,s}\bar{u}_s\}(t)$ and $\|\Delta_s(t)\| \leq \gamma_s\|e_s\|$.

Because there are m actuators, In total, C_m^l sets have the form s. Thus, for each l, exactly C_m^l NUIOs of the form (4.27) and (4.28) need to be designed for Case A. Similarly, C_m^l NUIOs of the form (4.29) and (4.30 have to be designed for Case B.

Another assumption is needed.

Assumption $A3$: An $n_0 \geq 0$ exists such that $CN_{j,s}^{n_0}G_{j,s} \neq 0$.

Remark 4.5.1 *If assumption A3 is not true and an NUIO given by (4.27) and (4.28) (or (4.29) and (4.30)) exists, the state estimation error can be shown to approach zero no matter whether actuators related to $\bar{u}_s(t)$ are faulty or not, which means this observer is insensitive to all actuator faults, and thus is not useful for fault isolation. This observation is the reason $A3$ is needed for fault isolation.*

In order to present answers for $FIP1$, $FIP2$, and $FIP3$, a concept, which is called the *NUIO Induced Fault Isolation Index (NUIOIFIX)* is defined below.

Definition 4.5.1 *System (4.2) or (4.1) is said to have an NUIO Induced Actuator Fault Isolation Index (NUIOIAFIX) equal to l if and only if, for all sets of the form s, systems given by (4.21) satisfy all the conditions in Theorem 4.1 for Case A (or Theorem 4.3 for Case B), and in addition, assumption A3 is satisfied. l is the largest number that has this property.*

For simplicity, AFIX will be used in the sequel to stand for NUIOIAFIX.

Remark 4.5.2 *If there is no $l > 0$ such that $AFIX = l$, system (4.2) or (4.1) does not have any AFIX. For consistency, $AFIX = 0$ is used to denote this situation. Because the AFIX is defined related to Case A or Case B, the AFIX for Case A might be different from that for Case B.*

Suppose $AFIX > 0$. As in Chapter 2, only the idea of a GOS can be used to design a bank of residuals $r_s(t), s \in 2^{S_I}$ such that $r_s(t)$ is sensitive to all faults, or fault groups, except those related to s. If the idea of the GOS is used, the following results are obtained.

Theorem 4.5 *If NUIOs of the form given by (4.27) and (4.28) for Case A (or (4.29) and (4.30) for Case B) are used to actuator fault isolation and under assumptions A41 and A42, the maximum number of actuator faults that can be simultaneously isolated is equal to AFIX.*

Proof. The proof is only provided for NUIOs designed for Case A because it is almost the same for Case B. $AFIX = 0$ and $AFIX > 0$ are considered separately.

For the case $AFIX = 0$, the fact that no single actuator fault can be isolated and only fault detection is possible needs to be proven. In order to isolate one single fault, m residuals based on m NUIOs of the form given by (4.27) and (4.28) for Case A must be designed such that each residual is insensitive to only one actuator fault, but sensitive to all other actuator faults. If this condition is met, by definition, $AFIX \geq 1$, which contradicts $AFIX = 0$. This contradiction proves that single fault isolation is impossible.

The following shows that the theorem is also true for the case when $AFIX > 0$.

Suppose $AFIX = l > 0$, by the definition of the $AFIX$, for any set $s = \{i_1, \cdots, i_l\}$, an NUIO given by (4.27) and (4.28) for Case A exists such that all the conditions in Theorem 4.1 are satisfied with $\bar{B}_{j,s}$, $\bar{u}_s$, $D_{j,s}$, and $d_{aug}(t)$ being treated as B_j, u, D_j, and $d(t)$ in (4.2), respectively. If actuators related to $\bar{u}_s$ are all healthy, then $e_s(t) = w_s(t) - E_s y(t) - x(t)$ will tend to zero for any u_s (whether it contains faulty actuator inputs or not). Suppose l actuator faults are present and u_{s^0} is the faulty actuator group with $s^0 = \{i_1^0, \cdots, i_l^0\}$, then $\bar{u}_{s^0}^H = \bar{u}_{s^0}$. If no sensor is faulty, according to Theorem 4.1, $lim_{t\to\infty} e_{s^0} = 0$. For any other actuator group u_s with $s \neq s^0$, $\bar{u}_s^H \neq \bar{u}_s$, which together with the definition of the AFIX cause e_s to generally not tend to zero under assumption A3. If the residuals are defined as $r_s = \|Ce_s\|$, $lim_{t\to\infty} r_{s^0} = 0$ and $lim_{t\to\infty} r_s \neq 0$ for $s \neq s^0$. The residuals satisfy the requirement of

GOS, and thus can be used to simultaneously isolate l faults.

To design a GOS to isolate $l+1$ faults, any combination of $l+1$ inputs must be able to be treated as unknown inputs. For all sets of the form of s, assume that an NUIO of the form given by (4.27) and (4.28) can be designed such that all the conditions in Theorem 4.1 are satisfied. Then, $AFIX = l+1$ by definition, which contradicts the fact that $AFIX = l$. This contradiction proves that isolating $l+1$ faults is impossible. This completes the proof. ∎

Actually, Theorem 4.5 has already provided solutions for $FIP1$ and $FIP2$ at the same time. This point is shown more clearly in the following corollaries.

Corollary 4.5.1 *For the isolation of actuator faults, assume that NUIOs of the form given by (4.27) and (4.28) are used for Case A, and that NUIOs of the form (4.29) and (4.30) are used for Case B. If $AFIX = 0$, actuator fault isolation is impossible; if $AFIX = 1$, isolating one single fault is possible; if $AFIX = l > 1$, actuator fault isolation is possible from one single fault up to l faults.*

Corollary 4.5.2 *For the isolation of actuator faults, assume that NUIOs of the form given by (4.27) and (4.28) are used for Case A, and that NUIOs of the form (4.29) and (4.30) are used for Case B. The maximum number of actuator faults that can be simultaneously isolated is $AFIX$.*

Theorem 4.5 shows that at most $AFIX$ actuator faults CAN BE ISOLATED. Suppose that the number of faults is n_f. Now, the problem is designing a fault diagnosis scheme that is able to achieve fault isolation, that is, how is $FIP3$ solved. To develop such a scheme, the following result is needed.

Theorem 4.6 *Under assumptions A41 and A42, and suppose that NUIOs of the form given by (4.27) and (4.28) for Case A (or (4.29) and (4.30) for Case B) are* *designed such that all the conditions in Theorem 4.1 for Case A (or Theorem 4.3 for Case B) are satisfied. If $n_f < AFIX$, the number of residuals (i.e., $r_s(t)$), which are insensitive to the n_f faults, is at least $C_{m-n_f}^{AFIX-n_f}$.*

Proof. Because only n_f faults exist, the number of sets of the form s, which include the faulty actuator group, is equal to $C_{m-n_f}^{AFIX-n_f}$. According to Theorem 4.1 for Case A (or Theorem 4.3 for Case B), all $C_{m-n_f}^{AFIX-n_f}$ NUIOs result in residuals $r_s = \|Ce_s\|$ tending to zero as $t \to \infty$. This completes the proof. ∎

Remark 4.5.3 *Generally, if $n_f < AFIX$ faults exist, exactly $C_{m-n_f}^{AFIX-n_f}$ residuals are insensitive to the faults. Therefore, once the number of those residuals (i.e., g_{num}) is obtained, the number of faults, n_f, can be determined by solving $C_{m-n_f}^{AFIX-n_f} = g_{num}$.*

The following result in Theorem 4.7 is useful in fault isolation.

Theorem 4.7 *Under the assumptions of Theorem 4.6, suppose that only g_{num} residuals, denoted by $r_{s_1}, \cdots, r_{s_{g_{num}}}$, are found to tend to zero. Define $S_j = \{i_1^j, \cdots, i_{AFIX}^j\}, 1 \leq j \leq g_{num}$ and let $S_F = \bigcap_{j=1}^{g_{num}} S_j$. If $C_{m-n_f}^{AFIX-n_f} = g_{num}$, S_F can be denoted as $S = \{i_1, \cdots, i_{n_f}\}$, and the n_f faulty actuators are the $i_1 \cdots i_{n_f}$th actuators.*

Proof. According to Theorem 4.6, at least $C_{m-n_f}^{l-n_f}$ residuals should approach zero if n_f faults occur. Because only g_{num} residuals approach zero and $C_{m-n_f}^{AFIX-n_f} = g_{num}$, the faulty actuator group must and can only be included in the g_{num} different sets defined by $S_j = \{i_1^j, \cdots, i_{AFIX}^j\}, 1 \leq j \leq g_{num}$. This fact equals to $\{i_1, \cdots, i_{n_f}\} \in S_j$ for any $1 \leq j \leq g_{num}$ and $S = \{i_1, \cdots, i_{n_f}\}$, which completes the proof. ∎

Now assuming that $AFIX > 0$ and $n_f < AFIX$, the actuator fault detection and isolation scheme is given in the steps below:

- Step 1 Fault detection can be performed as shown in Subsection 4.5.1.

- Step 2 Compute $AFIX$. If $AFIX = 0$, no fault can be isolated. If $AFIX = m$, then let $l = m - 1$ and go to Step 3. If $0 < AFIX < m$, then $l = AFIX$, go to Step 3.

- Step 3 Fault isolation

  1. For each s, design an NUIO of the form given by (4.27) and (4.28) for Case A (or (4.29) and (4.30 for Case B).

  2. Define residuals $r_s(t) = \|Ce_s\|/N_{normal}(t)$, where $N_{normal}(t)$ is chosen such that $r_s(t) \leq 1$ when only actuators corresponding to s are possibly faulty, otherwise, $r_s(t) \geq 1$. The threshold is chosen as 1.

  3. After faults are detected, denote the fault detection time as T_{detect}, choose a fault isolation time interval(FITI) as $(T_{detect}, T_{detect}+\Delta)$ where Δ is suitably large to perform fault isolation.

  4. Count the number of residuals that are below the threshold and have no tendency to grow. This number is denoted as g_{num}.

  5. If $g_{num} = 0$, more than l actuators are faulty and exact fault isolation is impossible except for the case $l = m - 1$. Stop.

  6. If $g_{num} = 1$, $n_f = l$ and l actuators have faults. If r_s is the only residual that is under the threshold, the $i_1 \cdots i_l$th actuators are faulty. Fault isolation is achieved.

  7. If $g_{num} > 1$, solve $C_{m-n_f}^{l-n_f} = g_{num}$ for n_f. If no integer solution exists, the number of faults occurred cannot be determined and fault isolation is

impossible at the moment. Stop. If an integer solution of n_f exists, the number of faults is equal to the integer solution of n_f.

8. If the number of faults n_f is determined and $C_{m-n_f}^{l-n_f} = g_{num}$ sets exist whose corresponding residuals are under the threshold, denote the sets $S_j = \{i_1^j, \cdots, i_l^j\}, 1 \le j \le g_{num}$, and let $S_F = \bigcap_{j=1}^{g_{num}} S_j$ and if $S_F = \{i_1, \cdots, i_{n_f}\}$, then, the faulty actuators are the $i_1 \cdots i_{n_f}$th actuators.

Remark 4.5.4 *Similar remarks can be made about this scheme as in Chapter 2.*

4.6 An Example and Simulation Results

In this section, a Lorenz's chaotic system with multi-inputs is chosen as an example to show the effects of our NUIOs and NUIO based fault diagnosis scheme. The system is described as

$$\begin{aligned} \dot{x}_1 &= -10x_1 + 10x_2 + 4u_1 + u_3 \\ \dot{x}_2 &= 28x_1 - x_2 - x_1x_3 + 3u_1 + u_3 \\ \dot{x}_3 &= x_1x_2 - \frac{8}{3}x_3 + u_1 + u_2 + u_3 \\ y &= Cx \end{aligned} \tag{4.31}$$

where $C = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $x_1 \in [-30, 30]$.

Let

$$B = \begin{pmatrix} 4 & 0 & 1 \\ 3 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}, B_1 = \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}, B_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, B_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Using the technique developed in [116], (4.31) can be represented by a TS fuzzy system described by the following two rules.

Plant Rule i:

IF $x_1(t)$ is M_1, THEN

$$\begin{cases} \dot{x} = A_1 x(t) + Bu(t), i = 1, 2 \\ y(t) = Cx(t) \end{cases}$$

where $M_1(x_1) = \frac{(1+\frac{x_1}{30})}{2}$ and $M_2(x_1) = \frac{(1-\frac{x_1}{30})}{2}$, and

$$A_1 = \begin{pmatrix} -10 & 10 & 0 \\ 28 & -1 & -30 \\ 0 & 30 & -8/3 \end{pmatrix}, A_2 = \begin{pmatrix} -10 & 10 & 0 \\ 28 & -1 & 30 \\ 0 & -30 & -8/3 \end{pmatrix}.$$

Given $(x(t), u(t))$, the final outputs of TS fuzzy systems are inferred as follows:

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{2} M_i(x_1(t))\{A_i x(t) + B_i u(t) + D_i d(t)\} \\ y &= Cx(t) \end{aligned} \tag{4.32}$$

where $h_i = M_i$ and $M_1(x_1) + M_2(x_1) = 1$.

4.6.1 The Design of NUIOs and Their Effects on State Estimation

For the purpose of fault isolation, NUIOs are designed to treat u_1, u_2, and u_3 as unknown inputs. Directly designing NUIOs is not easy because system (4.31) is nonlinear. However, if the system is represented as (4.32), NUIOs can be designed using the NUIO given by (4.29) and (4.30) for Case B, note that x_1 is not available.

When u_1 is treated as an unknown input, the NUIO is

$$\begin{aligned} \dot{w}(t) &= \sum_{j=1}^{2} M_j(\hat{x}_1(t))\{N_{j,1} w(t) + G_{j,1} u(t) + L_{j,1} y(t)\} \\ \hat{x}(t) &= w_1(t) - E_1 y(t) \end{aligned} \tag{4.33}$$

where $N_{j,1}, G_{j,1}, L_{j,1}, j = 1, 2, \cdots, r$ and E_1 are computed using Matlab's LMI toolbox as

$$E_1 = \begin{pmatrix} -0.4286 & -1 \\ -0.2857 & -1 \\ 0.1429 & -2 \end{pmatrix}, G_{1,1} = G_{2,1} = \begin{pmatrix} -1 & -0.8571 \\ -1 & -0.5714 \\ -1 & -0.7143 \end{pmatrix},$$

$$N_{1,1} = \begin{pmatrix} -626.8568 & -632.9997 & -503.5772 \\ -598.2019 & -654.6305 & -534.8363 \\ -592.9472 & -624.2329 & -567.9628 \end{pmatrix},$$

$$N_{2,1} = \begin{pmatrix} -626.8522 & -572.9951 & -529.2911 \\ -598.2066 & -594.6352 & -491.9795 \\ -592.9472 & -564.2330 & -559.3913 \end{pmatrix},$$

$$L_{1,1} = \begin{pmatrix} 231.6 & 1747.9 \\ 254.1 & 1806.4 \\ 244.2 & 1786.8 \end{pmatrix}, L_{2,1} = \begin{pmatrix} 252.4 & -1739.3 \\ 265.1 & -1660.7 \\ 260.1 & -1709.6 \end{pmatrix}.$$

When u_2 is treated as an unknown input, the NUIO is

$$\begin{aligned} \dot{w}(t) &= \sum_{j=1}^{2} M_j(\hat{x}_1(t))\{N_{j,2}w(t) + G_{j,2}u(t) + L_{j,2}y(t)\} \\ \hat{x}(t) &= w(t) - E_2 y(t) \end{aligned} \tag{4.34}$$

where $N_{j,2}, G_{j,2}, L_{j,2}, j = 1, 2, \cdots, r$ and E_2 are computed using Matlab's LMI toolbox as

$$E_2 = \begin{pmatrix} -0.7143 & 0 \\ -0.5714 & 0 \\ -0.1429 & -1.0000 \end{pmatrix}, G_{1,2} = G_{2,1} = \begin{pmatrix} -1.0000 & -0.4286 \\ -1.0000 & -0.1429 \\ -1.0000 & -0.2857 \end{pmatrix},$$

$$N_{1,2} = \begin{pmatrix} -305.4703 & -279.0418 & -118.3822 \\ -351.6746 & -375.5317 & -216.2318 \\ -213.7523 & -212.4666 & -434.7161 \end{pmatrix},$$

$$N_{2,2} = \begin{pmatrix} -305.4704 & -279.0419 & -161.2399 \\ -351.6744 & -375.5316 & -190.5169 \\ -213.7523 & -212.4666 & -443.2875 \end{pmatrix},$$

$$L_{1,2} = \begin{pmatrix} -111.9441 & 21.4286 \\ -127.2871 & -12.8571 \\ -125.0111 & 4.2857 \end{pmatrix}, L_{2,2} = \begin{pmatrix} -118.0666 & -21.4286 \\ -123.6135 & 12.8571 \\ -126.2356 & -4.2857 \end{pmatrix}.$$

When u_3 is treated as an unknown input, the NUIO is

$$\begin{aligned} \dot{w}(t) &= \sum_{j=1}^{2} M_j(\hat{x}_1(t))\{N_{j,3}w(t) + G_{j,3}u(t) + L_{j,3}y(t)\} \\ \hat{x}(t) &= w(t) - E_3 y(t) \end{aligned} \tag{4.35}$$

where $N_{j,3}, G_{j,3}, L_{j,3}, j = 1, 2, \cdots, r$ and E_3 are computed using Matlab's LMI toolbox as

$$E_3 = \begin{pmatrix} -0.8000 & 0.6000 \\ -0.6000 & 0.2000 \\ -0.2000 & -0.6000 \end{pmatrix}, G_{1,3} = G_{2,3} = \begin{pmatrix} -1.0000 & 0.6000 \\ -1.0000 & 0.2000 \\ -1.0000 & 0.4000 \end{pmatrix},$$

$$N_{1,3} = \begin{pmatrix} -383.1578 & -337.9578 & -276.3925 \\ -172.6436 & -190.2436 & -149.9157 \\ -257.1262 & -243.3262 & -264.9811 \end{pmatrix},$$

$$N_{2,3} = \begin{pmatrix} -383.1735 & -373.9735 & -324.3951 \\ -172.6278 & -202.2278 & -125.9132 \\ -257.1262 & -267.3262 & -276.9811 \end{pmatrix},$$

$$L_{1,3} = \begin{pmatrix} -205.8216 & 430.4432 \\ -92.4006 & 189.0679 \\ -151.1667 & 313.8667 \end{pmatrix}, L_{2,3} = \begin{pmatrix} -237.0284 & 408.8568 \\ -94.7938 & 205.8542 \\ -167.9667 & 311.4667 \end{pmatrix}.$$

The simulation results for the above three NUIOs are presented in Fig. 4.1 to Fig. 4.3, respectively.

As expected, when the system is free of faults, the simulation results in Fig. 4.1 to Fig. 4.3 show that all the three NUIOs can estimate the system states asymptotically.

4.6.2 Fault Diagnosis Based on the Design of the NUIO

Three NUIOs of the form given by (4.29) and (4.30) can be designed for $s = \{1, 2\}$, $s = \{1, 3\}$, and $s = \{2, 3\}$, respectively, and satisfy all the conditions of Theorem

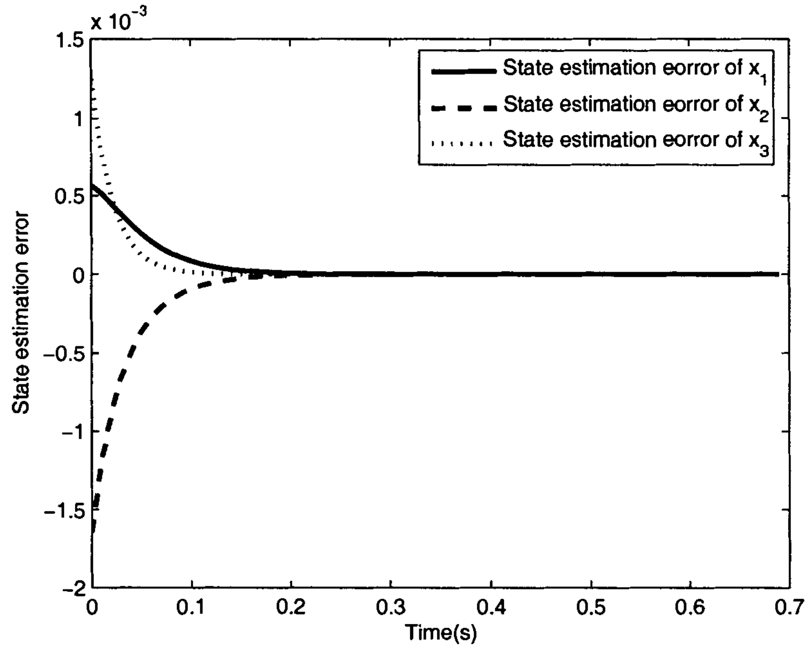

Figure 4.1: The state estimation errors for the first NUIO

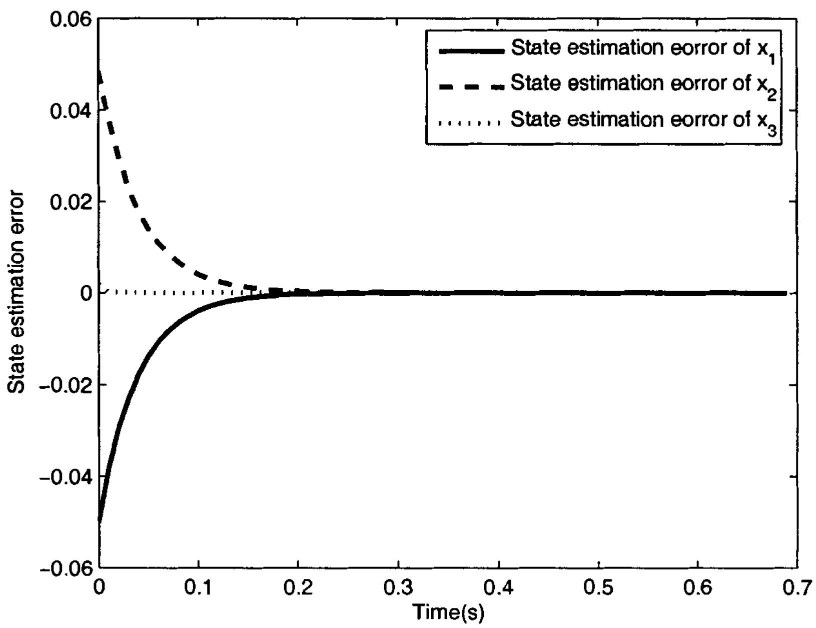

Figure 4.2: The state estimation errors for the second NUIO

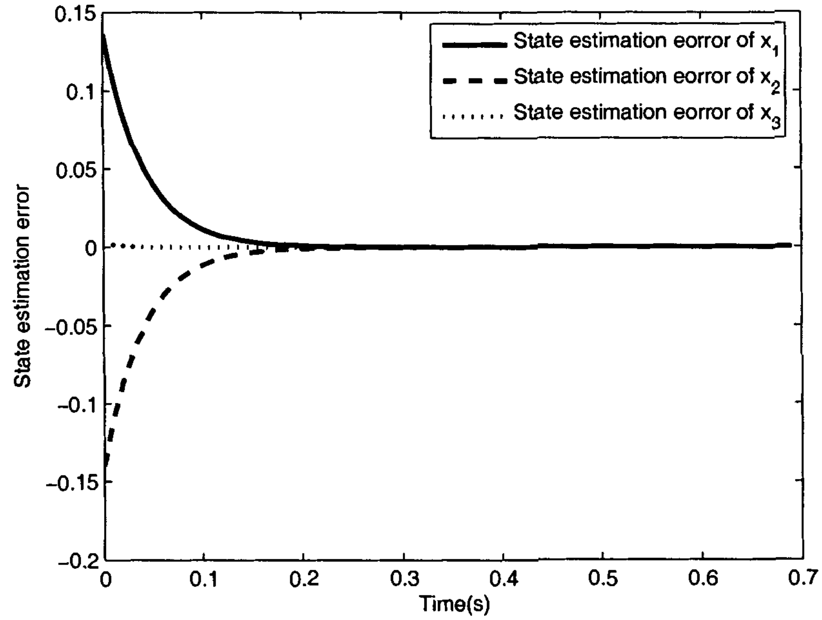

Figure 4.3: The state estimation errors for the third NUIO

4.3. However, $AFIX \neq 2$ because Assumption $A3$ is not satisfied by any of the three NUIOs. This claim is shown using the fact that $E_s(I + E_sC) = 0$ for any s. In fact, $AFIX = 1$. According to Theorem 4.5, only one actuator fault can be isolated. Because fault detection is easier, only simulation results for fault isolation are presented in Fig. 4.4. The first actuator becomes faulty at $3s$: i.e., $u_1(t) = 0$ after the time of $3s$.

In the simulation, $N_{normal}(t) = 0.005$. At $3.52s$, r_2 exceeds the threshold, faults are detected. For $FITI = (3.52s, 3.7s)$, Fig. 4.4 shows that residuals r_2 and r_3 exceed the threshold on the FITI, while residual r_2 stays close to zero. Using the proposed fault isolation scheme in the previous section, a decision can be made that one actuator is faulty and that the faulty actuator is the first one related to u_1. Thus, correct decision is made.

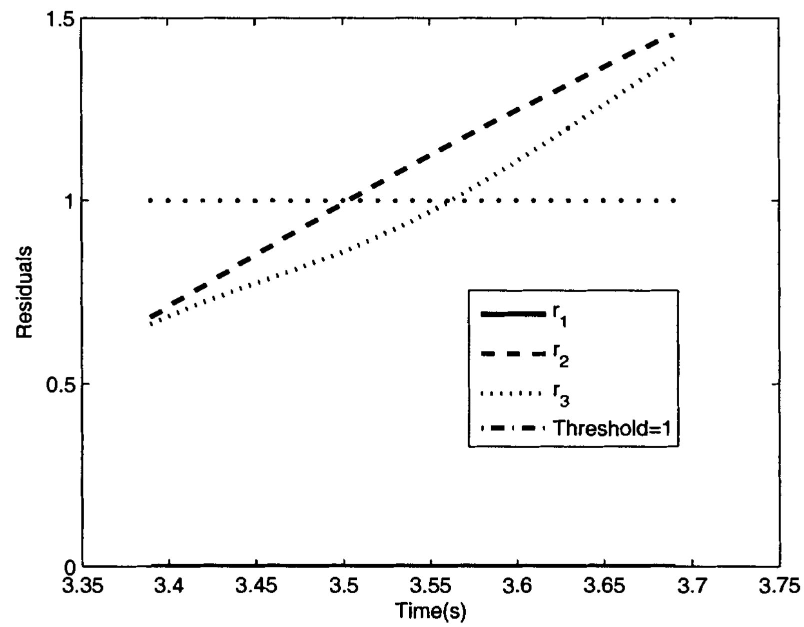

Figure 4.4: Fault isolation of a single actuator fault

4.7 Conclusions and Discussions

For nonlinear systems that can be represented by TS fuzzy systems, two types of nonlinear unknown input observers (NUIOs) were designed for two cases. One case is where the premise variables do not depend on the unmeasured state variables; the other case is that the premise variables depend on the unmeasured state variables. For both types of NUIOs, sufficient conditions for the existence of NUIOs were established, and the LMI based ones were proposed to ease observer design.

After the NUIOs were designed, fault detection and isolation problems for nonlinear systems described by TS fuzzy systems were then studied using NUIO design technique, and solutions for the problems were provided. A fault detection scheme was proposed using the NUIOs designed, which solved $FDP1$ and $FDP2$. To solve fault isolation problems, a bank of NUIOs were designed. Based on this bank of

NUIOs, results were obtained on the possibility of isolating single and/or multiple faults, which provides solutions for $FIP1$ and $FIP2$. $FIP3$ was solved by providing a fault isolation scheme.

A Lorenz's chaotic system with multi-inputs was chosen as an example to show the effect of NUIOs designed and the proposed fault diagnosis scheme. Simulation results show that accurate state estimation is achieved when NUIOs are used, and actuator faults can be isolated successfully.

Because uncertain nonlinear systems that can be represented by TS fuzzy systems are only a limited class of nonlinear systems, much work is needed for more general uncertain nonlinear systems. Finding sufficient conditions that are less conservative for NUIO design based on TS fuzzy systems is another future research topic.

Chapter 5

Output Estimator Based Fault Diagnosis for Uncertain Linear Systems

In previous chapters, fault diagnosis schemes are proposed based on the robust observer design for systems with matched non-parametric uncertainties. In order to deal with unmatched uncertainties, this chapter abandons the idea of observer design, and proposes to use output estimator design to carry out fault diagnosis for a class of linear systems with both matched and unmatched uncertainties.

5.1 Introduction

Chapter 2 and 3 showed that either UIOs or SMOs can be designed to completely remove the effect of the unknown inputs under the condition that the unknown inputs are matched. Chapter 4 also requires the uncertainties to satisfy certain matching conditions.

If the uncertainties do not satisfy the matching conditions, neither UIOs or SMOs used in previous chapters can be designed such that the effect of the unknown inputs can be removed completely. In order to design fault diagnosis schemes that are not affected by unknown inputs, a new design methodology is needed.

In this chapter, in order to solve the challenging fault diagnosis problem for systems with unmatched uncertainties, a novel idea is proposed. It abandons the idea of designing a whole state observer that is invariant to unknown inputs because it is sometimes very restrictive. Instead, the idea advocates the design of an output estimator invariant to unknown inputs because it is sufficient for the purpose of fault diagnosis.

No research has been found on the design of fault diagnosis schemes which are invariant to the unmatched unknown inputs. Furthermore, no systematic study has been carried out to solve all fault diagnosis problems raised in Section 1.2. The main purpose of this chapter is to present a detailed study of those problems for a class of linear systems with unmatched unknown inputs using an output estimator design based on sliding mode approaches.

The rest of this chapter is arranged as follows. In Section 5.2, the system of interest is described, and a canonical system structure is developed through state and input transformations, which is suitable for sliding mode output estimator(SMOE) design. In Section 5.3, SMOEs are designed based on the canonical system derived in the last section, and their properties are investigated. Section 5.4 discusses all fault diagnosis problems raised in Section 1.2 for both actuator and sensor faults using the output estimator design technique. In Section 5.5, an example is given to show the effect of the output estimator based fault diagnosis scheme in terms of actuator fault detection, isolation and estimation. Conclusions and discussions are presented in the last section.

5.2 System Formulation and A Canonical System Structure

5.2.1 System Formulation

The uncertain linear systems considered are of the following form:

$$\begin{aligned} \dot{z} &= Az + Bu + D_1 d_1(t) + D_2 d_2(t) \\ y &= Cz \end{aligned} \tag{5.1}$$

where $z \in R^n$ is the state vector, $y \in R^p$ is the output vector, $u \in R^m$ is the input vector, and $d_1 \in R^{q_1}$ and $d_2 \in R^{q_2}$ are bounded unknown input vectors which may consist of system uncertainties and/or disturbances.

The following assumptions are made.

Assumption $A51$: A, B, C, D_1, D_2 are known.

Assumption $A52$: B and $D = (D_1\ D_2)$ are both full column rank.

Assumption $A53$: rank $(CD_1) =$ rank D_1.

Assumption $A54$: For any complex number s with $Re(s) \geq 0$, the rank $\begin{pmatrix} sI_n - A & D_1 \\ C & 0 \end{pmatrix} = n + rankD_1$.

Assumption $A55$: rank $C(D_1\ D_2) =$ rank (CD_1).

Remark 5.2.1 *As proved in [114], for any unknown input vector $d_1(t)$ to be a matched unknown input vector, that matched conditions $A53$ and $A54$ must be satisfied. If $d(t) = (d_1^T\ d_2^T)^T$ is defined, $d(t)$ is not a matched unknown input vector because of $A55$. This fact means no UIOs or SMOs used in Chapter 2 and Chapter 3 can be designed such that the state estimation error is invariant to $d(t)$ for systems of the form (5.1)*

5.2.2 A Canonical System Structure

Theorem 5.1 presents a canonical system structure for system (5.1) and the conditions under which system (5.1) can be transformed into the canonical form.

Theorem 5.1 *Under assumptions A51 and A52, the assumptions A53,A54, and A55 are necessary and sufficient conditions for system (5.1) to be transformed through a state transform T and an unknown input transform $T_d = \begin{pmatrix} I_{q_1} & T_{d,2} \\ 0 & I_{q_2} \end{pmatrix}$ into the canonical form*

$$\begin{aligned} \dot{x} &= \bar{A}x + \bar{B}u + \bar{D}_1\bar{d}_1(t) + \bar{D}_2\bar{d}_2(t) \\ y &= \bar{C}x \end{aligned} \tag{5.2}$$

where $x = (x_1^T\ x_2^T\ x_3^T)^T = Tz$ with $x_1 \in R^{n-p-l}$, $x_2 \in R^l$, and $x_3 \in R^p$; $\bar{d}(t) = T_d d(t)$ with $d(t) = (d_1^T\ d_2^T)^T$; and the system matrices are defined according to x as

$$\bar{A} = TAT^{-1} = \begin{pmatrix} \bar{A}_{11} & \bar{A}_{12} & \bar{A}_{13} \\ 0 & \bar{A}_{22} & \bar{A}_{23} \\ \bar{A}_{31} & \bar{A}_{32} & \bar{A}_{33} \end{pmatrix}, \bar{B} = TB = \begin{pmatrix} \bar{B}_1 \\ 0 \\ \bar{B}_3 \end{pmatrix},$$

$$\bar{D}_1 = \begin{pmatrix} 0 \\ 0 \\ \bar{D}_{1,3} \end{pmatrix}, \bar{D}_2 = \begin{pmatrix} \bar{D}_{2,1} \\ 0 \\ 0 \end{pmatrix}, \bar{C} = CT^{-1} = (0 \quad 0 \quad I), \tag{5.3}$$

with $\bar{A}_{11} \in R^{(n-p-l)\times(n-p-l)}$ and $\bar{A}_{22} \in R^{l\times l}$ being Hurwitz.

Proof. Necessity: Let $\bar{D} = (\bar{D}_1\ \bar{D}_2)$. Clearly, $\bar{D} = TDT_d^{-1}$, which implies $TD_1 = (0\ 0\ \bar{D}_{1,3}^T)^T$. Because $A52$ implies that $rank(TD_1) = rank(D_1) = q_1$, $rank(\bar{D}_{1,3}) = q_1$. Subsequently,

$$rank(CD) = rank(\bar{C}TD) = rank(\bar{C}TDT_d^{-1}) = rank(\bar{C}\bar{D}) = rank(\bar{D}_{1,3}) = q_1.$$

On the other hand,

$$rank(CD_1) = rank(\bar{C}TD_1) = rank(\bar{D}_{1,3}) = q_1.$$

In conclusion, $rank(CD) = rank(CD_1) = q_1$, which implies that $A53$ and $A55$ are satisfied.

Now, $A54$ needs to be proved. Clearly,

$$\begin{pmatrix} T & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} SI - A & D_1 \\ C & 0 \end{pmatrix} \begin{pmatrix} T^{-1} & 0 \\ 0 & I \end{pmatrix} = \begin{pmatrix} sI - \bar{A} & TD_1 \\ \bar{C} & 0 \end{pmatrix}. \tag{5.4}$$

Note that $TD_1 = (0\ 0\ \bar{D}_{1,3}^T)^T$, and using the definition of $\bar{A}$ and $\bar{C}$ and the facts that $\bar{A}_{11}$ and $\bar{A}_{22}$ are Hurwitz, for any complex number s with $Re(s) \geq 0$, it follows that

$$rank \begin{pmatrix} sI - \bar{A} & TD_1 \\ \bar{C} & 0 \end{pmatrix} = n + rank(\bar{D}_{1,3}) = n + rank(D_1).$$

This fact together with (5.4) proves $A54$ is satisfied.

Sufficiency: If $A53$ and $A54$ are satisfied, according to [145], a state transform $x1 = T1z$ exists such that

$$\begin{aligned} \dot{x1} &= \begin{pmatrix} A_{11}^1 & A_{12}^1 \\ A_{21}^1 & A_{22}^1 \end{pmatrix} x1 + \begin{pmatrix} B1_1 \\ B1_2 \end{pmatrix} u + \begin{pmatrix} 0 \\ D1_2^1 \end{pmatrix} d_1(t) + T1D_2 d_2(t) \\ y &= (0 \quad I_p)\, x1 \end{aligned} \tag{5.5}$$

where A_{11}^1 is Hurwitz and $T1D2 = (\,(D2_1^1)^T \quad (D2_2^1)^T\,)^T$.

Note that $T1D1 = (0 \quad (D1_2^1)^T\,)^T$ and by the definition of $T1D2$, the following equations hold: $CD_1 = CT1^{-1}T1D1 = (0\ I)T1D1 = D1_2^1$ and $CD = CT1^{-1}(T1D1\ T1D2) = (D1_2^1\ D2_2^1)$. $A53$ implies that a matrix E exists such that

$$D2_2^1 = D1_2^1 E. \tag{5.6}$$

Now, by choosing $T_d^{-1} = \begin{pmatrix} I_{q_1} & -E \\ 0 & I_{q_2} \end{pmatrix}$ and letting $\bar{d}(t) = T_d \begin{pmatrix} d_1(t) \\ d_2(t) \end{pmatrix}$, the following is obtained:

$$\begin{aligned} \dot{x1} &= \begin{pmatrix} A_{11}^1 & A_{12}^1 \\ A_{21}^1 & A_{22}^1 \end{pmatrix} x1 + \begin{pmatrix} B1_1 \\ B1_2 \end{pmatrix} u + \begin{pmatrix} 0 \\ D1_2^1 \end{pmatrix} \bar{d}_1(t) + \begin{pmatrix} D2_1^1 \\ 0 \end{pmatrix} \bar{d}_2(t), \\ y &= (0 \quad I_p)\, x1. \end{aligned} \tag{5.7}$$

Let $B_D = (B1_1 \; D2_1^1)$. If the pair (A_{11}^1, B_D) is not controllable, a nonsingular matrix Ts exists such that

$$T_s A_{11}^1 T_s^{-1} = \begin{pmatrix} \bar{A}_{11} & \bar{A}_{12} \\ 0 & \bar{A}_{22} \end{pmatrix}, T_s B_D = \begin{pmatrix} \bar{B}_1 & \bar{D}_{2,1} \\ 0 & 0 \end{pmatrix}.$$

Note that A_{11}^1 is Hurwitz, so are $\bar{A}_{11}$ and $\bar{A}_{22}$. Now, let $T = \begin{pmatrix} T_s & 0 \\ 0 & I_p \end{pmatrix} T1$ and $x = Tz$. Define $TsA_{12}^1 = (\bar{A}_{13}^T \; \bar{A}_{13}^T)^T$, $A_{21}^1 = (\bar{A}_{31} \; \bar{A}_{32})$, $\bar{A}_{33} = A_{22}^1$, and $\bar{B}_3 = B1_2$, the canonical form (5.2) is reached using (5.7). This completes the proof. ∎

Remark 5.2.2 *If the pair (A_{11}^1, B_D) is controllable, the block related to x_2 is not in the canonical system structure.*

Remark 5.2.3 *The advantage of this canonical system structure is that it separates the matched and unmatched unknown inputs (i.e., $\bar{d}_1(t)$ and $\bar{d}_2(t)$) so that they can be dealt with separately, which eases the task of designing an asymptotic output estimator.*

5.3 A Sliding Mode Output Estimator

In the fault diagnosis literature, output estimators were often designed based on observers with certain desirable properties, and focus was often on the design of the observers. In this section, output estimators are designed directly using the sliding mode technique without the design of observers. Two types of output estimators are designed. One is suitable for actuator fault diagnosis, and the other is good for sensor fault diagnosis.

5.3.1 Sliding Mode Output Estimator for Actuator Fault Diagnosis

The following assumption is needed:

Assumption $A56$: CB is of full column rank; i.e., $\bar{B}_3$ is of full column rank.

Assuming only actuator faults can occur, the task of the output estimator based actuator fault diagnosis is to ensure that the output estimation error is invariant to all unknown inputs and certain group of inputs.

Denote $\bar{B}_3 = (\bar{B}_{3,1} \ \cdots \ \bar{B}_{3,m})$, and for any $s = \{i_1, \cdots, i_l\} \in 2^{S_I}$, where $i_j \in \{1, 2, \cdots, m\}$ for any $1 \leq j \leq l$, let $\bar{B}_{3,s} = (\bar{B}_{3,i_1}, \cdots, \bar{B}_{3,i_l})$. If one takes away all columns of $\bar{B}_{3,s}$ from $\bar{B}_3$, the remaining columns of $\bar{B}_3$ constitute a new matrix denoted by $\bar{B}^c_{3,s}$.

It follows from (5.2) and (5.3) that

$$\begin{aligned} \dot{x}_2 &= \bar{A}_{22}x_2 + \bar{A}_{23}y, \\ \dot{y} &= \bar{A}_{31}x_1 + \bar{A}_{32}x_2 + \bar{A}_{33}y + \bar{B}_{3,s}u_s + \bar{B}^c_{3,s}\bar{u}_s + \bar{D}_{1,3}\bar{d}_1(t). \end{aligned} \tag{5.8}$$

Note that $\bar{A}_{11}$ and $\bar{A}_{22}$ are Hurwitz and $d(t)$ is bounded. The following SMOE can be designed.

$$\begin{aligned} \dot{\hat{x}}_2 &= \bar{A}_{22}\hat{x}_2 + \bar{A}_{23}y \\ \dot{y}_s &= A_3(y_s - y) + \bar{A}_{31}\mu 1_s + \bar{A}_{32}\hat{x}_2 + \bar{A}_{33}y + \bar{B}_{3,s}\mu 2_s + \bar{B}^c_{3,s}\bar{u}^H_s + \bar{D}_{1,3}\mu 3_s \end{aligned} \tag{5.9}$$

where A_3 is any Hurwitz matrix, and y_s and $\hat{x}_2$ are the estimates of y and x_2, respectively. Let P be a positive definite symmetric matrix such that $A_3^T P + PA_3 < 0$, the sliding mode terms are defined as

$$\mu 1_s = \begin{cases} -\rho_1 \frac{\bar{A}^T_{31}Pe_s}{\|\bar{A}^T_{31}Pe_s\|}, & \|\bar{A}^T_{31}Pe_s\| \neq 0 \\ 0, & \|\bar{A}^T_{31}Pe_s\| = 0, \end{cases}$$

and

$$\mu 2_s = \begin{cases} -\rho_2 \frac{\bar{B}^T_{3,s}Pe_s}{\|\bar{B}^T_{3,s}Pe_s\|}, & \|\bar{B}^T_{3,s}Pe_s\| \neq 0, \\ 0, & \|\bar{B}^T_{3,s}Pe_s\| = 0, \end{cases}$$

and

$$\mu 3_s = \begin{cases} -\rho_3 \frac{\bar{D}_{1,3}^T P e_s}{\|\bar{D}_{1,3}^T P e_s\|}, & \|\bar{D}_{1,3}^T P e_s\| \neq 0, \\ 0, & \|\bar{D}_{1,3}^T P e_s\| = 0, \end{cases}$$

where $e_s = y_s - y$, and ρ_1, ρ_2 and ρ_3 are chosen such that $\rho_1 > \|x_1\|$, $\rho_2 > \|u_s\|$, and $\rho_3 > \|\bar{d}_1(t)\|$.

The property of the designed output estimator is given in Theorem 5.2.

Theorem 5.2 *Under assumptions A51 to A56, if $\bar{u}_s^H = \bar{u}_s$, u_s and y are bounded, and ρ_1, ρ_2 and ρ_3 are chosen such that $\rho_1 > \|x_1\|$, $\rho_2 > \|u_s\|$, and $\rho_3 > \|\bar{d}_1(t)\|$, the SMOE given by (5.9) can ensure that e_s exponentially approaches zero, and thus is invariant with respect to u_s and $d(t)$.*

Proof. The following is derived:

$$\begin{aligned} \dot{\tilde{x}}_2 &= \bar{A}_{22}\tilde{x}_2, \\ \dot{e}_s &= A_3 e_s + \bar{A}_{31}(\mu 1_s - x_1) + \bar{A}_{32}\tilde{x}_2 \\ &+ \bar{B}_{3,s}(\mu 2_s - u_s) + \bar{D}_{1,3}(\mu 3_s - \bar{d}_1(t)). \end{aligned} \tag{5.10}$$

Because $\bar{A}_{22}$ is Hurwitz, positive definite matrices $P2$ and $Q2$ exist such that $-Q2 = P2\bar{A}_{22} + \bar{A}_{22}^T P2$. Let $-Q = PA_3 + A_3^T P$ and choose a Lyapunov function $V = e_s^T P e_s + \tilde{x}_2^T P2 \tilde{x}_2$. Differentiating the Lyapunov function, the following is obtained:

$$\begin{aligned} \dot{V} &= -e_s^T Q e_s + 2e_s^T P\bar{A}_{31}(\mu 1_s - x_1) + 2e_s^T P\bar{A}_{32}\tilde{x}_2 \\ &+ 2e_s^T P\bar{B}_{3,s}(\mu 2_s - u_s) + 2e_s^T P\bar{D}_{1,3}(\mu 3_s - \bar{d}_1(t)) - \tilde{x}_2^T Q2 \tilde{x}_2. \end{aligned} \tag{5.11}$$

Using the definition of $\mu 1_s$, because $\bar{A}_{11}$ is Hurwitz and u_s, y and $\bar{d}_1(t)$ are bounded, x_1 is bounded. Hence, ρ_1 can be chosen such that $\rho_1 > \|x_1\|$. Clearly, $2e_s^T P\bar{A}_{31}(\mu 1_s - x_1) \leq 0$, $2e_s^T P\bar{B}_{3,s}(\mu 2_s - u_s) \leq 0$, and $2e_s^T P\bar{D}_{1,3}(\mu 3_s - \bar{d}_1(t)) \leq 0$. Using these three inequalities, it follows from (5.11) that

$$\dot{V} \leq -e_s^T Q e_s + 2e_s^T P\bar{A}_{32}\tilde{x}_2 - \tilde{x}_2^T Q2 \tilde{x}_2. \tag{5.12}$$

Note that for any positive k, the following is always true.

$$2e_s^T P\bar{A}_{32}\tilde{x}_2 \leq e_s^T Pe_s/k + k\|\bar{A}_{32}\tilde{x}_2\|^2.$$

If k is chosen such that $\bar{Q} = Q - P/k > 0$, the following is obtained:

$$\dot{V} \leq -e_s^T\bar{Q}e_s + k\|\bar{A}_{32}\tilde{x}_2\|^2 - \tilde{x}_2^T Q 2\tilde{x}_2. \tag{5.13}$$

Because $\|\tilde{x}_2\|^2$ exponentially converges to zero, (5.13) implies that e_s exponentially approaches zero despite the presence of unknown inputs. ∎

Remark 5.3.1 *Unlike in [30], x_1 cannot be estimated because of the presence of the unmatched disturbance $\bar{d}_2(t)$. Therefore, in the output estimator design, x_1 is treated as an unknown input vector.*

5.3.2 Sliding Mode Output Estimator for Sensor Fault Diagnosis

Assuming only sensor faults can occur, the task of the output estimator based sensor fault diagnosis is to ensure that the output estimation error is invariant to all unknown inputs and certain group of outputs.

Similarly, to develop an SMOE for sensor fault diagnosis, denote $\bar{C} = (\bar{C}_1^T \cdots \bar{C}_p^T)^T$ and $\bar{C}_s = (\bar{C}_{i_1}^T, \cdots, \bar{C}_{i_l}^T)^T$. If all rows of $\bar{C}_s$ are taken away from $\bar{C}$, the remaining rows of $\bar{C}$ constitute a new matrix denoted by $\bar{C}_s^c$.

Also denote $x_{3,s}$ as a vector consisting of the $i_1 \cdots i_l$th components of x_3 and $\bar{x}_{3,s}^c$ as a vector consisting of the remaining components.

It follows from (5.2) and (5.3) that

$$\begin{aligned}\dot{x}_{3,s}^c &= \bar{A}_{31,s}^c x_1 + \bar{A}_{32,s}^c x_2 + \bar{A}_{33,s}^{c,1} x_{3,s}^c + \bar{A}_{33,s}^{c,2} x_{3,s} \\ &\quad + \bar{B}_{3,s}^c u + \bar{D}_{1,3,s}^c \bar{d}_1(t).\end{aligned} \tag{5.14}$$

$\bar{A}_{33,s}$ and $\bar{A}^c_{33,s}$ are defined the same way as $\bar{C}_s$ and $\bar{C}^c_s$, whereby

$$\bar{A}^c_{33,s} = (\bar{A}^{c,1}_{33,s}\ \bar{A}^{c,2}_{33,s}).$$

All other notations are defined the same way as $\bar{C}_s$ and $\bar{C}^c_s$.

Remark 5.3.2 *Unlike the output estimator design for actuator fault diagnosis, for sensor fault diagnosis, x_2 cannot be estimated asymptotically once sensor faults occur. Hence, some design freedom is lost because its estimation can no longer be used. Therefore, only the differential equation in (5.14) can be used.*

Define y_s and y^c_s the same way as $x_{3,s}$ and $\bar{x}^c_{3,s}$. Denote the actual system output vector as y^H. $y^H = y = x_3$ if all the sensors are healthy. Here, an estimator is designed for y^c_s based on (5.14) by treating x_1, x_2, $x_{3,s}$ and $\bar{d}_1(t)$ as unknown inputs.

As in the last subsection, an estimator for y^c_s is given as

$$\begin{aligned}\dot{\hat{x}}^c_{3,s} &= A_3(\hat{x}^c_{3,s} - y^c_s) + \bar{A}^c_{31,s}\mu 1_s + \bar{A}^c_{32,s}\mu 2_s + \bar{A}^{c,1}_{33,s} y^c_s + \bar{A}^{c,2}_{33,s}\mu 3_s \\ &\quad + \bar{B}^c_{3,s} u + \bar{D}^c_{1,3,s}\mu 4_s \end{aligned} \tag{5.15}$$

where A_3 is any Hurwitz matrix, and $\hat{x}^c_{3,s}$ is the estimate of y^c_s. Let P be a positive definite symmetric matrix such that $A_3^T P + PA_3 < 0$; then the sliding mode terms are defined as follows

$$\mu 1_s = \begin{cases} -\rho_1 \frac{(\bar{A}^c_{31,s})^T P e^c_s}{\|(\bar{A}^c_{31,s})^T P e^c_s\|}, & \|(\bar{A}^c_{31,s})^T P e^c_s\| \neq 0, \\ 0, & \|(\bar{A}^c_{31,s})^T P e^c_s\| = 0, \end{cases}$$

$$\mu 2_s = \begin{cases} -\rho_2 \frac{(\bar{A}^c_{32,s})^T P e^c_s}{\|(\bar{A}^c_{32,s})^T P e^c_s\|}, & \|(\bar{A}^c_{32,s})^T P e^c_s\| \neq 0, \\ 0, & \|(\bar{A}^c_{32,s})^T P e^c_s\| = 0, \end{cases}$$

$$\mu 3_s = \begin{cases} -\rho_3 \frac{(\bar{A}^{c,2}_{33,s})^T P e^c_s}{\|(\bar{A}^{c,2}_{33,s})^T P e^c_s\|}, & \|(\bar{A}^{c,2}_{33,s})^T P e^c_s\| \neq 0, \\ 0, & \|(\bar{A}^{c,2}_{33,s})^T P e^c_s\| = 0, \end{cases}$$

and

$$\mu 4_s = \begin{cases} -\rho_4 \frac{(\bar{D}^c_{1,3,s})^T P e^c_s}{\|(\bar{D}^c_{1,3,s})^T P e^c_s\|}, & \|(\bar{D}^c_{1,3,s})^T P e^c_s\| \neq 0, \\ 0 & \|(\bar{D}^c_{1,3,s})^T P e^c_s\| = 0, \end{cases}$$

where $e^c_s = \hat{x}^c_{3,s} - y^c_s$, and ρ_j with $1 \leq j \leq 4$ are chosen such that $\rho_1 > \|x_1\|$, $\rho_2 > \|x_2\|$, $\rho_3 > \|x_{3,s}\|$, and $\rho_4 > \|\bar{d}_1(t)\|$.

The property of the designed output estimator is given in Theorem 5.3.

Theorem 5.3 *Under assumptions A51 to A56, if $(y^c_s)^H = y^c_s$, u and x_3 are bounded, and $\rho_j, 1 \leq j \leq 4$ are chosen such that $\rho_1 > \|x_1\|$, $\rho_2 > \|x_2\|$, $\rho_3 > \|x_{3,s}\|$, and $\rho_4 > \|\bar{d}_1(t)\|$, the SMOE given by (5.15) can ensure that e^c_s exponentially approaches zero, and thus is invariant with respect to y_s and $d(t)$.*

Proof. Because $(y^c_s)^H = y^c_s$ implies $x^c_{3,s} = y^c_s$, the theorem can be proved similarly to Theorem 5.2. ∎

5.4 Solutions of Fault Diagnosis Problems

For simplicity and clarity, in this section, only solutions for actuator fault diagnosis problems are provided because sensor fault diagnosis is performed in exactly the same way. Note that all the results obtained are under the assumption that only actuator faults can occur.

Actuator fault diagnosis is accomplished by examining (5.8) and (5.9). In order to provide solutions for actuator fault diagnosis problems in an efficient and clear way, let $U = (\bar{A}_{31} \ \bar{D}_{1,3})$ and $V_s = (U \ \bar{B}_{3,s})$. Additionally, a concept called the *Output Estimator Induced Actuator Fault Isolation Index (OEIAFIX)* is defined.

Definition 5.4.1 *System (5.1) is said to have an Output Estimator Induced Actuator Fault Isolation Index (OEIAFIX) equal to l if and only if for any s, one always has $rank(V_s) = rank(U) + l$. l is the largest number that has this property.*

Remark 5.4.1 *If there is no $l > 0$ such that $OEIAFIX = l$, then system (5.1) does not have any OEIAFIX. For consistency, $OEIAFIX = 0$ is used to denote this situation.*

The *Output Estimator Induced Actuator Fault Isolation Index (OEIAFIX)* has the following property.

Lemma 5.4.1 *For system (5.1), $0 \leq OEIAFIX \leq p - rank(U)$ is always true.*

Proof. From Remark 5.4.1 and the definition of OEIAFIX, $OEIAFIX \geq 0$ is always true. On the other hand, because the matrix V_s has p rows, $rank(U) + l \leq p$, which proves the lemma. ∎

For simplicity, AFIX is used to stand for OEIAFIX. The following result is obtained.

Theorem 5.4 *Under assumptions A51 to A56, assume that only actuator faults can occur and the SMOE given by (5.9) is used to perform actuator fault isolation. If $AFIX < m$, the maximum number of actuator faults that can be simultaneously isolated is equal to the $AFIX - 1$.*

Proof. Two cases need to be considered: 1)$AFIX \leq 1$, and 2) $AFIX > 1$.

For the case $AFIX \leq 1$, it needs to be shown that no single actuator fault can be isolated. When $AFIX = 0$, $rank(U\ \bar{B}_3) = rank(U)$, which implies that, for any fixed $l > 0$ and s, the output estimation error, e_s, resulting from using (5.9) is invariant to any bounded input vector u because all u can be incorporated into the channels of $\mu 1_s$ and $\mu 3_s$, i.e., the output estimation error will be insensitive to all actuator faults. Because all possible output estimation errors are insensitive to all faults, no faults can be isolated. Moreover, no actuator faults can be detected. When $AFIX = 1$, for any $s = \{i_1\}$, $rank(V_s\ \bar{B}_3) = rank(V_s)$, which implies that the output estimation error,

ey_s, resulting from using (5.9) is invariant to any bounded input vector u because all u can be attenuated by $\mu 1_s$, $\mu 2_s$ and $\mu 3_s$, i.e., the output estimation error is insensitive to all actuator faults. Because all possible output estimation errors are insensitive to all faults, no faults can be isolated. Moreover, no faults can be detected when $l = 1$. However, $l = 0$, the SMOE given by (5.9) (without the term $\mu 2_s$) can be used to detect faults because actuator faults can not be attenuated by $\mu 1_s$ and $\mu 3_s$. The output estimation error will be sensitive to actuator faults, and thus actuator faults can be detected.

For the case $AFIX > 1$, it is needed to prove that isolating actuator faults is possible.

Assume exactly $AFIX - 1$ actuator faults are present. Let $l = AFIX - 1$; then, for any set s, e_s, resulting from (5.9) is invariant to any actuator faults within the actuator group corresponding to u_s. However, it is sensitive to any actuator fault outside the actuator group corresponding to u_s because the definition of $AFIX$ ensures such actuator faults lie in independent channels of $\mu 1_s$, $\mu 2_s$ and $\mu 3_s$. Because there are $AFIX - 1$ actuator faults, the arguments above imply that only one set of the form s exists with output estimation error tending to zero. For all other sets, the resulting output estimation errors do not approach zero in general because they are sensitive to those actuator faults. Therefore, $AFIX - 1$ actuator faults can be isolated because only one output estimation error tends to zero while the rest do not.

Now, it is needed to show that isolating more than $AFIX - 1$ actuator faults is not possible. If $AFIX - 1$ actuator faults occur, and $l = AFIX - 1$ is chosen, by the definition of $AFIX$, all output estimation errors resulting from (5.9) are sensitive to actuator faults. Because $AFIX < m$, only the fact that more than $AFIX - 1$ actuator faults have occurred can be concluded, however they cannot be isolated. If

$l \geq AFIX - 1$ is chosen, for all s, the resulting output estimation errors from (5.9) approach zero because all the inputs are attenuated by $\mu 1_s$, $\mu 2_s$ and $\mu 3_s$. In such a situation, no faults can be isolated. ∎

In the proof of Theorem 5.4, fault detection using the SMOE given by (5.9) (without the term $\mu 2_s$) is not possible for $AFIX = 0$, but is possible for $AFIX > 0$. This fact solves $FDP1$ and $FDP2$.

Theorem 5.4 has also given solutions for both $FIP1$ and $FIP2$ at the same time, which are shown more clearly in the following corollaries.

Corollary 5.4.1 *Under the assumptions of Theorem 5.4, if $AFIX \leq 1$, actuator fault isolation is impossible; if $AFIX = 2$, only a single fault can be isolated; if $AFIX > 2$, actuator fault isolation can be performed for one up to $AFIX - 1$ faults.*

Corollary 5.4.2 *Under the assumptions of Theorem 5.4, if $AFIX > 1$, the maximum number of actuator faults that can be simultaneously isolated is $AFIX - 1$.*

For a system like (5.1) and under the condition that only actuator faults can occur, it was proved that at most $AFIX - 1$ actuator faults can be isolated at the same time if $AFIX < m$.

In the remainder of this section, a design method is introduced that uses as few SMOEs as possible to detect the faults, determine the number of faults, and also isolate and estimate the faults.

Theorem 5.5 provides theoretical support for the possibility of designing actuator fault diagnosis schemes based on SMOEs.

Theorem 5.5 *Under the assumptions of Theorem 5.4, assume that there are n_f actuator faults and $n_f \leq AFIX - 1$. A bank of SMOEs can be defined to generate a group of residuals such that each residual is only insensitive to faults in a particular actuator group but sensitive to all other faults outside the actuator group.*

Proof. Let $l = AFIX - 1$. For any s, if all the actuator faults are inside the actuator group corresponding to u_s, e_s approaches zero exponentially, which means e_s is insensitive to any actuator faults inside the actuator group corresponding to u_s.

On the other hand, if any actuator fault lies outside the actuator group corresponding u_s, $\bar{u}_s^H - \bar{u}_s \neq 0$, which makes the term $\bar{B}^c_{3,s}(\bar{u}_s^H - \bar{u}_s) \neq 0$. Because of the definition of $AFIX$, the nonzero term $\bar{B}^c_{3,s}(\bar{u}_s^H - \bar{u}_s)$ cannot be attenuated by $\mu 1_s$, $\mu 2_s$ and $\mu 3_s$. Therefore, e_s is sensitive to any faults outside the actuator group corresponding to u_s. ∎

Theorem 5.6 serves as a foundation for determining the number of faults.

Theorem 5.6 *Under all conditions in Theorem 5.5, if the number of faults is* $0 < n_f \leq AFIX - 1$, *the number of output estimation errors* ($e_s(t)$ *with* $s = \{i_1, \cdots, i_{AFIX-}\}$), *which are insensitive to the* n_f *faults is equal to* $C_{m-n_f}^{AFIX-1-n_f}$.

Proof. Because only n_f faults occur, the number of sets of the form $s = \{i_1, \cdots, i_{AFIX-}\}$, which include the faulty actuator group, is equal to $C_{m-n_f}^{AFIX-1-n_f}$. According to Theorem 5.5, all output estimation errors corresponding to these $C_{m-n_f}^{AFIX-1-n_f}$ sets are insensitive to the n_f faults and any other output estimation errors are sensitive to the faults. This completes the proof. ∎

Remark 5.4.2 *If* $n_f < AFIX - 1$ *faults occur, exactly* $C_{m-n_f}^{AFIX-1-n_f}$ *output estimation errors are insensitive to the faults. Therefore, once the number of those output estimation errors* (g_{num}) *is obtained, the number of faults* n_f *is found by solving* $C_{m-n_f}^{AFIX-1-n_f} = g_{num}$

The following result is useful in fault isolation.

Theorem 5.7 *Under the assumptions of Theorem 5.5, suppose that only* g_{num} *output estimation errors, i.e.,* e_{S_j} *with* $S_j = \{i_1^j, \cdots, i_{AFIX-1}^j\}$ *and* $1 \leq j \leq g_{num}$, *approach* *zero and an integer n_f exists such that $C_{m-n_f}^{AFIX-1-n_f} = g_{num}$. Define $S_F = \bigcap_{j=1}^{g_{num}} S_j$. Then, S_F can be denoted as $S_F = \{i_1, \cdots, i_{n_f}\}$, and the n_f faulty actuators are the i_1th to the i_{n_f}th actuators.*

Proof. According to Theorem 5.5, at least $C_{m-n_f}^{AFIX-1-n_f} = g_{num}$ output estimation errors approach zero if n_f faults occur. Because only g_{num} output estimation errors approach zero and $C_{m-n_f}^{AFIX-1-n_f} = g_{num}$, n_f actuator faults have occurred. If the faulty actuator group is denoted as $u_{i_1}, \cdots, u_{i_{n_f}}$, this faculty actuator group must and can only be included in the g_{num} different groups defined by $u_{S_j}, 1 \leq j \leq g_{num}$, which are insensitive to the faults. Therefore, by definition, it follows that $\{i_1, \cdots, i_{n_f}\} \in S_j$ for any $1 \leq j \leq g_{num}$ and $S_F = \{i_1, \cdots, i_{n_f}\}$. This completes the proof. ∎

Assume $n_f \leq AFIX - 1$ faults occur and $S_F = \{i_1, \cdots, i_{n_f}\}$. Then, exactly $C_{m-n_f}^{AFIX-1-n_f}$ residuals are insensitive to the faults. According to Theorem 5.7, the fault isolation can be performed. To estimate the faults, a certain set $s^{js} = \{i_1^{js}, \cdots, i_{AFIX-1}^{js}\}$ with the smallest output estimation error is picked. Because $e_{s^{js}}$ tends to zero, and if $\dot{e}_{i_1^{js}, \cdots, i_{AFIX-1}^{js}}$ is assumed to tend to zero, according to (5.10), the faults can be estimated using a low-pass filter as

$$u_{i_j}^{fe} = LPF(\mu 2_{s^{js}}(i_j)) - u_{i_j}^{H}, 1 \leq j \leq n_f \tag{5.16}$$

where $\mu 2_{s^{js}}(i_j)$ is the element in $\mu 2_{s^{js}}$ that corresponds to the index i_j, and LPF denotes a low-pass filter.

An actuator fault diagnosis scheme is given in the steps below.

- Step 1 Compute $AFIX$.

- Step 2 If $AFIX \leq 1$, no fault can be isolated, stop. If $1 < AFIX < m$, let $l = AFIX - 1$, go to Step 3.

- Step 3 Fault detection and isolation

  1. For each set s, design an SMOE given by (5.9).
  2. Define residuals $r_s(t) = \|e_s\|/N_{normal}(t)$, where $N_{normal}(t)$ is chosen such that $r_s(t) \leq 1$ when only actuators corresponding to s are possibly faulty, otherwise $r_s(t) > 1$.
  3. The threshold is chosen as 1.
  4. Fault Detection: If any of the C_m^l residuals is larger than the threshold at any time constant, faults are detected. Otherwise, no fault is detected.
  5. After faults are detected, denote the fault detection time as T_{detect}, choose a fault isolation time interval (FITI) as $(T_{detect}, T_{detect} + \Delta)$ with Δ suitably large, and perform fault isolation on the FITI.
  6. Count the number of residuals that are below the threshold, and denote the number as g_{num}.
  7. If $g_{num} = 0$, more than l actuators are faulty and exact fault isolation cannot be achieved. Stop.
  8. If $g_{num} = 1$, $n_f = l$ and l actuator faults occurred. If r_s is the only residual that is under the threshold, the i_1th, to the i_lth actuators are faulty. Fault isolation is done. Stop.
  9. If $g_{num} > 1$, solve $C_{m-n_f}^{l-n_f} = g_{num}$ for n_f. If no integer solution for n_f exists, the number of faults occurred cannot be determined and fault isolation cannot be accomplished at this moment. Choose a larger Δ, and go to Step 3.6. If an integer solution of n_f exists, the number of faults is equal to $n_f < l$.

10. If the number of faults $n_f < l$ is determined and $C_{m-n_f}^{l-n_f} = g_{num}$ residuals are under the threshold. Denote the corresponding sets as $S_j = \{i_1^j, \cdots, i_l^j\}, 1 \leq j \leq g_{num}$.

11. Fault Isolation for $g_{num} > 1$: Let $S_F = \bigcap_{j=1}^{g_{num}} S_j$. If $S_F = \{i_1, \cdots, i_{n_f}\}$, the faulty actuators are the i_1th to the i_{n_f}th actuators.

12. Fault estimation: Pick up a set $s = \{i_1^j, \cdots, i_l^j\}$ which corresponds to the smallest residual; then use (5.16) to estimate the faults by letting $i_1 = i_1^j, \cdots, i_l = i_l^j$.

Remark 5.4.3 *No such SMOE based fault diagnosis scheme has been proposed in the literature to deal with unmatched unknown inputs.*

5.5 An Example and Simulation Results

To show the effectiveness of the fault diagnosis scheme, a linearized model of a tailless jet fighter in [113] is taken as an example with an added unmatched uncertainty.

The model is given as.

$$\begin{aligned} \dot{z} &= Az + Bu + Dd(t) \\ y &= Cz \end{aligned} \tag{5.17}$$

where $z = (\alpha, \beta, p, q, r)^T$ whose α is the angle of attack, β is the sideslip angle, p, q, r are the roll rate, pitch rate, and yaw rate, respectively. The control u is defined as $u = (\delta_{el}, \delta_{er}, \delta_{pflap}, \delta_{amtl}, \delta_{amtr})^T$ with $\delta_{el}, \delta_{er}, \delta_{pflap}, \delta_{amtl}, \delta_{amtr}$ being defined as the deflections of the left and right elevons, the pitch flap, and the left and right all moving tips, respectively. The system matrices are defined as follows:

$$A = \begin{pmatrix} -0.6344 & 0.0027 & 0 & 0.9871 & 0 \\ 0 & -0.0038 & 0.1540 & 0 & -0.9876 \\ 0 & -8.2125 & -0.7849 & 0 & 0.1171 \\ -0.5971 & 0 & 0 & -0.5099 & 0 \\ 0 & -0.8887 & -0.0299 & 0 & -0.0156 \end{pmatrix},$$

$$B = \begin{pmatrix} -0.0459 & -0.0459 & -0.0395 & -0.0133 & -0.0133 \\ -0.0047 & 0.0047 & 0 & 0.0031 & -0.0031 \\ 3.783 & -3.783 & 0 & 1.8255 & -1.8255 \\ -2.5115 & -2.5115 & -1.9042 & -0.9494 & -0.9494 \\ -0.0453 & -0.0453 & 0 & -0.2081 & 0.2081 \end{pmatrix} D = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix},$$

and

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

For this system, $AFIX = 3$ can be verified. Therefore, at most 2 actuator faults can be isolated. Because $m = 5$ and $AFIX - 1 = 2$, in total $C_5^2 = 10$ sets exist, i.e., $s = \{1,2\}$, $s = \{1,3\}$, $s = \{1,4\}$, $s = \{1,5\}$, $s = \{2,3\}$, $s = \{2,4\}$, $s = \{2,5\}$, $s = \{3,4\}$, $s = \{3,5\}$, and $s = \{4,5\}$.

In the simulations, the first and the second actuators become stuck after $5s$. In total, ten SMOEs are designed. $N_{normal}(t) = 0.00001$ is used, and the simulation results are plotted in Fig. 5.1 to Fig. 5.3.

Because the faults are abrupt, fault detection is fast, i.e., $0.37s$ after the faults occurred because r_{25} in Fig. 5.2 exceeds the threshold. If $FITI = (5.37s, 7s)$, Fig. 5.1 and Fig. 5.2 show that only r_{12} stays below the threshold on the chosen interval $FITI$. According to Step 3.8 in the fault diagnosis scheme, $g_{num} = 1$, which leads to a conclusion that two faults have occurred, and that the faulty actuators correspond

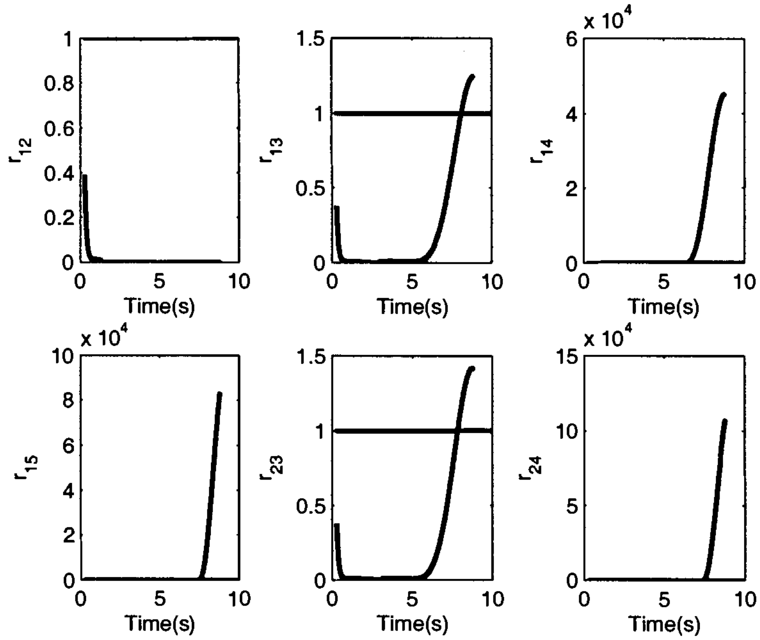

Figure 5.1: Aircraft fault detection and isolation – The first six residuals

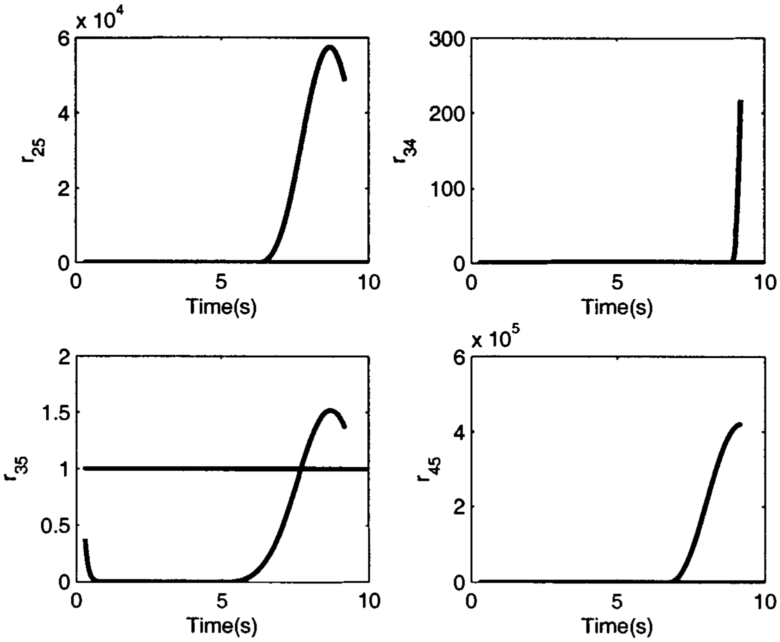

Figure 5.2: Aircraft fault detection and isolation – The other four residuals

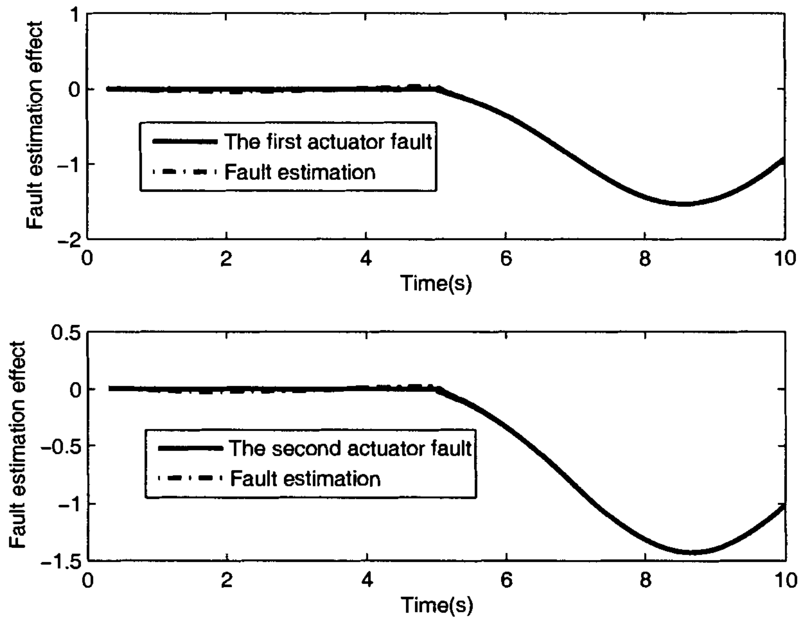

Figure 5.3: Aircraft actuator faults and their estimations

to r_{12}, i.e. the first and the second actuators. Thus, a correct fault isolation decision is made. Fig. 5.3 shows the actuator faults can be estimated accurately.

As shown in Fig. 5.1, r_{15} exceeds the threshold at $6.58s$. If $\Delta < 6.58$, r_{12} and r_{15} are below the threshold, which leads to a wrong fault isolation conclusion. Therefore, the choice of $FITI$ is important for fast and correct fault isolation.

The choice of $N_{normal}(t)$ also plays a very important role in correct fault isolation. For example, if $N_{normal}(t) = 0.001$, it follows from Fig. 5.1 that r_{12}, r_{13}, and r_{23} are below the threshold, which causes $g_{num} = 3$. Based on $g_{num} = 3$, fault isolation cannot be achieved because $C_{5-n_f}^{2-n_f} = g_{num}$ does not have an integer solution.

5.6 Conclusions and Discussions

In this chapter, for a class of systems with unmatched unknown inputs, a systematic and detailed study was carried out on how to design output estimators for actuator or sensor fault detection, isolation and estimation.

Firstly, a canonical system structure was developed. It separates the matched and unmatched unknown inputs explicitly and is in a very suitable form for sliding mode output estimator (SMOE) design. Secondly, based on the particular system structure, SMOEs were designed and proved to be invariant to both matched and unmatched unknown inputs. Thirdly, given that the proposed SMOEs are used for fault diagnosis, solutions were provided to all the fault diagnosis problems.

Finally, the simulation results for the linearized tailless jet fighter model showed that the fault diagnosis scheme detected and isolated abrupt faults successfully. Simulations also show that accurate fault estimation is possible.

Although direct output estimator based fault diagnosis schemes have strength in dealing with situations that are impossible for existing observer based fault diagnosis schemes, they are designed to complement the existing observer based fault diagnosis schemes rather than replace them completely. Whenever fault diagnosis scheme design based on the observer design is possible, observer based fault diagnosis schemes should be used to accomplish fault diagnosis.

Because only a class of linear systems is considered, extensions to more general linear systems and nonlinear systems remain to be investigated.

Chapter 6

Actuator Fault Diagnosis for Uncertain Linear Systems Based on High-order Sliding-mode Robust Differentiator (HOSMRD)

In Chapter 5, output estimator based fault diagnosis schemes were proposed for systems with unmatched non-parametric uncertainties by abandoning the idea of observer design. This chapter aims to extend the research in Chapter 5 to a more general class of linear systems by designing a fault diagnosis scheme using the estimates of both the outputs and their high order derivatives as well.

6.1 Introduction

Many observer based fault diagnosis schemes (including those schemes in Chapter 2 through Chapter 4) proposed for systems subject to unknown inputs operate on three assumptions: firstly, the system under consideration is at least detectable; secondly, the unknown inputs satisfy certain matching conditions; and thirdly, the relative degrees from the generalized input vector, including both known and unknown inputs, to the outputs are no larger than one, which is often implicit. For example, the existence conditions of the SMO in [30] are actually a certain type of matching conditions imposed on the unknown inputs, which imply a relative degree one requirement. High order sliding mode observers do not need the relative degree one condition, thereby requiring less restrictive matching conditions, and thus they can be used to solve more challenging fault diagnosis problems (see [129] and the references listed therein). Obviously, when the system under consideration is not detectable, observer based fault diagnosis schemes can no longer be used. For systems that are detectable but do not satisfying the relaxed matching conditions in [129], whether observer based fault diagnosis schemes can be designed or not is unclear. These observations with the idea of designing direct output estimators motivate the research in this chapter to develop a new fault diagnosis scheme that does not rely on observer design.

Although some results have been reported on fault diagnosis for nonlinear systems [27, 76, 81], fault diagnosis on linear systems subject to unknown inputs has not been studied for the case that none of the three assumptions mentioned earlier are met. This observation is the reason why uncertain linear systems are studied in this chapter before attempting fault diagnosis for nonlinear systems.

The goal of this chapter is to address this challenging fault diagnosis problem in order to develop a novel actuator fault diagnosis scheme for a general class of linear systems subject to unknown inputs that can work either with or without those three assumptions. In order to achieve this goal, as in Chapter 5, fault diagnosis based on observer design is abandoned. Instead, fault diagnosis based on estimator design of the outputs and their high order derivatives is employed. To overcome the difficulty caused by high relative degrees, the recently developed high-order sliding-mode robust differentiators (HOSMRDs) (see [130] and references listed therein) will be used to estimate the output derivatives.

The remainder of this chapter is arranged as follows. In Section 6.2, the system of interest is described, and a particular input/output relation suitable for actuator fault diagnosis is derived. In Section 6.3, an HOSMRD is introduced and its properties that were obtained in [130] are briefly reviewed. In Section 6.4, the problems listed in Section 6.2 are solved. In Section 6.5, an example is given to show the effect of the actuator fault diagnosis scheme on actuator fault detection, isolation, and estimation. Finally, conclusions and discussions are presented in the last section.

6.2 Preliminaries

6.2.1 System Description and Fault Diagnosis Problem Formulation

Consider a class of linear systems with unknown inputs in the following form

$$\begin{aligned} \dot{x} &= Ax + Bu + Dd \\ y &= Cx \end{aligned} \tag{6.1}$$

where $x \in R^n$ is the state vector, $y = (y_1\, y_2 \cdots y_p)^T \in R^p$ is the output vector, $u \in R^m$ is the input vector, and $d \in R^q$ is a bounded unknown input vector which may consist of system uncertainties and/or disturbances.

The following two assumptions are needed.

Assumption $A61$: Matrices A, B, C, D are known.

Assumption $A62$: Matrices B and D are both of full column rank, C is of full row rank.

Remark 6.2.1 *The SMO in [30] requires two conditions to ensure its existence: 1) the invariant zeros of (A, D, C) must have negative real parts, and 2) rankCD =rank$D$ = q, which implies that the relative degrees from d to the outputs are one. These two conditions, which are called matching conditions in this chapter, are also required by the UIOs in [27, 70, 69]. The latter condition is recently removed in [129], where relaxed matching conditions are allowed. In this chapter, no such conditions have been assumed. Moreover, the system is not necessarily required to be detectable.*

For system (6.1), all fault detection, isolation, and estimation problems raised in Section 1.2 will be studied for actuator faults.

6.2.2 An Input/Output Relation

In order to derive an input/output relation that can be used for actuator fault diagnosis and that does not involve the derivatives of either the known inputs in u nor the unknown inputs in d, a generalized input vector is defined as $u_d = (u^T \ d^T)^T$ as well as a new input distribution matrix $B_d = (B \ D)$. Additionally, the concept of relative degree from the generalized input vector u_d to the ith output y_i with $1 \leq i \leq p$ is introduced.

Definition 6.2.1 *For the system in (6.1) and any $1 \leq i \leq p$, r_i is said to be the relative degree from the input vector u_d to the ith output y_i if $C_iA^jB_d = 0$ for $1 \leq j \leq r_i - 2$ and $C_iA^{r_i-1}B_d \neq 0$, where C_i is the ith row of C.*

Remark 6.2.2 *If $C_iB_d \neq 0$, the relative degree is one. If $D = 0$, the resulting relative degree is from the input vector u to an output. If r_i is infinity, clearly, y_i is not affected by either u or d.*

Because of Remark 6.2.2, another assumption is needed.

Assumption A63: For any $1 \leq i \leq p$, r_i is finite.

Under assumption A63, the following are derived:

$$\begin{aligned} y_1 &= C_1x, \dot{y}_1 = C_1Ax, \cdots, y_1^{(r_1)} = C_1A^{r_1}x + C_1A^{r_1-1}B_du_d, \\ &\vdots \\ y_p &= C_px, \dot{y}_p = C_pAx, \cdots, y_p^{(r_p)} = C_pA^{r_p}x + C_pA^{r_p-1}B_du_d. \end{aligned} \tag{6.2}$$

Defining $O = (C_1^T \;\cdots\; (C_1A^{r_1-1})^T \; C_p^T \;\cdots\; (C_pA^{r_p-1})^T)^T$. Now, select all the independent rows from O in the following manner: first, pick $C_1, \cdots, C_p$ because C is of full row rank, second, find all the rows from $C_1A, \cdots, C_pA$, which together with $C_1, \cdots, C_p$ form another set of independent rows of O; continue until no dependent rows can be found. Finally, use all the independent rows obtained to form a new matrix as $T_O = (C_1^T \;\cdots\; (C_1A^{l_1})^T \; C_p^T \;\cdots\; (C_pA^{l_p})^T)^T$, which is of full row rank and has the same rank as O.

Note, because T_O is of full row rank, T_c can be chosen such that $T = (T_O^T \; T_c^T)^T$ is nonsingular. Now, let $w = (w_1^T \; w_2^T)^T = Tx$ with $w_1 = T_Ox$. Clearly, w_1 consists of the outputs and their derivatives.

Define matrices Y_d, M, N_u, and N_d as

$$\begin{aligned} Y_d &= \begin{pmatrix} y_1^{(r_1-1)} \\ \cdots \\ y_p^{(r_p-1)} \end{pmatrix}, M = \begin{pmatrix} C_1A^{r_1} \\ \cdots \\ C_pA^{r_p} \end{pmatrix} T^{-1}, \\ N_u &= \begin{pmatrix} C_1A^{r_1-1}B \\ \cdots \\ C_pA^{r_p-1}B \end{pmatrix}, N_d = \begin{pmatrix} C_1A^{r_1-1}D \\ \cdots \\ C_pA^{r_p-1}D \end{pmatrix}. \end{aligned} \tag{6.3}$$

Partition M according to $w = (w_1^T \; w_2^T)^T$ such that $M = (M_1 \; M_2)$. It follows from (6.2) that

$$\dot{Y}_d = M_1w_1 + M_2w_2 + N_uu + N_dd. \tag{6.4}$$

Let $M_{2,d} = (M_2 \ N_d)$ and choose $M_{2,d}^{\perp}$ such that $M_{2,d}^{\perp} M_{2,d} = 0$ and $rank(M_{2,d}) + rank(M_{2,d}^{\perp}) = n$. Denote $Y_{io} = M_{2,d}^{\perp} Y_d$, $M_{io} = M_{2,d}^{\perp} M_1$, $N_{io} = M_{2,d}^{\perp} N_u$. The following relation is obtained

$$\dot{Y}_{io} = M_{io} w_1 + N_{io} u \tag{6.5}$$

The above relation involves only the inputs, the outputs, and their high order derivatives. Consequently,it is called an input/output relation. The fault diagnosis will be performed based on this relation.

Remark 6.2.3 *(6.5) is obtained by treating w_2 as an unknown vector. Some detectable faults in some traditional FDI schemes [27, 70, 69, 30] may become undetectable for systems where w_2 is available. When w_2 can be estimated, by letting $M_{2,d} = N_d$ and $M_{io} = M_{2,d}^{\perp} M$ and replacing w_1 with w, the problem of fault detectablity loss can be easily fixed. Because w_2 is often not available through observer design if the system under consideration is either undetectable or not satisfying the relaxed matching conditions in [129], treating w_2 as an unknown vector allows us to be able to develop a novel actuator fault diagnosis scheme based on (6.5) for those challenging systems where observer based fault diagnosis might not be possible.*

6.3 High Order Sliding Mode Robust Differentiators (HOSMRDs)

Because HOSMRDs will be used to obtain high order derivatives of the outputs, an HOSMRD is introduced and its properties obtained in [130] are listed in this section.

6.3.1 An HOSMRD

Let $f(t) = f_0(t) + n(t)$ be a function on $[0, \infty)$, where $f_0(t)$ is an unknown base function with the nth derivatives having a Lipschitz constant L, and $n(t)$ is a bounded Lebesgue-measurable noise with unknown features. The problem of HOSMRD design is to find real-time robust estimations of $\dot{f}_0(t), \ddot{f}_0(t), \cdots, f_0^{(n)}(t)$ that are exact when $n(t) = 0$. An HOSMRD proposed in [130] has the following form:

$$\begin{aligned}
\dot{z}_0 &= v_0, v_0 = -\lambda_0|z_0 - f(t)|^{n/(n+1)} sign(z_0 - f(t)) + z_1 \\
\dot{z}_1 &= v_1, v_1 = -\lambda_1|z_1 - v_0|^{(n-1)/n} sign(z_1 - v_0) + z_2 \\
&\vdots \\
\dot{z}_{n-1} &= v_{n-1}, v_{n-1} = -\lambda_{n-1}|z_{n-1} - v_{n-2}|^{1/2} sign(z_{n-1} - v_{n-2}) + z_n \\
\dot{z}_n &= -\lambda_n sign(z_n - v_{n-1})
\end{aligned} \tag{6.6}$$

where $\lambda_0, \lambda_1, \cdots, \lambda_n$ are positive design parameters.

6.3.2 The Properties of the HOSMRD

With respect to the HOSMRD given by (6.6), the following three results are proved in [130].

Theorem 6.1 *If $n(t) = 0$ and all the parameters are chosen properly, after a finite transient, the following equalities are true:*

$$z_0 = f_0(t); z_i = v_{i-1} = f_0^{(i)}(t), i = 1, 2, \cdots, n. \tag{6.7}$$

Theorem 6.2 *If $|n(t)| = |f(t) - f_0(t)| \leq \epsilon$ and all the parameters are chosen properly, after a finite transient, the following inequalities are obtained:*

$$|z_i - f_0^{(i)}(t)| \leq \mu_i \epsilon^{(n-i+1)/(n+1)}, i = 0, 1, \cdots, n$$

$$|v_i - f_0^{(i+1)}(t)| \leq \nu_i \epsilon^{(n-i)/(n+1)}, i = 0, 1, \cdots, n-1 \tag{6.8}$$

where $\mu_i, i = 0, 1, \cdots, n$ *and* $\nu_i, i = 0, 1, \cdots, n-1$ *are some positive constants which are only dependent on the parameters of the differentiator.*

Consider the discrete-sampling case, when $z_0(t) - f(t)$ is replaced by $z_0(t_j) - f(t_j)$ on $[t_j, t_{j+1})$ with $\tau = t_{j+1} - t_j$.

Theorem 6.3 *Let* τ *be the constant sampling time. If* $n(t) = 0$ *and all the parameters are chosen properly, after a finite transient, the following inequalities are obtained:*

$$\begin{aligned} |z_i - f_0^{(i)}(t)| &\leq \mu_i \tau^{n-i+1}, i = 0, 1, \cdots, n, \\ |v_i - f_0^{(i+1)}(t)| &\leq \nu_i \tau^{n-i}, i = 0, 1, \cdots, n-1. \end{aligned} \tag{6.9}$$

6.4 Fault Diagnosis Based on the Input/Output Relation and the HOSMRD

The fault diagnosis problems are solved in this section. For simplicity, all notations not defined are the same as those in Chapter 2. Several new notations are introduced as follows. Denote $N_{io} = (N_{io,1} \ \cdots \ N_{io,m})$. For any $s = \{i_1, \cdots, i_l\} \in 2^{S_I}$ with $1 \leq l \leq m$, define $N_{io,s} = (N_{io,i_1}, \cdots, N_{io,i_l})$, where $i_j \in \{1, 2, \cdots, m\}$ for any $1 \leq j \leq l$ and N_{io,i_j} is the i_j-th column of N_{io}. If all columns of $N_{io,s}$ are taken away from N_{io}, the remaining columns of N_{io} constitute a new matrix denoted by $\bar{N}_{io,s}$.

6.4.1 Actuator Fault Detection and Generalized Actuator Fault Isolation Index

To solve the fault detection and isolation problems, two concepts are introduced: 1) *Input-Output Relation Induced Actuator Fault Isolation Index* (IORIAFIX), and 2) actuator fault detectability.

Definition 6.4.1 *System (6.1) is said to have an Input-Output Relation Induced Actuator Fault Isolation Index (IORIAFIX) equal to* l *if and only if for all sets of the form* $s = \{i_1, \cdots, i_l\}$, $rank(N_{io,s}) = l$. l *is the largest number for which this rank condition holds.*

Remark 6.4.1 *In previous chapters, different concepts of the actuator fault isolation index (AFIX) were defined under certain matching conditions and/or the relative degree one requirement. Those concepts are not suitable here because all the conditions needed are missing. Therefore, in order to provide concise answers to the fault diagnosis problems, the concept of IORIAFIX is introduced. For simplicity, AFIX is still used to stand for IORIAFIX.*

If $AFIX = m$, clearly, all the inputs can be reconstructed using sliding mode technique based on (6.5), making actuator fault diagnosis almost trivial. For this reason, in the remaining part of this chapter, only the case where $AFIX < m$ is studied.

Definition 6.4.2 *For System (6.1), actuator faults are said to be detectable if residuals based on the measured variables can be designed such that they will approach zero (or enter a small neighborhood of the origin) when no actuator fault is present, but will not approach zero (or enter a small neighborhood of the origin) for at least one type of actuator fault.*

Remark 6.4.2 *The fault detectability concept introduced here is very weak because all working fault detection schemes assume this fault detectability. It actually allows certain types of faults to remain undetected. One reason for this is that designing a residual that is sensitive to all types of faults is very difficult if not impossible. Another reason is that a necessary and sufficient condition for fault detectability can be derived.*

To perform fault detection, an estimator for Y_{io} is designed as

$$\dot{\hat{Y}}_{io} = H(\hat{Y}_{io} - \hat{Y}_{io,HOSMRD}) + M_{io}\hat{w}_{1,HOSMRD} + N_{io}u^H \tag{6.10}$$

where H is chosen to be any Hurwitz matrix, and $\hat{Y}_{io,HOSMRD}$ and $\hat{w}_{1,HOSMRD}$ are the estimates of Y_{io} and w_1 obtained using HOSMRDs given by (6.6).

Theorem 6.4 *Given that assumptions A61 through A63 hold and that the assumptions in Theorem 6.1 are satisfied, and assuming that only actuator faults can occur and that the HOSMRDs given by (6.6) are used to estimate $\dot{y}_1, \cdots, y_1^{(r_1)}, \cdots, \dot{y}_p, \cdots, y_p^{(r_p)}$. Then, $lim_{t\to\infty}(\hat{Y}_{io} - \hat{Y}_{io,HOSMRD}) = 0$ when no actuator fault occurs, i.e., when $u^H = u$.*

Proof. Because all assumptions in Theorem 6.1 are satisfied, it is possible to obtain the exact estimation of $\dot{y}_1, \cdots, y_1^{(r_1)}, \cdots, \dot{y}_p, \cdots, y_p^{(r_p)}$ after the transient periods, which implies $\hat{Y}_{io,HOSMRD} = Y_{io}$ and $\hat{w}_{1,HOSMRD} = w_1$. Because $\hat{Y}_{io,HOSMRD} = Y_{io}$ and $\hat{w}_{1,HOSMRD} = w_1$ after the transient periods, it follows from (6.5) and (6.10) that

$$\dot{\tilde{Y}}_{io,HOSMRD} = H\tilde{Y}_{io,HOSMRD} + N_{io}(u^H - u) \tag{6.11}$$

where $\tilde{Y}_{io,HOSMRD} = \hat{Y}_{io} - \hat{Y}_{io,HOSMRD}$. Finally, because H is Hurwitz and $u^H - u = 0$, the theorem is proved using (6.11). ∎

Based on Theorem 6.4 and by defining $r(t) = \|\hat{Y}_{io} - \hat{Y}_{io,HOSMRD}\|$, the actuator fault detection can be performed as follows:

$$Fault\ detection\ strategy \begin{cases} lim_{t\to\infty}(\hat{Y}_{io} - \hat{Y}_{io,HOSMRD}) \neq 0 & Failure \\ Otherwise & No\ Failure \end{cases}$$

Obviously, the above strategy solves $FDP2$. $FDP1$ is solved by Theorem 6.5.

Theorem 6.5 *If all the assumptions of Theorem 6.4 are satisfied, actuator faults are detectable using* $r(t) = \|\hat{Y}_{io} - \hat{Y}_{io,HOSMRD}\|$ *resulting from (6.10) if and only if* $AFIX \geq 1$.

Proof. Sufficiency. Because $AFIX \geq 1$, $N_{io,j}, 1 \leq j \leq m$ are all nonzero. Assume that $l \leq m$ actuators are faulty and that they correspond to $N_{io,s}$; therefore $u_s^H - u_s \neq 0$. Using (6.11),

$$\dot{\tilde{Y}}_{io,HOSMRD} = H\tilde{Y}_{io,HOSMRD} + N_{io,s}(u_s^H - u_s). \tag{6.12}$$

Because all columns of $N_{io,s}$ are nonzero, infinite low frequency fault signals exist such that $u_s^H - u_s \neq 0$ and $N_{io,s}(u_s^H - u_s) \neq 0$. Using this fact and the fact that H is Hurwitz, (6.12) ensures $lim_{t\to\infty}(\hat{Y}_{io} - \hat{Y}_{io,HOSMRD}) \neq 0$ for at least one type of actuator fault. This fact together with Theorem 6.4 proves that actuator faults are detectable by definition.

Necessity. If $l = 1$, (6.12) becomes

$$\dot{\tilde{Y}}_{io,HOSMRD} = H\tilde{Y}_{io,HOSMRD} + N_{io,j}(u_j^H - u_j). \tag{6.13}$$

In order to use $r(t) = \|\hat{Y}_{io} - \hat{Y}_{io,HOSMRD}\|$ to achieve single actuator fault detection, it is required that $r(t)$ should not approach zero for some $u_j^H - u_j \neq 0$. For this to happen, $N_{io,j} \neq 0$ is required. In order for any single actuator fault to be detectable, $N_{io,j} \neq 0$ with $1 \leq j \leq m$ are needed, which implies that $AFIX \geq 1$. This completes the proof. ∎

Remark 6.4.3 *If $AFIX = 0$, at least one column of N_{io} is a zero column. Faults that occurred in the actuator corresponding to the zero column are not detectable using (6.10). $AFIX \geq 1$ guarantees that faults in all actuators are detectable.*

6.4.2 Actuator Fault Isolation

In order to solve the fault isolation problem, a concept called actuator fault isolatability is introduced.

Definition 6.4.3 *System (6.1) is said to have actuator fault isolatability with respect to l faults if a bank of residuals based on the measured variables can be designed such that they can be used to isolate at least one of the l actuator faults.*

Similar comments to those of Remark 6.4.2 can be made here. Because $AFIX < m$, in order to perform fault isolation, a bank of C_m^l estimators has to be designed for Y_{io} which takes on the following form:

$$\dot{\hat{Y}}_{io,s} = H(\hat{Y}_{io,s} - \hat{Y}_{io,HOSMRD}) + M_{io}\hat{w}_{1,HOSMRD} + N_{io,s}\mu_s + \bar{N}_{io,s}\bar{u}_s^H \tag{6.14}$$

where $s = \{i_1, \cdots, i_l\} \in 2^{S_I}$ H is chosen to be any Hurwitz matrix, and $\hat{Y}_{io,HOSMRD}$ and $\hat{w}_{1,HOSMRD}$ are the same as defined in the last subsection.

The sliding-mode term μ_s is defined as

$$\mu_s = \begin{cases} -\rho\frac{N_{io,s}^T Pe_{y,s}}{\|N_{io,s}^T Pe_{y,s}\|}, & \|N_{io,s}^T Pe_{y,s}\| \neq 0, \\ 0, & \|N_{io,s}^T Pe_{y,s}\| = 0, \end{cases}$$

where $e_{y,s} = \hat{Y}_{io,s} - \hat{Y}_{io,HOSMRD}$, and ρ is chosen such that $\rho > \|u_s\|$. P is a symmetric positive definite matrix such that $H^T P + PH < 0$.

Theorem 6.6 *Under assumptions A61 through A63 and the assumptions in Theorem 6.1, assume that only actuator faults can occur and all the signals remain bounded* *after the occurrence of faults. Then,* $lim_{t\to\infty} e_{y,s} = 0$ *if the HOSMRDs given by (6.6) is used to estimate* $\dot{y}_1, \cdots, y_1^{(r_1)}, \cdots, \dot{y}_p, \cdots, y_p^{(r_p)}$ *and* $\bar{u}_s^H = \bar{u}_s$.

Proof. Because the assumptions in Theorem 6.1 are satisfied, $\hat{Y}_{io,HOSMRD} = Y_{io}$ and $\hat{w}_{1,HOSMRD} = w_1$ after the transients. As a result, when $\bar{u}_s^H = \bar{u}_s$, it follows from (6.5) and (6.14) that

$$\dot{e}_{y,s} = He_{y,s} + N_{io,s}(\mu_s - u_s). \tag{6.15}$$

By choosing $V = e_{y,s}^T P e_{y,s}$ and differentiating it along (6.15), the following is obtained:

$$\dot{V} = e_{y,s}^T(H^T P + PH)e_{y,s} + 2e_{y,s}^T PN_{io,s}(\mu_s - u_s). \tag{6.16}$$

Because of the definition of μ_s and the boundness of u_s, ρ can be chosen large enough such that

$$\dot{V} \leq e_{y,s}^T(H^T P + PH)e_{y,s} + 2\|N_{io,s}^T Pe_{y,s}\|(\rho - \|u_s\|) \leq e_{y,s}^T(H^T P + PH)e_{y,s} \tag{6.17}$$

Because H is Hurwitz and $H^T P + PH < 0$, $lim_{t\to\infty} e_{y,s} = 0$ follows from (6.17) immediately. ∎

Remark 6.4.4 *Theorem 6.6 implies that* $e_{y,s}$ *is invariant to any actuator faults contained in* u_s. *This property is very important in the design of the fault isolation scheme.*

Based on Theorem 6.6 and by defining $r_s(t) = \|e_{y,s}\|$, solutions for $FIP1$ and $FIP2$ are obtained in Theorem 6.7.

Theorem 6.7 *Under assumptions* $A61$ *through* $A63$ *and the assumptions in Theorem 6.1, assume that only actuator faults can occur and all the system signals remain* *bounded after the occurrence of faults, and that the HOSMRD given by (6.6) is used to estimate* $\dot{y}_1, \cdots, y_1^{(r_1)}, \cdots, \dot{y}_p, \cdots, y_p^{(r_p)}$*, system (6.1) has actuator fault isolatability with respect to* l *faults with a bank of residuals chosen as* $r_s(t)$ *resulting from (6.14) if and only if* $AFIX \geq l+1$.

Proof. Sufficiency. Assume the number of faults is l. By assumption, $l < m$. Assume the faults correspond to a particular set $s_0 = \{i_1^0, \cdots, i_l^0\}$. For this set, $\bar{u}_{s_0}^H - \bar{u}_{s_0} = 0$. According to Theorem 6.6, $lim_{t\to\infty} r_{s_0}(t) = 0$. However, for any other set $s = \{i_1, \cdots, i_l\}$ such that $s \neq s_0$, without loss of generality, one can assume $i_1 = i_1^0, \cdots, i_j = i_j^0$ and $i_{j+1} \neq i_{j+1}^0, \cdots, i_l \neq i_l^0$ with $j < l$, $\bar{u}_s^H - \bar{u}_s = u_{i_{j+1}^0,\cdots,i_l^0}^H - u_{i_{j+1}^0,\cdots,i_l^0} \neq 0$. This fact together with (6.5) and (6.14) leads to

$$\dot{e}_{y,s} = He_{y,s} + N_{io,s}(\mu_s - u_s) + N_{io,i_{j+1}^0,\cdots,i_l^0}(u_{i_{j+1}^0,\cdots,i_l^0}^H - u_{i_{j+1}^0,\cdots,i_l^0}). \tag{6.18}$$

Because $AFIX \geq l+1$, $N_{io,j}, 1 \leq j \leq m$ are all nonzero and $rank(N_{io,s}\ N_{io,i_k^0}) = l+1, j+1 \leq k \leq l$. These facts imply that infinite types of actuator faults exist such that $N_{io,i_{j+1}^0,\cdots,i_l^0}(u_{i_{j+1}^0,\cdots,i_l^0}^H - u_{i_{j+1}^0,\cdots,i_l^0})$ cannot be attenuated by the sliding mode term μ_s. Among all possible faults, infinite faults with low enough frequencies exist such that $lim_{t\to\infty} r_s(t) \neq 0$ because H is Hurwitz. This argument holds for all $s \neq s_0$. Because the choices of possible faults are infinite, at least one type of actuator fault exists such that $lim_{t\to\infty} r_s(t) \neq 0$ for all $s \neq s_0$. Based on the above arguments, obviously, the l actuator faults can be isolated according to the residual that approaches zero. This completes the sufficiency proof.

Necessity. For l faults to be isolated using the bank of residuals $r_s(t)$, $AFIX \geq l+1$ needs to be proven. Suppose $AFIX \leq l$. Then,a set $s = \{i_1, \cdots, i_l\}$ exists such that $s \neq s_0$ and $rank(N_{io,s}\ N_{io,i_{j+1}^0,\cdots,i_l^0}) = l$. These facts together with (6.18) imply $lim_{t\to\infty} r_s(t) = 0$. Because $lim_{t\to\infty} r_{s_0}(t) = 0$ is true, the isolation of l faults is impossible because one and only one residual is allowed, which contradicts the assumption that l faults can be isolated. Thus the necessity is proved. ∎

Based on Theorem 6.6, Theorems 6.8 and 6.9 can be proved, which can be used to determine the number of faults and to isolate them.

Theorem 6.8 *Assuming that the conditions in Theorem 6.6 are satisfied, if the number of faults is $0 < n_f \leq AFIX - 1$, the number of residuals($r_s(t)$) that are insensitive to the n_f faults, is at least $C_{m-n_f}^{AFIX-1-n_f}$.*

Theorem 6.9 *Under the assumptions of Theorem 6.8 and assuming that n_f actuator faults have occurred, uppose that the number of residuals approaching zero, is equal to g_{num}, and that the residuals are denoted as r_{s_j}, $1 \leq j \leq g_{num}$, where $s_j = \{i_1^j, \cdots, i_{AFIX-1}^j\}, 1 \leq j \leq g_{num}$. Let $S_F = \bigcap_{j=1}^{g_{num}} s_j$. If $C_{m-n_f}^{AFIX-1-n_f} = g_{num}$, S_F has exactly n_f elements, and $S_F = \{i_1, \cdots, i_{n_f}\}$ determines that the n_f faulty actuators are the i_1th to the i_{n_f}th actuators.*

6.4.3 Actuator Fault Estimation

Assume that $n_f \leq AFIX - 1$ faults occur and $S_F = \{i_1, \cdots, i_{n_f}\}$ is determined according to Theorem 6.9. To estimate the faults, pick up a certain set $s_{min} = \{i_1^{min}, \cdots, i_{AFIX-1}^{min}\}$ with the smallest residual. Because $ey_{s_{min}}$ tends to zero, and if the derivative of $ey_{s_{min}}$ is also assumed to tend to zero, according to (6.15) and the idea of using low-pass filter to estimate the equivalent control, the following approach is proposed to estimate the faults, where the ith actuator fault is defined as $u_i - u_i^H$.

$$u_{i_j}^{fe} = LPF(\mu_{s_{min}}(i_j)) - u_{i_j}^H, 1 \leq j \leq n_f \tag{6.19}$$

where $u_{i_j}^{fe}$ is the estimate of the i_jth actuator fault, $\mu_{s_{min}}(i_j)$ is the element in $\mu_{s_{min}}$ that corresponds to the index i_j, and LPF denotes a low-pass filter.

Based on the above, the solution to $FEP1$ and $FEP2$ is that estimating the shape of the actuator faults is possible and can be achieved using (6.19).

6.4.4 The Complete Fault Diagnosis Strategy

The overall fault diagnosis strategy is summarized in the steps of the following algorithm:

- Step 1 Compute $AFIX$.
- Step 2 If $AFIX \leq 1$, no fault can be isolated based on the input/output relation and only fault detection is possible. The fault detection can be performed using (6.10) and $r(t) = \|\hat{Y}_{io} - \hat{Y}_{io,HOSMRD}\|$. Stop.
- Step 3 Perform fault detection and isolation for the case $1 < AFIX < m$ in the following manner:

  1. For each set $s = \{i_1, \cdots, i_{AFIX-1}\}$, design an estimator for Y_{io} given by (6.14) based on (6.5) and HOSMRDs given by (6.6).
  2. Define residuals $r_{N,s}(t) = r_s(t)/N_{normal}(t)$, where $N_{normal}(t)$ is chosen such that $r_{N,s}(t) \leq 1$ when only actuators corresponding to s are possibly faulty, and $r_{N,s}(t) > 1$ otherwise.
  3. The threshold is chosen to be 1.
  4. If any of the C_m^{AFIX-1} residuals is larger than one at any given time, faults are detected. Otherwise, no fault has been detected.
  5. Once faults are detected, denote the fault detection time as T_{detect}. Choose a fault isolation time interval (FITI) as $(T_{detect}, T_{detect} + \Delta)$ with Δ suitably large, and perform fault isolation on the FITI.

6. Count the number of residuals that are below the threshold, and denote it as g_{num}.

7. If $g_{num} = 0$, more than $AFIX - 1$ actuators are faulty and exact fault isolation can not be achieved. Stop.

8. If $g_{num} = 1$, $n_f = AFIX - 1$ and $AFIX - 1$ actuators are faulty . If $r_{N,s}$ is the only residual that is under the threshold, the i_1th to the i_{AFIX-1}th actuator corresponding to this particular s are faulty. Fault isolation is accomplished. Stop.

9. If $g_{num} > 1$, then solve $C_{m-n_f}^{AFIX-1-n_f} = g_{num}$ for n_f. If there is no integer solution for n_f, then the number of faults occurred can not be determined and fault isolation can not be performed. If there is an integer solution of n_f, then it is concluded the number of faults is equal to the integer solution of n_f.

10. If the number of faults $n_f < AFIX - 1$ is determined and there are $C_{m-n_f}^{AFIX-1-n_f} = g_{num}$ sets such that their corresponding residuals are below the threshold. Denote these sets by $s_j = \{i_1^j, \cdots, i_{AFIX-1}^j\}, 1 \leq j \leq g_{num}$, and compute $S_F = \bigcap_{j=1}^{g_{num}} s_j$ and if $S_F = \{i_1, \cdots, i_{n_f}\}$, then the faulty actuators are the i_1th actuator,$\cdots$,and the i_{n_f}th actuator.

- Step 4. Perform fault estimation by picking up $s_{min} = \{i_1^{min}, \cdots, i_{AFIX-1}^{min}\}$ which corresponds to the smallest residual. Then, use (6.19) to estimate the faults.

Remark 6.4.5 *The use of HOSMRDs in fault diagnosis is the first reported in the literature. Moreover, no observer based fault diagnosis schemes have been proposed in the literature that are capable of dealing with linear systems not necessarily detectable, with unmatched unknown inputs and with high relative degrees.*

6.5 An Example and Simulation Results

To show the effectiveness of the proposed fault diagnosis scheme, the following system is chosen:

$$\begin{aligned} \dot{x} &= Ax + Bu + Dd \\ y &= Cx \end{aligned} \tag{6.20}$$

where $d = 0.01cos(t)$ and the system matrices are shown below.

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, D = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

This system is not detectable and does not satisfy the relaxed matching conditions in [129]. Simple computation shows that the relative degrees from the generalized inputs to the first output, the second output, and the third output are all $2, 2, 2$. For this system, $AFIX = 2$, and according to Theorem 6.7, isolating one actuator fault is possible. In the simulations, an incipient fault occurs at $t = 3s$ and takes form as $u_{1f} = u_1 - u_1^H = 0.02(t-3), t > 3$. $N_{normal}(t)$ is chosen as 0.01, and $r_1(t)$, $r_2(t)$, and $r_3(t)$ are used to denote the normalized residuals corresponding to $s = \{1\}$,$s = \{2\}$ and $s = \{3\}$. The fault detection and isolation simulation results are plotted in Fig. 6.1.

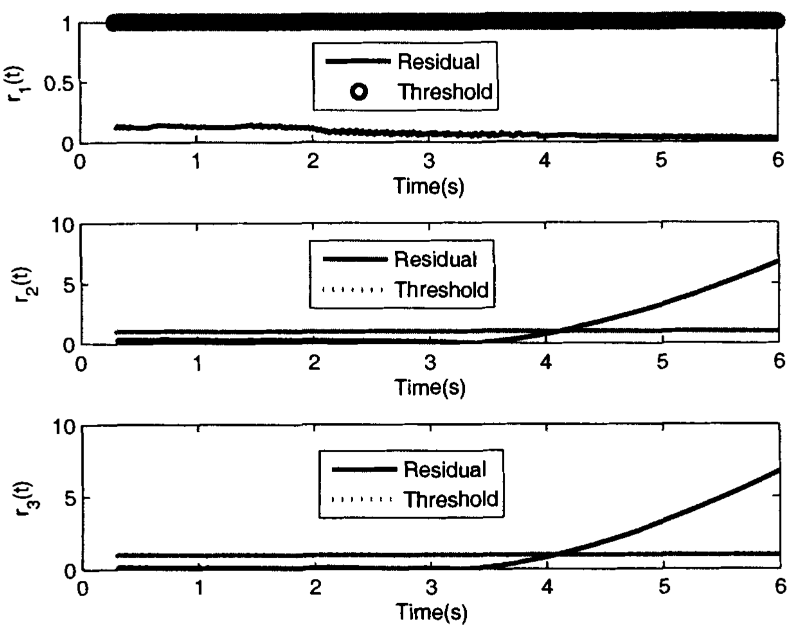

Figure 6.1: Actuator fault detection and isolation

Actuator fault is detected at $t = 4.09s$ because $r_3(t) > 1$ at that moment. Now, choose FITI as $(4.09s, 5s)$ and monitor the residuals on this interval, $r_1(t)$ is far less than one, while $r_2(t) > 1$ and $r_3(t) > 1$. According to the fault diagnosis scheme, the first actuator is faulty while the second and the third one are healthy. The faulty actuator is isolated and correct fault isolation decision is made.

The fault estimation results are plotted in Fig. 6.2, which shows a very promising performance in estimating the shape of the fault.

6.6 Conclusions and Discussions

This chapter addressed the fault diagnosis problem for two challenging situations in linear systems: 1) when the system is not detectable, and 2) when the unknown inputs do not satisfy certain matching conditions. In such situations, designing observers to

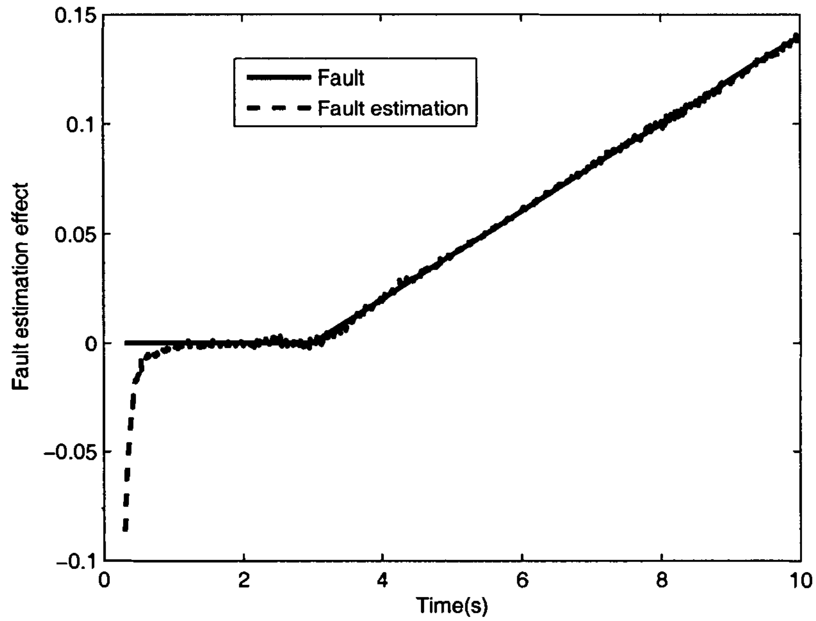

Figure 6.2: Actuator fault estimation

achieve asymptotic estimation of all system states is very difficult if not impossible. By abandoning the idea of designing observers to estimate all the system states, an input/output relation was derived, which involves only the measured outputs and their derivatives to perform fault diagnosis. Based on the relation and on the use of the recently developed high-order sliding mode robust differentiators (HOSMRDs), fault detection, isolation, and estimation problems were studied, and answers were provided in terms of a concept called *Input-Output Relation Induced Actuator Fault Isolation Index (IORIAFIX)*

The input/output relation based actuator fault diagnosis scheme using HOSMRDs offers a new way to carry out fault diagnosis that is different from observer design based approach. Its strength lies in its capability to deal with difficult situations where observer based fault diagnosis might fail to be applicable. The relation between the observer based approach and the method in this chapter needs to be fully explored in future research.

The need for using high order differentiators is a limitation when the measurement noises are present. The method of designing new input/output relation based fault diagnosis schemes, which do not depend on high order output derivatives, is a topic of future research.

Chapter 7

Adaptive Sensor Fault Detection and Isolation in Unknown Linear Systems

In previous chapters, only non-parametric uncertainties were considered, and the system parameters were assumed to be known. In this chapter, linear systems with parametric uncertainties (i.e., linear systems with unknown parameters) are considered, and the adaptive sensor fault detection and isolation problems are studied.

7.1 Introduction

In both the literature and in previous chapters of this thesis, a common assumption for robust fault detection and isolation (FDI) schemes is that the system (or the nominal system) parameters were known. If the system parameters are unknown, most robust approaches cannot be used any more. Instead, adaptive approaches are needed to solve FDI problems. Since this chapter deals with the adaptive approach, only those fault diagnosis results using adaptive approaches will be discussed.

Two types of adaptive FDI schemes have been proposed in the past. One class of approach assumes the systems (or nominal systems) are known. The schemes in [93, 94] belong to this class, where adaptive observers were designed to perform FDI under the assumption that the faults can be modelled by $\Theta^T u$, where Θ is a constant unknown parameter matrix. The adaptive FDI scheme designed for SISO systems in [95] is also of the same class, where faults are linearly parameterized and persistent excitation conditions are required. The schemes in [97, 98, 99] belong to this class too, where faults are parameterized and a compact convex region to which the unknown parameter vector θ^* belongs needs to be determined, additionally, some knowledge about faults has to be used.

The other class of adaptive FDI schemes deal with systems with unknown parameters. The works in [100, 102, 103, 110, 131] belong to this class. Although nonlinear systems were considered in [100, 103, 131], those adaptive schemes might not be applicable to the unknown linear systems considered in this chapter because they were designed under various system restrictions. The only adaptive FDI schemes for unknown linear systems in the literature were proposed in [102] and [110]. The FDI scheme in [102] was based on a PI adaptive observer and was designed for SISO linear systems, while the FDI scheme in [110] was based on an adaptive output estimator design and was proposed for MISO linear systems. The FDI scheme in [110] was only proposed for actuator fault diagnosis, and can only solve the fault isolation problem for constant actuator faults. For general unknown MIMO linear systems, solving the fault detection and isolation problems with adaptive approaches is a large open research topic.

Most of the above mentioned adaptive FDI schemes (except the one in [110]) rely on adaptive observer design. Their main idea is to first estimate all the state variables, and then use the estimated state to estimate the outputs in order to generate suitable residuals. However, adaptive observer design is not possible for undetectable linear systems. Even for the systems that are detectable, the adaptive observer design problem is only well solved for unknown systems with single output ([86, 83, 87, 88, 89, 102]). As for general unknown MIMO linear systems, the problem remains open because none of the existing adaptive observers designed for MIMO systems are applicable to such systems [84, 85, 90, 91, 92]. Moreover, for the purpose of fault diagnosis, observer design is not always necessary because all that is needed is to estimate all the outputs rather than all the states. Therefore, output estimators rather than observers are sometimes preferred in fault diagnosis, see Chen and Saif [13, 110].

Realizing that the fault diagnosis problems of general unknown MIMO linear systems remain a fruitful area of research, the purpose of this chapter is to make some contributions in this direction. By taking advantage of the fact that only output estimators are needed for fault diagnosis, a sensor fault detection and isolation scheme is proposed based on the design of adaptive output estimators for general unknown MIMO linear systems in this chapter. Firstly, an MIMO system is decomposed into a group of MISO systems and then a transfer function description for each MISO system is presented. Secondly, based on each transfer function and for each output, an output equation is obtained by filtering the corresponding output and all the inputs properly, which is suitable for output estimator design. Thirdly, using the derived output equations, adaptive output estimators are designed for all outputs. Finally, based on the designed output estimators, the adaptive sensor fault diagnosis problems are solved.

The remainder of the chapter is arranged as follows: In Section 7.2, the MIMO system model is introduced and the problem of interest is formulated. In Section 7.3, an MIMO system is decomposed into a group of MISO systems and a transfer function description for each MISO system is presented. In Section 7.4, in order to design output estimators, an output equation is derived for each MISO based on its transfer function. In Section 7.5, adaptive output estimators are designed based on the output equations derived. Based on the designed adaptive output estimators, the difficult sensor fault detection and isolation problem is solved completely in a straightforward manner. Simulation results are presented in Section 7.6 to show the effectiveness of the proposed adaptive FDI method. Finally, conclusions and discussions are made in the last section.

7.2 Systems of Interest and Problem Formulation

Consider MIMO systems described as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{7.1}$$

where $x(t)$, $y(t)$, and $u(t)$ are the system state vector, output vector, and input vector respectively, and $x(t) \in R^n$, $y(t) = (y_1(t) \ \cdots \ y_p(t))^T$, $u(t) = (u_1(t) \ \cdots \ u_m(t))^T$. A, B, and C are all unknown matrices.

Assumption $A71$: n, m, and p are known.

Under only one assumption, $A71$, the adaptive observer design is extremely difficult, if not impossible, because no adaptive observer has been found for the system under consideration. Hence, the idea of fault diagnosis based on adaptive observers has to be abandoned. Instead, the idea of an adaptive output estimator design for fault diagnosis is proposed, which leads to the following problem formulation.

Adaptive Sensor Fault Detection and Isolation Problem: Assume that assumption $A71$ is satisfied and only sensor faults can occur. Design an fault diagnosis scheme based on the design of adaptive output estimators such that it can solve the fault detection and isolation problems; i.e., $FDP1$, $FDP2$, $FIP1$, $FIP2$, and $FIP3$, adaptively once they occur.

Remark 7.2.1 *If systems given by (7.1) are not detectable, no observers can be designed to estimate all the states asymptotically, and thus existing observer based fault diagnosis schemes can not be applied. However, designing output estimators for such systems is possible and output estimators are often sufficient for the purpose of fault diagnosis. In the problem formulation, the latter fact is the reason why systems given by (7.1) are not required to be observable or even detectable. It is the idea of designing output estimators rather than state observers that leads to an elegant solution to the Adaptive Sensor Fault Detection and Isolation Problem.*

7.3 System Decomposition and Related Transfer Function Description

Estimating all the outputs directly from the MIMO system (7.1) is very difficult, which is the motivation for transforming the difficult MIMO output estimator design problem into several simpler MISO output estimator design problems by decomposing the MIMO systems into a group of MISO systems.

Let $C = (C_1^T \ \cdots \ C_p^T)^T$ and $B = (B_1 \ \cdots \ B_m)$. Obviously, a MIMO system given by (7.1) can be decomposed into p MISO systems, where for $1 \leq j \leq p$, the jth MISO system is of the following form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y_j(t) &= C_j x(t). \end{aligned} \tag{7.2}$$

Because (C_j, A) is not necessarily detectable, it is not appropriate to design any observer for the MISO system defined by (7.2). In order to estimate the outputs without designing observers for the MISO system defined by (7.2), the input-output relation of $u(t)$ and $y_j(t)$ described by the following transfer function is used.

$$y_j(s) = \sum_{l=1}^{m} G_{jl}(s)u_l(s), \tag{7.3}$$

where

$$G_{jl}(s) = C_j(sI - A)^{-1}B_l = \frac{b_{jl,n-1}s^{n-1} + \cdots + b_{jl,1}s + b_{jl,0}}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}, 1 \leq l \leq m, \tag{7.4}$$

and $s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 = det(sI - A)$.

For convenience, define $a(s) = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$ and $b_{jl}(s) = b_{jl,n-1}s^{n-1} + \cdots + b_{jl,1}s + b_{jl,0}$.

7.4 Output Equations for MISO Systems

For each $1 \leq j \leq p$, based on (7.3) and (7.4), and inspired by [83] and [111], the following state space realization can be given for (7.3):

$$\begin{aligned} \dot{x}_j &= \bar{A}x_j - ay_j + b_{j1}u_1 + \cdots + b_{jm}u_m \\ y_j &= x_{j,1} \end{aligned} \tag{7.5}$$

where $x_j = (x_{j,1} \ \cdots \ x_{j,n})^T$ and

$$\bar{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad a = \begin{bmatrix} a_{n-1} \\ \vdots \\ a_0 \end{bmatrix},$$

$$b_{jl} = \begin{bmatrix} b_{jl,n-1} \\ \vdots \\ b_{jl,0} \end{bmatrix}, \quad 1 \le l \le m. \tag{7.6}$$

Note that (7.5), which is observable, is *not* the same as (7.2), which might not be observable. It is also crucial to know that the outputs for both (7.5) and (7.2) are the same. Therefore, (7.5) can always be used to estimate y_j regardless of whether (7.2) is observable or not. This observation implies that the original system (7.1) is not required to be observable for the purpose of output estimation.

For each $1 \le j \le p$, in order to estimate y_j, first the state estimate for (7.5) needs to be derived. To do so, $u_l, 1 \le l \le m$ and y_j are filtered by $m+1$ $n-$dimensional filters defined as

$$\dot{\lambda}_l = A_0\lambda_l + e_n u_l, \quad 1 \le l \le m, \tag{7.7}$$

$$\dot{\eta}_j = A_0\eta_j + e_n y_j, \tag{7.8}$$

where $A_0 = \bar{A} - k(1 \ 0 \ \cdots \ 0)$, $k = (k_1 \cdots k_n)^T$ is chosen such that A_0 is Hurwitz, and for any $1 \le i \le n$, $e_i = (e_{i,1}, \cdots, e_{i,n})^T \in R^n$ is defined by $e_{i,i} = 1$ and $e_{i,j} = 0$ for $j \ne i$.

After some matrix manipulations, the following relationships are obtained:

$$\begin{aligned} a(A_0)e_n &= a - k \\ b_{jl}(A_0)e_n &= b_{jl}, \quad 1 \le l \le m \end{aligned} \tag{7.9}$$

where $a(A_0)$ and $b_{jl}(A_0)$ with $1 \le l \le m$ are matrix polynomials with $a(s)$ and $b_{jl}(s)$ with $1 \le l \le m$ being defined as earlier.

Now, the estimate for x_j is formed as

$$\hat{x}_j = \sum_{l=1}^{m} b_{jl}(A_0)\lambda_l - a(A_0)\eta_j. \tag{7.10}$$

Using (7.5) and (7.7)-(7.10), the estimation error $\varepsilon_j = (\varepsilon_{j,1}, \varepsilon_{j,2}, \cdots, \varepsilon_{j,n})^T = x_j - \hat{x}_j$ satisfies $\dot{\varepsilon}_j = A_0\varepsilon_j$. Denote $\xi_{ji} = A_0^i\eta_j, 0 \le i \le n-1$, $\xi_{jn} = -A_0^n\eta_j$ and $v_{li} = A_0^i\lambda_l, 0 \le i \le n-1, 1 \le l \le m$. Then (7.10) can be rewritten as

$$x_j = \xi_{jn} - \sum_{i=0}^{n-1} a_i\xi_{ji} + \sum_{l=1}^{m}\sum_{i=0}^{n-1} b_{jl,i}v_{li} + \varepsilon_j, \dot{\varepsilon}_j = A_0\varepsilon_j. \tag{7.11}$$

Clearly, all the $\xi-$ and $v-$signals and their derivatives are explicitly available:

$$\begin{aligned} \xi_{jn} &= -A_0^n\eta_j \quad \dot{\xi}_{jn} = A_0\xi_{jn} + ky_j \\ \xi_{ji} &= A_0^i\eta_j \quad \dot{\xi}_{ji} = A_0\xi_{ji} + e_{n-i}y_j, 0 \le i \le n-1 \\ v_{li} &= A_0^i\lambda_l \quad \dot{v}_{li} = A_0v_{li} + e_{n-i}u_l, 0 \le i \le n-1, 1 \le l \le m \end{aligned} \tag{7.12}$$

Remark 7.4.1 *Based on the expressions in (7.12), the derivatives of $\xi-$ and $v-$signals can be computed by the right-hand side of the differential equations, which means that there is no need to differentiate the $\xi-$ and $v-$ signals to obtain their derivatives.*

Note the estimate given by (7.10) cannot be applied directly because the parameters $a_i, 0 \le i \le n-1$ and $b_{li}, 0 \le i \le n-1, 1 \le l \le m$ are unknown.

7.5 Adaptive Fault Detection and Isolation

For each $1 \le j \le p$, under a no fault scenario, it follows from (7.5) and (7.11) that

$$\dot{y}_j = \xi_{jn,2} - (\xi_{j(2)} + e_1^T y_j)a + \sum_{l=1}^{m}(v_{l(2)} + e_1^T u_l)b_{jl} + \varepsilon_{j,2} \tag{7.13}$$

where $\xi_{jn}^T = (\xi_{jn,1}, \xi_{jn,2}, \cdots, \xi_{jn,n})$, $\xi_{(2)} = (\xi_{j(n-1),2}, \cdots, \xi_{j0,2})$, and $v_{l(2)} = (v_{l(n-1),2}, \cdots, v_{l0,2})$ are computed by (7.12).

The output equation given by (7.13) is desirable because all the outputs are already isolated. If an estimate for each y_j can be established based on (7.13), the sensor fault detection and isolation is straightforward.

In model based fault diagnosis, in order to detect and isolate faults, often quantities called residuals are generated and monitored. To this end, for each $1 \leq j \leq p$, an estimate for the output y_j will be constructed based on (7.13). Because the parameter vectors a, b_{jl} with $1 \leq l \leq m$ in (7.13) are unknown and $\varepsilon_{j,2}$ is not available, in order to construct an estimate for the output y_j, the unknown parameter vectors have to be replaced by their estimates, and the term $\varepsilon_{j,2}$ is not considered. Therefore, by utilizing the adaptive technique, an estimate of the output y_j is given as

$$\dot{\hat{y}}_j = -c_{y_j}(\hat{y}_j - y_j) + \xi_{jn,2} - (\xi_{j(2)} + e_1^T y_j)\hat{a}_{y_j} + \sum_{l=1}^{m}(v_{l(2)} + e_1^T u_l)\hat{b}_{jl} \qquad (7.14)$$

where $\hat{a}_{y_j}$ and $\hat{b}_{jl}, 1 \leq l \leq m$ are the estimates of a and $b_{jl}, 1 \leq l \leq m$, and $c_{y_j} > 1$ is a positive design constant.

The update laws for the unknown parameter vectors are given as

$$\begin{aligned} \dot{\hat{a}}_{y_j} &= \gamma_{a_{y_j}}(\xi_{j(2)} + e_1^T y_j)^T(\hat{y}_j - y_j) \\ \dot{\hat{b}}_{jl} &= -\gamma_{b_{jl}}(v_{l(2)} + e_1^T u_l)^T(\hat{y}_j - y_j), 1 \leq l \leq m \end{aligned} \qquad (7.15)$$

where $\gamma_{a_{y_j}}$ and $\gamma_{b_{jl}}$ with $1 \leq l \leq m$ are positive design constants.

Indeed, (7.14) and (7.15) constitute an adaptive estimate for y_j. By letting $j = 1, 2, \cdots, p$, all the outputs (i.e., $y_1, y_2, \cdots, y_p$) can be estimated adaptively based on (7.14) and (7.15). By defining the residuals as $r_j(t) = \hat{y}_j - y_j, j = 1, 2, \cdots, p$, the following result is obtained.

Theorem 7.1 *Under Assumption A71, assume that no actuator faults occur and also that all the inputs and measured outputs are still bounded after the occurrence of sensor faults. For any $1 \leq j \leq p$, if the jth sensor is healthy, and y_j is estimated adaptively using (7.14) and (7.15), $lim_{t\to\infty} r_j(t) = 0$ is true.*

Proof. It follows from (7.13) and (7.14) that

$$\dot{r}_j(t) = -c_{y_j} r_j(t) - (\xi_{j(2)} + e_1^T y_j)(\hat{a}_{y_j} - a) + \sum_{j=1}^{m} (v_{l(2)} + e_1^T u_l)(\hat{b}_{jl} - b_{jl}) - \varepsilon_{j,2} \quad (7.16)$$

where ε_j satisfies $\dot{\varepsilon}_j = A_0 \varepsilon_j$ and A_0 is Hurwitz.

By choosing a Lyapunov function as

$$V_j = \frac{1}{2} r_j^2 + \frac{1}{2\gamma_{a_{y_j}}} (\hat{a}_{y_j} - a)^T (\hat{a}_{y_j} - a) + \sum_{j=1}^{m} \frac{1}{2\gamma_{b_{jl}}} (\hat{b}_{jl} - b_{jl})^T (\hat{b}_{jl} - b_{jl}) + \varepsilon_j^T P_0 \varepsilon_j \quad (7.17)$$

where P_0 is the positive definite solution of $P_0 A_0 + A_0^T P_0 = -I$.

By differentiating the above Lyapunov function with respect to time t as well as using (7.16), the following is obtained:

$$\begin{aligned} \dot{V}_j &= -c_{y_j} r_j^2 + \frac{1}{\gamma_{a_{y_j}}} [\dot{\hat{a}}_{y_j}^T - \gamma_{a_{y_j}} (\xi_{j(2)} + e_1^T y_j) r_j](\hat{a}_{y_j} - a) \\ &+ \sum_{j=1}^{m} \frac{1}{\gamma_{b_{jl}}} [\dot{\hat{b}}_{jl}^T + \gamma_{b_{jl}} (v_{l(2)} + e_1^T u_l) r_j](\hat{b}_{jl} - b_{jl}) \\ &- r_j \varepsilon_{j,2} - \varepsilon_j^T \varepsilon_j. \end{aligned} \quad (7.18)$$

By substituting (7.15) into the above equation, the following is reached:

$$\begin{aligned} \dot{V}_j &= -c_{y_j} r_j^2 - r_j \varepsilon_{j,2} - \varepsilon_j^T \varepsilon_j \\ &= -(c_{y_j} - 1) r_j^2 - (r_j + \frac{1}{2}\varepsilon_{j,2})^2 - (\varepsilon_{j,1}^2 + \frac{3}{4}\varepsilon_{j,2}^2 + \varepsilon_{j,3}^2 + \cdots + \varepsilon_{j,n}^2) \\ &\leq -(c_{y_j} - 1) r_j^2 \\ &\leq 0 \ (since \ \ c_{y_j} > 1). \end{aligned} \quad (7.19)$$

Because $\dot{V}_j \leq 0$, $V_j(t)$ is bounded. Hence $r_j(t)$, the estimates $\hat{a}_{y_j}$ and $\hat{b}_{jl}$ with $1 \leq l \leq m$ are all bounded. Because $y_j(t)$ and $u_l(t)$ with $1 \leq l \leq m$ are bounded, all $\xi-$ and $v-$ signals are bounded. Therefore it follows from (7.16) that $\dot{r}_j(t)$ is bounded. From (7.19), $\int_0^\infty r_j^2 dt$ is bounded. This fact together with the boundness of $\dot{r}_j(t)$ proves that $lim_{t\to\infty} r_j(t) = 0$. This completes the proof. ∎

Remark 7.5.1 *The approach taken to adaptively construct the output estimates is quite different from other adaptive observer based techniques proposed in the literature. The main advantage here is that the original systems are no longer required to be detectable. Another attractive feature of the proposed approach is that the output estimates are constructed in a decoupled way, which makes the sensor fault isolation almost trivial.*

Theorem 7.1 serves as a foundation for adaptive fault detection and isolation. Based on it, both adaptive fault detection and adaptive fault isolation become almost trivial, which will be treated in the following subsection for clarity.

7.5.1 Adaptive Sensor Fault Detection and Isolation

If no actuator faults can occur and all sensors work normally, according to Theorem 7.1, all the residuals (i.e., $r_1(t), \cdots, r_p(t)$) tend to zero. Hence, a fault is declared if any of the p residuals is nonzero. Moreover, if no actuator faults can occur, Theorem 7.1 shows that, for any $1 \leq j \leq p$, $lim_{t\to\infty} r_j(t) = 0$ when the $j-$ sensor is normal, regardless whether other sensors are faulty or not. This fact implies that if $lim_{t\to\infty} r_j(t) \neq 0$, the $j-$th sensor is faulty. Based on this observation, if a common threshold is chosen for all residuals, sensor fault isolation can be performed very easily through simply monitoring all the p residuals. The number of residuals that exceed the threshold is the number of sensor faults; and the sensors corresponding to those residuals are faulty. To be specific, adaptive sensor fault detection and isolation is accomplished using the following procedure.

- Step 1 Solve equations (7.14) and (7.15) to obtain $\hat{y}_j$ for $j = 1, 2, \cdots, p$.

- Step 2 For $j = 1, 2, \cdots, p$, compute the residual $r_j(t) = \hat{y}_j - y_j$.

- Step 4 Choose a common threshold ϵ for all $r_j(t), j = 1, 2, \cdots, p$.

- Step 5 For each $1 \leq j \leq p$, compare the residual $r_j(t)$ with the threshold ϵ. If any residual exceeds its corresponding threshold, that sensor is detected to be faulty.

- Step 6 Count the number of residuals that exceed the threshold to determine the number of sensor faults. All the sensor faults are isolated as those that correspond with the residuals.

Remark 7.5.2 *Note that the fault detection is not limited to sensor fault detection. If all sensors are known to be normal, it can also be used to detect actuator faults. If it is not known whether the sensors or actuators are faulty a priori, the algorithm can still be employed to indicate faults.*

Remark 7.5.3 *Unlike fault detection, fault isolation is only applicable to sensor faults. As for actuator faults, a different algorithm has to be developed. Adaptive actuator fault isolation is much more difficult (as can be shown in the algorithm obtained in the next chapter for unknown MISO linear systems, which can only be used to isolate constant actuator faults with the help of fault isolation design functions).*

7.5.2 A Discussion on Threshold Selection

Theoretically speaking, thresholds (ϵ) can be chosen arbitrarily small. However, in practical situations, because other unconsidered uncertainties may exist, too small a

ϵ may lead to too many false alarms. On the other hand, too large ϵ may increase the missed detections. A trade-off has to be made on the choice of a suitable threshold.

In the following, some insights will be provided on threshold selection through investigating the relation between the design constant c_{y_j} and the threshold ϵ.

Denote $M_j(t) = -(\xi_{j(2)} + e_1^T y_j)(\hat{a}_{y_j} - a) + \sum_{j=1}^{m}(v_{l(2)} + e_1^T u_l)(\hat{b}_{jl} - b_{jl}) - \varepsilon_{j,2}$. Using (7.16),

$$r_j(t) = r_j(0)e^{-c_{y_j}t} + e^{-c_{y_j}t}\int_0^t e^{c_{y_j}\tau}M_j(\tau)d\tau. \tag{7.20}$$

Assume that $|M_j(t)| \leq M_{j0}$, it follows from (7.20) that

$$|r_j(t)| \leq (r_j(0) - \frac{M_{j0}}{c_{y_j}})e^{-c_{y_j}t} + \frac{M_{j0}}{c_{y_j}}. \tag{7.21}$$

In steady state, $|r_j(t)| \leq \frac{M_{j0}}{c_{y_j}}$. Based on this inequality, faults cannot be detected if $\epsilon > \frac{M_{j0}}{c_{y_j}}$. With a fixed M_{j0}, the upper bound of $|r_j(t)|$ decreases as c_{y_j} increases, which implies the missed detections might increase as c_{y_j} increases. Therefore, c_{y_j} should be chosen as small as possible to reduce the missed detection rate.

7.6 An Example and Simulation Results

In this section, a linearized model of a tailless jet fighter taken from [113] is first used to illustrate the use of the proposed adaptive FDI scheme to detect and isolate sensor faults. Simulation results are then provided to show the effectiveness of the proposed FDI scheme. The model is described as

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{7.22}$$

where $x = (\alpha, \beta, p, q, r)^T$ with α being the angle of attack, β being the sideslip angle, and p, q, r being the roll rate, pitch rate, and yaw rate respectively, and

$u = (\delta_{el}, \delta_{er}, \delta_{pflap}, \delta_{amtl}, \delta_{amtr})^T$ with $\delta_{el}, \delta_{er}, \delta_{pflap}, \delta_{amtl}, \delta_{amtr}$ being the deflections of left and right elevons, the pitch flap, and the left and right all moving tips, respectively.

The system matrices are defined as follows.

$$A = \begin{pmatrix} -0.6344 & 0.0027 & 0 & 0.9871 & 0 \\ 0 & -0.0038 & 0.1540 & 0 & -0.9876 \\ 0 & -8.2125 & -0.7849 & 0 & 0.1171 \\ -0.5971 & 0 & 0 & -0.5099 & 0 \\ 0 & -0.8887 & -0.0299 & 0 & -0.0156 \end{pmatrix},$$

$$B = \begin{pmatrix} -0.0459 & -0.0459 & -0.0395 & -0.0133 & -0.0133 \\ -0.0047 & 0.0047 & 0 & 0.0031 & -0.0031 \\ 3.783 & -3.783 & 0 & 1.8255 & -1.8255 \\ -2.5115 & -2.5115 & -1.9042 & -0.9494 & -0.9494 \\ -0.0453 & -0.0453 & 0 & -0.2081 & 0.2081 \end{pmatrix},$$

and $C = I_5$.

The MIMO system, (7.22), is decomposed into five MISO systems given below.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y_j(t) &= C_j x(t), j = 1, 2, 3, 4, 5 \end{aligned} \tag{7.23}$$

The transfer functions of the MISO systems are as follows:

$$\begin{aligned} y_1(t) &= \frac{-0.0459s^4 - 2.539s^3 - 2.03s^2 - 0.9955s + 1.028}{a(s)} u_1(t) \\ &+ \frac{-0.0459s^4 - 2.539s^3 - 2.033s^2 - 0.9977s + 1.027}{a(s)} u_2(t) \\ &+ \frac{-0.0395s^4 - 1.9317s^3 - 1.5442s^2 - 0.7550s + 0.7801}{a(s)} u_3(t) \\ &+ \frac{-0.0133s^4 - 0.9546s^3 - 0.7633s^2 - 0.3764s + 0.3878}{a(s)} u_4(t) \\ &+ \frac{-0.0133s^4 - 0.9546s^3 - 0.7659s^2 - 0.3789s + 0.3873}{a(s)} u_5(t), \end{aligned} \tag{7.24}$$

$$
\begin{aligned}
y_2(t) &= \frac{-0.0047s^4 + 0.6181s^3 + 0.8639s^2 + 0.7464s + 0.1415}{a(s)} u_1(t) \\
&+ \frac{0.0047s^4 - 0.5287s^3 - 0.6932s^2 - 0.5865s - 0.0789}{a(s)} u_2(t) \\
&+ \frac{0}{a(s)} u_3(t) \\
&+ \frac{0.0031s^4 + 0.4925s^3 + 0.7781s^2 + 0.6933s + 0.1970}{a(s)} u_4(t) \\
&+ \frac{-0.0031s^4 - 0.4925s^3 - 0.7781s^2 - 0.6933s - 0.1970}{a(s)} u_5(t), \quad (7.25)
\end{aligned}
$$

$$
\begin{aligned}
y_3(t) &= \frac{3.7830s^4 + 4.4333s^3 - 0.1107s^2 - 4.1192s - 3.3640}{a(s)} u_1(t) \\
&+ \frac{-3.7830s^4 - 4.4450s^3 - 0.6363s^2 + 3.2693s + 2.6943}{a(s)} u_2(t) \\
&+ \frac{0}{a(s)} u_3(t) \\
&+ \frac{1.8255s^4 + 2.0738s^3 - 1.6427s^2 - 3.7800s - 3.0058}{a(s)} u_4(t) \\
&+ \frac{-1.8255s^4 - 2.0738s^3 + 1.6427s^2 + 3.7800s + 3.0058}{a(s)} u_5(t), \quad (7.26)
\end{aligned}
$$

$$
\begin{aligned}
y_4(t) &= \frac{-2.5115s^4 - 3.5867s^3 - 2.2785s^2 + 0.3951s + 0.6426}{a(s)} u_1(t) \\
&+ \frac{-2.5115s^4 - 3.5849s^3 - 2.2786s^2 + 0.3961s + 0.6430}{a(s)} u_2(t) \\
&+ \frac{-1.9042s^4 - 2.7160s^3 - 1.7256s^2 + 0.3011s + 0.4863}{a(s)} u_3(t) \\
&+ \frac{-0.9494s^4 - 1.3576s^3 - 0.8633s^2 + 0.1477s + 0.2437}{a(s)} u_4(t) \\
&+ \frac{-0.9494s^4 - 1.3583s^3 - 0.8635s^2 + 0.1494s + 0.2444}{a(s)} u_5(t), \quad (7.27)
\end{aligned}
$$

$$
y_5(t) = \frac{-0.0453s^4 - 0.1965s^3 - 0.7803s^2 - 0.7881s - 0.5235}{a(s)} u_1(t)
$$

$$
\begin{aligned}
&+ \frac{-0.0453s^4 + 0.0214s^3 + 0.5011s^2 + 0.5916s + 0.4187}{a(s)} u_2(t) \\
&+ \frac{0}{a(s)} u_3(t) \\
&+ \frac{-0.2081s^4 - 0.4595s^3 - 0.9585s^2 - 0.7916s - 0.4704}{a(s)} u_4(t) \\
&+ \frac{0.2081s^4 + 0.4595s^3 + 0.9585s^2 + 0.7916s + 0.4704}{a(s)} u_5(t), \qquad (7.28)
\end{aligned}
$$

where $a(s) = s^5 + 1.949s^4 + 2.239s^3 + 0.7881s^2 - 0.09934s - 0.3748$.

To design and test the proposed adaptive FDI scheme, both A and B are assumed to be unknown. Thus, all the parameters in all transfer functions are unknown.

The following filters are needed for adaptive sensor fault detection and isolation:

$$
\begin{aligned}
\dot{\eta}_j &= A_0\eta_j + e_5 y_j, \quad \xi_{ji} = A_0^i \eta, 0 \le i \le 4, \quad \xi_{j5} = -A_0^5 \eta_j \;\; j = 1,2,3,4,5 \\
\dot{\lambda}_j &= A_0\lambda_j + e_5 u_j, \quad \upsilon_{ji} = A_0^i \lambda_j, 0 \le i \le 4, \quad j = 1,2,3,4,5 \qquad (7.29)
\end{aligned}
$$

where A_0 is defined as before, but in $R^{5\times 5}$.

7.6.1 Output Estimates for Fault Detection and Isolation

Based on (7.14) and (7.15) in Section 7.5, the adaptive estimates for the outputs are given as:

$$
\begin{aligned}
\dot{\hat{y}}_j &= -c_{y_j}(\hat{y}_j - y_j) + \xi_{j5,2} - (\xi_{j(2)} + (1\;0\;0\;0\;0) y_j)\hat{a}_{y_j} \\
&+ \sum_{l=1}^{m} (\upsilon_{l(2)} + (1\;0\;0\;0\;0) u_l)\hat{b}_{jl}, j = 1,2,3,4,5 \qquad (7.30)
\end{aligned}
$$

where $\hat{a}_{y_j}$ and $\hat{b}_{jl}, 1 \le j, l \le 5$ are the estimates of a and $b_{jl}, 1 \le j, l \le 5$, and $c_{y_j} > 1$ is a positive design constant.

The update laws for the unknown parameter vectors are given below

$$
\begin{aligned}
\dot{\hat{a}}_{y_j} &= \gamma_{a_{y_j}}(\xi_{j(2)} + (1\;0\;0\;0\;0) y_j)^T(\hat{y}_j - y_j) \\
\dot{\hat{b}}_{jl} &= -\gamma_{b_{jl}}(\upsilon_{l(2)} + (1\;0\;0\;0\;0) u_l)^T(\hat{y}_j - y_j), 1 \le j, l \le 5 \qquad (7.31)
\end{aligned}
$$

where $\gamma_{a_{y_j}}$ and $\gamma_{b_{jl}}, 1 \leq j, l \leq 5$ are positive design constants.

With the help of the above estimates, fault detection and isolation can be performed as in Section 7.5.

7.6.2 Simulation Results

In the simulations, five residuals are monitored to carry out the adaptive fault detection and isolation.

In all simulations, the design parameters are chosen as $k = (17.5\ 120\ 402.5\ 659\ 420)^T$, and all the $c-$ constants and the $\gamma-$ constants are chosen to equal 2. The first three sensors are faulty at and after $5s$. Two types of faults are considered, which are defined as follows.

- Case A The sensors have scaling errors, i.e., $y_j(t) = 0.5x_j(t), j = 1, 2, 3$, where $x_j(t)$ is the real output, and $y_j(t)$ is measured by the sensor.

- Case B The sensors have drifting faults, i.e., $y(t) = x(t) - 0.02([1\ 0\ 0\ 0\ 0] + [0\ 1\ 0\ 0\ 0] + [0\ 0\ 1\ 0\ 0])(t-5)$, where $x(t)$ is the real output vector, and $y(t)$ is the output vector measured by the sensors. The measured output values provided by the first three sensors are drifting away slowly from their real values.

The results for Case A are presented in Fig. 7.1, while those for for Case B are shown in Fig. 7.2, where the threshold is chosen as 0.001. Fig. 7.1 clearly shows that scaling sensor faults can be successfully detected very quickly with the chosen threshold and the faulty ones are the first three. Similar conclusion can be made about the results presented in Fig. 7.2. Clearly, satisfactory FDI performance has been achieved.

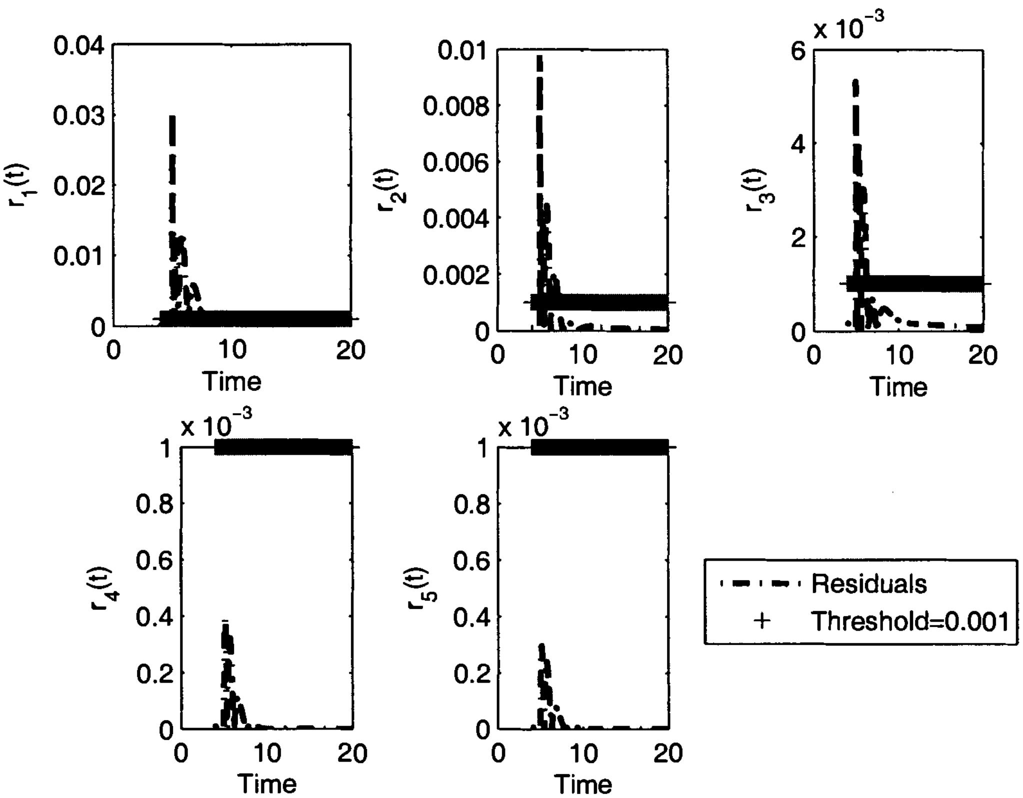

Figure 7.1: Fault detection and isolation for Case A

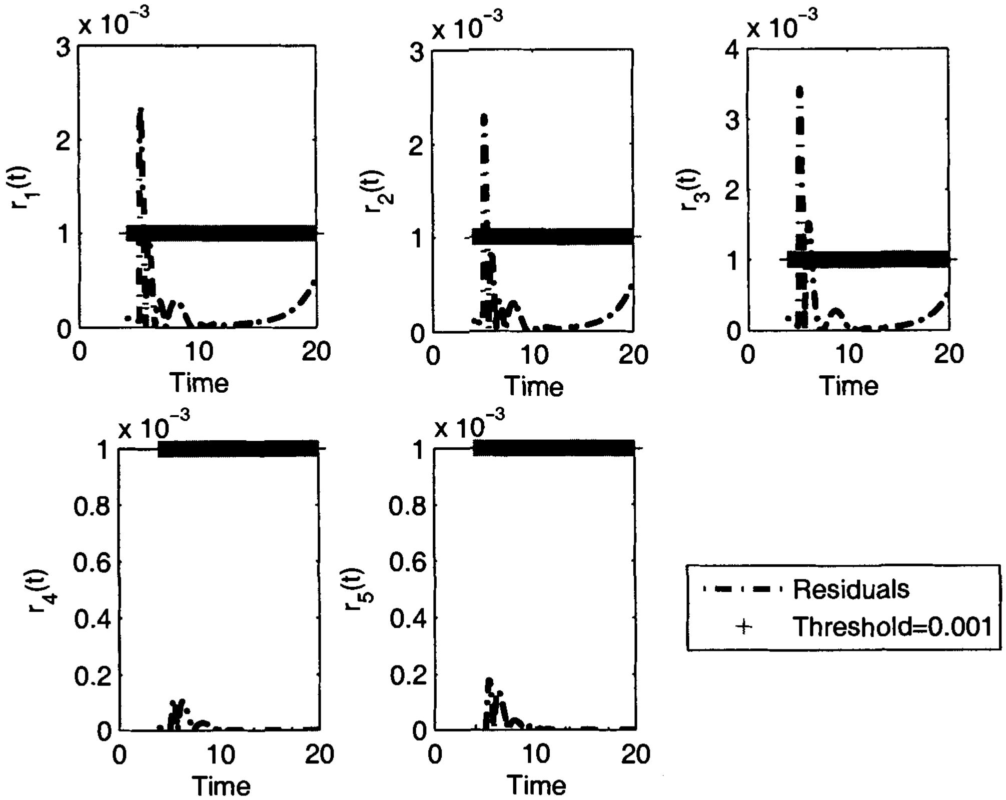

Figure 7.2: Fault detection and isolation for Case B

7.7 Conclusion and Discussions

In this chapter, an adaptive sensor fault detection and isolation problem was studied and solved for the considered class of unknown MIMO linear systems. One novelty of the proposed approach is that by directly designing an output estimator design rather than a state observer for the original systems, the original system is no longer required to be observable or detectable, which is a must for observer based fault diagnosis. Another novelty of the proposed approach is formulating the difficult fault diagnosis problem for a MIMO system as a group of simpler fault diagnosis problems for a group of separate MISO systems. This formulation was achieved by decomposing the MIMO system with p outputs into p MISO systems. The simulation results show that the designed adaptive fault detection and isolation scheme works well in sensor fault detection and isolation for both scaling error sensor faults and drifting sensor faults.

Although the sensor fault detection and isolation problem is solved well for the considered class of unknown MIMO systems, a similar solution of the actuator fault isolation problem has not been achieved for the same class of systems. Thus, one future research topic is to solve the actuator fault isolation for unknown MIMO linear systems. The results in the next chapter represent an effort in this direction. Extensions of these results to nonlinear MIMO systems with unknown parameters are also interesting for future investigation.

Chapter 8

Adaptive Actuator Fault Detection, Isolation, and Accommodation in Unknown Linear Systems

Unlike previous chapters, this chapter solves not only actuator fault detection and isolation problems, but also actuator fault accommodation problem. Because adaptive actuator fault detection, isolation, and accommodation for general unknown MIMO systems are very difficult to solve, this chapter only considers MISO systems with unknown system parameters.

8.1 Introduction

In Chapter 7, adaptive sensor fault detection and isolation problems have been completely solved for a class of MIMO systems with unknown system parameters. Because detailed review on adaptive fault diagnosis was already provied in Chapter 7, only results related to fault accommodation and active fault diagnosis are reviewed.

Recently, results were obtained on fault accommodation by using adaptive approaches (see for example, the work in [132, 133, 134, 135]). Inspired by model reference adaptive control, a series of results which successfully solved the adaptive actuator fault accommodation problems for unknown linear systems with multiple faults were reported (see [136, 137, 138, 139] and the related reference list therein).

Note that one community of the researchers has mostly studied fault detection and isolation, but not the fault accommodation problem, while the fault-tolerant control community has not been concerned with fault diagnosis [140?]. A few works address both FDI and accommodation problems as a whole. The most recent effort in this direction was in [141], where uncertain nonlinear systems with known nominal nonlinearities were considered under the assumptions that the full states are measurable and the faults can be approximated by online approximators, as in [97].

Every one of the fault diagnosis schemes mentioned above uses a passive approach to detect and isolate faults. To enhance the performance of fault detection and isolation, active fault diagnosis is sometimes preferred, where, if permitted, specially designed auxiliary signals or test signals are used. Some works related to active fault diagnosis are in [142] and the references listed therein. One disadvantage of active fault diagnosis is that, in certain cases, the system's performance will be degraded because of the auxiliary signals. Thus, these signals may not be allowed. An example is in automotive applications [143], where passive detection of faults in an Exhaust Gas Recirculating (EGR) valve is a challenging task, whereas, active diagnosis of the valve is relatively simple. However, active means of EGR valve diagnosis generally lead to higher vehicle emission levels and therefore are not desirable. To deal with this disadvantage of active fault diagnosis, a novel idea is proposed in this chapter, which integrates the auxiliary signal design into the controller design. Here, the auxiliary signals do not degrade the system performance when there is no fault, but they enhance fault detection and isolation.

The objective of this chapter is to present new adaptive schemes to solve both FDI and accommodation problems for a class of unknown MISO systems with multiple actuator faults. The method developed in the last chapter of using direct output estimators is used here to perform fault diagnosis. Also proposed is a novel idea of controller design for fault isolation, i.e., some fault isolation design functions are introduced in the controller designed for the healthy system in order to efficiently solve isolation problem when faults occur. To accommodate actuator faults, the faulty actuators are taken off and the control is realized using an adaptive controller which uses the healthy actuators. The controller design for the healthy system, as well as the controller which accommodates faults after they are detected, are designed using the controller backstepping design method [111].

The remainder of the chapter is arranged as follows: In Section 8.2, the system model is introduced and the problem of interest is formulated. In Section 8.3, an output formula is derived for output estimator design. In Section 8.4, by using the controller design for fault isolation and the backstepping design, a controller for the healthy system is proposed. A group of fault isolation design functions, which are only used for fault isolation, are introduced in the controller design. In Section 8.5, a new adaptive fault detection scheme is proposed. In Section 8.6, the actuator fault isolation problem is studied. Unlike fault detection problem, only constant actuator faults are taken into account using the adaptive technique, a group of output estimators are designed for all possible fault combinations in order to isolate multiple faults. The method of designing fault isolation design functions is discussed and a universal isolatability theorem is presented at the end of the section, which forms a solid basis for fault isolation in the studied class of systems. In Section 8.7, an adaptive fault accommodating controller is proposed and designed. Simulation results are presented in Section 8.8 to show the effectiveness of the proposed adaptive FDI and accommodation methods. Finally, concluding remarks are presented in the last section.

8.2 Problem Formulation

Consider MISO systems described as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{8.1}$$

where $x(t)$, $y(t)$, and $u(t)$ are the system state vector, output, and input vector, respectively, and $x(t) \in R^n$, $y(t) \in R$, $u(t) = (u_1(t) \ \cdots \ u_m(t))^T$. A, B and C are all unknown matrices.

The transfer function of a system of the form (8.1) is denoted as

$$y(s) = \sum_{j=1}^{m} G_j(s)u_j(s) \tag{8.2}$$

where $G_j(s)$ with $1 \leq j \leq m$ are defined as

$$G_j(s) = \frac{b_j(s)}{a(s)} = \frac{b_{jp_j}s^{p_j} + \cdots + b_{j1}s + b_{j0}}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}, 1 \leq j \leq m; p_1 \geq \cdots \geq p_m. \tag{8.3}$$

Remark 8.2.1 *Although in practice, MISO systems are not prevalent, practical examples, like the Boeing 747 airplane model used in [139], do exist. Because an MIMO system can be decomposed into several MISO systems, the FDI research on MISO* *systems might provide direct solutions to the FDI problems for some MIMO systems, which could serve as a starting point and a possible new way to solve general MIMO FDI problems.*

Adaptive Fault Detection, Isolation, and Accommodation (FDIA) Problem: Assume that all the parameters in the transfer function of (8.1) are unknown, and also that only actuator faults can occur. Design an adaptive control system such that it can solve the fault detection and isolation problems as well as accommodate adaptively the faults once they occur.

The above problem is solved in three steps. As a first step, adaptive fault detection is performed. After detecting the fault(s), adaptive fault isolation over a fixed time period is performed. Finally, once the fault is identified, the faulty actuators are accommodated adaptively.

Remark 8.2.2 *Based on the above problem formulation, two tasks are performed: adaptive FDI, and adaptive fault accommodation. The adaptive fault accommodation problem was considered in a series of work [136, 137, 138, 139] for systems in state space and transfer function description, but the FDI problem was not addressed. On the other hand, [134] and [135] have reported some work on the problem as described above. Their method can work well for a single actuator fault case in completely known systems. If the system has uncertainties and/or multiple actuator faults occur, whether their method can work or how to extend their results, remains to be seen.*

8.3 An Output Formulae for Output Estimator Design

As in [111], represent the system (8.2) as

$$\begin{aligned} \dot{x} &= \bar{A}x - ay + b_1 u_1 + \cdots + b_m u_m \\ y &= x_1 \end{aligned} \tag{8.4}$$

where

$$\begin{aligned} \bar{A} &= \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, a = \begin{bmatrix} a_{n-1} \\ \vdots \\ a_0 \end{bmatrix}, \\ \bar{b}_j &= \begin{bmatrix} b_{jp_j} \\ \vdots \\ b_{j0} \end{bmatrix}, b_j = \begin{bmatrix} 0_{(n-p_j-1)\times 1} \\ \\ \bar{b}_j \end{bmatrix}, \ 1 \le j \le m. \end{aligned} \tag{8.5}$$

Remark 8.3.1 *When $m = 1, u_1(t) = u(t)$, (8.4) is the same as (2.1) in [111].*

In the remainder of this section, the state estimate will be derived for (8.4). To do so, $u_j, 1 \le j \le m$ and y are filtered with $m+1$ $n-$dimensional filters of the following form:

$$\begin{aligned} \dot{\eta} &= A_0\eta + e_n y \\ \dot{\lambda}_j &= A_0\lambda_j + e_n u_j, \quad 1 \le j \le m \end{aligned} \tag{8.6}$$

where $A_0 = \bar{A} - k(1\ 0\ \cdots\ 0)$ and $k = (k_1 \cdots k_n)^T$ is chosen such that A_0 is Hurwitz. For any $1 \le i \le n$, $e_i = (e_{i,1}, \cdots, e_{i,n})^T \in R^n$ is defined by $e_{i,i} = 1$ and $e_{i,j} = 0$ for $j \ne i$.

After some matrix manipulations, it can be shown that

$$\begin{aligned} a(A_0)e_n &= a - k, \\ b_j(A_0)e_n &= b_j, \quad 1 \le j \le m \end{aligned} \tag{8.7}$$

where $a(A_0)$ and $b_j(A_0), 1 \le j \le m$, are matrix polynomials with $a(s)$ and $b_j(s), 1 \le j \le m$ being defined in (8.3).

Now the state estimate is formed as

$$\hat{x} = \sum_{j=1}^{m} b_j(A_0)\lambda_j - a(A_0)\eta. \tag{8.8}$$

Using (8.4) and (8.6) to (8.8), the estimation error $\varepsilon = x - \hat{x}$ is verified to satisfy $\dot{\varepsilon} = A_0\varepsilon$. Using $\xi_i = A_0^i\eta, 0 \le i \le n-1$, $\xi_n = -A_0^n\eta$ and $v_{ji} = A_0^i\lambda_j, 0 \le i \le p_j, 1 \le j \le m$, (8.8) can be rewritten as

$$x = \xi_n - \sum_{i=0}^{n-1} a_i\xi_i + \sum_{j=1}^{m}\sum_{i=0}^{p_j} b_{ji}v_{ji} + \varepsilon, \dot{\varepsilon} = A_0\varepsilon. \tag{8.9}$$

Clearly, all the $\xi-$ and $v-$signals and their derivatives are explicitly available:

$$\begin{aligned} \xi_n &= -A_0^n\eta, \dot{\xi}_n = A_0\xi_n + ky, \\ \xi_i &= A_0^i\eta, \dot{\xi}_i = A_0\xi_i + e_{n-i}y, 0 \le i \le n-1, \\ v_{ji} &= A_0^i\lambda_j, \dot{v}_{ji} = A_0v_{ji} + e_{n-i}u_j, 0 \le i \le p_j, 1 \le j \le m. \end{aligned} \tag{8.10}$$

Remark 8.3.2 *Based on the expressions in (8.10), the derivatives of $\xi-$ and $v-$signals can be computed by the right-hand side of the differential equations, differentiating the $\xi-$ and $v-$ signals to obtain their derivatives is not necessary.*

Based on (8.9), an output formula, on which output estimators can be designed for fault diagnosis, is derived as

$$\dot{y} = \xi_{n,2} - (\xi_{(2)} + e_1^T y)a + \sum_{j=1}^{m}(v_{j(2)} + V_{sgn,j}u_j)\bar{b}_j + \varepsilon_2 \tag{8.11}$$

where $\xi_n^T = (\xi_{n,1}, \xi_{n,2}, \cdots, \xi_{n,n})$, $\xi_{(2)} = (\xi_{n-1,2}, \cdots, \xi_{0,2})$, $v_{j(2)} = (v_{jp_j,2}, \cdots, v_{j0,2})$, which are computed by (8.10). For notational convenience, $V_{sgn,j} = (1 - sgn(n - p_1 - 1))(1 + sgn(p_j - p_1))(1\ 0 \cdots 0)$ is introduced.

8.4 Controller Design for the Healthy System

In this section, an adaptive controller is designed under the condition that the systems considered are healthy. This controller is used in the healthy system to achieve the desired control objectives. A novel idea is proposed: designing the controller with an outlook towards the FDIA task. Specifically, designing a controller which can assist in fault isolation is advocated. As such, the controller is designed to not only achieve the control objective, but to also provide flexibility which can be used for fault isolation. This task is achieved by introducing a group of fault isolation design functions as additive terms in the redundant inputs to the system.

The control objective is: For a given reference signal $y_r(t)$, design a controller for system (8.2) such that all closed-loop signals are bounded and the output of the system can track the reference signal asymptotically.

All actuators are used to accomplish the control objective as long as they are healthy. Any undefined notation in this section is defined as in Section 8.3.

Denote $Z_m(s) = \sum_{i=1}^{m} b_i(s)$. Note that $p_1 \geq \cdots \geq p_m$, one can denote $Z_m(s) = Z_{m,p_1}s^{p_1} + \cdots + Z_{m,1}s + Z_{m,0}$. Let $Z_m = (0^T_{(n-p_1-1)\times 1}\ \bar{Z}_m^T)^T$ with $\bar{Z}_m^T = [Z_{m,p_1}\ \cdots\ Z_{m,0}]$. To realize the control objective, the following assumptions are needed:

Assumption A81: $Z_m(s)$ is Hurwitz and $Z_{m,p_1} \neq 0$. Moreover, the sign of Z_{m,p_1} is known.

Assumption A82: The reference signal $y_r(t)$ and its first $n - p_1$ derivatives are known and bounded. In addition, $y_r^{(n-p_1)}(t)$ is piecewise continuous.

Remark 8.4.1 *Assumption A81 is equivalent to minimum phase and known high-frequency gain sign requirements, which are basic assumptions in adaptive control of linear systems. Assumption A82 is also required in [111].*

Obviously, (8.4) can be rewritten as

$$\begin{aligned} \dot{x} &= A_0 x + (k-a)y + b_1 u_1 + \cdots + b_m u_m, \\ y &= x_1. \end{aligned} \tag{8.12}$$

Now, let $u_1(t) = u(t)$, $u_2(t) = f_2(t) + u(t)$, $\cdots$, $u_m(t) = f_m(t) + u(t)$, where the functions $f_2(t)$, $\cdots$, $f_m(t)$ are bounded design functions which are mainly introduced for fault isolation. Therefore, they are called *fault isolation design functions* (FIDFs). Because FIDFs are introduced for fault isolation and they will not affect the control performance, the method of choosing these functions is discussed later in the fault isolation section.

The reason that all the controls include the same term $u(t)$ is that the backstepping design will be used, which is only applicable for SISO systems in [111]. By regarding $u(t)$ as a new input, the MISO systems can be viewed as SISO systems, hence, the backstepping design in [111] can be applied.

Remark 8.4.2 *Although different adaptive controllers such as the model reference adaptive controller could be designed for the considered MISO systems, integrating the FIDF design with those adaptive controllers needs further investigation. However, as long as FIDFs can be introduced into those adaptive controllers in the same manner, the FDI problem can be solved in exactly the same way because the FDI strategy does not depend on specific controller design.*

Define $\rho = n - p_1$, first considered is the case $\rho > 1$. It follows from (8.12) that

$$\dot{x} = A_0 x + (k-a)y + b_1 u(t) + \sum_{i=2}^{m} b_i (u + f_i(t))$$

$$
\begin{aligned}
&= A_0 x + (k-a)y + Z_m u(t) + \sum_{i=2}^{m} b_i f_i(t), \\
y &= x_1.
\end{aligned}
\tag{8.13}
$$

In order to design $u(t)$, the following filters are defined:

$$
\begin{aligned}
\dot{\eta} &= A_0\eta + e_n y, \quad \xi_i = A_0^i \eta, 0 \le i \le n-1, \xi_n = -A_0^n \eta, \\
\dot{\lambda}_1 &= A_0\lambda_1 + e_n u, v_{1i} = A_0^i \lambda_1, 0 \le i \le p_1, \\
\dot{\bar{\lambda}}_j &= A_0\bar{\lambda}_j + e_n f_j, \bar{v}_{ji} = A_0^i \bar{\lambda}_j, 0 \le i \le p_j, \quad 2 \le j \le m.
\end{aligned}
\tag{8.14}
$$

If $u(t)$ is treated as $u_1(t)$ and $f_j(t)$ as $u_j(t)$ for $2 \le j \le m$, according to Section 8.3,

$$
\begin{aligned}
x &= \xi_n - \sum_{i=0}^{n-1} a_i \xi_i + \sum_{i=0}^{p_1} Z_{m,i} v_{1i} + \sum_{j=2}^{m} \sum_{i=0}^{p_j} b_{ji} \bar{v}_{ji} + \varepsilon, \\
\dot{\varepsilon} &= A_0 \varepsilon.
\end{aligned}
\tag{8.15}
$$

By defining $v_{1(2)} = (v_{1p_1,2}, \cdots, v_{10,2})$ and $\bar{v}_{j(2)} = (\bar{v}_{jp_j,2}, \cdots, \bar{v}_{j0,2})$ for $2 \le j \le m$, where $v_{1p_1,2}$ is defined as the second component of v_{1p_1}, deriving the following is easy:

$$
\dot{y} = \xi_{n,2} - (\xi_{(2)} + e_1^T y)a + v_{1(2)} \bar{Z}_m + \sum_{j=2}^{m} \bar{v}_{j(2)} \bar{b}_j + \varepsilon_2.
\tag{8.16}
$$

Using (8.12)-(8.16), adaptive backstepping design developed in [111] can be applied to design a controller for (8.12). Because the backstepping design is quite standard now, the adaptive controller for (8.12) is directly given as

$$
\begin{aligned}
u(t) &= -c_\rho z_\rho - d_\rho z_\rho \left(\frac{\partial \alpha_{\rho-1}}{\partial y}\right)^2 - z_{\rho-1} - \beta_\rho \\
&+ \frac{\partial \alpha_{\rho-1}}{\partial y} \omega^T \hat{\theta} + \frac{\partial \alpha_{\rho-1}}{\partial \hat{\theta}} \tau_\rho - \sum_{k=2}^{\rho-1} z_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \Gamma \frac{\partial \alpha_{\rho-1}}{\partial y} \omega,
\end{aligned}
\tag{8.17}
$$

and the update law for the unknown parameters is given as

$$
\begin{aligned}
\dot{\hat{\theta}} &= \tau_\rho, \\
\dot{\hat{Z}}_{m,p_1}^{inv} &= \gamma sgn(Z_{m,p_1}) z_1 [c_1 z_1 + d_1 z_1 + \xi_{n,2} - \dot{y}_r + \bar{\omega}^T \hat{\theta}].
\end{aligned}
\tag{8.18}
$$

The notations in the above equations are defined as

$$z_1(t) = y - y_r, \; z_i(t) = v_{1p_1,i} - \alpha_{i-1}, \;\; 2 \le i \le \rho, \tag{8.19}$$

$$\begin{aligned} \tau_1 &= \Gamma\bar{\omega}z_1(t), \tau_2 = \tau_1 - \Gamma\frac{\partial\alpha_1}{\partial y}\omega z_2 + \Gamma e_{n+1}z_1z_2, \\ \tau_i &= \tau_{i-1} - \Gamma\frac{\partial\alpha_{i-1}}{\partial y}\omega z_i, \end{aligned} \tag{8.20}$$

$$\begin{aligned} \alpha_1 &= -\hat{Z}^{inv}_{m,p_1}[c_1z_1 + d_1z_1 + \xi_{n,2} - \dot{y}_r + \bar{\omega}^T\hat{\theta}], \\ \alpha_2 &= -c_2z_2 - d_2z_2(\frac{\partial\alpha_1}{\partial y})^2 - \beta_2 + \frac{\partial\alpha_1}{\partial y}\omega^T\hat{\theta} - \hat{Z}_{m,p_1}z_1 + \frac{\partial\alpha_1}{\partial\hat{\theta}}\tau_2, \\ \alpha_i &= -c_iz_i - d_iz_i(\frac{\partial\alpha_{i-1}}{\partial y})^2 - z_{i-1} - \beta_i + \frac{\partial\alpha_{i-1}}{\partial y}\omega^T\hat{\theta} \\ &+ \frac{\partial\alpha_1}{\partial\hat{\theta}}\tau_i - \sum_{k=2}^{i-1} z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma\frac{\partial\alpha_{i-1}}{\partial y}\omega, \;\; 3 \le i \le \rho, \end{aligned} \tag{8.21}$$

$$\begin{aligned} \beta_2 &= -k_2v_{1p_1,1} - \frac{\partial\alpha_1}{\partial y}\xi_{n,2} - \sum_{j=1}^{2}\frac{\partial\alpha_1}{\partial y_r^{(j-1)}}y_r^{(j)} \\ &- \frac{\partial\alpha_1}{\partial\xi_n}(A_0\xi_n + ky) - \sum_{i=0}^{n-1}\frac{\partial\alpha_1}{\partial\xi_i}(A_0\xi_i + e_{n-i}y) \\ &- \sum_{i=0}^{p_1-1}\frac{\partial\alpha_1}{\partial v_{1i,2}}(v_{1i,3} - k_2v_{1i,1}) - \sum_{j=2}^{m}\sum_{i=0}^{p_j-1}\frac{\partial\alpha_1}{\partial\bar{v}_{ji,2}}(\bar{v}_{ji,3} - k_2\bar{v}_{ji,1}) \\ &- \frac{\partial\alpha_1}{\partial\hat{Z}^{inv}_{m,p_1}}\gamma sgn(Z_{m,p_1})z_1[c_1z_1 + d_1z_1 + \xi_{n,2} - \dot{y}_r + \bar{\omega}^T\hat{\theta}], \\ \beta_i &= -k_iv_{1p_1,1} - \frac{\partial\alpha_{i-1}}{\partial y}\xi_{n,2} - \sum_{j=1}^{i}\frac{\partial\alpha_{i-1}}{\partial y_r^{(j-1)}}y_r^{(j)} \\ &- \frac{\partial\alpha_{i-1}}{\partial\xi_n}(A_0\xi_n + ky) - \sum_{j=0}^{n-1}\frac{\partial\alpha_{i-1}}{\partial\xi_j}(A_0\xi_j + e_{n-j}y) \\ &- \sum_{l=0}^{p_1-1}\frac{\partial\alpha_{i-1}}{\partial v_{1l,i}}(v_{1l,i+1} - k_iv_{1l,1}) - \sum_{q=1}^{i-1}\sum_{l=0}^{p_1}\frac{\partial\alpha_{q-1}}{\partial v_{1l,l}}(v_{1l,q+1} - k_qv_{1l,1}) \end{aligned}$$

$$
\begin{aligned}
&- \sum_{q=1}^{i}\sum_{j=2}^{m}\sum_{l=0}^{p_j} \frac{\partial \alpha_{q-1}}{\partial \bar{v}_{jl,q}}(\bar{v}_{jl,q+1} - k_q \bar{v}_{jl,1}) \\
&- (1 + sgn(i-\rho))\sum_{j=2}^{m}(1 + sgn(p_j - p_1))\frac{\partial \alpha_{\rho-1}}{\partial \bar{v}_{jp_j,\rho}} f_j \\
&- \frac{\partial \alpha_{i-1}}{\partial \hat{Z}^{inv}_{m,p_1}} \gamma sgn(Z_{m,p_1}) z_1 [c_1 z_1 + d_1 z_1 + \xi_{n,2} - \dot{y}_r + \bar{\omega}^T \hat{\theta}], \quad 3 \le i \le \rho, \qquad (8.22)
\end{aligned}
$$

where $\hat{\theta}$ is an estimate of $\theta = (-a^T \ \bar{Z}_m^T \ \bar{b}_2^T \ \cdots \ \bar{b}_m^T)^T$, and $\hat{Z}^{inv}_{m,p_1}$ is an estimate of $\frac{1}{Z_{m,p_1}}$, $\omega^T = (\xi_{(2)} + e_1^T y \ \ v_{1(2)} \ \ \bar{v}_{2(2)} \ \cdots \ \bar{v}_{m(2)})$ with $e_1 = (1 \ 0 \cdots \ 0)^T \in R^n$, and $\bar{\omega}^T = \omega^T - e_{n+1}^T v_{1p_1,2}$, where e_{n+1} is defined in the same way as e_i but in $R^{n+\sum_{j=1}^{m} p_j + m}$. For all $1 \le i \le \rho$, c_i and d_i are the positive design constants, Γ and γ are the positive design matrix and constant, respectively. With $u(t)$, all the controls can be computed as $u_1(t) = u(t)$, $u_2(t) = f_2(t) + u(t)$, $\cdots$, $u_m(t) = f_m(t) + u(t)$.

Now, the case $\rho = 1$ is considered. The control law takes a much simpler form:

$$
u(t) = \alpha_1 = -\hat{Z}^{inv}_{m,p_1}[c_1 z_1 + d_1 z_1 + \xi_{n,2} - \dot{y}_r + \bar{\omega}^T \hat{\theta}], \qquad (8.23)
$$

and the update laws for the unknown parameters are given as

$$
\begin{aligned}
\dot{\hat{\theta}} &= \tau_1 = \Gamma \bar{\omega} z_1(t), \\
\dot{\hat{Z}}^{inv}_{m,p_1} &= \gamma sgn(Z_{m,p_1}) z_1 [c_1 z_1 + d_1 z_1 + \xi_{n,2} - \dot{y}_r + \bar{\omega}^T \hat{\theta}], \qquad (8.24)
\end{aligned}
$$

where $\omega^T = (\xi_{(2)} + e_1^T y \ \ v_{1(2)} \ \ \psi_{2(2)} \ \cdots \ \psi_{m(2)})$ with $\psi_{j(2)} = \bar{v}_{j(2)} + (1 + sgn(p_j - p_1))(1 \ 0 \cdots 0) f_j$ for $2 \le j \le m$ and $(1 \ 0 \cdots 0)^T \in R^{p_1+1}$, and the definition of $\bar{\omega}$ is the same.

When the controller for the healthy system is applied, the following stability result is obtained.

Theorem 8.1 *When the system has no fault, application of the controller designed for the healthy system results in closed-loop signals that are bounded. Furthermore, the* *output will track the reference signal $y_r^*(t)$ asymptotically; i.e., $lim_{t\to\infty}|y(t) - y_r^*(t)| = 0$.*

Proof. It follows from assumptions A81 and A82 that all the requirements needed by the adaptive backstepping controller in [111] are also satisfied. Based on these observations, the theorem is proved in the same manner as in [111]. ∎

Remark 8.4.3 *Note that f_j for $2 \le j \le m$ affect the controller directly through either $\omega(\rho = 1)$ or $\beta_\rho(\rho > 1)$. Moreover, according to (8.14), they affect the controller design indirectly through $\bar{v}_{ji}$ for $2 \le j \le m, 0 \le i \le p_j$. However, Theorem 8.1 proves that the auxiliary signals introduced in the controller do not lead to the loss of the asymptotic tracking property.*

Remark 8.4.4 *The controller given by (8.17) to (8.22) is a new one because FIDFs are introduced at the controller design step for the purpose of fault isolation.*

8.5 Adaptive Fault Detection

In this section, an adaptive fault detection scheme is provided for any type of actuator fault.

Under a no-fault scenario, (8.11) can be derived and is rewritten as

$$\dot{y} = \xi_{n,2} - (\xi_{(2)} + e_1^T y)a + \sum_{j=1}^{m}(v_{j(2)} + V_{sgn,j}u_j)\bar{b}_j + \varepsilon_2 \tag{8.25}$$

where $u_1(t) = u(t)$, $u_2(t) = f_2(t) + u(t)$, $\cdots$, $u_m(t) = f_m(t) + u(t)$, and for notational convenience, $V_{sgn,j} = (1 - sgn(\rho - 1))(1 + sgn(p_j - p_1))(1\ 0 \cdots 0)$ is introduced.

Remark 8.5.1 *Obviously, $1 - sgn(\rho - 1) \neq 0$ is equivalent to $\rho = 1$, i.e. $p_1 = n - 1$, which means that the terms $V_{sgn,j}u_j, 2 \le j \le m$ disappear when $\rho > 1$. By introducing $V_{sgn,j}u_j$, both the case $\rho > 1$ and $\rho = 1$ can be dealt with in a united way, but not separately as in the controller design.*

In model based fault detection, in order to detect faults, often quantities called residuals are generated and monitored. Residuals large in magnitude would then indicate a fault. To this end, an estimate of the output is constructed based on (8.25). Because the parameter vectors a, $\bar{b}_j, 1 \leq j \leq m$ in (8.25) are unknown and ε_2 is not available, in order to construct an estimate of the output, the unknown parameter vectors have to be replaced by their estimates and the term ε_2 is not considered. Therefore, by using the adaptive technique, an estimate of the output is given as

$$\dot{\hat{y}} = -c(\hat{y} - y) + \xi_{n,2} - (\xi_{(2)} + e_1^T y)\hat{a} + \sum_{j=1}^{m}(v_{j(2)} + V_{sgn,j}u_j)\hat{\bar{b}}_j \tag{8.26}$$

where $\hat{a}$ and $\hat{\bar{b}}_j$ for $1 \leq j \leq m$ are the estimates of a and $\bar{b}_j$ for $1 \leq j \leq m$, and $c > 1$ is a positive design constant. The update laws for the unknown parameter vectors are given as

$$\begin{aligned} \dot{\hat{a}} &= \gamma_a(\xi_{(2)} + e_1^T y)^T(\hat{y} - y), \\ \dot{\hat{\bar{b}}}_j &= -\gamma_{b_j}(v_{j(2)} + V_{sgn,j}u_j)^T(\hat{y} - y), 1 \leq j \leq m, \end{aligned} \tag{8.27}$$

where γ_a and $\gamma_{b_j}, 1 \leq j \leq m$ are positive design constants.

Now, by defining a residual as $r(t) = \hat{y} - y$, the result in Theorem 8.2 is obtained.

Theorem 8.2 *Assume that no faults occur, and that the control law designed for the healthy system is applied. If (8.26) with (8.27) is applied to estimate the output signal,* $lim_{t\to\infty} r(t) = 0$.

Proof. It follows from (8.26) and (8.25) that

$$\dot{r}(t) = -cr(t) - (\xi_{(2)} + e_1^T y)(\hat{a} - a) + \sum_{j=1}^{m}(v_{j(2)} + V_{sgn,j}u_j)(\hat{\bar{b}}_j - \bar{b}_j) - \varepsilon_2. \tag{8.28}$$

where ε satisfies $\dot{\varepsilon} = A_0\varepsilon$ and A_0 is Hurwitz.

Choose a Lyapunov function as

$$V = \frac{1}{2}r^2 + \frac{1}{2\gamma_a}(\hat{a} - a)^T(\hat{a} - a) + \sum_{j=1}^{m} \frac{1}{2\gamma_{b_j}}(\hat{\bar{b}}_j - \bar{b}_j)^T(\hat{\bar{b}}_j - \bar{b}_j) + \varepsilon^T P_0 \varepsilon \quad (8.29)$$

where P_0 is the positive definite solution of $P_0 A_0 + A_0^T P_0 = -I$. By using (8.28) and differentiating the above Lyapunov function with respect to t, the following is obtained:

$$\begin{aligned} \dot{V} &= -cr^2 + \frac{1}{\gamma_a}[\dot{\hat{a}}^T - \gamma_a(\xi_{(2)} + e_1^T y)r](\hat{a} - a) \\ &+ \sum_{j=1}^{m} \frac{1}{\gamma_{b_j}}[\dot{\hat{\bar{b}}}_j^T + \gamma_{b_j}(v_{j(2)} + V_{sgn,j}u_j)r](\hat{\bar{b}}_j - \bar{b}_j) - r\varepsilon_2 - \varepsilon^T\varepsilon. \end{aligned} \quad (8.30)$$

By substituting (8.27) into the above equation, the following is derived:

$$\begin{aligned} \dot{V} &= -cr^2 - r\varepsilon_2 - \varepsilon^T\varepsilon \\ &= -(c-1)r^2 - (r + \frac{1}{2}\varepsilon_2)^2 - (\varepsilon_1^2 + \frac{3}{4}\varepsilon_2^2 + \varepsilon_3^2 + \cdots + \varepsilon_n^2) \\ &\leq 0 \ (since \ \ c > 1). \end{aligned} \quad (8.31)$$

Based on the inequality, $V(t)$ is bounded. Hence $r(t)$, the estimates $\hat{a}$ and $\hat{\bar{b}}_j, 1 \leq j \leq m$ are all bounded. Because $y(t)$ and $u_j(t), 1 \leq j \leq m$ are bounded, all ξ and $v-$ signals are bounded. Hence, the boundness of $\dot{r}(t)$ follows. From (8.31), $\int_0^\infty r^2 dt$ is concluded to be bounded, which together with the boundness of $\dot{r}(t)$ proves that $lim_{t\to\infty} r(t) = 0$. This completes the proof. ∎

According to Theorem 8.2, $r(t) = \hat{y} - y$ tends to zero if no faults occur. Hence, a fault is declared if $r(t)$ is nonzero. Based on this observation, adaptive fault detection is accomplished as follows:

$$\begin{aligned} if\,|r(t)| &\leq \epsilon, \quad no \ \ fault \ \ occurs, \\ if\,|r(t)| &> \epsilon, \quad faults \ \ occur, \end{aligned} \quad (8.32)$$

where ϵ is a pre-specified threshold.

Because $lim_{t\to\infty} r(t) = 0$ has been proved, theoretically ϵ can be chosen arbitrarily small. However, in practical situations, because other unconsidered uncertainties may exist, too small an ϵ may lead to too many false alarms. Conversely, too large ϵ may increase the number of missed detections. A trade-off is needed on the choice of a suitable threshold.

In the following, the relationship between the output estimate design constant c and ϵ is briefly discussed. By denoting $M(t) = -(\xi_{(2)} + e_1^T y)(\hat{a} - a) + \sum_{j=1}^{m}(v_{j(2)} + V_{sgn,j} u_j)(\hat{\bar{b}}_j - \bar{b}_j) - \varepsilon_2$ as well as using (8.28), the following equation is obtained:

$$r(t) = r(0)e^{-ct} + e^{-ct}\int_0^t e^{c\tau} M(\tau)d\tau. \tag{8.33}$$

Assume that $|M(t)| \leq M_0$. Then, from (8.33)

$$|r(t)| \leq (r(0) - \frac{M_0}{c})e^{-ct} + \frac{M_0}{c}. \tag{8.34}$$

In steady state, $|r(t)| \leq \frac{M_0}{c}$. Based on this inequality, faults cannot be detected if $\epsilon > \frac{M_0}{c}$. With fixed M_0, the upper bound of $|r(t)|$ decreases as c increases, which implies the missed detections might increase as c increases. Therefore, c should be chosen as small as possible to reduce the missed detection rate.

8.6 Adaptive Fault Isolation

As in previous chapters and for notational convenience, let $s = \{j_1 \cdots j_h\} \in 2^{S_I}$, where $1 \leq h \leq m-1$ and $j_g \in \{1, 2, \cdots, m\}, 1 \leq g \leq h$.

Unlike the fault detection case, fault isolation cannot be solved for any type of actuator fault using an adaptive technique. To carry out fault isolation, the type of faults must be restricted. To be specific, the actuator faults are assumed to be modelled as

$$u_j(t) = \bar{u}_j, \; t \geq t_j, j \in 1, 2, \cdots, m \tag{8.35}$$

where $\bar{u}_j$ is a constant and t_j is the instant at which the jth actuator has failed.

Remark 8.6.1 *This type of fault can often occur in practice. For example, in aircraft control systems, one may encounter these faults [136, 134, 135]. Because the adaptive control literature has shown that an adaptive technique works satisfactorily for slow time-varying parameters as well, an reasonable expectation is that the adaptive fault detection scheme might also work well for slow time-varying faults. However, the constant actuator fault assumption is quite restrictive. For other types of faults, adaptive fault isolation might be possible if the design technique here is combined with those reported in [97] and [141]. This approach is left as a future research topic.*

Assume at least one actuator is healthy. Suitable formulas for state estimation and the system output are derived for the case when h actuator faults occur, where h satisfies $1 \leq h \leq m-1$. These formulas are used for isolating the faults. Suppose that the j_1th to the j_hth actuator have faults with each $j_g \in S_I$ for $1 \leq g \leq h$. Then, by the definition, $u_{j_g}(t) = \bar{u}_{j_g}$ for $1 \leq g \leq h$. For analysis, $n-$dimensional filters are defined as

$$\dot{\bar{\lambda}}_{j_g} = A_0 \bar{\lambda}_{j_g} + e_n \bar{u}_{j_g}, 1 \leq g \leq h. \tag{8.36}$$

Accordingly, define

$$\bar{v}_{j_g i} = A_0^i \bar{\lambda}_{j_g} \quad 0 \leq i \leq p_{j_g}, 1 \leq g \leq h. \tag{8.37}$$

It is easy to show that $\bar{v}_{j_g i}$ satisfies

$$\dot{\bar{v}}_{j_g i} = A_0 \bar{v}_{j_g i} + e_{n-i} \bar{u}_{j_g}, \; 0 \leq i \leq p_{j_g}, 1 \leq g \leq h. \tag{8.38}$$

Noting that $u_{j_g}(t) = \bar{u}_{j_g}$ for $1 \le g \le h$ and using (8.8), when faults occur, the state estimate can be formed as

$$\bar{\hat{x}} = \sum_{j \neq j_1, \cdots, j \neq j_h} b_j(A_0)\lambda_j + \sum_{g=1}^{h} b_{j_g}(A_0)\bar{\lambda}_{j_g} - a(A_0)\eta. \tag{8.39}$$

By defining the estimation error as $\bar{\varepsilon} = x - \bar{\hat{x}}$, and using (8.36)-(8.39), $\dot{\bar{\varepsilon}} = A_0\bar{\varepsilon}$. Similar to Section 8.3, the following relationship is obtained:

$$x = \xi_n - \sum_{i=0}^{n-1} a_i\xi_i + \sum_{j \neq j_1, \cdots, j \neq j_h} \sum_{i=0}^{p} b_{ji}v_{ji} + \sum_{g=1}^{h}\sum_{i=0}^{p} b_{j_g i}\bar{v}_{j_g i} + \bar{\varepsilon}, \quad \dot{\bar{\varepsilon}} = A_0\bar{\varepsilon}. \tag{8.40}$$

Using (8.36)-(8.40), the actual output satisfies

$$\dot{y} = \xi_{n,2} - (\xi_{(2)} + e_1^T y)a + \sum_{j \neq j_1, \cdots, j \neq j_h} (v_{j(2)} + V_{sgn,j}u_j)\bar{b}_j + \sum_{g=1}^{h} \bar{v}_{j_g(2)}\bar{b}_j + \bar{\varepsilon}_2, \tag{8.41}$$

where $V_{sgn,j}, 2 \le j \le m$ are defined in the last section.

Because all $\bar{u}_{j_g}, 1 \le g \le h$ are constants and A_0 is Hurwitz, $\bar{\lambda}_{j_g} = \theta_{j_g} + o_{j_g}(t)$ for $1 \le g \le h$, where each θ_{j_g} is an unknown constant vector since the fault $\bar{u}_{j_g}$ is unknown, and each $o_{j_g}(t)$ tends to zero exponentially. These facts together with the definition of $\bar{v}_{j_g}$ imply that θ_s and $o_s(t)$ exist such that $\sum_{g=1}^{h} \bar{v}_{j_g(2)}\bar{b}_j = \theta_s + o_s(t)$, where θ_s is an unknown parameter and $o_s(t)$ tends to zero exponentially. Substitute the expression of $\sum_{g=1}^{h} \bar{v}_{j_g(2)}\bar{b}_j$ into (8.41), the following equation is obtained:

$$\dot{y} = \xi_{n,2} - (\xi_{(2)} + e_1^T y)a + \sum_{j \neq j_1, \cdots, j \neq j_h} (v_{j(2)} + V_{sgn,j}u_j)\bar{b}_j + \theta_s + o_s(t) + \bar{\varepsilon}_2 \tag{8.42}$$

As in Section 8.5 and based on (8.42), an estimate for the output of the system as can be constructed as

$$\begin{aligned} \dot{\hat{y}}_s &= -c_s(\hat{y}_s - y) + \xi_{n,2} - (\xi_{(2)} + e_1^T y)\hat{a}_s \\ &+ \sum_{j \neq j_1, \cdots, j \neq j_h} (v_{j(2)} + V_{sgn,j}u_j)\hat{\bar{b}}_j + \hat{\theta}_s, \end{aligned} \tag{8.43}$$

where $\hat{a}_s$ and $\hat{\bar{b}}_j$ are the estimates of a and $\bar{b}_j$ respectively, and $c_s > 1$ is a positive design constant.

The update laws for the unknown parameter vectors are given as

$$\begin{aligned}
\dot{\hat{a}}_s &= \gamma_{a_s}(\xi_{(2)} + e_1^T y)^T(\hat{y}_s - y) \\
\dot{\hat{\bar{b}}}_j &= -\gamma_{b_j}(v_{j(2)} + V_{sgn,j}u_j)^T(\hat{y}_s - y), \quad j \neq j_1, \cdots, j \neq j_h \\
\dot{\hat{\theta}}_s &= -\gamma_\theta(\hat{y}_s - y)
\end{aligned} \tag{8.44}$$

where all $\gamma-$s are positive design constants.

For all $1 \leq h \leq m-1$ and all $s = \{j_1 \cdots j_h\} \in 2^{S_I}$, $\sum_{h=1}^{m-1} C_m^h$ adaptive estimations for the output can be constructed, which are given by (8.43) and (8.44).

Define a group of residuals as $r_s(t) = \hat{y}_s - y$ for all s. For notational convenience, define $s_1 - s_2$ as a set that includes all elements in set s_1 that are not in set s_2. The following result, as described in Theorem 8.3, is obtained.

Theorem 8.3 *Assume that $l \leq m-1$ actuator faults have occurred at the same time, but l is not known. Also assume that the output y and the inputs are bounded. If, for all s, (8.43) with (8.44) are applied to estimate the output signal, and if the j_1^fth to the j_l^fth actuators are faulty and let $s_f = \{j_1^f, \cdots, j_l^f\}$, the following equations hold true:*

- *For s_f,*

$$lim_{t\rightarrow\infty} r_{s_f}(t) = 0. \tag{8.45}$$

- *For $s \neq s_f$,*

$$\begin{aligned}
\dot{r}_s &= -c_s r_s - (\xi_{(2)} + e_1^T y)(\hat{a}_s - a) \\
&+ \sum_{j\neq j_1,\cdots,j\neq j_h} (v_{j(2)} + V_{sgn,j}u_j)(\hat{\bar{b}}_j - \bar{b}_j) + (\hat{\theta}_s - \theta_{s_f} + DE(t)) \\
&- o_{s_f}(t) - \bar{\varepsilon}_2,
\end{aligned} \tag{8.46}$$

where $DE(t) = \sum_{j \in S_{c,s} - S_{\wedge,s}} (v_{j(2)} + V_{sgn,j} u_j)\bar{b}_j - \sum_{j \in S_{c,f} - S_{\wedge,s}} (v_{j(2)} + V_{sgn,j} u_j)\bar{b}_j$ *with* $S_{c,s} = S_I - s$, $S_{c,f} = S_I - s_f$ *and* $S_{\wedge,s} = S_{c,s} \cap S_{c,f}$.

Proof. Let $h = l$ and $j_1 = j_1^f, \cdots, j_l = j_l^f$, it follows from (8.42) that

$$\dot{y} = \xi_{n,2} - (\xi_{(2)} + e_1^T y)a + \sum_{j \neq j_1^f, \cdots, j \neq j_l^f} (v_{j(2)} + V_{sgn,j} u_j)\bar{b}_j + \theta_{s_f} + o_{s_f}(t) + \bar{\varepsilon}_2. \quad (8.47)$$

Using (8.43),

$$\begin{aligned} \dot{\hat{y}}_{s_f} &= -c_{s_f}(\hat{y}_{j_1^f, \cdots, j_l^f} - y) + \xi_{n,2} - (\xi_{(2)} + e_1^T y)\hat{a}_{s_f} \\ &+ \sum_{j \neq j_1^f, \cdots, j \neq j_l^f} (v_{j(2)} + V_{sgn,j} u_j)\hat{\bar{b}}_{j;s_f} + \hat{\theta}_{s_f} \end{aligned} \quad (8.48)$$

where $\hat{a}_{s_f}$ and $\hat{\bar{b}}_{j;s_f}, 1 \leq j \leq m$ are the estimates of a and $\bar{b}_j, 1 \leq j \leq m$, and $c_{s_f} > 1$ is a positive design constant.

It follows from (8.44) that

$$\begin{aligned} \dot{\hat{a}}_{s_f} &= \gamma_{a_{s_f}} (\xi_{(2)} + e_1^T y)^T (\hat{y}_{s_f} - y), \\ \dot{\hat{\bar{b}}}_{j;s_f} &= -\gamma_{b_{j;s_f}} (v_{j(2)} + V_{sgn,j} u_j)^T (\hat{y}_{s_f} - y), \;\; j \neq j_1^f, \cdots, j \neq j_l^f, \\ \dot{\hat{\theta}}_{s_f} &= -\gamma_\theta (\hat{y}_{s_f} - y), \end{aligned} \quad (8.49)$$

where $\gamma_{a_{s_f}}$ and $\gamma_{b_{j;s_f}}$ are positive design constants.

Now, by defining a residual as $r_{s_f}(t) = \hat{y}_{s_f} - y$, it follows from (8.48) and (8.49) that

$$\begin{aligned} \dot{r}_{s_f} &= -c_{s_f} r_{s_f} - (\xi_{(2)} + e_1^T y)(\hat{a}_{s_f} - a) \\ &+ \sum_{j \neq j_1^f, \cdots, j \neq j_l^f} (v_{j(2)} + V_{sgn,j} u_j)(\hat{\bar{b}}_{j;s_f} - \bar{b}_j) \\ &+ (\hat{\theta}_{s_f} - \theta_{s_f}) - o_{s_f}(t) - \bar{\varepsilon}_2. \end{aligned} \quad (8.50)$$

Choose a Lyapunov function as

$$\begin{aligned} V &= \frac{1}{2}r_{s_f}^2 + \frac{1}{2\gamma_{a_{s_f}}}(\hat{a}_{s_f} - a)^T(\hat{a}_{s_f} - a) \\ &+ \sum_{j\neq j_1^f,\cdots,j\neq j_l^f} \frac{1}{2\gamma_{b_{j;s_f}}}(\hat{\bar{b}}_{j;s_f} - \bar{b}_j)^T(\hat{\bar{b}}_{j;s_f} - \bar{b}_j) \\ &+ \frac{1}{2\gamma_\theta}(\hat{\theta}_{s_f} - \theta_{s_f})^T(\hat{\theta}_{s_f} - \theta_{s_f}). \end{aligned} \tag{8.51}$$

By differentiating V with respect to t and using (8.50) and (8.49),

$$\begin{aligned} \dot{V} &= -c_{s_f}r_{s_f}^2 - r[o_{s_f}(t) + \bar{\varepsilon}_2] \\ &\leq -(c_{s_f} - 1)r_{s_f}^2 + \frac{[o_{s_f}(t) + \bar{\varepsilon}_2]^2}{4}. \end{aligned} \tag{8.52}$$

Since, by definition, both $o_{s_f}(t)$ and $\bar{\varepsilon}_2$ tend to zero exponentially, they are bounded. This fact together with (8.52) implies that V and hence r_{s_f}, $\hat{a}_{s_f}$, and $\hat{\bar{b}}_{j;s_f}$ are all bounded. This boundness together with the boundness of y and the healthy system inputs proves that $\dot{r}_{s_f}$ is also bounded. From the exponential convergence of $o_{s_f}(t)$ and $\bar{\varepsilon}_2$, $\int_0^\infty [o_{s_f}(t) + \bar{\varepsilon}_2]^2 dt$ is bounded. Using this fact and (8.52), $\int_0^\infty r_{s_f}^2 dt$ is bounded. Noting that $\dot{r}_{s_f}$ is bounded, it follows immediately that

$$lim_{t\to\infty} r_{s_f}(t) = 0. \tag{8.53}$$

Then, for any $s \neq s_f$, it follows from (8.43) and (8.47) that

$$\begin{aligned} \dot{r}_s &= -c_s r_s - (\xi_{(2)} + e_1^T y)(\hat{a}_s - a) \\ &+ \sum_{j\neq j_1,\cdots,j\neq j_h} (v_{j(2)} + V_{sgn,j}u_j)\hat{\bar{b}}_j - \sum_{j\neq j_1^f,\cdots,j\neq j_l^f} (v_{j(2)} + V_{sgn,j}u_j)\bar{b}_j \\ &+ (\hat{\theta}_s - \theta_{s_f}) - o_{s_f}(t) - \bar{\varepsilon}_2. \end{aligned} \tag{8.54}$$

According to the definitions of $S_{c,s}$, $S_{c,f}$ and $S_{\wedge,s}$, clearly

$$\dot{r}_s = -c_s r_s - (\xi_{(2)} + e_1^T y)(\hat{a}_s - a)$$

$$\begin{aligned}&+\sum_{j\in S_{c,s}}(v_{j(2)}+V_{sgn,j}u_j)\hat{\bar{b}}_j-\sum_{j\in S_{c,f}}(v_{j(2)}+V_{sgn,j}u_j)\bar{b}_j\\&+(\hat{\theta}_s-\theta_{s_f})-o_{s_f}(t)-\bar{\varepsilon}_2.\end{aligned}\tag{8.55}$$

Note, $S_{c,s}=S_{\wedge,s}\cup(S_{c,s}-S_{\wedge,s})$ and $S_{c,f}=S_{\wedge,s}\cup(S_{c,f}-S_{\wedge,s})$, the following is obtained:

$$\begin{aligned}\dot{r}_s&=-c_sr_s-(\xi_{(2)}+e_1^Ty)(\hat{a}_s-a)\\&+\sum_{j\in S_{c,s}}(v_{j(2)}+V_{sgn,j}u_j)(\hat{\bar{b}}_j-\bar{b}_j)\\&+\sum_{j\in S_{c,s}-S_{\wedge,s}}(v_{j(2)}+V_{sgn,j}u_j)\bar{b}_j-\sum_{j\in S_{c,f}-S_{\wedge,s}}(v_{j(2)}+V_{sgn,j}u_j)\bar{b}_j\\&+(\hat{\theta}_s-\theta_{s_f})-o_{s_f}(t)-\bar{\varepsilon}_2\end{aligned}\tag{8.56}$$

By the definition of $DE(t)$, (8.56) is the same as (8.46). ∎

Based on (8.46) in the above theorem, Theorem 8.4 provides a result on universal isolability.

Theorem 8.4 *Under the conditions of Theorem 8.3, no matter how many constant faults have occurred, they can always be isolated by the proper choice of fault isolation design functions; i.e.,* $f_j(t), 2\le j\le m$.

Proof. From (8.46), r_s does not tend to zero if $DE(t)$ is not vanishing and changes with time fast enough so that $\hat{\theta}_s$ cannot track it asymptotically. By definition, $DE(t)=\sum_{j\in S_{c,s}-S_{\wedge,s}}(v_{j(2)}+V_{sgn,j}u_j)\bar{b}_j-\sum_{j\in S_{c,f}-S_{\wedge,s}}(v_{j(2)}+V_{sgn,j}u_j)\bar{b}_j$. For any $s\neq s_f$, $S_{c,s}-S_{\wedge,s}$ and $S_{c,f}-S_{\wedge,s}$ cannot be both empty, but $S_{c,s}-S_{\wedge,s}\cap S_{c,f}-S_{\wedge,s}$ is empty. These facts together with the definitions of $v_{j(2)}$ and FDIFs imply that there are infinite choices of FDIFs which can make $DE(t)$ a non-vanishing time-varying signal that changes with time fast enough. Therefore, the FDIFs can always be chosen such that only the residual corresponding to the fault combination can tend to zero, and all other residuals r_{s_f} do not. By checking which residual tends to zero, isolating the faulty actuators can always be achieved. ∎

The above theorem states that any $l < m$ actuator faults which occur at the same time can always be isolated as long as the FDIFs are chosen properly. As argued in the above proof, to guarantee actuator fault isolation, the FIDFs $(f_j(t), 2 \leq j \leq m)$ have to be chosen such that for any $s \neq s_f$, the corresponding $DE(t)$ does not vanish and changes with time fast enough so that $\hat{\theta}_s$ cannot track it asymptotically.

In order to determine a method for choosing $f_j(t), 2 \leq j \leq m$, the definitions of $v_j, 2 \leq j \leq m$ are rewritten as

$$\begin{aligned} \dot{\lambda}_j &= A_0\lambda_j + e_n(u(t) + f_j(t)), \\ v_{ji} &= A_0^i\lambda_j, 0 \leq i \leq p_j, 2 \leq j \leq m. \end{aligned} \tag{8.57}$$

Note that $DE(t)$ depends on $v_{ji}, 0 \leq i \leq p_j, 2 \leq j \leq m$. To make it a non-vanishing time-varying signal with suitably fast changing rate, $v_{ji}, 0 \leq i \leq p_j, 2 \leq j \leq m$ has to change fast enough and be non-vanishing. To this end, from (8.57), $f_j(t), 2 \leq j \leq m$ have to be chosen as non-vanishing time-varying functions with a rate that changes fast enough. This fact excludes the possibility to choose $f_j(t), 2 \leq j \leq m$ as either constant or slow time-varying signals.

Based on the discussions above, $f_j(t), 2 \leq j \leq m$ have to be chosen as non-vanishing time-varying functions with fast enough changing rate. Actually, infinite choices of $f_j(t), 2 \leq j \leq m$ exist, which means that they can be chosen almost arbitrarily as long as they are non-vanishing time-varying functions with fast enough changing rate. Three choices are given below as examples:

$$\begin{aligned} (a)\ f_j(t) &= df_j\ sin(\omega_j t), 2 \leq j \leq m; \\ (b)\ f_j(t) &= df_j\ cos(\omega_j t), 2 \leq j \leq m; \end{aligned}$$

$$(c)\ f_j(t) = df_j\ sin(\omega_{1j}t)\ cos(\omega_{2j}t), 2 \leq j \leq m, \tag{8.58}$$

where $df_j, 2 \leq j \leq m$ are signal magnitudes that can be chosen, and $\omega_j, \omega_{1j}, \omega_{2j}, 2 \leq j \leq m$ are frequencies that are chosen suitably large and are different from one another. For example, $\omega_j \neq \omega_k$ if $j \neq k$.

Remark 8.6.2 *Because the filters designed act like low-pass filters, FIDFs with too large of $\omega_j, \omega_{1j}, \omega_{2j}, 2 \leq j \leq m$ will be blocked by the filters, and thus will not have much effect on fault isolation. In this case, the scenario is similar to choosing all FIDFs as zeros. Moreover, too large of $\omega_j, \omega_{1j}, \omega_{2j}, 2 \leq j \leq m$ may also cause a stability problem of the closed-loop systems especially in presence of faults. Therefore, for the purpose of fault isolation, $\omega_j, \omega_{1j}, \omega_{2j}, 2 \leq j \leq m$ should be chosen suitably large in order to ensure $f_j(t), 2 \leq j \leq m$ are non-vanishing time-varying functions with fast enough changing rate.*

Remark 8.6.3 *If $u_1(t) = \cdots = u_m(t) = u$ is taken for healthy system controller design, as in [138], and if $b_1 = \cdots = b_m$, $DE(t) = 0$ for all t. In such a case, all residuals tend to zero; hence, fault isolation cannot be achieved. By introducing FDIFs, this difficulty is easily addressed. Simulation studies show that there may be other situations which can make $DE(t)$ tend to zero when the adaptive technique is applied. In such cases, the use of FDIFs is also a good approach to solve the isolation problems.*

The adaptive fault isolation is performed by following the steps of the following algorithm:

- Step 1 Choose proper FDIFs which are different from one another. For example, those in (8.58).
- Step 2 Solve equations (8.43) and (8.44) to obtain $\hat{y}_s$ for all sets in 2^{S_I}.

- Step 3 For $1 \leq h \leq m-1$, compute $r_s(t) = \hat{y}_s - y$.
- Step 4 Choose a constant T.
- Step 5 Compare all residuals on the time interval $[dt, dt+T]$, where dt is the time instant when the faults are detected.
- Step 6 The residual which tends to zero corresponds to the fault combination, which determines both the number of actuator faults and the faulty actuators.

Remark 8.6.4 *In simulation studies, it was discovered that in order to avoid erroneous isolation of the faulty actuators, the residuals must be compared over a certain time interval to determine which one has a tendency to zero. Too small a T may increase the rate of wrong isolation decisions. Too large a T is often undesirable because the control system needs to return to normal performance as quick as possible.*

Remark 8.6.5 *If the same technique employed in Section 8.5 is used, similar relationship is established between the adaptive output estimator design parameters (i.e., $c_s, s \in 2^{S_I}$) and the residuals.*

8.7 Adaptive Fault Accommodation

The task of actuator fault accommodation is to achieve the same control objective even when actuator faults occur. The same controller structure as that of the controller for the healthy system is used for the adaptive accommodating controller. The design of the adaptive accommodating controller is based on the exact isolation of the faults. The idea is to take the faulty actuators out of operation and to use the remaining actuators to achieve the control objective.

Assume that l actuator faults have been isolated and the faulty combination is $j_1 \cdots j_l$. Denote the index of the healthy actuators as $j_{l+1}, \cdots, j_m$. For simplicity, and with a slight abuse of notation, denote $u_{j_{l+i}}$ and $b_{j_{l+i}}$ as u_i and b_i, respectively for $1 \leq i \leq m-l$ and let $u_1 = u$ and $u_i = u + f_i(t)$ for $2 \leq i \leq m-l$. Then, by replacing m with $m-l$ and defining all notations in the no-fault controller given in Section 8.4 accordingly, the adaptive fault accommodation controller can be given exactly by (8.17)-(8.22) for $\rho > 1$ and (8.23) with (8.24) for $\rho = 1$, which is not be repeated here.

The controller given by (8.17)-(8.22) or (8.23) with (8.24) can always be designed no matter the number of faulty actuators as long as at least one actuator is healthy. The problem is under what assumptions it can still achieve the control objective. According to Section 8.4, if Assumption $A81$ is still satisfied with the healthy actuators, the control objective can be maintained by using the adaptive fault accommodation controller. This assumption can be further weakened by assuming that $A81$ is satisfied by at least one subset of healthy actuators.

Remark 8.7.1 *Directly switching off faulty actuators and turning on the adaptive accommodation controller may result in non-smooth system behavior. One way to overcome this problem is to choose the controller design constants to ensure $u(t)$ continuous at the switching point.*

Remark 8.7.2 *Note that the system has to work under faulty conditions until all faults are isolated and the adaptive accommodating controller is turned on. Also, the adaptive fault accommodation scheme heavily depends on the FDI scheme of this chapter. If faults cannot be isolated, other accommodation schemes, for example those proposed in [136, 138, 137, 139], should be explored. A better adaptive FDIA scheme for MISO systems may be created by combining the FIDF based FDI scheme in this chapter with the adaptive accommodation controller proposed in [136, 138, 137, 139].*

8.8 An Example and Simulation Results

In this section, it is first illustrated on how to use the proposed FDI scheme to detect and isolate actuator faults and how to design adaptive accommodating controller for the lateral dynamics model of a Boeing airplane used in [139]. Some simulation results are then provided to show the effectiveness of the proposed FDI scheme and the adaptive accommodating controller. In horizontal flight at $40,000ft$ and a nominal forward speed of $774ft/sec$, the linearized lateral dynamics model of a Boeing airplane used in [139] is described as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{8.59}$$

where $x(t) = (\beta, y_r, p, \phi)^T$ with β, y_r, p, ϕ being the side-slip angle, the yaw rate, the roll rate, and the roll angle respectively. $u = (u_1, u_2, u_3)^T$ is the control vector, which consists of three control signals to represent three rudder servos: $\delta_{r1}, \delta_{r2}, \delta_{r3}$. A, B and C are

$$A = \begin{pmatrix} -0.0558 & -0.9968 & 0.0802 & 0.0415 \\ 0.598 & -0.115 & -0.0318 & 0 \\ -3.05 & 0.388 & -0.465 & 0 \\ 0 & 0.0805 & 1 & 0 \end{pmatrix}, B = \begin{pmatrix} 0.00729 & 0.01 & 0.005 \\ -0.475 & -0.5 & -0.3 \\ 0.153 & 0.2 & 0.1 \\ 0 & 0 & 0 \end{pmatrix},$$

and $C = (0 \quad 1 \quad 0 \quad 0)$.

This MISO system has three inputs and one output, and its transfer function is of the form:

$$\begin{aligned} y(t) &= \frac{-0.475s^3 - 0.2479s^2 - 0.1187s - 0.05633}{s^4 + 0.6358s^3 + 0.9389s^2 + 0.5116s + 0.003674} u_1(t) \\ &+ \frac{-0.5s^3 - 0.2608s^2 - 0.1223s - 0.05832}{s^4 + 0.6358s^3 + 0.9389s^2 + 0.5116s + 0.003674} u_2(t) \\ &+ \frac{-0.3s^3 - 0.1564s^2 - 0.07467s - 0.03549}{s^4 + 0.6358s^3 + 0.9389s^2 + 0.5116s + 0.003674} u_3(t). \end{aligned} \tag{8.60}$$

To test the proposed adaptive FDIA schemes, all parameters are assumed unknown and the signs of b_{13}, b_{23}, and b_{33} are known to be negative. It is easy to check Assumption A81 is true, $p_1 = p_2 = p_3 = 3$, and $\rho = 4 - p_1 = 1$.

The following filters are needed for the healthy controller design, adaptive fault detection and isolation, and adaptive fault accommodation.

$$\begin{aligned}
\dot{\eta} &= A_0\eta + e_4 y, \quad \xi_i = A_0^i\eta, 0 \le i \le 3, \quad \xi_4 = -A_0^4\eta, \\
\dot{\lambda}_1 &= A_0\lambda_1 + e_4 u, v_{1i} = A_0^i\lambda_1, 0 \le i \le 3, \\
\dot{\bar{\lambda}}_j &= A_0\bar{\lambda}_j + e_4 f_j, \bar{v}_{ji} = A_0^i\bar{\lambda}_j, 0 \le i \le 3, j = 2, 3,
\end{aligned} \tag{8.61}$$

where A_0 is defined as before but in $R^{4\times4}$, u is defined later, and f_2 and f_3 are determined using (8.58).

8.8.1 Healthy Controller Design

As in [139], two reference signals are chosen as $y_{r,1}(t) = 0.01 - 0.01e^{-3t}$ and $y_{r,2}(t) = 0.03(3sin(0.1t) - 0.1cos(0.1t) + 0.1e^{-3t})/9.01$. Obviously, assumption A82 is satisfied for both reference signals.

Because $\rho = 1$, according to 8.4, the following controller for the healthy system is derived:

$$u(t) = -\hat{Z}_{3,3}^{inv}[c_1 z_1 + d_1 z_1 + \xi_{4,2} - \dot{y}_r + \bar{\omega}^T\hat{\theta}], \tag{8.62}$$

with the update laws for the unknown parameters given as

$$\begin{aligned}
\dot{\hat{\theta}} &= \Gamma\bar{\omega}z_1(t), \\
\dot{\hat{Z}}_{3,3}^{inv} &= -\gamma z_1[c_1 z_1 + d_1 z_1 + \xi_{4,2} - \dot{y}_r + \bar{\omega}^T\hat{\theta}],
\end{aligned} \tag{8.63}$$

where $\hat{\theta}$ is an estimate of $\theta = (-a^T \ Z_3^T \ b_2^T \ \bar{b}_3^T)^T$ with $a^T = (a_3 \ a_2 \ a_1 \ a_0)$, $b_j^T = (b_{j3} \ b_{j2} \ b_{j1} \ b_{j0}), 1 \le j \le 3$ and $Z_3 = b_1 + b_2 + b_3$. $\hat{Z}_{3,3}^{inv}$ is an estimate of $\frac{1}{Z_{3,3}}$ with

$Z_{3,3} = b_{13} + b_{23} + b_{33}$ and $sgn(Z_{3,3}) = -1$. $\omega^T = [\xi_{(2)} + e_1^T y \; \upsilon_{1(2)} \; \psi_{2(2)} \; \psi_{3(2)}]$ with $\psi_{j(2)} = \bar{\upsilon}_{j(2)} + (1 \;\; 0 \;\; 0 \;\; 0)f_j$ for $j = 2, 3$ and $\bar{\omega}^T = \omega^T - e_5^T \upsilon_{1p_1,2}$ with e_5 defined in the same way as e_i but in R^{16}. c_1 and d_1 are positive design constants, Γ and γ are the positive design matrix and constant, respectively. With $u(t)$, all the controls are computed as $u_1 = u(t)$, $u_2 = f_2(t) + u(t)$, $u_3 = f_3(t) + u(t)$.

8.8.2 Construction of an Output Estimate for Fault Detection

Based on (8.26) and (8.27) in Section 8.5, an adaptive output estimate is given as

$$\dot{\hat{y}} = -c(\hat{y} - y) + \xi_{4,2} - (\xi_{(2)} + (1\;0\;0\;0)y)\hat{a} + \sum_{j=1}^{3}(\upsilon_{j(2)} + (1\;0\;0\;0)u_j)\hat{\bar{b}}_j, \qquad (8.64)$$

where $c > 1$ is a positive design constant, $\hat{a}$ and $\hat{\bar{b}}_j, 1 \leq j \leq 3$ are the estimates of a and $\bar{b}_j, 1 \leq j \leq 3$, which are updated as

$$\begin{aligned} \dot{\hat{a}} &= \gamma_a(\xi_{(2)} + (1\;0\;0\;0)y)^T(\hat{y} - y), \\ \dot{\hat{\bar{b}}}_j &= -\gamma_{b_j}(\upsilon_{j(2)} + (1\;0\;0\;0)u_j)^T(\hat{y} - y), 1 \leq j \leq 3, \end{aligned} \qquad (8.65)$$

where γ_a and $\gamma_{b_j}, 1 \leq j \leq 3$ are positive design constants.

By defining $r(t) = \hat{y} - y$, the fault detection is performed by using the logic given in Section 8.5.

8.8.3 Construction of Output Estimates for Adaptive Fault Isolation

Based on (8.43) and (8.44) in Section 8.6, the adaptive estimates of the output used for fault isolation are given as

$$\dot{\hat{y}}_s = -c_s(\hat{y}_s - y) + \xi_{4,2} - (\xi_{(2)} + (1\;0\;0\;0)y)\hat{a}_s$$

$$+ \sum_{j \neq j_1, \cdots, j \neq j_h} (v_{j(2)} + (1\ 0\ 0\ 0)u_j)\hat{\bar{b}}_j + \hat{\theta}_s \tag{8.66}$$

where $\hat{a}_s$ and $\hat{\bar{b}}_j$ are the estimates of a and $\bar{b}_j$ respectively, and $c_s > 1$ is a positive design constant. The update laws for the unknown parameter vectors are given as

$$\begin{aligned} \dot{\hat{a}}_s &= \gamma_{a_s}(\xi_{(2)} + (1\ 0\ 0\ 0)y)^T(\hat{y}_s - y) \\ \dot{\hat{\bar{b}}}_j &= -\gamma_{b_j}(v_{j(2)} + (1\ 0\ 0\ 0)u_j)^T(\hat{y}_s - y), \quad j \neq j_1, \cdots, j \neq j_h \\ \dot{\hat{\theta}}_s &= -\gamma_\theta(\hat{y}_s - y) \end{aligned} \tag{8.67}$$

where all γ−s are positive design constants.

For this example, note that $h = 1$ and $h = 2$. $C_3^1 + C_3^2 = 6$ output estimates are needed in total. With the help of the above estimates, fault isolation is performed as in Section 8.6.

8.8.4 Adaptive Accommodating Controller

For this example, if there are only fewer than 2 actuator faults, Assumption A81 is always satisfied after switching off the faulty controllers. Therefore, as discussed in Section 8.7, the adaptive accommodation controller can always be designed .

8.8.5 Simulation Results

In the simulations, initially a controller designed for the healthy system is being used and the adaptive fault detection scheme is also running in parallel in order to detect possible faults. After faults are detected, the adaptive isolation scheme is switched on for a time period to isolate the faults. After the faults have been successfully isolated, the faulty actuators are turned off and the adaptive accommodating controller is switched on.

In all simulations, the initial conditions are given as $x(0) = (0\ 0.005\ 0\ 0)^T$, $\eta(0) = \lambda_1(0) = \bar{\lambda}_2(0) = \bar{\lambda}_2(0) = (0\ 0\ 0\ 0)^T$, $\hat{a}(0) = (0.6\ -0.9\ -0.5\ -0.003)^T$, $\hat{b}_1(0) = (-0.43\ -0.23\ -0.1\ -0.05)^T$, $\hat{b}_2(0) = (-0.5\ -0.2\ -0.1\ -0.05)^T$, $\hat{b}_3(0) = (-0.3\ -0.15\ -0.075\ -0.035)^T$, $\hat{Z}_3(0) = (-1.2\ -0.6\ -0.23\ -0.12)^T$, $\hat{Z}_{3,3}^{inv}(0) = -1$, $\hat{y}(0) = 0.005$, $\hat{y}_1(0) = \hat{y}_2(0) = \hat{y}_3(0) = 0.005$, $\hat{y}_{12}(0) = \hat{y}_{13}(0) = \hat{y}_{23}(0) = 0.005$, $\hat{a}_1(0) = \hat{a}_2(0) = \hat{a}_3(0) = \hat{a}_{12}(0) = \hat{a}_{13}(0) = \hat{a}_{23}(0) = \hat{a}(0)$, $\hat{\hat{b}}_j = \hat{\hat{b}}_{j;1}(0) = \hat{\hat{b}}_{j;2}(0) = \hat{\hat{b}}_{j;3}(0) = \hat{\hat{b}}_{j;12}(0) = \hat{\hat{b}}_{j;13}(0) = \hat{\hat{b}}_{j;23}(0) = \hat{b}_j(0)$ for $1 \le j \le 3$.

The design parameters are chosen as $k = (3.15\ 3.65\ 1.8375\ 0.3375)^T$, $c_1 = d_1 = 100$, $c = c_1 = c_2 = c_3 = c_{12} = c_{13} = c_{23} = 10$, $\Gamma = 2I$, and all $\gamma-$ constants are equal to 2. Additionally, $f_2(t) = 0.02sin(3t)$ and $f_3(t) = 0.02sin(2t)$.

In all simulations, the second actuator is faulty at, and after, $50s$. For simplicity, four cases are defined as follows:

- Case A The reference signal is chosen as $y_{r,1}(t)$ and a constant fault is considered.
- Case B The reference signal is chosen as $y_{r,1}(t)$ and a time-varying fault is considered.
- Case C The reference signal is chosen as $y_{r,2}(t)$ and a constant fault is considered.
- Case D The reference signal is chosen as $y_{r,2}(t)$ and a time-varying fault is considered.

For fault detection, all four cases are considered with $u_2(t) = 0.03$ as a constant fault and $u_2(t) = 0.5(u(t) + f_2(t))$ as a time-varying fault. In order to show the power of FIDFs, two groups of fault detection simulation results are provided in Fig. 8.1 and Fig. 8.2. Fig. 8.1 shows the results when FIDFs are used, while Fig. 8.2 shows the results when FIDFs are not used (i.e., when $f_2(t) = f_3(t) = 0$). In both figures, plots labelled with A, B, C and D are results for Case A, Case B, Case C, and Case D, respectively.

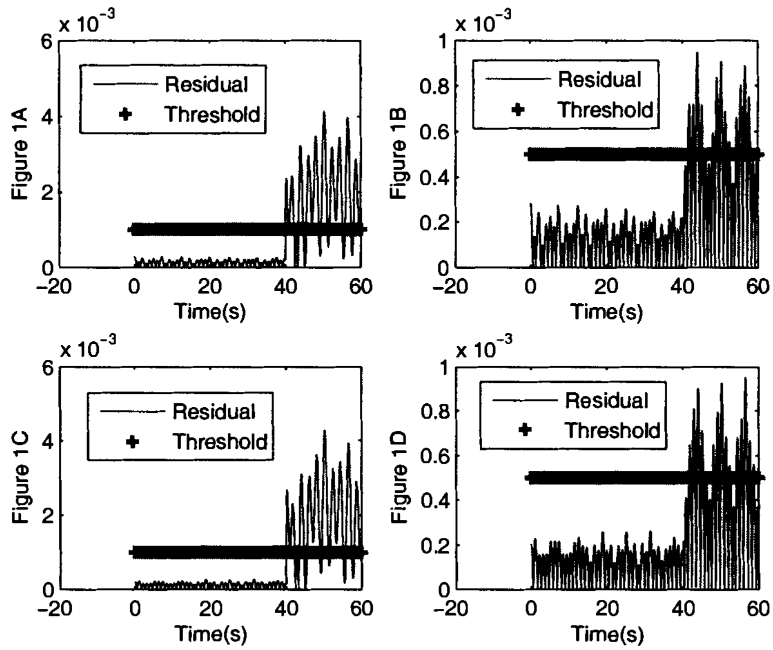

Figure 8.1: Fault detection with FIDFs

Fig. 8.1 clearly shows that both the constant and time-varying fault is successfully detected with the chosen thresholds. Moreover, faults are detected within 0.05s for Case A, 1.55s for Case B, 0.05s for Case C, and 1.59s for Case D, respectively. However, as shown in Fig. 8.2, only constant actuator faults for Case A and Case C are detected while time-varying faults for Case B and Case D cannot be detected with the same thresholds. This result implies that, with FIDFs, the missed fault detection rate is reduced.

For fault isolation, only Case A and Case C are considered, which correspond to constant fault cases. The simulation results for Case A and Case C are presented in Fig. 8.3 and Fig. 8.4. In both figures, the solid thin lines represent results when

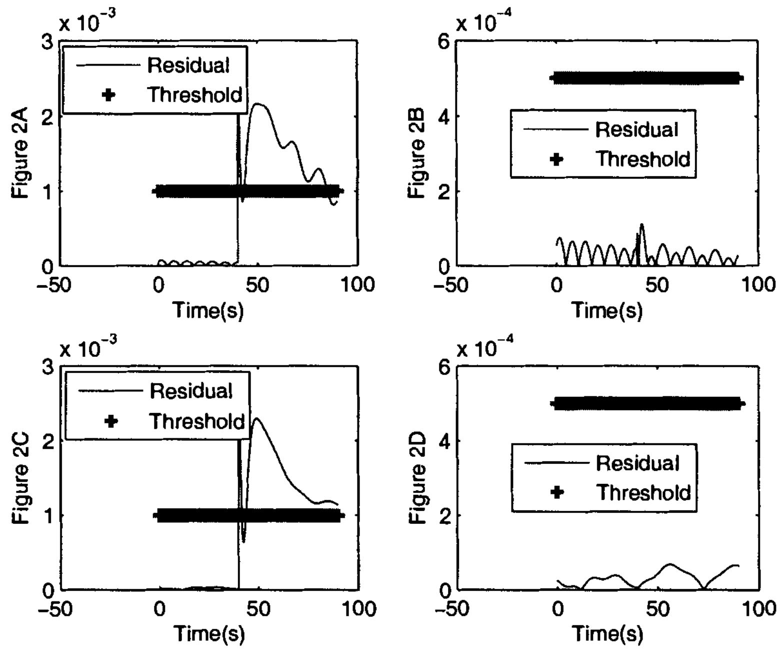

Figure 8.2: Fault detection without FIDFs

FIDFs are used while thick dashed lines represent results when FIDFs are not used.

By comparing the six residuals $r_1(t)$, $r_2(t)$, $r_3(t)$, $r_{12}(t)$, $r_{13}(t)$, and $r_{23}(t)$ shown in Fig. 8.3 on the time interval $[50.05s, 70s]$, amongst the six residuals shown by thin solid lines, only $r_2(t)$ has a tendency to approach zero, while all the other residuals do not have such a tendency. This behavior, according to the isolation scheme, leads to a correct decision that only one actuator is faulty and that actuator is the one corresponding to $u_2(t)$. However, because those residuals represented by thick dashed lines all have the tendency to approach zero, no isolation decision can be made when FIDFs are not used. Same observations can be made based on Fig. 8.4. For both Case A and Case C, Fig. 8.3 and Fig. 8.4 show that correct fault isolation decisions can be made using the FIDF adaptive fault isolation scheme, however, without using FIDFs, adaptive fault isolation cannot be achieved for this given example. In conclusion,

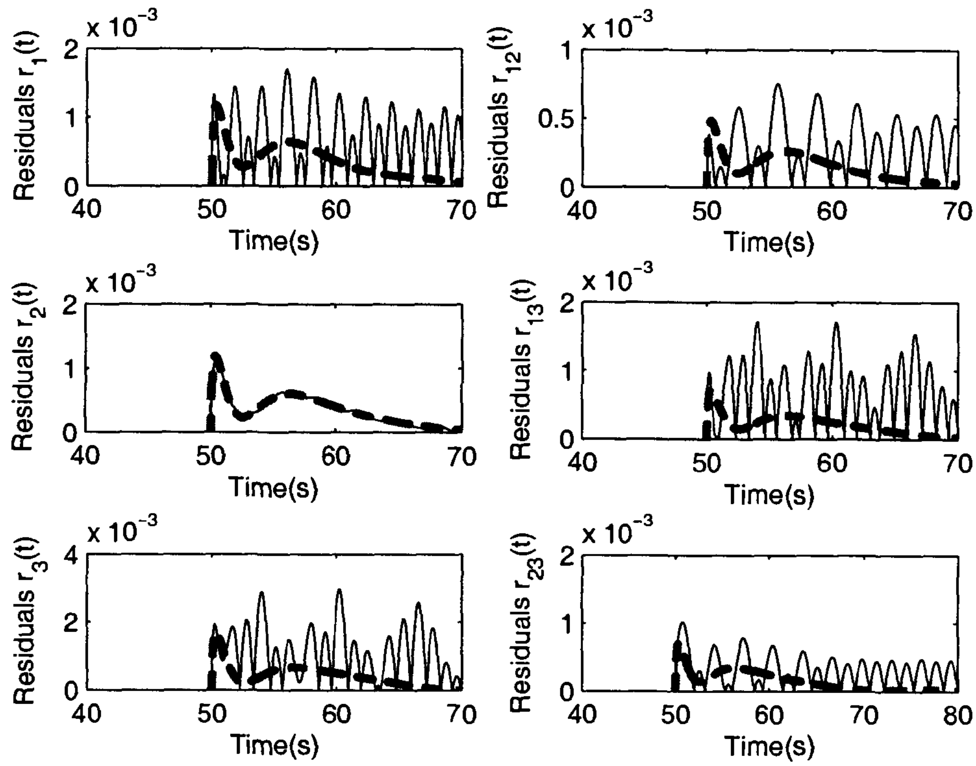

Figure 8.3: Fault isolation for Case A: Thin solid lines: with FIDFs: Thick dashed lines: without FIDFs

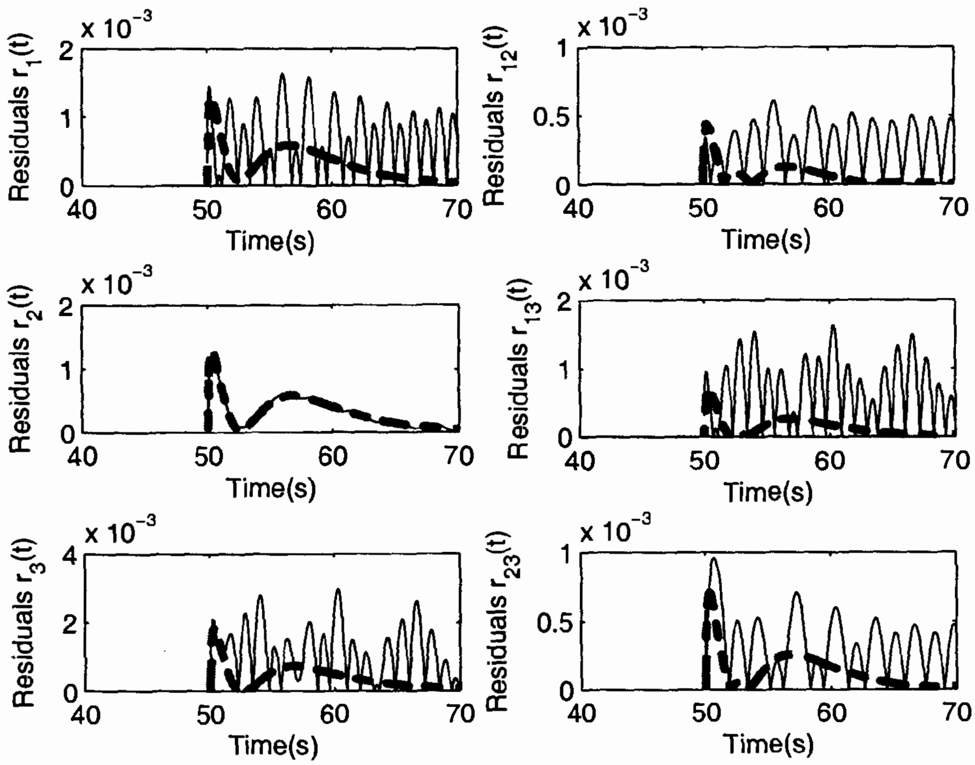

Figure 8.4: Fault isolation for Case C: Thin solid lines: with FIDFs: Thick dashed lines: without FIDFs

FIDF based adaptive fault isolation scheme really works well and FIDFs are preferred for obtaining correct fault isolation decisions.

The simulation results of the adaptive accommodation scheme for both Case A and Case C are presented in Fig. 8.5. The upper three plots are for Case A, while the lower three plots are for Case C. From the reference tracking error plots, the tracking performances are degraded during the presence of faults, but return to normal after the faults are isolated within $20s$ and the accommodating controller is switched on.

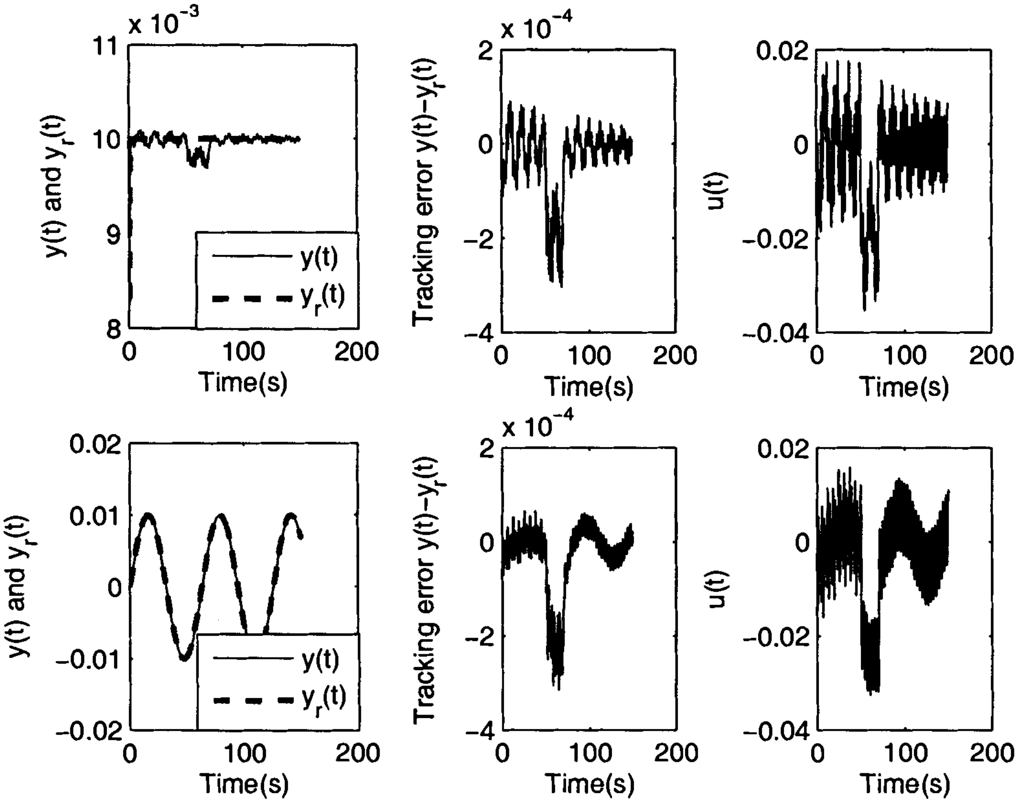

Figure 8.5: Adaptive fault accommodation

In order to show that the FIDFs can be chosen rather freely, isolation results for Case A with $f_2(t) = 0.02cos(3t)$ and $f_3(t) = 0.02cos(2t)$ are presented in Fig. 8.6. Based on these simulation results, a correct decision can again be made that only one actuator is faulty and that actuator is the one corresponding to $u_2(t)$.

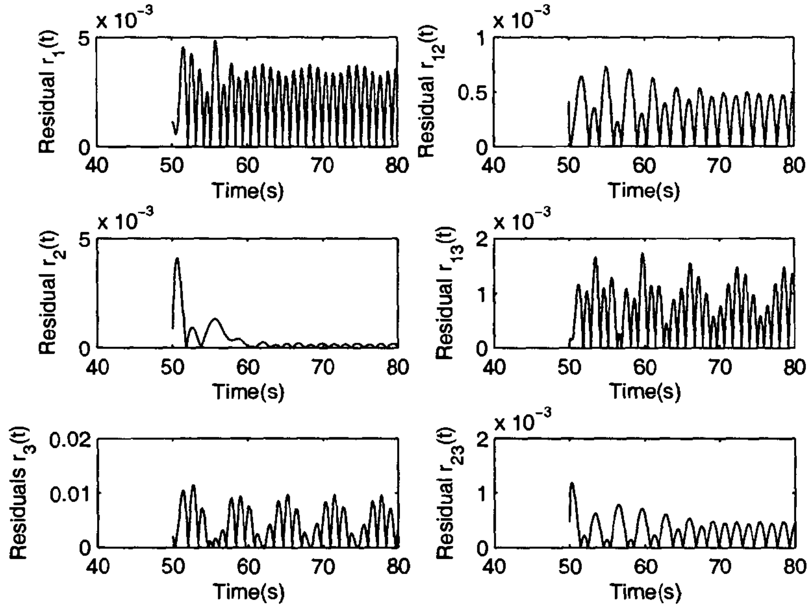

Figure 8.6: Adaptive fault isolation with another group of FIDFs

Simulations were also conducted on the effect of the adaptive fault isolation scheme for isolating very slow time-varying faults. The results for the fault $u_2(t) = 0.002(t - 50)/7$ for $t \geq 50s$ are given in Fig. 8.7. From Fig. 8.7 that only one actuator is faulty and that actuator is the one corresponding to $u_2(t)$, which are correct decisions. This result means the adaptive fault isolation scheme, although proposed for constant faults, may also work well for slow time-varying faults.

8.9 Conclusion and Discussions

In this chapter, an adaptive FDIA problem was studied and solved for a class of unknown MISO linear systems. One novelty of the idea was that of controller design with an eye towards the fault isolation problem. This design led to proposing the

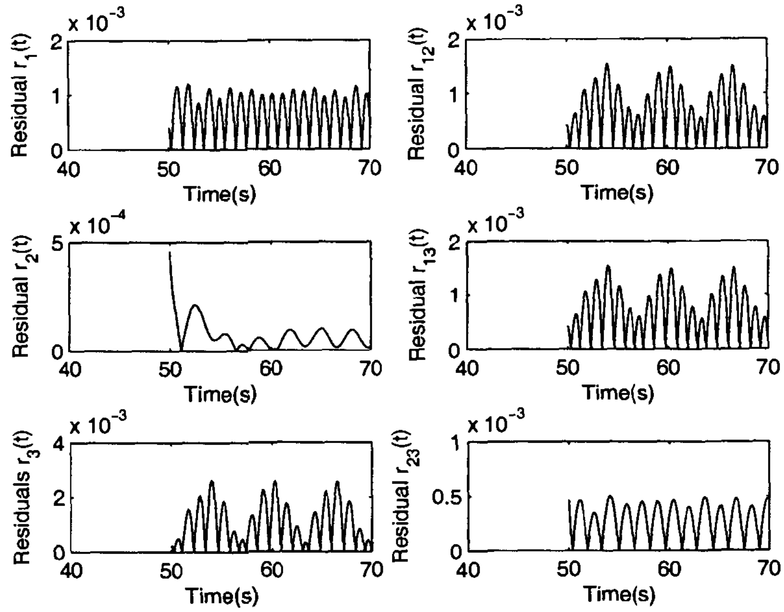

Figure 8.7: Adaptive fault isolation for a slow time-varying fault

use of fault isolation design functions, FIDFs, in the control law in this chapter. The simulation results show that the FIDF based technique works well in both fault detection for any types of actuator faults and in fault isolation for constant or very slow time-varying faults.

One limitation of the proposed adaptive FDIA is that it only works for constant actuator faults. Future research is needed in this direction.

Chapter 9

Conclusions and Future Works

9.1 Conclusions

In this thesis, fault detection, isolation, and estimation problems have been formulated and addressed in a systematic way for several classes of systems subject to various types of complexities such as nonlinearities, and nonparametric and parametric uncertainties. In order to deal with the nonparametric and parametric uncertainties encountered in these problems, both observer and direct output estimator based fault diagnosis schemes have been proposed by using robust and adaptive approaches. The key results are listed below.

- Robust observers, that is, UIOs and SMOs, have been used to solve the fault diagnosis problems for Lipschitz nonlinear systems as well as nonlinear systems represented by TS fuzzy systems. The fault isolation problems, which are solved less satisfactorily in the literature, were the main focus of all the related research. UIOs and SMOs based fault diagnosis schemes were proposed, whose main novelty lies in the fault isolation strategy. Related results were presented in Chapter 2, Chapter 3, and Chapter 4. They demonstrate that robust observer based fault diagnosis schemes are powerful in dealing with matched non-parametric uncertainties and can be designed in a careful way for certain types of nonlinear systems.

- If the uncertainties are not matched, observer based fault diagnosis becomes very challenging, if not impossible, because robust observers such as UIOs and SMOs used in this thesis may not be able to be designed. In order to deal with this difficult situation, a novel idea, which advocates direct output estimator design and abandons the observer design, was proposed. Using this idea and based on a new canonical system form derived from system decomposition, robust output estimator based fault diagnosis schemes were developed for a class of linear systems with both matched and unmatched non-parametric uncertainties. For this class of systems, neither UIOs nor SMOs can be designed to be completely decoupled from the unknown inputs. The results given in Chapter 5 prove that the idea of using direct output estimator design to perform fault diagnosis does work and can be used to solve more difficult fault diagnosis problems.

- Direct output estimator design based fault diagnosis was extended to a more general class of linear systems, which are with unmatched uncertainties and high relative degrees, and are not even detectable. A novel idea, which uses the design of output estimators and output derivative estimators, was proposed. Based on an input-output relation involving only the inputs and the outputs and their high order derivatives, a high-order sliding mode differentiator (HOSMD) based actuator fault diagnosis scheme was designed. The use of HOSMDs in fault diagnosis is a first in fault diagnosis. The results shown in Chapter 6 demonstrate that very challenging fault diagnosis problems, which are not possible to solve using an observer design, become solvable by employing the design of output estimators and output derivative estimators.

- Direct output estimator design based fault diagnosis was used in sensor fault detection and isolation problems. A novel sensor fault diagnosis scheme was proposed using adaptive approaches for a class of MIMO linear systems with unknown parameters. The results presented in Chapter 7 show that the direct output estimator design is very promising and powerful in sensor fault diagnosis for the considered class of unknown MIMO linear systems. The fault detection and isolation problems were solved completely under very weak conditions. The sensor fault diagnosis scheme is even new for known MIMO linear systems because it does not require the system under consideration to be detectable.

- Direct output estimator design was further employed for actuator fault diagnosis problems, which, compared with sensor fault diagnosis, are much harder. Another novel idea, that is, integrating the fault isolation design functions introduced solely for fault diagnosis purpose into controller design, was proposed, without which, the idea of direct output estimator design alone can not solve the fault isolation problem. A novel adaptive actuator fault diagnosis scheme was designed only for a class of unknown MISO linear systems, which solved the fault detection problems completely, and achieved constant actuator fault isolation and accommodation. The results provided in Chapter 8 demonstrate that the idea of direct output estimator design for fault diagnosis together with the idea of integrating fault isolation design functions into controller design does work well in actuator fault detection and constant fault isolation. They also demonstrate actuator fault diagnosis is very difficult because parametric uncertainties are present.

In summary, the research in this thesis demonstrates that robust approaches are powerful in dealing with non-parametric uncertainties, while adaptive approaches have strength in dealing with parametric uncertainties. It also reveals both the strength and the weakness of observer based fault diagnosis. Moreover, this thesis offers a new way of performing fault diagnosis; that is, output estimator based fault diagnosis.

9.2 Future Works

This thesis has solved fault diagnosis problems for several classes of systems with only limited types of complexities. Because many other types of challenging system complexities need to be dealt with, the opportunities in the research of fault diagnosis are still many. Some of these opportunities are closely related to the results in this thesis and are listed below.

- **Fault diagnosis for linear systems with more challenging complexities:**
  - Extend UIO, SMO, or output estimator based fault diagnosis to linear systems with both parametric and non-parametric uncertainties.
  - Design UIO, SMO, or output estimator based fault diagnosis to more classes of linear systems such as time-delay systems and algebraic-differential systems.
- **Fault diagnosis for nonlinear systems with more challenging complexities:**
  - Design UIO and SMO based fault diagnosis for systems with more general nonlinearities.

- Extend UIO and SMO based fault diagnosis to nonlinear systems with both parametric and non-parametric uncertainties.
- Investigate the design of UIO and SMO based fault diagnosis for more classes of nonlinear systems, such as time-delay systems and algebraic-differential systems.
- Employ output estimator design to carry out the fault diagnosis research proposed above.

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