CROSS-SECTIONAL AND MULTIVARIATE TESTS OF CAPM AND FAMA-FRENCH THREE-FACTOR MODEL

by

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Abstract

In this project, I test the the mean-variance Capital Asset Pricing Model (CAPM) and the Fama-French Three-Factor Model. I employ two datasets which consist of 25 portfolios formed on size and the book equity to market equity ratio and 11 portfolios formed on dividend yield. I also divide the whole period into two to consider the sub-period effects. I employ the cross-sectional tests as well as the multivariate time-series tests for both of the models. The results do not unambiguously show that one model fits better than the other. Moreover, the two sub-period results are inconsistent with each other and with the results from the whole period.

Keywords: Cross-Sectional; Time-Series; Multivariate Tests; CAPM; Fama-French Three-Factor Model

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1. Introduction

1.1 The Theory

Of all the sub-fields in financial economics, asset pricing theory occupies the first position of importance, among which, Mean-Variance Capital Asset Pricing Model (CAPM) has taken the central part ever since when William Sharpe (1964), John Lintner (1965) and Jan Mossin (1966) finally shaped and developed it theoretically as a way of thinking about how risks of an asset would affect its return. The central implication of CAPM is that, the expected return of an asset will depend on two things: the market risk premium and the asset's beta versus the market. What is more, the market betas are sufficient to calculate the expected returns. The simplicity of its implications and the easily testable characteristics, plus its informative and intuitive predictions are the main attractions of CAPM, no wonder why after nearly 40 years, it is still widely used by both academics and practitioners.

The basis of CAPM is on the "mean-variance" efficient model of Markowitz (1959). In that model, the author assumed a risk averse investor, who only cares about the expected return and variance of an investment in single period, tries to maximize his expected return by choosing among portfolios in a ideal capital market. Although this "mean-variance" model assumes a very simplified and perfect world, but it provides us the key conditions to derive the basic CAPM form.

When Sharpe (1964) and Lintner (1965) further assumed that, all investors have the same estimate of portfolios, they all have access to the same portfolios and they can borrow or lend at the same risk-free interest rate, the CAPM story finally takes shape. The idea is that, all mean-variance efficient portfolios are combinations of the risk-free interest (borrowing or lending) and the risky tangent portfolio, which just accommodates the Tobin's (1958) two funds "separation theorem". Since all investors have the same tangent portfolio and same risk-free interest rate, and they all want to hold the same mean-variance efficient portfolio, it must be that they are just holding the market portfolio in order to equalize the supply and demand for this portfolio. Therefore, the market portfolio is mean-variance efficient, so that the CAPM equation can be described as, under the mean-variance efficient frontier condition:

$$E(r_j) = r_f + [E(r_m) - r_f]\beta_j \tag{1}$$

Where r_m is the market portfolio return, r_f is the risk-free interest rate. Define β_j as to be $\beta_j = Cov(r_j, r_m) / \sigma^2{}_m$, here $Cov(r_j, r_m)$ is the covariance between the return on portfolio $_j$ and return on the market portfolio, $\sigma^2{}_m$ is the variance of the return on market portfolio. So β_j measures the sensitivity of the assets return to the return on the market portfolio. In the other way, β_j measures the systematic risk of the portfolio that cannot be diversified away. Therefore, from this point of view, CAPM implies only the nondiversified risks can be priced.

However, Sharpe-Lintner CAPM still relies on its two further assumptions in addition to Markowitz's (1959) model. In 1972, Fischer Black eliminated the risk-free lending/borrowing interest rate, instead, he employed an unrestricted short-sale of risky assets, then still derives the key idea of the Sharpe-Lintner CAPM, which is, mean-variance efficient market portfolio. On the other hand, Lintner (1969) and Merton (1987) raised an idea to allow for heterogeneous beliefs. To derive more specific results, Lintner further assumed an exponential utility functions for investors, he concluded that because investors have available fewer desirable securities, they diversify less and tend to hold higher risky portfolios rather than market portfolios.

Others tried to relax the other original assumptions of Markowitz's (1959) mean-variance efficient model. Mayers (1973) considered non-traded assets as well as human capital but did not make explicit adjustments for illiquidity. A more nonmarketable assets case has been studied by Pastor and Stambaugh (2003), they concluded that stocks whose returns are more sensitive to market liquidity have higher returns than those have low sensitivity to market liquidity.

Merton (1969, 1971, and 1973) studied the cases with multi-periods, continuous-time framework. In the Merton's (1973) intertemporal capital asset pricing model (ICAPM), he put a different assumption on the investors' preference, which is to maximize his wealth at the end of current period by choosing among portfolios. Therefore, in his model, investors concern not only the expected returns and variance, but also the covariance of returns in each state. He ended up with an idea of "multi-factor efficient". Rubinstein (1976), Lucas (1978) and Breeden (1979) developed an intertemporal consumption-based model (CCAPM), in which the investors care about maximized consumption each period.

As a general consideration of multi-factor efficient, in the 1990s, Fama and French provided empirical evidence to support their famous Fama-French three-factor pricing equation, in which they postulated that the expected return of a portfolio can be explained by the sensitivity of its return to three factors: the market excess return, the difference between the return on a portfolio of small stocks and return on a portfolio of large stocks (SMB) and the difference between the return on portfolio of high book-to-market stocks and return on portfolio of low book-to-market stocks (HML). More specific, the pricing equation of the expected on portfolio j is:

$$E(r_j) = r_f + [E(r_m) - r_f]b_{mj} + E(r_{SMBj})b_{SMBj} + E(r_{HMLj})b_{HMLj} \tag{2}$$

Where $E(r_m) - r_f$, $E(r_{SMBj})$, $E(r_{HMLj})$ are expected premiums.

To sum up, the standard CAPM connects the expected return with portfolio betas in mean-variance efficient portfolio settings, but only under several very strict and unrealistic assumptions can it show that the market portfolio is efficient. This makes its fundamental theory, as well as its applications, questionable at best. Therefore, it's necessary to test the model against data.

1.2 Empirical Tests

As discussed earlier in the CAPM theory, it has a few intuitively important implications that can be tested. First of all, the expected returns of any assets depend entirely on their betas, and this relationship is linear. Second, the market portfolio is MV efficient, and when there is risk-free asset, the CAPM market portfolio is the tangency portfolio. This implies the market portfolio satisfies the MV efficient condition, which is given by equation (1) in the above section 1.1. Therefore, the beta premium which is just the market premium should be positive and close to excess market return. Third, all securities must be plotted on the Security Market Line (SML) on the mean-beta plane, so that the intercept of SML which is also the zero-beta return should be the risk-free return and, the slope of SML should be the market premium. Most of these tests are either time-series or cross-sectional, or both.

It was Jensen (1968) that first developed a time-series test on Sharpe-Lintner's CAPM model. Followed by the same idea, Black, Jensen and Scholes (1972) developed their famous time-series test. For each asset, they regressed the excess return of an asset (exclude risk-free rate from an asset's return) on excess mark return. The time-series univariate test is testing whether the intercept is different from zero for each portfolio. That is their famous "Jensen's Alpha", which now is still used as a major method for performance measurement of portfolios.

Early cross-sectional tests of Sharpe's CAPM model mainly focused on the intercept as well. They regressed the historical return of individual asset on its estimated beta, test whether the intercept is the risk-free asset. But as Black, Jensen and Scholes (1972) suggested, they run a slightly different cross-sectional regression model, that is, instead of regressing portfolio returns on portfolio betas, they used the excess return of portfolio to do the regression. In this way, theoretically if CAPM is right, then all intercept terms from the regression should be zero, and all slope terms should be the excess returns of market portfolio. Their cross-sectional tests are based on this. Similar tests have been done ever since, such as in the earlier time, Douglas (1968), and then Miller and Scholes (1972), Fama and MacBeth (1973), or Blume and Friend (1973). They keep on finding the same interesting result, that is, the CAPM predicted positive relationship between excess returns and betas is flatter than the true model. In other words, the intercept term is higher, the slope term is lower than the true model would suggest.

In the time-series side, they also find the same results that the CAPM prediction is "flatter", such as Blume and Friend (1970) and Stambaugh (1982). More recently, Fama and French (1992) also confirmed this.

Actually, there are a few problems associated with these tests. First among all is the use of ex post returns (also as to be the historical returns) for ex ante returns (also as to be the expected return). They definitely are not the same but one can only observe the realized returns. The other problem happens when one tries to estimate the beta for individual asset. There will be big measurement error in these betas. This can somehow explain why people keep on finding a "flat" prediction of SML to some extent. The last problem is in the OLS cross-sectional regression residuals. Since the error terms are autocorrelated and heteroskedastic, so that their t-statistics are problematic. This would challenge the significance level of the regressions.

Consequently, to fix the problem of measurement errors, people begin to employ grouped data, where they sort the individual assets into different portfolios either by size, capitalization or prices to reduce estimation errors in beta. On the other hand, to correct the suspect t-statistics from the normal cross-sectional regressions, Fama and MacBeth (1973) suggested a univariate cross-sectional method based on the OLS estimator.

More recently in addition to the univariate t-tests, based on BJS (1972) time-series testing method, Gibbons, Ross and Shanken (1989) jointly tested the hypothesis that all intercepts are close to zero. The jointly multivariate F-test is used to measure the deviations from the SML of portfolios.

To the question of whether or not the expected returns of assets would depend solely on their betas linearly, Fama and MacBeth (1973) included the unsystematic risks (the residual variance from regressions of returns on market return) and the squared market betas to test for the linearity of beta relationship They found a consistent result with what CAPM suggests. However, other empirical studies have shown many contradictions. Banz (1981) documented a size effect on the cross-sectional average returns. Bhandari (1988) showed a positive relationship between the leverage and the average return. Stattman (1980), Rosenberg, Lanstein and Reid (1985) found that average returns are positively related to the ratio of a firm's book value to its market value (BE/ME), based on the U.S stocks evidence and Japanese stocks evidence. In 1983, Basu documented an earnings-to-price ratio (E/P) effects on the cross-sectional average returns. DeBondt and Thaler (1985) documented a long-term reverse effect on average returns. Jegadeesh and Titman (1993) found a persistent momentum effects on returns. All these patterns that are not explained by CAPM are called anomalies.

In the 1990s, Fama and French provided empirical evidence to claim that CAPM is inaccurate. Fama and French (1992) showed that there is no relationship between average returns and betas so that CAPM is rejected. Furthermore they confirmed the size and book-to-market equity effects on returns. Fama and French (1993, 1994, 1996a, 1996b, 1998) documented the famous new Fama-French three-factor model, in which they also suggested a way of doing the time-series testing on their pricing equation. The intercept term in their tests measures the pricing errors of Fama-French's pricing equation. Therefore, the time-series tests are still testing whether the alphas are equal to zero. They asserted when portfolios are sorted by firm specific attributes, the FF three-factor model outperforms CAPM because the absolute pricing errors (the absolute average of intercept term) of CAPM are 3 to 5 times bigger than those from FF three-factor model.

However, as one of the major comments, Ferson and Harvey (1999) criticized the FF three-factor model because they found evidence of time-varying alphas in an industry setting. Under the same settings, Grauer (2000) found the absolute pricing errors of CAPM are smaller. Therefore, Kothari, Sloan and Shanken (1995) doubted whether the three-factor model would really dominates CAPM, because a useful pricing model cannot just be valid for limited set of portfolios, but should be worked under much wider and looser conditions. They put forward the idea of "survivor bias", which means there is data selecting bias when choosing the high book-to-market portfolios. As another critique of the FF model, MacKinlay (1990), Black (1993) and MacKinlay (1995) argued that all the anomalies of CAPM are the result of data mining. They claimed that for these tests, people just try to add more variables that can be worked out better to gain more explanatory power. These tests are not really based on theory. Furthermore, the Fama-French three-factor model still cannot explain the continuation of short-term returns (the momentum effects).

Finally, as Roll Richard (1977) pointed out, strictly speaking, ever since the first extensive tests of the MV CAPM appeared in the early 1970's, none of them would really test the CAPM. The reason is because one couldn't really include all marketable assets as the market portfolio. Even though people employed many market proxies, but they are not the true market portfolio, then the betas measured against an inefficient portfolio are meaningless measures and cannot be used to accept or reject the CAPM.

The validity of CAPM versus Fama French three-factor model is still hotly discussed. Therefore I would like to test CAPM and FF model employing cross-sectional and multivariate tests. In Section 2, I introduce the data that used in this paper. Section 3 presents the regression results as well as the testable implications. Section 4 briefly summarizes the major findings and draws a conclusion about this project.

2. The Data

I will test against two datasets in this project as my main objective. One is a 25-portfolio data formed on size and the book-to-market equity ratio, the other one is a 11-Portfolio data formed on dividend yield. These datasets are collected from an unity of New York Stock Exchange (NYSE), American Stock and Options Exchange (AMEX), and National Association of Securities Dealers Automated Quotation System (NASDAQ) stocks.

The 25-portfolios data is constructed as the intersections of 5 portfolios formed on size (market equity, ME) and 5 portfolios formed on the ratio of book equity to market equity (BE/ME). They are monthly average value weighted returns that was created by CMPT_ME_BEME_RETS using the 200601 Center for Research in Security Prices (CRSP) database. The size breakpoints for year t are the NYSE market equity quintiles at the end of June of t. ME is market cap at the end of June. BE/ME is book equity at the last fiscal year end of the prior calendar year divided by ME at the end of December of the prior year. Firms with negative BE are not included in any portfolio.

The 11-Portfolios data is also the value weighted monthly returns formed on Dividend-to-Price ratio (D/P) that was created by CMPT_DP_RETS using the 200512 CRSP database, including utilities and financials. D/P is computed when the portfolios are constructed, at the end of June. The dividend yield use to form portfolios in June of year t is the total dividends paid from July of t-1 to June of t per dollar of equity in June of t. The portfolios are sorted by 10 deciles plus one zero/negative divided yield portfolio, totally 11 portfolios. Firms with zero dividends are in only the Dividends = 0 portfolio.

Rm-Rf, the excess return on the market, is the value-weight monthly returns on all NYSE, AMEX, and NASDAQ stocks (from CRSP) minus the one-month Treasury bill rate (from Ibbotson Associates). Rm-Rf includes all NYSE, AMEX, and NASDAQ firms. SMB and HML for July of year t to June of t+1 include all NYSE, AMEX, and NASDAQ stocks for which we have market equity data for December of t-1 and June of t, and (positive) book equity data for t-1[1].

All the datasets are downloaded on March 2nd, 2006 from the Kenneth R. French's data library[2]. The data contains periods from January 1931 to December 2005,[3] 900 months altogether. I will also consider the period effects on CAPM, so that the whole 900 months will be divided into two sub-periods, one of which is from January 1931 to June 1963, totally 390 months; the other one starts from July 1963 to December 2005, totally 510 months.

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[1] See Fama and French, 1993, "Common Risk Factors in the Returns on Stocks and Bonds," *Journal of Financial Economics*, for a complete description of the factor returns.

[2] Since French seems to keep on modifying the datasets every once a while, it would be necessary to mark the exact date version of the data. The data website address is: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[3] I drop the periods from 1927 to December 1930 since there are missing data in this interval.

The reason for this division is not only to follow and to replicate what Fama and French (1996) did in their empirical tests (from July 1963 to 1993), but also more importantly, to study the out-of-sample application of CAPM. A good model should be valid for forward looking as well as for backward looking.

3. The Results[4]

To begin the analysis, first let's look at APPENDIX TABLE A1 and APPENDIX TABLE A2, which are the summary statistics of average excess returns and standard deviations. APPENDIX TABLE A1 shows the data of 25 portfolios formed on size and BE/ME from the whole period of Jan 1931 to Dec 2005, totally 900 months. APPENDIX TABLE A2 shows data of the 11 portfolios formed on dividend yield also for this total 900 months.

In Appendix Table A1, one can see clearly that small stocks tend to have higher excess return in average than big stocks, also high BE/ME value stocks have higher excess returns in average than the low BE/ME ones. Therefore, there seems to have a relationship between the excess returns and size, BE/ME effect, on which I will elaborate in part 3.4 the Fama French model. Likewise in APPENDIX TABLE A2, one can observe a positive dividend yield effects on returns. That is, dividend-paying stocks outperform non-dividend-paying stocks, and high dividend yield stocks seem to earn higher returns, on average, than low dividend yield stocks.

[4] See APPENDIX for all the detailed tables

3.1 Time-Series Tests of the CAPM

For each asset (portfolio), the Black, Jensen and Scholes (1972) run a time-series regression of the excess returns of the asset (portfolio) on the excess market returns. The model can be described as the following regression:

$$r_{jt} - r_{ft} = \alpha_j + \beta_j (r_{mt} - r_{ft}) + e_{jt} \qquad (3)$$

where r_{jt} is the return on portfolio j at time t, r_{ft} is the risk-free interest rate at time t, and r_{mt} is the market portfolio return at time t. The time-series univariate test is testing H0: $\alpha_j = 0$ for each portfolio j.

Generally speaking, as suggested by all panels in APPENDIX TABLE A3, the time-series model has a relatively strong explanatory power for excess return of portfolios, since all the average R^2's are above 0.72.

In all the 25-portfolio regressions, for the whole time period and two sub-periods, the average R^2 are 0.76, 0.79 and 0.73, the regression seems to fit the 1931 to 1963 sub-period best. Moreover, the explanatory power tend to increase from small size low BE/ME assets to big size high BE/ME assets (R^2 seems to have a somewhat weakly increasing trend). The slope loading β_j's, which capture the sensitivity of the asset's return to variation in the market return, are strongly statistically significant for all the portfolios (t-stats for β are relatively high for all 25 portfolios). It suggests that market portfolio is a fairly important explanatory variable. On the other hand, the intercepts α's turn out to reject CAPM since most of the α's are distinguishable from zero. For the sub-period July 1963 to 2005, the average of absolute intercepts, known as the pricing bias, is 31 basis points higher than zero. As a contrast, the pricing bias for sub-period 1931 to June 1963 is the lowest, at a 22 basis points higher than zero. According to all the t-statistics or associated P-values, under a significant level of 0.05, there are 11 rejections out of 25 portfolios for the whole sub-period 1931 to 2005, and 13 rejections for sub-period July 1963 to 2005. However, one can hardly find any rejections for the sub-period 1931 to June 1963, which may suggest that all alphas are close to zero! Together with the relative average R^2, sub-period January 1931 to June 1963 seems to support the CAPM best, sub-period July 1963 to 2005 seems to support CAPM worst; others all have intercepts that are distinguishable from zero which leave us large unexplained returns.

In the 11-portfolio regressions, I find the results are similar to the 25-portfolio estimations. The β 's are all statistically significant, yet most of the α 's are far from zero. The estimation for sub-period 1931 to 1963 once again shows her strong explanatory power of CAPM: The average estimation R^2 is 0.89 which is the highest one among the three (other two are 0.83 for whole period and 0.75 for period from July 1963 to 2005); Under the significance level of 5%, there are only one rejection case in the sub-period 1931 to 1963; and the average absolute pricing error is only 13 basis points higher than zero, which is also the lowest among the three time periods. On the other hand, estimation for sub-period July 1963 to 2005 is still the one that supports CAPM worst.

As a comparison across datasets, the 11-portfolio data shows a better support of CAPM than the 25-portfolio data, since the regression fit (R^2) is on average higher and the absolute pricing errors are on average lower.

3.2 Cross-Sectional Tests of the CAPM

3.2.1 OLS cross-sectional tests

The Black, Jensen and Scholes (1972) cross-sectional test is based on the OLS estimator. It is the regression of the portfolio excess returns on the betas of those portfolios. The model can be described as:

$$\bar{r}_j - \bar{r}_f = \gamma_0 + \gamma_1 \hat{\beta}_j + e_j \tag{4}$$

Where $\bar{r}_j$ and $\bar{r}_f$ are time-series average rates of return on risky asset j and the risk-free asset. $\hat{\beta}_j$ is collected from the above time-series regressions (equation (3)) of the portfolios. Theoretically, if CAPM is right, then all γ_0 should be zero, and γ_1's should be the excess returns of market portfolio. The tests are based on this.

As shown in APPENDIX TABLE A4, according to the overall R^2 level, the portfolio betas seem to have less explanatory power. Thus the model does not capture most of the variations in the average return on the portfolios on cross-sectional regressions.

The following TABLE 1 shows the summary of all the cross-sectional tests for both the 25-portfolio data and the 11-portfolio data in all time periods.

Let's first investigate within data across time. Among all periods in the 25-portfolio estimations, the big t-stats value indicates that almost all γ_1 are relatively significantly different from zero, yet only in period July 1963 to December 2005, the slope estimation is negative. For intercept term γ_0, the first two periods show us the insignificant intercepts are close to zero, whereas in period July 1963 to December 2005, it shows a rejection, which suggests a distinguishable intercept from zero! It seems that all the discrepancy and inconsistency happened during the second half of the whole time period.

The 11-portfolio estimations however show us another story. The results of the full time period and the second half (from Jul 1963 to Dec 2005) have the same patterns, which refers to the insignificant γ_1 and significant γ_0. This is a totally rejection of CAPM. In sub-period 1931 to 1963, it shows a reversed pattern. Its significant γ_1 and insignificant γ_0 can be a support of CAPM.

Table 1 Estimation Summary of Cross-sectional Tests for all time periods

The regression is: $\bar{r}_j - \bar{r}_f = \gamma_0 + \gamma_1 \hat{\beta}_j + e_j$

Where $\bar{r}_j$ and $\bar{r}_f$ are time-series average rates of return on risky asset j and the risk-free asset. $\hat{\beta}_j$ is collected from the BJS time-series regressions of the portfolios.

Time Period		25-Portfolios		11-Portfolios	
		Estimation Coefficient	P-value for t-statistics	Estimation Coefficient	P-value for t-statistics
Jan1931-Dec2005	Slope γ_1	0.68	0.04	0.15	0.44
	Intercept γ_0	0.11	0.78	0.60	0.01
Jan1931-Jun1963	Slope γ_1	0.74	0.02	0.56	0.04
	Intercept γ_0	0.25	0.55	0.43	0.11
Jul1963-Dec2005	Slope γ_1	-0.54	0.06	-0.08	0.39
	Intercept γ_0	1.30	0.00	0.63	0.00

Furthermore, in order to investigate across datasets, let's look at the SML below.

Theoretically if CAPM is true, there must be a Security Market Line (SML) on the mean-beta space, showing clearly the linear relationship between the excess return of portfolios and associated betas. As shown in the Appendix graph A1.

First, it can be seen from the graph that, in almost all the cases there are many big outliers, yet the confidence interval does not cover many of the observations. That is consistent with the regression fit (R^2) predicts. Second, as mentioned earlier, γ_1 should be the estimated excess market return. But in period from July 1963 to December 2005 for both the 25-portfolio and the 11-portfolio, the estimated slopes γ_1 are negative! They predict a negative market excess return! Since if the excess market return is negative, then the risk-free rate of return would dominate the market return, one can easily choose to invest in risk-free return to get higher benefit instead of market return, and then the market return wouldn't be mean-variance efficient! So from this point of view, this negativity prediction of γ_1 suggests rejection of CAPM.

One thing should be noticed is that the t-statistics in the model are problematic because all the error terms are correlated with each other, but in an OLS setting, the error terms must be independently identical distributed. Therefore, in the following part, I will apply the Fama-MacBeth (FM) cross-sectional method to correct the suspect t-statistics problem.

3.2.2 The Fama-MacBeth Cross-Sectional tests of the CAPM

As mentioned early above, there is a problematic t-statistics for both the slope and intercept coefficients in the OLS（BJS）cross-sectional test of CAPM since all the error terms are correlated. I now employ Fama-MacBeth (FM) month-by-month test of CAPM to correct the t-statistics. The estimation model is, for every month, I run the following cross-sectional regression:

$$r_{jt} - r_{ft} = \gamma_{0t} + \gamma_{1t}\beta_{jt} + e_{jt} \tag{5}$$

Fama and MacBeth (1973) used to have the β_{jt}'s estimated once a year based on a five-year moving window. More recently, instead of using different estimates of β 's each month, people simply use one β for each portfolio, that is, we use the full-period time-series estimation $\beta_j = \beta_{jt}$ suppose that beta is stationary over time according to the CAPM assumption, this would be consistent with the cross-sectional tests. Then I can use the standard deviations of γ_{0t} and γ_{1t} to generate the sampling errors of γ_0 and γ_1. The univariate corrected t-test would be:

$$t(\bar{\gamma}_j) = \frac{\bar{\gamma}_j}{s(\gamma_j)/\sqrt{T}} \tag{6}$$

Where $\bar{\gamma}_j$ is the average of the r_{jt}'s, and $s(\gamma_j)$ is the standard deviations. T is the number of time series.

There are 900, 390 and 510 γ_0's and γ_1's been estimated for these three different time periods (1931 to 2005, 1931 to June 1963 and July 1963 to 2005) respectively. As shown in the following summary TABLE 2, I provide the means of those slope coefficients γ_1 and intercept coefficients γ_0.

Table 2 Result for Cross-Sectional Tests of the CAPM

Notice, t-statistics are corrected by Fama MacBeth Cross-Sectional Method

Time Period		25-Portfolios		11-Portfolios	
		Average of Time-Series Estimation Coefficients	P-value for Corrected t-statistics	Average of Time-Series Estimation Coefficients	P-value for Corrected t-statistics
Jan1931-Dec2005	Slope γ_1	0.68	0.07	0.15	0.61
	Intercept γ_0	0.11	0.75	0.60	0.01
Jan1931-Jun1963	Slope γ_1	0.74	0.16	0.56	0.23
	Intercept γ_0	0.25	0.54	0.43	0.18
Jul1963-Dec2005	Slope γ_1	-0.54	0.10	-0.08	0.80
	Intercept γ_0	1.30	0.00	0.63	0.02

Let's look at all the time periods for all the datasets as a whole. Compare to the above TABLE 1, it's interesting for find the averages of all the time series of the γ_{0t} 's and γ_{1t} 's are equal to γ_0 and γ_1 that estimated from the OLS cross-sectional tests provided when $\beta_{jt} = \beta_j$. What is more, after the correction of t-statistics, one can find in almost all the cases, slope term γ_1 's are statistically insignificant, which means under the null, they are close to zero. For intercept term γ_0, there still have three significant cases, there are in 25-portfolio July 1963 to 2005, 11-portfolio 1931 to 2005 and 11-portfolio July 1963 to 2005. If the CAPM is true, we must have intercept γ_0 to be close to zero and slope γ_1 to be the positive excess market return. Therefore, after the t-statistics correction, the cross-sectional univariate test shows that the CAPM is questionable.

3.3 Time-Series Tests of Fama-French Three-Factor Model

As an empirical alternative to CAPM, Fama-French Three-Factor model suggests that, as a result of data mining, the expected return of an asset is determined by its sensitivity to three factors. Therefore, the regression model can be described as:

$$r_{jt} - r_{ft} = \alpha_j + b_{mj}(r_{mt} - r_t) + b_{SMBj} r_{SMBj} + b_{HMLj} r_{HMLj} + e_{jt} \qquad (7)$$

where r_{SMBj} and r_{HMLj} are the returns on the SMB and HML portfolios. SMB is the returns on a portfolio of small minus big stocks; HML is the returns on portfolio of high minus low. The intercept term α_j measures the abnormal performance of Fama-French's testing equation. Therefore, the FF time-series tests are still testing whether the alphas are equal to zero.

The estimation result that I find seems to be consistent with what Fama and French did in their 1996 paper[5]. First, an overall average of over 0.90 R^2 indicates that the FF model is a good description of returns of the 25 portfolios formed on size and BE/ME as well as the 11 portfolios formed on dividend yield. It captures much of the return variations better than CAPM does. Moreover, the estimated sensitivities or loadings on market portfolio, SMB and HML are almost statistically significant (not shown in

[5] See Fama and French, 1996, "Multifactor Explanation of Asset Pricing Anomalies," *Journal of Finance*, for a complete estimation description.

APPENDIX TABLE A5). The argument on why SMB and HML play an important role in explaining variations in excess returns in addition to market portfolio is that, SMB and HML actually mimic much of the underlying risk factors concern to investors, therefore, "it absorbs most of the anomalies that have plagued the CAPM" (Fama French 1996). For the α_j, we still have some unexplained deviations from zero, for example, in most cases of 25-portfolio data, portfolio R1 (the large negative unexplained return in the smallest size interacted with lowest BE/ME), and in portfolio R21 (large unexplained positive return in the biggest size interacted with lowest BE/ME); as a comparison, in most cases of 11-portfolio data, portfolio R1 (large negative unexplained return in the portfolio of no dividend yield).

However, the result shows a relatively similar period selection bias to the OLS time-series regressions of the CAPM. In the 25–portfolio dataset, the sub-period 1931 to 1963 seems to support the FF model relatively more, yet the sub-period July 1963 to 2005 seems to support FF model relatively less. The reason is in the sub-period 1931 to 1963, under a 5% significant level, I can only reject two cases that α_j=0 whereas in the sub-period July 1963 to 2005, there are 8 rejections, though the regression fit (R^2) are very close to each other.

3.4 Cross-Sectional Tests of Fama French Three-Factor Model

The OLS cross-sectional test of Fama French three-factor model follows the same idea as the cross-sectional test on CAPM. However, as I argued above, this OLS cross-sectional regression does have a problematic t-statistics, it is necessary to fix the t-tests by Fama MacBeth cross-sectional method. This can be described as, for each month t, I run the following regression:

$$r_{jt} - r_{ft} = \gamma_{0t} + \gamma_{1t}\beta_{jt} + \gamma_{2t}b_{SMBjt} + \gamma_{3t}b_{HMLjt} + e_{jt} \tag{8}$$

Again, I use the full-period time-series estimation $\beta_j = \beta_{jt}$, $b_{SMBj} = b_{SMBjt}$ and $b_{HMLj} = b_{HMLjt}$. The corrected t-test for intercept term is:

$$t(\bar{\gamma}_0) = \frac{\bar{\gamma}_0}{s(\gamma_0)/\sqrt{T}} \tag{9}$$

Where $\bar{\gamma}_0$ is the average of γ_{0t}'s, and $s(\gamma_0)$ is the standard deviations. T is the number of months. The intercept term γ_0 measures the abnormal performance of Fama-French's pricing equation. Therefore, it is still testing on whether the intercept gammas are equal to zero.

The following TABLE 3 provides the summary of OLS cross-sectional tests of Fama-French three-factor model and corrected t-statistics using Fama MacBeth cross-sectional method.

Look through the table 3 as a whole, suggested by the t-tests, the abnormal pricing biases for 25-porfolio data in all periods are significant, these huge positive γ_0's leave us an unsatisfactory performance of FF model. As a contrast, for the 11-portfolio data, all the insignificant t-tests clearly show a support of the FF model. Furthermore, the pricing abnormal biases in sub-period 1963 to 2005 are the lowest among the three periods.

Therefore, it shows almost inconsistent features with the above Fama French time-series regressions for both the 25-portfolio data and 11-portfolio data.

Table 3 Result for Cross-sectional Tests of Fama French Model for all time periods

Notice, t-statistics are corrected by Fama MacBeth Cross-Sectional Method

Time Period	25-Portfolios		11-Portfolios	
	Intercept γ_0	P-value for Corrected t-statistics	Intercept γ_0	P-value for Corrected t-statistics
Jan1931-Dec2005	1.84	0.00	0.87	0.13
Jan1931-Jun1963	1.35	0.00	0.94	0.11
Jul1963-Dec2005	1.26	0.00	0.63	0.22

Here in the cross-sectional tests of FF model I just emphasize on the γ_0's as the major argument instead of looking at all the intercept terms and slope terms, this would be more straightforward for the following comparison of the CAPM and FF three-factor model.

3.6 Estimation Summary

of all Cross-Sectional Tests for all 6 time periods

TABLE 4 below shows all the estimation summary of the OLS cross-sectional tests together with the Fama MacBeth cross-sectional t-tests corrections on intercept γ_0 of CAPM and the Fama-French three-factor model.

Table 4 Summary of all Cross-Sectional Tests

Notice, t-statistics are corrected by Fama MacBeth Cross-Sectional Method

Time Period	Cross-Sectional Test	25-Portfolios Intercept γ_0	25-Portfolios P-value for Corrected t-statistics	11-Portfolios Intercept γ_0	11-Portfolios P-value for Corrected t-statistics
Jan1931-Dec2005	CAPM	0.11	0.75	0.60	0.01
	FF three-factor model	1.84	0.00	0.87	0.13
Jan1931-Jun1963	CAPM	0.25	0.54	0.43	0.18
	FF three-factor model	1.35	0.00	0.94	0.11
Jul1963-Dec2005	CAPM	1.30	0.00	0.63	0.02
	FF three-factor model	1.26	0.00	0.63	0.22

Table 4 actually shows two interesting features that are worth of being noticed. The first point is that, for most of the time periods in both of the datasets, CAPM predicts a relatively smaller intercept than what FF model predicts. These γ_0's actually measure the deviations from the zero-beta rate of return, so the smaller pricing deviations of CAPM show an inconsistency with what Fama and French claimed in 1996 that FF model dominates CAPM in all respects. The second point is that, 25-portfolio data seems to support more of CAPM than FF model since the intercept γ_0 for CAPM are insignificant in most of the time periods, whereas for FF model, they are all significant; 11-portfolio data seems to support more of FF model than CAPM since γ_0's for FF model are insignificant in all of the periods, whereas for CAPM, two out of three periods have significant intercepts. Data selection seems to lead to biases in the cross-sectional tests.

3.7 Estimation Summary of all Time-Series Regressions and Multivariate Tests for all 6 time periods

Here I will show the comparison of CAPM and FF three-factor model, which has been a hotly debate since Fama French (1992) until now. In addition to the univariate t-tests on intercepts and slopes, I will add one more critique which is the multivariate test.

Based on Jobson and Korkie (1982, 1985), Gibbons, Ross and Shanken (1989, Volume I, Chapter 8) (hereafter GRS) developed the multivariate tests for CAPM. Define α as to be a vector that contains all the intercepts for all N-portfolios from Black, Jensen, and Scholes (BJS) time-series regression, i.e. $\alpha = (\alpha_1,...,\alpha_n)^T$. Define $\varepsilon_t = (\varepsilon_{1t},...,\varepsilon_{nt})^T$ which contains all the error terms. Assume ε_t is jointly normally distributed, so that $E(\varepsilon_t) = 0$, further assume $Cov(r_{mt}, \varepsilon_t) = 0$, then $E(\varepsilon_t \varepsilon_t') = \Sigma$ is the variance-covariance matrix. The jointly F-test is given by:

$$J = \frac{T-N-1}{N}(1+\frac{\hat{\mu}_m^2}{\hat{\sigma}_m^2})^{-1}\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha} \tag{10}$$

With the hypothesis:

H0: all α_j=0

HA: all $\alpha_j \neq 0$

Where $\hat{\mu}_m^2$ and $\hat{\sigma}_m^2$ are the average excess return and standard deviation of the market portfolio. Under the null hypothesis, J is an unconditional central F distribution with N degree of freedom in the numerator and T-N-1 degrees of freedom in the denominator. This multivariate test is used to measure the deviations of portfolios from the SML in the mean-beta plane.

Campbell and MacKinlay (1997) in their book "The Econometrics of Financial Markets" suggest a way to do the multivariate test for Fama French Three-Factor Model, which is very similar to the GRS multivariate test of CAPM. The only difference is, instead of having just one explanatory variable for measuring excess return of portfolio—the excess market return, now in FF three-factor model there are three variables. So the scalar $\hat{\mu}_m$ becomes the vector μ_K that contains all the average excess market return, average SMB and average HML; the scalar $\hat{\sigma}_m$ becomes the variance-covariance matrix that contains all the standard deviations of market return, SMB and HML. Therefore, the joint test becomes:

$$J_1 = \frac{T-N-K}{N}[1+\hat{\mu}_K'\hat{\Omega}_K^{-1}\mu_K]^{-1}\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha} \tag{11}$$

Here K is the number of explanatory factors, and obviously in here it's 3. The null hypothesis is still testing on all α_j =0. Under the null, J_1 follows the same unconditional central F-distribution as J does.

TABLE 5 below shows all the summary of pricing errors of both the time-series tests of the CAPM and the Fama French three-factor time-series regressions, together with their multivariate tests and associated P-values.

Table 5 Summary of all Average Absolute Pricing Errors and Multivariate Tests

In bracket under GRS multivariate tests are associated P-values

Time Period	Time-Series Regressions of	25-Portfolios: Average Absolute Pricing Error	25-Portfolios: GRS Multivariate Tests	11-Portfolios: Average Absolute Pricing Error	11-Portfolios: GRS Multivariate Tests
Jan1931-Dec2005	CAPM	0.23	3.45 (0.00)	0.12	2.01 (0.02)
	FF model	0.16	3.31 (0.00)	0.11	3.46 (0.00)
Jan1931-Jun1963	CAPM	0.22	1.63 (0.03)	0.13	1.64 (0.09)
	FF model	0.20	1.64 (0.03)	0.15	2.64 (0.00)
Jul1963-Dec2005	CAPM	0.31	4.40 (0.00)	0.14	1.43 (0.16)
	FF model	0.10	3.38 (0.00)	0.09	1.80 (0.05)

Fama French (1996) claimed that FF three-factor model would outperform CAPM in all aspects. One of their evidence is the average absolute pricing errors. As the above table 5 shows, in all the 25-portfolio estimations, time-series tests show the CAPM have higher average absolute pricing error than FF model. Only in period 1931 to June 1963, they get close (0.22 versus 0.20). The same patterns apply to the 11-portfolio estimations in period 1931 to 2005 and period 1963 to 2005 though the difference between the CAPM pricing errors and the FF pricing errors are smaller. The only exception is the period 1931 to 1963. Under this situation, FF model (0.15) has a higher pricing error than CAPM (0.13).

Let's look through all the multivariate tests. In all the 25-portfolio estimations, the GRS tests are significant under 5% level, which would suggest that, both CAPM and FF three-factor model are not true. However, in the 11-portfolio case, except for the whole period of 1931 to 2005, other sub-periods all have insignificant GRS tests of the CAPM. Whereas for FF model, the only period shows relatively insignificant GRS test is period 1963 to 2005. In the sub-period 1963 to 2005, both CAPM and FF three-factor model hold true.

4. The Conclusion

To summarize, a comment on all the cross-sectional and time-series multivariate tests of MV CAPM is necessary.

First, I confirm several well-known results in my project. 1) Some of the tests still support CAPM, and portfolios formed on dividend yield settings show a better support of CAPM than the size and BE/ME data. 2) With respect to the Fama French three-factor model, CAPM are outperformed in some cases. 3) However, as the t-stats shows, both of the models have unexplained deviations and both of them do not have satisfactory alphas, which shows a model failure for both of them.

Second, there are many critiques associated with the cross-sectional tests of the CAPM in section 3.3. First of all, the CAPM predicts that the slope term γ_1 is equal to the average excess market return, however, Fama and French (1992) test whether it is equal to zero. Second, the estimated γ_1 in some sub-periods is negative. Although some would argue it shows the CAPM is problematic, this can result from other reasons. Appendix Table A3 shows, the betas have a small spread especially in the period 1963 to 2005. So if one does the cross-sectional tests of CAPM using this small spread of betas, the slope coefficient may be biased and the power of the tests's low.

I also have reported some maybe questionable but interesting finding with respect to the interaction of several time periods between two datasets. First, in the cross-sectional tests, CAPM seems to outperform the FF model in terms of the smaller abnormal performance biases (the intercept term); second, in most of the cross-sectional and multivariate tests, estimation for sub-period July 1963 to 2005 is the one that supports CAPM worst no matter for 25-portfolio data or 11-portfolio data, but supports FF three-factor model best; whereas for sub-period 1931 to June 1963, the results suggest a strong support of both CAPM and FF model for both 25-porfolio data and 11-portfolio data. Since in Fama French (1996), they employed a dataset from July 1963 to 1993 to conclude CAPM was dominated by FF model, I wonder if there would be a period (data) selection bias.

Finally, as suggested by the Roll (1977)'s critique, empirically, my project still concludes that "tests of the CAPM are ambiguous at best". (See Grauer (2002), "Asset Pricing Theory and Tests", *The International Library of Critical Writings in Financial Economics*)

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Appendix

TABLE A 1

Summary statistics of average excess returns and standard deviations

25 portfolios formed on intersections of 5 portfolios formed on size (market equity, ME) and 5 portfolios formed on the ratio of book equity to market equity (BE/ME). They are monthly average value weighted returns. Panel A shows the summary for means, panel B shows the summary for standard deviations.

Panel A: Summary Statistics for Means

		BE/ME Quintiles				
		Low	2	3	4	High
SIZE	Small	0.88	1.27	1.48	1.67	1.86
	2	1.01	1.32	1.45	1.53	1.65
	3	1.09	1.25	1.32	1.38	1.54
	4	1.01	1.09	1.28	1.35	1.50
	Big	0.92	0.91	1.02	1.11	0.59

Panel B: Summary Statistics for Standard Deviations

		BE/ME Quintiles				
		Low	2	3	4	High
SIZE	Small	12.44	10.85	9.18	8.70	9.67
	2	8.09	7.98	7.43	7.68	8.76
	3	7.76	6.59	6.82	6.83	8.75
	4	6.23	6.25	6.44	7.12	9.16
	Big	5.44	5.22	5.58	7.03	11.24

TABLE A 2

Summary statistics of average excess returns and standard deviations

11 portfolios formed on dividend yield (sorted by 10 deciles plus one zero/negative dividend yield portfolio). They are monthly average value weighted returns. Panel A shows the summary for means, panel B shows the summary for standard deviations.

Panel A: Summary Statistics for Means

	Zero/Negative Dividends
mean	1.14

	10 Deciles									
	Low10	Dec2	Dec3	Dec4	Dec5	Dec6	Dec7	Dec8	Dec9	Hi10
mean	0.97	1.01	0.93	1.04	0.93	1.01	1.08	1.21	1.19	1.12

Panel B: Summary Statistics for Standard Deviations

	Zero/Negative Dividends
Standard deviation	8.84

	10 Deciles									
	Low10	Dec2	Dec3	Dec4	Dec5	Dec6	Dec7	Dec8	Dec9	Hi10
Standard deviation	6.26	5.67	5.44	5.40	5.62	5.51	5.21	5.90	5.97	6.61

TABLE A 3

The Time-Series Regressions on CAPM

This table reports the (BJS) time-series regression results. The model is:

$$r_{jt} - r_{ft} = \alpha_j + \beta_j (r_{mt} - r_{ft}) + e_{jt}$$

Panel A shows the time-series regression over 900 months, from January 1931 to December 2005 on 25 portfolios formed on size and BE/ME.
Panel B shows the time-series regression over 390 months, from January 1931 to June 1963 on 25 portfolios formed on size and BE/ME.
Panel C shows the time-series regression over 510 months, from July 1963 to December 2005 on 25 portfolios formed on size and BE/ME.
Panel D shows the time-series regression over 900 months, from January 1931 to December 2005 on 11 portfolios formed on dividend yield.
Panel E shows the time-series regression over 390 months, from January 1931 to June 1963 on 11 portfolios formed on dividend yield.
Panel F shows the time-series regression over 510 months, from July 1963 to December 2005 on 11 portfolios formed on dividend yield.

The time-series univariate test is testing H0: $\alpha_j = 0$ for each portfolio j in each time period. Portfolios written in bold are rejection cases under the 5% significance level.

Panel A: Estimation Summary of 25 Portfolios from January 1931 to December 2005

Portfolios	β_j	t-statistic for β_j	α_j	t-statistic for α_j	Associated P-value for α_j's	R^2
R1	1.66	31.45	-0.53	-1.85	0.07	0.52
R2	1.50	33.96	-0.04	-0.17	0.86	0.56
R3	1.39	43.32	0.25	1.41	0.16	0.68
R4	**1.33**	**43.80**	**0.48**	**2.89**	**0.00**	**0.68**
R5	**1.40**	**37.77**	**0.62**	**3.05**	**0.00**	**0.61**
R6	1.26	47.18	-0.14	-0.96	0.34	0.71
R7	1.30	55.05	0.15	1.18	0.24	0.77
R8	**1.20**	**54.76**	**0.34**	**2.83**	**0.01**	**0.77**
R9	**1.24**	**53.90**	**0.40**	**3.18**	**0.00**	**0.76**
R10	**1.37**	**48.38**	**0.43**	**2.76**	**0.01**	**0.72**
R11	1.30	64.38	-0.09	-0.78	0.44	0.82
R12	**1.13**	**75.48**	**0.19**	**2.35**	**0.02**	**0.86**
R13	**1.17**	**73.50**	**0.24**	**2.73**	**0.01**	**0.86**
R14	**1.14**	**61.84**	**0.31**	**3.12**	**0.00**	**0.81**
R15	**1.41**	**54.32**	**0.29**	**2.04**	**0.04**	**0.77**
R16	1.07	77.04	-0.01	-0.16	0.87	0.87
R17	1.10	92.76	0.05	0.71	0.48	0.91
R18	**1.11**	**76.08**	**0.23**	**2.94**	**0.00**	**0.87**
R19	**1.20**	**65.17**	**0.25**	**2.46**	**0.01**	**0.83**
R20	1.47	53.24	0.21	1.40	0.16	0.76
R21	0.96	101.37	-0.03	-0.60	0.55	0.92
R22	0.92	96.20	-0.01	-0.24	0.81	0.91
R23	0.99	71.75	0.06	0.75	0.45	0.85
R24	1.15	58.21	0.03	0.26	0.79	0.79
R25	1.16	20.28	-0.49	-1.56	0.12	0.31

Panel B: Estimation Summary of 25 Portfolios for Sub-Period Jan 1931 to Jun 1963

Portfolios	β_j	t-statistic for β_j	α_j	t-statistic for α_j	Associated P-value for α_j's	R^2
R1	1.78	19.60	-0.65	-1.09	0.28	0.50
R2	1.66	22.28	-0.41	-0.84	0.40	0.56
R3	1.58	31.10	0.11	0.34	0.74	0.71
R4	1.52	32.56	0.33	1.07	0.29	0.73
R5	1.62	26.98	0.53	1.34	0.18	0.65
R6	1.15	30.22	0.03	0.13	0.90	0.70
R7	1.37	38.13	0.20	0.87	0.38	0.79
R8	1.31	38.94	0.22	0.99	0.32	0.80
R9	1.40	40.81	0.25	1.12	0.26	0.81
R10	1.57	36.14	0.28	0.98	0.33	0.77
R11	1.26	47.16	0.08	0.48	0.63	0.85
R12	1.14	56.10	0.17	1.27	0.21	0.89
R13	1.28	63.49	0.18	1.39	0.17	0.91
R14	1.27	49.95	0.16	0.96	0.34	0.87
R15	1.67	45.63	-0.07	-0.30	0.76	0.84
R16	0.96	70.10	0.03	0.37	0.71	0.93
R17	1.11	74.26	0.10	0.99	0.32	0.93
R18	1.19	61.22	0.16	1.23	0.22	0.91
R19	1.37	54.74	0.00	-0.02	0.99	0.89
R20	1.76	44.19	-0.05	-0.19	0.85	0.83
R21	0.94	93.67	0.01	0.09	0.93	0.96
R22	0.90	83.59	-0.08	-1.10	0.27	0.95
R23	1.07	59.89	0.00	0.01	0.99	0.90
R24	1.38	54.58	-0.19	-1.13	0.26	0.88
R25	1.37	12.98	-1.38	-2.00	0.05	0.30

Panel C: Estimation Summary of 25 Portfolios for Sub-Period Jul 1963 to Dec 2005

Portfolios	β_j	t-statistic for β_j	α_j	t-statistic for α_j	Associated P-value for α_j's	R^2
R1	1.46	28.94	-0.44	-1.95	0.05	0.62
R2	1.24	28.25	0.25	1.26	0.21	0.61
R3	**1.09**	**30.05**	**0.36**	**2.26**	**0.02**	**0.64**
R4	**1.00**	**28.79**	**0.60**	**3.92**	**0.00**	**0.62**
R5	**1.03**	**27.10**	**0.70**	**4.14**	**0.00**	**0.59**
R6	1.45	38.35	-0.28	-1.67	0.10	0.74
R7	1.18	38.49	0.12	0.85	0.40	0.74
R8	**1.04**	**37.55**	**0.44**	**3.58**	**0.00**	**0.74**
R9	**0.98**	**34.92**	**0.52**	**4.20**	**0.00**	**0.71**
R10	**1.06**	**31.86**	**0.55**	**3.76**	**0.01**	**0.67**
R11	1.37	43.24	-0.22	-1.54	0.12	0.79
R12	**1.11**	**48.32**	**0.21**	**2.06**	**0.04**	**0.82**
R13	**0.97**	**41.52**	**0.28**	**2.73**	**0.01**	**0.77**
R14	**0.91**	**36.99**	**0.44**	**3.99**	**0.00**	**0.73**
R15	**1.00**	**32.47**	**0.58**	**4.25**	**0.00**	**0.67**
R16	1.26	52.99	-0.05	- 0.51	0.61	0.85
R17	1.07	55.59	0.01	0.09	0.93	0.86
R18	**0.98**	**45.19**	**0.30**	**3.10**	**0.00**	**0.80**
R19	**0.91**	**39.29**	**0.45**	**4.32**	**0.00**	**0.75**
R20	**1.00**	**33.77**	**0.43**	**3.26**	**0.00**	**0.69**
R21	1.01	58.89	-0.06	-0.81	0.42	0.87
R22	0.95	56.97	0.04	0.48	0.63	0.86
R23	0.85	41.16	0.10	1.13	0.26	0.77
R24	0.78	32.10	0.21	1.93	0.05	0.67
R25	0.83	26.98	0.20	1.48	0.14	0.59

Panel D: Estimation Summary of 11 Portfolios from January 1931 to December 2005

Portfolios	β_j	t-statistic for β_j	α_j	t-statistic for α_j	Associated P-value for α_j's	R^2
R1	1.51	73.01	-0.17	-1.55	0.12	0.86
R2	1.07	72.63	-0.05	-0.61	0.54	0.85
R3	0.99	87.45	0.04	0.60	0.55	0.89
R4	0.94	77.86	0.00	0.00	1.00	0.87
R5	0.94	84.44	0.10	1.68	0.09	0.89
R6	0.96	72.82	-0.01	-0.19	0.85	0.86
R7	0.94	71.04	0.08	1.13	0.26	0.85
R8	**0.88**	**68.02**	**0.19**	**2.69**	**0.01**	**0.84**
R9	**0.99**	**64.04**	**0.25**	**2.92**	**0.00**	**0.82**
R10	**0.97**	**55.64**	**0.24**	**2.52**	**0.01**	**0.78**
R11	1.00	42.48	0.15	1.16	0.25	0.67

Panel E: Estimation Summary of 11 Portfolios for Sub-Period Jan 1931 to Jun 1963

Portfolios	β_j	t-statistic for β_j	α_j	t-statistic for α_j	Associated P-value for α_j's	R^2
R1	1.53	52.32	-0.21	-1.12	0.27	0.88
R2	1.00	49.71	-0.12	-0.93	0.36	0.86
R3	0.95	64.34	0.11	1.11	0.27	0.91
R4	0.89	61.47	-0.12	-1.29	0.20	0.91
R5	0.92	74.28	0.15	1.82	0.07	0.93
R6	0.99	62.58	-0.02	-0.21	0.83	0.91
R7	0.97	58.19	0.06	0.51	0.61	0.90
R8	**0.89**	**53.73**	**0.23**	**2.08**	**0.04**	**0.88**
R9	1.09	53.55	0.22	1.61	0.11	0.88
R10	1.12	50.96	0.17	1.19	0.24	0.87
R11	1.27	46.00	-0.06	-0.31	0.75	0.85

Panel F: Estimation Summary of 11 Portfolios for Sub-Period Jul 1963 to Dec 2005

Portfolios	β_j	t-statistic for β_j	α_j	t-statistic for α_j	Associated P-value for α_j's	R^2
R1	1.48	48.54	-0.14	-1.06	0.29	0.82
R2	1.18	55.08	0.00	0.02	0.99	0.86
R3	1.06	59.96	-0.02	-0.23	0.82	0.88
R4	1.02	51.40	0.09	1.02	0.31	0.84
R5	0.97	49.55	0.07	0.76	0.45	0.83
R6	0.91	40.99	-0.01	-0.06	0.96	0.77
R7	0.88	41.28	0.10	1.08	0.28	0.77
R8	0.86	41.11	0.16	1.76	0.08	0.77
R9	**0.82**	**36.46**	**0.28**	**2.77**	**0.01**	**0.72**
R10	**0.72**	**28.90**	**0.30**	**2.73**	**0.01**	**0.62**
R11	**0.55**	**17.42**	**0.32**	**2.32**	**0.02**	**0.37**

TABLE A 4

The Cross-Sectional Test of the CAPM

This table reports the OLS cross-sectional regression results. The model is:

$$\bar{r}_j - \bar{r}_f = \gamma_0 + \gamma_1 \hat{\beta}_j + e_j,$$

Where $\bar{r}_j$ and $\bar{r}_f$ are time-series average rates of r_j and r_f. $\hat{\beta}_j$ is collected from the above time-series regressions in TABLE III.

Panel A shows the cross-sectional regression over 900 months, from January 1931 to December 2005 on 25 portfolios formed on size and BE/ME.

Panel B shows the cross-sectional regression over 390 months, from January 1931 to June 1963 on 25 portfolios formed on size and BE/ME.

Panel C shows the cross-sectional regression over 510 months, from July 1963 to December 2005 on 25 portfolios formed on size and BE/ME.

Panel D shows the cross-sectional regression over 900 months, from January 1931 to December 2005 on 11 portfolios formed on dividend yield.

Panel E shows the cross-sectional regression over 390 months, from January 1931 to June 1963 on 11 portfolios formed on dividend yield.

Panel F shows the cross-sectional regression over 510 months, from July 1963 to December 2005 on 11 portfolios formed on dividend yield.

Panel A: Estimation Summary of Cross-Sectional Test on 25 Portfolios from Jan 1931 to Dec 2005. The regression R^2 is 0.17. In bracket is the associated standard error.

	Estimation Coefficient	t-stat	Associated P-value
Slope γ_1	0.6820 (0.3144)	2.17	0.04
Intercept γ_0	0.1091 (0.3931)	0.28	0.78

Panel B: Estimation Summary of Cross-Sectional Test on 25 Portfolios from Jan 1931 to Jun 1963. The regression R^2 is 0.21. In bracket is the associated standard error.

	Estimation Coefficient	t-stat	Associated P-value
Slope γ_1	0.7418 (0.3020)	2.46	0.02
Intercept γ_0	0.2542 (0.4133)	0.62	0.55

Panel C: Estimation Summary of Cross-Sectional Test on 25 Portfolios from Jul 1963 to Dec 2005. The regression R^2 is 0.15. In bracket is the associated standard error.

	Estimation Coefficient	t-stat	Associated P-value
Slope γ_1	-0.5411 (0.2695)	-2.01	0.06
Intercept γ_0	1.2981 (0.2896)	4.48	0.00

Panel D: Estimation Summary of Cross-Sectional Test on 11 Portfolios from Jan 1931 to Dec 2005. The regression R^2 is 0.07. In bracket is the associated standard error.

	Estimation Coefficient	t-stat	Associated P-value
Slope γ_1	0.1509 (0.1865)	0.81	0.44
Intercept γ_0	0.5991 (0.1920)	3.12	0.01

Panel E: Estimation Summary of Cross-Sectional Test on 11 Portfolios from Jan 1931 to Jun 1963. The regression R^2 is 0.43. In bracket is the associated standard error.

	Estimation Coefficient	t-stat	Associated P-value
Slope γ_1	0.5552 (0.2287)	2.43	0.04
Intercept γ_0	0.4317 (0.2455)	1.76	0.11

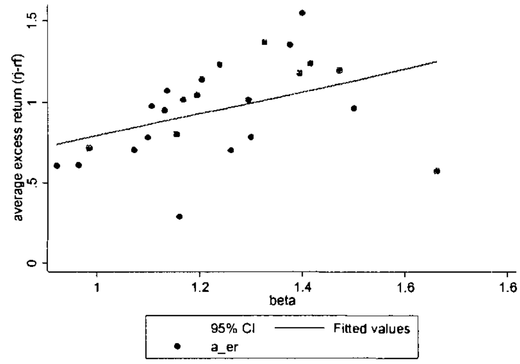
Panel F: Estimation Summary of Cross-Sectional Test on 11 Portfolios from Jul 1963 to Dec 2005. The regression R^2 is 0.15. In bracket is the associated standard error.

	Estimation Coefficient	t-stat	Associated P-value
Slope γ_1	-0.0800 (0.0876)	-0.91	0.39
Intercept γ_0	0.6250 (0.0857)	7.29	0.00

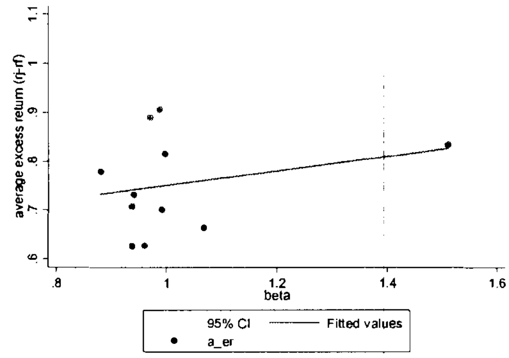
GRAPH A 1

Scatter Plot the Values of $\bar{r}_j - \bar{r}_f$ versus β_j's with Fitted Regression Line

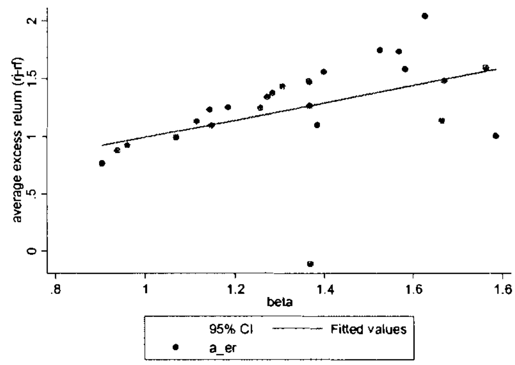
On the vertical axis is the average excess return, which is $\bar{r}_j - \bar{r}_f$ for every portfolio j. On the X-axis are the associated portfolio betas.



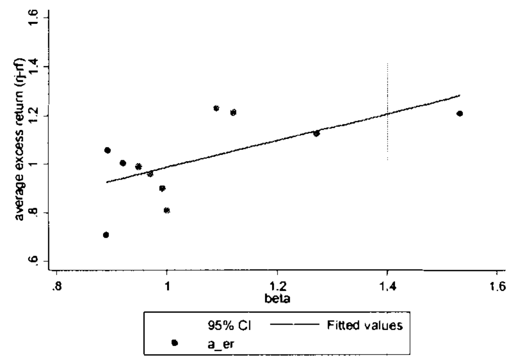
25 Portfolios, from Jan 1931 to Dec 2005



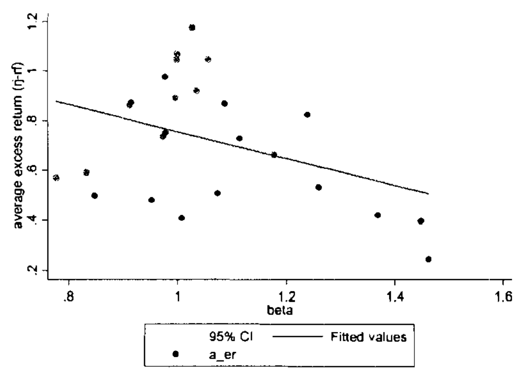
11 Portfolios, from Jan 1931 to Dec 2005



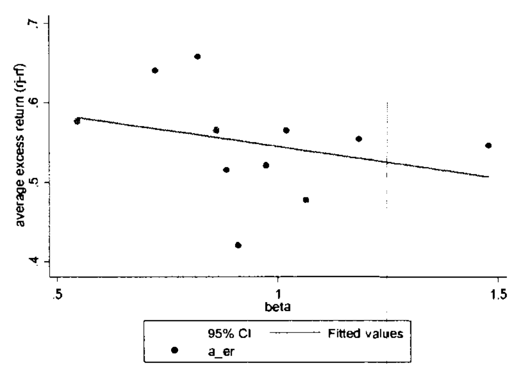
25 Portfolios, from Jan 1931 to Jun 1963



11 Portfolios, from Jan 1931 to Jun 1963



25 Portfolios, from Jul 1963 to Dec 2005



11 Portfolios, from Jul 1963 to Dec 2005

TABLE A 5

The Time-Series Regressions of the Fama-French Three-Factor Model

This table reports the Fama French time-series regression results. We run the regression:

$$r_{jt} - r_{ft} = \alpha_j + b_{mj}(r_{mt} - r_t) + b_{SMBj} r_{SMBj} + b_{HMLj} r_{HMLj} + e_{jt}$$

This time-series test is testing H0: $\alpha_j = 0$ for each portfolio j. Portfolios written in bold are rejection cases under the 5% significance level.

Panel A shows the time-series regression over 900 months, from January 1931 to December 2005 on 25 portfolios formed on size and BE/ME.
Panel B shows the time-series regression over 390 months, from January 1931 to June 1963 on 25 portfolios formed on size and BE/ME.
Panel C shows the time-series regression over 510 months, from July 1963 to December 2005 on 25 portfolios formed on size and BE/ME.
Panel D shows the time-series regression over 900 months, from January 1931 to December 2005 on 11 portfolios formed on dividend yield.
Panel E shows the time-series regression over 390 months, from January 1931 to June 1963 on 11 portfolios formed on dividend yield.
Panel F shows the time-series regression over 510 months, from July 1963 to December 2005 on 11 portfolios formed on dividend yield.

Panel A: Estimation Summary of Fama French Three-Factor Model on 25 Portfolios from January 1931 to December 2005

Portfolios	b_{mj}	b_{SMBj}	b_{HMLj}	α_j	t-statistics for α_j	Associated P-value for α_j	R^2
R1	**1.32**	**1.31**	**0.44**	**-0.91**	**-3.67**	**0.00**	**0.65**
R2	**1.09**	**1.68**	**0.38**	**-0.46**	**-2.99**	**0.00**	**0.83**
R3	1.08	1.18	0.47	-0.11	-1.06	0.29	0.88
R4	0.98	1.20	0.60	0.07	0.95	0.35	0.94
R5	0.97	1.36	0.92	0.07	0.86	0.39	0.93
R6	**1.09**	**1.05**	**-0.27**	**-0.23**	**-2.62**	**0.01**	**0.90**
R7	1.06	0.99	0.21	-0.09	-1.32	0.19	0.94
R8	0.97	0.86	0.37	0.07	1.08	0.28	0.94
R9	0.98	0.81	0.57	0.07	1.15	0.25	0.95
R10	1.05	0.92	0.85	-0.02	-0.28	0.78	0.96
R11	**1.17**	**0.78**	**-0.19**	**-0.15**	**-2.26**	**0.02**	**0.93**
R12	1.02	0.50	0.09	0.08	1.28	0.20	0.92
R13	1.03	0.41	0.35	0.05	0.77	0.44	0.93
R14	0.96	0.46	0.52	0.06	0.95	0.35	0.93
R15	1.17	0.49	0.92	-0.11	-1.47	0.14	0.94
R16	1.07	0.28	-0.36	0.06	1.14	0.26	0.93
R17	1.03	0.22	0.14	-0.04	-0.66	0.51	0.92
R18	1.01	0.21	0.33	0.09	1.31	0.19	0.91
R19	1.06	0.18	0.61	0.01	0.14	0.89	0.92
R20	**1.25**	**0.32**	**1.01**	**-0.19**	**-2.20**	**0.03**	**0.93**
R21	1.03	-0.16	-0.24	0.08	1.90	0.06	0.95
R22	0.96	-0.19	0.00	0.02	0.43	0.67	0.92
R23	0.98	-0.22	0.32	-0.02	-0.27	0.79	0.90
R24	**1.08**	**-0.17**	**0.73**	**-0.19**	**-3.01**	**0.00**	**0.93**
R25	**1.04**	**0.00**	**0.75**	**-0.74**	**-2.46**	**0.01**	**0.37**

Panel B: Estimation Summary of Fama French Three-Factor Model on 25 Portfolios for Sub-Period January1931 to June 1963

Portfolios	b_{mj}	b_{SMBj}	b_{HMLj}	α_j	t-statistics for α_j	Associated P-value for α_j	R^2
R1	1.14	0.94	1.10	-0.90	-1.70	0.09	0.61
R2	**1.13**	**2.07**	**0.29**	**-0.83**	**-2.58**	**0.01**	**0.81**
R3	1.16	1.26	0.40	-0.16	-0.69	0.49	0.87
R4	1.03	1.39	0.52	0.03	0.19	0.85	0.95
R5	0.88	1.61	1.02	0.16	0.99	0.32	0.94
R6	0.98	1.07	-0.10	-0.17	-0.99	0.32	0.86
R7	1.09	1.14	0.12	-0.02	-0.20	0.84	0.94
R8	1.00	1.02	0.26	0.00	0.04	0.97	0.95
R9	1.01	0.96	0.47	0.04	0.37	0.71	0.96
R10	0.99	0.98	0.92	0.03	0.28	0.78	0.96
R11	1.12	0.76	-0.03	-0.06	-0.53	0.60	0.93
R12	1.04	0.53	-0.01	0.07	0.67	0.51	0.94
R13	1.13	0.46	0.15	0.09	0.86	0.39	0.95
R14	1.00	0.59	0.36	0.02	0.23	0.82	0.95
R15	1.18	0.42	0.98	-0.21	-1.63	0.10	0.95
R16	1.02	0.12	-0.21	0.02	0.26	0.79	0.94
R17	1.03	0.29	0.07	0.04	0.45	0.65	0.95
R18	1.05	0.34	0.17	0.08	0.76	0.45	0.94
R19	1.10	0.16	0.57	-0.07	-0.56	0.58	0.94
R20	1.25	0.35	1.07	-0.18	-1.18	0.24	0.94
R21	1.05	-0.05	-0.25	0.03	0.68	0.50	0.98
R22	0.96	-0.10	-0.10	-0.05	-0.80	0.43	0.96
R23	0.97	-0.22	0.34	0.02	0.24	0.81	0.93
R24	1.11	-0.12	0.71	-0.21	-1.81	0.07	0.95
R25	**1.07**	**0.13**	**0.66**	**-1.45**	**-2.12**	**0.03**	**0.33**

Panel C: Estimation Summary of Fama French Three-Factor Model on 25 Portfolios for Sub-Period July 1963 to December 2005

Portfolios	b_{mj}	b_{SMBj}	b_{HMLj}	α_j	t-statistics for α_j	Associated P-value for α_j	R^2
R1	**1.07**	**1.37**	**-0.32**	**-0.46**	**-4.39**	**0.00**	**0.92**
R2	0.96	1.31	0.06	0.01	0.17	0.87	0.94
R3	0.92	1.10	0.29	0.03	0.49	0.63	0.95
R4	**0.90**	**1.03**	**0.45**	**0.19**	**3.07**	**0.00**	**0.94**
R5	**0.98**	**1.08**	**0.68**	**0.15**	**2.24**	**0.03**	**0.94**
R6	**1.12**	**0.98**	**-0.40**	**-0.20**	**-2.68**	**0.01**	**0.95**
R7	1.03	0.88	0.18	-0.12	-1.73	0.08	0.94
R8	0.98	0.75	0.42	0.09	1.40	0.16	0.93
R9	0.98	0.71	0.59	0.08	1.30	0.19	0.93
R10	1.08	0.84	0.78	-0.02	-0.25	0.80	0.95
R11	1.08	0.73	-0.46	-0.07	-0.94	0.35	0.95
R12	1.06	0.50	0.22	0.01	0.09	0.93	0.90
R13	1.02	0.43	0.51	-0.07	-0.94	0.35	0.89
R14	1.01	0.39	0.66	0.00	0.01	0.99	0.90
R15	1.11	0.52	0.84	0.03	0.32	0.75	0.89
R16	**1.05**	**0.37**	**-0.45**	**0.15**	**2.10**	**0.04**	**0.94**
R17	**1.10**	**0.20**	**0.26**	**-0.17**	**-2.10**	**0.04**	**0.88**
R18	1.08	0.17	0.51	-0.01	-0.19	0.85	0.88
R19	1.04	0.22	0.62	0.06	0.81	0.42	0.88
R20	1.17	0.25	0.82	-0.08	-0.86	0.39	0.86
R21	**0.96**	**-0.27**	**-0.39**	**0.20**	**3.56**	**0.00**	**0.94**
R22	1.04	-0.22	0.14	-0.01	-0.18	0.86	0.90
R23	0.98	-0.22	0.31	-0.04	-0.47	0.64	0.84
R24	1.00	-0.21	0.62	-0.11	-1.68	0.09	0.88
R25	**1.07**	**-0.09**	**0.78**	**-0.23**	**-2.29**	**0.02**	**0.79**

Panel D: Estimation Summary of Fama French Three-Factor Model on 11 Portfolios from January 1931 to December 2005

Portfolios	b_{mj}	b_{SMBj}	b_{HMLj}	α_j	t-statistics for α_j	Associated P-value for α_j	R^2
R1	**1.35**	**0.67**	**0.11**	**-0.33**	**-3.83**	**0.00**	**0.92**
R2	1.11	0.00	-0.28	0.05	0.63	0.53	0.88
R3	1.04	-0.08	-0.18	0.11	1.91	0.06	0.91
R4	0.99	-0.16	-0.12	0.07	1.07	0.28	0.89
R5	0.97	-0.16	0.04	0.12	2.00	0.05	0.90
R6	0.98	-0.18	0.13	-0.03	-0.39	0.70	0.87
R7	0.94	-0.14	0.19	0.04	0.61	0.54	0.87
R8	**0.88**	**-0.12**	**0.17**	**0.15**	**2.28**	**0.02**	**0.86**
R9	0.95	-0.10	0.37	0.14	1.90	0.06	0.87
R10	0.92	-0.07	0.45	0.10	1.28	0.20	0.85
R11	0.88	0.04	0.73	-0.11	-1.11	0.27	0.82

Panel E: Estimation Summary of Fama French Three-Factor Model on 11 Portfolios for Sub-Period January 1931 to June 1963

Portfolios	b_{mj}	b_{SMBj}	b_{HMLj}	α_j	t-statistics for α_j	Associated P-value for α_j	R^2
R1	**1.25**	**0.53**	**0.43**	**-0.34**	**-2.42**	**0.02**	**0.93**
R2	1.10	0.04	-0.26	-0.11	-0.92	0.36	0.88
R3	1.04	-0.06	-0.19	0.13	1.45	0.15	0.93
R4	0.96	-0.07	-0.14	-0.10	-1.10	0.27	0.92
R5	**0.95**	**-0.17**	**0.02**	**0.18**	**2.34**	**0.02**	**0.94**
R6	1.01	-0.11	0.01	0.00	-0.01	0.99	0.91
R7	0.94	-0.02	0.07	0.06	0.51	0.61	0.90
R8	**0.87**	**0.03**	**0.03**	**0.22**	**2.02**	**0.04**	**0.88**
R9	0.99	-0.01	0.25	0.20	1.60	0.11	0.89
R10	1.02	0.01	0.24	0.16	1.12	0.26	0.88
R11	1.05	0.15	0.47	-0.11	-0.73	0.47	0.89

Panel F: Estimation Summary of Fama French Three-Factor Model on 11 Portfolios for Sub-Period July 1963 to December 2005

Portfolios	b_{mj}	b_{SMBj}	b_{HMLj}	α_j	t-statistics for α_j	Associated P-value for α_j	R^2
R1	1.26	0.68	-0.23	-0.12	-1.32	0.19	0.92
R2	**1.10**	**-0.06**	**-0.34**	**0.20**	**2.28**	**0.02**	**0.88**
R3	1.03	-0.09	-0.18	0.10	1.23	0.22	0.89
R4	1.05	-0.23	-0.07	0.16	1.91	0.06	0.86
R5	1.05	-0.14	0.17	-0.01	-0.11	0.91	0.85
R6	1.03	-0.20	0.27	-0.13	-1.40	0.16	0.82
R7	1.02	-0.20	0.35	-0.06	-0.78	0.43	0.84
R8	1.01	-0.19	0.40	-0.03	-0.45	0.65	0.86
R9	0.98	-0.13	0.48	0.02	0.25	0.80	0.83
R10	0.90	-0.07	0.58	-0.01	-0.16	0.87	0.78
R11	0.75	0.01	0.76	-0.11	-1.05	0.30	0.64