

**THE INTRA- AND INTERSTATE EXCESS MARKET
RETURN FOR LOW- AND HIGH-VOLATILITY LEVEL**

by

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APPROVAL

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Degree: **M. A. (Economics)**
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ABSTRACT

This paper provides a method to estimate the market risk premium that accounts for shifts in investment opportunities by explicitly modeling the volatility level. This method decomposes the market risk premium into two components, the expected risk premium within the volatility state and risk premium associated with an unexpected change in volatility state. I find that the expected risk premium within the state is negatively related to the market volatility, which is consistent with empirical observations. Whereas the positive relationship between estimated market risk premium and market volatility is consistent with the theoretical relationship. Finally, evidences show the simple historical average of excess market returns overstate in the high-volatility state and understate in the low-volatility state. Therefore, different market risk premium should be used accordingly for different volatility level.

Keyword: Excess market return; Risk premium; Volatility; Markov-switching model

DEDICATION

TO MY FAMILY, especially my mother

To Grace

For their unconditional support, throughout my academic career

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1. Introduction

The market risk premium is a number frequently required for the solution of many investment and corporate finance problems, especially for asset pricing models. The most common practice to estimate the market risk premium is using the simple average of historical excess market returns. Unfortunately, the accuracy of this method to measure market risk premium is questionable. A substantial body of research shows that excess market returns vary over time. French, Schwert and Stambaugh (1987) find evidence of a positive relation between the expected risk premium on common stocks and the predictable level of volatility. They also suggest future work in expected risk premiums and predictable movements in volatility should be tested. Merton (1980) uses the theoretical relationship between expected returns and the contemporaneous variance of returns to estimate the market risk premium. The empirical research of the theoretical relationship has failed to find a significant positive relationship between expected returns and the level of market volatility. Scruggs (1998) suggests that the failure of finding positive relationship between expected market returns and market volatility is probably a result of not controlling for shifts in investment opportunities. Mayfield (2004) provides a method of estimating the market risk premium using the equilibrium relationship between volatility and expected returns by controlling the discrete shift in the level of market volatility. This method can successfully obtain the positive relationship between market volatility and expected market risk premium. However, the estimation and computation

process have shown complicated. In this paper, I develop a method to estimate the market risk premium by controlling for the shifts in investment opportunities based on the relationship between expected market returns and the market volatility.

The volatility of excess market returns during the past century has varied significantly. French et al. (1987) shows that volatility and expected risk premiums have fluctuated widely over the period of 1927 to 1987. Schwert (1989a, b) studies historical variations in market volatility and relates the fluctuations to changes in economic and financial market conditions. Hamilton and Lin (1996) go beyond Schwert's analysis and find out economic recessions are the single largest factor, accounting for over 60% of the stock market's volatility. Mayfield (2004) suggests that over half of the measured risk premium is associated with the risk of future changes in market volatility level. Similar to Mayfield's (2004) findings, my results suggest that by controlling the shift in the level of market volatility, simple historical average of excess market returns usually overstate in the low market volatility period and understate in the high market volatility period.

In my analysis, market risk is characterized by periodic episodes of high market volatility followed by a return to a lower, more typical, level. I assume that the evolution of volatility follows a Markov process, and I model the market risk premium as a function of the underlying process governing the evolution of two volatility states. Many of the researchers have used a two-state Markov-switching model to describe the time series properties of market returns, including Schwert (1989a), Turner et al. (1989), Pagan and Schwert (1990), Hamilton and Lin (1996), Kim, Morley and Nelson, (2000) and Mayfield (2004). In much of this research, they have shown that the addition of a third volatility state is not statistically significant. Also, most of the research concludes

that market volatility increases during recessions. My analysis of historical average excess market return controlling for the two state volatility levels follows Merton (1980), which failed to find a positive relationship between average returns and market volatility. My estimation model incorporates the risk premium associated with the jump between low and high volatility states, which reconciles the empirical observation that market risk premiums are lower in periods of high volatility with the theoretical intuition that risk premiums should be positively related to the level of market volatility.

Hamilton's (1989) Markov-switching model proposes a formal statistical representation of two different economics phases. Hamilton and Lin (1996) use Hamilton's estimation method as the base to find the relationship between stock market volatility and business cycle. Pagan and Schwert (1990) compare several statistical models for monthly stock return volatility on U.S. data from 1834 to 1925. Although their result suggests that the Hamilton and the GARCH models produce weak explanations of the data, the usefulness of both estimation methods on monthly stock volatility cannot be denied. Mayfield (2004) also uses Hamilton's method to describe time series properties of the market risk premium. My estimation model maps directly into the standard empirical framework for estimating time variation in market volatility which follows the Markov-switching process. As a result, the time series parameters – specifically the transition probabilities between the two different volatility states – can be estimated using Hamilton's Markov-switching model.

Using Hamilton's model, my analysis shows that the excess market returns are drawn from two significantly different distributions: a low market volatility distribution, from which about 85.67% of the excess returns are drawn, and a high market volatility

distribution, from which are about 14.33% of the excess returns are drawn. In the low market volatility state, the mean annualized excess return is 7.33%. While in high market volatility state, the mean annualized excess return is -26.2%. The weighted average of both volatility states' annualized excess returns is 2.71%, which is consistent with the simple historical average excess return. My estimation model allows me to map directly into the estimated probabilities for the two conditional states from the Markov-switching model. I decompose the unconditional market risk premium into two state-dependent risk premiums, as well as into premiums required for intrastate diffusion risk and interstate jump risk. My estimates for the annualized state-dependent risk premium in the low-volatility state are 1.62% and high-volatility is 6.33%. My analysis suggests that most of the state-dependent risk premium is associated with the risk of future change in the level of market volatility.

The remainder of the paper is structured as follows. Section 2 presents the analytical model of risk premium with discrete volatility states. Section 3 describes the empirical framework used to identify and estimate the parameters of the model and report the results. In Section 4, I go into a further discussion and interpretation of my results. Finally, Section 5 summarizes the main findings of the paper.

2. A two-state model of the market risk premium

My market risk premium estimation begins with the assumption that the variance of market returns follows a two-state Markov process. To represent the two different states of economy, I will define $s \in (L, H)$ where L to be the low market volatility level state and H to be the high market volatility level state. The variance of returns at each instant is given by the following notations:

$$\sigma_s^2 = \begin{cases} \sigma_L^2 & \text{if } s = L \\ \sigma_H^2 & \text{if } s = H \end{cases} \quad (1)$$

Where σ_L^2 is the variance of returns in the normal low-volatility state and σ_H^2 is the variance of returns in the abnormal high-volatility state. To concentrate the estimates on the risk premium associated with the jump between two volatility states, the investors are assumed to know the current volatility with certainty and face a possibility of a change in volatility at each point in time. Following the Markov process, the probability of a change in volatility is a function of the current state.

$$\pi^s = \begin{cases} \pi^L & \text{if } s = L \\ \pi^H & \text{if } s = H \end{cases} \quad (2)$$

π^L is the probability of switching out of the low-volatility state. Once investors are in the low-volatility state, they face this π^L probability of switching to the high-volatility state. Similarly, π^H is the probability of switching out of the high-volatility state.

Whenever, the investors are in the high-volatility state, they face this π^H probability of

switching to the low-volatility state. These transition probabilities can be viewed as the uncertainty of the future being faced by the investors while they are at a certain state. In this environment, investors are compensated for the current volatility of excess market returns as well as the uncertainty associated with a change in volatility.

Before looking at the risk premium model, it is necessary to examine the decomposition of expected market return. In the market, investors realize expected return will not be the same as expected within-state return because wealth changes when the economy changes state. Therefore, the expected return on the market is given by the following equation:

$$E[R_t] = \mu_t + \pi_t J_t \quad (3)$$

where $E[R_t]$ is the expected market returns at time t , μ_t is the expected return conditional on staying in the current state, π_t is the transition probability of a switch from the current volatility to the other volatility at time t to time $t + 1$ and J_t is the percentage change in wealth associated with the change in volatility.

Expression (3) shows the expected market return is not equal to the expected intrastate return, which is consistent with the trend for historical returns on the market. When economy is in low-volatility state, investors expect a reduction in wealth when the economy enters the high-volatility state. For this reason, in the low volatility state, the expected return on the market is less than the expected intrastate return. Similarly, investors will expect an increase in wealth when the economy reenters into the low-volatility from the high-volatility state. For this reason, the overall expected return on the market will be greater than the expected intrastate return. Figure (1) is illustrated in a way to show the relationship between the state-dependent expected market returns

and intrastate market returns. For each state, the slope of the line labeled “Expected market return” shows the required returns and the slope of the line labeled “Expected intrastate market return” shows the expected returns conditional on the economy remaining in the current state. The jump in wealth associated with a change in volatility state is represented by the vertical line segments at the boundary of low- and high-volatility state.

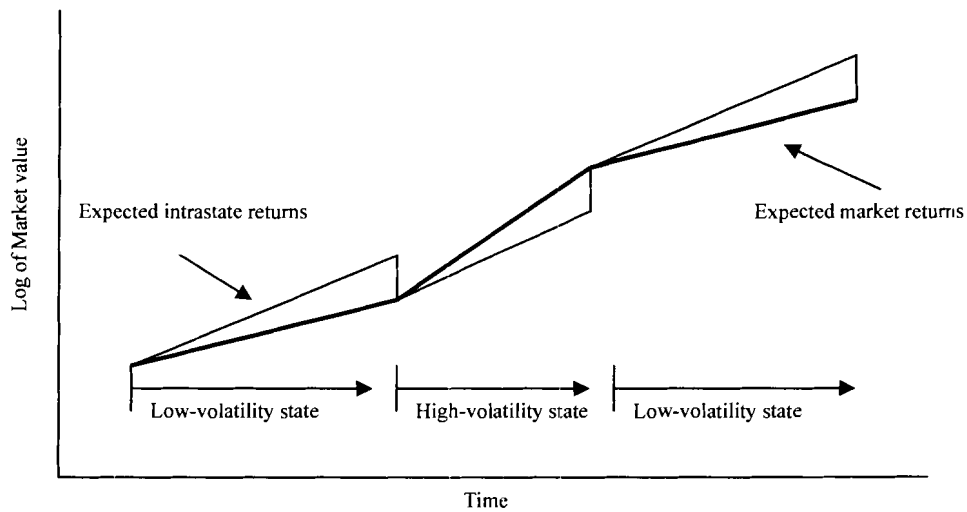


Figure 1 – Expected Market returns versus expected intrastate returns¹.

The vertical axis is the log of market value and the horizontal axis represents the time. The economy is initially in the low-volatility state, then enters into the high-volatility state and reenters into the low-volatility state. The slope of the bold line is the required return in each volatility state. The slope of the thin line represents the expected return jump conditional on two volatility states. The vertical line segment at the boundary of the states represents the jump in wealth associated with the switch between the two states.

To model the excess market returns into the expected risk premium within the volatility state and uncertainty risk premium from the change in volatility state, I derive Eq. (3) into the following form.

$$E[R_t] - R_t^f = \mu_t - R_t^f + \pi_t J_t \quad (4)$$

¹ “Expected Market returns versus expected intrastate returns” is taken from Mayfield (2004).

By subtracting Eq. (3) using R_t^f the contemporaneous risk-free rate of return, I obtained the state-dependent expected market risk premium that is decomposed into two components. The first term $\mu_t - R_t^f$ on the right side of Eq. (4) is the component that accounts for the diffusion risk of the current volatility state, which I refer to as the intrastate market risk premium. The second term $\pi_t J_t$ on the right side of Eq. (4) is the component that captures the jump risk from the unexpected change in volatility state, which I refer to as the interstate market risk premium. Because there are only two volatility states, no uncertainty exists over the magnitude of the future change in volatility. Therefore, uncertainty exists only over the time at which level of volatility will change. In the market, investors are only interested in the market risk premium for the volatility state that they are currently at. In order to analyze the state-dependent market risk premium, the intra- and interstate risk premium component must also depend on the state of the economy. For this reason, I revise Eq.(4) into a state-dependent s expression rather than time-dependent t expression, As a result, Eq. (4) becomes the following:

$$E[R_s] - R_s^f = \mu_s - R_s^f + \pi^s J_s \quad (5)$$

The relationships of expected market risk premiums and expected intrastate risk premium from Eq. (4) and (5) will follow the analysis for the expected market returns in Eq. (3). In the low-volatility state, the expected intrastate risk premium is greater than the required market risk premium. In high-volatility state, the expected intrastate risk premium is less than the required market risk premium. If the expected interstate risk premium is sufficiently large, then the market risk premium is positive in high-volatility state even though the historical expected intrastate market risk premium is negative. This model provides a plausible explanation for reconciling the empirical observation that

expected intrastate risk premium is lower in periods of high volatility with the theoretical intuition that expected market risk premium should be positively related to the level of market volatility.

3. Model Estimation

This section presents the results from the estimated model on market risk premium.

3.1. Data

The model described in Section 2 is estimated using data from the Canadian Financial Markets Research Centre (CFMRC) Summary Information Database. I use TSX composite monthly total return index² over the period from February 1956 through November 2004 to represent the monthly market returns. The monthly total return index is preferred over the daily total return index because investors might not perceive the current volatility state within days, which might lead to delays in recognizing the switch between the two volatility states. For this reason, the daily market returns might not be representative enough to show the switch between volatility states. Therefore, excess market returns are calculated as the TSX monthly returns minus the risk-free rate using the contemporaneous yield on 30 day Treasury bills.

The average annualized excess market returns over the sample period is 2.71% and the annualized standard deviation of excess market returns is 15.69%. The largest and

² The S&P/TSX Composite Total Return Index is based on the prices and distributions (stocks, dividends, etc.). The ten sectors being included are Energy, Materials, Industrials, Consumer Discretionary, Consumer Staples, Health Care, Financials, Information Technology, Telecommunications Services and Utilities Sectors.

smallest monthly excess market returns are 14.61% and -26.52%, respectively. The reported skewness measure is -0.9601, indicating that large negative returns are more frequent than large positive returns. Finally the measure of excess kurtosis is 3.68 indicates that large returns occur more frequently than would be the case if returns were normally distributed. These findings are consistent with Mayfield (2004), where time variation in market volatility produces excess kurtosis in stock returns and large negative excess market returns happen more frequently than positive market returns. Table 1 summarizes the historical statistics for monthly excess returns within the period from 1956 through 2004.

Table 1 – Descriptive statistics for monthly excess returns, 1956 – 2004

Excess returns are calculated by the monthly returns on S&P/TSX in excess of the contemporaneous yield on one-month Treasury bills. Data are obtained from Canadian Financial Markets Research Centre (CFMRC) Summary Information Database.

Statistics	
Mean (annualized)	2.71%
Standard deviation (annualized)	15.69%
Maximum (monthly)	14.61%
Minimum (monthly)	-26.52%
Skewness (monthly)	-0.9601
Excess Kurtosis (monthly)	3.68
Number of observations	586

3.2 Methodology

To estimate the components of the market risk premium in each volatility state, I need to use a three-step process. In the first step, I use Hamilton³ (1989) Markov-switching model to estimate the moments of the two state-dependent return distributions σ_s and the transition probabilities π^s that govern the dynamics of the underlying volatility process. In addition, the implied probability of low-volatility state, transition probability of low- to high-volatility state and transition probability of high- to low-volatility state for each month in the sample period are also estimated from Hamilton's (1989) Markov-switching model. To classify the months into low- and high-volatility states for the sample period, I use the monthly implied probability of low-volatility state from Hamilton's (1989) method. If the implied probability of the low-volatility state for the month is greater than 0.5, then I categorize this month into low-volatility state. Similarly, if the probability of the low-volatility state for the month is less than 0.5, then I categorize this month into high-volatility state. To estimate the two components on the market risk premium model, it is also necessary to obtain the implied transition probabilities for each month t under the Markov process. These transition probabilities with respect to time and states are as follows:

$$\pi_t^s = \begin{cases} \pi_t^L & \text{if } s = L \text{ and } t = \text{month within the low - volatility state} \\ \pi_t^H & \text{if } s = H \text{ and } t = \text{month within the high - volatility state} \end{cases} \quad (6)$$

The notations in Eq. (6) for the implied transition probabilities are as follows. If the

³ I applied Hamilton's Markov-switching model econometrics package for estimating some of the parameters. The econometrics' package is on Dr. James D. Hamilton's website at <http://weber.ucsd.edu/~jhamilto>.

current month t is at the high-volatility state, then π_t^H is the probability of next month $t+1$ in the low-volatility state. Similarly, if current month t is at the low-volatility state, then π_t^L is the probability of next month $t+1$ in the high-volatility state. These estimated transition probabilities are the uncertainty that the investors are facing within the month. Therefore, the implied transition probability for the month in low-volatility state is π_t^L and high-volatility state is π_t^H . In the second step, I can use these estimated parameters to estimate the market risk premium components using the following expression.

$$R_t - R_t^f = \alpha + \beta\pi_t^s + \varepsilon_t \quad (7)$$

Where $R_t - R_t^f$ is the actual monthly excess market return, π_t^s is the current month's implied transition probability, α is the estimated intrastate risk premium, β is the estimated change in wealth associated with a jump to the other volatility state and ε_t is the error term with zero mean and variance σ^2 . To estimate α and β for the two states, the monthly excess market returns for the periods in the state, as well as the monthly state-dependent transition probabilities are used. At low-volatility state, the two parameters are estimated using the monthly excess market returns and implied transition probability π_t^L for low-volatility state's months. Similarly, the high-volatility state's parameters are estimated using the monthly excess market returns and implied transition probability π_t^H for high-volatility state's months. Because the monthly implied transition probabilities are state-dependent, introducing a state dummy variable S_t into Eq. (7) will be more convenient to obtain the estimates for the two volatility states.

$$S_t = \begin{cases} 1 & \text{if the month is within the high - volatility state} \\ 0 & \text{if the month is within the low - volatility state} \end{cases} \quad (8)$$

Combining Eq.(7) and (8), I can derive the following dummy variable model for estimation:

$$R_t - R_t^f = \alpha + \beta_1 S_t + \beta_2 S_t \pi_t^H + \beta_3 (1 - S_t) \pi_t^L + \varepsilon_t^4 \quad (9)$$

Using this expression, I can estimate the state-dependent expected intrastate risk premium and percentage change in wealth that involves a jump between the volatility states. In the low-volatility state, the estimated expected intrastate risk premium is α , and percentage change in wealth is β_3 . In the high volatility state, the estimated expected intrastate risk premium is $\alpha + \beta_1$, and percentage change in wealth is β_2 . In the third step, I can use the estimated parameters to map directly into Eq. (5) as follows:

$$\begin{array}{ll} \text{In the low - volatility state} & \mu_L - R_L^f = \alpha \quad J_L = \beta_3 \\ \text{In the high - volatility state} & \mu_H - R_H^f = \alpha + \beta_1 \quad J_H = \beta_2 \end{array} \quad (10)$$

With Eq. (10) and the transition probabilities π^s for the two states, I can use the expression given by Eq. (5) to calculate the expected market risk premium as well as the intrastate and interstate components of the risk premium for each volatility state.

3.3. Results

Table 2 reports the estimated results from the first and second steps of my three-step process for estimating the market risk premium. Panel A provides the transition probabilities of the two states from applying the Markov-switching model to my sample of returns. In the Appendix, I also displayed the implied low-volatility state probability

⁴ This dummy variable model gives the same results as doing two separate estimations for each state independently. To solve the heteroskedasticity problem that occur, I use ordinary least square regression with robust standard error.

and the implied transition probabilities for the months of my sample period. These probabilities are estimated using the method described in Hamilton (1989). Panel B reports the estimated values of the parameters α , $\alpha + \beta_1$, β_2 and β_3 using the expression from Eq. (9). Table 3 reports the implied decomposition of the monthly market risk premium by calculation using the mapping from Eq.(10), the transition probabilities from Table 2, Panel A and the expression given by Eq.(5). Finally, Table 4 reports the implied decomposition of the annualized market risk premium using the values from Table 3.

Table 2 – Estimated Parameters

Estimates are based on 586 monthly excess market returns from February, 1956 through November, 2004. Panel A reports the transition probabilities estimates for the two-states Markov switching model. Panel B reports the estimates from Eq. (9) for the expected intrastate risk premium and the percentage in wealth associated with the two states. Standard errors are reported in parentheses. The reported values are rounded to nearest 4 decimal places.

Volatility state	Estimated Parameters		
	Panel A	Panel B	
Low-volatility ($S = L$)	$\pi^L = 0.064$ (0.0184)	$\alpha = 0.0131$ (0.0018)	$\beta_3 = -0.1841$ (0.0371)
High-volatility ($S = H$)	$\pi^H = 0.1943$ (0.0663)	$\alpha + \beta_1 = -0.0491$ (0.0096)	$\beta_2 = 0.2783$ (0.0854)
Log-likelihood Value	1566.88		
Number of Observations	586		

Table 3 – Implied Risk Premium decomposition (monthly)

The monthly intrastate and interstate market risk premium is calculated base on Eq. (6). The monthly total market risk premium is the addition of the intra- and interstate market risk premium. The unconditional mean is calculated using the State Probabilities and the market risk premiums.

Volatility state	Risk Premium decomposition			
	State Probability	Intrastate	Interstate	Total
Low-volatility ($S = L$)	0.7512	0.01314	-0.01178	0.001352
High-volatility ($S = H$)	0.2488	-0.0488	0.054069	0.005273
Unconditional mean		-0.00227	0.004601	0.002328

Table 4 – Implied Risk Premium decomposition (annualized)

The annualized intrastate, interstate and total market risk premium is calculated base on the monthly value from Table 3. The unconditional mean is calculated using the State Probabilities and the market risk premiums

Volatility state	Risk Premium decomposition			
	State Probability	Intrastate	Interstate	Total
Low-volatility ($S = L$)	0.7512	0.15763	-0.14140	0.016225
High-volatility ($S = H$)	0.2488	-0.58555	0.64883	0.063278
Unconditional mean		-0.02728	0.055209	0.027932

Panel A of Table 2 reports the point estimates of the transition probabilities π^L and π^H that indicate 0.064 and 0.1943 probability of switching out of the low- and high-volatility states, respectively. If the market is in low-volatility, it is not likely that it will switch to high-volatility state. But in the high-volatility state, the probability of reentering low-volatility state is relatively higher. Base on the estimated state probabilities, the expected durations of the low- and high-volatility states are approximately 35.9 and 6.5 months, respectively. These results are consistent with

previous studies that use the Markov-switching model to describe the time series properties of returns on the US market, including Schwert (1989a), Turner et al. (1989), Pagan and Schert (1990), and Mayfield (2004).

Panel B of Table 2 reports the estimate parameters from the dummy variable model Eq. (9) for the expression of market risk premium Eq. (5). The estimated values for the percentage change in wealth associate with the jump between volatility states J_L and J_H are -0.1841 and 0.2783, respectively. These estimates on the jump parameters are significantly different from zero. From these estimates, it suggests wealth will decrease when switching into high-volatility state from low-volatility state and wealth will increase while reentering low-volatility state. The estimated values for the expected monthly intrastate risk premium in low- and high-volatility are 0.0131 and -0.0488, respectively. These estimated expected intrastate risk premiums for the two states are significantly different from zero. These expected intrastate returns are consistent with the average excess market returns on the historical data for the two states. When the market is in high-volatility state, there will be a negative average excess market returns. While the market is in low-volatility state, there will be positive average excess market returns.

Table 3 and 4 reports the implied decomposition of the market risk premium. The first column of both tables report the unconditional probability of each volatility state based on the transition probabilities presented in Panel A of Table 2. The second and third column of Table 3 shows the monthly intrastate and interstate components of the two state-dependent risk premiums and Table 4 shows the annualized intrastate and interstate risk premiums for the two states. Finally the fourth column of Table 3 and 4 show the monthly and annualized state-dependent market risk premium for each volatility

state. For each component of the risk premiums, the unconditional estimate is calculated as the state probability weighted average of the two state-dependent results. These unconditional components of risk premiums are reported in the forth row of Table 3 and Table 4. Based on the estimated transition probabilities, the unconditional probability of the economy being in the low- and high-volatility state is 0.7512 and 0.2488, respectively. The calculated annualized market risk premium for low-volatility state is 1.62%, where the annualized intrastate market risk premium is 15.8% and the reduction from the annualized interstate market risk premium is -14.1%. This shows the market risk premium in low volatility state is mostly associated with the intrastate market risk premium. Moreover, the calculated annualized market risk premium for high-volatility state is 6.33%. The intrastate risk premium is -58.6% and the interstate risk premium is 64.9%. This shows the market risk premium in high volatility state is mostly associated with the risk premium from the change in volatility state. The unconditional market risk premium is equal to 2.79% where most of the risk premiums are due to the unexpected risk from the future. This suggests one of the factors that cause the market risk premium to fluctuate is from the investors' expectation on the probability of switching volatility states in the future.

4. Further Discussion and Interpretation of Results

To estimate the state-dependent market risk premium using my three-step method, it is a necessary to classify each month into one of the two volatility states for my sample period during February 1956 through November 2004. This process relies heavily on the Markov-switching model using Hamilton's (1989) method. This method has been used in much research⁵ to estimate the stock market's volatility and has proven to explain the historical data of the market volatility. On this two-state estimation method, I define low-volatility periods for those months with the implied probability of being in the low-volatility state greater than 0.5. Based on this criterion, there are 14 low-volatility periods and 13 high-volatility periods during the sample period from 1956 to 2004. Of the 586 months in the sample, 502 months are categorized as low volatility and 85 months are categorized as high volatility. Therefore, about 85.67% of the sample period is in low-volatility state and 14.33% of the sample period is considered high-volatility state.

Figure 2 plots the historical excess market returns on which the model is estimated along with the identified high-volatility period represented by the shaded areas. Visual inspection of the figure suggests that the average duration of high-volatility periods is

⁵ Paper that used Markov-switching model to estimate the two-states volatility, includes Schwert (1989a), Turner et al. (1989), Pagan and Schwert (1990), Hamilton and Lin (1996), Kim, Morley and Nelson, (2000) and Mayfield (2004). I would like to thank Dr. Robert A. Jones to help me verify the historical causation of the switch between volatility states during my sample period.

shorter than low-volatility periods. The average duration of low-volatility periods is 35.86 months and the average duration of high-volatility periods is 6.54 months during the sample period from 1956 through 2004.

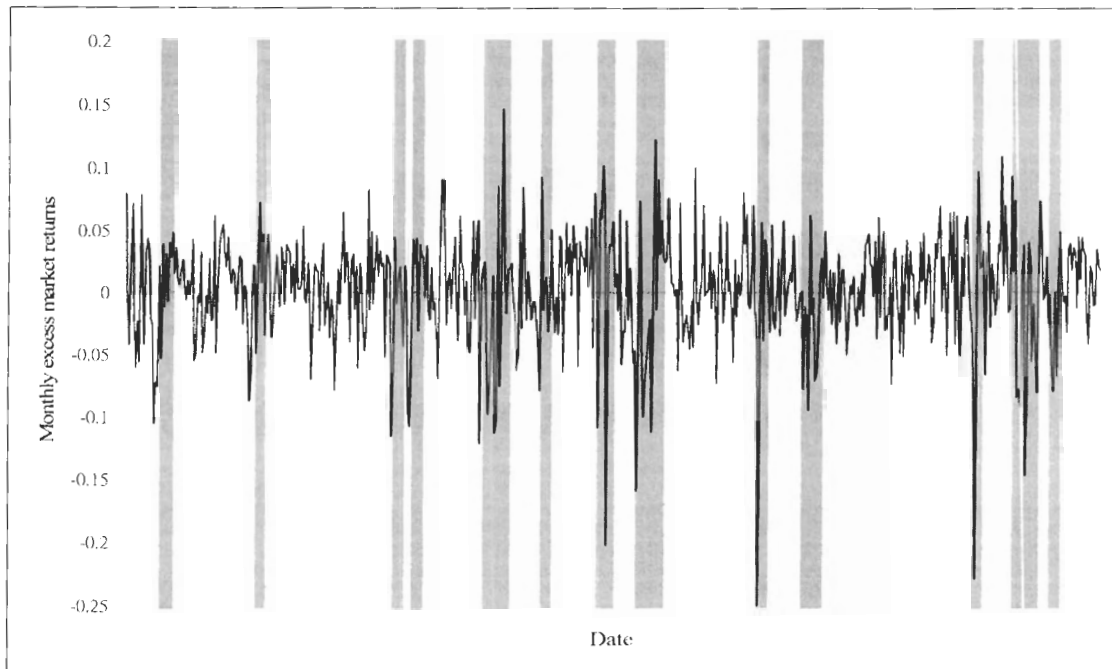


Figure 2 – Monthly excess market returns from 1956 through 2004

The solid line plots the monthly excess returns for the period 1956 through 2004. The shaded areas correspond to the high-volatility episodes identified by the low-volatility state implied probability less than 0.5.

By observing the historical excess market returns for both low- and high-volatility period, the first month's market risk premiums are frequently negative in high-volatility periods and positive in low-volatility periods. One explanation for this case is pointed out by Turner et al. (1989) and Mayfield (2004). They suggest investors do not have perfect knowledge of the current state and so they must infer the volatility from the market risk premium they observe. When the economy is in low-volatility state, the market risk

premium's fluctuation is small and determining whether the economy has switched to the high-volatility state is easy. However, the state recognizing process for the investors is more difficult when the economy is in high-volatility state. In the high-volatility state, small market risk premium occurs as well as large market risk premium. So when small market risk premium does occur investors cannot reveal the economy has switched to low-volatility state and they will need to learn that the economy has returned to the low-volatility state. This investors' environment is consistent with the findings from this paper. When the investor realize there is an increase in market value that yields a positive excess market return over a longer period of time, then it is most likely the market has entered the low-volatility state. Within this state, the investors are facing an uncertainty of switching to high-volatility state in the future and they will expect a negative change in wealth during the transition period of the jump. Therefore, the interstate risk premium enters the market risk premium's expression in the low-volatility state negatively. If the investors enter high-volatility state where 60% of the causation is by the economy recession as suggested in Hamilton and Lin (1996), they will expect a decrease in market value that yield a negative expected intrastate risk premium. In this state, the investors will face an uncertainty of reentering the low-volatility state that yields a positive percentage change in wealth during the transition period. Therefore, the interstate risk premium enters the expression for the high-volatility state's market risk premium in a positive manner. These results are consistent with the time path of expected returns depicted by Figure 1 in the theoretical discussion of the model. In addition, returns during the transition between volatility states are also generally consistent with those depicted in Figure 1.

My results also suggest the market risk premium depends on the level of the transition probability for the two states. If the transition probability for switching out of the low-volatility state is high, then magnitude of interstate risk premium might exceed the expected intrastate risk premium. This might lead to a negative market risk premium at the transition period. Likewise, if the transition probability of reentering into low-volatility state is relatively high then the interstate risk premium might again exceed the expected intrastate risk premium and leads to a large positive market risk premium for the corresponding transition month. In this environment, investors are expecting an increase of wealth when high-volatility state switches to low-volatility state. Therefore, investors will still invest into the market at high-volatility state even if the expected market risk premium is negative. From the Markov-switching model, the estimate for the transition probability of switching out of low-volatility state is 6.4%, which yields a market risk premium of 1.62% in the low-volatility state. Meanwhile, the estimated transition probability of switching out of the high-volatility state is 19.4%, which yields a market risk premium of 6.33%. These findings show the market risk premium is positively related to the market volatility level, which is consistent with the theoretical relationship between the market risk premium and volatility.

5. Conclusion

The objective of this paper is to present a method to estimate the market risk premium that incorporates shifts in investment opportunities, specifically the volatility level. In this paper, I have used Canadian market monthly returns from February 1956 through November 2004 to estimate the market risk premium. My result for the annualized market risk premiums in low-volatility state is 1.62% and in high-volatility state is 6.33%. The historical simple average market risk premium is 2.71%. These numbers suggest the market risk premiums in the two volatility states are differ from the historical average risk premium. In the low-volatility state, historical average market risk premium will overstates the market risk premium. While in the high-volatility state, historical average market risk premium understates the market risk premium. For this reason, simple historical average of market risk premium cannot explain the historical relationship between market risk premium and volatility, as well as the theoretical relationship between market risk premium and volatility. Much empirical research has tried to find a positive theoretical relationship between the market risk premium and market volatility. However, many were unsuccessful in supporting the theoretical relationship when the shift in investment opportunities is not controlled for.

My market risk premium model controls for the shift in investment opportunities by decomposing the market risk premium into two components – the intrastate risk premium and interstate risk premium. Using this decomposition, it is possible to explain the risk

premiums that are required for intrastate diffusion risk as well as the interstate unexpected jump risk. The investors that are facing these risks must be compensated by the market risk premiums. Theoretically, investors' should have a bigger compensation if the market is more volatile. This is because investors' are facing more uncertainty on the level of growth in market value. But empirically, investors' suffer larger negative returns when the market is a high-volatility environment. For that reason, historical average market risk premium in high-volatility state is negative. My estimated results suggest the expected intrastate risk premium is positive when the market is in low-volatility state and negative in high-volatility state. This finding on the expected intrastate risk premium component explains the historical relationship of market risk premium and volatility level. When I incorporate the interstate risk premium component with the expected intrastate risk premium, the expected market risk premium is larger in high-volatility state than in low-volatility state. This finding on the expected market risk premium explains the theoretical relationship of the market risk premium and volatility level.

In conclusion, my three steps method reconciles the empirical finding that market risk premiums are lower in periods of high volatility with the theoretical intuition that expected market risk premiums should be positively related to the level of market volatility. The empirical relationship is explained by the relationship between expected intrastate risk premium and volatility level. Nevertheless, the theoretical relationship is explained using the expected market risk premium. The results for the expected market risk premium in the two volatility states are very different. Hence different market risk premium should be used for different volatility states instead of applying the average historical market risk premium for both states.

Appendix

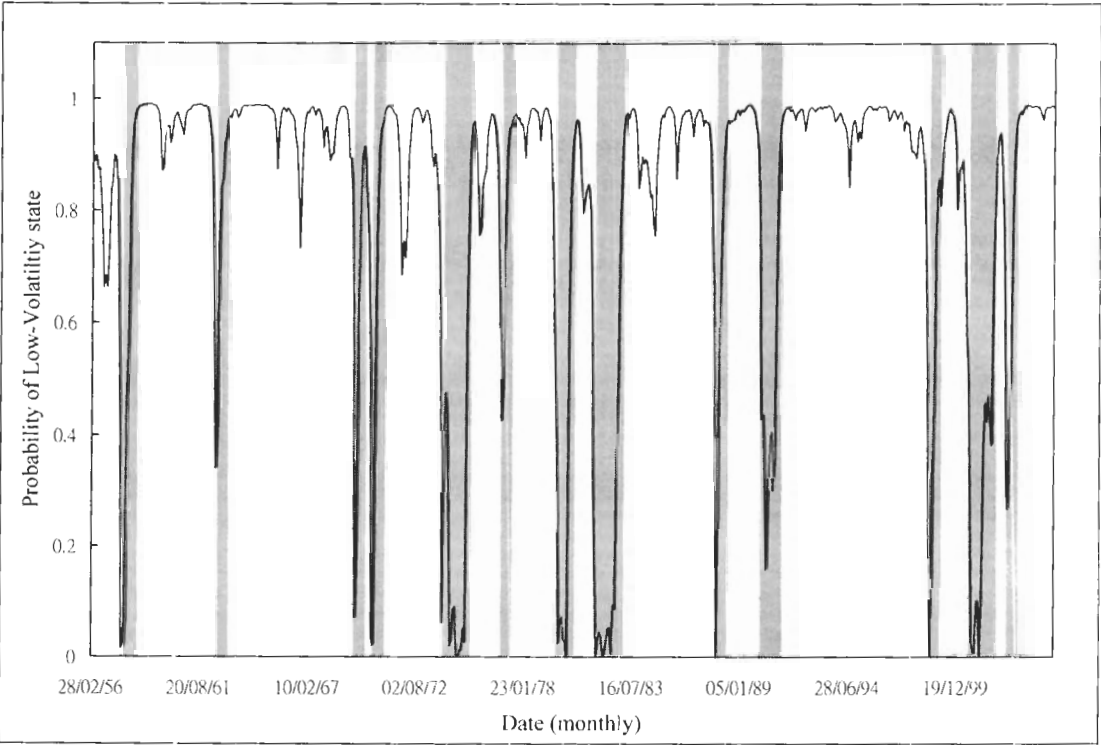


Figure 3 – Implied probability of low-volatility state from 1956 through 2004

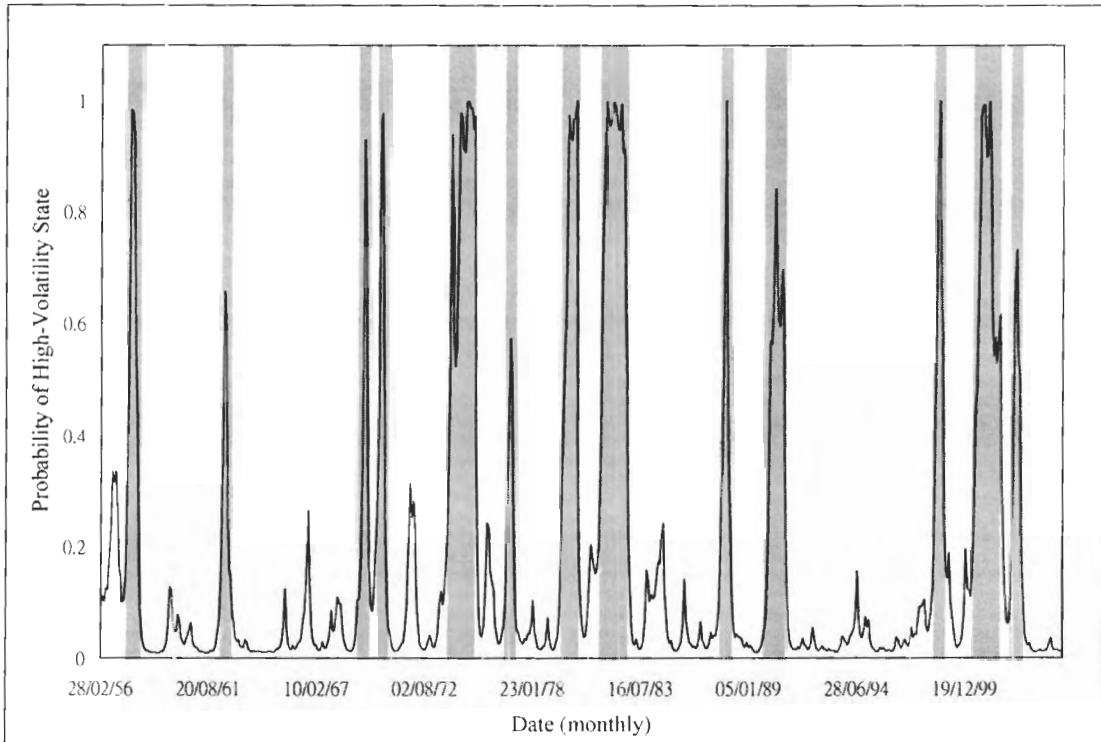


Figure 4 – Implied probability of high-volatility state from 1956 through 2004

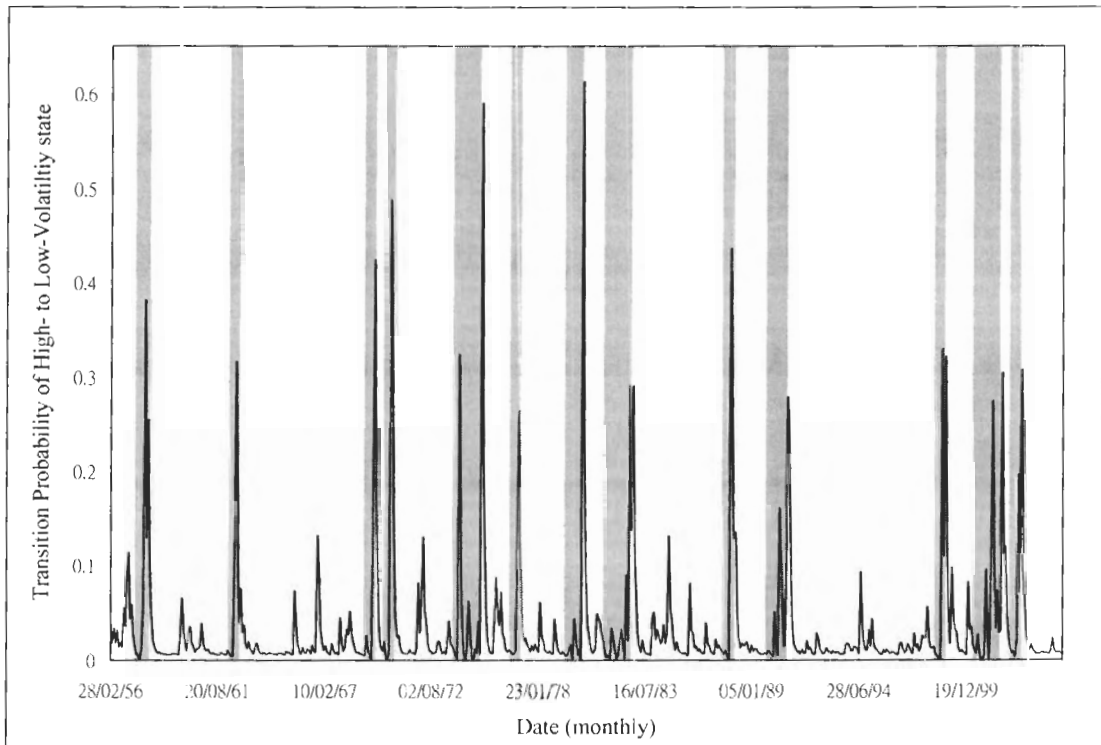


Figure 5 – Implied monthly transition probability of high- to low-volatility state from 1956 through 2004

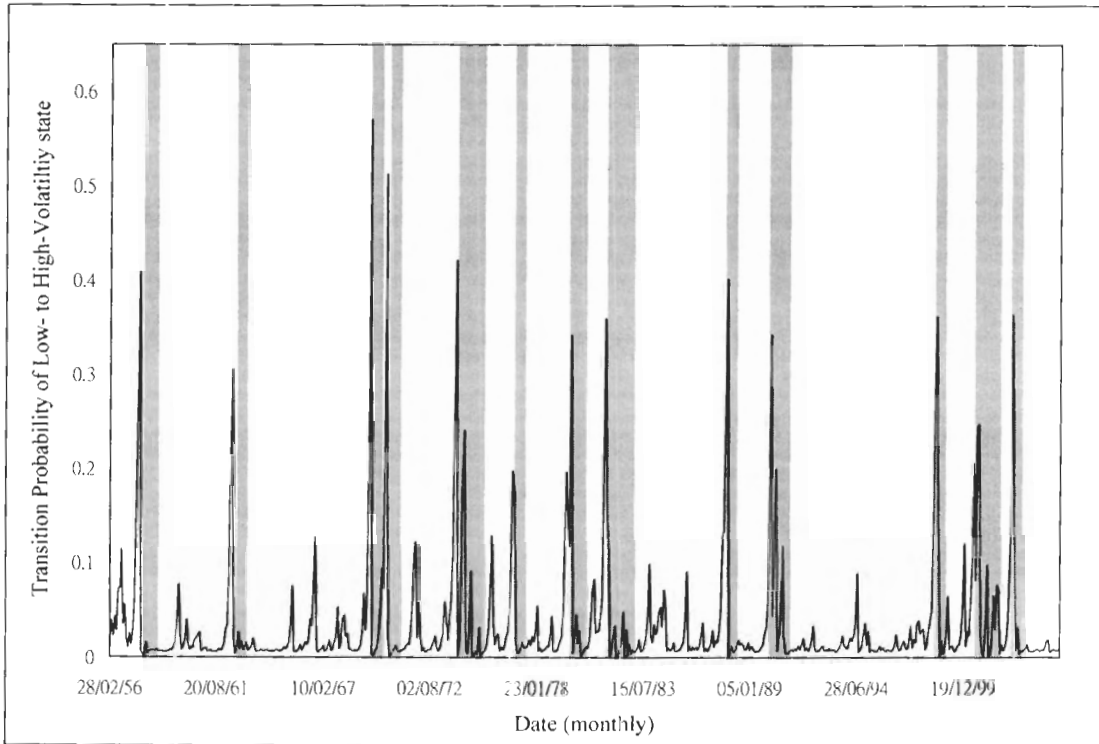


Figure 6 – Implied monthly transition probability of low- to high-volatility state from 1956 through 2004

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