ENERGY INCOME AND OPTIMAL INVESTMENT POLICY FOR THE ALBERTA HERITAGE SAVINGS TRUST FUND

by

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ABSTRACT

The present paper illustrates the impact of the inclusion of the economic value of government energy income in the optimal asset allocation decision for the Alberta Heritage Savings Trust Fund. Existing investment policy does not consider exogenous income when determining asset mixes. The result is an over-allocation of capital to energy stocks, higher volatility and lower expected utility among all government assets. Two tests are conducted to examine the differences in asset allocations when energy resources are considered to be a nontradable asset in an expanded portfolio and when the allocation decision only involves financial assets. The first test assumes that the existing asset mix is ideal. An optimiser is calibrated to produce parameter estimates that result asset weights consistent with existing industry sector weights as of March 31, 2005. The second test uses parameter variables estimated using historical data. Associated Sharpe Ratios are compared to determine whether the investor receives an economic benefit from the new portfolio weights.

When compared to existing industry weights within the financial portfolio, the optimal portfolio mix decision will always exclude energy stocks except in those instances where the nontradable asset is smaller than the optimal allocation to this industry sector. In the latter case, the values are highly improbable and so I conclude that the Alberta Heritage Savings Trust Fund should not be investing its funds in the energy sector. I also show that over allocation prevents the fund from more effectively diversifying its holdings.

Keywords: ***Alberta Heritage Savings Trust Fund, Asset Allocation, and Nontraded Assets.***

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1 INTRODUCTION

Investors are impacted in their savings-consumption decisions by both state dependent earnings as well as from income derived from savings. In practice, investment managers have tended to separate the investment policy impacting an individual's investment portfolio from his human capital and other illiquid assets. The resulting asset mix decision fails to consider variables exogenous to the investment portfolio and their impact upon total wealth. Rather than the investment portfolio acting to hedge state variables impacting earnings, the asset allocation is done without reference to nontradable assets. This may function to increase an investor's exposure to these risks.

In the case of the Alberta Heritage Savings Trust Fund (AHF), the fund invested approximately 8% of its total liquid assets in the energy sector in 2005. If the AHF incorporated into the asset mix decision a reference to nontradable energy income, one would expect to see the investments contained in the fund act to hedge state factors impacting this exogenous income. The resulting asset allocation would likely not include any energy component. In order to demonstrate the impact of this nontradable asset I test to see if its inclusion will alter the asset mix both assuming that the asset allocation in the AHF is optimal (as at March 31, 2005) and then by determining optimal weights within the fund by estimating mean-variance-covariance parameters using historical data. In both cases, once a constraint representing the nontradable energy asset is added to the portfolio, the allocation to energy disappears. These optimal weights for the expanded portfolio are then tested to assess whether the investor would benefit from the change. Sharpe Ratios are calculated to compare the change in risk adjusted returns to the expanded portfolios. In each instance, the optimal portfolios that constrain investment in energy stocks have higher Sharpe Ratio than those portfolios that fail to constrain allocation in the fund's investment policy.

The paper proceeds as follows: in Section Two I review the literature exploring nontradable assets and portfolio choice; in Section Three the methodology by which I test for the impact of this nontradable asset is detailed. In Section Four the results of the two tests are discussed and I comment on the findings of the tests.

2 LITERATURE REVIEW

Much has been written about illiquid assets and their impact on portfolio choice. While most of the research in this area has been focused on portfolio choice with tradable assets, fewer have focused on illiquid or non-tradable assets. One of the earlier papers to focus on exogenous income was Joseph Liberman's landmark study on human capital's influence on the financial capital markets (1980). The context of his study was Sharpe and Lintner's single period Capital Asset Pricing Model. Liberman relaxed assumptions of liquidity to include human capital to see if its inclusion would influence individual utility maximization in portfolio composition choices. He concluded that human capital had no impact on both capital asset pricing and portfolio composition. Despite this conclusion, many academics hold that Liberman's conclusions were erroneous and that human capital should influence investment policy.

Supporting the notion that human capital should be included as part of a portfolio optimisation program is Campbell and Vinceira. In their book, Strategic Asset Allocation (2002) they discuss the impact of these flows in optimal asset mix decisions. They focus their discussion on the variability of these human capital flows and their correlation with risky asset returns. They note that positive correlation of labour income to risky assets will affect an investor's optimal asset mix. Bodie, Merton and Samuelsen (1992) recognized the variability of risk in human capital. They noted that an individual's ability to adjust labour income increases their ability to take on risk.

Expanding the study to include illiquid assets aside from human capital, Fraser, Jennings and King (2000) argue that individuals should incorporate the impact of Social Security benefits as part of their asset mix decision. They demonstrate that excluding these government benefits from the asset allocation decision results in decreased portfolio efficiency and sub-optimal financial portfolios. They compared their estimated present value of Social Security benefits to both an investors' financial portfolio and their "expanded portfolio". The expanded portfolio included the impact of these government retirement benefits. In order to estimate the present value of the benefits they likened them to the benefits received from an inflation-indexed treasury. Using varied allocation models, Fraser et al. determined that the impact on the optimal asset mix was statistically significant.

In his paper, "Portfolio Choice and Asset Pricing with Nontraded Assets" Lars Svensson (1988) argues that nontraded assets are an integral component in determining optimal portfolios. He notes the existence of nontraded assets arises from a variety of sources – including a government's right to future tax receipts. The problem varies from standard continuous time portfolio problems in that, in addition to portfolio income, the investor has exogenous stochastic income stemming from a nontraded asset. He relates the portfolio problem with the nontraded asset by adding a state variable to model the risk in this asset. He notes that it is inherently difficult to price a claim to income and therefore this makes the process of finding optimal portfolios even more difficult. He states that "knowing the implicit value of the claim facilitates very much the interpretation of the optimal portfolio of traded assets."[1] The value of this nontraded asset is subject to the value of three variables: level of wealth, the state variable and time. With the nontradable income included, the optimisation problem changes from one focusing on liquid assets to one that forces the investor to choose risky assets subject to current wealth levels, state variables and consumption. Influences on optimal portfolios with exogenous nontraded income include the income hedge portfolio and a state variable hedge portfolio. The latter is a portfolio, which minimizes variance of the random variable and portfolio return. He finds that the optimal portfolio in continuous time can be written as a linear combination of the riskless asset, tangency portfolio, the income hedge portfolio and the state variable hedge portfolio.

Stephan Cauley, Andrey Pavlov and Eduardo Schwartz (2005) examine the impact of nontradable assets and investment policy in the case of individuals who have the bulk of their wealth invested in their home. Their paper "Home Ownership as a Constraint on Asset Allocation" attempts to measure the impact the constrained asset allocation will have on post retirement wealth. They find that the individual would need a material increase in their net worth to realize the utility of wealth if their real estate was a liquid and divisible asset. The reason is that allocation to equity securities is constrained by the wealth invested in the home. The larger the ratio of real estate costs to wealth, the greater the constraint. This results in higher volatility of wealth and reduced expected utility. The importance of the paper to this study on optimal asset allocations for the AHF is reflected in the view that the portfolio of assets to be allocated extends

[1] Svensson, 1988, page 11

beyond the investment portfolio to the entire portfolio of assets that impacts the investor's total wealth and future consumption.

Hyeng Keun Koo discusses the investor's assessment of nontradable exogenous income and its relationship to optimal consumption and asset allocation policies in the paper entitled, "Consumption and Portfolio Selection with Labour Income: A Continuous Time Approach" (1998). Koo demonstrates that constrained liquidity and unhedged income risk results in lower consumption as compared to a situation where financial assets do not have the liquidity constraint. From this relationship an optimal investment policy is derived for risky assets. This is modelled by way of a function. The first term of this function describes the proportional mean-variance efficient portfolio while the second term describes the exogenous income hedge portfolio. This second term is critical as it has the lowest correlation with exogenous income growth rate.

Lucie Tepla's work "Optimal Hedging and Valuation of Nontraded Assets" (2000) takes the position that exogenous nontradable income cannot be perfectly hedged in absence of a replicating portfolio. As a result the optimal investment policy for traded assets becomes dependent upon the hedge portfolio and the valuation of the nontraded asset. The difficulty in determining optimal investment policy for traded assets is hampered by difficulties in valuing the nontraded asset.

Rui de Figueiredo (2003) notes that illiquid assets are important elements in investment portfolios, as they tend to have lower correlated return behaviour relative to liquid assets. Consequently, these assets become important in diversifying investment portfolios. In his paper, "Asset Allocation with Illiquid Investments: A New Approach" he notes that illiquid assets tend to be poorly represented in portfolios due to the difficulty in marking to market and therefore determining both returns and standard deviation or risk. This also means that covariance parameters are difficult to estimate. He focuses on mechanisms used for the valuation of these assets and attempts to construct comparisons to traded assets. He notes that practitioners tend to treat tradable and nontradable assets as separate portfolios. As a consequence, optimal asset sizes and asset correlations are ignored. He states that, "investors who construct their holdings with a mental division between portfolios often wind up in a place they did not want, or expect to be."[2] Recognizing diverse methods for valuing illiquid investments, he takes the position that smoothing techniques act to disguise the inherent risk return relationship. Instead, he suggests that "unsmoothing" techniques will better represent the higher volatility of the illiquid asset.

[2] Figueiredo, 2003.

3 METHODOLOGY

The purpose of this paper is to both explain the rationale for considering exogenous income in optimal portfolio decisions, and secondly, to illustrate the impact of this income on the optimal asset mix for the Alberta Heritage Savings Trust Fund (the AHF).

The paper will initially discuss the need to include illiquid or non-tradable assets in the portfolio optimisation process and detail the rationale for its inclusion. Next, I will estimate a portfolio mix for the AHF. I differentiate between the fund's financial portfolio (which is represented by the portfolio's liquid assets) and the expanded portfolio, which includes exogenous nontradable income from government energy revenues. To simplify the optimisation process the financial portfolio will also exclude US and Non-North American securities.

I estimate a range of values to account for the present value of this future income. In order to project a value for the expanded portfolio I estimate the present value of future cash flows accruing to the Government of Alberta by averaging recent flows and discounting them by a range of values in order to test the sensitivity of the optimisation of the expanded portfolio. This range of values is then assigned parameter estimates for return, variance and covariance using the TXS Total Return Energy Index as a proxy. For the nontradable asset, the return parameter takes less importance in the asset mix decision process as the lower and upper bounds are fixed to the weight of the assumed value of the exogenous income. Of critical importance are the variance-covariance relationships with the other portfolio assets.

I assess the impact of the nontraded asset by testing for optimal assets weights with the nontraded asset functioning as a constraint in the allocation process. To do this I take two approaches:

(1) Assume that the existing asset weights as of March 31, 2005 in the financial portfolio are optimal and representative of the tangency portfolio. Given this assumption, the mean parameters are adjusted to make them $\Sigma \mathbf{x}_m$ compatible with current asset weights. $\Sigma \mathbf{x}_m$ compatible means are weights that will comprised the tangency portfolio as follows:

$$\mathbf{x} = 1 / (a - r_z c)[\, \Sigma^{-1}\boldsymbol{\mu} - r_z\, \Sigma^{-1}\boldsymbol{\iota}] \quad \textbf{(1)}$$

Where:

$\mathbf{x}$ = weights in the tangency portfolio

$\boldsymbol{\mu}$ = mean returns

$a = \boldsymbol{\iota}'\Sigma^{-1}\boldsymbol{\mu}$

$c = \boldsymbol{\iota}'\Sigma^{-1}\,\boldsymbol{\iota}$

(Note: bold types indicate column vectors)

In order to assume the existing weights of the financial portfolio reflect conditions of optimality, $\boldsymbol{\mu}$ is adjusted while other factors are held constant. These means are determined as follows:

$$\boldsymbol{\mu} = r\,\boldsymbol{\iota} + [(\mu_m - r) / \sigma^2]\, \Sigma \mathbf{x}_m \quad \textbf{(2)}$$

Thus, the existing weights are assumed to be representative of optimality conditions resulting in corresponding means that must plot on the security market line.

(2) Assume the existing asset mix is sub-optimal. I estimate the optimal asset allocation using historical mean-variance-covariance parameter estimates – that is determine an optimal weights for the existing financial assets before examining the impact of the nontraded asset constraint. In this instance, parameter estimates based on historic data are used in formula (1) to determine the tangency portfolio.

The nontradable asset constraint and its associated parameters are then added to both of these models and new asset weights estimated for the financial portfolio. This will allow comparisons between the asset allocations within the financial portfolio for the impact of the nontradable asset. These optimal portfolios are compared to ones that include the additional provision for energy stocks. In each case related Sharpe Ratios are compared to assess the benefits accruing to the investor.

In addition to the general budget constraint, each of the above processes will be subject to the following constraints:

(1) No short sales are allowed;

(2) The investor cannot borrow to purchase more securities;

To test the resulting optimal mix policy, Sharpe Ratios are calculated for both the expanded portfolios containing investment constraints and the expanded portfolios that exclude them. The Sharpe Ratio measures the price of risk and portfolio performance relative to the Capital Market Line (see (2) above). It is denoted as follows:

$$\text{Sharpe Ratio} = (E(r) - r_f) / \sigma_p \quad \textbf{(3)}$$

This ratio measures performance as a function of excess return per unit of risk (or return adjusted for risk). As a consequence, higher values are preferred to lower values. If we use Sharpe Ratios to compare investment strategies, a strategy resulting in a higher Sharpe Ratio would be considered superior for investment outcome for that period of time. In relation to the tests of optimal investment policy, Sharpe Ratios for both the constrained and unconstrained cases will be compared to determine if the optimal expanded portfolio results in superior risk adjusted returns to the investor (the Government of Alberta). Ultimately, this should confirm whether or not the investor benefits from this policy.

4 DISCUSSION AND RESULTS

4.1 Nontraded Assets and Portfolio Choice

When the Alberta Heritage Savings Trust Fund was established in 1976, its primary goals were to produce income to offset diminishing future energy revenues; reduce the need for government borrowing; improve Albertans' quality of life; and help diversify the economy from the resources that founded it.[3] To achieve these three goals, the fund was allocated to three separate divisions. The Alberta Investment Division largely invested in other Alberta crown corporation debt and equity securities. Canada Investments Division invested in the debt securities of other provincial governments and provincial government crown corporations. These securities tended to be purchased at reduced yields. The Capital Projects Division invested in provincial projects that included infrastructure, medical research, and other investments that were deemed to serve the public good. No financial returns were necessarily contemplated when these investments were made.

In 1980 the Province of Alberta created the Commercial Investment Division and the Energy Investment Division were created. The former was to invest in a broad array of listed Canadian securities and the latter was intended to promote expansion within the energy industry. The fund was again restructured in 1997. At that time, two investment portfolios were created –

[3] Warwick and Keddie. 2000, page 2

the Transition Portfolio and the Endowment Portfolio. The transition portfolio was to temporarily hold all of the financial assets connected with the former three-division structure of the fund while the endowment portfolio held all tradable securities. As assets within the Transition Portfolio were sold, the proceeds were forwarded to the Endowment Portfolio and then invested in tradable assets. The Transition Portfolio was wound up in the first half of the 2002-2003 fiscal year.

The AHF's existing investment policy and the legislation creating the AHF not only conduct the operations of the fund as a separate entity from the Alberta Government, but also as a stand-alone entity in terms of investment policy. The current allocation of financial resources suggests that the investor's savings-consumption function is separate from the investment objectives of the fund. The stated mission of the fund is:

> "To provide prudent stewardship of the savings from Alberta's non-renewable resources by providing the greatest financial returns on those savings for current and future generations of Albertans."[4]

Maximizing returns subject to acceptable levels of risk dominate the fund's policy objectives. The AHF appears to ignore total government wealth when optimal asset allocations are estimated. This concept is noticeable by its absence in its stated goals:

> "The five main outcomes of the Fund, as outlined in the 2004-07 business plan remain as follows:
>
> 1. Maintain nominal value of assets at a 5-year planning horizon.
> 2. Achieve budgeted cumulative income forecasts during a 5-year planning horizon.
> 3. Preserve the real value of assets over a long-term horizon of 20 years.

[4] Alberta Heritage Savings and Trust Fund 2005 Annual Report, page 3

4. The Heritage Fund policy asset mix is expected to generate a total real rate of return of 4.5% at an acceptable level of risk over a moving five-year period.

5. The market rate of return is expected to be greater than a passively invested benchmark portfolio by 0.50% per year (after fees are deducted) by adding value through active management."[5]

Indeed, The Alberta Heritage Savings Trust Fund Act directs that:

(i) Investments made under the endowment portfolio must be made with the objective of maximizing long-term financial returns....

(ii) Subject to the regulations, when making investments the Provincial Treasurer shall adhere to investment and lending policies, standards and procedures that a reasonable and prudent person would apply in respect of a portfolio of investments to avoid undue risk of loss and obtain a reasonable return that will enable the endowment portfolio and the transition portfolio to meet their respective objectives.[6]

The economic perspective taken by the fund in its operation does not include an assessment for financial risk on other government income (i.e. their non-tradable interest in energy). Since the AHF is then run on a self-standing basis, the ability of the fund to hedge the volatility of energy prices and its negative impact on provincial income is impaired. Thus, the optimum asset mix decision does not consider the government's single largest income producing asset – energy, despite the fact that the province's economy is dominated by this industry sector. Indeed, investing liquid assets in the sector results in a sub-optimal asset allocation that adds, rather than diversifies risk.

[5] Alberta Heritage Savings and Trust Fund 2005 Annual Report, page 3
[6] The Alberta Heritage Savings Trust Act, Sec. 3 subsection 2 and 3

Modern Portfolio Theory takes a different viewpoint. The efficient frontier is formed by a horizontal parabola when plotted on a chart as a function of expected mean portfolio returns and the standard deviation of those returns. In theory, this frontier consists of the portfolios of all available assets. However, since nontraded assets are difficult to value, practitioners have generally excluded them from the portfolio optimisation process. The Theory also demonstrates that uncorrelated assets will tend to produce better risk-adjusted returns than those portfolios that have correlated assets. The higher the concentration of a portfolio in a single asset class, the greater the risk of loss to the portfolio. This supported the conclusion that optimal asset diversification based on mean-variance relationships will produce superior risk adjusted returns.

The view that financial portfolios are optimised in the absence of stochastic income variables is in stark contrast to that of recent views on financial economics (as presented in the Literature Review). Svensson's paper "Portfolio Choice and Asset Pricing with Nontraded Assets" explicitly considers government revenues as one example of a nontradable asset that should impact investment policy. There are many examples of these other forms of wealth influencing the asset mix decision. For example, in the case of an individual, the existence of human capital should impact the optimal asset allocation due to its return and covariance relationship with other risky financial and non-financial assets. State factors impacting the exogenous income from employment should be offset by assets that have low correlation to the industry in which the employment income is earned. Consequently, an individual's industry sector allocations will be influenced by the industry in which the individual is employed.

In contrast to the AHF, the Government of Norway takes the view that transfers to their Petroleum Fund from government energy royalties amounts to a purchase of financial assets with energy assets. The values of future cash flows to the government from oil and gas royalties are therefore incorporated into their fund's investment policy. Svein Gjedrem, Norwegian Central Bank Governor notes "The increase in the value of the Petroleum Fund does not represent a wealth increase, but a deliberate conversion of petroleum wealth into financial assets."[7] This belief both recognizes the non-renewable nature of oil and gas resources but also implicitly recognizes the view that future energy royalties are effectively illiquid assets from a financial management perspective. Conversion is possible but liquidity is severely constrained (i.e. a government can allow an increase in the amount of oil/gas that is extracted annually but the ability to increase the extraction rate is limited). Norway acknowledged the impact of their non-financial oil assets in setting up their investment policy for their petroleum fund. Since they view fund contributions as an asset exchange, then naturally, the Fund becomes an opportunity to diversify their domestic economy from negative impacts of the unpredictable movements in the price of oil. This concept is incorporated in the fund's investment policy [8] and commented on by Central Bank Governor Svein Gjedrem.

> "During the next years, we will see a substantial transformation of wealth from petroleum to financial assets. This represents ... a diversification of risk and transfer of wealth into high yielding assets. The current investment strategy of the Petroleum Fund is based on diversifying the market risk over international equities and fixed income products in twenty-seven countries in more than 1750 individual companies and more than 450 bonds. This does not protect the fund from negative portfolio effects resulting from a global downturn. However, the

[7] Gjedrem, 2001
[8] See http://odin.dep.no/fin/english/topics/pension_fund/strategy/bn.html

strategy protects the portfolio from significant problems in a single enterprise, sector or country."[9]

While the Norwegian Central Bank governor's comments do not directly indicate the impact of oil and gas as a nontradable asset on their petroleum fund, though they do emphasize that this government has identified the need to diversify their holdings in such a way as to minimize the impact of cyclical changes in the energy sector from that of the rest of the economy. As a consequence, oil and gas companies only comprise approximately 3% of this fund's total holdings. In contrast, the AHF's allocation to the Energy Sector amounted to more than 8% of the financial portfolio. Despite this, it is not certain whether the Norwegian government goes as far as to formally quantify the impact of exogenous income on their petroleum fund and set investment policy targets based on these results.

Clearly, the investment portfolio influences the consumption-savings decisions of their owners – the Government of Alberta. As of March 31, 2005 the AHF had contributed $27.6 billion to provincial revenues and a little more than $1 billion in the 2005 fiscal year.[10] These transfers represent investment income and are explicitly incorporated into Government budget revenue estimates along with capital flows resulting from the local energy industry.[11] By comparison the energy industry contributed more than $9.7 billion dollars to the provincial treasury in the same year. The investor's consumption is thus financed both by financial assets and illiquid assets. This being the case, the investor's tradable assets are therefore impacted by exogenous state variables impacting it.

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[9] Gjedrem, 2001
[10] Alberta Heritage Savings Trust Fund 2005 Annual Report, page 6
[11] Alberta Finance 2005-06 Fiscal Plan, page 35

Michael Hoffman of the University of Alberta recognized this and took a similar view in his paper, "The Economic Impact of the Alberta Heritage Savings Trust Fund on the Consumption-Savings Decision of Albertans." He wanted to assess the success of two of the stated goals of the AHF – to save for the future and to improve the quality of life in Alberta. He reasoned that the existence of the fund should result in a lower future tax burden and therefore positively impact personal disposable income. His empirical test results failed to show a connection to individual Albertans' consumption-savings decisions. Despite the negative results for Hoffman's tests for individuals, they are certainly positive at the government level as shown above. The connection between savings (in the form of the AHF) and consumption (annual transfers to general revenue) create a consumption-savings function link for the government. Operating the fund as a stand-alone entity without reference to this relationship will limit its effectiveness.

4.2 Comparison of Optimised Financial Portfolios

I now examine the AHF's optimal asset mix in the context of the expanded portfolio as compared with the fund's financial portfolio.

To determine the optimal asset mix including illiquid assets I first estimate a present value for this asset. Econometric models were not used due to diverse stochastic state variables impacting value. Oil prices tend to follow a random walk[12] and extraction rates tend to have the characteristics of both increasing and decreasing volumes over the course of the non-renewable asset's existence. There are other factors that would also need to be held constant in order to arrive at an estimated value. Since the goal of the exercise is to illustrate the impact of the nontradable asset on the portfolio's asset mix, a simple annual average is calculated using data compiled from the Alberta Department of Energy and the Canadian Association of Petroleum Producers (see Table 1).[13] These values act as proxies for future projected annuities accruing to the provincial government and are discounted at values ranging from 5% through 12% resulting present values between $170 billion and $70 billion (see Table 2). In trying to price the nontradable income, it should be noted that on a cash flow basis, typical yields on Canadian energy trusts have tended to range from 4% - 5.5%. While noting the disparity between the range of the discount rates and trust unit yields, the importance of the range of values lies in the fact that

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[12] Fassano, 2000, page 3.
[13] Taylor, Severson-Baker, Winfield, Woynillowicz, Griffiths, 2004.

it will allow for an examination of the sensitivity of the optimal asset mixes given a spectrum of assumed values for the nontradable income. While these amounts is debatable, they will still serve to illustrate the impact of nontradable exogenous income both in its magnitude and as it declines in value as resources are used.

Holding all things constant, financial assets will only be invested in the Energy Sector once the nontradable asset's value declines below a level consistent with the total optimal allocation to energy. In a real world scenario, this will only hold when depletion results in minimal energy reserves or when substitutes result in lower unit prices. The resulting impact to government investment policy would then dictate a corresponding increase in investment in energy assets within the government's investment portfolio.

4.2.1 Results from the Optimisation Calibrated to Existing Weights within the AHF

The Canadian Equity portion of the AHF had a weight of approximately 21% in energy as of March 2005. This closely paralleled the weight of energy in the TSX Composite Index, which was approximately 24.7% of the index at that time. If it is assumed that the existing weights within the Financial Portfolio (consisting of the Canadian Fixed Income portfolio and the Canadian Equity portfolio) are optimal, the monthly mean returns are adjusted to ones that are Σx_m compatible (or "equilibrium compatible means" as Black-Literman call them). All other historical parameter estimates are held constant. These parameters are estimated from industry sector indices data provided by the CFMRC TSX Database (University of Toronto). These parameter estimates are determined using monthly total return data spanning from December 1987 through to March 31, 2005. Sector indices data is not available from this database for periods prior to December 1987. March 31, 2005 corresponds to the fund's year-end and the period that is being examined for asset allocation inefficiencies.

Each pair of portfolios (constrained by the nontradable income and unconstrained by the nontradable income) is compared using the varied weights based on differing present value estimates for the nontradable income. As the weight allocated to nontradable income changes, the optimal asset mix is impacted by return-variance-covariance parameters. Accordingly, Health Care stocks only receive any allocation once the Nontradable Asset value drops to six percent of the total value of the financial portfolio. In all cases, except that of the extreme low value, when the nontradable asset is added to the portfolio's total value the Energy sector no longer receives any allocation (see Table 3 in Appendix). Only when the nontradable asset is assumed to have a value less than $500 million does energy receive any allocation.

One might argue that minimal amounts of liquid assets relative to the value of a nontradable asset would negate the importance of optimal asset allocation policies in an expanded context. Yet, this assertion ignores two important facts:

(i) If strong covariance relationships between liquid and illiquid assets exist, state variables negatively impacting illiquid assets will also negatively impact liquid ones. It reduces the financial portfolio's ability to hedge the risk associated with state variables;

(ii) It limits the optimal risk-adjusted return to the investor. This is discussed below.

The portfolios are then tested to verify whether the investor receives a benefit from the investment policy. Assuming a risk free rate of 2.5%, a Sharpe Ratio is then calculated for each pair of expanded portfolios – one with a constrained energy allocation and one without the constraint (in accordance with existing policy). For each of these pairs of optimised portfolios, the Government receives the benefit of an increase in the overall risk-adjusted return when investment in the energy sector is constrained. This is a direct result of the decreased over-allocation to this sector. Table 4 illustrates this – the first row exhibits the Sharpe Ratio for the Constrained Portfolio and the second row exhibits the Sharpe Ratio for the Unconstrained Portfolio. The third row presents the differences between the two (the increase of the ratio of

Constrained Portfolios as opposed the Unconstrained Portfolios). In each case, the differences result in positive values indicating improved risk-adjusted returns. Indeed, as the assumed value of the nontradable income decreases, the positive value of the Sharpe Ratio differences increases. This is consistent with Cauley, Pavlov and Schwartz's findings in the case of an individual homeowner. They determined that the larger the ratio of home value to total wealth the greater the negative impact of the constraint of home value on investor utility. This becomes important in the case of the AHF as our illustration excludes both US and non-North American equity securities. As a consequence, holding values and other factors constant, increasing the value of the financial portfolio relative to nontradable income should result in greater positive differences and larger investor benefits.

This increase in the risk-return relationship is illustrated in Figure 1. The optimal policy shown in the second graph portrays the reality of the advantage realized by the investor. At each point in the efficient frontier, the investor realizes a superior risk-return relationship as compared with the first graph (the sub-optimal policy – energy stocks unconstrained by nontradable energy income).

4.2.2 Results from the Optimisation Based on Historic Mean-Variance Relationships

As described in the Methodology, the second test attempts to optimise the portfolio mix using historical parameter estimates. As with the first test, the parameter variables are estimated from data derived from the CFMRC TSX Database (University of Toronto). In this instance mean parameters are not adjusted. Holding the parameter estimates constant, optimal asset weights are calculated for the financial portfolio (consisting of the Bond Portfolio and the Canadian Equity Portfolio). Adding the weights for nontradable income to the financial portfolio the expanded portfolio again provides the basis for the asset mix comparison. This asset allocation is then compared to an optimised portfolio that adds the exogenous income constraint.

As with the first test, the allocation to energy is constrained by the extreme weight of the nontradable asset and its high correlation to it (see Table 5). In contrast to the first test, energy stocks now receive no allocation when nontradable income is assumed to amount to only 6% of the financial portfolio. This is a consequence of the optimal mix allocating only 2.6% of the total assets to energy. In this case, the present value of the nontradable income would need to be less than $165 million before the energy sector receives any allocation.

Again, as with the first test, each of the pairs of portfolios (constrained and unconstrained) is tested to verify that investor receives a benefit from the investment policy of limiting allocation to energy. As with the first test, Sharpe Ratios are calculated for each pair of portfolios (2.5% risk free rate is assumed). In each instance, the expanded portfolios subject to the nontradable income constraint produce higher risk-adjusted returns than do the portfolios subject to optimal policies with the nontradable income added (see Table 6). Once more, this is a direct result of the decreased over-allocation to the energy sector. The positive Sharpe Ratio difference is lower than in the case of the first test. This is a consequence of the lower optimal allocation to energy (7.6% in the first test as compared with 2.6% in the second test). Yet, as with the first test, the value of the positive differences increases as the relative value of the nontradable income constraint decreases.

5 CONCLUSION

The Alberta Heritage Savings Trust Fund operates separately from its investor – the Government of Alberta. Its policy of asset allocation does not consider exogenous income accruing to the investor/government through income derived from energy and hence is ineffective at hedging risks to this income. As a consequence, its investment policy seeks its own optimum value without reference to the state factors impacting the investor's income. The ensuing asset mix is then a sub-optimal result that increases risk to the government rather than allowing it to diversify the risks that affect its economy. Two tests to determine optimal asset allocations for the AHF incorporating a constraint representing nontradable income results in no allocation to energy stocks. These outcomes also result in superior risk-adjusted returns to the investor.

Following a year when energy stocks have represented the majority of the increase in the TSX Composite Index it becomes difficult to imagine any asset allocation decision that would exclude the energy sector from a major institutional portfolio. Nevertheless, the perspective of the investor (the Government of Alberta) should provide the focus for the asset allocation process for the AHF rather than the fund acting independently from the state factors impacting government revenues. The result would be a fund that is better able to diversify its asset mix away from the non-renewable resource income that founded it.

APPENDIX

Table 1. Revenue from Oil and Gas Production 1995 – 2002 (millions)

REVENUE SOURCE	1995	1996	1997	1998	1999	2000	2001	2002
Natural Gas Royalty	1,389	1,099	1,393	1,750	1,519	2,441	7,038	3,809
Crude Oil Royalty	1,227	1,146	1,486	969	487	1,072	1,466	933
Bonus Bids and Sales of Crown Leases	1,093	630	994	1,136	479	743	1,133	916
Income Taxes	836	1,914	773	723	794	1,762	2,103	3,508
Royalty Tax Credit	-325	-319	-257	-239	-259	-188	-141	-103
TOTAL	4,220	4,470	4,389	4,339	3,020	5,830	11,599	9,063

Source: Pembina Institute, 2004 based on figures supplied by the Alberta Dept. of Energy and the Canadian Association of Petroleum Producers

Table 2. Estimated Present Values – Government of Alberta Oil and Gas Revenues (millions)

Discount Rate	5.2%	10.0%	12.0%	n/a
Estimated Present Value (in millions)	$170,000	$90,000	$73,000	$400
Percentage of Expanded Portfolio	96.4	93.4	92.0	6.0

Table 3. Fund Allocations – Optimiser Calibrated to Existing AHF Weights

	Actual Weights	Optimal Mix with the Nontradable Asset Constraint			
Optimal Allocations	**2005**				
As a Percentage of Expanded Portfolio	%	%	%	%	%
Non-tradable Energy Income	–	**96.4**	**93.4**	**92**	**6**
Bonds & fixed income	63.6	2.5	4.5	5.5	63.6
Consumer goods	3.9	0.2	0.3	0.4	3.9
Energy	**7.6**	**0**	**0**	**0**	**1.6**
Financials	12	0.4	0.8	1	12
Health Care	0.4	0	0	0	0.4
Industrials	2.2	0.1	0.2	0.2	2.2
Information technology	2.2	0.1	0.2	0.2	2.2
Materials	5.5	0.2	0.4	0.5	5.5
Telecommunications	2.2	0.1	0.2	0.2	2.2
Utilities	0.4	0	0	0	0.4

	Actual Weights	Optimal Mix with the Nontradable Asset Constraint			
Optimal Allocations	**2005**	**96.4**	**93.4**	**92**	**6**
As a Percentage of Financial Portfolio	%	%	%	%	%
Bonds & fixed income	63.6	69.0	67.8	68.4	67.7
Consumer goods	3.9	5.6	4.5	5.0	4.1
Energy	**7.6**	**0**	**0**	**0**	**1.7**
Financials	12	11.1	12.1	12.5	12.8
Health Care	0.4	0	0	0	0.4
Industrials	2.2	2.8	3.0	2.5	2.3
Information technology	2.2	2.8	3.0	2.5	2.3
Materials	5.5	5.6	6.1	6.3	5.9
Telecommunications	2.2	2.8	3.0	2.5	2.3
Utilities	0.4	0.4	0.4	0.4	0.4

Source for Actual Weights: Alberta Heritage Savings Trust Fund, Annual Report for the Year Ended March 31, 2005

Table 4. Sharpe Ratio Comparison – Existing AHF Weights with Nontradable Income

Energy Income – Assumed Portfolio Weight	**96.40%**	**93.40%**	**92.00%**	**6.00%**
Sharpe Ratio – With Income Constraint	0.212559	0.223342	0.228151	0.358864
Sharpe Ratio – Without Income Constraint	0.211547	0.221582	0.226081	0.355552
Investor Benefit With Income Constraint	0.001012	0.001760	0.002070	0.003312

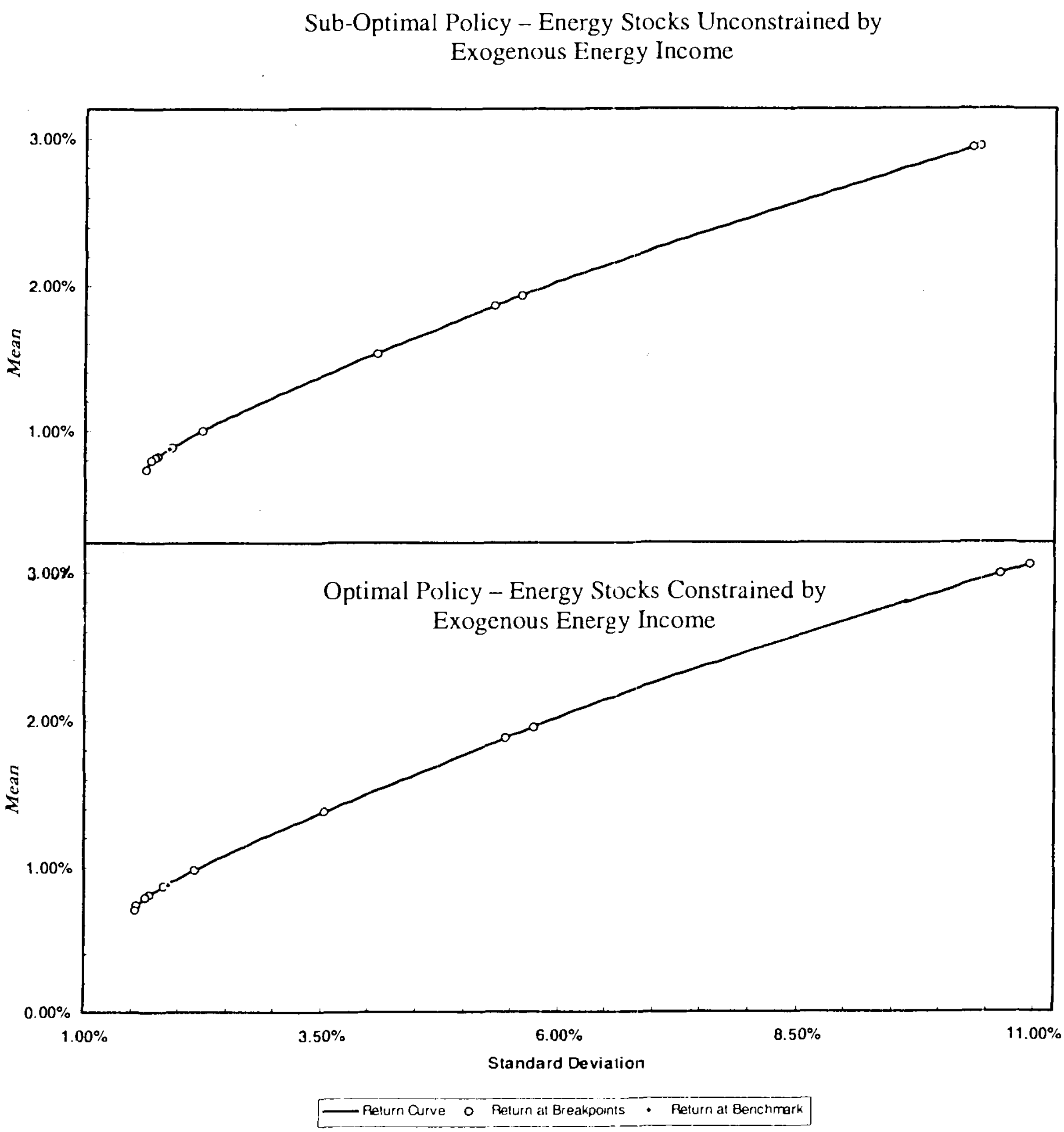
Table 5. Fund Allocations – Optimiser Calibrated Using Historical Data

	Optimal Weights	Optimal Mix with the Nontradable Asset Constraint			
Optimal Allocations	**2005**				
As a Percentage of Expanded Portfolio	%	%	%	%	%
Non-tradable Energy Income	**–**	**96.4**	**93.4**	**92**	**6**
Bonds & fixed income	75.8	0	0	0	73.1
Consumer goods	3.2	0	0	0	1.8
Energy	**2.6**	**0**	**0**	**0**	**0**
Financials	10.8	0	1	1.8	10.6
Health Care	0	0	0	0	0
Industrials	0	0	0	0	0
Information technology	0	0	0	0	0
Materials	0	0	0	0	0
Telecommunications	3.9	3.6	5.6	6.2	4.5
Utilities	3.7	0	0	0	4
	Actual Weights	Optimal Mix with the Nontradable Asset Constraint			
Optimal Allocations	**2005**	**96.4**	**93.4**	**92**	**6**
As a Percentage of Financial Portfolio	%	%	%	%	%
Bonds & fixed income	75.8	0	0	0	77.8
Consumer goods	3.2	0	0	0	1.9
Energy	**2.6**	**0**	**0**	**0**	**0**
Financials	10.8	0	15.2	22.5	11.3
Health Care	0	0	0	0	0
Industrials	0	0	0	0	0
Information technology	0	0	0	0	0
Materials	0	0	0	0	0
Telecommunications	3.9	100.0	84.8	77.5	4.8
Utilities	3.7	0	0	0	4.3

Table 6. Sharpe Ratio Comparison– Historical Data

Energy Income – Assumed Portfolio Weight	96.40%	93.40%	92.00%	6.00%
Sharpe Ratio – With Income Constraint	0.147663	0.153271	0.159665	0.427631
Sharpe Ratio – Without Income Constraint	0.146479	0.153080	0.155568	0.427446
Investor Benefit With Income Constraint	0.000184	0.000191	0.000410	0.000185

Figure 1. Comparison of Efficient Frontiers Under Current and Optimal Investment Policies



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