

**Mathematics Teacher Tension:  
Arising in, and Through,  
their Attempt to Change Practice**

**by  
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## Ethics Statement



The author, whose name appears on the title page of this work, has obtained, for the research described in this work, either:

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## Abstract

While much research is devoted to *what* it is teachers do, there is far less known about *why* teachers do the things they do. This is particularly true in the area of mathematics teacher change where, despite an abundance of literature on ways to think about and facilitate change in mathematics teaching practice, a lack of meaningful change in practice is an ongoing concern. This dissertation explores this gap through a qualitative analysis of tension experienced by fourteen teachers engaged in implementing change in their mathematics practice. Viewing teachers as tension managers, whose actions are shaped by an undercurrent of uncertainty, offers insight into the ‘why’ behind their actions; it allows for a focus on the *process* of change in practice, rather than the product.

The study uses theoretical constructs of teacher change and teacher agency to position teachers as arbiters of change, responsible for their own growth. Using a hermeneutic phenomenology approach, data collection was conducted in three distinct phases and comprised interviews with, written reflections by, and classroom observations of, groups of teachers at various stages of change. Using a form of emergent coding, data was first analysed for contexts which held potential for change. These were then re-examined for tension using emotion and hedging as indicators of uncertainty.

The results indicate that teachers experience internal and external tension that can both trigger and impede meaningful change in mathematics teaching practice. This is dependent not only on the context, but also on the quality of tension, as two types (useful tension and productive tension) are identified and explored for their potential to impact change. Furthermore, the data supports the view that managing tension in change is an agential response. Two management strategies are articulated: living with tension and resolving tension. Finally, the presence of unacknowledged virtual tension was hypothesized as an impediment to the achievement of meaningful change.

**Keywords:** teacher tension; teacher agency; teacher change; mathematics teaching practice; hermeneutic phenomenology

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# Chapter 1. Introduction

I shudder to think that I begin my dissertation with a cliché, but this whole process has very much been a journey. I started out in one place, ended up in another, and was very much changed in the process. And not only was writing my thesis a journey, it was a hero's journey in which one is faced with seemingly insurmountable obstacles, one receives help from unexpected sources, and, against all odds, one slays the proverbial dragon. Bloodied but unbroken, one comes out at the end with hard-won insights, ideas, and possibilities all neatly encapsulated in less than 100 000 words. At least, that was what it felt like to me as I transitioned from an elementary classroom teacher to a mathematics education researcher.

I cannot remember a time when I did not want to be a teacher. I was one of those fortunate children who loved school and for whom learning came easily. At the end of grade one, I remember waking up on the first day of summer holidays and getting ready for school. I was heartbroken to discover that school was over for the next two months, forever for a six-year-old. Not surprisingly, I quickly switched from student mode to teacher mode to fill the void and that summer I was teacher to my classroom of stuffed animals. The next several summers found me roping as many of my six brothers as I could into being my students. I would create mathematics worksheets for them, have them take turns reading aloud, and then send them out for recess so I could get my marking done. It was bliss for me (not so much for my brothers perhaps).

Despite this early start, I did not pursue a teaching degree until later in life. It was only then that I was able to bring that same enthusiasm into an actual classroom with real students. It turned out that I was a good teacher, if the measure is how much my students, their parents, and my colleagues liked me. Still, after the first two or three years, I began to feel dissatisfied with my teaching. Up until then, I had been content with a teacher-centred pedagogy that, in retrospect, looked remarkably similar to how I had taught as a six-year-old. I did a lot of teaching and a lot of grading, but there did not seem to be a lot of learning happening. This came to a head when I found myself scolding my students for their misbehaviour while walking to the gym. I remember telling them, "If you don't quit fooling around, we're going to go back and do math instead!" It was shocking to realize that I viewed mathematics learning in my classroom as a punishment.

That was when I began to seek change. At first, I looked to professional development to find ways to improve student learning, particularly in mathematics where my teacher-centred pedagogy seemed most problematic. I simply did not know any other way to teach mathematics; that is, until I attended a mathematics workshop led by Dr. Peter Liljedahl. His session gave me a glimpse of what a mathematics classroom truly could be. I was hooked and I sought out more of his sessions, and others like his, to help me transform my mathematics classroom. When it was announced that a master's program in numeracy was to be offered by Dr. Liljedahl and his colleagues at Simon Fraser University, I immediately signed up. This was the prelude to my journey and, little did I know, would result in the end of my career as a classroom teacher.

Like all journeys, there must be a beginning. If I had to put a pin on a figurative map and label it 'My journey started here', it would likely be a conversation with a teacher named Carly. I was in the last year of my master's and out of the classroom for the first time, working as a differentiation lead teacher for my district. Among other things, this offered the opportunity for me to work with other teachers in their classrooms, observing, modelling, advising; really, just providing whatever assistance they requested.

I had been working closely with Carly. An experienced teacher, she had decided that it was time to change her mathematics teaching practice. No longer satisfied with many of the teaching methods that she felt had served her well throughout her career, she reached out to me to help her find ways to meet the myriad learner needs she was encountering in her classroom. She was eager, so eager for everything I could offer, and we both learned a great deal as she implemented, reflected on, and improved my suggestions. Yet, motivated as she was, she described the ongoing process of change as a struggle filled with uncertainty as she second-guessed everything she was trying to do. In one of our last conversations, she used an analogy to describe that struggle that has stuck with me; she said she felt she was an elastic band, stretching and then bouncing back and then trying to stretch a bit further without breaking. It was a beautiful metaphor, not only for what I had been witnessing in her mathematics classroom, but also for what I had felt in my own.

I left teaching that year to begin studying mathematics education as a doctoral student. I had become so enamoured by the mathematics thinking and learning that I experienced as a master's student that, when the opportunity to continue my studies arose, I knew it was something I wanted to do. But this is where I faced my first seemingly insurmountable

problem – choosing a thesis topic. What did I want to devote the next few years of my life to studying and researching? With the support of my supervisor, I began by focusing on that which captured both my experience and my passion: working with teachers. My desire for change had led me to develop an interest in teacher professional growth and development, not only that of my colleagues, but my own as well. I was able to envision a different way of being in my classroom and I had persisted in searching for ways to realize that way of being. Like a pilot who makes minute adjustments to her horizon indicator, I constantly fine-tuned my practice. A little too much reliance on summative testing? Time to restore equilibrium by embracing formative assessment. Discouraged by my students' struggles with whole number operations? Time to explore non-standard algorithms. As an early adopter of innovative practices, my willingness to implement change had led very naturally to an unofficial leadership role in my district as colleagues asked me to share my experience and then to an official one as the district lead teacher. In so doing, I found myself increasingly called upon to provide professional development and I recall being intrigued by the responses to my sessions. Some teachers would walk away happy to have a specific new idea to try out, while others left with the intention of changing their entire mathematics teaching practice. Still others left apparently untouched by anything they had encountered during the session.

It was these teacher responses to professional development that became the early focus of my studies: who changed, why they changed, and how they changed. In looking at instances of literature on mathematics teacher change, I noticed a tendency towards deficit thinking: teachers do not change, change hurts, and teachers are stubbornly resistant to change because they want to cling to their old ways (Cavanagh, 2006). While this may have matched particular instances of teachers whom I had mentored, in general, this was not my experience. I could not help but think back to that conversation with Carly. Most of the teachers I mentored were more like her; they wanted to change, they worked hard to change, but, also like Carly, some felt barriers or forces that pulled and pushed and generally interfered with their attempts. In time, I came to see those barriers or forces as *tension*. Tension in teaching is not a novel idea. Starting with Berlak and Berlak (1981), researchers have been interested in understanding the tension that can plague the practice of teaching.

Pursuing a doctorate afforded me the opportunity to think more deeply about tension in the teaching of mathematics; it proved to be the second pin in the map of my journey. The

decision to leave my rewarding teaching career was difficult but it made possible the move from mere curiosity about mathematics teacher tension to actually exploring it deeply. There is a movie called *Mindwalk*, in which a character, in explaining why she left her career at its height, said that she now had time to think a thought through to its end. That is what I now had, and I chose to spend the time thinking about Carly, elastic bands, and tension.

In what follows, I describe that thinking and the conclusions to which it led. The next two chapters serve to narrow and refine my research focus. Each contains a literature review of various elements that guided and shaped my research questions and my interpretation of the data. And, while I do draw on other literature, my predominant focus is literature in mathematics education.<sup>1</sup> Chapter 2 frames Carly's unseen barriers or forces as tension. This overview of the tension literature looks closely at Berlak and Berlak's earlier study, and those that have followed since, to find out what is already known about tension in mathematics teaching and where I found room for further research. The eventual aim is to describe to the reader the early understanding that guided my research and led to the gradual development of my research questions. Chapter 3 details the theoretical constructs which I will use in my analysis. In particular, I focus on the literature around teacher change and agency, and their connection with tension.

Chapter 4 moves from the theoretical to the practical as I describe my methodological perspective and the methods I used in data collection and analysis. The fifth chapter describes tension experienced by thirteen teachers as they implemented change in their mathematics teaching practice, while Chapter 6 delves more deeply into the tension experienced by one teacher in particular. Chapter 7 brings it all to a close, as I address my research questions and offer a look forward as I consider certain implications of my results and the potential avenues for further research.

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<sup>1</sup> A strong desire to remain grounded in mathematics education literature led me to create a table comparing my literature sources. See Appendix A.

## Chapter 2. Conceptualizing Tension

*I ask researchers to meet us where we are. Show an awareness of what has gone before. When you bring new terms into the lexicon, tell us how they are similar to or different from the terms with which we are familiar. (Crosswhite, 1987, p. 269)*

Teaching is an act that is very familiar to teachers. They move around their classrooms doing what they do without really having to think of what it is that they are doing. Much of teaching becomes an autonomous action, an operation in the Leont'ev (2009) sense. Students whispering in the corner? Without thinking, without planning, teachers customarily find themselves moving towards the students and the whispering is subdued. Roth (2002) suggested that teachers begin to think and reflect only when there is some sort of breakdown – such as rather than decreasing, the whispering increases – that forces the situation into their consciousness. It is then that a teacher must decide what to do. Should she shush the whisperers? Ignore them? Ask them to repeat their whispers aloud? Simultaneously, the teacher must also be thinking of what to do next. What if the students persist when shushed? What if they do not willingly share their whispers? This decision-making takes place in the briefest moment in time; an observer in the classroom may only note that the teacher told a group of students to be quiet.

Layered in with the autonomous actions of teaching, however, is a complex undercurrent of uncertainty with which teaching is imbued (Berlak & Berlak, 1981). These authors were, I believe, the first to suggest that one way to represent that complexity is through the language of tension. This would allow for simplifying the complexity “without over-generalizing or distorting the nuances and problems of school life” (p. 107). Like Carly second-guessing the changes in her mathematics teaching practice, teachers can be beset by uncertainties around their decisions. And, as with the whispering above, each potential solution gives rise to further uncertainties that ultimately affect their actions. As Herbst (2003) suggested, “The notion that while acting in a certain way, a teacher may be coping with tension is crucial to making sense of why those actions take place” (p. 205). The language of tension, then, reflects the complexity of teaching while offering insight into teachers' thinking and actions.

In the following, I accede to Crosswhite's above request and provide an overview of the literature regarding tension in teaching, which I think of as fingerprints on my phenomenon

of interest.<sup>2</sup> I begin with a general understanding of tension to show an awareness of what has come before and then move to describing how I further developed my own understanding. I end with a description of the metaphors that encapsulate the thinking around tension in teaching mathematics.

## 2.1. How the literature describes tension

*“Tension” is another way to describe an idea of play: that flexible sensation of possibility we encounter when we experience a constructed world or an object of loose fit, when we feel the opposition of forces, as when we move against a device designed to strengthen our muscles through resistance. The tension – the play of the possible we find in the machine – makes us respond with our own strengths. That tension and play in our own growing and changing informs the private and public construction and reshaping of ourselves. Too much play and we lose direction and power, too little play and we have no direction or motion. We are locked into no pattern or one pattern; either way is a loss. This dynamics of play seems pervasive: I know of no bright person who is not steadily caught in the midst of tense transitions, nor of any person who is not a learner because of them. (Carr, 1998, p. 196)*

In Carr’s metaphor connecting tension and play, there is a sense of tension as **choice**, as **compromise**, and as **endemic**. This was echoed throughout the literature where there was surprising homogeneity in the understanding of tension, with many of the studies sharing the elements found in Carr’s depiction. His notion of **choice** appeared frequently, as in Katz and Rath’s (1992) description of tension as “a situation that offers a choice between at least two courses of action” (p. 376) or Sparrow and Frid’s (2001) view of tension as “a decision between two equally important choices” (p. 452). This understanding of the choices being equally valued did not feature in Carr’s metaphor, but recurred elsewhere in the literature where choices were referred to as “competing, worthwhile aims” (Ball, 1993, p. 373) and “competing, highly prized values” (Cuban, 1992, p. 6). Carr’s notion of **compromise**, that “either way is a loss” (p. 196), was threaded throughout the research where the collective understanding was that whichever choice a teacher makes “sacrifices the advantages of the alternative” (Katz & Rath, 1992, p. 376), while Lampert (1985) further suggested “the conflicted teacher is her own antagonist; she cannot win by choosing” (p. 182).

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<sup>2</sup> See Appendix B for a table of tension in teaching previously identified in the literature.

Another area in which the research coalesced was in regard to the ubiquitousness of tension in teaching (e.g., Berry, 2007a; Carr, 1998; Horn, 2012; Lampert, 1985). Tension is considered “**endemic** and inescapable” (Mason, 1988, p. 164), since “teaching is evidently and inevitably uncertain. No teacher can be sure how a lesson will go or exactly what a student will learn” (Floden & Buchmann, 1993, p. 373). I suggest this positions tension as an unknown variable in teaching practice. This is important to consider, as it suggests that there can never be a ‘recipe’ for teaching that could be applied across all practices and all contexts.

This does not mean that all of the tension in a teacher’s practice is present all of the time (e.g., Berlak & Berlak, 1981; Carr, 1998; Sparrow & Frid, 2001). As Berry (2007a) suggested, “there is ebb and flow between the tensions such that they may all well exist at once, but rise to the surface in different ways at different times depending on the situation and the way that it may be ‘played out’” (p. 140). I suggest, then, that tension has an idiosyncratic nature and further raise the possibility that no two teachers encounter precisely the same tension in precisely the same manner. Nor, for that matter, will an individual teacher. Related to this is the notion that tension, while isolatable, is not independent of other tension (e.g., Berry, 2007a; Katz & Raths, 1992; Lampert, 1985; Mason, 1988). Consequently, “they are discussed separately, yet are in fact related in complex ways that reflect the complexity of teaching situations” (Sparrow & Frid, 2001, p. 453). An image of an interconnected web of tension comes to mind. What happens when a teacher finds a way to manage tension? What happens when yet more tension is added?

Tension, then, proves useful as a way for teachers to describe their own experiences of practice. Therefore, much as Carly did, acknowledging tension helps teachers better understand what hinders the changes they are trying to implement, as “letting them [tension] out into the open means that they can be robbed of their numbing effect and turned instead into potent sources of energy” (Mason, 1988, p. 164). To develop their classroom practice, then, it would be helpful for teachers to recognize and define tension, as “in the process of renaming what they know through their experience, the teachers critically reflect on – and thus begin to renegotiate – their ideas about teaching and learning” (Freeman, 1993, p. 488). This notion of using tension as a means for reflection on practice was echoed throughout the literature (e.g., Berry, 2007a; Jaworski, 2006;



Jones, 1995; Lampert, 1985), while Adler (2001) suggested, “teachers can use a language of dilemmas to reflect on and transform their practices” (p.1).<sup>3</sup>

While tension can serve as a language for describing and reflecting on individual practice, it is also beneficial in providing a language for discourse within the community of teachers (e.g., Adler, 1998; Ball, 1993; Barbosa & de Oliveira, 2008; Berlak & Berlak, 1981). Indeed, the language of tension is “a powerful explanatory and analytic tool, and a source of praxis for mathematics teachers” (Adler, 1998, p. 26). Additionally, the use of tension language enables communities of teachers “to identify, recognise, talk about and act on the tensions their practice. It can bring those obscured aspects of practice to light” (Adler, 1998, p. 32).

Similarly, Horn (2012) utilized the phrase ‘underlying assumptions’, to suggest that uncovering these ‘obscure practices’ illuminates the connections between the nature of mathematics and schooling that can potentially be used to frame discussions. She offered the example of a teacher experiencing tension regarding how much mathematics her students can “take in” (p. 26). The underlying assumption being that students’ “capacity to take in mathematics was restricted, mainly because one can only remember so much before one gets ‘full’” (p. 26). This notion that the main cognitive activity of mathematics is ‘remembering’ could potentially act as the basis for discourse. This discourse, Jones (1995) suggested, is an essential component of growth.

## 2.2. Problem versus tension

*Who the teacher is has a great deal to do with both the way she defines problems and what can and will be done about them. The academician solves problems that are recognized in some universal way as being important, whereas a teacher's problems arise because the state of affairs in the classroom is not what she wants it to be. (Lampert, 1985, p. 180)*

Although a problem and tension bear similarities, there are differences worth noting. Cuban (1992) saw problems as “fairly routine, structured situations that produce some level of conflict because a desired goal is blocked” (p. 6). What is important to note is that these problems have solutions. Implicit, then, in a problem is that there is a satisfying solution and, should the problem recur, the solution can be successfully reapplied. It is

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<sup>3</sup> Adler (2001) uses both dilemma and tension as terms to describe the uncertainty inherent in teaching. See section 2.4.1 for further discussion.

when confronted with problems for which no satisfactory solution can be reached that tension emerges (Lampert, 1985). However, tension encompasses the inner turmoil teachers experience when faced with contradictory alternatives for which there are no clear answers (Berry, 2007a). They appear as problems; they even feel like problems. However, they are far messier and complex.

Lampert (1985) provided an illustrative example of the tension she herself experienced upon choosing where to sit her students during mathematics lessons. Describing her students as 'allergic' to peers of the opposite sex, she placed her boys on one side of the room near a blackboard and the girls on the other – a problem accompanied by a satisfactory solution. But it was not quite so simple. To curtail the inevitable misbehaviour of a large group of boys, she found herself remaining in close proximity to them, at least until she became aware that this led to another problem – her presence near the boys had inadvertently put the girls at the 'back' of the room. The boys could see and hear better and she was more likely to respond to their questions. Switching to the blackboard near the girls would improve the girls' learning opportunities, but at the cost of classroom management. No matter which arrangement she chose, it would be to the detriment of some of her students. Whether she chose to promote classroom order or equal pedagogic opportunity, either the girls or the boys would miss something she wished them to learn. Outwardly appearing as a simple 'problem', the thought process entailed in managing her tension demonstrated the complexity involved. And no satisfactory solution emerged that could then be applied to successive lessons.

But here lies a curious thing: what is a tension for one may simply be a problem for another (Adler, 2001; Ball, 1993; Chazan & Pimm, 2016). Chazan, in Chazan and Pimm (2016), spoke of sharing Lampert's seating tension with teachers who "were simply not captured by the story; the dilemma did not have quite the same force for them that it did for the author" (p. 20). For those teachers, it was not a source of tension; it was a problem that had a satisfactory solution that could be applied. Undoubtedly then, the reverse might also be possible; what is a problem for one may be tension for another. This again speaks to the idiosyncrasy of tension.

This is why creating a distinction between problems and tension is helpful. Teachers live and work in a can-do culture where problem solving is viewed as a prized ability (Cuban, 1992; Lampert, 1985). Repeated failures in solving a problem can lead to a debilitating

sense of guilt and feelings of professional incompetence (Ball, 1993). Cuban (1992) suggested that, “to distinguish between problems that can be solved and dilemmas that require “satisficing”<sup>4</sup> can reduce guilt. We can pursue ways of reframing those dilemmas to get unstuck from familiar ‘solutions’ and create better compromises” (p. 8).

### 2.3. Solving versus managing

*As he goes about teaching, at any given moment, Mr. Scott is pulled and pushed towards numbers of alternative and apparently contradictory behaviours. One set of alternatives is whether to allow Steven to discuss the football cards or to chastise the child, or in one way or another remind him that he must complete his maths – but at a given moment Mr. Scott cannot both remind and overlook. (Berlak & Berlak, 1981, p. 132)*

The literature suggests researchers are united in their agreement that tension in teaching is inescapable; it lies “in the heart of the pedagogical process” (Byers, 1984, p. 36). With one exception, they are also united in their view of tension as unresolvable. It is Berlak and Berlak (1981) who were alone in their regard of tension as problems with potential solutions. They offered a categorization of tension intended to be useful “to citizens, researchers, parents and professionals for clarifying differences over schooling practices, and for engaging in collaborative inquiries into the origins and consequences of present patterns of schooling and the possibilities and desirability of change” (p. 3). Implicit in their work was the understanding that this would contribute to resolving the tension.

Building on Berlak and Berlak’s early work on tension, others have come to see tension as something that is managed rather than eliminated (e.g., Adler, 2001; Berry, 2007a; Byers, 1984; Cuban, 1992). This may seem like a fine point, but the research is unanimous in its view that managing is not akin to solving; rather, it is a matter of compromise and ongoing management of tension (Katz & Raths, 1992). Furthermore, this suggests a usefulness in managing tension in that, “embracing rather than trying to resolve pedagogical dilemmas gives teachers a power to shape the course and outcomes of their work with students” (Ball, 1993, p. 394). Lampert (1985) agreed, offering her view of a teacher as a “dilemma manager who accepts conflict as endemic and even useful to her work rather than seeing it as a burden that needs to be eliminated” (p. 192). Also

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<sup>4</sup> Cuban (1992) defines ‘satisficing’ as sacrificing in order to satisfy. The term satisfice was first coined by Herbert Simon (1957, pp. 204–205).

suggesting that teachers could work within rather than against their tension, Fecho, Collier, Frieze, and Wilson (2010) argued that to attempt to eradicate tension is to misunderstand its purpose and possibility. They suggested that to experience tension is to “enter a state of wobble, one that asks them to pay attention to the issues at hand and to author a response. The goal is not to remove oneself from that tension but instead to enter into a dialogue that, like the cables on a suspension bridge, uses tension for support and equilibrium” (p. 446).

Recognizing the unusualness in advocating for managing over solving, Lampert (1985) noted, “This way of submerging the conflict below an improvised, workable, but superficial resolution is, of course, quite different from what many cognitive psychologists or curriculum experts would advocate (p. 189). She suggested this is because most equate classroom management with the ability to control behaviour, in that to manage means to control others. Lampert used ‘manage’ in a different sense, where to manage meant to be able to find a way to do something, which “can also mean to contrive to do it, implying that the capacity for invention or improvisation is a necessary part of the manager's repertoire (p. 193). Lampert saw in this the potential for teachers to continue to act or even to thrive in adverse situations, which casts the managing of tension in a positive light. Similarly, Wheeler (1988) suggested the term ‘manage’ is helpful “because it doesn’t have to meet very stringent conditions. ‘I managed to’ tells us just enough; I might have tackled whatever I was doing clumsily, inefficiently, long-windedly, unimaginatively, etc., but at least I ‘managed to’ so I must have ‘managed it’” (p. 304).

Offering the image of walking a tightrope as a metaphor for tension, Cuban (1992) suggested that the managed tension will resurface:

These good enough bargains among values that we strike have to be renegotiated again and again because they are so deeply embedded in who we are and the practice of teaching, administration, and research. Thus, more often than not, we end up managing recurring dilemmas, not solving problems. (p. 7)

This notion of recurring tension was apparent in the work of Lampert (1985) and, long before her, in Dewey (1922) who wrote, “no matter what the present success in straightening out difficulties and harmonizing conflicts, it is certain that problems will recur in the future in a new form or on a different plane” (p. 285).

Several researchers acknowledged that admitting that tension is unsolvable can be socially and pedagogically unacceptable, and potentially viewed as a sign of weakness (e.g., Ball, 1993; Byers, 1984; Cuban, 1992; Lampert, 1985). This may make it more difficult for teachers to be willing to share and reflect on the tension in their practice. Ball (1993) commented that, initially, she “thought people might scoff at my labeling this a dilemma. I thought others might just tell me what I should have done – which, while possibly helpful, would underestimate the complexity of my dilemma” (p. 396). Lampert (1985) agreed, noting that, “sorting out problems and finding solutions that will make them go away is certainly a more highly valued endeavor in our society” (p. 193). She went on to add that, “the work of managing dilemmas requires admitting some essential limitations on our control over human problems” (p. 193). I suggest this has methodological implications for studies focusing on tension, since methods that utilize direct discussions or questions regarding tension in teaching may not prove fruitful. A more indirect approach may be necessary. Additionally, Lampert’s use of the possessive ‘my’ in conjunction with dilemma is telling. Again, it suggests that tension is not universal. It is possible it interacts with the individual values, goals, and desires as a teacher and even perhaps as a person.

## **2.4. Deepening my understanding of tension**

Cooper’s (1917) account of studying under renowned biologist, Louis Agassiz, is a poignant essay on the power of truly coming to understand a phenomenon of interest. In it, he described the experience of being asked to study a single, dead fish placed in front of him. In his description of the task, we see the fish becoming figuratively closer and clearer to him. This drawing in is echoed in his words, as he first refers to ‘a’ fish, then ‘the’ fish, and finally ‘my’ fish. Similarly, Hofstadter’s (1997) account of undertaking a geometric exploration follows the same pattern, as he moves from describing ‘a’ triangle, to ‘the’ triangle, to ‘my’ triangle. I think that is what happens when one really notices, thinks, and reflects; it is akin to a physical movement of bringing something in closer and closer. It becomes personal; I can see that fish on the table, in my hands, and then in my head. And that is how my dissertation process has felt – at first, tension was ‘a’ thing out there, then it became more tangible, while now I feel like I am turning it over and over, looking closer and closer still, as tension becomes ‘my’ thing.

The literature I had been reading on tension made tension 'a' thing; I understood its essence, but not in any meaningful, insightful manner. To gain a better understanding of tension, I now go deeper into the specific literature that had a profound impact on shaping my approach to studying tension, particularly that of Berry (2007a), Lampert (1985), and Berlak and Berlak (1981). Accompanying this will be brief references to some of my own early studies on tension in which I marked my own fingerprints on my phenomenon of interest. I refer to them here, as they were essential in guiding me towards my research questions. In this next subsection then, I unpack how my understanding of tension grew closer and deeper as it became 'the' tension.

#### **2.4.1. Tension versus dilemma versus contradiction**

What my review of the literature made clear was the variation in terminology for describing tension in practice. 'Tension' and 'dilemma' were by far the most common terms and, an unusual aspect worth mentioning, is how often the terms were used interchangeably. One exception is Chazan and Pimm (2016) who felt "it important to keep these two ideas distinct" (p. 21). However, this distinction was not apparent elsewhere. Within one article, a reader finds phrases, such as "tensions within my practice" and "dilemmas existed in my own practice" (Berry, 2007b, p. 117 & p. 129, respectively). Likewise, in Adler (1998), are found "educators face the tension" (p. 24) and "she faced the dilemma" (p. 30).

In general, although authors were careful to operationalize the dominant construct, whether it was dilemma or tension, I found no piece (other than Chazan and Pimm) which offered an understanding of both. I gained the sense that, for those who wrote of dilemmas and tensions (plural), they are choices, whereas tension (singular) is the result of having to choose. But that is, at best, a guess. This is, however, supported by Byers (1984), who suggested that tension emanates from dilemmas. His was the only paper to utilize tension (singular) in its entirety.

Confusion arises, though, when considering the understanding of the word 'dilemma' as a choice between two alternatives, which are, or appear to be, equally unfavourable ("Dilemma", 2019). Facing these kinds of choices is often referred to as 'being on the horns of a dilemma' or, more colloquially, 'being between a rock and a hard place'. This fits with the understanding mentioned earlier of the choice being "between at least two courses of action" (Katz & Raths, 1992, p. 376), but it conflicts with the general understanding of the

choices being equally valued, as in “competing, worthwhile aims” (Ball, 1993, p. 373) or “competing, highly prized values” (Cuban, 1992, p. 6). Noting that almost all the studies referenced Lampert (1985), I returned to her work to locate the source of the shift.

Magdalene Lampert was a teacher-researcher whose doctoral dissertation focused on unsolvable dilemmas in teaching practice (Lampert, 1982). Arguing against the prevailing norm that problems in teaching were solvable, she instead suggested that, from the teacher’s point of view, trying to solve many common pedagogical problems leads to practical dilemmas. It was not a matter of making the correct decision between conflicting alternatives; rather, it entailed managing the dilemma by “submerging the conflict below an improvised, workable, but superficial resolution” (Lampert, 1985, p. 189). This notion of teachers as dilemma managers, Lampert noted, was at odds with the current understandings at the time:

There are, of course, many incentives for teachers and scholars to want to eliminate conflict and to think of classroom problems as solvable. If pedagogical problems could be separated one from another rather than entangled in a web of contradictory goals, then they could be solved in some sort of linear progression – shot down like ducks coming up in a row at a penny arcade. (Lampert, 1985, p. 193)

Recognizing that some teachers do solve their dilemmas by choosing, Lampert argued choosing was not the only way. To do so she offered what she called a “technical definition” of a dilemma as, “an argument that presents an antagonist with two (or more) alternatives, but is equally conclusive against him whichever alternative he chooses” (p. 182), which she felt focused more on deliberation about alternatives rather than choosing between them.

Here, then, is the genesis of a dilemma as choosing between alternatives that are in and of themselves not undesirable; though the *consequences* of the alternatives might be. Think back to Lampert’s seating problem in which she felt the pedagogical need to position herself near the mischievous boys in her classroom but, in so doing, found herself less attentive to the needs of the girls. Her alternatives were classroom order or equal opportunity, both of which are desirable goals in a classroom. It was the potential consequences of her decision that could be seen as unfavourable: if she moved closer to the girls, she undermined classroom order; if she remained near the boys, the gender equity that she desired was diminished. There was no one ‘right’ solution and, rather than

'solving' this problem, she 'managed' it by reorganizing her seating plan. The initial conflict of managing rowdy boys continued to exist, but it was placed temporarily at bay.

While this may explain the rationale behind the use of 'dilemma', it still does not explain the use of, nor interchangeability with, 'tension'. Are they the same thing? Is it necessary to distinguish their usages? These questions arose again for me when I conducted a study to identify the tension faced by a secondary mathematics teacher implementing journal writing for the first time (see Rouleau, 2018). I interpreted the results through the lens of activity theory (Engeström, 1987) where the term 'contradiction' is used in place of 'dilemma' or 'tension'. In this theory, contradictions are defined as, "historically accumulating structural tensions within and between activity systems" (Engeström, 2001, p. 137). Yet, even then it is important to note that general education studies utilizing activity theory sometimes used the terms 'contradiction' and 'tension' interchangeably (e.g., Barab, Barnett, Yamagata-Lynch, Squire, & Keating, 2002; Mukute & Lotz-Sisitka, 2012), while studies in mathematics education that utilized activity theory tended to substitute 'tension' for 'contradiction' entirely (e.g., Page & Clarke, 2010). I suggest this occurred as the term 'contradiction' in mathematics is generally understood to mean a logical contradiction which is solvable. This conflicted with the understanding of contradictions in activity theory as both endemic to activity and unsolvable. It may have seemed prudent to some mathematics education researchers to avoid confusion by substituting the term 'tension'.

Like many other researchers in activity theory, I too began my journal-writing study with the understanding that the terms tension, dilemma, and contradiction are interchangeable. However, in the process of explicating their interconnectedness through Engeström's levels, I came to see them as distinguishable. Tension is the affective surface marker of a system in which deeper contradictions or dilemmas lie, much like the dying off of bees can be an indicator of wider underlying environmental concern. Tension, therefore, is an indicator of something more complex that we could be looking to understand. This understanding, in combination with my developing awareness of the potential for tension to be non-binary, cemented my decision to use 'tension' (singular), in my further studies.



### **2.4.2. Towards a non-binary sense of tension**

Very early in my studies, I came across the work of Amanda Berry that informed a great deal of my thinking about tension and to which I connected personally. At the time, I was teaching my first mathematics method course to pre-service elementary teachers. Suffering from a good dose of imposter syndrome, I found myself filled with uncertainty regarding how and what I was teaching. This was not unlike Berry, originally a high school biology teacher, who used her self-study to describe the uncertainty she encountered in her first year as a teacher educator of prospective biology teachers. So, although our subjects differed, there was an overlap in our experiences that proved useful in exploring tension in teaching mathematics.

Initially intent on studying how various types of personal assumptions (e.g., deeply held, surface) played out in her pedagogy, Berry found herself moving towards the notion of tension for two reasons. First was the personal vulnerability she experienced in conducting a self-study. She recognized that reflection on one's own practice was as revealing of self as it was of practice. This manifested as a tension for Berry, as she thought through the decision of whether to present the findings of her study in her teacher educator voice. Second was noticing that the pedagogical approach she adopted was challenging her students. Her teaching style was influenced by the belief that prospective teachers need to learn about teaching for themselves, rather than learning to reproduce another's style. She gradually came to recognize this as a tension, as she questioned the validity of her approach when confronted by students who valued being told what to do. For Berry, the move from assumptions to tension offered a useful way to conceptualize and communicate her practice and study.

Berry's initial approach to data analysis was to identify that which she experienced as problematic in her practice. Borrowing Dewey's (1922) notion of a problem as an intellectual difficulty, Berry defined problems as situations that, "caused me doubt, perplexity or surprise and that led me to question otherwise taken-for-granted aspects of my approach" (p. 27). Thinking of these problems as tension, she suggested, allows tension "to serve both as a language for describing practice and as a frame for studying practice" (p. 166). Her study resulted in her establishing the following six tension pairs:

1. *Telling and growth*

- between informing and creating opportunities to reflect and self-direct;
- between acknowledging prospective teachers' needs and concerns and challenging them to grow.

2. *Confidence and uncertainty*

- between making explicit the complexities and messiness of teaching and helping prospective teachers feel confident to progress;
- between exposing vulnerability as a teacher educator and maintaining prospective teachers' confidence in the teacher educator as a leader.

3. *Action and intent*

- between working towards a particular ideal and jeopardising that ideal by the approach chosen to attain it.

4. *Safety and challenge*

- between a constructive learning experience and an uncomfortable learning experience.

5. *Valuing and reconstructing experience*

- between helping students recognise the 'authority of their experience' and helping them to see that there is more to teaching than simply acquiring experience.

6. *Planning and being responsive*

- between planning for learning and responding to learning opportunities as they arise in practice. (pp. 32–33)

Similar to Berry, there is a strong tendency in the literature to present tension as generated between pairs of binary opposites (e.g., Adler, 2001; Berlak & Berlak, 1981; Jaworski, 1999). Berry (2007b) suggested that tension is “expressed in terms of binaries in order to capture the sense of conflicting purpose and ambiguity held within each” (p. 120). This is understandable if one thinks of tension, as many do, as choosing between conflicting options. For instance, Jaworski (1999) described tension, experienced by secondary mathematics teachers, in terms of having to choose between investigative and didactic teaching approaches. However, there are occasions where tension is framed as a single entity that is not located ‘between’ two conflicting ideas. For example, Ball (1993),

expressed a personal tension she experienced in representing negative numbers to young students. However, rather than depicting this as a dichotomy, she showed evidence of numerous persistent uncertainties encompassed within a single tension. This suggests that, rather than binary choices, tension can occur from myriad choices or uncertainties.

It is not surprising, then, that I began my own research thinking of tension as 'between', as connections between choices or situations or forces, some of which themselves were interconnected. Indeed, I used Berry's notion of binary tension pairings to frame some of my own studies (see Rouleau & Liljedahl, 2015, 2016). Yet I encountered tension in my participants' narratives that appeared more complex than what could be understood in terms of dichotomies; rather, it appeared as a collection of uncertainties. Akin to the dictionary definition of tension as the state of being stretched tight ("Tension", 2019), I began to envision experiencing tension as being pulled by many competing uncertainties. In addition to tension as dichotomous, then, I also saw tension as a web or series of interconnections. Some may indeed manifest as tension between opposing forces, while others appear more as a collection of uncertainties that revolve around a specific conflict or situation. For example, a teacher says she is experiencing tension with assessing mathematical understanding. Is the tension what to assess? How to assess? Is it because parents prefer summative assessment? Is it because students have test anxiety? Is it a combination of these? All of these?

My earlier study, that I mentioned (designed to identify tension faced by a secondary mathematics teacher implementing journal writing), provided some insight into these questions. The results were unexpectedly rich, as they allowed me to see instances of tension that were binary, like Berry (2007a), and instances of tension that were more in keeping with Ball (1993). Not only did I develop an understanding of what happened during the journal implementation, the immersion in activity theory helped me develop a non-dichotomous understanding of tension. Like Cooper (1917), I was getting closer to 'my' tension.

### **2.4.3. Towards a dialectical sense of tension**

Although Berlak and Berlak (1981) published the first comprehensive study of tension in teaching, I did not, in the beginning, appreciate the influence their work would have on my dissertation. Despite reading their seminal work very early on in my studies, I did not really

‘read’ it. I skimmed through it, felt I understood the critical components well enough and was content to cite their work as the cornerstone of the literature on tension – work on which others, like myself, founded their own research. Variants of *“Building on the work of Berlak and Berlak (1981) who identified sixteen dilemmas that illuminated the relationship between every day school events and broader social, economic, and political issues, it was Lampert (1985)”* (Rouleau, 2017, p. 2989) appeared in almost all my publications. I went no deeper.

Upon reflection, it is likely that I was initially drawn more to the work of authors like Lampert and Berry who wrote vivid depictions of tension from their own practices. Theirs were self-studies of teaching with which I felt an immediate kinship. I had lived, and in the case of Berry, was still living the tension they described. Berlak and Berlak, on the other hand, wrote about other teachers’ experiences of tension in a process removed from personal experience. Perhaps Lampert described it best:

When I consider the conflicts that arise in the classroom from my perspective as a teacher, I do not see a choice between abstract social goals, such as Excellence versus Equality or Freedom versus Standardization. What I see are tensions between individual students, or personal confrontations between myself and a particular group of boys or girls. (1985, p. 181)

In favouring Lampert’s and Berry’s layer of intimacy, though, I initially overlooked the richness that was in the Berlaks’ work. This is important, as it was they who set me on a path towards a different understanding of tension.

Looking for language to characterize what they had noticed during a six-month stint observing in classrooms drew the Berlaks to the language of dilemmas. When asked if the classrooms they visited were more ‘teacher-centred’ or ‘student-directed’, or whether admin was more sensitive to children’s emotions and needs or more concerned with academics, they realized they could not easily formulate an answer that represented their observations, and often found themselves saying ‘yes’ and ‘no’ and sharing the observations that led to both conclusions. There was a sense in which generalizations, such as “open-classroom teachers believe children have the right and competence to make learning decisions” (p. 22), were both true and not true. So, they sought to develop terminology that would adequately represent the complexity they observed in teachers’ classrooms without distorting its nuances.

To that end, the Berlaks utilized the term 'dilemma' to portray the unceasing interaction of internal and external forces at play – those “contradictions that are simultaneously in consciousness and in society” (p. 124). Their work resulted in a taxonomy of sixteen dilemmas that illuminated the relationship between everyday school events and broader social, economic, and political issues. For instance, one classroom tension that relates to a societal tension is noted in their observation:

On the one hand, teachers are pulled towards the view that all children deserve equal shares and, on the other, towards the idea that some students merit more than others. Throughout history the criteria for deciding who deserves more or less of these resources and how much more or less one deserves have been a source of strife. (p. 159)

These sixteen dilemmas were categorized as three distinct sets which served to describe and explain what the Berlaks had observed in and across the schools and teachers they studied. The four *control* dilemmas comprised uncertainty over locus and extent of control of students e.g., “whole child vs. child as student”. The eight dilemmas in the *curriculum* set captured contradictions over transmission of knowledge, and ways of knowing and learning, such as “knowledge of content vs. knowledge as process”. The remaining four *societal* dilemmas consisted of contradictions that were implicit in the way children are dealt with in all forms of institutional life, both within and outside of school. An example of a societal dilemma was “equal allocation of resources vs. differential allocation” of which the “equal shares” quotation in the preceding paragraph is an example.

Berlak and Berlak saw in this language of dilemmas the means of representing the diverse and contradictory patterns of schooling: “Dilemmas do not represent static ideas waiting at bay in the mind, but an unceasing interaction of internal and external forces, a world of continuous transformation” (p. 133). They referred to these forces as the ‘internal’ and ‘external’ dialectic and suggested that, because teachers are capable of becoming aware of these forces, they are capable of altering their own actions. These dilemmas are a source both for action and reflection and for a means for transformation and growth.

This is an understanding of tension as a significant and necessary component to growth and development in the dialectical sense. In mathematics education, tension is framed as the affective result of a teacher having to deliberate between competing, worthwhile aims (e.g., Lampert, 1985). Studies have produced lists of tension that impact mathematics education (e.g., Adler, 2001; Liljedahl, Andrà, Di Martino, & Rouleau, 2015; Thomas &

Yoon, 2013), that have resulted in a collective depiction of mathematics teachers as “dilemma managers who accept conflict as a continuing condition with which persons can learn to cope” (Lampert, 1985, p. 192). Along with this has emerged the understanding that managing recurring tension is not akin to solving it; rather, it is a matter of compromise and “satisficing” – sacrificing in order to satisfy (Cuban, 1992, p. 8).

Prior to this, Dewey (1922) had proposed a different perspective on tension, suggesting it is “the *sine qua non* of reflection and ingenuity” (p. 301) – that conflict is a necessary precursor for development and growth. I suggest this view is often overlooked in mathematics education, where tension is seen, at best, as useful for reflection on practice and, at worst, as something to be tolerated. Dewey’s was a dialectical view of development, as a process driven by tension. In this view, tension is more than useful; it is productive, in that it has the capability to produce something.

This dialectical understanding of tension also developed from my earlier study regarding journaling in a secondary mathematics classroom. To conceptualize tension as an ending point where a mathematics teacher utilize skills of ‘satisficing’, means overlooking the growth that occurs in overcoming tension. In my study, the teacher ‘managed’ his tension by discontinuing mathematics journaling, but it was a trade-off that stalled not only his development, but also that of his students. Had he better understood his tension as a point of growth, it may have encouraged him to reflect on how he could alter the journaling implementation, rather than abandon it.

In looking back through the mathematics education literature, I noticed only two instances where researchers explicitly spoke of tension in the dialectical sense (see Adler, 2001; Jones, 1995). I had to go back to Berlak and Berlak (1981), whose dialectical sense of dilemmas was founded on understandings of Marx and Mead, before I found reference to tension as necessary for development. And, although many tension studies in mathematics education make reference to Berlak and Berlak (1981), it seems that the understanding of tension as necessary for growth and development has been lost in the interim.

#### 2.4.4. Metaphors for tension

When communicating a phenomenon of interest, one of the first obligations is to be clear about it oneself, yet I struggled to conceptualize tension. Much like I put a pin in the map at the start of this dissertation journey, I was trying to put a pin in tension. I wanted a rigid construct that I could offer up as *the* definition of tension. This presupposed that there was one coherent definition that would suit all situations and cover all circumstances. The impossibility of that was brought home to me during a discussion with colleagues in which we attempted to construct a definition of angle. Despite this seemingly simplistic task, we met with difficulty at every turn. Each definition we came up with worked in only some instances and fell apart in others.

Skemp (1987) suggested that there is often an advantage to having choice around the definition of a concept. He offers the example of describing a shape using the term ‘cuboid’. If that term is unfamiliar one could try again, perhaps calling it a ‘rectangular block’. This allows one to classify the same idea in different ways and, as Skemp argued, “it can help us to emphasize that aspect of a complex idea which is most relevant to particular circumstances (p. 53). Perhaps this is why I was coming across so many metaphors for tension in the literature; they were proving a useful tool for researchers for making a complex term accessible for specific contexts.

Metaphors permit the understanding of one kind of experience in terms of another (Lakoff & Johnson, 1980). More than just linguistic expressions, they are the means through which we conceptualize our experience; that is, most concepts are partially understood in terms of other concepts (Lakoff & Núñez, 2000). As we attempt to make sense of the perceived similarities inherent in metaphor, we shift our understanding as new connections between the concepts are considered. For instance, ‘mathematics is language’ is a metaphor through which we might gain a new understanding of mathematics beyond numbers when we start to think about mathematics in terms of syntax and semantics/grammatical notation. As Pimm (1988) suggested:

The power of this metaphoric adjectival construct is that it creates links between the new and the old setting, by highlighting (or, from an alternative philosophical position, creating) certain commonalities. The extended meaning may also have a certain retrospective effect on the old setting, by drawing attention to certain features of it that may not have been noticed

previously or considered of importance. It certainly has the effect of altering the balance between the various connotations of the concept. (p. 33)

These are experiential connections and who we are as individuals matters, as “metaphor is as much a part of our functioning as our sense of touch, and as precious” (Lakoff & Johnson, 1980, p. 239). As the use of a metaphor may mean different things to different people, to negotiate meaning requires, “finding the right metaphor to communicate the relevant parts of unshared experiences or to highlight the shared experiences while deemphasizing the others. Metaphorical imagination is a crucial skill in creating rapport and in communicating the nature of unshared experience” (p. 231).

While bearing underlying similarities, each metaphor I encountered helped me understand more fully the complexity inherent in tension, an understanding that I could then apply to my own work. Some metaphors reinforced my previous understanding, others provided clarity and new ways of thinking about tension. As Lakoff and Johnson suggested, “metaphors are capable of creating new understandings and, therefore, new realities” (p. 235). In what follows, I share four metaphors that helped me capture different aspects of tension. See Table 2.1 for a list of the metaphors, the connections I noticed, and the understandings I developed as a result.

### **Tension is friction (Tsing, 2005)**

Tsing’s metaphor arose from her ethnographic study of capitalism’s interactions with nature and culture in Indonesia in which the complicated, messy process resulted in new cultural forms. For her, friction was caused by the rubbing together of the global and local influences. Looking past the initial irritation this caused, she instead saw the productivity generated:

A wheel turns because of its encounter with the surface of the road; spinning in the air it goes nowhere. Rubbing two sticks together produces heat and light; one stick alone is just a stick. (p. 5)

This productive view of friction can be applied to tension that arises in teaching, which, while challenging, also has creative potential. For teachers, tension that results from the “rubbing together” of disparate teaching methods, philosophies, or policies may spark insights for teachers that they might not otherwise have had. For example, overhearing colleagues discuss new assessment practices in the teachers’ lounge may create the friction needed eventually to adjust one’s own practice. Similarly, as researchers, our



conceptual frameworks for making sense of the lived experiences of teachers are altered by the friction generated through our own encounters with teachers whose pedagogy is informed by personal biographies, cultural contexts, and educational backgrounds that are often quite dissimilar to our own. I find this to be a productive view of tension in that it can often result in new and unanticipated insights into the process of teaching both for the researcher and for the teacher.

### **Tension is walking a tightrope (Cuban, 1992)**

Despite noting a lack of community between researchers and teachers, in his article, Cuban suggested both groups share the commonality of tension in practice, which he described as walking a tightrope: “We invent a tightrope to walk, knowing that to cross the tightrope juggling the competing claims will still leave us uneasy” (p. 7). At first glance, his tightrope metaphor speaks only to the uneasiness of tension: experiencing tension is a matter of precarious balance along a predefined path. For most of us, tightropes are terrifying and something to be avoided. In Cuban’s metaphor, though, I was able to see other, more compelling ideas inherent in tension. For instance, he argued that managing tension is an “art form, filled with doubt but at least free of corrosive guilt” (p. 8). In so doing, he suggested that we learn to create “better compromises and more elegant tightropes” (p. 8). What is important here is that it is teachers who ‘invent’ the path; it is their decisions that shape it. Teachers are not passive responders to tension; rather, they might use it as a means of growth. There is also an aesthetic sense to this that I appreciated, as Cuban saw managing tension as an art, one where teachers developed skill to create more elegant ways of dealing with tension. It brings to mind the grace and poise of acrobats as they work with the tension of the rope. Related to this is the idea that tension is recurring. When we watch acrobats cross over a tightrope, they always return. And this time the wind is blowing from a different direction and the sun is facing a different way. From this, I take the notion that tension, once managed, will also return and teachers will experience it in a new form and, possibly, need to find a new way to manage it.

### **Tension is play (Carr, 1998)**

Whereas Cuban’s (1992) tightrope constricted space, I feel Carr’s metaphor of play, “that flexible sensation of possibility” (p. 196), expands it. In describing tension in his own teaching practice, Carr likened tension to play, in the sense of freedom or room for

movement; it is the space in or through which a thing can or does move (“Play”, 2019). Its connection to tension lies in the ability of both to overwhelm or to suppress growth. In drawing a comparison to muscle resistance training, Carr suggested, “Too much play and we lose direction and power, too little play and we have no direction or motion” (p. 196). This is an experiential connection that can be made by those who have been surprised by an overly weighted barbell that was impossible to lift or its opposite, where the lack of weight sent the barbell flailing through the air. Tension too can be imagined having these limits. Like Goldilocks’ porridge, there is an amount of tension that is preferable; too much tension and a teacher is overwhelmed, too little and the teacher has nothing to push against. Either way is a loss, as there is no scope for development. Tension, then, is necessary for growth; we need to feel it to respond with our own strengths. I find similarity in Carr’s metaphor of play and Carly’s elastic band metaphor, where she felt herself stretching, being pulled back, and then stretched a little further. Inherent in both is the usefulness and necessity of resistance. Also inherent in both is the notion of just the right amount of tension: What happens when an elastic band is stretched too far? What happens when it is stretched for too long?

### **Tension is navigating between Scylla and Charybdis (Chazan & Pimm, 2016)**

In their commentary on the work of Adler (2001), Chazan and Pimm used this metaphor not only to describe dilemmas, but also to work through their own tension regarding its definition. For Chazan and Pimm, dilemmas are Scylla and Charybdis, the two mythic sea monsters guarding the Strait of Messina. To pass by meant sailing perilously close to one or the other:

Crucially, there is no alternative in two distinct senses if one is to reach one’s destination (goal): there is no other way round and there is no ideal path between them that somehow keeps them both at bay. You have to engage to some degree with one or the other (as Odysseus manages to in Homer), though this engagement has costs (for Odysseus, the loss of six of his crew). (p. 20)

Theirs is a sense of dilemma in the dictionary definition as choosing between two equally unfavourable options. In coming to terms with Lampert’s (1985) understanding of dilemmas as comprising incompatible worthwhile options, Chazan, in Chazan and Pimm, suggested, “each of these monsters perhaps represents not honoring one of one’s competing commitments” (p. 22). It is not the choices that are undesirable for teachers; it

is the consequences. And, much like Odysseus successfully navigating the pass between Scylla and Charydbis, teachers find a way to survive and keep their losses to a minimum.

Chazan and Pimm's metaphor also speaks to the enduring nature of tension suggesting:

As a teacher, one comes back the next day and they are still there, but, taking up your metaphor of classroom rapids (and bearing Heraclitus in mind), the monsters have moved. Nothing stays still. The dilemma does not go away, simply because I have decided to act in a certain way, this time. (p. 29)

Like Scylla and Charydbis, whose mythology has not waned with time, tension is always present in a teacher's practice; there is no way to avoid it. Additionally, the emphatic brevity of "Nothing stays still" draws attention to the idea of movement. Similarly, to Cuban (1992), this feels like movement in constricted space; there is not enough room to sail between the two monsters without engaging with one or the other to some degree. And sailing between implies sailing past which leads me further to imagine this movement as forward motion. Odysseus could have turned back, but in sailing forward and successfully navigating the Strait, he was one step closer to reaching home. Likewise, should a teacher choose to engage with tension, the result may be growth, and, in this, I see a dialectical sense of tension.

**Table 2.1. Metaphor Connections and Understandings**

	Metaphor	Connections	Key understanding
Tsing (2005)	friction	irritation; between objects	productive
Cuban (1992)	tightrope	unease; balance; constricted space; individual; repeat/return	recurring
Carr (1988)	play	open space; optimal resistance; flexible	necessary for growth
Chazan & Pimm (2016)	Scylla and Charydbis	unavoidable; choice; survival; constricted space; forward motion	endemic/enduring; dialectic

## 2.5. Summary

While much research is devoted to *what* it is teachers do, there is far less known about *why* teachers do the things they do. Attending to tension can help to bridge that gap. Tension is an affective state associated with uncertainty, in which one yearns for a

resolution that may or may not be attainable. It is the impetus for growth and change as one chooses from two or more competing, yet worthwhile pathways. Positioning teachers as tension managers, whose actions are shaped by an undercurrent of uncertainty, offers insight into the 'why' behind their actions. It allows for a focus on the process of teaching rather than solely on the product.

That tension is inherent in teaching is not surprising. Teaching is a complex act and teachers frequently find themselves having to make difficult choices. Not to be conflated with a problem, tension is considered something to be managed, not solved. This means that a teacher takes into consideration alternative solutions none of which is the 'right' solution or 'best' practice, yet she must do something. And that 'something' is a temporary solution that works (or not) in a given situation, as tension is idiosyncratic: each teacher has her own experience of tension that may change with context.

The value in tension is twofold. From a practical point of view, Adler (2001) pointed out, "The value in identifying key teaching dilemmas and naming them is that they can then become objects of reflection and action" (p. 49). This holds true both for the teacher trying to improve her practice *and* for the researcher trying to understand that process. As a researcher, my understanding of the work of teaching may well be enhanced by exploring how teachers experience and make use of tension.

This leads to the second reason why tension proves valuable; it holds within it the potential for a dialectical transformation of practice. This is an under-researched area that is worth exploring. In the literature, tension has been identified and described, but there has not been a closer examination of how teachers manage instances of tension that they experience and its impact on change in practice.

With this in mind, my objective is to embark on that 'closer examination' of tension in change. I am particularly interested in tension that accompanies teacher-initiated change in mathematics teaching practice. Accordingly, the next chapter explores the connections among change, agency, and tension.

## Chapter 3. Theoretical Constructs

*A good theory enables us to penetrate beyond the observables to the heart of the matter. (Skemp, 1987, p. 143)*

A theoretical framework situates, shapes, and informs the research; it moves one from simply describing into questions of ‘why’ and ‘how’ instead. Reminiscent of Cooper’s fish, frameworks are essential for “deep understanding, not just ‘for this’ understanding” (Lester, 2010, p. 70). In essence, I need to understand fully the phenomena I am studying and not limit myself to finding temporary solutions to immediate problems. Lester suggested that a research framework, “helps us develop deep understanding by providing a structure for designing research studies, interpreting data resulting from those studies, and drawing conclusions” (p. 70).

With this in mind, I began to search for theoretical frameworks and perspectives that might be a good fit for thinking about tension in teaching mathematics. I found this process of exploration integral in shaping and formulating my research focus. I found myself asking, “What will this construct help reveal about tension that is not already known?” I was searching for a framework that would help me access Lester’s “deep understanding” of tension in teaching mathematics.

However, just as I was looking to find *the* definition of tension in the previous chapter, I was looking for *the* framework for studying tension. In so doing, I was forgetting that, while choosing a framework is essential, there is not necessarily one perfect frame. Instead, I found two different constructs which informed and shaped my study: teacher change and teacher agency. In the following, I highlight the key aspects of each and how they connect and offer insight into tension in teaching mathematics.

### 3.1. Teacher change

I have not forgotten about Carly and elastic bands. Tension explained the ‘stretching’ she felt, but how does that connect to change? When I first began exploring tension, I got caught up in categorizing and labelling all sorts of tension found in mathematics teaching (e.g., Rouleau 2017, 2018, 2019). Although this was useful in developing my understanding of tension, it had not brought me any closer to my earlier questions of who

changed, why they changed, and how they changed. To do that, I first needed to spend some time thinking and reading about change.

I believe most teachers change all the time. To be a teacher means to live in a constant state of flux – changes in room assignments, grade assignments, changes in administration – this is the norm for a teacher. These, though, are minor changes, what Cuban (1988) described as first-order changes and which fit quite nicely with dictionary definitions of change, e.g., “The action or process of making or becoming different” (“Change”, 2019). Not better, not improved, just different. What I am interested in are second-order changes, those which bring about different ways of teaching, learning, and thinking (the different way of being in the classroom that I desired in Chapter 1). Inherent in that view is the assumption that the changes are necessarily for the better; I find myself agreeing with Jenkins (2003), who suggested that change is not just to make different, but also continually to improve. However, as Lerman (2000) pointed out, the notion of improvement is problematic, as it leads to questions of who decides what constitutes improvement. A researcher? A principal? A school district? Parents? A government agency? The teacher herself? In siding with the last, I use the notion of improvement to refer to teachers’ efforts to achieve self-determined changes in their practice.

That is not the only problem though. As with tension and dilemma, there is interchangeability in change terminology, where the words ‘development’ and ‘growth’ are also frequently used. Following Clarke and Hollingsworth (1994), I use ‘development’ when referring to teacher in-service; ‘change’ when referring to an observable process with the goal of improvement; ‘growth’ both to encompass the change process and to invoke a notion of improving. For example, as a result of attending a professional *development* program, a teacher decides to *change* how she questions her students. Her goal is to ask questions that elicit deeper mathematical thinking. In so doing, she comes to the realization that the kinds of questions she used to ask actually interfered with her students’ learning. Her own realization constitutes *growth*.

### **3.1.1. Perspectives on change**

There are two bodies of change literature which, while interconnected, focus on differing aspects of teacher change. The first is organizational change, where the shared assumption is that change is not entirely an individual phenomenon; it is orchestrated by

the school context, which can serve to constrain or nurture growth, and by the promotion of school-wide changes in practice that teachers are strongly encouraged, if not mandated, to implement (Richardson & Placier, 2001). It is from the latter that a deficit view of teacher change usually arises, as any opposition to the change is often construed as recalcitrance (McLaughlin, 1987). The second is change associated with the individual, where a degree of autonomy and personal choice is assumed. Here, change is viewed as voluntary and takes place over the career of a teacher as she engages in a variety of professional experiences: discussions with colleagues, teacher materials, workshops, or professional development. Underpinning these experiences is the belief that the power over what teachers choose to change is in their own hands (Richardson & Placier, 2001).

Clarke and Hollingsworth (1994) described these two ways of thinking about change through the use of six categories of change perspectives: training; adaptation; systemic restructuring; personal development; local reform; and growth (see Table 3.1). Although Clarke and Hollingsworth advocate the change perspective of growth in design and development of mathematics teacher education, they acknowledge that an awareness of the various perspectives is beneficial both for the design of professional development and for research. As a means of conceptualizing change, the categories are useful in informing the design of professional development, as identifying the differing change perspectives of the teacher participants provides insight into the teachers' expectations and experiences of growth. From a research perspective, the authors suggest these categories offer alternative ways to interpret teachers' motivations and actions that move beyond a deficit view of teacher change. The six change perspectives are outlined below.

## **1. Change as training**

Clarke and Hollingsworth (1994) described this perspective with the adage "change is something that is done to teachers" (p. 154). This was the predominant thinking of early versions of professional development models that focused on the teacher as lacking in skills, knowledge, or ability, all of which could be addressed through the proper intervention (Guskey, 1986). The teachers were positioned as passive receptors of information. More colloquially known as 'sit and get', the ineffectiveness of models that operate from a deficiency stance has been well-documented. Unfortunately, this model persists. Noting that teachers are seldom involved in planning for change, Roettger (2003)

observed, “The change is handed to the teacher to perform without any meaningful dialogue expressing why” (p. 33).

## **2. Change as adaptation**

Bearing similarities to Cuban’s (1988) first-order changes, in this perspective, teachers change in response to something; they adapt their practices to changed conditions. For instance, teachers may find it necessary to alter their practice in response to stimuli such as changes in class sizes or availability of material resources. Although these may be made willingly, the changes they make are a passive response to external influences from the teachers’ environments, not from their own personal desire for change. Clarke and Hollingsworth referred to this change as an ‘adaptation’ which, in evolutionary theory, is the mechanism by which organisms adjust to new environments or to changes in their current environment.

## **3. Change as systemic restructuring**

In this perspective, teachers enact the change policies of a system that operates outside the borders of a classroom or school. This is true of national or provincial curriculum changes, developed externally to the teachers, yet intended to be implemented by them. Clarke and Hollingsworth suggested, “While the purpose of the initiated change may be system-wide curricular reform, we would suggest that consequent changes in individual teachers' practices is inevitable” (p. 158).

## **4. Change as personal development**

Describing teachers as reflective practitioners (see Schön, 1983), Clark and Hollingsworth suggested this perspective encompasses those that seek to change in an attempt to improve their performance or develop additional skills or strategies. This is second-order change, as teachers take from their learning experience new ways of thinking or teaching. From this perspective, even in traditional sit-and-get professional development, the teacher may take away elements they find relevant to their practice.

## **5. Change as local reform**

Similarly, to change as personal development, in this perspective, teachers also change for reasons of personal growth. The difference lies in the object targeted for change. Moving beyond personal development, here, the teacher is a reformer who actively seeks



out assistance in implementing reform in their local environment, whether with the assistance of an expert or by targeted professional development that addresses the initiative they wish to implement. Again, this is internally driven change, as the teacher herself desires the reform.

## 6. Change as growth or learning

Predicated on theories of learning that advocate for ownership and control of one's learning, in this perspective, teachers are themselves learners working within a community of other learners. This implies a continual change process with the teacher in control of the direction and form of the learning. Internally motivated, this change is thought of as organic and intrinsic as the teacher learns, reflects, and responds metacognitively to their professional practice.

**Table 3.1 Change Perspectives (adapted from Clarke and Hollingsworth, 1994)**

Richardson & Placier (2001)	Clarke & Hollingsworth (1994)			
	Change Perspective	Location of Change Initiative	Teacher Role	Object of Change
Organizational Change	Training	External	Subject	Teacher
	Adaptation	External	Respondent	Environment
	Systemic Restructuring	External	Implementer	Curriculum
Individual Change	Personal Development	Internal	Reflective Practitioner	Teacher
	Local Reform	Internal	Reformer	Environment
	Growth or Learning	Internal	Learner	Teacher

Over 45 years ago, Jackson (1974) argued the need for a professional growth approach. He suggested, “The motive for learning more about teaching is not to repair a personal inadequacy as a teacher, but to seek greater fulfillment as a practitioner of the art” (p. 26). And, although the response has been slow, there has been a research shift towards perspectives of change as growth – such as that of Clarke and Hollingsworth (1994) – in which teachers are viewed as continuous learners and architects of their own change, rather than as passive receptors (Roettger, 2003). However, change continues to be problematic in terms of its actual achievement in practice (Chapman & Heater, 2010). This has resulted in the development of models of professional growth in an attempt to understand the change process. In what follows, I briefly introduce several models of professional growth that appear in mathematics education research, before closely examining the transformational change model introduced by Chapman and Heater.

### **3.1.2. Transformative change**

There are a variety of models for thinking about teacher change in mathematics. For example, in a special issue of the *Journal of Mathematics Teacher Education* on change, Goos and Geiger (2010) used zone theory to offer a socio-cultural perspective of change that extended Vygotsky’s (1978) concept of the zone of proximal development (ZPD) to incorporate the social setting, goals and actions of participants, while Walshaw (2010) used the discourse of the reflective practitioner (along with psychoanalytical theory) to examine change in mathematics pedagogy. Still others utilized social constructivism (de la Cinta Muñoz-Catalán, Yáñez, & Rodríguez, 2010); critical collegueship (Males, Otten, & Herbel-Eisenmann, 2010); actor-network theory (Boylan, 2010); communities of practice (Hunter, 2010). All offered variants on three elements which Llinares and Krainer (2006) identified as being common across contemporary mathematics education research on change. The first is the acknowledgement of reflection as an important mechanism for change. The second is the growing awareness of the social dimension of teacher change, while the third is the increasing attention to the organisational context of teachers’ work and the extent to which it provides resources to support change.

Acknowledging the variety of ways to think about change offered in the mathematics education literature, Chapman and Heater (2010) argued that the literature still falls short of its goal. Instead, they suggested, “We still need to understand the motivation for change, opportunities for teachers to change themselves to promote learning of

mathematics, and how all this relates to the learning of mathematics” (p. 446). Basing their own work on the notions of the reflective practitioner (Schön, 1983), inquiry (Dewey, 1938/1991), and noticing and researching from the inside (Mason, 2002), Chapman and Heater (2010), theorized a method of framing teacher change in which the teacher takes responsibility for her own change. With a focus on change in mathematics pedagogy, the authors’ conception of change is that it involves “significant transformation in the teacher’s thinking and practice” (p. 445). Similar to Clarke and Hollingsworth’s change as growth perspective, it requires that teachers engage as metacognitive learners to:

1. examine their own experience of work on themselves while addressing the question of how to support students in learning;
2. attend to experience so as to develop sensitivities to others and to be awake to possibilities;
3. focus on problems and experiment with situations;
4. engage in introspective and interspective observations (p. 447).

This four-stage cyclical process is what Chapman and Heater suggested is essential for a type of change they refer to as foundational change. As its name implies, foundational change suggests a change in the very foundation of identity and practice. The changed teacher “thinks in new ways and is empowered to transform her teaching as a whole” (p. 456). Along with foundational change, the authors also described two types of less transformative change: instrumental change and conceptual change. Instrumental change is thought of as surface change, where new techniques or strategies are implemented, which gives the appearance of change without altering any of the underlying structure of practice or how the teacher thinks of practice. Conceptual change, on the other hand, also includes changes in techniques and strategies, but with this type of change, teachers are able to understand why a new technique might work. This increases the scope of the change, as it allows for use of the technique beyond the context in which it was learned (see Table 3.2).

**Table 3.2      Types of Change (adapted from Chapman and Heater, 2010)**

Types of Change	
Instrumental	Surface change in which new techniques or strategies are implemented, without altering the structure or thinking of practice.  Professional development leaders are viewed as experts with the right answers.
Conceptual	New techniques are implemented with an understanding of why a technique works and when and how to apply it beyond the situation in which it was learned.  Professional learning is viewed as collaboration with others and professional development leaders are viewed as mentors to facilitate understanding.
Foundational	Deep change in the foundation of practice and the thinking on which it is built.  Learning as a self-authoring way of knowing in which one's own knowledge and experience are important to the learning process. Requires a change in self, not just practice.

Chapman and Heater's (2010) transformational change model is appealing because it takes into account the teacher voice that has so often been missing. A difficulty inherent in change literature is that it is often written *about* teachers, without adding their collective voice to the discussion (Richardson & Placier, 2001). This was pointed out much earlier by Klein (1969), who noted, "studies of change appear to be taken from the perspective of those who are the change agents seeking to bring about change rather than of the clients they are seeking to influence" (pp. 498–499). Accordingly, Chapman and Heater's model is based on the assumption that, "meaningful change can occur when the process is initiated and rooted in the teacher's experience based on a tension in self and/or practice that is personal and real to him or her" (p. 456). I agree, and suggest that teacher voice can bring insight to the daily frustrations and needs of the classroom and the changes that result. Furthermore, listening to those concerns can help researchers better understand the apparently inconsistent behaviour they may observe. If teachers are thought of as "dilemma managers" (Lampert, 1985), what might be construed as instrumental change could be recast as a rational decision that weighed the practicality of the change against its potential consequences.

This is where tension comes in to play. Chapman and Heater's transformational change model suggests that growth occurs when teachers recognize and seek to manage tension.

Whether or not growth occurs requires more than reflection; it also requires recognition of the inherent tension that can hinder or nurture change. Yet, I suggest that it is also possible for a teacher to engage in introspective metacognition on tension that does not result in change in the improvement sense.

Consider the metaphor of an open door. Standing at the threshold, one has a choice. Do I go in or not? Choosing to move across inevitably leads to something new and choosing not to move across might indicate a desire to remain in the comfort (or discomfort) of sameness. It is tension that brings a teacher to the doorway and, as she stands there thinking through her decision, it is her choice whether or not to cross. Either way, the tension is managed, but it is possible that only one way brings improvement.

This image of standing at a threshold brings to mind Hegelian dialectics in which individuals cannot expect to progress without experiencing these moments of decision in the metaphorical doorway. To remain inert is to avoid the interplay between stability and change that leads to transformation. Hegel conceptualizes individual growth as a dialectical process in which individuals generate opposition to themselves simply by trying to change. Growth is achieved by bringing that opposition into themselves and overcoming it (Jackson, 2012). For Hegel, there can be no life which is not characterized by these encounters and individuals cannot progress without experiencing these moments of tension. From tension arises negative moments from which power is derived, “by looking the negative in the face, and tarrying with it. This tarrying with the negative is the magical power that converts it into being” (Hegel, 1807/1977, p. 19). However, individuals must see the positive within the negative moment of development if they are truly to grow, as “seeing the positive within the negative moment allows an individual to grow more effectively; the negative is not thrown away, but fully felt and understood so that the lessons of this “contradiction” can be wholly appreciated” (Jackson, p. 69). This brings us back to Chapman and Heater’s notion of transformational change where meaningful change occurs in processes that begin with experiencing a personal tension. This suggests extending Llinares and Krainer’s (2006) three elements of change (reflection, social dimension, and context) in mathematics education to include the presence of tension.

I am interested in that connection between tension and change. The literature acknowledges that tension is often apparent when teachers are involved in changing up

practice, for instance when implementing new forms of assessment or new curriculum (e.g., Dietiker & Riling, 2018; Cooney, 2001). What is left unaddressed is how that tension affects change. Sztomptka (1994) suggested growth is a “change of”, not just a “change in” (p. 27). I found myself questioning what might be the characteristics of tension that bring about or impede growth in the “change of” sense? What is happening in that metaphorical doorway? This led me to consider the notion of teacher agency.

## **3.2. Teacher agency**

Through my own experience both as a teacher and in working with teachers, I have come to see teachers as responsible for their own growth. Like Carly, most teachers know who they are, who they might want to be, and if, how, and when that might happen. This vision of teachers led quite naturally to the construct of teacher agency which, “involves teachers finding alternative ways of knowing the truth about themselves” (Pignatelli, 1993, p. 420). In what follows, I begin by describing agency in general before outlining two educational models for understanding teacher agency. I end with a look at literature from mathematics education that focuses on dance as a metaphor for teacher agency.

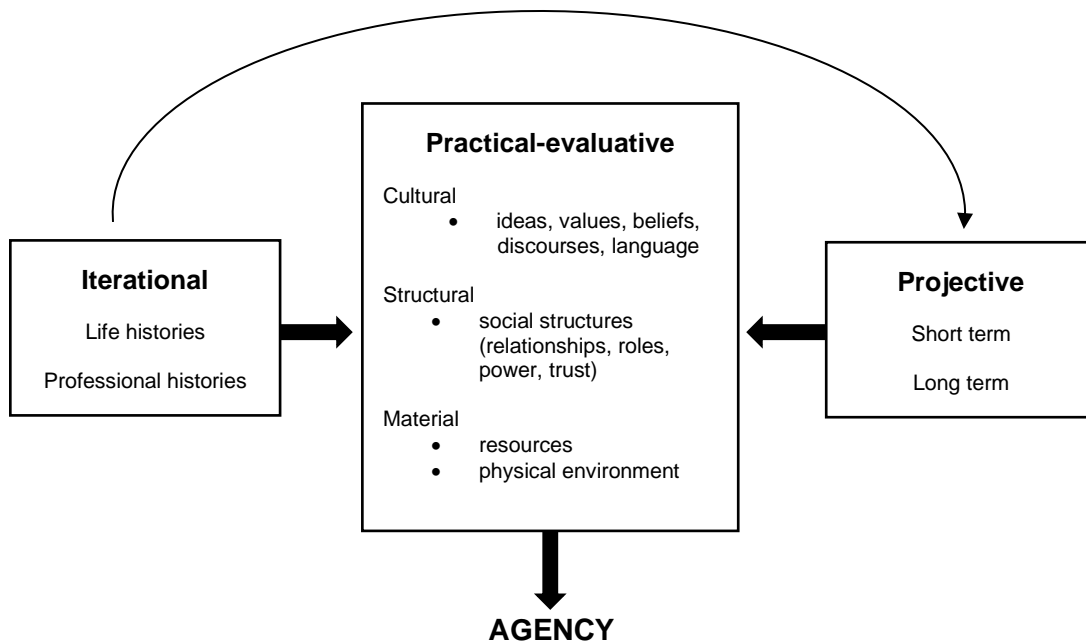
### **3.2.1. Human agency**

The origins of teacher agency lie in the broader sociological conceptualization of human agency, which attempts to address the question of what power humans have to impact the society in which they live. Often described as the relationship between structure and agency, this question captures a central tension in sociological theory regarding the limits of human action and the potential for change and resistance (Hollis, 1994). A structuralist view of structure and agency is founded on Marxist understandings of the power and nature of societal institutions, in which humans are thought to have very little personal agency (Althusser, 1971/2002). Rather, humans are viewed as functions of the structure which is sustained and reproduced through their actions. Critics of the structuralist view (e.g., Giddens, 1989; Pignatelli, 1993) argue that structure and agency are equally important in determining or shaping social action in that they are symbiotically related, “Humans are purposive actors, who virtually all the time know what they are doing (under some description) and why” (Giddens, 1989, p. 253). Accordingly, structure only exists in and through the activities of human agents.

The result of these early understandings of agency was the tendency to view agency as an innate capacity, that is, something humans can have or possess (Priestley, Biesta, & Robinson, 2015). This led to definitions of agency not far removed from that of the dictionary: the ability or capacity to act ("Agency", 2019). For instance, the "capacity for autonomous action ... [independent] of the determining constraints of social structure" (Calhoun, 2002, cited by Priestley et al., 2015, p. 22); or as the capacity of individuals who "critically shape their responses to problematic situations" (Emirbayer & Mische, 1998, p. 971). Priestley and colleagues (2015) suggested that while these definitions seek to capture the nature of agency, all are potentially problematic, as "they may be taken to suggest an overly individualistic view of agency as human capacity, seeing agency as something that people possess to varying degrees as a result of their personal attributes" (p. 22). Instead, the authors suggested an alternative conceptualization of agency, as an emergent phenomenon where agency is not understood as a possession of a human, rather it is something that is achieved (Priestley et al., 2015). This occurs, "through the interplay of personal capacities and the resources, affordances and constraints of the environment by means of which individuals act" (p. 19). What this interplay allows for is the possibility for an individual to have agency in one situation but not in another, as achievement in one situation does not necessarily mean it will be achieved in other situations as well (Biesta & Tedder, 2007). This also makes it possible to understand fluctuations in an individual's agency over time.

Priestley and colleagues described this as an ecological approach to agency which is both relational and temporal. The relational aspect refers to how humans function in their social and material environments while the temporal aspect comprises three dimensions. First is that agency is rooted in *past experience*, which means more experienced individuals may more readily achieve agency. Second, agency is oriented to the *future* via goal setting and envisioning future possibilities where individuals with limited aspirations are less likely to achieve agency than those who can imagine multiple pathways forward. Third, agency is acted on in the *present* and shaped both by what is thought possible and that which is actually possible given existing resources and constraints. Building on the work of Emirbayer and Mische (1998), Priestley and colleagues referred to these three temporal aspects as the *iterational*, the *projective*, and the *practical-evaluative* dimensions of agency. The result was a model for understanding agency that suggested that "the achievement of agency is always informed by the *iterational*, the past experiences,

including personal and professional biographies; orientated towards the *projective*, the future, both with regard to more short-term and more long-term perspectives; and enacted in the *practical-evaluative*, the here-and-now, where such enactment is influenced by what we refer to as cultural, material and structural resources” (Biesta, Priestley, & Robinson, 2015, p. 627, italics mine) (see Figure 3.1). It is important to reiterate that agency is always achieved in concrete and specific situations that occur in the practical–evaluative dimension. This dimension of agency achievement focuses on the practical: that is, what is practically possible and feasible in this particular situation; and the evaluative: that is the way in which one evaluates both the situation’s current issues and the possibilities for action.



**Figure 3.1** A model for understanding the achievement of agency (adapted from Priestley et al., 2015)

### 3.2.2. Teacher agency in general education

With its foundation in human agency, Priestley and colleagues developed their model with the intention of understanding the achievement of *teacher agency*, which had begun to receive increasing attention in educational research because of its strong connection to change. When teachers’ sense of agency is empowered, they are more likely to consider their practice as a “meaningful profession rather than just a ‘job’” (p. 149). They perceive



themselves as active, reflective learners who are motivated to implement and develop their expertise (Toom, Pyhältö, & O'Connell-Rust, 2015). This strengthens their desire to be a particular kind of teacher and contributes to their ongoing professional development (Tao & Gao, 2017). Teacher agency also offers an alternative to recalcitrant views of change. Rather than pawns in the reform process, teachers are active agents who remain true to themselves and may resist change that conflicts with their professional understandings (Lasky, 2005).

Priestley and colleagues described an example of Scottish teachers achieving agency while working on a project to develop curriculum. Although most teachers in their study initially self-reported a lack of confidence, confusion about the aims of the intended curriculum, and a general sense of disempowerment, their involvement with the project led the majority of teachers to achieve greater agency.

These teachers were not simply implementing policy – this was not a case of teachers being manipulated into becoming agents of change. Nor were they being offered *carte blanche* to do whatever they wished. Instead, the affordances offered by the specification of goals and processes enabled them to become genuinely agentic<sup>5</sup> as they actively developed and adapted the curriculum to meet both curricular goals and local needs. (p. 142)

Their agency was achieved as they were encouraged to envision a future curriculum that incorporated knowledge from their past experience and was shaped by what was currently possible with the resources at hand.

Acknowledging the dearth of empirical studies on teacher agency, Pantić (2015) developed a model for studying teacher agency that draws on structuration theory (Giddens, 1989); relational theory (Archer, 2000); the ecological approach of Biesta and Tedder (2007). Originally developed to study teacher agency in social justice, Pantić's model of teacher agency comprises four aspects: sense of purpose, competence, autonomy, and reflexivity, which are summarized below. See Table 3.3 for a bulleted outline of Pantić's model.

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<sup>5</sup> Coined by Bandura (2001), the term 'agentic' is used in social cognition theory to describe people who are intentional, proactive, self-reflective and self-regulated. Along with the more recognizable terms 'agential' and 'agentially', both 'agentic' and 'agentially' are terms used in agency literature.

### *Sense of purpose*

This aspect of teacher agency involves teachers' commitment to and understanding of new policies, and this includes a willingness to take on the role of implementers as well as pushing back when new policy conflicts with their own beliefs. Indeed, one of the basic assumptions of teachers acting agentially is that they believe such agency is part of their professional identity. They are motivated to pursue a purpose because they believe it is worthwhile for its outcome.

### *Competence*

Competence focuses on teacher practice as a core expertise. Agentic teachers also understand the influence societal forces have on schooling. They view student ability as transformable and look to tension in teaching to explain learning difficulties rather than see them as problems within students. This means seeking out new and creative ways of working with students and requires knowledge of how to influence a desired outcome in practice.

### *Autonomy*

Most simply put, autonomy is the perception that one has the ability to make a difference within a given environment. Factors that affect autonomy include levels of confidence and control and the amount of trust and power in teachers' relationships with students, parents, and administration. Although it is through individual teachers that policy is implemented, it often requires collective agency for systemic change, therefore interpersonal interactions and relationships are considered critical. Autonomy is also impacted by teachers' involvement in decision making at a broader, systemic level. Inherent in all aspects of autonomy is the notion of individual and collective efficacy, in which teachers' capacity to have an effect is shaped by their belief they can do something or achieve a worthy outcome and that they are supported in that goal.

### *Reflexivity*

Reflexivity is the capacity for teachers' to articulate and share their professional knowledge and use it to justify their actions. In so doing, they are able to reevaluate their own practice and motivate others. This occurs through critically and openly reflecting on their own

assumptions, practices and pushes them to explore alternatives. They see potential in their classrooms and schools as sites for transformation.

**Table 3.3 Aspects of Teacher Agency (adapted from Pantić, 2015)**

Aspect of teacher agency	Characteristics
<b>Purpose</b> (commitment, motivation)	Teachers' perceptions of their moral roles, sense of identity, and motivation as agents of new policy. Teachers' own understanding of new policies.
<b>Competence</b> (practice as core expertise)	Teachers' understanding of broader social forces that influence schooling. Teachers' pedagogical practice, including: <ol style="list-style-type: none"> <li>1. Replacing deterministic views of ability with a concept of transformability.</li> <li>2. Demonstrating how the difficulties students experience in learning can be considered dilemmas for teaching rather than problems within students.</li> <li>3. Modelling new creative ways of working with and through others.</li> </ol>
<b>Autonomy</b> (individual and collective efficacy and agency, relationships, contextual factors)	Teachers' beliefs about individual and collective efficacy. Levels of confidence, control, and resilience. Collaboration and collective agency for new policies. Levels of power and trust in teachers' relationships. Perceptions of school cultures and the principal's leadership. Perceptions of teachers' roles as school and system developers and participation in decision-making. Broader education policy and socio-cultural contexts.
<b>Reflexivity</b> (reflexive monitoring of own action and social contexts)	Teachers' capacity to articulate practical knowledge and justify actions. Teachers' meaning-making of the structures and cultures in their schools as sites for transformation. Critical and open reflection on their assumptions, practices, and exploration of alternatives.

While the model developed by Priestley and colleagues offers a broad method for understanding how teacher agency is achieved, I suggest that Pantić's model provides a more fine-grained method for understanding that process. This latter model also brings to mind self-determination theory, which addresses motivation and behaviour change. This theory is particularly concerned with human flourishing through satisfaction of the basic

human needs for competence, autonomy, and relatedness (Ryan & Deci, 2017). Both self-determination theory and agency acknowledge the impact of contextual conditions, but only agency speaks to one's capability of transforming those conditions. Ontologically, this appeals to me since, like Chapman and Heater's transformational model, teacher agency places strong emphasis on teachers bearing responsibility for their own change. Change is not something that happens to them, it is something they choose (or not) to make happen. What teacher agency adds is the understanding that change depends on context. Priestley and colleagues suggested teachers' development into agents of change results from "the interplay of individuals' capacities and environment conditions" (p. 3). It is necessary to consider what the teacher brings to a situation and what the situation brings to the teacher.

Additionally, Chapman and Heater introduced the role of tension in foundational change, but tension also connects with teacher agency. Priestley and colleagues suggested:

Teachers achieve agency when they are able to choose between different options in any given situation and are able to judge which option is the most desirable, in the light of the wider purposes of the practice in and through which they act. (p. 141)

This emphasis on choosing between different desirable options suggests a connection between agency and tension. When teachers are faced with a choice of desirable options, tension arises as choosing one necessarily excludes the other and, in the choosing, agency is achieved. Agency can be restricted, however, if the options are limited. It is not present at all if there are no available options nor if the teacher goes about her routine without consideration of any alternatives. This is similar to foundational change which, does not occur without tension as its impetus.

### **3.2.3. Teacher agency in mathematics education**

At the heart of change in mathematics itself is a very different view of mathematics. Rather than a fixed collection of facts and procedures, mathematics is seen as a dynamic body of knowledge that is continually enriched through conjecture, exploration, analysis, and proof (Smith, 1996). This is an active view of learning mathematics that changes what teachers do to teach and what students do to learn. For those teachers and students accustomed to mathematics teaching as telling, this move can affect their sense of agency and create tension (Brown & Redmond, 2008).

For Pickering (1995) this tension can be seen as a tension between human agency and the agency of the discipline. And, in setting out to develop a theory to account for the emergence of science and technology, Pickering (1995) identified a third type of agency, that of material agency. Human agency is irreducibly bound with both material agency (physical reality) and disciplinary agency (conceptual systems). Tension between the human and non-human agencies resulted in what Pickering called the “dance of agency”. For example, Pickering studied the work of mathematician Sir William Rowan Hamilton who developed the mathematical system of quaternions. He identified “free moves” when Hamilton used his own agency, which are “tentative and revisable trials that carry with them no guarantee of success” (p. 127) and “forced moves”, which are those times in which the “agency of the discipline” took over, such as when Hamilton needed to follow standard procedures of mathematical proof in order for his ideas to undergo accepted methods of verification. It is in negotiating this back-and-forth interplay between human and disciplinary agency that “the dance of agency” can be conceptualized as occurring.

Limiting her focus to the dance of agency that occurs between disciplinary and human agency, Boaler drew on Pickering’s “dance of agency” metaphor to describe mathematics reform:

Pickering challenges the duality of different agencies in the development of conceptual advances, arguing that mathematics work takes place at the intersection of agencies. Teaching similarly, is as an action that takes place at the intersection of knowledge and thought. Just as mathematics learners need to engage in a dance of agency, so to do teachers. (Boaler, 2003, p. 12)

To apply Pickering’s notion of movement between human and disciplinary agency to the teaching of mathematics rather than the discovery of mathematics suggests teachers who follow traditional mathematics teaching practice privilege disciplinary agency and those who instead facilitate and guide students in novel ways of learning mathematics can be seen as privileging human agency (Brown & Redmond, 2008). This reflects the common perception that change in mathematics shifts the agency from the teacher to the student, which Boaler suggested is only partly true. In her study of the relationship between teaching and learning and its connection to practice, she suggested that, more importantly, the shift in agency can move from the teacher to the discipline of mathematics. She describes this shift in the teaching practice of a teacher in her study, Ms. Conceptual:

When students were working on problems and they asked 'is this correct?'- she rarely said 'yes' or 'no', nor did she simply ask 'what do you think?' instead she would ask questions such as: 'have you tried it with some different numbers?' 'can you draw a diagram?' or 'how is this example related to the last one we saw?' (p. 8)

Boaler suggested that in employing this practice, Ms. Conceptual was moving the focus from herself (*don't ask me*) to the discipline of mathematics. Boaler found this interesting, as opponents of change often cast reform teaching as removed from the mathematics and traditional teaching as inherently mathematical. In thinking through teaching practice with the metaphor of the dance of agency, Boaler found that traditional teachers are actually less likely to shift agency to the discipline of mathematics, rather it is the teacher and the textbook that hold the agency. As Boaler noted, "We consider classrooms such as Ms. Conceptuals to be *more* mathematical, because the teacher positions the discipline of mathematics as the authority from which students should draw" (p. 8).

While there are relatively few studies connecting teacher agency and mathematics, most who did used a sociocultural perspective of agency like Boaler (2003) or Norén (2015), who studied student agency in multilingual classrooms in Sweden. Interestingly, many of the studies referred to Pickering's (1995) metaphor of the dance of agency. For example, Wagner (2007) used the metaphor to connect student agency and discourse, while Grootenboer and Zevenbergen (2009) used it to show how practicing teachers used their sense of agency around particular mathematical ideas and their collective knowledge as a group to solve a task. Brown and Redmond (2008) also studied practicing teachers and suggested that collegial discourse about practice influences teachers' reconceptualization of agency in the mathematics classroom. I suggest focus on the dance of agency occurred because Pickering's metaphor equates the agency of the discipline with that of the individual. In mathematics education, primacy has traditionally been placed on disciplinary agency, as traditional practices have created contexts in which learners surrender their agency to the discipline and follow the procedures and patterns set out before them (Wagner, 2007). On the other hand, the dance of agency allows both for disciplinary agency *and* for consideration of human agency in the mathematics classroom, a necessary component in the movement toward change in mathematics teaching practice (Boaler, 2003). This allows for a focus on subject matter without loss of human agency. As Wagner (2007) suggested:

Dance is about relationship. However, the relationship itself cannot be observed directly. We see only the dance steps. As we see and feel the moves, we learn something about the relationship. In mathematics, there is a dance of agency between humans and either conventionality or common necessity. This relationship expresses itself in the language that flows among people doing mathematics. If language is the dance step, then awareness of language allows us to understand the relationships between the actors in our mathematics. Though it is important to participate in the dance when we are learning it, at times there is value in attending more closely to the steps themselves. (p. 48)

Implementing changes in mathematics teaching practice involves learning a new dance with new steps for both the teacher and the student. Brown and Redmond suggested this makes apparent tension that exists between traditional mathematics and new practices, which can affect the success of the latter's implementation. The authors argued, "What is needed if teachers are to move their pedagogy beyond transmission are opportunities for reconceptualizing agency in the teaching of mathematics" (p. 102). Agentic mathematics teachers recognize the need to shift agency from themselves to the student or to the discipline and the new ways of knowing and doing mathematics in order to achieve change.

In what follows, I revisit Pantić's four aspects of teacher agency from a mathematics education perspective. Although originally developed to study agentic inclusive pedagogical practice in social justice classrooms, she suggested her model would serve to explore other areas of teacher agency. Her emphasis on change in pedagogical practice makes this a good fit for focusing on the change in mathematics teaching practice, as does her recognition that achieving agency is an emergent phenomenon affected by the affordances and constraints of a teacher's environment. Related to this, I further expand on Pantić's model to incorporate the notion of transference, that dance of agency that arises in the particular environment of mathematics teaching and learning. (See Table 3.4 for a summary.)

### *Sense of purpose*

Pantić's suggestion that agentic teachers both commit to and understand new policies is also important in a mathematics classroom where, perhaps more than in any other subject, teachers navigate the norms around teaching and learning practices in order to implement change. Klein (1999) suggested that students arrive with constituted knowledge of what mathematics is and how teaching mathematics is done. These norms are difficult to

displace and, as Klein noted, “This is not solely a cognitive knowing but it comprises conscious and unconscious aspects of experiences and feelings. Our students come to us with constituted predispositions to learn in certain ways; these particular ways feel “right” and pleasurable and are difficult to change” (p. 86). These are referred to as classroom social norms and sociomathematical norms that influence classroom practice (Yackel & Cobb, 1996). Classroom social norms affect normative interactions in the classroom in general, and sociomathematical norms affect normative understandings that are specifically related to the teaching and learning of mathematics. Yackel and Cobb offered an example to distinguish the two. Social norms might require that students are expected to explain their thinking in mathematics while sociomathematical norms decide what constitutes an acceptable mathematical explanation. An agentic mathematics teacher is aware of these norms and the ways in which change might disrupt them. Despite the difficulties that this may bring, they are motivated to change their mathematics teaching practice because they believe it is a worthwhile outcome to pursue. Chapman and Heater (2010) suggested this required “not only a desire by the teacher to change but also the belief that alternatives that are more beneficial are possible” (p. 456). Inherent in this desire is Pantić’s notion that acting as critical change agents is part of mathematics teachers’ professional identity. They willingly undertake the role of arbiter of change.

### *Competence*

While teacher practice as a core expertise also plays a role in agentic mathematics teachers, competence in teaching mathematics combines knowledge about teaching mathematics with knowledge of mathematics. Calling this “mathematical knowledge for teaching”, Ball, Thames, and Phelps (2008) suggested that, while knowing and being able to use the knowledge in practice is vital, a deep understanding of mathematics is equally important in teacher competency. This intertwining of knowledge and practice is what enables agentic mathematics teachers to find creative ways of working with students to influence desired outcomes. As changes in practice are implemented, a subsequent shift in knowledge may be necessary to accompany those changes and maintain a strong sense of agency.

Pantić’s societal forces also come into play in a mathematics classroom. This was apparent in the work of Berlak and Berlak who recognized the relationship between tension in the classroom and in broader social, economic, and political issues. Like any



classroom, there is no mathematics classroom where it is possible to be removed from the influences of the social world and the vast array of discourses it comprises. Related to this is recognizing how classroom interactions position students. Like Pantić, Klein (1999) suggested achieving agency requires the recognition that learning difficulties might arise from the practice of teaching itself, as it, “positions learners in ways that can authorise and empower or alienate and prevent them from acting in powerful ways” (p. 89). Sfard (2008) described this as, “The distinction between difficulty experienced despite instruction and difficulty that develops because of instruction” (p. 23). For example, Lampert (1985) showed a strong sense of agency in her recognition that tension with her seating arrangement resulted in empowering the boys in her classroom while simultaneously alienating the girls. An agentic mathematics teacher, like Lampert, recognizes, and takes into consideration, the impact societal forces has on the teaching and learning of mathematics.

### *Autonomy*

Autonomy in mathematics change varies little from Pantić’s model where individual and collective self-efficacy plays an important role. Teachers’ beliefs about their capability to succeed are essential in determining whether they achieve that success and feeling supported in achieving that goal is critical. Chapman and Heater (2010) also suggested the necessity for mathematics teachers implementing change to control the direction and the form of the learning. Implicit in this is the confidence and resilience necessary to carry out new practices. Autonomy also requires that mathematics teachers acknowledge their own role in the process of change; although they are part of a broader, systemic effort, they take ownership of the process. They do need to feel however, a sense of collective efficacy, that their decision to implement change is supported by others within their local and global systems (Brown & Redmond, 2008). This implies the presence of Pantić’s levels of power and trust in teachers’ relationships.

### *Reflexivity*

Two key factors in reflexivity in mathematics change are the engagement in introspective and interspective observations (Mason, 2002). Introspective observations, “in which an inner witness observes the self caught up in the action” (p. 85), align with Pantić’s notion of teachers’ critically and openly reflecting on their own practice. Interspective observation,

“in which people share observations as witness to each other, yielding objectivity from negotiated subjective information” (p. 85) closely matches with Pantić’s model where reflexive teachers articulate and share their knowledge. Combined, introspective and interspective observation suggests reflecting on one’s mathematics teaching and communicating with others about that teaching contribute to reflexivity. Chapman and Heater suggested there is a metacognitive aspect to this as mathematics teachers learn, reflect, and respond to their practice. This enables them to view their practice as a site for transformation.

### *Transference*

This aspect of agency in mathematics change involves the counterintuitive notion of giving away agency. During change this comes in the form of teachers giving agency to the new ways of knowing and doing mathematics. In traditional classrooms, although agency is thought to reside in the discipline, it actually resides with the teacher and occasionally shifts to the textbook. With change comes the recognition that agency shifts to the students *and* to the discipline as the teacher incorporates novel ways of doing and learning mathematics (Boaler, 2003). Rather than diminish teacher agency, this heightens it, as teachers see benefits for both themselves and their students.

**Table 3.4 Aspects of Teacher Agency in Mathematics Teacher Change**

Aspect of teacher agency	Characteristics in mathematics teacher change
<b>Purpose</b> (commitment, motivation)	Teachers’ perceptions of their roles, sense of identity, and motivation as agents of change. Teachers’ own understanding of the changes they implement and awareness of the mathematical norms change will disrupt. Teachers as arbiters of change who see the outcome as worthwhile.
<b>Competence</b> (mathematical knowledge for teaching as core expertise)	Teachers’ understanding of broader social forces at play in a mathematics classroom (e.g., gender issues, socioeconomics). Teachers’ own mathematical knowledge for teaching. Teachers’ understanding of how classroom interactions position students in a mathematics classroom. Teachers’ recognition that learning difficulties might arise from the practice of mathematics teaching itself rather than from within the students. Teachers’ modelling of new creative ways of working with and through others.

Aspect of teacher agency	Characteristics in mathematics teacher change
<b>Autonomy</b> (individual and collective efficacy and agency, relationships)	Teachers' beliefs about individual and collective efficacy. Levels of confidence, control, and resilience in deciding the form and direction of the change. Collaboration and collective agency for change in mathematics teaching practice. Levels of power and trust in teachers' relationships. Teachers' perceptions of local and global support. Teachers' perceptions of their roles as participants and developers of the change process.
<b>Reflexivity</b> (introspective and interspective observations)	Teachers' metacognitive capacity to critically reflect on their practice. Teachers' capacity to communicate practical knowledge to, and justify actions with, others. Teachers' meaning-making of the structures and cultures in their schools as sites for transformation.
<b>Transference</b> (shift in agency)	Teachers' capacity to shift agency from themselves to their students and to new ways of doing and learning mathematics.

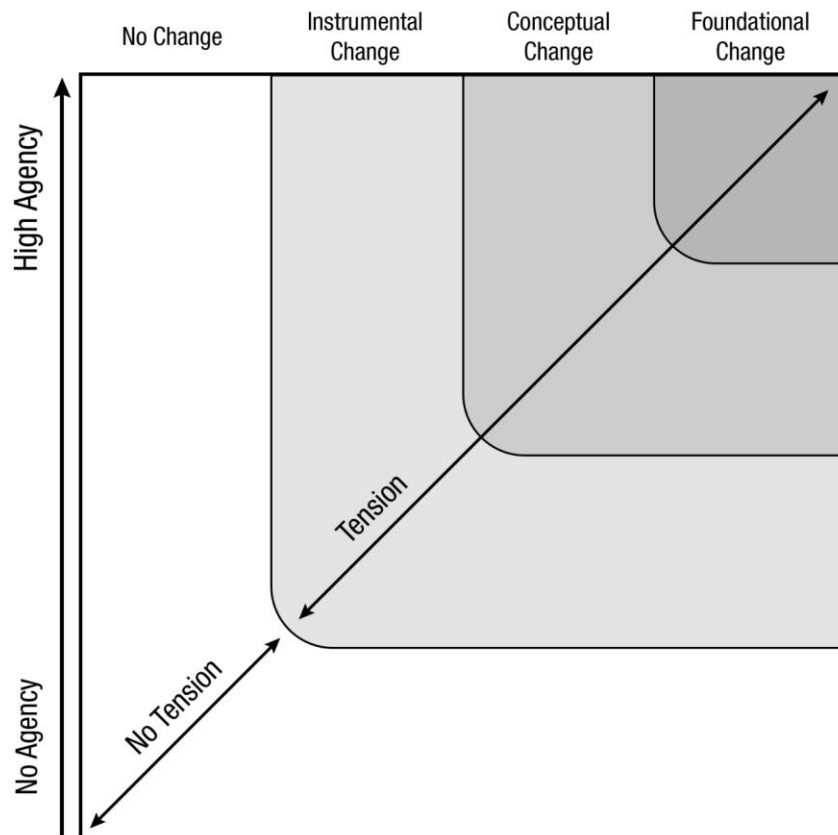
### 3.3. Summary

Change in the professional growth sense as advocated by Clarke and Hollingsworth (1994) is bottom-up change, in that the change is sought and implemented through the teacher's own initiative. This implies that the teacher is in control of the direction and form of the change and that it requires a metacognitive response to reflection on practice. Chapman and Heater (2010) add to this understanding the necessity of tension as a driver of this process as teachers seek to recognize and manage tension. Transformational change occurs at the juncture of tension and metacognitive reflection and is characterized by deep change in the foundation of teacher practice and the thinking on which it is built.

Agency is also a bottom-up approach. Priestley et al. (2015) suggested that teacher agency is "at the heart of what it means to make education good" (p. 149). What agency offers to an understanding of teacher change is that, while change is driven by tension, it is also shaped by what the teacher decides to do about the tension. A teacher might engage in introspective reflection on tension that does not result in growth and change in

practice. As seen from the perspective of agency, this can be an agentic action. The teacher has considered the alternatives and made a metacognitive decision; this is an act of agency that occurs irrespective of whether change follows.

This leads me to several conclusions. First, I suggest that agency offers a way to think about how and why teachers respond to tension in change. A teacher's experience of tension makes apparent the opportunity and/or options for growth and change, and agency sheds light on the process of managing that tension. Second is that, while agency can be achieved independent of change; change, and transformational change in particular, cannot be achieved without agency. Finally, a teacher with high agency has the potential to be anywhere on the change spectrum, from no change, all the way to foundational change; it is the level of tension that drives them upward on the change spectrum. This suggests there is a potential tension tipping point, in which a certain amount of tension is necessary for growth to occur and an insufficient amount may impede growth. (See Figure 3.2).



**Figure 3.2** Relationship among tension, change, and agency

### 3.4. Research questions

I am interested in exploring this tension tipping point where teacher agency exists, and a variety of change is possible. In Chapter 1, I wrote of my interest in teacher change: who changed, why they changed, and how they changed. I wanted to understand what Carly was experiencing that led her to compare change in her mathematics teaching practice to being stretched like an elastic band. A focus on tension provides a way to think about her experience of change.

The value in studying tension is twofold. From a practical point of view, identifying and describing tension allows it to become a source of praxis. This holds true both for the teacher trying to improve her practice *and* for the researcher trying to understand that process. As a researcher, therefore, my understanding of change in mathematics teaching practice may be enhanced by exploring how teachers experience and make use of tension. This leads to the second reason why tension proves valuable; it holds within it the potential for a dialectical transformation of practice. This is an under-researched area that is worth exploring as there has not been a close examination of how teachers manage instances of tension that they experience and its impact on change in practice. Thus, tension in changing mathematics teaching practice is my phenomenon of interest.

More specifically, I am interested in tension that arises during a particular type of change, that which occurs during the implementation of a “Thinking Classroom” (see section 4.2) and within the context of teacher agency. This provides a means for looking at specific instances of change common to a group of teachers and shifts my phenomenon of interest away from *what* is changing to *how* it is changing.

Accordingly, my study focuses on tension in change and is organized around these two questions:

1. What are the kinds of tension experienced by teachers who are implementing a Thinking Classroom in their mathematics classroom?
2. How do teachers manage tension that arises as they implement a Thinking Classroom in their mathematics classroom?

## Chapter 4. Methodological Considerations

Although I am no longer a practicing elementary teacher, my ‘teacherness’ infuses my very being. I could no more stop being a teacher than I could stop the sun from shining. For me, being a teacher does not hinge on whether or not I have a classroom of children; rather, it is an embodiment of my disposition, attitudes, and beliefs. And, even though I now identify as a mathematics education researcher/educator, I retain a lifetime membership to the ‘teacher club’. Therefore, when I am conducting research with teachers, I move in and out of the dual roles of a teacher and a researcher; I am, at the same time, both an insider and an outsider. This is a complementary rather than competitive pairing, as my research informs my teaching and my experience as a teacher also informs my research (Ainley, 1999).

Being an insider and an outsider, however, is not without both benefits and challenges. Having been a teacher provides an immediate kinship with the participants – all practicing teachers – as, although we may have taught different grades and/or subjects, we have a shared understanding and experience of the nature of teaching. This includes both the downsides of teaching as well as its positive aspects. This is particularly important in my study, as I was interested in tension in teaching mathematics, which required delving into the struggles teachers encounter when teaching the subject. From my own experience, I know that teachers can be hesitant to share the downsides of teaching with outsiders until an element of trust has been established. Being a teacher myself, however, helped to bypass that hesitancy. Being an insider also gave me personal insight into the changes the teachers were attempting in their mathematics teaching practice. Indeed, I had experienced many of the same changes myself as a classroom teacher. (See section 4.2 for an explanation of the changes implemented.)

The challenge arose as a result of my need also to fulfil my role as a researcher – as an outsider. A familiarity with the classroom might make me less likely to question what feels second nature to me. It was necessary to be aware of my own assumptions and to ask questions an outsider might pose rather than assume I already knew the answer. I had to remind myself that, although we had shared experiences, their experiences were not mine. In short, maintaining my perspective as an outsider was an essential requirement for my role as a researcher.

This awareness of my dual roles was an integral aspect of my study. It was necessary for me to place my subjectivity in the foreground in order to separate what belonged to me rather than to the participants. As Finlay (2008) suggested, researchers need to bring a “critical self-awareness of their own subjectivity, vested interests, predilections and assumptions and to be conscious of how these might impact on the research process and findings” (p. 17). This manifested as an ongoing process of self-reflection as I worked to separate my own experience from that of the participants in an attempt to balance the dual roles. Indeed, much of my earliest writing was reflections on my own experience in conducting research. I came to see this self-reflective way of writing as a way of working through my own tension, an emptying of myself so I could better understand the participants’ experiences while maintaining an awareness of my own biases and presuppositions. In so doing, I was hoping to avoid falling prey to ‘navel gazing’ – that preoccupation with a researcher’s own emotion and experience that can impact the research process and privilege the researcher over the participant (Finlay, 2012). I did not want to discount my experience and my role as an insider, but I did want to ensure there was balance.

In searching for a methodology that acknowledged these dual roles of insider and outsider, I found myself drawn to hermeneutic phenomenology. It most closely aligned with my perspective and, although I did not employ a full phenomenological approach, I drew on its techniques to gather and analyze my data, the bulk of which comprised interviews with groups of teachers at various stages of change. Accordingly, I begin this chapter by describing hermeneutic phenomenology before moving into a detailed account of my methods.

## **4.1. Hermeneutic phenomenology**

Phenomenology is a project that is driven by fascination: being swept up in a spell of wonder, a fascination with meaning. The reward phenomenology offers are the moments of seeing-meaning or “in-seeing” into “the heart of things” as Rilke so felicitously put it. Not unlike the poet, the phenomenologist directs the gaze toward the regions where meaning originates, wells up, percolates through the porous membranes of past sedimentations – and then infuses us, permeates us, infects us, touches us, stirs us, exercises a formative affect. (van Manen, 2007, p. 12)

Starting with my earliest studies, I felt the fascination van Manen described as I read through my interview transcripts and attempted to glean insights into, and from, the participants' experiences. Consequently, when I first read this quote, I was immediately drawn to phenomenology and, with further reading, to hermeneutic phenomenology in particular. With its focus on broad phenomena that span universal contexts, however, hermeneutic phenomenology proved too elaborate a methodology for my study. Whereas my interest was in the dual phenomena of tension and change, phenomenology focuses instead on "what all participants have in common as they experience a phenomenon" (Creswell, 2007, p. 58), which is not my goal. Still, the tenets of hermeneutic phenomenology appealed to me and guided my interviews and their analysis. In particular, its focus on language as a pathway to deeper understanding, and on the importance of the researcher's relationship with the participant, the reader, and the data.

The philosophy of phenomenology was established by Edmund Husserl to challenge the Cartesian dualism of reality as being something 'out there' or completely separate from the individual (Lavery, 2003). It is essentially the study of lived experience with an emphasis on the world as lived by an individual, not the world or reality as something separate from the individual. Phenomenology, then, provides descriptions of lived experience and asks that the researcher sets aside – 'brackets' – her biases in order to engage in the experience without preconceived notions about what will be found in the investigation (van Manen, 2014). What hermeneutics adds is the art and science of interpretation and, thus, also of meaning – not meaning in the sense of something final and stable, but meaning that is continuously open to insight and reinterpretation (Henriksson & Friesen, 2012).

Hermeneutic phenomenology combines both philosophies in an interpretive process that seeks to bring understanding to lived experience through the medium of language. In so doing, it requires openness and reflexivity from the researcher in embracing his or her biases, which are considered essential to the interpretive process. "Hermeneutic phenomenology, in short, is as much a disposition and attitude as it is a distinct method or program for inquiry" (Henriksson & Friesen, 2012, p. 1). These are ontological presuppositions that I suggest mediate the type of knowledge and understanding that will be acquired. For example, researchers who believe, like Heidegger (1927/1962) and Gadamer (1976), that it is necessary to embrace their biases, the epistemological assumptions they make will be markedly different than if they believe, like Husserl



(1952/1980), that they can bracket out their preconceived notions in their description of the phenomenon.

Ontologically, then, I was drawn to the notion of reflexivity inherent in hermeneutic phenomenology. This is where the use of my empathy and my relevant prior experience as a teacher could act as an aid in the collection and interpretation of the data. I would be required to give considerable thought to my own experience as a teacher, and to make explicit the ways in which that experience related to my research. My own personal assumptions and philosophical basis from which I interpret the data are considered essential to the process.

Another benefit of hermeneutic phenomenology is its attention to language – to notice how one uses it and how others use it as, “in a deep sense our language contains the story of who we are as a people. It is reflective of our desires, our regrets and our dreams; in its silences it even tells us of what we would forget” (Smith, 1999, p. 39). For my research, I wanted to be able to read and then tell those stories in a meaningful way and to do so required that I attend closely to the words my participants chose (or did not choose). As Smith suggested, “Every hermeneutical scholar should have a good etymological dictionary at her or his side” (p. 39). I found this emphasis on explanation appealing. When I analyze a participant’s words, I am not looking to convince a reader that this is the ultimate answer, I am only wanting to share with the reader my interpretation. I want room for the reader to nod and say “yes”; or say “that made me think of...”. This allows for the possibility that not all interpretations will resonate with every reader, yet that does not negate the findings as untrue or untrustworthy. Accordingly, the reader also comes to the researcher’s text with an openness of heart and mind:

It is necessary to allow yourself the luxury and the risk of getting lost in this huge forest of the text – and then suddenly noticing something vaguely familiar, or glimpsing something moving out of the corner of your eye, or following a rocky side trail of traces and footprints that stop at the sheer edge of a cliff. (Jardine, 1992, p. 5)

Rashotte and Jensen (2007) suggested that this relationship between the researcher and the reader is one of three key relationships of hermeneutic phenomenology, “Both the researcher and the reader must *leap into* the text with a hermeneutic attitude – that is, with a sense of attentiveness, empathy, sensitivity, carefulness, respect, reflection, engagement, conscientiousness, awareness” (p. 104). There is a sense of reciprocity in

this in that, as a researcher, I invite the reader into the teachers' experiences while the reader must be willing to be drawn into the implications of my interpretation and themselves and, in so doing, become part of the experience themselves (Jardine, 1992). Hermeneutic phenomenological studies, then, "aim to create a sense of resonance in the reader. Resonance means that the reader recognizes the plausibility of an experience even if he or she has never personally experienced this particular moment or this kind of event" (van Manen, 2014, p. 240).

The second relationship is between the researcher and the participants. I have to consider my own influence on the research process and the social interactions therein.

As soon as I begin an interchange of looks with another person, and I sense them as looking toward me in a certain way (as they see me looking toward them in the same way too), a little ethical and political world is created between us. (Shotter, 2005, p. 104)

For the duration of the interview then, the teachers and I inhabited a shared world where "we are present to each other as who we are, at least to a minimal extent, we can see into each other" (p. 104). I was foremost a researcher asking teachers to describe their experience of change. Yet I wanted to make clear to the participants that I was, or had been, a practicing teacher myself and could relate to the experiences they described. In our shared world, they gave to me the story of their change and I gave to them the sense of being fully understood.

The final relationship is between the researcher and the data. Rashotte and Jensen (2007) describe the necessity of moving away from a linear process of data collection, coding and then analysis and instead, acknowledge the actual circularity of the interpretive process. This is referred to as a 'data analysis spiral', in which each aspect of the process is revisited (Creswell, 2007). Within this spiral is an element of discovery as opposed to proving; the researcher is not seeking out evidence to support their hypothesis. Instead the data is used to create meaning from the participants' experience of the phenomenon.

In the end, however, I believed that to use hermeneutic phenomenology in my study would be the equivalent of using a sledgehammer on a pin nail. Very likely its use would have obfuscated that which I was looking to examine more closely. I was not looking to examine commonalities in large life themes; I was narrowing in on a very specific aspect of teachers' lives – the tension they experience as they go about changing their mathematics

teaching practice. Having felt from the start that my own role in the research cast a looming shadow over the data collection and analysis, hermeneutic phenomenology helped me recognize the power of incorporating the self into the interpretive process. My own experiences were not to be disregarded; they were to become part of the interpretation, and thereby impact my relationship with the reader, the participants, and the data.

## **4.2. The study**

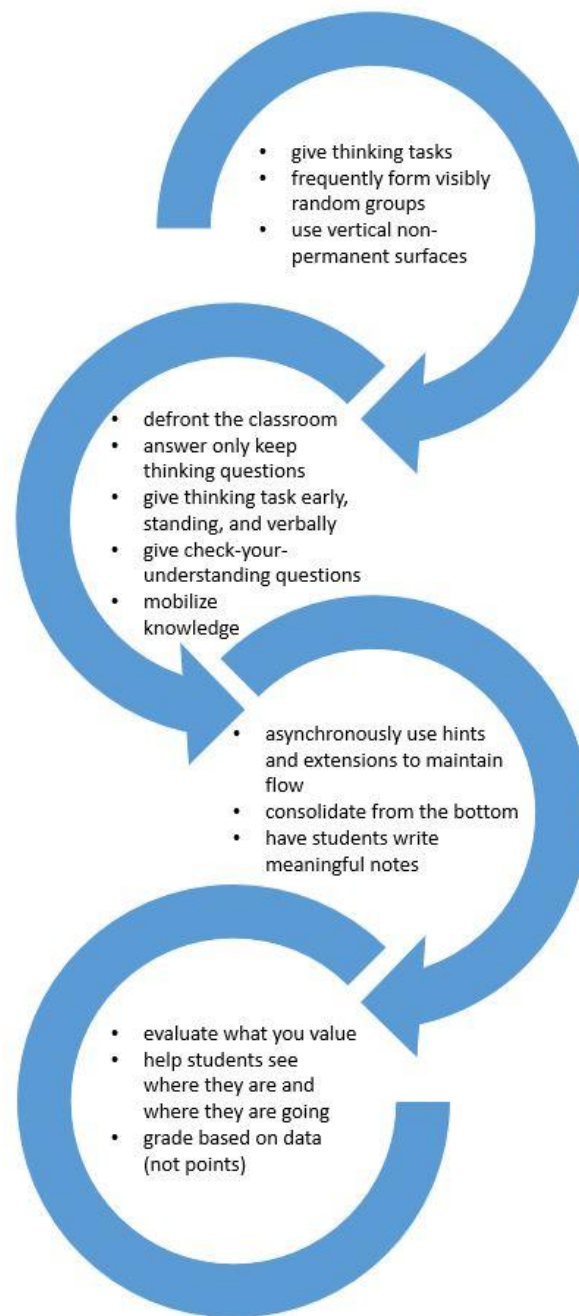
This study entailed three phases of data gathering, each with a different group of participants. Common to each was the participants were all practicing teachers who were, or had been involved in, changing their mathematics teaching practice. Furthermore, for all the participants, these changes had been influenced by Dr. Peter Liljedahl of Simon Fraser University, either through contact in a master's program or through attending one or more of his mathematics professional development sessions.

Liljedahl's (2016) research revolved around what he calls a "Thinking Classroom" in which students are encouraged to engage deeply with mathematical content. The goal in a Thinking Classroom is to create:

A classroom that is not only conducive to thinking but also occasions thinking, a space that is inhabited by thinking individuals as well as individuals thinking collectively, learning together and constructing knowledge and understanding through activity and discussion. (p. 362)

Noting that institutional norms often act as a barrier to student thinking, Liljedahl developed what he described as a pseudo-sequence of fourteen practices that subvert traditional classroom norms (see Figure 4.1). For example, in a Thinking Classroom, a key practice is that students are expected to work at a non-permanent vertical surface such as a whiteboard or blackboard. This is novel in a mathematics classroom where most student work is completed on paper at their own desks. Two other practices, considered foundational to a Thinking Classroom, are rich tasks and visibly random grouping. Rich tasks involve problem-solving tasks which engage the students and from which rich mathematics emerges. Visibly random grouping is a strategy designed to improve collaboration and engagement in mathematics classrooms (Liljedahl, 2014). Through the use of visibly random methods such as numbered cards, all the students within the classroom are randomly assigned to heterogeneous groupings. Its affordances include an

increased tolerance for fellow classmates, the elimination of classroom social barriers, and increased knowledge transfer between students, which is accompanied by a decreased reliance on the teacher for answers. This is an atypical grouping strategy in classrooms, where the norm is self-selected affinity groupings and/or teacher-selected homogenous groupings.



**Figure 4.1** Thinking classroom practices organized in a pseudo-sequence (Liljedahl, in press, with permission)

Critical to Liljedahl's work is that the teachers are not just told about the practices in the workshops, rather the sessions are designed so that teachers are introduced to the them through modeling and immersion. Positioned as learners, Liljedahl described the teachers in his session as having "experienced a different way in which their classroom could look and how their students could behave" (2016, p. 384). The result is a significant uptake of the practices in actual classroom practice as its affordances are immediately apparent to the teachers who experience them firsthand.

The teachers I chose to study had been exposed to a Thinking Classroom and had, or were attempting to, incorporate its practices into their own mathematics teaching practice. Although I expected that change would also occur outside of these practices, a focus on change regarding the Thinking Classroom practices offered a concrete way to approach a conversation with the teachers about change. It also provides a means for looking at specific instances of change that are common across all my teachers' practice. This homing in changes the focus from *what is changing* to *how it is changing*. This was further enhanced by the fact that I had, as a former elementary teacher, implemented these same practices in my own mathematics teaching practice. This provided a common ground between myself and the teachers, as I anticipated that I had lived some of the same tension they were experiencing during their implementation of a Thinking Classroom. This positioned me as an insider, both as a fellow teacher and through a shared experience of change in mathematics teaching practice.

The first phase of data gathering occurred during the spring of 2017, when I interviewed eight of fourteen teachers who had just finished a two-year master's program in numeracy. This program utilized a cohort model in which the teachers proceed through the program as a unit, completing the same courses at the same time. I had several connections to this particular group of teachers. First, I had completed the same master's program myself three years earlier, which meant I was familiar with their learning experience and instructors and second, I was the site assistant for their current program. In this role I offered support for any questions they had regarding the program and/or their assignments. Additionally, I had occasionally guest-lectured in their courses and would also drop in on their classes from time to time. Wanting to ensure a wide range of grades and experiences for my study, I set a goal of interviewing five participants. To that end, I decided to invite all fourteen teachers in the cohort to participate in my study. Eight agreed and I decided it was worthwhile to interview all of them. Four were elementary teachers

(grades 3 to 7) and four taught at the secondary level (grades 8 to 11). All taught in a large urban city and their teaching experience ranged from 4 to 18 years. (See Table 4.1 for further details.)

Data gathering in phase one also included two writing assignments the eight teachers had completed as part of the assessment for a course taught by Dr. Liljedahl. This course focused on mathematics curriculum and instruction and one of the goals of the course was that “students will be expected to enact the ideas regarding mathematics instruction developed in the course within their own classrooms and to report back on their experiences in doing so” (course outline) and the assignments were designed to help meet this goal. The first assignment was in the form of online discussion posts written by the teachers as they implemented Liljedahl’s Thinking Classroom and were available for their peers to read and comment on. The intent was to provide a forum for encouragement and support as the teachers worked through the implementation. I also had access to a reflective essay they wrote after reading Boaler’s (2002) *Experiencing School Mathematics: Traditional and Reform Approaches to Teaching and Their Impact on Student Learning*. This assignment asked the teachers to ‘read’ themselves into Boaler’s comparison study of a traditional school and a reform school by reflecting on aspects that resonated with their own practice. Although both the online discussion posts and the essay were written to satisfy the requirements of the course, I viewed both as a potential source of insight into the changes the teachers had made. Having completed the same assignments myself when I was in the course, I was cognizant of their reflective value. Both were a place to park my thoughts and worries as I worked through change in my mathematics teaching practice; I anticipated similar responses in the teachers’ writings.

The second phase occurred throughout the September 2017 to June 2018 school year during which I interviewed and observed in the classrooms of five teachers about to start a series of three full-day professional development sessions led by Dr. Peter Liljedahl on building a Thinking Classroom. These sessions were spread throughout the school year: September 2017; December 2017; and April 2018 in a school district in a different province. I felt the travel worthwhile as, while Phase 1 provided a rich reflection on tension in change, I anticipated that this phase would allow me the opportunity to observe the teachers’ attempts to implement change as it was occurring. While interviewing is useful in providing an opportunity to talk of experience in general, I felt observing offered the opportunity to be part of explicit experiences that could then form the basis for a

subsequent interview. Accordingly, I arranged both to interview and to observe in these teachers' classrooms, with the interviews taking place immediately after the observation whenever possible. (See Table 4.1 for further details.)

I used my observation notes as prompts to stimulate a commentary by the teachers on their thought processes. For example, a discussion regarding a teacher's method of randomly grouping her students began with me recounting what I had noticed as she grouped her students. In particular, I attended to the students' actions, such as trading cards to get partners they preferred, actions of which the teacher was perhaps unaware. It is not that I was hoping to 'catch' the teacher off-guard, more that I was looking to re-examine the incident from the teacher's point of view for traces of tension. Incidents in which I noticed students' behaviour which could be cast as positive (i.e., students cheering at the mention of random grouping) served the same purpose, as the teachers would often recount the pathway they travelled until such behaviour became the norm.

Similar to the Phase 1 participants, I had a connection with the Phase 2 participants: they all worked for the same school district where I had previously been employed as an elementary teacher. However, although this was a small school district, it encompassed a large northern region, and I had never taught at the same school, at the same time, as any of the five participants. This was a matter of convenience sampling in two regards: I knew these teachers were about to embark on a professional development journey that could potentially result in change in mathematics teaching practice and my familiarity with their school district allowed me ease of access – I was someone whose name was familiar, and their administration would be more readily willing to grant me access to their schools and classrooms. Indeed, it was the district's mathematics lead teacher who provided the contact information of the five teachers who would be participating in the upcoming sessions and all five agreed to participate. Their teaching experience ranged from a first-year teacher to fourteen years of teaching experience and all were middle-school or secondary teachers (grades 7 – 12).

What Phase 2 made clear was the benefit of observing the changes as they took place in real time. Rather than recalling an event from the past, the interviews could focus on what the teacher was experiencing now. It provided an immediacy to the tension and subsequent interviews could focus on ways that the teachers had been attempting (or not) to manage it. I found myself wishing that I could visit their classrooms more frequently, but

the distance was prohibitive. Midway through Phase 2, however, I became aware that there was a local teacher who was about to implement a Thinking Classroom and was willing to have a researcher in his classroom. And thus began Phase 3.

This teacher, Nicolas, had attended one full day professional development session with Dr. Liljedahl in the spring of 2017 and had planned to implement the Thinking Classroom practices in the upcoming 2017/2018 school year. An unexpected change in teaching assignment delayed his implementation until January of 2018, which is when I began conducting face-to-face interviews with him, immediately after observing a lesson. In all, I observed eight classroom lessons and conducted six interviews with Nicolas. Again, there was a researcher/participant connection, as I had been present as an assistant at the professional development session Nicolas had attended.

While my intention with Nicolas was to replicate Phase 2, but with more frequent interviews and observations, there was an unexpected bonus. Of his own initiative, Nicolas began emailing me almost daily with accounts of his mathematics lessons. This was his method of reflecting on his practice and keeping a record of his teacher moves to which he could later refer – a digital journal so to speak. These twenty emails, and my emailed replies, became part of the data as they chronicled Nicolas’ struggles and thoughts about the changes he was attempting to implement. Additionally, like the other participants, Nicolas was aware that I had made similar changes in my own mathematics teaching practice and these emails frequently included questions he had for me. (See Appendix C for an example of an email exchange.)

Together, the data gathered in Phases 1 and 2 are used in Chapter 5 to identify and describe tension and agency in change in general. Data gathered in Phase 3 with Nicolas permitted a more detailed examination in Chapter 6 of change as it was enacted and tension that arose.

**Table 4.1 Participant Information**

Teacher	Grade and Subject	Years of Experience	Background	Data
Phase One				
Alison	9 math/science	6	BEd	One phone interview Discussion posts Reflective essay
Amy	3/4 elementary generalist;	5	BEd French major	One in-person interview Discussion posts



Teacher	Grade and Subject	Years of Experience	Background	Data
	French immersion			Reflective essay
Corey	6 elementary generalist	9	BA Linguistics major; BEd	One in-person interview Discussion posts Reflective essay
Kate	0.5 math lead; 0.5 grade 10 math	17	BEd; Post baccalaureate in curriculum and differentiation *First time teaching secondary math	One in-person interview Discussion posts Reflective essay
Kelly	8-11 math	10	BSc Mathematics major; BEd	One in-person interview Discussion posts Reflective essay
Leah	10-11 math	10	BSc Biology major	One in-person interview Discussion posts Reflective essay
Lily	5 elementary generalist	18	BEd	One in-person interview Discussion posts Reflective essay
Sara	6/7 elementary generalist	4	BA Economics major; BEd	One in-person interview Discussion posts Reflective essay
Phase Two				
David	7-8 math/science/social	4	BS Human Kinetics; BEd; MEd Organizational Studies *First time teaching mathematics	Two phone interviews Two in-person interviews Three classroom observations Field notes
Diane	7-8 math/science/social	4	BEd	One phone interview Two in-person interviews Two classroom observations Field notes
Jamie	8 math/science/social	14	BEd Elementary	Two phone interviews Two in-person interviews Three classroom observations Field notes
Nadia	10-11 math; French immersion	1	BEd French major; Mathematics & Chemistry minors *First-year teacher	Five in-person interviews Four classroom observations Field notes
Sam	7-8 math/science	14	BEd Physical Education major	One phone interview Two in-person interviews Two classroom observations Field notes
Phase Three				
Nicolas	6/7 elementary generalist; Francophone	5	BSc Mathematics (France); BEd (Canada)	Six in-person interviews Eight classroom observations 20 emails Field notes

#### 4.2.1. Data gathering

I knew without a doubt that my research method would involve semi-structured interviews. By talking with the teachers, I could ask about, and listen to, what they themselves had to say about their lived experience of teaching. As Bean (2006) wrote, “There is no substitute for prolonged and focused conversations between trusted parties to discover what is important to the interviewees and how respondents understand key elements in their own lives” (p. 361). This fits with a hermeneutic phenomenological approach, as the interviews were the means for obtaining insight into the teachers’ experiences of change in teaching. This was true as well with the data collected in the form of online discussion posts, written essays, and Nicolas’ emails. These last three had the added benefit of being static forms of data gathering: the teachers were not trying to guess what it was I wanted to hear, which can occasionally occur in interview settings (Kvale & Brinkmann, 2009).

Where I veered from hermeneutic phenomenology was in my approach, as hermeneutic phenomenology requires asking the participant directly about their experiences with the phenomenon, which in my study was tension. However, as Lampert (1985) pointed out, admitting to tension may be difficult as “sorting out problems and finding solutions that will make them go away is certainly a more highly valued endeavor in our society” (p. 193). So, I purposefully chose *not* to introduce the term ‘tension’ directly in my initial interviews, in case there was a reluctance to admit any kind of struggle. Instead, I asked broader questions about aspects of teaching and made use of follow-up questions whenever the teachers spoke of struggle or uncertainty. (See Appendix D for a list of guiding questions and potential follow-up questions.) For example, when asked about her own experiences as a student, one teacher described having a sit-down conversation with a university professor where together they decided an appropriate grade for an assignment, and how much she valued that experience. My follow-up question regarding whether she would consider a similar practice with her own students indicated tension, as she spoke of feeling like she was not allowed to do that, that she alone was the arbitrator of the mark a student received.

Drawing on both my status as an insider and my familiarity with the teachers, I approached the interviews with a spirit of genuine inquiry and camaraderie. Yet, interviews are such peculiar contrivances. Despite their guise as a conversation between interested parties, the actuality is that interviews really serve one purpose, that of the researcher (Kvale &

Brinkmann, 2009). For myself, the purpose was to gain insight into the teachers' worlds from their point of view and from their experience. To do so meant approximating an everyday conversation, but one in which the balance of power was firmly on my side – I initiated the conversation; I steered its direction through the asking of questions; I decided when to press further; I also chose when the conversation was over. The success of the interview lay in my own ability to orchestrate this pseudo-conversation.

My opening stance for each interview was that we would be having a conversation about teaching and, although I had a list of questions, I chose to keep the interview as informal as possible. To that end, after conducting the first three Phase 1 interviews, I memorized the questions so as not to have to refer to a sheet a paper. This helped maintain the flow of the interview, as I realized early on that stopping to check my sheet for another question often interrupted the cadence of the interview. I wanted the teachers to talk freely about their experience, rather than anticipate or wonder about what question was next on my sheet.

Initially, I had expected that some of the data gathered would come from the notes I had written during the interview. In the quietness that surrounds a listener, there is often room for insights, thoughts, and connections that I knew I would want to recall later. So, I started out writing notes, (and it also made me feel comfortable in giving me something to do with my hands). But I quickly noticed that it tended to make some teachers uncomfortable, especially if I did not take care to explain what I was writing. The notetaking also interfered with the natural progression of the interview, as sometimes I would note hesitations in the teachers' flow of speech – almost as if they were waiting for me to catch up. Or perhaps they had said something that required some form of tacit acknowledgment, and I had missed the cue. My notetaking made visible that we were engaged in an interview rather than a conversation, and so I discontinued it altogether during the interview.

As mentioned, I did not directly refer to tension in teaching, but that it was my phenomenon of interest was made clear at the close of the first interview with each teacher. Therefore, teachers in Phase 2 (and Nicolas in Phase 3), with whom I had subsequent interviews, were aware I was interested in tension in changing mathematics teaching practice. This proved to be beneficial, as in between interviews the teachers were attempting to implement the various practices of a Thinking Classroom. I would start the subsequent interviews by inquiring about what changes they had attempted since we last spoke, and

they willingly shared any struggles or successes they had encountered. They knew I had a sympathetic ear and could relate to, and was interested in, their experience of change.

To be honest, conducting interviews proved not difficult for me. As a quiet, private person, I have always tended towards listening and asking the next question that would compel the talker to carry on, rather than have to bear my share of the conversation. What was difficult, was not allowing myself to become so immersed in the interview as to lose sight of its purpose. As Smith (1999) suggested, “There is a certain quality of self-forgetfulness as one gives oneself over to the conversation itself” (p. 38). Indeed, after conducting my first few interviews, I wrote myself a reminder to “be cognizant of when I fall too deeply into the conversation” and that “I needed to stay outside myself”. I needed a dual ‘me’, one involved in developing the interview and one involved in observing the event, like the birds in *Rig Veda* (O’Flaherty, 1982):

Two birds, fast-yoked companions,  
Both clasp the self-same tree;  
One eats of the sweet fruit;  
While the other looks on without eating. (Book 1, Ch. 164, st. 20)

My slipping from an impartial observer to that of a feasting colleague was most noticeable when looking back through my earliest transcripts. My insider role as a former teacher meant that there were often assumptions on the part of the teacher regarding shared knowledge, which most often presented in some variation of “You know what I mean?”. For example, during one early interview, a teacher was describing feeling held back by the colleagues around her and then said, “I tried to do my own thing, you know, you’ve done it too”. Rather than a follow-up question asking her to describe what ‘doing her own thing’ might look like, I answered affirmatively. In retrospect, when relistening to the recording, I knew I had been caught up in the moment and forgot that my purpose was data gathering. I quickly became alert to those “you know” statements. I would respond by saying either, “No, I don’t know”, or by acknowledging that I did know, but that I needed it said aloud for the recorder. In effect, I was creating fictional moments, for if it had been left at ‘you know’, there would have been no data for analysis.

Phase one of data gathering consisted of one interview with each of the eight teachers that lasted between 60 to 75 minutes. One interview was conducted over the phone due to distance, but the remainder were in-person. Of the latter, five took place in the teachers’ classrooms, one in a university classroom, and one in the teacher’s home. In each

instance, the teacher was the one to choose the location, as I wanted to conduct the interview where the teacher felt most comfortable. Phase two started with an introductory phone interview with each of the five teachers followed by two to four more classroom observations and subsequent in-person interviews in their classroom, excepting two occasions in which the follow-up interviews occurred by phone due to the teachers' schedule. Phase 3 consisted of eight classroom observations in Nicolas' classroom and six in-person interviews conducted immediately afterwards in his classroom.

All of the interviews were digitally recorded (audio only) on two devices (for redundancy) and were transcribed in their entirety. In attending to features of the interview discourse, I included non-verbal communication such as pauses and instances of laughter, as they might be indicative of instances of uncertainty or nervousness. For clarity and readability, I removed "ums", "uhs", and most slang interjections such as "like", and changed informal contractions such as "gonna" to "going to". I chose to note changes in intonation through the use of punctuation, e.g., exclamation marks to denote excitement or emphasis. I left intact the online discussion posts and essays written by the Stage 1 teachers, but I did make a few grammatical changes to the emails composed by Nicolas, whose first language is French. For example, in one email Nicolas wrote, *"I ended up adding one solution presented by a student that I had not think of and I thought was nice to provide as an example of student provided solution and alternate way to reach the solution."* When using this excerpt in the analysis, I decided on three minor changes for clarity: I changed "think" to "thought" and added an indefinite article in front of "student" and "alternate": *"I ended up adding one solution presented by a student that I had not **thought** of and I thought was nice to provide as an example of **a** student provided solution and **an** alternate way to reach the solution.* Although the transcripts, once completed, became my preferred record of the interview data, I did, on occasion revisit segments of the audio recordings, particularly when a participant spoke about the same topic in a subsequent interview. For example, during her third interview Nadia described an altercation with a parent. During her final interview several months later, she again spoke of the incident. Curiosity drove me to relisten to her earlier interview so I could compare changes in intonation.

Although it was not always possible, I tried to complete the transcriptions prior to conducting the next interview, whether with the same teacher or a different one, as I found each interview informed the next. This was particularly true when reading through responses to the final question "Is there anything else you would like to tell me?" In

preparing the list of questions, I had drawn on my own experience of change. In asking if there was anything else, I essentially opened up the interview to their singular experience. This turned out to be an especially rich move, as the teachers became more at ease as the interview progressed; it was my sense that they forgot they were being recorded and were speaking freely. Their responses to this last question often informed my next interview. For example, mid-way through my Phase 1 interviews, when asked if she had anything else to tell me, one of the teachers spoke of the difficulty in preparing for a substitute teacher. Preparing the transcript of that interview brought it to mind and I used it as a follow-up question in my next interview.

#### **4.2.2. Data analysis**

My formal data analysis process comprised two different, yet intertwined, modes of interpretation. The first was an aesthetic response to the teachers' interviews and other data; the second an explicit categorization of responses through the lenses of tension and agency. In keeping with hermeneutic phenomenological methodology, the first stage did not involve counting word usage or applying codes, rather it involved submersing myself in the data through reading and rereading hard copies. Indeed, I often referred to it as "swimming with my data", as there was a sense of being underwater – in that dreamy feeling of sensory deprivation – from which I would emerge slowly, gradually becoming aware of the world around me. At the risk of sounding melodramatic, I had a real sense of being one with the data.

During this first stage, I jotted notes and comments in the margins that occurred to me as I read. Sometimes it was to bring attention to a particular phrase or passage; for instance, a teacher detailing a decision they made. Other times it was to connect what I was reading in one document with a related thought or idea in another. This occurred both within a single teacher's data sources and for data from two or more teachers. For example, I pencilled a note (*this sounds just like Alison!*) beside Amy's recall of a parent interaction. This was an iterative process, as I found myself rereading early data sources whenever I completed transcriptions of new interviews and would then make new notes and connections.

In the next stage, I moved towards developing a formal coding system. Despite my focus on tension in change, I was not, however, coding for tension at this point. Rather, I was

looking for broader categories in which I could later situate tension. To that end, I used a form of emergent coding, in which some of the codes were drawn from the text rather than established *a priori* (Creswell, 2007). This is in keeping with a hermeneutic phenomenological approach where “instead of discovering meaning, meaning is more explicitly seen as emerging from the interaction between the data and the researcher” (Sullivan, 2012, p. 11). For example, some of the *a priori* codes I anticipated were related to a Thinking Classroom, e.g., problem solving, group work, and collaboration while others, like parents, substitute teachers, and homework were emergent. I began by uploading all of my data sources into NVivo, a qualitative data analysis computer software package produced by QSR International. However, for the initial coding, I coded each teacher’s data set individually. This was done to make the task of coding less overwhelming, as it portioned the data into manageable-sized chunks. In the end, though, this proved to have been a fortunate decision, as I frequently referred back to these individually coded data sets during both further analysis and the writing process.

This first attempt at coding was a broad sweep through the data as I coded for overarching categories. For example, the initial codes that emerged from Lily’s data were assessment; beginning teaching; goals; grouping; homework; math is different; parents; pedagogy; problem solving; students; vertical surfaces. While there was overlap between codes of individual teachers, no two teachers shared all the same codes. I coded each data source line by line and often found myself applying two or more codes to the same phrase/sentence. For example, the phrase “*The students would be in random groupings and be given a problem that they would have to collaborate on and come to an answer together.*” was assigned codes of problem solving, grouping, and collaboration.

I then consolidated all the data from all the teachers. This created a rather large data set with broad categories. To reach a more fine-grained classification of the data, I decided to recode each category, item by item. For example, all of the items in the category of students were further sorted into related subsets, e.g., student engagement, student ability. The latter was then further sorted into low and high achieving students. This was an arduous, but necessary process, and required frequent revisiting of the original data sources as I clarified the best fit for each item. The data set was no smaller, but this reclassification permitted a closer examination of each code and its implicit properties.

### ***Operationalizing tension***

At this point I had a large data set of categories of items that reflected teachers' thoughts on assessment, students, etc., and I now turned my attention to analyzing for tension. This is where hermeneutic phenomenology's emphasis on attention to language proved helpful. I began by creating a list of words and phrases that could conceivably denote tension such as "decide", "uncertain", "unsure", "worry", and "I don't know". I also drew on the metaphors with which tension was described and so I expanded my list to include words like "stretch", "irritate", "juggle", and "navigate". With my list in hand, I returned to the database to examine the teachers' experiences for instances of tension. To do so, I attended closely to the words they used, particularly when they hedged or evinced emotion.

Hedges, a linguistic device first identified by Lakoff (1973), are "words whose job is to make things fuzzier" (p. 471). Consisting of words such as 'mostly', 'might', 'somewhat', 'kind of', and 'sort of', their use in a proposition can convey uncertainty or vagueness (Rowland, 1995). Consider the following two sentences: (1) Timed drills *are* an effective teaching practice and (2) Timed drills *might be* an effective teaching practice. Uttered by a teacher, the first sentence leaves no room for doubt. There is no apparent tension, and if part of an already established classroom routine, the teacher is unlikely to feel the need to reflect on and question the use of timed drills. The addition of the hedge '*might be*' softens the unequivocal first statement and if the teacher who utters the second sentence uses timed drills, it may be with a sense of conflict or unease. Depending on the context, the hedge '*might be*' may be indicative of tension.

I also attended to the explicit and implicit emotions in the data. Hannula (2006) understands emotions as affecting motivation and therefore as directing behaviour by affecting both a person's goals and choices. Emotions constitute a feedback system for goal-directed behaviour, and thus shape a person's choices. This suggests that emotions are not only a by-product of teachers' choices, but they guide, influence, and anticipate such choices. Additionally, although teaching is necessarily affective, its emotions are often intensified in times of change (Zembylas, 2005). Change affects not only teachers' work, but also how they feel about their work. There is an emotional engagement in the process of change and "the way one changes is guided by emotions, as is any decision-making process" (Reid & Zack, 2010, p. 372). This emotional investment can result in



tension when teachers' practices are challenged or new expectations are imposed (Kelchtermans, 2011). As teachers work to manage the tension, their decisions are often visible through emotions (De Simone, 2015). Hence, emotions are a critical aspect of teachers' lived experiences, and useful in revealing the underlying tension that may have instigated the change (Rouleau, 2019).

Although I did not undertake a theoretical analysis to hedging or emotions, attending to these aspects of the teachers' language helped identify experiences of tension. For example, in a search for variants of 'scare', the phrase *"I was like, wow, this is so different and a little bit scary"* was highlighted. In further reading of the context in which the phrase was uttered, it was apparent this was an emotive response to the changes the teacher was attempting in her classroom, which provided evidence to support my interpretation that she experienced tension while questioning the decisions she was making.

These instances of tension were coded within the larger data set. This required a closer look at the context in which each tension occurred, and entailed frequent revisits of each teacher's individual data set as I pieced together all the occasions in which the teacher spoke or wrote of experiencing the tension and its related aspects. For example, Nadia spoke of her tension with assessment during several of her interviews. Once I identified it as a tension, I returned to each of her interviews to fully capture all the occasions in which she described the difficulties she experienced, and how she managed the tension. I would then write a paragraph describing her tension and her response, using her own words whenever possible. These early writings became the basis for Chapter 5.

### ***Winnowing the data***

I now had many explicated instances of tension but my interest in teacher change led me to incorporate only those that were in some way related to implementing change. For example, I had initially assigned a code of tension with student understanding to excerpts such as one from Alison, who wrote in her essay about a connection between her own students and the students she was reading about. She wrote, *"I would say the vast majority of my students would conduct the same interview – realizing that they had learned the correct procedure months ago but did not know how to apply it. Their overall understanding of the mathematics behind the procedure was not there."* She goes on to relay her struggles with getting students to learn mathematics for understanding, not for a grade. While undoubtedly a tension, I came to the realization that this was a tension that neither

originated, nor was amplified by the implementation of a new practice. Thus it, and others like it, were not included in the final results.

This winnowing process resulted in twenty-four instances of tension experienced by the teachers. This was the result of a gradual narrowing of the data as I read, coded, combined, and revisited the data. I was now ready, however, to look more closely at these instances of tension and to find connections between them. I felt this was necessary if I was to present the results of twenty-four instances tension in a coherent fashion. Accordingly, I looked for resemblances in the tension that did not diminish their singular power. This re-sort resulted in two broad classifications of external and internal tension, with the latter comprising sub-categories of intrapersonal, interpersonal, and pedagogical tension.

### **4.3. Summary**

Hermeneutic phenomenology was a methodology that was well-suited to my study. It took into account my status as an insider and an outsider, allowing me to draw on the strengths of each as I interviewed participants and prepared and analysed the data. In the introduction, I referred to writing my thesis as a hero's journey, and it was never more apparent than when I was involved in making methodological decisions. I encountered numerous obstacles – from choosing how many participants to interview, to how to prepare the data – and overcoming each was a point of growth. Indeed, my own change from a teacher/researcher to a researcher/teacher was a mirror for my study of tension in teacher change. Calling it a “dialogic journey”, Smith (1999) suggested that hermeneutic studies provide a sense of the researcher's own transformations. As much as I thought I would be studying others, I found that I was studying myself just as often and deepening my own self-understanding through the art of conducting research.

In the next two chapters, I provide the results of that research. Chapter 5 details the nineteen instances of tension experienced by the teachers as they implemented change in their mathematics teaching practice while Chapter 6 delves more deeply into five singular instances of tension experienced by Nicolas.

#### 4.4. Coda: On the participants becoming ‘my’ teachers

In the next chapter, a curious reader may wonder why I refer to the participants in my study as ‘my teachers’. In explanation, I found preparing and analyzing the data was a surprisingly intimate experience, especially as it was so one-sided; I feel that I got to know my participants despite revealing little of myself to them. For example, I remember spending several days transcribing and coding one of Sam’s interviews: his voice echoing over and over again in my head. By sheer coincidence, I met him shortly after at an educational event and greeted him warmly. He looked at me rather surprised. Although I had interviewed him, we really did not know each other all that well. I had simply carried on in the moment with the conversation that I had been having with him in my head while transcribing. He had become ‘my’ teacher, not from a sense of ownership, but from this sense of a deepening (albeit one-sided) relationship. Just as tension moved from *a thing* to *the thing* to *my thing*, I noticed that the way I addressed the participants followed a similar pattern – first I wrote about *a teacher*, then *the teacher*, and finally, in the analysis, *my teacher*. This was not so much a decision I made; rather, it was a decision to continue using once I became aware that it was happening.

## Chapter 5. Teachers' Experience of Tension in Changing Mathematics Teaching Practice

*Out of the quarrel with others we make rhetoric; out of the quarrel with ourselves we make poetry. (W. B. Yeats, Anima Hominis)*

This chapter details the types of tension experienced by my teachers as they attempted to implement change in their mathematics teaching practice. My intent is to explicate the kinds of tension mathematics teachers experience and lay the groundwork for further exploration.

I broadly divided tension in two categories: internal and external. Internal tension exists within teachers as they experience uncertainty and doubt regarding the choices they have made or need to make regarding their practice. External tension is tension from without, as its genesis is policies imposed on teachers' practice by outside forces such as district or government mandates. Unlike internal tension, in which the power to respond lies within the teacher, external tension implies a power imbalance in which teachers have no choice but to comply; they cannot reason with an externally mandated requirement. Rather, the course of action is imposed upon them. The categories of internal and external tension are then further subdivided (see Table 5.1).

In keeping with a phenomenological approach to qualitative data analysis, I have provided my interpretation of the data and attempted to weave excerpts from my teachers with the theory outlined earlier.<sup>6</sup> It is through their voice that tension is described, but it is through mine that their words will be interpreted.

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<sup>6</sup> Excerpts taken from data other than interviews will indicate their source e.g., online discussion post, essay, or email. All names are pseudonyms.

**Table 5.1 Sources of Tension**

Internal Tension - genesis lies within a teacher	
Intrapersonal	Motivated change
	Resulted from change - self doubt about implementation - pull of previous practice - physical limitations
Interpersonal	Parents - critique of practice - critique of character
	Students - engagement - ability
	Colleagues - sharing ideas - sharing space
Pedagogical	Assessment
	Classroom management
	Group management
	Homework
External Tension - genesis lies outside a teacher's control	
Assessment and reporting	
Curriculum	
Institutional norms	
Substitute teachers	
Classroom support	

## 5.1. Internal tension: Tension from within

Internal tension is tension that exists within teachers as they experience uncertainty and doubt regarding the choices they have made or need to make regarding their practice. Lampert (1985) fittingly calls this “an argument with oneself” (p. 182). While outside influences do play a role, the genesis of internal tension lies within the teachers as they struggle to determine the best course of action and endures as they live out the consequences of their decision. While tension is inherent in the activity of teaching in general; it is acutely felt in times of change (Dietiker & Riling, 2018). This was true for my teachers who experienced internal tension that was both intrapersonal and interpersonal, as well as pedagogical.

### 5.1.1. Intrapersonal tension

While all tension can be considered personal in that it is borne in, and borne by, the teacher, I single out tension that emerges from conflict with aspects of one’s self as intrapersonal tension. If thought of in terms of agency, teachers are in a constant state of becoming (Roth, 2002) that is constructed from their past experiences and shaped by their present. For teachers involved in change in mathematics teaching practice, tension arises as they struggle to reconcile a new conception of teaching and learning with their previous experiences as learners and teachers. This next subsection highlights the intrapersonal tension that both motivated, and resulted from, the changes my teachers implemented.

#### ***(a) Intrapersonal tension that motivated change***

For most of my teachers, the decision to implement change in their mathematics classroom stemmed from dissatisfaction with their current practice. Most had learned mathematics as learners in traditional classrooms and had simply gone on to replicate that for their own students. As Kelly recalled, *“There was nothing during my journey to becoming a mathematics teacher that made me think of another way to teach math”*. Their collective desire to move away from the notion of teaching as telling and learning as listening (and remembering) so permeated their interviews that I originally coded these excerpts as ‘Tension with being ‘that teacher’’. However, it was this intrapersonal tension between who they were and who they wanted to be that impacted their projective agency and led to change in their mathematics teaching practice. They could envision other

possibilities and were motivated to bring about a future that was different from their past and their present.

Many of the teachers described similar situations, where tension with their teaching practice drove them to seek out professional development. For example, Amy's tension began with the feeling that, *"I was boring, like they just weren't getting from me what they needed"*. Furthermore, in an act of reflexive agency, she wrote in her essay that she felt her traditional mathematics practice was harming, rather than helping, her students:

My practices resulted in increased anxiety and frustration amongst my students; damaged their mathematical confidence; removed their desire to think deeper and search for understanding; as well as robbed my students of conceptual experiences. Valuing speed and accuracy comes at a great cost for students and gives them little mathematical benefit.

To manage this tension, Amy acted agentially, as she sought out professional development *"for some new ideas"* that would better engage her students. Instead, she experienced a student-centred teaching style that *"completely transformed my pedagogy"*. No longer content with her product-oriented mathematics classroom where students worked individually to develop procedural fluency, she turned to a process-oriented model that valued conceptual understanding and collaboration. Although she may have started out seeking instrumental change, her act of agency resulted in foundational change.

For other teachers, it was attending professional development that caused the tension with their teaching practice. Kelly described the same sort of experiential learning from professional development as Amy, but added, *"I never questioned it [her practice] until my eyes were opened, when I saw another way. Since then, I have felt my teaching pedagogy do a complete 180° shift."* Although she had willingly attended the professional development session, her attendance was not due to prior tension with her own practice; it was more a matter of convenience and opportunity: *"It was our district ProD and it was a math topic. I was there because I was a math teacher."* Describing herself as a typical, traditional mathematics teacher, the experience provoked tension that inspired her to implement changes in her teaching. As she explained, *"It was confounding to learn that something I was doing in my class was actually taking away from students' learning. It really makes you think about and reflect on what you are doing as a teacher."* Like Amy, the traditional teacher she once was appeared to be something to avoid as she

emphasized a new sense of purpose, *“I knew I never wanted to be that teacher”*. This was an introspective observation for Kelly as she inwardly reflected on her own actions.

For both Amy and Kelly, there is a sense that the tension precipitated an internal reaction outside their control; it happened *to* them. Their use of expressions like *“transformed”* and *“eyes opened”* suggests they experienced foundational change, as if the core of who they were as a teacher had been unexpectedly altered. They may have set out to change some particular things about their practice, but also ended up changing themselves.

For other teachers, this alteration appeared to be a more purposeful decision. Sam speaks of being at a *“crossroads”* where tension with his teaching style caused him to ponder two choices: seek out professional development or quit teaching. In the end, he chose the former, saying, *“I’m going to try out for one more year and I’m going to become better”*. No mention of transformational experiences; rather, this was a deliberate response to an intrapersonal tension. He was not happy with who he was as a teacher and he set out to change that. His was a practical-evaluative achievement of agency shaped by what was currently possible and motivated by a strong sense of purpose: *“I’m going to become better”*. This notion of purposeful deliberation appears again with David, a new teacher assigned to teach mathematics. Having never planned to be a mathematics teacher, he first turned to colleagues for advice on what to do:

I asked them, how do you teach math? How can I make this fun? And they’re ...like, I hate to say it, but they’re older teachers, and they have very traditional views on math, and they kind of do it how I was taught math. They just work on the problem on the board, show them how it’s done, and get them to practice, practice, practice until they get it. And I knew that’s not how I wanted to do it. That’s not who I wanted to be.

Although David had not yet developed a mathematics pedagogy, he knew who he did not want to be as a teacher. This created intrapersonal tension, which motivated him to sign up for a series of professional development sessions that focused on progressive teaching practices in mathematics. And, over time, he implemented the strategies he learned in his classroom. Again, there is less a sense of an unexpected transformation and more of a determined decision to take a different path.

### ***(b) Intrapersonal tension that resulted from change***

Although the changes my teachers implemented in their mathematics classrooms were self-motivated, that does not mean they were easy to make. The uncertainties they



experienced and the doubt the change created caused intrapersonal tension, as the teachers questioned the decisions they made. In particular, three areas of tension arose: tension with self-doubt; tension from the pull of previous practice; tension with physical limitations.

### **(1) Tension with self-doubt**

There is no tried-and-true playbook for implementing change in teaching and this is particularly true for mathematics which has proved especially resistant to reform (Hiebert, 2013). With change, then, sometimes comes the uncertainty that one is doing 'it' right or that one should be doing 'it' at all. For my teachers, this 'it' took various forms and tension was evident as they wrestled with their doubts. This sense of uneasiness calls to mind Cuban's (1992) metaphor of tension as walking a tightrope. My teachers were teetering between self-doubt and certainty about their ability to implement the changes they were making. For example, Kate questioned her ability to respond flexibly in the moment, "*with questions or interventions that move students forward*", as she was unsure whether she possessed "*a strong enough repertoire of open-ended knowledge*". She later added, "*While I am making inroads with this, it is also a cause of anxiety*". According to Pantić (2015), this worry regarding competence can have a deleterious effect on agency. The changes Kate is implementing might be at risk should she eventually decide she is unable to develop the skill of questioning.

The incorporation of open-ended problems also created tension for Corey, who admitted, "*I don't know what I'm doing. I'm afraid I'm going to make a mistake but as teachers we learn that that's not a reason for not doing something, right?*" That neither teacher mentioned specific methods they were utilizing to alleviate their self-doubt is indicative of a willingness to live with tension regarding their competence. Perhaps they expect it will lessen over time as they gain experience? Additionally, Corey's switch from the singular 'I' pronoun to the collective 'we' suggests that this willingness might come from a sense that she, along with any other teachers trying to change their mathematics teaching practice, is part of a reform effort that is larger than herself. That Corey included me in this effort was evident as she ended with the interrogative, "*right?*".

For others, tension arose in their uncertainty regarding the future impact the changes will have on their students. In critically reflecting on the changes, Sara worried that, "*In training them to think and learn in a new way, I'm setting them up for failure when they go on to*

*their next teacher and back to traditional learning*", while Alison questioned her students' future success with a looming standardized assessment: *"I find it difficult in teaching in this new approach. I'm questioning if I am putting my students at a disadvantage when it comes to the PAT<sup>7</sup> exams."* While neither teacher gives any indication that this tension will impede the changes in their practice, it appears to be an ongoing concern. Again, there was a noticeable absence of potential resolution to this tension; it was just endured. This suggests that some intrapersonal tension appears to be an unpleasant yet tolerated companion to change. This tolerance might be explained by the iterative dimension of agency. The past experiences my teachers may have had with overcoming struggles in change may provide the reassurance they will again, with time, prevail through this tension. Like walking a tightrope, they will get better with practice.

## **(2) Tension from the pull of previous practice**

Traditional mathematics teaching practices comprise universally accepted classroom social norms, such as teacher-led examples, individual seat work, and silent practice, that are especially difficult to displace. Such a strictly controlled environment offers the illusory appeal that serious learning is taking place. This notion is embedded in the mathematical backgrounds of my teachers for whom the pull of traditional practices lingered. This created tension for those attempting to suppress these desires and for those who succumbed, both of which impact change. For example, Lily recalled that in her early teaching career she believed that, *"The quieter the class, the more I thought learning was happening"*. She had come to recognize that this is not the case, yet acknowledged, *"I do on occasion go back to this method because of a bad day or I am not prepared. When I do go back to this traditional method, I am aware that it was not a good teaching day for me or the students."* This created tension, as she realized that her decision, while satisfying her immediate needs, had unintended consequences – *"not a good teaching day"* – for both her and her students. The rarity of her use of the *"traditional method"* makes this an infrequent tension that she appeared willing to live with, although she acknowledged, *"This is an area for me to continue to grow and look to change"*. This is a reflexive response, as it indicates a critical awareness of the impact culture has on her practice.

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<sup>7</sup> PAT is an acronym for Provincial Achievement Tests. These are standardized, government-mandated assessments written by students in grades 3, 6, and 9.

Interestingly, this notion of being unprepared or having an off-day appears to be the impetus for several others who also return to more traditional methods to satisfy their own needs. As Kelly recounted in her discussion post:

So today I sort of reverted. I have not been feeling great and I needed something quick and easy to put together for a lesson. I started the class with a review/notes of all the topics we have been doing. We did some examples together on the board then I gave them a "worksheet" at desks with chairs. This class has rarely come into the room to see desks and chairs set out that are available to sit in. But today I caved. I was hoping for some quiet time while they worked.

This backfired for Kelly, as she later admitted, *"for the most part I spent the rest of class going from one student to another with hands up helping them with problems"*. Like Lily, her tension lay in knowing that her decision to 'revert' had had unintended consequences both for herself and for her students. It appears that the challenge of implementing change can occasionally nudge teachers towards that which was once familiar and therefore seen as easier. Hoping for respite, they instead experience tension.

This return to the familiar also occurred for several teachers not because it was easier, but rather because they missed the reassurance of traditional teaching. This created tension for them, as they struggled to suppress this need. Leah mentioned wanting to be sure she was covering the content since she implemented the changes in her classroom:

I still occasionally like to start by demonstrating something new and then having students do similar problems or problems connected to what was demonstrated. This comforts the 'traditional' teacher in me, but I do feel like it is cheating or missing the point.

For Leah, the pull of previous practice resulted in conflicting feelings; she is both comforted and discomfited by her choice to occasionally shift away from the changes she implemented. She is a "conflicted teacher", as described by Lampert (1985) and as "her own antagonist; she cannot win by choosing" (p. 182).

A need for reassurance is also apparent with Diane, who mentioned occasionally returning to her previous teaching practices:

I really want to make sure that everybody's learning. When they're quiet and they're all looking at me I know I have their attention. I'm not sure if everybody is paying 100% attention when they're working in the problem-solving groups.

When speaking later of year-end assessments she added, *"I know I don't need to do it [teach traditionally]. I know I shouldn't. They all did so well that it solidified for me that the*

*way I was doing it was already working.*” Despite knowing the changes she had implemented were successful in terms of student learning, Diane still occasionally returned to traditional teaching for the familiar feedback it provided – e.g., the students are all looking at me, therefore I can be sure they are learning – that she did not appear to get from her new teaching style. For both Leah and Diane, I see this as a temporary waver in their sense of purpose; they are momentarily uncertain about the changes they are implementing. So, like the tension around self-doubt, their tension is also connected to ‘it’. However, this time the ‘it’ is not the change from traditional teaching, but the change back. And, like Lily and Kelly, the relative infrequency with which they experience this tension perhaps makes them willing to live with it, as neither mentions a means to manage their tension.

### **(3) Tension with physical limitations**

While all my teachers spoke of fatigue and tiredness in general, in only one case did worry over physical limitations result in tension. Like the others, Corey had implemented new practices in her classroom that required changes not only in the physical movements of her students but, in her own as well. She mentioned, *“Physically the vertical learning can be challenging for me. I struggle to stand for the whole day, so I have to make sure I’m doing a mix of things throughout the day.”* During the interview, she let this thought be and then came back to it unexpectedly about ten minutes later, as she further added, *“I just don’t want to be that teacher”*. When asked to clarify, she explained:

Because I struggle to stand. [pause] I don't ever want to be that teacher that sits at the desk all day, because that's not effective at all. I think if it's this bad, I'm 43, what am I going to do five years from now? Six years from now? How's it going to look? That's something that keeps me up at night. How am I going to best serve these kids when I can't move around the room? So, yeah, it's a concern. That's one of the reasons I might not always be a classroom teacher; it might not be an option for me physically, to do a really good job of it.

There are two particular things to note here. First, Corey’s use of *“that teacher”* suggests she had developed a schema of what a teacher is and is not. This allows her, like Kelly and David, to know what kind of teacher she does *not* want to be, and despite the tension that results from worries over her physical limitations, she does not veer from that. Second, it is interesting that while Corey does later mention solutions such as a *“motorized scooter”* or *“mixing things up”*, moving away from the new practices that are taxing her physically is not mentioned. Similar to Sam, it seems she would rather leave the profession than

return to her previous teaching practices. This suggests a high level of reflexivity and sense of purpose as both teachers hold true to their deeply held principles.

### **5.1.2. Interpersonal tension**

While *intrapersonal* tension can be thought of as an argument with oneself (Lampert, 1985), *interpersonal* tension emerges as conflict involving relationships. The intensity of these relationships can lead to a variety of interpersonal tension, as those involved negotiate the boundaries of their relationship. According to Palmer (1998), “teaching is always done at the dangerous intersection of personal and public life where weaving a web of connectedness feels more like crossing a freeway on foot” (p. 12). This was true for my teachers in whose responses can be found tension with parents, with students, and with colleagues that affected their sense of agency. And, although these relationships exist outside a teacher, they are categorized as internal tension as the power to respond (or not) lies within the teacher. Whether parents, students, or colleagues, all have little or no power over teachers, so teachers are able to make their own decision regarding the best course of action in dealing with the tension.

#### **(a) Parents**

Parents and teachers have a common interest in the growth and development of children, which necessarily involves close connections between home and school. However, despite sharing broadly similar goals, tension can arise as parents and teachers find themselves at odds with each other around methods for achieving those goals. This is particularly true in mathematics where pervasive sociomathematical norms regarding the teaching and learning of mathematics are often an obstacle for any teachers attempting to incorporate new practices. Parents have an expectation of what mathematics looks like that may well conflict with what teachers believe mathematics looks like. This creates tension for teachers as they try to balance their desire to implement new pedagogical practices with their desire to maintain supportive relationships with the parents of their students. This was true for the majority of my teachers who experienced tension with parents regarding their practice. For some, this tension moved beyond their practice to focus on their character.

## (1) Tension with parents' critique of practice

Three areas of practice emerged as the main sources of tension between parents and teachers: homework, grades, and types of learning activities. For example, Diane recalled tension she experienced regarding homework, *"One parent really wants homework all the time. She [the parent] really wants them to do multiple problems, doing the exact same problem, using the same formula over and over again."* Diane's use of repetitive language (*"same"* and *"over and over"*), along with her double use of *"really"* emphasizes how problematic this is for her, as she placed little value on homework saying, *"I don't see the purpose of sending them home twenty problems just to do on their own"*. Her word choices also make clear how important homework is to the parent and that serves to magnify her tension. That her autonomy has been impinged is clear; she still has agency, but exercising it now has consequences. Diane managed the tension by verbally reassuring the parents of their child's progress, while holding fast to her 'no homework' policy.

A focus on grades was also a frequent source of tension around autonomy. This is exemplified by Sara who mentioned her discomfort with a few parents who questioned their child's grade. While acknowledging the parents' desire to support their child, Sara preferred that they look past the grade, as *"the number grade doesn't tell the whole story"*. For Sara, this tension was especially poignant in the moment of actually grading students' work:

I don't let it affect me much, but it's always in the back of my mind. Oh, this kid didn't do well. I might potentially be hearing from their parents. Or when I get to report cards, right? If I've given a kid a B in math, I'm like, well, I know that parent is going to come and talk to me to say, 'Why are they getting a B? What do we need to do to get an A?'

Although her use of the hedge *"much"* tacked on to the end of *"I don't let it affect me"*, suggests a lingering tension, Sara has worked to manage this tension through two proactive means that speak to a heightened sense of agency. First, throughout the year she utilizes an online reporting system, *"so the parents see more of the smaller stuff"*. Second, before sending report cards home, she has her students write a self-reflection of their achievement and provides them an opportunity to ask her questions regarding specific grades:

That has resulted in almost no parents coming to talk to me about report cards because their kids are actually able to say, 'Well, I didn't do well on this' or 'I didn't understand this' so they can kind of explain it to their parents.

When asked about her approach for those parents who still do question the grade, Sara replied, *"I know I shouldn't, but I send home worksheets; they [the parents] just want to feel like they're helping"*. She went on to explain that she does not find worksheets useful for developing conceptual understanding in mathematics. Sara's is a limited agentic response. She is acting out of genuine concern for the needs of the parents despite her own misgivings about the action itself. Additionally, during the interview, Sara's opening clause *"I know I shouldn't"* captured my attention, as it was accompanied by a quick look in my direction that I made a note of at the time. Similar to Corey earlier (whose use of *"right"* was meant to include me), Sara was drawing me in, looking at me expectantly, as if to check whether this was a shared understanding between us. She may have previously been able to live with this tension (sending home worksheets), but having to say it in front of me caused it to resurface. She then adds a plausible rationale (parents want to help), as if she feels compelled to explain her action to me.

The learning activities the teachers choose to incorporate within their practice were also a source of tension between parents and teachers. A proponent of collaborative problem solving and hands-on manipulative use, Lily recalled being asked, *"Why is my kid playing games in class? They're not learning by playing games."* This created tension for Lily, as she had come to appreciate the *"greater engagement, deeper understanding, and perseverance"* that her students had displayed since she moved away from the traditional workbook model adopted by the majority of teachers in her school. She further added, *"This caused some push back from parents who enjoyed the scaffolding method in Jump Math [workbook program]"*. Recognizing that, in this instance, unfamiliarity breeds contempt, Lily attempted to maintain her autonomy by inviting parents into her classroom, so they could experience the learning first-hand. Kelly responded to a similar parental concern regarding her new teaching methods by switching agency for the changes from herself to the curriculum. In this dance of agency, she told parents, *"that it's the new curriculum and this is the new way of teaching, and just asking them to give it a chance"*.

The collective actions the teachers took regarding tension are mostly indicative of a desire to resolve the tension through proactive methods, such as frequent progress reports, pictures of students engaged in math, and frequent parent contact. This suggests a practical-evaluative approach to reducing tension around the exercising of agency. In so doing, the teachers are attempting, as Kate suggested *"to get all the parents on board"*. Indeed, winning over the parents appeared to be the main method for resolving tension,

as variations of the phrase 'on board' appear multiple times when teachers spoke of parents. However, there were indications that some teachers were also content to live with tension. For example, with Sara, who mentioned printing off worksheets to keep parents happy despite her antipathy for worksheets, or with Alison, who prepared homework to satisfy one parent despite her no homework policy.

## **(2) Tension with parents' critique of character**

This tension was evident with Amy who described how her excitement in sharing her practice with parents was tempered by the response of a few who took issue not only with her practice but also her character. She reached out to the other teachers in a discussion post:

This week I had parent-teacher interviews and was excited to explain the new practices I've adopted in my classroom: vertical surfaces, student collaboration, random groups, student discovery etc. The majority of the parents were on board with this new approach, but some parents looked at me like I was "slacking" and not doing my job properly. One mother even told me that I wasn't teaching what was on the curriculum. Has anyone else felt like they need to "overly" validate these new practices to parents or has everyone you've told been receptive to this new approach?

Amy's tension arose from her desire to implement new practices while maintaining her relationship with her parents. Her use of "overly" suggests a strong commitment to the practices she is implementing. The tension she was experiencing was not going to weaken her goal; if anything, she was looking for ways to strengthen it. This is an autonomous reaction, as she wants not only to validate her practice, but "to overly validate" it and thereby perhaps make it impenetrable to further judgement. Interestingly, her question for her peers focused on ways to support her practice, not her character. A possibly rationale for this can be found in an excerpt from Alison who, upon noting, *"I have one parent that just dislikes me in general"*, went on to explain that, *"She [the parent] is totally afraid of math and has never been successful in math so she is just angry in general about math"*. As Amy also has one parent who is not "on board" with the mathematics, perhaps both Alison and Amy are able to overlook the personal attacks because they view it not as dislike of themselves but dislike of mathematics. This is suggestive of a dance of agency. Here, the focus is put back on the discipline which, in this instance, includes mathematics pedagogy rather than just mathematics itself.



It is also possible this is bolstered by their teaching experience, which contributes to a strong sense of iterational agency. Although tension with parents was heard in a *majority* of teachers' interviews, it came with a general sense that it occurred with a *minority* of parents. Whether parental tension was with practice or character, most teachers described single instances of tension and the word 'outlier' made frequent appearances. For example, in describing tension with a parent, Lily recalled that during her encounter, "*I kept saying to myself 'outlier'".* Indeed, in answering Amy's discussion post query, three of the five peer responses began with a reminder that this parent was an outlier before proceeding to offer suggestions on how to validate her practice. It was as if there was a collective acknowledgement that tension with parents does happen, and will continue to happen, but that it is mitigated by the fact that the majority of parents are 'on board'. One parent attacking your character or practice cannot diminish that.

This becomes more apparent with Nadia, a first-year secondary mathematics teacher, who described an encounter with a parent. Early in her first semester of teaching, Nadia had called a parent with a concern about their child's attitude. Calling it a "*traumatizing experience*", Nadia described the situation:

So, I called her, and I said, 'This is what's going on', and then she went off on me. Literally, I didn't get a word in for 15 minutes and she just attacked my character and told me that I'm so young, I have no life experience. That they should never have put me, a new teacher, in charge of this group of kids and all this stuff and how she was going to take him out of the French immersion pod because I'm such a horrible teacher and I'm trying crazy ideas in my classroom and all these other things and I [pause] it was really, really hard for me because I'd never been attacked like that. And it's one thing to not agree with how I do something. It's another thing to attack me as a person.

In reflecting on the phone call in an interview shortly after the incident, Nadia questioned whether the parent would have responded the same to an older, more experienced teacher, as she explained, "*I just feel like she only spoke to me that way because of my age. Because I'm 24 years old and I'm a new teacher.*" Although Nadia continued her practice of contacting parents, her lack of experience prevented her from viewing this parent as an 'outlier', and she began to fear contact with all parents:

I'm more afraid now than I used to be because now, I'm like, this conversation could go anyway. Who knows? Like, I have no idea. But I really try not to let it get in the way because I do think frequent parent contact is very important and I think that calling is even more effective than email because you can have a conversation, you can hear tone, you can get information across that you can't always get across in an email.

Nadia was not swayed from her goal of maintaining parent contact, but she was also initially unable to compartmentalize the tension as an outlier in the same manner as the more experienced teachers. However, there was a shift in her agency that was readily apparent during an interview that occurred four months after the incident. When asked her current thoughts on the situation, Nadia responded, *"It was a learning experience for me, and I learned how to deal with a difficult situation and not to take it personally"*. Noticeably absent is the raw emotion that coloured her earlier descriptions and in its place is an analytical reflection of the growth that occurred, which suggests reflexivity may have played a role in her agency. When asked how she was able to achieve that growth, she provided two reasons: *"Thankfully, that is the only difficult parent experience I've had so far"* and *"I've had many parents say, 'You're the only one who contacts us.' They seem to really appreciate that"*. Like the experienced teachers, Nadia has come to see it as an isolated incident, an 'outlier' and she feels strengthened by the support of the 'on board' parents.

### **(b) Students**

The relationship between students and teachers is one of reciprocity governed by classroom social norms and sociomathematical norms. The expectation is that the teachers are there to teach and the students are there to learn. The reality, however, is not quite so clear cut. Students whose primary goal for schooling seems to be anything but learning can disrupt that expectation, as can teachers who change what learning looks like in their classrooms. For my teachers, the changes they were implementing did result in tension with students, as some struggled to adapt to these unexpected ways of learning mathematics. This was particularly noticeable in the areas of student engagement and student ability.

#### **(1) Tension with student engagement**

Common to every teacher in my study was their desire to have their students collaborating in mathematics. This came with the expectation that the students work together not only to understand the mathematics, but also to explain their thinking and share their strategies. Equally important was a focus on ensuring individual members of the group understand the collective thinking of the whole. This was apparent while observing a group of three students working in David's mathematics classroom. I had watched as two members of the student group explained their thinking to the third and then called David over saying,

*"Hurry, Mr. B, before she forgets!"* Tension, however, can arise when students are reluctant, or outright refuse, to collaborate. As Sara explained:

Because it is more work to have to constantly communicate clearly your reasoning and justify methods to someone else. This style of learning is often seen as more work because they are constantly being forced to think and understand. Some students find this exhausting and would prefer traditional methods simply because it is easier to do mindless individual tasks.

In introducing collaboration, the teachers have changed the classroom social norms around the kind of learning the students are expected to do. Some are compliant, but not surprisingly some are not, and this can create tension for the teacher. Kate mentioned her tension with students who are reluctant to collaborate noting that some of her students disliked sharing their ideas:

One interesting problem was how ticked off a couple of my students were about having to share their thinking and strategy out loud while solving the problem: "But it's MY strategy, and I don't want them to know it".

Kelly recalled a similar experience and offered the suggestion that the students *"are so entrenched with 'fairness' that they see collaboration as cheating somehow"*. For the teachers who experienced this tension, most showed a willingness to live with this tension in the hope that it would lessen over time. As Sara mentioned, *"Hopefully they will start to relax the more often we work like this"*. Her use of the pronoun 'we' is notable, as it suggests that the teacher is integral in the success of the collaboration process. According to Pantić (2015), this speaks to the role competence plays here. Sara sees the difficulties her students are experiencing as being tension with the new practice rather than problems within her students. It is not that her students cannot engage collaboratively; it is more that they, including Sara, need more time working on the skill.

But for other teachers this tension goes beyond a reluctance to share ideas, their students outright refused to engage in any form of collaboration. David spoke of his concern for some of his students:

I have some kids that'll just kind of stand there and not want to participate. I don't know if they're nervous or if they're just anxious about sharing their ideas in the group? But I try and push them in and encourage them, but they still kind of are the ones that just want to stand off the side. So, I just [pause] I try and I don't know. I don't know how to get them involved.

In his interview, David mentioned collaboration as being integral to the changes he was trying to make in his classroom. He wanted this to succeed and so looked for ways to

manage the tension he was experiencing. He offered two plausible reasons for his students' reluctance (nervous; group anxiety) and posed them in the form of a question, as if looking for affirmation from me. I suggest this is indicative of limited autonomy; it is as if I agreed these were potential causes, then he could possibly form concrete solutions and not have to give up on collaboration. Jamie also shows determination in the face of students' reluctance to join groups and refused to alter her plans, *"I just keep doing what they need; not what they want"*. Like David, she remained insistent that the students join their groups. For both teachers, this is a competence aspect of agency. They are aware that there are broader forces at play (i.e., nervousness at trying new practices) that can interfere with change.

Nadia and Kate both dealt with their tension by allowing unwilling students some choice on collaboration. As Nadia observed:

There are always students who never want to collaborate. They're just kids sometimes that like to be by themselves and like to work by themselves and that's fine. I try to push them to go out of their shell a little bit, but not too far because I want them to feel that they can be themselves and express themselves.

This is a tension she was trying to manage in order to maintain her teacher/student relationship. This modeling of creative ways to work with students is a competence response that reveals a strong sense of agency. My teachers were trying to find ways to accommodate all of their students while still implementing the changes they desire. This *"leave them alone"* approach seemed to be working for Kate too, as she had experienced some success with the technique. She shared her thoughts in a reply to one of the numerous discussion posts regarding what to do with students who refused to collaborate:

I definitely have a couple kids who just won't engage. I have been using the 'leave them alone' technique, and sometimes there's success and sometimes there isn't. I try to keep telling myself that if I had just given them textbook practice, these non-engagers would likely not have done that either and, remind myself that overall, I feel I have more participation than I did before. I think the non-engagers just become more obvious to us when the rest of the class is up on the board. They stand out more than when they did seatwork, when everyone would be doing individual textbook practice. I think that's why it's harder to just leave them alone.

There are two notions to consider in Kate's response to her peers. Again, there is a sense of an 'outlier' in that there are just a few students responsible for the tension. In her phrase, *"overall, I feel I have more participation"*, there is a sense that Kate was able to see a collective benefit to the changes she was implementing. This suggests that had the benefits been limited to a few individual students, Kate might have been less willing to

continue with the changes in practice she had implemented. This was apparent in several teachers' replies. For example, when asked why the resistance does not cause her to abandon the collaboration model, Alison responded, "*Because it is, you know, just the few odd students*". The second notion to consider is that the new practice (collaboration) is making the previously invisible behaviour visible, as Kate explained, "*They stand out more than when they did seatwork*". This is important, as now that it is visible, some of the teachers felt compelled to manage the resulting tension.

## **(2) Tension with student ability**

While meeting the diverse needs of their learners was often the reason my teachers were moving to incorporate open-ended problems and collaborative groupings in their rooms, the path to that goal was not without tension. As above, this again is a competence aspect of agency that presented as an 'outlier' tension, as teachers mentioned tension regarding specific students or groups of students. As Kelly recalled:

I always lie awake at night thinking about the kids, the specific students, like meeting their needs. It's like okay, I didn't get a chance to check in with him today, did he actually get it? Or they're dealing with so many other social and personal issues that's taking them away from their learning, I'm worried about them even passing my course by the end. I'm continuously thinking what can I do? What else can I do?

This appeared as a binary tension for Kelly that resulted from her desire to implement new practices while meeting the needs of her students. This was a tension shared by many of the teachers as they reflected on the kinds of students they taught.

Most of the teachers were surprised to find their *high-performing* students challenged by the changes they were implementing; they had assumed (hoped) these students would naturally adapt and be successful. But, in effect, the teachers were asking the students to change too. And considering that high-performing students have felt particularly successful in the 'old' system, it is not unexpected that they may be a source of tension when that system undergoes change. Diane referred to this when asked how her students enjoyed the practices she had initiated:

I find almost all the higher achieving ones find it a little. [pause]. How do I put it? Out of the norm of what they are used to doing in other academic classes. They want to sit alone, and they want to get it done, and they want it to be over with. Whereas, I just keep giving them more and more, where they're not done. They're not going to be done until the end of class. And they're not going to work alone.

And I think they find that they want to complete something and be done. But I think it's good for them. It's worth it.

Like Jamie earlier, Diane believed she was giving the students what they needed, not what they wanted which speaks to her high sense of competence. Her ending sentence (“*It’s worth it*”) is suggestive of two things. First, it indicates her desire to live with the ensuing tension as her high-performing students push for a return to the classroom social norm. Like Sara’s earlier willingness to live with her students’ reluctance to collaborate, Diane also displays a sense of purpose in her agency, in that she believes the outcome she is pursuing is worthwhile. Second, although the preceding sentence speaks to the worth of collaboration for students, her use of “*it’s*” and “*it*” in the final sentence points at a more universal application, one that benefits both her and her students. Perhaps it is that dual benefit that makes her content to live with the tension. Hegel would suggest individuals must see the positive within the negative moment of development if they are truly to grow.

Kelly also describes living with the same tension with her high-performing students and contrasts it with the response of the rest of her students:

The students who traditionally did well in my class – they liked the notes, they liked just being able to sit and do their work – some of them hated it; they absolutely hated it. They’re like, just give me a textbook; I want to sit down and just do some work. And the others, who never liked learning that way, they’re like, ‘Oh this is awesome!’ They’re up and moving and they don’t have to sit in their seat and they’re thinking and sharing ideas. It was almost the opposite. It almost feels like no matter what you do, you can’t please everybody. But now I’ve pleased a different set of students. And I think the other ones might come around. I can see they need to be convinced.

This is a similar response to Diane but on a collective basis, with Kelly choosing to put the needs of the many before the wants of a few. Indeed, this was how most of the teachers responded to this tension with high performers; they were living with it because they felt the overall benefits were worthwhile. Kelly’s choice of the word ‘convince’ is also interesting. Again, there is a willingness to work on managing tension over a protracted period of time. Yet, later in the subsection on tension with colleagues, having to ‘convince’ someone is shown to contribute to tension.

Tension related to their *low-performing* students permeated the teachers’ responses and, while most spoke positively of how the changes that they had implemented benefited most of their struggling students, they remained concerned about those few who continued to struggle, their ‘outliers’. Like the high-performing students, tension originated in the

changes the teachers had implemented to which the low-performing students struggled to adapt. Unlike the high-performing students, the teachers were not content to live with this tension, they were actively looking for agentic solutions that would allow them to continue with their new practices while supporting their low-performing students; they were looking to increase their competence. For example, Alison wrote a discussion post around her concerns for a low-performing student that ended with a plaintive question, *“How do I help her be successful in this new teaching style?”*

Like with Kate and her reluctant collaborators, tension arose because the introduction of a different way of teaching and learning had helped make visible what was once hidden. Lily made this clear as she described her tension about her implementation:

The only conflict I have about this is sometimes how to reach my weaker kids. I do feel that they miss out and that they're not getting the individual attention that they might need. I bring them to me, and they work beside me but then I see that as a disadvantage because sometimes their peers can help them better than I and also, I don't always want to pull out these kids. I'm just not sure what is the best route for them. It's more obvious to me, it's just more obvious that I know they are flailing. There's no doubt. I don't think it was just as in my face before; it's so much more in my face now that I have to go 'ooh'.

Lily's change in practice had brought to the forefront a previously hidden or perhaps ignorable tension that she had been living with. Now confronted with its existence, she was looking for a way to manage it. She was not content with her current method, as it involved separating the student from their peers. Other teachers also pulled students aside and were equally unhappy with this as a permanent strategy. For Amy, this was because, *“I don't want to make it that obvious that he [her student] needs help”*. Similarly, after sharing a list of ideas she was considering trying with her low-performing students, Corey ended by adding, *“But whatever I do, I don't want it to look too obvious what we are up to, so discretion will be key”*. This suggests that compounding tension with low-performing students are two desires: to keep any potential strategy hidden from general view and to come up with a way that keeps students with their peers.

### **(c) Colleagues**

As Hargreaves (2001) points out, collegial relations among teachers are a peculiar combination of closeness and distance. Despite working in adjacent classrooms in densely populated buildings, most teachers' workdays are spent in relative isolation from other adults. While close bonds can and do develop, for the most part teachers form casual

relationships “where ‘weak ties’ (Granovetter, 1973) or weaker collegial forms of help, assistance and social acceptance tended to prevail” (Hargreaves 2001, p. 516). This lack of connectivity perhaps accounts for the distinct lack of tension my participants experienced with colleagues, as many acknowledged that they seldom had the opportunity to interact with colleagues in any real, meaningful way. And those that did mention close contact with colleagues, mostly described positive experiences such as Kelly who described her colleagues as very supportive:

I have lots of support with my colleagues. We try different things together. Like, right now I'm team teaching with one teacher for one block and it's just incredible for professional development – to be able to bounce ideas off of each other consistently.

Still there is tension that appeared that is worth exploring and it can be expressed as tension around sharing ideas and tension around sharing space.

### **(1) Tension in sharing ideas with colleagues**

Collegial tension appeared in Sam's attempt to collaborate with his colleagues. His description reveals tension around the creation of cross-curricular projects as he recalled, *“Sometimes it's hard to energize or convince other teachers to jump on board”*. When asked why he felt compelled to convince his colleagues, Sam replied that it is beneficial not only for him but for them as well and added:

I think it makes sense, because looking at all the subject outcomes, a lot of them are similar. I think it's a waste of time to break it into separate projects. But they have to find value in it, too. It can't just be thrown together. If they don't believe in projects, that kind of shuts my thoughts down about this.

In recognizing that he cannot force collaboration, Sam was forced to live with the tension around his desire for cross-curricular projects. This impacts his autonomy, as this kind of collaboration requires a willing partner and, until Sam is able to *“convince”* someone of their value, he has to put cross-curricular projects on hold.

The expression ‘on board’ crops up again in Kate's description of an adversarial relationship with a colleague that resulted in tension regarding Kate's attempts to innovate:

I used to be very shy – except when I'm about talking about teaching – then I wasn't. So, in staff meetings, I'd be proposing new ideas and asking, ‘Why don't we do this?’ Some teachers were on board, but this woman was never, ever on board. I could tell everyone the sky was blue and she would argue it because I



said it. She'd roll her eyes at me at staff meetings; it was like that. We had a very challenging relationship.

Unfortunately, for Kate, this strongly traditional teacher wielded a great deal of power and Kate's ideas were usually ignored. When asked how she went about managing the tension, she responded:

I 'niced' her to death. It was the only way to do it. I ignored a lot of it and I would be overly kind to her. I'm sure it was because I was doing things that were different and she didn't understand. She was, 'Let's do the spelling test on Friday. Here's your math booklet.' It took a good, say, seven or eight years, but I niced her to death.

Like in Sara's earlier tension with student engagement (*"Hopefully they will start to relax the more often we work like this"*), Kate's answer displayed a willingness to live with tension over a protracted period of time while actively attempting to manage it. She continued to share her ideas, while simultaneously implementing them within her own classroom, which suggests the aspect of agency most strongly at play here is the sense of autonomy. Agentic autonomy arises from the belief that the teacher has the power to make a difference and, although this particular colleague never did implement any of Kate's ideas, she became less vocally resistant which allowed ideological room for Kate and her other colleagues to make change.

Interestingly, Kate experienced more tension with sharing ideas with colleagues. This time, she had moved schools and into a new position as a mathematics coach. Here her role was to support teachers as they implemented a new mathematics curriculum; a role which included classroom visits. Noting the reluctance with which most teachers received her requests to co-teach lessons, she described the result of one such visit:

I did manage to convince a secondary teacher to let me come in and lead a problem-solving lesson with her students. I did random groupings and encouraged them to talk about their strategies. It was a success and the students were really engaged. The teacher's response? 'It's nice to have some fluff days here and there.'

For Kate (and Sam), part of the tension likely stems from first having to *"convince"* a colleague of the benefit of a new practice. This also occurred earlier in Kelly's attempt to 'convince' her high-performing students to accept the changes she was implementing. As Mason (2002) reminds us, "Intentionally disturbing others is at best, likely to be steadfastly resisted and at worst, to backfire. No one likes to be told to change" (p. 143). It appears that this resistance is what both Kate and Sam have come up against. For Kate, this comes

with the added pressure of her role as a mathematics coach and she is stung by the backhanded compliment of her colleague: *"It's nice to have some fluff days here and there"*. She later said that she felt it undermined her professionally and positioned the pedagogy she modeled as unnecessary, both of which could affect her sense of autonomy. This remains an unresolved tension for Kate, as she has not been back to that teacher's classroom. She did, however, reach out to her master's course peers for advice in a discussion post:

So, here's my question: What do you do when you encounter a colleague like this? When you're trying to effect change, that you know will benefit students, while honouring our code of ethics and our individual professional autonomy? I have a feeling we are all going to encounter this moving forward, as more and more people see what is happening in our classrooms. Just something I'm wrestling with a bit.

I see this as a desire to manage the tension, perhaps not with the current colleague but with any future collegial tension she may encounter. Again, it is a sense of proactive management of tension. Interestingly, although the phrase 'on board' does appear several times in situations of tension with colleagues, I found no use of the idea of the notion of 'outlier'. This was despite the teachers' recognition that tension with colleagues was infrequent.

## **(2) Tension in sharing space with colleagues**

While only one teacher mentioned tension with sharing their physical environment with colleagues, I felt its inclusion worthwhile as it is tension that may be singular to mathematics classrooms. Leah, who implemented collaborative problem solving in her secondary mathematics classroom, was aware that this practice generated more noise than usually heard in the other mathematics classrooms in her school. During problem-solving activities, her students worked collaboratively in small groups at vertical surfaces. This quite naturally led to a louder classroom as the students engaged in mathematical discussion with their group members, students in other groups, and the teacher. This was different from what occurred in most mathematics classrooms in her school:

I have to remember that sometimes teachers who are used to being at the front while the kids are sitting down, and working or not, quiet at their desks. I mean there are teachers at the school that actually grade students on that and so there's quite a different culture depending on the area of the school you're in or the teacher in the classroom.

Leah's suggestion that the culture is different "*depending on the area of the school you're in*" implies that, while noise may be tolerated in other disciplines, it is considered unacceptable in a mathematics classroom. In disrupting this norm, she experienced tension with maintaining the change in her teaching style while maintaining the relationship with her colleague:

The teacher next door is often concerned with how loud they [her students] are. He will remind me every time his students have a test and I try to be a little bit more conscious of it [pause] when they have a test.

This is tension that Leah appeared content to live with. She continued with problem solving while respecting her colleague's need for quiet during critical times in his own classroom. In the interview, Leah paused before tacking on the phrase "*when they have a test*". This suggests a compromise that satisfied Leah's sense of autonomy at least.

### **5.1.3. Pedagogical tension**

Whereas intrapersonal and interpersonal tension arose from uncertainty around 'if' they should change and 'who' it ultimately affected, pedagogical tension arose as the teachers wrestled with the 'how' aspects of change that impacted them personally. While not unexpected, the change from a teacher-centred approach to a student-centred approach required changes in how that learning would now be assessed. Moreover, asking students to be collaborative participants in their own learning required structural changes to classroom dynamics. The result was that the changes my teachers had implemented created pedagogical tension around assessment, classroom management, group management, and homework.

#### **(a) Tension with assessment**

Assessment was the almost universal answer when asked what my teachers found difficult in teaching mathematics. Most indicated they felt a mismatch between the new practices they were implementing and their usual assessment practices. Although this may be indicative of a lack of competence, my teachers were able to show reflexive agency as they viewed their assessment practices with a critical lens. Lily demonstrated this sense of agency when she wrote in her discussion post:

I feel my teaching practice has changed a lot, but this has not transferred to assessment. I am more aware of how students are doing and can predict quite

accurately how they will do on the test but I'm hoping to learn about less traditional forms of assessment.

This notion of already knowing ('predicting') how well the students can perform appeared in several responses. It appears that some of the changes they have implemented have made visually assessing the students easier, yet the teachers still turned to traditional testing for proof. Kate wrote of her urge to test in a discussion post:

I am curious about how well they ALL actually understand the measurement conversions though and I am fighting my urge to give them a little paper/pencil, check in quiz. I have a feeling I can predict who would do well, and who would struggle with it, if I did.

Having watched her students work through measurement conversions in small groups, Kate's emphasis on "*ALL actually understand*" indicates that, while she knows what they can do as a group, she wants to know how they perform individually. And to gain that understanding, she felt the urge to revert to traditional testing. This move from assessing individual learning to assessing collaborative learning appeared to be the biggest stumbling block for the teachers. As David mentioned:

The thing that I struggle with is the idea of doing things in groups and kind of having a way to evaluate them working through problems and stuff. [pause] It's hard, I don't know how to really assess math because growing up for me, it all depended on the individual marks you got on tests and assignments.

David's reliance on, and familiarity with, traditional assessment practices to measure learning caused tension as he tried to impose this system on an unconventional classroom. This does not appear to be tension David, nor the other teachers, were willing to live with. In a discussion post, Amy wrote of trying to think beyond traditional assessment as she questioned the need for written proof and reached out to her peers in a discussion post:

I tried to get students in random group as much as possible and working on vertical erasable surfaces as much as possible. This being said, when I told a coworker about my success this week, she brought up a good point: What about assessment? Of course, as a teacher monitoring your students, you have a good idea of their comprehension, but do you need written (permanent) proof of their ability? What strategies do you have to assess your students' abilities who are working in groups on erasable surfaces?

Her peers replied with suggestions of group tests, exit slips, anecdotal notes, and checklists and commiserated with her struggle. Variations of 'it's so hard' permeated the discussion as they shared their assessment experiences. It was as if the teachers were

involved in an iterational development of agency as a collective, rather than as individuals. As each shared their past experiences, I had the sense that any agency achieved in one teacher could have a domino effect on the practical-evaluative agency of the rest. For example, Kate wrote in a discussion post that she did indeed offer a paper-and-pencil test, which she quickly regretted:

On my first assessment attempt, I put nine questions on the board and had them working individually. DISASTER! Some were able to do some of the questions, lots of kids got totally stalled. So, I put them into random partners and sent them to the boards to collaborate on all the questions together, then took pictures of what they had done, as their assessment. Even then, had a couple kids totally disengaged, totally refusing to take part in any way. For the most part though, it was a success. [...]I went to my next block and didn't even bother having them do it on their own – went straight to working in partners. Things went much more smoothly.

Kate appeared to have success when she switched her assessment to match the way in which the learning happened. This notion of better matching the assessment style to the learning style appeared to be the goal of most of the teachers, likely as they saw it as potentially easing their tension around assessment. This was apparent in Jamie:

I just, I need to know how do I get away from those paper tests? [...] I've got a few boys especially who are awesome, awesome problem solvers. And really like to talk through their math with me. Yet, if they sit down and put it on paper, it does not match. And I say, why is written work so important? Why is it? Because that's the way it's always been done? Ideally, I would like to sit down and have an interview with them. But I don't see how that's possible at this point right now. I just don't have the time. Do you know what I mean?

What is interesting here is Jamie was able to pinpoint the disparity between what her students know and their inability to demonstrate that knowing using traditional assessment. She even has a potentially 'ideal' method but is unable to implement it. This appears to add an additional element to her tension; she knows what to do but is not able, or perhaps willing, to do it, which indicates a lack of competence and/or autonomy. She offers the justification of lack of time and appeals to me, as a fellow teacher, to support her in that justification. Interestingly, Mason (1988) suggested that what he chose to do with his time as a teacher was indicative of the things he *really* wanted to do. Perhaps this is also true for Jamie. Time is a commodity and the tension she felt around assessment had not yet caused her to reallocate that commodity to it.

### ***(b) Tension with classroom management***

A well-managed classroom has long been considered the hallmark of an effective teacher. (Carter & Doyle, 2006). And although, as they further suggested, we have come to see *how* a teacher keeps order is as important as *whether* a teacher keeps order, a preference for students working silently on-task lingers. Sam recalled his first classroom evaluation:

I was applauded by my principal when he came because my class was so well-behaved. I had the old school projector with notes up on it. I was moving the paper down, and kids were silently taking notes. We're going through it like that. That was my first year of teaching.

Implicit in Sam's statement is that this situation was valued and should be replicated, not only for subsequent evaluations, but as an overall management strategy. One of Lortie's (1975) premises is that "occupations shape people" (p. 55). This was true for Sam who retained the style of teaching valued in his first year long into his career. Indeed, his tension with the 'shape' he had been moulded into was the eventual impetus for his decision to change. Even Nadia, a first-year teacher at the time of this study, referred to the importance that was placed on effective classroom management during her studies:

You hear in school, in education, classroom management is the key. If you don't have classroom management under wraps by two weeks, you are in trouble. Like, you need to be strict and only give them a little bit and make them sit and listen.

It is little wonder then that implementing new practices that emphasized student-centred collaboration on open-ended problems created tension for many of my teachers – no longer would the students be sitting quietly working in their seats. In her essay, Kelly likened it to a loss of control:

This [off-task behaviour] is one aspect of the open-ended learning that sometimes deters me from actually doing it. Students who are not self-disciplined enough to make the class time productive make me feel as if I have lost control over the learning environment [...] This makes me think of our discussion in class about the students at the end of the 'hockey stick'. Although one-third or more of the students being off task produces a very long blade to a hockey stick.

Kelly was alluding to a metaphor of a hockey stick that she used to ease her tension around classroom management. The shaft of the stick represented the majority of her students, while the blade represented those few who were likely to cause management

issues.<sup>8</sup> This allowed her to live with tension between maintaining a sense of control and implementing her new practices. And, although Kelly did mention that her hockey stick had “*a very long blade*”, there was again a sense that this tension was managed from an outlier perspective.

Amy also worried about potential misbehaviour arising as she moved away from telling students everything they needed to know to solve a problem before assigning it:

I was nervous to implement it because I wasn't sure that my students were going to get it. I was afraid that they were not going to get it, they were going to be off task, and they were going to misbehave because I feel like if students don't get something that's like the downfall of the classroom management. So, I've been afraid to implement some of the activities because I'm afraid that my students don't have the skills to do them.

Amy had a singular method for managing this particular tension; she discussed her fears about the potential misbehaviour with *her students*:

So, we did it and it went so well. I don't know why, like, they loved it. I told them we're going to do this, I'm nervous about it, and they were so cute though, 'You can do it Miss U'. They were encouraging me and then they loved it. So, maybe the next activities that I'm not really that confident in introducing, I'll tell them, I'll share that with them.

Amy's switch from 'I' in the first paragraph to 'we' in the second paragraph suggests a shift from the sense that she was doing something *to* her students to doing something *with* her students. Her agency in this lies not in her authority as the teacher nor in the task, but in her capacity as an expert who is willing to include her students in her struggle to implement change that improves their mathematics learning. Essentially, she engaged in a dance of agency with her students.

Like the previous tension with parents and colleagues, this suggests getting the students 'on board' with the changes as a way of managing the tension. It was not wholly effective however, and once again there was a sense of the outlier as Amy mentioned that during more recent activities, “*The majority of the students are engaged and learning at their own pace, but unfortunately, despite my best effort to make my lessons as engaging as possible, some students are still off task*”.

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<sup>8</sup> This metaphor was introduced to the teachers by Dr. Liljedahl to help them reconceptualize student behaviour and needs.

This notion that this is an enduring tension was also seen in Sara who had similar struggles, *“I think the honeymoon period has worn off and the initial enthusiasm at doing something new is waning a bit. It's getting more challenging for me to keep my class on task.”* This suggests an ebb and flow to tension, as both teachers and students adjust to the changes in the classroom.

### ***(c) Tension with group management***

While group management can be seen as a subset of classroom management, and indeed I originally coded it as such, the frequency with which my teachers referred to struggles with group management made evident that this tension was distinct from general classroom management. In a Thinking Classroom, students are expected to work collaboratively in visibly random groups, both of which are unusual in a mathematics classroom. For my teachers, this change resulted in tension from competence aspects specific to group management such as group design, group size, and group dynamics.

Some of the tension centered around decisions regarding group design. In her discussion post entry, Lily questioned the optimum number of students for a group:

Today, I went from groups of 2 using vertical surfaces and random groups to groups of 3. I noticed that some of my weaker students struggled. This was not that different from groups of 2 however, now there were sometimes 2 other people in the group that knew what they were doing so the pace was actually faster than it would have been in a group of 2. I discussed this with my husband, [a secondary mathematics teacher], who suggested giving the weak student the pen.<sup>9</sup> However, giving a student a pen does not mean they are learning. Yes, they can write down what is being told to them but if the pace is too fast for them, then they just become a scribe. Is a scribe learning? In a group of 2, then the pace could be slower because there was only one other person to talk to. It can become more intimidating when you are with 2 others that know what they are doing.

Lily's tension with group design was driven by concern for her weaker students. She appeared to want those few students who struggle to both be part of a group and to learn, a balance she has not yet achieved. Despite this being an outlier situation, Lily did not appear willing to live with this tension. She agentically sought help from teachers both in and out of her cohort. Interestingly, this post was one that generated a lengthy discussion that involved the majority of the cohort. They were seeking non-trivial solutions to a seemingly simple question: Should I have two students in a group or three? Suggestions

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<sup>9</sup> In a Thinking Classroom, students in a collaborative working group share one writing implement.



included going back to groups of two and/or frequent rearranging of groups. Others, like Sara, supported the option of giving the weaker student the pen:

I do agree that encouraging the weaker students to the pen can be a good idea. I usually do my best to get the one student in my class that is waaaaaay below grade to be the scribe for her group. Even though she is usually lost and has little to contribute herself, I feel that it's beneficial to have her working with her peers and at the same level as them, and that any information she absorbs (no matter how little) is beneficial to her. It also helps her build some self-esteem knowing that she is contributing to her group in some way. The alternative is that she is working on something separate from the class.

Sara's suggestion appeared aimed towards making living with the tension more palatable. First, she suggested some learning "*no matter how little*" is better than none. Like Kate's tension with student engagement, Sara viewed it as an overall benefit to the student, so she was willing to continue with the practice. Second, she alluded to the general reluctance to have weaker students working separately from their peers (see subsection 5.1.2). This suggests that the tension from pulling low-performing students away from their peers takes priority over tension that results from having them collaborate in groups.

Kate offered a different approach. In her discussion post, she suggested just letting the group dynamics play out and see what happens:

There is always going to be a student or two like this. I'm not saying ignore them (of course not!) but I know that our instinct as teachers is to hover, encourage, and mother these kinds of students a bit. I wonder what would happen if we backed off – would their peers pick them up?

I see this as relying on her past experiences of interacting with students ("*hover over them*") in groups to inform her current situation (back off, instead). This shows Kate developing a practical-evaluative response to agency. Based on what she has previously learned, she will apply it in a new form. Similarly, in a later discussion post, Lily reported back that she had again tried groups of three:

On Tuesday, the group of 3 with the weak student worked much better. This time the student was with more caring and aware students. I informed their group that student A (the weak student) had to hold the pen the entire time. As well, student B and student C were to work at the pace of student A. Student B and C talked one at a time to student A which helped. So, this time it was 2 helping 1 rather than 2 vs 1. It was really lovely to see, and I learned a lot about student B and C through the process. I will try this again with the new random group and see what happens.

Lily appeared to have managed the tension for the moment and she attributed part of the success to the presence of *“caring and aware students”*. I suggest her concern for her weaker student remained as an underlying tension though, as she appeared uncertain that another group dynamic would yield the same success; she hedged by adding she will *“see what happens”*.

Whatever the size, this notion of having students functioning successfully in a group was another area of tension for my teachers. For collaboration to occur, the teachers needed the students to remain with their groups and actually work together. This starts with having students find their groups, which proved difficult, as Leah wrote in her discussion post:

It's painful to watch 15-year olds trying to find their group, especially first thing in the morning. Some of them just wander around holding up their card, but not saying anything.<sup>10</sup>

In response, Kate offered a potential management strategy, one that she was given by another teacher in the cohort:

I was having the same problem watching the 15-year-olds randomly hope that someone would find them. I agree that it's totally painful to watch them try to make their groups. Corey suggested that you put numbers on each piece of board, or place you want them to work, and then their cards match to the numbers. I've tried this with my class, and it is MUCH more efficient and less painful to watch.

This may have alleviated the tension of finding the groups, but another tension arose as some students tried to alter the group makeup by subverting whichever sorting method the teacher used. As Kelly described in her discussion post:

At first, I was not happy when students were switching their cards to be with certain people. I have a feeling some students were bullied to give up their cards. I should have re-sorted as soon as I realized this, but I just let it go. I realize now, as I am reflecting on it, that this switch in cards may have been the reason why I had one group completely shut down and not communicate at all. I had to sit with them the entire time to prompt them into communicating.

Underlying this tension of card switching was Kelly's concern for her students. In hindsight she recognized that in allowing the switching to go unchecked, she may have inadvertently affected the learning of some of her students. It appears that she had been willing to ignore the initial tension of card switching, up until the point that it impacted other students. For

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<sup>10</sup> In a Thinking Classroom, one of the methods for visibly randomizing the collaborative groupings is through the use of randomly distributing numbered cards, with groups comprising members who have the same number.

help in managing the latter tension she reached out in a discussion post. Leah replied with a similar tension around creating the groups:

After being worried about my students trying to switch cards to get a certain partner, I found some students just deciding to work together even though they knew that their cards didn't place them in the same group. They didn't even see the need to try to get the same cards. I had to hand put a couple of students into their proper groups.

Again, there appears to be initial tension (with card-switching) that is impacted by further tension (outright ignoring the cards). While it is unclear how Leah dealt with the initial tension, her response to the second is clear; she personally intervened and moved students to their assigned groups. This is a practical-evaluative response driven by Leah's sense of purpose. She believed having the students work collaboratively was an outcome worth pursuing.

Student behaviour in a group was a concern for other teachers as well. In a discussion post, Lily wrote that she has had to resort to removing a student from a group over concerns about behaviour towards peers: *"At times I have removed a student from a group and have them work at their desk with a pencil and paper which they seem to not like"*. Likewise, throughout most of the teachers' responses there appeared to be zero-tolerance for behaviour that negatively impacted other students. Their desire for student safety and well-being outweighed their desire for student collaboration. As Kelly mentioned:

Any time any students are ever negative about someone else, the first time they're like, "I'm not working with them", I kick them out of the room right away. They realize that's not acceptable; they can't be saying that.

For teachers like Lily and Kelly, removing the students who are the source of the group dysfunction was a way to manage the tension that resulted when group collaboration conflicted with student safety. Considering that many of the teachers earlier expressed a general reluctance to remove students from collaborative activities, (see 'low-performing students'), I suggest this removal would be the source of further tension as the teachers struggle with their desire to have every student participating in a group.

Other kinds of behaviour also introduced tension into implementing group work. Sara described students *"wandering away"* from their assigned groups. In a discussion post, she described her effort to counter this by modeling what group work might look like:

I still had kids wandering away from their groups, so at the end of the lesson I demonstrated with two kids what a productive group looked like, and what an unfocused group member looked like. In the middle of our demo problem, while my group members were trying to explain a solution to me, I wandered over to another student and started talking to them about basketball. The class literally burst out laughing because they realized how ridiculous it was. We also talked about what a group could do when they think they're done (see what others are doing, extend the problem, think about what I might ask them next).

Like Amy's earlier tension with classroom management, Sara was involved in a dance of agency as she worked towards getting all the students on board as a means of alleviating tension with group management. It is possible she saw the "*wandering away*" as resulting from a lack of awareness of what collaboration entails rather than from deliberate attempts to be disruptive and so she modelled her expectations. Other teachers also mentioned situations where students' (mis)behaviour in groups was understandable, as Leah described:

So, I've got a couple different situations. I've got one student in the first block who does have some legitimate concerns with anxiety and we're working on it. And it sometimes means that she'll sit at the table on her own for a minute before she'll get up and join the group. Or, maybe it means that she'll kind of stand away from the group and not be too involved. It's a work in progress. I have a couple of students who I know aren't super comfortable because of abilities so that's still, again, that's a work in progress. Some of them will come in early for block A and I'll give them some of the notes ahead of time. So, I have been able to do that with some of them. My students who aren't all that interested, I still need more advice for how to handle some of those situations.

Here again, it appears the need to balance students' well-being with the implementation of collaborative group work created tension that Leah, and other teachers, were trying to manage. This time, however, it was the safety and well-being of individual students that was being affected, not that of the rest of the class. Leah's agentic response was to incorporate several methods to reduce the tension, yet her addition of "*that's a work in progress*" suggests it is an ongoing tension she continued to try actively to manage. Students who are disengaged seem to be the main source for her tension and she does not appear to have any methods to deal with it. Yet, despite the tension involved in collaborative groupings, neither Leah nor any of the other teachers suggested giving up on group work as a potential solution. In a discussion post from Leah, there can be seen a potential reason for this:

Although I do find it difficult to manage 10 groups, I can't imagine it would have been any easier to manage them as 30 individuals doing seat work.

Leah saw it as exchanging one tension for another, which I see as an agentic response. While developing methods for overcoming student resistance in general are important, most of my teachers appeared aware that not all forms of resistance should necessarily be overcome. Brodie (2009) suggested that such resistance can be a sign of healthy interaction in a classroom, albeit uncomfortable for teachers trying to implement change. Accordingly, my teachers agentially sought ways to accommodate the needs of individual reluctant students, while maintaining their own sense of purpose and autonomy.

#### ***(d) Tension with homework***

Tension with homework preceded the changes my teachers had implemented. Most mentioned common issues such as unfinished homework, how to grade homework, and volume of homework assigned. However, with the changes being implemented, new tension arose for those teachers determined to stop assigning homework and for one teacher who grappled with wanting to move away from assigning homework at all, yet still valued homework as a method for tracking individual student understanding.

Those who made the shift to little or no homework appeared to do so because it better suited the kind of classroom they were trying to maintain and, more importantly, helped address tension around student engagement. As Kelly explained:

They were really excited when I told them today that it was not my intent to assign any homework for the course. So, if they at least showed up every day and participated and tried their best they should have no trouble being successful in the course.

This is a variation of 'getting on board' that the teachers had found successful in managing tension in other instances too. Kelly was willing to sacrifice homework to secure students' engagement with mathematics. Similarly, to Sara managing her tension with group dynamics, this prioritizing suggests agentic competence as Kelly appeared aware of her ability to influence the outcome she desired. This prioritizing was also apparent from Alison:

I feel like I've moved away from everything that I don't want to do any more. Like, I used to do notes and textbook, not all the time, but quite a bit, so I feel like I've totally taken that away. And I used to do homework checks and I've totally not done that anymore. I feel like I haven't stuck with anything that I didn't feel valuable.

Her rationale has a reflexive aspect, as she has thought through her mathematics pedagogy and decided what is most important. This is reminiscent of Mason's (1988) view

of time as a commodity in a classroom. Her strong sense of agency in this regard was apparent, as Alison had decided that some of the things she had been spending time on were not “valuable” and so she “moved away” from them. This likely increased the time she had to spend on those aspects of pedagogical change she was trying to instil.

For the few teachers who did assign homework, most offered the rationale that it was for practice. For example, Lily mentioned, *“I do believe kids need to practice. So, I give homework, but the homework is more like the minimum that I want them to learn.”* Only one teacher mentioned assigning homework with the intention of evaluating student understanding. Nadia was a first-year secondary teacher who worried about her ability to assess individual student’s understanding during the collaborative group work she was implementing. She stated in her first interview, *“But in math, how do I know if they got this or not? So, I assign homework.”* This created tension for her, as she realized that the information gleaned from her students’ homework was not timely. This was apparent during her second interview when asked about homework: *“But I need to know at the end of the lesson. I don't want to know three days later when the homework's due. So that's another thing that I'm working on.”* She was actively looking for a way to ease this tension, as she had begun to accept that homework’s main use may be as a mark to justify grades to parents. In her final interview near the end of her first year, Nadia mentioned:

Not all of them do it on a regular. I give them a mark out of 10, but the 10 is, is it done or is it not done, right? And it's in PowerSchool, but it's not included in their grade. It's just so if a mom comes in and asks, "Why is so-and-so failing?" And we have 10 homework assignments and she did two of them, that's why. That's part of it. So, it's just for me to be able to track, and parents to be able to track, are you trying on your own? Are you making an effort?

There is a strong iterational sense to her developing agency regarding the use and effectiveness of homework. Although new to teaching, she was building on her previous experiences and trying to find a way to ease the tension that stemmed from her desire to check her students’ understanding.

**Table 5.2 Summary of Internal Tension in Mathematics Teacher Change**

	Elements	Emotions	Strategies for managing tension
Internal tension: Intrapersonal tension			
Intrapersonal tension that motivated change			
Teaching style	Who I am as a teacher and who I want to be	Boring Unhappy Transformed Determined	Professional development Anti-goal
Intrapersonal tension that resulted from change			
Pull of previous practice	Being unprepared Seeking reassurance	Dissatisfied Uncertain	Endure, as it rarely happens Anti-goal
Self-doubt	Am I doing 'it' right? How will the changes impact my students' future?	Doubt Anxiety Uncertain Worry	Endure and learn with time
Physical limitations	Classroom mobility	Struggling Worry Concern	Motorized scooter "mixing things up"
Internal tension: Interpersonal tension			
Parents' critique of practice	Not teaching curriculum, playing games, not enough homework, better grades	Frustration Uncomfortable Concern Responsive	Supplemental homework Worksheets Verbal reassurances Being proactive (pictures, frequent progress reports, frequent contact, inviting parents to class) Get parents on board
Parents' critique of character	Unprofessional (slacking), disliked, inexperienced	Determined Judged Traumatized Afraid	Outlier Don't take it personally Get parents on board
Student engagement	Compliance	Tiring Supportive Determined	Wait them out Outlier Allow some leeway for certain students
Student ability	High- and low-performers	Surprised Worried Uncertain Determined	Outlier Worth the outcome Wait them out Pull-out students Convince high performers

	Elements	Emotions	Strategies for managing tension
Sharing ideas with colleagues	Lack of willing collaborators Lack of respect	Frustrated Ignored Perseverance Undermined	'Nice' her to death Convince colleagues Get colleagues on board
Sharing space with colleagues	Noise	Respectful	Compromise
Internal tension: Pedagogical tension			
Assessment	How to assess collaborative learning	Unsure Struggling Hard Regret	Occasionally reverting to traditional methods Alternative methods (group tests, exit slips, anecdotal notes, checklists)
Classroom management	Off-task behaviour	Loss of control Nervous Challenged	Outlier Share concerns with students Get students on board
Group management	Group design Group size Group dynamics	Uncertain Concern Perseverance Painful Worry	Outlier Ignore the behaviour Pass the pen Groups of two Frequent rearranging of groups Remove student from group Model appropriate behaviour Prep certain students Allow some leeway for certain students Get students on board
Homework	Wanting to move away from homework, yet still finding it valuable	Satisfied Worry Uncertainty	Onboarding Endure

## 5.2. External tension: Tension from without

External tension arises as teachers encounter policies or expectations imposed on their practice by outside forces such as district policies or government mandates. Some of the imposed mandates my teachers encountered conflicted with the changes they were trying to implement in their classrooms. This was particularly noticeable in five areas: assessment and reporting; curriculum; institutional norms; preparing for substitute teachers; administration.



### 5.2.1. Tension with assessment and reporting

It was necessary to distinguish what I came to see as personal tension with assessment from systemic tension with assessment and reporting out. The former involves a struggle between the teacher and her own ideas around assessment while the latter involves a struggle between the teacher and imposed practices such as standardized testing or report cards.

While all assessment is imposed in some manner or another, my teachers were trying to match their own classroom assessments to the changes they had been implementing and the systemic assessment was at odds with this. Most felt that the systemic assessments were not keeping up with their changes in teaching. For example, Sara worried about how her students would fare on their up-coming provincial Foundational Skills Assessment (FSA) as she wrote in her essay:

I have found that it is difficult for students to transition from progressive learning and instruction to standard tests such as the FSAs. They are entirely unfamiliar with the type of testing scenario that occurs suddenly in the middle of the year. As curricula and teachers move from the traditional to the progressive styles, so must assessment.

Although Sara had no agency in the creation of the FSA, nor in choosing to deliver it, she did mention strategies she had incorporated to help her students, such as test-taking practice and targeted review. Although this is demonstrative of good teaching practice in general, it also suggests that tension can drive teachers to seek agency even in environments where agency is limited or non-existent.

This was especially noticeable around the issue of reporting out, where there was tension for many of my teachers. Most had moved towards assessing their students formatively and sharing that assessment with them regularly. It led them to question the need for reporting out, as the students (and teachers) already had a good understanding of the students' abilities, as Alison explained:

We were talking about report cards the other day and I'm kind of like, why do we have report cards anymore? I upload the kids marks on their assignments and projects. I feel like it's very transparent as to what the student is getting in which outcomes and they can check that online at any time. So, I feel that the report card is kind of redundant because they should know where they are.

This is interesting, as Alison is transferring the agency of the static report card to the dynamic online reporting system, which allows for an ongoing view of the students' learning that she prefers. Inherent in this is that the students are also involved, as they choose (or not) to engage with this new assessment. More importantly, this would impact other stakeholders, which Alison referred to when asked if she felt report cards would be made redundant in her lifetime:

No, I don't think it will happen. I think that parents are too stuck. I think they're still not even aware of what assessment is yet, so I feel like they're not going to get to the stage of not wanting a report card.

This is tension that Alison will continue to live with, as her projective agency is impacted by her lack of autonomy in this particular context. Her reference to parents as being not just stuck but *"too stuck"* suggests that, unlike with previous parental tension, which was managed by "getting parents on board", she cannot envision a future without traditional report cards.

For others, the impact of formative assessment on report cards created tension around what actually to report. Most felt that their new practices had provided them with a better holistic understanding of their students' performance, but scant practical evidence, as Amy wrote in a discussion post, *"I don't have very much in terms of "evidence", but I have a perfect idea of how each of my students are doing"*. She further wrote about the requirement for evidence leading teachers to *"pad their markbook"*, which suggested teachers translate the *"perfect idea"* of how a student is doing into a grade. This created tension in preparing the report card, as my teachers worried about having to justify the marks they reported. Kate wrote about this in her discussion post:

I feel like I know where everyone is at (somewhat), and I could write their reports, but am a bit worried if someone questions me or asks me to justify what I write. I don't have anything on paper!!

Kate's addition of the qualifier *"somewhat"* suggests the tension is not limited to the reporting, but may, for her, also include the assessment itself. Her response also reveals a limited sense of agency, as she feels she has to answer to *"someone"*. Like Alison, Kate's autonomy is diminished in the context of reporting out.

### 5.2.2. Tension with curriculum

What to teach has been the mainstay of education since traditional schooling evolved. Generally decided upon by those outside the influence of the classroom teacher, curriculum has been criticized because of its lack of connection to *how* to teach. Although recent curriculum updates have attempted to address this disconnect, tension still arises as teachers worry about covering its content in a thorough and timely manner. For example, Corey wrote of this tension in her essay, *“I think we all feel this pressure in the classroom – it seems there simply aren’t enough hours in the day to get through all of the curriculum”*. As the curriculum itself is outside her realm of control, Corey’s autonomy is affected as she doubts her ability to accomplish her professional task of teaching it.

The constraints of covering curriculum are particularly noticeable when trying to implement change. For my teachers, one of the changes they had been implementing in their mathematics teaching practice was privileging process rather than product. This necessarily takes more time as the focus moves to learning curriculum rather than teaching curriculum. Kate said that felt confident she had made this shift:

I am no longer concerned with ‘getting through everything’ and moving on to a new concept, even if many of my students are struggling with something. Building understanding has become more important than covering it all.

I suggest her use of the phrase *“I am no longer concerned”* does not indicate that this has been an easy shift; rather, it simply indicates that there has been one. She had decided to let go of the need for *“getting through everything”* and to focus instead on *“building understanding”*. Indeed, she later referenced this in a discussion post about the challenges of covering particular curriculum:

I’m feeling very successful with all this so far. Mind you, I’ve only been doing the fun math up until now. Next week, I plan to try and get back to my measurement conversion unit. Without being able to use a textbook, or any practice sheets (not that I relied much on them), it’s going to be challenging for this particular unit.

Her addition of *“mind you”*, along with its qualifier that *“I’ve only been doing the fun math up until now”*, indicates awareness of the difficulty in building understanding in all content. This willingness to implement change that affected their ability to cover the curriculum was evident in many of my teachers. Kelly mentioned her own struggle:

I haven’t figured out how to teach all the curriculum in this way yet. I’m just making what can work, work right now in the capacity that I have. I’ve realized I can’t bite

everything off. I can't do everything at once. But if I can make little changes every year, like, so I've been doing this vertical learning for at least the last three years now and I've taught bits and pieces of pre calc eleven with it.

This is tension that Kelly was willing to live with as she built her own capacity and competence. It had an impact on how she implemented change, as she felt it was necessary to “*make little changes*”. Although this speaks to her own autonomy in controlling the decisions around the changes, it also suggests that some of the changes she is making may only rise to the level of instrumental or conceptual change.

### 5.2.3. Tension with institutional norms

Social and sociomathematical norms influence classroom practice, with the former affecting normative interactions in the classroom in general, while the latter affects normative understandings that are specifically related to the teaching and learning of mathematics (Yackel & Cobb, 1996). Beyond the classroom, however, lay institutional norms, which also influence mathematics teaching practice. These norms refer to the expectations of behaviour that are acceptable within an institution (Liu & Liljedahl, 2012). They “take the form of cultural theories, ideologies, and prescriptions about how society works or should work” (Meyer, Boli, & Thomas, 1987, p. 9). Institutional norms generally lie outside teachers’ control and tension can arise when these norms interfere with changes in classroom practice.

Leah addressed this in her essay:

I think that many Canadians hold a similar idea about what a stereotypically ‘good’ classroom looks like. I have teachers and administrators come into my classroom during lessons and the louder and messier the group activity appeared to be, the more I felt that I needed to justify what my students and I were doing. Perhaps this is more about my own insecurity regarding what others are thinking; but either way, I think it brings up the point that there is a societal norm that needs to be challenged.

Leah’s mention of her “*own insecurity*” suggests this tension affects her competence. She responded to tension by feeling the need to justify her practice, which suggests a limited autonomy but, in pointing out the possibility that it is norms that are getting in the way, she appeared intent on reasserting it. She does not mention who is responsible for challenging the norms though, and inherent in the vagueness is a limited sense of agency. It is unclear if she realizes that she is one those already challenging the norms through the changes in her practice.

Also lying outside my teachers' control were the supplemental resources that were and were not allowed to be utilized in the classroom. Interestingly, it was the same resource – textbooks – that caused the tension. Alison wrote in her essay of tension she experienced around her mathematics department's insistence that all her school's mathematics teachers use the same textbook and workbooks:

Overall, the mathematics department insisted you use the textbook and have students work in notebooks provided by the school. I had very little freedom to plan lessons differently, but sometimes I would try a student-centered activity, typically with good response.

Her mention of *"little freedom to plan"* clearly indicates that this tension interfered with her autonomy as it brings to mind Carr's (1998) metaphor of tension as play. Alison's 'play' was being restricted and she was feeling powerless. She lived with the tension by occasionally trying out different activities. As this mention of imposed resources applied to earlier in her career, during her interview, Alison was asked to further clarify her current textbook usage. She said that her department continued to mandate the resources and added:

I feel like I have continued traditionally with implementations of new ideas as well. I haven't, like it's not that I don't bring out the textbook, because I still do but I feel like generally to implement a variation of different things is always a success like textbook questions, some notes, some problem solving, some projects, like just a variation.

This juxtaposition of introducing new ideas via traditional methods suggests that Alison is in the process of instrumental or conceptual change. One conjecture for this might be that the limited agency she experienced in her choice of resource has affected her ability to achieve foundational change. Another might be that she is simply not ready. Either way, it suggests a practical-evaluative agency shaped by what Alison believes is possible and what is actually possible in her particular circumstance. That she is experiencing tension around this is evident. When asked why she is trying to change her practice, she paused and replied, *"Because kids enjoy it more"*. While enjoyment might not affect their learning of mathematics, it does affect their engagement. Her pause and hesitation in replying suggests she recognized that the limited change she is implementing had a worthwhile effect on her students', even though she has not fully embraced foundational change.

David experienced the opposite effect of mandated resources. For new teachers, textbooks and their accompanying guides can be useful in that they offer established

concept timelines and guidance on how to proceed through the curriculum. New teachers also count on the established norms that textbooks bring to the classroom. This was true for David who mentioned that he had started out his first year by handing out textbooks:

I have this stigma, like, I get a vibe from the school district which is kind of going away from textbooks. So, at the beginning of the year, I almost felt bad using a textbook. I tried not to even look at it.

Although David switched to the word “*vibe*” midway through, his initial choice of the harsher word “*stigma*” suggests the impact of his district’s decision on his autonomy was quite severe. His choice whether or not to use the textbook was not only removed from him, even referring to it caused him to feel badly. This created tension for David in trying to change how he taught mathematics. He saw potential in the textbook:

I just find that even in the textbooks there are some decent problems and I was like, “Okay. I’m going to put them in random groups. I’ll give them their white board or whatever.” But they work on those problems really well. So, I’ve been doing that a lot more too, which I find is not only helping the group part, but it’s helping with getting through curriculum.

His earlier reference to “*at the beginning of the year*” suggests a developing sense of agency, as David continued to live with the tension. He decided the textbooks do serve a purpose in his mathematics classroom and he eventually incorporated their use into his new practices. Although there is tension around their usage, his pinpointing of two areas where textbooks are “*helping*” suggest he has found an acceptable rationale for their use as a way to live with the tension.

#### **5.2.4. Tension with preparing for substitute teachers**

Preparing for substitute teachers<sup>11</sup> (or TOC – teacher on call) can be a cause of tension, as it requires myriad decisions such as what tasks to prepare and how to ensure support for particular students. In traditional classrooms, classroom social norms actually help ease the tension, as substitute teachers are expected to already have an idea of how

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<sup>11</sup> Although substitute teachers are also colleagues, tension with them would not be considered an internal, interpersonal tension. There is no sense of relationship, as teachers rarely meet the substitute teachers who come into their classrooms. They also generally have little or no choice in who the substitute is. This is a decision external to the teacher, as substitute teachers are assigned to cover classrooms by administration.

mathematics classrooms are run. This is not the case in classrooms where there have been changes in practice that disrupt the norms, as Corey wrote in a discussion post:

I will say that a difficulty I had with being away was trying to plan for a TOC to teach the same way I've been teaching with my class. I felt it required so much explanation that it wasn't a great idea to do it while I was gone out of my class. Has anyone had to have a TOC in their class and given them directions to teach this way yet? I just didn't want to force it onto a TOC who might not be comfortable.

In not expecting the substitute teacher to teach in a manner that *“might not be comfortable”*, Corey’s response indicated a potential strategy for managing her tension, lower her expectations. That she reached out to her peers for other solutions suggests that the tension was actually ongoing. Indeed, it was the volume of replies describing similar tension that made me decide that tension with substitute teachers warranted its own section.

Kate posted a response that she had defaulted to worksheets for her substitute teacher:

I was really struggling with what to leave for the TOC. If I had been there, I planned to have the kids generate their own methods for finding all of these things, in vertically random groups, and then getting them to generate the formulas, if they could. However, since I wasn't there, I decided to leave some practice worksheets that reviewed the area and perimeter piece for the 2D shapes. I felt totally guilty about doing this!!!

Like Corey, Kate’s attempt to manage the tension by not expecting the substitute teacher to adopt her changes in practice resulted in further tension. There is a wavering sense of autonomy here, as Kate’s initial decision is accompanied by guilt. This is interesting as, even though her careful listing of all the things she would have done had she been there offered a plausible rationale for her decision to leave worksheets, she still felt guilty. This speaks to a strong sense of purpose in implementing change, as she was motivated to want to enact the changes even when she was not physically present.

Sara did try having a substitute teacher incorporate the changes she had been making. She wrote in her discussion post:

It turned out fine. The only glitch was that when I got back to the school for the afternoon, I saw that she had written both the problem and the extensions on the board for them to see. Not the end of world, especially considering it was a TOC.

Sara’s use of the phrase *“it turned out fine”* suggests she had initially experienced tension in that she had worried that it would not be fine at all. She decided to do it anyway and

offered two reasons why she was willing. First was because she felt confident her students were capable of managing the change themselves: *"I know that my class knew what they should be doing at this point, so I told her to just ask them if she had any questions"*. This suggests that Sara was able to transfer agency from herself to her students for the new practices in her classroom. The second reason was that Sara did have autonomy over the choice of her replacement: *"She's a retired teacher that I know is incredibly competent, knows the kids, and is up for trying new things"*. This likely made Sara more willing to expect the substitute teacher to incorporate the changes. That this expectation was slightly lowered though can be seen in Sara's ending phrase, *"especially considering it was a TOC"*.

### 5.2.5. Tension with administration

While tension with substitute teachers was notable for its frequent appearance in the data, tension with administration was notable because of its *infrequent* appearance. Only one teacher, Corey, mentioned feeling unsupported in the changes she was trying to make. Most of my teachers felt the opposite. For example, Alison described her administration as, *"very supportive and very open to trying anything or doing anything and if I have an idea, they say go for it"*. Still, while support from administration was not a strong source of tension for my teachers, Corey's tension with her administration was noteworthy, as it was the first tension experienced by any of my teachers that effectively halted the changes being implemented.

Corey explained the effect on her and her grade partners:

The problem happened when our admin changed. We got told you can't teach math that way and we were like why? The reasoning they gave was that students were doing too much 'self-teaching' and that we as teachers were not teaching math anymore. We weren't told much else except that we needed to teach differently. Myself, and a few of the other teachers, were wondering whether it had been parental complaints that had been the reason for it, but none of us ever found out the actual truth behind it. So, when that happened, we were all really angry. I tried to stand my ground with a couple of other teachers that were doing it. I felt our method of teaching mathematics was shut down in this way, simply because it wasn't how things had been done in the past.

Corey has no agency in this situation. Her attempts to find out the rationale behind the decision suggest an attempt at regaining some limited autonomy. Perhaps if she had found the rationale palatable, she would have acquiesced in the demands without



resorting to anger? Her reference to *“it wasn’t how things had been done in the past”* shows awareness that she had disrupted norms.

**Table 5.3 Summary of External Tension in Mathematics Teacher Change**

External tension			
	Elements	Emotions	Strategies for managing tension
Assessment and reporting	Mismatch between imposed assessments and teaching style Questioning the need for report cards and how to provide evidence for grades	Worry Frustration	Teach test-taking strategies Padding gradebooks
Curriculum	Covering the curriculum Time spent on process vs product	Pressure Doubt Worry Confident	Make small changes Justify actions
Institutional norms	What makes a good classroom? Use of textbooks	Insecure Limited Bad	Use the textbook under certain conditions
Preparing for substitute teachers	How to explain new classroom norms to an outsider	Unsure Guilty Confident	Worksheets Give students responsibility Lower expectations
Administration	Told how to teach	Frustration Confusion Anger	Discontinue change in practice

### 5.3. Summary and conclusions

Tension that my teachers experienced while attempting to change their mathematics teaching practice was readily identifiable in the data. While not every teacher experienced every tension, what I have provided is a general overview of the kinds of tension experienced in change in mathematics teaching practice. I also do not presume to have identified every kind of tension; these are the instances of tension that I noted.

In all I was able to identify and elucidate fourteen instances of internal tension and five instances of external tension. Internal tension was referred to as an “an argument with oneself” (Lampert, 1985, p. 182) and its genesis lies within the teacher. It was further

subdivided into intrapersonal tension (where the conflict is between self and self); interpersonal tension (where the conflict is between self and others); pedagogical tension (where tension occurs between self and aspects of practice). I identified four instances of intrapersonal tension that motivated the change in practice, or resulted in tension with self-doubt, the pull of previous practice, or physical disabilities. Interpersonal tension accounted for six instances of tension equally distributed among tension with parents, students, and colleagues. Pedagogical tension was the final category of internal tension and it accounted for the remaining four instances: with assessment, classroom management, group management, and homework. The five instances of external tension captured tension between my teachers and forces outside their control such as tension with assessment and reporting; curriculum; institutional norms; preparing for substitute teachers; administration.

Although the mathematics educator in me wanted to find clear-cut patterns and connections between agency and tension, such as all external tension correlates with limited agency, this proved not to be the case. Still there were several findings worth noting. First of all, how my teachers managed tension was clearly agentic. Although I was not always privy to *all* the options they had considered in reaching a decision, *that there were* options and *that they made* agentic choices from those options was clear. Second, there were fluctuations in their agency, which resulted in individuals showing both high and limited agency depending on the context. This fits with the ecological agency of Priestley and colleagues in which agency is not considered an innate quality of an individual; rather, it is something the individual achieves (or not) through interplay with their environment. For example, Leah's managing of tension around student groupings (see subsection 5.1.2) was highly agentic as she physically moved and regrouped her students, while her insecurity with not meeting institutional norms provided evidence of limited agency (see subsection 5.2.3).

I now turn to two specific findings that emerged from the analysis. I begin by exploring strategies for managing tension in change and highlight two particular ways in which my teachers managed tension: resolving and living with tension. I follow that with a closer look at the tension my teachers experienced with not wanting to be 'that teacher', using Skemp's (1986) account of anti-goals as a lens.

### 5.3.1. Strategies for managing tension in change

Teachers as tension managers was an apt description of what I noticed as my teachers dealt with experiences of tension. Although doubt and uncertainty appeared to be constant companions to the changes they enacted, a number of strategies they used to manage tension could be discerned. Some examples include ameliorating tension with parents by providing supplemental homework sheets and verbal reassurances; occasionally reverting to aspects of traditional practice to manage external tension with assessment; managing tension with student ability by allowing leeway for particular students.

What was also interesting was the discourse they used to describe those strategies. It emerged as a source of praxis that Adler (1998) described as useful in talking about mathematics teaching practice. For example, my teachers frequently spoke of “outliers” and “onboarding”. The usage of these terms appeared to be methods my teachers used to confront and work with tension they were experiencing. This also led me to consider the possibility of nuances to managing tension as I found myself using phrases like ‘resolve’ and ‘live with’ to distinguish the strategies they mentioned. While the term ‘manage’ broadly described my teachers’ response to tension, it was apparent from the data that there were variants in how they managed their tension.

#### ***Resolving tension: Onboarding***

Resolving was a form of managing tension in which my teachers actively chose to address tension they were experiencing. This was not a solution, more of an alleviation of some aspect of tension that was within their power to address. With its focus on choosing from options, I suggest that resolving tension is an agentic action. Although the resolution may only offer temporary respite from tension, it is the result of the teachers choosing to do *something*. For example, when Amy experienced tension with her “*boring*” mathematics practice, she actively sought out professional development in order to resolve it.

While resolving tension is similar to what Lampert (1985) more broadly described as managing tension, I suggest the difference lies in its intent. For Lampert, managing tension required choosing an option – to put the boys at the front of the class – that she knew was a short-term solution as she recognized this was “a temporary respite that would prevent the underlying conflicts from erupting into more serious, distracting discord” (p. 185). I argue that Amy was not looking to manage her tension so that she was temporarily not

boring. Rather, teachers like Amy, who are looking to resolve tension, are hoping that the option they choose might eventually result in an abatement of the tension. While both managing and resolving tension might result in temporary relief, only in the latter is there hope of a more permanent diminishment.

For my teachers, the strategy most utilized in resolving tension in change was 'onboarding'. It required a strong iterational and practical-evaluative sense of agency as the teachers used their past and current experiences to anticipate the needs of their students, parents, colleagues, and administration. For example, onboarding as a resolution strategy appeared frequently in tension with parents. My teachers spoke of using methods such as frequent parent contact as a way *"to get all the parents on board"* with the changes they were making. Once they could convince the parents of the merits of the changes they were implementing, my teachers' tension decreased.

I came to view resolving as a proactive approach to minimizing tension, as the teachers anticipated those areas which experience tells them it may result in. That does not mean they were able to avoid tension, rather that they temporarily found a way to keep it at bay and, in so doing, kept change alive in their mathematics classrooms. Each new way they found added to their repertoire of tools, which could then be used to resolve recurring tension. This desire to resolve tension required constant innovation, reminiscent of Cuban's (1992) metaphor in which teachers are learning to create more "elegant tightropes". In working to resolve tension, teachers are developing skill to create more elegant ways of dealing with it.

### ***Living with tension: Outlier***

Living with tension was a form of managing it where there was little or no action taken by my teachers to resolve it. For example, Kate questioned her ability to respond flexibly in the moment *"with questions or interventions that move students forward"*, as she was unsure whether she possessed *"a strong enough repertoire of open-ended knowledge"*. This was an intrapersonal tension that she appeared willing to live with, as she did not refer to any strategies for its resolution.

Although Pickering (1995) suggested passivity is the antithesis of human agency, I argue that living with tension can be an agentic response. For example, both Amy and Alison experienced tension with parents critiquing their character. Rather than take it personally,

they engaged in a dance of agency, where they allowed for the possibility that the critique was more a dislike of mathematics than themselves. Their 'choice' of action was to ignore the behaviour. Likewise, my teachers' willingness to live with tension from their high-performing students dislike of the new practices being implemented was also an agentic response. There was a strong sense of purpose that the changes were worthwhile, so they 'chose' to do nothing about tension.

I suggest living with tension is a necessary agentic response that also helps keep change alive. Had the teachers felt compelled to manage their tension, it might have been detrimental to the changes they were implementing. This was noticeable for those teachers who experienced tension from the pull of previous practice. To manage tension, they moved away from the changes they had been implementing. Fortunately, this was a temporary withdrawal, and they resumed the changes and continued to live with tension. Had they worked to manage the tension; it may have been to the detriment of the changes.

The strategy my teachers most often used in living with tension was the notion of outlier. This was a form of compartmentalization where the source of tension was considered an anomaly, an 'outlier' in their words. Whether it was a parent, a colleague, or a student, my teachers saw the source as a small, negative part of a larger, positive whole. Although this meant that the tension remained, this outlier perspective allowed them to live with the tension and carry on with the changes they were implementing. I suggest those teachers who are successful with change despite the myriad tension they experience are those who are able to position the source of tension as an outlier. For them, tension with a student does not magnify into tension with *all* students, quite the opposite, actually.

### **5.3.2. Not wanting to be 'that teacher'**

In Chapter 2, I wrote of the dialectical connection between tension and change where tension is necessary for change to occur. This is a productive view in that tension is seen as capable of producing something – the “potent sources of energy” as described by Mason (1988, p. 164). Change itself, can create further tension as both intrapersonal and interpersonal forms arise from conflicts in change. This was true for many of my teachers for whom the changes they made led to new tension with, for example, students for whom the changes altered traditional mathematical norms.

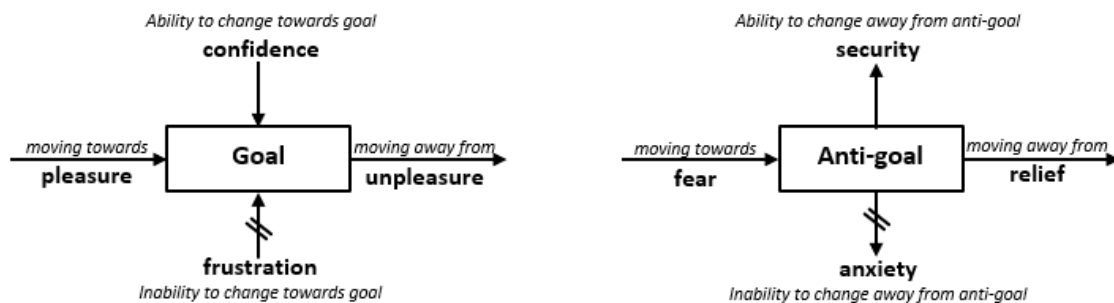
It was apparent that some of the changes my teachers implemented made visible what was previously invisible, which resulted in new tension arising. For example, asking her students to work together collaboratively made Kate very aware of the disengagement of some of her students as she explained, *"They stand out more than when they did seatwork"*. This previously invisible behaviour was now very visible, and Kate felt compelled to do something about it. Similarly, the changes Lily implemented made obvious the struggles of her low-performing students: *"it's just more obvious that I know they are flailing. There's no doubt. I don't think it was just as in my face before."* It is not that my teachers were completely oblivious to the behaviour prior to the changes; rather, now the tension that the behavior created made it difficult to ignore. This is tension in the productive sense, as my teachers searched for ways to ease it, which in turn may lead to more change.

Tsing (2005) conceived of productive tension as friction – the contact of a wheel with the surface of the road or the rubbing two sticks together – which can help explain how tension and change work to make invisible behaviour visible. Prior to the change, teachers like Kate and Lily only had one stick and it did not rub up against anything. Struggling or disengaged students are easy to ignore when everyone is working individually at their desks. Requiring all students to work collaboratively at vertical surfaces rendered it visible. This change introduced a new stick – something for that lone one to rub up against – and it created tension. Additionally, there was a sense that my teachers were surprised when the two sticks created tension. Lily described it best when she commented that, *"it's so much more in my face now that I have to go 'ooh'"*. My teachers were not expecting that the change might be revelatory.

On the other hand, there were occasions when my teachers deliberately sought out another stick to rub up against. Implementing change is challenging and, occasionally, my teachers found themselves moving towards that which was once familiar and therefore seen as easier or better. Most of my teachers were motivated to change their mathematics teaching practice because of an intrapersonal tension between who they were as a teacher and who they wanted to be. Having been taught in traditional mathematics classrooms most now eschewed that teaching style and were attempting to incorporate a more student-centred classroom instead. This was made apparent by the frequency with which the phrase 'that teacher' appeared in the data. It seemed that my teachers had a well-developed schema of who and how they did *not* want to be in the classroom. Who

they wanted to be was their goal; who they did not want to be (that teacher) became an anti-goal.

Skemp (1979) used goals and anti-goals as a means of making sense of human actions. His framework was built on fundamental ideas in psychology and linked emotions to goals which a learner may wish to achieve, and also to anti-goals which a learner wishes to avoid (see Figure 5.1). For example, a teacher might have a goal of fostering student discourse and an anti-goal of teaching as telling. When she moves towards her goal, she experiences pleasure and when she moves away from her anti-goal, she feels relieved. Skemp emphasized that goals and anti-goals were not simply opposite states; rather, a goal was something that increased the likelihood of success, while an anti-goal was something to be avoided along the way.



**Figure 5.1 Emotions associated with goal states (adapted from Skemp, 1979)**

Interestingly, what my data showed is that rather than avoiding anti-goals, my teachers were sometimes drawn to them; they sought out the stick to rub against. This was apparent in Lily and Kelly who would revert to traditional teaching when they had a bad day or felt unprepared. Similarly, Leah and Diane returned to traditional teaching for the reassurance that it provided. With traditional teaching, they felt certain they were covering the content and that students were learning. When speaking later of year-end assessments, Diane added, *“I know I don’t need to do it [teach traditionally]. I know I shouldn’t. They all did so well that it solidified for me that the way I was doing it was already working.”* This suggests that anti-goals serve another purpose. Teachers might purposefully move towards an anti-goal in order to experience the relief it brings when they move away. In essence, they are reconnecting with their anti-goal in order to affirm the changes they are making in their mathematics teaching practice.

I suggest that reconnecting with anti-goals might be useful for two reasons. First, recognizing what one does not want to be brings into sharper relief what one does want. Having that clarity might enable teachers to seek out actions and changes that will help them reach that goal. For example, a teacher who realizes she does not want to be ‘that teacher’ who only uses unit tests for assessment may look for learning opportunities to broaden her assessment practice. Finally, anti-goals also prove useful in keeping change alive. Teachers who find themselves pulling back from the changes they have implemented, find in the emotional reconnection with their anti-goal the encouragement or reinforcement needed to continue with the change.

In this chapter, I identified and described tension my teachers experienced in change, along with exploring ways they managed tension. In the next chapter, I take a closer look at the experience of Nicolas, a middle school teacher, as he implements change in his mathematics teaching practice during a four-month period. This will allow for a better understanding of how tension in change is experienced and managed over time.



## **Chapter 6. Nicolas' Experience of Tension in Changing Mathematics Teaching Practice**

As described in Chapter 4, one aspect of my research involved observing multiple mathematics lessons in one teacher's classroom. Over a period of four months, I observed eight mathematics lessons in Nicolas' classroom and was able to conduct interviews with him immediately afterwards on six occasions. An unexpected bonus was a series of twenty email exchanges, initiated by Nicolas, in which he described his daily successes and frustrations in relation to lessons that I had not observed. What made Nicolas singular as a participant was that he was about to implement change in his mathematics teaching practice, in particular, the first phase of a Thinking Classroom. (see section 4.2) Specifically, he wanted to implement vertical non-permanent surfaces, visibly random groupings, and rich tasks. This meant I was able to be in his classroom to observe both the initial implementation and how it continued to unfold over time. So, while Chapter 5 offered an overview of tension and agency in change in general, this collection of classroom observations, interviews, and emails permits a more detailed examination of change as it was enacted.

This chapter begins with a brief description of Nicolas: a summary of the background information I obtained during my first interview with him. The intent is to provide the reader with a sense of who Nicolas is as a teacher and details of the context in which he teaches. I follow that with a description of a specific lesson in which the three practices mentioned above were used for the first time. This is written similarly to an account-of (Mason, 2002), in which I attempt to document the lesson through my first-person account.<sup>12</sup> This is a deliberate choice on my part, as I want to provide the reader with an initial account of Nicolas' actions in the classroom devoid of my analysis and/or explanations. The intent is to allow the reader to gain a contextual understanding of the elements of the change Nicolas implemented, as they come into play in the eventual analysis. I treat the initial follow-up interview in the same manner, this time offering the reader an account-of Nicolas' immediate thoughts and perspectives after the initial implementation without any accompanying analysis.

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<sup>12</sup> For Mason, an account-of is generally used to describe to a single event within a brief episode, rather than an entire lesson.

The final section in this chapter analyzes tension Nicolas experienced during his implementation. I begin with a table outlining tension he had in common with my other teachers. This is followed by a deeper examination of five instances of tension singular to Nicolas, which are written in the form of a narrative to provide context and to allow for a temporal sense of tension. In keeping with a hermeneutic phenomenology approach, they are a means of describing Nicolas' lived experience as I document his attempts to manage tension during the duration of the change.

## **6.1. Nicolas: Introductory interview**

Nicolas taught in a small Francophone school with less than 100 students from kindergarten to grade 7. Typically, Francophone schools serve students whose first language is French, but students here spoke mainly English in their home. At school, however, all instruction was in French, with the exception that all the students have one English class taught entirely in English. At the time of the study, Nicolas was in his fifth year of teaching and had only ever taught at this school. During his first three years, he taught part-time in a combined grade 6/7 classroom. In his fourth year, he was excited to land what he called his "*dream job*", namely serving as the full-time mathematics specialist for the entire school. This abruptly changed midway through his fourth year when he was asked to replace the grade 6/7 teacher who had fallen ill. Feeling he had no choice in the matter, Nicolas took the assignment. The mathematics specialist position was cancelled shortly thereafter but, as the ill teacher never returned, Nicolas became the official grade 6/7 full-time classroom teacher.

The mathematics specialist position was exciting for Nicolas for two reasons: his background and his personal teaching goal. Nicolas had obtained a university degree in mathematics in France before moving to Canada, where he worked for fifteen years in the private sector in a mathematics-related field. Finding the stress and time constraints of his job untenable, he decided to pursue a teaching degree, as he felt teaching would allow him to utilize his strengths in mathematics and French. The second was because his goal as a teacher was to become a pedagogical expert in both those subjects. Being a mathematics specialist gave him the chance to think about the pedagogical aspect of mathematics and try new strategies and techniques.

Some of these he learned during his time studying to be a teacher. He had earned his Education degree at a Canadian university that promoted co-operative learning and project-based inquiry and expected their student teachers to enact those philosophies during their practicums. Noting that this was the antithesis to his own traditional schooling – *“I come from France, it’s very traditional”* – Nicolas was surprised by how much he enjoyed his university experience and even received an award for his summative co-operative learning portfolio. However, his time as a practicum student was more challenging than he had expected. Anticipating that he would eventually want to teach mathematics to secondary students, Nicolas had been placed in a Francophone secondary school and found it leaned heavily towards traditional teaching. Although encouraged by his mentor teacher to try new things, he found the preparation daunting, especially as he realized he could *“just take the textbook, open page 20, and do the lesson”*. After completing his practicum, he moved into teaching elementary students for pragmatic reasons; the school was near his home.

During the spring of his fourth year of teaching, Nicolas had attended a mathematics professional development workshop led by Dr. Liljedahl. He had gone with the intention of finding a few new ideas, but instead found he was inspired to change his entire mathematics teaching practice. The three changes he focused on were using visibly random grouping, vertical non-permanent surfaces, and rich tasks, in order to foster students thinking and working collaboratively. The inspiration to make these changes was driven by his goal as a teacher – he wanted all of his students to succeed. For Nicolas, this meant two things: he wanted his students *“to be academically ready and also to be happy in the classroom”*. By December of his fifth year, he had been using random groupings daily to assign his students seating and was ready to move to a full implementation. I was made aware of this opportunity to observe change in action and Nicolas willingly opened his classroom up for my research.

My first classroom observation and follow-up interview took place in January of Nicolas’ fifth year. His whiteboards were not in place yet, so this was more of an introductory observation to meet his 24 students and see his classroom. In the next section, I offer an account-of our second meeting, which was the first time Nicolas used the three practices. In it, I briefly describe both what I observed and what Nicolas said to me and his students as he moved about his classroom during the lesson.

## 6.2. An account-of the first observation and follow-up interview

*He had the whiteboards up! Nicolas said my coming had motivated him finally to get them up, mostly because he did not want to show me a boring, traditional lesson. He mentioned he was worried about disappointing me.*

*Nicolas had prepared spaces for the students to work in pairs. The spaces were located in a u-shape around the room with four pairs at the front, four at the side, and four at the back windows. Each space had a wall-mounted whiteboard with playing card taped to it, (which Nicolas said was an organizing strategy), along with a single whiteboard marker and eraser. Though his students were used to being assigned seating by random grouping, he said they had never used the whiteboards before. He mentioned his surprise that one student already asked to work at the window – he had not even begun the class yet. He said he read in Liljedahl's chapter<sup>13</sup> that this would happen, but that it still surprised him.*

*I had never before watched a teacher initiate vertical surfaces. Nicolas started off by pairing his students (two students were absent). He had playing cards laying face down on a side table and called up students a few at a time to choose one card at random. Then they went and stood at a whiteboard that had a matching card. At the end, there were three cards left over and the last student to choose, Darien, looked through all three cards before choosing one. In all there were ten pairs of two students and two students without partners.*

*When all the students were standing at their assigned areas, Nicolas told them they were to work together and that meant each pair had only one whiteboard marker to share. Two students were still without partners. He gave the following problem orally:*

*There is a 5 km race around a 1 km racetrack. Gabriel runs the race in 30 minutes, Ryan runs it in 45 minutes. When will they meet?*

*Two pairs solved the problem in less than one minute by creating a table. In both cases, Nicolas asked the students to explain their thinking to him before he moved away and circulated around the room. After a few minutes, he returned to the two pairs and gave*

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<sup>13</sup> This is in reference to Liljedahl (2016).

*them extensions: If the race were longer, when would be the next time they meet? How many laps they did run each time? Other pairs had now also finished, and Nicolas gave them these extensions as well. The first two pairs had now finished the extensions and Nicolas asked them about the shape of the racetrack. How long would each side be if it was a square? A pentagon? A hexagon?*

*Two students caught my attention. I noticed Cole, who left the room almost immediately with his instructional aide and did not return. I also noticed Darien, who spent more time away from his partner than with him (and Nicolas spoke to Darien after noticing he had pushed a boy in a different pairing). The rest of the students were noisy, but stayed at their assigned work area and were writing on the vertical surfaces.*

*When all the pairs had finished the initial problem, Nicolas addressed the whole class. He explained the answers to the problem and its extensions before assigning a new problem: Find the largest multiple of 8 less than 1000. He had no extension for this problem and those groups who found an answer were given another problem similar to the first one, this time using three different racers and times. These problems were not debriefed.*

*After about 30 minutes, Nicolas had the students return to their desks. He asked them what they thought about working together on vertical surfaces. The students replied: "Okay"; "Fun"; "Too much screaming, we shouldn't be shouting answers"; "I feel smarter even when I just write on the board". Another added, "We write things which are not needed, but we still look smart". One student asked, "Are we racing?" to which Nicolas replied, "No". When Nicolas asked for a show of hands for who wanted to try working again at vertical surfaces, all but two students put up their hands with one student adding, "But calmer".*



I was able to interview Nicolas immediately after the lesson and he stated he was happy with its outcome. When I mentioned the student who felt smarter at the board, he said, *"I thought you'd like that! It's amazing; the mere act of standing there made him feel smarter!"* Nicolas said he was also pleased with the student who noticed that he [the student] had written things that were not needed:

I really emphasized this part. I told him what you said is very important. It is the trace of your thinking that is only on your draft, not your completed work. So, the

fact that you write things that aren't needed, means actually you're thinking. You try things, maybe they don't work, that's okay.

Nicolas' response to my mention of how quickly the students solved the first problem was laughter, followed by:

Yes, I couldn't believe it! Too damn fast, that one! We had been working on common multiples in class last week and most of them couldn't get an answer on some basic questions!

He said the speed caught him off guard and surprised him:

It was a happy surprise. Like, they actually, most of them started to do something. Which is what the theory says about this system. But I didn't believe it that much. I was like, 'Wow! It actually worked!'

Nicolas said his surprise stemmed from the fact that when working individually, some of his students need a lot of coaxing: *"If they had their pen and paper, I can name quite a few who would not have started at all and I would have to be on them"*. That he had not anticipated this outcome was apparent, *"So it took me some time to figure out what else, right and then I could extend the problem"*.

When I asked if he felt any students had not been successful with the vertical surfaces, Nicolas referred to the two students that I had also noticed:

Darien is autistic. It caused a bit of perturbation when he kept going everywhere and he pushed someone which caused more problems. But the fact that Darien was here to me is a success. Because there is another student Cole who couldn't even stay. It was too hard.

He said that the combination of the noise level and Cole's communication difficulties might have prevented him from engaging with the class, but added, *"I have no solution at this point"*. He alluded to Darien's presence as a minor victory, despite the pushing.

Nicolas said his method of visibly random grouping was also new. His usual method was to greet his line-up of students each morning and give them each a playing card that indicated their seat for the day. Never having had them work together in random groups, he decided that a different method might be needed as, *"I didn't want there to be a gigantic mess when they all come together to get their cards. It made a very nice beginning that way."* He said it made worth the time he had spent the previous evening prepping for the lesson.

When asked what he would change for next time, Nicolas was quick to reply:

Classroom management. I am going to put some ground rules first. I only relied on my ground rules that I used in the past, like my clapping hands and ringing my bell. And, of course, they have one rule that works well – be at your desk. Those ground rules showed some weakness here.

He added that it took him too long to get his students back together.

I asked Nicolas why he wanted to try implementing vertical surfaces. He said that he had two reasons. The first was to leverage what he had learned in a professional development course about the power of working at vertical surfaces. He felt this would further develop himself as a teacher:

Because I want to do things better, but basically, I'm not convinced until I try. It's funny you know. They always sit and I'm standing. Right? And it's always harder to stay seated. So, I know that, but I wanted to see it.

And the second was to try out something different: *“That was really what I was thinking of. In a way, it's more trying out something different in the classroom rather than absolutely looking for something better”*. He further added his intention to try vertical surfaces in other subjects too:

One thing I thought is to try to do that, and not just in math. I think that this whole way of working where we go from standing to sitting and we can go think with small groups, go back to your desk is so dynamic. To me, this could be leveraged all over the curriculum. All over what we do. When I put those boards up, I said, “Look, they're going to be used at every opportunity I can, for every subject I can.”

Nicolas mentioned that, *“It requires a lot of the teacher”*, but added how much value he found in the lesson:

It was absolutely worth it. Just the fact that they all started to work, and how some of them communicated the answers was beautiful. This actually works. This thing is good.

Near the end of the interview, Nicolas said, *“So, it turned out very well in at the end. Even though I was just a bit scared.”*

### **6.3. Nicolas' tension**

While our conversations over the course of classroom observations, interviews, and emails indicated, as they would in any teacher's practice, an abundance of tension, I focus

here on tension that emerged as Nicolas implemented a Thinking Classroom. Tension arose for Nicolas, as this implementation also involved a shift in pedagogy as he moved from a teacher-centred mathematics classroom towards a student-centred one. This disruption of norms caused a series of cascading tension, as each change further impacted other aspects of his practice. In Table 6.1 below, I identify tension Nicolas had in common with my other teachers.

**Table 6.1      Nicolas' Experience of Tension in Changing Mathematics Teaching Practice**

	Examples	Strategies for managing tension
Internal tension: Intrapersonal tension		
Intrapersonal tension that motivated change		
Teaching style	I do too much traditional way of here is the method, example, exercise. I would say that my most difficulty as a teacher is to change my default behavior. Absolutely, my own default behavior, which is, the way I was at school, actually.	Professional development Having me in his classroom
Intrapersonal tension that resulted from change		
Self-doubt	I don't master enough the content to just, you know, kind of figure out what to do, what to give. As a teacher, most of the time you're alone. Like you get no feedback, no nothing, you know. So, it's very hard to actually improve, very hard to even have a picture of how you teach.	Endure and learn with time
Internal tension: Interpersonal tension		
Student engagement	I still think that some students are not working at vertical surfaces and that bothers me. You know, I'd like to have 100% engagement. And I don't. And that bothers me. I have one student who doesn't want to work this way. He was doing like Kumon and these kinds of things, and he's super-fast at computing. Right? So, he keeps wanting to go back to the thing that he knows. Like he knows fraction addition, subtraction, multiplication, division. And he doesn't want to think, he just wants to show me how fast he is. And I keep telling him, "Look, it's not fast, it's not fast." He did not want to do anything. He really resisted. I just had to work on him. I had to remind him about the rules of engagement, that it's about working together and helping each other.	Outlier Positive feedback Reinforce rules



	Examples	Strategies for managing tension
Student ability	<p>At first, I thought why don't their instructional assistants be with them? But they [the students] don't really like it. Like, Darien doesn't like it because it humiliates him, right? It really makes obvious that he is a bit different. So, then it's a bit of a dilemma, you know. Maybe they [instructional assistant and student] still work in the group, but then he would just feel too much singled out. So, I don't know. It's a difficulty.</p> <p>So, I said, no, you join, you join. That's what I did. Where the other times, they seem to do their own thing, separate of the classroom. They have a tendency to leave the classroom to do their own work. So, I said explicitly, please don't leave, please you're going to join a group and you're going to make an effort to work together. The two autistic kids did not participate, so far, the whole vertical surfaces is not working for them.</p> <p>One teacher assistant asked about Cole (autistic) if he were to participate and I said yes. She tried to devise a way to make him work but then they disappeared from the classroom (I am not sure why. Later I learned that he absolutely refused to enter into the activity).</p> <p>Darien is autistic. It caused a bit of perturbation when he kept going everywhere and he pushed someone which caused more problems. But the fact that Darien was here to me is a success. Because there is another student Cole who couldn't even stay. It was too hard.</p> <p>I observed that the best computation student may go forward and leave the other one a bit lost.</p>	<p>Outlier</p> <p>Positive feedback</p> <p>Convince</p>
Internal tension: Pedagogical tension		
Classroom management	<p>I have to raise my voice, you have to make sure everyone is listening, that annoys me a bit, that I have to do that.</p> <p>I think it was a very good explanation, but I don't think they listen much when a peer explains the easy problems.</p> <p>I had some misbehavior today. One girl tagged a student all over his face and arm with a white board pen.</p> <p>I don't move enough after having given a problem. I have a tendency to stay in the middle of the class while I should rather walk around and provide encouragement.</p>	<p>Endure</p>

	Examples	Strategies for managing tension
Group management	GD refused to participate as he does not like it (the same one as yesterday who ended up with his face on the table hiding). I forced him to join, he picked a card, got a boy that he never got along with and fought with since he joined the school 3 years ago. He told me that no chance he would be with him. I allowed him to pick another card. Another one complained: Why can he take another card and not me? I told him that justice is about not treating everyone equally sometimes (It was not as well put. I think I said something like "This is the way it is sometimes, as for you, you stick with your group, you don't pick another card"). Then GD got a group with a very nice buddy, joined him and worked happily.	Outlier Positive feedback Reinforce rules for majority Allow some leeway for certain students
External tension		
Assessment and reporting	I have so much stuff to do, like, right now it's the end of the semester so I need enough data to fill my report cards. Like I have to give my report cards to the principal on Friday.	Temporarily revert to traditional methods
Curriculum	I will not finish the curriculum. I mean I say, well, who cares, right? Because probably, maybe they will get better if we keep doing that [implementation]. They actually work, they are engaged to think. Who cares about like the area of a circle? But I'm not confident in that yet to really do that step. You know, because I think "Uh..." I'm not sure. You know the, the default and most, uh, secure for me position is to just sometimes go back to the book, because at least I know I will have completed the curriculum. I feel so much pressure to finish the curriculum that right now, for instance, you know today, I still haven't progressed on the curriculum of the year. We have the transformations; we have the geometry and stuff. Still haven't finished.	Use the textbook sometimes
Substitute teachers	I like to go back to the different style of the classroom when I'm away. When I have substitutes, it's not for the substitute to figure out the system and they [students] trick him [substitute]. They just don't care about the system. They go wherever they want, and it's much harder for the substitute because then they get the two boys making a mess, together. So, what I do is I plan for it. I just say, "Look. I use a system." And sometimes I say, "Don't use it. Just be careful and move them as soon as you see them make trouble."	Lower expectations Temporarily revert to traditional methods

I now turn to describing the five instances of tension that were singular to Nicolas. The first two are instances of internal pedagogical tension: tension with notetaking and tension with choosing problems. I focus my attention here, as these two particular instances of tension appeared frequently during the interviews as Nicolas talked through the difficulties he was having. This allows for the opportunity to examine more deeply how a teacher copes with tension over time. The last three are also instances of internal tension and could be considered related to, or a subset of, tension experienced by my other teachers. The first was an intrapersonal tension from the pull of previous practice and the second and third were both interpersonal: tension with getting students to listen and tension with a parent/colleague's critique of practice.

### **6.3.1. Tension with notetaking**

Nicolas' described his previous mathematics lessons as typically involving direct instruction, during which he would explain and notate a concept and offer a worked example, while his students copied the information into their notebooks before moving into individual practice. He valued this combination of direct instruction and notetaking, as he felt it provided a means for in-class learning and for students later to complete their homework and study for tests. This form of classroom instruction is common in Canadian classrooms, with teacher-led instruction considered critical and notetaking as something students do (or not). This was not the case for Nicolas; it was clear he considered notetaking an integral part of the learning process, not an afterthought.

This is more readily understood when considering Nicolas' educational background. He was educated in France, where notetaking is considered an essential part of the learning process. French students learn to take comprehensive notes in which they reflect on their learning and use the notes to make meaning for themselves (Omer, 2003). Nicolas spoke of his own need to write notes as a way to "*put a framework on my own thinking*" and used the French idiom "*decanter*" to describe notetaking as an intellectual process. Accordingly, for him, there is no privileging of direct instruction over notetaking; notetaking is part of how he teaches, as that is how students solidify their learning.

Tsing's (2005) tension metaphor of friction was useful here in thinking about how Nicolas' previous practice was 'rubbing up' against his new practice. Changing to a student-centred practice, where students spent most of the mathematics lesson working collaboratively at

vertical surfaces, brought tension with how to incorporate notetaking into this new practice. This was interesting to observe over time, as each of his attempts to manage his tension resulted in new tension. There was a strong dialectical sense to this process as he incorporated what he had learned into his next attempt.

For the first month, Nicolas discontinued notetaking altogether. However, concerned about what his students were learning and wanting some guidance for himself, he had begun to lean toward the notion of having his students take notes after working at vertical surfaces. This differed from tension my other teachers experienced with the pull of previous practice in that Nicolas was not reverting to a previous practice; he was trying to integrate the old with the new. He started by asking his students their opinion:

I asked them, "You want to go back to me writing on the board – the old-fashioned way?" They're saying, "Sir, we just write without understanding anything. It was just useless." But I needed to do that anyway because I was so lost about how to teach fractions. I didn't know anything, you know, where they were [pause] I had to go back to some grounding point for me as a restart. And I knew it was inefficient, but I had no other choice but to make some notes so that at least I would put a framework on my own thinking as to what are we even learning. So, it helped me. And I knew I had to do that. I knew it was useless, um, not so useful, but I wanted to do it anyways.

Nicolas' switch from "*useless*" to "*not so useful*" indicates that, despite notes being "*inefficient*", he still found some value in the practice. He was aware that notetaking fulfilled his own needs, but tension arose as his students made clear they did not find value in the process. In engaging his students' opinion on notetaking, he extended his agency to include their input in his decision. The cultural divide is apparent here as the students made it clear that notetaking, for them, was not about learning "*it was just useless*". Nicolas, however, for whom notetaking is intertwined with understanding, proceeded with it as it helped him develop his own understanding of how to teach fractions.

Taking their feedback into account, however, Nicolas decided he would prepare a handout<sup>14</sup> with problems and solutions they had completed during that day's lesson. He wrote of his intent in an email:

So my thought is that for maximum effectiveness, a teacher would provide soon after the vertical surface work a written trace of the thinking done, of the solution(s) achieved and related homework so that students get to leverage and "gel" on their

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<sup>14</sup> See Appendix E for a sample of a teacher-prepared handout.

learning (I found the idiomatic French word for it “decanter” (settle, decant) – which actually provides better connotations of the actual intellectual process to me).

These handouts satisfied Nicolas’ concern for his students’ understanding and, implicit in this, it also satisfied his need to frame his own thinking. This was better than no notes, and even an improvement on his previous notetaking method, as he found value in his students’ solutions as he added, *“I ended up adding one solution presented by a student that I had not thought of”*. He was accustomed to learning from making notes and so, in preparing notes for his students, he learned something he had *“not thought of”*. Nicolas’ use of *“gel”* and *“decanter”* are also indicative of how he views notetaking as a useful tool for developing understanding. Notetaking is an opportunity to process what was learned while problem solving at vertical surfaces. He is deeply committed to this idea and has compartmentalized the learning that occurs at the vertical surfaces as something that needs to be reinforced through the modality of notetaking. And, if his students were unwilling or unable to take notes, creating them for the students was a possible solution.

This new method also had its drawbacks. Nicolas tried creating the notes for his students for over one month before he noticed, *“I always end up putting the work on me. I still end up defaulting to: Okay, I have to prepare some more stuff.”* In an email, he compared his new and old methods:

Obviously one tension is that it is more work for me to produce the correction of the problems given during vertical surface than using class time to write on the board and get them to copy it.

So again, Nicolas sought out his students’ opinion, asking them if they found the handouts useful:

I asked them, so, was it useful, did you read it? And most of them said, yes. Right? I’m not quite sure that they read it, or all read it. But, most of them, you know, seemed to be willing to keep getting that. So, that’s what they said. But then I thought why after vertical surfaces wouldn’t they open their workbook and redo the same problems by themselves? And keep a track of it. Their track. Because then it would force them to actually write down what’s important instead of me doing the work. Plus, it will be a bonus for them. They rethink, you know, they ingrain their thinking ... plus, bringing some accountability for those who didn’t do anything [during the vertical surface work]? And I thought, why don’t I do that?

There is a lot going on here. Nicolas suspected that the students do not really make good use of his handouts. He appeared to accept their reassurances that they did find the notes useful but his addition of *“So, that’s what they said”* suggests he was aware that what his students *do* does not necessarily match what they *say*. This created tension for him

because preparing the handouts was time consuming. His response to this tension was to rethink his notetaking method. He came up with a new method that shifted agency to his students. This eased his tension, as his need for notes that “*ingrain their thinking*” was met and his workload lightened. That there is differing cultural expectation around notetaking is evident in Nicolas’ surprise, even after five years of teaching in Canada, that his students do not automatically “*open their workbook*” and create notes after problem-solving activity. Creating his own “*track*” of his learning is what he would have done as a student. His surprise also reveals that he believes that his students have learned something worth “*ingraining*”, even if they do not see the need to document their learning. This is important as it is possible that it is why he is willing to continue with a Thinking Classroom despite his tension with notetaking.

Initially, Nicolas allowed his students the choice of whether or not to write notes. After the vertical surface work, he would write out the day’s problems on the board while explaining the solution(s). The students could choose what and how much to write.<sup>15</sup> To encourage the notetaking, he told his students they could access their notes during tests. This movement towards student-driven notetaking suggests that he recognized that notetaking needed to be meaningful *for his students* in order for it to be successful. This was a move towards implementing notetaking with an understanding of *why* it worked.

This was a short-lived change as the meaningful notes led to another tension: students, whom he believed most needed to write the notes, did not:

I was inconsistent, because I said, “Look, we’re going to write notes; it’s going help you. You will be able to have your notes during the test and everything.” And then some said, “Oh, I don’t need the notes.” Some, I know they get it, so it’s okay. Some, I’m not sure they get it. You know, Sophie, for example, she didn’t write anything. That’s terrible, so I said you’re all going have to write notes. So, then I made it mandatory.

Although Nicolas used the word “*inconsistent*”, I suggest that what he was trying to do was meet the individual needs of his students, which speaks to his competence. Additionally, allowing the notes to be accessed during a test was part of his strategy to get his students to write notes they found useful. This was a transference of agency that might have given the students their own sense of purpose so that notetaking became something they did for themselves rather than for him, but what he did not appear to consider was his

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<sup>15</sup> See Appendix F for a sample of teacher-board notes and student-created notes.

students' (in)ability to write meaningful notes. I suggest it is likely they were unaccustomed to the kind of notetaking Nicolas envisioned and, consequently, they produced poor quality or, in some cases, no notes at all. In so doing, it interfered with what he saw as the benefit of notes – learning. To ease this tension, he mandated the notetaking. He would write out the problems and the solutions which, at a minimum, the students were expected to copy. This restored his sense of purpose and he was gratified at the response of his students:

The silence during the notetaking was amazing! Right? They seem to be actually quite engaged with the notetaking! So, for sure I threaten them with a test! Right? For sure it was a big deal! But I never had so much silence in notetaking time. So, that's a good thing, okay?

Nicolas had a pattern of management that started because of his tension with no notes. He tried teacher-created notes, then student-created notes, then mandated student-created notes. Each new resolution grew out of his experiences with the previous resolution which suggests the influence of iterational agency in developing his competence. Interestingly, each tension resolution resulted in a temporary state of contentment or happiness. Teacher-created notes felt good because he came to value students' own solutions, student-created notes because it put the accountability back on the students, and mandated notes because his students fully engaged with the process. Threaded throughout each was his sense of purpose – that notes helped to “*décanter*” the mathematics for the students.

This dialectical tension with notetaking led Nicolas to a deeper insight about himself, as he wrote in an email:

Another thing I wanted to tell you is that I realized that as teacher, there can easily be a big discrepancy between effort produced and learning of the kids. It dawned on me. This is a huge tension for me, I can spend a lot of time doing something and yet the benefit to the learning may be very small. This is the reason why I now try to choose my focus and give myself room to be good enough or (barely good enough) in some parts of my work (why would I spend time and energy when there is minimal learning gain for the students?) and spend a lot of energy in others (like the time dedicated to vertical surface work and talking – I hope that this time is and will be very valuable for the learning of the students).

Nicolas' capacity to metacognitively reflect on his practice was apparent here. His use of “*realized*” and “*dawned*” demonstrated a sense of reflexive agency which appeared to be a learning moment for him. He would be the one to choose where he would be “*good enough*” and where he would spend time and energy, which again brings to mind the notion of time as a commodity. What drove the choice of how to best use his time was his

focus on student learning. Despite his difficulties with notetaking, he would continue implementing a Thinking Classroom, which suggests he viewed it as important for student learning. There was a sense that he had improved notetaking to a point where it was “*good enough*” and could turn to ensuring the success of the vertical surface work.

Through exploring Nicolas’ tension with notetaking, I was able to hypothesize the existence of unacknowledged or *virtual* tension that reverberates underneath other tension. I use virtual in the sense that it was present in essence or effect although it was not formally recognized nor admitted (“Virtual”, 2019). Nicolas himself knew that he was experiencing tension with notetaking. He worked hard to figure out how he could make notetaking fit but did not call into question *if* he should make it fit. Eventually, perhaps, he would have realized that he needed to rethink his whole idea of notetaking, as I suspect that mandated notetaking would eventually result in a new tension, but he has not done that yet. He is still looking for surface level changes rather than actualizing or naming the deeper, underlying tension that his efforts around notetaking have made apparent. Like someone suffering from a sore finger who suddenly discovers that along there has been a splinter lodged under the skin, dealing with the surface tension is a temporary solution. Discovering the splinter – the underlying, virtual tension – is what leads to foundational change.

### **6.3.2. Tension with choosing problems**

Problem solving is an integral practice of a Thinking Classroom, which focuses on using rich problem-solving tasks. In particular, the problems used early in the implementation need to be highly engaging tasks that encourage student collaboration. This may cause tension for teachers as they experiment with finding the right kind of problems to foster this collaboration. This was true for Nicolas, whose decision to use simple problems created tension as he attempted to enact this aspect of a Thinking Classroom. His willingness to live with the tension resulted in his experiencing tension in other areas of his practice.

Similar to Kelly (see subsection 5.1.1), Nicolas had willingly attended a professional development session not because of tension with his practice, but because it was a mathematics workshop and, at the time, he was a mathematics specialist. However, the



experience provoked tension, as he realized there was a lack of problem solving in his practice:

I thought, I don't do enough problem solving in the classroom. I do too much traditional way of here is the method, example, exercise. I do a lot of that and I don't do enough problem solving.

Although Nicolas acknowledged that he does *"too much traditional"* teaching, his phrasing of *"I don't do enough problem solving"* does not suggest a transformational change. Rather, it indicates that there is an unstated amount of problem solving that he wants to include in his practice, not to replace this traditional way, but to balance it. So, he came away from the experience determined to implement *"enough"* problem solving.

Nicolas had decided to use simple problems as a means of ensuring all of his students' engagement. However, he realized the problems he was giving were too easy, *"So the problems are easy. But then I'm thinking, it's too easy. They don't learn anything. It's just, they get it immediately."* For instance, he orally gave this problem to his students: *"If three quarters of a number is equal to eight, then what is nine quarters of the number?"*, which the students quickly finished. Possibly, he was motivated to continue living with the tension of easy problems as, despite this ease of solving, he saw a richness in the solutions his students shared. He wrote about this in an email regarding a different problem:

The richness of the solutions was immense compared to if I had to do a lesson beforehand. I would have taught the "reduction to unity" method which is one I learned in grade 5. This method never came about in the vertical surface work and for the better, to find the 100% when you know 10% of a number and use my method would have really prevented student thinking. I still intend to eventually teach my method as it is the general way to find the solution to the 3rd problem, as well as a good way to think about a problem.

One of the benefits of a Thinking Classroom is precisely what Nicolas experienced here – students' exposure to, and use of, alternative problem-solving methods. Although his desire to teach *"my method as it is the general way"* prioritized his own mathematical knowledge, he appreciated the thinking in the solutions that emerged during problem solving. And, as this benefit emerged through the use of easy problems, he had little incentive to move towards richer tasks. It is interesting that, despite his use of powerful descriptors such as *"richness"*, *"immense"*, and *"for the better"* to describe his students' work; and acknowledging that his method *"would have really prevented student thinking"*; he still intends to teach his *"reduction to unity"* method. It suggests a virtual tension of control is underlying his unwillingness to move away from simple problems.

The use of easy problems, however, continued to be an issue for three reasons. The first issue involved the whole class debrief of the students' solutions, a critical practice of a Thinking Classroom. Termed 'consolidate to the bottom', the teacher leads the debriefing by focusing on the basic mathematics that emerges from students' solutions and continuing on to the more challenging mathematics. The intent is to cognitively engage all learners in some mathematical aspect of the problem. Although Nicolas generally led the debrief after a problem-solving session, he often had the students share their explanations as well. He found, however, that his students were disinterested in listening to student explanations of the problems they considered easy. In describing one student explain his groups' work, he said, *"I think it was a very good explanation, but I don't think they listen much when a peer explains the easy problems"*.

A second issue was his students also began to compete to see who finished first. This led to additional tension with classroom management, as those who finished quickly had nothing to do. The tension with some students racing to finish appeared in the opening lesson (*Are we racing?*) and continued throughout the implementation. Nicolas recalled one student who complained, *"If it's not a race, what's the point"*. This led Nicolas to admonish all his students:

I made it very clear. It's not a competition; it's not about speed; it's not to race. You don't shout [out the answer] to allow space for the others. And we are reflecting on this as a class.

One potential way to alleviate this tension could have been to pose more difficult problems. However, rather than changing the type of problems he posed, Nicolas shifted his agency to include the students and put part of the responsibility back on them by *"reflecting on this as a class"*. I suggest his willingness to live with this tension was likely because, although he admonished his entire class, he actually felt it was one student responsible for the competitiveness. He stated, *"There is one who absolutely wanted to race, absolutely not the majority"*. Like my other teachers, he sees this student as an outlier and is thus willing to endure the tension.

The final issue arose as, although they may not all have been racing, the students finished quickly. This led to classroom misbehaviour as the students waited for Nicolas to provide extensions and/or the next question. My field notes indicated that the room got louder, and students started wandering away from their groups, whenever too many groups were waiting for direction from him. Perhaps not connecting behaviour to the quality of the task

given, he frequently expressed worry about why his students were “*fooling around*”. This suggests a limited competence in this area, as Nicolas located the source of tension in his students’ behaviour rather than in the teaching practice which led to the behaviour.

About midway through the implementation, I asked Nicolas if he had considered giving his students an open-ended problem that might keep them engaged for longer periods of time to which he responded, “*No, never done that*”. When prompted to explain, he added:

Because I don't have one ready. I'd have to look for one. For a good one. But I just, it's one thing after the other. So, for the open problem, I just don't have the bandwidth to look for it. And because it's not my top priority. Making an open problem in the third trimester, won't change my mandate to complete the program [implementation]. Even though it's a good thing, right. Right now, you know, the time for me to look for one, make sure I prepare it, it's not worth it. And I'm not taking it [the time], basically.

This is an agentic response, as he weighed the outcome against his sense of purpose. He could look for more difficult problems, but as he (and his students) were already experiencing benefits from problem solving, he likely saw little reason for “*taking the time*”. His acknowledgment that there would be a benefit to using richer problems was tempered by pragmatics, as he said, “*It would certainly change things, and uh, if you have one to give it to me, I'd take it. If you do, if you do my work.*” Again, the theme of time as a commodity is apparent. Despite knowing that richer tasks “*would certainly change things*”, Nicolas was not willing to allocate time to finding them. As with his tension with notetaking, there is the sense that he believes the problems are “*good enough*” so he chooses to focus his efforts elsewhere.

Similarly, to notetaking, I see a pattern of management that occurred as Nicolas lived with the tension with choosing problems. Both patterns of management resulted in new tension arising, but the difference was their origin. In attempting to manage tension with notetaking, each new tension was a direct result of the previous resolution (teacher-created notes, student-created notes, mandated notes). With choosing problems new tension arose because he made no attempt to resolve the original tension. As he was willing to live with choosing easier problems this caused tension with students competing, being off-task, and not listening.

### 6.3.3. Tension from the pull of previous practice

Although this tension was also experienced by my other teachers, I felt it was worth exploring here, as it manifested differently and with far more frequency in Nicolas' practice. Recall how my other teachers experienced this tension: Lily and Kelly reverted to more traditional practice when they had had a bad day or felt unprepared, while Leah and Diane found comfort and reassurance in traditional practice (see subsection 5.1.1). For all four, this was tension they appeared willing to live with, as they rarely succumbed to the pull. However, this was something that Nicolas fought against on what appeared to be a daily basis. Beset by tension with covering the curriculum before the end of the school year, his emails documented his ongoing tension with a strong desire to revert to seatwork instead of using the practices he had been implementing. Interestingly, it was his students who prevented his retreat:

Today, students requested to start with vertical surfaces, as we were building the agenda of the day. I had forgotten about my commitment to make one vertical surface work a day and I wanted to move on as we are behind on pretty much everything for the trimester. Upon the request of the students, I then remembered about vertical surfaces and I tried to delay it saying that I would prefer to do a math lesson first, they all opposed it. One even justified saying, "For once we actually want to do work, why would not you let us do it?" I asked the classroom if there was a majority in favor and it was close to 100% (actually, maybe it was – I am not sure). So, I gave in.

This is similar to the onboarding strategy many of my other teachers used with parents, colleagues, and students, except, in Nicolas' classroom, it was reversed. Here it seems that his students are already on board and they are trying to convince *him*. His willingness to follow the lead of the majority indicates a sense of shared agency with his students. However, his use of the phrase "*So, I gave in*". suggests the control was given up unwillingly and that would affect his autonomy. This indicates that his willingness to give up control is context dependent, as it contrasts with his willingness to give up control when asking his students' opinion on notetaking. It also brings to mind the image of a tug-of-war or similar battle, inherent in which there are winners and losers. This was apparent in another email Nicolas sent:

Today, vertical surface work was again requested by the students. I still fought to explain a few things. I spent time explaining to the students how to solve it [homework problem] and I wanted them to take their explanation further by verbalizing it and so I wanted to teach the class. It went well, even though when I spoke for a very short mini lesson, a few students expressed frustration about "when are we going to go to the board".

The battle imagery continued here with Nicolas feeling like he “*fought*” to teach his frustrated students. There was a war going on here between Nicolas and his students, but also between Nicolas and himself, as he had to manage tension between wanting to change his practice yet feeling like he was also meeting his professional responsibilities. There is a wavering in his sense of purpose, as his motivation to change was hindered by his uncertainty that the changes help him meet those professional responsibilities. His past experience had proven that direct instruction satisfied the latter, so drawing on his iterational agency led to a practical-evaluative response: a default to direct instruction.

This battle continued, even as Nicolas wrote many emails acknowledging the benefits in the changes he had been implementing. For instance, upon noticing that his students had misapplied a percentage strategy, he wrote:

I have to say that vertical surfaces allow very easily to find out misconceptions. I probably would not have noticed the misconception so easily if another way of working was chosen. I see much more the work of my students this way, since when they work on their desk, I circulate and look at their work way less (it is too easy for me to get going on my own stuff while I “assume” that they actually do some work – how naive!).

His inclusion of “*how naïve*” indicates his awareness that some of his students were not really working before. This was made clear to him after the implementation when he realized how much more of their work he saw. This bears a similarity to Kate’s tension in which the changes she was making made previously invisible behaviour, visible, as she remarked, “*They stand out more than when they did seatwork*” (see subsection 5.1.2). For Nicolas, the changes he implemented were making visible behaviour that he had once overlooked. This visibility led to two benefits. First, he could more readily identify his students’ misconceptions so it follows that he could therefore more readily address them. Second, he could now easily see whether his students were working whereas before, he just assumed they were. His use of the word “*naïve*” to describe himself suggests reflexive agency as he thinks through his own previous actions.

Nicolas also made frequent mention of how engaged his students were after he implemented the changes. He mentioned one student who, while she still struggled to participate, was more engaged than ever before, as he wrote in an email: “*She actually requested more problems to do once we finished the first one. This really felt magical to see her engaged this much!*” He also forwarded me a parent email exchange in which he was able to reassure them that their child was doing well:

Here is what I am able to report to a parent today thanks to vertical surfaces. It is about a child who is typically very disturbing. I said that his behavior with respect to vertical surfaces is excellent – that he does not want to stop solving math problems. I have to say that if someone else would report such change, I would not believe it (I'd say the teacher exaggerated to look good). As I write it, I still feel like a fraud since it sounds so surreal, but I witnessed it today, so I cannot state it differently!

Nicolas' experience with the implementation was going so well that he it actually surprised him. His use of the words *"fraud"* and *"surreal"* show his genuine astonishment at the success. This echoed his experience of the first time he used vertical surfaces when he spoke of his *"happy surprise"* that the system he was told would work, did indeed work. Nicolas was able to witness the benefits in his struggling student and share that with the child's parents.

But it was not just one or two students who were engaged, Nicolas was made aware by his students' daily pressuring of him to incorporate vertical surfaces and from his own observations, that it was this was a benefit for the majority of his students. As he explained, *"It was simpler for me to get them to work"*. Furthermore, he felt they were now more engaged than they were before:

What I really like is that they are, most of them are mostly engaged all the time. And that's huge. Because, even though sometimes they are not all engaged, or they fool around, if they were seated, they would fool around as much, and they would be way less engaged. I'm sure, I wouldn't have that, the ratio of engagement.

Nicolas had many more such positive experiences with the changes which caused me to expect that the pull of previous practice would diminish as the benefits accrued. In speaking of trying out the implementation after the first lesson, he had mentioned that *"I'm not convinced until I try"*. The success he had been experiencing might have been enough to convince him. However, this was not the case and he *"fought"* with the tension to return to his previous practices through to the end of my observation.

Nicolas' rationale for wanting to return to his traditional teaching practices was his feeling *"we are behind on pretty much everything"*. This made his previous practice appealing as direct teaching offers a teacher the satisfaction of feeling that they have covered the content. This was demonstrated frequently during our time together through his usage of variations of: *"You have to be sure that you cover the curriculum"*. This pressure made him less willing to allocate what he referred to as his *"bandwidth"* to other aspects of his pedagogy. Like my other teachers, he is exerting his agency in choosing how to spend his

commodity of time. What was interesting was his answer to whether he felt “*behind*” in other years (when he utilized a teacher-centred approach to teaching). In reply he offered an unequivocal, “*Absolutely! Every year I run out of time to teach.*” This disconnect is interesting, apparently no matter which teaching method he uses, he runs out of time. This suggests that falling behind is a function of something other than the teaching method.

#### **6.3.4. Tension with getting students to listen**

Although this tension could be classified as tension with classroom management, Nicolas’ repeated references to his tension with wanting students to listen made it worth distinguishing as a subset of classroom management. This tension may arise as one by-product of a student-centred classroom is a change in the locus and volume of classroom discourse. As students engage collaboratively with mathematics, and with one another, sound levels naturally rise, as more voices are speaking simultaneously in all parts of the classroom. And, although teachers are an integral part of this discourse, they spend less time in direct instruction and more time talking with students in groups. This can be disconcerting for teachers when they first attempt to change from a teacher-centred classroom to a student-centred classroom. Speaking directly to a group of quietly seated students provides the sense that students are listening. This was made apparent earlier by Diane (see subsection 5.1.1) when she commented, “*When they’re quiet and they’re all looking at me I know I have their attention. I’m not sure if everybody is paying 100% attention when they’re working in the problem-solving groups.*” However, quietly looking at the teacher does not necessarily mean listening is occurring, as Nicolas seemed well aware:

You know, even though it is easy to fool ourselves, you know, “Yeah, yeah. They all are following.” But this morning for instance, I talked for ten minutes and whatever I was saying interested me, so at least I was engaged with myself. But then I really was suspect of how many are really listening to me. So, I say to myself, maybe N, maybe a few of the top students, are fully listening. But even there, you don’t really know. I can’t really fool myself anymore!

Like Diane, Nicolas had decided to engage in direct instruction with his students at their desks, something he had done regularly prior to the implementation. His inclusion of “*I really was suspect of how many are really listening to me*” and “*I can’t really fool myself anymore*” indicated his awareness that some of his students were not listening that morning, and likely never were before either. This was made clear to him after the

implementation when he struggled with getting his students to listen when standing. Again, this suggests that the implementation shed light on previously invisible behaviour. It is not that his students were listening any differently now; he had just become more aware of it, hence his phrase, *“I can’t really fool myself anymore”*.

Nicolas’ tension with listening stems from *“my need to control the class. To make them listen. I make them listen; it’s very intense for me.”* It was the difficult process of ‘making’ his students listen that created tension for him. It was noticeable the first time he utilized vertical surfaces (see section 6.2) when he realized that he would need some new *“ground rules”*, as the ones he had been using (clapping hands, ringing bells, sitting at desks) showed *“some weakness”* and were ineffective with the changes he had made. He mentioned that he found he had to talk louder, *“I have to raise my voice, you have to make sure everyone is listening, that annoys me a bit, that I have to do that”*. I suggest his need to make his students listen arises as control for Nicolas involves management of cognition. Students cannot learn, cannot *“decanter”*, if they are not listening.

The aspect he was struggling with was the whole-class debriefing portion of the implementation. This was when all the students would gather together, and Nicolas would highlight the relevant mathematical content. As a group they would move together so he could utilize the students’ work in his explanation. His struggle was to find a way *“to manage to get them together to listen to me, to go back together. There has to be more structure.”* To provide that structure, Nicolas said he modeled for his students how to listen. The first step was to have them learn to turn their bodies towards him when he spoke and this was nonnegotiable: *“I’m very tough, when I speak, you listen, you turn to me, you turn yourself to me, so I make them turn to me. It’s a mandate I put.”* This focus on *making* students listen and *making* them turn came at a cost to him:

It’s too negative for them. I have this tendency of being assertive, and then it’s my default mode, and maybe stress, a bit angry, then it’s disrespectful to the child. So, it’s not easy, but I found that when I’m assertive, you know, I can control things. But you know, it can’t be just that, because it’s not healthy. Not for me, I’m drained, and not for them because it’s not how they grow either.

Although control is connected to autonomy, Nicolas’ awareness of *“it’s not healthy”* makes it clear that the kind of control he was seeking may limit his agency. While it is tempting to make the assumption that he was *“assertive”* before the implementation, it is an unhelpful assumption, as what matters now is that, in this new context of change, his tension with



students' listening resulted in what he believed to be a negative learning environment. This was incompatible with his goal for students "*to be happy in the classroom*" and it also put the changes he was implementing at risk, as living with the tension may be untenable. His approach to managing the tension by teaching the students how to listen could prove effective, but he felt pressured by the time required:

It needs to be modeled and taught. I think it is exactly the right thing, we're going to model listening. And I have to say, it's very hard for me to say, okay, let's model this, you know. It means I have to take the time; I have to make that a priority of the classroom and, you know, right now, my priority is not that.

For now, Nicolas appeared to be willing to live with the tension. He was feeling the pressure of time and, as mentioned previously, he felt pressure to cover the curriculum; it had "*priority*" over spending the time teaching his students how to listen.

This tension with getting students to listen had a second aspect though: getting students to listen to each other. This was a requirement not only during collaborative group work but when students were offering whole group explanations of their mathematical thinking.

When students explained what they found to the whole classroom, I found that students speaking were not listened to (not clear speaking, student not talking but not listening either), I feel compelled to re-explain or clarify explanations on top of what students say or explain.

Here Nicolas does not attempt to "*make*" his students listen to each other. Rather, he turns the focus of listening back onto himself, as he re-explains what he believes they missed. This again speaks to his need for controlling the learning, but I further suggest this is because he has more experience with getting students to listen to *him* rather than to each other. And although listening to him is not without its own tension, it is a practical-evaluative sense of agency built on prior experiences, as he mentioned having unsuccessfully tried out student practices such as think-pair-share:

As soon as I give a little bit of room, they can abuse it. And that's my trick, that, how can I give them more freedom, so that they learn how to respect that freedom, work within it, without immediately abusing it. And that's very hard. It's very hard, because even when it's talks, it's usually them and me, right? If they're free to talk and give their opinion, it's very hard for me to do a, "Okay, you talk together now, and then you talk to the classroom", you know, this whole think-pair-share. Very difficult.

Control again seems critical here. Nicolas was willing to give up some control, but experience taught him that the outcome might not be worth it. He wanted his students to

learn to make the right decisions with the freedom he was offering, but did not seem to have a way to go about instilling that skill. Repeating his students' explanations appeared to provide a way to live with the tension.

### **6.3.5. Tension with a parent/colleague's critique of practice**

Nicolas had an unusual situation in that the parent of one of his students was also employed as an instructional assistant in his classroom. Although assigned to another child, she instinctively watched over her own son, as he had difficulties as well. Nicolas mentioned that her son was *"weak, with an attentive disorder. Doesn't get to work, you know, doesn't work. Always fools around. And has weaknesses in math."* He described one instance when the parent/colleague's assigned child was absent, and so she joined a group that her own son was working in:

This instructional assistant has her child in my class. And her child (Robert) has difficulties but no instructional assistant. And she keeps advocating for him. So humanly, it's human, she has a tendency to watch over her son. And he [her son] was working with (Riley), and she told me that (Riley) just completely ignored him, completely ignored her, and did the whole thing by himself. And I didn't see it; I saw none of it! But she saw everything and said, "That's not acceptable. They were supposed to work together, and they didn't. These things [vertical surfaces] don't work."

Although Nicolas had not seen the behaviour, he trusted her and felt compelled to take action. Likely this occurred as they had a good relationship and he valued her feedback. This can be seen as an interspective aspect of his reflexive agency as he listened to her observations and "yield[ed] objectivity from negotiated subjective information" (Mason, 2002, p. 85):

And we have very good parent-teacher relationship. So, she feels very free to tell me what she thinks, because I always welcome it a lot when she tells me because it helps me to adapt.

Her critique of his practice though was focused on his use of vertical surfaces. She wanted him to return to his old way of teaching:

She told me that she thought that with vertical surfaces her son was not learning as much or was not getting much. And I have to say, it might be true. Because her son has extreme difficulty to be engaged in anything, to think. It's probably some difficulty like attention deficit, and when it's too intellectually complex he just blanks out. And my old way, which I still do sometimes, is to basically go on the projector and then write on my paper, and project exactly what I write, step by step, how to do things. And with this way, which was my first method all the way last year, he

said that, “With Mr. X [Nicolas], we can understand because it was very step by step and you could just copy exactly and reproduce.” But by not having that kind of, exactly shown step by step, he is so lost that he gets nothing. And even if he doesn't understand step by step, which he doesn't, at least he does something. Which is you know, it's better having something, even if you don't understand, at least you produce something, you know, even if you don't make the connections, as opposed to just blank out because you understood nothing, and you get no possible tool to move ahead.

Nicolas, as evinced in the previous tension, was already feeling the pull of his previous practice, and now it was reinforced by a parent/colleague who felt the “*old way*” better served her own child's needs. His response of “*it might be true*” indicated a willingness to agree despite his awareness that the student did not understand mathematics the old way either. Despite this being an outlier situation (in that no other parent or colleague had complained) it was almost as if he welcomed this tension, as it supported what he was already thinking. Or perhaps the fact that this outlier had a dual role as parent *and* colleague gave her opinion more credence.

When I asked what he intended to do, Nicolas said that he planned to continue having her son work collaboratively at vertical surfaces. However, he added that he was considering making some changes, “*I might introduce back something where I do show things myself. I might show some solutions myself.*” Although this speaks to his sense of autonomy in that he was willing to work collaboratively to address a parent/colleague's concerns, it may also be that he momentarily lost sight of his sense of purpose. He had had enough positive experiences to believe that this change might benefit a struggling student, but this tension caused him to forget or overlook that potential. It also suggests the possibility of another virtual tension. I argue that his willingness to move away from his new practices despite the overall success of his students, stems from the fact that he does not see it as teaching. Teaching, for Nicolas, means “*my old way, which I still do sometimes, is to basically go on the projector and then write on my paper, and project exactly what I write, step by step, how to do things*”. Teaching in a Thinking Classroom requires a different way of teaching, one which moves away from teaching as telling. Although unacknowledged, it is this underlying tension regarding his belief about the true nature of teaching that makes him willing to reverse course.

That pulling back from a Thinking Classroom was unwarranted was made clear when the parent/colleague sent Nicolas an email shortly after my last observation. In hindsight, she had come to recognize the benefits:

This learning method with the whiteboard is an excellent exercise to learn how to communicate orally in front of the teacher and the students. This teaching method, when used frequently will allow students to practice for “épreuve d'art oratoire” (a French speaking contest) and the science fair. For some students, this method is very hard initially but upon doing it regularly it will become a regular routine. My son kept complaining about this method because he felt exposed in front of everybody and was afraid to make mistakes and be embarrassed. As for me I thought this method was difficult for special need students like my son as the teaching is abstract and “in the air”. There is not enough visual support and one has to grasp information given orally and use it as it comes, which is difficult for some students. But this method develops meaning, self-confidence, and the ability to express oneself in front of others, so I changed my mind about this, and I would like this method to become a daily practice.

This excerpt brings to mind two things: transferring agency and onboarding. Perhaps Nicolas’ agentic response to her, in taking her concerns seriously, provoked her own sense of agency. The level of trust they had may have allowed her to step back from her parent role and look at the practice instead from a teaching perspective. It took some time for this to happen, but eventually she realized the benefits of what he had been trying to do for her son and the rest of the students. Over time, he was able to get her ‘on board’, in the same way my other teachers did with their parent tension. But it came at a cost, as he originally adjusted his Thinking Classroom to accommodate her request for more direct instruction.

## **6.4. Summary and conclusions**

Nicolas’ desire to implement change in his mathematics teaching practice was the result of attending a professional development workshop. What he experienced there led him to want to implement the ideas he learned into his own classroom. He was no longer satisfied with a traditional classroom and it was this tension between how he taught and how he wanted to teach that led to change. He started with visibly randomizing his students’ seating plan before incorporating problem solving and student collaboration at vertical non-permanent surfaces.

Nicolas was a self-described traditional teacher and his past was present, even as he struggled to renounce and surpass it. The changes he was implementing required not only physical changes, but an accompanying paradigm shift in his thinking. This shift was accompanied by tension as he contended with jettisoning some of his old beliefs and/or attempted to meld the old and the new. Like my other teachers, he lived with uncertainty about what he was doing and how he was doing it. He also experienced interpersonal

tension with student engagement and student ability and pedagogical tension with classroom and group management. And, he too managed external tension from assessment, curriculum, and substitute teachers.

Singular to Nicolas were five instances of tension. He experienced tension with how to utilize notetaking into his new practice and with the kinds of problems he chose for his students. He also had tension with getting his students to listen, which brought to the forefront his need for control. Feeling pressure from his need to ensure he covered the curriculum, he frequently found himself fighting the urge to revert to his traditional practices. This tension was magnified by the presence of a parent/colleague in his classroom who questioned the effect the changes were having on her child.

Nicolas found ways to manage tension he was experiencing. Similarly, to my other teachers, viewing the source of tension as an outlier helped occasionally as did the notion of onboarding. Onboarding for Nicolas had a slight variation, however, as once he effectively had his students on board they, in turn, worked to get him on board whenever he decided he did not have enough time to allow students to work collaboratively at vertical surfaces. He also relied on rule enforcement as a means of exerting control and thereby managing tension. He recognized he had a strong need for control yet many of the practices he was implementing required a lessening or transference of that control to students. He sometimes transferred that control to the students by polling them thereby encouraging them to take initiative and demonstrate agency, but other times he did not. Nicolas would also revert to some of his traditional practices to assuage his tension.

The positive feedback Nicolas received from his students and/or from his own observations of the change also helped manage his tension. As noted in the discussion of Hegel in Chapter 2, individuals must see the positive within the negative moment of development if they are truly to advance. This was certainly true for Nicolas who frequently described his surprise at his students' engagement despite his ongoing worry about students not listening or not taking notes. Similarly, his experience with notetaking was a succession of tension followed by positive moments: Teacher-created notes felt good because he came to value students' own solutions, student-created notes because it put the accountability back on the students, and mandated notes because his students fully engaged with the process. Had his negative moments of worry been followed by negative feedback, it is unlikely he would have been willing to continue with the changes. This

notion of finding the positive within the negative is not unlike my other teachers who, upon temporarily reverting to their traditional teaching practice, felt reassured about the changes they had made. This reassurance helped keep the change alive for them, as it did for Nicolas.

I now turn to two other areas of interest. The first was the notion of patterns of tension management. In tension with notetaking, Nicolas cycled through a series of tension where each arose from the resolution of the previous tension. In tension with problem solving, his pattern of management focused on managing tension that arose because he made no attempt to resolve the original tension. This made clear that certain tension, while isolatable, is not independent of other tension (e.g., Berry, 2007a; Katz & Raths, 1992; Lampert, 1985; Mason, 1988). It also made clear that there were different traits or qualities of tension. Related to this is the second area of interest: the presence of virtual tension. My time spent observing and talking with Nicolas allowed me to notice underlying tension of which he was not necessarily aware. In what follows, I elaborate further on these two areas of interest.

#### **6.4.1. Qualities of tension**

Nicolas was experiencing tension with notetaking. He knew what he was doing was not working and he demonstrated agency as he strove to adjust his practice. His previous practice had been to have his students copy notes as he wrote them on the board. He discontinued this practice with the onset of the Thinking Classroom implementation as he focused on establishing routines with getting the students to work collaboratively. Eventually, however, this omission created tension for him as he valued notetaking as a method for decanting the learning. Wanting to re-introduce notetaking, he polled his students on their thoughts regarding the practice. They made clear they found little value in notetaking so, to assuage his tension, he decided to create handouts of notes for the students. This quickly resulted in a new tension as he found the preparation time prohibitive and suspected that his students really did not utilize the notes as he felt they should. To resolve this new tension, he decided to have the students create their own notes immediately after an activity. He was pleased with this decision as it was also fit more closely to his own schema of notetaking, in which notes are created by the learner as an aid for their own learning. However, perhaps unaware that students need guidance in how to take notes, Nicolas was disappointed with his students' efforts. He resolved this

tension by mandating that the students copy, at a minimum, the problems and solutions that he wrote on the board. Whether or not they chose to add more to the notes was left to the students' discretion. Observing in his classroom, I saw this as a creative process of tension management.

In working through the analysis of Nicolas' tension with notetaking, however, I found myself thinking of kinds of tension. Not just in categories such as pedagogical or external but also in traits or qualities of tension. For example, he had decided to attend Dr. Liljedahl's professional development session because he knew he relied too much on traditional practices and wanted some new ideas. I came to see this as *useful* tension; tension that prompts teachers to action – to change surface aspects of their mathematics teaching practice. Instead, what Nicolas experienced in the session created a *productive* tension, in that it inspired him to think about his practice in an entirely new light and made him willing to put in the effort to make changes. This caused me to reimagine my original conception of the connections among tension, agency, and change (see Figure 3.2) as a matrix (see Figure 6.1 below).

Agency	High Agency	No Change	Instrumental Change	Conceptual Change	Foundational Change
	Moderate Agency	No Change	Instrumental Change	Conceptual Change	Conceptual Change
	Limited Agency	No Change	Instrumental Change	Instrumental Change	Instrumental Change
	No Agency	No Change	No Change	No Change	No Change
		Tension			

Productive Tension
  Useful Tension
  Tension
  No Tension

**Figure 6.1** Tension, change, and agency matrix

Rather than thinking of tension solely in terms of amount or level, what this new figure offers is a representation of the qualitative differences in tension. I chose to depict this as a colour gradient to make clear the connection among the quality of tension, the kind of change, and the level of agency. While the amount of tension teachers experience undoubtedly impacts their practice, I suggest it is the *quality* of the tension experienced that plays a pivotal role in the type of change they achieve.

*Productive tension* is tension in the generative and creative sense in that it has the quality of producing something significant through effort or work (“Productive”, 2020). It can lead to conceptual and/or foundational change depending upon the degree to which it impacts a teacher’s way of thinking about the teaching and learning of mathematics. At a minimum, productive tension leads to conceptual change, which is an understanding of why new practices being implemented work and how that applies to other contexts. When tension is highly productive, it can lead a teacher to make deep changes to the very foundation of their practice. Whether paired with moderate or high agency, productive tension occasions the introspective and interspective thinking that enables teachers to view their mathematics teaching practice as a site for transformation.

*Useful tension* is tension that is capable of being put to beneficial use (“Useful”, 2020). This is the kind of tension that often compels teacher to seek out change; it is the itch that first makes teachers aware that the state of their practice is not how they wish it to be. For some teachers, learning a new technique or strategy manages the tension and they achieve instrumental change. For others, like Nicolas in the professional development workshop, this initial useful tension can lead to a productive tension as they find their thinking altered.

I also found myself thinking of tension that is neither useful nor productive; it is just experienced and does not result in change. Corey experienced this sort of tension when she wanted to make changes, but did not have the approval of her administration (see subsection 5.2.5). While this likely started as a useful, or even productive tension, her limited agency left her feeling all the negative aspects of tension and none of its benefits.

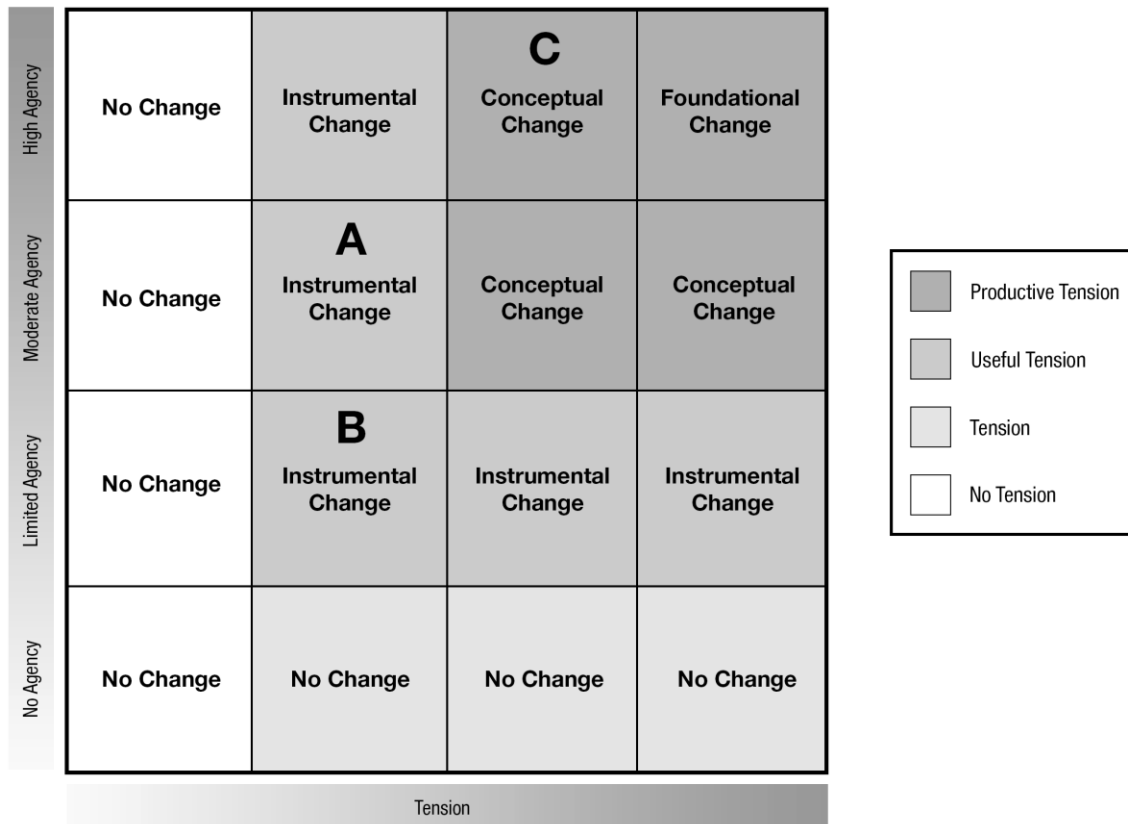
Agency, then, plays an important role as, accompanying these tension traits are various degrees of agency. Teachers who achieve high agency display a strong sense of many or all of the aspects of agency: purpose, competence, autonomy, reflexivity, and



transference, while teachers who achieve limited agency fall short in every aspect or miss some entirely. For example, in the previous chapter, I suggested that Leah's insecurity with not meeting institutional norms was evidence of limited agency (see subsection 5.2.3). Her competence was affected by her insecurity and her need to justify her practice to her administrator suggested a lack of autonomy. Her mention of the importance of challenging societal norms without mentioning who is responsible for instigating the challenge might also suggest a lack of autonomy and reflexivity as it is unclear if she realizes that she, herself, is already challenging the norms. Taken all together, this suggested a limited agency for Leah, in the context of coping with institutional norms.

Thinking of tension in teacher change in mathematics practice through the lens of this matrix is useful in explaining and understanding the decisions teachers make. For example, a teacher who is experiencing tension, but makes no changes, very likely also has minimal agency. Should that teacher find, or be offered, the opportunity to achieve agency, I would argue that change, whether instrumental, conceptual, or foundational, could occur. Similarly, a teacher who has no tension, but high agency will not change unless there is an opportunity for tension to be introduced that will compel the teacher to want to change. Other example scenarios include a highly agentic teacher who experiences foundational change as she manages a productive tension or instrumental change as she manages a useful tension. Rather than teachers as resisters of change, what this matrix offers is a view of teachers as rational decision makers who, when faced with tension, consider the practicality of innovations in their mathematics teaching practice and make agentic choices.

Nicolas' management of his tension with notetaking can be considered through this lens. He had a strong sense of purpose regarding notetaking and showed autonomy and transference in his polling of students for their views. His strategy of a teacher-created handout, however, was an instrumental change led by a useful tension. I see it as a useful rather than productive tension, as although he valued students decanting their thinking, he chose a strategy where he was the one doing the decanting. Indeed, he remarked on how he had learned a new solution in preparing one of the handouts. He had implemented a strategy without a full understanding of why it would or would not work, yet his polling of his students' opinion on the change was highly agentic. Accordingly, I would suggest he achieved moderate agency. See Box A on Figure 6.2 to track his pattern of management.



**Figure 6.2 Nicolas' pattern of managing tension with notetaking**

Inevitably, preparing a handout led to new tension and Nicolas decided the students should create the notes. His decision to decrease his workload was an agentic action but in attempting to recreate the notetaking of his own schooling, he showed little awareness of the mathematical norms this change would disrupt. Allowing the students to use the notes on future tests showed that he recognized that the notes needed to be meaningful for his students, yet I would suggest this was still an instrumental change as he again displayed little understanding of why or why not this strategy would work. Furthermore, this is suggestive of limited agency as he was seemingly unaware of the difficulties his students would experience and did not pre-teach notetaking skills. He saw the problem as stemming from his students rather than from his own (lack of) teaching. See Box B.

This created more tension as his students balked at writing notes. I see this as a productive tension that increased his agency as it caused Nicolas to reflect critically on what he was doing and how it affected his students. The result was he moved to a blend of teacher-created board notes for the students to copy (and use as a model for notetaking) and student-created notes. This was a move towards implementing notetaking with an

understanding of *why* it worked, which suggests conceptual change. Foundational change may have been achieved had he questioned or talked about the difference between his own and his students' conception of notetaking. See Box C.

In Chapter 2, I wrote of a metaphorical doorway in which one has the choice to cross the threshold. Sameness lies on one side, while the unfamiliar lies on the other. It is tension, in general, that brings teachers, like Nicolas, to the doorway, but as they stand there thinking through their options, it is useful and productive tension that nudges them across the threshold. This is an agentic view of tension management that positions teachers as arbiters of their own change.

#### **6.4.2. Virtual tension**

My time spent observing and interviewing Nicolas also allowed me to notice underlying or *virtual* tension, of which he was not necessarily aware. For example, I hypothesize there was a virtual tension of control, of both the mathematics and of his time, that complicated his attempts to implement a Thinking Classroom. He tended to choose problems that allowed him to show his method for solving. Necessarily, this resulted in choosing problems that had single algorithmic solutions rather than the open-ended rich tasks required for a Thinking Classroom. Searching for those tasks would have required giving up control over how he chose to spend his time, something he was unwilling to do. He appeared aware of his need for control yet, at the same time, seemed unaware of how that need contributed to the tension he was experiencing. Likewise, with notetaking. Nicolas' desire to introduce notetaking stemmed from his own background in which notetaking was a highly valued endeavour that decanted the learning. He worked hard to figure out how he could make notetaking fit but did not call into question *if* he should make it fit. Eventually, perhaps, he would have realized that he needed to rethink his whole idea of notetaking, but he has not done that yet. He is still looking for surface level changes rather than actualizing or naming the deeper, underlying tension that his efforts around notetaking have made apparent.

This virtual tension occasionally appeared when teachers attempted to blend previous practice with a Thinking Classroom. It is one thing to willingly incorporate change; it is another entirely to weave it into existing practice. In instrumental change, teachers choose to simply incorporate a new strategy and leave everything else as is. In conceptual

change, teachers make an attempt to weave the new practice in with the old by making small alterations to either. Foundational change requires a move away from the old as teachers alter the very foundation of their practice and thinking. It is in the former two, that the phenomenon of virtual tension was more readily apparent.

I conceptualized as a virtual tension that which seemed paradoxical to me: Nicolas' pride in describing his students' multiple solutions, yet his insistence on showing preference for his own method or Jamie's description of her progressive assessment practices that allow for students to demonstrate their understanding followed by her later comment "*I know they got it, because they got 86% on the test*" that appeared to devalue her new assessment practices. I would sometimes jot these paradoxes down to ask about in a later interview. For example, I wrote the following field note during my second visit with Nicolas:

He [Nicolas] has students who finish quickly wait until all the others are finished before he poses the next math problem. There is a tension here between the need to challenge some students while keeping the class moving forward as a cohesive whole. (Field notes, Feb 2018).

This created tension for me as the students who finished early often began to misbehave. From my own experience, I knew that this kind of off-task behaviour can often derail changes a teacher is attempting to make. What surprised me during the subsequent interviews, however, was how infrequently these incidents emerged as tension for the teachers. These were not hidden or subconscious tension that the teachers immediately claimed kinship with upon my making them visible. A question to Nicolas regarding his early finishers brought a reply that was notable for its absence of tension. This was not one of which he was unaware, rather it simply was not a tension for him at all and my mention of the incident evoked no recognition or emotional response. In a sense, then, some tension that I observed was virtual tension. It was real enough in my purview, but did not emerge from the teachers, nor did the teachers give any indication that they were aware the tension existed. Although change might still be occurring, I would argue an unrecognized virtual tension impedes foundational change as, teachers, like Nicolas with his tension with notetaking, attempt to manage related tension without understanding the underlying force that keeps them recurring.

This led me to consider absence of tension, in general, rather than its presence. Tension can be useful for those who accept the conflicts and use them to shape identity and practice (Lampert, 1985). It is tension that often propels teachers towards professional

development and provides the impetus to improve their practice (Rouleau & Liljedahl, 2015). If tension is considered a necessary precursor for change, its absence then necessarily implies that change will not occur. If Nicolas experienced an absence of tension regarding students standing idle, how would he come to recognize the need to change that aspect of his practice?

I suggest a way to achieve this is to deliberately introduce productive tension. Berlak and Berlak (1981) note that because a person is capable of being made aware of tension, they are capable of altering their practice. It is important to keep these two notions distinct; a teacher educator can provide the opportunity for change, but the agency of change lies with the teacher. "Effective change is something that people do to themselves; more radically, but more aptly when investigated closely, change is something that happens to people who adopt an enquiring stance towards their experience" (Mason, 2002, p. 143). However, for this change to happen, the teacher must become aware of possibilities which were not previously available via the deliberate introduction of tension. I could tell Nicolas that there is a tension around early finishers, but until he personally experiences a similar situation himself, perhaps as a learner in a professional development environment, he is unlikely to reflect upon his own practice of harnessing his class so that all move forward at the same pace.

The state achieved after having gone through an experience of productive tension may be preferable to the one before the tension experience, thus justifying the deliberate exposure to potentially negative tension experiences. Introducing tension can challenge teachers' understanding of their practice thereby providing an environment for learning in which their practice model is expanded. It does not guarantee a change in practice, but it does awaken awareness and provide opportunity for the introspective and interspective reflection that Mason suggested is necessary for change in practice to occur. From this vantage, the introduction of productive tension is a way to stimulate and challenge teachers to make sense, to see another perspective, to make obvious what is hidden – effectively to alter the focus of their attention. In so doing, they become their own agents of change.

## Chapter 7. Conclusions and Reflections

There is a story told by Leo McGarry in an episode of *The West Wing* that resonates with me:

This guy is walking down a street when he falls in a hole. The walls are so steep, he can't get out. A doctor passes by, and the guy shouts up, "Hey you, can you help me out?" The doctor writes a prescription, throws it down in the hole and moves on. Then a priest comes along, and the guy shouts up "Father, I'm down in this hole, can you help me out?" The priest writes out a prayer, throws it down in the hole and moves on. Then a friend walks by. "Hey Joe, it's me, can you help me out?" And the friend jumps in the hole. Our guy says, "Are you stupid? Now we're both down here." The friend says, "Yeah, but I've been down here before, and I know the way out." ("Noel", *The West Wing*, 2000)

As a teacher, who both wanted and needed to change, I have been in such a hole. I have been inundated with well-intended prescriptive measures that missed the mark. I have felt unheard and ignored. Then someone jumped down in the hole with me and helped me find a way out. And I have been forever grateful. This thesis has been my opportunity to jump back down in the hole, not only to help others, but also to linger for a bit myself – to examine that hole to see what can be learned from it.

In what follows, I describe those learnings as I briefly revisit the conclusions reached in the previous two chapters. In so doing, I formulate a response to my research questions, before moving on to highlight some implications for mathematics teacher education. I then look forward to the potential for future research, before ending with a final personal reflection.

### 7.1. Identifying and describing tension in change

The literature in Chapters 2 and 3 describes strong connections among tension, agency, and change, in that change is driven by tension and it is also shaped by what a teacher decides to do about the tension. To understand that process more fully, therefore, requires first attending to tension that is present in change. In my study, I chose to focus on a particular type of change (see section 4.2), which allowed for examining more closely tension common to teachers enacting the same change. Accordingly, my first research question is a query regarding tension teachers experience during a particular type of

change: *What are the kinds of tension experienced by teachers who are implementing a Thinking Classroom in their mathematics classroom?* I view this question as trying to understand those steep walls in the hole, walls that I imagine as lined with tension. Identifying and describing that tension provides a little light and is the first step towards a teacher finding her own way out.

That teachers experience tension in change was readily apparent in the data, and I was able to identify twenty-four instances of tension that were then categorized according to the source of the tension: internal or external tension. These are summarized in Table 7.1 as questions that characterize the tension category. This was necessary as I do not presume to have identified every kind of tension, nor will every teacher experience every tension. What I provide is an overview of the kinds of tension teachers are likely to experience during change in mathematics teaching practice.

### **7.1.1. Tension sources**

The forms of tension I identified were categorized as internal or external tension. Internal tension exists within teachers as they experience uncertainty and doubt regarding the choices they have made or need to make regarding their practice. While outside influences do play a role, the genesis of internal tension lies within teachers as they struggle to determine the best course of action and endures as they live out the consequences of their decision. Internal tension accounted for nineteen of the twenty-four identified instances of tension, which indicates the origin of most tension in changing mathematics teaching practice comes from within. This is consistent with the literature, where tension is most often depicted as an “argument with oneself” (Lampert, 1985, p. 182).

I further subdivided internal tension into three types: intrapersonal, interpersonal, and pedagogical. Intrapersonal tension exists when the conflict is between competing aspects of one’s self as teachers wrestle with their identity and their beliefs about the teaching and learning of mathematics. Intrapersonal tension can both motivate and result from change. For example, some teachers come to recognize and articulate who, and how, they *do not* want to be in the mathematics classroom. They are motivated to change as they do not want to be considered ‘that teacher’. This intrapersonal tension between who they are and who they want to be can lead to change in their mathematics practice; who they want to be as a teacher becomes their goal, while who they are, or want to avoid becoming,

becomes an anti-goal (Skemp, 1979). Intrapersonal tension can also result *from* change, as teachers struggle with questions such as why they are implementing change and/or if they should be changing their practice.

Interpersonal tension stems from conflict with the self and others, and arises as teachers negotiate the boundaries of their relationships with stakeholders such as students, parents, and colleagues. When a change in practice is involved, maintaining these relationships can lead to tension as teachers feel the need, for example, to convince others that the change is worthwhile. Pedagogical tension encompasses tension that emerges as a conflict between the self and aspects of practice. Teachers experiencing pedagogical tension find themselves questioning how they can effectively implement the change and how they blend the new changes in with their previous practice.

External tension is tension from without as its genesis lies outside a teacher's control. It arises as teachers encounter policies or expectations imposed on their practice by outside forces such as district policies or government mandates. This leads teachers to question how they will meet their professional obligations while attempting to implement change in their practice.

**Table 7.1      Summary of Tension Sources**

<b>Internal Tension</b> – genesis lies within a teacher
<b>Intrapersonal</b> - emerges from conflict with competing aspects of self  - can both motivate, and result from, change  What kind of teacher do I want to be? Why am I doing this? Should I be doing this? How can I keep on doing this? Is this really better than what I used to do?
<b>Interpersonal</b> - emerges from conflict in relationships with others  How do I convince others that change is worthwhile? How do I convince others to support the changes I am making? How do I convince others to engage with the changes?
<b>Pedagogical</b> - emerges from conflict between self and aspects of practice  How do I do this? How do I blend new practices with my previous practice?
<b>External Tension</b> – genesis lies outside a teacher's control
emerges from expectations imposed on teaching practice by outside forces  How do I meet my professional obligations without sacrificing the changes I am implementing? Am I meeting my professional obligations? How can I accommodate or circumvent outside influences?



### 7.1.2. Tension quality

The results of this study also suggest that, in addition to kinds of tension, there is variance in the quality of the tension itself. To return to the hole analogy, although the walls of the hole might appear uniformly lined with tension, it appears that some of the tension might jut out a little more than others and provide a better toe-hold for change to occur. Along with tension in general, I further distinguished useful tension and productive tension. Together, it is the quality of tension and the degree of agency achieved that determine the type of change that occurs (see Figure 7.1).

*Productive tension* is generative and leads to conceptual and/or foundational change depending upon the degree to which it impacts a teacher's way of thinking about mathematics teaching practice. When productive tension is accompanied by moderate or high levels of agency it can lead to conceptual change; when tension is highly productive and paired with high agency, it can lead to foundational change. Whether paired with moderate or high agency, productive tension occasions the reflexive thinking that enables teachers to view their mathematics teaching practice as a site for transformation.

*Useful tension* is beneficial in that it is the kind of tension that often compels teachers to seek out change. As it prompts teachers to change only surface aspects of their mathematics teaching practice, however, useful tension, even when paired with high agency, at best results in instrumental change. One of the advantages of useful tension, though, is that it creates an opportunity for more productive tension to take hold; a teacher who makes a surface change in her mathematics teaching practice might find it leads to further change. An accumulation of useful tension and accompanying instrumental changes can become the source of praxis that Adler (1998) said is necessary for transformation of mathematics teaching practice.

In addition, there is tension that is neither useful nor productive as it is paired with a lack of agency. Instead, this tension is just experienced and does not result in change. Moreover, as tension is a necessary precursor to change, where there is an absence of tension, there is also no change regardless of the level of agency.

High Agency	No Change	Instrumental Change	Conceptual Change	Foundational Change
Moderate Agency	No Change	Instrumental Change	Conceptual Change	Conceptual Change
Limited Agency	No Change	Instrumental Change	Instrumental Change	Instrumental Change
No Agency	No Change	No Change	No Change	No Change
	Tension			

Productive Tension
  Useful Tension
  Tension
  No Tension

Figure 7.1 Tension, change, and agency matrix

7.2. Managing tension in change

My second research question arose from my curiosity with how teachers contended with the myriad tension they experienced. Specifically, I sought to answer the question: *How do teachers manage tension that arises as they implement a Thinking Classroom in their mathematics classroom?* That teachers manage tension in their mathematics teaching practice is not a novel idea. It was Lampert (1985) who first wrote of the value of seeing tension as something to be managed rather than solved. This was a valuable contribution to mathematics education, as the notion of managing tension resulted in a new understanding of some tension as potentially useful and even necessary in mathematics teaching practice. Managing tension allows for the possibility that some conflicts cannot be solved and the challenge becomes finding a way to maximize the potential of tension without being overwhelmed by its presence.

But ‘manage’ is a broad term. How do teachers actually manage tension? To respond to this question, my natural inclination was to review the literature on tension, which was

surprisingly unhelpful. While, variants of 'manage' appeared most frequently with the occasional substitution of the word 'cope' or 'resolve', what was lacking was any description of the *kinds* of strategies teachers used to manage tension. In fact, Lampert's (1985) study ended with a series of questions regarding tension in teaching mathematics, one of which was "What different kinds of strategies are used in classrooms to cope with unsolvable problems?" (p. 194). Yet I could not find an answer to her query in the literature.

So, as I identified the ways in which my teachers managed tension while making changes in their mathematics teaching practice, I looked for nuances in the data. At first, I thought of these as a dichotomy of proactive and reactive responses to managing tension, but that did not really encapsulate what I was noticing. While my teachers did display instances of both, the focus on 'action' in this dichotomy felt too rigid and narrow. Instead, I was seeing instances of tension management where there was action in response to tension or instances of little or no action in response to tension. During the process of analyzing the tension my teachers experienced, I came to see the former as resolving tension and the latter as living with tension. Both appear to be critical elements in managing tension in change. Where they were able, the teachers made small changes necessary, and where they were not, they found ways to live with the tension.

Resolving was a term already associated with tension in the literature and I found interesting the distinction between solving and resolving. Solving connotes a sense that there is a definite answer that would be applicable across all teaching practices. The meaning of resolve is close to the meaning of solve, but with the difference that resolve is used more generally to conclude a problem. The conclusion reached with resolving something may be one of many choices, and it may not please everyone, but it concludes the problem, even if only temporarily. Resolving is a form of tension management in which teachers actively attempt to alleviate some aspect of tension they are experiencing that is within their power to address. While the resolution may only offer a temporary respite, it is an agentic action as the teachers are choosing to do *something* in the hope of a more permanent abatement of tension. The *something* that teachers in my study most often used to resolve tension was a strategy described as 'onboarding', in which the teachers worked to convince others of the merits of the changes they were implementing. This was an effective way to keep tension temporarily at bay and, in so doing, keep change alive in their mathematics classrooms.

Living with tension describes those instances of tension where teachers take little or no action. It may be that they do not know what to do, or more typically, that they make a choice not to take action. This is also an agentic response to tension as it was clear in my study that there were options and that the teachers made the decision that to live with tension was the optimal choice in particular contexts. The strategy most often associated with living with tension was positioning the source of the tension as an 'outlier'. The outlier strategy is a form of compartmentalization in which the source of tension is considered an anomaly and thus as a small, negative aspect of a much larger, positive whole. This is significant as, although this means that the tension remains, an outlier strategy allows teachers to live with tension and continue with the changes they are implementing.

Complicating teachers' attempts to manage tension is the presence of virtual tension, which is an underlying, unacknowledged tension that interferes with attempts to change mathematics teaching practice. Although the results of my study only hinted at its presence, virtual tension was noticeable from a researcher viewpoint when teachers were attempting to blend old and new practice and it appeared to affect the choices teachers made. For example, a teacher who is determined to continue with a particular aspect of their previous practice may encounter a series of tension as they decide how to alter the new practice to accommodate their previous practice and/or to alter their previous practice to fit with the new. Undermining this effort is a virtual tension that calls into question the merit of pursuing this blend of practice. Until recognized and reflected upon, this virtual tension will continue to exist unabated, creating further tension and complicating the change process. This suggests that, until virtual tension is actualized by the teacher, it is neither useful nor productive; neither resolved nor lived with. It is simply in the background preventing further growth.

### **7.3. Implications for mathematics teacher education and research**

Whether we, as mathematics teacher educators and researchers, express surprise and disappointment at a teacher's modification or accommodation of change, or whether we assume that such alterations are themselves a significant aspect of change, reveals a good deal about how we view change in mathematics teaching practice. Many professional development models comprise a top-down approach in that it is something

that is done to teachers, and teachers who imperfectly implement prescribed change are viewed, at best, as unable and, at worst, as recalcitrant. They are not using, or are not using correctly, the prescriptions and prayers tossed down to them in the hole. Such assumptions do not take into consideration the tension that might enable or constrain teacher change. They also reveal the inadequacy of change theories, where the teacher is positioned as a thoughtless appropriator of change.

There is, however, a growing body of research that seeks to alter those assumptions about teacher change where, instead of being a bystander in change, teachers are thought of as agentic beings responsible for their own growth and change. It is here that I situate my research on tension in changing mathematics teaching practice. Attending to tension in change is a bottom-up approach, one that seeks to make sense of the ways in which teachers exert control over their own growth and change. As a mathematics teacher educator, I can help show them the way out of the hole, but it is the teachers themselves who choose (or not) to do the climbing.

For teachers who are experiencing tension in changing mathematics practice, the research suggests there is value in using that tension as a means for discursive reflection on practice, as Mason (1988) stated, “letting them [tension] out into the open means that they can be robbed of their numbing effect and turned instead into potent sources of energy” (p. 164). While this discourse could include naming and describing tension, I suggest it could also include strategies for both living with and resolving it. And, while it is tempting to consider a prescriptive approach where mathematics teacher educators directly introduce discourse around strategies such as outliers and onboarding, a bottom-up approach means that these strategies arise from a teacher’s own discourse as part of a community of learners. This is a necessary element as, while it is reassuring for a teacher to know that others face the same difficulties, imposed strategies seldom result in foundational change. Rather than proposing what a teacher *should* do, it is more helpful to create opportunities for discussion which elicit suggestions of what a teacher *could* do (Pimm, 1993).

Attention might also be paid to the opportunities a teacher has for achieving agency. What agency offers to an understanding of change in mathematics teaching practice is that, while change is driven by tension, it is also shaped by what the teacher decides to do about the tension. Agency, therefore, offers a way to think about how and why teachers

respond to tension in change. The findings suggest that teachers who are experiencing high levels of tension but lack agency are unlikely to enact changes in practice. For change to occur, it may be necessary to occasion opportunities for achievement of agency. From the teacher as agent perspective, however, an increase in agency does not ensure that change will occur nor, if it does occur, that the change will necessarily appear as an improvement in the eyes of a researcher and/or mathematics educator. Taking into consideration the combination of tension and agency a teacher experiences can help mathematics teacher educators and researchers better understand the apparently inconsistent behaviour we observe; what might be construed as no change or instrumental change could be recast as a rational decision that weighed the practicality of the change against its potential consequences.

Related to this is the potential for mathematics teacher educators to occasion tension. Just as no change occurs without agency, there is also no change without tension. When confronted with an absence of tension, in order to stimulate change, a mathematics teacher educator may need to introduce tension. This would involve raising awareness of other ways of being in the classroom through discussion or activity which, when juxtaposed with a teacher's own experience, creates a useful or productive tension. This leads to an ethical dilemma in that we should not be wanting to 'make' teachers change. Change in this connotation feels negative and unhealthy. Mason (2002) stated, "No one likes to be told to change" (p. 144), before adding that change is more effective when it is something people, upon reflection, do to themselves. Introducing tension provides a new experience for this enquiring stance to reflect and act upon. So, rather than setting out to 'change' teachers, introducing tension is a means of raising teachers' awareness of aspects of practice they may (or may not) want to consider changing.

Finally, I want to point at two aspects of my study that have implications for research in mathematics education. First off, the more recent turn in mathematics education towards teacher change through inquiry, in which teachers are "encouraged to reflect on their practice and to change it where it is appropriate" (Llinares & Krainer, 2006, p. 429), has positioned teachers as active constructors of their knowledge. This is reflected in contemporary mathematics teacher education research where variants of three themes are common across the change literature. The first is the acknowledgement of reflection as an important mechanism for teacher change. The second is the growing awareness of the social dimension of teacher change and the third is the increasing attention to the

organisational context of teachers' work and the extent to which it provides resources to support change (Llinares & Krainer, 2006). As a result of my study, I would recommend extending this trio to include a fourth element: developing an understanding of the tension inherent in teacher change. Useful and productive tension is the driving force behind change; it is a significant and necessary component to growth and development in the dialectical sense. As such, it needs to be taken into consideration when studying mathematics teacher change and/or designing opportunities for professional growth.

The second aspect of my study that affects research in mathematics education also involves extending a model. In Chapter 3, I adapted Pantić's (2015) aspects of teacher agency, so it could be used to identify and describe teacher agency in mathematics teacher change. This is important as mathematics, more than any other discipline, is constrained by norms and expectations that can affect teachers' agency. To understand how agency is achieved in mathematics teacher change then, requires a model that takes into account its unique characteristics. Furthermore, I expanded the model to incorporate the notion of transference, which is a dance of agency that arises in the particular environment of mathematics teaching and learning. In mathematics education, primacy has traditionally been placed on disciplinary agency, as traditional mathematics practices have created contexts in which learners surrender their agency to the discipline as they follow prescribed procedures. Yet the reality is that agency exists with the teacher and occasionally with the textbook. During change in mathematics teaching practice, however, an agentic teacher will shift agency from themselves to the student and to the discipline. This involves the counterintuitive notion of giving away agency to achieve agency. As there is currently no model for understanding mathematics teacher agency, the model I propose may lay the groundwork for further studies.

## **7.4. Questions and areas for future study**

Writing a thesis is an exercise in managing tension; the entire process consists of a series of decisions that involve choosing from a myriad of worthwhile possibilities. In considering methodologies, theoretical constructs, and research questions, each decision altered the path of my journey and I arrived where I did because of the choices I made along the way. As much as I longed for it, there was no predetermined path. Each decision I made marked the next step forward and each option I ignored marked potential avenues for exploration.

I continue to wonder, however, about those unexplored avenues. For example, a possible limitation of my study was that I selected teachers who were already willingly implementing a specific change in their mathematics teaching practice; they wanted to establish a Thinking Classroom. In choosing this path, I excluded teachers who are seeking less specific change and those not really interested in change at all. How would their experience of tension differ? How would their experience of tension management differ? What methodology would be required to tease out these differences? How could the results better inform the design of professional development for teachers?

I also have questions about occasioning tension. Deliberately introducing tension provides an experience that can become the object of discussion and reflection and potentially lead to growth and change. But how should that deliberate introduction be accomplished? Herbst, Nachlieli, and Chazan (2011) have shown that breaches of instructional norms depicted in animated videos can provoke teachers to talk about their practice. Mason (2002) uses the notion of protasis<sup>16</sup> to stimulate teacher reflection that, while not always fruitful, may lead to deeper analysis of practice. Both of these methods are discussion-based. Would introducing tension by having teachers participate in activities that replicate classroom practices (potentially worth changing, such as whole-class multiplication drills) be another possibility? How would those activities be designed? Are there specific practices that might be targeted?

Virtual tension is also a potential avenue for further exploration. I find myself with many questions regarding it, such as 'Are there types of virtual tension?' My conjecture would be that virtual tension would comprise broad categories like time or control, that cut across many tension sources. Would this conjecture hold true? Related to this is the question of whether there are qualities of virtual tension? As with tension in general, does the quality of virtual tension affect the degree of change possible? Additionally, Engeström (1987) postulates the presence of a primary contradiction from which other contradictions arise. How would a primary contradiction compare with a virtual tension?

Finally, while undoubtedly some tension may be applicable to teaching in general, there is the potential for some tension to be considered singular to mathematics practice. Although I did not explore the change literature in other disciplines, from both the data and

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<sup>16</sup> Mason uses protasis as an aphorism, which is a pithy statement that contains a general truth. For example, "Teaching takes place in time; learning takes place over time".



from personal experience, it is apparent that change in mathematics practice is more difficult than change in general and would give rise to tension that other disciplines might not have such as covering the curriculum, notetaking, and homework. For example, Leah experienced tension when her collaborative classroom generated more noise than other mathematics classrooms in her school. That this might not be a tension in other disciplines is apparent as she stated, *“there’s quite a different culture depending on the area of the school you’re in”*. This implies that, while noise may be tolerated in other disciplines, it is considered unacceptable in a mathematics classroom. As Klein (1999) suggested, perhaps more than in any other discipline, teachers must navigate the norms around the teaching and learning of mathematics in order to implement change. I suggest, therefore, that we should think of tension in changing mathematics teaching practice as a special case. One of the earlier metaphors for tension was that of play, in which there is a sense of freedom or movement. I feel this might be an apt metaphor for tension for many disciplines, where the possibility for change is perhaps less restrictive and therefore potentially overwhelming. In mathematics, however, a more pertinent metaphor might be that of a vise where one feels pressure and is hemmed in by the constraints of the discipline itself. This suggests a different feel to the tension in changing mathematics teaching practice and warrants further study.

## **7.5. Personal reflections**

I began this thesis with thoughts of a journey in mind and so, at the close, I return to that metaphor. The Annette that began that journey is not the Annette who is typing this sentence and I am both saddened and grateful for that. I recognize that my early naïveté brought with it an unbridled enthusiasm that carried me far along the journey and allowed me to work through tension that I experienced throughout the research and writing process. In so doing, I found parallels between what my teachers were experiencing in change in mathematics teaching practice and what I was experiencing during the research process. Reflecting on my own tension drew me inward and allowed me to connect more deeply with what my teachers were experiencing. In what follows, I highlight two instances of my own tension and how I grew from it.

## Tension with collaboration

Throughout my time in the doctoral program I have had the good fortune to be involved in many collaborative research projects<sup>17</sup>. I saw each as an opportunity to hone my research and writing skill and I entered wholeheartedly into each new project. Some of them lay outside my phenomenon of interest, but others were directly related. It was with the latter that I began experiencing tension. With each new collaboration, I felt like I was contributing ideas that I had planned to pursue as part of my thesis. This created tension for me, even though I was aware that these ideas were likely generated *because* of the collaboration. In an attempt to resolve the tension, I chose to opt out of collaborative projects where the subject matter overlapped my phenomenon of interest. I began to realize, however, that such opting out was not the right choice for me. I was giving up opportunities to work with experienced researchers in an area in which I was passionately interested. I was giving up noticing how they thought through methodologies, how they went about considering frameworks, how they analysed data. I would likely have been successful in guarding my ideas, but if I gave up collaborating, I gave up a great deal more.

I chose to highlight my tension with collaboration as it came as a surprise. Part of the transformation of my own mathematics teaching practice was the realization of the power of collaboration in fostering rich learning opportunities. As a classroom teacher, I had come to value student collaboration and, like the teachers in my study, worked to make collaboration a regular part of my mathematics teaching practice. When I moved into teaching adults, I brought that same mindset with me and instilled collaboration as the foundation of all my work with adult learners. Upon reflection, what my own tension with collaboration made clear was, what I had previously thought of as transformational change in my own practice was more likely conceptual change; I talked the talk about collaboration, but I did not walk the walk. It was not until I temporarily chose to step away from collaboration that I truly understood its power.

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<sup>17</sup> See for example: Andrà, Rouleau, Liljedahl, & Di Martino (2019); Kontorovich & Rouleau (2018); Rouleau, Ruiz, Reyes, & Liljedahl (2019); Sinclair & Rouleau (2018); Zazkis & Rouleau (2018).

## **Tension with maintaining perspective**

I recall having to address the issue of 'potential risk to participants' when applying for ethics for this study. It was a simple enough answer, I thought; there would be minimal risk. In fact, in my own experience as an interviewee, being part of a research interview was beneficial, as it gave me the opportunity to reflect on my teaching practice. What I had not anticipated was the effect of the research on the researcher. Coding and analyzing the data was an arduous process, and I was immersed for extended periods of time in the struggles of my participants. I found myself becoming burdened by their collective tension, by their doubts and uncertainties. I was surrounded by the raw words, thoughts, and reflections of a group of dedicated, caring human beings, who met with challenge and uncertainty on a daily basis in a determined effort to improve the learning of their students. It was so discouraging.

Furthermore, I was teaching pre-service elementary mathematics methods throughout the duration of my thesis writing. I would look at those fresh, eager faces after having spent a day pouring through a teacher's transcript of her first tension-filled experience with a parent. 'Which of them will experience the same?' I would wonder, knowing full well the answer was likely to be ALL of them. This created tension for me as I struggled with the uncertainty of how best to prepare new teachers for what I knew lay ahead, yet without overburdening them. Borrowing from my own research, my resolution to this tension was to use a proactive approach. As reflection is critical in making use of tension, I made reflective discourse and writing a norm in my classroom, in the hope that it would become habit-forming. I was front-loading skill that I knew the pre-service teachers would need later.

I chose to include this particular tension because, in resolving it, I made use of my own research. I found this very motivating. While I had long been aware that the research process had benefited me personally, I was now able to see its potential in benefiting others. And, although coding and analyzing was still arduous, seeing that potential made the burden worth bearing.

## **7.6. Afterword**

The end of my thesis journey coincided with the beginning of a pandemic. Our entire way of life has been vastly altered and, like people everywhere, teachers are struggling to adapt to the abrupt changes. Seldom does a day go by without my being contacted by teachers with whom I have worked, teachers whom I have taught, or teachers who have participated in my research. Some are checking in on me, but most are looking for help. There is a whole community of us down in that COVID-caused hole right now. And, by 'us', I include mathematics education researchers and educators, as we also adjust to this new way of being in the world. I am heartened by the collective response of teachers and educators who are trying to meet the needs of their learners, and by researchers who are trying to make sense of this new norm.

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## Appendix A: Literature Sources by Field

Literature Source		
Mathematics Education	General Education	Other
1. Adler (1998, 2001)	1. Archer (2000)	1. Althusser (1971)
2. Ainley (1999)	2. Barab, Barnett, Yamagata-Lynch, Squire, & Keating (2002)	2. Archer (2000)
3. Andrà, Rouleau, Liljedahl, & Di Martino (2019)	3. Bean (2006)	3. Bandura (2001)
4. Ball (1993)	4. Berlak & Berlak (1981)	4. Calhoun (2002)
5. Ball, Thames, & Phelps (2008)	5. Berry (2007a, b)	5. Cooper (1917)
6. Barbosa & de Oliveira (2008)	6. Biesta & Tedder (2007)	6. Creswell (2007)
7. Boaler (2002, 2003)	7. Biesta, Priestley, & Robinson (2015)	7. Emirbayer & Mische (1998)
8. Boylan (2010)	8. Carr (1998)	8. Engeström (1987, 2001)
9. Brodie (2009)	9. Carter & Doyle (2006)	9. Gadamer (1976)
10. Brown & Redmond (2008)	10. Cuban (1988, 1992)	10. Giddens (1989)
11. Byers (1984)	11. Dewey (1922, 1938)	11. Granovetter (1973)
12. Carter & Richards (1999)	12. Fecho, Collier, Friese, & Wilson (2010)	12. Hegel (1807/1977)
13. Cavanagh (2006)	13. Finlay (2008, 2012)	13. Heidegger (1927/1962)
14. Chapman & Heater (2010)	14. Floden & Buchmann (1993)	14. Hollis (1994)
15. Chazan & Pimm (2016)	15. Freeman (1993)	15. Husserl (1952/1980)
16. Clarke & Hollingsworth (1994)	16. Guskey (1986)	16. Jardine (1992)
17. Cooney (2001)	17. Hargreaves (2001)	17. Klein (1969)
18. Crosswhite (1987)	18. Henriksson & Friesen (2012)	18. Kvale & Brinkmann (2009)
19. de la Cinta Muñoz-Catalán, Yáñez, & Rodríguez (2010)	19. Jackson, P. (1974)	19. Lavery (2003)
20. De Simone (2015)	20. Jackson, J. (2012)	20. Leont'ev (2009)
21. Dietiker & Riling (2018)	21. Jenkins (2003)	21. Meyer, Boli, & Thomas (1987)
22. Goos & Geiger (2010)	22. Katz & Rath (1992)	22. Mukute & Lotz-Sisitka (2012)
23. Gregg (1995)	23. Kelchtermans (2011)	23. O'Flaherty (1982)
	24. Lasky (2005)	24. Rashotte & Jensen (2007)
		25. Ryan & Deci (2017)
		26. Shor (2005)
		27. Simon (1957)

Literature Source		
Mathematics Education	General Education	Other
24. Grootenboer & Zevenbergen (2009) 25. Hannula (2006) 26. Herbst (2003) 27. Herbst, Nachlieli, & Chazan (2011) 28. Hiebert (2013) 29. Hofstadter (1997) 30. Horn (2012) 31. Hunter (2010) 32. Jaworski (1999, 2006) 33. Jones (1995) 34. Karaağaç & Threlfall (2004) 35. Klein (1999) 36. Kontorovich & Rouleau (2018) 37. Lakoff (1973) 38. Lakoff & Johnson (1980) 39. Lakoff & Núñez (2000) 40. Lampert (1982, 1985) 41. Lerman (2000) 42. Lester (2010) 43. Liljedahl (2014, 2016, in press) 44. Liljedahl, Andrà, Di Martino, & Rouleau (2015) 45. Liu & Liljedahl (2012) 46. Llinares & Krainer (2006) 47. Males, Otten, & Herbel-Eisenmann (2010) 48. Mason (1988, 2002) 49. Norén (2015)	25. Lortie (1975) 26. McLaughlin (1987) 27. Omer (2003) 28. Palmer (1998) 29. Pantić (2015) 30. Pignatelli (1993) 31. Priestley, Biesta, & Robinson (2015) 32. Richardson & Placier (2001) 33. Roettger (2003) 34. Schön (1983) 35. Smith (1999) 36. Tao & Gao (2017) 37. Toom, Pyhältö, & O'Connell Rust (2015) 38. Tsing (2005) 39. Windschitl (2002) 40. Zembylas (2005)	28. Sullivan (2012) 29. Sztomptka (1994) 30. van Manen (2007, 2014)



Literature Source		
Mathematics Education	General Education	Other
50. Page & Clarke (2010) 51. Pickering (1995) 52. Pimm (1988, 1993) 53. Reid & Zack (2010) 54. Roth (2002) 55. Rouleau (2017, 2018, 2019) 56. Rouleau & Liljedahl (2015, 2016) 57. Rowland (1995) 58. Sfard (2008) 59. Sinclair & Rouleau (2018) 60. Skemp (1979, 1987) 61. Smith, J. (1996) 62. Sparrow & Frid (2001) 63. Thomas & Yoon (2013) 64. Wagner (2007) 65. Walshaw (2010) 66. Wheeler (1988) 67. Yackel & Cobb (1996) 68. Zazkis (2000) 69. Zazkis & Rouleau (2018)		

## Appendix B: Sources of Tension in the Literature

Tension in Mathematics Education Literature	Tension in General Education Literature
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Berlak & Berlak (1981)	<p>"Sixteen dilemmas that relate the daily problems of schooling to the social and political problems of the society at large."          "to provide a language for examining the macro in the micro, the larger issues that are embedded in the particulars of the everyday schooling experience" (p. 3).</p>							
Control Set <i>over the locus and extent of control over students</i>	Whole child v. child as student (realms)	Teacher v. child control (time)	Teacher v. child control (operations)	Teacher v. child control (standards)				
Curriculum Set <i>contradictions in how teachers, transmit knowledge &amp; ways of knowing &amp; learning</i>	Personal knowledge v. public knowledge	Knowledge as content v. knowledge as process	Knowledge as given v. knowledge as problematical	Learning is holistic v. learning is molecular	Intrinsic v. extrinsic motivation	Each child unique v. children having shared characteristics	Learning is individual v. learning is social	Child as person v. child as client
Societal Set <i>contradictions related to equity and social relations</i>	Childhood continuous v. childhood unique	Equal allocation of resources v. differential allocation	Equal justice under law v. ad hoc application of rules	Common culture v. sub-group consciousness				

Byers (1984)	Dilemmas in a mathematics classroom.
The desire to keep ideas as simple and straightforward as possible within the framework of the course. However, there is also the opposed desire to explain as completely as possible the phenomena under consideration.	
The goal of technical mastery versus that of theoretical understanding.	
<p>*Dilemma experienced by learners - homeostasis (comfort, safety, self-preservation) versus a set of forces which demand change and development (desire to please teachers and parents to the desire for success and the need for self-esteem) p. 37          "Many problems which teachers encounter in the attitudes of their students - from rebelliousness to passivity - may be understood as the students' reaction to the tension which arises out of this conflict." (p. 37).</p>	

Mason (1988)	Tension in mathematics education.
Keeping control	Keeping a class under control yet providing opportunity for individuals to explore, to express their own ideas.
Time	Taking the time necessary for students to really understand a topic yet exposing them to everything that is expected.
Confidence vs challenge	Using methods to instill confidence that remove the challenge (e.g., simplifying tasks).
Product vs process	Wanting students to participate in mathematical thinking and to take initiative but their attention is on learning what they are told they have to learn, on being able to do the questions.

Autonomy	Gattegno's "the subordination of teaching to learning" - getting students to work for themselves rather than for the teacher.
Intervening	When should I intervene (join a group, guide, check in) and when should I stay back?
Didactic contract	The teacher's task is to foster learning, but it is the student who must do the learning. Often, they wish to invest a minimum of energy in order to succeed. Everything the teacher does to make students produce learning behaviour the teacher expects, tends to deprive the student of the conditions necessary for producing the behaviour.

Katz & Rath (1992)	Dilemmas in general teacher education - while not a comprehensive list, these are enduring and especially nettlesome.
Coverage versus mastery emphases	All teacher educators face conflicting pressures to emphasize either coverage or mastery of the content and skills to be taught. The more content and skills pupils cover, the less they can master, and vice versa (p. 377).
Evaluative versus affective emphases	Teacher educator's role is to evaluate the progress of candidates [...] Another part of the role, is the obligation to address learners' needs for support and encouragement (p. 378).
Emphasis on current versus future needs of candidates	Teacher educators may be disposed to do something for their candidates that will satisfy them at this time without considering what is best for their students in the long term (p. 379).
Thematic versus eclectic approaches	Education programs that are designed around coherent themes versus programs where faculty are encouraged to do their own thing (p. 380). "The horns of this dilemma re whether to organize teacher education programs around a theme, philosophy, or model or to give faculty license to take an eclectic approach in which each faculty member advocates a preferred philosophy or pedagogical model" (p. 381).
Emphasis on current practice versus innovative practice	Teacher education programs can prepare candidates to perform successfully in today's schools or to preparing candidates for schools that would represent improvements over today's schools (p.381). "Thus, a teacher education program faculty can choose to focus on helping candidates acquire competence in the current standard practices of the schools available to them or program faculty may give priority to helping candidates learn the most recently developed innovative practices - ones that are rarely seen in today's schools." (p.382)
Specific versus global assessment criteria	A teacher education program might define its objectives concretely and specifically, making clear to candidates precisely what is expected of them. Alternatively, it might define its objectives in ways that make use of broadly defined constructs (p.383).

Cuban (1992)	A core trilemma of three overlapping cultural values
University values of reflection, rigorous analysis, and scientifically produced research versus values within a professional school of applying disciplinary knowledge to practical situations in order to prepare the next generation of teachers, administrators, and researchers versus values within schools where action is prized and the knowledge that is admired is concrete, relevant, drawn from experience, and applied to the practical dilemmas of teaching and learning.	

Ball (1993)	Self-study of teaching mathematics to third graders.
Representing the content	What are the hooks that connect the child's world with particular mathematical ideas and ways of thinking?
Respecting children as mathematical thinkers	Respecting children's thinking even while helping students to acquire particular tools, concepts, and understandings. "Very difficult to figure out what some students know or believe - either because they cannot put into words what they are thinking or because I cannot track what they are saying" (p. 387).

Creating and using community	Using the classroom as a learning community, a community of mathematical discourse where the teacher is not the authority Striving "to be a learning <i>community</i> and also to be <i>learning</i> community." (p. 388). "The dilemmas inherent in trying to use the group to advance the individual and vice versa, all while keeping one's pedagogical eye on the mathematical horizon, are not trivial" (p. 398).
Trying to be "intellectually honest" in teaching mathematics	Creating a practice that is honest to mathematics and honoring of children "To do this productively, I must understand the specific mathematical content and its uses, bases, and history, as well as be actively ready to learn more about it through the eyes and experiences of my students." (p. 394).

Gregg (1995)	Tension that underlies the beliefs and practices of the school mathematics tradition in high-level and low-level mathematics classrooms.	
Justifying the school mathematics tradition - High level	Both teachers and students view school mathematics as inherently unmotivating.	
Engaging students in proofs and developing students' reasoning abilities - High level	Attempts to engage students in proofs conflicts with the emphasis on procedures, form, conventions, and rules.	
Emphasizing form and rules - High level	The emphasis on form and procedures in school mathematics helped students achieve their successes (helped them to appear competent), but it also contributed to their failures (difficulties).	
Assessing students' in nontesting situations - High level	Ego involvement results in some students being afraid to ask questions in class for fear of looking dumb in front of their peers.	
Employing tests as measures of students' understanding - High level	Test scores as the primary measure of students' understanding.	
Engaging students in general mathematics - Low level	Students see no intrinsic or extrinsic reasons for engaging in the study of mathematics.	
Maintaining control and the constitution of mathematics - Low level	Adherence to rules and procedures exacerbates control of classroom management problems.	
Proceduralizing and decomposing mathematics - Low level	Reducing mathematics to the association of certain procedures with certain types of problems be successful contributed to students' difficulties on tests.	
Accounting for student difficulties – Low level	Difficulty in accounting for students' incorrect answers.	

Jones (1995)	Tension/dilemma in becoming a better mathematics teacher.	
Getting a perspective on the nature of mathematics	Within a teacher	A teacher's view of mathematics directly affects the way in which he or she teaches mathematics.  Either math is a discipline composed of rigid rules and correct answers or the belief that mathematics can be found in almost anything involving patterns, interpretation of symbols, or logic (p. 230).
	Between teachers	E.g., those who disagree on the importance or meaning of problem solving.
	Between teacher/admin and community	Teachers or admin who wish to begin offering low-level classes for students who have difficulty in mathematics and members of the community who believe that such an action is only a short-term solution.

	Between teacher and students	Goal for student is to try to pose and resolve their own questions but reality is students who simply want to be told what to do.
Teaching for conceptual and procedural knowledge	The tension that teaching for conceptual knowledge is preferred yet takes too much time, will not foster basic skills. Also, tension arises from lack of teacher's own conceptual knowledge and their ability to teach it.	
Managing competing responsibilities	Tension occurs between the enjoyable aspect of helping students develop their understanding of mathematics and job-related teacher's duties (time).	

Carr (1998)	Tension in teaching at the university level (all of his tension headings start with "tensions created by").	
What the teacher values	What the teacher values, the university too often does not value.	
Teacher's definition of knowledge	The research university tends to define "the advancement of knowledge" primarily in the sense of recognized scholarly research whereas a teacher's definition may be more the more immediate advance experience of "the advancement of knowledge" in the classroom and laboratory.	
Economies of time	The better one teaches the more teaching asks of one's time, and the less it returns.	
Primary audience for teaching	Students as expert consumers of teaching are very quick "to bestow contempt or admiration according to a very few critical standards".	
Rewards for excellent teaching	The great teacher is often rewarded by larger classes and less time for research and reflection; and by a reputation for attending to students. The poor teacher is often rewarded by fewer students and more time for research and reflection.	
Standards of excellence	Apart from the other challenges of teaching, the successful teacher is also challenged to document and construct a credible public case for his or her own growth and skill.	
Shallow evaluations of teaching	Opportunities for critical, formative evaluations of teaching quality are rare. The challenge is to remember that a great teacher is always more than the sum of the evaluations; and so is a poor one.	
Constructed situation of teaching	What should our structures of teaching be? What is it about the classroom that makes us think it is a good way to change human lives? What do we want to have happen there? What kind of transformation do we believe we can cause in these often-barren spaces?	
Striving to lead a private life	Teaching requires the teacher to bear the burden of maintaining a living balance in one life.	

Carter & Richards (1999)	Dilemmas of constructivist mathematics teaching in middle school.	
What ideas to pursue	Deciding what to teach.	
To tell or not to tell	Figuring out what to tell students directly and what to push them to figure out on their own.	
Time	Resolving the conflict between their commitment to student exploration and their felt need to cover material.	

Jaworski (1999)	Dilemmas in the mathematics classroom through a social constructivist lens.
To inculcate or to elicit	Concerns interaction between students and the teacher with regard to the construction of knowledge. The student's task is to construct mathematical knowledge. The teacher's task is to support and challenge this construction. It might also be described as a conflicting intersection of two paradigms: an objectivist paradigm in which the required curriculum and its examination structures are based, and a constructivist paradigm in which the teaching is situated (p. 164).
Didactic or Investigative	Wanting to implement an investigative approach to teaching without being certain how to do so.

Zazkis (2000)	Extends Adler's dilemma of code-switching between languages to code-switching between the mathematical and everyday registers of English in a monolingual mathematics classroom.
Modelling	To model appropriate mathematical usage of concepts without 'talking too much'.
Mediation	To encourage students to express their ideas but also to develop appropriate communication skills for those ideas in the mathematical register.
Transparency	A balance between emphasizing appropriate language per se and utilizing mathematical language in mathematical activity or problem situations.

Adler (2001)	Dilemmas in multilingual mathematics classrooms.
Code-switching	Responsible for helping students understand and pass mathematics [...] yet also responsible for ensuring students are competent in mathematical English. <i>If they stick with English, students often don't understand. Yet if the "resort" to [primary language] they must be "careful", as students will be denied access to English and to being able to "improve" (p. 2).</i>
Mediation	Tension between validating diverse learner meanings and at the same time intervening so as to work with learners to develop their mathematical communicative competence (p. 3). <i>While it is important for learners to explore, explain and argue their interpretations and ideas, they easily "lose track". Coming up against important conventional mathematical meanings requires her intervention (p. 3).</i>
Transparency	Tension between implicit and explicit language practices. <i>"Explicit mathematics language teaching appears to be a primary condition for access to mathematics" yet "There is always the problem [...] of 'going on too long', of focusing too much on what is said and how it is said." (p. 5) "... when students are involved in task-based activity and generate informal ways of speaking mathematically [...] mathematical descriptions are partial or quasi-mathematical [...] sometimes 'they do it right but say it wrong'." (p. 4).</i>

Sparrow & Frid (2001)	Dilemmas of beginning teachers in primary mathematics.
Beliefs versus recommendations for teaching	Exposure to ideas and pedagogical practices that could be said to fit within a constructivist learning perspective [...] that are sufficiently different to cause conflicts.
Learning with understanding versus learning to perform	Tension between teaching for understanding and teaching to demonstrate correct responses.
Risk taking versus playing safe with familiar teaching approaches	Tension of trying different or non-traditional teaching strategies to reach pedagogical goals.
Less able students versus the rest of the class	Tension of concentrating on the middle or teach to the less able.

Windschitl (2002)	Dilemmas in a general constructivist classroom.			
Conceptual	Disconnections Between Theory and Practice	Which Constructivism?	Internalizing a Constructivist Epistemology	
Pedagogical	Student Understanding as the Focus of Classroom Practice	Managing Classroom Interaction and Discourse	Understanding Content	Assessing Students' Knowledge
Cultural	Understanding Classroom as Culture	Disjunctures Between School Culture and the Lives of Students		
Political	Confronting issues of accountability with various stakeholders in the school community; negotiating with key others the authority and support to teach for understanding.			

Herbst (2003)	Used the theory of the didactical contract to analyze tension in implementing a novel task.
Where to direct students' activity	<i>On the one hand, the teacher may be compelled to maintain students' attention to the product that was explicitly set as an expectation of their work. On the other hand, the teacher may be compelled to seize the possibilities for the development of new ideas that students' work offers to him or her. p. 206</i>
How to represent mathematical objects	<i>On the one hand, a teacher may be compelled to identify precisely which features of the representations used task are relevant to the mathematical ideas targeted by the task. On the other hand, the teacher may be compelled to maintain a certain degree of vagueness regarding what in those representations is relevant so that students are reserved the opportunity to mathematize, to make deliberate choices instrumental to their inquiry. p. 206</i>
How to elicit students' conceptual actions that are instrumental for the task	<i>On the one hand, the teacher may be compelled to give students unambiguous directions and constraints that indicate to them what they are expected to do and think about as they work on the task. On the other hand, the teacher may be compelled to maintain a productive ambiguity about directions and constraints, keeping the task open for students to come up with actions that make sense to them as being instrumental in completing the task.</i>

Karaağaç & Threlfall (2004)	Used Activity theory to analyse a tension of a secondary mathematics teacher in a private school.
Beliefs and practice	Between his view on how math should be taught and his own classroom practice.

Berry (2007a)	Dilemmas as a teacher educator in biology.
Telling and growth	<ul style="list-style-type: none"> <li>• between informing and creating opportunities to reflect and self-direct</li> <li>• between acknowledging prospective teachers' needs and concerns and challenging them to grow.</li> </ul>
Confidence and uncertainty	<ul style="list-style-type: none"> <li>• between making explicit the complexities and messiness of teaching and helping prospective teachers feel confident to progress.</li> <li>• between exposing vulnerability as a teacher educator and maintaining prospective teachers' confidence in the teacher educator as a leader.</li> </ul>
Action and intent	<ul style="list-style-type: none"> <li>• between working towards a particular ideal and jeopardising that ideal by the approach chosen to attain it.</li> </ul>
Safety and challenge	<ul style="list-style-type: none"> <li>• between a constructive learning experience and an uncomfortable learning experience.</li> </ul>

Valuing and reconstructing experience	<ul style="list-style-type: none"> <li>• between helping students recognise the 'authority of their experience' and helping them to see that there is more to teaching than simply acquiring experience.</li> </ul>
Planning and being responsive	<ul style="list-style-type: none"> <li>• Between planning for learning and responding to learning opportunities as they arise in practice.</li> </ul>

Barbosa & de Oliveira (2008)	Tension experienced by elementary teacher in the practice of mathematical modelling.
Students' involvement	Tension of choosing a theme that would guarantee that the students would develop and participate.
Understanding of the activity of modelling by students	Tension in relation to students understanding the activity so that it guaranteed his/her accomplishment of it.
Students' comprehension of mathematical content	Tension about what the students knew about mathematical ideas and algorithms.
Deciding what to do	Tension of which decisions to be made in certain moments of the class.

Page & Clark (2010)	Used Activity Theory to analyze tension in primary mathematics classrooms.
Tension within the subject	Personal experiences as a learner of mathematics have resulted in a lack of confidence in teaching mathematics. Wants math to be fun and enjoyable but has very few enjoyable personal experiences with math.
Tension between subject and community	Collaborating with colleagues (community) who are more advanced. Wanting practical resources as compared to philosophical discussions.
Tension between community and object	Tension between the student (community) perceptions of mathematics and the object of creating numerate mathematicians.
Tension between rules and object	The struggle to teach the curriculum (rules) and simultaneously developing mathematics lovers (object).

Horn (2012)	Paradoxes in school mathematics.
Of control	On the one hand, schools are bureaucratic institutions. Central offices make decisions about curriculum and policy which teachers are meant to implement. At the same time, teachers work in relative isolation and with a high degree of autonomy. Loosely coupled control structures minimize the need to coordinate with colleagues and, as a result, the demands remain low for a technical language for teaching (p. 26).
Of student participation	In many parts of the world, students' attendance is compulsory. At the same time, those in authority often act with a presumption of voluntary attendance, positioning students who do not wish to attend as deviant, "unwilling," or "unmotivated" (p. 26).
Of timescales for learning	The timescales of schooling may not always align with the timescales of meaningful learning (Lemke, 2000). Whether it is the fifty-minute lesson or the nine-month academic year, teachers, who greatly depend on the emotional rewards of their work, look for success in these units of time when, in some cases, students' development may take place on different timescales. Timescales dictate pace and normalize certain patterns of development, positioning different learners as "fast," "slow," "ahead," or "behind" (p. 26).
Of the individual in the crowd	Classrooms are among the most crowded settings in which people spend time, with more people per square foot than even jails, yet teachers are expected to respond to individual learners, with "adaptation" and "individualization" being highly valued practices. Thus teachers' time and attention becomes a precious commodity. Expeditious systems of reward and evaluation manage to both conserve this resource and sort students, while potentially devaluing deeper and more meaningful student learning (p. 26).



Thomas & Yoon (2013)	Uses Schoenfeld's (2011) ROG to analyse tension of a secondary high school teacher.
To prepare students for success on future tasks	Between wanting his students to know everything and wanting them to develop that knowledge actively.
To engage in student-centred learning	Between wanting his students to construct their own understandings yet that takes too much time.
To fulfil the requirements of curriculum, time and assessment	Between speeding up his teaching to cover the material and quality of teaching.
To respect students' cultural influences on their learning	Between various cultural norms - quiet classrooms, teacher-led instruction, shared mistakes/problems vs his student-centred approach.

Liljedahl, Andrà, Di Martino, & Rouleau (2015)	Used and expanded Berry's (2007a) tension pairings to analyse tension of a fictional mathematics teacher aggregate.
Confidence and uncertainty	Confidence in becoming a teacher undermined by the reality of the classroom.
Intent and action	Desire to teach progressively versus the actual implementation of those practices.
Tradition and innovation	Traditional teaching practice versus progressive teaching practice.
Safety and challenge	Between wanting decrease students' mathematics anxiety and wanting to push them further.
Valuing and reconstructing experience	The difficulty in reconstructing student teacher experiences seen as valued.
Telling and growth	Between creating opportunities for students to construct their own knowledge and the transmission model of teaching.
Conforming and personal convictions	Feeling the pressure to conform to school norms and personal beliefs about the teaching of mathematics.
Time and results	Between teaching for deep understanding and the pressure for immediate results.

Rouleau & Liljedahl (2015)	Used Berry's (2007a) tension pairings to analyse tension of an elementary mathematics teacher.
Telling and growth	Between the desire to avoid telling and instead focus on growth of student understanding through experience.
Confidence and uncertainty	Between exposing a weakness in mathematics and maintaining the respect of students.
Action and intent	Between assessing students in a way that best matches the way content was learned yet relying on summative assessment that interferes with that aim.

Rouleau & Liljedahl (2016)	Used Berry's (2007a) action and intent tension pairing to analyze pre-service teachers' intention to implement timed mathematics drills.
Action and intent	Between the desire to use drills as a means of practice and assessment and the recognition of the harm these practices can inflict.

Rouleau (2018)	Used Activity Theory to analyze a secondary mathematics teachers journal-writing implementation.
Between subject and tool	Between the desire to have students writing to learn in mathematics and the ineffectiveness of its implementation.
Between rules and community	Challenging students' expectations of mathematics.
Between subject and object	Maintaining a relationship with students while engaging them mathematically.

## Appendix C: Phase 3 Email Exchange

**Thursday, Feb 8/18**

**Subject: Vertical surfaces today**

Hi Annette,

Today I tried again vertical surfaces with another "least common multiple" problem, but of different style. (minimal square surface filled by a rectangular tile - as opposed as "when do you meet again"). I added two letters A and B to the back of the cards and then stated that the letter B would start with the pen. This visibly randomized pen choosing strategy worked well.

One autistic child refused to come in the classroom. Another one joined a group, tried a bit, then left the classroom. I left the single persons alone - I did not tried to make them join a group. One child (the one who did not like it yesterday) tried a bit then called in sick and went to the office (but she has been not well since the beginning of the day - so it sounds half true). Everyone else was calm and engaged.

One group shared their solution to everyone else. The students asked for another question after we shared on the first one, but some did not want a word problem, just some arithmetic. I suspect the language made the entry to the work a bit difficult, (few had understood on the first go the word "pavé" which is the French for tile. So, they wanted an easier entry. So, I gave them a fraction addition and it went well as well.

Upon discussing, all liked it stating that it was better than yesterday - calmer, except one who found it "boring" (one student who has the habit of sleeping on her desk - I suspect that because one has no choice but to actually be engaged - that surely is a more annoying than sleeping in !). Some students looked at her with the look saying "how could you find it boring?". I did state that she had the right to her opinion.

Overall, huge success. I'll keep at it tomorrow!

Thank you for your advises full of insight yesterday, I hope you feel better!

Regards,

Nicolas

**My response:**

That's wonderful! Keep it going!

Try to ensure you don't have too many singles - once you let one student work alone, others sometimes want to as well... and the collaborative benefits diminish.

I'm so happy that this has been a positive experience for you and for your students.

**Friday, Feb 9/18**

**Subject: Vertical surfaces**

Hi Annette,

I tried Friday as well, it went well, a bit more noisy than Thursday.

The beauty is that one student girl who sat in the group of 3 the very first time we did it, was happy to be in a team of two and actually requested more problems to do once we finished the first one. This really sounded magical to see her engaged this much!

In order to minimize the groups of one as your suggestion, I amended the set of cards based on the number of absent students. This way, I get either one or none groups of one.

I had some issue with one group where a very competitive student could not stand to be with a less proficient one and kept getting an extra pen for himself. I strictly forbid it. He ended up complying and I later congratulated him privately for sticking to his group and working better with his partner.

I am going to try to work with vertical surfaces every day and email you each time my impressions. This way, maybe we can keep a log of how it goes and see trends and areas for improvements.

Regards,

Nicolas

**My response:**

This memory log is a good idea and will help you look back later.

As for the competitive boy, privately praising him was awesome - you are working to build a culture where they respect and tolerate the strengths and weaknesses of everyone in the classroom - not just those who are like themselves. The competitive boy needs to understand that you value the collaboration as much as the speed/correctness. And you showed him that!

## Appendix D: Potential Guiding Interview and Follow-Up Questions

1. Where do you teach?  
Describe your school.  
What grade/level do you teach?  
Describe your support structure (colleagues, admin, PLC).
2. Did you always plan to be a teacher?  
Are you surprised to find yourself here?  
What led you to it?  
What are your main objectives/goals as you teach?
3. Describe a time when you made a change in your practice.  
What has been the easiest to implement?  
What has been the hardest?  
Is there something you want to implement that you have not yet? Why?  
Is there something you are doing that you want to stop but have not been able to? Why?  
Is there something you will always see yourself doing now? Never doing?  
What has been your students' reactions to change? Colleagues? Parents?
4. What are you best at in teaching math?  
Is there a specific thing that is more of a challenge for you in teaching math?  
What is the hardest mathematical topic to learn or teach at your school level?
5. If I was able to compare your classroom in your first year of teaching and now, what differences would I see?  
Compare how you teach to how you were taught.  
Describe something that you believe to be sound practice but that maybe research or other people tell you, you shouldn't be doing, but you do it anyways.  
Are there any differences between what you do and what other teachers do?
6. Describe a time when you felt pressure from the ministry, curriculum, administration, colleagues, parents, or students to do something.  
Did you agree with it?  
Did you change? Why?  
Is there something you are doing (or not doing) right now that others are telling you needs to change? How do you resist this?
7. Tell me about your curriculum.  
Does it fit with what you teach?  
Does it fit with what you think kids should know?  
Does it fit with how you teach?
8. What are your thoughts around assessment? (Describe your approach.)  
Does it affect what you teach?  
Does it affect how you teach?
9. Is there anything else you would like to tell me?

## Appendix E: Teacher-Prepared Handout

### Correction du travail sur tableaux – (fractions, leçon 19)

Élena vient d'acheter une nouvelle télévision. Elle a payé 392 \$. Elle a eu de la chance car le magasin d'électronique lui a fait un rabais de 20 %. Comme elle habite en Colombie-Britannique, elle a dû payer des taxes de 12 %. Retrouve le prix initial de la télévision avant taxe et avant rabais.\*

Première étape : Calculer le montant avant taxe

*Si on considère que le montant initial représente 100 % du prix, le montant avec taxe représente 112 % (100 % + 12 %)*

112 %  $\rightarrow$  392 \$ (montant avec taxe)

100 %  $\rightarrow$  ? (montant sans taxe)

On connaît 112 % du montant, 1 % du montant est donc  $\frac{392}{112} = 3,5\$$ .

On connaît maintenant 1 % du montant, on cherche 100 %, ce qui est donc  $3,5 \times 100 = 350\$$ .

Le prix avant taxe est donc 350 \$

Deuxième étape : Calculer le montant avant rabais

*Puisque le rabais est de 20 %, 350 \$ représente 80 % du prix avant rabais*

80 %  $\rightarrow$  350 \$ (prix avec rabais)

100 %  $\rightarrow$  ? (prix initial)

On connaît 80 % du prix, 1 % du prix est donc  $\frac{350}{80} = 4,375$

On connaît 1 % du prix, on cherche le 100 %, ce qui est  $4,375 \times 100 = 437,5\$$

Le prix avant taxe et avant rabais est donc 437,5 \$

Ryan veut acheter une nouvelle carte mémoire pour son ordinateur. Mais elles ont récemment augmenté de 4 %. Elles coûtent maintenant 26 \$. Quel était leur prix avant augmentation ?†

*Le prix a augmenté de 4 % donc 26 \$ représente  $100+0,04=104$  % du prix initial*

104 %  $\rightarrow$  26 \$ (prix avec augmentation)

100 %  $\rightarrow$  ? (prix avant l'augmentation)

Il faut donc diviser 26 par 104 pour trouver le 1 % :  $\frac{26}{104} = 0,25$

Puis multiplier le résultat par 100 pour trouver le 100 % :  $0,25 \times 100 = 25\$$

(On peut aussi faire plus rapidement :  $\frac{26 \times 100}{104} = 25\$$ )

Le prix de la carte mémoire avant l'augmentation est donc de 25 \$

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\*. Exercice adapté de <http://www.alloprof.qc.ca/8V/pages/m1033.aspx>

†. Exercice tiré de [http://www.capte-les-maths.com/pourcentage/les\\_pourcentages\\_p10.php](http://www.capte-les-maths.com/pourcentage/les_pourcentages_p10.php)

## Devoirs – (fractions, leçon 19)

Un DVD est soldé pour 16,5 \$ avec un pourcentage de rabais de 25 %. Combien coûtait le DVD avant les soldes ? \*

Une paire de skis coûte 156 \$ après une augmentation de 20 %. Combien coûtait la paire de ski avant l'augmentation ? †

Une belle montre coûte 224 \$ après un rabais de 20 % et des taxes de 12 %. Quel était le prix de la montre avant le rabais et sans les taxes ? *Un rabais est une réduction du prix.*

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\*. Exercice tiré de [http://www.capte-les-maths.com/pourcentage/les\\_pourcentages\\_p10.php](http://www.capte-les-maths.com/pourcentage/les_pourcentages_p10.php)

†. Exercice adapté de <http://fabie.info.pagesperso-orange.fr/math/cours7.htm>



## Appendix F: Teacher Board Notes and Student-Created Notes

- ① Si  $\frac{3}{4}$  d'un nombre est 8 alors quel est  $\frac{9}{4}$  du nombre?
- ② Calcule 10% de 110 (sans opération!)
- ③ Un cœur bat 10 fois en 10 secondes. Combien de fois bat-il en 2 mn?
- ④ Anouk, au cinéma, part au bout de 20 mn d'un film qui dure 3 h. Quelle fraction du film a-t-elle vu?

- ⑤ Noahmy a reçu 100\$.  
Son oncle lui donne 50% de plus.  
Puis son grand-père lui donne 50% de plus.  
Combien a-t-elle?
- ⑥ Une imprimante doit imprimer tous les nombres naturels entre 0 et 100. Elle peut imprimer un maximum de 3 chiffres par seconde. Combien de temps durera l'impression?

4 avril 2018

① Si  $\frac{3}{4}$  d'un nombre est 8 alors quel est  $\frac{9}{4}$  du nombre?

② Calcule 10% de 110 (sans opération!)

③ Un cœur bat 10 fois en 10 secondes. Combien de fois bat-il en 2 minutes.

④ Anouk au cinéma, part au bout de 120 minutes d'un film qui dur 3 heures. Quelle fraction du film a-t-elle vu?

⑤ Noahmy a reçu 100\$. Son oncle lui donne 50% de plus. Son grand-père lui donne 50% de plus. Combien a-t-elle?

⑥ Un imprimante doit imprimer tous les nombres naturels entre 0 et 100. Elle peut imprimer un maximum de 3 chiffres par secondes. Combien de temps durera l'impression?

Réponses

①  $\frac{3}{4} = 8$      $3 \overline{) 24}$     Donc,  $\frac{9}{4}$  de ce nombre est 24.  
 $\frac{9}{4} = ?$      $3 \times 8 = 24$

② 10% de 110 = 11 (Tu bouge la virgule un fois vers la gauche)  
NOTE!:

③ ♥ bat 1x par secondes    Donc, un cœur bat 120 fois dans 2 minutes.  
 $2m = 120s$

④ 120m = 2h    Donc, Anouk a vu  $\frac{2}{3}$  du film.  
 $\frac{2h}{3h} = \frac{2}{3}$

⑤ Noahmy → 100\$

Oncle donne 50%+

Grand-père donne 50%+

Oncle donne 50\$

$$100 + 50 = 150$$

Grand-père donne 75\$

Donc, Noahmy a 225\$ en tout.

50% de 100 = 50

50% de 50 = 25

50% de 150 = 75

$$\begin{array}{r} 100 \\ + 50 \\ + 25 \\ \hline 175 \end{array}$$

$$\begin{array}{r} 100 \\ + 50 \\ + 75 \\ \hline 225 \end{array}$$

