Three essays on occupational choice, financial market frictions, learning and dynamic incentives

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Abstract

This thesis consists of three essays studying the ramifications of financial market frictions. The first two chapters focus on occupational choices in the presence of learning and market frictions (credit constraints and weak enforcement respectively). A common theme of the second and third chapters is strategic default and dynamic incentives in a microfinance market.

In the first chapter, I develop a dynamic occupational choice model combining financial constraints with learning, i.e. the exploration of the agent's own entrepreneurial ability. The occupational choice is between entrepreneurship and a fixed-wage job. What I find is that, if learning does take place, financial constraints not only postpones entrepreneurship, but also cause a long-run effect in reducing the number of entrepreneurs in the economy. In other words, learning perpetuates the welfare loss caused by borrowing constraints. Using PSID data, I find evidence consistent with learning. The observed business entry and exit patterns cannot be explained by borrowing constraints alone, but can be explained by my model with both borrowing constraints and learning.

In the second chapter, I investigate the impact of loan enforcement on the experimenting time of the micro-entrepreneurs who rely on loans for their working capital in each period, and the impact on their expected lifetime payoff. Like in the first paper, I assume agents learn their entrepreneurial abilities by running business projects. I find that experimentation is less with default possibility than without. Besides, the possibility to default strategically leads to a lower expected lifetime payoff.

In the third chapter, I analyze the efficacy of dynamic incentives. Shapiro (2015) points out an inherent fragility of dynamic incentives in microfinance without collateral or long-term loans. He shows that the dynamic incentive mechanism unravels for all except a single value of initial beliefs. In this chapter I show that his concern is overcome by a small (yet crucial) modification of the environment by introducing any proportion of "commitment type" borrowers who never default by nature. I prove that all inefficient equilibria in Shapiro's model are ruled out by adding an infinitesimal proportion of commitment-type borrowers. Moreover, a unique efficient equilibrium exists. In the unique equilibrium the loan terms become more favourable over time, and the proportion of non-defaulters converges to one.

Keywords: Occupational choice; Learning; Dynamic incentives; Credit constraints; Strategic default; Microfinance; Commitment type

Dedication

This work is dedicated to my parents, Hua Zhou and Carol Qu, who first taught me to read, write and explore the world. Thank you Dad for urging me to speed up and make progress. And thank you Mom for reminding me to slow down, trust God and give thanks.

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Chapter 1

Learning, Liquidity Constraints and Occupational Choice

1.1 Introduction

An extensive literature investigates the reasons behind the occupational choices of individuals, for example, why certain individuals run their own business while others choose fixed-wage jobs. The majority of occupational choice literature models the expected profitability of business projects as known to the agent who makes the choice. In contrast, in this chapter, I assume that individuals' entrepreneurial abilities are not known precisely ex-ante, but are revealed over time as business projects are carried out. I will refer to this as learning, i.e. the exploration of the agent's own ability. My approach is in line with a growing body of research that recognizes that the outcome of a career, a business, or a research project cannot always be correctly anticipated. (See Bernardo & Welch (2001), Bergemann & Hege (2005), Antonovics & Golan (2012), Kerr *et al.* (2014) and Konon (2016).)

In this chapter, I model the interaction between borrowing constraints and learning and their influences on the occupational choice of individuals.This takes into account the liquidity constraints faced by agents who consider to start their own business. This is consistent with the prevalent evidence of liquidity constraints in the developing world (e.g. Paulson & Townsend (2004), Paulson *et al.* (2006), Nguimkeu (2016) etc) as well as the developed world (e.g. Evans & Jovanovic (1989), Buera (2009), Fairlie & Krashinsky (2012) etc). [1]

I find that the presence of learning amplifies the welfare loss generated by the borrowing constraints. Specifically, while the liquidity constraints can postpone the time to start business by an individual who knows that running a business is the optimal occupation, liquidity

[1] One rare exception is Hurst & Lusardi (2004) which argues that liquidity constraints are not an important factor affecting the decision to start a business by showing with PSID data that business entry changes little with wealth except for the top of wealth distribution. Fairlie & Krashinsky (2012) however argues that the flat relationship between entry and wealth is mainly driven by those who become entrepreneurs after experiencing an involuntary job loss.The authors show that the positive relationship between wealth and business entry is restored when controlling for job loss.

constraints are more likely to prevent business (re)entry permanently if the entrepreneurial ability is only revealed by running a business.

In this chapter, I build a dynamic model in which forward-looking agents make occupational choices and choices on saving, borrowing and consumption for finite periods of time. The decisions today matter for the future through two channels. First, savings made today help overcome the borrowing constraints in the future, which is similar to Buera (2009) and Nguimkeu (2016). Second, the occupational choice determines whether more information about the entrepreneurial ability, which affects future decisions, will be revealed. While Buera (2009) implies the borrowing constraints' effect on welfare works mainly through the intensive margin, i.e. delaying the time agents start their business, my paper suggests that borrowing constraints not only affects *when* people become entrepreneurs but also have a long-term effect on *who* will ever start a business at all. In my model, when agents can only learn their entrepreneurial ability by experimenting, borrowing constraints will cause some people to choose the fixed-wage job and never run a business, while the same people optimally would eventually become entrepreneurs if borrowing constraints were absent. To get some intuition, the incentive to explore is stronger for younger people, because the information obtained from learning is likely to be useful for a longer time. For some liquidity-constrained individuals, since it takes time to grow out of the constraints, by the time they could potentially have saved enough to open the business, they would already have lost the appetite to learn more, i.e. running a business no longer yields the highest expected payoff, unless they are optimistic enough, i.e. their ex-ante belief of their own entrepreneurial ability is high enough. Thus those individuals who do not have high enough belief will never start a business even if they actually have high ability.

Since my model suggests that learning would exacerbate the welfare loss caused by borrowing constraints, it is worth investigating whether learning indeed shapes individuals' occupational choices in addition to credit constraints. Given ample evidence of the credit constraints, my empirical work will focus mainly on finding the evidence for learning.

Using PSID data from 1984 to 1994, I investigate the relationship between the likelihood to start a business, and wealth levels of households, ages of heads, and the duration of time without a business. Consistent with the prediction of credit constraints, richer households are more likely to start a business. However, there are some additional facts that cannot be explained by credit constraints alone, but can be understood if learning is considered. First, the difference in the likelihoods of starting a business between the wealthy and less wealthy households is smaller for the young than for the old. This observation is explainable by a model with learning, because learning incentive, i.e. the incentive to explore one's entrepreneurial ability,[2] can counteract the difference in the likelihood of entering business caused by wealth. When one gets older, the learning incentive diminishes, and the difference

[2] Alternatively, the quality of a business project.

caused by wealth stands out. A model without learning, however, cannot explain this observation. On the contrary, because households are selected into their favourable occupation over time, we expect the difference of entry likelihood between the wealthy and the less wealthy to be smaller among the old than that among the young if learning is not present, which is the opposite to what the data suggest. Second, in the data, the young are always more likely to enter a business and less likely to exit than the old, regardless of wealth. These facts are again consistent with the stronger learning incentive among the young who explore by doing. In addition, even among the wealthiest, an entrepreneur who has run a business for a short duration is more likely to exit than one who has a long duration in business, suggesting that those who exit are those who have learned that entrepreneurship is not their best option.

Learning, or experimentation, is receiving increasingly more attention in the literature in fields of occupational choice, e.g. Antonovics & Golan (2012) and Konon (2016), entrepreneurship, e.g. Kerr *et al.* (2014), and firm learning, e.g. Jovanovic (1982) and Arkolakis *et al.* (2018). However, the modeling of the interaction between credit constraints and learning in occupational choice is rare. One exception is Nyshadham (2013). However, while that paper assumes learning and found some evidence for it, in the model occupational choice in a given period is made only by picking the occupation that yields the highest expected payoff in that period. In other words, agents are myopic and the incentive to "learn" is not incorporated as one reason to run a business. In comparison, I model perfectly forward-looking agents. As it turns out, agents' incentives to explore plays an important role in making model predictions and explaining the data.

The rest of this chapter is organized as follows. Section 1.2 introduces and analyses the model, and gives numerical examples. Section 1.3 discusses some empirical analysis. Section 1.4 concludes.

1.2 The model

1.2.1 The setup

Each agent has two occupational choices, taking a fixed-wage job or being an entrepreneur, i.e. running his own business. Agents have different initial wealth levels. Loans are available to finance a business project. However, the amount of money a household can borrow is limited. Specifically, there is a restriction that a household with initial wealth z can borrow at most $(\lambda - 1)z$ (with $\lambda \geq 1$). The borrowed amount, as well as the initial wealth, can be used to buy capital for the business project. Thus the maximum amount of capital a household can invest in the project is λz.

An entrepreneur can have either high ability or low ability, which is fixed throughout the lifetime. Assume it is only possible to run a business when the agent invests more than k_m dollars every period. The fixed cost k_m can be interpreted as rent, insurance, etc.

There are two stages a project could go through, and the turning point is the occurrence of a "breakthrough". For example, if the project is a restaurant, the breakthrough might be the moment when it is recommended by someone relatively influential. Before the breakthrough (stage I), the project yields its regular output $f(k)$ (where $k \geq k_m$ is the amount of capital invested). The breakthrough happens with probability q to those who have high entrepreneurial ability, and never happens to those with low ability. In the very period that a breakthrough happens, there is a bonus $G > 0$ in addition to the regular output. In every period after the breakthrough (stage II), the bonus comes with probability q' where $1 \geq q' \geq q$.

Ex-ante the agent knows ϕ, the objective probability that one has high entrepreneurial ability (given the observable characteristics), but what he does not know is if he has high or low ability himself . He will update his belief every time the project is run. Let us use p_τ for the belief that one has high ability at the beginning of the τ-th period. If the project yields a "bonus", then the borrower knows instantly that he is a high ability entrepreneur, i.e. $p_\tau = 1$ for all $\tau > t$; otherwise, his belief will be updated according to the Bayes rule:

$$p_{t+1} = \frac{p_t(1-q)}{p_t(1-q) + (1-p_t)}$$

where p_t is the borrower's belief that he has high ability at the beginning of period t, and p_{t+1} is the updated belief after the project is run. If the fixed-wage job in period t is chosen, then the belief is not updated, i.e. $p_{t+1} = p_t$. Assume $p_1 = \phi$ for all agents.

I assume default is impossible,(this assumption will be justified in the next section by a minimum-capital restriction) and the borrower has means to save. The interest rate R therefore is equal to the risk-free interest rate. [3] Agents are risk-neutral, so $R_f = 1/\delta$, where δ is the borrower's discount factor. If the agent does not choose entrepreneurship, he has an income of s from the fixed-wage job.

The agent's problem is to choose an occupation, which is indivisible, and to choose saving/borrowing in each period so as to maximize the discounted value of life-time utility.

1.2.2 Model Analysis

In this section, I focus on modeling agent's learning incentive and its interaction with borrowing constraints. And I will leave the discussion of the occupational switching patterns to the next section where some simulation results will be shown.

[3] I assume that the interest rates for borrowing and saving are the same.

Parameter Assumptions

First, assume $f(k^*) - Rk^* < s < f(k^*) - Rk^* + q'G$, where $k^* = \arg\max_{\{k\}}\{f(k) - Rk\}$. This means that the outside option is bigger than the regular output, but smaller than the expected output of a good project.

Second, as long as the capital invested is more this minimum k_m, a borrower can always repay, even just with regular output. [4] That is, $f(k) - Rk \geq 0$ for all $k \in (k_m, k^*]$. The agents are not allowed to start a business if they cannot invest at least k_m. This ensures that as long as an agent is allowed to run his own business, no involuntary default will happen.

The Learning Incentive

I assume the time horizon is finite with T periods, in order to model that the learning incentive changes with life cycle. As time approaches the end, the learning incentive decreases. I assume away borrowing constraints here and focus on the "learning" incentive. Let p^c_τ be the cutoff belief, above which the borrower chooses entrepreneurship at period τ, The proposition below shows that p^c_t is strictly smaller than p^c_{t+1}, the cutoff probability one period later. The proposition describes that the longer an individual expects himself to live (or work), the stronger motivation he has for learning about his own ability (or the project quality), and is therefore less demanding on the ex-ante belief about the project quality.

Proposition 1. *Without borrowing constraints*[5]*, for any $t \geq 0$, $p^c_t < p^c_{t+1}$, i.e. the cut-off belief to have high entrepreneurial ability, above which agents choose entrepreneurship, increases as time approaches the end.*

Proof. See appendix. □

If time horizon is instead infinite, then one easily sees that the cutoff belief does not change over time. In fact, we can show that when there are τ periods left, the cut-off belief of having a project, above which one chooses entrepreneurship, is:

$$p^c_{T-\tau} = \frac{(1-\delta)(s - f(k^*) + k^*)}{q[(1-\delta)G + \delta(1-\delta^{\tau-1})(f(k^*) - k^* + q'G - s)]}$$

.

A few implications can be derived from this proposition. First, it suggests that, without borrowing constraints, once an agent chooses the fixed-wage job, in which learning is impossible, he has no incentive to become entrepreneur in the future, as long as the environment

[4]Trivially, a borrower would not be able to repay if "too much" is invested, because of diminishing returns. But no rational agent would do it.

[5]This means that agents can always run their business projects with capital amount k^*.

does not change. Second, all else equal, agents with more periods left will be more likely to become entrepreneurs because of stronger learning incentives. Thus if there is exogenous shock affecting life expectancy or expected number of years left in career, it may affect the agent's occupational choice too.

Indeed, as will be discussed in section 1.3, PSID data suggest that, the young households are more likely to start a business than the old, given wealth level and duration of not having a business. Similarly, using Thai data, Nyshadham (2014) found that young people are more likely to switch from the fixed-wage job to the entrepreneurial occupation than old people. Compared to the old, young people will have bigger learning incentive, because they expect to have more future periods to benefit from the information obtained through experimenting.

Borrowing constraints' effect on learning

Now we impose borrowing constraints such that the maximum amount of money an agent can borrow in any period t is a multiple $\lambda - 1$ of their wealth z_t,i.e. $k_t \leq \lambda z_t$. Now in addition to p^c_τ, the cutoff belief when agents are not facing borrowing constraints, define $q^c_\tau(z)$ as the cutoff belief for an agent with wealth z at time τ if they face borrowing constraints. In other words, with borrowing constraints, an agent with z at the start of τ would choose entrepreneurship if and only if he believes his ability to be high with at least a probability of $q^c_\tau(z)$. The next proposition suggests that borrowing constraints discourages experimenting since the cutoff belief would be higher with borrowing constraints than without.

Proposition 2. *Given wealth z and number of periods left $T - \tau$, the cutoff belief of having a "good" project, above which an agent chooses entrepreneurship, is (weakly) higher when borrowing constraints exist than otherwise, i.e. $q^c_\tau(z) \geq p^c_\tau$. Strict inequality holds if the borrowing constraint is binding in the current period, i.e. if $\lambda z < k^*$.*

Proof. See appendix. □

Proposition 2 suggests that, when facing borrowing constraints, some agents who would otherwise choose entrepreneurship in any period instead stay in the fixed-wage job. In fact,this is common to all models with borrowing constraints. However, if we combine the two propositions above, we will realize an additional channel through which borrowing constraints effects entrepreneurship. With only borrowing constraints, initially poor agents with high ability will eventually become entrepreneurs as long as they can save enough during their lifetime, to make entrepreneurship feasible and more favourable than the outside option. Thus borrowing constraints do not cause any long-run misallocation of entrepreneurs without learning.

In comparison, we know that with learning, occupational misallocation would take place if a good entrepreneur mistakenly believed his ability to be low and permanently exited because of a series of bad shocks. However, when borrowing constraints exist in addition to learning, there would be additional long-term occupational misallocation among the poor who are more wealth constrained. The reason is that the incentive to learn diminishes over time. As one takes time to save to overcome borrowing constraints, entrepreneurship might already become a less favourable option than a fixed-income job as time passes. Now illustrate this point with a simple example.

Imagine a model with only two periods and a breakthrough happens immediately when a high-ability entrepreneur runs a business. Assume that one has an initial belief p_1 that he is a high-ability entrepreneur, and p_1 is above the cutoff belief for the rich, but below that for the poor when there are two periods left. So a poor person with p_1 would not run a business in period 1, while a rich person with p_1 would. In period 2, even if the initially poor person is no longer constrained, it is possible that his belief, which is still p_1, is below the cutoff when there is only 1 period left, in which case this person would never start a business. If the person actually has high ability, a breakthrough would happen in the first try, but an initially poor person would permanently miss it while an initially rich person would not. Therefore, there is a permanent difference in the proportion of business owners between the rich and poor. On the contrary, if learning does not exist, this misallocation would disappear in period 2 as long as the high-ability entrepreneurs who are initially poor can save enough to prefer entrepreneurship by then. The low-ability would never enter a business regardless of their initial wealth. Thus the difference between the rich and poor is more likely to be temporary without learning than with.

The next proposition suggests that the effect of borrowing constraints on entrepreneurship is not only temporary, as suggested in Buera (2009),[6] but can also be permanent when learning is present. Now assume that the initial belief of agent i to have high ability, p^i, is taken from a distribution with Probability Density Function (PDF) $f_p(p^i) > 0$ for all $p^i \in (0, 1)$, and the initial wealth distribution has PDF $f_z(z) > 0$ for all $z \in (0, \bar{z})$ with $\bar{z} > 0$. [7]

Proposition 3. *a) Let T be the time horizon. If learning does not exist, for any given distribution of initial wealth, there exists a time period $\tau > 0$ such that the proportion of*

[6]The feature that credit constraints are overcome over time and only cause a delay of entry is also seen in Midrigan & Xu (2014).

[7]The heterogeneity and continuity of initial belief in $(0, 1)$ and of initial wealth in $(0, \bar{z})$ are sufficient conditions for the strict inequality in Proposition 3 b) to hold. (As we can see from example 1, the result can still hold without at least one of these assumptions.) First, with this assumption, there must exist some low-wealth agents who would not start a business initially with borrowing constraints though they would without, because of the continuity of wealth distribution in $(0, \bar{z})$. So borrowing constraints at least delay the time to start a business for some agents. Second, the continuity of initial belief in $(0, 1)$ (and therefore, the continuity of p_t in $(0, 1)$ for all t) ensures that there will always be agents who would drop out completely under borrowing constraint and learning ("BCL") while they would not do so under learning only ("L"), because their business was delayed. Alternatively, instead of assuming the heterogeneity of initial belief, we could assume the heterogeneity and continuity of the outside option s within a certain range.

entrepreneurs in the economy with borrowing constraints will be equal to the fraction of good entrepreneurs in the economy, for all $t \in [\tau, T]$, when T is large enough. [8] *b) However, if learning exists and the initial belief of agent i to have high ability, p^i, is taken from a distribution with Probability Density Function (PDF) $f(p^i) > 0$ for all $p^i \in (0, 1)$, then there exists $\tau' > 0$ that the proportion of entrepreneurs is strictly lower with borrowing constraints (i.e. in the "BCL" economy) than without (i.e. in the "L" economy) for all $t \in [\tau', T]$.*

Proof. See appendix. □

According to proposition 3, borrowing constraints can result in permanently lower proportion of entrepreneurs in a learning environment, while the proportion of entrepreneurs under borrowing constraints will eventually catch up if learning does not exist. In other words, borrowing constraints have bigger impact on agents' occupational choices when learning is present.

Example 1. *In this simulation, there are* 30000 *agents of different wealth levels. Among them,* 84% *have high ability and* 16% *have low ability. And every agent starts with the initial belief of* 84% *that he has high ability. For agents with "good" projects, they experience different sequences of "luck" when running their business.*[9] *All agents have the same outside options $s = 0.085$, and a good project always yields a bonus with probability* 0.5*, regardless of the breakthrough. That is, $q = q' = 0.5$. I assume that $\lambda = 2$, or the maximum amount one can borrow is equal to the wealth of the agent, and the wealth itself can be invested in the project through capital.*

According to the simulation, when the project types are known and borrowing constraints are present (treatment BC), the agents with good projects will eventually overcome the borrowing constraints, as we can see from figure 1.1. So in this case the borrowing constraints' effect on entrepreneurship is only temporary as in Buera (2009) and the percentage of population that are entrepreneurs increases almost monotonically over time until it reaches 84%*, the proportion of good projects in the economy. (And we can call as "efficient percentage"). When the agents are not facing borrowing constraints but do not know project types ex ante (treatment L), the percentage of entrepreneurs ends up around* 65% *at period 3 and changes no more. With this particular parameter configuration, everyone chooses entrepreneurship initially, and the agents who eventually quit consist of all those with bad projects and some who have good projects but never experienced good outputs in the initial trials. Thus when ability is unknown, there are around* 19% *of the population who suffer from occupational misallocation if borrowing constraints does not exists.*

[8] This ensures that constrained agents will escape borrowing constraints over the life time, and that enough experiments will be carried out to eliminate overconfidence that cannot last.

[9] The simulation includes 252 different high/low output sequences (for good projects) in total, and 100 wealth levels from 0.001 to 0.100 with an increment of 0.01 for each sequence. So we have $100 \times 252 = 25200$ agents with good projects and 4800 agents with bad projects.

When we combine learning and borrowing constraints (treatment BCL), the proportion of entrepreneurs in the population changes non-monotonically as time passes and ends up at 42%. *The gap between the proportions of entrepreneurs in L and BCL treatments is persistent, whereas the proportion in BC eventually catch up with that without borrowing constraints or learning. This result is consistent with Proposition 3 that borrowing constraints combined with "learning" will have permanent effect on the agents' occupational choices. And as we can see, borrowing constraints cause misallocation for an an additional* 23% *of the population if learning exists.*

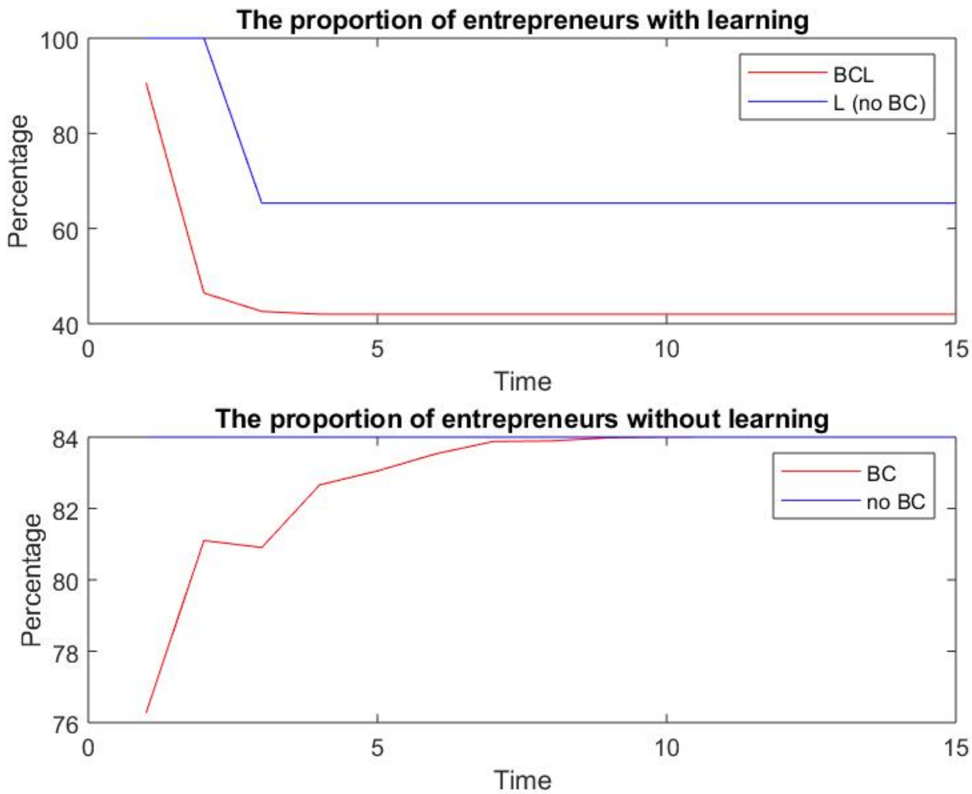

Figure 1.1: Proportion of entrepreneurs in the population (example 1)

The effect of learning on borrowing constraints

The analysis above implies that the existence of learning matters because it determines whether borrowing constraints have a long-run or short-run effect on the proportion of business owners in the economy. Now I explore another fact when learning interacts with borrowing constraints. This fact will also be used later when I use PSID data to find evidence in favour of the existence of learning.

Proposition 4. *Suppose $T = 2$ and that the old has 2 periods left while the young has 1 period. If both learning and borrowing constraints exist, then there exists $\epsilon > 0$, such that for any production function $f(k)$ satisfying $-\epsilon < f''(.) \leq 0$, the young have a (weakly) lower cutoff wealth, above which they will choose to run a business, than the old, all else equal. In comparison, the cutoff is the same for both the young and the old if learning does not exist.*

This proposition implies that wealth matters less for younger households in their decision of becoming entrepreneurs, all else equal. This is because for the young, their learning incentive is strong, and their benefit of learning offsets part of the income loss of running a business with less capital invested because of borrowing constraints. Therefore, all else equal, the young do not require as high initial wealth to start a business as the old to optimally choose entrepreneurship. I will prove this proposition analytically for $T = 2$, and do a numerical example with more periods.

Proof. See appendix. □

Now I use a numerical example to show that this is also true with $t > 2$.

Example 2. *Assume that wealth constraints exist and $\lambda = 2$. Once a breakthrough happens, a project yields a bonus in every period after it (including the very period that a breakthrough happens), i.e. $q' = 1$. I vary the life expectancy of agents (or the total number of periods left at the beginning) from 1 period to 50 periods, and numerically calculate the cutoff wealth levels. The probability that a breakthrough happens to an entrepreneur with high ability is 0.6 for all agents ($q = 0.6$), and the objective probability for a random agent to have high entrepreneurial ability is $\phi = 0.7$ (which is also the ex-ante belief which every agent starts with). The regular output given input k is $f(k) = k^{0.5} - 0.3$ and the bonus output value $G = 0.3$.*

In this particular example, if life expectancy is $T < 5$, no one would choose entrepreneurship regardless of wealth. However, once there are at least 5 periods left, agents with wealth level above certain thresholds would optimally become entrepreneurs. In figure 1.2, I plot the relationship between cutoff wealth levels and life expectancy. The red line plots the cutoff wealth for a good entrepreneur to enter if learning does not exist, and the blue line shows the corresponding cutoffs if learning exists. We can see that the threshold of wealth decreases with life expectancy (or increases with age) if learning exists, but not if learning does not exist. Instead, if learning does not exists, the cutoff wealth level is constant except for life expectancy short enough. In fact, the volatility of the cutoff in the BC treatment for that range of life expectancy is caused by the concavity of production function. If we replaced the production function with a linear one, the cutoff would be constant with life expectancy.

Proposition 4 implies that, if learning is present, wealth level may play a bigger role in the business entry for the old (or those with shorter life expectancy) than for the young. This

Figure 1.2: Cutoff level of wealth for entrepreneurship to be optimal (example 2)

is because a weaker learning incentive among the old makes the current-period monetary payoff more crucial for occupational choice. This difference does not exist if learning does not exist.

Learning and exit

In a world without borrowing constraints or learning, those with high entrepreneurial ability (or high-quality business projects) would enter a business once and for all. And without macro demand shock, it would be hard to observe business exits. Even if exits indeed happen because of macro shock, exiting pattern would be independent of duration in business.

In contrast, if learning exists, and business quality (or entrepreneurial ability) reveals over time, households would experiment for some periods and exit if no breakthrough comes. Eventually, households who ever started a business would either learn that they are good, or have exited because of continual low outputs. Thus with learning and no borrowing constraints, households with long duration in business are not expected to exit.

Proposition 5. *Learning can cause the probability of business exiting to decrease with duration including when borrowing constraints do not exist.*

I show this through another simulated example.

Example 3. *In this simulated example, borrowing constraints does not exist. Though agents start with the same ex-ante belief of having good projects ($\phi = 0.7$), they differ not only in the actual quality of projects (whether it is good or bad), the sequence of "luck" if they have a good project, but also their outside option value, s. Figure 1.3 plots the relationship between duration in business and the proportion of entrepreneurs exiting a business among those who have survived this duration. As we can see in this simulation, in general, the longer one has had a business, the less likely he exits.*

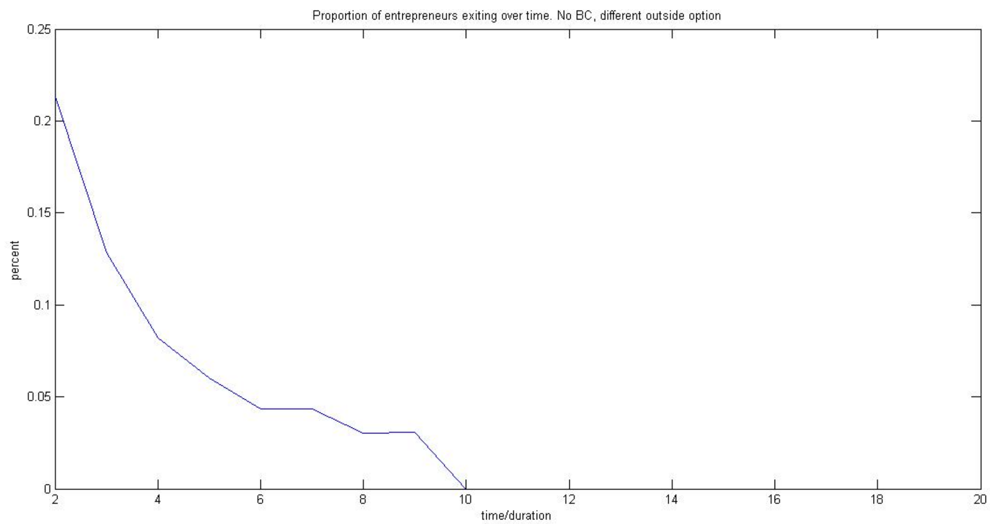

Figure 1.3: Proportion of entrepreneurs who switch out over time (example 3)

If borrowing constraints exist, there is another possible reason why the probability of exiting a business can fall with duration in business. That is, as one has owned a business longer and accumulated more wealth, more capital can be invested into the project, and a single low output is less likely to cause someone to exit the business. In comparison, for someone who is relatively new in the business, it is more likely that he has not yet accumulated much wealth, and a bonus output is more crucial for him to maintain enough wealth to be able and willing to continue with his business.

Therefore, focusing on the wealthiest population, who are least likely to be affected by the wealth constraints, will enable us to interpret the declining trend of exit probability with duration as a result of learning.

To summarize, if some or all of the following are observed from the data, it is consistent with a model where both learning and borrowing constraints are present, but unlikely to fit a model without learning.

First, let us focus on the richest population, who are less likely to be impacted by the borrowing constraints than the rest of the population. If among them, the younger are less likely to exit and more likely to enter, a more plausible explanation is the stronger learning incentive of younger households. (Proposition2, 4.)

Second, if even among the rich, the probability of exiting a business decreases with duration of having a business, then it is most likely to be caused by selection through learning. As households engage in the business activities, the low ability entrepreneurs drop out over time, and there is a larger chance that those who survive are good entrepreneurs who are unlikely to exit. (Proposition 5.)

Third, only with learning is it possible to observe that wealth makes a smaller difference in the likelihood of entering a business among the young than among the old. This is because a stronger learning incentive among the young can make having a business a more attractive option even with less wealth (or less capital to invest into it), and the role of wealth is thus less important for occupational choice. (Proposition 4.)

More details about predictions will be given in section 1.3.3.

1.3 Data Analysis

1.3.1 Data and descriptive analysis

I use a 11-year panel of the PSID data from 1984 to 1994, which contain information on occupations, family business, educational levels, assets, income etc. I calculate wealth as the sum of home equity, money on the checking/saving account and the value of business/farm. These data are available for 1984, 1989 and 1994. For each of the three years, I divide the households into five (almost) equal-sized groups based on the distribution of the total wealth. (For example, those in the first group have total wealth in the top 20%.) Furthermore, for each of the three years, if a household is in the top 40% of the wealth distribution, then it belongs to the “wealthy” category. Thus there are in total three measures of total wealth, and three measures of whether a household is “wealthy”.

The proportion of households who have a business is relatively constant throughout this 11 years, with an average of 10.7 percent. The wealthier households are more likely to be an entrepreneur (see Figure 1.4). For example, among the households that are in the top 40% of wealth distribution in 1984, 19.65 percent reported to have businesses in 1994, whereas the percentage is only 8.38 for those who are in the bottom 60%. Besides, given the duration in business, the wealthy exit business less frequently than the less wealthy. On the other hand, the wealthy are more likely to start a business, given the duration of time without a business (see Figure 1.5 and Figure 1.6). All these facts are consistent with the existence of borrowing constraints.

On average, households with younger heads are more likely to be entrepreneurs. For example, among the households with heads younger than 40 years old in 1984, 11.42% reported having businesses in 1994, compared with 10.63% among those with heads older than 40 years old. This difference, though not significant, is consistent with the model implication that younger households are more likely to run businesses because of a stronger learning incentive. Figure 1.4 plots the proportion of households who reported to own businesses, for years 1984 to 1994, grouped by whether they were young or wealthy in 1984. Besides, as we can learn from Figure 1.5 and Figure 1.6, young households are less likely to quit a business and more likely to start a business regardless of wealth level, consistent with the stronger learning incentive among the young.

It is also evident from Figure 1.5 and Figure 1.6 that the longer one stays in one occupation, the more unlikely it is to switch to another. In other words, households with a long tenure in businesses are more likely to continue running a business in the next period. On the other had, households with a long time without a business are less likely to start a business. The fact that business exit likelihood decreases with duration in business even among the wealthiest (the top 20%, see Figure 1.7), who are least likely to be facing financial constraints, is another evidence for learning.

Furthermore, I explore whether the prediction of proposition 4 applies to the data, in other words, whether wealth is indeed less important for the young to become entrepreneurs than the old. I get 2471 observations of households switching from having no business in the past (since 1984) to having a business for the first time. [10] I find that among those households who switch from non-business to business for the first time, the majority (around 77%) belong to the "wealthy" category if the head of the household is over 40 years old in 1984, while only about a half are in the "wealthy" category if the head is younger than 40 years old in 1984. This is robust to which of the three wealth measures is used. [11] It suggests that wealth is a less important factor in occupational choice for the old than for the young. Similar result holds if we have more detailed age categories, for example, if ages are divided based on "younger than 30","31-50" and "older than 50".

1.3.2 Regressions

Firstly, I investigate the relationship between the probability of entering business and the age of the household head, wealth level, and the duration of time that a household does not have a business. I adopt the complementary log log model, a model used extensively in survival analysis.

Let $\lambda(t|X)$ be the probability for a household with characteristics X to enter a business by time t given that the households does not have a business in the past $t-1$ periods.

[10] Those who reported having a business in 1984 are excluded.

[11] Among all the sampled households who ever had experience without a business, that is, 14970 in total, 50-60% of older households are "wealthy", while the proportion of the wealthy among the younger households is 20-30%.

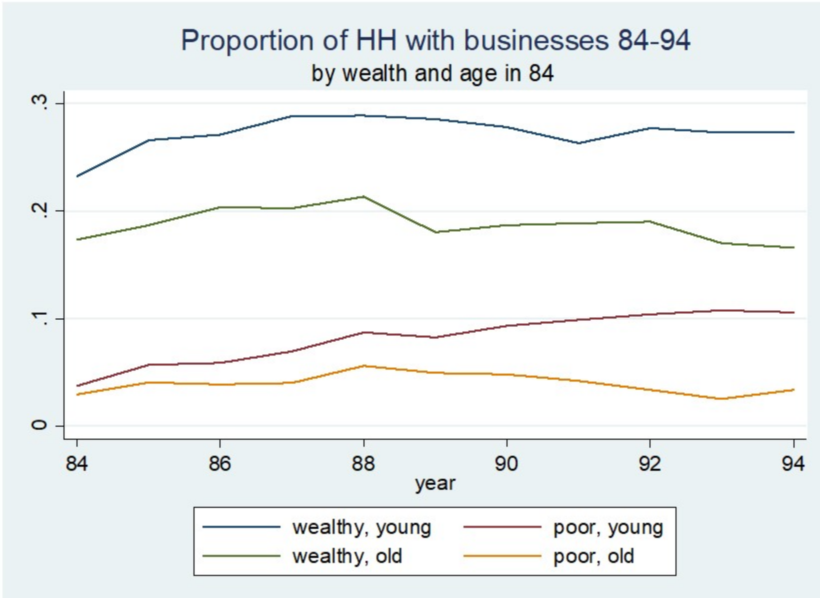

Figure 1.4: Proportion of entrepreneurs from PSID data

Household characteristics X can include age of the head (when they start the current spell), wealth level, whether the household has business experience before the current spell, etc. In complementary log log model, we have

$$1 - \lambda(t|X) = [1 - \lambda_0(t)]^{\exp(X'\beta)} \tag{1.1}$$

for any period t, where $\lambda_0(t)$ is the “baseline hazard function”. [12]

The equation 1.1 is equivalent to

$$\log(-\log(1 - \lambda(t|X))) = \alpha(t) + X'\beta \tag{1.2}$$

where $\alpha(t) = \log(-\log(\lambda_0(t)))$. Notice that the LHS of 1.2 is an increasing function of $\lambda(t|X)$.

On business entry, the baseline regression model I use is

$$\log(-\log(1 - \lambda(t|X)) = \alpha + \beta_w W + \beta_y Y + \beta_{wy} W \times Y + \gamma_t$$

where $W = 1$ if the household belongs to the wealthy category (in the top 40% of the wealth distribution), and $W = 0$ otherwise; and $Y = 1$ if the head is younger than 40 years old in

[12] This model can be derived from the continuous-time proportional hazard model $\theta(\tau|X) = \theta_0(\tau)\exp(X'\beta)$ by grouping time τ into intervals. Thus it applies although in reality the households make decisions in continuous time, instead of once a year. Besides, this model allows the baseline hazard rate to be varying within each time interval.

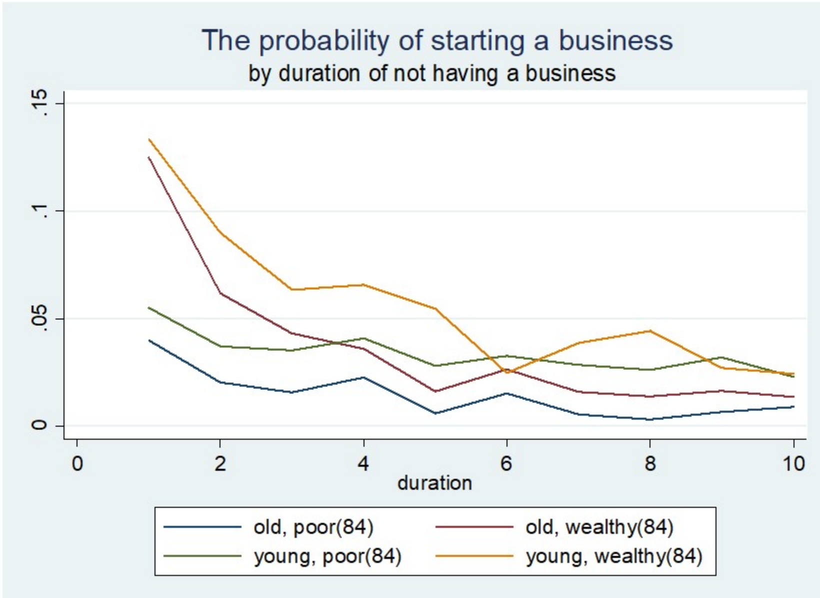

Figure 1.5: Proportion of households switching into business (PSID data)

1984, and $Y = 0$ otherwise. And in my regressions $t \geq 2$. Therefore, the difference between the rich and the poor among the young is measured by $\beta_w + \beta_{wy}$, and the that among the old is measured by β_w. Thus if wealth affects the old and the young differently, β_{wy} would be significantly different from zero. From γ_t we can calculate the likelihood of starting a business if an old, less wealthy household has t periods without business.

Second, I also checked the opposite side: the relationship between business exit and the duration in business, wealth level and age. The regression is similar, with the baseline model

$$\log(-\log(1 - \lambda'(t|X)) = \alpha' + \beta'_w W + \beta'_y Y + \beta'_{wy} W \times Y + \gamma'_t$$

where $\lambda'(t|X)$ is the probability of exiting a business, for households with characteristics X and whose business already survived t periods.

In the baseline regressions I use the total wealth of 1984 as the wealth measure, since 1984 is the first year of the panel and the wealth of 1984 better captures the idea of "initial wealth". I also run robustness checks using total wealth of 1989 and 1994. For both business entry and exit, I run regressions for all households, households in the top 40% of the wealth distribution, and households with wealth in the top 20%.

1.3.3 Predictions

A model with borrowing constraints and a model with both borrowing constraints and learning would share some common predictions. However, the existence of learning does make a difference in the some predictions. The details are as follows.

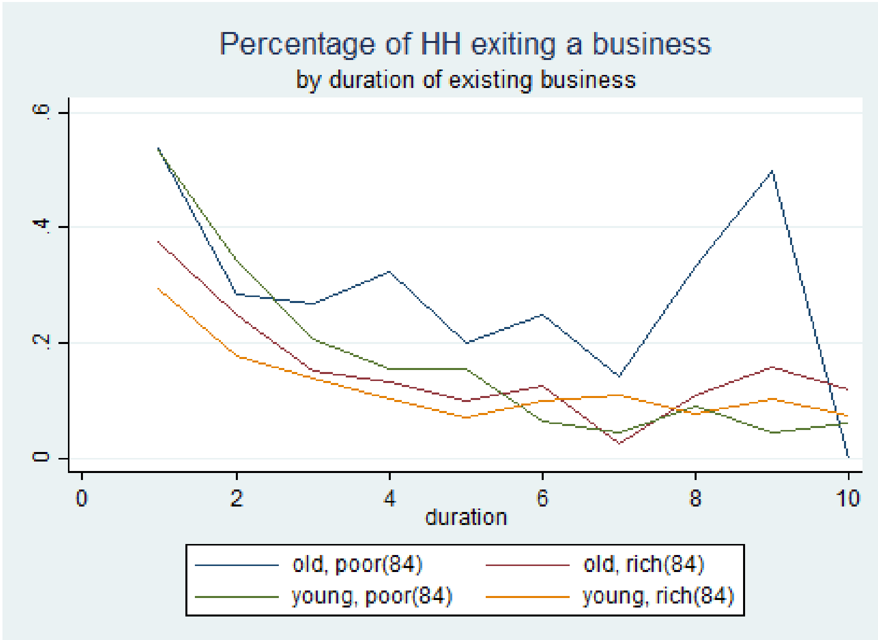

Figure 1.6: Proportion of households exiting business (PSID data), by wealth and age

First, if learning and/or borrowing constraints exist, then we expect the following results:

1) We expect $\beta_w > 0$ and $\beta'_w < 0$, as long as borrowing constraints exist. In other words, wealthy households are more likely to start a business and less likely to exit a business.

2) If learning exists, then even for the rich, the business exiting rate can eventually decline with duration of having a business. In other words, even if we run a regression only among the wealthy ($W = 1$), γ'_t should be negative for $t > 1$ and in general falls with τ. And if this is true even among the richest (say, the top 20%), then it is consistent with the existence of learning, according to Proposition 5.

3) Regardless of learning, the selection into business predicts that if one does not have business for a long time, he probably would never start a business, so the estimated coefficient γ_t should be negative at least for t big enough.

4) We expects that $\beta_y > 0$, or the younger more likely to enter a business. This can simply be a result of selection, i.e. the old probably have already settled down in the more favourable occupation, so it is harder to observe them switching from other occupations to running a business.

The prediction 4) can be observed if borrowing constraints alone exist, in which case young households with high entrepreneurial ability would enter when they have enough savings, while the old households without a business probably have low entrepreneurial ability, and would never start a business. However, if the gap between the old and the young exists among the richest who are likely insulated from borrowing constraints, (say, the top 20%,) then a more plausible explanation is the combination of learning and macro

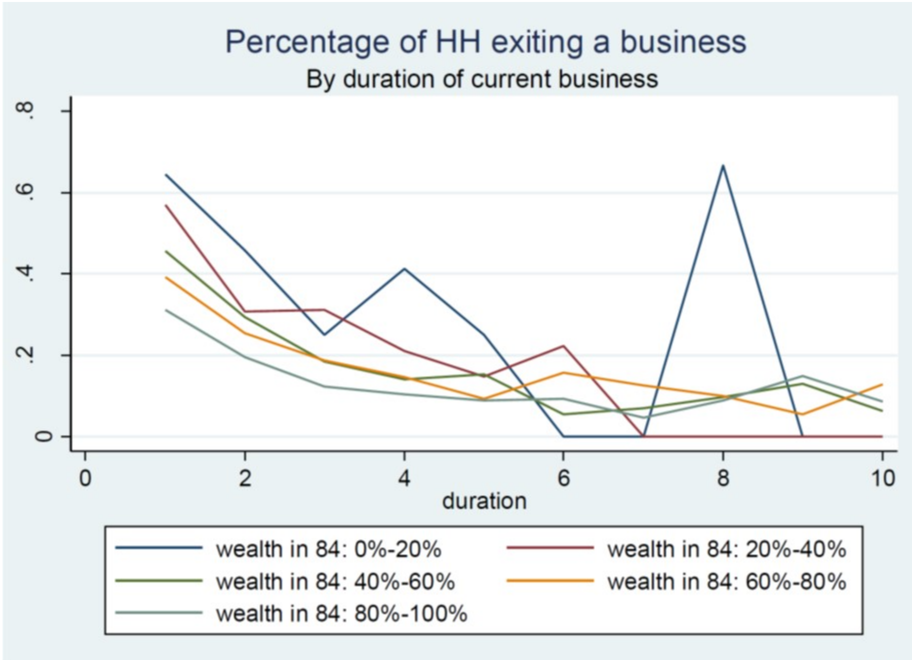

Figure 1.7: Proportion of households exiting business (PSID data), by wealth distribution

shock. For example, young people are more interested in learning by running a business, so they are more inclined to enter if an opportunity opens up.

5)If learning exists, $\beta'_y < 0$ at least for those who are wealthy. In other words, among the wealthy, the young are less likely to exit than the old. This is because the young have a stronger learning incentive, which makes them more willing to continue experimenting given all else equal. Among the poor, the sign of β'_y might be more ambiguous because of borrowing constraints. For example, a bad shock may force young households, who have not yet accumulated enough wealth, to exit, (e.g. if the minimum capital requirement can no longer be satisfied,) even if they would optimally continue to run a business if allowed. Besides, compared with the old, the young are more likely to be in the pre-breakthrough stage and encounter regular output more often (since $q \leq q'$), which also causes more exits among the young. This can happen with or without a breakthrough. Thus if a negative β'_y is observed, it can only be explained by the difference in the learning incentives of the young and the old.

6) In addition, if β_{wy} is negative given β_w positive, a model with learning would be more convincing than a model without, since a negative β_{wy} is consistent with that learning incentive makes wealth level less important for business entry. The explanations are as follows.

β_{wy} is zero if the effect of wealth on business entry is the same for older and younger households. If β_{wy} is negative, it means that the difference of the likelihood to enter a business between the rich and the less rich is smaller for the young. This can happen when learning exists. As Proposition 4 implies, the stronger learning incentive of the young makes them more tolerant with the amount of capital it takes for entrepreneurship to be more desirable than the outside option. As a result, wealth level can have a smaller influence on the likelihood of entering a business for the young.

In fact, we might also observe a positive β_{wy}. One reason is that the old are less likely to change their occupations according to the selection effect, thus it is possible that wealth makes less impact on the business entry of the old. [13] If learning does not exist, then what makes a difference in the effect of wealth for different age groups is the selection effect, and we expect to observe a positive β_{wy} instead.

Table 1.1: The likelihood of business entry (using wealth in 84)

	(1) All	(2) top 40%	(3) top 20%
entry			
2 years ($\hat{\gamma_2}$)	-0.558***	-0.611***	-0.754***
	(-7.80)	(-6.64)	(-6.09)
3 years ($\hat{\gamma_3}$)	-0.802***	-0.979***	-0.964***
	(-9.84)	(-8.87)	(-6.83)
young ($\hat{\beta_y}$)	0.757***	0.367***	0.405***
	(8.22)	(5.57)	(4.21)
wealthy_84 ($\hat{\beta_w}$)	1.016***		
	(10.79)		
young&wealthy_84 ($\hat{\beta_{wy}}$)	-0.389***		
	(-3.44)		
N	44820	17641	8227

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Column (1) shows results for all households, Column (2) for households in the top 40% of wealth distribution, and Column (3) for the top 20% percent.

Households with t continuous years without business are on average $\exp(\hat{\gamma_t}) - 1$ more likely to start a business than those having no business for 1 year.

1.3.4 Results

The results of the baseline regressions are presented in table 1.1 and table 1.2. The way to interpret the reported estimates is as follows. For example, for the business entry, $\hat{\gamma_2} = -0.558$, it means that those with 2 continuous years without a business are on average

[13] And it is hard to track the old individuals' choices from the beginning. What we observe is only a period of time.

Table 1.2: The likelihood of business exit (using wealth in 84)

	(1) All	(2) top 40%	(3) top 20%
exit			
2 years ($\hat{\gamma_2'}$)	-0.587***	-0.530***	-0.524***
	(-8.60)	(-5.89)	(-4.39)
3 years ($\hat{\gamma_3'}$)	-1.031***	-0.973***	-1.013***
	(-10.94)	(-8.16)	(-6.41)
young ($\hat{\beta_y'}$)	-0.0941	-0.269***	-0.437***
	(-1.05)	(-3.89)	(-4.50)
wealthy_84 ($\hat{\beta_w'}$)	-0.494***		
	(-5.54)		
young&wealthy_84 ($\hat{\beta_{wy}'}$)	-0.169		
	(-1.49)		
N	6462	4587	3089

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Column (1) shows results for all households, Column (2) for households in the top 40% of wealth distribution, and Column (3) for the top 20% percent.

Households with t continuous years running a business are on average $\exp(\hat{\gamma_t}) - 1$ more likely to end a business than those having a business for 1 year.

$\exp(-0.558) = 57.24\%$ as likely (or 43.76% less likely) to start a business as those with 1 year without a business. The estimates in exponential form are also reported in the appendix.

As we can see, the results from the baseline regressions are consistent with predictions (1) to (6), and robust to the wealth measure used. In particular, the results suggest that the possibility of learning cannot be ignored. In tables 1.1 and 1.2, highlighted in green are the results which are consistent with a model with both credit constraints and learning, but unlikely to be observed in a model without learning. First, $\hat{\gamma_\tau'} < 0$ for $\tau > 1$ and $\hat{\gamma_3'} < \hat{\gamma_2'} < 0$ even among households in the top 20 percent of the wealth distribution. This means that the possibility to exit a business decreases with the time duration of having a business even for the richest, consistent with the hypothesis that learning causes a households to exit when they realize that entrepreneurship is not the most suitable occupation. Second, $\hat{\beta_y} > 0$ even among the wealthiest households, i.e. the young are more likely to enter a business than the old, consistent with bigger learning incentives among the young (Proposition 1). Third, $\hat{\beta_y'} < 0$ for all households, or the younger households are less likely to exit, which can be also explained by the stronger learning incentives of the young. In fact, all else equal, the young households are 64.6% as likely to exit a business as the old.

In addition, I find that the effect of wealth on business entry is smaller for the young than for the old, i.e. $\hat{\beta}_{wy} < 0$. Specifically, among the young, the wealthy are 87.2% more likely to start a business than the less wealthy,[14] while among the old, the wealthy are 176.2% more likely to start a business. [15] As argued in the previous subsection, this is unlikely to happen if learning does not exist. When learning exists, however, the incentive to explore makes wealth a less important factor for whether to start a business, as suggested in proposition 4, and this incentive is stronger for the young than for the old.

I also conducted three robustness checks (see appendix) and all the results above still hold. In two robustness checks, I use wealth levels of year 89 and year 94 respectively, instead of year 84. In one other check, I include the actual wealth (instead of only the indicator variable of whether a household is wealthy or not) as a regressor.

1.4 Conclusions and discussions

Does learning play a part in the occupational choices between entrepreneurship and a fixed-wage job? According to the PSID data, the answer is affirmative, as suggested by the relationship between business exiting and tenure of having a business, as well as the dif-

[14]The calculation is $exp(1.016 - 0.389) - 1 = 187.2\% - 1 = 87.2\%$.

[15]The size of business owned by a household, measured by the number of employees, does not differ significantly with age, given the 760 available observations among the self-employed in 1993. This data is not available in other years.

ference in the occupational switching patterns between the younger and older households, neither of which can be fully explained without learning. If learning does exist, then credit constraints do more than simply postponing the time to start a business, but may also have a long term impact as it would deter some potentially good entrepreneurs from starting a business at all. In other words, providing more credit to young households would not only allow them to start a business earlier and be more immune to negative economic shocks, but also encourage more people to explore which occupation is best for them and give rise to more entrepreneurs in the economy in the long run.

However, the current research has its limitations. For example, I assume that the old and the young face the same credit constraints, which may not be true in practice. Instead, the young may be either more or less constrained than the old. On the one hand, the young may receive larger loans because they have larger future wealth than the old; On the other hand, however, they may be allowed to borrow less because they may not have built enough credit. With this confounding factor, we cannot directly interpret the observed entry/exit difference between the old and the young as an evidence for learning. In addition, the future access to loans can depend on the current and past performance of the business. An empirical study on the factors affecting credit constraints would be beneficial. If credit constraints are age and history dependant, this can also alter the interaction between learning and the borrowing constraint. For example, when a forward-looking agent makes occupational choice, he would take into account both the benefit of information gathering and the impact on loan access in the future. This could be interesting for future research.

Chapter 2

Repeated Loans, Enforcement and Learning

2.1 Introduction

Microfinance plays an important role in lending to poor people in many developing countries. Unlike traditional loans, microfinance usually does not require collateral from the borrowers, but instead relies on other mechanisms, such as group lending and dynamic incentives, to motivate repayment. The need for using these mechanisms is largely caused by the lack of enforcement on repayment in the developing world. A large body of theoretical and empirical literature, including Besley & Coate (1995), Godquin (2004), Abbink *et al.* (2006), Kurosaki & Khan (2012), Allen (2016), Shapiro (2015), etc, has explored whether these mechanisms are effective in preventing strategic default. In this chapter, however, I study an aspect that has received little attention, that is, the impact of weak enforcement on learning (i.e., experimenting) of micro-entrepreneurs. I base my analysis on a model with infinite horizon where dynamic incentive mechanism is used. I find that the lack of enforcement has a negative effect on learning. In other words, micro-entrepreneurs would experiment less when it is possible to default strategically. In terms of welfare, a shorter experimenting time is associated with a smaller expected lifetime payoff.

While the assumption of strategic default is commonly seen in microfinance literature, learning is not often discussed. However, as reviewed in Chapter 1, learning is receiving increasing attention in the recent years in the fields of occupational choice, entrepreneurship, firm learning, etc. Therefore, it is natural to incorporate the assumption about learning in a model where agents rely on microfinance to run a business, and explore how the lack of enforcement impacts learning and welfare.

In this chapter, I build a model where agents can run business projects where qualities are unknown ex-ante but are revealed over time from the outputs they yield. In other words, an entrepreneur can gain information about his project by carrying it out over and over. And the quality of a project remains fixed. Without access to saving technology, one need to borrow to finance their working capital in each period. There is no collateral needed for borrowing. However, dynamic incentives are what lenders use to harness the default behaviour. That is, a borrower is forever cast out once they default, which is the common rule of many microfinance institutions. I analyze from this model whether a borrower experiments more when there is perfect enforcement, or when he can strategically default and faces dynamic incentives. Alternatively, we can interpret the comparison as between the experimenting time between the "rich" who use their own money to run their business (or who have collateral to ensure that they would repay) and the "poor" who rely on borrowing to run a business.

I find that the possibility to default hurts efficiency because it causes the agents to experiment less than optimal, which results in long-run misallocation of occupations. The difference in the experimenting time under different enforcement regimes lies in the difference in the benefit of "quitting" one's project (either after defaulting or repaying) compared to "waiting" for one more period. First, when enforcement is perfect, quitting today means that one still repays the loan due today, but does not pay anything tomorrow. However, when strategic default is possible, one can avoid paying current period's loan (including interest rate) right away, which is preferred to escaping the same amount in the next period, because agents are not fully patient. Second, the interest rate equals the risk free interest rate with perfect enforcement because of competition, but since lenders have to break even, the possibility of strategic default makes the interest no lower than that in all periods, which means a larger amount of payment to avoid when one quits, making quitting more attractive. Third, when one defaults today and is excluded from the loan market, he does not pay anything today or tomorrow. But if one wait till tomorrow (and default if there is still no breakthrough), he makes a payment today with certainty, and he also repays tomorrow in case a breakthrough occurs (by construction). Therefore, defaulting enables one to not only escape his payment due today, but also the expected payment he will make tomorrow. In other words, more than one payment is expected to be saved by defaulting right away instead of waiting for another period. Under perfect enforcement, however, an entrepreneur saves only one payment, namely, the payment due tomorrow, if he quits today compared to waiting until tomorrow.

The results can be extended to a model with a probability of $\theta \in [0, 1]$ that a borrower can default strategically in any period. In other words, θ can be interpreted as an indicator of the weakness of enforcement in the economy, and a smaller θ means stronger enforcement. We can show that both the experimenting time and an agent's expected lifetime payoff decrease with θ.

The rest of this chapter is organized as follows. Section 2.2 introduces the general setup of the model. Section 2.3 analyzes the equilibrium. Section 2.4 compares a model with default possibility to a model without in terms of the experimenting time and expected lifetime payoff. Section 2.5 generalizes the model to one with the default probability taken from a continuous range $[0,1]$. Section 2.6 concludes.

2.2 General Setup

2.2.1 The Project

Assume that there are business projects available. Each agent can run one project at a time. If a project is run by an entrepreneur with high ability, it can go through two stages, with the turning point called a "breakthrough" (for example, when some influential people visit a good restaurant and make positive comments about it). When working capital k is invested into the project at the beginning of a period, it will always at least yield "regular output" of $f(k)$ at the end of that period. Moreover, in every period before a breakthrough (i.e. in the first stage), there is a probability q that a breakthrough, marked by the first time the project yields a "bonus" G, would come. After that, the project will keep yielding a payoff of $f(k)+G$ with working capital k. In contrast, if the owner has low entrepreneurial ability, a breakthrough will never occur, and the project only yields the regular output $f(k)$ whenever k is invested into it. An entrepreneur's ability is not known by anyone before he runs a business for the first time. [1] However, the occurrence of a breakthrough is made known to all.

2.2.2 The borrower

The agents concerned in this chapter do not have access to saving technology and need to rely on the loan to purchase working capital each period. There is a continuum of potential borrowers with different outside option values (the amounts they will get in a period if they are not running business projects). Let the outside option value of borrower i be s_i. Assume that outside option values are observable to all. Thus lenders can offer loan contracts based on borrowers' outside option values.

As mentioned before, no borrower knows his exact ability at the beginning and the prior belief for a borrower to have high ability starts with is p_0. A borrower's belief is updated each period after the project outcome is realized. Define p_{it} as the belief that borrower i has high ability at the beginning of period t. If a breakthrough occurs for borrower i by the end of period t, his belief will jump to $p_{it+1}=1$. Otherwise, it is updated according to the Bayesian rule:

$$p_{it+1} = p'_{t+1} = \frac{p_{it}(1-q)}{1-p_{it}+p_{it}(1-q)}$$

[1] Alternatively, instead of assuming that the entrepreneurial ability is the object to be learned, we can assume that learning is about the project quality (which can be either "good" or "bad"), and that one can only run one project in his lifetime.

where p_t' is the updated belief of having high ability after t periods' experimentation without a breakthrough.

While no collateral is needed for borrowing, a borrower's access to future funds hinges on his record of having never defaulted. Once a borrower defaults, no further loan will be issued to him.

2.2.3 The lender

Assume that the lending market is competitive, which implies zero profit for all lenders while no profitable deviation is possible. In any period t, each lender makes offer(s) of loans and sets loan terms including interest rate R_t and loan size k_t. Assume that lenders cannot commit to loan contracts for longer than one period, as assumed in Ghosh *et al.* (1999) and in Shapiro (2015). Define R_f as the risk-free interest rate.

2.2.4 Timing

The timing is as follows: First, at the beginning of each period, lenders make offers simultaneously (each offer includes a loan size and an interest rate). Second, borrowers choose a loan offer, get the money, and invest it into his project. Then the output of the project is realized. After that, a borrower chooses either to repay the loan and proceed to the next period, or to default and never get another loan.

2.2.5 Parameter assumptions

In this section we make a few assumptions about the parameters. In addition, define $k_{max}(R)$ as the value of k that satisfies $f(k) - Rk + G = 0$. That is, given the interest R, k_{max} is the largest amount of capital above which a borrower will not be able to repay even with the bonus output.

Assumption 1. *For all s_i, $\delta(f(k^*) - R_f k^* + G) > (1-\delta)f(k^*) + (1-\delta)G + \delta s$.*

The inequality above requires the bonus G to be big enough that any borrower already with a breakthrough will not find it optimal to default under any loan term. [2] It also implies $s_i \leq f(k^*) - R_f k^* + G$, i.e., the maximum profit for an entrepreneur with high ability exceeds the outside option.

Assumption 2. *For all s_i, $s_i < \bar{s}$, where $\bar{s}$ is a constant such that $\mathcal{H}(\bar{s}) = \frac{R_f k^*}{(f(k^*)+G)q}$ with $\mathcal{H}(s) = \frac{(1-\delta)(s - f(k^*) + R_f k^*)}{q\delta(f(k^*) - R_f k^* - s) + qG}$.*

[2]We can show that this inequality is equivalent to $f(k^*) + G + \frac{\delta s}{1-\delta} < \frac{\delta(f(k^*) - R_f k^* + G)}{1-\delta}$, that is, the condition for defaulting to be less favorable than repaying and remaining as a borrower forever even when one gets the highest possible gain from default.

This condition rules out credit rationing on the equilibrium path, which happens if the interest rate in a period in equilibrium becomes too high. We will be ready to prove that this is a sufficient condition for no credit rationing in equilibrium after proposition 8 is established. The proof can be found in the appendix. [3]

This condition also implies that $f(k^*) - R_f k^* < s_i$. [4] In other words, for all borrowers concerned in the paper, the maximum profit from a bad project (after repayment) is less than their outside option values. It also implies that $f(k^*) - R_f k^*/\delta < s_i$. Since a necessary condition for the existence of an equilibrium in which borrowers with s_i never default (in the observable type case) is that $f(k^*) - R_f k^*/\delta \geq s_i$,[5] This assumption ensures that all borrowers will eventually default in equilibrium if no breakthrough happens.

2.3 Model Analysis

Let's start by analyzing the case of perfect enforcement. This case serves only as a benchmark. If default never happens, interest rate will always be the risk free interest rate R_f. Therefore, lenders will always set loan size to $k^* \equiv \arg\max_k \{f(k) - R_f k\}$. Borrowers who always repay will choose to exit and never return if they have been borrowing for many periods without a breakthrough. We can calculate the threshold belief of having high entrepreneurial ability above which agents with s will choose running business projects instead of picking the outside option. The cutoff belief can be calculated as $p^c = \frac{(1-\delta)(s-f(k^*)+R_f k^*)}{q\delta(f(k^*)-R_f k^*-s)+qG}$. [6] As we can see, the threshold decreases with δ and q, and increases with s. Intuitively, all else equal, a more patient borrower would wait longer before exiting when there is no breakthrough, so would a borrower with a lower outside option value. Besides, when the probability that a breakthrough happens to a borrower with high ability increases, the cutoff decreases, because the probability to have high ability and that for a breakthrough to happen to a good entrepreneur are complementary to each other, in the sense that the product of the two is the ex-anti probability that a breakthrough happens to anyone who is still learning.

[3] The main result of this paper still hold without this assumption, but the math would be much more cumbersome.

[4] This is because if $s_i \to f(k^*) - R_f k^*$, $f(s_i) \to 0$ and the condition does not hold. Since $f(s)$ increases with s, if this assumption is true, $s_i > f(k^*) - R_f k^*$ must hold.

[5] which comes from $f(k^*) + \delta s/(1-\delta) \geq f(k^*) - R_f k^* + \delta \lim_{t\to\infty} \{p_t q(\frac{f(k^*)-R_f k^*+G}{1-\delta}) + (1-p_t q)(f(k^*) + \frac{\delta s}{1-\delta})\} = f(k^*) - R_f k^* + \delta(f(k^*) + \frac{\delta s}{1-\delta})$, that is, a borrower always find it optimal not to default even after infinite number of periods without a breakthrough.

[6] To see this, note that at the threshold belief the value from quitting equals the value from not quitting. Therefore, $s/(1-\delta) = f(k^*) - R_f k^* + pqG + \delta((1-pq)\frac{s}{1-\delta} + pq\frac{f(k^*)-R_f k^*+G}{1-\delta})$, which gives $p^c = \frac{(1-\delta)(s-f(k^*)+R_f k^*)}{q\delta(f(k^*)-R_f k^*-s)+qG}$.

[7] This cutoff belief, together with the Bayes' rule of belief updating, enable us to compute the period in which a borrower would exit if no breakthrough happens, as a borrower would exit as soon as his belief to have high ability drops below the cutoff.

From now on we assume that the borrowers are able to default strategically. In the case with observable outside option values, separate contracts can be offered to borrowers with different outside option values. Let us now focus on borrowers with outside option value s. Since another observable information is whether a breakthrough has occurred, lenders can also vary loan terms accordingly. By assumption, a borrower would never default once a breakthrough happens. Thus it is obvious that for borrowers who already had a breakthrough, competition among lenders will drive the interest rate equal to the risk free interest rate R_f, and the loan size equal to $k^* \equiv \arg\max_k f(k) - R_f k$.

2.3.1 Definition of equilibrium

Focusing on the symmetric outcomes where all lenders offer the same contract, we define an equilibrium as a sequence of loan terms $\{R_t, k_t\}_{t=0}^{\infty}$ for those without a breakthrough, and borrowers' defaulting time T such that, for a given s:

1) Given the loan terms set by the lenders, it is optimal for borrowers to default at T if no breakthrough happens.

2) Given the current and future loan terms offered by all other lenders, no lender can make a profitable deviation by changing either R_t or k_t at any t.

In addition, we focus on equilibria with the following off-equilibrium-path feature: If every borrower without a breakthrough defaults at t in equilibrium, but for some reason (e.g, by mistake), a borrower without breakthrough does not default at $t, ...j-1$ $(j > t)$, and still applies for a loan at j, then at the end of j, the borrower will optimally default unless a breakthrough happens. [8]

2.3.2 Constructing equilibrium

First, if in a symmetric equilibrium where all lenders offer the same contract, a fraction ϕ of existing borrowers will repay by the end of period t given the current and (correctly anticipated) future loan terms, [9] then we can show that all lenders offer $R_t = R_f/\phi$ as a result of free entry. In other words, the interest rate enables the lender just to break even.

[7] The fact that the cutoff belief decreases with q, however, does not mean that one waits longer before quitting when q is high. As we can see, p_t drops faster with a higher q. Thus even if the threshold is lower, a borrower may experiment for fewer periods when q increases.

[8] This assumption is needed to rule out possible equilibria with the following (or similar) off-equilibrium features: If by mistake a borrower passed the optimal default time t, then at $t+1$ he would optimally choose to wait (for one or more periods) instead of defaulting right away, even if a breakthrough still does not happen.

[9] or equivalently, each borrower with these characteristics repays with ex-anti probability ϕ,

Second, define $k^* \equiv \arg\max_k f(k) - R_f k$, we show that each borrower's (equivalently, all borrowers') expected profit from the loan is maximized at k^*, if $f(k^*) - R_f k^* + G \geq 0$, i.e. a borrower is able to repay in full even when a breakthrough happens at R_t and k^*. If a borrower is unable to repay in full when a breakthrough happens, credit rationing will arise at t. Assumption 2 allows us to focus on the case without credit rationing on the equilibrium path (see appendix for the proof.) However, credit rationing is possible off equilibrium path. (For example, when a borrower should default but does not, there will still be a loan offered to him in the next period which might include credit rationing.)

Lemma 1. *In equilibrium, if default happens only at t for borrowers with outside option value s, the following series of interest rates and loan sizes are offered to those without a breakthrough.*

$$R_j = \begin{cases} R_f & j < t \\ \frac{R_f}{p_{j-1}q} & j \geq t \end{cases} \tag{2.1}$$

where R_f is the risk-free interest rate.

$$k_j = \begin{cases} k^* & j < t \\ \min\{k^j_{max}, k^*\} & j \geq t \end{cases} \tag{2.2}$$

where $f(k^j_{max}) - R_j k^j_{max} + G = 0$.

From Lemma 1 we can see that, before the optimal defaulting time arrives, the interest rate is the risk-free interest rate, reflecting the fact that no one would default. On the equilibrium path, after period t, we should not find any borrowers still repaying who have not had a breakthrough, because t is the maximum experimenting time. However, it is necessary to specify the whole contingent plan of the lender including the cases in which borrowers are not behaving optimally. For any borrower without a breakthrough who still can and does borrow at $j > t$, the interest rate given above indicates that he will surely default at the end of j unless a breakthrough happens by then.

In addition, free-entry also means that the expected per-period payoff for every existing borrower without a breakthrough is maximized at the breaking-even interest rate. Since no one would default before period t, the per-period payoff for a borrower without a breakthrough in any $j < t$ is $f(k_j) - R_j k_j = f(k_j) - R_f k_j$, which is maximized at k^*. Then at $j \geq t$, every borrower without a breakthrough has a prior probability to repay the loan equal to the probability that a breakthrough happens at j, i.e. $p_{j-1}q$. When a borrower can repay in full if a breakthrough happens, the expected per-period payoff at j is $f(k_j) - p_{j-1}q(R_j k_j) + p_{j-1}qG = f(k_j) - R_f k_j + p_{j-1}qG$, which is also maximized at k^*. However, if the interest rate R_j is too high, then a borrower would not be able to repay in full with k^* even under a breakthrough. In this case, lenders will offer the maximum possible loan, $k^j_{max} < k^*$ such that the borrower can repay in full. Under the Assumption 2, however, credit rationing will not occur on the equilibrium path, i.e. $k_t = k^*$.

Now in order to find an equilibrium, we utilize two necessary conditions (given below) for the existence of an equilibrium in which every borrower without a breakthrough defaults at (and only at) the end of t.

First, in period t, a borrower without a breakthrough prefers to default right away than to wait another period and default then (if still no breakthrough comes):

$$\begin{aligned} f(k^*) + \frac{\delta s}{1-\delta} \geq \\ f(k^*) - R_t k^* + \delta\Big\{ p'_t q \max[\{0, f(k^*) - R_{t+1}k^* + G\} + \frac{\delta(f(k^*) - R_f k^* + G)}{1-\delta}] \\ + (1 - p'_t q)[\min\{f(k^*), f(k^{t+1}_{max})\} + \frac{\delta s}{1-\delta}]\Big\} \end{aligned} \tag{2.3}$$

The right-hand side reflects the possibility of credit rationing at $t+1$. Specifically, for those who do not default at t and have no breakthrough by the end of $t+1$, the loan terms for $t+1$ are $R_{t+1} = R_f/(p_t q)$, and $k_{t+1} = k^*$ if $f(k^*) - R_{t+1}k^* + G > 0$, [10] and $k_{t+1} = k_{max}$ otherwise.

Second, at $t-1$, borrowers without a breakthrough prefer to wait until t than default right away.

$$\begin{aligned} f(k^*) + \frac{\delta s}{1-\delta} \leq \\ f(k^*) - R_f k^* + \delta\Big\{ p'_{t-1} q[f(k^*) - R_f k^* + G + \frac{\delta(f(k^*) - R_f k^* + G)}{1-\delta}] \\ + (1 - p'_{t-1} q)[f(k^*) + \frac{\delta s}{1-\delta}]\Big\} \end{aligned} \tag{2.4}$$

Notice that now p'_t and R_t are both functions of t. Thus the conditions (2.3) and (2.4) are only conditions on t given the model parameters. In fact, once the equilibrium defaulting time t is decided, everything else, including the interest rates in all periods, defaulting fractions, etc. will be pinned down. Now let us further assume that $\mathcal{G}(\tau) \equiv -R_\tau k^\tau_{max} + \delta\Big\{ p'_\tau q \frac{\delta(f(k^*)-R_f k^*+G)}{1-\delta} + (1-p'_\tau q)[f(k^{\tau+1}_{max}) + \frac{\delta s}{1-\delta}]\Big\}$ decreases with τ. When this assumption holds, the net gain to continue (compared to defaulting) decreases no matter whether there is credit rationing in the current period for those without a breakthrough. [11]

[10] In this case, we can show that $k^* \leq k_{max}$.

[11] Without this assumption, the net gain of continuing still decreases over time when there is no credit rationing, as shown in the proof of Lemma 2 in the appendix. When there is credit rationing in a given period, the profit in that period is zero for those who encounter a breakthrough and repay the loan. Obviously, the value to continue, $\delta\Big\{ p'_\tau q \frac{\delta(f(k^*)-R_f k^*+G)}{1-\delta} + (1-p'_\tau q)[f(k^{\tau+1}_{max}) + \frac{\delta s}{1-\delta}]\Big\}$, decreases with τ, as a borrower becomes more and more pessimistic and the loan size for the next period keeps shrinking. However, the

The two conditions (2.3) and (2.4) are crucial to find an equilibrium. We can show that if there are multiple values of t satisfying (2.3) and (2.4), the biggest t among them is the unique equilibrium defaulting time for borrowers without a breakthrough. And we assume that (2.4) is automatically satisfied for $t = 1$ given that a borrower is willing to participate.

Proposition 6. *The largest t that satisfies both (2.3) and (2.4) is the equilibrium defaulting time for borrowers without a breakthrough.*

The proof is completed in two major steps. First, when (2.3) and (2.4) hold at t, we can show that t is not only a "locally" optimal defaulting time but also "globally" optimal, given loan terms constructed based on that default only happens at t. In other words, it is better to default at t than not only at $t-1$ or $t+1$, but also at any other period (or never) if no breakthrough happens. Then we can define equilibrium candidates as those associate with t values satisfying both (2.3) and (2.4). Second, among all candidates, only the one with the largest t value, t_{max}, will survive as an equilibrium allocation. Because it is under defaulting time t_{max} and the correspondent loan terms that borrowers receive the highest expected life-time payoff, while lenders always break even. In fact, in a candidate equilibrium with defaulting time $t' < t_{max}$, a lender can make a profitable deviation. [12]

Proof.

Lemma 2. *When both (2.3) and (2.4) are satisfied at t, it means that when loan terms are reflecting that default happens at period t, according to (2.1) , t is indeed the optimal time to default for those without a breakthrough.*

The proof is left in the appendix. To prove that (2.3) and (2.4) means that it is better for those without a breakthrough to default at t than at any other period (or never), first, we show that under the interest rates given in (2.1), if a borrower prefers to default at t than at $t+1$ (i.e. (2.3) holds at t), he will also prefer to default at any $t+m$ than at $t+m+1$ ($m \geq 0$). Second, we prove that if (2.4) holds (which means that the borrower prefers to default at t than at $t-1$), he will also prefer to default at $t-m$ than at $t-m-1$ for any $m \geq 0$ given the loan terms.

Proof. See appendix. □

Therefore, from conditions (2.3) and (2.4), we get a set of equilibrium candidates. In any candidate equilibrium, the following must be true: 1) the loan terms reflect that default happens at t in the way described in page 29; 2) Both conditions (2.3) and (2.4) hold at t, i.e. the optimal defaulting time for borrowers without a breakthrough is t given these loan terms.

amount one can escape by defaulting, $R_\tau k_{max}^\tau$ also decreases since it is equal to $f(k_{max})^\tau + G$. Under our assumption, the first effect dominates the second. The purpose to have this assumption is that, if it holds, then we can show that for any t, if defaulting at t is better than waiting till $t+1$, defaulting at $t+1$ is also better than waiting till $t+2$.

[12]The result that only the candidate with t_{max} survives not only holds under free entry, but is also guaranteed with only two competitors.

One more criterion is yet needed to select the equilibrium allocation(s). That is: Given the current loan terms described in page 29, there is no way for any lender to still increase his profit. Now to be ready to evaluate the candidates according to this criterion, let us propose two lemmas. First, notice that when there are multiple values of t satisfying (2.3) and (2.4), they must be consecutive integers with no break in between.

Lemma 3. *If there are multiple values of t satisfying both (2.3) and (2.4) , with the smallest value t_1 and the largest value t_2, then for any $t \in [t_1, t_2]$, both conditions (2.3) and (2.4) also hold.*

Lemma 2.3.2 is true because (2.3) becomes slacker while (2.4) tighter whenever t increases. Thus since (2.3) holds at t_1, it also holds at any $t > t_1$. And since (2.4) holds at t_2, it holds for any $t < t_2$ as well. So for any $t \in [t_1, t_2]$, both (2.3) and (2.4) satisfy.

Second, let us compare the expected lifetime payoff for the borrowers under different equilibrium candidates.

Lemma 4. *When conditions (2.3) and (2.4) hold for both t_1 and t_2 with $t_1 \leq t_2$, the allocation with t_2 as the defaulting time yields higher lifetime payoff for the borrowers than the allocation with t_1 the defaulting time.*

Lemma 4 is proven by comparing two allocations in which default happens in two consecutive periods respectively.

Proof. See Appendix. □

Therefore, since free entry selects the allocation that maximizes the borrower's profit, the only allocation that survives as an equilibrium is associated with the largest t that satisfies both (2.3) and (2.4). In the unique equilibrium, default happens only at this t, and this fact is reflected in the equilibrium loan terms.

□

To summarize, we can decide a set of equilibrium candidates for an equilibrium by solving conditions (2.3) and (2.4). The largest t that satisfies both conditions is the unique defaulting time for borrowers without a breakthrough in equilibrium. Once the defaulting time is determined, the equilibrium loan terms for all periods will also be determined.

2.3.3 Existence of equilibrium

Now we can prove the existence of a pure-strategy equilibrium by showing that there exist at least a t at which both conditions (2.3) and (2.4) hold. And notice that there cannot be more than one equilibrium for the reason we stated before.

Proposition 7. *A unique equilibrium exists, in which there is a $t > 0$ such that a borrower defaults at and only at t if no breakthrough happens by then.*

Proof. Assume that a pure-strategy equilibrium does not exist. In other words, there does not exist a t for which both (2.3) and (2.4) hold. And since we can show that (2.3), which gets slacker as t increases, holds in the limit ($t \to \infty$) because of Assumption 1, there is a $t_1 > 0$ such that (2.3) holds for all $t \geq t_1$, but fails for $t < t_1$. (If $t_1 = 1$, (2.3) holds in every period.) Also, because (2.4) is satisfied for at least $t = 1$ but fails at $t \to \infty$ (also because of parameter assumption (1)), there exists a $t_2 > 0$ such that (2.4) holds for all $t \leq t_2$ but fails for all $t > t_2$. Therefore, any $t \in [t_1, t_2]$, as long as $[t_1, t_2]$ is nonempty, satisfies both conditions. If no equilibrium exists, it must be true that $t_1 > t_2$. In that case, for any $\tau \in [t_2, t_1)$, τ fails (2.3) and $\tau + 1$ fails (2.4). But this is impossible at least for t_2. Because if (2.3) fails at $\tau = t_2$, we have

$$\begin{aligned}\frac{\delta s}{1-\delta} < -R_\tau k^* + \delta\Big\{p'_\tau q[\max\{0, f(k^*) - R_{\tau+1}k^* + G\} + \frac{\delta(f(k^*) - R_f k^* + G)}{1-\delta}]+ \\ (1 - p'_\tau q)[\min\{f(k_{max}(R_{t+1})), f(k^*)\} + \frac{\delta s}{1-\delta}]\Big\} \\ < -R_f k^* + \delta\Big\{p'_\tau q[\max\{0, f(k^*) - R_{\tau+1}k^* + G\} + \frac{\delta(f(k^*) - R_f k^* + G)}{1-\delta}]+ \\ (1 - p'_\tau q)[\min\{f(k_{max}(R_{t+1})), f(k^*)\} + \frac{\delta s}{1-\delta}]\Big\} \\ \leq -R_f k^* + \delta\Big\{p'_\tau q[f(k^*) - R_{\tau+1}k^* + G + \frac{\delta(f(k^*) - R_f k^* + G)}{1-\delta}]+ \\ (1 - p'_\tau q)[f(k^*) + \frac{\delta s}{1-\delta}]\Big\}\end{aligned}$$

But this means that (2.4) holds for $\tau + 1$. Therefore we have shown a contradiction. □

2.4 Comparison with a perfect-enforcement model

2.4.1 Experimenting time comparison

The next proposition explores the effect of strategic default on experimentation.

Proposition 8. *When strategic default is possible and the outside option value is observable, a borrower defaults (weakly) earlier than the time he would quit if strategic default is impossible.*

In other words, if a borrower who has not experienced a breakthrough exits the loan market at the end of period t when strategic default is impossible, then the same borrower would default no later than t if he could default strategically.

Proof. We can write down a necessary condition for exiting the market after repaying the loan at the end of period t in a model without strategic default:

$$\frac{\delta s}{1-\delta} \geq \delta p'_t q \frac{f(k^*) - R_f k^* + G}{1-\delta} + \delta(1 - p'_t q)(f(k^*) - R_f k^* + \frac{\delta s}{1-\delta}) \quad (2.5)$$

The inequality above says that the borrower would rather quit today (and get the discounted future payoff given in the left-hand side) than to wait for another period and quit if and only if there is still no breakthrough in the next period (and get the discounted future payoff given in the right-hand side).

Now we prove this proposition by contradiction. When strategic default is possible, if a borrower with no breakthrough has not yet defaulted, and will not default at t, even without a breakthrough, the following condition must be satisfied:

$$\frac{\delta s}{1-\delta} \leq -k^* R_t + \delta\Big\{f(k^*) + p_t' q[-k^* R_{t+1} + G + \frac{\delta(f(k^*) - R_f k^* + G)}{1-\delta}] + (1 - p_t' q)\frac{\delta s}{1-\delta}\Big\} \quad (2.6)$$

Now we compare the right-hand side of (2.5) (or $RHS_{2.5}$) and that of 2.6 (or $RHS_{2.6}$). [13]

$$\begin{aligned} &RHS_{2.5} - RHS_{2.6} \\ &= k^* R_t + \delta[f(k^*) - f(k^*)] + \delta p_t' q k^* R_{t+1} - \delta k^* R_f \\ &> \delta k^* R_t + \delta p_t' q k^* R_{t+1} - \delta k^* R_f \\ &= \delta(k^* R_t - k^* R_f) + \delta p_t' q k^* R_{t+1} > 0 \end{aligned} \quad (2.7)$$

Therefore, (2.5) and (2.6) contradict. [14]

□

So the possibility of strategic-default will cause the borrowers without a breakthrough to exit earlier and experiment less, which means fewer projects remaining in the long run. As one main result of this chapter, the possibility of strategic default will discourage learning (experimenting) and cause more long-run misallocation of occupations.

To get some intuition why this is true, we can compare the costs and benefits of "waiting" for one more period at the end of t if a breakthrough has not happened by then. Instead of defaulting, if a borrower runs the project for one more period, and quit/default at the end of the next period unless a breakthrough comes, he can potentially benefit by what he will learn. If a breakthrough happens at $t+1$, the borrower will be able to enjoy a higher payoff (profit) for all future periods starting $t+2$, and the possibility of strategic default does not make a difference in this respect. Also, obviously, for period $t+1$ alone, instead

[13] $RHS_{2.6} = -k^* R_t + \delta p_t' q \frac{f(k^*) - R_f k^* + G}{1-\delta} + \delta(1 - p_t' q)(f(k^*) + \frac{\delta s}{1-\delta})$. Notice that the difference between $RHS_{2.5}$ and $RHS_{2.6}$ lies in: 1) There is an extra cost of $k^* R_t$ occurred right away at t if the borrower choose not to default (in 2.6 when default is possible); 2)In 2.6 we can see that at $t+1$, the loan is repaid with probability $p_t' q$, that is, a repayment is made if and only if a breakthrough occurs.

[14] Notice that the proof holds for any R_{t+1} and any $R_t \geq R_f$, which implies that the result of the proposition can apply more broadly.

of his outside option, the borrower gets the payout of his project, which (before the loan is repaid) is not affected by strategic default either. So for this comparison, the possibility of strategic default matters only through the loan payments at t and $t+1$ as explained below.

When strategic default is possible, compared to defaulting at t, "waiting" means not only forgoing the amount of money one could otherwise seize by defaulting at t, but also repaying again at $t+1$ with a chance of $p'_t q$ (the possibility that a breakthrough happens at $t+1$) . In other words, the "cost of waiting" has an expected present value of $k^* R_t + \delta p'_t q k^* R_f$. In comparison, under perfect enforcement, "waiting" rather than quitting at t leads to only one extra loan payment at $t+1$, which has a present value of $\delta k^* R_f$, and is lower than $k^* R_t + \delta p'_t q k^* R_f$.

The difference in the waiting cost can be broken down into three parts. First, unless the borrower is fully patient(i.e. $\delta = 1$, escaping what is due today (t) is more attractive than avoiding the same amount of payment tomorrow $(t+1)$. And strategic default makes it possible to skip the payment right away at t. Second, when one can default strategically, the interest rate is never lower than under perfect enforcement. A higher interest to pay makes it more attractive to avoid the payment. Third, under perfect enforcement, "quitting" frees a borrower from the duty to repay only once, but possibly twice (with probability $p_t q$) under strategic default. Mathematically, these three components correspond to $\delta < 1$, $k^* R \geq k^* R_f$ and $p'_t q > 0$ respectively and explain the inequalities in (2.7). Now we use a numerical example to illustrate the result of proposition 8.

Example 4. *Let's assume that $f(k) = \sqrt{k}$, $p_0 = 0.9$, $q = 0.2$, $\delta = 0.9$, $G = 3$ and $R_f = 1/\delta$. So $k^* = \delta^2/4$, $f(k^*) = \delta/2$. Varying s, we can numerically compute the equilibrium defaulting time for a borrower with outside option s and the optimal exiting time if strategic default is impossible. Figure 2.1 depicts the range of values of t satisfying both (2.3) and (2.4), Thus the upper bound (the black line) shows the equilibrium defaulting time when strategic default is possible. The figure also shows the optimal experimenting time under perfect enforcement (the red line), which is calculated by finding the smallest t s.t. $p'_t < p^c(s)$ for given s.*

From figure 2.1 we can conclude that when strategic default is possible, a borrower can never experiment longer in equilibrium than when strategic default is impossible. Instead, in this particular example, a borrower almost always defaults earlier than when he would quit under perfect enforcement.

2.4.2 Welfare comparison

Now we investigate whether an shorter experimenting time means inefficiency and compare a borrower's expected lifetime payoff when strategic default is possible with that under perfect enforcement. After all, shorter experimentation time is not a bad thing by itself, and sometimes it can mean that agents get settled sooner in their favorable occupations.

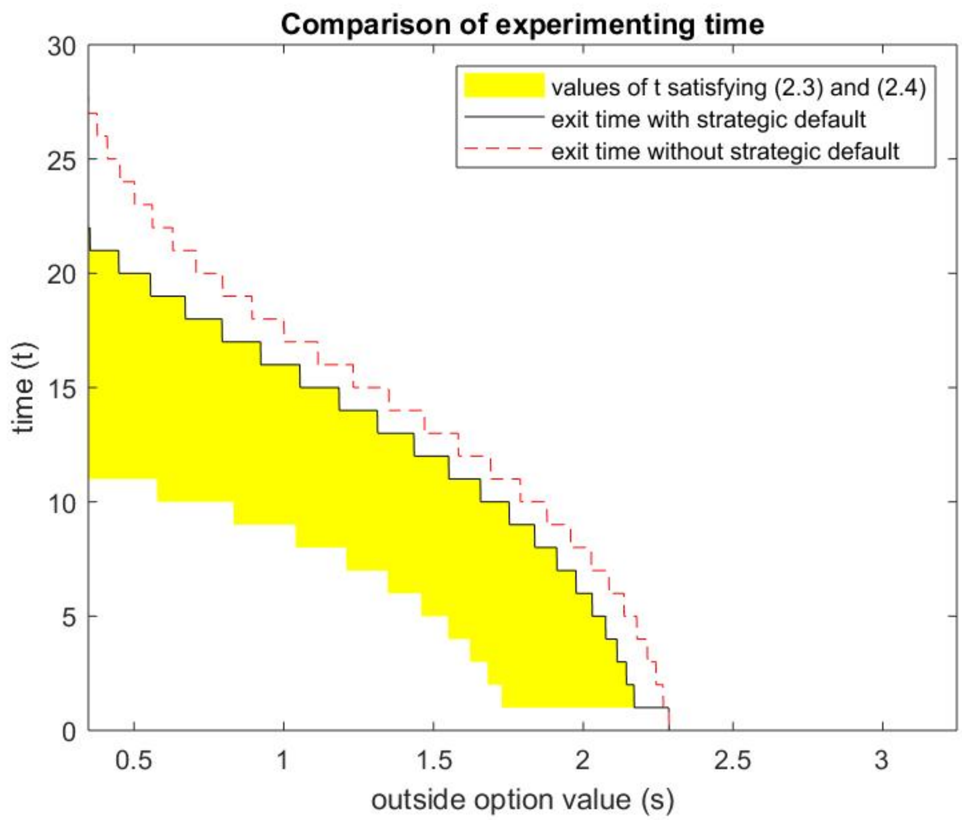

Figure 2.1: Comparison between the equilibrium defaulting time with default and the exiting time under perfect enforcement

However, we will find that the possibility to escape loan payment does not lead to a higher payoff.

Proposition 9. *A borrower gets a lower expected lifetime payoff when strategic default is possible than what he would get if strategic default is impossible.*

Proof. We already know that the experimenting time with the possibility of default cannot exceed that under perfect enforcement. Suppose that in equilibrium, a borrower defaults at the end of t if no breakthrough happens. And when default is impossible, no borrower will quit earlier. Notice that borrowers' behaviour and loan terms are exactly the same in both worlds before t. So we can simply start from period t to compare the payoffs of a borrower with s. When strategic default is impossible, at the beginning of t, the borrower's present value of expected lifetime payoff is greater or equal to $\Pi_1 = f(k^*) - R_f k^* + p'_{t-1}q(G + \delta(f(k^*) - k^* R_f + G^*)/(1-\delta)) + (1 - p'_{t-1}q)(\delta s/(1-\delta))$, which is his expected payoff if he quits at the end of t unless a breakthrough occurs within period t. In comparison, at the beginning of t, a borrower's expected lifetime payoff is $\Pi_2 = f(k^*) - p_{t-1}qk^* R_t + p'_{t-1}q(G + \delta(f(k^*) - k^* R_f + G^*)/(1-\delta)) + (1 - p'_{t-1}q)(\delta s/(1-\delta))$. Notice that $\Pi_1 = \Pi_2$ because $p_{t-1}qR_t = R_f$. In other words, the possibility for a borrower to escape his loan payment is fully incorporated into a high interest rate. This means, only in the best possible situation in which the experimenting time is the same regardless of default possibility will the lifetime expected payoff be unaffected. If the equilibrium experimenting time with strategic default is shorter than that under perfect enforcement, then the expected lifetime payoff will be strict smaller with the possibility to default. □

From the proof we can see that, in the current setup, the loss of the welfare does not come from worse loan terms (i.e. smaller loan size and/or larger interest rate). In fact, there is no credit rationing on the equilibrium path; The interest rate, though higher because of default possibility, just offsets the a borrower's expected again from defaulting. Thus all the welfare loss is caused by less-than-optimal experimentation. In our previous simulation example, the highest welfare loss caused by the possibility of strategic default is 1.49%, as we can see from figure 2.2. What we can also learn from the figure is that the percentage loss of welfare does not change monotonically with the outside option. On one hand, given the experimenting time with default possibility (t) and that without (T), the larger the outside option, the smaller is the loss simply because the difference of the payoffs from the project (with bonus) and the outside option is smaller. In other words, although a high-ability person is less likely to end up running a project because of less experimentation, he suffers a smaller loss if the outside option is not so bad. On the other hand, notice that experimenting time in either world decreases with s. When experimenting time with default possibility (t) decreases while there is no change T, we can see a jump in the welfare loss

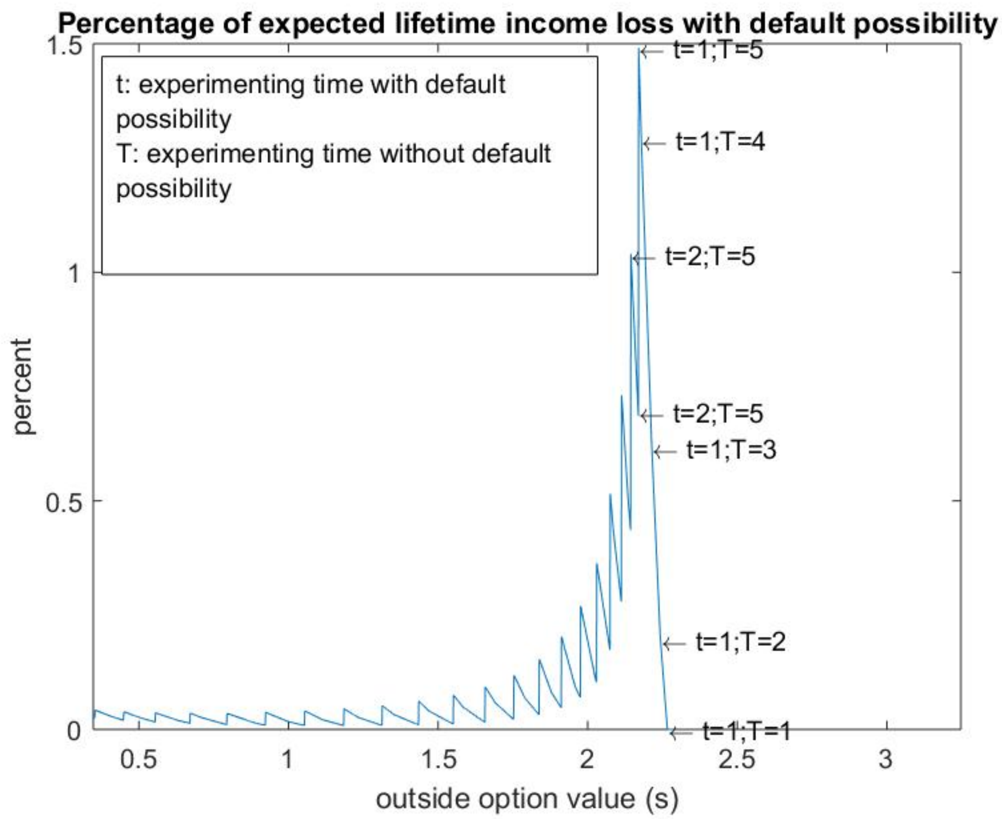

Figure 2.2: Expected lifetime income loss with strategic default compared to without strategic default (in percentage)

because of the additional loss caused by one more "missed" period of experimentation. [15] See Figure B.1 in the appendix for the plot of lifetime incomes with and without strategic default possibility.

Another question one may ask is whether default possibility also causes less participation in projects, since borrowers always get weakly lower expected lifetime payoffs with default possibility, as we have just shown. In the current setup, however, participation is not affected. To see this, we show that whoever participates with default possibility will also participate without, and vice versa.

First, notice that the payoff of not participating, i.e. that from the outside option, is not affected by the possibility to default. If an agent chooses to participate when default is possible, and get a lifetime expected payoff of Π, he can also at least earn Π if it is impossible to default, and thus is better off if he participates. Second, for a marginal agent who is just indifferent between participating and staying out under perfect enforcement, he will exit at the end of the first period unless a breakthrough happens. When strategic default is possible, the same agent would also default immediately after the first period in case of no breakthrough, if he indeed chose to participate. From the proof of 9, we know that in this case the payoff from participation is the same regardless of default possibility, because in either case a borrower without breakthrough exits at the same time. Therefore, whoever participates under perfect enforcement will also participate when default is possible. In summary, participation is not affected by the possibility of strategic default. In the numerical example, we can see that any borrower with outside option value smaller or equal to 2.286 will participate regardless of the possibility to default.

However, remember that we are only focusing on the case without credit rationing on the equilibrium path. Otherwise, if credit rationing happens even for the marginal borrower who would default after the first period without a breakthrough, participation would be smaller with strategic default than without.

2.5 Generalization

In this section, instead of assuming that default is always or never possible, I assume that it is possible with probability $\theta \in [0, 1]$ in each period. We can interpret $1 - \theta$ as the degree

[15] When experimenting time without default possibility (T) goes down while t does not change, the welfare loss also shrinks. In this simulation, the percentage loss of income peaks at the first moment when $t = 1$. The reason why the loss is smaller when he experiments longer is because the extra information gained decreases as one experiments longer. So the loss of information, and thus loss of welfare, is also greater for each lost period in experimentation if he would otherwise experiment for only 5 periods, than if he would experiment for 15 periods, for example.

of enforcement, or an indicator of the strength of institution. The economy has perfect enforcement of repayment when $\theta = 0$ and no enforcement when $\theta = 1$. [16]

Although the value of θ is known ex-ante and fixed throughout all periods, whether or not default is possible is independent each period, and known only at the end of the period after the project outcome is realized, if $\theta \in (0, 1)$. In other words, now the timing is as follows: First, as before, lenders make offers simultaneously at the beginning of each period. Second, each borrower chooses a loan offer, and invest the borrowed amount of money into his project. Then the output of the project is realized. After that, whether default is possible is revealed (if $\theta \in (0, 1)$). Lastly, if default is impossible, borrowers repay and either quit and choose the outside option in the future, or proceed to the next period of borrowing. Otherwise, a borrower chooses either to repay the loan and proceed to the next period, or to default and never get another loan.

I will show in this section that a larger θ always leads to a (weakly) lower expected lifetime payoff, and a shorter experimenting time.

The analysis of the generalized model is similar to that with $\theta = 1$. First, to find the equilibrium defaulting time, I establish analogs to conditions (2.3) and (2.4). Then I show that the largest t value satisfying these two analog conditions is again the equilibrium defaulting time, at which all borrowers without a breakthrough would default it it turns out possible. Finally I analyze the effect of θ on the experimenting time and expected payoff. [17]

Definition 1. *Given that no breakthrough has happened by the end of t (today), define $W_t(\theta)$ as the present discounted value of the expected future payoff by repaying today, proceeding to the next period, and defaulting as soon as possible in the future unless a breakthrough happens or the borrower optimally quits borrowing.*

[16] Partial enforcement, though on the lender's side, has been modeled by Kovrijnykh (2013), in which the contract can be voided and renegotiated in each period with some probability before the lender invests in the borrower's project. That paper finds that partial commitment may lead to higher welfare than either full or no commitment. Like in their paper, we can interpret θ as the probability of the lender's request being rejected when he goes to the court after being defaulted upon.

[17] To avoid further mathematical complication, I assume away the possibility of credit rationing on and off equilibrium path in the generalization section.

According to the definition,

$$W_t(\theta) = \delta\Big\{p'_t q[f(k^*) - R_{t+1}k^* + G + \frac{\delta(f(k^*) - R_f k^* + G)}{1-\delta}]$$
$$+ (1 - p'_t q)[(\theta(f(k^*) + \frac{\delta s}{1-\delta})$$
$$+ (1-\theta)(f(k^*) - R_{t+1}k^* + \max\{\frac{\delta s}{1-\delta}, W_{t+1}(\theta)\})]\Big\} \tag{2.8}$$

At the end of each period, if default is impossible, a borrower decides between continuing and quitting after repaying. In a pure-strategy equilibrium, there is a period t such that a borrower without a breakthrough would default as soon as possible once period t arrives.

Now for the earliest default to happen at t in equilibrium, we can generalize the conditions (2.3) and (2.4) to:

$$f(k^*) + \frac{\delta s}{1-\delta} \geq f(k^*) - R_t k^* + W_t(\theta) \tag{2.9}$$

where $R_t = \frac{R_f}{1-(1-p_{t-1}q)\theta}$, and

$$f(k^*) + \frac{\delta s}{1-\delta} \leq f(k^*) - R_f k^* + W_{t-1}(\theta) \tag{2.10}$$

Condition (2.9) says that when allowed, borrowers without a breakthrough will not wait till $t+1$ to default, and (2.10) is the condition for the borrowers not to prefer defaulting at $t-1$ to waiting till t, even if it is possible to default at $t-1$. We can now prove the following lemmas, which are mostly generalizations of those made for the case where $\theta = 1$.

Lemma 5. *Given θ, $W_t(\theta)$ (the value of the expected future payoff by repaying today, proceeding to the next period, and defaulting as soon as possible in the future unless a breakthrough happens or the borrower optimally quits) decreases with t.*

Lemma 6. *For $\theta \in [0,1]$, if there exists a t that satisfies (2.9) and (2.10), then if loan terms are based on the anticipation that the earliest default happens at t if possible, borrowers without a breakthrough will indeed optimally default as soon as possible from period t onward, if not quitting after repaying when default is impossible.*

Lemma 7. *For any t, $W_t(\theta)$ is a decreasing function of θ.*

Lemma 8. *For any $\theta \in [0,1]$, there exists a unique equilibrium in which borrowers will default as soon as possible from period t_θ on, if not quitting earlier before default is possible, where t_θ is the largest value of t that satisfies both (2.9) and (2.10).*

The proofs to lemmas 5 to 8 can be found in the appendix.

Now we are ready to investigate the effect of the probability to default on experimenting time and welfare in the following proposition.

Proposition 10. *Let $0 \leq \theta < \theta' \leq 1$. If the probability to default is θ in each period, a borrower will experiment (weakly) longer before default and get a (weakly) higher expected lifetime payoff than if the probability to default is θ'.*

Proof. See appendix. □

Therefore, all else equal, when it is more difficult to default in an economy, it can cause more experimentation, higher expected payoffs of borrowers who ever run the projects. Participation, however, does not vary according to θ, which can be shown in a similar way as at the end of section 2.4.2, as long as there is no credit rationing on the equilibrium path.

2.6 Conclusions and discussions

In this chapter I investigate the implication of strategic default on the exploration of one's entrepreneurial ability and the welfare of agents who either run business projects or choose their outside options. I find that strategic default hinders experimentation, lowers the expected payoff from starting a business project and causes long-term occupational misallocation. In other words, better enforcement will make potential entrepreneurs more willing to discover their ability and give them higher lifetime payoffs. Allowing borrowers the "freedom" to default is harmful, not because of a higher interest rate which is compensated by the possibility to walk away without repaying, but because it stifles the incentives for entrepreneurs explore their own abilities. When the enforcement in an economy is weaker and default is more likely, fewer businesses will survive in the end and more individuals will stuck in the (maybe less favorable) outside option.

Though we have been assuming observable outside option values, the result that default possibility cause less experimentation can also hold when the outside option is unobservable to the lender. To see this, it is important to recall that the proof of proposition 8 does not rely on any particular form of R_t and R_{t+1}, the interest rates for period t and $t+1$ respectively. When outside option values are unobservable, we no longer have a segmented market as before where each borrower is offered with an individual contract. Instead, there may exist a separating equilibrium where in at least one period, more than one contracts are available for every borrower to choose from. [18]In this case, there will be further welfare loss for some type(s). Specifically, some type(s) will suffer from a smaller loan size than under observable outside option values, in order to prevent other borrowers from picking the contract intended for them. Therefore, there is further welfare loss arising from asymmetric information, and this additional loss only occurs when strategic default is possible.

[18]However, an equilibrium may not always exist.

In addition, as mentioned before, we have been restricting our analysis to cases where there is no credit rationing along the equilibrium path. When credit rationing occurs on the equilibrium paths, it may bring additional welfare loss from smaller loans, and this may further hurt the incentive to participate or continue with experimentation.

A caveat is that the result is not applicable for analyzing involuntary default. Throughout this chapter I assume that borrowers always have the ability to repay. This assumption is suitable for the current setup with dynamic incentives, whose purpose is to prevent strategic default. However, if only involuntary default can happen but not strategic default, the possibility to default, especially if there is limited liability, can on the other hand encourage young entrepreneurs to take risks and experiment. It would be interesting for future research to study the effect of default in an environment where both types of default are possible.

Chapter 3

Dynamic Incentives in Microfinance with Commitment-type Borrowers

3.1 Introduction

As discussed in Chapter 2, microfinance is an important source of credit for people living in the least developed countries in the world. Since the traditional goal of microfinance is to help the most impoverished population who generally do not have the assets needed for collateral backed loans, microfinance relies extensively on other, non-traditional, forms of "collateral", such as social capital (through joint liability) and dynamic incentives (through promising future, or expanded, access to credit), to motivate borrowers to repay their loans when formal enforcement mechanisms are weak.

In this chapter, I study the dynamic incentives in a repeated lending setting, where the availability of future loans is conditional on the repayment of the current loan. The effectiveness of dynamic lending without collateral has been questioned by Shapiro (2015), who suggests that whether dynamic incentives work hinges crucially on the lender's initial belief about borrowers' repayment behaviour, which in equilibrium is self-fulfilling. Shapiro shows that the economy almost surely ends up in an equilibrium with eventually worsening terms unless starting at a single particular value of beliefs. In the end, even the most patient borrower defaults because of the progressively worse future loan terms, i.e. higher interest rates and smaller loan sizes. On the other hand, the worsening loan terms are justified by the decreasing repayment rates over time. Consequently, no loans will be given in the long run. Only one among the infinite number of equilibria exhibit loan terms improving over time and is called the "efficient equilibrium", because only in that equilibrium do a fraction of borrowers continually obtain loans, and it gives the highest total lifetime payoff to the borrowers. However, I show that a small yet crucial modification of the setup, i.e. introducing a type of borrowers who always repay by nature (henceforth called "commitment type" borrowers) in the model economy, eliminates this problem by ruling out all inefficient equilibria and restores the efficiency of dynamic incentives.

Except for the existence of the commitment type borrowers, the setup proposed in this chapter is adopted from Shapiro (2015) and features no collateral, access to future loans contingent on repayment history, no availability of long term loan contracts, heterogeneous discount factors among borrowers, and a so-called "loan term monotonicity" (LTM) condition[1]. I show that Shapiro's market unraveling argument is fragile to an infinitesimal perturbation because all inefficient equilibria vanish when an arbitrarily small fraction of commitment-type borrowers is added, leaving only a unique equilibrium, in which dynamic incentives work as intended and the loan terms improve (with higher loan size and lower interest rate over time).

The reason why any positive fraction of commitment type borrowers prevents inefficient equilibria is that, no matter how few they are, if a large part of the other borrowers default strategically, the commitment type becomes the dominant population, making the repaying proportion high and the loan terms attractive. Working backwards, before this happens, the borrowers who rationally anticipated this would not default in the first place unless they are impatient enough. So the inefficient equilibria with progressively worsening loan terms cannot be sustained. Therefore, even a small fraction of borrowers who always repay is helpful for the coordination between the lender and other borrowers and brings a "positive externality" to all borrowers.

This chapter is related to a large body of literature on the efficacy of dynamic incentives. Among the theoretical works,Albuquerque & Hopenhayn (2004) and Ghosh *et al.* (1999) suggest that the equilibrium loan will exhibit progression in scale over time, which reinforces the dynamic incentives. Although a progressive loan size in equilibrium is also predicted by the current paper, in those two papers the mechanism behind this prediction is either availability to long-term contracts [2] or information asymmetry among lenders,[3] which is very

[1]That is, where the loan size, the repaying borrower's profit, and the borrower's relative gain from defaulting all strictly increase and are continuously differentiable with q, the belief of fraction of loans that are repaid. This condition is satisfied under various market structures and missions of the microfinance institution (MFI). For example, in this chapter I will assume a single non-profit MFI who maximizes each repaying borrower's payoff.

[2]Albuquerque & Hopenhayn (2004) suggest that an optimal contract in a firm financing context would include progressive short-term loans, which are used to finance the working capital. The lender has the right to liquidate the firm upon default. Under expanding short term loans, the equity value of the firm grows as the long-term debt gets paid down and the working capital approaches the efficient level, which relaxes the borrowing constraint and allows larger short-term loans.

[3] Ghosh *et al.* (1999) assume that moneylenders do not share repayment information of the clients. If a borrower defaults, access to future loans will be denied by the current lender, but the borrower can instead apply for a new loan from another lender. They show that a unique equilibrium exists with micro-rationing for the new clients. In other words, any borrower who approaches a lender for the first time receives a smaller loan size than an old borrower.

different from that of the current paper. Among the empirical studies, Karlan & Zinman (2009) conducted field experiments in South Africa and find "large and significant" effect of dynamic repayment incentives in reducing default rate. Giné *et al.* (2010) ran a lab (framed field) experiment among owners and employees of microenterprises in Peru, and find significantly higher repayment rates in treatments with dynamic incentives than analogous treatments under either individual or joint liability.

However, the limitations of dynamic incentives in different context are not overlooked by the literature. In the context of sovereign debt, Bulow & Rogoff (1989) discuss the limitation of dynamic incentives without collateral. They predict that a small country will renege on the payment of sovereign debt in at least some states, and use it to initiate a series of cash-in-advance contracts to insure itself, which leads to no loans in equilibrium. Giné *et al.* (2010), while finding dynamic incentives effective in preventing default, conclude that dynamic incentives can cause borrowers to take on too little risk compared to what is socially optimal. Carlson (2017) investigates the impact of dynamic incentives in the digital credit market in Africa and finds that while initial default is more likely under larger initial loans, a steeper loan ladder causes more later defaults. In other words, progressive lending shifts the timing of default instead of mitigating overall default risks. Carlson (2017) also conducts a structural estimation of the model and suggests that a combination of larger initial loans and a moderately steep loan ladder can maximize the lender's overall profit.

This chapter also contributes to a group of studies discussing how the existence of behavioral agents changes the equilibrium outcome. Akerlof *et al.* (1985) show that a small deviation from rationality can have first-order effects on the equilibrium outcome and the distribution of income, and can further cause a first-order loss in social surplus in an economy with distortions. Haltiwanger *et al.* (1985) suggest that non-maximizing agents can have a disproportionally large effect on equilibrium in situations exhibiting synergistic effects. [4] In the finance literature, the impact of noisy traders on equilibrium price is discussed by De Long *et al.* (1990) and Bhushan *et al.* (1997), among others. In the microfinance literature, Ghosh *et al.* (1999) introduces a proportion of myopic borrowers who always default. The authors show that an equilibrium without macro-rationing exists only when the fraction of myopic borrowers is large enough, which is different from the current paper in which the proportion of commitment-type borrowers can be infinitely small for a unique equilibrium to exist.

The assumption of "commitment-type" borrowers is supported by the experimental literature. For example, Fischbacher & Föllmi-Heusi (2013) find that, in an experiment in which a subject's payoff is determined by the self-reported result of a secret throw of a die, about 39% of subjects report the outcomes honestly despite the potential to increase the

[4] That is, in these situations, an agent is better off when the number of other agents who choose the same action as him increases.

monetary payoff by lying. The main results hold even when it is impossible to track which subject reports what,[5] thus reputation concerns do little to explain their honest behaviour. Charness & Dufwenberg (2006) conduct an experiment based on a sequential trust game between two players. They show that allowing the second mover to make an unenforceable promise improves cooperation between the players. The second mover's promise to cooperate enhances the first mover's trust, as well as the second player's own trustworthy behaviour, even when breaking the promise would give the second player a higher monetary payoff.

The rest of the chapter is organized as follows. Section 3.2 describes the model and gives theoretical predictions. Section 3.3 gives simulations of the model. Section 3.4 concludes.

3.2 Model

Consider an infinite-horizon model where each agent can run a project in every period. The output of the project is given by the production function $f(k) = Ak^{\alpha}$ where k is the amount of capital invested into the project, $\alpha \in (0, 1)$ and $A > 0$. Assume that capital depreciates fully at the end of each period, and saving technology is unavailable. Therefore, agents need to borrow every period to finance their projects. If an agent does not borrow in a period, he cannot run his project and receives a payoff of 0. Furthermore, borrowers differ in their discount factor δ. Lenders do not observe the value of δ for each borrower, but they know the distribution of δ in the economy.

Assume that there is a non-profit lender who seeks to maximize the welfare of each repaying borrower.[6] The lender picks the loan terms, i.e. k_t and interest rate R_t offered to borrowers who are running their projects for the t-th period. A borrower who borrows from lender at t, therefore, invests k_t into his project at t and is required to repay $k_t R_t$ before the next period starts. Borrowers may however default strategically. A borrower can never get a loan in the future once he defaults. This is the dynamic incentive I investigate in this chapter. I assume that no renegotiation is possible, as it is the common practice among MFIs (see Laureti & Hamp (2011) and Alexander (2000)). In addition, lenders cannot commit to long term contracts.

Let the total mass of borrowers be 1. Now I assume there exists a fraction $m > 0$ of borrowers who *never* default and are called the *commitment type* borrowers. (We can call the rest the "non-commitment type" or "strategic type".) Later I will show that the existence of these borrowers plays a crucial role in this chapter. I assume nothing about the discount factors of these borrowers since the discount factors do not affect the analysis

[5] So the experimenter cannot even use Bayesian updating to update his belief about any subject's likelihood of lying.

[6] The same assumption is made by Shapiro (2015).

of this chapter. The model analyzed by Shapiro (2015) can be regarded as a special case where $m = 0$.

Definition 2. *An equilibrium of the model is defined as a sequence of beliefs about repaying borrowers, $\{q_t\}_{t=0}^{\infty}$, loan terms $\{k_t\}_{t=0}^{\infty}$ and $\{R_t\}_{t=0}^{\infty}$ and borrowers' strategy $T(\delta)$ (optimal time to default for borrowers with discount factor δ) such that the rational expectation conditions for both the borrowers and the lenders hold and borrowers are behaving optimally. In other words, an equilibrium requires: 1) Based on the current loan terms and correctly anticipated future loan terms, $T(\delta)$ is indeed the optimal defaulting time for borrowers with discount factor δ; 2)Based on correct expectations about borrowers' behaviour, at any period t, the lender breaks even and every repaying borrower's profit is maximized under interest rate R_t.*

Define p_t as the equilibrium fraction of repaying borrowers at period t. In equilibrium, $q_t = p_t$ because of rational expectations. Since the lender will break even, and every repaying borrower's profit is maximized, the following conditions will be satisfied in equilibrium:

$$R_t = 1/q_t = 1/p_t$$

and

$$k_t = \arg\max_k f(k) - R_t k$$

. [7]

Therefore, there is an one-on-one relationship between k_t, R_t and q_t (or p_t). We can therefore write the interest rate R_t and the loan size k_t as functions of q_t, though in fact $q_t(t \geq 0)$ is endogenously determined and affected by the loan terms. From now on, k_t and q_t will be used interchangeably with $k_t(q_t)$ and $R_t(q_t)$. And the path of q_t in an equilibrium contains all the information for the equilibrium paths of k_t and R_t. Given the production function $f(k) = Ak^{\alpha}$, we have $k_t = q_t^{\frac{1}{1-\alpha}} = p_t^{\frac{1}{1-\alpha}}$ in the equilibrium we study.

Let δ_t be the cutoff discount factor at t below which the borrower will prefer to default in period t. Also define $\epsilon(q_t) \equiv \frac{R(q_t)K(q_t)}{F(K(q_t))}$. For example, with $f(k) = Ak^{\alpha}$, we have $\epsilon(q_t) = A$.

[7] This outcome, however, is not an equilibrium under free entry. In fact, a pure-strategy pooling equilibrium does not exist because it is always profitable to propose a new contract that attracts only borrowers who repay under the new loan contract. That is, for any contract with terms $\{R_t, k_t\}$ at t with $R_t > R_f$, there always exists a profitable deviation $\{R_t', k_t'\}$ with $R_f < R_t' < R_t$ and $k_t' < k_t$ such that $f(k_t') - R_t'k_t' > f(k_t) - R_t k_t$. The new contract $\{R_t', k_t'\}$ will attract only those who will repay under it (because otherwise they would be better off defaulting under $\{R_t, k_t\}$ instead of under $\{R_t', k_t'\}$). So a lender can make a profit under $\{R_t', k_t'\}$ because $R_t' > R_f$. And obviously, there is no separating equilibrium in which one contract is intended for those who would repay under it, and one for those who would not, because no lender would be willing to offer the second contract.

Assume that $\delta \in [\delta_{min}, \delta_{max}]$ with $\delta_{max} > \epsilon(q_t)$ for any $q_t \in [0,1]$ and $\epsilon(1) > \delta_{min}$. [8] Now we can show that the following conditions will also be satisfied on the equilibrium path.

$$\begin{cases} \delta_t = \epsilon(q_t) \cdot \frac{F(K_t)}{F(K_{t+1})} \\ q_{t+1} = \frac{(1-\Phi(\delta_{t+1}))\cdot(1-m)+m}{(1-\Phi(\delta_t))\cdot(1-m)+m} \end{cases} \tag{3.1}$$

The first equation derives from the marginal borrower's indifference between defaulting at t and $t+1$, i.e. $F(K_t) - R_t K_t + \delta_t F(K_{t+1}) = F(K_t)$. The right-hand side of the second equation calculates the actual proportion of repaying borrowers in $t+1$ based on the cutoff discount factors in t and $t+1$. [9] At $t = 0$, the second equation in (1) is

$$q_0 = (1 - \Phi(\delta_0)) \cdot (1 - m) + m \tag{3.2}$$

Now I show how we can computationally derive the equilibrium paths of δ_t and q_t using both equations in (3.1) as follows. Note that for any q_0, we can solve for δ_0 with (3.2), and then K_1 (and q_1) with the first equation in (3.1). Next we get δ_1 from the second equation in (3.1), and then again K_2 (and q_2) from the first equation ....Therefore, for each q_0 we can unwrap a unique path of q_t, δ_t and the loan terms R_t, k_t corresponding to it by repeatedly using (3.1). In other words, each q_0 contains the information of a whole $\{q_t, \delta_t\}_{t=0}^{\infty}$ path.

Now we define what is a feasible path of $\{q_t\}_{t=0}^{\infty}$.

Definition 3. *A $\{q_t\}_{t=0}^{\infty}$ path is feasible if $m \leq q_0 \leq 1$ and $\frac{m}{(1-\Phi(\delta_{t-1}))\cdot(1-m)+m} \leq q_t \leq 1$ for all $t > 0$. That is, the proportion of repaying borrowers is at least that when only the commitment type repay, and at most that when all borrowers repay.* [10] *Otherwise, the $\{q_t\}_{t=0}^{\infty}$ path is infeasible.*

If a q_0 is the initial belief in an equilibrium, the whole trajectory of corresponding q_t should be feasible. Otherwise, if at some t, the computation result yields some q_t with either $q_t > 1$ or $q_t < \frac{m}{(1-\Phi(\delta_{t-1}))\cdot(1-m)+m}$, the corresponding q_0 cannot be sustained in an equilibrium. [11] On the other hand, if the sequence of q_t generated from a certain q_0

[8] Like in Shapiro (2015), without assumption of $\delta_{max} > \epsilon(q_t)$ for any $q_t \in [0,1]$, it is possible that no type of borrowers are patient enough for dynamic incentives to work. The other assumption $\epsilon(1) > \delta_{min}$ ensures that some types will eventually default.

[9] It is derived as follows. Assume the mass of all borrowers is 1. The strategic population remaining in the loan has a mass of $(1-m)(1-\Phi(\delta_t))$ at the beginning of $t+1$, and the repaying strategic population at $t+1$ has a mass of $(1-m)(1-\Phi(\delta_{t+1}))$, while the commitment population always has a mass of m. Thus the mass of total repaying population at $t+1$ is $(1-m)(1-\Phi(\delta_{t+1})) + 1 + m$, which counts for $\frac{(1-\Phi(\delta_{t+1}))\cdot(1-m)+m}{(1-\Phi(\delta_t))\cdot(1-m)+m}$ of the existing population. Note that when $m = 0$, the second equation becomes the same as (6) in Shapiro (2015).

[10] Note that at the beginning of period t, the mass of existing borrowers is $(1 - \Phi(\delta_{t-1})) \cdot (1 - m) + m$.

[11] In other words, this initial belief q_0 can only be supported by future loan terms based on unrealistic beliefs, e.g. "more than all" borrowers repay, or even some of the commitment type default.

never goes out of the boundary of feasible q_t, the trajectory of $\{q_t, \delta_t\}_{t=0}^{\infty}$ from this q_0 is an equilibrium trajectory.

Therefore, we can computationally pin down the value(s) of q_0 on the equilibrium path(s). If a unique equilibrium exists, then there is a unique q_0 associated with a feasible $\{q_t\}$ path. The number of equilibria equals the number of feasible $\{q_t\}_{t=0}^{\infty}$ paths.

The next proposition is the main result of this chapter. Unlike Shapiro (2015) which predicts an infinite number of equilibria, all but one of which are featured by loan terms eventually deteriorating,[12] this chapter shows that any positive fraction of commitment type borrowers will rule out all such equilibria, and the unique equilibrium left will be the one in which loan terms improve over time. Corresponding to this unique equilibrium is a unique path of q_t ($t \geq 0$) increasing with t. Computationally, if $m > 0$, any $\{q_t\}$ path derived using (3.1) will turn out unfeasible unless we start with the unique equilibrium q_0. In contrast, if $m = 0$ (which degenerates to Shapiro's model), though the q_0 that leads to a feasible and monotonically increasing path of q_t is unique, the q_t path started by any q_0 lower than that is also feasible, and will eventually decrease with t.

Proposition 11. *For any $m > 0$, there exists a unique equilibrium. In equilibrium, the loan term improves over time and the repaying fraction strictly increases over time. Moreover, $(q_t, \delta(t))$ to $(1, \epsilon(1))$.*

With details left in the appendix, the proof is completed by four steps. We can show: First, an equilibrium exists. Second, q_t converges to 1 in any equilibrium. The fact that $m > 0$ is crucial for this result to hold. In essence, I utilize the fact that the mass of population remaining repaying is equal to both $\prod_{\tau=0}^{t} q_\tau$ and $(1 - \Phi(\delta_t))(1 - m) + m$ after period t. Since the latter is always greater than $m > 0$, the two cannot equal to each other when $t \to \infty$ unless $q_t \to 1$. Otherwise, there always exists a $\delta > 0$ and a sub-sequence of $\{q_t\}_{t=0}^{\infty}$, $\{q_{f(t)}\}_{t=0}^{\infty}$, that satisfies $q_{f(t)} < 1 - \delta < 1$ for all $t \in \mathbb{N}$, and in that case, $\prod_{\tau=0}^{t} q_\tau \leq \prod_{\tau=0}^{t} q_{f(\tau)} \to 0$ when $t \to \infty$. Therefore, $\prod_{\tau=0}^{t} q_\tau$ and $(1 - \Phi(\delta_t))(1 - m) + m$ can only equal to each other in all periods if $q_t \to 1$ whenever $m > 0$. When $m = 0$, in comparison, this contradiction does not exist anymore, and it is possible that q_t converge to a value smaller than 1.

Third, q_t increases with t along any equilibrium path. To see this, we can first show that once q_t start declining, it will never increase again. [13] Therefore, if q_t does not increase monotonically, it can only converge to 1 from above when $m > 0$, which is impossible. As

[12]In Shapiro (2015), there is a unique efficient equilibrium in which $q_t (t \geq 0)$ increases monotonically with t. This unique efficient equilibrium matches with a unique initial belief q_0^*. In any other equilibrium, the corresponding q_0 is lower than q_0^*. In equilibria with q_0 low enough, (say, below a certain thresholds $\bar{q_0}$,) q_t decreases monotonically with t. In the other equilibria (with q_0 between $\bar{q_0}$ and q_0^*), q_t first increases then decreases with t. See the simulated results in figure 3.2 and figure 3.1.

[13]This is the same as in Shapiro (2015).

we can see, the result from the third step eliminates all the declining and inverse-U-shape equilibrium paths of q_t[14]. Fourth, the equilibrium is unique. To show this, I first prove the uniqueness of equilibrium in which q_t always increase with t, which, combined with the result from step 3, is sufficient for proving the uniqueness of equilibrium.

In comparison, if $m = 0$, results of three last steps no longer hold. In fact, as Shapiro (2015) shows, infinite number of equilibria exist, only one of which is "efficient" with dynamic incentives working as intended and a fraction of borrowers forever remaining in the market.

Regardless of the value of m, as long as $m \in (0, 1)$, the cutoff discount factor and loan terms always converge to the same values as those under the efficient equilibrium when $m = 0$, as we will see in the simulation results in Section 3.3.

There is an immediate implications of the uniqueness of equilibrium. That is, the existence of commitment type makes a difference to the potential benefits from renegotiation. Imagine that the borrowers who default are suddenly allowed to enter the pool for the second time and receive different loans than those who had not defaulted. Without commitment-type borrowers, it is possible that those who drop out for the first time and have discount factor $\delta > \epsilon(1)$ will repay for all periods when given a second chance, because of the multiplicity of equilibria. With the commitment type borrowers, however, the uniqueness of equilibrium ensures that all who default initially have discount factor $\delta \leq \epsilon(1)$, and will default and exit sooner or later even when given a second chance. In other words, with commitment type borrowers in the economy, even an opportunity comes for the previously excluded borrowers to obtain an loan again, it does not affect the number of projects existing in the long run, and the incentive for the borrower to renegotiate with the lender is less strong than if there is no commitment type.

Next I show that not only does the existence of commitment-type borrowers rule out all inefficient equilibria, but the repayment rate also increases with the proportion of such borrowers in the economy, associated with a larger loan size and lower interest rate.

Proposition 12. *The equilibrium repayment rate q_t for any $t \geq 0$ increases with m, the initial fraction of commitment-type borrowers, when $m > 0$. When $m = 0$, q_t in any equilibrium is lower than when $m > 0$.*

Proof. See appendix. □

Proposition 12 suggests that the commitment-type borrowers bring positive externality to all borrowers through improved loan terms. Since both $f(K_t)$ and $f(K_t) - R_t K_t$ increase with q_t in the model, with a larger m, each borrower can at least make the same default choice as with a smaller m, and earn a larger profit. Thus it is guaranteed that all borrowers are better off with a larger proportion of commitment-type borrowers.

[14] Those paths are seen in the model without the commitment type.

3.3 Simulations

The simulations are based on the same setup as in Shapiro's numerical example. Like assumed earlier in this chapter, the loan terms are set according to $R_t = 1/q_t$, and $K_t = argmax_K\{f(K) - R_tK\}$. Besides, the production function takes the form $f(K) = \sqrt{K}$, and the discount factors of the borrowers (except the commitment type) are uniformly distributed between 0 and 1. What is in addition to Shapiro's simulation is a fraction m of commitment type borrowers.

Figure 3.1 and figure 3.2 show the equilibrium paths of q_t and δ_t respectively when $m = 0$. We can see that without commitment type there exist three kinds of equilibria, in which the repayment rate increases, decreases, and first increases and then decreases respectively. In fact, only a unique equilibrium belongs to the first category, which is called the "efficicient equilibrium" by Shapiro. In contrast, both the second and the third categories have infinite number of equilibria, all of which are inefficient and predict that no loan would be given in the long run.

When $m > 0$, the equilibrium is unique. Figure 3.3 and figure 3.4 present the corresponding equilibrium q_t and δ_t trajectories when $m = 0.01$, $m = 0.1$ and $m = 0.2$.

We can see that in all three cases, q_t converges to 1, and δ_t to 0.5. This is consistent with proposition 1 that the unique equilibrium q_t converges to 1. And according to the first equation of 3.1, δ_t always converges to $\epsilon(1) = 0.5$ when q_t converges to 1, regardless of m. Notice also that when $m = 0$, q_t and δ_t in the (unique) efficient equilibrium converge to 1 and 0.5 respectively too.

Moreover, we observe in the simulations that the more commitment-type borrowers the economy has, the higher q_0 (which implies better initial loan terms) and q_t for all $t > 0$, which implies bigger loan size and lower interest rate in all periods, consistent with proposition 12. This is however accompanied by a higher δ_0, meaning that initially a bigger proportion of the non-commitment type would default. Intuitively, with a higher m, the temptation to default in the initial period increases, while the loan terms still converge to the same values in the long run regardless of m, thus the access to future loans becomes relatively less attractive, resulting in more non-commitment borrowers defaulting initially.

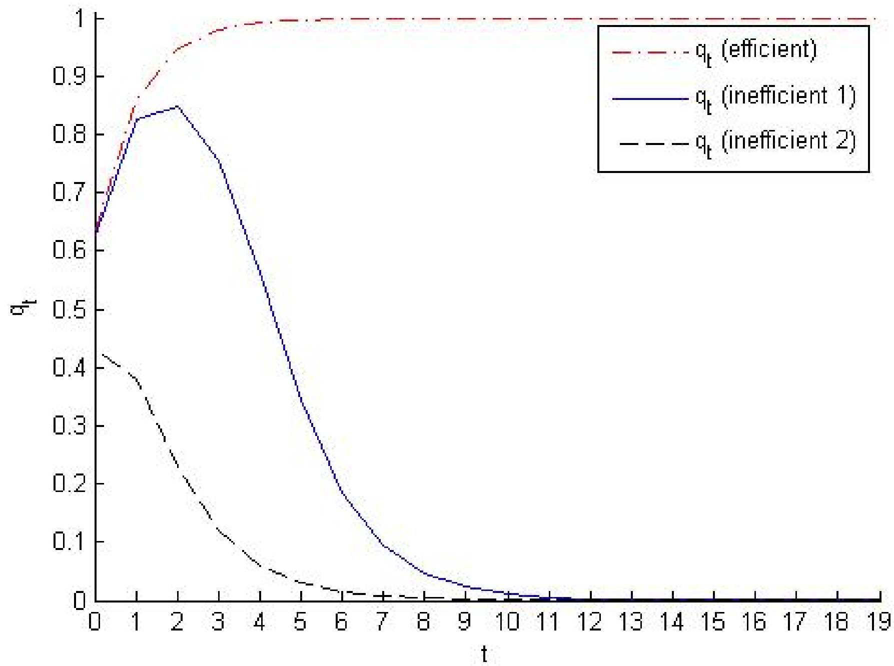

Figure 3.1: Repayment rates in 3 types of equilibria in Shapiro (2015)

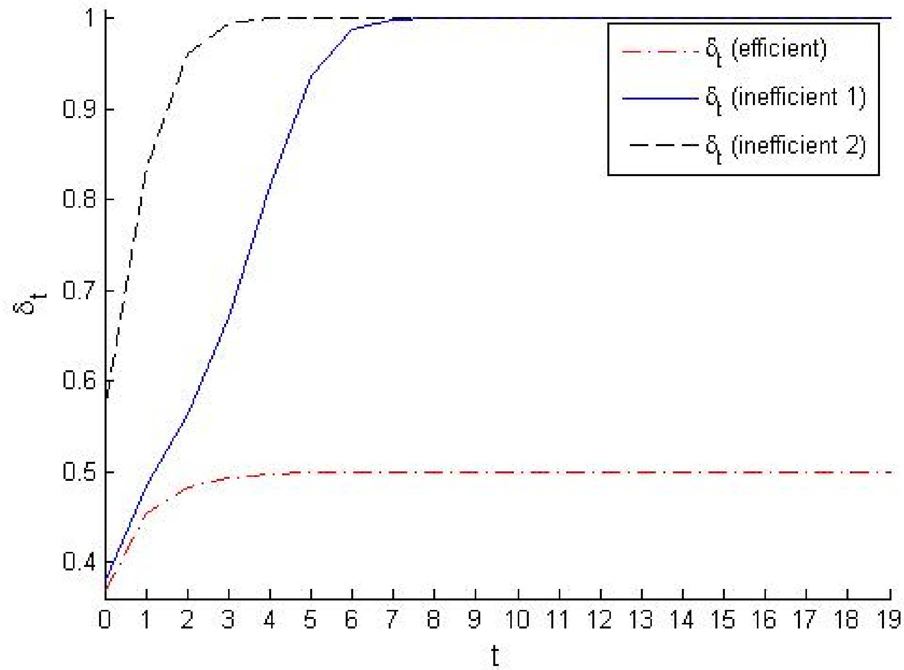

Figure 3.2: Cutoff discount factors in 3 types of equilibria in Shapiro (2015)

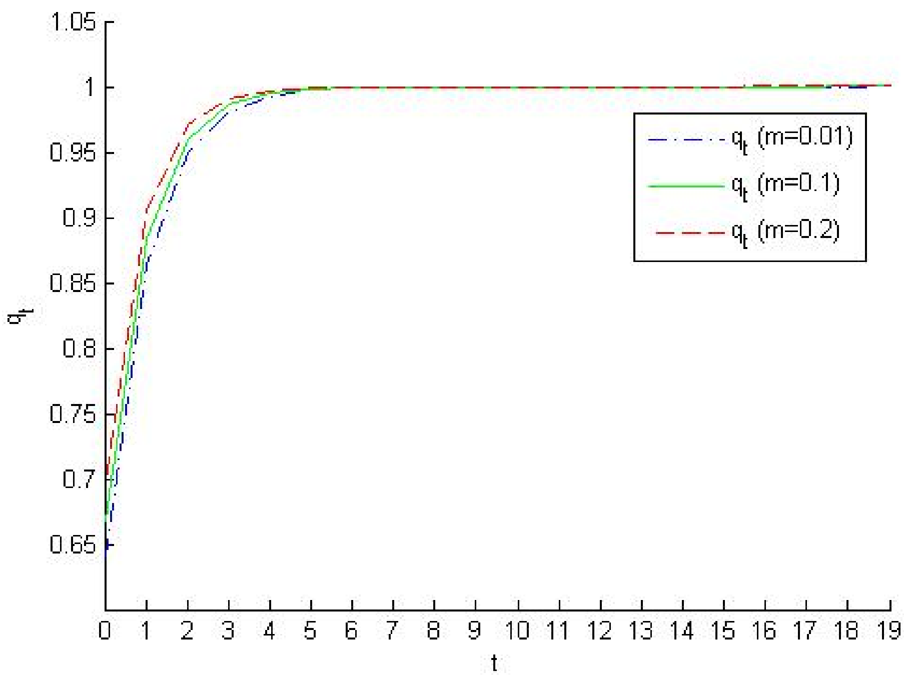

Figure 3.3: Equilibrium repayment rates with $m = 0.01$, $m = 0.1$, and $m = 0.2$

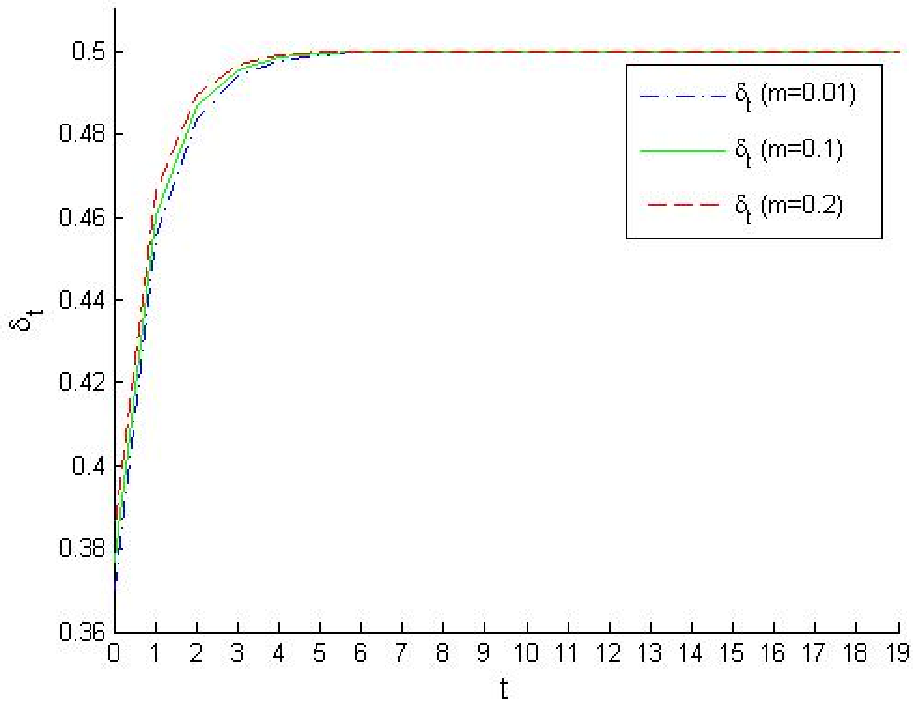

Figure 3.4: Cutoff discount factors with $m = 0.01$, $m = 0.1$, and $m = 0.2$

3.4 Conclusions

A small seed of promise-keeping people can benefit the whole economy, at least in the context of microfinance when both collateral and long-term contracts are lacking. In such context, the dynamic incentive mechanism can be fragile without the commitment type borrowers. As Shapiro shows, in all equilibria except one, the loan terms eventually deteriorate. In these inefficient equilibria, even the most patient borrowers default at some point, which in turn justifies the worsening loan terms. As a result, no loan is given in the end. Therefore, it is a self-fulfilling prophecy if the borrowers expect worsening loan terms and the lender anticipates a high current default rate.

On the other hand, if it is common knowledge that at least some commitment type borrowers exist in the economy who always repay, this “prophecy” cannot be fulfilled. This is because as more strategic borrowers leave the market, the proportion of borrowers of the commitment type increases. If the proportion of strategic borrowers who repay indeed approaches zero, the repayment rate, no matter how small initially, will get close to one, which translates into very attractive loan terms. Expecting this, strategic borrowers patient enough will choose not to default, making the deteriorating loan terms unsustainable.

Therefore, in order to make dynamic incentives work effectively when we cannot rely on other mechanisms like commitment to long-run contracts or informational asymmetry among lenders, it is important to have at least some borrowers who always honour their debts, and make the public aware of it. The existence of those borrowers would induce the coordination between borrowers and the lender, let the productive projects be continually carried out, and enhance the efficiency of the loan market.

A limitation of this study is that my results hinge crucially on the assumption of LTM (loan term monotonicity). Although this condition can be satisfied under a range of market structures and production functions, it can be easily violated as well, for example, under the assumption of a standard perfectly competitive market. [15] However, since this paper's main purpose is to uphold the validity of dynamic incentives given the concerns in Shapiro (2015), and Shapiro's arguments also only apply to settings with LTM, it is beyond the scope of the current research to consider alternative assumptions.

[15] Though some variations of Bertrand competition can sustain the loan terms of the current paper as an equilibrium outcome, even in these setups there exist other equilibria in which LTM does not hold.

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Appendix A

Appendix to Chapter 1

A.1 Proof of Proposition 1

Proof. First we can show this is true for $t+1=T$. Let $V_\tau(\pi)$ be the value function at the beginning of period τ given belief π. Therefore, $V_\tau(\pi)=\max_{E,S}\left(VE_\tau(\pi),VS_\tau(\pi)\right)$, where $VE(.)$ and $VS(.)$ are the value functions for choosing entrepreneurship and the fixed-wage job respectively. In the last period, obviously $p^c_{t+1}=p^c_T=\frac{s-f(k^*)+Rk^*}{G}$. At $t=T-1$, given $p_t=p$, the expected payoff (including current and future) of choosing entrepreneurship is $VE_t(p)=f(k^*)-Rk^*+pqG+\delta pV_T(1)+\delta(1-p)V_T(p')$, where $p'=\frac{p(1-q)}{p(1-q)+(1-p)}$ and $VS_t(p)=s+\delta V_T(p)$ if choosing the fixed-wage job. If $p\geq p^c_{t+1}$, $VS_t(p)=s+\delta max(f(k^*)-Rk^*+pqG,s)=f(k^*)-Rk^*+pqG+\delta s\leq f(k^*)-Rk^*+pqG+\delta s<f(k^*)-Rk^*+pqG+\delta pV_T(1)+\delta(1-p)\max s,f(k^*)-Rk^*+p'qG=VE_t(p)$. Therefore, if $p_{T-1}\geq p^c_T$, $VE_{T-1}>VS_{T-1}$. Thus $p^c_{T-1}<p^c_T$. In other words, if at period $T-1$ the occupational choice is the fixed-wage job occupation, then at period T, the agent will also choose subsistence.

Then we prove this is generally true for any $t\in[1,T]$. Suppose we have proven that $p^c_{t'}<p^c_{t'+1}$ for all $t'>t$, we now show $p^c_t<p^c_{t+1}$. From $p^c_{t'}<p^c_{t'+1}$ we know that once the agent chooses subsistence at any $\tau\geq t'$, he will never choose entrepreneurship again. At t, if $p_t=p\geq p^c_{t+1}$, we have $VE_t(p)=f(k^*)-Rk^*+pqG+\delta pqV_{t+1}(1)+\delta(1-pq)V_{t+1}(p')$ and $VS_t(p)=s+\delta(f(k^*)-Rk^*+pqG)+\delta^2pqV_{t+2}(1)+\delta^2(1-pq)V_{t+2}(p')$. Since $V_{t+1}(p')\geq s+\delta V_{t+2}(p')$, which holds with equality for $p'\leq p^c_{t+1}$, and $V_t(1)=f(k^*)-Rk^*+qG+\delta V_{t+1}(1)$, we have $VE_t(p)-VS_t(p)\geq f(k^*)-Rk^*+pqG-s+\delta pq(f(k^*)-Rk^*+qG)+\delta(1-pq)s+\delta(f(k^*)-Rk^*+pqG)$ (*). Since $f(k^*)-Rk^*+qG>s$, the RHS of (*) increases with p. Call the RHS $\Psi(p)$ Therefore, $VE_t(p)-VS_t(p)\geq\Psi(p^c_{t+1})=VE_t(p^c_{t+1})-VS_t(p^c_{t+1})$. Now if we can show $VE_t(p^c_{t+1})-VS_t(p^c_{t+1})>0$ (**), then it is sufficient to prove $VE_t(p)>VS_t(p)$ for all $p\geq p^c_{t+1}$, and therefore $p^c_t<p^c_{t+1}$. Condition (**) is indeed satisfied because $VE_t(p^c_{t+1})=f(k^*)-Rk^*+p^c_{t+1}qG+\delta p^c_{t+1}qV_{t+1}(1)+\delta(1-p^c_{t+1}q)\frac{s(1-\delta^{T-t})}{1-\delta}=f(k^*)-Rk^*+p^c_{t+1}qG+\delta p^c_{t+1}q(V_{t+2}(1)+\delta^{T-t}(f(k^*)-Rk^*+G))+\delta(1-p^c_{t+1}q)(\frac{s(1-\delta^{T-t-1})}{1-\delta}+\delta^{T-t}s)=VS_{t+1}+\delta^{T-t+1}p^c_{t+1}q(f(k^*)-Rk^*+G)+\delta^{T-t+1}(1-p^c_{t+1}q)s>VS_{t+1}(p^c_{t+1})+\delta^{T-t+1}s=VS_t(p^c_{t+1})$. □

A.2 Proof of Proposition 2

Proof. Let $VE_\tau(z,p)$ and $VS_\tau(z,p)$ be the value functions for an agent with wealth z and belief to have high ability p to choose entrepreneurship and the alternative occupation at period τ respectively, when borrowing constraints do not exist. And let VE'_τ and VS'_τ be the corresponding value functions when borrowing constraints exist.

First let's show $q^c_t(z) \geq p^c_t$ for $t = T$. In the last period, for a risk neutral agent with belief p, he would choose entrepreneurship only if $f(k_T) - Rk_T + pqG \geq s$, i.e.$p \geq \frac{s - f(k_T) + Rk_T}{qG}$. When the agent is not credit constrained, $k_T = k^*$, otherwise, $k_T \leq k^*$, therefore, $p^c_T \leq p'^c_T$. When the borrowing constraint is binding, in other words, when borrowers prefer to borrow more but cannot, $k_T < k^*$. In such cases $p^c_T < p'^c_T(z)$.

Then I will show that if there exist some t, such that for all z and all $t' > t$, $p^c_{t'} \leq p'^c_{t'}(z)$ holds, then the result can be extended to period t too, i.e. $p^c_t \leq p'^c_t(z)$ for all z. Since we assumed $R = 1/\delta$, all who face borrowing constraints would weakly prefer to save all income to overcome current and future wealth constraints. But when borrowing constraints are absent, agents is indifferent between saving and consuming today. So in this proof we assume that agents in both cases save all income and consume it in the end of the last period. Therefore, with borrowing constraints, $VE'_t(z,p) = \delta pq V'_{t+1}(f(k) - Rk + zR + G, 1) + \delta(1 - pq)V'_{t+1}(f(k) - Rk + zk, p')$ and $VS'_t(p,z) = \delta V'_{t+1}(Rz + s, p)$. Without borrowing constraints, the only difference is that now $k = k^*$ for all periods. Therefore, $VE_t(p,z) = \delta pq V_{t+1}(f(k^*) - Rk^* + zR + G, 1) + \delta(1 - pq)V_{t+1}(f(k^*) - Rk^* + Rz, p')$ and $VS_t(p,z) = \delta V_{t+1}(Rz + s, p)$.

When $p = p^c_t$, that is, when the current belief is at the unconstrained cutoff level, by proposition 11 we know $p^c_t < p^c_{t+1}$. Therefore, without borrowing constraints, if the agent chooses the fixed-wage job at t, he will strictly prefer the fixed-wage job in the future. Thus $VS_t(z,p) = z + s/(1-\delta)$. Now we can prove $VS'_t(z,p^c_t) = VS_t(z,p^c_t)$. This is true because $p^c_{t'} \leq p'^c_{t'}(z)$ for all z at any $t' > t$. Thus with borrowing constraints, the agent with z and $p = p^c_t$ will also stay in the fixed-wage job for a lifetime if he chooses subsistence at t.

On the other hand, $VE'_t(z,p) \leq VE_t(z,p)$ holds for the following reason. For any paths of occupational choice and wealth level possible under borrowing constraint, we can exactly replicate them when borrowing constraints does not exist. However, not all wealth paths and occupational choice paths feasible without borrowing constraints can be achieved when borrowing constraints exist. In other words, there are more options without borrowing constraints and the agent must do at least as well without constraints as with constraints.

Now since $VE_t(z,p^c_t) = VS_t(z,p^c_t)$, $VS'_t(z,p^c_t) = VS_t(z,p^c_t)$ and $VE'_t(z,p^c_t) \leq VE_t(z,p^c_t)$, we have $VE'_t(z,p^c_t) \leq VS'_t(z,p^c_t)$. Similarly we can show that agents with borrowing constraints will strictly prefer subsistence if $p < p^c_t$. Thus $p^c_t \leq p'^c_t(z)$. When in the current period the borrowing constraint is binding, $VE'_t(z,p^c_t) < VE_t(z,p^c_t)$, so in that case $p^c_t < p'^c_t(z)$. □

A.3 Proof of Proposition 5

Proof. First, I will compare the economies "L" and "BCL", which are equivalent except for the existence of borrowing constraints. Let's compare two risk-neutral learning agents with the same initial wealth, same entrepreneurial ability, same initial belief about the probability to have high ability (p_1), and same numbers of periods (T) left. One of them lives in the L economy (called A_L) and the other the BCL economy (called A_{BCL}). If both agents have low ability, then in the long run both agents will be in the fixed-wage job. Now we discuss what happens if both agents have high ability.

Because everything other than borrowing constraints is the same in the two economies, I assume A_L and A_{BCL} face the same success/failure sequence when they run their own business. In the long run, if A_{BCL} ends up as an entrepreneur, he must have learned that his ability is high. Otherwise, sooner or later, successive low outputs would push his belief below the cutoff belief (even the cutoff without borrowing constraints) and he would give up the business option forever.

Define period t_1 as the first period that the project of A_{BCL} yields high output. In other words, at t_1 agent A_{BCL} knows he has high ability for the first time. Then A_L, A_{BCL}'s counterpart in L economy, must also receive the first ever high output no later than t_1. First, in period 1, A_L must be an entrepreneur. Because otherwise, he would always stay in the fixed-wage job, and so would A_{BCL} whose cutoff belief is always at least as high as A_L's. Second, also as suggested by proposition 1, A_L must either work as an entrepreneur for all periods in his lifetime, or until he gives up forever. (Because if he chooses subsistence in some period, he will never switch back into entrepreneurship, since his belief will no longer be updated while the threshold belief is increasing.)

Therefore, if A_L runs the project for the k-th time at all, he would do it in the k-th period, while A_{BCL} either runs the project for the k-th time in period $k' \geq k$ or never run it for the k-th time. Again let p^c_τ and p'^c_τ be the cutoff beliefs of A_L and A_{BCL} at time τ respectively. By propositions 1 and 2 we know $p^c_k \leq p^c_{k'} \leq p'^c_{k'}$. Thus for any $k > 0$, if A_{BCL} is willing to run his project for the k-th time, A_L will not quit entrepreneurship before the k-th period.

Now assume that for both agents, the first high output comes when the project is run for the n_1-th time. (Note that $n_1 \leq t_1$ and at t_1 A_{BCL} runs his project for the n_1-th time.) From the above analysis we know A_L certainly runs a business and realizes that he has high ability at period n_1. Therefore, in the long run, if A_{BCL} ends up as an entrepreneur, so will A_L. On a macro level, the entrepreneur population in L economy cannot be smaller than in BCL economy.

The opposite, however, is not true. Again assume agent A_L is willing to run a business for the k-th time given $k-1$ failures. (And the first high payoff will come during the k-th trial.) A_{BCL} may have exited permanently before running a business for the $(k-1)$-th time. If not, another possibility is that borrowing constraints delay the time A_{BCL} finishes running the project for $k-1$ times. In that case, the k-th trial can only happen at a $k' > k$, with $p^c_k < p^c_{k'} \leq p'^c_{k'}$. If after the first $k-1$ failures, the belief of the A_L and A_{BCL} (who start with the same initial belief) falls above p^c_k and below $p^c_{k'}$, then A_{BCL} will never run a business again and never discover his true ability, while A_L will run a business for the k-th time, discover that his ability is high, and remain an entrepreneur for the rest of lifetime. Since the initial belief p^i has pdf $f(p^i) > 0$ for all $p^i \in (0, 1)$, the continuity of belief updating function ensures that the density of any belief between 0 and 1 is positive at any t. So such agents A_L and A_{BCL} exist. For example, if A_{BCL} does not have enough initial wealth to meet the minimum requirement, or $\lambda z < k_{min}$, while his counterpart in economy L starts his carrier as an entrepreneur, then if the two agents have certain initial belief, there is a positive probability that A_L ends up with entrepreneurship while A_{BCL} does not. Thus on a macro level, with a continuum of agents, the economy L will end up with more entrepreneurs than the economy BCL.

In comparison, in economy BC every agent with high ability will eventually become an entrepreneur. We can show this with $T \to \infty$. In this case the cutoff wealth for a high-ability agent to run a business is the same for all periods. If an agent with high ability does not start as an entrepreneur, he will save his income from the outside option, and his wealth will eventually surpass the cutoff wealth. Since under the minimum capital restriction, $f(k) - Rk \geq 0$ always holds, an agent's wealth cannot decrease over time, so this agent will remain an entrepreneur for the rest of life time. Thus for T long enough, the proportion of entrepreneurs in the BC economy will eventually become the proportion of high-ability entrepreneurs in the economy, as the constrained agents save to overcome borrowing constraints. □

A.4 Proof of Proposition 4

Proof. (1) Assume the extreme case where $f''(k) = 0$, i.e. $f(k)$ is a linear function of k. Assume a two-period model. If learning does not exist, an agent would simply maximize the expected payoff in each period, thus the cutoff wealth is the same in every period. In other words, households will choose entrepreneurship if and only if $f(\lambda z) - R\lambda z + pqG \geq s$

(2) Now suppose that instead everyone starts with ex-ante belief p that his project is good, and the probability for a breakthrough to happen to a good project is q. The old have only one period left, so they start with $t = 2$. The young start with $t = 1$ and have two periods left. At $t = 2$, only the current-period payoff matters for the occupational choice. Therefore, if $f(k^*) - R_f k^* + pqG < s$ or $p < \bar{p} < p(s - f(k^*) + R_f k^*)/(Gq)$, no one with p would become an entrepreneur. Now we focus on the case if $p \geq \bar{p}$ and $p < s/G$. At $t = 2$, the threshold wealth $\bar{z}$ for starting a business is such that $f(\lambda\bar{z}) - R_f\lambda\bar{z} + pqG = s$. Now I show that with $\bar{z}$, the young strictly prefers running a business.

With $z = \bar{z}$, if the agent runs a business at period 1, the expected present value of lifetime payoff is $VE \geq \Pi(\lambda z) + pqG + \delta pq(\Pi(k'_H) + q'G) + \delta(1 - pq)(\Pi(k'_L) + pqG)$ where $\Pi(k) = f(k) - Rk$ and $k'_H = \lambda(\Pi(\lambda z) + G + Rz)$ and $\Pi(k) = f(k) - Rk$ and $k'_L = \lambda(\Pi(\lambda z) + Rz)$. If the agent chooses the outside option, then $VS = s + \delta(\Pi(k'_s) + pqG)$, where $k'_s = \lambda(Rz + s)$. So it would be sufficient to prove $VE > VS$, if we could show $\Pi(\lambda z) + pqG + \delta pq(\Pi(k'_H) + q'G) + \delta(1 - pq)(\Pi(k'_L) + pqG) > s + \delta(\Pi(k'_s) + pqG)$. Since the production function is linear, we have $pq\Pi(k'_H) + (1 - pq)\Pi(k'_L) = \Pi(k'_s)$. Thus the inequality to prove becomes $\Pi(\lambda z) + pqG + \delta pqq'G + \delta(1 - pq)pqG > s + \delta pqG$.

Replacing s with $\Pi(k) + pqG$, we now need to show $\delta pqq'G + \delta(1 - pq)pqG > \delta pqG$, which is equivalent to $\delta pqq'G - \delta pqpqG > 0$, or $q' > pq$. This is indeed true, because $q' > pq$.

Therefore, in a two-period model, if an old agent with z is indifferent between entrepreneurship and the outside option, a young one with the same wealth and belief will strictly prefer entrepreneurship. In other words, the cut-off wealth level, above which one chooses to run a business, is lower for the young than for the old, because the learning incentive partially offset the business income loss due to a lower wealth. $\square$

A.5 Regression results with estimates in exponential form

Table A.1: The likelihood of business entry (using wealth in 84, coefficients in exponential form)

	(1)	(2)	(3)
	All	top 40%	top 20%
entry			
2 years ($\exp\hat{\gamma_2}$)	0.572***	0.543***	0.470***
	(-7.80)	(-6.64)	(-6.09)
3 years ($\exp\hat{\gamma_3}$)	0.448***	0.376***	0.382***
	(-9.84)	(-8.87)	(-6.83)
young ($\exp\hat{\beta_y}$)	2.131***	1.443***	1.500***
	(8.22)	(5.57)	(4.21)
wealthy_84 ($\exp\hat{\beta_w}$)	2.761***		
	(10.79)		
young&wealthy_84 ($\exp\hat{\beta_{wy}}$)	0.678***		
	(-3.44)		
N	44820	17641	8227

Exponentiated coefficients; t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A.2: The likelihood of business exit (using wealth in 84, coefficients in exponential form)

	(1) All	(2) top 40%	(3) top 20%
exit			
2 years ($\exp \hat{\gamma_2'}$)	0.556***	0.588***	0.592***
	(-8.60)	(-5.89)	(-4.39)
3 years ($\exp \hat{\gamma_3'}$)	0.356***	0.378***	0.363***
	(-10.94)	(-8.16)	(-6.41)
young ($\exp \hat{\beta_y'}$)	0.910	0.764***	0.646***
	(-1.05)	(-3.89)	(-4.50)
wealthy_84 ($\exp \hat{\beta_w'}$)	0.610***		
	(-5.54)		
young&wealthy_84 ($\exp \hat{\beta_{wy}'}$)	0.844		
	(-1.49)		
N	6462	4587	3089

Exponentiated coefficients; t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

A.6 Robustness checks

A.6.1 Regressions with wealth in 89 and 94

Table A.3: The likelihood of business entry (using wealth in 89)

	(1) All	(2) top 40%	(3) top 20%
entry			
2 years ($\hat{\gamma_2}$)	-0.516***	-0.729***	-0.706***
	(-7.69)	(-8.14)	(-6.39)
3 years ($\hat{\gamma_3}$)	-0.787***	-0.992***	-1.001***
	(-10.37)	(-9.72)	(-7.75)
young ($\hat{\beta_y}$)	0.703***	0.321***	0.445***
	(7.68)	(5.32)	(5.66)
wealthy_89 ($\hat{\beta_w}$)	1.041***		
	(11.03)		
young&wealthy_89 ($\hat{\beta_{wy}}$)	-0.379***		
	(-3.46)		
N	51805	20306	8975

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A.4: The likelihood of business entry (using wealth in 94)

	(1) All	(2) top 40%	(3) top 20%
entry			
2 years ($\hat{\gamma_2}$)	-0.486***	-0.642***	-0.657***
	(-7.09)	(-7.44)	(-6.17)
3 years ($\hat{\gamma_3}$)	-0.711***	-0.849***	-0.911***
	(-9.16)	(-8.73)	(-7.38)
young ($\hat{\beta_y}$)	0.543***	0.287***	0.433***
	(5.59)	(4.86)	(5.85)
wealthy_94 ($\hat{\beta_w}$)	1.077***		
	(10.94)		
young&wealthy_94 ($\hat{\beta_{wy}}$)	-0.255*		
	(-2.24)		
N	47075	20344	9674

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A.5: The likelihood of business exit (using wealth in 89)

	(1) All	(2) top 40%	(3) top 20%
exit			
2 years ($\hat{\gamma_2'}$)	-0.619***	-0.555***	-0.585***
	(-9.66)	(-6.53)	(-5.21)
3 years ($\hat{\gamma_3'}$)	-1.054***	-1.028***	-1.135***
	(-11.86)	(-9.09)	(-7.50)
young ($\hat{\beta_y'}$)	-0.0581	-0.266***	-0.381***
	(-0.68)	(-4.13)	(-4.51)
wealthy_89 ($\hat{\beta_w'}$)	-0.578***		
	(-6.71)		
young&wealthy_89 ($\hat{\beta_{wy}'}$)	-0.207		
	(-1.94)		
N	7178	5219	3824

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A.6: The likelihood of business exit (using wealth in 94)

	(1) All	(2) top 40%	(3) top 20%
exit			
2 years ($\hat{\gamma_2'}$)	-0.642***	-0.644***	-0.619***
	(-9.59)	(-7.55)	(-5.67)
3 years ($\hat{\gamma_3'}$)	-1.048***	-1.021***	-1.203***
	(-11.33)	(-9.17)	(-7.90)
young ($\hat{\beta_y'}$)	0.0208	-0.199**	-0.348***
	(0.23)	(-3.15)	(-4.22)
wealthy_94 ($\hat{\beta_w'}$)	-0.497***		
	(-5.48)		
young&wealthy_94 ($\hat{\beta_{wy}'}$)	-0.221*		
	(-1.99)		
N	6708	5090	3744

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

A.6.2 Regressions including actual wealth

Table A.7: The likelohoof of business entry (using actual wealth)

	(1)	(2)	(3)
	year=84	year=89	year=94
enter_1year			
2 years	-0.557***	-0.509***	-0.485***
	(-7.78)	(-7.59)	(-7.08)
3 years	-0.799***	-0.777***	-0.711***
	(-9.81)	(-10.23)	(-9.16)
Young	0.757***	0.705***	0.544***
	(8.22)	(7.70)	(5.60)
wealthy × Old	0.977***		
	(10.36)		
wealthy × Young	0.592***		
	(9.31)		
total wealth 84	0.000000415***		
	(5.82)		
wealthy × Old		0.940***	
		(9.85)	
wealthy × Young		0.603***	
		(10.74)	
total wealth 89		0.000000675***	
		(11.83)	
wealthy × Old			1.025***
			(10.36)
wealthy × Young			0.791***
			(13.87)
total wealth 94			0.000000294***
			(7.18)
Observations	44820	51805	47075

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A.8: The likelohoof of business exit (using actual wealth)

	(1)	(2)	(3)
	year=84	year=89	year=94
exit			
2 years	-0.581***	-0.607***	-0.627***
	(-8.51)	(-9.47)	(-9.37)
3 years	-1.022***	-1.027***	-1.019***
	(-10.83)	(-11.55)	(-11.01)
Young	-0.0950	-0.0603	0.0408
	(-1.06)	(-0.71)	(0.45)
wealthy × Old	-0.397***		
	(-4.27)		
wealthy × Young	-0.601***		
	(-8.22)		
total wealth 84	-0.000000639***		
	(-3.33)		
wealthy × Old		-0.354***	
		(-3.89)	
wealthy × Young		-0.603***	
		(-8.66)	
total wealth 89		-0.00000117***	
		(-6.25)	
wealthy × Old			-0.213*
			(-2.19)
wealthy × Young			-0.485***
			(-6.88)
total wealth 94			-0.00000134***
			(-7.32)
Observations	6462	7178	6708

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Appendix B

Appendix to Chapter 2

B.1 Proof of Lemma 2

Proof. We already know (2.3) and (2.4) are necessary conditions for t to be the optimal defaulting time, if the loan terms are constructed based on lenders' anticipation that default happens only at t. Now we prove that (2.3) and (2.4) are also sufficient conditions for t to be the optimal defaulting time given such loan terms.

Let $V(\tau)$ be discounted value of expected lifetime income at the beginning of period one if a borrower with s defaults at period τ, if loan terms reflect the anticipation that borrowers default at t if no breakthrough occurs.

First we show that $V(t) \geq V(t+1)$ implies that $V(t+m) \geq V(t+m+1)$ for any $m \geq 0$. Given Lemma 1, the condition for the borrower to prefer to default at $t+m$ to $t+m+1$ is

$$\begin{aligned} R_{t+m}k^* + \frac{\delta s}{1-\delta} \geq \\ \delta\Big\{p'_{t+m}q[f(k^*) - R_{t+m+1}k^* + G + \frac{\delta(f(k^*) - k^*R_f + G)}{1-\delta}] \\ + (1-p'_{t+m}q)(f(k^*) + \frac{\delta s}{1-\delta})\Big\} \end{aligned} \quad \text{(B.1)}$$

if there is no credit rationing at and before $t+m+1$. Because $R_{t+m+1} = R_f/(p'_{t+m}q)$, the RHS also equals $\delta[f(k^*) - k^*R_f + p'_{t+m}q[G + \frac{\delta(f(k^*)-k^*R_f+G)}{1-\delta}] + (1-p'_{t+m}q)(f(k^*) + \frac{\delta s}{1-\delta})]$. And we can see the value increases with p'_{t+m}, and thus decreases with m.

At the same time, the value of the LHS of (B.1) increases with m because R_{t+m} is inversely proportional to p_{t+m-1}, and p_{t+m-1} decreases with m.

When $m = 0$, (B.1) becomes the condition (2.3) for t. So (B.1) holds for $m = 0$. When $m > 0$, (B.1) should hold with strict inequality, because the LHS increases while the RHS decreases. Therefore we have $V(t) > V(t+1) > V(t+2) > V(t+3)$... when the interest rates offered correctly reflects the borrower's optimal behaviour both on and off the equilibrium path.

If the first period of credit rationing happens at period $t+n+1$ with $n \geq 1$, then from above we know that $V(t+m+1) < V(t+m)$ for all $0 \leq m < n$. At $t+n$, the condition of preferring to default right away instead of waiting till $t+n+1$ is

$$R_{t+n}k^* + \frac{\delta s}{1-\delta} \geq \delta\Big\{p'_{t+n}q\frac{\delta(f(k^*) - k^*R_f + G)}{1-\delta} + (1-p'_{t+n}q)(f(k_{max}t+n+1) + \frac{\delta s}{1-\delta})\Big\} \quad \text{(B.2)}$$

which is more relaxed than B.1. Therefore, $V(t+n) > V(t+n+1)$ if the earliest rationing happens at $t+n+1$ for $n \geq 0$. The condition above also implies

$$R_{t+n}k_{max}^{t+n} + \frac{\delta s}{1-\delta} \geq \delta\Big\{p'_{t+n}q\frac{\delta(f(k^*) - k^*R_f + G)}{1-\delta} + (1-p'_{t+n}q)(f(k_{max}t+n+1) + \frac{\delta s}{1-\delta})\Big\} \quad \text{(B.3)}$$

because $k_{max}^{t+n} > k^*$. By assumption $\mathcal{G}(\tau) = -R_\tau k_{max}^\tau + \delta\Big\{p'_\tau q\frac{\delta(f(k^*)-R_fk^*+G)}{1-\delta} + (1-p'_\tau q)[f(k_{max}^{\tau+1}) + \frac{\delta s}{1-\delta}]\Big\}$ decreases with τ. Thus it is also true that

$$R_{t+n}k_{max}^{t+m} + \frac{\delta s}{1-\delta} \geq \delta\Big\{p'_{t+m}q\frac{\delta(f(k^*) - k^*R_f + G)}{1-\delta} + (1-p'_{t+m}q)(f(k_{max}t+m+1) + \frac{\delta s}{1-\delta})\Big\} \quad \text{(B.4)}$$

for all $m > n$, i.e., $v(t+m) > V(t+m+1)$ holds for all $m > n$.

To summarize, $v(t) > V(t+1)$ implies $V(t+m) > V(t+m+1)$ for all $m \geq 0$.

Now we show that $V(t) \geq V(t-1)$ implies that $V(t-m) > V(t-m-1)$ for all $m > 0$. Given the loan terms, the condition for the borrower to prefer to default at $t-2$ than waiting till $t-1$ is

$$f(k^*) + \frac{\delta s}{1-\delta} \leq f(k^*) - k^*R_f + \delta[p'_{t-m-1}q(f(k^*) - k^*R_f + G + \delta\frac{f(k^*) - k^*R_f + G}{1-\delta}) + (1-p'_{t-m-1}q)(f(k^*) + \frac{\delta s}{1-\delta})] \quad \text{(B.5)}$$

Compared to (2.4) (for t), (B.5) is less strict because the RHS of (B.5) is bigger while the LHS the same. The reason why RHS of (B.5) is bigger than of (2.4) is that $f(k^*) - R_fk^* >$

$f(k^*) - R_t k^*$ and $p'_{t-m-1} > p'_{t-1}$ for $m > 0$. In other words, a bigger weight is put on the future income if a breakthrough does happen, which is higher than if it does not, and the future income if a breakthrough happens is also bigger in (B.5) when $m > 0$ than in (2.4) because before t the interest rate is lower ($R = R_f$) in (B.5) and makes the borrower gain more from the project.

Therefore, if (2.4) satisfies, (B.5) also holds for all $m > 0$. This implies that $V(t) \geq V(t-1) > V(t-2) > V(t-3)$....

In conclusion, if (2.3) and (2.4) both hold, then t is the optimal defaulting period if no breakthrough happens under interest rates given in page 29, according to the (correct) anticipations that default happens only at t. Therefore, (2.3) and (2.4) are sufficient and necessary conditions for defaulting at t to be optimal for those without a breakthrough, given the lenders' anticipation that default happens only at t. □

B.2 Proof of Lemma 4

Proof. Assume that any $t \in [t_1, t_2]$ satisfies both 2.3 and 2.4. Thus it suffices if we show a candidate equilibrium defined by t (as its unique defaulting time of borrowers without a breakthrough) yields a lower expected lifetime utility than a candidate defined by $t+1$ for $t \in [t_1, t_2)$. Since in these two candidate equilibria, (let's call them A and A' respectively,) borrowers' behaviour and the loan terms are exactly the same before t, so we can just start the comparison from period t.

At period t, if a breakthrough happens, the borrower defaults in neither equilibrium, and gets $f(k^*) - R_t k^*$ in period t under A and $f(k^*) - R_f k^* > f(k^*) - R_t k^*$ under A'. From $t+1$ on the payoffs will be the same again in these two candidate equilibria. So if a breakthrough happens at t, A gives a lower total payoff than A'.

If a breakthrough does not happen in period t, then the borrower will default under A and get a lifetime payoff (starting t) of present value $f(k^*) + \delta s/(1-\delta)$. Under A', however, the same borrower does not default at t but will do so at $t+1$ unless a breakthrough happens by then. Thus under A' the present value of the lifetime payoff (starting t) is $f(k^*) - R_f k^* + \delta p'_t q \frac{f(k^*) - R_{t+1}k^* + G}{1-\delta} + \delta(1 - p'_t q)(f(k^*) + \frac{\delta s}{1-\delta})$. Because (2.4) is satisfied at $t+1$, so $f(k^*) + \delta s/(1-\delta) \leq f(k^*) - R_f k^* + \delta p'_t q \frac{f(k^*) - R_{t+1}k^* + G}{1-\delta} + \delta(1 - p'_t q)(f(k^*) + \frac{\delta s}{1-\delta})$. Therefore, the expected payoff under A' is larger than that under A when a breakthrough does not happen in t.

To summarize, the lifetime expected payoff in a candidate equilibrium where a borrower without a breakthrough defaults at t is strictly less than that in a candidate where default happens at $t+1$, if both t and $t+1$ satisfy (2.3) and (2.4). Therefore, when multiple values of t satisfy both (2.3) and (2.4), the candidate equilibrium defined by the longest experimenting time (largest t) yields the highest expected lifetime payoff to the borrower. □

B.3 Proof for no credit-rationing to happen in equilibrium

Proof. Let t be the period that a borrower with s would default if a breakthrough does not come, and t' be the period he would exit if default is impossible. We already know that $t \geq t'$. Therefore, $R_t = \frac{R_f}{p_{t-1}q} < \frac{R_f}{p_{t'-1}q}$. Since without the possibility to default, a borrower would exit as soon as his belief falls below the cutoff $p_s = f(s) = \frac{(1-\delta)(s-f(k^*)+R_f k^*)}{q\delta(f(k^*)-R_f k^*-s)+qG}$, thus $p_{t'-1} > p_c = f(s)$ and $R_t < \frac{R_f}{p_{t'-1}q} < \frac{R_f}{f(s)q}$. There will be no credit rationing on the equilibrium path as long as $f(k^*) - R_t k^* + G > 0$, or $R_t < \frac{f(k^*)+G}{k^*}$. Thus a sufficient condition of it is $\frac{R_f}{f(s)q} < \frac{f(k^*)+G}{k^*}$ or $f(s) > \frac{R_f k^*}{(f(k^*)+G)q}$. □

B.4 Proofs of lemmas 5, 6, 7 and 8

(1) lemma 5

Proof. Define $V_t^w(\theta)$ as the present discounted value of future payoffs by repaying today and proceed to tomorrow (and may or may not default in future periods even if possible, and may or may not quit after repaying when default is impossible). Thus we have:

$$V_t^w(\theta) = \delta\Big\{p_t'q[f(k^*) - R_{t+1}k^* + G + \frac{\delta(f(k^*) - R_f k^* + G)}{1-\delta}] + (1-p_t'q)[(\theta(f(k^*) + \frac{\delta s}{1-\delta}) + (1-\theta)(f(k^*) - R_{t+1}k^* + \max\{\frac{\delta s}{1-\delta}, V_{t+1}^w(\theta)\})]\Big\} \tag{B.6}$$

And $V_t^w(\theta) \geq W_t(\theta)$ for all t. If $t^* \leq t+1$, the interest rate at $t+1$ is $R_{t+1} = \frac{R_f}{1-(1-p_t q)\theta}$. In this case,

$$\begin{aligned} V_t^w(\theta) = W_t(\theta) = \\ &\delta\Big\{f(k^*) - (1-(1-p_t q)\theta)R_{t+1}k^* + p_t'q(G + \frac{\delta(f(k^*) - R_f k^* + G)}{1-\delta}) \\ &+ (1-p_t'q)[\theta\frac{\delta s}{1-\delta} + (1-\theta)\max(\frac{\delta s}{1-\delta}, W_{t+1}(\theta))]\Big\} = \\ &\delta\Big\{f(k^*) - R_f k^* + p_t'q(G + \frac{\delta(f(k^*) - R_f k^* + G)}{1-\delta}) \\ &+ (1-p_t'q)[\theta\frac{\delta s}{1-\delta} + (1-\theta)\max(\frac{\delta s}{1-\delta}, W_{t+1}(\theta))]\Big\} \end{aligned}$$

with $W_{t\to\infty}(\theta) = \delta(f(k^*) - R_f k^*) + \frac{\delta^2 s}{1-\delta} < \frac{\delta s}{1-\delta}$.

If $t^* > t+1$, $R_{t+1} = R_f$, then we also have $W_t(\theta) = \delta\Big\{f(k^*) - R_f k^* + p_t'q(G + \frac{\delta(f(k^*)-R_f k^*+G)}{1-\delta}) + (1-p_t'q)[\theta\frac{\delta s}{1-\delta} + (1-\theta)\max\{\frac{\delta s}{1-\delta}, W_{t+1}(\theta)\}]\Big\}$.

Because $W_{t\to\infty}(\theta) = \delta(f(k^*) - R_f k^*) + \frac{\delta^2 s}{1-\delta} < \frac{\delta s}{1-\delta}$, there exists a $\hat{t}$ such that for all $t > \hat{t}$, $W_t(\theta) = \delta[f(k^*) - R_f k^* + p'_t q(G + \frac{\delta(f(k^*)-R_f k^*+G)}{1-\delta}) + (1-p'_t q)\frac{\delta s}{1-\delta}]$. Since p'_t decreases with t and $G + \frac{\delta(f(k^*)-R_f k^*+G)}{1-\delta} > \frac{\delta s}{1-\delta}$, for those t, W_t decreases with t.

Now assume that $\hat{t} > 1$ is the last period in which $W_{\hat{t}}(\theta) > \delta s/(1-\delta)$, so first of all, $W_{\hat{t}}(\theta) > W_{\hat{t}+1}(\theta)$. Now we can show that if for some t, $W_t(\theta) > W_{t+1}(\theta')$ with $W_t \geq \delta s/(1-\delta)$, then $W_{t-1}(\theta) > W_t(\theta)$. This is because that $W_{t-1}(\theta) - W_t(\theta) > \delta((p'_t - p'_{t-1})q(G + \frac{\delta(f(k^*)-R_f k^*+G)}{1-\delta} - \frac{\theta\delta s}{1-\delta}) + (1-p'_{t-1}q)(1-\theta)(W_t(\theta) - \max(W_{t+1}(\theta), \delta s/(1-\delta)))) > 0$. □

(2) lemma 6

Proof. According to lemma 5, $W_t(\theta)$ decreases with t. Thus (2.9) becomes slacker and (2.10) tighter when t increases. Therefore, if (2.9) holds for period t, it also holds for all periods after t, and (2.10) holds for all periods before t. Assume that in equilibrium, the earliest default happens at t if possible. In any period after t, say t', it is easy to see the condition for a borrower without breakthrough to default if possible at the end of t' is the same as the condition (2.9) for t', i.e., $f(k^*) + \delta s/(1-\delta) \geq f(k^*) - R_t k^* + W_{t'}(\theta)$. Therefore, if (2.9) holds for t, defaulting at t is preferred to waiting till $t+1$. In other words, without a breakthrough, a borrower prefers defaulting as soon as it is possible.

On the other hand, for any period before $t-1$, say t'', the condition for defaulting in the next period to be better than defaulting right away is $f(k^*) + \delta s/(1-\delta) \leq f(k^*) - R_f k^* + V^w_{t''}(\theta)$, which is slacker than the condition $f(k^*) + \delta s/(1-\delta) \leq f(k^*) - R_f k^* + W_{t''}(\theta)$ ((2.10) for period $t''+1$) because $V^w_{t''}(\theta) > W_{t''}(\theta)$. Since condition (2.10) holds at $t''+1$, the borrower would not default even without a breakthrough in any period before t.

So (2.9) and (2.10) are sufficient conditions for that borrowers without a breakthrough will optimally choose to default at the end of t if possible, given that the loan terms are constructed according to this anticipation. □

(3) lemma 7

Proof. Assume $\theta > \theta'$. Since $W_{t\to\infty}(\theta) = \delta(f(k^*) - R_f k^*) + \frac{\delta^2 s}{1-\delta} < \frac{\delta s}{1-\delta}$ for all θ, there exists a T such that for all $t > T$, $W_t(\theta) < \frac{\delta s}{1-\delta}$ and $W_t(\theta') < \frac{\delta s}{1-\delta}$. Thus $W_{t-1}(\theta) = W_{t-1}(\theta') = \delta\Big\{f(k^*) - R_f k^* + p'_t qG + (1-p'_t q)\frac{\delta s}{1-\delta}\Big\}$ for all $t > T$. Assume $t = T$ is the earliest period that $W_t(\theta) = W_t(\theta') < \frac{\delta s}{1-\delta}$, then $W_{T-1}(\theta) = W_{T-1}(\theta') > \frac{\delta s}{1-\delta}$. Since from lemma 5, W_t decreases with t, so for any $t \leq T-1$ and for $\theta'' \in \{\theta, \theta'\}$, $W_t(\theta'') = \delta\Big\{f(k^*) - R_f k^* + p'_t qG + (1-p'_t q)[\theta'' \frac{\delta s}{1-\delta} + (1-\theta'')W_{t+1}(\theta'')]\Big\}$ (*). Therefore, for any $t < T-1$, if $W_{t+1}(\theta) \leq W_{t+1}(\theta')$ and $\theta' > \theta$, we can see from $(*)$ that $W_t(\theta) \leq W_t(\theta') > \frac{\delta s}{1-\delta}$ is also true. □

(4) lemma 8

Proof. First, we can show by contradiction that at least an integer $t > 0$ exists such that both (2.9) and (2.10) hold, like in the case of $\theta = 1$. Again remember that (2.9) is more slack and (2.10) more strict as t increases. If there is no pure-strategy equilibrium, there is no t that satisfies (2.9) and (2.10) at the same time. Like when $\theta = 1$, this means that there must be a t' such that (2.9) fails at t' and (2.10) fails at $t' + 1$, which implies $f(k^*) - R_t k^* + W_t(\theta) > f(k^*) - R_f k^* + W_t(\theta)$ and is impossible.

Second, we show that when there are multiple values of t satisfying (2.9) and (2.10), the largest t will be selected as the unique earliest defaulting time in equilibrium. To do this, we again start with proving that the candidate equilibrium with later default yields a higher expected payoff. Assume there are two candidate equilibria with the earliest default happening at t and t' respectively, and $t < t'$. Then every period within $[t, t']$ satisfies both (2.9) and (2.10). Assume A and A' are two candidate equilibria with the earliest default happening at t and $t + 1$ respectively. As in the case of $\theta = 1$, it is sufficient to compare the payoffs in A and A' starting from t if a breakthrough has not happened by the end of $t - 1$. First of all, if a breakthrough happens at t, then under A the borrower gets a lower payoff at t than under A', and the future payoffs (after t) are the same in the two candidate equilibria.

Second, if a breakthrough does not happen at t and default turns out possible, the lifetime payoff at the beginning of t is $f(k^*)+\delta s/(1-\delta)$. Under A', the same borrower will not default immediately at t but will do so at $t+1$ if that is possible (unless a breakthrough happens). Thus under A' the present value of the lifetime payoff starting t is $f(k^*) - R_f k^* + \frac{\delta s}{1-\delta}$. Because A' is an equilibrium, which means (2.10) holds at $t + 1$, the payoff under A' is bigger in this case than under A.

If a breakthrough does not happen at t and default is impossible, obviously the expected lifetime payoffs under the two candidate equilibria are again the same after t, and the payoff at t is bigger under A' than under A.

To summarize, the expected payoff under A' is bigger than under A. Similar to the case when $\theta = 1$, we can show that under A, but not under A', the lender can pose a profitable deviation, and thus A cannot survive as an equilibrium. Therefore, only the candidate with the latest default time will be selected as the unique equilibrium. □

B.5 Proof of proposition 10

Proof. First, we prove by contradiction that a larger probability to default leads to shorter experimentation. When $\theta' > \theta$, if the earliest possible default happens earlier in equilibrium under θ than under θ', there must exist a τ such that (2.10) fails at $t = \tau$ with defaulting probability θ but holds with θ, because (2.10) is getting tighter as t increases. This is possible only if $W_{\tau-1}(\theta) < W_{\tau-1}(\theta')$. However, by lemma 7, $W_{\tau-1}(\theta) > W_{\tau-1}(\theta')$. Therefore, a smaller possibility to default cannot cause the earliest default to happen sooner. Now let us compare the welfare. First, if the earliest default happens at the same time in either case, (say, at τ), then from period 1 to period $\tau-1$, the borrower gets exactly the same per-period payoff under θ and θ' given the same "luck". At the end of period $\tau - 1$ after repayment is made, a borrower's expected future payoff is $W_{\tau-1}(\theta)$ and $W_{\tau-1}(\theta')$ respectively if a breakthrough has not yet happened by then (because default will happen in the next period if possible in both cases), and $\delta(f(k^*) - R_f k^* + G)/(1 - \delta)$ if it has. By lemma 7, $W_{\tau-1}(\theta) \geq$

$W_{\tau-1}(\theta')$. So the expected lifetime payoff is greater under θ than under θ' if the earliest default happens at the same time in either case.

Otherwise, if in equilibrium, the earliest default happens at period τ under θ and τ' under θ' with $\tau' < \tau$. Then again, the payoff of each period between period 1 and $\tau' - 1$ is the same under θ and θ' given the same sequence of "luck". At the end of $\tau' - 1$, the borrower has the expected future payoff $W_{\tau'-1}(\theta')$ after repaying if the probability that one can default is θ'. Under θ, however, the expected future payoff at the end of $\tau' - 1$ (if without a breakthrough by then) $V^w_{\tau'-1}$ is higher than $W_{\tau'-1}(\theta)$. This is because even without a breakthrough, it is not optimal to default at the end of the following period (τ') either. So we have $V^w_{\tau'-1}(\theta) > W_{\tau'-1}(\theta) \geq W_{\tau'-1}(\theta')$. Therefore, in this case, the expected lifetime payoff under θ' is greater than that under θ' as well. $\square$

B.6 Lifetime income comparison

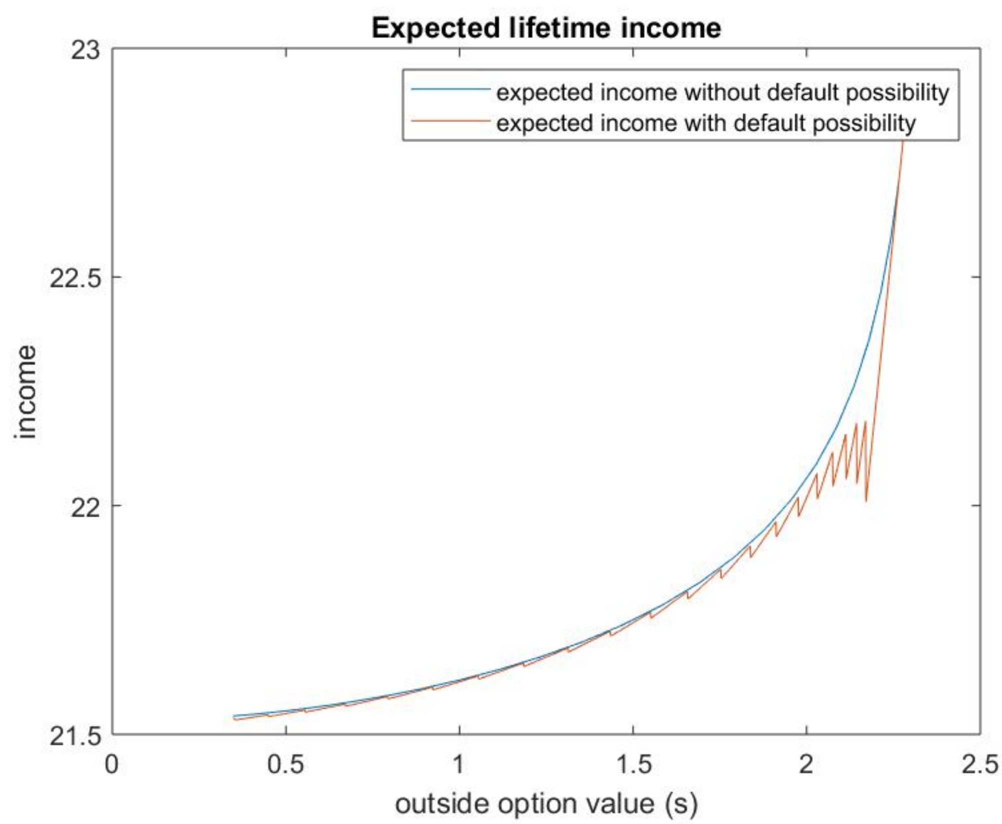

Figure B.1: Comparison of lifetime expected incomes with and without strategic default possibility

Appendix C

Appendix to Chapter 3

C.1 Proof of Proposition 11

Proof. The proof will be completed in four steps.

1) An equilibrium exists.

Let A be the set of all $q_0 \in [0,1]$ that generate infeasible trajectories of q_t of type A. For any trajectory of type A, there exists a $t_1 > 0$ such that $q_{t_1} > 1$ and $q_{t'}$ is feasible for any $t' < t_1$. In other words, any q_0 in A generates feasible q_t path until it goes beyond the upper bond. A is non-empty because if $q_0 = 1$, q_0 is in A. This is because $q_0 = 1$ implies $\Phi(\delta_0) = 1 - \frac{q_0 - m}{1-m} = 0$ by (3.2) and thus $\delta_0 = \delta_{min}$. By the assumption that $\delta_{min} < \epsilon(1)$ and the first equation in (3.1),we have $F(K_0) < F(K_1)$, which implies $q_1 > q_0 = 1$ because $F(K(q_t))$ increases with q_t. Therefore $t_1 = 1$ and $1 \in A$.

Let B be the set of all $q_0 \in [0,1]$ that generate infeasible q_t trajectories of type B. For any type B trajectory, there exists some $t_2 \geq 0$ s.t. $q_{t_2} < \frac{m}{(1-\Phi(\delta_{t_2-1}))\cdot(1-m)+m}$ if $t_2 > 0$ (or $q_{t_2} < m$ if $t_2 = 0$) and any $q_{t''}$ with $t'' < t_2$ is feasible if $t_2 > 0$. In other words, any q_0 in B generates a feasible path of q_t until it falls out of the lower bond. For any $m > 0$, B is also non-empty because if $q_0 < m$, $q_0 \in B$. In contrast, if $m = 0$, $B = \emptyset$. [1]

By definition $A \cap B = \emptyset$. Since $A \cup B$ is the set of all $q_0 \in [0,1]$ that generates infeasible trajectories, if we can show $(A \cup B)^c$ is non-empty, then there exists at least one equilibrium path. In the following paragraph I will prove that $(A \cup B)^c$ is indeed non-empty.

First, both A and B are open sets if $m > 0$. This is because q_t is a continuous function of q_0 according to (3.1) and (3.2), and because of the strict inequalities for q_{t_1} and q_{t_2} in the definitions of A and B. Also from (3.1) and (3.2) we can see that q_t is a monotonically increasing function of q_0, thus $\inf A \notin A$ and $\sup B \notin B$. Now we show $\sup B \notin A$ and $\inf A \notin B$. Let $q_0' = \sup B$. If $q_0' \in A$, there exists a $\epsilon > 0$ such that any q_0 with $|q_0 - q_0'| < \epsilon$ also belongs to A, since A is an open set. This neighbourhood of $\sup B$ of course intersects

[1] Because now a q_0 is in B only if it leads to some negative q_{t_2} in the future. However, from the second equation of 3.1 that we can see that this is impossible because now $q_{t+1} = \frac{1-\Phi(\delta_{t+1})}{1-\Phi(\delta_t)} \geq 0$.

with B. So it belongs to both A and B, contradicting with $A \cap B = \emptyset$. Thus $\sup B \notin A$. Similarly, if $\inf A \in B$, there must exist some $q_0'' \in A$ close enough to $\inf A$ that is also in B, which again leads to a contradiction. Therefore, $\inf A$ and $\sup B$ are both in $(A \cup B)^c$ and $(A \cup B)^c$ is non-empty. In other words, a feasible trajectory with $q_0 \in (A \cup B)^c$ does exist, and so an equilibrium exists with $q_0 \in (A \cup B)^c$ and all q_t $(t > 0)$ derived from (3.1).

In comparison, without the commitment type, i.e. when $m = 0$, A^c includes all the equilibrium q_0. One can prove that an equilibrium exists by showing that A^c is non-empty. (See Shapiro (2015).)

2) q_t in any equilibrium converges to 1.

The next step is to prove that on any equilibrium path, q_t must converge to 1. We will see that this result holds only when $m > 0$, i.e. when the commitment type exists.

Suppose a feasible sequence q_t does not converge to 1, then there exists a $\sigma > 0$, a subsequence of q_t, namely, $q_{f(t)}$, and a $T > 0$, such that $q_{f(t)} < 1 - \sigma$ holds for all $t > T$.

For $N > T$,

$$\prod_{t=0}^{N} q_{f(t)} = \prod_{t=0}^{T} q_{f(t)} \cdot \prod_{t=T+1}^{N} q_{f(t)} < (1-\sigma)^{N-T} \prod_{t=0}^{T} q_{f(t)}$$

.

Since $0 < 1 - \sigma < 1$,

$$\lim_{N\to\infty} (1-\sigma)^{N-T} = 0$$

.

Combining the above equation with $q_{f(t)} \geq 0$, we have

$$\lim_{N\to\infty} \prod_{t=0}^{N} q_{f(t)} = 0$$

and therefore,

$$\lim_{N\to\infty} \prod_{t=0}^{N} q_{(t)} = 0$$

.

Now because in equilibrium

$$(1 - \Phi(\delta_{N+1}))(1-m) + m = ((1-\Phi(\delta_0))(1-m) + m) \prod_{t=0}^{t=N} \frac{(1-\Phi(\delta_{t+1}))(1-m)+m}{(1-\Phi(\delta_t))(1-m)+m}$$

$$= \prod_{t=0}^{N+1} p_t = \prod_{t=0}^{N+1} q_t$$

so

$$\lim_{N\to\infty} (N(1-\Phi(\delta_{N+1}))(1-m)+m) = 0$$

.

This contradicts with $(1-\Phi(\delta_{N+1}))(1-m)+m \geq m > 0$. Therefore, q_t in any equilibrium must converge to 1.

Note that only when the commitment type exists, i.e., when $m > 0$, does the above contradiction work. In fact, without this assumption (when $m = 0$), q_t in an equilibrium may converge either to 1 or 0 as shown by Shapiro (2015).

Now from either equation of 3.1 we know if $q_t \to 1$, $\delta_i \to \epsilon(1)$. Thus (q_t, δ_t) in any equilibrium converges to $(1, \epsilon(1))$ if $m > 0$.

We know from the previous steps that an equilibrium exists and any equilibrium converges to $(1, \epsilon(1))$. Now I prove the uniqueness of the equilibrium. As Shapiro (2015) shows, when the commitment type does not exist, i.e. when $m = 0$, the equilibrium is not unique. However, the equilibrium with increasing q_t is indeed unique. In the following steps, I first prove that q_t increases with t in any equilibrium when $m > 0$. Then I generalize Shapiro's proof on the uniqueness of the equilibrium with expanding q_t to any $m \geq 0$, and thus establish the uniqueness of equilibrium when $m > 0$.

3) Along any equilibrium path, q_t increases with t.

Lemma 1. *If there exist t s.t. $q_t \geq q_{t+1}$, then $q_{t+s} > q_{t+s+1}$ for any $s > 0$. (Shapiro (2015))* [2]

The lemma above suggests that once q_t does not increase, it will keep declining forever. We have shown that q_t in any equilibrium converges to 1, thus q_t either increases monotonically with t, or increases before decreasing. However, if for some τ, q_t decreases for all $t > \tau$, it means that q_t converges to 1 from above. In that case the sequence of q_t is not feasible. Thus q_t can only increase monotonically when $m > 0$.

In contrast, if $m = 0$, we do not have the result of step (2). Instead, q_t can instead converge to either 0 or 1. It is possible then to have q_t that eventually declines with t and converges to 0, so the result of step (3) does not hold either.

4) The equilibrium is unique. Like in Shapiro (2015), suppose there are two equilibrium trajectories (q_t, δ_t) and (Q_t, Δ_t) where $q_0 > Q_0$. We have:

$$\begin{pmatrix} Q_{t+1} - q_{t+1} \\ \Delta_{t+1} - \delta_{t+1} \end{pmatrix} \approx \begin{pmatrix} \frac{\partial q_{t+1}}{\partial q_t} & \frac{\partial q_{t+1}}{\partial \delta_t} \\ \frac{\partial \delta_{t+1}}{\partial q_t} & \frac{\partial \delta_{t+1}}{\partial \delta_t} \end{pmatrix} \begin{pmatrix} Q_t - q_t \\ \Delta_t - \delta_t \end{pmatrix}$$

Using the first equation of (1) we can prove $\frac{\partial q_{t+1}}{\partial q_t} > 0$ and $\frac{\partial q_{t+1}}{\partial \delta_t} < 0$. (See Shapiro (2015) for details.) Now we rewrite the second equation of (1) as $\Phi(\delta_{t+1}-1)(1-m)-m+q_{t+1}((1-\Phi(\delta_t))(1-m)+m) = 0$ and use to the implicit function theorem to derive:

$$\frac{\partial \delta_{t+1}}{\partial q_t} = -\frac{(m+(1-m)(1-\Phi(\delta_t)))\frac{\partial q_{t+1}}{\partial q_t}}{\phi(\delta_{t-1})(1-m)} < 0$$

[2]This lemma is directly taken from Shapiro (2015). (See page 83 "Proof of Theorem 3.6" ii).) The existence of commitment-type borrowers does not change this result or the proof.

and

$$\frac{\delta_{t+1}}{\delta_t} = q_{t+1}\frac{(1-m)\phi(\delta_t)}{(1-m)\phi(\delta_{t+1})} - \frac{(m+(1-m)(1-\Phi(\delta_t)))\frac{\partial q_{t+1}}{\partial \delta_t}}{\phi(\delta_{t-1})(1-m)}$$

What follows will be the same as in Shapiro (2015). (See page 82.) Since the first term converges to 1 in equilibrium and the second term is negative and bounded away from zero given that $\frac{\partial q_{t+1}}{\partial \delta_t}$ is bounded away from zero. Therefore, we have $\frac{\delta_{t+1}}{\delta_t} > 1+\mu$ for some $\mu > 0$. Thus

$$\Delta_{t+1} - \delta_{t+1} \approx \frac{\partial \delta_{t+1}}{\partial q_t}(Q_t - q_t) + \frac{\partial \delta_{t+1}}{\partial \delta_t}(\Delta_t - \delta_t)$$

Since $Q_t < q_t$ and $\Delta_t > \delta_t$ (which can be derived from $Q_0 < q_0$, see Proposition 5.3 in Shapiro (2015)), we can establish that

$$\Delta_{t+1} - \delta_{t+1} > \Delta_t - \delta_t$$

when both trajectories approach $(1, \epsilon(1)$. Therefore, the two trajectories cannot converge to the same equilibrium outcome.

To summarize, from steps (1) to (4) we can see that when $m > 0$, a unique equilibrium exists, in which q_t increases monotonically and converges to 1. □

C.2 Proof of Proposition 12

Proof. Assume there are 2 economies with population 1 and the fraction of commitment-type borrowers in economy 1 and 2 are m_1 and m_2 with $m_1 > m_2$. Repaying population at period $\tau \geq 0$ for the two economies are q_1^τ and q_2^τ respectively. Cutoff discount factors at τ are δ_1^τ and δ_2^τ respectively. First I prove that $q_1^0 > q_2^0$. Suppose not. Then from the second equation of (1) we have $m_1 + (1-m_1)(1-\Phi(\delta_1^0)) \leq m_2 + (1-m_2)(1-\Phi(\delta_1^0))$ which is equivalent to $\frac{\delta_1^0}{\delta_2^0} \geq \frac{1-m_2}{1-m_1} > 1$. From the first equation of (1) this implies $q_1^1 < q_2^1$.Again using the second equation of (1) and we have $\frac{m_1+(1-m_1)(1-\Phi(\delta_1^1))}{m_1+(1-m_1)(1-\Phi(\delta_1^0))} < \frac{m_2+(1-m_2)(1-\Phi(\delta_1^1))}{m_2+(1-m_2)(1-\Phi(\delta_1^0))}$ and therefore, $m_1 + (1-m_1)(1-\Phi(\delta_1^1)) > m_2 + (1-m_2)(1-\Phi(\delta_1^1))$, or $\frac{\delta_1^1}{\delta_2^1} > \frac{1-m_2}{1-m_1} > 1$. Thus from the first equation of (1) we can see $q_1^2 < q_2^2$. This process repeats and we always have $\frac{\delta_1^t}{\delta_2^t} > \frac{1-m_2}{1-m_1} > 1$ for any t, which contradicts to the fact that δ_1^t and δ_2^t converge to the same strict positive value. So it has to be true that $q_1^0 > q_2^0$.

Now I will prove that $q_1^t > q_2^t$ for any t.Again we show this by contradiction. Since we already know that $q_1^0 > q_2^0$, let's assume that $q_1^\tau > q_2^\tau$ for all $\tau \leq t'$ and $q_1^{t'+1} \leq q_2^{t'+1}$. By the LTM condition and the first equation of (1) we can see $\delta_1^{t'} > \delta_2^{t'}$. To justify a (weakly) better loan term for economy 2 at $t'+1$, it also has to be true that $\delta_1^{t'+1} > \delta_2^{t'+1}$.

To see this, let $\gamma_i = m_i/(1-m_i)$, $i = 1,2$. The second equation of (1) becomes $q_i^t = \frac{\gamma_i+(1-\Phi(\delta_i^t))}{\gamma_i+(1-\Phi(\delta_i^{t-1}))}$. A higher q_i^t means a lower defaulting population $d_i^t \equiv 1 - q_i^t$. So $d_{t'+1} = \frac{\Phi(\delta_i^{t'+1})-\Phi(\delta_i^{t'})}{\gamma_i+(1-\Phi(\delta_i^{t'}))} = \frac{1}{1+\frac{\gamma_i+(1-\Phi(\delta_i^{t'+1}))}{\Phi(\delta_i^{t'+1})-\Phi(\delta_i^{t'})}}$. If $\delta_1^{t'+1} \leq \delta_2^{t'+1}$,we have $d_1^{t'+1} < d_2^{t'+1}$ because $\gamma_1 > \gamma_2$

and $\delta_1^{t'} > \delta_2^{t'}$. This implies $q_1^{t'+1} > q_2^{t'+1}$ and contradicts with $q_1^{t'+1} \leq q_2^{t'+1}$. Therefore $\delta_1^{t'+1} > \delta_2^{t'+1}$.

Now by applying the first equation again we can see $q_1^{t'+2} < q_2^{t'+2}$ and then again $\delta_1^{t'+2} > \delta_2^{t'+2}$.... And this process can always be repeated and $q_1^{t+2} < q_2^{t+2}$ for all $t > t'$. Combining this with the second equation of (1) we know that for any $t > t'$, we have $\frac{m_2+(1-m_2)(1-\Phi(\delta_2^{t+1}))}{m_1+(1-m_1)(1-\Phi(\delta_1^{t+1}))} > \frac{m_2+(1-m_2)(1-\Phi(\delta_2^{t}))}{m_1+(1-m_1)(1-\Phi(\delta_1^{t}))} ... > \frac{m_2+(1-m_2)(1-\Phi(\delta_2^{t'}))}{m_1+(1-m_1)(1-\Phi(\delta_1^{t'}))} > \frac{m_2+(1-m_2)(1-\Phi(\delta_1^{t'}))}{m_1+(1-m_1)(1-\Phi(\delta_1^{t'}))}$. Since both δ_1^t and δ_2^t converge to $\epsilon(1)$, $\frac{m_2+(1-m_2)(1-\Phi(\delta_2^{t}))}{m_1+(1-m_1)(1-\Phi(\delta_1^{t}))}$ will converge to $\frac{m_2+(1-m_2)(1-\Phi(\epsilon(1)))}{m_1+(1-m_1)(1-\Phi(\epsilon(1)))} < \frac{m_2+(1-m_2)(1-\Phi(\delta_1^{t'}))}{m_1+(1-m_1)(1-\Phi(\delta_1^{t'}))}$. [3],thus a contradiction is shown. Therefore, $q_1^t > q_2^t$ for all $t \geq 0$. □

[3]Let $f(\phi) = \frac{m_2+(1-m_2)(1-\phi)}{m_1+(1-m_1)(1-\phi)} = (\frac{\gamma_1+1}{\gamma_2+1})(\frac{\gamma_2+1-\Phi}{\gamma_1+1-\phi}) = (\frac{\gamma_1+1}{\gamma_2+1})(1 - \frac{\gamma_1-\gamma_2}{\gamma_1+1-\phi})$, since $\gamma_1 > \gamma_2$, $f'(.) < 0$. So $f(\delta_1^{t'}) > f(\epsilon(1))$