SECTOR DIVERSIFICAITON AND ITS EFFECT ON RISK EXPOSURE OF CANADIAN REAL ESTATE INVESTMENT PORTFOLIO

by

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Abstract

The objective of this project is to use sector diversification method to mitigate the downside risk of a Canadian real estate investment portfolio under Value-at-Risk measurement. During real estate bubble burst, Canadian real estate investors are unable to sell their assets due to the illiquid market condition.

In the first half of this paper, we attempt to set up an appropriate portfolio risk measure by applying GJR-GARCH model and t-copula methods, associated with Extreme Value Theory, which will improve model accuracy.

We discover that using 10% of the portfolio value to short Canadian commercial bank stocks can result in considerate VaR reduction. The research result provides a more affordable way for individual investors to hedge their risks in Canadian real estate market.

Keywords: Canadian Real Estate, Canadian Commercial Bank, Copula VaR

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Glossary

REIT	Real Estate Investment Trust
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
VaR	Value-at-Risk
ES	Expected Shortfall
HPI	Home Price Index
EVT	Extreme Value Theory

1: Introduction

Real estate industry accounts for 12.65% of Canada's GDP in 2018, which is the highest among all industries and its related industries have made up over 20% of Canadian GDP (Statistics Canada, n.d.). It attracts tremendous amount of investment from either individuals and financial institutions all over the world. Investing in real estate is being considered as worthy and attractive since it is a good way to earn a passive income with inflation protection. Also, with the help of mortgage financing, you can leverage your investment and could potentially earn higher and substantial return. But investing solely in one sector could be extremely dangerous since your investment could be exposed to a huge amount of downside risk. For individual investors of real estate, they might face market risk, liquidity risk and regulation risk. During market downturn, they might not be able to stop their loss by selling their assets due to the illiquidity.

Real estate market has always been considered as high bubbles in Canada. Starting in late 2010s, right after the crash of U.S. subprime mortgages, the global housing market start to recover from the previous mess. At the same time, foreign investments and speculative activities in major urban areas of Canada caused a dramatical increase in the real estate market. With the concern of future bubble burst, many methods such as non-resident speculation tax and vacancy tax, on controlling the price increase of real estate prices has been implemented, but none of them are effective (Tencer, 2019). Public news also shows a trend that some U.S hedge funds are worried about the

Canadian real estate bubble and decide to short Canadian banks in order to hedge the downfall (BNN Bloomberg, 2019). These news provide us with the idea of our research.

Therefore, for investors in Canadian real estate market, it is reasonable for them to worried about the potential loss of their investments, even if they've already benefited from the booming of Canadian real estate market. From risk management perspective, the downside loss of their portfolio is reflected in the VaR of their portfolios, where the extreme loss is reflected in the ES of their portfolios.

This project will try to evaluate whether forming a more diversified portfolio by adding commercial banks' stocks will reduce the risk exposure on a Canadian real estate investment portfolio. The risk exposures of the portfolios above are measured by their 6-month 95% VaR and Expected Shortfall. And they are modelled by 15-year monthly historical price data and Monte Carlo Simulation.

2: Literature Review

There has been a considerable amount of research on using Copula-based VaR for measuring the downside loss of diversification in portfolio management, but usually in agricultural portfolio. Nguyen-Huy et al. have used Copula-based VaR to measure the downside risks of geographical diversification in Australian wheat farming portfolio management (2018). They concerned that the value of the agricultural commodities will suffer from climate variability such as extreme drought and heat events, so they proposed their own diversification strategies of geographical spread, and applied Copula VaR to measure the downside risks before and after the spread. They use 26-year data from Australia's three key production zones of wheat and try to assess their Copula VaR individually and collectively. After statistical analysis, they concluded that through geographical diversification, the portfolio can gain a relatively large risk reduction. They also discussed that they have spent little effort to prove how valid the VaR is, since they have applied a bunch of well-accepted and statistically robust mathematic models based on the well-established statistical theories.

Back in 2012, Mandal & Lagerkvist, from Swedish University of Agricultural Sciences, posted their research on the comparison of traditional Guassian VaR and Copula VaR. They used a 3-asset-class based, 250 firm's operation data for 18 years to examine that. They suggest that Copula-based VaR is a refinement of the traditional VaR methodology, since traditional VaR can only be effective with normal-distributed returns and has limited usefulness on empirical returns, which is more likely to be asymmetric and leptokurtic. Their also conclude that using Copula-based VaR will not alter the conclusion under the condition of using traditional VaR (Mandal & Lagerkvist, 2012).

Both researches has applied classic Markowitz mean-variance approach on determining the optimal weight of each asset, where we use mean-VaR approach in our project and pick the weight that has marginal risk reduction as the optimal weight.

We decided to conduct our research as the extension of Nguyen-Huy et al.'s work, from Australian agricultural portfolio to Canadian real estate portfolio, considering both industries are asset-backed and alike. And we focus on market bubble burst rather than climate risks in our research.

3: Model Accuracy Adjustment

3.1 GJR-GRACH model

There are two commonly used models to handle the asymmetric volatility, both developed from the original GARCH model, the EGARCH and GJR-GARCH. According to research, these two GRACH models generally perform better than the original GARCH model because it includes a leverage effect (i.e. asymmetric volatility) that can enhance the robustness of forecast (Dahlvid & Granberg, 2017). We prefer GJR-GARCH over EGRACH because of its main advantage of directly modeling variance without using the natural logarithm on return variance, which makes it easier to implement.

Here is what a typical GJR-GARCH model looks like:

$$\boldsymbol{\sigma_t^2 = \omega + (\alpha + \gamma \mathrm{II}_{t-1})\mathcal{E}_{t-1}^2 + \beta\sigma_{t-1}^2}$$

Where:

II_{t-1} is the indicator function, $\mathrm{II}_{t-1}(\mathcal{E}_{t-1}) = \mathcal{E}_{t-1}$ for $\mathcal{E}_{t-1} > 0$ and $\mathrm{II}_{t-1}(\mathcal{E}_{t-1}) = 0$ otherwise. $\boldsymbol{\gamma}$ is the scale of the asymmetric volatility.

The GJR-GRACH (1, 1) model can be presented in the equation below (Asgharian, 2016):

$$\boldsymbol{\sigma_t^2 = \omega + \alpha_1 \eta_{t-1}^2 + \alpha_2 I_{t-1} \eta_{t-1}^2 + \beta\sigma_{t-1}^2}$$

Where:

$\boldsymbol{\omega}$ is the intercept for the variance. $\boldsymbol{\alpha_1 \eta_{t-1}^2}$ is the variance that depends on previous lag error terms. $\boldsymbol{\alpha_2}$ is the scale of the asymmetric volatility. $\boldsymbol{I_{t-1}}$ is a dummy variable that is only activated when the previous innovation is negative.

3.2 Extreme Value Theory

In financial area, fat-tailed distribution is frequently observed in portfolio return data. Traditional VaR methods, including variance-covariance method and historical simulation method can failed in these scenarios. For real estate risks analysis, long-term VaR forecast is desired based on the investment horizon and the fat tail interference can be aggravated. Extreme Value Theory (EVT) is introduced to overcome this issue. EVT is considered as an efficient tool to improve the estimation of extreme quantiles and the downside risk of a return distribution. In 1975, Pickland introduced the Generalized Pareto Distribution, the density function of GPD is shown as below (Odening & Hinrichs, 2003):

$$G_{\xi,\beta}(y) = \begin{cases} 1 - \left(\left(1 + \frac{\xi_y}{\beta}\right)^{-\frac{1}{\beta}}\right) & ;\xi \neq 0 \\ 1 - \exp\left(-\frac{y}{\beta}\right); \xi = 0 \end{cases}$$

Where:

ξ is the shape parameter, which has an important impact on the character of the Generalized Pareto Distribution. β is the scale parameter.

Extreme Value Theory can be used as an approximation of the stochastic behavior of a process with extreme outcomes. EVT and Peaks Over Threshold (POT) can be combined and applied for risk measurement by using the formula below:

$$VaR_p = \mu + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{n}{N_\mu}(1-p)\right)^{-\xi} - 1\right); p \geq F(\mu)$$

Where μ is the threshold value, n is sample size, p is the defined probability and N equals the number of violation over threshold. Applying EVT to VaR can produce a more accurate approximation of the fat tail of the distribution.

3.3 Copula and choice of t-Copula

Copula is a statistical measure that represents the joint distribution function of multiple random variables with their respective marginal density function. Copula function has been proven effective in the construction of joint distribution function, correlation analysis and handling leptokurtic distribution.

Practically, the distribution of returns on financial assets are more likely to be leptokurtic rather than normally-distributed, which will make it more peaked and has fatter tails. So simply applying the traditional statistical processing on normal distribution will decrease the effectiveness and accuracy of analysis on financial assets returns. A Student's t-Copula can be applied to account for the tail dependencies and resolve the tail risks of the returns, even if correlation coefficients between two assets equal to zero.

We would expect to get more accurate VaR and ES measure using Student's t-copula, but there will be no guarantee that there will be reduction in risk exposure through diversification at this point, which is left to prove.

4: Data and Methodology

4.1 Data Selection

We choose securities in Canadian commercial banking sector to include in our portfolio. It is somewhat correlated to real estate sector, which we will further discuss and verify in the following sections. The intuition of our inclusion is that commercial banking sector is also considered as a medium to long-term investment which will provide investors with stable return because of its relatively low beta and P/E ratio. Furthermore, biggest banks within a country might be the last resort during a financial crisis since it is the core of a country's financial system. So, we think commercial bank stock might be a good source of risk reduction.

And we choose to use these securities' 15-year monthly price quote as our sample.

4.1.1 Canadian Real Estate Sector

The value of real estate investment is tied to several factors, such as the status of real estate market (supply and demand), the economy, etc. Real estate investment values are not likely to experience rapid fluctuation within short-term, and investors usually set their investment horizon to medium to long-term.

4.1.1.1 MLS® Home Price Index (HPI) for Canada

The MLS® Home Price Index for Canada is a seasonally-adjusted monthly data series developed by the Canadian Real Estate Association (CREA) that reflects the

benchmark price of various houses in Canada. We select the HPI for Greater Vancouver area specifically in our study.

As mentioned, a home price index could be used as a financial stability or soundness indicator to measure risk exposure, and as an input into individual's decision making on whether to invest in residential property (Fenwick, 2013). This is consistent with our purpose of this research.

4.1.1.2 Canadian Apartment Properties REIT (CAR-UN.TO)

This is an internally managed unincorporated open-end real estate investment trust founded in 1997. And the company it belongs to is CAPREIT, a Toronto-based company that owns interests in residential rental properties located in urban areas of Canada and the Netherlands. It invests in ERES so that it could indirectly invest in Netherlands.

4.1.1.3 Home Capital Group (HCG.TO)

Home Capital Group is a holding company operating through its principal subsidiary, Home Trust Company. Home Trust is a federally regulated trust company offering deposit, mortgage lending, and credit card issuing services. It conducts business purely within Canada. When its stock price decline sharply from 2015 to 2017, and nearly collapsed in 2017 due to customers trust issues, it seems to find its way back afterwards.

4.1.2 Canadian Commercial Banking Sector

Empirically, there is no direct or causal relationship between real estate sector value and commercial banking sector value. However, there might be indirect effects that movements in one sector can affect the other. And both sectors can provide a relatively stable dividend income over year.

For example, interest rate and mortgages would be seen as factors that might impact the prices of the two sectors. Because most real estates are financed by mortgage loans, while majority of commercial banks' revenues come from lending out mortgages and charge for interests. As interest rate increases, mortgage value would decrease. At the same time, mortgages become more expensive for home buyers, which might restrict the growth of housing price. However, banks seem to benefit from the increase interest rate spread since they borrow at short-term rate and lend at long-term rate.

Another factor is housing price, when housing price fall, consumer would more likely to default on their mortgage loans, which could harm banks' profitability and hence drive its stock price down. Furthermore, there will be fewer funds available in the market for saving and debt repayments, causing further impact on commercial banks. Vice versa. (Investopedia, 2015) This can also be explained by the increasing correlation of each asset class during market downfall, which makes it possible to decrease the downside loss by long one and short the other.

Given all the clues above, the prices of the two sectors tend to move in uncertain direction when facing certain economic events at the same time, so we would like to add banking sector for our diversification strategy and test for its effect.

4.1.2.1 Bank of Montreal (BMO.TO)

Originating in Quebec, doing business as BMO Financial Group, BMO is the oldest bank in Canada. Its equities are actively traded on both the Toronto Stock Exchange (TSE) and the New York Stock Exchange (NYSE). BMO hasn't missed any dividend payment since 1829, when dividend serves as a stable income for BMO's shareholders (over 4% per year). BMO's stock price fluctuates within a narrow range (around $90 - $110) in the past three years. Since we are investing in Canadian real estate, we would only pick BMO's Canadian portion of stocks. And when we talk about BMO's stock, we are referring to BMO's common shares outstanding.

4.1.2.2 Royal Bank of Canada (RY.TO)

Being known as the largest bank in Canada by revenue and market capitalization, RBC is a multinational financial services company. Its equities are also actively traded on TSE and NYSE, so we only include its common shares outstanding listed on TSE into our data. Its dividend yield (around 4% per year) and price pattern (around $90 - $110) is similar to BMO's

4.2 Methodology

Our methodology is to model the 95% Confidence Level Value-at-Risk and Expected Shortfall of our original portfolio and different scenarios of a more diversified portfolio with two commonly accepted data processing method, Monte Carlos simulation (forward-looking) for multi-asset portfolios and historical estimation (backward-looking) for our base investment in HPI, using GJR-GARCH model and Student's t Copula. The

result would be 6-step 95% VaR and ES for each scenario, where you can adjust the length of the step, here we use one month as the length of step. Then we will compare the results between different scenarios and determine if there is a reduction in risk exposure after sector diversification, mainly by the amount of VaR. (MathWorks, n.d.)

First, we transfer the price quotes of the securities into returns. Then we apply a suitable GARCH model to eliminate the autoregression and heteroskedasticity of the return series. Here we apply GJR-GARCH (1, 1) model because of the asymmetric distributions of the data series. The filtered residuals will be used for further processing.

Then we construct the sample CDF of each asset return using a Gaussian kernel estimate for the interior and a generalized Pareto distribution estimate (maximum likelihood estimation) for the upper and lower tails.

When data are all from multivariate distributions with complicated relationships among the variables, calibration of t copula will be conducted at this stage. A Student's t copula is then fit to the data and used to induce the correlation between the simulated residuals of each asset.

Finally, the simulation assesses the Value-at-Risk and Expect Shortfall of each portfolio over a six-month horizon, this selection of time length is because investment in real estate are usually long-term, short-term portfolio value fluctuation does not matter.

4.3 Assumptions

We assume that the investors in our case are advocacy of real estate investment and solely invest in real estate market and are highly risk-averse. Their investment objective is to protect the value of their principals and have a stable return over years. The market

is the perfect market where there is no information asymmetry, and no transaction costs associated with trading, and we can long or short any kind of securities. Moreover, the investors are only familiar with traditional investment tools, and do not want to use derivatives.

Before processing our data, we are trying to determine the optimal weight of HPI in our portfolio. We run a control test, keeping the long position of HPI. For the rest of the portfolio, we will either long or short the additional security, and look at the expected return next year and VaR of different combination. And we visualize the result into *Figure 4.1 to 4.4*.

Figure 4.1 Portfolio returns with different HPI weight when long the other security

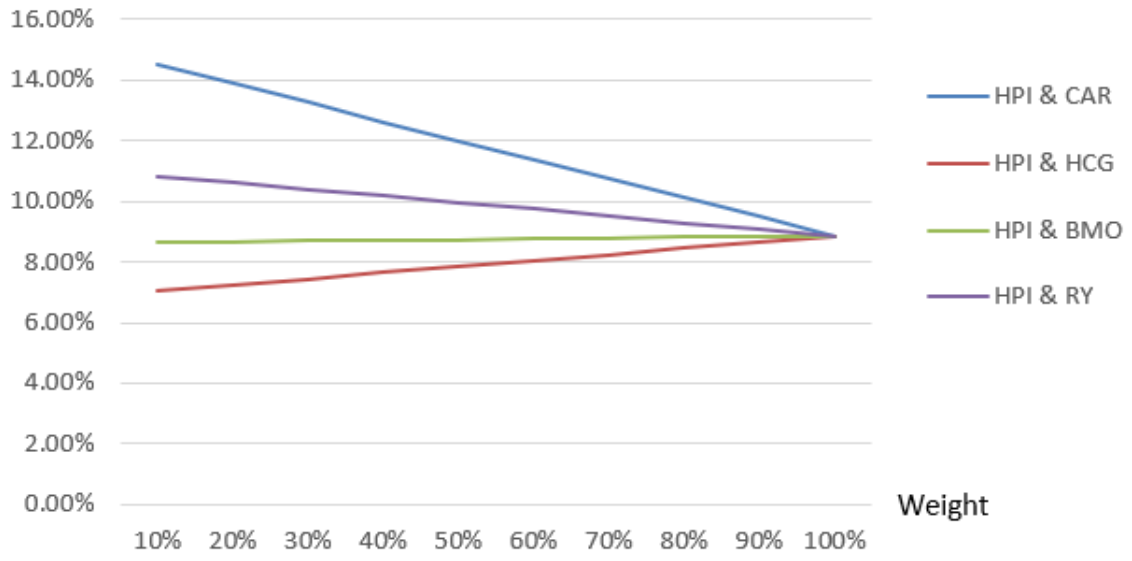


Figure 4.2 Portfolio VaRs with different HPI weight when long the other security

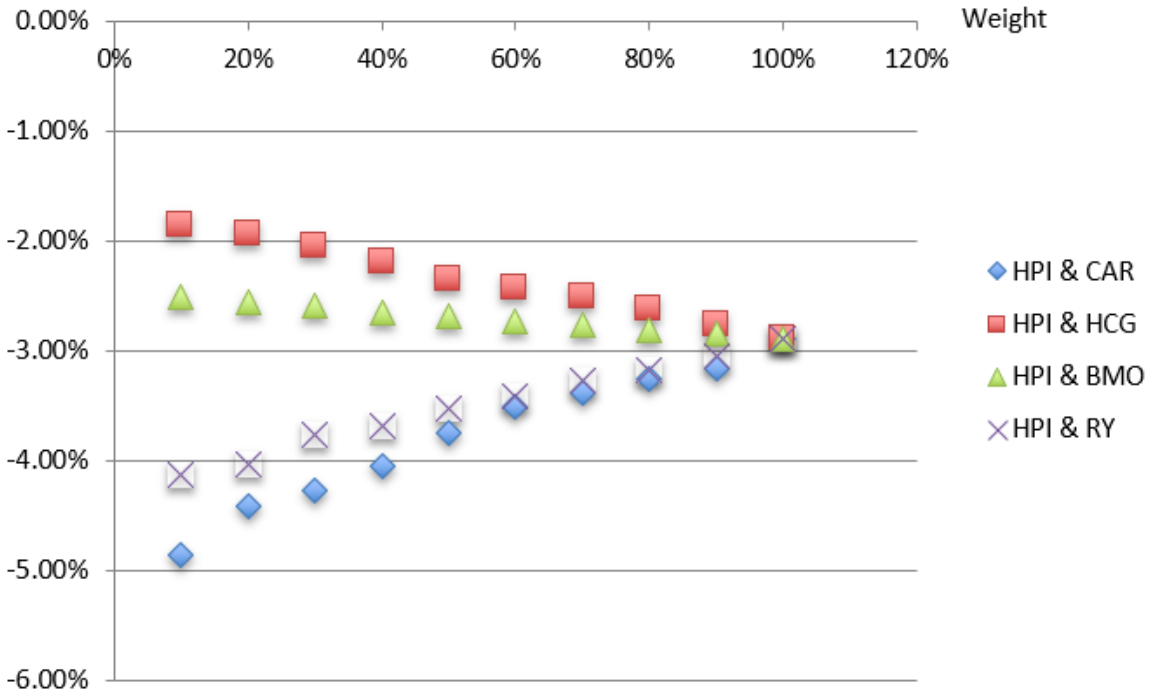


Figure 4.3 Portfolio returns with different HPI weight when short the other security

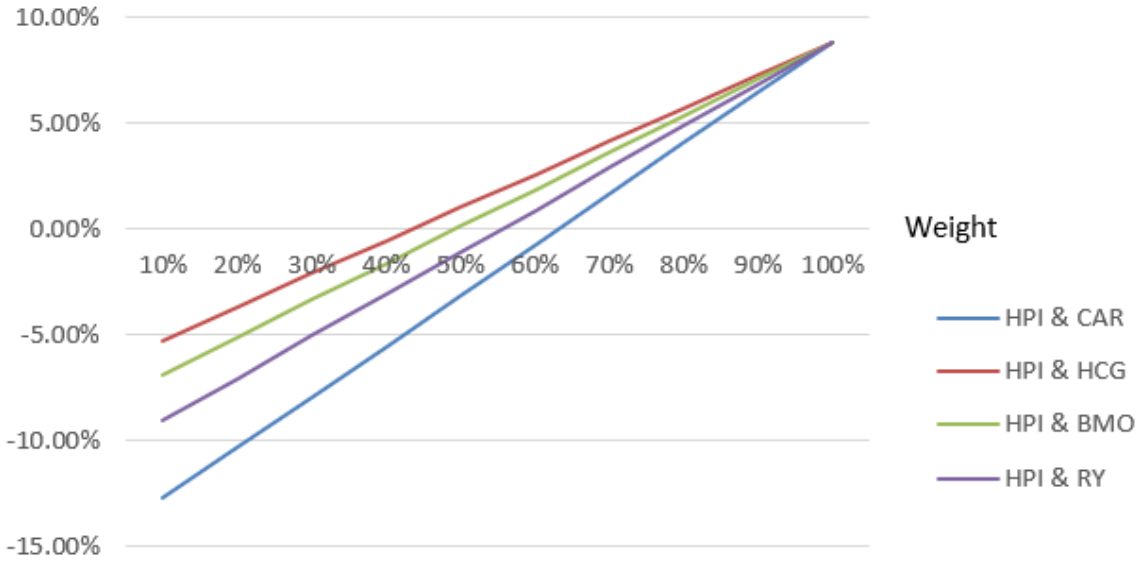


Figure 4.4 Portfolio VaRs with different HPI weight when short the other security

As we can see in *Figure 4.1 and Figure 4.2*, long position on both securities will generally increase the expected return of those portfolios as less portion is assigned to HPI, but at the same time will increase the downside risk of investors' investment portfolios either since the VaR also goes up. Our purpose is to find a way to reduce the tail risks of our investment while maintaining the expected return of our portfolio, so the "long-long" position is rejected.

As for the "long-short" position in *Figure 4.3 and Figure 4.4*, we notice that if we assign 10% of our portfolio value to short the other security, there will be a significant decrease in VaR yet the expected return only decreases slightly. And then the relationship reverse. That is, as more portion of portfolio value is assigned to short the other security, the expected return will decrease considerably without decreasing the VaR. Thus, we infer that the "90% long and 10% short" portfolios gives the highest marginal reduction

on risk exposure without affecting the stability of portfolio returns and this allocation is suitable for our scenario setting.

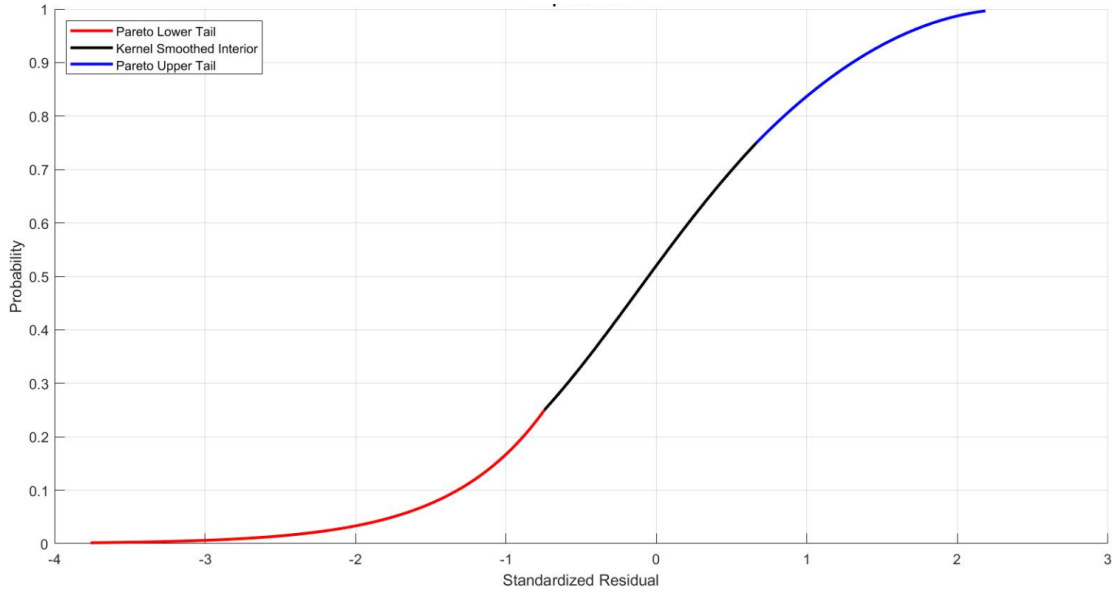
We decide to set up the following scenarios according to our analysis above:

1. Only long in HPI (a 1-asset, 1-sector investment).
2. Long 90% in HPI and short 10% in CAR (a 2-asset, 1-sector portfolio).
3. Long 90% in HPI and short 10% in HCG (a 2-asset, 1-sector portfolio).
4. Long 90% in HPI and short 10% in BMO (a 2-asset, 2-sector portfolio).
5. Long 90% in HPI and short 10% in RY (a 2-asset, 2-sector portfolio).

5: Empirical Results

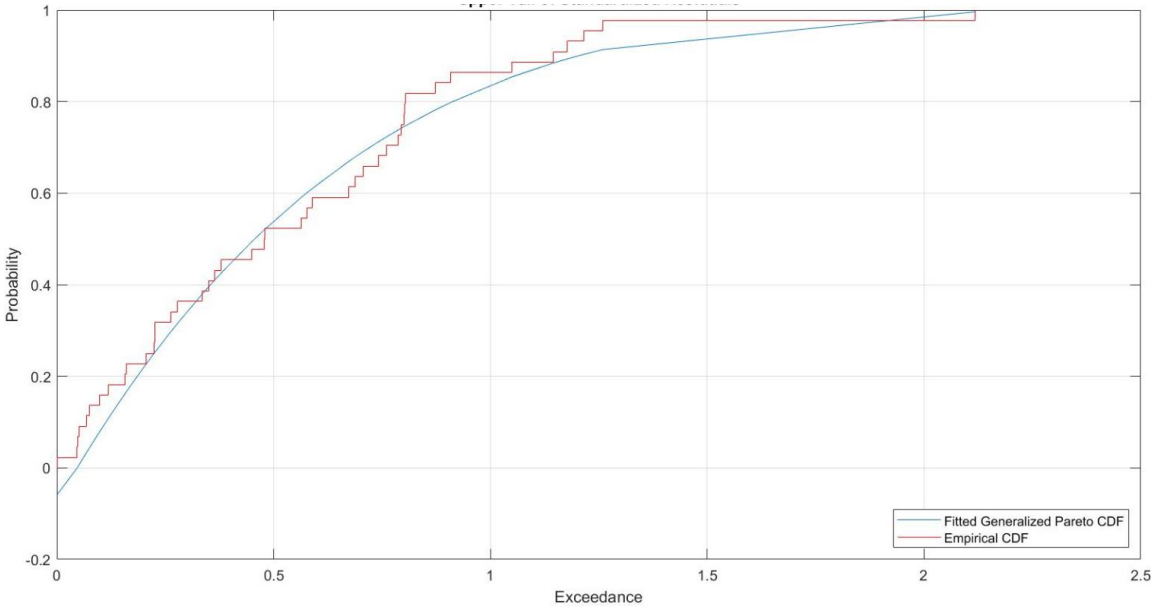
5.1 Modelling of tail risks

Figure 5.1 Empirical CDF of the standardized residuals



To smooth the empirical CDF of each stock by using the standardized residuals from GJR-GARCH process, find upper and lower thresholds such that 25% of the residuals is reserved for each tail. This step optimizes the negative log-likelihood function to estimate the tail parameters (β and ξ) of the Generalized Pareto Distribution. The lower tail (red) and upper tail (blue) regions are suitable for extrapolation.

Figure 5.2 Upper tail fit of the standardized residuals



The graph above provides a visual assessment of the GPD fit by plotting the CDF fitted by the GPD along with the empirical CDF of the upper tail exceedances of the residuals.

5.2 Risk Exposure

Table 5.1 95% 6-month Student's t Copula VaR and ES under different scenarios

	Scenario 1	***Scenario 2***	***Scenario 3***	***Scenario 4***	***Scenario 5***
VaR	-2.89%	-1.09%	-1.40%	-0.71%	-0.81%
ES	-3.91%	-2.89%	-3.19%	-2.51%	-2.59%

The results are close to what we expected, the more diversified two-asset portfolio do decrease the risk exposure of a single security investment, by at least a half, which is effective. Within real estate sector, the Canadian Apartment Properties REITs shows a

better risk reduction than Home Capital Group. The result also ensures the usefulness of adding commercial banking investment into our portfolio, with the present of a more superior risk reduction.

6: Conclusion

Real estate sector contributes a large percentage of Canada's current national economy. If real estate market cools down, real estate investors suffer from both loss in market value and illiquidity. An efficient method to measure and mitigate real estate market risk is important for individual investors. This study introduces an alternative VaR approach to measure the portfolio risk by combining GJR-GARCH, copula and EVT theory to improve the accuracy of tail modelling, as well as a diversification strategy of adding Canadian commercial bank stock into the investment portfolio. Thus, investors can have a better outlook of the market and diversify their investment in advance.

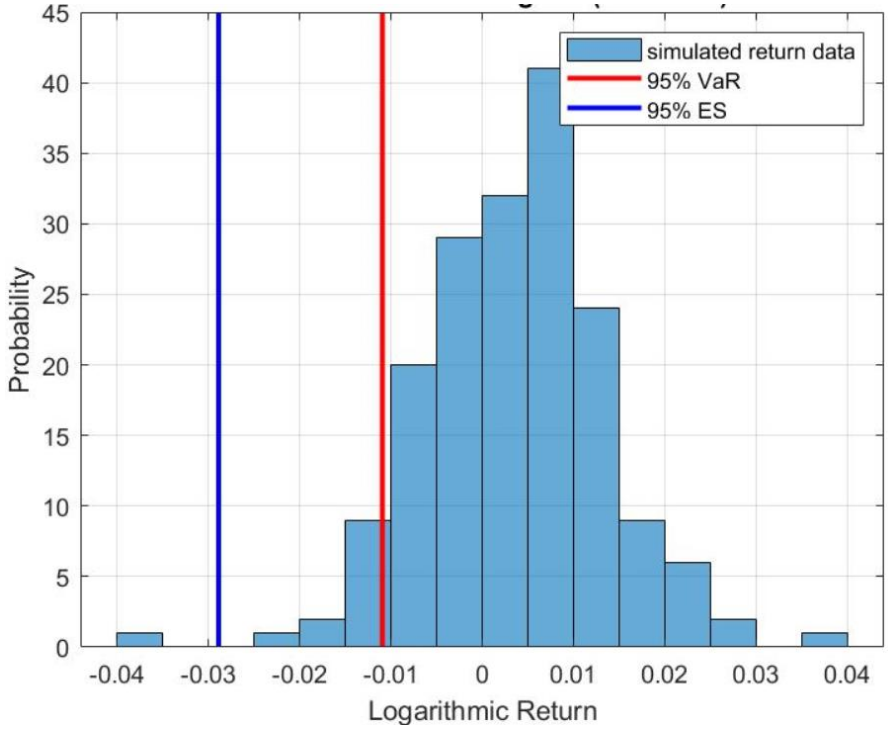
Certain hypothetical portfolios consisted of properties price index and other financial instruments is set up for empirical analysis using the Student t's Copula VaR model. VaR results from different portfolio combination are compared in order to select the more efficient method for hedging. Analysis shows that portfolio diversification by shorting stock of commercial banks, mortgage issuers and REITS all have varying degrees of tail risk mitigation, where shorting 10% of the portfolio value in commercial banks stocks gives the greatest reduction on risk exposure. For further research, it could be meaningful to include more securities to form a more diversified portfolio and test for the reduction on VaR and combine more financial sectors to intensively analyze sector diversification effect.

Moreover, it could be feasible to combine property derivatives into investors' portfolios to reduce risk exposure. There have been some successful practices on managing European real-estate risk using property derivatives and structured swaps

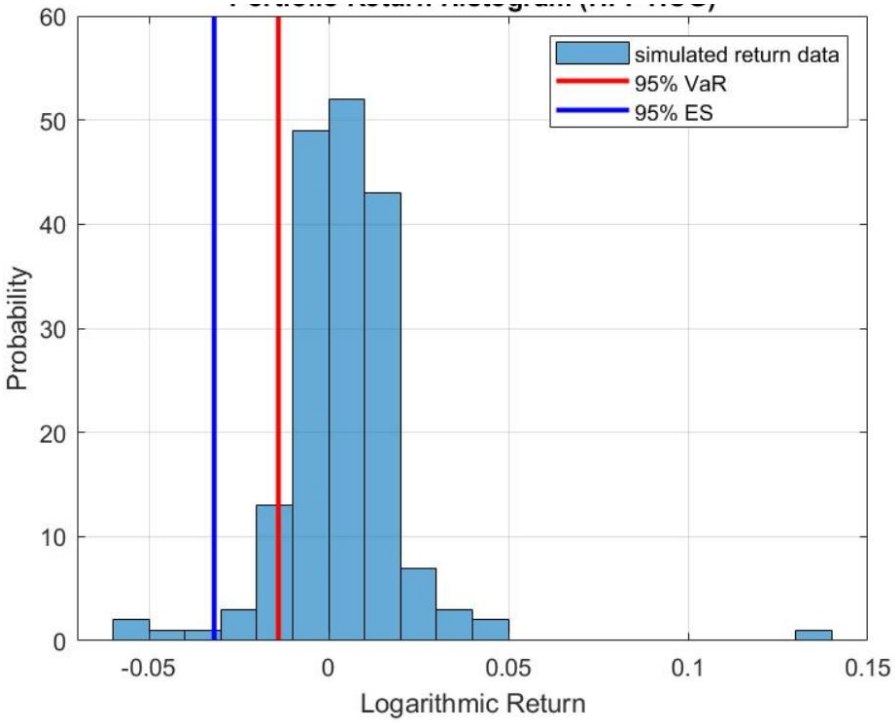
designed by specialized financial institution (Fabozzi et al., 2009). However, these financial tools experience low trading liquidity and are usually large in contract value since the market for them is still in its early stage. With the and expansion and development of the real estate market around the world, there will be a foreseeable increase in demand for risk management tools for real-estate risks. We can also expect to see more available and affordable hedging tools for real-estate risks, eventually, the popularization and mature of these financial products among not only institutional investors but also individual investors.

Appendices

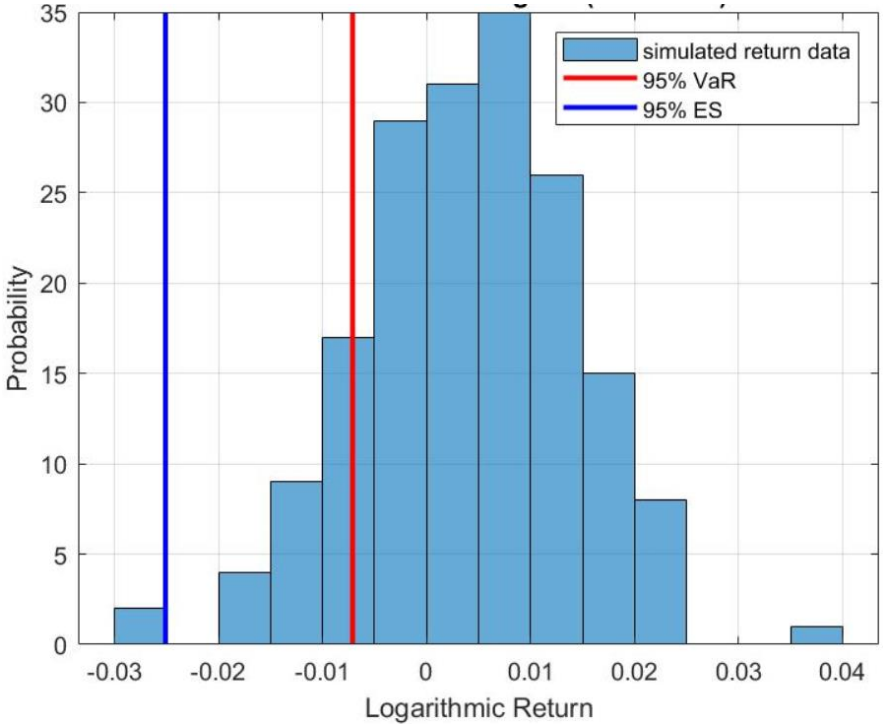
Appendix 1 Copula Distribution of Returns in Scenario 2



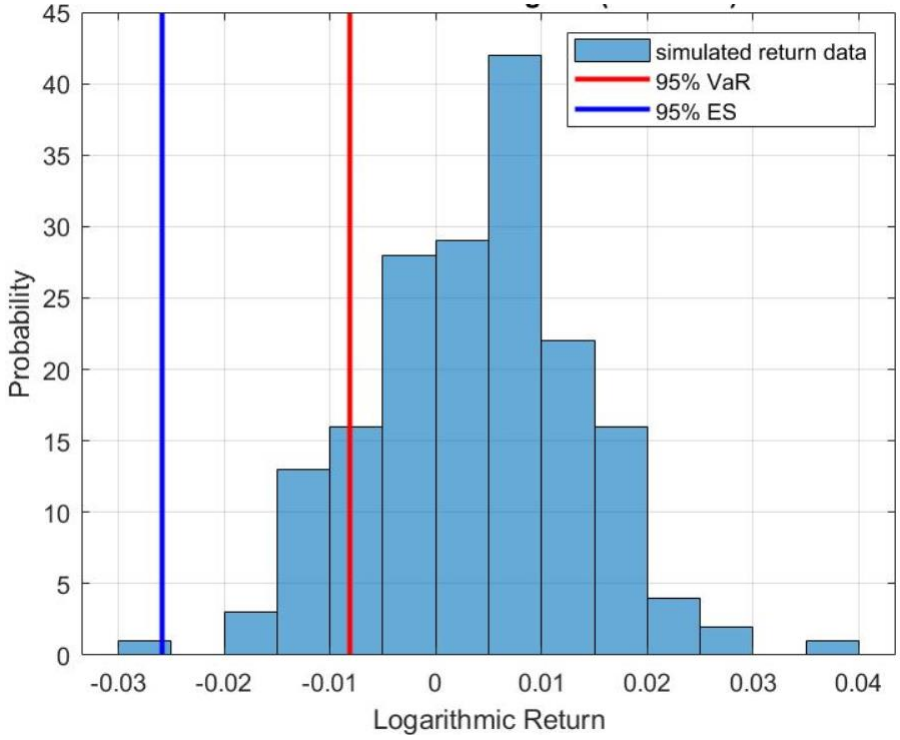
Appendix 2 Copula Distribution of Returns in Scenario 3



Appendix 3 Copula Distribution of Returns in Scenario 4



Appendix 4 Copula Distribution of Returns in Scenario 5



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