COMMODITIES AND THEIR ROLE IN PORTFOLIO OPTIMISATION BETWEEN 2002-2019

by

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Abstract

In this research, it is investigated whether adding a commodity index, a passive investment, creates more value to the portfolio of bonds and stocks. Since the study time frame includes financial crisis, I separated time into the pre-crisis period, period including the financial crisis, post-crisis period and the full period.

My finding was that the commodity index did not offer that much to the betterment of the portfolio's performance, especially after the financial crisis to the present. The index improved the return and risk of the portfolio only in the pre-crisis period. The commodity index is highly volatile, and it is not recommended for mean-variance investor's portfolio.

Also, for portfolio optimisation, mean-variance method has been used, and efficient frontiers have been generated.

Keywords: Commodity; Portfolio Optimisation; Alternative Investment

Dedication

I dedicate this project to my dear family for their support and patience.

Acknowledgements

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1. Introduction

It has been a while that alternative investment has come into attention for the purpose of portfolio diversification. This is also consistent with modern portfolio theory. In fact, the thought of alternative investments came into play in 2008 and 2009 when markets experienced downturns.

Commodities are also considered as an alternative investment in a portfolio along with other alternative investment asset classes like private equity, REITs and TIPs, especially in times when stocks are underperforming. The use of commodity investments is doubled by private investors and asset managers in the last decade, mostly due to diversification benefits and that they are seen as a hedge against inflation. The main discussion about adding commodities to a portfolio of mixed assets of equity and bond is due to a couple of reasons mainly that the diversification of this asset class significantly changes between different kinds of commodities, another thing is that the prices of commodity are not driven by basic rules like for bonds and equities, but instead they are affected by world demand and supply chain.

This study investigates adding a global commodity index to also a global portfolio of traditional equities and bonds due to potential benefits of adding commodity in the portfolio investigated by peer-reviewed articles. Then I figure out if adding the alternative investment to the portfolio shows diversification benefit by calculating return, risk and correlation of the chosen traditional assets, which consists of US bonds and equities plus ex-US bonds and equities, with the same portfolio with commodity index to see if adding commodity helps to improve the performance of the portfolio. I will then further use mean-variance optimisation to find the optimal portfolio by generating efficient frontier and also find out about the effect of adding commodity to the portfolio

on the efficient frontier. The difference between my research with the peer-reviewed ones is the extended period of the study to present and also a different universe of assets.

2. About the Commodity Index

In this study, the Goldman Sachs Commodity Index monthly total return has been used as the measurement of its performance. The index is unlevered with only a long position for commodity futures, and it is considered in first-generation commodity indices. It consists of 24 commodity groups, including energy (largest sector), precious metals, industrial metals, agriculture and livestock.

Table 1: *S&P GSCI constituents*

Commodity	2019 weights (%)		Commodity	2019 weights (%)
Agriculture			**Energy**	
Chicago Wheat	2.77		Crude oil	26.42
Kansas wheat	1.15		Heating Oil	4.24
Corn	4.36		RBOB gasoline	4.48
Soybeans	3.14		Brent Crude oil	18.61
Coffee	0.72		Gasoline	5.56
Sugar #11	1.54		Natural Gas	3.11
Cocoa	0.32		**Industrial Metal**	
Cotton#2	1.41		Aluminium	3.89
Livestock			Copper	4.45
Lean Hogs	1.27		Lead	0.78
Live Cattle	3.48		Nickel	0.76
Feeder Cattle	1.27		Zinc	1.28
			Precious Metals	
			Gold	3.72
			Silver	0.42

3. Literature Review

Commodity investment increased from $170 bn to $410 bn (almost doubled) from 2007 to 2013 (Croft, 2013). To further explain the reason for the expansion of adding alternative investment to a traditional bond-stock portfolio, many authors explored the use of hedge funds, real estates, private equity, commodities, TIPS and others. (Bond, S. A., Hwang, S., Mitchell, P., & Satchell, S. E., 2007) showed that by adding real estate instead of private equity and hedge funds to the core asset can significantly reduce the risk and that adding which alternative asset depends on the situation of the market; in a bear market, hedge funds are preferred while in a bull market commodities are preferred.

(Garay, U. & Ter Horst, E., 2009) found out that private equity provides higher returns than traditional assets since the invested firms can be affected by investors to achieve the increased return. (Schmidt, 2006) concluded that for an unconstrained investor with the goal of minimum variance and maximum performance, mixed-asset portfolio of private equity and stocks is of preference. The (Karava, N, Georgiev, G, 2002) study also showed that a portfolio of hedge funds and managed futures and traditional asset classes like bond and stock would improve the portfolio return and reduce the risk whether under past market environments or forecasted return. (Emmrich, O., & McGroarty, F., 2013) found that since 2007 adding commodity like gold to a traditional portfolio will lead to investment with significant adjusted returns. Since the problem of investing private equity in the portfolio of traditional assets is its illiquidity, (Aigner, P., Beyschlag, G., Friederich, T., Kalepky, M., & Zagst, R., 2012) showed that investment with listed private equity solves the problem of illiquidity and also improves the risk and return of the portfolio. (Roll, 2004) demonstrated that adding TIPS to a portfolio of bond and U.S equity would increase the

performance of the portfolio by analysing the correlation between TIPS and nominal bond and equity returns.

I will further expand the literature for the studies which studied the success of adding commodity as an asset class in alternative investments to traditional asset portfolio. As for (Gorton, G. & Rouwenhorst K., 2006) study, including commodity futures in which the index is equally weighted does have diversification benefit to the portfolio of equity and bond. (Huang J.-Z. and Zhong Z., 2013) tested time-varying correlation and diversification gains of three asset classes; commodity, REITs and TIPS, and for this, they used the DCC model. (Erb C.B. and Harvey C.R., 2006) showed that a portfolio with commodity futures might have equity-like return by shifting the portfolio towards commodity futures with positive spot and role returns. (Jensen, G.R., Johnson, R.R., Mercer, J.M., 2000) supported that since the Sharpe ratio for commodities is less than bonds and stocks, therefore stand-alone commodity indexes performance is lower than them. (Daskalaki, C., & Skiadopoulos, G., 2011) also studied the diversification benefit of including commodity in a traditional asset portfolio in 2 ways: 1- they used spanning tests that comply with mean-variance and non-mean-variance approaches 2- by studying the diversification benefits of this asset class of alternative investment through out-of-sample analysis. They concluded that in-sample diversification benefits happen just in the case of applying higher moments pf assets' returns distribution.

However, (You L., Daiglar R.T., 2011) tested individual futures contract from the point of ex-ante and ex-post stability for optimal MV portfolios. They mentioned that if the ex-ante and ex-post results are not stable through the time, investors are better off with traditional portfolios. (Conover C., Jensen G.R., Johnson R. and Mercer J., 2010) demonstrated that regardless of investment style, allocating 10% or more of the portfolio to commodities reduce the risk to a great extent. So since

by growth of commodity markets, the correlations between commodity and bond and stock returns go higher, (Beckmann, J., Belke, A., Czudaj, R., 2014) showed that monetary policy can help to raise this correlation. (Wolff, 2015) concluded that because of the financialization of commodity in recent years, investment should be done in the commodity index rather than in selective commodity group. They also found out that for most asset allocation strategies that They implemented, aggregate commodity indices and industrial metals yield the best performance. They concluded that livestock and agriculture commodities do not yield gains in portfolio, and that difference comes up in obtaining benefits by adding commodities as other optimisation approaches and asset classed are used. (Creer, 1978) mentioned that the unleveraged commodities are less risky than stocks since the value of the stock may decrease to zero and discussed that a combination of stock, commodity and debt lead to a constant rate of return and he advised the risk-averse investor to use the commodities futures to contribute to the risk-adjusted return.

4. Data

For this study, I used equity and bond indices, along with a commodity index. For equity and bond index, the USA and ex-USA indices were applied. For bond indices, I also separated the indices for corporate and government bond index again for USA and ex-USA. The data was retrieved from the Bloomberg terminal. So, the global portfolio is chosen as the universe.

For equity index, S&P 500 and MSCI ACWI ex-USA were used as proxies of equity in order as USA equity and ex-USA equity. Also, as bond indices, S&P Investment Grade (IG) corporate bond and Bloomberg Barclay US Treasury Total Return (Unhedged USD) represent USA corporate and USA government bond indices.

Furthermore, S&P International Corporate Bond Total Return Bond and Morningstar Global ex-USA Government Bond were applied for proxies for corporate and government global ex-USA Bond Indices. Also, the data for the commodity index is S&P GSCI total return which indicates the direction of price and inflation in the global economy for commodities.

The data for all indices are monthly for the date range of September 2002 until September 2019. All the data is US-denominated. The chosen date range was due to limitation of data for Morningstar Global es-USA Government Bond.

5. Methodology

5.1. Analysis of Returns, Volatility and Correlation

In this study, a global portfolio including separate government and corporate bond indices along with equities have been chosen, the purpose of adding more assets is to diversify the portfolio, and also to investigate the effect of global assets in the portfolio.

By using the mentioned data above, I calculated the historical return of each asset. all data were annualised. Risks (standard deviations) and expected returns were calculated using historical returns. Since the data was monthly, the data time frame includes the 2008 financial crisis; therefore, the period is broken into pre-crisis, crisis and post-crisis along with the full period for further analysis. The pre-crisis period was chosen from September 2002 to September 2007, the crisis period from September 2007 to September 2009 and the post-crisis period from September 2009 to September 2019. Table 2 shows the returns and volatility of all seven indices in the full period.

Table 2: *Expected return and volatility for the full period*

Index	Annualized Return	Annualised Volatility
MSCI ACWI ex-USA	0.056	0.18
S&P 500	0.076	0.14
Bloomberg Barclay US Treasury	0.038	0.044
S&P Investment Grade (IG) Corporate Bond	0.048	0.056
S&P International Corporate Bond	0.053	0.098
Morningstar Global ex-USA Government Bond	0.046	0.0834
S&P GSCI	-0.0034	0.23

As (Daskalaki, C., & Skiadopoulos, G., 2011) has also investigated, the commodity index has a lower return and higher volatility compared to stocks and bonds. In fact, the general trend of the commodity index is negative. Therefore, considering only the commodity index does not contribute to a good performance. S&P 500 outperforms other indices in the full period. Government bonds are less volatile than corporate bonds with higher returns. Also, the correlation between commodity index and other indices are calculated. For the correlation calculation, the following formula to calculate the correlation was applied:

$$\rho_{ij} = \frac{COV_{ij}}{\sigma_i \sigma_j}$$

In which σ_i and σ_j are the volatilities of asset i and asset j.

If the correlations between the commodity index return and other chosen indices of bonds and equities are low or negative, it is concluded that there may be a diversification benefit of adding the commodity index to the portfolio.

Table 3: *Correlation matrix for the full period*

Security	MSCI ex-USA	S&P 500	Barclay US Treasury	S&P Investment Grade Corporate Bond	S&P International Corporate Bond	Morningstar Global ex-US Government Bond	S&P GSCI
MSCI ex-USA	1.00						
S&P 500	0.87	1.00					
Barclay US Treasury	-0.23	-0.31	1.00				
S&P Investment Grade Corporate Bond	0.32	0.17	0.64	1.00			
S&P International Corporate Bond	0.63	0.42	0.24	0.57	1.00		
Morningstar Global ex-US Government Bond	0.35	0.15	0.51	0.55	0.83	1.00	
S&P GSCI	0.49	0.36	-0.19	0.09	0.39	0.21	1.00

Table 3 indicates the correlations between the return of seven assets in the full period. There is a low correlation between the commodity index, S&P Investment-grade corporate bond, and Morningstar global ex-US government bond. There is also a negative correlation between commodity index and Barclay US Treasury. S&P GSCI Index has the highest correlation with MSCI ex-US equity. So due to high correlation of S&P GSCI with other assets, there can't be diversification benefit by having the commodity index in the portfolio.

Table 4: *Expected return and volatility for the pre-crisis period*

Index	Annualized Return	Annualised Volatility
MSCI ACWI ex-USA	0.19	0.14
S&P 500	0.1	0.11
Bloomberg Barclay US Treasury	0.04	0.05
S&P Investment Grade (IG) Corporate Bond	0.05	0.052
S&P International Corporate Bond	0.1	0.096
Morningstar Global ex-USA Government Bond	0.08	0.09
S&P GSCI	0.14	0.25

According to table 4 in the pre-crisis period, the return of MSCI ACWI ex-USA is way better than the S&P 500 with return almost double in the period. The volatility of the commodity index is high. Therefore, it may be concluded that adding commodity to this portfolio will make no better improvement in performance in this period. This is the only period that the return of the S&P GSCI is higher than the S&P 500.

Table 5. shows the correlation between asset returns in the pre-crisis period to investigate further the upside of having commodities in the portfolio.

Table 5: *Correlation matrix for the pre-crisis period*

Security	MSCI ex-USA	S&P 500	Barclay US Treasury	S&P Investment Grade Corporate Bond	S&P International Corporate Bond	Morningstar Global ex-US Government Bond	S&P GSCI
MSCI ex-USA	1.00						
C	0.836	1.00					
Barclay US Treasury	-0.2	-0.31	1.00				
S&P Investment Grade Corporate Bond	-0.057	-0.14	0.94	1.00			
S&P International Corporate Bond	0.242	-0.011	0.515	0.57	1.00		
Morningstar Global ex-US Government Bond	0.212	-0.024	0.566	0.599	0.974	1.00	
S&P GSCI	0.017	-0.25	0.121	0.083	0.212	0.19	1.00

Though according to correlations in the above table, there is a low correlation between commodities and the assets of the portfolio and also the correlation between commodity and S&P 500 is negative.

So, the portfolio performance can be improved by having S&P GSCI in it. I will further see if that helps with the efficient frontier in portfolio optimisation section.

Table 6: *Expected return and volatility for the time period including the financial crisis*

Index	Annualized Return	Annualised Volatility
MSCI ACWI ex-USA	-0.16	0.44
S&P 500	-0.17	0.32
Bloomberg Barclay US Treasury	0.07	0.08
S&P Investment Grade (IG) Corporate Bond	0.029	0.16
S&P International Corporate Bond	0.054	0.18
Morningstar Global ex-USA Government Bond	0.1	0.15
S&P GSCI	-0.16	0.49

The next studied period is the time period including crisis, in which the returns have dropped significantly except for government bonds that stayed high in the crisis (both Barclay US Treasury and Morningstar global ex-US government bond). This is because the prices have fallen. Also, the volatilities of assets have increased dramatically, especially equities, corporate bonds, as well as the commodity index. This shows in a crisis, where investors prefer to invest in government bonds as they could keep their return and volatility in a good position rather than corporate bonds and equities. The commodity index is exceptionally volatile, which means that in this period, it seems it cannot offer diversification benefits as people tend to invest in safer assets like bonds and especial commodities like gold. Since the majority of the GSCI index is energy, even energy commodities are not considered safe in the period. Another important thing to deal with in the crisis period is the increased correlation among assets. This result is also consistent with findings of (Daskalaki, C., & Skiadopoulos, G., 2011), regarding no diversification benefit of including

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commodities in the mean-variance investors' portfolios in sub-prime crisis period. The return of

S&P GSCI outperforms S&P 500 and is almost the same as MSCI ex-US.

Table 7: *Correlation matrix for the time period including crisis*

Security	MSCI ex-USA	S&P 500	Barclay US Treasury	S&P Investment Grade Corporate Bond	S&P International Corporate Bond	Morningstar Global ex-US Government Bond	S&P GSCI
MSCI ex-USA	1.00						
S&P 500	0.931	1.00					
Barclay US Treasury	-0.11	-0.12	1.00				
S&P Investment Grade Corporate Bond	0.649	0.523	0.32	1.00			
S&P International Corporate Bond	0.729	0.58	0.303	0.783	1.00		
Morningstar Global ex-US Government Bond	0.477	0.366	0.17	0.624	0.88	1.00	
S&P GSCI	0.654	0.561	-0.251	0.252	0.479	0.22	1.00

The correlations between commodity index total return with the assets are in line with what was

concluded with return and variance relations. These correlations have become stronger. Only the

correlation between commodity index with government bonds stayed low. The correlation between

equities and US treasury bond is negative. It can be concluded that during the crisis due to this

high correlation with all the studied assets, adding commodity does not bring diversification

benefits and investor's preference is for safe assets like gold and government bonds.

Table 8: *Expected return and volatility for the post-crisis period*

Index	Annualized Return	Annualised Volatility
MSCI ACWI ex-USA	0.03	0.16
S&P 500	0.11	0.13
Bloomberg Barclay US Treasury	0.029	0.037
S&P Investment Grade (IG) Corporate Bond	0.048	0.038
S&P International Corporate Bond	0.03	0.095
Morningstar Global ex-USA Government Bond	0.015	0.078
S&P GSCI	-0.043	0.19

In the post-crisis period, the returns of government bonds have dropped. Though the return of US corporate bond has increased and has gone back almost to the pre-crisis level. The volatility of commodity index is higher than equities, and it has a negative return, this means there is no way it can contribute to the performance of the portfolio.

According to the correlation table, the correlation between commodity index and equities are still high since the crisis period. Only the correlation between commodity index and Barclay US treasury and S&P Investment Grade Corporate bonds are low and negative. These also show that commodity cannot help at this period for having upside in the portfolio.

Table 9: *Correlation matrix for the post-crisis period*

Security	MSCI ex-USA	S&P 500	Barclay US Treasury	S&P Investment Grade Corporate Bond	S&P International Corporate Bond	Morningstar Global ex-US Government Bond	S&P GSCI
MSCI ex-USA	1.00						
S&P 500	0.86	1.00					
Barclay US Treasury	-0.34	-0.41	1.00				
S&P Investment Grade Corporate Bond	0.19	0.05	0.73	1.00			
S&P International Corporate Bond	0.76	0.56	0.043	0.46	1.00		
Morningstar Global ex-US Government Bond	0.37	0.17	0.38	0.52	0.72	1.00	
S&P GSCI	0.59	0.55	-0.39	-0.041	0.46	0.23	1.00

5.2. Mean-Variance Optimisation

For this study, mean-variance optimisation is used so that the vectors of returns of assets and matrix of variance-covariance will be the entry to the optimiser. The goal of the optimisation is that by generating many portfolios, the efficient frontier is achieved which will be the best combination of returns of assets with their related volatilities. Each vector of weights produced by the optimiser has return and risk. The return and risk of each portfolio are calculated in the following way:

$$r_p = \sum_i W_i \, (r_i)$$

where: $\sum_i W_i = 1$

$$Var\ _p = \sum_i W_i^2 \sigma_i^2 + \sum_i \sum_j W_i W_j \sigma_i \sigma_j \rho_{ij}$$

The only constraint I considered in the optimiser is the lower bound of 5% and upper bound of 30% for the assets weights in the portfolio to let the portfolio be diversified. The efficient portfolio is the one with higher return and minimum risk.

5.3. Portfolio Optimisation

As the first step in optimisation, the vector of returns of the six assets of which two are equities and four are bonds along with the variance-covariance matrix of the same assets are entered to the optimiser. Then the efficient frontier is generated for the feasible portfolio with the output of weights, returns and risks of those portfolios for each period.

As shown in figure 1 for the full period, portfolios without commodities have a return range of 4.8% to 5.9% both with volatilities of 5.1% and 11.1%.

As the commodity is added to the portfolio the range of return of the portfolio changes to 4.4% to 5.6% with volatilities of 5.2% to 11.3%, also as shown in the figure, it becomes more volatile and the efficient frontier shifts down.

Therefore, adding the commodity index to the portfolio in the full period does not help its performance, since the volatility of the commodity index is more than the other assets, and its return is negative.

Figure 1: *Efficient frontier for the full period (30/9/2002 - 30/9/2019)*
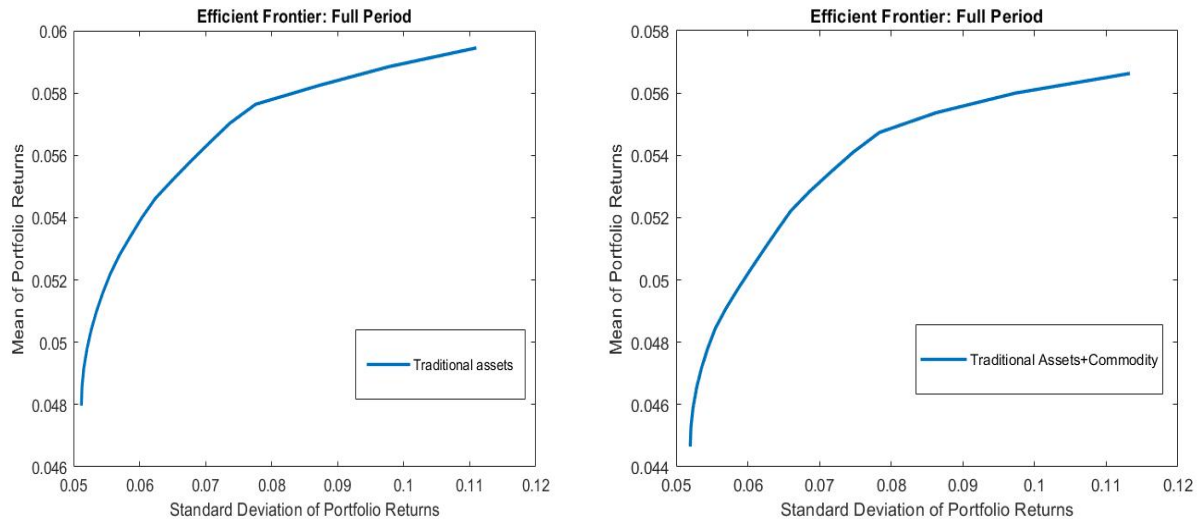
In the pre-crisis period, the range for the return of portfolios is between 7% to 12%, both with volatilities from 4.5% to nearly 8%. The maximum return is achieved by more weights in equities and also ex-US corporate bond. As the commodity return index is added to the portfolio, the return range changes to a minimum of 7.25% and a maximum of 13.25% with volatilities from 4.3% to 8.8%. Therefore, adding the commodity in this period contributes to the portfolio performance and the maximum return is increased by 10%. Therefore, for having more return than the traditional asset portfolio, there should be at least 13% of commodity index in this period.

The efficient frontier also shifts up. This is consistent with the conclusion in correlation section that commodity return index had low correlation with other assets and it can be seen that adding commodity to the portfolio rises the volatility (especially for higher returns), but it will also improve the performance of it.

Figure 2: *Efficient frontier for the pre-crisis period (30/9/2002-30/9/2007)*
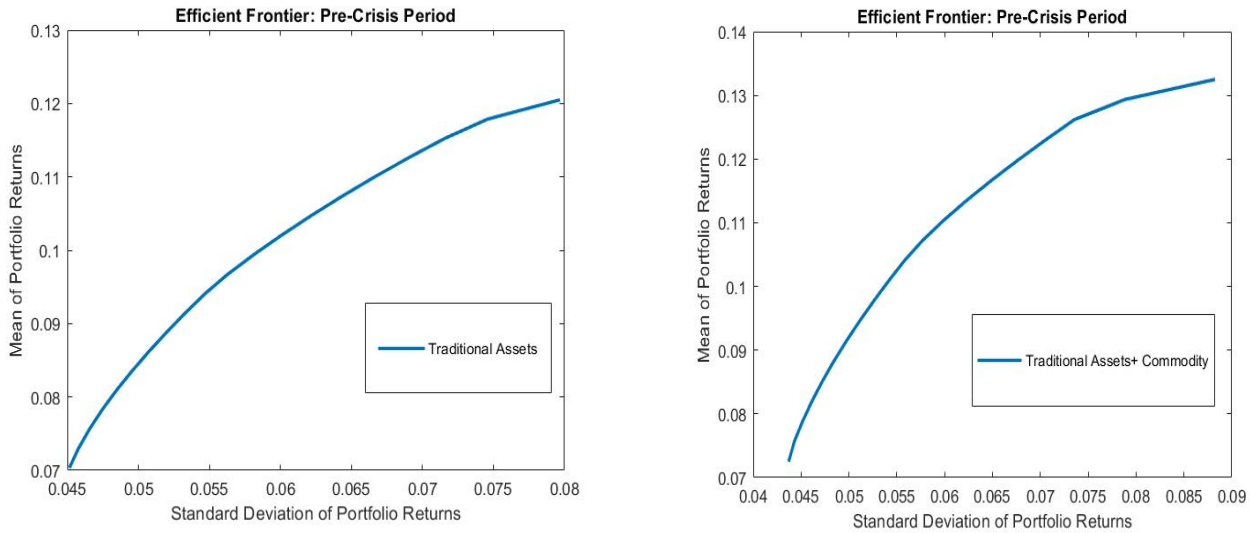
In the crisis period, the returns of portfolios range from 4.56% to 5% with volatilities of 11.9% to 12.7%. In this period the volatilities have increased compared to the pre-crisis period, and unlike the previous period, optimiser mostly allocates weights to Barclay US Treasury and ex-US bonds for achieving maximum return. When the commodity is added to the portfolios, it changes the returns range from 3.61% to 3.97% with volatilities of 12.3% to 13%. So, using commodity index does not help the performance of a portfolio in the crisis. This is due to the high volatility of commodity in crisis and the strong correlation between commodity and other assets. Also, it can be due to the negative sentiment of the investor to avoid more risky investments and to invest in gold. The efficient frontier in this period shifts to the lower right.

Figure 3: *Efficient frontier for the period including the crisis (30/9/2007 - 30/9/2009)*
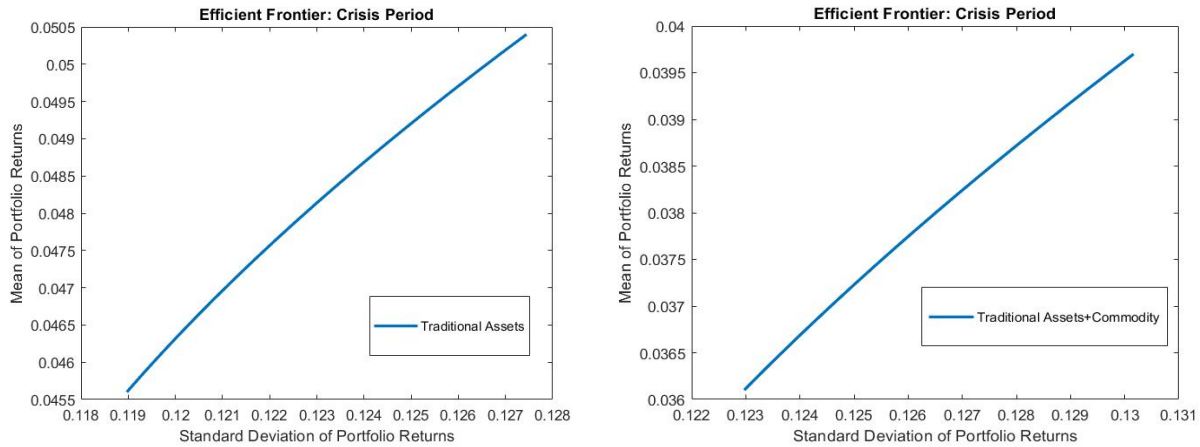
In the post-crisis period, which includes the bullish market, the return of the portfolio varies between 3.9% to 5.9% with volatilities of 4.2% to 8.5%. The portfolio return rise compared to the crisis period is due to the huge increase in S&P 500 return in this period. The returns of government bonds have decreased compared to the crisis, which shows that investors' risk tolerance has become higher, and they shifted more to invest in corporate bonds and equities in this period.

By adding commodities to the portfolio, as shown in the figure, the efficient frontier shifts down, since the return of the commodity index is still negative also after the financial crisis period.

The portfolio returns ex-post ranges between 3.3% and 5.5% with volatilities between 4.26% and 8.31%. Therefore, the portfolio risk is not changed that much in this period by adding commodity to it. It just decreases the return because the volatility of the commodity index has dropped a lot in this period. The returns of ex-ante and ex-post portfolios have not yet gone back to the return of the same portfolios in pre-crisis and the only period in which adding commodity to the portfolio improves its performance, is pre-crisis period. That's also for the reason that as early discussed, the correlations between commodity index return and equities' return have stayed high since crisis period and this can be due to the change in risk behaviour of the investor, since investors are less

risk-averse and also, economic conditions. The other reason is the financialization of the commodity market since financial crisis.

Figure 4: *Efficient frontier for the post-crisis period (30/9/2009 - 30/9/2019)*
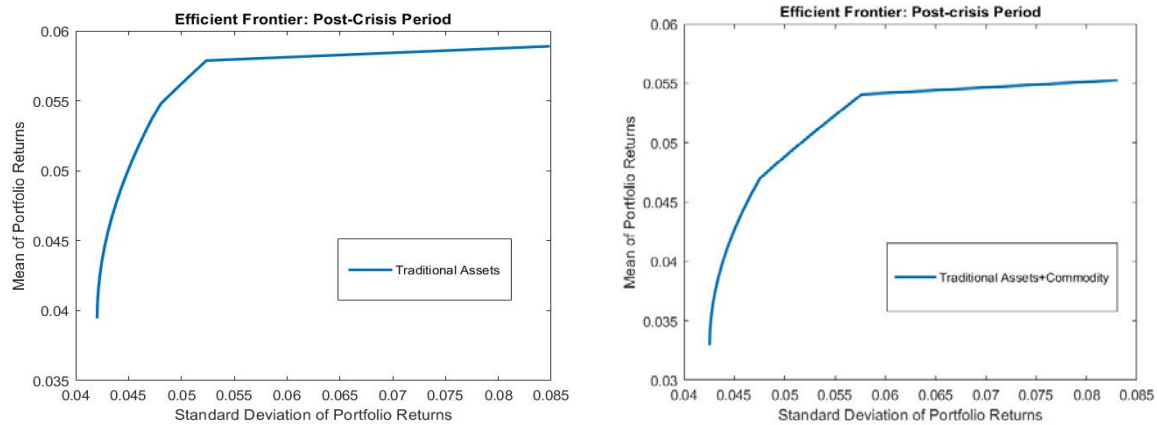
6. Results and Discussions

6.1. Financialization of Commodity Markets

The peak of commodity index (S&P GSCI) prices was in August 2008 (commodity boom period), and since then, the general trend of the index is decreasing to the present. In 2008, financialization distorted commodity prices (Ing-Haw Cheng and Wei Xiong, 2014). Also, according to (Olson E., Vivian A.J.,Wohar, M.E., 2014) Correlation depends on global economic conditions which depends on energy price. Therefore, investors who care less about risk-return are better off with including commodities in their portfolios.

The money inflows to commodity indices have caused commodity financialization. As (Croft, 2013) has mentioned the investment has become almost double from 2007 to 2013. The larger

correlation of S&P GSCI and MSCI ex-US rather than the S&P 500, which encompasses frontier and emerging markets could be the more growing demand for commodities from these economies.

6.2. Cumulative Total Return Analysis of the Assets

Figure 5: *Cumulative total return of assets*
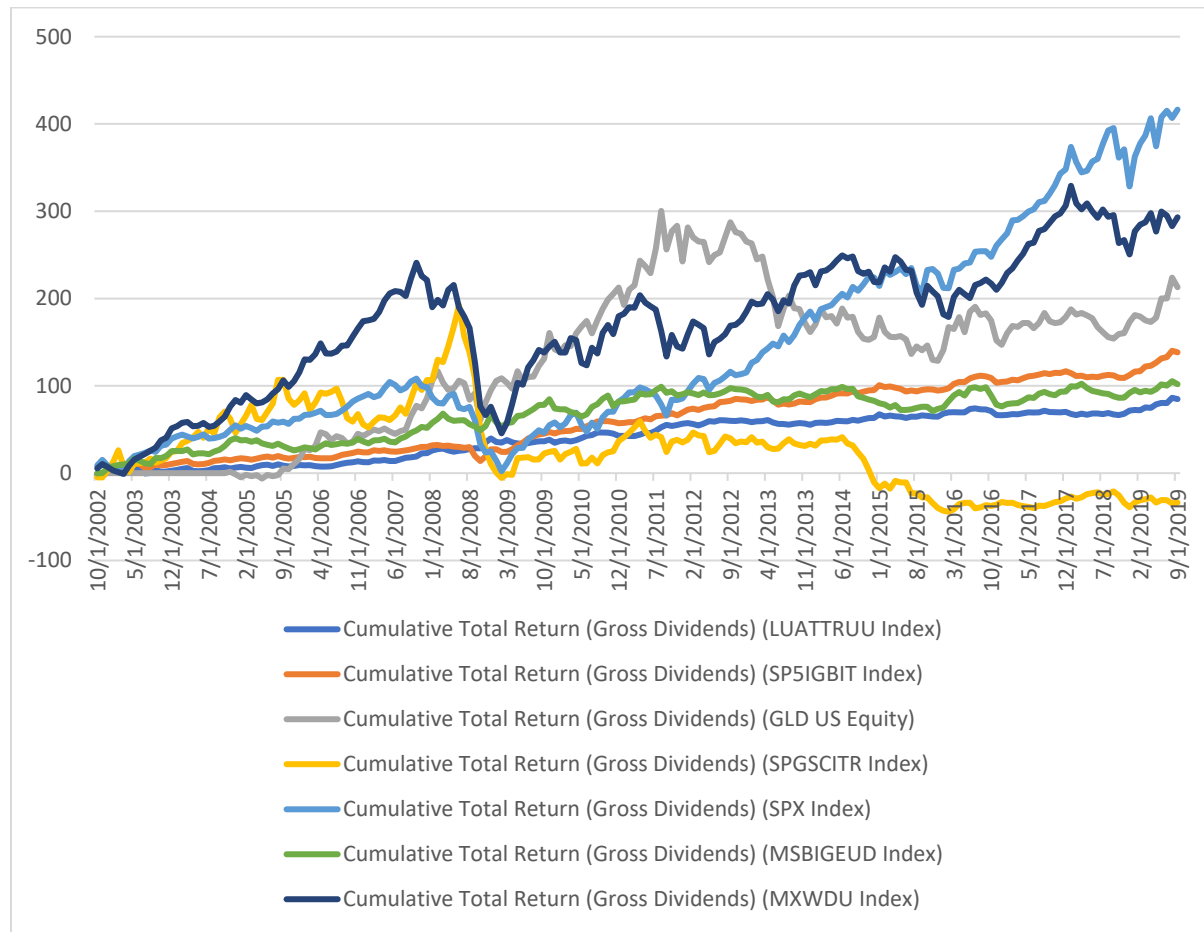
According to figure 5, the decreasing trend of the commodity index is because of its high correlation with oil. As the graph shows after the financial crisis, the cumulative total returns of equities and corporate bonds have increased, and the investment in safe assets has shrunk since we have had a bullish market. Even it seems investments in gold is not anywhere as like as the period

of the financial crisis and a few years after that. The S&P GSCI index has underperformed all the indices, the only time it outperformed almost all the assets was during year 2008 (financial crisis) and the S&P 500 has outperformed all the indices for the last couple of years.

7. Conclusion

In this study, a portfolio of global traditional assets was chosen to see if adding commodity index to it makes the performance better. As past studies had shown adding commodity to traditional asset portfolio has diversification benefits due to low and negative correlations between assets and commodity. The results of this study show that except for the pre-crisis period in which correlation between the commodity index chosen in this study and assets was low, and it improved the performance of the portfolio to some extent, there was no performance betterment in other periods. This is due to the strong correlation since the last financial crisis between the commodity index and assets; however, the correlation between the commodity index and the US government and corporate bonds has stayed low. The main reason for the strong correlation and yielding no benefit diversification could be the financialization of commodity market and also the dependence of the commodity index to oil which is function of economic conditions. Due to these reasons and high volatility of commodity, investment in the commodity is not offered for mean-variance investors.

 The optimiser used for achieving the results in this study is based on mean-variance optimisation. For generating the efficient frontiers, portfolios of best-expected returns along with their related volatilities are selected. Adding the commodity index to the portfolio resulted in shifting the efficient frontiers. Both ex-ante and ex-post portfolios are not equally weighted.

Further research can be the inclusion of an index which is less reliable on energy (specifically, oil in this study) or an index which is not overweight in one sector. Another way of extending the study is to consider individual commodity futures like base metals to track their effects on portfolio performance.

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Appendices

Table 10: *Ex-ante portfolios weights in the full period*

MSCI ex-USA	S&P 500	Barclay US Treasury	S&P IG Corporate Bond	S&P Int'l Corporate Bond	Morningstar Global ex-US Government Bond
0.05	0.097	0.3	0.3	0.05	0.203
0.05	0.117	0.3	0.3	0.05	0.183
0.05	0.138	0.3	0.3	0.05	0.162
0.05	0.158	0.3	0.3	0.05	0.142
0.05	0.178	0.3	0.3	0.05	0.122
0.05	0.198	0.3	0.3	0.05	0.102
0.05	0.218	0.3	0.3	0.05	0.082
0.05	0.238	0.3	0.3	0.05	0.062
0.05	0.259	0.3	0.291	0.05	0.05
0.05	0.278	0.293	0.279	0.05	0.05
0.05	0.288	0.262	0.3	0.05	0.05
0.05	0.3	0.239	0.3	0.061	0.05
0.05	0.3	0.199	0.3	0.101	0.05
0.05	0.3	0.158	0.3	0.142	0.05
0.05	0.3	0.118	0.3	0.182	0.05
0.05	0.3	0.078	0.3	0.222	0.05
0.05	0.3	0.05	0.263	0.287	0.05
0.118	0.3	0.05	0.182	0.3	0.05
0.193	0.3	0.05	0.107	0.3	0.05
0.3	0.3	0.05	0.05	0.25	0.05

Table 11: *Ex-ante returns and volatilities related to each portfolio weights in the full period*

Volatility Vector	Return Vector
0.0512	0.0480
0.0513	0.0486
0.0516	0.0492
0.0520	0.0498
0.0527	0.0504
0.0535	0.0510
0.0545	0.0516
0.0556	0.0522
0.0570	0.0528
0.0586	0.0534
0.0604	0.0540
0.0624	0.0546
0.0650	0.0552
0.0678	0.0558
0.0707	0.0564
0.0737	0.0570
0.0776	0.0576
0.0871	0.0582
0.0977	0.0588
0.1110	0.0595

Table 12: *Ex-post returns and volatilities related to each portfolio weights in the full period*

Volatility Vector	Return Vector
0.0520	0.0447
0.0521	0.0453
0.0524	0.0459
0.0529	0.0465
0.0535	0.0472
0.0544	0.0478
0.0554	0.0484
0.0569	0.0491
0.0586	0.0497
0.0604	0.0503
0.0622	0.0510
0.0641	0.0516
0.0660	0.0522
0.0687	0.0528
0.0716	0.0535
0.0747	0.0541
0.0784	0.0547
0.0863	0.0554
0.0973	0.0560
0.1133	0.0566

Table 13: *Ex-post portfolios weights in the full period*

MSCI ex-USA	S&P 500	Barclay US Treasury	S&P IG Corporate Bond	S&P Int'l Corporate Bond	Morningstar Global ex-US Government Bond	S&P GSCI
0.05	0.069	0.3	0.3	0.05	0.181	0.05
0.05	0.090	0.3	0.3	0.05	0.160	0.05
0.05	0.111	0.3	0.3	0.05	0.139	0.05
0.05	0.132	0.3	0.3	0.05	0.118	0.05
0.05	0.153	0.3	0.3	0.05	0.097	0.05
0.05	0.174	0.3	0.3	0.05	0.076	0.05
0.05	0.195	0.3	0.3	0.05	0.055	0.05
0.05	0.217	0.3	0.28	0.05	0.050	0.05
0.05	0.240	0.3	0.260	0.05	0.05	0.05
0.05	0.255	0.280	0.265	0.05	0.05	0.05
0.05	0.263	0.240	0.3	0.05	0.05	0.05
0.05	0.28	0.221	0.3	0.050	0.05	0.05
0.05	0.3	0.204	0.3	0.050	0.05	0.05
0.05	0.3	0.169	0.3	0.081	0.05	0.05
0.05	0.3	0.127	0.3	0.123	0.05	0.05
0.05	0.3	0.085	0.3	0.165	0.05	0.05
0.05	0.3	0.05	0.278	0.222	0.05	0.05
0.080	0.3	0.05	0.170	0.3	0.05	0.05
0.159	0.3	0.05	0.091	0.3	0.05	0.05
0.3	0.3	0.05	0.05	0.2	0.05	0.05

Table 14: *Ex-ante portfolios weights in the pre-crisis period*

MSCI ex-USA	S&P 500	Barclay US Treasury	S&P IG Corporate Bond	S&P Int'l Corporate Bond	Morningstar Global ex-US Government Bond
0.05	0.242	0.3	0.3	0.05	0.058
0.077	0.223	0.3	0.3	0.05	0.050
0.107	0.193	0.3	0.3	0.05	0.050
0.136	0.164	0.3	0.3	0.05	0.050
0.165	0.135	0.3	0.3	0.05	0.050
0.193	0.111	0.3	0.3	0.05	0.050
0.219	0.092	0.3	0.29	0.05	0.050
0.244	0.073	0.3	0.28	0.05	0.050
0.270	0.054	0.3	0.277	0.05	0.05
0.290	0.050	0.300	0.260	0.05	0.05
0.3	0.053	0.283	0.25	0.068	0.05
0.3	0.061	0.219	0.27	0.100	0.05
0.3	0.069	0.154	0.29	0.132	0.05
0.3	0.079	0.106	0.3	0.166	0.05
0.3	0.088	0.062	0.3	0.200	0.05
0.3	0.101	0.050	0.26	0.238	0.05
0.3	0.115	0.05	0.208	0.277	0.05
0.300	0.144	0.05	0.156	0.3	0.05
0.300	0.197	0.05	0.103	0.3	0.05
0.3	0.3	0.05	0.05	0.25	0.05

Table 15: *Ex-ante returns and volatilities related to each portfolio weights in the pre-crisis period*

Volatility Vector	Return Vector
0.0451	0.0703
0.0458	0.0730
0.0466	0.0756
0.0475	0.0783
0.0485	0.0809
0.0496	0.0835
0.0507	0.0862
0.0520	0.0888
0.0533	0.0915
0.0547	0.0941
0.0563	0.0967
0.0581	0.0994
0.0601	0.1020
0.0621	0.1047
0.0643	0.1073
0.0666	0.1099
0.0690	0.1126
0.0716	0.1152
0.0746	0.1179
0.0797	0.1205

Table 16: *Ex-post returns and volatilities related to each portfolio weights in the pre-crisis period*

Volatility Vector	Return Vector
0.0437	0.0725
0.0443	0.0757
0.0452	0.0788
0.0461	0.0820
0.0472	0.0851
0.0484	0.0883
0.0497	0.0914
0.0511	0.0946
0.0526	0.0978
0.0542	0.1009
0.0558	0.1041
0.0577	0.1072
0.0599	0.1104
0.0623	0.1136
0.0649	0.1167
0.0677	0.1199
0.0706	0.1230
0.0736	0.1262
0.0788	0.1293
0.0882	0.1325

Table 17: *Ex-post portfolios weights in the pre-crisis period*

MSCI ex-USA	S&P 500	Barclay US Treasury	S&P IG Corporate Bond	S&P Int'l Corporate Bond	Morningstar Global ex-US Government Bond	S&P GSCI
0.05	0.200	0.3	0.3	0.05	0.050	0.05
0.06	0.218	0.3	0.26	0.05	0.050	0.05
0.09	0.199	0.3	0.25	0.05	0.050	0.06
0.12	0.180	0.3	0.24	0.05	0.050	0.06
0.15	0.161	0.3	0.23	0.05	0.050	0.06
0.17	0.142	0.3	0.22	0.05	0.050	0.07
0.2	0.122	0.3	0.21	0.05	0.050	0.07
0.23	0.103	0.3	0.2	0.05	0.050	0.08
0.25	0.084	0.3	0.184	0.05	0.05	0.08
0.28	0.065	0.300	0.172	0.05	0.05	0.08
0.3	0.057	0.273	0.18	0.05	0.05	0.09
0.3	0.07	0.150	0.28	0.050	0.05	0.1
0.3	0.1	0.086	0.3	0.053	0.05	0.11
0.3	0.11	0.050	0.28	0.085	0.05	0.12
0.3	0.13	0.050	0.23	0.121	0.05	0.13
0.3	0.14	0.050	0.17	0.157	0.05	0.13
0.3	0.16	0.05	0.110	0.194	0.05	0.14
0.300	0.17	0.05	0.052	0.23	0.05	0.14
0.300	0.18	0.05	0.050	0.15	0.05	0.22
0.3	0.2	0.05	0.05	0.05	0.05	0.3

Table 18: *Ex-ante portfolios weights in the crisis period*

MSCI ex-USA	S&P 500	Barclay US Treasury	S&P IG Corporate Bond	S&P Int'l Corporate Bond	Morningstar Global ex-US Government Bond
0.05	0.050	0.3	0.25	0.05	0.300
0.05	0.050	0.3	0.24	0.061	0.300
0.05	0.050	0.3	0.23	0.071	0.300
0.05	0.050	0.3	0.22	0.082	0.300
0.05	0.050	0.3	0.21	0.092	0.300
0.05	0.050	0.3	0.2	0.103	0.300
0.05	0.050	0.3	0.19	0.113	0.300
0.05	0.050	0.3	0.18	0.124	0.300
0.05	0.050	0.3	0.166	0.134	0.3
0.05	0.050	0.300	0.155	0.145	0.3
0.05	0.050	0.300	0.14	0.155	0.3
0.05	0.05	0.300	0.13	0.166	0.3
0.05	0.05	0.300	0.12	0.176	0.3
0.05	0.05	0.300	0.11	0.187	0.3
0.05	0.05	0.300	0.1	0.197	0.3
0.05	0.05	0.300	0.09	0.208	0.3
0.05	0.05	0.3	0.082	0.218	0.3
0.050	0.05	0.3	0.071	0.229	0.3
0.050	0.05	0.3	0.061	0.239	0.3
0.05	0.05	0.3	0.05	0.25	0.3

Table 19: *Ex-ante returns and volatilities related to each portfolio weights in the crisis period*

Volatility Vector	Return Vector
0.1190	0.0456
0.1193	0.0459
0.1197	0.0461
0.1201	0.0464
0.1204	0.0466
0.1208	0.0469
0.1212	0.0471
0.1217	0.0474
0.1221	0.0476
0.1225	0.0479
0.1230	0.0481
0.1234	0.0484
0.1239	0.0486
0.1244	0.0489
0.1249	0.0491
0.1254	0.0494
0.1259	0.0496
0.1264	0.0499
0.1269	0.0501
0.1274	0.0504

Table 20: *Ex-post portfolios weights in the crisis period*

MSCI ex-USA	S&P 500	Barclay US Treasury	S&P IG Corporate Bond	S&P Int'l Corporate Bond	Morningstar Global ex-US Government Bond	S&P GSCI
0.05	0.050	0.3	0.2	0.05	0.300	0.05
0.05	0.050	0.3	0.19	0.058	0.300	0.05
0.05	0.050	0.3	0.18	0.066	0.300	0.05
0.05	0.050	0.3	0.18	0.074	0.300	0.05
0.05	0.050	0.3	0.17	0.082	0.300	0.05
0.05	0.050	0.3	0.16	0.089	0.300	0.05
0.05	0.050	0.3	0.15	0.097	0.300	0.05
0.05	0.050	0.3	0.14	0.105	0.300	0.05
0.05	0.050	0.3	0.137	0.113	0.3	0.05
0.05	0.050	0.300	0.129	0.121	0.3	0.05
0.05	0.050	0.300	0.12	0.129	0.3	0.05
0.05	0.05	0.300	0.11	0.137	0.3	0.05
0.05	0.05	0.300	0.11	0.145	0.3	0.05
0.05	0.05	0.300	0.1	0.153	0.3	0.05
0.05	0.05	0.300	0.09	0.161	0.3	0.05
0.05	0.05	0.300	0.08	0.168	0.3	0.05
0.05	0.05	0.3	0.074	0.176	0.3	0.05
0.050	0.05	0.3	0.066	0.184	0.3	0.05
0.050	0.05	0.3	0.058	0.192	0.3	0.05
0.05	0.05	0.3	0.05	0.2	0.3	0.05

Table 21: *Ex-post returns and volatilities related to each portfolio weights in the crisis period*

Volatility Vector	Return Vector
0.1230	0.0361
0.1233	0.0363
0.1236	0.0365
0.1240	0.0367
0.1243	0.0369
0.1247	0.0370
0.1250	0.0372
0.1254	0.0374
0.1257	0.0376
0.1261	0.0378
0.1265	0.0380
0.1269	0.0382
0.1273	0.0384
0.1277	0.0386
0.1281	0.0388
0.1285	0.0389
0.1289	0.0391
0.1293	0.0393
0.1297	0.0395
0.1302	0.0397

Table 22: *Ex-ante portfolios weights in the post-crisis period*

MSCI ex-USA	S&P 500	Barclay US Treasury	S&P IG Corporate Bond	S&P Int'l Corporate Bond	Morningstar Global ex-US Government Bond
0.05	0.090	0.3	0.3	0.05	0.210
0.05	0.101	0.3	0.3	0.05	0.199
0.05	0.112	0.3	0.3	0.05	0.188
0.05	0.122	0.3	0.3	0.05	0.178
0.05	0.133	0.3	0.3	0.05	0.167
0.05	0.144	0.3	0.3	0.05	0.156
0.05	0.155	0.3	0.3	0.05	0.145
0.05	0.165	0.3	0.3	0.05	0.135
0.05	0.176	0.3	0.300	0.05	0.124
0.05	0.187	0.300	0.300	0.05	0.113
0.05	0.198	0.300	0.3	0.05	0.102
0.05	0.209	0.300	0.3	0.050	0.091
0.05	0.219	0.300	0.3	0.050	0.081
0.05	0.23	0.300	0.3	0.050	0.07
0.05	0.241	0.300	0.3	0.050	0.059
0.05	0.252	0.298	0.3	0.050	0.05
0.05	0.265	0.29	0.300	0.050	0.05
0.050	0.277	0.27	0.300	0.05	0.05
0.050	0.29	0.26	0.300	0.05	0.05
0.25	0.3	0.05	0.3	0.05	0.05

Table 23: *Ex-ante returns, and volatilities related to each portfolio weights in the post-crisis period*

Volatility Vector	Return Vector
0.0420	0.0394
0.0421	0.0405
0.0421	0.0415
0.0423	0.0425
0.0425	0.0435
0.0427	0.0446
0.0430	0.0456
0.0434	0.0466
0.0438	0.0476
0.0443	0.0487
0.0448	0.0497
0.0453	0.0507
0.0459	0.0517
0.0466	0.0528
0.0473	0.0538
0.0481	0.0548
0.0495	0.0558
0.0509	0.0569
0.0524	0.0579
0.0849	0.0589

Table 24: *Ex-post portfolios weights in the post-crisis period*

MSCI ex-USA	S&P 500	Barclay US Treasury	S&P IG Corporate Bond	S&P Int'l Corporate Bond	Morningstar Global ex-US Government Bond	S&P GSCI
0.05	0.052	0.3	0.3	0.05	0.198	0.05
0.05	0.064	0.3	0.3	0.05	0.186	0.05
0.05	0.076	0.3	0.3	0.05	0.174	0.05
0.05	0.089	0.3	0.3	0.05	0.161	0.05
0.05	0.101	0.3	0.3	0.05	0.149	0.05
0.05	0.114	0.3	0.3	0.05	0.136	0.05
0.05	0.126	0.3	0.3	0.05	0.124	0.05
0.05	0.138	0.3	0.3	0.05	0.112	0.05
0.05	0.151	0.3	0.300	0.05	0.099	0.05
0.05	0.163	0.300	0.300	0.05	0.087	0.05
0.05	0.175	0.300	0.3	0.05	0.075	0.05
0.05	0.188	0.300	0.3	0.050	0.062	0.05
0.05	0.2	0.300	0.3	0.050	0.05	0.05
0.05	0.215	0.285	0.3	0.050	0.05	0.05
0.05	0.229	0.271	0.3	0.050	0.05	0.05
0.05	0.244	0.256	0.3	0.050	0.05	0.05
0.05	0.258	0.24	0.300	0.050	0.05	0.05
0.050	0.273	0.23	0.300	0.05	0.05	0.05
0.050	0.287	0.21	0.300	0.05	0.05	0.05
0.2	0.3	0.05	0.3	0.05	0.05	0.05

Table 25: *Ex-post returns and volatilities related to each portfolio weights in the post-crisis period*

Volatility Vector	Return Vector
0.0426	0.0329
0.0426	0.0341
0.0427	0.0353
0.0429	0.0364
0.0431	0.0376
0.0435	0.0388
0.0439	0.0400
0.0443	0.0411
0.0449	0.0423
0.0454	0.0435
0.0461	0.0447
0.0468	0.0458
0.0476	0.0470
0.0492	0.0482
0.0508	0.0494
0.0525	0.0505
0.0542	0.0517
0.0559	0.0529
0.0577	0.0541
0.0831	0.0553