OPTIMUM PORTFOLIO CONSTRUCTION CONSIDERING VALUE AT RISK USING GENETIC ALGORITHMS AND MONTE CARLO SIMULATION

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ABSTRACT

OPTIMUM PORTFOLIO CONSTRUCTION CONSIDERING VALUE AT RISK USING GENETIC ALGORITHMS AND MONTE CARLO SIMULATION

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Determining the best portfolio out of set of alternative investment opportunities to optimize risk-adjusted return and value-at-risk simultaneously is a challenging issue for many practitioners. In recent years, the application of non-conventional methods for portfolio optimization problems has grown in importance in the investment industry. As an effective alternative to traditional optimization techniques for handling the computationally complicated portfolio optimization problems, many nature-inspired optimization methods have emerged and have been developed by researchers. In this thesis, a novel algorithm is suggested to construct a promising portfolio in terms of Mean return- VaR and Sharpe ratio-VaR from a limited number of securities from a set of available equities. The algorithm consist of three stages of refining. The first stage is to select 60 stocks out of all the securities in S&P500 index based on fundamental factors using factor analysis. In the second stage, the proposed algorithm employs a Markowitz' mean-variance optimization model to refine the quality of initial population of portfolios of 30 stocks and improve the convergence behaviour of the algorithm. And in the third stage, a state-of-the-art genetic algorithm is applied to determine an optimized portfolio of assets in terms of risk-adjusted return and value at risk. The novel genetic algorithm developed in this research benefits from an innovative solution representation which make GA searches over both discrete and continuous variables in the problem of optimizing stock and industry selection and weight allocation. In this study, the outperformance and effectiveness of the proposed algorithm are demonstrated by comparing annual return, annual volatility, Sharpe ratio, Jensen's alpha and beta of a constructed portfolio with the S&P 500 index and Mean-Variance constructed portfolio. The robustness of our evolutionary algorithm is verified by evaluation of the results in both in-sample and out-sample data.

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Chapter 1

1. Introduction

This study is aimed at finding a solution for selecting a limited number of stocks from S&P 500 index while trying to maximize the risk-adjusted return and minimizing Value-at-Risk of a portfolio. In a capitalization-weighted index like the S&P 500 index the exposure to overvalued stocks will be increased while those undervalued stocks will carry less weight due to their lower market cap. Although index investing will provide us with diversification, we can achieve diversification with a smaller portfolio of stocks while benefiting from different advantage. However, in capital market, there exists many equities in different industry sectors with different characteristics, and construction of a profitable portfolio with limited capital can be challenging.

In constructing an optimized portfolio, we will face several constraints, such as limiting number of stocks from industry sectors, and also limiting weight allocation to an individual stock. The aim of this study is to provide a novel approach to select limited number of stocks from a large pool of stocks in a way that it maximize the risk-adjusted return and minimizes VaR of the portfolio while taking into account constraints that limit our portfolio exposure to a certain industry sector or individual stock.

Since VaR is a non-linear function of return, a small change in the portfolio allocation would have considerable impact on the VaR of the portfolio, hence the classic models of the portfolio optimization are incapable of optimizing the cost function stated in this paper.

The problem considered in this research is a mixture of the optimal selection of stocks ,which is an integer programing problem where decision variables in the constraints restricted to be either 0 or 1, and optimized weight allocation of stocks where variables are not necessary discrete, and a non-linear parameters in the objective of the portfolio. Therefore, we face a mixed-integer programing (MIP) problem in this study. Since MIP problems are categorized into non-convex problems finding optimal solution with traditional optimization methods will be a difficult task with substantial memory and time consumption requirement.

Therefore, the main objective of this paper is to present a novel metaheuristic approach to tackle the presented portfolio optimization problem with cardinality constraints, a well-known NP-hard problem due to the non-linearity of objective functions and integrality of constraints.

This approach helps an investor to automatically create her own portfolio from a pool of assets with the aim of achieving an active return and be able to beat the index.

1.1 Portfolio Diversification:

In this paper, our goal is to make an optimized diversified portfolio of 30 stocks from the 5 best performance companies (obtained from factor analysis) in each of 12 sub-industry of the S&P 500 index. To achieve this goal, our approach is to have at least one security from each sub-industry in the S&P 500 but not more than five. We would select the five top performance equities out of the 10 largest market cap companies in each sub-industry utilizing factor analysis scoring and then optimize the portfolio using genetic algorithms considering mean of returns and Value at Risk in a weighted cost function. The following shows our steps to find an optimal portfolio of S&P 500 securities:

i. Categorization of the S&P 500 stocks (1-Retailing, 2-Software, 3-Technology & hardware, 4-Healthcare, 5-Consumer service, 6-Semiconductors, 7-Pharmaceutical & biotechnology, 8-Transportation, 9-Household & personal goods, 10-Capital goods, 11-Diversified financials, and 12-Food, beverage and tobaccos.)
ii. Rank stocks in each sub-industry based on their market capitalization.
iii. Choose 10 largest market cap stocks in each sub-sector.
iv. Run factor analysis on 10 selected stocks in each sub-industry.
v. Choose five stocks in each sub-industry with the highest score obtained from factor analysis
vi. Define objectives and constraints for the portfolio optimization
vii. Use genetic algorithms model to make an optimized portfolio of 30 stocks
viii. Compare the result with generated portfolio by mean-variance optimization method to see if any improvement happened or not.

Conventionally, based on the study by (Fisher 1970) it was assumed that 95% of the benefit of diversification will be achieved by holding 32 stocks in a portfolio. However, it was shown in another study by (Surz 2000) that by having 60 stock portfolios about 90% of available diversification will be captured. Moreover (Campbell 2001) proposed idiosyncratic risk in 50 stocks can became negligible.

In other words, by increasing the number of stocks to more than 30 stocks, one might not benefit greatly from diversification, and there might be no significant difference in diversification benefits of 60 stocks over 30 stocks. Therefore, substantial number of stocks needed to be added to an approximately 30 stocks portfolio to completely remove idiosyncratic risk whereas by this way you might trade off active management advantages with complete diversification of portfolio. While increasing the number of stocks in a portfolio can mitigate unsystematic risk however cost of having a portfolio with large number of securities (i.e. transaction costs) is considerable. Therefore, the ideal range of securities in a portfolio should be between 30 to 50 stocks in order to eliminate idiosyncratic risk (Busetti 2005). In this study, we chose a portfolio consist of 30 assets.

1.2 Portfolio Optimization

Of the most common ways of portfolio constructions, market cap weighted (equally weighted), inverse volatility, equal risk and maximum diversification methods can be mentioned. These are methods with different assumptions considering characteristics of securities available in the portfolio, and the main weakness of these methods is picking risk or return over other one. The most popular solution for the aforementioned weakness, is using portfolio optimization.

Portfolio optimization is the process of selecting and combining the best assets to make a portfolio, out of many available securities, according to the specified objectives, which could be maximizing the return given a certain level of risk or minimizing the risk of the whole portfolio having a specified return goal (or more specifically, having the highest possible sharp ratio, which is measurement of the excess return for an unit of a risk), this usually follows some constraints, such as asset weights, and/or numbers of trades.

Portfolio optimization originated from Modern Portfolio Theory saying that investors desire to have the highest possible return for the lowest possible inherent risk. Modern Portfolio Theory argues that risk and return are two dependent factors in a portfolio which they should be assessed to get the maximum return for a given level of risk. Mean-variance optimization has become base of the modern finance theory (Markowitz, Portfolio selection. 1952).

Since the introduction of Modern Portfolio Theory, many have applied portfolio optimization with various objectives and constraints on their portfolios to reach an optimal asset allocation. Also, some other approaches have been used to overcome restrictions in the real-world investments, such as transaction limits and costs. One of these approaches is using genetic algorithms method to make an optimal portfolio.

Chapter 2

2. Literature Review

Portfolio optimization is a challenging task for investors to find the best risk-adjusted investment which can meet their desired level of risk and return. The most popular solution for portfolio optimization is mean-variance model, minimizing the risk level (variance of assets considering covariance between them) of a portfolio for a given level of return (defining as the mean return of the assets), or maximizing the return of the portfolio for a given level of risk, which was introduced by (Markowitz, Portfolio selection 1952)

Although Markowitz model has been the foundation of most portfolio selections and researches, but the model, neither contains cardinality constraints (to impel each portfolio to have a certain number of assets) nor uses bounding constraint (to limit the available amount of money to invest in each asset) (Alberto Fernández 2005). To overcome these limitations, some methods such as Constrained Optimization (CO), Quadratic Programming (QP), Linear Programming (LP) and Second-Order Cone Programming (SOCP) have been developed to provide an exact solution for the optimization which most of the time, they work based on linear assumptions and a single objective (Davidsson 2011).

However, in the current complex financial environment, sometimes; these assumptions and objectives are costly (or even inapplicable) to be utilized to solve the portfolio optimization with the mentioned methods. Of solutions proposed as an alternative to these mentioned methods, is using metaheuristics approaches. Metaheuristics is an approach found to solve complex portfolio optimization problem more competently than classical optimization problems. Metaheuristic which were first introduced by (Glover 1986) are methods to find near-optimal solution for an NP-hard optimization problem with relatively low computational costs and close to classical approach results.

One of the metaheuristic solutions for this problem would be using genetic algorithm (GA) which is simply process of generating n random variables to look for the best solutions (Holland 1992). Using GA would firstly help to solve the optimization model by looking both forward and backward inductions, besides, GA can overcome large computational issues existing in the classic model (Yang 2006) . This stochastic technique can solve non-linear optimization problems, with various characteristics such as optimization problems containing non-smooth and non-continuous objectives or continuous and integer variables (Chi-ming Lin 2007)

A genetic algorithm is a natural based selection principle that has been used in the field of finance increasingly recently to find the optimum solution for different purposes, and one of its main usages is in investment portfolio optimization. A GA works with an iterative method by manipulating a population of a constant size including chromosomes. Each chromosome, formed by a set of genes, has a solution for the defined problem. In each iteration, a new the population is generated using genetic operators which are selection, crossover, and mutation. Eventually, every chromosome competes with other chromosomes, and consequently; this competition specifies a winner chromosome which is the optimum solution for the problem (Benbouziane 2012).

The application of GA has been increased in the finance field as the effectiveness of its results has been proven by much research. (Andrea Loraschi 1996) successfully used GA to find optimum weights of different assets in stocks portfolio to minimize the risk of the portfolio for the predefined level of return, and it was shown that the approach is even effectively applicable when there are multiple equilibriums.

Many researches have been conducted to use GA in the area of investment portfolio optimization. It has been shown that GA would give use more desirable performances over the traditional models in managing index funds tied to the benchmark indices even when the market is flat (Kyong Joo Oha 2005). To rebalance the asset-liability matching portfolio of insurance companies, GA was tested to optimize a portfolio with the sensitivity to interest rate changes and consequently, it gave a better allocation for different risk situation (Zhang 2010).

Multi-objective GA techniques, combining with fuzzy logic, were used to optimize portfolio with real-world constraints such as floor and round-lot constraints to obtain improvement in performance of the Vector Evaluated Genetic Algorithm (VEGA) and the result was positive (Prisadarng Skolpadungket 2007) . The efficiency of GA to solve portfolio optimization with different risk measures and tendencies was observed and also it suggested that small portfolios (around 30 assets) would perform better than those of bigger ones (Tun-Jen Chang 2009). Another observation was that using single objective GA approach to optimize VaR of a portfolio would give efficient portfolio with returns distributed within the given range and potentially minimum risk for desired level of return, while multi-objective evolutionary algorithm could give us desired return in the range in a shorter time (Vladimir Ranković 2013).

Chapter 3

3. Objective Function

For the objective function of optimization, usually one of Sharpe ratio or a risk factor (such as Value at risk) has been used in the previous researches.

To find the fittest species, minimizing risk and concurrently maximizing return (in other words, maximizing sharp ratio assuming risk-free rate is zero) was used as the objective function by (Sinha 2015) to calculate the fitness value of the chromosomes using GA.

Sharp ratio was introduced by (Sharpe, Mutual fund performance 1966) in the attempt to evaluate and predict the performance of mutual fund. Sharp ratio is additional compensation of an asset over risk free investment for one unit of risk of the asset, hence higher ratio, gives you more return for a unit of the risk. It is a good measurement for comparing different assets and portfolios, but it becomes problematic when the distribution is not normal, since standard deviation would not be a good factor to gauge the risk in a non-normally distribution.

(Vladimir Ranković 2013) defined the objective function as maximizing return and minimizing Value-at-Risk at the same time to find the fitness value using GA.

Value-at-Risk (VaR) is a main measure of risk for different industries, specifically financial sector. VaR is the maximum loss can occur in a given time horizon for a given confidence level. For an example, if a portfolio has VaR of $50 million at one month with 99% confidence level, it means that with the chance of 99%, the loss would not be more than $50 million over any given month, in the other words, there is 1% chance that the loss would exceed $50 million over any particular month.

Technically, VaR is a quantile of a portfolio's profit/loss for a specific horizon and a given probability. The variance of a portfolio can be approximated analytically, while analytical estimation does not work for VaR, unless it is assumed that portfolio return or value distribution can be perfectly estimated using some theoretical distribution. However, in reality, especially in highly volatile market or periods, when asymmetry increases in data of financial assets (more specifically, stock returns) accurate approximation becomes impossible even by theoretical distribution. Considering that, Normally, minimizing VaR is very complex, which usually classical optimization methods are unable to solve the optimization problem, however, researchers have shown metaheuristics methods (popularly genetic algorithms) are efficient for solving this complex portfolio VaR optimizing (Vladimir Ranković 2013).

In our paper, we not only concentrated on sharp ratio in our main objective function, as our primary goal is to have a higher risk-adjusted return, but also; we added VaR to the objective, with less concentration, to limit the portfolio loss. For another objective, we tested mean return and VaR as our main objective with different concentrations. With these objectives, we also would take into account both ex-post effects in sharp ratio, and ex-ante effects in VaR using Monte Carlo simulation. The code used for simulation of assets paths (Monte Carlo simulation) is based on the code written in (Goddard 2006).

3.1 Mean-Variance

Different objective functions have been used in literature. The most popular one is the mean-variance portfolio framework which was first introduced by (Markowitz, Portfolio selection. 1952) explaining that a portfolio can be optimized by maximizing the return and minimizing the risk simultaneously using deterministic algorithm solution. The framework can be formulated as the following:

Minimize :

$$Objective\ \sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{m} w_i w_j \sigma_{ij} \quad (3.1)$$

Subject to :

$$r_p = \sum_{i=1}^{n} w_i r_i \quad (3.2)$$

$$\sum_{i=1}^{30} w_i = 1 \qquad (3.3)$$

$$0.1 \leq w_i \leq 1\ ;\ i = 1, 2, 3, \ldots, 30 \quad (3.4)$$

Where σ_p represents the portfolio total risk, σ_{ij} denotes covariance between the ith security and the jth security. r_i represents the expected return of each security in the portfolio,w_i is the weight of each asset in the portfolio consists of n assets and r_p is the expected portfolio return. Solving the above equation for a range of different r_p will offer set of optimal portfolios where risk is minimized for a given level of expected return. This efficient set of points will shape a curve known as efficient frontier that lies between the global minimum variance portfolio and the maximum return portfolio

With specified return, the risk can be minimized, or with having the highest tolerable risk of investor, the return can be maximized by the Markowitz model. The framework is easy to apply and needs the least inputs (return and variance) to work.

3.1.1 Markowitz Model Limitation

As the financial market has become more complex the Markowitz model has shown more limitations in practice. For examples, the model would become problematic practically considering transaction cost, or when number of securities is large, or constraints are many; as the computational complexity increases.

Markowitz model tries to simplify the real word to more focus on theoretical aspect of portfolio optimization. However, in practice, many realistic constraints must be taken to consideration by portfolio managers in the process of portfolio creation and optimization. The important realistic constraints can be categorized into cardinality constraints, round-lot constraints, floor constraints and trading constraints (Skolpadungket 2007). These constraints might be placed based on the instructions given by portfolio investors or might be set for practical reasons such as transaction costs and execution efficiency (Jize Zhang 2018). Solving large-scale problem with these realistic constraints (e.g. minimum and maximum number of securities) is NP-hard (non-deterministic polynomial-time hard). Which means that finding the solution in a reasonable amount time will require methods other than mixed integer nonlinear programing and other deterministic approaches (Skolpadungket 2007) .

In this study we enhanced our model with cardinality constraint that restrain the number of stocks to be traded from a specific industry and also ceiling constraint that determine the minimum and maximum weight that can be held by a security in our portfolio.

3.2 Proposed Fitness Function

An alternative solution for this problem (cardinality constraints) would be using genetic algorithms for our project as we have several constraints such as integer variables and 50-days forward VaR on returns in cost function.

After selecting securities, we are required to find the optimum weights for securities in a way that they optimize our cost functions. To achieve this goal, Sharpe ratio which was first introduced by (Sharpe, The Sharpe Ratio 1994) is used in the objective function and also Monte-Carlo predicted portfolio 50 days VaR is incorporated into the objective function.

Maximize:

$$Objective=\left(\frac{R_p}{\sigma_{portfolio}}\right)+\frac{Portfolio\ VaR_{50days}}{10} \quad (3.5)$$

Or

$$Objective=\lambda \times R_p + (1\text{-}\lambda) \times PortfolioVaR_{50days} \quad (3.6)$$

Subject to:

$$\sum_{i=1}^{30} w_i=1 \quad (3.7)$$

$$\sum_{i=1}^{60} u_i=30 \quad (3.8)$$

$$w_i \leq 0.1\ ;\ \forall(i) \quad (3.9)$$

$$1\leq u_j \leq 5\ ;\ \forall(j) \quad (3.10)$$

$$0.1 \leq w_i \leq 1\ ;\ i= 1, 2, 3 ,\dots, 30 \quad (3.11)$$

Where R_p is the average daily return of the portfolio and λ is the weight given to each parameters in objective function. w_j is the weight of a i^{th} security in a portfolio which is a continuous variable that takes a value between 0.1 and 1. u_i is a binary variable that takes value 1 if i^{th} security (from 60 stocks) is included in 30-stock portfolio and takes 0 otherwise.

Since, VaR in this problem is the least value of the given portfolio return in 50 days with 0.05 probability our aim is to maximize this value.

3.2.1 Penalty Function

In an unconstrained optimization problem, genetic algorithm performs a search in specific regions of solution space in order to find promising solutions. However, in case of constrained optimization problem, every random point may not locate in feasible region since it might violate the equality or non-equality constraints. In order to make GA to search over feasible solution space, we need to convert them to equivalent unconstrained optimization problem. Application of penalty functions (exterior functions) are one of the common approaches for constraints handling. Penalty function is to quantify the amount of infeasibility, in other words, it measures how far the solution is from the feasible region. Thus, penalty function is zero if all constraints are satisfied in a solution (Michalewicz 1996).

In this study, for the purpose of transferring the above optimization problem to the unconstrained problem, we incorporated the constraints into the objective function using penalty functions. We changed both quality and non-equality constraints in the following way:

$$\sum_{i/1}^{30} w_i = 1 \rightarrow Min\left(\sum_{i/1}^{N} w_i = 1\right)^2 \qquad (3.12)$$

$$w_i \leq 0.1\ ;\ \forall(i) \rightarrow \quad \sum_1^{30}(Max\ (0,w_i\text{-}0.10\ ))^2 \quad (3.13)$$

$$1 \leq u_j \leq 5\ ;\ \forall(j) \rightarrow \quad \sum_1^{30}\big(Max\ (0,\ 1\text{-}u_j)\big)^2 \qquad (3.14)$$

Therefore, the objective function in case of weighted Sharpe-VaR is transformed to the following equation:

$Minimize$:

$Objective=$

$$\left(\frac{R_p}{\sigma_{portfolio}}\right)+\frac{Portfolio\,VaR_{50days}}{10}+$$

$$10^2\times\left[\left(\sum_{i/1}^{N} w_i = 1\right)^2\right]+\sum_{1}^{30}(Max\ (0,w_i\text{-}0.10\ ))^2\ +10\times\sum_{1}^{30}\big(Max\ (0,\ 1\text{-}u_{\mathrm{j}})\big)^2 \quad (3.16)$$

The coefficient of penalty functions is determined in tuning process, which is required in inclusion of each penalty function based on the value of the main cost function.

Chapter 4

4. The proposed Algorithm

4.1 Factor Analysis Approach

Based on "Arbitrage pricing theory", expected return of an investment security can be attributed to various macroeconomic factors (Ross, The arbitrage theory of capital asset pricing 2013). As the theory does not specify the factors (unlike CAPM), we can expand the model to include various factors, considering nature of market and industry. Macroeconomic factors (inflation, surprises, etc.), statistical factors (using principal components analysis), and fundamental factors (industry characteristics, valuation ratios, technical indicators, etc.) are main factors used currently in the market (Jennifer Bender 2013). Fama & French model was a very first model considering three factors (market, size, value) to connect the return of the stocks to these three factors (Eugene F. Fama 1992) which later other factors have been added to consider wider spectrum of elements affecting the return. Popularity of using fundamental factors to understand return/risk characteristics of a stock, leads to the creation of the multi-factor Barra risk models.

Factor investing is an approach to choose appropriate investments opportunities (undervalued securities) out of available securities utilizing factors explaining securities' risk and return for the long-term equity portfolio. There are several factors that have shown long-term risk premium and exposure to systematic source of risk such as value, growth, ESG, quality, leverage, momentum, sentiment, and etc. In this project, five of these factors have been chosen to evaluate stocks performance in each sub-industry of the S&P 500 index.

4.2 Security Selection Scoring System

Now, it is necessary to select securities from each sub-industry based on a scoring system. Our scoring system evaluate securities based on several selective factors including value, momentum, quality, ESG and leverage. These factors chose as our stocks are mainly from mature large companies. In the process of initial stock portfolio selections, after allocating scores to each security, those with highest score would be selected to be in our hypothetical portfolio.

In our approach, securities are firstly ranked in each industry based on their market capitalization size, and then the top 10 securities have been chosen from each sub-industry of the S&P 500, this has been done to reduce the size risk in the portfolio. However, the asset ranking has also been done using genetic algorithm by (Lai 2006), where financial indicators of assets including return on capital employed (ROCE), price/earnings ratio (P/E Ratio), earning per share (EPS) and liquidity ratio are encode into the genes and then quality of the asset are evaluated using genetic algorithm.

For purpose of ranking ten stocks in each sector, factor model analysis is applied considering five equity risk premium factors: Value, Momentum, Quality, Leverage, and ESG. The data used for factor analysis is based on the securities data on December 12, 2018. For each factor, three major components have been considered to be representative of the factor, and they have been given equal weight, for an example for value factor, three components are price-to-earning value ratio, dividend yield and ROIC spread (ROIC-WACC)

$$ValueScore = 33.33\% \, . \frac{P}{E} Score + 33.33\% \, .DividendYield + 33.33\% \, .ROICspreadScore \tag{4.1}$$

In the following, elements which have been used to score each factor are described.

Value	
Metric	**Explanation**
P/E ratio	It shows relative value of a share price to a company's earnings per share. It is representative of how market prices the company for $1 of its income. Lower PE ratio give some sense of cheaper price. To score this ratio, we used earning yield which is reverse of the ratio.
Dividend yield	It shows how many percentages of a stock paid as divided, higher yield means that shareholder's return is higher.
ROIC spread	Return on invested capital is an accounting term to calculate returns go back to investors, calculating as Net Operating Profit After Tax (NOPAT) divided by sum of debt and shareholder's equity. ROIC spread is excess of return after weighted average cost of capital of the capital (WACC), higher ROIC spread is better sense for investors.

Momentum	
Metric	**Explanation**
3 months return	It is an indicator of a stock return in a short period of last 3 months.
6 months return	It is an indicator of a stock return in last 6 months.
12 months return	It is an indicator of a stock return in the longer period of last 12 months. Considering all three returns, take the volatility into account as well.

Quality	
Metric	**Explanation**
ROE	Return on equity (ROE) represents the amount of income could be generated for $1 investment of a shareholder. Higher ROE gives shareholders the sign of better investment.
Free cash flow yield	It is one of the indicators of cash generation quality, calculating as free cash flow generated by the company divided by the stock price. The higher yield shows ability of the company in producing cash.
Asset turnover	Asset turnover, calculating as sales over total assets, giving us a picture of efficiency of total assets in making sales for a company, put another way, it shows how much a company can make revenue from $1 of its total assets. The higher quality of the assets, more sales can be supplied.

Leverage	
Metric	**Explanation**
Net interest coverage ratio	Coverage ratio is indicator of a company's ability to cover its interest payable to its creditors, calculating as EBIT (earnings before interest and tax) over net interest.
Net debt-to-EBITDA ratio	Net debt-to-EBITDA shows how long it takes to pay its current debts (after using cash to pay part of the debt) with the current operating income or EBITDA (earnings before interest, taxes, depreciation and amortization.
Total debt-to-equity	Total debt-to-total equity shows how leverage is a company, and how its capital structure is.

ESG	
Metric	**Explanation**
ESG disclosure	It is a rating out of 100. As ESG disclosure is not yet a mandatory requirement in behalf of regulatory, if a company has disclosed it and has efficient explanations and contribution, it gives advantage over others and can get higher score.
Independent director	It is a percentage of independent directors from the board, higher rate shows more independency.

Based on performance of each company on each factor and allocating appropriate weight to each factor, we score the company on each factor and rank companies. By doing the same steps for each factor and having suitable weight on each factor (shown in the formula below), finally the companies are ranked using weighted average of scores in each factor.

$$
\begin{aligned}
&TotalScore = \\
&25\% \, . \, ValueScore + 20\% \, . \, MomentumScore + 20\% \, . \, QualityValue + \\
&20\% \, . \, LeverageValue + 15\% \, . \, ESGValue
\end{aligned}
\tag{4.2}
$$

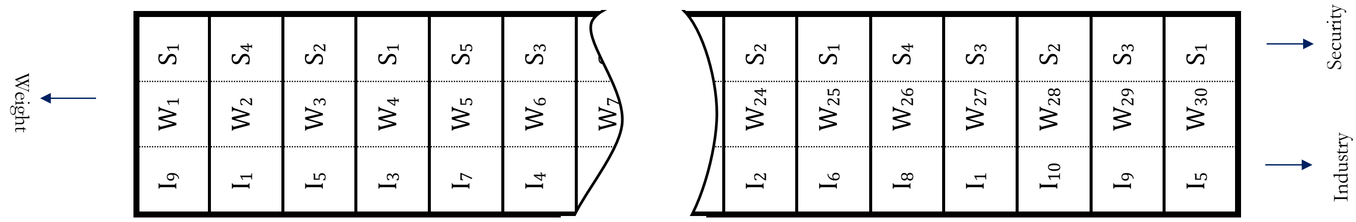
4.3 Proposed Genetic Algorithm

4.3.1 Solution Representation

The first step in implementation of genetic algorithm is to construct a proper solution representation in the form of a gene. In this section we explained our proposed solution representation which encoded every possible solution to the optimization problem as a chromosome, where sum of all allocated weight percentages is equal to one.

4.3.2 Proposed Solution Representation

In the following solution representation industry selection, security selection, and weight allocations are encoded into a chromosome. In this representation, every gene in every chromosome is represented by a triplet *(I, W, S)* where W denotes the weight of S^{th} security of industry I^{th} in our portfolio. I is the index of sub-industries in our portfolio which can take a binary number between [1 12] and S can also be a number between [1 5] which is the indicator of the security index in that particular sub-industry. In this study, we included 30 genes in each chromosome which can be interpreted as a portfolio of 30 stocks. This solution representation is inspired from (Defersha 2010) , however, it is used in completely different context.



4.3.3 Initial population generation:

The initial population consists of 250 chromosomes, where each chromosome represents a possible asset allocation solution.

For generating random binary numbers for industry index, we have generated binary random numbers using uniform distribution with the restriction that a particular industry sector index can be repeated for maximum five times.

Moreover, we have applied the same procedure for generating random numbers for security index, however; this time with no restriction on frequency of the index numbers.

For weight allocation to each gene in initial population, the following mean-variance optimization has been applied to find the optimum weights for each security, while the allocated weight to each security is limited to be 10%.

Minimize:

$$Objective\ \sigma_{portfolio} = \sqrt{W \times Covariance_{portfolio} \times W'} \quad (4.3)$$

Subject to:

$$r_p = \sum_{i=1}^{n} w_i r_i \quad (4.4)$$

$$\sum_{i=1}^{30} w_i = 1 \quad (4.5)$$

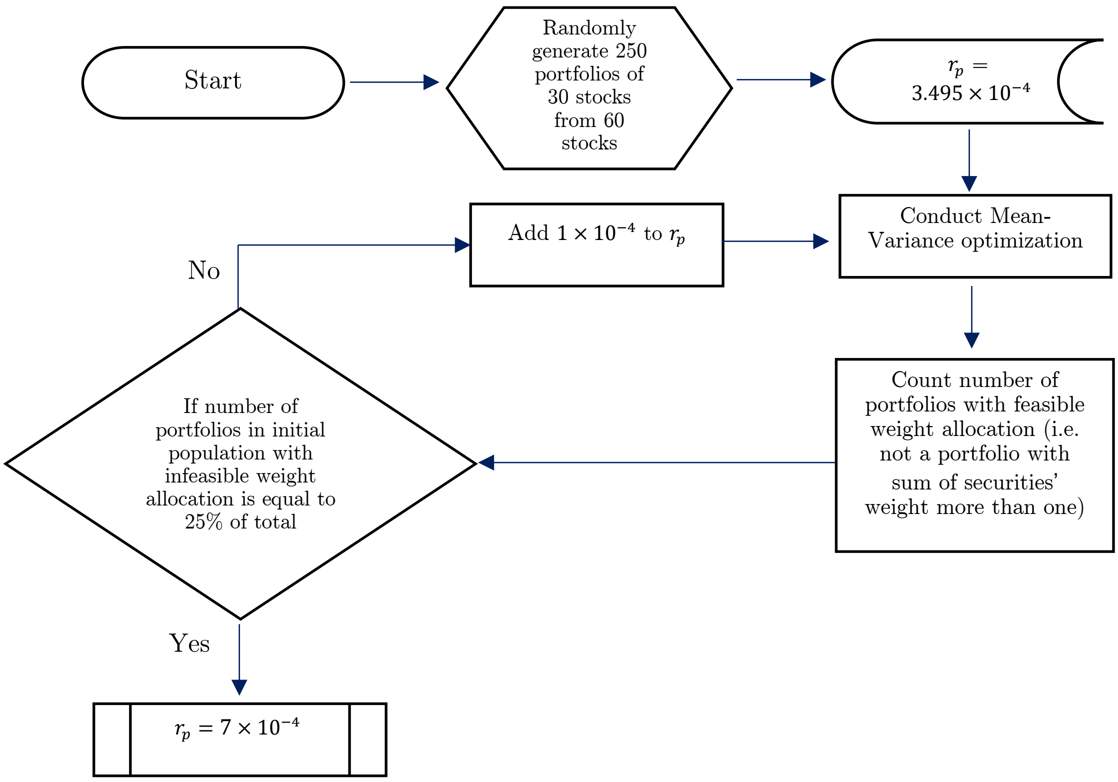
$$0.1 \leq w_i \leq 1\ ;\ i = 1, 2, 3, \ldots, 30 \quad (4.6)$$

Where W is the vector of portfolio security's weight. The main purpose of avoiding random weight allocation and determining weights optimally, is to dramatically improve convergence rate of proposed algorithm. However, the main issue is to determine the suitable amount of required return r_p as one of the mean-variance constraints. If a low number is chosen for r_p (e.g. The S&P 500 daily return, 3.495×10^{-4}), computational cost would increase due to slow convergence rate. On the other side, if we choose a high number as r_p (e.g. 9.0×10^{-4}), mean-variance optimization would not be able to find feasible solution for every chromosome (i.e. each portfolio created by random number assignment to stock and industry index) in initial population. To overcome this issue, mean-variance optimization is conducted several times on initial portfolios while each time r_p got a value from a range of possible portfolio daily return. If the r_p of chosen portfolio, which is a constraint for initial mean-variance optimization is not realistic, the mean-variance method may not be able to give optimum feasible weights for every portfolio security in initial population.

Each time, Mean-Variance optimization is executed with different r_p, number of portfolios in initial population (from total 250 of portfolios) where weight constraint (i.e. sum of securities weight in a portfolio must be equal to one) have been violated, have been counted.

In this study, we have searched for a r_p that Mean-Variance optimization be able to allocate feasible weights to at least 75% of portfolios in initial population. Evidently, if a low r_p is chosen Mean-Variance optimization is able to assign feasible weights to all portfolios in initial population, however; this initial population of portfolios will lead to a slow convergence rate since GA aims to increase return of portfolios as its objective function. On the other hand, if a high r_p is chosen, Mean-Variance optimization is able to find optimum feasible weights just for few portfolios from initial pool of portfolios. Although, these infeasible solutions gradually will be eliminated by GA evolution process, GA algorithm starts with less diversified pool of feasible portfolios might result in premature convergence of GA.

The main reason why this approach has been applied to construct initial pool of portfolios, is to improve the quality of initial population which greatly has improved our proposed GA convergence behavior. In this study, we have employed Mean-Variance optimization while we have restricted the required return constraint to be 0.0007. As a result, in constructing of our initial population process, the total weight of securities in only 66 out 250 portfolios, were greater than one. However, these changes with each random generation of 250 portfolios. In the following flowchart the initial population creation is depicted.



Initial population creation process flowchart

4.3.4 Selection Operators:

In our GA approach, we have applied k -way tournament selection operator, which was first introduced in (Goldberg 1989). The selection procedure has been done by holding tournament among k randomly selected chromosomes and choosing the best fitness as a winner of the competition. The copy of the winner chromosome is then copied into the mating pool, and all chromosomes participated in the tournament will be returned into initial population. The procedure continues until the size of the mating pool reach to the current population size.

4.3.5 Crossover Operators:

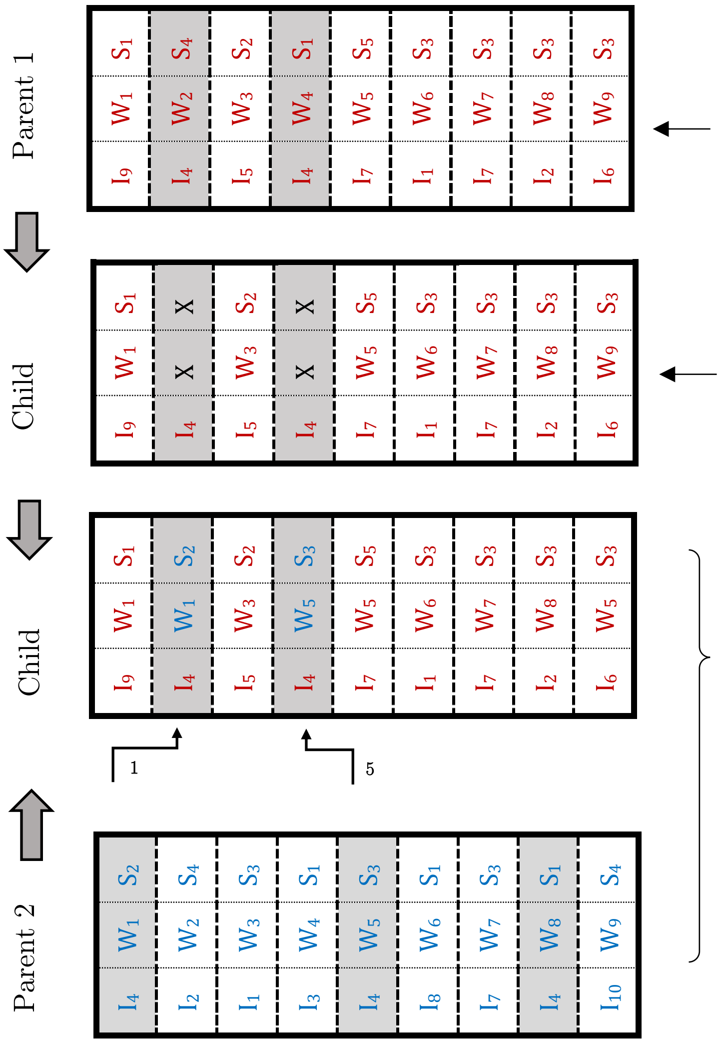
After the selection of chromosomes for mating pool, for the purpose of evolution and enriching the population with chromosomes with better fitness, crossover operators have been applied. The crossover operator for our proposed algorithm can be categorized as industry sector crossover operator.

4.3.5.1 Industry Crossover Operator

In this method, the sub-industry allocation information of parent 1 will be preserved by far in the new offspring. This operator, initially would select industry (e.g. Capital goods) randomly from one parent, then based on the number of associated genes with this sub-industry in the other parent, there would be two possible scenarios:

- Scenario A: when number of associated genes to the selected industry sector in both parents are equal
    - In this case, simply the associated genes are exchanged between two parents
- Scenario B: when number of associated genes to the selected industry sector is not equal in both parents

Security crossover operator in Scenario B



Step 1:

Selecting the sub-industry index (here, I4) randomly and then choosing the parent with lower number of associated gene to the selected industry index as parent 1

Step 2:

Copy all the genetic information of parent 1 to the new offspring except the security and weight attribution of genes associated with the selected industry index.

Step 3:

Final step is to copy security and allocated weight properties of genes with same industry index in parent 2 to offspring starting from first gene (here, gene 1 and 5) until the new offspring until all the empty places in the offspring is filled.

4.3.6 Mutation Operator

For the purpose of maintaining diversity in the mating population and also in order to avoid being trapped in local minima, we have needed to perform mutation with low pre-specified probability on the offspring resulted from crossover operation. This would help us to preserve diversity by exploring the whole solution space.

In this study, three different mutation operators have been used:

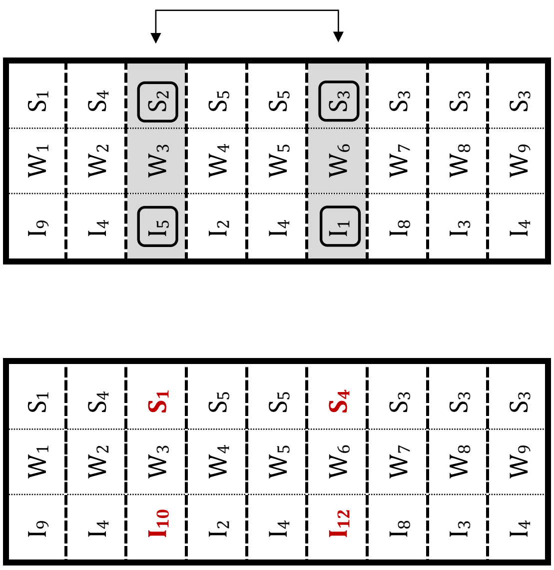
4.3.6.1 New portfolio mutation operator

A chromosome in a population may undergo new portfolio mutation with a low probability. The role of this operator is to create is to create a new portfolio by assigning random numbers for security and industry index to each gene while randomly allocation weight to each security using following formula:

$$Weight\ allocation = \begin{cases} \alpha_i = Uniform\ Random\ Number\ in\ [0,1]\ interval\ \ i=1,2,3,\ldots,30 & (4.7) \\ w_i = \dfrac{\alpha_i}{\sum_{i=1}^{30}\alpha_i}\ \ i=1,2,3,\ldots,30 & (4.8) \end{cases}$$

4.3.6.2 Security-Industry mutation operator

Security-Industry mutation operator is applied with low probability on few chromosomes. The role of security-Industry mutation operator is to alter security and sub-industry properties of 10 randomly selected genes in a chromosome.



Step 1:

Selecting 10 genes randomly.

Step 2:

Random integer numbers are assigned to the security and industry indexes in a way that to avoid any identical genes.

4.3.6.3 Weight mutation operator

This operator is also applied with low probability of few chromosomes in a population. This operator is responsible to randomly change allocated weights of all securities in a chromosome. The method for allocating new weights is similar to the equation explained in 4.3.6.1.

4.4 Steps of the Proposed approach

1. Choosing 10 securities with highest market cap in each industry from the S&P 500 index (total of 120 stocks)
2. **First refinery process:** Selecting the best 5 stocks in each sector using factor analysis (total of 60 stocks)
3. Creating initial population of 250 chromosomes, each contains a portfolio of 30 stocks created from selected 60 stocks
   a. Randomly assign industry and security index to each gene
   b. **Second refinery process:** Find optimum weight allocation for each created portfolio in step 3.a using Mean-Variance method optimization
4. Calculate fitness value of chromosomes in the initial population
   a. Applying 30-tornoument selection operator to create mating pool
   b. Applying industry crossover operator explained in **4.3.5.1** with probability of 0.70
   c. Applying new portfolio mutation operator explained in **4.3.6.1** with very small probability (0.25) in order to avoid unnecessary delay in convergence to optimal solution
   d. Applying security-industry mutation operator explained in **4.3.6.2** with probability of 0.5 in order to avoid being trapped in local minima and preserve population diversification.
   e. Applying weight mutation operator explained in **4.3.6.3** with probability of 0.15.
   f. Simulate 50 days forward portfolio path for each chromosome using Monte Carlo simulation
   g. Find 50- days VaR on portfolio return with 95% confidence level is
   h. Calculate fitness value of each chromosome
5. Repeat Steps “a” to “h” for 3500 generations to find the best solution

Following pseudocode is presented in order to clarify the approach mechanism.

Proposed Algorithm Pseudocode

```
1: for N˙C (1) ⟶ N˙C (Maximum Number of Chromosomes=250) do
2:     while there is any two identical genes N˙C (1) continue
3:         if it is a first row then assign random integer numbers between 1 to 5 for stock index
4:         if it is a third row then assign random integer numbers 1 to 12 for industry index
5:         end while
6:         if it is a second row then find optimized weight by executing mean-variance optimization
while the required return is restricted to 7×10^-4 for daily return.
7: end for                                                        Initial Population Creation
--------------------------------------------------------------------------------------------
8: for g (1) ⟶ N˙C (Maximum Number of Generations=3500) do
9:         for N˙C (1) ⟶ N˙C (Maximum Number of Chromosomes=250) do
10:                Extract stocks, associated weights and industry index from Chromosome
11:                Calculate Fitness function of Chromosome
12:        end for                                                Fitness Function Calculation
--------------------------------------------------------------------------------------------
13:        for N˙C (1) ⟶ N˙C (Maximum Number of Chromosomes=250) do
14:                for t (1) ⟶t (Maximum Number selected for Tournament=30)
15:                        Choose the best chromosome in terms of Fitness
16:                        Place the winner chromosome in the mating pool
17:                end for
18:        end for                                                Selection Operation
--------------------------------------------------------------------------------------------
19:        for C˙R (1) ⟶ C˙R (Maximum Number of Chromosomes=250) do
20:                Randomly choose two chromosomes from mating pool
21:                Randomly select one industry index and count number of associated genes in
each chromosome
22:                if random number is less than Security Crossover Probability (0.70) then
23:                    Operate Security Crossover in Section 4.3.5.1 and create two new
chromosomes
24:                end if
25:        end for                                                Crossover Operation
--------------------------------------------------------------------------------------------
26:        for N˙C (1) ⟶ N˙C (Maximum Number of Chromosomes=250) do
27:                if random number is less than New portfolio mutation Probability (0.25) then
28:                        Create a random chromosome using the approach explained in 4.3.6.1
and replace it with current chromosome
29:                end if
30:                if random number is between New portfolio mutation (0.25) and
31:                Security-Industry mutation (0.75) then
32:                        while there is any two identical genes N˙C (1) continue
33:                        Randomly change the industry index of 10 genes as explained in 4.3.6.2
34:                        end while
35:                end if
36:                if random number is greater than Security-Industry mutation (0.75) and
37:                weight mutation probability (0.9) then
38:                        Change weights of chromosome using the approach explained in 4.3.6.3
39:                end if
40:        end for
41: end for
                                                                  Mutation Operation
```

Chapter 5

5. Results

5.1 Analysis of constructed portfolios

5.1.1 Portfolio Sharpe ratio – VaR as fitness function (10SHV)

In the following table, the selected equities and their corresponding weights obtained by two methods are presented:

- The following results are obtained as weighted Sharpe-ratio VaR is considered as objective function:
  - $Minimize\ Objective function = -\ SharpeRatio - \frac{\left(Portfolio\ return\ VaR_{50days}\right)}{10}$ (5.1)
  - In the GA column, securities selection and their optimal weight by our proposed method is presented
- In the MV column, allocated optimal weights to 60 stocks obtained by mean-variance optimization are presented. As stated before, these 60 stocks are selected by refining the S&P 500 index using factor analysis.

Fitness Function: Weighted Sharpe ratio- VaR			
Equity	**GA**	**MV**	**Sub Industry Sector**
HD UN Equity	1.4919%	0.8616%	Retailing
LOW UN Equity		0.7409%	
TJX UN Equity	0.5355%	0.7637%	
TGT UN Equity		0.4571%	
ROST UW Equity	0.5641%	0.7562%	
MSFT UW Equity	6.2521%	2.5786%	Software
V UN Equity	8.3609%	2.6303%	
MA UN Equity	6.7900%	7.2785%	
IBM UN Equity		0.3720%	
ACN UN Equity		0.8401%	
AAPL UW Equity	1.2482%	1.2419%	Tech & Hardware
TEL UN Equity		0.5691%	
APH UN Equity		0.7940%	
HPQ UN Equity	0.8939%	1.0814%	
HPE UN Equity		0.5821%	

UNH UN Equity	7.7244%	2.3094%	Healthcare
DHR UN Equity		0.8974%	
CVS UN Equity		0.4890%	
ANTM UN Equity	9.3040%	9.9657%	
CI UN Equity	8.2159%	1.5814%	
SBUX UW Equity	0.2685%	0.6815%	Consumer Goods
LVS UN Equity		0.5061%	
HLT UN Equity	1.2482%	1.0057%	
RCL UN Equity		0.7391%	
CMG UN Equity	0.5464%	0.8236%	
NVDA UW Equity		1.4190%	Semiconductor
MU UW Equity	0.2685%	2.1430%	
AMAT UW Equity		0.5506%	
LRCX UW Equity	0.4403%	0.8312%	
AMD UW Equity	0.3466%	2.5717%	
JNJ UN Equity	3.6760%	0.8881%	Pharma and Biotechnology
AMGN UW Equity	0.7544%	0.9905%	
ABBV UN Equity	0.1366%	1.1131%	
BMY UN Equity		0.5020%	
GILD UW Equity		0.4835%	
UPS UN Equity		0.4472%	Transportation
NSC UN Equity	0.5464%	1.0485%	
LUV UN Equity		0.5743%	
KSU UN Equity		0.6783%	
AAL UW Equity		0.3505%	
PG UN Equity	2.3389%	0.6804%	Household and Personal Goods
EL UN Equity	9.0097%	3.9988%	
CL UN Equity		0.5420%	
KMB UN Equity	0.5464%	0.5801%	
CLX UN Equity	8.9350%	1.3155%	
BA UN Equity	8.8025%	30.0442%	Capital Goods
UTX UN Equity		0.6090%	
LMT UN Equity	1.2482%	0.6950%	
GE UN Equity		0.1650%	
RTN UN Equity	0.6231%	0.7364%	
GS UN Equity		0.3726%	Diversified Financials
MS UN Equity		0.4838%	
BLK UN Equity		0.5194%	
SPGI UN Equity	6.7900%	1.2124%	
COF UN Equity		0.4714%	
PM UN Equity		0.5133%	Food, Beverage and Tobacco
MO UN Equity		0.4271%	
KHC UW Equity		0.3032%	
GIS UN Equity		0.3212%	
BF/B UN Equity	0.7544%	0.8713%	

5.1.2 Portfolio Sharpe ratio – VaR as fitness function (20SHV)

In the following table, the selected equities and their corresponding weights which are obtained by two methods are presented:

- The following results are obtained as weighted Sharpe-ratio VaR is considered as objective function:
  - $$Minimize\ Objective function\text{=-}\ SharpeRatio - \frac{\left(Portfolio\ return\ VaR_{50days}\right)}{20} \quad (5.2)$$
- In the GA column, securities selected and their optimal weight by our proposed method is presented
- In the MV column, allocated optimal weights to 60 stocks obtained by mean-variance optimization are presented. As stated before, these 60 stocks are selected by refining the S&P 500 index using factor analysis.

Fitness Function: Weighted Sharpe ratio- VaR

Equity	GA	MV	Sub Industry Sector
HD UN Equity	1.2482%	0.8118%	Retailing
LOW UN Equity		0.6967%	
TJX UN Equity	0.1527%	0.7177%	
TGT UN Equity		0.4289%	
ROST UW Equity	1.2482%	0.7115%	
MSFT UW Equity	7.3378%	2.4902%	Software
V UN Equity	8.2914%	2.5308%	
MA UN Equity	8.5773%	7.2696%	
IBM UN Equity		0.3480%	
ACN UN Equity		0.7933%	
AAPL UW Equity	2.3389%	1.1831%	Tech & Hardware
TEL UN Equity		0.5360%	
APH UN Equity		0.7493%	
HPQ UN Equity	0.5825%	1.0205%	
HPE UN Equity		0.5488%	
UNH UN Equity	8.1673%	2.1893%	Healthcare
DHR UN Equity	1.2482%	0.8433%	
CVS UN Equity		0.4581%	
ANTM UN Equity	9.2274%	9.9660%	
CI UN Equity	6.7900%	1.4865%	
SBUX UW Equity	1.1906%	0.6371%	Consumer Goods
LVS UN Equity		0.4759%	
HLT UN Equity		0.9427%	
RCL UN Equity		0.6957%	
CMG UN Equity	2.2836%	0.7769%	
NVDA UW Equity		1.3668%	Semiconductor
MU UW Equity	1.2366%	2.0722%	

AMAT UW Equity		0.5185%	
LRCX UW Equity		0.7908%	
AMD UW Equity	0.8939%	2.5537%	
JNJ UN Equity	3.9122%	0.8328%	
AMGN UW Equity	0.1527%	0.9287%	
ABBV UN Equity	1.1906%	1.0515%	Pharma and Biotechnology
BMY UN Equity		0.4695%	
GILD UW Equity		0.4541%	
UPS UN Equity		0.4204%	
NSC UN Equity	1.6175%	0.9848%	
LUV UN Equity		0.5407%	Transportation
KSU UN Equity		0.6354%	
AAL UW Equity		0.3304%	
PG UN Equity	1.1906%	0.6351%	
EL UN Equity	9.0097%	3.8014%	Household and Personal Goods
CL UN Equity	0.4760%	0.5082%	
KMB UN Equity		0.5457%	
CLX UN Equity	7.7244%	1.2364%	
BA UN Equity	8.9350%	32.7741%	
UTX UN Equity		0.5727%	
LMT UN Equity		0.6498%	Capital Goods
GE UN Equity		0.1546%	
RTN UN Equity	0.4760%	0.6906%	
GS UN Equity		0.3489%	
MS UN Equity		0.4567%	
BLK UN Equity		0.4882%	Diversified Financials
SPGI UN Equity	3.1993%	1.1518%	
COF UN Equity		0.4437%	
PM UN Equity	0.1527%	0.4795%	
MO UN Equity	0.1785%	0.4006%	
KHC UW Equity		0.2841%	Food, Beverage and Tobacco
GIS UN Equity		0.3006%	
BF/B UN Equity	0.8074%	0.8195%	

5.1.3 Portfolio weighted Mean-VaR as fitness function (10MEV)

In the following table, the selected equities and their corresponding weights which are obtained by two methods are presented (with more weight given to the mean):

- The following results are obtained as weighted Sharpe-ratio VaR is considered as objective function:
  - $$Minimize\ Objective function = -\ 10 \times mean - \left(Portfolio\ return\ VaR_{50days}\right) \quad (5.3)$$
- In the GA column, securities selected and their optimal weight by our proposed method is presented
- In the MV column, allocated optimal weights to 60 stocks obtained by mean-variance optimization are presented. As stated before, these 60 stocks are selected by refining the S&P 500 index using factor analysis.

Fitness Function: Weighted Mean- VaR

Equity	GA	MV	Sub Industry Sector
HD UN Equity	0.5464%	0.7148%	Retailing
TJX US Equity		0.5694%	
TGT US Equity		0.6456%	
ROST US Equity		0.3591%	
EBAY US Equity		0.6155%	
MSFT US Equity	6.2521%	2.3934%	Software
V US Equity	8.3609%	2.5304%	
MA US Equity	4.2344%	10.0852%	
IBM US Equity		0.2685%	
CAN US Equity	1.5900%	0.6678%	
AAPL US Equity	4.7551%	1.0382%	Tech & Hardware
APH US Equity	0.1453%	0.4342%	
KEYS UN Equity	0.7544%	0.6371%	
HPQ US Equity	3.8464%	0.8099%	
HPE US Equity		0.4454%	
UNH US Equity	8.2159%	2.2251%	Healthcare
MDT US Equity		0.7325%	
CVS US Equity		0.3699%	
ANTM US Equity	9.3040%	24.4975%	
CI UN Equity	6.8915%	1.2212%	
LVS UN Equity		0.5544%	Consumer Goods
YUM US Equity		0.3732%	
CCL US Equity	0.5464%	0.8048%	
HLT US Equity	0.3466%	0.5386%	
CMG US Equity		0.7190%	
INTC US Equity	2.2904%	0.9059%	Semiconductor
QCOM US Equity	3.6760%	1.3261%	
MU US Equity		0.3677%	

AMAT US Equity	1.4919%	0.5306%	
LRCX US Equity	9.1502%	1.2063%	
JNJ US Equity	0.6231%	0.7829%	Pharma and Biotechnology
AMGN US Equity	0.6298%	0.8050%	
ABBV US Equity	1.8073%	0.8955%	
BMY US Equity	0.3466%	0.3853%	
CELG US Equity		0.3673%	
UPS US Equity	0.5438%	0.3526%	Transportation
DELTA TB Equity		0.7665%	
LUV US Equity		0.4515%	
UAL US Equity		0.5120%	
KSU US Equity		0.2399%	
PG UN Equity	0.3466%	0.5767%	Household and Personal Goods
EL UN Equity	9.0735%	5.1256%	
CL UN Equity		0.4359%	
KMB US Equity	1.2167%	0.4908%	
CLX US Equity	3.9316%	1.3348%	
HON US Equity	7.9477%	21.8461%	Capital Goods
LMT US Equity		0.4808%	
CAT US Equity		0.5357%	
RTN US Equity		0.1181%	
DE US Equity		0.5738%	
AXP US Equity		0.2587%	Diversified Financials
MS UN Equity	0.4237%	0.3588%	
BLK UN Equity		0.3767%	
SPGI US Equity		1.0246%	
COF US Equity		0.3627%	
PEP UW Equity		0.4010%	Food, Beverage and Tobacco
PM UN Equity		0.3386%	
MO UN Equity		0.2295%	
GIS UN Equity		0.2489%	
BF/B US Equity	0.6309%	0.7361%	

5.2 Results comparison

In the following table, the results of proposed GA algorithm with different objective functions and Mean-Variance optimization are compared. These results are achieved based on the in-sample data of securities from December 11, 2016 to December 12, 2018. Also, performance of S&P500 index based on different metrics are presented. GA algorithm is first executed for each defined objective function, and the return of best portfolio generated by GA approach is considered as required level of return for the portfolio constructed by Mean-Variance optimization to compare.

* Required level of return for the portfolio constructed by Mean-Variance optimization

** Risk free rate is assumed to be equal to zero

IN-Sample Results — December 11, 2016 to December 12, 2018							
Objective Function	10.Sharpe-VaR (SHV)		20.Sharpe-VaR (20SHV)		10.Mean-VaR (10MEV)		S&P 500 Performance
Method Used	Genetic Algorithm	Mean-Variance	Genetic Algorithm	Mean-Variance	Genetic Algorithm	Mean-Variance	
Annual Return	27.04%	27.04%	27.87%	27.87%	**29.09%**	29.09%	8.36%
Annual Volatility	**12.73%**	15.36%	13.17%	15.62%	16.27%	14.62%	12.00%
$\mathrm{VaR}_{0.95}^{\mathrm{Return}}$ Annual (*Variance − covariance*)	21.01%	25.34%	21.73%	25.78%	**26.84%**	24.12%	19.80%
Sharpe Ratio**	**2.1239**	1.7607	2.1162	1.7835	1.6173	1.1631	0.6969

In the following, the performance of the constructed portfolio by proposed GA algorithm with different objective functions and Mean-Variance optimization are evaluated on the out-sample data from December 13, 2018 to December 12, 2019.

OUT-Sample Results — December 13, 2018 to December 12, 2019							
Objective Function	10.Sharpe-VaR (SHV)		20.Sharpe-VaR (20SHV)		10.Mean-VaR (10MEV)		S&P 500 Performance
Method Used	Genetic Algorithm	Mean-Variance	Genetic Algorithm	Mean-Variance	Genetic Algorithm	Mean-Variance	
Annual Return	18.79%	20.56%	20.13%	20.01%	**26.21%**	18.07%	19.54%
Annual Volatility	**15.14%**	17.89%	15.64%	18.16%	18.90%	17.37%	14.34%
VaR	4.547%	6.561%	**4.255%**	5.651%	5.369%	7.866%	-
Sharpe Ratio**	1.3933	1.2879	1.4491	1.2360	**1.6173**	1.1631	1.3629
Portfolio Beta	**0.8560**	1.1319	0.8959	1.1375	1.1886	1.0251	1.00
Jensen's alpha	0.2464	0.1581	0.2530	0.1973	**0.2572**	0.1970	0.1954
Holding Period Return (251 Days)	21.09%	23.04%	22.67%	22.45%	**30.566%**	20.198%	19.54%

5.2.1 Average annual return

The average annual returns of **10SHV, 20SHV** and **10MEV** constructed portfolios by proposed GA, which are calculated using following formula, are **18.79%, 20.13%** and **26.21%** respectively.

$$AAR = e^{(PortfolioDailyReturn \times 251)} - 1 \qquad (5.4)$$

These annual returns are calculated based on out-sample prices of securities between **December 13, 2018** to **December 12, 2019**. Two of GA constructed portfolios provided reasonable active return comparing with **19.54%** annual return S&P 500 index for the same period. However, the 10MEV portfolio has higher annual return than 10SHV and 20SHV portfolios, due to the higher weight given to mean return in cost function.

5.2.2 Holding Period Return (251 Days)

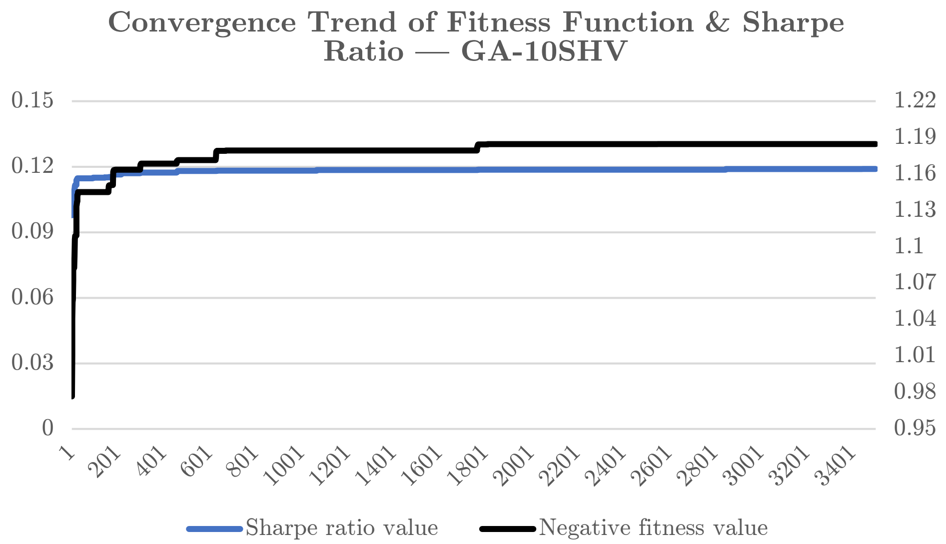
The holding period return of GA-10SHV, GA-20SHV and GA-10MEV constructed portfolios by proposed GA are 21.09%, 22.67% and 30.566% respectively.

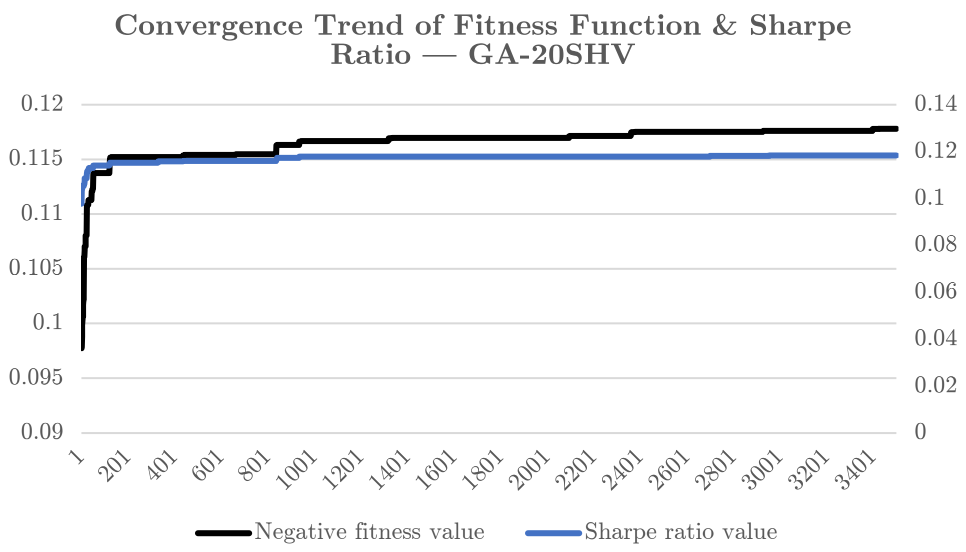
These holding period return is between December 13, 2018 to December 12, 2019. While, the holding period return of GA-portfolios were higher than S&P500 index and portfolios built by mean-variance, evidently GA-10MEV provided the highest holding period return among all portfolios.

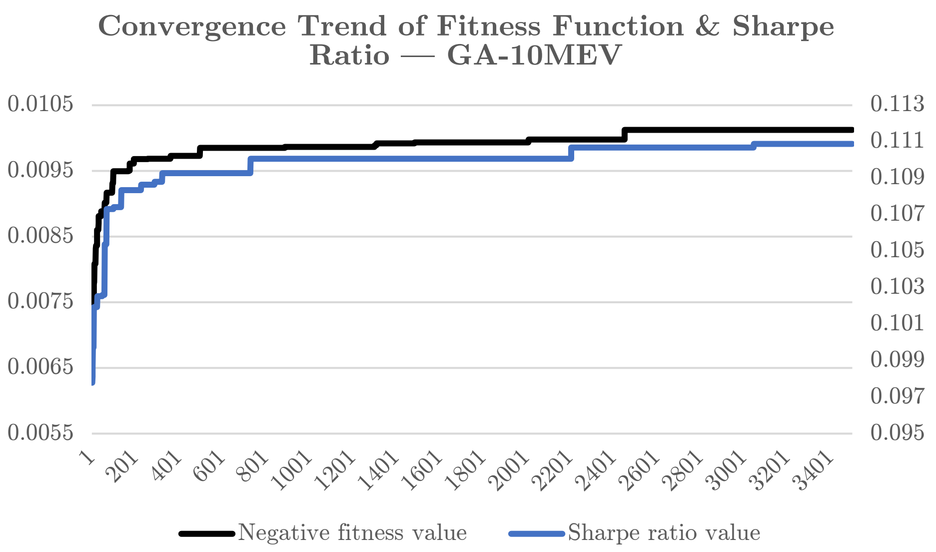
5.2.3 Sharpe ratio of portfolio

Sharpe ratio is the greatest contributor to fitness function in GA-10SHV and GA-20SHV algorithms. While, VaR amount of portfolio affects the fitness function value, more weight is given to Sharpe ratio in GA-10 SHV and GA-20SHV approaches. Interestingly, although, Sharpe ratio is not considered in GA-10MEV approach, generated portfolio by this method has the highest Sharpe ratio among all the portfolios.

As Sharpe ratio is a tool to gauge the risk-adjusted performance of a portfolio, Sharpe ratio of GA constructed portfolios are compared with Sharpe ratio of S&P index which is 1.3629. The higher Sharpe ratio of GA constructed portfolios comparing with both S&P index and Mean-variance- portfolio's Sharpe ratio confirms the fact that proposed algorithm with each objective function is able to provide higher risk-adjusted return than portfolios created by conventional methods.





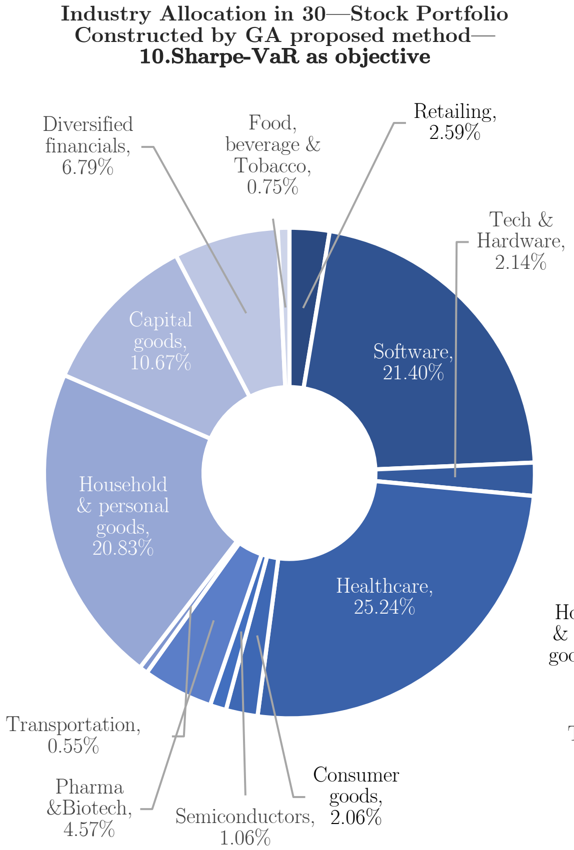


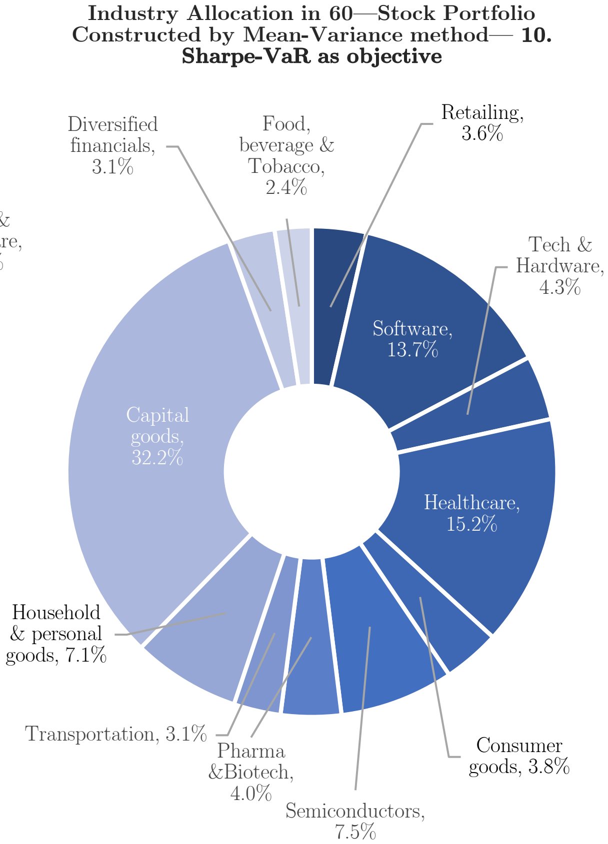
5.2.4 Beta of portfolio

The beta of portfolio, which is the weighted sum of securities betas, is a tool for volatility measurement of portfolio relative to the market changes. In another word, it evaluates the portfolio systematic risk. The beta of constructed portfolios by GA-10SHV, GA-20SHV and GA-10MEV methods in this study, are 0.8560, 0.8959 and 1.1886 respectively. This indicates the constructed portfolios have lower systematic risk than entire market. Additionally, the lower beta of GA-constructed portfolios than beta of Mean-variance constructed portfolios proves that the proposed algorithm in this paper, is effective in lowering portfolio exposure to systematic risk and diversifying asset allocation.

5.2.5 Portfolio Volatility

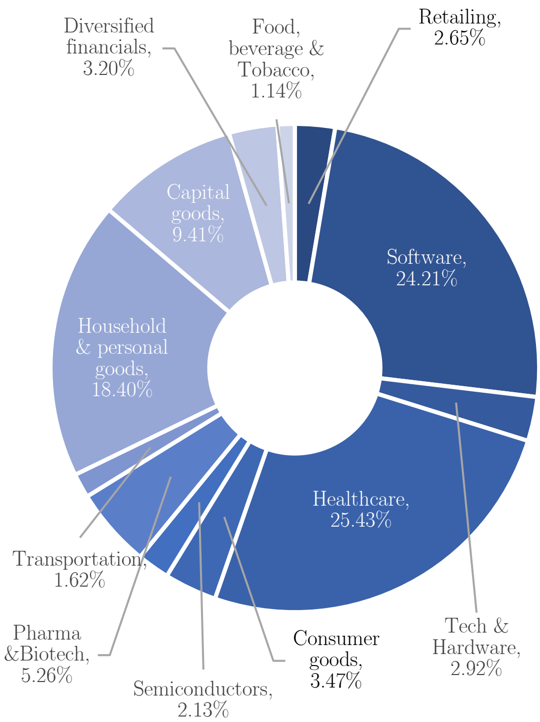
In order to validate the robustness of our algorithm, we compared the volatility of our GA constructed portfolios with 60-stocks portfolios optimized by mean-variance method. As mentioned earlier, initial pool of 60 stocks that we obtained after refining the S&P 500 securities by factor analysis, are used by our proposed algorithm to construct a 30-stocks portfolio of assets with different objective functions. We applied mean-variance approach on 60 stocks portfolio, while restricting the required return to the best daily return obtained from GA approaches with different objective functions.



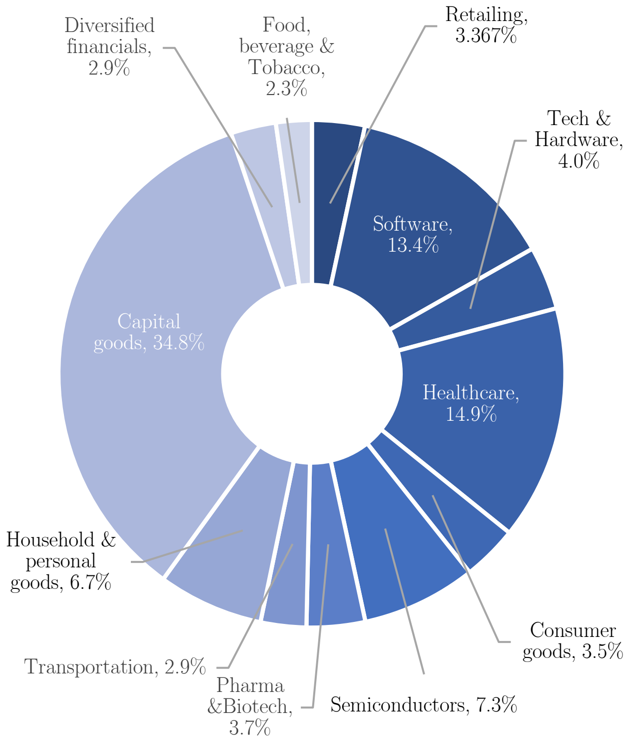


Evidently, portfolio constructed by GA-10MEV approach is riskier than GA-10SHV and GA-20SHV approaches since less weight is given to volatility in cost function of GA-10MEV approach. On the other hand, the results show that GA-10SHV and GA-20SHV approaches achieved significantly less volatile portfolios than MV-10SHV and MV-20SHV portfolios. The annual volatility of 60-stock portfolio obtained by MV-10SHV was 17.89% while the volatility of our 30-stock GA-10SHV portfolio was 15.14%, the lowest volatility among all portfolios. Clearly, this result indicates that optimized portfolio by mean-variance can provide the higher return as our constructed portfolio but in the expense of increasing portfolio risk significantly. Additionally, GA-10MEV was unable to provide less risky portfolio than MV-10 MEV. In the above and following figures, the industry allocation distribution of GA-constructed versus MV-constructed portfolios are illustrated.

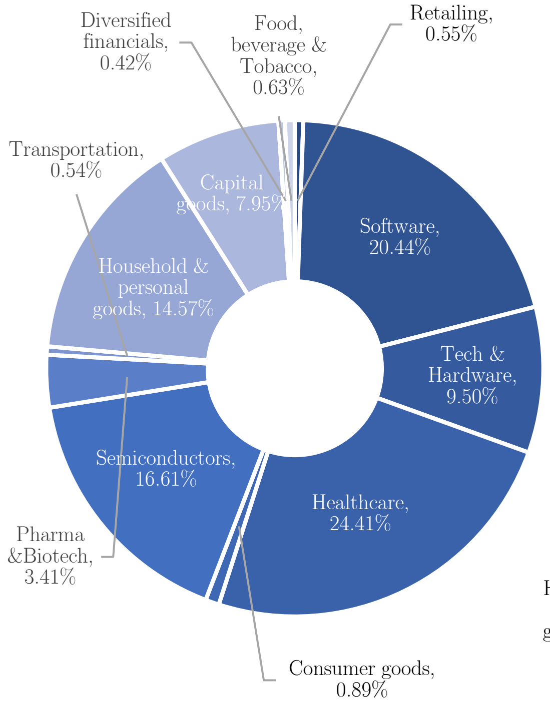
Industry Allocation in 30—Stock Portfolio Constructed by GA proposed method— 20.Sharpe VaR as objective



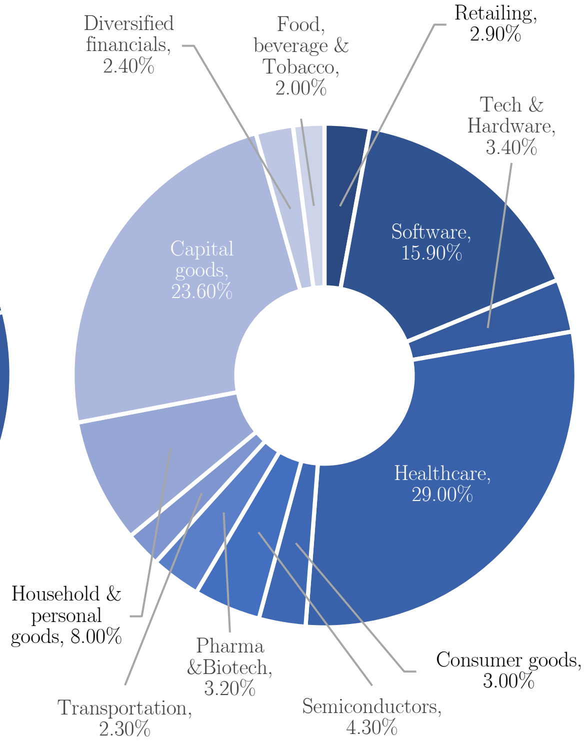
Industry Allocation in 60—Stock Portfolio Constructed by Mean - Variance Method — 20.Sharpe VaR as objective



Industry Allocation in 30—Stock Portfolio Constructed by GA proposed Method — 10.Mean-VaR as objective

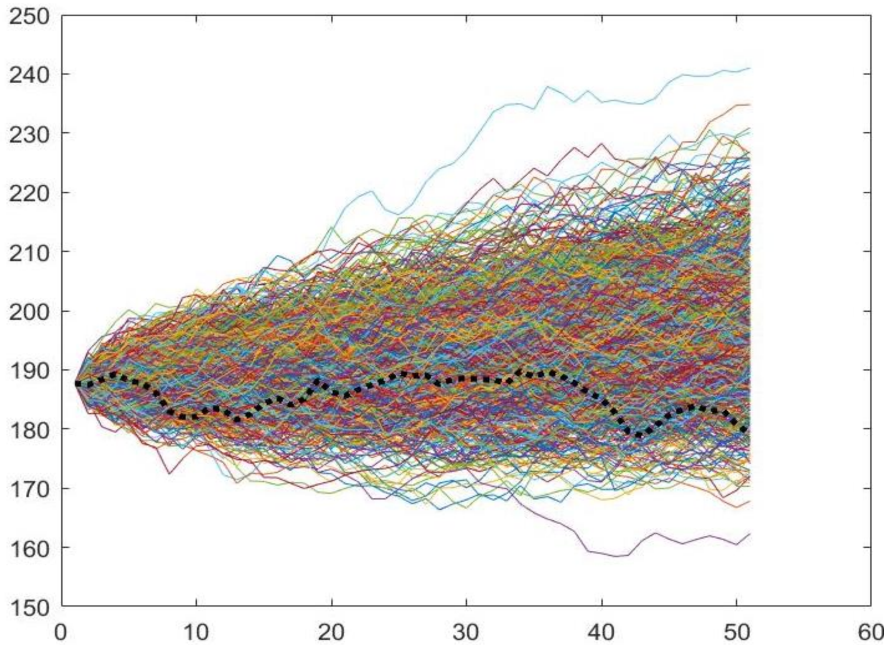


Industry Allocation in 60—Stock Portfolio Constructed by Mean - Variance Method — 10.Mean-VaR as objective

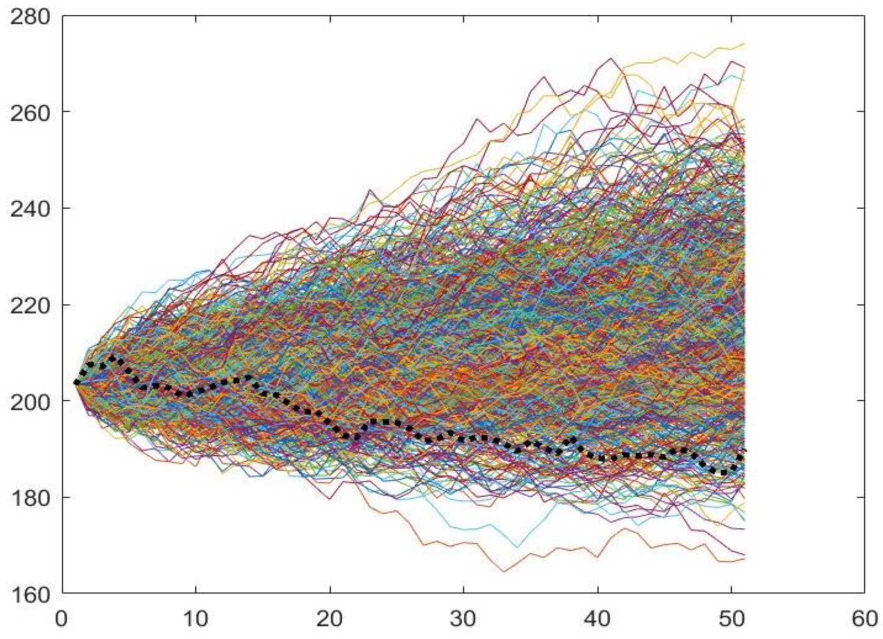


5.2.6 Portfolio Value at Risk

For the purpose of demonstrating the improvement attained in portfolio's Value at Risk (VaR) using our proposed algorithm, 50-days VaR on return of GA-constructed portfolios are compared with 50-days VaR on return of Mean-Variance constructed portfolios. 50 days forward path of assets are simulated using Monte Carlo simulation by assuming that securities prices evolve over time according to a Geometric Brownian Motion process. After executing the simulation, the GA-10SHV portfolio VaR on return with 95% confidence level was **0.0455** while MV-10SHV portfolio had **0.0656.** This indicate that GA vas successful at optimizing VaR of portfolio. In the following the 50-days portfolio price path simulation is illustrated.

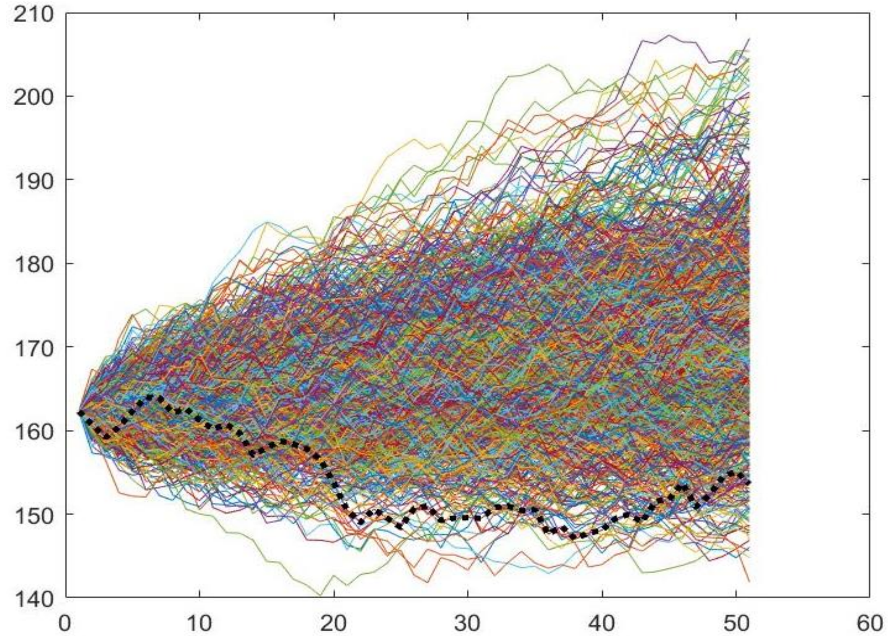


10SHV-Proposed GA 50-days portfolio price path simulation. Portfolio VaR is depicted using dotted black line

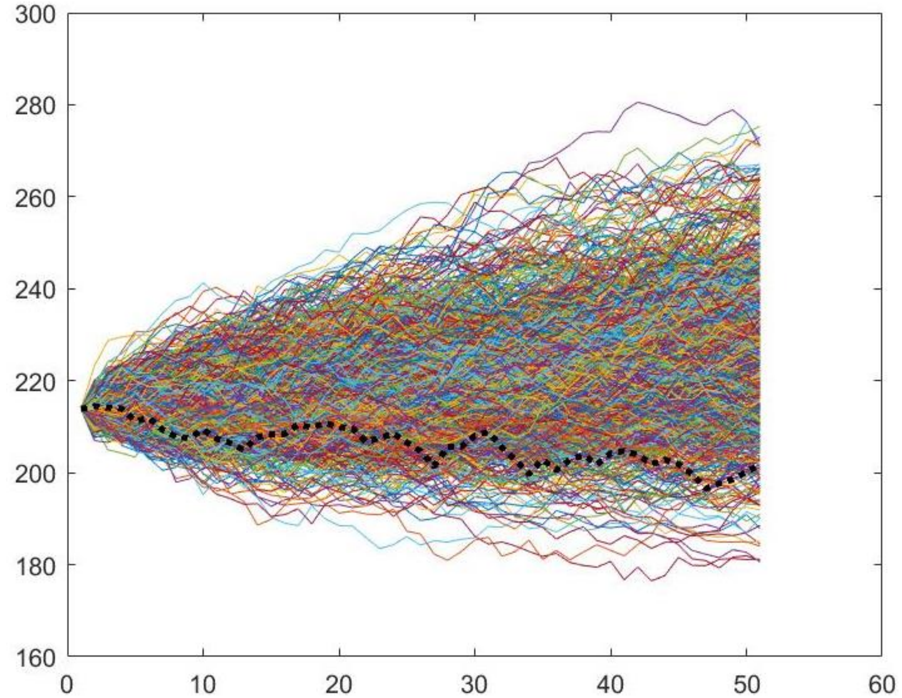


10SHV-mean variance 50-days portfolio price path simulation. Portfolio VaR is depicted using dotted black line

Also, the VaR on 50-days return with 95% confidence level for both GA-10MEV and MV-10MEV are compared. As illustrated in below, the proposed method in this study was successful in improving portfolio VaR on 50-days return from **0.0537** which is obtained by Mean-variance approach to **0.0565**.

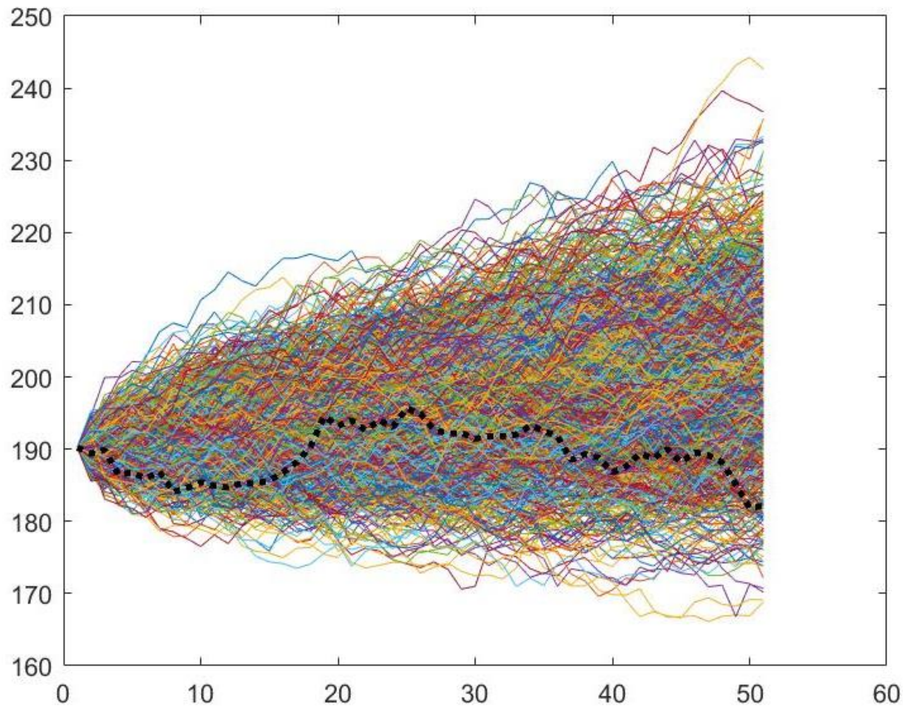


10MEV-Proposed GA 50-days portfolio price path simulation. Portfolio VaR is depicted using dotted black line

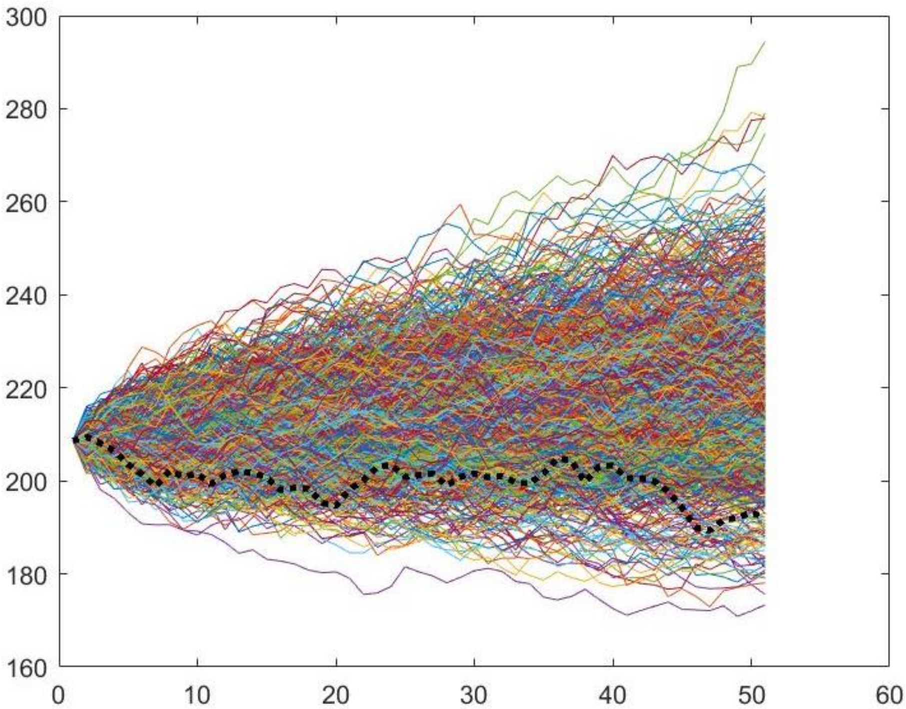


10MEV- mean variance 50-days portfolio price path simulation. Portfolio VaR is depicted using dotted black line

Also in the following, the portfolio VaR on return of both GA-20SHV and MV-20SHV which are **0.0425** and **0.0787** is depicted. The improvement obtained by GA method in all three objective functions indicates that the portfolios constructed by our proposed algorithm with 95% probability provide higher return than the portfolios constructed by mean-variance method.



20SHV-Proposed GA 50-days portfolio price path simulation. Portfolio VaR is depicted using dotted black line



20SHV-mean variance 50-days portfolio price path simulation. Portfolio VaR is depicted using dotted black line

Chapter 6

6. Research Outline

6.1 Summary and Conclusion

The optimization problem discussed in this paper contains cardinality and integer (discrete) constraints that turn mathematical formulation of this problem to a difficult task. Even though, if we would be able to model the above problem mathematically, but owing to the fact that the model will have mixed-integer constraints and non-linear objective, and considering that it would be categorized into NP-hard problem, it requires application of evolutionary algorithms to solve the problem in reasonable amount of time.

In this study, a metaheuristic method is proposed to tackle the comprehensive portfolio optimization which considers the optimum selection of securities, industry allocation, and portfolio weight allocation simultaneously. We used genetic algorithm approach for this specific problem with solution representations that contains 30 genes where each gene in a chromosome represents a security from a specific industry with an allocated weight.

Also, an innovative fitness functions are also employed to further improve promising solutions by considering both weighted Sharpe ratio-VaR and weighted Mean-VaR objectives.

As a result, after running for 3500 generations, we achieved promising result by comparing our solution with Mean-Variance optimization and S&P500 index. We used the return of GA solution as a required return constraint for Mean-Variance optimization, and then compared the volatility of the constructed portfolio by our proposed method with optimum volatility given by Mean-Variance optimization. As a result, constructed portfolio by our proposed algorithm gave us less risky portfolio than the one created by conventional Mean-Variance approach, while both portfolios provide investors with same annual return.

6.2 Future Research and Recommendations

For further research, a range of weights can be given to two parameters in both Sharpe-VaR and Mean-VaR objective functions in order to evaluate the results and find the optimum weights for each parameter. Also as GA is able to tackle more complex problems, inclusion of more realistic constraints into optimization model is suggested

Finally, in factor analysis, factor weights can be optimized by GA based on the historical data. In another word, the factor weights can be found in a way that construct the best 60-stocks portfolio based on in-sample data as an input for GA algorithm. This might improve the results that obtained without factor weights optimization.

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8. Appendices

8.1 Factor Analysis Scoring Results on December 2018

Retailing Sub-industry

	Scoring Weight	30%	20%	20%	15%	15%		
Ticker	**Name**	**Value**	**Momentum**	**Quality**	**Leverage**	**ESG**	**Factor Score**	**Rank**
AMZN UW Equity	Amazon.com	(1.97)	(0.34)	(0.51)	0.02	0.18	(0.73)	10
HD UN Equity	Home Depot	0.52	0.65	0.45	(0.20)	0.37	0.40	4
BKNG UW Equity	Booking Holdings	0.52	0.11	(0.44)	(0.96)	(0.39)	(0.11)	9
LOW UN Equity	Lowe's Cos	0.17	1.88	(0.16)	(0.37)	0.35	0.39	5
TJX UN Equity	TJX Cos	(0.41)	1.92	(0.02)	1.04	(0.05)	0.41	3
TGT UN Equity	Target	0.16	5.34	0.10	(0.22)	1.51	1.33	1
DG UN Equity	Dollar General	(0.25)	2.05	(0.26)	(0.09)	(0.38)	0.21	8
ROST UW Equity	Ross Stores	0.07	2.31	0.02	1.22	(0.75)	0.56	2
ORLY UW Equity	O'Reilly Automotive	0.32	1.65	0.69	(0.31)	(1.56)	0.28	7
EBAY UW Equity	eBay	0.86	(0.09)	0.11	(0.12)	0.72	0.35	6

Retailing Sub-industry

	Scoring Weight	30%	20%	20%	15%	15%		
Ticker	**Name**	**Value**	**Momentum**	**Quality**	**Leverage**	**ESG**	**Factor Score**	**Rank**
MSFT UW Equity	Microsoft Corp	(0.01)	4.15	(0.15)	(0.22)	0.01	0.77	3
V UN Equity	Visa	(0.09)	2.20	(0.29)	(0.00)	0.26	0.39	5
MA UN Equity	Mastercard	0.39	3.61	0.65	0.90	0.45	1.17	1
ORCL UN Equity	Oracle Corp	0.44	1.46	0.09	(0.25)	(1.12)	0.24	7
CRM UN Equity	salesforce.com	(1.18)	0.61	(0.65)	(1.04)	(0.51)	(0.59)	10
ADBE UW Equity	Adobe	(0.40)	2.28	(0.27)	0.07	0.31	0.34	6
PYPL UW Equity	PayPal Holdings	(0.57)	0.08	(0.54)	(0.01)	0.99	(0.12)	9
IBM UN Equity	International Business Machine	1.50	0.25	0.86	(0.12)	0.39	0.71	4
ACN UN Equity	Accenture PLC	0.57	2.39	0.93	1.02	0.31	1.04	2
FIS UN Equity	Fidelity National Information	(0.66)	2.49	(0.64)	(0.35)	(1.10)	(0.04)	8

Tech & Hardware Sub-industry

Ticker	Name	Value	Momentum	Quality	Leverage	ESG	Factor Score	Rank
	Scoring Weight	30%	20%	20%	15%	15%		
AAPL UW Equity	Apple	(0.25)	3.06	0.80	(0.35)	0.98	0.79	2
CSCO UW Equity	Cisco Systems	(0.31)	(0.88)	(0.37)	(0.19)	0.38	(0.31)	8
TEL UN Equity	TE Connectivity Ltd	0.05	0.62	0.22	(0.12)	(0.16)	0.14	4
APH UN Equity	Amphenol Corp	(0.62)	1.31	(0.01)	(0.03)	0.11	0.09	5
MSI UN Equity	Motorola Solutions	(0.73)	0.48	(0.05)	(0.06)	0.29	(0.10)	7
HPQ UN Equity	HP	2.21	0.03	1.68	0.43	0.38	1.13	1
GLW UN Equity	Corning	(0.44)	(0.55)	(0.92)	0.02	0.61	(0.33)	9
HPE UN Equity	Hewlett Packard Enterprise Co	1.04	0.86	(0.75)	(0.26)	(0.29)	0.25	3
KEYS UN Equity	Keysight Technologies	(0.64)	2.72	(0.48)	(1.26)	0.03	0.07	6
ANET UN Equity	Arista Networks	(0.31)	(1.78)	(0.13)	1.81	(2.33)	(0.55)	10

Healthcare Sub-industry

Ticker	Name	Value	Momentum	Quality	Leverage	ESG	Factor Score	Rank
	Scoring Weight	30%	20%	20%	15%	15%		
UNH UN Equity	UnitedHealth Group	0.58	0.65	1.05	0.26	0.23	0.59	2
ABT UN Equity	Abbott Laboratories	(0.99)	(0.01)	(0.66)	(0.11)	0.97	(0.30)	9
MDT UN Equity	Medtronic PLC	(0.36)	0.35	(0.65)	(0.17)	0.13	(0.18)	7
DHR UN Equity	Danaher	(0.54)	1.35	(0.39)	0.89	(1.92)	(0.12)	5
CVS UN Equity	CVS Health Corp	1.18	1.08	1.08	(0.37)	0.40	0.79	1
SYK UN Equity	Stryker	(0.37)	(0.57)	(0.28)	0.08	(0.55)	(0.35)	10
ANTM UN Equity	Anthem	0.65	(0.25)	0.21	(0.44)	0.13	0.14	4
BDX UN Equity	Becton Dickinson and Co	(0.75)	0.04	(0.84)	(0.02)	0.85	(0.26)	8
CI UN Equity	Cigna	1.29	0.36	0.79	(1.04)	0.35	0.52	3
ISRG UW Equity	Intuitive Surgical	(0.71)	0.42	(0.32)	0.91	(0.58)	(0.14)	6

Consumer Service Sub-industry

Ticker	Name	Value	Momentum	Quality	Leverage	ESG	Factor Score	Rank
	Scoring Weight	30%	20%	20%	15%	15%		
MCD UN Equity	McDonald's	0.07	(0.39)	(0.31)	(0.25)	0.47	(0.08)	9
SBUX UW Equity	Starbucks	(0.11)	0.97	1.48	0.49	0.00	0.53	1
LVS UN Equity	Las Vegas Sands	0.76	2.04	0.34	(0.26)	(1.20)	0.49	2
MAR UW Equity	Marriott International Inc/MD	(0.56)	1.44	(0.12)	(0.38)	(0.17)	0.01	8
YUM UN Equity	Yum! Brands	0.67	(0.60)	0.02	(0.38)	0.47	0.10	6
CCL UN Equity	Carnival	0.95	(1.32)	(0.26)	0.43	0.05	0.04	7
HLT UN Equity	Hilton Worldwide Holdings	(0.81)	2.15	0.31	(0.27)	0.93	0.35	4
RCL UN Equity	Royal Caribbean Cruises Ltd	0.22	0.79	(0.87)	(0.15)	0.53	0.11	5
CMG UN Equity	Chipotle Mexican Grill	(0.62)	2.43	0.13	1.20	(0.13)	0.48	3
MGM UN Equity	MGM Resorts International	(0.57)	1.47	(0.73)	(0.44)	(0.94)	(0.23)	10

Semiconductor Sub-industry

Ticker	Name	Value	Momentum	Quality	Leverage	ESG	Factor Score	Rank
	Scoring Weight	30%	20%	20%	15%	15%		
INTC UW Equity	Intel	(0.12)	0.48	(0.33)	0.29	1.12	0.21	7
NVDA UW Equity	NVIDIA	(0.07)	2.29	0.34	0.53	0.33	0.63	4
AVGO UW Equity	Broadcom	0.14	0.60	(0.00)	(0.31)	(2.01)	(0.19)	8
TXN UW Equity	Texas Instruments	0.05	0.22	0.20	0.47	0.40	0.23	6
QCOM UW Equity	QUALCOMM	(1.15)	1.60	(1.32)	(0.60)	0.55	(0.30)	10
MU UW Equity	Micron Technology	2.10	1.49	1.11	1.15	(0.15)	1.30	1
AMAT UW Equity	Applied Materials	0.12	2.72	0.20	0.04	0.25	0.66	3
LRCX UW Equity	Lam Research	0.04	3.98	0.17	(0.49)	0.40	0.83	2
ADI UW Equity	Analog Devices	(0.35)	0.27	(0.65)	(0.28)	0.00	(0.22)	9
AMD UW Equity	Advanced Micro Devices	(0.75)	3.62	0.30	(0.80)	(0.88)	0.31	5

Pharma Sub-industry

Ticker	Name	Value	Momentum	Quality	Leverage	ESG	Factor Score	Rank
	Scoring Weight	30%	20%	20%	15%	15%		
JNJ UN Equity	Johnson & Johnson	(0.33)	(1.11)	0.00	1.67	1.51	0.16	4
MRK UN Equity	Merck & Co	(0.50)	(0.21)	(0.32)	0.30	0.38	(0.16)	7
PFE UN Equity	Pfizer	0.84	(1.91)	0.37	0.19	(0.48)	(0.10)	6
AMGN UW Equity	Amgen	(0.30)	1.45	(0.44)	(0.83)	0.45	0.05	5
TMO UN Equity	Thermo Fisher Scientific	(1.39)	0.43	(0.59)	(0.23)	(0.48)	(0.55)	10
ABBV UN Equity	AbbVie	1.37	0.29	0.58	(0.63)	(0.48)	0.42	1
LLY UN Equity	Eli Lilly & Co	(0.75)	(0.54)	(0.39)	0.18	0.09	(0.37)	9
BMY UN Equity	Bristol-Myers Squibb Co	(0.07)	1.75	0.21	(0.22)	0.31	0.38	2
CELG UW Equity	Celgene Corp	0.72	(1.03)	(0.03)	(0.46)	(0.82)	(0.19)	8
GILD UW Equity	Gilead Sciences	0.43	(0.10)	0.61	0.04	(0.48)	0.17	3

Transportation Sub-industry

Ticker	Name	Value	Momentum	Quality	Leverage	ESG	Factor Score	Rank
	Scoring Weight	30%	20%	20%	15%	15%		
UNP UN Equity	Union Pacific	0.46	(1.37)	(0.23)	(0.11)	0.30	(0.15)	10
UPS UN Equity	United Parcel Service	(0.53)	1.76	1.38	(0.11)	0.44	0.52	2
CSX UW Equity	CSX	0.59	0.08	(0.24)	(0.11)	(1.21)	(0.05)	7
NSC UN Equity	Norfolk Southern	(0.58)	1.44	(0.37)	(0.11)	1.48	0.25	3
FDX UN Equity	FedEx	(0.12)	(0.57)	(0.11)	(0.11)	0.38	(0.13)	9
DAL UN Equity	Delta Air Lines	(0.04)	0.21	0.13	(0.07)	(0.33)	(0.01)	6
LUV UN Equity	Southwest Airlines	0.69	0.84	0.30	(1.15)	(1.05)	0.11	5
UAL UW Equity	United Airlines Holdings	(0.03)	0.21	0.34	(0.10)	(0.93)	(0.05)	8
KSU UN Equity	Kansas City Southern	(1.15)	5.11	(0.58)	(0.11)	0.30	0.59	1
AAL UW Equity	American Airlines Group	0.72	(1.75)	(0.62)	1.96	0.60	0.12	4

Pharma Sub-industry

Ticker	Name	Value	Momentum	Quality	Leverage	ESG	Factor Score	Rank
	Scoring Weight	30%	20%	20%	15%	15%		
PG UN Equity	Procter & Gamble	(0.09)	2.86	(0.41)	0.52	0.32	0.59	2
EL UN Equity	Estee Lauder Cos	(0.68)	3.32	(0.39)	1.12	(1.05)	0.39	3
CL UN Equity	Colgate-Palmolive	0.74	(0.50)	0.55	0.05	0.28	0.28	4
KMB UN Equity	Kimberly-Clark	1.14	1.40	0.14	(0.93)	0.94	0.65	1
CLX UN Equity	Clorox	0.62	(0.66)	0.96	(0.74)	0.36	0.19	5
CHD UN Equity	Church & Dwight Co	(0.07)	(0.57)	(0.05)	0.05	(0.20)	(0.17)	7
COTY UN Equity	Coty	(1.66)	3.13	(0.79)	(0.08)	(0.64)	(0.14)	6

Transportation Sub-industry

Ticker	Name	Value	Momentum	Quality	Leverage	ESG	Factor Score	Rank
	Scoring Weight	30%	20%	20%	15%	15%		
BA UN Equity	Boeing	0.55	(0.41)	0.66	(0.35)	0.67	0.26	4
HON UN Equity	Honeywell International	(1.09)	1.00	(0.04)	(0.08)	0.00	(0.15)	10
UTX UN Equity	United Technologies	0.06	1.68	(0.21)	(0.10)	0.19	0.32	3
LMT UN Equity	Lockheed Martin	0.08	1.25	1.36	0.13	0.73	0.68	1
MMM UN Equity	3M	0.42	(0.92)	0.39	(0.15)	(0.12)	(0.02)	7
GE UN Equity	General Electric	(1.33)	3.70	(1.59)	0.48	0.00	0.09	5
CAT UN Equity	Caterpillar	0.96	(1.86)	(0.21)	(0.36)	0.54	(0.10)	9
RTN UN Equity	Raytheon	(0.11)	1.92	0.11	1.34	0.19	0.60	2
NOC UN Equity	Northrop Grumman	0.31	0.81	0.30	(0.08)	(1.60)	0.06	6
DE UN Equity	Deere	0.16	1.15	(0.78)	(0.81)	(0.60)	(0.09)	8

Diversified Financials Sub-industry

Ticker	Name	Value	Momentum	Quality	Leverage	ESG	Factor Score	Rank
	Scoring Weight	30%	20%	20%	15%	15%		
BRK/B UN Equity	Berkshire Hathaway	(0.09)	0.35	0.21	0.04	(1.94)	(0.20)	10
AXP UN Equity	American Express	(0.61)	(0.13)	0.24	(0.25)	1.22	(0.01)	8
GS UN Equity	Goldman Sachs Group	0.69	1.82	(0.98)	(0.54)	(0.25)	0.25	4
MS UN Equity	Morgan Stanley	0.44	2.33	0.02	(0.46)	(0.30)	0.49	3
BLK UN Equity	BlackRock	(0.18)	2.29	(0.30)	(0.95)	(0.22)	0.17	5
CME UW Equity	CME Group	(0.20)	(0.58)	(0.36)	1.58	(0.01)	(0.01)	7
SPGI UN Equity	S&P Global	0.11	3.82	1.66	0.87	1.20	1.44	1
SCHW UN Equity	Charles Schwab	(0.46)	1.77	(0.46)	0.04	(0.28)	0.09	6
ICE UN Equity	Intercontinental Exchange	(0.52)	0.15	(0.35)	(0.01)	0.26	(0.16)	9
COF UN Equity	Capital One Financial	0.82	1.97	0.32	(0.33)	0.31	0.70	2

Food, Beverage and Tobacco Sub-industry

Ticker	Name	Value	Momentum	Quality	Leverage	ESG	Factor Score	Rank
	Scoring Weight	30%	20%	20%	15%	15%		
KO UN Equity	Coca-Cola	(0.78)	0.46	(0.86)	(0.17)	0.02	(0.34)	8
PEP UW Equity	PepsiCo	(0.51)	0.66	0.43	(0.11)	0.17	0.08	6
PM UN Equity	Philip Morris International	0.64	0.92	0.55	0.01	0.09	0.50	2
MO UN Equity	Altria Group	1.40	0.24	1.27	0.53	0.34	0.85	1
MDLZ UW Equity	Mondelez International	(0.51)	0.37	(0.72)	0.35	0.38	(0.11)	7
STZ UN Equity	Constellation Brands	(0.32)	(1.00)	(0.39)	(0.31)	0.09	(0.41)	9
KHC UW Equity	Kraft Heinz	1.37	(0.68)	(1.18)	0.20	0.34	0.12	5
MNST UW Equity	Monster Beverage	(0.69)	0.21	0.24	(1.00)	(1.74)	(0.53)	10
GIS UN Equity	General Mills	0.08	1.04	0.58	(0.25)	0.36	0.36	3
BF/B UN Equity	Brown-Forman	(0.67)	1.40	0.09	0.75	(0.05)	0.20	4