Modeling the Risks and Damages from a "Potential" Invasive Plant Species: Yellow Starthistle in British Columbia

by

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Abstract

Yellow Starthistle (Centaurea Solstitialis) is an annual invasive weed introduced to Western United States from the Mediterranean region. It favours sunny areas and responds aggressively to human disturbances such as road development, firebreaks and animal grazing. It also benefits from longer growing seasons and increased levels of CO_2 disproportionately more than native plants. Yellow starthistle (YST) is not yet known to occur in Canada but has been sighted in Washington and northern Idaho. I use a bioeconomic model to produce five study cases of the effects of YST on ranching in BC: (i) a baseline scenario without YST; (ii) a counterfactual scenario where YST is allowed to invade unimpeded; (iii) with the stimulating effects of climate change; (iv) a case where the model is augmented by a hazard function to mimic YST's invasion risk, (v) and the same scenario augmented by climate change. I use an exponential probability distribution for invasion that has been derived from statistical analyses of YST biological characteristics and time to invasion of a representative sample of herbaceous invasives in North America. A representative ranching operation is used as a study site with rangelands being the dominant type of land-use. Producers are assumed to maximise their profit subject to the function of YST spread and the probability of a YST invasion. I found that YST could have significant impacts on ranch operations: severe reductions in yearly profits (-62%) in case of unimpeded invasion, -80% with the climate change catalysis. I found that persistent populations occupying between 19% and 25% of a representative ranch could be expected. Hazard-augmented model showed that the risk of invasion could be internalised through relatively moderate reductions in stocking rate (-19%) and more significant reductions stocking rate in case of climate change-catalysed invasion (-51%) from the business as usual scenario. I analyse these numbers in more detail through sensitivity studies by concentrating on long-term profitability. I conclude with a discussion of the policy implications of our research for addressing invading species risks prior to invasion, beginning with the cost-effectiveness advantages of early detection.

Keywords: yellow starthistle, bioeconomic modeling, economic valuation, exotic species, invasive plant impacts

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List of Acronyms

AU	Animal Units
AOGCM	Atmosphere–ocean general circulation models
BAU	Business as Usual
BCCA	British Columbian Cattlemen's Association
CC	Climate Change
CFIA	Canadian Food and Inspection Agency
EDDR	Early Detection and Rapid Response
ENGO	Environmental Non-Governmental Organisation
NPV	Net Present value
SFU	Simon Fraser University
YST	Yellow Starthistle

1.   Introduction

Biological invasions have the capacity to drive global environmental change. Charles Elton, a British ecologist, and others before him, was the among first to bring to the scientific forefront the risks of invasive alien species (IAS) to indigenous ecosystems (Ricciardi, Jones, Kestrup, & Ward, 2010). Since then, the concept of IAS has entered the mainstream – social institutions and decision-making authorities have become cognizant of the risks of invasive species (Didham, Tylianakis, Hutchison, Ewers, & Gemmell, 2005). As a result, policy-makers are now aware of the impact of invasive species and many developed countries have mechanisms in place to counter, with various degrees of success, the spread of IAS (Perrings et al., 2002). However, the magnitude of government's response cannot be determined solely as a function of the amount of biological knowledge that a given society possesses about invasive species and recipient ecosystems. This is because not all alien species become invasive and cause serious impacts. In fact, most introduced species fail to establish sustainable populations and many of those that do become established do not cause serious impacts (Ricciardi et al., 2010).

Since neither economic or ecological systems operate in isolation, if I wish to create realistic impact assessments, a broader perspective is necessary than that provided exclusively by ecologists or biologists. Multidisciplinary efforts by economists, public policy makers, biologists and ecologists allow for management solutions that prevent and control IAS in a manner that minimises public expenditure (Keller et al, 2009). Given that scientific research is one of the primary methods of informing public policy design (Howlett, 2003), this research project attempts to fill the knowledge gap by analysing impacts from a potentially invasive species in British Columbia – yellow starthistle (*Centaurea solstitialis* or YST). Studying the potential impacts of an invader allows the responsible government authorities to plan their budgets and the details of their response to new invaders ahead of time. Since prevention is considered the most cost-effective method of IAS control (Leung et al., 2002), advance knowledge of a potential invasion will increase the effectiveness of preventative actions even further. Moreover, this research will allow range managers to understand the tradeoffs between agricultural practices such as the cattle stocking rate and invasion risk.

Analyses in this paper make use of the mathematics of dynamic optimization. Statistical techniques are used to analyse biological characteristics of species, combined with methods such as survival analysis, to give a more complete picture of the potential evolution of the British Columbian rangelands with impact from YST. Modelling results are presented in monetary terms where the representative ranch and the effect of the potential invasion on the ranch's profits are examined in five scenarios: (1) without YST, (2) with YST, (3) with YST & climate change, (4) with risk and lastly, (5) with risk & climate change. Results are supplemented with a number of sensitivity studies on variables with high uncertainty or variables deemed comparatively more important in a policy context.

2.   Background and Literature Review

IAS are a major source of economic damages to agriculture, industry, tourism, recreation and biodiversity. Pimentel et al. (2005) estimated that the total damages and control costs amount to $137 billion USD per year for all IAS in the U.S. Just the herbaceous alien species cause a total of $27 billion USD per year (in 2005 dollars) in the form of lost crop production on top of $3 billion spent on control of the aforementioned species per year. In Canada, no country-wide, comprehensive study has been yet taken akin to the one by Pimentel et al. However, a study by Colautti et al. (2006) focused on a subset of ten species and found that they cause between $13.3 and $34.5 billion CDN of damages in the form of lost production and control costs per year. Using Colautti et al.'s estimates by averaging the range of damages produced by the ten selected IAS, I obtain a reduction of 1.65% of 2006 GDP. This is a fairly large number considering that the Canadian GDP grew by 2.6% between 2005 and 2006 and by 1.5%, on average, since the 1960s (Statistics Canada 2009).[1]

Moreover, IAS are changing ecosystems by decreasing levels of ecosystem resilience to shocks and through removal of important functional links between organisms while diminishing biodiversity (Chapin et al., 2000).[2] Furthermore, Holling & Meffet (1996) and Selkoe et al. (2015) draw attention to potential regime shifts where due to an influence of one or multiple invaders, or human overuse of an ecosystem, the entire ecosystem may switch from a high-productivity, high-biodiversity equilibrium to a low-productivity, low-biodiversity equilibrium. Nicholls et al. (2011), for example, demonstrate how invasive zebra mussels led to an ecosystem shift in Lake Ontario. Knowler (2005) explored the impacts and management implications of a regime shift in the Black Sea resulting from invasive jelly fish and the anchovy losses that followed; Li et al. (2016) explored how seriously management options of Argentinian hake are affected by changes in resilience metrics of the given ecosystem.

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[1] Note that control costs and government spending is still part of GDP calculation, the purpose of the number is only for comparison, and should not be interpreted as a dampener of potential GDP growth.

[2] Functional links refer to the number of connections between species at differing trophic levels. For example, a   predator – prey link represents one such connection with predator being on a higher trophic level.

2.1. IAS in British Columbia

In British Columbia, Frid et al. (2009) examine seven invasive herbaceous species and estimated that in 2008, the range of economic damages was in the order of $1 to $20 million CAD (2006 dollars) for each one of the species.[3] Moreover, by 2020 these damages are projected to increase to between 5 and 60 million dollars per species, with the total mean damages for all seven species projected to be $65 million in 2008, increasing to $139 million by 2020, assuming absence of management. Moreover, the authors point out that these estimates are likely underestimates since economic data were not available for the full range of impacts (Ibid, 2009). Such impacts include biodiversity loss which may be a result of human activity or a result of IAS, what is clear however is that biodiversity loss is positively correlated with invasive species presence (Didham et al., 2005).

2.2. Potential Invasion: Yellow starthistle

A habitat that's under invasion pressure from yellow starthistle is southern British Columbian rangelands, which are very well-suited for it climatically and biogeographically. Northern Idaho, for example, is home to 243,000 ha of YST-infested rangeland that expanded from only 10 ha in the 1950s (Ditomaso, Kyser, & Pitcairn, Michael J., 2006). Because of its unpalatability to cows and toxicity to horses[4], YST is a highly noxious agricultural weed. It is responsible for annual damages of $17.1 million USD to cattle production in the state of California (Eagle, Eiswerth, Johnson, Schoenig, & Cornelis van Kooten, 2007). Almost $8 million of the aforementioned sum represents livestock forage losses; ranchers' out-of-pocket expenditures on YST control amount to $9.45 million per annum. In another study in Idaho, Julia et al. (2007), using an input-output model, found that direct and secondary costs due to YST's infestation in N. Idaho's rangelands represent $12.7 million per year, with direct costs estimated to be $8.2 millions and secondary costs of $4.5 million 2005 dollars per year. Furthermore, YST depletes soil moisture reserves; DiTomaso (2005) estimates that on invaded sites,

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[3] These species include purple loosestrife (Lythrum salicaria), diffuse knapweed (Centaurea diffusa), hawkweed (Hieracium sp.), cheatgrass (Bromus tectorum), Scotch broom (Cystius scoparious), Eurasian watermilfoil (Miriophyllum spicatum) and Dalmatian toadflax (Linaria dalmatica).

[4] YST contains toxins that cause a so-called chewing disease that is deadly for horses

soil moisture reduction is equivalent to 15%-25% of mean annual precipitation. Water reductions caused by yellow starthistle invasion raise the cost of agriculture and are estimated between $3 million and $15 million annually (Ibid 2005). Water conservation in the Sacramento watershed due to *C. solstitialis'* alteration of the water regime is estimated to cost from $16 to $75 million dollars per year (Gerlach, 2004).

A range of indirect losses are estimated to result from loss of biodiversity, soil moisture regime changes and losses to tourism that are harder to quantify but are estimated by experts to be in the "tens of millions of dollars" (Ditomaso et al., 2006). To make matters worse, interior BC and southern Alberta are in the same ecozone as the current infestations of YST in Washington and Idaho (Montane Cordillera; (Zouhar, 2002)). Given YST's effective propagation via wind, animal vectors, trade, and given the catalytic effects of climate change, there exists a significant risk of spread and establishment of *C. solstitialis* in BC (Costello, Mcausland, Costello, & Mcausland, 2003; Ditomaso et al., 2006; Dukes, Chiariello, Loarie, & Field, 2011; Reid, 2006). Lastly, yellow starthistle has become an established invader in the states directly south of British Columbia's border – Washington and Idaho, thus presenting an immediate risk of spread into southern BC. As of Summer 2016, sightings of YST have been recorded south of BC in Washington's Okanogan County (Scott, L. personal communication).

2.3. Effects of Climate Change on YST

According to Hellmann et al. (2008) and Dukes & Mooney (1999), IAS possess biological characteristics that make them considerably more competitive than native species. However, the range of interactions between native and invasive species subject to climate change is harder to envisage. Such consequences consist but are not limited to the following: increase in the range of climatic constraints on invasive species, altered impact of existing invasive species, as well as altered effectiveness of management strategies (Hellmann et al., 2008). More specifically, climate change is likely to increase the climatic range and the abundance of any given IAS thus multiplying its impact on the economy. YST happens to be one of the weeds that benefits heavily from increased CO_2 concentrations and nitrates deposition. Dukes et al. (2011) found that the success of YST was highly improved by elevated levels of warming, atmospheric CO_2, exposure to fires and nitrates. Across all treatments, YST grew six times larger when exposed to higher levels of CO_2 and three times larger in response to nitrate deposition (Ibid 2011).

Growth was measured as aboveground biomass, stem diameter and height. Interestingly, burning doubled *Centaurea* establishment suggesting that this plant would perform well in areas that could see an increase in fire frequency because of climate change.

Nitschke & Innes (2008) who modelled fire potential in the Okanagan, a warmer, drier climate will likely result in a 30% increase in fire season length. Furthermore, Toews & Allen's (2009) study of groundwater recharge systems in the Okanagan basin suggests that spring precipitation will probably increase, and summer precipitation will decrease thus leading to drier summers. This is likely to help YST compete with other species since its seed germination and growth trajectories are highly dependent on spring rain events when they grow the fastest while being highly tolerant to dry summers (Ditomaso et al., 2006). As its name suggests, *Centaurea* **solstitialis** ("of summer solstice"), has high survival rates in summers and grows well in arid climates like that of south Okanagan. Its fitness in such climate are, in part, due to its leaf structure, deep taproot and the shape of YST's winged stems help it dissipate heat like a radiator (Ibid 2006).

2.4. Propagule Pressure, Trade and Risk of Invasion

Successful establishment of sustainable populations of IAS outside their range is positively related to propagule pressure[5], which in its turn depends on the magnitude of human disturbance in a given geographic region. This means that the probability of a species becoming established is increased by the magnitude of the introduction effort (including both the number of individuals released and the frequency of introduction attempts) (Kolar & Lodge, 2001). The pattern described above is not only intuitively obvious but also quantitatively confirmed by more than 70 studies of invasive species propagation throughout the globe over the past three decades (Costello & McAusland, 2014). Those species that didn't become established despite high propagule pressure didn't exhibit invasive characteristics such as high reproductive rate, single parent or vegetative reproduction, ability to tolerate a wide range of habitats or ecological

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[5] Propagule pressure (also known as introduction effort) is a composite metric that consists of a number of individuals of a species released into a region where they are alien. It combines estimates of the absolute number of individuals released and the frequency of release events (Lockwood, Cassey, & Blackburn, 2005).

conditions (eurytopy), ability to feed on a variety of sources (polyphagy), early maturation and small body size (Kolar & Lodge, 2001).

Given increasing international trade and movement of people, IAS that possess biological characteristics that may make them invasive are moving ever more rapidly and in greater quantities throughout the world (Costello & McAusland, 2014). Trade in both manufactured and agricultural goods serves as a vector of introduction although the damage from establishment and invasion is heavily weighted toward agricultural damage (Ibid, 2014). Levine & D'Antonio (2003) used trade value of cumulative imports as a predictor of a number of invasions in the U.S. and were able to generate exotic-species accumulation curves. Assuming no change in regulations and shipping technology (e.g. packaging, inspection), the authors were able to use the generated species-accumulation curves to make forecasts about rates of future invasions in the United States. Moreover, Costello & Mcausland (2003) showed that greater protectionism reduces the rate at which exotic species are introduced but introduces price distortions that increases the amount of crops available for damage by exotic pests as well as the area of disturbed land available for propagation of such pests.

2.5. Cow-Calf Operations & Forage Production in BC

Given the fact that the literature on economic damages cited in section 2.2, used the ranching sector as a proxy to compute the costs of invasion I will also adopt the ranching sector in British Columbia as a proxy to evaluate the potential damages from YST's invasion. However, it is important to consider its key characteristics before engaging in a modeling exercise.

Due to favourable environmental conditions, British Columbia's grassland region is an important producer of meat and dairy. It lies to the west of another traditionally significant ranching region – southern Alberta. According to BC Cattlemen Association the total economic contribution of the industry is estimated at $600 million annually, or 0.25% of the province's GDP and employs over 9000 people (BCCA, 2016). In Alberta, that is also facing the risk of invasion, ranching contributes to than $6.8 billion to the province's GDP (Heigh, 2011). However, due to the fact that the grasslands east of the Rockies have a different evolutionary history than the grasslands in BC, because of the historic presence of large ruminants such as bison (Milchunas, Lauenroth, Chapman, & Mohammad, 1989), they may present stronger competition against yellow starthistle.

Although it should be noted that even if the establishment of YST is less likely in Alberta, the economic size of its ranching sector is more than ten times larger than that of BC. As a result, the economic costs arising from Alberta's invasion could be scaled proportionally, even if the potential impact is a fraction of that in BC.

BC's forage and grazing resources total more than 10 million ha, 85% of which consists of Crown range (Rothwel, 2005). These resources comprise of forage crops, improved pasture, native range and community pastures. The primary source of feed consists of native grasses and forbs with little or no grain supplementation. About 90% of British Columbia's range resource is grazed by cow-calf and yearling operations for spring, summer and fall forage where crown range provides over 60% of cattle's total forage requirements with the remaining 40% produced on private rangeland and irrigated pasture (Wikeem, McLean, Bawtree, & Quinton, 1993).

Cow-calf operations represent the first step in the production process of beef. Cows are selected for their mothering ability[6], quality of beef and other desirable traits. Mating with select bulls takes place in early summer with the peak of the calving period taking place in spring. On most ranches, the entire production process takes place entirely outside, on open rangelands where cows range and calves nurse. When the calves reach the weight of 500-600 pounds (227-272 kg), they are weaned from their mothers and overwinter outdoors on a forage-based diet (Anonymous, 2006).

Most ranchers in the southern interior manage livestock using altitudinal migration that accesses several vegetation types depending on the season and precipitation. The Bunchgrass, Ponderosa Pine, Interior Douglas-fir and Montane Spruce zones are the key grazing zones in the southern interior region. Due to high variability of topography, terrain, biogeoclimatic zones, soil types, annual precipitation and degree of forest canopy closure, southern interior BC has a wide spectrum of forage yield (Ibid 1993). The geographical focus of the study, the southern interior BC, has an average forage yield of 680 kg/hectare (Wikeem, 1993).[7]

Finally, as it has been described earlier, YST has a predominantly forage-reducing effect, but it is also known to affect other, higher value crops such as vineyards and orchards. Unfortunately the losses from those types of agricultural operations have

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[6] Mothering ability refers to heifers' propensity to be caring and nurturing toward their calves.

[7] Table 4 and Figure 1 in the Appendix show a more detailed forage yield distribution and the derivation of the aforementioned yield value.

not yet been quantitatively assessed for those sectors (Ditomaso, Kyser & Pitcairn, 2006). Thus, I will continue using ranching as a proxy that has established a strong foothold in the literature on valuating invasive species impacts.

2.6. Livestock Production and the Role of Stocking Rates

(Holechek, Pieper, & Herbel, 1998) emphasise that stocking rates have more influence on forage productivity than any other cow-calf management factor. This is because ruminants such as cows have specific nutrient requirements and will avoid grasses, forbs and shrubs that don't meet those requirements or are simply unpalatable (e.g. YST). By increasing the stocking rate, a manager risks increasing the proportion of unpalatable plants on their range because weeds and invasive plants that cows avoid will have additional room (and therefore less competition) as well as more nutrients (cow manure) to spread more aggressively. Furthermore, more than 30 long term grazing management studies in United States and Canada have demonstrated that long term financial returns from livestock production depend on healthy ecological fundamentals such as forage production, the status of watersheds as well as soil stability, all of which are strongly affected by grazing intensity (Holechek, Gomes, Molinar, & Galt, 1998).

Moreover, in a worldwide review of literature of grazing effects of large herbivores, Milchunas et al. (1989) and Jones (2000) concluded that an evolutionary history where grazing animals were present was the most important factor in determining the negative impacts of grazing on forage productivity. This issue stems from evolutionary history where grasses west of the Rockies evolved in the absence of large ruminants such as bison (*Bison bison)* and as a result were more disturbed by ranching than grassland ecosystems east of the Rockies. The effect was particularly drastic in semiarid grasslands. The lack of mutualism between grasses and ruminants has been hypothesised by Dukes (2002) to have been responsible for the quick spread of invasive plants west of the Rockies including YST. In view of the evidence above, it is clear that grazing intensity regulated via stocking rates influences the health of grassland vegetation, with more vigorous, competitive grass communities being essential to maintain and enlarge a plant community's biological resistance to a yellow starthistle invasion (Lass, L. W., McCaffrey, J.P., Callihan, 1999).

2.7. Existing Modelling Research on YST

Despite a wealth of research on established invasive species that attempts to measure economic damages as well as the cost and application of control and prevention options, there are few studies that evaluate scenarios of a "potential" invasion (Colautti et al., 2006; Finnoff, Shogren, Leung, & Lodge, 2007; Lodge et al., 2006; Maddox, Mayfield, & Poritz, 1985; Perrings et al., 2002; Pimentel et al., 2005). Regarding YST specifically, there are no studies that evaluate future damages from a potential invasion anywhere in Canada. However, Bradley, Oppenheimer, & Wilcove, (2009) have explored the spread of the five most damaging invasives in Western U.S., including YST. They found that when accounting for climate change, as forecasted by an ensemble of ten atmosphere-ocean general circulation models (AOGCMs) combined with bioclimatic envelope models, YST's range will likely expand thus expanding the invasion risk north-westward (Ibid, 2009). Unfortunately, their analyses don't include any economic or management components and their spatial invasion scenarios fall short of including any Canadian provinces.

Studies by Epanchin-Niell, Haight, Berec, Kean, & Liebhold, (2012) and Leung et al. (2002) explore prevention and other management options more generally. However, their recommendations are very broad and lack specificity in regard to particular invasion scenarios and specific species. Generalising preventative and control responses to invasive alien species is a discerning modelling approach since it allows for insight into how maximum social welfare could be achieved. Unfortunately this approach ignores the differing probabilities of invasion associated with dissimilar alien species (Barbier, Edward, Gwatipedza, Knowler, & Reichard, 2011). Leung et al. (2002) address the issue of choice of prevention versus control and arrive at a conclusion that in most cases prevention is the more cost-effective option. However, the authors state that the results of their model are highly contingent on the context such as a species in question, management region, ecosystem type, and other physical and social characteristics of the landscape in question. Epanchin-Niell et al. (2012) analyse the problem of optimal surveillance allocation to prevention of invasives, glossing over the issue whether a particular alien species has the right biological characteristics to become invasive, with the only proxy for invasiveness being the rates of growth of the alien species. Their results show that optimal surveillance effort depends on population establishment and growth rates, sample sensitivity, eradication effort and damage costs.

Thus, given the body of knowledge on the impacts of IAS on agriculture, the availability of data on trade in grains and cereals that could contain YST seed, the presence of a ranching industry that would be the most likely target to incur costs from invasion, as well as the similarity of climates between British Columbia and the American states with existing YST infestations, it would be revealing to model the potential evolution of the invasion. Using bioeconomic modelling, a type of modelling that combines biological characteristics of a resource (stock) with the dynamic optimisation techniques commonly used in economics, I can derive the optimal outcomes of a productive system in question. Such a model can then be used to derive insights and answer important questions about a counterfactual reality where, for example, YST is allowed to invade unimpeded, or what management actions can be taken to reduce the risk of invasion.

3.   Research Questions

After perusing research on the costs from YST in the U.S., the nature of operations of the cow-calf industry and ecological characteristics of YST and the region of BC where invasion might occur, I constructed a bioeconomic model to answer the following questions:

(1) What are the potential costs of invasion of *C. solstitialis* on the economic welfare of ranchers in southern interior BC, and what is a privately optimal response to the invader? [8]

(2) What effect does climate change have on the solution values in (1)?

(3) What are the optimal levels of cattle stocking on ranch grasslands that minimises the risk of introduction of *C. solstitialis*?

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[8] "Privately" means without the involvement of any governmental or non-governmental authorities that could be involved to help the ranchers share the costs of managing the invasion.

4.   Methodology

To answer the above research questions, I developed a mathematical model, borrowed from a class of models commonly called 'bioeconomic optimisation' models. This modelling framework, in the context of exploitation of renewable resources, was first developed by Clark, (1976), who specified how dynamic optimisation could be used to answer questions of optimal resource use over time. The main component that distinguishes static from dynamic optimisation models is the presence of a dynamic constraint. This constraint is in the form of a differential equation, which, unlike a static optimisation problem with a constraint exemplified by some linear or non-linear function, outlines a functional relationship between some physical quantities (e.g. quantity of foraged grass) and the rates of change of the resource in question.[9] Dynamic models can be used to answer the question of optimal levels of resource allocation over a defined period of time (Clarke & Reed, 1994; Knowler, 2002). Given this distinction, the structure of dynamic optimisation models is the same as that of static models: an objective function is maximised or minimised subject to a constraint that defines the solution space.

When constructing dynamic optimisation models, the analyst must ensure the logical coherence of the model. In other words, they must select control (variables that I can influence) and state (independent) variables that represent as accurately as possible the multitude of relationships between ecological, economic and logistic systems, among others. Control variables (e.g. cattle offtake[10], or the weed control effort by a rancher), and variables that I cannot control (e.g. beef and feed prices) are combined in an objective function that is then maximised or minimised depending on the objectives of the analyst and the form of the objective function and the constraint. This modelling

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[9] In this case, a differential equation constraint exemplifies the logistic growth of Yellow Starthistle thus showing the change in the rate of growth of the species as a function of time, or the state of propagation of species as it develops and spreads over time (Roche, Thill, Shafii, & Roche, 1997).

[10] Offtake refers to the number of cattle in AUs that are removed from the herd and sold to the backgrounding operations where they are fed a grain diet ranging typically between two weeks and two months.

exercise seeks to maximise the profits of a representative ranch (or industry[11]) by finding optimal levels of control variables (cattle offtake & YST control effort) and associated state variables (herd size, invaded area). Finally, all monetary values used in the analysis are strictly from the perspective of the representative ranch and as a result do not include indirect costs such as losses to biodiversity and tourism. Thus, the monetary impacts of YST revealed by the model may be an underestimate since they ignore costs from a "social perspective", i.e. those costs that don't accrue directly to the rancher but are born by society at large. Such costs could include loss of tourism revenue due to loss of biodiversity and infestation size (Eagle et al., 2007) as well as losses due to changes in the hydrological cycle (Enloe, DiTomaso, Orloff, & Drake, 2004).

The iteration of a bioeconomic model that I developed is enhanced with a number of functional elements and 'sub-models': (i) it incorporates survival (or hazard) analysis; (ii) a Principal Component Analysis (PCA) is used to generate the survival and hazard functions[12], and (iii) it includes a damage function that represents impact of the invader as a function of area invaded. The incorporation of survival analysis builds on theoretical work by Kiefer (1988) and practical applications in natural resource economics by Reed & Heras (1992). Knowler & Barbier (2005) and Barbier, Gwatipezda, Knowler & Reichard, (2011) explore the inclusion of hazard analysis within optimisation problems as a way to gauge the risk of an invasive species becoming established in a new environment. Li, Villasante & Zhu (2016) use the same approach to model the risk of an ecosystem exposed to a potential regime shift between drastically different levels of productivity.

4.1. YST-Free State of Rangelands ("no YST")

I assume a basic cost structure following the example of Rosen (1987) which was extended further by Widanage, (2012):

$$C(X, N; L) = p_m N + p_a L + p_h X + p_r g \frac{R_0}{z} \tag{1}$$

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[11] In which case it will be multiplied by the number of ranches in the given province and adjusted for productivity differentials, among other things.

[12] The derivation of the hazard rates and results of the PCA analysis can be found in the Appendix.

where p_m, p_a, p_h, p_r are the costs of marketing, land lease, holding cattle over winter, and the price of supplementary feed respectively[13]; X, N and L are variables representing herd size, offtake and land area. Herd size and offtake are represented in animal unit equivalents (AUE) where one cow-calf unit is equal to one animal unit (AU). R_0 is the ungrazeable residual that consists of unpalatable native or non-native invasive plants (e.g. hound's tongue); z is the number of hectares of forage required per animal per season. Lastly, the g parameter stands for seasonal forage consumption per animal. The last term of the cost expression, $\left(p_r g \dfrac{R_0}{z} \right)$, represents the total annual expenditure on the forage that has to be bought on the market to replace forage lost from the ungrazeable residual. Noy-Meir (1975) estimates that unpalatable residual can reach close to five percent of the rangeland.

It is now possible to formulate the present value of the profit expression which consists of offtake (in animal units) multiplied by the price of a calf less the costs defined earlier,

$$\Pi = e^{-\delta t} \left(pN - (p_m N + p_a L + p_h X + p_r g \frac{R_0}{z}) \right) \tag{2}$$

where δ is the rate of discount and t the time horizon.

Specifying growth of the cattle herd as logistic, I get

$$g(X;L) = rX \left(1 - \frac{X}{a\,(L-R_0)} \right) \tag{3}$$

where X is the size of the cattle herd, a represents maximum stocking rate of the grassland in question (AUs/ha), r represents the intrinsic rate of growth of the cattle herd (%AU), and L is the average size of the BC ranch (ha). Thus, $a(L-R_0)$ represents the maximum carrying capacity in animal units. The use of the logistic growth to represent

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[13] P_m includes loading, trucking and other costs related to the transportation and sale of an animal at a cattle auction.

the growth of the herd is adopted from Rosen, (1987) and Hein, (2010). The basic profit-maximisation problem takes the following form:

$$\max_{N} \int_{0}^{T} e^{-\delta t} \left[pN - (p_m N + p_a L + p_h X + p_r g \frac{R_0}{z}) \right] dt = \max_{N} \int_{0}^{T} e^{-\delta t} V(N, X) dt \tag{4}$$

$$s.t. \ \dot{X} = rX \left(1 - \frac{X}{a(L - R_0)} \right) - N \tag{5}$$

Following Conrad and Clark (1987), I derived the current value Hamiltonian along with the first order conditions for the above optimal control problem:

$$\tilde{H} = \left(pN - (p_m N + p_a L + p_h X + p_r g \frac{R_0}{z}) \right) + \mu \left(rX \left(1 - \frac{X}{a(L - R_0)} \right) - N \right) \tag{6a}$$

$$\frac{\partial \tilde{H}}{\partial N} = p - p_m - \mu = 0 \tag{6b}$$

$$\dot{\mu} = \delta \mu - \frac{\partial H}{\partial X} = \delta \mu + p_h - \mu \left(r \left(1 - \frac{X}{a(L - R_0)} \right) - \frac{rX}{a(L - R_0)} \right) \tag{6c}$$

$$\frac{\partial \tilde{H}}{\partial \mu} = \left(rX \left(1 - \frac{X}{aL} \right) - N \right) = 0 \tag{6d}$$

Solving this system of equations yields the following steady-state optimal solutions for optimal cattle offtake N^* and herd size X^*.

$$X^* = \frac{1}{2} \frac{\left(a(L - R_0)((\delta - r) p_m + (r - \delta) p - p_h) \right)}{(p - p_m) r} \tag{7a}$$

$$N^* = \frac{1}{4} \frac{a(L - R_0)\left((\delta - r) p_m + (r - \delta) p - p_h \right)\left((-r - \delta) p_m + (\delta + r) p + p_h \right)}{(p - p_m)^2 r} \tag{7b}$$

A few observations can be made on the above solutions. First, in order for these results to make economic sense, $P > P_m$, i.e. the price of 1 AU of cattle has to be higher than the marketing cost. Secondly, one can observe the dampening effect of the ungrazeable residual on the solution ($-R_0$). Note also the importance of the stocking rate, a, it

multiplies both solutions thus representing the multiplicative contribution of the stocking rate on the size of the available forage, i.e. the higher the stocking rate, the higher the herd size and the amount of offtake.

In the following section I present the first extension of the model by introducing the effect of unchecked yellow starthistle expansion on private rangeland and the required control effort.

4.2. With Unchecked YST Invasion ("+YST")

Note that the herd size (X) is bounded by the maximum carrying capacity of the pasture aL. Thus, the sustainability constraint for the pasture can be described by the following expression:

$$X \leq \frac{a(L - R_0)}{z} \tag{8}$$

where z is the required number of hectares to feed one cow-calf unit per grazing season[14].

Given that the forage demand for a representative ranch per grazing season per animal is g, the total required forage is gX. With the invader present in this version of the model, some amount of forage (in hectares) will be lost to the YST infestation. The amount of forage available from grazing is therefore:

$$g\left(\frac{L - U - R_0}{z}\right) \tag{9}$$

Where U is the area of the infestation. The remaining forage now has to be met from elsewhere:

$$R = g\left(\frac{U + R_0}{z}\right) \tag{10}$$

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[14] Note that cow-calf and animal unit (AU) are used interchangeably throughout this text.

This problem includes a new dynamic constraint that represents the growth of YST in the form of a logistic growth equation. If no management of the infestation takes place, the propagation of most invasive species can be described using a logistic growth function as specified by Shigesada and Kawasaki (1997) and adopted within bioeconomic literature by a large number of authors such as Barbier, Gwatipedza, Knowler & Reichard (2011), Carrasco, Mumford, MacLeod, Knight, & Baker, (2010), Eiswerth & Johnson, (2002), Frid, Knowler, Myers, Scott, & Murray, (2013) and Knowler & Barbier, (2005). Moreover, Pitcairn, Schoenig, Yacoub, & Gendron (2006) have plotted herbarium collections data in California enhanced by survey data at the county level yielding a logistic expansion curve which strongly supports my postulation in regard to YST's propagation pattern. The maximisation problem now consists of selecting optimal quantities of offtake $(N^{\#})$ and labour-days $(D^{\#})$ for controlling YST during the grazing season. The extended model consists of the following form of a maximisation problem with two constraints:

$$\max_{N,D} \int_0^T e^{-\delta t} \left[pN - (p_m N + p_a L + p_h X + wD + p_r R + cU) \right] dt =$$

$$\max_{N,D} \int_0^T e^{-\delta t} G_0 \left(N, X, D, U \right) dt \tag{11a}$$

$$s.t. \quad \dot{X} = rX \left(1 - \frac{X}{a(L - U - R_0)} \right) - N \tag{11b}$$

$$\dot{U} = yU \left(1 - \frac{U}{L} \right) - T(D) \tag{11c}$$

Where y represents the intrinsic rate of growth of YST and L is the maximum potential extent of the invasion (can be thought of as U_{max}). $T(D)$ represents the removal effort of YST by a representative rancher and is assumed to have the following functional form:

$$T(D) = U(1 - e^{-\beta D}) \tag{12}$$

The above equation has the form of a production function that represents control effort in person-days (D) needed to control (diminish) the yellow starthistle infestation,

exponentiated by the search parameter, β (Widanage, 2012).[15] Moreover, I assume that the output from this effort has an asymptotic limit U and and the control function exhibits diminishing returns given the increasing difficulty in locating and controlling plants in progressively remoter locations.

The above two-constraint optimal control problem can be solved following Conrad & Clarke (1987). The steady-state solution consists of two state ($X^{\#}$, $U^{\#}$) and two control variables ($N^{\#}$, $D^{\#}$):

$$X^{\#} = \frac{1}{2} \frac{a(L - U^{\#} - R_0)\big((\delta - r)p_m + (r - \delta)p - p_h\big)}{(p - p_m)r} \tag{13a}$$

$$N^{\#} = \frac{1}{4} \frac{a(L - U^{\#} - R_0)\big((\delta - r)p_m + (r - \delta)p - p_h\big)\big((-r - \delta)p_m + (\delta + r)p + p_h\big)}{(p - p_m)^2 r} \tag{13b}$$

$$U^{\#} = \frac{L(e^{-\beta D^{\#}} + y - 1)}{y} \tag{13c}$$

$$D^{\#} = Too\,long\,to\,be\,analysed,\,only\,numerical\,solutions\,exist. \tag{13d}$$

Solutions for optimal herd size and off-take with no YST control have the identical form as solutions in the first version of the model except for the addition of the area invaded '-$U^{\#}$' which dampens the values of both solutions due to its sign. One can also observe that the $U^{\#}$ solution has exponential form and should therefore be sensitive to the values of both β and $D^{\#}$. However, $D^{\#}$ cannot be analysed algebraically due to the length of the solution, only the numerical solution is presented in this paper.[16]

4.3. Including the Risk of YST Invasion ("+YST Risk")

Let $V_o(N^{\#}, X^{\#}, D^{\#}, U^{\#})$ represent the value function in a post-invasion state with optimal steady-state values for offtake ($N^{\#}$), control effort ($D^{\#}$), herd size ($X^{\#}$) and total invaded area ($U^{\#}$), derived in the previous section. Moreover, I assume that YST becomes invasive at a random time-period, τ, after which ranchers face new costs in the form of

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[15] This formulation is similar to the approach found in fisheries' literature describing harvest when searching for fish occurs.

[16] The algebraic solution can be found in the supplemental materials.

reduced forage and additional control costs due to the size of the infestation. Following Clarke & Reed (1994) I a new formulation of the optimisation model:

$$J = E\left\{\int_0^\tau e^{-\delta t}V(N,X)dt + \int_\tau^\infty e^{-\delta t}G_0\left(N^\#,X^\#,D^\#,W^\#\right)dt\right\} \tag{14}$$

where the expected value of the objective function is dependent on the probability distribution of the random event of invasion which takes place at τ. The first term indicates the NPV for the optimally managed rangeland in the non-invaded state, while the second term represents the value once invaded, where G_0 represents the post-invasion state and the associated utility that is greatly reduced in comparison to the prior state (Clarke & Reed, 1994). The risk of invasion is modeled using a survival function to represent the ecosystem's likelihood of surviving in the non-invaded state into the next time period, t.[17]

Moreover, I assume that southern BC is subjected to increasing propagule pressure from the U.S. through natural expansion of YST from the relevant border states, as well as trading pressure in potentially contaminated hay and feed from YST-affected states. Given that yellow starthistle has been detected in the states immediately adjacent to the border with British Columbia (EDDMapS, 2017; Scott, L., personal communication), and given that trade in animal feed & seeds is projected to increase according to the Freight Analysis Framework (FAF, 2017) from the five states with active YST invasions (ID, WA, CA, OR, MN) the risk of invasion should increase as well.[18]

Thus, if τ is the moment of ecosystem invasion then the cumulative distribution associated with invasion, denoted $F(t)$, is $F(t) = \Pr(T < t)$. The cumulative survivor function has the following form $S(t) = \Pr(T \geq t) = 1 - F(t)$. Following Reed & Heras (1992) I use a hazard function as a way of transforming this maximisation problem from a stochastic to deterministic formulation, by introducing a new state variable, $y(t)$, where:

$$y(t) = -\ln S(t) \tag{15}$$

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[17] Please refer to the Appendix for a chapter documenting the details of the derivation of the survival function.

[18] Based on the U.S.' Freight Analysis Framework (FAF) database, with forecasts up to 2045, I calculated a 15.71% average growth for a five-year period in the tonnage of hay, cereal grains and seeds from California, Idaho, Montana, Oregon and Washington (Anonymous, 2017).

so that

$$\dot{y}(t) = \frac{-S'(t)}{S(t)} = h(t) \tag{16}$$

I assume that the hazard rate, *h(t),* depends on offtake, *N*, (and the resulting stocking rate) as well as the biological characteristics of the invasive plant *(β_k)* according to

$$h(t) = \varphi\big(\alpha(N), \beta_k, t\big) \tag{17}$$

and the dynamics of the y variable are given by the following differential equation

$$\dot{y} = h(t) = \varphi\big(\alpha(N), \beta_k, t\big); \quad y(0) = 0 \tag{18}$$

where $\alpha = \left(\dfrac{\tilde{N}}{N}\right), N > 0$ acts as a risk scaling function dependent on offtake. $\tilde{N}$

represents the offtake amount that was solved using the following sustainable stocking rate expression:

$$\tilde{a} = \left(\frac{\tilde{X} - \tilde{N}}{L}\right), \tilde{L} > 0 \tag{19}$$

where $\tilde{a}$ is the sustainable stocking rate derived from maximum carrying capacity using Holechek, et al.'s (1998) recommended forage utilisation rate of 35% for semi-arid grassland ecosystems and forage yields from the Okanagan region (Wikeem et al. 1993). $\tilde{X}$ and $\tilde{L}$ represent the average BC herd size and ranch area respectively. According to Holechek, et al. (1998), a lower utilisation rate has been shown to be sustainable for ranchers over longer periods of time, especially when overstocking risks to long-term financial returns were accounted for. Thus, risk-neutral offtake with no invasion $(\tilde{N})$, is used as a numerator so that in the case where $N = \tilde{N}$, the risk of invasion doesn't change but when $N < \tilde{N}$, the risk is scaled upwards.[19] Thus, given the

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[19] Please refer to Figure 3 in the Appendix for a graphed function.

form of the scaling function, offtake below $\tilde{N}$ augments the hazard rate through larger herd size (and therefore stocking rate), ultimately leading to an increase in invasion risk.

Inversely, offtake above $\tilde{N}$ reduces the likelihood of invasion. The use of the scaling function follows work by Barbier et al. (2011) and Knowler and Barbier (2005), where the fully-specified hazard relationship is influenced not only by invasive plant characteristics, $\varphi(\beta_k)$, but also by the control variable, *N*. Due to limited data, I replicated the approach developed by the authors above. Similarly, I represent the hazard function as $h(t) = \varphi(\alpha, \beta_k, t) = \varphi(\beta_k)\alpha(t)$, so that the function $\varphi(\beta_k, \alpha, t)$ is in fact a product of two functions, $\varphi(\beta_k)$ and $f(\alpha(t))$. Such a formulation implies that the underlying hazard rate based on the inherent invasiveness of YST is distinct from the effect of lower offtake (and therefore higher stocking rate) on the overall likelihood that YST will become invasive.

Evidence for the relationship between quantity of livestock on rangeland and presence of ungrazeable species is based on research by Milchunas et al. (1989) and Jones (2000) who discovered that there is a positive relationship between the quantity of large ruminants such as cattle and the number of established invasive species.[20] Thus, the probability of YST becoming invasive will depend on the ranch management decision in regard to the stocking rate. Lastly, Pratchett and Gardiner (1991) stress the fact that that maintaining moderate stocking rates is especially important in arid and semiarid regions where recurrent droughts will amplify the effects described above, thus increasing the potential not only for YST to invade at a faster pace but for other, non-palatable alien or native invasives.

Following Clarke & Reed (1994), the expression containing two integrals in equation (14) can be simplified, and after integrating the second term of equation by parts, it can be evaluated in terms of y(t) yielding the following expression:

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[20] Reiterating findings from the literature review: light to moderate stocking rates promote biodiversity of grasses to a certain level but heavy stocking rates create niches for invasives to establish (Holechek, Pieper, et al., 1998). Since YST is unpalatable to cattle at later stages of its development, heavier stocking rates will lead to a higher density in undesirable species (More on this in Lit Review, section 2.6)

$$J* = \int_0^\infty e^{-\delta t - y(t)} \Big[V(N,X) - G\big(N^{\#}, X^{\#}, D^{\#}, U^{\#}\big) \Big] dt \tag{20}$$

where $N^{\#}$, $X^{\#}$, $D^{\#}$, $U^{\#}$, are steady-state values derived in section 4.2 (equation 13) where YST was allowed to invade unimpeded ("post-invasion"). Thus, the maximisation problem becomes:

$$\max_N \int_0^\infty e^{-\delta t - y(t)} \Big[V(N,X) - G\big(N^{\#}, X^{\#}, D^{\#}, U^{\#}\big) \Big] dt \tag{21a}$$

$$s.t. \quad \dot{X} = rX\left(1 - \frac{X}{a(L - R_0)}\right) - N, \quad N(0) = N_0 \tag{21b}$$

$$\dot{y} = \varphi\big(\alpha(t), \beta_k, t\big); \quad y(0) = 0 \tag{21c}$$

The current value Hamiltonian and the first order conditions are derived below.

$$H = e^{-\delta t - y}\Big(V(N,X) - V_0(N^{\#}, X^{\#}, D^{\#}, W^{\#})\Big) + \lambda_1\left(rX\left(1 - \frac{X}{a(L - Ro)}\right) - N\right) + \lambda_2\big(\varphi(\beta_k)f(\alpha)\big) \tag{22}$$

$$\frac{\partial H}{\partial N} = -\varphi(\beta_k)\big(f_N(\alpha)\big)e^{-\delta t - y}\Big(V(N,X) - V_0(N^{\#}, X^{\#}, D^{\#}, W^{\#})\Big) +$$

$$+ e^{-\delta t - y}\big(V_N(N,X)\big) - \lambda_1 + \lambda_2\varphi(\beta_k)(f_N(\alpha)) = 0 \tag{23}$$

$$-\frac{\partial H}{\partial X} = \delta\lambda_1 - e^{-\delta t - y}\big(V_X(N,X)\big) - \lambda_1\left(r\left(1 - \frac{X}{a(L - Ro)}\right) - \frac{rX}{a(L - Ro)}\right) = 0 \tag{24a}$$

$$\frac{\partial H}{\partial \lambda_1} = rX\left(1 - \frac{X}{a(L - Ro)}\right) - N \tag{24b}$$

$$\frac{\partial H}{\partial \lambda_2} = \varphi(\beta_k)f(\alpha) = \varphi(\beta_{yst})f(\alpha) \tag{24c}$$

Given the above FOCs, I can now solve this system and derive optimal levels of N^{**} and X^{**}. However, due to the length of algebraic solutions, it is hard to derive any analytical insights without parametrisation. In the following section I parametrise the solutions and compare outputs from all three models and calculate present values of profits, control efforts and their costs, invaded area size and a number of other results that add colour the analysis.

4.4. Incorporating Climate Change

Lastly, I incorporate the effects of climate change without changing the fundamental structure of the model. I accomplish that by increasing the YST spread rate thus making it relatively more virulent than the average rates observed in the literature. Bradley, Oppenheimer & Wilcove (2009) and Dukes, Chiariello, Loarie & Field (2011) found that biomass growth and climatic range of YST increase in line with projections of climatic warming and the rise in atmospheric CO_2. Dukes, Chiariello, Loarie & Field (2011) found that some of YST's biological characteristics increase by more than sevenfold (e.g. aboveground biomass – 755%), when examining monocultures of YST subjected to a 700 ppm CO_2 level. However, growth responses in mesocosm experiments where YST competed with resident species were much more modest with supplemental CO_2 causing only a 24% increase in aboveground biomass. Still, YST growth responses to CO_2 and nitrate deposition far exceeded those of resident species.[21]

Furthermore, based on the predictions of a number of atmosphere–ocean general circulation models (AOGCMs), Bradley, Chiariello, Loarie & Field (2009) reason that by 2100 future climate warming may expand YST's range by nearly one hundred percent. Given such variability in results, it is hard to say accurately how much faster YST will expand under conditions of climate change. Especially since growth rate increase is a dynamic process, changing from year to year as environmental conditions vary. Based on observations in the field, Duncan et al. (2004) have cited YST expansion rates of 46% per year in Oregon or 275% higher than the rate found in Washington state.[22] Given the variety of the above findings I took the midpoint of Bradley et al.'s projections and assumed that YST will increase its spread rate by roughly fifty percent under climate change. In order to take into account the uncertainty associated with the derivation of the YST spread rate parameter I included it in sensitivity studies in section twelve and mention its limitations in the last section.

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[21] Mesocosm is an enclosed and self-sufficient (but not necessarily isolated) experimental environment that is on a larger scale than a laboratory microcosm (one species focus).

[22] Yearly spread rate was derived only from twelve years of observation which should warrant healthy skepticism that this number is likely an overestimate.

4.5. Parameter Values & Sources

I used economic and biological data from publicly available sources such as the BCCA, Statistics Canada, Government of BC, Government of Alberta, BC Livestock auctions, peer-reviewed papers and theses (refer to Table A.2 in the Appendix for the full list of sources). Biological data on IAS characteristics and time to invasion was borrowed from Barbier, Gwatipedza, Knowler & Reichard (2011). It was populated with one additional data point that describes the biological characteristics of *Centaurea solstitialis*. YST was coded using the same methods as in the original paper (Knowler, personal communication). Principal Component Analyses (PCA) were replicated to match the results described by the original paper and then re-done with the added data point. Detailed results from the Principal Component Analysis and Survival Analysis are explained in section I in the Appendix. The parameters below were used to calculate steady-state values as well as net present values (NPVs), their values and sources can be found in Table A.2 in section II of the Appendix.

Table 1.   Glossary of Economic and Biological Parameters

Description	Parameter	Description	Parameter
Growth Rate of YST (%)	y	YST infested area (ha)	U
Herd Size (AU)	X	Annual Sales (offtake; AU) ($)	N
Max Stocking Rate (AU/ha)	a	Size of representative ranch (ha)	L
Growth Rate (AU/year)	r	Selling Price of 1 AU	P
Hectares/AU/year	z	Feed Demand per Animal per Season (kg)	g
Land rental Cost ($)	P_a	Marketing & Trucking ($)	P_m
Cattle Holding Cost ($)	P_h	Replacement Feed (per ton) ($)	P_t
Catchability coefficient	β	# of Person Days for YST Control (days)	D
Daily Wage Rate ($)	w	Interest Rate (%)	δ
YST Hazard Risk	γ	Threshold offtake	$\tilde{N}$

5. Results

In this section I present results from the modelling exercise for a private ranching enterprise under five scenarios: (1) a scenario without invasion ("no YST"); (2) a counterfactual scenario with unimpeded YST invasion ("+YST"); (3) a counterfactual scenario with unimpeded YST invasion and climate change ("+YST & CC"); (4) a scenario including the invasion hazard derived from survival analysis ("+Risk"); and finally, (5) including the invasion hazard derived from survival analysis with climate change ("+Risk & CC").

5.1. Representative Ranch Results for Each Model Scenario

Table 2 shows the results from all five modelling scenarios arranged based on the development described in the Methodology section. These solutions imply optimality of variable values given the form of the objective function, the constraints and the parameter values. It consists of the net present values (NPV_{100}) of profits, using a 50 and 100-year horizon, control costs, offtake values, herd size, stocking rate, YST-infested area, and the number of person days required to control yellow starthistle.

One should treat the first three scenarios (No YST, +YST and +YST & CC) as explorations of the counterfactual state of southern BC where YST was allowed to invade unimpeded. The results from the other two scenarios (+YST Risk and +YST Risk & CC) represent changes in solutions as a result of integration of YST invasion risk into the baseline model (no YST). It is important to consider this difference because in the case of +YST an +YST & CC, the reduction in profits, for example, is more severe due to extra control costs, lost forage and lost revenue. Scanning Table 2, one can observe the reduction in the values of variables that are fundamental to the operation of a representative ranch. As a result of lower herd size and offtake, annual profit decreases with the presence of YST and at an even higher rate when the catalytic effects of climate change on YST growth are incorporated in the model. On the other hand, the decrease in profits is considerably smaller when only invasion risk is integrated in the model. As outlined previously, this is due to the fact that the risk of invasion enters the objective function through a discount rate and not a separate dynamic constraint like in the +YST

and +YST & CC cases. In the following section, I explore the results of each model scenario in greater detail.

Table 2. **Optimal Values for Management of a Representative Ranch under Five Scenarios with Assumptions on Climate Change Impacts and Invasion Risk**

Variables	No YST	+YST	+YST & CC	+YST Risk	+YST Risk & CC
Offtake (N^*)	47.1 AU	37.9 AU	34.7 AU	39.6 AU	24.9 AU
Herd Size (X^*)	95.6 AU	77 AU	70.5 AU	77.7 AU	47.1 AU
Infested area (U^*) (ha)	-	91.6	123.8	-	-
Person Days (D^*)	-	55.7	69.5	-	-
Profit Per Year	$25,778	$9,922	$5,262	$23,925	$18,694
Stocking Rate(AU/ha)	0.193	0.16	0.142	0.157	0.1
% Land Cover YST	0%	18.5%	25%	0%	0%
NPV of profits (50 yrs)	$473,233	$182,148	$103,300	$439,214	$343,183
NPV of profits (100 yrs)	$512,079	$197,100	$111,780	$475,268	$371,354
Control Cost Per Year[1]	-	$8,912	$11,123	-	-
Reduction in Profit due to Ctrl Cost	-	34.6%	43.2%	-	-
Reduction in Profit due to Lost Grazing	-	26.9%	36.4%	-	-
NPV of Direct Costs (100 yrs)[2]	-	-$314,979	-$400,299	-	-
NPV of Control Costs (100 yrs)	-	-$177,039	-$220,965	-	-

[1] All NPV values are computed using a 5% discount rate.
[2] Control costs refer to the additional labour required to reduce the size of the YST infestation.
[3] Direct costs refer to the total of control and grazing costs imposed by the YST infestation. Grazing costs consist of replacement feed costs and losses from reduced availability for cattle grazing.

5.1.1. YST-free state of rangelands (no YST)

In the YST-free scenario, steady-state solutions of herd size equal to 96 AU and offtake of 47 AU. The resulting stocking rate is 0.19 AU/ha which is very close to BCCA's (2017) factsheet which states that the average head count of a BC herd was 109 AUs as of 2011. Given an average ranch size of 495 hectares, I calculated a stocking rate of 0.22

AU/ha. The baseline model results are close to the data reported by BCCA, 0.22 AU/ha is only 14% higher than the baseline rate found in the no YST scenario. Thus, it is reasonable to proceed by augmenting the baseline model with the yellow starthistle invasion in order to study its effects. It is important to note, however, that instead of using single year prices I decided to keep the 5-year average of cattle prices due to the smoothing effects of the average and the fact that it's more likely to capture a larger portion of the cattle cycle.[23]

5.1.2. With Unchecked YST Invasion (+ YST)

Given the rate of yellow starthistle spread observed in northern Washington (Duncan et al., 2004), and assuming no response from the appropriate governmental agencies or ENGOs, YST is expected to invade 92 ha of the representative BC ranch. The results from this scenario with unchecked invasion revealed that invaded area could reach 19% of total ranch area and may lead to total direct losses of $314,979 due to reduced grazing, increased spending on replacement feed, with out-of-pocket control costs reaching -$177,039 over a 100-year period, assuming a discount rate of 5%. Total direct costs (control & grazing reduction) represent 31% (18% and 14%, respectively) of revenue, an amount that could be thought of as an 'invasive fee'.

5.1.3. Integrating Climate Change (+YST & CC)

Inclusion of the catalytic effects of climate change on the speed of growth of yellow starthistle has a very strong negative effect on results. For example, the invaded area of a representative ranch increases by 35% (124 ha), followed by an increase in the number of person-days spend on control (+25%), thus raising the total direct costs by 27% (-$400,299) when compared to the "+YST" scenario. The graph below, as well as the sensitivity studies in the following section, show that varying YST spread rates result in significant changes to the solutions. However, they are not as drastic as the changes produced by the movements in the price of beef or the price of feed. Thus, a full range of variation, consisting of four 15% increments (total of 60%) in the speed of YST spread changes the NPV_{100} value of profits by a total of $160,695 in the +YST scenario and by

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[23] An approximate time period of 9-10 years over which the number of beef cattle is alternatively expanded and contracted as a result of price fluctuations and lagged decision to expand or reduce the herd by the ranch managers.

$228,134 in the +YST & CC scenarios. Furthermore, as shown on the Figure 1 below, the negative effects of the YST growth rate on the NPV_{100} of profits are more pronounced at the higher end of the spread rate (top 30% of the variation in parameter) where profits fall to -$13,975. This means that the difference between the first and the second 15% increase in the YST spread rate results in a 59% decrease in profits while the difference between the third and the fourth 15% increment causes a 131% decrease in the NPV_{100} of profits (from $45,406 to -$13,975 – a decrease greater than 100%). This suggests that at a high end of YST spread rates, i.e. in case of a strong climate change effect, controlling YST becomes a serious burden on long-term ranch profitability. Figure 1 on the next page demonstrates that effect particularly drastically in the +YST & CC scenario where the distance between data points increases at the top 30% ra Finally, it is important to take note of this non-linearity in the effect of YST spread rate increase since this is where a preventative intervention would be most effective in case such a high rate of YST propagation becomes reality.
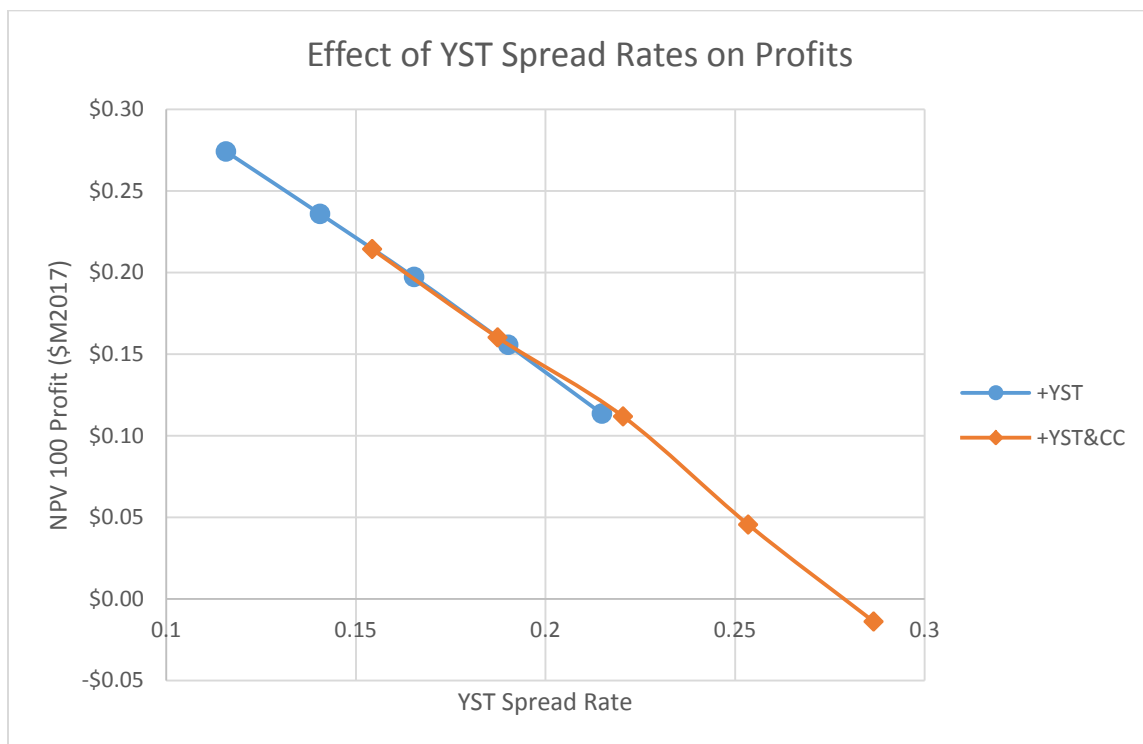

Figure 1.   Effect of Yellow Starthistle Spread Rate Variation (15% increments) on NPV of Profits: stronger effect of non-linearities on the NPV of profits in the high end of the spread rate values (top 30%)

Thus, the addition of the climate change effect to the "+YST" scenario resulted in a drastic drop in profits when compared to the BAU scenario ("no YST") – a reduction

from \$25,778 annual profit to only \$5,262, a drop of 80%. The reason for such a drastic decrease is due to non-linearities in the computation of steady-state values. A faster-spreading invasion leads to a higher proportion of land that is no longer suitable for ranching. This causes the rancher to reduce the herd, while simultaneously increasing spending on control and on replacement feed. This process represents a vicious circle for the rancher which leads to the reductions in stocking rate and profits described earlier. Finally, the effect of higher spread rates is bounded by the search function that dictates the effectiveness of YST control. As I show in the next section, because the search function exhibits diminishing returns, there is a point after which it is not optimal to increase the quantity of labour allocated for control. However, it is important to note that as the technology of weed control evolves, so will the effectiveness of search and eradication of YST. For example, as drone technology becomes more widespread and cheaper it can have considerable effects on the value of steady state solutions through an increase in effectiveness in the search function (Crutsinger, Short, & Sollenberger, 2016). Given the fact that the 0.287 rate of spread is the highest rate I use to represent the effect of climate change, rates as high as 0.46 have been observed in Oregon (Duncan et al., 2004).

5.1.4. Integrating Invasion Risk (+Risk)

Introducing the hazard rate into the bioeconomic model represents a way of internalising risk of invasion into the dynamic profit-optimisation model and exploring how steady-state values change as a result. Following initial mathematical formulation by Reed & Heras (1992) who used the fish stocks to represent a state variable, herd size is the terrestrial equivalent of stock variable. Ranjan, Marshall, & Shortle (2008) for example, adopt a similar approach to Reed & Heras (1992) where fish stock and fishing effort are state and control variables. Unlike Ranjan, Marshall, & Shortle (2008) who formulate the risk of invasion as a function of both stock maintenance and preventative effort (mitigating the impact of IAS arrival), I concentrate only on the influence of invasion hazard via the stock variable (herd size).[24]

As expected, including the hazard rate of YST invasion in a separate model scenario resulted in a slight decrease of both the herd size and the resulting offtake from

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[24] In fact, compared to the formulation in this model, Ranjan, Marshall, & Shortle (2008) added an additional constraint in a form of a function that represents the effects of mitigation efforts aimed at increasing the ecosystem resilience.

the "no YST" scenario.[25] In Table 2. shown in section 5.1, the risk of YST invasion reduced offtake and NPV of profits by 19% and 7%, respectively, from the noYST scenario. It should be noted, however, that profitability falls slower than decreases in herd size and offtake. This is due to the accompanied decreases in costs associated with smaller herd size and offtake, lower overwintering costs, holding costs and marketing costs among others. Overall, this finding can be generalised: at higher levels of herd size, marginal profitability of a ranching enterprise exhibits diminishing returns, a result consistent with diminishing marginal productivity observed in most industries, including ranching (Holechek, et al. 1998).

It is important to note that when comparing the "+Risk" to the "noYST" scenarios the reductions in NPV_{100} values are quite small. Only a 7% decrease in profits allows for the risk created by YST biological characteristics and historic invasion timelines to be negated. However, the decrease in annual profits between the models mentioned above, is multiplied almost fourfold when climate change is included. As a result, without the effect of climate change, one could make a strong argument that it makes economic sense to take a cut in profits in the current time period in order to ensure a longer-term health (and therefore profitability) of the rangeland. In a scenario where invasion risk is not negated by decreases in stocking rate, then the ranch manager will unknowingly increase the probability of invasion in each consecutive season. This behaviour means that future revenues would collapse sooner. Moreover, another caveat of this finding is that current stocking rates on BC private ranches are already quite conservative. Thus, assuming that there is no effect on YST spread rate from climate change, ranch managers should concentrate on controlling stocking rates in areas where their effect is greatest (more on this in section 8 on Policy and Management Recommendations).

5.1.5. Integrating Invasion Risk & Climate Change (+Risk & CC)

The results change appreciably in the "+Risk & CC" scenario when the climate change-influenced steady state solutions from the "+YST & CC" scenario are integrated within the risk model: offtake decreases by 47% and the NPV_{100} of profits falls by -28%. Integrating the climate effect inflates the value of the residual *G(.)* function thus resulting

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[25] A chapter detailing the derivation of the hazard rate of YST invasion can be found in section I. of the Appendix entitled "PCA, Survival and Hazard Analyses".

in the subtraction of a greater value from the revenue expression leading to an overall reduction of profits.

Moreover, another effect of removing the YST growth and control constraint from the model formulation effectively removes the control variable that influences the spread of YST (days of control effort) thus bounding this model to be solely influenced by one control variable: offtake. The result of this effect is a greater spread between the offtake values when comparing "+YST" and "+YST & CC" scenarios to "+Risk" and "+Risk & CC" scenarios: 38 AUs and 34.7 AUs vs 39.6 AUs and 24.9 AU respectively. Although ranchers could undertake other preventative efforts such as a more active IAS surveillance regime which would hypothetically decrease the magnitude of the effect of the sole control variable, this method is beyond the scope of this study. Moreover, this result demonstrates that a crude method of risk control that focuses on offtake only may be inadequate thus requiring to be bundled with other prevention methods (see section 7 – Policy and Management Recommendations).

Another caveat of removing the YST growth and control constraint from the "+Risk" scenarios pertains to how to properly compare the results between "+Risk" scenarios and the +YST" scenarios. Because, for example, the "+YST & CC" scenario exhibits significantly lower NPV_{100} profits than "+Risk & CC", but the reduction in profits is due to the extra costs included in the "+YST" models, and especially the diminishing effectiveness of the search and eradication function. The absence of control effort in the "+Risk & CC" scenario altogether means that the only lever of control of the risk of YST invasion is through the stocking rate and not mechanical or chemical control of the infestation like in the "+YST" scenarios. Thus, despite a decrease in offtake in the "+Risk & CC" scenario that is almost two times larger than in the "+YST & CC" scenario, the NPV_{100} of profits is actually more than threefold higher (232) in the "+YST & CC" scenario. This is because YST control efforts and extra replacement feed (when compared to the "+YST" and "+YST & CC" scenarios) are no longer used in the calculation of profits in the "+Risk & CC" scenario which leads to higher profit values. Consequently, comparisons should be made between the "+Risk" scenarios and the "+YST" scenarios on the basis of revenue and not profits. When one applies this approach, larger differences emerge when comparing revenues: -16% and -47% lower in case of "+Risk" (vs "+YST") and "+Risk & CC" (vs "+YST & CC") scenarios respectively.

5.1.6. Model Comparison

A full breakdown of differences between the five model scenarios are summarised in Figure 2. Horizontal bars represent differences with the "no-YST" scenario. For example, assuming that climate change will have no impact on the spread rate of YST, the invasion is expected to result in a 7% reduction in the NPV of profits from the YST-free scenario. As I mentioned earlier, the greatest reduction in profits are from the "+YST" and "+YST & CC" scenarios due to the fact that the objective function includes additional costs in the form of control and supplementary feed expenditures. This graph allows for quick comparisons between scenarios to be made: first, it is clear that the effect from unimpeded YST invasion trumps potential profit reductions generated by the inclusion of risk into ranchers' management actions. Second, the reductions in profit due to the catalytic effects of climate change on the YST spread rates in both "+Risk & CC" and "+YST & CC" scenarios are substantial.
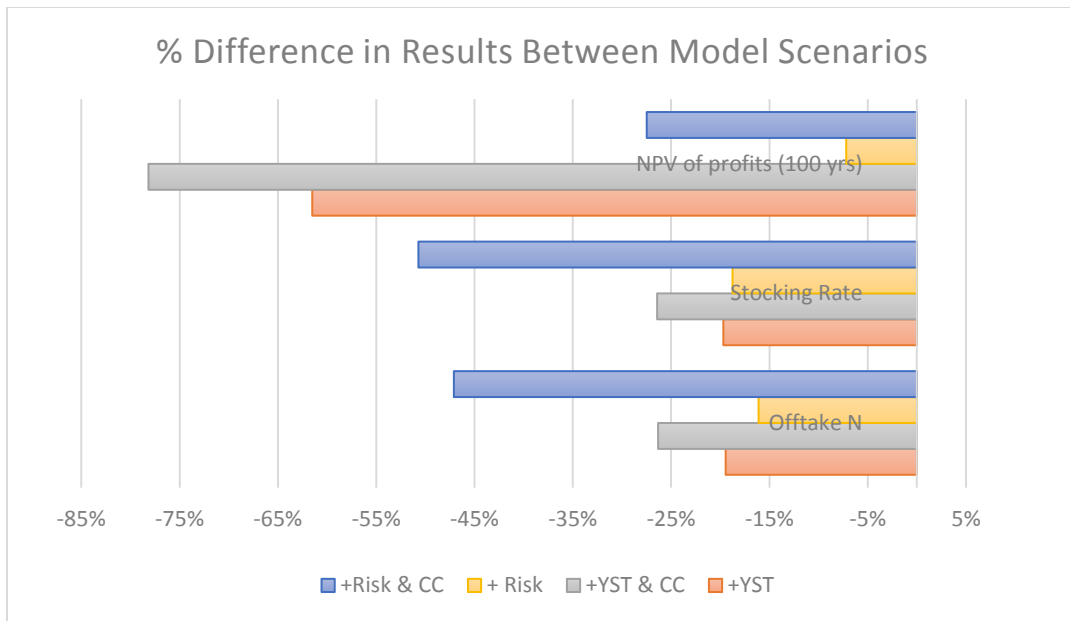

Figure 2.   Comparison between model scenarios: variation in key metrics of ranch operation among four scenarios, with "no YST" representing a baseline scenario (0%)

5.2. Industry-wide Results

As of the 2011, there were 4,086 ranches in British Columbia. However, it won't be reasonable to assume that all of them will be invaded with YST. Thacher et al. (2010)

report that 85% of all counties in California report YST infestation. However, applying this number as an expected percentage of invaded ranches in BC is inappropriate since California is quite different climatically and biogeographically from British Columbia. Moreover, California had a YST infestation problem since the 1950s, with first accounts of YST appearing in 1890s (DiTomaso, 2005). As a result, a more realistic assumption is to adopt the proportion of YST infestations in a northern state such as Washington where infestation levels are reported at approximately two-thirds of all counties (Thacher et al., 2010) and apply it to ranchers in BC. Moreover, to lend further support to the assumption that approximately two-thirds of ranchers in BC would be affected by infestation levels found in the "+YST" and "+YST & CC" scenarios, more than half of all cattle in BC is reported to be in the regional districts on the edge of the border with Washington and Idaho (BCCA, 2017). Consequently, due to the lack of better data, I assume that 66% of BC ranchers could incur the magnitude of damages computed by the model. Since these results are derived from steady-state values shown earlier for the representative ranch, the same analytical implications apply to the industry-wide data. The benefit of industry data is in showcasing the potential total effect of the YST invasion as exemplified by the magnitudes of direct costs, losses of revenue to ranchers and losses of tax revenue to the government. Given the parsimonious assumptions outlined above, I created

Table 3.   Industry-wide results: comparison of five model scenarios

Variables	No YST	+YST	+YST & CC	+ Risk	+ Risk & CC
Profit per year	$105,327,475	$62,136,285	$49,441,615	$64,517,446	$56,528,535
Control Costs per year	N/A	$24,276,288	$55,885,859	N/A	N/A
NPV of profits (100 yrs)	$2,092,355,685	$1,234,352,282	$982,169,619	$1,281,654,632	$1,122,952,983
NPV of direct costs (100 yrs)	N/A	-$858,003,403	-$1,110,186,066	N/A	N/A

6. Sensitivity Analyses

Sensitivity analyses are typically conducted to test the robustness of the model and to explore scenarios where uncertainty of specific parameters is especially high. More specifically, sensitivity analysis involves (1) finding any unexpected relationships between inputs and outputs (searching for errors), (2) finding out whether specific variables have a disproportional effect on the results of the model, and (3) to ensure that variables whose parameters carry a high degree of uncertainty are thoroughly analysed. I selected the following parameters of interest, either due to their structural importance to the model, cyclicality, stochasticity or uncertainty of estimation: the price of cattle (P), the price of replacement feed (P_r), the intrinsic growth rate of YST (y), the growth of the herd (r), the maximum carrying capacity of pasture (a), the daily wage of a unit of labour as well as the intrinsic growth rate of YST under climate change.

The results of variation of wages, prices of beef, carrying capacity, and the spread rate of YST are summarized in the three box plots below. Overall, price of beef has the largest effect on the results of the model, with herd growth rate in second place. A strong effect of replacement feed variation is observed only in the "+YST" scenario. Wages and spread rate of yellow starthistle have moderate effects on the results, with changes in carrying capacity having the lowest effect on the outputs of the model. Finally, the results from all five scenarios behaved as expected for variables that exhibited cyclicality (beef prices) or carried uncertainty of estimation (e.g. maximum carrying capacity). That is, following parameter manipulation, the results generally moved in the logical direction as predicted by the structure of the model (e.g. higher feed prices result in higher costs, higher rates of YST spread result in higher area invaded, lower herd size and offtake, etc). A more detailed discussion is presented in the next section.

The following figures (3-5) have been graphed by varying the parameters mentioned earlier by 15% increments and recording the changes in key outputs (e.g. NPV_{100} of profits, herd size and offtake). More specifically, I use 70% of a parameter value such as wage, for example, as a lower bound, with 130% of the value being the upper bound and 100% of the representing the baseline solution presented in the Results section. I graph the results from sensitivity analysis tables in section VI of the Appendix using scatterplots connected by lines in order for the reader to be able to

grasp the most relevant information about the sensitivity studies outputs (e.g. spread of results, increase or decrease of the effect from parameter manipulation).
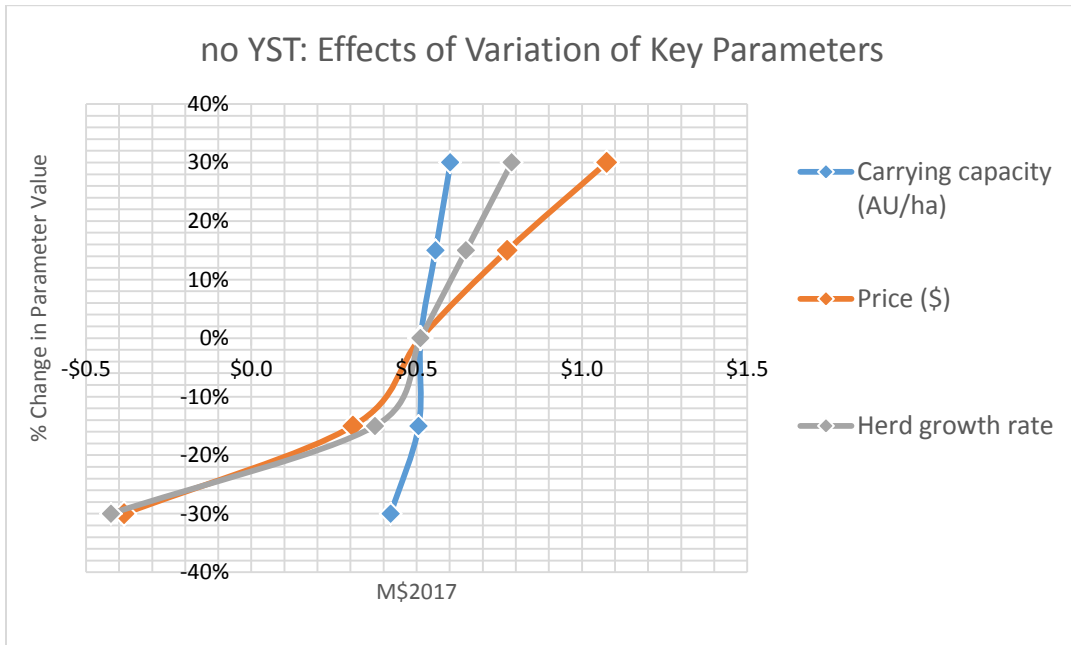
Figure 3. "No YST" Scenario: this figure shows changes in results as a function of 15% variation in selected parameters (shown in the legend), starting at the 70% of the value to a maximum of 130%
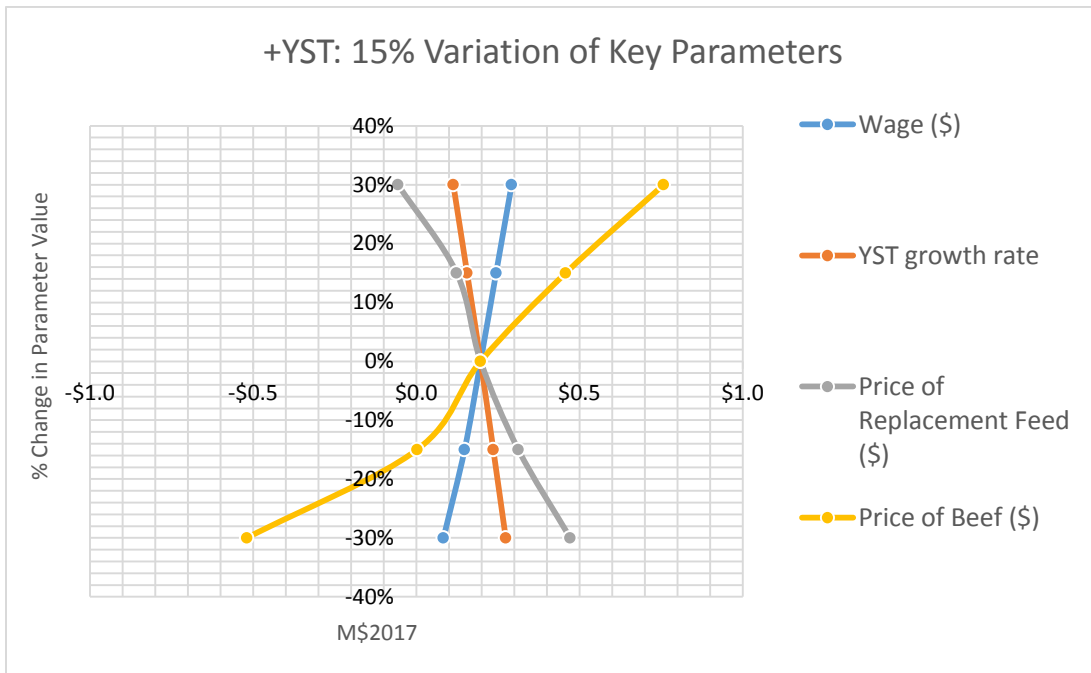
Figure 4.　　"+YST" Scenario: this figure shows changes in results as a function of 15% variation in selected parameters (shown in the legend), starting at the 70% of the value to a maximum of 130%
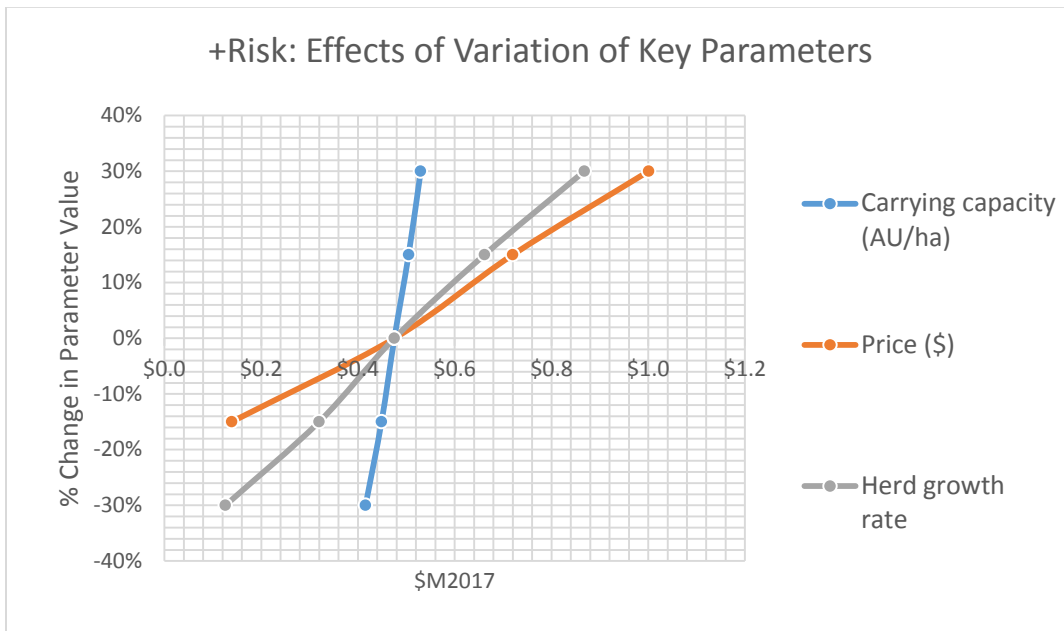
Figure 5.　　"+Risk" Scenario: this figure shows changes in results as a function of 15% variation in selected parameters (shown in the legend), starting at the 70% of the value to a maximum of 130%

Thus, based on the total range of the values shown by the length of the price line, the maximum selected variation (60%) in the price of beef has the largest effect on the results across all three scenarios. Moreover, non-linearities exist where, for example, a 30% drop in price from the baseline scenario, which is not uncommon in livestock markets, given the cyclical nature of the industry, results in losses of $896,000 in the "no YST" scenario. However, a 30% increase in price in the same scenario results only in $562,900 rise in profit. A reduction of 30% in the price of beef produces no solution in the "+Risk" scenario but a 15% decrease in price results in NPV_{100} profits of only $138,907.[26] Unsurprisingly, ranch profitability suffers the most in the "+YST" scenario where a 30% decrease in price leads to losses of $519,463. Such a large decrease in profitability is due to the loss in revenues which leads to a decrease in labour-days spent on control and a subsequent growth of YST area to 166 ha, effectively doubling in size from the "+YST" scenario's original value. A reduction in forage area and an increased

[26] Note that the model returned an imaginary number (i) when using a parameter that was 30% lower than that used in the default scenario. This means that that particular parameter was outside of the solution space as defined by the constraints of this optimisation problem ("no solution" reported in Table 1.1.3 in the Appendix).

need for additional feed creates a vicious cycle where lower prices lead to lower herd size and therefore lower offtake; lower offtake leads to depressed revenues which in turn reduces the amount of money that can be spent on search and control of YST. YST can therefore spread even further thus decreasing forage availability and increasing requirements for feed replacement which brings up the costs even more.

However, the magnitude of variation in NPV_{100} profits depends on the model scenario. For example, because the highest output occurs only in the "no YST" scenario, the highest variation in the effect of price of beef on the steady state values is present only in that scenario. The total range of NPV_{100} of profits is $1,458,899 resulting from a 60% variation in the price of beef. This represents the largest variation in the effect of any given parameter of all model scenarios. The "without invasion" ("no YST) scenario underlines the strength of the effect of price variation and is consistent with the phenomenon of cattle cycles where price fluctuations drive the size of the North American herd size as a whole (Holechek et al. 1998).[27] One caveat of this finding is that the demand for meat in the U.S. and in Canada's main export markets has significant repercussions on ranchers' stocking decisions and as a result on the spread of invasive rangeland weeds such as YST. Furthermore, the effect of lower prices has a totally opposite effect in the case where YST has established permanent infestations. Lower prices leave land managers such as ranchers with diminished resources to control the infestation. For example, a 30% decrease in prices leads to an 81% increase in YST-infested area (92 ha to 166bha) and a 30% increase in beef prices results in a 112% decrease in infested area (92 ha to 43 ha). Thus, the economic health of the ranching industry is positively related to ranchers' ability to control YST and maintain the health of rangelands. Eagle et al. (2007) mentions a finding in a similar vein where time response to invasion (a function of a given rancher's prosperity) was a key determinant in whether ranchers (as well as local governments) were able to stop YST spread early on, thus keeping the county from having permanent infestations.

Finally, the 'price effect' on YST control, and IAS propagation more generally, is interesting because it underlines the effect of the wider economic forces, outside of a given ecosystem. For example, Costello & McAusland (2014) showed that propagule pressure and therefore the probability of introduction is a function of trade and the

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[27] My informal interviews have confirmed the fluctuating nature of ranch profits where ranch owners have stated that it's very common to have positive profits some years and negative in other years.

broader macroeconomic climate where periods of economic expansion are likely to generate an uptick in unintended introductions and invasions. Consequently, in the case of the "+Risk" scenario, the YST invasion risk is also a function of trade, the underlying macroeconomic climate and consumer preferences. Thus, in the case where YST has established invasive populations, an increase in beef price increases ranchers' ability to control YST once it has invaded, but in another case, similar to Costello & McAusland's (2014) findings, an increase in price may lead to an increase in propagule pressure and therefore the probability of invasion.
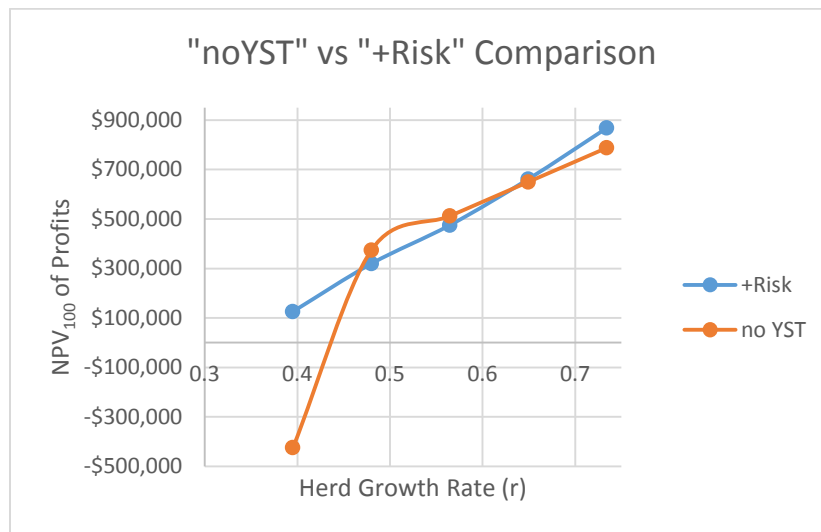
Figure 6.　　Variation of Herd Growth Rate: Variation is greatest in the "no YST" scenario due to a lack of a dampening effect of the hazard of invasion present in the "+Risk" scenario

　　　　The second largest effect on the profitability of the ranch in the "no YST" and "+Risk" scenarios stems from the variation in the growth rate of the herd (Figure 6). For example, in the "no YST" scenario, it results in the range of $1.2 million dollars from a 60% variation in the growth rate of the herd. Interestingly, in both scenarios, there are significant non-linearities at the bottom 30% of the variation. For example, at 85% of the herd growth rate, the NPV_{100} of profits is very close to the "+Risk" scenario (Figure 6). However, at 70% of the parameter value, profits fall into the negative for the "no YST" scenario (-$383,967) while "+Risk" scenario is still in the positive ($126,034). This effect is a result of the way I integrate risk within the objective function where I specify offtake amount to be inversely related to risk. In other words, when risk is a function of offtake (on top of YST's biological characteristics) the optimisation procedure is influenced by the dampening effect of YST risk and the offtake threshold function. One caveat of this

finding is that the integration of invasion risk reduces the spread of the variation in NPV_{100} of profits at the lowest 30% (70%-100%) of the variation in herd growth rates.

The third largest effect on the profitability of the ranch in the "+YST" scenario is the effect of changes in the price of supplementary feed. Unfortunately, it's impossible to compare this effect to other scenarios because the role of replacement feed in the "no YST" and "+Risk" scenarios is extremely small because YST is not yet present in those models. However, in the "+YST" scenario, a 60% swing in the price of feed changes the value of the NPV_{100} of profit by \$527,843. Furthermore, the price of replacement feed depends on the precipitation and fire regimes in a given year and, as a result, varies greatly from year to year (Wikeem, McLean, Bawtree & Quinton, 1993). Consequently, a 30% price swing is a realistic scenario which can make a fundamental difference in the profitability of a ranching enterprise and its ability to control YST.

Lastly, another variable of interest is the spread of YST under the influence of climate change. Figure 7 demonstrates that there are diminishing returns to labour days spent on search and control of YST. This is due to the functional form of the search function (diminishing returns to search assumption) as well as the value of the search parameter that I found in a study on control of hound's tongue in the Interior of BC by Widanage (2012). An important caveat of this observation is that at the higher range of the spread rate, the effectiveness of control decreases and becomes largely ineffective. Assuming no technological growth in the effectiveness of search and eradication and given elevated spread rates, YST-invaded areas may become very difficult to keep in check and likely impossible to reduce according to this scenario.
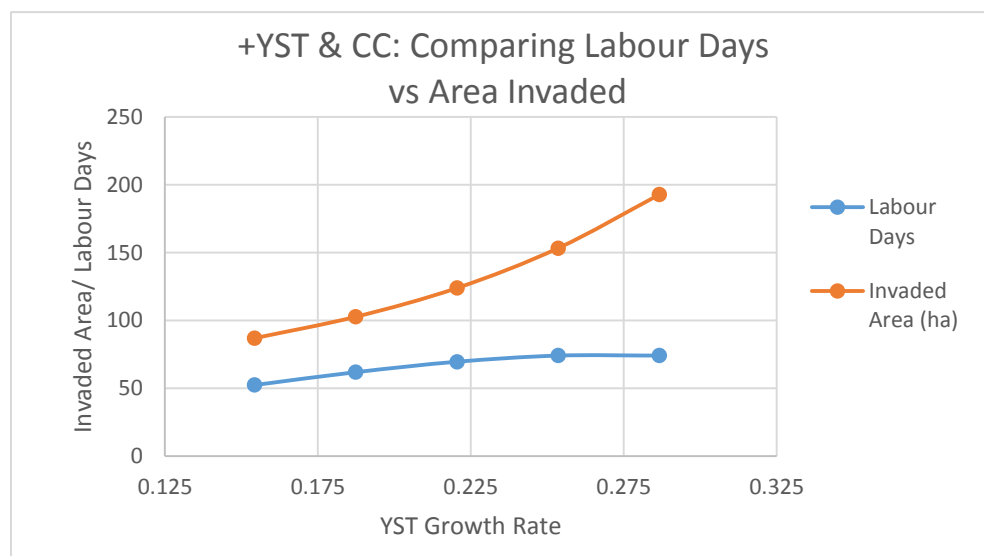
Figure 7.   Labour Days and Area Invaded: greater spread rates (100%-130%) of
the variation show diminishing returns of effectiveness of search and eradication function

7.   Discussion

My findings are within the range of values reported in the literature. For example, Hartmans, Zhang, & Michalson (1997) report an 80% reduction in revenues due to declines in range productivity and costs of control efforts in Idaho, based on expert opinion. Eiswerth & Van Kooten (2002) found that forage losses are 6-10% of revenue for minimal infestations and 22-28% for moderate infestations, with moderate infestation representing 30% cover. Furthermore, Eagle et al. (2007) reported a range of 13-15% of control and forage costs as a percentage of revenue based on surveys of ranchers' expenditures in three California counties. The same authors found that control costs ranged from 36-49% of the total direct costs; whereas in my analysis that number stands at 56%. This discrepancy could be due to the selection of the search and eradication parameter that I borrow from a study by Widanage (2012), when compared to the level of effectiveness of eradication encountered in survey. It could also result from differences in control and replacement feed costs that BC ranchers face, compared to those in California. Furthermore, in the 11 years since Eagle et al.'s study, inflation and the growth in feed prices would have further contributed to increasing the amount of total direct costs found in this model. Another example of the magnitude of control costs is found in in the U.S. where Pimentel, Zuniga & Morrison (2005) claimed that the annual costs from invasive weeds on pasture stand at around \$6 billion, with most costs (83%) resulting from control efforts as opposed to grazing losses and other types of weed damages (e.g. damages from thorns). However, according to the above-mentioned authors, the percentage of control costs is most likely an overestimate due to the scarcity of data when estimating control costs as well as simplifying assumptions (e.g. how negative effects are proportional to the size of the infestation) (Ibid., 2005).

Furthermore, given the parameter values I use for cattle prices, wages, effectiveness of ranchers' ability to locate and destroy YST (the search parameter β), as well as sixteen other parameters, it's not optimal to reduce the infestation area on the representative ranch below 92 ha. One important caveat is that reducing the YST-invaded area below this amount would result in revenue losses that will undermine the profitability of the ranch. This result is consistent with highly invasive species such as kudzu that was estimated to cause anywhere between \$100 to \$500 million per year and will most likely never be eradicated in the United States (Forseth & Innis, 2004). Thus,

unlike eradication of the invasion, perpetual control becomes the only cost-effective option for highly invasive species like YST once the invasion has occurred. Moreover, Ditomaso, Kyser & Pitcairn (2006) have documented that eradication is not a cost-effective option in states with heavy yellow starthistle infestations (California, Idaho, Washington) where only control (biological, mechanical, cultural and chemical) can achieve a level of cost-effectiveness at this point of the invasion process. As a result, according to the same authors, current levels of infestations in California can be managed with a mix of (1) perpetual control, (2) ecosystem preservation (e.g. for recreation and biodiversity) and (3) preventing reinvasion of yellow starthistle in cleared areas. As a result, given the aforementioned findings, the magnitude of potential direct costs (control & replacement feed) on the cattle industry over the next one-hundred-year period (-$858,003,403), prevention may indeed be a cost-effective option.

Incorporating the hazard rate into the model produced results that are consistent with my expectations as well as research in resource economics that uses similar methodology. For example, Ranjan et al. (2008) also used Reed & Heras' (1992) methods to integrate invasion risk in a fisheries context where invasion risk was a function of fish stock health. Ranjan et al. reported that integrating risk led to reductions in fishing effort in order to keep the stocks healthier. However, having had informal conversations with ranchers in the Okanagan valley, BC, I have found that they may be reluctant to decrease the stocking rate as much as the "+Risk + CC" model indicates. Moreover, as the sensitivity studies have demonstrated, the effects of price on the decision to stock the range are very strong and may supersede the management implications of implementing a 51% reduction in stocking rate. Moreover, due to the uncertainties regarding the catalytic effects of climate change on the YST invasion, I would approach the above number with care.[28] However, given the results of the "+Risk" scenario, an 18.8% reduction in the stocking rate led to only a 7% reduction in yearly profits and may be well worth implementing as a risk management approach. Moreover, even though ranchers are not concerned about the potential effects of YST at the moment, their concerns might change once YST starts to establish initial populations in British Columbia.

Decreasing the stocking rate to levels outlined by the "+Risk" scenarios has a reductive effect on the probability of invasion. However, it is important to address two

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[28] Please refer to the next chapter for more information on the limitations of this model.

spatial considerations: (1) cattle distribution on the pasture and (2) the issue of accounting for vectors of invasion. Supported by conversations with ranchers in the Okanagan valley and by existing research by Epanchin-Niell et al. (2010), the heterogeneity and the state of neighbouring landscapes play an important role during the process of invasion. Unsurprisingly, the range's border regions tend to be the most likely pathways of invasion from neighbouring farmers, especially when a neighbour is slow at controlling the spread of invasive species or doesn't control it at all due to the nature of their agricultural activity (e.g. beekeeping). Thus, to account for the heterogeneity of the landscape, Holechek, Pieper & Herbel (1998) suggest using salt blocks, additional feed placement and water sources strategically, in order to distribute cattle according to ranchers land management plans. The aforementioned methods may be used to distributed cattle away from areas that are neighbouring landscapes with no natural borders such as ravines or mountains that could prevent the spread of YST. This way, lower-stocked border regions could promote healthier pasture thus serving as barriers against YST invasion.[29]

Another important consideration in maintaining grassland health and diversity, according to Holechek, Pieper & Herbel (1998), is ensuring that the timing of grazing occurs when grassland productivity is highest. As a result, if the ranch manager wants to graze the border regions of their pasture, thus potentially weakening the grassland competition against YST, it would be best to do this following major rain events but leave border regions understocked during periods of drought. Keeping conservative stocking rates in periods of drought is especially important for preventing drought-tolerant species such as YST. This management regime is referred to as "opportunistic management" because it is aimed at semi-arid and arid ecosystems which tend to have less predictable succession states and thus require a more nuanced and careful management approach. Westoby, Walker, & Noy-Meir (1989) have developed this rangelands management framework to manage the difficulty in maintaining equilibrium grazing conditions under arid dry ecosystems.

The second spatial issue is more complex since invaders don't always spread in distinct, wave-like patterns, they can sometimes jump between states and countries through trade (McAusland & Costello, 2004). Canadian Food Inspection Agency (CFIA), for example, conducts yearly inspections of granaries and feed importers to search for

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[29] This method would also be successful in preventing other IAS from invading rangelands.

new invaders (Canadian Food Inspection Agency, 2017). To pinpoint areas at a higher risk of invasion would require combining bioeconomic modelling with more rigorous spatial analysis which is outside of the scope of this paper. However, given the information outlined above, ranches located next to hay distribution centers or other types of feed distributors could find it useful to maintain healthy grassland communities on the boundaries of their rangelands according to the stocking rate specification of this research project in order to maximise their preventative actions.

8.   Management Recommendations

Prevention, control or eradication of invasive species is considered a public good because such efforts reduce the negative externalities of YST, among other invasive species. The government of British Columbia has developed an invasive species Early Detection and Rapid Response (EDRR) plan that explicitly acknowledges the acceptance of early response protocols as the most cost-effective method for controlling invasive species (IMISWG, 2014). As its name suggests, the main principle of early detection and rapid response protocols is to detect the species early enough before it establishes populations that are too expensive to be eradicated completely (due to size of infestation), thus presenting only perpetual long-term control as a cost-effective management option (Leung et al., 2002). Long-term control, as previously mentioned, can ramp up extensive costs, especially given the indefinite time horizons of such efforts. However, YST is not on the list of priority invaders that trigger the EDDR protocol (IMISWG, 2016), and as a result has no quantitative risk assessment (for BC or any other Canadian province). In the current instance of EDRR, YST compelled only a brief mention in a form of a reiteration of its economic impacts and other indirect damages in the United States. In contrast, a number of not-for-profit organisations such as Central Kootenay Invasive Species Society (CKISS), Okanagan-Similkameen Invasive Species Society (OASIS), and regional districts like Thompson Nicola (TNRD) have all recognised YST as a Category 1 invader in their respective environments (Thompson-Nicola Regional District, 2015; Wikeem, 2007a). Such a discrepancy in risk assessments exemplifies a problem with the risk assessment component of the provincial EDRR protocol which tends to place a heavier weight on the biological characteristics of an invasive species (Wikeem, 2007b).

Provincial ministries (Forests, Lands, Natural Resource Operations & Rural Development), responsible federal authorities (CFIA) and environmental NGOs (e.g. ISCBC, OASISS) should consider the ranges of costs of invasion outlined in this paper. Governmental authorities outlined above, as well as regional invasive species councils could benefit from organising their invasive species budgets toward targeted, species-specific, preventative actions. For example, preventative actions could entail integrating economic impacts from YST, and other IAS more generally, into the process outlined in

the EDRR. At the minimum, making these results accessible to the relevant stakeholders using the infrastructure of governmental platforms would generate a positive externality as a result of greater public awareness and official recognition of the risks of invasive species such as YST.[30] Furthermore, it is especially important to focus on prevention because given the results outlined in the "with invasion" and "with climate change" scenarios ("+YST" and "+YST & CC"), economically optimal management strategies involve only perpetual control. Perpetual control, in the context of optimal control modelling, means keeping the invasion at 91 ha or 124 ha respectively (in the case of "+YST & CC"), and not eradicating the invasion because it would be highly cost-ineffective. As a result, monetary damages outlined in this study should be accounted for by decision makers when evaluating the allocation of funds to EDRR early enough to ensure detection and feasible eradication of YST populations.

When considering the long-term repercussions of allowing YST to invade British Columbia's Okanagan region, ranchers could organise a preventative response that could decrease the risk of invasion without the involvement of governmental agencies or ENGOs. In the long-run, ranchers would benefit if they carry out the stocking rate reductions outlined at the beginning of this chapter. The recommended stocking rate would be between 0.16 AU/ha and 0.095 AU/ha (+Risk & CC), representing an 19-51% reduction in stocking rates from the business as usual scenario (0.22 AU/ha). The aforementioned reductions could be revised further down through opportunistic management actions that take into account the spatial dimension, local climate and the landscape of an average BC ranch. It is certain, however, that they would lead to losses in income outlined in the Results chapter.

In the case where federal and provincial authorities would consider engaging in proactive management of potential invasions through a different mechanism than EDDR, they may also consider expanding existing programs such as the British Columbia AgriStability Enhancement Program.[31] Through this act, in 2017, the BC government helped producers who have suffered from loss of income due natural events like the winter freeze, excessive moisture and wildfires (Anonymous, 2018). As a result, I would recommend that federal and provincial governments responsible for delivering similar

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[30] Yellow starthistle is listed under a regulated pest category on CFIA's website, however, the species information web page does not include any impacts from an economic perspective.

[31] Governmental programs may have overlapping mandates and in some cases may wish to use different tools due to segmentation of budgets (Wikeem, 2007a).

types of financial aid to livestock producers recognise that there is a longer-term benefit to helping ranchers who are willing to take proactive steps in reducing the risk of the YST invasion. AgriStability, for example, could incentivise ranchers to use opportunistic management techniques outlined earlier by helping them recover the lost revenue from diminished stocking rates.

However, given the management recommendations outlined above, it is important to consider the limitations of this type of modelling work. It is noteworthy that the notion of a steady-state solution in the context of a dynamic optimisation model implies that it is the best possible outcome over time given the set of parameters of the model, i.e. price of feed, average size of the ranch, YST spread rate, and every other parameter within the model. Moreover, dynamic optimisation models generate solutions via a construct called "the solution space". However, solution space takes the boundaries based on the author's assumptions about the natural world and how to represent mechanisms within by choosing the most relevant functional forms (e.g. logistic growth of the herd and YST). Those functional forms and their parameters are derived from the literature and carry uncertainties of estimation that have not been explicitly accounted for in this model. Moreover, optimal control models overstate (1) human rationality and the assumption that every business owner is a perfect profit optimiser, (2) may carry uncertainties within the computed parameters and lastly (3) ignore uncertainties associated with stochastic (random) noise that's ubiquitous in natural phenomena.[32] This model doesn't explicitly account for uncertainties with respect to the last two points. Furthermore, this model doesn't consider indirect costs and benefits to ecosystems such as reduction in water availability that is correlated with increases in fire intensity, decrease in biodiversity and tourism revenue as well as revenue from honey producers who use yellow starthistle infestations as a source of feed for bees (Eagle et al., 2007). Lastly, YST spread rate varies greatly across the literature due different climate and ecozones. It is for that reason that I selected the spread rates reported in Washington state due to its geographic proximity to British Columbia. However, they may be on the low end of the range due to the fact that YST arrived to Washington relatively recently (listed as noxious weed in 1988). Moreover, at that time YST reached Washington state, policies to slow the spread of invasives were

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[32] Ritten, Bastian, & Frasier, (2010) have shown that it is possible to integrate stochasticity related to precipitation in their work. This model, however, ignores stochasticity for simplification.

already implemented by state and federal governments aided by local ENGOs (Ditomaso, 2005).

9.   Conclusions

This project aimed to answer two types of questions pertaining to measuring the costs of invasion in a counterfactual reality: (1) where YST was allowed to invade unimpeded, and (2) where YST invasion risk could be controlled through management actions. In both cases, the principal units of analysis were the effect of YST invasion on ranch profits, the magnitude of control costs and the size of invaded area. My estimates indicate that allowing YST to invade could generate up to $858,003,403 of direct costs to the BC ranching industry over a 100-year period and reduce annual profits by 62%, in comparison to the case where the invasion never occurs. YST could be found on as much as 246,888 hectares of private rangeland and improved pasture, representing over 19% of total cattle grazing area. Furthermore, these results are within the range of findings reported in similar studies on yellow starthistle or similar species found in the academic literature, but with a level of specificity and detail that has not yet been explored previously.

A few important management implications emerge from studying the effects of YST in the British Columbia context: (1) the magnitude of damages and perpetual presence of YST (due to cost-ineffectiveness of eradication) stresses the value of prevention and public awareness of the effects of the invasion, and (2) maximising the role of ranchers as stewards of rangelands in generating some level preventative efforts. The results of this study show that it is possible to minimise the risk of YST invasion by controlling the amount of cattle offtake in tandem with lowering the stocking rate. Assuming no effect from climate change, the offtake rate would need to be lowered by an estimated 19%, resulting in a marginally lower annual profit. Given the magnitude of damages mentioned earlier, such a small reduction of profits may appear to be justifiable.

Finally, my sensitivity analysis shows that the results of my modelling are influenced very strongly by fluctuations in beef prices. As a result, decisions regarding the stocking rate are similarly strongly influenced by beef prices, among other factors, albeit to a much lesser degree. Furthermore, from informal conversations with ranchers in the Kootenay region of BC, fluctuating profitability in the ranching industry may present a psychological barrier to consciously lowering the stocking rates below current

rates. Given this issue, I recommend controlling the stocking rate by accounting for spatial features of a given rangeland, e.g. keeping stocking rates lower in areas bordering other land-users, especially high-risk ones (e.g. non-agricultural operations). To conclude, ranchers are on the forefront of fighting biological invasions and have to deal with the resulting costs first hand. It is in their interest to keep their rangelands YST-free, and to do that, the results outlined in this study may be helpful for improving operating procedures while capitalising on the local knowledge of the landscape.

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Appendix

I.   PCA, Survival and Hazard Analyses

I follow previous analyses by Barbier et al. (2011) and Knowler & Barbier (2005) who adopted a duration model to represent invasion hazard by analysing the effect of biological characteristics of herbaceous North American plants on the period of time that it has taken for some of these plants to become invasive. According to Kiefer (1988), the "spell" refers to a number of periods before "failure" occurs. In this model the spell refers to the number of periods before invasion of the ecosystem by the yellow starthistle occurs. To reiterate the assumption from the Methods section, I assume that BC rangelands are experiencing propagule pressure from two sources (1) natural expansion of YST in N. Washington and (2) the increase in trade in seeds and cereals with states that have out of control infestations. The baseline hazard rate $\varphi(\beta_k)$, is derived from performing a Principal Component Analysis (PCA) of plant attributes and using the resulting factor scores as independent variables in an exponential survival model. The sample database consists of 107 entries where 78 (72.9%) were invasive and 29 (27.1%) were non-invasive species that have been introduced to North America since record-keeping began. The database was expanded by including Yellow Starthistle using the coding methodologies described in the glossary of variables attached to the sample database (Knowler, personal communication). Moreover, for consistency, I ensured that the same sources were used to code YST and that the data sources were consistent with the year of creation of the database (2008).[33] Lastly, my coding scheme was verified by a professional biologist. Before carrying the analyses on the YST-updated database, I replicated the existing PCA and survival analysis and ensured that I am getting the same results which can be found in the supplementary materials for this project.[34] PCA generated a total of eight Principal Components (also referred to as factors), explaining 100% of the variance. Although there is no single rule on how many components to keep, I used two tests: the scree plot and the eigenvalues greater than 1 rule, both of which are regarded as the most common decision rules when deciding on a

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[33] Updating the database to the current year would have involved re-checking every plant species in order to verify whether it has become invasive in any of the dozens of different biomes throughout the world.

[34] Can be found in the supplementary materials upon request.

number of factors (Tabachnick & Fidell, 1996, p. 646). The four factor scores explained 66% of total variation. FS1 (continents & global) explains 17.6% of total variance, FS2 (annual & flower) explains 16.4% of the total variance, FS3 (polyploidy, self compatibility & abiotic) explain 16.1% of total variance and FS4 (germination requirements) explains 15.8% of the total variance. The four factor scores for each plant were then used as explanatory variables in the exponential survival model. Table X.x shows the coefficient breakdown. FS4 was dropped from the estimation of the hazard rate due to its insignificance given a high p-value (0.68).

i.   Table A.1 – Survival regression results using PCA scores

Survival Analysis & Hazard Rate				
Variable	Value	Std. Error	Z-Score	P-value
Intercept	5.307	0.121	43.968	0.0000
FS1	-0.2691	0.107	-2.509	0.0121
FS2	0.2686	0.118	2.278	0.0227
FS3	-0.2508	0.106	-2.368	0.0179
FS4	-0.0486	0.119	-0.409	0.6827
N	107			
Phi(average)	0.0051			
Phi (YST)	0.008			

The hazard rate of any given species in the sample becoming invasive is the product of the vector of three principal components' mean and the coefficients of the survival model. In case of an exponential hazard model, it is defined as $\varphi = e^{-\beta x_i}$ where β is the vector of coefficients and x_i is the mean vector of factor scores. Table 2 computes the mean value of φ to be 0.0051 which suggests that the probability of any given species becoming invasive in B.C. has a risk of 0.51%. However when the factor scores are

used for a plant species with the characteristics of YST, it faces a risk of 0.8% which is substantially higher than the average risk of 0.5% for herbaceous species. Thus, although the values are small the difference nevertheless represents 60% in heightened potential for invasion when compared to the average hazard of the sample.

ii.　　Propagule Pressure and Risk of Invasion

Yellow starthistle has been detected in the states immediately adjacent to the border with British Columbia (EDDMapS, 2017), and given that trade in animal feed & seeds is projected to increase according to the Freight Analysis Framework (FAF, 2017) from the five states with active YST invasions (ID, WA, CA, OR, MN), so is the risk of invasion.[35] Thus, I assume that there is growing propagule pressure from infested areas immediately south of the border and an additional risk from imports in cereals and seeds where YST seeds are most likely to be present. The hazard rate computed in the previous section is scaled by the amount of offtake chosen by ranch managers. It is represented by the *f(α)* function that scales the risk of a plant being invasive down if *f(α)* < 1 and up if *f(α)* > 1. For example, if offtake is greater than 55.9 AU/season (as per "no YST" scenario), the risk of invasion is being diminished, similarly, if the offtake rate less than 55.9 AU/season, the risk of invasion by YST is increased. I assume that a one-third reduction in offtake increases risk threefold in a linear fashion akin to Barbier et al. (2011). Based on the above assumption I formulate a risk scaling function described in section seven.

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[35] For more details on the FAF projections on hay, cereal and seed trade volumes to British Columbia, please see figure 2 in the Appendix.

iii.   Survival & Hazard Analyses Results (graphed)

IAS Survival Analysis
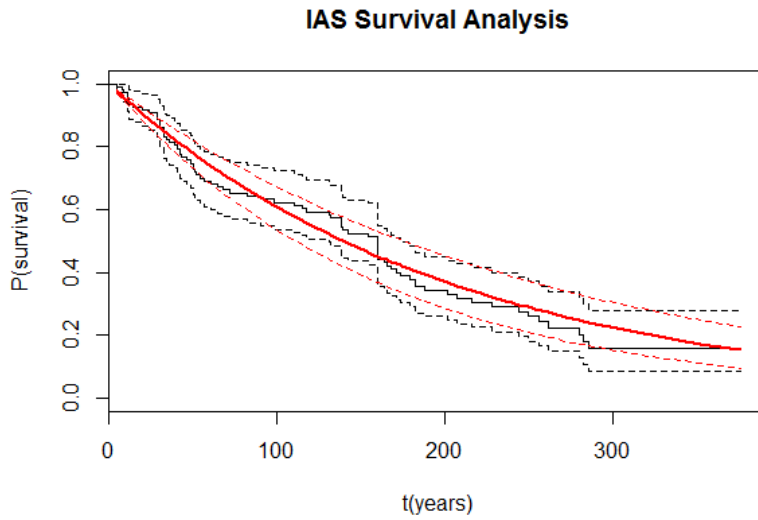
Figure A1   Probability of ecosystem being IAS-free over 300 years

Hazard Rates Over Time
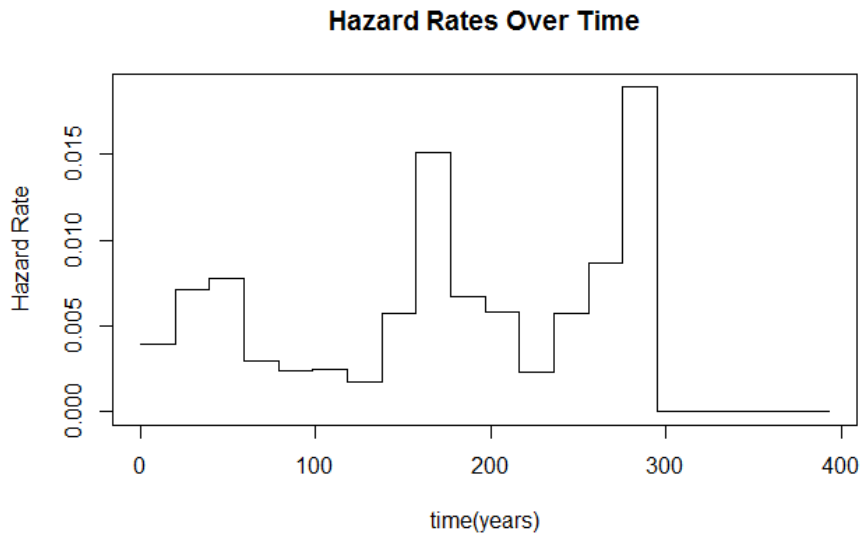
Fig. 6 – Hazard rate of invasion over 300 year period: increase of hazard rate over 300 year period.

II. Table A.2 Parameters for Empirical Analysis

Variable Description	Variable/ Parameter	Value	Source
Intrinsic growth rate of Yellow Starthistle	*y*	*0.1684*	Lass, L.W., McCaffrey, J.P., Callihan, R.H. (1999)
Size of the herd (cow-calf) per season	*X*	**State Variable #1**	N/A
Growth rate of the herd	*r*	*0.5647*	Statistics Canada, (2015)[36]
Annual Sales	*N*	**Control Variable #1**	N/A
Net selling price of animal	*P*	*C$1342.98*	Statistics Canada, (2017)
Land rental cost	P_a	*C$20/ha*	Rupananda (2012)
Marketing & trucking costs	P_m	*C$40/cow-calf*	Rupananda (2012)
Hectares of rangeland required per animal per season	*z*	*9.6*	Holechek, Pieper & Herbel (1998)
YST PCA & survival analysis risk	*γ*	*0.008*	See "Estimation of the hazard function"
Size of representative rangeland area	*L*	*495.21 ha*	BCCA (2017)
Maximum stocking rate	*a*	1.5987 AU/ha	Wikeem et al. (1993)

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[36] See Table A.4 for the details of the growth rate calculation

Yellow Starthistle infested land area	U	*State Variable #2*	N/A
Holding cost	P_h	*$65.15 per cow-calf*	Rupananda (2012)
Price of replacement feed	P_t	*$180 per tonne*	Alberta Agriculture and Forestry, (2017)
Wage rate	w	*$160/day*	Rupananda (2012)
Interest rate	δ	*0.05*	Rupananda (2012)
Feed demand per cow-calf unit per grazing season	g	*2324.48 kg*	Alberta Agriculture and Forestry, (2017)
Number of person-days required for controlling YST	D	*Control Variable #2*	N/A
Catchability coefficient (aka control effectiveness)	β	*0.0026*	Rupananda (2012)
Threshold offtake	$\tilde{N}$	*53.41*	Statistics Canada, (2015)[37]

III. Table A.3 *Centaurea solstitialis* Spread Rates Calculation and Sources

Variable	Calculation on a yearly basis (annual spread rate)	Geographic Region	Source
YST growth rate: y_1	*$((10-1)/1)^{(1/15)}=1.1577$*	*California*	(Bisson, H. 1999)

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[37] Derivation can be found in section VI of the Appendix.

YST growth rate: y_2	$((1-0.1338)/0.1338)^{(1/12)}=1.1684$	*Washington*	**(Duncan, C. 2001)**[38]
YST growth rate: y_3	$((0.95-0.001)/0.001)^{(1/12)}=1.46$	*Oregon*	(Duncan, C. 2001)
YST growth rate: y_4	$((600-0.025)/0.025)^{(1/46)}= 1.25$	*Idaho*	(Lass, L. W. et al. 1999)
YST growth rate: y_5	*1.13-1.17*	*Western U.S.*	(C. A. Duncan et al., 2004)

IV. Table A.4 – BC Cow-Calf Parturition Rate Calculation

Year	**Calves Born** (100,000s)	**Beginning-of-year inventory**	**Calving Rate**
2006	291.30	540.20	0.54
2007	328.90	519.30	0.63
2008	288.60	514.30	0.56
2009	263.20	494.00	0.53
2010	261.20	447.10	0.58
2011	238.20	443.00	0.54
Average			**0.5647**

Note that growth rates are calculated using the following formula: Calves/(Cows+Bulls+Steers). They are then averaged over the 6 years that these data were made available from Statistics Canada (2015).

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[38] Due to the geographic proximity of the Washington State to British Columbia, the similarity of climates between the two interior regions and the growth rates confirmed in the literature (Duncan et al., 2004), I am only using the Washington rate of YST propagation in my model.

V. Table A.5 – Southern Interior Forage Production

Low (kg/ha)	High (kg/ha)	Average Value (kg/ha)	Condition	Grassland type & Geographical area	Source
Source of all forage data: Wikeem et al. (1993)					
400	900	650	excellent	near Kamloops	McLean & Marchand (1964)
		1000	Not Reported	Penticton	Marchand (1964)
250	500	375	poor to excellent	near Kamloops	McLean & Marchand (1968)
		1300	excellent	north of Kamloops	Wikeem et al. (1989)
475	2700	1100	excellent	Southern Interior	McLean & Marchand (1968)
200	1000	475	Not Reported	Ponderosa Pine near Kamloops	McLean & Marchand (1968)
		800	Not Reported	Ponderosa Pine near Penticton	McLean et al. (1971)
		660	Not Reported	bluebunch wheatgrass near Williams Lake	Wikeem & Newman (unpubl. Data)
		110	poor	lower grasslands	McLean & Marchand (1968)
		280	fair	lower grasslands	McLean & Marchand (1968)
		450	good	lower grasslands	McLean & Marchand (1968)
		620	excellent	lower grasslands	McLean & Marchand (1968)
335	800	567.5	good/excellent	Skookumchuck Prarie near Cranbrook	McLean & Smith (1973)
500	800	650	Not Reported	Douglas fir near Cranbrook	McLean & Smith (1973)

			Not	Idaho fescue near	
		900	Reported	Similkameen	McLean & Smith (1973)
		450	Not Reported	Idaho fescue near Similkameen	McLean et. al (1970)
273	675	474	Not Reported	pinegrass	McLean et. al (1971)
100	725	412.5	Not Reported	Douglas fir near Kamloops	Tisdale (1950)
Average:		**580.14 kg/ha**			

VI.   Calculating the *a,* the *a~ and N~* parameters

a = 0.8(1000lb cow-calf unit) = 800 pounds or 362.874 kg.

Average forage production for interior BC: 580.14 kg/ha. *a*=580.14/362.874=1.5987 AU/ha

$\tilde{a}$*=0.35*(a).* I can now use this value to find the threshold $\tilde{N}$ value from equation (19).

Solving equation (19) for $\tilde{N} = \tilde{X} - L\tilde{a}$ I get a threshold value of 55.9.

VII.   Sensitivity Analyses Tables

Table A.6 – "No YST" model with 15% variation in carrying capacity, price of beef and herd growth rate

"no YST"					
Herd size (X), AUE	Offtake (N), AUE	Labour Demand (D), person days	Infested Land (U) hectares	NPV of profits for 100 years, ($)	Stocking Rate, AU/ha
Carrying capacity, a (AU/ha)					
66.94	33.00	0.00	0.00	$421,595.11	0.14
81.30	40.07	0.00	0.00	$506,279.50	0.16

95.64	**47.14**	**0.00**	**0.00**	**$512,079.22**	**0.19**
109.99	54.21	0.00	0.00	$557,311.84	0.22
124.32	61.27	0.00	0.00	$602,487.87	0.25
Price, P ($)					
-14.96	-8.62	0.00	0.00	-$383,967.41	-0.03
50.45	26.58	0.00	0.00	$307,536.54	0.10
95.64	**47.14**	**0.00**	**0.00**	**$512,079.22**	**0.19**
128.73	60.25	0.00	0.00	$774,746.76	0.26
154.00	69.16	0.00	0.00	$1,074,931.40	0.31
Herd growth rate, r					
-24.53	-10.01	0.00	0.00	-$424,205.11	-0.05
83.89	35.78	0.00	0.00	$374,259.60	0.17
95.64	**47.14**	**0.00**	**0.00**	**$512,079.22**	**0.19**
104.33	58.35	0.00	0.00	$649,898.87	0.21
111.01	69.47	0.00	0.00	$787,718.54	0.06

Table A.7 – "+YST" model with 15% variation in the wage rate (control effort), price of beef and YST growth rate.

"+ YST"					
Herd size (X), AUE	Offtake (N), AUE	Labour Demand (D), person days	Infested Land (U) hectares	NPV of profits for 100 years, ($)	Stocking Rate, AU/ha
Wage, w ($)					
84.30	41.56	61.12	55.30	$292,141.04	0.17
80.86	39.86	58.58	72.23	$244,491.32	0.16
77.03	**37.97**	**54.83**	**91.55**	**$197,099.99**	**0.16**
72.31	35.65	52.33	114.35	$148,019.04	0.15
124.80	19.73	46.97	132.52	$83,355.57	0.14
Price of Beef, P					
-9.68	-5.58	44.78	166.13	-$519,462.99	-0.02
35.33	18.61	48.42	141.04	$172,450.53	0.07

77.03	**37.97**	**54.83**	**91.55**	**$197,099.99**	**0.16**
112.13	52.48	60.31	60.67	$457,755.39	0.23
139.87	62.81	62.95	43.18	$758,133.47	0.28
YST growth rate, y					
80.64	39.75	39.95	73.31	$274,071.88	0.16
78.97	24.46	48.05	75.71	$235,889.01	0.16
77.03	**37.97**	**54.83**	**91.55**	**$197,099.99**	**0.16**
74.36	35.08	62.58	104.25	$155,561.71	0.15
71.16	21.33	66.59	120.00	$113,376.80	0.14

Table A.8 – "+YST & CC" model with 15% variation in carrying capacity, price of beef and price of feed

+YST&CC					
YST growth rate, y					
77.88	38.39	52.35	86.90	$ 214,159.45	0.16
74.68	36.81	61.86	102.69	$ 160,200.49	0.15
70.47	**34.73**	**69.52**	**123.84**	**$ 111,780.20**	**0.14**
64.42	31.76	74.05	153.21	$ 45,405.67	0.13
56.41	27.81	74.05	192.65	-$ 13,974.99	0.11
Price of Beef (P)					
-9.68	-5.58	44.78	166.13	-$519,462.99	-0.02
35.33	18.61	48.42	141.04	$2,450.53	0.07
77.03	**37.97**	**54.83**	**91.55**	**$197,099.99**	**0.16**
112.13	52.48	60.31	60.67	$457,755.39	0.23
139.87	62.81	62.95	43.18	$758,133.47	0.28

Table A.9 – "W/ Risk" model with 15% variation in carrying capacity, price of beef and herd growth rate

Herd size (X), AUE	Offtake (N), AUE	Labour Demand (D), person days	Infested Land (U) hectares	NPV of profits for 100 years, ($)	Stocking Rate, AU/ha
"+ Risk"					
Carrying capacity, a (AU/ha)					
63.45	31.73	0.00	0.00	$415,714.56	0.13
71.67	36.16	0.00	0.00	$448,717.13	0.14
77.65	**39.55**	**0.00**	**0.00**	**$475,267.78**	**0.16**
84.52	43.30	0.00	0.00	$504,666.39	0.17
90.08	46.41	0.00	0.00	$529,680.26	0.18
Price, P ($)					
no solution					
31.76	17.22	0.00	0.00	$138,906.54	0.06
77.65	**39.55**	**0.00**	**0.00**	**$475,267.78**	**0.16**
106.82	52.18	0.00	0.00	$719,995.78	0.22
124.72	59.33	0.00	0.00	$1,000,657.94	0.25
Herd growth rate, r					
43.91	16.39	0.00	0.00	$126,034.80	0.09
63.85	28.18	0.00	0.00	$319,765.36	0.13
77.65	**39.55**	**0.00**	**0.00**	**$475,267.78**	**0.16**
87.77	50.68	0.00	0.00	$661,561.75	0.18
95.52	61.66	0.00	0.00	$867,841.41	0.19