DB versus DC: A Comparison of Total Compensation

by

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Abstract

Employer-sponsored pension plans play an important role in providing employees with adequate retirement income. They are expensive and carry some important risks. The employer and its employees share these costs and risks differently depending on the plan design. In this project, two designs are studied, a defined benefit (DB) plan and a defined contribution (DC) plan. They are analyzed in a simple common business setup under the same stochastic economic scenarios generated from a calibrated VAR model. The employer's total compensation budget is assumed to be constrained so that higher pension contributions are associated with lower salary increases, and vice versa. The two types of plans are compared based on the total compensation, defined as the value of wages and retirement income, received by 25 cohorts of new employees. On an adjusted basis, we find that the two types of plans provide equivalent total compensation to their members.

Keywords: Pension plan; DB plan; DC plan; salary; contribution; retirement income; total compensation

Dedication

To my beloved Laoba, Laoma, mother, husband and my adorable daughter!

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Chapter 1

Introduction

1.1 Background and Motivation

During the past decades, comparisons between defined benefit (DB) pension plans and defined contribution (DC) plans has been a hot topic in the literature. Over the last 30 years, a gradual shift away from DB plans to DC plans occurred in several countries, especially in the United Kingdom and the United States (Broadbent et al. 2006), and to some extent in Canada. For instance, 41% of Canadian employees were covered by a DB plan in 1991, and that proportion was down to 30% in 2006. The main factors contributing to this shift include the low portability of DB plans; a changing investment climate producing lower stock returns and falling interest rates since 2000 which put a huge pressure on employers sponsoring defined benefit plans. Also, current generations are reluctant to contribute more in order to eliminate funding deficits generated by previous cohorts who did not save enough to support the cost of their retirement.

By transferring the investment risk to the employees, DC plans have lower volatility of costs. This reduction in the risk to the employer comes at the expense of a greater risk to the employees upon retirement. With a DC plan, retirees bear some significant risks. They might not have saved enough during their working life, resulting in low income during retirement. If they consider buying an annuity, they may have to buy it at a time when annuities are expensive (e.g. when interest rates are low). If they choose not to annuitize, they face the risk of living longer than their retirement account was designed for, this is the longevity risk.

Although DC plans can outperform DB plans in periods of economic growth where markets are buoyant, they don't perform as well during more turbulent periods like the financial crisis of 2008. A recent survey by Willis-Towers-Watson showed that employees with DC plans had to work longer into their retirement years in order to support their lifestyle. Employers may have to decide whether to keep employees at a declining productivity and lower

engagement or pay out a severance in order to terminate long-term employees. As a result, a shift from a DB plan to a DC plan does not fully transfer the risk from the employer to the employees.

Some employers that have converted from a DB plan to DC plan are considering converting back to a DB plan. Brown and McInnes (2014) mention two U.S. employers in the public sector, the States of Nebraska and West Virginia, that had converted to DC converted back to DB, at least partially, because of concerns over the level of income to retirees.

Many studies compare pension plans, see, for example, the bibliography cited in Brown and McInnes (2014) and Sanders (2016). DB and DC plans have been compared in terms of the risks involved and who bears those risks, the costs (contribution rates) and their volatility, the level of retirement income provided by each type of plan. The comparison is even affected by the type of employer. Private sector employers have different concerns than public sector employers when it comes to sponsoring pension plans. Therefore, the perceived advantages of each type of plan in one sector may not apply in the other sector.

Retirement benefits are an important part of negotiated compensation packages. Although their main goal is to secure adequate retirement income at a reasonable cost, they can help attract and retain qualified employees. To Quote Brown and McInnes (2014), "Retirement is expensive and someone has to cover the costs". We add that securing retirement income is a risky business and someone must bear those risks.

The main goal of this project is to compare the total compensation, that is, salaries and retirement benefits, received by members of each type of pension plans, DB and DC. The comparison is done in an arbitrarily chosen business setup in which an employer has a fixed budget, indexed to inflation, to spend on salaries and pension plan costs.

1.2   Outline

The setup of the project is as follows. Chapter 2 provides some background on pension plans focusing on studies comparing DB and DC plans. The VAR model and a description of the economic scenario generations are introduced in Chapter 3. Chapter 4 describes the design of the defined benefit (DB) plan studied here and its funding rules in our budget-controlled environment. Simulations are used to project contributions made to the plan and salaries earned during the working lives of 25 cohorts of new entrants. Chapter 5 compares the salaries earned and the lump sum retirement funds that cohorts of employees in a DC plan would get under the same economic scenarios. In Chapter 6, present values are adopted

as measures to compare the total compensation received by different cohorts of employees between DB plan and DC plan. Chapter 7 summarizes our findings and briefly discusses possible extensions for future work.

Chapter 2

Literature Review

Samwick and Skinner (1998) compared actual retirement benefits provided by DB plans with those provided by DC plans. Using simulated earnings histories, they found that randomly selected DC plans from 1995 outperformed DB plans from 1983, by providing substantially higher average and median benefits. The DB plans were selected at a time when they covered most workers with a pension plan. DC plans were selected shortly after they covered more workers than DB plans. They claim that their result is robust to changes in equity returns, productivity and earnings uncertainty. However, they did find evidence that DC plans are riskier than DB plans in the sense that there is more variability in the level of retirement benefits. DC plans also provided worse benefits in the lower 20% of the cases.

There are limitations of their study, some discussed by the authors, that could affect the conclusion. First, one can argue that averages and medians are not good measures to compare an individual's pension benefits under different pension plans. As proposed by Blake, Cairns and Dowd (2001), Value-at-Risk (VaR) measures which are percentages of plan members who will receive more or less than a certain benefit is much more meaningful than the mean or median. Second, this research was done two decades ago when stocks and bonds provided relatively higher rates of returns. These results may not apply in the current economic conditions of low interest rates and improved longevity of retirees.

One inspiring idea from this paper is that they pointed out that what matters is the total compensation package of the workers including wages and all benefits received from the employer. Studying this total compensation question is the main goal of this project.

Blake, Cairns and Dowd (2001) used simulations to study the ratio of DC pension to DB pension for several asset models and a range of asset allocation strategies. They found that DC plans can be exposed to extreme risk compared to a DB benchmark. However, the DC plan possessed the advantage of offering greater portability than a DB plan by allowing members to easily transfer their pension fund when changing jobs. Value-at-risk

is adopted as a robust and well-established risk measure to compare pension ratios. Unlike estimates of the moments, VaR statistics are not driven by extreme outcomes. They found that VaR estimates were most sensitive to the choice of asset allocation strategy, less to the choice of asset return parameters and least to asset return models. Another of their conclusions is that a static asset-allocation strategy with high proportion of equity in the investment portfolios delivered better results than any of the dynamic strategies including lifestyle strategies. This is an important finding given that lifestyle strategies were the cornerstone of many DC plans. Nevertheless, they admitted that this might be a tentative conclusion and further investigation is needed since with limited amount of historical data, all asset models adopted in this paper produced high mean equity returns. If the dataset or parameter values of the asset models were changed, the results may be different to some extent.

Blake, Cairns and Dowd (2003) compared different distribution programmes of retirement income with a conventional life annuity. Some countries have no mandatory regulations forcing pensioners to purchase life annuities, or only impose retirees to purchase annuities at a specific age. Only few pensioners are voluntarily willing to annuitize their funds. Strong bequest motive, self-perceived poor health condition and willingness to manage the asset themselves for sophisticated individuals may be key reasons for those reluctant pensioners. As a result, alternative decumulation strategies exist such as Equity-linked annuities (ELA) and Equity-linked income-drawdown annuities with bequest. These two decumulation strategies together with a conventional life annuity are compared in terms of expected discounted utilities, which are determined by the individuals' levels of risk aversion. The authors determined the optimal asset allocations by maximizing the expected discounted utility function. The main conclusion drawn is that the optimal choice of distribution programme is determined mostly by the proportion of assets invested in equity, and is totally insensitive to the plan member's bequest motive.

McCarthy (2003) adopted a lifecycle model to study the performance of a defined benefit plan bearing the investment and wage risks, and examined the tradeoffs between defined contribution plans and defined benefit plans. The author emphasized wage risk and modeled individuals' wage instead of projecting wages by fixed formulas. Since the retirement benefit of a DB plan is determined by final salaries of the member, it makes DB plans much more sensitive to wage variability than DC plans. Under the assumptions made in the paper, the author concludes that DC plans, with the flexibility of investing assets in stock market, would be preferred by young worker. Since most of young workers' wealth is human capital, equities provide opportunities to diversify away from human capital. As a worker ages, more is known about the final wage which lowers the uncertainty regarding the benefit provided by DB plans. As a result, older workers would benefit from choosing DB

plans since it would provide a valuable diversification of wealth out of financial markets. Other interesting findings include the fact that DB plans were more valuable to workers when wage variability was low, equity return was low and for those individuals who were financially wealthier.

Over the past two decades, with the emergence of Hybrid plans, studies have started to explore the costs, risks and intergenerational risk sharing among traditional plans and modern plans.

Bovenberg et al. (2007) compared costs and benefits of collective pension schemes versus individual schemes in terms of saving and investing behaviors over the life cycle. They found that an advantage of collective pension schemes is that they can relieve borrowing constraints and allow intergenerational risk sharing. However, this ignored the diverse specific preferences and circumstances of different individuals by imposing uniform rules on heterogeneous participants and introducing other constraints. They also use a simple setting in which wage income is riskless.

Cui et al. (2011) studied welfare aspects of intergenerational risk sharing by conducting comparisons among collective plans and the optimal individual DC pension scheme. They study the life cycle consumption that optimizes a discounted utility function for specific pension schemes. Their model assumes that employees earn a flat real labor income throughout their working careers. The paper concludes that collective pension schemes with IRS would be welfare improved over the optimal individual benchmark, and that welfare gains for a new entrant did not come at the cost of other cohorts. In short, the schemes with IRS were zero-sum games in market value terms but were positive-sum games in welfare terms. As a result, the recognition of the welfare aspects played an important role. They also agreed with former papers by Campbell and Nosbusch (2007) and Gollier (2008) that schemes with IRS would be more aggressive due to their enhanced capacity of risk-bearing.

Blommestein et al. (2009) analyzed the trade-off between volatility in contributions and uncertainty of benefits from the perspective of a plan member for a variety of pension plans. It included a traditional DC plan as a benchmark and other types of pension arrangements. They adopted the funding ratio and the replacement rate as variables to indicate the level of contributions required and the standard of living provided by benefit payments respectively. The arrangements with higher stability in funding ratio led to a greater variability in replacement ratio, in other words, members who had greater security of consumption level while active would face volatile consumption changes around retirement age. The simulation results showed that hybrid plans had higher efficiency and sustainability in risk sharing which ensures a more predictable pension benefit while maintaining some security in contri-

bution costs. The authors do mention that although it is the employer's contribution rate that tend to fluctuate for Defined Benefit plans, implicitly employees are also paying when contributions increase if the total compensation is fixed. However, they do not evaluate the pension designs based on the total compensation received by the employees.

Hoevenaars and Ponds (2008) demonstrated that value-based generational accounting is an important and useful decision making tool when analyzing pension funds with intergenerational risk sharing. In order to avoid the problem of valuation in an incomplete market, they assumed that the real wage growth is zero. Results revealed the zero-sum feature in value terms, which implies that any policy change will inevitably lead to value transfers among generations of participants. For illustration, a less risky asset mix or a reallocation of risk bearing from flexible benefits to flexible contributions would benefit older members at the expense of younger members.

Chapter 3

Economic Scenario Generator

3.1 The VAR Model

The vector autoregressive (VAR) model is a flexible and easily implemented stochastic process that captures the linear interdependencies between multiple time series. It explains the evolution of each variable as a linear function of its past value as well as the past values of the other variables in the model, and random innovations.

As in Li (2017), we adopt a first-order VAR process to model our selection of economic variables. The m-dimensional VAR process can be written as:

$$z_{t+1} = \Phi z_t + P\epsilon_{t+1}, \tag{3.1}$$

where z_t is a $(m \times 1)$ vector consisting of mean-adjusted variables and $\epsilon_{t+1} \overset{i.i.d}{\sim} N(0, I)$ is a $(m \times 1)$ vector of innovations. Φ and P are both $(m \times m)$ matrices. Φ is the correlation coefficient matrix of the VAR model. P is the Cholesky decomposition of the contemporaneous variance-covariance matrix Σ of the innovations. More precisely, P is a lower triangular non-singular matrix with positive diagonal elements such that the variance-covariance matrix is equal to the following product

$$\Sigma = PP^T. \tag{3.2}$$

Consequently,

$$E\left(P\epsilon_t(P\epsilon_t)^T\right) = \Sigma. \tag{3.3}$$

The four economic variables used in this project are inflation, short-term interest rate, long-term interest rate and equity return. They are described in more detail in the next section. Denoting the vector of our four economic variables at time t by x_t, the VAR model

that will be estimated from past data and used to generate future economic scenarios is given by Equation (3.1) where

$$z_t = x_t - \mu, \tag{3.4}$$

and μ is the mean vector of the process.

The model can be rewritten as

$$x_{t+1} - \mu = \Phi(x_t - \mu) + P\epsilon_{t+1}. \tag{3.5}$$

3.2 Economic Variables

The comparisons made between DB plans and DC plans in this project rely on future projections of four economic variables. The salaries paid to workers and the contributions made to the pension plans will depend on the total budget of the employer. This budget will be assumed to increase at a rate consistent with inflation. Assets will be allocated between fixed-income securities and equities. The actuarial liabilities of the plans will be valued at a rate that is a function of short-term and long-term interest rates, as well as equity returns. The assumptions, plan designs and the business setup are discussed in the next two chapters.

In this section, we describe the four variables used in the VAR model. We follow the notation introduced in Li (2017) and use a tilde to distinguish monthly rates from the corresponding annual rates.

On account of the monetary policy adopted by the Bank and the Government of Canada in 1991, and renewed regularly since then, which aims at keeping inflation within a target range of 1 to 3 percent, data prior to 1991 are excluded. A dataset of rates at an annual frequency would contain too few data points to reliably estimate the parameters of the VAR model. At the highest frequency readily available, that is daily values, the manipulation of the data set would become tedious and unnecessary. Consequently, monthly data from May 1991 to April 2016 are used in this project.

The percentage change in the Consumer Price Index(CPI) is a widely used indicator of inflation. Inflation rates are calculated from monthly CPI data available in CANSIM CPI Table 326-0020. Here, the instantaneous monthly rate of inflation during period (t-1, t], $\tilde{\lambda}_t$, is defined as:

$$\tilde{\lambda}_t = ln \frac{CPI_t}{CPI_{t-1}}, \tag{3.6}$$

where CPI_t is the value of the Consumer Price Index at the end of month t. The annualized rate of inflation is $\lambda_t = 12\tilde{\lambda}_t$.

The yield on 1-month Treasury bill is used as a proxy for the short-term interest rate. The historical data for this variable was obtained from CANSIM Table 176-0043. The yield quoted for month t, which we denote by P_t^1, is quoted as an annual effective rate, and as a percentage. We get the instantaneous monthly short-term rate, $\tilde{y}_t^1$ using the following transformation:

$$\tilde{y}_t^1 = \frac{1}{12} ln \left(1 + \frac{P_t^1}{100} \right). \tag{3.7}$$

As a proxy for the long-term interest rate, we arbitrarily choose the yield on a 10-year zero-coupon bond. These historical yields are extracted from the website of the Bank of Canada, where yields on zero-coupon bonds with maturity dates ranging from three months to 30 years are available on a daily basis. The 10-year annual effective yields, i_t^{120}, were retrieved at a monthly frequency, using values quoted on the first trading day of each month. The superscript stands for 120 months until maturity. For consistency, the annual effective yields are transformed into instantaneous monthly rates, $\tilde{y}_t^{120}$. The monthly rate at time t is:

$$\tilde{y}_t^{120} = \frac{1}{12} ln \left(1 + i_t^{120} \right). \tag{3.8}$$

For the equity return, we use the total return on Canadian equities which includes the capital gains and the distribution of dividends over time. The monthly Canadian dividend yields, DY_t, and the S&P/TSX composite index values, CI_t, at the end of month t were retrieved from CANSIM Table 176-0047 (where the index for year 2000 is set at 1000). We define the instantaneous monthly rate of return on Canadian equities during month t as:

$$\tilde{\pi}_t^S = ln \left[\frac{CI_t}{CI_{t-1}} + \frac{DY_t}{12} \right], \tag{3.9}$$

3.3 Historical Data

Plots of historical annualized rates for the four economic variables from 1991 to 2016 are presented in Figure 3.1. Recall that the historical values consist of monthly rates observed at a monthly frequency. These observed monthly rates have been multiplied by 12 to get corresponding annualized values and are therefore shown in the same scale as the one most commonly used in practice.

Overall, there is no obvious trend in inflation and in equity returns. Inflation rates are less volatile than equity returns. The fact that the annualized inflation rates appear to be quite volatile given Canada's monetary policy is due to the measurement period of one month. The monetary policy has a target of 1 to 3 percent inflation per annum over a medium horizon. Aggregating inflation rates over a period of a few months would reveal that the monetary policy has indeed kept inflation around its target value since 1991. Large negative equity returns in 1998 and 2008 reflect some financial crisis during those periods. As for bond yields, one notices a downward trend during the chosen period and much smaller fluctuations over short periods. This may suggest that bond yields exhibit some strong auto-correlation. Finally, the yields on short-term bonds have been more volatile, and tended to be lower, than those for long-term bonds since 1991.

3.4 Estimation

The VAR model was estimated using the R package MTS. First the model was estimated without any constraints. The estimated parameters were obviously similar to, but not exactly the same as, the corresponding parameters shown in Table 2.1 of Li (2017). Removing the U.S. equity data from the model produced only slightly different estimates for the reduced model. The estimates of the last row of the correlation coefficient matrix, Φ, suggested that the dependency of the equity return for a given month on past economic variables (including equity itself) is not important. Therefore, equity returns can be modeled by a White Noise process independent of other variables. It was decided to make this a feature of the VAR model by forcing the last row of the matrix Φ to be 0 when estimating the model. This was done by using the VAR command in the MTS package.

Key statistics and estimated parameters of the VAR model are summarized in Table 3.1. Note that since the observations were collected at a monthly frequency, the parameters are appropriate to project economic scenarios at a monthly time step.

Figure 3.1: Historical data (annualized rates)
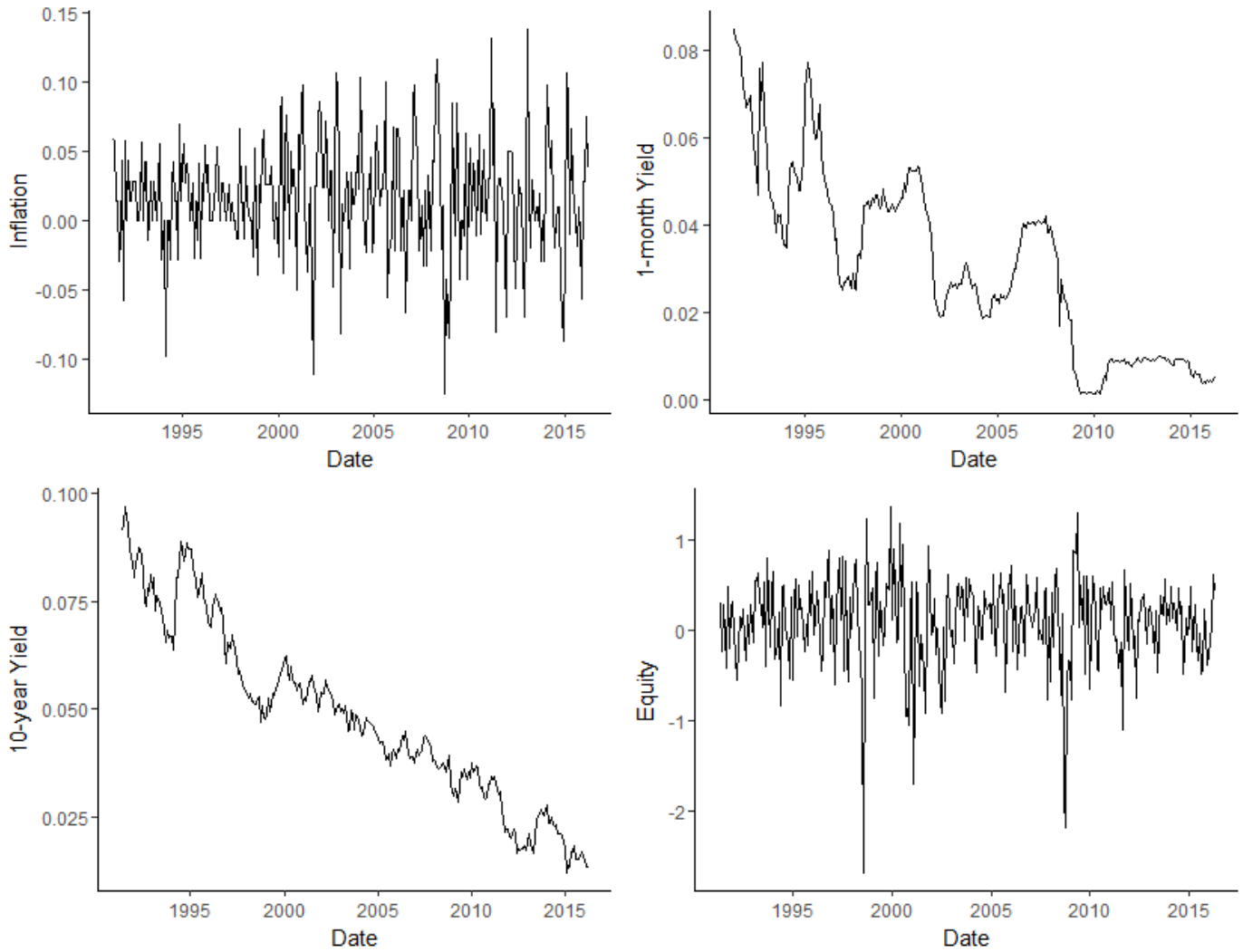
Annualized rates for the four economic variables used in the VAR model. The historical data was retrieved from CANSIM tables and the Bank of Canada website at a monthly frequency from May 1991 to April 2016. The monthly rates were calculated using (3.6)-(3.9) and multiplied by 12 to get the corresponding annual rates.

Table 3.1: Summary statistics and estimated VAR model

a) Summary statistics	$\tilde{\lambda}_t$	$\tilde{y}_t^1$	$\tilde{y}_t^{120}$	$\tilde{\pi}_t^S$
μ	0.00148	0.00257	0.00403	0.00658
σ	0.00338	0.00171	0.00173	0.04199
b) Correlation matrix, Φ	$\tilde{\lambda}_t$	$\tilde{y}_t^1$	$\tilde{y}_t^{120}$	$\tilde{\pi}_t^S$
$\tilde{\lambda}_{t+1}$	0.15818	0.09065	-0.09744	0.01010
$\tilde{y}_{t+1}^1$	-0.00065	0.95112	0.02995	-0.00003
$\tilde{y}_{t+1}^{120}$	0.00082	0.02354	0.97101	-0.00023
$\tilde{\pi}_{t+1}^S$	0	0	0	0
c) Covariance matrix, Σ	$\tilde{\lambda}_t$	$\tilde{y}_t^1$	$\tilde{y}_t^{120}$	$\tilde{\pi}_t^S$
$\tilde{\lambda}_t$	1.0832×10^{-5}	9.1654×10^{-8}	7.1008×10^{-8}	1.5816×10^{-5}
$\tilde{y}_t^1$	9.1654×10^{-8}	7.8612×10^{-8}	7.6624×10^{-9}	-4.5870×10^{-7}
$\tilde{y}_t^{120}$	7.1008×10^{-8}	7.6624×10^{-9}	3.6238×10^{-8}	-5.3583×10^{-7}
$\tilde{\pi}_t^S$	1.5816×10^{-5}	-4.5870×10^{-7}	-5.3582×10^{-7}	1.7621×10^{-3}
d) Cholesky matrix, P	$\tilde{\lambda}_t$	$\tilde{y}_t^1$	$\tilde{y}_t^{120}$	$\tilde{\pi}_t^S$
$\tilde{\lambda}_t$	3.2912×10^{-3}	0	0	0
$\tilde{y}_t^1$	2.7848×10^{-5}	2.7899×10^{-4}	0	0
$\tilde{y}_t^{120}$	2.1575×10^{-5}	2.5311×10^{-5}	1.8743×10^{-4}	0
$\tilde{\pi}_t^S$	4.8057×10^{-3}	-2.1238×10^{-3}	-3.1251×10^{-3}	4.1530×10^{-2}

Historical equity returns have the highest average and largest volatility among the four variables. Inflation is weakly correlated with its immediate past value. Based on the last column of panel b), inflation depends more on the past equity return than the bond yields do. The second and third elements on the diagonal of Φ are fairly close to 1 suggesting, as expected, that bond yields in consecutive months are highly correlated. Panel (c), with its small values in rows 2 and 3 relative to other values in Σ, confirms that bond yields are relatively stable over short periods.

The environment in which the DB plan is assumed to operate (and described in the next Chapter) makes the contribution rates and salary increases quite sensitive to extreme returns on equity. Further, it is known that when projecting DB plans over a long period, funds may end up so large that contribution holidays will occur over long periods, or the plan may end up seriously underfunded requiring unreasonably large contribution rates. Since the goal is to compare DB and DC plans in normal operating conditions, it is deemed appropriate to limit the number of extreme scenarios. With the estimated parameters of the

Table 3.2: Calibrated Variance-Covariance Matrix and its Cholesky Decomposition

c) Covariance matrix, Σ	$\tilde{\lambda}_t$	$\tilde{y}_t^1$	$\tilde{y}_t^{120}$	$\tilde{\pi}_t^S$
$\tilde{\lambda}_t$	$1.0832\text{x}10^{-5}$	$9.1654\text{x}10^{-8}$	$7.1008\text{x}10^{-8}$	$7.9082\text{x}10^{-6}$
$\tilde{y}_t^1$	$9.1654\text{x}10^{-8}4$	$7.8612\text{x}10^{-8}$	$7.6624\text{x}10^{-9}$	$-2.2935\text{x}10^{-7}$
$\tilde{y}_t^{120}$	$7.1008\text{x}10^{-8}$	$7.6624\text{x}10^{-9}$	$3.6238\text{x}10^{-8}$	$-2.6791\text{x}10^{-7}$
$\tilde{\pi}_t^S$	$7.9082\text{x}10^{-6}$	$-2.2935\text{x}10^{-7}$	$-2.6791\text{x}10^{-7}$	$4.4052\text{x}10^{-4}$
d) Cholesky Matrix, P	$\tilde{\lambda}_t$	$\tilde{y}_t^1$	$\tilde{y}_t^{120}$	$\tilde{\pi}_t^S$
$\tilde{\lambda}_t$	$3.2912\text{x}10^{-3}$	0	0	0
$\tilde{y}_t^1$	$2.7848\text{x}10^{-5}$	$2.7899\text{x}10^{-4}$	0	0
$\tilde{y}_t^{120}$	$2.1575\text{x}10^{-5}$	$2.5311\text{x}10^{-5}$	$1.8743\text{x}10^{-4}$	0
$\tilde{\pi}_t^S$	$2.4028\text{x}10^{-3}$	$-1.0619\text{x}10^{-3}$	$-1.5626\text{x}10^{-3}$	$2.0765\text{x}10^{-2}$

VAR model found in Table 3.1, scenarios with long periods of contribution holidays and with unreasonably large salary increases was occurring frequently enough that any comparison would be doubtful. Also, a number of scenarios would require contributions so large (as much as 80% of the salary) that the plan would not be allowed to continue without serious modifications. Any comparison based on conditions that would not be allowed to exist would be meaningless. Therefore, the VAR model was calibrated such that the extreme conditions described above would be rare enough not to bias our comparison of the two plans. By trial and error, it was determined that reducing the volatility in equity returns by about 50% would generate acceptable economic scenarios for the entire horizon considered. In order to maintain the same contemporaneous correlation between variables, the covariance terms involving equity are divided by 2 and the variance of the equity return is divided by 4. The resulting variance-covariance matrix, Σ, and its Cholesky decomposition, P, are given in Table 3.2. Together with the estimated matrix Φ in panel (b) of Table 3.1, we will refer to this model as the calibrated one.

3.5   Simulated Results

Future economic scenarios are generated for a period of 60 years using the calibrated VAR model presented in the previous section. Let the innovation vector at future time h months from now, in a given scenario, be denoted by ϵ_h. Each vector consists of four independent Standard Normal variates. A total of $n = 10,000$ such vectors are generated for 720 months, $h = 1, 2, \dots, 720$.

The mean-adjusted economic variables, z_h, $h = 1, 2, \ldots, 720$, are obtained as follows:

$$z_h = \Phi^h z_0 + \sum_{i=0}^{h-1} \Phi^i P \epsilon_{h-i}. \tag{3.10}$$

The initial value, z_0, is the mean-adjusted vector of the last observed values for inflation, short-term interest rate, long-term interest rate and equity return. In April 2016, the vector of economic variables was $x_0 = (0.003122, 0.000416, 0.001162, 0.035727)$ and the mean-adjusted vector was $z_0 = (0.001643, -0.002149, -0.002864, 0.029143)$.

Projected monthly rates, $x_h = (\tilde{\lambda}_h, \tilde{y}_h^1, \tilde{y}_h^{120}, \tilde{\pi}_h^S)^T$, are obtained by simply adding the mean vector, μ. That is

$$x_h = z_h + \mu, \tag{3.11}$$

where μ consists of the values found in panel a) of Table 3.1.

The economic variables that will be used in the next chapters are the effective rates per annum at an annual frequency. Therefore, the instantaneous monthly values, x, must first be converted into appropriate effective annual rates.

The instantaneous annual rate of inflation for year t, λ_t, is simply the compounded effect of the monthly rates of inflation during that year. That is,

$$\lambda_t = \sum_{h=12(t-1)+1}^{12t} \tilde{\lambda}_h, \tag{3.12}$$

for $t = 1, 2, \ldots, 60$. Finally, the corresponding annual effective rate of inflation for year t is:

$$r_t^{inf} = e^{\lambda_t} - 1. \tag{3.13}$$

Similarly, the instantaneous and annual effective equity returns in year t, $t = 1, 2, \ldots, 60$, are

$$\pi_t^S = \sum_{i=12(t-1)+1}^{12t} \tilde{\pi}_i^S \tag{3.14}$$

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and

$$r_t^S = e^{\pi_t^S} - 1 \tag{3.15}$$

respectively.

The returns on the fixed-income portfolio are obtained differently. We assume that the assets invested in short-term instruments pay a rate consistent with 1-month Treasury bills, in any given month, and that they are reinvested in a similarly way at the beginning of each month. The annual effective rate of interest earned in year t on those assets, i_t^{12}, is given by

$$i_t^{12} = exp\left(\sum_{h=12(t-1)+1}^{12t} \tilde{y}_h^1\right) - 1. \tag{3.16}$$

Assets invested in long-term instruments are assumed to earn a rate consistent with the yield on 10-year bonds prevailing at the beginning of the year. The annual effective rate of interest on long-term investments throughout year t, i_t^{120}, is therefore equal to

$$i_t^{120} = exp\left(12\tilde{y}_{12t}^{120}\right) - 1 \tag{3.17}$$

for $t = 1, 2, ..., 60$.

The fixed-income portfolio is invested 20% in short term instruments and 80% in long-term ones and is rebalanced at the beginning of each year. The annual effective return in year t earned on that portfolio is:

$$r_t^B = 0.2i_t^{12} + 0.8i_t^{120}. \tag{3.18}$$

Figures 3.2 and 3.3 illustrate the expected values and standard deviations, respectively, of the simulated annual effective rates that will be used in the next chapters to compare DB and DC plans. They confirm that equity is the most volatile economic variable with the highest expected value. Since the volatility of the equity returns was lowered in the calibrated model, the standard deviation of the equity return in our projections is lower than the one observed in the historical data. The expected value and standard deviation for the two bonds, 1-month and 10-year, have similar patterns; with the 10-year bond offering a higher expected return. Those two series, on average, revert to their long term expected levels in about 20 years from now. It is also worth noting that the simulations start in a low interest rate environment.

Figure 3.2: Average of 10,000 annual rates generated by the calibrated model
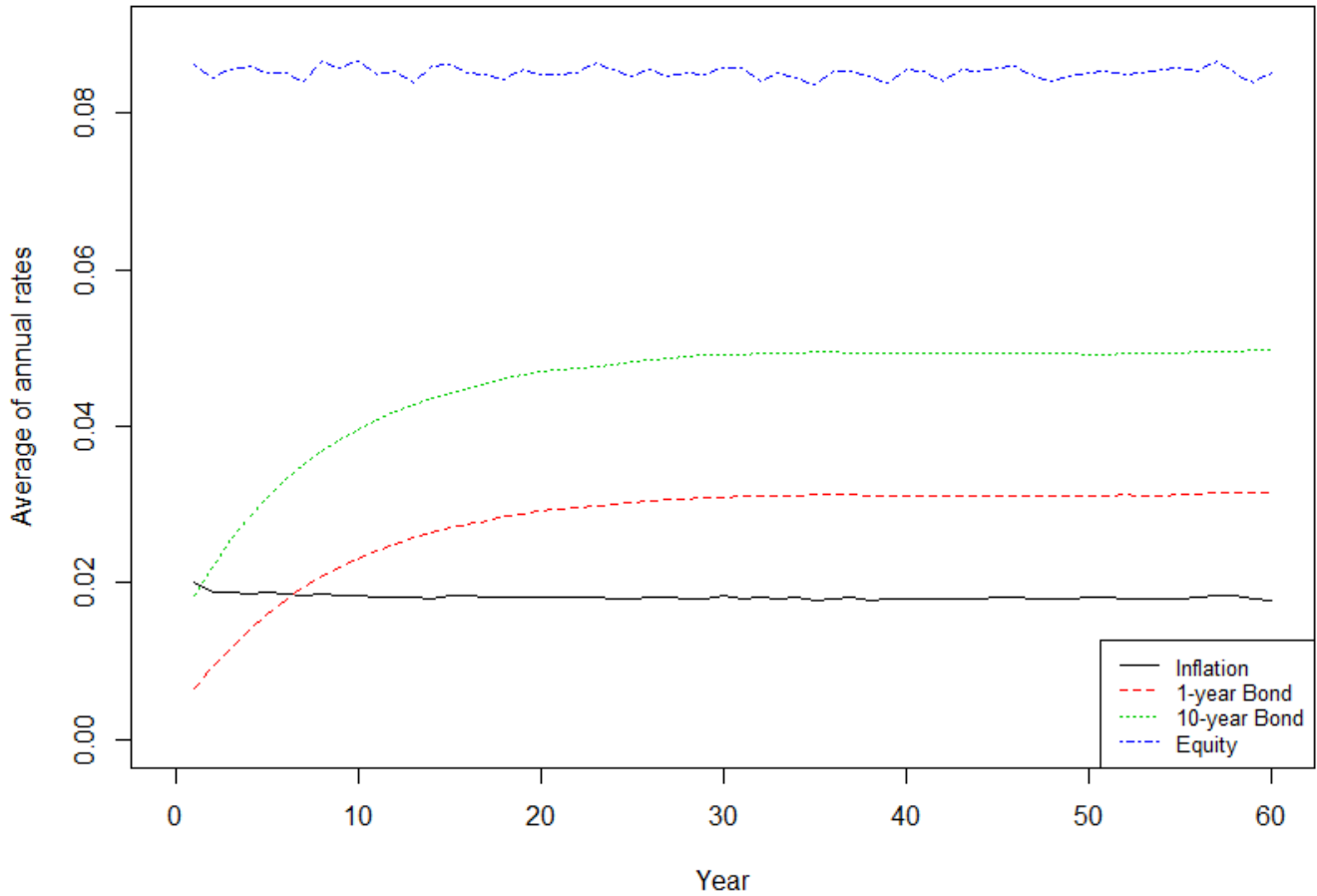
The four economic variables are simulated at a monthly frequency for 60 years using (3.10) and (3.11) with the covariance matrix found in panel b) of Table 3.1 and the Cholesky matrix in panel d) of Table 3.2. Each of the 10,000 paths starts with the observed monthly rates in April 2016 for the economic variables, $x_0 = (0.003122, 0.000416, 0.001162, 0.035727)$. Corresponding annual rates are calculated using (3.12)-(3.17).

Figure 3.3: Standard deviation of 10,000 annual rates generated by the calibrated model
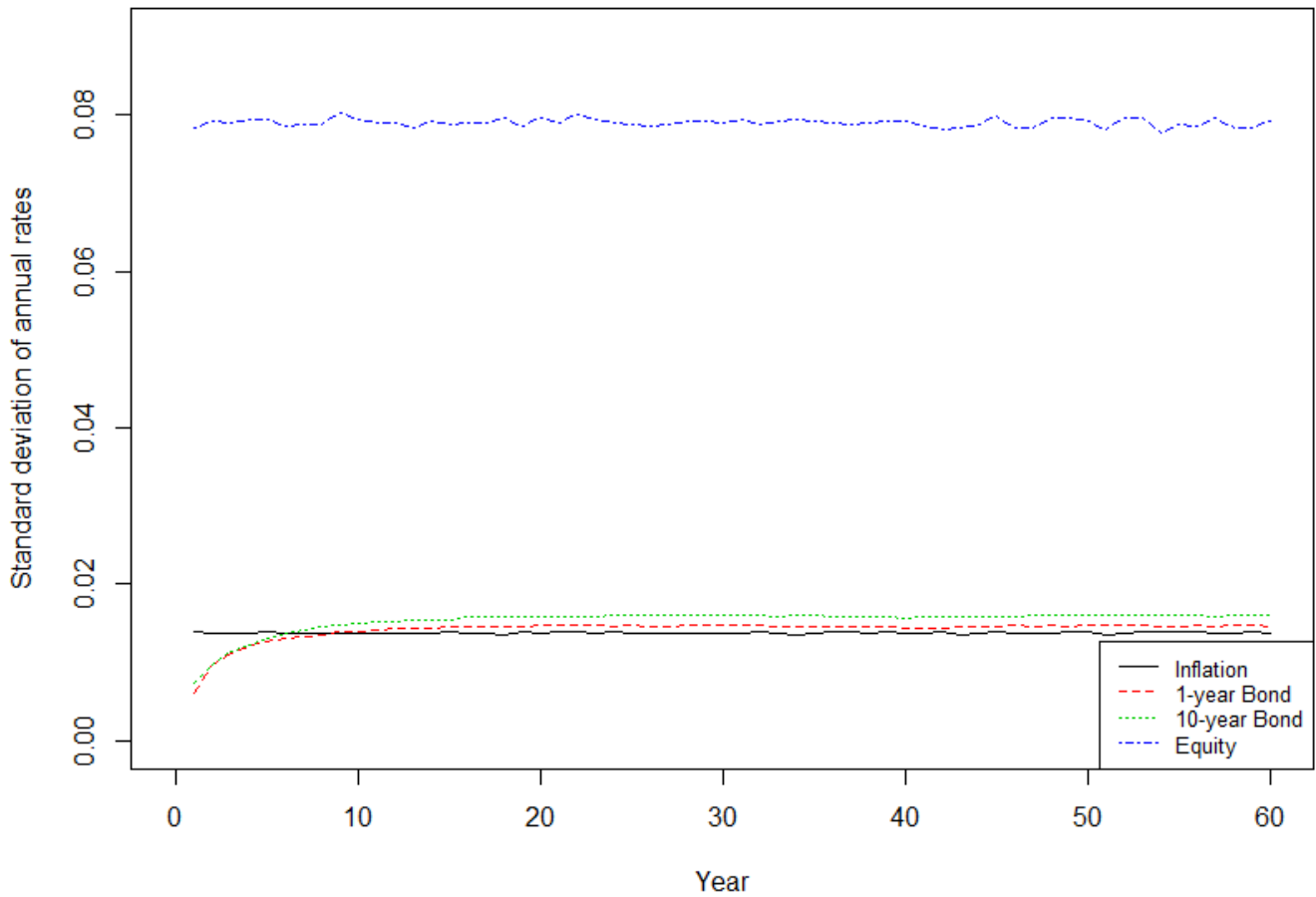
The four economic variables are simulated at a monthly frequency for 60 years using (3.10) and (3.11) with the covariance matrix found in panel b) of Table 3.1 and the Cholesky matrix in panel d) of Table 3.2. Each of the 10,000 paths starts with the observed monthly rates in April 2016 for the economic variables, $x_0 = (0.003122, 0.000416, 0.001162, 0.035727)$. Corresponding annual rates are calculated using (3.12)-(3.17).

Chapter 4

Defined Benefit Plan

4.1 Definition

A DB plan is a pension plan that promises its members a retirement income based on retirement age, years of service, salary and possibly other factors like, for example, entitlement to social security benefits. The retirement income may be partially or fully indexed to inflation. They come in many forms. In fact, the Pension Benefits Standards Act (1985) defines a DB plan as "a pension plan that is not a defined contribution plan". An analysis of the range of DB plan designs is beyond the scope of this project.

Put simply, a typical DB plan guarantees its members some retirement benefits, calculated by a pre-determined formula, which are funded through contributions generally made by the employer and investment income on the pension funds. The level of contributions varies depending on the investment performance and the value of the actuarial liabilities relative to the fund balance. Since the employer pays the contributions, it is a well-known fact in the industry and the literature on DB plans that it is the employer who bears this investment risk. The employer also bears other risks like the longevity risk, a risk not considered in this project.

4.2 Investment risk

This investment risk is usually studied for its impact on the costs to the employer and on the funding ratio of the plan. Although the employer is the one directly and perhaps mostly impacted by the variable contributions, the plan members might also be impacted through lower future salaries, and to some extent through lower retirement income (since the benefit is a function of the salary earned).

In some, if not most situations, an employer who is required to made larger contributions to a DB plan will have less room in its budget to cover the salaries of its employees and make capital investments. This is certainly the case in some public sectors where the budget of the employer is likely to increase at a rate linked to inflation. There are also examples of this in the private sector, where large pension deficits and financial hardships (not unlikely to arise in the same periods) have forced employers to cut jobs, reduce salaries and pension benefits. One can find many examples of DB plan members who saw their pension benefits reduced. For example, a number of employees in the automobile industry and the aerospace industry, in municipalities, in universities lived through periods of salary freeze (some having to settle for salary cuts) over the years. Even retirees for these employers were affected in some cases, being forced to accept lower benefits than promised at retirement.

Although the benefits in a DB plan are generally considered guaranteed, this is not always the case. They are guaranteed as long as the employer has the financial means to pay any deficits (over a period of time) and make the required ongoing contributions. Stelco, Nortel and recently Sears are examples of bankruptcies where employees lost part, if not most, of their pension benefits.

Our goal is to study how much of the investment risk is borne by the members of DB plan and compare it to the investment risk that a DC plan member would bear in the same economic environment. For convenience, we will consider that the employer sponsoring the representative DB plan described in the next section operates in a simple controlled financial environment. The analysis will focus on the salaries and the defined benefits that DB plan members will get under stochastically generated future economic scenarios. The risks that these members would see their retirement benefits reduced because of financial hardship of the employer, a plan termination or bankruptcy are ignored. Hence, the pension determined at time of retirement is indeed guaranteed in our model.

4.3   Representative DB plan

The DB plan selected for our analysis is a 5-year final average salary. The benefit provided is 2% per year of service of the average salary over the last 5 years of employment. This plan would provide a replacement ratio of approximately 70% to a member retiring after 35 years of service. A 70% replacement ratio, i.e. the ratio of retirement income to final salary, is generally considered satisfactory to maintain one's standard of living after retirement (see for example, Aitken (1996)).

4.3.1 Assumptions

This section lists the key demographic and economic assumptions made in our model. The valuation assumptions are presented in Section 4.4.1.

The demographic assumptions are:

1. All employees enter the plan when they start working for the employer at age 30.

2. The employer hires 100 new employees at the start of every year, all aged 30.

3. Retirement is mandatory at age 65 with no early retirement allowed.

4. There are no decrements while active, i.e. until retirement at age 65.

5. Retirees experience mortality rates based on the CPM2014 Public Sector Mortality Table. The mortality rates are unisex rates arbitrarily calculated as a weighted average of 40% of male's mortality rates and 60% of female's rates at each year of age. The limiting age is set at 116.

6. The DB plan membership is stationary from time 0. It consists of 100 active employees at each age between 30 and 65, and a distribution of retirees consistent with the assumed mortality table.

The main economic assumptions are:

7. The investment returns on the plan assets are net of any administration fees and management expenses.

8. All assets are invested conservatively, 60% in equity and 40% in fixed-income with rebalancing happening annually.

9. Contributions are made and benefits are paid at the beginning of each year.

10. The starting salary of newly hired employees at time 0 is $S_0 = 65000$.

11. The salary scale in any given year is based on a merit increase, m, of 1% per year of service.

12. Annual salary increases are negotiated every 3 years coinciding with the plan valuation. The rate is level and guaranteed until the next negotiation.

13. The operating budget of the employer increases annually at a rate equal to the rate of inflation experienced in the previous year.

14. The employer is not allowed to keep carrying forward a surplus or deficit. Therefore, any operating surplus experienced by the end of a 3-year cycle must be amortized fully over the next 3-year cycle.

4.3.2 Notation

The notation used in this section is summarized below.

- e: the entry age, which is 30.

- r: the retirement age, which is 65.

- $n_{x,t}$: the number of members aged x at time t.

- as_t : the annual salary rate of increase at time t. It is the rate used to adjust each employee's salary at the beginning of year t. This rate is generally positive but can be negative in some cases.

- $S_{x,t}$: the annual salary paid to an active employee aged x at time t. Note that $S_{e,0} = S_0 = 65,000$.

- TS_t: the salary mass in year t. The sum of all salaries paid to active employees in year t.

- $FAE_{x,t}$: the 5-year final average earnings for a member aged x at time t.

- $b_{x,t}$: the annual retirement benefit for a member aged x at time t, actual benefit for retirees, projected for active employees.

- Tb_t: the total benefits paid to all retirees at time t.

4.3.3 Salaries

The salary paid to a member in a given year is his/her salary earned the previous year adjusted by the total annual salary rate of increase, as_t. This salary rate of increase includes the merit increase, m, which is fixed at 1%, and the adjustment to the overall salary scale which depends on the financial situation of the employer and the DB plan. The determination of this salary rate of increase, which is done every three years, is discussed in Section 4.5.

The salary of an employee aged x at time t can be obtained recursively. Starting with the salary scale at time 0, which is by assumption,

$$
S_{x,0} = \begin{cases} S_0 \times (1+m)^{(x-e)} & e \leqslant x \leqslant r-1 \\ 0 & \text{otherwise,} \end{cases} \tag{4.1}
$$

we obtain all future salary scales, for $t = 1, 2, \ldots$, as follows:

$$S_{x,t} = \begin{cases} S_{e,t-1} \times \frac{(1+as_t)}{(1+m)} & x = e \\ S_{x-1,t-1} \times (1+as_t) & e < x \leqslant r-1 \\ 0 & \text{otherwise.} \end{cases} \quad (4.2)$$

The salary of new entrant at time t can also be calculated as $S_{e,t} = \frac{S_{e+1,t}}{1+m}$.

Past salary scales are needed to determine the benefits of those currently retired at time 0 and those retiring in the next 5 years. Here, we assume that the salary scale has been increasing annually at a rate of 2%. This rate is equal to the Bank of Canada's target inflation rate.

So, for $t = 1, 2, \ldots$,

$$S_{x,-t} = \begin{cases} S_{x,0} \times (1.02)^{-t} & e \leqslant x \leqslant r-1 \\ 0 & \text{otherwise} \end{cases} \quad (4.3)$$

The total salaries paid to all active members at time t is

$$TS_t = \sum_{x=e}^{r-1} n_{x,t} \times S_{x,t}, \quad (4.4)$$

4.3.4 Benefits

The 5-year Final Average Earnings for a retiree aged x, $x \geqslant r$, at time t is given by the average of the 5 appropriate annual salaries calculated with the expressions above. For example, the FAE of a member retiring at age 65 at time 0, is

$$FAE_{65,0} = \frac{1}{5} \sum_{i=1}^{5} S_{65-i,-i}. \quad (4.5)$$

In general, for $x \geqslant 65$ and $t = 0, 1, \ldots$, we have

$$FAE_{x,t} = \frac{1}{5} \sum_{i=1}^{5} S_{r-i,t-(x-r+i)}. \quad (4.6)$$

The retirement benefits paid to the retirees of our representative DB plan are:

$$b_{x,t} = 2\% \times (r - e) \times FAE_{x,t}, \quad (4.7)$$

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for $x \geqslant r$ and $t = 0, 1, 2, \ldots$

The total amount of retirement benefits paid to retirees at time t is

$$Tb_t = \sum_{x=65}^{115} n_{x,t} \times b_{x,t}. \tag{4.8}$$

4.4   Actuarial Valuation

The employer of this DB plan is required to have an actuarial valuation done every three years. In this section, the assumptions that will be made to determine the financial situation of the plan are discussed. The funding method and the rules regarding the amortization of any actuarial surplus or deficit are presented. Each valuation will determine the contributions that the employer will have to make to the pension fund for the next three years.

The Entry Age Normal (EAN) funding method, a commonly used actuarial funding method will be adopted here. Under this method the value of the projected benefits for an employee is allocated as a level percentage of salary between entry age and retirement age. The total annual contribution has two components, the normal cost for the year and the amortization of unfunded actuarial liabilities.

4.4.1   Valuation Assumptions

Valuing the actuarial liabilities of the plan requires a number of assumptions about the future. These assumptions can be classified into demographic ones and economic ones. They are commonly set at conservative levels compared to the expected experience of the plan.

For this project, the demographic valuation assumptions are exactly the same as the assumed experience. There is no conservatism built into these assumptions. For example, there is no decrement until the mandatory retirement age of 65 and retirees' survival is based on the CPM2014 Public Sector Mortality Table as described in Section 4.1.1.

The valuation of the plan liabilities will be a function of two key economic assumptions. The first one concerns future salary increases. Each year, employees get the merit increase, $m = 1\%$, as well as an adjustment for inflation which is assumed to be 2%. So, for valuation purposes, salaries increase at a rate of $s = 3.02\%$, i.e. $s = (1.01)(1.02)\text{-}1$, at the beginning

of each year. The second economic assumption is the selection of the valuation rate. The future cash flows, contributions and benefits, will be discounted at a rate consistent with the expected future market returns at the time of the valuation, less a small spread to value the plan somewhat conservatively.

When determining the future expected market return at a given valuation time, the equity return will be set at its long-term average. For simplicity, it will be calculated as the average of the equity returns for the given year over the 10,000 scenarios. This average rate of return is very stable.

As for the return on the fixed-income investments, the assumption is that the rate will start at the last observed return on fixed-income and will revert back towards some long-term average assumed to be 5%. For valuation purposes, fixed-income assets are assumed to earn the average of the current value and the 5% reversion level. This rate is path-dependent and may fluctuate significantly within and between scenarios. Specifically, the valuation rate, iv_t, is obtained as follows:

$$iv_t = 0.6 \times \bar{r}_t^S + 0.4 \times \left[(0.5)(r_t^B) + (0.5)(0.05)\right] - sp, \tag{4.9}$$

where $\bar{r}_t^S$ is the average of the 10,000 simulated equity returns for year t, and sp is the valuation spread. Different values of spread ranging from 0 to 2% were tried in order to choose one that produced stable results over the projection period of 60 years. A 50 basis points spread was chosen, so $sp = 0.005$.

4.4.2 Actuarial Liabilities

The actuarial liability at time t is the value of assets that the plan should have accumulated to cover the accrued benefits of the plan members. This value is determined using the valuation assumptions in effect at time t.

Using the prospective method, the actuarial liability at time t, AL_t, is the difference between the present value of all projected future benefits, $TPVFB_t$, and the present value of the projected future normal contributions, $TPVFNC_t$. Mathematically,

$$AL_t = TPVFB_t - TPVFNC_t. \tag{4.10}$$

The present value of future benefits at time t in respect of a member aged x is

$$PVFB_{x,t} = \begin{cases} \frac{b_{x,t} \times \ddot{a}_r}{(1+iv_t)^{r-x}} & 30 \leqslant x \leqslant 64, \\ b_{x,t} \times \ddot{a}_x & 65 \leqslant x \leqslant 115, \\ 0 & \text{otherwise.} \end{cases} \qquad (4.11)$$

where the annuity factors, $\ddot{a}_r$ and $\ddot{a}_x$ are valued at the valuation rate, iv_t, using the mortality table for retirees. Recall that the retirement age is $r = 65$.

An n-year life annuity due valued at an interest rate of i is calculated

$$\ddot{a}_{x:\overline{n}|} = \sum_{t=0}^{n-1} \frac{{}_tp_x}{(1+i)^t} \qquad (4.12)$$

where ${}_tp_x$ is the probability that someone age x will survive to age $x + t$. The value of a whole life annuity due on a life aged x, $\ddot{a}_x$, is obtained by using the above expression with $n = 115 - x$.

Finally, the total present value of future benefits for all members is simply the sum of the $PVFB$ for both actives and retired members. That is,

$$TPVFB_t = \sum_{x=30}^{115} n_{x,t} \times PVFB_{x,t}. \qquad (4.13)$$

4.4.3 Normal Cost

Under the EAN method, the Normal Cost (NC) is a level percentage of salaries contributed to the plan in respect of benefits accruing during the year. The level percentage of salaries, U_t, is calculated such that the present value of future benefits is equal to the present value of normal costs for a new entrant at the time of the valuation. The normal costs for all active employees is that same percentage of their salaries.

The level percentage U_t is calculated as follows:

$$U_t = \frac{b_{e,t} \times (1 + iv_t)^{r-e} \times \ddot{a}_r}{S_{e,t} \times \ddot{a}^s_{\overline{r-e}|}}. \qquad (4.14)$$

The annuity in the denominator of the above expression, $\ddot{a}^s_{\overline{r-e}|}$, is one where the payment increases at the assumed rate of salary increase s at the beginning of each year, and being

discounted at the current valuation rate, iv_t. That is

$$\ddot{a}^s_{\overline{r-e}|} = \sum_{i=0}^{r-e-1} \frac{(1+s)^i}{(1+iv_t)^i}. \tag{4.15}$$

The Normal Cost for an active employee aged x at time t is given by

$$NC_{x,t} = U_t \times S_{x,t}. \tag{4.16}$$

The total Normal Cost for the DB plan in year t is the sum of the Normal Costs for all active workers, which is

$$TNC_t = \sum_{x=30}^{64} n_{x,t} \times NC_{x,t}. \tag{4.17}$$

Having determined the Normal Cost for an employee aged x at time t, $NC_{x,t}$, we can obtain the present value of future Normal Costs for that employee as follows:

$$PVFNC_{x,t} = NC_{x,t} \times \ddot{a}^s_{\overline{r-x}|}. \tag{4.18}$$

Finally, since the Normal Cost is a level percentage of salary assumed to increase annually at a rate of s, the total present value of Normal Costs at time t is given by

$$TPVFNC_t = \sum_{x=30}^{64} PVFNC_{x,t} \times n_{x,t}. \tag{4.19}$$

With the calculated values for $TPVFB_t$ and $TPVFNC_t$, the actuarial liability, AL_t, is known from Equation 4.10.

4.4.4   Pension Fund

The contributions to the DB plan are pooled into a fund and the benefits to the retirees are paid out of the fund. The assets available at the beginning of the year are invested 60% into equity and 40% into fixed-income. The effective annual rate of return earned on those assets in year t, for $t = 1, 2, \ldots$, is:

$$r_t^m = 0.6r_t^S + 0.4r_t^B, \tag{4.20}$$

where the annual effective rates r_t^S and r_t^B are the returns on equity and fixed-income investments respectively. They are defined in Chapter 3 and generated using the calibrated VAR model.

We will call the rate r_t^m the market return for year t since it represents the rate actually earned by investing the plan's assets in the market.

Denoting the total contributions made to the fund at time t by C_t, the fund evolves according to the following recursive equation for $t = 1, 2, \ldots$:

$$F_t = (F_{t-1} + C_{t-1} - Tb_{t-1}) \times (1 + r_{t-1}^m). \tag{4.21}$$

We will assume that the plan is fully funded at time 0. Otherwise the plan would start with a surplus or a deficit which would affect the comparison with the DC plan. So, the initial fund is

$$F_0 = AL_0. \tag{4.22}$$

4.4.5   Unfunded Actuarial Liability

The plan's experience over any three-year period (between valuations) is unlikely to match the assumptions made when calculating the Normal Cost. For example, the market return will likely be different than the valuation rate. As a result, the accumulated pension fund will not remain equal to the actuarial liability after time 0. The difference between the actuarial liability and the fund value on a valuation date is called the unfunded actuarial liability (UAL). We have, for $t = 1, 2, \ldots$,

$$UAL_t = AL_t - F_t. \tag{4.23}$$

This UAL represents a surplus or a deficit which must be amortized over a period of time. Different rules or regulations exist regarding the amortization of the UAL. We will use a 15-year period to amortize any actuarial surplus or deficit that exists on any valuation date. The annual amount needed to amortize the UAL over 15 years is a special contribution that will be made to the pension fund each year for the 3 years following a valuation. At the end of those 3 years, a new UAL will be calculated and a new special contribution will be determined. This cycle will be repeated every 3 years.

Denoting by AP_t the amortization payment determined at valuation time t, a special contribution made to the plan in years t, $t+1$ and $t+2$, we have

$$AP_{t+s} = \begin{cases} \dfrac{UAL_t}{\ddot{a}_{\overline{15}|iv_t}} & s = 0; t = 0, 3, 6, \ldots \\ AP_t & s = 1, 2; t = 0, 3, 6, \ldots, \end{cases} \tag{4.24}$$

where the 15-year annuity is valued at the valuation rate prevailing at time t, iv_t.

Since the total contributions made to the fund at time t are

$$C_t = TNC_t + AP_t, \tag{4.25}$$

the recursive formula (4.22) for the pension fund can be written as:

$$F_t = (F_{t-1} + TNC_{t-1} + AP_{t-1} - Tb_{t-1}) \times (1 + r_{t-1}^m). \tag{4.26}$$

Note that since the unfunded actuarial liability can represent a surplus or a deficit, the special contribution, AP, can be positive or negative; and can even make the total contribution, C, negative.

4.5 Business Setup

The business setup is such that the employer will have a limited budget to cover all salaries of active members, pay the Normal Costs and make any special payments to the DB fund required to amortize the UAL. The plan contributions were discussed earlier in this chapter.

The employer will negotiate salary increases with its employees every 3 years. A level annual rate of salary increase will be guaranteed for those 3 years. In doing so, the employer will take into consideration its expected budget over the next 3 years, the newly calculated contribution rate for the plan and the existing operating surplus or deficit since the last valuation. The main reason an operating surplus or deficit arises is that the actual annual budget increases are linked to inflation which is not know at the time the guaranteed salary increases are negotiated. Finally, the employer is required to balance its budget and expenditures over each 3-year cycle.

The operating budget of the employer is adjusted for inflation at the beginning of each year. Our projections start with a balanced budget, no existing operating surplus or deficit at time 0 and as mentioned earlier no unfunded actuarial liability. The budget at time t is denoted by B_t and given by

$$B_t = B_0 \times \prod_{i=1}^{t}(1 + r_i^{inf}) \tag{4.27}$$

for $t = 1, 2, \ldots$

Whenever the budget needs to be projected into the future, either implicitly or explicitly, it will be done by increasing the last known actual budget at the target rate of inflation of 2% per annum.

4.5.1 Expenses

The available budget is intended to cover the expenditures, namely all salaries, total plan contributions and any amounts needed to eliminate any outstanding operating surplus or deficit. These expenditures, or simply expenses at time t, E_t, are equal to the sum:

$$E_t = TS_t + C_t + AOpD_t \tag{4.28}$$

where TS_t is the total salaries defined in (4.4), C_t is the total contributions defined in (4.28) and $AOpD_t$ is the amount needed to amortize the operating surplus or deficit over 3 years. If there is an existing operating deficit, the employer is assumed to borrow money to cover it temporarily and required to repay it; the amount $AOpD_t$ will be positive. If there is a surplus, the employer will be allowed to spend it over the next 3 years and $AOpD_t$ will be negative. By assumption, we have no surplus or deficit at time 0, so $AOpD_0 = 0$.

As mentioned above, the operating surplus or deficit arises when the expenses do not match the actual budget. The operating deficit at a valuation time t will be the sum of the annual deficits for the previous 3 years. That is, for $t = 3, 6, \ldots,$

$$OpD_t = \sum_{i=1}^{3} [TS_{t-i} + TNC_{t-i} + AP_{t-i} + AOpD_{t-3} - B_{t-i}]. \tag{4.29}$$

Any operating surplus or deficit will be amortized over 3 years starting immediately which gives

$$AOpD_{t+s} = \begin{cases} \dfrac{OpD_t}{\ddot{a}_{\overline{3}|r_{t-1}^m}} & s = 0; t = 0, 3, 6, \ldots \\ AOpD_t & s = 1, 2; t = 0, 3, 6, \ldots. \end{cases} \tag{4.30}$$

The discount rate used in the 3-year annuity is the current market rate which means that the employer borrows (or invests) money at the rate last earned on the assets of the pension fund.

4.5.2 Salary Increase

In this business setup, the employer attempts to balance its budget by offering salary increases that can be afforded based on projections made at time t. We will assume that the actuarial valuation of the DB plan has been completed when salary increases are being negotiated (or, shall we say, determined). Similarly, the operating surplus or deficit for the 3-year period that just ended is known. And consequently, its impact on the budget for the next 3 years is known. The budget for the coming year, however, will only be known at the start of that year and is therefore not used in determining the salary increases.

At the beginning of year t, the employer starts by assuming that its annual budget for each of the next 3 years, $i = 0, 1, 2, \ldots$, will be

$$\hat{B}_{t+i} = B_{t-1} \times (1 + 0.02)^{i+1} \tag{4.31}$$

where B_{t-1} is the known budget for last year, $\hat{B}$ is a projected budget using a 2% inflation rate.

In an ideal world where no UAL and no operating deficit would ever occur, an employer should be able to increase its salary mass by 2% each year. Each employee, except newly hired ones, would get a salary increase of 3.02%, including the 1% merit increase. Unfortunately, it is unlikely that experience will match all assumptions, generating UAL and operating deficits. So, salary increases will have to be set at levels different than the expected 3.02%. For example, consider an employer with an operating deficit at the end of a 3-year period. To balance its budget again, salaries should increase at a rate lower than 3.02%. This will tend to lower the actuarial liability (at least compared to its value had the salaries being increased by 3.02%) and produce an actuarial surplus. So next valuation, chances are that the employer would be able to afford to increase salaries by more than 3.02%. This would tend to increase the actuarial liability at the following valuation and generate an actuarial deficit. The next salary increase would have to be lower than the target. The pattern would repeat itself and produce unstable funding ratios and very volatile salary increases. This approach was tried and indeed produced unstable results.

To help smooth the results, the employer will consider projected results over a slightly longer term. When determining the rate of salary increase at valuation time t, the employer will aim for a projected balance budget in 4 years, i.e. in year $t + 3$. Let $\Delta_{t+3}(ns)$ be the difference between the projected budget and the projected expenses in year $t + 3$ assuming annual salary increases at a new negotiated rate ns. This difference is the projected operating

surplus for year $t + 3$ and is given by

$$\Delta_{t+3}(ns) = \hat{B}_{t+3} - \hat{E}_{t+3}(ns) \tag{4.32}$$

where $\hat{E}_{t+3}(ns)$ are the projected annual expenses for year $t + 3$ assuming salary increases of ns per annum, calculated using (4.37) with corresponding projected components.

The goal is to find the value of ns that makes this projected operating surplus equal to 0. We obtain a value for ns by linear interpolation between two carefully chosen rates, ns_1 and ns_2. Starting with $ns_0 = .0302$, we calculate ns_i, $i = 1, 2$, as follows:

$$ns_i = ns_{i-1} + 0.00058 \times \frac{\Delta_{t+3}(ns_{i-1})}{1,000,000 \times 1.02^t}. \tag{4.33}$$

The factor 0.00058 in the above formula is approximately the change in the rate of salary increase necessary to eliminate a projected surplus of $1,000,000$ at the end of the first 3-year cycle. Because the projected surplus or deficit is very sensitive to small changes in the rate of salary increase and with a non-linear relationship, it is important to select values of ns carefully. Since the impact of a salary increase on the projected surplus is proportional to the salary mass, the adjustment factor needs to reflect inflation. Hence, the 1.02^t factor that appears in the denominator.

The annual rate of salary increase awarded to all employees in years t to $(t+2)$ is defined as

$$ns = ns_2 - \frac{\Delta_{t+3}(ns_2) \times (ns_2 - ns_1)}{\Delta_{t+3}(ns_2) - \Delta_{t+3}(ns_1)}. \tag{4.34}$$

This rate, ns, becomes the rate awarded in years t, $t + 1$ and $t + 2$ (i.e. as in (4.2) for those years).

4.6 A Deterministic Scenario

This section illustrates how the salary increases are determined. Using a deterministic scenario based on the expectations for future economic variables, results are shown at three key stages leading to the determination of the salary increase for years 3, 4 and 5.

In Table 4.1, the assumed business setup is presented. At time 0, a newly hired employee earns a salary of 65,000. The total rate of salary increase, for merit and inflation, is 3.02% for the next two years.

Using the initial valuation rate, the total present value of benefits and contributions are calculated. It is determined that the plan has a 1.925 billion actuarial liability. Since the plan is assumed to be fully funded, the value of F_0 is also 1.925 billion.

The expenses for the first year are the sum of the salaries, TS_0, and all contributions, TNC_0. The budget is balanced in the first year, so B_0 is equal to the expenses (309,476,205).

Table 4.2 shows the results known to the employer immediately after the actuarial valuation performed at the end of the third year, i.e. just before $t = 3$. At that time, the valuation rate is up to 6.12%. With the higher salaries, the actuarial liabilities are up to 2.043 billions. The pension fund received annual contributions, paid the benefits to retirees and earn the market return allowing it to grow to 2.037 billions. The plan has an unfunded actuarial liability of 6,622,291 was will have to be amortized over 15 annual special payments of 640,094.

Also, the employer ends up with an operating deficit because its budget increased at inflation which was less than the assumed 2% included in the salary increases. To eliminate this operating deficit, the employer must pay 799,167 in each of the next three years.

Table 4.3 shows the results of the budget planning done by the employer and leading to the negotiated salary increases for the next three years. The actual budget at time 2 is assumed to grow at rate of 2% per year. As explained in the previous section, the goal is to set the salary increase such that the budget will be balanced starting at time 6, that is after amortizing the UAL and eliminating the operating deficit realized at time 3. This is achieved by solving for the salary increase that will make the projected deficit, $\hat{B}_6 - \hat{E}_6$, equal to 0. Since most values beyond time 2 in this table depend on the salary increase awarded in the next three years, there is no explicit solution. The linear interpolation method described in the previous section is used to approximate the desired salary increase. Awarding salary increases of 3.2% will result in a projected deficit at time 6 of 4.16 which is close to 0.

Table 4.1: Business setup assumed at time 0, DB plan under a deterministic scenario

Time, t	0	1	2	3	4	5	6
Starting Salary, $S_{e,t}$	65,000	66,300	67,626				
Salary Increase, as_t	0	.0302	.0302				
Salary Mass, TS_t	270,791,791	276,207,627	281,731,780				
iv_t	0.056757						
$TPVFB_0$	2,481,573,831						
Normal Cost rate, U_0	0.1429						
TNC_t	38,684,413	39,458,102	40,247,264				
$TPVFNC_0$	507,545,906						
AL_0	1,974,027,925						
F_0	1,974,027,925						
UAL_0	0						
AP_0	0						
Budget, B_t	309,476,205						
Expenditures, E_t	309,476,205						
OpD_0	0						
$AOpD_0$	0						

Table 4.2: Information known after valuation at time 3, DB plan under a deterministic scenario

Time, t	0	1	2	3	4	5	6
Starting Salary, $S_{e,t}$	65,000	66,300	67,626				
Salary Increase, as_t	0	.0302	.0302				
Salary Mass, TS_t	270,791,791	276,207,627	281,731,780				
iv_t	0.056757			0.058869			
$TPVFB_t$	2,481,573,831			2,558,119,837			
Normal Cost rate, U_t	0.1429	0.1429	0.1429	0.1347			
TNC_t	38,684,413	39,458,102	40,247,264				
$TPVFNC_t$	507,545,906			498,344,561			
AL_t	1,974,027,925			2,094,854,227			
F_t	1,974,027,925			2,088,080,689			
UAL_t	0			6,773,538			
AP_t	0			646,052	646,052	646,052	
Budget, B_t	309,476,205	314,583,590	320,780,160	327,195,765			
Expenditures, E_t	309,476,205			327,511,512			
OpD_t	0			2,281,020			
$AOpD_t$	0			803,888	803,888	803,888	

Table 4.3: Projections used to determine salary increase, ns, at time 3, DB plan under a deterministic scenario

Time, t	0	1	2	3	4	5	6
Starting Salary, $S_{e,t}$	65,000	66,300	67,626	69,152	70,713	72,309	
Salary Increase, as_t	0	0.0302	0.0302	0.032795	0.032795	0.032795	0.0302
Salary Mass, TS_t	270,791,791	276,207,627	281,731,780	288,090,237	294,592,200	301,240,907	307,265,725
iv_t	0.056757			0.058869			0.058869
$TPVFB_t$	2,481,573,831			2,557,303,136			2,715,574,901
Normal Cost rate, U_t	0.1429	0.1429	0.1429	0.1347	0.1347	0.1347	0.1347
TNC_t	38,684,413	39,458,102	40,247,264	38,805,324	39,681,128	40,576,699	41,374,687
$TPVFNC_t$	507,545,906			498,344,561			532,853,510
AL_t	1,974,027,925			2,094,854,227			2,182,663,197
F_t	1,925,965,816			2,088,080,689			2,220,650,992
UAL_t	0			6,773,538			-37,987,795
AP_t	0			646,052	646,052	646,052	-3,666,632
Budget, B_t	309,476,205	312,066,074	318,213,056	327,195,765	333,739,680	340,414,474	347,222,763
Expenditures, E_t	309,476,205			328,339,536			347,185,885
$\hat{B}_t - \hat{E}_t$	0						6.80852
OpD_t	0			2,281,020			6,313,357
$AOpD_t$	0			803,888	803,888	803,888	2,230,902

4.7 Projected Salary Increases and Contribution Rates

Figure 4.1 illustrates the range of salary increases that the employer might be able to afford based on our 10,000 stochastic economic scenarios. Starting at a value of 3.02%, the annual rate of salary increase occasionally goes up to 10% in some scenarios. On the downside, the employer has to cut salaries by 4% in a number of scenarios. Those extreme cases only happen in 1% of the best or worst scenarios generated. The long-term mean of the rates of salary increase stays around the 3% level which is the salary increase assumed for the first 3 years of our projection period. The middle-quartile range indicates that most salary increases will fall between 1% and 5%. These results seem quite reasonable.

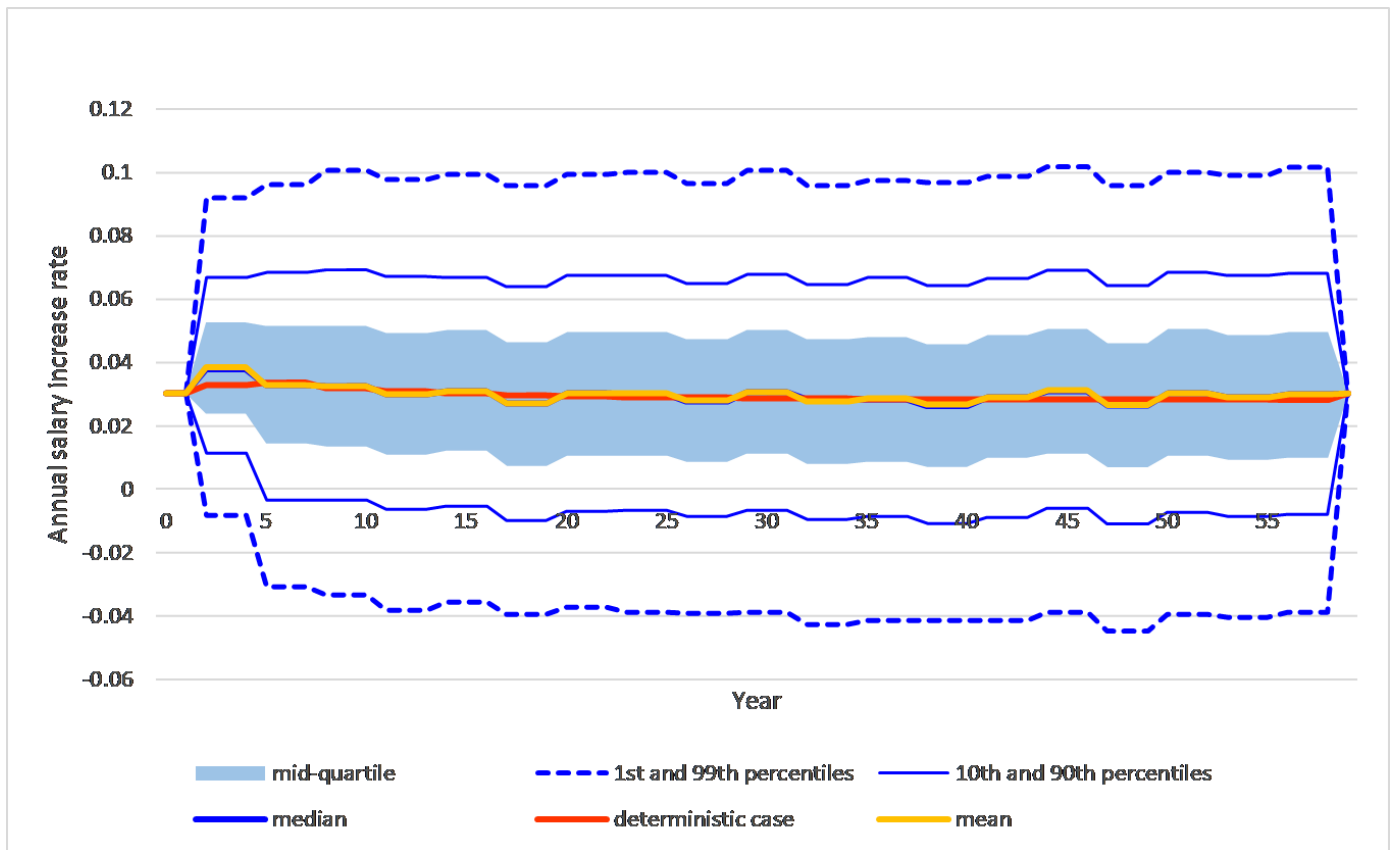Figure 4.1: Projected Salary Increases for Sixty Years

Figure 4.2 shows the Normal Cost rates over our projection horizon of 60 years. The starting Normal Cost percentage was calculated to be 13.4%. The Normal cost rate steadily declines to a level around 11.5% over the first 15 years and remains stable at that level until the end of the projection horizon. The reduction in the Normal Cost percentage is partly due to the choice of a conservative valuation rate, which enables the pension fund to carry more assets than what would be just necessary to cover the expected costs of future benefits and partly due to the fact that we expect fixed-income returns to increase from their current low levels towards higher long-term means.

Figure 4.2: Projected Normal Cost Rates for Sixty Years
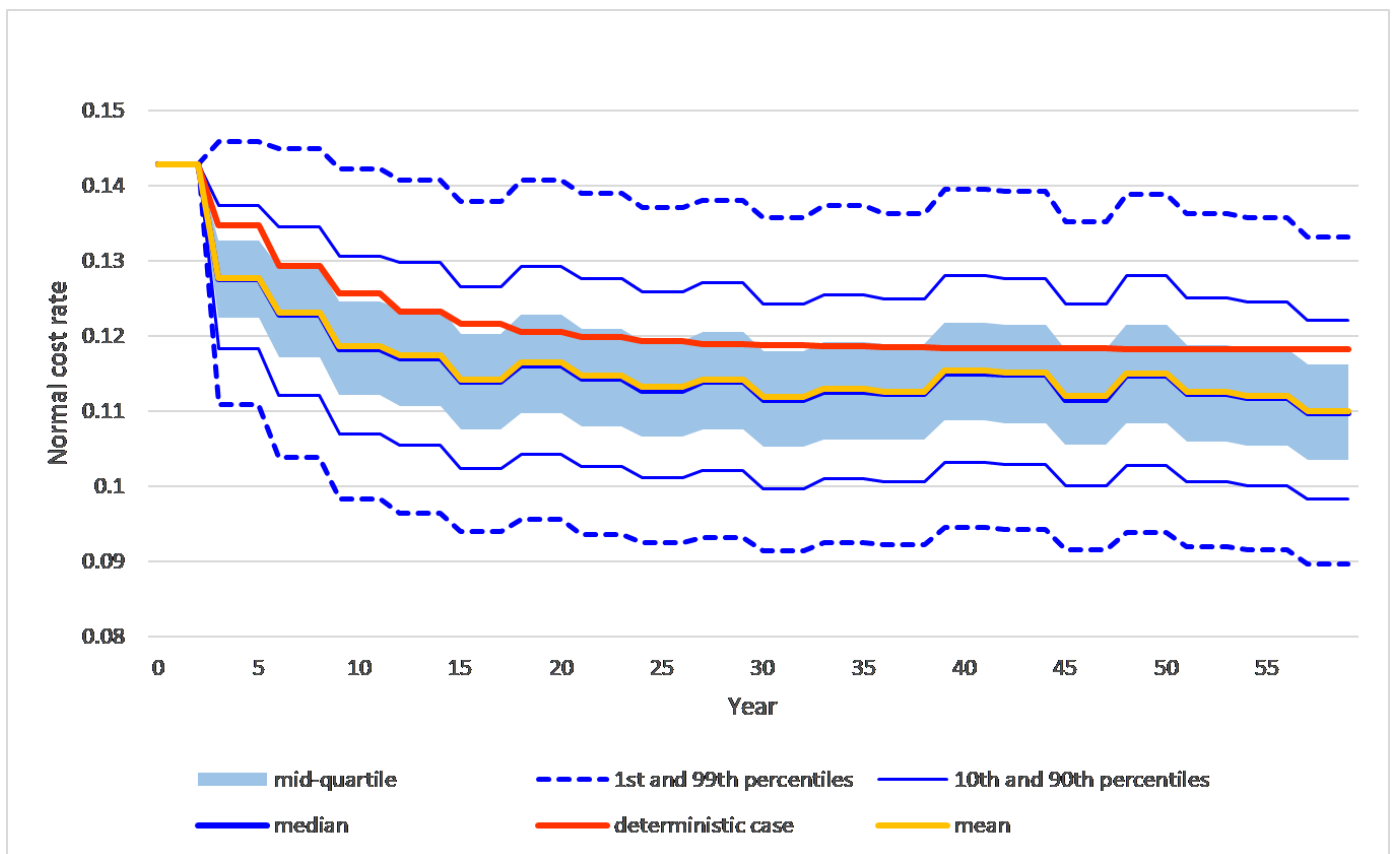
Chapter 5

Defined Contribution plan

5.1 Definition

A Defined Contribution (DC) plan is a pension plan where the contributions are of a set or defined amount. The contribution amount is often a fixed percentage of an employee's salary. Each plan member has an individual account in which contributions are deposited and investment earnings are credited. DC plans may also be called a Money Purchase Plans (MPP).

The retirement income of a member depends on the amount accumulated in his/her individual account at the time of retirement. The member has a number of options at retirement ranging from continuing to manage the account and draw income from it to using the balance in the account to buy an annuity from an insurance company.

DB and DC pension plans face many of the same risks. The level and impact of a given risk may vary between the two types of plans. How the existing risks are shared between the employer and the employees is arguably one of the most discussed topic in the pension literature. In the next section, we discuss briefly the most important risks involved with a DC plan.

5.2 Risks with a DC Plan

Unlike DB plan members, participants of a DC plan will enjoy stable contribution rates. However, the DC members will bear most of the risks associated with having an individual account. With DB plans, the members essentially pool the risks and therefore share them with other members, past, present and future throughout built-in intergenerational transfers. DC members owning their individual accounts without the risk of having to cover

any deficits in the plan; and having the opportunity to manage their funds successfully are advantages cherished by some employees.

5.2.1 Investment Risks

The member bears the investment risk during the accumulation period, i.e. while actively employed and contributing to the plan. The member is usually offered a number of options to invest the funds available in his/her account. Therefore, the member is bearing the risk associated with market fluctuations in relation to his/her investment strategy. Depending on the financial literacy of the member, this risk may be quite significant.

The DC member is also impacted by the timing of market volatility and market corrections. The volatility in the financial market is known to vary over time and market corrections tend to happen more frequently than some would like to admit (for example, in Figure 3.1 we can see large negative market returns in 2003, 2008 and more recently in 2012). A member who sees his DC account depleted because of a market crash early in his/her career will have time to recover with many years of future contributions and investment earnings. The member who is near retirement when a market crash happens will not be so lucky. The depleted account might force the member to retire with a low income or work longer.

5.2.2 Interest Risk

A DC plan member who wishes to buy a life annuity at time of retirement, or later during retirement, will face the risk of having to do so when interest rates are low, and therefore annuity prices high. This is the interest risk. This is not a risk borne by DB members, provided the plan remains solvent. In addition, a DC plan member interested in buying a life annuity will generally not be able to do so at the lower group rate effectively offered to DB participants (McCarthy 2003).

5.2.3 Other Risks

DC plan members who choose not to buy a life annuity will face the risk of outliving their assets. This is the longevity risk. Compared to DB plan members, inflation will have a greater effect on DC members. The rate of inflation during employment and after retirement may affect DB and DC members differently. Some DB plans offer retirement incomes that are partially indexed to inflation. These other risks are beyond the scope of this project.

5.3 Representative DC Plan

5.3.1 SFU Pension Plan

SFU offered a non-contributory DB plan from 1964 to 1973. In March 1973, the plan was amended to a DC plan that provided a fully vested and portable individual account to existing members. Plan members at the conversion date (called the Closed Group) are entitled to the greater of a life annuity purchased with their individual account or the benefit determined by the old formula. The cost of the formula benefit was funded by a surcharge of 2.24% for 2 years, then 2% for 5 years, of the basic salary of members in the Closed Group. The surcharge was paid by levies from the individual accounts of the DC plan members.

Some of the key features of the SFU Academic Pension Plan taken from the SFU 2016 Annual Report of Academic Pension Plan (SFU, 2016) and the SFU Pension Plan for Academic Staff Summary (SFU, 2013) are summarized below.

- Contributions
  The University makes a compulsory contribution of 10% of a member's basic salary less the University's required contributions to the Canada Pension Plan (CPP). Each plan member has an individual account called Money Purchase Account (MPA). A plan member is permitted to make voluntary contributions to his/her MPA up to a certain percentage (18% in 2016) of his/her earnings less the amount of contribution made by the employer. The portion of the MPA due to the employer's contributions cannot be withdrawn until retirement. The member can withdraw any voluntary contributions, with the corresponding investment earnings, while employed at SFU.

- Retirement
  Normal retirement date is the first day of September following attainment of age 65. Early retirement is allowed on the first day of any month after attaining age 55. Retirees at normal retirement date have the option to purchase several different types of annuities at rates quoted by Sun Life Financial or leave the funds in their MPA until age 71.

- Investment choices
  Individuals have a number choices when deciding on an investment strategy. They can allocate assets in a range of investment funds, or choose one of the Investment Funds offered. The default fund is the Balanced Fund which has a target allocation of 65% equities and 35% fixed income. The Balance Fund earned annual returns of

6.8%, 6.3% and 11.5% in year of 2016, 2015 and 2014 respectively.

Members of the SFU DC plan are starting to retire in larger numbers after spending their academic careers as SFU employees. Due to recent market performances and a low-interest rate environment, some members retiring are finding that the benefits provided by their MPA are inadequate. This is certainly a strong reminder that the members of a DC plan are the ones bearing the investment and interest risks. However, to conclude that DC plans are most expensive and riskier than DB plans on this basis is ignoring the possible outcomes for future generations of plan members. It should also be pointed out that one should expect to contribute more than 10% of salary to adequately fund his/her retirement.

In this project we undertake not only to compare the benefits provided by both plans in a wide range of economic situations but also to study the impact that given environments may have on the salaries earned by members while employed. For our comparative analysis, a representative DC plan with characteristics similar to an existing one, the SFU Academic Plan, was chosen.

5.3.2   Design and Assumptions

The SFU plan forms the basis for the selection of the representative DC plan used in this project. We assume a fixed 10% contribution rate of basic salaries paid entirely by the employer. For simplicity, there are no voluntary contributions from the members.

The representative member is investing the assets available in the MPA in a fund consisting of 60% equities and 40% fixed-income. The asset mix is rebalanced at the beginning of every year. This investment strategy is consistent with the one adopted by the plan sponsor of the DB plan studied in Chapter 4.

The demographic assumptions, where appropriate, are the same as those described in Section 4.3.1.

5.4   Business Setup

For a fair comparison, the budget of the employer is exactly the same as the one for the employer offering our representative DB plan. That is, the budget increases annually with inflation. The employer is not allowed to carry any surplus or deficit. Consequently, every

year the full budget is spent on salaries and DC contributions. As was the case with the employer sponsoring the DB plan, there are no other expenses to be covered with this budget. This setup makes the projections of future salaries and DC accounts a fairly simple exercise.

Under the design of our representative DC plan, in any given year t, 110% of total salaries would use up the full budget for that year. The amount available for all salaries in year t, TS_t^{DC}, is

$$TS_t^{DC} = \frac{B_t}{1.1} \tag{5.1}$$

where B_t is the budget amount described in Section 4.5.1.

With 100 members at each age and a 1% merit increase per year of service, the total salaries in year t is also given by

$$
\begin{aligned}
TS_t^{DC} &= \sum_{x=e}^{r-1} n_{x,t} S_{x,t}^{DC} \\
&= 100 \sum_{i=0}^{34} S_{e,t}^{DC} (1+m)^i \\
&= 100 S_{e,t}^{DC} \ddot{s}_{\overline{35}|m}
\end{aligned}
\tag{5.2}
$$

where $S_{x,t}^{DC}$ is the salary earned by an employee aged x in year t.

The annuity certain is valued at a rate of $m = 0.01$ which gives $\ddot{s}_{\overline{35}|m} = \frac{(1+m)^{35}-1}{m} = 41.6603$.

Substituting (5.2) into (5.1) and solving for the salary earned by a new entrant at t, we get

$$S_{e,t}^{DC} = \frac{B_t}{1.1 \ddot{s}_{\overline{35}|m}}. \tag{5.3}$$

The annual salary of a member of the DC plan who is aged x at time t is

$$S_{x,t}^{DC} = S_{e,t}^{DC} (1+m)^{x-e} \tag{5.4}$$

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and the amount contributed to the MPA of this member is

$$C_{x,t}^{DC} = 0.1 \times S_{x,t}^{DC}. \tag{5.5}$$

The contributions will accumulate in the MPA for 35 years and provide a lumpsum at the time of retirement of the member. Denoting the fund available at time $t+s$ in the MPA of a member who was hired at time t by $F_{t,s}^{DC}$, we have the following recursive equation:

$$F_{t,s+1}^{DC} = [F_{t,s}^{DC} + C_{e+s,t+s}^{DC}] \times (1 + r_{t+s+1}^m) \tag{5.6}$$

where $F_{t,0}^{DC} = 0$ for all t, that is new entrants start with no funds available in their MPA.

5.5 A Deterministic Scenario

Unlike the complex actuarial valuation required for a DB plan, a DC plan account is simple and straightforward to track. The numerical results for an individual account of our representative DC plan, under the same deterministic scenario that was used to illustrate the DB plan results in Section 4.6, up to year 6 are shown in Table 5.1. The table shows the balance in the MPAs for the first two cohorts of new employees.

Again, the budget amounts are exactly same as those obtained when studying the DB plan. The salary of a newly hired employee in a given year is determined by Equation (5.3). The contribution amount which is 10% of the salary is deposited into the MPA at the beginning of the year. The fund balance earns the market return, r^m, for that year.

Some of the key numbers in Table 5.1 are calculated as follows:

The starting salary at time 0 is $S_{e,0}^{DC} = 66,992 = \frac{306,999,561}{(1.1)(41.6603)(100)}$,

The salary at time 1 of a new entrant at time 0 is $S_{e+1,1}^{DC} = 68,779 = 66,992(1.016503)(1.01)$,

The accumulated fund at time 1 for a new entrant at time 0 is $F_{0,1}^{DC} = 13,956 = 6699(1.056549) + 6878$ and the starting salary of a new entrant at time 1 is $S_{e,1}^{DC} = 68,098 = 66,992(1.016503)$.

Table 5.1: Individual Accounts for first 2 cohorts of employees in the DC plan under a deterministic scenario

Time, t	0	1	2	3	4	5	6
Inflation, r_t^{inf}	0.016503	0.019698	0.018867	0.018734	0.018628	0.018539	0.018463
Market return, r_t^m	0.054186	0.055734	0.057159	0.058411	0.59513	0.060484	0.061340
Budget, B_t	309,476,205	314,583,590	320,780,162	326,832,208	332,954,982	339,157,134	345,444,753
New entrant at 0:							
Salary, $S_{e+t,t}^{DC}$	67,532	69333	71,406	73,481	75,606	77,784	80,019
Contribution, $C_{e+t,t}^{DC}$	6753	6933	7141	7348	7561	7778	8002
Fund balance, $F_{0,t}^{DC}$	6753	14,053	21,976	30,581	39,927	50,082	61,113
New entrant at 1:							
Salary, $S_{e+t-1,t}^{DC}$	0	68,647	70,699	72,753	74,857	77,014	79,227
Contribution, $C_{e+t-1,t}^{DC}$	0	6865	7070	7275	7486	7701	7923
Fund balance, $F_{1,t}^{DC}$	0	6865	14,317	22,411	31,206	40,764	51,152

Chapter 6

Comparison of DB and DC Plans

The SFU Faculty Association is considering the possibility of switching from the existing DC plan to the B.C. College Pension Plan, which is a jointly sponsored DB plan. A report prepared by PBI Actuarial Consultants Ltd (2015) compares the retirement income for existing members based on their current age and retirement age under a number of deterministic scenarios. It is suggested that the B.C. College Pension Plan would reduce investment risk, inflation risk and longevity risk. It concludes that in most scenarios, most members would do better in the College Pension Plan than in their current DC plan. The deterministic scenarios and the analysis of retirement benefits only are two of the limitations of the study. The motivation of this project is to use a stochastic model and take future salaries into account.

At the time of a pension plan conversion, existing members may be given the option to stay in the old plan. New employees, however, are required to join the new plan. The financial situation of the old plan may make it more or less advantageous for existing members to stay in it. For this reason, our comparison of the two plans will exclude existing members.

The focus of our comparison is on the first 25 cohorts to join the plans after time 0. We will project their experience with the employer and the pension plan until the last cohort reaches retirement age, after 35 years of service. Therefore, the projections will run for 60 years.

We consider two employers operating in the same conditions with the same available budget to cover salaries and all contributions to the pension plan. The only difference is the type of pension plan they sponsor, one offers a DB plan, the other a DC plan.

Employees join the plan offered by their employer at the time they get hired. Since contributions vary over time and differ between plans, the employees of the two employers earn different salaries. However, the salary scale is based on a 1% merit increase in both

cases. The new employees in the DB plan may join at a time when Normal Costs are low and consequently, start at a relatively higher salary. Conversely, they may start when there is a funding deficit requiring higher contributions and have to settle for a lower salary. So future cohorts may be affected differently than the initial cohort hired at time 0.

In this chapter we analyze how well the cohorts in one group do compared to corresponding cohorts in the other group. The total compensation received by a cohort is the measure of choice here to compare the two plans. Total compensation is arbitrarily defined as the value at time of retirement of all salaries earned during the 35-year career plus the equivalent lumpsum available in the pension plan at time of retirement. For this purpose, the salary accumulation is done at a rate that can be interpreted as a preference rate of the employee for future cash flows. We consider two rates, the first is a low discount rate corresponding approximately to the average inflation over all the projections. A second discount rate of 6%, which is closer to the average market returns, is also considered. Other rates and approaches, for example, using an expected utility function, can be used to measure the value of the total compensation received by a plan member.

The comparison will focus on the difference between total compensations for members of the two plans. These differences will be cohort-specific as later cohorts may be affected differently than earlier ones. We will use the concept of Value-at-risk (VaR) to compare the plans. The graphs presented in this chapter will all show the mid-quartile range as well as four quantiles, namely, 1%, 10%, 90% and 99%, of the 10,000 generated results. Although not particularly relevant to one employee per se, means and medians are also shown in each graph.

Finally, the difference obtained between the two plans for the deterministic scenario will also be included in each graph. We call the difference in the deterministic case the baseline. It is important to recognize that the two plans do not start exactly on the same basis. The DB plan starts as a mature and fully funded plan with assets above the expected liability of the plan. Recall that the actuarial liability is valued at a conservative level. This excess funding over the expected cost of the benefits was contributed in part by past cohorts who have long left the employer. This represents an advantage to existing employees of the DB plan. The DC members start their own MPA from scratch when they get hired with no subsidy from other employees. The distribution of results obtained from the 10,000 simulations are informative in themselves but also in relation to the baseline.

Before presenting the results for the total compensation, it is interesting to look at key intermediary values and major components of the total compensation. It will provide a better understanding of where the differences in compensation come from. We will discuss the

distributions of the starting salaries of the 25 cohorts, the value of their retirement funds at age 65 and their total compensations.

6.1   Comparison of Starting Salaries

Figure 6.1 shows key values of the distribution of the 10,000 simulated differences in the starting salaries of the 25 cohorts. The DB plan members contribute about 13% of the salaries at time 0 which is more than the 10% contributed by the DC plan members. Consequently, their salaries are less. After about 10 years, the DB Normal Cost rates remain fairly stable around 11.5%, which is still higher than the 10% contribution rate of the DC members. However the DB plan tends to generate a surplus each year, partly due to the increasing trend in the valuation rate (recall that the projections start in a low interest rate environment). This plan surplus, in our model, is redistributed in the form of higher salaries which explains that starting salaries of DB employees on average exceed those of the DC employees in later years.

Figure 6.1: Difference in starting salaries by cohort between DB and DC plans
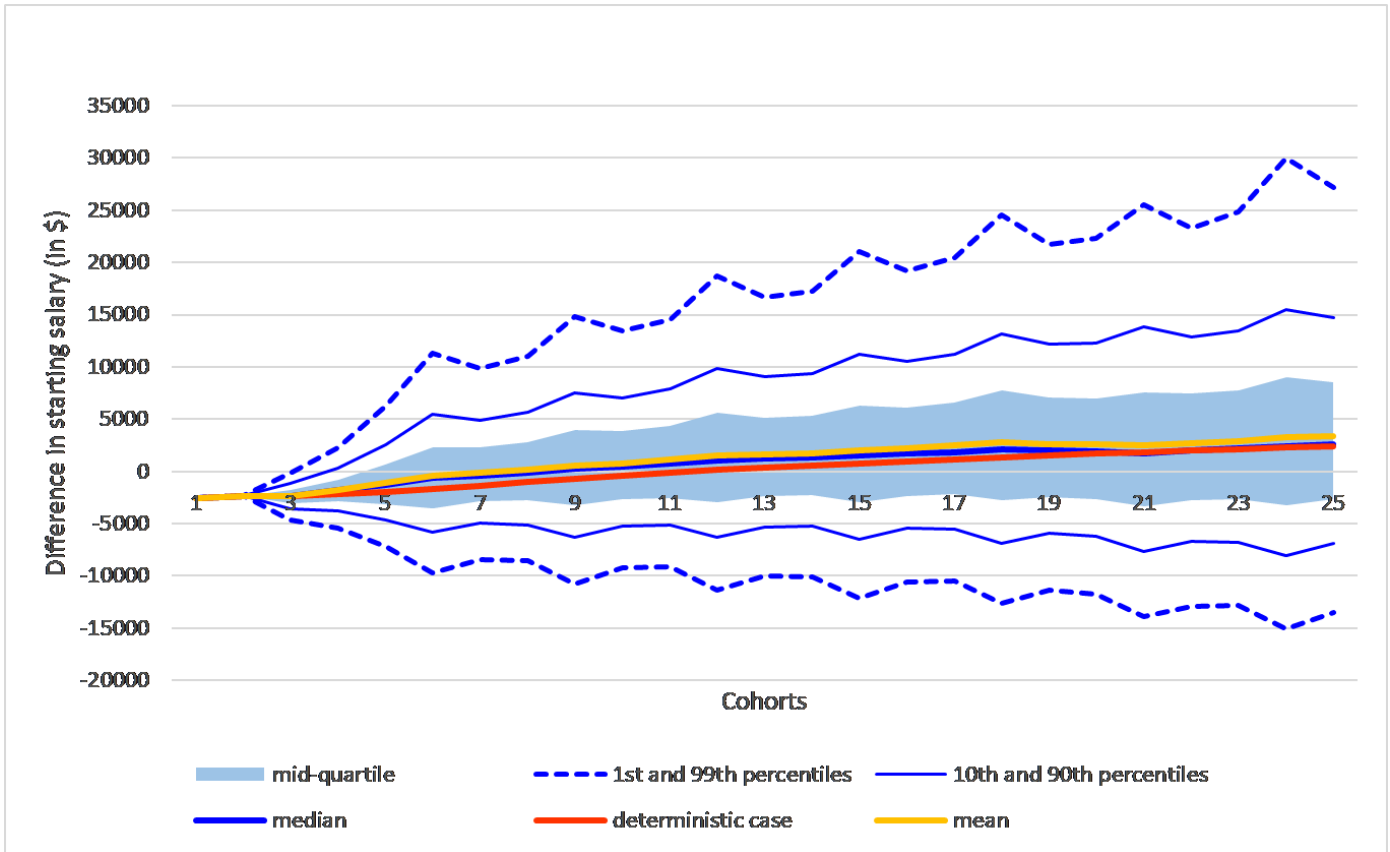
6.2   Comparison of Retirement Benefits

The amount available in the individual account of a DC plan member is, in some sense, the present value of his/her retirement benefits. This lumpsum can be managed to draw income or used to purchase a life annuity. A DB plan member does not have an account balance at retirement. The member instead has the right to receive a defined benefit for life, provided the plan remains funded. The value used for comparison will be the discounted value of his/her life contingent payments at a rate equal to the valuation rate prevailing at the time of retirement. The fair market value of the annuity would need to take into account a number of factors not modeled here, like market rates used by insurance companies to price annuities, the risk of plan insolvency, the risk bankruptcy of the employer, the management fees associated with maintaining the DC account, etc. The plan valuation rate determined in each scenario at the time a member retires, is a reasonable rate that can be used to approximate the present value of the benefits and study the volatility associated with the annuity payments.

Figure 6.2 presents the results of 10,000 simulations of differences between lumpsums of the two plans. We see that in more than 75% of the scenarios, the DB members will receive a higher value of retirement benefits.

Figure 6.2: Difference in values of retirement benefits by cohort between DB and DC plans
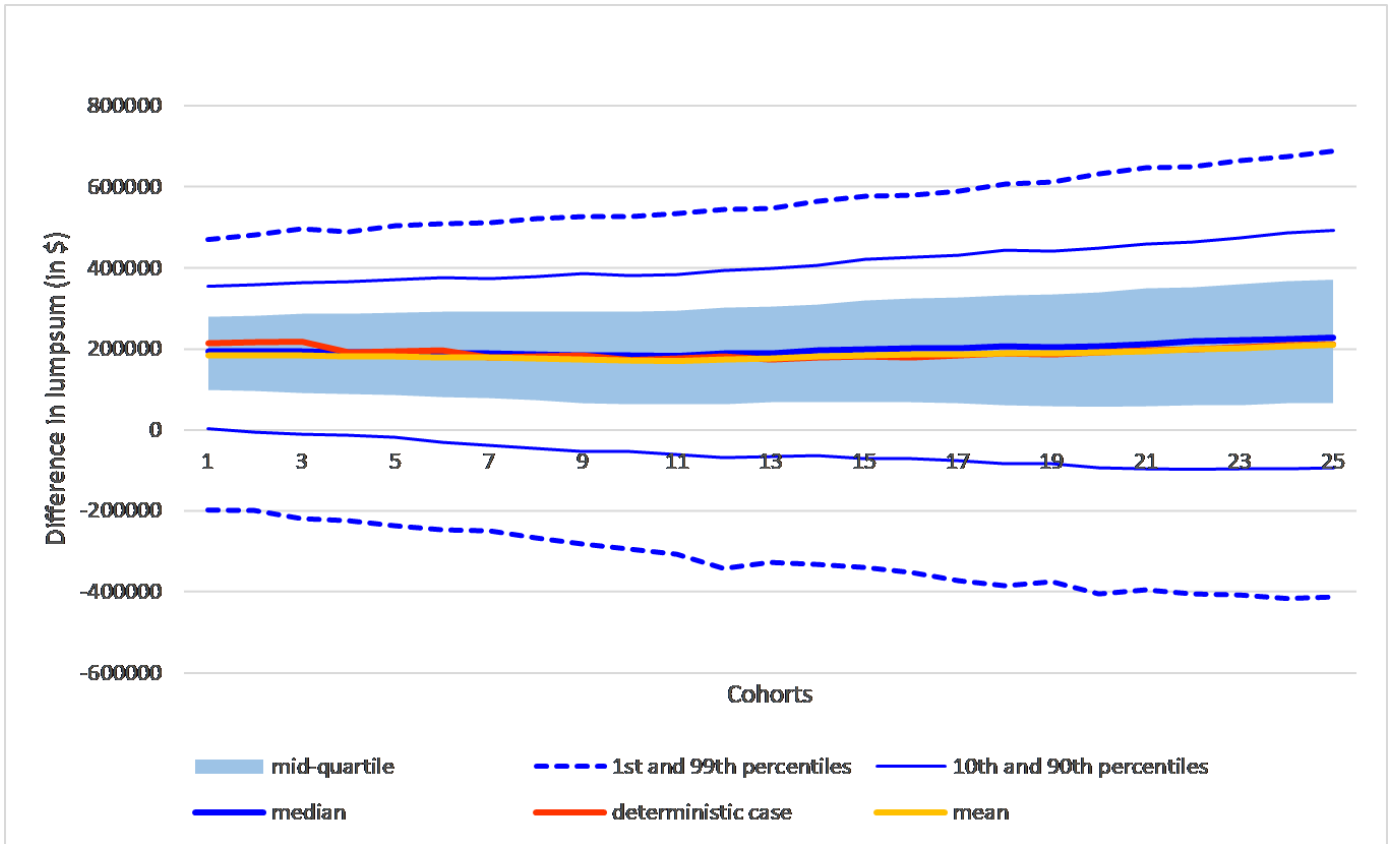
6.3   Comparison of Total Salaries

Looking only at the value of the retirement benefits ignores the important component of an employee that is the wages earned while actively employed. Providing adequate retirement benefits are not very appealing if the employee must work for a salary placing him/her under poverty level for 35 years. Conversely, a plan providing lower retirement benefits may be more than adequate overall if employees are given higher wages allowing them to make additional contributions to supplement their retirement income, for example in a registered retirement savings plan (RRSP).

Figure 6.3 and Figure 6.4 show the difference in the value of all salaries received while working, by cohort, between the DB members and DC members. In Figure 6.3 the salaries are accumulated at 1.8%, the approximate average of the rate of inflation. Salaries are accumulated at 6% in Figure 6.4.

Figure 6.3: Difference in accumulated values of salaries at 1.8% by cohort between DB and DC plans
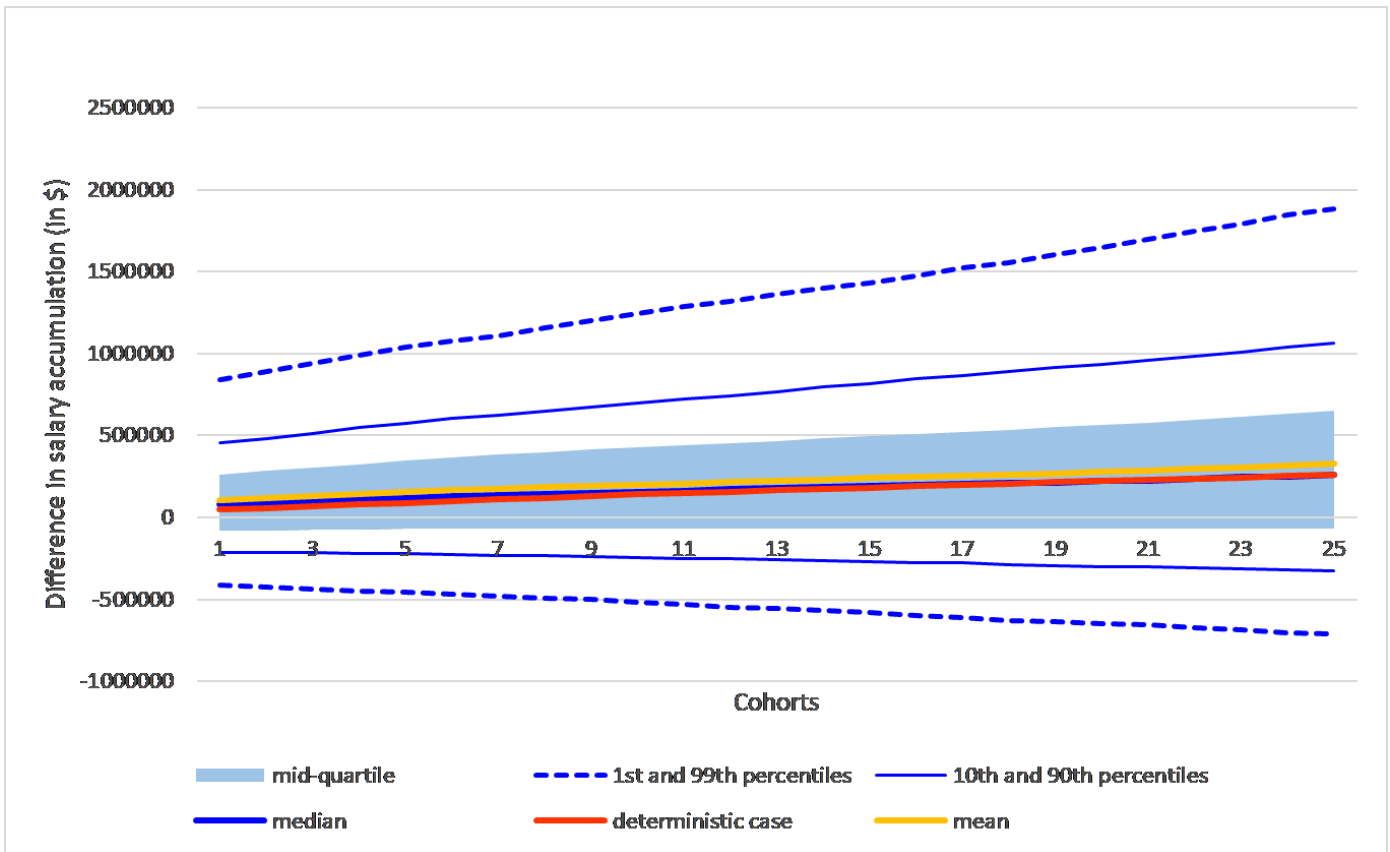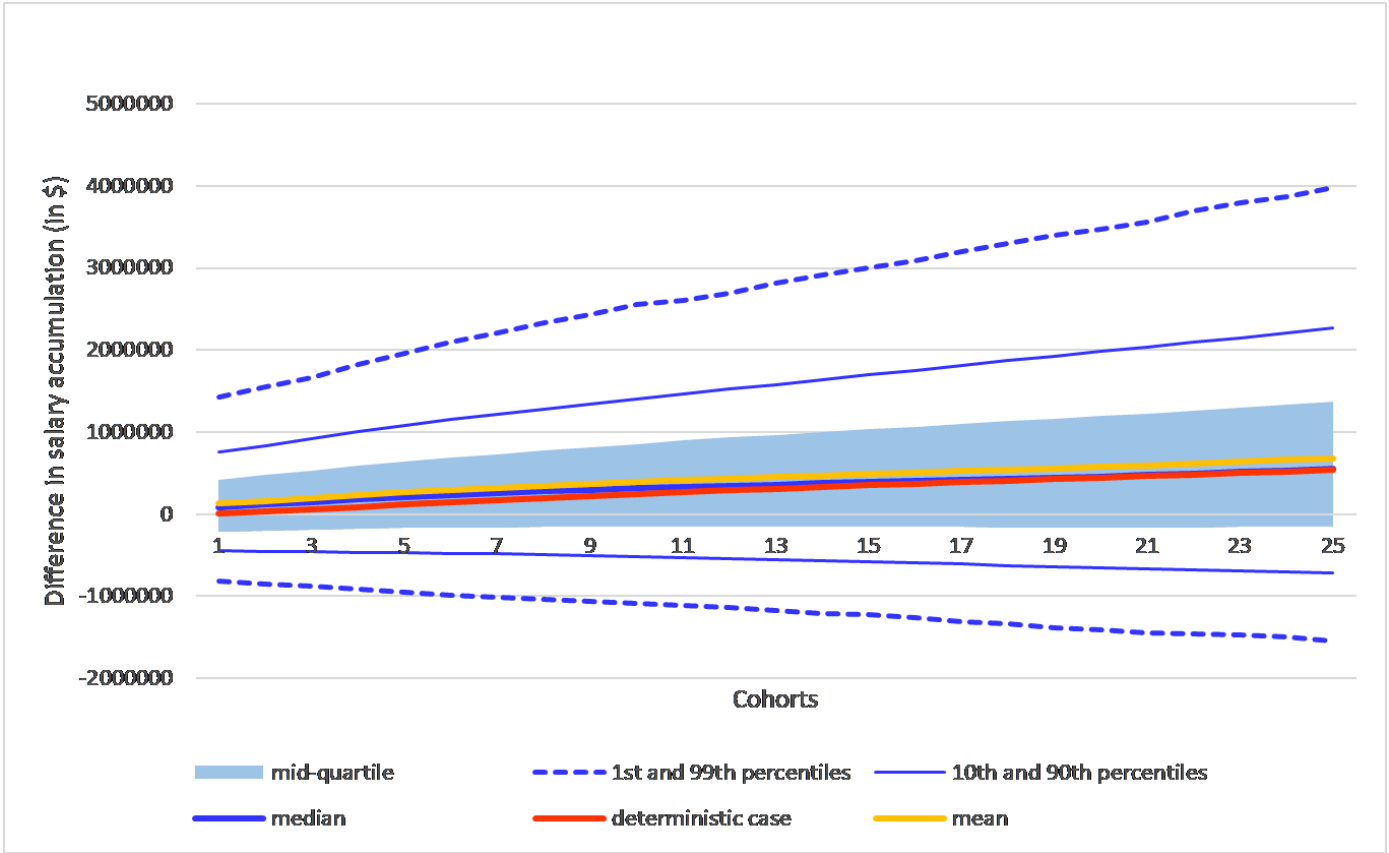
Figure 6.4: Difference in accumulated values of salaries at 6% by cohort between DB and DC plans

6.4   Comparison of Total Compensation

In the next two figures, the value of the retirement benefits are added to the accumulated value of the salaries. Each sum is for a specific scenario and a particular cohort. Figure 6.5 is for the case where salaries are accumulated at 1.8%. Figure 6.6 corresponds to salaries accumulated at 6%.

Figure 6.5: Difference in values of total compensation by cohort between DB and DC plans, 1.8% case
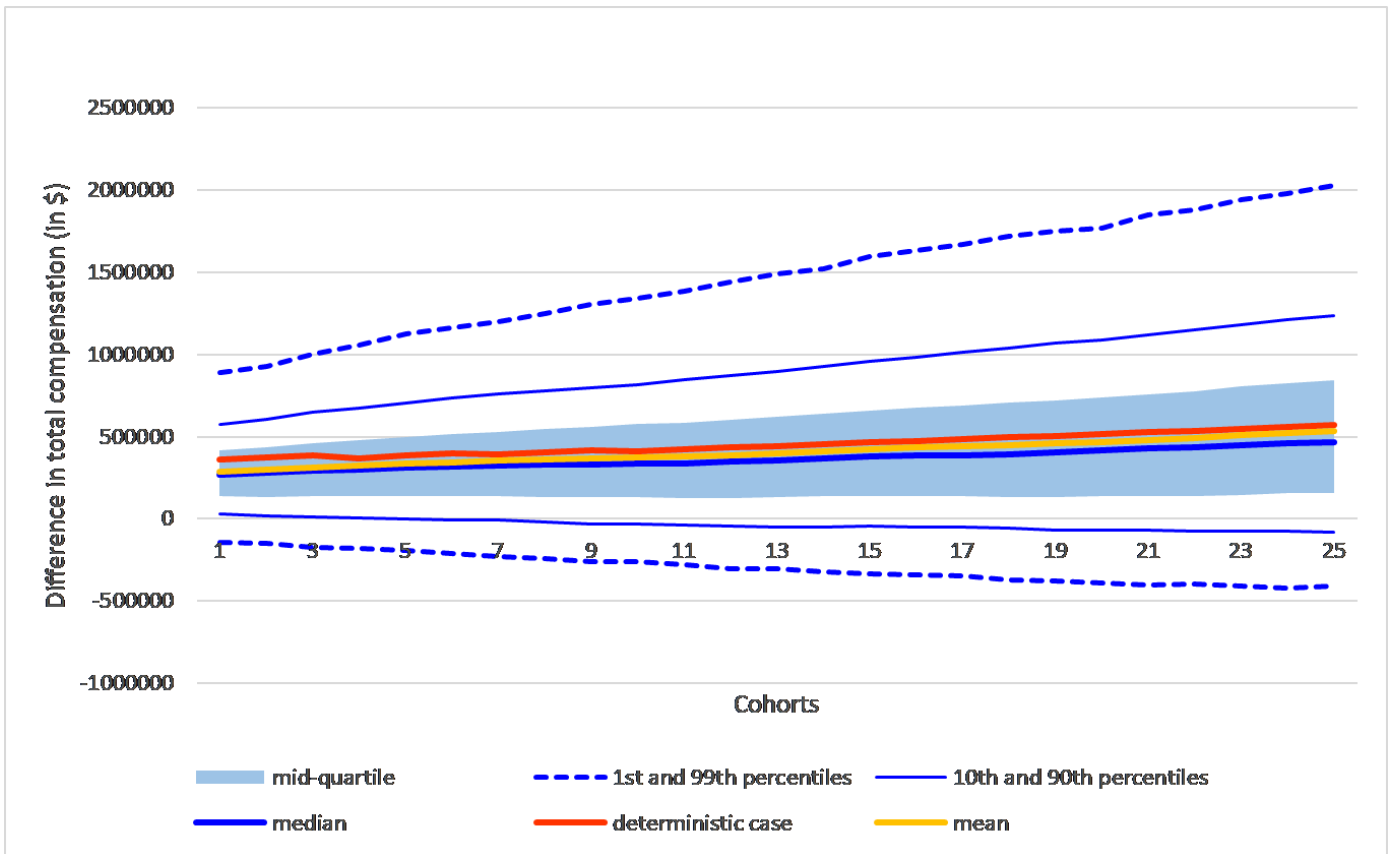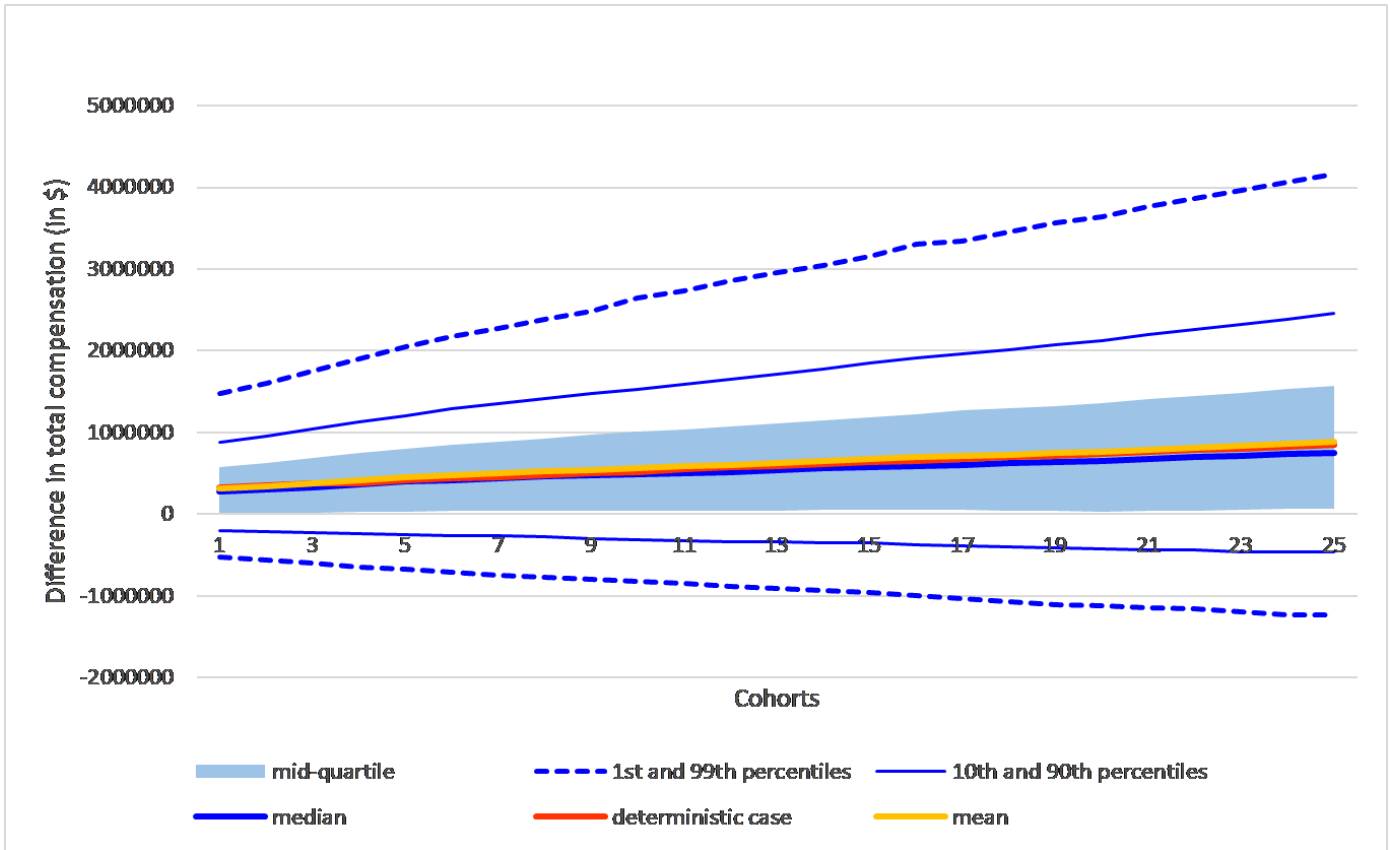
Figure 6.6: Difference in values of total compensation by cohort between DB and DC plans, 6% case

Based on the results shown in Figure 6.5 and Figure 6.6, our projections seem to indicate that the DB members would be better compensated overall than DC members. This is true at rates of 1.8% and 6%, perhaps suggesting that it would be the case at other preference rates and possibly at whatever rate would be deemed a fair market rate or even when one would consider expected utilities.

6.5   Comparison of Total Compensation Relative to Baseline

As mentioned earlier, the DB plan in our model is a fully funded plan at time 0, partly or mostly due to contributions in excess of the expected cost of benefits made by past cohorts of employees. Together with the fact that the valuation rate tends to increase over time in our projections, allows the plan to slowly release emerging surpluses to later cohorts of employees. In other words, the setup implicitly favors DB members. Had the projections started in a high interest rate environment, the conclusion might have been different. As an attempt to correct for this slight bias in favor of DB members, the baseline results (i.e. the

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deterministic case) are subtracted from the projected results. Differences in total compensation, baseline-adjusted, with salaries accumulated at 1.8% and 6% are displayed in Figure 6.7 and Figure 6.8 respectively.

Figure 6.7: Baseline-adjusted difference in values of total compensation by cohort between DB and DC plans, 1.8% case
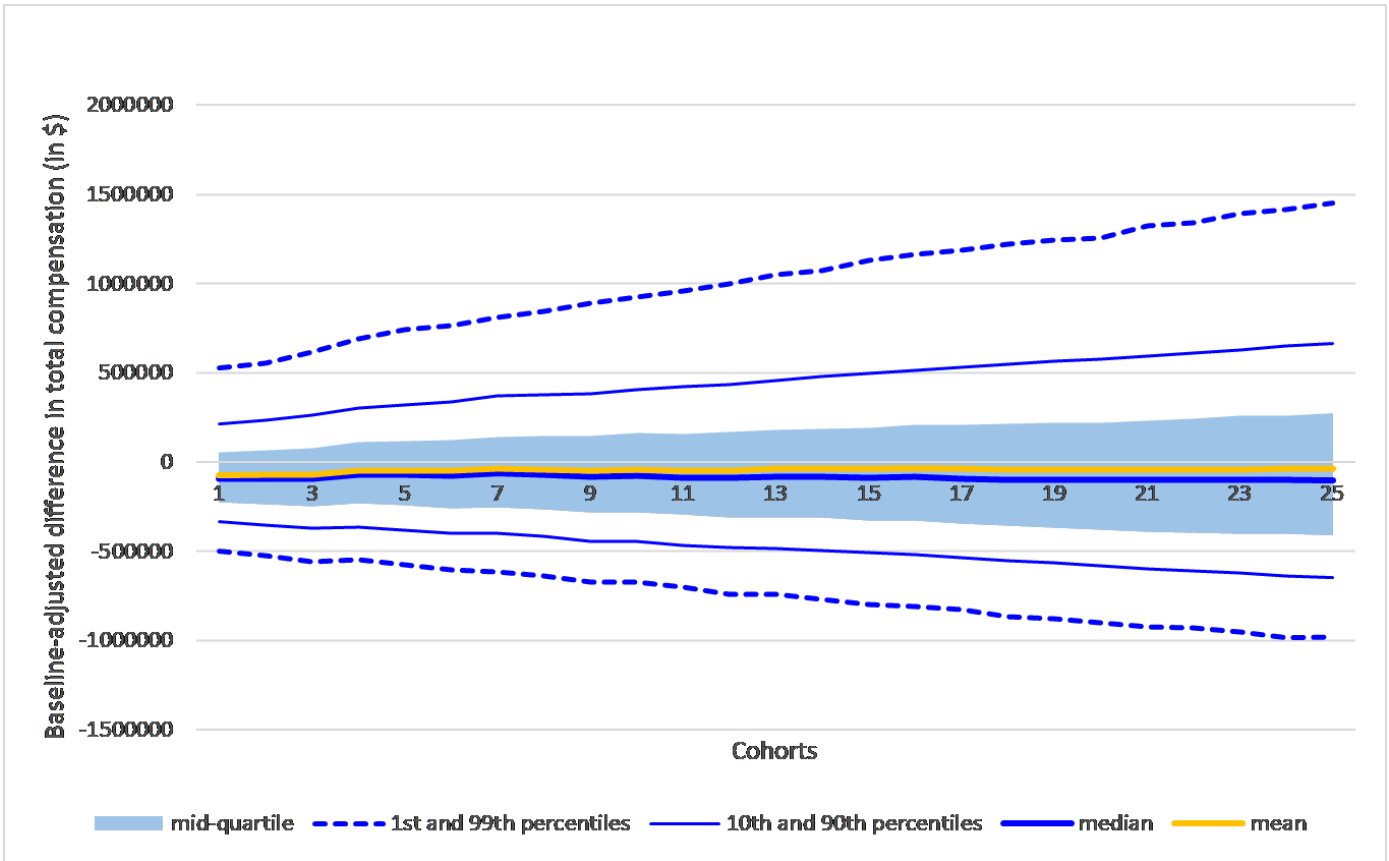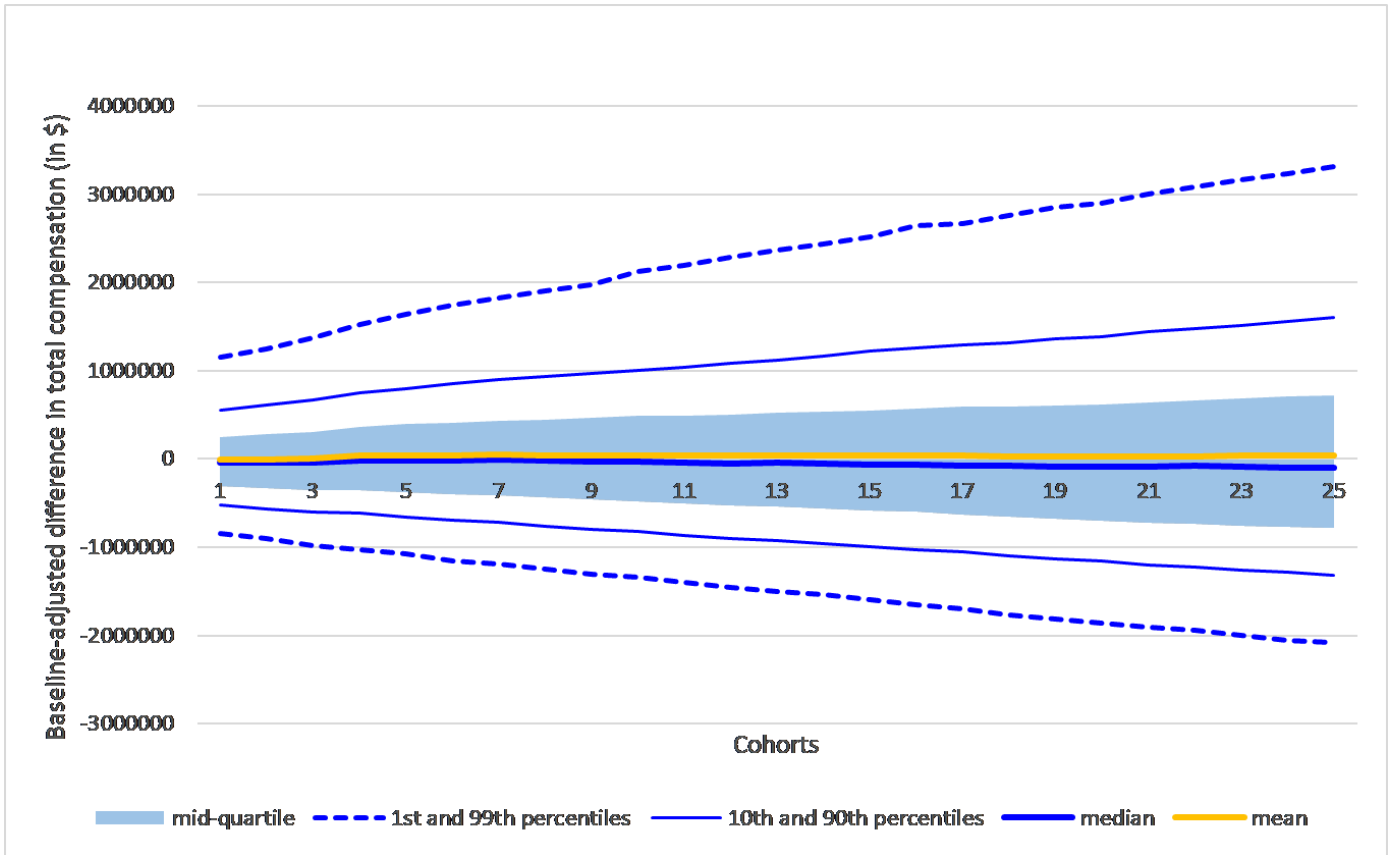
Figure 6.8: Baseline-adjusted difference in values of total compensation by cohort between DB and DC plans, 6% case

The baseline-adjusted results suggest that employees of either plan are likely to receive more or less equivalent total compensations. The differences in compensation are due to future economic environments, some favoring one type of plan over the other.

Chapter 7

Conclusion

This project was inspired by the possible transfer of the SFU Faculty Association DC plan to a jointly sponsored DB plan. Pure DB and DC plans have been the subject of many studies in the literature but their findings are far from showing some agreement as to which plan is better.

Here, the framework used to compare these two types of plans, although simplistic, is novel in the pension literature. The total compensations that the members of each type of plan would received throughout their working careers and during their retirement years form the basis of the comparison. Stochastic economic scenarios are generated by a four-dimensional VAR model and used to project the future financial situation of the employer and its impact on the wages and retirement incomes of the employees. The employer has a set budget, indexed to inflation, to be spent entirely on salaries and contributions to the pension plan. As a result, increasing the required pension contributions would adversely impact the current and/or future salaries of the employees.

A deterministic scenario corresponding to the average projections of the four chosen economic variables serves as the baseline results. In this baseline case, DB members do better than the corresponding DC members. Two main reasons explain the difference. First, the DB plan is valued conservatively and is assumed to be fully funded at the start of our projections. Secondly, the current low interest rate environment which is expected to revert back towards higher long term levels tend to lower the actuarial liabilities of the DB plan, therefore releasing funds to the employer which are then slowly redistributed to employees in higher salaries. Those are advantages not offered to DC plan members. Had the projections started with an underfunded DB plan and/or in a high interest rate environment, the DB plan members would have been at a disadvantage.

To correct for the bias due to the starting conditions of our projections, the baseline results are subtracted from the stochastically generated ones. With this correction, we find

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that the two types of plans offer similar total compensations to their members, with comparable averages and distributions.

The simple setup in which the two types of plans are compared in this project can be extended in many ways. Future work could better reflect differences that exist in practice, some in favor of the DB plan, others in favor of the DC plan. We mention a few suggestions for future work.

The life annuity promised to the DB member was valued at a rate that would not be available to the DC member. One could project market prices of life annuities and use those prices when DC members choose to buy an annuity at retirement.

Investment performances were assumed to be identical between the two types of plans. This is unlikely to be the case in practice. A DB plan can negotiate lower management fees and can rely on financial experts to manage the assets of the plan. A DC member, on the other hand, may choose to invest more aggressively or adopt a life-cycle investment strategy to increase expected returns.

The risk aversion of a member could be taken into account when considering his/her wages and retirement incomes. Using a utility function would better reflect preferences of different cash flows over time .

Many DB plans are underfunded, a comparison of the two types of plans assuming different initial funding ratios for the DB plan would be interesting.

Another possible extension, although more difficult to model, would be to allow the employer more flexibility with regard to its budget. For example, the employer could lay off some employees or delay hiring new ones when necessary.

Studying total compensations offered by different hybrid pension plan designs could also be the subject of future research.

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Appendix A

Mortality table used for retirees

Figure A.1: CPM2014 Public Sector Mortality Table

	Public				
Age	Male	Female	Age	Male	Female
18	0.00062	0.00015	67	0.00893	0.00671
19	0.00069	0.00017	68	0.00973	0.00734
20	0.00076	0.00018	69	0.01064	0.00803
21	0.00083	0.00019	70	0.01169	0.00880
22	0.00088	0.00020	71	0.01290	0.00966
23	0.00094	0.00022	72	0.01431	0.01064
24	0.00098	0.00022	73	0.01593	0.01177
25	0.00100	0.00024	74	0.01781	0.01307
26	0.00104	0.00025	75	0.01999	0.01459
27	0.00107	0.00026	76	0.02251	0.01638
28	0.00108	0.00027	77	0.02544	0.01847
29	0.00110	0.00028	78	0.02884	0.02091
30	0.00111	0.00030	79	0.03279	0.02377
31	0.00113	0.00031	80	0.03735	0.02711
32	0.00113	0.00033	81	0.04261	0.03097
33	0.00111	0.00035	82	0.04864	0.03544
34	0.00111	0.00039	83	0.05552	0.04057
35	0.00111	0.00042	84	0.06333	0.04645
36	0.00111	0.00045	85	0.07217	0.05316
37	0.00113	0.00048	86	0.08213	0.06079
38	0.00116	0.00052	87	0.09331	0.06943
39	0.00120	0.00056	88	0.10583	0.07916
40	0.00126	0.00060	89	0.11981	0.09009
41	0.00133	0.00065	90	0.13540	0.10233
42	0.00142	0.00069	91	0.15277	0.11602
43	0.00153	0.00074	92	0.17209	0.13131
44	0.00164	0.00079	93	0.19358	0.14836
45	0.00176	0.00086	94	0.21749	0.16738
46	0.00190	0.00092	95	0.24273	0.18859
47	0.00203	0.00100	96	0.26845	0.21225
48	0.00216	0.00108	97	0.29400	0.23865
49	0.00231	0.00118	98	0.31895	0.26591
50	0.00246	0.00128	99	0.34308	0.29261
51	0.00264	0.00140	100	0.36639	0.31779
52	0.00284	0.00152	101	0.38908	0.34076
53	0.00308	0.00167	102	0.41159	0.36161
54	0.00334	0.00185	103	0.43454	0.38072
55	0.00363	0.00206	104	0.45879	0.39894
56	0.00393	0.00230	105	0.47904	0.41916
57	0.00425	0.00257	106	0.49928	0.43937
58	0.00458	0.00285	107	0.51950	0.45956
59	0.00494	0.00316	108	0.53970	0.47973
60	0.00531	0.00348	109	0.55987	0.50988
61	0.00570	0.00381	110	0.58000	0.53000
62	0.00612	0.00418	111	0.60000	0.55000
63	0.00658	0.00460	112	0.62000	0.57000
64	0.00707	0.00507	113	0.64000	0.59000
65	0.00762	0.00558	114	0.66000	0.61000
66	0.00824	0.00612	115	1.00000	1.00000