The Relationship In Between Mathematics Students' Self and Group Efficacies in a Thinking Classroom

by

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Abstract

Self-efficacy has been thoroughly studied in its connections to mathematics education and student learning. However, the modern secondary mathematics classroom is shifting towards increased group activities and tasks. *The Thinking Classroom* model is a particular means of increasing student engagement through group work. As classrooms evolve, group efficacy has become an increasing factor in student achievement. This study is an exploration of the possible relationships between self and group efficacy in a *Thinking Classroom*. Students were observed during group work and then given a questionnaire to measure their self and group efficacies and any possible variation. A select group of students was then interviewed. After the interview, the results were analyzed for common themes of particular efficacies. The results suggest that the *Thinking Classroom* model can lead to more positive group efficacy, which then leads to more positive self-efficacy.

Keywords: Mathematics Education; Self-efficacy; Collective efficacy; Group efficacy; Thinking classroom

To every mathematics student who believes that they “can’t do it”.

You can.

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List of Acronyms

BC	British Columbia
IB	International Baccalaureate
PC	Pre-Calculus
SFU	Simon Fraser University
VNPS	Vertical Non-Permanent Surface
VRG	Visibly Randomized Group

Chapter 1. Introduction

When I was young I joined a swim club. It started off casually enough when I was ten or eleven, swimming two or three times a week, but in a few short years I became pretty serious. I learned all of the competitive swim strokes (freestyle, backstroke, breaststroke and butterfly), flip turns and how to dive for a race. I also learned how to stretch properly before and after practice, eat well as an athlete and be a good teammate. After a while I started swimming every day of the week, sometimes twice a day. Our team had swim meets twice a month that often involved travel – either in my home province of Alberta, or within neighbouring provinces and states. Towards the last few years of my (albeit brief) career I was very focused on my goal – qualifying, and eventually medalling at Provincial championships. My coach, a former Olympic medallist himself, helped me to achieve this goal.

Why am I writing this? Because I wanted to outline the complex factors that went into attaining my goals. Of course, there were countless hours both in and out of the pool training physically, but just as importantly as training my body, I also learnt how to prepare mentally for races. I remember the first time we were taught to visualize our races, when I was fourteen or fifteen. I found it difficult at first – when I closed my eyes I mostly saw nothing, just darkness behind my eyes, but over time and with practice I was able to see myself gliding down the pool seamlessly. I could see my best times and my qualifications. I could see myself touch the wall before my rivals. It worked. Visualization was a key factor in my success – if I could believe in my goal and see it take place, it could become reality. I eventually did qualify for Provincials, and have a silver medal to my name.

The mind is powerful. In athletics, psychology is playing a bigger and bigger role in preparation towards success as we learn more about its potential positive and sometimes devastatingly negative effects. This is true not only in sports as our beliefs, confidence and mindset play into our success or failure in anything we do.

"Whether you think you can, or you think you can't, you're right."

Henry Ford

I have been teaching mathematics in secondary schools for nearly eight years now. As I started to hone my own teaching practice, my focus gradually shifted away from what my own actions were in the classroom and more and more to those of my students. How were they behaving, thinking (or not thinking!), questioning and responding to their work in the classroom. My experiences indicated that students had an overwhelming desire to learn and find success, but were often inhibited by many factors. Generally speaking, students *wanted to learn*. So why was it that many were not?

There are, of course, countless reasons why a student may not engage in their learning on any given day. However, in my observations, student's mental health was certainly near the top of this list. Only in the last few years have I really started to pay attention to the psychological states of my students. Now that I had begun paying particular attention to students mental well-being I started to notice the effects on student learning – both positive and negative.

Being a teenager is difficult. On any given day students come into the classroom with a variety of different challenges and concerns. Some students come into the classroom with a positive outlook and believe that they are able to meet whatever learning challenges I may present to them that day. Others, come in with a more negative attitude, not believing in themselves. This could be for a variety of reasons; social, emotional, behavioural or based on past experiences. However, these beliefs have strong effects on the student learning on that day.

As I became a more experienced teacher and had worked more with students, I began to take note as they showed their beliefs about their skills through their body language, sometimes even putting them into words. Hearing "I can't do this" or "I'm bad at math" is likely a common enough experience for most teachers, but in particular those who teach mathematics. A particularly negative stigma around mathematics and the learning of mathematics has developed over the past few decades. When asked, many people will readily admit to their own difficulties with mathematics as a student or self-profess that they too were “bad at math”. I am often surprised when people who I know to be good problem solvers have these sentiments. I observed all of these factors in my own students as well. I found myself wondering how I could encourage a more positive outlook and help them to see their own potential.

So where did these negative beliefs begin? The younger students that I taught seemed excited and positive towards learning mathematics. This did not, of course, fade for all students, but as students got older I noticed the excitement decline for many. These same students also maintained the idea that even if there is a particular skill or set of knowledge that they do not presently know, they are able to learn it through time and effort. This is known as the “growth mindset”, discussed in detail in Carol Dweck’s 2006 book ”Mindset: The new psychology of success”. This mindset that a person *can* learn and will get smarter has been shown to be paramount in their learning (Dweck, 2006). However, in general, as students got older I noticed more and more had a negative outlook.

At some point in their life, many students are met with challenge, or perhaps an event which can shift their personal beliefs. Beliefs that once were positive sometimes shift to negative ideations of ability. In particular, in mathematics class I saw many very capable students who maintained that they were unable to accomplish a given task. However, once given a starting point, the majority of these students were able to find success. This discord between how the students thought they would perform, versus what I saw them actually accomplish, always fascinated me. As a teacher this lack of self-awareness was puzzling. Although the majority of these observations were with students who claimed inability, there were also converse situations; students who thought they knew everything, but regularly showed that they did not. Truly, the psychology of some students did not align with what they were actually demonstrating in my classroom. I found myself wondering whether these beliefs stemmed from previous experiences and were being projected into new ones.

Early in my teaching career I noted the positive effects of group work in the mathematics classroom. Although my own high school classrooms had been set up as rows of individuals working quietly, I knew that opportunities to discuss and think gave rich opportunities for student growth. I have always tried to find moments for group work, whether for short moments during lessons in a “think, pair, share” activity, or for more in depth learning – perhaps in a discovery model. I have also seen immense value in student journaling and since my student practicum in April 2008 have found ways to incorporate this reflective task in my classroom.

After some time in my own classroom I began looking for new strategies – ways to engage my students in their own learning and include group work in meaningful learning opportunities. After some discussion with a colleague, I learnt about the Master’s program for Secondary Mathematics Teachers at Simon Fraser University. After some research into the program I decided to apply in the fall of 2015. I was excited to learn new teaching strategies and to question my own practice. I was also excited to meet like-minded educators from around the lower mainland and share a common experiences.

Although there are countless positive things I took away from the program as a whole, two stood out in particular for me in my own teaching. First was our course with Dr. Peter Liljedahl on the teaching and learning of mathematics. In this course we were introduced to the “thinking classroom” model, which I will explore in more detail in Chapter 2. The most impactful aspect of this model was the continued emphasis on the effectiveness of group work. Although I had frequently used partner work in my classroom over my years of teaching, I had not used groups of more than two on many occasions. Dr. Liljedahl showed us through both our own experience how groups of three or four students helps to contribute varied ideas, opinions and insights. In particular, when students are given a challenging task that has a low entry point with further extensions, Dr. Liljedahl showed that students are more likely to stay engaged and interested in their work when working together.

I found this intriguing. In our course we were often exposed to this style of teaching as students ourselves. I found myself completely engrossed in the tasks. Of course, it is worth mentioning that I was already (almost) always excited to think mathematically, whether in a group or not, but this new format felt exciting. There were times when I was stuck in my work and would gain insight from my group members so that we could make progress. It was effective. There was lively discussion amongst my peers and our understanding was always deeper than it would have been individually. I found myself employing group work in this manner almost immediately in my own classroom and was astounded by the impact. Students were talking with each other about mathematics. They were engaged in lively debates and were often asking each other about their reasoning or questioning other's work. I was excited about the change that I saw in my students.

Although I see the positive impact on my students when working in groups, this is not the sole focus in my classroom. I still think that it is important for students to work individually. I hope that my students will take their experiences in their groups and then construct their own understanding. Individual work is a key component of my classroom as well, but where and how the learning takes place has shifted more towards the group components. I now see individual work as a time to solidify and concretize understanding. Group work can be an effective place to establish and improve understanding, but it can be difficult to assess and evaluate individual's understanding. I am still looking for methods to assess particular learning outcomes during group work, but currently evaluation is mostly done individually.

Secondly, our course on research methodology with Dr. David Pimm was very significant for me. In particular, his introduction of the term *noticing*, as used by John Mason gave a specific term to observing students. As teachers we of course make observations about our students every day, but simply by labeling and noting particular behaviours we can then act on what we observe. In particular I started to take note of student activity in my classroom and began to make a record of what I would notice. When students showed particular behaviours of either engagement or disengagement I would write it down in my day planner. This was done primarily during group work, although sometimes during individual work time as well. Among my observations were physical actions such as slouching versus standing upright, facing towards their whiteboard or away, holding a pen or not, among others. Communication was also noted; the language that was used, whether students spoke frequently or not as often, students asking each other questions, and any other means of communication that I saw. Once student actions were labeled as either more or less engaged I was able to see what was more effective in my classroom for different purposes.

One of my most frequent observations was a stark difference in students when working in a group versus alone. Some students would be active participants in a group setting, and for example take the whiteboard pen or engage in lively debate with their group members. However, these same students would sometimes disengage when working alone and sometimes seem as though they had no idea where to start on a given task. The opposite was also true; students who would fly through their individual problems would not seem as positive and motivated in their groups. I found this dissonance in group versus individual engagement to be utterly fascinating.

My new classroom model was leading me to be more excited about my teaching again, but I could not shake this observation of student variance from individual to group work. Generally, students seemed to believe in themselves more when working in a group. This ultimately was visible as a student’s group efficacy – that is, how he or she thought they would perform in a group. My observations led me to believe that student’s efficacy was more positive in a group than it was when they were working alone. In other words, students seemed to think they would be more able to complete a mathematics task when working in a group then by themselves. I wondered if this was true.

All of this experience culminated in my curiosities of psychological impacts of students learning individually versus in a group. I wondered about why students seemed to feel so much more positively when working with others than on their own. I wondered why the belief of how they would perform so strongly impacted their actual performance, which eventually led me to my main research question:

> *What is the relationship between student self and group efficacy in a mathematics classroom?*

Do students with more positive self-efficacy have more or less positive group efficacy? This answer would help me relate what I was observing in the psychology of my students when working alone versus in a group. In addition to my main research question I hoped to find insight into several other sub-questions:

- Does the **type** of mathematics problem **posed** have any effect on efficacy?
- What effect do the group members have on group efficacy?
- Is there a particular type of student who has more positive self than group efficacy?
- Is there a particular type of student who has more positive group than self-efficacy?

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Chapter 2. Related Literature

2.1. History of self-efficacy

Self-efficacy is defined as a person’s beliefs in their ability to perform a particular task, or achieve a specific goal (Bandura, 1977). The construct was first introduced as part of a theory of behavioural change that united several psychological domains. This new theory became known as Albert Bandura’s social cognitive theory. In short, this theory posited that people learn by observing, imitating and modeling others (Bandura, 1977). Social cognitive theory also introduced the idea of efficacy whereby people make judgments on their own ability. Based on Bandura's theory, self-efficacy is “an individual's convictions (or confidence) about his or her abilities to mobilize the motivation, cognitive resources, and courses of action needed to successfully execute a specific task within a given context” (Stajkovic & Luthans, 1998, pp. 62). Self-efficacy represents a positive belief in one’s abilities, but does not necessarily reflect the ability to actually achieve results.

Decades of research have shown that positive self-efficacy is linked to positive outcomes in various facets of life (Gallagher, 2012). Those identified as having more positive self-efficacy (through questionnaires, for example) have been shown to have higher motivation and set higher goals for themselves (Bandura, 2000). With this comes positive visualization of themselves achieving these goals making them more likely to actually do so; thus creating a positive feedback loop. Higher self-efficacy has been linked to better self-regulation and a general feeling of having control over events (Bandura, 2000). Conversely, those who fail to exercise control over their thoughts may have a weakened ability to cope with life’s stress, possibly leading to anxiety and mental health concerns (Bandura, 2000). People with lower self-efficacy tend to avoid difficult tasks and are therefore less likely to experience new successes (Bandura, 2000). Positive self-efficacy has been tied to the willingness to seek treatment of mental illness and other psychological ailments (Gallagher, 2012). All of this is to say that more positive self-efficacy allows people to face everyday challenges with a positive outlook and believe that they will be able to cope with whatever life may bring.

Self-efficacy therefore has numerous connections to agency, and has a positive impact on our lives. Human agency refers to our ability to influence outcomes and exercise our free will. Bandura (2000) hypothesized that efficacy beliefs formed the foundation of human agency. If people did not have a sense of agency that they could produce particular results or outcomes, then they had little or no incentive to act (Bandura, 2000). Self-efficacy beliefs have been shown to be primarily based on past performance, and this experience is used to inform current efficacy expectations. These beliefs, in turn, influence persistence in a task. It is important to note that efficacy is not constant – it can be changed for the better or worse depending on developing attitudes, experiences and outcomes.

Attitude has also been studied in relation to self-efficacy (Liu & Koirala, 2009). There are, however, differences between self-efficacy and attitude towards a task. While self-efficacy is defined as a person's belief about their ability to complete a task, attitude relates more to a person's general sense about the task (Liu & Koirala, 2009). For example, a person may believe that a task is important, enjoyable and worthwhile, but not believe in their own skill to do so. This exemplifies a positive attitude, but low self-efficacy. Of course, the opposite can also be true, where a person has a negative attitude, but high self-efficacy.

It seems a natural extension then to consider whether or not a person's enjoyment of a task relates to self-efficacy. If attitude towards a particular task correlates with self-efficacy, then enjoyment should too. If a person is having fun, then their persistence and perseverance will increase (Csikszentmihalyi, 2008). In his book, *Flow: The psychology of optimal experience*, Csikszentmihalyi discusses his idea of flow – a state that is the perfect balance of appropriate challenge for one's skill level. Most people have experienced flow, perhaps during sports activities, performing arts or working on a puzzle or game. Usually these people would agree that what they were doing was fun and that they became so engaged in their experience that they lost track of time. Csikszentmihalyi would argue that in order to achieve flow, one must have an appropriate balance between challenge and skill. Once experienced, flow is a state that is desirable; that is, those in flow want to stay in flow.

This idea of flow can naturally extend into the mathematics classroom. Through the experience of flow, students may feel swept up in their experience and engage more deeply with their task. In his research into the thinking classroom, Peter Liljedahl (2016) has connected flow with mathematics education and argues that having flow is an optimum experience for many learners. Students were observed in a state of flow while working on carefully selected problems over the course of several weeks. Student engagement was maintained, even though they were working on the same problem because students were able to make complex discoveries through their work. Bandura (1997) showed that efficacy beliefs affect engagement with an activity as well as a willingness to increase effort. More recently, Mesurado, Richaud and Mateo (2015) have shown that “self-efficacy has a positive effect on flow and engagement” (p. 295). Although this particular study was not specific to mathematics and looked more generally at post-secondary students, it seems likely that research in mathematics education would yield similar results. Self-efficacy, flow and engagement seem to be intimately connected and are deserving of further research.

2.2. Mathematical self-efficacy

As self-efficacy research increased and diversified, specific areas were studied in more depth. Mathematical self-efficacy, or how one perceives their own abilities in particular to mathematics, started to grow in research popularity. There have been countless studies of the relationship between self-efficacy and mathematics since the early 1980s and how this relationship may be related to other aspects of self or future self.

As described in section 2.1, self-efficacy is related to one’s attitude towards a task. Although it is possible to have a discord in attitude towards mathematics and self-efficacy, a positive attitude usually helps students persist through difficulties and ultimately may increase their self-efficacy (Liu & Koirala, 2009). It seems intuitive, but Liu & Koirala (2009) showed that there is a direct relationship between attitude towards mathematics and mathematics achievement – that is, the better the attitude, the better the achievement. In addition, people who say that they enjoy mathematics also tend to achieve higher grades (Liu & Koirala, 2009). These indicators of positive outlook are related to agency and a sense of ability – if one feels that they can influence an outcome then they are more likely to have more positive self-efficacy. Ma and Xu (2004) also showed that attitude towards mathematics was a significant predictor of achievement in mathematics. Attitude and its relation to mathematics has been widely studied, and a relationship between the two has been noted. However, attitude and self-efficacy are not the same, and the findings of attitude towards mathematics do not necessarily carry over to self-efficacy.

Confidence is closely related to self-efficacy, but is generally considered more global – that is, less specific to the task at hand. Hackett and Betz (1981) found that self-efficacy was a more effective predictor of mathematical performance than confidence and further is a more accurate predictor of mathematics-related educational and career choices. Moreover, mathematics anxiety, an increasingly discussed psychological construct, has been shown to be a result of low mathematics self-efficacy, according to social theory (Hackett & Betz, 1989). Self-efficacy has been shown to help determine whether or not someone will attempt a task in the first place (Bandura, 1986). Expectations of success or failure, which help determine one's self-efficacy, have been shown to be an important factor towards mathematical performance, and ultimately, career choice (Hackett & Betz, 1989). Indeed, people who identify as having low self-efficacy may not even start a task in the first place, as they do not think they will succeed – clearly their performance will be poor. It follows then, that low self-efficacy in mathematics may cause someone to choose a career path elsewhere, regardless of skill. In these studies, Hackett and Betz argue that self-efficacy is a situation or problem-specific measure, as opposed to confidence or anxiety, which are usually more global measures.

As Bandura noted, both content and context play a large role in the development of self-efficacy beliefs. According to Pajares (1996), this specificity is sometimes overlooked in mathematical self-efficacy research. There have been some very specific content and context studies done, but the results cannot be generalized to say anything larger about mathematical self-efficacy. For example, in 2003, Nielsen and Moore conducted a study of over 300 students in Grade 9 in Melbourne, Australia. Their study designed a Mathematics Self-Efficacy Scale (MSES) based on the particular curriculum of this grade in Australia. The scale included nine items on a Likert scale and asked students to respond to how they would estimate their ability. The questionnaire did not ask students to solve problems but rather how they thought they would perform. Nielsen and Moore administered the questionnaire twice, once when the context of the problems was discussed in the mathematics classroom, and then again in a test setting. The questions were content specific, for example, trigonometry and curve sketching. The findings were that students reported significantly higher self-efficacy in the classroom than in the context of testing. This validated Bandura's (1997) findings that self-efficacy is both content and context specific. In particular, Nielsen and Moore's research showed that students with less positive mathematical self-efficacy showed greater differences in their scores from the classroom to testing contexts, with more positive self-efficacy in the classroom.

Self-efficacy measures will therefore vary, depending on the task that is assigned. If students know the task will be very difficult, their self-efficacy will be reported as lower, whereas a simple task will result in higher self-efficacy. Pajares and Miller (1997) conducted a study on the effect of varied assessment on student self-efficacy. In their study, they gave two groups of students different formats of a test on the same material; one was a multiple-choice test, and the other an open-ended format. Interestingly, the groups showed no significant difference in their self-efficacy scales. However, the group who took the multiple-choice test had higher scores than those on the open-ended test. This finding suggests an issue of calibration – that is, the accuracy of the students' judgment of their own abilities. If a student has good calibration, she will be able to accurately predict her outcome on a particular assessment. Poor calibration of mathematical performance suggests that a student cannot predict an outcome because of inaccurate alignment with efficacy beliefs. Pajares and Miller (1997) found that there may be inaccuracies with students' self-perception of their ability to perform on the mathematical assessments provided, regardless of their format. Hackett and Betz (1989) have also found that students consistently overrate their ability to solve mathematics problems. Perhaps students have developed trust in their teacher or educator to give them appropriate level of challenge on an assessment, but further research is needed to identify the causes of poor calibration.

Nielsen and Moore (2003) discussed this issue of a self-efficacy scale. Content specific scales seem ineffective as it is unreasonable to perform a measurement of self-efficacy for every individual task or learning outcome. They noted that it "seems unlikely that a student with high self-efficacy for one algebra problem would have low self-efficacy for another algebra problem" (Nielsen & Moore, 2003, p. 129). Such adherence to content specific diagnostics seems time consuming and potentially costly for research. Nielsen and Moore (2003) therefore suggested that an improved and efficient mathematical self-efficacy tool might ask questions about more general mathematics abilities; for example questions pertaining to algebra, geometry or fractions independent from one another. This maintains some content specificity, but is a more practical means of conducting research. Similarly, Nielsen and Moore (2003) discuss both the location of the assessment conducted and to where the mathematics discussed would take place. Many students do not manage well on examinations due to time constraints and the lack of availability of supporting materials, but perform quite well in other scenarios. Self-efficacy has been shown to vary whether it pertains to a testing scenario or work within the regular mathematics classroom (Nielsen & Moore, 2003). These content and context specific requirements make research in mathematical self-efficacy challenging as every study must choose to measure very specific content accurately, or general content less accurately, and will pertain to the particular context in which it is set.

2.3. Changing self-efficacy

According to Bandura (1997), there are four methods to improve self-efficacy. The most powerful means of doing so is by a person's own previous experience, referred to as mastery experience. Experiencing success in a particular domain will increase self-efficacy. Secondly, vicarious experience, or watching another person have success has also been shown to increase self-efficacy beliefs. This can be why modelling behaviours of others can be so important in our self-beliefs. Thirdly are social persuasions from others around us. This can be feedback from a family member, friend, teacher or peer that has some evaluative component – for example, being told that you have improved in a particular domain or that you have done something well by another. Lastly, emotional and physiological states can influence self-efficacy, such as anxiety, fatigue and low mood. Improving these states has been shown to improve self-efficacy (Bandura, 1997).

These four means of self-efficacy improvement can be practiced in the classroom. Student mastery and vicarious experiences, social persuasions and feedback from peers and teachers are all a part of classrooms. Thus, students are experiencing Bandura's four forms of self-efficacy improvement on a near daily basis, assuming that they are receiving the positive versions of these means. If students can identify and practice these four means then self-efficacy has been shown to be enhanced over time (Usher, 2009). Teachers can also help improve self-efficacy through effective classroom structures and activities (Usher, 2009). Student behaviours can also positively influence self-efficacy and an improvement in self-regulation has also been shown to enhance self-efficacy (Usher, 2009).

There is evidence of student experiences changing affect and efficacy beliefs through classroom practices. In his case study of one student, Sara, Liljedahl (2015) observes that a mathematics student with initially low self-efficacy transforms to more positive efficacy beliefs. Liljedahl (2015) proposes the presence of what he calls an "affect system". Liljedahl suggests that several affective variables, such as beliefs, emotions, and efficacy are all connected in a dynamic way. Liljedahl notes, however, that since beliefs "are not held logically – they are felt not reasoned" (p. 3) these developed systems of affect are not organized in a logical means. He states there are two sources of initially developing this affect system, the first being experience-affect. That is to say, a particular experience may result in an affect reaction – for example a bad test experience may cause a student to feel poorly. The second source of developing an affect system is affect-affect. For example, an already low self-efficacy belief may cause higher anxiety. In research, Liljedahl found that by changing a student's experience through the *thinking classroom*, he was able to note significant affect changes through the experience-affect means. These changes were noted through qualitative interviews. Interestingly, through his case study, his participant specifically noted a change in group-efficacy and noted the feeling that "we are stronger together" (pg. 9). Clearly, this is one of the first indications that the *thinking classroom* model can positively change efficacy beliefs. So then, what is this *thinking classroom*?

2.4. The thinking classroom

The stereotypical traditional mathematics classroom is easy to picture. Imagine a board at the front of a room with rows of students taking notes, guided by a teacher at the front of the room. This picture, although perhaps not as prominent as it once was, is often how people have experienced at least some of their mathematics education. There has been much research done on new approaches to the teaching of mathematics. Dr. Peter Liljedahl of Simon Fraser University has spent over ten years researching classroom and teaching practices in British Columbia and has devised several strategies to help develop what he has called the *thinking classroom* (Liljedahl, 2016a).

A *thinking classroom* is defined as "a classroom that is not only conducive to thinking but also occasions thinking, a space that is inhabited by thinking individuals as well as individuals thinking collectively, learning together and constructing knowledge and understanding through activity and discussion" (Liljedahl, 2016a, p. 364). Liljedahl's work builds on decades of research on similar ideas that focus on getting students to work in groups, engage in problem solving and to construct their own knowledge. An emphasis on discourse and discussion provides rich learning opportunities that do not typically occur in traditional classrooms, such as collaborative dialogue and resolving of conflict amongst peers (Yackel, Cobb, & Wood, 1991). Others have done much research into constructivist teaching approaches and how best these can be implemented in the classroom (O'Shea, & Leavy, 2013). Upon learning about Liljedahl's *thinking classroom* as a pedagogical approach, I wanted to engage my students in the construction of their own mathematical ideas and shifting to this more dynamic classroom model intrigued me. There are several key shifts that took place in my own classroom from a more traditional to the *thinking classroom* that I will outline here.

2.4.1. Tasks

One of the first and simplest shifts suggested in Liljedahl's work is to start each class with an engaging task. Initially, these tasks should be non-curricular, highly engaging tasks that students want to solve (Liljedahl, 2016a). Tasks should have a low floor and a high ceiling (Liljedahl, 2016a). That is that any student can start on the task without much background knowledge, but the task can be extended to higher complexity as appropriate. In addition to making the current task more complex, extensions or modifications to the task can be offered as students start to understand the topic better. Extending a task as needed should be based on the direction the students have been taking and may vary from group to group. Liljedahl argues that oral delivery of tasks is most effective to get students focused and collaborating as quickly as possible. If any part of the task is unclear, students will need to immediately clarify when they begin working together, and this process can mitigate any quiet starting moments.

Tasks should also be highly collaborative. A good task is one that compels students to talk to each other (Liljedahl, 2008). By having students engage with these approachable, non-curricular and interesting tasks, the classroom dynamic rapidly shifts. Students become more autonomous and willing to take risks in their mathematical thinking. For me, the shift was sudden and apparent – students did not want to stop working. This simple way of starting my classes, which was quickly dubbed our "warm-up" by students, was a way to engage students in mathematical thinking and encourage collaboration through a task they genuinely enjoyed.

After some time of developing this new way of starting class, after two to three weeks, non-curricular tasks can be replaced by curricular tasks (Schoenfeld, 1985). By this time students have adjusted to this new way of working on a task to start the class and will comfortably transition into tasks involving the curriculum, or the prescribed learning outcomes of the course. These tasks can be taken from class notes, or textbooks, so long as they are presented in the same manner as previously.

2.4.2. Visibly-randomized groups

Group work has become common in progressive classrooms (Davidson & Lambdin Kroll, 1991) as it has many positive effects on a learning environment. Students who work in cooperative classroom environments usually outperform students working on similar material in individual settings (Stevens & Slavin, 1995). Group work has been shown to benefit both low and high achieving students (Van den Eeden & Terwel, 1994). High achievers benefit by explaining concepts to other's in their group, while low achievers have more opportunity to ask questions within their peer group. Schoenfeld (1983, 1989) has found that cooperative learning has several positive effects particularly on student's abilities in problem solving. Discussion is a necessary part of group work and students will develop their communication skills. Teachers are able to circulate and guide groups in their direction, suggesting strategies that will further their understanding. Working closely with peers also gives students insights to how others are perceiving a problem and that their difficulties are likely not alone. Given all of the possible benefits of group work, it seems natural that it is having a resurgence of interest in the classroom. With this interest, there is a need to have some pre-designed structure for cooperative learning with clear guidance from the teacher as to the expectations of his or her students. Possible pre-designed structures have been previously researched by many, but in particular Jo Boaler (1997), whose suggested models often assigned students specific roles within a group.

I have often employed group work throughout my classes, but prior to beginning my Master's education I would usually leave it to students to self-select their groups. In doing so, I inadvertently allowed students to create groups based on social persuasions, which is common in many classrooms (Urdan & Maehr, 1995). However, grouping based on social reasons can be problematic. In my experience it can lead to exclusion and negative psychological impacts on students – there are typically some students that feel 'left out'. In addition to this psychological effect, working with the same, or similar students every time does not give students opportunity to learn to collaborate with new people. Anecdotally, being exposed to new perspectives in their mathematical work can help to encourage students to be open to new ideas and approaches. Although it was clear that group work was an effective pedagogical tool, I was not sure how best to assign the groups.

When I began learning from Dr. Peter Liljedahl at Simon Fraser University in the fall of 2015, one of the first strategies he introduced was the use of visibly random groups. His strategy was to use playing cards to do this, although I have since learned that there are many other ways to have the same effect. By having students draw a card at random from a deck, it becomes transparent that there has been no strategic selection of groups made by the teacher. Liljedahl has since shared his work on visibly random groups (VRG), and there have been some excellent findings. Not only do VRGs lead to many of the outcomes teachers hope to achieve with group work, but they also have several other positive effects on the learning environment. It has been shown that frequent randomization can fundamentally transform the social structure of the classroom within three weeks (Liljedahl, 2016a, 2016b, 2014).

In his work on the thinking classroom, Liljedahl includes a chapter on “The Affordances of Using Visibly Random Groups in the Mathematics Classroom” (Liljedahl, 2016). His main findings show that VRGs can lead to:

- Students becoming more agreeable to work in any group.
- The elimination of social barriers in the classroom.
- An increase in mobility of knowledge and understanding between students and student groups.
- A decrease on reliance of teacher knowledge and validation.
- An increase on reliance of student groups; both inter and intra-group.
- An increase in student enthusiasm and engagement in their mathematics tasks.

As stated above, the typical goal of group work is to enhance and foster all of the above, but many teachers try to create these outcomes by creating the ‘perfect’ groups. Instead, by consistent using VRGs, we can quickly and seamlessly create an environment in which students feel supported by each other and can develop their autonomy in mathematics.

2.4.3. Vertical non-permanent surfaces

As group work became more abundant in progressive mathematics classrooms, student workspace also evolved to best support the physical needs of groups. Moving away from a traditional set-up of rows of desks has allowed for more movement, discussion and collaboration in the classroom. Liljedahl’s work with the thinking classroom states that student work is best done on vertical non-permanent surfaces (VNPS) (Liljedahl, 2016). VNPS can be any surface – typically whiteboards, blackboards or windows – that students can use for their work. In their non-permanency students have been found to lose some of their inhibition around starting their work, and it was found that most groups begin working sooner (Liljedahl, 2016). A major component of working at VNPS is that students stand which gives less opportunity to get tired or complacent in their work. Also, student groups should only be given one pen to increase discussion amongst the group (Liljedahl, 2016)

It was also found that a sense of sharing and group knowledge was fostered by the use of VNPSs. By having student work up in a visible sense, both teachers and students could easily look around the room and quickly see what other groups were doing. This made it easier for the teacher to check in for progress, but also made it easier for other groups to gain knowledge from one another. According to Liljedahl, “the use of vertical non-permanent surfaces will increase eagerness to start, increase discussion, participation, and perseverance amongst the group members, and facilitate the mobility of knowledge between groups” (Liljedahl, 2016a, 2016b, pp.5). In the past few years VNPS has taken off in mathematics education and become somewhat of a hot discussion point at many conferences and in teacher discussions. In my experience, VNPS effectiveness at engaging students and increasing participation and perseverance cannot be understated.

2.5. Collective efficacy

While self-efficacy concerns an individual's beliefs of how they will perform, group, or collective efficacy is how one perceives a collection of people to perform a given task. Bandura defined human agency as a person's feeling of having the capacity to influence their situation (Bandura, 1977). This idea of agency also related to groups and how an individual may have influence in a group situation. Bandura's theory "… approaches the enhancement of human agency, whether in individual or collective form, in terms of enablement. Equipping people with a firm belief that they can produce valued effects by their collective action and providing them with the means to do so are the key ingredients in an enablement process" (Bandura, 1977, p. 477).

Collective efficacy was also first discussed by Bandura and related to his ideas about human agency. Since people do not live in isolation, it seemed a natural extension to consider how they might perceive their abilities when working together. Group efficacy is essentially the combined self-efficacy of individuals acting within a group (Bandura, 1977). According to Bandura (2000a), group agency, or a group's belief in their influence over outcomes, and their collective efficacy were based on several factors. "People's shared beliefs in their collective efficacy influence the types of futures they seek to achieve through collective action, how well they use their resources, how much effort they put into their group endeavor, their staying power when collective efforts fail to produce quick results or meet forcible opposition, and their vulnerability to the discouragement that can beset people taking on tough social problems" (Bandura, 2000b, p. 77).

Collective efficacy has been measured through two main approaches (Bandura, 2000b). The first is to measure the individual's perceived self-efficacy at their particular task within the group. This method of measurement considers the individuals within the group, as opposed to the whole. The second method is a more holistic measure of the group as a whole from each member within the group. This second method incorporates the group dynamics and reflects the fact that individual members may take on more than one task or role within the group. While these have been the two main approaches, a third possible approach would be the group coming to a consensus as a whole as to their efficacy through discussion. Bandura deemed this third method as problematic due to potential social influences within the group of a final measurement – perhaps a stronger or persuasive group member will sway the measure in one direction or another. However, if group efficacy is deemed as the unit of analysis, this third method may be effective as well.

Depending on the task a group may face, one measure of collective efficacy may be more appropriate than another. Bandura (2000b) used the examples of sport teams: a gymnastics team's success is based on the sum of individual performances, whereas a soccer team's success relies on the team working together and the intricate system of their play. In the former example, a more individual measure of self-efficacies within the larger group may be more appropriate, whereas the latter example would be more appropriate to use the individual's perception of the group efficacy as a whole.

In addition to the method of appraising collective efficacy, Bandura (2000b) discussed the effects of particular individuals. If it is known that there are outliers in a particular group, either very strong or very weak abilities at a given task, then a group efficacy will likely be swayed by the presence of this individual. For example, if it is known that one member of a soccer team is a star player, then the perceived group efficacy will be higher than if this player were absent. Information that a group may have about the other individuals in their group will affect their perceived group efficacy.

2.6. Changing collective efficacy and the research question

Mathematical self-efficacy has been heavily researched for over forty years and much is known about its relationship to student achievement, future career choices, appropriateness of assessment choices, and student calibration among others.

However, pedagogy is shifting from a more individualistic approach to a ‘thinking classroom’ where collaboration, discussion and group work are paramount. With this shift comes a need for new research. It is clear that there is a need to learn more about mathematical group efficacy and in particular, how does this relate to mathematical self-efficacy?

The relationship between self and group efficacy is largely unknown in mathematics. However, there has been some preliminary research done in some other fields of study, for example in computer science. Wang and Lin (2006) analyzed data on both self and collective efficacy from groups of three students while working in a digital environment. The results of their study indicated that groups composed of those with higher self-efficacy had higher collective efficacy as well. Also, higher collective efficacy was directly proportional to a group’s use of higher-level cognitive skills as well as having positive effects on group discussion. While this study was specific to a computer-supported learning environment, I wonder whether or not there would be similar findings in other scenarios. There is clearly opportunity for research in this area. Mathematical self-efficacy is widely researched, and group efficacy has been discussed for decades, but how are the two related?

Knowing that self-efficacy is such a strong indicator of so many successes in mathematics makes it imperative to understand in our students. This imperative, coupled with a rising use of group work makes the need to understand collective efficacy increasingly important. As already discussed, there is a dearth of literature on mathematical group efficacy. This led me to want to conduct research on these topics. My key research question is: what relationship exists between secondary mathematics student’s self and group efficacy, if any?

In relation to this individual versus group efficacy, I am also hoping to investigate the following secondary research questions:

- Are students with particular mathematical experiences prone to having higher self than group efficacy? If so, what might these experiences have been?

- Are students with different particular mathematical experiences prone to having higher group than self-efficacy? If so, what might these experiences have been?

- What could teachers do to increase student efficacy in both individual and group tasks?

Chapter 3. Methodology

This chapter describes the methodology used in this research study in order to answer the research questions. The research undertaken was qualitative in nature.

3.1. Course Descriptions

At the time this research took place, mathematics education in British Columbia (BC) was under reform. In the fall of 2016 all subjects from kindergarten to grade 9 saw changes in both the curriculum covered and pedagogy. Grade 10 to 12 changes are forthcoming, and as of the current writing are set to have optional implementation in fall of 2018. One of the primary shifts of the curriculum change is a move away from long lists of prescribed learning outcomes to more of an emphasis on the “Big Ideas” – larger concepts that ideally learners can apply well beyond the classroom. This shift is also expected to allow teachers more time and freedom for exploration and inquiry in their classrooms. These changes will affect all BC courses.

The courses in BC mathematics are currently mostly predetermined until a student reaches their later high school years. Students move from Math 8, to Math 9 and then typically to Foundations of Math 10 and Pre-Calculus, however students in Grade 10 can choose an alternative Apprenticeship and Workplace 10 course – this course is intended for those students planning to go into the trades. After successful completion of a Grade 10 mathematics credit, students require one more mathematics course for graduation. In Grade 11 most students choose between Pre-Calculus 11 and Foundations 11, while a few still choose Apprenticeship and Workplace 11. This year is where students are forced to make a choice as to their specific mathematics course. The intent of these courses is to prepare students for post-secondary or opportunities beyond high school, and these influences should be the reason for their course selection. Of course, this is not always the case, and many students choose their course for other reasons; parental pressure, peer pressure, teacher choice, schedules, just to name a few. In the Grade 12 year students have the continuation courses from Grade 11 available, as well as the addition of Calculus 12. The senior courses – in particular Pre-Calculus 11, 12 and Calculus 12 are currently very curriculum heavy. These are demanding courses in which the enrolment typically drops throughout the year due to high workload and difficult material.

I have taught most courses from the BC curriculum, with the exception of Foundations 11, 12 and Calculus 12. My typical teaching load contains a combination of Math 9, Math 10, Pre-Calculus 11 and 12 as well as courses from the International Baccalaureate (IB) program, which I will describe in more detail in section 3.1.2. These courses, in particular those more senior ones, tend to have motivated students who hope to pursue higher education in a field that involves mathematics. The purpose of the "Pre-Calculus" pathway is just that – to prepare students to one day take calculus.

At the time the research was conducted I was teaching both Pre-Calculus 11 and 12, as well as IB Math Grade 11 and 12. I saw these courses as split into two mathematics pathways as they lead to different graduation diplomas; Pre-Calculus and International Baccalaureate. The data collected was from all four of my teaching courses at the time – both grade 11 and 12 of each of these pathways. These two pathways will be discussed in more detail in the next sections.

3.1.1. Pre-Calculus (PC)

Pre-Calculus is a pathway offered as part of the graduating coursework for those students interested in attending post-secondary school in a field related to Mathematics, Sciences, Commerce, Medicine or Engineering. As described by the British Columbia Ministry of Education:

> "This pathway is designed to provide students with the mathematical understandings of critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus."

Pre-Calculus courses are offered at Grade 11 and 12 only, and students elect into these courses. The courses are often considered the more difficult mathematics courses in the graduation program. Content tends to be more abstracted and less applied than other course's curriculums. Topics in the curriculum include:

Pre-Calculus 11	**Pre-Calculus 12**
Quadratic Functions & Equations Trigonometric Equations Radical, Rational and Absolute value Equations Systems of Equations Linear & Quadratic Inequalities Sequences & Series.	Transformations Polynomial Functions Trigonometry (Equations, Functions and Identities) Exponents & Logarithms Function Operations Permutations & Combinations

3.1.2. International Baccalaureate (IB)

The IB program is an internationally developed curriculum that is only offered at those schools who have applied to be accredited an IB school. According to IB:

> "The IB's programmes are different from other curricula because they: encourage students of all ages to think critically and challenge assumptions, develop independently of government and national systems, incorporating quality practice from research and our global community of schools encourages students of all ages to consider both local and global contexts develop multilingual students."

At my school, and many other schools, IB is considered a challenging program with fewer than 100 students enrolled in the entire diploma program for any graduating year. The program focuses much of its direction on student inquiry, giving many opportunities for students to direct their own learning. Students in this program tend to be academically focused and have a very good work ethic. These students can also be anxious about their marks and feedback in the classroom. The mathematics curriculum in these courses is very dense and is covered at a rapid pace, leaving less time for exploration in the classroom. The curriculum covered at our school is approximately divided as follows:

Grade 11 (Year 1)	**Grade 12 (Year 2)**
Quadratics	Probability
Polynomials	Statistics
Radical, Rational, Absolute value functions	Probability Distributions
Sequences & Series	Vectors
Transformations	Differential Calculus
Logarithms	Integral Calculus
Trigonometry	

The Grade 12 students must also complete an Internal Assessment on a topic of their choice. This assessment is submitted in the form of a paper, typically six to ten pages in length. The paper should investigate a topic of interest to students in relation to the mathematics studied in the course and is worth 20% of their final mark. At the end of Year 2 (Grade 12) students write two comprehensive exams covering both years of study. These exams are worth 80% of their final mark.

3.2. Classroom Structure & Culture

The culture in my classroom is something that I work hard to develop with my students. I always maintain a very positive and student-centred environment where students are welcome to investigate and offer ideas without judgment. I try to facilitate a supportive learning environment and coach students on helpful language to use with their peers. I think of myself as a kind, patient, and caring teacher and strive to model this with my students. I hope that in doing so they will always feel supported in my classroom and may become more willing to offer their opinions, ideas and conjectures.

At the same time, I am clear with students that constructive criticism and questioning is a part of doing mathematics. During group activities, I expect my students to work with each other and question each other’s ideas in a polite and helpful way. I try to model this as I move from group to group. I help coach students in appropriate language when working with one another. I have worked hard to develop and maintain these practices in my classroom, but I feel that it is important for students to have this environment so that they feel less anxious and more courageous in their studies of mathematics.

From the beginning of the school year, most of my classes have had a similar structure. Typically, I will start the day by getting students into visibly random groups (VRG) of two to four students. The number of students in the group depends on the activities duration and complexity, as well as the number of students present on the given day. This is done by having students draw a card from a standard deck. Next I will present a problem, either verbally or visually on the overhead projector, or both. The problems vary from day to day, and can depend on where we may be in our learning, the time we have to devote, or even my mood. Problems are sometimes based on curricular content and may come from a textbook, or they may be a more open task to work on reasoning and problem-solving skills.

There are assigned stations in the classroom from A (ace) to 8, corresponding with the card that they drew at random and students will 'meet' their group members at their group's assigned whiteboard. The classroom is structured so that there are whiteboards or windows around the outside of the entire room. Students are expected to work on their vertical non-permanent surface (VNPS) with their VRG on the task. Groups are given one whiteboard marker and eraser to use on their surface. The VNPS allows for both myself as the teacher, but also the other groups to see work that is being presented. This allows for interaction amongst groups, and teacher, as well as forces groups to be accountable for their work. In addition, the non-permanence of the surface lowers anxiety in groups and helps them start on their task more efficiently. Students then begin working on their problem for anywhere from fifteen to forty minutes, depending on its complexity and how the work is proceeding.

As students work through their tasks I circulate through the room looking for groups that may be finished, stuck or have questions about the problem. I try not to interfere if group members are moving forward ('forward' here is taken to be in any direction, not necessarily the ‘right’ one). Any groups that may have a solution are usually asked to reason why it may be the solution, or are given an extension to the original problem. Groups that may be stuck might have suggestions on an approach or be asked to look at another group’s board for hints. The activity is usually wrapped up once all groups have had some success with the problem. I may then ask select groups to share their findings, I may highlight some board work, or I may summarize the central ideas at the front of the room. Some summative discussion is offered, either by me or the students, or both.

After our work at the whiteboards the class usually moves to a more traditional lesson, whereby I will teach on the overhead projector through a series of definitions and examples. Students are invited to share their thinking throughout and are sometimes invited up to the board to work through examples, either partially or as a class. The lessons are relatively interactive with opportunities to pause, discuss and question. Over the course of the year, students begin to feel more comfortable with one another and are more open to class discussions.

The class will sometimes have fifteen to twenty minutes near the end, which is time for individual practice on concepts covered that day. This time is sometimes spent with individual students going over ideas for clarification, or handing back assessments. This time is valuable as I can address any particular concerns or needs of the class, either as a whole, or on more particular cases. Depending on the length of the first two parts of the class, there is sometimes no time at the end of class at all. This will vary in its purpose and length from day to day.

3.3. Setting and Participants

The setting of this classroom was a relatively affluent public school on Vancouver's North Shore in British Columbia. The school is situated in West Vancouver and hosts students from Grades 8 to 12, with ages ranging from twelve to nineteen. The school's total population is approximately 1400 students and it employs approximately 100 staff, including teachers, administrators and support staff. The school offers a wide variety of programs ranging from arts and sports academies, a robotics program and a carpentry ace-it program to name a few. The school is also an International Baccalaureate (IB) school, which will be further described in section 3.2. The school is often self-described as a comprehensive secondary school, because of its wide variety of programs that meet the needs of a variety of learners. Because of this variety, the student population is also widely varied in terms of their skills, background knowledge and motivation.

The participants in this study were all students from Grade 11 and 12 mathematics courses. Students were all enrolled in what are considered the more academic mathematics courses at our school, either: Pre-Calculus or IB Mathematics, Standard Level – these courses will be described in more detail in section 3.2. Within these courses student participants were varied in their skill set and were not targeted for participation. Classes were introduced to the research and voluntarily decided whether or not to participate. A total of 104 students took part in the study; 39 males and 65 females. The following tables give a more comprehensive breakdown of course and gender of both participants and non-participants:

	Males	**Females**	**Total**
Pre-Calculus 11	8	10	18
Pre-Calculus 12	19	31	50
IB Math 11	6	17	23
IB Math 12	6	7	13
Total	39	65	104

Table 1: Participant gender, as a function of course

	Males	Females	Total
Pre-Calculus 11	23	31	54
Pre-Calculus 12	37	46	83
IB Math 11	10	17	27
IB Math 12	7	9	16
Total	77	103	180

Table 2: Total student enrolment as a function of course

As the above tables show, over half of the invited students chose to participate with the majority of them being female. Participants were mostly from the Pre-calculus courses, although a higher proportion of students from the IB courses chose to participate. Possible reasons for this will be offered in chapter 4. As the classroom teacher I had already established relationships with student participants – some for nearly three years. Students had been experiencing my classroom for a minimum of five months at the start of the research and had become comfortable with both myself as the teacher and my classroom dynamics.

3.4. Data

The data included my observations during class, a series of two identical questionnaires and follow-up interviews with selected students. This research did not attempt to alter or disrupt the natural flow of the existing classroom setting, but rather it was designed to qualitatively observe student work and have students self-report their own sense of efficacy.

3.4.1. Observations

Informal observations were made throughout most group and individual tasks during class. Once the research commenced, I began making observations of students during their work at the whiteboards and how they interacted together. I looked for behaviours that might indicate their feelings towards their work. Positive engaged behaviour might include actions like: facing towards the whiteboard, looking towards a group member as they spoke, actively writing at the board, smiling, talking to group members about the problem, etc. Negative behaviour might include actions like: facing away from the whiteboard, looking away while a group member spoke, walking away from the task, using a cellphone, crossed arms, etc. Gestures and behaviours were noted when observed. These observations were all recorded in a small notebook.

In addition to physical behaviours, indicators of student emotion were observed – whether students were generally happy, sad, angry, bored or frustrated during both individual and group work were recorded. In particular I was looking for a change in student emotional state from individual to group work. Whether student emotions were consistent was important to help sense their feelings toward group and individual work, in particular if there were inconsistencies from one to the other. While I was not able to keep track of 25 – 30 students simultaneously, I attempted to move around the room and spend some time with each group every class. Any behaviour that was noticeable was noted in the same notebook.

3.4.2. Questionnaire

I designed a questionnaire to evaluate students' feelings towards their mathematical work in both individual and group settings. Some of the questions were adapted from Nielsen and Moore (2003), and many were inspired by Bandura (1983). However, the scale was designed to assess group efficacy, which has not been previously studied in mathematics education, thus a new questionnaire had to be built. The desire was for a general scale of efficacy to be used, as opposed to a time or context specific set of questions. Although self-efficacy models have been shown to be more accurate when content and context specific (Bandura, 1997: Nielsen & Moore, 2003), and context in particular has been shown to be important for lower self-efficacy students (Nielsen & Moore, 2003), the hope of this research was to identify students with more positive group or self-efficacy in general, not measure student efficacy itself. Thus, the context was accepted to be the current mathematics classroom model in effect on any given day. Some content specific questions were included at the end of the questionnaire, so as to place the students clearly in the realm of mathematics.

The questionnaire starts with twelve questions that target student self-efficacy, then twelve for group efficacy. The scale for these first two sections was designed as a Likert scale ranging from 1 to 5, where 1 is “strongly disagree” and 5 is “strongly agree”. The last section targets specific mathematical skills of students with six specified areas, again individually, then in a group. These questions again range from 1 to 5, where 1 is “very poorly” and 5 is “very well”. An excerpt from the questionnaire can be seen below and it can be found in its entirety in Appendix A.

Questions in the first part of the questionnaire were asked twice: firstly for ‘individual work’ and secondly for ‘group work’. These questions were developed based on previous research questionnaires aimed to measure self-efficacy, however they were extended to include group efficacy measures. The first section was designed to measure overall sense of efficacy of students, as well as the possible discrepancy between their self and group efficacy scores.

Some sample items include:

- *I/We am/are good at mathematics.*
- *I/we am/are good at mathematics in this course.*
- *I believe I/we can do well on a mathematics task.*
- *I am confident that I/we can solve any problem.*

Questions in the second part of the questionnaire were also asked twice: firstly if completed ‘by yourself’ and secondly if completed in a ‘random group’. These questions were all based on particular mathematics skills that students at this level would be expected to be able to do. The second section was designed to measure particular mathematical content efficacy, as previous research indicates that content improves accuracy of efficacy scores (Bandura, 1977). Again, these were asked twice to look for changes from self to group efficacy.

Some sample items include:

- *Graphing a function.*
- *Solving an equation algebraically.*
- *Working without a calculator.*
- *Solving a new problem that is unfamiliar.*

This questionnaire was administered twice to ensure some consistency in student answers, although some students were absent at either the first or second administration. The duration from the first to second questionnaire was approximately six weeks. The questionnaire was administered in our usual classroom where other students may have seen one another's answers. This potential viewing of questionnaire item responses may have caused some unrest for students, as their peers may have judged or commented on their answers. Although a private setting may have been preferable for student privacy, it was impractical given the number of participants and the repetition of the questionnaire.

One of the main risks of this research was on student emotion and well-being. Although this research was deemed as minimal risk, I was acutely aware that having students reflect on their own abilities may cause some anxiety or discomfort. Some questions asked students to reflect on their feelings about their mathematical ability and some students may have found this disheartening, particularly those with lower efficacy scores.

3.4.3. Interviews

Semi-structured interviews were conducted with six questions pre-determined, but with flexibility to move off script as necessary. The six structured interview questions were as follows:

1. On reading your questionnaire, you indicated that you felt differently working in a group than you do on your own. Why do you think this might be?
2. What do you think it is that a group adds to your mathematical ability?
3. What do you think it is that a group takes away from your mathematical ability?
4. Does the type of problem you are working on in your group make a difference? Why or why not?
5. Do the group members you are working with make a difference? Why or why not?
6. Does randomizing the group members affect how you think you will do on a task? Why or why not?

As the interviewer, I did inquire when students gave an answer that required clarification or offered more insight if probed. All interviews went off script at some point in time. The interviews lasted anywhere between five and ten minutes, were recorded and then later transcribed in full. Students were all interviewed individually in my classroom either before class in the morning, during the lunch hour, or after class in the afternoon at an agreed upon time.

3.5. Analysis

Once collected, questionnaires were given a total score for each section: the first two sections out of 60 and the last two out of 30 possible points. A point was given to an answer that implied higher efficacy. For example, a student answer of five to the statement “I always contribute to my group in a positive way” was awarded five points. However, some questions needed to be reverse scored, for example, a student answer of five to the statement “I worry that I will let my group members down”, was awarded only one point. Each section was tallied and students were given totals for each section, with higher total scores implying higher efficacy of that type.

After all totals were calculated, three groups of student scores emerged: those with similar self and group efficacy, those with higher self than group efficacy and those with lower group than self-efficacy. I was particularly interested in those students that had some variation between their self and group efficacy scores, so I looked for a discrepancy of more than five points in the those sections of the questionnaire. A total of 41 students were identified as having this discrepancy consistent across both questionnaires administered. Twelve students were selected of these 41 for interview, based on a higher difference between the two types of efficacy scores. Eleven of these students agreed to be interviewed. Students were contacted for interview approximately three months after the beginning of the study.

I created a student profile for all eleven students that were asked and consented to interview. The profiles were based on my previous work with each student leading up to the interview itself. The purpose of the profile is to add some background context to the student’s personality traits observed individually and in groups, as well as their assessed mathematical skills. The profile was later edited to include the student’s final standing in their mathematics course. These profiles are included in Chapter 5.

Upon completion of the interviews and transcription, I looked for themes in student answers. Students were grouped in terms of their scores: either higher self-efficacy or higher group efficacy scores. Trends across these groups were then identified and further examined. Similar word choice and behaviour were noted in each group of students. These observations are further discussed in the next chapter.

Throughout the interviews there were times when I offered possible terms to help students identify their thoughts. This was done to clarify student meanings and not intended to lead them. For example, when students were asked to distinguish between problem types, they would often describe an algebraic or algorithmic sort of problem, and so these terms were offered to help clarify.

Chapter 4. Results and Discussion

4.1. Questionnaire Results

The first questionnaire was administered to a total of 88 students and the second questionnaire was administered to a total of 85 students approximately one month later. Two questionnaires were administered to see if the results varied, and if so, further questioning would take place to ensure valid results. The individual results from each questionnaire can be found in Appendix B.

Participating students came from Pre-Calculus and IB Math courses, as discussed in Chapter 3. A larger proportion of the students who chose to participate in the study were enrolled in the IB courses offered at our school, likely because many of these students had a past relationship with me as their classroom teacher. I had known many of the students for at least one and a half years prior to commencing the study and many felt happy to participate in something that would help with my work. Also, these students tend to be more inquisitive in their nature, and were eager to learn about themselves as well as mathematics education. Many of the IB students asked about the research and wanted to learn more. To a smaller degree this was also true of many of the participating students in Pre-Calculus courses.

Table 3 below shows the mean scores on the first two sections of the two rounds of questionnaires. Each of these sections had a total possible score of 60, where a higher score reflected higher efficacy. This section of the questionnaire looked at general efficacy sentiments, separate from both context and content, except for specifying that it was in relation to mathematics. Questions in this section were deliberately left vague to gain a general idea of students' sense of mathematical efficacy. The context was left to student interpretation and did not specify whether the mathematics in question were on a test, in homework, classroom activities, at a job, etc. The only specifications were whether the mathematics were to be individual or in a group. The questions for each of the first two sections were the same, but they differed from individual to group work, and they were shuffled in their order. The full questionnaire is available in Appendix A. The total scores for both self-efficacy and group efficacy are out of 60 possible points.

	Self-Efficacy (SE)	**Group Efficacy (GE)**	**Difference**
Questionnaire 1 *(n = 88)*	42.17	41.58	GE1 – SE1 = 41.58 – 42.17 = - 0.59
Questionnaire 2 *(n = 85)*	42.86	42.82	GE2 – SE2 = 42.82 – 42.86 = - 0.04
Difference	SE2 – SE 1 = 42.86 – 42.17 = 0.69	GE2 – GE1 = 42.82 – 41.58 = 1.24	

Table 3: Mean Scores on Questionnaires 1 and 2 in the first two section

The results from the table show a very small change in results from Questionnaire 1 to 2. This indicated to me that student results could be considered reliable and did not change drastically over the relatively short period of time of one month. This signified that I did not need to administer the questionnaire a further time, as the results from the first two were deemed accurate. The only discernable change from Questionnaire 1 to 2 was that efficacy scores increased slightly, a difference of 0.69 and 1.24 on self and group efficacies respectively. This mean was only calculated to show that there was not a significant difference on each questionnaire.

Although small changes, it is possible that student efficacies had increased slightly in the month since the first administration of the questionnaire. Although this was not the intent of this study, it was interesting to see this small change. It is possible that this change is insignificant, but without further questionnaires we cannot tell if this trend would continue. However, as Bandura (1977) noted, there are effective means of increasing efficacy. These students were becoming more adept and efficient with both their individual and group mathematics, and may have felt more positively at the time. Also, the particular day of administration and external effects may have altered results slightly. The second questionnaire was administered shortly before Spring Break after several months of consecutive study. Students may have been feeling differently about their studies than the month prior because of their mental state.

The second two parts of the questionnaire had a total possible score of 30, and higher scores again reflect higher efficacy. The questions in this section focused on particular mathematics skills, so content was specified, but context was not. The content was not specific to particular questions, but rather certain domains or skills in mathematics. For example, questions about algebra were asked, but specific equations to solve were not provided.

	Self (S)	**Group (G)**	**Difference**
Questionnaire 1 (n = 88)	21.10	23.20	G1 – S1 = 23.20 – 21.10 = 2.11
Questionnaire 2 (n = 85)	21.34	23.71	G2 – S2 = 23.71 – 21.34 = 2.37
Difference	S2 – S1 = 21.34 – 21.20 = 0.24	G2 – G1 = 23.71 – 23.20 = 0.51	

Table 4: Mean Scores on Questionnaires 1 and 2 in the second section

Similarly to the first section of the questionnaire, the results in Table 4 revealed that scores on the second administration were slightly higher than the first. This is likely due to similar reasons as outlined above. However, unlike the first section of the questionnaire, once content was specified students tended to rate their group efficacy higher than self-efficacy. Overall, students stated higher group efficacy than self-efficacy when the content was specific to mathematical domains. This result was not surprising – many people tend to feel strength in numbers. Students echoed this in class observations as they were able to discuss ideas and come to consensus within a group as opposed to feeling stuck on their own. It was surprising to me, however, that this difference was not as visible on part 1 of the questionnaire when the content was not specified.

There was also more variation in their individual scores in part 2: where a student might rate themselves highly in one content area, another area might be ranked quite low. This variation was also true within group efficacy, although less so. It is likely that students attributed the skills of their group members to heighten the areas that they felt they were lacking. Students are generally familiar with their own strengths and weaknesses and as such can better predict their ability on particular tasks. However, without knowing the members of their group, they do not know their strengths or weaknesses. Logically, students seemed to assume that if their ability was low in a particular area, others would be on average better and bring up their ability. This variation in particular content areas from self to group efficacy scales is further discussed in section 4.2.

4.2. Questionnaire Implications

The first part of the questionnaire looked at a general sense of efficacy. Context was not specified and left for students to interpret. There were no mentions of setting, question type, or purpose for doing any mathematics. The purpose of this research was not to differentiate between any mathematical contexts, and so this was left unspecified to the students. The differences on part 1 for questionnaires 1 and 2 were -0.59 and -0.04 as shown in Table 3 with a mean of -0.315.

This shows a very slight increase of self over group efficacy scores because of the negative values. Again this is in a general setting and was somewhat surprising. My initial predictions were that students would identify higher group than self-efficacies overall, which was not supported by this data. Although this mean difference is very small, it represents a decrease of 0.5% from self to group efficacy in part 1 of the questionnaire. I believe that this may be in part to several students who felt very strongly that they had higher self than group efficacy. These responses skewed the data in a negative direction. Eliminating outliers may give a more accurate sense of the participants overall sense.

Moreover, the mathematical content was left unspecified for the first portion of the questionnaire. As discussed in Chapter 2, most research in mathematical self-efficacy has found it to be highly related to specific content, but the aim of this research was to relate self-efficacy to group efficacy. As such, it was important to include some general questions that could be carried from individual to group scenarios. These questions were important to distinguish a sense of general mathematical efficacy from students. The overall scores from part 1 and 2 did correspond – that is, a student who had more positive self-efficacy in part 1 also had higher self-efficacy in part 2 and vice versa.

The second part of the questionnaire looked at content specific questions. Student responses to these questions generally corresponded with their responses to the first part. However, there were some interesting exceptions. Some students who demonstrated a large difference in general efficacies in part one did not show a large difference when the content was specified. The reverse was also true. The content specificity seemed to change the effect to which students reported strong variation in their efficacy scores – their judgments seemed more accurate and closer to the mean. This result is in line with Bandura's (1977) contention that accurate self-efficacy measures needed to be content specific. This doesn't necessarily mean that the general measures are inaccurate, but rather that the content specific questions may be more accurate. Some interesting cases of students with these variations will be looked at more closely in Chapter 5.

The larger difference in mean scores in part 2 also shows that students overall identified a larger difference in self and group efficacy when content is specified. This difference was 2.11 and 2.37 on questionnaires 1 and 2, respectively, with a mean of 2.24 show a higher overall group efficacy of all 88 students questioned. This result suggests that when content is specified students believe they will perform better in a group. This difference of 2.24 is somewhat significant out of a possible total score of 30. It represents an average efficacy increase of 7.5%.

4.3. Interviews

Eleven of the participating students were selected and consented to be interviewed. The interviews were semi-structured and were based on the six questions as listed in section 3.4.3. The purpose of the interviews was to gain further insight into students' feelings about both individual and group work in mathematics. All interviews went off script at some points to ask further questions about a comment made or to clarify an idea. Excerpts from the interviews are discussed in Chapter 5. The interview questions attempted to distinguish how students may feel differently when working in a group as opposed to alone.

The interviews lasted between five and eleven minutes. Some students were able to articulate their rationale for their identified efficacies whereas others struggled to put their sentiments into words. At times terminology was offered to students to help them describe their feelings and they were free to either agree or disagree. The interviews were particularly useful as qualitative data. On analysis of the interviews, it was clear from similar word choice and behaviour that two groups emerged: those with higher self than group efficacy and those with higher group than self-efficacy. The interviews were able to probe more as to why students may have reported one over the other. Each group is henceforth examined.

4.3.1. More positive group efficacy

Seven of the students interviewed reported a more positive group efficacy than self-efficacy, whereas the other four reported more positive self than group efficacy and will be further discussed in section 4.3.2. The students selected for interview had the following overall mean scores on the efficacy scales:

	Self (S) Part 1	Group (G) Part 1	Difference G1 – S1	Self (S) Part 2	Group (G) Part 2	Difference G2 – S2
Student 1	30	44	14	13.5	21	7.5
Student 2	26	34	8	12.5	20.5	8
Student 3	38.5	45	6.5	21	26.5	5.5
Student 4	44.5	47.75	3.25	17	23.5	6.5
Student 5	33.5	43	9.5	17	27	10
Student 6	34	47	13	17	29.5	12.5
Student 7	31	41	10	18.5	26	7.5

Table 5: Mean scores of interviewed students with more positive group efficacy

The positive difference in group and self-efficacy scores on both parts 1 and 2 of the questionnaire indicated higher group than self-efficacy scores. This pattern was true for these students in both the non-context-based and context-based questions. The students in this group were of varying ability in their classes – some were not strong students and were in the C range, but some were also quite strong in ability and were earning A's. However, none of these seven students were at the top of the class. They generally were working hard to earn whatever grade they were getting in their course. Mathematics did not come as easily to these students, and the interviews were able to get more detail about how they felt about their mathematical efficacy in general.

Several themes emerged amongst this group of students in interview. The first, and most common sentiment was that students thought that they gained knowledge from their classmates in group work. This 'strength in numbers' sentiment was common across all seven students in this grouping. They all believed that there were content areas that they were not as knowledgeable about that their classmates could help them with. These students also shared the concern about getting 'stuck' on a mathematics problem, and with a group they felt there would be a member to help them get unstuck. None of this surprised me as the researcher as I have witnessed these sentiments many times.

What was more surprising was the feedback around students with anxiety. Four of these seven students stated that they often felt anxious problem solving on their own. With the addition of a group they felt more comfortable and confident because of the support from their classmates. This theme of mathematics anxiety led me to designate this as a subgroup within the students with more positive group efficacy. These four students shared the feeling of group support, but additionally described symptoms of mathematical anxiety when working alone. That is not to say that their anxiety dissipated completely in a group, but students indicated that it lessened or improved overall.

A word that came up in several interviews was “fun” during group work. Student enjoyment and engagement has been shown to increase when working in a group (Liljedahl, 2016) and students echoed this in the interview. Enjoyment of the task would increase engagement, which in turn seemed to increase student group efficacy. This is an important observation as efficacy has been linked to many predictors of success in student learning. Although group efficacy has not been as studied in this regard, it seems tied to self-efficacy and one will impact the other. Perhaps group work can increase engagement and overall efficacy for some students who report higher group efficacy than self-efficacy. More research needs to be done to say for sure. This theme also came up in interview with students with more positive self-efficacy.

4.3.2. More positive self-efficacy

Four of the students interviewed reported a more positive self-efficacy than group efficacy. The scores in Part 1 were out of 60 possible points and measured general self and group efficacy beliefs. The scores in Part 2 were out of 30 possible points and measured self and group efficacy on particular mathematics tasks. The students selected for interview had the following overall mean scores on the efficacy scales:

	Self (S) Part 1	Group (G) Part 1	Difference G1 – S1	Self (S) Part 2	Group (G) Part 2	Difference G2 – S2
Student 8	45.5	37	-8.5	23.5	20	-3.5
Student 9	44	29.5	-14.5	25.5	22.5	-3
Student 10	46.5	31	-15.5	23.5	23	-0.5
Student 11	55	34	-21	28.5	22.5	-6

Table 6: Mean scores of interviewed students with more positive self-efficacy

The negative difference in group and self-efficacy scores on both parts 1 and 2 of the questionnaire indicated higher self than group efficacy scores. This pattern was true for these students in both the non-context-based and context-based questions. The students in this group were all of high mathematical ability and generally had very good marks in their classes. Not all were hard working, but all four had good mathematical insights and were gifted in mathematics. The table shows that the scores on part 1 had larger differences between self and group efficacies than part 2. Again, part 2 had content specific questions, and according to Bandura (1977) this will yield more accurate efficacy scores. The scores are less dramatic in part 2, but are still in line with part 1 results.

Several themes emerged from interviews with these four students. The first and most prominent theme was that the students interviewed felt the group members may slow them down. As these four students believed they would be able to tackle most types of mathematics questions on their own, they thought that the addition of group members would hinder their progress. Students stated that they would need to slow down to explain their thinking as they worked. This focus on efficiency was interesting and was not anticipated, but all four students mentioned time in their interview. Most group problems in our classroom do not have a time limit or a particular goal to reach, so it was counterintuitive that students felt the need to rush. Perhaps just for their own interest they wanted to push further in the question and felt less able to do so in a group.

One of the four students stated that she became more confused in a group. Other students would present ideas in discord with her own and she found it difficult to have to find resolution around this difference. She stated that this confusion would then cause her to question her own understanding and thinking and would then not be able to progress in a question. This student's interview, along with others, is analyzed in more detail in Chapter 5.

Chapter 5. Case Analysis

In this section I will present three different groups of students that emerged from interviews. Each of the students profiled in this section clearly had discrepancy in self and group efficacy scores and the interviews served to find some rationale. For each group of students I will include student profiles to provide some background of the student's characteristics and previous assessments in their course. Then I will highlight some of the interview results and analyze the discussion within each subgroup. Students of each subgroup did not necessarily come from the same class and would have rarely, if ever worked together on tasks due to the use of VRGs.

Note: Student names have been changed to maintain anonymity.

5.1. Strength in numbers: James, Laura, Teresa, Bill

These four students all showed more positive group than self-efficacy. This was apparent in classroom observation as well as their questionnaire scores. When interviewed, these students had an overarching theme of gaining more knowledge and confidence when working with classmates. As the researcher, this was my initial intuition for most students – a "strength in numbers" mentality. As an individual we are left only with our own knowledge and ideas, but with a group we can gain knowledge from those around us. All four of these students echoed this sentiment in some way during interview.

5.1.1. Student Descriptions

James

I have known James for the past two years. James tends to be a quiet and reserved student who is generally focused and on-task during class time. In groups, James varies between a leader and an observer, depending on the other members in his group. He tends to work diligently on tasks and asks good questions until he understands an idea thoroughly. James is one of the few students in his Grade 12 class who would consistently work quietly on his own when given the choice. He finished Pre-Calculus 12 with an A (86%).

Laura

Laura is a student in the full IB Diploma program and has been in my class for eight months. Laura is a social student who is often distracted by chatting in class time. Laura started the year with a strong mathematical background, but did not put forth much effort throughout the year. Her marks declined and her self-confidence seemed to follow suit. She sat with other strong mathematics students, but did not like to ask questions when she was unsure of a concept. In groups, similar attitudes were noticeable – she did not involve herself much with problems and was shy to ask questions. She finished IB Math 11 (Standard Level) with a C+ (69%).

Teresa

Teresa and I have been working together for two years. Teresa is very anxious when it comes to school in general, and mathematics in particular – she is nervous about her grades. She is usually more interested in an end result and whether or not she is right, rather than deeper understanding of a concept. Teresa is also very competitive, and this has shown itself in our group work. She will often race other groups or exclaim that they ‘win’ if her group is perceived to reach an answer first. This has dissipated somewhat throughout the year. Teresa does not catch on to concepts quickly and repetition seems to be key for her to reach understanding. She finished Pre-Calculus 11 with a B (73%).

Bill

Bill is a well-known student around the school. He has been in my classroom for eight months, but I have known him longer through his socialization. Bill seems to have higher than average self-confidence in general and has a good sense of self. He has an excellent sense of humour and generally remains on-task in group settings. Bill does not work well individually, unless he already has a good grasp of a concept, however he is very open to working with others on an idea that is new or less familiar. Bill is as comfortable taking control of a group as he is to listen to other’s ideas and let them take charge. Bill is a good team member and other students clearly like to work with him. He finished Pre-Calculus 12 with an A (86%).

5.1.2. Interview Results & Analysis

Support from classmates

The overwhelming sentiment from this group of students was that they felt stronger, or more confident in a group. With support from their fellow classmates, students felt they could have some validation in their work as they went through a problem and have group members confirm or question anything unclear. Here, James echoes this:

> James: I'm more confident in getting the right answer in a group.
>
> Me: Why do you think that is?
>
> James: Uh, well, getting the right answer, there's collaboration, so there's different points of view, so I guess you have a higher chance of getting the right answer.

Laura spoke about this added support as well; to her, groups add a more varied knowledge base from what an individual might have.

> Laura: Like, in a group, it's like you can ask each other things and just team work, so you can all share what you think and like, (…) how to do it.
>
> Me: [Could you] elaborate just as what do you think it is that a group adds to your math ability? Specifically.
>
> Laura: Um. Specifically? Uh… I guess like team work, and it's just you can all share your ideas and if you're unsure you can just ask and they can like, explain it to you if you don't understand something.

Teresa felt very strongly about this issue. She mentioned several times in interview how group work would add a wider knowledge base and therefore add to the general knowledge of the group. For her, this led to a more positive group efficacy:

Me: What do you think it is that a group could add to your mathematical ability? What might be good about working in a group?

Teresa: I think just like, all your brains working together. So say if someone doesn't know how to do it, but they know how to do like one part of it that another person doesn't know how to do. Then you can be like, I know that you have to do this and maybe the other person didn't know that you had to do that, but they knew how to do the other thing that the other person didn't know how to do. So then you end up figuring it out, because you each have little pieces that you remembered.

Later on she again noted:

Teresa: I think if you're going to work on your own, you don't have all the brains combined, (…) so say if you forgot that extra information you wouldn't have like, them to help you. So you'd be like, trying to figure it out but you can't remember and there's nothing you can do and you can't figure it out, but then they know. So just in a group it's a lot easier.

Bill also spoke about this validation:

Me: So the reason you're here [for interview], is because on reading your questionnaires that you filled out in class, you indicated that you felt differently when you work on your own versus when you do in a group. So my question is just why do you think that would be?

Bill: Um, I feel like when I'm working in a group there's more, there's either, my answer is like validated and I know that the process I'm going through is correct, or somebody says no, actually this is how you do it and I feel like when I'm working alone a lot, or maybe if I do something wrong, I'm not going to be able to catch myself as much.

Again, this feeling of 'strength in numbers' was my intuitive guess as to how most students would feel about their group efficacy. However, as we will see in the next two subgroups, students gave various reasons for their efficacy beliefs. All four of these students spoke several times about the benefits of group work of providing varied ideas and being able to validate each other's work as the group progressed. This was the most notable commonality in this group, but a few other patterns also emerged.

Group Member Preference

Another common theme across interview in this group was their feelings towards their group members. All four students believed that their group members would affect their efficacy, but for different reasons.

Both James and Laura felt that similar strength in the subject area would increase the group's overall efficacy, although James notes that no matter who the group members are, you must collaborate regardless.

> James: Well, the success of the group, if it's randomizing then it could be a gamble. You could get you know, if it's a group of 5, you could get 4 people who are fairly strong in math, or you could get 4 people who are fairly weak in math, so the chance of like solving a question, it's like, it varies on your people. But, [the] collaboration process is always constant. So, you'll still have to collaborate with whoever you get and I think that that, that's an important thing.

> Laura: Um… sometimes. I feel like it's better if it's like most people don't know what's going on so you can like figure it out together, than if there's like one person who knows it really well, and they're like, oh I can do everything for you.

Teresa spoke about a different aspect of group work – that of having international students in her group. This was something that I had not specifically thought of effecting efficacy beliefs, but she references varied skill sets and mathematical values from other countries. A large proportion of her class that year had been international (approximately 25%) and most of these students came from countries that placed emphasis on procedural mathematics. These students had very strong skillsets, but were not as able to explain their process; either because they did not know, or their English language skills were still developing.

> Me: Do the group members you are working with matter? Why or why not?

Teresa: I think so. But then some of them are so nice and so supportive. Like one of the girls, she's so nice to me and I love her, she's so nice. And she's always so supportive and she always explains it to me really well. But then some of the guys are like, they just do it. And I'm like 'Oh, how did you do that?', and they're like, what? Like they don't know how to explain it to me.

Me: Uh huh. So their English isn't as strong.

Teresa: Yea. And then when you're with one of your friends. Like how Adam and Nolan and I like always want to be together because we always work really well. And I know Adam will be able to explain it to me, or I'll be able to explain it to Nolan, or vice versa. Like we'll be able to explain it to each other because we're more comfortable with each other. Right, so. I think when you're with people you know better, like I think you're going to work better with them.

Me: Ok. You think that's always the case?

Teresa: Um. I think it helps, I think sometimes like with us being with the International students I think it helps them probably, because maybe they learn off of us too and obviously we learn better math off of them, sometimes, if they can explain it. But, I think for the cases it's better to be with people you're closer with, but it's also really helpful for them. So it goes either way.

Teresa did not specifically reference efficacy in this excerpt – she spoke more of preference. This was common in interview as students were not always clear as to the difference between preference and efficacy. Generally though, there was a subtext of preference meaning more success. Teresa mentioned how she and her friends "work really well", which indicates that she thinks they would have more success on a problem. This is indicative of higher efficacy in this particular group – a chosen group of friends that have proven to be successful in the past.

It was not surprising that a chosen group of students that have proven to work well would lead to higher group efficacy. If you know that the people in your group have knowledge that is a good foil of your own and that they have something to offer the group, then logically your sense of efficacy would be higher. Knowing who your group members were seemed to effect the efficacy that students held. Especially once students had become familiar with their classmates. This sentiment is echoed later on in other subgroups as well.

Finally, Bill discusses the engagement of group members effecting the group efficacy. He indicates that group members that do not engage and just “sit back” will lower his efficacy.

> Me: Ok. The next question is if you’re going to work on any kind of problem, do the group members you’re working with make a difference?
>
> Bill: Well, they, it definitely does, because sometimes you’ll get people in a group who are the people who just sit back and watch other people do the questions. Maybe they are getting something out of it, I don’t know. I don’t know because I’m not them, but um, I think that as long as the people in your group are very collaborative people and I think that they need to be open people, um, to criticism really, because maybe you’re doing it wrong. I feel in this class there’s a lot of collaborative people and are really good, which is why this year specifically, I’ve really enjoyed working more in groups.

Problem Preference

All students were asked whether the ‘type’ of problem was important to their group efficacy. The first instinct of students was a need to clarify what ‘type’ of problem meant. All four of these students eventually came to the conclusion that there were two types of problems that we did in groups; those that were more prescriptive, curricular problems, and those that were more open or unfamiliar problems. Here are some excerpts from their interview about their feelings working in groups on these types of problems.

> Me: So, if in the first case, with the word problem, would you rather be in a group, or not be in a group, or does it matter?
>
> James: I think I would uh prefer to be in a group if it’s like uh an elaborate word problem.
>
> Me: And if it’s a solve for x problem?
>
> James: Then it’s simply solve for x, then it’s just a process.

Me: Uh huh, so would you rather be in a group or by yourself, or does it matter?

James: For solve for x, I would just prefer alone.

Here, James talks about a preference towards group work for a “word problem” and individual work for a “solve for x” problem – this same sentiment is observed several times throughout all three groups and will be mentioned again later in the chapter. Although this does not necessarily say directly that his efficacy would be more positive for one or the other, I think it is a fair assumption that preference indicates that he would feel more confident, thus have higher efficacy. James was not the only student to feel this way, not just in this group of students, but across all interviews. This was not a surprising response to me, as my experience shows that students tend to view curricular problems to be more algorithmic in nature and may not benefit as much from other opinions, despite this not necessarily being true.

Bill discusses more particular content areas of mathematics as opposed to only the two types of curricular or word problems. Specifically, he references graphing functions and that in this situation, he would prefer a group. Although my questioning did not strive to connect content to efficacy, Bill did this naturally by acknowledging his own strengths or weaknesses and extending this to his efficacy beliefs.

Me: My next question is if you were about to solve a problem and you could work either alone or in a group, you could choose, does the type of problem you are doing make a difference?

Bill: Um… for me personally, I think this is just because of my own preference in math, if it’s something in terms of graphing, or that type of math, I’d like to work more in a group, in terms of just solving an equation, I think that’s easier to just sit down and do it on paper in front of yourself. But, with concepts that get a little more abstract and you can approach them in different ways, I like the group aspect.

Bill mentions “concepts that get a little more abstract” here. To me, this is again interpreted as less prescriptive problems that do not have an obvious approach. These may come across as word problems, but in general are certainly less curricular kinds of problems.

5.2. Mathematical Anxiety: Alex, Amy, Sarah, Tina

Similar to the first subgroup, these students reported higher group than self-efficacy. Although there were some clear similarities between the two groups, there were some marked differences in the reasons stated for their preferences towards group work.

5.2.1. Student Descriptions

Alex

She is a creative student taking the full IB diploma program at our school and has been in my classes for two years. Alex will be the first to admit that mathematics is not her strength and I have observed much anxiety related to subject over these two years. Alex works very hard in this course and has worked with a private tutor over the duration of our time together. She tends to collaborate very well – notably this class was very small (only 16 students) and had all known each other for at least two years. Alex often grasped mathematical concepts well, but did not always have the underlying background skills or knowledge to be ‘successful’ in a traditional assessment. She finished IB Math 12 (Standard Level) with a predicted score of 5, or a B (76%).

Amy

I have known Amy for the past year. She is enrolled in the full IB diploma program at our school. Amy is an incredibly determined student who takes school very seriously. She shows signs of mathematics anxiety, but this appears to have decreased somewhat throughout the school year. Amy’s main strengths lie in other areas, and she works very hard for her success in mathematics. She works very well with classmates, so long as they are also focused and committed to their work. She would frequently come in for extra help, and did not always show good insight to a new question when one arose.

She was quick to get defeated and hers was usually the first hand in the air with a question. She finished IB Math 11 (Standard Level) with a B (75%).

Sarah

Sarah is in the full IB Diploma program and has been in my course since the fall. Sarah is a very social student who collaborates effectively with classmates. She shows signs of mathematics anxiety, but has seemed to improve throughout the year. She has stated more than once throughout the year that her confidence in mathematics has improved. She often shies away from new problems, but enjoys problems that she recognizes. Sarah is not shy to take control in a group setting and will voice her opinions readily. She finished IB Math 11 (Standard Level) with a B (73%).

Tina

She is taking the full IB Diploma program at our school. In my work with her since the fall, Tina's abilities in mathematics seem greater than she believes and I have often noticed very good insights from her during class in both group and individual settings. Tina's main struggle is algebraic concepts from past years that she has not yet grasped, but she understands new ideas well. Tina works very well in groups and often takes charge in group settings. Her communication is clear and her reasoning is sound. She finished IB Math 11 (Standard level) with a C – (61%).

5.2.2. Interview Results

Anxiety & Low Self-Confidence

These four students all showed some degree of anxiety related to mathematics in our classroom. They frequently mentioned their uncertainties when working through mathematics problems. Here, Amy says why the group may help with uncertainty:

> Amy: I think a group adds confidence. I think that a group adds clarification, with math and I actually think it's helpful to, like, have someone else explain it to you just to see the other ways that a person might approach a question. So that's different

than the way you might teach it and the way that someone else might teach it is different, it could be more useful.

Although not explicit, she describes how the added group members will become a reliable way of decreasing any uncertainty about her individual work by offering alternate approaches and explanations. This certainly would decrease anxiety in an anxious student. Sarah reiterates similar feelings about the combination of her low self-confidence and how a group can help to support her insecurities in her work:

> Sarah: Like I've never really been 100%, sure of myself when I was doing math. Like I'm not very confident in it just because I know that's not my strongest thing. So I underestimate myself and my capacity to actually do well. Which is kind of sad, but I feel like when I'm in a group there's just more brains. Or ideas could kind of just come together.

All four students in this group are female. In my observations, girls tend to underestimate their mathematics ability, or rate themselves lower in terms of self-efficacy. Female students have been shown to display higher anxiety in mathematics classes than male (Gallagher & Kaufman, 2004; Bieg, Goetz, Wolter & Hall, 2015). Although, I did not explicitly analyze gender differences in self and group efficacies, this was definitely consistent with my observations. It was not uncommon to hear girls make comments about feeling intimidated or uncertain in their work and their response would be to withdraw in the group work. Sarah and Amy both summarize this nicely here:

> Sarah: Yea. I mean sometimes I tend to talk less because of the people I'm with. Where it's like, well they obviously know more than me, so like why am I even talking?
>
> Amy: Well, depends on who the group members are, because if you have someone who's very strong in math and they don't care to explain anything, or they're somewhat condescending, um, with their tone or how they're explaining something, like 'what, you don't know this?', so that's, I think that could be bad.

Both of these comments touch on the importance of the group members the student has been paired with. I will look more specifically at this later in this section, but all four students in this group commented several times about the importance of who their group was comprised of. Sarah even goes so far as to talk about having her group members "make fun of" her if she does something wrong. Clearly the group members can contribute to or alleviate anxiety.

> Sarah: Yea, I think that some people maybe, they feel that they're afraid to do something wrong, because the other people, like, might make fun of them or say something like that, so I think that's something that could go wrong [in group work].

Group Member Preference

A common trend with both groups with more positive group efficacy was their strong sense of support from group members. All four of the students that showed anxiety trends spoke about this at some point in the interview. However, these students distinguished clearly between wanting a group to help them feel supported and more confident in their work, not only to help with ideas.

> Tina: In a group it's more, if I don't know stuff it's really nice because then I have support from other people. Which is good.

> Alex: Uh, I think probably because like with a group you have combined intelligence. Like there's people there if you get stuck on something it helps to like have somebody like 'that's the way you do it', and then you're like 'oh, yea that's the way you do it'.

Something less expected that came out of interview with this group of students was that they were almost fearful of making mistakes in front of their peers. In particular, if their group members were deemed to be of a higher skill level than they were, these students were more apprehensive to engage with a question. Amy discussed at length how she viewed herself, not only in our classroom, but within the school body at large. The school is considered quite academic and competitive, and taking an IB level course, the students are very driven and generally achieve very highly. Some students can seem, at times, conceited or incredibly sure of themselves – they outwardly demonstrate high efficacy:

Amy: If it's a group and everybody is feeling like this one person is kind of condescending, then the other group members might not say anything either, and so everyone's just kind of going with what that one person's saying.

From the start of the school year Amy clearly felt intimidated, adding to her overall anxiety, and lessening her self-efficacy. She spoke about her transition to our school, a move that was made with strong academics in focus. Amy wanted to earn an IB Diploma, and our school was the closest place where she could do this. Interestingly, she spoke of her skill set within those of her classmates and felt of a lower ability, in mathematics in particular. Here, she speaks about this tendency to sort herself within our class and how that ranking changes her self-efficacy.

Note: Amy speaks about the difference between SL and HL here. For clarification, SL is "Standard Level" and HL is "Higher Level", which is of more difficulty.

Me: So what I'm hearing is that you sort of see it like a tier of skill level? And you're sort of thinking of where you are on that tier, and as long as you're within a reasonable level of that tier, you're okay. If we start working within sort of the top skill then you're going to feel less confident. If we are working within the bottom skill then you're going to feel more confident?

Amy: Yea. But it's also then like when you're asking a question with someone who's like, of a better understanding of math, you might be completely off base and you have no idea if it's even related.

Me: Ok. And what do you think about, like if we did this on the first day of school and there weren't courses yet? Like, if there was no HL and there was no Foundations 11 and there was no SL 11 and it was just Math 11. How would that make you feel? Because it's interesting, you're talking about an interesting thing that happens in a lot of countries and provinces and places around the world where they do streaming, um, some places they do it really a lot and I'll use for example the UK, they stream like, eight different levels, and in some countries they don't really do it at all and students take whatever course they're interested in. That's kind of what you're talking about.

Amy: Yea. I think that if it were the first day and I think you can quickly figure out who's confident in math and who's not. So I think that the same kind of thing would apply. Yea, I think the same kind of thing would happen.

Me: Ok, so it would just vary a bit on the individual group that day if you were with so-and-so?

Amy: Yea. Or also I think it depends like on the school. Like, for me personally if I would have done that at my old school I would have been a lot more confident in the math and my math abilities, but now just because there is like the possibility that someone is in HL math or whatever, then it's going to have an effect on me.

Me: So because it's a more academic school? So it just makes it a bit more competitive?

Amy Yea. And just stressful.

Both Sarah and Amy spoke about this idea, of rating themselves within the classroom early on and choosing either a leading role or to sit back as an observer.

Sarah: Cause at first I'd feel like 'I'm just going to sit back, because I know this person's like stronger than this person', but now I've become more used to it, so I think that I ask them more questions about it. I, ask them more question about it when I don't understand. Or like I let them start.

Given these sentiments, of ranking themselves in their environment quickly; both in the classroom at large and within a given group, both self and collective efficacy of these students shifted. Firstly, their self-efficacy seemed to increase in relation to their confidence – when they felt of a higher rank within the classroom or group they were more likely to talk about taking initiative and attempting a problem. It is clear that there is a strong relationship between group members and the student's self-efficacy within that group. What is still unclear from these particular interviews is how that change in self-efficacy directly impacts the group's efficacy on a case by case analysis.

Here, Tina speaks about clear variations between unknown group members and tasks, and their effect on how confident she would feel approaching the problem. She clearly states that ideally she would know both her group and her task, and that this would make her feel the most confident – in other words her efficacy is more positive.

Me: So, if we can think of an example of where we were getting into groups and we were going to do a problem and you didn't know what the problem was and you didn't know who your group members were, how would you feel? Would you feel confident? Or not so confident?

Tina: No.

Me: No, not confident?

Tina: No. It's better if you know the people. Because then you can feel like you can be honest with them. And be like 'I have no clue what we're doing here, can you explain it?' But if you don't know them and you say that then, it's weird.

Me: So what if you knew the problem, so I had shown you the problem ahead of time and you didn't know who your group members were?

Tina: That's a little bit easier, because at least you have an idea of what you're doing. So you can either take charge or hopefully somebody else can and you can help them with it. Kinda feels a little bit easier to be part of it.

Me: Ok. And then last one. What if you know the question ahead of time and you know who your group members are.

Tina: That's the best one. That's good.

Problem Preference

Similarly to the first group of student's with more positive group efficacy, this group clearly distinguished between two types of problems; those more algorithmic and those more open or unfamiliar. Again, there is clear evidence that students have a strong preference to work on unfamiliar problems – again called 'word problems' in their groups. Their self-efficacy was generally lower than group efficacy on those particular problems.

Me: Ok. My next question is just does the type of problem you're working on in your group make a difference?

Sarah: Like, word problems for example, I really like to do them in groups. Because I think the first thing that I don't understand in word problems is how to set it up. But once I've set it up, I know exactly like how to do the calculation for example. It's just like the set up that I have problems with.

Tina: Yea. Word problems. I'm so bad at those, so that's when I like to do it as a group. Because then it's like everyone goes, so like this is what we're dealing with and you kind of all lay it out. A lot easier doing word problems.

Amy: Umm.. Yea. I would say so. Because I think that, like for me personally, I think that if it's like a word problem or something that needs more sleuthing skills to figure it out I think it's easier to do in a group, whereas like something like, some sort of equation or algebraic something you could do yourself because you don't really need as much guidance as like it's this way or it's wrong kind of thing.

Although I didn't notice at first, all four students in this sub-group are in the IB program. This was not surprising, as the IB program is a very demanding one. Students in this program frequently talk about the demands that are put on them and their extremely heavy workloads. It is not uncommon when these students discuss openly how stressed they are. I frequently hear them talking about how little sleep they get nightly or observe other unhealthy habits – poor eating, lack of exercise, anxiety to name a few.

It then follows that these students must find means of coping with the academic demands while in the program. Many students form very strong friendships and often rely on these friends to help with school work in addition to simple socializing. As such, a higher group efficacy is in line with what these students have become accommodated to – relying on their friends and classmates to help them 'survive' IB – a commonly touted phrase.

5.3. High Ability: John, Michelle, Alan

This was the only subgroup of students that reported a more positive self than group efficacy. Despite this, there were some themes consistent with the first two groups that emerged through interview. However, there were also some marked differences, as discussed in this section.

5.3.1. Student Descriptions

Alan

Alan has been in my class for two years. He is an insightful mathematics student who seems to thrive on interesting or novel ideas. He has often shown great interest in problem-solving tasks and more than once has come back to a problem a day or two after the class has finished working on it. He does not always work well in groups and has sometimes been seen isolating himself from his partners to further work on a task. His work tends to be quite scattered and difficult to follow, but he usually ends up towards a well understood and communicated final answer. Alan does not do well with work he considers repetitive or easy. He finished Pre-Calculus 11 with a B (73%).

John

John is currently in Grade 11 and has been in my class since Grade 10. John is an International Student from China studying in Canada until graduation. John's mathematics skills are very good and he consistently scores at the top of the class on traditional assessments. John's English skills are developing – he has become much more confident in his spoken English over the time that I have known him. John works very well in groups, but gets discouraged if his group members are off task. John is very mathematically talented and has learnt several topics prior to our class covering them – he tends to be a leader in groups. This often means that he is able to solve a problem more quickly than the rest of his group members, but he is willing to slow down or explain his thinking when asked. John is hard-working and polite, with a good sense of humour. He finished Pre-Calculus 11 with an A (98%).

Michelle

I have known Michelle for the past year. Michelle is a quiet and thoughtful student with excellent mathematical insights. She works well on her own, but is not shy to ask either friends or the teacher when she cannot figure out a problem. Michelle is very resourceful using past examples, ideas or books to help her find new strategies for unknown problems. Michelle worked well in groups and always collaborated in a respectful manner. She would usually be shy to take lead in a group, given her nature, but often ended up taking control as she would have good insights. She finished Pre-Calculus 12 with an A (99%) and won the course award for the school.

5.3.2. Interview Results

Individual efficiency, or group inefficiency

The students in this subgroup shared the common viewpoint that being in a group set them back in their work in some way. Although all three rationalized that there could be times a group may be helpful, overall their preference was strongly to individual work. Not only their preference, but they believed they would be more successful on whatever mathematics task if working alone. There were several reasons for this.

All three students stated at one point or another during interview that being in a group introduced some form of doubt in their work. These three students are all very confident in their process and have strong opinions and ideas. When working with others, they needed to pause and wrestle with other's opinions and either agree or disagree respectfully. This challenged the student's thought clarity and sometimes led to frustration or confusion. Here, Michelle echoes this in response to why she indicated a preference to working individually than in a group:

> Michelle: Um, I think it's cause like when I'm working in a group my train of thought gets interrupted and I get really lost when I'm doing math, so if I'm like doing it in a group and people are asking questions or shouting out ideas, then I get lost if I'm not doing it by myself.

John had a slightly different take, in that he didn't feel the need to have a group as he was confident on his own.

> John: I think I'm not familiar with everyone in the class and I don't really know what their math level is at. Cause I think I can handle most of the questions, but not everyone's the same, so that's why.

But Alan agreed with Michelle's sentiments – the group could lead to tension. He sometimes dealt with this by separating himself from his group, which was consistent with my observations in the classroom. Alan felt he lost his train of thought, or "focus" when dealing with a group. The clarity that he felt individually disintegrated through conversation and other's ideas.

> Me: What is it that you think a group takes away from your math ability?
>
> Alan: Sometimes I lose focus. Like, depending on the group member. It is sometimes good to explain stuff and then you understand it better. But sometimes your group member, sometimes when you're with a group you get someone who feels very strongly that this is right and this is the way to do it and you're on the other side and you don't and it's kind of hard to and you separate yourselves and um, you kind of just want to do your own thing, because the other group member is, uh, is thinking completely different than you.

Another common theme amongst these students was their concern about time. They felt more efficient when working individually and this led them to feel as though group work was a waste of time.

> Alan: I do feel sometimes it's, you don't, your group's not engaged and it takes, a lot of time, or I feel like it's a waste of time, sometimes.
>
> Michelle: If like, eventually you get to the same answer, then I guess it's the same, but sometimes you're working in a group and it takes so long you don't end up finishing the problem. Then that's not as good.

Notably, both of these students are also heavily involved in other activities outside of school – primarily in athletics. It is my experience that student's with busy lives value their time and tend not to waste it. In line with this, these students all tend to be efficient in their work and not succumb to typical distractions of the classroom. They don't socialize as much as others and would have what most would describe as good work habits.

Lastly, this group also indicated some frustration with always being seen as the leader in a group, or the person who should have all the answers. Their other group members would deem them as having higher ability, and thus would sit back and let them take over. This observation correlates to the previous two groups examined saying the converse sentiment – that is that they would let the stronger students take over.

> Me: What do you think it is that a group takes away from your math ability?
>
> John: Probably most of them when they have the question they just look at it and like not participating in the question, so I think that would be a problem.
>
> Me: Ok. So if the group members are not involved?
>
> John: Yea. And most of them, like some of them just, they just don't even care about the question. They just want to sit on their phone or chat with their friends.

Problem Preference

One area of inconsistency amongst this group of students was their efficacy beliefs when it came to problem type. As the other groups, these students distinguished between two types of problems; curricular and non-curricular, or word problems. The inconsistency was whether or not the problem type affected their preference to be in a group or not.

John was in agreement with the other two subgroups – he would prefer to be in a group for a word problem, or what he called a "logic problem". He refers to consulting a "handout" for curricular problems, in other words these problems are repetitive and follow algorithms similar to those in the notes or a textbook. This sort of problem was not truly a problem as it followed a clear set of instructions that was pre-determined. John felt that logic problems were where he may benefit from being in a group.

John: Yea. Oh yea, it's not a math problem. It's more like a logic problem.

Me: Ok. And if it was more of a 'math' problem? Like something from our textbook for example?

John: Oh that would be easy.

Me: Right, so you wouldn't need a group for that?

John: Yea I think if you're confused about something you can just check the handout.

On the other hand, Alan and Michelle both maintained their preference to work alone as they felt they would have more success. Here, they both refer to a non-curricular problem they had worked on earlier that week, dubbed the 'pumpkin problem'. In this problem they worked in groups on whiteboards. Although the problem ultimately came down to solving an equation, it was not clear how to approach the problem at first. Both Alan and Michelle indicate their self-efficacy was still more positive than their group efficacy on these "word problems".

Me: What about with our problems, like, for example the pumpkin problem we did yesterday, or one of those kinds of questions?

Alan: Yea, I felt like if I was individually I would have been able to organize my thoughts better just on paper, instead of scrambling all over the whiteboard.

Michelle: Yea. I think I do better on those by myself as well. Just for the same reason. I get distracted and then confused. But, I mean, it can be nice to like bounce ideas off of other people, but I usually do it by myself first and then work in the group.

Alan goes on further about more particular contents within the pumpkin problem. Here he references that this problem did not need a "formula" and that you "had to really think about what was actually happening". This sort of *non-curricular* problem was an area where Alan shined. He was a very good thinker, but not as successful when it came to curricular work. He enjoyed puzzles and mathematical thinking and often relished the opportunity to work through an interesting problem. But he does acknowledge his weaknesses – if he feels “uncomfortable” with a topic then a group would benefit. He is placing specific content areas within the open tasks. It is possible that Alan can think through a problem, but may get held up when a problem is set up and needs to be solved using a particular algorithm or some sort of background knowledge.

> Alan: Well, for that one it was like you had to really think about what was actually happening, it wasn't putting it in a formula. But you really had to think about what you had to do for the formula. That was the biggest part of the problem. And, I guess, definitely when I feel uncomfortable with a topic I feel like it's really good having group members, and sometimes I don't know, it feels good when you're comfortable with a topic and you have group members, it feels like you're, it gets your confidence with math and almost makes you understand the subject more.

Here again Alan speaks about his areas of weakness and the potential good a group may offer:

> Alan: Oh, yea. And when I don't know certain things about a question about how we should approach it, then it's really good having a group member, um, but sometimes with different topics if it's got something that's got a lot of, I don't know, more individual thinking and you need to really, sort of think outside the box and you have to kind of discuss to your group members about what you're thinking, sometimes it really slows you down.

These three students were not in agreement about the effect of a group depending on the problem type. All three had slightly different feelings about whether or not the group could increase their overall abilities. Further research with this subgroup of students could add insight into any overall patterns, or whether it truly is variable by individual.

Group Member Preference

Similar to the previous two groups interviewed, this group agreed that the group members would impact their collective efficacy. However, each student gave a slightly different reason for why this was.

Alan chose to focus on whether or not his group members were engaged. To him, participation was of the utmost importance, regardless of specific skill level. He thought that his group would find the most success if all students were interested in completing the task or solving the problem. He also specifies that particular skills, or content knowledge would be beneficial. In response to whether or not the group members made a difference in how he thought his group would do:

> Alan: Yea. Um. I almost feel like, yea it makes a pretty big difference. I mean I guess if they're engaged or not, or if they were I don't know, didn't want to do the question or not. Or, if they had the skills to help with the group.

John, on the other hand, focused on group cohesion. He had experienced group settings where two members were friends and one was not, or for some reason there was sort of an 'odd man out'. This was, to him, the least effective situation to be in. In his perspective, either all or none of the members of the group should be friends.

> Me: So you're saying then that the group members matter, it's best if it's someone you know, because then you don't need to break the ice?
>
> John: The best is when it's everyone you know, or everyone you don't know.

John also spoke about the uncertainty of both the group and the problem being posed. He was clear that his group efficacy would be more positive when he knew who his group members were ahead of time. Further, he discussed that if he knew what the problem was ahead of time he would just "start doing it", without the help of his group. His measure of strong self-efficacy suggests that he thinks he will have more success on his own. After discussing his preference to have group cohesion – either all friends or all unknown I asked how he thought his group would do on a problem.

> Me: How would you feel about how you would do with the problem? Let's say you don't know what the problem is.
>
> John: I'll be more confident, because everyone can communicate more easily, because of friendship and relationship.
>
> Me: Ok, and let's say you did know what the problem was, ahead of time, and you were in a random group. Would that make a difference about how you feel?
>
> John: Not really. I'll just look at the problem right away and start doing it.

Michelle brought up a different point about group member preference. She thought that her group would have the most success based solely on their abilities. Regardless of friendship, or engagement, skill level was the most important to her in her group efficacy. That said, to increase her efficacy she believed that she would first need to be familiar with the possible members of her group and have some background of what their skills are. This indicates that a longer and more in depth relationship with her classmates could increase or decrease her group efficacy beliefs as she would have more experience to draw from. This is in line with Bandura's (1977) theory of ways to increase efficacy – past experience informs Michelle's judgment about how her classmates would perform.

> Me: Next question, do the group members you are working with make a difference? Why or why not?
>
> Michelle: Yea (giggles). Um, if they know what's going on, and they're like stronger in math, I find that I like working in a group more, cause I feel like I get less confused and if someone asks me a question that, I don't know, sometimes you have people who are not as good at math ask questions that are very confusing for me and that makes me more confused, but yea I think some people can add more to group work.
>
> Me: So you're thinking about their ability? Like their skills?

Michelle: Yea.

Alan strengthens this case, adding that he would prefer group members at the "same level" as himself.

Alan: Or, I don't know. It feels. I feel like I do better if it was a couple people that were kind of, I guess at the same level as me. And, um, but it obviously would be hard to do that, and like, if you have a really a smart person, or a really skilled person, you're going to have usually like a lower skilled person.

5.4. Themes across all three groups

After examining the three subgroups independently there were some clear differences, as detailed above. However, there remained some common themes amongst all students interviewed, irrespective of what group they were in.

5.4.1. Group Member Preference

Ten out of the eleven students interviewed indicated that the members of their group impacted their group efficacy beliefs. The reasons for this impact varied from student to student and group to group – there were no observable trends. However, there were common answers that recurred within several interviews.

1. **Engagement**

   Five of eleven students interviewed hinted that their preference was to be with group members who were engaged. They believed that this engagement would increase their group's ability to perform, and thus the student's group efficacy. A partner's willingness to attempt a problem, make mistakes, and change an approach was described as very valuable to a group.

2. **Friendliness**

   Five of eleven students also mentioned that their preference was to have friendly group members. In contrast, some students also mentioned that they did not want group members that were “condescending” or took over the group. Willingness to work together and pause as needed would fall under this category as well. Students did not necessarily want to work with their friends, but rather the members of their group should be friendly. This key ingredient to positive efficacy beliefs – group friendliness – is one in which I try very hard to foster in my classroom. If students are supportive and friendly to each other it is clear that they are more likely to work well together in a productive way. The interview responses echoed this sense of importance in fostering a cohesive and friendly classroom environment.

3. **Skill Level**

   Lastly, three of eleven students interviewed discussed their preference to have their group members be of similar skill level to them. This surprised me, as I initially thought that this number would be higher. Students valued more affective and emotional values rather than knowledge based ones in their groups. Still, similar skill level was mentioned across two of three groups and was more valued by stronger students. It seemed that students with higher skill sets did not want to be slowed down by students with a lower skill set.

5.4.2. Problem Preference

Across all three groups students indicated a preference for a group in the case of word problems. The students generally agreed that these problems required more unpacking, or gave them more pause and in turn they felt the most benefit from group support. Although some students did not identify this type of problem as a "word problem", they all used common language around less curricular types of problems. Not only did they show preference for a group for word problems, but the questionnaire results indicated that collective efficacy was more positive for a variety of mathematics problems. Students did not speak to other mathematical concepts in interview (e.g. graphing functions, solving equations, etc.), perhaps because this is a less frequent group task in my classroom.

Where there was more variance was in terms of curricular problems. The students with more positive self-efficacy had a strong preference for individual problems and their self-efficacy was consistently higher for this sort of problem. The other two groups of students, with more positive group efficacy varied more on these problems. Some students distinguished between problems that they felt they knew, or could "solve", and those they didn't. This distinction further strengthens Bandura's (1977) argument that content was important in identifying efficacy beliefs. Students wanted to know more about the type of problem prior to attempting it individually. If they knew it was a problem that they were familiar with, then they were comfortable alone, if not, then they would prefer a group. This makes sense and is not surprising. Curricular problems tend to be algorithmic in nature and students alluded to the 'right' or 'wrong' way to approach them. They could easily look up an algorithm if they are unsure, or a similar problem to guide them. This circumstance is significantly different than more open logic problems where they could not easily research how to approach the problem.

On analysis, there were several interesting similarities both within groups and across groups. However, there were still various reasons stated for differing self and group efficacy beliefs. Efficacy beliefs changed for most students depending on the type of task (content). Efficacy beliefs also changed depending on the group members assigned for the task. I think Michelle said it best, when she assessed these efficacy variances:

Michelle: Yea, I think it depends on the person, some people work better in groups and some people work better alone. I don't think there's one hard and fast rule.

Chapter 6. Conclusions

I started to focus on student efficacy nearly two years ago. As I have been refining my use of the *thinking classroom* and incorporating increased group work, I have noticed a change in my student's behaviours. Students seem more positive and engaged when working on mathematics tasks in groups, as compared to working individually. As a result of my research on self and group efficacies I have a more thorough understanding of what may impact these psychological states of my students. In this chapter I will discuss my findings and their implications, as well as their limitations. Further research will also be offered.

6.1. Findings: Answering the research question

Of the 104 students who participated in the initial questionnaires, approximately half of all students indicated more positive group than self-efficacy when no content was specified (n = 48). The other 56 students indicated either no efficacy change from self to group (n = 10), or a more positive self than group efficacy (n = 46). The numbers were split fairly even, indicating that when no content was specified – that is, the questions were about broad mathematics skills or results generally – there was not an obvious trend in efficacy beliefs towards either individual or group work.

However, once the type of mathematics problem was specified – for example graphing, solving word problems – the number of students who indicated more positive group efficacy increased to 70 students, from the initial 48. Almost all of the students with more positive group than self-efficacy in the content-free section continued indicated the same when content was specified with only eight of the 48 changing their efficacy beliefs. It was clear that content specificity changed student's beliefs. This finding indicated that in this sample of students there was a tendency towards more positive group than self-efficacy, only once content was specified. This result was in line with previous research showing that specifying content played a large role in self-efficacy research (Bandura, 1986; Pajares, 1996; Nielsen & Moore, 2003). The majority of students were consistent across efficacy types; if they indicated more positive self-efficacy without content specified, they would say the same once content was specified. I would be curious to know if this content and self-efficacy correlation is only true in mathematics, or perhaps extends to other subject areas. This is a possibility for further research in other content areas.

This pattern of results implied that self and group efficacies are related in the mathematics classroom; those students with a tendency towards more positive efficacy of one kind, will likely keep those beliefs, despite some changes in content, environment or time. Although there was some variation in self-efficacy to group efficacy beliefs, most students continued to indicate either more positive self or group efficacy over several questionnaires and also in the interviews. I hypothesize that this consistency means efficacy beliefs could be difficult to change, particularly at the secondary school level. Interviews were able to shed light on some more detailed nuance of the research and why student beliefs tended towards one efficacy type as being more positive than the other.

After the interviews with the selection of students who showed either very positive self-efficacy or group efficacy, it was clear that their efficacy beliefs were impacted based on several factors. The two emerging themes were the type of problem being posed (***problem preference***) and the group members that the student was working with (***group member preference***). These two themes were consistent with all students interviewed, regardless of their initial efficacy beliefs. It was generally the case that more knowledge about the problem type and group members was preferable for students – this was interpreted as their group efficacy becoming more positive in that context. Students felt that knowing their group members in particular would give them a good sense of their group's ability. This was an indication that group efficacy beliefs could, in fact, change, and were dependent on having more information about the task's content and context.

So then what else can be said about the research question:

What is the relationship between student self and group efficacy in a mathematics classroom?

It seems generally true that students with a more positive self-efficacy had less positive group efficacy and students with less positive self-efficacy had more positive group efficacy. That is, there is often an inverse relationship. More research needs to be conducted to verify if this is true, as this observation of inverse relationship is based only on the questionnaire results. It was generally observed that the strength of one efficacy type could predict the other. In most cases, students with very positive self-efficacy had a larger difference in their group efficacy on their questionnaire results and vice versa. So, there is a relationship between self and group efficacy, and the current questionnaire results helped to further this understanding.

The interviews further elucidated why efficacy beliefs may potentially change through the Thinking Classroom model. The interviews were a crucial component completed in order to shed insight on my secondary research questions:

1. Does the type of mathematics problem posed have any effect on efficacy?

I decided to refer to this as ***problem preference***. It was clear that there was an effect on student's efficacy beliefs based on the type of problem posed. Students generally identified two categories of problem that they may encounter in our classroom; those that could be tackled in an algorithmic means and those that could not be. Many students echoed that the algorithmic problems did not necessitate a group as they could simply look up how to tackle these problems. All students interviewed, there was an increase in preference towards groups when the problem was more of an open, unknown task – many students referred to these as "word problems". This preference was predominant in interview, but also exemplified in questionnaires section 2; where the type of mathematics problem was specified, group efficacy scores went up, or self-efficacy scores went down. That is, the group efficacy did not necessarily improve, but rather the difference between group and self-efficacy increased. Students believed their group would do better than they would do individually, even if they still believed they may perform poorly in both cases. It would be interesting to see if the converse is also true; when not given a word problem, does self-efficacy increase as compared to group efficacy? My sense is that although self-efficacy may increase, I do not think that group efficacy would also decrease, rather that students may indicate similar efficacies for both individual and group work when given a non-word problem. Further research would help to elucidate if this is true.

2. Do the group members have any effect on group efficacy?

After analysis, this was referred to as ***group member preference***. Again, it was clear from the interviews that almost all students felt the group members would impact their efficacy beliefs, to varying degrees. Interestingly, students felt knowing their group members would increase their efficacy beliefs, regardless of who the group members actually were. Students seem to rank themselves and their peers in terms of ability within their class from very early on in their interactions together. Despite these judgments, and whether or not the students in their group were deemed to have higher or lower mathematical ability than the individual, most students thought their group efficacy would improve if they knew their group members. Students did specify the qualities they hoped a group member would have: be engaged, be a good listener, and complement their own skills/knowledge. These qualities were valued by both students perceived to have higher ability, but also those who were good at collaborating and communicating. This was particularly interesting to me, as I thought students may prefer to be paired with the "smart" kids. This prediction was not the case across the board. Again, their group efficacy increased in the situation of having a good team – one where all group members contributed and were actively engaged in the task.

Providing opportunities for students to refine their skills in: communication; collaboration and social awareness has always been at the forefront of my teaching. The current results suggest that these opportunities should take place more often in my classroom. If all students are able to work effectively, most of the time, then not only will the classroom have a more positive environment, but also it will have the effect of increasing group efficacy. If group efficacy can become more positive, then students will hopefully have more positive learning experiences and construct more meaningful mathematical ideas than before. This is also in line with the new BC curriculum, and its new focus on the core and curricular competencies.

3. Is there a particular type of student who has more positive self than group efficacy?

Those students who were interviewed and had a more positive self-efficacy tended to be students with high mathematical ability. These are the types of students who may be dubbed as “smart”, or even “gifted” mathematically. That is not necessarily to say that their marks would reflect this, although that was sometimes the case. These were generally students who were good with problems and puzzles and did not need (or want) the help of others to either confirm their work or confuse them along the way.

4. Is there a particular type of student who has more positive group than self-efficacy?

Conversely, these were students who generally had to work harder to find success in mathematics. They were not always seen as mathematically “smart” amongst their peers. Again, this was not necessarily reflected in their marks as many of these students had great work ethics to meet learning outcomes at a high level. However, these students generally liked working with their peers to check their work, or help when in a situation they were unsure of.

These students also had the tendency towards higher anxiety. Students anxious about their solutions or work seemed to find some comfort in their peer group, being able to feel more confident once ideas were verified with others. It is possible that further research into the impacts of increasing group efficacy may be able to impact mathematics anxiety.

6.2. Implications

6.2.1. What I have learnt as a teacher

Like all teachers undergoing professional learning, I was hoping to takeaway specific practices and advice that I could employ in my own classroom and share with other educators. Through my research I have strengthened my own belief in what I was already doing. Providing ample opportunity for various types of work; both individual and group work, as well as more open tasks and more prescriptive ones, gives students plenty of opportunity to experience different educational scenarios and become more engaged in their learning. In turn, it can help them to develop a good sense of their own efficacies. Through my research I have learnt that there is a relationship between self and group efficacy.

I’ve learned that there is a relationship between self and group efficacy and that students with very strong self-efficacies tend to have less positive group efficacies and vice versa. This inverse relationship can inform my teaching practice as I strive to provide learning opportunities for students to work both individually and as a group, so as to give space for learners to not only work in their preferred way, but also to have them work in their less preferred way. My hope is that students will maintain their more positive efficacy in whichever means they indicate and that this may trickle down to the other means. In other words, a student with more positive self-efficacy can take that sense into a group and a student will more positive group efficacy can take that into their individual work. Over time, I think that this will help students reach their potential and more so, believe in their potential in any scenario.

This research has also taught me the value of giving students opportunities to choose both how they learn and are assessed. Students with more positive efficacy either towards self or group will likely benefit from working within that setting both in learning and assessment situations. Where possible and appropriate, providing these opportunities to students will only benefit their outlook and thus their performance.

Giving students the freedom to choose the avenue in which they feel the most confident should only serve to increase their ability. This is an opportunity for my own professional growth as well. I already do a good job of providing ample opportunities for various learning environments, but can improve when it comes to assessments. Something that I will take away from this research is that assessing a student when they have more positive efficacy will possibly give them a higher chance of success. This is another gap in the research that would be interesting to pursue: the relationship between self and group efficacy and assessment.

6.2.2. What I hope that my students have learnt

Students' conceptions or misconceptions of their abilities throughout this research was surprising. Although this research did not attempt to measure the validity of efficacy scores, I did find it surprising at times what students had to say about their mathematical skills. In fact, this is what motivated the research in the first place.

However, I hope that the students involved in my research, in particular those who participated in the interviews, began to think more about their beliefs. Moreover, I hope that these students can start to think about the effects of group work not just on their actual success, but also on how they may view their success. Students are not always thinking about their thinking – their metacognition – but are usually very able to if it is pointed out. In having them complete questionnaires, and participate in activities where they are observed and then asked about their views, my hope is that they can become more reflective on their perceptions and behaviours both in and out of a group.

Moving forward in my classroom, I have begun having students complete writing tasks in a journal by providing weekly prompts. Sometimes these prompts are mathematical in nature and have to do with course content, but often they are more about social-emotional aspects of learning. I have asked students to reflect on how they approach individual versus group work. By conducting this research I have become much more aware of the efficacy relationships within students and in turn hope that they can reflect on these relationships. By openly discussing what efficacy is with my students, and its larger effect on learning I hope to provide students with deeper insight into their own individual learning.

My next step is to continue my work in having students refine their ability to evaluate their self and group efficacy so that it can be more accurate. Content specificity is important in this, and I would like to develop a practice of having students reflect both before and after a mathematics task on how they think they will do. From there, I hope to find ways to help increase student efficacy so that they can have a more positive outlook towards mathematics. It is my belief that group work within an engaging thinking classroom model can increase efficacy. This is because students may find that their efficacy is generally higher in a group than individually and this may permeate to the individual level over time. It would be interesting to follow a group of students over a longer period of time and note if a thinking classroom improved self-efficacy. My hunch is that it would.

If nothing else, I hope that my students learnt something about themselves. By understanding their own learning tendencies they can then apply that in future situations. Students who feel stronger in a group can then gravitate towards working collaboratively. Students who feel stronger individually can then work on their own. There are often times when students will need to adapt to the opposite of their preference, but a better understanding of themselves will no doubt inform their future selves as well.

6.2.3. What I hope other teachers could learn

I think this research can serve as a powerful reminder to all teachers of the effects of their students' psychological states. A student will not learn if they are not in a headspace to do so – this is something that surely all teachers have observed at some time. By being mindful of the impact of student efficacy beliefs, teachers can help to provide a situation in which the student believes they can find success. This is evidence of the role of the teacher to provide varied learning opportunities. Mathematics classrooms can best serve students by giving opportunities for both group and individually centered learning. By varying the learning environment, teachers can help students develop both their group and self-efficacy, and whichever preference students may have will potentially improve the other.

This study provides further evidence of the effectiveness of the *thinking classroom* model. Bandura noted there are four means of changing efficacy beliefs, and positive work within a *thinking classroom* serves to provide previous and vicarious experience, as well as feedback and opportunities for positive social/emotional states. Hypothetically, by using VNPS and VRG, students will engage in their learning and potentially develop a more positive group efficacy Teachers already using this model will vouch for the positive effects that it can have in a classroom. Interestingly, the effects of the Thinking Classroom can further provide opportunity to improve the psychological well-being of our students, as observed in my research.

Ultimately, a large part of a teacher's role is to empower students in their learning, and this can be achieved by becoming more aware of their students efficacy beliefs. If teachers are to learn about efficacy and its effects on learning, then they can impart this knowledge to their students. The more students can understand their own tendencies and preferences, the more they can understand about themselves. This can be a critical step towards changing past behaviours and beliefs – becoming aware of them in the first place. If teachers can help students to become aware of potentially negative efficacy beliefs, then it is my hope that they can also help to improve those same beliefs. I hope that further research can come up with some specific strategies on how to do this, but in the meantime it will have to suffice to know that there are relationships. Simply being aware of this is a good place to start for both teachers and students.

6.3. Strengths & Limitations

As this is the first known study looking at self and group efficacy in mathematics learning, the limitations are many. This research did not set out to measure a change in student efficacy overtime, but rather to distinguish if there is a relationship between efficacies in the first place. In this section, I will outline the limitations of the study and suggest possible means of improvement for future studies.

First and foremost this was a small study conducted in a very specific environment. The number of students who completed the questionnaires in either administration was large (n=104), but only eleven of these students were selected to interview. Such a small sample for interview surely limits the breadth of conversation and idea that students may present. In addition to having only a small number of interviews, the selection of the interviewees was particular. Only those students who showed greater variation from self to group efficacy scores were selected. It may be that students with similar efficacy scores would have similar interview results, or vastly different. This eliminated a large section of possible interviews as they were deemed less dramatic for this initial study.

The students in this study were all selected from a very specific population. All were students in a public school in West Vancouver, British Columbia. This municipality is widely known to be of high socioeconomic standing with the average household income totalling nearly $100,000 in 2016 (Statistics Canada, 2017), over $30,000 more than neighbouring Vancouver. Given the economic stability of such students, many were privy to home support by various means; typically the parents were very much in support of their child's education. This often meant outside private tutoring in mathematics, or other subject areas. Extra support in this way would likely lessen the student's mathematical anxiety and increase their confidence, thus potentially effecting their efficacy beliefs. In addition to mathematics support, some of these students also had support through outside of school counselling or therapy, again potentially effecting their efficacy beliefs.

Not only would the extra support provided by more wealthy parents potentially effect efficacy, but it also meant that the students involved in the study were not particularly diverse. All students were of similar age (16 to 18 years), similar cultures (most students were of Caucasian background) and similar socioeconomics. Within those interviewed, it was noted that both genders were represented; four males and seven females. Surely, this narrow field of participants could have impacted the results.

Another concern with the study is that the primary investigator also served as the classroom teacher for these students. All students had been working with the teacher for a minimum of five months prior to the first questionnaire administration and at least seven months prior to the interview. These established relationships lead to the potential for bias in the results. Although it was acknowledged in the pre-questionnaires and pre-interview scripts, it was possible that students did not respond truthfully for fear that it may have negative consequences for their education. I did not have this sense in interview, but it is impossible to know. An impartial interviewer would have eliminated this concern. As both the classroom teacher and the main researcher, it is also difficult to analyze these results in an unbiased manner. Particularly, as I have worked with several of these students for a number of years, past knowledge and observations are likely to be taken into account. Analysis of only the data collected as detailed in section 3.4 is difficult to distinguish from past classroom work. Moreover, as the classroom teacher, students may be hesitant to give true responses to some of the questionnaire or interview items. Despite being reassured at the beginning of each type of data collection that research participation would not have any effect on their grades or standing in the course, students may worry about saying the ‘wrong’ thing, or being viewed in a particular light. During some interviews it was clear that student’s struggled with word choice so as not to appear in a certain way. This may have been mediated were the interviewer someone other than their classroom teacher.

In addition to the pre-established teacher relationship, students also had established relationships with each other. This certainly impacted their ideations of group efficacy, which in turn may have impacted their self-efficacy beliefs. Although this could be viewed not as a limitation, but rather part of the Thinking Classroom model and its effects on efficacy, it certainly could have had implications on the results. To further understand the relationship between self and group efficacy, similar studies should be conducted in groups of students that have no established relationship.

Finally, the questionnaire used for initial screening of student efficacies was designed without prior knowledge of its own effectiveness. Because this was the first time this tool was used for research purposes, it is possible that the questionnaire was not designed in a valid manner. The questions were designed based on previous knowledge of self-efficacy, but without a base to work from for group efficacy, the scale may be inaccurate. It is also possible that the scale designed was not sensitive enough to note significant variations in respondents. Another possible improvement on the questionnaire would be to re-administer the questionnaire after students had done different types of problems. An exposure to a wider variety of tasks may nuance the questionnaire results, and therefore give varied results. Similarly, students had only worked in specific groups on the days that both the questionnaire and interviews were given and this may have impacted the results.

In addition to the aforementioned limitations, this study certainly had its strengths. Most notably, this is the first known study of its kind, as there is no prior research done connecting self and group efficacies in secondary mathematics classrooms. As such, this study was unique and necessary to establish whether a relationship exists between efficacy types and is worthy of further research. Further, the tool developed is the first known of its kind and was certainly a strength of this research. The initial questionnaire was paramount to identifying students' initial efficacy beliefs and establish which students would offer valuable and insightful interviews. Certainly another strength of this study is that the relationship between self and group efficacy is only beginning to be understood. This study can highlight and direct future areas of research that will now be known to be valuable and interesting for mathematics education.

6.4. Further Directions for Research

Evidently, further research is necessary to better understand the relationship between self and group efficacy beliefs in mathematics students. This initial study served to establish that there are relationships between the two and that the research is certainly worthy of further pursuit. In particular, some specific avenues would be of interest.

Further research into the development of an effective tool to measure self and group efficacies would be an excellent starting point. Without affirming the validity of the responses to the questionnaire, it cannot be known for certain that the initial indications of efficacies are in fact accurate. Research into question types, format of the scale and variation in responses are all valuable in further research in efficacy scores. Once this tool has been developed in a robust and reliable manner, then it could be used in several other avenues to research many other aspects of efficacy dynamics.

Since this research took place a few months into the school year, participants were already familiar with their classmates and had been working together for at least five months – in several cases much longer. I wondered what interview results would indicate with a group of students that had never worked together. This is an excellent opportunity for further research. Research in a group of students who had not previously worked together would be important to help establish self and group efficacy beliefs in an unbiased environment. This would ensure that students were unfamiliar with one another's abilities, and thus had not had the chance to rank themselves amongst their classmates in terms of their perceived mathematical abilities. Similarly, research with an unbiased interviewer would possibly yield more accurate results in interview, as the students would have no incentive not to give truthful responses.

Further research as to the problem type ***(problem preference)*** and its impact on self and group efficacy beliefs would be useful. There was clearly an indication that more complex mathematics tasks widened the gap between self and group efficacy in this research. Based on this study, problem type may be the largest factor impacting efficacy beliefs, but further study would be needed to determine to what extent this is the case. Work could be done with particular sets of types of mathematics problems and brief interviews before, during and after group work. Similarly, further research on the group type ***(group preference)*** would be beneficial in further understanding the relationship between self and group efficacy. This could be done by pre-determining groups based on particular characteristics of students and again administering interviews before, during and after the group work.

Once established relationships between self and group efficacy have been noted by researchers, it would be beneficial to connect our learning with mathematics anxiety. Mathematics anxiety has been widely researched for decades, and its connection with self-efficacy is well established, but to my knowledge this connection has not been extended to group efficacy. Perhaps, if group efficacy can be increased, it may be that this also improves mathematics anxiety. Further research in the connection of these two psychological domains of the learning of mathematics would certainly help future students and teachers alike in their ability to confidently and happily think and work mathematically.

Assessment would be another area to connect with student efficacies. That is, does assessment vary when students work alone versus in groups, and in particular how do students perceive these abilities? This research did not attempt to validate student work as correct or incorrect, and thus did not attempt to say whether or not student efficacy measures were accurate. It could be that student efficacy perceptions were not at all accurate in these measures. Thus, by connecting the research of self and group efficacies to assessment of particular knowledge content or problem solving tasks, it would be possible to say whether or not the student abilities were in line with their sense of efficacies.

Lastly, and perhaps most importantly, as a practicing classroom teacher I would be most interested to research effective means of improving the accuracy and overall efficacies of students. If students can believe in their own abilities – both in and out of groups – then I believe they will have more confidence and success in mathematics. Further research as to means of improving efficacy beliefs would be of most practical use to a classroom teacher such as myself. As Bandura (1977) noted, there are four means of changing efficacy: mastery experience, vicarious experience, social persuasions, and emotional and physiological states. By researching practical means of applying these concepts into a mathematics classroom and studying their direct impacts on both self and group efficacy, teachers could learn effective strategies for use in their classroom. Specific tools and strategies could then be shared out in professional development settings and may serve to improve students overall mathematics education.

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Appendix A Efficacy Questionnaire

Participants Name: ____________________________________ *Date:* _____________________

The following questions ask you to estimate your mathematics ability. On a scale of 1 to 5, please rate how much you agree with each of the following statements by circling.

During Individual Work

1.	I am good at mathematics.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
2.	I am good at mathematics in this course.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
3.	I believe I can do well on a mathematics test.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
4.	I feel confident when I have to use mathematics outside of school.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
5.	I believe I can understand the content in a mathematics course.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
6.	I believe I will be able to use mathematics in my future career when needed.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
7.	I feel that I will be able to do well in future mathematics courses.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
8.	I worry I will not be able to understand the mathematics.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
9.	I worry that I will not be able to get a good mark in my mathematics course.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
10.	I am afraid to give an incorrect answer during my mathematics class.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
11.	I get nervous when asking questions in mathematics class.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
12.	I feel stressed when listening to mathematics instructors in class.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree

During Group Work

1.	I always contribute to my group in a positive way.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
2.	I am confident that our group will solve the problem.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
3.	Working in a group makes me more confident in mathematics.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
4.	Working in a group helps me understand concepts.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
5.	I worry that our group will not answer the problem.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
6.	I feel stressed when listening to my group members' thoughts and questions.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
7.	I get nervous when asking questions during group work.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
8.	I believe that our group can understand the content in this mathematics course.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
9.	Our group could solve any problem in this course.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
10.	Our group could solve any kind of mathematics problem.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
11.	I worry that I will let my group members down.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree
12.	I am afraid to give an incorrect answer during group work.	**1** Strongly Disagree	**2** Disagree	**3** Neutral	**4** Agree	**5** Strongly Agree

If you were going to work on the following kinds of mathematical problems **by yourself** in this class, rate how you think you would do.

1.	Graphing a function.	**1** Very Poorly	**2**	**3**	**4**	**5** Very Well
2.	Solving an equation algebraically.	**1** Very Poorly	**2**	**3**	**4**	**5** Very Well
3.	Working without a calculator.	**1** Very Poorly	**2**	**3**	**4**	**5** Very Well
4.	A geometry or trigonometry problem.	**1** Very Poorly	**2**	**3**	**4**	**5** Very Well
5.	Solving a word problem.	**1** Very Poorly	**2**	**3**	**4**	**5** Very Well
6.	Solving a new problem that is unfamiliar.	**1** Very Poorly	**2**	**3**	**4**	**5** Very Well

If you were going to work on the following kinds of mathematical problems in a **random group** in this class, rate how you think you would do.

1.	Graphing a function.	**1** Very Poorly	**2**	**3**	**4**	**5** Very Well
2.	Solving an equation algebraically.	**1** Very Poorly	**2**	**3**	**4**	**5** Very Well
3.	Working without a calculator.	**1** Very Poorly	**2**	**3**	**4**	**5** Very Well
4.	A geometry or trigonometry problem.	**1** Very Poorly	**2**	**3**	**4**	**5** Very Well
5.	Solving a word problem.	**1** Very Poorly	**2**	**3**	**4**	**5** Very Well
6.	Solving a new problem that is unfamiliar.	**1** Very Poorly	**2**	**3**	**4**	**5** Very Well

Do you approve being re-contacted for an interview if requested? YES / NO

If YES, please provide your e-mail address so that I may contact you and let you know that an interview is requested. Please print clearly.

E-mail: ____

If you have any questions or concerns, you can address them in the space below or ask the researcher at any time. You may withdraw from this study at any time.

Appendix B Questionnaire Results

Course	Q1-Ind	Q1-Gr	Q1-Ind.S	Q1-Gr S	Q1G-Q1S	Q1GS-Q1IS	Q2 - Ind	Q2 - Gr	Q2 - Ind. S	Q2 - Gr S	Q2G-Q2I	Q2GS-Q2IS	Mean QG-QS	Mean QGS-QIS
PC 11	40	51	14	18	11	4	42	48	19	21	6	2	8.5	3
SL 11	51	42	17	16	-9	-1	51	38	22	19	-13	-3	-11	-2
PC 11					0	0					0	0	0	0
SL 11	41	43	22	24	2	2	43	48	23	26	5	3	3.5	2.5
SL 11	42	49	20	25	7	5	39	42	19	21	3	2	5	3.5
SL 11	28	44	13	18	16	5	32	44	14	24	12	10	14	7.5
PC 12	32	41	12	19	9	7	39	43	16	23	4	7	6.5	7
PC 12	45	38	24	20	-7	-4	46	36	23	20	-10	-3	-8.5	-3.5
SL 11	38	50	19	23	12	4	38	49	17	23	11	6	11.5	5
PC 12	33	34	15	17	1	2					0	0	0.5	1
PC 12	35	43	17	17	8	0					0	0	4	0
SL 11	49	40	26	23	-9	-3	47	40	25	27	-7	2	-8	-0.5
PC 12	38	43	18	18	5	0	38	38	17	17	0	0	2.5	0
PC 12					0	0	52	43	24	24	-9	0	-4.5	0
PC 12	40	37	20	20	-3	0	39	34	22	23	-5	1	-4	0.5
PC 12	60	60	30	30	0	0	60	56	27	30	-4	3	-2	1.5
PC 11					0	0	49	49	21	23	0	2	0	1
PC 12	47	46	22	24	-1	2	45	48	18	24	3	6	1	4
SL 11	30	36	19	25	6	6	30	30	17	20	0	3	3	4.5
PC 12	48	30	25	21	-18	-4	40	29	26	24	-11	-2	-14.5	-3
SL 11	39.5	39	21	26.5	-0.5	5.5	40	40.5	21.5	26	0.5	4.5	0	5
PC 11	45	31	24	23	-14	-1	48	31	23	23	-17	0	-15.5	-0.5
PC 12	54	36	26	28	-18	2	53	50	27	26	-3	-1	-10.5	0.5
PC 12	58	51	26	30	-7	4					0	0	-3.5	2
PC 12	40	31	24	20	-9	-4	35	29	25	23	-6	-2	-7.5	-3
SL 11	42	47	14	13	5	-1	33	42	13	13	9	0	7	-0.5
PC 11	46	42	30	30	-4	0	45	39	29	29	-6	0	-5	0
PC 12	49	43	18	21	-6	3	46	44	19	20	-2	1	-4	2
PC 12					0	0					0	0	0	0
PC 11	46	50	24	28.5	4	4.5	44.5	43.5	19.5	17	-1	-2.5	1.5	1
SL 11	48	49	22	25	1	3	49	48	19	23	-1	4	0	3.5
PC 11					0	0	33	38	17	24	5	7	2.5	3.5
PC 12	21	32	13	13	11	0	28	35	12	16	7	4	9	2
SL 12	25	33	12	20	8	8	27	35	13	21	8	8	8	8
PC 12	41.5	48.5	18.5	24	7	5.5	41.5	46	16.5	24.5	4.5	8	5.75	6.75
SL 12	41	47	19	23	6	4	39	43	18	22	4	4	5	4
PC 12	38	41.5	14	23	3.5	9	35.5	39	15.5	21	3.5	5.5	3.5	7.25
PC 12	43	42	24	27	-1	3					0	0	-0.5	1.5
PC 12	13	20	10	17	7	7	15	27	9	14	12	5	9.5	6
SL 12	51	45	19	27	-6	8	47	42	21	27	-5	6	-5.5	7
PC 12	39	45	20	26	6	6	38	45	22	27	7	5	6.5	5.5
PC 12	45	48	25	25	3	0					0	0	1.5	0
PC 11					0	0	44	49	25	25	5	0	2.5	0
PC 12	50	53	28	29	3	1					0	0	1.5	0.5
PC 11	45	44	17	22	-1	5	44	51.5	17	25	7.5	8	3.25	6.5
SL 11	39	41	22	24	2	2	40	46	23	24	6	1	4	1.5
PC 12					0	0					0	0	0	0
PC 11	41	38	22	22	-3	0					0	0	-1.5	0
SL 12	48	46	26	26	-2	0	47	44	24	20	-3	-4	-2.5	-2
PC 12	41	30	23	22	-11	-1	25	29	17	19	4	2	-3.5	0.5
SL 11	35	41	16	26	6	10	32	45	18	28	13	10	9.5	10
PC 12	48	50	23	23	2	0	48	47	22	24	-1	2	0.5	1

PC 12	50	46	28	25	-4	-3	55	51	28	24	-4	-4	-4	-3.5
PC 12					0	0	48	46	24	24	-2	0	-1	0
PC 11	42	45	17	20	3	3	44	48	16	19	4	3	3.5	3
PC 12	54	49	26	26	-5	0	55	53	27	28	-2	1	-3.5	0.5
PC 11	53	46	23	25	-7	2	52	47	23	24	-5	1	-6	1.5
PC 12	34	37	20	22	3	2	34	36	21	22	2	1	2.5	1.5
PC 12	47	44	27	18	-3	-9	53	40	27	27	-13	0	-8	-4.5
SL 11					0	0	44	44	24	27	0	3	0	1.5
PC 12	37	30	21	23	-7	2	37	38	22	25	1	3	-3	2.5
SL 11	40	29	27	21	-11	-6	43	38	28	28	-5	0	-8	-3
SL 12					0	0					0	0	0	0
PC 12	59	52	26	30	-7	4	60	56	27	30	-4	3	-5.5	3.5
SL 12	37	42	18	23	5	5					0	0	2.5	2.5
PC 12	46	39	23	23	-7	0	49	40	27	23	-9	-4	-8	-2
SL 11	33	43	16	30	10	14	35	51	18	29	16	11	13	12.5
PC 12					0	0	53	49	27	28	-4	1	-2	0.5
SL 11	34	35	21	24	1	3	38	38	24	26	0	2	0.5	2.5
SL 11	36	34	24	26	-2	2	35	40	17	25	5	8	1.5	5
PC 12	57	55	27	27	-2	0	55	51	27	27	-4	0	-3	0
PC 12	36	35	19	21	-1	2	38.5	40	20	22	1.5	2	0.25	2
SL 11	24	34	13	20	10	7	26	34	13	18	8	5	9	6
SL 11	41	44.5	20	24	3.5	4	43.5	39.5	21.5	23	-4	1.5	-0.25	2.75
SL 12	45	47	20	23	2	3	46	46	20	24	0	4	1	3.5
SL 11	59	45	28	29	-14	1	58	53	27	29	-5	2	-9.5	1.5
PC 12	43	45	26	24	2	-2	52	46	28	28	-6	0	-2	-1
PC 12	40	45	23	22	5	-1					0	0	2.5	-0.5
SL 11	50	45	24	24	-5	0	47	44	24	25	-3	1	-4	0.5
PC 12	32	30	15	20	-2	5					0	0	-1	2.5
PC 11	34	35	15	16	1	1	34	42	16	21	8	5	4.5	3
PC 12	53	53	26	28	0	2	52	52	26	27	0	1	0	1.5
PC 11					0	0	51	48	28	28	-3	0	-1.5	0
PC 11	44	50	24	27	6	3					0	0	3	1.5
PC 11					0	0	43	45	16	20	2	4	1	2
PC 12	27	32	14	14	5	0					0	0	2.5	0
SL 12	36	36	21	25	0	4	32	34	20	18	2	-2	1	1
PC 12	43	41	20	20	-2	0	41	41	21	26	0	5	-1	2.5
SL 11	31	40	19	27	9	8	31	42	18	25	11	7	10	7.5
SL 12	49	39	23	30	-10	7	48	46	23.5	29	-2	5.5	-6	6.25
SL 12	47	45	23	27	-2	4	48	44	25	26	-4	1	-3	2.5
SL 11	29	33	21	24	4	3	28	24	13	21	-4	8	0	5.5
PC 12	38	27	20	22	-11	2	42	41	24	26	-1	2	-6	2
PC 12					0	0	39	38	15	18	-1	3	-0.5	1.5
PC 12	37	48	19	24	11	5	38	43	19	25	5	6	8	5.5
PC 11	53	32	27	27	-21	0	57	36	30	18	-21	-12	-21	-6
SL 12					0	0	46	50	20	26	4	6	2	3
PC 11	49	49	21	26	0	5	48	49	19	25	1	6	0.5	5.5
SL 12	50	35	24	22	-15	-2						0	-7.5	-1
PC 12	37	35	15	14	-2	-1					0	0	-1	-0.5
PC 12					0	0	49	53	28	23	4	-5	2	-2.5
SL 12	56	52	27	28	-4	1	54	51	27	29	-3	2	-3.5	1.5
PC 12	49	42	25	20	-7	-5					0	0	-3.5	-2.5
SL 12	51	48	27	29	-3	2	54	49	29	29	-5	0	-4	1
Mean	42.17	41.59	21.1	23.2	-0.59	2.108	42.86	42.82	21.34	23.71	-0.04	2.371		