Reduction of Mathematics Anxiety through use of Vertical Non-Permanent Surfaces and Group Discussion

by

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Abstract

Mathematics anxiety is a highly relevant issue in mathematics education. Research into pre-service teachers' mathematics anxiety has indicated a number of worthwhile strategies to help minimize its effect on the nervous learner. Of particular interest is the *Thinking Classroom* model. However, while the research is comprehensive on pre-service teachers, it is sparse when concerning middle-school aged children. I have implemented a number of these strategies into a middle-school program designed for anxious youth. The current study is an exploration into how the *Thinking Classroom* model helps students lower their anxiety surrounding mathematics. During the study, students were asked to create a written reflection piece called a *Math Autobiography*, given a MARS-R several times and then interviewed. While not all students exhibited a decrease in anxiety, most students reported at the end of the study that they felt their abilities had increased in mathematics and felt more positive about the subject.

Keywords: mathematics anxiety; MARS-R; Math Autobiography; Thinking Classroom; middle school

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List of Acronyms

SFU	Simon Fraser University
LAC	Library and Archives Canada
MMP	Morning Math Problem

Chapter 1.

Introduction

Much like many of the students I have taught in my career, mathematics anxiety has been my constant and unwelcome companion on my journey through education. I have several distinct negative memories of mathematics that can I trace back to be the sources of my anxiety and issues with the subject. The earliest and most distinct that I can recall is when I was being removed from my fourth grade classroom due to bullying issues (I was the one being bullied). This, in itself, was a stressful enough event; however, I distinctly remember the topic of mathematics that was to be covered right before my unceremonious exit to a different class, that of learning the multiplication tables. The class that I was transferred into had already covered this topic, and was utilizing it in all their subsequent mathematics lessons. Since I had missed this important topic, I struggled for the rest of the year. I felt for the rest of my elementary and middle school career that I was a weak mathematics student because of having missed the teaching of this basic concept. I knew that I had missed something essential, and I would be forever broken as a mathematics student as a result. I looked on in envy at my classmates who could recall what 7x8 was without even a pause, whereas I had to think about groups of 7s and what they added to in a much slower fashion.

This significant event in my mathematics education career impacted me greatly, as I took my fear and anxiety of mathematics to higher levels. Wanting to be a doctor, or astronaut or both (à la Star Trek), meant that I had to be good at mathematics to do the higher-level sciences I needed. I ceased looking for meaning in mathematics classes, as I could see no purpose in the learning of mathematics other than as a vehicle for my future goals as an astronaut-doctor. This was how I viewed mathematics, as a game with rules, that as long as I played by those rules, I would “win”. I began to excel at this game of mathematics around middle school the real meaning of what I was doing was still hidden, but I was good at it. The issue now was in seeing an unfamiliar question. It would throw my mind into an anxious mess. I would end up with even more anxiety if I was struggling with “codifying” the mathematics rules. The result would inevitably be that I would lose sleep or break down into tears. This was because, in my mind, a failure to

understand mathematics meant a closing of all doors of science to me. This fear motivated me to “learn” the mathematics, spending long hours doing as many questions as I could and following the step-by-step notes the teacher gave out. This method of learning mathematics followed me into my undergraduate degree in science, as biochemistry needed a lot of calculus for the thermodynamics of chemistry to make sense. My worst memories of university were of calculus, as my methods for learning mathematics I had perfected in high school required more time than any of my high school courses ever did and were even less effective in higher-level mathematics.

When I decided to become a teacher in 2008, having realized that an astronaut-doctor career would be prohibitively expensive and time-consuming, I knew that mathematics teachers were “in-demand”. Thus, I would take a mathematics education minor to branch out and be more employable with a math/science education background. Then something happened during my Professional Development Program. A course called Education 411 “Mathematics for Secondary Teachers” taught by Dr. Peter Liljedahl was unlike any mathematical experience I had ever had. Randomized groups, group work and mathematics questions unlike any I had ever experienced. This course was simply pass/fail, but I worked harder for this course than I did for any of my other education courses. I began to realize something after successfully completing the course. I had missed something very essential to mathematics along the way, some beauty and enjoyment that I had been blind to for my entire life. I was however not able to discern what it was that had made me change my perceptions, thinking it must have just been the different type of questions. I took note of every one of the problems and made sure to squirrel them away for future, undetermined use.

When I started my teaching career, I taught science and mathematics in almost equal measure. Initially, I taught mathematics the way I had been taught and learned. Notes that covered the topic, practice on what I had just taught in class and homework to reinforce concepts we had just learned in class. However, I began to notice that my students were not engaged with the material or, worse, avoided mathematics as much as possible, ignoring my carefully prepared, photocopied notes as soon as they had finished writing them down. At first, I had thought this was just the way mathematics was after all, it was how I had experienced it, and it is how I remember classmates of mine reactions to it. There were only ever the one or two students who seemed to genuinely enjoy mathematics, and that was certainly evident in the mathematics classes I taught

now as well. However, I was troubled by the fact that I had experienced mathematics in a very different way due to the Education 411 course. I wanted my students to have a similar change of heart and begin to see mathematics for the interesting and beautiful art form that it is. I tried various methods to accomplish this, to very little success in each case. I tried implementing something like what I had experienced in Education 411, but students seemed to rebel at the idea of doing questions like this. Unsure of how to remedy this, due to inexperience or lack of courage, I relegated the problems I had learned in that course to a “try this if you’re bored” activity. Some students seemed to take these questions up and enjoy them, but more often than not, students would never even think to attempt them.

This tension of trying to get my students to enjoy mathematics and take more pleasure in it began to rise in my teaching as I taught more middle-school and high-school mathematics courses. Sometimes the students whom I taught were classified as being at-risk of dropping out of school and sometimes my students were more academic and bound for university or college. Despite the difference in socio-economics or level of class I would always encounter students who would always loudly declare to me that they hated mathematics or were indifferent towards it. They would fold their arms or sit quietly, waiting for my lessons to end or quietly take notes, never really engaging in the lesson or the materials. On tests, my most anxious students would rush through their work, leaving all my meticulously crafted open-ended answers blank. I began to see in these students the same feelings of mathematics anxiety that I had struggled with my entire life. For some, this anxiety was acting as a major roadblock, preventing them from ever experiencing success in mathematics. As a result of this lack of success, they were dissuaded from ever attempting anything more difficult in terms of mathematics, opting for lower streams of mathematics. Other more motivated students would constantly ask for help and attention, asking for more examples or complaining bitterly when they were presented with a question that they had not previously seen before. In both cases, it seemed that many of my students entered a heightened state of anxiety when they entered my classes. I began to search for ways that I could help these students feel better about their mathematics and relieve their anxiety towards the subject so that they might improve their skills in the subject and risk taking higher and more difficult mathematics classes.

My thinking turned to sources of student anxiety. Notes must have been the thing, if I made clear and concise notes with meticulously detailed steps, my students' anxiety would be lessened, and they should aid in letting the students take some risks with their mathematics as, if my notes were thorough enough, they would be able to apply them to unique situations with little trouble. This was unsuccessful in the way I attempted it; there was no change in the anxious student behavior in mathematics. I then tried limiting the time that notes and lessons took place in class to only 20-30 minutes, focusing on what I had determined the fundamentals of what it was for students to learn that day. I would ensure that each student kept up during note-taking time with examples for them to try as part of the notes. This again was unsuccessful; I still had all the same anxious behavior I had wanted to diminish. I created sophisticated homework-tracking sheets with three levels of achievement in the hope that my weaker students could be enticed to complete the most basic questions for a completion mark. The easiest questions completed would get students a five or six out of ten for homework completion. These questions were the most basic kind and could be easily completed within class time. The harder questions were in the other two categories and would get students higher homework completion marks. My thoughts were that if I could just get my most anxious or struggling learners to do a little mathematics homework and feel as though they had achieved a mathematics goal, then they might be further encouraged in their capabilities. The results were mixed; I found no major difference in students' attitudes, although some appreciated the homework tracking sheets, I would find more and more of them half filled out in students' notebooks or on the floor after class as the course progressed.

I then considered that students' anxiety must be a result of the teacher. The dour or intimidating mathematics teacher is a common trope in popular culture, thus I endeavored to try and be its opposite. In this vein, I attempted at being the more "fun" mathematics teacher. My regular teaching style is fairly open and hopefully engaging for students. For mathematics, I tried dialing up the "fun" a few notches for mathematics class by being more "jokey" and gregarious with students and the material. This seemed to have a more positive influence on the students' attitudes towards mathematics class, but all the same behaviors that indicated that mathematics was still making my students uncomfortable were still present. I also began to feel as too much "performer" and not enough as an educator, which bothered me as I felt I was losing the forest for the trees

by focusing too much on the delivery rather than on the subject matter. I truly realized that this approach was not having the intended benefit when one day in a class conversation a student remarked that they hated mathematics but they liked me as the teacher of their hated subject. This made me reconsider a great many things. While a nice compliment to receive from a student, I had to reflect on whether or not what I was doing was sufficient to change these students' attitudes towards the subject. Added to this, I wondered how I might help the students actually like mathematics in the way that I had learned to while I was in my teacher training courses. The question of how I could alleviate my students' anxiety and perceptions of the subject of mathematics became a major focus of my professional development.

Like in many things regarding teaching, collaboration with my fellow teachers on issues or problems can yield tremendous results. However, conversations with my colleagues were varied on the best methods to helps students feel better about their mathematics. "Our job wasn't to make the kids like it, just to learn it". I rebelled at this notion; there is beauty in mathematics for students to learn, there had to be a way to do both. "Workbooks are really good at skill building and help the students approach it at their own pace". This too did not sit well with me, as I hold the pedagogical view that teachers are supposed to be a vital part of the learning process, placing students in a workbook, no matter how well designed would not engender in them a love of learning or relieve their anxiety when it came time to apply their knowledge. "You just can't reach some kids when it comes to math". I rejected this out of hand, as had this been the case applied to my early schooling, I would not now be a teacher of science and mathematics. It seemed as though Lockhart's Lament (Lockhart, 2009), where mathematics is only viewed as a procedural and utilitarian process, was an inescapable reality of mathematics education for many of my fellow mathematics teachers. However, my mind kept coming back to the experiences of the education course I had taken with Dr. Peter Liljedahl and the question of how I could recreate the feelings of accomplishment and autonomy that I had experienced in my students.

I had always intended to go back to Simon Fraser University and enroll in the Masters of Science program in Mathematics Education. The Master's program had been introduced at the tail end of the 411 course that had influenced me so much over the six years of teaching that I knew it would assist me in my goal of improving not just the students' anxiety surrounding mathematics, but the whole of their outlook and attitudes

concerning it as well. Enrolling in the program, I had some old familiar feelings of anxiety surrounding mathematics and learning it again. I began to doubt my abilities in mathematics again and would indulge my mind in dark suspicions that I was an outsider among the excellent mathematics teachers that would enroll in this program and that I was only playing at the role of being a mathematics teacher and not worthy of being in such company. I started to realize that this must be how a lot of my students feel every day in my classes as it seemed that my core identity as a mathematics learner has changed very little since my days in grade 4. However, during the course of my Master's program, I was introduced to some new concepts that had a dramatic effect not only on my outlook on mathematics, but also had shown to be shockingly effective at engaging highly anxious mathematics students as well in a program called *Imagine*.

The use of vertical non-permanent surfaces, randomized groups, and open-ended questions, which has been termed "The Thinking Classroom" (Liljedahl, 2016) seems, on its surface, such a simple thing to change in a mathematics class. However, the effects it had on me and my colleagues in the course and our teaching cannot be understated. I had that same feeling of appreciation and elation for mathematics that I had experienced in the Education 411 course. With encouragement from our program supervisor, Dr. Peter Liljedahl, we implemented or experimented with these *Thinking Classroom* elements in our own mathematics classes. At the time I was working at Fraser Valley Distance Education School (FVDES), which can make implementation of this difficult. At this time, though, FVDES had been running a program called *Imagine* for anxious middle school students in grade 8 and 9. I gave these students some problems in the style of the *Thinking Classroom*, fully expecting students to not respond more positively than anything else I had tried previously. I had never been so pleased to be proven wrong as students responded to these elements of a *Thinking Classroom* in the way that I had searched and hoped for in my practice.

Something about this *Thinking Classroom* had allowed the students to be autonomous in their work, take risks and open up their ideas and conceptions to what mathematics was and seemingly increased their enjoyment of the subject. These students were able to have fun with mathematics in this context and were beginning to ask good questions of the type that I had always hoped for in my regular classes. Additionally, students were starting to talk to each other about mathematics more fluently than I had experienced previously. I pondered what would happen if students were

exposed to being randomly grouped with other students, given open-ended tasks and working on vertical non-permanent surfaces (whiteboards) on a daily basis and not the periodic one that had framed my initial foray into the *Imagine* program. The limited exposure that my first experiment with the *Imagine* program had seen students' anxious behaviors diminish and had reportedly been much enjoyed by the students, so much so that they had requested more of these kinds of questions from me. This powerful effect must be measured. In this thesis, I present research based on the effects on students' anxiety when exposed to the *Thinking Classroom* regularly in a blended-learning environment called the *Imagine* program.

Chapter 2. Related Literature

I am neither the first to be interested in the topic of mathematics anxiety, nor the first to wish to find ways in which to mitigate its negative effects. The issue of mathematics anxiety is a well-researched field to which many valuable contributions have been made over the years. In this chapter, I review some of the most relevant literature on the topic of mathematics anxiety. First, I will review the literature that shows what mathematics anxiety looks like in students and the various ways in which mathematics anxiety manifests itself. Finally, I will end with a review of literature of methods and techniques that are suggested to potentially ameliorate students' anxiety towards mathematics.

2.1. Understanding Mathematics Anxiety

In order to help students with their mathematics anxiety, we must understand what its root causes are. Understanding the genesis of students' mathematics anxiety will help us create learning situations which can most positively affect the outlook of the anxious mathematics learner.

2.1.1. Sources of Mathematics Anxiety

Identifying what mathematics anxiety is and how it manifests itself in students is a difficult problem and one that has been debated for a number of years. Cemen (1987) defined mathematics anxiety as "the state of discomfort created when students are asked to perform mathematical tasks". This is a broad category and definition as it brings into question which mathematical tasks are most "uncomfortable" and what levels of "discomfort" prevent a student from being able to operate within a mathematical or problem solving-framework. In a study dedicated to creating a measure of students' anxiety scale, it is stated that mathematics anxiety "involves feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (Richardson et al., 1972 pg.551). The researchers in this study go on to create categorizations of different scenarios regarding mathematical situations and has students rate their own feelings regarding the individual mathematical scenarios. This has become known as the Mathematics Anxiety Rating Scale (MARS), which will be discussed as a diagnostic tool

for mathematics anxiety in a later section. These categories included specific situations that could arouse anxiety in people while they perform it, such as adding 2-digit and 3-digit numbers while someone looks over your shoulder. Not addressed, however, is why such an anxiety would come to exist or what its primary source might arise from. Identifying the symptoms of anxiety does not seem to deal with the "why" of a student's anxiety. Thus, it seems to appear that the discomfort and tension that students feel when learning or performing mathematics can be traced to some key issues that they may have faced in their formative learning years. Jeff Evans (2000) identifies some of the possible origins of mathematics anxiety in his study on how adults view mathematics. In this study he creates a model which links how adults failed to develop confidence in their mathematics abilities and their mathematical thinking in their earlier years learning mathematics. He identifies socialisation, critical incidents and affective characteristics as three primary parts to his model. *Socialisation* is defined by Evans as how the students perceive themselves and are perceived by their peers in a mathematics class or environment. *Critical Incidents* are those events and experiences which may have impacted how the student learned mathematics. The final part, *affective characteristics*, is how the 'internalisation' of socialisation and critical incidents has impacted the student's views of and feelings towards mathematics. With this model, it becomes salient to identify how and why anxious mathematics learners' beliefs about themselves are formed. These beliefs can manifest themselves as symptoms which may help identify what has caused the student to believe about themselves in regards to their own abilities in mathematics.

Sheila Tobias (1990) called this "mapping the terrain" when trying to discover the sources of students' beliefs about themselves and mathematics. In it she identifies the students' histories through the use of "Mathematics Autobiographies" in which students describe specific scenarios and incidents that have led them to believe what they do about their own abilities in mathematics. Tobias lists several common themes that she has identified as mathematics anxiety in autobiographies. She lists 'passivity' as a symptom of mathematics anxiety. This symptom is characterized by the feeling that the student is aware of their lack of knowledge or understanding, which when presented with a unique mathematical scenario or problem ends up causing a "sudden death" in which the student now feels completely incapable. This is generally typified by the "myths" that are prevalent in mathematics such as innate abilities or rare insights being the only

paths to true mathematical understanding. A student who feels failure in mathematics will experience this “sudden death” and then feel either that they hadn’t worked hard enough and let everyone down or that they are pre-destined to failure as they did not possess the mathematical mind. Tobias goes on to list ‘ambiguity’ as a large source of student anxiety with words such as ‘root’, ‘plane’ and ‘multiply’ having a different meaning in regular discourse than in mathematics. This ambiguity is prevalent in mathematics and causes a great deal of anxiety for students when trying to understand mathematical concepts. This creates a disconnect in the minds of the students who think that “logically” a word that means one thing in everyday use should mean the same thing in mathematics. This is rarely the case and creates anxiety around this ambiguous use of terminology. She adds to the symptoms of anxiety a fear of having missed vital information, known as “the dropped stitch”. This “dropped stitch” creates an anxiety in a student that he or she will not be able to progress in their mathematics after missing some crucial information. Tobias cites that this is primarily due to the sequential nature by which mathematics is often presented or taught. Included in the symptoms of anxiety is a fear of social isolation from being perceived as either too good or too bad at mathematics. Tobias feels that this fear can be considered an especially gendered issue with girls being the victims of expectations that they shouldn’t be good at mathematics. This in turn makes girls choose to avoid mathematics. However, a study on the predictors of mathematics anxiety and achievement found no statistically significant differences between either sex (Meece et al., 1990) in their anxiety and later achievement and valuation of mathematics, although certain other studies still found strong gender differences in achievement and enrollment in mathematics courses (Hembree, 1990). The study by Meece et al. (1990) determines that mathematics anxiety is a direct result of the “achievement expectancies” and the perceptions of the mathematics efficacy that a student has of themselves, finding that the more “positive achievement” that a student expected of themselves to do, the less anxiety they were likely to feel. What is not addressed though is what might cause these expectancies to be low in the first place and thus a cause of the student’s anxiety. Tobias (1990) lists another symptom of mathematics anxiety, a distrust of intuition, which causes students to mistrust their own feelings and abilities. This leads into another symptom of mathematics anxiety, self-defeating self-talk, in which the mathematics anxious person convinces themselves that they are already incapable due to the previous existence of the just listed symptoms. This is similar to the paper by Meece et al. (1990), who found

that positive views of self-efficacy correlate to reduced feelings of mathematics anxiety. All these symptoms become clear with reflective pieces authored by students in the form of *Math Autobiographies*, which seem to give an enormous amount of qualitative data on this subject of which the need is great for examining the origins of mathematics anxiety and the affective domain as it pertains to mathematics (Evans, 2000).

A different study which sought to find the generative causes and some relief to mathematics anxiety compared mathematics anxiety to that of test anxiety (Hembree, 1990). This study found that mathematics anxiety is related to a generalized anxiety for which there is no correlation between performance and level of anxiety. Similarly, it seems that in a more recent study it was found that a more negative affective perception of oneself is a root cause of mathematics anxiety (Hannula, Kaasila, Laine & Pehkonen, 2005). This study identified a number of affective areas such as perceived support from family and how hard they worked. They then categorized the students based on their most prevalent affective areas. They found that students who were categorized as "hopeless" had extremely high levels of mathematics anxiety and their perceptions regarding their efficacy were very low despite working hard at learning mathematics. This "hopeless" group believed that they are incapable of learning mathematics. It seems that Tobias' symptoms of mathematics anxiety are supported by more current research into the sources of anxiety and potential relief as this clearly parallels the self-defeating self-talk and distrust of intuition outlined in her book.

A study conducted by Hobden and Mitchell (2011) into the origins of pre-service teachers' perceptions of mathematics and the origins of those perceptions using *Math Autobiographies* showed many of the same categorizations of students fears that Tobias had listed in her book and provides evidence of sources of students' mathematics anxiety and possible reasons for their negative beliefs regarding the subject. This study also provides support to the usefulness of qualitative data that *Math Autobiographies* provide. In this study, the authors coded 'typical' responses that they found in the autobiographies. Student belief about subject, environmental factors, personal factors and critical incidents were the four broad categories that were created. In these broad categories were sub-categories that listed more specific details that students' responses would be coded to. Similar to Tobias' symptoms of anxiety, these have to do with students detailing their experiences in mathematics that have affected their current perceptions of the subject. There exist many parallels between these coded categories

when compared to Tobias' book, as the authors of this study found that prevalent themes arose after an analysis of their data. As an example, the study identified that poor perceptions of their [the pre-service teachers] mathematics ability contributed to their breakdown of mathematical learning. This is similar to the self-defeating self-talk as outlined by Tobias (1990) as a primary symptom of mathematics anxiety and a hindrance to further mathematical success. This study further supports the idea that the genesis of mathematics anxiety occurs fairly early on in students' lives as these negative memories begin to impact their future learning by fueling their "attitudinal negativity" (Klinger, 2006). This then compounds the student's issues with mathematics by increasing negative affective behaviors such as mathematics avoidance, poorer self-image and efficacy, heightened anxiety and withdrawing from the subject entirely (Klinger, 2006).

2.1.2. Maintenance of anxiety into Avoidance and Negative Feelings

Having examined some of the origins and symptoms of mathematics anxiety, it becomes pertinent to examine how mathematics anxiety is maintained and how it begins to interfere in students' mathematics education. These "sources" or "symptoms" of anxiety and tension can have a cumulative effect on the mind of a mathematics anxious person and begin to create in those same students an avoidance of mathematics that can later have the effect of preventing "a student from passing fundamental mathematics courses or prevent his pursuing advanced courses in mathematics or the sciences" (Richardson et al., 1972 pg.552). The effect of a negative outlook was established in the previous section as being correlated to a higher level of mathematics anxiety (Klinger, 2006) and a more "hopeless" outlook in which the student begins to feel that there is no way in which they can become better at mathematics (Hannula, 2005).

Sheila Tobias (1990) discusses in her book that mathematics avoidance is especially prevalent in the gender expectations of society. Since girls are not "supposed to be good at math", this causes in the minds of those young mathematics learners that their struggles are not a normal part of learning or achieving in mathematics, and that rather they are incapable due to their gender. While there has been quite the push to change this perception (Evans, 2000), it still remains an insight into how anxiety is maintained and causes mathematics avoidance in the future choices that a person might make. Similarly, the "myth" of a special "math mind" causes students to believe that they

are incapable at being good at mathematics if they do not possess this attribute, thus causing similar avoidance in both genders. It is posited that the myth of a “math mind” is promulgated by the “stories” told regarding mathematics and mathematicians (Mendick, 2005). These stories of heroic mathematicians coming to sudden realizations that have changed the course of history contribute to this myth and the effects that such an idea can have as these present an image of mathematics as immutable and perfect, which may seem unattainable to the anxious student.

In examining students’ reasons for mathematics avoidance, pre-service elementary school teachers have been a subject of great interest as they are among the first to introduce children to mathematics (Hannula, 2005). Thus, attempting to reduce these teachers’ negative feelings towards mathematics and prevent the spread of this negative affect to students is of great importance. Many pre-service teachers exhibit a higher than average mathematics anxiety than other college majors (Hembree, 1990) which makes them an ideal population to find the causes of their mathematics anxiety. Pre-service teachers have developed their own anxiety and mathematics avoidance from their previous negative experiences in school and maintained them well into post-secondary education (Kaasila, 2000) and as a result avoid mathematics classes. For many of these pre-service teachers, a negative self-perception about their abilities or a “tragic backstory”, similar to the plot used in literary tragedies, regarding their past experiences in mathematics maintains an overall negative view of mathematics (Kaasila, Hannula & Laine, 2012). These stories surrounding these pre-service teachers have become a part of their identity as well, ensuring mathematics avoidance and anxiety is also a part of their personalities. Maintenance of these negative stories and perceptions of their own abilities comes from the perpetuation of the different “math myths” (Tobias, 1990). These myths are similar to those already mentioned by Tobias of the ability to do mathematics as an innate ability and encompass things such as the notions that higher level mathematics is too difficult for normal people and that you can live life perfectly fine without any mathematics. With these types of engrained myths, and the negative stories that students create about themselves with these types of mathematical myths in mind, it is small wonder that mathematics anxiety is maintained as strongly as it is in many people.

The types of mathematics avoidance can be broken down into two categories, “Global” and “Local” avoidance (Ashcraft & Faust, 1994). A global-avoidance of

mathematics entails a complete avoidance of mathematics and mathematical contexts due to the aggregation of the subject's negative experiences in mathematics. A person who globally avoids mathematics will avoid areas of study that might have mathematics integrated e.g. engineering or physics degrees. People who are globally avoiding mathematics will also be less fluent and knowledgeable about mathematics, and as a result not develop any strategies within the "mathematical domain" (Ashcraft & Faust, 1994). Local avoidance of mathematics is a response to the stress of doing mathematics in the moment. Rushing a mathematical task as to be disposed of it sooner is a local avoidance strategy and a manifestation of mathematics anxiety. During local-avoidance the subject tries to avoid the distressing stimuli of solving mathematics problems by answering the question as quickly as possible. This causes a large number of errors in the task due to this rushing. The student exhibiting local avoidance is attempting to expend minimal cognitive resources as to also avoid the distressing stimuli of doing mathematics (Ashcraft & Faust, 1994). These types of avoidance speak loudly of the beliefs that subjects hold regarding their abilities in mathematics.

Belief about self and capability strongly maintains ones views on mathematics (Hannula et al., 2005). These beliefs become engrained through the daily procedures of mathematics class, such as emphasis on timed tests, right answers and ambiguous language (Tobias, 1990). Mathematics anxious students who do not experience success under these types of conditions and environments begin to form negative opinions about school mathematics that are not changed if the conditions are not changed. This causes students to avoid higher level mathematics classes as they will not expect to do better later on as they have already found that they are not routinely successful in these activities. These beliefs may also be formed by the societal expectations of either gender or class (Tobias, 1990; Evans 2000) in which people are told by greater society that they are simply not good at mathematics as a result of their gender or among the lower social-economic class. This leads to a belief that that they had no control over their own mathematical destiny and thus this belief perpetuates itself and maintains a level of mathematics anxiety and avoidance.

2.1.3. Diagnostics for Student Anxiety

Since symptoms and maintenance of students' anxiety are so powerful as to become part of a student's identity (Kaasila et al., 2006) then it stands to reason that there must

be some tool or tools that can act as a reliable diagnostic for sources of students' anxiety towards mathematics. It appears from the literature that there have been many different diagnostics created that help with this effort.

It is always an effort to quantitatively measure how much anxiety a student feels regarding mathematics. These efforts have led to two widely regarded surveys or questionnaires that place a number on the severity of these feelings. The Mathematics Anxiety Scale or MAS (Fennema & Sherman, 1976) and Mathematics Anxiety Rating Scale or MARS (Richardson & Suinn, 1972) are two such surveys that have been developed to help diagnose students' anxieties towards mathematics. The MAS was developed with nine domain-specific, "Likert-type" scales that measure students' attitudes towards items where the student will rate statements such as "I enjoy doing mathematics" with either a "strongly agree", "agree", "neither agree or disagree", "disagree" and "strongly disagree". This gives insight into the student's perception of one of nine domains depending on the question type. Two relevant examples of the nine domains of the MAS are regarding statements about the mathematics teachers and how their teacher views them as learners and also a mathematics anxiety scale. These are only two of the nine domains; there are seven others that have to do with motivation, gender bias and parental support. The MAS measures "feelings of anxiety, dread and nervousness associated with bodily symptoms" and was validated at US high schools. As part of the MAS, the nine domains highlight gender differences, such that comparisons using the MAS will produce gender differences in anxiety when measured (Evans, 2000). The Mathematics Anxiety Rating Scale was originally a 98-item questionnaire that was composed of "brief descriptions of behavioral situations" (Richardson & Suinn, 1972). An example of such a situation would be "Walking into a math class". A student would then be asked to rate how anxious this made them feel on a Likert type scale, similar to the MAS. This would range from a 1-not at all, 2-a little, 3-a fair amount, 4-much and 5-very much. The MARS was validated with large groups of female education students at a university in Missouri. The question of which one is a better measure of students anxiety is answered from an analysis of the two's traits and analysis. Since the MARS specifically measures anxiety and is less likely to produce gender differences by design it would seem that this is a more ideal measure. Its drawback is that it lists situations that are specific to college mathematics; however it does so across mathematical contexts, which appears to make it a more appropriate

scale to use if one is going to measure the variation in anxiety across different mathematical situations (Evans, 2000).

The issue with the use of the 98-item MARS and its seeming focus on college-level mathematics and the fact that it was originally developed in 1972 is dealt with the identification of key categories of situations. Reduction of the number of items while also maintaining the validity of the survey as a tool for measuring anxiety is also important as the validity of the original MARS had been questioned a number of times (Hopko, 2003). The identification of categories of items or scenarios and then reducing the MARS to less items (such as 24 or 12) that deal just as effectively with rating the anxiety surrounding that particular mathematical context has been the focus for a number of years and has helped update the MARS and allowed the creation of a reduced-item Mathematics Anxiety Rating Scale-Revised or MARS-R. This MARS-R has been confirmed to have valid internal consistency and correlates well with the full 98-item MARS and other anxiety measures (Hopko, 2003). This MARS-R may be the ideal measure for students younger than college age due to its brevity and confirmed validity and usefulness in rating mathematics anxiety across different contexts.

As stated before, the use of *Math Autobiographies* has been used to help students begin to alleviate their negative affect towards the subject by getting them to evaluate their reasons for feeling the way they do. This makes the use of *Math Autobiographies* a good diagnostic tool to examine students' origins of anxiety and provide qualitative data in regards to the affective domain. Shelia Tobias (1990) describes in her book "Overcoming Math Anxiety" the use of *Math Autobiographies* in mathematics anxiety clinics in order to gauge the students' experiences with mathematics and begin to describe the symptoms (as discussed above) of students' anxiety. This is the start to providing tools in which a teacher might diagnose the root causes of anxiety for a student and begin to provide some form of mathematical therapy, as we will discuss in the next section. *Math Autobiographies* can provide a window into the identities of the mathematics anxious learner by having themselves cast as the protagonist and describe the way in which the student had been taught, their experiences in learning and the way they dealt with or were supported in their mathematics learning (Kaasila, 2006). Writing a *Math Autobiography* begins to help the teacher and the student know the "Mathematics Identity" of a person and includes how the anxious mathematics learner interacts with mathematics both inside and outside of

school (McCulloch et al., 2013). There are two kinds of “Identities” identified within a *Math Autobiography* in a study conducted on pre-service elementary school teachers by McCulloch et al. (2013). The “Actual Identity” is one in which stories detail their real experiences doing mathematics. For example, a story regarding a time when a student had failed a test they hadn’t expected to, and how that made them feel is part of their “Actual identity”. The “Designated identity” is where the student or author of the autobiography has “designated” their ability of themselves or their future expectations of mathematics e.g. “I don’t have a math mind and I will never be good at math” is an example of a designated identity. This can provide valuable insight into how the student perceives themselves and allow the teacher to know what areas a student or students may feel most anxious in mathematics and how better to support these students (McCulloch et al., 2013).

A study undertook in South Africa by Hobden and Mitchell (2011) on the use of *Math Autobiographies* and how they describe students’ relationship to mathematics and anxiety created a coded system that broke down a number of common themes that appeared in students’ autobiographies. These themes were broken down into 4 categories and provided qualitative data on how the students felt regarding mathematics, themselves and their relationships to the subject. This study found that many students who were the most mathematics anxious learners also listed negative experiences and highlighted the role of memory in how students view mathematics (Hobden & Mitchell 2013). Thus, it appears that *Mathematics Autobiographies* can provide an excellent qualitative diagnostic tool for identifying the origins of student anxiety, helping students to begin a reflective process of examining why their feelings towards mathematics are as they are and allowing the teacher insight into the background of the students they teach mathematics to.

2.2. Mitigating Mathematics Anxiety

With tools such as the MARS-R to provide a quantitative measure of the intensity of anxiety felt surrounding common situations in mathematics and the *Math Autobiography* to provide a qualitative insight into the origins of the negative affect that many students exhibit, it becomes all the more salient to wish to examine methods in which these negative memories and feelings can be mitigated and possibly relieved while maintaining the necessary rigor of learning mathematics.

2.2.1. Therapeutic Approach to Negative Mathematics Memories

There are many methods and suggestions within the literature that prescribe some method of promoting anxiety relief for mathematics. Sheila Tobias (1990) suggests specialized mathematics clinics that the mathematics anxious attend in order to address both the academic and emotional side of mathematics anxiety. The purpose here is to possibly create the first "positive" mathematical experience for these mathematics anxious students. These clinics must focus on parts of mathematics other than simply testing as mathematics anxiety is more than just a form of test anxiety (Hembree, 1990). Thus it falls to more of dealing with the socio-emotional side of mathematics as well.

Research has been done into the usage of *Math Autobiographies* as a form of "Math Therapy" to allow students to examine their own and other's anxieties surrounding the learning of mathematics. One type of method of attempting to assuage prospective students' closely held negative experiences in mathematics is through the use of "Narrative rehabilitation" (Kaasila, 2004 pg.52) in which students share their experiences in mathematics in a small group setting. This is meant to have students reflect on their experiences and hopefully demonstrate to the student that their interpretation of past events might not be totally correct and potentially have them "search new aspects into their mathematical past and future" (Kaasila, 2004 pg.53). Similarly, an approach called "Bibliotherapy" in which students read other past or current students' mathematical autobiographies is meant to produce an affective change by having the reader more closely relate to the struggles of that person by identifying their struggles in a different narrative. The result is hopefully to have the reader gain a new perspective of their experiences and "gain hope" surrounding their learning of mathematics. This form of empathy is meant to help the student or anxious mathematics learner not feel alone in their individual struggles with mathematics and thus help the student to re-interpret their previous negative experiences and hopefully move the student away from negative behaviors and beliefs that are affecting their current feelings towards mathematics (Kaasila, 2002). A study into the use of bibliotherapy identified that the largest change in students, as identified from their writing, was the identification that others are facing the same issues as them and was cathartic. The author of this study cautions that the application of bibliotherapy is limited due to the fact that two students might have had extremely different experiences than one another in their history with mathematics. Thus, a student who had always struggled with mathematics, in reading an

autobiography where the students only talks of their love and ease with the matter might have the opposite than intended effect and cause a reinforcement of the struggling student's negative identity. However, the premise of the bibliotherapy of having students identify with each other in mathematics struggles and thus learn to empathize not only with their fellow students, but also their own experiences and gain a different, more positive perspective on such struggles, is a positive one. Having students realize that they "are in the same boat" (Kaasila, 2002) as everyone else when approaching mathematics seems to be able to have a therapeutic effect on the negative memories that some students may have.

Similar approaches towards the handling of negative memories from school time to assuage anxiety have been suggested as well, but have the common theme of promoting a more social atmosphere than the normal individualistic environment found in a typical mathematics class. In a study of how pre-service elementary teachers changed over the course of a mathematics education course provides more insight into the role that negative memories play in the attitudes held regarding mathematics and how these memories may be mitigated. It demonstrates how a reflective approach to teaching mathematics, bibliotherapy and working with other students were central facilitators for change in the views of four pre-service elementary school teachers (Kaasila et al., 2006). This study identified that the central facilitators for change were in: the handling and reflecting of experiences learning mathematics, exploring with concrete materials (such as manipulatives) and working in pairs. This study concludes that it is difficult to change students' perceptions of themselves and mathematics but that "handling recollections of past experiences" through the use of reflective practices such as bibliotherapy can provide some affective change. Additionally, exploring mathematics more deeply with concrete materials allowed the pre-service teachers to become more familiar with mathematics and potentially "feel better" about them. Also emphasized is the group work or paired learning aspect, which heavily influenced the views of these teachers of themselves as mathematics learners (Kaasila et al., 2006). This move is credited with having the pre-service teachers change from an "ego-defensive" mode, in which they are expecting to do poorly and are continually "bracing" themselves for bad news, to a "socio-dependant" mode in which the students are now needing more help, but have an expectation of "positive emotion" from teacher feedback. This move, while not the most ideal solution, is stated by the authors as an important step towards the

ideal mode of getting the students to become “task-oriented” in which students are willing to challenge themselves and are curious regarding the nature of the mathematical task presented without the need for external validation.

Kaasila et al. (2006) make an interesting conclusion in considering the difficulty of changing student negative views from ego-defensive or socio-dependent to ultimately task-oriented) of mathematics using only the methods of reflective writing to deal with their negative memories. They ponder the effect that could potentially be had if students were presented positive experiences in mathematics in the form of a type of mathematical discovery known as the “AHA!” experience (Liljedahl, 2004). The authors posit the interest of such a study in moving the students towards “task-orientation” and how effective it might be for relieving the anxiety students feel and allowing the students to see “the beauty of mathematics”. This promotion of positive experiences in mathematics is thought to be a therapeutic approach to the relief of mathematics anxiety as well (Hannula et al., 2007). In this way, it may assist in the repair that negative experiences and memories have on the mathematics anxious learner, however it is thought that such a method of providing positive experiences must be coupled with some type of reflection, as reflection will result in “new comprehensions”, beliefs and attitudes (Hannula et al., 2007). The providing of positive experiences in mathematics is the subject of the next sub-section.

2.2.2. Providing Positive Experiences in Mathematics

Having discussed the literature of how handling negative memories regarding students’ negative mathematical experiences is powerful for the relief of mathematics anxiety, it becomes essential for a review of the literature to determine ways to provide students positive experiences in mathematics and thus change their beliefs regarding not only mathematics, but of themselves as mathematical learners. The question of how one provides positive experiences in mathematics is not easy to answer. Regardless of how mathematics is taught, some students may bring negative memories and experiences that may compound or bias them to future mathematics learning, as we have already reviewed. The simplest method of starting out a mathematics class correctly is for the teacher to be a positive presence in the room. It was found that simply greeting students at the door at the start of class by name increased students’ on task behavior by a large margin (Alday & Pakurar, 2007). This is however part of an interpersonal experience,

that we will discuss in the next section regarding the effects of providing caring mathematics instruction.

It becomes important to provide students with positive mathematical experiences, and not simply just interpersonal ones, although the interpersonal can never be removed from the equation as this isolation that students feel may be a major source of anxiety felt in the mathematics class (Kaasila, 2002). Consequently, this brings us to the conclusion of the study by Hannula et al. (2007) in their regards to the effect of providing students with opportunities for mathematical discoveries called “AHA!” experiences. These were outlined in an unpublished dissertation by Dr. Peter Liljedahl (2004). These “AHA!” experiences are sudden moments of mathematical illumination that appear after much work on a particular problem. Since mathematics at its core is fundamentally about problem solving (Liljedahl, 2004), then it stands to reason that incorporating or facilitating “AHA!” experiences can be extremely beneficial to the affective domain of anxious mathematics learners (Liljedahl, 2005). These “AHA!” experiences can form the foundation of positive experiences that may help to alleviate students’ mathematical anxiety and potentially change their self-image surrounding mathematics. In a study of both pre-service secondary school mathematics teachers and elementary school teachers, both groups were enrolled in a mathematics course and were given problems verbally, rather than having been written down with detailed instructions. These teacher-students would then work in groups on these problems with the emphasis on talking and “playing with the problem” rather than on note taking. Students would then reflect on their experiences, especially any particular moments that could have been defined as “AHA!” moments. The tasks that were given out as the problems were classified as “Good Problems”. These would be defined as problems that are accessible, can be approached a multitude of ways, introduce important mathematical ideas and lead to rich exploration which leads to more “Good Problems” (Liljedahl, 2004). From anecdotes provided by the students in the study, it appears that peer interaction was listed as essential to problem solving. The results of analyzing the journals were that 26 out of 34 secondary students mentioned having an “AHA!” moment, and 36 out of 38 among the elementary students. It appears that the environment and problem type utilized in Liljedahl’s study is ideal for increasing the likelihood or “invoking” of an “AHA!” experience and creating fond memories of mathematics and mathematical problems.

These memories have the potential to become central to the mathematical identities of students and form the basis for their future views concerning mathematics.

Given that it seems that group work on rich mathematical tasks can “invoke” the “AHA!” experience and how upon reflection, students cited that it can be transformative, it would appear that the “AHA!” experience is very different from any other type of mathematical experience (Liljedahl, 2004). In a study on the impact of the “AHA!” experience on the affective domain of students on pre-service elementary school teachers it was found that there exists a link between the occurrence of these “AHA!” experiences on “changing the beliefs and attitudes of resistant mathematics learners” (Liljedahl, 2005 pg.220). The role of positive emotion in changing these students’ beliefs was also a prominent feature of this study. The pre-service elementary school teachers had deemed themselves to be mathematics incapable and/or phobic prior to this study, and were only taking the course in question to satisfy their mathematics requirements. These pre-service teachers had already established a pattern of “Global Avoidance” (Ashcraft & Faust, 1994) as discussed earlier and could be qualified as “resistant” mathematics learners. These students were given assignments which they were expected to complete in groups of 3-5 and wrote a final reflective piece in which they would discuss an “AHA!” experience. There were two groups of “AHA!” that needed to be defined; those of a mathematical discovery (and therefore of interest) and those of teaching “AHA!” where they realized how better to teach a concept. Students who listed an “AHA!” experience almost always described a sense of accomplishment, which is a positive change to note in mathematics. Liljedahl (2005) mentions that other literature describes that short term change is prone to relapse into old negative ways of thinking about mathematics and for significant change, there must be sustained and “successive success”. However, Liljedahl (2005) challenges this notion, highlighting the data of the effect that the positive emotion of a single “AHA!” experience had on subjects in the aforementioned study. His rationale is that the experience of “AHA!” is much more powerful than other emotions created in other types of problem solving. This makes the fostering and creation of mathematical discoveries or “AHA!” moments powerful changers of students’ beliefs regarding themselves and thus need to be included in mathematical contexts if any alleviation of students’ mathematics anxiety is to be achieved.

Liljedahl (2005) ends his discussion of the study citing what factors appear necessary to increase the chances of these powerful and illuminating “AHA!” experiences occurring. Increasing the chance of an “AHA!” moment occurring is centered around placing emphasis on talking rather than details e.g. group work and discussion and orally delivering problems. This peer interaction requires a great deal of time, thus more class time is needed to dedicate to the group problem solving process. Removing the teacher from the conversation as the influence of the teacher “changes the nature of the conversation” (Liljedahl, 2005). Finally, Liljedahl states that increasing the amount of information “in the air” but not explicitly linked together increased the chances for an “AHA!” to occur. This seems to point to a potential therapeutic method of alleviating students’ mathematics anxiety, in which group work on rich mathematical tasks where information is freely being passed around can act as a potential mathematics intervention that much of the literature discusses as necessary for the relief of mathematics anxiety. It also bears close resemblance to the “Math Clinics” that Sheila Tobias discusses in “Overcoming Math Anxiety”. After a review of the literature, these “AHA!” experiences are potentially the best and most accessible means to providing the mathematics anxious learner positive experiences in which to change their beliefs about their own abilities and that of the nature of mathematics itself.

2.2.3. Caring Instruction and its Role in Mathematics Anxiety

In the theme of providing positive experiences in mathematics, the role of the teacher cannot be understated. Whereas the provision of occasioning “AHA!” experiences requires the peer-interaction, and rich mathematical tasks, the literature states that there is much more that the teacher can do and foster in terms of providing students with anxiety relief and positive experiences within the social context of a mathematics classroom.

Some students’ most negative memories of mathematics are of their teachers or school related experiences. In the paper by Hobden and Mitchell (2011), their coding of mathematics autobiography themes demonstrated a strong trend for the memory of their past mathematics teachers. These teacher interactions featured prominently under the category of environmental factors. When analyzed, it appears that negative experiences with their past mathematics teachers mention that either the teachers were not knowledgeable in their areas, knew too much regarding the subject and couldn’t relate it

to students, or their unprofessional behavior in which significantly negative incidents, such as humiliating students for not knowing the answer, were significant themes when the teacher was mentioned in the *Math Autobiographies*. Whenever the teacher was viewed as an authoritarian through their behavior, it appears that it disempowered students (Hobden & Mitchell, 2011). These memories of past negative teacher interactions and apparent "Learner dissatisfaction" seemed a direct contributor to the mathematics anxiety felt by the pre-service elementary school teachers and their "dis-enchantment" with mathematics. This certainly aligns with what Tobias (1990) mentions in her book when discussing the gender differences in mathematics achievement and enrollment at higher levels. With young girls being told by their teachers about certain mathematics myths such as the natural inability for women to do mathematics or the "math mind", the result is a dis-enchantment that causes an avoidance of mathematics. When a student is faced with a hostile learning environment, where teachers are perceived as dis-interested or left to "struggle-alone", the result always seems to be an increase in mathematics anxiety and the students to "switch-off" in their interest (Rooney, 1998).

From the literature, it appears that the interaction with the mathematics teacher is extremely important to the young mathematics learner. Thus, it appears that caring instruction in mathematics is something that ought to be considered when attempting to lower the anxiety of students and prevent a student from "switching-off" (Rooney, 1998) in their interest in mathematics. This can be accomplished in a number of ways. Steele and Arth (1998) suggest that providing positive instruction and assessment is key to convincing students that they can do mathematics, and change their beliefs. We covered a method of providing such an environment in the last section of which they seem to parallel in their paper on lowering anxiety in the mathematics curriculum, but the question of what else can be done to provide this positive instruction remains.

This brings to the issue of what "Caring Mathematics Instruction" (Jansen & Bartell, 2013) looks like and the role it has in forming students' perspectives surrounding mathematics. It was found that when "at-risk" students (those students who were at a higher than average risk of dropping out of school) perceived that their teachers cared about their instruction, would put more effort into their school work than they would have otherwise (Muller, 2001). This seems to parallel a study conducted on affecting the on-task behavior of students by simply greeting them by name at the door of the classroom

as they entered (Allday & Pakurar, 2007). In this study, greeting students increased the occurrence of on-task behavior from 45% to 75%. This seems to provide some evidence that when a student perceives the teacher as engaged or caring, that it can have a "discriminative stimulus" that shows the teacher's attention is on the student (Allday & Pakurar, 2007).

In studying what students perceive as caring instruction, Jansen and Bartell (2013) label two types. There is when teachers provide "academic care", which is attention to the technical aspects of teaching and learning. In academic care a "shared intellectual space" is cultivated and where it is apparent that teachers "work to share their understanding of a concept" with students. Then there is "interpersonal care", where the teacher demonstrates that they care about the feelings and perspectives of the student and acknowledge those feelings. These two types of provided care could be broken down into 4 dimensions that students felt were the most important: 1. Teach such that every student's learning matters, 2. Communicate high expectations, 3. Create a welcoming and inviting classroom community and 4. Engage students in learning mathematics. In the case of teaching such that every student's learning matters, this can be difficult with 30 students, but this concept is further broken down by the authors of this study which is to "reach every student", thus accomplishing the need that the teacher is responsive to the needs of individual students as this communicates to each student that they are "worth the teachers time" (Jansen & Bartell, 2013). Communicating high expectations is similar to what was discussed previously in providing positive experiences in mathematics where the "AHA!" experience (Liljedahl, 2005) is cultivated can provide challenging and positive learning experiences. High expectations are viewed by students as a form of caring as it means to the student that the teacher has not given up on them and communicates the idea that the teacher still believes in their students. Supplementary to this notion of high expectations, students view having multiple chances to redo work as essential to this idea as they note that they wish to "learn from their mistakes". In creating a welcoming classroom, it is very much similar to the phenomena of welcoming students at the door first thing in the morning, that if students feel that they are in a safe place to ask questions without summary judgement of the question itself, and then they feel that they can take more intellectual risks (Jansen & Bartell, 2013). This includes the perception of teachers caring about their students as people, and not just as students. Finally, engaging students in learning mathematics is a

key perception. Teachers must be engaging and use engaging instructional methods. The types they are similar to previously discussed methods of providing positive learning experiences in mathematics (Liljedahl, 2004, 2005) in group work and “good problems”. There is a caveat, however, that providing only caring instruction is not effective mathematics teaching (Jansen & Bartell, 2013). The students must perceive that they are being taught the material effectively and clearly see that the teacher has a mastery over the material and teaching it in order for the academic care to be effective.

2.3. Research Questions

It seems fairly clear now that students’ sources of anxiety are their previous negative experiences in mathematics (Evans, 2000) which have come to define their identities in mathematics (Kaasila et al., 2006). This creates in their minds self-defeating notions that only reinforce themselves. While there have been studies centered around the reduction or amelioration of anxious feelings in adults (most notably pre-service teachers), with findings that these adults formed their negative perceptions of mathematics early in their school lives, there seems to be sparse literature regarding the effect of relieving anxiety in younger students. Studies into effective methods of ameliorating mathematics anxiety in younger students would be valuable as it could inform better instructional practices of mathematics as mathematics anxiety is a major obstacle for many students.

A study that had been conducted on elementary school children in an attempt to relieve their mathematics anxiety was on a one-on-one “cognitive tutoring” with a teacher for eight weeks (Supekar et al., 2015). To measure the effect of this cognitive tutoring a specialized computer based mathematics program had students perform arithmetic problems and measured their competency and speed at these tasks. During this time the subject student’s brain activity was measured with a focus on the “stress” response. This study found that the intensive cognitive tutoring reduced the activity in the children’s amygdala, and thus their stress response more than the control group which had not undergone one-on-one cognitive tutoring. The authors of this study credit the reduced anxiety response to repeated exposure to the stressful stimuli of what the subject children feared most, “normalizing” their brain patterns.

One-on-one tutoring is not always an option for all students, or the teacher due to the realities of the classroom. Brainwave activity measurement tools are also not readily

available to teachers in order to measure the supposed decrease in students' stress responses. Additionally, this study also does not address the affective domain of these students, nor their attitude towards the subject of mathematics. Nor does it address what reasons these young children may have had for their "aberrant" brain scans prior to their tutoring. Thus is behooves us as educators to find a more practical method of measuring and ameliorating mathematics anxiety.

From the literature there appears to be a gap in the research of mathematics anxiety in implementing a classroom based approach to relieving students' mathematics anxiety at an early stage. It is clear from Liljedahl (2005) that the positive affect of the "AHA!" moment can provide potentially powerful relief for mathematics anxiety. It is also clear from the literature that helping students gain positive memories surrounding mathematics and changing their beliefs surrounding their abilities is also of paramount importance when trying to alleviate students' mathematics anxiety. It seems that a model of teaching called "The Thinking Classroom" (Liljedahl, 2016) is very effective at encompassing all the discussed methods of anxiety relief. A "Thinking Classroom" is a space that is conducive to thinking, occasions thinking and is inhabited with thinking individuals, and it may be the best method for having students experience positive mathematical experiences and dealing with student anxiety towards mathematics. The "Thinking Classroom" model as outlined by Liljedahl (2016) entails students standing and working in randomized groups at large vertical non-permanent surfaces (such as whiteboards). The problems or tasks are delivered verbally by the teacher and are defined as "Good Problems" (Liljedahl, 2005). The work the students do at the boards is managed in both information and difficulty called "Flow" by the teacher. From observations of this *Thinking Classroom* model, students are more eager to start working on mathematics problems, actively work on the mathematics problem, discuss ideas and participate in more intra-group collaboration. All teachers who participated in this study mentioned how much improved the "classroom-community" was by the implementation of this *Thinking Classroom*.

Through a review of the relevant literature it appears that providing a safe and welcoming environment for students in mathematics class, positive mathematical experiences like an "AHA!" experience, reflection on past experiences and caring instruction are all extremely important factors in reducing students' mathematics anxiety. The *Thinking Classroom* model incorporates many of these factors, and has been

demonstrated to increase the levels of community and student engagement in mathematics classrooms; this leads me to my research question:

What effect will there be on the students' mathematics anxiety and their perceptions of mathematics when the 'Thinking Classroom' model is implemented in a classroom full of anxious mathematics learners in grade 8 and 9?

Chapter 3. Methodology

In order to explore the research question from the previous chapter, I collected qualitative and quantitative data through collecting assignments, administered surveys and interviews in the aforementioned *Imagine* program for anxious youths. To best describe the methodology in my research, I will first detail aspects of the program prior to my involvement and then changes in how the program was run after I began my involvement. I then move on to describe typical features of the *Imagine* program and the implementation of the *Thinking Classroom* intervention I utilized to reduce my students' mathematics anxiety. Finally, I list the descriptions of the students in the *Imagine* program and the tools and timeline I used to identify and assess the students' mathematics anxiety.

3.1. Setting: Blended Learning in the *Imagine* Program

The setting for this research was in a program called *Imagine* which was hosted by Fraser Valley Distance Education School (FVDES), which is one of the largest public distance education and online schools in British Columbia. The *Imagine* program had been running for two years prior to my involvement. It had the supervision of two teachers of mixed disciplines to help students achieve success in their online studies by supporting their learning and keeping them up to date. The program was intended to utilize a "blended-learning" approach in which students would be enrolled in all the core online courses (science, mathematics, socials and english) but came onto site two mornings a week for extra support with their online studies. The blended/online philosophy of instruction was to make available all units and lessons at the start of the students' enrollment in the course, and allowed students to work at their own pace throughout the year, with the ultimate deadline being the end of the school year. For some students, this system worked well; for others, a program like *Imagine* provided some structure to help move students forward in their studies at a regular pace but provided flexibility to their instruction that might not be have been present in a regular school. The primary intention of *Imagine* was the successful re-integration of students back into the regular school system with skills and strategies for coping with the academic and social pressures that they would face on a daily basis in a regular school.

The year that I became involved in the program, I undertook some reorganizing of how the program was to be run with my co-teacher. My co-teacher and I organized the program and supported each other in lessons that would benefit our students. Most prominent of these changes undertaken were that students would come in three mornings a week instead of two for periods of time of three and a half hours. During the students' on-site time, portions of it would be allotted each day to a specific focus on one or more of their core academics.

When students would get into class at our 8:30 am start time, they would invariably log in to their computers and wait for class to begin. FVDES is a school with a large budget dedicated towards technology, so each student in the *Imagine* program could be guaranteed of having a dedicated desktop computer in a computer lab designed specifically for both the *Imagine* program and tutoring time for other students enrolled in other online programs at FVDES. Their online mathematics course comprised the bulk of their instruction and assessment in mathematics 8 and 9. Lessons were typically narrated videos with some digital interactions acting as reinforcement of the concepts taught and online quizzes and tests acting as their summative assessment. As a teacher I had always felt that a well-planned lesson, engaging teaching methods and the integration of technology would be beneficial when combined with a regular type of classroom instruction. I had thought that this method of instruction would be especially effective in assisting students learn essential mathematical skills and reduce anxiousness towards the subject. However, recent studies that I had undertaken at Simon Fraser University had turned my thinking around from the primacy of direct and technology-assisted instruction to a more social and group method of learning mathematics known as the *Thinking Classroom* (Liljedahl, 2016). The potential for having a powerful effect on reducing negative student feelings of anxiety towards the subject of mathematics when approaching mathematics utilizing the *Thinking Classroom* model was something that I was extremely interested in implementing. It was my intention for these students to be placed in a situation that would not only motivate them to interact with their peers, but also to engage more deeply with mathematics through robust tasks in randomized groups which would then be discussed in a group setting. This would in turn provide students with positive memories of success in mathematics (Liljedahl, 2005) which could have a net positive effect on their affective domain and help give students skills that would translate into their routine online mathematics courses.

Fortunately, the geography of the classroom where the *Imagine* program was situated was favourable to have students be away from their computers as the computer lab adjoined a separate classroom that was also dedicated to the use of *Imagine*. This separate room possessed an open floored area with sufficient amounts of whiteboard space for students to do their group work on, and thus was an ideal space to implement the *Thinking Classroom*. Regardless of whichever subject was the primary focus of the day, every morning students in *Imagine* would be randomly sorted into groups and participate in a 45-60 minute session of solving a rich, open-ended mathematics problem.

3.2. Participants

As is the nature of specialized programs, enrollment in the *Imagine* program was variable throughout the year. It was not uncommon for students to be enrolled and then, two months later, find placement in a different school for a variety of reasons. It was also not uncommon for students to be enrolled much later than September. However, the number of students in *Imagine* was maintained around 18 students throughout the year. The composition of *Imagine* was Grade 8 and 9 students of ages 13-16 from various socio-economic backgrounds and skill levels. All students were residents of Chilliwack and students of the Chilliwack School District. The typical students generally referred to the *Imagine* program were those who were identified by their parents or prior teachers as being highly anxious regarding school for a variety of academic, social and emotional reasons**.** This anxiety drove these students out of a traditional school setting and into the *Imagine* program. Given that the focus of the program centered on anxious students, it was a significant goal for me to work on reducing these students' anxiety surrounding what is generally considered one of the most difficult subjects for students, which is mathematics. This is why the first activity every morning was to have students engaged in a rich mathematical task with their peers.

3.3. Setting: The Intervention

To understand better my students' anxiety towards mathematics and help them face their own feelings regarding it, I had to implement a number of diagnostic tools useful in assessing mathematics anxiety. In attempting to help alleviate their anxiety towards

mathematics and improve their affective domain, I had to foster a culture that would help students engage more deeply with mathematics than they had before.

3.3.1. Math Autobiography

To begin a study and reflection of the students' mathematics anxieties, I had the students complete an assignment at the start of the year called the *Math Autobiography*. This was a half to one page written assignment that detailed their experiences and history with mathematics classes. This would give me a sense of their individual histories surrounding mathematics and provided some information about the genesis of their anxieties. It also would provide them with an introspective activity to unpack why they felt the way they did about mathematics. The assignment would count for one part of their online english courses, in which they were doing a unit in composing paragraphs and grammar. The information that students divulged in these assignments provided me with a starting point for each individual's mathematics anxieties and the history behind them. Utilizing these *Math Autobiographies*, I was able to tailor my approach when speaking to my students about mathematics and their general feelings towards the subject.

3.3.2. MARS-R

At the start of the year, or whenever a new student arrived in class, I administered the MARS-R and gave a brief explanation of what it was and why I gave it to them. Students would fill them out with the caveat to be honest when answering. I then collected their responses to the MARS-R and logged the data into an Excel file for easier comparison. I administered the MARS-R twice more, once in January and then again in June, to get a sense of how the students' anxiety was responding to our classroom routine. Each time, students were again asked to be honest in how they filled it out as I would be using it to focus on certain areas in our *Morning Mathematics Problems*.

3.3.3. The 'Morning Mathematics Problem'

At the start of the year, I informed my students that we would be doing a mathematics problem every morning the *Imagine* class met. This problem ran for the first hour of class and this would become known in the class as the *Morning Mathematics Problem* (MMP). Students were randomly sorted into groups of 2-3 students, and then verbally given a

“rich” or “robust” mathematical problem. In this case, many of these questions had been given to me by Dr. Peter Liljedahl, found on his website or issued through the British Columbia Association of Mathematics Teachers (BCAMT) email listserv. These questions had been explored by me and other teachers beforehand in order to help guide students through their own explorations of the mathematics problems. These “robust” tasks could have also been described as being “low-entry” types of problems which students at all skill levels could contribute and participate in finding a solution to. Students would then begin an exploration of the problem on vertical non-permanent surfaces (whiteboards) that were placed around the room and attached to the walls. Essential to the process of students undertaking these problems was the

Figure 1: Example of students in their groups solving problems at the whiteboard

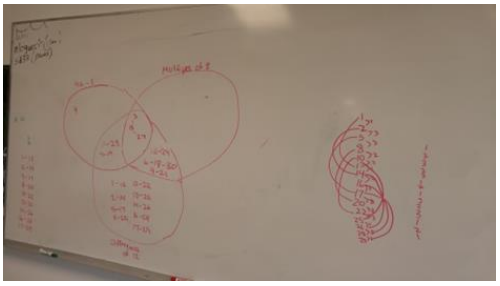

Figure 2: Example of student work on the whiteboard: Question was “The Following pairs of numbers were categorized together [4, 11] and [3, 21] what other numbers fall into these categories?

fostering of group discussion towards a solution. Students were encouraged to talk and discuss with one another the possibilities that the problem presented and ideally come to a common solution as a group. Additionally, I moved around from group to group, ensuring that students were given some guidance or taught a skill to move them along in their thinking and progress in the question. Most often, I changed an aspect of the problem that would make it a more demanding or difficult task, which would be met at first with groans of frustration, but often curiosity at the “new” problem. Occasionally, I made a point of encouraging the students to view their peers work on the whiteboards if they were stuck on a difficult part of the problem or having trouble understanding some of the concepts.

In the process of selecting problems for my students to explore a number of considerations had to be made. Initially, the types of problems that were chosen were those that I felt would be the most interesting to my students and foster an interest in the mathematical concepts inherent in those problems. I also selected problems that would

tie in to some of the curricular competencies found in the British Columbia Mathematics 8 and 9 curriculums. After setting up the culture of the MMP's and utilizing data from the completed MARS-R's, I selected questions on the basis that I felt explored student anxieties that the class seemed to be expressing. Once I had felt that students had sufficiently explored the problem for the day or if time was running short, I called the students in from their explorations and ask each group to explain what they had found. If discussion began to flag during this time, I would then point out groups that I felt had discovered something interesting by highlighting their work on the whiteboards. This was to further facilitate discussion amongst the class as a whole and assisted students in talking with each other regarding their solutions. This discussion or "debrief" generally lasted anywhere from 10-15 minutes, which at the end students were encouraged to write something down about the problem or take a digital photograph of the work done with their phones. This was to help them remember how to solve it later when they would have to return to explore the question more at a later date for an assignment. I would also take pictures of their work left on the boards and post it to the *Imagine* class website. This final part of the routine was to assist students in the completion of an assignment known as a 'Producible' for their mathematics class.

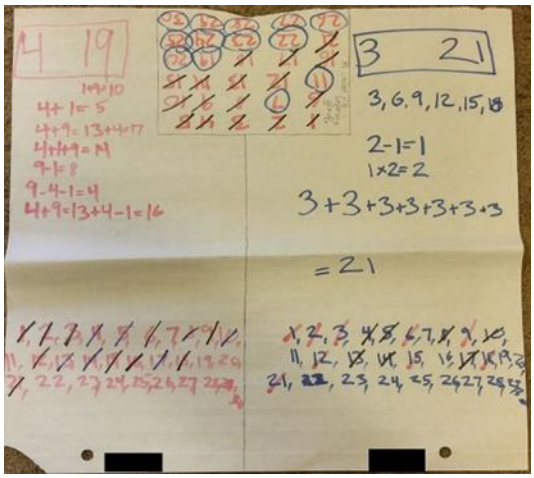

Figure 3: Example of "Placemat" 'Producible' done by two students in Imagine

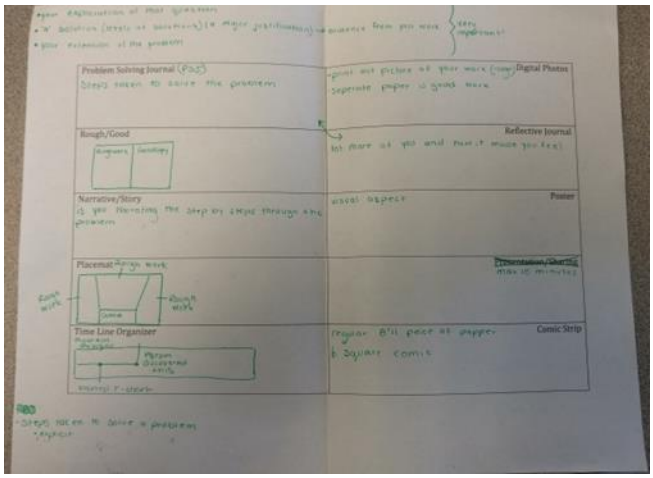

Figure 4: Example of 'Producible' list

Students were tasked with exploring problems that had been done in the MMP sessions in one of a variety of ways called 'Producibles'. A 'Producible' is a written product of a student's efforts and exploration of one of the mathematics problems that were assigned during the MMP. It was meant to make the student explore one of the

questions in much greater detail and depth. It was also meant to help me assess how well they had understood the underlying concepts that had been presented in the question. I had learned about this method of assessing students' learning and understanding of a problem through my Master's program from Dr. Peter Liljedahl. I felt that it would be very successful for such a group of diverse learners, especially in helping to try and alleviate their symptoms of anxiety towards mathematics as they would be able to take control of their own learning and produce tangible evidence of their mathematical success.

These 'Producibles' were presented to the students as opportunities to demonstrate their understanding of the underlying mathematics of one of the questions, but to also to try and encourage them to generate their own extensions of the problem which they could then solve on their own. I typically mentioned that the extensions that they could come up with should be in the same style that I modeled when helping the groups with their MMP. At first, I had attempted to get students to generate these for the sake of their learning, but after multiple suggestions from students, I then agreed to make them worth an omission of some other minor assignment from their online mathematics courses.

Close to the end of the school year, certain students were selected for interviews based on changes observed in the MARS-R's, attitude in class, and willingness to participate in the MMP's. This is covered in more detail in the Data and Analysis section.

3.4. Data and Analysis

The data that I collected through this study would be a mix of both qualitative and quantitative. Analysis and comparison of this data should generate some common themes that would help my understanding of my students' mathematics anxiety.

3.4.1. Data: Math Autobiography

To start any conversation into the level of anxiety with students towards their feelings regarding mathematics, it was essential to get a sense of the history of the student and their experiences with mathematics. As I mentioned in chapter two, *Math Autobiographies* offer a "mapping of the terrain" (Tobias, 1990) of students' feelings

towards the subject and can be powerful tools for helping students with introspection towards their feelings about mathematics (Kaasila, 2006). Having just met my students, I needed to know how they viewed mathematics at the very start of the year, thus the *Math Autobiography* was an essential start to my intervention in reducing these students' potentially negative feelings, such as anxiety, towards mathematics. It was presented as a cross-curricular project between their mathematics course and english courses. They were given credit for the composition in their english course and it provided me with insight into how these students felt towards mathematics and some of the events in their past that might explain these feelings. Students were given the outline (**see Appendix A. Math Autobiography Outline**) and asked to "tell me the history of you and math".

3.4.2. Math Autobiography Analysis

Analysis of the *Math Autobiographies* needed to be handled with care as sensitive information could have been presented in these compositions. I opted to analyze these in the same vein as a study using *Math Autobiographies* done in South Africa (Hobden & Mitchell, 2011) and look for significant events that these students reported that would have been influential in their current outlook on mathematics. Further analysis of the *Math Autobiographies* would be done when trying to draw parallels between the students and their initial MARS-R surveys.

3.4.3. Data: Math Anxiety Rating Scale-Revised (MARS-R)

To get a sense as to whether my intervention and efforts were having an effect into the students' affective domain, it became essential to utilize a diagnostic tool such as the Mathematics Anxiety Rating Scale (MARS) in order to evaluate both my own practices during this study and to measure the relative effect that my methods were having on the students. As pointed out in chapter two, the MARS is an effective measure of the variation in students' affective domain across mathematical contexts (Evans, 2000). Since the population of students that I was dealing with were grade 8 and 9 students, it was necessary to reduce the number of items from the original MARS. Students were instead given the MARS-R, which reduced the 98 category items on the original MARS to only 15. In each of these categories students would be given a likely mathematical scenario (e.g. thinking of an upcoming mathematics test the day before) and then asked to rate their anxiety of the scenario from 1 to 5 when given the prompt of "When I am … I

feel…". 1 being no anxiety and 5 being a large amount of anxiety. This MARS-R has been demonstrated to have similar efficacy in assessing anxiety in subjects (**see Appendix B. MARS-R**) as the MARS (Hopko, 2003); this meant it was an excellent tool for assessing my students' anxieties as they pertained to mathematics. I gave the MARS-R to my students once in September (or their first day in class), once in January, after the winter break, and then a final time in June. The final MARS-R in June was modified slightly to include a 16th category (**see Appendix C. MARS-R year-end**) that I placed in at the suggestion of my program supervisor Dr. Peter Liljedahl. The new category was "Listening to or solving a word problem". This was done in order to gauge the students' relative anxiety surrounding the types of open-ended problems that we had done throughout the school year.

3.4.4. MARS-R Analysis

Analysis of the MARS-R would be made on a quantitative basis both on a global (total individual score) basis and a by-category score basis. The maximum possible score for an individual's MARS-R is 75 and the minimum is 15. A score of around 50 and above would be considered a moderately high score in this study and indicate that the student is highly anxious in relation to mathematics. Anything below a score of 30 would demonstrate a low level of anxiety towards mathematics. These global scores would be compared at the middle and end of the year to the initial September MARS-R score. This was done to determine whether the students were exhibiting any changes in their relative anxiety over the course of the year. Since the global scores could mask any changes that students might have had in the individual MARS-R categories, an analysis of the students' changes in each category was also prudent as a student might have experienced an increase in one category, but a decrease in another, which would be masked by the total score, but revealed by a per-category comparison. I would take the data from the MARS-R scores in September and January and then utilize this data to choose problems based on what the class as a whole felt anxious about. For example if the class felt that interpreting algebraic statements was an area of significant anxiety, I would try and choose open-ended tasks where one of the goals would be to derive a generalized mathematical expression to explain the mathematical phenomena. This was an attempt to help reduce the specific anxiety category toward mathematics and hopefully lower their overall global score by the June assessment. The final MARS-R

survey given in June would determine which students I would interview based on whether the students' anxiety increased, decreased, or stayed the same. I would interview these students to get a sense of how their perceptions had changed in mathematics during the course of our year.

3.4.5. Data: Interview

Interviews would be essential to provide an endpoint to this study on the efficacy of this particular intervention. It also provided good points of contrast to the *Math Autobiography* students had written at the start of the year. This comparison helped identify what had changed for the students over the course of the year having done an open-ended mathematics problem every morning on vertical non-permanent surfaces. It also provided qualitative data as to whether or not the intervention was successful in reducing students' mathematics anxiety and whether or not it had provided any skills that would translate into their regular mathematics courses. All interviews were audio-recorded in a semi-structured interview format. Students were given a list of the eight interview questions that we would discuss (**see Appendix D. Interview Questions**). The questions in the interview could be broken down into two categories; the first four were general questions regarding how they felt they had changed over the year which would allow the answers to the more specific questions emerge more naturally, and then the last four were more direct questions asking specifics regarding why they felt that had changed and what the main causes of that change might have been. If there was an interesting response that was not strictly based on the questions, I would attempt to follow it up with other probing questions in order to elicit more data regarding the students' change (or lack thereof) in the interview. Selection of my students for interviews were based on how the individual student's June MARS-R's compared to their initial September MARS-R's. The three categories that I looked for when choosing my students for interviews were those who exhibited one of the following three changes: I would choose at least two students who exhibited no change in their anxiety as indicated by their global MARS-R scores, at least two who exhibited a decrease in their global MARS-R's scores, and at least two students who exhibited an increase in their global MARS-R scores. By the end of the selection, nine students were chosen based on these three categories. Additionally, I selected certain subjects whom I had identified as having

possibly unique or strong opinions on the subject of mathematics, the MMP and their experiences over the year.

3.4.6. Interview Analysis

Identifying themes was an essential part of working with accounts of incidences that by focusing on them we can "enter a specific state of inquiry" (Mason, 2004) that tries to tell the whole story without expectation getting in the way of analysis. I transcribed the interviews in their entirety and then read through the transcriptions multiple times looking for themes that the students may have had in common and were relevant to my research questions. Through the reading of these transcriptions, I would highlight prominent features that students mentioned had affected them and explanatory notes in the margins. My aim was to ascertain information that would be specific to each student's experience throughout the year as it pertained to their views of mathematics and how it had changed from when they had started in the *Imagine* program. A case was written for each student from this information and correlated to their *Math Autobiography*, MARS-R's and my field notes throughout the year (see chapter four). I analyzed all nine cases using the principles of analytic induction.

Analytic induction offers a specific form of inductive analysis that begins deductively, by formulating propositions or hypotheses, and then examines a particular case in depth to determine if the facts of the case support the hypothesis. If it fits, another case is studied and so forth (Patton, 2002).

3.5. Field Notes

Throughout the year, I wrote down, in my daily lesson planner and elsewhere, notes regarding how each problem went and interesting or significant interactions with students during and outside class time. I collected significant data on each student's attendance and final grades in their regular online course work and also pictures of each group's board work on our MMP's. These notes acted as compliments to the *Math Autobiographies*, MARS-R surveys, and interviews and provided a more complete picture of the progress some of my students had made throughout the year in their feelings towards mathematics. Some of these observations and data are mentioned in the cases in chapter four.

3.6. Being a teacher and a researcher

I had to take great pains throughout the course of my research to ensure that I always had the best in mind for my students' education regardless of the effect it might have had on my research. The MMP represented a real risk for some of my more socially shy and introverted students for whom this research and intervention may have been the most benefit. However, the many benefits that I saw my students take away from this intervention, as I will relate in later chapters, encouraged me to carry on. No mention of anxiety, or any of my analytical tools were ever mentioned during our times in the morning when we would solve the mathematics problems, which was part of my effort to ensure that my students did not realize the MMP was supposed to assist them beyond simply improving their mathematics skills and solving interesting problems. In this way, I tried not to influence the data I was collecting, thus presenting the truest possible data in their *Math Autobiographies* and MARS-R's of their relative anxiety and causes. Whenever I gave out the MARS-R's, I emphasized to my students that they be completely honest, as it would help me to pick problems in the morning. At this point, my students were blind to the secondary reason of distributing this survey to them of my research into their mathematics anxiety. However, the danger was ever present that my students would not take the questionnaire seriously. This possibility presented a potential limitation in my research and will be dealt with in more detail in chapter six. This made viewing the class as a whole, rather than on a case by case basis, an unreliable notion, which is why I have chosen to analyze the data as a set of cases. Throughout the year, my students were aware I was doing my Master's degree in secondary mathematics education. However, it wasn't until time came for the interviews that I had to send home consent forms and explain to my students what my main area of interest that I was focusing on was a reduction in their mathematics anxiety through what we had done throughout the year. This created a risk that the responses I received from my students in the interviews could be biased in order to please me as their teacher and help me in my research. I had built a very strong rapport with my students through the course of the year and I have no reason to believe that my students would not tell me the truth as to their feelings and thoughts regarding the MMP's and its effects. Nevertheless, the danger of this bias is present. This presented a limitation to my study, where I take everything my students said to be true, even though that might not be the case. In the interviews, I constantly emphasized the need for them to be honest and not

solely focus on just the ‘MMP’s, but their whole view towards mathematics. Since giving interviews was voluntary on the part of the student, it may seem that this narrowed the type of student who would volunteer. As it stands, I interviewed nine out of eighteen enrolled students who had been in *Imagine* since the start of the year. To get over this particular limitation, to gain as comprehensive a picture as possible from the interviews, I selected students who to me represented a good spectrum of experiences from the year and had indicated the previously mentioned changes in their MARS-R’s throughout the year. As a result, I am confident that my selection of interviews was a good representation of the class as a whole of the changes and reasons for those changes that they may have experienced throughout the year.

Chapter 4. Results by Cases

As I mentioned previously, I had given out the *Math Autobiography* assignment and administered the MARS-R three times to the entire *Imagine* class. Thus, it becomes relevant for me to discuss prominent features and changes I observed of the entire class before I discuss a case by case analysis of the students whom I chose to interview at the end of the school year that best represented these broad changes and observations. This look at the class as a whole will provide a good background from which we can begin our more specific analysis with the particular student cases. What follows are descriptions of behavior, broad trends and changes that I observed in my class's mathematics anxiety levels over the course of the school year. Included are my notes of the individual's engagement and discussion from work done on the boards, excerpts from students' *Math Autobiographies*, discussion into their MARS-R results both on an overall score and a by-category basis and a recounting of prominent features from my interviews with the case students. All student names mentioned are pseudonyms to maintain the relative anonymity of my students.

4.1. The Class as a whole

Tthis class had exhibited some interesting changes over the course of the year. A large number of my students cited their enjoyment of the MMP and were often disappointed if some other event took precedence over it. Most students felt that it provided an new and relevant source of skills which helped them in their regular online mathematics courses as well. However, when it comes to their reported anxiety towards mathematics, there is quite the spectrum of results.

From the students' *Math Autobiographies*, it comes as no surprise for children to be reluctant to divulge information about something that they may not have ever thought about before. Students' most common response to being asked to tell the story of them and mathematics could be paraphrased as "I don't have a history with mathematics" or they "cannot remember what it was like to take mathematics before" or that they had "no feelings positive or negative to remember". Indeed, it seems that little if any reflection had ever been done before by these students on their past affective experiences in mathematics, which made starting their autobiographies, something of a chore for them.

However, once prompted with advice, which was to write anything that stuck out for them about their mathematical experiences, students were able to provide a map of why they currently felt the way they did about mathematics. I asked them to tell me about any significant events, significant teachers, students or memories that they recalled. I received 15 autobiographies out of a class of 18 students, each of varying levels of detail, but all retold some significant and important events that my students had experienced in their past mathematical lives. There were two kinds of autobiographies received from my students; those with extremely negative associations on mathematics, and those who had neutral to positive associations surrounding mathematics.

Out of the 15 autobiographies, I received 10 that could be listed as having a negative perception of mathematics or listing some significant negative incident. Of those 10, 6 of the autobiographies listed negative incidents with previous mathematics teachers as being a turning point in their thoughts on mathematics. It seems that the most common theme amongst these six autobiographies considered to have a negative view of mathematics and/or mathematics class fell under what Hobden and Mitchell (2012) described in their study as "critical incidents" that caused a transition of attitude from positive to negative. A common theme amongst these autobiographies seemed to be a benign view of the subject itself, but definite anxious or negative thoughts and opinions at the classroom environment. What follows is an excerpt from a particular example.

> "Okay, I really do love math however from switching schools I haven't learned a lot because I barely attended school…"

It then goes on to describe an incident where their previous mathematics teacher called this particular student a liar and ripped up their homework making them do it again. It was this incident which was the cause of this student switching schools and subsequent enrollment in the *Imagine* program. Another student describes how they perceived that they were always scolded by their teacher when asking for mathematics help.

> "I would ask for help and he would practically scold me for asking for help, it's not my fault that I don't understand this and you're a teacher and you're there for help, not to scold me!"

This student had a particularly negative view of mathematics that was detailed in their autobiography, citing a number of views that related their distrust of mathematics and

also detailed many negative perceptions regarding the usefulness of mathematics outside of a classroom context. This student's autobiography also related a positive interaction with a teacher who was perceived as being very helpful. However, this positive interaction mentioned about a teacher did not seem to override the negative or anxious feelings this student had towards the subject.

Another student cited the fact that they perceived their mathematics teachers to practice favouritism and a strong sense of injustice at this action. The student felt that the resultant social isolation in class was a prominent feature in their autobiography. These are only a few of the examples of the negative critical incidents that caused students' transition of attitudes. Positive incidents or memories surrounding mathematics were very sparse. Strangely enough, 8 of the 10 negative autobiographies indicated that mathematics itself was rarely the issue, rather, events that could have occurred in almost any other class. One student started their autobiography by saying:

> "Honestly, I have always liked math but it was the teachers. They treated me like I was stupid…"

This student's sentiment was echoed as a fairly strong theme amongst all the negative autobiographies students had written. The perception of how they had been treated had made them appear "dumb" or incapable to their peers in mathematics had given them extremely anxious feelings when approaching the topic, even though they themselves held no blame for the subject itself. Only 2 of the 10 negative autobiographies blamed mathematics itself for their dislike or anxiety surrounding it, citing specific examples from their past mathematical experiences (like division or fractions) and the difficulty in working with these objects as the sources of their frustration and anxiety.

The benign/positive autobiographies shared interesting themes as well. In each case where the students described in their autobiography as having mostly positive interactions with mathematics, there was almost always an environmental factor, such as positive social or familial support detailed in the students accounts. The most positive of the autobiographies that stood out the most had details about friends who helped in and out of class or positive teacher interactions. In these accounts, the teacher was perceived as being helpful or caring and good at teaching mathematics. In each of these, students relayed that working with their families or friends on their routine

mathematics work had helped them enjoy the subject and each described how they now enjoyed working on the subject.

Each autobiography provided a wealth of information about the students' pasts in mathematics and sources of their possible anxieties surrounding the subject. However, only a few students specifically mentioned the word 'anxiety' or even having it in regards to mathematics. The measure of my students' anxiety necessitated a numerical indicator for the strength of certain specific anxieties in mathematics. The MARS-R showed a good correlation to specific categories that related to many of the incidents detailed within my students' *Math Autobiographies*. However, there did not seem to be any single category or categories that were common amongst all my students.

The students were given the MARS-R in September or on their first day of enrollment in the *Imagine* program. These would form the baseline of their anxiety accounts and act as the comparison at the end of the school year. Looking at the initial data MARS-R scores, several features became very apparent. Students regularly would rate categories in the MARS-R highest if they had mentioned specific negative events in their autobiography. A student who was in a split grade 5/6 class mentioned a pertinent memory which correlated with a score of 4 out of 5 in the MARS category of "Getting stuck in a math problem" and a 3 out of 5 in both "walking into a math class" and "listening to the teacher explain a problem on the whiteboard":

> "The teachers always focused on the grade fives, I practically re-did all the grade five math from the year before and whenever the grade six's got to do work of our actual grade level, we were sent out into the hall to do it on our own with no explanation of how to do it…"

These strongly rated categories and seemingly related events relayed in the *Math Autobiography* were a common theme amongst all students in the class. One particular student, who will be discussed in one of the cases later on, would consistently tell me before the MMP that they hated mathematics and didn't want to do it. This same student had also described a strong dislike of mathematics and an inability to perform the necessary steps to perform mathematics correctly in their *Math Autobiography*. This negative self-image of their perceived inability to do mathematics was evident in their initial MARS-R score as well, rating above a score of 3 in eleven out of fifteen categories of the MARS-R. It seems that any student's *Math Autobiography* that exhibited similar themes of negative self-talk or demonstrated perception of "mathematics as a hostile

other" (Hobden & Mitchell, 2011) would also produce a very high score on their initial MARS-R indicating a high degree of anxiety surrounding mathematics. One student, who was mentioned above as having been "scolded" for asking for mathematics help and a negative perception of the usefulness of mathematics, had very strong scores (4 and 5) for all categories of the MARS-R that had something to do with the teacher or listening to another person about something in mathematics. Curiously, this student scored 1's on all categories involving the use of mathematics texts.

Those students who would list the most negative events or views in their *Math Autobiographies also had the* highest rated initial MARS-R's. Students who relayed an indifference towards mathematics seemed to have a moderate to high levels of anxiety in most of the categories of the MARS-R, except for those of an evaluative nature. One student wrote "it's interesting but tedious… I am quite indifferent towards it".This student rated a 4 in "thinking about an upcoming test" category and had an overall score of 40 on their initial MARS-R.

Those students who relayed the most positive memories or had a generally positive view of mathematics had the lowest MARS-R scores. Of the three positive autobiographies, each one had a MARS-R score of less than 30. Strong familial memories of parents helping students were often mentioned in these MARS-R's, as well as positive talk about their past mathematics teachers.

> "The reason that I am so good at mathematics though is because of my family" or "I recall a time when my mom helped me with my multiplication" and "[My previous teacher] always made it really exciting for me and also made it easier"

These were common themes amongst the students who scored the lowest on the MARS-R, providing just as much of a background to these students' thoughts and feelings toward mathematics as the negative autobiographies.

The MARS-R scores were a bit of surprise. I had at first expected overwhelmingly high anxiety scores given the nature of the *Imagine* program; however, the scores among my students were not as high as I would have conjectured. Despite this, there existed strong opinions towards the nature of mathematics and its relevance among the

students and seemed to be a prominent negative view among these students prior to the start of the MMP intervention.

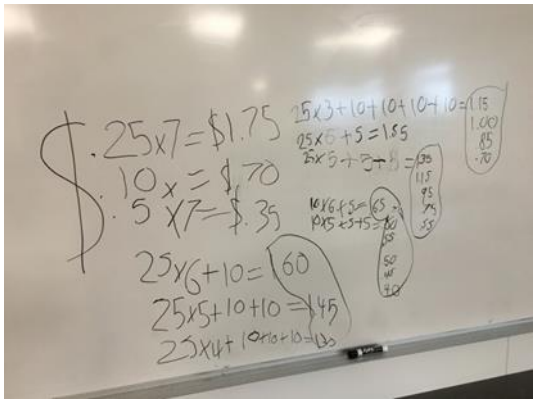

Figure 5: Example of Whiteboard work done in October. Question was "I have 7 silver coins in my pocket, how much do I have?" Note: disorganization and lack of detail in solution

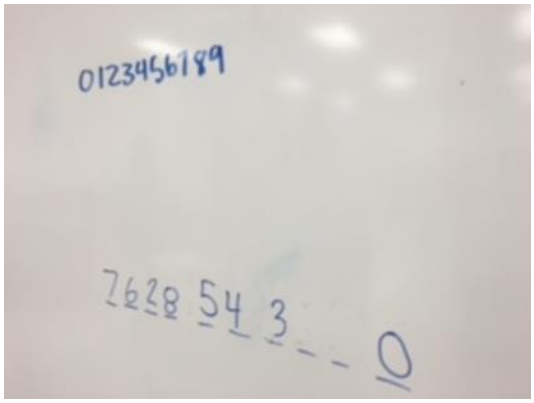

Figure 6: Example of Whiteboard work done in November. Question was "What is a 10 digit number that is divisible by 1 after the first digit, 2 in the first two digits…" Note lack of detail in both processes and solution

When students would be randomly sorted into groups to do an open-ended task at the whiteboards, it was met with resistance at first. It was resisted especially by those students who had listed many negative events in their autobiographies and by those who had listed negative social experiences in mathematics class, such as being perceived as dumb by their peers. Many of the *Imagine* students were displeased that this would be the normal routine every morning. However, after several weeks of placing students into groups and undertaking mathematical tasks, there started to be a different feeling in the class. Students, at the start of the year, felt that they were in competition with each other as the whiteboards would generally show other groups' progress. There was also the question of "cheating" by the mere act of looking at another board. After a few weeks of the MMP, these students were working together with students whom they might not normally interact with in a different class and beginning to argue and discuss mathematics with each other and myself. It is difficult to measure the "feel" of a class, but the cohesion amongst this group of students seemed to be increasing and students were beginning to look forward to the MMP. Interesting phenomena began to crop up though. Students' work on the whiteboards started off as rudimentary and basic, exhibiting an unwillingness to commit many details to a public forum. Conversations with students were brief and indicated that they were not confident in their reasoning or answers to the task at hand.

Students' work on the board was at first sparing. However, as the year progressed, students began to commit more detail to the whiteboards and started discussing and even arguing around solutions. 'Producibles' that were assigned for "bonus" work were spare in submissions from all students and those 'Producibles' that were submitted varied in quality depending on the student who submitted it. This too began to change as the year progressed. It started to become very clear that there were certain types of problems that the students enjoyed more than others. Problems such as the "7 silver coins" problem (Fig.#6), in which students are tasked with determining what the value of 7 "silver" coins could be, and the 13 marbles question, where students are tasked with using a seesaw to find the one heavier marble using the fewest possible measurements,were some of the most successful questions in terms of student engagement. Whereas logic problems, in which there was a human protagonist or a lack of numerical values, were questions that students seemed disengaged by and would often elicit strange or silly answers indicating, I felt, a lack of engagement or interest. When I questioned my students regarding this noted issue, my students would say that the fact that there were numbers in the first type of questions made it easier for them to work the problems rather than those types of questions that they perceived had no logic to them (as I would not permit changes to the internal logic of those questions). However, even the problems that students did not enjoy were being worked on well and thoroughly and would inevitably lead them to create some singular or interesting solutions and strategies.

By the end of the school year, students were often looking forward to starting the MMP, reminding me that it was time to start the problem of the day or asking whether or not today's problem was going to be a good one. Disappointment always followed a break in routine now if the morning problem were replaced for some other event. Students' board work had become more sophisticated and robust conversation amongst the students groups dominated during these times. Students would often move off and go and discuss others' solutions and work and argue their positions. The debrief after a problem would often involve all students, and even those introverted students who at the beginning of the school year had indicated a negative perception of mathematics in their autobiographies were contributing valuable insights into the solutions of the morning problem.

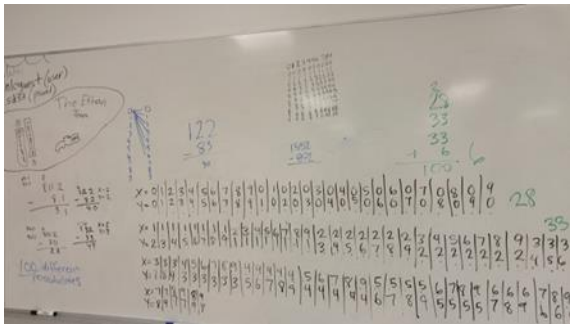

Figure 7: Example of end of year solution on whiteboard to "Smudge" Problem 1X2/8Y=? What are the possible values of X and Y?

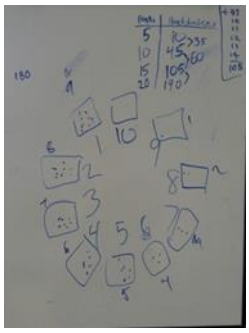

Figure 8: 10 people gather around and shake hands with everyone

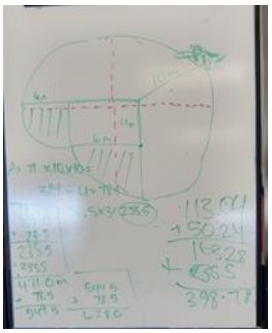

Figure 9: Tethered Goat Problem

There were certain students, however, that, no matter with whom they were partnered or how much I entreated them throughout the year to stand up and engage with their groups and the question, would never really engage with the problem. Almost all these students' MARS-R's scores increased in anxiety throughout the year, both in January and June, when compared with their initial MARS-R. Most of these students declined an interview except one and I will discuss this student in the case study section. However, the common thread with these students was that they were always "on the outside, looking in". They contributed very little to the group discussion or did not ever speak that much in any setting in the classroom and rarely held the whiteboard pen to write the group work on the board. The frustration that was felt by those students who were actively working with the problem with students who were often off-task or disengaged in their groups was always palpable during the problem. Good faith efforts were made by the other, more engaged students to bring these students into the problems. However it seemed that even this could not engage these students.

Near the end of the year, students started opting to work on their online mathematics work on the whiteboards as well when they were stuck on problems in the style of our MMP's. I had been encouraging my students to try this with their normal work and it seems that in the final "push" to get students' online coursework completed before the end of the year, students started to embrace this behavior almost of their own accord.

The average score out of 75 for all students who took the initial MARS-R (including those students who enrolled later in the year) was 36, which is not a significantly high level of anxiety in this classroom, especially considering the focus of

the *Imagine* program on anxious youth. The final MARS-R given out at the end of the year showed a minor decrease in the classroom's overall anxiety to a score of 34. These averages erase individual changes in students' attitudes and outlooks, and thus must be considered on an individual basis as there seemed to be three key categories of change for students over the course of the year. 7 out of the 18 students that remained in the program had decreases in their overall anxiety in various categories, 7 out of 18 had increases their overall anxiety and 4 students remained unchanged in their MARS-R scores over the course of the year. There appears to be no standard category that increased or decreased over the year. Those students who reported an increased MARS-R score ranged from an increase of 3 to 20 points, and those students who reported a decrease ranged anywhere from a decrease of 3 to 21 points. Of all the MARS-R data, there was no one category that saw a consistent change across the students. It seems that the changes experienced were as variable as the students themselves. In the final MARS-R given in June, I had added a 16th category to get a sense of how the students felt in terms of their anxiety towards doing mathematics problems on the whiteboard which was "Listening to or solving a word problem". Only 3 students of 18 rated a score of 4 or more, and these students were also among some of those with the largest increases in their overall anxiety from the beginning of the year. Every other student rated 3's or less in this category.

Since there was such a diversity of results from the MARS-R, it stands to reason that an examination of the three types of identified students mentioned above would be necessary to get a sense of what worked to help reduce certain students' mathematics anxiety and what might have caused some students' mathematics anxiety to increase. Also of interest would be those students whose anxiety remained unchanged throughout the year and whether they had any benefit to having participated in the MMP. This brings me to the case studies below with interview data that sheds light on the changes observed for the *Imagine* class as a whole.

4.2. Case Studies

In order to get a clearer view of why the class showed the trends they did over the course of the year, it was important to study cases of students who showed the three types of changes as reported in their MARS-R scores. What follows are accounts of 9 of the students who I identified from both their MARS-R's and from their behavior and

engagement exhibited throughout the year to fall into one of the three categories. I will start with some background on each student, what drove them to the *Imagine* program, some specifics on their *Math Autobiography* (if it had been completed by the student) and changes seen in their MARS-R's. I will then conclude with a thematic analysis from their interviews. All students' names are a coded pseudonym to maintain student anonymity.

4.2.1. Cases where students showed a decrease in Anxiety

Case 1: 'Jonathan'

'Jonathan' had responded particularly well to the *Imagine* program. The previous year in *Imagine*, Jonathan had barely been able to make it through the doors of the classroom due to his anxiety, much less be an active participant in the regular classroom setting at the time. When *Imagine* had started for the year of this study, Jonathan had made a lot of progress towards dealing with his generalized anxiety and coming into class on time and being more of a participant. As an overall student, Jonathan was above average in his ability to communicate and think logically and critically. Jonathan's biggest challenges were a tendency to procrastinate until almost the point of no return on assignments and a tendency to be quiet and let others take the lead on a lot of different activities. At one point in the year, Jonathan was failing most of his core online academic courses; this was simply the result of not having handed in or completed many of his assignments. Jonathan was aware of this fact and after multiple conversations with both him and his parents he was able to complete all of his missing work. Jonathan was an exceedingly thoughtful student who would think deeply on his studies and answers before verbalizing them.

Jonathan's *Math Autobiography* could definitely be described as a negative one, citing many negative experiences. His autobiography details a number of critical incidents with teachers that appear to have formed the basis for Jonathan's mathematics anxiety. In it, Jonathan identifies himself as an average student, who sometimes struggled a lot, developing anxiety towards mathematics at around grade 3. Jonathan cites a memory of teachers "picking" on him when he would ask for help or of teachers getting angry with Jonathan because they had just explained what to do. A rather severe incident retells a grade 5/6 split class, in which Jonathan was in grade 6. In this class the

only grade 6's were him and another student. According to Jonathan, he was expected to learn the grade 6 mathematics on his own with no explanation as a matter of convenience for teaching the majority of grade 5 students their mathematics. Whether or not this is the true telling of events, it has definitely formed the perception that has stuck in Jonathan's mind since then. Jonathan then describes entering grade 7 having learned none of the concepts, and then makes known a major source of his anxiety "when the teacher first explains how to do the math we're working on, I never understand it."

Looking at the major sources of Jonathan's anxiety, they start to make sense given the background offered in his *Math Autobiography*. On his initial MARS-R Jonathan's overall score was 35. Jonathan gave a score of 3 for "Walking into a math class" and "Watching a teacher explain a problem on the whiteboard". A score of 4 was given to the category "Being given a homework assignment with many difficult problems" and to "Getting stuck in a math problem". Most of the other categories were fairly low, being mostly 2's. In the January MARS-R, Jonathan's score on had decreased by 11 points to a total score of 25. The "Walking into a math Class" and "Watching a teacher explain a problem on the whiteboard" was now a 2 and a 1 respectively and the "Being given a homework assignment with many difficult problems" and "Getting stuck in a math problem" was now rated as a 2 and a 3. Many of the other categories also dropped a point. This score remained constant in the final June MARS-R at 24 with a new decrease of 1 point in "Starting a new math problem". In the 16th category, that I had added to the June MARS-R, Jonathan had scored that a 1 out of 5.

At first, Jonathan was not a fan of the MMP's. It seemed that randomizing Jonathan into groups at the start pushed him out of his comfort zone when he would have rather worked alone or with a member of *Imagine* whom Jonathan had known from the previous year. However, Jonathan is a good student and tried to engage with the problems at my request. At first he was not the most vocal member of the group but gradually started being more engaged with the mathematics tasks that we did every morning. It seemed that Jonathan's comfort with listening to the questions that I posed verbally started increasing. There were times when I would see Jonathan and his group standing around and when I would ask what they were thinking, Jonathan would shyly tell me that they had not understood the question, at which point I would re-explain and write some helpful pointers on their board. This would generally get Jonathan and his

group off to form a solution. Over time, Jonathan seemed to start having an easier time asking me for clarification, and I could begin to tell that he was enjoying this intervention of the MMP. This continued throughout the year, and Jonathan surprised me halfway through the year when time came for the “debrief” of the question and he confidently explained their solution to the task on behalf of his group. Jonathan also defended it against another student who had felt that the solution was incorrect.

Due to all of the above observations, I decided to ask Jonathan for an interview to let me know why his anxiety in mathematics had dropped the way it had, as it seemed that he had benefitted from the events and activities of the past 10 months. Jonathan was comfortable in the interview and very sincere in all his answers. This is typical of Jonathan as we had had conversations throughout the year on various topics unrelated to mathematics that both he and I had enjoyed. When I asked him what had changed for him this year in mathematics, his response was that he is “definitely more comfortable with it” and that he felt like he was getting a better understanding of it and that it seemed like it was “easier to do now”.

When I asked Jonathan about his favourite part of mathematics, he replied that it was definitely the MMP as it had “really helped me in math”. This was not a surprising response as Jonathan had begun requesting us to start the problem if I was falling behind in the morning routine of attendance and/or setting up the day. I had asked Jonathan why this was his favourite aspect; he replied that it was because they were working together in groups made it easier to verbally express his thoughts about mathematics, which he identified as a hard thing to do. Given how much more verbal Jonathan had become during the MMP’s, I had to agree with his view on this. I then asked what Jonathan’s least favourite aspect was in mathematics, to which he replied quickly with “algebra” as there were “so many different steps and I just get really confused”. Most of Jonathan’s algebra experience would have been with his online coursework, and this particular category in his MARS-R remained constant at a score of 2 out of 5. Jonathan went on to state that he felt that he was better at mathematics than he was at the start of the year, citing the MMP’s as the reason for this view. I asked Jonathan to explain what exactly about the problems we had done had helped him with, to which Jonathan was unable to thoughtfully respond, however Jonathan had mentioned later on in our interview that he had gotten a lot more confident in himself and “[didn’t] really have anxiety with it [math] anymore” as a result of the MMP’s.

After this first part of the interview, we came into the specifics of his MARS-R changes that I had observed in Jonathan and asked his view on why they had changed the way they did. When I asked about why he felt that getting stuck in a mathematics problem category had changed the way it did Jonathan responded

> "I used to get really anxious with it, like Oh I don't know how to do this and then freak out a little bit, but now it's not really that, I just kind of get frustrated…"

Jonathan, on the subject of why his rating of the category of watching a teacher explain a problem on the whiteboard had decreased, relayed more background than was covered in his *Math Autobiography*, with his 5/6 split class, where the teacher would explain things on a whiteboard and then accuse Jonathan of not having paid attention and when I asked why that had changed again, he cited the MMP's, crediting it with the following

> "… a big part of it is listening to other people explain them [the problems] and I feel there's so many different ways of people interpreting things has kind of helped me understand what they all mean and you know like diagrams and what we're writing and stuff, it's just made me understand it better"

Keeping with this theme of listening to people's different explanations, when I asked him about the benefit of randomizing the groups, Jonathan mentioned that there were some people whom they were not always happy about working with, but that it was an overall good thing for him as he would always try to make things equitable in the groups, regardless of who he was partnered with.

> "…you're not with specific people every time, it's you know, different ways of doing a problem, I think with certain people, we wouldn't all be doing it exactly the same way depending on who you're with, you're going to have a different outcome, like a different way of getting to your outcome and it's interesting, and that can open you up to more ways of doing things again."

Jonathan again made mention of how he tried throughout the year to engage all of his group members equally, but elaborated that when he was partnered with certain people that refused to do anything, that he just couldn't work with that.

Jonathan also mentioned that the open nature of the whiteboards and group work helped him when he would get stuck in a problem from time to time, allowing his group to "catch-up" if they were stuck by viewing some other groups' work. This also allowed Jonathan to feel some measure of relief when viewing others' work to help re-

affirm that they were on the right track or that at least, everyone else was thinking along similar lines. Additionally, when asked if the MMP had helped with his mathematics skills, Jonathan responded affirmatively, explaining that it had opened him up to be more experimental when figuring out problems, and that he was no longer afraid of trying something else to see if he either got the same answer or at least a similar one. I queried Jonathan to see if he was talking only about the MMP or his online mathematics course material as well, and Jonathan responded that he was talking about both. To see if my students' opinions of mathematics had changed, I asked them in the interviews of whether they could think of a favourite problem; Jonathan at first said that he didn't think he'd be able to think of one, but then immediately recanted and mentioned that he really liked the "Fly on a Coin Problem" in which a fly lands on a grid of coins arranged in a 4x4 pattern and needs to touch each coin exactly once by only hopping vertically and horizontally. Jonathan mentioned that he was going to join a group and complete a producible, but at the time had more pressing work. I asked him why he liked that problem so much, to which his reply was that he thought it was neat and he came up with a lot of good solutions.

Case 2: 'Ellie'

'Ellie' had had a difficult year in *Imagine* for reasons unrelated to the *Imagine* program. It seemed, however, that it was a very positive place for her to be when she was there. Ellie's attendance throughout the year was spotty, having missed 37 out of the 85 total *Imagine* classes that were held. Most of those absences were from the first half of the school year and were mainly due to issues not related to school. Because of this, my co-teacher and I made it a point to welcome her warmly on those days that she made it into school, and always ended the days with a hope to see her next time. The other students in *Imagine* were always happy to see her as well, and verbalized it as well on those days that she arrived. Ellie is a very intelligent young lady; her ability to write and verbalize in all her academics was always above average. Initially, she had enrolled as a grade 8 student in *Imagine*, due to having not passed her previous year in school, but both my co-teacher and I saw that her aptitude was far above that of the grade 8 level and with our administrators' permission, promoted her to grade 9, where she would be more challenged by the material. She proved to be more than equal to the task despite having "missed" a year of school and seemed extremely grateful for the opportunity that we had given her.

Ellie's *Math Autobiography* was very short and very limited and could be classified as relatively benign with respect to in her view of mathematics. There was not much detail or thought put into its composition, although given conversation with Ellie later on, this might have been because she did not want to elaborate too much of her personal life to me at the time. In it, Ellie stated that she has always been on good terms with mathematics and that she had "never had any problems with it and they [math] had never had problems with [her]". Most telling was when she stated in her autobiography that "I don't exactly love it, but I don't dislike it at all." She goes on to say that she also thought mathematics was "pretty fun" and recounted a time when she was in grade 5 and she tried her sister's grade 8 mathematics work and thought it was "pretty easy stuff". Ellie then finishes her autobiography stating that "she and math have a pretty good relationship".

This relatively benign *Math Autobiography* felt like it was missing some details and that maybe Ellie was not being a hundred percent forthcoming with details. However, Ellie's MARS-R has clues to what her true feelings surrounding mathematics might have been. Her initial MARS-R had a score total score of 38 and she had rated 4's in a number of categories related to mathematics class, but not that of mathematics itself. Ellie rated a 4 in the categories of "Walking into a math class", "Watching the teacher explain a problem on the whiteboard", "Picking up a textbook" and "listening to another student explain a math formula". Each of these categories is related more to the environment of a mathematics classroom, and not so much about the mathematics itself. It is more than entirely possible that Ellie is telling the truth in her autobiography, that the actual subject and she do have a "good relationship". It seems that perhaps Ellie has made a crucial distinction between the environments that she engaged the subject in versus the actual subject itself. Ellie had a large increase in her reported levels of anxiety in her January MARS-R, with an increase of 22 points to 52. This was a bit of a surprise as when she was present she had been rather engaged in class. However, by this time she had missed 50% of the *Imagine* classes and had fallen significantly behind on all her studies. Almost all categories had increased by a point of 1 or 2 except for "watching the teacher explain a problem on the whiteboard" which had fallen by 1 point. It seemed that Ellie's mathematics anxiety had increased to a much more severe level. However, by the time of the June MARS-R, Ellie's score had fallen to an overall score of 25, appreciably lower than her initial MARS-R. "Walking into a math class" was rated as

a 3, but “Watching a teacher at the whiteboard” had fallen to a score of 1. Each category had fallen by 1 to 2 points except for “starting a new math problem” which had increased by 1 point from the September MARS-R. The 16th category I had added to the final MARS-R of listening to or solving a word problem she ranked as a 1 out of 5.

Ellie was one of the more introverted students that I taught, opting to avoid direct eye contact when I talked to her and being very quiet in her responses. When performing the MMP, it was apparent that Ellie was extremely uncomfortable interacting with certain students in the classroom, and would have preferred to not be up at the whiteboards or engaging in classroom discussion. Attendance issues also plagued Ellie and she was often late to class, coming in half way through a problem or she would miss entire weeks of class. She was gradually falling further and further behind in her studies. Whenever Ellie came in late during the MMP, I would not fault her or even pay attention to that fact, I would simply hand her a playing card (handing out cards was how I randomly sorted my students) and tell her to find her group and get them to fill her in on the question. I would then make sure that I went over to her group a little later to ensure that she had been filled in on the problem. Almost every time she had been filled in on the problem by her group and eventually I didn’t even need to check on her or her group as it started becoming self-evident that she was engaged in the problem. It was through the course of the year that Ellie’s talents for problem solving and unique solutions became more and more evident. Her explanations for her group’s solutions became increasingly verbal and thought out. Attendance was still an enormous issue, and a parent-teacher meeting about it helped remedy it for the last 3 months of school. During this time, Ellie exhibited her best growth and academic capabilities both in the MMP and her online course work. Ellie never presented her group’s findings in the final debrief; however, it was always clear how the other students in her group deferred to her when they got stuck on their explanations to the class.

Again, due to all the above factors, it became evident that I should ask Ellie for an interview to get her input and perspective as to how she felt about the whole process as it seems that she benefitted greatly from something in the second half of the year. Whether the change that was exhibited was a result from the activities we had done in class, or some other external factor became an important investigation when interviewing Ellie.

When I asked Ellie for an interview, I had expected her to be unwilling as she had always been a very private person, but she had recently been opening up both to my co-teacher and myself during class time about her likes and dislikes. She was pleased that I decided to ask her for an interview and it was here that I finally got a better image of the student that she had once been at the start of the year. When asked what had changed for her in mathematics, she mentioned that she was anxious about everything, especially mathematics, almost breaking down trying to do it, but that now it was one of her favourite subjects. This was quite the change from the autobiography she had written. I asked her for reasons that this change had occurred and she talked about some rather severe bullying that she had undergone the year previous that had caused her great mental anguish. As she still didn't elaborate about many of the details, and being that she obviously felt strongly about those experiences, I didn't pursue this issue any further. However, these incidents might explain why Ellie was so withdrawn from her classmates and teachers at the start. I then asked her what her favourite aspect of mathematics was this year, to which she responded that the MMP's were her favourite and that module tests were her least favourite as they are not easy. She went on to say that she had always had a problem with the stress of a testing situation, but her ability to deal with it had improved considerably this year. Ellie said the following when I asked her to explain why:

> "I don't get to interact with a lot of people and the morning math problems really help… help that and I get to interact with more people, I get to talk and I get to solve things with others which I've never got to do a lot before, so it's really interesting."

I asked Ellie if she felt as though she had improved this year in mathematics, to which she responded in very strong terms that she had due to learning "all this new stuff" which when I asked her to elaborate on what "stuff" meant, she meant the skills and strategies that we had gone through in the MMP's. She went to explain that doing such problems in a 30-45 minute block had helped her get faster and more confident in her answers and methods.

> "…before it would it get so trapped in my mind and now I can just… I don't know what happened but now I could just really use my brain and before I couldn't!"

I asked Ellie if she felt better about mathematics class as a whole, and not just mathematics, to which she replied yes, but could not point to one specific factor that

made it this way, but rather a day-to-day progression. We then started discussing her MARS-R scores and the changes that had been exhibited throughout the year. I didn't ask about the mid-year increase, as meetings with her parents had accounted for perhaps a reason why that increase had happened as more of an indicator of her general level of anxiety, so I focused on her decrease from the start to the end of the year. I asked about her largest decrease in the category of "Watching the teacher explain a problem on the whiteboard". Again, Ellie credited the particular situation that the MMP afforded her.

> "I really thank uhh… the morning math problems in taking us in for the math courses as, before, in other schools the teachers always like just hand out pieces of paper and say 'okay this is what we're doing'… that's it and not a lot of times would they write on the board math problems, and when they did they would always call someone up and I did not like that…"

When I pointed out the fact that I had been similarly calling students up to the boards and asking them to explain their answers in the after problem debrief, Ellie stated that the key difference was the fact that it was in groups that were like "in their own little worlds" while working on the problem and that no one person was the center of attention for the whole problem. I found this interesting, and tried to follow up with more questions, however it seemed Ellie had said all that she intended to on the subject. I moved on to why Ellie felt that she had had such a decrease in the "Interpreting algebraic statements" category of the MARS-R. Ellie revealed a critical incident that she had not discussed in her *Math Autobiography* where she had been bullied by a group of her "friends" who would do anything to call her out for being wrong whenever she would do mathematics. This seemed to tie in very closely with her MARS-R scores for algebraic statements and listening to another student explain a mathematics formula. She goes on to state that even before meeting those people, algebra made her very nervous due to its complexity. When I asked her, why she felt better about it, she responded with the following:

> "I've learned that when I'm wrong that doesn't necessarily mean, like, I'm a dumb person, like before, I would think oh if you're wrong you're dumb and now I don't think that because I've seen many people in this class including me be proven wrong and no one calls me dumb, they're not a dumb person"

On this topic of seeing her classmates struggle in the same way as her, Ellie talked about the relief she felt at the randomized groups as she is always afraid that in group selection that no one would want to work with her. She also talked about the fact that

hearing other people's opinions, people whom she might not normally associate with in mathematics problems, really helped her skills and level of anxiety. Ellie also mentioned that the ability to see each other's work on the whiteboards helped her immensely when she and her group might have gotten stuck, but also reinforced to her that she wasn't "dumb" if she couldn't get to an answer, as she could always see if some of her peers were also struggling.

> "…when you see someone on the same boat as you, you're like oh okay I'm not the only one, like, oh this person is having trouble with this math problem and I am too so we're both okay."

Case 3: 'Toni'

'Toni' was a curious case whom I hadn't expected to include in this category. When I met her at the start of the year and discussed the concept of the MMP, she would always let me know that she "hated" mathematics and was terrible at it. She would mention this to me every morning when we'd start the problem and as I sorted her into the small groups. She would make a habit of sitting during the problem (each student was encouraged to stay standing), even after I would ask her to stand. Rather than badger her on this, I would ask her about the problem we were doing. At first, her answers were short and she always stated that she didn't understand the problem because she didn't like mathematics and was terrible at it. I would then make an effort to re-explain the problem and help her and her group with some advice and pointers. Toni struggled quite a bit with her academics throughout the year. She did not seem all that interested in school apart from the social interaction aspect. Her attendance was immaculate though, she only missed one day of class due to illness, which considering her attitude towards school was an interesting feature of this student.

Toni never completed a *Math Autobiography* for me, as any written output was difficult to obtain from her over the course of the school year. This made understanding her past history with mathematics more difficult. However, Toni had told me verbally that she had felt that her previous teachers were very mean to her, and that her mathematics teacher at her previous school had gone out his way to humiliate her in mathematics class. Toni had a tendency to exaggerate for the sake of a story, so this account might be not as severe as the telling: however, it could be considered a type of oral *Math Autobiography*. This unfortunately means that analysis of the specific incidents is not

possible, but it definitely speaks to having negative associations in regards to mathematics.

Toni's initial MARS-R certainly reinforced that her views towards mathematics was one of anxiety and negativity. Her total score was a 53, with most categories having a rating of 3 and above. Her most anxious categories rated at 5 were in the "Taking a test in a math course", "Thinking about an upcoming math test one day before" and "Looking through the pages in a math text". There didn't seem to be any one area of mathematics that was not anxious for her. Her lowest score in a category was "Walking into a math class" at a score of 2. When time came for the January MARS-R, Toni's score had fallen slightly to 47, with a notable decrease of 2 points in "Listening to another student explain a math problem". All other decreases were 1 point in various categories of the MARS-R, however her lowest rated category of walking into a mathematics class had dropped to a 1 from the initial MARS-R. There were two increases of 1 point in categories related to getting a test back and doing homework. The June MARS-R was a total surprise, as Toni reported a score of 32 on her final MARS-R which was a drop of over 21 points from her initial score. The most notable decreases were of 3 points in the "Looking through the pages in a math text book" and "Taking a math test" categories. Almost every category saw a decrease of 1 or 2 points except for "reading or interpreting graphs or charts" which increased by 1 point from the beginning of the school year. The 16th category that I had added in the Final MARS-R of "Listening to or solving a word problem", Toni had rated a 2 out of 5.

Toni was difficult to teach at times, as it seemed that past experiences in school had not been very positive and had set in her mind that school and academics were an opponent out to get her. In trying to teach her mathematics, science or English, she would constantly need attention or help through an assignment rather than trying it out on her own. The MMP's were, at the start, a difficult journey for her. It seemed that the exposure of being up at boards with other students who might rely on her proved a great source of anxiety. When I would come around to whatever group she was in, she would loudly tell me how much she didn't understand the question and tell me how much she was holding back her group. I would then engage with her and her group on this line of thinking. I would ask her group if they thought this was true, to which her group would emphatically respond that she was not. I would then ask them to make sure that

everyone understood what the question was and why they were doing what they were doing in terms of solving the problem. Looking back on this, it could have gone very negatively with a different set of students; however, my co-teacher and I had built a very safe classroom environment. Toni would grumble about this and tell me that it was still a "dumb thing to do in the morning". This continued for some time and I was starting to think about what else I could do to reach Toni, until I started catching Toni giving advice or suggesting different paths that maybe they could explore on different questions and being an interactive member in general. I never called her out on this in front of the group; rather I would opt to privately tell her what a great thing it was for her to participate the way she did. I would usually ask her if she still hated mathematics, to which she normally replied "of course!" This isn't to say that Toni made a total turnaround it was still a chore to get her to participate fully when paired with certain students but it seems that she had undergone a change in her willingness to participate. I gradually had to intervene less with Toni as the year went on, and only when she was paired with certain members of the class that she was more socially interactive with outside of class. Unfortunately, Toni was ultimately unsuccessful in her core academics and will have to repeat the school year.

Toni represents a unique perspective on a student who, while ultimately unsuccessful at their normal academics, clearly benefitted on an emotional level in their relationship to mathematics. Asking her for an interview would provide an interesting perspective into the mind of a reluctant mathematics learner.

Toni agreed to the interview without hesitation and spoke openly and frankly regarding her experiences throughout the year. Our conversation was very easy as we had developed a good student-teacher rapport over the last 10 months of schooling. When I asked Toni as to what had changed for her this year in mathematics, she did not understand what I meant; I elaborated that she hated doing anything mathematical at the start of the year and I wanted to know if that had changed. She responded that, yes she had changed and she credited it to "feeling like [she] was in a safe environment" and she wouldn't be judged if she got something wrong. She also cited the amount of people helping her had increased, which has overall made her feel better about the subject. I asked for clarification on whether it was just having two teachers that made the difference, to which Toni explained that the students were also a big help. When asked

about what her favourite aspect of mathematics was, she stated it was the MMP's as it really "helped her brain get moving" in the mornings and "wake her up". Toni mentioned that her least favourite aspect was anything having to do with variables, like algebra or formulas.

I asked Toni whether she felt better about Mathematics class as well, to which she also responded yes. I found this interesting as, at this time, she was failing her online mathematics class. Asking her what had changed she responded with the following.

> "it's just having people be able to help me […] for the last couple months I went to [previous middle school] and all the teachers there weren't helpful at all. If I asked him for help [teacher], he didn't do anything to help me, it was more just like, he explained it again the same way instead of explaining it in a different way that I would understand it."

As we continued our interview, almost all of Toni's responses to the interview questions were of a similar theme, that seeing and feeling like the support that she needed was there if she needed it and not being judged for needing it had helped her feel more positively towards the mathematics subject. She also mentioned that the undemanding schedule of online course work was also a large benefit. In discussing her MARS-R changes, she credited the online environment and the MMP's equally. She also made mention of the fact my co-teacher and I seemed to actually enjoy teaching students and had a "better attitude".

When I asked Toni if she had a favourite problem, she had some trouble coming up with a specific question at first; however, she did mention that she enjoyed "the more logical ones". When pressed about this definition she cited "The Marble Problem" as her least favourite in which students had to find the slightly heavier marble among 12 identical marbles using a see-saw as measuring tool. To her this was not a logical problem as the premise seemed ridiculous. Since this represented a unique position on types of mathematics problem, I asked her what the kinds of problems were she enjoyed the most; she answered that the Numeracy tasks were what she enjoyed most. These tasks had been designed by Dr. Peter Liljedahl in which students were presented with some data or real-world issue that they had to work with and come up with a solution to (**see Appendix E. Ski trip fundraiser**). Toni stated that these types of problems helped

bring a real world relevance to mathematics which she credited with helping her in her regular online coursework. I recalled her experience when doing “The Ski trip Fundraiser” problem, which Toni was a very verbal contributor to the fairness of division of funds. It was one of the turning points I had noted for Toni’s attitude towards the MMP. I asked about why she felt this way and I mentioned to her that the marbles problem seemed more akin to problems she would typically see in a mathematics textbook. To which she replied

> “…it’s just easier for me to see it in my head, and having all the numbers actually there… all I really look for in word problems are the numbers… (laughs)”

It seems that having concrete numbers that represented some real quantity helped Toni a great deal, and created some connection to material that wasn’t present before. I then asked Toni about the randomized groups and working at the whiteboards. Toni mentioned how the randomized groups helped her actually get work done, as there were some people that she didn’t work well with in class. She enjoyed the whiteboards aspect, but for the kinesthetic rationale that standing “helps the blood get moving to our brains”. She mentioned that looking at other students’ work on the whiteboards still felt strange, as it had been engrained into her that looking at others’ work was cheating. However, she did credit the standing at the whiteboard environment with something unique. She mentioned that it was easy to see me helping other students with similar questions to hers, which made her feel better about being stuck on a question and also helped her as she could listen to my advice to other students without having to ask me for clarification personally.

4.2.2. Cases where students show no change in anxiety

There were students whose MARS-R indicated that their anxiety had not changed throughout the year. When discussing no change in MARS-R scores, I am considering a “no change” category to be within 3 or 4 points of their initial score whether high or low. These cases were not limited to those students who rated very low anxiety at the beginning, but rather seemed to be a situation that could not be classified in any one specific student type. What follows are three specific case studies on this particular absence of change.

Case 1: 'Freya'

Freya is an excellent student for whom most academics came easily. She had come from online schooling the year previous and she enjoyed the freedom that online schooling provided for her in terms of flexibility. Her enrollment in the *Imagine* program was interesting, given our focus, but not unwelcome as she was a very friendly and engaged student who would often help others in class and ensure that any one in her group, regardless of activity, was keeping up at the same pace as her. She started the year by telling me how much she loved mathematics; however, after the first week of MMP, she told me how little she enjoyed these problems and how frustrated she got that they didn't have specific answers or set answers that she could easily solve. She was also frustrated with me that I would not give the answer to our daily problems, and my insisting that the students continue working on them on their own. We had many conversations throughout the year as to what the purposes of mathematics were and what actually constituted mathematics. For example, to her mathematics was just about following steps and finding the 'hidden' solution. To this I would challenge her to think of mathematics as more of a problem-solving art. However, despite her initial dislike of the MMP, she was always an engaged personality and a welcome presence in class.

Freya's *Math Autobiography* could be considered very positive with quite a number of positive associations listed when reading it. She began with, "I have always liked math" as her opening line. She went on to explainin how all her teachers were very good and helpful and then went on to say that she would sometimes get bored with the subject and felt like it was "the worst thing on earth". This is quite the change in perspective within the first two sentences. However, she quickly went on to describe her relationship with her best friend who was a "math nut". In this relationship, Freya describes how involved her friend was in mathematics and that "whenever she needed help, she'd ask her friend." She went on to describe how easy mathematics was with her friend's help and the enjoyable hours she spent doing homework and assignments with her. Freya then started describing how she (Freya) would help other students with their mathematics and how she would try her utmost to "not make them feel dumb". She then finished her autobiography by saying she had a lot of fun helping people that she'd made a lot of good friends this way.

Given her autobiography, Freya's MARS-R showed an unsurprising a score of 21 overall. Her highest rated categories were 3's in both "thinking of an upcoming math test" and "being told how to interpret algebraic statements". Every other category was a 2 or a 1 in score. Conversations with Freya had already indicated that she felt no anxiety when thinking about the subject, and her *Math Autobiography* and MARS-R reinforced that these were her true feelings again. Her January MARS-R showed a minor decrease in score to an overall score of 19, now no score was higher than a 2 in each category. Her final MARS-R decreased once more to a score of 18. Though this is a decrease overall, it is only a 4-point drop from her initial and from scores of 2 to 1. Her highest categories at the start of "thinking of an upcoming math test" and "being told how to interpret algebraic statements" now rated at a 1 and a 2 respectively.

Freya was a great personality to have in class, but was at first reluctant to the more open nature of the MMP. After some time doing them, she asserted herself as a group member who tried to ensure that all students had a say, and that their group had some solution or method to solve the daily task. Freya created a few *Producibles* that I will use as exemplars for future students in my mathematics courses as well. At first, she struggled with the concept of extending her problem solving; in her words, "why would I want to make the question harder when I've already solved it?" and this was evident in the first *Producible* that she created. However her second and third *Producibles* started becoming far more in depth and extensions into generalized forms of the questions (as I had been pushing my students to do in the MMP) were starting to take a more mature approach. It was becoming clear that Freya had gotten over whatever issues that she had had initially with the 'MMP, and now relished this activity for its opportunities for mathematical exploration.

Due to the positive nature of her autobiography, low scores on her MARS-R throughout the year and excellent work in our MMP, it became necessary to find if Freya had felt there was any benefit to the MMP intervention, or whether she had simply done it as a matter of being a good student, which she had already proven at the beginning the year that she was.

When asking Freya what had changed for her this year in mathematics, she surprised me with the following comments.

> "I think I started putting more of me into math, not just, oh this is the math question, I'm just going to do it and I'll finish it like that, I try to go more into the math question and try to see if I can do more with it and I know especially the morning math problems have helped with that a lot. I also like Math a lot more this year, I used to like math, but not, I mean I liked it, just not, I didn't love it. Now I like math a lot more."

Considering where Freya had started out, this was an exceptional response that she enjoyed mathematics even more than she did when she started, which considering her *Math Autobiography* was no mean feat. Asking her what her favourite part of mathematics was this year, she mentioned that the MMP were definitely her favourite because of the interaction that she experienced with other people working on the whiteboards and my method of making the question harder as they progressed through the problem. I inquired as to why my making the problem harder for her had made her enjoy it so much more, when at the start she was so resistant to it. She felt that it was the fact that because I didn't tell them the answer made the students including her have to think about it more than she would have had to otherwise. When asking about her least favourite aspect of mathematics, she mentioned that textbooks are not her favourite, due to the "rote" nature of teachers assigning questions and students having to complete them.

I then asked if she felt that she was better at mathematics since starting and whether she felt better about mathematics class. Freya responded that she definitely felt that she was better at mathematics than at the start, crediting again the MMP and that mathematics class was even better, especially since my co-teacher had started doing separate lessons in mathematics, rather than the blended-approach we had tried throughout the year. I asked Freya about her MARS-R and why she felt it had remained relatively constant; she told me how she never really felt anxious in mathematics, and re-told me about her friend who loved mathematics that had been in her *Math Autobiography*. I followed up with a related question, asking if the MMP had helped her with her mathematics skills, to which she replied that it definitely had because it had introduced skills and strategies to mathematics problems that she had not known about before we had learned about them in doing the problems. I then asked her if she had a favourite problem; Freya quickly remembered a mathematics problem that I had taken and adapted from the Waterloo mathematics contests, where $\frac{1X2}{8Y} = ?$ and X and Y are

simply unknown digits from 0 to 9. I asked students to find me solutions to this problem. Freya and her group had exhausted all the possibilities in the 45 minute block of time, writing them on the board (I used it as an example of student board work in chapter three). I remember this problem as Freya and her group had kept asking for more time when I tried to bring an end to our MMP.

I asked Freya if whether or not the group dynamic of our problems had helped her. She responded that it had as sometimes, depending on who was in the group it had made the question a little more difficult. I am not sure whether she was referring to those students who would 'opt out' of trying to help or those students whose skill levels were a little lower. However, she mentioned this:

> "…I think it still has, cause then you're not by yourself and you're not the only one who has ideas because everyone has different ideas and those ideas all come together to sometimes make a solution."

When I asked about if randomizing the groups had helped with what she said, she didn't think it made that much of a difference, except that it again introduced her to different people and ideas that she may not have had by herself or in her preferred group. I asked her why the "different people, different ideas" was so important:

> "Because not every math problem has to get solved with the same technique, the same type of solving, it's always different 'cause math problems are all so different…"

Moving on from this idea, I asked Freya about the whiteboards and how they helped. Freya responded that she enjoyed the process of writing down mathematics solutions on a whiteboard immensely as it was a larger canvas to write ideas down on and the process that you could write something down and then immediately erase it if it wasn't any good, which helped get ideas out for her. I then asked her if looking at her fellow classmates' work and struggles ever helped her. Freya then talked about how if she was ever stuck and could see not 'What' they were doing but the 'Type' of thing they were doing, that would let her try something new and help her and her group find a solution. She mentioned that seeing another group not struggling when she was stuck helped her realize what paths or strategies she should implement for solving solutions which in the end made her feel less alone when she struggled.

Case 2: 'Karla'

'Karla' was an interesting case as she was a late arrival to our program from a private school. She was extremely shy and introverted, but was a seemingly strong student at her academics. Karla did not seem to enjoy the MMP at all. She would arrive late enough most days to have just missed the final debrief for the mathematics problem. This became such a common occurrence that I had to speak to her one day after her mother had told us that she had been coming late specifically to avoid the MMP. In the conversation with her regarding this, she at first denied that was the case, but then realized that her mother had informed us of this fact, and then was more honest about her feelings surrounding the group problem. In it, she told me that she felt very "exposed" when doing the problem and felt as though she was going to "bring down the group" as she felt that she wasn't very good at mathematics and didn't want to be blamed for their group's inability to solve the MMP. She was also afraid of being labeled as "that weird kid". I reassured her that a lot of students felt the same way about working in groups, including myself (as I was in the middle of my own Master's program). She seemed surprised that I would experience this as well as her peers. I then challenged her to make a real effort of it in the mornings, to be an active participant and to not be afraid to take risks with her fellow students, as I was sure that she had meaningful contributions to make to the group. This seemed to have the intended effect as she started coming on time for a number of months and while still shy and introverted in her groups, seemed to be trying to take a more active role in the problem.

Karla's *Math Autobiography* could be considered a negative one with some minor positive associations. In it, Karla relates that she was fairly good at mathematics getting A's and B's in her earlier grades. However, she mentioned that she never remembered liking mathematics. She then recounted her experiences in private school, where she felt as though the mathematics teachers would pick favourites, which in turn would cause her to feel that if she asked questions the "class favourites" would bully her or think she was "dumb". Given my conversation with her previously, this gives some insight into the genesis of her reluctance to participate in the group problems. She then recounted her entrance into middle school where she was doing very well at mathematics, but had to leave for a good portion of the year due to health issues. She then went into how she now felt that she had missed all of her previous grade of mathematics instruction, which

in turn was causing her to panic when thinking about how much work she would have to do to catch up in her knowledge deficit.

Karla's MARS-R reflects her feelings described in her *Math Autobiography* and those that she expressed to me in that critical conversation regarding her lateness habits. Overall, her score was 45, which is fairly high compared with the class average. A lot of her categories were rated at 3 or higher, but her most anxious categories were "Thinking about an upcoming math test one day before", "Taking a test in a math course" and "Being given a homework assignment of many difficult problems", each of which she rated at a 4. She also rated "Getting stuck in a math problem" at a 5. Most of these categories seem to relate to her feelings of a knowledge deficit that she perceives that she has from her having missed school. The social aspect to mathematics class categories, such as "Walking in to math class" and "Listening to another student explain a math formula" were also high, ranking mostly at 3's. Her January MARS-R showed a good amount of improvement in her MARS-R score, dropping to a 36 overall. Her largest decrease at this time was in the "Getting stuck in a math problem" which dropped by two points to a 3 and "Listening to another student explain a math formula" which also dropped two points to a score of 1. Most other categories at this time showed a drop in score, except for "Reading a formula in a science text" which increased by 2 points to a score of 4. It seemed that Karla would be included in the overall decreased anxiety category, however, by the June MARS-R, her score had gone back up to a 42 point overall score. This increase seemed limited to a few categories though, such as "Reading a formula in a science text", "Thinking about an upcoming math test one day before", "Taking an test in a math course" and "Being given a homework assignment of many difficult problems" which had all gone up to a score of 5. These categories were already highly rated at 4's and 3's and hadn't changed from the September to January MARS-R (except for "Reading a formula" which increased from January again to a 5). The decreases that had occurred in January such as "listening to another student explain a math formula" had stayed however, and there were a few one point decreases in some other categories. It seems that these decreases were eclipsed by the major increases in those 4 categories in her overall score. In the new 16th category of "listening to or solving a word problem" Freya had rated a score of 3 out of 5.

Karla had taken my advice and started engaging more with the MMP's, but was one of the quieter participants. Whom she was partnered with seemed to affect how

deeply she would engage with mathematics problems and the problem-solving process. I would have to go to her group if I knew if she was partnered with certain people to ensure that she was being involved in the process or that she had understood the problem and its parameters, otherwise she would be content to wait patiently for the morning problem to end. Gradually, it seemed as though she was becoming more and more confident with the process of problem solving, but around spring Karla started missing an inordinate amount of schooling due to various health issues. Meetings and conversations with her parents and counsellors dictated a reduced workload in her online courses, and she started coming in late again to class. I had asked her if she was coming in late to avoid the problems in a non-accusatory manner. However, she insisted that now it had nothing to do with the problems, but rather some other issues that for confidentiality purposes I cannot go into here. A positive note is that when she would arrive late she would find a group that had a smaller number of members and join without my prompting or if she missed the problem entirely, she would ask what the problem was so that she might try it on her own. Karla ended up passing the year in mathematics with a C-.

For all reasons, especially the decrease in her January MARS-R and then her increase back to her initial score in June, I had thought it important to interview Karla to get her perspective on why the changes had occurred the way they did, and if she felt there was anything positive that she would carry forward with her into next year.

In our interview, she answered my first question of how she felt she had changed in mathematics this year with a self-reported major decrease in her anxiety levels towards the subject. This I found a surprising given her most recent MARS-R had gone back up to a score of 42, a similar score as to when she started the year. I asked her about this phenomenon and she insisted that the “overwhelming” sense of anxiety that she felt in class when I would introduce a problem or if a teacher were explaining things at a whiteboard had disappeared entirely. She mentioned that the reason she felt that such strong anxiety at the start of the year was due to her previous excellence in mathematics.

> “I would get a hundred percent with all my tests and then for some reason I just started to feel like nervous because I wanted to keep my grade that high all the time, like, I felt like I had an obligation”

She mentioned that now that feeling had lessened. When asking her what her favourite and least favourite aspects of mathematics had been this year, she mentioned that her favourite aspect had been the MMP, even though she “didn’t like them at first”. I mentioned the incident of her coming in late to specifically avoid them, to which she revealed that there were more reasons than just that. I asked her why she would want to avoid the problems.

> “Because I just didn't want to look stupid in front of my peers! […] it's hard to explain… I couldn't really… like, when you explained it to me [the question], I didn't understand at all…”

I asked her if that was different now, to which she responded that she did understand verbal explanations a little better now, although it was still hard when another student explained it to her, but that she really liked it when a peer would explain it to her as she enjoyed getting multiple explanations on the same problem. She then listed that algebra was her least favourite aspect of mathematics as it involved a lot steps that were strewn with potential for error. When asking her if she felt that she was better at mathematics now than at the start, she answered that she felt more comfortable with the concepts as the MMP had gotten her to think about mathematics in a more “problem-solving way” which had helped her in her regular mathematics work as well. Interestingly, she mentioned that the skills learned in the MMP had not only crossed over in helpfulness in her regular online mathematics, but also into her personal life and interactions, as it had made her learn to “take a step back and think about it”. ‘It’ in this case is in reference to peer interactions that she labels as “drama”. Added to this, she says that she does feel better about mathematics class, but that looking towards next year and returning to the regular school system, was making her wonder about the classroom situation and whether she would fall victim to the same circumstances that had driven her to *Imagine* in the first place. She credited *Imagine* with having an environment that was, in her words, very safe. I asked her if this was the reason for why her MARS-R score had increased from January, to which she replied that it was the uncertainty of not knowing if the public school system would be similar to her unpleasant experience in private school. She didn’t provide much else on the increase in testing anxiety, but I could infer from her answers that it had mainly to do with her re-entry into a regular school system.

Karla’s view on her skills in mathematics was that she felt that she had improved a great deal due to her newfound ability to “think in segments” and the fact that when

they were solving a mathematics problem I would always come around to change the questions or add to them. When queried about whether she had a favourite mathematics problem, she mentioned that the “one of paper ones” also known as the numeracy tasks were her favourite types of problems, especially the “Race around the World” task (**see Appendix F. Race around the world**). In this task, students had to use a map and “travel around the world heading only east and stopping on each continent.” This was one particular task that I remember her engagement being very high. During this task she was suggesting different possible paths and actively comparing measurements that would get her group the “shortest” path around the world. She cited that it was the fact that she could write on paper that made it one of her favourites. This helped broach the topic of how she felt about our working on the whiteboards. She responded that she found it harder to work on whiteboards, but couldn’t list a reason as to why that was so. She also mentioned that working on whiteboards was more stressful because it was so open and everyone could see her, but admitted that looking around at other students’ work on the whiteboards had helped her get “unstuck” from a problem on more than one occasion. She mentioned that seeing the students’ work made her feel less alone when working on the mathematics problems.

> “you don't feel so alone and you feel just like you can say ‘hey I don't get this either so maybe we can… maybe two groups come together and do something’…”

I then asked her if working in groups had helped her mathematics skills to which she responded that it definitely had due to the fact that she was exposed to more perspectives in mathematics than if she was by herself, again citing the fact that it made her feel less alone in the effort of solving a problem. She especially enjoyed the randomizing of the groups as it helped her talk to people that she might not have normally interacted with. She also liked the fact that randomizing prevented favoured or select groups from forming, which she said made her feel better as no one could purposely exclude someone else. Interestingly, she did mention that sometimes “some people” in *Imagine* would be disappointed in being partnered with certain students, but that she credited the randomization with preventing any one student, regardless of social view or skill level with becoming the “odd person out”.

Case 3: 'Nick'

'Nick' was a unique case as he started the year as almost a different person than he ended it. Nick was quick to make friends in the *Imagine* program, but quickly alienated those friends through some ways or means that my co-teacher and I were not privy to. Nick was very vocal about his thoughts and methods of doing school work or the MMP. He would constantly tell the class about how smart he was and how he was always smarter than his teachers. I don't believe I ever heard him talking about how much smarter he was than his classmates, but it seemed to be implied as whenever I would view him working with his group in the MMP. He would inevitably be the only one writing on the board with the other group members off to the side not participating. This was regardless of whom he was partnered with. When I would ask him to trade pens or let someone else write, he would protest loudly, stating things like he didn't want to give up control or "no one else" could solve the problem like he could. This was frustrating at times for his group members as they were obviously not too happy about their diminished role in the morning problem. However, Nick would often provide some singular insights and solutions to questions, that while not always accurate or even satisfactory, showed creativity that if it had had more thought or perspectives put behind it, would have been very good solutions to the MMP.

Nick's short *Math Autobiography* could be considered relatively benign in regard to his views of mathematics. He would refer to school mathematics as 'normal people math' and states that he never disliked mathematics, but disliked how he perceived that the teachers would treat him as "stupid". Nick goes on to state that he had to "dumb" himself down such that he could appear "normal" and prevent his teachers from identifying him as being different. In his autobiography, Nick said that to this day, he never had to work to his full potential "because he never had too [sic]". Nick credited his move to online education as a turning point where he could finally "think" about what he was doing in school instead of just cruising through his studies without thinking and "getting A's". Nick ended his autobiography by stating that school "sucked" until he enrolled at FVDES's online education program.

Nick's MARS-R scores indicate a low level of anxiety throughout the year. Nick's initial MARS-R had an overall score of 26. Most of his categories had a score of 1 except for a few. Those categories that scored above a 1 all had to do with test taking or waiting

to receive a test back. His two highest rated categories were “waiting to get a math test returned in which you expected to do well”, which was rated at a 4 and “walking into a math classroom”, which was rated at a 5. This student had told me that this was more to do with normal school anxiety and not so much the fact that it was a mathematics class. Nick’s January MARS-R showed a small increase in his overall score to 29, with some very interesting individual category changes. The “Walking into a math class” category had fallen to a score of 1, whereas there had been some minor increases in the categories relating to testing. One notable increase was in the category of “Reading a formula in a science text” which had increased by two points to a score of 3. Nick’s final MARS-R stayed at 29, relatively unchanged except for his “Taking a math test” category which had increased to a score of 5, with other small decreases to his initial MARS-R scores. The “Walking into a math class” category remained at a score of 1. Interestingly, Nick had written explanations for his scores in the margin of the MARS-R, stating that his score of 5 was that due to “generalized anxiety”.

Nick was an interesting person to have in the *Imagine* program. In the beginning, we were following a more blended approach to lessons, allowing students to move at their own pace, which he seemed to enjoy. He also established himself as being one of the most welcoming and gregarious students in the class at the beginning, but over time this aspect of his personality faded. In his studies, Nick fell behind schedule relatively soon into the school year, insisting that the way he worked was always a “last minute” panic in which he would get 100% on everything. His participation in the MMP was always enthusiastic. In discussion or clarification, he would be very verbal and often times challenging to the parameters of whatever question I set for the class to work on. Most often, he would take issue with some small wording or turn of phrase in the question and then ignore most of the parameters of the question to pursue his own exploration. I thought that perhaps these explorations might add something to the overall solutions that each group might achieve. However, it seemed to have the effect of causing Nick’s group members to be pushed to the side and unable to contribute. He sought constant validation of his efforts during the task; when I suggested a path of inquiry or sticking to the original parameters of the question until his group had come up with a solution to that, I was met with an exasperated Nick stating that he had already solved that. I would then ask a member of Nick’s group what that solution was, only to be met with shrugs. His presentations during the debriefs were always interesting; as he

sometimes had trouble verbalizing his thoughts such that the rest of the students could understand them. He was almost unsuccessful in his academics this year due to his procrastination habits. My co-teacher and I also had made a pedagogical shift to having discrete lessons in the last two months of school to help students, which Nick was not pleased about and routinely threatened that he would transfer to the online versions of the courses only if we insisted he participate in these "normal" classes. Nick ended up with a B as his final grade in mathematics.

Since his MARS-R had changed but in ways that caused his score to remain relatively the same, and since he had had such a different experience of the MMP, I felt it important to get Nick's perspective on the events of his year in *Imagine*.

Nick began his interview by stating that he had always loved and enjoyed mathematics and that he felt that he was stronger in mathematics and more confident in his answers now at the end of the year than he had been at the start. Again, as in the case of 'Karla', this was surprising, as Nick's MARS-R score was not markedly different in score from the start of the year. I asked what had precipitated this perceived change, to which he answered that it was just that he was more confident now. We moved along to what his favourite or least favourite aspects of mathematics had been this year. Nick stated that his favourite aspect was that he could work at his own pace rather than be in a classroom situation. His reasons for this was that the only people seeing his work were myself, my co-teacher and him, as opposed to his last school where everything was peer-marked. This came from his fear that people were going to judge him as "not being smart enough" and not wanting to be perceived as "stupid". I insisted that no one here would think of him in that way, however it still bothered him to think that someone else might judge his work who wasn't a teacher. I asked what his least favourite aspect of mathematics was; he said that the MMP was his least favourite, but he still loved doing them, as he loved everything that was mathematics. His main complaint was that he would get frustrated with the problems and especially my perceived lack of appreciation for how great his answer was:

> "I find them enjoyable and challenging but I get frustrated with them so easily, mainly because I think I've come up with a great answer then you come over and go that's wrong…"

I mentioned to Nick that I had never outright stated he was wrong and he relented on this point, which puzzled me as to why he would say it. He then elaborated that he was most frustrated that when he got an answer, I would change the parameters of the question for his group making his first answer “null and void”. I asked what he would have preferred me to do rather than changing the question to make it a little more difficult, to which his answer was to “give him a totally different question”, but one that still utilized the same mathematics behind it. I asked for clarification as it sounded somewhat like what I had been doing; I asked Nick if changing the quantity or challenging him to find a different solution wasn’t what he was asking for. He responded that he wanted a different question that used the same sort of mathematics. He then complained that my advice or changes to the problem would always come on the heels of his having already explored that in the question. His chief complaint was that I would give my attention to other students once I had given my extension to the problem to him and his group.

> “It's not fair! They're like, it always happens when I've got a question and I ask you the question and then you basically give me the same answer that I already had in my **head** and hasn't helped me at all and then you just walk away and I'm like oh what am I supposed to do!”

I then asked Nick if he was better at mathematics now than at the beginning of the school year. He stated that he was, but that it was only natural given that it was the end of the school year, and he got better at mathematics every year regardless as he inevitably picked up new skills and experiences in mathematics. I asked what skills he felt he had acquired; unfortunately, Nick could not think of any at the time of the interview. I moved on from there asking if he felt better about mathematics class now than he did at the start of the year. Nick stated that he never had a problem with mathematics class, just the way they were run and stated that he had really enjoyed how the *Imagine* program had run its class as it was pretty “free and easygoing”. Nick cited that he really liked my co-teacher, stating that he’d never had a good mathematics teacher before her.

We then got into his MARS-R scores, in particular his decrease in the area of “walking into a math class”. Nick felt like he was repeating himself, but mentioned that he didn’t feel like we would label him as stupid and that no one else would look at his work. I asked if the MMP had helped him with his skills, after some thought he replied that it

had helped him look at mathematics in a “happier” light. This is because he did look forward to the morning activity to help “wake him up”. When asking about what his perspective was on the group work at the whiteboards, Nick’s opinions were mixed. He felt that the groups did not help his abilities and he usually resented the other people in his group interfering in his solutions. He did not like the randomized nature of the groupings either, as he felt he always got stuck with the same people over and over. Admittedly, the randomizing did have Nick partnered with the same people a number of times, but overall it was fairly randomized who he worked with. He states that he enjoyed group work more when he got to pick his group as he believed that he could pick people that he works with better than the randomizing. However, Nick loved working on the whiteboards, as it afforded him more freedom to write and was a better posture for thinking, citing it was a better “mind-body” connection. It also afforded him the opportunity to stand back and view his total work. To Nick, this benefit outweighed the chance that students might see his work and judge him for it as everyone else was too busy on their whiteboards to notice his work.

4.2.3. Cases where students anxiety increased

In this study, it becomes ever more salient to examine those students who seemingly went in the opposite intended direction in terms of anxiety. It is important to see what, if any, benefit these students gained from my intervention and methods. Then it might be possible to re-evaluate these methods, in order to better help these types of students in the future. In this section, I examine three students who showed an overall increase in their mathematics anxiety according to their MARS-R scores.

Case 1: ‘Vance’

Vance’s inclusion in this category was a surprise as he had had a seemingly good experience in the *Imagine* program and had responded particularly well to the MMP. Vance was a strong student, whose easygoing nature and amiable attitude would help him work well with a diverse range of students. He would arrive early to class and always be ready to dive right into the day’s activities, whatever they were. He was a dedicated student and an ideal candidate for the type of learning environment that *Imagine* provided as he never fell behind schedule in his online submissions and assignments. It had come to our attention midway through the school year that Vance’s

calm and collected demeanor was something of a front for the benefit of his teachers and class as his parents and counsellor had come to us to ask about a reduction in his assignment workload due to him having a rough time at home regarding the amount of work that he had to do. He would spend hours and hours making sure each assignment was perfect before he submitted them. This was causing him an enormous amount of stress as there was quite an increase in the volume of assignments and subjects from his previous grade. He was always an excellent group member in the MMP. He would constantly seek his group's consensus or understanding before continuing on in the question and ensure that no one student was left out of the process.

Vance's *Math Autobiography* could be considered a very positive one, with many positive associations. He admitted in it that he found mathematics difficult at times, but considered it a "great subject". He mentioned in his autobiography that he used to consider himself very bad at mathematics, especially algebra and division. He went on to say that he now felt like he loved mathematics and was good at it, citing his averages in his previous mathematics classes. He did mention how he gets "sad" if he gets less than 80% on a test though, as he was "not used to getting that grade". He credited his skills and competencies with mathematics to his family's support of him. He then described how his father, mother and siblings would individually help him solve mathematics problems if he was struggling. He stated that his whole family is great at mathematics and thanks them for their help. He then goes on to describe a mathematics game that he played with his mother using a deck of cards to help with his multiplication and how much he enjoyed this time with his mother.

Vance's initial MARS-R score was 25 overall, indicating a fairly low level of mathematics anxiety with no categories rated over 3. The highest-rated categories at a score of 3 were "Thinking about an upcoming math test one day before", "Taking a test in a math course", "Waiting to get a math test returned in which you expected to do well" and "Getting stuck in a math problem". His next MARS-R in January showed an increase of 8 points to an overall score of 33. No individual category was rated at a 5, but now "Waiting to get a math test returned in which you expected to do well" was rated as a 4 and there were a number of other single point increases in a variety of other categories that were initially rated at a score of 1. The biggest increase was in "Reading a formula in a science text" which had increased by two points to a score of 3. All his other highest

rated categories from his initial MARS-R stayed constant at 3. In his June MARS-R, Vance showed an increase from his initial MARS-R by 11 points to a score of 36 overall. Vance now had several high-rated categories. Both “taking a test in a math course” and “Getting stuck in a word problem” had increased by 1 point from his initial MARS-R to a score of 4 and “Thinking about an upcoming test had increased by two points to a score of 5. Almost each category had increased in scores for anxiety. There were no decreases in any individual category. In the 16th category that I had added in the June MARS-R, he rated a score of 1 out of 5.

Vance had seemingly enjoyed the *Imagine* class and was always an active participant. He would be sorted into his group and start working on the problem immediately, always endeavoring to get the most out of the problem that he could. He would occasionally call me over to verify that his group was on the right track or describe his solutions. I would then change the question slightly as to make it more difficult and have his group extend their learning. Vance was one of the first students that I challenged to begin creating his own extensions of the questions. I had told him that he has seen what I normally do to a question to make it more difficult, and I wanted him to start trying that on his own with his group if I wasn’t immediately available. He seemed pleased by this challenge and afterwards when I would check on his group to see what extensions I could give, I would see that he had already changed the problem in a way that I would have done as well. Sometimes, his extensions were naïve, such as simply changing the numbers in the question to recreate what was essentially the same problem without changing any of the parameter to make it more difficult. However, with my advice, his extensions to problems started becoming more and more mature, where he would extend the problems into patterns and make real challenges from the initial mathematics problems. At one point in the year, I had ordered mathematics contests from the University of Waterloo, and had been encouraging everyone in the *Imagine* program to try taking them. Vance was one of the first people to volunteer to take it and scored the best out of everyone in the class (but did not make national honors by about 5 points). From every qualitative observation of Vance that I could have made through the year, I did not expect his MARS-R to indicate such an increase in his anxiety levels.

For all of the above observations, I felt it very necessary to interview this student as I had predicted that his score would remain relatively constant or fall like in the case of ‘Freya’. I had gotten something wrong, so it seemed, and thus I needed to talk to

Vance and get his insights as to why the indicated changes were not what I would have surmised from my observations.

In our interview, I started by asking him what had changed for him this year in mathematics. He stated that he found it really difficult at the beginning of the year, especially the MMP, citing that the format made it difficult for him; however, he said that it gradually became easier as he got better at problem solving. When I asked him what his favourite and least favourite aspects of mathematics were, Vance responded that his favorites were both the MMP and algebra, stating that he liked anything with "numerical expressions or values". When I queried him regarding this unusual response regarding algebra he said it was easy for him, so that made it his favourite. Vance's least favourite part of mathematics was word questions, which was a bit of a surprise as he had just mentioned that the MMP was one of his favourite things. I asked him about this discrepancy and he clarified that it was the "typical" word problems that you would find in any mathematics course or text book that he disliked.

I continued on this front, asking how the MMP were different than the ones found in a textbook. Vance said that the fact that I say the questions verbally made it easier for him to understand, rather than just reading a mass of words on sheet of paper or screen. He also mentioned that, in the MMP, he perceived me as being immediately available for help. When I asked Vance as to whether he was better at mathematics now he replied that he was definitely better, primarily because of the MMP as they had taught him how to problem solve, which had translated over into his regular mathematics work. In contradiction to his MARS-R scores, Vance stated that he felt better about mathematics class as well, due to the fact that he now felt that he could talk to anyone, students and teachers, about mathematics and not keep to himself about it. He enjoyed getting many perspectives and ideas from the morning problems and that had helped him in his being able to explain his reasoning behind his solutions.

When I asked Vance about his MARS-R scores, he was very surprised to learn of the increase in his MARS-R scores. I asked him if he felt if the MARS-R wasn't an accurate reflection of his feelings towards mathematics. To which he responded:

> "Well, umm, I really enjoy math, but it's just, tests in general I usually find scary because, I dunno, I'm just worried of like getting a bad grade, because I never usually gotten a bad grade, so I'm afraid of getting my first bad grade."

He goes on to state that he's always been afraid of tests and that maybe this MARS-R was a result of that anxiety as he feels he is more stressed about tests now than in prior years.

When I asked Vance about the effect of the MMP on his mathematics skills, he reiterated that they had definitely helped him with his mathematics skills, especially in the area of problem solving. When I had asked him if he could recall a favourite problem, after some thought, he listed a problem that we had done recently as a class where there were 5 voters voting for two parties, each voter numbered 1-5 had that many votes (voter 1 = 1 vote, voter 2= 2 votes etc.) and the students had to arrange the votes that any person changing their vote caused the other party to win. I had observed that Vance had really engaged with this problem, coming up with some clever ways of solving it. He stated that his enjoyment came from the challenge of this question, and cited that the "easy format" or open-ended nature of the problem had helped place it as his favourite one.

When I asked him about the randomized groups and whiteboard aspects of the MMP, Vance stated that working in groups had really helped him with his mathematics abilities because he had more than just his thoughts going into the question

> "…because instead of just my thoughts being in one question, I can get 3 or 4 different perspectives on one idea, so every single math [question] it's helped me with my overall math because it gives me a more perspectives than just one."

When we got into the randomized nature, Vance mentioned that sometimes it was more difficult with certain people than others, but stated that in the groupings where there was more diversity in skill level, he learned more from the question as the variety of perspectives and ideas was greater than if all the students in his group were at the same level. He liked the randomizing as it increased the chances of getting all skill levels in a group, and thus a better chance to learn more.

In regard to the whiteboards, Vance stated that, by being able to look around and see everyone else's work, he was able to increase the number of perspectives for a question. He then credited this being able to see everyone's mathematical perspectives on the whiteboards with broadening his exploration of the MMP and making his regular mathematics work easier. I asked him if it ever helped him when he was struggling:

> "Yeah, I think so, because it's like we're all in it together instead of just one piece of paper and one person just writing it down right?"

Case 2: 'Luther'

'Luther' was an interesting case and would have been ideally suited for a program like *Imagine* had he been able to maintain a steady work ethic. However, my co-teacher and I had been warned by his previous teacher and by Luther himself that he was a serial procrastinator and vehemently disliked mathematics. Luther was an engaging and enjoyable student, who had no issue relating to his peers or teachers in class, but was prone to bouts of insecurity during which he would seek validation from me and my co-teacher. These bouts would normally occur near end-of-term report cards, when Luther would be reminded that he had fallen behind due to his course inactivity. He had been dealing with some major issues at home from which he had taken solace in discussing with me and my co-teacher, and occasionally he would use these issues as his reason for not having completed some module or assignment. Given the nature of the things that he was dealing with, we tried not to push Luther too hard on this front, fearing that if he no longer considered *Imagine* a safe space to come to, then we might lose him entirely. While his attendance was relatively good, he would be absent from classes for weeks at a time, showing up once or twice during that time and with a story of what had recently occurred to keep him away.

Luther's *Math Autobiography* was very long; it seemed that he was eager to tell me how much he disliked mathematics. His autobiography listed some major critical incidents that had affected his view of mathematics and his ability in it. Interestingly, he listed his experiences in mathematics grade by grade, citing a different incident with his teacher for that year. Most often Luther would talk about how bad a teacher had been at teaching a mathematics concept so that he didn't learn it very well. In one particular incident, he talked about how as a joke he had "taken a bite" off the corner of a mathematics test, and how the teacher had made him redo the entire test despite, in his words, having aced it the first time. Luther continued to list off different mathematics subjects that he had difficulty with from grade to grade, but mentioned how his father was "amazing at math" and was able to help him with his homework. In another incident, he talked about learning fractions and enjoying it a lot more than his other mathematics experiences due to his ability to apply it to his everyday life. He ended this anecdote with talking about how it didn't matter that he was good at fractions that year anyway, as the

teachers went on strike and he never received a grade for his efforts. Luther went on to describe coming to FVDES (the year before his enrollment in *Imagine*), and talked about it as a turning point for his mathematics, even though he was perpetually behind. He felt this way because he had been introduced to "so many new things", such as finding the area of a circle.

Luther's initial MARS-R score was fairly high, being 45 overall. Many of the individual categories were rated at a 3 or higher, and only a few were rated lower than that. His highest categories were "Reading a formula in a science text", "Thinking about an upcoming math test one day before" and "Waiting to get a math test returned in which you expected to do well" which were all rated at a 4. "Picking up a math textbook to begin working on a homework assignments", "Being given a homework assignment of many difficult problems" and "Getting stuck in a math problem" were all rated at a score of 5. The January MARS-R showed a minor decrease in Luther's overall mathematics anxiety as it had dropped 5 points from his initial score to a score of 40. Notable decreases had occurred in the categories of "Getting stuck in a math problem", "Taking a test in a math course" and "Reading and interpreting graphs or charts" which had each decreased by two points. "Reading a formula in a science text" had decreased by three points to a score of 1. There were some notable increases in this score as well, "Looking through the pages in a math text" had increased by two points to a score of 5 and "Listening to another student explain a math formula" had increased by three points to a score of 4. His final MARS-R taken in June showed an increase of 10 points from his initial score to a score of 55 overall. Some categories remained unchanged from the start of the year. Notable increases were in the "Listening to a student explain a math formula" and "Being told how to interpret algebraic statements" both of which had increased by four points to a score of 5. Two categories had decreased by two points each; "Getting stuck in a math problem" and "Picking up a textbook to work on a math assignment" were now at a score of 3. Luther rated the 16th category in the June MARS-R at a score of 5 out of 5

Luther's MARS-R scores were somewhat surprising as I had observed him to be relatively engaged in the MMP and I had developed a good rapport with this student. However, at this time, there had been a quite a number of negative life experiences that had been plaguing Luther and affecting him even more in his school life. At the start of

the year he had enrolled in *Imagine* with the intention of “only trying it”. His opinion changed after the first class and he was seemingly excited and enthusiastic about the prospect of coming to *Imagine*. he was at first a distracted group member when it came to the MMP, but over time he seemed to get more involved with the actual solutions to the problem and less about entertaining his group members. About halfway through the school year, Luther came to me about having students create their own MMP, and tossed his idea at me of “how many circles could fit in a square?” or vice versa. I had thought this a pretty good problem and I told him as much, which seemed to surprise him that I would say that. It seemed that Luther’s confidence was increasing in mathematics throughout the year as well. He would often opt to try his own online work on the whiteboards if he was stuck, stating that it helped “wake him up”. When I had been encouraging students to take the Waterloo mathematics contests, Luther had surprised me yet again by being the first in his grade to volunteer. He opted to work on a whiteboard instead of a piece of paper, which I allowed. He scored about the same as the 4 others in his grade who also took the contest that day, which really surprised him as he considered himself “the dumbest one in [his] grade”. Luther started missing a lot more school towards the end of the school year, and started getting more and more despondent due to several personal events having to do with his home life. This was compounded by the fact that it was starting to look like that he would have to repeat the grade due to not having completed enough of his academic work. Luther did manage to pass his grade after completing some major assignments, but it was very close. He finished with a C- in mathematics.

For all the details listed above, I felt it necessary to interview Luther and get his views. It had seemed that he had enjoyed the MMP, but struggled with the regular mathematics and seemed to resent its having to exist. He also seemed to have improved his attitude towards mathematics overall, but then regressed in his view despite being an active member of the MMP.

When I asked what had changed for him in mathematics, his answer was immediate that he disliked it more now than before and felt it was more of the same as last year just harder and that he resents it. He did state that he enjoyed the mathematics that made him think such as reducing fractions or finding patterns, but that was it. He was only referring to his mathematics course, so I asked him about the MMP, which he said were “really fun” and unlocked a whole new way of thinking about mathematics for

him. He also mentioned that he learned a lot of skills that helped him in his regular mathematics. This puzzled me a little so I then asked him what his least favourite aspect of mathematics was. He disliked intensely “unsolvable” problems. He then went on to describe one of our earlier MMP’s we had done about 2 boys and a man trying to get across the river. He cited that one because he couldn’t figure it out. He likened this question to the order of operations BEDMAS routine. I asked why this was and he stated that he liked doing mathematics his own way, which some of the MMP’s allowed, but things like BEDMAS were very methodical. I then asked if he felt better about mathematics now than he did at the start of the year, to which he emphatically replied that he definitely felt better about it now as he felt his skills had increased due explicitly to the MMP’s. He stated that he never really thought about mathematics questions before and now he was thinking up his own MMP’s for questions that he would struggle with in his regular online mathematics work, which would help him get over his mental blocks.

When I asked if he felt better about mathematics class, Luther stated that he felt worse about it now, as he was so used to how *Imagine* did things, he feared having to go back to a regularly paced classroom environment and not “get the chance to know what he was doing”. On this front, we started talking about his MARS-R scores throughout the year. I asked about the listening to another student category increase; Luther then explained that he sometimes didn’t understand what another student’s explanation of the problem was, and that would make him anxious because he kept imagining a “what-if it had been for marks scenario”, which was the cause of his anxiety. He went on to state that from the MMP’s, he realized how much he didn’t understand and that has caused the increase in the MARS-R. I asked him about the decrease in getting stuck in a mathematics problem:

> “I don't know why I feel better about it now, I just feel like if I'm getting stuck on it I'm thinking about it more and that way I feel that if I get actually stuck on it I'll get the right answer because that means I'll be thinking about it more and I won't just be speeding it through and I'll be making sure that I've got all my stuff right”

When I asked him about whether the MMP had helped him with his mathematics skills, he said that it had with so much more than just mathematics. This was a distinctive answer similar to ‘Karla’, so I asked for clarification. He goes on to describe how it has helped him be a better strategist when it comes to his computer games where resource

management is a key game mechanic. I then asked if he had a favourite question that we'd done. Luther listed three different questions we had done throughout the year, stating that each one was one of his favourites because he had found some good and interesting answers to them.

I then asked Luther about the overall increase in his MARS-R, as everything he had been telling me should have indicated a decrease. We went off the record where he described a large number of negative life events that had been affecting him. He then confirmed that the June MARS-R score was more of a reflection of his world at the moment and less of his view on mathematics. I thanked him for his honesty and went on to ask about working in randomized groups and whiteboards and whether that had helped with his mathematics abilities. He mentioned that he preferred working in groups of two, as three seemed like he couldn't contribute too meaningfully, but that he liked the randomizing and thought it was a very good thing because "it gives everyone a chance to work with different people and build relationships in the class…".

However, he mentioned that he wouldn't have minded working with his friends more often and stated that randomizing and picking your group were both good options. On the topic of the whiteboards, Luther stated that he felt like it was still cheating,looking at other students' work regardless of what I say, as he doesn't want to pick up bad habits to take into future classes that were not run like *Imagine*. When pressed, Luther stated that he "puts everyone above him [in skill level] in math", which encouraged him to "reach" and get better at mathematics, but when I asked if there might be any benefit looking at the whiteboards of other students, he said:

> "…just looking other students work, I think it would help that because I would feel like if I think that everyone's doing better than me and then I've got let's say I'm behind [in a question] on the whiteboard, got it turned around doing the question and then like everyone shows their questions… and then [it shows that] not everybody is better than me… that would make me think that I'm reaching what I wanted to reach and that is to be doing better than what I would normally be doing"

Case 3: 'Penny'

'Penny' was an extremely reluctant participant in anything the *Imagine* program tried to do as a class. Similar to 'Toni', she was a very reluctant mathematics learner and a firm opponent to participating in the MMP. Penny had been held back a grade level in

mathematics, but was at grade level for every other subject. As a result of this, over the course of the year, she was working on trying to catch up and complete two grades of mathematics. During the course of the year Penny had missed about 40% of the *Imagine* classes; her parents would always email us telling us that their daughter would be missing school that day. The reasons were always varied. Interestingly, Penny never fell too far behind in her schoolwork, but did not excel in any one area, always maintaining a passing grade in all her subjects. My efforts to get her to participate in the MMP were always met by exclamations of "I didn't understand the question" or various other reasons why she didn't want to participate.

Penny's *Math Autobiography* can be considered very negative in terms of views towards mathematics and lists a number of influences from her home life and experiences that reinforce her views of mathematics as being a malevolent "other". She started by describing mathematics as very hard for her and making her confused. She then described how her parents were not very good at mathematics and would consistently tell her that mathematics had changed from when they were younger and was "foreign" now. She then described that she felt she has a disorder in mathematics because she "sucks" at it. She then listed two separate incidents with teachers, one negative one where in her grade 8 year the teacher scolded her for asking for help and another more positive one from grade 5 where the teacher had helped her and she had achieved a good mark in mathematics that year. She then went on to describe how meaningless learning school mathematics is, stating that "no one is going to stop you in the street and solve an algebra problem!" She then stated that she had no desire to go into any career that would involve mathematics, and then ponders if being a veterinary technician requires mathematics. She finishes the autobiography by detailing a learning assistance block she had been put into for mathematics, and how it didn't help her with that subject. She then wished that she was good at mathematics, describing how she was made fun of by people for not being capable in the subject. She ended again by questioning the real-world relevance of mathematics and how much it bugged her that she could not figure out mathematics no matter how hard she worked and wondered how people could just be so good at it without seemingly trying.

Penny's MARS-R scores seem to describe her feelings towards mathematics as stated in her *Math Autobiography*. Her initial MARS-R score was a 52 overall. Ten out of fifteen categories were rated at a 4 or a 5 and every other category was rated at a 1 or a

2. Interestingly, each low-rated category had to do with reading or using mathematics text books, except for “Walking into a math class” which was also rated at a 1. Her January MARS-R showed a large increase to a score of 65 overall. Almost every category that had been low-rated (belonging to the textbooks or graphs) noted an increase of two to three points each. However two categories did see a decrease of one point. Both “Watching a teacher explain a problem on the whiteboard” and “Listening to another student explain a math formula” decreased from a score of 5 to a score of 4. Penny’s June MARS-R showed a decrease from January to a score of 54 overall. This was a mild increase from her initial MARS-R, but her inclusion in this category has to do with the January increase mostly. “Listening to another student” and “Watching a teacher explain…” were still one point lower than at the start, but all other categories had not changed except for “Picking up a math textbook to begin working on a homework assignments” which had gone up 4 points from her initial score. In the 16th category that I had added on the June MARS-R of “Listening to or solving a word problem” she rated a score of 4 out of 5.

I had not been able to create a good rapport or engage Penny meaningfully through the course of the year, despite my efforts. Her overall attitude towards teacher support seemed to be dismissive and it appeared that Penny only wanted to be left alone to work on her online materials. This was in whichever activity the class might have been engaged in, not simply the MMP. Penny’s attendance in the first half of the year was fairly poor; however, on those days where she was absent, she would submit online assignments from home, meaning that she was working on her school work, just not in the environment of *Imagine*. After some calls and emails with her mother regarding the repeated absences, Penny’s attendance improved in the latter half of the school year, although it seemed that Penny was not happy to be disturbed from just simply working on her computer. I kept trying to engage her in the MMP, but she would consistently gradually edge her way away from whichever group she was in and find her friend in the class and talk to her, pulling her off task as well. When I tried to re-engage her in meaningful conversation about the problem, it generally proved fruitless as she would barely respond with anything except for “I don’t understand”. She passed the school year with a C- overall, but only for the grade that she had been held back in. Unfortunately, she is still a year behind in her mathematics.

For all these reasons, I felt it necessary to talk to Penny, as she had not responded well to the classroom activities, the MMP, or anything that the *Imagine* program had tried to offer. I felt it necessary to find out what I could have done differently to have engaged her in the MMP or what she felt had been missing from this intervention.

Penny agreed to the interview, seemingly surprised that I would want to talk to her about the year. I asked Penny what had changed this year for her in mathematics, to which she replied that not much had changed because she's always been bad at mathematics. She also mentioned that, whenever she felt like she was about to "get the hang of something", that the class would inevitably move on to something else and frustrate her further. Asking her about her least favourite aspect, she replied "Everything!" and that there was nothing that she viewed in mathematics favourably, but she had no real reason why. She went on to say that she felt that she was at the same skill level as she was in the beginning, as she feels that no matter what she does, no matter how much help she gets, she never "levels up". I then decided to ask her if she felt better about mathematics class, to which she replied that she does, but only a little. Her reasons were for the small class size that *Imagine* offers. However, she felt that it has not helped her in mathematics. She went on in this vein talking about how she dislikes a large class as she often feels forgotten and a small class makes her feel better about herself as she won't be "judged as hard".

This was interesting so I pursued this; I asked her why this was such a source of anxiety, to which she replied:

> "I don't know it is I feel like if I mess up someone's gonna get upset with me and try to push me to my limit where I can't really do anymore and I'm trying my best."

We moved on to her MARS-R and the reasons for the perceived changes. However, when I asked her why she had had such a large increase in January, she declined to say anything regarding it. I urged her to verbalize her feelings, to which Penny said that she perceived *Imagine* to be too mathematics-focused, which had kept her anxiety high all year and this was monumentally different from her experience in other schools where she perceived the focus to be on other academics. She said that with this being the focus here, and her seeing how good all the other students in *Imagine*

were at mathematics made her feelings of inadequacy increase. She then said that she wished that she could be good at mathematics and “pass it all the time”.

I asked her if she had never witnessed her fellow students struggling or not being very good with mathematics as well, to which she replied that she had not seen that. This was surprising to me, given everything I had observed about the *Imagine* class, I told her that there were many people who struggled with the mathematics just as much as she did and then I asked what would have helped her understand her peers’ struggles.

> “I don't know… maybe working on a complicated question as like a whole class or something?”

When I pointed out that this was what we had been doing every morning all year, she said that the questions from the MMP were just too different from the actual work in her online courses, so she couldn’t bridge that gap. I asked her if there were any questions that she had enjoyed from the MMP’s and she mentioned the one we had done just recently where you had to flip pairs of adjacent coins in a row of 9 to get them all to the same side. She said that she enjoyed the challenge of that one and had to push herself to figure out how to do it.

I then moved on to ask her about how working in groups and at the whiteboards had helped her. She said she disliked working in groups as it made her more aware that some people were better at mathematics and that she felt as though she was holding them back. Regardless of the group, she always felt that she wasn’t smart enough. She also disliked the randomizing aspect as it would occasionally put her with “smarter” people. She would have preferred to have been put in groups that were at her skill level as she felt that would have helped her more as then she would not be nervous to get a question wrong and limit the judgement on her. This feeling of judgement was also linked to her dislike of working on whiteboards as she felt that students would view her work and judge her for her inability. She did mention that she never felt as though myself or my co-teacher ever judged her negatively, just her fellow students.

Interestingly, I asked her if she had ever looked around at her peers work during the MMP to which she said she had. She then said that it helped her a little bit seeing the other students work:

> "Because I didn't feel like I was getting it wrong and when I see someone [struggling] and I'm kinda struggling it made me feel, okay well everyone isn't perfect and I can see they're struggling a bit too…"

4.3. Summary

As can be seen from the descriptions above, most students took some benefit away from the MMP intervention. From the case studies, those students whose anxiety fell over the year felt better at their academic mathematics and more confident in their abilities to do mathematics. Those students who had maintained the same overall score of anxiety did see notable decreases in some key individual categories related to the social context of mathematics, but increases in other areas and those students whose anxiety increased possibly increased as a result of factors beyond the control of the classroom which no mathematical intervention could have prevented. Each student's experience with the MMP came about as a result of their engagement with the problems throughout the year and personal factors. Several themes become apparent across case studies though and these will be discussed and analyzed more in chapter five.

Chapter 5. Results across Cases

In the previous chapter, I discussed the *Imagine* class' experience as a whole and then individual students' thoughts and experiences with doing open-ended tasks on vertical non-permanent surfaces every morning called the MMP. It becomes apparent that there exist some strong parallel themes amongst the different case studies. What follows is an analysis of several of those common themes, the sorts of changes to students perceptions of mathematics and an analysis of whether or not there has been a lasting change in these students' affective domain.

5.1. Common Threads to students anxiety relief

From the previous chapter, it has become apparent that those students who have had negative experiences in a mathematical context have developed anxious feelings and a negative affect towards the subject of mathematics. Each student in the case study whose autobiography described some negative experiences or trends had a MARS-R score that was above 30, indicating that they were more anxious about mathematics. In the case of 'Jonathan', we saw his autobiography include an anecdote of how isolated his teacher's methods had made him feel, and how he had felt his skills in mathematics to be lacking due to this experience. His MARS-R verified that teacher interactions and skills were an area of anxiety rating high scores in categories related to teachers and mathematics problems. This was similar for each other case discussed in which the student's *Math Autobiography* indicated a negative perception of mathematics. This theme mirrors a study finding that elementary school teachers who had negative experiences and memories from their earlier mathematics schooling affected their views on it as pre-service teachers (Kaasila, 2000). Students' prior negative memories of mathematics had affected their current views on the subject causing them to dislike the subject more for the associations that they have brought from their past experiences. Utilizing MARS-R in conjunction with their *Math Autobiographies*, provides insight into how severely these memories are affecting a student's anxiety towards the subject. In the case studies, it's clear that those students with the most negative views of mathematics can offer an abundance of negative experiences or no positive experiences in their *Math Autobiography*.

Conversely, the cases of ‘Freya’ and ‘Vance’ demonstrate what the existence of positive memories associated with the learning of mathematics can achieve, especially when those memories are of close friends and family in the support of the student’s learning. Both Freya and Vance described extremely positive social and familial support when describing their earlier experiences learning mathematics. In Freya’s case, this came in the form of a strong friendship created in mathematics class, and for Vance, it was a warm retelling of the support he received from his family when he struggled with his mathematics. Their MARS-R’s indicate a low level of anxiety that correlates to these positive memories and associations. This seems to align with previous findings that students’ beliefs about themselves and the subject are prone to the influence of their emotional experiences and can fall into positive, neutral or negative categories (Hannula, 2005). These positive, neutral or negative categories are reflected in students’ initial MARS-R scores and can provide a window into specific areas that might be affecting individual students or the class as a whole. This relationship between the MARS-R and the events recorded in the *Math Autobiographies* is strong, and helped students begin the reflective process and guided my teaching and selection of mathematics problems to help reduce these students’ noted areas of anxiety.

Looking to which themes connect the students who experienced a decrease in their anxiety, from my observations and the interviews it seems that all students benefited from the MMP in one way or another. However, Jonathan, Ellie and Toni all started in very different places in terms of their MARS-R scores and their *Math autobiographies*. Their personalities are all very different from one another, so identifying any one particular character trait that would cause such a response to the MMP interventions does not seem to be possible.

One theme that is prevalent among the three students who experienced a decrease in their anxiety is the amount of “outreach” or interaction that I had had with the student throughout the year. It has been known that greeting students at the door every morning does potentially increase student success in remaining on task for longer periods of time (Allday & Pakurar, 2007). It is also known that having students understand that you care about their success and are visibly supportive will make it more likely that students will respond to you with greater academic efficacy and an improved view of their academics (Jansen & Bartell, 2013). It is possible that responses to some of my questions regarding whether the students feel better about mathematics class from

this group indicate that they do feel that their teachers care about their success and their actual learning of the concepts that are being taught. Given that Jonathan and Toni both candidly discussed negative teacher experiences with me show that they perceived themselves to be in a safe environment created in the *Imagine* program. This perhaps allowed for them to participate in taking risks in the MMP and allowed them to witness and receive mathematics help from myself and my co-teacher. This appears to have made some inroads in repairing Jonathan and Toni's mathematics anxiety. In Ellie's case, it seems that she benefited similarly, as she stated in her interview that the MMP really made her value mathematics more as she saw her teachers in the MMP value more than worksheets. She also credited the more social dynamic with helping repair some of the anxiety that she had developed from her prior class's peer bullying that had caused her to leave that school in the first place. Additionally, each student had made a realization from looking around the room that they could help themselves and their groups by viewing other groups' work on the whiteboards as it was being done to help them along in their solutions. This seemed to have an effect on these students for reassuring them that even though they were currently having troubles with their problem, so too were the others in their class. Although Jonathan, Ellie and Toni seemed to be the most aware of this, and derived the greatest benefit of the nine students I interviewed, each case study student reported somewhat similar realizations to varying degrees. I will discuss this similar theme and its lack of effect in the cross-case analysis of the 'no change' group and the increase in anxiety group.

The question of whether the students who had little or no change in their reported MARS-R's benefitted from the MMP intervention is answered in the case studies of Freya, Karla and Nick. The common theme in this group is that each student reported that they took away some new skill or increased sense of confidence from having been a part of the MMP. This indicates that there has been a positive impact to their affective domains, even if their reported anxiety from the MARS-R has not changed in any significant way.

Since Freya's MARS-R started low at 21 points overall, it would be difficult for her score to go any lower. However, it seems from my observations of Freya throughout the year and her interview that she derived some benefit from the MMP's in a variety of areas. Freya had mentioned how she now felt a much deeper appreciation for mathematics and that she "put more of herself" into the subject. Even starting at a place

where she had a fondness for the subject, she ended up enjoying it even more as a direct result of having done an open-ended problem or task every morning. Even more telling is that she feels that her skills have improved a great deal this year due to the fact that she learned a whole variety of new skills and techniques in the MMP's that she utilized in her regular mathematics as well. This speaks to an increased confidence in Freya that despite her consistently low MARS-R score shows that the MMP has had a net positive effect on her.

The influence on Karla's affective domain is also a positive one in regard to the MMP, despite her MARS-R remaining consistent. Karla's MARS-R remained constant in her overall score.However, she saw significant decreases in two categories of "getting stuck in a math problem" and "listening to another student explain a math problem". These decreases were erased by single-point increases in a variety of other categories. However, it seems that there was an effect on how Karla felt about different aspects of mathematics. Given that she was not the most social of students in *Imagine,* and had tried to purposefully miss the MMP's specifically due to a fear of being the person to bring down the skill level of her group, these category decreases are all the more important and relevant. Speaking to Karla regarding her feelings surrounding mathematics, she indicated that she now felt more confident in mathematics, similar to Freya's feelings, and that the skills she learned in the MMP had transferred over to her regular mathematics and allowed her to think about mathematics in "a more problem-solving way". This speaks to an increased confidence in Karla in this regard, even though she admitted that she still felt anxious when in group situations. This part of her anxiety is shown to have at least decreased in the MARS-R, but due to her thinking forward to the next year in mathematics, other categories increased to erase any single decrease in the MARS-R.

This theme of benefit to affective domain in mathematics despite no change in the MARS-R score is more difficult to see in terms of Nick, whose confidence in his own abilities seemingly never wavered. However, he did note quite a mistrust of his teachers and their competencies in his autobiography and in the interview, citing how his previous teachers had not known what they were doing and as a result treated him as "stupid". In the interview, Nick had talked about how often that he would have wanted to call me over and get a new or different problem and was often disappointed with my responses, stating that he had already thought of those but not placed them on his group's

whiteboard. He also mentions in his interview how he had never had good mathematics teachers prior to the *Imagine* class. A trust in his mathematics teachers as competent caring individuals is definitely a positive change in his affective domain as being able to trust that the authority figure in your classroom wants to help you is part of the interpersonal care (Jansen & Bartell, 2013) so integral to caring instruction. This in turn should help him start from a better place with his future instructors and his fellow classmates, as his MARS-R decreased by 4 points in the category of "Walking into a math class".

Those students whose MARS-R increased show several themes that are worth analyzing; each student whose anxiety increased had external factors that affected their MARS-R's in a negative way, each student seemingly derived some benefit from the MMP and the environment it created, and each student was not quite able to get over their anxiety regarding some regular aspect of a mathematics classroom environment (e.g. testing). Starting with external factors, the case of Luther shows a particularly strong influence of home life and its effect on a student's mathematics anxiety. Luther derived a great deal of enjoyment and benefit from the MMP's as indicated in his interview. He stated that he enjoyed certain types of the MMP's and the social aspect of them as well. He states that now when he approaches his regular mathematics and gets stuck, he tries to approach them as though they were a MMP instead, using the skills that he derived from them. He also took a large chance in volunteering for the Waterloo mathematics contest, despite his reservations regarding his own abilities as compared to the other students who also volunteered. This all speaks to a positiveimpact on his affective domain. In fact, were one to look at his January MARS-R, we would see that the trajectory that he was on indicated that he was feeling less anxious about mathematics, however his large increase in the June MARS-R indicates a large amount of anxiety surrounding the subject. Luther did indicate during his interview that his increase in anxiety might have more to do with the number of severe negative life events and non-school related stresses that he was experiencing. He also indicated that he was thinking about how next year he might not be in the *Imagine* program and given that he stated that the MMP's gave him insight into several of his perceived inadequacies in interpreting his peers mathematics explanations, this might account for the increase. Nevertheless, he stated that he will carry forward very positive memories of having done

mathematics this year, which can concievably act as ballast against whatever mathematical forces he encounters in his future education.

External factors such as parental influence also had an impact on those students whose anxiety increased. Both Penny and Vance had heavily listed their parents as major influences on their views of mathematics in their respective *Math Autobiographies*, however the difference between the views held by Penny's parents and Vance's parents definitely influenced their overall perceptions. External forces in this case can come in the form of parental influence and support. Penny's *Math Autobiography* clearly stated how her parents had helped her form her adversarial view of mathematics, whereas Vance's parents were detailed as warm and supporting of Vance's struggles with learning mathematics. This would seem to be represented by both students' initial MARS-R scores, as Penny's started at 52 out of 75 and Vance's was 25 out of 75. This seems to confirm that parental views are a very strong determinant of children's view of their surroundings (Frome & Eccles, 1998), especially in regards to mathematics. However, the reason for both of these students' increase in anxiety over the year is due to different external factors.

Penny had the most averse reaction to the MMP intervention out of all the students in *Imagine* and indicated that she still sees no point to doing mathematics and reports no increased enjoyment or ease of mind when approaching the topic. This is in stark contrast to Vance who by his own account and from my observations had thoroughly enjoyed both the MMP's and *Imagine*. In his interview, Vance insists that he is a better mathematics student and has a deeper appreciation for the mathematics as a result of the MMP's. Despite these two contrasting opinions and perspectives regarding the MMP, both students MARS-R increased. The increase appears to be due to external factors that were not affected by the *Thinking Classroom* that was fostered by the MMP.

The peer influence of the MMP was lost on Penny, but was of benefit to Vance. As indicated in their separate interviews, Penny viewed each student in the *Imagine* program as more competent in mathematics, rightly or wrongly throughout the course of the year. Vance reported in his interview that being exposed to his peers thinking and methods was a direct benefit to his views on mathematics. This means that the increase in MARS-R scores must be due to different external factors. Penny's negative views could be accounted for by the repeated absences that were seemingly sanctioned by her

parents without excuse. Given that Penny felt that the *Imagine* program was so mathematics focused, and she herself and her parents had the view of mathematics being "foreign", it is possible that these absences were a result of trying to miss the environment of the MMP entirely as Penny would often work from home on her assignments as though she were simply an online student. This is an strong external factor that may have inhibited any effect that the MMP might have had on Penny's overall mathematics anxiety and her affective domain. Vance's increase was due to a very different external factor. He started off with a low MARS-R score of 25, and then began to increase in his January and June MARS-R's. Vance's "external" factor in this case that may have contributed to his increased anxiety is his unrelenting fear of doing poorly on mathematics tests. These were areas of large increases on his MARS-R and also mentioned in his *Math Autobiography* and stated in his interview. He fears getting "less than 80%" on his tests, and identified this as the reason why his MARS-R increased. The MMP had little to do with reducing test anxiety, as the assessments in the online program that Vance was enrolled in were all traditional mathematics tests of multiple choice and short answer questions.

These external factors may have played a large role in how these students responded to the 'Morning Math Problem' intervention. For Penny, there were some positives from the intervention as indicated from her interview. As her attendance improved, she participated more in the MMP, and her large January increase on her MARS-R from a score of 52 to 65 came back down to a score of 54 in June. She could also list a favourite problem, which as we will discuss later is a potential sign of a positive impact on her affective domain for mathematics. However, her parents' views towards the subject still remain as an obstacle to a permanent change outside the classroom. Vance's case shows the influence that routine features of a mathematics classroom, such as tests, can play in the role of mathematics anxiety. Where Penny's anxiety increase was a result of her parents' negative views, a fear of failure on tests, and perhaps disappointing his parents seemed to be the main cause of Vance's anxiety increase. Over the course of the year it appeared that Vance was steadily becoming more anxious regarding mathematics as indicated in the MARS-R. Vance seemed to be unaware of this increase as he was surprised by his trend and stated in his interview that he disagreed with the assessment. While he may disagree, the MARS-R efficacy in describing student mathematics anxiety is well established, and while his anxiety

surrounding certain parts of mathematics may have increased, Vance will take forward many positive associations and skills from the MMP's that should help him with his regular mathematics. However, his desire to get above a certain mark in mathematics and on tests will continue to be a source of anxiety for him.

5.2. Students' Perceptions of Mathematics

A theme which appears to be recurring among all the student cases is that of their perspectives on mathematics and how they were affected by the MMP's. In almost every case the students mentioned that getting a different view of the problem from working with others in doing a mathematics problem had helped them broaden their ways of thinking about a problem. For most cases, this seemed a positive influence in students as it had given them a plurality of ways of thinking about a mathematics problem instead of focusing solely on a single method of finding a solution. This theme is exemplified by almost all of the case studies where students reported that the interaction with their peers in randomized groups had helped them to view mathematics as more of a dynamic problem solving activity, rather than a solo activity where they bore sole responsibility for a correct solution. This focus on the importance of perspectives might have been due to an increased intuition in mathematics felt by my students from exposure to so many perspectives when problem solving. This increased intuition means that an increase in confidence in their own abilities follows which is an essential component of lowering students' mathematics anxiety (Steele, 1998).

As examples of this type of new-found confidence from the case studies, Jonathan felt that being partnered with different people each time there was a new question to explore provided an opportunity to work with people he might not have normally and see that problems could be solved in a different ways. Freya had felt that her mathematics abilities had been helped by the presence of students at all skills levels, whose different opinions and skills could "all come together and form a solution", because it had made her realize that "not every problem could be solved with the same technique". Karla, whose nervousness at being thought of as bringing down the group, credited the experience of having multiple perspectives as having helped her skill level as she wanted to "… take all the explanations [she] could get". Even those students whose anxiety increased discussed how the various perspectives offered by other students had been impactful to them. Vance, whose surprise at being included in the

increased anxiety category, had told me that his feeling of personal efficacy had increased due to having worked with such a variety of students in his class. That in this type of work, he had increased his ability to communicate his mathematical ideas and solutions more articulately due to having been exposed to so many different perspectives on the MMP's than just his own.

Luther's realization that other students' explanations were difficult for him to understand also fits this theme, as his mature realization that his own understanding of the mathematics behind a question or concept may be inadequate. To him the realization of this had been a source of anxiety, however, it is my belief that had he been able to admit this to his peers, they might have made more of an effort to help increase his level of knowledge, and as a result lower his anxiety towards mathematics. For him, working with other people was more of a benefit for building a stronger community in class. Added to this, he stated that he disliked those questions where there was only a single method of doing mathematics (e.g. BEDMAS). As working in the environment provided by the MMP afforded multiple methods of solving the same mathematics problem, this demonstrates the importance of having students provide other perspectives in a problem solving environment to help lower student anxiety.

Penny, whose negative affect towards mathematics had been almost unaltered through the course of the year, had mentioned how her working in randomized groups had been a large source of anxiety for her as she had felt that each student was smarter than her. Her inability or unwillingness to see the problems from other students' perspectives or even to venture her own due to her anxiety of being "judged" had caused her to be blind to this and perhaps had an effect on how different people have different methods of solving problems. The question of why there is such an importance on being exposed to many student perspectives may be in part to a type of "diffusion of responsibility" for any one student having to find a correct method. With such a variety of students' opinions and views at their disposal now, each student no longer feels that he or she have sole responsibility for being incorrect or attempting to find the correct answer (Steele, 1998). This helps students feel a sense of relief whenever they are doing mathematics as they have come to the realization that there exists multiple and diverse ways of solving a problem and it's always easier to find multiple solutions than just a single one.

An interesting theme that came up when I asked the students I interviewed as to whether or not they had a favourite problem was that each student could recall in some detail a favourite problem or one that they had enjoyed doing. While there was no "common" problem or problem type across cases, each student could, in thinking of their favourite questions, recall the fine details of the problem, their solutions, and their rationales for their solutions. These were still present in their minds even after a fairly long period of time had elapsed since they had done the problem. This was the general phenomenon that I had also observed in the classroom as whole when I began to note the change in the overall affect of the students in the *Imagine* program. That was student engagement of the problems even beyond the classroom. It started becoming more common in class that students would tell me that they had talked about one of the MMP's with their family at dinner or had tried to stump their parents or older siblings with it. This speaks to a positive association of the memories and feelings involving mathematics and of having worked these problems and found solutions to them, which in mathematics can be a difficult thing to achieve.

In some cases, students were surprised by their own recall of a problem; for example, Jonathan had not thought he could recall a problem, but then immediately remembered a problem and specific details regarding it including who he had been in his group. Ellie could remember having "a lot of fun" with her remembered problem, also recalling specifics of who she worked with and details of the problem. Toni, who had strong opinions regarding mathematics and her ability, listed her least favourite problem, but then listed what had been a very positive problem for her, citing its real world relevance and how that had been impactful for her in mathematics. Each of these cases are examples of a diminished anxiety according to their MARS-R's scores; each student talks about how they had discovered something or had fun finding a solution. This "AHA" effect (Liljedahl, 2004) that each student had experienced has had anoticeable effect on their views and feelings of mathematics, which also seems to have had a positive impact on their feelings towards "regular" mathematics. Even those student cases who did not experience a significant change in their anxiety or experienced an increase in their anxiety according to their MARS-R could recall a favoured question from the MMP's.

Freya, Karla and Nick, whose MARS-R's indicated relatively no change in their mathematics anxiety could each recall a favourite problem and describe why they enjoyed it. In these three cases, the students, even if they couldn't remember the

specifics of the problem itself, could remember what the problem was, and why they had enjoyed it. In Freya, Karla and Nick's case, they associated their favourite remembered problems with positive feelings of having understood the question and its solution. In the case of Freya, she remembered her favourite problem in that she and her group had thoroughly explored "every" possible solution. In the case of Karla, who still struggled with the social dynamic of the MMP's, recalled her favourite problem as one in which she contributed highly to her group's solution. Nick's favourite problem was unique in that it was one that he had been unable to solve immediately, but since his opinion of his own abilities was so high, finding a problem such as this had given him a very positive "AHA" type experience when I finally did explain the correct answer. Most telling were the students who had an increased MARS-R anxiety score. Each could recall a favourite problem that they would describe as having found an interesting solution or method for solving. Vance's description was of a more recent problem that we had done in class, but he had discovered a number of solutions that had set his groups answer apart from the class. Luther, despite his increase in reported anxiety, listed 3 separate problems in which he had felt a sense of accomplishment when he had personally discovered something significant about the problem with his group. This seems to be critical for him in his supposed change in attitude towards mathematics, despite his protests that he feels worse about it.

Most surprising was the admission by Penny, whose views on mathematics were still overwhelmingly negative (for reasons that I will analyze later), could recall a problem that she had enjoyed despite it being a mathematics problem. It was a more recent problem, so it is probable that she felt compelled to find a favourite problem and listed the most recent one we had done as a class. However, she mentioned her reasons for enjoying it in the same theme as every other student case. That was she had enjoyed the challenge of the question and had discovered something in the process of attempting to find a solution for it. I would posit that if she had approached more of the problems in the year the way she had that particular question and had this positive affective experience, she may well have been included in one of the other categories. It seems that most students, regardless of final MARS-R outcome had had a "transformative" experience in the form of 'AHA' moments in the MMP. Their identification of a favourite problem is interesting for a regular mathematics class, as students can often feel confident when approaching the textual problems found in a regular mathematics class,

but it is not common, in my experience, for students to recall fond memories of the process they had undergone to solve one of those kinds of problems. It seems to provid support that the impact on the affective domain, and not simply just that concerning anxiety, of discovering something in these problems was enough for even resistant students to form positive memories that they can later associate with their future mathematics learning (Liljedahl, 2005).

Feelings of isolation were a prominent theme when I interviewed students. In most of the negative autobiographies, students would commonly cite experiences of being isolated socially or academically through the environments in their mathematics classrooms or through significant negative experiences. While these experiences may be also present in other contexts for these students, it seems from the *Math Autobiographies* that they had been especially impactful in a mathematical context. In Jonathan's case, he felt removed from his grade 5/6 classmates and made to struggle alone. While this had the effect of causing him to feel that he had missed essential information, the feeling of removal from his peers was a prevalent starting theme in his *Math Autobiography*. Ellie had described in her interview how she had always felt isolated in a group of people and didn't get to interact with other people very often. She then lists how she would feel that other students would judge her as being "dumb" as an even larger source of her previous negative feelings towards the subject. Even though her views of mathematics were benign, the social isolation that she felt in general seemed to cross-over into her feelings regarding mathematics class and affected her academically. Toni was adamant about her dislike for mathematics, and as I had found out in the interview, she had felt humiliated by her previous mathematics teacher's methods of getting answers from students by pointing students out and calling them individually to the board to solve a mathematics problem. She credited her change in view of mathematics class due to the "safe environment" of the *Imagine* program and the positive support that she received from my co-teacher and me whenever she had questions.

These feelings of isolation in a mathematical context were prevalent even in those students whose anxiety remained constant or went up. Karla had mentioned in her interview that the perceived favouritism she had witnessed in her previous class had made her feel like an outsider in mathematics class. This had contributed to a large part of her overall mathematics anxiety. From the point of view of Nick, it seemed that he

wanted be on the outside due to his own high opinion of his abilities. However this seemed to have had its genesis in the feeling that his previous mathematics teachers had considered him dumb. Added to this, Nick mentioned in his interview a great amount of anxiety through peer marking. The reason for this was that he didn't want to be thought of as dumb by his peers for any mistakes on the minor assignments or quizzes. This could be considered as a form of isolating himself in order to prevent the perception that Nick is not good at mathematics. Vance mentions in his autobiography that the help he received from his family had helped him not feel alone when learning mathematics, which had contributed to his initial positive views of mathematics. Luther had a different view of mathematics. He mentions in his interview how his default position is to view every student as automatically being better at mathematics than him. He uses this isolated position to motivate himself to get better at mathematics; however, it does cause a significant amount of anxiety when he begins to perceive that he is not as talented at others at mathematics. Penny consistently mentions how she always feels like she is the only one who is not good at mathematics. In her interview, she mentions a very interesting perspective that every student in the *Imagine* program is good at mathematics, or at least better than her. This sense of isolation in skill is a tremendous source of negative feelings towards the subject for her, as she feels that she is the only one struggling and wishes that she too could join the ranks of those students who are "just naturally good at math".

A part of the relief that each student felt towards mathematics and mathematics class had to do with the amelioration of these feelings of isolation. Not feeling like they struggled alone was the complementary and therapeutic theme to this sense of isolation that each student had felt in mathematics. Each student had cited a different aspect of the MMP's that had helped them get over the isolation they had felt in mathematics, but the importance of this newfound sense of "group" could not be ignored in the students' responses. Jonathan had mentioned that looking around the room at the other students' white boards had helped him feel less alone when he started to struggle with solutions. He also credited the group work itself with helping to feel better about mathematics overall with "I feel there's so many different ways of people interpreting things has kind of helped me understand what they all mean…". Ellie had stated that as a result of the social aspect of the MMP's she had expanded the number of people that she could talk to and interact with. She revealed that she had been bullied by her previous "friends" in

mathematics class whenever they found an opportunity, which had caused her a great deal of anxiety. Her anxiety regarding being singled out or not being picked for groups had been much reduced by her inclusion in randomized groups. Also her participation in the MMP's had allowed her to see other students struggling in mathematics as well which had allowed her to realize that if she was struggling then it wasn't a symptom of her being "dumb" as much as it seemed to be that the question itself was hard. She sums up her feelings memorably "…oh this person is having trouble with this mathematics problem and I am too so we're both okay". This is a positive turn around for a student like Ellie to identify her struggle in her fellow students, allowing her to feel relief towards her anxiety as it is no longer a "personal failing" in struggling with the question but something that all her peers can be clearly perceived to be dealing with as well.

For Toni, her relief came from the "safe environment" which had allowed her to feel as though she wouldn't have been judged as harshly for her incorrect answers in mathematics. She had mentioned in the same vein that the connection to the other students through the MMP had also made it so that she didn't feel as though she was the only one struggling with mathematics and that even the most accomplished mathematics students had some struggles similar to hers. Her realization that she was not unique in her mathematics struggles came about as well due to my visibility during the MMP's, moving from group to group helping students. She mentioned that this had helped her also realize that other students needed help and she wasn't the only one in need. In those students whose anxiety remained the same, the result was still a sense of relief when they realized that everyone had their own struggles with mathematics, same as them. Freya was already coming from a place where she had helped students with their mathematics (according to her autobiography), but worried that she would make them feel dumb were she not careful. The MMP provided an opportunity for her to work collaboratively and perhaps get over this fear. Karla had felt some amount of relief from her inclusion in the problems through the randomizing aspect as this had helped her not fear that she would be always selected last as it removed the personal choice from the students. Those students whose anxiety had increased mentioned similar themes of feeling less alone when struggling with the mathematics as a result of the MMP despite the indication on their MARS-R of increased anxiety towards mathematics. Vance had stated that from being able to work and see everyone's solutions on the

whiteboard that he felt like everyone in the class was “in it together”, as opposed to struggling independently or alone.

Luther hadn’t felt too much of a relief in his mathematical struggles from the environment afforded by the MMP. He had felt that it was a good way to build relationships in class, however, he never identified as feeling that he was not alone in struggling with his mathematics. If anything, it seems as though he views that his isolation had increased both socially and academically. This may have had to do with his self-admitted habit of immediately placing every other student’s skill level above his own in order to motivate himself. Penny provides an excellent perspective on the power of students realizing that their mathematics struggles are shared amongst their peers. She had increased in anxiety in a significant way during the course of the school year, however, when asked what would have helped her understand that her perception of every other *Imagine* student being better at mathematics than her was incorrect, she mentions that “doing a math problem as a whole class” as a solution. This indicates a bit of disconnect in her experience in the MMP as this had been the entire focus of the MMP’s. However her mention of this provides support to the idea that had Penny been made to feel less alone through such an intervention (or had she realized that she had been doing that), she would have felt less isolated in her personal experience of mathematics.

5.3. A lasting change in the students’ affective domain?

The question of whether or not these students will carry forward a different and healthier attitude and outlook towards mathematics is a clearly defined theme and a primary goal of the *Imagine* program. In most of the case studies, the students mentioned that they had a different view of mathematics at the end of the year and felt more skilled at mathematics. Most credited this to having done the MMP’s.

In many of those cases, they state that the MMP’s had given them skills that had helped them in their regular mathematics work as well. This is most strongly evident in those students whose anxiety decreased according to the MARS-R, of which the student Toni might be the exemplar, as her extreme dislike of all things mathematics had changed significantly, potentially as a result of the methods utilized throughout the year in problem solving. Both Jonathan and Ellie also mention significant shifts in their

attitudes and feelings towards mathematics despite their relatively average grade in their online mathematics course. These changes in perceptions and attitude all signal that these students have indeed made a positive affective change in their views on mathematics.

Similarly, in the cases of those students whose anxiety remained relatively constant, there too appears to have been lasting changes in their outlook on the subject as a result of the MMP. Freya started from a relatively good place in terms of her views of mathematics and despite that, benefitted almost as much as those students whose anxiety had decreased as indicated in her interviews. Stating things like “I feel like I’m putting more of myself into mathematics” and her insistence that she is a better at mathematics specifically due to the MMP’s which have also given her many positive memories that she will carry forward with her into her future mathematics classes is potentially indicative of a positive change in her affective domain.

Even in the cases of students such as Karla and Luther show a positive and potentially lasting change in their affective domains. In their interviews, they stated that that thinking about what mathematics might look like for them the next school year makes them anxious and slightly fearful of the different environment, but not of the mathematics itself. Karla fears that her re-entry into a regular school system might mean a return to the favouritism and “hierarchical” bullying that she had faced in her previous mathematics class where her views on her personal efficacy at mathematics was very low. Similarly, Luther fears some of the strategies utilized in the MMP such as viewing other students’ whiteboard work as an unhealthy habit that may get him into trouble in future mathematics classes that are not like the *Imagine* program. This fear is compounded by his method of viewing every other student as more capable in mathematics than him as his primary source of motivation. These students indicated that they have improved in mathematics and that they feel more confident in it. Whether this confidence in their abilities can prove lasting is a matter for the learning environment that they find themselves in the following school year and how strongly they are attached to the positive memories from having participated in the MMP’s and *Imagine*. The fact that positive memories have been created for these students is clear from both their behavior and responses to the interview questions.

Students for whom their anxiety increased also show a change in their affective domain. In the case of Vance, his primary anxiety was for the sake of testing and doing poorly on summative type assessments. However, he mentioned in his interview how much he enjoyed the problem-solving aspects of the MMP and stated that his improvement and increased enjoyment were a direct result of the MMP's. The issue with Vance was that his overwhelming desire to not do poorly on tests may undermine his future confidence in his mathematical abilities should he encounter a series of poor test results. Again, the association and memories of the confidence he feels regarding his mathematics abilities from *Imagine* and the MMP's may carry him through these potential difficulties. Nick and Penny provide an interesting obstacle when trying to determine whether they've had any reasonable or noticeable change in their affective outlook regarding mathematics. Nick indicated no change in his MARS-R and Penny noted an increase. Nick already feels very confident in his abilities surrounding mathematics, and my observations are mixed as to whether his own views are as warranted as they are, however, Nick seems to have had some positive memories and experiences to take away from the MMP's and *Imagine*. Specifically, one such experience is the enjoyment of interacting with teachers that do not attempt to make him feel "dumb". This alone might be the most lasting change for Nick as it will potentially remind him that not every mathematics teacher is his adversary. Penny might be the least changed by her experience from the MMP's and *Imagine*. Her views remained relatively static throughout the year, and her interview indicated that she feels almost exactly the same in her abilities and outlook as she did at the start of the school year. In this case it is difficult to state with any certainty that any lasting change has been made in to her outlook and attitude towards mathematics, for reasons which we have discussed in earlier sections of this chapter. The most positive changes that I can hope for in her is that she realizes that her mathematics teachers are a support rather than an obstacle to her mathematics learning and that she did remember one of the open-ended mathematics problems when asked about a favourite one. This might indicate a small but positive change in her outlook.

5.4. Summary

It seems that the result for most students who were part of *Imagine* and participated in the MMP is an overall more positive image and view of mathematics as an enjoyable

activity that they can participate in. Those students who experienced the greatest decrease in their anxiety were those students who were more engaged with the problems and had been able to perceive that their fellow students working with them shared in their fears and struggles regarding mathematics as much as they did. Those students who had no significant anxiety change benefitted in that they had made some positive memories and associations with mathematics, which perhaps had an effect in increasing their confidence in themselves regarding the abilities. For those students whose anxiety had increased, they show that they may have still benefitted from the MMP as well, just not in the realm of anxiety reduction. These students benefit in different ways in their affective domains, with skills or increased sense of confidence. For many of the students whose anxiety increased, external factors beyond the reach of the *Imagine* program and my intervention are shown in interviews to be the a factor for the increase of anxiety.

Each student interviewed potentially takes with them positive memories and associations in regards to mathematics and their own abilities which should help each in their resilience when they inevitably encounter some adversity in their future mathematics learning.

Chapter 6. Conclusions

The end result the implementation of the *Thinking Classroom* under the guise of the MMP is one that showcases the power of a more social method of performing mathematics. In this section, I discuss my response to the research question, account for the changes my students underwent and propose some potential research considerations for a future study of this kind.

6.1. Responding to the Research Question

The Research Question was: *What effect will there be on the students' mathematics anxiety and their perceptions of mathematics when the "Thinking Classroom" model is implemented in a classroom full of anxious mathematics learners in grade 8 and 9?*

The factors that I have found that affected students' mathematics anxiety were very different from those I had anticipated prior to starting this study. I had previously thought that mathematics itself was the source of anxiety; however, a comparison of the *Math Autobiographies* that I had received at the start of the year and the students' MARS-R category scores provided a window into the affective domain of my students that showed I was mistaken. This comparison demonstrated that far more factors were at play than just a simple fear of numbers and arithmetic. From the literature, we know that the anxiety or negative affect that students have in regard to mathematics has its genesis in prior negative events. My students' *Math Autobiographies* correlate with this phenomenon. Those students who had the most negative autobiographies indicating strong negative memories associated with mathematics scored higher than other students on the MARS-R than those students whose *Math Autobiographies* indicated benign or good experiences in their prior mathematics histories. This indicates that strong negative memories surrounding mathematics had already taken root and was perhaps actively causing my students to have negative perceptions of mathematics and their abilities within it. This provided a starting point for the study from which I was able to measure the change in their anxiety and other effects throughout the year to their views of mathematics.

As a result of the extensive problem-solving regimen known as the MMP they had undertaken as a class, there is some validity in the idea that randomized groupings, open-ended problems and group discussion or *The Thinking Classroom* can lead to potentially positive outcomes for students in regard not only to their opinion of mathematics, but also in their self-reported efficacy and comfort in using problem solving skills to enhance their understanding and approach to regular academic mathematics. There were many different effects on the learners in the *Imagine* program from our implementation of the *Thinking Classroom* model that became known as the MMP. Most notably, students were more engaged with mathematics than I had observed from any of my previous mathematics classes, and the level of community that had been built in the classroom from our engaging in the 'MMP was also very positive.

The effects on students' anxiety and affective domain were also very encouraging, even in those cases where there was no measured change or an increase in MARS-R score. Each student took something positive away from the MMP that may have changed their views of what mathematics was. This was evidenced from my observations of the class as a whole, my conversations with students throughout the year and the interview data that I collected from my nine case studies. Each student interviewed, regardless of their final MARS-R score, had many positive things to note regarding the daily task of the MMP. Case study Penny might be the only exception to this assertion for reasons that we had discussed in the previous chapter. However, the positive feedback from the other eight case studies is definitively indicative of a positive affective change to the students in the *Imagine* program, potentially due to the unique exhilaration provided by the "AHA!" experience that the *Thinking Classroom* model increases the chance of occasioning.

The nine case studies provide a window into what these effects are and how the students' views of mathematics may have changed as a result of the MMP. When there was a reported decrease in overall mathematics anxiety, it was a significant and powerful event for the students. Jonathan, Ellie and Toni, who made up the case studies for the students with a decrease in their anxiety all had significant decreases in their mathematics anxiety as evidenced by their MARS-R scores and now reported that they enjoyed mathematics more than they had ever due expressly to the MMP. Their attitudes towards mathematics had obviously changed to view mathematics not as an academic obstacle but as an enjoyable pursuit, one that they now felt they were more

able to participate in. The most surprising student to have had a decrease in anxiety was Toni, who at the start of the year was the most mathematics-resistant student I had ever met. Her change in attitude towards mathematics and her now-stated enjoyment of the subject provides evidence for the effect that “AHA!” moments and the *Thinking Classroom* model can have on reducing the anxiety of middle school mathematics students. While this increased positive affect and decreased anxiety towards mathematics is evident from this studies data, there did not appear to be a net positive effect on these students’ final grades. Jonathan and Ellie both ended the year with a C- in their online mathematics and Toni ended up not passing the year overall. Thus, it seems that there was no clear positive effect of the MMP on their regular online course work, even though these students claimed that the MMP was critical to their perceived increased efficacy and skill development.

Those students who had no change in their anxiety reported positive effects on their attitudes towards mathematics and their views of the subject. Freya had started from a place of fondness for the subject yet, despite this, the MMP ended up being her favourite part of mathematics and had potentially helped her change her view of the subject from an individualistic timed task to a more cooperative problem-solving form. Karla had reported that the MMP had intimidated her, but that as she became more accustomed to it she began to feel more confident in her abilities and herself in the subject, believing that she was more capable than before. Nick was the hardest to tell if there had been an effect, positive or otherwise, as Nick’s faith in his abilities seemingly never wavered. However, his MARS-R’s did indicate a decrease in his anxiety walking into the mathematics class, and his perception of myself and my co-teacher may have improved his opinion of mathematics teachers. Thus, there is some evidenced positive change in Nick’s affective domain.

Those students that had an increased score in the MARS-R indicating an increase in anxiety also provided surprisingly positive feedback regarding the effect that the MMP had on them overall. Vance and Luther, while both having an increased MARS-R score reported how the MMP had been their favourite aspect of mathematics. Vance stated that he felt he was better at mathematics as a result of the daily tasks and that his overall skill level had increased. This increase in self-efficacy is a positive effect. Luther had similar things to say, despite his large increase in MARS-R score, his observed behavior towards mathematics, as evidenced by his enthusiasm for the

‘Morning Math Problems’ and his willingness to attempt the Waterloo mathematics contest speaks to a positive change in his perceptions of his abilities in mathematics and potentially a longer lasting positive change in his affective domain towards the subject. Penny is the student upon whom I and the MMP’s were not able to have much of an effect. Her resistance to mathematics remained strong. However, as I noted in the results and analysis question, she may have been beginning to show the start of a more positive affective change with her ability to remember the finer details of her favourite mathematics problem.

The students from my case studies exemplify the general trend and categories of students and their feelings regarding mathematics as from my year-long observations of the *Imagine* classroom in action, the move from the students’ poorer attitudes at the start of the year when starting the MMP, the resistance, the constant calls for teacher help, lack of perseverance and other anxious behaviors had been changed in more positive directions. At the start of the year, students would ask for constant clarification, unable to move forward in the question unless I had vetted their progress. Whiteboards reportedly helped students get past their reluctance to take risks. This was due to the impermanent nature of the whiteboards as mentioned by many of the interviewed students as a freeing experience that allowed them to experiment and test their ideas more freely. Surprisingly, one of the most positive effects had on Nick and Luther were their love of moving their mathematics from paper to standing at the whiteboards as it allegedly assisted them in their problem solving methodology.

The open-ended problems helped the students develop and learn comprehensible and approachable strategies that they had reported had a cross-over into their regular mathematics. Close to the end of the year, I had been able to remove myself almost entirely from their spaces and have them work in close and thorough consultation with each other on a solution to the given task. The conversations that students had with each other regarding solutions to the problem had also improved in both substance and strategy indicating an increased willingness to take risks with their mathematics which is a sure sign that their insecurity and anxiety towards mathematics had diminished. This change was significant for this class as before, students would passively go with whoever had some idea or had been viewed as the one “good at math”. After some months of the ‘Morning Math Problem’, most, if not all students would contribute actively to the solution or argue with one another over some potential avenue

of discovery. This observation was supported from my interviews with my students. Jonathan, who potentially had benefitted the most from the MMP had cited the group work with his classmates during the morning problems as being transformative in that it had forced him to both argue his ideas, and also critique others. This is an extremely positive effect that appears to be widespread throughout the class and indicates a positive change in the student's affective domain.

Students' perceptions of mathematics were also fundamentally changed as was evidenced from their engagement with their regular mathematics work and from the interview data I collected. Luther, despite his increase in anxiety near the end of the year, had started working on his online mathematics course at the whiteboards early in the year. He had also mentioned in his interview that if he had ever found himself "stuck" in one of his online problems, he would try and rephrase the question or approach it as though it were done in the style of the 'Morning Math Problem'. This indicates a shift in perception for this one student that I believe may typify a similar change in the rest of the class towards mathematics. This type of perception change was also evidenced from Freya who now felt that she was putting more of herself into mathematics instead of simply going through the motions and completing work. Again, I believe Freya's case exemplifies this change in perspective regarding the purpose of mathematics for most of the students in the *Imagine* program.

The special nature of the *Imagine* program prevents any one factor from being the definitive answer to why these students reported a reduction in anxiety and improved attitude towards mathematics. The data does seem to give strength to the approach of conducting mathematics utilizing the *Thinking Classroom* model. Giving students open-ended problems to work on in groups empowers these students and gives them strong positive memories from their individual or group "AHA!" moments. These moments of illumination now form some of the basis of their mathematical identities which has caused them to have newly reported self-efficacy and confidence with mathematics. This may prove vital to these students in the future when approaching mathematics. The whiteboards provided a window into the struggles of their fellow students and made them feel that their individual struggles were not unique to them, which increased their confidence. Similarly, the students were able to see me as the teacher in action answering questions; providing some relief to them that I was available for help should they need it. From these results, it is my contention that the effect of the *Thinking*

Classroom (Liljedahl, 2016) on mathematics anxiety is a positive one, even though it may not reduce it for every student. It is demonstrably an effective tool for teachers to utilize to help their students change their perceptions surrounding mathematics.

6.2. Accounting for Change

What were the specific reasons for the changes that the students exhibited? I have done an accounting of the behaviors that I had seen my students undergo as a result of the MMP's in chapter four. In chapter five, I provided some accounts for why students had changed in the way they had. The question of whether or not all these changes can be directly attributable to only the MMP would be an extraordinary claim. What seemed evident from the interviews in the case studies is that students had experienced interconnectedness with their classmates through the MMP. Each student had mentioned how they had felt that by looking around at each other's work on the whiteboards, they were able to see that if others were struggling, then they too had reason to struggle, thus alleviating a fear that it was only their inability that was causing their struggles. This seemed to have a benefit for preventing more negative self-perceptions from taking root in these students' mathematical identities and worked towards alleviating their already previously ingrained negative identities regarding their mathematical abilities. This may account for some of the changes in anxiety for certain students whose anxiety was reduced in certain areas.

The commonalities between those case studies where there was an overall anxiety decrease are interesting to note. The factors that account for these changes between three very different students are difficult to string together. The only commonality appears to be the MMP and the good that they took away from having done them throughout the year. In the case of Ellie, she had not been present for a lot of the year, but had stated that the problems and the social climate they provided had been impactful enough that her anxiety regarding mathematics class had diminished. Jonathan and Toni had similar things to say regarding this social climate. For these students, it may be that the MMP was the most impactful due to various reasons of preference and it simply creating an environment that was most therapeutic for these three students' mathematics anxieties.

The students whose anxiety stayed constant or increased according the MARS-R present an interesting picture as to the effects of the *Thinking Classroom* model of which the MMP imaged itself after. For those students whose anxiety remained constant, there were decreases in categories such as “Getting stuck in a math problem” and “Listening to another student explain a math formula” and “Walking into a math class”. These decreases were erased by increases in other areas of the MARS-R. Coupled with the interviews it begins to paint a picture that demonstrates what areas of anxiety are improved by participation in something like the MMP. For these students, the collaborative environment and daily exposure to open-ended tasks had the intended effect of decreasing these students’ anxieties as evidenced by the decreases in these particular items in the MARS-R. However, external factors such as test anxiety (most notably exhibited by case study Vance) and other issues that students face in their lives outside of the classroom had the effect of increasing the other categories of the MARS-R. As an intervention for mathematics anxiety, the MMP effected changes in these students’ specific categories, but could not act as a “cure-all” for all anxieties surrounding mathematics.

Prior to completing this section, I must address Penny whom I had identified as the most resistant and the least changed throughout the year. Her lack of change in anxiety or affective domain must be examined in order to account for why the MMP was ineffective in her case. The student whom I felt mirrored Penny’s negative mathematical sentiment at the start of the year was Toni, as she was equally resistant to doing mathematics but more verbal regarding her dislike. The large difference at the end of the year between the two students presents a window into what factors might make the *Thinking Classroom* model more effective for students like Penny. Toni never missed a single class of *Imagine* and, though reluctant at first, began to participate more actively in the MMP as the year progressed. This had a large impact on her as evidenced by her observed change over the year, MARS-R and interview. When Penny was present, she was always on the outside of her group, disengaged from the task. Despite my interventions and attempts, she would undergo “local avoidance” strategies in order not to participate. This might account for part of her lack of change.

Also accounting for this lack of change is the fact that Penny was absent a great deal of the year from our classroom. In order for anything classroom based to be effective, the student must attend. Ellie had had similar attendance issues however, and

she was potentially one of the most positively impacted by the MMP out of all my students. Thus we cannot account for this lack of change solely on the basis of attendance. Ellie demonstrates what Liljedahl (2005) discussed in his paper that, in order for an affective change, it does not need to be constantly reinforced, and that singular "AHA!" moments can be extremely impactful. However, the reasons for the lack of attendance may provide more insight. Penny's autobiography mentioned how her parents viewed mathematics as "foreign" and this echoed her insights in the interview. Her absences were always to avoid the "mathematical focus" that *Imagine* had started as she had been submitting work online from home during *Imagine* time. This makes it difficult to change her view as there is reinforcement from her parents justifying her decision to avoid more mathematics. This is unlike Ellie's absences which were due to other, non-academic reasons; however, it was clear from parent-teacher meetings that her parents valued mathematics learning. This home reinforcement and "justification" for avoiding mathematics seems to have acted as a nullifying factor for Penny of which the MMP was unable to effectively counter.

6.3. Research Considerations

With any study it is possible to draw conclusions based on extraordinary events and assume that those events would occur normally in different contexts. With this in mind, the *Imagine* program represents an extraordinary set of circumstances with which to conduct this study and potentially introduce a bias in my results. Each student was enrolled in this program due to anxiety and avoidance of the regular school system; this makes drawing wholesale conclusions surrounding the effects observed in these students difficult and potentially biased. However, the success in changing the attitudes and perceptions of most of these students in regards to their mathematics cannot be ignored. Thus, it becomes essential to discuss possible refinements to the study to minimize issues and discuss the usefulness of such an approach in a regular classroom context to help students ameliorate their anxiety towards mathematics.

A significant issue with this study is that students had extra time devoted to mathematics in a "Math Clinic" format which had been called the MMP. This makes extrapolation to a regular classroom context difficult as most students' exposure to mathematics in BC public schools is once a day for approximately 70 minutes, whereas these students were given mathematical tasks every morning for 45 minutes to an hour

and then would work on their online mathematics later on. While the MMP was cited by many of the case study students as their favourite aspect of mathematics, there were two very different mathematical contexts that these students were operating under. It would be interesting to see the effect on students' anxiety in a regular classroom context if the *Thinking Classroom* model of teaching and learning mathematics were implemented.

Another limitation of this study was that this was a small scale study with only 18 students, in a single classroom, who dealt with only two teachers throughout the course of the year. What might be illuminating for future research considerations is to have the *Thinking Classroom* model implemented into a regular mathematics classroom and measure the students' anxiety in a similar fashion to how it was done in this study. In a typical classroom context, the implementation of such a model has had great benefits (Liljedahl, 2016). Thus were we to compare the effects on students' anxiety that were enrolled in a mathematics course in which this model was implemented by the teacher versus that of a typical everyday mathematics classroom in which students are seated and taking notes, may provide us greater insight into what other factors affect students' self-perceptions and anxiety surrounding mathematics. It might also give us information into how much more effective a *Thinking Classroom* is at minimizing students' mathematics anxiety in addition to its other already determined benefits. The comparison of these two theoretical classes MARS-R data and interviews with selected students would eliminate many confounding variables found in my study and potentially give even stronger evidence that the *Thinking Classroom* can help students beat their mathematics anxiety through the more collaborative environment that the model provides. As further refinement, it might be even better if this theoretical study were to have the same teacher for both classes. One class would be taught in the typical "fronted classroom" method and the other to be taught utilizing the *Thinking Classroom*. This might eliminate the effect that an individual teacher might have on students' affective domain, as teachers can have a massive impact on students' perceptions across many different contexts.

The tools utilized for this study to measure and assess students' mathematics anxiety are well established and provided a wealth of information into the changes these students experienced. However the question of whether or not the changes that the students underwent in *Imagine* will be permanent in their effect on their mathematics

anxiety is one that must be considered. The skills and positive self-image built up during the MMP sessions affecting their future mathematics learning have not been measured in terms of their longevity for the obvious reasons that no one can predict the future. It is hoped that the positive experiences they had from their “AHA!” moments and realizations of their struggles being collective rather than individualistic will prevent students from sliding back into negative self-perceptions when they inevitably struggle with their mathematics again. However, this study does not address this aspect. It would be of great interest to create a longitudinal study of the students of the *Imagine* program and see what perceptions they have of themselves at the end of their next year of schooling and so on to gauge how effective in the long term the MMP intervention was at ameliorating their anxiety. This could be coupled with determining which students had exhibited a change in their anxiety during their time in *Imagine* and their enrollment in future higher level mathematics courses to see if their experiences during this time had any effect on this aspect.

Another consideration for future research on this topic is of focusing on more affective factors than on only anxiety. In this study, my initial consideration was only that of mathematics anxiety and its relief, this necessitated that I utilize a quantitative measurement tool, known as the MARS-R, to identify whether or not my interventions were having the desired effect of reducing my student’s anxiety. Unexpectedly, during the course of this study, it has become apparent that the use of the MARS-R is not without its issues. The MARS-R data that I collected from my students was “noisy” and inconsistent. I could not say for certain whether my students would score reliably on the questionnaire from one day to the next. This is perhaps an issue with its having been developed with university students, thus making it an unreliable measure for middle-school aged children for whom emotions are potentially in a state more apt to change. This could also be a potential issue with any Likert type rating system and not unique to the MARS-R. This makes the MARS-R an unsophisticated tool as it seems that the MARS-R data did not match to my other two data sources. Much more reliable and accurate data was collected from the use of the *Math Autobiographies* and the student interviews that I believe more accurately reflects the change in both anxiety and affective domain that my students exhibited. The cases of Vance and Luther demonstrated that the MARS-R was incorrect in determining the feelings of these students surrounding their outlook on mathematics. So much so that Vance disagreed with my assertion that

his anxiety had increased during our interview, and Luther divulged that his MARS-R was more a reflection of the troubles in socio-emotional areas. Coupled with this, both insisted that they felt better regarding mathematics than they had at the beginning of the year, which also contradicts their MARS-R data. If these students' results can be called into question, as I have no reason to doubt Vance or Luther's assertions about their affect towards mathematics, then it calls into question the accuracy of the other scores that I collected and how accurate it represents students and the changes in their anxieties towards the subject.

Potential solutions to this issue could be in the creation of more emotionally relevant question types such as the 16th category that I included on the final MARS-R which queried the students on how they felt "listening to or solving a word problem". If the MARS-R were to be made a more nuanced tool of measuring anxiety, the inclusion of more or altered categories that demonstrate how students had positive effects beyond just a relief in their mathematics anxiety such as an increased appreciation for mathematics, or a new-found fondness for problem solving would be a positive move. This could include teacher developed categories for an updated MARS-R that may include statements along the lines of "When I approach the teacher for mathematics help", which would yield some quantitative data into the relief that some students had in their affective domain. This could also be accomplished, perhaps, by the integration of something like the MAS (Fennema & Sherman, 1976) which measured nine different domains of students' feelings towards the subject, instead of just anxiety and would provide a clearer map of the individual student's affective domain.. The MARS-R is a good measure of mathematics anxiety over mathematical contexts (Evans, 2000); however from this study it seems that there are other factors that must be considered in terms of relief for middle-school aged students.

A few ways in which this study might have yielded some better data or results were in the implementation of certain measurement items. The use of the *Math Autobiography* in this study never went beyond getting a sense of the student's causes of their mathematics anxiety. Some form of "Bibliotherapy" (Kaasila, 2006), in conjunction with everything else going on the class, might help reinforce the fact that students are not alone in their struggles (Kaasila, 2002) more than the students viewing each other's work on the whiteboards during the task. It has been demonstrated by Kaasila et al. (2006) to be an effective tool at handling students' negative memories

rather than trying to supplant them as I attempted with the positive experiences provided by the *Thinking Classroom* model.

Finally, in the last MARS-R I gave out, I had included a new category “Listening to or solving a word problem”; this was to gauge how anxious students felt regarding the very nature of the MMP. I wish that I had had this category included at the start of my study to see if the students had had any change in this area, as most of them had indicated a very low score in their anxiety surrounding this topic. It would be interesting to see if this had always been the state of things or whether this had been one of the positive effects of the MMP. For this study, I can only surmise that it was one of the positive changes, as it is not something that I have heard students having an easy time or being comfortable with to the degree indicated on many of the MARS-R’s.

6.4. Potential integration into a regular classroom

Given the obvious factor of this study having had students do an “extra” hour of mathematics problem solving outside of their regularly scheduled time to work on their online mathematics courses, it becomes extremely relevant to discuss how to implement something like the MMP intervention into a regularly scheduled class. Fortunately, this is something that has already been implemented multiple times by other teachers such as in Liljedahl’s “Building Thinking Classrooms” (2016) study in which the use of open-ended tasks, randomized groups and vertical non-permanent surfaces was assessed for its effectiveness and found to create improved classroom cultures and student engagement in mathematics. This is the challenge of having the *Thinking Classroom* implemented into an environment with curricular competencies and the pressures of summative assessment. It becomes too easy to remove the open-ended tasks and replace them with the usual text book questions. The open-ended tasks or “good problems” are an essential part to the change in attitude that students had towards mathematics in my study and if it is to still be successful at lowering students’ anxiety and changing their emotions and attitudes towards mathematics we cannot lose sight of this. Having attempted throughout the year in the *Imagine* program to include curricular type problems met with some success, I was able to develop some “good problems” as defined by Liljedahl (2004) that met with certain curriculum standards and allowed the students robust exploration. However, not all of my questions were as well met by my students.

This issue of a “lack” of curricular “good problems” is dealt with by a new concept of “Smudge” mathematics, which is when you “smudge-out” strategic parts of a question and ask the students to explore possibilities that these new ambiguities present. This idea of smudge mathematics problems has been tested and shared a number of times on Twitter with the hashtag #smudgedmath. Teachers have shared their successes on social media with this concept of creating curricular type problems that allow all the qualities of “good problems” or open-ended tasks. Some of my most successful MMP’s were with smudge mathematics problems (see Fig.5 in chapter four for one such question). The danger again is overuse of the smudge-math, but utilizing this approach to creating open-ended problems for students to solve seems to be a good step in integrating the *Thinking Classroom* while addressing curriculum.

Another approach which I have recently utilized in addition to ‘smudge math’ to solve the issue of a lack of curricular problems was to take my old lessons, which I had traditionally started with the “big idea” of what I wanted students to learn and then start the students with problems that are solved, then ask the students to figure out what I had done in order to move from the left side of the equation that I had written to the right side. For example, in a Foundations of Mathematics and Pre-calculus 10 summer course I have just recently taught, I taught polynomial expansion utilizing this “Big Idea” with a question on the board already written as $(x+3)(x+7) = x^2+10x+21$. I then asked the students to determine what it was that I had done. This ambiguity and search for meaning then turned into a very constructive classroom environment where students attempted to solve what it was that had occurred. Many had re-created the traditional FOIL method without ever mentioning this particular method of expansion. This is not to say that I have always succeeded in this endeavor; however, implementing the *Thinking Classroom* fully into a mathematics class is not only possible, but is advisable.

One thing that must be kept in mind, which only my experience in attempting to implement the *Thinking Classroom* into a mathematics classroom has lead me to conclude, is that if I were to implement the *Thinking Classroom* model, I would have to make time for non-curricular problems regularly and not a sole focus on trying to create curricular problems. The *Thinking Classroom* is much more energy intensive than a routine mathematics course, and students can get burnt out if all the questions addressed are only curricular. I would introduce non-curricular problems regularly in-between sessions in a regular mathematics class as these are the types of problems

that seem to “refresh” students and get them into the right mathematical frame of mind. In a way, I would think of it as after doing short pass drills in a physical education class; at some point, the students would want to just play a game of basketball. At some point, students will want to just solve a genuinely interesting problem that has no curricular competency or ‘hidden’ academic agenda.

Utilizing the tools that were used in this study into a regular mathematics classroom could be very valuable to any mathematics teacher who is interested in learning more about his students. The use of the MARS-R and the *Math Autobiographies* can bring a great deal of relevant information to the teacher, giving the mathematics teacher much needed insight into the lives of his students that he would not normally be privy to. It also could allow a ‘humanizing’ effect on mathematics by having the students reflect upon their past learning and their reasons for feeling about mathematics the way they do. A course of “Bibliotherapy” might also help the students air out any negative memories associated with learning mathematics and act as a starting point to build empathy between them, thus relieving their anxiety. It can also inform the teacher on his students’ views mathematics, their fears and hopes and anxieties. This can help the teacher to modify their approach to that student or the class lessons in general in order to build skills in those areas students feel most deficient in.

6.5. How I’ve Grown

Self-reflection is a valuable and necessary part of growth as a teacher as in order to improve individual pedagogy and more effectively teach students. It is my thought that as teachers, we must be willing to evaluate our practices and perceived efficacies in order to not succumb to our unknown biases in education as these biases can prevent us from ever adopting more effective teaching practices. Through the conducting of the research for this thesis, I have done an enormous amount of reflection on how I’ve not only grown as a teacher, but as a learner and researcher.

6.5.1. As a researcher

Over the course of this study, I feel as though I have done a great deal of growth as an education researcher. Given my science background, I started this program with a much narrower view as to what constituted “good” research. I felt that the only worthwhile data

to collect was quantitative with large enough sample sizes to make statistically significant generalizations and draw my conclusions from the trends evident in the data. I had felt that qualitative data was a good addendum to add flavor and context to numerical data and potentially lay groundwork for future study. Now, I've come to realize that while there is a place for quantitative data in education research, it is not the be all and end all. As a researcher, I've come to appreciate the immensely useful role that qualitative data provides and how its collection and analysis can be immensely more illuminating for a teacher in education pedagogy. I have my program supervisor and my excellent professors to thank for helping me shed my preconceptions regarding what constitutes research. Through the writing of this study, I realize how far I have come, and how much further I have to go. Admittedly, at points in composing this study, writing, reading and analyzing literature and data, I felt as though I was feeling my way through a dimly lit hallway, unsure of whether I was going in the right direction. However, after going through the process of writing this study part by part, receiving the excellent feedback of my supervisors, I am now more keenly aware of how to research literature, collect good data and perform inductive analysis, all essential aspects to writing an academic research paper.

6.5.2. As a learner

Admittedly, I undertook this Master's Program with a high level of mathematics anxiety. Mathematics had always given me cause for worry, as I looked upon it as the purest science, one that only a blessed few could properly function in and work with. Now, after much coursework in and around not just the teaching of mathematics, working with other teachers and also learning new and interesting concepts as part of the mathematics courses, I can now feel confident in describing myself as a confident and life-long learner of mathematics and not just a teacher of them. Part of this new-found outlook comes from the great autonomy in assignments and methods that was provided to me by the professors in this Master's program and my program supervisor. I was truly allowed to explore mathematics for the first time, rather than simply learn the algorithms. However, I always knew that should I need advice or guidance or simply reassurance that I was on the right path, help would always be freely given. Everything that I had wished to learn in order to teach students mathematics more effectively was modeled in a way that was never explicitly mentioned, but was always fostered in how the courses and assignments

were conducted. From this modeling, I know now what it is that I want my mathematics students to experience in my mathematics courses and what it was that I had always felt was missing from my own pedagogy.

6.5.3. As a teacher

I had always endeavored to never be the teacher who taught the same subject the same way for 30 years. Rather, I have endeavored to be a teacher whose practice evolved to adapt to changing social and academic moors to best educate my students. Having seen the ineffectiveness of some of my previous strategies for teaching mathematics, I branched out in my attempts to creatively deal with common issues in the classroom. Having suffered from mathematics anxiety most of my schooling career, I had always made it my mission to try and reduce these negative feelings towards mathematics such that no one would have to suffer as I did at the hands of this "ruthless" subject. In conducting this study with the *Imagine* program and experiencing firsthand the effectiveness of the *Thinking Classroom* on not only students' mathematical anxiety, but their engagement, enjoyment and skills, I have come to learn that while some traditional methods of mathematics education are still valid and very useful, there are some things that can be updated in ways to help better engage students.

Additionally, having gone through the academic research, I now realize that there is a plethora of ideas and models for testing and implementation to help students achieve more in mathematics. I need not be an island unto myself for inventing methods for helping students learn mathematics. I can rely on the hundreds of keen minds publishing research papers describing the new things being tried every day. In this way, I've grown as a teacher that I can draw upon a great deal more sources for inspiration and ideas than I had ever done previously in my teaching practice before.

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Appendix A. Math Autobiography Handout

Mathematical Autobiography

The purpose of this assignment is to have you reflect on your experiences with mathematics.

This will set the stage for all the (possibly different) ways you will experience mathematics in this class. The autobiography will also help me in knowing how you feel about mathematics and what you have experienced in mathematics classes and will help me create the course for you.

The audience is just me, your teacher, and in some sense you.

Assignment: Write a half to one page mathematical autobiography. You should include some of your early experiences, include some experiences with teachers (in any grade), both good and bad ones, how your attitude towards mathematics has changed over the years (if it did), why you like it or don't like it, what feels good about doing math or learning math or teaching math, what is scary for you, what is exciting for you.

I am also curious to know what you believe mathematics is. How would you define mathematics to someone who doesn't know anything about it?

Appendix B. MARS-R

Mathematics Anxiety Rating Scale – revised[1]

When I am … I feel ANXIOUS	Not at all	A little	A fair amount	Much	Very much
1. Looking through the pages in a math text	1	2	3	4	5
2. Walking into a math class	1	2	3	4	5
3. Reading a formula in a science text	1	2	3	4	5
4. Thinking about an upcoming math test one day before	1	2	3	4	5
5. Watching a teacher explain a problem on the whiteboard	1	2	3	4	5
6. Being told how to interpret algebraic statements	1	2	3	4	5
7. Picking up a math textbook to begin working on a homework assignment	1	2	3	4	5
8. Taking an test in a math course	1	2	3	4	5
9. Reading and interpreting graphs or charts	1	2	3	4	5
10. Starting a new math problem	1	2	3	4	5
11. Being given a homework assignment of many difficult problems	1	2	3	4	5
12. Waiting to get a math test returned in which you expected to do well	1	2	3	4	5
13. Getting stuck in a math problem	1	2	3	4	5
14. Starting a new chapter in a math book	1	2	3	4	5
15. Listening to another student explain a math formula	1	2	3	4	5

[1] The revision of the revised scale is based upon the work of Derek R. Hopko Confirmatory Factor Analysis Of The Math Anxiety Rating Scale–Revised, *Educational and Psychological Measurement* 2003; 63; 336

Appendix C. MARS-R Year end version

Mathematics Anxiety Rating Scale – revised[1]

When I am … I feel ANXIOUS	Not at all	A little	A fair amount	Much	Very much
1. Looking through the pages in a math text	1	2	3	4	5
2. Walking into a math class	1	2	3	4	5
3. Reading a formula in a science text	1	2	3	4	5
4. Thinking about an upcoming math test one day before	1	2	3	4	5
5. Watching a teacher explain a problem on the whiteboard	1	2	3	4	5
6. Being told how to interpret algebraic statements	1	2	3	4	5
7. Picking up a math textbook to begin working on a homework assignment	1	2	3	4	5
8. Taking an test in a math course	1	2	3	4	5
9. Reading and interpreting graphs or charts	1	2	3	4	5
10. Starting a new math problem	1	2	3	4	5
11. Being given a homework assignment of many difficult problems	1	2	3	4	5
12. Waiting to get a math test returned in which you expected to do well	1	2	3	4	5
13. Getting stuck in a math problem	1	2	3	4	5
14. Starting a new chapter in a math book	1	2	3	4	5
15. Listening to another student explain a math formula	1	2	3	4	5
16. Listening to or solving a word problem	1	2	3	4	5

[1] The revision of the revised scale is based upon the work of Derek R. Hopko Confirmatory Factor Analysis Of The Math Anxiety Rating Scale–Revised, *Educational and Psychological Measurement* 2003; 63; 336

Appendix D. Interview Questions

Description of Possible Interview Questions

The following questions may be asked during interviews of the selected participants. The goal of the interviews is to gather information on student thoughts on how the class was conducted, the use of vertical surfaces, random groupings, and open-ended problems and the effects these had on their general feeling (anxiety) towards mathematics. Follow-up interview questions will be predicated on individual responses to the initial interview questions

Possible Interview Questions:

1. What has changed for you this year in math? [You may get questions about how math is different. That's fine. But then you follow this with, "So, what about you? How do you feel you are different in and with math this year?"]
2. What has been your favourite and least favourite aspects of math this year?
3. Are you better at math at the end of the year than you were at the beginning?
4. Over the past 10 months, would you say that you feel better about Math Class? Why or Why not?
5. (This will be student specific) You noted and overall (increase/decrease) in the area(s) of the MARS-R category _____, why do you think this was the case?
6. Has the Early Morning Math problems that we've done throughout the year helped you with your Math Skills? Can you think of a favorite problem?
7. Do you think working in groups has helped you with you Math abilities? Why or Why not? What about the randomized nature of the groups?
8. What about writing on the White boards has this helped you? Does seeing your classmates math struggles on the whiteboards to be like your own help you to understand you are not struggling alone?

Appendix E. Ski Trip Fundraiser Numeracy Task

SKI TRIP FUNDRAISER

The grade eight ski club is going to Grouse Mountain. Each person tried their best to raise money for their trip. Below is a chart that shows how much money each person raised, and their individual cost, depending on whether they need rentals or lessons.

Determine whether they have raised enough money for their trip. What would be a fair way to share the money that was fundraised among the people listed below? All of the money raised must be applied to the cost of the trip, and every person must go on the trip, even if it means that they may have to put in their own money to do it.

Name	Amount Raised	Rental Cost	Lift Ticket	Lesson Cost
Alex	75	20	40	40
Hilary	125	10	40	40
Danica	50	30	40	0
Kevin	10	40	40	40
Jane	25	0	40	0
Ramona	10	0	40	40
Terry	38	30	40	0
Steve	22	40	40	40
Sonia	200	20	40	0
Kate	60	25	40	0

Appendix F. Race Around the World Numeracy Task

RACE AROUND THE WORLD

You have just entered a race around the world. The rules of the race are very simple:

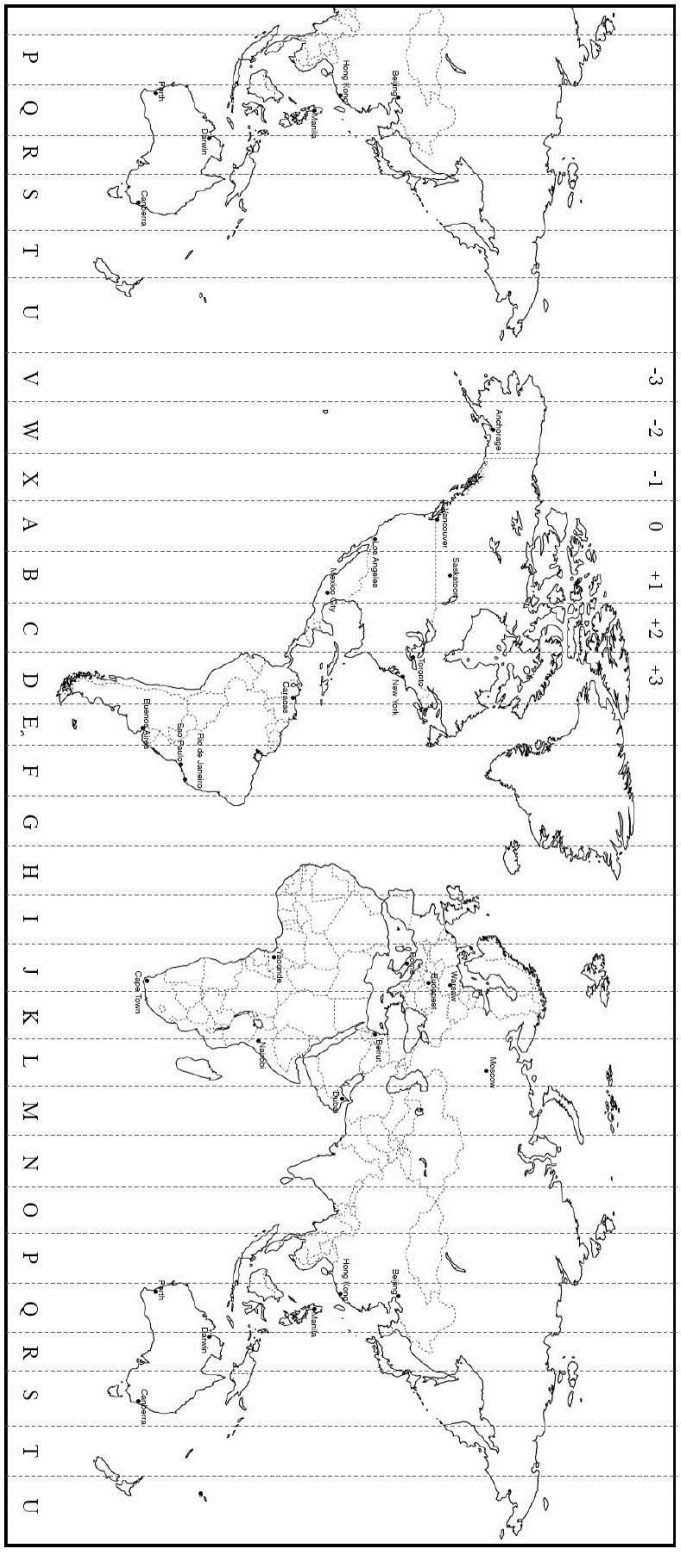
- you must start and finish in Vancouver.
- you must visit one major city (marked) on each continent except Antarctica.
- Vancouver does not count as your North American city.
- Your airline ticket only allows you to travel east.

Your goal is to get back to Vancouver in the shortest amount of time.

To help you calculate your time please keep these simple rules in mind:

- flight paths can be seen as straight lines between cities.
- 1 cm of travel on the map takes an airplane 2 hours to fly.
- airplanes depart each city on every even hour **local time**. That is, they leave at 2:00, 4:00, 6:00, ...
- the dotted vertical lines on the map are time zones. Every time you cross one of these lines while travelling east you should advance your clock by one hour.

Good luck – and may the best team win.


-3 -2 -1 0 +1 +2 +3
P Q R S T U V W X A B C D E F G H I J K L M N O P Q R S T U
Hong Kong
Beijing
Manila
Perth
Darwin
Canberra
Anchorage
Vancouver
Saskatoon
Los Angeles
Mexico City
Toronto
New York
Caracas
Rio de Janeiro
Sao Paulo
Buenos Aires
Cape Town
Yaounde
Nairobi
Beirut
Budapest
Warsaw
Moscow
Dubai