Using patterns-of-participation approach to understand high school mathematics teachers' classroom practice in Saudi Arabia

by

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Abstract

During the past decade, the Saudi Arabian education system has undergone major changes. Government agencies involved in education have introduced new policies, standards, programs, and curricula. The recent changes in the education system motivated me to conduct this study. The focus of this research is to describe and understand high school mathematics teachers' current practices in Saudi Arabia.

This research includes four cases of teachers currently teaching high school mathematics in Saudi Arabia. Using the Patterns of Participation concept (PoP) as the main framework, I identified some of the significant practices, or figured worlds, from the teachers' sense of their practices. Some of the figured worlds that emerged are mathematics, the textbook, reform, responsibility for students' achievement, and relationship with others. Mathematics, as it has always been, remains an influential figured world for mathematics teachers. Reform and the textbook are becoming as influential because of the current changes in the education system in Saudi Arabia. While some participant teachers are developing a new understanding of what mathematics is and what it means to teach it, they also indicated that they are mostly still using traditional teaching strategies rather than reform teaching strategies.

In addition, I conducted a cross-case analysis to connect the findings from each case in order to gain some understanding of how high school mathematics teachers in Saudi Arabia respond to the shared or common circumstances they are facing in the current reform movement. I identified and described six common themes from the cross-case analysis. These themes are useful for showing the range of mathematics high school teachers' practices in Saudi Arabia and the ways in which their practices differ. Participant teachers responded differently to the shared or common circumstances they face in the current reform movement. I found more differences than similarities in the current teaching practices of the participant teachers.

Keywords: Mathematics; Teachers' practices; Patterns of Participation (PoP); Reform; Textbooks.

Dedication

This thesis is dedicated to my beloved parents; without your support, my success wouldn't have been possible.

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PoP	Patterns of Participation

Chapter 1.

Introduction

1.1. Motivation and research goal

Teaching is a complex practice that involves constant and dynamic interaction between teacher, students, subject matter, and much more. When education organizations implement reform initiatives, one of the main goals is often to change teachers' classroom practices. In the past, educators viewed changing the curriculum as an endeavour to change the content of instruction more than the teachers' classroom practices. However, most recent curricula reform has focused on promoting and implementing teacher practices that promote students' understanding of mathematics alongside changes in content (Cohen & Ball, 1990; Tirosh & Graeber, 2003).

Mathematics education reform has been the focus of researchers over the past several decades. Many researchers describe mathematics reform as movement away from a traditional approach or teacher-centered instruction towards a more student-centered or learner-centered approach. In a learner-centered practice, students are active participants in their learning. Teachers provide students with opportunities to investigate, communicate, and make connections within mathematics concepts and with the world around them (Ball, 1994; Simon, 1995; Boaler, 2002; NCTM, 1989, 1990, 2000; Ross et al., 2003; Smith & Star, 2007).

Prior to 2007, the Saudi Arabian education system experienced very few changes. This was also true for mathematics teaching practices, which were very traditional. Teachers relied on traditional mathematics textbooks and focused on delivering mathematical content and knowledge (Al Sheki, 2011; Al Balawi & Al Rajeh, 2012). Prior to the current reform, using the textbook was enough for teachers to have acceptable teaching practices.

However, since 2007, the Saudi Arabian education system has undergone major changes. One of the major changes happened in 2011 when the Ministry of Education in Saudi Arabia introduced new mathematics textbooks, the primary resource for teachers.

The Ministry sees this initiative as a major step towards creating change in teaching practices.

Recent government initiatives intended to improve the quality and quantity of education in Saudi Arabia are evidence of the urgency for education reform, especially with the number of young people increasing. Government agencies involved in education have introduced new policies, standards, programs, and curricula with the expectation that teachers will incorporate the changes seamlessly; however, they have failed to take into consideration existing teacher practices.

As a mathematics teacher from Saudi Arabia, I understand the great amount of pressure mathematics teachers feel to quickly alter their own practices in order to adjust and adapt to changes that resulted from education reform. However, despite their common experience concerning education reform, the Ministry of Education does not understand the difficulties teachers face since the changes began.

The education reform movement has included many changes to teachers' roles. Researchers are calling for additional examination of the changes to gain a better understanding of teachers' classroom practices. The recent changes in the education system have motivated me to conduct this study. This research project evolved from my background as a mathematics teacher and my personal interest in the improvement of mathematics education in my country. I am fascinated by what Saudi mathematics teachers are experiencing now and hope to gain some level of understanding of their current practices. Therefore, my main research goal is to describe and understand high school mathematics teachers' current practices in Saudi Arabia. Additionally, the result of this research could provide insight into what Saudi educators do not understand about high schools' mathematics teaching in Saudi Arabia.

1.2. Theoretical perspective

This research presents four cases of teachers currently teaching high school mathematics in Saudi Arabia. Using the Patterns of Participation concept (PoP) (Skott, 2010, 2011, 2013, 2014a, & 2014b) as the main framework, I identify some of the significant practices, or figured worlds, from the teachers' sense of their practices as a mathematics teacher. In addition, I conduct a cross-case analysis to connect the findings

from each case in order to generate a broad understanding of high school mathematics teachers' experience during the current reform movement.

In this study, the PoP approach serves as a lens to interpret and understand Saudi high school mathematics teachers' current practices. The PoP framework identifies teachers' practices as being how teachers narrate and position themselves in relation to multiple, and sometimes conflicting, figured worlds (Skott, 2013). Figured worlds are imagined communities that function dialectically and dialogically as if in worlds. They constitute sites of possibility that offer individuals the tools to impact their own behaviour within these worlds (Holland et al., 1998; Skott, 2013).

Traditionally, most research in education that focuses on studying teachers' practices adopt an acquisitionist approach, especially those studying teachers' beliefs and knowledge in relation to teachers' practices (Skott, 2013). Recently, more researchers, including Skott (2010, 2013), adopt participationism as a metaphor for human functioning to understand teachers' practices. "The origins of participationism can, indeed, be traced to acquisitionists' unsuccessful attempts to deal with certain long-standing dilemmas about human thinking" (Sfard, 2006, p. 153). Skott presents PoP as a coherent, participatory framework that is capable of dealing with matters usually faced in the distinct fields of teachers' knowledge, beliefs, and identity. Therefore, PoP is a theoretical framework that aims to understand the relationships between teachers' practices and social factors. Skott (2010, 2011) initially developed the PoP framework in relation to teachers' beliefs. However, in order to develop a more coherent approach to understand teachers' practices, Skott (2013) extended the framework to include knowledge and identity.

The social approach of research in mathematics education has progressively promoted the notion that practice is not only a personal individual matter; it is in fact situated in the sociocultural context. Although the relationships between individual and social factors of human functioning have generated much debate in mathematics education, it is mainly in relation to student learning (Skott, 2013). To a considerable degree, PoP adopts participationism as a metaphor for human functioning more than mainstream belief research. Therefore, PoP draws on the work of participationism researchers, specifically Vygotsky, Lave and Wenger, and Sfard.

"The intention of PoP is to take this one step further by limiting the emphasis on acquisition and include a perspective on the dynamics between the current practices and the individual teacher's engagement in other past and present ones" (Skott, 2013, p. 557). This framework focuses mainly in understanding what practices and figured worlds are significant for the teacher and how the teacher engages in those figured worlds. A teacher's engagement with these figured worlds inform and adjust the interpretations s/he makes to her/himself and the way s/he engages in on-going interaction in the classroom.

1.3. Why study teachers' practices

The work of teachers is complex and involves a diverse range of practices. Teaching is not simply a matter of being in classrooms and delivering lessons to students; teaching involves a complex set of practices that takes place both inside and outside the classroom (Bransford, Darling-Hammond, & LePage, 2005).

Teachers' practices include strategies used in their everyday professional activities. Teachers must work effectively with their students to incorporate and structure curricular materials and theoretical concepts into actual classroom practices. In order for teachers to offer an environment for effective learning, they must do more than merely stand in front of a classroom and lecture. Moreover, although teachers spend most their school time in the classroom, classroom teaching is just one aspect of their profession (Bransford, Darling-Hammond, & LePage, 2005).

Teachers recognize that teaching includes instructional planning, assessing students' learning, and interacting with students (Bransford, Darling-Hammond, & LePage, 2005). Therefore, in their practices, teachers continue to learn and apply a variety of different approaches that incorporate different types of knowledge (Fishman & Davis, 2006; Goos & Geiger, 2010). Studying teacher practices can help researchers understands teachers' learning opportunities for teaching practices, which can have positive impact on students' learning experiences (Bransford, Darling-Hammond, & LePage, 2005).

Studying teachers' practices is important for understanding and improving educational processes. Generally, "teachers are the cornerstone of nearly all formal

instructional system" (Fishman & Davis, 2006, p. 535). Many researchers consider teachers as influential individuals in their classroom because the decisions they make affect all aspects of classroom instruction and learning (Cooney, 1994; Simon, 1995). They are also the most significant element in educational innovations (Fishman & Davis, 2006). Hill, Rowan, and Ball (2005) noted teachers' practices structure students' learning environment and greatly impact student motivation and achievement. They view teachers as the main school-related factor affecting student achievement.

Over the past three decades, the mathematics education community from a variety of countries has been involved in a great deal of reform activity. Educators see students as responsible of their learning and thus, students need opportunities to investigate, reason, and make sense of mathematical concepts and problems, in addition to making connections between mathematics and the world around them. The teacher's role has changed from traditional teaching practices to teaching practices that facilitate discussions and engage students in the learning process (Ball, 1994; Simon 1995; Simon, 1994; Tzur, Simon, Heinz, & Kinzel, 2001; Ross et al., 2003; Smith & Star, 2007).

Cohen and Ball (1990) highlighted the importance of the teachers' role in education reform. They stated, "Teachers are cast as the key agents of improvement" (p. 233). Moreover, they see teachers simultaneously as the targets and the central agents of reform efforts. In addition, they claim that teacher practice plays a role in limiting or altering the intentions of reforms. However, despite the emphasis on the teachers' role in education reform, researchers know little about the practices of teachers engaging with reform and specifically about mathematics reform. A deeper understanding of how teachers engage with reforms in their teaching of mathematics could lead to improvements in the teaching of mathematics at all school levels.

1.4. Organization of the thesis

This dissertation has seven chapters. Chapter one introduces the topic of this dissertation. Chapter two is a general overview of the education system in Saudi Arabia and a detailed description of the changes that mathematics teachers in Saudi Arabia are experiencing. Sections include a brief history, the main features, and a criticism of the Saudi education system. An overview of the most recent reform initiatives in the Saudi

Arabian education system is also included; it highlights the initiatives that have a direct impact on high school mathematics teachers practice.

In chapter three, I outline the conceptual framework that guided my study. The chapter includes information about Patterns of Participation (PoP) as a theoretical framework and outlines its potential for explaining and understanding mathematics teachers' classroom practices. It also contains an explanation of the connection between PoP and other theories it draws from and describes its usefulness and limitations as a framework to understand the role of the teacher for emerging classroom practices.

For this dissertation, I decided not to include a traditional literature review chapter. Instead, the relevant academic literature is contained in chapters two and three. I believe organizing the literature this way fit more with the nature of my research since I am using PoP as the main framework. The use of this framework guided the organization of this theses. To make it clear for the readers, I included the literature review about theories related mathematics teachers' classroom practices in chapter three with the details and explanation of the framework and I included literature review about education system in Saudi Arabia in chapter two.

In chapter four, I explain in detail the research methods and the methodology implemented for this study. It covers the main research questions and describes the practical steps I went through during the design of my research including recruiting participants, means and modes of data collection and analysis techniques.

Chapter five comprises of four case studies of high school mathematics teachers. The chapter presents each individual participant's case separately to capture the unique essence of their experience. The aim of the presentation of every case is to develop a deeper understanding of the participant teachers' significant practices and figured worlds and how each teacher engages with these figured worlds.

Chapter six presents the results of my cross-case analysis. The goal of the cross-case analysis was to connect the findings from each case in order to generate a broad understanding of high school mathematics teachers' practices during the current reform movement. It also provided an opportunity to examine how each teacher contributed to my general understanding of high school mathematics teachers' practices

in Saudi Arabia. The last chapter, chapter seven, contains the conclusions from this research, including suggestions for future research and the implications of this study.

Chapter 2.
Overview of The Education System in Saudi Arabia

This chapter provides a general overview of the education system in Saudi Arabia. Sections include a brief history, the main features and a criticism of the Saudi education system. In addition, there is an overview of the most recent reform initiatives in the Saudi Arabian education system that highlight the initiatives that could have a direct impact on high school mathematics teachers' practices.

Saudi Arabian society is very conservative and gender segregation is part of everyday life. When we talk about the history of education in Saudi Arabia, we must separate boys' education from girls' education since gender segregation is part of the education system and the introduction of girls' formal education was much later than education for boys. Because of the nature of this research, this chapter provides more details about the history of girls' education in Saudi Arabia than it does boys' education.

2.1. The Kingdom of Saudi Arabia

The Kingdom of Saudi Arabia is the largest Arab state in Western Asia spreading over 2,150,000 km^2 and covering almost 80% of the Arabian Peninsula. It is the second largest country in the Arab world (after Algeria). According to the Central Department of Statistics and Information, the total population in Saudi Arabia in 2014 was 30.8 million people. Around 20,702,536 are Saudi citizens. More than fifty percent of Saudi Arabia's population is under the age of 25.

Abdulaziz Al Saud founded The Kingdom of Saudi Arabia in 1932 after a 30-year campaign to combine most of the Arabian Peninsula. This campaign placed the Al Saud family in a leading position to rule the country from their traditional origin of Najd province in the centre of the country. The royal Al Saud family has ruled the country since its birth and all of Saudi Arabia's kings have been sons of the kingdom's founder, king Abdulaziz. Saudi Arabia is known as the birthplace of Islam; it is also the home of Islam's two holiest places: Masjid al-Haram, in Makkah, destination of the annual Hajj pilgrimage, and Medina's Masjid an-Nabawi, burial site of the prophet Muhammad (peace be upon him).

2.2. A brief history of education in Saudi Arabia

Early education in Saudi Arabia used the traditional educational system kuttab. In kuttab, students mainly memorized the Qur'an and learned basic reading, writing, and arithmetic; students also learned moral education. The learning instruction was very traditional relying firmly on memorization instead of intellectual inquiry and critical thinking (Nolan, 2011). However, students attending kuttab schools included only a small number of males from privileged families. In 1924, 20 years after king Abdulaziz started his campaign to unite the kingdom the Directorate of Education was established. This formed the base of the first modern educational system in Saudi Arabia. Its creation was to enable the expansion of formal education in the country (AlSadan, 2000).

The government founded the first public boys' school in Saudi Arabia in 1925. In 1939, only 2,319 students were enrolled in schools in the entire kingdom. At that time, illiteracy was widespread through most of the Arabian Peninsula. According to the UNESCO estimation, in 1960, only 5% of the population enrolled in schools to obtain basic literacy skills and knowledge (Nolan, 2011). Under the supervision of the Directorate of Education, the number of elementary schools increased from four schools in 1925 to 306 in 1952 and included 40 000 students and 1500 teachers. In 1934, for the first time, free public education became accessible for most boys around the kingdom (AlZaid, 1990).

In 1953, the government completely transformed the national education system and the Ministry of Education replaced the Directorate of Education. The goals of the new Ministry were to expand and develop the national school system and to give it a modern foundation similar to that of the Western world. The Ministry of Education established regional educational offices throughout the country to serve as local representatives. They were responsible for administering and managing education in their districts (AlSadan, 2000; AlZaid, 1990). At that time, there were no qualified people to work as teachers in public elementary schools; therefore, the government hired teachers from other Arab countries, mainly Egypt and Syria.

Before 1937, secondary-level education in Saudi Arabia did not exist. The first boys' high school opened after the establishment of the Foreign Mission preparatory school. The primary purpose of this school was to prepare male Saudi students to

continue their higher education in Egyptian universities. Accordingly, the curricula were in line with the Egyptian curricula and most of the teachers were Egyptian (AlZaid, 1990).

The economic growth in the oil sector during the 1950s and 1960s, which created great industrial progress in Saudi Arabia, resulted in the need for an immediate reform of the elementary and secondary education system. As a result, the Ministry of Education opened additional elementary and secondary schools and criticism of the old system increased. At that time, the shortage of qualified experts to lead the reform meant that other Arab countries, primarily Egypt, provided support and guidance. In response to the criticisms and lack of firsthand knowledge, the government created the Saudi Supreme Committee of Education in 1963 to oversee a large-scale reform of the Saudi education system (AlZaid, 1990). One of the major results of this reform was the establishment of the intermediate school level, which covers three years of education between the elementary and secondary levels (AlZaid, 1990).

2.3. The history of girls' education in Saudi Arabia

When the government introduced formal education in 1924, enrolment was restricted to boys. Official education for girls in Saudi Arabia was introduced later than education for boys. Until 1960, the education of girls was almost unheard of except within some wealthy families (AlMunajjed, 1997; Alharbi, 2014). Alharbi (2014) indicates that cultural and social factors delayed the establishment of public education for girls in Saudi Arabia. At that time, the education for girls was secondary. Culturally, Saudis believed that official education for girls' conflicted with a girls' primary job as wives and mothers. In addition, conservative religious people, who had a powerful and respectful status in society, considered public schools for girls a threat to the Saudi society. Schools were a place where young Saudi girls would be exposed to Western culture and education, which would lead to the expansion of voices calling for openness and an erosion of traditional values.

With the increase of political pressure from external organizations and governments, the Saudi government decided to establish girls' formal education. However, to overcome the social opposition within Saudi Arabia to girls' formal education, the government announced that girls' schools would focus on teaching the

fundamentals of Islam and other subjects that are consistent with conservative views, such as home economics and childcare. Moreover, the government established girls' schools separate from boys' schools. The separation was not only with respect to school buildings but also in curriculum and management. The Ministry of Education did not oversee girls' schools. Instead, the government established the Presidency of Girls' Education as a separate and independent organization to supervise and manage girls' education at all levels. A committee of trusted members of religious people led and controlled the Presidency of Girls' Education to ensure that girls' education followed Islamic laws (AlMunajjed, 1997; 2009).

Saudi Arabian society was, and still is, an extremely conservative society, characterized by great pride in its traditions and culture. It is particularly challenging for conservative societies to disregard its traditional values and keep pace with modern changes. For the first two years after the establishment of girls' education, some religious leaders who were against girls' education tried to close some public girls' school (Nolan, 2011). As such, it is easy to understand why women's education did not become acceptable or receive considerable attention.

Shortly after the introduction of education for girls, families began to see the benefit of sending their daughters to school and acceptance of girls' education increased. As a result, the number of schools for girls began to increase. The number of elementary schools for girls in 1960 was fifteen with a student enrollment of 5,180; by 1970, it had increased to 377 elementary, 31 intermediate and five secondary schools with a total enrollment of 126, 230 students (Nolan, 2011).

In 1980s, basic formal education became available to all Saudi boys and girls and the number of girls and boys in public schools was almost equal (Hamdan, 2005). Opportunities for girls' education were no longer limited to basic education. Higher education also became more accessible and many young women registered for and graduated from colleges and universities (AlMunajjed, 2009).

The General Presidency for Girls' Education did not have the same prestigious status as the Ministry of Education and religious and conservative scholars controlled it almost completely. This control guaranteed that women's education did not stray from the primary purpose of woman education, to prepare young girls to be wives and

mothers, and to give them the skills for socially acceptable jobs such as teaching and nursing (Hamdan, 2005). “At that period of time in Saudi Arabia, boys were the only ones who appeared to benefit from girls’ education because schools were preparing girls to be good wives for their husbands. Therefore, girls at that time were taught a different curriculum from boys because the purpose of schooling was different” (Alharbi, 2014, p. 2023).

Girls’ education remained under the supervision of the General Presidency for Girls’ Education until 2002, at which time the Ministry of Education took over the task. The elimination of the General Presidency for Girls’ Education came at a time when public pressure was high due to the death of fifteen girls in a school fire in 2001. Reports suggested that religious police hindered the firefighters during the rescue and thus contributed to the high death toll (Hamdan, 2005). Many people considered the elimination of the General Presidency for Girls’ Education a major decrease of power and control by the religious conservative party over girls’ education. One of the main results of the elimination of this institution was the creation of a united school curriculum for both boys and girls in the subjects of mathematics, science, and language arts (Nolan, 2011).

Today, the educational system in Saudi Arabia is more advanced and comprehensive than that of its early years. The literacy rates in the country clearly demonstrate the results of the changes to education. Illiteracy rates among adults in 1972 were at 70 percent; today, they are at 10 percent (UNESCO, 2013). Moreover, according to the UNICEF 2015 State of the World's Children Report, literacy rate among young Saudi males (15-24 years) is 99% and 97% among female youth.

2.4. Education in Saudi Arabia: Main Features

Along with the universally recognized purpose of education, developing students’ skills and knowledge and satisfying the needs of the society, Saudi education emphasizes the strength of students’ Islamic knowledge. According to the Saudi Official Policy of education, published in 1980, the central purpose of the Saudi education system is the continuance of the Islamic educational heritage. This is mainly because Saudi Arabia's political, social, and economic rules are founded on the essentials of Islam. Islamic religion has greatly influenced Saudi education; the curriculum of all

educational levels embraces Islamic ideals and serves as the core subject of the curriculum (AlSalloom, 1991). According to the Saudi Official Policy of education (1980), the first principle of the general foundations on which education is based is strengthening faith in God and Islam, and in Muhammad (peace be upon him) as prophet and messenger of God.

Though girls and boys are educated separately throughout Saudi Arabia, both genders receive the same basic formal education consisting of: kindergarten, elementary, intermediate and secondary school. Public education in Saudi Arabia is free for all students at all levels including higher education; private schools and some private universities charge fees. Early childhood education, including kindergarten, is not mandatory in Saudi Arabia. Therefore, public kindergarten classes are reserved for the children of employees at the Ministry of Education, such as teachers. As such, most people who wish to send their children to kindergarten enroll their children in private kindergarten schools (AlIssa, 2009).

The school year at all levels consists of two semesters, each fifteen weeks long. The number of classes per week varies from 28 to 38. The length of each class is 45 minutes. Schools start at 7:00 am and end at a time between 12:00 and 2:00 pm. Passing an exam at the end of the school year is required in order to move up a grade or level. Students who fail the exam are required to take another test in the failed subject area before the new school year starts. If the student fails a second time, the student stays at the same grade level (The Ministry of Education in Saudi Arabia, 2015).

In Saudi Arabia, only elementary schooling is compulsory. Elementary school consists of six grades serving students between the ages of six and twelve. It focuses on teaching students the basics of the Islamic culture and values, Arabic language, social studies, mathematics and sciences. Students also start to learn English as a second language at grade four. In elementary schools, there is an automatic progression system. In intermediate and high schools, students move to the next level by passing an examination. The weekly lesson timetable below shows the weekly classes for every school subject in every grade in public schools (Alissa, 2009; Alotabi, 2014).

Table 1. Number of classes for each grade in every subject in elementary public schools

Subject	Number of classes for each grade per week						
	Grade1 Semesters		Grade2	Grade3	Grade4	Grade5	Grade6
	1	2					
Islamic studies	9	9	9	9	9	9	9
Arabic language	13	11	9	9	9	8	8
Mathematics	2	4	4	4	5	5	5
Science	-	1	2	2	2	3	3
History	-	-	-	-	1	1	1
Geography	-	-	-	-	1	1	1
Civic	-	-	-	-	1	1	1
Home economic	2	2	2	2	2	2	2
Fine arts	2	1	2	2	1	1	1
English language	-	-	-	-	2	2	2
Total	28	28	28	28	33	33	33

Source: The Ministry of Education in Saudi Arabia (2015), (AlIssa, 2009).

Intermediate education includes grades 7-9 for students between the ages of thirteen and fifteen. It focuses on furthering students' study of the values of Islamic culture and general education including mathematics and sciences. Table 2 shows the number of weekly classes for every subject in every grade in public schools.

Table 2. Number of classes for each grade in every subject in public intermediate schools

Subjects	Number of classes for each grade per week		
	Grade7	Grade8	Grade9
Islamic studies	8	8	8
Arabic language	6	6	6
Mathematics	4	4	4
Science	4	4	4
History	2	2	2
Geography	2	2	2
Civic	1	1	1
Home economic	2	2	2
Fine arts	1	1	1
English language	4	4	4
Total	34	34	34

Source: The Ministry of Education in Saudi Arabia (2015), (Alissa, 2009).

Secondary education consists of three grades, 10 to 12, for students between the ages of sixteen to eighteen. In the first year (grade 10), students learn general

information about all subjects such as Islamic studies, social studies, Arabic and English literature, mathematics, physics, chemistry, biology and computer sciences. After grade 10, students have a choice of two streams to follow for the remaining two years (grades 11 and 12): liberal arts and scientific major. Students taking the liberal arts stream do not take mathematics or sciences. Students in the scientific major continue taking mathematics and sciences but stop taking social studies and psychology. Table 3 below shows the number of classes for each grade in every subject in secondary public schools.

Table 3. Number of classes for each grade in every subject in public secondary girls' schools

Subjects	Number of classes for each grade per week				
	Grade10	Liberal arts		Science	
		Grade11	Grade12	Grade11	Grade12
Islamic studies	5	5	5	5	5
Arabic language	6	9	9	6	6
Mathematics	5	-	-	6	7
Physics	2	-	-	3	3
Chemistry	2	-	-	3	3
Biology	2	-	-	2	2
Computer science	2	2	2	2	2
Psychology	-	2	-	-	-
Sociology	-	-	2	-	-
History	2	2	2	-	-
Geography	2	2	2	-	-
Civic	1	1	1	1	1
Home economic	2	2	2	2	2
Library and research	1	1	1	1	1
English language	4	4	4	4	4
Total	36	30	30	35	36

Source: The Ministry of Education in Saudi Arabia (2015), (AlIssa, 2009).

2.5. Criticism of the Saudi education system

Since its creation in 1924, the formal education system in Saudi Arabia has received constant criticism from education researchers for its poor structure. Reforming the educational system and raising the standard of schools remains the biggest challenge facing the Ministry of Education. According to a document published in 2004

by the Saudi Ministry of Education, “the education system with its tools and methods has not had the desired effect on students’ behavior and has not contributed to the vision of the present circumstances in relation to the immediate and distant environments. This makes it imperative to provide clear vision and mature recognition of the contents of the education system that will fulfill the society’s needs and aspirations” (Maroun, Samman, Moujaes, & Abouchakra, 2008, p. 24). A World Bank report from 2012 states that even with the very high average income per capita, the oil states, such as Saudi Arabia, provide a lower quality education than most other Middle Eastern and North African countries (World Bank, 2012).

In his book, Educational reform in Saudi Arabia, AlIssa (2009) provides a critical analysis of the present education system in Saudi Arabia. He claims that the education system in Saudi Arabia has failed to produce generations who are successfully able to embrace globalization and openness to other cultures as well as keep up with scientific and technological progress. He also blames the education system for the lack of critical thinking skills among Saudi school graduates. In his effort to answer the question about why most reforms and educational development attempts in Saudi Arabia have not produced noticeable results, he identifies three main factors.

The first factor according to AlIssa (2009) is a lack of clear vision at the central administration level at the Ministry of Education about the needs of the country with respect to the nature of education reform and what changes are needed to meet those needs. The second factor is the conservative nature of Saudi society, which is resistant to the idea of change in general, and more specifically changing the education system. Saudi Arabia’s education system has continued to reflect mainly conservative party ideology despite efforts to expand and modernize education. The third factor is the centralized structure of the education system. Conventionally, the educational system in Saudi Arabia maintained an extremely centralized and highly bureaucratic structure within each sector. Issues like funding, teacher recruitment, textbooks, instruction, curricula, and educational policy all flow from a central bureaucracy. The Ministry of Education requires teachers to follow the textbook and the teacher's guide, which can restrict differences in individual teaching styles and approaches (AlIssa, 2009).

These three factors negatively affect the secondary school education in Saudi Arabia. According to the Saudi Official Policy of Education (1980), one of the objectives

of the secondary school level is opening opportunities to qualified students and supporting them to continue with higher education after high school as well as preparing students who do not plan to pursue higher education for the labour market. However, most students who graduate from high school are not adequately prepared for either continuing with higher education or joining the labour market (AlSalloom, 1991). Iqbal and Zenchenkov (2014) also indicated that many students in Saudi Arabia who complete high school are not intellectually and professionally ready for their lives after high school.

Critics claim that many first-year university students in Saudi Arabia could not make the leap from their weak public school education into the university's various undergraduate courses. Since many first-year students arrive with very poor academic and social skills, all universities in Saudi Arabia recently adopted a mandatory preparatory one-year program. The preparatory program is one way to bridge the gap between the public-school education system and the higher education system in Saudi Arabia. The main purpose of the program is to create a smoother transition from the public-school system to the teaching and learning setting of the high education system. It helps the student to engage and adopt the academic, social, and research aspects of university life (AlAqeeli, 2014).

The program focuses on enhancing the students' skills and knowledge in subjects such as mathematics, sciences, English and computers. It also focuses on developing other skills such as communication skills, critical thinking and problem-solving skills, leadership skills and self-learning skills. All students must complete this one-year prerequisite prior to enrolling in the university's various undergraduate programs (AlKathiri, 2014).

The inability of high schools to prepare students for life after graduation influences the labour force as well as universities. According to the view of some Saudi researchers, one of the main reasons for the high level of unemployment among young Saudi's is the low quality of the education system and the high level of foreign labour working in Saudi Arabia (Al-Dosary, Rahman, & Shahid, 2005; Iqbal & Zenchenkov, 2014). Quality is a fundamental element in ensuring that graduates of the educational system participate actively to the country growth instead of being a burden on it. The quality of education for those graduating from formal schools is low and does not equip students with adequate skills to join the workforce (Al-Dosary, Rahman, & Shahid, 2005;

Iqbal & Zenchenkov, 2014). The unemployment rate among males is 10.6% and it is higher among females (Iqbal & Zenchenkov, 2014). Some researchers indicate that the education system in Saudi Arabia has not succeeded in providing young Saudis with the required knowledge and the essential employable skills and attitudes that enable them to enter the workplace and develop individually and within their communities (Al-Dosary, Rahman, & Shahid, 2005; Iqbal & Zenchenkov, 2014).

Because most Saudi graduates lack adequate knowledge and skills required by companies today, many private businesses depend on hiring skilled foreign workers (Al-Dosary, Rahman, & Shahid, 2005; Iqbal & Zenchenkov, 2014). According to the World Bank report (2012), Saudi Arabia is the world's second largest sources of workers' remittances after the United States. Researchers Al-Dosary, Rahman and Shahid (2005), Maroun, Samman, Moujaes and Abouchakra (2008) and Iqbal and Zenchenkov (2014) emphasize that the skills of Saudi students are not currently adequate to meet the needs of the present local labour market.

Al-Dosary, Rahman and Shahid (2005) acknowledge that most Saudi students with international education are easily finding employment opportunities, while most of the locally educated Saudis are finding it very difficult to get appropriate jobs. This situation is because Saudi students lack employability skills. Also, Maroun, Samman, Moujaes and Abouchakra (2008) emphasize that "the Saudi graduates from all levels of the education system lack training in "soft" business skills such as leadership, team motivation, project management, problem solving, communication, and negotiation" (p. 6).

Criticism of the Saudi education system mainly targets school curriculum and traditional teaching practices. Critiques indicate that most school curriculum are rigorous and promote traditional teaching methods that employ rote memorization. According to Al Sheki (2011), in many mathematics classrooms, teachers are still using traditional teaching strategies rather than reform teaching strategies. Common teaching methods in schools do not focus on motivating young students to learn more and foster their curiosity to develop their communication skills and problem-solving abilities (Rugh, 2002). In mathematics classrooms, teachers mostly lecture on mathematics concepts and students repeatedly complete routine tasks that have little meaning and provide

minimal challenge (Al Balawi & Al Rajeh, 2012). This is partly because most teachers do not have the appropriate academic and pedagogical skills (Alissa, 2009).

Critics also claim that the Saudi educational curriculum focuses mainly on deepening students' knowledge about religious studies and literature and does not provide enough consideration to other subjects such as mathematics and science (Iqbal & Zenchenkov, 2014; Alissa, 2009). If we go back to tables 1 and 2, we realize that students at elementary and intermediate schools spend more time taking Islamic studies and Arabic language studies compared to mathematics and science.

Professional development for teachers is another issue of concern when we talk about the Saudi education system. Saudi teachers in general and mathematics teachers in particular have very limited professional development opportunities (Al Sheki, 2011). For example, Al Balawi and Al Rajeh (2012) explored the reality of mathematics teachers' professional development in ten school districts in Saudi Arabia. The study revealed that most of the 626 teachers participated in the study relied on their personal efforts to seek personal growth and performance improvement. Teachers relied mostly on self-accessed resources such as books and journals, communication with other teachers, and communication with their supervisors as the main means of professional development. Most of the participant teachers did not have the chance to join any formal professional development or workshops for the past three years.

In Saudi Arabia, the reality of students' mathematics achievement in schools is not satisfying. According to Trends in International Mathematics and Science Study (TIMSS, 2011), Saudi students are falling behind students in most other nations in mathematics achievement. TIMSS 2011 international result in mathematics indicates that only 55% of grade four students reach low international benchmark and only 2% reach the advanced international benchmark. Grade eight result are even worse than grade four with only 47% of students reaching the low international benchmark and only 1% reaching the advanced international benchmark. According to TIMSS 2011, low international benchmark for grade four students indicates, "Students have some basic mathematical knowledge. Students can add and subtract whole numbers. They have some recognition of parallel and perpendicular lines, familiar geometric shapes, and coordinate maps. They can read and complete simple bar graphs and tables" (p. 87). In addition, low international benchmark for grade four students indicates, "students with

low have some knowledge of whole numbers and decimals, operations, and basic graphs" (p. 113). According to World Bank report (2012) the average achievement for Middle East countries is far below the world average in mathematics and science achievement and countries like Saudi Arabia scored particularly low on mathematics compared to other countries in the region.

These unproductive and undesirable outcomes of the formal education system in general and mathematics education in particular require further research in Saudi Arabia. In Saudi Arabia, mathematics education as a field is under represented compared to other educational fields. There is a lack of research regarding mathematics teachers' practices specifically for teachers of high school mathematics (Almaraee, 2003). According to Almaraee (2003), one of the main obstacles for researchers in mathematics education in Saudi Arabia is the shortage in studies about school mathematics at all school levels. Generally, there are not enough documents available about high school mathematics education and most of the available documents are very broad and superficial (Almaraee, 2003).

In recent years, the Ministry of Education has made great effort to develop and implement comprehensive reform programs with the aim of improving the quality of the public education system. In 2011, the Ministry of Education's budget had more than the tripled since 2004, with approximately 25% of the national budget devoted to education (Nolan, 2011). Although funding is central to education development, when it occurs in isolation from the other dimensions of the education-reform strategy agenda, its impact is not effective (Alissa, 2009). In the next section, I provide an overview of the most recent reform initiatives in the education system in Saudi Arabia with a special highlight of these initiatives that may have a direct influence on high school mathematics teachers' practices.

2.6. Recent reform initiatives in the education system in Saudi Arabia

The recent reform movement in Saudi Arabia started during the reign of king Abdullah bin Abdulaziz who ruled the country from August 2005 until 2015. Under his rule, Saudi Arabia witnessed a unique development in the education sector. The king gave priority to improvement in education. One of the most important initiatives was the

implementation of king Abdullah bin Abdulaziz's project for the development of public education, commonly known by the Arabic acronym "Tatweer" in 2007. The project's goal was to improve the overall quality of the public education system.

One of the practical steps to implementing changes in the education system was the creation of The Tatweer Company for Education Services "T4edu" in 2012. According to their website, T4edu is in charge of implementing the new K-12 education development strategy, which was directed by the Tatweer project. T4edu works very closely with the Ministry of Education to design and implement education projects that aim to improve the quality of teaching and learning at public schools. Some of the projects that the company works on designing and implementing are: to improve the quality of science, technology, engineering, and mathematics learning (STEM project); to develop English language learning; the project for teacher training and professional development; and the special needs students learning project. Many more projects cover a variety of areas in education. While the list of projects is available on their website, the T4edu does not provide any details about these projects and no documents explaining their design or implementation (T4edu, 2015).

My purpose in talking about the recent reform movement in Saudi Arabia is to highlight actual changes or initiatives that have happened in the education system in Saudi Arabia, especially those that may have an impact on high school mathematics teaching and learning. The four main changes that I consider related to high school mathematics teaching and learning are; (1) the expansion of the higher education system; (2) the introduction of Tatweer high schools; (3) the introduction of new mathematics textbooks; and, (4) the introduction of the standardized testing for high school students.

2.6.1. The expansion of the higher education system

One great achievement of the reform movement is the expansion of the higher education system. The higher education system embarked on a rapid expansion, growing from only seven public universities to 23. In addition, there is the establishment of colleges and technical and health institutes in less than ten years. Moreover, the higher education system witnessed the introduction of 33 private universities and colleges (Alamri, 2011).

In order to provide high school graduates with more opportunities to continue their higher education, the government established the King Abdullah Foreign Scholarship Program in 2005. The program provides Saudi students with the resources to attend the best universities around the world, with most students attending universities in the USA, Canada, the United Kingdom and Australia (Alamri, 2011; Taylor & Albasri, 2014). Students are able to complete their Bachelor's degree, Master's degree, Doctorate degree or Medical Fellowship.

The purpose of this scholarship program is "to prepare and qualify Saudi human resources in an effective manner, so that they will be able to compete on an international level in the labor market and the different areas of scientific research, and thereby become an important source of supply of highly qualified individuals for Saudi universities as well as the government and private sectors" (Taylor & Albasri, 2014, p. 110). Around 200 000 students have joined the program and approximately 54% of them study at American universities (Taylor & Albasri, 2014). However, according to Alamri (2011), Saudi Arabia is fourth behind China, India, and South Korea with respect to the number of its citizens who study outside their home country.

In 2010, the Ministry of Higher Education began sponsoring students who had been refused entrance to public universities, allowing them to attend private universities in Saudi Arabia. This step provides students who cannot afford the tuition and additional fees of the local private universities the opportunity to continue their higher education (Alamri, 2011). The expansion of the higher education system through the increased number of universities and the introduction of the scholarship program has encouraged many high school students to continue with higher education instead of joining the labour market immediately after high school. Current high school students in Saudi Arabia have easier access to higher education compared to high school students ten years ago. As a result, preparing high school students for post-secondary education has become an essential part of teachers' instruction.

2.6.2. The introduction of Tatweer high schools

There are two types of high schools in Saudi Arabia, traditional schools, which follow the two-semester system that is applied at other educational levels, and Tatweer

schools, which follow the credit system. After finishing grade 9 (intermediate level) students can choose to join either a traditional high school or a Tatweer high school.

Before the implementation of the Tatweer school project, all schools were traditional. The introduction of Tatweer schools is the greatest change to the high school system since the enactment of the King bin Abdulaziz's Project. The aim of these schools is to improve the quality and relevance of education services and introduce a more modern system of instruction. According to Meemar (2014), Tatweer schools have good building facilities and equipment, extracurricular activities, special training and professional development of teachers. When the project started, the Ministry of Education converted a few schools in every major city to Tatweer schools. The number of Tatweer schools increases every year.

Students in the Tatweer schools are able to choose their courses and receive credits for each successfully completed course. Moreover, students have the option of taking summer courses. However, they cannot take mathematics in the summer nor take two mathematics courses in the same semester. Despite the differences, students at both traditional and Tatweer schools use the same textbooks for all school subjects, including mathematics.

2.6.3. The introduction of new mathematics textbooks

In 2010, the Ministry of Education in Saudi Arabia introduced new mathematics textbooks. The new textbooks replaced the previous mathematics textbooks, which Saudi high schools had been using for more than 30 years.

In Saudi Arabia, textbooks hold the status of clearly reflecting official curriculum. The expectation is that all teachers follow the textbooks and cover all topics by the end of the school year. The textbook is usually the primary and sometimes only resource for teachers. The Ministry of Education is the main authority in the country that issues textbooks used at all school levels (Al-Abdulkareem & Hentschke, 2004). The Ministry of Education distributes textbook series free of cost as a classroom resource; each student receives his or her own textbook. Within the Ministry of Education, the body in charge of textbook publication is the Centre for Educational Development (CED). Saudi experts who work for the CED within the Ministry of Education designed and developed

all previously introduced mathematics textbooks. Before I talk about the change in mathematics textbooks in Saudi Arabia, it is important to review some research that studied the role of textbooks in mathematics teachers' practices. The following section presents some research about teachers' use of mathematics textbooks. The literature presented below is not specific to the Saudi Arabia setting, but it is relevant to research about teachers' use of textbooks.

Mathematics teachers and the use of textbooks

Traditionally, curriculum materials or textbooks have been a center agent of policies to regulate mathematics practices in ways that parallel instruction with the reform perspective (Remillard, 2005). Textbooks are often the main resource for students and teachers in the classroom, offering the everyday materials of lessons and guiding the activities teachers and students do. As a result, educational policy makers use textbooks as an essential means to decide what students learn. Textbooks are a vital part of curriculum materials for directing students' acquisition of certain culturally appreciated concepts, procedures, intellectual dispositions, and ways of thinking (Battista & Clements, 2000).

The terms curriculum materials, curriculum, and textbook resources that are in print, and frequently published, are the materials geared for teachers to use with students in the classroom (Remillard, 2005). Research on teachers' curriculum use focuses on understanding how teachers "interact with, draw on, refer to, and are influenced by" curricular materials when designing their lessons (Remillard, 2005, p.212). While effective student learning is one expected outcome of textbook use, the development of teachers' techniques and practices is an additional desired outcome. Researchers have only recently started shedding light on the impact of curriculum materials on teachers, and how teachers use them (Davis & Krajcik, 2005; Remillard, Herbel-Eisenmann & Lloyd, 2009).

The focus of how teachers interact with and use curriculum materials has not been always significant in curriculum studies. Historically, research about school curricula relied mainly on examining the textbooks to restructure the contents of classroom practices (Love & Pimm, 1996). Reform efforts in mathematics education are the product of curriculum development supported by standards adopted by the National Council of Mathematics Teachers (NCTM, 2000). Teachers have since faced the

demand of applying new curriculum materials and adopting new conceptual and pedagogical approaches to teach new standards-based curriculum. Standard-based curriculum requires students to answer questions with high levels of cognitive demands that emphasize conceptual understanding and connection of many mathematical ideas, rather than traditional procedural skills. As a result, Remillard (2005) calls for more research in order to understand teachers' use of reform-based textbooks.

Studies suggest that the way teachers use textbooks influences their individual teaching practices and could possibly shape their pedagogical and content knowledge. Textbooks can have an impact on mathematics teachers' instructional practices in their classrooms (Robitaille & Travers, 1992; Stein & Kim, 2009). Textbooks are still the visible curriculum in the majority of classrooms. Teachers use textbooks to identify the topics to be covered and to choose problems and questions to make topics concrete (Ball & Cohen, 1996). For mathematics teachers, textbooks do not only characterize a considerable proportion of the content, sequence and objectives of the curriculum, they also influence how teachers present certain mathematical topics. In addition, teachers' interaction with the textbooks is the primary source, which directs learning. While teachers plan their lessons, they have to decide whether the material presented in the textbooks is adequate to teach. The textbook materials motivate the teachers' thinking and learning. Teachers have to understand the strategies required to teach certain concepts in the textbooks (Remillard 2000; Remillard & Bryans, 2004). According to Ball (1994), teachers are continually constructing new knowledge from their classroom experiences and interactions with the textbooks and other curriculum materials.

A number of studies have found that in-service teachers have inadequate content and pedagogical content knowledge for different ranges of mathematical subjects. Some researchers suggest that curricular materials have the potential to effectively influence the knowledge of practicing teachers. Some educators argue that curriculum materials have the potential to enrich the knowledge of practicing teachers (Ball & Feiman-Nemser, 1988; Ball & Cohen, 1996; Remillard, 2000). Researchers that support this view often regard a new textbook as a major factor to change teachers' instructional practices (Cohen & Ball, 1990). Educators sometimes assume that by offering new curriculum materials, which use the reform approach in mathematics teaching, teachers will follow textbooks, change their instructional practices in the way teachers anticipated, and as a result improve students' mathematics learning (Cohen & Ball, 1990; Remillard

& Bryans, 2004).

However, some studies suggest that using new curriculum materials does not necessarily lead to changes in teacher practices. Manouchehri and Goodman (2000) observed teachers' reactions to the implementation of new standard-based mathematics textbooks in United States and found that changes in teachers' practices are not the result of the introduction of new textbooks and other materials. They concluded that teachers do not necessarily change their teaching practices merely based on interaction with new materials.

Teachers also influence the curriculum materials they use by working as a filter through which they include their own interpretation of the curriculum content (Cohen & Ball, 1990; Love & Pimm, 1996; Remillard, 2005). Teachers differ in their level of textbook implementation. Several contextual and social aspects affect the way teachers interact with and use curriculum materials. Some teachers use their experience with other textbooks to alter the lessons, enhance the lessons with additional activities or materials such as worksheets, or selectively pass over entire lessons (Love & Pimm, 1996; Drake & Sherin, 2006). Teachers also draw on their experiences as students to reflect on their use of the curriculum materials (Cuoco, 2001). Drake and Sherin (2009) indicate that every teacher has his/her own "curriculum vision". Teachers develop this vision as they interact with the curriculum while they read, evaluate, and adapt curriculum materials before, during, and after instruction (Drake & Sherin, 2006, 2009).

In her 2005 article, Remillard reviews the research literature on teachers' use of curriculum materials and addresses the issue of fidelity by identifying four distinct perspectives on how researchers conceptualize the term "use": following or subverting, drawing on, interpreting, and participating with (p. 217). Every one of these perspectives has a distinct notion of curriculum materials and the teacher's role related to them. The "following or subverting" perspective regards curriculum materials as embodying visible and broad images of practice. It focuses on examining the degree to which teachers follow these guidelines with fidelity. According to this perspective, the teacher's only role is to enact a planned curriculum. The "drawing on" perspective views curriculum materials as one of several available resources and teachers are active designers of the enacted curriculum, and they have agency over the curriculum. The "interpreting" perspective perceives curriculum use as interpretations and representations of tasks and

concepts. According to this view, teachers are supposed to draw upon their experience to make meaning. In addition, fidelity of implementation is impossible. Finally, the "participating with" perspective considers curriculum materials as artefacts or tools that teachers use to design the enacted curriculum. Teachers design the enacted curriculum through a dynamic process of collaboration with the curriculum materials or the textbook.

The old mathematics textbooks in Saudi Arabia

Some studies indicated that the way teachers use textbooks can influence their individual teaching practices and could possibly shape their pedagogical and content knowledge. Textbooks can have a major influence on mathematics teachers' instructional practices in their classrooms (Robitaille & Travers, 1992; Stein & Kim, 2009). Textbooks are one means through which some policy makers have offered support and encouragement to teachers to change their instructional practices in their classrooms. The Ministry of Education has used textbooks as an influential tool to improve teachers' practices and students' learning of mathematics. Therefore, introducing new textbooks is an important part of any education movement in Saudi Arabia. In 1994, the Ministry of Education introduced a new mathematics textbook in all boys' high schools across Saudi Arabia. Girls' school continued with the existing textbook with the reasoning that the General Administration of Girls' Education, which was independent from the Ministry of Education, managed boys' schools. However, the introduction of the new mathematics books in high school in 2011 included both girls' and boys' schools.

According to the cover of the old mathematics textbooks used in high schools, the General Administration of Girls' Education approved the textbooks as the official textbooks for girls' schools. Saudi experts at the Ministry of Education developed these textbooks. The names of the authors are not included in the textbooks. Each grade has two textbooks; one for each of the two semesters. For each semester, the textbook has between four and five chapters.

Every chapter in the textbook includes six to nine lessons. The chapters do not contain an introduction and all the lessons follow the same structure. Each lesson begins a definition of a mathematical concept or a theory followed by a solved example and then an exercise. The same pattern repeats itself until the end of the lesson, which includes a list of exercises. The textbooks do not rate the level of difficulty of the

exercises and there is no indication of what mathematical skills the exercise is focusing on. Every chapter ends with a small summary that mostly includes a list of the exact definitions or theorems presented in the chapter with no further explanation followed by general exercises similar to those presented after every lesson. The last part of every textbook includes answers to most of the exercises included in the textbook.

The new mathematics textbooks in Saudi Arabia

In Saudi Arabia, one of the major reform initiatives targets the existing mathematics curriculum. In 2010, the Ministry of Education introduced new mathematics textbooks. The Ministry sees this initiative as a major step towards creating change in teaching practices. McGraw Hill Education Learning Company publishes the new mathematics textbooks.

A group of experts and specialists from the Ministry of Education in Saudi Arabia worked on the translation, editing and adapting the American version of the textbooks. The group of experts included specialists in: mathematics, curriculum and instruction, psychology, evaluation and assessment, educational technology, design and production, Arabic language, and English language. The group of experts also included experienced mathematics teachers and educational supervisors. According to the General Director of Curricula at the Ministry of Education, experts adjusted around 20% of the original American version, mainly to adapt to local culture. The group of experts also reorganized the content of the textbooks to enhance the scope and sequence in the grade 1-12 mathematics curriculum (Ministry of Education, 2015).

As with the old textbooks, each grade has a textbook for each of the two semesters. All six high school textbooks have the same introduction outlining the objectives of the textbooks; these objectives are the same as those on the Ministry of Education's website (Ministry of Education, 2015). The Saudi textbooks include the names of the original authors and consultants of the original American version as well as the names of the group of Saudi experts who translated and adapted the American version.

The new high school mathematics textbooks in Saudi Arabia have a different title than the original American ones. The title of the original American grade ten textbook is Geometry; the current textbooks for Saudi Arabia, one for the first semester and the

other for the second, are Mathematics 1 and Mathematics 2. The Saudi grade eleven textbooks are Mathematics 3 and Mathematics 4 for semester one and two textbooks, respectively. For grade twelve, the title for the Saudi textbooks are Mathematics 5 and Mathematics 6 for semester one and two textbooks, respectively, whereas the original American versions are titled Precalculus and Algebra 2.

According to the introduction of the new textbooks, the mathematics textbooks aim to: (a) help students develop higher-order mathematics thinking skills; (b) develop ways of mastering these skills; (c) construct a strong conceptual foundation in mathematics that enables students to apply their knowledge; (d) make connections between related mathematical concepts and between mathematics and the real world; and, (e) apply mathematics logically to solve problems from daily life.

By 2013, all grades in Saudi Arabia had received the new textbooks. The Ministry of Education introduced the new textbooks gradually in 2010, starting with grades one, four, and seven. The 2011-2012 school year saw the new textbooks introduced in grades two, five, eight and ten. The Ministry of Education then introduced the new textbooks for the 2012-2013 school year to grades three, six, nine, and eleven. Finally, grade twelve students began using the new textbook at the beginning of the 2013-2014 school year.

The new textbook for each semester has four chapters; each chapter is divided into lessons. Every chapter starts with a “get ready for the chapter” lesson. This lesson starts with the title, and then outlines previously covered skills and concepts, the purpose, and the learning outcome of the chapter. It also includes an image illustrating how students can make a brochure, which helps them organize the information included in the chapter. The “get ready for the chapter” lesson also includes a quick test section and a quick review section. Both these sections include questions and examples related to skills and concepts that students should already know.

Each lesson in the textbook begins with the title written in both Arabic and English, previously covered skills and concepts, learning outcome of the lesson, and the main mathematical vocabulary used in the lesson in both Arabic and English. According to the introduction of the teacher's guide, including the main mathematical vocabulary

used in every lesson in English helps prepare high school students for university since most universities in Saudi Arabia use English books and resources.

The purpose section presents information usually relates to real-life situations and sometimes requires the students to answer questions that follow the information. Every lesson contains three parts: (a) instruction, (b) performance, and (c) assessment. The instructional part contains systematic explanations, definitions, theories and examples of the concepts or skills being presented. Sometimes this part includes a section with the title and a real-life example. In this section, the textbook presents an example from real life where students can apply the concept presented in the lesson. The performance part contains two sections: (a) check your understanding and (b) practice and problem solving. The assessment part contains three sections: (a) higher order thinking problems, which requires the use of complex thinking skills; (b) a test practice; and, (c) a cumulative review.

Every chapter has a quiz in the middle of the chapter, a study guide and review, an end of chapter test, and a cumulative practice test, which contains questions and problems from all the chapters presented in the textbooks from the two semesters of the same year. Another noticeable feature of the lessons is the use of the margins. Every lesson has little boxes in the margins, which are usually titled with the following: (a) guidelines for study, which provide some general information about the concept presented; (b) guidelines for the test, which provide some tips for the test; and, (c) caution, which are warnings for the student about common mistakes; (d) real life connection; this provides general information about some every day concepts presented in the lesson and its relation to mathematics concepts; and, (e) reading mathematics, which are tips about how to read mathematics writing, including mathematics symbols.

A critical analysis of mathematics textbooks in Saudi Arabia

To provide a general examination of the nature of the old and new mathematics textbooks, I conducted a critical analysis of one chapter in each textbook. The aim of this analysis is not to evaluate the two textbooks or to focus on the difference on the mathematical content in the old and new textbook. The purpose is to provide a general understanding of the look and voice of the textbook and explore how the reader perceives it. This examination of the two textbooks also helps me to understand some aspects of the general teaching perspective the textbooks reflect. In order for the

analysis to be reasonable, and to avoid the misperception that the differences uncovered in the analysis of the two textbooks is due to the differences of the main mathematical content of the two chapters, I chose to analyze two chapters that have the same title in the old and new textbooks. As such, I chose to analyze the chapter entitled "Trigonometry" from the new and old grade eleven mathematics textbooks. Both chapters include an introduction to trigonometry.

Although Saudi teachers do not officially use the old textbooks in their classroom, it is important to include an analysis of the old textbooks in this study because the old textbooks were the official curriculum document for more than 30 years. As such, they have had a great influence in shaping the culture of mathematics teaching practices in Saudi Arabia. All teachers who participated in this study had learned from these old textbooks in school when they were students and had experience teaching from these textbooks.

In addition, researchers such as Manouchehri and Goodman (2000) indicate that when mathematics teachers adopt a new curriculum or textbooks, changes in their practices do not necessarily occur. Manouchehri and Goodman (2000) conclude that even after the implementation of new textbooks, their experience teaching from previous textbooks continued to influence teachers' practices. Therefore, a critical analysis of the old and new textbooks provides a general examination of the nature of the two textbooks. This examination is important to the later stages of my research when looking at how the participant teachers experienced teaching the two textbooks and how this experience may inform their practices in classrooms. More details about the analysis are included in Appendix C, including the framework I used and a detailed analysis. In the next section, I include my conclusions of the analysis of the two chapters.

While completing the analysis of the chapter entitled "Trigonometry" from the old textbook (I refer to it as Chapter 1) and the chapter entitled "Trigonometry" from the new textbook (I refer to it as Chapter 2), I carefully read each word to understand its significance. This analysis helped to develop a broad understanding of the general teaching perspective of the two textbooks.

It is noticeable that the structures of the lessons in the two chapters have many differences, and as a result, the two mathematics textbooks are quite different. Lessons

in the old textbooks seem simple, starting with a short introduction that is very straightforward, followed by explanations and the main notion (definition or theory) and then ending with an example and a few exercises.

Comparatively, lessons in the new textbooks start by clearly stating the objectives of the lesson and the mathematics vocabulary used in the lessons written in both Arabic and English. Before the introduction of any definition of new mathematical concept, the lesson presents the purpose section. In this section, the reader engages with information usually related to real-life situations and usually requiring the reader to answer some questions. The accompanying exercises presented in the purpose section aim to guide students to new notions.

The new textbook also provides the reader with many sidebar comments about the presented concept and how to apply it when problem solving. The examples that show each step of the solution come after the main notion (definition or theory) followed by check your understanding exercises. The structure of the old textbook does not provide the reader with as much information about the presented mathematical notion. For example, the lessons do not ask the reader questions in relation to the purpose or the importance of the mathematical concept. Therefore, the old mathematics textbooks emphasize teacher-directed instruction by allowing the teacher to be the main source of information in the classroom. On the other hand, the new mathematics textbooks encourage a teaching style where students develop and discover mathematical concepts. The textbooks offer examples and exercises that explain why and how learning mathematics is useful.

Chapter 2 emphasized mathematical communication more than Chapter 1. Chapter 2 reflects mathematics teaching that supports students' development of the use of mathematics language and fosters familiarity with mathematical vocabulary. While Chapter 1 explains mathematics vocabularies only by presenting the mathematical definition of the concepts, every lesson of Chapter 2 starts with a presentation of the mathematics vocabulary used in the lessons. Most lessons in Chapter 2 have little boxes in the margins titled mathematics language. These boxes usually include information that demonstrates the correct use of specialized mathematical terminology and notation. Sometimes, they also explain the difference between the use of a particular word in everyday life and in mathematics. For example, in one of these boxes, the authors

comment about how everyday language uses the word “relation” in different ways and as a mathematical concept.

When analyzing the use of imperatives, I noticed a greater variety of linguistic choices by the authors of the new textbooks as compared to the old one. This variety indicates the authors are trying to communicate with the reader using diverse methods of presenting different mathematics ideas. Using a limited number of imperatives, such as find, write, proof, solve, and notice as in the case of the Chapter 1, could limit the reader’s view about the presented mathematical content. In these imperatives, the reader is labelled “scribbler” because s/he is expected simply to follow direction and complete a task without engaging in critical mathematical thinking (Rotman, 1988, 2000). However, the use of a greater variety of linguistic choices in the new textbook could encourage mathematics teaching that focuses on developing students’ mathematical thinking (O’Keeffe & O’Donoghue, 2015).

Most of the imperatives found in the two textbooks are in the exercise sections. Most exercises in Chapter 1 start with the imperative find, which is an “exclusive" imperative that addresses the reader as “scribbler” (Rotman, 1988, 2000). Most of these exercises are conducive to promoting teaching that focuses on procedural rather than conceptual knowledge. In addition, exercises that start with the imperative find are “close-ended” problems. Close-ended exercise focus primarily on finding an answer that is a number or figure. Moreover, close-ended exercises do not allow students to explain their thinking processes (Nam Kwon & Park, 2006).

Exercises in Chapter 2 use the imperatives find, write, prove, and solve as well as discuss, compare, write using your own words, and construct. These exercises encourage students to discuss and describe, verbally or in writing, mathematical objects and concepts which enrich their conceptual knowledge. The diversity of the imperatives used in the new textbooks may result in teachers using a larger variety of methods, which allow students to demonstrate their knowledge about the mathematical concepts presented using both verbal and nonverbal mathematical representation.

Every lesson in Chapter 2 includes exercises and activities at different difficulty levels as well as a higher order thinking problems section. This section includes different exercises, which fall under five sub-titles: (1) open-ended problem, includes a question

or problem, which has multiple correct solutions and more than one strategy to obtain the answers; (2) challenge, includes a question with higher order thinking skills; (3) find the mistake, which presents two imaginary students' answers of a certain problem and the reader must determine which one is right and correct the mistakes; (4) justifying, which provides a statement about a mathematics concept and the students must justify and explain it; and, (5) writing, where students must use their own words to explain a mathematical concept and how to apply it to solve problems. Most problems in this section are "open-ended" and engage students with genuine mathematical ideas and encourage exploration and discussion providing teachers with valuable information that can inform their teaching while eliciting several responses (Capraro, An, Ma, Chavez, & Harbaugh, 2012; Nam Kwon & Park, 2006).

Since the mathematical discourse includes not just language, but also visual semiotic resources, I decided to compare the two textbooks in terms of the use and nature of non-linguistic features. Chapter 1 contains only two graphs along with the written mathematical symbols. No other images or representations are in the chapter. On the other hand, Chapter 2 employs many mathematical representations, generic drawings and photographs. Images and mathematical representations can help to enhance students' conceptual knowledge. The new mathematics textbooks attempt to reflect teaching practices that support students' development of mathematical meaning by relating mathematics to real situations using mathematical representations of real situations.

From my analysis of Chapter 1 and Chapter 2, I find the style of writing for both textbooks to be quite authoritative; the use of modal verbs in both chapters supports this idea as they communicate a high degree of certainty. This language could reflect teaching practices with a traditional absolutist view. Teaching mathematics with an absolutist notion of mathematics recognizes mathematics as a subject with a broad collection of firm and impeccable concepts and skills (Romberg, 1992). Alternatively, as Ernest (1991) describes it, a set of unrelated, but utilitarian rules and facts. This viewpoint could reflect teaching that emphasizes memorization of rules and formulas and procedural knowledge.

Generally, the analysis of the old and new textbooks revealed that the new textbook is more engaging, providing readers with rich mathematical ideas and based on

constructivist principles. The authors made a noticeable effort to encourage the readers to develop and discover mathematical concepts. The authors try to get mathematics to make sense to the reader by offering examples and exercises that explain why and how learning mathematics is useful. On the other hand, the old textbook mainly presents mathematics knowledge, provides direct examples and offers exercises that are similar to the provided examples.

2.6.4. The introduction of the standardized testing for high school students

As mentioned earlier, recently the higher education system in Saudi Arabia experienced an intense expansion. Therefore, more high school students than ever before have the desire to continue their education by attending four-year colleges and universities. The increased number of high school students who are pursuing higher education makes it more competitive to get into top universities and graduate schools. Applicants to higher education institutions must be ensured a fair and transparent admissions system. Post-secondary education institutions face a complex task determining which criteria most precisely predicts success of an applicant's future academia. While selection criteria vary among institutions, cognitive tests such as standardized ability test are one of the main criteria for the admission choices in many of higher education institutions. Recently, two standardized tests became part of university admissions: The General Aptitude Test (GAT) and the Standard Achievement Admission Test (SAAT).

Since mathematics is a major subject in the two tests, the introduction of standardized testing for high school students is one of the recent reform initiatives that have a direct impact on high school mathematics teaching and learning. For decades, post-secondary education institutions focused on only a student's overall high school average as the criterion for admission. Until recently, this system received no criticism; however, this is no longer the case. Today, there is much negativity to relying solely on high school grade averages. Therefore, the National Centre for Assessment in Higher Education (NCAHE) developed the General Aptitude Test (GAT) and the Standard Achievement Admission Test (SAAT). Postsecondary institutions began to include the two tests scores along with high school percentage in their admission requirements to ensure acceptance of capable students. In fact, universities consider only 30% of the

school grades that the students get at secondary school examinations while 70% from the GAT and SAAT (Siddiek, 2011).

Standardized testing: SAT, GAT and SAAT

Although the idea of standardized testing is relatively new to the Saudi education system, standardized testing has been used to evaluate the accumulated knowledge of an applicant pool since early times. The earliest evidence of standardized testing comes from Imperial China. The system of civil service examinations is one of the most noteworthy contributions from Chinese history to the world. The history of Chinese civil service examinations is attached to the history of the civil service itself. The civil service examination system originated during the Han dynasty (206 B.C. – 221 A.D.). The original purpose for the Chinese civil service examinations was to guarantee that appointees to civil service positions possessed the right abilities, talent, and education and their hiring was not because of inherited privilege (Menzel, 1963; Elman, 1991; Miyazaki, 1976). This idea was heavily influenced by Confucian ideals, which focused on the principle that moral guidance, courtesy, and filial piety could maintain a thriving government and social system (Menzel, 1963).

Deciding who the most qualified person is for a particular job can be a complicated task, which may require applicants to take tests, which measure their skills and knowledge in a particular area. Colleges and universities face the same perplexing task when accepting new students. The ancient Chinese notion of choosing the most qualified people based on the result of examinations has found its way into many education systems around the world. In some places, universities and colleges applicants strive to demonstrate their academic abilities by taking tests. Universities and colleges use the results, along with high school GPA and other indicators to determine students' academic readiness.

In the United States, the SAT is the most commonly used test by high school students as a reliable, effective measure of their readiness. The College Board in the United States developed the SAT; it also administers it to interested parties. It is a non-profit organization founded in 1900 by a gathering of 12 colleges and universities. Its aim was to develop excellence and fairness in education by offering fair and reliable tests, particularly by expanding access to higher education and facilitating the application process for students (College Board, 2011b).

The story of the conception of the SAT goes back to 1923 when Carl Brigham became the chairperson of the College Board organization. He was interested in adopting a standardized test for the selection of the United States military members for use to college admission. In 1926, the SAT was born under its original name the Scholastic Aptitude Test (Lemann, 2004).

According to some researchers such as Atkinson, Geiser and Lemann, the SAT, originally called Scholastic Aptitude Test, was not originally established to assess post college skill performance and subject knowledge taught in school; rather, it was designed for the purpose of measuring students' intelligence (Atkinson, 2004; Atkinson & Geiser, 2009; Lemann, 2004). It was originally a test for measuring IQ to assess the intelligence of the employees of the American Army. The original purpose of the test was to measure the "innate ability," or "inborn academic aptitudes" (Lemann, 2004; Epstein, 2009).

According to the College Board, the SAT measures students' abilities in critical reading, mathematics reasoning, and writing skills that the students have developed during their in and out of school experience and tests their ability to apply that knowledge to participate positively in higher education institutions. Students who take the SAT benefit from the experience some of the most comprehensive performance feedback of any admission test. Students who score very well have access to excellent scholarship opportunities. Companied with GPAs and high school records, SAT scores help admission officers choose the most qualified applicants based on fair comparison (College Board, 2011b).

Throughout the years and in spite the many changes, the claim that SAT is a valid tool to measure students' general analytic ability has been abiding. However, some educators remain skeptical of the possibility that anyone can design a test that can measure students' innate abilities and "not family background or the quality of education" (Lemann, 2004, p. 11). As such, one of the main critiques of the SAT is that students differ in their mental abilities, which are often associated with their background. Some educators, Lemann, Atkinson, and Gilroy to name a few, argue that the SAT provides some students more of an advantage over the content of the test than others do because of its design and presentation. It limits education equity and hinders access to higher education for otherwise qualified students (Gilroy, 2007).

The SAT has been criticized for being vulnerable to the impact of elements related to students' socioeconomic status, such as schooling excellence and SAT coaching (Atkinson, 2001; Atkinson, 2004). While some researchers, such as Powers and Rock (1999), argue that coaching has minimal effects on SAT performance and it only marginally improves test scores, others indicate that SAT tests can be highly coachable (FairTest, 2007). Many test-preparation providers promote average score enhances of 100 points on the SAT; however, research suggests coached students have an average increase of 30 SAT scores (Briggs, 2009). The level of effectiveness of coaching programs on SAT performance does not prevent some educators from arguing that SAT coaching is not reasonably available for all test taker, which provides unfair advantage to some test takers over others (Epstein, 2009; Stringer, 2008).

Another common criticism of the SAT describes the test as not being strong enough in predicting students' academic performance (Geiser, 2009; Stringer, 2008; Atkinson & Geiser, 2009). Stringer (2008) notes that the test is failing to measure students' actual knowledge, which influences the ability of the test to predict future success in college. One of the main reasons behind not considering the test as a valid predictor of college success is that the test does not take into account some other elements that influence students' achievement in higher education such as motivation and study skills (Stringer (2008).

The ongoing controversy over the SAT set off the SAT-Optional Movement, which first began in 1969 when Bowdoin College in Main offered its applicants the option of submitting the SAT score with their application (Epstein, 2009). The SAT-Optional movement was further fueled when, in 2001, the president of the University of California at that time, Richard Atkinson, proposed that the University of California, one of the largest universities in the USA, no longer require students to submit SAT scores when applying (Gilroy, 2007; Epstein, 2009). The SAT-Optional movement has gained more followers since Atkinson's proposal in 2001 (FairTest, 2007).

With the increase of the SAT-Optional movement in the United States, Saudi Arabia has begun questioning the use of aptitude testing. For decades, post-secondary education institutions used only the overall high school average of a student to decide on admission acceptance. Until recently, this system received no criticism; however, this is no longer the case. Today, there is much negativity to relying solely on high school

grade averages. As a result, he National Center for Assessment in Higher Education (NCAHE) developed a new aptitude test, the General Aptitude Test. Postsecondary institutions began to include the General Aptitude Tests (GAT) scores along with the Standard Achievement Admission Test (SAAT) and high school percentage in their admission requirements to ensure acceptance of capable students.

The story behind the creation of the GAT and SAAT shares the same main purpose as introducing the civil service examination and the SAT, which is choosing the most qualified individuals. However, the story of the creation of the Saudi standardized tests has its own specific details. For most Saudi universities and colleges, especially the elite ones, a high school percentage was not enough for making admission decisions. Therefore, most Saudi universities established their own admission test to evaluate applicants along with the high school percentage. This movement had an impact on the application process, which became complicated for those applicants who applied to more than one university and for those who did not live close to the university to which they applied. This system vanished with the creation of GAT and SAAT.

For Saudi universities and colleges, the reason behind not depending on only high school percentages for evaluating university applicants was because of the way this percentage is obtained. 70% of high school students' percentage is calculated based on student scores gained on the work in the twelfth grade only. In addition, this percentage includes the evaluation of different classroom teachers who usually develop and grade the tests that high school students take at schools. These tests are different in their quality in terms of strength and reliability, which may influence the scores of students coming from different schools.

The National Center for Assessment in Higher Education (NCAHE) was established in 2001, a movement in educational reform in Saudi Arabia. Its mission is to "assure fairness and equal opportunity in higher education and contribute in the efficiency of higher education institutes based on solid scientific grounds" (NCAHE, 2015). It administers and develops different standardized tests, including the GAT. The NCAHE first administered the General Aptitude Test (GAT) and the Standard Achievement Admission Test (SAAT) at girls' schools in 2010. According to the NCAHE, the GAT and the SAAT are both required for admission to all institutions of higher learning in Saudi Arabia (NCAHE, 2015).

The General Aptitude Test (GAT) is an aptitude test used to assess the level of general ability in verbal and quantitative areas mastered over time. Its design is to measure general comprehension, as well as analytic and quantitative abilities in language and mathematics. NCAHE claims that the GAT is not a subject-oriented test based on specific standards related to particular subject materials (NCAHE, 2015). The GAT measures students' abilities in "reading comprehension, logical relations, problem-solving behaviour, inferential abilities, inductional abilities" (NCAHE, 2011, p. 2). The test consists of 120 multiple-choice questions. It has two main components, the verbal, which includes 68 questions and the quantitative, which has 52 questions. The verbal section contains questions in three areas: sentence completion, analogy, and reading comprehension. The questions in the quantitative section consist of 40% arithmetic, 23 % algebra, 24% geometry, and 13% interpretation of graphs and tables (NCAHE, 2015).

SAAT is an admission test that covers five basic subject areas: Biology, Chemistry, Physics, Mathematics, and English proficiency. Questions in the test cover the subjects in even percentage; each section is valued at 20%. SAAT focuses on high school material within the scientific curriculum. The distribution of questions on the SAAT is 20% for each subject of the grade 10 curriculum and 40% for each subject of the grade eleven and twelve curriculum. The SAAT test consists of 130 items in five basic subject sections.

Since the SAAT design is supposed to be consistent with high school curriculum, GAT coaching is more common than SAAT coaching. Joining professional coaching institutes has become more popular for the test takers. However, the NCAHE claims that students do not need to join any professional coaching institutes to prepare for the test. According to the NCAHE, the GAT is not an achievement test; "GAT is based on skills related to logical thinking, analysis and relationship. These skills have been acquired by test-takers throughout their education and through exposure to different experiences in life" (NCAHE, 2011, p. 3). Therefore, what students need to do in order to be prepared is familiarize themselves with the test by reading the booklets published by the NCAHE (NCAHE, 2011).

According to the descriptions of the GAT and the SAT in terms of their purpose, it is notable that these two tests share the same purpose. However, content and sections of the two tests are not the same. The early versions of the SAT are more similar in term

of its content and sections to the GAT. While the SAT is moving towards becoming more an achievement test than an aptitude test, it is not expected that the GAT could move in the same direction since the NCAHE is already offering an achievement test (SAAT) for university admission.

Despite the relatively short history of standardized testing in Saudi Arabia, these tests have been greatly criticized (Siddiek, 2011). According to Siddiek (2011), many Saudi educators are dissatisfied by the tests provided by the NCAHE. These tests are not statistically valid in terms of measuring students' abilities. These tests do not provide an accurate picture of the degree of achievement these students gained through their 12 years in general education. Siddiek (2001) indicates that the test is mentally and physically exhausting for the students, parents and educators. Students spend a long time preparing and taking the test since most of them take it more than one time. He also notes that some students earn around a 90% high school average, but they get only 60% in the tests provided by the NCAHE.

This kind of result tells us a lot about these tests and about the whole education system in Saudi Arabia. Students come to take the test with the assumption that they have already acquired the basic knowledge requirements in school subjects to prepare them to take such tests. Unfortunately, this is not the case; the poor performance of schools results in students not intellectually and psychologically prepared to take such test. "Actually we have to improve the teaching performance then the testing would come next" (Siddiek, 2011, p. 64).

2.7. Summary

In this chapter, I provided a general overview of the education system in Saudi Arabia and a detailed description of the changes that Saudi mathematics teachers are experiencing. This overview includes a brief history, the main features, and a criticism of the Saudi education system. An outline of the most recent reform initiatives in the Saudi Arabian education system is also included; it highlights the initiatives that have a direct impact on high school mathematics teachers' practices. The recent changes in the education system require examination to gain a better understanding of teachers' current classroom practices. The next chapter will present the main framework used in this study to interpret and understand Saudi high school mathematics teachers' current practices.

Chapter 3.
Theoretical Framework

As mentioned in the previous chapter, the Saudi Arabian education system has undergone major changes in the past decade. Government agencies involved in education have introduced new policies, standards, programs, and curriculum with the expectation that teachers incorporate the changes seamlessly, without consideration of existing practices. The aim of this research is to gain a better understanding of high school mathematics teachers' current practices during the current reform movement.

In this research, I use the Patterns of Participation (PoP) theory as the main framework to guide my research. Skott (2010, 2011, 2013, 2014a, 2014b) presents PoP as a framework that aims to understand the role of the teacher for emerging classroom practices. Instead of relying on a traditional approach to understanding classroom practices by analyzing teachers' beliefs, this framework applies a participatory approach to look for patterns in the participation of individual teachers in many social practices at the school and in the classroom.

This chapter presents PoP as a theoretical framework and outlines its potential for explaining and understanding mathematics teachers' classroom practices. The chapter also explains the connection between PoP and other theories it draws on. Furthermore, the chapter describes its usefulness and limitations as a framework to understand the role of the teacher for emerging classroom practices.

3.1. Patterns of Participation: moving away from a belief–practice approach

Skott's main motivation in developing the PoP framework was to overcome the conceptual and methodological problems of belief research (Skott, 2009, 2010). In this chapter, I explain how the emergence of PoP was an attempt to meet the conceptual and methodological challenges of the belief–practice approach for understanding teachers' roles in classrooms. Later in the chapter, I present the extended of the use of framework to include teachers' knowledge and identity.

Over the last 25 years, there has been a significant amount of research on teachers' beliefs (Skott, 2010). Many researchers support the view that the beliefs of students and teachers have a great influence on the way both behave in a school environment (McLeod, 1992; Thompson, 1992). As a result, numerous researchers have conducted studies on the crucial role of beliefs in learning and teaching mathematics. The studies assume that issues resulting from beliefs play a major role in mathematics learning and instruction (McLeod, 1992).

Scholars focusing on belief research assume that teachers' beliefs are a major determinant for teachers' practices in the classroom. As a result, researchers in this area argue that influencing teachers' beliefs could play a crucial role in changing teachers' classroom practices (Wilson & Cooney, 2002; Lerman, 2002). Skott (2009, 2013, 2014a, 2014b) indicates that this research approach was, and continues to be, strongly built on the assumption that teachers' beliefs are a major barrier to educational change, and that research about teachers' beliefs has the potential to find solutions to "the problems of implementation of the new and more process-oriented approach to mathematics instruction" (2009, p. 28).

However, belief research has many unresolved issues and many conceptual and methodological challenges (Skott, 2010, 2011, 2013; Skott et al., 2011; Skott 2014a, Skott 2014b). One of the primary conceptual challenges researchers encounter when studying beliefs is the diverse meanings of the term (Pajares, 1992; McLeod & McLeod, 2002; Skott, 2014a, 2014b). Pajares (1992) argues that to define beliefs is not clear-cut and is dependent on the definer; users of the word belief often use attitudes, judgements, opinions and perceptions, among others, synonymously. "Defining beliefs is at best a game of player's choice. They travel in disguise and often under alias [such as] attitudes, values, judgments, axioms, opinions, ideology, perceptions, conceptions, conceptual systems..." (Pajares, 1992, p. 309).

Moreover, Skott (2014a, 2014b) explains four major aspects of the conceptual challenges of belief research. First, beliefs are used to explain a person's mental structure, "which are subjectively true for the person in question" (p. 6). Second, "there is an element of affect to beliefs. Beliefs then are value-laden and characterized by a certain degree of commitment" (Skott (2014a, p. 6). The third major aspect is the stability of beliefs. Beliefs are mostly treated as being a stable structure. People are supposed to

have the same beliefs regardless of the situations they are encountering. In addition, people are not expected to change their beliefs unless they encounter a significant new experience. The last aspect is beliefs are expected to control people's actions, engagement and participation in different social settings. "To sum up, the notion of beliefs is used in the literature about mental reifications that are acquired on the basis of comprehensive, previous social experiences and that are characterised by considerable degrees of conviction, commitment, stability and impact" (Skott, 2014a, p. 6).

Part of understanding the meaning of the term belief is understanding the nature of beliefs and how they are structured and organized. One way to do this is to perceive beliefs as a cognitive structure, and as dynamic in nature, and are therefore subject to reform as individuals change and reassess their beliefs in response to new experiences (Thompson, 1992). Green (1971) also explained the nature of beliefs arguing that there are three dimensions to belief systems. First, beliefs have "a quasi-logical" structure where some beliefs are held as "primary" beliefs and others as "derivative" beliefs, which means that a certain belief can never be independent from all other beliefs. Second, beliefs inside the system can be classified as either "central" or "peripheral" where the central ones are held strongly and the peripheral ones are more apt to be reviewed and changed. Third, beliefs cluster into relatively independent groups (Green, 1971 as cited in Thompson, 1992). This clustering nature of beliefs makes studying and understanding teachers' beliefs an extremely difficult task (Thompson, 1992).

Liljedahl, Oesterle and Bernèche (2012) shed light on the contradictions in research regarding the stability of belief. "Authors had no difficulty allowing the ideas that beliefs are stable and beliefs can (and do) change, to coexist within their work, whether these constructs were stated explicitly or existed implicitly within the empirical evidence of their research" (p. 113). In addition, in their analysis of literature on research about beliefs, Liljedahl, Oesterle, & Bernèche (2012) indicate that beliefs are continually changing and researchers could not find evidence that indicates the stability of the beliefs systems (Liljedahl, Oesterle, & Bernèche, 2009). As a result, beliefs cannot be clearly defined and no single correct clarification could be found in research about its nature (Pajares, 1992; McLeod & McLeod, 2002).

Another major challenge when studying beliefs is the difficulty in distinguishing them from knowledge. Ernest (1989) suggests that while knowledge is the cognitive

outcome of thought, belief is the affective outcome; however, he states that beliefs also possess a small, but very important cognitive component. Thompson (1992) argues that distinguishing beliefs and knowledge is very complex, but that the distinction is important for researchers to address since teachers may consider their beliefs to be knowledge.

Thompson (1992) summarizes three features that distinguish beliefs from knowledge. First, beliefs usually hold different levels of conviction while knowledge is a certainty. The second feature is that beliefs "are not consensual". That is, beliefs are debatable whereas knowledge is usually unquestionable. Pajares (1992) also considers knowledge to the result of objective facts whereas beliefs come from personal evaluations and judgements. Therefore, the concept of knowledge is "somehow purer than belief and closer to the truth or falsity" (Pajares, 1992, p. 310). Finally, although there is common agreement about procedures for assessing and judging the legitimacy of knowledge, there exist no such criteria when looking at beliefs. Beliefs are often held or supported for reasons that can be described by lack of agreement over how they can be assessed (Thompson, 1992).

Furinghetti and Pehkonen (2002) highlight the strong relationship between knowledge and beliefs, claiming that beliefs are part of a person's subjective (personal) knowledge. They argue that knowledge can be separated into two types; "objective (official) knowledge that is accepted by a community and subjective (personal) knowledge that is not necessarily subject to an outsider's evaluation" (p.43). Furinghetti and Pehkonen (2002) also argue that knowledge has what they called "truth-property" meaning that knowledge is always valid with a probability of 100%. On the other hand, the probability for belief rarely reaches 100%.

Recent research has also indicated that we can identify beliefs and knowledge as subcategories of our opinions and principles. "Beliefs and knowledge can profitably be viewed as complementary subsets of the things we believe" (Leatham, 2006, p. 92). Consequently, people cannot easily distinguish between what is true and what they believe to be true. Additionally, in his attempt to clarify between the two concepts of teachers' beliefs and teachers' knowledge, Kuntze (2011) includes teachers' beliefs as aspects of what he calls "teachers professional knowledge". According to Kuntze's (2011) model of mathematics teachers' professional knowledge, teachers' beliefs are integrated beliefs as an aspect of professional knowledge.

As other human systems, beliefs and teaching practices have a complicated, two-way causal connection that is contextually determined (Thompson, 1992). Researchers of mathematics have found various levels of consistency between mathematics teachers' beliefs and their instructional practices. While some researchers found mathematics teachers' beliefs were consistent with their practices, others described mathematics teachers whose beliefs were inconsistent with their teaching practices.

Thompson (1984) suggests that teachers' beliefs about mathematics teaching play an important role in determining a teacher's view of how to teach mathematics and what kind of practices they use during instruction. Moreover, in her 1985 study, Thompson presented the case of a teacher with declared beliefs about mathematics and the teaching of mathematics that were consistent with her practice. Thompson explained that the consistencies between the teacher's beliefs and practices were a result of the teacher's ability to reflect on her own practice. Stipek, Giwin, Salmon, and MacGyvers (2001) conducted another study that indicates substantial consistency between teachers' beliefs about mathematics and their observed classroom practices. Participants in this study included twenty-one grade four, five, and six teachers. The findings show that traditional beliefs about mathematics were associated with more traditional classroom practices.

However, other studies reveal inconsistencies between teachers' beliefs and their classroom practices when they saw variation in the degree of consistency with these concepts. Thompson (1984) presents a study about a teacher whose stated beliefs about mathematics instruction were inconsistent with her actual classroom practices. During an interview, the teacher stated that mathematics instruction should encourage students to actively participate in class discussions. However, the researcher described the participant teacher's classroom practices as being dominated by lectures and routinized seatwork classroom activities. Similarly, Cooney (1985) interviewed a teacher who expressed that problem solving should be the essence of mathematics learning. However, he found the participant teacher's classroom practice to be extremely controlling and not favourable to problem solving. Finally, in her qualitative research, Raymond (1997) also indicates that mathematics beliefs and practices of first year elementary teachers were not consistent.

The conceptual challenges facing beliefs research have created some methodological challenges. The problem of having a clear definition to the concept of beliefs creates a challenge of how to operationalize it (Skott, 2014a, 2014b). Therefore, researchers have questioned the methodologies use for belief attribution. Those researchers imply that researchers and teachers may have different perceptions and understandings of the concept of belief (Speer, 2005). This methodological problem is due to the inherent difficulty of describing teachers' beliefs, which leads to the need to employ multiple sources and use a mix of methodologies when conducting a research. Skott (2014a) also states, “The methods used in the field do not provide access to what people really believe or at least not to beliefs that matter for the situation at hand” (p. 6). Lester (2002) also emphasizes the methodological challenges of studying teachers’ beliefs. He indicates that researchers may be involved in a circular argument of inferring beliefs from the nature of mathematical activity while trying to explain the same activity with regard to a principle construct of beliefs.

With more than two decades of belief research, the field has not yet reached the expectations of researchers (Skott, 2009, 2013, 2014a). Despite the rich research on belief, we have not yet found answers to some basic questions (Gates, 2006). Although some researchers, such as McLeod and McLeod (2002), note there has been significant advancement in the study of beliefs and affect in mathematics learning, the progress is more noticeable in relation to theoretical aspects. Researchers continue to call for extensive studies that ensure quality instruction progress exists. However, Skott (2009, 2013) views the call for further study a negative sign. “To a large extent, then, belief research is still conceived of as a promising field of study. Phrased negatively, however, its still-promising character suggests that after 20 years of persistent effort, the field has still not lived up to the expectations of its founders” (Skott, 2009, p. 28).

The challenges and complexity associated with belief research has led some researchers, such as Skott (2009, 2010, 2011, 2013, 2014a, 2014b) and Gates (2006), to call for more social approaches to belief research. Gates (2006) indicates that there is a need to take a social approach when studying teacher belief systems because it will shift focus from cognitive constructs. A change towards sociological constructs will balance existing views about the nature and genesis of beliefs. Skott (2010) also supports this view; he takes a context-practice approach by adopting of Patterns of Participation framework provides, which he sees as providing a more coherent and

dynamic understandings of teaching practices. Furthermore, this approach may help to resolve some of the conceptual and methodological problems of a belief–practice approach while maintaining an interest in the meta-issues that constitute the field of beliefs. The PoP framework challenges dominant traditional belief research by questioning the very notion of beliefs and its acquisitionist theoretical foundation (Skott, 2010).

3.2. Patterns of Participation: towards a more social approach for understanding teachers' practices

Traditionally, the disciplines of psychology and mathematics have dominated research on mathematics education (Kilpatrick, 1992). Towards the end of the 1980s, theoretical frameworks in mathematics education research started to take a more social approach to interpret mathematics teaching and learning (Lerman, 2000). The shift focuses on studying how the social and cultural practices of education shape individual learners, and conversely, how learners influence the shaping of these social and cultural practices. Inspired by Vygotsky's work, mathematics education research started adopting sociocultural theory that highlighted the essentially social character of the higher mental functions (Lerman, 2001).

The rising influence of Vygotsky's work has attracted attention to the social context of learning. It has introduced the field of mathematics education to the importance of anthropology, sociology and cultural psychology (Lerman, 2000). The turn towards social aspects in research acknowledges that meaning, thinking, and reasoning are products of social activity and that mathematics teaching and learning is best understood in relation to sociocultural contexts in which it is learned. Lerman (2000) explained that recent research calls for a sociocultural, discursive psychology in order to allow for the connection between the actions of individuals and groups in the classroom with history and culture. By acknowledging this connection, researchers can understand mathematics teaching and learning at a particular moment through the zoom of a lens (Lerman, 2001).

With the advance of cognitive psychology, the move in mathematics education research towards a unified theory of learning permitted a shift from stimulus response models, in which learners see learning as automatic reactions to stimuli, towards

meaning-based models such as constructivism, which endorses that learners produce knowledge and build meaning based upon their experiences (Von Glasersfeld, 1994).

As disciplines, such as anthropology and sociology, joined the search for a comprehensive theory of learning, highlighting the more widespread tradition of individual knowledge construction, the theories have expanded to incorporate the role of culture and context in this process. The result is the emergence of social constructivism theory. Social constructivism emphasizes that experience is the base for reconstructing knowledge within an interactive environment that supports development in both the individual and the social group (Ernest, 1994; Simon, 1995).

In the last two decades, researchers working with the theory of situated learning have offered a new perspective on the nature of knowledge and how people acquire it. These researchers (such as Greeno, Collins, & Resnick, 1996; Lave, 1996; Lave & Wenger, 1991) have clearly rejected the idea behind many educational perspectives, that knowledge can be separated from the social situations in which it is developed and obtained (Hughes & Greenhough, 1998). Lave (1996), for example, views learning as "an aspect of participation in socially situated practices" (p. 150). This approach presents learning as something that is essentially situated in the everyday social practices that people engage in; that knowing and learning are constructed through participation in a social discourse and practices (Greeno, Collins, & Resnick, 1996).

Boaler (1999) argues that situated learning theory presents learning in a different perspective than other learning theories because it focuses mainly on the social and cultural activity that is taking place within the community. She explains that in mathematics education, some theories of learning that have been widely applied, such as behaviourism and constructivism, focus on individual learners and on mathematics as a subject. While behaviourism focuses mainly on "the repetition of appropriate mathematical behaviours" (p. 260), constructivism understands mathematical learning as the process of how the individual construct mathematical knowledge (Simon, 1995). On the other hand, situated learning theory suggests, "the behaviours and practices of students in mathematical situations are not solely mathematical, nor individual, but are emergent as part of the relationships formed between learners and the people and systems of their environments" (Boaler, 1999, p. 260). Boaler (2000) adds, "in theoretical terms, constructivism posits a view of learning as the individual mind being influenced by

the social world, whereas situated theories propose that learning is a social phenomenon constituted in the world" (p. 5).

Packer and Goicoechea (2000) argue that sociocultural and constructivist perspectives presume different, and incommensurate, ontological assumptions; therefore, they offer different views of a single phenomenon. They claim that learning from the sociocultural perspective is the process of human change and transformation, whereas from the constructivist perspective, learning is only part of that larger process. Situated learning perspectives draw more attention to the identity formation of learners. Moreover, it emphasizes how learning is not only an epistemological, but also an ontological practice. In other words, the sociocultural perspectives describe learning as more than just changes in knowledge; it entails broader changes in being. "Sociocultural conception of identity addresses the fluid character of human being and the ways in which identity is closely linked to participation and learning in a community" (Packer & Goicoechea, 2000, p. 229).

Understanding the relationship between individual and social features of human learning has generated an ongoing argument in research in mathematics education (Skott, 2013). Some researchers support the necessity of consistently using one family of theoretical frameworks. Researchers who advocate this approach, such as Sfard (2008), highlight the importance of the social character of individuals' meaning-making and emphasize the importance of participation in learning. On the other hand, researchers such as Cobb (1994), Cobb and Yackel (1996), and Lester (2005) suggest that embracing ideas from a variety of theoretical sources is useful to provide a better understanding and offer richer explanations. By following this approach, a coordination of cognitive and social perspectives on knowing and coming to know could be obtained (Cobb & Yackel, 1996).

The social approach of research in mathematics education has progressively promoted the notion that practice is not only a personal individual matter; it is within a sociocultural context. Researchers must interpret practice relatively, between individuals and social settings. Although the relationships between individual and social factors of human functioning have generated much debate in mathematics education, it is mainly in relation to student learning (Skott, 2013). Therefore, PoP is a theoretical framework that aims to understand the relationships between teachers' practices and social factors.

The PoP framework elaborates on the view that teachers' practices in classrooms are not simple expressions of their desire and personal resources; it also views their practices as adaptations to social conditions in which they work. As noted by Skott (2013), the "teacher contributes to classroom interaction by re-engaging in other past and present practices, possibly reinterpreting and transforming them in the process" (p. 548). The PoP framework presents a useful tool to understand the teachers' position for emerging classroom practices that takes into account the multiple perspectives of student learning in educational research.

3.3. Patterns of Participation: adopting participationism as a metaphor for human functioning

In mathematics education, researchers have discussed extensively the acquisition metaphor and participation metaphor as two basic metaphors underlying theories of learning. Sfard (2008) compares the nature of mathematics learning using two different theories: the acquisitionists and the participationists. She explains that while acquisitionists view learning as the result of the learners' individual endeavour to arrive at a coherent understanding of the world, participationists' vision recognizes that learning arises essentially from one's attempt to make sense of other peoples' view of this world. Sfard (2008) points out that according to acquisitionist theory, learning mathematics can occur without the participation of others, but participationists view mathematics as a form of discourse (any form of communication) that requires learning to originate through communication with others and adjusting one's discursive ways to those of other people. In other words, participationists' theory indicates that mathematics learning starts through participation in collective mathematical discourses, exists in various learning environments (home, community, or school) and gradually increases knowledge of how to individualize the discourse as individuals communicate mathematically with themselves and others.

Skott (2010) claims that connections exist between the focus of belief research about individuals and the acquisitionist metaphor for learning in constructivism. While belief research contradicts the restricted view of understanding teaching and learning concerning cognitive aspects, the basic definitions and characteristics of the concept of beliefs indicate that it agrees with the constructivist emphasis on the individual. Although researchers mainly consider beliefs the result of experiences that consist of long-lasting

engagement in social interaction, when beliefs are constructed, researchers usually consider them as completely individual constructs. Some participatory theoretical views challenge constructivism acquisitionist because there are a few exceptions to the principle that beliefs are entirely the property of the individual.

PoP is a theoretical framework developed in line with several other social approaches to research in mathematics education. It aims to develop a more coherent understanding of the teacher's role for learning and life in mathematics classrooms. This alternative framework emphasizes the emergent nature of classroom practices. To a considerable degree, PoP adopts participationism as a metaphor for human functioning more than mainstream belief research. Therefore, PoP draws on the work of participationism researchers, specifically Vygotsky, Lave and Wenger, and Sfard.

The participationist approach of learning has grown from the sociocultural tradition, which is mainly based on Vygotsky's theory of human learning (Sfard, 2001, 2008). According to Sfard (1998), "'participation' is almost synonymous with 'taking part' and 'being a part,'" and it views learning as a process of becoming a part of a greater whole, part of a certain community" (Sfard, 1998, p. 6). This theory rejects the acquisitionists' view of individuals acquiring knowledge. Rather, participationist researchers view the learner as an emerging practitioner trying to get access to "a well-defined, historically established form of human doing" (Sfard, 2008, p. 78). In other words, learning is a gradual transformation of the individual from participation in a collectively applied activity to a similar form of doing, but in which s/he is able to perform on ones' own accord (Sfard, 1998).

Participationists' view of learning is parallel with Vygotsky's sociocultural theory. The main argument in Vygotsky's sociocultural theory is social interaction plays a fundamental role in the development of cognition. Vygotsky's theory describes learning as a social process that has historical and cultural relativity. It stresses the role of social interaction as being crucial to the individual's cognitive, social, and cultural development. Sociocultural theory rests on the idea that people do not exist in isolation; their existence is the result of their constant interaction with others and with their environment to develop higher orders of thinking and being. According to sociocultural theory, in a social context, the individual and their social world construct knowledge collaboratively. Both the individual and the social world have equally interrelated roles in learning

development. Therefore, the sociocultural perspective emphasizes the interdependence of individual and context instead of viewing one as an influence on the other. Sociocultural theory is not a theory of the social or cultural aspects of human life, but rather, a theory of human cognitive development (Confrey, 1995; Wertschdel Rio & Alvarez, 1995).

Although the connection between the PoP framework and Vygotsky's sociocultural theory is not stated explicitly in Skott's work, we can see the influence of Vygotsky's theory on the framework of PoP. PoP adopts Vygotsky's theoretical view stresses the fundamental role of social interaction in the development of function. PoP follows a participationist approach of learning, which grows from the sociocultural tradition. However, PoP does not draw on every aspect of Vygotsky's sociocultural theory. One main aspect of Vygotsky's sociocultural theory is the Vygotskian idea social tools mediate learning. However, PoP theory does not apply this concept clearly.

Social practice theory, in the work of Lave and Wenger, is one of the current sociocultural approaches, which have a more direct influence on the origins of the PoP approach. Social practice theory emphasizes the role of social practice in human learning. One major aspect of social practice theory is that individuals achieve learning through lived experience in the world through communities of practice (Lave & Wenger, 1991). According to the theory of communities of practice, learning is a result of the learner's participation in social practice and continual adjustment to the unfolding circumstances and activities that take place in a particular context; learning occurs within a specific social and physical environment. According to Lave and Wenger (1991), communities of practice exist everywhere from educational institutions, work organizations, and people's homes. From their involvement in communities, people accumulate different practices due to combined learning, which leads to an end-result of a specific practice. These practices, which develop over time, define the kind of community and are therefore communities of practice. Learning does not just involve the learners obtaining the offered knowledge. It actually involves all participants in an ongoing practice of apprenticeship. The apprentices and their masters interact in the learning environment as co-learners. Lave and Wenger (1991) note that most learning happens in connection to collaborating apprentices.

A main aspect of Lave and Wenger's communities of practice theory is their identification of what communities they define and identify as a community of practice. Wenger, McDermott, and Snyder (2002) define communities of practice as "groups of people who share a concern, a set of problems, or a passion about a topic, and who deepen their knowledge and expertise in this area by interacting on an ongoing basis" (p. 4). Members of a certain community of practice will have a common understanding of their actions and what they represent in their personal and community lives.

Based on this definition, a community of practice is not merely a work group or task force. It is not a gathering of people assigned to a particular group. Members of communities of practice usually become members based on their interest in the domain and their willingness to contribute to the practice (Bozarth, 2008). Therefore, a typical classroom is not necessarily a community of practice. However, Skott's framework of PoP defines the classroom as a community of practice. In his analysis of teachers' practices in the classroom, Skott (2011, 2013) argues that teachers and students are members of one community. Skott also tries to understand teachers' pattern of participations through "multiple, simultaneous actual and virtual communities of practice" (Skott, Moeskær Larsen, & Hellsten Østergaard, 2011, p. 33) by identifying the classroom as one of these communities. Therefore, it seems that Skott's perception of the concept of community of practices is not exactly in line with that of Lave and Wenger.

Wenger's (1998) view of the notions practice and participation has influenced Skott's ideas of PoP. Wenger perceives practice as embedded in a community, which embraces a shared goal, common engagement, and a united repertoire. Practice "is a set of frameworks, ideas, tools, information, styles, language, stories, and documents that community members share" (Wenger et al., 2002, p. 29). Practice is a way of talking about mutual, historical and social resources, frameworks and views that can maintain mutual engagement in action. Learners in a community share a basic body of knowledge that generates a shared foundation, which allows members to work together productively (Wenger 1998).

The notions of practice and participation are basic constructs in the origins of PoP. The term practice in PoP holds the same meaning as when researchers use it in social practice theory, as a social phenomenon. Skott et al. (2011) consider practice as

an outcome of individual and communal meaning-making and agency that emerges in the local social environment. “It is embedded in broader social situations, but the emphasis on emergence means that we regard it as an empirical question how and to what extent for instance a school culture, the students’ family backgrounds, national or local educational regulations, or recommendations for reform play a role for the practices that evolve” (p. 32). This understanding of the concept of practice leads to the conclusion that teachers’ practices are not directly linked to any individual in the school or classroom community. Teachers’ practices in classrooms are results of their classroom interaction by re-engaging other past and present experiences, and reinterpreting and transforming these experiences in the process. A teacher’s practice is influenced and bound together through re-engagement in other essential discourses and practices through the meaning they place on the social interaction itself.

Teachers are constantly interpreting their students’ individual and collective actions, drawing on a variety of other social practices to do so. Some of the teachers’ practices in the classroom are discursive in a direct verbal way, while others are not. In addition, some of these practices are virtually connected to the classroom, which means they do not physically exist in the classroom or at the school (Skott, 2009). PoP views teachers’ social interaction in a certain community as one piece influencing other pieces of social interactions. In every interaction, the ‘pieces’ shape a ‘fluctuating pattern' that shows the shifting impact of different, previous practices and the dynamic relations between them.

In order to understand teachers’ practices in the classroom, PoP works at two levels of analysis. At one level, it pieces “together the pattern in the teacher’s contribution to individual classroom episodes,” while at different level “it looks across individual episodes and builds on longitudinal studies to discern patterns of patterns, i.e. to point to trends and developments in the recurrent and possibly routinized ways in which the teacher engages with the students and the contents” (Skott, 2013, p. 548).

Skott has built the theory of PoP using some aspects of Sfard’s (2008) theory of commognition. Sfard (2008) developed a participationist theory that emphasizes that cognition and communication are different manifestations of what is essentially a same social phenomenon. She defines thinking as “an individualized version of (interpersonal) communicating” (p. 81). The term commognition encapsulates both inter- and intra-

personal communication, which generate human thinking. Different types of communication are discourses, and these discourses are constantly developing and increasing in complexity. According to the commognitive framework, learning is an individualizing discourse, where one becomes more able to communicate within the discourse, with others as well as with oneself (Sfard, 2008).

Inspired by Sfard's ideas, Skott et al. (2011) point out that in research we need to sustain the processual emphasis on what is commonly referred to as beliefs. However, rather than viewing the teacher's meaning-making and input to the engagement of the mathematics classroom as the consequence of an enactment of objectified beliefs, Skott (2011) notes that researchers should analyze them as the concurrent interaction in a variety of mathematical, meta-mathematical, and other wider social practices.

Skott also applies Sfard's critique of acquisitionist on the reliance of objectification. Objectification is the process that converts human engagement in social discursive processes into separated entities in their own right. Sfard (2008) explains the two stages of objectification: reification and alienation. Reification refers to replacement of talk about processes and actions into talk about objects. It observes actions, but talks about them as objects. Alienation is presenting the objects in an impersonal way independently from the combination of concepts that give rise to them.

Skott (2010) notes that, in educational research, researchers view beliefs as being built on objectification. Researchers mostly interpret teachers' beliefs as a product of the reification process. Researchers usually solidify teachers' experiences into object-like entities, treating them as properties that will take on a life of their own and have major influence on practice. Belief research also focuses on the alienation of the reified objects. While researchers view beliefs as an embedded personal construct, they also regard beliefs as relatively depersonalized entities and are supposed to be apparent in teachers' practices without taking into account the teachers' experience of the present situation. Skott adopted Sfard's view that "objectifies the process of objectification itself, and uses the term objectifications to point to an independent entity as well as the process" (Skott, 2010, p. 195).

PoP also gets inspiration from Blumer's (1969) theory of symbolic interactionism in order to understand how a teacher's immediate social interaction connects to his/her

engagement in past and present practices (Skott, 2013). Mead and Blumer (1969) presented the symbolic interactionism theory as an alternative to the uncritical behaviourism approach that was common in sociology during that era. The basic theory of symbolic interactionism has become commonly known as the Chicago School of Interactionism (Dingwall, 2001). Blumer (1969) highlighted the interpretive process in the construction of meaning of the depth and diversity of social experience as it was lived. He attempted to understand the participant's world and elaborated on Mead's explanation of the `I' and `me' to understand the dynamic and processual nature of human behaviour. The basic philosophy of symbolic interactionism is that humans should be regarded in the context of their environment and that individuals and the context in which they exist in are inseparable (Blumer, 1969).

Blumer (1969) identified three basic assumptions underlying symbolic interactionism. First, humans, individually and collectively, act on the origin of the meanings that things have. Individuals do not react directly to things; however, they connect meaning to things and act based on meaning. Second, meanings emerge in the process of social interaction among individuals. Meaning for a person comes out of the ways in which other individuals take action to define them. Third, meanings are modified through an interpretive process that is constantly changing, and is vulnerable to redefinition, removal and alteration. According to symbolic interactionism, meanings changes according to the context for the individual. The nature of our actions is best understood through individual interpretation of reality in every social context. Therefore, understanding human participation requires looking for a means to understand the meaning of a situation from the perception of the individual and societal groups.

From this perspective, PoP takes into account that a “teacher negotiates classroom practices by interpreting the students’ and her own possible contributions to the interactions symbolically” (Skott, 2013, p. 550). PoP analyzes teachers’ specific practices in relation to other classroom practices that teachers engage in simultaneously. The teacher also takes the attitude of individual and generalized others and relates it to practices generated from other social interactions such as interactions with other teachers, and from meetings with the parents or the school management.

3.4. The extended PoP framework: including knowledge and identity

Skott initially developed the PoP framework in relation to teachers' beliefs. However, in order to develop a more coherent approach to understand teachers' practices, Skott (2013) extended the framework to include knowledge and identity. Skott (2013) notes that research on teachers has mainly focused on studying three relatively distinct domains: teachers' knowledge, beliefs, and identity. This leads to some incoherence that negatively influences the understanding of the teachers' role in classrooms. Skott presents PoP as a coherent, participatory framework that is capable of dealing with matters usually faced in the distinct fields of teachers' knowledge, beliefs, and identity.

3.4.1. PoP's understanding of teachers' knowledge and practice

While research on teachers' mathematical knowledge is mainly concerned with the specifics of teachers' content preparation, since the 1980s, it has also investigated some meta-issues such as teachers' conceptions of mathematics and its teaching and learning in the classroom. The teacher-knowledge perspective has become one of the main approaches to thinking about teachers and their practices. In mathematics education, research on teacher knowledge has been popularized since Shulman's hypothesized model in the 1980's of teacher professional knowledge. Although Shulman proposed seven different categories of teacher knowledge, two of these categories of knowledge have influenced the direction of research about mathematics teachers: content knowledge and pedagogical content knowledge (Ball et al., 2008; Petrou & Goulding, 2011).

Shulman indicates that in order to provide a comprehensive understanding of mathematics teachers' knowledge; researchers should examine all the different categories of teacher knowledge, not only separately but also in terms of their relationship in supporting teacher learning and practice (Petrou & Goulding, 2011). Ball is one of the researchers who worked on developing Shulman's model. However, Ball et al. (2008) point out that there are possible limitations for research when applying the model to study teachers' knowledge of practice. The diversity in curricula and associated classroom implementations leads to difficulties in defining different concepts

used in the model. Furthermore, the model does not take into account the role of cultural variability across and among teachers and students.

While teachers' pedagogical knowledge and mathematical content knowledge are important, teachers' mathematics knowledge for teaching is also very important. Hill, Rowan and Ball (2005) defined mathematical knowledge for teaching as the "mathematical knowledge used to carry out the work of teaching mathematics" (p. 374). Similarly, teachers should have the knowledge required to teach mathematics effectively. Mathematics teachers should have the knowledge necessary to do things such as explain mathematical concepts, interpret and understand students' work, and use the textbook effectively.

The National Council for Teachers of Mathematics (NCTM) emphasizes the importance of such knowledge for teachers. The NCTM (2000) states, "teachers must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks" (p. 17). Mathematics teachers should have the appropriate knowledge to be able to become successful facilitators of students' learning. In addition, teachers should have knowledge of what sorts of instructional opportunities allow students to learn mathematics effectively and able to decide suitable tasks that would challenge all students, regardless of their mathematical background, in the learning of mathematics (NCTM, 2000).

Skott (2013) notes that focusing solely on constructs of different types of knowledge when examining teachers' knowledge for teaching mathematics provides a limited view of what really happens in mathematics classrooms. Therefore, the teacher-personal perspective, which includes other aspects such as beliefs and identity, should be joined to the teacher-knowledge perspective. Furthermore, with the limitations related to research on mathematics teachers' knowledge, Skott (2013) indicates that these studies suggest that there are types of mathematical knowledge and ways of knowing that are essential to teachers' practices. Unfortunately, teachers cannot acquire these by simply taking a standard university course. Therefore, "a more processual and participatory understanding is needed of what it means to know" (Skott, 2013, p. 551).

3.4.2. PoP's understanding of teachers' identity and practice

The interest of research in teachers' identity is more recent compared to research on teachers' beliefs and knowledge. Research of teacher identity has generally adopted a social participationist approach. It highlights the processual. Researchers generally view identity "as fluid and always in the making, as tales of being and becoming as they relate to simultaneous engagement in multiple, social practices" (Skott, 2011, p. 212). In this regard, researchers consider identity as a more dynamic construct than knowledge and beliefs (Sfard & Prusak, 2005; Skott, 2011; Wagner & Herbel-Eisenmann, 2009). Skott's view of identity is in line with the work of Holland et al. (1998) and Wenger (1998).

PoP adaptation of Holland et al.'s notion of figured worlds

Holland et al.'s (1998) theory of identity draws from different schools of thought, mainly from the work of Vygotsky and Bakhtin. Holland et al.'s (1998) theory is a sociocultural practice theory of identity and self. It sheds the light on identity forming in process or activity. Holland et al. view identity as a dynamic co-constructed cultural phenomenon; identity forms in response to specific contexts and through time. Identity is self-understandings of who we think we are. According to Holland et al. (1998), a person's sense of self mediates people's behaviour. "People tell others who they are, but even more important, they tell themselves and then try to act as though they are who they say they are" (p. 3). Therefore, individuals construct identities through participation in cultural activities, which permit them to engage in identity construction. Identities develop as a result of conceptual and practical performances of the self.

The aim of Holland et al.'s theory is to establish an understanding of a person's developing identity and its link with activity, or as they name it "identity in practice". Holland et al. (1998) explain the constructs of "practiced identities" in relation to the notion of figured worlds. Figured worlds are imagined communities that function dialectically and dialogically as if in worlds. They constitute sites of possibility that offer individuals the tools to impact their own behaviour within these worlds. Holland et al. (1998) define figured worlds as "socially and culturally constructed realm[s] of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others" (p. 52). On the other hand, figured worlds are socially organized and reproduced phenomena that

provide "a context of meaning and action in which social positions and social relationships are named and conducted" (Holland et al., 1998, p. 60). Figured worlds are formed upon the individual's co-constructed understanding through interaction with others rather than on innate, individual nature. Through figured worlds, individuals incorporate a system of interpretations drawn from their social interactions. Therefore, figured worlds are different from beliefs system, which are constructed mainly on an individual's feeling.

Holland et al. (1998) describe figured worlds by four fundamental features. First, figured worlds are "historical phenomena" that are constructed by participants and at the same time forming participants. Second, figured worlds are "social encounters in which people's positions matter" and they are "located in times and places" (p. 41). Third, figured worlds are reproduced via the individuals' reconceptualization of their roles and who they recognize themselves to be, as individuals or participants of a social group. Figured worlds imply a range of rules, norms, expectations, and notions that restrict and allow particular types of participation. Fourth, figured worlds are encountered in everyday social realities and lived through cultures practices and activities. Through these four features of figured worlds individuals are offered "a context of meaning and action in which social positions and social relationships are named and conducted" (Holland et al., 1998, p. 60). Therefore, figured worlds "gather us up and give us form as our lives intersect them" (Holland et al., 1998, p. 41).

Figured worlds is one of the four contexts that Holland et al. (1998) suggest as sites where "practiced identities" are produced. The second context is positionality, which refers to the positions provided to an individual in different figured worlds. It is an analytically distinct counterpart to figuration. When an individual is positioned, s/he is not really engaged in self-making, but restricted to varying levels of accepting, declining, or negotiating the identity being offered to them. The third context of identity, "space of authoring", is Bakhtin's interpretation of the regular world of an individual or group. It is people's ability to make choices and respond to how they are being socially identified by other people. The fourth context of identity, "making worlds", is inspired by Vygotsky's construct of play in children's development. Holland et al. (1998) consider social play as an essential element for people to develop various social competencies from which new imagined figured worlds can emerge.

Holland et al.'s notion of figured worlds serves as an appropriate lens for the PoP framework in order to understand mathematics teachers' practices. The PoP framework focus on exploring and identifying significant practices and figured worlds in which mathematics teachers participate and upon which they draw on. The PoP framework "suggests attempting to understand how teachers draw on and renegotiate their participation in a range of their past and present practices and figured world as they engage in classroom interaction" (Skott, 2014a, p. 21).

The PoP framework views teachers' identity as being how teachers narrate and position themselves in relation to multiple, and sometimes conflicting, figured worlds. Teachers do not position themselves merely by the contents of their verbal actions, "but by how [they move] in the classroom, how [they]—possibly unreflectively—react to disruptive behaviour, and the assertion with which [they] address their colleagues in staff meetings" (Skott, 2013, p. 551). However, this view of teachers' identity leads to empirical issue of understanding "how the significance, meanings, and mutual relationships of these narratives relate to her contributions to the practices that emerge in the classroom" (p. 551). To address this issue, Skott (2013) adopts a situated perspective on identity, which perceives identity as not equal to the self, but as something that focuses on the changing versions of the me that occur through contact with others.

PoP inspiration by Wenger's (1998) conception of identity

Skott's view of identity resonates with Wenger's (1998) conception of identity, which is to some extent similar to Holland et al.'s (1998) view. Wenger (1998) extended his practice-based theory of learning by emphasizing the concept of identity. He conceptualizes learning as an aspect of identity and the development of identity as the result of learning. In addition, he emphasizes the role of social practices in the developing of identity. "Learning transforms our identities. It transforms our ability to participate in the world by changing all at once who we are, our practices, and our communities" (p. 226). "Identity is a becoming", therefore identity is lived, negotiated and constructed through a process of social interactions in our communities. According to Wenger (1998), the process of creating identity is inescapable and continuous. How we construct knowledge about our identity, and how we interpret our position, are negotiated in the course of our social interaction with others.

Wenger's (1998) work about identity inspired the theory of PoP with respect to the role of teachers' participation in a variety of social settings in and out of the classroom in forming teachers' identity. Similarly, Wenger's (1998) framework proposes that learning and identity are identical processes and that to learn is the process of becoming, and that is identity. This view supports the PoP framework in developing a coherent approach to understanding teachers' practices.

3.5. Patterns of Participation: teachers' practices and educational reform

The National Council of Teachers of Mathematics considers learning as the intellectual work of a community of learners, and provides a very different view of the role of the teacher in the mathematics classroom. The documents call for decreasing the traditional teaching activities of telling and showing mathematics that students require and encouraging a teaching style that enhances learning experiences by focusing on activities which include problem solving, reasoning, communicating, and building meaningful links among mathematical ideas (Ball, 1994; Simon, 1995; Simon, 1994; Tzur, Simon, Heinz, & Kinzel, 2001). The Professional Standards for Teaching Mathematics (1990) promotes changes in mathematics teaching practices and calls for a move that would transfer classrooms into mathematical communities instead of viewing classrooms as a group of individuals.

Simon (1994) calls for a strong research foundation on teacher development that supports reform efforts in students' mathematical development. In Simon's (1994) view, understanding the process of learning mathematics is important; however, understanding the process of teaching mathematics in reform-oriented approaches is equally important. The Professional Standards for Teaching Mathematics (1990) suggests that research in mathematics teaching education should focus on providing a set of standards that

> promotes a vision of mathematics teaching, evaluating mathematics teaching, the professional development of mathematics teachers, and responsibilities for professional development and support, all of which would contribute to the improvement of mathematics education as envisioned in the Curriculum and Evaluation Standards (p. vii).

The 2000 NCTM Standards document, Principles and Standards for School Mathematics, suggests that effective mathematics teaching requires teachers to have a solid knowledge of mathematics; an understanding of what students know and how students learn; and a belief in reform-oriented mathematics teaching and learning in an attempt to form rich learning environments. Therefore, understanding how students learn cannot be achieved by only understanding how they experience mathematics. It is very important to also understand how the other main players in the students' educational world, especially teachers, define students' learning experience. It is about how the social and cultural practices of education create learners, and how individuals are agents in the shaping of these practices.

Given the wide acceptance of these recommendations, there is consensus in the field that teachers should change their traditional way of teaching mathematics. To do this, there is a need to develop a theory that allows us to understand the teacher's role in the classroom. Skott (2013, 2014a, 2014b) presents PoP as a potential framework to understand the practices of teachers as influenced by current reform efforts in mathematics education. PoP takes into account the role of reform as a social aspect for the teachers' practices that evolve (Skott, 2013).

3.6. Skott's empirical use of Patterns of Participation

Skott used PoP as the framework to analyze three teacher cases. For all three cases, Skott used a combination of research methods including questionnaire, semi-structured interviews, stimulated recall interviews, the observations from teachers' meetings and video-recorded mathematics lessons.

Skott (2009, 2010) presents the case of Larry, a novice teacher who, despite his enthusiasm about current reform efforts in mathematics education, starts his teaching career by working at a conservative private school. The researcher interviewed Larry for the first time during the first week of his teaching career. During the interview, Larry talked about his educational background and his opinion about his teaching career. Larry also commented on three sets of written materials including a sample of a grade 5 student's work; the other two written samples were comments from an experienced teacher regarding mathematics instruction. Six months after the first interview, the researcher visited Larry again for two and a half weeks. During that time, the researcher

conducted a number of interviews with Larry. Larry was asked to comment on selected video clips from his classrooms.

According to Skott (2009, 2010), Larry's struggle to adopt his new career as a teacher working in a conservative school puts him in a position of professional isolation in his school. Larry's main struggle is due to his effort to create balance between his beliefs about student involvement in what he considers genuine mathematical activities and his colleagues' emphasis on students' command of skills and their performance on standardized tests. Skott (2009, 2010) claims that there is a conflict in Larry's participation in the actual community of practices in his school and his participation in a virtual community of teaching practices, which is based on Larry's pre-service learning experience. As a result of his engagement in the virtual community of teaching, Larry views his role as teacher as being the facilitator of mathematical learning. Larry's engagement in the actual community of practices in his school challenges his view about the role of teacher as facilitator. The actual community of teachers' practices in his school does not fit with facilitating students' mathematical learning, but with students' test performance. According to Skott (2009, 2010), this conflict has a significant impact on Larry's classroom practices.

Skott (2009) explains that the mainstream belief research may interpret how the conflict manifests itself in the classroom by describing Larry as being inconsistent or unable to enact his beliefs. However, Skott (2010) interprets the conflict that clearly appears in Larry's classroom beliefs and practices relationship as a manner of shifts in his participation in a range of social practices. Skott (2009) suggests that belief research should focus on examining the actual and virtual communities of practice and on teachers' contexts as they come out from participations in those communities. This approach, according to Skott (2009), provides researchers with different perspectives to understand the relationship between the teacher's educational and mathematical preferences and his/her evolving classroom practices. This approach will "contextualise the act of teaching in intersubjectively established and continually re-generated settings and suggests that we acknowledge the simultaneous existence of multiple, possibly conflicting, actual and virtual communities of a teacher's practice" (Skott, 2009, p. 44).

Another study done by Skott, Moeskær Larsen, and Hellsten Østergaard (2011) presents the case of Susanne. Susanne graduated from university with a Bachelor's

degree in sport. Later, she started teaching mathematics, science and English at primary and lower secondary school without having a degree in education. Since she found teaching very interesting, she decided to enroll in a two years' college program for second-career prospective teachers. At college, Susanne specialized in mathematics; the program she was taking at the college focuses on studying the subject itself alongside completing courses about educational issues. While she was in college, Susanne continued to teach at the primary level at her school. However, after her graduation from college, she started teaching mathematics to grades 5 and 6 students.

The data collected about Susanne spanned over two years. The researchers conducted the first semi-structured interview with Susanne during her final term of her teacher education program. They also video-recorded six mathematics lessons from Susanne's grade 3 classroom. Researchers also visited Susanne again 18 months later, which was around a year after her graduation. They video-recorded six more lessons and conducted two more semi-structured interviews, one before the lesson observations and one after having observed four lessons.

Skott et al. (2011) find that Susanne placed herself in relation to three disconnected sets of social practices by using her educational and professional experiences. The first set of practice consists of a public description of school mathematics that is characterized by central accounts of the subject in official curricular documents, which is usually inspired by reform ideas about quality teaching in mathematics. The second set of practices is related to Susanne's main experiences as student when she was in secondary school and during her pre-service teacher education with mathematics teaching and learning. The last set of practices is about Susanne's adjustment to her position at her school.

Skott et al. (2011) explain that according to mainstream belief research, Susanne's practices in her classroom could be the result of Susanne having a traditional view of mathematics teaching and learning and that her classroom practices are consonant with her beliefs. However, moving away from the mainstream belief approach, Skott et al. (2011) propose interpreting Susanne's teacher contributions to classroom practices from the viewpoint that they consist of her shifting participations in a variety of simultaneous practices. Some of these practices depend on instant physical interaction, whereas others are mainly theoretical. "Some of them are currently active,

while others consist of the re-engagement in prior practices; and some are positively laden while others are used primarily negatively to make other ones stand out in relief. They mutually structure each other as resources for the practices that emerge in the mathematics classroom" (p. 52). The most significant social practices for Susanne in her mathematics classroom are traditional and reform approaches to teaching mathematics and her school culture and its organizational structures.

In his 2013 paper, Skott presents PoP as a more developed framework. He demonstrates the empirical use of his more developed framework by presenting the case of Anna. Anna is a new mathematics teacher who teaches lower secondary level classes. Data collection include a questionnaire, three major interviews, one of them based on stimulated recall method, observation and video recordings of 30 lessons from three two week periods in three consecutive terms, some informal interviews after lessons observation, observations of two teacher meetings between Anna and her three closest colleagues, and samples of Anna's students' work. The first interview conducted with Anna was directly after her graduation from teacher education program. The remainder of the data was collected during and after the end of her second semester working as a full-time teacher.

Skott's (2013) analysis of Anna's case suggests that there are four major practices or figured worlds to Anna's classroom practice as a novice mathematics teacher: 'relationing', 'the reform', 'mathematics', and 'teaming'. Anna's conversation with the other teachers in her team is limited to discussion about students' social problems in school. Since she is confident professionally as well as mathematically, she does not discuss with her team issues related to mathematical teaching. Anna's professional isolation in relation to mathematics teaching is due to her perception that other teachers in her school do not generally share the practices she wants to endorse in her classroom. Anna is concerned about building a strong trusting relationship with her students. Anna usually makes use of reform ideas about mathematics learning in her classroom practice. She promotes mathematical communication by encouraging her students to explain their understandings and describe how the mathematical content relates to their everyday life.

Skott (2013) concluded that developing a dynamic and contextual understanding of Anna's teaching practices requires the use of multiple methods that provide easy

access to all the practices and figured worlds that are vital for Anna's classroom interaction. Skott (2013) explains that researchers may face difficulties in gaining easy access to some of these practices and figured worlds, especially those related to the teacher's learning experience in school or to the teacher's pre-service program. Skott (2013) also suggests that more investigation is needed in order to understand "how certain modes of [teachers] participation become dominant, transformed, or subsumed by others, and how their robustness or permeability and susceptibility to change influences the teacher's contribution to the practices that evolve at the instant" (p. 557).

3.7. Patterns of Participation: the promise and limitations

In classrooms, students and teachers interact in several simultaneous practices. Some of these practices are directly related to the teaching and learning of mathematics while others are not. Some of them are discourse related to an explicit verbal feature, while others are not. They relate to communities that are not actually present in the classroom or at the school. Understanding the teachers' role in the classroom entails understanding the complex relationship between these simultaneous practices (Skott, 2013). PoP is a promising framework, which aims to understand the complexity of teachers' practices in classroom.

The teacher contributes to the constant creation of classroom practices. S/he engages in varied actions such as "repeating procedural explanations, solving disciplinary problems, ensuring a student's position in the classroom community, and taking a child's problematic home situation into account" (Skott, 2011, p. 213). Within all of that, patterns from the teacher's previous experience in social engagements are enacted, integrated, merged, and sometimes changed beyond recognition as they meet, combine with, convert, and further develop those that are associated with the immediate social circumstances (Skott, 2013).

From this viewpoint, teaching is not acting according to pre-reified constructs of knowledge and beliefs. It is a meaning-making process in which the teacher constantly manoeuvres between diverse types of participation in different past and present practices. PoP takes on a dynamic perception on classroom practices as well as on the teacher's contributions to them. PoP intends to outline the nature of these practices and to identify the patterns of the teacher's participation in them. PoP is an attempt to answer

questions related to “if and how certain modes of participation become dominant, transformed, or subsumed by others, and how their robustness or permeability and susceptibility to change influences the teacher’s contribution to the practices that evolve at the instant” (Skott, 2013, p. 557).

In mathematics education research, there is, to some extent, an unexpected disengagement between research on teachers’ beliefs, knowledge, and identity. This disconnect in research hinders the development of coherent understandings of the teachers’ role for classroom practices and for student learning. Researchers could use PoP as a coherent, participatory framework that has the potential to address issues usually faced within in distinct fields of teachers’ knowledge, beliefs, and identity. However, PoP does not connect the analyses of teachers’ knowledge, beliefs, and identity by regulating the use of theoretical views across the acquisition–participation part. As an alternative, it employs a participatory approach and looks for patterns in individual teacher’s participation in different social practices. Therefore, PoP is a framework that could enrich research, especially for those who are interested in understanding and theorising about mathematics teaching.

It is important to note that PoP is not the only theory that aims to understand and theorize about mathematics classroom practices from the view that individual’s meaning-making and practices are dynamic and evolving. For example, Wagner and Herbel-Eisenmann’s (2009) research emphasizes the importance of social interaction in the development of mathematical understanding. However, their research uses interpersonal positioning theory in order to understand teacher-student interaction in the mathematics classroom. The method Wagner and Herbel-Eisenmann (2009) use focuses more attention on the narratives at play in mathematics classrooms than outside classrooms.

The PoP framework is also in line with Herbst and Chazan (2003, 2011) who aim to understand mathematics teachers’ actions in the classroom. These authors introduced the theory of practical rationality of mathematics teaching. According to their theory, “teacher’s actions in an instructional situation are modulated by a practical rationality, a feel for the game” (Herbst & Chazan, 2011, p. 218). The theory of practical rationality does not aim to characterize the individual practitioner, but rather understand a collective phenomenon, a characteristic of the practice of mathematics teaching

(Herbst & Chazan, 2011). While PoP theory shares the same purpose as practical rationality, the methodology they apply is different. The data collection technique used in Herbst and Chazan's study relies mainly on video representations of breached instances of instructional situations; they then study how teachers engage and react with the presented situations.

Mainly, PoP research applies methodical triangulation including instance interviews that contain stimulated recall, observations, and document analyses, which is similar to the methods used in belief research. However, although the methods used in the two fields (beliefs and PoP) are the same, the intentions behind the use of a combination of the methods is vastly different. "In patterns-of-participation research we do not assume that one might get better access to the true character of contextually and temporally stable constructs like beliefs. Different methods are used exactly because they may shed light on decidedly different forms of practice and decidedly different modes of participating in them" (Skott et al., 2011, p. 34).

As a framework, PoP directs the research questions, shapes the research design, and controls data gathering and analysis. The research questions for PoP include asking about the role of teacher's stories of themselves as professional in their classroom engagement, the impact of teachers' relation to educational discourses such as the reform adoption, and the connection between how teachers engage with mathematics in and outside classroom contexts. The questions are, however, dependent on the person-in-practice. In addition, the design of the research should, as much as possible, permit access to teachers' practices and figured worlds beyond the classroom. "The unit of analysis may then be described as the teacher-in-multiple-practices- and-figured worlds as they relate to classroom interaction" (Skott, 2013, p. 552).

PoP also has its limitations. The main limitation of the theory, in my opinion, is a methodological one. According to PoP, in order to understand teachers' immediate actions in the classroom, we should be able to have access to all the practices and figured worlds that are likely to be significant for immediate classroom interaction. This presents a major difficulty since the researcher cannot have access to all the past and present practices that are significant for the immediate classroom interaction.

Another limitation of this framework is the difficulty in identifying an individual's patterns-of-participation in settings over a short time. PoP adopts a participant approach to learning and knowing. PoP views the practices in the mathematics classroom as an ever-evolving result of individual and communal acts of meaning-making on the part of the teacher and the students. Therefore, according to this view, applying the PoP framework requires close observation for a long period in order to identify the PoP through the ever-evolving process of teaching practices.

Finally, this framework does not shed light on the influence of the PoP of students' mathematics learning experience. It seems that this framework aims to theorize about mathematics teaching without giving enough attention to the mathematics learning experience from the students' perspective. This theory does not explain explicitly the value of Patterns of Participation in relation to students' learning, which is the main purpose of teaching.

3.8. Summary

In this chapter, I presented the conceptual framework that guided my study. The chapter includes information about PoP as a theoretical framework and outlines its potential for explaining and understanding mathematics teachers' classroom practices. It also contained an explanation of the connection between PoP and other theories it draws from and described its usefulness and limitations as a framework to understand the role of the teacher for emerging classroom practices. In the next chapter, I present in detail the research methods and the methodology implemented for this study. I will introduce the main research questions and describe the practical steps I went through during the design of my research.

Chapter 4.
Research Methods

The purpose of this chapter is to introduce the main research questions and to explain in detail the research methods and the methodology implemented for this study. The aim of this study is to investigate high school mathematics teachers' practices during the current reform movement in Saudi Arabia. Recognizing that teaching is a multi-layered continuous learning practices, this study looks at many experiences and contexts to shed light on their impact on teachers' current teaching practices. The research design incorporates data sources that access participant teachers' current and past experiences to understand how these experiences influence teachers' immediate practices in the classroom.

Two factors influenced the selection of means and modes of data collection and analysis for this dissertation. The first is the framework for this study. Using PoP as the main framework informs the research design and influences the collection and analysis of the data. During the design phase, I tried as much as possible to collect rich data that allow me to get access to practices and figured worlds beyond the participant teachers' classroom actions. The second factor is the limited access to data sources. The fact that I was doing my study in Canada and conducted my research in Saudi Arabia provided me with limited access to engage with the participants. Due to this circumstance, I did not have a long period of time to collect the data. Therefore, during my visits with teachers in their schools, I tried to collect as much data as possible.

4.1. Research questions

Using the information presented in the previous chapters, the research questions for this study are:

1- What are the figured worlds, or significant practices, to the participant teachers' sense of their practice as mathematics teachers and how does each teacher engage with these figured worlds?

To answer this question, I present each individual's case separately to capture the uniqueness of their experiences. For every case, I identify the figured worlds that

contribute to the teacher's sense of her practice as a mathematics teacher and explain how the participant teacher engages with these figured worlds.

2- How do high school mathematics teachers in Saudi Arabia respond to the shared or common circumstances they are facing in the current reform movement?

In order to answer this question and to gain a better understanding of high school mathematics teachers' practices during the current reform movement, I conduct a cross-case analysis to connect the findings from each individual case. I identify common themes from the participants' cases. For each theme, I describe the similarities and differences between the practices of the four participating teachers

The following section includes a description of the study's practical design steps including recruiting participants, means and modes of data collection and analysis techniques. The choice for research methods for this study provided a way to create a thick description (Creswell, 2012) of the teachers' experiences to gain a better understanding of their teaching practices. This thick description includes information from transcripts of observations and interviews, field notes, coding of data, and memo writing.

4.2. Type of Qualitative Design: A collective case study

The focus of the study is to understand high school mathematics teachers' practices during the current reform movement in Saudi Arabia. A qualitative research methodology is the best and most appropriate approach for this study. According to Creswell (2013), qualitative research is described as "an inquiry process of understanding based on a distinct methodological approach to inquiry that explores a social or human problem. The researcher builds a complex, holistic picture, analyzes words, reports detailed views of participants, and conducts the study in a natural setting" (p. 300).

For this research study, I used a multiple case study methodology to present four cases. Each case represents the experience of a female mathematics high school teacher in Saudi Arabia to develop a deep understanding of their teaching practices in current girls' high school classroom.

Case study research is an examination and analysis of a single case or a group of collective cases intended to shed the light on the complexity of the phenomena of study (Stake, 1995). Creswell (2013) defines case study as a qualitative approach that "explores a real-life, contemporary bounded system (a case) or multiple bounded systems (cases) over time, through detailed, in-depth data collection involving multiple sources of information" (p. 97). Creswell (2013) explains a bounded system as the case is separated out for study in terms of some type of physical boundaries such as time and place. Similarly, Yin (2003) defines case study as "an empirical inquiry that investigate a contemporary phenomenon within its real life context, especially when the boundaries between a phenomenon and context are not clear …and the researcher has little control over the phenomenon and context" (p. 13).

My study is a "collective case study" (Stake, 1995) because I studied more than one teacher (case). The indications made from a multiple or collective case study are more robust and reliable compared to a single case study (Yin, 2003). Collective case studies provide an opportunity to examine the experience of high school mathematics teachers and explore the differences and similarities that occur among individuals and different contexts (Stake, 1995). A collective case study allows me to look beyond the individual case to the phenomenon, in this case mathematics teaching in high school during current reform movement. The collection of data from multiple sources offered a rich, thick description with several dimensions and realities offering deeper insight. The study included within-case analyses to provide an embedded exploration of each specific case and a cross-case analysis to create a deeper understanding.

Using case studies as a research method allows me to learn from the participants by describing and interpreting the meaning of their teaching experiences. Stake (1995) explains the main motivation and aim of using a case study methodology by saying:

> The real business of case study is particularization, not generalization. We take a particular case and come to know it well, not primarily as to how it is different from others but what it is, what it does. There is emphasis on uniqueness, and that implies knowledge of others that the case is different from, but the first emphasis is on understanding the case itself (p. 8).

Every teacher participated in my research has her own unique experience, but by looking at each of their experiences individually and as a whole, I am able to gain a

deeper understanding of the practices of teaching high school mathematics in Saudi Arabia.

I chose to use a case study design as my research method to investigate individual teachers' practices because it has a level of flexibility that is not readily offered by other qualitative approaches such as narrative inquiry or phenomenology. Case study research provides details about participants that are not normally easily obtained by other research designs. While narrative inquiry focuses mainly on closed interpretations of stories told by participants and phenomenology focuses on interpreting participants' descriptions of their lived experience, case study allows the researcher to investigate the research questions in more detail. The data collected in case study is a lot richer and comes from multiple sources, which allows for greater depth in data collection. Therefore, the case study design allowed me to obtain a comprehensive understanding of rich data gathered about the research participants in a natural setting. Using a case study design not only helped me to describe the data in a real-life setting, but also helped to explain the complexities of participants' teaching practices in real-life situations.

4.3. Recruiting participants

Recruiting the right participants is the basis of effective research. As such, I recruited participants who were fully willing to participate and provide me with invaluable responses. I used purposeful sampling (Patton, 2002; Creswell, 2012) to select participants for my study. The selection criteria for teachers participating in the study included: participants are high school mathematics teachers currently working in public schools in the Eastern region in Saudi Arabia. Participants must have a minimum of five years teaching experience to ensure that participants have had experience using the new and the old mathematics textbooks in their teaching. In addition, because participation is voluntary, participants must be interested in participating in the study and willing to have me in their classes. Finally, participants must be female. This criterion is the result of the gender segregation in schools in Saudi Arabia.

Patton (2002) highlights the significance of purposeful sampling by selecting "information-rich cases" (p. 242) that lead to a rich, dense understanding of the research questions. The criteria for selecting participants support the selection of "information-rich cases" of participants whom I can learn most from and can provide me with deep insight and understanding regarding current high school mathematics teachers' practices in Saudi Arabia.

I recruited participants mainly by using a snowball (or chain) sampling method. Snowball sampling is a technique for finding research subjects where an existing participant provides the researcher the name of another subject, and so on (Weiss, 1994; Patton, 2002; Creswell, 2012). This technique helped me to maximize the number of participants recruited over a limited time. I contacted teachers who agreed to participate and fit the purpose and criteria for my research directly.

4.3.1. Narrowing the focus of my research

Initially, I collected data from ten participant teachers who fit the criteria and agreed to participate in my research. However, in this study, I only include and use data recorded from four out of the ten teachers who participated in my study. The selection of those four teachers came in two phases. In phase one, four out of ten were eliminated from the focus of my study because they were not comfortable with the audio recording during the interviews and classroom observation. I was only able to take notes for data collection.

However, after transcribing the data collected from other teachers, I realized that the audio recording provides me with more in-depth data about the participating teachers. By relying merely on note taking, I was not able to record all the information discussed during the interviews. For example, it is extremely difficult to identify the figured worlds that are significant to a teacher's practices without at least having a full and accurate record of what the teacher said. Therefore, I decided not to use the data I collected from those teachers in this study.

The second phase occurred after transcribing the data and conducting the initial analysis. During this phase, I realized that, with four participants' teachers, every two teachers appeared to have similarity in the figured worlds that are significant to their

practices. Therefore, I selected one teacher case of every two similar teaches cases. The result, I ended up with four participant teachers to be the focus of my research. Details and information of the four participants are below.

4.3.2. Participants

Data presented in this study is about four teachers: Abeer, Noha, Maram and Huda. The teachers represent different levels of experience and teach different grades in high schools. The following table presents some basic information about the participants.

Table 4. Basic information about the participants

Participants	Degree	Years of experience	Grade currently teaching	Number of lessons currently teaching
Abeer	B.Ed specialization in mathematics.	Six years teaching high school only	11	24 lessons per week
Noha	B.Ed specialization in mathematics	Four years teaching middle school and nine years teaching high school (13 years total)	10 & 11	22 lessons per week
Maram	BSc specialization in mathematics	11 years teaching high school only	11	18 lessons per week
Huda	BSc specialization in mathematics	Two years teaching middle school and six years teaching high school (8 years total)	11 & 12	18 lessons per week

4.4. Data sources

I incorporated a triangulation of the data from a variety of resources including interviews, observations, and a review of written documents. In PoP, the main purpose of methodical triangulation “is not that it enables one to ‘locate’ or specify one and the same construct with greater accuracy. Quite the opposite, different methods provide at least some access to different social practices and figured worlds, which may turn out to be significant for the interpretation of the teacher’s interaction with the students” (Skott, 2013, p. 557). According to Skott (2013), researchers mostly use triangulation in order to verify and confirm findings of their studies. Patton (2002) also notes, “multiple sources of information are sought and used because no single source of information can be trusted to provide a comprehensive perspective” (p. 306). In the triangulation technique, “the strengths of one approach can compensate for the weaknesses of another approach”

(Patton, 2002, p. 306). According to Skott (2013, 2011), triangulation is used to enrich research findings by shedding light, as much as possible, on diverse practices and figured worlds teachers engage with.

Merriam (1989) indicates that researchers, for the most part, do not use all sources of collected data equally. She states, “one or two methods of data collection predominate; the other(s) play a supporting role in gaining an in-depth understanding of the case” (p. 137). In my research, data is drawn from two primary sources and four minor sources. The two primary sources are interviews and classroom observations. The minor sources for data collection are data from informal observations of staff-room communication between the participant teachers and their colleagues, a copy of the teachers’ lesson planning notebooks, some of their worksheets and test samples and the official mathematics textbooks used in high schools.

I considered interviews and classroom observations as primary sources because they were more useful in my analysis to identify and understand the significant practices or figured worlds in the participants’ practices as mathematics teachers. Therefore, I mostly relied on these two sources for data analysis. The field notes prepared from the observations and the interviews were among the primary sources of the data. All the data I collected is in Arabic including the interviews, the observations and all the written documents I gathered from the teachers.

4.4.1. Data gathering

In order to collect my data, I travelled from Canada to Saudi Arabia twice. I scheduled interviews and observations ahead of my travel dates, based on the participants’ schedules and my travel schedule. I visited each participant teacher during each visit at her school. The first visit, phase one, took place during the first week of March, and the second phase occurred seven months later.

For phase one, I went to Saudi Arabia to gather data over a six-week period. During the first visit, I conducted two interviews and attended two lessons for each of the ten teachers participating in my research. For each teacher, I conducted the first interview before attending any of her classes. The second interview took place after I had attended two classes of that teacher. When phase one finished, I went back to

Canada and started transcribing and did some initial coding. After the initial coding, as mentioned, I narrowed the focus of my research to four teachers. I also felt there was a need to meet these four teachers again, clarify some issues, and ask some additional questions. Phase two occurred seven months later when I went back to Saudi Arabia and had a three-week period to gather further data. During this trip, I only visited the four teachers I chose for the focus of my research, conducted a third interview and attended two more classes for every teacher.

4.4.2. Primary sources of data

Interviews

Qualitative interviewing is a powerful tool that researchers can use to gather rich data. It provides the researcher with deep and detailed data that enables a profound understanding of participants' experiences. Through interviewing, the researcher can understand how participants describe and interpret their experiences, and the meaning they make of those experiences (Weiss, 1994). “Interviewing gives us access to the observations of others. Through interviewing we can learn about places we have not been and could not go and about settings in which we have not lived” (Weiss, 1994, p. 1).

As a researcher, I had carefully designed my study to acquire as much relevant data as possible and to ensure the ethical guidelines were properly met. I talked to all participant teachers by phone to arrange the best day and time to visit them at their schools. My awareness that research depends a in large part on how I communicate with the participants led me to try to build mutual trust by addressing any concerns or issues the participants had before we started the interview (Weiss, 1994; King & Horrocks, 2010). Although I emailed the participants the information document explaining my research and the informed consent form before our first meeting, some teachers were unclear about the nature of my role as a researcher. Some teachers seemed to think that my role was to evaluate their work. I think this unclear expectation is because none of the participant teachers had experience participating in research before and did not know others who had experience participating this kind of research. I explained to the teachers that I was there to learn from their experience not to assess their work.

All of the interviews conducted for this research study were one-on-one and face-to-face semi-structured interviews. I interviewed every teacher three times. Every interview took between 50-60 minutes. The format of semi-structured interview is appropriate for finding depth of meaning and gaining insight and understanding of the teachers' experience. The semi-structured format of an interview allowed me to get detailed information in a style that has an informal nature and is slightly conversational. The informal nature of the interviews style offers the opportunity to understand thoroughly the answers provided (King & Horrocks, 2010). The interview structure employs open-ended questions that are informal and engaging for the participant, while making them feel comfortable to express their point of view and speak their minds (Patton, 2002; King & Horrocks, 2010).

In order to build a relationship of trust with the participants, I spent some time at the beginning of the first interview getting to know the participant. I asked them to tell me about their background and to explain the outline of their profession history (Weiss, 1994; King & Horrocks, 2010). During this stage, I felt that I had to share something about myself with the participants. Some participants asked me about my experience studying in Canada. I limited the sharing I had with the participants to issues not related to my research and avoided talking about issues related to any questions I asked the participants. During the interviews, some teachers asked for my opinion before giving their view about certain issues I asked them about. For example, when I asked participants about their experience teaching the new textbooks, some answered by asking me what I thought of the new textbooks. I always avoided giving the teachers any specific answer and I replied by saying that they are the experts in this regard because they have the experience using these textbooks in their teaching. I explained to the teachers that their participation is valuable to my research and makes my research possible.

Some of the questions I asked were structured to ensure the minimum necessary data from each participant. This structure guided the interview through the list of questions that I generated prior to meeting the participants (Weiss, 1994). However, some of the questions I asked were non-structured questions that emerged from the data as it developed during the interviews. When a participant teacher brought up a relevant issue or a topic I had not anticipated, I asked follow-up questions based on this new topic. The interview protocol was to guide the participants to highlight their

mathematics teaching experience in high school, as well as their understanding of what grounded their teaching practices in and out of the classrooms. I designed the protocol for the interviews to create prompts for each question to help keep me on track during the interview (King & Horrocks, 2010; Creswell, 2012). Prompts helped me to remember the main issues I planned to ask about while at the same time allowed for unexpected data to emerge by taking the participants in several different directions (Weiss, 1994). The interview protocol is included in Appendix A.

During the first interview, I started by asking the participants basic background questions about themselves such as years of experience, classes and grades have and currently teaching etc. as a way of warming up the participant. I asked the participants about their school and university experience and how they made the decision about what to study at university. Then I moved to more concrete questions about reform and the textbooks change. For the second interview, I asked them about everyday practices in relation to their profession. I tried to situate the question within a practice context; for example, I asked questions such as, “Walk me through the process you use to prepare for you lesson today?” and “Think about what you taught today; how would you describe the lesson?” I also asked questions related to the lessons I attended using stimulated recall technique. For the third interview, I asked questions to clarify some issues raised from the first and second interviews and asked any questions I missed during the first visit.

During the second and third interviews, I used the stimulated recall technique. The purpose of using this technique is to encourage teachers to reflect and elaborate on their views of mathematics teaching while referring to their actual classroom practices. Skott (2014b) suggests using stimulated recall technique to invite “teachers to think aloud about relevant classroom process” (p. 21). The main focus of the stimulated recall is to shed light on how the participant teachers’ experience the teaching situation and to uncover the thinking processes associated with their actions. Using the stimulated recall technique, the researcher could possibly access the thought processes, which cannot be reached merely by relying on traditional observation techniques (Calderhead, 1981).

I applied this technique by playing a small part from the lessons I observed and asked the teachers to comment on what is happing in the audio recording. I used verbal prompts when needed to encourage the teachers to reflect more deeply about what they

were hearing themselves doing. In order to increase validity, I tried to minimize the time delay between the event and the recall interview. After I observed one or two lessons for every teacher, I listened to the recording of the lessons and chose a small clear part from the audio to use during the second interview.

The parts I chose to replay during the stimulated recall are where the teacher performed a certain practice in the classroom, such as writing on the board or asking students to open their books. I chose these parts because I was interested in asking participants to explain and reflect on their decision making. I wanted to learn the perspectives of participants, their interpretation of their practice and their thinking at that moment.

All interviews were audio-recorded and transcribed. I was also able to take some field notes during the interview. I tried to focus on things that may not come across on the recording when played back while transcribing such as the body language expressed by the interviewee. I focused on observing the participants' behaviour during the interview to include the emerging relationship between the participant and myself in my notes, as recommended by Charmaz (2006). The note taking during the interviews also focused on recording my thoughts, questions, and concerns that were raised while the interviewee was talking so I could ask about them later without cutting the flow of the interviewee's speech (King & Horrocks, 2010; Creswell, 2012). All notes were in Arabic.

At the beginning of my data gathering process, especially after the first interview, I was not sure if the teachers were feeling safe and confident enough to open themselves during the conversation. However, I felt that the comfort level rose after every meeting with the teachers. Moreover, at the end of the data gathering process, I was amazed at how willing teachers were to talk to me about their experience and express their concerns and sometimes frustrations.

Classroom observations

Observation is valuable during qualitative research because it allows the researcher to get direct access, perceptions, and embedded views into the examined phenomena, settings, and participants (Creswell, 2012). I visited and observed the participants in the classrooms where they taught. These observations provided me the

opportunity to see the participants as they are teaching in their classrooms. I was able to see them being mathematics teachers through the experiences they have told me about, as well as through perceiving their experiences in the classrooms while they were happening.

I observed four lessons for every participant; two lessons during each visit to the school. Observations lasted from 40 to 45 minutes, the duration of a complete mathematics class. During the observations, I did not participate in classroom activity in any way; I sat in a location that caused the least amount of disruption to the classroom environment, as requested by each teacher. I placed an audio recording at the front of the class to primarily capture the teacher's voice.

During the observations, I took field notes to document the teachers' activities throughout the lesson. The notes were descriptive, focusing on recording and describing the teaching activities; however, after every lesson I wrote some reflective field notes to record my personal thoughts and insights about the lesson (Creswell, 2012). All notes were in Arabic.

To capture as much information as possible during the classroom observations, I developed an observation protocol for recording information. Teaching is a complex and dynamic activity, and throughout a lesson various things could happen simultaneously; therefore, the observation protocol helped me to understand the complexity of classroom dynamics and capture some major aspects of the teachers' practices. The protocol focuses on things such as how the teacher structures the classroom environment for learning, how the teacher starts the lesson and how she ends it, the strategies and procedures during instruction, how the teacher sets up activities, the way the teachers offered instructions and explanations, and how feedback is provided to learners. The protocol also focuses on how the teacher and the students use the textbook and other materials, the language used by the teacher and the students, and students' level of engagement during different parts of the lesson. The observation protocol is included in Appendix B.

In chapter five, I provide an analysis of a short classroom episode; this is a short period of time from a lesson I observed. Though I observed four lessons per participant teacher, I chose each episode based on two criteria. First, it had to show active

classroom interaction between the participant teacher and her students. Second, it had to clearly display many figured worlds.

4.4.3. The minor data sources

The minor sources for data collection are a copy of the teachers' lesson planning notebooks, some of their worksheets and tests samples, the official mathematics textbooks used in high schools and data from informal observations of staff-room communication between the participant teachers and their colleagues.

The documents I received from the participants include a copy of their lesson planning notebooks and some of their worksheets and tests samples offer a source of text data for my research. As Creswell (2012) noted, these types of documents "provide the advantage of being in the language and words of the participants, who have usually given thoughtful attention to them" (p. 223). These documents revealed some insights and relevant information about the participant teachers' teaching strategies, the use of curriculum materials, and students' assessment. According to Merriam (1998), one important advantage of documentary data is they are "objective sources of data compared to other forms" (p. 126). While the presence of the researcher could affect the collection of interview and observation data, the collection of documents relevant to the study does not create intrusive measures into the research setting (Merriam 1998).

Another data source I investigated is the official mathematics textbooks the teachers used in their teaching. In Saudi Arabia, textbooks hold the status of clearly reflecting official curriculum. All teachers are supposed to follow the textbooks and cover them by the end of the school year. To understand the mathematics textbooks used in Saudi Arabia, I conducted a partial critical analysis of the old and new mathematics textbooks used in high schools. Although Saudi teachers do not officially use the old textbooks in their classroom, it was important to do an analysis of the old textbooks. The old textbooks were the official curriculum document for more than 30 years. As such, they have had a great influence in shaping the culture of mathematics teaching practices in Saudi Arabia. All teachers who participated in my study had learned from these old textbooks in school when they were students and had experience teaching from these textbooks.

Although results of this analysis are not directly used in my research, it provided a general examination of the nature of the two textbooks, which helped me understand how the participant teachers experienced teaching the two textbooks and how this experience may inform their practices in classrooms. Details about the analysis are included in Appendix C.

On the day of the interviews, I arrived at the school early so I had the opportunity to move around, explore the school, and spend some time in the staff room. A staff room is usually the perfect place to observe a school culture. Teachers interact constantly and talk about different issues. During this time, I was able to have informal observations of staff room communication between the participant teachers and their colleagues.

I tried to focus on noticing the topics of conversation in the staff rooms, especially the professional communications that are related to teachers' practices. For example, I paid attention to teachers' conversations about their classroom performance, teaching strategies, lesson planning, curriculum materials, students learning and assessment, students' behaviour, professional development, school administration, and the General Administration of Education. I used field notes to record my observations and document interactions between teachers in the staff room.

4.5. Transcribing

Careful transcription of audio data is an essential primary step in data analysis (King & Horrocks, 2010; Creswell, 2012). In order to undertake a transcription, I listened to the interview tapes many times to become more familiar with the data every time I listened to it. While the process of transcription is usually described as a technical task involving transforming the spoken words into written data, the challenge I faced with transcription was mainly related to the difference in languages. Transcription that involves translation from one language to another presents a particularly complex and challenging task.

To overcome this challenge, I adopted Kvale's (1996) view that "transcripts are not copies or representations of some original reality; they are interpretative constructions that are useful tools for given purposes" (p. 165). Therefore, during the transcribing process, instead in focusing on the question "What is the correct

transcription?" I tried to focus on "What is a useful transcription for my research purposes?" (Kvale, 1996, p. 166). Since I am writing my research in English, I decided it would be more useful to the purposes my research to write the transcripts in English, which entails a direct translation of the spoken language. When it comes to translating when writing a transcript, there is no one right technique that should be followed or adopted. Researchers should make choices to cope with the situation and come up with their own technique (Nikander, 2008).

The technique I used is directly translating what the participants said; however, occasionally I had to interpret what the participants said. Sometimes translating directly what the participant said literally from Arabic to English produced sentences that make no sense and have no meaning. The most challenging part was translating cultural expressions from Arabic to English. In some cases, I was not able to find the cultural equivalent expressions in English language, which express the same meaning. In this case, I had to interpret the meaning in terms of the second language and culture. However, to ensure validity in terms of accurate interpretations, I included the original Arabic text in the transcripts beside the translated / interpreted parts. This way I can also ensure that the findings of my research are a result of analyzing the primary data instead of the reconstruction of it.

4.6. Data analysis

Because my study is a collective case study, I applied two stages of analysis: the within-case analysis and the cross-case analysis. For the within-case analysis, each case is first treated as a comprehensive case in itself with in-depth exploration a standalone entity. As recommended by Yin (2003), I used theoretical propositions to lead the analysis using PoP as my main theoretical perspective. I present all cases analysis individually in a separate chapter (chapter five).

Chapter five offers my response to the first research question. After completing the within-case analysis, I began cross-case analysis in order to answer the second research question. The purpose of the cross-case analysis is to identify themes that are consistent across individual cases and across data sources. I present and clarify the themes from the cross-case analysis in chapter six.

4.6.1. Within-case analysis

The first stage of analysis is the within-case analysis. The purpose of this stage is to present every case individually by adopting PoP to identify the significant practices or figured worlds of participant teachers and how they engage with these figured worlds. By adopting PoP as the main framework for data analysis, three methodological challenges could arise. Skott (2013) explains these challenges as

> First, we need an approach that views instruction as continuous transformations of teachers' modes of participation in the classroom in view of broader social practices and figured worlds at the institution in question and beyond. Second, an interpretive stance is needed that views these practices as well as shifts in the teachers' engagement in them from the perspective of the teachers themselves. Finally, it is not apparent at the outset what practices and figured worlds are significant for the teacher in question, and the design needs to be so flexible as to allow for new and unexpected ones to turn up. (p. 552)

As recommended by Skott (2013, 2011), to overcome these challenges, I use techniques and procedures to analyze data inspired by grounded theory approach including the use of coding schemes, constant comparisons, and memo writing. Since the data analysis process is the core of grounded theory, I use grounded theory as a method of data analysis techniques while disregarding the "objectivist connotations" that are usually affiliated with the theory (Skott, 2013, p. 552). Also, as I mentioned before, I am using PoP as the main framework for my study; therefore, I am approaching the data with a theoretical perspective, which to some extent, guided the coding process. The way I engaged with the data was not as open as recommended by the grounded theory approach; I adopted the grounded theory coding technique for data analyses without adopting the philosophy behind the grounded theory. Grounded theory approach recommends not conducting any pre-research literature review nor relying on any theoretical framework for data collection and analysis; this is not the style I used in this research.

The coding process for the individual cases occurred first to capture the essence of a participant's experience. This allowed me to develop a deep understanding of every participant teacher's significant practices and figured worlds. I engaged with the data with the intention of identifying the significant practices or figured worlds of participant teachers and how these teachers engage with these figured worlds. However, utilizing

analysis procedures inspired by grounded theory within my research allows me to organize the data through subsequent reasonable stages. The aim is to reduce the collected data to a manageable size and identify patterns in order to develop a way to communicate with the data.

According to Charmaz (2000, 2006), coding helps the researcher to engage with the data and ask questions that help to gain new perspectives with regard to the phenomena under examination. Coding in grounded theory consists of many different stages. In this dissertation, I applied the two fundamental and basic stages of coding identified by Charmaz (2000, 2006); the open or initial coding and the focus or selective coding. Through the coding process, I was able to organize, group and reflect on the data. The process includes isolating patterns and categorizing the data to identify practices and figured worlds that are significant for the participant teachers and how they engage with these figured worlds.

Documents analysis was part of the data analysis for this study. According to Bowen (2009), when researchers use documents analysis as a part of their data collection method, they usually do not provide enough detail with regard to the technique followed and the results of the documents analysis especially when documents analysis is used in combination with other qualitative research approaches as a source of triangulation. This gap in research leads in most cases to ineffective use of documents analysis in qualitative studies (Bowen, 2009).

Analyzing the written documents I received from the teachers including a copy of their lesson planning notebooks and some samples of their worksheets and tests, was challenging. The challenge is because the lesson planning notebooks for every teacher is a big document with over 200 pages. Therefore, doing line-by-line coding would be extremely difficult and a time-consuming process. Therefore, I tried to find a useful and practicable technique to analyze these documents.

Analysis of the written documents happened in two steps. The first step involved skimming or a superficial and rapid examination of the documents (Bowen, 2009). I went through the material quickly in order to get some insights of its essence and identify how it was organized. I observed how the teacher organized every lesson and if there was anything that the teachers emphasized in her lesson plans. During this step, I made

observational notes, including emergent ideas and themes that connected to other forms of data. The second step involved taking a closer look at selected lessons from every participant teacher's lesson plan notebook and performing coding. The lessons I selected are the lessons I observed teacher giving.

I started the coding process with open or initial coding by analyzing the interview transcripts, observation transcripts, field notes and the selected lessons from every participant teacher's lesson plan notebook. According to Charmaz (2006), during the initial coding stage, the researcher should ask questions such as "what is this data a study of?", "what does the data suggest", and "from whose point of view "the data comes from" (p. 47). I followed Charmaz's (2006) recommendation by doing the initial coding quickly and with "spontaneity" to "preserve actions" (p. 48). I examined the data by outlining actions or incidents within it line-by-line. During this stage, I tried to use gerunds to understand what was happening in the data. By looking at teacher actions and words, I was able to assign a code that seemed to identify what was happening. Most of the coding at this level was descriptive, summarizing the significance of the points. At the same time, I did not ignore any underlying assumptions and implied actions and meanings that emerged.

During this stage, I coded some phrases and sentences with more than one code and included them in more than one category. This process was one of the ways that helped me understand how different figured worlds could support or conflict with each other. For example, the sentence "provide students with proper mathematics understanding to help them in the test" coded as students' evaluation and effective mathematics teaching. These two codes ended up in two different categories.

I used a constant comparative process of data analysis throughout this stage. This continuous process of producing and testing codes is fundamental in the grounded theory approach. Constant comparison is a way of maintaining a connection between the codes and the data. Every time I generated a new code, I compared it to the other data I coded to check consistency in applying the codes. At this stage, coding involved a progression toward saturation by looking for more instances that represent the same code and continuing to look for new information about the code until the concept or the code is saturated. The constant comparison process eventually led to saturation (Charmaz, 2000, 2006).

Following this, selective or focused coding was undertaken (Charmaz, 2000, 2006). It involved exploring the codes and examining relationships between codes and comparing them. Open or initial codes were refined and grouped according to connections and similarities. At this level, I focused on codes that appeared to potentially represent the figured worlds in the participant teacher's practice. I selected and identified codes to form core categories that led to identification of participant teachers' significant practices and figured worlds.

Memo writing also occurred during the process of coding and constantly comparing the data. According to Charmaz (2000, 2006) memos are a process of engaging with data and analyzing it and eventually writing about it. Memos are about codes and the developing categories, a link between categories, the gaps, and my thoughts about the usefulness of the categories. During the two stages of coding, I kept expanding and developing my memos, which represent my research diary to develop ideas. I used memo writing mostly during the constant comparison process and forming the core categories, which represent the figured worlds. Memos provided a means of recording my thoughts and insights about the core categories. They were helpful in writing about the relationship between different figured worlds with which the teachers engaged.

4.6.2. Cross-case analysis

The second and final step of data analysis included cross-case analysis. The purpose of the cross- case analysis was to identify themes that are consistent across teachers and across data sources. I analyzed the data comparatively across the four teacher cases for the cross-case analysis. I compared core categories generated from every participant's data across cases to identify similarities and differences. In order to compare core categories, I examined relationships between codes across cases and gathered codes that were comparable. I found some codes from every case belonged together under one theme. For example, the code "relying on the textbook" was a code from one case and "resistance of the textbook" was a code from another case; these two codes belong under the theme "the role of the textbook".

After comparing core categories and codes across cases, several key themes emerged related to the second research question. I present every theme, in chapter six,

by describing similarities and differences among the four teachers' experiences in relation to that theme.

The cross-case analysis was a process of interrelating findings from each case in order to generate a broad understanding of how high school mathematics teachers in Saudi Arabia respond to the shared or common circumstances they are facing in the current reform movement. The cross-case analysis provided an opportunity to examine the phenomenon of high school mathematics teaching in Saudi Arabia. According to Stake, (1995) cross-case analysis allows the researcher to see beyond the individual cases, to the phenomenon and offers a new perspective to examine the phenomenon by bringing the findings from the individual cases experiences to the research questions. The cross-case analysis helped me understand each teacher's case, contributing to my understanding of each participant's teaching experience and therefore answering my second research question. Finally, the cross-case analysis also provided insights to propose questions for future research and investigation about high school mathematics teachers' practices in Saudi Arabia.

4.7. Summary

In this chapter, I explained in detail the research methods and methodology for this study. The chapter also includes the main research questions and a description of the practical steps I took during the design of my research including recruiting participants and the means and modes of data collection. In this chapter, I also explained the two stages of analysis: the within-case analysis and the cross-case analysis. The next chapter, I present four case studies of high school mathematics teachers in Saudi Arabia and provide the results of the within-case analysis.

Chapter 5.
Individual Cases

In this chapter, I present each individual case separately to capture the unique essence of their experience. The goal of presenting every case is to develop a deeper understanding of the participant teachers' current practices. This chapter outlines the answer to the first research question, What are the figured worlds, or significant practices, to the participants' teachers' sense of their practices as mathematics teachers and how does each teacher engage with these figured worlds?

As a teacher positions herself in relation to her profession as a mathematics teacher, she draws on several, often incompatible, figured worlds. Her engagement with these figured worlds does not only appear in her verbal communication, but also by the choices she makes in her all other actions related to her profession, such as her immediate reaction to certain student behaviors or the way she expresses her view when engaging in a conversation with her colleagues (Skott, 2013).

5.1. Abeer's case

Abeer is a high school teacher with six years of experience working at the same school. She works at a high School in Aldammam city located in the Eastern Province of the Kingdom of Saudi Arabia. She graduated with a Bachelor of Education Degree with a specialization in mathematics. The education courses Abeer had in university focused on general issues related to teaching, such as lesson planning and classroom management. She does not have any experience taking educational course related to teaching mathematics specifically. She has been teaching from the new textbooks for three years. When I met Abeer she was teaching 24 lessons per week to students in grade 11.

Abeer works in a Tatweer school, which follows the credit system. Before the 2013-2014 school year, the school's entrance requirement was a 90% overall grade average upon completion of the Intermediate School Certificate. Students who finished middle school with an overall grade average of less than 90% were unable to register at the school. However, starting in 2014, the school changed this policy and now accepts all local students with no entry requirement. The school building is relatively new and is

located in a quiet middle-class community. The school has 478 students. The mathematics department has five teachers; two of them have more than 15 years of experience.

5.1.1. Abeer's classroom

Abeer has 30-33 students in every class. Her students sit in groups that do not remain static; she changes who is in each group every week. She forms new groups every Sunday (the first day of the school day in Saudi Arabia) by assigning students randomly. The groups sit in a square formation with four to five students in each group. In her classroom, students work individually as well as in groups. The textbook is always a part of her lesson. She usually starts her lesson with an activity that students do within their groups. She often chooses the activity from the textbook, but sometimes she comes with a different activity than what is in the textbook. Abeer rarely assigns her students homework, but when she does, her students know that they are not required to work on it at home. Every Thursday, which is the last school day of the week, the lesson starts with a quick quiz. The quiz is usually about what they have been doing in class during the week.

5.1.2. Significant practices or figured worlds in Abeer's case

After six years of teaching, the data generated about Abeer suggest that there are five significant practices or figured worlds to Abeer's sense of her practices as a mathematics teacher. These figured worlds are mathematics, the textbook, the reform, students' achievement, and her relationship with others in her school including other mathematics teachers, the school principal and the school inspectors.

Mathematics

Abeer's sense of her practices has developed from her experiences as a student and a teacher of mathematics. Abeer tries to create a learning environment that is different from her own experience learning mathematics in school. "My mathematics class is not like my own experiences in school; we had to sit in rows, quietly, and work individually on long algorithms exactly like the teacher demonstrated". As a student, teachers told her learning mathematics is about finding the right answer to a problem.

However, her notion of mathematics learning as a teacher is quite different. “I always knew that learning mathematics has more to it than finding the right answer”.

In Abeer’s classroom, effective communication is vital as both a learning process and an outcome. Sharing ideas and clarifying understanding is important in the communication process to help students build meaning of the mathematics concepts. Abeer’s students know that even if there is one right answer to a problem, the way they go about finding the answer or the way they explain the answer is very valued in her classroom.

When Abeer decided to become a mathematics teacher, she knew there were two ways to do so, the “easy way” and the “hard way”. “It would be an easy choice to teach mathematics as a subject where there is a right and a wrong answer and that's all there is to it”. Abeer decided to take the harder, more challenging way. Abeer explained that by motivating students to communicate their mathematical thinking and provide time for students to discuss and hear the mathematical ideas of other students, she makes her job as a teacher “more difficult”. She has to pay attention to everything the students say and always be ready to respond to their ideas.

Doing mathematics in Abeer’s classroom is not about finding the right answer, it is about applying thinking skills and being able to explain how to find an answer. She endorses students' ability to think reflectively about their answers. Abeer’s ultimate goal for her students learning experience is to get her students to get to a point where they are sure of their answer and they can explain how they got it.

In her classroom, Abeer often invites her students to think of multiple approaches to solving mathematical problems because, as she indicated, it is very important to student learning of the mathematical concepts. Having students look for multiples ways of finding solutions and using multiple strategies provides Abeer with more opportunities to encourage her students to talk about their work and explain how they reached a solution.

When Abeer uses the mid-chapter quiz, the end of chapter test, and the cumulative practice test, she divides the questions from of the book between the different groups in the class. Every group works on some of the problems, and then the groups exchange their work. Abeer refers to this practice as a highly effective method to

help students work on as many problems as possible. It also gives students the opportunity to share and organize their thoughts without leaving their desks, and become responsible for their own learning.

The textbook

Abeer has a strong appreciation for the textbooks she is currently using in her teaching. The implementation of the new textbooks, according to Abeer, was a necessary step. For her, the old textbooks were insufficient in providing the knowledge and incentives for teachers to reflect on their practices.

Abeer uses the new textbook as a tool to reflect on her practice. She stated that when she plans her lessons, and before she chooses a particular part from the textbook to use in her lesson, she asks herself, "What do I actually want to get out of this? What do I need my students to take away from this? How can I add to this to enrich students' learning experience? By answering these questions, I can imagine different scenarios". It is evident that Abeer engages deeply with the textbook during her planning for the lesson phase.

In her classroom, the textbook has an active presence. Abeer often invites her students to engage with the textbook. At the beginning of the lesson, she asks the students to read the parts of the lesson including previously covered skills and concepts, learning outcome of the lesson, and the major mathematical vocabulary used in the lesson. She gives them few minutes to discuss what they read in their groups. Then, she asks them to read the purpose of the lesson section, which presents information usually related to real life situations and sometimes requires the students to answer questions that follow the information. She usually discusses this part with the students and this discussion leads to the introduction of the main concept of the lesson.

During the lesson, Abeer also refers her students to the textbook many times. The students read most of the instruction part of the lesson. Sometimes, Abeer reads certain parts of the textbook and the students follow along in their textbooks; other times, she asks one student to read aloud from the textbook, and in some cases, the students she asks students to read sections and discuss the information in their groups.

Unlike other mathematics teachers in Abeer's school, Abeer insists that her students work on the section of the textbook that contains higher order thinking problems in every lesson. According to her, these problems give her students the opportunity to encounter different levels of mathematical thinking. However, when working with this part, she tries to facilitate the students' thinking by asking questions and making comments that can lead the students to the right answer.

Abeer indicated that the part of the textbook, which contains higher order thinking problems, provides her with a new and different perspective about student learning. "One of the new things that got my attention about this part of the new textbook are the questions where students are asked to write using their own words, their understanding of a certain mathematics concept. I never thought of using writing in my mathematics classroom before". She also encourages her students to incorporate drawing and symbolism in their writing to help express their ideas.

Abeer uses writing as a learning tool and as an assessment tool. For Abeer, writing helps students to gather and organize their thoughts. It encourages them to think clearly, to find out what they know and what they do not. It is also a good opportunity for her to assess students understanding. It gives her some access to her students' thoughts and the way they understand mathematical concepts. Classroom discussion often follows writing time, and students are encouraged to talk about their ideas and discuss with the rest of the class.

She also explains that writing is a difficult task for most of her students and she needs to motivate them to be able to work through a task like this. When she started to incorporate writing in her classroom, most of her students resisted. To make the task easier for them, she allowed her students to work in pairs to do the writing. She asked them to share thoughts with their partner and come up with one piece of writing together. However, now that her students know the expectation to do writing in mathematics class, they mostly do the writing activity individually. Although it was not an easy approach to take in mathematics classroom, according to Abeer "it is worth it".

The reform

Abeer is very enthusiastic about the current reform movement in Saudi Arabia. Her enthusiasm shows from her involvement in any professional development activities

offered to her. She indicated, “most teachers don’t like to participate in any activity related to professional development. For me personally, I am always willing to do so; I always make sure to tell the school principle and the school inspector that I am always ready for these kinds of activities”. For Abeer, these activities provide her the opportunity to find new ideas, meet different teachers, and talk about mathematics teaching. Abeer had the chance to attend only one workshop after the introduction of the new textbooks. The workshop was only one day, which in Abeer’s opinion was not enough.

For Abeer, the new mathematics curriculum created a new language to talk about mathematics teaching and learning. For example, teachers started talking about learning strategies such as cooperative learning, and flipped classroom. Abeer learned about these teaching strategies from the workshop she attended and from her inspector. Using learning strategies has become a big part of Abeer’s every day planning process. She learns about effective ways of engaging these strategies in her class from her own experience by reflecting on her own practices.

Cooperative learning is a big part of Abeer’s classroom practices. She began adopting this teaching strategy after the implantation of the new textbooks. She indicated that cooperative learning offers motivational tools to encourage students to participate and be active. “It makes my math classroom more alive”. She applies cooperative learning strategies mostly when learning new concepts because it encourages students to share their understanding, their thinking strategies, and their difficulties with each other.

Every week Abeer divides her students into groups of four to five people. The group stays together during the whole week. She also assigns different roles among students in every group. The group organizer regulates team discussion, keeps the group focused on the task and encourages everyone to share their ideas. The group writer takes notes about the group discussion and ideas or solutions. The group speaker has the responsibility of presenting the group’s thoughts, ideas or solutions to the class. Finally, the group timekeeper makes sure the group is aware of time constraints. The roles change every day; students assume the role that the person to their right had the day before. The different roles she assigns to group members helps her students function effectively and get involved. “When I started to apply cooperative learning in my classroom, students had a hard time engaging with it. I was frustrated and it seemed like

a waste of time. Applying the roles strategy helps them get more involved in respect the group work".

In her classroom, Abeer draws on the message of the reform by giving her students more control over their own learning. She tries not to be the person with total authority all the time. She creates an atmosphere where students share responsibility for what is happing in the classroom. She is aware of how influential cooperative learning is in creating a thought provoking and interactive environment in the classroom.

Reform for Abeer means change. Abeer re-defined her role as teacher in the classroom after she was inspired from the reform ideas. When she talks about reform, she usually explains how the current reform movement has changed some aspects of her practice. Abeer described the first two years of her teaching career as being very traditional. "I was the person who had the mathematics and my job was simply to present what I know in the class. I used to start my lesson by presenting the concept and writing on the board the definition and the formula, and then I would work few examples and then give them a worksheet and tell them to do just as I did then I would correct them really quick".

Abeer's relationship with other staff members in her school

Abeer appreciates her experience working at her school. She is dedicated to the development of her school. She realizes the importance of a school community to create a positive learning environment for everyone. She indicated that schools are for not only sitting and learning for students; "everyone who works at this school learns from this experience". Abeer values communication with other teachers in the school and considers it an important source for her personal learning experience as teacher.

At her school, Abeer meets with other mathematics teachers every week or two to discuss issues related to students and school activities, but they also discuss issues related to mathematics teaching. Abeer and the other teachers do not design specific lessons together at these meetings, but they do share ideas and express their challenges in relation to teaching certain mathematics concepts. "I remember before I started teaching the trigonometry unit, I asked other teachers lots of questions and received some good ideas".

After the implementation of the new textbooks, teachers at Abeer's school started inviting other teachers to their classrooms to observe a lesson almost every month. Abeer is often excited about this part of her practice at her school. She enjoys having teachers in her class as much as she enjoys being in their classrooms. The trusting relationship Abeer has with other teachers in her school allows her to look at her practice as a professional and reflect more empirically and critically about her own practice. Moreover, the relationships raise her confidence in delivering effective teaching practice, especially when she started teaching at the school and, as she described, "the teacher with the least teaching experience".

Abeer also draws on the principal's support for a learning community among teachers. According to Abeer, although the principal of the school was not a mathematics teacher, she appreciates mathematics as a subject and realizes its importance. The principal is convinced that more communication between teachers can actually lead to improved teaching and learning. Therefore, she engages teachers in setting the agenda for the meetings, and helps teachers to coordinate their schedules, so that they find a suitable time to meet, observe each other teaching and offer each other feedback on their observations.

A school inspector is a part of the education system in Saudi Arabia. Inspectors in every subject visit schools two to four times a year to evaluate teachers' practices. For Abeer's practice, the inspector's visit is usually a challenging part of her practice. Her general view of what is valuable in her teaching does not seem to match the inspector's view. Abeer argues that the inspector is a former teacher with significant experience, but has no experience teaching the new mathematics curriculum. "The inspector cares about how much I give and whether I am following the plan we received from the Ministry of Education. I don't follow their plan; I usually make my own plan which I always adjust during the year". In the interviews, Abeer did not consider the lack of agreement about her view of what is valuable in her teaching practice with the inspector view as a major problem. According to Abeer, the choices she makes provide her students with the most effective learning experience. She is content with the support she receives from the principal and other teachers in her school.

Students' achievement

In the interviews, Abeer appears strongly committed to her students' achievement. Sharing students' main interest and understanding their biggest concerns is significant to her philosophies of her role as a high school mathematics teacher. For Abeer, teachers must meet the needs and address the concerns of their students. Abeer constantly engages with her students' emotional state of being under pressure to achieve in high school and then move on to college. The effort Abeer makes to support her students as they go through this tough experience is strongly related to her mathematics teaching. She expressed that students need to struggle to understand the mathematical concept; that this struggle would contribute to their achievement. "I play a significant role in how my students perform, I know that, and I am trying to give my students more control of the learning situation. I sometimes let them struggle to figure it out by themselves".

Homework is not part of Abeers' everyday practices in classroom. She relies on the weekly quizzes more than on homework to keep her students connected to what they learn. For Abeer, weekly quizzes encourage practice and review; they provide students more opportunities for feedback and positively impact students' study time. Quizzes in Abeer practices are useful tools to enhance learning and assess students' achievement. They also help her to assess the effectiveness of her instruction in classroom. She explains, "Looking at the questions students answer incorrectly helps me determine where there might have been a gap in instruction and figure out how students got confused".

Homework, on the other hand, does not seem to be an effective tool in Abeer practice to assess students' achievement or the effectiveness of her instruction. When I asked Abeer about homework, she answered, "I rarely give homework. Homework doesn't necessarily mean more learning or lead to higher achievement". Abeer's goal in her teaching practices is to help her students develop a desire for learning. An important part of her practice is to watch her students' progress and when she gives her students homework, she has no insight into how the homework is completed.

Abeer's eagerness for her profession of teaching mathematics is more than matched by her effort to help her students to be successful in the standardized tests. Abeer is concerned about her students' achievement in the aptitude test students taken

in high school. In her classroom, she often refers to the aptitude test and giving students tips to achieve better scores. “To survive the standardized tests, students don’t need to do basic operations fast, as most teachers emphasis. I need to support their understanding of the mathematics concepts, so they can apply this understanding to situations other than the classroom”. Abeer encourages her students to ask questions related to the aptitude test. She collects students’ questions during the week in a small box she keeps in the classroom. She refers to these questions during the lessons when possible.

Abeer relies on the collaborative working environment she has at her school to reflect on students’ achievements. During her group meetings, Abeer talks with other teachers about improving students’ achievement and helping them achieve not just at school, but also on the standardized tests taken in high school.

5.1.3. Classroom episode

Mathematics, students’ achievement, the reform, and textbook

In this episode, Abeer is introducing the mathematical concept of proof by induction to her students. After she wrote the title of the lesson on the board, the following discussion took place:

A: Can you tell me, using a simple language, what does proof mean in mathematics?

Students started to respond to this question randomly at the same time.

S1: Is it confirmation?

S2: Demonstrate something is true.

S3: Making sure a mathematics formula is right.

S4: Testing…. Testing if something is true.

A: O.K, I think I hear some good answers. I like the word demonstration.

A: So, how do we demonstrate a mathematical statement is true?

S1: We use proof.

S2: We rely on what we already know.

S3: We use something we know is true until we reach the statement we want to proof.

Abeer was nodding her head after she heard every answer and she pointed at the third student and said:

A: This is right. We have to make sure that the starting point is true. O.K., today we are going to learn a special type of proof in mathematics… proof by induction, which is a method to prove a statement that is true for every natural number. But, before we go further discussing the mathematical definition of this poof, we will play a game.

Abeer then put a pile of dominos on each group's table. She then said: "In your groups, I want you to construct patterns of dominos, so that when the first domino is pushed, the entire design collapses". Abeer moved around making sure every group was able to create such a design. After five minutes, she asked the groups to create patterns of dominos, so that when the first domino is pushed, not all the dominos collapses. Abeer is still moving around helping the groups and making sure they all did what she asked. Abeer then got the class to pay attention to her again. The discussion then continued:

A: What is the different between the two patterns you constructed?

S: One works and the other doesn't.

A: But what makes the first one work and the second not?

After few seconds of silence in the class, Abeer said:

A: Discuss in your groups. I want you to think of the conditions that make the first pattern work and the second not.

After every group came up with the conditions that made the patterns of dominos collapses when the first domino is pushed, and after discussing the conditions as a whole class, they came to an agreement that two main conditions are required to make

the design work. First, each domino must be able to knock over the following domino, and second, the first domino must be pushed first. Abeer then introduced the notion of proof by induction. She copied the formal definition from the textbook. She emphasized the similarity between the two required conditions to make the dominos design work and the two main steps to do proof by induction.

A: proof by induction is in a metaphor a pattern of dominos. The aim is to make all the pieces fall over, where if the nth piece of domino falls, so will the (n+1)st. piece. However, if you don't push the first domino, none of the pieces will fall over.

After a few seconds of silence, Abeer asked.

A: Is everyone able to understand the metaphor I am talking about?

No answers from the students.

A: OK, let's try to explain this in a different way. In proof by induction, first we must show that that the statement is true when n=1. With reference to the dominos metaphor, we are making sure that the first domino falls down. The next step is we assume that the statement is true when n=k and try to prove that the statement is true when n=k+1 is true. With reference to the dominos metaphor, we are showing that every time a domino falls, it makes the following domino fall too.

In the previous episode, we see that Abeer is engaging with many figured worlds, mathematics, students' achievement, the reform, and the textbook. These figured worlds support one another as well as contribute to Abeer's classroom practices. According to Abeer, learning mathematics is about communication in the classroom. Abeer started by asking her students to 'use a simple language to explain what proof is in mathematics'. She also used her student's word "demonstration" to keep the conversation going. Abeer is pushing her students to be more confident in communicating their thoughts, ideas and understanding in the classroom. When I asked her about this part of the lesson, she indicated that if she wants her students to give her a formal mathematical definition, she would ask for it. However, during this incident, she was trying to make her students understand that talking about mathematics is not always about giving a formal definition.

The textbook mentions the dominos metaphor; however, the textbook does not suggest using it as an activity for students to work on. Although Abeer started with the example mentioned in the textbook (the dominos), she decided to use it as an activity in her lesson. When I asked Abeer about her decision to have her students work on the dominos pattern, she replied by saying that this way her students would have an opportunity to conduct investigations that help deepen their mathematical understanding. This investigation provides an original way for students to develop their mathematical reasoning and communication skills, thus supporting the message of the reform by giving her students more control over their own learning.

When I asked Abeer about the part where students were struggling to understand the dominos metaphor in relation to proof by induction, she indicated that she chose to really emphasize this metaphor in her lesson, so her students could understand the part that the induction step plays in the whole proof. She expected the struggle students had to understand the connection between the dominos activity and notion of proof by induction. This struggle can contribute to students understanding of mathematical concepts, which in turn can contribute to students' achievement. In her comments on the struggle, Abeer said, "For a student to progress in mathematics, teachers need to focus on building the mathematics concept before they go into procedures".

5.1.4. Abeer the teacher

In terms of PoP, Abeer is developing her sense of being a mathematics teacher through her engagement and re-engagement in the figured worlds of mathematics, the textbook, the reform, students' achievement, and her relationship with others in her school. Abeer has a membership in these figured worlds, which interact with each other through Abeer's actions and participation in classroom.

Abeer has highly dedicated to her job and a willingness to deal with the challenges and the responsibilities involved. She always wanted to become an educator and many of her experiences learning mathematics at school greatly influenced. Her main motivation for becoming a mathematics teacher was that she wanted to be able to share with her students the amazing rewarding experience learning mathematics can have. According to Abeer, interacting with the students is the best part of being a

teacher. The most exciting moments are when you can actually see students start to understand mathematics concepts.

Her commitment to her profession shows in her commitment to constantly improve her practice. “I am always thinking about how to improve what I can do in class with the 45 minutes I spend with my students every day”. The main challenge Abeer faces in her practice is not related to her mathematical content knowledge; the challenging part in her opinion is “to simplify the mathematics down or to make things that seem so complicated a lot simpler”. Making things easier for her students does not prevent her from giving her students some challenging mathematics problems. She is a teacher who “enjoys challenging my students to grow up, and really be in charge for their own academic growth”.

For Abeer, teaching mathematics is not about knowing whether a student can find a correct answer. Her understanding of her role as a mathematics teacher is to make sure that the student understands why they solved it the way they did. Mathematics learning in Abeer’s classroom is more than just memorizing a series of steps that students observe her doing. Abeer’s role in her classroom is more like concept facilitator, where questions motivate students to think and experience the mathematical concepts at hand. In her classroom, students are encouraged to use different forms of language to explain what they mean, to express their thoughts using oral and written forms.

5.2. Noha’s Case

Noha is a high school teacher with thirteen years of experience teaching middle and high school. She is currently working at a high school in Al Khobar city located in the Eastern Province of the Kingdom of Saudi Arabia. She graduated with a Bachelor of Education Degree with a specialization in mathematics. The education courses Noha had in university focused on general issues related to teaching, such as lesson planning and classroom management. She does not have any experience taking educational courses related to teaching mathematics specifically. After she graduated from university, she started teaching mathematics at a middle school. After four years, she moved to a high school. She has nine years of experience teaching mathematics at the high school.

Noha taught from the old mathematics textbooks for middle and high school. When the new mathematics textbooks were introduced to high schools in 2011, they were implemented first to grade ten classes. Noha was teaching grade ten that year; as such, she was among teachers who used the new textbooks in the first year of implementations in high schools. She has been teaching from the new textbooks for three years. When I met Noha, she was teaching 22 lessons per week to students in grades 10 and 11.

Noha works in a traditional public school, which follows the two-semester system. The school building is relatively old and is located in a quiet middle - to low - class community. The school accepts all local students with no entry requirement. The school has 512 students. The mathematics department has four teachers.

5.2.1. Noha's Classroom

Noha has 35-37 students in each class. In her classroom, students are usually quiet and calm, sitting in neat rows of two tables that face the front of the classroom. Normally, Noha starts her lesson by checking students' daily homework. She then reviews previous material. The classroom environment emphasizes getting work done. Noha plans her lessons every day and she makes sure to follow the plan very carefully. A measure of time on task indicates that the lesson is going very well and that students are doing what they are supposed to do in her class. According to her, the most effective way to teach mathematics is to use the classroom board to introduce a mathematics concept, explain different mathematics procedures in relation to the presented mathematical concept, and then get students to practice these procedures individually. In her classroom, the official mathematics textbook is never used. Instead of the textbook, Noha designs a notebook that she and her students use during the lessons. This notebook replaces the official textbook in her classroom.

5.2.2. Significant practices or figured worlds in Noha's case

After thirteen years of teaching, the data generated about Noha suggest that there are six significant practices or figured worlds to Noha's sense of her practices as a mathematics teacher. These figured worlds are mathematics, the textbook vs the

notebook, the reform, students' achievement, relationship with students, and voluntary work.

Mathematics

According to Noha, mathematics is a body of knowledge centered on specific concepts, and learning these concepts means knowing how to use them. For Noha, mathematics is all about doing; if you are able to do mathematics, then you know mathematics. She explained,

> I see it as a body of knowledge we use in order to solve problems. Or you could say it's a tool that enables people to do things to get answers. The main components of mathematics' knowledge are things like a set of rules and formulas and a set of methods and procedures that allow us to find out the correct answers. You need to know what is the method or procedure that goes with every rule or formula.

During a typical class session, Noha spends 10-15 minutes on whole-class instruction in order to introduce the new concept by using the board. Then she does an exercise that demonstrates how to use the concept and explains very clearly the methods and procedures to do the exercise. The students' main role during this part of the lesson is to listen carefully to the teacher. Noha makes sure while she is presenting the new material that the students are paying attention to what she is doing by saying phrases like "listen carefully to what I am saying" or "focus your attention on me". If she notices that any student is not paying enough attention to her, she calls them by name to get their attention.

After introducing the new material, she gives her students a few minutes to copy the notes from the board into their notebooks. Then she asks the students to do a similar exercise to the one she did. The exercise is usually written in the notebooks that she prepared for the students to work on during the lesson. Students work on the exercise individually or with their neighbour. The exercises are usually short and closed.

Noha reported that she usually gives her students a lot of practice during class time. While her students work on the exercises, she moves around the classroom and provides students with direction about how to do the exercises. At this time, Noha's job is "to let them (students) know what they really have to put down". She explained that moving around provides her with a good sense of what the students are doing. Noha's students usually do not have many difficulties doing the exercises because when she is

presenting the material on the board, she usually models exercise, so that students are prepared to work a similar one without her assistance.

According to Noha, a basic part of understanding mathematics involves memorization and repetitive practice. She clarified why memorization plays an important role in mathematical understanding by saying,

> Some facts in mathematics need merely to be accepted as true and memorized, I can't explain some mathematics to my students in a way that they really understand. Maybe some people would not agree with my view, but I really see that there is a place for memorization of basic facts in mathematics learning.

However, she also added that memorization and being able to remember is not the ultimate goal in mathematics. She noted "most often in a mathematics classroom, students start with rote memorization. They should be able to gradually come to understanding by practicing".

Noha argued that although her teaching style is traditional, her approach plays an irreplaceable role in helping all students, regardless of their level of ability and learning style, to gain high level of conceptual understanding of mathematics and acquire strong mathematics problem-solving and reasoning skills. Her teaching approach focuses on three aspects: first, memorization of facts, rules and formulas; second, repetitive drills and practice of basic computation; and third, procedural skills practice and training. This approach, according to Noha, helps students to build a strong foundation of basic mathematics knowledge and skills, such as a deep understanding of mathematical concepts and fluency in using different mathematical procedures and methods. Students with such a foundation of basic mathematics knowledge and skills have the ability to do problem solving and reasoning. She declared,

> My teaching strategy is simple and direct, if you explain mathematical methods and procedures clearly, students would gain an understanding of them. Then if you make students to do a number of similar exercises, students will eventually know how to use these methods because the act of repeating a method or a procedure they learned would help students remember it later.

The textbook vs the notebook

One notable practice in Noha's classroom is the absence of the textbook. Neither Noha nor her students use the textbook during the lesson. Noha explained her history of

using the official textbook in her classroom by saying, “during my first year of teaching, I based much of my classroom activities on the textbook. In my second year, I used very little from it. Finally, in my third year, I got rid of it altogether and I haven't used the textbook during my lessons since. I started to rely on the notebook I design”. After using the textbook as a main source for her practices for two years, Noha realized the textbook's deficiencies and substituted with an alternate version of the textbook. “The textbook failed to arouse my students’ interest and keep them on track”.

Noha designs a notebook each year to use with the students in her classroom. This notebook replaces the textbook. During the summer, when schools are closed, she plans her notebook. She organizes the notebook by chapters and lessons based on how they appear in the official textbook. Every lesson in her notebook starts with an empty line for students to write the title and date. Then there is an empty rectangle for students to use to write the main definition, theory or formulas after Noha writes them on the board. The last part for each lesson within the notebook contains a written example and unsolved exercise. Noha takes some of the exercises included in the notebook from the textbook while others she creates. At the beginning of the school year, Noha photocopies the notebook and distributes one to each of her students. Students rely on their notebooks entirely during the lessons, when they study, and for all their homework.

According to Noha, the notebook provides learning situations that guarantee keeping students engaged in learning activities during the lesson. She added, “Without a textbook, I can create lessons that engage students by relating mathematics to their needs. Lessons become clearer when I present the topic in an organized way, using a language that my students understand”. Noha also indicated that she does not always follow her previously designed notebook, sometimes she makes changes based on student needs, “My notebook is peppered with activities that allow me to evaluate my students’ understanding right away and adapt quickly to their needs”.

Noha talked about her notebook very proudly and she does not intend to change this aspect of her teaching practices. She revealed that before the introduction of the new textbooks, the school inspector used to praise her for organizing and designing the notebook, however, after the introduction of the new textbooks the inspector asked her to stop using the notebook and start using the new textbook in her classroom. Noha refused to give up this aspect of her teaching practices even if this means that she starts

to get a relatively low evaluation marks from the inspector. Noha complained that forcing teachers to mainly rely on one specific textbook in their teaching undermines the teacher's professional judgment regarding appropriate mathematical activities that meet the needs of all students.

In the notebook, Noha develops many problems and exercises to supplement the suggested textbook activities. Noha does not only rely on the activities presented in the textbook in her classroom practices. She noted that she perceives herself as a teacher who is responsible for classroom dynamics; in order to create a positive classroom dynamic, the teacher has the right to alter the textbook activities as a way of addressing issues in the classroom. When I asked her about what issues she is talking about, she indicated that one of the main issues she considers when choosing any activity for her classroom is closing the gap between "low achieving students" and "high achieving students". She mostly picks activities that allow the low achieving students to participate and engage in the activity.

Noah rejected the old textbooks because she found them old and outdated. She argued that the old textbooks did not consider the learner or provide a rich learning opportunity. Noha also rejected the new textbooks because she finds them loaded with large masses of data that student cannot comprehend. According to Noha, students usually find it challenging to understand the relevance of so much data to their personal lives. She also finds the reading level of the new textbook too difficult.

Noha described the new textbooks as "balloons" looking full from the outside, but actually empty on the inside. She indicated that the new mathematics textbooks came with a new "advertised" perspective on mathematics teaching. It promotes mathematics instruction that allows deep conceptual understanding to take place in the classroom and provides opportunities for students to answer their own questions by investigating meaningful real-world tasks that employ different mathematical concepts into one problem. This advertisement in ,Noha's opinion, is not successful since they do not explain how a teacher can create this environment in her classroom. She added, "In these textbooks, all they did is load them with so many activities and examples that could distract student and the teacher from focusing on leaning the main concept". The use of metaphors that negatively describes the new textbooks indicates that Noha's

perception of the new mathematics textbook is mainly negative; therefore, the new textbooks do not have an effective impact on her teaching practices.

The reform

Over several years, the school inspectors recognized Noha as an excellent teacher of mathematics because she represented the culturally accepted values of effective mathematics instruction. However, after the reform movement started, inspectors no longer appreciated her teaching practices. Noha indicated that when the reform movement started, especially with the introduction of the new textbooks, the school inspector told her she needed to reconsider her role as a mathematics teacher with regard to student learning and choosing mathematical activities. The school inspector also asked Noha to stop using the notebook in her classroom and to mainly use the new textbook for her classroom activities.

Noha complained that the reform curriculum materials, including the new textbooks, new teacher guide and the circulated notes of recommendations that teachers receive regularly from the Ministry of Education, do not prescribe or describe practices for teachers, but rather offer new visions of mathematics teaching practices. She noted, “according to the new vision, I am not supposed to be a source of knowledge, I am supposed to be a facilitator to help students develop their own knowledge. When I asked the inspector how I am supposed to do that, she said you need to let your students read from the textbook and try to learn by themselves. I simply don’t agree with that view”.

Noha explained that her role in the classroom is more valuable than the textbook. Her students consider her the main source of information because she knows them more the textbook does. Her role as a teacher involves understanding what her students know and what they need to know, and providing the environment for them to learn.

Noha argued that students in the mathematics classroom usually find it hard to understand what it is they need to know and why they need to know it. She talked about common questions from students in mathematics classroom such as “Why do I need to learn this?” or “When will I ever need to use this in my life?” Noha answers her students honestly - “you need it in the test”. Noha implied that her teaching practices work in the short term. She admitted that if a student can “do” a problem, it does not necessarily

mean that the student “understands” the mathematics in the problem. However, for her, it is a part of the ugly truth about mathematics teaching and learning.

Noha described her teaching practices as being realistic. She stated, “You may consider my teaching model as being old school, which assumes that students essentially need to learn mathematics facts, however, I consider my teaching model as being realistic, providing students with what they need”. She is convinced that her students are used to learning this way. According to her, her “realistic” way of teaching represents the culture of mathematics teaching and learning in Saudi Arabia and changing this culture in high school would create confusion and chaos, which students cannot handle. If the chaos happens, no one would be happy, including students, parents, school principals and even the school inspector.

When describing what she means by realistic, she explained that her teaching practices are the result of her own adaptation to existing circumstances; those existing circumstances have not changed enough in a way that allow teachers to make effective changes. She claimed that teachers face so many obstacles if they decide to change their practices. She noted,

> In high school, we don’t have the tools and ability to teach mathematics as a subject of figuring things out or making sense of things. The content is getting harder and more abstract. And we don’t have the tools and resources to teach this way. For example, although many chapters in the textbooks rely on using a graphing calculator, they are simply unavailable at most schools. We don’t have computers in our school. Also, I am not allowed to bring any electronic device that has a camera to my classrooms and let the students use them. I could bring a couple of iPads to use in my class, but I am simply not allowed.

Although the new teacher guide and school inspectors suggest that teachers should implement group work in their classrooms, Noha uses group work in her classroom very rarely. She justified her practice by noting when she puts her students in groups, often one student in the group works out the problem while the others observe or sometimes go off task. In this case, the teacher’s focus shifts from doing mathematics to helping students develop social communication skills. Although these social skills are important, they should not be the focus in a mathematics classroom. She finds group work time and energy consuming with no direct real results to students’ mathematics learning.

Students' achievement

According to Noha, there is a strong connection between successful and effective teaching and student achievements. Noha indicated that teaching must lead to improvement in students' academic performance; students' achievement is a measure of successful and effective teaching. She stated, "Student achievement is always the result of successful mathematics teaching. A teacher can never be considered successful if her students' achievement is low". She argues that to be successful, teachers should make a tangible impact on student achievement.

Student achievement appears to be the main goal of Noha's job as a mathematics teacher. In her practices, she relies mainly on two sources to evaluate student achievement, written tests and homework. She indicated that written tests are the main official evaluation method used in high school. She added, "In reality, students' learning experience in high school is mainly evaluated based on how well they do on tests. Students encounter different types of written tests (such as) quizzes, final, midterm, and standardized tests". She argued that educators have yet to come up with other reliable and effective ways to evaluate and assess student achievement. Besides the midterm and final exams, Noha gives her students a quiz at the end of every chapter. The end of the chapter quiz helps her assess the effectiveness of her instruction, as well as students' understanding of the concepts taught. Noha also explained that she does not support weekly testing because it destroys students' interest and motivation to study for tests.

Noha pointed out that she relies on homework as a daily formative assessment tool in class in order to measure the level of student knowledge and understanding of the previous lesson. Homework is an instrument that demonstrates to Noha how well her students understand the material. If most of the students are unable to do the homework or experience great difficulty completing it, she can modify and adjust classroom instruction to decrease the amount of confusion or struggle.

In Noha's classroom, almost every lesson begins with whole-class correction of homework on the board. To make sure that her students do the homework themselves instead of copying it from someone else, she randomly picks a different student to write her answers on the board; students are not aware of who will be providing their answers. She said, "Every student in my classroom expects that I ask her to write the homework

answers on the board. When they can't do the homework, they tell me before the class starts, so they don't copy it from someone else".

Noha's expressed that she not only worries about her students' ability to succeed on school tests, she voiced unease regarding her students' abilities on standardized tests. She explained that while teachers are not required to provide any preparation help to students to succeed on standardized tests, she understands that her students want to achieve high scores on all types of exams. Therefore, she considers it part of her job to help prepare her students, especially for the General Aptitude Test (GAT). She signified that students in high school, especially in grade 11, feel a tremendous amount of pressure to perform well on the examinations and the weeks before the test are a very stressful time for her students and for her.

In order to help her students achieve better marks in the GAT, Noha conducts workshops for students at her school to help them prepare. Besides the workshops, during her regular lessons Noha refers to the GAT when she is teaching a topic normally found on the test, such as probability and statistics. She provides her students with tips on how to correctly answer test questions. Noha mentioned that after she started organizing the workshops, she realized the importance of going back to the mathematics basics during her regular lessons. Noha noted that concepts such as fractions, decimals, and area are topics that students need to review regularly.

Relationship with students

In Noha's teaching practices, it is crucial to connect with her students in a positive way. She said, "A positive teacher-student relationship can make my classes run easily. Without it, nothing will. Students need to feel that their teacher cares about them". She explained that a teacher and students who use good and open communications skills are able to easily establish a positive relationship in the classroom. Noha also makes sure to demonstrate respect towards her students by using a kind voice and appropriate language when speaking with students. According to her, teachers who treat their students with respect will have active learners in their classroom. Offensive and disrespectful teachers lack the ability to control the classroom and students' behaviour.

It is very important to Noha that her students know she cares about them. She explained some of the strategies she uses, such as stressing the things that she and her students have in common. She noted, “I always explain to my students that I have the same goals as they have and I make it clear to them that my job is to help them achieve their goals”. She also communicates positive expectations letting her students know that she is proud of them. Noha likes to show her kind side to her students by using terms of endearment when calling her students in classroom. Terms like sweetie, honey, and dear are used a lot by Noha.

It is also important to Noha to show interest in her students by getting to know them a little bit more and allowing them to discuss their concerns with her. She listens to her students and encourages them to share ideas and concerns; even those not related to mathematics. Noha explained that many of her students share their ideas, concerns and dreams about what they are planning after high school. She recounted, “Some students are unsure of what they want to do with their lives after graduating high school. They come to me because they want someone they trust to listen to them”. Noha’s students feel a strong personal connection to her; they talk with her frequently. She offers them constructive guidance and sometimes she just praises them to increase their confidence and self-esteem.

Noha is a very enthusiastic and energetic teacher. She makes sure to bring her kind personality and excitement to the classroom. She explained that with the teachers’ energy and enthusiasm, lessons become more dynamic and students pay more attention. “When I come to the classroom showing my excitement, students respond and listen to what I say, even though the mathematical concept might be challenging”.

Noha uses an incentive system using points to motivate her students to participate and engage in class. She uses a notebook to keep track of the points. When she gives her students a task to do, she rewards every student who finishes the task one point. When a student collects five points, the student gets ¼ of a mark. According to Noha, the technique helps keep her students excited and energetic during the lesson.

Voluntary work

Noha is a very active teacher. She is willing to do any work that could benefit students. She has no problem volunteering to do extra work even if not related to

mathematics teaching. She explained, “I continue to volunteer in my school whenever there's a chance. Only few teachers in my school do volunteer work. All I want is to help create a more positive and productive school environment for all students”. Noha revealed that most teachers at her school never give their time beyond the school day in any way and never consider doing any extra-curricular activities.

While I was walking with her to the teachers’ room, Noha showed me some posters on the walls that she designed and printed as a part of her volunteer activities. The posters were about topics not related to mathematics, such about the benefits of eating healthy food and the importance of time management skills.

Noha is also one of the few teachers who agree to go on field trips with students. Field trips are very rare in girls’ high schools because of the cultural restrictions in Saudi Arabia. Noha feels obligated to support taking her students on field trips because “students need to do something different once in a while”. One of the common places Noha takes her students is to university campuses. Noha finds these field trips helpful, because they introduce students to university life and encourage students to explore their interests in a university environment.

A major part of Noha’s volunteer work is designing and conducting free workshops for students at her school. Noha is one of few teachers in the district who conducts such workshops. The workshops focus on offering students skills and knowledge to help them score better on the GAT test. The workshops are open to all grade 11 and 12 students attending her school, not only the students in her classes. She offers this workshop twice a year; however, to minimize the number to students attending every workshop, students can attend only one workshop a year. Noha is not happy that some private institutes are trying to take advantage of the importance of this test and offer paid courses to teach students skills they learn at school.

Noha indicated that during the workshops, she helps students understand the design of the GAT exam by highlighting how it is different from tests they usually take in school. She explains the mathematics facts, rules and formulas that students must know. She has created a list of the most important formulas that students should memorize. She also collects all the GAT questions she can find and organizes them in a folder with their answers. She updates this folder yearly by including more questions.

Noha gives each participating student a copy of the folder and uses it frequently during the workshop. She also does mini timed practice tests to train her students for the test environment. She said, “I keep an eye on the GAT test every semester. I know there are some questions that the test makers include very frequently, my students come to me all the time to tell me that many problems we did at the workshop were on the test”.

Noha’s finds that the nature of the GAT test requires her to train students to quickly and efficiently obtain answers; therefore, her instructions during the workshops tend to be procedure-oriented. She mentioned that she mostly trains her students to perform mathematical procedures that enable them to find answers to problems according to set rules. She explains that procedure-oriented instruction helps students to do well on tests, especially on the purely procedural parts of the test. According to Noha, the workshops help students feel more confident about doing the real test by becoming more familiar with its format.

5.2.3. Classroom episode

Textbook, reform, mathematics and students’ achievement

This episode is part of one of Noha’s lessons that I observed. After checking the homework on the board, Noha announced the title of the lesson. In this episode, Noha introduced the concept of conditional probability.

Noha: Our lesson today is conditional probability. We have learned about probability before. Who can tell me what is the basic probability formula?

Student1: It is the part divided by the whole?

Noha: Yes, you are right. The part divided by the whole or in other words, it is the ratio of a possible outcome in an event to the total number of possible outcomes.

(Noha now writes on the board)

Probability of an Event = number of outcomes in event/ the total number of possible outcomes = Part/whole.

Noha: Today, we are to going to learn conditional probability. We are going to learn the definition of conditional probability and its formula.

Noha: Now, what is the definition of conditional probability? It is when I have an event that actually already happened and I want to see the probability of another event in relation to the previously happened event.

Therefore, (now Noha is writing on the board while speaking) conditional probability is the probability of an event occurring given that another event has already occurred.

Noha: Now, when you have a question where you are asked to find a probability of an event and you are given information about another event related to it, it means that this question is a conditional probability question.

Noha: Now, how do we find the formula of the conditional probability? Let's demonstrate every word from the definition I have on the board and translate it into mathematical symbols. Probability starts with the letter P. "An event occurring" let's call it B. "Under the condition" how can we demonstrate this part mathematically? As a line, /. Another event has already occurred; let's call it A.

(Noha writes on the board P(B/A).)

Noha: How do we read this?

(Noha is pointing at P (B/A) on the board.)

Noha: The probability that B occurs, if it is known that A has already occurred. We find the conditional probability by dividing the probability of the intersection of B and A by the probability of the event that has already occurred, which is A.

(Noha writes the final formula on the board).

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Noha: conditional probability problems are common in General Aptitude Test (GAT). You need to memorize this formula and remember always when you get a

question asking about the probability of an event in relation to an event that has already occurred to use this formula. Conditional probability problems can usually be recognized by words like “given that” and “if you know that”.

Noha: Anyone have any questions?

A few students: No.

Noha: Now, write the formula in your notebook.

Noha gives students a few seconds to right down the formula.

In the previous episode, Noha is engaging with mathematics, students’ achievement, reform, and textbook. Her engagement with these figured worlds inform and adjust the perceptions she makes and the way she engages in this episode. We see that her engagement with the figured worlds mathematics, students’ achievement and textbook support one another and restrict her engagement with the reform.

We see that Noha is the sole source of knowledge, and students in her classroom receive knowledge from the teacher. Most of the time during this episode Noha is talking and the students are listening. Even when she asks questions during the introduction of the mathematical concept, she does not give the students time or opportunity to answer or even think about the questions she asks. She simply asks questions and immediately answers them. The students are simply there to learn through lectures and direct instruction. Students are to accept the knowledge she provides them without any questioning.

Noha does not allow the textbook to share with her the role of being a source of knowledge in the classroom. She never uses or refers to the textbook during the lesson. She does not encourage her students to use the textbook to find information. She makes sure the students copy down the formula of conditional probability in their notebooks, even though the formula is in the textbook in a very clear way.

In this episode, we can see that Noha is experiencing resistance to change; she is refusing to change her practices to follow some of the reform recommendations and allowing the students to take a more active part in their learning experience. She is resisting giving up some of her authority to the textbook.

It is clear from this episode that in Noha's practices, memorizing plays an important role in learning mathematics. According to this episode, the meaning of understanding the concept of conditional probability in Noha's practices is about memorizing the formula. She relies on memorization as a key learning strategy and one of the very basic techniques in her teaching practices. She highlights the importance of remembering the formula and provides students with tips to help them memorize it. Noha actively discourages her students from thinking about mathematical relationships by telling them the rules and formulas that they should remember. For Noha, doing mathematics is about remembering the correct formula and applying it to find the correct answer in order to do well on tests.

The last part of the previous classroom episode demonstrates that Noha considers memorizing to be an appropriate and valuable strategy to help students do well on standardized tests. In this part of the lesson, Noha integrates test strategies into her lesson. She provides classroom instruction that incorporates tips, set range of knowledge, and skills that she considers valuable for helping students achieve higher scores on tests. Noha feels enormous pressure to raise her students' scores on high-stakes tests. Therefore, teaching for the test has become part of her practices in order to increase her students' success on school tests and on standardized tests.

5.2.4. Noha the teacher

Using PoP, I attempted to understand Noha's teaching practices through her simultaneous engagement in different figured worlds. The figured worlds that are significant to Noha's practices are: mathematics, the textbook vs the notebook, the reform, students' achievement, relationship with students, and voluntary work. According to PoP, Noha encounters these figured worlds through her practices every day and during the course of classroom activities. These encounters motivate her to re-construct the meanings of her role as a mathematics teacher.

Noha is a very active teacher and has a strong commitment towards her teaching practices. It is very important to Noha to build a strong relationship with her students. She demonstrates interest in extending her relationships beyond the classroom by voluntarily participating in extra-curricular activities with her students. An important part of her commitment towards her teaching practices is her students' achievements. She is

experiencing huge stress to help her students achieve well in school tests and raise their scores on standardized tests.

Although Noha has a strong sense of duty and obligation toward her students and cares about them, her negative perception of reform and the new mathematics textbooks is a result of many factors. These factors include the shortage of resources and professional support she received during the process of implementing new curriculum, the poor communication between Noha and other teachers at her school about mathematics teaching and the professionally weak relationship between Noha and the inspector.

Noha does not have open and positive communication about mathematics teaching with other teachers at her school or with the school inspector. Noha's relationship with other teachers is mainly friendly. In teachers' room, she rarely discusses issues related to mathematics teaching with other teachers. They talk about other issues, but not about mathematics teaching. She commented, "In our school, most teachers are independent. I just don't talk with other teachers about what I do".

Generally, Noha rejects reform ideas about mathematics teaching. The textbook is one of the main reform changes with which Noha has difficulties. The new textbook does not seem to have an impact on her teaching practices. She has the sense that the new textbook is replacing her role as the center of knowledge and information in the classroom. This feeling prevents her from engaging positively with the textbook and allowing the textbook to change some aspects of her teaching practices. The new textbooks did not encourage her to change her practice to focus more on helping students to build a strong conceptual understanding as intended by the textbooks developers. On the other hand, some reform changes such as standardized tests reinforce her practices about using a procedure-oriented approach.

Although Noha seems to reject reform ideas about mathematics teaching, some parts of the interview indicate that she seems to admit that there is another way to teach mathematics. This way could work if there were more resources available for teachers. She is also afraid of the chaos that could result from any change she could make because she feels there is a lack of support.

5.3. Maram's Case

Maram is a high school teacher with eleven years of experience teaching high school. She is currently working at a high school in Al Khobar city located in the Eastern Province of the Kingdom of Saudi Arabia. She graduated with a Bachelor of Science degree with a specialization in mathematics. She does not have a degree in education; she has never taken any university education courses. After she graduated from university, she started teaching in a private school. She worked there for three years and she taught grades 10 to 12. After that, she received an offer to teach in a public high school. She has been teaching in the public high school for eight years.

Maram was not among the teachers who used the new textbooks in the first year of implementation since she was teaching grade 11 at that time. She has been teaching from the new textbooks for two years. When I met Maram, she was teaching 18 lessons per week to students in grade 11.

Maram works at a traditional public school, which follows the two-semester system. The school building, located in a middle to lower class community, is old, but in a good shape. It accepts all local students with no entry requirements and has 592 students. The mathematics department has 5 teachers.

5.3.1. Maram's classroom

Maram's classrooms are relatively crowded for two reasons. First, there are around 42-48 students in her classrooms, which is more than the average number of students in a regular high school classroom in the Al Khobar district. Second, the classrooms in Maram's school are small. Students' desks are in rows of three facing the front of the classroom. She occasionally assigns her students homework. Once every week or two, she gives her students a quiz, or as she calls it, a "timed test". She mostly relies on lecturing to introduce the new concepts, but tries to use additional teaching techniques to encourage deeper and meaningful discussions in her classroom. She follows the textbook sometimes, but not always. For example, she might ask students to read a lesson introduction from the textbook for a particular chapter, but then ignores the textbook introduction other times.

5.3.2. Significant practices or figured worlds in Maram's case

After eleven years of teaching, the data generated about Maram suggests that there are five significant practices or figured worlds to Maram's sense of her practices as a mathematics teacher. These figured worlds are mathematics, the textbook, the reform, students' achievement, and social network engagement.

Mathematics

Maram is very passionate about mathematics. She expressed her love for mathematics as a subject by saying, "As far back as I can remember, I have always loved mathematics. It is just something I enjoy. When I was a student at school, mathematics classes made me happy and I was relaxed in mathematics classes because I was doing something that makes sense to me".

Although Maram loves mathematics as a subject, she does not consider that main objective of her job as making her students love mathematics; she sees her job as making her students do mathematics. For Maram, teaching mathematics is about helping students built a strong understanding of mathematical concepts and practice mathematical procedures. According to her, the best way to learn mathematics is to encourage students to talk about mathematical concepts, explain their thinking process and do as many exercises as they can. She stated, "Practicing mathematics problems is a major part of learning mathematics. You can't learn to do mathematics without actually doing it a lot".

Maram noted that it is very common in high school to see that most students do not like mathematics. "I think students mostly don't like mathematics because they don't like that they have to struggle to be able to do mathematics. I always tell my students not to expect mathematics to be easy. It is actually hard". Maram explained that struggling is normal while doing mathematics. Students usually expect to know how to solve the problem as soon as they read it. She encourages her students to not get frustrated when they do not know how to solve the problem and look for ways to help themselves by drawing a picture, looking for the important words in the problem, and trying to remember a similar problem.

In Maram's classroom, the central goal of learning mathematics is for students to acquire the ability to apply their understanding of mathematical concepts to successfully solve problems. Although Maram evaluates her students based on content knowledge and not the effort they make, she stated that the more effort students put into the class work, the more likely they are to understand the mathematical content.

According to Maram, most of her students do not expect mathematics to be meaningful and make sense. Students are mostly content working with mathematical symbols and doing routine problems without ever grasping a real understanding of the problem. Most students perceive their roles mainly as acquirers who memorize formulas and rules and apply them repeatedly to similar problems. According to Maram, students' view of mathematics learning is a direct result of traditional teaching of mathematics, which is still the most common way of teaching mathematics, especially in high school. Although Maram was comfortable learning mathematics in a traditional environment when she was a student, where her teachers wrote examples on the board and she copied them down, as a teacher she tries to do something different. She does not consider her teaching style far from the traditional style, but tries to find ways to move away from the traditional approach.

Maram is always trying to find new ways to help her students reach a deep understanding of the concepts behind the problems they are working on in class. She insists that students can effectively articulate the mathematics that they are working on. She usually asks students to explain what strategies they use to get the answer. Engaging students in a conversation about the mathematics they are working on is a tool she uses to assess the students understanding. If students cannot correctly articulate verbally what they are doing, they do not really understand the mathematics.

For Maram, discussion is important in learning mathematics because it helps students to interpret and grasp new ideas differently. One of the exercises Maram uses with her students to engage in a rich mathematical conversation is to have them imagine there is a person in the class who does not know the meaning of the concept they are working with. Students have to explain the concept to this person in an easy and clear way. The purpose of this exercise is to encourage students to talk about mathematics concepts. She stated, "I want them to use a language they can understand and not stick with the textbook definition. I tell them I don't care about the grammar or what language

they are using. I just want them to come up with their own definition". According to her, this exercise helps her students to engage in the discussion and practice their use of the mathematical language. It also gives the students the confidence to talk about mathematics concepts.

Students in Maram's classroom use calculators all the time, regardless of the fact that the school inspector does not approve of it. Most students in her classroom have their calculator ready on their desks. She considers calculators as a useful tool to promote the higher-order thinking and reasoning needed for problem solving. She allows her students to use calculators to simplify tasks, so students spend less time on repetitive and dull calculations and more time on understanding and explaining their thinking. She finds calculators help reduce students' frustration and increase their confidence about their mathematics abilities. She noted, "In high school, many students have difficulties doing basic calculations, but I don't want that to hinder their ability to participate actively and learn. When they use calculators, they expect me to ask them to explain how they got the answer". Therefore, Maram uses calculators as a tool to improve students' mathematical communication in the classroom.

Another important issue regarding mathematics teaching that Maram brought up for discussion is that students in her classrooms come with different mathematical backgrounds and have very different needs; therefore, her students in her classroom, regardless of their abilities, deserve to have their learning needs met. She talked about having two different types of students: "strong" students and "weak" students. Her perception of these two types appears to be in terms of the students' abilities to acquire knowledge and their willingness to make an effort to acquire knowledge. According to Maram, strong students work to gain the knowledge and understand the material; they do not expect the teacher to do the work for them. These students usually have a strong mathematical foundation. Weak students, she finds, do not try hard enough to gain knowledge and they usually come to class with a poor mathematical background. Maram clarified that when she plans her lessons, she does not target one type of student, "strong" or "weak"; rather, she tries to use the "teaching to the middle" approach. She said, "Mainly, the approach I use is intended to offer instructional environments that support all my students on a daily basis for the sake of improving their foundation in mathematics". She admits that her teaching practice may not support the

strong students to reach their full potential, but it could help the weak students to improve.

The reform

Current reform ideas in education remind Maram of the teacher she wanted to be when she started her teaching career. When she began teaching, she was determined to be a “non-traditional mathematics teacher”, meaning a teacher who does not rely on the traditional lecture format of teaching. However, Maram considers her approach to mathematics teaching to be more on the traditional side. “I don’t consider myself far from traditional teaching; most of the time I am traditional and present mathematics as facts and procedures, but I am willing to learn to do new things”. She implied that she feels she is under pressure to be a good teacher. When I asked her to explain to me her view of a good teacher, she replied that a good teacher is the one who has a positive impact on student understanding, quality of learning, and student achievement.

Generally, Maram supports reform and articulated clearly that changing how teachers teach mathematics in schools is a necessary step. She supports reform recommendations regarding concrete exploration and meaningful representation of mathematical concepts. However, she finds some reform ideas too challenging and hard to translate into practices. She explained that making a shift in the classroom from focusing on procedural to conceptual learning and incorporating problem solving strategies to be extremely challenging for teachers. She finds that reform recommendations and new curriculum materials provide teachers with visions and do not offer explanations about how to transfer these visions into practices.

According to Maram, in high school, mathematics is presented in a very abstract and formal way. It is very difficult, and sometimes impossible, for teachers to create a learning environment for students where they can experience mathematics in a meaningful way that is related to real world. She mentioned that reform recommendations emphasize the importance of using manipulatives, visual representations, and hands on activities to help students develop conceptual understanding of mathematical ideas; however, she finds this recommendation difficult to apply in high school mathematics. “Most topics are very abstract; I don’t know how to teach grade 11 curriculum using manipulatives. I don’t know if I can find appropriate manipulatives materials; materials that can add value to my lessons. And even if I find

the appropriate manipulatives, I am afraid my students will not be able to see the mathematics concepts that the manipulatives are demonstrating". Although Maram did not recall using manipulatives in any of her classes lately, she is questioning the effectiveness of using manipulatives to build abstract understanding of mathematical concepts by first exploring relationships with physical objects. It seems that she does not know how to use manipulatives and therefore, hesitates to use them in her lessons.

Maram also finds reform recommendations to be misleading, sometimes providing teachers with mixed messages about best and effective practices. She explained that reform focuses on ensuring that high school students are equipped for university. According to Maram, most parents, high school students, and teachers interrupt "preparing students for university" to help students to achieve higher grades. She stated,

> In recent years, there is too much emphasis on preparing students for university. This emphasis forces teachers to adopt what I call teaching for entering university practice or teaching for the tests; […]it is the practice that focus on doing routine problems and never having a proper understanding of the principles behind it. Perhaps it is fair to say that with this practice students can be successful in the short-term, doing well in tests, but in the long run, we are not actually helping them.

Maram's opinion is that teachers should not be concerned with students getting high grades as much as helping students to achieve deep and real understanding.

In general, Maram struggles to adopt reform-oriented teaching. She supports some of the ideas of reform, but is having a hard time applying them. She also seems unsure that changing her teaching approach would help students learn. She noted, "My worst fear is to waste students' time trying to do something new and it doesn't work. In high school, there is no time to fail as a teacher. If you can't help your students learn and provide them the environment to learn, there is no time to fix it later".

The textbook

Maram expressed that before the introduction of the new textbooks, she was very excited about using different textbooks in her teaching. She finds that the new textbooks are generally better than the old ones at providing more opportunities for student engagement and participation. She likes that the new textbooks offer different levels of exercises that aim to develop mathematical problem solving and

communication skills. She understands that the purpose of introducing the new textbooks is to encourage teachers to develop learning environments where their students have more time and room to reflect, discuss and investigate on their own. While she understands the vision of the new textbooks about what school mathematics should be, she still admits that mathematics in her classroom and in all her colleagues' classrooms "Is still generally taught using lecturing, whole class teaching, and regular testing".

Maram explained the reason why the new textbooks did not have the expected effect in her teaching practices could be due to the lack of support and preparation she received during their implantations. She expressed the complaint explicitly that the Ministry of Education did not put enough consideration into teachers' preparation of the use of the new textbooks. She implied that teachers were anticipating great support and professional development related to the new textbooks based on the Ministry of Education's manner of promoting the books.

Using open-ended problems in her teaching is a new practice that Maram started to use after the implementation of the new textbooks. However, Maram usually makes changes to the challenging open–ended problems presented in the textbook when she presents them to her students. Most of the time, she re-writes the problems or activities, so they contain more structure, direction and clues to help students engage more with the problem. This is her way to personalize the textbook material to fit more with her students' needs and abilities. According to Maram, some problems in the new textbook require more analysis and discussion on the part of the students than students are accustomed to, especially with open–ended problems. Her students feel "nervous and not comfortable when the problem is open and they don't seem quite sure where it is going; they even give up and stop trying". To avoid losing her students' interest to work on challenging problems, she provides them with tips and directions. She explained that providing direction is more effective than having students explore with no direction; "It saves classroom time, maintains students' attention and helps to keep their confidence level about their mathematical abilities".

Maram also talked about one aspect of the new textbooks that was not available in the old textbooks, making mathematics relevant to students' lives. She explained that although most lessons in the new textbooks have a section that relate the mathematical

concept presented in the lesson to everyday life, Maram thinks that the new textbooks are not successful in making mathematics relevant to students lives. As a teacher, she finds making mathematics relevant to students' lives in high school is a very difficult task to achieve and, according to her, this is a fundamental reason for weak mathematics performance and motivation among high school students.

One of the techniques that Maram is trying to effectively implement in her classroom is group work. Using group work is one of the new textbooks recommendations. She usually asks students to do group work with their neighbors. Therefore, groups usually contain three students. Maram explained, "Three is the perfect number for group work, so all students in every group can participate. When the group is bigger a few students tend to control the situation". Maram does not change members of the groups very often because at her school, classrooms have little space to work with and it is challenging to arrange students differently. In addition, she finds when students know each other, they work together better. They get used to each other and get comfortable working together and the result is it makes working in groups easier. It is important to her that students engage with each other when doing group work, "I always stress to them when they do group work that they need to share thoughts and truly work together".

Maram mainly uses group work to create conversation among students. In their groups, she encourages students to explain their thoughts and discuss their understanding of the mathematics they are working on. She pointed out that students learn about their mistakes and correct erroneous understandings or incomplete procedures when they discuss mathematics with their classmates.

Maram also uses group work to learn for herself about students' understanding. When her students are doing group work, she makes sure to move around and "listen to what they are doing and discussing". She explained, "I use it as a way for me to know what my students are thinking, but sometimes when I discover that most of them have stopped working on the mathematics and have begun socializing, I end the group work and I go for a whole class discussion".

From her experience teaching with the old and new textbooks, Maram explained that she has learned that any textbook is merely as good as the teacher who uses it.

While during her first years of teaching, she would follow the textbook entirely without even thinking about making any changes, now she understands that “The textbook is just a tool”. She elaborated “[the textbook is] possibly a very important tool, but still a tool that I can use the way I find appropriate; …there is no complete and perfect textbook; not the old one and not the new one”. When planning her lessons, she considers the textbook as a guide, not a mandate for instruction; sometimes she follows the structure of the lesson presented in the textbook and sometimes she creates her own instruction.

Students’ achievement

Maram explained that the main goal of high school mathematics teachers is to create a supportive environment where students can learn the necessary concepts for academic achievement. However, Maram finds that for most mathematics teachers this view of academic achievement does not go beyond the classroom and the written tests given to students. She understands students’ achievement in a way that is different from how other teachers, and students, understand. She indicated that most teachers and students limit student achievement to their grades. “Teachers and students are not concerned with learning as much as they are concerned with achieving higher scores”. In her opinion, the grading system used in schools, which is mainly based on written tests, does not reflect or communicate the level of actual academic progress or achievement that a student has developed during her time in school.

Maram also stated that teachers usually evaluate their teaching practices based on their students’ grades. This practice leads teachers to focus on increasing their students’ grades by teaching for the test. Maram blamed reform recommendations for not trying to adjust teachers’ view about students’ grades and academic achievement. She also claimed that some reform changes, such as introducing standardized tests, have emphasized the culture of testing in schools. She explained that introducing standardized testing in the Saudi education system has created a milieu that prevents teachers and students from valuing the importance of deep understanding of mathematics. Maram does not provide any type of training for her students to prepare for taking standardized tests such as the GAT test because she does not support the educational purpose of the tests. She also stressed the importance of adjusting the common view of students’ academic achievement in order to make a real change in how teachers teach in schools. “New reform recommendations did not come with new ways

to assess students' achievement rather than testing. We can't change how we teach our students if we don't change how we evaluate them. It is all related".

Maram wishes to be able to eliminate some of the mandatory written tests students take in her classroom, such as the mid-term and final. She stated, "If it were up to me I would not do the mid-term, but as a teacher I have no control of that". She also indicated that she does like the idea that students study for these tests. These tests put teachers and students under stress. She finds that teachers are under increased pressure and valuable time is lost from actual learning in order to train students to take these tests.

Maram does not disregard the idea of using written tests to assess her students. She uses timed written tests regularly. She gives the students exactly 10 minutes to do a problem related to what they are studying in class. She does a timed test once every week or two. She does not tell the students to prepare for the test beforehand. "Sometimes I don't do the test before the lesson starts. When I have time before the end of the lesson and we finished all the activities planned for the lesson, I give students a test". Her timed tests are usually simple, but thoughtful. She finds that, "A one-question test can be very informative". If most students get the answer wrong, the next class she would spend some time discussing the relevant concept again.

Specifying how much time students have to complete the test makes it easier for her and the students to understand the mechanism of the test. It also helps students to prepare for testing generally. She also explains to her students that the task she usually asks them to do in these tests requires 10 minutes or less to complete. If students need more time to do it, it means they need to work more and practice more to achieve full understanding. She finds that short, timed tests to be the most effective tool to evaluate the amount of learning taking place in her classroom. According to Maram, the timed test is a quick way of gathering information on how well her students are meeting their learning objectives and engaging with presented material. She finds that the timed test does not only motivate students to keep current with the material they are learning in class, but also rewards them for being active learners.

When I asked Maram about her thoughts on which teaching practices could improve students' achievement in mathematics, she indicated it is difficult to name

certain practices. She explained the best teaching practices are the practices that meet the needs and strengths of students. She expanded by saying advocates of traditional teaching claim that focusing on the teaching of basic mathematics knowledge and skills will lead to improvements in students' academic achievement; however, advocates of reform-oriented teaching claim that placing problem solving at the center of the teaching instructions can improve students' academic achievement in mathematics. She finally claimed, "Perhaps the best practice is a mix of both".

Social network engagement

Maram is very active on social media, especially on Twitter. She is a social media enthusiast with a passion for sharing ideas about mathematics and mathematics teaching. She uses Twitter to interact with others who share the same interests. She described her social media interaction, especially on Twitter, as her best hobby. She likes to post mathematics problems and get people's responses. She discusses their ideas about the problems she posts, corrects their answers and finally provides the right answer. According to Maram, this interaction helps her to learn more about the thinking process when engaging with mathematics. She enjoys responding to the questions people ask and the comments they make.

In her Twitter bio, she identifies herself as a person who loves mathematics and does not mention that she is a mathematics teacher. She stated, "I didn't want to describe myself in my Twitter bio as a mathematics teacher because my Twitter reflects me as a person not merely as a teacher; of course being a teacher is part of who I am, but it is not all of who I am". She indicated that she mainly uses Twitter to "Share the love of mathematics with others" and do "Fun mathematics stuff without being restricted to an official school curriculum". She gets most of the problems she posts from different recourses though mostly online. It is very rare that she uses some of these problems with her students in her classroom. The reason, she explained, is there is not enough time to cover all the curriculum material and do more extra activities. Although some of her students follow her on Twitter, they rarely engage with her online or try to solve any of the problems she posts.

While Maram stated that she mainly uses social media to share her love of mathematics, she also uses social media to connect with other mathematics teachers. Maram follows several mathematics teachers; teachers, both teachers of mathematics

and other subjects, from different schools follow her. Twitter allows her to keep in touch with some mathematics teachers she met outside of her school.

In her school, Maram does not interact with other mathematics teachers very often. She stated, “When the new textbooks were introduced, we used to talk; recently, we don’t talk that often, maybe because I am the only teacher who teaches grade 11. Other teachers teach grades 10 and 12; I don’t have a person that I talk with regularly about what I do”. She also stated that the school inspector usually visits her once a year and mostly gives her positive comments. Therefore, her social media interaction makes her feel that she is not working in isolation. She described her connections in Twitter networks as sources of learning new things related to mathematics teaching. Sometimes she uses Twitter as a resource to compare classroom teaching techniques and learning styles and finds new ideas for lesson plans. Her social media interaction also helps to keep her in touch about issues regarding education in Saudi Arabia. “On Twitter, people usually discuss new regulations and policies coming from the Ministry of Education or the school district”.

5.3.3. Classroom episode

Mathematics, the reform, the textbook and students’ achievement

Students in Maram’s classroom have been learning about arithmetic and geometric sequences. In this episode, Maram started by writing the following question on the board:

Explain in writing, using your own words, how you can determine whether a given sequence is arithmetic or geometric or neither?

Maram: In your groups, answer the question on the board.

Students started to read and discuss the question. Maram waited for around three minutes before she started to circulate among the groups. She was prompting and directing students’ work, asking the same questions and making the same comments to each group such as “start by identifying the mathematical concepts in the question. What do you know about them?”

After around 10 minutes, Maram spoke to the whole class.

Maram: Can I have your attention for a minute? When I say in the question, explain in writing using your own words, I expect to see more than formulas. I want to see words.

A student: Do we need to provide examples of both?

Maram: Do what you think is enough to answer the question on the board.

Maram was moving around watching what students were doing. She commented on their work mainly by encouraging them to write more words, making comments like, “write your thinking process, what are the things you see and test in a sequence to determine its type?”

After about 15 minutes, she said, Remember, I need the answer written down in a clear and organized way. You have 5 more minutes and then it will be time to share your writing with the class.

In this episode, Maram engaged with mathematics, reform and the textbook during her classroom interaction. The question she used in this activity is from the higher order thinking problems in the textbook. Maram was relying on group work to engage students in a writing activity. Maram started by writing the activity on the board and asked the students to start working on it without giving a detailed explanation. Students seemed accustomed to working in groups. They immediately started engaging with the activity in their groups. Maram indicated that students are more willing to engage in a group activity when the activity is something they are not used to doing in mathematics classroom, such as writing.

Maram explained that the reason she provided significant direction to students was that asking students to write is not “A typical classroom activity” in her class. She always expects to see her students struggle when she introduces an activity that is not part of her regular practices. At the beginning, students were not sure how to approach the question. The comments she was giving the groups, such as “I expect to see more than formulas” and “what is the things you see and test in a sequence to determine its type,” encouraged them to use more words to answer the question. “I wanted them to give me something more than the formulas that are already in the textbook”.

The reason Maram used this activity in this episode was not to make her students document information, but as a way to deepen students' conceptual understanding. Although Maram finds writing helps to deepen students' conceptual understanding, she does not rely on writing activities very often for many reasons. According to her, writing activities are very time consuming because students are not familiar with doing writing in mathematics. She explained, "I know they hate to write things down in mathematics class. I can't rush an activity like this". She also mentioned that writing is an activity that is not always appropriate with the topics she teaches in high school. She stated, "I can't always incorporate writing activities in my lesson plans, but I thought this question is very doable and they [students] can write down good explanations".

Although in this episode, Maram followed the reform recommendation for teaching mathematics by choosing a writing activity from the textbook, most of the time she provided students with tips and directions to guide their thinking which indicated that her engagement with mathematics restricted her engagement with reform.

5.3.4. Maram the teacher

PoP suggests that Maram regenerates meanings of her participation in classroom practices by drawing from different figured worlds. Through her engagement in the figured worlds of mathematics, the textbook, the reform, students' achievement, and social network engagement, she constructs meaningful practices for the classroom context.

Maram is concerned about her practices and is trying to be a good teacher. She is trying to find ways to improve her students' leaning experience. She indicted that there is a need to change school mathematics. She supports reform ideas in education, but at the same time, is struggling to adopt reform-oriented teaching. Although she is trying to adopt some changes in her practices, such as incorporating group work and unfamiliar activities such as writing, she still relies on traditional methods of teaching mathematics. Her engagement in social media suggests that she is willing to learn new things to improve her practices.

According to Maram, her view of the teachers' role and responsibility about students' achievements does not coincide with most mathematics teachers. Maram finds that for most mathematics teachers the view of academic achievement does not go beyond the classroom or the written tests given to students. Maram's main goal teaching high school mathematics is to create a supportive environment where students can learn the necessary concepts for academic achievement. While she does not disregard the idea of using written tests to assess her students, she would like to see a reduction in their use of written tests for evaluating students.

Maram is approaching the way she is using the new textbook cautiously, looking for ways it fits with her students' needs and abilities. She understands that the purpose of introducing the new textbooks was to encourage teachers to develop learning environments where their students have more time and room to reflect as well as discuss and investigate on their own. She admitted that the new textbooks did not have the expected effect on her teaching practices.

5.4. Huda's case

Huda is a high school teacher with eight years of experience teaching middle and high school. She is currently working at a high school in Dammam city located in the Eastern Province of the Kingdom of Saudi Arabia. She graduated with a Bachelor of Science degree with a specialization in mathematics. She does not have a degree in education; she has never taken any university education courses. After she graduated from university, she started teaching in a middle school (grades 7-9). She worked there for two years. After that, she began teaching high school. She has been teaching in high school for six years.

When the new mathematics textbooks were introduced to high schools in 2011, Huda was among the teachers who used the new textbooks in the first year of implementation since she was teaching grade 10 at that time. She has been teaching from the new textbooks for three years. When I met Huda, she was teaching 18 lessons per week to students in grade 11 and 12.

Huda works at a traditional public school, which follows the two-semester system. The school building is relatively new and is located in a middle class community.

The school accepts all local students with no entry requirements. The school has 483 students. The mathematics department has 4 teachers.

5.4.1. Huda's classroom

Huda has 30-33 students in every class. In her classroom, students have to sit in assigned seats in neat rows of two tables that face the front of the classroom. The classroom atmosphere is serious and orderly most of the time and no one is permitted to when she is talking. She expects her students to work from the moment class begins right to the end. Students must hand in all work on time; she does not accept excuses. During class, she keeps her voice low and she rarely smiles. She reprimands students who talk in class while she is teaching. She occasionally assigns her students with homework, and when she does, she starts her lesson by checking it. She then moves to reviewing previous material. She knows the curriculum content very well and is able to explain the concepts clearly. She follows the textbook very carefully; she holds the textbook in her hand most of the time and students follow along in their own textbooks.

5.4.2. Significant practices or figured worlds in Huda's case

The data generated about Huda suggests that after eight years of teaching there are five significant practices or figured worlds to Huda's sense of her practices as a mathematics teacher. These figured worlds are mathematics, the textbook, the reform, students' achievement, and relationship with students.

Mathematics

Huda was a straight A student in mathematics when she was at school. She never had problems learning mathematics and that was the main reason she chose her specialization and decided to become a mathematics teacher. There was never any doubt that she would choose mathematics as a degree. However, when she started teaching she realized that knowing mathematics is not enough to be able to teach it. She explained, "When I started teaching, I thought I had what it takes to be a good mathematics teacher, but as soon as I started teaching, I became aware of the fact that knowing mathematics and teaching mathematics are two different things".

She knew that she was not ready to become a teacher and she had to rely on herself to learn how to teach mathematics. Huda still relies mainly on her teaching experience to improve her teaching practices. She gains knowledge about mathematics teaching though monitoring her practice. She remarked, “My experience helps me learn more about students’ thinking and understanding. This makes me constantly change how I introduce and explain mathematical concepts”. She also relies to some extent on her own experience as a student to learn about mathematics teaching. At the beginning of her career, her own experience as a student somewhat influenced her teaching style. Emulating the practices of good teachers by recalling the effective teaching methods she experienced as a student was a valuable resource for her at the beginning of her teaching career.

Huda indicated that since becoming a teacher, she does not get to see other teachers teach any more. She claims that a teacher observing other teachers is an effective tool for professional development as it improves teaching practices. She explained that teachers often are stuck in their own routines and procedures; observation offers an opportunity to see how other teachers teach, others’ effective teaching strategies and how someone else deals with the same problems teachers face every day.

Huda has heard from teachers outside her school that some schools have implemented peer observation as a collaborative professional development technique She talked to teachers at her school to convince them to incorporate this technique at her school. Her idea was to use teacher observation as a means of sharing instructional techniques among teachers. Her plan was for mathematics teachers to work as a team by inviting each other for classroom observations, and meeting regularly, every week or two, to share ideas about lesson plans, teaching strategies, as well as ways to handle behavioural problems. However, teachers at her school did not get excited about the idea because teachers feel nervous when others observe them. She stated, “Teachers in my school thought this would increase their stress level; maybe because observation tends to be associated with evaluation”. She wishes that teachers at her school would interact with each other and learn from each other.

Huda indicated that mathematics is a subject where the teacher is a significant factor in students learning. It is her opinion that in subjects other than mathematics, a

good teacher can only motivate uninterested students to become interested to learn the subject while she is teaching them. However, when students move to other teachers, they may lose interest in the subject again. Huda explained, “Mathematics is different because in mathematics we teach skills; we don’t merely provide information like most other subjects. We help our students to gain skills that stay with them for the rest of their lives, but only good mathematics teachers can do that”.

Huda argued that most of the time, mathematics is taught poorly in schools. Teachers teach mathematics as a subject where there is usually a “one definite correct answer”. She explained that they put a lot of emphasis on finding the correct answer, not the method for reaching the solution by encouraging students to make use of ready-made mathematical formulas. She also finds that teachers present mathematics to students as a subject where practice matters a lot. Her view is that this is the wrong way to present mathematics and that it limits mathematics learning. Huda explained, “Practice is important, but it is not the only thing that makes you better in mathematics… I don’t do a lot of practice in my classroom. Students don’t need to do the same problems repeatedly to understand a particular concept”.

Huda also criticized mathematics teaching practice that presents memorization as an effective method for learning mathematics. She argued that memorization might have some effect on mathematics learning, but it is not a significant component. Depending on memorization to learn mathematics leads to useless and ineffective mathematics learning; the learning is temporary and impossible to apply and transfer into different settings. She justified that understanding should be the ultimate goal of learning mathematics; and by understanding, she means students being able to use and apply what they learn in many different ways rather than applying one skill in one situation repeatedly.

According to Huda, the primary source of the misrepresentation of mathematics in school is ineffective teachers. She finds that most mathematics teachers start their career unprepared and what is worse is they continue their career with no effective professional development. Teachers have to rely on themselves to improve their practices and their main source for improving their practices is their own teaching experience. She provided herself as an example of a teacher who started her teaching

career unprepared. She reflects on her own practices as a source, possibly her only one, of professional development.

In Huda's view, another common source of the misrepresentation of mathematics is the textbook. She explained that mathematics teachers are restricted to a textbook that they have to follow. Teachers do not have the freedom to choose how to introduce their lessons or present a mathematics problem that present the solution in a way that is different from the textbook. She explained, "In Saudi Arabia, following a textbook is the only way teachers know how to teache mathematics because this is how they learn it".

Huda also claimed that the widespread misrepresentation of mathematics in schools is a major source for the common "mathematics phobia" among students. She explained many students, especially in high schools, fear mathematics and find it an extremely tough subject that they cannot master. According to Huda, students view mathematics as a subject that has a very low tolerance level for errors; you either do it perfectly and find the exact answer or your work is worthless. She finds that this phobia of mathematics prevents students from trying to improve their mathematics skills. She stated, "What I find very disturbing is that students in high school are not embarrassed to admit that they are bad at mathematics… they just accept that as a fact and they don't try to change it".

Huda explained that her students generally view most of what they are learning in mathematics class as not interesting nor particularly useful. She admitted that sometimes her view about what she is teaching is not different from that of her students. She said most of the topics students learn in her classroom do not seem acutely relevant to students' lives outside of classroom; some topics are more problematic and it is hard to make them relevant to students' lives or needs. However, she noted other topics might seem interesting or somewhat relevant to students' lives. She explained, "I might try to convince my students that learning about probabilities is relevant, but how I am supposed to convince them that learning the binomial theorem is relevant or at least interesting". She also expressed that mathematics in high school is abstract for the most part, not practical or concrete and the higher level you get in to, the more abstract it gets.; therefore, it is difficult for many students to relate to it and understand its value.

The reform

Huda remarked that almost everyone who cares about mathematics agrees that reform of mathematics learning in public schools is a necessity. However, the current reform trend does not appear to be the solution; the current reform has focused heavily on changing the content of the mathematics curriculum, but not on teaching instruction. She claimed that reform method of instruction does not appear to improve significantly over the traditional method in mathematics classrooms. She stated, “Although we (mathematics teachers), are now using different textbooks, but our teaching instruction is still generally the same”.

According to Huda, the current reform movement has put mathematics teachers under a significant amount of pressure. Teachers are expected to change their practices and produce higher mathematics achievement even though there are no effective professional development programs offered for teachers. She explained, “Recently, the school inspector became more critical and expects me to do more with less… I am expected to be creative and teach in a different way and all I was given is a new textbook”. She complained that teaching mathematics is becoming more difficult. Beside working long hours at school, she spends long time preparing lessons at home and marking students work. The school principal also asked her to offer students training sessions for the GAT test, which she refused to do. She indicated that she does not know about the structure of the GAT test and she has never seen what types of questions are on the test of. She simply does not consider it as a part of her job because it is not part of the school curriculum.

Huda agrees with the foundation of reform about mathematics learning, which is helping students develop a strong conceptual understanding and not merely procedural knowledge. However, she noted that many students come to high school with poorly laid foundations in mathematics. Their knowledge of mathematics comes in the form of memorized formulas and ad hoc processes. These students are not able to apply logical processes when doing mathematics. “Many of my students haven’t even mastered very basic concepts before moving on to more difficult ideas in mathematics. How are they supposed to fully understand pre-calculus topics while they aren't even comfortable with basic mathematics concepts like fractions or calculating percentages?” She explained further that mathematics is really background dependent; you need to learn one thing

before you can learn another in order to advance and successive mathematical concepts build upon other concepts.

Huda suggested that regardless of reform recommendations, every teacher knows what her students need. Students come to her classroom with different levels and backgrounds and everyone should have the same opportunity to learn mathematics and be good at it. Some students just cannot reach the level of deep conceptual understanding, but they can still do well with adequate procedural understanding. Her goal is to help all students learn as much and as deeply as they possibly can. She noted, “We should not expect all our students to be at the same level”.

The textbook

Huda relies heavily on the textbook while planning her lessons and even during classroom instruction. She mostly follows the textbook to plan her lessons; during classroom instruction, she refers to the textbook while introducing the mathematical concept. She holds the textbook in her hand most of the time and students follow in their textbooks. She indicated that unlike the old textbooks, the new textbooks provide organized units of work, which give her all the plans and lessons she needs to cover a topic in a detailed and logical way.

Huda finds the new textbooks provide opportunities to encourage students to learn how to read a mathematics textbook and make them active learners. She explained that encouraging students to read form the textbook during the lesson does not make the teacher the focus of lesson and makes students feel like partners. She explained that she relies more on the new textbook during instruction than she did on the old one. She stated, “I am trying to help my students learn how to read a mathematics textbook because I think it is very important. When they go to university, they will need to do that… many students read a mathematics textbook as if it's a story book, and this actually doesn't work. They need to learn how to make links between previous and current learning while they are reading”. She stated that the new textbooks provide a meaningful context for students’ learning, which helps ensure students stay engaged in leaning activities.

She also mentioned that the new mathematics textbooks opened her eyes about an important aspect of mathematics learning that she has never considered before,

which is mathematics as a language. She explained that mathematics language has a nature that is very different from commonly used languages. The way we explain and write mathematics has a special logical framework that applies special rules, terms and vocabulary. She declared that students in high school lack the ability to explain their thinking using a clear mathematics language. She commented, “Many times I have had my students put the homework answers up on the board. And, I ask them to explain their solutions like, what steps did they take to reach the final answer. Most of them do not like to talk because they struggle to explain their thinking”. Huda hopes that by making reading from the textbook part of students’ every day activities in the classroom, they can be able to explain their understanding of mathematical ideas by using a correct mathematical language to clarify their explanation.

Although Huda seems like she is enjoying teaching mathematics using the new mathematics textbooks, she declared having negative experiences using the new textbooks. She pointed out that when the new textbooks came out, teachers started to learn more about the significance of using group work. Teachers were encouraged by the textbook and the school inspectors to use group work in their everyday instructions. After reading about the value of group work in the teacher guidance book, she became very interested in applying it in her classroom. She thought it would engage students and allow them to take greater responsibility and ownership for their learning. She was hoping it would provide opportunities for peer teaching and cooperative learning. However, Huda described her experience after applying group work in her classroom as unsuccessful. She stated, “It was an awful experience and a total waste of time. The classroom was a big mess and I lost control of my students”. She ended up giving students a great deal of guidance and direction because students failed to control the situation.

Although Huda follows the textbook very carefully, sometimes she decides to ignore certain parts such as “real world connection”. She finds the new textbooks to be overly concerned with making the mathematical concepts relevant to students’ real world. Most lessons in the textbook have a section entitled “Real- Life Connection” which provides general information about some everyday concepts presented in the lesson and its relationship to mathematics concepts. Huda claimed that most of the examples provided under this section are superficial and not related to the mathematics students

do in school. “These examples confuse students instead of making understanding the mathematical concepts easier”; therefore, she rarely includes them in her lesson plans.

In addition, most of the time Huda does not include higher order thinking problems such as open-ended problems presented in the textbook in her lessons. According to Huda, these problems require the use of complex thinking skills that most of her students do not have. She pointed out that only 3-4 students in her class would engage in these difficult problems and the rest of the class just get lost. Based on her experience, when the level of difficulty increases in the mathematical problems, students become disinterested and perform poorly. Therefore, every time she gives her students one of the higher order thinking problems, she has to offer her students a huge amount of assistance because students usually fail to deal with the problem by themselves. She also stated, “Whenever I give them a difficult problem, I have to lead them to the answer and sometimes I had to give them too many hints to the point where finding the answer becomes really pointless”.

Relationship with students

In general, Huda considers herself a “firm” teacher; she is trying to balance her relationship with her students to maneuver between too strict and too friendly. She stated, “I am a firm teacher and I don’t think this is a bad quality in a teacher; my students know that they have to adhere to my guidelines for classroom behaviour; but I don’t think I am too strict, but at the same time I am not too friendly”.

Huda explained that the behaviour in a classroom has a great impact on how students, colleagues, principals, school inspectors and even parents see your professional competencies as a teacher. Without classroom management, teachers’ practice would not appear to be as effective. She also considers it as an indicator for teachers assessing their overall self-performance as a teacher. She elaborated, “If your class is in chaos, you know you are doing something wrong”. According to Huda, a problem with students’ discipline in a classroom leads to low evaluation for teachers’ performance by school inspectors and principals.

In her teaching practices, Huda is aware of her tendency to have a total control of the classroom and classroom management is an important aspect of her practices. She explained that although it is essential to establish respect and open communication with

students, it is also important to build structure and organization in the classroom. Having fair expectations guarantees that students learn to give priority to what is important, listen attentively and participate effectively. It is critical for Huda to have this classroom environment to create successful lessons with effective mathematical discussion and meaningful mathematical tasks. Without such, students would only learn a portion of what they could really learn. She mentioned that the planned lesson would not succeed if the classroom did not have structure; without classroom management, “nothing will be accomplished”.

Huda also pointed out that addressing discipline in the classroom helps to prepare students to be in a good mindset and behavior before a productive lesson can take place. She also affirmed that not establishing discipline in classroom is a source of career-related stress for teachers. She stated, “There are always a few students who play around and use school as amusement. Teaching becomes difficult when you don’t control them; without control of your class, you can have a difficult battle all the time”.

Students’ achievement

Huda mentioned that the school system in Saudi Arabia officially relies on grades to determine students’ achievement, which is a culturally accepted tool to determine teachers’ effectiveness. She indicated that although teachers are not officially evaluated based on students’ achievement, culturally, students’ achievement has a big influence on how teachers evaluate their own practices and how others evaluate them. She explained that students, parents, education officials and even teachers use students’ achievement in tests as an indicator to measure teachers’ effectiveness. She stated, “We all expect that good effective teaching should produce students with high achievement which is measured mainly by their test scores”.

She also added that because teachers have the most direct, constant contact with students and total control over students’ grades, teachers might feel pressured to help students score higher grades. She claimed that most teachers both implicitly and explicitly do certain practices to help students to achieve high marks. Some of the common practices include giving students easy and direct questions in tests, offering students worksheets before the test that have exercises similar to those on the test, and offering students loads of extra credit to improve their grades. According to Huda, these

practices lead to grade inflation where a student's grade does not reflect their actual skills and knowledge.

Huda reported that she tries to be aware of her practices when it comes to students' assessment because she understands that students usually try to receive higher grades with the least amount of time and effort possible. She cited that students are only interested in passing the examinations with high grades, no matter how they achieve those grades. Huda also revealed that she relies on giving her students surprise quizzes every two or three weeks in order to assess her students' actual knowledge. She explained that students do mid-term and final tests and they know the dates for these two tests beforehand, therefore, they do not need to know the dates for surprise quizzes. She uses this strategy to make sure that students are keeping up with what is happening in class and to encourage them to work consistently through the semester.

Huda also talked about the most important element, in her opinion, to improve students' achievement, a good classroom management. She explained that what the teacher does to structure and regulate the learning environment in the classroom is the most important element for increasing student achievement; classes that are poorly organized and managed negatively influence student achievement. She noted that having a suitable environment for all students to learn is her main purpose of implementing effective classroom management, so all students can reach their full learning potential.

5.4.3. Classroom episode

Mathematics, the textbook, the reform and relationship with students

In the lessons leading up to this episode, Huda was teaching her students about geometric series and sequence. Huda started the lesson with a general review of basic concepts related to geometric series such as testing whether a sequence is a geometric sequence or not, first term (a), common ratio(r), number of terms (n), finding the n^{th} term (an), and the sum of n terms (Sn). Then she introduced the new concept by saying;

Huda: The geometric series that we were working on were all finite; today, we are going to learn about infinite geometric series. We will learn about the sum of an infinite geometric series, but before we learn that, we need to know when a geometric

series is considered converges and when a geometric series is considered diverges. Let's look at the textbook now. Open your textbook page number 87.

While students started opening their textbooks, Huda said: Remember we will start working with infinite geometric series; which means geometric series with an infinite number of terms. OK, let's read what the textbook is saying about convergent and divergent geometric series. Who would like to read?

A few students raised their hands (about 7 students). Huda chose one student to read.

Huda said direct to the student to the student: Start reading what is in the blue section under the title Basic Concept.

Student 1: Convergent geometric series; If the absolute value of r is less than 1, then the partial sum comes up to a finite value. Divergent geometric series; If the absolute value of r is bigger or equal to 1, then the partial sum does not come up to a finite value.

Huda: OK, thank you. Let's think about what this means. I will give you an example. Let's consider the geometric series (Huda writes the series on the board) 3+6+12+24+… what do you notice about the numbers?

One student raised her hand and Huda nodded to her to speak.

Student 2: They diverge

Huda: That's right. They diverge. Then this series is a divergent one. Now, see this series; (she writes the series on the board) 4+2+1+1/2+1/4+… what do notice about the numbers?

A few students raise their hand and Huda pointed at one of them to answer.

Student 3: They converge.

Huda: Then, we say this series is a convergent one. But the textbook gave us a rule we can easily use to determine if an infinite geometric series is convergent or divergent. Going back to the textbook, it says an infinite geometric series is considered

convergent, if the r is less that 1 and bigger than -1 and an infinite geometric series is considered divergent, if r is bigger or equal to 1.

Huda: Now, let's read example 1 from the textbook; who would like to read?

In the previous episode, Huda engaged with mathematics, textbook, reform, and relationship with students. Her engagement with these figured worlds informs how she engages in her classroom practices. We see that Huda relies on the textbook to introduce the new concepts of divergent and convergent infinite geometric series. During the episode, she held the textbook in her left hand and used her right one to write on the board. Students were reading and following along in their textbook. She ignored the examples provided in the textbook and gave the students another example to understand the different between the convergent and divergent geometric series. When I asked her about that, she said, "In the examples offered in the textbook, it is not easy to see the difference between convergent and divergent geometric series. You need to start with obvious examples when you first introduce a concept".

Although Huda allows the textbook to have some level of authority in her classroom, she does not let the textbook replace her role entirely. She decides to ignore certain parts of the textbook if she does not think they are good. She also comments and provides more explanation than the textbook offers. She controls who speaks in the classroom; students never speak before she gives permission.

Huda tries to give her students a role in introducing new concepts by asking them to read the new material. By doing that, she is trying to follow the reform recommendation of students to take a more active part in their classroom learning. Although "students reading" is part of her everyday practices in the classroom, most students did not want to participate in this activity; I noticed that only a few students raised their hands showing their willingness to read. Huda explained that she knows that students are not comfortable with reading the new material aloud because reading mathematics is different from other reading; the main difficulty in reading mathematics is notational. She finds that most students are afraid of making errors while reading mathematical notations.

When introducing the two concepts of convergent and divergent geometric series, Huda tended to focus more on the procedural understanding rather than the

conceptual understanding, which conflicts with reform ideas. She told the students directly they needed to rely on the rules indicated in the textbook about how to determine if the geometric series is convergent or divergent by finding the ratio. She seemed to give more priority to the procedures rather than the conceptual understanding of the mathematical material.

5.4.4. Huda the teacher

In terms of PoP, Huda re-constructs meanings of her role as a mathematics teacher through her simultaneous engagement in different figured worlds. The figured worlds that are significant to Huda's practices are: mathematics, the textbook, the reform, students' achievement, and relationship with students. These figured worlds provide the contexts for meaning for Huda's classroom practices.

Huda mostly works in isolation. Since she started her teaching career, she has been mainly independent and has relied on her personal efforts to learn about mathematics teaching. In order to improve her teaching practices, she has relied mostly on reflecting on her experience and gains knowledge by monitoring her practice. For Huda, the teacher is a significant factor in students' learning of mathematics. She mainly blames ineffective teaching practices, such as emphasizing memorization, for the widespread misrepresentation of mathematics in schools, which she considers a major source of the common "mathematics phobia" among students.

Although Huda is trying to follow some of the reform recommendations about mathematics learning, which is helping students to develop a strong conceptual understanding and not merely procedural knowledge, she finds that focusing on procedural knowledge can make learning easier for students with learning difficulties. For Huda, teaching mathematics in high school is a very challenging task because students cannot relate to most of the topics they learn in mathematics class.

It is very important to Huda to have her students participate in the learning of the new concepts in her class. Having students read from the textbook is an important aspect of her understanding of her role, the students' role and the textbook's role. By having students read the new martial form the textbook, she does not see herself as the main source of knowledge in the classroom. She asserts that the textbook and the

students both have a part in the learning process. However, Huda is still the main authority in the classroom because she decides who can talk, when to use the textbook and what part of the textbook to focus on. Her job is to make sure that she manages the classroom environment, so that everyone in the classroom understands and respects their roles. How well she manages student behaviour seems to be very crucial to her success as a teacher.

5.5. Summary

In this chapter, I described the individual and cross-case findings related to the study's first research question. For each individual case, I identified the significant practices or figured worlds for each participant teacher and explained how each teacher engaged in their figured worlds. I also included an analysis of a short classroom episode for each case study. I picked episodes that show an active classroom interaction between the participant teacher and her students. It is important to clarify that I am not claiming that the figured worlds I identify for every case are the only figured worlds that contribute to the teacher's sense of her practice as a mathematics teacher. In the next chapter, I present the results of my cross- case analysis in order to connect the findings from each case study.

Chapter 6.
Cross-Case Analysis

In the last chapter, I presented the four individual case studies. In this chapter, I present the results of my cross-case analysis. The purpose of the cross-case analysis is to identify common themes from the participants and within the data as well as gain further insight into high school mathematics teachers' practices during the current reform movement. The themes that emerged relate to my second research question, How do high school mathematics teachers in Saudi Arabia respond to the shared or common circumstances they are facing in the current reform movement? For each theme, I describe the similarities and differences between the practices of the four participating teachers.

While PoP theory allowed me to employ a case study approach and conduct an in-depth investigation into the teaching practice of each participant, PoP did not provide me with tools to look across cases. Therefore, I distanced myself from the PoP theory when I conducted the cross-case analysis. I felt a need to conduct a cross-case analysis to connect the findings from each case study because I began this investigation with the goal of gaining some understanding of high school mathematics teachers' experiences during the current reform movement. During the cross-case analysis, my goal was to connect the findings from each case in order to generate a broad understanding of high school mathematics teachers' practices during the current reform movement. It also provided an opportunity to examine how each teacher contributed to my general understanding of the practices of high school mathematics teachers in Saudi Arabia.

6.1. Themes from cross-case analysis

In what follows, I identify and describe six common themes from the cross-case analysis. These themes are: changing teachers' practices, factors influencing change, the role of the textbook, conceptual vs procedural understanding, classroom environment, and student assessment.

6.1.1. Changing teacher' practices

The first theme, changing teachers' practices, connects to a major element of the education reform agenda in Saudi Arabia, which is to encourage change in teaching practices. One of the central goals of the reform initiatives is to improve students' learning through a change in teacher practices. As I explained in chapter 2, the new mathematics curriculum in high schools provides some guidelines regarding suggested changes in the classroom for teachers. Two of the Ministry of Education's suggested guidelines are encouraging students to engage actively in the learning process by solving real-life mathematical problems and explaining their mathematical reasoning; and helping students construct a strong conceptual foundation in mathematics that enables students to apply their knowledge in different contexts.

All four teachers in this study have had difficulty interpreting reform recommendations and transferring them into practice. Moreover, they all reported struggling with changing their teaching practices. However, each of them expressed a different kind of struggle. Abeer completely supports the changes the current reform movement, though she did express that she was experiencing struggles with implementing the changes in her classroom. However, Noha suggests that she rejects the proposed changes from the Ministry of Education. Maram shows that there is a need to change school mathematics, but she finds most reform ideas too confusing, making it very challenging to implement change into her practices. In Huda's case, the current reform trend does not appear to encourage her to implement any real change in her practice.

Reform for Abeer means change. When she talks about reform, she usually explains how the current reform movement has changed some aspects of her practice. Abeer describes the first few years of her teaching career as being very traditional. At the beginning of her career, her understanding of her role as teacher was limited to simply presenting mathematics concepts in class, working through a few examples and then giving students a worksheet and telling them to do just as she explained. However, since the reform, Abeer feels her teaching is also about giving her students more control over their own learning. She has been working towards not being the person with total authority in the classroom all the time by creating an atmosphere where students share responsibility for what is happening. She seems aware of how influential cooperative

learning is in creating a thought provoking and interactive environment in the classroom. Abeer re-defined her role as teacher in the classroom based on the reform ideas.

A second change that Abeer has noticed since of the introduction of the new curriculum is that teachers are using a new language to talk about mathematics teaching and learning. She explains that she and other teachers at her school have started talking about new learning strategies such as cooperative learning. Incorporating learning strategies has become a big part of Abeer's every day planning process. She is trying to learn effective ways to include strategies in her class and draws from her own experience by reflecting on her own practices.

Abeer's response indicates that her motivation to change her practice led her to use harder, more challenging ways to teach mathematics. By motivating students to communicate their mathematical thinking and provide time for students to discuss and hear the mathematical ideas of other students, Abeer notes that she has made her job as a teacher more difficult. She explains the struggles she faces stems from her constant search for new ideas and new ways to implement change in her classroom.

Noha thought very differently about accepting reform ideas about mathematics teaching. Generally, Noha rejects reform ideas about mathematics teaching. She is not convinced about the purpose or value of the changes and did not have clear understanding of how and why she needed to implement change in her practice. She complains that the reform curriculum materials, and the circulated notes of recommendations that teachers receive regularly from the Ministry of Education, do not prescribe or describe practices for teachers, but rather offer a vague new vision of mathematics teaching practices.

Noha's main struggle with change comes her impression that changing her practices is compulsory and she has no choice, but to make changes. Noha explains that during her career, school inspectors have recognized Noha as an excellent teacher of mathematics because she represented the culturally accepted values of effective mathematics instruction. However, after the reform movement started, she does not see her teaching practices as appreciated any more. Noha explains that when the reform movement started, especially the introduction of the new textbooks, the school inspector told her she needed to reconsider her role as a mathematics teacher with regard to

student learning and choosing mathematical activities. The school inspector asked her to stop using the notebook in her classroom and to use the new textbooks as the main resource for her classroom activities.

Noha has fallen back into her comfort zone in her teaching practices. She is content with her professional practices and refuses to make any changes to it. She refers to her teaching practices as being “realistic”. By realistic she means that her teaching practices are the result of her own adaptation to existing circumstances. She explains that the existing circumstances have not changed enough in a way that allow teachers to make effective changes. She also asserts her “realistic” way of teaching represents the culture of mathematics teaching and learning in Saudi Arabia; changing this culture in high school will create confusion and chaos that students cannot handle.

She points out that if the chaos occurs, no one will be happy, including students, parents, school principals and even the school inspector. She views her teaching practices as the result of her own adaptation to existing circumstances; those existing circumstances have not transformed in a way that allow teachers to make effective changes. She claims that teachers face so many obstacles if they decide to change their practices.

Maram suggests that there is a need to change school mathematics. She supports reform ideas in education, but at the same time, is struggling to adopt reform-oriented teaching. It is clear from Maram’s case that there is a problem of the inconsistency between her positive reactions towards change and the fact that she is not able to bring effective change into her classrooms. Although she mentions that she has tried to adopt some changes in her practices, such as incorporating group work and unfamiliar activities such as writing, she still relies on traditional methods of teaching mathematics.

One of the changes Maram has been trying to adopt in her classroom is group work; but it seems like group work in her classroom become monotonous practice because she usually just asks students to do group work with their neighbours. Although she follows some reform recommendations to change her practices, she modifies the practices to fit with her regular teaching practices. For example, she usually makes changes to the challenging open–ended problems presented in the

textbook when she presents them to her students. Most of the time she re-writes the problems or activities with more structure, direction and clues to help students engage more with the problem and solve the problem with no challenge.

Maram is not confident in her ability to implement a new teaching approach in her classroom. She is afraid that implementing change would result in using classroom time ineffectively. She is also unsure that changing her teaching approach would help students learn. She finds some reform ideas too challenging and hard to translate into practices. She signals that making a shift in the classroom from focusing on procedural to conceptual learning, and incorporating problem solving strategies, to be extremely challenging for teachers. She finds that reform recommendations and new curriculum materials provide teachers with visions and do not offer explanations about how to transfer these visions into practices. Maram also finds reform recommendations to be misleading, sometimes providing teachers with mixed messages about best and effective practices.

Change is not one of Huda's main concerns. For her, the current reform trend does not appear to be enough to encourage real and actual change in mathematics teaching practices. She explains that she sees the current reform as focusing heavily on changing the content of the mathematics curriculum, but not on teaching instruction. She claims that reform method of instruction does not appear to improve significantly over the traditional method in mathematics classrooms.

The main struggle for Huda is the pressure she faces to make changes in her classroom. In her view, the current reform movement put mathematics teachers under a significant amount of pressure to implement change, but at the same time does not offer teachers with practical ways to implement any real change. She does not have a clear understanding of what aspect of her teaching practices she needs to change.

6.1.2. Factors influencing change

From the section above, it is clear that participant teachers have difficulties understanding, supporting or implementing change in their practices. The second theme explains some of the factors contributing to actual and effective changes in mathematics teaching practices among participants including factors that support or prevent change.

These factors are: teachers' views about their role, teachers' concerns about students, and teachers' professional support and training.

Teachers' views about their role

All four teachers perceive their role as being the main source of mathematical knowledge, which students will acquire by attentive listening and following of teachers' orders. Participant teachers put great emphasis on ensuring that learning in their classrooms is well structured. However, Abeer and Maram are more inclined to regard students as active participants in the process of learning in classroom, while Noha and Huda see their first priority as maintaining order in the classroom and having a controlled learning environment.

Abeer identifies her role in classroom more like a concept facilitator, where she poses questions to prompt students thinking about the mathematical concept. In her classroom, students are encouraged to use different forms of language to clarify their understanding; they are also pushed to express their thoughts using oral and written forms. While Maram mainly relies on lecturing to present the new concepts, she tries to apply additional teaching techniques to encourage richer and meaningful discussions in her classroom. For Maram, teaching mathematics is about assisting students build their own understanding of mathematical concepts and practice mathematical procedures.

In Abeer's classroom, effective communication is vital as both a learning process and an outcome. Sharing ideas is significant in the communication process to help students build meaning of the mathematics concepts. For Abeer, doing mathematics is not about knowing whether a student can find a correct answer; it is about helping student understand why they solved it the way they did. In Maram's classroom, the central goal of learning mathematics is for students to acquire the ability to apply their understanding of mathematical concepts to successfully solve problems. According to Maram, the best way for students to learn mathematics is to have them talk about mathematical concepts, explain their thinking process and do as many exercises as they can.

Although both Abeer and Maram perceive the role of the teacher to be the main source of mathematical knowledge, encouraging effective communication and classroom discussion is also a valuable aspect in their understanding of their role as teachers and

of their understanding of students' role. This understanding positively influences their view of the purpose of making change in their practices. Even though they are not content with the change they are trying to make and are confused about how to make the effective change, they still support the idea of change.

Both Noha and Huda try to maintain a total control of the learning process in their classrooms. For them, maintaining order in the classroom and controlling every aspect of the learning environment is their main role as teacher. Noha considers her role in the classroom as most valuable. Her students consider her the main source of information because she knows her students' needs more than the textbook does. Her role as a teacher entails identifying what her students know, what they need to know and designing an environment for learning. Huda also reports that mathematics is a subject where the teacher plays a significant role in students learning. Huda expects students in her classroom to follow the structure she plans for the lessons. In Huda's opinion, without classroom management, teachers' practices would not be as effective and students learning would not take place. The view that both Noha and Huda share about the role as teachers could contribute to their failure to recognize and support the need for change in teachers' practices. Both teachers are afraid of losing control of the learning environment and dealing with any chaos that could result from applying any change in their classrooms.

Noha's role as a mathematics teacher evolves around helping students do mathematics. According to Noha, mathematics is a body of knowledge centered on specific concepts, and learning these concepts means knowing how to use them. For Noha, mathematics is all about doing; if you are able to do mathematics, then you know mathematics. In Noha's view, an essential factor of understanding mathematics involves memorization and repetitive practice. This view of learning mathematics may also be preventing her from moving away from her traditional style of teaching mathematics.

Concerns about students

Three of the teachers express some concerns related to their students' learning if they change some aspects of their teaching practices. While Abeer is the only teacher who seems confident in her students' knowledge and abilities to learn, and has no problem to challenge her students, the other three teachers express their worry that

students may not have sufficient background knowledge or the ability to learn using a new approach.

Noha argues that although her teaching style is traditional, her approach helps students of all abilities and learning styles acquire strong mathematical skills. She justifies that in high school, the mathematical content is getting harder and more abstract and if teachers try to teach mathematics as a subject of figuring things out or making sense of things, the result will be confusion and chaos, which students cannot handle.

Huda also talks about the negative impact change can have on students learning. For example, Huda mentions her experience of applying group work in her classroom as unsuccessful. She describes it as a terrible experience and a total waste of time and the classroom was a big mess; she ended up giving students a great deal of guidance and direction because students failed to control the situation. Another experience Huda identifies as negative was from when she tried to give her students higher order thinking problems, such as open-ended problems. According to Huda, because these problems required the use of complex thinking skills that most of her students do not have, students became disinterested and performed poorly. Her conclusion is higher order thinking problems confuse students instead of making understanding the mathematical concepts easier.

Maram also seems unsure that changing her teaching approach would help students learn. According to Maram, high school mathematics is mostly very abstract and formal way. It is very difficult, and sometimes impossible, for teachers to create a learning environment for students where they can always experience mathematics in a meaningful way connected to real world. She notes her worst fear is wasting students' time trying to do something new and it does not work. She clarifies that in high school, teachers cannot make mistakes; if a teacher fails to provide students with the suitable environment to learn, there is no time to repair the damage in students' learning.

Support and training

All four teachers signal different levels of disappointment about the support and professional development opportunities they have had through their teaching career. Some teachers reveal that they never had any support or professional development

opportunities, others indicate the support or professional development they have received was not enough.

Abeer is one of the teachers who had some professional development opportunities during her teaching career, but according to her, these opportunities were not enough. For example, after the implementation of the new textbooks, she had the chance to attend only one one-day workshop introducing the new textbooks. She expresses her wish to receive additional professional development opportunities. However, Abeer acknowledges the positive support system she has at her school. Abeer meets with other mathematics teachers every week or two to discuss issues related to students and school activities, but they also discuss issues related to mathematics teaching. Moreover, every month teachers invite other teachers to their classrooms to observe a lesson. Abeer is often excited about this part of the practice at her school. She enjoys having teachers in her class as much as she enjoys being in their classrooms. The trusting relationship Abeer has with other teachers in her school allows her to look at her practice as a professional and reflect more empirically and critically about her own practices. Teachers in Abeer's school value communication with each other and are committed to supporting each other. This style of environment has encouraged Abeer to reflect meaningfully on her teaching practices. According to Abeer, the supportive relationship with other teachers in her school is a real factor that promotes her reflective teaching practices.

The three other teachers complain explicitly about the lack of support, ineffective professional development, and inadequate resources offered to teachers. Noha, Huda and Maram are teachers who mostly work in isolation. Since starting their teaching careers, they have been mainly independent and have relied on their personal efforts to learn about mathematics teaching. None of the participants reported having open and positive communication about mathematics teaching with other teachers at their schools. They do not interact with other mathematics teachers very often and rarely discuss issues related to mathematics teaching with others at school. Maram's case had one exception; her social media interaction provides her with some professional support. She uses her connections in Twitter networks as sources of learning new things related to mathematics teaching. The lack of support, and effective professional development, these teachers have received could be considered a factor hindering an effective change in these teachers' practices.

6.1.3. The role of the textbook

All four teachers rely on the textbook in their teaching practices. However, each one of them has a different relationship with the textbook and a different approach to using it.

In Abeer's classroom, the textbook has an active presence; she often invites her students to engage with it. At the beginning of every lesson, she asks the students to read sections from the textbook related to the lesson; the sections include previously covered skills and concepts, learning outcomes of the lesson and the major mathematical vocabulary. The students read most of the instruction part of the lesson. During the lesson, Abeer also refers her students to the textbook many times. Sometimes, Abeer reads parts of the textbook and the students follow along; other times, she asks a student to read aloud from the textbook or has students read and discuss the information in their groups.

Abeer also engages deeply with the textbook during her lesson planning. She reflects on every part of the textbook and thinks deeply about its purpose to enrich students' learning experience. Abeer has a strong appreciation for the textbook she is currently using in her teaching. The textbook shares her teaching philosophy; mathematics learning is not only rote rules and procedures, but also an exploration, investigation, and deep understanding of mathematical concepts. The textbook plays an essential role in her teaching pedagogy by offering new pedagogical instruction such as giving students higher order thinking problems and encouraging the employment of different teaching strategies such as group work.

For Maram, the textbook is the primary source of information for deciding how she presents mathematical content. However, the textbook does not have a prominent position in her classroom. She follows the textbook sometimes, but not always. For example, she sometimes asks students to read a lesson introduction from the textbook for a particular chapter, but ignores the textbook introduction other times. Maram clarifies that during her first years of teaching, she followed the textbook entirely without even thinking about making any changes; now she sees the textbook as a tool that she can use as she finds appropriate. When planning her lessons, she considers the textbook as

a guide, not a mandate for instruction; sometimes she follows the structure of the lesson presented in the textbook and sometimes she creates her own instruction.

Maram approaches the way she uses the new textbooks cautiously, looking for ways it fits her students' needs and abilities. Most of the time she re-writes the problems or activities with more structure, direction and clues to help students engage more with the problem. This is her way to personalize the textbook material to fit more with her students' needs and capabilities. According to Maram, some problems presented in the new textbooks require more analysis and discussion on the part of the students than students are use to, especially with open–ended problems. To avoid losing her students' interest to work on challenging problems, she provides them with tips and directions. She finds that providing direction is more effective than having students explore with no direction.

Huda makes most of her decisions and sets the mathematical priorities in her day-to-day teaching practices based on the textbook information. She relies heavily on the textbook during classroom instruction and when preparing her lessons. During classroom instruction, she holds the textbook in her hand most of the time and students follow along in their own textbooks. She often refers to the textbook while introducing the mathematical concept.

Huda respects the information in the textbook, but she is cautious not to allow students to see the textbook as a replacement for the teacher. She ignores certain parts of the textbook if she does not think they are good. She also comments and provides more explanation than the textbook offers. In addition, sometimes she asks students to read from the textbook. She considers that encouraging students to read from the textbook during the lesson removes the teacher from being the focus and makes students feel like partners. Students do not see her as the only source of knowledge in the classroom, but she is still the most dominant source because she decides what parts from the textbook students read and when. In addition, she often comments about what students read and provides additional explanation above what is in the textbook. For Huda, a student reading from the textbook is an important aspect of her role, the students' role and the textbook's role. Huda, the textbook and the students all have a part in the learning process. However, Huda is still the main authority in the classroom

because she decides who can talk, when to use the textbook and what part of the textbook to focus on.

In Noha's case, the textbook has a unique status. In her classroom, the official mathematics textbook is never used. Instead of the textbook, Noha designs a notebook each year that she and her students use during the lessons. This notebook replaces the official textbook in her classroom. Noha uses the textbook to help her design her notebook; she uses the textbook as a guide to identify and order the topics and as a source for some exercises. For the notebook, Noha develops many problems and exercises to supplement the suggested textbook activities. Most of the time, Noha does not rely on the activities presented in the textbook in her classroom practices. She notes that she perceives herself as a teacher who is responsible for classroom dynamics; in order to create a positive classroom dynamic, the teacher has the right to alter the textbook activities as a way of addressing issues in the classroom. According to Noha, the notebook provides learning situations that guarantee keeping students engaged in learning activities during the lesson.

Noah rejected the old textbooks because she found them old and outdated. She argues that the old textbooks did not consider the learner nor provide a rich learning opportunity. Noha also rejects the new textbooks because she finds them loaded with large masses of data that students cannot comprehend. According to Noha, students usually find it challenging to understand the relevance of so much data to their personal lives. She also finds the reading level of the new textbook too difficult. In general, Noha finds that forcing teachers to rely mainly on one specific textbook in their teaching undermines the teacher's professional judgment regarding appropriate mathematical activities that meet the needs of all students.

6.1.4. Conceptual vs procedural understanding

All four teachers emphasize the importance of helping students to build a strong mathematical conceptual understanding. However, they have different views into how to apply this aspect in their practices.

Abeer highlights the importance of conceptual understanding as a guide to procedural fluency. She explicitly says that students need to struggle to understand

mathematical concepts, and that struggle contributes to their ability to do procedural parts more easily. For Abeer, doing mathematics is not about knowing whether a student can find a correct answer. Her understanding of her role as a mathematics teacher is to make sure that the student understands why they solved it the way they did. Mathematics learning in Abeer's classroom is more than just memorizing a series of steps that students observe her doing. Abeer's role in her classroom is more like concept facilitator, where she poses questions to motivate students to think and experience the mathematical concepts at hand.

In order to help her students to build a strong conceptual understanding, Abeer relies on a variety of strategies in her classroom including using different forms of language to explain what they mean as well as encouraging students to express their thoughts using both oral and written language forms. She also uses writing activities where she asks student to use their own words, about their understanding of a certain mathematics concept. She encourages her students to incorporate drawing and symbolism in their writing to help express their ideas.

Noha's views about the importance of conceptual understanding are contrary to those of Abeer. She argues that mastering the procedural skills eventually leads to conceptual understanding. Noha explains that although her teaching style is traditional, her approach plays an irreplaceable role in helping all students, regardless of their level of ability and learning style, to gain a high level of conceptual understanding of mathematics and acquire strong mathematics problem-solving and reasoning skills. According to Noha, her teaching approach is simple and direct, focusing on three aspects: first, memorization of facts, rules and formulas; second, repetitive drills and practice of basic computation; and third, procedural skills practice and training. This approach, according to Noha, helps students build a strong foundation of basic mathematics knowledge and skills, such as a deep understanding of mathematical concepts and fluency in using different mathematical procedures and methods. She finds that students with such a foundation of basic mathematics knowledge and skills have the ability to do problem solving and reasoning.

Noha uses the same teaching approach in her volunteer work conducting free workshops for students preparing for the GAT test. Her instructions during the workshops tend to be procedure-oriented. Her response indicates that she mostly trains

her students to perform mathematical procedures that enable them to find answers to problems according to set rules. She explains that procedure-oriented instruction helps students to do well on tests, especially on the purely procedural parts.

Maram and Huda have similar views about conceptual understanding in the mathematics classroom, which fall in the middle of Abeer's and Noha's views. Both Maram and Huda support the central foundations of reform about mathematics learning, which is helping students develop a strong conceptual understanding and not merely procedural knowledge. However, they both find that making the shift in the classroom from focusing on procedural to conceptual learning and incorporating problem solving strategies to be extremely challenging and hard to translate into practices. They both find that mathematics in high school is abstract for the most part, not practical or concrete; therefore, it is difficult for many students to reach a deep conceptual understanding, especially those students with learning difficulties and with a weak mathematics foundation. As a result, they both find the best teaching practice is the one that incorporates a mix of both procedural and conceptual learning.

6.1.5. Classroom environment

Each teacher has their own unique classroom environment, the result of her teaching practices. Likewise, every participant teacher has a different perspective when it comes to influencing the classroom environment.

Abeer tries to create a classroom environment for her students that is different from her own experience of learning mathematics in school. According to her, the classroom environment she had as a student was a teacher centered environment where the teacher controlled every aspect of the classroom. In her classroom, she tries to create a more students centered environment using interactive approaches such as small groups and cooperative learning. She is trying to not to be the person with total authority all the time and creates an atmosphere where students share responsibility for what is happing in the classroom. She is aware of how influential cooperative learning and hands on activities are for creating a thought provoking and interactive environment in the classroom and making the classroom more alive.

Noha's classroom is more a teacher-centered environment. Her classroom remains orderly, students are usually quiet, and she maintains full control of the classroom and activities. The delivery of lesson material is her main duty. Her main role in the classroom is to create an environment where students acquire a common set of skills and knowledge. For her, the most effective way to teach mathematics is to use the classroom board to introduce a mathematics concept, explain different mathematics procedures in relation to the presented mathematical concept, and then get students to practice these procedures individually.

Noha's classroom is an emotionally safe environment where she does not relinquish her role as teacher. She has the ability to find the right balance between being emotionally open without losing the boundaries and hierarchy between her and her students. In her teaching practices, it is essential to connect with students in a positive way. It is very important to Noha that her students know she cares about them. She identifies some of the strategies she uses, such as stressing the things that she and her students have in common. She tries to make it clear to her students that her job is to help them achieve their goals. She also communicates positive expectations letting her students know that she is proud of them. She creates a positive classroom atmosphere by building positive relationships with students using humor and terms of endearment when calling on her students in the classroom.

Maram's classroom is a mix of both a teacher centered and a student-centered environment. While she tries to create an engaging environment where students actively participate in lessons, the classroom environment does not diminish her role as a teacher in the learning process. She mostly relies on lecturing to introduce the new concepts, but at the same time, tries to use additional teaching techniques to encourage deeper and meaningful discussions. She works to engage and involve students, but at the same time, knows when to intervene and what kind of interventions enable her students to discover their own way of understanding. When her students are working, she knows when to include more structure, direction and clues to help students engage more with the problem.

Maram recognizes that students can learn from each other and that rich learning happens when students have the opportunity to discuss, practice and get feedback. However, the classrooms in Maram's school are small and crowded. She considers the

school building as a main factor hindering her attempts to create an engaging classroom environment. In her classroom, students' desks are in rows of three facing the front; she does not have the space to arrange her classrooms and students' in a way that helps her to create a more engaging environment.

The environment in Huda's classroom is more of a teacher-centered environment. She manages he classroom well, the atmosphere is serious and orderly most of the time and no one is to interrupt when she is talking. She expects her students to work from the moment class begins right to the end. According to her, this classroom environment offers an effective, efficient, and safe place where students want to learn. It also prevents distracting actions and behavioral problems, which allows for effective instructional and learning time. In her classroom, governing student behavior and actions helps to optimize opportunities for student learning. It is critical to Huda she balances her relationship with her students between too strict and too friendly. In her classroom, building structure and organization is very important. By having fair expectations of students, Huda teaches that they learn to give priority to what is important, listen attentively and participate effectively.

6.1.6. Students' assessment

The Ministry of Education controls the examination system in Saudi Arabia. Teachers administer one midterm and one final test, which make up 80% of the students' grade in mathematics. Teachers use the remaining 20% of the final grade to assess students on homework, assignments, projects, classroom participation, and quizzes. All four teachers choose to rely on written tests and quizzes as the primary form of assessment in their classrooms to evaluate students on the remaining 20% of their final grade.

Abeer relies on the weekly quizzes to keep her students connected to what they have learned. Noha gives her students a quiz at the end of every chapter. She does not support weekly testing because, according to her, it destroys students' interest and motivation to study for tests. Maram regularly uses timed quizzes once every week or two. She gives the students exactly 10 minutes to do a problem related to what they are studying in class. Huda relies on giving her students surprise quizzes every two or three weeks in order to assess her students' actual knowledge.

While all four teachers are required to use the mid-term and final as summative assessments, Noha and Huda also use quizzes as a tool for summative assessment. Both of them explain that they give a quiz at the end of a chapter to assess students' actual knowledge. On the other hand, Abeer and Maram use quizzes a formative assessment tool to learn about student achievement, monitor progress and plan further instruction. According to Maram, the timed test is a quick way of gathering information on how well her students are meeting their learning objectives and engaging with presented material. For Abeer, weekly quizzes encourage practice and review; they provide students more opportunities for feedback and positively impact students' study time. Teachers do not use other forms of summative assessment tools besides written tests such as projects.

Of the four teachers, only Noha uses homework as a reliable source for assessment. She uses homework as an everyday formative assessment tool. Noha finds that homework helps her measure the level of student knowledge and understanding of the previous lesson. Most of the time, Noha does not mark homework; homework has no weight on students' final grade. Noha uses homework as an indicator of the students' level of understanding of new material. She clarifies that while quizzes measure the level of student knowledge and understanding after the learning occurred, homework assesses students' understanding during the learning process. Usually, when Noha notices that most of the students experience great difficulty completing the homework, she will modify and adjust classroom instruction to decrease the amount of confusion or struggle.

Abeer and Maram note that they use other tools for formative assessment in classrooms. Abeer uses writing as an assessment tool. Sometimes, she incorporates exercise questions where she asks students to explain in writing, using their own words, their understanding of a certain mathematics concept. For Abeer, writing helps students gather and organize their thoughts. It also gives her some access to her students' thoughts and the way they understand mathematical concepts. In Abeer's classroom, writing time often precedes classroom discussion, where students are encouraged to talk about their ideas and discuss with the rest of the class. Maram also notes that engaging students in a conversation about the mathematics they are working on is a tool she uses to assess the students understanding. According to Maram, if students cannot

correctly articulate verbally what they are doing, they do not really have a deep understanding of the mathematics concepts.

One important point all participant teachers draw attention to in relation to students' assessment in high schools is the focus on high-stakes standardized testing and its impact in narrowing the public understanding of accountability. Teachers participated in this study mention that these tests put teachers, students and parents under pressure to care about test scores rather than on real learning.

Maram and Noha stress the importance of adjusting the common view of students' academic achievement in order to make a real change in how teachers teach in schools. The day-to-day decisions teachers make in assessment and evaluation are critical to improving instructional practices and enhancing student learning. Participating teachers argue that there is a need to make changes in how teachers assess student learning in school. They suggest that eliminating some of the mandatory written tests students take in schools could create effective change in teaching practices.

6.2. Summary

In this chapter, I presented the results of my cross- case analysis. I identified common themes from the participants and within the data as well as gained further insight into high school mathematics teachers' practices during the current reform movement. The cross-case analysis provided me an opportunity to examine the diversity of practices and orientations of the participant high school mathematics teachers. It revealed that participant teachers are responding differently to the shared or common circumstances they are facing in the current reform movement. In the next chapter, I provide a summary of the analysis to respond directly to the research questions. In addition, I reflect on the findings of the analysis. I also present the contributions and implications of the study, and some suggestions for further research.

Chapter 7.
Conclusion

In this chapter, I summarize the analysis and respond directly to the research questions presented in chapter 3. Then, I briefly present the contributions of the study to research about mathematics teachers' practice in general and to the practices of mathematics teachers in Saudi Arabia specifically. Finally, I offer some suggestions for further research and take a quick look back to reflect on my experience doing this research.

7.1. Responding to my research questions

In Saudi Arabia, the changes to the education system were, for the most part, focused on specific areas, such as increasing access to education and administrative structures. This led to some education areas experiencing little change for long periods; this was evident in mathematics, where teaching practices remained very traditional until only recently. In traditional mathematics teaching practice, teachers relied on traditional mathematics textbooks and focused on simply delivering the mathematical content knowledge (Al Sheki, 2011; Al Balawi and Al Rajeh, 2012). In that context, the textbook was enough for teachers to have an acceptable teaching practice.

However, during the past decade, the Saudi Arabian education system has undergone major changes. Government agencies involved in education have introduced new policies, standards, programs, and curriculum. I began this investigation with a hope of gaining some understanding of what Saudi high school mathematics teachers are experiencing now. The goals of the research questions were to uncover, describe, and understand high school mathematics teachers' current practices in Saudi Arabia.

In this study, I use the Patterns of Participation (PoP) approach as a lens to interpret and understand Saudi high school mathematics teachers' current practices. The PoP framework identifies teachers' practice as being how teachers narrate and position themselves in relation to multiple figured worlds (Skott, 2013). Figured worlds are imagined communities that function dialectically and dialogically as if in worlds. They

constitute sites of possibility that offer individuals the tools to impact their own behaviour within these worlds (Holland et al., 1998; Skott, 2013).

In order to respond to the questions of this research, I presented four cases of teachers currently teaching high school mathematics in Saudi Arabia. In Chapter five, I presented each case separately to capture the uniqueness of each participant's experiences. Using the Patterns of Participation concept (PoP) as the main framework, I identified some of the significant practices, or figured worlds, from the teachers' sense of their practice as a mathematics teacher and explained how the participant teacher engages with these figured worlds. In the next section, I present a summary of findings of the four case studies. Then, I reflect about the findings from the four case studies.

7.1.1. Responses to the first research question

What are the figured worlds, or significant practices, to the participant teachers' sense of their practice as mathematics teachers and how does each teacher engage with these figured worlds?

The first case study is about Abeer. Using PoP as the main framework, the data generated about Abeer suggest that there are five significant practices or figured worlds to Abeer's sense of her practice as a mathematics teacher. These figured worlds are mathematics, the textbook, responsibility for students' achievement, the reform, and her relationship with others in her school.

Mathematics is one figured world that is significant to Abeer's teaching practice. An essential part of Abeer's sense of her practice has developed from her experiences as a student learning mathematics. Since she started her career as a teacher, she has tried to provide her students with a learning environment that is different from her own experiences of learning mathematics in school, which she descried it as being "very traditional, focusing only on finding the right answers". Learning mathematics in Abeer's classroom is not only about finding a correct answer; it is also about applying thinking skills and being able to explain how to find an answer.

The textbook is another figured world Abeer engages with in her teaching practice. Abeer values the new textbooks greatly. She considers the new textbook "the best thing that's happened to mathematics learning [in Saudi Arabia] in a very long

time…they help teachers to use more reform-oriented practices". Moreover, the textbook has an active presence in her classroom; she often invites students to engage with the textbook. Abeer uses the new textbooks as a tool to reflect on her practice and learn new ways of teaching mathematics.

An additional figured world that is significant to Abeer's teaching practice is reform. Abeer is very passionate about the current reform movement in Saudi Arabia. Reform for Abeer means change. Abeer re-defined her role as teacher in the classroom after being inspired by the reform ideas. She explained that for her the main message from the reform is, "I am not supposed to be the only source of knowledge; I think learning is a shared responsibility …students should be more engaged in the learning process and contribute more effectively to the shared understanding in the classroom". When she talks about reform, she usually explains how the current reform movement has changed some aspects of her practice.

Abeer's relationship with others in her school is another substantial figured world to her practices. Abeer has trusting relationships with the school principal and other teachers in her school. These positive relationships allow her to look at her practice as a professional and reflect more empirically and critically about her own practice. Abeer appreciates communication with other teachers in the school and considers it an important source for her personal learning experience as a teacher. The principal at Abeer's school contributes to this rich learning environment by engaging in setting the agenda for the teachers' meetings, and helping teachers to coordinate their schedules, so that they find a suitable time to meet, observe each other teaching, and offer each other feedback on their observations. However, Abeer has a challenging relationship with the inspector. Her general view of what is valuable in her teaching rarely matches the inspector's view.

The last figured world that is significant to Abeer's practices is her strong commitment to her students' achievement. She closely monitors students' achievement by relying on weekly quizzes. For Abeer, weekly quizzes encourage practice and review; they provide students more opportunities for feedback and positively impact students' study time. Understanding students' concerns and sharing their interests is essential to Abeer's view of her role as a high school mathematics teacher. In addition, Abeer cares about her students' achievement in the aptitude test students taken in high school. In her

classroom, she regularly refers to the aptitude test and gives students tips to achieve better scores.

The second case study is Noha's. From the PoP framework, six significant practices, or figured worlds, emerged as part of Noha's sense of her practice as a mathematics teacher. These figured worlds are mathematics, the textbook, students' achievement, the reform, relationship with students, and voluntary work**.**

The first significant important figured world to Noha's teaching practice is mathematics. She views mathematics as a body of knowledge that is centered on specific concepts, and learning these concepts means knowing how to use them. In Noha's teaching practice, mathematics is all about doing; if you can do mathematics, then you know mathematics. For her, "the main components of mathematics' knowledge are things like a set of rules and formulas and a set of methods and procedures". Consequently, a fundamental part of learning mathematics in Noha's classroom involves memorization and repetitive practice.

The textbook is the second figured world that is significant to Noha's practice. In Noha's classroom, the new textbook is absent. The textbook is one of the main reform changes with which Noha has difficulties. Noha rejects using the new textbooks because she finds them loaded with large amounts of data that students cannot grasp. The new textbook does not seem to have an impact on her teaching practice. Noha replaces the textbook with a notebook she designs each year to use with the students in her classroom. Noha is very proud of her notebook and she does not intend to change this aspect of her teaching practice.

Third in the figured worlds for Noha is reform. Generally, Noha has a negative view about reform ideas. Because of the reform ideas, some of her teaching practices, such as designing a notebook, are no longer appreciated. She refers to her teaching as being "realistic". According to her, her "realistic" way of teaching represents the culture of mathematics teaching and learning in Saudi Arabia and changing this culture in high school would create confusion and chaos, which students cannot handle.

Student achievement is another figured world that Noha draws on in her teaching; it appears to be the main goal of Noha's job as a mathematics teacher. In Noha's practice, there is a strong connection between successful and effective teaching

and student achievement. She measures her success as a teacher and the effectiveness of her practice through her students' achievement. Noha also sees herself as responsible for all mathematics related exams. Therefore, she considers preparing students for standardized tests part of her practice.

Another figured world that Noha draws on in her teaching practice is her relationship with students. Connecting positively with her students is essential in Noha's practice. She relies on open communications skills, respect, kind voice, and appropriate language to establish a positive relationship in the classroom. This positive relationship "can make classes run easily" and "make students feel safe and more comfortable with each other and with a teacher who they feel cares about them".

Voluntary work is another figured world that is significant to Noha's practice. Noha is a very active teacher and she is willing to do any work that could benefit students. She demonstrates interest in extending her relationships beyond the classroom by voluntarily participating in extra-curricular activities with her students. A major part of Noha's volunteer work is designing and conducting free workshops for students at her school. The workshops focus on offering students the skills and knowledge to help them score better on the GAT test.

The third case study is about Maram. Using PoP, the data analysis revealed five figured worlds to Maram's sense of her practice as a mathematics teacher. These figured worlds are mathematics, the textbook, the reform, students' achievement, and social network engagement.

The first figured world I identified about Maram's practice is mathematics. Generally, Maram is very passionate about mathematics. Her love for mathematics as a subject by has developed since she was a student. Being in a classroom learning mathematics made her feel like "doing something that makes sense to me". Although Maram loves mathematics as a subject, she does not consider that main objective of her job as making her students love mathematics; she sees her job as making her students do mathematics. In Maram's classroom, the central goal of learning mathematics is for students to acquire the ability to apply their understanding of mathematical concepts to successfully solve problems.

Reform is another figured world that is significant to Maram's teaching practice. Current reform ideas in education remind Maram of the teacher she wanted to be when she started her teaching career. Maram's plan was to adopt a "nontraditional mathematics" approach to teaching by relying less on lectures and more on interactive learning activities. However, Maram found this approach to be challenging and hard to translate into practice.

The textbook is another figured world that Maram draws on in her teaching. In Maram's teaching practice, the textbook is nothing more than a tool. She expressed that all textbooks are merely as good as the teacher who uses it. She considers it as "a very important tool" and she uses it in a way she finds "appropriate". The introduction of the new textbooks influenced some aspects of her teaching practices, such as group work. However, in Maram's view, the new textbooks did not have the expected effect on her teaching practice, which she attributed to the lack of support and preparation she received during their implantation.

Another figured world that is significant to Maram's teaching practice is student achievement. In Maram's practice, the main goal of high school mathematics teachers is to create a supportive environment, so students can learn the necessary concepts for academic achievement. For Maram, academic achievement goes beyond the grades and the written tests given to students. In her view, the grading system used in schools, which is mainly based on written tests, does not reflect or communicate the level of actual academic progress or achievement that a student has developed in school.

The last significant figured world to Maram's practices is her social network engagement. Maram is very active on social media, especially Twitter. She is a social media enthusiast with a passion for sharing ideas about mathematics and mathematics teaching. On Twitter, she likes to post mathematics problems and get people's responses. She discusses their ideas about the problems she posts, reviews their answers, and finally provides the right answer. According to Maram, this interaction helps her learn more about the thinking process when engaging with mathematics. Her engagement in social media suggests that she is willing to learn new things to improve her practices.

The last case study is about Huda. Using PoP, the analysis suggests five significant practices, or figured worlds, to Huda's sense of her practice as a mathematics teacher. These figured worlds are mathematics, the textbook, the reform, student achievement, and relationship with students.

The first figured world that Huda draws on in her teaching practice is mathematics. Even though she received a strong foundation in mathematics knowledge as a student, when she started teaching, she realized that knowing mathematics is not enough to be able to teach it. Her teaching practice made her "aware of the fact that knowing mathematics and teaching mathematics are two different things". According to Huda, mathematics is a subject where the teacher is a significant factor in students learning "because in mathematics we teach skills; we don't merely provide information like most other subjects.".

The second figured world that is significant to Huda's sense of her practice as a mathematics teacher is reform. Although Huda stated that almost everyone who cares about mathematics agrees that reform of mathematics learning in public schools is a necessity, she finds that the current reform trend does not appear to be the solution. According to her, the current reform has focused heavily on changing the content of the mathematics curriculum, but not on teaching instruction.

The textbook is another figured world that is significant to Huda's practice. Huda relies heavily on the textbook in her teaching practice. She mostly follows the textbook, and in her classroom, she holds the textbook in her hand most of the time. Although Huda follows the textbook very carefully, sometimes she skips over parts such as "real world connection and higher order thinking problems". According to Huda, these parts of the textbook require the use of complex thinking skills that most of her students do not have.

Another figured world that Huda draws on in her teaching practice is her relationship with her students. Huda describes herself as a "firm" teacher; she is always trying to balance her relationship with her students to maneuver between too strict and too friendly. In her teaching practice, Huda is aware of her tendency to have total control of the classroom and classroom management is an important aspect of her understanding of effective teaching.

Students' achievement is the last figured world that is significant to Huda's practices. According to her, the school system in Saudi Arabia officially relies on grades to determine students' achievement, which is a culturally accepted tool to determine teachers' effectiveness; therefore, students' achievement has a big influence on how teachers evaluate their practices and how others evaluate them. She suggested that many teachers, both implicitly and explicitly, do certain practices to help students to achieve high marks.

7.1.2. Reflection on the findings from the four cases

To answer my first research question, I used PoP as the main framework to identify some of the significant practices, or figured worlds, from the teachers' sense of their practice as a mathematics teacher and explained how the participant teacher engages with these figured worlds. According to PoP, being a mathematics teacher means developing a sense of teaching practice. A teaching practice is built from the meanings that a teacher attaches to her engagement and re-engagement in many figured worlds. These figured worlds work within a complex relationship, where they either co-operate or counteract each other.

It is important to clarify that using PoP was a little challenging. PoP, as a framework, makes sense to me conceptually, but in practice, I found it hard to apply. PoP provided a strong conceptual view of how I could study and understand mathematics teachers practice; however, in terms of using it as an analysis tool, the theory focuses more on identifying the figured worlds or significant practices and explaining how a teacher engages in a figured world by analyzing what the teachers are doing instead of analyzing what their beliefs are regarding their teaching practice. Therefore, although PoP offers a strong conceptual understanding of teachers' practices, I find the practical use of the framework does not fully reflect its philosophy.

While recognizing that we can never gain a complete understanding of another's experiences, the framework offers a fair picture of the participant teachers' experience and an increased understating of their practices. The PoP framework helped me see how participant teachers are different in their practices even though they share many commonalities, such as using the same textbook.

Also, in analyzing the data, I identified and explored some of the figured worlds with which teachers engage in more depth. It is important to clarify that I am not claiming that the figured worlds I identified for every case are the only figured worlds that contribute to the teachers' sense of their practice as mathematics teachers.

Three figured worlds appeared for each of the participant teachers, mathematics, textbooks, and reform. Through participation in these figured worlds, teachers can re-conceptualize their practices and adjust how they understand themselves as individuals and as members of their communities. These figured worlds work in a very complex system where they could support, and sometimes restrict, one another as every teacher contributes to classroom practice (Skott, 2013). Even though these figured worlds are significant to all participant teachers, every teacher engaged in these figured worlds differently, as explained in chapter five.

In looking at all of these cases, it becomes clear that mathematics, as it has always been, remains an influential figured world for mathematics teachers. Reform and the textbook are becoming as influential because of the current changes in education system in Saudi Arabia. While some participants' teachers are developing, a new understanding of what mathematics is and what it means to teach it, participant teachers also indicated that they are mostly still using traditional teaching strategies rather than reform teaching strategies. The traditional methods in mathematics teaching can be characterized as a mode to deliver information; the teacher provides information, mostly from textbooks, in a teacher-centered, lecture-style learning approach.

The participating teachers are at very different places in their engagement with reform. They each have different degrees of engagement with reform and each one is trying to make sense of reform in terms of their own experience and practice. The new textbook for participant teachers represents the most important reform effort. They identified the new textbooks as a potential vehicle for helping teachers develop more reform-oriented practices. However, the textbook, as a figured world, means different things to different teachers. The textbook for some teachers presents only the mathematical knowledge, but for other teachers, the textbook is more than that. It presents a new way of teaching mathematics. In this way, we see that the new textbook has fractured teachers' practices by creating additional views on how teachers understand their role as mathematics teachers. At the same time, teachers divided the

textbook and engage with different parts of the textbook in different ways. It is possible that participant teachers acted upon parts of the textbook that filled their needs, and passed over the rest.

7.1.3. Responses to the second research question

How do high school mathematics teachers in Saudi Arabia respond to the shared or common circumstances they are facing in the current reform movement?

To answer this question, I conducted a cross-case analysis to connect the findings from each case study in order to generate a broad understanding of high school mathematics teachers' experience during the current reform movement. The purpose of the cross-case analysis was to identify common themes from the participants and within the data as well as gain further insight into high school mathematics teachers' practices during the current reform movement. I identified and described six common themes from the cross-case analysis. These themes are: changing teachers' practices, factors influencing change, the role of the textbook, conceptual vs procedural understanding, classroom environment, and student assessment. To respond to the second research question, I present a summary of themes that emerged from the cross-case analysis and provide a reflection on the findings.

The first theme that emerged from the cross-case analysis is changing teachers' practice. This theme relates to a major element of the education reform agenda in Saudi Arabia, which is to encourage change in teaching practice. The four participant teachers in this study indicated they have experienced difficulty interpreting reform recommendations and transferring them into practice; they all stated they have struggled with their attempt to make effective change in their teaching practice. However, the struggles were different in every case. Abeer's responses indicate that her decision to change her practice led her to use harder, more challenging ways to teach mathematics. The struggles she faces stems from her constant search for new ideas and new ways to implement change in her classroom.

In contrast, generally, Noha rejects reform ideas about mathematics teaching. She is not convinced about the purpose or value of the changes and did not have clear understanding of how and why she needed to implement change in her practice. Noha's

main struggle with change is the result of her impression that changing her practice is compulsory and she has no choice. During parts of the interview, she admitted there are other ways to teach mathematics, but she is struggling to know what they are.

Maram supports the need to change school mathematics, but she finds most reform ideas too confusing, making it very challenging to implement change into her practice. It is clear from Maram's case that there is a problem inconsistency between her positive reactions towards change and the fact that she is not able to bring effective change into her classrooms.

In Huda's case, the current reform trend does not appear to encourage her to implement any real change in her practice. The main struggle for Huda is the pressure to make changes in her classrooms. In her view, the current reform movement put mathematics teachers under a significant amount of pressure to implement change, but at the same time does not offer teachers with practical ways to implement any real change.

The second theme identifies some of the factors that contribute to actual and effective change in mathematics teaching practices among participants. This theme is the result all participant teachers expressing difficulties understanding, supporting or implementing change in their practices. These factors are: teachers' views about their role, teachers' concerns about students, and teachers' professional support and training.

The first factor is teachers' views about their role in the classroom. All four teachers generally perceive their role as being the main source of mathematical knowledge, which students will acquire by attentive listening and following teachers' instructions. Participant teachers put great emphasis on ensuring that learning in their classrooms is well structured. However, Abeer and Maram are more inclined to regard students as active participants in the process of learning in classroom, while Noha and Huda see their first priority as maintaining order in the classroom and having a controlled learning environment. This view that both Noha and Huda share about the role as teachers could contribute to their failure to recognize and support the need for change in their practices.

The second factor is teachers' concerns about students learning. Three of the teachers expressed some concerns related to their students' learning if they change

some aspects of their teaching practices. Abeer is the only teacher who seems confident in her students' knowledge and abilities to learn, and has no problem challenging her students. The other three teachers expressed worry that students may not have sufficient background knowledge or the ability to learn using a new approach. These concerns could prevent teachers from using a variety of strategies and techniques to promote students' engagement and in-depth learning.

The third factor is support and training. All four teachers communicated different levels of disappointment about the support and professional development opportunities they have had through their teaching career. Some teachers revealed that they never had any support or professional development opportunities, others indicated the support or professional development they have received was not enough. All teachers, except Abeer, said they feel isolated and work in schools where the culture of work is mainly individualized. Despite the education reform taking place, the reality for teachers is there aren't enough resources and opportunities to develop professionally to create positive change. The lack of support, and effective professional development, these teachers have received could be considered a factor hindering effective change in these teachers' practices.

The third theme involves the role of the textbook. All four teachers rely on the textbook in their teaching practices. However, each one of them has a different relationship with the textbook and a different approach to using it. Abeer has a strong appreciation for the textbook. She engages deeply with the textbook during her lesson planning. In her classroom, the textbook has an active presence. She often invites her students to read and engage with it.

For Maram, the textbook is the primary source of information for deciding how she presents mathematical content. However, the textbook does not have a prominent position in her classroom. She approaches the way she uses the new textbook cautiously, looking for ways it fits her students' needs and abilities.

Huda makes most of her decisions and sets the mathematical priorities in her day-to-day teaching practice based on the textbook information. Huda relies heavily on the textbook and respects the information in the textbook, but at the same time, she is cautious not to allow students to see the textbook as a replacement for the teacher.

In Noha's case, the official mathematics textbook is never used in her classroom. Instead of the textbook, Noha designs a notebook she and her students use during lessons. She organizes the notebook by chapters and lessons based on how they appear in the official textbook and she includes some exercises from the textbook in her notebook. Noha asserts that teachers should have the freedom to choose the materials they find best for their students.

The fourth theme correlates to teachers' tendency to focus on conceptual understanding vs procedural understanding. All four teachers emphasize the importance of helping students build a strong mathematical conceptual understanding. However, they have different views into how to apply this aspect in their practices. In Abeer's practice, conceptual understanding is the core of mathematics learning. In her view, conceptual understanding is the guide to procedural fluency; students need to struggle to understand mathematical concepts, and that struggle contributes to their ability to do procedural work more easily.

Noha's views about the importance of conceptual understanding are contrary to those of Abeer. She argues that mastering the procedural skills eventually leads to conceptual understanding. Maram and Huda have similar views about conceptual understanding in the mathematics classroom, which fall in the middle of Abeer's and Noha's views. Both Maram and Huda support the central foundations of reform about mathematics learning, which is helping students develop a strong conceptual understanding and not merely procedural knowledge. They both find the best teaching practice is the one that incorporates a mix of both procedural and conceptual learning.

The fifth theme is classroom environment. Each teacher has their own unique classroom environment, the result of her teaching practice. Likewise, every participant teacher has a different perspective when it comes to influencing the classroom environment. Abeer tries to create a classroom environment for her students different from her own experience learning mathematics in school. She tries to create a more student centered environment using interactive approaches such as small groups and cooperative learning. Noha's classroom is more a teacher-centered environment. Her classroom remains orderly, students are usually quiet, and she maintains full control of the classroom and activities. She has the ability to find the right balance between being

emotionally open without losing the boundaries and hierarchy between her and her students.

Maram's classroom is a mix of both a teacher centered and a student-centered environment. While she tries to create an engaging environment where students actively participate in lessons, the classroom environment does not diminish her role as a teacher in the learning process. The environment in Huda's classroom is more of a teacher-centered environment. She manages her classroom well; the atmosphere is serious and orderly most of the time and no one interrupts when she is talking.

The last theme is students' assessment. According to the examination system in Saudi Arabia, teachers must administer one midterm and one final test, both of which make up 80% of the students' final grade in mathematics. Teachers have the freedom to use the remaining 20% of the final grade in the way they find fits with their practices. Teachers mostly use the remaining 20% to assess students using homework, assignments, projects, classroom participation, and quizzes. All four teachers choose to rely on written tests and quizzes to evaluate students on the remaining 20% of their final grade; however, each participant teacher uses different approaches when administering tests. For example, Abeer gives her students weekly quizzes to keep them connected to what they have learned.

Noha is against weekly testing because, according to her, it destroys students' interest and motivation to study for tests. Instead, gives her students a quiz at the end of every chapter. Maram usually uses timed quizzes once every week or two. She gives students exactly 10 minutes to complete a problem related to what they have been studying in class. Huda relies on giving her students surprise quizzes every two or three weeks in order to assess their actual knowledge. Noha and Huda use these quizzes merely as a tool for summative assessment. They both give quizzes to assess students' actual knowledge. On the other hand, Abeer and Maram use quizzes a formative assessment tool to learn about student achievement, monitor progress and plan further instruction. Of the four teachers, only Noha uses homework as a reliable source for assessment. She uses homework as an everyday formative assessment tool to measure the level of student knowledge and understanding of the previous lesson.

7.1.4. Reflection on the findings of the cross-case analysis

The main goal of the cross-case analysis was to gain further insight into high school mathematics teachers' practices during the current reform movement. The themes that emerged from the cross-case analysis relate to my second research question, How do high school mathematics teachers in Saudi Arabia respond to the shared or common circumstances they are facing in the current reform movement? Even though this study focused on the teaching practices of only four individuals, their words and actions provide some general understanding of the current practices of high school mathematics teachers.

While PoP theory allowed me to employ the case study approach and conduct in-depth investigation into the teaching practice of every individual participants, PoP did not allow me or provide me with tools to look across cases. Therefore, I distanced myself from the PoP theory when I conducted the cross-case analysis. Because I began this investigation with a goal of gaining some understanding of high school mathematics teachers' experiences during the current reform movement, I felt a need to conduct a cross-case analysis to connect the findings from each case study.

The cross-case analysis provided me an opportunity to examine the diversity of practices and orientations of the participant high school mathematics teachers. The themes that emerged are useful for showing the range of mathematics high school teachers' practices in Saudi Arabia and the ways in which their practices differ. As discussed in the themes above, I found that participant teachers respond differently to the shared or common circumstances they face in the current reform movement. I found more differences than similarities in the current teaching practices of the participant teachers.

The changes made in the education system as part of the current reform have resulted in all participant teachers having difficulties understanding, supporting or implementing change in their practices. They feel pressured, and sometimes forced, to adopt changes in their teaching without the opportunity to decide what changes are appropriate in their classrooms. Obstacles to teachers changing their professional practices appear when teachers feel overwhelmed with reform policies coming from the outside. However, each participant experienced the idea of change differently. While

some teachers prefer to fall back to their comfort zone of existing routines and refuse to make any changes in their teaching practices, others are trying to adopt reform-oriented teaching, but have not been able to implement these changes effectively.

In additions, participant teachers could not see the value of some reform recommendations such as teaching higher-order thinking problems and applying mathematical concepts to real life situations. Teachers cannot adopt change in their practices if they cannot see or understand the value of the change. Moreover, when teachers cannot see the significance of changing certain aspects of their practices in order to follow reform recommendations, then they will not engage with those reform recommendations in ways that encourage essential change.

The Ministry of Education's aim for introducing the new textbooks was to encourage teachers to adopt more reform-based instruction. When the Ministry of Education introduced the new textbooks, it emphasized the importance of teachers following the textbooks and teaching from it, regardless of what opinions teachers had about the textbook.

The teachers in this study have had different reactions to the new textbooks; some of them reject relying on the textbooks while others try to adopt the new textbooks and use it how they see fit within their practices. In general, it is fair to say that the new textbooks have not had the effect that the Ministry of Education expected on mathematics teaching practices in high school. Forcing teachers to mainly rely on one specific textbook in their teaching undermines the teacher's professional judgment regarding appropriate mathematical activities that meet the needs of all students.

According to the participating teachers, the current reform movement in Saudi Arabia has not made any changes to student assessment methods in schools. Participant teachers still rely on written tests as the primary form of assessment in their classrooms. Not changing the way teachers assess students' achievement in the classroom is one of many factors in hindering any attempt to introduce real change in how teachers teach mathematics in high schools. In fact, some of the changes introduced recently, such as standardized testing, have had a negative impact on teachers. These tests put teachers, students, and parents under pressure to care about test scores rather than on real learning.

While teachers in this study seem to understand that teaching mathematics is about helping students build a strong mathematical conceptual understanding, they still have a strong tendency to focus more on procedural knowledge. Focus on memorizing facts or procedures, with no understanding of the underlying conceptual meaning, is still the most common practice in mathematics high schools. Participant teachers have mostly failed to adopt teaching approaches that shift attention from procedural knowledge to deep conceptual understanding. Some of those teachers are afraid of the chaos that could result from changing their teaching practices. Furthermore, some reform changes, such as standardized tests, reinforce the practice of relying on a procedure-oriented approach.

The negative perception participant teachers have about reform and the new mathematics textbook is a result of the shortage of resources and professional support teachers have received. Participant teachers mostly work in isolation without positive collaboration. The lack of support, ineffective professional development, and inadequate resources have created a work environment that has no encouragement or support for implementing new practices.

7.2. Contributions of the study

The work of teachers is deeply complex and involves various practices. This study contributes to the body of research that aims to explore and understand mathematics teachers' practices using a participationist approach. PoP adopts participationism as a metaphor for human functioning more than mainstream belief research, which mostly adopts acquisition metaphors. According to the participationism view, practice is not only a personal individual matter; it is part of a sociocultural context. This approach views teachers' practices as adaptations to social conditions in which they work. From this viewpoint, teaching is not pre-reified constructs of knowledge and beliefs. It is a meaning-making process, in which the teacher constantly maneuvers between diverse types of participation in different past and present practices.

This study also contributes to the body of research aiming to understand the practices of teachers as influenced by current reform efforts in mathematics education. PoP takes into account the role of reform as a social construct in the evolution of teachers' practices (Skott, 2013). Although there is great emphasis in research about the

teachers' role in mathematics education reform, little is known about the practice of teachers who are experiencing mathematics education reform. Examining how teachers engage with the reforms in their teaching of mathematics could lead to increased improvement in the teaching of mathematics at all school levels.

This study fills a gap in the existing research about high school mathematics teachers' practices in Saudi Arabia. Research about teachers' practices in Saudi Araba is limited. My search of the literature resulted in very few studies related to mathematics teaching in general and no studies directly addressing high school mathematics teachers during the current reform movement. This study has documented Saudi high school mathematics teachers' experiences and their response to the current changes in the education system. It contributes to knowledge in the field by providing teachers' viewpoints and voices on the complexities of educational reform.

The reform documents in Saudi Arabia state that teachers are to create learning environments where students can develop a deep understanding of mathematics concepts, acquire skills in higher order thinking, and construct and solve problems. Moreover, the guidelines suggest teachers promote students' understanding by applying a variety of strategies and techniques to create a classroom environment in which teachers and students communicate their ideas and investigate problems framed in meaningful contexts. This view of the mathematics classrooms represents a significant departure from the traditional teaching practices in Saudi Arabia. The changes incorporate assumptions about mathematics teaching and learning. My Study indicates that participant teachers do not have a clear conception of how to transfer this reform view into practice. This study has shed some light on high school mathematics teachers' struggles and challenges, which play an important part in shaping teachers' professional conduct and their thoughts on professional practices and improvement. Teachers work very hard and put much effort into understand reform in order to enrich students' learning experience.

7.3. Implications of the study

The findings of the study provide educators and policy makers in the Ministry of Education in Saudi Arabia with some important information.

The first implication of this study is it provides important information about mathematics teaching, which can be useful for the development of mathematics teachers' education programs in Saudi Arabia. This study uses PoP to identify and explore figured worlds that are significant for high school mathematics teachers' classroom practices. Applying PoP provides some understanding of the complexity of current mathematics teaching practice in Saudi Arabia especially during the current reform movement. Conducting such investigations is important to understand current teachers' practices because such an understanding is crucial for the improvement of pre-service and in-service mathematics teacher education programs that are responsive to challenges in mathematics classrooms.

The second implication is that the Ministry of education in Saudi Arabia should take into account how to develop mathematics teachers' sense of being active agents. Findings from this study suggest that even though teachers are aware of the reform's recommendations when it comes to mathematics teaching and learning, they do not consider themselves as active agents in the educational reform. My Study indicates that participant teachers see themselves as a target of the reform, not as an active agent in the process of change. Developing teachers' sense of being active agents for change could stimulate their enthusiasm and willingness to implement reform recommendations in their teaching practices.

The third implication of my study is that teachers need to be convinced about the value and benefit of the reform recommendations. Teachers in this study teach in ways they think are the best for meeting their students' needs. They engage with the new textbooks and teaching recommendations that come with the new textbooks only in the ways they see fit for providing students with the most effective learning experience. According to Ball (1990), reforming mathematics teaching and learning in schools "requires something that likely goes beyond written texts. It requires changed views of what mathematics is and what it means to know and do mathematics as well as changed assumptions about students and how they learn" (p. 258). Teachers need evidence that the approach to teaching contained in the reform textbooks provides a much-enriched learning experience for students. Reform does not work if it is imposed on teachers. If teachers are not convinced of the value and effectiveness of recommended changes, they will be unwilling to implement them actively.

The fourth implication of this study is related to the role played by the Ministry of Education in reform implementation. Although teachers have a central role to play when it comes to reform implementation, they cannot be held solely responsible for achieving reform recommendations. What the Ministry of Education in Saudi Arabia has done is not enough. Implementing new textbooks and waiting for teachers to implement changes in classrooms does not solve the problems. Reform will not succeed unless it comes with the creation of a school environment in which teachers can work effectively.

The last implication of this study is the importance of professional development and teachers' collaboration in improving teachers' practices. Participant teachers in this study demonstrated a need for professional development. Despite the education reform that is taking place in Saudi Arabia, the reality for many teachers is there aren't enough resources and opportunities to fully understand reform recommendations. The Ministry of Education should provide teachers with reform-oriented professional development activities that support teachers in creating positive change. Mathematics education reform is not only about offering new teaching guidelines related to teacher practices; it should also offer the opportunity for all teachers to learn new practices. This study also indicates that strong teachers' collaboration, as the case in Abeer's school, can offer the environment of support and inspiration required for the professional development of teachers and the improvement of their practices. Fostering teachers' collaboration in schools could provide opportunities for continual growth and improvement.

7.4. Future research

Findings from this study suggest many avenues for further exploration. Some suggestions for further research include:

Researchers should continue examining teachers' practice. Future research should expand data collection to include teachers who are teaching in other school levels such as elementary and middle schools. Mathematics teachers' practices may vary depending on the level they are teaching. Teaching different school levels requires different teaching practices and classroom engagement.

In addition, researcher should look at why collaboration activities are not common among teachers and explore ways to foster teachers' learning activities that

help teachers improve their classroom practices. From Abeer's case, this study shows the role of collaboration in providing encouragement and supporting new practice.

This study did not look closely at the impact of the new textbook on teachers' pedagogy and practices, but the study found that, for some teachers, the textbook is playing a considerable role in shaping teaching practice. Therefore, more research is needed to explore the role and the impact of the textbook.

In the context of educational reform especially in relation to mathematics learning and teaching, change is always challenging. Reform often creates new challenges for teachers. Future research should look into the challenges teachers in Saudi Arabia face and the strategies they use to deal with and overcome these challenges.

Finally, this study found that participant teachers have different levels of motivation about reform. Future research should investigate the issue of teacher motivation during the current reform movement in Saudi Arabia and explore which factors contribute to enhancing teachers' motivations.

7.5. Personal reflection

> We cannot live other people's lives, and it is a piece of bad faith to try. We cannot but listen to what …they say about their lives.... Whatever sense we have of how things stand with someone else's inner life, we gain through their expressions, not through some magical intrusion into their consciousness. It's all a matter of scratching surfaces. (Geertz, 1986, p. 373)

Teaching mathematics is a very complex and demanding profession. Most mathematics teachers work long hours in school and at home and have many challenging demands in their practices. When mathematics teaching is observed from a distance, many details that go into the daily practices of teachers can go easily unnoticed. My conversations with the participating teachers show a group of teachers who are caring and very hard working. They strive to ensure the welfare of their students and put forward their best efforts in fulfilling their responsibilities. In this dissertation, I tried as much as possible to represent the richness and complexity of the data I collected about their teaching practices.

This research has greatly affected the way I see and understand high school mathematics teaching in Saudi Arabia. This research experience has changed my understanding of what it means to teach and learn mathematics in Saudi Arabia and has enabled me to appreciate more deeply the challenges that mathematics teachers face in their ever day practices. Before conducting this research, I expected to see more similarity in the teachers' practice since those teachers work in a homogeneous environment under relatively similar circumstances. This experience has been the most exciting learning journey. I hope this work contributes to promoting more research and understanding of mathematics teaching and learning in my country.

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Appendix A.

Interview protocol: list of possible interview questions

1- Think about your experience of learning mathematics as a student. In what ways does this experience influence your teaching practice?

2- How would you describe the preparation you received in college before you began your career as a mathematics teacher?

4- When your students ask you about the purpose of learning mathematics in school, how do you respond?

5- How do you explain to your students the connection between school mathematics and their everyday life outside the classroom?

6- How would you describe your experience as a mathematics teacher since you started your career?

7- How would you describe your experience as a mathematics teacher during the last three years?

9- Think about what you taught today, can you walk me through the process you used to prepare for a lesson? Describe how the lesson went?

10- Are your lessons similar or would you say that there are different types of them?

11- Describe the characteristics of a successful mathematics class you have had.

12- Thinking about yourself as a mathematics teacher at the beginning your career and now, how have you changed?

13- What do you do to improve your teaching practices? Can you tell me about any new strategies or techniques you have started using recently to improve your teaching practice?

14- How would you describe recent reform initiatives?

15- Have recent reform initiatives in any way affected your teaching practices?

16- Do you find that students' understanding of mathematics has changed? If yes, how do you describe this change? If no, what changes should the Ministry of Education and mathematics teachers make in order to change students' understanding of mathematics in a positive way?

17- If someone asks to compare the old textbooks with the new textbooks, how do you reply?

18- Think about the lesson you taught today. How does using the new textbook make the lesson different then using the old textbook?

19- Are you satisfied with using the new mathematics textbooks in your teaching?

20- To what extent do the new mathematics textbooks affect your practice in mathematics teaching?

21- In your opinion, what are the positive and negative features of the new textbooks?

22- To what extent do you follow the structure of the textbook when planning your lesson?

23- Do you always follow your planned lesson when you are in the classroom? How often do you make changes in your lesson plan when teaching?

24- When was the last time you had the opportunity to take any professional development sessions?

25- Compare the professional development sessions you had before and after the implementation of the new textbooks.

Appendix B:
Classroom observation protocol

- Lesson organization
    - How does it start?
    - How does it progress?
    - How does it end?
- Classroom managements
    - Description of the classroom setting
    - Time management
    - Maintaining order of tasks and activities
- Teaching activities
    - What activities are included?
    - How long does it take?
    - Teacher’s role during the activity
- Types of teaching activities
    - Whole class activities
    - Group activities
    - Individual activities
- Teaching strategies
    - Displaying activities and tasks
    - Organizing activities

Teaching methods

Teacher's use of materials

- Textbook use

  How teachers/ students use the textbook

  When teachers/ students use the textbook

  What exercise/examples teacher chose from the textbook

  The structure of the textbook and the structure of the lesson

  Other resources and materials used

- Teacher's language

  Instructional language

  Questions language

  Feedback techniques

- Students' level of engagement

  During tasks (groups and individual)

  During instruction

  During discussion

- General observations:

Appendix C:
Textbook analysis of the Saudi mathematics textbooks

Textbooks play a fundamental role in education and are at the center of mathematics classroom practice. Although the authors of textbooks generally regard the student as the primary reader, teachers rely heavily on textbooks in their day-to-day practice. For most teachers, textbooks determine and control how lessons are framed, crafted, and delivered (Love & Pimm, 1996). In many cases, textbooks become the real curriculum that is filtered through the actual learning experience of teachers and students.

In 2010, the Ministry of Education in Saudi Arabia introduced new mathematics textbooks. The new textbooks replaced the previous mathematics textbook which had been used in Saudi high schools for more than 30 years. For the purpose of understanding the mathematics textbooks used in Saudi Arabia, I use a critical analysis framework that is based on the work of Love and Pimm (1996), Herbel-Eisenmann and Wagner, (2007), Herbel-Eisenmann (2007), Remillard (2005), Morgan, (1996), and Rotman (1988, 2000). The framework focuses on highlighting the look and voice of the text, and how the reader perceives it. I apply this framework on one chapter from the old and the new textbooks.

Although Saudi teachers do not officially use the old textbooks in their classroom, it is important to include an analysis of the old textbooks in this study. The old textbooks had been used as the official curriculum document for more than 30 years. As such, they have had a great influence in shaping the culture of mathematics teaching practices in Saudi Arabia. All teachers who participated in my study had learned from these old textbooks in school when they were students and had experience teaching from these textbooks. Also, researchers such as Manouchehri and Goodman (2000) indicate that when mathematics teachers adopt a new curriculum or textbooks, changes in teachers' practices do not necessarily occur. Manouchehri and Goodman (2000) concluded that even after the implementation of new textbooks, teachers' practices continue to be influenced by their experience teaching from previous textbooks. Therefore, a critical analysis of the old and new textbooks provides a general examination of the nature of

the two textbooks. This examination is important to the later stages of my research when looking at how the participant teachers experienced teaching the two textbooks and how this experience may inform their practices in classrooms.

The framework for the textbook analysis

Love and Pimm (1996) comment on the shortage of research in mathematics education related to the use of texts. They indicate that the reason may be the difficulty in gathering data on the use of textbooks. Herbel-Eisenmann and Wagner (2007) also refer to the lack of appropriate theoretical frameworks that provide deep insight about textbook use. They argue that there is a need for a framework that sheds light on how teachers use textbooks in classrooms. Remillard (2005) points out that in shaping the enacted curriculum, the teacher plays the most influential role compared with the textbook. While Remillard (2005) discusses the need for in depth research related to how teachers use curriculum materials, Herbel-Eisenmann and Wagner (2007) assert that understanding teachers' use of textbooks is not possible without an analysis of the textbooks. Furthermore, van Dormolen (1986) indicates that when teachers interact with the textbook to plan their lessons, they need to make decisions. These decisions are based on "an analysis of the text, on the abilities of the student, on the teachers' goals and objectives and on any other relevant circumstances" (p. 142). Therefore, textbook analysis is an important step for understanding teachers practice in classroom (Herbel-Eisenmann and Wagner, 2007).

Mathematical texts differ from text in other subjects. The use of symbols in mathematical texts is one of the subject's main distinctive features (Pimm, 1987). Mathematical texts present more concepts per word, sentence, and paragraph than any other subject (Schell, 1982). This feature of mathematics writing is one reason for the need to developed content-specific literacy skills in mathematics; this process will enable individuals to be able to read a mathematics textbook (Österholm, 2006). Morgan (1996) claims that mathematical texts are different in relation to "their subject matter, in the relationships between author and readers, and in the formation of argument" (p. 2). Rotman (2000) describes mathematics writing as words and phrases drawn from natural language mixed with marks, signs, symbols, diagrams and figures. This mix in the mathematical texts does not only follow different syntactical rules, but also different grammatical rules, as compared to natural language. Therefore, to conduct a critical

analysis of mathematical texts, researchers should pay attention to the presentation of all these features of mathematics texts (Herbel-Eisenmann & Wagner, 2007).

The approach offered by Love and Pimm (1996), Herbel-Eisenmann and Wagner, (2007), Remillard (2005), and Rotman (1988, 2000) highlights features of textbooks that are not directly related to the content, emphasizing the voice and look as well as structure of the textbooks. Otte (1986) argues that when we examine a text, we should consider it as both an “objectively given structure of information” and a “subjective scheme”. An “objectively given structure of information “requires looking at the physical structure of the text, or as Love and Pimm (1996) explain, “what can be seen when looking at such material” (p. 379). Examining textbook materials as “subjective scheme” focuses on the interaction between an actual reader and the textbook and how the reader perceives the text (Herbel-Eisenmann, 2007). It encompasses the readers’ cultural values, meanings, and perspective that mediate the readers’ interpretation of the objective structure (Remillard, 2005).

Love and Pimm (1996), Herbel-Eisenmann and Wagner, (2007), Remillard (2005), and Rotman (1988, 2000) use an analysis approach to focus mostly on written curriculum materials as objectively given structure. This approach allows researchers to examine the potential of the textbook materials for assisting or hindering the ideological and epistemological goals of the NCTM's Professional Standards for Teaching Mathematics (1991) (Herbel-Eisenmann, 2007).

In mathematics education, the conventional analysis of mathematics textbooks as an objectively given structure has examined features such as “mathematical ideas, their forms of representation, and their organization for student learning” (Herbel-Eisenmann, 2007, p. 347). Little research has focused on analyzing the language used in the textbooks. Morgan (1996) points to the role of mathematics language used in textbooks in student learning. Students’ experience with the language of mathematics textbooks influences students’ writing and the teachers’ reading of student writing. Also, Herbel-Eisenmann (2007) indicates that examining the language patterns in textbooks provides a means to investigate the ideological and epistemological issues of the textbook.

The voice of the text refers to how the text communicates to the reader, the subject it communicates about and how the text positions the reader (Love & Pimm, 1996; Remillard, 2005). It is how the text talks to the reader in order to guide their actions. Generally, the main questions when studying a text's voice is, “who is speaking” (Herbel-Eisenmann, 2007) and “to what extent [the author(s)] acknowledge their presence in the writing? And what pronoun(s) do they use to refer to themselves and the reader” (Love & Pimm, 1996, p. 380). Herbel-Eisenmann and Wagner (2007) explain that in creating a text, writers make conscious and unconscious choices, and that textbooks as examples of text have agency with respect to how they can structure relationships.

In her (2007) paper, Herbel-Eisenmann develops a framework to examine the voice of a text. She analyzed the interpersonal function in a text by drawing on three linguistic forms: imperatives, personal pronouns, and modality. Moreover, she described the ideational and textual aspects that relate to the construction of the reader. The most important aspects in examining the ideational function of the text “include (a) who is involved in doing what kinds of processes; and (b) the depiction or suppression of agency” (p. 351). In order to understand the textual function, the ways in which the text maintains consistency should be investigated; “e.g., modes of reasoning and the features of the text that preserve continuity” (Herbel-Eisenmann, 2007, p. 351).

Examining imperatives is one important aspect to understanding a text’s interpersonal function. It is significant to look at imperatives because they implicitly engage readers in the construction of mathematics and address them as members of the mathematics community (Morgan 1996; Herbel-Eisenmann, 2007). Rotman (1988) analyzed semiotics in mathematics and draws attention to the distinction between imperatives. He distinguishes between exclusive imperatives (such as write, put and find) and inclusive imperatives (such as explain, prove and consider). In exclusive imperatives, the reader is labelled “scribbler” because s/he is expected simply to follow direction, while in inclusive imperatives, the reader is “thinker” because s/he is expected to reflect on and interact with the world. However, Rotman (1988) explains that mathematical activity is often done in isolation. Therefore, a person can be both a scribbler and a thinker. The thinker pictures worlds and the scribbler is the agent of the thinker performing in these worlds.

Pronouns are another important aspect to understanding a text's interpersonal function. The first-person pronouns I and we, as well as the second-person pronoun you, are important pronouns to the construction of the interpersonal function of the text (Herbel-Eisenmann, 2007). The first person pronouns I and we show the author's personal involvement with the text and engagement with the reader (Morgan, 1996). The pronoun I is used to indicate an actual person practicing mathematics. The pronoun we draws readers into the picture and invites them to share in the activity as well as to be engaged in and persuaded by a presented argument (Herbel-Eisenmann, 2007). Pimm (1987) indicates that the use of the pronoun we presents some ambiguity with regard to whom we refers to. Morgan (1996) notes that the absence of a first-person pronoun fails to show the presence of human beings in the design of the text. The use of the second person pronoun you also engages the reader with the text because it communicates directly to the reader. However, the author can use the pronoun you in a general sense and not in reference to any particular person (Remillard, 2005). An author can also use the pronoun you to inform the readers about themselves, giving the author total control of the common knowledge (Herbel-Eisenmann & Wagner, 2007).

Modality refers to the level of certainty used in the writing. Modality of the text could be examined through the "use of modal auxiliary verbs must, will, could, etc., adverbs certainly, possibly, or adjectives e.g., I am sure that... " (Morgan, 1996, p. 6). The use of the adverb certainly indicates that mathematics is a subject that people have no doubt about; and the use of the adverb possibly indicates that there is more than one possibility when dealing with mathematics (Herbel-Eisenmann & Wagner, 2007). Another way to express modality in a text is through the use of hedges. Lakoff (1973) defines hedges as "words whose job is to make things fuzzier or less fuzzy" (471). He indicates that truth and falsity are a matter of degree. Regular language sentences can be more or less true or more or less false by using hedges.

Herbel-Eisenmann and Wagner (2007) indicate that pictures influence the reader's experience of the text just as written texts do. Recently, authors have been using more pictures and illustrations in mathematics textbooks (Remillard, 2005). Love and Pimm (1996) state that visual representations and images contained in mathematics textbooks have a range of purposes. Some of the visual representations are related to the mathematical ideas or instructional activities while others are unnecessary and serve no purpose. Herbel-Eisenmann and Wagner (2007) examine the role of images in

mathematics textbooks by comparing the use of generic drawings and particular photographs and by paying attention to the roles played by the people in the images.

The old mathematics textbooks in Saudi Arabia

In Saudi Arabia, textbooks hold the status of clearly reflecting official curriculum. All teachers are expected to follow the textbooks and cover them by the end of the school year. The Ministry of Education is the main authority in the country that issues textbooks, which are, used in all school level education systems (Al-Abdulkareem & Hentschke, 2004). The Ministry of Education distributes textbook series for free as a classroom resource; each student receives his or her own textbook. Within the Ministry of Education, the body in charge of textbook publication is the Center for Educational Development. All previously introduced mathematics textbooks have been written and developed by Saudi experts who work for the Center for Educational Development within the Ministry of Education.

The Ministry of Education has used textbooks as an influential tool to improve teachers' practices and students' learning of mathematics. Therefore, introducing a new textbook is an important part of any education movement in Saudi Arabia. In 1994, a new mathematics textbook was introduced in all boys' high schools across Saudi Arabia. Girls' school continued with the textbook that was introduce before 1994 with the reasoning that they were managed by the General Administration of Girls' Education which was independent from the Ministry of Education. However, one change of the recent reform movement is that girls' schools were put under the Ministry of Education, which was already managing boys' schools across the country. Therefore, the introduction of the new mathematics books in high school in 2011 included both girls' and boys' schools.

According to the cover of the old mathematics textbooks used in high schools, these textbooks were approved by the General Administration of Girls' Education to be used as the official textbooks for girls' schools. Saudi experts at the Ministry of Education developed these textbooks. The names of the authors are not included in the textbooks. Each grade has two textbooks; semester one and semester two textbooks. For each semester, the textbook has between four and five chapters.

Every chapter in the textbook is divided into six to nine lessons. The chapters do not include any introductions. The lessons in the textbooks follow one structure. It starts by presenting a definition of a mathematical concept or a theory followed by a solved example and then followed by an exercise. The same pattern repeats itself until the end of the lesson with includes a list of exercises. The textbooks do not rate the level of difficulty of the exercises and there is no indication of what mathematical skills the exercise is focusing on. Every chapter ends with a small summary that mostly includes a list of the exact definitions or theories presented in the chapter with no further explanation followed by general exercises similar to those presented after every lesson. The last part of every textbook includes the answers to most of the exercises included in the textbook.

The new mathematics textbooks in Saudi Arabia

In Saudi Arabia, one of the major reform initiatives directly addresses existing mathematics curriculum. In 2010, the Ministry of Education introduced new mathematics textbooks which is usually the primary, and sometimes only, resource for teachers. The Ministry sees this initiative as a major step towards creating change in teaching practices. The new mathematics textbooks in Saudi Arabia are based on the curricula published by McGraw Hill Education Learning Company.

A group of experts and specialists from the Ministry of Education in Saudi Arabia worked on the translation, editing and adaptation of the American version of the textbooks. The group of experts included specialists in: mathematics, curriculum and instruction, psychology, evaluation & assessment, education technology, design and production, Arabic language, and English language. The group of experts also included experienced mathematics teachers and educational supervisors. According to the General Director of Curricula in the Ministry of Education, around 20% of the original American version was adjusted by the experts mainly to adopt local culture. The group of experts also reorganized the content of the textbooks to enhance the scope and sequence in the grade 1-12 mathematics curriculum (“Ministry of Education”, 2014).

The new high school mathematics textbooks that are used in Saudi Arabia have a different title than the original American version. For grade ten, Saudi textbooks are entitled Mathematics 1 and Mathematics 2 and are used separately in semester one and

two. The original American version is titled Geometry for grade ten textbooks. The Saudi grade eleven textbooks are titled Mathematics 3 and Mathematics 4 for semester one and two textbooks. The original American version is titled Algebra2. For grade eleven, the Saudi textbooks are titled Mathematics 5 and Mathematics 6 for semester one and two textbooks, whereas the original American version is titled Precalculus and Algebra 2. The Saudi textbooks include the name of the original authors and consultants of the original American version as well as the names of the group of Saudi experts who translated and adapted the American version.

According to the Ministry of Education in Saudi Arabia, the new mathematics curriculum aims to (a)help students develop higher-order mathematics thinking skills, (b) develop ways of mastering these skills, (c)construct a strong conceptual foundation in mathematics that enables students to apply their knowledge, (d) make connections between related mathematical concepts and between mathematics and the real world, and (e) apply mathematics logically to solve problems from daily life (“Ministry of Education”, 2013).

The Ministry of Education introduced the new textbooks gradually in 2010 starting with grades one, four, and seven. The 2011-2012 school year saw the new textbooks introduced in grades two, five, eight and ten. The new textbooks were then introduced to grades three, six, nine and eleven for the 2012-2013 school year. Finally, grade twelve students began using the new textbook at the beginning of the 2013-2014 school year. Therefore, by 2013, all grades in Saudi Arabia had received the new textbooks.

Each grade has two textbooks; semester one and semester two textbooks. In high school, the textbook for each semester has four chapters; every chapter is divided into lessons. All six textbooks for high school have the same introduction outlining the objectives of the textbooks; these objectives are the same as those on the Ministry of Education’s website (“Ministry of Education”, 2013).

Every chapter starts with a “get ready for the chapter” lesson. This lesson starts with the title, and then outlines previously covered skills and concepts, the purpose, and the learning outcome of the chapter. It also includes an image illustrating how students can make a brochure which helps them organize the information included in the chapter.

The “get ready for the chapter” lesson also includes a quick test section and a quick review section. Both these sections include questions and examples related to skills and concepts that students are expected to know.

Each lesson in the textbook begins with the title written in both Arabic and English, previously covered skills and concepts, learning outcome of the lesson, and the major mathematical vocabulary used in the lesson in both Arabic and English. The purpose section presents information usually related to real life situations and sometimes requires the students to answer questions that follow the information. Every lesson contains three parts: (a) instruction, (b) performance, and (c) assessment. The instructional part contains step-by-step explanations, definitions, theories and examples of the concepts or skills being presented. Sometimes this part includes a section with the title and a real life example. In this section, the textbook presents an example from real life where the concept presented in the lesson can be applied. The performance part contains two sections: (a) check your understanding, and (b) practice and problem solving. The assessment part contains three sections; (a) higher order thinking problems, which requires the use of complex thinking skills, (b) a test practice, and (c) a cumulative review.

Every chapter has a mid-chapter quiz in the middle of the chapter, a study guide and review, an end of chapter test, and a cumulative practice test which contains questions and problems from all the chapters presented in the textbooks from the two semesters of the same year. Another noticeable feature of the lessons is the use of the margins. Every lesson has little boxes in the margins which are usually titled with (a) guidelines for study; this provides some general information about the concept presented, (b) guidelines for the test; this provides some tips for the test, (c) caution; these are warnings for the student about common mistakes, (d) real life connection; this provides general information about some every day concepts presented in the lesson and its relation to mathematics concepts, and (e) reading mathematics; these are tips about how to read mathematics writing, including mathematics symbols.

Partial textbook analysis

In order to provide a clearer picture about the old and new textbooks, I will apply the framework I presented previously, which is based on the work of Love and Pimm

(1996), Herbel-Eisenmann and Wagner, (2007), Herbel-Eisenmann (2007), Remillard (2005), Morgan, (1996), and Rotman (1988, 2000) to one chapter of the old and new grade eleven mathematics textbook.

The purpose of this analysis is not to evaluate the two textbooks or to focus on the difference on the mathematical content presented in the old and new textbook. The purpose is to provide a general understanding of the look and voice of the text and how the reader perceives it. This examination of the two textbooks can also help me to understand some aspects of the general teaching perspective the textbooks reflect. In order for the analysis results to be reasonable and to avoid the misperception that the differences uncovered in analysis resulting from the two textbooks is due to the differences of the main mathematical content of the two chapters, I chose to analyze two chapters that have the same title in the old and new textbooks. As such, I chose to analyze the chapter entitled “Trigonometry” from the new and old grade eleven mathematics textbooks.

This analysis focuses only on the student editions, not the teachers’ guides. The reasoning behind this choice is that the student edition is the only one that both the teacher and the student read and use most of the time in the classroom. Since there is no electronic version available for the old textbooks, I used a paper version only for analyzing the old textbook. However, I used both electronic and paper versions for analyzing the new textbooks because the electronic version allows for easy searching of particular words or symbols more efficiently.

The old grade eleven mathematics textbooks have five chapters in the semester one textbook; these are matrices, groups, Analytic geometry, Vectors, and trigonometry. The semester two textbook has four chapters: complex numbers, exponential and logarithmic functions, mathematical induction, and probability and statistics. The grade eleven new mathematics textbooks have four chapters for both semester one and semester two textbooks. The chapters for the semester one textbook are: functions and inequalities, matrices, polynomials and polynomial functions, and inverses and radical functions and relations. The semester two textbook chapters are: rational functions and relations, sequences and series, probabilities, and trigonometry.

In this analysis, I examined the two chapters word-by-word, sentence-by-sentence, and section-by-section to identify and classify particular linguistic forms Love and Pimm (1996), Herbel-Eisenmann and Wagner, (2007), Herbel-Eisenmann (2007), Remillard (2005), Morgan (1996), and Rotman (1988, 2000) highlighted. I examined imperatives, pronouns, and modality from the text. I also investigated the visual images appearing in the chapter. For the purpose of clarity and brevity, I will refer to the chapter entitled “Trigonometry” from the old textbook as Chapter 1 and the chapter entitled “Trigonometry” from the new textbook as Chapter 2.

Imperatives

Chapter 1 includes thirty-four pages with six lessons while Chapter 2 has seventy-two pages with eight lessons. Questions and imperatives are the two most commonly found forms in the two chapters. Chapter 1 has 123 questions and 148 imperatives. Chapter 2 has 265 questions and 352 imperatives. Both chapters include a section with exercises after every lesson. Chapter 2 contains more sections with exercises than Chapter 1; it includes a mid-chapter test, an end of chapter test, a cumulative test, and a guide for studying section which includes more exercises. This could explain the high number of pages, questions and imperatives found in Chapter 2.

The only imperatives in Chapter 1 are find, write, prove, solve, and notice. The most common imperative in Chapter 1 is find, with 96 instances; the least frequently used imperative in Chapter 1 is notice, with only two instances. Imperatives found in Chapter 2 include find, use, describe, write, graph, solve, identify, explain, prove, select, see, choose, construct, compare, draw, round, collect, fold, check, and illustrate. . The most commonly occurring imperative in Chapter 2 is find, with 42 instances; the least common imperatives in Chapter 2 are collect, with two instances and fold, with one instance. According to Morgan (1996), imperatives impact the roles and relationships between the author and reader. All imperatives in Chapter 1 and most imperatives in Chapter 2 direct the reader to do mathematical activities. They allow the authors to address the reader using an authoritative voice about the material of the chapter.

Other imperatives found only in chapter 2, but less often, include consider with 4 instances and suppose with 12 instances. These imperatives “implicate the reader, who is addressed implicitly by the imperative form, in the responsibility for the construction of

the mathematical argument" (Morgan, 1996, p.6). The imperatives consider and suppose occurred mostly in the instruction part of the lessons.

Another feature of the two chapters is the high frequency of exclusive scribbler imperatives, such as find, solve and write compared to inclusive thinker imperatives, such as explain, prove, compare and illustrate. Of the 148 imperatives in Chapter 1, only 15 were inclusive thinker imperatives in form of prove; and of the 352 imperatives in Chapter 2, 62 were inclusive thinker imperatives in form of explain, prove, compare, construct, describe and illustrate. The imperative illustrate is the most common inclusive imperative in Chapter 2.

Although Herbel-Eisenmann (2007) indicates that the use of exclusive scribbler imperatives are more common than inclusive thinker imperatives in most mathematics school textbook, I expected to find more inclusive thinker imperatives in Chapter 2 since the introduction of the new textbook states that the textbook aims to develop higher-order mathematics thinking skills. Also, Rotman (1988, 2000) explains that actual practice in the mathematics classroom may not emphasize the use of inclusive imperatives. For example, a thinker imperative may get transformed into a scribbler imperative when the teacher does not direct students to explain their results. Therefore, the use of inclusive and exclusive imperatives cannot be determined merely by looking at the textbooks.

Pronouns

According to Morgan (1996), first person pronouns I and we show the "author's personal involvement with the activity portrayed in the text" (p. 5). The first person pronoun we is found eight times in Chapter 1 while there are no instances of the use of I. In Chapter 2, I found 9 instances of pronoun we, mainly in the instruction sections. The use of we in the two chapters seems to refer to a mathematics community rather than to a relationship between the authors and reader (Herbel-Eisenmann & Wagner, (2007).

The first person pronoun I was found in Chapter 2 twenty times. The first person pronoun I is limited to the learning outcomes or objectives section of every lesson. For example, the learning outcomes indicated in the first lesson "Trigonometric Functions in Right Triangles" are: (a) I find the values of trigonometric functions of acute angles (b) I

use trigonometric functions to solve a right triangle. The pronoun I was not found in the instruction, performance, or assessment sections.

Although Morgan (1996) states that the use of the pronoun I models an actual person practicing mathematics, the use of pronoun I in Chapter 2 does not seem to serve this purpose. Since the pronoun I was not found in the instruction, performance, or assessment sections where students actually engage with the textbook, the restricted use of the pronoun I to indicate the learning outcomes does not have a noticeably effective impact on the reader. The absence of the pronoun I in Chapter 1 also indicates the texts inability to perceive the reader as practicing mathematics. The two chapters' inability to employ the use of first person pronouns I and we effectively conceals the existence of human beings in the text which distances "the author from the reader, setting up a formal relationship between them" (p. 6).

The second-person pronoun you is entirely absent in Chapter 1. This indicates a weak construction of roles and relationships between readers and authors (Herbel-Eisenmann, 2007). The text fails to address the reader and build a relationship between author and reader or between reader and subject matter (Morgan, 1996).

I found twenty-seven instances of the second-person pronoun you in Chapter 2. Although this may indicates that the author is trying to construct a relationship with the reader, some cases of the second-person pronoun you in Chapter 2 come in the form by looking at…, you can see or you can found. In these cases, the authors are telling the readers exactly what they have to see or find and directing the readers' perception by identifying what the reader should perceive from the text. The authors are "addressing an individual reader personally and directing her attention with a degree of authority" (Morgan, 1996, p. 6).

Modal verbs

The language used in the two chapters contains modal verbs that communicate a high degree of certainty. The modal verbs that mostly occur in the texts are can, must, and will. The use of hedges was entirely absent from the two chapters. The authors provide the mathematical knowledge with strong conviction implying that mathematics is something that people are always sure about. The chapters do not give readers the opportunity to questions the offered information. Burton and Morgan (2000) state that

"the naive assumption might be that because mathematics is about certainty, mathematical writing would always have the same absolute modality" (p. 438). Herbel-Eisenmann and Wagner (2007) criticize the high degree of certainty in mathematics textbook writing which presents mathematics with an absolutist image that implies mathematics is not dependent on human particularities.

Passive voice

Another noticeable linguistic feature is the use of the passive voice. I found 9 instances of passive voice in Chapter 1 and 13 instances in Chapter 2. According to Burton & Morgan (2000), the use of passive voice can be seen as a way to hide human agents in mathematical texts. Sometimes, the use of passive voice is not merely a choice about style, but is a choice about whether to represent or to hide the function of the agent in the process.

In most cases in the two chapters, I found the use of the passive voice completely unnecessary. For example, when the authors talk about trigonometric identities in Chapter 1, they use the phrase "more trigonometric identities can be proven…". In this example, the authors have chosen not to present the reader as the person proving the trigonometric identities. In Chapter 2, when the authors explain converting angles between radians and degrees, they use the passive voice; "angles can be measured in units of either degrees or radians…". The sentence would provide clearer meaning if it was written, "you can measure angles in units of either degrees or radians". This way, the sentence would clarify the role of the reader when measuring angles.

Images

Since the mathematical discourse includes not just language, but also visual semiotic resources, I decided to compare the two textbooks in terms of the use and nature of non-linguistic features. Chapter 1 has only two graphs; both are for unit circles. No tables or other images are included. All the writing is in black and white except for the titles and headings which are in red.

Chapter 2 is more colorful with black, green, blue, red and yellow and has more images. Most images that appear in the chapter are unit circles, triangles, tables and

graphs of functions. Other images presented in the chapter include generic drawings and particular photographs. The chapter has 15 photographs and 29 generic drawings. Most of the photographs and generic drawings are not related to mathematical ideas or instructional activities. They contain pictures of everyday objects that are mentioned in word problems in the chapter. For example, one of the problems explains that a wave surfer rotates at a 540 angle in the air; the reader is asked to draw the 540 angle in its standard position. On the same page, there is a photograph of a wave surfer.

Only four images have pictures of real humans; the others are regular objects from everyday life such as a tree, a boat and a calculator. No images exist in the chapter about humans doing mathematics or investigating a mathematical idea. It is clear that the authors are trying to connect the mathematical concepts to the readers' everyday life by using images of objects from everyday life. However, in some cases, images are unnecessary and serve no purposes (Love & Pimm, 1996).

It is also noticeable that all the photographs in the chapter represent Saudi Arabia's environment. For example, one photograph shows an image of a Saudi man wearing a traditional Saudi outfit walking in a street that looks like a typical Saudi area. The authors try to reflect the image of Saudi society in the textbook by including photographs that the reader will identify as being part of their environment and culture. The generic drawings, however, do not seem to reflect Saudi society. In one image, for example, we see a drawing of a blond person playing in a theme park. It seems that the generic drawings included in the Saudi textbook are taken from the original English version of the McGraw Hill textbooks.

Teaching perspective

While completing the analysis of Chapter 1 and Chapter 2, I carefully read each word to understand its significance. This analysis helped me to develop a broad understanding of the general teaching perspective of the two textbooks.

It is noticeable that the structures of the lessons in the two chapters have many differences, and as a result, the two mathematics textbooks are quite different. Lessons in the old textbooks seem simple, starting with a short introductory that is very straightforward, followed by explanations and the main notion (definition or theory) and then ending with an example and a few exercises.

Comparatively, lessons in the new textbooks starts by clearly stating the objectives of the lesson and the mathematics vocabulary used in the lessons written in both Arabic and English. Before the introduction of any definition of new mathematical concept, the lesson presents the purpose section. In this section, the reader engages with information usually related to a real life situations and usually requires the reader to answer some questions. The accompanying exercises presented in the purpose section aim to guide students to new notions.

The new textbook also provides the reader with many sidebar comments about the presented concept and how it can be applied in problem solving. The examples that show each step of the solution come after the main notion (definition or theory) followed by check your understanding exercises. The structure of the old textbook does not provide the reader with enough information about the presented mathematical notion. For example, the lessons do not ask the reader questions in relation to the purpose or the importance of the mathematical notion as do the lessons in the new mathematics textbooks. Therefore, the old mathematics textbooks emphasize teacher-directed instruction by allowing the teacher to be the main source of information in the classroom. On the other hand, the new mathematics textbooks encourage a teaching style where students develop and discover mathematical concepts. The textbooks try to get mathematics to make sense to the reader by offering examples and exercises that explain why and how learning mathematics is useful.

Mathematical communication is also emphasized throughout Chapter 2 more than in Chapter 1. Chapter 2 reflects mathematics teaching that supports students' development of the use of mathematics language and fosters familiarity with the mathematical vocabulary. While Chapter 1 explains mathematics vocabularies only by presenting the mathematical definition of the concepts, every lesson of Chapter 2 starts with a presentation of the mathematics vocabulary used in the lessons. Most lessons in Chapter 2 have little boxes in the margins titled mathematics language. These boxes usually include information that demonstrates the correct use of specialized mathematical terminology and notation. Sometimes, they also explain the difference between the use of a particular word in everyday life and in mathematics. For example, in one of these boxes, the authors comment about the different way the word "relation" is used in everyday language and as a mathematical concept.

When analyzing the use of imperatives, I noticed a greater variety of linguistic choices by the authors of the new textbooks as compared to the old one. This variety indicates the authors are trying to communicate with the reader using diverse methods of presenting different mathematics ideas. It also offers the reader a richer environment to learn and understand the presented mathematical content. Using a limited number of imperatives, such as find, write, proof, solve, and notice as in the case of the Chapter 1, could limit the reader's view about the presented mathematical content. The new textbook encourages mathematics teaching that focuses on developing students' mathematical communication.

Most of the imperatives found in the two textbooks are used in the exercise sections. Most exercises in Chapter 1 start with the imperative find which is an "exclusive" imperative that addresses the reader as "scribbler". Most of these exercises are conducive to promoting teaching that focuses on procedural rather than conceptual knowledge. Also, exercises that start with the imperative find are "close-ended" problems. Nam Kwon & Park (2006) indicate that closed-ended exercises focus primarily on finding an answer that is a number or figure. Also, closed-ended exercises do not allow students to explain their thinking processes.

Exercises in Chapter 2 use the imperatives find, write, prove, and solve as well as discuss, compare, write using your own words, and construct. These exercises encourage students to discuss and describe, verbally or in writing, mathematical objects and concepts which enrich their conceptual knowledge. The diversity of the imperatives used in the new textbooks may result in teachers using a larger variety of methods which will allow students to demonstrate their knowledge about the mathematical concepts presented using both verbal and nonverbal mathematical representation.

Every lesson in Chapter 2 includes exercises and activities at different difficulty levels as well as a higher order thinking problems section. This section includes different exercises which fall under five sub-titles: (1) open-ended problem; includes a question or problem which has multiple correct solutions and more than one strategy to obtain the answers. (2) challenge; includes a question with higher order thinking skills, (3) find the mistake; presents two imaginary students' answers of a certain problem and the reader must determine which one is right and correct the mistakes, (4) justifying; provides a statement about a mathematics concept and the students must justify and explain it, and

(5) writing; students must their own words to explain a mathematical concept and how to apply it to solve problems. Most problems in this section are “open-ended” and engage students with genuine mathematical ideas and encourage exploration and discussion providing teachers with valuable information that can inform their teaching while eliciting several responses (Capraro, An, Ma, Chavez & Harbaugh, 2012; Nam Kwon & Park, 2006).

As mentioned above, Chapter 1 contains two graphs along with the written mathematical symbols. No other images or representations are used in the chapter. On the other hand, Chapter 2 employs many mathematical representations, generic drawings and particular photographs. Images and mathematical representations can help to enhance students’ conceptual knowledge. The new mathematics textbooks attempt to reflect teaching practices that support students’ development of mathematical meaning by relating mathematics to real situations using mathematical representations of real situations.

From my analysis of Chapter 1 and Chapter 2, I find the style of writing for both textbooks is quite authoritative; the use of modal verbs in both chapters supports this idea as they communicate a high degree of certainty. This language could reflect teaching practices with a traditional absolutist view. Teaching mathematics with an absolutist notion of mathematics recognizes mathematics as a subject with a broad collection of firm and impeccable concepts and skills (Romberg, 1992). Or as Ernest (1991) describes it, a set of unrelated, but utilitarian rules and facts. This viewpoint could reflect teaching that emphasizes memorization of rules and formulas and procedural knowledge.

Conclusion

Textbooks are important tools for mathematics teaching in classrooms. As such, textbook analysis is an important step for understanding teachers’ practice in the classroom. In this part I used frameworks from the literature of mathematics textbook analysis to do a partial textbook analysis of one chapter from the old and new mathematics textbooks used in Saudi Arabia. Mathematics textbooks play an important role in mathematics education as they determine and organize the mathematical content

of classroom teaching and construct classroom lessons with the examples, exercises, problems and activities they offer.

Love and Pimm (1996) explain that while authors write textbooks with the assumption that students are the main readers, teachers' use of the textbook greatly influences students' experiences with the textbook. Teachers, like students, are readers of the textbooks they use in mathematics classrooms and their use of the textbook influences their teaching practice. Pimm (2009) states "materials and texts are at best seen as one starting point; they usually require teachers to be thoughtful, aware, and autonomous to use them successfully" (p. 196). Therefore, examining and understanding the nature of the textbooks could offer insights about the nature of teachers' practices in the classroom.

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