

Competition and efficiency: application to tax haven, profit shifting and platform competition

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Abstract

The first chapter considers the tax information exchange agreement (TIEA) as a way to draw Pareto improvement between off-shore tax havens and non-haven countries. We find that establishing a TIEA can make the non-haven country increase the tax rate so that both tax evasion and welfare increase. This is because non-haven country government catches tax criminals with greater probability so it can handle greater volume of tax evasion under higher tax rate. Because not only the non-haven country's welfare but also the capital inflow into the tax haven are increased, tax havens would sign TIEAs with non-haven countries voluntarily in anticipation of greater future payoff.

The second chapter, I, jointly with Dr. Mongrain and Dr. Ypersele, develop a continuous fiscal competition model in which two countries compete over multinational firms (MNF) by varying the tax rates and the tightness of profit shifting control. Being loose on profit shifting decreases the tax base of one country but at the same time it brings two benefits. First, the country attracts more MNFs for given tax rate. Second, loose control of one country allows the other country to set high tax rate by alleviating the pressure of tax competition. Since the tax rates of the two countries are strategic complements, both countries can achieve efficiency gain by not actively controlling international profit shifting.

The third paper studies the competition in the video game platform markets where improving the quality standard may decrease the number of software developed. The disadvantage of the quality upgrade in one platform is spread to its competitors through 'porting,' and the platforms tend to increase the quality standard by extra amount. Our model shows that the quality competition in the competitive market is excessive because the platforms fail to internalize the inter-platform externality.

Keywords: applied microeconomics; public economics; industrial organization; efficiency; tax competition; tax evasion; tax haven; profit shifting; platform competition; indirect network externality

Dedication

*To my wife, Hanna, for the motivation,
to my parents for the endless love and support,
to my brother and sister for the patience,
and to my lovely cat, Reno, for waking me up every morning.*

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Table of Contents

Approval	ii
Abstract	iii
Dedication	iv
Acknowledgements	v
Table of Contents	vi
List of Figures	viii
1 Tax information exchange agreements for pareto improvement	1
1.1 Introduction	1
1.2 The tax evasion model	4
1.3 Tax payer behavior	7
1.4 Optimal tax rate with information exchange	8
1.5 Composition change in enforcement	13
1.6 Welfare and Fairness	17
1.7 Conclusion	18
2 The price may actually be right; potential benefits of profit shifting	20
2.1 Introduction	20
2.2 The Model	22
2.2.1 Profit Shifting	23
2.2.2 Location Decisions	25
2.3 Government's decision	27
2.3.1 Fiscal Best Response Functions	27
2.3.2 Shape of the best response functions	29

2.4	Equilibrium	32
2.4.1	Symmetric equilibrium tax rate	32
2.4.2	Asymmetric equilibrium tax rate	33
2.5	Optimal profit shifting control	35
2.6	Conclusion	37
3	Platform competition in the presence of cross-platform indirect network effect	38
3.1	Introduction	38
3.2	Platform competition model	41
3.2.1	Demand	42
3.2.2	Supply	42
3.2.3	Market clearing condition	45
3.3	Cooperative standard setting	46
3.3.1	Quantity decision	46
3.3.2	Cooperative quality decision	47
3.3.3	Cooperative portability decision	48
3.4	Competitive standard setting	49
3.4.1	Quantity decision	49
3.4.2	Competitive quality decision	50
3.4.3	Competitive portability decision	52
3.5	Asymmetric platforms	53
3.6	What happened in 7th generation?	54
3.7	Conclusion	56
	Bibliography	56

List of Figures

Figure 1.1	GDP of tax havens	3
Figure 1.2	Tax payer group change after a TIEA	18
Figure 2.1	Allocation of Firms	25
Figure 2.2	Best response tax rates when $C < 1$	31
Figure 2.3	Symmetric equilibrium tax rate	33
Figure 2.4	Asymmetric equilibrium tax rates	34
Figure 2.5	Change of Δ^e when α_1 increases	35
Figure 3.1	Portability and Sony's game division profit	39
Figure 3.2	Optimal quality choices	51
Figure 3.3	Cumulative video game operating income	55

Chapter 1

Tax information exchange agreements for pareto improvement

1.1 Introduction

International tax evasion that utilizes off-shore tax havens significantly shrinks the tax bases of non-haven countries. For instance, approximately \$2 trillion dollars of the United States' assets are held by the Caribbean Banking Centers, which include Bahamas, Bermuda, Cayman Islands, Netherlands Antilles and Panama, according to the 2011 U.S. department of Treasury reports. Zucman (2013) also estimates that 8% of the global financial wealth of households is held in tax havens and three-quarters of the hidden wealth is unrecorded. With growing awareness of international tax evasion and flourishing tax havens¹, the Organization for Economic Cooperation and Development (OECD) and the G-20 nations declared the war against tax evasion and have taken action since 2000. This paper develops a model of establishing Tax information exchange agreements (TIEAs), one of OECD countries' effort to fight against tax evasion, to present how the agreements can induce the mutually beneficial outcome for both the non-haven countries and the tax havens.

There are different types of tax havens around the world and the individuals' use of the tax havens also vary.² One typical use of tax havens is the tax avoidance. Individuals try to reduce the tax burden by profit shifting which could be sometimes beneficial as shown by

¹The tax havens exhibited average growth rate of 3.3 percent between 1982 and 1999 compared to 1.4 percent world average. See Hines (2005).

²One conventional and widely-used definition about the tax havens, on which this paper is based, is OECD's. According to OECD (1998), four key factors of tax havens are 1. no or only nominal taxes, 2. lack of effective exchange of information, 3. lack of transparency, and 4. no substantial activities.

Hong and Smart (2010), Desai et al. (2006a) and Desai et al. (2006b). Their intuition is that the non-haven countries may gain from the existence of tax havens because the individuals eventually benefit the non-haven countries by conducting more investment with the saved cost. However, these papers tend to overlook the illegal tax evasion that is undertaken at the same time; some individuals or firms simply resist to declare the income generated in the tax havens because the tax havens do not share banking information with other organizations or jurisdictions. Our paper focuses on the harmful tax evasion that utilizes the banking secrecy and the non-transparency of low-tax jurisdictions.³

Although the tax havens may have some benefits on the non-haven countries, they are generally considered harmful in terms of world welfare because non-haven countries waste real resources to protect their tax base. Slemrod and Wilson (2009) show that eliminating tax havens, either fully or partially, strictly improves welfare in non-haven countries. However, depriving tax havens of all their existing financial privilege is not an easy task, which confronts countries' sovereignty. Therefore it could be easier and more desirable to seek for Pareto improvement as the first step toward the fundamental solution, and we find that TIEA is one of such devices.

A TIEA is a bilateral agreement between two countries promising to exchange tax information relevant to the administration and enforcement of their domestic tax laws. The main purpose of TIEAs is to prevent tax havens from concealing the banking information of non-compliant tax payers because banking secrecy is another key element of tax havens⁴ other than low tax rates or stabilized legal system⁵. By giving the right to request tax information to administer its own tax laws, a TIEA greatly improves the efficiency of enforcement in the signing countries. As of 2014, 518 bilateral agreements have been made worldwide.

OECD publicly exerted substantial pressure on tax havens⁶, and we observe that the pressure was effective; all of the existing offshore tax havens have signed at least 12 TIEAs in order not to be marked as uncooperative jurisdictions⁷. The worrisome part was the freedom of choosing the partners; the tax havens could have neglected the rule by signing agreements with other tax havens or the countries with insignificant economic linkage. However, the

³Narrowing our scope onto the tax havens based on OECD definition excludes several tax havens such as Ireland, which are popular for profit shifting but still agree to provide the relevant tax information to other countries, from our list.

⁴See Schwarz (2009) for the advantage of holding banking secrecy.

⁵The empirical result in Dharmapala and Jr. (2009) supports that the tax havens tend to be better governed.

⁶See the final communique of the G20 summit in London on April 2, 2009

⁷Since May 2009 when Andorra, the Principality of Liechtenstein and the Principality of Monaco are removed from the list, No countries are marked as uncooperative tax havens by OECD.

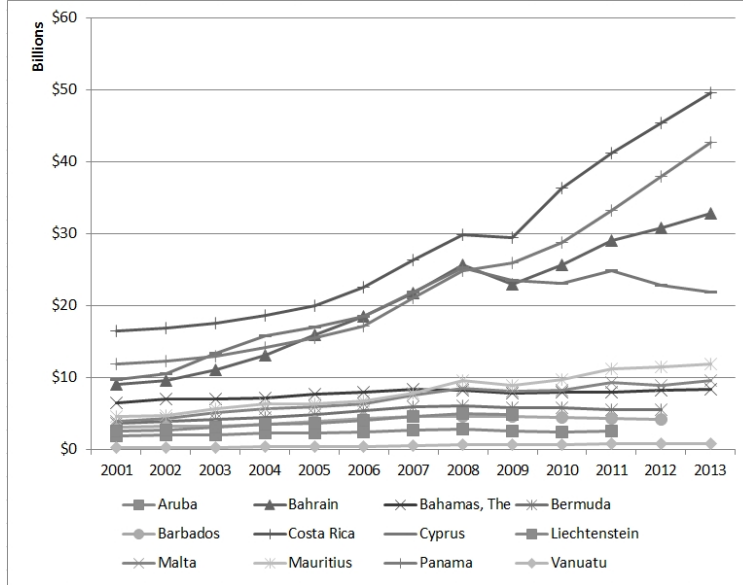


Figure 1.1: GDP of tax havens

evidence provided by Bilicka and Fuest (2014) shows that the tax havens actually signed the agreements with partners with which they have greater economic interaction instead of choosing other tax havens.

Then an interesting question arises: why did the tax havens make the agreements not only with their peers but also with close non-haven countries? One of the conventional answers to this question is the effectiveness of pressure⁸; the non-haven countries that are more linked to the tax havens economically can exert penalization more effectively. But if the non-haven countries had the power to punish havens for not signing the agreement, they would have capitalized the bargaining advantage, resulting in great loss of tax havens. However, GDPs of tax havens, although the shadow economy portion is not fully represented in the figure, show a temporary sink in 2009 when the entire world faced depression but bounced back after⁹. Also, according to Buehn and Schneider (2012) who estimated time series of OECD countries' tax evasion, the tax evasion of all countries showed the declining trend from 1999 to 2009 but then it started to increase in 2010. Given that 70 percent of entire TIEAs are established in 2009 and 2010, the recent increasing trend of tax evasion since 2009 looks odd. Therefore we suggest another hypothesis: considering more than 88 percent of TIEAs are

⁸See Bilicka and Fuest (2014) and Elsassyad (2012).

⁹Note that Cyprus is the only country that faced notable GDP decrease recently. Cyprus did not sign any TIEAs but it only established several double taxation conventions (DTC) with much delays. Considering DTCs are tax havens' preferred option over TIEAs, it is possible that Cyprus bore more pressure to avoid TIEAs. See Elsassyad (2012) for the different types of tax treaties.

made in between the non-haven countries and the tax havens and the tax havens did not perish after a number of TIEAs are established, TIEAs may not harm - or rather benefit - the tax havens so the havens did not mind choosing close non-haven countries as partners. In this paper, we show that the non-haven country raises the statutory tax rate after a TIEA so that the capital inflow into the tax havens would be increased.

There are a few studies on the process and determinants of tax information exchanges. Konrad and Stolper (2015) study the role of investors' expectations in tax havens' decision to sign a TIEA. They explain that investors' expectation about a tax haven's revenue pool creates a matching game structure; the tax havens that are expected to have large international investment remain tax havens while the ones with lower expectations cease to function as tax havens and sign the TIEA. Our paper can contribute to this study in two ways. First, we pay attention to the fact that some low tax jurisdictions that already signed TIEAs are still functioning as tax havens. Second, we suggest the possibility of costless instruments instead of costly international pressure to establish the TIEAs.

In the next section, we introduce a model with a non-haven country, a tax haven and the residents living in the non-haven country that can decide to participate in tax haven operation. In section 1.3, we derive the optimal tax evasion behavior of individuals. In section 1.4, we describe the optimal tax rate change with a TIEA and how it makes both the non-haven country and tax haven better off. Section 1.5 explores the composition change of enforcement after a TIEA. Section 1.6 analyzes the trade-off between welfare and fairness. Section 3.7 concludes.

1.2 The tax evasion model

We consider the perfect information game that consists of a non-haven home country, an off-shore tax haven that has zero tax and a mass of individuals who reside in the home country. At first, the home country federal government sets the domestic income tax rate t . Given the tax rate, each individual with taxable income 1 decides whether to hide a certain proportion of income in the off-shore tax haven. Let s denote the proportion of income hidden in the tax haven. When an individual submits a tax file, it is audited with probability a . Within the audited files, the revenue agency can detect the non-compliant tax file with the probability of $P(i)$ with $P(1) > P(0)$ where i is the indicator function that takes a value of 1 if tax information exchange agreement is established and 0 otherwise. When the revenue agency succeeds to identify a non-compliant individual, they collect fine $f > 1$ per every dollar

concealed¹⁰. To summarize, a non-compliant individual can successfully conceal s percent of income with the probability of $1 - aP(i)$, or he/she gets caught with the probability of $aP(i)$ and end up paying $f > 1$ per each dollar concealed. For now let us assume that the audit rate is fixed and the detection probability only depends on the tax information exchange.¹¹

The cost of income shifting process is $\theta + \frac{\nu s^2}{2}$ where θ represents the fixed setup cost, s represents the proportion of income hidden and ν is the marginal cost of hiding 1 percent of income. The fixed setup cost θ can be interpreted as the costs paid to lawyers and accountants for researching the relevant tax laws and havens. Each individual has different fixed cost, and we assume that θ is distributed uniformly with the density of $\psi < 1$.¹²The variable cost, which can be interpreted as accounting fees to research better income-shifting strategy, increases in the amount of income hidden. Both fixed and variable costs are fully wasteful so they will not be transferred to the tax haven.¹³

We are now ready to define the payoff functions of each player. The expected payoff of an individual who takes advantage of income shifting can be represented as:

$$\left(1 - t(1 - s(1 - aP(i)f))\right) - \left(\theta + \frac{\nu s^2}{2}\right) \quad (1.1)$$

Note that $1 - aP(i)f$ represents the expected marginal profit of concealing 1 percent of income. Therefore, increased a or establishing an tax information exchange agreement strictly reduces the payoff of non-compliant individuals *ceteris paribus* by reducing the marginal profit of concealment. We will assume that $aP(i)f < 1$ at any given (a, i) to make tax evasion attractive all time to a certain degree.

By adding up the after tax profit of all individuals, we can derive the consumer surplus:

¹⁰In the real world, detected tax criminals pay back the tax obligation at first and then pay additional penalty which is proportional to the tax obligation. In order to simplify the notation in the model, we make the fine include the pay back of initial tax obligation.

¹¹We will relax this assumption later to see the composition change of the optimal audit and effort.

¹²Each individual in the home country owns the same level of pre-tax income but differs in setup cost θ . We can let the income level vary instead and the results are mostly identical. We can therefore interpret the low participation cost as the high income which stands for the greater incentive to perform tax haven operation.

¹³Generally most of such costs are generated in the home country accounting offices, so we assume that the tax havens do not directly benefit from these costs.

$$\begin{aligned}
CS &= \underbrace{(1 - \alpha)(1 - t)}_{\text{compliant individuals}} + \underbrace{\alpha(1 - t(1 - s(1 - aP(i)f))) - \left(E[\theta|\theta \leq \Theta] + \frac{\nu s^2}{2}\right)}_{\text{non-compliant individuals}} \\
&= \underbrace{(1 - t(1 - \alpha s(1 - aP(i)f)))}_{\text{consumer surplus without cost}} - \underbrace{\alpha\left(E[\theta|\theta \leq \Theta] + \frac{\nu s^2}{2}\right)}_{\text{tax planning cost}}
\end{aligned} \tag{1.2}$$

where α denotes the share of individuals that participate in tax evasion and Θ denotes the setup cost of the marginal non-compliant individual. Accordingly, the total tax revenue net of the enforcement expenditure is

$$TR = t(1 - \alpha s(1 - aP(i)f)) \tag{1.3}$$

Note that $1 - aP(i)f$ in the home government's perspective is the marginal loss of tax revenue net of fine collection; the initial tax evasion is $t\alpha s$ but the government recovers $t\alpha s \cdot aP(e; i)f$ as the fine revenue.

The federal government maximizes the domestic welfare W which is the sum of consumer surplus and the tax revenue. We assume that each dollar of tax revenue has an advantage over the same amount of consumer surplus by $\lambda > 0$. The background of this assumption is the marginal cost of public funds; if the government loses tax collection due to tax evasion, they need to finance its budget from other distortionary sources such as labor or consumption. Based on this assumption, the welfare of a non-haven country can be represented as:

$$W = CS + (1 + \lambda)TR \tag{1.4}$$

Finally let us define the payoff of the tax haven. Although the tax haven does not collect any tax, it indirectly benefits from capital inflow that facilitates its economy.¹⁴ Therefore we assume that the haven's payoff H is defined by the amount of income shifted from the home country¹⁵:

$$H = \alpha s \tag{1.5}$$

¹⁴The benefit could be service fee for increased 'concealment service' or increased direct/indirect investment in the tax haven.

¹⁵Some literatures such as Bucovetsky (2014) assumes that the tax haven only benefits from fixed service fee, but adopting such setting does not affect our result qualitatively.

Note that the haven strictly prefers a greater amount of income shifting.

We consider the subgame perfect equilibrium of which the time line is as follows:

1. Tax rate, enforcement, and individuals' income shifting decision which are optimal without a TIEA are effective from the beginning.
2. The home country proposes a TIEA offer to the tax haven. *Status quo* is preserved if the offer is declined.
3. The home country federal government chooses the optimal tax rate given the new tax information availability.
4. Each individual decides whether to participate in income shifting and chooses optimal concealment s .

1.3 Tax payer behavior

We address individual's optimal behavior with and without a TIEA in this section. Individuals in the non-haven country maximize their after-tax income (1.1) by choosing s and deciding whether to participate in the tax haven depending on their fixed setup cost θ . Solving the first order condition derives each individual's optimal proportion of hidden income given t and tax information i :

$$s(t; i) = \frac{1}{\nu} t(1 - aP(i)f) \quad (1.6)$$

Since all individuals have the same variable costs of income shifting, the optimal proportion of hidden income is the same across all individuals regardless of their θ . Also, greater t increases the optimal proportion of hidden income, and establishing TIEA decreases it *ceteris paribus*. The optimal proportion of income shifting function, i.e., $s(t; i)$, depicts the intensive tax evasion of each individual.

Once an individual participates in a tax haven, the net benefit of income shifting without counting the participation cost is:

$$\Theta = ts(1 - aP(i)f) - \frac{\nu s^2}{2} \quad (1.7)$$

Each individual decides to participate in a tax haven when he/she earns positive gain from it, hence the individuals with $\theta \leq \Theta$ will actually conduct tax haven operation. As θ is

distributed uniformly in $[0, 1/\psi]$ with density ψ , the proportion of individuals that choose to participate is $\alpha = \psi\Theta$.¹⁶ Using the intensive income shifting function (1.6), the share of participating individuals given t and i can be defined as:

$$\alpha(t; i) = \frac{\psi}{2\nu} t^2 (1 - aP(i)f)^2 \quad (1.8)$$

Note that $\alpha(t; i)$ illustrates the extensive dimension of income shifting. Again, we observe that greater t facilitates participation while establishing a TIEA suppresses it. By multiplying intensive and extensive income shifting functions, we can derive the total income shifting function of:

$$H(t; i) = \alpha(t; i)s(t; i) = \frac{\psi}{2\nu^2} t^3 (1 - aP(i)f)^3 \quad (1.9)$$

which also represents the payoff function of the tax haven given t and i . Similarly, the higher statutory tax rate increases the total tax evasion in two channels. First, greater t increases intensive income shifting by raising the marginal benefit of hiding one percent of income. Second, a higher tax rate boosts the extensive income shifting by increasing the concealment's profitability thus attracting individuals with greater participation cost θ .

Finally we consider the impact of the information exchange. For any given t , the information exchange strictly decreases the intensive tax evasion $s(t)$ because increased probability of detection decreases the expected return from income shifting. Similarly, the information exchange strictly decreases the extensive tax evasion $\alpha(t)$ because the decrease of the expected return drives out the individuals with high θ . Therefore, the total tax evasion $H(t)$ strictly decreases after the information exchange for any given t .

1.4 Optimal tax rate with information exchange

In this section, we derive the equilibrium tax rate and analyze how it is affected by the information exchange. At first, inserting (1.6), (1.8) and (1.9) into (1.2) derives the consumer surplus function for given t and i :

¹⁶We will only consider the interior solution by setting ψ low enough.

$$\begin{aligned}
CS(t; i) &= 1 - t \left(1 - \frac{\psi}{4 \cdot 2\nu^2 m} t^3 (1 - aP(i)f)^4 \right) \\
&= 1 - t \left(1 - \frac{1}{4} (1 - aP(i)f) H(t; i) \right)
\end{aligned} \tag{1.10}$$

Note that the total tax evasion increases the consumer surplus for saving tax. However, because of the wasteful accounting cost, the net gain from tax evasion is reduced to a quarter of the original amount of tax saving.¹⁷

Similarly, we can derive the tax revenue function for given (t, i) using (1.6), (1.8), (1.9) and (1.3):

$$\begin{aligned}
TR(t; i) &= t \left(1 - \frac{\psi}{2\nu^2 m} t^3 (1 - aP(i)f)^4 \right) \\
&= t \left(1 - (1 - aP(i)f) H(t; i) \right)
\end{aligned} \tag{1.11}$$

Then the domestic welfare function for given (t, i) is:

$$\begin{aligned}
W(t; i) &= 1 + \lambda t - \left(\lambda + \frac{3}{4} \right) \frac{\psi}{2\nu^2} t^4 (1 - aP(i)f)^4 \\
&= 1 + \lambda t \underbrace{\left(1 - \frac{\lambda + \frac{3}{4}}{\lambda} (1 - aP(i)f) H(t; i) \right)}_{\text{net benefit of taxing}}
\end{aligned} \tag{1.12}$$

Note that the home country government takes the tax evasion more seriously when choosing t to maximize the welfare function (1.12) than the tax revenue function (1.11): $\frac{\lambda + \frac{3}{4}}{\lambda} > 1$. This is because the welfare function takes individuals' tax evasion cost, which is a pure loss, into consideration while the tax revenue function does not.

Let us discuss the effect of the tax information exchange on the non-haven country's domestic welfare.

Lemma 1. *The home country is always better off by establishing a TIEA.*

We first examine the immediate effect of a TIEA on the home country's domestic welfare before the home country changes its tax rate. For any given t , establishing a TIEA improves the non-haven country's domestic welfare by two channels. First, the agreement decreases the total tax evasion; recall that a TIEA strictly decreases the total tax evasion $H(t)$ for

¹⁷The amount of reduction depends on the variable cost of the tax evasion.

given t because fewer individuals participate and each individual evade less proportion of income. Second, the information exchange increases the detection probability so the non-haven country recovers the evaded tax with greater probability. Since any rational change of tax rate made after a TIEA would further increase the domestic welfare, the home country is always better off by establishing a TIEA.

Now let us analyze the marginal effect of the information exchange to see the change of the optimal tax rate. The following lemma compares the optimal tax rates with and without the information exchange:

Lemma 2. *Optimal tax rate increases when a TIEA is established: $t^*(1) > t^*(0)$.*

The first order condition of (1.12) with respect to t is:

$$\underbrace{\lambda - (\lambda + \frac{3}{4})(1 - aP(i)f)H(t; i)}_{\text{marginal benefit}} = t \underbrace{(\lambda + \frac{3}{4})(1 - aP(i)f)H_t(t; i)}_{\text{marginal cost}} \quad (1.13)$$

The left-hand side represents the marginal benefit which is the value of additional tax collection. The right-hand side represents the marginal cost of the tax increase which is the result of the increased tax evasion.¹⁸ As a direct effect, establishing a TIEA strictly increases the marginal benefit by decreasing the total tax evasion, $H(t; i)$, and the net loss of tax revenue, $1 - aP(i)f$. Indirectly, the marginal cost of tax increase, which is the increased tax evasion, strictly decreases after a TIEA for two reasons. First, the increased probability of detection makes the change in the tax evasion less important. Second, because a TIEA reduces the total volume of tax evasion proportionally, the total tax evasion becomes less sensitive to the tax change after the agreement: $H_t(t; 1) < H_t(t; 0)$ for given t ¹⁹. Because a TIEA increases the marginal benefit but decreases the marginal cost, the optimal tax rate rises.²⁰

Alternatively, using the fact that the increased marginal revenue and decreased the marginal cost by a TIEA are both resulted by the tax total evasion change, we can rewrite (1.13) as:

¹⁸Since marginal benefit is concave and marginal cost is convex, solving (1.13) yields $t^*(i)$, the optimal tax rate for given tax information availability.

¹⁹It can be easily shown that $H_t(t; 1) = \frac{3\psi}{2\nu^2}t^2(1 - aP(1)f)^3 < H_t(t; 0) = \frac{3\psi}{2\nu^2}t^2(1 - aP(0)f)^3$.

²⁰Formally, the change of $W_t(t; i)$ by the agreement is:

$$\Delta W_t = -(\lambda + \frac{3}{4})(1 + \epsilon_t)(-a\Delta P f \cdot H(t; i) + (1 - aP(i)f)\Delta H(t)) \quad (1.14)$$

where $\epsilon_t = \frac{\partial H(t)}{\partial t} \cdot \frac{t}{H(t)} > 0$ denotes the tax elasticity of total income shifting. Recall that $\Delta P > 0$ and $\Delta H(t) < 0$. Therefore, ΔW_t is strictly positive, which means that the optimal tax rate increases after a TIEA.

$$\underbrace{\lambda}_{\text{marginal benefit without evasion}} = \underbrace{\left(\lambda + \frac{3}{4}\right)(1 - aP(i)f)(1 + \epsilon_t)H(t; i)}_{\text{combined marginal cost}} \quad (1.15)$$

The left-hand side of (1.15) can be interpreted as the marginal benefit of tax increase without any tax evasion. The right-hand side represents the combined marginal cost of tax increase net of fine collection. Note that the combined marginal cost embraces both the current tax evasion, which accounts for the reduction of marginal tax revenue in (1.13), and further increase of tax evasion due to the tax increase. Also note that the combined marginal cost increases in t while the raw marginal benefit of tax is a constant. The tax rate is determined so the combined marginal of tax revenue is equal to the raw marginal benefit of tax.

Now we are in a position to characterize the change of total tax evasion in the new equilibrium. Although the total tax evasion decreases after a TIEA for any given t , the home country would prefer a higher tax rate with tax information in hand. We now show that the total tax evasion would rather increase in the new equilibrium because the higher tax rate enlarges the total tax evasion by increasing its profitability. The following proposition summarizes the equilibrium tax rate and welfare changes:

Proposition 1. *When a TIEA is established, the home country increases the tax rate to the level at which the total tax evasion is greater than status quo. The increased total tax evasion also makes the tax haven strictly better off.*

For ease of analysis, let us first assume that fine is collected from the detected individuals but it does not contribute to the tax revenue or welfare. Such assumption changes (1.15) to:

$$\lambda = \left(\lambda + \frac{3}{4}\right)(1 + \epsilon_t)H(t; i) \quad (1.16)$$

In this case, a TIEA strictly decreases the combined marginal cost by decreasing $H(t)$ for given t while the raw marginal benefit of tax remains constant. Since the marginal cost is lower than the marginal benefit at the previous tax rate $t^*(0)$, the home country increases the tax rate to the level at which the total tax evasion becomes equivalent to the previous level: $H(t^*(1), 1) = H(t^*(0), 0)$. This result implies that the tax haven would remain indifferent if the home country government does not consider the fine revenue.

Now we discuss the effect of increased chance of detection. A TIEA increases the detection probability so the home country can recover the lost tax revenue with greater probability, making $1 - aP(1)f < 1 - aP(0)f$. This lets a TIEA reduce the combined marginal cost in (1.15) further than in (1.16) for any given t . Therefore, the home country can increase the

tax rate by the additional amount to have greater tax evasion than before: $H(t^*(1), 1) > H(t^*(0), 0)$. Intuitively, the home country can handle the greater volume and its expansion of tax evasion thanks to the superior detection probability so the new equilibrium tax rate achieves greater tax revenue and domestic welfare even if the total tax evasion also becomes bigger. Since the total tax evasion increases after a TIEA, such outcome is preferred by the tax haven, too.

The strict Pareto improvement is possible because of the difference in payoff structure of the home country and the tax haven. The non-haven home country is mainly concerned about the tax revenue, and becomes better off because the government recovers lost tax revenue from its domestic residents with greater probability. However, the tax haven is not interested in the transactions between the government and the residents of the non-haven country; the tax haven only cares about the capital inflow which is initiated by tax evasion. Even if a non-compliant individual gets caught for tax fraud and pays penalty to the revenue agency, his/her investment in the tax haven still remains. Note that, as previously mentioned, the capital inflow to the haven would be greater after a TIEA since the greater tax pressure from higher home country tax rate forces more people to shift greater portion of income to the tax haven.

One caveat here is the constant elasticity assumption. The tax elasticity of total income shifting in our model is constant since the income shifting function is in Cobb-Douglas form. Although it is derived by general expected utility maximization setup, we can discuss the effect of relaxing the constant elasticity assumption. First, if the tax elasticity increases after a TIEA, then the total income shifting is even greater than the constant elasticity case, so the result in proposition 1 still holds. Second, if the tax elasticity decreases after a TIEA, then we need to consider the “elasticity of elasticity.” If tax elasticity of income shifting decreases less elastically than the net marginal loss of tax revenue, then the result of proposition 1 remains. The only case in which the income shifting decreases is where tax elasticity of income shifting decreases more elastically than the net marginal loss of revenue. In this case, the acceptance constraint binds so the home country needs to offer an inefficiently high tax rate in order to establish a TIEA. Therefore the home country government needs to compare the welfare of the two states: excessive tax rate under a TIEA or *status quo*.

1.5 Composition change in enforcement

In this section we relax the fixed enforcement assumption to see the composition change of enforcement. To prevent tax evasion, the federal government operates a revenue agency which decides the audit rate, a , and the amount of effort devoted on each tax file audited, e .²¹ The revenue agency can either increase the extensive chance of detection a and auditing more tax files, or increase the intensive detection probability $P(e; i)$ by paying more effort on each tax file. Assume that $P_e(e; i) > 0$ and $P_{ee}(e; i) < 0$ for given e and i so that greater effort increases the chance of detection in decreasing rate. Also, assume that $\Delta P(e) = P(e; 1) - P(e; 0) > 0$ and $\Delta P_e(e) = P_e(e; 1) - P_e(e; 0) \geq 0$ for any given e so establishing a TIEA increases the chance of detection and it does not decrease the productivity of effort.

The revenue agency operates under a finite budget M to to maximize the tax revenue given tax rate:

$$TR(t, a, e; i) = t(1 - (1 - aP(e; i)f)H(t, a, e; i)) - cae \quad (1.17)$$

$$cae \leq M \quad (1.18)$$

where M denotes the budget size and c denotes the marginal cost of enforcement. Note that this objective is different from the federal government's which is to maximize the sum of consumer surplus and tax revenue. The assumptions about the fixed budget and different objectives are based on the way the agencies operate in most countries²², and comparing the outcomes of different environments will produce meaningful implications. The primary difference lies on how serious each organization is about the tax evasion. The following lemmas explain the differences.

Lemma 3. *Without the budget constraint, the revenue agency chooses higher (a, e) than the socially optimal level.*

This is a common finding from comparing the two objective functions. Note that the welfare function in enforcement decision stage is:

²¹Since non-compliant tax payers use various accounting skills to exploit tax laws' loopholes, each audited file needs a certain amount of effort e to identify that the individual actually concealed income from taxation.

²²Generally a revenue agency is given the finite budget as the other departments of the government. And unlike the federal government and its congress which are relatively more sensitive to the residents' voices, the performance of the revenue agency is rated based on maximizing the tax revenue and minimizing the tax crime.

$$\begin{aligned}
W(t, a, e; i) &= 1 + t\left(\lambda - \left(\lambda + \frac{3}{4}\right)(1 - aP(e; i)f)H(t, a, e; i)\right) - (\lambda + 1)cae \quad (1.19) \\
cae &\leq M
\end{aligned}$$

Since the revenue agency does not value the consumer surplus, it ignores the increased consumer surplus of which the source is tax savings of income shifting. From the social planner's point of view, such emphasis on enforcement is not optimal. Note that with the extreme assumption of $\lambda = \infty$, which makes the tax revenue incomparably superior to consumer surplus, both the federal government and the revenue agency ignore the positive side of tax evasion on the consumer surplus thus making the same enforcement choices. The following lemma highlights the virtue of fixed budget in the general cases:

Lemma 4. *Under a fixed budget, the revenue agency chooses the same (a, e) as what the federal government would choose with the fixed budget.*

Proof. See Appendix. □

The solutions of the revenue agency's and the federal government's maximization problems without budget constraints only differ in the levels; revenue maximization demands greater amount of (a, e) in the same proportion compared to welfare maximization. Because the composition of (a, e) is solely determined by the relative productivity of them, the solutions of the two objective functions exhibit the same proportion of (a, e) combination. Therefore, introducing the fixed budget, which gets rid of the level effect, makes the optimal (a, e) chosen in two different objective functions identical. This explains why most enforcement agencies in the world operate within fixed budgets.

Let us consider the optimal audit and effort choices. Audit and effort are complementary in the sense that each instrument enhances the other; effort is more valuable when the agency audits more files, and audit is more valuable when detection probability increases by effort. At the same time, they are also substitutes because budgetary issue forces the agency to lower effort on each file reviewed when it increases the audit rate, and vice versa. To see this, we can make (1.17) an unconstrained maximization problem such as

$$TR(t, a(e), e; i) = t\left(1 - (1 - a(e)P(e; i)f)H(t, a(e), e; i)\right) - M \quad (1.20)$$

where $a(e) = \frac{M}{ce}$ is the budgetary relationship between a and e derived from (1.18). Note that $a_e(e) < 0$ and $a_{ee}(e) > 0$. Then the first order condition is:

$$TR_e(t, a(e), e; i) = t(a_e(e)P(e; i)f + aP_e(e; i)f)H - t(1 - a(e)P(e; i)f)(H_a a_e(e) + H_e) = 0 \quad (1.21)$$

Because a and e are tied in a budget constraint, increasing e always brings the opposite effect resulted by decreased a . First, spending more budget on e would increase the expected marginal revenue of fine one one hand, but smaller a decreases the expected marginal revenue fine on the other hand. Second, greater e reduces total tax evasion but it indirectly increases the total tax evasion by losing a . It turns out that, since H is also the function of $1 - aP(e; i)f$, the degree of trade-off between (a, e) in the expected marginal revenue and tax evasion are the same so we can regard them as one. The reduced form of (1.21) is:

$$TR_e(t, e; i) = 4tf(a_e(e)P(e; i) + aP_e(e; i))H(t, a(e), e; i) = 0 \quad (1.22)$$

Note that the optimal audit and effort given information, $a^*(i)$ and $e^*(i)$, are determined where increased marginal benefit of e equals the reduced marginal benefit of a : $aP_e(e; i) = -a'(e)P(e; i)$. This condition reveals several findings:

Lemma 5. *Optimal choices of audit and effort are independent of tax rate if the budget is fixed.*

When the tax rate goes up, clearly both audit and effort become more valuable so the revenue agency may try to increase both (a, e) accordingly if there is no budget constraint. However, when the budget is fixed, the tax rate is neutral on the choice of each instrument because increase in t makes both audit and effort equally valuable. Similar to the conclusion of lemma 4, the change of the tax rate only results in the level effect which is removed by the budget constraint. Therefore the optimal audit and effort do not depend on the tax rate.

Lemma 6. *The optimal effort choice does not depend on the budget size while the optimal audit choice does.*

Now let us consider the effect of information exchange on the enforcement's composition. The following proposition explains the composition change:

Proposition 2. *If information exchange does not improve the productivity of effort, i.e. $\Delta P_e(e) = 0$, it is optimal to reduce effort on each audited tax file and increase the audit rate instead. If information exchange improves the productivity of effort, i.e. $\Delta P_e(e) > 0$, it is optimal to increase effort only if the productivity increase is sufficiently big.*

The change of (26) with respect to i is:

$$\Delta TR_e(t, e) = 4tf \left(\underbrace{\left(a_e(e)\Delta P(e) \right)}_{<0} + \underbrace{\left(a\Delta P_e(e) \right)}_{\geq 0} \right) H(t, e; i) + \underbrace{\left(a_e(e)P(e; i) + aP_e(e; i) \right)}_{=0 \text{ for envelope theorem}} \Delta H(t, e) \quad (1.23)$$

Increased probability of detection by tax information immediately decreases total tax evasion *ceteris paribus*; hence, both a and e become less valuable. However, such force does not affect the optimal composition of (a, e) because a and e become equally less important as in lemma 5. This is represented by the envelope theorem $a_e(e)P(e; i) + aP_e(e; i)\Delta H(t, e) = 0$ in 1.23. Therefore, the optimal composition entirely depends on the productivity change of the effort.

Note that the revenue agency's e choice rises only if $a\Delta P_e(e) > -a_e(e)\Delta P(e)$, meaning that the information exchange dramatically improves the productivity of effort so it outweighs the crowding-out of auditing. Since $-a_e(e) = \frac{M}{ce^2} = \frac{a}{e}$, the condition can be rewritten as:

$$\rho > 1 \quad (1.24)$$

where $\rho = \frac{\Delta P_e(e)}{\Delta P(e) \cdot \frac{1}{e}}$.

Finally we compare the outcomes of fixed enforcement and variable enforcement cases.

Lemma 7. *Compared to fixed enforcement case, both the home country and the tax haven are better off when the revenue agency can change the composition of enforcement.*

As lemma 5 states, optimal choices for (a, e) do not depend on tax rate but on the tax information, so the optimal audit and effort functions that solve (26) can be written as $a(i)$ and $e(i)$ respectively. The welfare function subject to the federal government's maximization is

$$W(t; i) = 1 + \lambda t - \left(\lambda + \frac{3}{4} \right) t (1 - a(i)P(e(i); i)f)H(t, a(i), e(i); i) - (\lambda + 1)M \quad (1.25)$$

of which first order condition is:

$$\lambda - \left(\lambda + \frac{3}{4} \right) (1 + \epsilon_t) (1 - a(i)P(e(i); i)f)H(t, a(i), e(i); i) = 0 \quad (1.26)$$

These not only look similar to (1.12) and (1.15) but also induce the same result; the optimal tax rate under a TIEA increases both total income shifting and domestic welfare. The only difference lies in the increased expected marginal revenue of fine, $a(i)P(e(i); i)f$. The tax information increases expected marginal revenue of fine even without the composition change, and the revenue agency chooses a better combination of them to further increase the expected marginal revenue of fine. Therefore, the net loss is lower in flexible enforcement situation than in the fixed one: $a^*(1)P(e^*(1), 1)f > a^*(0)P(e^*(0), 1)f$. Given that, the federal government can further increase tax rate to take advantage of greater recovery of tax evasion. Since the net marginal tax revenue is greater than the fixed enforcement scenario, the home country can handle a greater volume of tax evasion so both optimal tax rate and tax evasion become greater, too. Therefore, both home country and tax haven are better off than in the fixed enforcement case.

1.6 Welfare and Fairness

We showed that the home country's welfare is improved in all cases. We need to consider, however, the possible drawback which is unseen in the welfare analysis: the fairness across the individuals. When the home country increases tax rate with a TIEA, it induces with more people to participate in the tax haven. Such action creates three different groups of individuals: always-compliant tax payers, always-non-compliant tax payers, and the ones who were once compliant but decide to participate newly in income shifting after a TIEA.

Figure 1.2 depicts the fixed cost distribution of three different groups when a TIEA is followed by tax rate increase. As shown in the previous sections, the number of non-compliant individuals increases because the ones with greater θ participate in income shifting after tax increase. These newly participating individuals exhibit different gain and loss pattern from the other two groups that do not change their behavior. Note that, if individuals do not value the public goods funded by the tax revenue, they are always worse off after a TIEA because they simply pay greater tax but they do not consider the increased public good provision valuable. Although the simplification limits the analysis of individual well-being in absolute terms, it is still possible to argue the fairness across the groups assuming that all individuals have equal access to the public goods with same marginal utility;

Proposition 3. *When a TIEA is followed by tax rate increase, always-compliant individuals suffer the most, the individuals who newly become non-compliant are next worse off, and always-non-compliant individuals lose the least among the three groups.*

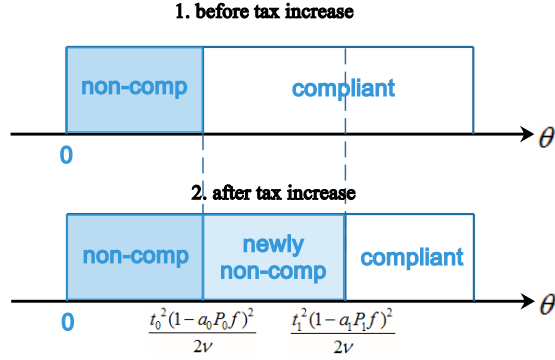


Figure 1.2: Tax payer group change after a TIEA

Proof. Let $j = A, B, C$ indicate always-non-compliant group, newly participating group and always-compliant group respectively. Define the wealth effect of a TIEA as $\Delta\pi^j = \pi^j(1) - \pi^j(0)$ where $\pi(i)$ is the equilibrium payoff of each individual given i . Recall in advance that $t^*(1)^2(1 - a^*(1)P(e^*(1), 1)f)^2/2\nu > \theta^B > t^*(0)^2(1 - a^*(0)P(e^*(0), 0)f)^2/2\nu$ where θ^B denotes the fixed cost of newly participating group; the gain from income shifting under increased tax rate is greater than the fixed cost under tax/enforcement combination under a TIEA but is smaller than that without a TIEA. We can easily show $\Delta\pi^B - \Delta\pi^C = t^*(1)^2(1 - a^*(1)P(e^*(1), 1)f)^2/2\nu - \theta^B > 0$ meaning that newly participating group is better off than always-compliant group after a TIEA. Also we can show that $\Delta\pi^A - \Delta\pi^B = \theta^B - t^*(0)^2(1 - a^*(0)P(e^*(0), 0)f)^2/2\nu > 0$ which means that always-non-compliant group is better-off than newly participating group after a TIEA. \square

Always-compliant individuals are the worst-off because they bear the entire burden of tax increase at all time. The individuals who newly become non-compliant are better than always-compliant individuals because they had a chance to participate in income shifting to attain positive gain after tax increase. Always-non-compliant individuals are even better off than the other two groups because they have low enough participation cost to always retain positive gains from income shifting. The result suggests that we need to be careful in tax-increasing TIEA as it could oppose fairness for squeezing the compliant group.

1.7 Conclusion

The tax havens erode tax bases of non haven countries by opening the channel of income concealment. Non-haven countries therefore should try to defend their revenue base by

enforcement activities and tax treaties with off-shore tax havens. Our analysis shows that establishing TIEAs would benefit not only the non-haven countries but also the tax havens. This result expects that the non-haven countries do not exert advantage in bargaining power, such as economic or political pressure, to make tax havens worse off.

There are a few research directions in which our model can be extended. First, our model made the enforcement budget fixed but one can relax this assumption and let the home country choose both the tax rate and enforcement budget. Especially, if the home country faces hardship in increasing tax rate for political reasons or the worry about individuals' relocation, it may consider decreasing the enforcement budget in order to satisfy the acceptance constraint and letting the haven sign the agreement. Enforcement budget as another endogenous policy instrument adds another dimension of synergy or trade-off to this model and may induce an interesting result.

Another direction could be to expand the model to multiple non-haven and haven countries' competition. Actually our model can be converted to multiple tax haven setup without any qualitative changes of results. As long as the non-haven country can prevent some tax havens from not signing TIEAs for free-ride²³ by imposing adequate sanction, the non-haven country increases the tax rate and the capital inflow to the havens increases as in our model with a single haven. A meaningful contribution could be made by the multiple non-haven countries setup as the non-haven countries now have choices of signing a TIEA with tax havens to fight against tax evasion or lower the tax or not signing them to allure non-compliant individuals. This will create similar competition environment as Peralta et al. (2006) with greater dimension.

²³There is an evidence that the efficiency of the TIEAs are reduced when the free ride is possible. See Johannesen and Zucman (2014).

Chapter 2

The price may actually be right; potential benefits of profit shifting

2.1 Introduction

Corporate tax collection is difficult, as firms have many tools at their disposition to reduce their tax burden. With international or multi-regional firms the challenge is even greater since firms can shift profits from high tax regions to low tax ones. Profit can be shifted by reorganizing debt or by altering internal firm prices to generate lower profit in a fiscally undesirable jurisdiction. An article in *The Economist*¹ titled “The Price isn’t Right” focuses on the fiscal cost of profit shifting. It is stated, for example, that US corporation are able to lower their effective tax rate by almost 20 percent, in great part by shifting profits. Due to the complexity of tax codes and the international aspect, monitoring and enforcing profit shifting is not a simple task, and is definitively a reason for the prevalence of fiscal leakage. In this paper, we shift the question away from monitoring and enforcing constraints and investigate if profit shifting can actually be beneficial even for a profit exporting region.

Many argued that profit shifting is harmful. The loss of fiscal revenue is an obvious reason to limit the extend of profit shifting. Other arguments have also been put forward, like Slemrod and Wilson (2009) who argue that profit shifting is undesirable because of the wasteful shifting cost. They also state that an increase of the shifting cost generally reduces the amount of shifting and may even increase the welfare. Some however, disagree and see some value in permitting some profit shifting. Hong and Smart (2010) argue that profit shifting can be beneficial if there are both mobile and immobile tax bases that cannot

¹The Economist, February 16th 2016

be differentiated by the tax authority. The intuition is that the government can use profit shifting similar to the preferential regime; adjusting the strictness of profit shifting control could be used as having two different tax rates on immobile firms and mobile firms. Similarly, Desai et al. (2006a) and Desai et al. (2006b) present both empirical and theoretical evidence that the profit shifting decreases the effective corporate tax rate and boosts the investment therefore it can benefit the host countries by increasing international investment.

While papers above are either based on static analysis or the single-country-assumption where competition between the host countries are not highlighted, there are several researches which are more focused on the strategic effects regarding profit shifting. As introduced in Keen and Konrad (2014), the tax competition model of Kanbur and Keen (1993) can be applied to profit shifting model where two countries compete for a single multinational firm. In this application, the country with smaller domestic tax base tries to undercut in order to attract the taxable profit of the multinational firm.² The competition for extensive margin is also studied in Ferrett and Wooton (2010) which develop a model in which two countries bid for one firm. Their result shows that the firm locates where it generates the higher welfare meaning that the outcome is efficient. Their result for efficiency, however, as shown in Kessing, Konrad and Kotsogiannis (2009), is not always satisfied when vertical fiscal externalities and free-riding problems in either country are considered.

Peralta et al. (2006) contribute to the tax competition literature in the presence of profit shifting by considering degree of profit shifting control as an additional dimension of competition. They introduced a tax competition model in which two asymmetric countries compete over a multinational firm by means of the corporation tax rates and profit shifting monitoring policies. In their analysis, choosing lax monitoring policy and allowing profit shifting could be beneficial to the bigger country if hosting the firm would increase domestic consumer surplus substantially; the firm would declare profit in the low tax jurisdiction, but the high tax country still benefits from the increased domestic economic activity. One important reason why profit shifting may be desirable is that it allows for discrimination between different tax base. By shifting profit, multinational firms who are also likely more mobile face a lower effective tax rate than local firms who must pay the higher statutory rate. The goal of this paper is to explore other reasons why allowing for some profit shifting may be beneficial.

We depart from Peralta et al. (2006) in two meaningful ways. In their paper there exists a single international firm shifting all profit. We model a mass of multinational firms that

²Bucovetsky (2015) also derives the similar results by adopting multiple firms which varies in the attachment to the home country.

may shift a portion of their original profit. This allows us to properly define two forms of tax base elasticity. Firm's location decision generates a tax base elasticity similar to the standard one present in virtually all tax competition models. Profit shifting generates a new form of tax base elasticity. Allowing for some profit shifting will essentially shift tax revenue from one base to the other. Relaxed regulations or monitoring of profit shifting leads to a loss of tax revenue for a given distribution of firms, but makes the region a more attractive location. Separating the two forms of elasticities also helps us investigate the strategic effects of relaxing profit shifting enforcement. Relaxing profit shifting regulations reduces the fiscal incentive to locate in a low tax region, making firms' location tax base more inelastic. It however increases the per-firm retained profit tax base elasticity. Consequently, tax rate may increase or decrease with more permissive regulation. If tax rate increases, it provides a new argument for more relaxed regulations. Second, we eliminate the local tax base to see whether the countries still have the incentive not to choose the strong monitoring policy when discrimination is not the reason.

In the next section we set up the model and characterize firms' location and profit shifting decision for a given menu of tax and enforcement policies. In section 3 we define governments' best fiscal reaction functions. In section 4 we solve for the tax competition equilibrium and look at how equilibrium tax rates vary with enforcement policies. In section 5, we investigate whether less than full effort at controlling profit shifting may be desirable even if such effort is costless. We then conclude. All proofs are in the appendix.

2.2 The Model

There exist two regions denoted 1 and 2. Each region sets a source base profit tax at a rate t_i . We will discuss the government's objective in detail later on. Governments can also control the cost of profit shifting via monitoring and well designed regulations, summarized by parameter α_i . To really focus on potential benefit associated with profit shifting, we assume that monitoring is costless.

The economy is composed of a unitary measure of firms. All firms are international and mobile. Firms decide where to locate their main revenue generating facility, which we refer to as the headquarter. Each firm generates A units of profits where the headquarter is located. Firms also operate overseas subsidiaries only for profit shifting purpose; each

firm generates zero profits in this location.³ In addition, firms are indexed by a location specific cost parameter c distributed on $[0, C]$, according to a density and a cumulative distribution function $f(c)$ and $F(c)$. For simplicity, we assume the function $F(c)$ to follow a uniform distribution. A firm of type c pays an additional un-observable cost c in Region 1 and $(1 - c)$ in Region 2. This c reflects the comparative advantage of one region in attracting a given firm. Region 1 has comparative advantage with low c firms. When $C = 1$, then the distribution of firms is symmetric and as many firms prefer Region 1 to Region 2. When $C < 1$, then more firms have a comparative advantage toward Region 1.

Let γ_i be the share of taxable profit a firm located in region i decides to shift to the other country. The cost to a firm of shifting a proportion γ_i of its total profits to the other region is $(1 + \alpha_i)g(\gamma_i)A$, where $g'(\gamma_i) > 0$ and $g''(\gamma_i) > 0$. We assume that $g'(1) \rightarrow \infty$, so firms never shift all profits. We also assume that $g'(0) = 0$, but $g''(0) > 0$. This implies that all firms shift some profit, unless α_i is arbitrary large. Finally, for simplicity we assume that $g'''(\gamma_i) = 0$.⁴ All those assumption together still allows for a large range of quadratic functions. The policy parameter $\alpha_i \in [0, \bar{\alpha}]$ is chosen by region i to make profit shifting more or less costly for firms. Define $\bar{\alpha}$ as the maximum cost level possible. When $\bar{\alpha} \rightarrow \infty$, governments can simply make profit shifting infinitely expensive. We also consider cases where policy can only do so much and arbitrary high cost to shift profit are not possible. In such a case $\bar{\alpha}$ would be finite.

2.2.1 Profit Shifting

Since firms only shift profits from a high tax region to a low tax one, it is important to distinguish whether the tax rate in region i is larger or lower than the one in region j . Denote the difference in tax rates by $\Delta = t_1 - t_2$. Imagine a firm located in the high tax Region 1 has to decide how much profit to shift to the other region. For any $\Delta > 0$, such a firm would shift profit according to

$$\gamma_1(\alpha_1, \Delta) = \arg \max_{\gamma_1} \left\{ \left[(1 - t_1) + \Delta(1 - \gamma_1) - (1 + \alpha_1)g(\gamma_1) \right] A \right\} \quad (2.1)$$

Since $g'(0) = 0$, all firms shift some profit, and since $g'(1) \rightarrow \infty$, no firm is willing to fully shift all its profits. The optimal amount of profit shifting $\gamma_1(\alpha_1, \Delta)$ is given by the first

³Normalizing subsidiaries profits to zero does not qualitatively influences the analysis. Having subsidiary profit $B < A$, would allow for the possibility of shifting profit from the subsidiary firm back to the main region of operation.

⁴Note that all results would be preserved as long a $g'''(\cdot)$ is not too negative.

order condition:

$$(1 + \alpha_1)g'(\gamma_1(\alpha_1, \Delta)) = \Delta. \quad (2.2)$$

Note that no profit shifting is done when $\alpha_1 \rightarrow \infty$. Profit shifting is increasing in the tax difference Δ , and decreasing with monitoring α_1 . Comparative static reveals that:

$$\frac{\partial \gamma_1(\alpha_1, \Delta)}{\partial \Delta} = \frac{1}{(1 + \alpha_1)g''(\gamma_1)} > 0, \text{ and } \frac{\partial \gamma_1(\alpha_1, \Delta)}{\partial \alpha_1} = -\frac{g'(\gamma_1)}{(1 + \alpha_1)g''(\gamma_1)} < 0.$$

We define $\varepsilon(1 - \gamma_1 | t_1)$ as Region 1's per-firm retained profit tax base elasticity with respect to its own tax rate, which is represented by:

$$\varepsilon(1 - \gamma_1 | t_1) = -\frac{\partial[1 - \gamma_1(\cdot)]}{\partial \Delta} \frac{t_1}{1 - \gamma_1(\cdot)} = \frac{t_1}{(1 + \alpha_1)g''(\gamma_1(\cdot))[1 - \gamma_1(\cdot)]} > 0. \quad (2.3)$$

Note that we express all elasticities in positive term, but retained profit are actually decreasing with domestic tax rate. Similarly, we can define the elasticity of the per-firm retained profit tax base with respect to its monitoring as $\varepsilon(1 - \gamma_1 | \alpha_1)$, where:

$$\varepsilon(1 - \gamma_1 | \alpha_1) = \frac{\partial[1 - \gamma_1(\cdot)]}{\partial \alpha_1} \frac{\alpha_1}{1 - \gamma_1(\cdot)} = \frac{\alpha_1 g'(\gamma_1(\cdot))}{(1 + \alpha_1)g''(\gamma_1(\cdot))[1 - \gamma_1(\cdot)]} > 0. \quad (2.4)$$

How tax rates influence the per-firm retained profit tax base elasticity is important to determine future results, in particular:

$$\frac{\partial \varepsilon(1 - \gamma_1 | t_1)}{\partial t_1} = \frac{\varepsilon(1 - \gamma_1 | t_1)}{t_1[1 + \gamma_1(\cdot)]} \left[1 + \frac{t_1}{1 - \gamma_1(\cdot)} \right] > 0. \quad (2.5)$$

As Region 1 increases its own tax rate, it faces a more elastic per-firm retained profit tax base. On the contrary, an increase in monitoring reduces the elasticity of the per-firm retained profit tax base with respect to its own tax rate:

$$\frac{\partial \varepsilon(1 - \gamma_1 | t_1)}{\partial \alpha_1} = -\frac{\varepsilon(1 - \gamma_1 | t_1)}{1 + \alpha_1} \left[1 + \frac{g'(\gamma_1(\cdot))}{g''(\gamma_1(\cdot))[1 - \gamma_1(\cdot)]} \right] < 0. \quad (2.6)$$

Additional monitoring increases the cost of shifting profits, and so firms require larger tax differential to be willing to shift profit. Consequently, they become less sensitive to variation in domestic tax rate.

Lastly, we define Region 2's per-firm acquired profit elasticity with respect to t_2 as $\varepsilon(\gamma_1 | t_2) = -\frac{\partial \gamma_1(\cdot)}{\partial \Delta} \frac{t_2}{\gamma_1(\cdot)}$, where:

$$\varepsilon(\gamma_1 | t_2) = \frac{t_2}{(1 + \alpha_1)g''(\gamma_1(\cdot))\gamma_1(\cdot)} > 0. \quad (2.7)$$

Also note that:

$$\frac{\partial \varepsilon(\gamma_1 | t_2)}{\partial t_2} = \frac{1}{t_2} \varepsilon(\gamma_1 | t_2) [1 - \varepsilon(\gamma_1 | t_2)] > 0. \quad (2.8)$$

Obviously, where $\Delta > 0$ no firm in Region 2 shifts profits. If Region 2 was the high tax region, then Δ would be negative and $\gamma_2(\alpha_2, -\Delta)$ would be define exactly the same way as $\gamma_1(\alpha_1, \Delta)$.

2.2.2 Location Decisions

When firms decide where to locate, they compare net profit associated with placing the headquarter in Region 1 versus Region 2. A firm with cost c locating in Region 1 generates the following profits:

$$\pi_1(c) = \left[(1 - t_1) + \Delta \gamma_1(\alpha_1, \Delta) - (1 + \alpha_1)g(\gamma_1(\alpha_1, \Delta)) \right] A - c \quad (2.9)$$

Similarly, one can also write the profit if the same firm was to locate the headquarter in region 2:

$$\pi_2(c) = (1 - t_2)A - (1 - c) \quad (2.10)$$

Define $\bar{c}(\Delta)$ as the productivity parameter so that a firm is indifferent between locating in one or the other region. Figure 1 below shows the allocation of firms across regions for a case where $\Delta > 0$.

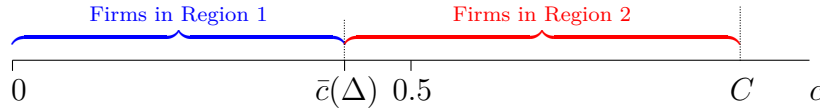


Figure 2.1: Allocation of Firms

Solving the indifference condition $\pi_1(c) = \pi_2(c)$ gives us $\bar{c}(\Delta)$:

$$\bar{c}(\Delta) = \frac{1}{2} - \left[[1 - \gamma_1(\alpha_1, \Delta)]\Delta + (1 + \alpha_1)g(\gamma_1(\alpha_1, \Delta)) \right] \frac{A}{2} \quad (2.11)$$

Firms with $c \leq \bar{c}(\Delta)$ chose Region 1. A reduction in Δ favours Region 1, as well as a reduction in monitoring α_1 . More specifically, we can show that:

$$\frac{\partial \bar{c}(\Delta)}{\partial \Delta} = -[1 - \gamma_1(\alpha_1, \Delta)] \frac{A}{2} < 0 \text{ and } \frac{\partial \bar{c}(\Delta)}{\partial \alpha_1} = -g(\gamma_1(\alpha_1, \Delta)) \frac{A}{2} < 0.$$

The second order derivative with respect to Δ is simply $\frac{\partial^2 \bar{c}(\Delta)}{\partial \Delta^2} = \left[\frac{\partial \gamma_1(\alpha_1, \Delta)}{\partial \Delta} \right] \frac{A}{2} > 0$. Region 1 loses firms at a faster rate when tax differential increases. Obviously, $\frac{\partial \bar{c}(\Delta)}{\partial t_1} = -\frac{\partial \bar{c}(\Delta)}{\partial t_2} = \frac{\partial \bar{c}(\Delta)}{\partial \Delta}$.

We can define Region 1's firms location tax base elasticity with respect to its own tax rate as $\varepsilon \left(\frac{\bar{c}(\Delta)}{C} \middle| t_1 \right) = -\frac{\partial \frac{\bar{c}(\Delta)}{C}}{\partial t_1} \frac{t_1}{\frac{\bar{c}(\Delta)}{C}}$, where:

$$\varepsilon \left(\frac{\bar{c}(\Delta)}{C} \middle| t_1 \right) = \frac{t_1 [1 - \gamma_1(\alpha_1, \Delta)] A}{1 - \left[[1 - \gamma_1(\alpha_1, \Delta)] \Delta + (1 + \alpha_1) g(\gamma_1(\alpha_1, \Delta)) \right] A} > 0. \quad (2.12)$$

Again we express all elasticities in positive term for more straightforward discussion. An increase in t_1 would obviously lead to a reduction in the number of firms located in Region 1. We can also define Region 1's firms location tax base elasticity with respect to its own tax rate as $\varepsilon \left(\frac{\bar{c}(\Delta)}{C} \middle| \alpha_1 \right) = -\frac{\partial \frac{\bar{c}(\Delta)}{C}}{\partial \alpha_1} \frac{\alpha_1}{\frac{\bar{c}(\Delta)}{C}}$, where:

$$\varepsilon \left(\frac{\bar{c}(\Delta)}{C} \middle| \alpha_1 \right) = \frac{\alpha_1 g(\gamma_1(\alpha_1, \Delta)) A}{1 - \left[[1 - \gamma_1(\alpha_1, \Delta)] \Delta + (1 + \alpha_1) g(\gamma_1(\alpha_1, \Delta)) \right] A} > 0. \quad (2.13)$$

We can define similar elasticities for Region 2. The firms location tax base elasticity with respect to its own tax rate define as $\varepsilon \left(1 - \frac{\bar{c}(\Delta)}{C} \middle| t_2 \right) = \frac{\partial \left(1 - \frac{\bar{c}(\Delta)}{C} \right)}{\partial t_2} \frac{t_2}{1 - \frac{\bar{c}(\Delta)}{C}}$ is given by:

$$\varepsilon \left(1 - \frac{\bar{c}(\Delta)}{C} \middle| t_2 \right) = \frac{t_2 [1 - \gamma_1(\alpha_1, \Delta)] A}{2C - 1 + \left[[1 - \gamma_1(\alpha_1, \Delta)] \Delta + (1 + \alpha_1) g(\gamma_1(\alpha_1, \Delta)) \right] A} \quad (2.14)$$

Note that unlike other elasticities, $\varepsilon \left(1 - \frac{\bar{c}(\Delta)}{C} \middle| t_2 \right)$ varies with C , i.e. $\frac{\partial \varepsilon \left(1 - \frac{\bar{c}(\Delta)}{C} \middle| t_2 \right)}{\partial C} < 0$. Since profit shifting flows only from high tax Region 1 to low tax Region 2, it implies that $\varepsilon \left(1 - \frac{\bar{c}(\Delta)}{C} \middle| \alpha_2 \right) = 0$.

2.3 Government's decision

Governments are assumed to maximize tax revenue. This is the most standard objective function employed in the literature. We can write the objective function of the government when $\Delta > 0$ as follows:

$$\Omega_1(\Delta > 0) = \int_0^{\bar{c}(\Delta)} t_1 [1 - \gamma_1(\alpha_1, \Delta)] Af(c) dc. \quad (2.15)$$

Similarly when $\Delta < 0$, total tax revenue are given by:

$$\Omega_1(\Delta < 0) = \int_0^{\bar{c}(\Delta)} t_1 Af(c) dc + \int_{\bar{c}(\Delta)}^C \gamma_2(\alpha_2, \Delta) t_1 Af(c) dc. \quad (2.16)$$

With simplification Region 1's objective function is:

$$\Omega_1(\Delta) = \begin{cases} \frac{\bar{c}(\Delta)}{C} t_1 [1 - \gamma_1(\alpha_1, \Delta)] A & \text{for } \Delta \geq 0 \\ \frac{\bar{c}(\Delta)}{C} t_1 A + \left[1 - \frac{\bar{c}(\Delta)}{C}\right] t_1 \gamma_2(\alpha_2, \Delta) A & \text{for } \Delta < 0. \end{cases}$$

Similarly, for Region 2, the objective function is:

$$\Omega_2(\Delta) = \begin{cases} \left[1 - \frac{\bar{c}(\Delta)}{C}\right] t_2 A + \frac{\bar{c}(\Delta)}{C} t_2 \gamma_1(\alpha_1, \Delta) A & \text{for } \Delta > 0; \\ \left[1 - \frac{\bar{c}(\Delta)}{C}\right] t_2 [1 - \gamma_2(\alpha_2, \Delta)] A & \text{for } \Delta \leq 0 \end{cases}$$

2.3.1 Fiscal Best Response Functions

We will now look at a region's optimal tax rate for a given foreign tax rate and a set of monitoring efforts. To simplify the notation, let FOC_i^+ denote $\frac{\partial \Omega_i(\Delta)}{\partial t_i} \Big|_{\Delta > 0}$ and FOC_i^- denote $\frac{\partial \Omega_i(\Delta)}{\partial t_i} \Big|_{\Delta < 0}$.

High tax Region 1 – The effect of a change in tax rate t_1 on Region 1's welfare when $\Delta > 0$ is given by:

$$\begin{aligned} FOC_1^+ = & \underbrace{\frac{\bar{c}(\Delta)}{C} (1 - \gamma_1(\alpha_1, \Delta)) A}_{\text{Gain from higher tax rate}} - \underbrace{\frac{\bar{c}(\Delta)}{C} [1 - \gamma_1(\alpha_1, \Delta)] \varepsilon \left(\frac{\bar{c}(\Delta)}{C} \mid t_1\right) A}_{\text{Loss from firms movement}} \\ & - \underbrace{\frac{\bar{c}(\Delta)}{C} [1 - \gamma_1(\alpha_1, \Delta)] \varepsilon (1 - \gamma_1 \mid t_1) A}_{\text{Loss from profit shifting}}. \end{aligned} \quad (2.17)$$

Based on (2.17), we can express the first order condition of Region 1 when $\Delta > 0$ as the following :

$$\underbrace{1}_{\text{Marginal benefit of tax}} = \underbrace{\varepsilon\left(\frac{\bar{c}(\Delta)}{C} \middle| t_1\right)}_{\text{Marginal cost from firms movement}} + \underbrace{\varepsilon(1 - \gamma_1 \mid t_1)}_{\text{Marginal cost from profit shifting}}. \quad (2.18)$$

Given all other parameters being constant, a tax revenue maximizing government aims to be on the top of the Laffer's curve. This implies that the elasticity on the tax base equal to one at the optimal. In our case, there are two elasticities to consider: i) the firms' location elasticity and ii) the per-firm retained profit elasticity. Intuitively, we can decompose the total tax base into two components: the firm mobility tax base and the profit shifting tax base, with each of their own elasticity. Increasing its own tax rate push firms to move away and remaining firm to shift more profit. The sizes of the elasticities in the equilibrium are described in the following lemma:

Lemma 8. *In any equilibrium where $\Delta \geq 0$, it must be the case that $\varepsilon\left(\frac{\bar{c}(\Delta)}{C} \middle| t_1\right) \leq 1$ and $\varepsilon(1 - \gamma_1 \mid t_1) \leq 1$.*

If we ignore the profit shifting completely, firms' location tax base elasticity with respect to t_1 would equal one.⁵ This would correspond to the top of the Laffer's Curve. When considering profit shifting, however, the firms location tax base elasticity must be less than one. Region 1 must also take into account the effect increasing taxes imposes on profit shifting too. The same reasoning apply to the per-firm retained profit tax base elasticity. We assume that the second order condition is satisfied. See the Appendix for the condition under which the second order condition are satisfied.

Low tax Region 2 – We now look at the tax effects in the low tax Region 2. We have that:

$$FOC_2^+ = \underbrace{\left[1 - \frac{\bar{c}(\Delta)}{C}\right] A + \frac{\bar{c}(\Delta)}{C} \gamma_1(\alpha_1, \Delta) A}_{\text{Gain from higher tax rate}} - \underbrace{\left[1 - \frac{\bar{c}(\Delta)}{C}\right] \left[1 - \gamma_1(\alpha_1, \Delta)\right] \varepsilon\left(1 - \frac{\bar{c}(\Delta)}{C} \middle| t_2\right) A}_{\text{Loss from movement of firms}} - \underbrace{\frac{A}{2(1 + \alpha_1)g''C} t_2}_{\text{Loss from profit shifting}} \quad (2.19)$$

⁵This would be similar to Mongrain and Wilson (2016).

Note that the last term can be also expressed as $\frac{\bar{c}(\Delta)}{C}\gamma_1(\alpha_1, \Delta)\varepsilon(\gamma_1 | t_2)A = \frac{\bar{c}(\Delta)t_2A}{(1+\alpha_1)g(\cdot)''C}$. The first order condition is then given by:

$$\underbrace{\left[1 - \frac{\bar{c}(\Delta)}{C}\right] \left[1 - [1 - \gamma_1(\cdot)]\varepsilon\left(1 - \frac{\bar{c}(\Delta)}{C} \middle| t_2\right)\right]}_{\text{Net marg. revenue: local firms}} + \underbrace{\frac{\bar{c}(\Delta)}{C}\gamma_1(\cdot) \left[1 - \varepsilon(\gamma_1 | t_2)\right]}_{\text{Net marg. revenue: foreign profit}} = 0. \quad (2.20)$$

Again the first order condition trade-off must take into account of the two tax bases, but now local and foreign firms contribute differently. When a local firm is taxed, it brings one unit of tax revenue. If the firm leaves, it only costs $1 - \gamma_1(\cdot)$ units because profits will be repatriated. Taxing foreign firms bring $\gamma_1(\cdot)$ unit of tax revenue, but profit shifting will contract. This effectively generates two weighted Laffer's curves.

The first order and second order conditions for $\Delta < 0$ where Region 1 sets lower tax rate than Region 2 are summarized in Appendix.

2.3.2 Shape of the best response functions

With the first order conditions in all cases, we look how the tax rate of one region affects the best response of the other region. Let $t_i^*(t_j)$ denote the best response tax rate of Region $i = 1, 2$ as a function of the other region. The following proposition states that the slope of the best response function is positive, which means that tax rates of two regions are strategic complements:

Lemma 9. *Taxes are strategic complements; $\frac{dt_i^*(t_j)}{dt_j} > 0$ for $i, j = 1, 2$ and $i \neq j$.*

As in most of the tax competition models, taxes are strategic complements. When one country increases its tax rate, the other region responds by increasing its own tax rate. Having that the best-response functions are upward-sloping, we also need to prove that the slope of best response functions are less than 1 with positive intercept for existence of the equilibrium. The following lemma shows that our best response functions exhibit the necessary properties.

Lemma 10. *The slopes of the high-tax-regions' best response functions are less than 1 and the intercepts are greater than 0.*

Although the best response functions have slopes which are positive and less than 1, they need additional attention because the functions are **discontinuous** around the 45° line. In order to understand what causes the discontinuity, let us first consider Region 1's best

response under full symmetry: $\alpha_1 = \alpha_2$ and $C = 1$. When Region 1 increases t_1 , the loss from additional profit shifted and from firms movement are the same whether Δ is positive or negative. The impact of firm mobility and profit shifting are identical in both regions. Therefore the best-response t_1 is the same around the 45° line and there is no discontinuity in this particular case.

Now suppose that Region 1 monitors profit shifting more aggressively, so $\alpha_1 > \alpha_2$. When $\Delta > 0$, profits flow from Region 1 to Region 2 and so monitoring α_2 is the relevant policy. When $\Delta < 0$, the flow of profits is reverse and monitoring α_1 is now the relevant policy. Therefore, the discontinuity arises at around $\Delta = 0$.

Another channel of discontinuity is C which can be interpreted as the relative number of firms favoring Region 1 over Region 2. With $C = 1$, Region 1 losses of firms and profits suffered when Δ is slightly positive exactly match the gains arising when Δ is slightly negative. In simple words, there as much to gain as there is to lose. With $C < 1$, Region 1 losses more firms and more profit with $\Delta > 0$ than it gains when $\Delta < 0$. The following two lemmas summarizes the effect of C on the best responses.

Lemma 11. *Lower C has no effect on high tax Region 1's tax decision, but increases best response t_1 of a low tax Region 1.*

In Region 1's point of view, a decrease in C reduces the importance of the foreign tax base. Where $\Delta > 0$, Region 1 only cares about the domestic tax base because there is no profit shifted from the foreign region. Therefore, the best response t_1 is neutral in C . On the contrary, C affects low tax Region 1's tax decision. Lower C reduces the foreign tax base relative to the domestic tax base. Region 1 is less interested in attracting foreign firms and profits, so Region 1 becomes less aggressive. A similar logic is applied to Region 2.

Lemma 12. *Lower C decreases best response tax rate of Region 2 for both $\Delta \geq 0$ and $\Delta < 0$. The degree of change by varying C is greater when $\Delta > 0$ than $\Delta < 0$.*

When Region 2 is the low tax Region, the intuition is the complete opposite to the case where Region 1 is the low tax region. Lower C means that fewer firms are favouring Region 2, making foreign profit shifting tax bases more important relative to the domestic firm tax base. Region 2 will then tend to be more aggressive. When Region 2 is the high tax country ($\Delta < 0$), relative attractiveness still matter. Lower C implies that fewer firms are loyal to Region 2 and so Region 2 fights harder to keep firms and profits.

Figure 2.2 shows the shape of best response functions of Region 1 and Region 2 for $C < 1$. We start with Region 1. Because Region 1 has a comparative advantage, it tends to be less

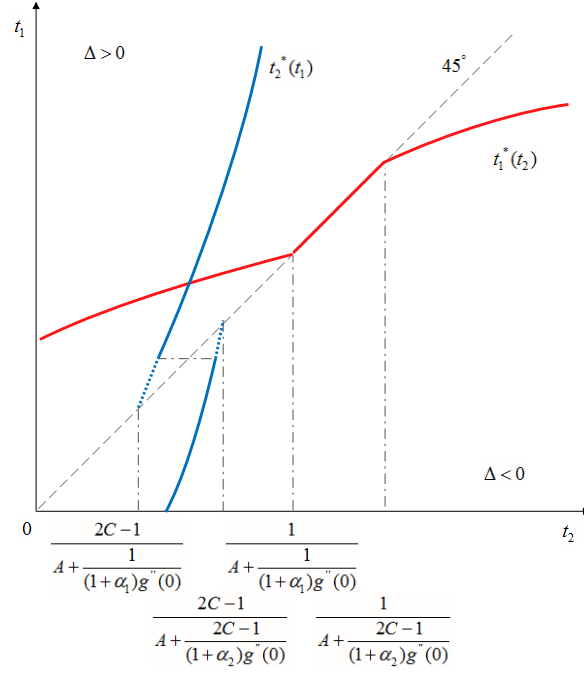


Figure 2.2: Best response tax rates when $C < 1$

aggressive especially when it faces a low tax region. For low value of t_2 , Region 1 is happy to set $t_1 > t_2$. However as stated in Lemma 3, the slope of the reaction function is less than one. As t_2 increases, t_1 also increases and so losing firms and profits become more costly. This explain why Region 1 keeps t_1 closer to t_2 , as t_2 increases. The tax gap then shrinks, up to $\Delta = 0$ where Region 1's best response becomes discontinuous around the 45° line. At that point, the best response for $\Delta > 0$ would suggest a $t_1 < t_2$ and so it is no longer the relevant response. On the right hand side of the 45° line, the best response t_1 must by definition be such that $\Delta \leq 0$. However, for an intermediate range of t_2 , Region 1 would like to set a higher tax rate. This implies that the best response t_1 follows the 45° degree line. When t_2 become large enough, Region 1 becomes satisfied with undercutting Region 2 and attracting firms and profit at a relatively high tax rate.

We now look at Region 2 who is more aggressive when $\Delta > 0$ than when $\Delta < 0$. The best response t_2 when $\Delta > 0$ is smaller than when $\Delta < 0$. Note that there are overlapping segments in best responses t_2 . Region 2 is better undercutting above a certain t_1 , but is better choosing to be a high tax region below a certain t_1 . The dotted segment of the best response t_2 is the dominated strategy by the other choice. Overall, the discrepancy of one

region's best response becomes larger as C gets farther from one. If $\alpha_1 = \alpha_2$, $C = 1$ makes the best response functions of both Regions continuous.

Monitoring obviously affects the reaction functions, but it also has an impact on the discontinuity. For $\Delta > 0$, an increase in α_1 moves the best response functions for both regions to the right. Stricter monitoring reduces the loss (gain) from profit shifting of Region 1 (Region 2), hence increases the incentive to raise the tax rate of both regions. Similarly, increase in α_2 moves the best response functions for $\Delta < 0$ of both regions to the right by reducing the loss (gain) from profit shifting of Region 2 (Region 1) when $\Delta < 0$. Because the best response tax rate of each region when $\Delta < 0$ are greater than $\Delta > 0$ for $C < 1$, we can make the best response functions continuous by increasing α_1 and/or decreasing α_2 . The condition for continuity of the best response functions is below.

Lemma 13. *For given $C \leq 1$, there exists a combination of α_1 and α_2 such that $\alpha_1 \geq \alpha_2$ which makes the best response functions of **both** Region 1 and 2 continuous.*

In all other cases, the best response functions are discontinuous.

2.4 Equilibrium

We now characterize the equilibrium tax rates taking profit shifting control policies as given. We start in the next section by solving the tax competition game in a symmetric environment. It has the advantage to be simple, so we can clearly understand the various forces. However, it is not that informative to talk about profit shifting, as there is none. We then continue with the more difficult case where tax rates are different.

2.4.1 Symmetric equilibrium tax rate

We start with symmetric countries: $C = 1$ and same monitoring. Let t_1^e and t_2^e denote the equilibrium tax rates of Region 1 and 2 respectively. Then the equilibrium in symmetric tax competition can be described as:

Proposition 4. *When two countries have the same comparative advantages ($C = 1$) and the same profit shifting control $\alpha_1 = \alpha_2 = \alpha$, there exist a unique Nash equilibria where they both choose the same tax rates $t_1^e = t_2^e = \frac{(1-\alpha)g''(0)}{1+(1-\alpha)g''(0)A}$.*

When the two region have the same comparative advantages and the same profit shifting control, they chose the same tax rates. When $C = 1$ and $\alpha_1 = \alpha_2$, best response functions

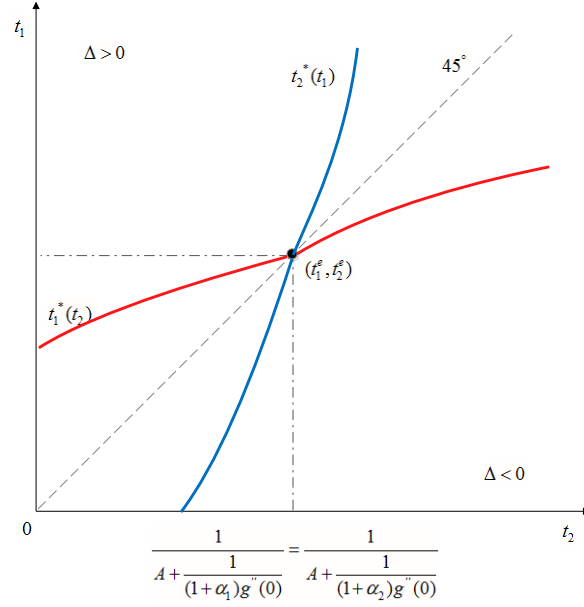


Figure 2.3: Symmetric equilibrium tax rate

for both Regions are continuous and cross on the 45 degree line as shown in Figure 2.3. Note that since $\frac{dt_i^*}{d\alpha_i} > 0$ then $\alpha_1 = \alpha_2 = 1$ results in the highest tax rate.

2.4.2 Asymmetric equilibrium tax rate

More interesting is the case where countries have different comparative advantages: $C \neq 1$. Without loss of generality, we assume that $C < 1$ so that more firms prefer Region 1 to Region 2. Under some conditions, stated in the Proposition below, there exist a unique set of equilibrium tax rates.

Proposition 5. *When Region 1 has greater comparative advantage such that $C < 1$, there exists an equilibrium set of tax rates (t_1^e, t_2^e) , where $t_1^e > t_2^e$ under the following conditions.*

- *Sufficient condition for the equilibrium: $t_1^e > \frac{2C-1}{A + \frac{2C-1}{(1+\alpha_2)g''(0)}}$;*
- *Necessary condition for the equilibrium: $\Omega_2(t_2^e, t_1^e)|_{\Delta > 0} > \Omega_2(t_2^*(t_1^e), t_1^e)|_{\Delta \leq 0}$.*

No pure strategy equilibrium exist where $t_1^e < t_2^e$.

The sufficient condition states that the intersection point of best response functions located above the discontinuous area of best response t_2 . This guarantees the existence of

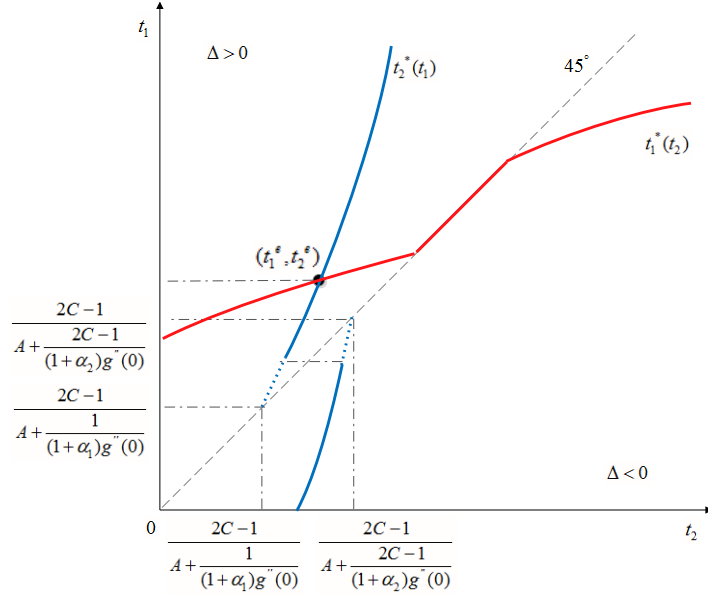


Figure 2.4: Asymmetric equilibrium tax rates

equilibrium. Figure 2.4 illustrates the equilibrium where the sufficient condition is satisfied. The necessary condition requires that if the intersection lies within the discontinuous area of $t_2^*(t_1)$, then Region 2's welfare at the intersection point (t_1^e, t_2^e) should dominate welfare obtained when $\Delta < 0$. Region 2 then chooses the best response for $\Delta > 0$ and the equilibrium exists. Note that less firms in favouring of Region 2 makes low tax Region 2 more aggressive therefore in equilibrium we have $t_1^e > t_2^e$.

In such asymmetric equilibria, we are also interested in $\frac{d\Delta^e}{d\alpha_1}$, i.e., how the tax gap of two regions changes as α_1 varies. The following lemma describes the sign of $\frac{d\Delta^e}{d\alpha_1}$ under a certain condition.

Lemma 14. *If increasing α_1 does not reduce the responsiveness of one region's best response tax rate with respect to another region's, the equilibrium tax gap increases as α_1 increases, i.e., $\frac{d\Delta^e}{d\alpha_1} > 0$.*

Imagine at first that α_1 does not change the slopes of best response functions in both regions. Figure 2.5 shows that in such cases the equilibrium tax differential Δ^e becomes greater when α_1 changes from α_1^l to α_1^h where $\alpha_1^l < \alpha_1^h$. This is because the points at which high tax Region 1's best response function cross 45 degree line shifts to a greater extent relative to the intersection point of low tax Region 2's responses. Region 1's comparative advantage

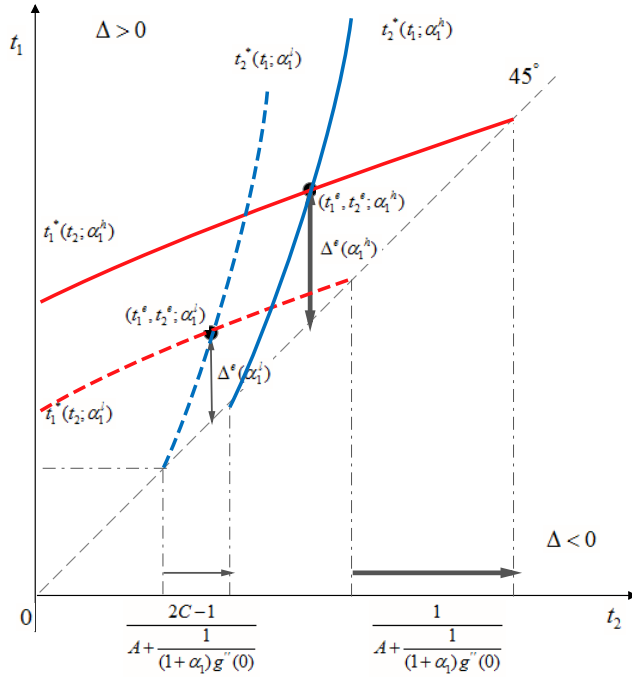


Figure 2.5: Change of Δ^e when α_1 increases

implies that an increase in α_1 pushes FOC_1^+ to a greater extent relative to FOC_2^+ . This result is also preserved when α_1 increases the slopes of the best response functions.

Note that the sign of $\frac{d\Delta^e}{d\alpha_1}$ becomes uncertain if increasing α_1 decreases the slope of best response functions significantly because reduced responsive of best response tax rate offsets the effect described above. In this case, the equilibrium tax differential could decrease if increasing α_1 decreases the responsiveness of one region's tax choice with respect to another's too much and outweighs the effects of different comparative advantages.

2.5 Optimal profit shifting control

In this section, we analyze the choice of the profit shifting control, α_1 . In 2.4.2, we found that Δ is positive in equilibrium when $C < 1$. In such equilibria, the profit shifting control of the low tax Region 2 does not mean anything because no profit shifting is made by the firms located in Region 2. Therefore, if the profit shifting incurs a minimal cost, low tax Region 2 will choose $\alpha_2 = 0$.

Now we consider the choice of Region 1. In order to demonstrate countries' incentives to be lax in profit shifting control, we set α_1 to its maximum level and show that such a choice may not be optimal even under the extreme assumption that α_i is costless. We assume that the condition in Lemma 14 holds so that increase in α_1 increases the tax gap in the equilibrium. The effect of α_1 , which is determined prior to the tax rate decision, on Region 1's objective function is:

$$\frac{d\Omega_1}{d\alpha_1} = \frac{\partial\Omega_1}{\partial t_1} \frac{dt_1}{d\alpha_1} + \frac{\partial\Omega_1}{\partial t_2} \frac{dt_2}{d\alpha_1} + \frac{\partial\Omega_1}{\partial \gamma_1} \frac{d\gamma_1}{d\alpha_1} + \frac{\partial\Omega_1}{\partial \bar{c}(\Delta)} \frac{d\bar{c}(\Delta)}{d\alpha_1}. \quad (2.21)$$

Note that $\frac{\partial\Omega_1}{\partial t_1} = 0$ from the first order conditions, and that $\frac{\partial\Omega_1}{\partial t_2} = 0$ because t_2 only affects Ω_1 via Δ . The remaining effects are based on $\frac{\partial\Omega_1}{\partial \gamma_1} = -\frac{\bar{c}(\Delta)}{C} t_1 A < 0$ and $\frac{\partial\Omega_1}{\partial \bar{c}(\Delta)} = \frac{(1-\gamma_1)t_1 A}{C} > 0$, meaning that profit shifting is harmful to high tax Region 1 while more firms are helpful. Using these, the equation above can be rewritten as:

$$\frac{d\Omega_1}{d\alpha_1} = \underbrace{\frac{\partial\Omega_1}{\partial \gamma_1} \frac{\partial \gamma_1}{\partial \alpha_1}}_{(+)\ \pi\ \text{shifting effect}} + \underbrace{\frac{\partial\Omega_1}{\partial \bar{c}(\Delta)} \frac{\partial \bar{c}(\Delta)}{\partial \alpha_1}}_{(-)\ \text{firm location effect}} + \underbrace{\left(\frac{\partial\Omega_1}{\partial \gamma_1} \frac{\partial \gamma_1}{\partial t_1} + \frac{\partial\Omega_1}{\partial \bar{c}(\Delta)} \frac{\partial \bar{c}(\Delta)}{\partial t_1} \right)}_{\text{tax competition effect}} \frac{d\Delta^e}{d\alpha_1}, \quad (2.22)$$

where $\Delta^e = t_1^e - t_2^e$ denotes the tax difference in equilibrium. The first two components in (2.22) are the direct effects of α_1 on Region 1's tax revenue. Profit shifting effect is always positive since increase in α_1 increases the tax revenue with less profit shifting. Firm location effect is always negative since increase in α_1 reduces the number of firms locating in Region 1 and the tax revenue. The third component in (2.22) is the indirect effect of α_1 , which is called *tax competition effect*. Because $\frac{\partial\Omega_1}{\partial \gamma_1} \frac{\partial \gamma_1}{\partial t_1}$ and $\frac{\partial\Omega_1}{\partial \bar{c}(\Delta)} \frac{\partial \bar{c}(\Delta)}{\partial t_1}$ are both negative, the sign of tax competition effect totally depends only on the sign of $\frac{d\Delta^e}{d\alpha_1}$. When the tax gap between the two regions grows with α_1 , then additional monitoring has a negative impact on Regions 1's welfare. The following proposition summarizes this result.

Proposition 6. *When full deterrence of profit shifting is feasible, i.e. $\alpha_1 = \infty$, fully deterring profit shifting is not optimal when relaxing profit shifting control reduces the gap between tax rates in the equilibrium.*

If profit shifting becomes infinitely costly, there will be no profit shifting hence α_1 has no direct impact on tax revenue. However, α_1 could still indirectly influence the tax revenue by changing the equilibrium tax gap which affects firms' location decision. Because greater Δ reduces the number of firms locating in Region 1, Region 1 is better off by being loose in profit shifting control if being loose in profit shifting control reduces the tax difference in

the equilibrium. This result is interesting because Region 1 may still allow some degree of profit shifting even if it can choose perfect deterrence with no cost.

In order to relax the extreme condition of $\alpha_1 = \infty$ which completely disallows profit shifting, let us assume that the regions face finite resource constraint so a certain degree of profit shifting occurs even under the maximum choice of $\alpha_1 = \bar{\alpha}_1$. The following proposition describes the condition in which *looser* profit shifting control is optimal for Region 1.

Proposition 7. *When full deterrence of profit shifting is not feasible, i.e., $\alpha_1 = \bar{\alpha} < \infty$ is the maximum feasible choice, further reducing α_1 below $\bar{\alpha}$ is optimal if the firm's movement effect and tax competition effect outweighs the profit shifting effect.*

Now the direct effects of α_1 persist because maximum α_1 cannot fully deter profit shifting. In this case, relaxing the tax competition and attracting firms create the incentive to be loose on profit shifting. Region 1 compares this to the loss created by increased profit shifting, and may decide to be loose if the former is greater than the latter.

2.6 Conclusion

Modelling continuous tax competition model in the presence of profit shifting and the enforcement elements reveals the two important tax bases: firms' location tax base and per-firm retained profit tax base. Relaxing the degree of profit shifting enforcement shifts the weight from the per-firm retained profit tax base to the firms location tax base. For this trade-off, choosing not to be strict in profit shifting control may allow the high tax region to choose a higher tax rate.

We also show that relaxing the profit shifting regulations can reduce the equilibrium tax gap between high tax region and low tax region. This is because the high tax region's tax choice is generally more sensitive to the profit shifting control. As all the downward forces on the tax rate choices, which are resulted by the location effect and per-firm profit shifting effect, are positively related to the equilibrium tax gap, choosing the lax profit shifting control may close the tax gap and lead to the higher tax revenue even for the high tax region.

Chapter 3

Platform competition in the presence of cross-platform indirect network effect

3.1 Introduction

Microsoft and Sony are two major platform companies in the video game industry that sold over 140 million consoles which account for 60% of the entire market. Considering that only three major companies are controlling the entire video game market of which size is even greater than the movie or music industry¹, the loss of \$8 billion dollars that both of Microsoft and Sony experienced in the first 6 years of the 7th generation video game console competition seems unusual. Their unprofitable outcomes were primarily due to the high console production cost that exceeded their retail prices². This paper claims that one of the factors that caused platforms' excessive competition was the high inter-platform indirect network effect between the platforms.

The main difference between the 7th generation consoles and the previous generations was the frequency of 'porting'. Porting refers to the process of adapting software titles developed for one platform to others. Until the 5th generation consoles, most of the video games were playable only in the 'lead platform.'³ In contrast, more than 80% of Playstation games

¹The U.S. video game console industry generated \$22 billion in 2008 and its size already surpassed U.S. movie and music industry in 2005 and 2007 respectively.

²Per-unit losses on the Xbox 360 were estimated at \$125 at launch. See Niu (2012), *How One of Microsoft's Greatest Consumer Successes Is a Financial Failure*.

³A lead platform refers to the system which the software is mainly developed for.

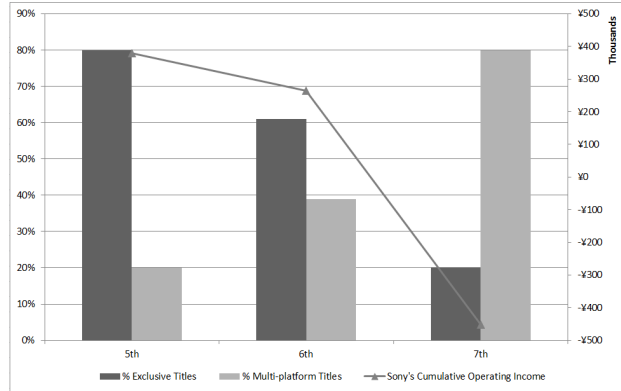


Figure 3.1: Portability and Sony's game division profit

became playable in the other platforms starting from the 6th generation consoles.⁴ Platforms are increasing 'portability' to attract software developed into their platforms because the number of video game software is directly related to the value of the their platforms. The scope of porting has widened in other similar platform business as well, e.g. smartphone's operating systems or web browsers⁵.

However, unlike the common belief that porting is beneficial, we observe a strong negative relationship between the rate of porting and platforms' profits as depicted in Figure 3.1⁶. The losses can be partially explained by the two-sided market theory. Parker and Van Alstyne (2005) and Rochet and Tirole (2003) show that platforms are willing to bear a certain degree of loss in one market to penetrate the other side of market. For example, Adobe distributed PDF reader at zero prices for greater use of PDF format and gained profit from increased sales of PDF writer. This theory explain console manufacturers' early losses in a certain degree⁷. However, unlike the previous generations that exhibited long-run profitability eventually,

⁴See Corts and Lederman (2009) that presents the empirical evidence of the growing inter-platform indirect network effect.

⁵One example in the smartphone OS is Blackberry which can run almost all Apps for Android. Also Microsoft introduced a tool to port Google Chrome extensions to its new web browser Edge. See Hoff (2015) and Callahan (2016).

⁶Microsoft and Sony seem to be aware of the problem as they started to limit portability by introducing distinct proprietary motion capture devices such as "Kinect" and "Move". Sony and Microsoft's standard controllers for 8th generation consoles have even more distinct features such as light bars, touchpads and inseparable Kinect camera. Their adoption of different input devices restricts the chance of porting because the software developers will need to burden additional costs in adaptation process.

⁷Until the 6th generation of video game consoles, the console manufacturers initially earned low or negative profits after the new console launches but they turned to positive net profit after one or two years by collecting software royalties.

Sony's loss in the 7th generation could not be recovered in 8 years⁸ in spite of their decent sales and market share. The particular hardship in the 7th generation requires an alternative theory for explanation.

There are two types of indirect network effects which are present in the video game industry: the indirect network effect *within* a platform and the indirect network effect *across* the platforms. The former means that the more popular platform tends to have more complementary software because software developers can expect greater potential sales in the platform with large consumer base. For example, Apple's iPhone which is one of the most popular cell phones has more selection of complementary accessories in the market compared to cell phones which are not very popular. On the other hand, the "cross-platform indirect network effect" means that one platform's hardware sales also affects the number of software in another platform. For instance, if more people own Microsoft Xbox 360, the number of games for Sony PlayStation 3 increases as well because greater number of games will be developed for Xbox 360 and then ported to Playstation 3.

Our paper is based on the traditional platform-component literatures regarding indirect network effect. The first model was developed by Katz and Shapiro (1985) who studied the competition in the presence of *direct* network effect by making preference exhibit network externalities. In their model, firms compete in quantity *a la Cournot* and firms choose either complete or zero interconnectivity. Based on their work, Chou and Shy (1990, 1996) and Church and Gandal (1992) studied *indirect* network effect and showed how the effect influences the number of components in each platform. After Chou and Shy (1990) showing that the traditional direct network theory can substitute for indirect network effect, later studies such as Cremer et al. (2000) and Hogendorn and Yuen (2009) translates the direct network effect model of Katz and Shapiro (1985) into the indirect network competition model. Cremer et al. (2000) analyze the Internet backbone market to show that the dominant platform has an incentive to lower the degree of interconnection to take advantage of the initial quality difference.⁹

The contribution of our paper is to consider the inherent quality competition as another dimension of the video game console competition. In our model, the platforms still compete by varying portability as in Cremer et al. (2000), but they also compete by varying

⁸The loss is deeper than other similar industry's examples considering the normal lifespan of video game consoles are 5 to 10 years.

⁹Baake and Boom (2001) produced the similar result as Cremer et al. (2000) in the sense that the dominant firm prefers incomplete interconnectivity. But Baake and Boom (2001) find that in spite of bigger firm's preference for limited interconnectivity, they will achieve complete connectivity in the end because the smaller firm can successfully prevent incomplete interconnectivity equilibrium through its quality choice.

objective qualities of their products. The introduction of vertical differentiation adds a new interaction between portability and quality choices. Our result shows that an increase in portability intensifies the pressure on the quality competition, so the platforms may choose lower portability to reduce the degree of quality competition. Such ‘quality competition effect’ of additional portability is absent if the quality improvement does not significantly reduce the number of software.

The rest of this paper is organized as follows. Section 3.2 develops the platform competition model with quality and portability choices in duopoly setting. Section 3.3 analyzes the equilibrium in which the platforms cooperatively choose quality and portability levels. Section 3.4 presents the equilibrium of competitive environment and compare the result to the cooperative case. The result shows that platforms tend to choose excessively high quality standard if hardware quality improvement curtails the software availability and non-zero inter-platform indirect effect exists among them. Moreover, unlike previous literatures in which only the platform with greater exogenous advantage tends to choose limited portability, our model shows that even platforms in a symmetric setting have an incentive to lower portability in order to avoid costly quality competition. Section 3.5 generalizes the model by allowing asymmetry of each platform’s initial advantage. Section 3.7 concludes.

3.2 Platform competition model

We consider the three-stage game which consists of two console manufacturers and a mass of consumers. The time line of the game is as below:

1. Platforms simultaneously choose their preferred level of portability, θ_1 and θ_2
2. Platforms choose the inherent quality levels for their consoles, l_1 and l_2 .
3. Platforms set their production levels q_1 and q_2 .

We assume that the portability decision takes place prior to the quality decision because portability decision usually takes a longer time than selecting quality.

3.2.1 Demand

Suppose there is a mass of consumers with idiosyncratic preference $\tau \in [\underline{\tau}, 1]$ on the gaming console where $\underline{\tau}$ is the constant lower bound.¹⁰ A consumer with τ who purchases a console from platform $i = 1, 2$ will obtain the surplus of:

$$u_i = \tau + \nu l_i + \mu s_i - p_i \quad (3.1)$$

where l_i denotes the average software quality for console i , s_i denotes the number of video game titles (i.e., software quantity) that can be played with console i , and p_i denotes the price of the console. ν and μ are marginal utilities of quality and quantity of the video games respectively. We assume that the marginal utilities ν and μ do not vary across individuals.

Naturally only the consumers with positive surplus make a purchase from either platform:

$$u_i = \tau - \hat{p} \geq 0$$

where $\hat{p} = p_i - \nu l_i - \mu s_i$ denotes the price adjusted for quality and quantity of software (also called *hedonic price*). This implies that the marginal consumer is characterized by the condition:

$$\tau = \hat{p}$$

Assuming that τ is distributed from $\underline{\tau}$ to 1, total demand for consoles in the market is

$$\int_{\hat{p}}^1 G(\tau) d\tau \quad (3.2)$$

where $G(\tau)$ denotes the cumulative distribution function of τ . Assuming uniform distribution of $\tau \in [\underline{\tau}, 1]$, we can rewrite Eq. (3.2) as:

$$(1 - \underline{\tau})(1 - \hat{p}) \quad (3.3)$$

3.2.2 Supply

Software

Let us assume that each console manufacturer can directly control the average quality level of its own platform's video games, l_i , by choosing the appropriate processing units for their

¹⁰We assume that $\underline{\tau} < 0$ for interior solution.

console hardware¹¹. There are three determinants of software quantity, s_i . First, s_i is positively correlated with the console quantity, q_i , because of the indirect network effects. Second, increasing the average quality of a platform, l_i , decreases the number of games in the platform because of the development cost pressure¹². Third, increased portability increases software quantity via porting.

Let $\tilde{\theta}$ denote the effective portability between platforms which is determined by the preferred portabilities of two platforms, θ_1 and θ_2 . The decision mechanism and assumptions will be discussed in section 3.3.3. The relationships explained above suggest:

$$n_i = \kappa q_i - F(l_i) \quad (3.4)$$

and

$$s_i = n_i + \tilde{\theta} n_j = \kappa(q_i + \tilde{\theta} q_j) - F(l_i) - \tilde{\theta} F(l_j) \quad (3.5)$$

where n_i denotes the number of software *originally developed* for platform i , s_i denotes the software quantity in platform i , and finally $0 < \kappa < 1$ denotes the degree to which console quantity affects the number of lead platform games. $F(l_i)$ with $\frac{\partial F}{\partial l} > 0$ and $\frac{\partial^2 F}{\partial l^2} > 0$ is the function that describes the negative relationship between average software quality and the number of video games; the number of video games is assumed to be decreased by the quality level in an increasing rate¹³. Note that $s_i \geq n_i$ at all time; this is because a certain portion of platform 1 video games will be converted to platform 2 and vice versa, so the total number of video games *available* in platform i is always greater than the number of games that are purely *developed* for either platform.

Because of porting, the software quantity s_i increases with q_i and with q_j . In addition, the negative effect of quality on quantity is also spread onto the competitor; if platform 1

¹¹When a platform adopts a high-end processing units for its console, the overall quality standard of software becomes high because software developers would create games that fully utilizes the superior computing resource.

¹²Increasing software quality standard, however, shrinks the quantity of software not only because average development time becomes longer but also only a limited number of renowned software developers can attract capital on a large scale and create mainstream games. Conventional low-quality video games, which is still playable in the superior hardware system, is apt to be fallen behind as they will fail to meet consumers' expectation.

¹³Until 6th generation video game competition, the developing mainstream games a huge burden with less than \$5 million dollars of total cost. However, the average cost of game development is almost quadrupled in the 7th generation. The geometrical cost increase, which made the average AAA games for the 7th generation video games require the blockbuster movie budget of 20 million dollars, is due to the extensiveness of video game development. If a developer wants to create a game with superior graphics, it needs to expand the team exponentially to manage equally good music, sound effects, voice acting and marketing.

raises its quality standard, it does not reduce only the number of platform 1 game software but also that of platform 2 software. Such inter-platform externality fades away only if $\tilde{\theta}$ converges to zero.

Hardware

The platforms play an output game in which they can also choose their quality standards and portability in advance. Platforms have perfect information about consumers' preference and software availability for any given levels of console production, game quality and effective portability. Based on the information, platforms choose their preferred levels of portability θ_i , software quality standard of its own console, l_i , and their production levels q_i . We assume that a consumer buys only one console from either platform at most.

Among the several possible Nash equilibria, there is only one type of equilibrium where both platforms attract positive number of consumers.¹⁴ In such an equilibrium, a new customer should view both platforms as perfect substitutes, meaning that the surplus of purchasing a console from platform 1 or 2 are equivalent so that:

$$u_1 = u_2$$

Using Eq. (3.1), this condition can be rewritten as

$$p_1 - \mu l_1 - \nu s_1 = p_2 - \mu l_2 - \nu s_2 = \hat{p} \quad (3.6)$$

Thus the price adjusted for quality and quantity of software, \hat{p} , is equal for both platforms.

Eq. (3.6) pins down the price of each console when both platforms have positive demand. In the production stage in which the prices are determined, the software quantity and software quality of each platform are taken into account from the previous stages. For given software quantity and quality, a platform can sell its console at higher price if the software quality and quantity are higher than its competitor's. In other words, a platform must set a lower price if its own software quantity and quality are lower than the competing platform. Everything being equal, each platform therefore wants to increase software quality and quantity in order to increase its price and profit. Knowing this price formation mechanism, the platforms derive the demand functions and maximize their profits. This condition

¹⁴Other types of equilibria are 'winner-takes-all' equilibria and the equilibrium with zero demand for both platforms.

on prices is useful to derive the demand functions and profit maximizing decisions of the console manufacturers.

3.2.3 Market clearing condition

Now we specify the market clearing condition of the console market to derive the supply functions of the console manufacturers. Given the total demand in Eq. (3.3) and the total supply $q_1 + q_2$, the market clearing condition is thus:

$$q_1 + q_2 = (1 - \tau)(1 - \hat{p}) \quad (3.7)$$

Eq. (3.7) illustrates the inverse relationship between the hedonic price and platforms' output levels. When a platform increases its own output, the equilibrium hedonic price faced by both platforms must fall because the consumers with lower willingness to pay need to buy the consoles for market to be cleared. Therefore the relationship between output, price and profit is similar to the standard Cournot game where each platform's increase production decreases the equilibrium market price as well as the other platform's profit.

Based on Eq. (3.5), (3.6) , and (3.7), solving for p_i gives the inverse demand function faced by each platform:

$$\begin{aligned} p_i &= 1 - (1 - \tau)(q_i + q_j) + \mu l_i + \nu s_i \\ &= 1 - (1 - \tau - \nu\kappa)q_i - (1 - \tau - \nu\kappa\tilde{\theta})q_j + \mu l_i - \nu(F(l_i) + \tilde{\theta}F(l_j)) \end{aligned} \quad (3.8)$$

Eq. 3.8 describes how the price of each console is given platforms' output, qualities and portability. Similar to the standard Cournot model, one platform's increased output will negatively affect the console prices of both platforms. And if a platform is equipped with video games of greater quality and quantity, the platform can sell its console at higher price.

One important feature of this demand function is the externality created by porting. Suppose platform 1 increases the quality such that consumers' increase in utility dominates the disutility caused by decreased number of games: $\mu > \nu F'(l_i)$. In this case, if platform 1 increases its software quality standard, the price of its own console increases directly. On top of this direct effect, platform 1's quality improvement also lowers the attractiveness of platform 2 console because platform 2 now has fewer game ported from platform 1.

Given the demand function, each platform maximizes its own profit:

$$\begin{aligned}
\pi_i &= (p_i - C(l_i))q_i \\
&= \left(1 - C(l_i) - (1 - \tau - \nu\kappa)q_i - (1 - \tau - \nu\kappa\tilde{\theta})q_j + \mu l_i - \nu(F(l_i) + \tilde{\theta}F(l_j))\right)q_i \quad (3.9)
\end{aligned}$$

The marginal production cost function with respect to quality, $C(l_i)$, is increasing and convex in the quality standard of each platform: $C' > 0$ and $C'' > 0$.

3.3 Cooperative standard setting

This section consider the case in which platforms cooperatively choose quality and portability levels while they are still competitive in quantity decision. Unlike the collusion in the production stage which limits the outputs to monopoly level, cooperation in quality and portability decisions leads to the superior outcome both in terms of platforms' profit and social welfare. We will therefore consider the cooperative standard decision scenario as the benchmark to highlight the disadvantage in competitive environment.

3.3.1 Quantity decision

In the production stage, each platform decides production level maximizing its profit (3.9) for given quality and portability. We assume that the platforms cannot cooperate in this stage. Solving the profit maximization problem in Cournot style with respect to the quantity produces:

$$q_i^{CO}(l_i, l_j, \tilde{\theta}) = \frac{2(1 - \tau - \nu\kappa)\omega_i(l_i, l_j, \tilde{\theta}) - (1 - \tau - \nu\kappa\tilde{\theta})\omega_j(l_i, l_j, \tilde{\theta})}{4(1 - \tau - \nu\kappa)^2 - (1 - \tau - \nu\kappa\tilde{\theta})^2} \quad (3.10)$$

where $\omega_i(l_i, l_j, \tilde{\theta}) = 1 + \mu l_i - \nu(F(l_i) + \tilde{\theta}F(l_j)) - C(l_i)$, the console price net of marginal production cost *excluding the quantity effects*¹⁵. Note that $\omega_i(l_i, l_j, \tilde{\theta})$ can be interpreted as the 'competitive advantage' of platform i for measuring the profitability of its console production *excluding the quantity effects*.

Eq. (3.10) shows that the equilibrium quantity is the difference of platform i and j 's competitive advantages; the equilibrium quantity increases when one platform's competitive advantage gets bigger compared to its competitor's. In order to understand the mechanism,

¹⁵'Quantity effect' refers to the negative effect of increased production onto the equilibrium price.

suppose that two platforms are in symmetric equilibrium. If ω_i increases *ceteris paribus*, i will increase its console production, and the console production of j , even if ω_j remains unchanged, must decrease because increased production of console i reduces the equilibrium price of both consoles *a la Cournot* hence reduces the profitability of j .

Using (3.10) and (3.9), the equilibrium profit function is

$$\pi_i^{CO}(l_i, l_j, \tilde{\theta}) = (1 - \tau - \nu\kappa)q_i^{CO}(l_i, l_j, \tilde{\theta})^2 \quad (3.11)$$

and it is easily seen that the equilibrium profit increases with the equilibrium quantity.

Since a platform's market share and profit depends on the difference of competitive advantages, a platform's quality setting strategy is focused on how to increase its own competitive advantage and, if possible, to decrease its competitor's advantage. On the one hand, raising quality l_i increases ω_i as consumers appreciate the increased average video game quality. On the other hand, increased l_i decreases ω_i because consumers dislike decreased quantity of video games and marginal production cost of consoles rises, too. Also note that increased l_j , the quality improvement of the competitor, only decreases ω_i since it reduces the ported video games without bringing any external benefits. The details of quality decision will be discussed in the next section.

3.3.2 Cooperative quality decision

When the platforms choose cooperatively to set their quality standards, the objective of the platforms is to maximize the joint profit by choosing the appropriate quality levels:

$$\max_{l_i, l_j} \left(\pi_i^{CO}(l_i, l_j, \tilde{\theta}) + \pi_j^{CO}(l_i, l_j, \tilde{\theta}) \right) = (1 - \tau - \nu\kappa)(q_i^{CO}(l_i, l_j, \tilde{\theta})^2 + q_j^{CO}(l_i, l_j, \tilde{\theta})^2) \quad (3.12)$$

Recall that $\partial\pi_j^{CO}/\partial l_i < 0$ from (3.10) and (3.11), so in the optimum $\partial\pi_i^{CO}/\partial l_i > 0$ needs to hold so that $\partial\pi_i^{CO}/\partial l_i + \partial\pi_j^{CO}/\partial l_i = 0$; the platform i 's marginal profit of quality improvement should be positive and equal to the marginal loss of platform j . Let l_i^{CO} be the quality that solves the cooperative profit maximization problem, then the optimality conditions are

$$(\alpha\omega_i - \beta\omega_j)\frac{\partial\omega_i}{\partial l_i} = -(\alpha\omega_j - \beta\omega_i)\frac{\partial\omega_j}{\partial l_i} \quad (3.13)$$

where $\alpha = 4(1 - \tau - \nu\kappa)^2 + (1 - \tau - \nu\kappa\tilde{\theta})^2$ and $\beta = 4(1 - \tau - \nu\kappa)(1 - \tau - \nu\kappa\tilde{\theta})$. The condition implies that platform 1's marginal profit of quality improvement should be equal to platform

2's marginal loss. Because the two platforms are symmetric, $\alpha\omega_i - \beta\omega_j = \alpha\omega_j - \beta\omega_i$ in the equilibrium so the optimality conditions can be reduced to:

$$\mu = C'(l_i) + \nu(1 + \tilde{\theta})F'(l_i) \quad (3.14)$$

for $i = 1, 2$. The LHS of Eq. (3.14) is the marginal benefit of increasing platform i 's quality equal to the consumers' marginal utility of game quality improvement. The RHS is the marginal cost associated with quality improvement which is the sum of marginal production cost and consumers' marginal disutility of decreased game quantity. Since $C'' > 0$ and $F'' > 0$, solving equation (3.14) produces an unique value of l_i^{CO} that can be denoted as $l^{CO}(\tilde{\theta}) = l_1^{CO}(\tilde{\theta}) = l_2^{CO}(\tilde{\theta})$. The finding is summarized in the lemma:

Lemma 15. *Increased portability reduces the optimal quality in cooperative decision.*

Increased $\tilde{\theta}$ makes quality improvement more costly. If $\tilde{\theta} = 0$, the cost of quality improvement is the decreased game quantity in a single platform. As $\tilde{\theta}$ becomes positive and larger, the game quantity of both platforms are reduced by quality increase of either platform. To summarize, increased $\tilde{\theta}$ enlarges the degree to which quality improvement decreases the total video game quantity. Because the cooperative platforms take both platforms' video game quantity into consideration, the cooperative quality choice needs to be decreased when $\tilde{\theta}$ increases. This is the internalization of the negative externality of quality improvement in the cooperative profit maximization problem. Therefore, both platforms' optimal qualities in cooperative environment decrease as the effective portability increases: $\partial l^{CO}(\tilde{\theta})/\partial \tilde{\theta} < 0$.

3.3.3 Cooperative portability decision

Portability, which determines the frequency of porting across competing platforms, can be unilaterally lowered by either party. High portability can be achieved only if such outcome is pursued by both platforms.¹⁶In the stage of portability setting, the equilibrium quantities are

$$q_i^{CO}(\tilde{\theta}) = \frac{1 + \mu l - C(l^{CO}(\tilde{\theta})) - \nu(1 + \tilde{\theta})F(l^{CO}(\tilde{\theta}))}{3 - 3\underline{\tau} - 2\nu\kappa - \nu\kappa\tilde{\theta}} \quad (3.15)$$

and the objective function to maximize the joint profit is:

¹⁶This assumption is based on the fact that adoption of unique controller, which is the general way to lower portability in video game market, is usually protected by patents hence cannot be easily copied by competitors. This differentiate our model from the other horizontal differentiation models such as Hotelling location model in which one can intentionally achieve small degree of horizontal differentiation by expecting and following competitor's move.

$$\max_{\tilde{\theta}} \pi_i^{CO}(\tilde{\theta}) + \pi_j^{CO}(\tilde{\theta}) = 2(1 - \underline{\tau} - \nu\kappa) \left(\frac{1 + \mu l - C(l^{CO}(\tilde{\theta})) - \nu(1 + \tilde{\theta})F(l^{CO}(\tilde{\theta}))}{3 - 3\underline{\tau} - 2\nu\kappa - \nu\kappa\tilde{\theta}} \right)^2 \quad (3.16)$$

Solving the first order condition yields the following lemma:

Proposition 8. *In the cooperative standard decision, both platforms select the maximum portability: $\theta_1^{CO} = \theta_2^{CO} = \tilde{\theta} = 1$*

The (3.16) is of the form of $\pi_i^{CO}(\tilde{\theta}) = \Psi(\tilde{\theta})\Phi(l^{CO}(\tilde{\theta}))$ where $\Psi = 2(1 - \underline{\tau} - \nu\kappa)/(3 - 3\underline{\tau} - 2\nu\kappa - \nu\kappa\tilde{\theta})^2$ and $\Phi(l^{CO}(\tilde{\theta})) = (1 + \mu l - C(l^{CO}(\tilde{\theta})) - \nu(1 + \tilde{\theta})F(l^{CO}(\tilde{\theta})))^2$. $\partial\Psi/\partial\tilde{\theta}$, which represents the direct portability effect, is positive because more porting increases the game quantity in both platforms and increases console sales and profit¹⁷. On the other hand, $\partial\Phi(l(\tilde{\theta}))/\partial\tilde{\theta}$, which is the indirect effect through quality change, is zero because of the envelope theorem. Because there is only positive effect with increased $\tilde{\theta}$, both platforms agree on the maximum portability: $\tilde{\theta} = \theta_1 = \theta_2 = 1$.

Intuitively, for cooperative platforms, increasing portability is always beneficial because it only increases the number of games in both platforms without any negative effects. The possible drawback, the indirect effect through the quality decision, is non-existent in the cooperative scenario because the negative effect of quality improvement is fully internalized at the quality decision stage. When portability is increased, the platforms do not neglect the negative effect of one platform's quality improvement on the other platform's profit so that they lower optimal quality levels accordingly. Knowing that, the platforms have no reason to limit the portability. This result contrasts with the competitive standard setting scenario which will be discussed in the next section.

3.4 Competitive standard setting

3.4.1 Quantity decision

Now suppose the platforms decide the quality and portability levels non-cooperatively. Optimal production levels are the same as in section 3.3.1 because the outputs are chosen non-cooperatively in both instances. The equilibrium output function for given quality and portability is

¹⁷Since $\nu < 1/2$ and $\kappa \leq 1$.

$$q_i^*(l_i, l_j, \tilde{\theta}) = \frac{2(1 - \tau - \nu\kappa)\omega_i(l_i, l_j, \tilde{\theta}) - (1 - \tau - \nu\kappa\tilde{\theta})\omega_j(l_i, l_j, \tilde{\theta})}{4(1 - \tau - \nu\kappa)^2 - (1 - \tau - \nu\kappa\tilde{\theta})^2} \quad (3.17)$$

where the term $\omega_i(l_i, l_j, \tilde{\theta}) = 1 + \mu l_i - C(l_i) - \nu(F(l_i) + \tilde{\theta}F(l_j))$ represents the competitive advantage of platform i as defined previously. Accordingly, the equilibrium profit functions for given quality and portability levels is:

$$\pi_i^*(l_i, l_j, \tilde{\theta}) = (1 - \tau - \nu\kappa)q_i(l_i, l_j, \tilde{\theta})^2 \quad (3.18)$$

so the profits still covary with output levels.

3.4.2 Competitive quality decision

In the quality setting stage, each platform maximizes its own profit (3.18). The first order condition tells us that the profit-maximizing quality in competitive standard decision scenario, l_i^* , solves

$$\mu + \sigma(\tilde{\theta})F'(l_i^*) = C'(l_i^*) + \nu F'(l_i^*) \quad (3.19)$$

where $\sigma(\tilde{\theta}) = \frac{(1-\tau-\nu\kappa\tilde{\theta})\tilde{\theta}}{2(1-\tau-\nu\kappa)}$, $\sigma'(\tilde{\theta}) > 0$ and $0 \leq \sigma(\tilde{\theta}) \leq 1/2$ is the degree to which the quality improvement of one platform decreases the opponent's competitive advantage. LHS and RHS of Eq. (3.19) is the marginal benefit and cost of quality improvement respectively. For interior solution, we assume that the net marginal cost of quality improvement, i.e. $C'(l_i^*) + \nu F'(l_i^*) - \sigma(\tilde{\theta})F'(l_i^*)$, increases with l_i^* . Based on Eq. (3.19) we can define the competitive quality function of each platform, $l_i^*(\tilde{\theta})$. By comparing (3.19) to (3.14), we can derive the following proposition:

Proposition 9. *Given effective portability $\tilde{\theta}$, platforms in competitive environment choose higher qualities than in the cooperative standard setting equilibrium: $l_i^*(\tilde{\theta}) \geq l_i^{CO}(\tilde{\theta})$ for $\tilde{\theta} \geq 0$. Moreover, $l_i^*(\tilde{\theta}) - l_i^{CO}(\tilde{\theta})$ increases with $\tilde{\theta}$ from $\tilde{\theta} = 0$.*

Proof. See Appendix □

There are two reasons why competitive quality levels are higher than the cooperative scenario. Firstly, the competitive platforms fail to internalize the negative externality of quality improvement. Therefore the marginal cost in competitive scenario is smaller than in

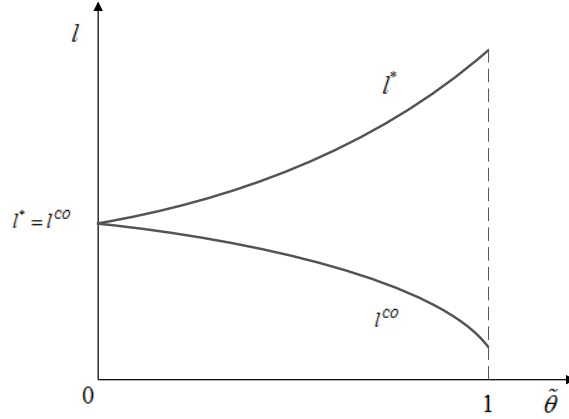


Figure 3.2: Optimal quality choices

the cooperative case. Secondly, the platforms benefit from the negative externality because a platform's market share and profit increase when the opponent's competitive advantage decreases. Hence the marginal benefit in the competitive case is greater than the cooperative case. A higher marginal benefit and a lower marginal cost lead to a higher profit-maximizing quality in a non-cooperative environment.

What interests us more is the relationship between the non-cooperative quality decision and portability:

Lemma 16. *Higher portability increases the optimal quality choices in non-cooperative environment.*

$\partial l_i^* / \partial \tilde{\theta} > 0$ because higher portability increases the marginal benefit of quality improvement; the degree to which one platform's quality improvement reduces the competitor's competitive advantage by decreasing ported game quantity becomes bigger. Therefore, platforms in non-cooperative environment tend to prefer high quality standard when effective portability is high. This prisoner's-dilemma-like result contrasts with the cooperative quality decision case where platforms behave in the opposite way for greater portability only increases the marginal cost of quality improvement. Because increased portability enlarges the gap between optimality conditions of the cooperative and competitive problems, the discrepancy between the two optimums deepens as shown in figure 3.2.

As a result, both sales and profit of each platform in the non-cooperative game is strictly lower than the cooperative outcome because they fail to internalize the negative externality during the profit maximization process.

3.4.3 Competitive portability decision

As previously assumed, each platform chooses its preferred level of portability $\theta_i \in [0, 1]$ simultaneously and the smaller value takes effect: $\tilde{\theta} = \min\{\theta_1, \theta_2\}$. In the stage of portability setting, the equilibrium quantities are

$$q_i^*(\tilde{\theta}) = \frac{(1 + \mu l_i^*(\tilde{\theta}) - C(l_i^*(\tilde{\theta})) - \nu(1 + \tilde{\theta})F(l_i^*(\tilde{\theta})))}{3 - 3\underline{\tau} - 2\nu\kappa - \nu\kappa\tilde{\theta}} \quad (3.20)$$

and the equilibrium profits are:

$$\pi_i^*(\tilde{\theta}) = (1 - \underline{\tau} - \nu\kappa) \left(\frac{(1 + \mu l_i^*(\tilde{\theta}) - C(l_i^*(\tilde{\theta})) - \nu(1 + \tilde{\theta})F(l_i^*(\tilde{\theta})))}{3 - 3\underline{\tau} - 2\nu\kappa - \nu\kappa\tilde{\theta}} \right)^2 \quad (3.21)$$

Recall that the profit is still of the form $\pi_i^*(\tilde{\theta}) = \Psi(\tilde{\theta})\Phi(l^*(\tilde{\theta}))$. As shown earlier, when platforms choose their qualities non-cooperatively, greater portability forces platforms to increase their optimal qualities: $\partial l_i^*/\partial \tilde{\theta} > 0$. This makes the indirect effect of portability $\partial \Phi(l^*(\tilde{\theta}))/\partial \tilde{\theta}$ non-zero unlike in the cooperative case. The following lemma specifies the sign of the indirect effect:

Lemma 17. *If platforms choose their standards non-cooperatively, the indirect effect associated with an increase in portability has a negative sign.*

Proof. In the cooperative case, $\frac{\partial \Phi(l^{CO}(\tilde{\theta}))}{\partial l^{CO}(\tilde{\theta})} = 0$ is satisfied at the quality decision stage. In the non-cooperative case, $\frac{\partial \Phi(l^*(\tilde{\theta}))}{\partial l^*(\tilde{\theta})} < \frac{\partial \Phi(l^{CO}(\tilde{\theta}))}{\partial l^{CO}(\tilde{\theta})} = 0$ because $l_i^*(\tilde{\theta}) \geq l^{CO}(\tilde{\theta})$ for given $\tilde{\theta}$ and $\partial^2 \Phi / \partial l^2 = -C' - \nu(1 + \tilde{\theta})F' < 0$. Therefore, it follows from $\frac{\partial \Phi(l^*(\tilde{\theta}))}{\partial l^*(\tilde{\theta})} < 0$ and $\frac{\partial l^*(\tilde{\theta})}{\partial \tilde{\theta}} > 0$ that $\frac{\partial \Phi(l^*(\tilde{\theta}))}{\partial \tilde{\theta}} = \frac{\partial \Phi(l^*(\tilde{\theta}))}{\partial l^*(\tilde{\theta})} \cdot \frac{\partial l^*(\tilde{\theta})}{\partial \tilde{\theta}} < 0$. \square

Because the quality chosen in competitive environment is already higher than in the cooperative case, further increase in quality negatively affects the equilibrium profit. As lemma 16 predicts, an increase in $\tilde{\theta}$ forces platforms to choose even higher qualities hence the indirect effect of portability through quality decision on profit is always negative. Let us call this *quality competition effect* of portability.

The following proposition summarizes our findings:

Proposition 10. *If platforms choose their standards non-cooperatively, the platforms prefer smaller degree of portability than they would choose in the cooperative environment.*

Proof. $\partial\Phi(l^*(\tilde{\theta}))/\partial\tilde{\theta} \leq 0$ shows $\partial\pi(l^*(\tilde{\theta}))/\partial\theta \leq \partial\pi(l^{CO}(\tilde{\theta}))/\partial\theta$. Then a standard revealed preference argument shows that $\theta_i^* = \arg \max_{\tilde{\theta}} \pi_i^*(\tilde{\theta})$ is smaller or equal to $\theta_i^{CO} = 1$. \square

Unlike the cooperative environment, the quality competition effect is negative in competitive game so the platforms may achieve imperfect portability; the quality competition effect explains the incentive of restricting portability even when the platforms have *symmetric* initial advantages. Note that the equilibrium profit in competitive game is always inferior to the cooperative outcome because of the excessive quality competition due to externality.

3.5 Asymmetric platforms

In this section, we expand the previous model to cover more general cases by allowing asymmetry in platforms' initial advantages. Consider the exclusive factor that is exogenously attached to each platform and gives positive surplus to consumers.¹⁸ Without the loss of generality, let us assume that platform 1 has superior exclusive features that gives the consumers extra surplus of $\Delta > 0$. Then the surplus of a consumer with the exclusive factor considered is

$$\begin{aligned} u_1 &= \tau + \mu l_1 + \nu s_1 - p_1 + \Delta \\ u_2 &= \tau + \mu l_2 + \nu s_2 - p_2 \end{aligned}$$

and equilibrium condition becomes:

$$p_1 - \mu l_1 - \nu s_1 - \Delta = p_2 - \mu l_2 - \nu s_2 = \hat{p} \tag{3.22}$$

Based on this we can derive the corresponding payoff functions and equilibrium outputs, qualities and portabilities. This section will only identify the key differences.¹⁹ The following lemma summarizes the result of the quality decision stage.

Lemma 18. *Both platforms with different exogenous advantages choose the same qualities in the competitive environment: $l_1^*(\tilde{\theta}) = l_2^*(\tilde{\theta})$.*

¹⁸In reality, this is the value of platform i 's exclusive features such as the brand value and the first party game titles that are only available in the specific platform.

¹⁹See Appendix for the full derivation.

Because the exogenous advantage is irrelevant from the qualities of either platform, the quality choices in competitive standard decision game do not depend on Δ . Therefore, both platforms choose the same qualities. The following corollary directly follows from the lemma and Eq. (44):

Corollary 1. *The platform with greater exogenous advantage exhibits greater output and profit levels than the smaller platform; $q_1^*(\tilde{\theta}) > q_2^*(\tilde{\theta})$ and $\pi_1^*(\tilde{\theta}) > \pi_2^*(\tilde{\theta})$ for any given effective portability $\tilde{\theta}$.*

Finally in the portability stage, we can establish the following proposition:

Proposition 11. *The platform with greater exogenous advantage prefers lower portability than the other platform: $\theta_1^* \leq \theta_2^*$. Moreover, its preferred portability decreases when the gap between the platforms' initial advantages increases.*

Proof. The output function of the superior platform is of the form $q_1^* = \Psi\Phi - \rho\Delta$ where $\Psi(\tilde{\theta}) = 1/(3 - 3\tau - 2\nu\kappa - \nu\kappa\tilde{\theta})$, $\Phi(l^*(\tilde{\theta})) = 1 + \mu l^*(\tilde{\theta}) - C(l^*(\tilde{\theta})) - \nu(1 + \tilde{\theta})F(l^*(\tilde{\theta}))$, and $\rho(\tilde{\theta}) = \frac{2(1-\tau-\nu\kappa)}{4(1-\tau-\nu\kappa)^2 - (1-\tau-\nu\kappa\tilde{\theta})^2}$. It can be easily showed that $\frac{1}{2} \frac{\partial^2 q_i^2}{\partial \theta \partial \Delta} = 2\rho'\rho\Delta + (\Psi\Phi\rho)' < 0$ Because ρ and $\Psi\Phi\rho$ are decreasing functions of Δ . Since $\partial^2 \pi / \partial \theta \partial \Delta < 0$, a revealed preferences argument shows $\theta_1^* \leq \theta_2^*$ and that the superior platform's incentive to limit portability decreases as Δ increases. \square

The bigger platform prefers the lower portability to differentiate its product from the smaller platform's.²⁰ In addition, considering the negative indirect effect of portability through quality competition, lowering portability gives additional benefit by relaxing the quality competition. This synergy of incentives in restricting portability implies that the dominant platform benefit much from the restriction of portability. The next section will shed light on the result with the example of the 7th generation console competition.

3.6 What happened in 7th generation?

In the 7th generation console competition between three major platforms (Microsoft, Sony and Nintendo), Nintendo was the obvious winner in terms of profit. From 2006 when Nintendo's 7th generation console Wii was launched, Nintendo made prominent profit as shown

²⁰This result is similar to Cremer et al. (2000) or Baake and Boom (2001), and our paper introduces additional incentive to restrict the portability which has the synergy with the incentive of dominant platform. Details will be discussed in the next section.

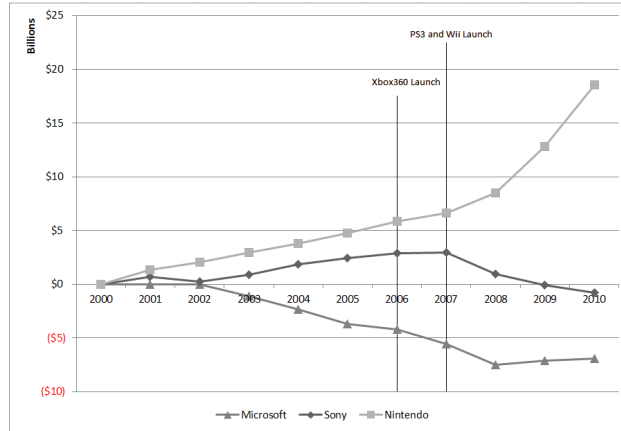


Figure 3.3: Cumulative video game operating income

in figure 3.3. Nintendo’s success contrasted with the huge loss that Microsoft and Sony experienced since 2005 and 2006 when their 7th generation consoles were launched respectively. This was because Nintendo Wii’s marginal production cost was not as high as Sony and Nintendo’s consoles.²¹

The main difference between Nintendo and other two platforms was low portability. In fact, Wii did not share many game titles with its competitors. Nintendo adopted a completely unique type of controller for Wii to limit the portability²² and its unique input method made it hard to port software from Wii to other platforms and vice versa. Consequently, the portability and the inter-platform indirect network effect between Nintendo and others were very low. As previously shown in section 3.4.2, the low portability allowed Nintendo to avoid the excess quality competition and low profit. The most persuasive explanation is the exogenous advantage discussed in the previous section. Nintendo had many famous first party game titles that can attract the consumers, hence Nintendo wanted to choose the lower portability to avoid costly quality competition.

Another possibility is that the platforms underestimated the negative relationship between software quality and quantity. Because the design of consoles takes considerable amount of time, it is possible that the design concept of Playstation 3 and Xbox was already determined in the middle of 6th generation video game competition when the development cost was not yet burdensome. If the harmful effect of quality on game quantity is underestimated, two symmetric platforms are likely to choose high portability since the expected

²¹Wii used the core electronic parts with substantially slower speeds compared to the other systems. The grade of parts can be directly comparable in the specification available in each manufacturer’s website.

²²Nintendo Wii used a controller that can recognize users’ movement in the 3rd dimension that Playstation 3 and Xbox 360 controllers could not capture.

quality competition effect is small. The superior platform, however, still prefers lower level of portability since it wants to take advantage of differentiation effect and choosing low portability makes the platform avoid the costly quality competition.

3.7 Conclusion

This paper showed the unique inefficiency that lies in the recent video game industry. When the quality and portability standards are to be set competitively, the platforms tend to choose excessively high qualities because the cost of quality increase is spread to the competitor platform. Therefore the platforms have an incentive to restrict the portability in order to reduce the degree of quality competition. When we allow asymmetry in the model, the superior platform has an incentive to further lower portability.

There are many directions of extension of this study. At first, analyzing three or more platforms' asymmetric competition at first would illuminate the strategic aspect of the high portability between Microsoft and Sony compared to Nintendo. Also, the empirical study could be feasible using console and software's sales and price data that relates convergence/divergence of compatibility across the platforms in each generation.

The more interesting application could be the introduction of the bargaining between the hardware manufacturers and software developers. Although the source of asymmetry in our paper is exogenous, we can endogenize the asymmetry in the model by considering the stronger voices of software developers in the 7th generation video game competition era. In software developers' point of view, high portability across the platforms are always better as the cost of porting is significantly lower than initial development cost. Since developers became bigger in order to cope with huge budget, it is possible that their bargaining power against the console manufacturers grew accordingly. If software developers can exercise influence over platforms, they will persuade platforms to set high portability. Obviously, if a platform is not heavily dependent on the third party software developers, it has no reason to increase portability and bear unprofitable quality competition. Considering that Nintendo was less dependent on the third party software developers by the help of their popular first party game titles, we can predict that Nintendo was able to choose low portability even if the third party developers demanded higher level. Endogenizing the bargaining power in the model would reveal meaningful interaction between software developers and hardware manufacturers.

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Appendices

Appendix of Chapter 1

Proof of Lemma 4: With a fixed budget, the revenue agency and the federal government would choose a and e to minimize some weighted average

$$\delta_1(1 - aP(e; i))fH(t, a, e; i) + \delta_2cae \quad (23)$$

subject to a budget constraint $cae \leq M$. Note that $\delta_1 = t$ and $\delta_2 = 1$ for the revenue agency and $\delta_1 = t(\lambda + \frac{3}{4})$ and $\delta_2 = \lambda + 1$ for the federal government. Using $a(e) = \frac{M}{ce}$, we can make (23) an unconstrained maximization problem such as

$$\delta_1(1 - a(e)P(e; i))fH(t, a(e), e; i) + \delta_2ca(e)e \quad (24)$$

Taking derivative of (24) with respect to e gives us the first order condition

$$\delta_1 \left((a_e(e)P(e; i)f + aP_e(e; i)f)H - (1 - a(e)P(e; i)f)(H_a a_e(e) + H_e) \right) = 0 \quad (25)$$

and reducing the functions gives us:

$$4\delta_1 f (a_e(e)P(e; i) + aP_e(e; i))H(t, a(e), e; i) = 0 \quad (26)$$

Note that the cost term δ_2cae is removed in the first order conditions because the choice of a and e are bound to the fixed budget; when they want to double the audit, they need to halve the effort in order to maintain the fixed budget. Since both δ_1 and δ_2 do not affect the

optimal choices of a and e , the federal government's and the revenue agency's enforcement decisions are identical under the fixed budget. ■

Appendix of Chapter 2

Proof of Lemma 8: Since $\varepsilon\left(\frac{\bar{c}(\Delta)}{C}\middle|t_1\right) > 0$ and $\varepsilon(1 - \gamma_1 | t_1) > 0$, (2.17) directly implies that $\varepsilon\left(\frac{\bar{c}(\Delta)}{C}\middle|t_1\right) < 1$ and that $\varepsilon(1 - \gamma_1 | t_1) < 1$. ■

Second Order Conditions in Section 3.1 – If we evaluate the second order derivative for High tax Region 1, in the neighborhood where the first order condition are satisfied we obtain that :

$$\begin{aligned} \frac{\partial^2 \Omega_1}{\partial t_1^2} = & -2 \frac{\bar{c}(\Delta)}{C} \frac{[1-\gamma_1(\cdot)]}{t_1} \left[\varepsilon(1 - \gamma_1 | t_1) - \varepsilon\left(\frac{\bar{c}(\Delta)}{C}\middle|t_1\right) \varepsilon(1 - \gamma_1 | t_1) \right. \\ & \left. + \varepsilon\left(\frac{\bar{c}(\Delta)}{C}\middle|t_1\right) - \frac{\partial^2 \bar{c}(\Delta)}{\partial t_1^2} \frac{t_1}{\bar{c}(\Delta)} \right] \end{aligned} \quad (27)$$

Given equation (2.18), we know that $\varepsilon(1 - \gamma_1 | t_1) - \varepsilon\left(\frac{\bar{c}(\Delta)}{C}\middle|t_1\right) \varepsilon(1 - \gamma_1 | t_1)$ is positive around the optimal solution. The following two term can be written as $\frac{1-\gamma_1(\cdot)}{\frac{\bar{c}(\Delta)}{C}} \left[1 - \varepsilon(1 - \gamma_1 | t_1) \right]$, which is also positive around the optimal solution. Consequently, second order condition are locally satisfied. We can rewrite Region 2's first order derivative as

$$1 - \frac{\bar{c}(\Delta)}{C} \left[1 + [1 - \gamma_1(\cdot)] \varepsilon\left(\frac{\bar{c}(\Delta)}{C}\middle|t_2\right) - \gamma_1(\cdot) [1 - \varepsilon(\gamma_1 | t_2)] \right] = 0 \quad (28)$$

The second order derivative is then given by:

$$\begin{aligned} \frac{\partial^2 \Omega_1}{\partial t_2^2} = & - \left[[1 - \gamma_1(\cdot)] \left[1 + \varepsilon\left(1 - \frac{\bar{c}(\Delta)}{C}\middle|t_2\right) \right] + \gamma_1(\cdot) \varepsilon(\gamma_1 | t_2) \right] \frac{\partial \bar{c}(\Delta)/C}{\partial t_2} \\ & + \frac{\bar{c}(\Delta)}{C} \left[\varepsilon\left(1 - \frac{\bar{c}(\Delta)}{C}\middle|t_2\right) + [1 - \varepsilon(\gamma_1 | t_2)] \right] \frac{\partial \gamma_1(\cdot)}{\partial t_2} \\ & - \frac{\bar{c}(\Delta)}{C} [1 - \gamma_1(\cdot)] \frac{\partial \varepsilon\left(\frac{\bar{c}(\Delta)}{C}\middle|t_2\right)}{\partial t_2} - \frac{\bar{c}(\Delta)}{C} \gamma_1(\cdot) \frac{\partial \varepsilon(\gamma_1 | t_2)}{\partial t_2}. \end{aligned} \quad (29)$$

The first and second line are both negative. Moreover $\frac{\partial \varepsilon(\gamma_1 | t_2)}{\partial t_2}$ is positive, making the last term negative. Finally, $\frac{\partial \varepsilon(\frac{\bar{c}(\Delta)}{C} | t_2)}{\partial t_2} = \varepsilon\left(\frac{\bar{c}(\Delta)}{C} | t_2\right) \left[1 - \varepsilon\left(\frac{\bar{c}(\Delta)}{C} | t_2\right)\right] + \frac{\partial \bar{c}(\Delta)}{\partial \Delta} \frac{t_2}{\bar{c}(\Delta)}$, which is positive around the optimum tax rate. Therefore the second order condition is locally satisfied for low tax Region 2 as well.

Low tax Region 1 and High tax Region 2 – The opposite case ($\Delta < 0$) can be summarized as below:

$$\begin{aligned}
 FOC_1^- = & \underbrace{\frac{\bar{c}(\Delta)}{C} A + \left[1 - \frac{\bar{c}(\Delta)}{C}\right] \gamma_2(\alpha_2, -\Delta) A}_{\text{Gain from higher tax rate}} & (30) \\
 & - \underbrace{\frac{\bar{c}(\Delta)}{C} \left[1 - \gamma_2(\alpha_2, -\Delta)\right] \varepsilon\left(\frac{\bar{c}(\Delta)}{C} | t_1\right) A}_{\text{Loss from movement of firms}} - \underbrace{\left[1 - \frac{\bar{c}(\Delta)}{C}\right] \gamma_2(\alpha_2, -\Delta) \varepsilon(\gamma_2 | t_1) A}_{\text{Loss from profit shifting}},
 \end{aligned}$$

where $\varepsilon\left(\frac{\bar{c}(\Delta)}{C} | t_1\right)$ is defined as above and $\varepsilon(\gamma_2 | t_1) = -\frac{\partial \gamma_2}{\partial t_1} \frac{t_1}{\gamma_2}$. Same interpretation from low tax Region 2 can be applied. The only difference is that γ_2 takes place instead of γ_1 since the profit shifting is conducted from high tax Region 2 to low tax Region 1. The location variables $\frac{\bar{c}(\Delta)}{C}$ and $1 - \frac{\bar{c}(\Delta)}{C}$ are switched from low tax Region 2 case, too. The first order condition are given by:

$$\underbrace{\frac{\bar{c}(\Delta)}{C} \left[1 - \left[1 - \gamma_2(\cdot)\right] \varepsilon\left(\frac{\bar{c}(\Delta)}{C} | t_1\right)\right]}_{\text{Net Marg. revenue - local firms}} + \underbrace{\left[1 - \frac{\bar{c}(\Delta)}{C}\right] \gamma_2(\cdot) \left[1 - \varepsilon(\gamma_2 | t_1)\right]}_{\text{Net Marg. revenue - foreign profit}} = 0. \quad (31)$$

For high tax Region 2, we have

$$\begin{aligned}
 FOC_2^- = & \underbrace{\left(1 - \frac{\bar{c}(\Delta)}{C}\right) \left(1 - \gamma_2(\alpha_2, \Delta)\right) A}_{\text{Gain from higher tax rate}} & (32) \\
 & - \underbrace{\left(1 - \frac{\bar{c}(\Delta)}{C}\right) \left[1 - \gamma_2(\cdot)\right] \varepsilon\left(1 - \frac{\bar{c}(\Delta)}{C} | t_2\right) A}_{\text{Loss from firms movement}} - \underbrace{\left(1 - \frac{\bar{c}(\Delta)}{C}\right) \left[1 - \gamma_2(\cdot)\right] \varepsilon(1 - \gamma_2 | t_2) A}_{\text{Loss from profit shifting}},
 \end{aligned}$$

which can be re-written as:

$$\underbrace{1}_{\text{Marginal benefit of tax}} = \underbrace{\varepsilon\left(1 - \frac{\bar{c}(\Delta)}{C} \mid t_2\right)}_{\text{Marginal cost from firms movement}} + \underbrace{\varepsilon(1 - \gamma_2 \mid t_2)}_{\text{Marginal cost from profit shifting}} \quad (33)$$

Proof of Lemma 9: We want to show that $\frac{dFOC_1}{dt_2}$ and $\frac{dFOC_2}{dt_1}$ are positive so that the best response t_1 increases as t_2 increases and vice versa. Note that both FOC_1 and FOC_2 do not contain t_2 and t_1 respectively, which means that the tax rate of one region affects another region's tax decision only through Δ which affects firms' mobility and profit shifting. Because $\frac{dFOC_1}{d\Delta} = -\frac{dFOC_1}{dt_2}$ and $\frac{dFOC_2}{d\Delta} = \frac{dFOC_2}{dt_1}$, showing $\frac{dFOC_1}{d\Delta} < 0$ and $\frac{dFOC_2}{d\Delta} > 0$ is equivalent to showing the strategic complementarity of two tax rates given the second order conditions satisfied.

Firstly, the first order derivative of high tax region 1 can be rewritten as:

$$FOC_1^+ = \frac{A}{C}(1 - \gamma_1)\frac{\bar{c}(\Delta)}{C} \left[1 - \varepsilon\left(\frac{\bar{c}(\Delta)}{C} \mid t_1\right) - \varepsilon(1 - \gamma_1 \mid t_1) \right]$$

Note that $(1 - \gamma_1)$ and $\frac{\bar{c}(\Delta)}{C}$ decreases in Δ . Also $\varepsilon\left(\frac{\bar{c}(\Delta)}{C} \mid t_1\right)$ and $\varepsilon(1 - \gamma_1 \mid t_1)$ increases in Δ . Therefore $\frac{dFOC_1}{d\Delta} < 0$ and $\frac{dFOC_1}{dt_2} > 0$. Also, developing $\frac{dFOC_2^+}{d\Delta}$ gives us:

$$\begin{aligned} \frac{dFOC_2^+}{d\Delta} &= \underbrace{\frac{\partial \gamma_1}{\partial \Delta} \frac{A}{2C}(1 - \gamma_1)t_2}_{>0} - \underbrace{\frac{\bar{c}(\Delta)}{C}t_2 \frac{\partial^2(1 - \gamma_1)}{\partial \Delta^2}}_{=0} \\ &\quad + \underbrace{(1 - \gamma_1)^2 \frac{A}{2C^2}}_{>0} + \underbrace{\frac{\bar{c}(\Delta)}{C} \frac{\partial \gamma_1}{\partial \Delta}}_{>0} > 0 \end{aligned}$$

Hence $\frac{dFOC_2}{d\Delta} > 0$ and $\frac{dFOC_2}{dt_1} > 0$. The same procedure can be applied to show the cases for $\Delta < 0$. Therefore the tax rates in two region are strategic complements. ■

Proof of Lemma 10: We first show that the intercept of the best response function is greater than 0. It can be easily shown that

$$FOC_1^+|_{t_1=t_2=0} = FOC_2^-|_{t_1=t_2=0} = \frac{A}{2C} > 0$$

Therefore Region 1's tax choice is strictly positive at $t_2 = 0$, and Region 2's tax choice is also strictly positive at $t_1 = 0$. Knowing this, the slopes less than 1 can be shown directly from the fact that all the points of intersection of the best response functions and 45° are less than 1. Therefore each region, especially when it is a high tax region, exhibits the best response function with the slope less than 1.

In order to ensure that low tax regions' best response functions exhibits the slope less than 1 too, we evaluate FOC_i at $t_1 = t_2 = 1$. For low tax region 2, this becomes:

$$FOC_2^+|_{t_1=t_2=1} = \frac{A}{2C} \left[(2C - 1)(1 - A) - \frac{1}{(1 + \alpha_1)g''(0)} \right]$$

For low tax region 1:

$$FOC_1^-|_{t_1=t_2=1} = \frac{A}{2C} \left[1 - A - (2C - 1)\frac{1}{(1 + \alpha_1)g''(0)} \right]$$

This is always negative when $A > 1$ given $\frac{1}{2} \leq C < 1$. Therefore, both countries' best response functions exhibit the slopes less than 1 when they are low tax regions. ■

Proof of Lemma 11: From (2.18), it can be easily shown that $\frac{\partial FOC_1^+}{\partial C} = 0$, hence C does not affect the tax decision of high tax Region 1. And (31) gives us

$$\begin{aligned} \frac{\partial FOC_1^-}{\partial C} &= \underbrace{-\frac{1}{C} \left[\frac{\bar{c}(\Delta)}{C} \left[1 - [1 - \gamma_2(\cdot)] \varepsilon\left(\frac{\bar{c}(\Delta)}{C} \mid t_1\right) \right] + \left[1 - \frac{\bar{c}(\Delta)}{C} \right] \gamma_2(\cdot) \left[1 - \varepsilon(\gamma_2 \mid t_1) \right] \right]}_{= 0 \text{ by FOC}} \\ &\quad + \underbrace{\gamma_2(\cdot) \left[1 - \varepsilon(\gamma_2 \mid t_1) \right]}_{< 0 \text{ when FOC is satisfied}} < 0 \end{aligned}$$

Therefore reducing C increases the optimal tax rate of low tax Region 1. ■

Proof of Lemma 12: From (2.20), we acquire

$$\begin{aligned}
\frac{\partial FOC_2^+}{\partial C} &= \underbrace{\left[1 - \frac{\bar{c}(\Delta)}{C}\right] \left[1 - [1 - \gamma_1(\cdot)] \varepsilon \left(1 - \frac{\bar{c}(\Delta)}{C} \middle| t_2\right)\right] \frac{\bar{c}(\Delta)}{C} \gamma_1(\cdot) \left[1 - \varepsilon(\gamma_1 \mid t_2)\right]}_{= 0 \text{ by FOC}} \\
&\quad + \frac{1}{C} \underbrace{\left[1 - [1 - \gamma_1(\cdot)] \varepsilon \left(1 - \frac{\bar{c}(\Delta)}{C} \middle| t_2\right)\right]}_{> 0 \text{ when FOC is satisfied}} > 0
\end{aligned} \tag{34}$$

Therefore low tax Region 2's optimal tax rate goes down when C decreases.

On the other hand, in the first order condition for high tax Region 2 shown in (33), C only affects the marginal cost from firms movement, i.e. $\varepsilon \left(1 - \frac{\bar{c}(\Delta)}{C} \middle| t_2\right)$. Note that $\frac{\partial \varepsilon \left(1 - \frac{\bar{c}(\Delta)}{C} \middle| t_2\right)}{\partial C} < 0$ as shown in (2.14). Since increased C increases the marginal cost of taxing, optimal tax rate of high tax Region 2 goes up when C decreases. ■

Proof of Lemma 13: We have

$$\begin{aligned}
FOC_1^+|_{\Delta=0} &= \frac{A}{2C} \left[1 - t_1 \left(A + \frac{1}{(1 + \alpha_1)g''(0)}\right)\right] \\
FOC_1^-|_{\Delta=0} &= \frac{A}{2C} \left[1 - t_1 \left(A + \frac{2C - 1}{(1 + \alpha_1)g''(0)}\right)\right]
\end{aligned}$$

and

$$\begin{aligned}
FOC_2^-|_{\Delta=0} &= \frac{A}{2C} \left[2C - 1 - t_2 \left(A + \frac{1}{(1 + \alpha_1)g''(0)}\right)\right] \\
FOC_2^+|_{\Delta=0} &= \frac{A}{2C} \left[2C - 1 - t_2 \left(A + \frac{2C - 1}{(1 + \alpha_1)g''(0)}\right)\right]
\end{aligned}$$

We also have that $FOC_1^+|_{\Delta=0} = FOC_1^-|_{\Delta=0}$ and $FOC_2^-|_{\Delta=0} = FOC_2^+|_{\Delta=0}$ when $1 + \alpha_1 = \frac{1 + \alpha_2}{2C - 1}$. Because $2C - 1 \leq 1$, $\alpha_1 \geq \alpha_2$ must hold when the condition above is satisfied. ■

Proof of Proposition 4: The first order conditions of Region 1 and 2 for any $\Delta \geq 0$ are respectively gives by:

$$At_1 + \frac{t_1}{(1 + \alpha_1)g''(0)} = 1 \quad (35)$$

$$At_2 + \frac{t_2}{(1 + \alpha_1)g''(0)} = 1 \quad (36)$$

Solving the equations above yields the symmetric equilibrium tax rates:

$$t_1 = t_2 = \frac{(1 - \alpha_1)g''(0)}{1 + (1 - \alpha_1)g''(0)A} \quad (37)$$

Computing the equilibrium tax rates when $\Delta \leq 0$, also gives us

$$t_1 = t_2 = \frac{(1 - \alpha_2)g''(0)}{1 + (1 - \alpha_2)g''(0)A} \quad (38)$$

These conditions require $\alpha_1 = \alpha_2$ and $C = 1$ so that best response functions are continuous.

Proof of Proposition 5: Imagine that $\Delta > 0$ with $C < 1$. Given Lemma 10 there exist a t_1^e and t_2^e , such that $t_1^e >_2^e$ and equations (2.18) and (2.20) are both satisfied whenever $t_1^e \geq \frac{2C-1}{A + \frac{2C-1}{(1+\alpha_2)g''(0)}}$, which correspond to the point where $t_2^*(t_1)$ is discontinuous. If the sufficient condition is not satisfied, the equilibrium may still exist. If the intersection lies within the discontinuous area of $t_2^*(t_1)$, then Region 2's welfare at the intersection point (t_1^e, t_2^e) should dominate welfare obtained when $\Delta < 0$. Region 2 then chooses the best response for $\Delta > 0$ and the equilibrium exists. Since $t^*(t_2)$ cross the 45 degree line above $\frac{2C-1}{A + \frac{2C-1}{(1+\alpha_2)g''(0)}}$, then there can be an equilibrium where $t_2^e > t_1^e$.

Proof of Proposition 6: $\frac{\partial \Omega_1}{\partial \gamma_1} \frac{\partial \gamma_1}{\partial \alpha_1}$, $\frac{\partial \Omega_1}{\partial \bar{c}(\Delta)} \frac{\partial \bar{c}(\Delta)}{\partial \alpha_1}$ and $\frac{\partial \Omega_1}{\partial \gamma_1} \frac{\partial \gamma_1}{\partial t_1}$ are 0 when $\alpha_1 = \infty$ while $\frac{\partial \Omega_1}{\partial \bar{c}(\Delta)} \frac{\partial \bar{c}(\Delta)}{\partial t_1} < 0$. Therefore, given that $\frac{d\Delta^e}{d\alpha_1} > 0$, $\frac{d\Omega_1}{d\alpha_1} < 0$.

Appendix of Chapter 3

Proof of Proposition 9

Proof. Since $0 \leq \sigma(\tilde{\theta}) \leq 1/2$, marginal benefit of competitive quality increase in Eq. (3.19) is always greater or equal to that of cooperative case in Eq. (3.14). In addition, the marginal cost in Eq. (3.19) is smaller or equal to that of Eq. (3.14) for $\tilde{\theta} \geq 0$. With greater marginal benefit and smaller marginal cost of quality improvement, the platforms in cooperative environment chooses higher qualities for any given effective portability. The difference of two first order conditions, $\nu(\tilde{\theta} + \sigma(\tilde{\theta}))F'(\cdot)$, increases as $\tilde{\theta}$ increases because $\sigma'(\tilde{\theta}) \geq 0$, and becomes zero when $\tilde{\theta} = 0$. Therefore $l_i^*(\tilde{\theta}) - l_i^{CO}(\tilde{\theta})$ is an increasing function of $\tilde{\theta}$. \square

Proof of Lemma 16

Proof. Equation (3.19) can be rewritten as

$$FOC = \mu - C'(l_i^*) - (\nu - \sigma(\tilde{\theta}))F'(l_i^*) \quad (39)$$

We want to show that $-\frac{\frac{\partial FOC}{\partial \tilde{\theta}}}{\frac{\partial FOC}{\partial l_i}} < 0$. Since the second order condition is satisfied, $\frac{\partial FOC}{\partial l_i} < 0$. Also we can easily show that $\frac{\partial FOC}{\partial \tilde{\theta}} > 0$. Therefore competitive quality choice increases with $\tilde{\theta}$. \square

Derivation of Asymmetric equilibrium

The asymmetric equilibrium condition implies that for the same software quantity and quality, platform 1's console price is higher than platform 2 console's by Δ . Equating the total demand and total supply as before, the inverse demand function can be derived as:

$$\begin{aligned} p_1 &= 1 - (1 - \underline{\tau} - \nu\kappa)q_1 - (1 - \underline{\tau} - \nu\kappa\tilde{\theta})q_2 + \mu l_1 - \nu(F(l_1) - \tilde{\theta}F(l_2)) + \Delta \\ p_2 &= 1 - (1 - \underline{\tau} - \nu\kappa)q_2 - (1 - \underline{\tau} - \nu\kappa\tilde{\theta})q_1 + \mu l_i - \nu(F(l_2) - \tilde{\theta}F(l_1)) \end{aligned}$$

The profit function of each platform becomes:

$$\begin{aligned}
\pi_1 &= \left(1 - C(l_1) - (1 - \tau - \nu\kappa)q_1 - (1 - \tau - \nu\kappa\tilde{\theta})q_2 + \mu l_1 - \nu(F(l_1) + \tilde{\theta}F(l_2)) + \Delta\right)q_1 \\
\pi_2 &= \left(1 - C(l_2) - (1 - \tau - \nu\kappa)q_2 - (1 - \tau - \nu\kappa\tilde{\theta})q_1 + \mu l_2 - \nu(F(l_2) + \tilde{\theta}F(l_1))\right)q_2 \quad (40)
\end{aligned}$$

Comparing Eq. (3.9) and Eq. (40), we find that the profit of inferior platform is unchanged from the symmetric case while the superior platform's profit is increased thanks to the advantage of exclusive feature.

Quantity decision

In the production stage, platforms decide the output levels that maximize their profit functions (40) for given levels of hardware quality and portability that are previously chosen. The equilibrium quantity of each platform is:

$$\begin{aligned}
q_1^*(l_1, l_2, \tilde{\theta}, \Delta) &= \frac{2(1 - \tau - \nu\kappa)\omega_1(l_1, l_2, \tilde{\theta}) - (1 - \tau - \nu\kappa\tilde{\theta})\omega_2(l_1, l_2, \tilde{\theta})}{4(1 - \tau - \nu\kappa)^2 - (1 - \tau - \nu\kappa\tilde{\theta})^2} \\
&\quad + \frac{2(1 - \tau - \nu\kappa)\Delta}{4(1 - \tau - \nu\kappa)^2 - (1 - \tau - \nu\kappa\tilde{\theta})^2} \quad (41)
\end{aligned}$$

$$q_2^*(l_1, l_2, \tilde{\theta}, \Delta) = \frac{2(1 - \tau - \nu\kappa)\omega_2(l_1, l_2, \tilde{\theta}) - (1 - \tau - \nu\kappa\tilde{\theta})\omega_1(l_1, l_2, \tilde{\theta})}{4(1 - \tau - \nu\kappa)^2 - (1 - \tau - \nu\kappa\tilde{\theta})^2} \quad (42)$$

$$- \frac{(1 - \tau - \nu\kappa\tilde{\theta})\Delta}{4(1 - \tau - \nu\kappa)^2 - (1 - \tau - \nu\kappa\tilde{\theta})^2} \quad (43)$$

where $\omega_i(l_i, l_j, \tilde{\theta}) = 1 + \mu l_i - C(l_i) - \nu(F(l_i) + \tilde{\theta}F(l_j))$ and the equilibrium profits are:

$$\pi_i^* = (1 - \tau - \nu\kappa)q_i^*(l_i, l_j, \tilde{\theta}, \Delta)^2 \quad (44)$$

Since $\Delta > 0$, the presence of asymmetry makes superior platform's market share and profit greater and inferior platform's smaller compared to symmetric case *ceteris paribus*.

Competitive Quality decision

Suppose platforms decide the quality levels of their products non-cooperatively taking the portability level given. The first order condition regarding l_i is:

$$\mu + \nu\sigma(\tilde{\theta})F'(l_i^*) = C'(l_i^*) + \nu F'(l_i^*) \quad (45)$$

where $\sigma(\tilde{\theta}) = \frac{(1-\underline{\tau}-\nu\kappa\tilde{\theta})\tilde{\theta}}{2(1-\underline{\tau}-\nu\kappa)}$ and l_i^* is the quality level that solves Eq. (45). Note that Eq. (45) does not include Δ , so l_i^* does not depend on Δ .

Portability decision

In the stage of portability setting, the equilibrium quantities are

$$q_1^* = \frac{(1 + \mu l^*(\tilde{\theta}) - C(l^*(\tilde{\theta})) - \nu(1 + \tilde{\theta})F(l^*(\tilde{\theta})))}{3 - 3\underline{\tau} - \nu\kappa - \nu\kappa\tilde{\theta}} + \frac{2(1 - \underline{\tau} - \nu\kappa)\Delta}{4(1 - \underline{\tau} - \nu\kappa)^2 - (1 - \underline{\tau} - \nu\kappa\tilde{\theta})^2} \quad (46)$$

$$q_2^* = \frac{(1 + \mu l^*(\tilde{\theta}) - C(l^*(\tilde{\theta})) - \nu(1 + \tilde{\theta})F(l^*(\tilde{\theta})))}{3 - 3\underline{\tau} - \nu\kappa - \nu\kappa\tilde{\theta}} - \frac{(1 - \underline{\tau} - \nu\kappa\tilde{\theta})\Delta}{4(1 - \underline{\tau} - \nu\kappa)^2 - (1 - \underline{\tau} - \nu\kappa\tilde{\theta})^2} \quad (47)$$

On the top of the direct and indirect effect of portability increase that have been discussed previously, there is an additional ‘differentiation effect’ in the asymmetric problem²³; the platform with greater exogenous advantage tries to reduce the portability in order to differentiate its product from the smaller platform’s. Although the small platform prefers relatively high portability, the dominant platform tries to limit the portability as it gains comparative advantage when $\tilde{\theta}$ decreases:

²³The differentiation effect is also mentioned in the previous researches. See Cremer et al. (2000) and Baake and Boom (2001).