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Joint tax evasion

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Abstract. Tax evasion analysis typically assumes that evasion involves individual taxpayers responding to some given policies. However, evading taxes could require the collaboration of at least two taxpayers. Detection depends on the costly avoidance activities of both transacting partners. An increase in sanctions leads to a direct increase in the expected cost of a transaction in the illegal sector, but it may also increase the incentive for the partners to cooperate in avoiding detection. The total cost of transacting in the illegal sector can fall, and tax evasion may increase. The policy implications of this phenomenon are considered. JEL Classification: H26

L'évasion fiscale collective. Dans les analyses de l'évasion fiscale, on suppose habituellement que le payeur de taxe fait face à un ensemble donné de politiques auxquelles il réagit. Pourtant, dans le cas des transactions marchandes, l'évasion fiscale n'est possible que si plusieurs agents coopèrent ensemble. La probabilité que l'évasion soit détectée dépend alors des efforts que chacun fait pour la cacher. Dans un tel contexte, de plus lourdes sanctions accroissent le coût espéré des transactions illégales, mais peuvent aussi, indirectement, accroître l'incitation pour les partenaires à coopérer pour cacher leur activité illégale. Il en résulte que le coût total des transactions illégales peut diminuer et l'évasion fiscale augmenter. Nous étudions les implications de ce phénomène.

1. Introduction

Most of the literature on tax evasion is presented in a principal/agent framework, with the government (principal) trying to provide the right incentives to each tax-

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payer (agent).[1] This type of analysis might be suitable for income taxes because the main strategic interaction is between the taxpayer and the government. However, there are many types of tax evasion that involve the participation of more than one taxpayer. Taxes on transactions, such as sales taxes, excise taxes on tobacco or alcohol, and taxes on trade are examples of taxes for which evasion often involves the collaboration of at least two taxpayers – a buyer and a seller. In fact, even income tax evasion might require at least the complicity of second parties, as when labour services are supplied in the untaxed sector. Our purpose in this paper is to investigate the determinants of tax evasion in settings where both agents to a transaction must collaborate to determine whether to undertake it in the illegal sector. Although our analysis involves tax evasion, it applies more generally to any form of criminal behaviour in which two agents collaborate in a criminal activity, including prostitution, the exchange of illegal goods (e.g., drugs), or a bank robbery performed by more that one agent. When two agents agree to evade taxes, they face the possibility of being detected and sanctioned. We suppose that the probability of this depends upon effort devoted to avoiding detection by the two transacting partners.[2] An increase in the sanction for tax evasion leads to a direct increase in the expected cost of a transaction in the illegal sector. However, a higher sanction may also facilitate cooperation between criminals by reducing the incentive to cheat. It may then be the case that a small increase in the sanction reduces the total cost of a transaction in the illegal sector and therefore increases tax evasion.

We construct a model in which a continuum of infinitely lived agents, differing only in their aversion to dishonesty, decide whether to undertake their transactions in the legal or the illegal sector. All agents undertake a large number of transactions each period – for simplicity, one with every other agent in the economy. This ensures that pairs of agents form lasting repeated relationships and that relationships span all combinations of honesty-types. For each transaction in each period, the pair of agents involved can choose which sector to use. We assume that no agent can force another one to transact in the illegal sector. Those who choose the legal sector in a given period obtain a sure benefit from the transaction, but have to pay a tax. Those

1 The classic analysis is by Allingham and Sandmo (1972). For general reviews of the traditional literature, see Cowell (1990) and Myles (1995). Tax evasion has been incorporated into an optimal non-linear income tax setting by Cremer and Gahvari (1996), Marhuenda and Ortuño-Ortin (1997), and Chandar and Wilde (1998). Some recent analysis has departed from the principal-agent setting by allowing taxpayers and tax collectors to collude. See, for example, Flatters and MacLeod (1995) and Hindriks, Keen, and Muthoo (1999). In this literature, there is no cooperation among taxpayers, which is the focus of our analysis.

2 When transacting in the illegal sector, individuals can cheat in several ways. One possibility is for an individual to provide less care than was agreed on in avoiding detection by the authority. This is the kind of cheating we are focusing on in this analysis. Of course, such cheating increases the probability of detection for all individuals involved in the transaction. Examples of such cheating are that an individual may publicly (rather than privately) consume a good, or that he may openly discuss the 'low' price he paid for the good. Another example is that individuals who have transacted in the illegal sector should also provide care so as to avoid being caught for other crimes, because observing one crime may reveal that other crimes have been committed.

who choose the illegal sector avoid paying taxes, but may be caught and sanctioned. They receive an uncertain benefit that depends on their aversion to dishonesty and on the level of crime enforcement undertaken by the government. The chances of getting caught engaging in an illegal transaction depend partly on the amount of costly avoidance effort that is provided jointly by the two parties to the transactions.

In our base case, the payoffs to participating in an illegal activity take the form of a prisoner's dilemma, and the two agents transacting in the illegal sector can potentially increase their payoff by simultaneously providing a high, or cooperative, level of avoidance effort. However, they will then expose themselves to potential deviation by their partner. Because contracts in the illegal sector are not enforceable, reputations and punishments must be relied on as mechanisms to enforce higher levels of effort. The possibility of cooperation enhances the payoffs from illegal activity. To enforce cooperation, agents will punish each other. Depending on the agents involved, the punishment may occur in either the legal or the illegal sector with low (non-cooperative) avoidance effort levels. For some agents – those with a higher aversion to dishonesty – the non-cooperative equilibrium in the illegal sector yields a lower expected payoff than that of the legal sector. Consequently, if one of them prefers the legal sector, they will transact in the legal sector for the duration of the punishment phase. On the other hand, if both prefer the illegal sector, they will keep on evading taxes with non-cooperative levels of avoidance for the duration of the punishment phase.

Under the assumptions we make, the resulting equilibrium takes the following form. Agents with high aversion to dishonesty pay taxes on all their transactions. Agents with low or medium aversion avoid taxes by transacting in the illegal sector with all agents willing to do so. Agents in the illegal sector cooperate with other agents in the illegal sector until one of them deviates. When one partner deviates, they enter the punishment phase of the strategy. Pairs of agents with low aversion to dishonesty remain in the illegal sector for the punishment phase, while those in which at least one of the two agents has a medium aversion to dishonesty go back to the legal sector. Because an agent's aversion to dishonesty is observable to other agents, and because agents are not willing to make a transaction in the illegal sector if they know that their partner will cheat, some agents who would prefer to trade in the illegal sector simply cannot do so. Indeed, some agents are unable to commit to behaving cooperatively in the illegal sector and, consequently, have to undertake all their transactions in the legal sector. This implies that in equilibrium, there is no deviation from cooperative behaviour in the illegal sector. In contrast to the standard literature, we find that it is not solely the willingness to participate in the illegal sector that determines which agents evade taxes, but also their ability to commit not to cheat. Some agents are left out of the illegal sector, despite their desire to transact in it, simply because they cannot commit to providing the cooperative level of avoidance effort.

When the government changes the level of the sanction, all payoffs in the illegal sector decrease, but in different proportions for different types of participant. An increase in the sanction can lead to a larger reduction in the deviation payoff than in the cooperation payoff. This can increase cooperation, thereby increasing tax evasion. Despite the direct impact of an increase in the sanction on the expected payoff of transacting in the illegal sector, tax evasion can increase with an increase in sanction because it is the ability to commit not to cheat that determines which agents evade taxes. By the same token, an increase in the tax rate can lead to an increase in tax evasion.

In the following section, we formulate the model and our assumptions, and set out the types of equilibria in avoidance effort and their results for the case where sanctions are such that payoffs in the illegal sector take the form of a prisoner's dilemma. In section 3, we analyse which levels of dishonesty will be sufficient to enable agents to commit to cooperative transactions in the illegal sector repeatedly. We establish precisely how transactions divide themselves between the legal and illegal sectors according to the aversion to dishonesty of the partners. We show that all transactions in the illegal sector will be accompanied by cooperative avoidance effort levels – no one will deviate in equilibrium. Moreover, we show that the number of transactions carried out illegally will increase in the sanction as well as in the tax rate provided the discount rate is high enough. In section 4, we extend our analysis to the case where sanctions are such that the payoffs to participating in an illegal activity no longer take the form of a prisoner's dilemma. This suggests some policy implications for the optimal level of sanctions. We conclude in section 5.

2. The model

We are interested in any economic activity involving two partners that can be undertaken either legally or illegally. We call the activity a transaction and, for simplicity, we call the partners the buyer and the seller. Naturally, since all transactions involve a buyer and a seller, there are equal numbers of buyers and sellers in this economy. We assume, for simplicity, that there is a continuum of sellers S and buyers B, and that their populations are normalized to unity. Within each group, agents differ only in their tolerance for engaging in illegal transactions – those that involve evading taxes. Denote this tolerance for dishonesty by θ, with $\theta \in [0,1]$. The distribution of θ for each group $k = S,B$ is given by the distribution function $F^k(\theta)$, where $F^{k'}(\theta) > 0$ for all $\theta \in [0,1]$. Agents in each population engage in many bilateral transactions with those in the other, and these may be in the underground (illegal) sector u or the legal one l. Our analysis focuses on representative types of transactions that can occur in each sector.

To facilitate the analysis, we make the extreme assumption that each seller from population S engages in a large number of transactions per period, one with every buyer in population B, and that both types of agents are infinitely lived. We can then treat each sequence of transactions between a given pair of agents as an infinitely repeated game in which lasting relationships between sellers and buyers determine the nature of the transactions. In particular, since the payoffs from transactions depend upon whether agents behave cooperatively or not, repeated relationships can give rise to cooperative behaviour's being sustained in equilibrium.[3]

All agents are risk neutral. They can undertake any given transaction in the legal or the illegal sector, provided the agent with whom they are transacting agrees. There will be some agents who conduct a portion of their transactions in sector l and the rest in sector u. We abstract from production and simply suppose that, for legal transactions, each seller and buyer receives a before-tax benefit per period of v_S and v_B, respectively. Those of type θ who transact in the illegal sector only get benefits of θv_S or θv_B, as well as incurring the chance of being caught. For transactions in sector l, a tax t_k per transaction is levied on each agents of type $k = S, B$. The net benefit an agent in population k obtains per transaction in sector l, denoted π_k^l, is therefore $\pi_k^l = v_k - t_k$.

Agents transacting in sector u pay no tax. Those who are detected evading the tax have sanctions s_S or s_B imposed on them. Although sanctions can vary for buyers and sellers, illegal transactions are detected on the spot. Thus, both agents are detected at the same time, so they share the same probability of detection. In fact, for the first part of our analysis, we shall assume, for simplicity, that sanctions are also the same for both types of agents ($s_S = s_B$). Later, we take up the case where sanctions can be asymmetric. Qualitatively similar results will occur if probabilities of sanctions are asymmetric, but the analysis is more complicated. Agents can reduce the likelihood of detection by providing some costly avoidance effort. For simplicity, we assume that effort can take only two values, high (H) or low (L), and that the cost associated with each of these choices is c and zero, respectively, the same for both populations. We assume that $c < t_k$; otherwise, legal transaction will always dominate illegal ones accompanied by high level of effort. The avoidance effort levels of the two individuals engaged in an illegal transaction combine to yield a probability that their transaction will be detected. If both choose effort H, then the probability is p_2; if both choose effort L, the probability is p_0; and if only one chooses effort H, the probability is p_1. It is natural to assume that as total avoidance effort increases, the probability that an illegal transaction will be detected decreases, so $p_0 > p_1 > p_2$.[4] Note the important point that the probability of detection depends only on current

3 An alternative, more complex model would assume that transactions occur randomly between agents in S and B. Kandori (1992a,b) has shown that the Folk Theorem for repeated games can be generalized to the case of a large community of individuals who are matched randomly in pairs each period. Even if two individuals are matched only once, cooperation can be enforced if their behaviour in previous matches is observable. In that case, an individual may want to cooperate because cheating now will trigger retaliation by future partners, whoever they may be. But obviously, for this to be implementable each tax evader's history needs to be observable by all agents. In this case, transgressions could be punished not only by one agent but by the entire market, making cooperation even easier to sustain. At the same time, one can argue that the government might also be able to observe the history of behaviour and enhance its enforcement accordingly. To rule out in the simplest way the possibility of illegal behaviour's being publicly observable, we adopt the environment described.

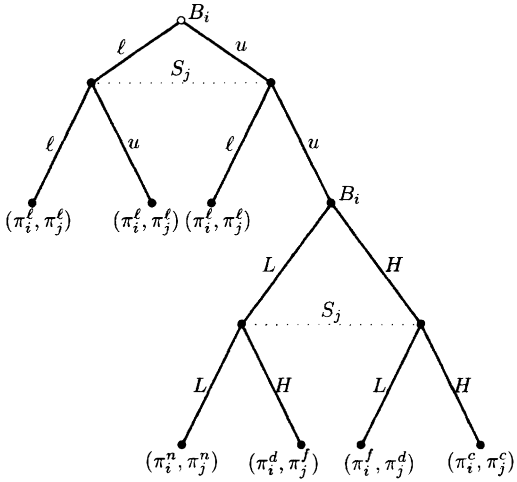
4 The impact of both agents' effort is assumed to be symmetric. Results similar to those derived below could be obtained in a generalized version of the current model, in which effort from each type has it own impact on the probability of detection.

avoidance effort. It does not depend either on past avoidance effort or on whether illegal behaviour has been detected in the past. This is obviously a strong assumption: it is conceivable that enforcement agencies monitor past criminals more intensively than they monitor those who have never been convicted. Nonetheless, the assumption is not uncommon in the literature and we adopt it for simplicity. By the same token, we assume that the sanctions s_S and s_B are independent of past convictions.

We assume that agents undertaking an illegal transaction choose their avoidance effort levels simultaneously. Two individuals providing maximal avoidance effort H are said to *cooperate*. Under cooperation, agent i in population k obtains a payoff $\pi^c_{\{i,k\}} = \theta_i v_k - p_2 s_k - c$. Alternatively, the two individuals may *not cooperate* and provide minimal avoidance effort L. Under no cooperation, the payoff of agent i in population k is $\pi^n_{\{i,k\}} = \theta_i v_k - p_0 s_k$. Because effort is chosen simultaneously, an individual may fool his cooperating partner and *deviate* from maximal to minimal avoidance effort. Because the fooled partner provides low effort, the payoff of agent i in population k who deviates is $\pi^d_{\{i,k\}} = \theta_i v_k - p_1 s_k$. That of the fooled partner, say j, is $\pi^f_{\{j,k\}} = \theta_j v_k - p_1 s_k - c$.

As mentioned, in each period all sellers of group S make one transaction with every buyer of group B. It is worth describing precisely the sequence of events and the information assumptions applying in each period. At the beginning of every period, each agent observes for every other agent with whom they transact: (i) their tolerance to dishonesty θ, and (ii) the level of avoidance effort exerted in all previous illegal transactions in which the two engaged. Given that information, agents from each group choose which of their transactions with the other group to undertake in the legal or illegal sectors. Since no agent can force another agent to engage in an illegal transaction, a transaction will be undertaken in sector u only if both agents prefer to do so. If one or both agents choose a legal transaction, each agent gets a payoff $v_k - t_k$ and the game moves to the next period. If both agents choose the illegal sector, the game moves to the next stage where avoidance effort levels are chosen. Both agents choose simultaneously and non-cooperatively their levels of effort (which can differ from that in previous periods and in other transactions in the same period), and probabilities of detection are determined. Next, the illegal transactions occur. Some of them are detected, sanctions are imposed, and agents' payoffs are determined. Since the probability of detection depends only on current-period avoidance effort, the sequence of transactions constitute a repeated game in which the only link between periods is the ability of agents to observe the past behaviour of their partners. This sequence of events is summarized in the game tree of figure 1.

As figure 1 indicates, the stage game of the repeated game involving a pair of agents includes, first, the choice of sector in which to transact and, second, if the transaction is in the illegal sector, the amount of avoidance effort exerted by each agent. The latter two-player subgame is referred to in what follows as the *effort subgame* of the stage game. The possible payoffs of this effort subgame in which each player chooses between cooperating and not cooperating are $\pi^z_{\{i,k\}}$, $z = c, n, d, f$. In figure 2, we present



(Payoff of B_i, Payoff of S_j)

FIGURE 1 The stage game

the effort subgame payoffs for a player i from population S and player j from population B.[5]

For each transaction in every period, agents must decide whether to transact in the illegal sector and, if so, how much effort to provide. The various parameter values and the size of the sanctions determine how transactions are divided between the legal and illegal sectors, and the nature of the equilibrium in the latter. In order to ensure that there is an interior solution with transactions divided between the legal and illegal sectors, we make the following assumption:

ASSUMPTION 1. *Interior solution:*

(a) $t_k > p_0 s_k > p_2 s_k + c$	*for* $k = S, B$
(b) $p_1 s_k + c > t_k$	*for* $k = S, B$
(c) $v_k - t_k > 0$	*for* $k = S, B.$

Part (a) implies that for $\theta_k = 1$, $\pi^c_{\{i,k\}} > \pi^l_{\{i,k\}}$, which says that for some agents – those with high enough values of θ – it is better to evade taxes if they can cooperate than to transact in the legal sector. This ensures that there will be some tax evasion in equilibrium. Part (a) also implies that for $\theta_k = 1$, $\pi^n_{\{i,k\}} > \pi^l_{\{i,k\}}$, meaning that for agents with sufficiently high values of θ, evading taxes under no cooperation dom-

5 It is easy to generalize this model to all crimes where two or more criminals need to interact (such as illegal drugs trafficking or prostitution), by setting tax to zero and benefit of transacting in the legal sector also to zero. Figure 2 will take the same form, and all results will apply.

	Player j	
	H	L
Player i H	$\pi^c_{\{i,S\}} = \theta_i v_S - p_2 s_S - c$ $\pi^c_{\{j,B\}} = \theta_j v_B - p_2 s_B - c$	$\pi^f_{\{i,S\}} = \theta_i v_S - p_1 s_S - c$ $\pi^d_{\{j,B\}} = \theta_j v_B - p_1 s_B$
Player i L	$\pi^d_{\{i,S\}} = \theta_i v_S - p_1 s_S$ $\pi^f_{\{j,B\}} = \theta_j v_B - p_1 s_B - c$	$\pi^n_{\{i,S\}} = \theta_i v_S - p_0 s_S$ $\pi^n_{\{j,B\}} = \theta_j v_B - p_0 s_B$

FIGURE 2 The effort subgame payoffs

inates a transaction in the legal sector. The implication of part (a) is that both cooperative and non-cooperative outcomes in the illegal sector are possible. Part (b) states that all individuals prefer to transact in the legal sector than to be fooled. Part (c) ensures that all individuals prefer to transact legally rather than not transacting at all, and, in addition, for $\theta_k = 0$, $\pi^l_{\{i,k\}} > \pi^c_{\{i,k\}}$. This ensures that some persons will transact in the legal sector. In what follows, assumption 1 is invoked as required to ensure an interior solution.

Next, it is useful to identify the circumstances in which the payoffs in the illegal sector constitute a prisoner's dilemma. The following assumption provides the characterization:[6]

ASSUMPTION 2. *Prisoner's dilemma:*

(a) $p_2 s_k + c < p_0 s_k$ *for* $k = S, B$
(b) $p_1 s_k + c > p_0 s_k$ *for* $k = S, B$
(c) $p_2 s_k + c > p_1 s_k$ *for* $k = S, B$

Part (a) says that the payoff under cooperation is larger than that under no cooperation. Part (b) implies that the best response to non-cooperation is to not cooperate. Part (c) ensures that there is an incentive to fool one's cooperating partner and deviate. If assumption 2 applies for both agents, the effort subgame constitutes a prisoner's dilemma.

We shall consider both the case where assumption 2 is satisfied and that where it is not. To start, we focus on the symmetric case where sanctions are the same for both sellers and buyers ($s_S = s_B = s$). Symmetric sanctions ensure that both types of agent either satisfy each of the parts of assumption 2 or do not. The analysis is more complicated if sanctions are asymmetric, and we return to this case in section 5. Under symmetric sanctions, we can identify three ranges of values of s over which the prisoner's dilemma does and does not apply. When $s = 0$, part (a) is not satisfied.

6 Mongrain (2001) shows that the same prisoner's dilemma structure for the payoffs can be obtained in a continuous effort model.

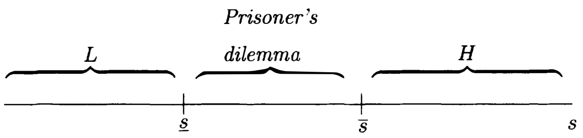


FIGURE 3 Choices in the effort subgame as a function of s

Both agents choose a low level of effort (which is also Pareto optimal), and the subgame does not exhibit the features of a prisoner's dilemma.[7] As s increases, this continues to apply until s reaches the point where (a) is just satisfied, say, $\underline{s}$. At this point, the prisoner's dilemma game becomes the relevant one. This continues to be the case until s becomes large enough that part (b) and/or (c) are not satisfied, say, $\bar{s}$. For all values of $s > \bar{s}$, the Nash equilibrium of the effort subgame is to cooperate (which is also Pareto optimal), so again the subgame is no longer a prisoner's dilemma. This is summarized in figure 3, which depicts the choices made by the players in the effort subgame for the various ranges of s.

Below, our basic analysis will be for the middle range $\underline{s} \leq s \leq \bar{s}$ in which assumption 2 is satisfied. The extension to the cases where it is violated on either side turns out to be straightforward and to yield standard results, as will be seen in section 4.

In the following two sections, we study the equilibrium of this repeated game for different types of agent and the level of illegal transactions this entails. For comparison purposes it is useful to summarize the results that would be obtained if the game described above were one-shot rather than repeated (e.g., if previous behaviour were not observable). In all cases, the one-shot game leads to equilibria that yield the same results as in the traditional tax evasion literature. If $s \leq \bar{s}$, so that the Nash equilibrium for illegal transactions involves low levels of effort, then only pairs of agents for which the benefit to each of transacting in the illegal sector with low effort exceeds the benefit of transacting legally ($\pi^n > \pi^l$) will evade taxes. All other pairs will transact legally. Since willingness to evade taxes determine the level of tax evasion, an increase in the sanction s will always reduce the size of the underground economy. Similarly, for the case where the Nash equilibrium is to provide a high level of effort ($s > \bar{s}$), the same intuition applies, except that high effort is provided. Consequently, in a one-shot game, if sanctioning is costless it is always marginally crime reducing to increase sanctions. The authorities will always prefer to set s to its maximum possible value.

3. The repeated prisoner's dilemma game

We now turn to the determination of which pairs of agents transact in the illegal sector and which pairs transact in the legal sector for the case where assumption 2 is satisfied. This involves specifying the circumstances that must apply for (repeat-

7 Note also that part (a) of assumption 1 is not satisfied, but this is inconsequential, since a high level of effort will never be exerted.

ed) equilibrium transactions in the illegal sector to entail cooperative behaviour. To anticipate our results, we shall show that only two sorts of transaction will occur in equilibrium – those in the legal sector and cooperative outcomes in the illegal sector. Thus, there will be no deviations from cooperative behaviour in equilibrium. There will be a marginal agent with $\theta_k = \bar{\theta}_k$ in each population $k = S, B$ such that transactions will be in the illegal sector only if both agents have $\theta_k \geq \bar{\theta}_k$. If at least one agent has $\theta_k < \bar{\theta}_k$, the transaction will be a legal one. Having characterized $\bar{\theta}_k$, we can then show how government policies affect the volume of illegal transactions. Paradoxically, increasing the sanction level s_k on either group can actually *increase* the number of illegal transactions. At the same time, increases in the tax rate t_k will increase the size of the illegal sector.

We begin by considering how certain pairs of agents can sustain a high level of effort, given that both choose to undertake the transaction in the illegal sector. The strategies for each agent and for each transaction in the illegal sector are assumed to be the following. All agents use the same trigger strategy with infinitely lasting punishment, where the form of punishment is discussed below. In any time period, for each transaction in the illegal sector, seller i in population S chooses a high effort level with every buyer j in population B who never deviated in any transaction with him in the past. At the same time, seller i in population S punishes every buyer j in population B who deviated in any illegal transaction with him in the past. Equivalently, for any transaction between agents i and j in the illegal sector, if buyer j in population B deviates in any time period, seller i in population S will punish him in *all* subsequent periods. This is one of an indefinite number of strategies that would lead to similar results. We choose to concentrate on this particular strategy because it is simple and standard in the literature on repeated games.

Punishments might take one of two forms. After agent j deviates, i might punish j by exerting low effort in all subsequent illegal transactions. As a result, all future illegal transactions between the pair would be non-cooperative equilibria. Alternatively, i might simply refuse to transact illegally, in which case all future transactions are in the legal sector. To choose between these two possible punishments, we follow Abreu (1988) and assume that agent i selects the punishment that imposes the worst payoff on the non-cooperating agent j. This necessarily involves punishment in the legal sector: if the non-cooperating agent is worse off in the illegal sector, then he can refuse to transact illegally.[8] In fact, allowing for the possibility that punishing agents agree to punish (non-cooperatively) in the illegal sector if they are better off in the non-cooperative equilibrium than in the legal sector does not affect our main results. In particular, as we show below, it does not affect the total number of illegal transactions, nor does it affect how that number varies with the severity of sanctions or the tax rate.

The decision to transact in the legal or the illegal sector is simple. If the two agents are in a punishment phase, the transaction is legal. If they are not in a

8 Abreu (1988) makes the case for the use of a Nash equilibrium, which inflicts the worse possible punishment on deviators. For a textbook discussion on punishments in infinitely repeated games, see Fudenberg and Tirole (1992, chap 5).

punishment phase, for each pair, a buyer (or seller) will choose the legal sector if the expected payoff is larger than that in the illegal sector. It is important to note that since the equilibrium payoffs in the effort subgame depend on θ, the choice of legal versus illegal sector will also depend on this observable variable. We begin by characterizing various ranges of values of θ that will allow us to determine which pairs of agents are able to cooperate in equilibrium in the illegal sector, which are unable to cooperate but would still prefer to transact noncooperatively in the illegal sector, and which would prefer transacting in the legal sector.

First, we establish the conditions under which agents i in population S and j in population B can commit in an infinitely repeated series of transaction to exerting the cooperative level of effort. An agent who can commit to cooperation is one who is better off being in a cooperative equilibrium indefinitely rather than deviating now and subsequently being punished forever, where the punishment is in the legal sector. Let δ_k be the discount factor for all agents of type $k = S, B$. Lemma 1 indicates which individuals in the two groups have the incentive to cooperate in equilibrium. The proofs of this and other propositions are given in the Appendix.

LEMMA 1. *An agent drawn from population k ($k = S, B$) can commit to cooperating forever if and only if $\theta \geq \bar{\theta}_k$, where*

$$\bar{\theta}_k = \frac{\delta_k(v_k - t_k) + (p_2 s_k + c) - (1 - \delta_k) p_1 s_k}{\delta_k v_k}. \tag{2}$$

The marginal agents in population S with $\bar{\theta}_S$ and in population B with $\bar{\theta}_B$ are the ones who can just commit to cooperating forever if transacting in the illegal sector. Therefore, cooperation in the illegal sector will occur for transactions between all pairs of agents i and j such that $\theta \geq \bar{\theta}_S$ for agent i in population S and $\theta \geq \bar{\theta}_B$ for agent j in population B. Conversely, if at least one agent to a transaction has $\theta < \bar{\theta}_k$ for $k = S, B$, then the transaction will take place in the legal sector.[9]

Having established which agents can commit themselves to cooperating if they choose the illegal sector, we now examine which agents will in fact prefer to make their transactions in the illegal sector.

LEMMA 2. *An agent i in population k ($k = S, B$) would prefer to cooperate in the illegal sector rather than transacting in the legal sector if $\theta \geq \hat{\theta}_k$, where $\hat{\theta}_k$ is given by*

$$\hat{\theta}_k = \frac{v_k - t_k + p_2 s_k + c}{v_k} > 0. \tag{3}$$

9 It should be noted that even if the sanction is zero for one of the two types of agents, those agents will still have an incentive to provide effort in hiding the crime, provided $t_k > c$. The reason is that, even if those agents cannot be sanctioned, they still want to provide the effort to keep the relationship in a cooperation state, since they share in the benefits of avoiding the tax. Our main results turn out to apply in this case.

It can be seen that $\hat{\theta}_k$ is decreasing in t_k and increasing is s_k.

Finally, it is useful to identify those in the population who would rather transact in the legal sector than in the illegal sector when there is no cooperation.

LEMMA 3. *An agent i in population k ($k = S, B$) prefers transacting illegally without cooperation to transacting legally if $\theta \geq \tilde{\theta}_k$, where $\tilde{\theta}_k$ is given by*

$$\tilde{\theta}_k = \frac{v_k - t_k + p_0 s_k}{v_k} < 1. \tag{4}$$

The following relationship applies among the three marginal agents of lemmas 1–3:

LEMMA 4. $0 < \hat{\theta}_k < \bar{\theta}_k < \tilde{\theta}_k < 1$.

The two middle inequalities in lemma 4 are important. The first implies that the number of agents in each population who would like to transact in the illegal sector if they could cooperate (those with $\theta \geq \hat{\theta}_k$) is larger than the number of agents who can commit to cooperating in the illegal sector (those with $\theta \geq \bar{\theta}_k$). This leaves us with some agents who would be better off in the illegal sector in a cooperative equilibrium, but cannot commit themselves to cooperating. Since, by the second inequality, those individuals also prefer the legal sector to the illegal sector without cooperation, they necessarily end up in the legal sector. The capacity to commit to cooperating is therefore a key factor. It is this capacity that ultimately determines who transacts in the legal or the illegal sector.

The second inequality is relevant for another reason. Those with $\bar{\theta}_k < \theta_k < \tilde{\theta}_k$ prefer the legal sector to the illegal sector without cooperation. Therefore, all those who cannot commit prefer to take their punishment in the legal sector. In proving lemma 1, we assumed that punishment would be in the legal sector. If punishment in the illegal sector had been allowed, these agents would still be punished in the legal sector, since they would refuse to transact non-cooperatively in the illegal sector. Therefore, lemma 1 would not be affected if punishment in the illegal sector were allowed. However, those with $\bar{\theta}_k < \tilde{\theta}_k < \theta_k$ prefer the illegal sector without cooperation to the legal sector. It is then possible that for θ_k high enough, agents would choose not to cooperate if the strategy profile entailed punishment in the illegal sector. In this case, the possibility of punishment in the illegal sector might result in a group of agents with the highest values of θ transacting non-cooperatively in the illegal sector. This possibility does not affect our results, since these persons will always be infra-marginal in the illegal sector. From now on, we assume, following Abreu (1988) and as discussed above, that maximal punishment is imposed, implying that it takes place in the legal sector.

We are now in a position to describe the equilibrium when assumption 2 (prisoner's dilemma in the effort subgame) is satisfied. All agents in each population with $\theta < \hat{\theta}_k$ prefer to trade in the legal sector, so they will make all their transactions

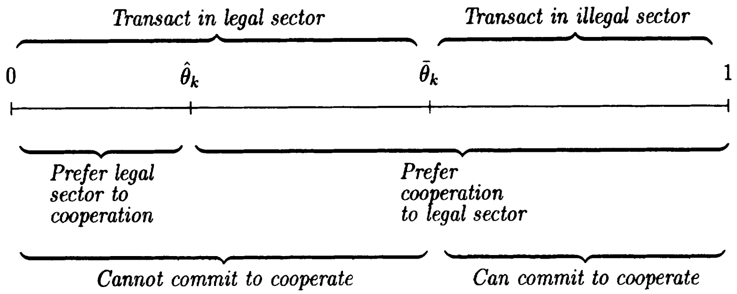


FIGURE 4 Equilibrium transactions

there. (Recall that they cannot be forced to trade in the illegal sector.) All agents in each population with $\hat{\theta}_k \leq \theta \leq \bar{\theta}_k$ would prefer to cooperate in the illegal sector rather than trade in the legal sector. They are not able to commit to cooperating in equilibrium, however, so any illegal trades must involve a non-cooperative (low avoidance effort) equilibrium. In these circumstances, they prefer the legal sector over the non-cooperative equilibrium in the illegal sector, so they all choose to transact in the legal sector. Finally, all agents in each population with $\theta \geq \bar{\theta}_k$ prefer to cooperate in the illegal sector rather than transact in the legal sector, and they can also commit to cooperating in equilibrium. So all agents in population S with $\theta \geq \bar{\theta}_S$ will transact in the legal sector with all agents in population B for which $\theta < \bar{\theta}_B$, and they will transact cooperatively in the illegal sector with all agents in population B for which $\theta \geq \bar{\theta}_B$. Since all agents transacting in the illegal sector do cooperate, punishment is never observed. This equilibrium is summarized in the following proposition and depicted in figure 4.

PROPOSITION 1. *When all players use the above-described trigger strategy with infinitely lasting punishment, all transactions between agents in population S with $\theta \geq \bar{\theta}_S$ and agents in population B with $\theta \geq \bar{\theta}_B$ are undertaken cooperatively in the illegal sector. All other transactions are in the legal sector.*

The size of the underground economy is consequently given by $[1 - F^S(\bar{\theta}_S)][1 - F^B(\bar{\theta}_B)]$, so an increase in either $\bar{\theta}_S$ or $\bar{\theta}_B$ will cause the size of the underground economy to diminish. Proposition 1 has a feature that cannot be found in the standard literature on tax evasion. Some agents in each population who would like to evade taxes are not able to do so (those with θ between $\hat{\theta}_k$ and $\bar{\theta}_k$). The reason is that these agents are too 'honest' (have a low tolerance for dishonesty θ) and are not able to commit to cooperating: they would always deviate from a cooperative equilibrium. Because honesty is observable by transacting partners, no agents from the other population want to trade with them in the illegal sector. In contrast to the standard literature, it is not solely the difference between the payoffs in the legal and illegal sectors that determines who is the marginal evading agent, but the ability of this agent to commit to cooperating in equilibrium. This difference has substantive implications for the effects of policy, as the following proposition demonstrates.

PROPOSITION 2. *If all players use the above-described trigger strategy with infinitely lasting punishment, then, in equilibrium:*

(a) If $\delta_k < (p_2 - p_1)/p_2$, an increase in the sanction s_k leads to a reduction in $\bar{\theta}_k$, which leads to an increase in the number of transactions that are made in the illegal sector.
(b) For all δ_k, an increase in the tax rate leads to an reduction in $\bar{\theta}_k$, which leads to an increase in the number of transactions that are made in the illegal sector.

Proposition 2(a) indicates that if the discount factor is low enough, an increase in the sanction s_k applied to either sellers or buyers can increase the number of agents who can commit to cooperating forever in the illegal sector, that is, $d\bar{\theta}_k/ds_k < 0$. The intuition is as follows. When s_k increases, the payoff from cooperating forever in the illegal sector decreases, while the payoff from taking the punishment phase in the legal sector stays the same. This makes it harder for an agent to commit to cooperating. At the same time, an increase in s_k leads to a reduction in the payoff from deviating larger than that from cooperating. This can lead to an increase in cooperation (a reduction in $\bar{\theta}_k$) if the discount factor is low enough.

Similarly, proposition 2(b) indicates that when t_k increases, the payoff from the punishment phase decreases so it becomes more attractive for an agent to cooperate. This result might be compared with that obtained in the standard model, where an increase in the tax rate causes a reduction in evasion if absolute risk aversion is decreasing with income (Myles 1995). Of course, since all households are risk neutral in our model, the comparison is of limited relevance. It is worth emphasizing that only one type of agent's sanction or tax needs to increase to cause the number of illegal transactions to go up. Moreover, the sanctions and the taxes applied to buyers and sellers may differ considerably.

4. Extensions

In this section we consider two extensions to the above analysis. First, we study what happens when the level of sanctions s is such that assumption 2 no longer applies. As we have seen, when s is outside the range $[\underline{s}, \bar{s}]$, the effort subgame is not a prisoner's dilemma. While proposition 2 no longer applies, it will turn out to be straightforward to characterize the effect of sanctions on the quantity of illegal transactions throughout the range of s. This will allow us to draw more general policy implications of the impact of a change in the level of sanctions. Second, we investigate the consequences when sanctions on the buyers and sellers are asymmetric. Although this complicates the analysis considerably, we are able to obtain some policy implication of adopting such a strategy.

4.1. *The impact of the sanction*

When assumption 2 is satisfied, the infinitely repeated nature of the game allows tax evaders to enforce a high level of effort on each other, a desirable outcome that is generally not possible in the one-shot prisoner's dilemma game. However, assumption 2 applies only if $\underline{s} \leq s \leq \bar{s}$. Consider what happens outside that range.

First, if $s < \underline{s}$, assumption 2(a) is violated. With a small sanction, the players' joint payoff is maximized when they provide low effort. This poses no problem, since the Nash equilibrium in that case also entails providing low effort. Thus, there is no need for the players to use a trigger strategy: they simply provide low effort when transacting in the illegal sector and there is no need for punishment. The unique subgame perfect equilibrium of the infinitely repeated game involves all illegal transactions being undertaken with low effort. Lemma 3 tells us that agents with $\theta \geq \tilde{\theta}$ are those who prefer an illegal transaction without cooperation to a legal transaction. Therefore, the size of the underground economy is given by $[1 - F^S(\tilde{\theta})][1 - F^B(\tilde{\theta})]$.[10] For $s < \underline{s}$, then, an increase in the sanction leads unambiguously to a reduction of the size of the underground economy, since $\tilde{\theta}$ is decreasing with s. On the other hand, an increase in t amplifies the size of the underground economy.

At the other extreme, when $s > \bar{s}$, Assumptions 2(b) and/or 2(c) are not satisfied. In this case, a high level of effort both maximizes payoffs and is the unique Nash equilibrium of the effort subgame. There is no need for a trigger strategy to enforce cooperation. Thus, the unique subgame perfect Nash equilibrium of the infinitely repeated game is the one in which all players provide high effort when transacting illegally. Since agents with $\theta \geq \hat{\theta}$ prefer to transact cooperatively in the illegal sector rather than to transact in the legal sector, the size of the underground economy is given by $[1 - F^S(\hat{\theta})][1 - F^b(\hat{\theta})]$.[11] This is decreasing with the sanction and increasing with the tax rate. For a given tax rate, there will eventually be a level of sanctions, denoted by s_m, at which the underground economy will have been eliminated. This corresponds with the Becker (1968) maximal sanction.

We are now able to characterize the potential impact of the sanction on the size of the illegal sector. Figure 5 presents the situation corresponding with the case in which proposition 2(a) is satisfied. There, we assume that the players do adopt the above-described trigger strategy when $\underline{s} \leq s \leq \bar{s}$, and that the condition $\delta_k < (p_2 - p_1)/2$ holds. Figure 5 presents the size of the illegal sector, denoted by μ, as a function of the sanction s. Recall that $\underline{s}$ is the value of s such that assumption 2(a) is satisfied with equality.[12] Similarly, $\bar{s}$ is the value of s such that either assumption 2(b) or assumption 2(c) is satisfied with equality, while the other may still be satisfied as an inequality.[13]

10 Note that when $s^S = s^B = s$, then $\tilde{\theta}_S = \tilde{\theta}_B = \tilde{\theta}$.

11 Again, when $s^S = s^B = s$, then $\hat{\theta}_S = \hat{\theta}_B = \hat{\theta}$.

12 In particular $\underline{s} = c/(p_0 - p_2)$, and if $s < \underline{s}$, assumption 2(a) is not satisfied.

13 In other words, $\bar{s} = \min\{c/(p_0 - p_1), c/(p_1 - p_2)\}$. Note that $\bar{s} > \underline{s}$. When $s > \bar{s}$, assumption 2 is violated.

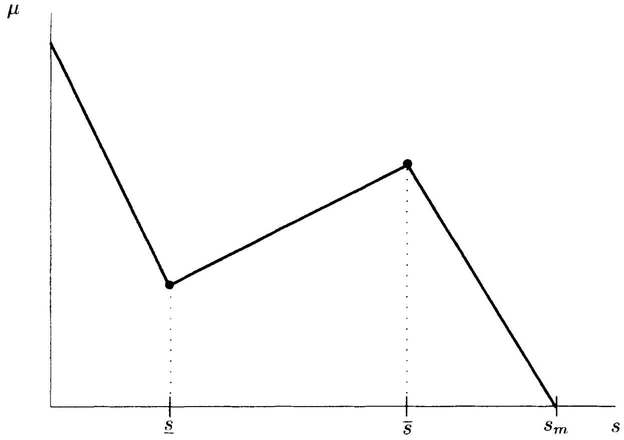


FIGURE 5 Possible impact of the sanction on the illegal sector

Suppose the government would prefer the size of the illegal sector to be as small as possible. As figure 5 indicates, the illegal sector is decreasing in s when $s < \underline{s}$ or when $s > \bar{s}$, but it is increasing in s between $\underline{s}$ and $\bar{s}$. Under such circumstances, and if there is no restriction on the size of the sanction, it is obviously optimal to set s at its maximal level, s_m, thereby eliminating all underground transactions. But if there is a limit on the sanction that could be imposed,[14] the government would not necessarily want to impose the largest permissible sanction. For example, if the limit to the sanction falls between $\underline{s}$ and $\bar{s}$, then the optimal sanction is $\underline{s}$. The simple policy implication here is that the largest permissible sanction is not necessarily that which minimizes the amount of crime.

4.2. Asymmetric sanctions

Suppose the government can impose different sanctions on the buyers and the sellers. Note, first, that when the sanctions are such that assumption 2 is satisfied for both agents and proposition 1 applies, asymmetric sanctions have the same impact as symmetric ones. That is, proposition 2(a) applies to changes in sanctions on one side of the market alone. Note that this result requires that sanctions be imposed on both sides of the market so that $\underline{s} \leq s_k \leq \bar{s}$ for $k = S, B$ (and that agents adopt trigger strategies). It is not sufficient to impose sanctions on one side only. By the same token, if assumption 2 is violated in the same direction for both types of agents – either $s_k < \underline{s}$ or $s_k > \bar{s}$ for $k = S, B$ – the same results as above apply. In these ranges, an increase in the sanction on either agent will reduce the size of the underground economy.

14 The reasons for this could be ethical, but also technological, informational, or, even simpler, a matter of costs.

Matters are more complicated if assumption 2 is satisfied for one type of agent and violated for the other. For example, consider the case where assumption 2 is satisfied for type-B agents, but violated for type-S agents. Take the extreme case where the sanction on the sellers is set to zero. Then, all sellers prefer to transact in the illegal sector, and because assumption 2(a) is violated, they will choose a low level of effort. In such a case, a type-B agent cannot expect cooperation to be enforced. Thus, the size of the illegal sector is determined $\tilde{\theta}_k$, the level of θ identifying an agent indifferent between transacting legally and transacting illegally without cooperation. Because all sellers prefer the illegal sector, the size of the illegal sector is $1[1 - F(\tilde{\theta}_B)]$, and an increase in s_B reduces the size of the illegal sector. At the same time, an increase in s_S from zero will also reduce the size of the illegal sector to $[1 - F(\tilde{\theta}_S)][1 - F(\tilde{\theta}_B)]$. A similar analysis can be performed for other combinations of the sanctions. The results will obviously be different, but they will have the same flavour.

A number of other questions might be raised once asymmetric sanctions are introduced. For one thing, if side payments were allowed in the infinitely repeated game, asymmetric sanctions might have the same impact as symmetric ones of the same total size. For another, if the two illegal traders are not detected at the same time, asymmetric sanction might be used to extract information from the agent who has been detected, analogous to a system of plea bargaining. These questions are beyond the scope of the present analysis, but would be interesting topics for further analysis.

5. Conclusion

The two main results of this paper are as follows. First, when tax evasion requires the complicity of two agents (e.g., a buyer and a seller), a key determinant of which transactions are in the illegal sector is the ability of each of the participating agents to commit to undertaking the cooperative level of avoidance activity. Indeed, some agents would like to evade taxes, but cannot because of their inability to commit to cooperate. In our model, ability to commit is determined by an agent's tolerance for dishonesty. More dishonest agents are better able to commit, since their payoffs from illegal activity are higher. Second, when the discount factor is low enough, an increase in the sanction can increase the ability of an agent to commit to cooperate, and can lead to more tax evasion.

Our analysis could be extended in several ways. The current model assumes that aversion to dishonesty (θ) as well as past deviations are observable. As is shown in Mongrain (2001), making these things costly to observe – in the limit unobservable – can help to explain the dynamics of recidivism. Indeed, individuals may then go back and forth from the legal to the illegal sector. In the same vein, individuals could search for partners in the illegal sector rather than meeting them randomly. If search costs are large enough, cheating is less likely to occur, because individuals willing to transact in the illegal sector would find it more difficult to seek each other out. It would also be possible to endogenize the size of the surplus that agents obtain when transacting. For example, an individual with a low aversion to dishonesty may decide to carry out relatively large transactions in the illegal sector. Finally, the current analysis is a positive one. It would be interesting to compare optimal deterrence policy – optimal probability of detection and sanctions – in the current multi-agent framework with those that obtain in the standard tax evasion model.

Appendix

Proof of lemma 1. For any agent i in population S, the discounted payoff of cooperating forever is given by $\pi^c(\theta_i)/(1-\delta_S)$, while the discounted payoff of deviating is $\pi^d(\theta_i) + \delta_S \pi^l/(1-\delta_S)$. Agent i will prefer cooperating forever if $\pi^c(\theta_i)/(1-\delta_S) \geq \pi^d(\theta_i) + \delta_S \pi^l/(1-\delta_S)$, or equivalently if $[\theta_i v_S - p_2 s_S - c]/(1-\delta_S) \geq [\theta_i v_S - p_1 s_S] + \delta_S[v_s - t_S]/(1-\delta_S)$. Therefore, all agents with $\theta \geq \bar{\theta}_S$ will want to cooperate, and all agents with $\theta < \bar{\theta}_S$ will want to deviate, where $\bar{\theta}_S = (\delta_S(v_S - t_S) + (p_2 s_S + c) - (1-\delta_S)p_1 s_S)/(\delta_S v_S)$. We can find $\bar{\theta}_B$ in the same way.

Proof of lemma 2. An agent i in population k for $k = S, B$ will prefer to cooperate in the illegal sector rather than transacting in the legal sector if $\pi_k^c(\theta) \geq \pi_k^l$, or if $\theta_i v_k - p_2 s_k - c \geq v_k - t_k$. Therefore, agent i will prefer the illegal sector if $\theta \geq \hat{\theta}_k$, where $\hat{\theta}_k = (v_k - t_k + p_2 s_k + c)/v_k > 0$. ■

Proof of lemma 3. An agent i in population k for $k = S, B$ will prefer to not cooperate in the illegal sector rather than transacting in the legal sector if $\pi_k^n(\theta) \geq \pi_k^l$, or if $\theta_i v_k - p_0 s_k \geq v_k - t_k$. It is apparent that agent i will prefer the illegal sector if $\theta \geq \tilde{\theta}_k$, where $\tilde{\theta}_k = (v_k - t_k + p_o s_k)/v_k < 1$.

Proof of lemma 4. Using (2) and (3), $\hat{\theta}_k < \bar{\theta}_k$ for $k = S, B$ if $(v_k - t_k + p_2 s_k + c)/v_k < (\delta_k(v_k - t_k) + p_2 s_k + c - (1-\delta_k)p_1 s_k)/(\delta_k v_k)$, or $p_2 s_k + c > p_1 s_k$, which we have assumed to be satisfied. Using (2) and (4), it is possible to show that $\bar{\theta}_k < \tilde{\theta}_k < 1$ for $k = S, B$ if $p_0 s_k > p_2 s_k + c > p_1 s_k$, which holds given assumption 2. ■

Proof of proposition 1. This is an immediate implication of lemma 4 and of the description of the equilibrium. ■

Proof of proposition 2. Using $\bar{\theta}_k$ from lemma 1, we obtain $\partial\bar{\theta}_k/\partial s_k = (p_2 - (1-\delta_k)p_1)/(\delta_k v_k)$, which is negative if $\delta_k < (p_1 - p_2)/p_1$. We know from proposition 1 that $\bar{\theta}_k$ determines which transactions are made in the legal or illegal sectors for each group. Since $F^{k'}(\cdot) > 0$, and since the number of transactions in the illegal sector is $[1 - F^S(\bar{\theta}_S)][1 - F^B(\bar{\theta}_B)]$, it follows that anything that decreases $\bar{\theta}_k$ also increases the number of illegal transactions. As for the tax, using equation (2) yields $\partial\bar{\theta}_k/\partial t_k = -1/v_k < 0$. ■

References

Abreu, D. (1988) 'On the theory of infinitely repeated games with discounting,' *Econometrica* 56, 383–96

Allingham, M.G., and A. Sandmo (1972) 'Income tax evasion: a theoretical analysis,' *Journal of Public Economics* 1, 323–38

Becker, G.S. (1968) 'Crime and punishment: an economic approach,' *Journal of Political Economy* 76, 169–217
Chandar, P., and L.L. Wilde (1998) 'A general characterization of optimal income tax enforcement,' *Review of Economic Studies* 65, 165–83
Cowell, F.A. (1990) *Cheating the Government: The Economics of Evasion* (Cambridge, MA: MIT Press)
Cremer, H., and F. Gahvari (1996) 'Tax evasion and optimal general income tax,' *Journal of Public Economics* 60, 235–49
Flatters, F., and W.B. MacLeod (1995) 'Administrative corruption and taxation,' *International Tax and Public Finance* 2, 397–417
Fudenberg, D., and J. Tirole (1992) *Game Theory* (Cambridge, MA: MIT Press)
Hindriks, J., M. Keen, and A. Muthoo (1999) 'Corruption, extortion and evasion,' *Journal of Public Economics* 74, 395–430
Kandori, M. (1992a) 'Repeated games played by overlapping generations of players,' *Review of Economic Studies* 59, 81–92
— (1992b) 'Social norms and community enforcement,' *Review of Economic Studies* 59, 63–80
Marhuenda, F., and I. Ortuño-Ortin (1997) 'Tax enforcement problems,' *Scandinavian Journal of Economics* 99, 61–72
Mongrain, S. (2001) 'Recidivism,' mimeo, Simon Fraser University
Myles, G.D. (1995) *Public Economics* (Cambridge: Cambridge University Press)