Real-World Applications in Math Class

by

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Abstract

Calls to connect school mathematics to applications in the real-world are ubiquitous. I examine the experience of senior high school students as they encounter a student-centred real-world application task applying logarithms and exponential functions in a murder mystery context. I observed students through the task, analyzed their written solutions, and administered a follow-up questionnaire. Four case studies illustrate the range and nuanced experiences of students completing the real-world task. During the real-world task students experienced prolonged motivation, they made sense of abstract mathematics through the context of the task, and they benefited from group interactions. This empirical study provides support for the claimed benefits from the literature for the inclusion of real-world applications in the teaching and learning of secondary mathematics.

Keywords: Mathematics education; real-world mathematics; applications; Realistic Mathematics Education; student-centred task; classroom experience

Dedication

For Sara, who believed in me the whole way.

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To the builders of the Masters of Secondary Mathematics program at Simon Fraser University, you have had a huge impact on my professional career. To my colleagues in the program, you have inspired me to be a better teacher through our discussions and reflections of our practices.

And to those who are on the thesis path, take it one step at a time. Find strength from the many before you who have struggled yet persevered where you are now!

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List of Acronyms

BC	British Columbia
BCIT	British Columbia Institute of Technology
CORD	Center for Occupational Research and Development
CCF	*Common Curriculum Framework*
PC12	Pre-Calculus 12
RME	Realistic Mathematics Education
TIMSS	Trends in International Mathematics and Science Study
WNCP	*Western and Northern Canadian Protocol*

Mathematical Variables and Constants in Murder Mystery Task

C	constant to be determined
E	temperature of environment
k	constant to be determined
t	amount of time body has been dead
T	temperature of body
T_A	temperature of environment
T_0	initial temperature of body

Chapter 1.

Introduction

I can remember sitting in Grade 12 mathematics class wondering who uses the mathematics that was being taught. At senior levels in high school advanced mathematics, connections to reality are seldom present. Now as a high school mathematics teacher, I am trying to make sense of curriculum topics and continually searching for their applicability in the world beyond the walls of the classroom.

In my Grade 12 mathematics class, I recall the particular lesson's topic was complex numbers. My teacher told me that electrical engineers use complex numbers. She was not sure how or for what purpose. Although she had been teaching mathematics for over 20 years and was a dedicated teacher, she could not explain how this topic was used in the real world. As a student, I wondered why I was learning what I was learning and wanted to understand the people and purposes for which it could be used.

During my undergraduate degree in engineering, the purpose of mathematics became more apparent because mathematics was embedded within science and engineering courses and topics were reinforced through workplace co-op placements. For example, while working at a landing gear designer and manufacturer, a team of engineers and I problem-solved the stabilization of a feedback control system which involved complex numbers! We were also applying algebra and graphing including polynomials and equation roots. I was working on a real-world application of the mathematics topics I had spent years studying in high school and university. Finally, I was catching glimpses of how the mathematics I had learned in high school was used by professionals in the workplace. Because I studied and briefly worked in a field that uses mathematics to achieve non-mathematical goals, I learned examples of industry contexts in which classroom mathematics topics are applied.

In my teacher training in Ontario, I studied mathematics education in a class of pre-service teachers with an applied mathematics bent. Some of my classmates had worked as engineers and were now looking to teaching for a second career. Others had university backgrounds in physics, biology, or chemistry. Our instructor had a background in computer science. Through class presentations and discussions, mathematics felt useful and valued as a tool for understanding situations, solving problems, and designing solutions for areas outside of mathematics. While on campus during this time, I organized workshops for a large robotics competition in which I helped high school students apply concepts from high school mathematics to create a line-following robot. My year of teacher training reinforced the belief that applied uses of high school mathematics need to be conveyed to the students.

When I became a high school teacher, my drive to further my knowledge of real-world mathematics applications hit a speed bump. There were topics in my courses for which I had no idea why students needed to learn them. I would hit this bump when students asked me “Who uses this?”, “When will I need to know this?”, “Why do we have to learn this?” - sometimes I had answers, often I did not. It also arose when I was planning units or lessons that jumped from one area of mathematics to another with no logical transition. A good motivator might be a story, video, or problem about the applications of the mathematics topic we were about to learn but I had difficulty finding resources about applications when I could not draw on my own experience. How could I come to know and share with students the applications of classroom mathematics that stemmed from areas beyond my expertise, such as those encountered by physicists, chemists, biologists, and beyond?

I was not sure how to extend my knowledge of applications of classroom mathematics. The mapping of a particular classroom mathematics topic to its real, authentic uses in careers was elusive. Experienced teachers had tricks and tips to address students’ “When will I use this?” One teacher told me his answer is, “For the unit test and then the exam.” Another felt that when students say “When will I ever use this?”, they are *really* saying, “I don’t understand. This is too hard. I would like to give up,” and thus the appropriate teacher response is to address the underlying cause of their comment. Another teacher gave his students the analogy of football players running a tire drill:

> Will you ever see a football player running through tires during a football game? No! So why do they do it in practice? It trains their mind and body to be agile. In the game, players will encounter situations where quick reflexes and decision making will be invaluable. Will you ever use the quadratic equation in life? No! So why do you practice in school? Well, what happens in math problems? You have a problem, not sure what to do, you have to make sense of given information and other known information, you break the problem into smaller parts, pick a strategy and solve each part until you eventually solve the whole problem. That's useful. Useful for planning a dinner party or a vacation through Europe or getting a job. So no, you will never use the quadratic equation in real life. And a football player will never run through tires during a game. But you will apply the thinking processes of mathematics all the time.

Each of these strategies seemed to work for its teacher. But even if they satisfied the student, I was not satisfied. They each side-step the core question: when does someone use this piece of mathematics? Even if in some cases students do not really mean what they are asking, I really meant it when I was in Grade 12. I really did want to know where mathematics is actually used. How does it all link together? What awaits on the road ahead? And intentional or not, students' cyclical question of "When will I use this?" spotlights an area of knowledge about mathematics that is missing from the tradition of teaching mathematics – how topics within mathematics are used by people in the real world. It is a question to which we, mathematics teachers, ought to have an answer as ambassadors of mathematics.

When a student asks, "Who uses this?" a teacher may search for a real, authentic application for the particular mathematics topic being studied. In my experience, the teacher is often not able to find one. Not to say it does not exist, but rather it is not easy for a mathematics teacher to locate. Surprisingly, it seems no one has undertaken the knowledge-creating endeavour of mapping typical topics from a high school mathematics sequence to their myriad of applications in real-world careers.

I have had first-hand experiences as an engineering student applying sinusoidal functions and algebraic topics including polynomials, roots, and like-terms to achieve non-mathematical goals in industry settings. Without these experiences, I too might believe that there are no applications and ascribe the purpose of mathematics to other aims such as training the mind to think. With my work experience in applying mathematics in industry settings, I hold on to the idea that classroom mathematics is connected to useful applications in careers today.

Unfortunately, I did not make many gains in finding applications of mathematics topics in the early years of becoming a teacher. Teaching is a busy profession and there is not time for everything. As the years passed, my pedagogy improved in many areas, my relationship-building skills with students and parents improved, I started a family, and life took on other priorities. Instead of focussing on applications of mathematics, my professional learning time concentrated on pure mathematics, simply because there are many organized student opportunities in pure mathematics. I trained students in pure mathematics problem solving and brought them to provincial competitions. I joined the provincial and then national marking committee for an international mathematics contest. Besides mathematics, I focussed on technology in the classroom and led some workshops in school and at conferences. Other extra-curricular activities that I led for students took up the rest of my professional time, so the question of how mathematics is used in society faded away.

The question returned when I began to pursue a master's degree in secondary mathematics education. A PhD student presented her research on mathematical modelling in one of our classes. This reminded me of my engineering experiences - where modelling was a central theme through course work and co-op work placements. I realized that the various instances where I had recognized the application of classroom mathematics in engineering involved the use of a mathematical model - mathematics was being used to describe a particular system, and through mathematics a solution was found.

I saw a thesis as an opportunity to rekindle my question about applications of classroom high school mathematics. I hoped that through the thesis process I could explore and gain understanding about how teachers can tie mathematics to the real world. I initially set out to map all key topics in high school mathematics curriculum and I predictably struggled finding these linkages. I pared down the scope and focussed on one application of high school mathematics. In exploring one application in-depth, I hope to understand if real-world mathematical connections are important to my students as they are for me and uncover further dynamics of implementing this shift to valuing and exploring real-world applications in the mathematics classroom.

Chapter 2.

Literature Review

In exploring real-world mathematics in the classroom, it will help to survey previous research in this field. First, I will distinguish applications from its cousin, mathematical modelling. Then I will briefly examine its history as a research field and identify the broad goals of including applications in mathematics education. I will look in-depth at a mature educational philosophy known as Realistic Mathematics Education (RME) and follow up with a handful of empirical classroom studies. As there are two sides to any coin, I will also examine criticisms, cautions, and problems of incorporating real-world connections in the classroom voiced by researchers. Finally, I will be well-positioned to put forward an informed research question which blends my professional interests with the needs of the mathematics applications research field.

2.1. Applications from Modelling

Two terms often heard in discussions around real-world mathematics are applications and modelling. This study involves a real-world *application* task rather than a *modelling* task. While these terms are often lumped together within the literature, I will look to distinguish the two here. A teacher designing an application task is asking, "Where can I use this particular piece of mathematical knowledge?" (Stillman, 2010). The teacher has some mathematics and then searches for a real-world context where it can be used to solve a problem. This real-world context serves, among other things, to illustrate the utility of that mathematics to students who must learn it. Simply put, an application is a mathematics solution looking for a real-world problem.

In contrast, mathematical modelling is a real-world problem looking for a mathematics solution. The student is provided with a problem within a real-world context. The student then decides which mathematics to use in order to model the situation mathematically. The student analyzes and solves the mathematics and then interprets and validates the results in the context of the described situation. The solution is then validated in the real world and the cycle repeats to further improve the model and its solution. The modelling cycle is illustrated in Figure 1.

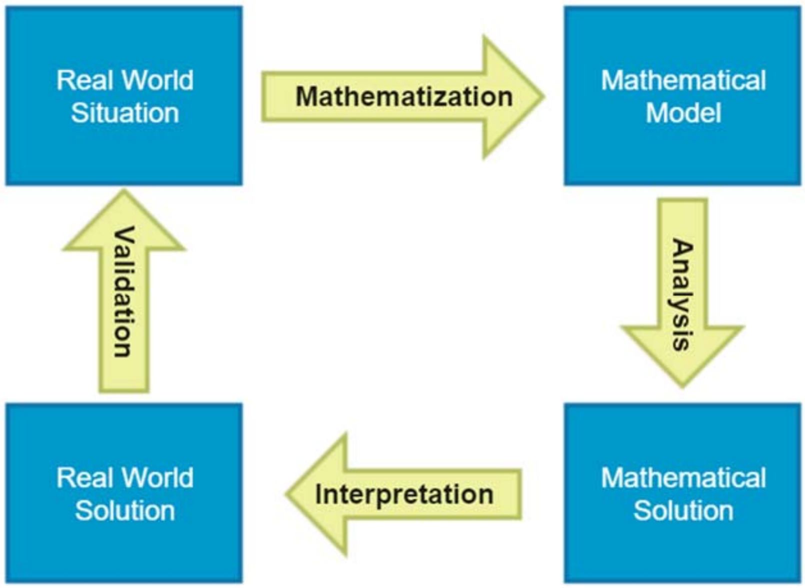


Figure 1. Modelling Cycle

Applications and modelling are easily confused because they both involve using mathematics to solve a problem within a context, illustrated in Figure 2. In modelling, a real-world situation is provided by the teacher and it is up to the student to mathematize it, that is, to come up with a mathematical model to solve the problem. It is as if the student is standing outside mathematics looking in and asking, "Where can I find some mathematics to help me with this problem?" (Niss, Blum, & Galbraith, 2007, p. 10). Different students may incorporate different mathematical topics into their model. Consider students who are asked how long it will take a ball to hit the ground when dropped from the top of the school. As a modelling task, some students may construct a linear model, others quadratic, others a novel method. As an application task, the teacher might have selected the same situation in order to illustrate a particular mathematics topic – perhaps square roots or quadratic functions or integration. Furthermore, the teacher would likely prescribe the mathematical model to use – perhaps $t = \sqrt{d/4.9}$ or $h = 9 - 4.9t^2$ or $a(t) = -9.8$. The differences between applications and modelling are subtle and at times fuzzy, which likely explains why they are often lumped together as “applications and modelling.”

Figure 2. Applications and Modelling both connect the Real-World and Mathematics

2.2. History and Goals

The call for including applications in mathematics courses has persisted for decades. The 14th International Commission on Mathematical Instruction traces the beginnings of applications and modelling as a research field to the 1960s (Blum, Galbraith, Henn, & Niss, 2007). In 1968, Hans Freudenthal organized the conference “Why to Teach Mathematics so as to be Useful?” In his opening address, he identified the weaknesses of teaching only pure mathematics, only applied mathematics, and the commonplace compromise of pure mathematics followed by applications. He argued the best approach “starts in a concrete context and patiently returns to concrete contexts” (Freudenthal, 1968). The subsequent work of Freudenthal in this field is detailed in section 2.3.

Since the 1968 conference, the field of applications and modelling has passed through a number of phases. The Advocacy Phase until 1975 called for serious inclusion of applications in curricula. In 1983, the first International Community of Teachers of Mathematical Modelling and Applications conference took place, and has recurred every other year since then. The Development Phase from 1975 to 1990 experienced actual development of curricula and materials (Blum et al., 2007).

Since 1990, the field has been in the Maturation Phase. Empirical studies of teaching and learning are being added to the theoretical perspectives but there is not yet an abundance of this sort of research available (Blum et al., 2007). This research seeks to narrow the gap.

In addition to appreciating the history of the research, it is important to articulate the goals of incorporating real-world connections into the classroom. Boaler (1993) noted that the abstract nature of mathematics can be seen as a “cold, detached, remote body of knowledge” (p. 13). Connecting mathematics to careers and contexts involving people humanizes the field. Blum et al., (2007) summarize three goals of using applications in the learning of mathematics:

- to develop a broad image of the nature and role of mathematics by demonstrating that it is used by people for a variety of purposes
- to help provide meaning and interpretation to mathematics concepts and processes
- to motivate students

The Dutch have been pursuing these goals for decades through an educational movement and philosophy known as Realistic Mathematics Education. The next section will examine the impact and advancement of the movement in the Netherlands and beyond.

2.3. Realistic Mathematics Education

Mathematics must be connected to reality, stay close to children, and be relevant to society in order to be of human value. Freudenthal 1977 (as cited in Hough & Gough, 2007)

2.3.1. What is RME?

RME is an educational philosophy that uses context as a route into mathematics and a medium through which learners develop understanding of mathematics (Hough & Gough, 2007). This is a reversal of a traditional approach where applications occur at the end of the teaching and learning sequence, once the topic has been taught and exercises practiced.

The term “realistic” in RME refers to contexts that the learner can imagine, as in “real in one’s mind.” RME originated in the Netherlands and the Dutch for “to imagine” is “zich realiseren.” Thus contexts can be taken from the real world or from fantasy worlds. RME views mathematics as a human activity as opposed to a body of knowledge to be transmitted to the learner. Mathematics must be real in the learner’s mind and relevant to society (Van den Heuvel-Panhuizen & Wijers, 2005).

2.3.2. RME in the Netherlands

The father of RME, Hans Freudenthal (1905-1990), was a professor of mathematics at the University of Utretcht from 1946 until his retirement in 1975. Freudenthal disagreed with the abstract approach of the “New Mathematics” movement that prevailed in the US during the 1960s. Freudenthal believed that instead of the learner grappling with a topic in abstraction from the outset, the learner should start with familiar contexts and progress through a series of carefully selected problems which naturally lead to abstraction (Case, 2005).

Three years after the landmark 1968 conference, Freudenthal founded the Institute for the Development of Mathematics Instruction - later renamed the Freudenthal Institute (Van den Heuvel-Panhuizen & Wijers, 2005). The Freudenthal Institute is the birthplace of RME and today the Institute continues to develop RME’s theory and curriculum (Hough & Gough, 2007).

By 1980, the influence of RME appeared in 5% of elementary school mathematics textbooks across the Netherlands. Ten years later, 75% of elementary school mathematics textbooks in the country had adopted an RME perspective. In the mid-1990s, RME was formally adopted into the elementary and secondary national mathematics curriculum (Case, 2005). Today, RME continues to play a strong role in the Dutch mathematics education system (Van den Heuvel-Panhuizen & Wijers, 2005). Applications and modelling are mandated at all levels of learning within the mathematics curriculum (Vos, 2013). Its principles are reflected in the curriculum: annual multi-week projects, secondary mathematics textbooks, and teachers’ pedagogy.

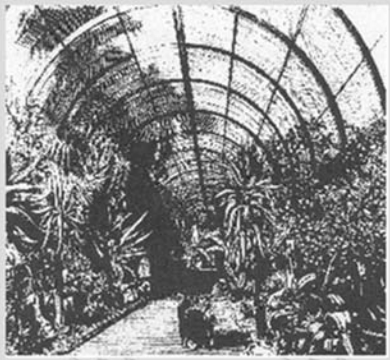
Vos (2013) details two undertakings in the Netherlands as examples of large scale RME initiatives in the country. One relates to the 1993 curriculum and the second is an annual team competition known as A-lympiad.

In 1993, a new middle school (Grade 7-8) and junior high school (Grade 9-10) curriculum was legislated for lower pathway students. The curriculum stressed usefulness with an emphasis on data modelling, 3D geometry, estimation, and information and computer technology. The use of variables was postponed and set theory abandoned. The curriculum called for a range of contexts to be used in examples and included open-ended problems. National exams incorporated questions that appealed to a daily life situation related to real-world photos. This pathway of Grade 8 Dutch students outranked Australia, England, Sweden, and the US in the Trends in International Mathematics and Science Study (TIMSS) 2003 (Vos, 2013).

The Mathematics A-lympiad is an annual full-day modelling competition organized by The Freudenthal Institute since 1989. The problems are complex, open-ended, and realistic, derived from real-life situations in areas such as politics, sociology, archeology, sports, and life sciences. The competition has grown from under 50 students in 1989 to over 4000 today. About 20% of Dutch schools participate and some other countries such as Denmark and Germany have joined the competition (Vos, 2013).

In teams of three or four, Grade 11 and 12 students engage in problems for a full day. The problems promote mathematics as an organizing activity where students act as policy consultants who incorporate numerical data and quantitative reasoning in their advisory role of a real-world issue explored in-depth. Students must justify assumptions and weigh options. The A-lympiad problems do not have a prescribed procedure or single correct answer (Vos, 2013). For instance, a problem might ask students to determine the best proportion of several types of plants for particular environments. The problem includes expert testimony about ecological diversity and objectives such as creating a formula to quantify diversity (Case, 2005). Part 1 of this A-lympiad is shown in Figure 3 and Part 2 can be found in the Appendix. The full set of A-lympiad problems are available from http://www.fi.uu.nl/alympiade/en/.

Diversity



Introduction

The plants and animals on our planet are having a hard time. Increasing numbers of species are dying out and the continued existence of many others is seriously threatened. Nature conservation institutions are trying to turn the tide both locally and globally. But it seems a hopeless struggle. In addition to the economic and financial problems there are also biological ones: favouring one species is often detrimental to another species. All these problems in maintaining as great a diversity of species as possible have fortunately not stopped the thinking in this respect. Two different aspects of diversity are considered in this assignment. Part 1 is about diversity where the species and numbers are examined. Part 2 is on the genetic relationships between species.

Part 1

In a rubber plantation the diversity of plants is of course less than in a natural forest. A measure needs to be found for this 'diversity'. In order to obtain insight into the conditions this measure has to satisfy, four photos of different combinations of five plants A, B, C, D and E, have been made. The following numbers of plants are displayed on each of the photos.

	A	B	C	D	E
photo 1	40	10	20	5	5
photo 2	40	20	-	25	30
photo 3	40	20	10	25	30
photo 4	40	30	10	-	20

Experts were asked to rank these photos according to decreasing diversity. They unanimously decided that photo 3 had the greatest diversity, and classified photo 2 ahead of photo 4. They did not all agree on the position of photo 1. Ultimately they came to the following order:

photo 3
photo 2
photo 4
photo 1
↓ decreasing diversity

Exercise 1
Specify the factors that were clearly considered to be important for 'diversity'.

Exercise 2
One of the ways of defining 'diversity' is to make it equal to the probability of getting two different species when arbitrarily selecting two items from the set. (Formally the selection is without putting the first one back, but here we will act as if it is put back.) Provide a formula for this definition of 'diversity'. Examine how the formula behaves, for example by looking at: the boundaries of the diversity; when the diversity has a maximum value for a collection of s species. Can you give a formula for this maximum diversity? What role does this maximum diversity play?

Exercise 3
Is the result of the definition of exercise 2 consistent with the result of the ranking of the photos? What will be the effect on diversity of thinning out one or more plants?

Figure 3. 1992-1993 A-Lympiad Problem Part I ("Diversity", 1993)

Some have pointed to the RME curriculum as an explanation for the Netherlands' success in TIMSS 2003 where Grade 12 students ranked first on reasoning, data analysis and social utility skills (Van den Heuvel-Panhuizen & Wijers, 2005).

2.3.3. RME Around the World

RME has been adopted in various educational settings worldwide including the US and England. According to de Lange (as cited by Shipulina, 2013), other nations that have adopted RME are Germany, Denmark, Spain, Portugal, South Africa, Brazil, Japan, and Malaysia.

In 1991, the Freudenthal Institute partnered with the University of Wisconsin-Madison to develop “Mathematics in Context” in the US, a Grade 5 to 8 mathematics curriculum for middle schools. The project was funded by the National Science Foundation to develop new curricula to reflect the 1989 National Council of Teachers of Mathematics curriculum standards that were consistent with RME. Mathematics in Context consists of 10 units per grade with sample problems, assessment materials, teacher guides, supplementary packs including estimation skills in authentic contexts and a collection of tasks for basic skills (Educational Development Centre, 2001). Mathematics in Context was implemented in several districts with research showing gains in achievement both in external and internal assessments (Romberg, Wabb, Shafer, & Folgert, 2005). The external assessment used National Assessment of Educational Progress and TIMSS items for number, geometry, algebra, and statistics/probability. The internal assessment was developed by the Freudenthal Institute and used non-curricular tasks in real-world contexts that were accessible by students at a variety of levels. Student responses were scored for computational strategies, explanations and descriptions, use of patterns, algebraic / geometric / measurement strategies, and justifications. Examples of the real-world contexts were playgrounds, baby formula, airships, and monkeys (Romberg et al., 2005).

Mathematics in Context was adapted in the UK. Funded by the Gatsby Foundation, Manchester Metropolitan University purchased Mathematics in Context in 2003 to develop a UK curriculum, textbooks, and videos known as “Making Sense of Maths.” In 2004, Making Sense of Maths was rolled out to 12 schools in the UK. In 2007, Manchester Metropolitan University partnered with The Freudenthal Institute to further develop materials for Making Sense of Maths including 10 booklets with British contexts. For instance, (1) how many Manchesters will fit into England? (2) find the length of a road in front of a car that cannot be seen by the driver. (3) draw to scale a hand span that you think is typical for a pianist (Hough & Gough, 2007).

Articles on RME have appeared periodically in *Mathematics Teacher*, a National Council of Teachers of Mathematics journal for middle and high school mathematics education. This is where I was first introduced to RME. I was searching for articles on real-world mathematics for a graduate course paper in EDUC 847 *Teaching and Learning Mathematics* in late 2014 and came across several articles on the topic.

RME was the beginning of an international trend moving toward the integration of realistic problems in mathematics education.

2.4. Empirical Studies

As noted by Blum et al. (2007), more empirical studies are needed on the implementation of applications in the classroom to complement the theoretical perspectives available. In this section, two such empirical studies will be described.

Gainsburg (2008) investigated the kinds of real-world connections secondary mathematics teachers make in practice. She found their real-world connections in mathematics classrooms are typically brief and require no construction of knowledge by the student. Furthermore, Gainsburg found that teachers often use real-world connections from everyday experience, not from workplace settings. Teachers prioritize contexts that appeal to student interest and usually think up the real-world connection themselves (Gainsburg, 2008).

Harvey & Averill (2012) point out that effectively incorporating real-world contexts is complex. While policy and curriculum often emphasize the importance of real-life contexts, the literature lacks examples of best practice. They go on to describe successful algebra lessons by a master teacher whom they observed using real-life contexts. Their analysis identifies the following key aspects of success:

- careful planning
- contexts introduced in an unhurried way with time spent on non-mathematical aspects of context
- ongoing referral to real-world context
- validity of mathematical solutions considered against real-world context
- teacher's questioning, passion for subject, depth of knowledge to develop real-world context, and relationship with students

Two algebra contexts were used. In the first context, students had to find a relationship between the span of a pre-fabricated bridge and the number of triangles needed to construct its sides. Students were shown a photograph of a local bridge (see Figure 4) and were told the triangles of this particular bridge were equilateral with side length 10 metres. Students had to then represent the relationship between the number of triangles and the bridge span using a table, a graph, and an algebraic rule. The second context involved carpeting a senior staff member's office at the school. Students could select various sizes for a central carpet square and then had to calculate the number of plain carpet squares to tile the rest of the room. The first context gave rise to linear relationship and the second a quadratic relationship (Harvey & Averill, 2012).

Figure 4. Bridge photo for real-world context in Harvey & Averill study (2012).

Harvey & Averill acknowledge that while the mathematics and context were woven together, the solution method is unlikely to be used in the real-world for these problems. The authors argue that it may only be necessary for teachers to use contexts that are “mainly faithful” to the real-world rather than having the same mathematics in the classroom.

The authors point to other successful elements such as positive teacher-student relationships, the teacher’s passion for the subject, and the teacher’s depth of knowledge to develop the real-world contexts (Harvey & Averill, 2012).

2.5. Criticism

Several researchers indicate problems associated with the calls for and implementation of real-world applications in the classroom. Beswick (2011) notes that prevailing opinion endorses linking classroom mathematics as closely as possible with the real-world yet cautions that this enthusiasm is occurring in advance of evidence of its effectiveness. She calls for research into how real-world contexts assist understanding of mathematics and which contexts are effective in which circumstances - accounting for factors such as learner characteristics and diversity. Beswick (2011) advises advocates of real-world context to ensure research is conducted to support beneficial claims.

Boaler (1993) puts forward a number of cautions and problematic assumptions with the typical use of real-world contexts in mathematics classrooms:

- contexts which attempt to motivate can instead hinder understanding if students are unfamiliar with the context (e.g. wage slip, household bills).
- the assumption that the introduction of contexts will influence student motivation but not impact the selection and accurate usage of the mathematical procedure. A teacher may expect real-world context to improve engagement but also expect students to approach the problem identically to the procedures learned in questions without context
- the assumption that students will correctly engage as if a “task were real while simultaneously ignoring factors that would be pertinent in the real life version of the task.” Boaler asserts that students do become skilled at engaging school mathematics real-world questions at the “right” level but this likely contributes to their inability to transfer to situations outside of school.

One example of the right or wrong level at which to engage a real-world question can be illustrated in the problem “How many ways can ten people sit around a circular table?” Kavousian reported some students insisting the answer should be 10! since in a real room there would be a difference between someone sitting at the spot in front of the window and the spot in front of the desk. The problem implicitly assumes the student will count permutations invariant under rotation. Kavousian terms this the “secret language” that teachers expect students to know, often without explicit instruction (Gerofsky, 2006).

Finally, it can be difficult to locate real-world uses of mathematics in authentic settings like the workplace. Smith (1999) argues that technological advances have led to the de-mathematization of the workplace since much of mathematics occurs within the technology’s “black box.” This position was supported in a study by Nicol (2002) where prospective teachers visited workplace sites in order to locate real-world mathematics. The prospective teachers observed and interviewed staff, and had to develop lesson plans based on their visits. They found it difficult to identify mathematics in the workplace and, when identified, had difficulty incorporating such mathematical ideas into teaching sequences at an appropriate level for their classes (Nicol, 2002).

2.6. Research Question

The literature makes it clear that the inclusion of real-world connections has the potential to serve students by boosting motivation, clarifying mathematical concepts, and building a broad view of mathematics (Blum et al., 2007). Groups around the world have worked on building curricula that connect mathematics to the real world, such as RME in the Netherlands, Mathematics in Context in the United States, and Making Sense of Maths in the UK (Case, 2005; Hough & Gough, 2007; Vos, 2013).

However, researchers are also cautioning the enthusiasm for real-world connections. It is not well understood how to best address problems that can arise when attempting to make real-world connections (Beswick 2011; Boaler, 1993; Gerofsky, 2006). Blum et al. (2007) have noted that more empirical research is needed to shed light on this gap in the literature.

Several classroom studies have focussed on the teacher's role in real-world connections (Beswick, 2011; Gainsburg, 2008; Harvey & Averill, 2012; Nicol, 2002). But what about the students' role? More research is needed to document the student experience. Understanding the student experience could potentially verify the theoretical goals of real-world connections and reveal insights not captured in studies that have focussed on the teacher experience.

The term "real-world" does not have a standard definition in research or practice. For my purposes, real-world refers to contextual details within a narrative that are based on, or inspired by, a situation in daily life or a workplace environment. There will be differences between the classroom activity and what actually takes place in the world. Thus a real-world task or real-world connection is a model of reality, with assumptions and simplifications. In my experience, students are quick to point out the differences between a real-world task and actual reality so with students I often substitute the expression "real-world-ish" to indicate it is not a perfect match.

Through this classroom empirical study, I plan to examine students as they learn through a real-world application of mathematics and have them reflect on the experience. My research question is:

How do students experience and perceive a real-world application in mathematics class?

Chapter 3. Methodology

This chapter describes the methodology of the study undertaken to address the research question. The research took place within a secondary school and in this section the school, classroom, and participants involved is described. The real-world task used for the research is presented along with a sample solution. Finally, the data - which includes my observations, student work, and student surveys - is detailed along with methods of analysis.

3.1. Theoretical Underpinnings

The research presented here is qualitative and grounded in an interpretivist epistemology. This type of research is concerned with how the social world is interpreted, understood, and experienced (Mason, 2004). My role as a researcher in this context is to encapsulate the students' subjective experience. As their classroom teacher with an already established role in the classroom, I am an insider teacher-researcher (Burke & Kirston, 2006).

The interpretive approach involves observation of people in the natural context, in this case, students in the classroom. The qualitative researcher seeks to discover and describe aspects of the social world which are not well captured through quantitative approaches. Epistemological and ontological assumptions are grounded in social constructivism which views people's reality created through social interactions and filtered through a subjective lens.

This interpretive qualitative research uses an inductive analysis approach. In inductive analysis, conclusions are drawn from empirical observation. Unlike the traditions of the natural sciences, hypotheses based on a priori theories are not tested. Rather, explanations are generated once data is captured. In this way, knowledge and understanding of how students experience real-world tasks is not restrained to my preconceptions as the researcher. Different researchers are likely to produce non-identical findings. A key measure of the trustworthiness of the findings is whether they represent the subjective world views of the participants based on feedback from participants (Thomas, 2003).

Since I have an established relationship with the participants as their classroom teacher, I am an insider researcher. This creates several advantages for interpretive research including that I have a greater understanding of the culture being studied, I will not alter the flow of social interactions unnaturally, I am in a better position to be able to judge the truth, and I have unhindered access to participants (Bonner & Tollhurst, 2002).

The drawbacks of insider research include that certain characteristics may be difficult to identify because the behaviour of participants within the setting is so familiar and taken for granted (Bonner & Tolhurst, 2002). Similarly, there is a risk of being so involved naturally in the setting that I could lose the research perspective by becoming a non-observant participant (Burke & Kirton, 2006). This risk is especially present in the complex role as classroom teacher where there are so many simultaneous decisions and factors to manage that it may be difficult at certain instances to dedicate sufficient attention to making observations as a researcher. This risk will be partially mitigated in the study since I have selected a task that students can undertake on their own with little instruction or guidance from me.

3.2. Setting and Participants

The setting for this research is my workplace, an affluent all-boys university prep school for Grades 1 through 12 with approximately 1200 students in Vancouver, British Columbia (BC). At the time of the study, I was in my sixth year as a teacher there. Mathematics classes for Grades 8 through 12 run from September to early June every other school day for 75 minutes. This study took place over two periods in May in my Pre-Calculus 12 (PC12) class.

At the school, every student is in one of three mathematics pathways referred to as provincial, honours, and advanced. Students in the provincial pathway take the mathematics course corresponding to their grade, ex. taking PC12 in Grade 12. Students in the honours pathway take the mathematics course corresponding to a year above their grade, ex. taking PC12 in Grade 11. The advanced pathway is similar to the honours pathway with the addition of enrichment topics and mathematics contest preparation. Typical class averages for each pathway are high 90s for advanced, high 80s for honours, and high 70s for provincial. Students in all pathways take the Pre-Calculus pathway of mathematics courses, while Foundations of Mathematics as well as Apprenticeship and Workplace Mathematics are not offered at the school. Out of 160 students in Grade 11, about half are in the provincial pathway, two fifths in honours, and the rest in advanced. The assignment of students to a pathway takes place at the start of Grade 8 based on student grades in Grade 7, teacher recommendations, and student or parent preference. Students can switch pathways in any grade according to their learning needs.

Eight students in my honours PC12 class were participants in this study. The remaining 12 students from the 20 in the class were not selected as participants for several reasons: They did not submit a consent form, they were partnered with someone who did not submit a consent form, they were absent from school on either of the days the study took place, or there was insufficient data for analysis.

As the teacher of PC12, I have considerable freedom in deciding the routines and expectations within the class as well as the delivery of lessons and design of assessments. The textbook, *Pre-Calculus 12* (McAskill et al., 2012), is used routinely as a source of exercises and problems. The course curriculum is laid out in documents by the government of BC. Both the textbook and provincial curriculum for PC12 are discussed in the next section.

3.3. Pre-Calculus 12

The curriculum from PC12 was born from the *Common Curriculum Framework*, a collaboration by several western provinces and northern territories. In 1993, the Ministers of Education from BC, Alberta, Manitoba, Saskatchewan, along with the territories signed a protocol which was later renamed the *Western and Northern Canadian Protocol* (WNCP) *for Collaboration in Basic Education Kindergarten to Grade 12*. WNCP published the original mathematics curriculum for K-9 in 1995 and 10-12 in 1996 under the title The *Common Curriculum Framework* (CCF). They released a new curriculum under the same name for K-9 in 2006 and 10-12 in 2008 (WNCP, 2008). The 2008 CCF describes the curriculum for PC12 which was implemented in the province of BC in September 2012. The research for this study took place in the 2014-15 school year, the third year of the new CCF curriculum for PC12.

The Ministry of Education website contains a 14-page document called *Pre-Calculus 12*, shown in Figure 5. This document has been extracted from the complete 114-page CCF document for Grades 10-12, also on the website. Unfortunately, the calls for real-world connections are made in the full CCF document and in the 24-page extracted Introduction, but not in the *Pre-Calculus 12* document. A teacher who downloads the curriculum for PC12 by selecting the *Pre-Calculus 12* document may not realize there are calls for real-world connections in the course located in other documents.

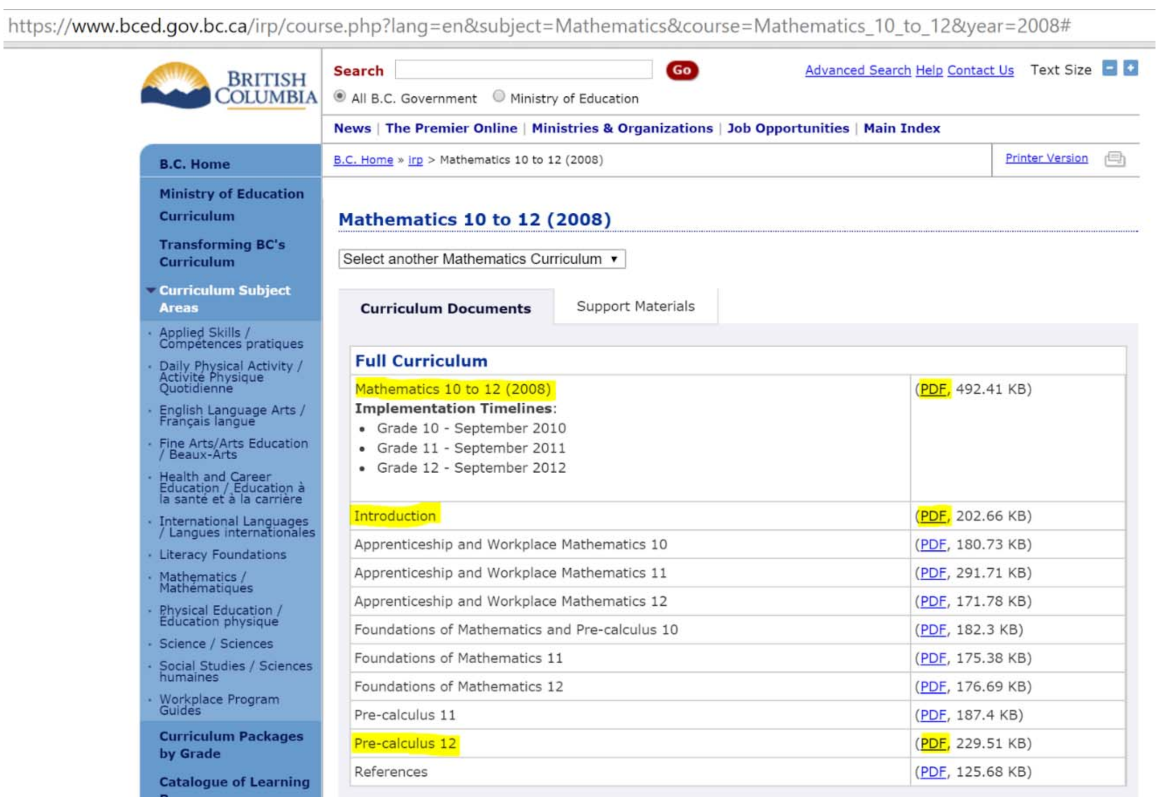


Figure 5. BC Curriculum Documents Screenshot from March 2016

The CCF for Grades 10-12 (contained in the *Introduction* document but not the *Pre-Calculus 12* document) identifies five main goals of mathematics education. One of these five goals is to prepare students to "make connections between mathematics and its applications" (WNCP, 2008, p. 4). To achieve the goals, seven mathematical processes are identified which "students must encounter … regularly" (p. 6). One of these processes is Connections: "When mathematical ideas are connected to each other or to real-world phenomena, students begin to view mathematics as useful, relevant and integrated. Learning mathematics within contexts and making connections relevant to learners can…increase student willingness to participate and be actively engaged" (p. 7).

The provincial curriculum document for *Pre-Calculus 12* contains a set of general and specific learning outcomes that make almost no explicit reference to connections between mathematics and the real world. Every specific outcome includes a list of optional achievement indicators. The only outcome with achievement indicators linked to the real-world is B10 "Solve problems that involve exponential and logarithmic equations." Indicators B10.5 through B10.8 relate to the real-world:

B10. Solve problems that involve exponential and logarithmic equations. [C, CN, PS, R]	10.1 Determine the solution of an exponential equation in which the bases are powers of one another. 10.2 Determine the solution of an exponential equation in which the bases are not powers of one another, using a variety of strategies. 10.3 Determine the solution of a logarithmic equation, and verify the solution. 10.4 Explain why a value obtained in solving a logarithmic equation may be extraneous. 10.5 Solve a problem that involves exponential growth or decay. 10.6 Solve a problem that involves the application of exponential equations to loans, mortgages and investments. 10.7 Solve a problem that involves logarithmic scales, such as the Richter scale and the pH scale. 10.8 Solve a problem by modelling a situation with an exponential or a logarithmic equation.

Figure 6. PC12 Learning Outcomes Related to Applications

The PC12 curriculum disappointingly contains few required connections to the real-world in its listed learning outcomes and achievement indicators. The other 148 Achievement Indicators grouped into 18 Specific Learning Outcomes and three General Outcomes do not make reference to the real world. The only other possible real world related indicator is B12.7: "Solve a problem by modelling a given situation with a polynomial function and analyzing the graph of the function" (WNCP, 2008, p. 101). However, the indicator does not specify real-world context for modelling and since the indicator is in a group of abstract indicators, the requirement here to make real-world connections is weak at best. In contrast to some of the real-world goals and processes stated in the introduction of CCF, the PC12 curriculum frames a predominantly abstract course.

McGraw-Hill Ryerson's *Pre-Calculus 12* textbook (McAskill et al., 2012) that I use in my class takes greater steps towards addressing applications and connections. Every chapter begins with a photograph depicting a real-world scene, a narrative previewing the chapter's content and a connection to the real-wold, a 'Did You Know?' box often with a historical fact, and a description of a career which uses mathematics (see Figure 7 for an example). The chapter is divided into several sections, and each section begins with a photograph and a couple of paragraphs suggesting connections between the photograph and mathematics as shown in Figure 8. The section contains a list of practice problems sorted into categories which often progress from abstract, simple exercises through to challenging word problems in a real-world context. This list is somewhat opposite to Freudenthal's urging that learners should start with familiar contexts and progress through a series of problems leading to abstraction (Case, 2005). The section ends with *Project Corner* which includes another photograph and problem often connected to the real world. Finally, every few chapters are grouped into a unit (see Figure 9). The unit ends with a Unit Project whose goal, according to the textbook, is to "connect the math in the unit to real life using experiences that may interest you" (McAskill et al., 2012, p. vi). The bulk of the text's explanation, examples, summaries, and exercises are abstract in nature and lack real-world context. However, given all the instances of real-world connections at the start and end of sections, chapters, and units, it is clear the editors have made efforts to include real-world contexts in the PC12 textbook.

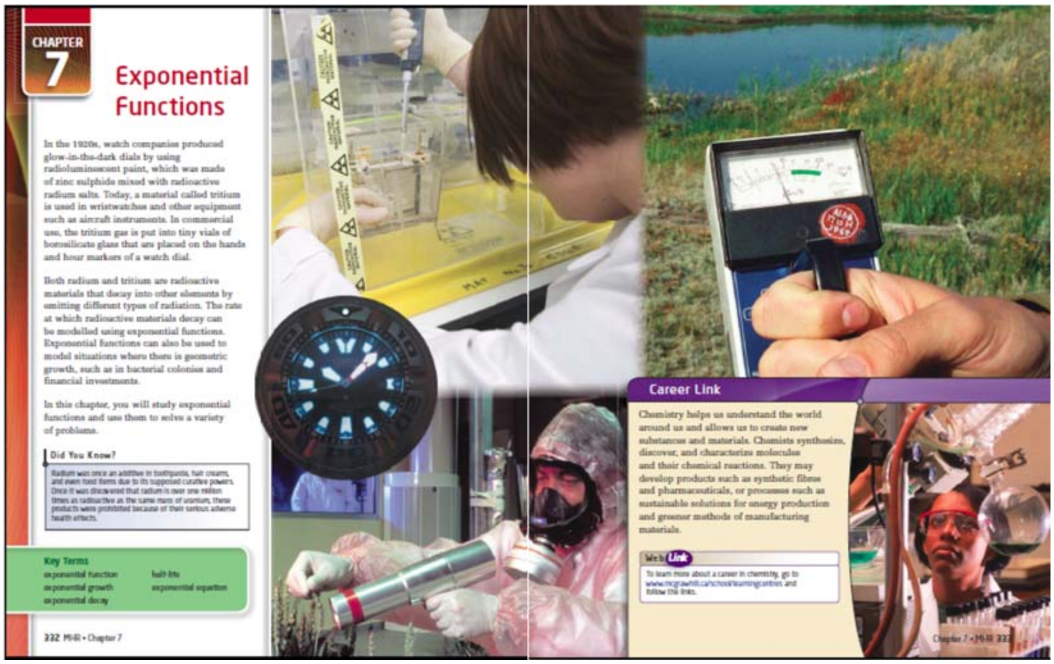


Figure 7. PC12 Textbook Real-World Features at Start of a Chapter

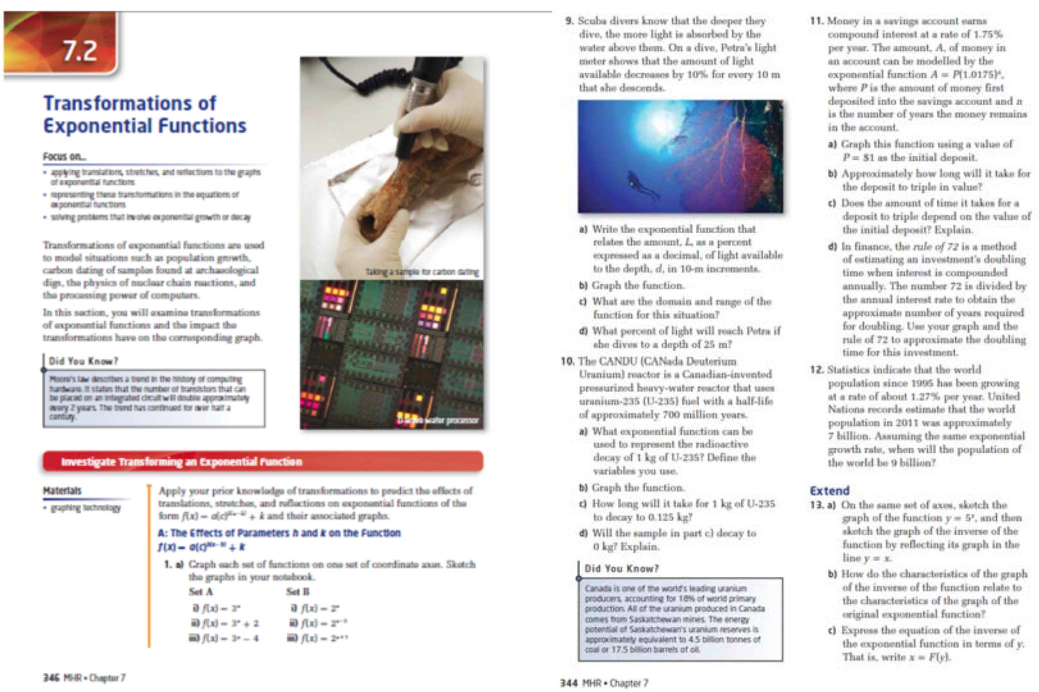


Figure 8. PC12 Textbook Real-World Features within a Section

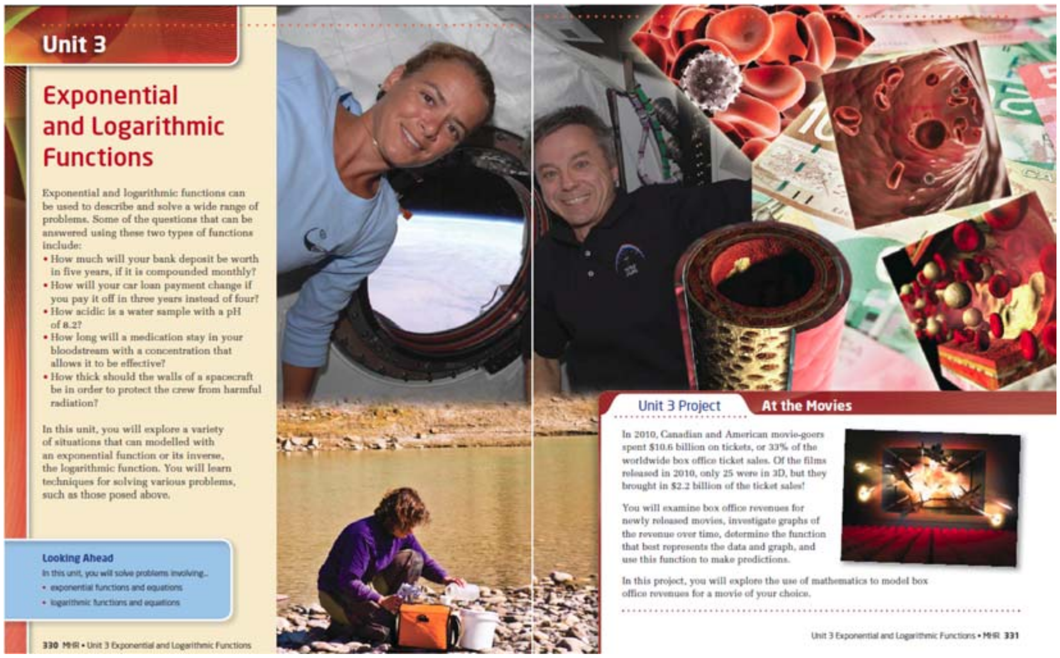


Figure 9. PC12 Textbook Real-World Features at Start of a Unit

During the 2014-15 school year, there were six teachers at my school delivering a PC12 course. All teachers follow the same ordering of the textbook units, are expected to include quizzes and a unit test for each chapter, and all students write a common end-of-year cumulative exam. Otherwise, each teacher is free to implement the course as (s)he chooses. The ordering of course topics from September to June is presented in Table 1.

As a teacher, I aim to balance the abstract mathematics at the core of PC12 as framed in the curriculum document with real-world contexts in line with the textbook. I also aim to complement a significant amount of direct instruction with student discussions, student explorations, and open-ended assignments. I regularly assign word problems for homework from the textbook that make real-world connections, such as banking interest or pH scales. I also include videos and assignments in the course, such as students researching real-world applications of sinusoidal functions in engineering fields.

Table 1. PC12 Course Topics Sequencing

Time Frame	Topics	Specific Outcomes	Duration
September to November	Permutations, Combinatorics, and the Binomial Theorem	C 1, 2, 3, 4	8 classes
	Function Transformations	B 2, 3, 4, 5, 6	8 classes
	Radical Functions	B 2, 3, 4, 13	6 classes
	Polynomial Functions	B 11, 12	6 classes
December to March	Unit Circle	A 1, 2, 3, 5	5 classes
	Trigonometric Functions	A 4, 5	7 classes
	Trigonometric Identities	A 5, 6	8 classes
	Function Operations	B 1	6 classes
April to June	Exponential Functions	B 2, 3, 4, 9 10	5 classes
	Logarithmic Functions	B 7, 8, 9, 10	6 classes
	Rational Functions	B 14	5 classes
	Exam Preparation		6 classes

The real-world task for the research presented here took place following the Exponential and Logarithmic Functions units. The task addresses the aforementioned provincial curriculum specific learning outcome B10, *Solve problems that involve exponential and logarithmic equations*; the processes of communication, connections, problem solving, and reasoning; and achievement indicators 10.5, *Solve a problem that involves exponential growth or decay* and 10.8, *Solve a problem by modelling a situation with an exponential or a logarithmic equation*.

3.4. Real-World Task

In what follows, I present the real-world task used in this research study. First, the real-world task called Murder Mystery is presented along with its solution. This is followed by the real-world task selection criteria and considerations.

3.4.1. Murder Mystery Task

This section describes the Murder Mystery task, its implementation in the classroom, and possible mathematical solutions to the task.

In the Murder Mystery task, students were invited to use mathematical analysis to solve a murder mystery. A handout described the fictitious murder that had recently taken place in a chemistry lab at the British Columbia Institute of Technology (BCIT). Students help solve the crime by determining the victim's time of death and identifying suspects. They were provided with:

- three body temperatures for the victim (normal 36.6°C, once the dead body was discovered 32.3°C, and one hour after the body was discovered 30.8°C)
- the temperature of the environment where the body was discovered (20°C)
- a formula $T = E + Ce^{-kt}$ to model body temperature (T) of the deceased body in relation to time (t) since death where *C* and k are constants to be determined and *E* is the temperature of the environment (20°C)
- Entry and exit times for six personnel into the lab where the murder took place

Students were asked to solve the time of the murder and generate a list of suspects based on the entry and exit times.

I provided students with a brief oral overview of the task. Students partnered up and every group received the handout shown in Figure 10 and Figure 11. Students could work at their desks or on the whiteboards at the front and back of the classroom. Each pair needed to write-up their own analysis and conclusion but were free to consult with other groups to solve the problem. After I had explained the task and distributed the handouts to each pair, the students had the remaining 60 minutes in class to complete the task.

Description: *Newton Cooling Murder Mystery with Numbered Suspects*

Jump To: Question | Information Fields

Question:

`B.C.I.T. Chemistry Lab, SW3-4695, Burnaby Campus,  4:46 am`

Police arrive on the scene of a suspected homicide. There is no sign of blood loss nor does it appear that the body has been moved since the time of death. A forensics expert immediately determines the temperature of the body to be 32.3°C.

A technician arriving for her shift identifies the body as that of Dr. Samuel Oliver Lewis. His medical records are accessed and reveal that he was in good general health (before his death). Records of his last full physical exam indicate that his normal body temperature was 36.6°C.

Exactly one hour after police arrived on the scene, the temperature of the body is taken again and found to have cooled to 30.8°C. Since there is no severe blood loss, the forensics expert knows that the body will cool, **from the moment of death**, according to Newton's Law of Cooling which says that, if the body has been dead for a length of time given by t, then the temperature T of the body will be determined by the formula:

$$T = E + Ce^{-kt}$$

where C and k are both constants to be determined, and E is the temperature of the environment.

The chemistry lab in `SW3-4695` is strictly controlled. Temperature is maintained at a constant 20°C. Access to the room is restricted to authorized personnel who enter and exit by swiping their ID card. The security system logs the ID number associated with each entry and exit.

Determine the time of death, rounded to the nearest minute. Express your answer using a standard 12-hour clock (**NOT** a 24-hour clock). Enter the hour in the first box below, and the minutes in the second box. Do **NOT** leave any blank spaces in either answer. Do **NOT** enter "am" or "pm" in either box.

Figure 10. Murder Mystery Handout, front page

The security system that controls access to the room records the time of each entry or exit from the lab and whose ID card was used each time. Police obtain a printout recording all entries and exits from the lab during the twelve hours prior to the discovery of the body. Use these records (provided below) to determine who was in the Chemistry Lab **at the time of death**. Your results will be used as the basis for an initial list of suspects in this investigation.

1) `Oscar Henderson entered at 5:51 pm and left at 3:43 am`
2) `Brenda Edgemont entered at 6:15 pm and left at 1:27 am`
3) `Carlo Sans entered at 12:11 am and left at 4:41 am`
4) `Natasha Milanova entered at 5:30 pm and left at 6:28 pm`
5) `Francesca Tartaglia entered at 5:01 pm and left at 7:26 pm`
6) `Ernie Heywood entered at 8:08 pm and left at 2:25 am`

1) Oscar Henderson (Occupational Health and Safety)	**2)** Brenda Edgemont (Biomedical Engineering)	**3)** Carlo Sans (Chemical Science)	**4)** Natasha Milanova (Nuclear Medicine)	**5)** Francesca Tartaglia (Food Technology)	**6)** Ernie Heywood (Environmental Health)

Enter in the box below the **number** associated with each person (from the security system records above) who was in the room **at the time of death**. If more than one person was in the room at that time, list the numbers associated with every person in the room. List the numbers **in order** and separated by commas. If none of the people in the security system records were in the room, enter the number **0** in the box. If someone entered or left the room at exactly the same time that Dr. Lewis died (to the nearest minute), **include** their number in the list of numbered suspects.

LIST OF SUSPECTS

BUILDING BETTER MATH

BCIT® BRITISH COLUMBIA INSTITUTE OF TECHNOLOGY

Figure 11. Murder Mystery Handout, back page

The Murder Mystery task description provides a key equation and several key data that allow the time of death to be determined. Three temperature-time data are combined with the equation to solve the time of murder. The problem affords many different algebraic solutions to arrive at the final answer. To begin, one possible solution is presented and alternatives are discussed afterwards.

Here is a summary of what is known:

$T = E + Ce^{-kt}$	body temperature equation where T is body temperature t is length of time body has been dead E is temperature of environment, given as 20 e is a mathematical constant (≈ 2.7182) C, k are constants to solve

1 T=36.6 at time of murder (t = 0)

2 T=32.3 when police arrive at 4:46 am (t = x)

3 T=30.8 one hour after body is discovered at 5:45 am (t = x+1)

All units of temperature are in degrees Celsius. The unit of *t* is not provided in the task, so here we assume it is hours. We also introduce *x* as the amount of time in hours it took the police to arrive from the time the victim was murdered.

Using E=20 and the data points 1, 2, 3, we generate the following equations:

1 $36.6=20+Ce^{-k(0)}$

2 $32.3=20+Ce^{-k(x)}$

3 $30.8=20+Ce^{-k(x+1)}$

The first equation simplifies to solve for C: *C* = 16.6. With *C* solved, the second and third equations form a system with two equations and two unknowns. There are many ways to go about solving this system of two equations. One approach is shown below. Since PC12 does not include learning about the natural logarithms (ln), the common logarithm (log) has been used in the solution.

2 $32.3 = 20 + 16.6e^{-k(x)}$ — We start with equation 2.

$12.3 = 16.6e^{-k(x)}$

*	$e^{-kx} = \frac{12.3}{16.6}$	We will need this later. Let's call this equation *.
3	$30.8 = 20 + 16.6e^{-k(x+1)}$	Now, let's move to equation 3.
	$16.6e^{-k(x+1)} = 10.8$	
	$e^{-k(x+1)} = \frac{10.8}{16.6}$	
	$e^{-kx-k} = \frac{10.8}{16.6}$	
	$e^{-kx} \cdot e^{-k} = \frac{10.8}{16.6}$	
	$\frac{12.3}{16.6}e^{-k} = \frac{10.8}{16.6}$	This line is achieved by substituting with equation *.
	$e^{-k} = \frac{10.8}{16.6} \cdot \frac{16.6}{12.3}$	
	$e^{-k} = \frac{10.8}{12.3}$	
	$e^{k} = \frac{12.3}{10.8}$	
	$k \cdot \log e = \log\left(\frac{12.3}{10.8}\right)$	
	$k = \log\frac{12.3}{10.8}/\log e$	We have now solved the constant *k.*
*	$e^{-kx} = \frac{12.3}{16.6}$	Here we return to equation * and will proceed to solve for *x*.
	$(e^{k})^{-x} = \frac{12.3}{16.6}$	
	$\left(e^{\log\left(\frac{12.3}{10.8}\right)/\log e}\right)^{-x} = \frac{12.3}{16.6}$	This line is achieved by substituting the value of *k* solved three lines up.

$$-x \cdot \log\left(e^{\log\left(\frac{12.3}{10.8}\right)/\log e}\right) = \log\left(\frac{12.3}{16.6}\right)$$

$$-x = \log\left(\frac{12.3}{16.6}\right) \Big/ \log\left(e^{\log\left(\frac{12.3}{10.8}\right)/\log e}\right)$$

$$x = -\log\left(\frac{12.3}{16.6}\right) \Big/ \log\left(e^{\log\left(\frac{12.3}{10.8}\right)/\log e}\right)$$

$$x \approx 2.3052$$

Therefore, the police arrived about 2.3052 hours (2 hr 18 min 31.2 sec) after the murder. Since the police arrived at 4:46 AM, the time of murder was approximately 2:27 AM.

Solving Time of Death - Alternate Approaches

A couple of alternate approaches are now discussed. At the point of arriving at a system of two equations above, e^{-kx} was isolated in one equation and substituted in the other. Alternatively, *x* could have been isolated and substituted - although this would have required more steps. Another alternative, *k* could have been isolated in one equation and substituted into the other to solve *x*. This is arguably more 'efficient' than the other options (isolating *x* or e^{-kx}) since it solves *x* without solving *k*, and only *x* is needed to determine the time of murder.

Another alternative approach is to define *t* differently from the start. As will be seen, it is advantageous to define t=0 as the time the body is discovered and t=x as the time of the murder:

1	T=36.6 at time of murder (t = x)	$36.6 = 20 + Ce^{-kx}$
2	T=32.3 when police arrive at 4:46 am (t = 0)	$32.3 = 20 + Ce^{0}$
3	T=30.8 one hour after body is discovered at 5:45 am (t = 1)	$30.8 = 20 + Ce^{-k}$

As before, *C* can be solved with one equation (now equation two). With *C* solved, k can be solved with one equation (equation three). With *C* and k solved, *x* can be solved with one equation (equation one). This approach avoids the 'mess' (and potential for error) of algebraic substitution. This method results in x≈-2.3052 indicating it took place about 2.3052 hours before t=0 which is at 4:46 AM. Therefore, the murder took place at around 2:27 AM, the same conclusion as above. This approach requires students to realize that time can be negative, a concept that is not addressed in the course.

Determining List of Suspects

The Murder Mystery narrative provides a list of all personnel that entered and exited the chemistry lab where the body was discovered during the twelve hours prior to the discovery of the body. Since personnel have to swipe in and out of the lab, the time of murder can be crossed with personnel entry and exits to establish an initial suspect list:

1) Oscar Henderson entered at 5:51 pm and left at 3:43 am
2) Brenda Edgemont entered at 6:15 pm and left at 1:27 am
3) Carlo Sans entered at 12:11 am and left at 4:41 am
4) Natasha Milanova entered at 5:30 pm and left at 6:28 pm
5) Francesca Tartaglia entered at 5:01 pm and left at 7:26 pm
6) Ernie Heywood entered at 8:08 pm and left at 2:25 am

Figure 12. Murder Mystery List of Suspects

Since the murder took place at 2:27 AM, it would appear that Oscar Henderson (who left at 3:43 AM) or Carlo Sans (who left at 4:41 AM) are possible suspects. Arguably, Ernie Heywood (who left at 2:25 AM) could be the primary suspect since he left right around the time of the murder and there are several factors in the model that could leave to a margin of error. There is no "right answer" to determining the list of suspects, so long as a logical argument is provided based on the mathematical analysis.

The task could have been implemented on a computer instead of a handout. BCIT has designed the task for students to complete it online with a computer. Once they have inputted a final answer, the computer grades the question and provides student with feedback. The teacher can view student results. I chose a paper and pencil implementation over the computer for several reasons. First, the distribution of student login information and instructions on how to navigate the online system would have consumed precious class time. Second, the students were accustomed to problem solving with paper and pencil as a medium and the introduction of a new medium (keyboard, monitor, and software) may have hindered their ability to solve the problem. Third, I saw no loss in switching to a physical handout since the task description consists of static text and graphics instead of say, animation or an interactive digital learning object. Fourth, and most importantly, I was interested in seeing the students' thinking as expressed through analysis write-up leading to their final conclusion. Using the online system, the only information I would have is the students' final answer without understanding what led them to it. For all these reasons, I used paper handouts for the activity.

3.4.2. Task Sources and Selection

Two sources of lesson materials for real-world connections were considered. BCIT publishes "Building Better Math", a database of problems that "show how [high school mathematics] concepts are used in real-world careers to solve problems in a wide range of leading industries, such as engineering, geosciences, health care, forensics, renewable resources, oceanography, and architecture" (BCIT, 2016b). Building Better Math targets three courses from the BC provincial curriculum including PC12.

BCIT is well positioned to identify real-world mathematics because it directly prepares students for various workplaces requiring mathematics. Furthermore, their mathematics department has twenty faculty members from a mixture of applied mathematics fields including "physics, chemistry, computer systems, aerospace, mechanical, biomedical, electrical, civil and quality engineering" (BCIT, 2016a).

The second source I drew from was an algebra textbook created by the Center for Occupational Research and Development (CORD). CORD is an American nonprofit organization established in 1979 that has created applied mathematics textbooks integrating academic, industry, and employability standards. The textbook I used, *Algebra 2: Learning in Context, Second Edition* (*Learning in Context*), was published in

2008 to fulfill the American Common Core Standards that require mathematics to be taught as “robust and relevant to the real world, reflecting the knowledge and skills needed for success in college and careers" (CORD Communications, 2014). The textbook is digital and includes workplace applications and real-world examples.

By leveraging CORD and BCIT to source real-world tasks, I overcome some of the challenges classroom teachers face designing authentic real-world tasks. Nicol (2002) found the prospective teachers had difficulty locating real-world mathematics when they visited workplaces and conducted interviews. Smith (1999) argued that workplaces have undergone de-mathematization due to much of the mathematics taking place through technology. Thus, leveraging the resources of institutions such as CORD or BCIT may be critical for high school teachers to successfully locate real-world contexts. These institutions have access to industry professionals and the time and funding required to locate real-world contexts for applications and modelling. By using a task designed by CORD or BCIT, I surmount the problems found by Gainsburg (2008) that real-world connections made by mathematics teachers were limited and from everyday experience instead of relating to workplace settings.

Access to Building Better Math and *Learning in Context* was provided by BCIT and CORD for the purpose of this research. After reviewing many tasks from Building Better Math and *Learning in Context*, I decided on one called *Newton Cooling Murder Mystery* (Murder Mystery) from Building Better Math. Building Better Math classifies each problem by difficulty and subject area. The Murder Mystery is classified as *advanced* with subtopic *exponential equations* and requiring two skills: *solving exponential equations* and *properties of exponents*. The problem also involves logarithms, which is part of the curriculum of PC12.

The task seemed to be the most challenging of those available through Building Better Math and my class had most recently completed the exponential and logarithms unit. Furthermore, the storyline is familiar from popular culture books, news stories, TV shows, and movies – trying to determine who committed a murder. This scenario is easy to imagine in accordance with RME, thus providing a realistic backdrop in which to conduct the mathematical problem solving. The sequencing deviates from RME since the task was introduced after the students had studied exponential functions and logarithms, while RME proposes realistic contexts from the start of the unit of study.

The extent to which the task meets the criteria proposed by Harvey & Averill (2012) is analyzed next. Harvey & Averill observed successful real-world context lessons and identified the following key characteristics: careful planning, time spent on non-mathematical aspects of context, referral to real-world context, validity of mathematical solutions against real-world context, teacher questioning, positive teacher-student relationship, teacher passion for subject, and teacher's depth of knowledge to develop real-world context.

Key Characteristic: careful planning

Many hours were spent in the task selection process. First, a set of sources were skimmed, looking for a high quality of task. I selected two sources, CORD and Building Better Math, as described earlier. From there, I reviewed over ten tasks and selected one to implement for the study. I was able to find a task of high quality for its storyline, the relevance of mathematics to course curriculum, the challenge of the mathematical task, and its authenticity of a plausible application of mathematics in the real world. Furthermore, the task involved low complexity to implement since it consisted of a handout for each pair of students.

Key Characteristic: contexts introduced with time spent on non-mathematical aspects

The context was embedded in the Murder Mystery task description. The task description balances mathematical and non-mathematical aspects of the context. Non-mathematical aspects include the story line about the murder, including its location and victim, pictures and entry/exit times of various characters, the police and coroner involvement, and the student's role to determine a list of suspects for the case. Mathematical aspects include the cooling equation. Key information that speaks to the storyline but is also required for the mathematical solution are the times of night and body temperature data. In the observed lesson by Harvey & Averill, because the teacher was leading the class, it was the teacher who decided how long to focus on non-mathematical aspects. In the current study, the students controlled how much attention was spent on these elements as they directed themselves through the task. The Murder Mystery task description balanced and integrated non-mathematical and mathematical elements. This can be seen in the back and forth between yellow and turquoise in Figure 13 and Figure 14 where non-mathematical elements are coloured yellow and mathematical elements are coloured turquoise. The task succeeded in promoting the non-mathematical aspects of the context.

Legend
Non-Mathematical Details
Mathematical Details

Question:

`B.C.I.T. Chemistry Lab, SW3-4695, Burnaby Campus, 4:46 am`

Police arrive on the scene of a suspected homicide. There is no sign of blood loss nor does it appear that the body has been moved since the time of death. A forensics expert immediately determines the temperature of the body to be 32.3°C.

A technician arriving for her shift identifies the body as that of Dr. Samuel Oliver Lewis. His medical records are accessed and reveal that he was in good general health (before his death). Records of his last full physical exam indicate that his norma body temperature was 36.6°C.

Exactly one hour after police arrived on the scene, the temperature of the body is taken again and found to have cooled to 30.8°C. Since there is no severe blood loss, the forensics expert knows that the body will cool, **from the moment of death**, according to Newton's Law of Cooling which says that, if the body has been dead for a length of time given by t, then the temperature T of the body will be determined by the formula:

$$T = E + Ce^{-kt}$$

where C and k are both constants to be determined, and E is the temperature of the environment.

Figure 13. Task with Mathematical Aspects (turquoise) and Non-Mathematical Aspects (yellow)

Legend
Non-Mathematical Details
Mathematical Details

The chemistry lab in SW3-4695 is strictly controlled. Temperature is maintained at a constant 20°C. Access to the room is restricted to authorized personnel who enter and exit by swiping their ID card. The security system logs the ID number associated with each entry and exit.

The security system that controls access to the room records the time of each entry or exit from the lab and whose ID card was used each time. Police obtain a printout recording all entries and exits from the lab during the twelve hours prior to the discovery of the body. Use these records (provided below) to determine who was in the Chemistry Lab **at the time of death**. Your results will be used as the basis for an initial list of suspects in this investigation.

1) Oscar Henderson entered at 5:51 pm and left at 3:43 am
2) Brenda Edgemont entered at 6:15 pm and left at 1:27 am
3) Carlo Sans entered at 12:11 am and left at 4:41 am
4) Natasha Milanova entered at 5:30 pm and left at 6:28 pm
5) Francesca Tartaglia entered at 5:01 pm and left at 7:26 pm
6) Ernie Heywood entered at 8:08 pm and left at 2:25 am

1) Oscar Henderson (Occupational Health and Safety)	**2)** Brenda Edgemont (Biomedical Engineering)	**3)** Carlo Sans (Chemical Science)	**4)** Natasha Milanova (Nuclear Medicine)	**5)** Francesca Tartaglia (Food Technology)	**6)** Ernie Heywood (Environmental Health)

Figure 14. Task with Mathematical Aspects and Non-Mathematical Aspects (cont.)

Key Characteristic: ongoing referral to real-world context

Harvey & Averill (2012) refer to the teacher connecting back to the real-world context in a teacher-led lesson. An example would be a teacher answering a student's question not just mathematically but also considering the real-world aspects. For the student-centred Murder Mystery task, the teacher was not in the spotlight to refer to the real-world context on an ongoing basis. The data are not available to determine if the students continually referred back to the real-world context. This might have been captured by analyzing a video recording of the pair and following up with survey or interviews. From the structure of the problem, I think students would need to return to the real-world context at various points in their problem solving to support interpretation and to guide where to go next. On the other hand, when I was solving the problem I converted the given real-world context information into three ordered pairs of time and temperature and solved for *C*, *k*, and time of murder *t* using a systems of equation approach. As an expert, because I could "see" where the problem was going, I did not refer back to the real-world context throughout my solution, apart from at the end to estimate whether my solution made sense. Thus it would be interesting to investigate this characteristic further in future research. In a high quality real-world student-centred task, how often do students refer back to the real-world context and for what purposes (e.g. to guide problem solving, when stuck, to validate solution)? And do stronger or weaker students refer back more or less often?

Key Characteristic: real-world validity of mathematical solutions

The mathematical solution in the Murder Mystery task is a value for t. This represents the relative amount of time that has passed since the murder (or relative to one of the other key events depending how students solved for *C* and k). The murder time as determined algebraically is a reasonable time of death in the real-world context of the Murder Mystery story. This time is then used to deduce a suspect based on who was in the lab at that time.

Teacher Characteristics: questioning, relationship, passion for subject, depth of knowledge to develop real-world context

The four remaining key characteristics identified by Harvey & Averill (2012) are teacher questioning, positive teacher-student relationship, teacher passion for the subject, and teacher's depth of knowledge to develop real-world context. In their study, Harvey & Averill evaluated these characteristics with a classroom teacher implementing a teacher-centric task – the teacher was leading the thinking process. These characteristics may play a suppressed role in my activity as it was implemented in a student-centred manner with a reduced presence of the teacher in the thinking process. In addition, it is harder for me to be unbiased in assessing these as I am both teacher and researcher.

Teacher questioning did not play a large role in my study as I mostly observed the students and let them interpret and try to solve the problem. I did not pose questions to the whole class; instead they worked through the problem in pairs. In the facilitator role, I might ordinarily have posed questions to prompt student thinking. For this study, however, I intentionally observed and let the students solve the task. This was a pragmatic decision so I could record observations for the purpose of this research. As researcher and classroom teacher, it is difficult to carry out both roles simultaneously. Certainly having separate researchers observe a real-world task facilitated by a teacher has its advantage, and this was the nature of the Harvey & Averill study. In the current study, teacher questioning was not recorded but my recollection is that it did not occur substantially. Thus, it was not likely relevant to the success of the task.

As for positive teacher-student relationship, it is hard for me to judge this in an unbiased way. I think that I had a very positive teacher-student relationship with this group of students. Some students remarked throughout the year that they “actually enjoyed” going to mathematics class this year. This class appreciated my sense of humour and we developed a number of “inside jokes.” Students hung around after class to chat. The following year when they were no longer my students, many sought me out in the halls to share updates about university admissions or just say hi. I really enjoyed teaching this group of students and think there was a positive teacher-student relationship, although I have no hard data collected on this metric.

Similarly, when it comes to teacher passion for the subject, I am passionate for mathematics. This is often a line I use when meeting parents - and I give examples that I’ve always loved mathematics pure and applied, and to this day continue to study mathematics in my spare time for enjoyment. In addition, I regularly attend mathematics education conferences, read up on mathematics education blogs, discuss and share mathematics problems or lesson ideas with colleagues, and am involved with mathematics clubs and contests. Furthermore, I enjoy going to work and am excited by helping students learn and discover mathematics.

I believe I have an expert level depth of knowledge to teach using real-world contexts. Learning mathematics within real-world contexts was an extensive component of my undergraduate education in engineering. I completed a dozen courses involving applications and modelling in various contexts. Furthermore, I used mathematical modelling approaches in aerospace, defence, and health industries at four companies accumulating 20 months of full-time professional work experience as a co-op student. In addition, I have led a workshop for middle school teachers to implement mathematical modelling in a hydraulics science unit and led a series of workshops for student teams from 20 high schools to learn about applications of mathematics in robotics.

While my knowledge and experience of the 'modelling cycle' is strong, as is my experience with Newton's Law of Cooling (the basis of the Cooling Equation in the Murder Mystery), I do not have expertise in human physiology nor forensics, the particular context of this task. Fortunately, the task did not require teacher expertise in these areas to be completed successfully.

On balance, the task herein supports the goals of real-world tasks identified by Blum et al. (2007) and satisfies the key aspects of success proposed by Harvey & Averill (2012).

3.5. Data and Analysis

The data for the eight participants in this study consisted of student profiles, my observations during the task, their written solutions, and survey responses. Since participants worked in pairs, each pair formed a case study that was analyzed.

I created a student profile describing the student based on my interactions and observations leading up to the task. The student profiles describe each student's abilities and characteristics as I viewed them during the previous eight months leading up to the Murder Mystery task. Since the students completed the task in pairs, the profiles also discuss the interpersonal relationship between the pair. The purpose of the student profiles is to provide a context in which to interpret the students' performances on the task and post-task survey responses – for instance, whether their engagement in the task matched or contrasted with their engagement in previous classes.

During the Murder Mystery task, the students selected a partner and a spot in the room to work through the task. After the initial instructions, I spent most of my time observing the groups and recording notes as a researcher. I circulated and observed the groups. At times, I focussed on one pair and jotted notes, recording their conversation or behaviours, while at other times I noticed interactions between groups. At the end of the class, I collected the written work each pair produced. In the following class, I distributed and collected a follow-up survey.

The medium for the students to submit their solution was pencil and paper. They were instructed to submit all their work once finished, including any rough work. The students were allowed to collaborate on one submission and not required to each submit a write-up. In the analysis, I will retrace their task solution process as captured through my observations and their written solution.

The Murder Mystery took the entire period, so student surveys were distributed in the following class. The surveys collected data about the Murder Mystery, as well as student views on the place of real-world connections in the mathematics classroom. The questions from the survey are shown in Figure 15.

The survey consisted mostly of open-ended questions requiring the students to compose responses. Two questions (#1 and #8) utilized a 5-point Likert scale. The survey responses will be analyzed by student pair, comparing and contrasting responses. Excerpts from the qualitative responses will be included to capture the student perspective through their words.

1. Did you find the Murder Mystery illustrated how mathematics might be used in the real world more so than typical math questions? Circle your opinion:

Strongly Disagree *Disagree* *Neutral* *Agree* *Strongly Agree*

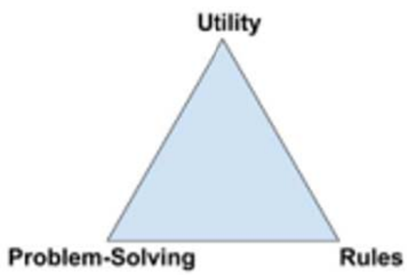
2. Why or why not? (for number 1)

3. What elements did you like / not like about the activity?

4. How did the activity influence your understanding of exponential functions or logarithms?

5. Some call mathematics that is separated from a science, business, or career, or real-world context 'naked'. Do you think mathematics should be taught 'naked'? Or should math courses include real-world (or real-world-ish) applications?

6. Place a dot in the triangle below to represent your view of mathematics. Utility means mathematics is application-oriented, a science relevant to society and life. Rules sees mathematics as a collection of rules or procedures. Problem-solving sees mathematics as the discovery of structure and problem-solving processes.



7. Does the inclusion of real-world-ish problems such as Murder Mystery influence your view of the role mathematics plays in society? Explain.

8. Real-world applications can boost student interest or motivation. Circle your opinion:

Strongly Disagree *Disagree* *Neutral* *Agree* *Strongly Agree*

Figure 15. Follow-up Survey Questions

Chapter 4. Case Analysis

This chapter presents four case studies, each representing the experience of one pair of students that participated in the Murder Mystery. Each case begins with student profiles to provide context about each student. The task results then examine the pair's written solution along with my observations. Third, student survey results are reviewed to understand the task experience through the lens of the student and in his own words. Finally, these three sections are considered in combination to synthesize the significance learned from each case study.

4.1. Daniel and Larry

4.1.1. Student Description

I will share some characteristics of Daniel and Larry based on my impressions as their teacher. These descriptions aim to provide a context for their task performance, analysed in later sections.

Larry is an introverted young man. He tends to be shy in any group size and stays quiet during most classes. Larry is a responsible student. He is attentive and tends to stay on-task. If the class has less structured study time, Larry reliably works through practice exercises without engaging in off-task behaviour. His homework record was close to perfect, and he was the only student all year to not have a single absence from class. If a peer engaged Larry in off-task conversation, Larry would politely respond but quickly return to the work at hand. Larry achieved A standing (defined as over 86%) consistently throughout the course, and finished with 91%.

Daniel is close to the opposite of Larry. Daniel is inconsistent. On one hand, he wanted to do really well in the course. At times, he would exhibit genuine resolve to succeed - preparing in advance for a unit test, setting up an extra-help appointment after school, and clarifying with me his understanding about errors he'd made on an assessment. On the other hand, he could be lazy and highly distractible and distracting. Daniel's homework was not consistently completed, he frequently arrived to class unprepared, and sometimes crammed for a test the night before. Daniel is very extroverted. He tends to talk first and think later, blurting out whatever pops into his head. He is a social butterfly and often the centre of an excited conversation or heated debate among several students at an inconvenient time - such as when there is another task to complete. During any type of unstructured setting, I would periodically remind Daniel to focus. It was not uncommon for Daniel to be working on the directed task one moment, and the next moment to be texting on his smartphone (which is not allowed) or starting up an off-topic conversation with the person next to him. Daniel finished the first two terms with 88% average (middle of class) and shot to the top of the class in the third term with a 98% average.

Daniel and Larry were friendly with one another but were not close friends. When students partnered up for the task, Larry had not been approached, nor had he approached anyone to be his partner. Daniel was the odd-man out from his group of friends, so he asked loudly who did not yet have a partner and Larry quietly indicated he did not.

4.1.2. Task Results

Daniel and Larry pulled their desks together and read over the task description on the handout. They worked constructively as partners listening as the other spoke and proceeding together through the problem solving process. They both worked steadily on the task for 60 minutes until they finished. I did not observe any off-task behaviour such as checking smartphones or engaging in conversations beyond those related to the task. They allowed themselves to be stuck and patiently continued to problem solve.

Only Daniel "held the pencil." At times, I would hear Larry make a suggestion and Daniel would cut him off, "Wait, wait, let me just write down the equation." Daniel may have found thinking about a problem, listening to someone else, and writing down their progress too much to process at once. In interrupting Larry, Daniel was advocating for his cognitive needs, albeit in an indelicate tone. On the downside, this may have hindered Larry's idea momentum. Daniel tended to control the direction of problem solving both because he held the pencil and because he was very blunt and insistent at times. However, Larry did not seem to mind or show any signs of frustration working with Daniel.

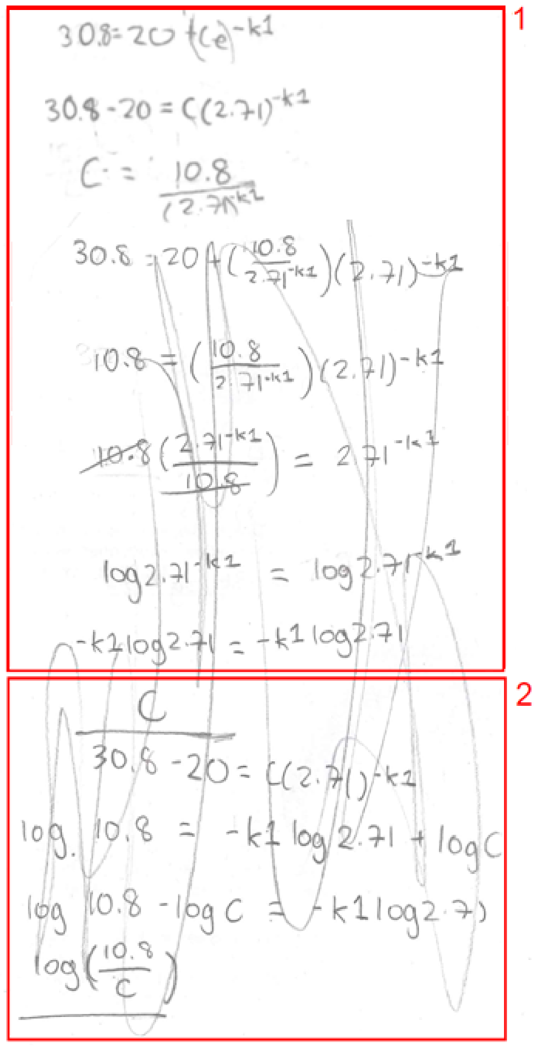


Figure 16. Daniel and Larry's First Page of Written Solution Showing Two Dead-Ends: Trying to Solve *k* (box 1) and Trying to Solve *c* (box 2)

The first page of their written solution is shown in Figure 16. As seen in box 1 of the figure, they begin by plugging in (time, Temp)=(1, 30.8) into the cooling equation correctly, $30.8 = 20 + Ce^{-k(1)}$. Next, they isolate for *C* and replace *e* with *e* truncated (not rounded) to two decimal places 2.71, $C = \frac{10.8}{(2.71)^{-k1}}$. I am not sure why they replaced *e* with 2.71 but it reminds me of some of my Grade 8 students who replace π with 3.14 even after they are shown there is a π button on their calculator that contains many more digits of the number or are shown how to leave their answer in terms of π. I suspect some students are more comfortable with the concrete appeal of a few finite decimals than the abstract idea of exact representation of an irrational number using a letter. Daniel and Larry might thus replace *e* with 2.71 to simplify one aspect of the equation. Another reason they may replace *e* with 2.71 is to distinguish e, a known constant, from the other two letters in the equation *C* and k which are unknown constants. Otherwise, *e* might be mistaken for a value that they must solve like *k* or *C*.

It seems at this point Daniel and Larry are not sure what to do to solve k. First, they substitute their expression for *C* back into the equation they used to isolate *C* to arrive at the identity $30.8 = 20 + \frac{10.8}{(2.71)^{-k1}} e^{-k(1)}$. This seems promising as they now have one equation with one variable, k. Unfortunately, when they simplify this equation further it becomes $-k1 \log 2.71 = -k1 \log 2.71$, an identity which does not enable solving k.

Having landed at a dead-end trying to solve for *k*, they backtrack to the equation $30.8 = 20 + Ce^{-k(1)}$ and instead try to solve *C*. This can be seen in box 2 of Figure 16. In their first step, they again replace *e* with 2.71: $30.8 - 20 = C(2.71)^{-k(1)}$. They employ some algebraic techniques such as taking the logarithm of both sides but after a few steps are not close to isolating *C* and give up this approach.

Daniel and Larry initially had some problem solving momentum but then came with two dead ends. At this point, another group - Kirk and Dylan - erupt with excitement declaring that they "found *C*." Another group goes over and after looking at what they've done exclaims, "Oh my God - it's so simple!" Daniel goes over to Kirk and Dylan's table where a number of students are now gathering around to understand how they've done it. Daniel returns to Larry and says, "We can solve for one variable and then substitute for the other."

Solving a system of linear equations with two variables and two unknowns is a skill these students studied in Foundations & Pre-Calculus 10 and have come across occasionally in Pre-Calculus 11 and this year in Pre-Calculus 12. However, in this case the students were not told to solve it using this technique but instead needed to recognize the opportunity on their own.

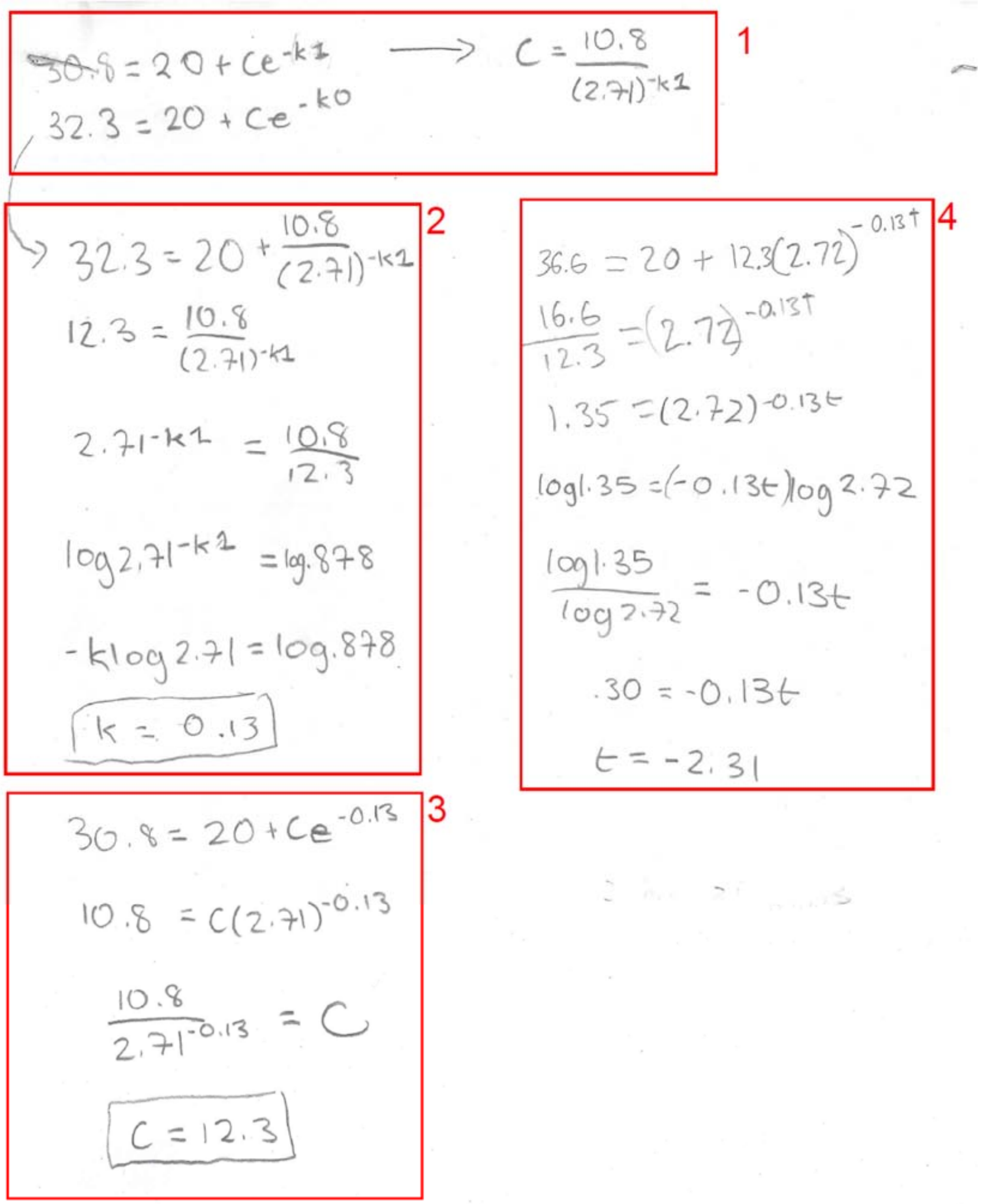


Figure 17. Daniel and Larry's Second Page of Written Solution

Daniel and Larry return to their desks with renewed energy, cross out their failed attempts, and restart on a new page which can be seen in Figure 17. Shown in box 1 of that figure, they start by rewriting their progress so far: $30.8 = 20 + Ce^{-k(1)} \rightarrow C = \frac{10.8}{(2.71)^{-k1}}$. Now, using (time, Temp)=(0, 32.3) they create a second equation $32.3 = 20 + Ce^{-k0}$.

Box 2 of Figure 17 shows their steps leading to solving k=0.13. First, they substitute their expression for *C* into the equation $32.3 = 20 + \frac{10.8}{(2.71)^{-k1}}$. By rearranging this equation, they solve for *k.* To do so, they correctly apply algebraic techniques including logarithms which they learned in the course during the Exponential & Logarithm Functions unit leading up to this task.

When they were finding *k*, their calculator would have indicated 0.13050691 and Larry and Daniel decided to round to two decimal places. There was no guidance in the instructions as to how far to round. In class, I promote exact answers unless told otherwise, and certainly to indicate rounding or truncation with an approximate equal symbol. Larry and Daniel's decision to use two decimal places for *e* is consistent with their use of 2.71 for e. Quantities provided in the task use one decimal place and so although undesirable to use only two decimals, it is a reasonable decision in the absence of any given instruction to do otherwise.

With *k* solved, Larry and Daniel substitute *k* into their first equation. They proceed to solve *C*, finding that C=12.3. These steps are shown in box 3 of Figure 17.

After two failed attempts to find *C* and *k*, Daniel and Larry have finally done it. I was observing them at this point and wrote down their subsequent conversation:

Daniel: Where do we go from here though? Do we plug it in? Or maybe we can graph it.

Larry: No, no. We have to figure out what *t* is.

Daniel: But we already figured it out.

Larry: ...

Daniel: Oh, I kind of see what you mean now. You're right!

In their next part of the solution, shown in box 4 of Figure 17, they solve for *t*. First, they substitute their solved values for *C* and *k* into the cooling equation along with (time, Temp)=(t, 36.6) to get $36.3 = 20 + 12.3(2.72)^{-0.13t}$. Again, they are able to accurately apply algebraic and logarithmic techniques previously studied to solve for *t*. In this set of steps, the pair has consistently rounded to two decimal places throughout their solution. They have used a rounded 2.72 for *e* instead of a truncated version 2.71 as they have done up until now. In this sequence, they have also rounded to two decimal places at every opportunity whereas, earlier they at times used three decimal places.

From here, the pair conclude the time of death is 2:27. This is a correct interpretation of their solved value $t = -2.31$ (-2.31 hours is 2 hours and approximately 19 minutes before the police arrived at 4:46AM, so the estimated time of death is 2:27AM). However, the pair did not justify in words or steps how they went from $t = -2.31$ to 2:27 which is inconsistent with their otherwise step-by-step solution. They did not indicate a list of suspects based on the time of death as the task asked, possibly because they overlooked this final task requirement.

4.1.3. Survey Results

On the survey, both Larry and Daniel had positive responses to the task. They agreed that the Murder Mystery task illustrated how mathematics might be used in the real world more than a typical mathematics question. Larry noted about the task, “They connect well because they’re relatable.” Daniel noted that murders happen, which is recognizing a plausible context was provided. Larry added that, “The plot was really interesting.” The pair agreed (4 on 5 point scale) with the statement, “Real-world applications can boost student interest or motivation.”

The pair identified different aspects of the task they enjoyed. Larry noted, “[The task] was challenging but once you figure it out you feel more accomplished than just solving a math question.” Daniel wrote, “I liked how we were allowed to collaborate with a partner.” Daniel also noted that he did not enjoy the amount of text laying out the task, “I thought the question was a bit wordy.”

Larry noted the activity required applying knowledge from “chapters 7 and 8” referring to Exponential and Logarithmic Functions and Equations. Daniel felt that the real-world context of the task provided insights to understanding a generic x-y relation, “This [task] helped enhance my understanding on making equations as well as the relationship between x and y.” Daniel states that “applying the real-life concepts should come after learning the base.” This reflects the teaching order in the current unit, where students learned Exponents and Logarithms.

4.1.4. Analysis

When Daniel reflected on the task, he noted, “This [task] helped enhance my understanding on making equations as well as the relationship between *x* and *y*.” One of the purposes of using real-world problems in the learning of mathematics is a vehicle through which to make sense of mathematics. Daniel considered that the real-world problem helped him understand the process of “making equations.” Instead of being given an explicit system of equations to solve, the task required Daniel to realize there were multiple equations, to construct them, and then to decide what to do with them. He also noted the task helped him understand the abstract notions of *x* and *y*. These variables on their own are abstract and their relationship is even more abstract. The task provided realistic meaning for *x* and *y* variables and their relationship: a body temperature decreasing over time, an idea that is imaginable by Daniel in the sense of RME. For Daniel, the task’s real-world context served as a means through which to understand mathematics.

One goal of including real-world applications is to motivate students (Blum et al, 2007). From my observations, the students engaged in the task for a sustained period, around 60 minutes. They asked each other questions, made suggestions on what to try next, and Daniel exhibited excitement and enthusiasm when he received the breakthrough about how to solve the constant *C*. My observations were consistent with the boys’ reflections of the task on the survey. Larry indicated he found the plot interesting and both boys agreed that real-world applications can boost their motivation. Larry noted about the task, “They connect well because they’re relatable,” which echoes RME’s position that mathematics ought to be imaginable or realizable for a student’s personal reality.

There were several factors contributing to Larry and Daniel’s perseverance through the task’s complexities. First, Larry and Daniel had an effective intragroup dynamic. They worked well as partners, Daniel dominant and Larry subordinate, Daniel talking first and Larry thinking first - but both contributing to the way forward. The excerpt of their conversation on page 50 indicates that while Daniel was the dominant partner, it is clear that he listened to Larry’s suggestions. Daniel reflected afterwards, “I liked how we were allowed to collaborate with a partner.” Larry gave Daniel the time to understand what he had said. They allowed themselves to be stuck and patiently continued to problem solve. Their ability to persevere when there was no clear path forward was supported by success of their respectful and collaborative intragroup dynamic.

A related factor that contributed to their perseverance was a helpful intergroup dynamic. Daniel and Larry sustained many wrong turns and were bound to eventually become deflated if their lack of success continued. A well timed eruption of excitement by one of the other groups enabled Daniel and Larry to overcome a roadblock - how to find C. They were then able to solve the mathematically more challenging k and eventually determine the time of death. In other words, their success navigating the task snowballed once they overcame their initial hurdle with the support of intergroup assistance. In his survey, Larry reflected, “[The task] was challenging but once you figure it out you feel more accomplished than just solving a math question.”

Daniel and Larry’s focus throughout the task is significant. Given Larry’s personality, I would entirely have expected him to remain focussed throughout the task. Daniel on the other hand is one of the most consistently distractible students in the class. He may start on some work only to be intensely engaged in a debate on a completely unrelated topic a few moments later. His focus throughout the task was uncharacteristic. Not once did I need to approach Daniel to remind him to get back on task. This may have been due to an intragroup dynamic where he was responding to and aligning with Larry’s focus, or it may have been due to the power of the application to draw students into the imaginable world of the task. It is likely a combination of these factors. The Murder Mystery task revealed another side of Daniel, someone who can be remarkably focussed and engaged mathematically.

4.2. Thomas and Kevin

4.2.1. Student Description

Thomas is fascinated by mathematics and science with advanced knowledge beyond high school curriculum. He consistently achieved perfect or near-perfect evaluations throughout the course. His work was always completed to an excellent standard often going above and beyond requirements on assignments. He managed class absences proactively and completed all homework throughout the year. He consistently participated, often in creative and outspoken ways such as using his arms to demonstrate the shapes of polynomial functions, performing a sine wave dance, or bluntly stating that my explanation was an oversimplification. At times, his comments were met with glares from classmates, as the comments were so far advanced and beyond the scope of the course (such as asking a question related to space-time or the geometrical implications of a hypercube). He also jumped on opportunities to complete challenge or bonus problems, and often posed his own recreational mathematic puzzles to me. His final course mark was 97%, the top in our class.

Thomas was the only student in my PC12 class who was enrolled concurrently in Calculus. He was in the highest of three levels of Calculus offered at our school: the Advanced Placement Calculus BC course, equivalent to a two-semester university course. Typically, students in my PC12 class may take one of the easier Calculus courses (Calculus 12 or Advanced Placement AB) after completing PC12. Thomas' enrollment in the most challenging Calculus course indicates his exceptionality. It also gave him a specific advantage in the Murder Mystery task because the underlying mathematical content of the task, Newton's Law of Cooling, is a topic Thomas studied in the Advanced Placement Calculus BC course.

The other student, Kevin, is very artistic and spends great portions of his time creating films, pursuing photography and painting. He is passionate about politics and philosophy. He is interested in big questions of mathematics and reads popular science magazines, which gave rise to discussions about the boundaries of science. Kevin demonstrated mixed emotions towards mathematics. While Kevin enjoys discussing the metaphysics of mathematics and likes the ideal of mathematics for its beauty and utility, he does not actually enjoy doing mathematics and was not diligent or enthusiastic in PC12. He handed in all assignments if they were for credit, but they were often not thoroughly completed. He presented as a bright young man who was putting in the bare minimum to get by in the course. He was very involved with creating films for various clubs and councils within the school and poured hundred of hours of his free time directing and editing movies.

From September to February, he prepared inconsistently for evaluations and often completed only some of the homework or none at all. After a slow and steady decline from September to January, he “hit rock bottom” in February with a 58% unit test. This is low and unusual for his pathway of mathematics where the class average typically sits between 88%–92%. This caused a much needed wake-up call and Kevin began trying harder in the course. From March until June, he consistently ranked among the top three students on each evaluation, substantially outperforming the class average with scores of 95% and higher on the remaining five evaluations, including the Logarithms and Exponential Functions units. His higher performance was not a result of a private tutor or attending extra-help sessions. Instead, he dedicated more time to the course. This activity took place in May, after Kevin became engaged.

4.2.2. Task Results

At the start of the task, another student asked his partner, “Where did the formula come from?” Thomas overheard and replied with authority, and loud enough for everyone to hear, “Calculus.” The student replied, “Okay, cool!” accepting Thomas’ answer. A little while later, Thomas said loudly to Kevin, “The formula is wrong - this is a simplification.” And several other times, Thomas criticized the cooling equation provided as unintuitive and confusing. Later, when this group finished, Thomas said, “We’re done – I don’t know how you solve with *that* equation.” I was concerned that Thomas was influencing others in the class to lose confidence that they could successfully solve the problem with the information provided.

Thomas and Kevin’s written solution is shown in Figure 18. Shown in Box 1, Thomas and Kevin start by noting the three given temperatures and times. Like Daniel & Larry, they first plug (time, Temp)=(60, 30.8) into the provided cooling equation, using minutes for the unit of time: $30.8 = 20 + Ce^{-k(60)}$. From here, they manipulate the equation to $\ln(C10.8) = -k60$. This manipulation contains one mistake: The correct version would be $\ln\left(\frac{10.8}{C}\right) = -k60$. Thomas was familiar with natural logarithm *ln* from his Calculus course but Kevin was not, which suggests Thomas was leading the problem solving.

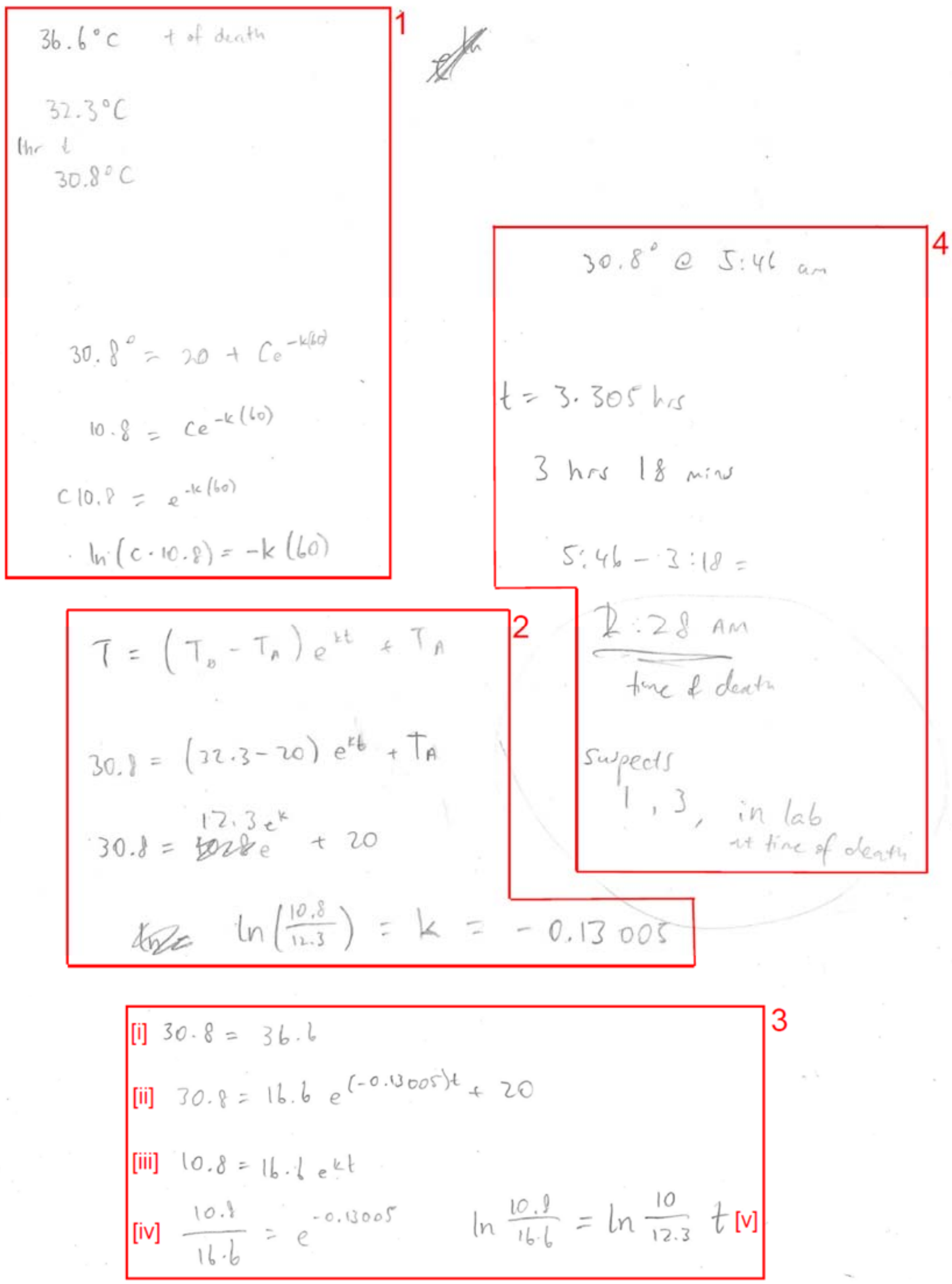


Figure 18. Thomas and Kevin's Written Solution

This is the same spot Daniel and Larry struggled. With no alternatives, Daniel and Larry continued working through the problem and eventually figured out what to do next. Thomas and Kevin opted to give up on the provided cooling formula and to rely instead on Thomas' knowledge from AP Calculus to attack the problem. Their next lines, shown in Box 2 of Figure 18, use the Newton's Law of Cooling equation from Advanced Placement Calculus BC. This reduces the complexity of the problem by eliminating the need to solve a system of equations. While the other students in the class were faced with the challenge of solving the unknown constant *C*, Thomas' equation used an expression in terms of given quantities for *C*:

$$T = (T_0 - T_A)e^{kt} + T_A$$

This cooling equation is almost identical to the equation provided in the task, except that *E* is replaced by ambient temperature T_A, and constant *C* is replaced with an expression in terms of the given information $T_0 - T_A$ where T_0 is the initial temperature. The main advantage of this formula for Thomas was that he was familiar with it and more confident with its use. It simplified the problem because they did not need to figure out how to solve for *C*, an area of struggle for the other groups.

From their new starting point, Thomas and Kevin again plugged in (time, Temp)=(1, 30.8), except this time changed the unit of time to hours instead of minutes as they'd used earlier, with initial temperature $T_0 = 32.3$ and ambient temperature $T_A = 20$. This resulted in the equation

$$30.8 = (32.3 - 20)e^{k} + 20$$

Thomas and Kevin successfully solve *k* in just two steps, facilitated by Thomas' knowledge of the natural logarithm:

$$\ln\left(\frac{10.8}{12.3}\right) = k = -0.13005$$

With *k* solved, the group could have simplified their cooling equation as follows:

$$T = (T_0 - T_A)e^{kt} + T_A$$

$$T = (12.3)e^{\ln\left(\frac{10.8}{12.3}\right)t} + 20$$

$$T = 12.3\left(\frac{10.8}{12.3}\right)^t + 20$$

Had they done this they could have then solved time of death by finding *t* by substituting (time, Temp)=(t, 36.6) in the last equation above. However, they did not simplify their cooling equation in this manner. Their work, shown in box 3 of Figure 18, is less clear.

I am unsure what they meant in step [i] by 30.8=36.6. Perhaps they substituted t=0 into the cooling equation or perhaps this is a line they started to write but abandoned and wrote the next line instead. In step [ii], they wrote $30.8 = 16.6e^{-0.13005t} + 20$. What is interesting here is they have changed their value T_0which changes the time of day that corresponds to the value of *t*. Earlier, they solved *k* using $T_0 = 32.3$ so t=0 corresponded to 4:46 AM. Now, they use $T_0 = 36.6$ so t=0 now corresponds to the time of the murder.

In steps [iii] to [v], Thomas and Kevin worked towards solving *t*. Step [iii] is correct. In step [iv], they accidentally omitted *t* from the right side exponent. They seemed to realize that it should be there since *t* returns in step [v]. In step [v], they take the natural logarithm of both sides and replace $k = -0.13005$ with the earlier expression $k = \ln\left(\frac{10.8}{12.3}\right)$, except they accidentally wrote 10 instead of 10.8. Again, their next step uses 10.8 in its calculation, correcting their written omission in step [v].

Their next steps are shown in Box 4 in Figure 18. They correctly solved *t* as $t = 3.305$. This represents the time (in hours) it took for the victim's body temperature to cool to 30.8 degrees at 5:46 AM. The murder took place when the victim's body temperature was 32.3 degrees (at $t = 0$). The group correctly subtracts 3.305 hours from 5:46 AM to arrive at the time of murder, 2:28 AM.

Thomas and Kevin finished the task within 35 minutes and at least 15 minutes earlier than any other group and reached the correct conclusion in a justified manner. This pair did not solve the problem in a manner I had anticipated. They opted to use a different equation than the one provided in the problem and used the natural logarithm which was outside the scope of PC12. They had the option to do so because Thomas had studied this topic previously in AP Calculus BC course, the only student in the course taking Calculus concurrently to PC12. Thomas and Kevin first tried to solve the problem as suggested by the task's instructions but as soon as they were unsure what to do next, they switched to Thomas' method. From their perspective, this was a good move because Thomas was familiar and confident with the method. It allowed them to solve the problem. I think Thomas and Kevin would have been able to solve it using the cooling equation provided by the task description had they persevered as other groups did. However, Thomas insisted that the provided method was confusing and he didn't understand why it was so complicated relative to the method he already knew. He opted to use a technique he was familiar with and ultimately it proved shorter and successful.

4.2.3. Survey Results

In looking at the survey, Thomas strongly agreed that the Murder Mystery task illustrated how mathematics might be used in the real world more than a typical mathematics question. He commented that, "Introducing real-world descriptions and context showed how mathematics can be used in the real world." He also strongly agreed that real-world applications can boost student motivation.

While Thomas agreed that the task was more realistic than typical mathematics questions, he stated that a different context would have been "even more realistic" and more "eye-opening." He wrote, "The Murder Mystery is only brushing the sides of what mathematics does in society. An even more realistic application of mathematics would be relevant, and more eye-opening. … An example would be structural load integrity calculations or cost/profit analysis." While having a strong sense of the utility of mathematics, Thomas indicated he views the nature of mathematics as problem solving more than utility or rules.

Thomas criticized the form of the cooling equation, responding that the activity "didn't really impact my understanding of exponential functions." He also commented that he "found the given equation/formula not very intuitive, which made the calculations awkward/unwieldy." This was seen in his written calculations where he tried to use the provided cooling equations, then started over using the cooling equation he learned in his calculus course.

According to him, Thomas teaching and learning of mathematics should first be done in the abstract before studying applications in real-world contexts. "This is similar to sciences…where we learn for example chemical theory and molecular studies before we apply the knowledge and create products."

While Kevin agreed to participate in the research, he did not return his survey.

4.2.4. Analysis

Thomas' experience with the Murder Mystery task was influenced by his previous knowledge with the underlying mathematics of the task. Combined with his outspoken manner, his previous study of Newton's Law of Cooling, and his view that the provided equation was complicated and unintuitive, he promoted doubt in the other groups by loudly stating criticisms of the task with an air of authority. Many of the groups persevered anyway but during the task I was concerned that Thomas' influence would adversely impact the outcomes of the others. Thomas can be seen as a negative intergroup dynamic, even though his group solved the problem relatively quickly.

In terms of working through the problem, Thomas' solution was consistent with what I expected - he outsmarted me! He is a brilliant young man. His unique advantage of having studied the relevant mathematics involved - Newton's Law of Cooling - made him confident from the start that he could solve the problem. In his survey, Thomas confirmed that the activity "didn't really impact my understanding of exponential functions."

Another aspect that Thomas highlighted is the possibility of unanticipated solutions. None of the solutions I had anticipated was the one that Thomas and Kevin used. During the problem solving process, the pair abandoned the suggested cooling equation and instead used methods Thomas had seen in another course. They finished the task quickly and efficiently, with a mostly clear written solution. While their final solution method was unanticipated, I think they demonstrated good problem solving. When they were not making progress with one method, they changed strategies and leveraged previous knowledge and applied it correctly to the task. Thomas is a strong problem solver and Kevin also enjoys a challenge. Their powerful and efficient approach is consistent with what I'd expect from these two. That is, I would expect them to find an unexpected solution!

Thomas and Kevin persisted. Thomas wrote, "[I] found the given equation/formula was not very intuitive, which made the calculations awkward/unwieldy." In response, they changed strategies. This was seen in his written calculations where he tries to use the provided cooling equations, then starts over using the cooling equation he learned in his calculus course. Thomas found the task's context realistic and believes such real-world contexts can boost motivation and engagement. I observed Thomas and Kevin to be 100% engaged in the task (along with their criticisms, they were attentive throughout the task). Thomas did note that perhaps even more realistic contexts could be used such as "structural load or profit analysis."

Finally, this group illustrates a difficulty in planning. When I planned the task, I believed the application of exponential functions and logarithms in the cooling equation would be unfamiliar. I felt the context of a murder was imaginable for the students and so I was pleased with the combination of illustrating an application of exponents and logarithms through an imaginable context. One of the elements that made the task so challenging is there was no clearly defined solution path. Much of the challenge was making sense of the constants C and k, variables t and T, and figuring out how to use those with the time and temperature data to decide the victim's time of death. While Thomas and Kevin may have experienced this briefly, they quickly resorted to known procedural knowledge that Thomas had learned in AP Calculus. Thus the task was less challenging, since he could leverage his previous experience solving similar problems. Thomas felt he did not advance his understanding of the mathematics topic through this task and may have hurt the confidence of others trying to do so. Students will have varying degrees of familiarity with the context or mathematics involved in a task and Thomas presents an illustration of how those challenges might manifest in the classroom.

4.3. Jonathan and Mike

4.3.1. Student Description

There is little that Jonathan and Mike have in common as learners. Mike is often sleepy during class. This interfered with his abilities to participate on a consistent basis and it was not uncommon for him to actually fall asleep during class. Mike's attendance was choppy, missing about 15% of classes throughout the school year. Three of the 20 students were absent more than he was, so Mike's large number of absences was not unique. Because he did not responsibly manage his absences, this likely contributed to his inconsistent performances on evaluations. His three terms and exam marks were 89%, 79%, 90%, and 64% - the most variance of any student in the class.

By contrast, Jonathan's grades had the least variance of any student in the class. Jonathan always prepared for evaluations and consistently performed well. His three terms and exam marks were 96%, 96%, 96%, and 97%. He also stood out as an artistic performer. He played prominent roles in the school's drama productions each year, and performed in dance competitions and musical theatre outside of school. His lively, optimistic, and go-getter personality contributed to a focussed and productive PC12 class. Jonathan was very mature and could get irritated if other students in the class were inattentive or acting immaturely. While his enthusiasm for mathematics was mixed, he was a reliably hard worker who put in his best effort for learning in class and at home.

Jonathan and Mike were courteous towards each other but were not friends. A more natural partner for Jonathan - one with whom he was friends with and shared a similar work ethic - was away the day of the task. Prior to this task, Jonathan and Mike had not worked together in the course.

4.3.2. Task Results

Jonathan and Mike did not collaborate well as a pair. Mike was fairly passive while Jonathan was left to try to solve the task. I observed Mike yawning and looking tired, which may have contributed to his lack of engagement, although this was also consistent with his typical behaviour.

Jonathan was confused by the appearance of *e* in the problem. When I explained to the class that *e* was an irrational number like π, and *e* was approximately 2.7, other students went about the task accepting this. Jonathan, however, seemed more distracted and put-off by the appearance of *e* since it was unfamiliar. In his survey afterwards, Jonathan wrote "[I] wasn't given the necessary information to even start the assignment such as what 'e' means," and he recommended "more guiding information to be able to complete the assignment." Jonathan was on unsure footing from the start and his partner was generally unhelpful. To make matters worse, Thomas' group was loudly voicing the idea that the provided cooling equation was 'wrong' and 'confusing', and referencing 'the natural logarithm' - a topic that Jonathan had not heard of before.

Jonathan and Mike's written work indicated several failed attempts and ultimately they reached an incorrect, final answer. The task might have been exasperating for Jonathan, who did not have an engaged partner. Thus Jonathan was shouldering the entire task. He kept running into dead-ends in his problem solving and was generally confused by the cooling equation. He felt like I had not equipped him with the knowledge to interpret the cooling equation.

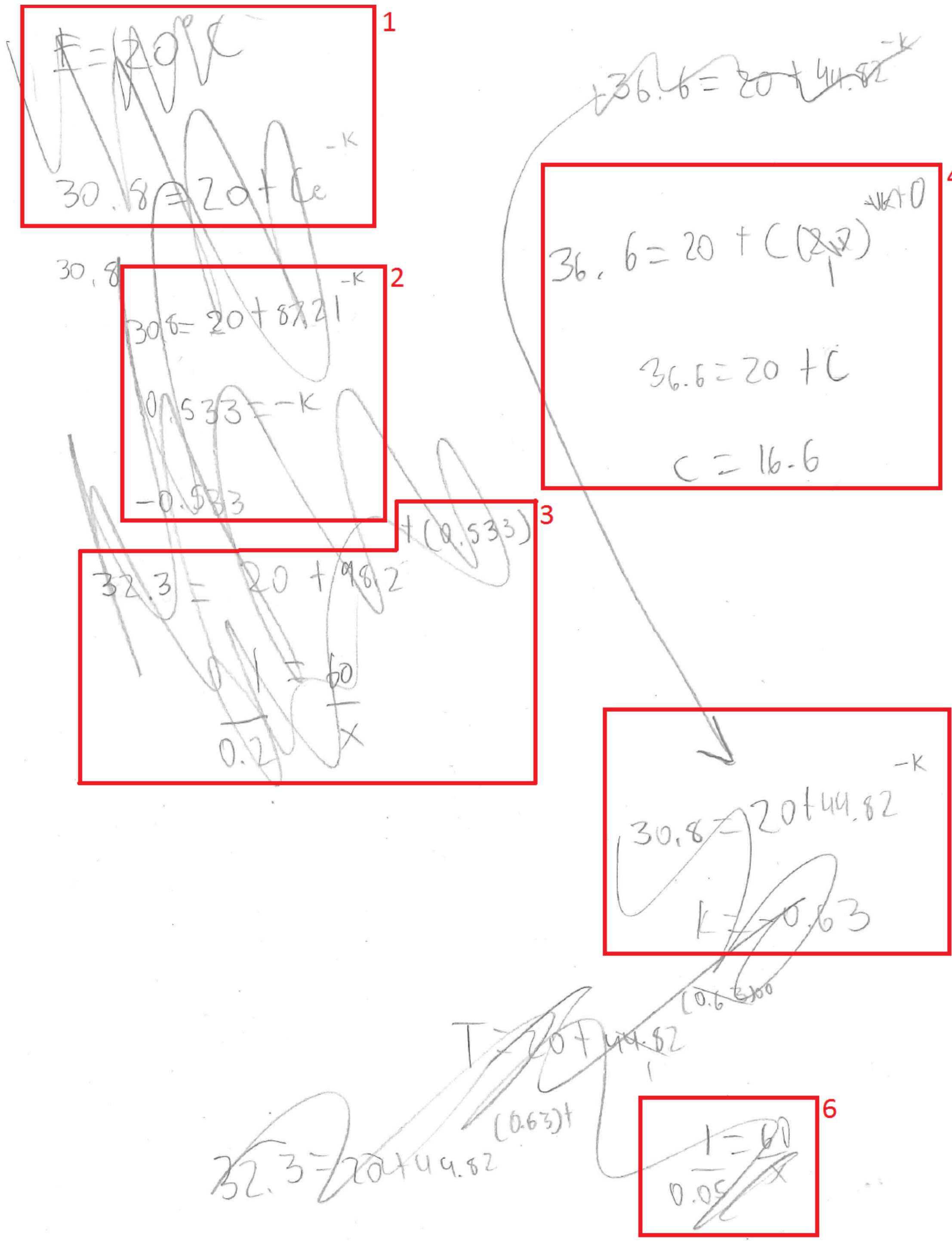


Figure 19. Jonathan and Mike’s First Page of Written Solution

Jonathan and Mike began their first attempt by recognizing that $E = 20$. They correctly used this value in the cooling equation throughout their solution. They also substituted $T = 30.8$ into the cooling equation except that they omitted variable t, $30.8 = 20 + Ce^{-k}$. This can be seen in Box 1 of Figure 19. This may be an accidental omission of *t*, or perhaps they were using t=1 since the temperature T=30.8 corresponds to one hour after police arrive. If so, t=0 would correspond to the time when the police arrived. (time, Temp) = (1?, 30.8) which is the temperature the police measured one hour after they arrive. Since *t* has disappeared, we can infer they have used t=1 (hour), meaning t=0 would correspond to the police arrival.

In Box 2 of Figure 19, their next line is $30.8 = 20 + 87.21^{-k}$. Where did *C* go? I infer that the group (incorrectly) believed C=Temp=32.3, the temperature when the police first arrived. They then multiplied C=32.3 by e=2.7 to arrive at 87.21 which they substituted for Ce. This is an error in order of operations, multiplying before exponentiation. It also demonstrates that they were unsure how to treat the constant *C*, and tried plugging in a different temperature value for *C* than *T*.

Their next line is 0.533 = -k and finally -0.533 presumably their solved *k* value. This is a correct solution for *k* from their equation $30.8 = 20 + 87.21^{-k}$, but I am unsure how they actually solved for it. Perhaps they wrote intermediate steps on another piece of paper, or perhaps Jonathan's mastery of algebra and logarithms was strong enough to complete it all in one step with a scientific calculator.

With *k* solved, they returned to the cooling equation which they wrote as $32.3 = 20 + 98.2^{t(0.533)}$, shown in Box 3 of Figure 19. They have plugged in E=20 and their solved value of *k*, k=0.533. They have also plugged in T=32.3 (the temperature when the police arrive) which they have not yet used for *T* although they did previously use this value incorrectly for *C* in order to find *k.* Interestingly, *t* appears in this equation. To be consistent with the above, they would have substituted t=0 since here *T* corresponds to the temperature when the police arrived. Previously they used the value Ce=87.21 (32.3 x 2.7) and here they've used Ce=98.2. My best guess is they used Ce=36.6 x 2.7 and erroneously copied the product as 98.2 instead of 98.82. This can be seen as consistent with their approach to *C* above in that in both cases *C* was replaced with the previous temperature measure compared with *T*. First they used T=30.8 and C=32.3. Then they used T=32.3 and C=36.6.

While their solution contained several errors up to this point, they landed on an equation $32.3 = 20 + 98.2^{t(0.533)}$ with only one variable *t*. We saw previously that they appeared to be able to solve equations of this form. However, I do not see that they ever solved *t* from this equation. Their next line contains the proportion $\frac{0.1}{0.2} = \frac{60}{x}$ which may be an attempt to convert between minutes and hours.

At this point, it appears they realized *C* is a value to solve rather than for substituting a body temperature, as they had been doing. I see this in Box 4 of Figure 19. Presumably this occurred when one group had the Aha! moment of solving *C* and this knowledge spread to the others. To solve *C*, they plugged E=20, t=0, and T=36.6 into the cooling equation. It is interesting that they use T=36.6 and t=0 since this means that time zero corresponds to the time of the murder, a departure from their previous attempts where t=0 corresponded to the time the police arrived. As we will see, Jonathan and Mike were also inconsistent with their handling of *t* going forward, which ultimately led to an incorrect answer.

In Box 5 of Figure 19, they repeat their solution for *k*, but with *C* replaced with the now correctly solved value of 16.6. They have repeated for the third time their order of operations error, multiplying C=16.6 and e=2.7 to arrive at 44.82 as the base. As earlier, they used T=30.8 and presumably t=1 corresponding to one hour after the police arrived. In one step, they jumped from $30.8 = 20 + 44.82^{-k}$ to a correct solution for -*k* and then k with k=-0.63. Without showing intermediate steps, I am not clear how they did this.

From here, they returned to the cooling equation, $32.3 = 20 + 44.82^{0.63t}$. They plugged in C=16.6 (correct), k=-0.63 (incorrect), and T=32.3 (temperature once police arrived) and have left time *t* in the equation. They surprisingly did not solve for *t* even though it is the only variable in the equation and they have demonstrated they know how to solve this type of equation. But again, a proportional statement appears shown in Box 6 of Figure 19. I am unclear how and what they are using that for, perhaps a conversion from hours to minutes as they do later on.

The solution up until now was crossed out except for the correct solution of *C*. The remainder of their solution they started on a new page shown in Figure 20. In Box 1, we see they first return to the cooling equation $30.8 = 20 + 16.6(2.7)^{-k}$ and, as before, *t* does not appear. We can again infer they have used t=1 since this temperature was taken one hour after the police arrived (time, Temp) = (1, 30.8). However, this is inconsistent (and thus incorrect) with how they previously defined *t* since in their solution of C=16.6 they used t=0 as the time of the murder and so t=1 here would be impossible.

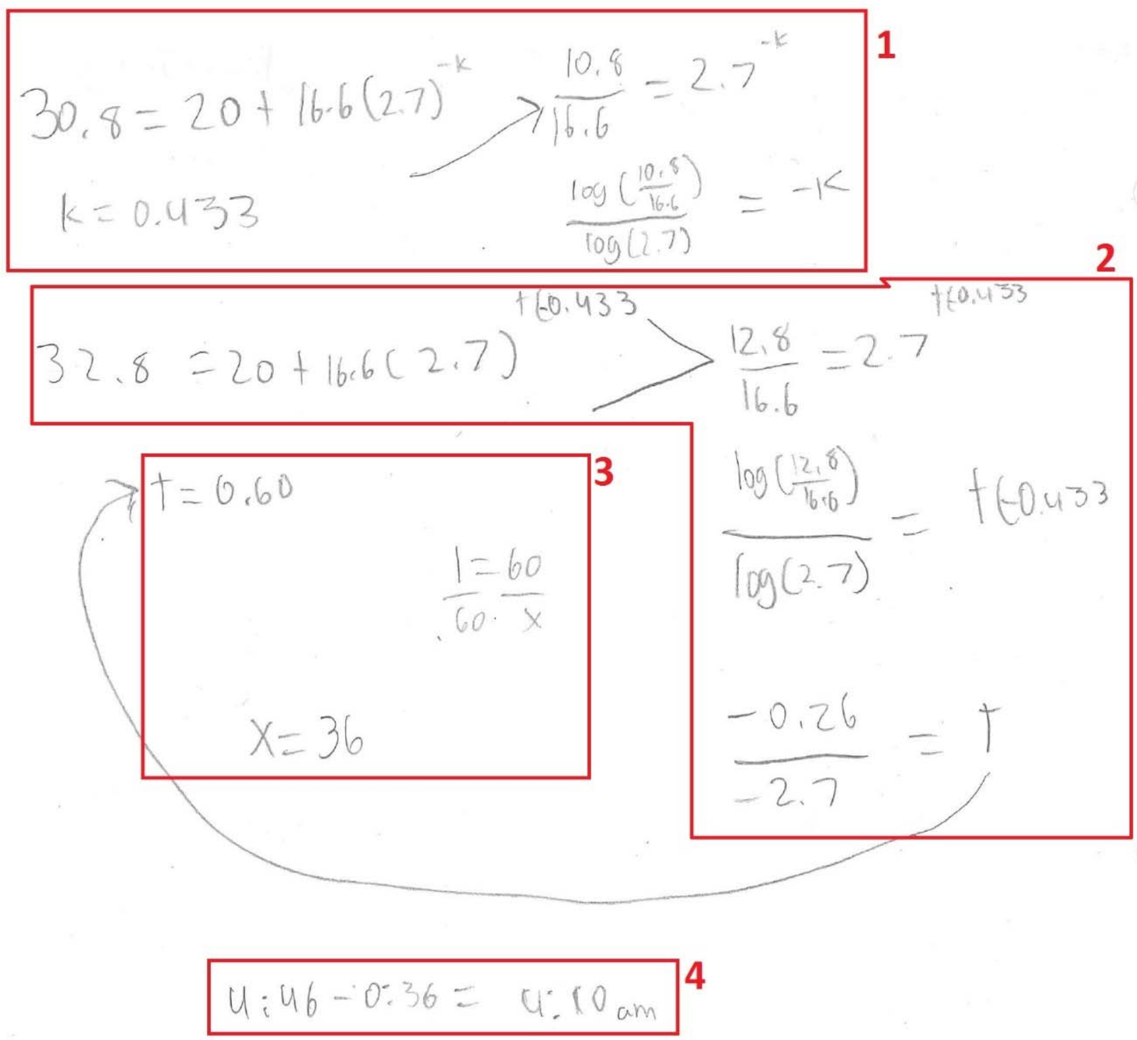


Figure 20. Jonathan and Mike's Second Page of Written Solution

In the rest of Box 1, they isolate for *k* in a series of steps. No longer do they suffer from order of operations misconduct and successfully isolate *k* in $30.8 = 20 + 16.6(2.7)^{-k}$.

This results in k=0.433 which is incorrect because their cooling equation was incorrect, as just noted.

With values for constants *C* and *k,* they returned to the cooling equation, shown in Box 2 of Figure 20. They plugged in C=16.6, k=0.433 (incorrect) and Temp=32.8: $32.8 = 20 + 16.6(2.7)^{t(-0,433)}$. As they were able to solve for *k* in the exponent, here they were able to solve *t* in the exponent and did so in a series of correct algebraic steps, including the application of logarithms to solve t=0.60.

In Box 3, it appears they used a proportional statement to convert from time in hours to minutes, so t=0.60 hours becomes 36 minutes. It is unclear what 36 minutes would signify; it is positive so presumably it would be 36 minutes after some reference time. The time of the temperature 32.3 substituted to solve *t* was the temperature taken when police arrived on the scene.

In Box 4, Jonathan and Mike subtracted the 36 minutes from 4:46, the time at which police arrived. This was the end of their solution, so I am not sure if they thought that 4:10 AM was the time of murder. They did not draw conclusions as to the murder suspects. They are the second pair that did not come up with a list of suspects (the other is Daniel and Larry).

Jonathan and Mike made many wrong turns in their problem solving of the task. First, they substituted certain temperatures for the constant *C* and misapplied the order of operations. Once they resolved those issues, they were still inconsistent in the relative value of time variable t. Ultimately, they did not reach a conclusion as to the murder suspects. In spite of their troubles and setbacks, their written solution displays a certain amount of resilience, as they kept trying to solve it and evolved their process. For the most part, they were able to correctly apply the knowledge of exponents and logarithms learned in the course but struggled with how to fit the pieces together and interpret the equation, its constants, and variables.

4.3.3. Survey Results

Jonathan commented that the mathematics for the task was “logical and necessary,” whereas in the textbook, such real-world questions “it seems like math applications are being forced to work in the context.” Mike wrote, “Most real-world-ish problems I see in textbooks are, to me, way too simple or unrelated for me to actually believe if people actually use mathematics to solve those kinds of problems, like asking to find the length of a ladder using cosine laws and such. Why would anyone need or want to find the length of the ladder especially using this kind of method?” While Jonathan agreed the Murder Mystery illustrated how mathematics might be used in the real world, Mike was not convinced. “In the real world, these kind of hands on, actual calculations by people don't really happen...people would just collect evidence like fingerprints and blood into their machines that do all these calculations for them.” Jonathan found the task sufficiently realistic while Mike did not.

This tension of whether a task is sufficiently realistic was discussed by Harvey & Averill (2012). They found that while a successful real-world task weaves mathematics and context, the solution method in the task is unlikely to be the actual solution method used in the real world for a similar problem. Harvey & Averill argue that it may only be necessary for teachers to use contexts that are “mainly faithful” to the real-world rather than having the same mathematics in the classroom.

Interestingly, Mike strongly agreed that real-world applications can boost student interest while Jonathan did not agree. So while Mike did not find the task realistic, he felt real-world tasks can boost motivation. Jonathan found the task realistic and did not agree that they boost motivation. With Mike and Jonathan feeling differently about this task’s authenticity and the ability of real-world tasks to generate interest, this pair illustrates the individual differences students will have in response to real-world tasks in mathematics.

Both students enjoyed aspects of the task. Jonathan liked the “general structure of the assignment” and “thought it was very creative and intriguing.” Mike liked that it was group work, “I personally prefer group work because most of the times, when I don't feel very confident, instead of struggling by myself, I can feel more involved and accomplish more than I would myself.”

Both students also felt that a solid mathematical foundation should precede the inclusion of real-world tasks. Jonathan wrote, “Introducing realworld [sic] applications should only be for higher level courses as they tend to be more complex and require actual interests in math.” Jonathan’s comment that applications “require actual interests in math” is consistent with his lack of agreement that real-world tasks increase interest. Mike wrote, “I think that we need to know and learn the most basic or pure aspects of mathematics before we can actually start learning courses that include real-world applications... we don't just use alphabets by themselves in real life, we just [use] words and sentences, but without knowing the basic forms...it would be much [more] difficult and not make sense to learn these real-world applications.”

Jonathan felt the inclusion of real-world tasks such as Murder Mystery would not influence his view of the role mathematics plays in society because at a high level he “already knew that science-based professions like coroner require the use of a lot of math.” He did think that such tasks would help him “understand some examples of the application of math.” Mike felt the task expanded his view of mathematics: the task “influenced my view that more [than] what I thought of mathematics were used in real life.”

4.3.4. Analysis

Jonathan and Mike are an interesting case. While they were not able to successfully rectify their misunderstandings, Mike did report positively on intragroup dynamics. He reported being more comfortable working with a partner and felt that this enabled him to progress where otherwise he would have been stuck.

Mike pointed out in his survey that in the real world, the calculations done by the students to solve the Murder Mystery would be done by machines. He makes an important insight. Even within real-world tasks, there are degrees of how real they are. The real-world context models the real world, and contains simplifications and assumptions. Since neither students nor most teachers have experience applying advanced mathematics in industrial settings, a teacher needs to rely on the subject matter expertise provided by the institute or authors of a particular task. On the other hand, “realistic” in terms of RME does not need to exist in the real world, so long as the context exists in the mind of the student.

The pair, like other groups, persisted through incorrect solutions and in the face of an uncertain path forward. Ultimately, they did not have a solid grasp of how the cooling equation operated and the meaning of its constants and variables. They never arrived at a correct solution.

The fact that Jonathan did not complete the task successfully is surprising. Jonathan consistently received top marks in the class. What made this task different from class assessments was there was no pre-defined route to solve the problem. He could not look back at notes or similar problems. Other pairs of students dealt with this uncertainty in different ways - drawing on other tools or procedures they knew or relying on each other to patiently problem solve; however, Jonathan was not able to successfully overcome this uncertainty. The task suggests that Jonathan's success in the course was accomplished through mimicking and memorizing. This finding aligns with Jonathan's view on the nature of mathematics as being more about rules than problem solving. I would not have predicted Jonathan's poor performance on this task, given he received some of the most consistently high marks in the course but I had not previously observed his discomfort with ambiguous, unstructured settings. As a researcher, I see how this task created value by revealing to me, and possibly also revealing to Jonathan, deeper characteristics of his learner profile. He sees mathematics as rules-based and therefore struggled with a task where the path forward was not clearly laid out.

4.4. Edward and Irfan

4.4.1. Student Description

Irfan is a memorable young man. There is a rigidity in the way he walks and talks. He was not well integrated with the other students and chose to sit away from them in the classroom. He avoided social interaction and generally seemed more comfortable speaking with adults than peers. He pursued his interests intensely and could be seen during his free time in the library reading. His knowledge in his areas of interest - cars and investing - were advanced for a young man of his age.

Irfan consistently completed his homework, never missing any all year. He was also very good about asking questions at the start of class about any homework questions with which he had trouble. There was one unit test where Irfan realized I had given him too many marks. When I thanked him for his honesty and told him it was fine, he became a little agitated and insisted I change the mark down to what it ought to be. Irfan's grades were consistent throughout the year in the 90 - 95% range.

Edward was not very well integrated in our class either, but for different reasons. Edward had spent most of his life in China and had a large social group of students of similar language and culture at the school but not in our class. Edward is a cheerful young man who was very optimistic about life in general. He has a great respect for teachers and always thanked me for the lesson and wished me well before he left class. Perhaps due to pressure at home, Edward was 'marks-obsessed'. Edward frequently pleaded to have his mark increased. He had a marks-focussed attitude towards learning and when an assignment was given, the first words out of his mouth were, "Is this for marks?" Edward did not have great study habits. He usually did the bare minimum during a unit and then studied hard for the unit test. He started the year with 95% in the first term and gradually slid down to 79%, finishing the course with 88% overall.

4.4.2. Task Results

Edward and Irfan were both loners in the class - neither boy was well integrated socially. Edward had friends outside the class, whereas Irfan tended to keep to himself throughout the school day. The classroom happened to be one of the largest in the school and the desks were very spread out. Almost the entire class clustered in half the room, while a small number of students preferred the other (quieter) half - Edward and Irfan were both in that half. They were not friends, were somewhat socially awkward, but were both respectful. They likely paired up because neither had an obvious or preferred partner and were in close proximity.

As discussed, there was a moment during the task when one group had a breakthrough and this caused other groups to cluster around to find out how this one group had solved for *C*. All the pairs analyzed to this point were part of that spread of knowledge.

Edward and Irfan were not. This may have been because they were happy to work on their own or perhaps they preferred to stay away from the unfolding social interaction. As a result, their written solution is unique.

Edward and Irfan had a draft solution, shown in Figure 21, which includes rough notes and wrong turns. They followed this up with a good copy of the solution which starts with the final answer and then justifies the process to arrive there.

Like Jonathan and Mike, Edward and Irfan erroneously replace *C* with a temperature value. While Jonathan and Mike eventually corrected this erroneous approach, Edward and Irfan never did. While Jonathan and Mike at first used different temperature values for *C*, Edward and Irfan consistently use C=32.3, the body temperature when police arrived.

Edward and Irfan's rough notes included snippets of interpreting and representing the information provided in the task description. In Box 1 of Figure 21, the pair appear to be interpreting and summarizing the given data from the problem. Their first line indicates that at 5:46 AM the victim's temperature had dropped from 32.3 to 30.8. Their second line indicates that the body temperature dropped from 36.6 when the victim was alive to 32.3 when police arrived.

In Box 2, they wrote "$30.8 = 20 \quad 32.3^{-kt}$", missing the plus symbol "+" between 20 and 32.3. The character in the exponent that follows -*k* appears to be *t*, although it is difficult to read. I am confident it is in fact a *t* both from the context and their later use of it shown in Box 3, t=1 hr.

So they took the cooling equation and set T=30.8, one hour after police arrived, as t=1. They correctly set E=20. They incorrectly replaced *Ce* with 32.3, the temperature when police arrived.

Their next step, in Box 4, is interesting. It seems they subtracted the exponent from the base. The power was 32.3^{-kt} where t=1 which they incorrectly simplified to 31.3^{-k}. Next, in Box 5, they took steps to isolate *k.* They correctly subtracted 20 from each side and arrived at $31.3^{-k} = 10.8$. Then they took the logarithm of both sides and correctly isolated for *k,* to arrive at k=-0.691. This *k* value is incorrect, however, due to their errors to this point.

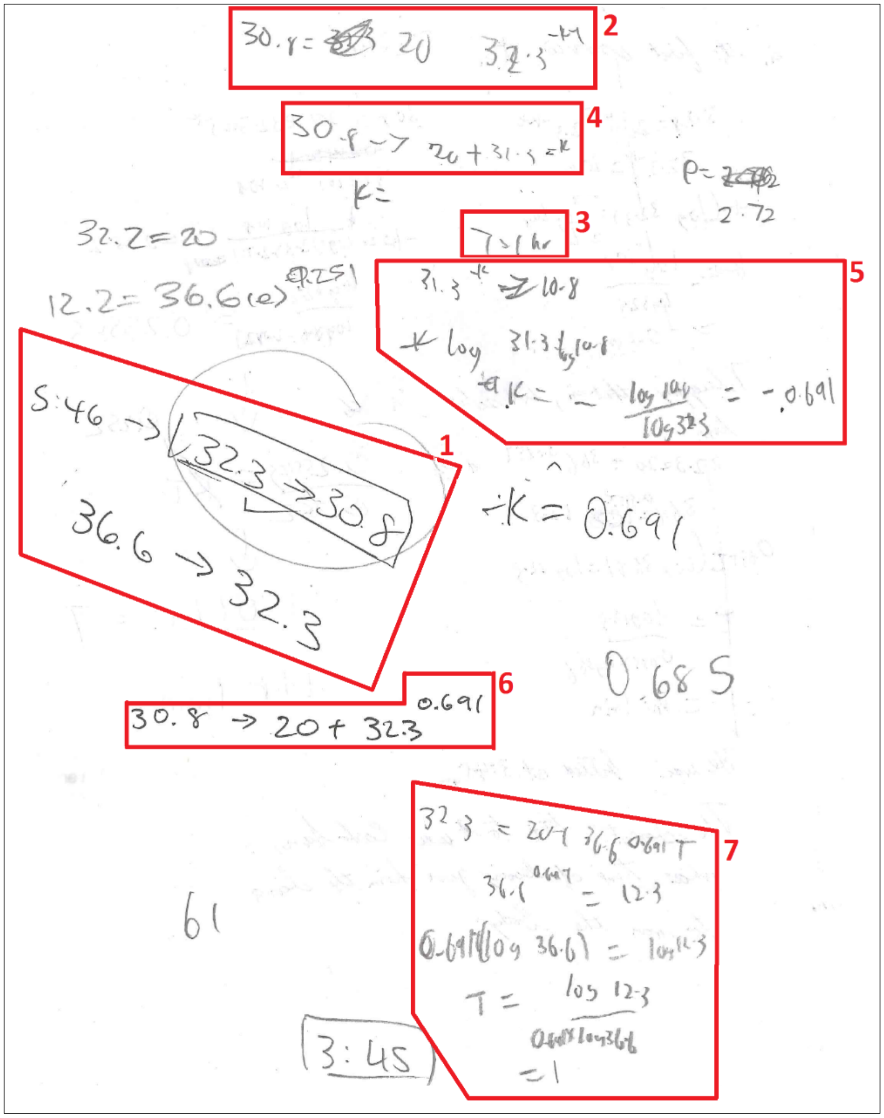


Figure 21. Edward and Irfan's First Page of Written Solution

In Box 6, they returned to the cooling equation above, which used T=30.8, E=20, Ce=32.3, and t=1 and substituted their solved value of *k,* -k=0.691. They left this and tried something else, shown in Box 7. They incorporated the temperature at time of murder into the cooling equation. They substituted T=32.3, E=20, Ce=36.6, -k=0.691, and capitalized time *t* (lower case) as *T* (upper case). Up until this point, they used a pair of temperatures as an (incorrect) approach to solve *k.* Then they switched to a different pair of temperatures to solve time *t*.

They followed a correct process which involved the application of logarithms to solve the exponential equation for time *t*, and arrive at t=log 12.3 / (0.691 log 36.6) = 1 (actual 1.008..) With *t* solved as t=1, they wrote 3:45 AM presumably reasoning that if *t* represents 1 hour, that indicates the murder took place 1 hour before the police arrival at 4:45.

They reached the bottom of the page, with 3:45 boxed as the last step. On the next page, shown in Figure 22, they started with their conclusion, “We believe that Carlo Sans is the primary suspect. We calculated that the time of death was 3:46 am. We got the value by calculating the value of *k* in $T = E + Ce^{-kt}$ in the first scenario, t=1 T=30.8 (shown in Box 1).”

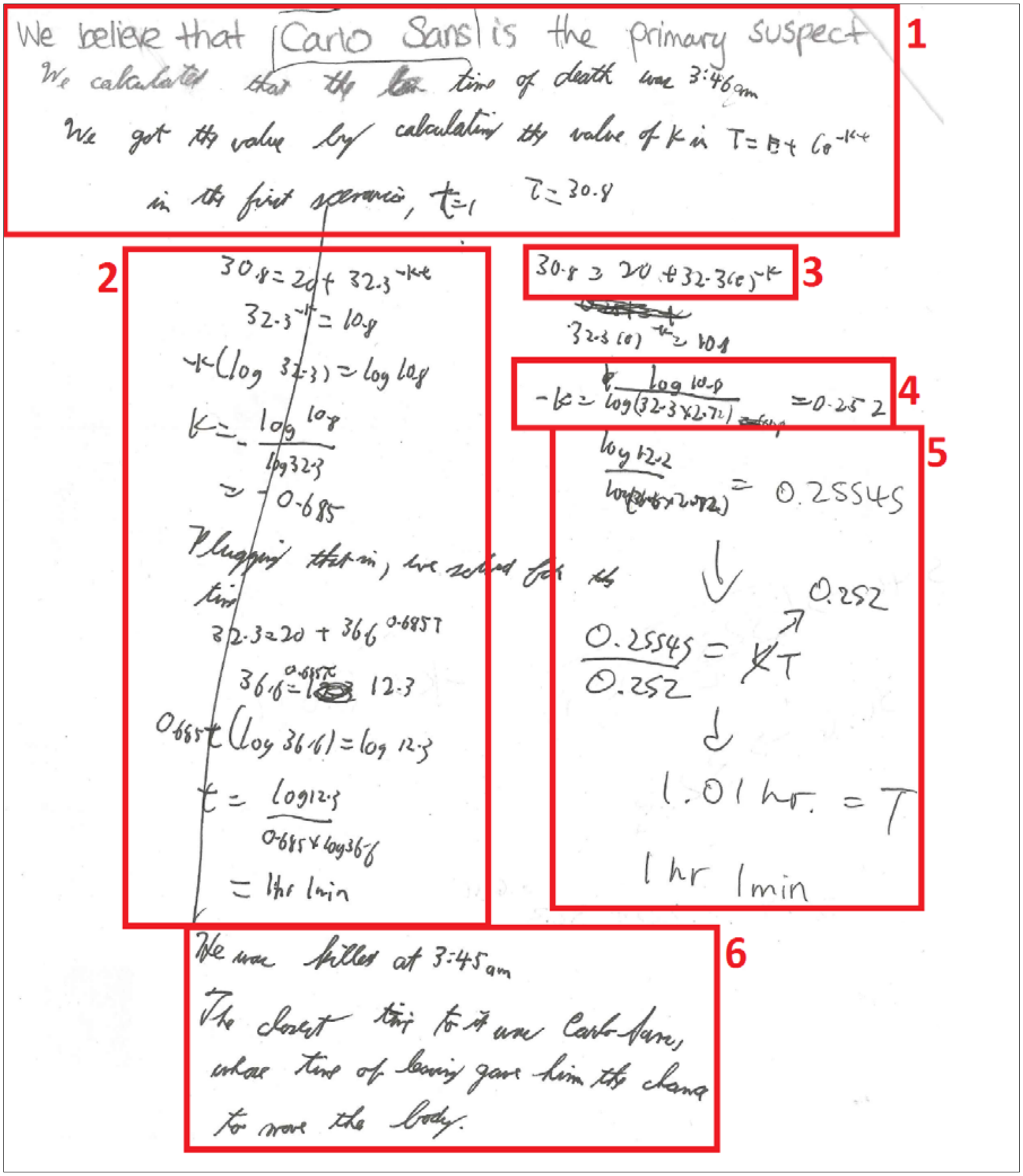


Figure 22. Edward and Irfan’s Second Page of Written Solution

It then appears they rewrote their draft solution with a more organized layout and all steps shown in Box 2, reaching k= -0.685 and t=1 hr 1 min. At this point, they must have realized they forgot *e* from their model. In Box 3, they restarted their solution and proceeded with *e* included. In Box 4, they correctly applied algebra and logarithms to isolate -*k* as $\frac{\log 10.8}{\log(32.3 \times 2.72)}$ where they replaced e=2.72. However, they made a mistake computing this expression with their calculator and arrived incorrectly at -k = 0.252. Their answer results from misplaced brackets, $\frac{\log 10.8}{(\log 32.3 \times 2.72)}$, equivalent to $\frac{\log 10.8}{(\log 32.3)(2.72)}$. The correct computation for their expression for -*k* would be $-k \approx 0.532.$

It is at first unclear what they did in their next steps, shown in Box 5. Upon comparing to their good copy solution that is crossed out (because it omitted *e*) we can infer their thinking. In their crossed out solution, they first use T=30.8 and C=32.3 to solve for *k.* Next, they use T=32.3 and C=36.6 to solve for t. In their current solution, they first use T=30.8 and C=32.3 to solve for *k.* Instead of repeating all these steps to do it again to find t, they simply replaced the *T* and *C* values in the isolated -*k* expression,

So -k = log 10.8 / log (32.3 x 2.72) becomes -kt = log 12.2 / log (36.6 x 2.72). They arrived at 0.25545, which should represent -*kT*. They wrote 0.25545 = kT and indicated that *k* is 0.252. This is a double notation error which ends up being correct (it should be 0.25545 = -kT and then replace -*k* with 0.252). Finally, they implicitly divided both sides by 0.252 to arrive at time = 0.25545/0.252 which they denoted with a capitalized *T.*

After solving for *k,* they solved for time using T=32.3 and C=36.6. This results in 1 hour and 1 minute which is correct with rounding. Incredibly, as a result of separate errors in both columns (in the left column they erroneously omitted *e*, in the right column they erroneously place *e* outside the argument of the logarithm), they have reached the same conclusion, t= 1 hour and 1 minute! This results from an embedded cancellation of *e* in their right column solution.

Having reached the same value of *t* of 1 hour and 1 minute using two methods (even though one of the methods had multiple errors!), they drew a conclusion about the time of murder, written in Box 6. Since police arrived at 4:45 AM and they solved $t > 0$, to be mathematically consistent they should have *added* their value of t to 4:45 AM. However, they *subtracted* t from 4:45 AM, presumably using the context of the question - the murder took place before the police discovered the body at 4:45 AM. Here, Edward and Irfan have used their understanding of the context to interpret their t value. And so their conclusion is, “He was killed at 3:45am. The closest time to it was Carlo Sans, whose time of leaving gave him the chance to move the body.”

4.4.3. Survey Results

Edward and Irfan had similar responses on their surveys. Both agreed or strongly agreed that the task illustrated how mathematics might be used in the real world. Edward noted, “I love...the interesting premise of the activity. The specific equations that we have to use for the solving of the problem. It made me learn a lot.” Irfan felt the activity was “useful for the field I am interested in, engineering...I want to be a materials engineer, which involves knowing the various breaking points and stresses. There, mathematics would play a major role.”

Edward commented that he “already realized that mathematics is very important in our lives even before.” He also noted that at first he did not understand the Murder Mystery but he “eventually realized how my understanding of exponential function played into this.” Edward’s view of mathematics includes a belief that mathematics is important in society.

Both boys felt that mathematics class should include real-world applications. Irfan notes, “There needs to be an understanding at both a technical and fundamental level,” and Edward strongly agreed that real-world applications can boost student motivation. He indicated, “It puts a different spin on a relatively ‘boring’ concept.” I interpret this to mean he felt that the topic of exponential functions and logarithms was boring until it was explored through the real-world Murder Mystery task.

4.4.4. Analysis

I was very pleased with Edward and Irfan’s experience with the task. They were collaborating with each other in a way that neither boy typically does. The social cohesion they experienced was in contrast to their social isolation in class. They responded positively to this opportunity to work together, given their overall positive feedback on the survey.

Edward and Irfan worked harmoniously as a pair, independently from the other groups. Their independence from the other groups is not surprising, given their weak social links in class. The lack of intergroup influence enabled Edward and Irfan to pursue their independent problem solving approach, ultimately to the detriment of their final solution as it contained some wrong turns. In particular, verifying progress with another group may have enabled them to realize that the constant *C* is not a temperature. However, since they reached a plausible solution, and verified it with a second method, the two were satisfied with their process. Both also reported enjoying the activity and supported the inclusion of real-world tasks in the learning of mathematics. As their teacher, I was pleased with their collaboration and had the opportunity to view a side of Edward and Irfan that was not typical in class.

The four case studies have illustrated a wide range of student experiences with the Murder Mystery task. Daniel and Larry interacted effectively and demonstrated focus and perseverance. Pivotal to their success was their dominant-subordinate dynamic, open communication, and collaboration with another group. Thomas and Kevin criticized the mathematics model provided and devised an original solution. They enjoyed the context and wanted to see even more realistic applications. Jonathan and Mike reported enjoying the task and working with a partner but struggled to make sense of constants, variables, and the cooling equation in a real-world task that did not have a previously learned set of steps to move through it. Edward and Irfan were isolated from the other groups and they solved the task with two methods which led them to be confident with their solution, although it was incorrect. Notably, Edward and Irfan experienced positive intragroup dynamics not typical for either one.

In the next chapter, I will reflect on the research findings and point to areas for further research.

Chapter 5. Conclusion

This thesis aims to contribute empirical knowledge to the research field of applications and modelling within mathematics education. The field has continued to mature since the seminal work of Freudenthal in the late 1960s. I have examined a real-world application in mathematics class through the experiences of students. Understanding the student experience adds an instrumental perspective to the research discussion. In the following section, I will highlight the key findings from this research. Subsequently, I will discuss implications of the research, limitations, and future work to be done.

5.1. Findings

Collectively, the experiences of the students captured by this research provide insights for mathematics education. The research question to be answered by this study is:

How do students experience and perceive a real-world application in mathematics class?

I observed the students on the whole to be motivated by the real-world task. This is one of the goals identified by Blum et al. (2007) for the inclusion of applications in mathematics. Students were motivated during the class as evidenced in my observations and their surveys. Students engaged in the task for a sustained period of 60 minutes, except Thomas and Kevin who finished quicker. Over this period, I saw excitement, enthusiasm, and perseverance. Larry noted in his survey that real-world tasks connected “because they’re relatable” and that he found the plot interesting. The ability to imagine and be intrigued by the Murder Mystery context may have contributed to this sustained attention.

Students exhibited motivation to keep going. Daniel and Larry produced many incorrect attempts but their patient intragroup dynamic allowed them to carry on when they were stuck. The class’ collective excitement discovering a solution for the constant *C* provided a motivational boost for Daniel and Larry and carried them forward to successfully solve the task. Their ability to sustain motivation seems to be sourced from the relatable nature of the real-world task and productive intragroup and intergroup dynamics. Finally, the reward stemming from the motivation to persevere through the task is summarized by Larry, “Once you figure it out you feel more accomplished than just solving a math question.”

Edward and Irfan successfully collaborated through the task in a way that neither typically does. This result may suggest that the inclusion of real-world tasks in the learning of mathematics can increase social cohesion among students. Both boys reported positively about the experience, enjoying the experience and the inclusion of real-world tasks. Mike also reported positive impacts of working with Jonathan. Although this group did not reach the correct solution, they persevered in their search. Daniel enjoyed working with Larry and these two exhibited an effective dominant-subordinate dynamic. Collaboration was a positive aspect of the students’ experience and perception of the real-world application.

Blum et al. (2007) identify another goal for using applications in mathematics education as providing meaning and interpretation to mathematics. Daniel indicated the real-world task clarified mathematical processes and concepts: “I think this question helped enhance my understanding on making equations as well as the relationship between x and y.” For Daniel, the task created deeper understanding of mathematics by necessitating work with equations and using a function with realistic (in the sense of imaginable) quantities: time and temperature in the place of x and y. The task also provided Daniel and Larry understanding of the utility of the mathematical process of solving a system of equations within a real-world context. They experienced a euphoric moment when they realized this process was necessary to move forward in the problem and solve one of the unknown constants.

The third goal identified by Blum et al. (2007) for using applications for the teaching and learning of mathematics is to develop a broad image of the nature and role of mathematics by demonstrating that it is used by people for a variety of purposes. One student wrote, “I learned that there are real life ways exponential functions can be used.” The sentiment was echoed by Thomas who wrote, “Simply introducing real-world descriptions and context showed how mathematics can be used in the real world." From his experience in AP Calculus, Thomas suggested the use of even more "eye-opening" applications such as structural load calculations or profit analysis. Applications have provided Thomas with a broad view of the role of mathematics, evident in his comment, "The Murder Mystery is only brushing the sides of what mathematics does in society." Irfan wrote that the activity was "useful for the field I am interested in, engineering." Students also noted their existing beliefs about mathematics. Mike felt that mathematics is performed by computers, not people, in real-world murder investigations. Irfan noted he already believed mathematics to be important in society. Jonathan echoed this existing belief mentioning he knows that science-based professions like those seen in the task require lots of mathematics.

While the case studies generally illustrate successful aspects of real-world tasks, it is important to note that not all students thrived. Jonathan, in particular, felt frustrated by the challenge. He felt that a high level of interest in mathematics was required to enjoy real-world applications. Jonathan's experience revealed that he views mathematics as rule-based and suggests part of his success in the course was achieved through mimicry of procedures rather than strong problem solving skills. Overall, Jonathan was the exception among the cases examined.

Students experienced the real-world task in a way that brought about collaboration. They experienced motivation to complete the task and persevere through dead-ends. Some students indicated the task illustrated how mathematics can be used in the real world, while others felt they already knew this. Not all students arrived at the "right answer" but the four case studies illustrate positive outcomes of including real-world tasks in the learning of mathematics.

5.2. Implications

This research has implications for teachers who include real-world tasks in mathematics education, as well as for researchers studying this field. Let's start with the two surprises:

Jonathan had 96% mark in the course yet was not able to complete the task. It is surprising that he struggled as much as he did. It suggested to me that some of his success in the course may be the result of mimicry and memorization rather than deep understanding. Jonathan views mathematics as the application of rules. This suggests that real-world tasks can serve as a tool to untangle ways of knowing – memorizing methods versus decision making in novel situations.

Irfan and Edward were both socially isolated in the class. Through the task, these two connected and successfully collaborated. They reported enjoying it and its real-world premise. This suggests that real-world tasks can support the development of successful interpersonal relationships.

This real-world application underlined certain consequences that arise in a learning task with minimal teacher guidance. The students read the problem and tried to solve it on their own as I stood back and observed. This unconstrained problem solving process meant that students went in directions I never expected. Thomas and Kevin solved the task using a different equation than the one provided. Also, intergroup dynamics influenced various groups, either positively by providing a breakthrough or negatively by casting doubt on whether the problem was possible to solve with the information provided.

The study also has implications for my own teaching practice. It was time consuming to search and sift through possible lessons that integrated real-world context and fit the PC12 curriculum. Some of this time can be cut down in the future now that I have found two high quality sources of real-world tasks, CORD and Building Better Math. I look forward to incorporating real-world tasks into my teaching through more units and topics. This research has helped me confirm and articulate the importance of including real-world tasks in the learning of mathematics. I have experienced the benefit of seeing my students through a different lens and providing a broader mathematical experience for them.

5.3. Limitations

One of the challenges of social research is drawing causal or generalizing conclusions. While this research has established that during the real-world task, participants experienced motivation, perseverance, and collaboration, the nature of this study cannot establish the causation of these attributes. It is not clear whether the motivation, perseverance, and collaboration resulted from the real-world aspects of the task or another aspect such as a challenging problem with no clear path forward.

While the data provide a rich narrative of the experiences of the student participants in the study, it cannot be generalized with certainty to *all* students. The value of this research comes from documenting the experience of this particular class and laying a foundation for future work, both in research and teaching practice.

A more practical limitation stems from absent student data. In particular, Kevin did not submit a follow-up survey. Thomas and Kevin's dynamic was one of the most interesting since they solved using a unique method and were first to finish. While much was ascertained from Thomas' survey, the picture is not complete without Kevin's viewpoint.

A final limitation relates to my dual role as teacher and researcher. During the data collection, it was challenging at times as a researcher to observe the class while concurrently managing the class as the teacher. It is also difficult to 'see' or evaluate aspects that may be immediately apparent to an outside observer. For instance, several aspects identified by Harvey & Averill (2012) as important for implementing real-world contexts relate to the teacher – such as passion for the subject, depth of knowledge, and teacher-student relationship. These are challenging for me to self-evaluate, and thus a limitation of the dual nature of teacher-researcher.

5.4. Future Research

The more I learn, the more I realize how much I don't know.
— Albert Einstein

This research is a single step in the journey to understanding how senior high school mathematics is applied in the real world. Many questions are raised by this research, leaving room for future research and contemplation by teachers.

Because the task was carried out in pairs, and the pairs could interact, there were fluid social dynamics at play within and across groups. Edward and Irfan were isolated from the class but nonetheless collaborated successfully with each other. They were satisfied with their own problem solving process and enjoyed the real-world aspects of the task. Was it the task that enabled positive social interactions? Or was it the chemistry of these personalities? More broadly, are inter- and intra-group social dynamics a result of the task selection, or reflective of the particular pairings of students in the class?

A second area for more study relates to the consequences of context and task familiarity. This will vary among students. A Murder Mystery scenario is *imaginable* by students because it forms part of pop culture from books, plays, TV, and other media. What is the best way to build familiarity for a real-world task whose industry setting context is less familiar? While this challenge was avoided in the current research through task selection, it is a vital piece for future research if mathematics educators are to provide students with an education that demonstrates that mathematics is useful in a wide variety of settings.

The seeds of this thesis were planted during my secondary education, as I experienced the disconnect of mathematics from the world in which I lived. As a researcher and teacher of secondary education, I am now equipped with the academic understanding of why it is important to connect mathematics to its applications, and have experienced first-hand the benefits of facilitating these connections in the classroom. In my future practice, I plan to continue to inquire and critically reflect on how to incorporate real-world contexts as a lens through which to understand and appreciate mathematics.

While my future is in the classroom with students, I know this work will be continued by the researchers whose work has been reviewed in this thesis and those that come after them. Efforts to improve mathematics education through real-world applications have been in progress for decades. Through the process of writing this thesis, I have gained admiration of all the well-written insightful research I reviewed. The state of mathematics education research today is strong with many committed, funded researchers tackling important questions such as those raised here – for the ultimate benefit of the teaching and learning of mathematics. It is a good time to be a mathematics teacher.

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Appendix

A-Lympiad Problem

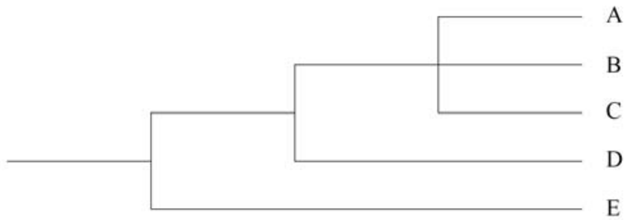
Part 2 of A-Lympiad problem is shown below. Final 1992-1993.

Up until now we have only looked at the ratios of numbers of species and plants. In principle every species is just as important for diversity. But there is also a case for considering that not all species are equally important. When it comes to the conservation of species, the genetic variation is of greater interest. There are scientists who have tried to develop certain criteria for this.
The current thinking with some biologists is: let's abandon the high ideal of conserving almost all species as unachievable and instead direct our efforts to preserving a limited number of species. These species must be chosen sensibly.

A new problem now arises: who decides, and in what way, which species get a place on the ark, so to speak? (Remember Noah in the Bible, Gen. 6, 14-22.)
In order to avoid being completely arbitrary, we have to try and devise a reasonably objective system which takes the importance of the species into account, as seen from a genetic point of view. This then has to lead to a (relative) value scale: for example from 0 percent to 100 percent, or by setting the lowest position to 1, and then setting the rest in relation to that.

Each species is given a position on the value scale. As it is not economically possible to preserve all species threatened with extinction, we will have to concentrate on the species that score the highest on the value scale. In addition, the difficulty of saving a particular species can also play a role.

People may have different opinions on which aspects have to be considered as important. But it is in any case an attempt to achieve an objective system.
Various biologists have already started doing preliminary exercises for developing such a system. They start with this hypothesis: it is desirable for the diversity of hereditary properties to be kept as great as possible (consider the need for resistant and productive races). They thereby use genealogical trees. Such a genealogical tree gives the level of genetic relatedness for a group of species.
Below is an example of a group consisting of five species A, B, C, D and E.



This genealogical tree is used to indicate that A and B are more closely genetically related than A and D for example. Thus if only two species can be saved, the combination of B and D is better than the combination of A and B, for example. Therefore D will be somewhat higher up the value scale than A. How much higher depends on the entire set.

Exercise 4

Design a value scale for this group of five species. One must be able to read from this which species you would first sacrifice if you 'were playing Noah'.

If (alas!) one species dies out, a new situation arises. A new value scale is then required. Try to make an overview in which the next 'victims' can be seen.

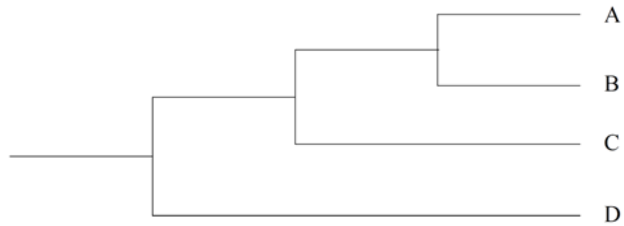
What is 'fair'?

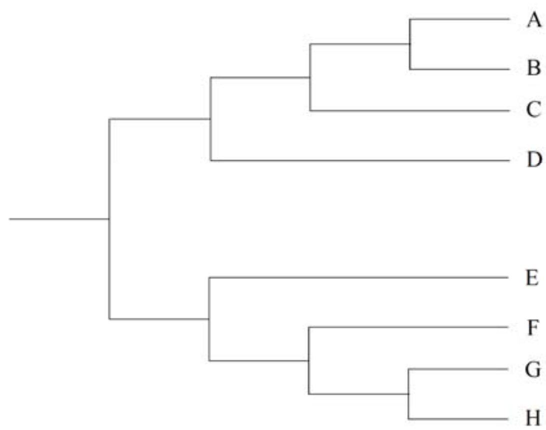
The value scale that you designed in exercise 4 is hopefully better than 'considering every species as equal', but it is still not ideal. In a group with many species a certain species can obtain an extreme score. A system that gives rather flatter results is better for such a case.

Exercise 5

Design one or more such systems and discuss the advantages and disadvantages with respect to the previous system, or the relative advantages and disadvantages of the new systems with respect to one another.

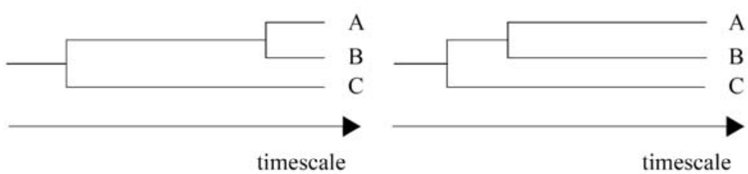
Clue: you can take the two trees below as a starting point for your discussion, and they can be compared to one another in the different systems.





Taking the time factor into account

If only the nodes and branches are considered, there is no difference between the following situations:

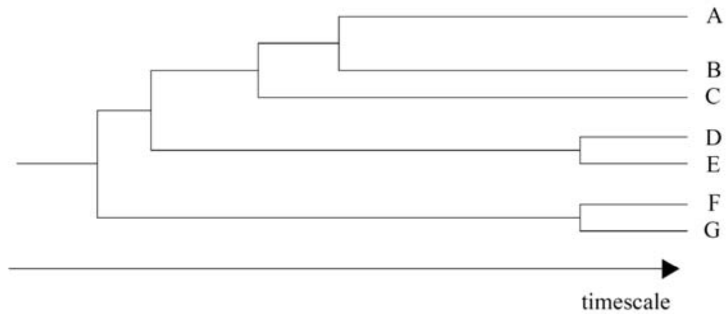


If you also take time into account and proceed on the basis that the differences between species become greater with time, then the situations are no longer the same. In the one case it seems reasonable that C is weighted more heavily than A or B. In the other case it is fairer to allocate roughly the same weight to A, B and C. A genealogical tree in which time also plays a role is called an evolution tree.

Exercise 6

Design a general *system* that also takes time into account.

Discuss this system by applying it to the following situation:



In conclusion

Put your ideas and findings into a report. It may be an ongoing account where you can choose to emphasise certain aspects yourself. It is thus not important if not all of the exercises have been tackled to the same depth.

Discussion of assessment of this task is found on pages 76 to 84 of *10 years of Mathematics A-Lympiad* (Haan & Wijers, 2000).