Observations in a Thinking Classroom

by

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Abstract

A Thinking Classroom is a classroom where students are engaged collaboratively in tasks designed to help with learning new concepts. It is a classroom where students are guided by the teacher and actively seek understanding from each other. Current research on Thinking classrooms is prescriptive in describing strategies for teachers to implement in order to break down existing classrooms norms and put in place new norms that are conducive to students working together and solving problems. I have implemented such a Thinking Classroom and in this thesis I look at what students and teachers are doing in a Thinking Classroom. Through analysis of classroom video, conclusions indicate that high mobility of students and ideas, autonomous behaviour in students, and a significant amount of class time spent on tasks were some of the observations that were noticed in a Thinking Classroom.

Keywords: thinking; classroom; mathematics; autonomy; tasks; culture; porosity; standing-biased

To Leanne, who encouraged and supported

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Table of Contents

List of Figures

List of Acronyms

BMI	Body Mass Index
VRG	Visibly Random Grouping
VNPS	Vertical Non-Permanent Surfaces

Chapter 1. Introduction

I have been teaching high school mathematics for twenty-two years, most of which have been at my alma mater, Slopeside Secondary School in North Vancouver. Over those years, like most teachers, I had refined my practice to a point where I believed that I was quite effective as a mathematics teacher. However, three years ago, I had an epiphany moment and I was inspired to embark on a radical change to my teaching, and as a result, to my classroom. This change was so radical that I hardly recognize anything from my old practice within my current classroom. This change created an environment in my classroom where students are engaged in mathematics, are intrinsically motivated to learn and seek understanding from others, and are genuinely enjoying mathematics. In this introduction, I would like to describe my classroom before the change, the impetus to my epiphany, and what my teaching and my classroom looks like today.

My Old Classroom

For most of my teaching years, I was refining my practice from a starting point that looked very similar to my own mathematics experience as a student. My students were sitting with partners in desks lined up in rows according to a seating plan. The seating plan was the tip of the iceberg for the control I was managing over my students. In the first week of classes, I would structure the seating plan alphabetically by the student's last name; then, over the course of the first few weeks, I would tweak the seating arrangement until I had it just right. After a few short weeks, all of the trouble-making students were either seated right next to me, or there was a large enough cushion of well-behaved and polite students surrounding them so that my class was quiet and peaceful. Once the seating plan was done, it saw very little change for the remaining months of the year.

My control over the class continued with other procedures that I had in place to deal with potential problems. To deal with tardy students, I had a system in place where I would shut and lock my door moments after the class-starting bell rang. I did this so that I could start my class without the constant interruption of students wandering in late. Late students needed to wait quietly outside my door until the time that I felt there was a natural break in my lesson and I let them in. My feeling here was that missing my wonderful lesson was a natural consequence for being late, and if students faced this consequence repeatedly, then they would "learn" to come to class on time.

In my attempts to improve student engagement, I put in place a reward system. If students went above and beyond by participating in front of the whole class or answering a particularly challenging question, then I would reward them with a 'stamp.' After students collected 10 stamps, they would earn a bonus mark. In hind sight, this system was fantastic for rewarding the students who were already engaging in the mathematics, but not so great at motivating the quiet and weaker students.

I placed a very high value on good note-taking and maintaining a neat and organized notebook. I can still hear myself saying such things as, “If it’s important enough for me to write it on my overhead, then it better be in your notebooks.” I would wander through my class performing random checks on student’s notebooks. If their notes were incomplete or even worse, if their notes were blank, then I would keep them in at break or at lunch to copy the notes down correctly from my overhead.

As I write this description, I cringe to think of how it must sound. I assure you, my class was not a horrible place. I was very good at explaining the mathematics content, I used humour and story-telling to keep the class enjoyable, and I was passionate about mathematics. I really wanted all of my students to learn, and I felt that everything I was doing was working together towards this goal. The common theme that permeated everything in my old classroom was control. I strived to have control of all aspects of my classroom and my students.

My lessons were also very structured. I would begin each class spending time going over student’s difficulties with their previous day’s homework. This was always a whole-class exercise, where I would wait for a student to ask a specific question that they were having difficulty with. Then, I would proceed to explain and demonstrate a good solution to the problem. This process would repeat over the course of the first fifteen to twenty minutes of the eighty-minute lesson. Next, I would decide on whether or not to perform a random homework check. During this check, I would award students a mark of 0, 1 or 2 depending on how much of the homework they had completed. After this, the lesson would begin. I liked to mark homework for two reasons. I felt that marking homework provided an incentive and motivation for students to actually do their homework, and I liked collecting data on student's work habits. For report cards at Slopeside Secondary, we are required to give a work habit mark along with a percentage mark to communicate student progress on the curriculum. For the work habit mark, I preferred an objective measure, and marking homework satisfied this preference.

For the lesson, I would explain the new topic for the day, show students how to do a variety of questions, and then I would put some questions on the board for the students to try. I now refer to this technique as the classic "now you try one" (Liljedahl, & Allan, 2013a, 2013b) instructional approach. During this time, I would walk through the class helping students in their learning. At least, this is what I would have told you was happening. In actuality, I was mostly managing behaviour, asking students to open up their notebooks, asking students to get back on task, or showing individual students how to do the problems. After about five minutes of this, I would ask for student volunteers to show their work on the boards. It was always the same group of students who would eagerly volunteer to show their steps and procedures on the boards before claiming their well-earned reward stamps. After a few rounds of the "now you try one" routines, I would give the students their homework, and expect them to work quietly at their desks for the remaining 10-15 minutes until the end-of-class bell.

For assessment, I gave bi-weekly quizzes, and about ten chapter tests over the course of a year. The tests were kept secure in my filing cabinet, because I re-used them from year to year in an effort to standardize my assessment. Students could view their tests by coming in to my class outside of class time, but they were not permitted to take their tests home. Before my big change, I did begin to experiment with allowing students to re-write tests, or to show understanding of basic concepts by writing what I called an ‘I’ test. The ‘I’ comes from the mark that they were earning because their progress was ‘Incomplete.’ If students could pass their I-test, then they would pass that particular part of the course. That was, however, the extent of my innovative teaching practice. As I stated earlier, I was a passionate dictator of mathematics, my students learned mathematics from how I told them that it should be done. I was the orator of mathematical knowledge, and as long as students both listened carefully and thought the way that I thought, then they would do well in my class.

After years of building classroom procedures and refining my instruction, I was always bothered by a certain ineffectiveness in most of my classes. By ineffective, I mean a general inattentiveness, lack of engagement, and apathy amongst most of my students. I’m not referring to my honours classes or my top students, but pretty much everyone else was uninterested, unmotivated, and uncommitted to learning mathematics. Near the end, I thought I just needed to perform better, be more funny, or provide more incentives to learn. I think I succeeded at all three of these self-improvements, but the general culture in my class was not changing.

The Change

In 2011, I was invited to participate as an executive member of the BC Association of Mathematics Teachers (the BCAMT), the specialist association for mathematics teachers in BC. One of my roles in this volunteer organization was to coordinate registration for all of their teacher conferences, a role that I still hold to this day. In late winter, 2013, I was coordinating registration on-site at one of our annual New Teachers Conferences. The keynote address was being given by Dr. Peter Liljedahl from Simon Fraser University, and it just happened to be in the same open area as where my registration table was set up. I was a captive audience member, and I soon realized that everything Dr. Liljedahl was sharing was directly related to my own experiences in my classroom.

Dr. Liljedahl presented on research that he had been conducting with one of his graduate students, Darien Allan (Liljedahl, & Allan, 2013a, 2013b). The research was on “studenting” which is essentially everything that students do in a classroom - the good and the bad. When student behaviour supports student learning, this is good and it is what we strive to attain in our classrooms. When student behaviour does not support learning, Liljedahl referred to this as “gaming,” and it is not a desirable behaviour in our classes. Liljedahl went on to show how common it is for students to be gaming in a classroom. One specific slide (see figure 1) that he discussed concerned his observations in a classroom where the teacher was using the “now you try one” instructional approach.

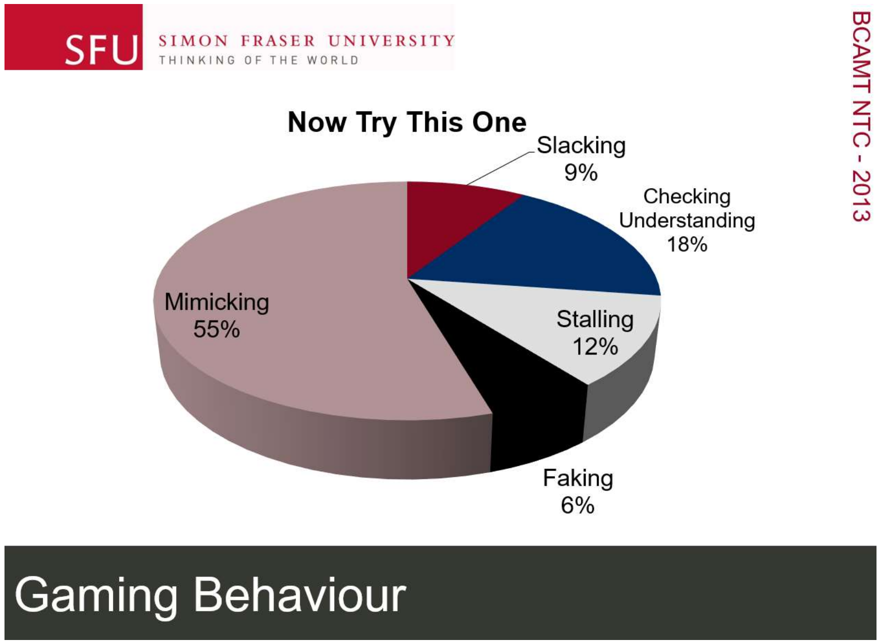


FIGURE 1. A SLIDE FROM LILJEDAHL'S PRESENTATION AT THE NEW TEACHERS CONFERENCE, 2013.

This slide resonated with me. I wasn't surprised by the results, for I could imagine each of my own students fitting into the categories described. In fact, the percentages attributed to each category were likely very similar to the percentages I observed in my own classroom. I knew my students who would rummage through their backpacks looking for paper or pens, ask to be dismissed for a drink of water or bathroom break, or ask if they could go to their locker to get their calculator. These students were classified as "stalling," and they did represent about 12% of my classes.

I also recognized my students who would slouch in their desks never lifting their pencils or opening their books. These students were in almost all of my classes; they were amotivated and apathetic, and they were about 9% of my students – these were the "slackers."

I had students in every class who pretended to participate in the activities. I can't say that I noticed this group as much, because they were the quiet ones who were earnestly moving their pencils, but the pencil's tips were not making contact with the paper. These were the students who were avoiding my attention by pretending to be engaged in the activity mostly waiting for the correct solution to be shown to them. These students accounted for about 6% of my class, and they were referred to as the "fakers."

The most staggering result in Liljedahl's observations were with the students that he classified as "mimickers." 55% of the students that he observed (and I expect a similar proportion of my own students) were copying the mathematics line-by-line from the teacher's demonstrated solution, substituting numbers where necessary, and never thinking about the underlying mathematics. Liljedahl suggested that these mimickers were not thinking, so they were not learning.

Because all of this student behaviour did not support student learning in this "now you try one" approach, Liljedahl classified all of this behaviour as "gaming." 82% of a typical mathematics class of students were *gaming* the lesson. Because my own observations were very much in agreement with Liljedahl's data, this suggested that about 80% of my students were not doing what I wanted them to do to learn mathematics. This was a profound epiphany moment, as I recognized all of these categories for gaming in my own instructional experience. I had been teaching for 19 years, and my culminated and refined teaching practices and procedures had been honed to a point where only about 20% of my students were engaged in thinking and learning. It was time for a change.

Fortunately for me, Liljedahl's keynote presentation also focused on strategies for breaking down classroom norms where gaming was prevalent, and transforming classrooms into ones where thinking, engagement, and problem solving were normal behaviours for students. Liljedahl suggested making three immediate changes to our teaching in order to begin the transformative process (Liljedahl, 2016):

1. Use visibly random groups (VRG) every day.

   Group work is a well-known strategy for improving student collaboration, communication, and learning; however, in most groups, students settle into roles quite quickly. Within the group, students quickly determine who will be the 'thinker' and the 'writer' and the 'slacker.' By randomizing groups every day, students are always required to re-think their group roles and are consistently given the opportunity to step up in their roles. By making this process visible, students trust the randomness and are more willing to work through difficult situations, knowing that it is only for a day and tomorrow will give them a different grouping.

2. Have students do all of their work in their groups at vertical non-permanent surfaces (VNPS).

   Liljedahl found that when students worked on VNPS (whiteboards or chalkboards), engagement increases, student work is made visible (this is good for teachers and students), and students are more willing to take risks with their thinking. This last point seems counterintuitive, but he suggested that it is the “non-permanence” of working at the whiteboards that increases student’s willingness to take risks. Because any potential mistake is easily erased, students are less afraid to engage in a problem.

3. Stop making students take notes.

   This is actually a natural consequence to implementing the first two strategies, but it was a big change for me. Liljedahl found that for most students, the act of taking notes was a proxy for learning – something that stands in the place of learning. Most students are not able to keep up with note taking, most students are not able to process the discussion in the class at the same time as taking notes, and most students never use their notes for learning later in the course.

My New Classroom

This New Teachers Conference was on a Saturday, and on the following Monday, I decided to *jump in with both feet* and implement these changes in all of my classes. On the first day, I had students in VRG’s. I did this by shuffling and distributing a deck of cards at the beginning of class, and then students would move into groups according to the rank on their cards.

Because of the data that I had seen on students working on the “Now you try one” type of problems, I avoided this teaching strategy. I began to experiment with giving students the problem that I would originally be expected to teach. I would give these problems (or tasks) with little to no introduction or guidance, and they would struggle and stumble their way through the task. At the end of this, we would re-group as a class and discuss their successes, failures and struggles. It was during this post-task discussion (the debrief) where my teaching would occur. I immediately found students to be more interested in the discussion and the learning likely due to the fact that they had all been engaged in the task prior to the discussion. In retrospect, I was beginning to experiment with problem-based learning (Allen, Donham, & Bernhardt, 2011) and more specifically with problematizing the curriculum (Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier, and Wearne, 1996).

Students were doing all of their work at my whiteboards (VNPS) in the class. Unfortunately, I did not have enough whiteboard space at the time, so I had some groups write on their desks with the non-permanent markers and one group write on a mirror that I 'borrowed' from the science lab. Early in this change, I was able to purchase sheets of Plexiglas that I had some students use as their VNPS. This worked fine for a short time until I was able to have more whiteboards installed in my room (see figures 2 and 3 for photograph examples).

FIGURE 2. STUDENTS WORKING AT VNPS IN FIRST WEEK OF IMPLEMENTATION.

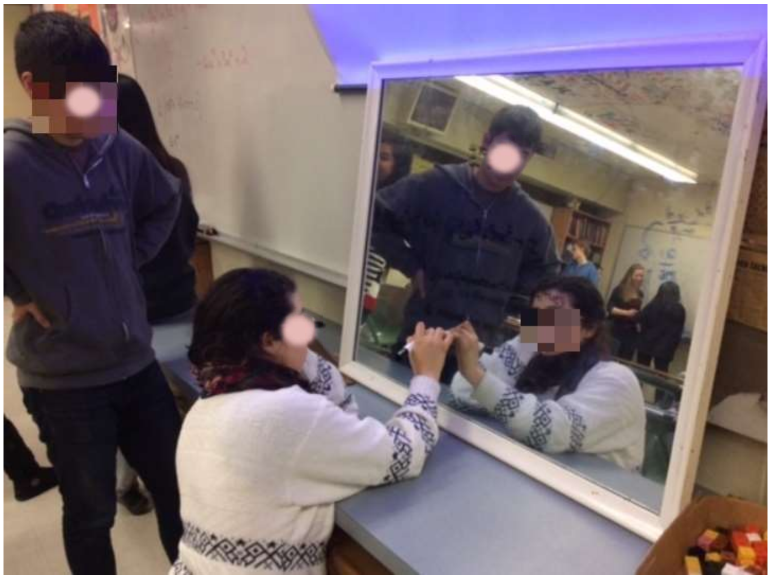


FIGURE 3. STUDENTS IN MY CLASS AFTER THE FIRST FEW WEEKS OF THE IMPLEMENTED CHANGES.

Lastly, I stopped making students take notes during my lessons. The 'no notes' was difficult for some students (and parents) to accept. This was understandable considering my previous beliefs and expectations around note taking. I did however maintain a website that documented our daily class activities and progress. To mitigate these student's concerns about 'no notes,' I offered to print out the website documents for these students. In the beginning, there were a handful of students that expressed this concern; and, after a few weeks of printing out these pseudo-notes, these students simply stopped asking for them.

The results from implementing these three changes were quite startling. I was emotional as I was witnessing whole-class engagement in mathematics, students enjoying their lessons, and a general culture shift in the classroom. After making these initial observations, I was convinced that I was going in the right direction with my changes in practice, but this was only the beginning.

Over the years I have kept steadfastly to the three changes that began the transformation, but I have continually made adjustments and new implementations in order to further improve the learning environment in my class. I have continued to avoid teaching using the “now you try one” strategy, and as a result, the content that I am teaching my students has become problematized (Hiebert et al., 1996). This means that almost all of the curricular content is delivered to my students through a series of tasks that they work on in groups before a whole class debrief and discussion. I have adjusted my assessments, I have changed the sequence in my lessons, and I have started using tasks that are not based on the curriculum of the day, or non-curricular tasks (Liljedahl, 2016), as a part of every lesson. All of these details will be described more in the following chapters, but the point is that my class and my teaching have changed dramatically over the past three years. As a result of all of this change, I now see my students engaged in mathematics tasks that elicit thinking, reasoning and mathematical dialogue for the duration of the entire 80-minute lesson. I am no longer the center of instruction and the source for all mathematics knowledge, rather I am a guide, task designer, and collaborator who works within the community of learners that is my class.

Since then, Liljedahl (2016) has encapsulated his research as Building Thinking Classrooms. In this, he defines a Thinking Classroom as a “classroom that is not only conducive to thinking but also occasions thinking, a space that is inhabited by thinking individuals as well as individuals thinking collectively, learning together, and constructing knowledge and understanding through activity and discussion” (Liljedahl, 2016, p. 364). Although this is how Liljedahl defines a Thinking Classroom, and it is what I am now witnessing in my classroom, most of Liljedahl’s work is devoted to how teachers can *build* these classrooms. Building these classrooms involves changing classroom norms (Liu, & Liljedahl, 2012), providing students with autonomy, and teaching through problems. In order to start teachers down this path of building Thinking Classrooms, Liljedahl suggests that teachers incorporate Vertical Non-Permanent Surfaces, Visibly Random Groups, and rich tasks which is very similar to how I began on my journey. Now, with my transformed mathematics classes, I wonder how Liljedahl’s “studenting” observations might look like in my students. I wonder what are the observable behaviours of the students and the teacher in a class that I find to be vastly improved over my past experiences in a traditional setting.

Even though, I am using Liljedahl’s term “Thinking Classroom,” I am not interested in investigating the act of *thinking* or how my students are *thinking*. To me, it is abundantly clear that my students are engaged in *thinking* like never before.

Liljedahl's research on Thinking Classrooms is focused on what teachers need to do in order to create new norms that are conducive to thinking, and I am interested in what students and teachers are doing in a Thinking Classroom. The topic for my research is *Observations in a Thinking* Classroom, and I wish to discover what is noticeable in a thinking classroom. To begin to answer this question, I need to first review what existing literature has to say around some components that are essential in a Thinking Classroom.

Chapter 2. Related Literature

Thinking – adjective – using thought or rational judgement; intelligent.

noun – the process of using one's mind to consider or reason about something. (Oxford Dictionaries, 2016)

These definitions come from Oxford's online dictionary, and describe what all teachers wish to see amongst their students in a typical classroom. Thinking is a precursor and an agent to learning, and more specifically, thinking is necessary to and a core component in learning mathematics. If students are not thinking about mathematics, then they are not learning mathematics. In my earlier years of teaching mathematics, I became more aware of the lack of 'thinking' that actually was taking place in my classroom. Students were still performing well on assessments, but they were largely reliant on memorized procedures and knowing arbitrary facts rather than fully understanding the reasoning and connectedness behind the mathematics. After implementing Liljedahl's strategies and making further changes to my teaching, I have seen a shift in student behaviour. I have begun to teach in a classroom where students are in a default state of 'thinking' and where students value the act of thinking to create their own understanding and learning.

In what follows, I will review literature on Building Thinking Classrooms for the purpose of aligning my current practice and experience with how the literature suggests for building a Thinking Classroom, and to introduce some terminology associated with Thinking Classrooms. I will review literature on classroom norms, as changing classroom norms is essential to building a Thinking Classroom. I will then review literature on autonomy, as it is an essential ingredient in a Thinking Classroom. I will finish with a review on problematizing the curriculum for the purpose of clarifying how this has evolved into an integral component in my Thinking Classroom.

Thinking Classrooms

Thinking Classrooms are classrooms where there is an expectation of students to create their own understandings, reason through problems, make connections and collaborate. Liljedahl describes a Thinking Classroom as a "classroom that is not only conducive to thinking but also occasions thinking, a space that is inhabited by thinking individuals as well as individuals thinking collectively, learning together, and constructing knowledge and understanding through activity and discussion" (Liljedahl, 2016, p. 364). Liljedahl's work is not focused on the "thinking" of the students, rather it is design-based research on teaching practices that are propitious to thinking. Knowing that having students thinking, collaborating, and engaged in mathematical discourse is a desired result, Liljedahl embarks on describing teaching practices that are conducive to either the building, or maintenance, of a thinking classroom (Liljedahl, 2016).

In his paper, Liljedahl describes the difficulty that teachers have in developing Thinking Classrooms because of existing classroom norms (Yackel, & Rasmussen, 2002). If students already have experience and expectations around thinking and problem solving, then amplifying or developing these attributes is not difficult. If, however, the culture in a classroom is one of direct instruction with no collaboration, then changing these classrooms towards Thinking Classrooms is quite difficult. Liljedahl found that the most effective way to find out how to change classroom culture was by adopting a contrarian approach. If something wasn't working, rather than slightly changing or adjusting the practice, he implemented the opposite approach. It is apparent that this radical approach is required to change a classroom into a Thinking Classroom because traditional (non-thinking) classroom norms are so entrenched in society. The classroom norms that permeate classrooms all around the world are so robust and entrenched that they go beyond the classroom and have become institutional norms (Liu & Liljedahl, 2012). "What the methods here offer is a violent break from these institutional norms, and in so doing, offers students a chance to be learners much more so than students" (Liljedahl & Allan, 2013a, pp. 257-264, 2013b).

Liljedahl outlines nine elements of mathematics teaching that foster, sustain, or impede thinking classrooms. These nine elements permeate most mathematics classrooms, and they are the controls that Liljedahl suggests teachers adjust to bring out student thinking in a classroom. These nine elements become Liljedahl's tools for building Thinking Classrooms.

1. *the type of tasks used, and when and how they are used*

   Classes need to begin with engaging collaborative problem solving tasks. These tasks are used to motivate students to want to talk to each other and promote a thinking culture. Once a Thinking Classroom is developed, then the tasks need to be shifted to be more aligned with the curriculum and "permeate the entirety of the lesson."

2. <u>the way in which tasks are given to students</u>

   Tasks need to be given orally. Sometimes diagrams may be needed, but the instructions need to be presented orally – like a story. This way students engage immediately with discussion rather than trying to decode text from a page.

3. <u>how groups are formed, both in general and when students work on tasks</u>

   Groups need to be visibly randomized on a daily basis. This method is shown to improve the dynamics of a class in the following ways:

   - Increase collaboration amongst all students in a class.
   - Break down social barriers.
   - Improve mobility of knowledge in the classroom.
   - Decrease reliance on teacher and increase reliance on other students for ideas.
   - Improve engagement in classroom tasks.
   - Increase student enthusiasm about mathematics.

4. <u>student work space while they work on tasks</u>

   Students need to work on vertical non-permanent surfaces. This also improves engagement and mobility of knowledge throughout the room. Mobility of knowledge, or porosity, describes how well thoughts and ideas move throughout a room. Traditionally, all mathematical ideas come solely from the teacher. In that traditional model, it can take some time for ideas to transfer to all students, and the mobility of knowledge or porosity is low. When students are working in random groups and on vertical surfaces, mathematical ideas move in three unique ways: students share ideas within a group or from group to group, groups can see mathematical ideas on other vertical surfaces from across the room, and the teacher can still offer mathematical ideas to students. In this system, knowledge moves throughout a room quite efficiently; and therefore, the porosity is considered to be high. Further, by making student work visible, it increases discussion and accountability on behalf of the students.

5. room organization, both in general and when students work on tasks

   Classrooms need to be de-fronted and the furniture arranged so that it promotes movement. Movement is necessary for students to collaborate and discuss with others. When students collaborate and discuss with others, knowledge and understanding is passed, co-constructed, or shared. In this way, when the room is organized to promote student movement, knowledge movement, or porosity, also increases.

6. how questions are answered when students are working on tasks

   Teachers need to refrain from answering questions that stop thinking in students, and only answer questions that promote further or deeper thinking. It is through implementing and continuing these practices that a Thinking Classroom develops and grows.

7. the ways in which hints and extensions are used while students work on tasks

When student's ability is low and the task is too challenging, they will become frustrated. When student's ability is high and the task is too easy, they will become bored. When both these variables match up, then students are in "flow" (Csikszentmihalyi, 1996). Hints and extensions need to be given to keep students in this perfect balance between challenge and ability, to keep students in flow.

8. <u>when and how a teacher levels their classroom during or after tasks</u>

   Levelling is the term that Liljedahl uses to describe how a teacher summarizes or debriefs after a task. He suggests that levelling needs to be done at the bottom. Wherever every group has achieved success, this is where the whole class discussion should focus on. Levelling needs to be a discussion that the whole class has already achieved success on. Alternatively, levelling to the top is when the teacher focuses on the group that has achieved the most in a task, and then tries to bring the rest of the class up to this level.

9. <u>and assessment, both in general and when students work on tasks</u>

   Assessment needs to "honour the activities of a thinking classroom through a focus on the processes of the learning more so than the products, and it needs to include both group work and individual work" (Liljedahl, 2016).

Thinking Classrooms are classrooms where students are expected to create their own understandings, reason through problems, make connections and collaborate; as opposed to classrooms where students are waiting to be told how to do mathematics at every step. These are expectations of students, but more importantly, they become the new classroom norms.

Liljedahl gives an excellent and effective recipe for changing classrooms norms and beginning to create a Thinking Classroom by narrowing the nine practices into three "blunt" elements that any teacher can implement if they wish to create a Thinking Classroom: "visibly random groups, vertical non-permanent surfaces, and beginning lessons with problem solving tasks." Of these three elements, the first two are necessary and highly effective, but they are playing a supportive role for the third element. Problem solving tasks are the quintessential ingredient in any Thinking Classroom. "By constructing a Thinking Classroom, problem solving becomes not only a means, but also an end. A Thinking Classroom is shot through with rich problems" (Liljedahl, 2016, p. 384).

Classroom Norms

Elaborating on Liljedahl's reference to classroom norms, classroom norms are an aspect of the culture of a classroom and "define the classroom participation structure" (McClain & Cobb, 2001, p. 237). In a traditional classroom, examples of these norms may include: students not contributing until called on by the teacher, the teacher being the only source for mathematics explanations, or students being expected to understand in only one way. In a more progressive classroom, examples of these norms may be quite different: students being expected to contribute to whole class discussions, students and teachers both being sources for mathematics explanations, or students and teachers valuing multiple strategies for understanding. When classroom norms become more specific to a mathematics class, they become sociomathematical norms.

Sociomathematical norms are important in the design of a Thinking Classroom, because they guide the class discussion and define the class culture. Yackel & Cobb (1996) describe some sociomathematical norms as “normative understandings of what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant” (p. 461). These norms are established through classroom discussions and support higher-level cognitive activity. When students are expected to describe their thinking around mathematically different understandings, or appreciate how one line of reasoning may be more sophisticated than another, they become “increasingly autonomous members of an inquiry mathematics community” (Yackel & Cobb, 1996, p. 474) and create a culture in the class that is conducive to thinking – a thinking culture.

Autonomy

Implicit in Thinking Classrooms is the high expectation/requirement for student autonomy. Autonomy can be defined as action that is chosen; action for which one is responsible (Deci & Ryan, 1987). Supporting autonomy is the idea that “an individual in a position of authority (e.g., an instructor) takes the other’s (e.g., a student’s) perspective, acknowledges the other’s feelings, and provides the other with pertinent information and opportunities for choice, while minimizing the use of pressures and demands" (Black & Deci, 2000, p. 742). Giving students autonomy in a classroom has been shown to improve student motivation, participation, and completion of work (Lewin, Lippitt, & White, 1939). Other positive results from autonomy include intrinsic motivation (Zuckerman, Porac, Lathin, Smith, & Deci, 1978), preference for challenging work (Harter, 1978), striving for conceptual understanding (Grolnick & Ryan, 1989), a sense of enjoyment and energy (Ryan & Deci, 2000), and self-confidence (Cordova & Lepper, 1996).

Given these positive results, how then should teachers build and maintain such autonomy? Stefanou, Perencevich, DiCintio, & Turner (2004) describe three types of autonomy supportive practices: organizational autonomy, procedural autonomy, and cognitive autonomy. Organizational autonomy includes allowing students to choose groups, due dates for assignments, and evaluation procedures. Procedural autonomy gives students opportunities to choose materials to use in class projects, choose the way competence will be demonstrated, and display their work in an individual manner. Cognitive autonomy allows students to discuss multiple approaches and strategies, find multiple solutions to problems, have ample time for decision making, debate ideas freely, and have less teacher talk time; more teacher listening time. "Although all are important for student motivation and achievement, … cognitive autonomy may be the salient feature of autonomy support as a motivator that leads to deeper involvement in learning and self-motivated scholarship" (Stefanou et al., 2004, p. 105).

Although all of these forms of autonomy promote many of the attributes that are seen in a Thinking Classroom. Educators should know that it is providing cognitive autonomy support that is essential for maximizing motivation and engagement (Stefanou et al., 2004). Students need to have freedom in how they think about their mathematics and in how they solve mathematical problems.

Problematizing the Curriculum

A Thinking Classroom is predicated on an assumption of problematizing the curricular content. Hiebert et al. (1996) describe problematizing as making the curriculum problematic, allowing "students to wonder why things are, to inquire, to search for solutions, and to resolve incongruities" (Hiebert et al., 1996, p. 12). Problematizing the curriculum is allowing students to engage in reflective enquiry as opposed to being told how to do the mathematics. This aligns nicely with a Thinking Classroom because teaching by problematizing the curriculum involves a teacher guided inquiry through posing open tasks, asking questions that bring out student reasoning, and encouraging discussion; so that, students will learn how to construct their knowledge by working through the problem (Cohen, 1988) and by collaborating with one another (Leinhardt, 1992).

Hiebert et al. (1996) bring together the ideas of problematizing the curriculum and problem solving in mathematics in their article, *Problem Solving as a Basis for Reform in Curriculum and Instruction: The Case of Mathematics*. They argue that problematizing the subject is the activity that most likely leads to the construction of understanding. This is very similar to Liljedahl stating that starting with a good problem solving task is essential to developing Thinking Classrooms (Liljedahl, 2016). Hiebert et al. draw a distinction between classical understandings of problem solving in mathematics and their view of problematizing the curriculum. Hiebert et al. see problem solving as treating all mathematics as problematic and contrasts this to three historic views of problem solving in mathematics: Problem solving makes mathematics useful, Problem solving engages students, and Problem solving is what mathematicians do.

Problem solving makes mathematics useful. Hiebert et al. believe that "real-life problems provide a legitimate context for problematizing mathematics. If students are engaged in solving a reflective activity, then the concern about an overemphasis on skills disappears. [His] concern rests with the narrowness of the approach. Real-life or everyday problems are one context, but only one context, for reflective inquiry" (Hiebert et al., 1996, p. 18).

Problem solving engages students. A common thought is that it is special tasks and interesting problems that engage students. Hiebert et al. disagree, and argue that if the culture in the class is developed so that students "problematize what they study, to define problems that elicit their curiosities and sense making skills" (p. 12), then all tasks can be engaging to students. Hiebert et al. believe that the reason for giving a task is not the task itself; rather it is to bring out the prior knowledge of the student and the conditions under which the task is completed (Hatano, 1988).

Problem solving is what mathematicians do. The thought here is that children might learn much like mathematicians and is in line with cognitive apprenticeship

(Brown, Collins, & Duguid, 1989) where learning is embedded in activity, “students engage a variety of problem situations, and artificial distinctions between acquiring knowledge and applying it are eliminated” (Hiebert et al., 1996, p. 19). Hiebert et al. agree with much of this perspective but caution about treating children too much like adults. The similarities between the mathematics that children work on and that of adults lies in their goal of understanding within a solution that makes sense.

Contrary to these three historic views of problem solving, Hiebert et al. suggest that all content can and should be problematized regardless of real world application, intrinsic student interest, or thinking like a mathematician. These three views are irrelevant. The important questions are “(1) has the student made the problem his or her own, and (2) what kind of residue is likely to remain” (Hiebert et al., 1996, p. 19). Hiebert et al. use the term “residue” to describe understanding that remains after the problem is resolved.

Hiebert et al. (1996) addresse problematizing learning in mathematics as one avenue for developing a problem solving culture within a class so that ideas are discovered collaboratively amongst students and guided by the teacher through open discussion. Hiebert et al., and later Liljedahl, recognize that this requires full scale change in teaching practice:

> Teaching mathematics as problematic requires changing the entire system of instruction. It is not achieved by injecting interesting problems into a curriculum that retains a distinction between acquisition and application. It is not achieved by

> adding problem solving into the mix of ongoing classroom activities. Rather, it is achieved by viewing the goal of instruction and the subject from a very different perspective… The culture of classrooms will need to change. (Hiebert et al., 1996, p. 19)

Hiebert et al. do not attempt to describe practical application of this 'change,' as their article is devoted to describing and evaluating a different way of envisioning problem solving. They provide an interesting complement to Liljedahl's Thinking Classrooms by suggesting that with the right classroom culture, all curricula can be problematized into rich tasks for students to engage in mathematics thinking and reasoning.

Summary

The motivation for this study is the need to better understand what student behaviour and teacher behaviour is observed in a Thinking Classroom. Yackel & Cobb (1996) suggest that a thinking culture can be cultivated by focusing on sociomathematical norms. These can be fostered by encouraging discussion around what is mathematically different, sophisticated, efficient, and elegant in student solutions. By making these the classroom norms, the culture is shifted to one where students have more cognitive autonomy. Stafanou (2004) agrees and adds that supporting cognitive autonomy is essential for maximizing motivation and engagement in students. Hiebert et al. (1996) proposes that the tasks given to students in a problematized learning environment do not necessarily need to be interesting or connected to real-life if a problem solving culture is developed in a class. When this culture is developed, then the curriculum can be problematized, and seemingly trivial tasks can become rich learning activities through open, collaborative discussion. Liljedahl's notion of a Thinking Classroom account for each of these. Liljedahl (2016) gives strategies for implementing a Thinking Classroom: visibly random groups, vertical non-permanent surfaces, and rich tasks among others. These strategies are intended to side-step the prohibitive existing classroom norms and introduce new norms that support thinking and a positive collaborative culture.

Liljedahl has defined the elements that are necessary to *build* a Thinking Classroom; his work focusses on what teachers *need to do* in order to create a Thinking Classroom. Having implemented all of these elements, I am now interested in what students are doing in such spaces, and what the resultant role of the teacher becomes once this space is created. In particular, I want to know *what student behaviour and what teacher behaviour is observed in a Thinking Classroom?* How does the physical layout of the classroom and the procedures of the lesson affect the porosity of the classroom? How effectively do students engage in mathematics when given cognitive and procedural autonomy? What is noticeably different in a Thinking Classroom?

Chapter 3. Methodology

The setting for this research was three sections of Pre-Calculus 11 classes and 1 section of AP Calculus AB that I taught in the spring term of 2015 at Slopeside Secondary School in North Vancouver, BC. These are senior academic mathematics classes with students aged 15 to 17 years old, and the school is located in a higher socioeconomic community. To describe the methodology behind this research, I will describe my classroom procedures and expectations, the classroom layout, and the setting and participants. I will then describe how I decided on collecting data and the lens through which I analyzed the data.

Classroom Procedures and Expectations

When describing my classroom procedures and expectations, I will first describe how an actual class runs, then I will describe how I promote a thinking culture in my class, and lastly I will talk about how assessment is addressed in my class.

When students arrive to class each day, their groups are randomly generated using an iPad application and projected onto a screen. The students are informed of where and with whom they are sitting by looking at the projected seating plan, they place their backpacks and jackets on the blue table at the side of the room, and then they make their way to their table. While students are sitting at their tables in their groups, I wander through the room welcoming each group to class and distributing a single whiteboard pen to each group. After all students are greeted and attendance is recorded, I read a First Peoples of Canada acknowledgement that recognizes and extends gratitude to our aboriginal community, for the class is taking place on First People's territory. I then begin class by presenting a non-curricular mathematics task usually in story form. Groups then move to their corresponding places at the whiteboards to discuss and work towards a solution for the task. This opening task can take anywhere from 10 to 30 minutes to complete. I debrief all tasks either by gathering the class around one group's board for a discussion, or by projecting a photo of a group's work for a discussion at their tables. This debrief includes discussion not only on solution strategies but also on extensions. In doing this, I am modelling a mathematician's disposition of inquiry and "what next." I will often say something like "now that we have figured this one out, what questions could we ask or how could we change the problem?"

After the opening task, the lesson continues with two or three tasks from the specific content of the current unit of study. It is important to point out here that each task is presented in a somewhat 'backwards' fashion. Traditionally, a mathematics teacher teaches by demonstrating a concept or a procedure, and then letting the students try it on their own – this was referred to earlier as the "now you try one" instructional strategy. In my class, each task is presented as an open problem with little to no teaching in advance – this is modelled after Hiebert et al.'s problematizing the curriculum (Hiebert et al., 1996). Students spend their time collaborating, struggling, and making their own sense of the mathematics before the whole class participates in a debrief. The debrief finishes each task and is usually teacher lead. During the debrief, I always begin the discussion from a specific group's work, sometimes asking for the student's to explain their thinking at different steps. Frequently, errors or mistakes are found and turned into learning moments for everyone.

I consider this somewhat 'backwards' because the teaching is happening at the end of the task and not the beginning. At the end of the eighty-minute lesson, there is a short summary of the big ideas for the lesson and page numbers for students to reference similar content in their textbooks are given. Homework is not specifically assigned; however, the importance of visiting new concepts outside of class is often discussed.

There are three other things I do to the way I run my classes that promote a thinking culture. I spend the first couple of weeks every year (about 5 lessons) just working on non-curricular mathematics tasks. After this introductory culture building period, I start every lesson with a non-curricular mathematics task to maintain this thinking culture. And lastly, I always model learning through understanding and sense-making both in small group and whole class discussions.

I start each year with non-curricular mathematics tasks, because these tasks can be more interesting for students, and students with weaker backgrounds in mathematics content are still able to achieve success and recognition, improving their self-confidence. I do this to build the thinking culture in my class. I use these non-threatening non-curricular problems for my students to build their communication and collaboration skills and to set my standards of expectation for the rest of the year. I use these tasks to talk about problem solving strategies, good group work, multiple solution strategies, communication, and mathematical reasoning. I think it is important to discuss these mathematical processes within tasks where students are finding success and enjoyment; and this is more likely to occur in tasks that are not related to curricular content.

This is also why I start every class with these types of tasks. I think students need to experience mathematics as an enjoyable and social activity, and I want all ability levels of students to be able to experience this in mathematics. Starting each class with a non-curricular task builds confidence and improves student affect towards mathematics, and promotes a thinking culture in my class.

I also model 'thinking' in every interaction that I have with my students. I'll ask questions like, "Why did you write this?", "I don't know if it is correct, can you explain what you are thinking?", and "Can anyone help me to understand this?". A thinking culture takes time to develop, but it also requires regular attention through all of the multitude of different interactions that I have with my students over the course of a year.

Assessment is another characteristic that helps to describe my classroom. Assessment is not only designed to help students learn, but it is also designed to promote a thinking culture in my class. Apart from the daily informal assessment that happens during the course of a class through small group and whole class discussions, I have numerous formal assessments. With the exception of four summative assessments in the form of unit tests, all of the other formal assessments are formative in design. All of my quizzes, which happen every couple of weeks, are partner quizzes. This catalyzes group discussion and sense making around the mathematics content. I assess students on their communication and students complete self-assessments and peer assessments on their own communication, collaboration and class participation three times over the course of a year. Students also produce a problem solving project each term where they get to showcase the solution to a favourite non-curricular problem that was worked on earlier in the year. As part of this project, they need to extend the problem and discuss their problem solving strategies and mathematical reasoning that helped them through to the solutions. The four summative unit tests are also designed to be somewhat formative. If students are unsatisfied with their result on a test, they can re-write the test after meeting with me to discuss where they have improved in their understanding. Students know what I value by how and what I evaluate. By evaluating my students on their collaboration, communication, problem solving, and curricular knowledge, I am making clear to my students that I value all of these aspects in my classroom; hence, I am promoting the thinking culture within my classroom (Liljedahl, 2016).

Classroom Layout

When I first began implementing Liljedahl's strategies and reshaping the norms in my class, I soon noticed that there were aspects to my existing class layout and structure that supported the old norms of the class and prohibited new Thinking Classroom norms from developing. The limited whiteboard space was the first aspect that I needed to address, but I also noticed that my classroom desks were prohibiting mobility in the class. In a Thinking Classroom, I knew that the movement of knowledge (porosity) is essential (Liljedahl, 2016). This porosity is aided by the public nature of student's work on the VNPS, but it is also hampered by classroom furniture. In my room the desks made it practically impossible for students (or teachers) to move quickly from one side of the room to the other. After a few months of implementing the new teaching strategies, I realized that I needed to change the furniture in my room.

I am grateful for the support of my administration, as they agreed to purchase 10 bistro-style tables with accompanying tall stools for my classroom. The classroom itself is average in size, by high school standards, with one door on the North wall, and the teacher's desk and computer station on the opposing wall beneath the only window in the room. There are no longer any student desks in this classroom. Instead, I have 10 tall circular bistro-style tables. Each table is about 32 inches in diameter and has 3 tall stools surrounding it (see figure 4). There is also one digital projector that is wirelessly connected to the teacher's tablet computer. This projector projects onto a screen located next to the teacher's desk.

FIGURE 4. A PHOTOGRAPH OF MY CLASSROOM WITH THE BISTRO TABLES AND STOOLS. GOPR1980

All four walls contain whiteboards for the students to work at and there is a small built in table extending halfway along the East wall. This table is where most students leave their books and bags when they enter the room.

The round tables were selected intentionally for both their height and their size. I wanted tall tables so that students could both sit and stand depending on how they chose. The tables were also selected for their surface-size. They are quite small with only 32 inches in diameter intended for three students at a time. I wanted small diameter tables for two reasons. First, I wanted to encourage the table space as a space for collaboration and not for independent work. The small tables accomplish this because it is not possible for all three students to have their books open at the same time on the limited space. The other reason why I chose small-diameter tables was that I wanted to maximize the space in my room for student movement and increased porosity. As stated above, when my room was filled with student desks, it was practically impossible for students to move unimpeded from one side of the room to the other, because there simply was not a free path. With the smaller tables, the room is now full of potential pathways for students to move and share ideas around the room. The stools that surround each table are small for the same reason, but they are also tall. Like the tables, I wanted to provide seating for students that is very similar to their standing positions. With tall tables and stools, the physical exchange from working and standing at the white boards and working and collaborating around the table is minimized. Students are able to move back and forth from working at the boards and working around the tables with minimum disruption. This way, there is very little transition time between the two classroom postures (standing and sitting) and therefore this minimizes distraction during class time.

Such a set up can be referred to as a standing-biased classroom (Dornhecker, Blake, Benden, Zhao, & Wendel, 2015). In this study, Dornhecker et al studied standing biased classrooms and their effect on academic engagement. Knowing that standing biased desks increase energy expenditure during class, the focus behind the research was on increasing mobility in students to reduce obesity. Obesity aside, "the findings indicate that students provided with stand-biased desks did not decrease in their academic engagement … when compared with their seated counterparts" (Dornhecker et al., 2015, p. 7).

Donnelly, Greene, Gibson, Smith, Washburn, Sullivan, DuBose et al (2009) investigated the relationship between increased physical activity, body mass index (BMI) and academic achievement in elementary aged students over a three-year period. This study found significant improvements to academic achievement among students who increased their activity within class (see figure 5)

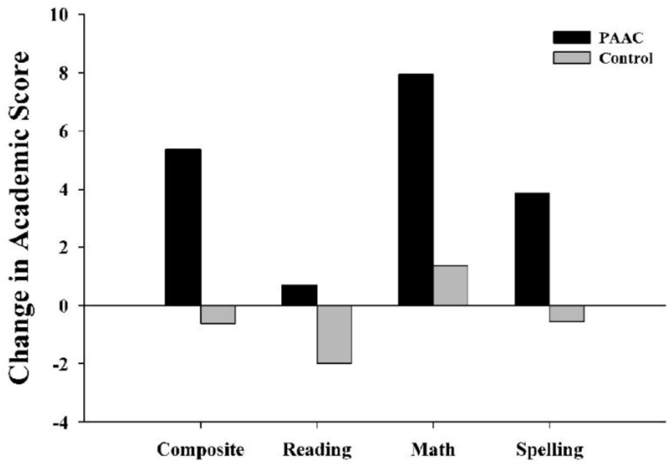


FIGURE 5

SHOWING CHANGE IN ACADEMIC ACHIEVEMENT FOR STUDENTS PARTICIPATING IN PHYSICAL ACTIVITY ACROSS THE CURRICULUM (PAAC), WHICH "PROMOTES 90 MIN/WK OF MODERATE TO VIGOROUS INTENSITY PHYSICALLY ACTIVE ACADEMIC LESSONS DELIVERED BY CLASSROOM TEACHERS" (DONNELLY ET AL, 2009)

These studies, as well as my personal experience, support the use of standing-biased tables such as my bistro tables and stools for a number of reasons. The tables increase the floor space in my room to encourage student and teacher movement and increase the porosity of the class. The standing-biased nature of the stools coupled with the lesson's activities increase the physical activity of the students and this has been shown to increase academic achievement (Donnelly et al, 2009). In addition, I have found that this layout contributes to an environment that feels more casual and comfortable for students. In this environment, I have found my students to be more willing to share stories and participate in groups discussions both around mathematics and extracurricular topics of interest.

Setting and Participants

The participants of the research study were students enrolled in the third term of Pre-Calculus 11 and AP Calculus AB at Slopeside Secondary. Pre-Calculus 11 is a course designed for students intending on taking programs of study at post-secondary institutions that require calculus. To this end, students in this course tend to be academically minded and focused on their studies. AP Calculus AB is equivalent to the first term of calculus offered at most post-secondary institutions. In fact, if students perform well on the internationally written AP Calculus exam, they can achieve actual university credit for the first term of calculus. For this reason, students in this class tend to be very focused and high achieving in their academic subjects.

Data were collected from three different sections of the Pre-Calculus 11 course and one section of AP Calculus. The class sizes for the Pre-Calculus 11 classes were 28, 29, and 29, and the class size for the AP Calculus class was 28. Students at Slopeside, and in these classes, come from a variety of ethnic backgrounds, and they tend to belong to families from higher than average socioeconomic status.

Video Evidence

To address the research question, I needed to collect data that showed student and teacher behaviour in a Thinking Classroom. My options were student interviews, third party teacher observations, and video recordings of lessons. I decided on the latter for two reasons. First, the collection of data (video recordings) is fairly unobtrusive to the teaching and learning environment. I have had many teachers visit my classroom over the past few years to view and learn from my teaching practices, and every time a stranger is in my room, the class behaves slightly differently. The second reason that I chose to use video recording for data collection was for the ease of collection and the long term availability of its analysis. I wasn't exactly sure of what behaviours I was looking for, and video evidence provided me the opportunity of re-watching the same segment of video multiple times to document various different student behaviours.

In order to collect video evidence in my classroom, I used a GoPro camera with a fisheye lens, and I tried a variety of different positions within my room. Initially, I placed the camera on my desk. This was convenient for turning the camera on and off; however, it was not close enough to any one group to capture conversations and details of board work. This position was also problematic, because students standing in front of the camera would block the views of the rest of the room.

I then moved the camera off to the side of the room nearer to one of the boards. Although this position improved the capture of student's work and conversation, it still posed a problem with students blocking views of the room. In the end, I decided on placing the camera in the Northwest corner on the top of a bookshelf. From this position, I was able to achieve an almost birds-eye view of the whole room. The camera captures activity and student behaviour throughout the class, but it is best at capturing activity on the near North and West walls. In these locations, the camera does a decent job of capturing audio for the near-group discussions and can capture some detail of the board work for the West wall. The camera records high definition video on to an SD memory card in 17.5 minute segments. So, for any given class, there are 5 different video files for capturing the 80-minute class.

The file naming format for the videos is set by the camera and appears as GOPR3141 and GP013141 for example. The files that start with "GOPR" are always the first 17.5 minutes of a recording session and the "3141" is the coded number for that recording session. The digits themselves identify a video segment within a larger recording session. The subsequent files within the same recording session would be named GP013141, GP023141, GP033141, and so on. In the results section of this paper, I have included various screen clippings from the video evidence. Each screen clipping has a caption that indicates which file the screen clip is taken from as well as a time stamp to indicate the location within said file.

The video recording took place over 6 days. During which I recorded one Pre-calculus 11 and the AP Calculus classes on each day; these represented 12 separate samples of class videos. My other classes this school year were Mathematics 9's, and I chose not to record these for two reasons. First, I wanted to have consistency in the age groups for my observations so that age and maturity wouldn't factor into the discussion. Second, although my two blocks of mathematics 9's were still working under a Thinking Classroom model; when considering the age for these students, I found that I was spending more time in these classes on management issues. Classroom management is more of an issue with junior mathematics classes than with senior classes, and I did not want this to cloud my observations of a Thinking Classroom.

Analysis

To analyze the video data and make observations on student and teacher behaviour in a Thinking Classroom, I used my experience as a mathematics teacher and made notes of behaviours and actions that I noticed. Noticing is a term used for the activity of observing and recognizing something. As a mathematics teacher with plenty of classroom experience, and as a professional in my field, I notice different things in a typical classroom than an ordinary person. Teacher noticing is the act of attending to and making sense of various events in a classroom, and thus becomes the lens through which the data for this research are analyzed. This process took some time and was not linear in nature. On many occasions, I would notice a behaviour after watching several videos, and then I would go back and re-view the earlier video to see if this behaviour occurred earlier and was not noticed.

Teacher noticing is a theoretical construct for research in mathematics education and teacher education. Sherin, Jacobs, & Philipp (2011) describe teacher noticing as encompassing the "processes through which teachers manage the 'blooming, buzzing confusion of sensory data' with which they are faced, that is, the ongoing information with which they are presented during instruction" (p. 5). They further describes two components to teacher noticing as attending to events and making sense of the events. "Teachers select and ignore on the basis of their sense-making; the way they respond shapes subsequent instructional events, resulting in a new and varied set of experiences from which teachers attend and make sense" (Sherin et al., 2011, p. 5). Schoenfeld (2011) believes that noticing is important because it can lead to change in teacher practice and because it is "intimately tied to [teacher] orientations (including beliefs) and resources (including knowledge)" (p. 231). In the context of this research, noticing is important because it will illuminate student and teacher behaviours in a Thinking Classroom. Through my eyes as an experienced teacher, my analysis of the video data seeks to categorize observations in a Thinking Classroom.

Using the theoretical construct of teacher noticing, I watched 16 hours (12 classes of 80 minutes each) of my classroom video recordings. I watched the entire video collection two times in sequence from first to last. In doing this, I noticed interesting student or teacher behaviour, I recorded the behaviour, the video file, and the time stamp for the location within the video. As my observations progressed, I became aware of repeated behaviours, and this led to some early categorizations. After categorizing behaviours, I would then go back through the video evidence and try to find other examples of those behaviours. Sometimes, as I was trying to make sense of a specific student behaviour, this would lead me to noticing something completely different and surprising. In what follows, I will describe and categorize the behaviour of both students and the teacher in a Thinking Classroom.

Chapter 4. Results

After analysis of the video data, I noticed eight observations of student and teacher behaviour that characterize this Thinking Classroom. The majority of these observations are student behaviour based, and I will discuss these first. I will also begin with the most apparent and most frequent observations and then finish with observations that were less obvious but perhaps equally profound. I noticed how students moved throughout the classroom. I separated this observation into two categories. Students moved as they worked within their groups (intra-group) and as groups worked with other groups (inter-group) in a Thinking Classroom. The fact that this was observable was not surprising to me, in fact, it was expected. However, I was surprised by the extent of this group work and the utter reliance that students placed on others for their learning. The other category for student movement looked at what students did when they needed help. The observations that I made of student behaviour when help is needed is strikingly different than what I would have observed in my earlier more traditional classroom. I also have a category for noticing how students make sense of and reify the mathematics that they are learning. This required more subtle observation where I needed to watch student actions, and listen carefully to student dialogue and group discussions. From my experience as a teacher in this room, when students are talking to others about a solution or people are coming to them and talking about their own solutions, students are in a process of sense-making. So, in observing students talking with one another around a mathematics task and how they gestured to their board work and to one another, I was able to conclude that students were making sense of the mathematics.

I could not help but notice the deviant behaviour of my students; however, I quickly learned that this behaviour was not all deviant. As a teacher with classroom experience in a generation where smart phones and technology infiltrated and in some cases took over the classroom, I cannot help but notice when students are using their smart phones. I have an instinctive reaction to seeing students on their phones that makes me want to grab the phone and throw it out the window. I found it really interesting in this video data when I was able to look more closely at what my students were doing with their phones. Sometimes, deviant behaviour was very short in duration and had no real consequence on the learning; other times, deviant behaviour was not really deviant.

Some other results are focused on observations that I made of my own actions and the general flow of the lesson. I found it interesting to see exactly what I did and what I attended to during a lesson, how I moved through the class, and where did I spend most of my time. I was also interested to see how much time I spent on tasks. Ever since I began this newer teaching approach, I have been aware of the fact that I cover fewer tasks in an eighty-minute lesson, but I was not able to quantify it. In my analysis, I determined just how much time each task was taking, and the result is surprising. I will also comment on what I noticed with respect to the general behaviour of the students in the class. In this Thinking Classroom, I give students plenty of cognitive autonomy, the freedom to act and think under their own direction and motivation. I will describe what this looks like in the video data. Lastly, I will discuss the problems or issues that I observed. I mentioned one already with noticing poor student behaviour around cell phones, but there are others. So I will finish with these issues that I noticed and their possible impact on the Thinking Classroom.

Student Movement

One thing made very clear when viewing the video data in a Thinking Classroom is that the students are always moving. This may be due to the fact that they are already standing at the whiteboards, and movement is more likely when there is no hindrance such as moving from a seated position at a desk. However, the students also appear to have good reasons for their movements. They may be seeking help from other students or sharing knowledge with other groups. Even within groups, students are moving to the whiteboards, and then passing off their pen and moving away from whiteboards. Students are always moving both for learning and also for social reasons. I have split the results in this section looking at how students are moving in their Group Work and then looking at how students are moving when they specifically require assistance. In the first section, the focus on group work is with how students and groups share mathematics ideas over the course of a task. Even when students have completed a task, they are still moving throughout the room and collaborating with others. In the second section, I focus on the observations I made when students required assistance.

Inter and Intra-group work

Group work and group communication includes student's getting help from other students, but it is much more. Group work can also include the sharing of mathematical ideas, discoveries, and observations. Within my data, there are two types of group work:

- Intra-group work is the collaboration and discussion that happens within a group – this is clear in every part of the video evidence.
- Inter-group is collaboration from group to group and is when the mathematics discussion goes outside of an individual group.

The latter is seen at various times in the video in two different forms: Sometimes, a single student goes to another group and discusses the mathematical ideas before coming back and sharing with his/her group. Other times, individual students from three or more groups will come together and form "super groups."

I have one section of video data from a calculus class where inter-group work is so prevalent it becomes difficult to distinguish what the original student groupings are. To try and show the student movement in pictures, I used a series of screenshots for each of the movers that I focused on. Each series of screenshots will have one student highlighted with a red and yellow circle to illustrate his position in the classroom. Before I begin the analysis of this class, it is important to see the initial grouping of the students. These groupings change so much over the course of the video, even I had difficulty knowing which student started in which group (see figure 5).

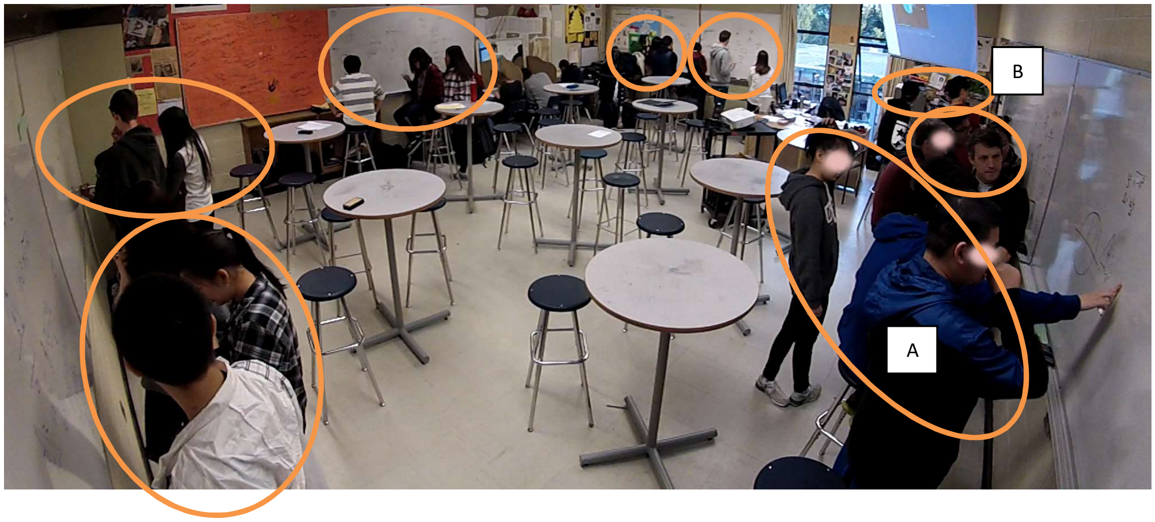
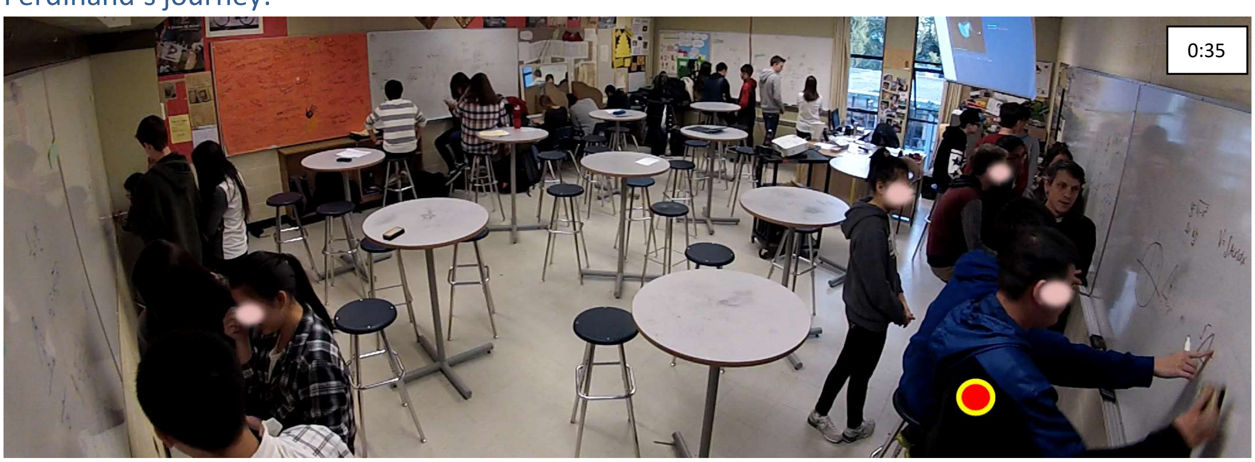


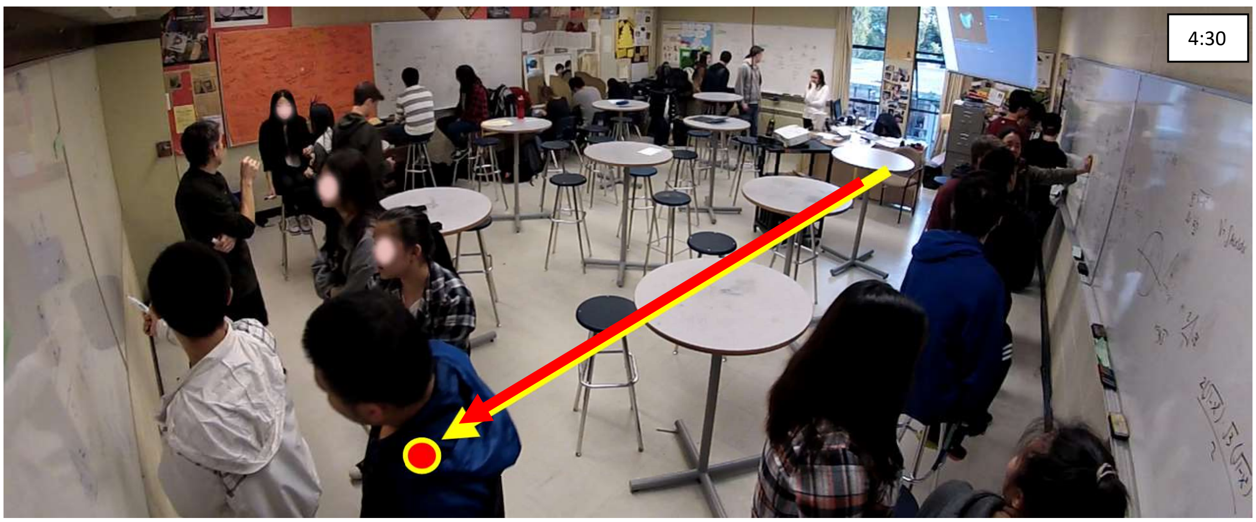
FIGURE 5. THE ORIGINAL GROUP CONFIGURATION BEFORE THE INTER-GROUP COLLABORATION BEGAN. GP011548

The original condition for this task has the class working in eight different groups on an applications of integration curricular task (see appendix A). Seven of these groups had three students and one group had just two students. In this segment of video, most of these groups move and change over seventeen minutes, but I will focus on the journeys of three different people: Ferdinand and Charles from group A and Devin from group B.

Ferdinand's journey:




3:50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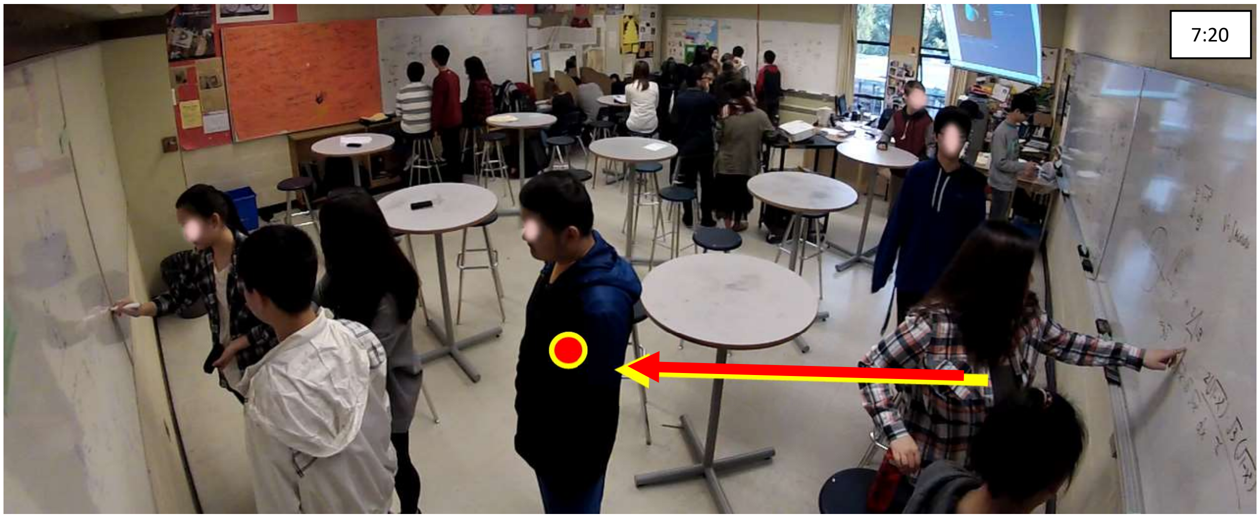
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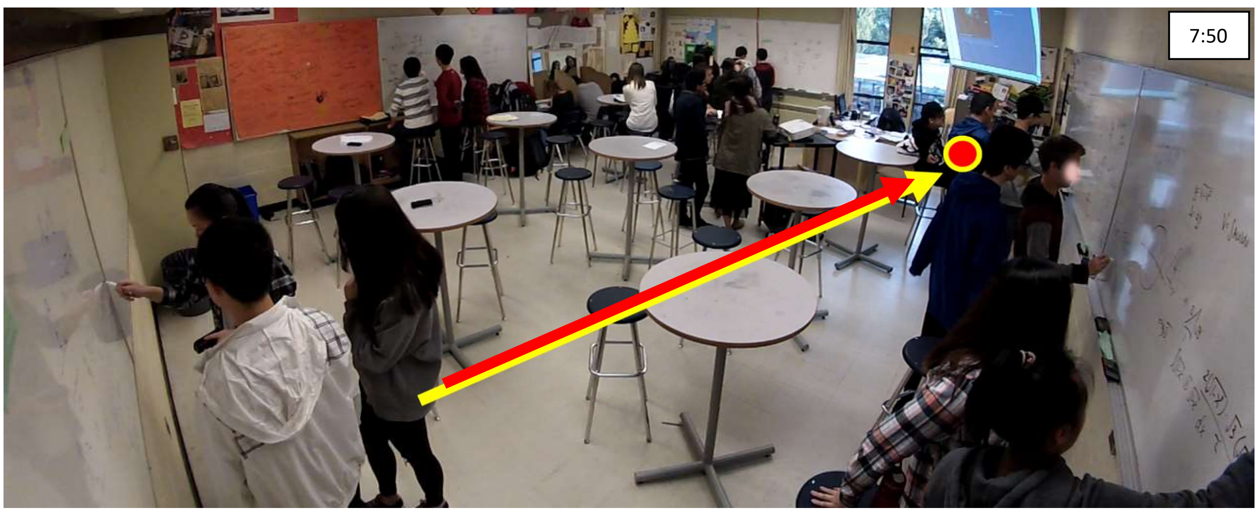
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FIGURE SERIES 6. THE RED DOT INDICATES FERDINAND'S TRAVELS OVER 11 MINUTES OF WORKING ON THE TASK. THE TIME STAMPS ARE IN THE UPPER RIGHT CORNER OF EACH PHOTOGRAPH. GP011548

Ferdinand is in a three-person group with Charles and Nelley (see figure series 6), and this video clip begins with myself in a discussion in Ferdinand's group apparently clarifying the task. I leave their group at the 2:30 mark, and they don't appear to make much progress. There is a small amount of writing, and a small amount of discussion, but not much progress. Ferdinand is the first to leave. He does this at the 3:50 mark. Ferdinand is quiet and not a very social student, so when he visits other groups, he tends to stand behind them and watch and listen to their work and discussions. The first group that he visits is the group to his left. At 4:30, he moves again; this time, he moves to the group at the bottom left of the screenshot and listens to a conversation that this group is having with the teacher. Then at 5:00, he re-joins his group, who haven't yet begun any journey of their own. They seem to make a small step of progress. This is evident because there is some writing, gesturing and some positive comments. Charles processes some of the work that Ferdinand brings in from his other group visits, and Charles acknowledges it saying "Ohhhh Ferdinand!" Charles then leaves the group for the first time (described below) and Ferdinand leaves at the 7:20 mark to observe the group at the bottom left of the photo again. At 7:50, Ferdinand moves again to discuss some progress in Devin's group at the right, top of the photo. It is not clear here how much productive discussion is taking place. The view from the camera makes it difficult to see if this is a productive part of his journey or if it is just a break for Ferdinand, but he returns to his group at the 9:45 mark. At this time, he and Charles make some real progress on the problem. Nelley is just watching for much of this, but when Charles leaves, Nelley and Ferdinand have a good discussion and seem to make sense of the problem.

Charles's Journey – a Super-Group forms:


9:00


10:00

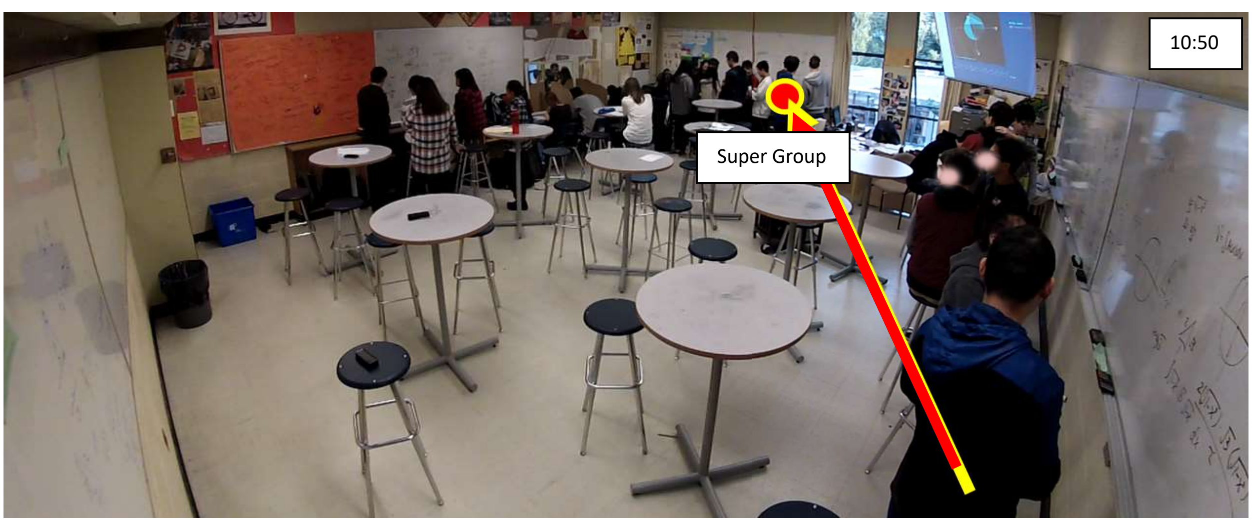

10:50
Super Group

FIGURE SERIES 7. THE RED DOT INDICATES CHARLES'S TRAVELS OVER 12 MINUTES OF WORKING ON THE TASK. THE TIME STAMPS ARE IN THE UPPER RIGHT CORNER OF EACH PHOTOGRAPH. GP011548

Charles begins his journey in a three-person group with Ferdinand and Nelley (see figure series 7). At the beginning, I am in a conversation with his group; and although Ferdinand leaves this group early for assistance, Charles and Nelley keep working on the problem for some time sharing ideas with the group to their left. After six and a half minutes (6:35), Charles begins moving around the room looking for assistance. He goes to Quentin's group at the far end of the room. Quentin is well known in this class as someone who understands mathematics deeply. Charles stays over with Quentin for a minute in a discussion where you can see both Charles and Quentin gesturing to work completed on Quentin's board. At 7:30 in the video, not only can you visibly see Charles becoming excited, you can even hear him exclaim, "Oh boy, you did it! He did it. It is correct." When he comes back to his board with a big smile on his face, he actually joins with his neighbouring group – this might be because Ferdinand is on his own walk and is not present at his board yet. At this point, Charles is quite energetic and excited about this task and his new found understanding. With this new found confidence, Charles now begins sharing his ideas with others. He is already working with the group to his left, and at the 9:00 mark, he moves down to his original group and shares some ideas with Nelley and a third person from another group. He is clearly sharing some mathematical insight here, as we can hear in his description:

"You do this on the interval 0 – 1 and you can multiply it by two and get the whole thing. It's x squared plus y squared equals one, right, so the radius is one, so instead of like doing the area twice for the negative… it's symmetrical, remember?"

I am noticing this as mathematical insight, because this is a middle step in setting up the integral for this task (appendix A). At the 10:00 mark, his partner, Ferdinand, catches up with him, and Ferdinand and Charles appear to make some good progress on the problem. The pair, Charles and Ferdinand, and the new pairing to their right, Nelley and the other girl, have completely changed positions and they are each discussing the other pairing's work. At this point, we can hear Charles in discussion with Ferdinand about the final details of the solution to the task. We can hear in the video Charles saying:

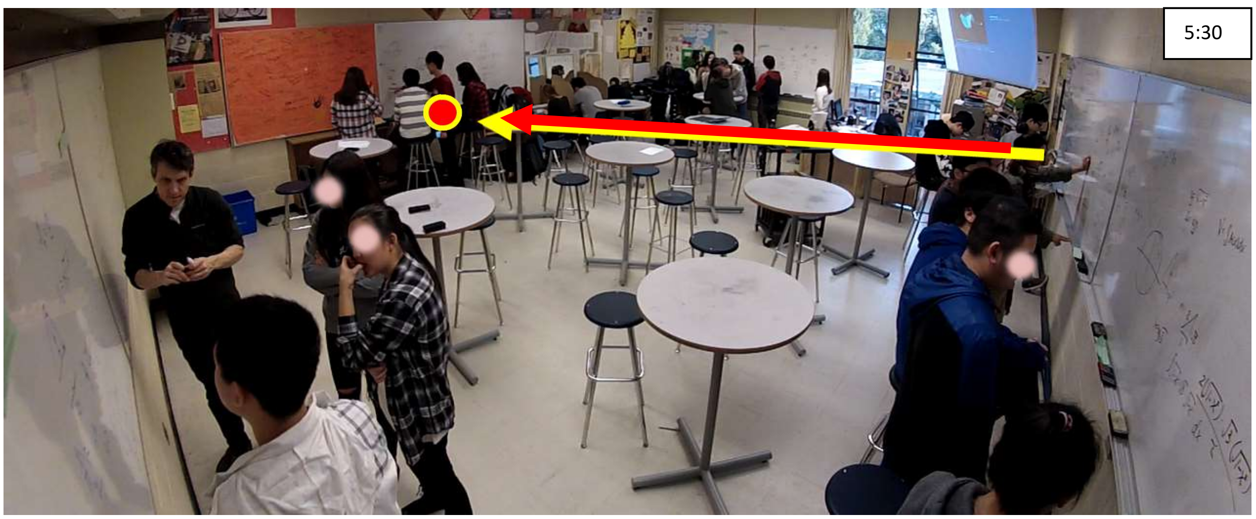
"2 root 3 times 2 over 3… this is the correct answer."

This is indeed the correct answer to the task in Appendix A, so it is clear that Ferdinand and Charles have reached understanding on the task; however, Charles's journey is not yet complete. He now (10:50) goes back to where he first found his inspiration – back to Quentin's group. When he moves to Quentin's group, there is another person working with Quentin. This other person is from a completely different grouping in the class. So, at this time, there is a group with 3 different group members – I call this a “super-group.” It is clear that they are still discussing the task because you can see all three gesturing to the work on the board. At this point, even though it is apparent that Charles has already achieved understanding on this task, he may just be checking in to see that he is still on the right track. Lastly, at the 11:45 mark, he returns to his neighbour group and discusses more of their work with his new understanding. It appears in the video that he actually corrects their work and helps them to make sense of the task.

I really like this example of Charles’s journey, because it not only shows how inter-group work helps students build their understandings, but it also shows what groups do once they achieve understanding. Halfway through Charles’s journey on this task, he and one of his partners understand how to do the problem; but, they don’t stop here. For the remaining 5 minutes (8:00 – 12:30), Charles is moving to other groups to both check his understanding and share his understanding.

Devin's Journey:


4:10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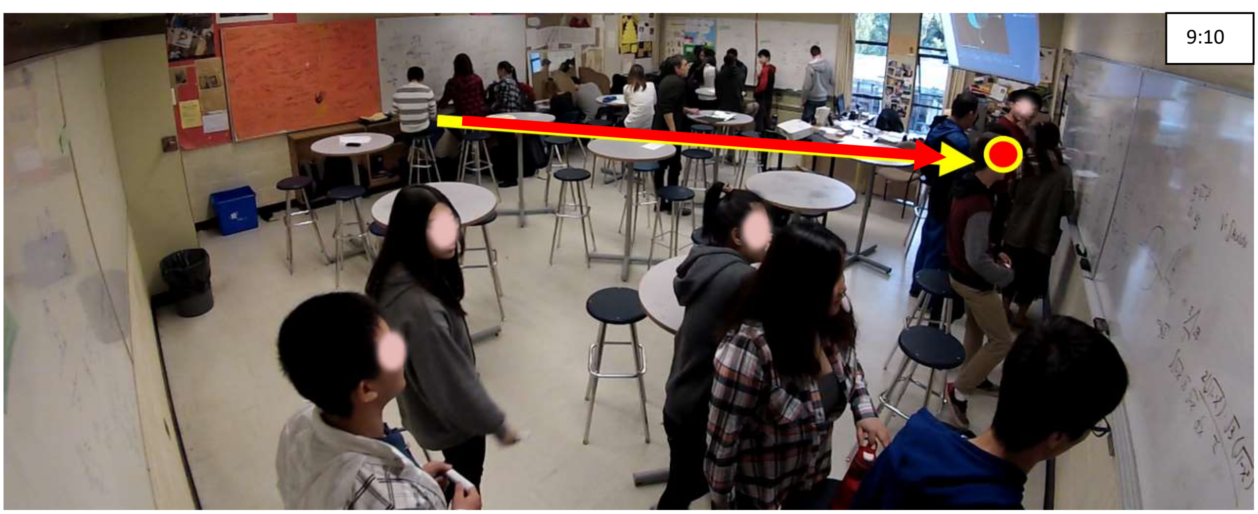
9:10

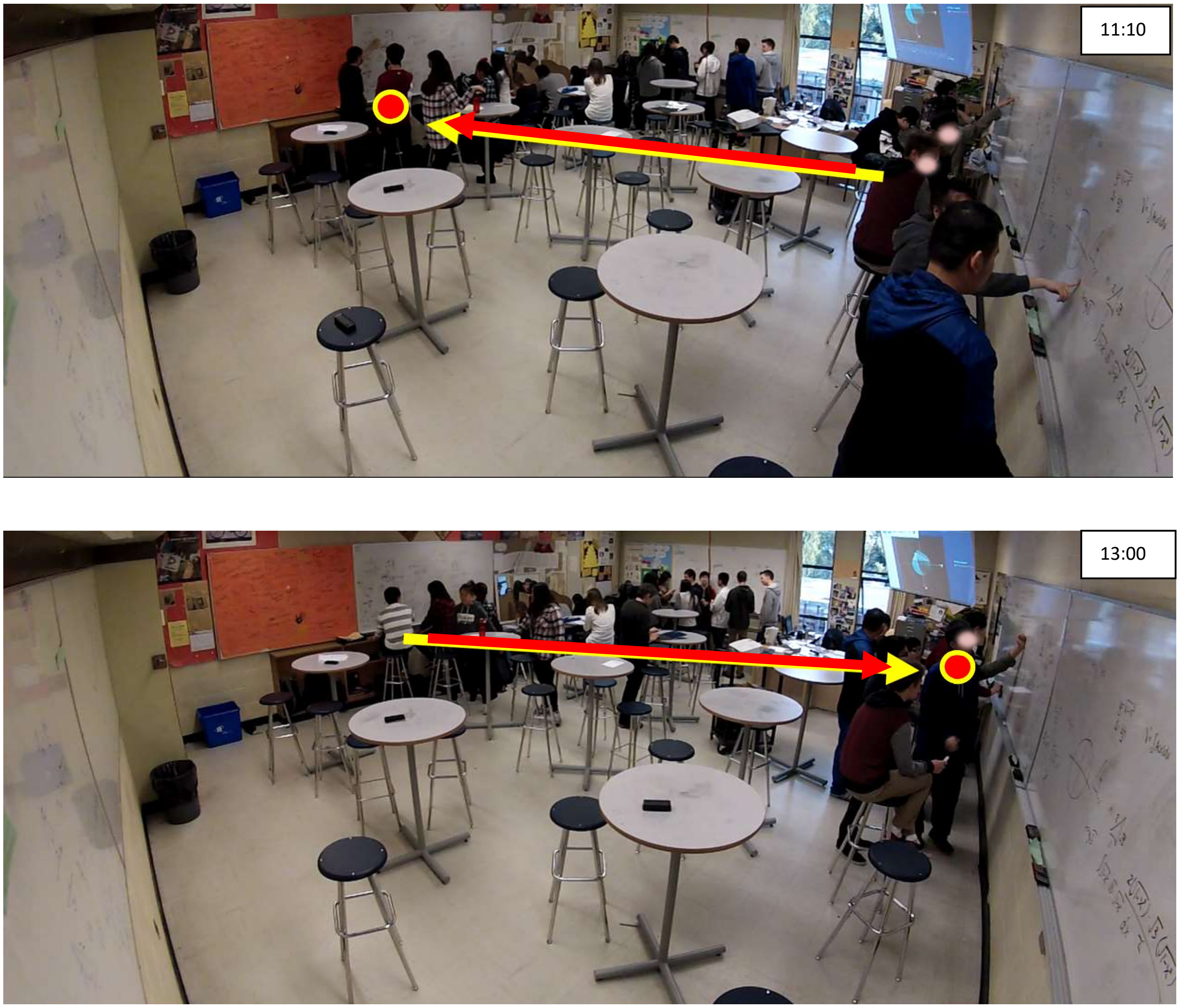

FIGURE 8. THE RED DOT INDICATES DEVIN'S TRAVELS OVER 13 MINUTES OF WORKING ON THE TASK. THE TIME STAMPS ARE IN THE UPPER RIGHT CORNER OF EACH PHOTOGRAPH. GP011548

Devin begins this task in a group with two other students, Stephen and William (See Figure 8). Their progress seems a little slow in the beginning. They all appear to be on their smart phones for a while, in fact Stephen seems to be lost in his smart phone for most of this video clip. After a short time (0:30), Devin begins making progress on the board and it looks like the whole group is working on the task. They are writing, gesturing, and discussing. At the 3:39 mark, Devin leaves his group and takes a look at Quentin's board. He does not interact with this other group, but he does look closely at their work, and then returns to his group about 15 seconds later. He spends the next 20 seconds stationed at his group, but still looking around the room at other boards. His two partners are engaged in conversation and writing on their board. At about the 4:10 mark, Devin fully joins his group and they all continue collaborating and writing on their board. At 5:30, Devin walks across the room and fully joins the group on the other side of the class. He even uses their pen and begins writing a solution on the board. At this point it looks like Devin is teaching this group how to do the task. The group is very attentive to what Devin is doing. At 9:10 and after a lengthy group discussion, Devin returns to his group. At this point there are two other people from other groups (one of these is Ferdinand) at Devin's board, and a super-group has formed. Ferdinand leaves pretty soon after, but Devin's group and their neighbouring group to the right continue in their collaboration. At 11:10, Devin notices that I am in a discussion with the group across the room, so comes to us to see what the conversation is about. I appear to be clearing up some confusion and enter into a conversation directly with Devin. After some direct discussion with me where Devin is seen gesturing with his hands, we can hear him exclaim "ohhhhhh!" as he indicates some new found understanding. At 13:00, Devin returns back to his group with this insight and begins erasing some of his group's work and fixing it up.

Group work is the means by which mathematics knowledge moves in a Thinking Classroom. It is not a one-directional teacher driven exercise; rather, the knowledge and ideas are moving amorphously. When a class is engaged in inter-group collaboration, the porosity (Liljedahl, 2016) of the classroom increases. On the surface, this appears random and inefficient, but after further observation and compared with a traditional model of information moving from a single source (the teacher), it is apparent that inter-group communication is a highly efficient mode of information and concept transfer.

These three student journeys highlight not only students moving in a classroom, but with the students, also knowledge and understanding moving in a Thinking Classroom through inter-group collaboration. These three students rarely relied on intra-group collaborations to make progress in the task. Although they all had their home base for intra-group discussions and work, almost all groups participated in inter-group collaboration at some point in the process. The examples above are not the only examples of people shifting and working with other groups. In a Thinking Classroom, the original groups are starting and finishing points only. What happens between these points is highly variable. It is interesting to see that people are moving not just to find ideas, clarifications or hints, but they also move to share ideas. Both Charles and Devin moved to other groups to share their ideas at times when they were finished in their own work on the task. The community of the Thinking Classroom exhibits reciprocity with knowledge transfer. Every member is responsible to search out knowledge if it is needed and to share-out knowledge once it is attained.

When Students Need Help

Most students experience some difficulties with learning in their classrooms. Typically, when students are 'stuck' on a problem, they will exhibit one or more of three positive actions: they will put their hand up and wait for the teacher to help, they will go to the teacher and ask for assistance, or they will try to get help from a peer. I consider the action of putting up one's hand and waiting for the teacher's assistance to be a 'stop-thinking' action, because usually students are no longer working on the mathematics when their hand is in the air. In prior experience, getting help from a peer is the least likely action, and I more often observed students exhibiting negative actions such as giving up and staring into space rather than going to their peers.

In this Thinking Classroom, I rarely see students asking me for assistance. If I am nearby, or if I join a group, then a conversation around difficulties ensues; but, I am rarely directly sought after for assistance. The norm in this Thinking Classroom is that students generally seek help from each other. For this reason, it is not common to see students raise their hands as they would in a traditional classroom. After analyzing all of my video data, there are only a few examples of students raising their hands for help. On most of these occasions, I am near and able to provide immediate assistance. It appears that the hand goes up almost because I am near and the student is signaling to me for help. On one occasion, seen below, Jeff and his group are working on the non-curricular race car task (see appendix B). He puts his hand up for help, and I am on the other side of the room (see figure 9).

FIGURE 9. JEFF'S HAND IS UP HOPING FOR HELP FROM THE TEACHER. GOPR1404, 8:50

In this example, Jeff's hand is up for 15 seconds and it goes back down when he notices that I am nowhere near for help. Work in the group stops for a short time and Jeff starts to look at the other group's board work (see figure series 20).

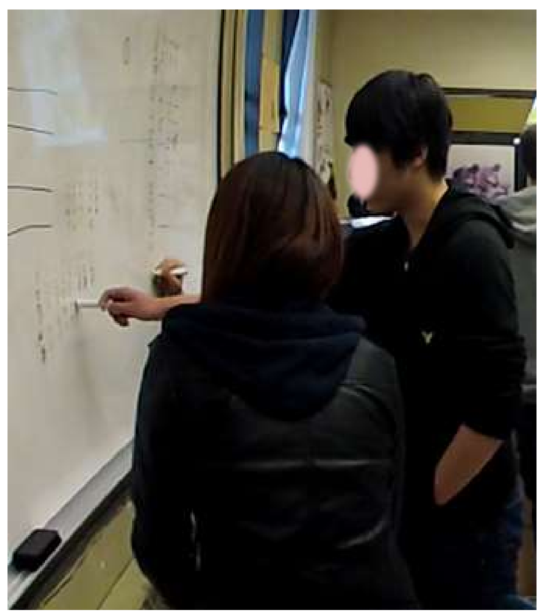

FIGURE SERIES 10. JEFF IS LOOKING AT THE OTHER GROUP'S WORK, AND THEN PICKS UP THE PEN TO CONTINUE WORKING ON THE PROBLEM. GOPR1404, 9:10, 9:45

For the next 30 seconds, Jeff and his partner are staring at their own work in contemplation. Then at the 9:45 mark, not even a minute after the initial hand went up, Jeff picks up his pen and continues to work on the problem (see figure series 10). In this example, Jeff and his partner were in need for assistance and Jeff put his hand up; however, this was not a 'stop thinking' behaviour. After a short time, Jeff clearly re-engages into the problem and continues to make progress.

When students are having difficulty in a Thinking Classroom, a more typical response is to seek help from others in the class. An example of students seeking help is captured in the screen shots below (see figure 11).

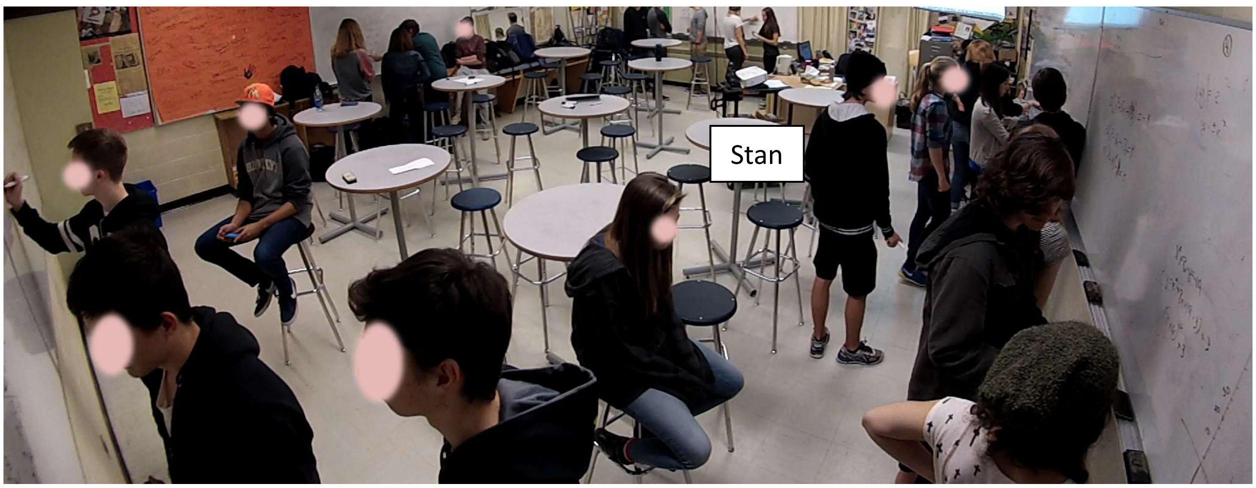

FIGURE 11. STAN ASKING FOR HELP FROM ANOTHER GROUP. GP021971, 0:20

In this video clip, Stan is stuck while working through a question about graphing systems of quadratic functions (see Appendix C), and after a quiet interaction with his partner, he goes over to the group on his left and says, "I don't understand where that positive six comes from. Help me. Help me." And then a girl in this group proceeds to help Stan understand. It is not possible to tell in this video portion what actual 'understanding' Stan achieves, but it is clear that Stan is engaged in seeking of understanding from other students.

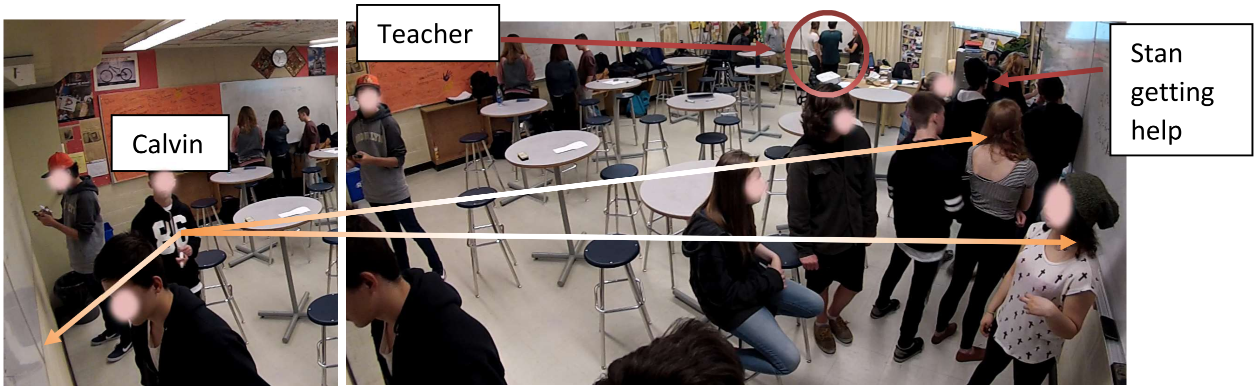

FIGURE SERIES 12. THE ARROWS INDICATE THE THREE DIFFERENT GROUPS WHERE CALVIN WENT FOR ASSISTANCE. GP021971, 0:51, 1:01

At almost the same time in the video, Calvin gets stuck. His next move is to look at other group's progress on their white boards (see figure 12). Calvin moves through three different groups before he finds the answer to his difficulty and gets back to work on his board. Through these two examples of student difficulty, the teacher does not play any role in providing assistance.

In the Thinking Classroom, I was struck by how seldom students went to the teacher for assistance. On occasion, students would put their hands up and ask for help, but this was very rare and most often resulted with the teacher not even noticing. In a Thinking Classroom, when you see students raising their hands, you can see that they are still engaged in the activity and the conversation around the task within their group. Sometimes, the hand shoots up, and then a few minutes later, the hand comes back down because the student was able to get assistance from someone other than the teacher. This demonstrates how in a Thinking Classroom, it is the whole class that is the source for learning and the source for ideas, not just the singular "Teacher at the front."

Students are still getting 'stuck' and having difficulties with the material, but their actions after being stuck are completely different from those in traditional classrooms.

When students have difficulties with mathematics in a Thinking Classroom, their first line of action does not involve seeking help from the teacher. First, students are always working in groups, so when an individual student is confused or doesn't know how to proceed in a problem, the first step undertaken is to discuss the issue with their partners. If the whole group is having difficulty with a task, the most common action that groups exhibit is looking at other whiteboards for ideas or inspiration. If looking at whiteboards doesn't quite fix the difficulty, then the next step taken is for a group member to visit another group and ask for assistance. They do this by looking at other group's board work, and by moving through the room and collaborating with others. This is not to say that the teacher does not help students. The teacher, as mentioned earlier, moves throughout the class and in and out of groups providing help where he believes it is needed. The students are demonstrating independence from the teacher in their learning and co-dependence on each other. This is another example of how a Thinking Classroom is a community of learners.

Deviant Behaviour

This behaviour falls under two categories: Behaviour that students participate in, but only for a short term, and behaviour that appears to be off task but, in reality, is not.

True Deviant Behaviour

In a Thinking Classroom, there is space for students to have mental breaks and then re-engage with renewed energy and enthusiasm (disengaged → re-engaged). In a traditional classroom, this behaviour is shunned and teachers spend a great deal of time structuring the class to avoid it. Examples of this type of behaviour are texting, chatting with others, distracting others, and doing something that is not part of the lesson.

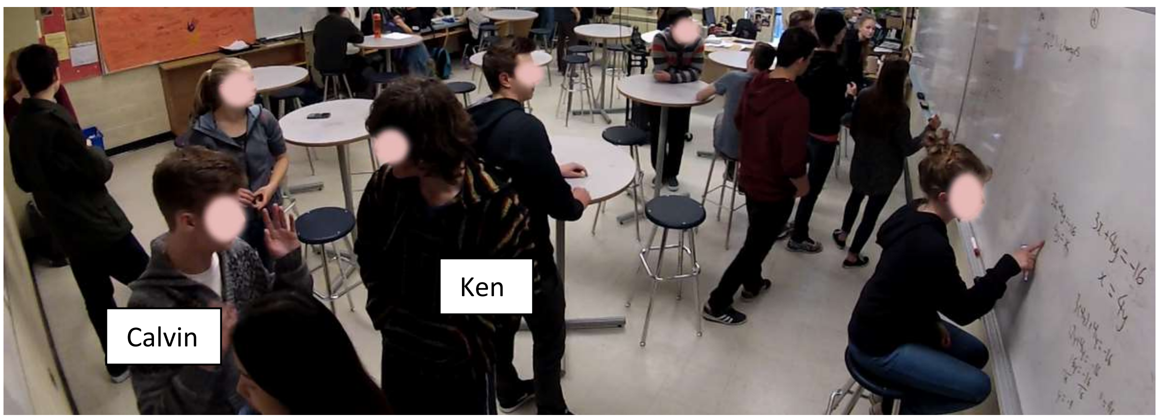

FIGURE 13. KEN HAS MOVED OVER TO ANOTHER GROUP TO BOTHER CALVIN. GP021977; 9:25

In the screenshot above (see figure 13) while working on a curricular task (see Appendix D), I see Ken coming over to say something humorous to Calvin. Calvin listens to what Ken has to say, they both laugh, and then Calvin gestures for Ken to leave so that he can get back to his task.

FIGURE SERIES 14. STAN DISENGAGING -> RE-ENGAGING AS HE CHECKS HIS PHONE. GOPR1971; 15:45,15:51

Here (see figure 14), we see Stan actively participating within his group working on a non-curricular problem solving task (see Appendix E) at the start of a class. They have been working on this task for approximately 3 minutes when we see Stan step back, and it appears that he is sending a text on his smart phone (15:45). Not more than 6 seconds later, the smart phone returns to his pocket, and he makes a contribution to the solution within his group. In all, this deviant behaviour is only seen as regular behaviour in one or two students in each class.

False Deviant Behaviour

The second category is an apparent deviant behaviour that is "learning in disguise." In figure series 15, the class is working on a curricular task (see Appendix C) and Vish appears to be transfixed on his device. At first, I assumed he was texting his friend or checking the scores for last night's game. After less than a minute, he bounds to the board, pen in hand, and starts participating in the mathematics of the problem. In the second picture, you can see his partner joining him and discussing what he found on his device. Although it is unclear exactly what he is using his device for, I believe that it is a graphing program used regularly in my classes (Desmos.com) that has shed some insight into this particular quadratic problem.

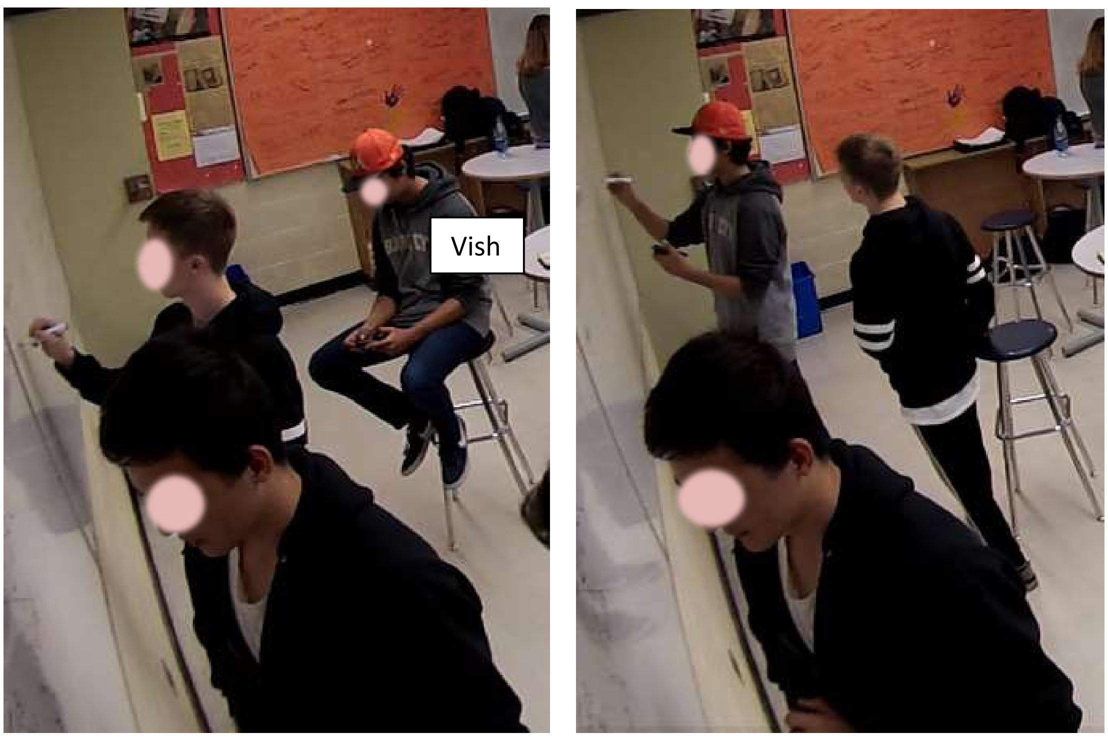

FIGURE SERIES 15. VISH APPEARS DISTRACTED BY HIS TECHNOLOGY, BUT HE IS ACTUALLY USING IT TO HELP MAKE SENSE OF THE MATHEMATICS. GP021971; 1:15,1:36

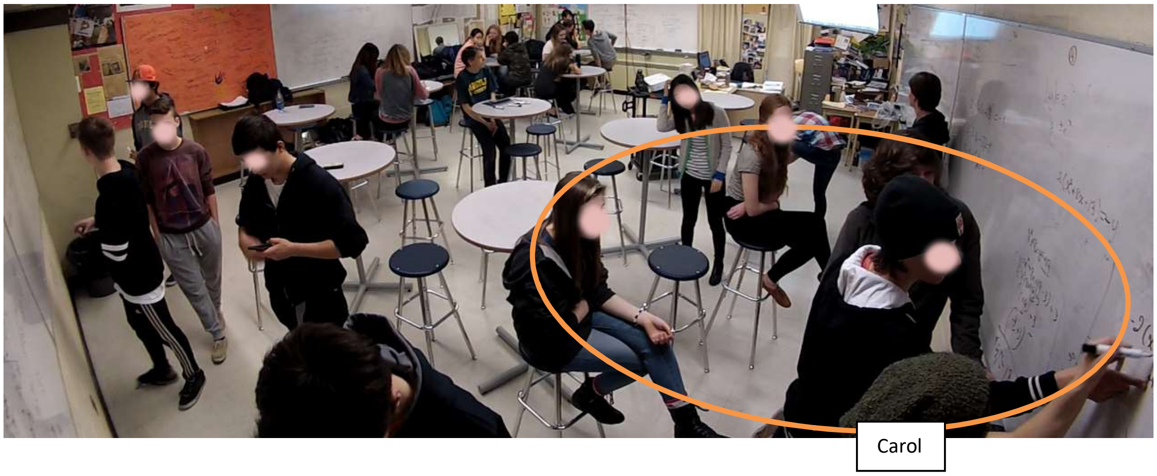

FIGURE 16. CAROL'S GROUP IS FINISHING THEIR WORK ON A PROBLEM. GP021971, 10:00

In figure 16, we see two groups that have come together, indicated in the orange circle, to work on the curricular task from Appendix C. They are nearing the end of the task and I announce that it is time to debrief, saying "Let's go over some of these graphing skills." At this point, Carol, at the bottom right of the photo, puts down the pen and picks up her smart phone that is sitting conveniently on the ledge of the whiteboard.

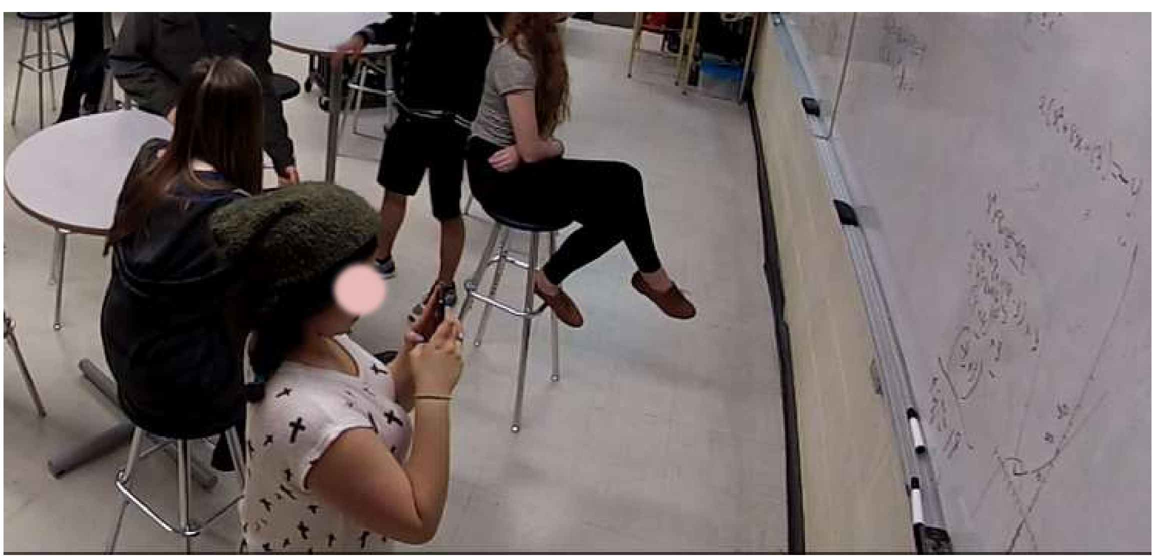

FIGURE 17. CAROL IS TAKING TWO PICTURES OF HER WORK WITH HER SMART PHONE. GP021971, 11:13

She proceeds to take two pictures of her group's work (see figure 17) before sitting at her table for the class discussion.

In another example from a different date, we see Jeff working with his partner on a radical equation task from the curriculum (see Appendix F). They are both quite engaged in the task and making good progress, as is evident by the neighbouring group attending to their work (see figure 13).

FIGURE 18. JEFF AND NANCY FINISHING UP THEIR WORK. GOPR1404, 28:37

You can also hear Jeff's partner, Nancy, acknowledging their progress:

Nancy: Ya, Ya, Ya, yeah….

Jeff: Ok, there we go.

Jeff Smiles.

Jeff: I feel so good about it…

FIGURE 19. JEFF COLLECTING EVIDENCE WITH HIS SMART PHONE. GOPR1404, 29:19

At this point, Jeff takes out his smart phone and snaps a picture (see figure 14). In a traditional classroom, students using cell phones in class is often perceived as deviant behaviour. In a Thinking Classroom, cell phones can be a student's tool for recording evidence of their learning.

In this classroom, students are always engaged in their mathematics at the whiteboards. The whiteboards work so well because they are vertical and non-permanent; however, these two situations conflict with their ability to take notes. In order for students to keep a record of their progress or their learning they can either use the teacher's website (where the teacher curates a collection of photos to show the class's progress) or they can collect their own photos. In the latter case students need to make decisions before taking these pictures. Students are not taking pictures without reason; otherwise, we would see the same students taking pictures all the time throughout a lesson. Instead, we are seeing students taking two or three pictures during the course of an 80-minute lesson. This suggests that these students are thinking about their work and deciding that this work is important for them to view at a later time. This is not the same as “note-taking” because in note-taking, students are often not exercising choice in the activity. With taking pictures, students are exercising their autonomy and deciding what is good evidence of their learning and what is worthwhile to keep for later viewing. What is not clear in these videos and would be interesting for further research, is how these students organize and use their collection of photos.

A student using their phone in a math classroom is normally considered deviant behaviour because most traditional mathematics classrooms have no need for a phone. Many teachers would consider the phone to be a distraction from learning. In a Thinking Classroom, students are often seen using their personal electronic devices for a variety of uses: some for learning and some not for learning. As evidenced above, two common uses for this technology as a tool for learning are as a computer to access mathematics applications and as a camera to document student’s work.

In a traditional classroom, a lot of teacher effort is devoted to preventing deviant behaviour in students. Seating plans are adjusted, students are warned and then warned again, detentions are given, and phones are confiscated. While observing a Thinking Classroom, I noticed that deviant behaviour still occurs. However, it appears that students engage in deviant behaviour almost as a break from their thinking and working. The behaviour is short and not consequential; in fact, I observed students reengaging with the material with more energy after their short break. Also, in many instances, the apparent deviant behaviour was not actually deviant. Many times, it appears that students are off task, but they are actually still engaged in the task, but under a different mode. From the video evidence, students appeared to be on their smart phones, but they were actually using a graphing program. On other occasions, it appeared like a student was going off to chat with her/his friends in another group, but in fact the students were sharing strategies with respect to the problem that they were working on. Students were also observed using their phone to take pictures in class, but they were not taking pictures for their leisure. They were actually taking pictures of their mathematics work for future reference. In a Thinking Classroom, students are given more freedom and room to explore, express, and share ideas. Some students may abuse this freedom, but the vast majority use it to express their autonomy and engage in their learning in a way that works best for them. It is important to notice this category of behaviour in a Thinking Classroom, because in a traditional classroom, this behaviour is often not acceptable. Teachers will spend a great deal of their resources either preventing or reacting to this type of behaviour.

Autonomous Behaviour

Student autonomy is a significant background contributor to a Thinking Classroom. Students need to experience and engage in mathematics at their own pace.

The most important type of autonomy for students to have is cognitive autonomy; this allows students to discuss multiple approaches and strategies, find multiple solutions to problems, have ample time for decision making, debate ideas freely, and have less teacher talk time; more teacher listening time (Stefanou et al., 2004). Autonomous behaviour is evident throughout these video observations, because the majority of class time is devoted to students working within their groups without teacher direction. Students are given the freedom to engage in the tasks with their own direction and design, as they are observed moving freely around the room seeking ideas and sharing learning with others. Even the layout of the classroom supports autonomy. The tables are small and circular serving two purposes: Being small, they provide more open floor space to encourage student movement during tasks. Being circular, they promote student discussion and collaboration because students are facing each other when they are seated together. Autonomy is an essential ingredient to a Thinking classroom; however, providing students with autonomy can sometimes lead to students abusing that autonomy. After observing the video evidence, there were some issues with student behaviour that I observed.

The issue that bothered me most was seeing students off task and lost in their smart phones. As mentioned earlier, I found students to be on task with their devices more frequently than off task, but the off task behaviour was frustrating to watch. In a Thinking Classroom, student autonomy is essential, and with autonomy, students have freedom with their technology devices. Some students use their devices off task as a break from their work, and I don't have an issue with this. A smaller group of students abuse this privilege and are on their devices too much.

FIGURE 20. GRETTA IS DISTRACTED BY HER DEVICE. GP021976

In the screenshot above (see figure 20), during a curricular task on quadratic systems of equations (see Appendix G), Gretta is seen texting with her smart phone. This was a frustrating video for me to observe, because she is so front and center in the picture and she is on her device for a very long time. The total task time was 23 minutes, and Gretta was on her device off and on but for an accumulated duration of 4 minutes. Off task behaviour on a device looks differently to on-task behaviour. During off-task behaviour, students are more intently focused on their device and less engaged with the environment around them. To make matters worse, within this time, I observed myself entering into the group for a conversation and then leaving again – apparently not noticing Gretta on her device. I suppose I was too focused on the work on the whiteboards. Normally, when I find students to be using their smart phones too much, I will quietly take the device away for the remainder of the class, and this usually proves to be effective.

Overall, these issues are minor and do not affect the culture in a Thinking Classroom. I believe students on smart phones is an unfortunate negative that I have to accept if I choose to give students cognitive autonomy in their work. Fortunately, I did not observe many students abusing this privilege. Most times, students would take a short break to check their phones and then reengage in the task – I believe this to be acceptable.

During most of the lessons, autonomous behaviour in students is observed when seeing students moving freely and in conversations with others on their own accord. The teacher is seldom seen giving specific direction with respect to behaviour or even student progress in tasks. The culture in these classes is vastly different from that of a traditional classroom. There is a casualness with which the students interact with each other, the teacher, and the mathematics, and there is an openness in the sense that there is little structure to each task that the students are working on. Providing cognitive autonomy to the students is requisite to sustaining this thinking culture. As a result, autonomous behaviour is observed everywhere, at all times, in all videos. If one values a classroom with high porosity, a classroom where knowledge moves around the room by means of all members in the room, then one needs to give students freedom to move and think when and how they wish.

Sense-making and Reifying

How are students making sense of abstract concepts and how are they making the mathematics real for them in a Thinking Classroom? For the purpose of this thesis, I consider students to be 'sense-making' when I observe them in discussion with others within a particular problem solving task. From my experience, when working in groups on a task, students exhibit one of two different social behaviours. They either collaborate productively on the task or they are off task. The former is observable when students are in discussion with others, using gestures, and writing on the whiteboards within the context of the task. When I observe this behaviour, I call it 'sense-making.' I know that it is not the only way that students reify and make sense of the mathematics, but it is one way and it is observable in the video data.

In the screenshot below (see figure 21), students are beginning to work on a non-curricular task (see Appendix E). The task is a challenging one that involves a box of marbles. In the beginning of the task, students are struggling to find ways to model this problem on the board. Some groups are off task, some groups are writing hesitantly at their white boards, but most people do not appear very engaged in the task. At the 11:35 mark of the video, I pick up a box of coloured pen caps and don't make any announcement, but proceed to shake the box loudly enough for the class to hear (see figure 22).

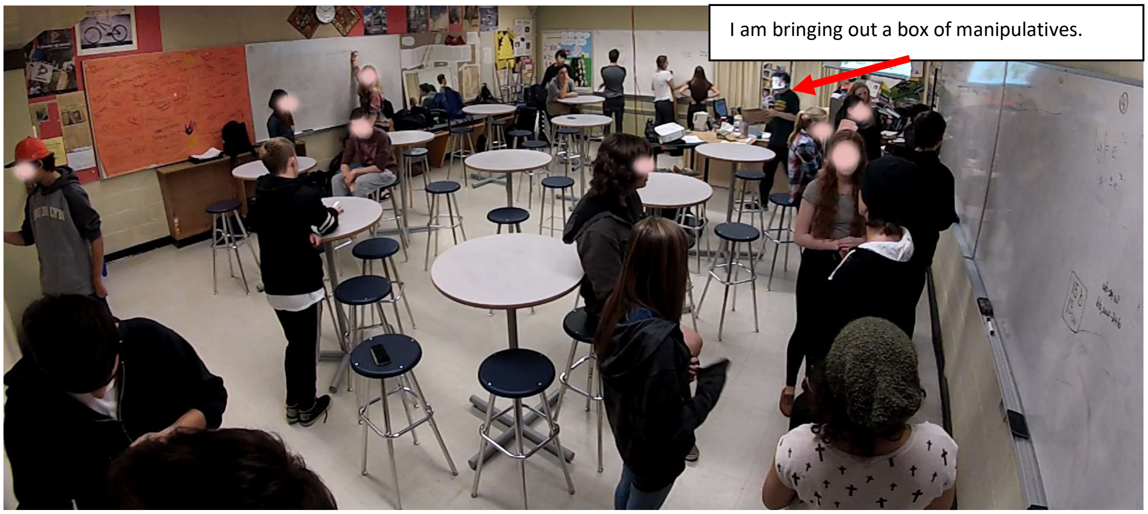

FIGURE 21. I AM BRINGING OUT A BOX OF MANIPULATIVES FOR THE CLASS TO USE. GOPR1971, 11:35

FIGURE 22. STUDENTS ARE RUSHING TO THE BOX OF MANIPULATIVES. GOPR1971, 11:50

Shortly after the class hears the shaking of the pen caps, you can hear students commenting with excitement.

“Oh, can we actually like use these?”

"Ohhhh, ohhh that's a game changer."

FIGURE 23. VISH HAS A BIG SMILE ON HIS FACE. GOPR1971, 12:05

The students are smiling (see figure 23) and the engagement level is obviously increasing as they suddenly have a new tool to help reify and make sense of this problem. Their steadfast sense making is made more evident later in the video when you can hear students say things such as:

"Ok, I understand how you do that…" (2:15)

"yeah, I did this wrong." (2:53)

In another class, while students were working on the quadratic systems task (see Appendix G), Wayne was observing another group's work for a while at their board. Then he moved in with a question:

Wayne: "At the very beginning, how did you get +4.9 and -3.7?" (GP021976, 13:35)

Doug: "Ok, so, this equation equals this equation… right?"

Wayne: “Ya, Ya, I understand that.”

Doug: “and then you just move it this way”

Girl from side: “you have to move them over in the right way.”

Wayne: “Ya, Ya, thanks.”

In this interaction, not only is Wayne seeking assistance from a peer, it is also quite clear in his language that he is seeking understanding. In a Thinking Classroom, final answers are not the end goal in student’s work; students are searching for understanding of the mathematics concepts.

In a Thinking Classroom students will use manipulatives, drawings, and discussion to help make abstract concepts concrete and make sense of the mathematics. In my experience with these classes, I frequently hear students saying things such as:

“I know what you did, but I don’t understand what you did”

“I am trying to make sense of this”

“I like this way because it makes sense to me”

In a Thinking Classroom, individual sense-making is highly valued. Students hear it frequently from their teacher, and then it becomes part of their language in the class and the group work. In the video, I heard this all the time, but I also saw students reifying the concepts through diagrams and working with manipulatives, and also through seeking out other ideas outside of their own group.

Teacher Movement

It is obvious from the video that I do not visit all groups with equal frequency. Some groups are never visited, and others seem to get an inordinate amount my attention. In a traditional classroom, a teacher is challenged to try to make contact with all students, so that the teacher can assess and provide intervention or assistance. In a traditional classroom, authentic individual intervention or assistance is next to impossible. In the Thinking Classroom, this problem is mitigated on two fronts. First, the teacher is able to see which groups require individual attention. And second, students themselves are able to get assistance from viewing other's work or from inter-group collaborations.

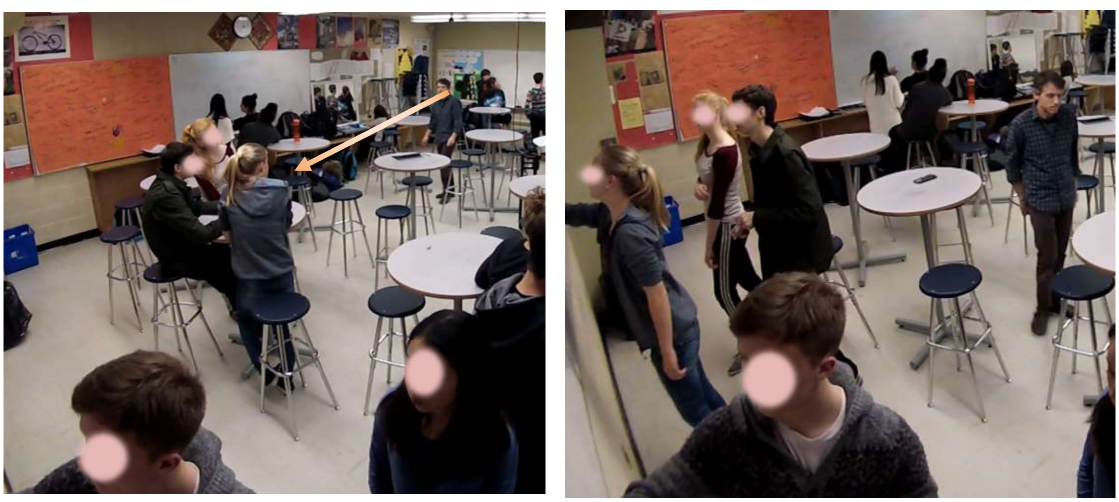

Figure Series 24. I am attending directly to a group in need of assistance. GP031977; 0:21, 0:40

In the video segment captured above (see figure series 24), the class is just beginning a new curricular task on solving quadratic systems (see Appendix H). All of the groups have moved to their board space except one. This group is apparently having some difficulty understanding the task. The teacher notices this group immediately and goes to them to provide assistance. After a few clarifying questions and short discussion, the group bounds to their board to begin work (see figure series 24). Because all the groups are working so visibly at the whiteboards, the teacher is able to notice immediately which students need assistance and move to mediate the problem. After this moment, the teacher moves around the room observing the student's very visible work.

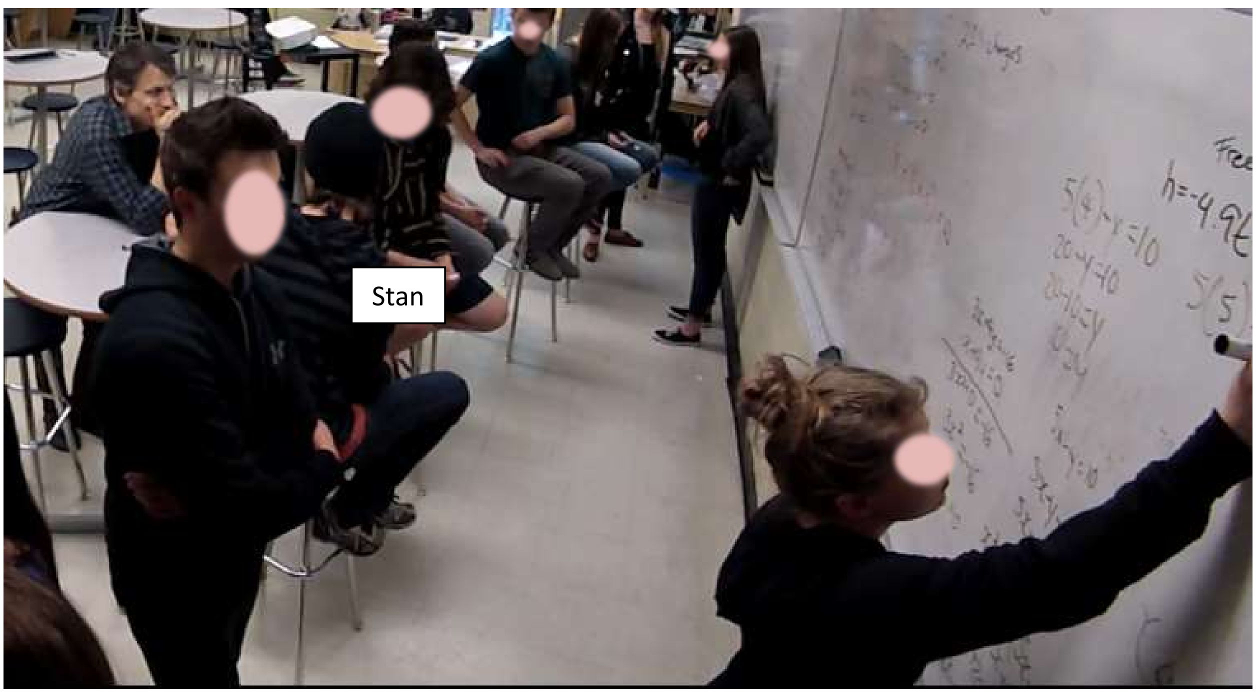

FIGURE 25. I AM CONVERSING WITHIN A GROUP. GP031977; 4:00

Every now and then, the teacher enters a group to have a short conversation about their progress (see figure 25). The class is still working on the curricular task from Appendix H that is asking students to find the ordered pairs for the solution. Stan and his group have made it pretty far through the task and have an answer of x = 5, 4. They have found the two x values for the two intersection points, but they are confused because Stan has written it as a single answer. Although the audio is difficult to hear in most groups, you can hear the interaction between myself and Stan at the 4:00 mark of the above video.

Stan: “I don’t think this is right.”

Teacher: “You can’t write x as the ordered pair."

Stan: “I am looking for an ordered pair aren’t I?”

Teacher: “[yes], so if the x is four, then what is y?”

In this small exchange, I begin by just being close to the group and staring at their board work. Stan notices me close by and starts by acknowledging that his result is not right. The group is unsure of how to answer the question, and I notice myself doing two things. First, I point out what the question is actually asking. Then, I provide a little nudge by asking the question, “if x is four, then what is y?”. The reason that I chose to highlight this exchange is because it shows the casualness with which I enter groups and participate in discussions. Because I am in a constant state of wandering, and observing, it is very natural for me to just enter a group and participate or push the discussion forward. In a Thinking Classroom, the teacher is accepted as part of the learning community more than being the leader and director of learning.

When I observe these videos and think back to how I used to teach mathematics, my movements and interactions with my students are very different. In my traditional classroom, I would wander the class only when they were working on questions that I had just showcased. When I interacted with students individually, I would be pointing out their error and telling them how to do it properly. Students were always sitting in desks and I was standing over them, talking down to them. In this Thinking Classroom, I am almost always wandering through the class and visiting groups of students. The only time I speak to the class as a whole is the two or three times that I debrief with the class on a particular task. My interactions with students are much different too. In a Thinking Classroom, the interactions are more like conversations where students and teachers share ideas and ways of thinking. And lastly, the visual representation of myself as the teacher in a Thinking Classroom is very different from the symbolically powerful representation of the teacher standing over the student or in front of the students. In a Thinking Classroom, I am interacting with groups of students as we all stand around work on a board. Sometimes it is even difficult to find me in the room, because I am working within groups, not standing over them. This last point may seem quite trivial, but I think it sends an important message to the students. Students are less likely to be intimidated because the teacher is speaking with them at their level, and also the teacher is usually speaking with a group of two or three students rather than just one.

In a Thinking Classroom, the teacher is part of the community of learners, wandering the class and interacting with groups when he feels it is necessary. The interactions almost appear like 'peer-to-peer' interactions where the teacher is observed gently suggesting or nudging the group on how to move forward in their thinking. In a Thinking Classroom, there are more interactions between teacher and students in small groups and the small groups allow the interactions to be both personal and personalized; thus, the quality of these interactions is higher.

Class Time Spent on Tasks

A significantly larger amount of time is allowed for each task because students are working on the tasks within a freedom of direction and constraints. This is a result of the autonomy provided in the class (Stefanou et al., 2004) and it is interesting to see students engage, disengage and re-engage within this time. In a traditional classroom structure, I remember planning to cover 6 to 8 questions in one class and then still giving students 15 to 20 minutes to work on their homework at the end of the class. In this Thinking Classroom, I typically plan for 1 non-curricular task and 2 curricular tasks in every class and there is no time set aside for homework.

To determine how much class time is spent on average, I analyzed ten randomly chosen different tasks from a variety of classes. The tasks are of two different types: Each class always begins with a non-curricular mathematics task, and then this is followed up with two, sometimes three curricular mathematics tasks. When measuring the class time spent on each task, I began recording the time when the whole class started their work at the whiteboards. I finished recording the task duration when I had completed the task debrief with the whole class. When using these two endpoint markers for ten different tasks, the average class time for each task was slightly over 21:00 minutes (see figure 26).

Task	Type	File(s)	Duration
Appendix E	Non-Curricular	GOPR1971, GP01971	17:05
Appendix C	Curricular	GP011971, GP021971, GP031971	30:10
Appendix I	Curricular	GP031971, GP041971	11:35
Appendix A	Curricular	GOPR1548, GP011548, GP021548	28:30
Appendix J	Non-Curricular	GOPR1981, GP011981	15:55
Appendix K	Curricular	GP021981	16:25
Appendix L	Curricular	GP031981	11:00
Appendix M	Non-Curricular	GOPR1980, GP011980, GP021980	33:55
Appendix N	Curricular	GP031980, GP041980	21:15
Appendix G	Curricular	GP021976, GP031976	25:00

FIGURE 26. TASK DURATIONS FOR 10 DIFFERENT TASKS.

In these 10 tasks the longest duration was 33:55 and the shortest was 11:00. Over the course of these tasks, I do see students dropping out and back in to engagement, but for the most part, I am struck by how much of the class is still engaged in mathematics activity just before the teacher begins the debrief.

FIGURE 27. STUDENTS ARE STILL ENGAGED 16 MINUTES INTO A TASK. GP031976, 1:00

In the above screen shot (see figure 27), one can see that the majority of the class is still engaged in mathematics discourse, and this is 1:30 before the debriefing, after a full 16 minutes of on-board student engagement.

In a Thinking Classroom, there is a significant amount of class time spent on each task. Because the teacher allows time for students to think deeply about the topics and dialogue with others before bringing the activity to a close, students are given a better chance to reach personal understanding in the mathematics. By spending over 21:00 minutes on average per task the students are given the message that learning takes time and should not be rushed. The students who finish their work early are responsible for sharing their thinking with others and developing deeper understandings. In the 80 minutes of a typical lesson, the students spend time working on one non-curricular task and only 2 – 3 curricular tasks. This is a significant difference from my traditional teaching experience. I remember introducing and modeling a question type and then giving the students time to try one on their own. This whole process may have taken about 15 – 20 minutes, but that was for two questions. I felt that I had to keep this pace in order to cover all of the question types; however, I see now that covering questions does not necessarily mean that students are learning and understanding the mathematics. When I watch this video evidence and hear students in discussion with others seeking understanding and sense-making, the extra time spent on each task seems well worth it. I have always known that learning takes time and should not be rushed, so this aligns well with the amount of time devoted to each task in a Thinking Classroom.

Chapter 5. Conclusions

I began experimenting with changing norms in my classroom 3 years ago. I was motivated to change after seeing evidence for the lack of student thinking in traditional mathematics classrooms like my own. Dr. Peter Liljedahl laid the groundwork for shifting my class from a non-thinking, teacher centered, traditional model to a classroom that is "not only conducive to thinking but also occasions thinking" (Liljedahl, 2016, p. 362). After implementing Visibly Random Grouping, Vertically Non-Permanent Surfaces, removing my focus on note taking, and teaching through problematizing the curriculum (Hiebert et al., 1996), my classroom eventually became a "space that is inhabited by thinking individuals as well as individuals thinking collectively, learning together, and constructing knowledge and understanding through activity and discussion" (Liljedahl, 2016, p. 362). After three years of implementing the aforementioned big changes as well as a plethora of smaller changes, I now look out on to my class of students and see something completely different from my earlier classrooms – it is unrecognizable.

I see students relying on each other to make progress in their understanding. I see students laughing and even shouting in exhilaration when they uncover a personal 'a-ha' moment. I see whole-class engagement in mathematics activity for long durations on single tasks. I see students discovering different solutions to tasks and valuing and finding elegance in the mathematically unique solutions. I see an increase in the porosity of the class by students moving throughout the room and exchanging knowledge with one another. The knowledge moves through the mingle and merging of groups and as student's eyes spy other's work on the vertical surfaces. My class is transformed, as it is now a Thinking Classroom.

After placing a video camera in my class for a few weeks in the Spring of 2015, I had the privilege of viewing mine and my student's actions so that I could make observations of student and teacher behaviours in a Thinking Classroom.

Answering the Research Question

The research question was: *What student behaviour and what teacher behaviour is observed in a Thinking Classroom?*

The most significant observation made in a Thinking Classroom was movement. This observation had two different, yet related, aspects. There was the physical movement of the students in the room, and partly related to this physical movement, there was also the movement of knowledge through the room. The physical movement was seen in students moving within their groups while working at the VNPS, and students moving to other parts of the room to collaborate with students or the teacher outside of their group. The layout of the room with the small tables and easy pathways enables and even encourages student movement; this was evident in every piece of video evidence and stands in stark contrast to the lack of movement seen in more traditional classrooms. Partly related to the physical movement is the movement of knowledge or the porosity of the classroom. Knowledge moves (porosity increases) when the students move, as it was observed when students were sharing their ideas with others in their travels. Often times the reason behind the movement was the searching out of ideas when students were stuck, but students were also seen to move to other groups to share or give knowledge to others in the class. Moving students was not the only vehicle for knowledge moving in a Thinking Classroom. Because of the vertical and public whiteboards (VNPS) that were used for all of the student's work, knowledge also moved across the room by students simply witnessing other group's work. The physical space of the classroom with the tall standing-biased tables with easy pathways between and the vertical surfaces encouraged student movement and increased the porosity of the room. The movement of the students and the movement of knowledge throughout a lesson is a key observation noticed in a Thinking Classroom.

Another observation in the Thinking Classroom was the desire for mathematics understanding and the interdependence among the students for their learning. Students were observed using manipulatives to help them understand concepts, asking for clarification, and not being satisfied with answers unless understanding was included. There is an independence from the teacher, for the teacher is no longer initiating all of the steps and processes. With the independence from the teacher, the students were observed to depend on each other for understanding and learning. This interdependency really makes the Thinking Classroom look like a community of learners where the teacher is simply an expert member within the community.

Another observation in a Thinking Classroom is the autonomous behaviour of the students. Students are given freedom to choose how to work and more importantly how to think. This autonomous behaviour is visible in seeing how students move, interact, dialogue, and even take breaks. The teacher places value on multiple solution strategies, so students feel at ease to explore their own ideas in mathematics and are often rewarded for doing so.

Possibly due to the students being given so much autonomy in their work, deviant behaviour was observed in the Thinking Classroom. Deviant behaviour included students distracting other students, students using their smart phones inappropriately, and students just not engaged in the tasks. Most of the time, when this behaviour was observed, it was for a very short duration. Deviant behaviour appeared to be a means for students to take mental breaks from the mathematical thinking. After joking with a partner, or checking a smart phone for texts, the student would engage back in to the activity. With the autonomy provided, the teacher rarely intervenes, and this appears to be quite effective. Other times, student's only appeared to be participating in deviant behaviour, and were actually still completely involved in the task.

Lastly, an observation made in this Thinking Classroom was the time with which students were provided for working on their tasks. This is another observation that starkly contrasts to what is seen in more traditional mathematics classes. The average duration for a mathematics task was about 21 minutes with some tasks lasting as much as 35 minutes. The time for a task was measured from the moment the students started working on the problem to the moment that the teacher had finished the problem's debrief. In a Thinking Classroom, students continue to be engaged in the task even when it is apparent that they are finished. Students were often seen visiting other groups and sharing their ideas or checking their own progress with others. In a Thinking Classroom, mathematics understanding is a valued commodity. You can hear it in the student's conversation and in the teacher debrief – students need to understand the mathematics if they hope to use the mathematics in variety of different circumstances. This understanding takes time. It is also apparent that with the additional time provided, students feel less rushed and less pressure; they now have the space to explore, reason, and understand the mathematics.

Contributions to Research and Teaching

This study describes observations and noticeable characteristics in a Thinking Classroom. The methods for developing Thinking Classrooms have been prescribed in Liljedahl's research (Liljedahl, 2016), but there is little research around what Thinking Classroom actually look like. With this study, we see specific student and teacher behaviours that are now associated with this model. It not only supports the work of Liljedahl but it also adds colour to his description of a Thinking Classroom.

Teachers will be able to use the results from this study to appraise their own classrooms in this context and ask themselves:

- Does the layout and furniture in my room enable student and knowledge mobility?
- Do my students move a lot through a lesson?
- Does knowledge move a lot through a lesson?

- Do my students have autonomy in their thinking and their work?
- Is learning teacher directed or student directed?

The study not only outlines my own steps in building this culture, but it also provides descriptions of what these classrooms look like and how they operate.

Limitations and Opportunities for Further Study

Being a small study where the researcher is researching his own teaching and student behaviour, there may be some concern regarding bias in the results. This possible bias was mitigated by using Teacher Noticing as the theoretical construct through which the data was analyzed. I kept the analysis to the observable behaviours of the teacher and students. I did not at any time comment on what I thought students to be thinking, or reasons behind their actions. Because of my relationship and existing knowledge of my students, this was possible but carefully avoided. All of the results were based on observations with actual video evidence to support it.

Another limitation to this study is the homogeneity of the age of the students. I chose to study my grade 11 and 12 classes so that I could focus on student behaviours and not be concerned about age having an influence on what I notice. This is a worthy reason; however, some might suggest that this behaviour only manifests in classes where students have a higher maturity.

Lastly, the idea of describing and containing these observations and results in a "thesis" format is a limitation. Putting these observations into a linear-thesis format is a challenge because the classroom doesn't behave in a linear fashion. There are so many student and teacher actions that are happening simultaneously, causing other actions, and depending on other actions that it is very challenging to pull individual observations out and study them in isolation.

Limitations aside, the conclusions reached provide suggestions for future studies. Student movement was observed to be a large part of a Thinking Classroom. Would a larger study want to quantify the amount of movement in a Thinking Classroom? Does a Thinking Classroom have any measureable impact on student learning?

There is an existing body of research around student movement and its effect on BMI and student achievement. Researchers may be interested to measure just how much students actually move in Thinking Classrooms. Considering the availability of inexpensive activity measuring devices, this type of research is quite achievable. Researchers may also be interested in how much students are actually thinking in a Thinking Classroom. What is the cognitive load on students?

In my three years of implementing a Thinking Classroom model, I have had students participate in large scale assessments including the Mathematics 10 provincial exam and the AP Calculus exam. There has been no noticeable decline in results for my students over these years; in fact, my AP students in the Thinking Classroom may have even out-performed my AP students in earlier years. This would be an interesting research topic to check the credibility of the Thinking Classroom model.

Lastly, considering the age group of the students in this study, it would be an interesting research topic to compare these results with those of students in lower grades. My experience tells me that the majority of the results would be repeated, but there may be new behaviours noticed with younger students in a Thinking Classroom.

What Have I Learned as a Teacher?

I feel that I have grown significantly as a teacher over the course of this study and also over the course of the three years of implementing a Thinking Classroom. By providing me with a third-person perspective of my teaching, the study has validated the methods that I am using in my classroom. Over the past three years of implementing a Thinking Classroom, I have learned to not be afraid to try new strategies in my teaching. I have learned that if the student and the learning is the focus of the change, then the experience is always fruitful.

Implementing strategies as radical as the ones needed for a Thinking Classroom always leaves room for teachers to doubt their decisions. Once my class was turned up-side-down and inside-out to begin the transformation towards a Thinking Classroom, I would frequently have concerns and doubts about it really benefitting my students. I could see the whole-class engagement on a daily basis, but I would always wonder about giving my students so much autonomy. The students have autonomy with how they spend their time in class and outside of class – is this too much? Do I need to control some of their behaviour? After spending time observing the video data, I was struck by how students dis-engaged and re-engaged on a regular basis. In fact, my students were behaving just how I behave in similar situations.

I remember one of my students, Ken, from when I actually taught the class. I remember thinking how frustrated I was with his distracted behaviour and how this was going to look horribly later when I was studying the video. In fact, when I finally got around to studying this particular video, Ken's behaviour wasn't as poor as I remembered. From my new third-person perspective, I saw Ken wandering the room, disrupting other students, and rocking my tables; but then, I saw him return to his group and make a really valuable contribution. This last part was not noticed when I was teaching, and it re-framed my thoughts on Ken. Ken, like many students, needs to move while learning, and his infractions, that I noticed while teaching, were not at all impactful on the other learners in the class.

Prior to my three-year journey of implementing a Thinking Classroom, I was a teacher that tried to control all aspects of my student's learning. I valued autonomy for myself as the teacher, but I never considered student autonomy. Over these three years, I have been releasing my control on almost all aspects of my student's learning. My students can choose how to engage in the mathematics, they can choose when to engage in the mathematics, and they can choose their solution strategies. They cannot choose their groupings; but because of the random grouping generator, I am not choosing their groups either. Their groupings are left to chance in the Visibly Random Group design. I have found that the more that I let go of the little things that I controlled in my earlier teaching, the more I am impressed with seeing how well students work and learn on their own.

What Have I Learned as a Researcher?

Over the course of analyzing the video data and describing the results for this study, I have learned that there is a lot of information that can be gleaned from a short segment of video. Video samples are an exceptional medium for observing what is happening in a classroom. First-hand observations can miss subtleties such as student's comments, gestures, or even gazes. With video, I have the ability to replay segments, and zoom in on segments to capture these subtler nuances in student behaviour. I have also learned that in trying to answer one research question, many other questions arise.

This research has showed me the value of observing my own teaching and classroom. During a lesson, I have so many things to attend to as the teacher. A small sample of the things I am thinking about in the middle of a lesson are pacing, student behaviour, assessment, and what groups need my attention. In the heat of the moment, it is very challenging to be a reflective practitioner. Video evidence affords me the opportunity to properly reflect on my practice and writing this research paper formalizes this reflection.

This research has also showed me that there is so much more to study in these classrooms. After analyzing all of my video evidence, I was struck by the number of other questions that I had that did not fit under this research. I would like to find out just how much movement students engage in during a typical class. I would like to know if there is measureable improvement in student achievement. I would like to know how much my standing biased tables affect student movement and engagement. I would like to see if all of my observations also manifest in lower grade classrooms.

This research has shown me that there are unique and observable behaviours of students and teachers within a Thinking Classroom. Before this study, I could describe how a Thinking Classroom is developed and how it might look. This research has now shown me that student movement, knowledge movement, time on tasks, autonomous behaviour, and how groups work are actions and behaviours that can be observed within a Thinking Classroom.

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Appendix A

The base of a solid is the region enclosed by the circle, $x^2 + y^2 = 1$. The cross sections perpendicular to the x-axis are equilateral triangles with one of the edges along the circle. Find the volume of the solid.

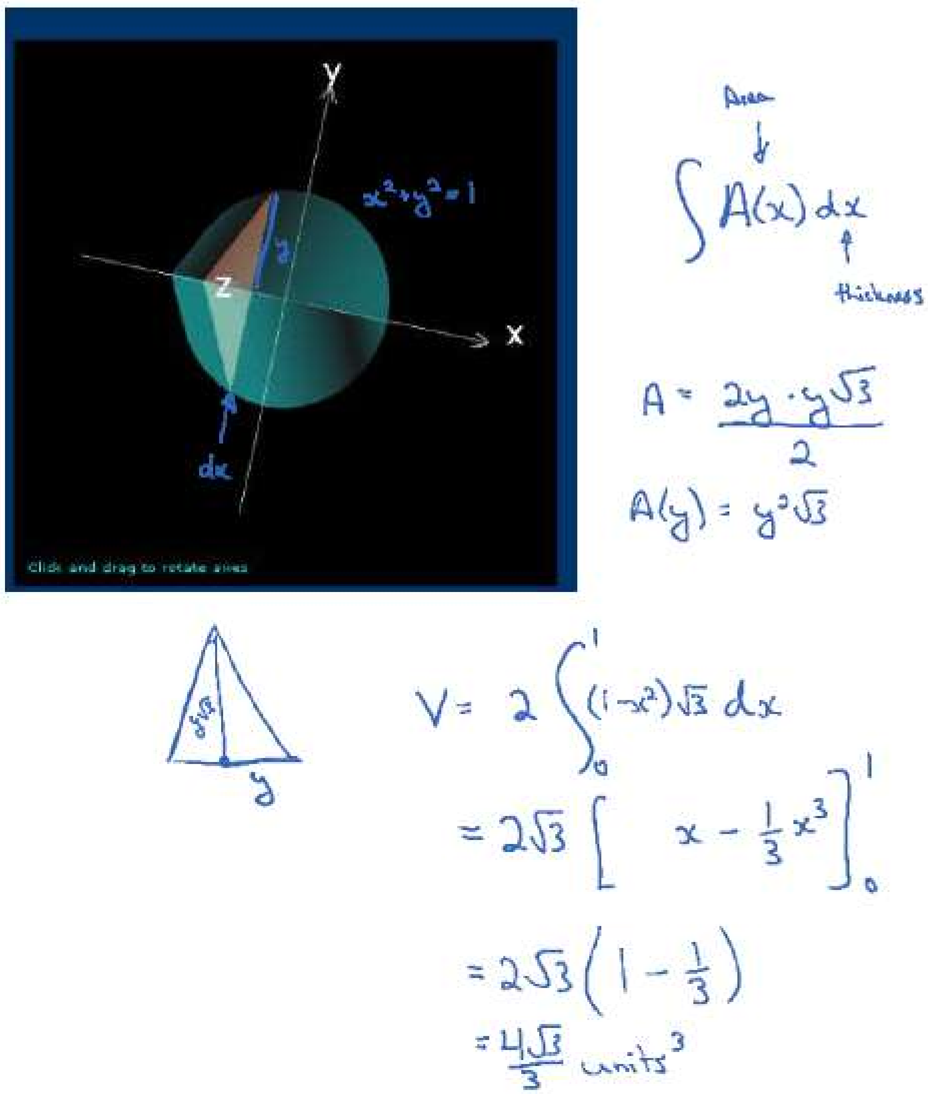


FIGURE 28. SAMPLE SOLUTION FROM CLASS NOTES

Appendix B

There are 25 race cars and a track that can only race 5 cars at a time. If there is no timer, and a car's performance never changes. Devise a strategy to determine the top three race cars (gold, silver, and bronze). How many races are necessary?

Appendix C

Solve the system graphically and verify your solution.

$$2x^2 + 16x + y = -26$$

$$x^2 + 8x - y = -19$$

(McAskill, B., Watt, W., Balzarini, E., Bonifacio, L., Carlson, S., Johnson, B., Kennedy, R., Wardrop, H., 2011)

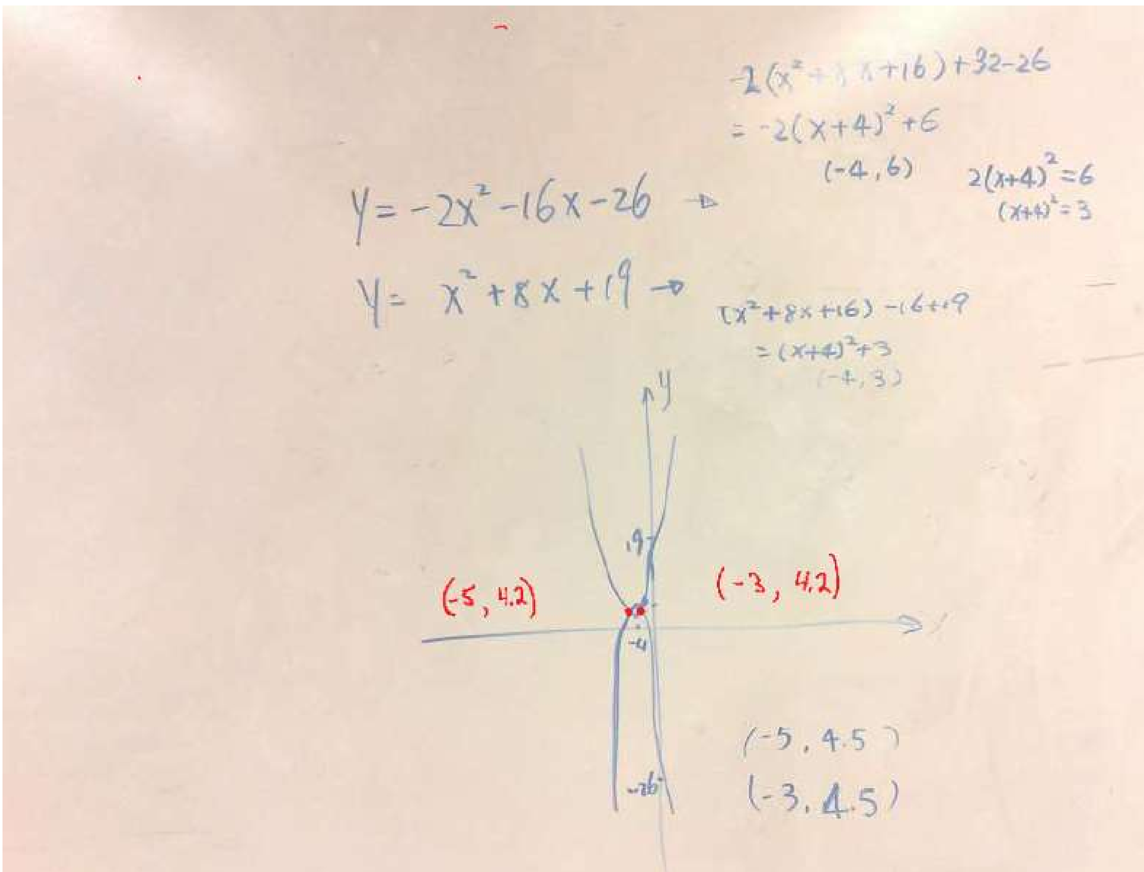


FIGURE 29. SAMPLE SOLUTION FROM CLASS NOTES.

Appendix D

Solve for x: $$3x + 4y = -16$$

$$x = 4y$$

(McAskill, B., Watt, W., Balzarini, E., Bonifacio, L., Carlson, S., Johnson, B., Kennedy, R., Wardrop, H., 2011)

Appendix E

In a box, you have 13 white marbles and 15 black marbles. You also have 28 black marbles outside the box. You repeatedly remove two marbles from the box and follow these instructions each time:

If the two marbles are different colours, then put a white marble back in the box.

If the two marbles are the same colour, then put a black marble back in the box.

Continue this until only one marble remains in the box. What colour is the last marble?

Appendix F

Solve: $n - \sqrt{5 - n} = -7$

(McAskill, B., Watt, W., Balzarini, E., Bonifacio, L., Carlson, S., Johnson, B., Kennedy, R., Wardrop, H., 2011)

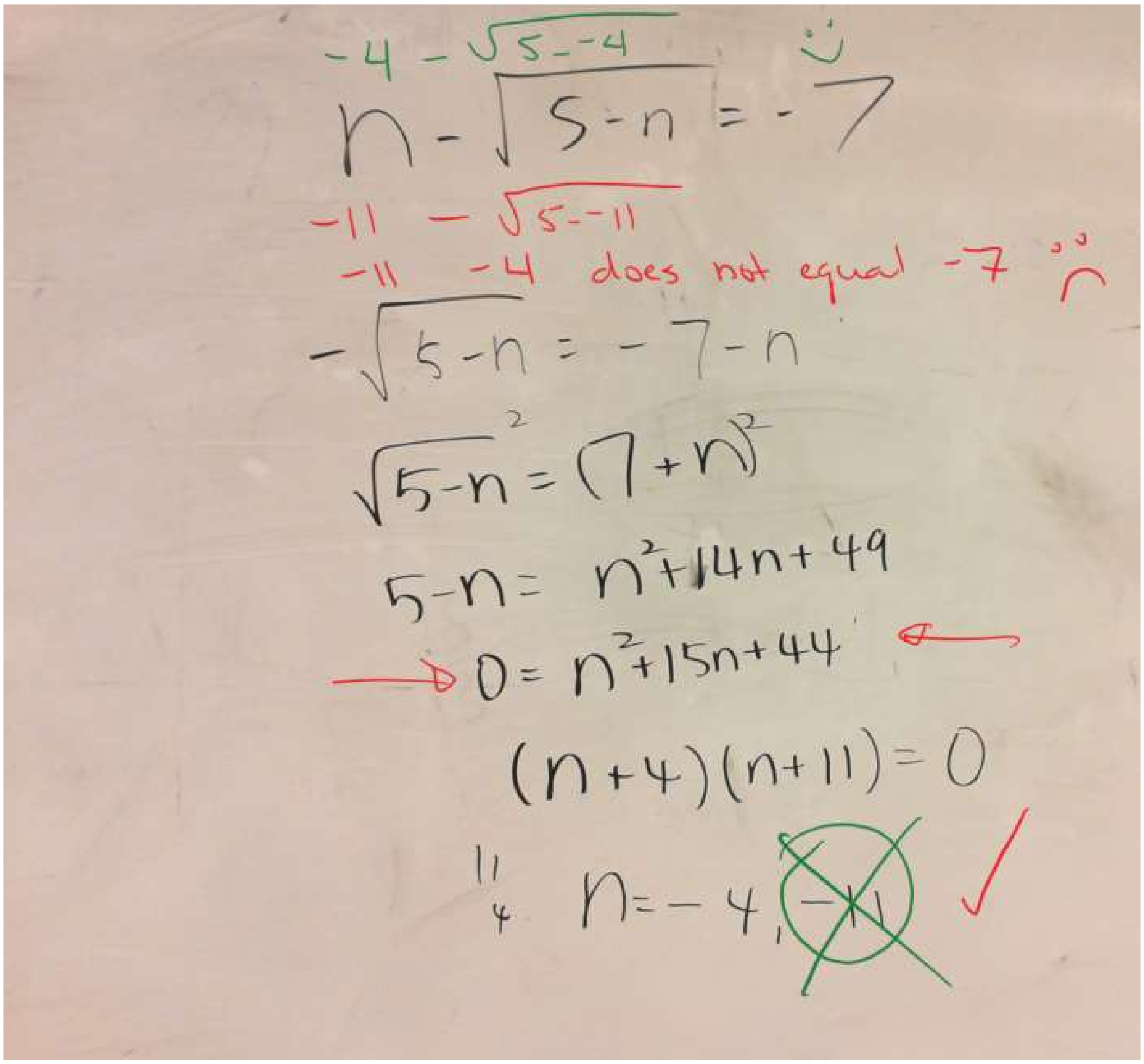


FIGURE 30. SAMPLE SOLUTION FROM CLASS NOTES.

Appendix G

A Canadian cargo plane drops a crate of supplies to aid-workers on the ground. The crate drops freely at first before a parachute opens to bring the crate gently to the ground. The crate's height, h, in metres, above the ground t seconds after leaving the aircraft is given by the following two equations.

$h = -4.9t^2 + 700$ represents the height of the crate during free fall.

$h = -5t + 650$ represents the height of the crate with the parachute open.

a) How long after the crate leaves the aircraft does the parachute open? Express your answer to the nearest hundredth of a second.
b) What height above the ground is the crate when the parachute opens? Express your answer to the nearest metre.
c) Verify your solution.

(McAskill, B., Watt, W., Balzarini, E., Bonifacio, L., Carlson, S., Johnson, B., Kennedy, R., Wardrop, H., 2011)

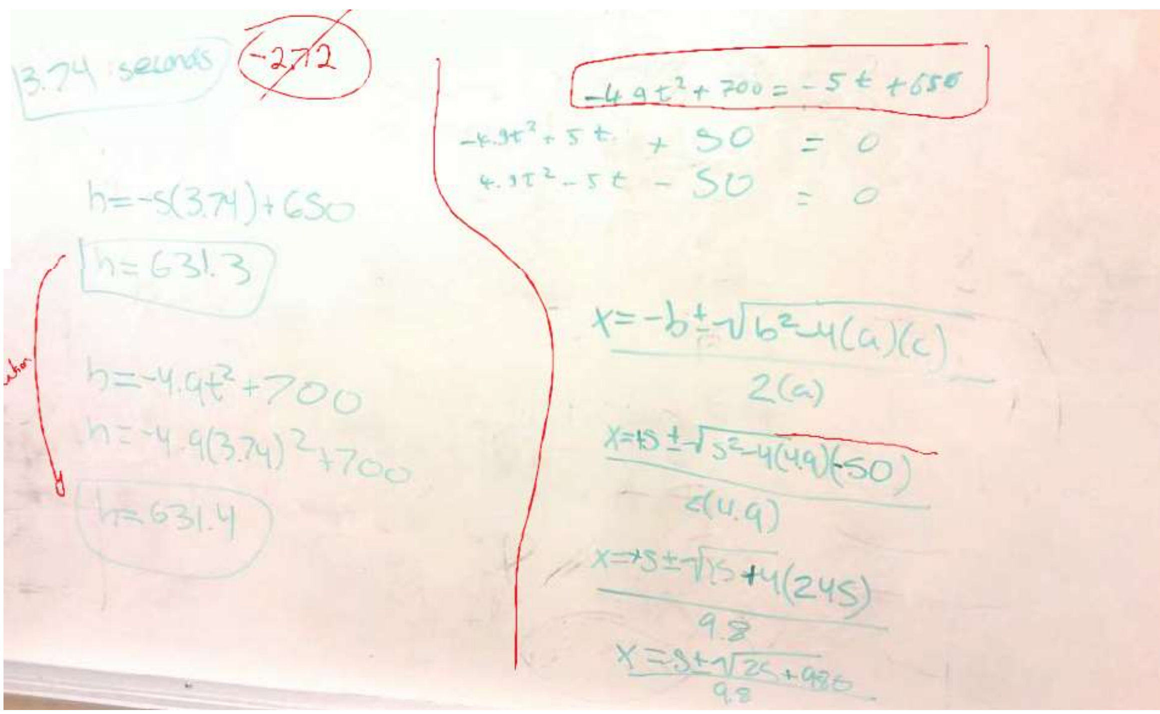


FIGURE 31. SAMPLE SOLUTION FROM CLASS NOTES.

Appendix H

a) Solve the following system of equations.

$$5x - y = 10$$

$$x^2 + x - 2y = 0$$

b) Verify your solution.

(McAskill, B., Watt, W., Balzarini, E., Bonifacio, L., Carlson, S., Johnson, B., Kennedy, R., Wardrop, H., 2011)

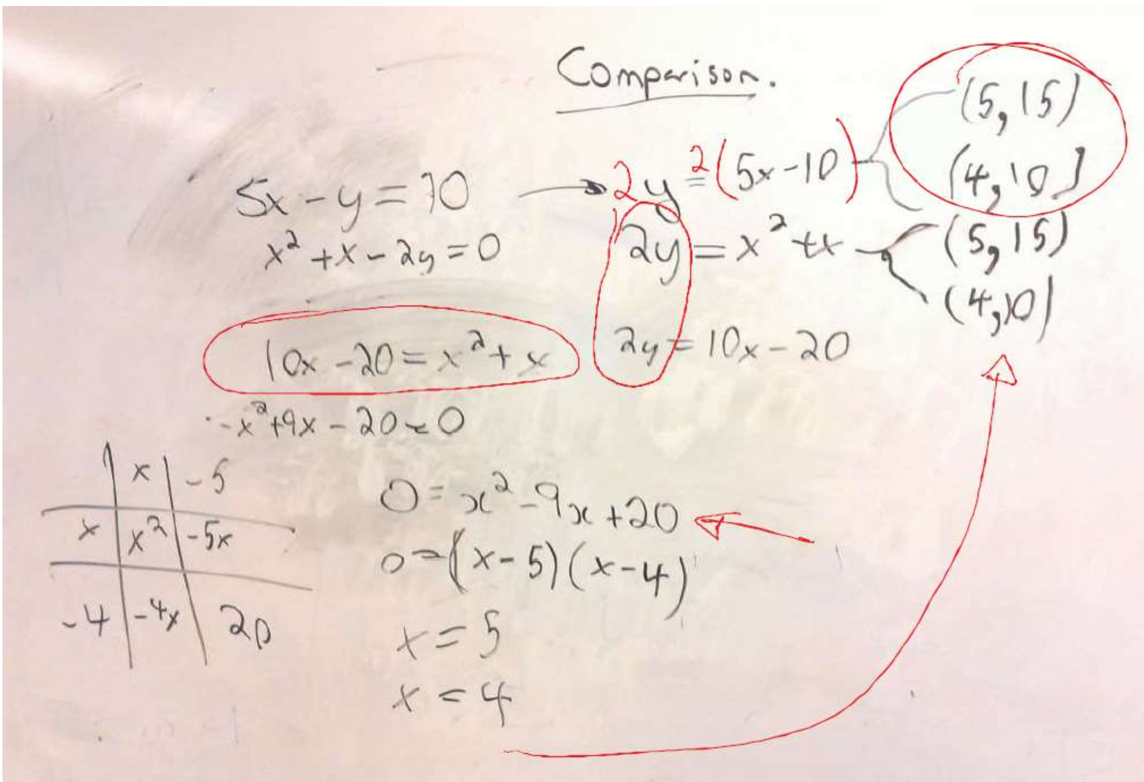
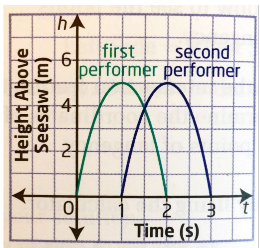


FIGURE 32. SAMPLE SOLUTION FROM CLASS NOTES.

Appendix I

Suppose that in one stunt, two Cirque du Soleil performers are launched toward each other from two slightly offset seesaws. The first performer is launched, and 1 s later the second performer is launched in the opposite direction. They both perform a flip and give each other a high five in the air. Each performer is in the air for 2 s. The height above the seesaw versus time for each performer during the stunt is approximated by a parabola as shown. Their paths are shown on a coordinate grid.



a) Determine the system of equations that models the performer's height during the stunt.
b) Solve the system graphically using technology.
c) Interpret your solution with respect to the situation.

(McAskill, B., Watt, W., Balzarini, E., Bonifacio, L., Carlson, S., Johnson, B., Kennedy, R., Wardrop, H., 2011)

Appendix J

Cups are randomly placed on a table; some are placed right-side-up and some are placed up-side-down. If you always select two cups at a time and change their orientation. Is it possible to make all of the cups right-side-up?

Appendix K

a) Graph $y < -2(x-1)^2 + 1$.
b) Determine if the point (2, -4) is a solution to the inequality.

(McAskill, B., Watt, W., Balzarini, E., Bonifacio, L., Carlson, S., Johnson, B., Kennedy, R., Wardrop, H., 2011)

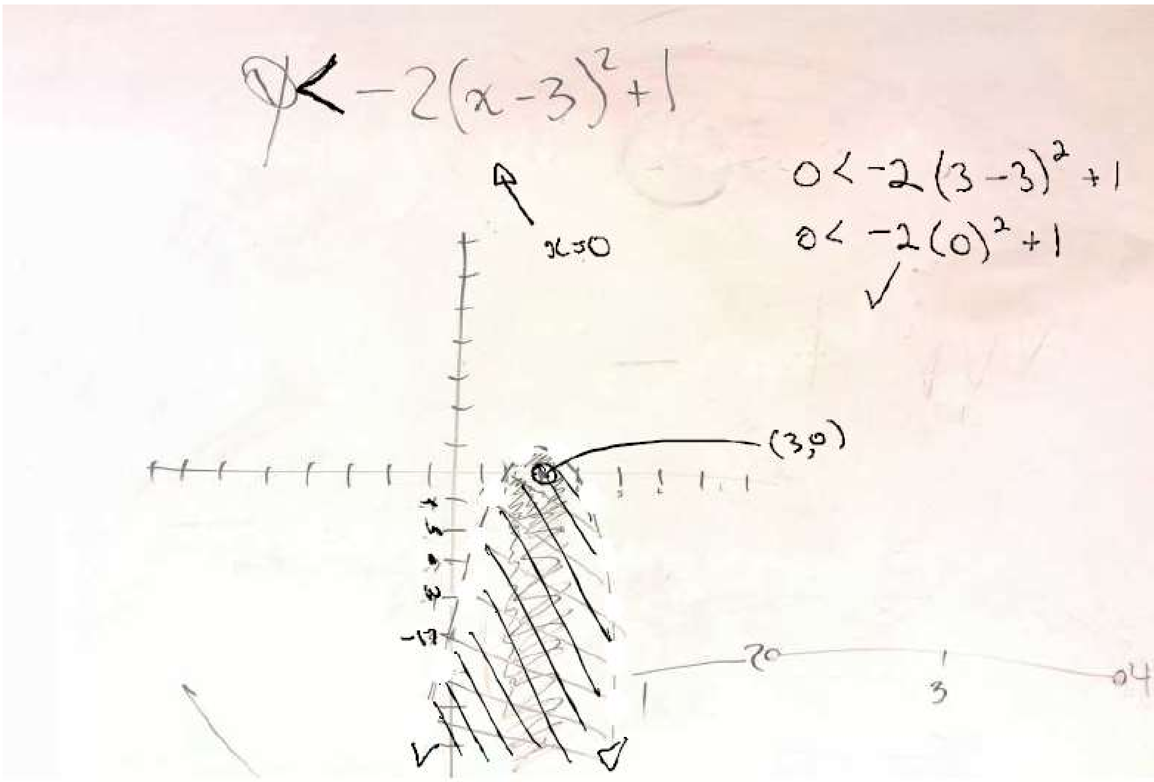


FIGURE 33. SAMPLE SOLUTION FROM CLASS NOTES.

Appendix L

a) Graph $y > (x-4)^2 - 2$.
b) Determine if the point (2, 1) is a solution to the inequality.

(McAskill, B., Watt, W., Balzarini, E., Bonifacio, L., Carlson, S., Johnson, B., Kennedy, R., Wardrop, H., 2011)

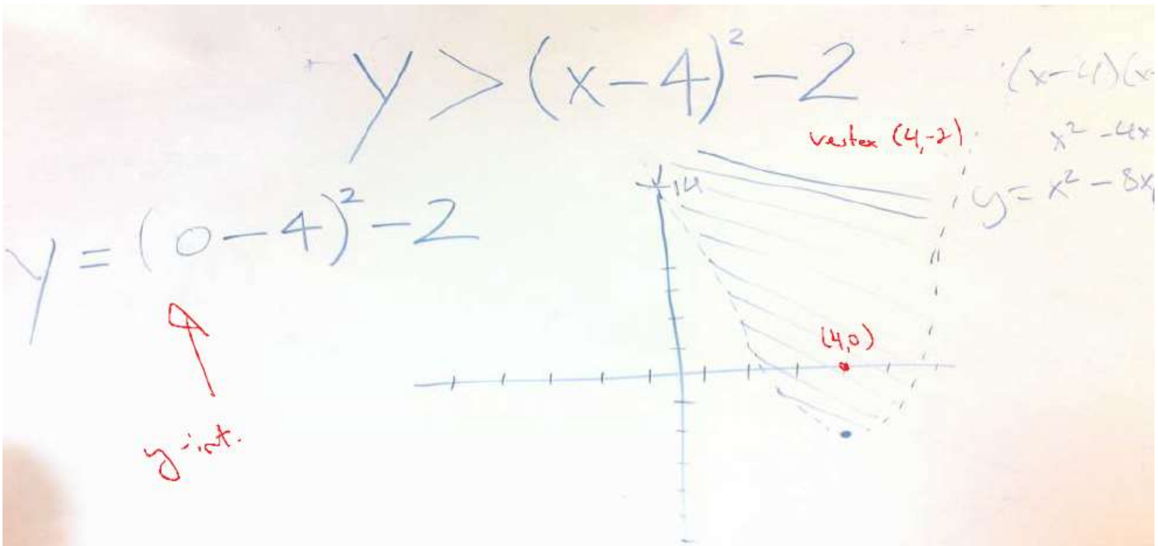


FIGURE 34. SAMPLE SOLUTION FROM CLASS NOTES.

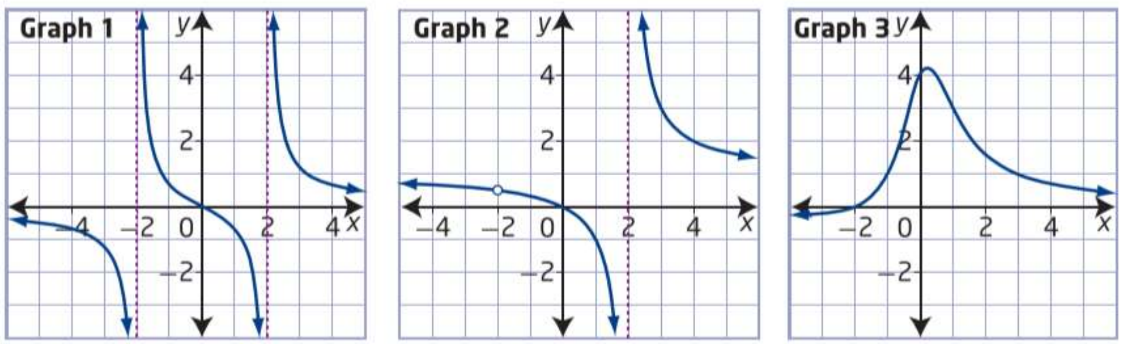
Appendix M

A round table spins with four deep pockets equally spaced around the table. Inside the pockets are cups that are either up or down. I can only determine the orientation of the cups once the table stops spinning by reaching both hands into two separate pockets and feeling the cups. At this point, I can change the orientation of any of the cups. As soon as my hands come out of the pockets, the table spins and then stops again. We need to come up with a plan to make all of the cups have the same orientation (all up or all down).

Appendix N

Match the equation of each rational function with the most appropriate graph. Give reasons for each choice.

$A(x) = \dfrac{x^2 + 2x}{x^2 - 4}$ $\quad B(x) = \dfrac{2x + 4}{x^2 + 1}$ $\quad C(x) = \dfrac{2x}{x^2 - 4}$



(McAskill, B., Watt, W., Balzarini, E., Johnson, B., Kennedy, R., Melnyk, .T., Zarski, C., 2012)