Evolution of Canadian Business Cycle with Terms of Trade

by

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Abstract

The terms of trade of commodity exporting small open economies are subject to large variations, and can be an important source of macroeconomic fluctuations. This paper quantifies the relationship between the terms of trade and the business cycle using a small open economy real business cycle model. I then use this model to explore the implications of terms of trade shocks as a source of business cycle fluctuations in Canada. Results suggest that terms of trade shocks have been increasingly important in Canada since the commodity price boom in 2002.

Keywords: Terms of Trade; Business Cycle

Dedication

To my wonderful parents, Ali Nawaz and Nasima Akther, my lovely sister Rifat and my girlfriend Naushin: for being a great encouragement and a boundless source of inspiration. I love you.

Acknowledgements

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Chapter 1

Introduction

"One of the most important forces powering Canada's economy today is the long-term strength in global prices for resources. For Canada, oil stands out. We have the world's third-largest reserves of crude oil. And we're number one in potash production. This means money in our pockets. In technical terms, it is a positive terms-of-trade shock. In non-technical terms, it's a gift. As the value of the resources we export goes up, and as the value of the products we import either remains low or rises more slowly-or, as our terms of trade improve-more wealth flows to Canadians."

Stephen Poloz, Governor of the Bank of Canada, 24 April 2014.

Trade openness makes small open economies subject to global shocks. Therefore, variation in the terms of trade is a natural candidate for explaining economic fluctuations in small open economies. Large and recurrent oscillations in the terms of trade are widely believed to be the main driving force of business cycle fluctuations in small open economies. The significant oil price increase in the 1970's, followed by abrupt declines in the latter decades, affected many industrial and developing countries. Terms of trade shocks affected the economies of the industrial nations mainly through the rising price of energy, as in [8] and [9].

One classic case of an industrialized nation being affected significantly by terms of trade variations is Canada. Given the fact that Canada's exports consist mainly of natural resources, terms of trade improvements began in 2002 when oil prices peaked at US $ 100 per barrel, gold prices went to US $800 an ounce, and the Canadian dollar rose above parity. As a result, due to higher commodity prices, an appreciating exchange rate, and increased contribution to global trade from the low-cost Asian manufacturers like China, Canadians could purchase a larger amount of imports using the proceeds from their exports. The terms of trade has not received much attention as a source of fluctuations in Canada. But

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in the last few decades, the terms of trade had noticeable impacts on consumption and investment. [1]

Until recently, Canada's terms of trade has been favorable. However, the economy experienced a deterioration in its terms of trade beginning in the last quarter of 2014. The major reason was plummeting oil prices, although other commodity prices declined as well.

Due to these terms of trade fluctuations, quantifying the effects of terms of trade shocks has become an important issue. Previous models have provided conflicting views about its effects on macroeconomic outcomes ([19],[11], and [4]). These previous studies employed various methods and data sets. The objective of this paper is to perform a quantitative analysis of terms of trade variations using a recent data set from 1981(1) to 2015(3), and to offer some useful insights about the Canadian economy.

I conduct an empirical analysis using [18]'s model to examine whether terms of trade shocks are an important source of economic fluctuations in Canada. Traditional RBC research has mainly focused on technology shocks, measured by Solow residuals, as the main driver of economic fluctuations. In contrast, my study explores whether and how international forces can be an important source of fluctuations over and above technology shocks. As an empirical extension to explore time variation in the role of terms of trade fluctuations, I split the data into two sub-samples. I find that terms of trade fluctuations have always been important, but their importance increased beginning in the 1990s.

The paper is organized as follows. Section 2 provides a brief review of the literature. In sections 3 and 4, I describe the data and present some stylized facts about the Canadian business cycle. Section 5 describes the model and characterizes the equilibrium. The calibration is presented in section 6. Section 7 reports the simulation results and provides an evaluation of the model. In section 8, robustness checks are presented. Section 9 provides a discussion of the terms of trade story for Canada. Finally, section 10 provides some concluding remarks.

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[1]See:http://www.statcan.gc.ca/pub/11-624-m/11-624-m2008018-eng.htm

Chapter 2

Related Literature

The implications of terms of trade shocks for small open economies has been a topic of controversy at least since the early work of [12] and [10], that has come to be known as the Harberger-Laursen-Metzler (HLM) effect. Both these papers used a simple static Keynesian approach. Later, [21] and [25] were among the first to examine the effects of terms of trade shocks in an inter-temporal optimizing framework.

My paper complements and builds on the literature investigating the macroeconomic effects of terms of trade shocks. Many papers in this literature have analyzed the macroeconomic consequences using calibrated business cycle models. A decade after Obstfeld's work, [19] attempted to assess the importance of terms of trade shocks as a source of business cycle fluctuations not only for small open developed economies, but also for developing economies. The key result of Mendoza's analysis is the importance of terms of trade shocks as a driving business cycle force in both advanced and developing small open economies. The paper argues that terms of trade shocks can account for nearly 50 % of the observed variability in output. However, when simulated with only terms of trade shocks, it accounts for about 88 % of the actual GDP in the benchmark developed economy.[1] In addition, to match features of a standard developing economy, [11] extends [19] by incorporating a richer production side in order to capture the distinctive features of developing economies. Using a variance decomposition method related to vector auto-regression model (VAR), the paper concludes that terms of trade shocks account for about 44 % of output volatility in Africa. In a model calibrated to Canada, [17] concludes that a 10 percent transitory decline in the terms of trade leads to an equal decline in output and a 20 percent decline in investment. The main contribution of this paper is empirical. I document the growing importance of terms of trade shocks for Canada using a calibrated small open economy real business cycle (RBC) model. The model's structure is based on [18], and I consider an updated dataset to capture the consequences of more recent terms of trade variations. [2]

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[1]Output and GDP are used interchangeably throughout the paper.

[2]See Section 3.

With the development of time series econometrics, other papers in the literature however reached different conclusions. Instead of a calibrated real business cycle (RBC) model, the other methodology that has been adopted to isolate the effects of international price shocks is the vector auto-regression approach (VAR). This literature typically finds that terms of trade shocks have less significant effects than implied by the structural business cycle models. For instance, using a panel VAR and data from 75 developing economies, [4] finds that a 10 % deterioration in terms of trade reduces GDP by 1 percent. Moreover, these trade shocks are successful in explaining only about 30 percent of volatility in real GDP growth. In an earlier study, [1] find that terms of trade fluctuations can account for only 6 % of the GDP forecast error variance in Canada. Motivated by the mixed evidence regarding the importance of international external forces in the literature , [13] estimates a canonical small open economy RBC model for five countries using Bayesian techniques. [3] Constructing a variance decomposition for each shock, they find that terms of trade shocks appear to be unimportant for generating business cycle fluctuations. They conjecture that this result might be due to the simple production structure in their model. Despite all these contrasting results using methodologies other than DSGE, there has been an exception in the literature. [6] estimate a VAR on Canadian data and find that shocks to external forces, such as interest rate shocks, shocks to US output as well as world export price shocks, contributes up to 74 % of the output forecast error variance output in Canada. However, the contribution of each shock is not reported which makes it difficult to assess the importance of individual shocks.

As my paper is interested in the business cycle fluctuations for a small open economy, I use [18]'s model to highlight empirically that terms of trade variations are important, at least for the last two decades in Canada.

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[3]These countries include: Canada, Australia, New Zealand, Chile and Mexico.

Chapter 3

Data

In order to analyze the performance of the theoretical economy, quarterly data is required to represent the equivalent of the variables in the model. The variables are output, consumption, investment, labor hours, terms of trade and net exports. The purpose of this section is to discuss how the data measurements are matched to the structure of the model described in chapter 5. However, for now, it is important to note that the model discussed in this study is a one-sector model.

Data for this study run from 1981(1) to 2015(3), and is chosen based on data availability.[1] All the data, except for the terms of trade, is obtained from the Canadian Socioeconomic Database (CANSIM). [2]

Gross Domestic Product (GDP) is the market value of all final goods and services produced in an economy during a given period of time and thus represents output in the model. GDP is generally calculated as the sum of consumption, investment, government expenditure, and net exports. However, the theoretical economy to be discussed in this paper is very abstract as it contains no government sector. Therefore, I construct GDP as a sum of consumption, investment and net exports.

Following [7] the final consumption expenditure from all sectors in the economy represents the consumption variable which also includes government expenditure.

[5] stated that investment for a one sector economy should correspond to the sum of gross fixed capital formation from all sectors, consumption of consumer durables, changes in inventories and net exports. Consumption of durable goods are included in investment rather than in consumption expenditure because they are seen as additions to the household's stock of capital. However, due to the unavailability of suitable data, investment measure does not include consumer durables and the resulting addition to output. Consumption of durable goods is a part of measure for consumption expenditure. Figure 3.1

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[1]Number in parenthesis refers to the quarter.
[2]For more details, see Appendix at the end.
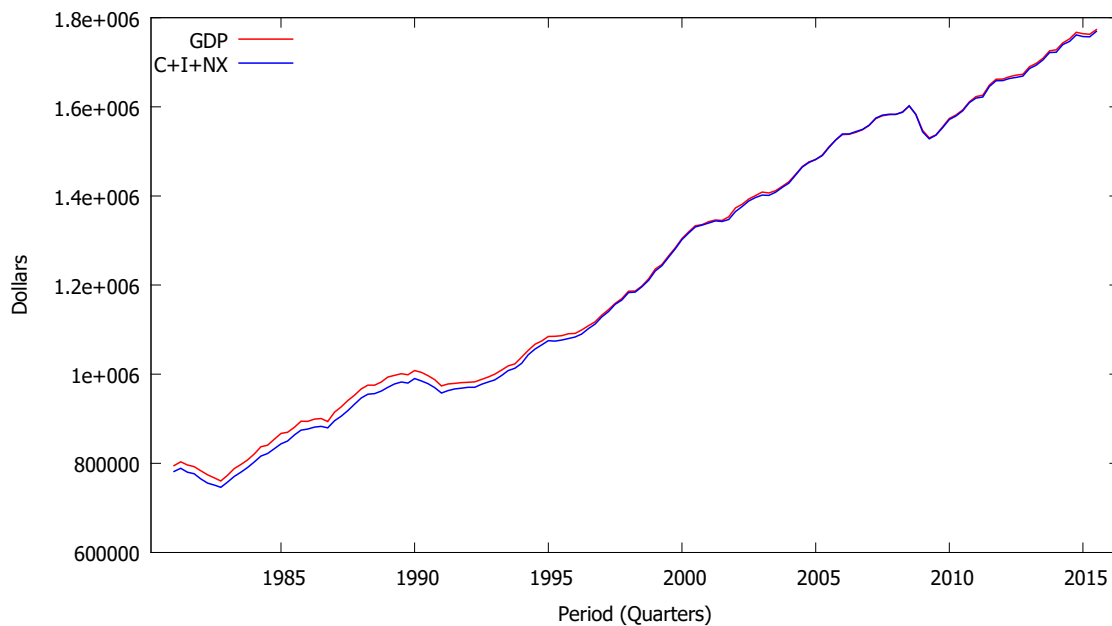
Figure 3.1: Output Construction (Data vs Constructed)

shows the sum of consumption, investment and net exports constructed against GDP from the data which shows that the two measures are very close to each other.

Labor input is a multi-faceted concept and can cover broad definitions. The main interest is in the intensive margin of labor input and hence, it is represented by total hours worked. Since there is no household production sector or farm sector, this is a measure of hours worked by all labor engaged in the production of goods and services in the non-farm business sector. An intensive margin such as total hours worked is presumed to be a better measure of labor input than an extensive margin such as civilian employment because it captures changes in weekly hours, changes in the proportion of part-time workers, overtime hours and annual leave.

Net exports are constructed by the standard textbook definition as the difference between exports of goods and services and imports of goods and services. Finally, the terms of trade is defined as the relative price of exports to imports.

Having constructed the data, I now turn to present some stylized facts about the Canadian business cycle.

Chapter 4

Stylized Facts of Canadian Business Cycle

Before developing the model, it is important to take a look at the Canadian data, in order to establish some stylised facts that the theoretical model will attempt to replicate. In Table 4.1, I present some unconditional moments from the data and some measures of variation of macroeconomic variables across time.

[15], found that "business cycles are all alike". This indicates that there are common features to business cycles, therefore stating that country specific peculiarities and factors from institutions such as central banks and governments do not influence business cycles. In order to evaluate the performance of the model discussed in this paper, the cyclical component needs to be extracted from the data. I apply the Hodrick-Prescott (HP) filter to extract the cyclical components.[1]

Figure 4.1 reports the amplitude of the fluctuations in aggregate variables in order to assess their relative magnitudes and measure the correlation of aggregate variables with real output to capture the extent to which variables display co-movement.

Table 4.1: Canadian Business Cycle Statistics

Variable	Std. Dev.	Rel SD.	Correlation with Output
Output	1.48	1	1
Consumption	0.67	0.45	0.74
Investment	5.21	3.52	0.88
Total hours worked	1.406	0.95	0.89
Trade Balance/Output	0.69	0.46	0.13

Given the period of analysis (1981(1)-2015(3)) and the above statistics in Table 1, some stylized facts of the Canadian business cycles are as follows:

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[1]Despite the controversy surrounding filtering methods, there is little evidence to reject HP filter. See [20], [24] and [2].
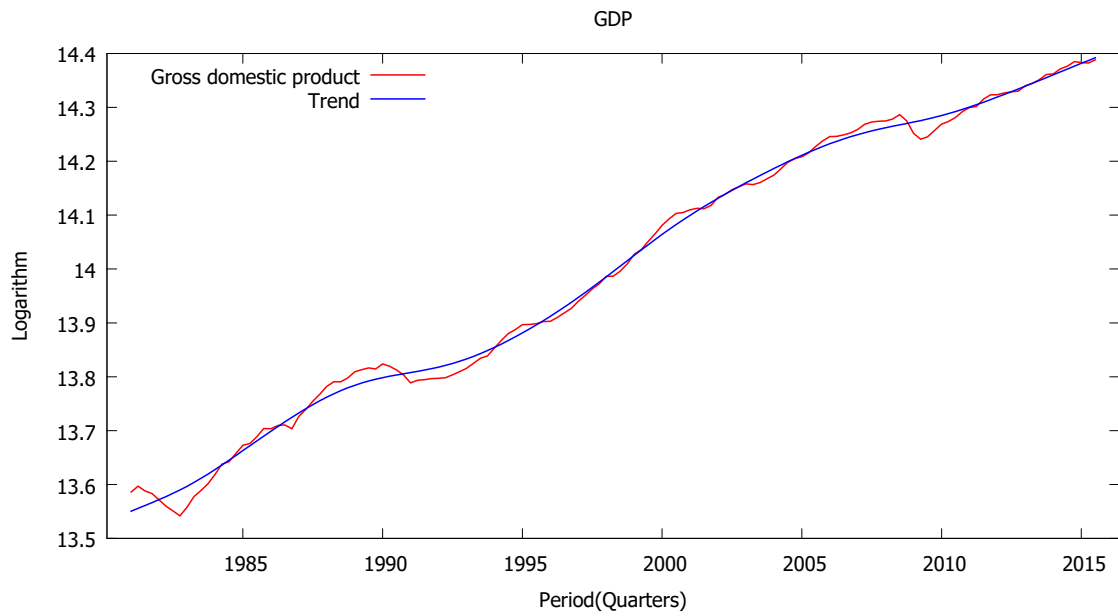
(a) Output and HP Trend
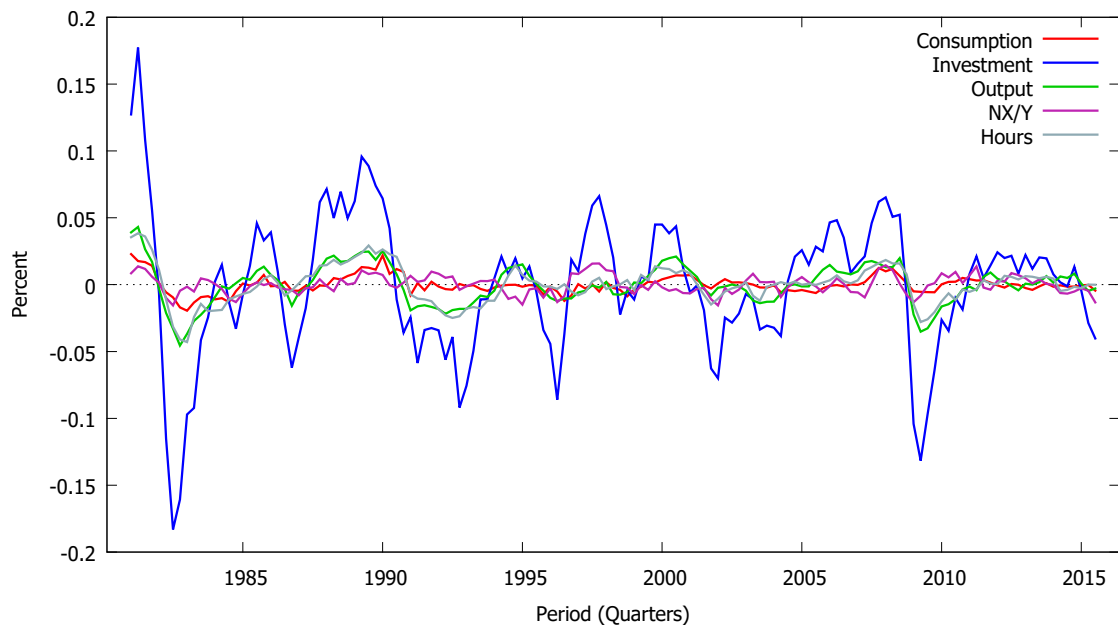
(b) Cyclical Component

Figure 4.1: HP Filtering

- Consumption is less volatile than output and is procyclical.

- Investment is more than twice as volatile as output and procyclical.

- Labor Hours is almost as volatile as output and procyclical as well.

- Trade Balance to output ratio is less volatile than output and weakly procyclical.

4.1 The Canadian Terms of Trade

The main objective of this paper is to identify the importance of terms of trade shocks for the Canadian economy. In this regard, I explore the time series fluctuations across time. Figure 4.2 shows the terms of trade - defined as the ratio of export prices to import prices for Canada between 1981(1) to 2015(3).[2] In the figure, the data line represents the level of the terms of trade in logs, while the trend line shows an HP-filtered trend. Figure 4.2(b) depicts the cyclical variation of terms of trade over time. [3]

Looking at the time series plot in Figure 3(a), it is obvious that Canada has experienced periods in which the terms of trade trend persistently decreased as well as periods in which the terms of trade trend persistently increased. Changes in the terms of trade is often large. The cyclical component shows that during the 1980s till the mid of 1990s Canada faced a sequence of terms of trade shocks that lead to deviations from the long-term trend of more than 2 %. Later in the early 2000s, the magnitude of the fluctuations changed dramatically. For instance during the mid 2000s, the trend terms of trade increased more than 4% above it's trend level. The increases in the terms of trade were mainly driven by rising commodity prices. Also, as Canada is a small open commodity exporting economy, the rise in the commodity prices were triggered by the accelerating growth in countries like India and China. During 2008 to 2010, the Great Recession led to the largest recorded drop in the Canadian export prices. However, the fluctuations became more stable after this period and is still below its trend due to the current bust in the oil prices. This substantial change in the terms of trade variations during the sample period suggests that significant changes in the terms of trade may have played a large role for Canada. It is clear from Figure 3(b) that terms of trade variations has been fluctuating with a greater magnitude in the last two decades. To quantify the effects of these large fluctuations in the last two decades, I conduct a sub-sample analysis. As will be discussed later, this analysis shows the importance of these changes in the Canadian economy for the last two decades.

These are the primary facts that characterizes the Canadian business cycle. I now turn to the theoretical model of the economy.

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[2]See Appendix A for the data source.

[3]However, it is important to note that the plot of the terms of trade suggests that using HP filtering arguably understates cyclical fluctuations. As can be seen from the figure, much of the variation during the past decade is attributed to the trend.

(a) Terms of Trade

(b) Cyclical Variations of TOT
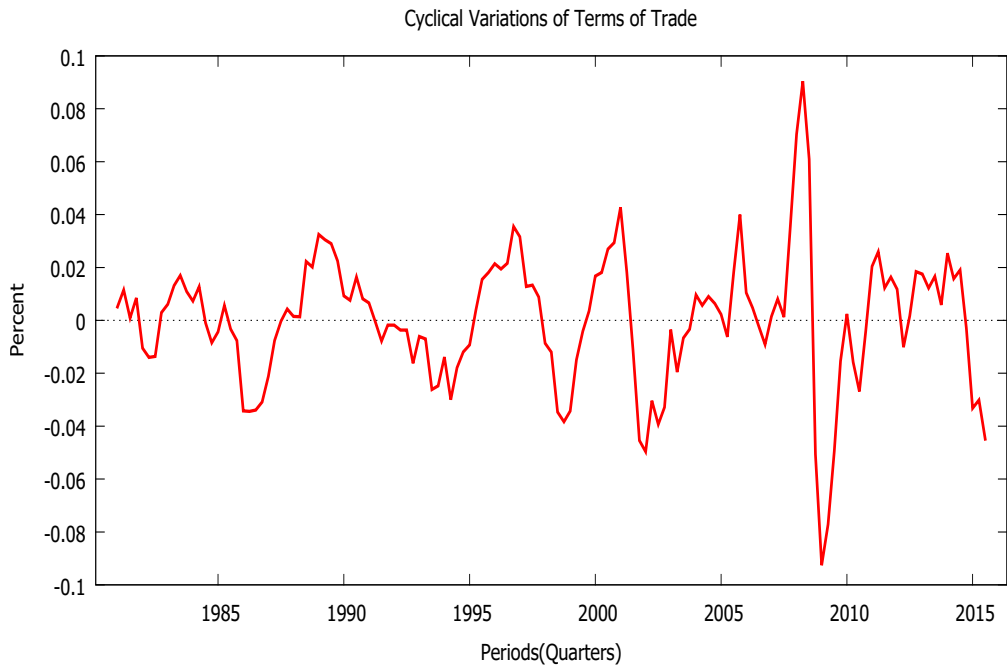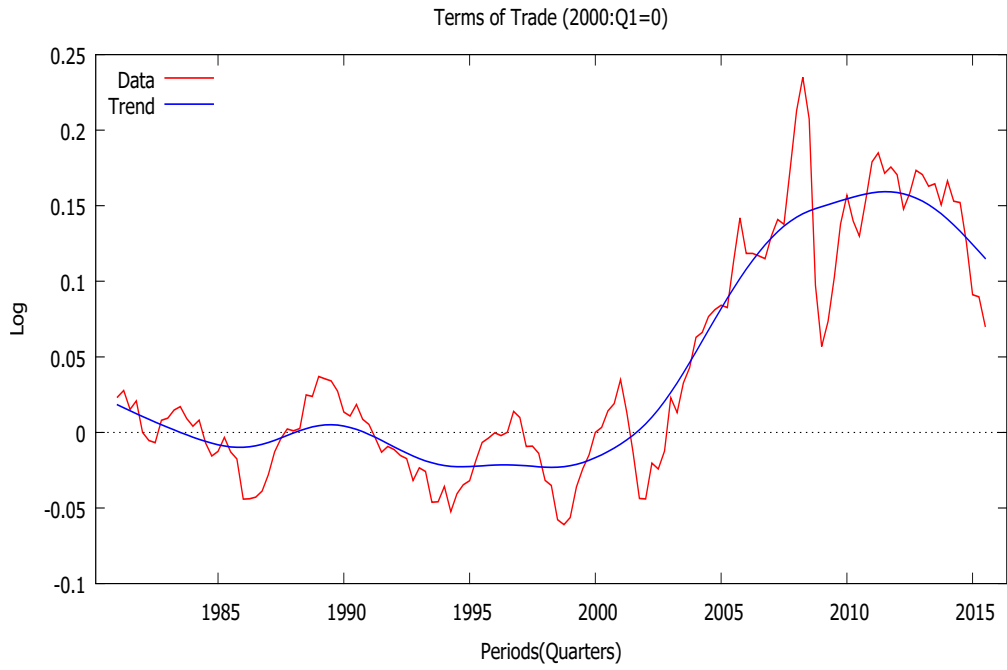
Figure 4.2: Terms of Trade Variation

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Chapter 5

Model

The theoretical environment follows the standard small open economy neoclassical model developed by [18] without any further modifications. The economy is populated by infinitely-lived identical households and an indeterminate number of competitive firms producing a single good. Households choose consumption, savings and labor supply to maximize their expected lifetime utility. The consumption basket is composed of two types of good, a home good and an imported good. In addition, they have access to two types of assets, physical capital and a one-period foreign bond transacted in international markets. The price of the bond is determined exogenously. Firms produce the single good with endogenous capital and labor as inputs. Time is discrete. The terms of trade is defined as the price of home good relative to the imported good.

5.1 Firms

I also assume that all firms are identical and normalize the number of firms to 1. Since, the firms are assumed to be small, their hiring decisions do not affect the wages they have to pay their workers or the rental price they pay for their capital. Production is characterized by constant returns to scale and thus, production of the tradeable good occurs through a Cobb-Douglas aggregate production function:

$$Y_t = Z_t K_t^\theta N_t^{1-\theta}$$

where Y_t is output, N_t is the labor, K_t is capital, $0 < \theta < 1$ is the capital share of income and Z_t is total factor productivity. The technology, Z_t follows a random process, and evolves exogenously according to the first-order autoregressive AR(1) process:

$$lnZ_t = (1 - \rho_Z)lnZ_{ss} + \rho_Z lnZ_{t-1} + \epsilon_t^Z$$

where ρ_Z is a positive constant between $0 < \rho_Z < 1$. This is the autocorrelation coefficient. The ϵ_t shocks are identically and independently distributed such as $\epsilon_t^Z \sim N(0, \sigma_{\epsilon_Z}^2)$. Profit maximization of any firm will be:

$$\pi_t = P_t^H Y_t - w_t N_t - r_t K_t$$

where P_t^H is the price of the home-produced good, r_t is the rate of return on capital, and w_t is the real wage.

Maximizing the firm's profits,

$$K_t : r_t = \theta P_t^H Z_t \left(\frac{K_t}{N_t}\right)^{\theta-1}$$

and

$$N_t : w_t = (1-\theta) P_t^H Z_t \left(\frac{K_t}{N_t}\right)^{\theta}$$

which implies that the optimal amounts of capital (K) and labor (N) to be hired by each firm, are such that the marginal products of the inputs equal their respective factor prices, r_t and w_t.

5.2   Households

The economy is populated by a time-invariant S identical and infinitely-lived individuals who gain utility out of consumption and leisure. Hence, accordingly every individual j will maximize the following discounted expected utility function:

$$\sum_{t=1}^{\infty} \beta^t \left(lnC_t - A_L \frac{N_t^{1+\psi}}{1+\psi} \right)$$

where C_t is the aggregate consumption, N_t is hours worked, ψ is the inverse of the labor supply elasticity, and A_L is a constant used to calibrate average labor supply in the model to match the data.

The household's aggregate consumption basket is composed of the following:

$$C_t = \frac{(C_t^H)^{1-\eta}(C_t^F)^{\eta}}{(1-\eta)^{1-\eta}(\eta)^{\eta}}$$

C_t^H are home produced goods and C_t^F are foreign-produced goods in the household's consumption basket. The parameters $1 - \eta$ and η denote the relative weights of home and foreign-produced goods in the household's consumption basket. Next, P_t denotes the consumer price index for the aggregate consumption C_t. This leads to:

$$P_t = (P_t^H)^{1-\eta}(P_t^F)^{\eta}$$

P_t^H and P_t^F are the prices of home and foreign-produced goods respectively. However, it can be easily deduced from the household optimization problem that ensures the demand for home and foreign goods:

$$C_t^H = (1 - \eta)\left(\frac{P_t}{P_t^H}\right)C_t$$

$$C_t^F = \eta\left(\frac{P_t}{P_t^F}\right)C_t$$

The household's budget constraint is as follows:

$$P_t C_t + I_t + Q_t B_{t+1} = w_t N_t + r_t K_t + B_t$$

where capital evolves as given below:

$$K_{t+1} = I_t + (1 - \delta)K_t - \frac{\mu}{2}K_t\left(K_{t+1} - K_t\right)^2$$

Here in the above equation, households have access to two assets: domestic capital and a one-period risk-free bond denominated in terms of foreign consumption goods to smooth their consumption. P_t is the relative price of consumption goods in terms of foreign goods (numeraire), B_t is the bonds that are transacted by the household in the international market and are also denominated in terms of foreign consumption goods, Q_t is the price of one-period riskless bond faced by the domestic economy, which is exogenous and following [23]:

$$\frac{1}{Q_t} = 1 + r^\star + \phi(e^{B_{t+1} - \bar{b}} - 1)$$

where $r^\star$ is the exogenous foreign interest rate and $\bar{b}$ is the steady state level of foreign debt holdings. The equation assumes that the price of debt is sensitive to the level of debt. In choosing the optimal amount of debt, the representative agent does not internalize the fact that the country faces an upward-sloping supply of loans.[1] Lastly, there is also an adjustment cost associated with the capital holdings which is denoted by μ.

The price of the foreign good is defined as the numeraire. Therefore, the terms of trade in the above model is defined as $tot = P_t^H$. This implies from the previous definition:

$$P_t = (tot)^{1-\eta}$$

And terms of trade (tot) follows an AR(1) process as given below:

$$lntot_t = (1 - \rho_{tot})lntot_{ss} + \rho_{tot}lntot_{t-1} + \epsilon_t^{tot}$$

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[1] This purpose of this assumption is only implemented to ensure stationarity of the bond holdings.

The following equations are derived from the household's utility maximization problem:

$$\frac{1}{C_t} = \lambda_t P_t \tag{5.1}$$

$$A_L N_t^\psi = \lambda_t w_t \tag{5.2}$$

$$\lambda_t \left[1 + \mu\left(K_{t+1} - K_t\right)\right] = E_t \beta \left[\lambda_{t+1}\left(r_{t+1} + (1-\delta) + \mu\left(K_{t+2} - K_{t+1}\right)\right)\right] \tag{5.3}$$

$$Q_t \lambda_t = \beta E_t \lambda_{t+1} \tag{5.4}$$

where λ_t is the Lagrange multiplier.

5.3   Competitive Equilibrium

A competitive equilibrium is defined as a sequence of quantities $\{Y_t, I_t, C_t, N_t, K_{t+1}, B_{t+1}\}_{t=0}^\infty$, prices $\left\{w_t, r_t, P_t^H, P_t\right\}_{t=0}^\infty$ and the stochastic processes . Given this, households and firms maximize at the equilibrium prices.

The following are the optimality conditions:

- Households:

$$C_t A_L N_t^\psi = \frac{w_t}{P_t}$$

$$\frac{1}{C_t P_t}\left[1 + \mu\left(K_{t+1} - K_t\right)\right] = E_t \beta \left[\frac{1}{C_{t+1} P_{t+1}}\left(r_{t+1} + (1-\delta) + \mu\left(K_{t+2} - K_{t+1}\right)\right)\right]$$

$$Q_t \frac{1}{C_t P_t} = \beta E_t \frac{1}{C_{t+1} P_{t+1}}$$

$$Q_t B_{t+1} + P_t C_t + I_t = P_t^H Y_t + B_t$$

$$K_{t+1} = I_t + (1-\delta)K_t - \frac{\mu}{2}\left(K_{t+1} - K_t\right)^2$$

- Firms:

$$Y_t = Z_t K_t^\theta N_t^{1-\theta}$$

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$$w_t = (1 - \theta) P_t^H Z_t \left(\frac{K_t}{N_t} \right)^\theta$$

$$r_t = \theta P_t^H Z_t \left(\frac{K_t}{N_t} \right)^{\theta - 1}$$

- Terms of Trade

$$P_t = (P_t^H)^{1-\eta}$$

5.4 Steady-State Relationship

From the model's stationary equilibrium conditions derived in the previous section, the following are the steady state equations:

$$c A_L N^\psi = \frac{w}{P^H}$$

$$Q = \beta$$

$$I = \delta K$$

$$Y = K^\alpha N^{1-\alpha}$$

$$\alpha \frac{Y}{K} = \frac{1}{\beta} + \delta - 1$$

$$\frac{1}{Q} = 1 + r^\star$$

$$(Q - 1)\overline{B} + C + I = Y$$

I normalize the steady state values for the home price and TFP to be 1. Therefore, $\mathbf{P^H = 1}$ and $\mathbf{Z = 1}$. Based on these normalizations and the parameters indicated in Table 1, the steady state values are computed. This exercise is analogous to that in [18].

Moreover, as is standard in the literature, I assume that households supply $\frac{1}{3}$ of their total time to market activity. Therefore to solve the system of equations:

$$K = \left(\frac{\frac{1}{\beta} + \delta - 1}{\alpha N^{1-\alpha}} \right)^{\frac{1}{\alpha - 1}}$$

Now plugging the above equation into the production function , we have:

$$Y = \left(\left(\frac{\frac{1}{\beta} + \delta - 1}{\alpha N^{1-\alpha}} \right)^{\frac{\alpha}{\alpha-1}} N^{1-\alpha} \right)$$

Next investment follows:

$$I = \delta \left(\frac{\frac{1}{\beta} + \delta - 1}{\alpha N^{1-\alpha}} \right)^{\frac{1}{\alpha-1}}$$

Consumption is found using the above equations:

$$C = (1 - \alpha) \frac{\left(\left(\frac{\frac{1}{\beta} + \delta - 1}{\alpha N^{1-\alpha}} \right)^{\frac{\alpha}{\alpha-1}} N^{1-\alpha} \right)}{A_L N^{1+\psi}}$$

Lastly, the trade-balance-to-output ratio implies the following steady state equation:

$$tby = \frac{Y - C - I}{Y}$$

Variables without time subscripts denote steady state values. I have also replaced the FOC's for the firm's problem into the above equations.

Chapter 6

Calibration

A simple and incomplete description of calibration is that it refers to setting the model's parameters to values that are consistent with empirical studies and indeed this is what this section does. However, the term calibration more precisely refers to the use of economic theory as the basis of restricting the model's structure (for example restricting preferences so that they are consistent with steady balanced growth) and at the same time using the implications of that structure in the measurement of data.[1] This means that while measurement helps us to establish a theory, simultaneously it must also be used in deciding what and how to measure.

Table 6.1: Calibration for the small open economy RBC model to match Canadian data

Parameter	Value	Description	
β	0.98	Discount factor	Standard in the literature
θ	0.4	Capital share	Standard in the literature
δ	0.0243	Quarterly depreciation rate	To match steady state $\frac{I}{Y}$
r^*	0.01	Quarterly world interest rate	As in [18]
μ	0.5	Capital adjustment cost parameter	To match the volatility of Investment
ϕ	0.001	Portfolio adjustment cost parameter	As in [23]
η	0.25	Import share of consumption	To match total imports-GDP ratio
ψ	0.455	Inverse Frisch Elasticity	Standard in the Literature
σ_z	0.006	Volatility of TFP Shock	Calibrated from the data
σ_{tot}	0.015	Volatility of TOT Shock	Calibrated from the data
ρ_{tot}	0.53	Persistence of TOT Shock	Calibrated from the data
ρ_z	0.96	Persistence of TFP Shock	Calibrated from the data

Some of the model parameters are calibrated and some of them are set based on previous literature. The parameters that have been calibrated to match the steady state of the Canadian economy are: the depreciation rate, capital share of income and the import share

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[1][5] provides detailed explanations on calibration.

of consumption.[2] The world interest rate is set following [18]. The parameter ϕ is set as a small value to ensure the stationarity of the model without having major impacts on the model. The inverse Frisch elasticity is set following [18]. Lastly, the capital adjustment cost parameter is set in order to match the volatility of investment.

In order to simulate the theoretical environment for comparing them to the Canadian economy, I obtain measures for the technology shocks and terms of trade shocks that are fed into the model. The simulated series are then de-trended using the HP filter before they are compared to actual data. Next using the data, TFP is measured as output minus share-weighted inputs:

$$ln\hat{Z}_t = lnY_t - \theta lnK_t - (1-\theta)lnN_t$$

Then, I begin by linearly detrending the measure of log TFP. As such, I regress the empirical measure of TFP on a constant and a linear time trend:

$$ln\hat{Z}_t = \zeta_0 + \zeta_1 t_t + u_t$$

After obtaining the estimates of ζ_0 and ζ_1, I take the measured residual,$\hat{u}_t$, and estimate an AR(1) process:

$$\hat{u}_t = \rho_z \hat{u}_{t-1} + \epsilon_t^Z$$

Following the above exercise, I obtain the following estimates: $\rho_z = 0.96$ and the standard deviation of the residual of 0.006. The above procedure is applied to the terms of trade shock process as well, which results in $\rho_{tot} = 0.53$ and $\sigma_{tot} = 0.015$. Table 6.1 reports the parameter values.

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[2]All the parameters are calibrated from the quarterly data (1981(1)-2015(3)).

Chapter 7

Results

In this section I first study the dynamic responses of the endogenous variables to terms of trade shocks. In particular, I examine the responses of output, aggregate consumption, investment , employment and trade balance-to-output ratio to fluctuations in terms of trade. In this regard, I present the impulse responses derived from the model. Second, I demonstrate that the model can successfully match some broad features of the Canadian macroeconomic data. Finally, I report the results of variance decompositions of each shock in the theoretical economy.

7.1 Impulse Responses

In order to generate the impulse responses, I use the recursive equilibrium laws of motion to examine the model's implications for terms of trade and technology shocks. Since each endogenous variable responds to a change in the stochastic component, it is important to see how the model responds to their fluctuations. The linear laws of motion are given by

$$x_t = Px_{t-1} + Qm_t,$$

$$y_t = Rx_{t-1} + Sm_t$$

and

$$m_t = Lm_{t-1} + \epsilon_t.$$

where x_t is the vector of state variables, y_t is the vector of control variables and m_t is the vector of exogenous stochastic variables.

The state variables are defined as $x_t = [\tilde{K}_{t+1}, \tilde{B}_{t+1}]$, the control variables as $y_t = [\tilde{Y}_t, \tilde{C}_t, \tilde{I}_t, \tilde{N}_t, t\tilde{b}y_t]$ and the stochastic variables as $m_t = [\tilde{Z}_t, t\tilde{o}t_t]$ where a tilde over a variable stands for log deviation of the variable from its steady state. The first order conditions that are found in section 5 are non-linear and so it is usually difficult, if not impossible, to solve the problem analytically. Linear models are relatively easier to solve. However, the

problem is in converting a non-linear model into a linear approximation which can then be solved and used to analyze the underlying non-linear system. A standard method for linear approximation is to log-linearize a model around its stationary state. My solution technique for finding the policy matrices follow methods of undetermined coefficients, and are discussed in length in the appendix.[1] In all cases, the response is to a single positive shock of 1 standard deviation that occurs in period 2.

7.2 Impulse Response Function Analysis

Figure 7.1 presents the response of the baseline model to a positive one percent transitory terms of trade shock. This shock increases the price of the economy's output relative to the price of consumption and investment goods. This makes households choose to increase investment and increase real wages, leading to an expansion in employment and therefore output. However, the response of employment dies out quickly and remains below its steady state level for several quarters. This is because a positive terms of trade shock increases the wealth of households. Also, as they receive disutility from working, households transform some of their wealth into leisure. Ouput increases initially, and later after 15 quarters decreases. This response of output is due to a decline in investment and the employment level later on. The intuition behind this is that since the agents realize that the shock is temporary, they bring forward production to take advantage of the temporary high relative export prices.
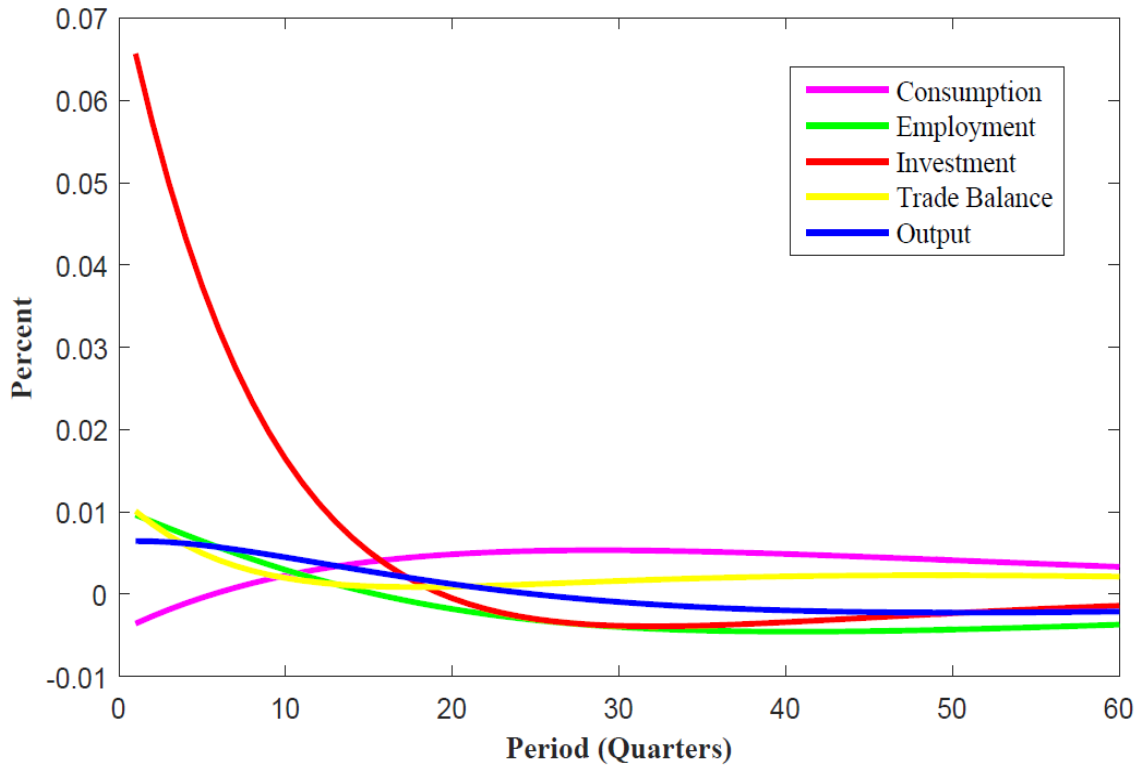
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[1]See [26] for an exposition.

Figure 7.1: Response to TOT

On the other hand, consumption follows an initial contraction. There are two offsetting factors in this contraction. First, an improvement in the terms of trade increases household wealth. But at the same time, they expect the relative price of the consumption good in terms of importables to fall in future. This expected decrease in the CPI implies an increase in the real interest rate. This feature in the economy encourages households to postpone consumption until prices return back to their steady state. It is important to note that the substitution effect of this higher interest rate is greatest immediately following the shock and later the income effects of the increase in wealth dominate the substitution effect. Both the small positive income effect and larger substitution effect will depress consumption. In addition to the substantial increase in output and decrease in consumption, trade balance increases. As the shock dissipates, consumers draw down the foreign assets that they accumulated during the improved trade balance and use it to finance future consumption. Lastly, the economy is subject to Harberger-Laursen-Metzler effect consistent with [19] and [22]. [2]

In conclusion, the response of the economy can be thought of as a standard consumption smoothing response to a transitory income shock. The economy produces more when relative

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[2]Harberger-Laursen-Metzler effect is the conjecture or result that a terms of trade deterioration will cause a decrease in savings due to the decrease in real income, and therefore that a real depreciation will cause an increase in real expenditure.

prices are high, and save part of the extra income to finance higher consumption in the future.

7.3 Model Fit to the Canadian Data

In this section, I discuss the outcome of simulating the artificial economy. Key summary statistics are presented in Table 7.1. 1000 simulations of 139 periods in length (same as the number of periods in time series from the actual economy) are computed. Each simulated time series has been filtered using the HP filter. The statistics reported in the table are averages of those computed for each of the 1000 simulations.[3] I compare some unconditional moments obtained from the data to those implied by the simulations of the artificial economy under two scenarios. The purpose of this exercise is to calculate the explanatory power of each shock. The simulation is conducted for each shock separately, one at a time.

Table 7.1: Summary Statistics: Actual data vs. Terms of Trade Shock vs. Technology Shock

Variables	Data	Terms of Trade Shocks	Technology Shocks
Output	1.48(1)	1.28(1)	1.52(1)
Consumption	0.70(0.45)	1.01(0.79)	0.77(0.51)
Investment	5.21(3.52)	6.76(6.69)	5.88(3.30)
Labor	1.40(0.94)	1.90(1.01)	0.52(0.29)
TB/Y	0.6(0.45)	0.55(0.54)	0.8(0.46)

As mentioned above, the simulation results with both the shocks are reported in column 3 and 4 of Table 3, respectively. When the economy is subject to terms of trade shocks, the benchmark model is quite successful in matching the key macroeconomic variables in the Canadian national accounts, with a few exceptions. In particular, output, labour hours, and the trade-balance to output ratio match the data quite well. There is a bit of exaggeration in the volatilities of investment and consumption. Investment is less volatile with the terms of trade shocks in comparison with the technology shocks. However, consumption seems to be reasonably close to the data. On the other hand, the trade-balance to output ratio and labor hours understate the volatilities relative to their empirical counterparts.

7.4 Forecast Error Variance Decomposition

The objective of this section is to assess the contribution of each shock to aggregate business cycle fluctuations by investigating the results of variance decompositions. Forecast Error Variance Decomposition (FEVD) seperates the variation in an endogenous variable into the

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[3]Relative standard deviation is shown in parenthesis.

component shocks. It shows the relative importance of each random innovation in affecting each variable over a particular forecast horizon.[4] The results are computed from the calibrated theoretical model. I explore the four macro aggregates: Output, Consumption, Investment and Trade Balance-to-Output ratio under both the shocks.

Table 7.2: Variance Decomposition

Variables	Technology Shocks	Terms of Trade Shocks
Output	58.17	41.83
Consumption	38.79	61.21
Investment	37.55	62.45
Trade-Balance to Output Ratio	44.59	55.41

Table 7.2 reports the variance decomposition for Canada. A number of interesting observations emerge from this exercise. Firstly, productivity shocks explain about 58 % of the variation in output. This implies that productivity shocks are the main contributor for output fluctuations. On the other hand, terms of trade shock dominates substantially in terms of explaining most of the behavior in Investment, Consumption and Trade Balance-to-Output ratio over the business cycles. Terms of trade shocks account for a significant amount of variation in comparison to productivity shocks, except for output.

These results are in line with [19] and [11], where agents hold full information about the persistence of terms of trade shocks, and are the main driving force for small open economies. In this paper, terms of trade fluctuations explain slightly higher than that shown in [19]'s analysis.

The large contribution of terms of trade shocks allows me to make some conflicting comparison with the results obtained using VAR. For example, [1] argues that terms of trade shocks accounts for only 6 % of the fluctuations in output for Canada. Similarly, [14] uses a monetary small open economy model and concludes that terms of trade shocks can only explain about 2 % of the variations of output in Canada and four other industrialized countries. On the other hand, [13] observes that the contribution of terms of trade shocks to output is almost zero. They argue that in order to accurately capture the notion of terms of trade shocks as a source of business cycle fluctuations, a richer production structure is required as in [19]. In comparison, despite using the similar production structure, I find that terms of trade shocks contributes a significant amount generating macroeconomic fluctuations in Canada.

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[4]This computation is done in Dynare. Dynare uses standard time series techniques for linear state space models. For forecast error variance dcompositions, see [16]: New Introduction to Multiple Time Series Analysis, Chapter 2.3.3.

Chapter 8

Sensitivity Analysis

In this section, I conclude that the results are robust to alternative parameter values. In particular, I test the sensitivity of the model's dynamic behaviour by resetting the values for (i) the inverse Frisch elasticity (ψ) and (ii) the parameter regulating the sensitivity of the risk-free interest rate to the foreign debt level (ϕ). These alternative parameter choices are performed one at a time, keeping the other parameters the same as in the benchmark model. In order to see the effects, figures 8.1 and 8.2 depict the impulse responses to a one standard deviation terms of trade shock in Canada using the above specified parameter values.

Benchmark model   With $\psi = 10$
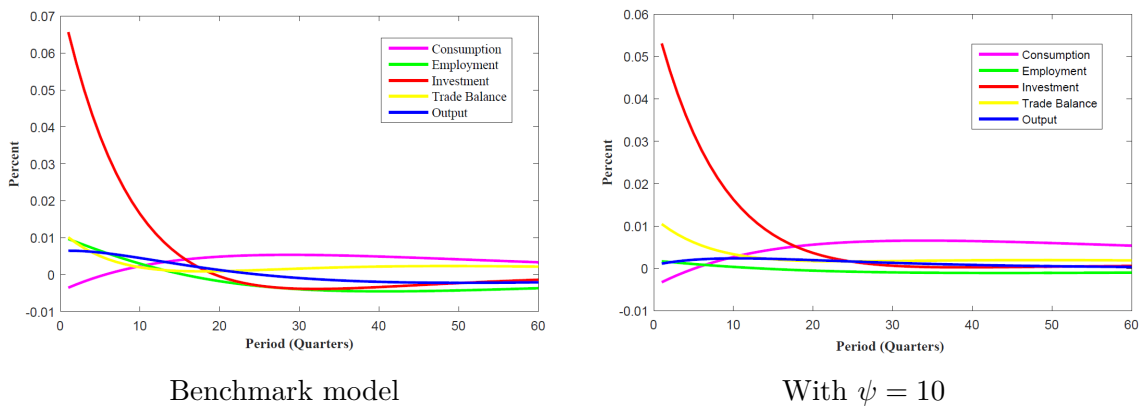
Figure 8.1: Responses to Terms of Trade Shocks with Alternative Parameter Values (ψ).

I increase the labor supply elasticity from 0.455 to 10. The business cycle effects are not seriously effected. This is because an increase in the elasticity decreases the desire of the households to alter their labor supply in response to the macroeconomic shocks. Intuitively, households now are less willing to respond to the shocks and engage in accumulating foreign assets. Figure 8.1 shows the response.

As described in chapter 5 of the paper, I have assumed an extremely low value of ϕ in order to lessen the effects of this parameter on the dynamic responses of the model. In contrast, other papers that have estimated the value of this parameter have tended to estimate higher values. A higher value of ϕ penalizes the economy for altering it's accumulation of foreign assets. Although setting the parameter to 10^{-4} decreases the time it takes to reach the steady state, the results generated by this alternative parameter choice are largely consistent to the benchmark model as can be seen in Figure 8.2.
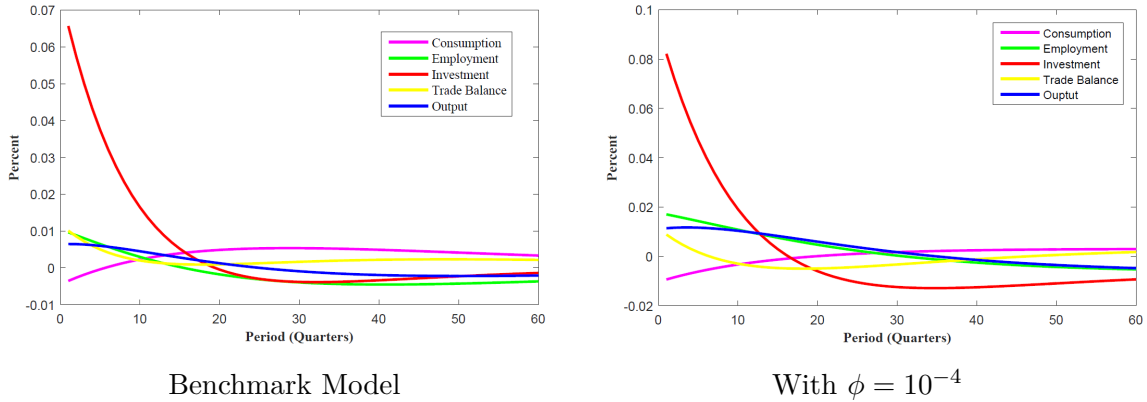

Benchmark Model   With $\phi = 10^{-4}$

Figure 8.2: Responses to Terms of Trade Shocks with Alternative Parameter Values (ϕ).

Finally a word on the asset market structure. This paper uses a conventional strategy of assuming exogenously incomplete asset markets. However, a natural question can therefore arise - How would complete asset markets change the importance of terms of trade fluctuations? According to [3] the main objective of incorporating a financial asset into the model is for smoothing out consumption. They argue that complete markets does not have significant implications when productivity shocks are transitory. In particular, the parameterization of the stochastic process imposes different business cycle implications under both the structure. Similarly, [23] also confirms that the dynamics are not different with complete asset markets. These results provide some degree of robustness to my results, since the terms of trade shocks here are transitory.

Chapter 9

The Terms of Trade Story for Canada

The most obvious question that comes to mind is what role did terms of trade variations play for Canada over the years ? Looking at Figure 3(b) it is clear that the cyclical variations began fluctuating more vigorously with time. Two reasons might explain these variations. First, policy changes in NAFTA in the 1990s might have had substantial effects on trade openness in Canada. Second, the commodity price boom and the Global Financial Crisis (GFC) had significant effects in the last two decades. Accordingly, I split the sample into two parts to quantify the growing importance of terms of trade shocks in Canada across time. The data series is divided into two samples: 1981(1) to 1995(4) and 1996(1) to 2015(3).

In terms of simulations, everything except the volatility of the terms of trade are kept constant in both the sub samples. I estimate the AR(1) process for the terms of trade shocks separately for both the sample periods to obtain the standard deviations. These parameters are then fed into the model described in section 5. The procedure for running the simulations is as follows, I draw a sample of length T, where $T = \{60, 78\}$ with their corresponding standard deviation from a normal distribution.[1] Finally, I generate the shocks with the standard deviation for each sub sample from two sampling distributions. This way I can observe how much of the increased variance in the macroeconomic aggregates are due to the terms of trade fluctuations.[2]

Table 9.1 reports the variance decomposition results of both the samples.

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[1]Note that 60 and 78 are the total number of quarters in both the sub-samples respectively.
[2]This is seen as the increase of volatility from 0.009 to 0.017 in the data.

Table 9.1: Sub Sample Variance Decomposition

| Sample | 1981(1)-1995(4) | | 1996(1)-2015(3) | |
Variables	TFP Shocks	TOT Shocks	TFP Shocks	TOT Shocks
Output	89.04	10.96	66.33	33.67
Consumption	76.48	23.52	16.89	83.11
Investment	79.18	20.82	19.21	80.79
Trade-Balance to Output Ratio	95.48	4.52	56.91	43.09

This exercise suggests some interesting observations. First, it is clear that terms of trade shocks have been a major driving force for the Canadian economy during the last two decades. In particular, the variation in the macroeconomic aggregates explained by the terms of trade shocks increased significantly for the second half of the sample conditional on the volatility of terms of trade shock. Both consumption and investment are driven highly by terms of trade shocks in the second half, such that there is an increase of almost 60 % for both the variables. In addition, terms of trade shocks also account for more than 40 % of the variation in the trade balance for the latter half of the time series. Lastly, the variance of output increased by 20 % in the second sub sample for the terms of trade shock although, the TFP shock still dominates.

These variations highlight an important feature of the Canadian economy that coincides with reality. Canadian economy has been strongly driven by the terms of trade shocks in the last two decades. The main reason is structural changes in the Canadian economy across time. In particular, Canada has become more resource dependent since the early 2000s. During this time, a demand-driven resource boom occurred, along with innovations in technology. There has been a coordinated global expansion, combined with the emergence of Chinese demand for natural resources that stimulated demand for most Canadian resource products. As a result of the rising demand for resources, particularly energy, the prices that Canadian producers received in international transactions has increased. For instance, one positive aspect worth mentioning is the oil sector, which became an important global resource for Canada due to the price hikes by 25% in 2002. This allowed Canada to gain comparative advantage in the resource sector which had substantial effects on the Canadian economy. Specifically, the Canadian economy became more sensitive to changes in terms of trade making it one of the main driving force for generating domestic business cycle fluctuations in the last two decades.

This exercise is roughly consistent with the historical macroeconomic observations of the Canadian economy. The increase in variance of the macroeconomic aggregates in the second half of the sample can be attributed mainly to the growing importance of terms of trade variations across time.

Chapter 10

Conclusion

This study has described how a small open economy neoclassical model can be tailored to analyze the role of terms of trade shocks in business cycles. By developing linear approximations to the model's equilibrium conditions, I have been able to quantify the response of the Canadian economy to unanticipated changes in terms of trade. Furthermore, using a variance decomposition method, I computed the contribution of terms of trade shocks to the macroeconomic fluctuations in Canada. Contrary to previous studies, the paper's main finding suggests that terms of trade shocks have a major contribution to the fluctuations of important macroeconomic aggregates. Most importantly, the paper conjectures that the importance of the terms of trade as a source of business cycle fluctuations has been increasing in recent years in Canada. In particular, these implications are consistent with the evolution of the Canadian economy since the commodity price boom of 2002.

Naturally, a number of extensions to this work deserve attention. First, it may be worthwhile to estimate the model for emerging small open economies, like Brazil or Argentina. Also, the results seem to be model dependent, and therefore, including a non-traded goods sector in order to make a richer production economy might produce interesting results. There could possibly be some spillover effects to the rest of the economy due to changes in the relative prices of the tradeables sector.

Finally, it is important to note that the economy here does not suffer from market failures. In other words, the outcome of the economy is Pareto optimum (given the asset market structure). Thus, a possible extension could be to include nominal rigidities in the theoretical economy. This would allow us to examine the implications of terms of trade shocks for monetary policy.

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Appendix A

Data Sources

This appendix describes the data sources for the Canadian quarterly time series used in this paper. All data except for term of trade are sourced from the Canadian Socio-economic Information and Management (CANSIM). The time period covered is from 1981(1) to 2015(3).

Output: Real Gross Domestic Product,Billions of Chained 2007 Dollars, Quarterly, Seasonally Adjusted; CANSIM Table 380-0064.

Consumption: Household Consumption Expenditures, Billions of Chained 2007 Dollars, Quarterly, Seasonally Adjusted + Government consumption expenditures and gross investment, Billions of Chained 2007 Dollars, Quarterly, Seasonally Adjusted; CANSIM Table 380-0064.

Investment: Gross fixed capital formation + Durable goods, Billions of Chained 2007 Dollars, Quarterly, Seasonally Adjusted; CANSIM Table 380-0064.

Labor Hours: Labour force survey estimates (LFS), actual hours worked at main job by North American Industry Classification System (NAICS), Seasonally Adjusted; CANSIM Table 282-0092.

Net Exports: Exports of goods and services, Billions of Chained 2007 Dollars, Quarterly, Seasonally Adjusted - Imports of goods and services, Billions of Chained 2007 Dollars, Quarterly, Seasonally Adjusted; CANSIM Table 380-0064.

Terms of Trade: I constructed a terms of trade index by dividing the exports of goods and services deflator by the imports of goods and services deflator. Sample - 1981(1) to 2015(3). Source- OECD (`www.oecd.org`)

Appendix B

Log Linearization

We consider an equation of a set of variables X_t. According to Uhlig (1999), we define $\tilde{X}_t = lnX_t - ln\overline{X}$. The tilde variables are the log difference of the original variable from the value $\overline{X}$. The original variable can then be written as

$$X_t = \overline{X}e^{\tilde{X}_t}$$

since

$$\overline{X}e^{\tilde{X}_t} = \overline{X}e^{lnX_t - ln\overline{X}} = \overline{X}e^{\frac{lnX_t}{\overline{X}}} = \overline{X}\frac{X_t}{\overline{X}} = X_t$$

Accordingly, taking the log linearizations of the three first order conditions derives to

$$(1 + \psi)\tilde{N}_t + \tilde{C}_t = \eta\tilde{P}_t^H + \tilde{Y}_t$$

$$\mu\overline{K}(\tilde{K}_{t+1} - \tilde{K}_t) = \tilde{C}_t + (1-\eta)\tilde{P}_t^H - E_t(\tilde{C}_{t+1}) - (1-\eta)E_t(\tilde{P}_{t+1}^H) + \beta\alpha\frac{\overline{Y}}{\overline{K}}E_t(\tilde{Y}_{t+1}) + \beta\alpha\frac{\overline{Y}}{\overline{K}}\tilde{K}_{t+1}$$
$$- \beta\alpha\frac{\overline{Y}}{\overline{K}}E_t(\tilde{P}_{t+1}^H) - \beta\mu\overline{K}\tilde{K}_{t+1} + \beta\mu\overline{K}E_t(\tilde{K}_{t+2}) \quad \text{(B.1)}$$

and

$$\tilde{Q}_t = \tilde{C}_t + (1 - \eta)\tilde{P}_t^H - E_t\tilde{C}_{t+1} - (1 - \eta)E_t(\tilde{P}_{t+1}^H)$$

The log-linearized capital accumulation rule is as follows:

$$\tilde{K}_{t+1} = \frac{\overline{I}}{\overline{K}}\tilde{I}_t = (1-\delta)\tilde{K}_t$$

The aggregate resource constraint:

$$\overline{QB}(\tilde{Q}_t + \tilde{B}_{t+1}) + \overline{C}(\tilde{C}_t + (1-\eta)\tilde{P}_t^H) + \overline{I}\tilde{I}_t = \overline{Y}(\tilde{P}_t^H + \tilde{Y}_t) + \overline{B}\tilde{B}_t$$

$$\tilde{Q}_t = \phi\tilde{B}_{t+1}$$

Finally, the production equation log-linearized as follows:

$$\tilde{Y}_t = \tilde{Z}_t + \theta\tilde{K}_t + (1-\theta)\tilde{N}_t$$

Lastly, the trade balance is just linearized:

$$\tilde{tby}_t = -(1-\overline{tby})\tilde{Y}_t + \frac{\overline{C}}{\overline{Y}}\tilde{C}_t + \frac{\overline{I}}{\overline{Y}}\tilde{I}_t$$

The two stochastic process:

$$\tilde{Z}_t = \rho_Z\tilde{Z}_{t-1} + \epsilon_t^Z$$

$$\tilde{tot}_t = \rho_{tot}\tilde{tot}_{t-1} + \epsilon_t^{tot}$$

Solving the Log-Linear System

This subsection provides a brief overview of the way the log-linear version of the model is solved once it has been divided into a set of equations with expectations and a set without expectations. The division of the model is important because we try to keep the dimension of the second (the expectational) equation small and if possible, to have the matrix C of full rank and hence, invertible. The model is divided into the sets of matrix equations

$$0 = Ax_t + Bx_{t-1} + Cy_t + Dm_t,$$

$$0 = E_t[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lm_{t+1} + Mm_t]$$

and a stochastic process

$$m_{t+1} = Nm_t + \epsilon_{t+1},$$

34

where $x_t = [\tilde{K}_{t+1}, \tilde{B}_{t+1}]'$, $y_t = [\tilde{Y}_t \; \tilde{C}_t \; \tilde{H}_t \; \tilde{tby}_t]'$ and $m_t = [\tilde{Z}_t \; \tilde{tot}_t]'$

The solution for this economy is given by a set of matrices P, Q, R and S that describes the equilibrium laws of motion,

$$x_t = Px_{t-1} + Qm_t$$

and

$$y_t = Rx_{t-1} + Sm_t$$

.