Mathematical Needs in the Physics Classroom

By

Pouyan Khalili

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B.Sc. (Honours), Simon Fraser University, 2007

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Name: **Pouyan Khalili**
Degree: Master of Science
Title of Thesis: Mathematical Needs in the Physics Classroom

Examining Committee:

Chair: **Cécile Sabatier,** Associate Professor

Rina Zazkis
Senior Supervisor
Professor

Howard Trottier
Supervisor
Professor

Peter Iiljedahl
Internal/External Examiner
Associate Dean Academics

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Abstract

This study focuses on the physics teachers' views on the difficulties students have in physics that are mathematical in nature. While research in physics education attends to these difficulties, it does not attend to the teachers' voices in identifying and handling these difficulties. Nine physics teachers, currently teaching at the high school level in the Lower Mainland of British Columbia, Canada, participated in my study. I designed two questionnaires that inquired into my participants' perspectives on the mathematical issues their students face in their physics classes, and possible remedies to overcome the identified problems. The results echo previous research in identifying the areas of difficulty (e.g., fractions, trigonometry), and add particular examples of problems that hinder students' success. Furthermore, the results reveal that the most common resolution to mathematical difficulties in the physics classroom is to value the understanding of mathematical processes rather than memorizing an algorithm and number crunching.

Keywords: mathematics education, physics education, mathematical applications, conceptual understanding, teacher perspective, physics curriculum

Dedication

I dedicate this study to my family, my friends, my teachers, and to my students. Although this study is a small contribution to the educational community, it is a contribution that I undertook with a tremendous quality of seriousness. It is the service of educators that enables life's integrities to flower into society's value systems. It is my responsibility to grow and to serve, and this study enables both of those purposes.

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My students, who endure many challenges in order to learn.

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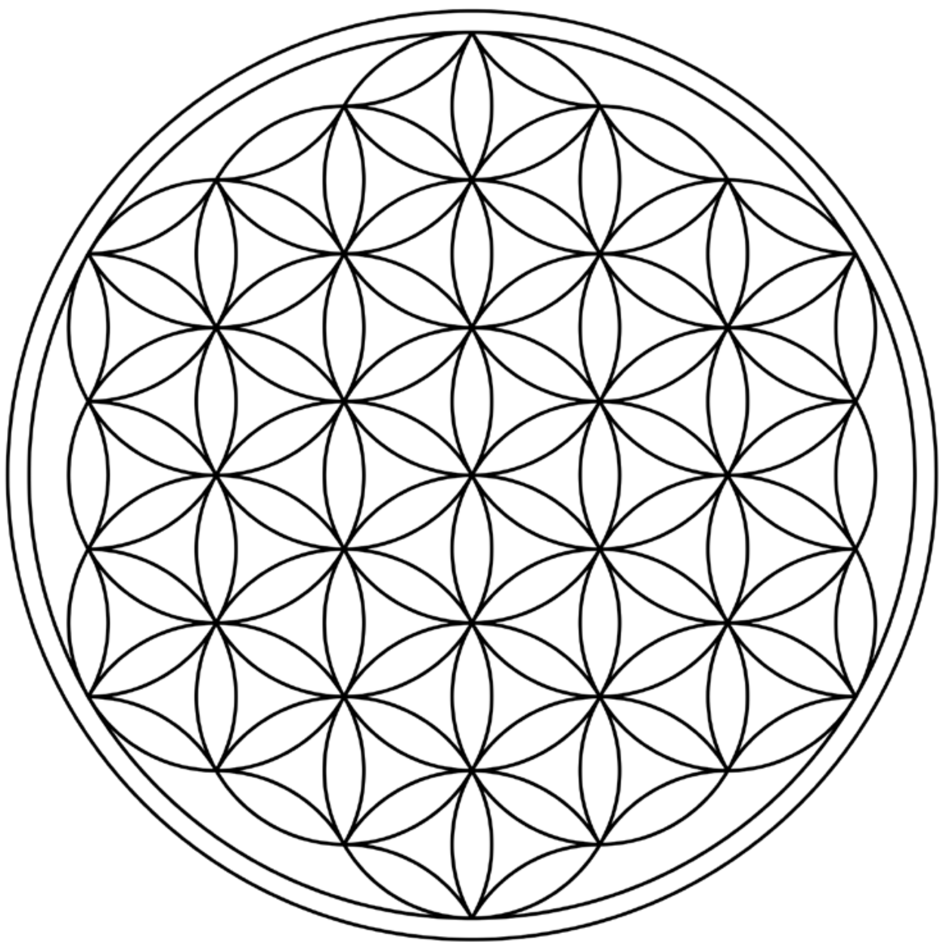

Chapter 1.

Introduction

The motivation for this study was based on three factors. The first was my own educational career as a student. The second was my experiences as a teaching assistant during my studies at the graduate level in the Department of Physics and the Department of Mathematics at Simon Fraser University (SFU). The third factor was my experiences as a mathematics and physics teacher at the high school level in the Lower Mainland of British Columbia, Canada. In what follows, I will give an overview of my personal educational background, followed by an account of the mathematical difficulties in my own education as well as my teaching career. My personal interest in this study, the research rationale, and the thesis organisation will conclude this chapter.

1.1. Personal Background

My high school education took place in North Vancouver, British Columbia. I was a good student of mathematics, interested in physics. After graduating high school with an even stronger interest in physics, I chose to study Mathematical Physics in the Department of Physics at SFU. After graduating with an Honours degree, I studied Quantum Chromadynamics (QCD) at SFU. I finished my coursework, which was heavily focused on quantum mechanics and particle physics. I started my thesis, but found that the long and intense hours of coding were not to my liking. Also, the most likely job for a person with a M.Sc. in QCD was in the financial sector, which was not an interest of mine. I transferred to the Department of Mathematics at SFU. I did coursework in Computational Fluid Dynamics, which also focused heavily on coding. Unfortunately, coding still had not grown on me or my career visions. During those 7 years, I steadily kept teaching assistant positions in both departments, helping undergraduate students in calculus and physics. Those experiences were very fruitful for me, both in terms of this study, which I will discuss below, but also to help me realize my passion for teaching. As a result, I transferred to the Department of Education at SFU. I completed my coursework and started doing research in Mathematics Education. I currently teach mathematics and physics at the high school level, and have been for the past 3 years.

1.2. Mathematical Difficulties

My realisation of the mathematical difficulties physics students encounter began with my own struggles in physics. In our undergraduate group at SFU, it always seemed that our biggest hurdle in physics was the lack of mathematical understanding. The topics that were covered in our physics classes heavily relied on calculus, and more often than not the calculus required in our physics classes was not taught to us in our mathematics classes yet. I recall taking electromagnetism in third semester of my year in physics; our teacher was showing us formulas with vector products (cross-products) and many of us were completely lost. We had never encountered vector products in mathematics, even though the physics professor expected us to know and use them. Up to that point, we had only taken the first two introductory courses in calculus, namely, differential calculus and integral calculus; we had not been exposed to vector calculus. In another physics class, we were expected to perform contour integrals, again having not seen them before in our mathematics classes. Transforms (Fourier, Laplace, Legendre, etc.) were peppered throughout our physics courses, but not in our mathematics courses. In quantum mechanics, the extent of the use matrix algebra was foreign to us, not to mention partial differential equations in general relativity. It was not only exposure, but adequate familiarity and practice with those mathematical concepts that hindered our physics achievements.

All in all, my learning as a student in physics class was impeded greatly by not having the necessary mathematical knowledge. It seemed that we were exposed to the mathematical concepts in physics class and expected to know how to use them, and sometime later formally learn them in mathematics class. Sometimes that learning came many semesters later, sometimes not at all. That was unfortunate, because I believe I could have concentrated more on physics concepts rather that struggling to learn the mathematical operations needed to solve the physics problems.

During my teaching assistant years, the mathematical difficulties were also the biggest hurdle for undergraduate physics students. I recall helping them more with arithmetic, algebra, trigonometry and geometry than the physics concepts. Their mathematical skillset was very limited coming straight out of high school. Even during my work in the Applied Calculus Workshop at SFU, it was evident that the students lacked very basic mathematical understanding in the areas of arithmetic, algebra, trigonometry and geometry.

Having been through all of those experiences and having taught at the senior high school level now, it is very clear to me that the biggest difficulties physics students have in physics class is their inability to perform mathematically. Difficulties such as understanding the processes involved in a formal solution, namely, data extraction, proper logic and reasoning, proper communication and representation of solutions, as well as correctly performing the necessary computations.

Hence the motivation of this study is to uncover, from the perspective of current teachers, which topics of the current physics curriculum do mathematical stumbling blocks appear the most, and if there are any remedies to these difficulties. In my own teaching, I strongly emphasise mathematical process while teaching physics concepts. I motivate the physics equations and prove them whenever possible. I use graphical alternatives for abstract problems. I put specific emphasis on the logical steps required and proper notation use. That approach has helped my students build a better mathematical arsenal, which is necessary in physics.

1.3. Personal Interest

My personal interest in this research is very clear: I wish to learn, from my own education and teaching, as well as the experiences of other physics teachers, if it is possible to teach physics at the high school level yet avoid or fix the many mathematical difficulties students face.

1.4. Study Rationale

In the pursuit of my personal interest and unlike most of the literature that report on tests administered to students, this study is based on teachers, their experiences and their approaches. What is needed is teachers' perspectives on the problem of mathematical difficulties in the physics classroom. The literature on the lack of mathematical ability based on testing is abundant and covered in the literature review section of this study. What is missing in the literature is the voice, experiences and suggestions of physics teachers.

1.5. Thesis Organization

This thesis is organised in 6 chapters. Chapter 1, the introduction, briefly describes my own education and career, the mathematical difficulties that are experienced in physics, my personal interest in pursuing the thesis topic, and the rationale for the necessity of this study.

Chapter 2 is the literature review. I examine the literature that relates to the mathematical difficulties encountered by physics students. I give particular examples of the difficulties and the associated studies that investigated those issues. I also make an account of the necessity of this study.

Chapter 3 is the analysis of curriculum, where I outline the current physics curriculum in British Columbia and describe the topics and the potential areas that give students great difficulty due to their lack of mathematical understanding. It is also where I indicate the topics I chose in order to create the questionnaires used in my research.

In Chapter 4, I describe my methodology in detail, which includes the participants in this study, the instruments, the tasks, the analysis method and the two questionnaires I created, namely *Teacher Questionnaire 1 and 2*, TQ1 and TQ2 respectively. TQ1 was a survey of the participants' education and teaching experience. TQ2 consisted of a set of problems I created based on the current curriculum. I also include the solutions to TQ2. I conclude with the thematic analysis of TQ2.

Chapter 5 is the complete analysis of the results, the participants' character profile, and the recurring themes in their responses. There is emphasis on particular solutions as well as alternative methods. This chapter also includes participants' suggestions on different teaching approaches based on their experiences.

Chapter 6, is the discussion section which covers the summary of the study, the contributions and limitations of the study, as well as my personal reflection on what I have learned by doing this research and what I would like to share with other teachers in the education community.

Chapter 2.

Literature Review

In this chapter, I describe the relevant literature in five parts: the conceptual challenges in physics, the role of mathematics in physics, required mathematical skills, traditional methods, and the motivation for this study. These parts are designed to provide an explanation of the vital role of mathematical understanding in physics and to illuminate the numerous skillsets students are required to learn and use effectively in physics problem solving. Further consideration goes into the traditional methods used to teach physics and possible new methods that can prove to be fruitful. And finally, I outline the main focus of previous studies and the need for this current study.

2.1. Challenges in Physics

Physics is commonly considered one of the most difficult subjects to undertake in high school. There are multiple reasons for this, namely, the precise logical reasoning required in the development of a solution to a problem, the large degree of abstract notions in the material, the necessarily high level of conceptual understanding required, and the extent of the essential mathematical sophistication. Each of the reasons outlined requires tremendous attention when investigating the many challenges a student may face in physics. In this section, I will outline some studies that have investigated the problem of conceptual understanding, as well as the instruments that were used in these studies, the results that were published and some possible remedies.

Typically, students taking a physics class find themselves in the traditional instructional environment. This atmosphere consists of one teacher reciting information to many students. Although this type of instruction can be fruitful for students' abilities to solve quantitative problems, it does not develop the conceptual understanding required nor does it remedy the common sense misconceptions of the students (Thacker, Kim & Trefz, 1994). In another study, Hake (1997) states that in the traditional passive-student model of the physics classroom, students' conceptual understanding undergoes no significant change even if the lesson is "delivered by the most talented and popular instructor" (p.64).

Many studies have been conducted on the problem of conceptual understanding in the physics classroom, see Clement (1981), Hestenes, Wells & Swackhamer (1992), Hake (1997), Thorton & Sokoloff (1998), Hoellwarth, Moelter & Knight (2000), to name a few. The studies address many possible causes of the conceptual difficulties students encounter in physics class, such as, preconceptions, prior misconceptions, underdeveloped ability to form concepts, unsuitable learning environments, ineffective instruction and presentation of the material.

Clement introduced the notion of *conceptual primitives* in dealing with higher-order concepts in physics. His motivation was to connect the ability to make mental constructs and the basic prerequisite for those higher-order concepts (1981). Clement states that *conceptual primitives* include "*key concepts* such as mass, acceleration, momentum, etc. as well as *fundamental principles* and *models* such as Newton's laws, the atomic model, etc." (p.66). His research shows that the origin of many difficulties with *conceptual primitives* lies in the student's preconceptions of the physics concepts before entering a formal course. Those preconceptions can originate from the student's personal experiences or from prior science courses. Although *key concepts* are typically dealt with using proper terminology and the associated physics definitions, *fundamental principles* and *models* require strong concept formation abilities. Often times, preconceptions develop into misconceptions and linger on for years and years:

> Difficulties at the qualitative level may go undetected because a student's superficial knowledge of formulas and formula manipulation techniques can mask his or her misunderstanding of underlying qualitative concepts.
>
> (Clement, 1981, p.66)

These persisting difficulties motivated many physics researchers to investigate class environments and instructional techniques. Perhaps the most notable is the studio style classroom. Studio style physics was introduced at Rensellaer Polytechnic Institute in 1993. The idea was to create an environment that eliminated the boundary between traditional lectures and laboratories. The aim was to promote active-learning instruction (Hoellwarth, Moelter & Knight, 2000). Although great efforts and resources were used to transform the traditional lecture to the studio style format at Rensellaer Polytechnic Institute, Cummings, Marx, Thornton & Kuhl (1999) concluded that the unfortunate reality is that the "standard studio style format is no more successful at teaching fundamental concepts of Newtonian physics than traditional instruction" (p.S44). The instruments they used to measure learning gains and arrive at their conclusions were the *Force Concept Inventory (FCI)* and the *Force and Motion Conceptual Evaluation (FMCE)*.

The Force Concept Inventory was developed by Hestenes, Wells & Swackhamer in 1992. It is a 30 question multiple-choice test based on a "systematic analysis of the basic concepts in introductory Newtonian mechanics" (Hestenes & Halloun, 1995, p.503). It was created to assess the student's conceptual understanding of the fundamental part of Newtonian mechanics, namely, forces. It is regarded as a key conceptual assessment instrument in the research involving students' understanding of Newtonian physics. Huffman & Heller (1995) state that "the FCI is one of the most reliable and useful physics tests currently available for introductory physics teachers" (p.138). The FCI test has no numbers, no equations and no calculations in any of the questions. It is a purely conceptual test; therefore, students cannot resort to using a calculator in order to answer the questions. That fact makes it a very attractive test to give to students to assess their conceptual understanding as well as reduce their dependency on the calculator.

The Force and Motion Conceptual Evaluation was developed by Thornton & Sokoloff in 1997. It is a 43 question multiple-choice test created to probe into the student's conceptual understanding of Newton's laws of motion. Similar to the FCI, it is also widely used by physics educators and researchers.

Although the implementation of studio style physics at Rensellaer Polytechnic Institute was not in itself a great tool for learning gains, Cummings *et al*. (1999) stated that the implementation of Interactive Lecture Demonstrations in the studio style environment did “generate significant gains in conceptual understanding with remarkable little instructional time” (p.S44). In their research, they also implemented Cooperative Group Problem Solving in the studio style format, which they found to be fruitful, stating that the use of this method “resulted in similar conceptual learning gains and seemed to also provide a mechanism which fostered improved quantitative problem-solving skills” (p.S44). Interestingly, Hoellwarth, Moelter & Knight (2000), also using the Force Concept Inventory and the Force and Motion Conceptual Evaluation, found that “the normalized gain in conceptual understanding was significantly larger for students in the studio sections” (p.1). The implementation of interactive-engagement methods can also be traced back to Hake’s (1997) six-thousand-student survey on mechanics. He concluded that the interactive-engagement formatted environment can increase the effectiveness of the mechanics course, both in the conceptual understanding of the student as well their as problem-solving abilities, well beyond that achieved in the traditional passive-student teaching practice.

Many different physics researchers have been concerned with the inability of their students to have sufficient concept formation abilities. As a result, great many studies have been performed, and innovative techniques have been implemented to remedy this immense problem, but no “silver bullets” have been discovered. Fortunately, methods that employ the removal of the lecture and laboratory boundary combined with interactive engagement are a step in the right direction, though not the focus of this study.

2.2. The Role of Mathematics in Physics

The research into mathematical skills as predictors of physics achievement has increasingly gained popularity in recent decades. The research can be dated back to the 1970’s and has not ceased receiving attraction from physics teachers in search of pinpointing their students’ mathematical difficulties in physics class, their students’ understanding of physics as well as improving their own teaching of the subject. The following summarises the necessity of mathematical ability in physics:

> Mathematics is the backbone of physics. It provides a language for the concise expression and application of physical laws and relations. A student's development as a physicist entails, in no small part, becoming increasingly comfortable with mathematics. As physics teachers, we share a responsibility to help our students develop fluency with the mathematics of physics. (Bing & Redish, 2009, p.1)

The problem of pinpointing the exact mathematical issues of students in the physics classroom is not an easy task to undertake. Developing an expert level of understanding in physics requires the ability to use the language of physics, namely, mathematics. The ability to understand and use appropriate mathematics skills effectively in the physics classroom is key in succeeding in physics, Bing & Redish (2009). Weak mathematical understanding and the inability to apply essential mathematical skills to diverse physics problems, can lead to a world of difficulty for students at all levels.

For all students, even prior coming to their first formal physics class, they are exposed to a minimum of 10 years of mathematics instruction and practice. It is in those years that they are being exposed to the language of mathematics, its formulations, rules, logic, functions, relevance, capabilities and to an extent its applications. In the physics unit of science 10, teachers cover basic kinematics. Students are shown formally, for the first time in the context of physics, the application of mathematics. The only real tools students have at that point is their common sense experiences and the mathematics they have learned. In no other realm of education, have they encountered the use of mathematics and its applications to a greater extent than in physics class. Mathematical skill becomes an important tool they must possess and employ effectively and efficiently. In fact, the most common preinstruction factor that relates to students' performance in physics is mathematics skill (Meltzer, 2002, p.1259). Without the timely and effective application of mathematics in the physics class, the student is lost.

Along with its rules, the logic inherent in mathematics makes it a tool that is unavoidable in science. In fact, Lindsay (1945) stresses that "logical thinking is essentially mathematical in character" (p.96). The physics student must be comfortable and well acquainted with the logical processes inherent in mathematics. It is difficult to solve a problem in elementary physics that does not rely heavily on mathematical understanding. Even the simplest kinematics problem with constant speed could require knowledge of addition, subtraction, multiplication, division, fractions, rates of change, dimensional analysis, and multiple applications of arithmetic and algebra. Even the slightest increase in difficulty in the problem can lead to the need, the understanding and the application of trigonometry and geometry.

From the extraction of useful data in a problem and the application of the relevant formulas, to the computations required, mathematics is at the core of most, if not every solution in physics. The link between physics and mathematics is evident; Buick (2007) put this very clearly, “it is essential that a physics student has a suitable mathematical background” (p.1073).

It is useful to make an analogy at this point, namely, in order to understand the structure and meaning of a poem in the English language, a prerequisite must be knowledge of the English language, its syntax and its proper applications. Similarly, to understand the structure and meaning of a physics problem, one must be well versed in the language of physics, namely, mathematics. Trying to capture the meaning of a physics problem and solve it without mathematics is similar to capturing the meaning of a poem without knowing the English language. The harmonious blend of science and art are illuminated in the application of mathematics in physics. The symmetries of Nature are a mathematical marvel. And more often than not, the mathematical solutions of physics are poetic.

The best test for the models in physics use the latest in mathematically derived technologies. From lenses used in microscopes and telescopes to simulations and detectors, mathematical concepts in design and fabrication are paramount. As a result, the use of sophisticated calculators and computer models have flowered in physics. Degrees in physics now require courses in optimization and programming. Both are based on mathematical logic and extend the students’ understanding and ability to create and test physical models. The evolution of mathematics links directly to the evolution of physics, and vice-versa.

2.3. Required Mathematical Skills

Correlation between mathematics skills and success in physics has been widely studied, see Hudson & Rottmann (1981), Hudson & Liberman (1982), Delialioglu & Askar (1999), and Omotade & Adeniyi (2013), to name a few. In those studies, there is overwhelming evidence suggesting that mathematics abilities strongly influence performance in physics. Among the factors that influence physics performance are these mathematics skills, outlined by Omotade & Adeniyi (2013):

> *#1. Computation skill:* number bases, problems, number in standard form, addition, subtraction, multiplication and division of fractions and decimal rate, ratio and proportion
>
> *#2. Algebraic process skills:* common factors, factorization of simple algebraic and quadratic expression, solving equations, simple equations and equations involving fractions, simultaneous equations and word problems leading to equations and variation
>
> *#3. Geometry skill:* geometrical constructions using ruler and compasses, finding angles between two lines, angles in a right angle triangle and using trigonometric ratios
>
> *#4. Measurement skill:* areas of plane shapes, volumes of common solids and areas and volumes of similar figures
>
> *#5. Tables and graphs interpretation skill:* interpretation of cost, travel and conversion tables and graphs, interpretation of statistical tables and graphs and interpretation of proportion graphs
>
> *#6. Probability and statistics skill:* probability became of major importance in physics when quantum mechanics entered the scene. A course on probability begins by studying coin flips, and the counting of distinguishable vs. indistinguishable objects. The concepts of mean and variance are developed and applied in the cases of Poisson and Gaussian statistics

As outlined above, each of those can typically be called upon to be applied in a physics problem. It is not only the application of each skill, but the combination of them which can hinder the physics students' ability to solve the problem. The variety of mathematical skills required simultaneously to solve even the most basic physics problem can be daunting for students. The ability to apply multiple concepts and theories in a single problem requires intuition and creativity. That type of mindset requires nurturing proper practices. As a result, the average student finds great many difficulties cultivating that type of approach having not been adequately exposed prior to taking physics classes. Redish (1994) explains why students describe physics as difficult:

> Physics as a discipline requires learners to employ a variety of methods of understanding and to translate from one to the other – words, tables of numbers, graphs, equations, diagrams, maps. Physics requires the ability to use algebra and geometry and to go from the specific to the general and back. This makes learning physics particularly difficult for many students (p.801).

The leap from using a particular skill to using many and combining them is a major difficulty for the student attending the physics class for the first time. On the previously mentioned list of mathematical skills, #1 can be generalized as arithmetic, and it is typically the first mathematical skill students are exposed to. Students attending their first formal physics class have had at least 10 years of experience with arithmetic, but that does not mean that they are fully comfortable with its application. Arithmetic can be the biggest difficulty for physics students (Lindsay, 1945). For #2 and #3, which for the purposes of physics problems, relates to algebra and trigonometry skills, Hudson and Rottmann (1981) showed that a reasonable predictor of the physics grade for students was a firm understanding and ability to perform algebraic and trigonometric operations. For #4, #5 and #6, Odili (1986) showed that measurement, everyday statistics and reading graphs and tables are most needed in the learning of secondary mathematics as correlated with achievement in sciences.

Furthermore, with the advent of technology, the requirement for physics students to be able to create models by programming has become a necessity. The programming courses they take are based in the mathematics of logic, recursion, graphing, and mapping. The physics students’ ability to model a system using mathematics is essential. Many computer-based tools are available now to learn and use by physics students, for example, Desmos, Wolfram Alpha, Maple and Matlab. The mathematics of computer algebra, symbolic and numerical computation, visualization and statistics are paramount in modern physics.

Mathematics has become a foundational tool for expressing scientific concepts and as such a firm understanding of mathematics, which is essentially logic and reasoning, is a prerequisite for scientific literacy as articulated by Oyedeji (2011). He further argues that mathematics is a critical part of the scientific endeavour, and that such an undertaking of the pursuit of knowledge cannot be without the refinement of logic and reasoning, which is a critical part of mathematics. This idea is further echoed by Omatade & Adeniyi (2013), “without mathematics, there is no science, and without science there is no modern technology and without modern technology there is no modern society” (p.391).

2.4. Traditional Methods

Instruction and instructional methods have always been a major topic of discussion in education. The content and delivery are the two tools that the teacher is equipped with to facilitate the learning of their students. Often times, teachers do not motivate a particular topic and simply recite the information to their students, expecting that they enthusiastically take the information as fact and be able to apply it to different situations. Unfortunately, this does not turn out to be the case most of the time.

In the sciences, specially in physics, teachers tend to give formulas without motivating or deriving them. They provide the student with the applicable formula for a particular range of problems and assign dozens of problems for them to do on their own. Furthermore, students are typically discouraged to solve the problems using their own methods and by applying their own creative logic (Epstein, 1941). Simple formulas in kinematics can be readily derived graphically using very basic rectangle and triangle area arguments, yet those formulas are just given to students to memorize and apply.

Without a motivated derivation of the formulas, students must resort to memorization. When it comes to problem solving, students have a formula sheet with dozens of formulas neatly laid-out for them, most even separated with the topic or chapter they apply to. Many students simply pick and choose, either from memory or trial-and-error, the formula that works for that particular problem (Redish, 1994).

Furthermore, many formulas can be derived from other basic formulas. For example, the formula for *Power* is typically given as $P = Fv$, where *F* indicates the force on the object and *v* the object's velocity. But the definition of *Power* is the rate of change of *Work* with respect to time, $P = \frac{W}{\Delta t}$, and the definition of *Work* is force multiplied by displacement, $W = F\Delta d$, given that information, one can easily derive the *Power* formula as follows: $P = \frac{W}{\Delta t} = \frac{F\Delta d}{\Delta t} = F\frac{\Delta d}{\Delta t} = Fv$. In that way, students do not need to memorize the final *Power* formula, because they can simply use prior definitions to understand where the formulas originate. Here is an example of a typical physics formula sheet:

PHYSICS FORMULAE

Vector Kinematics in Two Dimensions:

$v = v_0 + at \qquad \bar{v} = \frac{v + v_0}{2}$

$v^2 = v_0{}^2 + 2ad \qquad d = v_0 t + \frac{1}{2}at^2$

Vector Dynamics:

$F_{\text{net}} = ma \qquad F_{\text{g}} = mg$

$F_{\text{fr}} = \mu F_{\text{N}}$

Work, Energy, and Power:

$W = Fd \qquad E_{\text{p}} = mgh$

$E_{\text{k}} = \frac{1}{2}mv^2 \qquad P = \frac{W}{\Delta t}$

Momentum:

$p = mv \qquad \Delta p = F\Delta t$

Equilibrium:

$\tau = Fd$

Circular Motion:

$T = \frac{1}{f}$

$a_{\text{c}} = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$

Gravitation:

$F = G\frac{m_1 m_2}{r^2} \qquad E_{\text{p}} = -G\frac{m_1 m_2}{r}$

Electrostatics:

$F = k\frac{Q_1 Q_2}{r^2} \qquad E = \frac{F}{Q} \qquad E = \frac{kQ}{r^2}$

$\Delta V = \frac{\Delta E_{\text{p}}}{Q} \qquad E = \frac{\Delta V}{d}$

$E_{\text{p}} = k\frac{Q_1 Q_2}{r} \qquad V = \frac{kQ}{r}$

Electric Circuits:

$I = \frac{Q}{\Delta t} \qquad V = IR$

$V_{\text{terminal}} = \mathcal{E} \pm Ir \qquad P = VI$

Electromagnetism:

$F = BIl \qquad F = QvB$

$B = \mu_0 n I = \mu_0 \frac{N}{l} I \qquad \mathcal{E} = Blv$

$\Phi = BA \qquad \mathcal{E} = -N\frac{\Delta\Phi}{\Delta t}$

$V_{\text{back}} = \mathcal{E} - Ir$

$\frac{V_{\text{s}}}{V_{\text{p}}} = \frac{N_{\text{s}}}{N_{\text{p}}} = \frac{I_{\text{p}}}{I_{\text{s}}}$

Physics 12 Data Page 3

Figure 1: Physics 12 formula sheet

As is evident, students have to locate the chapter for the given problem, and apply one of many formulas to arrive at the solution. Whereas with inspection, one notices that some of those formulas are redundant and derivable from other ones. Providing students with fewer formulas and more exposure to derivations allows their logic and reasoning abilities to flower. It allows them to feel confident in knowing where some of the formulas originate from and can be more confident in applying them to different situations (Fiscarelli, Bizelli, Fiscarelli, 2013).

2.5. Motivation for this Study

Many studies have been done to find the correlation between mathematics pre-knowledge and learning gains in physics by administering a mathematical pre-test, see Hudson (1989), Meltzer (2002), Buick (2007), Champagne et al. (1980). These studies target the students' abilities as demonstrated by their performance on a test. However, the existing literature does not attend to teachers' perspectives. As such, the motivation of this study is to probe into the experiences of current physics teachers and find out their perspectives on the mathematical difficulties their students face. This study also inquires into the possible solutions those physics teachers have uncovered to remedy or workaround the difficulties their students face.

Chapter 3.

Analysis of Curriculum

This chapter is dedicated to presenting the current physics 11 and 12 curricula with explanations on specific topics that I chose to create my TQ2 problem set. Not all the topics in each curriculum were covered in TQ2, due to the constraints of the interviews and the fact that the mathematical skills required in various topics are similar. As such, specific problems were chosen to cover a wide range of mathematical skills while avoiding redundancy.

3.1. Current Curriculum

The current physics 11 and 12 curricula in British Columbia are packed with concepts. Many of those concepts have major mathematical components to them. Those mathematical components are the areas I chose to focus on in this research. They will be outlined below and in more detail in the chapters that follow. Below is the current curriculum with the prescribed learning outcomes for physics 11 in British Columbia. Following the curriculum, is a description of the mathematical requirements and the topics I chose to focus on in TQ2 for physics 11. Following physics 11, I proceed with a similar treatment for physics 12.

3.1.1. Physics 11 Curriculum

What follows is a list of topics and their associated prescribed learning outcomes for physics 11 from the Ministry of Education in British Columbia. Teachers are mandated to cover the topics below and assess their students' abilities accordingly.

1. **Skills, Methods, and Nature of Physics**

A1. Describe the nature of physics

A2. Apply the skills and methods of physics

2. **Wave Motion and Geometrical Optics**

B1. Analyse the behaviour of light and other waves under various conditions, with reference to the properties of waves and using the universal wave equation

B2. Use ray diagrams to analyse situations in which light reflects from plane and curved mirrors

B3. Analyse situations in which light is refracted

3. **Kinematics**

C1. Apply knowledge of the relationships between time, displacement, distance, velocity, and speed to situations involving objects in one dimension

C2. Apply knowledge of the relationships between time, velocity, displacement and acceleration to situations involving objects in one dimension

4. **Forces**

D1. Solve problems involving the force of gravity

D2. Analyse situations involving the force due to friction

D3. Apply Hooke's law to the deformation of materials

5. **Newton's Laws**

E1. Solve problems that involve application of Newton's laws of motion in one dimension

6. Momentum

F1. Apply the concept of momentum in one dimension

7. Energy

G1. Perform calculations involving work, force, and displacement

G2. Solve problems involving different forms of energy

G3. Analyse the relationship between work and energy, with reference to the law of conservation of energy

G4. Solve problems involving power and efficiency

8. Special Relativity

H1. Explain the fundamental principals of special relativity

9. Nuclear Fission and Fusion

I1. Analyse nuclear processes

3.1.2. Mathematical Requirements (Physics 11)

Starting with kinematics, emphasis is made on rounding-off numbers to the correct number of significant digits, fractions, algebraic manipulations, graphs, squares, square roots, averages, slopes, areas, vectors, trigonometry and solving systems of equations. All of those play a major role in solving problems in kinematics. These notions also arise in forces, momentum, energy, relativity and nuclear fission and nuclear fusion. In addition, wave motion and geometrical optics heavily rely on the ability to work with fractions, geometry, algebraic manipulation, the position of the negative symbol in fractions, division by zero, ratios, infinity, trigonometry, inverse functions and reciprocals.

I mainly focused on kinematics, wave motion and geometrical optics for the physics 11 portion of TQ2. Kinematics is typically taught before all of the aforementioned topics in physics, is the most basic topic but involves arithmetic, algebra, trigonometry and geometry. Wave motion and geometrical optics are two topics that give no end to mathematical difficulties for students, due to the additional necessity of fractional operations. The problems students are asked to solve in kinematics, wave motion and geometrical optics give a good sample of the mathematical skills required in physics 11.

3.1.3. Physics 12 Curriculum

What follows is a list of topics and their associated prescribed learning outcomes for physics 12 from the Ministry of Education in British Columbia. Teachers are mandated to cover the topics below and assess their students' abilities accordingly.

1. Experiments and Graphical Methods

A1. Conduct appropriate experiments

A2. Use graphical methods to analyse results of experiments

2. Vectors

B1. Perform vector analysis in one or two dimensions

3. Kinematics

C1. Apply vector analysis to solve practical navigation problems

C2. Apply the concepts of motion to various situations where acceleration is constant

4. Dynamics

D1. Apply Newton's laws of motion to solve problems involving acceleration, gravitational field strength, and friction

D2. Apply the concepts of dynamics to analyse one-dimensional or two-dimensional situations

5. Work, Energy and Power

E1. Analyse the relationship among work, energy, and power

6. Momentum

F1. Use knowledge of momentum and impulse to analyse situations in one dimension

F2. Use knowledge of momentum and impulse to analyse situations in two dimensions

7. Equilibrium

G1. Use knowledge of force, torque, and equilibrium to analyse various situations

8. Circular Motion

H1. Use knowledge of uniform circular motion to analyse various situations

9. Gravitation

I1. Analyse the gravitational attraction between masses

10. Electrostatics

J1. Apply Coulomb's law to analyse electric forces

J2. Analyse electric fields and their effects on charged objects

J3. Calculate electric potential energy and change in electric potential energy

J4. Apply the concept of electric potential to analyse situations involving point charges

J5. Apply principals of electrostatics to a variety of situations

11. Electric Circuits

K1. Apply Ohm's law and Kirchhoff's laws to direct current circuits

K2. Relate efficiency to electric power, electric potential difference, current, and resistance

12. Electromagnetism

L1. Analyse electromagnetism, with reference to magnetic fields and their effects on moving charges

L2. Analyse the process of electromagnetic induction

3.1.4. Mathematical Requirements (Physics 12)

Physics 12 typically starts with kinematics, dealing now with more general two dimensional problems, with more emphasis on rounding-off to the correct number of significant digits, fractions, algebraic manipulation, graphs, squares, square roots, averages, slopes, areas, vectors, trigonometry and solving systems of equations. All of those play a major role in solving problems in kinematics, but the addition of an extra dimension introduces a lot of mystery for students. Also, in physics 12, there is typically more emphasis on solving problems in variable form instead of purely numerical form. This poses problems for students who are not comfortable with variable manipulation and are not able to make sufficient use of their calculator to arrive at a numerical answer. Similar difficulties also arise in dynamics, momentum, torque, circular motion, gravitation, energy, electric circuits, electrostatics and electromagnetism.

Electric circuits is a new topic for students. This particular topic heavily relies on the ability to work with fractions, algebraic manipulation and the position of the negative symbol in fractions and ratios. This topic creates many mathematical difficulties for students, and as a result many physics teachers advise students to use their calculators, instead of performing the fractional arithmetic and algebra by hand. One consequence is the use of calculators can lead to irrational numbers with round-off mistakes.

The problems I created on TQ2 investigate the basics that physics teachers would like their students to know prior to coming to their class. As such, the physics 12 portion of TQ2 is mostly based on kinematics and electric circuits. Notions such as fractions, algebraic manipulation, graphical interpretation, squares, square roots, averages, slopes, areas, vectors, trigonometry and solving systems of equations should not be a mystery for physics students. Unfortunately, many of those topics are major stumbling blocks for many students. And while being occupied trying to learn, re-learn, memorize or implement different mathematical methods, students lose sight of the physics involved. As a result, more time is spent on the mathematics and the use of calculators as opposed to the learning and development physics concepts.

Chapter 4.

Methodology

In this study, the primary interest was to inquire into the mathematical difficulties students encounter in high school physics class. The aim of this work is to answer the following research questions:

1. From the teachers' perspective, what mathematical difficulties do students encounter when solving problems in high school physics?
2. What strategies do physics teachers employ to remedy or workaround these difficulties?

My interest was to see where these numerous mathematical difficulties reveal themselves in the physics classroom and how current teachers are dealing with them.

4.1. The Participants

The nine participants in this study were active mathematics and physics teachers at public school, private school and university in the lower mainland area of British Columbia. They were a convenience sample of my fellow graduate cohort peers as well as their acquaintances. All the physics teachers willing to participate were included in this study. The participants had varying degrees of physics background (see Table 4.1 and 4.2), but all have taught either or both physics 11 and physics 12. It was essential to interview physics teachers because they would have the most accurate responses for the research questions. Their invaluable experience teaching throughout the years to students from all academic backgrounds and abilities makes their responses of great value to this study. Moreover, the methods they have implemented in dealing with these difficulties could prove to be very useful for our academic establishments.

4.2. The Tasks

Numerous studies have been done on students' performances in the physics classroom in an attempt to find the mathematical difficulties they encounter, see Hake (2002), Oyelola (2011), Hudson & Liberman (1982), Buick (2007), to name a few. In this study I focused on the perspective of the teacher. In my literature review, research from the perspective of the teacher was lacking on this issue. Critical inputs from experienced physics teachers is very valuable and could give new insight into diagnosing the mathematical difficulties and perhaps even implementing new remedy methods.

It is worth noting that mathematical difficulties can take many forms, such as computational errors, calculator errors, inability to manipulate algebraically, etc. But irrespective of where these difficulties originate from, i.e., students learned the topics but insufficiently understood them (fractions), or students were not exposed to the topics (vectors). In this study the focus was to have the participants respond, not on the origin of these difficulties, but where these difficulties arose for students in their physics class.

4.2.1. The Questionnaires

I performed two interviews with the participating teachers; the first was an audio interview with questions from TQ1 found on the next page and in Appendix A. Both questionnaires were sent to the participants one week in advance. Each audio interview was done in private between the teacher and I. They lasted between 2-14 minutes. It was a biography of their educational career and courses they have taught. The TQ1 also dealt with the challenging aspects of physics that they have seen their students have difficulties, in general, and in terms of concept formation and problem solving. The TQ1 concludes with two questions about the possible solutions they have implemented and what they would suggest the educational system to implement in order to address these difficulties. Audio was chosen for the TQ1 to capture the tone of the participants' responses.

The questions in TQ 1 are general in order to get a wide scope of their experiences in the topic. It was audio taped because it allows the teacher to speak freely at the moment, being spontaneous and connecting thoughts on the spot, rather than to give them a written questionnaire in which they have to worry about constructing proper sentences, etc.

The following questions were asked in the audio interview:

1. What degree(s) do you have?
2. How long have you been teaching?
3. Have you taught mathematics at the high school level?
   -If yes, list the courses and the number of years you taught the course
4. Have you taught physics at the high school level?
   -If yes, list the courses and the number of years you taught the course
5. What is your favourite course to teach and why?
6. What aspects of physics do you believe to be the most challenging for students?
7. Where do you think physics students struggle the most in problem solving?
8. Where do you think physics students struggle the most in their way of concept formation?
9. What have you done in your classroom to remedy these problems?
10. What would you suggest be done in the education system to remedy these problems?

Questions 1-4 were designed to get precise numbers in terms of the length of the participants' teaching career as well as the courses they have taught. This background information proved to be useful in later analysis.

Question 5 was posed to get a sense of their preferences as a mathematics and physics teacher. I wanted to inquire into why they chose a particular favourite course to teach over other courses, because that helps in my analysis of their perspective in students' mathematical difficulties in physics as well as it made their view on recommendations to remedy the problems more attractive.

Questions 6-10 were general questions posed in order to open my research topic to them before getting into the specific problem set in TQ2. I wanted them to tell me what they thought were the most important issues their students have shown to have in physics, reflecting on their teaching experiences, and possibly get some useful techniques they have experimented with to avoid the mathematical stumbling blocks their students hit while learning and problem solving in their classrooms. What follows is a table summary of the participants' educational career:

Table 4.1: Teacher's education level and years of teaching experience

	B.Sc.	**B.Ed.**	**M.Sc.**	**M.Ed.**	**M.A.**	**Ph.D.**	**Years of Teaching Experience**
Jerry1	●	●		●		●	12
Estelle2	●						35
Elaine3	●	●	●				7
Kramer4	●	●					8
Newman5	●	●					5
George6	●	●		●	●	●	6
Frank7	●		●	●	●		1
David8	●	●					6
Larry9	●	●					4

As shown in Table 4.1, the number of years taught ranges from 1-35. This was a remarkable range and important to this study because it gave a wide range of responses to perform analysis on.

The more experienced teachers had many more classroom experiences and groups of students to rely on in answering the questions, taking into consideration the general patterns they have encountered and taught through. It is interesting to note that only 5 out of 9 participants actually majored in physics for their B.Sc., the others majored in either biology, chemistry or mathematics. Specifically, Elaine3, Kramer4, Newman5, George6 and Frank7, all majored in physics at the undergraduate level. This is of interest in many ways, but two questions in particular: Firstly, it will be fascinating to see their responses to question 5, which asks what their favourite course to teach is, as seen in Table 5.2. Secondly, analysis of their techniques to workaround or remedy mathematical issues and their recommendations to others could prove to be intriguing.

As mentioned previously, I performed two interviews with the participating teachers; the second was a video interview with problems from TQ2. Each video interview was done in private between the teacher and I. They lasted between 23-80 minutes. It was based on problem sets typically encountered in topics of both physics 11 and physics 12. Each problem had two parts; the first part, a general overview of the concept, and the second part, a particular aspect of that concept. Participants chose the problems they were comfortable answering based on whether or not they had ever taught that topic and if they had anything interesting to contribute given the questions surrounding this study. Participants were sent the questionnaires a week prior to the video interview in order to reflect on their experiences and be prepared to answer the problems during the interview. The TQ2 dealt with the challenging aspects of physics that they have seen their students have difficulties in, in general concept formation as well as mathematical difficulties. Participants used whiteboards to respond to the problems during the interview, this allowed for equations and diagrams to be elaborated on in their responses. It also allowed them to demonstrate different techniques, such as graphical methods, they have developed and used to remedy the mathematical difficulties their students encounter. Being able demonstrate their thoughts on a whiteboard allowed them freedom to precisely respond to the problems.

It is worth noting that some participant responses seem inconsistent for some problems, i.e., one participant felt that their students would not have difficulties with an aspect of a problem, whereas another participant may have felt differently on that aspect. This is a natural and common phenomenon due to the fact that students at different schools or grades or backgrounds do not always have exactly the same difficulties in a given physics problem. The interests in this study are the overarching themes that arose from the responses. Analysis is made on the commonalities of the responses and not necessarily single difficulties that small groups of students have had in the past.

What follows is the list of problems that the participants responded to. I created the problems after studying the physics curriculum and reflecting on my own experiences. Following each problem, a possible solution is presented, followed by explanations for what motivated me in creating the problems and my interest for each problem:

Initial questions posed in order to set the tone for the problems that follow:

-Where would students make mistakes?

-What different methods would they use in solving these problems?

Motivation: The aim of the above two questions was to have the participants have them in mind when reflecting on their experiences and answering what followed. Essentially, in every problem below, I was interested to know where they would anticipate their students making mistakes based on their experiences, as well as what different methods they have employed in order to avoid the mistakes their students typically make when dealing with those problems. Each problem below starts with a general problem which give the overall theme of the specific problem.

Problem #1

<u>General problem:</u>

What challenges do students face when dealing with distance vs. displacement?

Specific problem:

Jimmy makes a one-way trip from his home to the store:

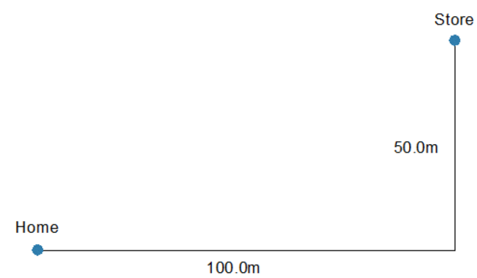

Figure 2: Distance vs Displacement

a) What is the total distance, *D*, travelled by Jimmy?

Possible solution:

$$D = 100.0m + 50.0m$$

$$D = 1.50 \times 10^2 m$$

b) What is his total displacement, $\vec{D}$?

Possible solution:

Note: vectors are represented by including an arrow on top of the variable, and magnitudes are represented by not including an arrow on top of the variable or by using double vertical bars (norm), as is the current convention in physics.

Magnitude:

$$\left\|\vec{D}\right\|^2 = D^2 = (100.0m)^2 + (50.0m)^2 = 12500m^2$$

$$\Rightarrow D = \pm\sqrt{12500m^2} = 112m$$

Direction:

$$\theta = \tan^{-1}(50.0/100.0) = 26.6^\circ$$

$$\therefore \vec{D} = 112m\ N26.6^\circ E$$

Motivation: The aim of this problem was to tackle the issue of significant figures, scalars, vectors, the recognition/application of the Pythagorean theorem, dealing with squares and square roots, angles and trigonometry. This is typically done at the start of physics 11 leading into kinematics. It is also typically covered in physics 12 as review.

In grade 11, students begin to learn about significant figures in chemistry and physics. It is a major hurdle for students, because of the rules they must be able to memorize and apply. For students, representing a number based on the precision of a measurement while also rounding-off can be difficult. They learn about the motivation for significant figures, but many just memorize the rules. In this problem, the students would need to realise that the final answer should have 3 significant figures, based on the information given in the problem.

Also, in grade 11, for the first time in their educational careers, students learn about quantities that have both magnitude and direction called vectors. The additional piece of information, namely the direction, proves to be a major difficulty for students. In terms of previous learning, they are first introduced to numbers, then numbers with units, and now numbers with units and direction. Dealing with only North, South, East and West can be fairly easy to understand, but it is when vectors are at particular angles and the use of trigonometry is involved, that students run into problems.

Although students are fairly familiar with the Pythagorean theorem, it seems to escape them when they initially encounter it in a non-mathematics class. They seem to not know that the Pythagorean theorem applies to all right-angle triangles, even in physics class, in problems dealing with distances and displacements. In this problem, that is made explicit in part b) of the problem.

Another difficulty, which lingers from mathematics class is computing squares and square roots. Students must remember that the possible mathematical answers to the square root of a number can be either positive or negative. Also, recognizing that they are dealing with physical quantities gives them many difficulties. Moreover, dealing with this new vector notion and the fact that the result of the Pythagorean theorem only gives the magnitude of the vector, students sometimes do not grasp the fact that the magnitude of a vector is always a positive quantity, and that the direction of the vector dictates the sign and the angle.

Another motivation for the inclusion of the vector in this problem is the necessity to perform trigonometric operations, which is a topic that is a major stumbling block for students coming into physics. The trigonometric ratios are typically memorized and not well understood. Students dealing with different triangles arising from physical situations have a hard time adjusting their notions of trigonometry to apply to different situations. The idea of ratios, which is examined in later problems, is still a topic that troubles many mathematics students, and naturally trigonometric ratios is one resulting symptom. The notion of the trigonometric function, and the object being operated on, leads to major understanding gaps, especially when dealing with inverse trigonometric functions, as is the case in this problem. Students typically do not have a good sense of what an inverse trigonometric function outputs, because they do not have a good concept of function notation in general. This proves to be a key mistake students make when dealing with vector directions.

The final part of the solution is the correct representation of the solution, namely the vector quantity being represented in its correct form:

- Arrow on the vector variable
- The correct units
- The correct number of significant figures
- The correct angle (with correct number of significant figures)
- The correct direction

As shown above, for a simple displacement problem, the number of mathematical notions is tremendous: rounding to correct significant figures, the recognition/application of the Pythagorean theorem, dealing with squares, square roots, angles and trigonometric functions. This fairly easy physics problem can give many students difficulties, and it is not necessarily the physics that is the stumbling block, it is the mathematical concepts and applications that give rise to difficulties.

Problem #2

General problem:

What challenges do students face when dealing with average velocity problems?

Specific problem:

Jane makes a one-way trip from home to the corner store. Jane walks East at an average velocity of 0.750 m/s for 21.5 s, and then runs East at an average velocity of 2.25 m/s for 25.8 s. What is Jane's average velocity on her trip?

Possible Solution:

$$\vec{V}_{avg} = \frac{Displacement_{total}}{time_{total}} = \frac{\vec{D}_{total}}{t_{total}}$$

$$\vec{D}_{total} = (0.750ms^{-1}\,[\text{East}]) \times (21.5s) + (2.25ms^{-1}\,[\text{East}]) \times (25.8s)$$

$$\vec{D}_{total} = 74.175m\,[\text{East}]$$

$$t_{total} = 21.5s + 25.8s = 47.3s$$

$$\Rightarrow \vec{V}_{avg} = \frac{74.175m\,[\text{East}]}{47.3s} = 1.57ms^{-1}\,[\text{East}]$$

Motivation: The aim of this problem is to address the misconceptions that arise from the word "average" in physics, especially in the kinematics context. The notion of averages is taught to students early in their mathematical career. It typically starts with the arithmetic mean. For example, the mean of 3,4,8,9 would be (3+4+8+9)/4 = 6, essentially the elements added together then divided by the number of elements. Students typically have a good notion of that concept. They are then taught to apply that knowledge to objects, for example, the average mass of a 55kg object and a 75kg object would be (55kg + 75kg)/2 = 65kg, which is basically the same procedure as before.

The subtlety comes when the elements that are to be averaged are dependant on multiple quantities, such as velocity which on displacement and time. Therefore, performing the previous procedure would not give the correct result when computing average velocity. The correct method is to calculate the sum of the numerators, namely the total displacement, and the sum of the denominators, namely the total time, as shown in the possible solution to problem #2. This gives great confusion to students because they have never really applied the notion of averages to cases that have variables with more than one dependent quantity.

A typical problem that gives great difficulty to students is that of zero total average velocity, for example, Jebediah runs to the store which is 1km away in 2 minutes and runs back in 2 minutes, what is his average velocity? Here, the total displacement is zero, because Jebediah returns to his original position, and therefore the average velocity is zero. Most students would say that the total displacement is 2km, the total time is 4min, and so the average velocity is 0.5km/min. The problems arising from the concept of vectors is evident here as well. The notion of averages is important in physics, and I included this problem to see if other teachers had experiences with their students struggling with this notion. As this is the first time students are exposed to scalars and vectors, they have great difficulty distinguishing between speed, which is a scalar with only a magnitude (size), and a velocity, which is a vector (size and direction). These types of problems give good exposure to this stumbling block.

Problem #3

<u>General problem:</u>

What challenges do students face when dealing with kinematics problems that rely on algebraic manipulation?

<u>Specific problem:</u>

a) Jimmy throws a tennis ball vertically downwards from his window. The ball travels for 15.0 m before hitting the pavement. Jane measures the speed of the ball as it strikes the ground to be 20.0 m/s. How much time did it take for the ball to travel from Jimmy's window to the pavement?

Possible Solution:

$$\left\|\vec{V_f}\right\|^2 = \left\|\vec{V_i}\right\|^2 + 2\vec{a}\bullet\vec{d} \Rightarrow \left\|\vec{V_i}\right\| = \pm\sqrt{\left\|\vec{V_f}\right\|^2 - 2\vec{a}\bullet\vec{d}}$$

$$V_i = 10.3ms^{-1} \text{ [downwards]}$$

$$\vec{V_f} = \vec{V_i} + \vec{a}t \Rightarrow t = \frac{\vec{V_f} - \vec{V_i}}{\vec{a}}, \text{ therefore } t = 0.991s$$

b) This time, Jimmy goes upstairs to his parent's bedroom, which is 5.0 m above his room and throws the tennis ball downwards from the window. Jane measures the travel time to be 1.25 s. What is the speed of the ball the moment before hitting the pavement?

Possible Solution:

$$\vec{d} = \vec{V_i}t + 0.5\vec{a}t^2 \Rightarrow \vec{V_i} = \frac{\vec{d} - 0.5\vec{a}t^2}{t}$$

$$\vec{V_i} = 9.87ms^{-1} \text{ [downwards]}$$

$$\left\|\vec{V_f}\right\|^2 = \left\|\vec{V_i}\right\|^2 + 2\vec{a}\bullet\vec{d}$$

$$\vec{V_f} = -22.1ms^{-1} \text{ [downwards]}$$

Motivation: The aim of the two parts of this problem is to address vector notation, coordinate systems for vector quantities, choice of direction of vectors, significant figures, algebraic manipulation and combination of kinematics formulas with positive/negative square roots.

In many parts of the physics curriculum, defining a coordinate system and being consistent within that coordinate system is a major difficulty. A teacher will often hear this phrase from physics students: gravity is always negative. The students take that as a known "fact". There is a lack of understanding that the negative symbol on a vector quantity indicates the direction of the vector; the sign of that vector is intimately related to the particular coordinate system the student has setup for the problem. Depending on the coordinate system, downwards may be positive, and in fact in some problems it is more convenient to choose the downwards direction as positive.

In particular, in the kinematics problems solved above, students will naturally pick the downward direction as negative because they associate that direction with gravity. But the errors come when they are using the vector equation, not understanding that the initial velocity must also be negative according to their choice of coordinate, since Jimmy is throwing the ball downwards. That implies they must take the negative square root instead of the positive square root, yielding -10.3m/s. That negative sign comes into play again in the next step when they are subtracting two negative velocities, and unless they realize that both velocities are negative, that simple calculation can be incorrectly done.

Another problem that students have is the algebraic manipulation:

$$\left\|\vec{V_f}\right\|^2 = \left\|\vec{V_i}\right\|^2 + 2\vec{a} \bullet \vec{d} \Rightarrow \left\|\vec{V_i}\right\| = \pm\sqrt{\left\|\vec{V_f}\right\|^2 - 2\vec{a} \bullet \vec{d}}$$

and

$$\vec{d} = \vec{V_i}t + 0.5\vec{a}t^2 \Rightarrow \vec{V_i} = \frac{\vec{d} - 0.5\vec{a}t^2}{t}$$

Many students still struggle with multiple variable manipulations in physics 11. They quickly want to plug in numbers and use their calculator to compute the answer. Many teachers, including myself, have gradually moved towards problems that only have variables and very little numbers, making the calculator less useful, and providing the opportunity for students to practice algebraic manipulation and feel comfortable with the mathematical rules of squaring and taking square roots of unknown quantities.

Another big hurdle many students face when coming into physics class is the inability to extract the useful data from word problems. In physics, it is not only the numbers that matter, but also the language used to describe those numbers.

For vectors, sometimes information is given with words such as: upward, southwardly, left, etc. In dynamics, forces can be described with: pulling, pushing, acting, stopping, etc. Quantities such as velocity can be described as: drop, throw, uniform, constant, etc. Students often need a lot of time to get used to the generally accepted terminology used in physics. This all adds another level of difficulty for students. As a result, a simple kinematics problem can be hidden in a long paragraph. Students who are usually struggling just to complete the calculations, also have to deal with terminology and data extraction before even writing down the first formula they need to use. Problems such as the one given above, are typical for physics 11 and 12, and kinematics is typically the first subject taught in those courses making the students rather disoriented at the level of complexity they encounter as soon as they come to physics class.

Problem #4

General problem:

What challenges do students face when dealing with the mathematical equations that describe reflections from curved mirrors?

Specific problem:

a) Jane is 170.0 cm tall and is standing at the focal length of a concave mirror. If the focal length of the mirror is 50.0 cm, how tall is her image?

Possible Solution:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

$$\frac{1}{d_i} = \frac{1}{50.0cm} - \frac{1}{50.0cm} = 0 \Rightarrow d_i \to \infty$$

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o} \Rightarrow h_i \to -\infty$$

b) Jimmy stands 50.0 cm in front of a convex mirror that has a radius of curvature of 20.0 cm. What is the magnification of Jimmy's image?

Possible Solution:

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}\Rightarrow\frac{1}{d_i}=\frac{1}{f}-\frac{1}{d_o}$$

$$\frac{1}{d_i}=\frac{1}{-10.0cm}-\frac{1}{50.0cm}$$

$$\Rightarrow d_i=-8.33cm$$

$$M=-\frac{d_i}{d_o}=-\frac{(-8.33cm)}{50.0cm}$$

$$\Rightarrow M=0.167$$

Motivation: The aim of these two problems was to probe into the mysteries surrounding students understanding of fractions, including the position of the negative symbol in fractions, the idea of division by zero and the concept of infinity.

Firstly, it may come as a surprise to the reader, but fractions remain an area of great concern even in senior physics classes. The fact that students routinely use calculators for almost every numerical calculation has rendered the concept of common denominators and fractional simplification almost extinct. Some students would have trouble computing a subtraction as simple as 1/50 - 1/50; they would quickly turn to the calculator because they would be afraid of a large number like 50 in the denominator.

Not surprisingly, an expression of the sort $1/x = 0$ is frightening to them because their calculators do not output a numerical answer in that case. The error message given to them by the calculator when trying to get a value for x is not of much help to them in this context. Many students would conclude that either $x = 0$ or that x does not exit. That mathematical stumbling block is of great concern in a subject like physics, where the concept of infinity can play a key role not only numerically, but conceptually, as is the case in this problem.

The image distance at infinity carries with itself not only a property of the lens, but also a property of an object being placed at a certain crucial distance from the lens. The interpretation of the phrase "the image is at infinity" is a valuable notion to understand in optics. None of that is easily attainable unless the student is capable of making the correct mathematical conclusion when faced with $1/x = 0$, as is the case in this problem.

Secondly, the difficulties of the negative symbol also make an appearance here. In my experiences, I have come across many students who cannot understand why the following statement is true:

$$-\frac{22}{7} = \frac{-22}{7} = \frac{22}{-7}$$

It is mysterious to them that the position of the negative symbol does not matter in this scenario. They believe that the fraction actually changes depending on the position of the negative symbol. Perhaps it is rooted in the fact that at an early age, we make the division operation analogous to physical objects, i.e., *22/7 is like asking how can 22 pizzas be divided among 7 people? Answer: each person would get 3.14 pizzas.* But if we switch that to 22/(-7), *it is like asking how can 22 pizzas be divided among negative 7 people?* The wording becomes a little strange. Maybe students still associate division with physical objects, and in this problem, it leads to more mathematical difficulties, such as finding common denominators with multiple numbers that are negative and positive.

Moreover, in these types of physics problems, the negative symbol actually signifies the orientation of the quantity of interest; a negative distance could signify the location an object or image in terms of its relationship with the lens. A negative height means that the image is inverted in relationship to the principal axis, and similarly a negative magnification dictates key properties of the image. So again the extra layer of physics complications comes from not only having the correct mathematical operations done and correct numbers attained, but also the interpretation of those numbers. As shown, a simple problem in optics combines many topics of mathematics and many conceptual challenges for physics students.

Problem #5

General problem:

What challenges do students face when dealing with refraction?

Specific problem:

a) A beam of light has a wavelength of 5.80×10^{-7} m in air and a wavelength of 4.30×10^{-7} m in liquid. If the beam enters the liquid at an angle of incidence of 20.0^o, what is the angle of refraction?

Possible Solution:

$$\frac{\sin\theta_a}{\sin\theta_l} = \frac{\lambda_a}{\lambda_l}$$

$$\frac{\sin 20.0^o}{\sin\theta_l} = \frac{5.80\times10^{-7}m}{4.30\times10^{-7}m}$$

$$\Rightarrow \sin\theta_l = 0.254$$

$$\Rightarrow \theta_l = \sin^{-1}(0.254) \Rightarrow \theta_l = 14.7^o$$

b) The beam of light from above travels through a liquid into plastic. Find the ratio of the index of refraction of the liquid compared to the plastic.

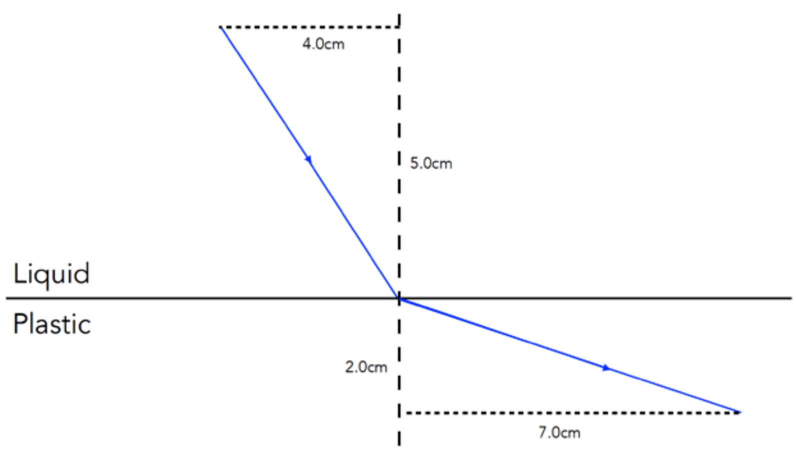

Figure 3: Refraction

Possible Solution:

$$\tan\theta_i = \frac{4.0cm}{5.0cm} = 0.80 \Rightarrow \theta_i = \tan^{-1}(0.80)$$

$$\Rightarrow \theta_i = 38.7^o$$

$$\tan\theta_p = \frac{7.0cm}{2.0cm} = 3.5 \Rightarrow \theta_p = \tan^{-1}(3.5)$$

$$\Rightarrow \theta_p = 74.1^o$$

$$\frac{\sin\theta_i}{\sin\theta_p} = \frac{n_p}{n_i} = \frac{\sin(38.7^o)}{\sin(74.1^o)} = 0.65$$

Motivation: The aim of these two problems was to investigate the difficulties students have with ratios, scientific notation, trigonometric ratios and functions. The first problem is a simple application of Snell's law. One of the problems students would encounter with that particular formula would be dealing with the ratio of two trigonometric functions with the added complication of scientific notation. Solving for the angle, as required in the problem is troublesome due to the inverse sine function computation. Even in physics 11, students do not have a conceptual understanding of the meaning of an inverse function and what it outputs, which in this case is an angle in degrees.

The second problem makes use of the tangent ratio to find the angle of incidence and the angle of refraction by the two triangles given in the problem. Usually those (or at least one) angles are provided for the students, but I purposely created the two right-angle triangles in order for trigonometric functions needing to be used to get those angles first. That would probe into difficulties that might appear in conceptual geometry and data extraction from diagrams.

Another stumbling block for students would be the fact that the problem asks for a ratio of two unknowns. Students have major difficulty when they are asked to solve a problem with more than one unknown involved in the answer. In this case, the application of Snell's Law produces the ratio of the indices of refraction of the two materials. Many students would not be able to grasp the idea that a final answer can be given as a ratio. They would know to apply Snell's law, but most likely stop midway because they would think they need at least one of the indices of refractions to solve the problem, in other words needing 3 out of 4 unknowns to get the last unknown.

Problems that probe into mathematical understanding of ratios, trigonometry and equations with multiple unknowns are commonplace in the physics 11, and become central in physics 12. Students with an inability to grasp those notions and apply their mathematical toolsets properly have great difficulty when solving problems in physics, specially in the optics unit.

Problem #6

General problem:

What challenges do students face when dealing with circuits?

Specific problem:

a) Jimmy builds a circuit containing three resistors in parallel. He measures the total resistance in his circuit to be $R_{eq} = 5.0\ \Omega$. If he measures the value of two of his resistors to be $R_1 = 10.0\ \Omega$ and $R_2 = 15.0\ \Omega$, what is the value of the last resistor?

Possible Solution:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\Rightarrow \frac{1}{R_3} = \frac{1}{R_{eq}} - \frac{1}{R_1} - \frac{1}{R_2}$$

$$\frac{1}{R_3} = \frac{1}{5.0\Omega} - \frac{1}{10.0\Omega} - \frac{1}{15.0\Omega} = \frac{1}{30.0\Omega}$$

$$R_3 = \left(\frac{1}{30.0\Omega}\right)^{-1} = 3.0 \times 10^1\Omega$$

b) Find the value of R_{eq}:

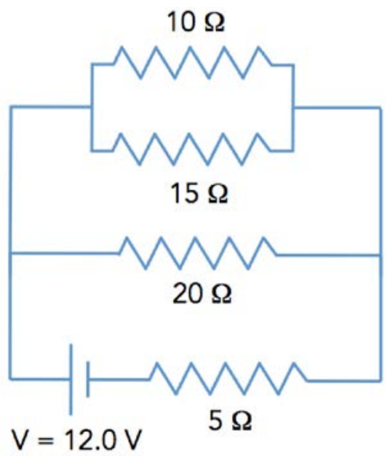

Figure 4: Circuit

Possible Solution:

$$\frac{1}{R}=\frac{1}{10\Omega}+\frac{1}{15\Omega}+\frac{1}{20\Omega}=\frac{13}{60\Omega}$$

$$\Rightarrow R=\left(\frac{13}{60\Omega}\right)^{-1}=\frac{60\Omega}{13}$$

$$R_{eq}=R+5\Omega=\frac{125\Omega}{13}=9.6\Omega\rightarrow 10\Omega$$

Motivation: The aim of these problems was further probing into ratios, significant figures, reciprocals, adding and subtracting fractions, and abilities to read diagrams. The formulas in the electric circuits unit are fairly straightforward, if you have a good ability to deal with ratios and reciprocals.

The first problem is a simple application of the equivalent resistance formula. Many teachers allow the use of calculators for these types of problems, because they don't want their students' time to be consumed with finding common denominators and making "silly" computational mistakes in addition and subtraction. Personally, I choose easy enough numbers for students to be able to perform those operations without the use of calculators. Often times, students end up with calculator round-off errors when taking reciprocals of decimals when computing the equivalent resistance.

For the second problem, deciphering the data given in a diagram proves to be a great challenge for many students. Understanding that the top three resistors are in parallel is a mystery to many physics students. I believe that problem can be traced back to their lack of visualization abilities and conceptual geometric thinking. Often times, circuit diagrams can be re-drawn to look simpler, but many students lack the ability to take that step to re-draw the circuit diagram. Unfortunately, that could lead to not applying the equation correctly and not getting the correct solution. The breaking down of diagrams, geometric or otherwise, is a major inability for mathematics and physics students. In physics, breaking down geometries is a key concept; for instance, it can lead to simplifications which allow one to see symmetries that can greatly reduce calculations, which can be very useful for students that lack the ability to do complicated arithmetic and algebraic computations.

4.2.2. Thematic Analysis

One of the challenges of this study was taking all the data from each teacher and trying to find common themes in their responses. It was interesting to listen to them explain their responses because they have varying years of education and teaching experience. In chapter 5, I grouped common themes for each problem because each question had its own reasons to be asked, and as a result common themes arose naturally. Although they can point to the same mathematical notion, a theme can give deeper insight, depending on the particular operation being done in the problem. After each problem, I give the response of every teacher that chose to respond. Then, I group their responses using the general themes that arose in terms of the mathematical difficulties their students face in solving problems, similar to those I used in my questionnaires.

Chapter 5.

Results and Analysis

This chapter will begin with tables indicating the courses the participants have taught along with their choice for favourite courses to teach. Table 5.1 is used to abbreviate the numerous courses that were given in the responses. Table 5.2 uses the abbreviations from Table 5.1 to display the courses each participant has taught in their teaching career, and highlights their favourite courses to teach. This was an important piece of data because it adds to the participants' responses (knowing what they prefer to teach and reasons why).

The responses to question 1-5 from TQ1 are summarized below in Table 5.1 and Table 5.2. Here are questions 1-5 from TQ1 once again:

1. What degree(s) do you have?
2. How long have you been teaching?
3. Have you taught mathematics at the high school level?
   - If yes, list the courses and the number of years you taught the course
4. Have you taught physics at the high school level?
   - If yes, list the courses and the number of years you taught the course
5. What is your favourite course to teach and why?

5.1. Data

What follows are tables used to identify course names and abbreviations, the participants' favourite courses to teach, the most common mathematical difficulties and possible remedies to those difficulties.

Table 5.1: Course names and abbreviations

Abbreviation	Course Name
M8	Mathematics 8
M9	Mathematics 9
M10 – FM&PC	Mathematics 10 – Foundations of Mathematics and Pre-Calculus
M10 – A&W	Mathematics 10 – Apprenticeship and Workplace
M10 – E	Mathematics 10 – Essentials
M10 – H	Mathematics 10 – Honours
M11 – A&W	Mathematics 11 – Apprenticeship and Workplace
M11 – E	Mathematics 11 – Essentials
M11 – F	Mathematics 11 – Foundations
M11 – PC	Mathematics 11 – Pre-Calculus
M11 – IB	Mathematics 11 – International Baccalaureate
M12 – A&W	Mathematics 12 – Apprenticeship and Workplace
M12 – F	Mathematics 12 – Foundations
M12 – PC	Mathematics 12 – Pre-Calculus
M12 – IB	Mathematics 12 – International Baccalaureate
P11	Physics 11
P12	Physics 12
P12 – AP	Physics 12 – Advanced Placement

As shown in table 5.1, the participants have taught 18 different courses, 3 of which are physics courses. Although there are only physics 3 courses, Table 5.2 will show a surprising result in terms of the number of participants that responded to their favourite courses to teach. Refer to Table 5.1 for the course names and levels. The dots (●) in the table represent the courses that each participant has taught at one point in their teaching career. The underlined stars (*) are the participants' favourite courses to teach.

Table 5.2: Courses the participants have taught in their career

	M8	M9	M10 – FM&PC	M10 – A&W	M10 – E	M10 – H	M11 – A&W	M11 – E	M11 – F	M11 – PC	M11 – IB	M12 – A&W	M12 – F	M12 – PC	M12 – IB	P11	P12	P12 - AP
Jerry1		●	●	●			●		●	●						*	*	
Estelle2	●		●	●			●		●	●		●	●	●		●	*	●
Elaine3	●	*	●	●			●	●		●				●		●	*	
Kramer4		●	●		●	●		●								●	*	
Newman5	●	●	●	●			●		●	●						*		
George6		●							●							●		*
Frank7	●		●													*		
David8		●	●							●	*			●	●	*		
Larry9			●				●		●	●							*	

As seen in Table 5.2, every participant chose a physics course as his or her favourite course to teach. Remarkably, even though only 3 out of the possible 18 courses above are physics courses, and considering the fact that the participants have taught numerous mathematics courses in their career, they still prefer teaching physics. The main themes that arose from the responses as to why they chose physics courses over other courses they have taught include:

- Students start seeing the underlying principals of motion in Nature
- The wide range of topics covered
- Motivated students that choose the course are scientifically orientated
- The course is fun to teach and exciting to experiment with
- Students are there because they want to, not because they have to
- Applications that arise from the course materials and concepts
- Being able to teach mathematics through a physics lens

It is certainly interesting to see the responses above. More emphasis is placed on the freedom the teachers have in physics class as well as the diverse course content in the curriculum as opposed to the maturity level of the students. In a sense, no participant explicitly responded that they enjoyed teaching physics class because the students are older or more mature. The excitement in the tone of their voices was evident in the audio recording when they explained the reasons for their choices. They emphasised that the material is much more interesting in physics class than in mathematics class because the students begin to see the applications of the mathematics in physics. In a sense they see for the first time, in an explicit context, the value of mathematics in Nature.

The teachers also expressed the value of hands-on experimentation in physics class as opposed to redundant and continuous problem solving in mathematics. They found that the course challenged them more in their pedagogy and as a result made the students more challenged by the material.

With the advent of technology, physics demonstrations and experiments have been made much more accessible to teachers and students. When students are able to visualize more concepts in physics as a result of these digital applications available on many different electronic devices, it makes the physics classroom a more exciting place to create conceptual understanding. That enthusiasm was evident in the participants' responses. They can improve on their instruction with strategically placed digital demonstrations. This allows for better flowing lessons and being able to cover more topics in depth like never before. This evident experimentation factor and freedom is an attractive notion for physics students and teachers alike.

Questions 6-10 in TQ1 were posed to get a general overview of the participants' thoughts on the difficulties students encounter in the physics classroom, as well as possible solutions they have tried and would suggest to the educational community.

Table 5.3 illustrates the general themes that arose in the participants' responses to question 6, 7 and 8:

6. What aspects of physics do you believe to be the most challenging for students?
7. Where do you think physics students struggle the most in problem solving?
8. Where to you think physics students struggle the most in their way of concept formation?

Table 5.3: Answers to question 6, 7 and 8

	Memorizing algorithm vs. interpreting problem	**Lack of a mental model**	**General mathematical difficulties**	**Deciphering words in order to apply proper equations**	**Too dependant on calculator instead of understanding**	**The scope of a physics vs. mathematics problem**
Jerry1	●	●		●		
Estelle2	●			●	●	●
Elaine3	●			●		●
Kramer4			●	●		
Newman5				●		
George6	●					
Frank7	●	●	●			
David8	●					
Larry9	●		●	●		

The two most common themes that arose from Question 6, 7 and 8 were:

1. The problem of memorizing algorithms as opposed to interpreting problems anew

2. The inability to decipher words in a problem in order to find the topic in question and apply the correct physics formulas to solve the problem

In terms of the first theme, teachers expressed concern that students coming from mathematics class begin to create patterns of problem solving. They start memorizing algorithms at an early age instead of treating each problem anew and interpreting the data given in the problem with a fresh set of eyes before associating it with a preconceived algorithm. This is a major problem in physics, because one particular physics equation can be applied in many different ways depending on the context. As a result, memorizing a particular algorithm that worked for a previous problem causes majors difficulties for students when faced with new problems. And when they are not successful at implementing their memorized algorithm for a new problem, they are discouraged and feel that they don't understand the physics, and fail to see that their quick and careless application of the previous solution is the stumbling block and not necessarily their conceptual understating. Teachers can typically identify this type of memorization in students' work because of the systematic way in which their mistakes occur. As an example, memorizing that the *Normal Force* in Newtonian mechanics caused by the interaction between two surfaces is "always" opposite gravity is a major mistake that is prevalent in dynamics.

For the second theme, there are a few factors. The language barrier and the fact that students are seeing physics terminology explicitly for the first time is an issue. The development of physics terms does take time, and many students are not accustomed to giving new meanings to words they have come to know in everyday language. The widespread use of Greek letters is troublesome for many students. The emphasis on units and direction is mysterious at first, and for many that difficulty lingers. All these result in an inability to decipher the meaning of the words used in physics problems in order to pick the correct formula to apply. Information extraction, whether in words or graphics, is a key stumbling block for physics students.

The participants' responses are shown in Table 5.4 for possible remedies for the general difficulties stated above. Here are questions 9 and 10 once again:

9. What have you done in your classroom to remedy these problems?
10. What would you suggest be done in the education system to remedy these problems?

Table 5.4: Answers to question 9 and 10

	Focus on conceptual understanding rather than numerical	**Focus on graphical representations**	**Focus on breaking down complex problems**	**<u>Focus on process rather than number crunching</u>**	**Focus on open-ended problems and experiments without specific instruction**	**Working on vertical surfaces: whiteboards**
Jerry1	●	●	●			
Estelle2				●		
Elaine3					●	
Kramer4		●		●		
Newman5			●			
George6				●		●
Frank7					●	
David8	●			●		
Larry9	●	●		●		

Table 5.4 shows the many different attempts that have been made in order to remedy the problems encountered in their classrooms. As shown, 5 out of 9 participants agreed that having students worry less about the numerical solutions, but focus on the variable form of the problem seems to be a successful approach. In other words, focus on mathematical process is key. This allows students not to be extremely reliant on their calculators, and be able to do algebraic manipulation in order to find general solutions.

After all, this approach is true to the nature of solving physics problems. Finding a general solution that can be applied to numerous situations is of greater value than to simply number crunch given a set of data points.

Here are responses from the interviews on the importance of focusing of mathematical process rather than number crunching:

> Estelle2: I don't give them numbers anymore. I work through how the process is supposed to be done, and the different processes required when solving different problems. That allows me to take the focus off the numerical answer and instead direct it towards the process.

> Kramer4: The numerically correct answer is not good enough in my class. Students need to show me how they got the answer, they need to show me their process. I care more about the process than the numerical answer. They have a hard time with that request, but ultimately it's for their own benefit.

> George6: I remove numerical values from a lot of my problems. Rather than having them solve to find a number, I have them solve the problem by manipulating an algebraic expression and arrive at a solution were they can show the relationship of one variable to another. So trying to get them to make general comments rather than worrying about number crunching. When students hurry to put numbers into the expressions, it can remove the ability to get insight into the physics of the problem.

> Larry9: I focus on my students solving problems with variables so they understand what's happening with the algebra.

Taking the focus away from numbers was expressed to be a successful approach, because it allowed the students to focus more on the process of solving the physics problem. Emphasis on why the steps should be taken and not just how they should be taken. The participants thought that when their students have a better understanding of the process, they are able to transfer that knowledge to other problems. As opposed to when students are dealing strictly with numbers and different problems become a brandnew endeavour with little to start with. When students gain confidence in dealing with variables rather than numbers and calculators all the time, it gives them the opportunity to develop their abilities to start a process and follow it, and this in the end a key skill to develop as a physics student.

5.1.1. Responses and Analysis

In this section, I present the responses to each problem in TQ2 in order. I follow the summary of responses with a section on the common themes that arose:

Problem #1:

General problem:

What challenges do students face when dealing with distance vs. displacement?

Specific problem:

Jimmy makes a one-way trip from his home to the store:

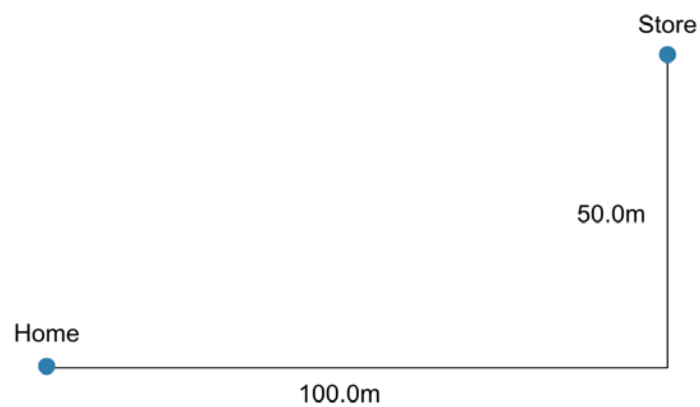

Figure 2: Distance vs Displacement

a) What is the total distance travelled by Jimmy?

b) What is his total displacement?

6 out of 9 participants responded to this problem. Their responses are illustrated below:

- My students would perform the following steps when computing the angle:

    i. $\tan(\theta) = x$

    ii. Given x, use calculator

    iii. θ = 26^o

  I think students would have an easier time plugging in numbers into the calculator if they remembered that $x^{0.5} = \sqrt[2]{x}$. They tend to mess up when pressing the square root button on their calculator, so if they knew to press "exponent 0.5" they would make less mistakes. (Jerry1)

- My students have memorized that "tangent is opposite over adjacent", but often they ask me when to use $\tan(\theta)$ as opposed to $\tan^{-1}(\theta)$. I explain it to them and say that when you are trying to solve for the angle θ in $\tan(\theta) = x$, think about "unlocking the angle", and you do that by taking the inverse tangent of both sides: if $\tan(\theta) = x$, then $\tan^{-1}(\tan(\theta)) = \tan^{-1}(x) = \theta$. That sometimes helps them, because it is the same idea as taking the inverse operation of multiplication in 3*x* = 5, namely division by 3, to find *x*. (Kramer4)
- When it comes to understanding what it means to have a tangent (or inverse tangent) function when finding the angle, they have no idea. They just plug the numbers into their calculator and press the inverse tangent button and write their answer. (Newman5)
- From a mathematical standpoint, I find that students are unwilling to draw a scaled diagram or are unwilling to trust what a scaled diagram would tell them. And when they are trying to do the trigonometry, they make all kinds of errors, from typing in the numbers into the calculator incorrectly, to finding the inverse tangent incorrectly, neglecting to make sure the calculator is in degrees, not knowing which angle they want to calculate and which trigonometric function is associated with that angle. I think trigonometry in general is a bit of a mystery to students. There is an inherent trust in the calculator and it shows when students write down the answer to an angle by writing down all the decimal places displayed in their calculator. (George6)

- My students would be able to solve the displacement using the Pythagorean theorem, but would struggle with the direction of the displacement vector because of their lack of ability in trigonometry. (Frank7)
- My students would have difficulty with the displacement calculation. Perhaps if you draw it out for them completely, they could apply the Pythagorean theorem:

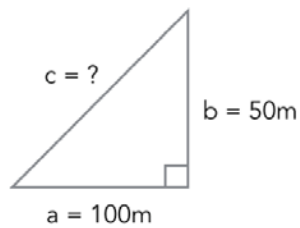

Figure 5: Pythagorean theorem example (1)

> But even so, if you remove the labels, 3 out of 15 students would not think to solve this with the Pythagorean theorem. (David8)

We notice that Frank7, George6, Newman5 and Kramer4 are all concerned about their students' lack of trigonometry ability. They feel that their students struggle in completing the vector portion of the problem because of struggles with trigonometry. They all agreed that the notion of tangents is mysterious to their students, and as far as answering the problems, their students rely heavily on the calculator and less on the mathematical meanings and mechanics of the trigonometry. Furthermore, their students memorize things like "SOH CAH TOA" and relate tangent to "opposite over adjacent" without really understanding what those words mean. The higher achieving students can relate those words to sides of a right angle triangle, but otherwise most are left in the dark about the concept, and therefore rely on their calculator.

That lack of understanding turns up time after time in physics, especially when students are asked to find angles using the inverse trigonometric functions. Not only do they struggle with the concept of inverses, the notation confuses them. Kramer4 chose to use the "unlocking the angle" metaphor to explain taking inverses and computing angles: $\tan^{-1}(\tan(\theta)) = \tan^{-1}(x) = \theta$. Newman5 has become accustomed to letting his students simply reach for the calculator as soon as they see an inverse function because she feels they have no idea about the concept of inverse functions. David8 mentions that even when students use the calculator, they are not fully sure what the output is, whether it is in degrees or radians. Furthermore, he is notices students copying down exactly what they see on the calculator, every digit, not realizing that truncation is required depending on the problem. David8 thinks that issue occurs because the students have so much faith in the calculator that they feel they need to write down everything that is displayed on the screen. Jerry1 is trying to find ways of teaching calculator shortcuts to reduce the number of mistakes students make. His method of teaching roots using decimal exponents is easier to use and is prone to less errors.

Recurring Themes:

- Weak trigonometry understanding and abilities
- Too much dependence on calculator

Problem #2:

General problem:

What challenges do students face when dealing with average velocity problems?

Specific problem:

Jane makes a one-way trip from home to the corner store. Jane walks East at an average velocity of 0.750 m/s for 21.5 s, and then runs East at an average velocity of 2.25 m/s for 25.8 s. What is Jane's average velocity on her trip?

9 out of 9 participants responded to this problem. Their responses are illustrated below:

- I only teach $V_{avg} = \frac{Displacement_{total}}{Time_{total}}$, I do not teach $V_{avg} = \frac{V_i + V_f}{2}$, but somehow students find it and make use of it in inappropriate situations. And I try to explain to them that if you have a constant slope in a velocity vs time graph (meaning constant acceleration), then that latter formula will work, but if it curves at all then that formula will not work. (Jerry1)

- You have to make the distinction that $V_{avg} = \frac{V_i + V_f}{2}$ only works in particular situations, namely when the acceleration is constant. I usually teach it graphically and also make an analogy with their marks; I asked them if this example is fair: You get $\frac{90}{100}$ on test #1 and $\frac{5}{10}$ on test #2, would your average test score be $\frac{90\% + 50\%}{2} = 70\%$? They realize that is an unfair mark and they see the problem with that reasoning, namely that the "out of" is not the same for both tests. One test is out of 100 and the other is out of 10. Similar to the problem of interest here, the "out of" is the time interval and they are not equal. So we work out that the fair and correct way of averaging the marks would be to total their earned marks and divide that by the total "out of" marks: $\frac{90+5}{100+10} = \frac{95}{110}$, and relating that back to average velocity would yield $V_{avg} = \frac{Displacement_{total}}{Time_{total}}$. (Estelle2)
- I think this is a concept problem, because things are typically weighted the same in mathematics, and so students have difficulties applying the concept to physics problems where things are typically not weighted the same. I usually give this problem: If you drive to school at 50 km/h and realize you forgot something at home and have to drive back, how fast would you have to drive back to have an average velocity of 100 km/h? We work through that as a class and it clarifies the subtlety of the average velocity formula. (Elaine3)
- My students say: I just add the two numbers and divide by two, right? I prefer not telling them this formula $V_{avg} = \frac{V_i + V_f}{2}$ at all. (Kramer4)
- I don't generally encourage $V_{avg} = \frac{V_i + V_f}{2}$, because I do find that they get lost using it. They use it at inappropriate times. Even in other problems, they would use that average velocity formula for no reason because they don't know what else to do with it. I tend to teach them $V_{avg} = \frac{Displacement_{total}}{Time_{total}}$ only. (Newman5)

- From a young age, when students are taught to find the average, it's the average between two quantities, so they tend to memorize: add the two quantities and divide by two, and in this case that would not be correct. (George6)
- My students would simply use $V_{avg} = \frac{V_i + V_f}{2}$ without realizing that in this problem the time intervals are different. You can only use that formula in certain situations. (Frank7)
- The formula $V_{avg} = \frac{V_i + V_f}{2}$ is one that I typically avoid because it implies equal weighting. At some point, I end up addressing it because it shows up in textbooks and websites. Usually I teach the solution to problems like this graphically. And by doing so, as a class we construct the correct equation of average velocity, namely total displacement divided by total time. (David8)
- My students would have trouble here in thinking about what the average velocity means here. Because of the two velocities here, they would turn to this formula on their formula sheet: $V_{avg} = \frac{V_i + V_f}{2}$, and then wonder what the initial velocity and the final velocity would be, and that's why they would add the two velocities and divide by 2. But of course, the different time intervals have an impact in this problem and so that would not be correct. (Larry9)

Although David8, Newman5, Kramer4 and Jerry1 generally avoid teaching the special case formula $V_{avg} = \frac{V_i + V_f}{2}$, there is a consensus among the rest of the participants to teach $V_{avg} = \frac{Displacement_{total}}{Time_{total}}$ first as the general formula, then if need be, mentioning the special case formula, while emphasising either graphically/algebraically the fact that it only works if acceleration is constant. Similar to Estelle2's example of average marks, David8 mentioned that if he needs to incorporate the special case formula into an example, he finds it useful to give this example: Wilhelm achieves 90% on a test worth 75% of his final grade and achieves 90% on a test worth 25% of his final grade. What is Wilhelm's final grade?

David8 says a typical solution from his students is: final grade = $\frac{90\%+80\%}{2}=85\%$

Essentially, his students are not recognising that the two tests are not weighted equally, and so apply the average formula without hesitation and arrive at the incorrect solution, not realizing that the above solution would be correct if the two tests had equal weighting. David8 works with the students and teaches them that the notion of average that must be considered is different than just applying $Mark_{avg}=\frac{Mark_1+Mark_2}{2}$. Rather, students must consider applying the correct weighting in the final grade calculation, namely $(0.90\times 75)+(0.80\times 25)=87.5\%$. David8 said that he avoids the special case average formula, but that students manage to find it in textbooks and websites regardless, and they tend to prefer it because it looks much simpler. He is then forced to give the above example to explain the special case formula in detail.

Estelle2, David8 and Jerry1 also use graphical means to convey the idea of average velocity to their students. Here are examples of the methods they mentioned; method one is showing how $V_{avg}=\frac{Displacement_{total}}{Time_{total}}$ can be equal to $V_{avg}=\frac{V_i+V_f}{2}$ if there is constant acceleration (constant slope) in a velocity vs. time graph. By calculating the area under the curve, and thereby computing the total displacement over the total elapsed time, they can arrive at the average velocity solution:

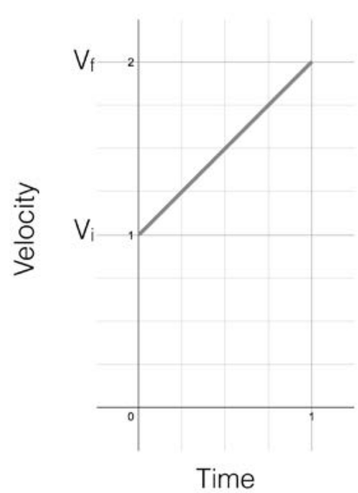

Figure 6: Velocity vs. Time (1)

According to $displacement = velocity \times time$, the displacement is calculated to be: $(1s \times 1 m/s) + (0.5 \times 1s \times 1 m/s) = 1.5m$. The total elapsed time is $1s$. Therefore, according to the general formula, $V_{avg} = \frac{D_{total}}{t_{total}} = \frac{1.5m}{1s} = 1.5 m/s$. The special case formula would also yield the same result here, because the slope of the line is constant which means the acceleration is constant: $V_{avg} = \frac{V_i + V_f}{2} = \frac{1m/s + 2m/s}{2} = 1.5 m/s$. That was a good example to show when the two formulas agree on the same final value. However, consider a very simple non-constant acceleration scenario:

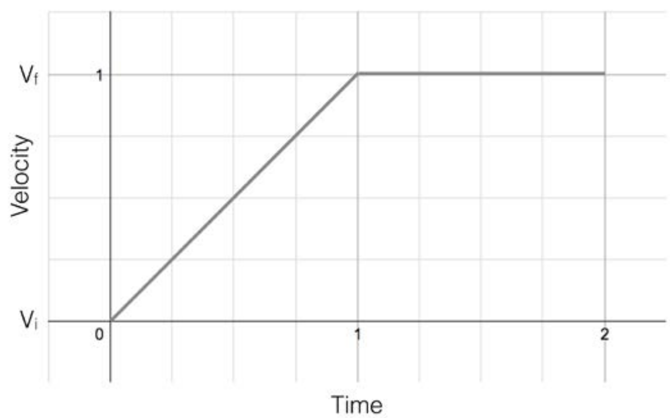

Figure 7: Velocity vs. Time (2)

The total displacement would be $(0.5 \times 1s \times 1 m/s) + (1s \times 1 m/s) = 1.5m$. The total elapsed time is $2s$. Therefore, according to the general formula, $V_{avg} = \frac{D_{total}}{t_{total}} = \frac{1.5m}{2s} = 0.75 m/s$. Whereas using the special case formula, $V_{avg} = \frac{V_i + V_f}{2} = \frac{0 m/s + 1 m/s}{2} = 0.50 m/s$. This simple yet effective example shows how the two formulas can differ. Estelle2, David8 and Jerry1 mention how important examples like the above are because students tend to understand kinematics better when they are exposed to the concepts graphically.

The idea of memorizing a formula from an earlier age came into my discussion with George6. He mentioned how we teach that the average of two numbers is the numbers added together divided by two, or in general that the average of a set of quantities is the addition of the quantities divided by the number of quantities. And we see this in many instances, for example if you have three individuals with masses $40kg$, $50kg$ and $60kg$, then their average mass would be $\frac{40kg+50kg+60kg}{3}=50kg$. In teaching this method, we neglect to teach that the concept of equal weightings must be understood for that calculation to be correct. In the physics examples, where equal weightings were not the case, such as Problem #2, due to the fact that the elapsed times are not the same, that formulation simply does not hold. This is the moment that a teacher can use analogies and graphical means to teach the difference to students instead of simply avoiding the special case formula, $V_{avg}=\frac{V_i+V_f}{2}$. One workaround is using graphical explanations, which turnout to be a wonderful tool not only in kinematics, but in many other topics in physics.

Recurring Themes:

- Misconceptions about the meaning and applications of averages
- Too much dependence on memorization
- Usefulness of using graphical means to convey concepts

Problem #3 (part a):

General problem:

What challenges do students face when dealing with kinematics problems that rely on algebraic manipulation?

Specific problem:

a) Jimmy throws a tennis ball vertically downwards from his window. The ball travels for 15.0 m before hitting the pavement. Jane measures the speed of the ball as it strikes the ground to be 20.0 m/s. How much time did it take for the ball to travel from Jimmy's window to the pavement?

4 out of 9 participants responded to this problem. Their responses are illustrated below:

- I find that my students have trouble using the formula $V_f^2 = V_i^2 + 2ad$, because of the square root involved in solving for either of the velocities. Since problem #3 requires specifying direction for the quantities and students are typically used to choosing the downward direction as negative, when they are solving for the final velocity, they have to square an unknown quantity, and a lot of them use the positive root. But according to their convention, that would mean that just before hitting the ground, the ball is actually moving upwards, which is not correct. I have to first review with them that when taking the square root of an unknown quantity, you must consider both the positive and the negative root. Specifically, in this scenario, they must also be consistent with the convention of direction they used, and so if they chose downwards to be negative, then they must pick the negative root for the final velocity. (Kramer4)
- It is a common occurrence in my physics tests that I write “math error.” (Kramer4)
- I deal with the negative and positive roots of an equation like $V_f^2 = V_i^2 + 2ad$ by front-loading the material. Basically, I do a lab where I have 3 scenarios: Initially, I drop a ball from a certain height, then I throw the ball down from the same height, and finally I throw the ball upwards and track it as it falls to the ground. In that way, students get an understanding of the up and down motion of a projectile in a mathematical sense. So when we see problems like #3, I ask them to apply either the positive or negative root depending on the context. In other words, in relation to the motion they observed during the front-loading lab. After all, most of them know the idea of positive and negative roots, for example the square root of 9 is either +3 or -3. Now, it’s just getting them to apply that logic to a physical situation. (Newman5)
- When my students dealt with equations that had squares in them, specially of the form $V_f^2 = V_i^2 + 2ad$ where there are two squares, it gave no end to arithmetic difficulties. Often times they would simply neglect the squares on the quantities, thinking that they had solved for velocity but in fact they solved for the velocity squared. (George6)

- One way I have tried teaching this problem is graphically. The neat thing about this approach is that students can typically work backwards to find the initial velocity, because they know the slope of the line is acceleration, for which they have the value to be the acceleration due to Earth's gravity near its surface. Some might say that by doing it this way I am avoiding the mathematics, but really I am avoiding the algebra by doing it geometrically. In terms of solving for displacement, one can simply find the area under the same graph. You can then solve two equations, both having squares in them using graphical methods.

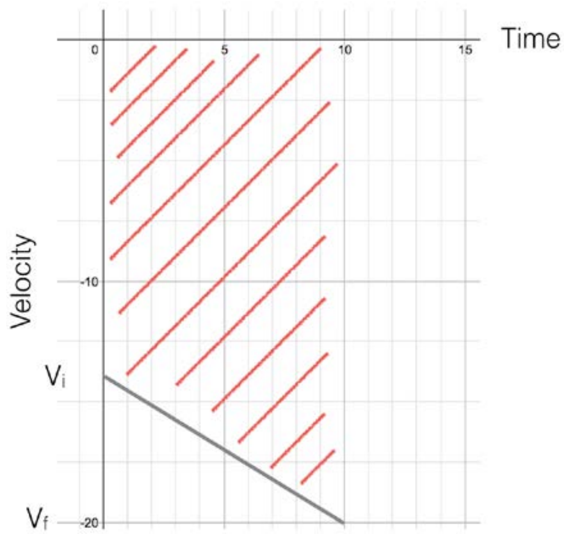

Figure 8: Area Under Velocity vs. Time Graph

- The slope of the line is nonzero; it represents the acceleration due to Earth's gravity near its surface. Students have worked with slopes a lot so they can calculate the initial velocity fairly easily. The graph also makes use of the negative region, which implies the direction that the ball is thrown. Also, the shaded region represents the displacement, and they get the idea that the area under a velocity vs time graph yields displacement. So they work with slope and area in the same graph.
- I feel like this approach allows me to introduce the importance of slopes and areas of graphs to motivate differentiation and integration. Students can be made aware of the physical significance of slopes and areas of graphs by doing kinematics. (David8)

Interestingly, two of the respondents of this problem, Newman5 and Kramer4, mentioned concerns about their students' difficulties in recognizing negative solutions to square root functions. This is particularly important in vector equations because it dictates the physical situation by providing the direction of motion. It is one those cases that the mathematics they may have learned and thought to be meaningless provides valuable information about the physics. All respondents agreed that solving problems using variables raised to a power was a difficulty for their students. It seems to them that applying their algebra skills to non-linear problems is a major stumbling block. And in a course like physics, those valuable mathematical skills typically cannot be avoided.

One workaround that David8 mentioned was the use of graphical techniques. He was the only respondent that teaches this alternate method. He knows the algebraic difficulties students have with equations of the sort $V_f^2 = V_i^2 + 2ad$. As a result, he resorts to teaching these types of kinematics problems utilizing a graphical approach, which he argues to be beneficial for their future calculus encounters. The idea here is that instead of pounding them with more algebra practice or side lessons, why not teach them an alternate method that is equally valid. It is not only beneficial for students that struggle with non-linear algebra problems, but it allows those who are comfortable with that type of problem to practice an alternate method to either check their algebra or to add to their arsenal of problem solving techniques.

Algebra skills are among those that are lacking with many mathematics students. Perhaps the physics class is not the best venue to address those difficulties, rather introducing basic graphs is a good alternative that some physics teachers may wish to pursue. After all, the task at hand is to apply mathematics to better understand the physics, and so tackling lingering algebra issues may take away from the time that should be devoted to the conceptual physics understanding. Perhaps the physics can help them understand the mathematics better too in the future.

Recurring Themes:

- Difficulties with squares and square root functions
- Usefulness of using graphical means to convey concepts

Problem #3 (part b):

b) This time, Jimmy goes upstairs to his parent's bedroom, which is 5.0 m above his room and throws the tennis ball downwards from the window. Jane measures the travel time to be 1.25 s. What is the speed of the ball the moment before hitting the pavement?

3 out of 9 participants responded to this problem. Their responses are illustrated below:

- When I do problems that use the formula $d = v_i t + \frac{1}{2}at^2$, I make it a point to review the fact that this is nothing but a polynomial of degree two and in this case *t* can be treated as the variable, and therefore the graph of the equation is a parabola, which is the *projectile* motion of the object. This is particularly helpful when they are dealing with the two roots of time in the equation when applying the quadratic formula, since the projectile achieves the same height at two different heights. (Kramer4)

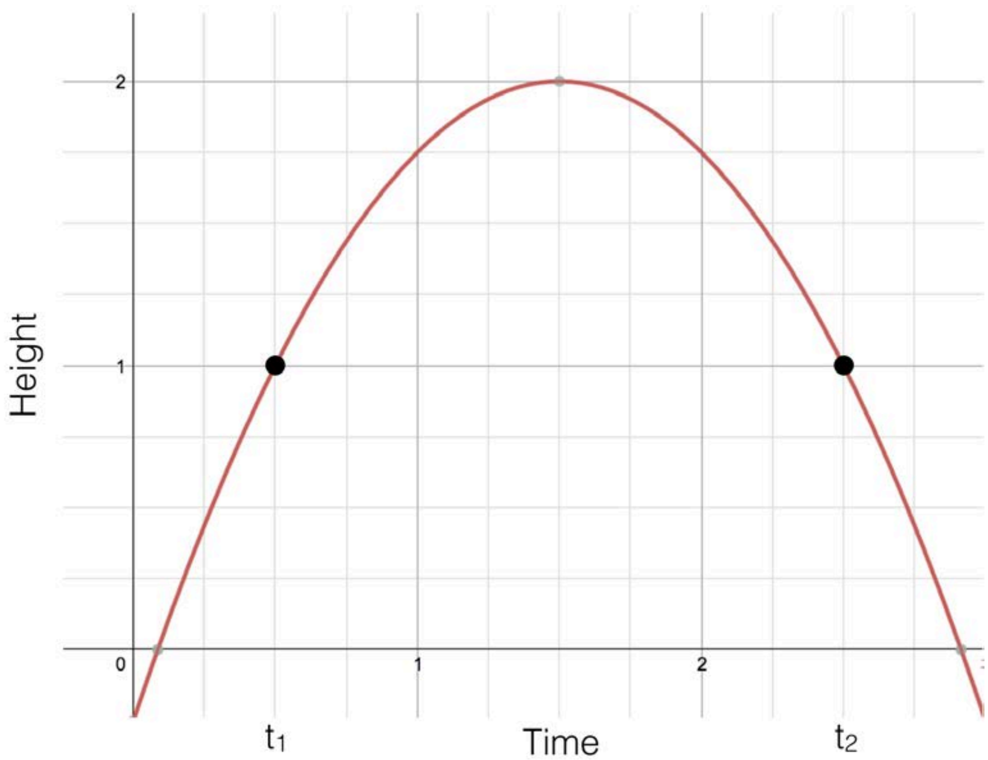

Figure 9: Projectile Trajectory

- Depending on the height, the equation has two solutions for time, one solution represents the ascent, and the other represents the descent. (Kramer4)
- Many of my students completely neglect the double root when they deal with equation of the sort $d = v_i t + \frac{1}{2}at^2$.
- Students take physics 11 typically having only finished mathematics 10. They have difficulty with equations of the sort $d = v_i t + \frac{1}{2}at^2$ because up to that point, they have only been asked to factor quadratic relations, so solving for time in that equation would be a huge difficulty for them. (David8)
- I would tell my students to use Desmos, the online digital graphical calculator, because they would not be comfortable solving a variable in terms of another and substituting it into an equation. In other words, they are not comfortable solving an equation and not getting a numerical answer at the end in physics 11. (Larry9)

All respondents resort to the graphical approach in solving this problem. Kramer4 directs his students to the parabolic nature of projectiles, and the logical argument that arises when plotting the data points. In other words, that the projectile travels through the same height twice in its journey (expect at the peak), and that same height corresponds to two different times. In a similar fashion, Larry9 allows students to plot the formula on Desmos and have the program show them the nature of the curve:

$$d = 5 \cdot t + .5 \cdot (-9.81)t^2 \longrightarrow$$

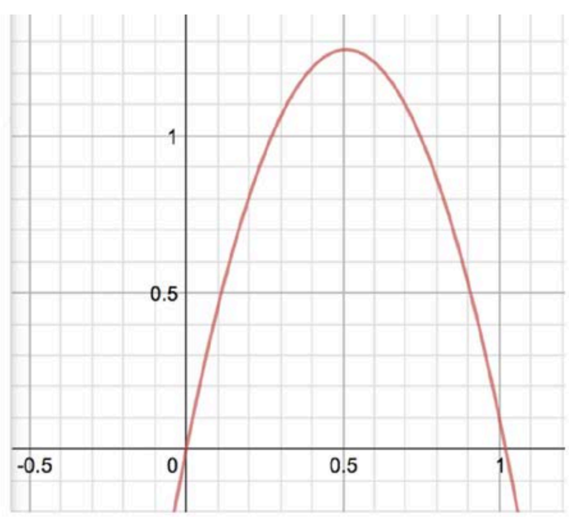

Figure 10: Desmos Generated Projectile Trajectory

In either approach, whether analog or digital, the teaching method is graphical. The participants argue that students that can visuals the motion can apply the mathematics more comfortably. David8 re-iterates the difficulties his students have with non-linear problems. And as mentioned previously in Problem #3, he resorts to a graphical method as well, though he does not specify what tools students use to solve these types of problems graphically. The comprehension of the motion in this problem is key in understanding the mathematical subtitles inherent in the quadratic formulation of the problem. With the aid of visuals, students can better understand the need for the double root in time. Depending on the details of the problem, either root may be called upon to be used, and making that distinction requires knowledge of both roots.

Recurring Themes:

- Usefulness of using graphical means to convey concepts

Problem #4 (part a):

<u>General problem:</u>

What challenges do students face when dealing with the mathematical equations that describe reflections from curved mirrors?

<u>Specific problem:</u>

a) Jane is 170.0 cm tall and is standing at the focal length of a concave mirror. If the focal length of the mirror is 50.0 cm, how tall is her image?

8 out of 9 participants responded to this problem. Their responses are illustrated below:

- I think my students would get to this step without a problem: $\frac{1}{d_i} = \frac{1}{50} - \frac{1}{50} = 0$, but understanding what this $\frac{1}{d_i} = 0$ means mathematically would give them trouble. Some students tend to make the mistake of saying $d_i = 50 - 50 = 0$ from the original fraction by-passing the fractional arithmetic. Some would recognise that $d_i = \frac{1}{0}$ and that this quantity is undefined, but to make the leap to the physical meaning, that is, that the image is located at infinity, would be problematic for them. (Jerry1)

- Some of my students would multiply both sides of this equation $\frac{1}{d_i} = 0$ by d_i and conclude that 1 = 0, and that $d_i = 0$. (Elaine3)
- My students cannot make the link between the graph of $\frac{1}{x}$ and solving the problem: $\frac{1}{x}$=something. In fact, some students struggle with recognizing that $\frac{1}{d_i}$ is the same equation as $\frac{1}{x}$ because they are so used to using the letter x as the variable. (Elaine3)
- I find that even if they understand how the graph of $\frac{1}{x}$ behaves, they don't understand how $\frac{4}{x}$ or even $\frac{1}{x+1}$ behave. There is a disconnect in them understanding the elements of graphs and translating those elements in a broader sense. (Estelle2)
- My students have trouble rearranging $\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$ to get $\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$. I allow the use of calculators and tell them "you are not in mathematics class, use decimal form" because I know they will make mistakes computing different operations on fractions. And, in terms of $\frac{1}{d_i} = 0$, some of my students would conclude that $d_i = 0$. Anytime we have symbols instead of numbers, they hate that. (Kramer4)
- For my students, using their calculator on $\frac{1}{d_i} = 0$ and getting an error would be troublesome, because they would not know the meaning in the physical sense. Furthermore, they would not know $\frac{1}{d_i} = 0$ implies that $d_i \to \infty$. But I have noticed

that rearranging $\frac{1}{f}=\frac{1}{d_i}+\frac{1}{d_o}$ this way: $d_i=\frac{1}{\frac{1}{f}-\frac{1}{d_i}}$ and asking them to plug that into their calculator makes sense to them. It is messy, but the calculator takes care of the mess. (Newman5)

- I find that $\frac{1}{d_i}=0$ gives students trouble, because the calculator is not much help to them in this case. (Frank7)
- In this situation $\frac{1}{d_i}=0$, I expect to get an answer $d_i=0$ from my students. Fractions, especially when the variable is in the denominator, cause great difficulties for students. (David8)
- My students have difficulty multiplying and dividing by zero like in $\frac{1}{d_i}=0$. They also have trouble dividing by negative numbers $\frac{10}{-2}$. They don't have the concept that the negative symbol can be placed beside the numerator or the entire fraction and it would mean the same thing mathematically. (Larry9)

Every respondent identifies the difficulties their students have in solving variable equations that involve fractions. No matter what techniques students use, calculator or no calculator, the meaning of an unknown in the denominator is a point of great difficulty in the physics classroom. Even though in the mathematics classroom, the idea of unknowns in the denominator is taught using different analogies, in the physics classroom the physical significance of the unknown in the denominator and solutions to problems that involve those unknowns greatly depend on the mathematical understanding of students. All respondents agree that the notion of infinity and $\frac{1}{x}=0$ seems to be a mystery to their students. It seems that their students just do not have the ability to compute some algebraic expressions that deal with zeroes. As a result, students simplify the problem and just flip the entire equation and get rid of the fraction. Of course, this ignores the infinity aspect and neglects the physics concept. Some students can make sense of the behaviour of $\frac{1}{x}$ graphically, but cannot extend that knowledge algebraically. This type of problem is of major concern in physics, and students who do not have the mathematical knowledge to solve and interpret the results in the physics classroom often find themselves lost. Fractions are a part of just about every topic in physics, as they should be, since they are a common mathematical tool.

Recurring Themes:

- Difficulties with arithmetic and algebraic operations of fractions
- Struggles with algebraic expressions with division by zero and infinity

Problem #4 (part b):

b) Jimmy stands 50.0 cm in front of a convex mirror that has a radius of curvature of 20.0 cm. What is the magnification of Jimmy's image?

4 out of 9 participants responded to this problem. Their responses are illustrated below:

- At this step $\frac{1}{d_i} = \left(-\frac{1}{10}\right) - \frac{1}{50}$ my students opt to use the calculator, even at the grade 12 level, because they will make mistakes in computing the common denominator and finding the solution. Practically speaking, associating the negative sign with the numerator cuts down a lot of errors; using this $\frac{-1}{10}$ instead of $-\frac{1}{10}$ or $\frac{1}{-10}$, even though mathematically they are all identical. (Jerry1)
- Although I tell my students to do $-\frac{1}{10} - \frac{1}{50}$ with a calculator, they would be able to do it by hand because the numbers 10 and 50 are simple enough to work with. But, if it was something like $-\frac{1}{11} - \frac{1}{33}$, they would divide and round-off horribly. Then they will give you an answer that is close to what you should get, but I think

the thing that bothers me is the use of the calculator, the round-off error, and if it's a multiple choice problem with fractions as answers, they would try to convert back and find that their answers, which is not there in an exact sense. (Elaine3)

- I believe that you should still be able to do the physics stuff without knowing all the numbers. If you throw all the numbers at the beginning, the physics is lost. I try to make them solve the mathematics without numbers until the very last steps. (Elaine3)
- As much as I love mathematics, I find that it is standing in the way of my students' physics understanding. I do not take many marks off in a problem if they make mathematical mistakes, instead I look at their physics concept formation. I feel that you should be able to do the physics without actually putting the numbers in. (Estelle2)
- I find that students do not have a good understanding of the notion of $-\left(\frac{A}{B}\right)=\left(\frac{-A}{B}\right)=\frac{A}{(-B)}$, and so the optics unit is one of the biggest sources of surprise as a teacher in terms of mathematical difficulties experienced by students. (George6)
- I have come to the point with fractions that when students give answers of the form $\frac{-5}{-2}$, I tend to accept it. (David8)

It is remarkable to see the view that the respondents had on this problem. In all the responses, there is a consensus that students have great difficulty in computing fractions. As a result, the participants allow the use of calculators because they place great importance on the physics and not necessarily the mathematics. Jerry1 and George6 are concerned that their students do no understand $-\left(\frac{A}{B}\right)=\left(\frac{-A}{B}\right)=\frac{A}{(-B)}$.

Jerry1 teaches his students to change the placement of the negative to the numerator whenever possible so that the fraction looks familiar enough to solve mathematically. Typically, when introducing fractions at a young age, analogies such as 3/4 are like asking how many slices would 4 people receive from 3 full pizzas. But you can see the shortcoming of that way of teaching fractions if you have 3/(-4). The analogies we teach are not always well suited for higher-level mathematics. By allowing calculators, Jerry1 and Elaine3 avoid the difficulties students have with finding common denominators and proceeding with the computation.

Recurring Themes:

- Misconceptions about the placement of the negative symbol in fractions
- Too much dependence on the calculator in computing fractions

Problem #5 (part a):

General problem:

What challenges do students face when dealing with refraction?

Specific problem:

a) A beam of light has a wavelength of 5.80×10^{-7} m in air and a wavelength of 4.30×10^{-7} m in liquid. If the beam enters water at an angle of incidence of 20.0^{o}, what is the angle of refraction?

3 out of 9 participants responded to this problem. Their responses are illustrated below:

- In the mathematical context, students are used to working with ratios. But when it gets to a physical situation, they have difficulties using their mathematical knowledge and applying it to the physics. (Newman5)
- Many students can say “SOH CAH TOA” but have little understanding how those ratios are related to each other or how they can apply those ratios in a physics problem. Some simply memorize one particular triangle and say things like “If I want to calculate this angle, it’s always tangent”, not understanding how the fundamentals of trigonometric ratios apply in different orientations. (George6)
- A major problem for my students is not being able to recognize whether or not the output from their calculator makes physical sense, or whether an angle should be in radians or degrees. Many have difficulties realising that the settings on the calculator need to be adjusted prior to the computation, something they

should be familiar with from mathematics class. They have a lack of insight into what the angle should be approximately, which means they need more practice using the trigonometric ratios. (Frank7)

The difficulties with trigonometric functions extend well into the physics class. Whether it is the student's lack of understanding what the trigonometric ratios mean and where they originate from, as mentioned by George6, or how to apply them in a physics problem that does not explicitly give you a right-angle triangle, as mentioned by Newman5, or whether to use degrees or radians, as mentioned by Frank7. The general notion of basic trigonometry is the link between their responses. The nature, application and meaning of a trigonometric ratio is missing. These concepts are supposed to be covered in mathematics class, and applied to physical situations in the physics. Students should be familiar with these concepts and their use depending on the physics. Unfortunately, students are uncomfortable with trigonometric functions, and usually resort to using the calculator instead of grasping the fundamentals of these important functions.

Recurring Themes:

- Difficulties in trigonometric concepts
- Too much dependence on calculators

Problem #5 (part b):

b) The beam of light from above travels through a liquid into plastic. Find the ratio of the index of refraction of the liquid compared to the plastic.

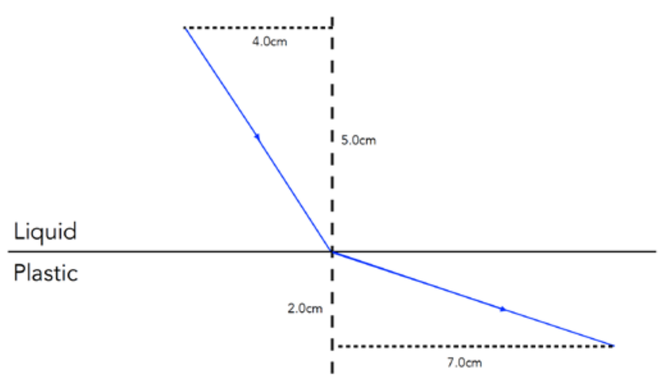

Figure 3: Refraction

6 out of 9 participants responded to this problem. Their responses are illustrated below:

- Some of my students do not appreciate the fact that the tangent is simply a ratio of the sine and cosine function, and this causes difficulty when they are trying to apply different ratio relationships in trigonometry. Furthermore, most students when dealing with relations of the form $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ would not be able to distinguish if the answer that the calculator outputs is in radians or degrees, because they do not have a good understanding of the notion of angles and trigonometric ratios. You are hoping they have the physical intuition required to interpret the mathematical output, but that is not the case all the time. (Jerry1)
- When you ask students to find the final value given a ratio, they don't usually have problems with that. It's when you ask them to give their answer in a ratio that causes difficulties for them. I suspect it has to do with the fact that two variables are involved now, instead of the usual single variable, in order to solve and state the answer to the problem. (Jerry1)
- My students have difficulties with inverse functions in general, therefore $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ causes them trouble. They would be able to input it into their calculator, but would not know the mechanics behind the computation. (Elaine3)
- Typically, in mathematics class, I give them nice functions to plot, whereas in many instances in physics the data is not very nice and students have trouble in concept formation and plotting. I wish they saw more applications in mathematics class when learning topics like trigonometry so that their first exposure is not in physics class. For example, in mathematics class, when students are exposed to which curves are classified as functions and the rules for a curve to be a function, rather than simply stating the rules, perhaps teachers should give a simple kinematic motion example of distance vs time and how a non-function does not only break the function rules, but is actually physically impossible. In that way students are not simply memorizing a rule, rather they have an understanding of where the rule originates and how it is applied to a physical situation. (Estelle2)
- I find that I have to walk them through the process of what a ratio is. (Newman5)

- Even though many students are able to do ratios in the mathematics class, they get stuck when ratios are given in a different context, such as with Snell's law. It seems they forget what to do when they are trying to apply their mathematical knowledge in a physics problem. (Frank7)
- Asking students to write two numbers (that they have already computed) in a ratio is a stumbling block for a lot of them. (David8)
- My students would have no problem solving $\tan(\theta) = \frac{4}{5}$ using their calculator, but they would have no idea what the ratio or the inverse functions actually means in terms of the triangle that is represented in the problem. (David8)

Jerry1, Newman5, Frank7 and David8 are concerned about their students' ability to represent solutions in a ratio. They agree that their students' lack the fundamental understanding of ratios and that hinders their understanding in physics class. Estelle2 mentions that trigonometry is a great obstacle for students because it is based on ratios. Combining that with the fact that in these types of problems, students are also required to compute inverse functions, proves to just be insurmountable for most.

Elaine3 and David8 agree that the meaning of inverse functions ought to be taught at greater depth in the math class. Typically, students simply learn to punch numbers in their calculator, not really understating why and what output they should expect. For example, the tangent of an angle is equal to the ratio of the opposite and the adjacent sides of that angle in a right-angle triangle, and the inverse tangent of the ratio of the opposite and the adjacent sides yields the angle. That back and forth logic of functions and their inverses is a mystery to many students. Students tend to rely entirely too much on the calculator and less on knowing what outputs would make sense, as emphasised by David8.

Recurring Themes:

- Difficulties with ratios
- Too much dependence on calculator
- Difficulties with inverse functions

Problem #6 (part a):

General problem:

What challenges do students face when dealing with circuits?

Specific problem:

a) Jimmy builds a circuit containing three resistors in parallel. He measures the total resistance in his circuit to be $R_{eq} = 5.0\ \Omega$. If he measures the value of two of his resistors to be $R_1 = 10.0\ \Omega$ and $R_2 = 15.0\ \Omega$, what is the value of the last resistor?

3 out of 9 participants responded to this problem. Their responses are illustrated below:

- I tell my students to compute $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$ by plugging $R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$ into their calculator. I do not ask them to solve for common denominators because they make many mistakes by doing that. (Newman5)
- The anxiety level of students goes up as soon as they encounter fractions. The problem lies in the mathematics and not the physics. (George6)
- I think they partly don't expect to be doing fractions by hand in physics class. When they are in a math class, they expect to be doing mathematics by hand, but in physics class they expect to be doing physics rather than mathematics. They separate science and mathematics. (Frank7)

Frank7 states that in a sense his students believe that physics class is somehow disconnected from math class, in terms of computations. As a result, George6's students have great difficulties in applying their mathematical knowledge to physics problems. In their words, when the mathematics is buried in the physics problem, or when they encounter physical quantities represented in fractional form, their ability to perform the necessary computations plummets. Not surprisingly, Newman5 allows his students to jump for the calculator as soon as they can, because he knows that the mathematical difficulties his students encounter with fractional simplification is too great and takes away from the main physical problem.

Recurring Themes:

- Difficulties with fractions
- Too much dependence on calculator

Problem #6 (part b):

b) Find the value of R_{eq}:

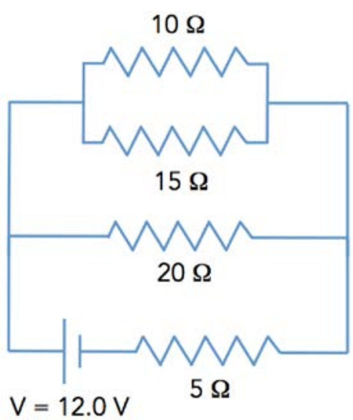

Figure 4: Circuit

2 out of 9 participants responded to this problem. Their responses are illustrated below:

- It seems that students all of a sudden forget how to do fractions in physics class, even though they have seen it much earlier in their mathematics career. As soon as it is in a different context, they forget how to do it.
- Frequently, students reach for a calculator far too soon. They don't realize that a calculator is not needed for $\frac{1}{R_{eq}} = \frac{1}{30}$ because their answer will not be an exact decimal. And so they round off their answers, which makes it not exact, and makes it harder for them to verify their answers at a later time. (Frank7)
- In my class, I allow them to use calculators to solve these fractions because this is not a mathematics course; they are allowed to use tools like calculators in order to arrive at the correct physical answers. (Larry9)

Frank7 outlines problems with fractions in the physics context. His view is that students have difficulties applying prior knowledge in different context. And when they can make progress, their calculator can be a barrier in problems, such as calculating the exact equivalent resistance due to round-off errors. Larry9 is not too concerned with those small errors, because he feels they are more mathematical in nature rather than physical, and since he is teaching them physics, it does not bother him too much.

Recurring Themes:

- Difficulties with applying prior knowledge to a new context
- Too much dependence on calculator

General Thoughts:

It is remarkable to see that the *too much dependence on the calculator* theme was present on several occasions. It is evident that students are too comfortable neglecting to do the arithmetic and algebraic computations by hand, knowing they can always plug it into their calculator. Perhaps teachers are being too permissive with this because they know how much students depend on their calculators. Perhaps they know that there is very little time to cover the entire physics curriculum, let alone worry about the mathematics involved. Maybe they know that weak mathematical skills take time to remedy and the physics classroom is not the best venue. Or maybe they recognise that the problem is too deep rooted and that students can get away with using digital devices to carry them through these courses. Whatever the possibilities are, these mathematical difficulties are not going away. Resorting to graphical means of teaching physics may be a good alternative for some topics and for some students, but it seems a general overview of this problem is needed in the current educational system.

Chapter 6.

Discussion

Chapter six gives the summary, contributions and limitations of this study, as well as what I have personally learned in completing this work and what I hope to share with my colleagues and the educational community at large.

6.1. Summary

The two questions that drove this research were:

1. From the teachers' perspective, what mathematical obstacles do students encounter when solving problems in high school physics?
2. What strategies do physics teachers employ to remedy or workaround these difficulties?

My interest was primarily to seek these answers from the vantage point of current physics teachers in high school, because it directly related to me as a physics teacher and it was lacking in the literature. In what follows I provide a brief summary of the results of this study.

6.1.1. Mathematical Difficulties

Many issues were mentioned by the participants when asked about their students' mathematical difficulties in physics class. Their responses were based on teaching many different groups of students with varying degrees of experience and backgrounds.

The two overarching difficulties were:

- Memorizing an algorithm as opposed to interpreting the problem
- Deciphering words in order to apply the proper equations

Physics students tend to want to apply memorized recipes when faced with new problems, no matter the context or topic. They lack the ability to interpret the new data and apply mathematical logic and reasoning in the proper manner. It is worth noting that the dependence on calculators appeared in several recurring themes of the problem set response analysis, but this is merely a symptom of the main two overarching difficulties above. Students who are incapable of deciphering the words tend to want to memorize an algorithm and therefore resort to using their calculator and number crunching to get some sort of an answer. For those students the calculator becomes a sort of saviour.

For instance, in a displacement problem, if the unknown is the not the hypotenuse of a right-angle triangle, they will still treat the unknown as c^2 in the Pythagorean theorem. As an example:

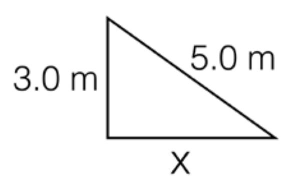

Figure 11: Pythagorean theorem example (2)

Students would start with the formula $c^2 = a^2 + b^2$, and they perform this incorrect computation: $X^2 = (3.0\text{ m})^2 + (5.0\text{ m})^2$ because they have memorized that the unknown side is "always" c^2, rather that following the reasoning for the formula.

In addition, students have difficulties understanding the words in problems and gathering the useful data needed to solve the problem. Therefore, when faced with physics problems, they have difficulties extracting key terms and applying the proper equations to solve the problem. This links with the previous response; in essence they lack the ability to decipher the problem and this leads to applying a memorized algorithm in an attempt to solve a new problem. This way of generalizing a memorized recipe and applying it to a problem can take many forms, for instance some students can make errors in interpreting schematic data incorrectly by memorizing a specific rule and applying it to other problems. They memorize that resistors that look parallel use the parallel rule for resistors. In the circuit problem, students would notice that the 4 resistors “look” to be in parallel, and they would apply the equivalent resistor formula:

$$\frac{1}{R_{eq}} = \frac{1}{10\Omega} + \frac{1}{15\Omega} + \frac{1}{20\Omega} + \frac{1}{5\Omega}$$

But the 5Ω resistor is not in parallel with the rest of the resistors, it just appears that way because of the schematic that has been provided:

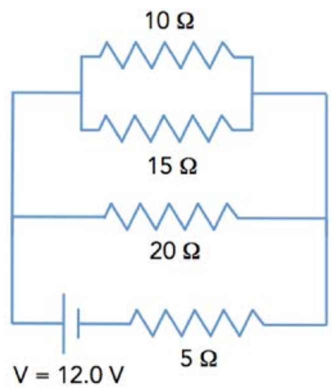

Figure 4: Circuit

6.1.2. Possible Remedy

The most common remedy to mathematical difficulties in physics, according to the participants, was the shift in focus from brute force number crunching to focusing on mathematical processes. The participants expressed that when they focus on explaining and working through the process, the logic and the reasoning of a physics problem, their students have a greater opportunity to understand the mechanism rather than a recipe. Applying a preconceived recipe leads to the urge to scramble for numbers to plug into equations just to get a final answer. Once the student understands the process of arriving at a sensible solution, they can apply that train of thought to decipher the requirements in a new problem and apply the process.

In Figure 11: Pythagorean theorem example (2) on page 81, focus would be given on the process of identifying the hypotenuse of a right-angle triangle first, then applying the formula. In the circuit on page 82, the focus would be either re-drawing the circuit to make the position of the resistors more evident or tracing the path of current in the circuit to show that the 5Ω resistor is not in parallel with the rest of the resistors. Once that has been established, the formula for parallel resistors can be applied. Therefore, in both cases, identifying the problem at hand as not just another problem they may have seen, but to analyze it anew, then apply the correct formulas in proper places seems to be a plausible remedy. Mathematical process also includes the student's ability to logically represent their train of thought and to communicate their thinking using correct terminology and recognised formulations. Ultimately, the student should know why each step in a solution is needed, not just how to do each step.

6.2. Contributions of Study

While in prior studies, mathematical competence has been identified as an obstacle for students' success in physics, those studies have not attended to the voices of teachers. This study filled the literature gap by focusing on the mathematical problems students face in physics classes according to their physics teachers. The participants have seen group after group of physics students, from varying backgrounds and understandings and their responses were arrived at from years of direct teaching experience. The education, experience and feedback of the participants provided a very valuable data set to analyse and contribute to this field of research. Their valuable responses allowed for the itemization of the most common problematic issues and possible remedies to those problems. Taking the lens of teachers in this study advances the research in a new realm, after all, teachers have equal share in these problems.

6.3. Limitations of Study

The limited number of participants was naturally an issue in this study. With only nine participants, it is challenging to generalize this study to a larger number of physics classes. Due to the time constraints of each interview, not all aspects of each problem were fully developed. If a longer interview time was available, more questions could be posed and more topics could be investigated. The limited number of participants is also a limitation when considering the scope of this study. Attempting to generalise this study to different physics curricula in other provinces or countries could prove to be challenging. As curriculum and topic focus change, so does the pedagogy, and that could have an impact on the problematic mathematical areas experienced by students in the physics class.

The results of this study would be more impactful if it was feasible to interview more physics teachers, from a larger region. The more physics teachers that can contribute their expert opinion, the larger the data set becomes and this in turn would increase the scope of the study. Perhaps it is the current curriculum that leads to the mathematical difficulties that we encounter in physics class. Perhaps a transformation is needed in our pedagogy, and therefore our curriculum. A larger study may reveal that our current physics educational value system is in need of urgent attention. Perhaps it will reveal to us that if we want to create better physics students, be need to create better mathematics students.

6.4. Related Issues

In this study, the mathematical difficulties students face in the physics classroom was given from the perspective of physics teacher. As a result, it raises the issue of the students' perspective of the mathematical challenges they face in their physics classrooms compared to those presented in this study, which can be a study on its own. And although this study presented possible remedies that the participants have implemented in their own practice, it is unclear to what degree their suggestions actually work in general. It may prove to be fruitful to implement their ideas in a larger study by other physics teachers in their classrooms in order to verify their validity.

Furthermore, the issue of the *placement* of the mathematical instruction is a key point that would benefit from more research. If the mathematical difficulties are precisely identified, a great question arises, namely, where and when is the most effective place for the instruction of these mathematical notions in the academic career of the student?

Would it be in a mathematics class before the student takes the physics course? Would it be a unit of mathematics review before the start of the physics material? Or would it be in the physics course before a particular topic where the physics teacher starts using the mathematical techniques? In essence, how can the mathematics teacher ensure that the students completing their course is mathematically ready for their first physics course? And how can the physics teacher verify that those students have the mathematical ability to perform the necessary operations required before worrying about teaching them techniques to enhance their abilities of concept formation? All of these questions are valuable to this field, though out of the scope of this study. However, the current study may be a good foundation towards the exploration of these issues.

6.5. Final Comments

I hope to use the results form this study directly in my own practice. I will suggest to my students and propose to teachers that strengthening the understanding of the mathematical process is more valuable than the numerical results and the memorization of algorithms. Our assessment pieces should be heavily based on problems that reveal the details of process rather than final answers. It should be based on the application and understanding of the methods rather than the application of memorization. Being able to decipher the problem, extract the useful data and proceed with the logic and reasoning required by a sound mathematical process is crucial.

I have currently implemented some of these ideas in my classes. I allow my students to create their own experiments in physics to verify particular formulas they have learned. They then present to me their thinking by documenting the process of the problem creation, solution and presentation. It has proved to be fruitful thus far, but it is still in its the early stages, and already I have found ways to make the experience more rewarding for both the students and myself. In the future, I hope to compile my learnings from this research as well as the methods I have employed in my classroom as a result of this study to the educational community. I hope to share the insights that revealed themselves in this study with my fellow physics teachers. The field of physics should no longer be hindered by avoidable mathematical difficulties, specially so early on at the high school level.

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Appendix A.

Teacher Questionnaire 1

1. What degree(s) do you have?
2. How long have you been teaching?
3. Have you taught mathematics at the high school level?
   -If yes, list the courses and the number of years you taught the course
4. Have you taught physics at the high school level?
   -If yes, list the courses and the number of years you taught the course
5. What is your favourite course to teach and why?
6. What aspects of physics do you believe to be the most challenging for students?
7. Where do you think physics students struggle the most in problem solving?
8. Where to you think physics students struggle the most in their way of concept formation?
9. What have you done in your classroom to remedy these problems?
10. What would you suggest be done in the education system to remedy these problems?

Appendix B.

Teacher Questionnaire 2

List of problems:

-Where would students make mistakes?

-What different methods would they use in solving these problems?

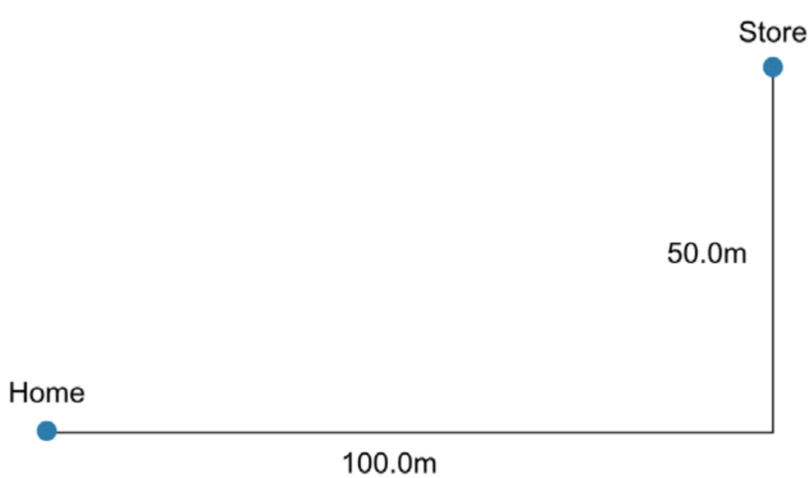
Problem #1

General problem:

What challenges do students face when dealing with distance vs. displacement?

Specific problem:

Jimmy makes a one-way trip from his home to the store:



a) What is the total distance travelled by Jimmy?

b) What is his total displacement?

Problem #2

General problem:

What challenges do students face when dealing with average velocity problems?

Specific problem:

Jane makes a one-way trip from home to the corner store. Jane walks East at an average velocity of 0.750 m/s for 21.5 s, and then runs East at an average velocity of 2.25 m/s for 25.8 s. What is Jane's average velocity on her trip?

Problem #3

General problem:

What challenges do students face when dealing with kinematics problems that deal with algebraic manipulation?

Specific problem:

a) Jimmy throws a tennis ball vertically downwards from his window. The ball travels for 15.0 m before hitting the pavement. Jane measures the speed of the ball as it strikes the ground to be 20.0 m/s. How much time did it take for the ball to travel from Jimmy's window to the pavement?

b) This time, Jimmy goes upstairs to his parent's bedroom, which is 5.0 m above his room and throws the tennis ball downwards from the window. Jane measures the travel time to be 1.25 s. What is the speed of the ball the moment before hitting the pavement?

Problem #4

General problem:

What challenges do students face when dealing with the mathematical equations that describe reflections from curved mirrors?

Specific problem:

a) Jane is 170.0 cm tall and is standing at the focal length of a concave mirror. If the focal length of the mirror is 50.0 cm, how tall is her image?

b) Jimmy stands 50.0 cm in front of a convex mirror that has a radius of curvature of 20.0 cm. What is the magnification of Jimmy's image?

Problem #5

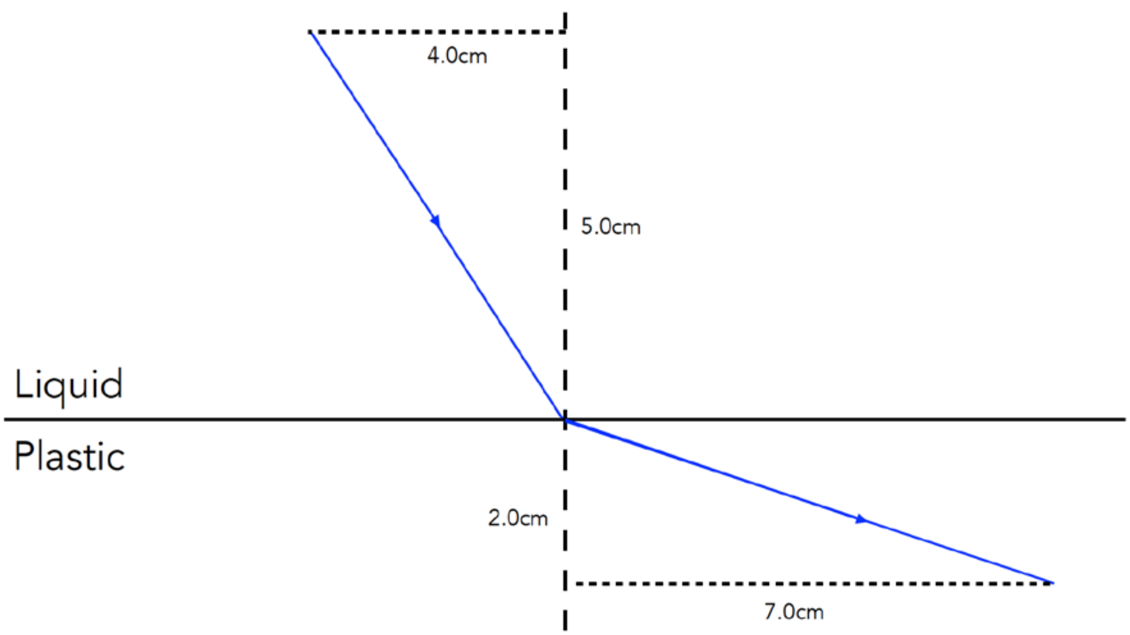
General problem:

What challenges do students face when dealing with refraction?

Specific problem:

a) A beam of light has a wavelength of 5.80×10^{-7} m in air and a wavelength of 4.30 $\times 10^{-7}$ m in liquid. If the beam enters water at an angle of incidence of 20.0^{o}, what is the angle of refraction?

b) The beam of light from above travels through a liquid into plastic. Find the ratio of the index of refraction of the liquid compared to the plastic.



Problem #6

General problem:

What challenges do students face when dealing with circuits?

Specific problem:

c) Jimmy builds a circuit containing three resistors in parallel. He measures the total resistance in his circuit to be $R_{eq} = 5.0\ \Omega$. If he measures the value of two of his resistors to be $R_1 = 10.0\ \Omega$ and $R_2 = 15.0\ \Omega$, what is the value of the last resistor?

d) Find the value of R_{eq}: