

**REALIZED GARCH: EVIDENCE IN CSI 300  
DURING A HIGH-VOLATILITY PERIOD**

by

Te-Wei Shih

Bachelor of Arts in Economics, National Dong Hwa University, 2013

Bachelor of Business Administration in Accounting, National Dong Hwa University, 2013

and

Ran Duan

Bachelor of Financial Engineering, Sichuan University, 2014

PROJECT SUBMITTED IN PARTIAL FULFILLMENT OF  
THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE IN FINANCE

In the Master of Science in Finance Program  
of the  
Faculty  
of  
Business Administration

© Te-Wei Shih and Ran Duan, 2015

SIMON FRASER UNIVERSITY

Fall, 2015

All rights reserved. However, in accordance with the *Copyright Act of Canada*, this work may be reproduced, without authorization, under the conditions for *Fair Dealing*. Therefore, limited reproduction of this work for the purposes of private study, research, criticism, review and news reporting is likely to be in accordance with the law, particularly if cited appropriately.

# Approval

**Name:** **Te-Wei (Thomas) Shih**

**Ran Duan**

**Degree:** **Master of Science in Finance**

**Title of Project:** **REALIZED GARCH: EVIDENCE IN CSI 300  
DURING A HIGH-VOLATILITY PERIOD**

**Supervisory Committee:**

---

Dr. Andrey Pavlov  
Senior Supervisor  
Professor of Finance

---

Dr. Phil Goddard  
Second Reader  
Instructor

**Date Approved:** \_\_\_\_\_

## **Abstract**

Numerous studies have suggested the application of GARCH and its extensions to model volatility of stock prices and indices. However, the performance of these models is not well established during the period of unusually high volatility. In this paper, we compare three GARCH specifications namely, standard GARCH, EGARCH, and Realized GARCH, in their ability to model volatility during the recent Chinese stock market debacle. In addition, three models are applied to the quantile forecast of Value-at-Risk (VaR). Normal distribution, student's t distribution as well as skewed student's t distribution are used. While all specifications perform in a similar fashion during normal periods, we document that Realized GARCH model with skewed student's t distribution outperforms the others during the high-volatility period from January 2015 to October 2015.

**Keywords: GARCH, EGARCH, Realized GARCH, High Frequency Data, Realized Volatility, VaR Forecast**

## **Acknowledgement**

We would like to express our sincere gratitude to our supervisor, Dr. Andrey Palvov, who offered us valuable feedback, encouragement and patience to support us throughout this paper. He also provided us with stimulating comments and suggestions. With his rich research experience, he guided us in the right direction through the course of this research. In addition, we would like to give special thanks to Dr. Phil Goddard for his support to our project as the second reader.

# Table of Contents

<b>Introduction</b> .....	<b>1</b>
<b>I. Literature Review</b> .....	<b>2</b>
<b>II. Methodology</b> .....	<b>5</b>
2.1 Standard GARCH Mode .....	5
2.2 EGARCH model .....	5
2.3 Realized GARCH model.....	6
2.3.1 Quadratic Variation Theory and Realized Volatility.....	6
2.3.2 The Leverage Function.....	8
2.3.3 Structure of Realized GARCH model .....	8
2.4 Distribution.....	10
<b>III. Empirical Result</b> .....	<b>11</b>
3.1 Data and Basic Analysis .....	11
3.1.1 Basic Analysis .....	11
3.1.2 Realized Volatility.....	12
3.2 Fitting Results .....	13
<b>IV. VaR Forecast</b> .....	<b>17</b>
<b>V. Conclusion</b> .....	<b>21</b>
<b>VI. References</b> .....	<b>22</b>

## **Introduction**

In recent years, with the popularity of high-frequency trading, intraday financial data is available in a variety of markets. This paper connects high frequency data with the realized measure of volatility using GARCH models.

The data used in this paper is the Shanghai Shenzhen CSI 300 index from January 2015 to October 2015. In 2015, China's stock market has fallen sharply, resulting in high volatility of stock price. In this paper, we compare GARCH, EARCH and Realized GARCH model in their ability to model volatility and to forecast VaR during this period of unusually high volatility. The conclusion is that the Realized GARCH model with skewed student's t distribution performs better in volatility estimation and VaR forecast during this unusual period.

This paper is organized as follows. Chapter I introduces the literature review pertaining to the GARCH model and its extensions. Chapter II introduces specific form of GARCH, EGARCH and Realized GARCH model. The leverage function and distributions of the standardized error term will also be discussed. Chapter III & IV give the empirical results for the comparison of estimation and VaR forecast with different models. Chapter V gives the summary and conclusion.

# I. Literature Review

It is essential to model the dynamics of volatility because the financial volatility changes over time. In reality, the volatility of financial data usually possesses the following characteristics:

- Volatility cluster exists which means that large changes tend to be followed by large changes, and small changes tend to be followed by small changes. (Mandelbrot, 1963)
- Volatility changes continuously in a fixed range over time.
- Volatility reflects to the rise and fall of return in a different way and grows more when the stock price falls, which is called the ‘leverage effect’.
- The distribution of volatility usually has a fat tail and skewness. Thus, the assumption of normal distribution is normally not applicable.

The first model on the estimation of volatility was the ARCH (Autoregressive Conditional Heteroskedasticity), published in the seminal paper by Engle (1982). Later, Bollerslev (1986) introduced GARCH (Generalized ARCH) which uses daily asset returns to extract information about the present and future level of volatility. ARCH/GARCH models are rapidly applied to the empirical research because they are able to accurately describe the characteristics of the volatility.

In the GARCH model, the GARCH equation describes the feature of volatility clustering well. In addition, compared to the ARCH model, GARCH model better

reflects the distribution with fat tail. However, there are some weaknesses in the GARCH model. First, volatility is insensitive to the direction of the price change. In practice, the volatility tend to be larger in the case of decreasing price. Secondly, volatility is only stationary for very restricted values of the parameters.

In order to overcome the weaknesses of GARCH model, Nelson (1991) established EARCH model to describe the leverage effect in the real financial data. The EGARCH introduces new coefficients that allow the sign and the magnitude of return to have different effects on the volatility. In addition, the new model uses exponential form to measure conditional variance, thus solving the problem of the parameter restriction. On the basis of above advantages, the EGARCH model is widely used in empirical research.

In recent years, high-frequency financial data are available. The empirical test of Dacorogna (2001) has presented that intraday data is superior to daily returns. However, neither GARH nor EGARCH model are suited for this situation where volatility changes rapidly. Then, a series of paper put forth the importance of realized measures of volatility (Andersen& Bollerslev 1998; Barndorff-Nielsen& Shephard 2001; Comte& Renault 1998).

In 2012, Hansen, Huang and Shek established Realized GARCH model. This model introduces measurement equation which connects the realized measure of volatility to the latent volatility. Besides, the model also uses leverage function to differentiate between the signs of the return change. The Realized GARCH model initially



assumes a standard normal distribution for the standardized error term. However, distributions like the student's t and skewed student's t can also be included and have also been discussed in this paper. In addition, the Realized GARCH model has another advantage that the conditional volatility used in this model is able to measure the return volatility completely, including both trading and non-trading time.

All models described above initially assume a Gaussian distribution for the standardized error term. However, both heavy tails and skewness should be considered in financial data. When analyzing data, t-distribution is considered more precise than normal distribution because they are more spread out and the tails decrease more slowly. (Bollerslev 1987) In 1994, Hansen introduced skewed student's t distribution using a non-centrality parameter. Skewed student's t distribution considers both the fat tail and skewness of the standardized error term.

## II. Methodology

### 2.1 Standard GARCH model

In the standard GARCH model, the conditional variance,  $h_t$ , is determined by  $h_{t-1}$  and  $r_{t-1}^2$ . The volatility specification of the GARCH (1,1) used in this paper is given by:

$$h_t = \omega + \beta h_{t-1} + \alpha \varepsilon_{t-1}^2 \quad (2.1.1)$$

$$z_t \sim N(0,1) \quad (2.1.2)$$

with  $h_t$  denoting the conditional variance,  $\omega$  the intercept and  $\varepsilon_t^2$  the residuals from the conditional mean equation.

In the standard GARCH model, in order to get positive variance and variance stationarity, there are some requirements on parameters:

$$0 \leq \beta \leq 1 \quad (2.1.3)$$

$$\beta + \gamma < 1 \quad (2.1.4)$$

### 2.2 EGARCH model

The EGARCH (1,1) model of Nelson (1991) is defined as:

$$\log(h_t) = \omega + \beta \log(h_{t-1}) + [\eta_1 z_{t-1} + \eta_2 (|z_{t-1}| - E|z_{t-1}|)] \quad (2.2.1)$$

$$z_t \sim N(0,1) \quad (2.2.2)$$

The formulation allows the sign and the magnitude of  $z_t$  to have separate effects on the volatility.

The EGARCH model shows some significant differences from the standard GARCH model:

- Volatility can react asymmetrically to the good and bad news.
- The parameter restrictions for strong and covariance-stationarity coincide.
- The parameters are not restricted to positive values.

## 2.3 Realized GARCH model

### 2.3.1 Quadratic Variation Theory and Realized Volatility

During the market debacle, stock prices often exhibit extreme jumps. Jump diffusion models are common for identifying these kinds of jump variations (Barndorff-Nielsen & Shephard 2004; Bollerslev & Diebold 2007). According to the articles, asset return could be expressed as:

$$dp(t) = \mu(t)dt + \sigma(t)dW(t) + k(t)dq(t) \quad (2.3.1)$$

where  $\mu(t)$  and  $\sigma(t)$  are the drift and instantaneous volatility,  $W(t)$  is a standard Brownian motion, and  $q(t)$  is a Poisson counting process, with the corresponding time-varying intensity function  $\lambda(t)$ .  $\lambda(t)$  is the intensity of arrival process for jumps, with corresponding jump size  $k(t)$  for any time  $t$  given that  $dq(t) = 1$ .

In the model, the last term captures the characteristic of jump diffusion in financial data. The overall variance is determined by the number of jumps and their respective sizes. Quadratic Variation Theory then splits the total variation into a continuous simple path part and a jump part. The total quadratic variation can then be represented as:

$$QV_t = \int_0^t \sigma^2(s)ds + \sum_{s=1}^{q(t)} k^2(s) \quad (2.3.2)$$

Realized volatility could be used as a proxy for the unobserved quadratic variation represented about (Andersen, Bollerslev, Diebold & Ebens 2001). If the frequency (M) of intra-daily sampling increases, then the quadratic variation could be written as:

$$\lim_{M \rightarrow \infty} RV_t = \int_0^t \sigma^2(s)ds + \sum_{s=1}^{q(t)} k^2(s) \quad (2.3.3)$$

Assuming that the frequency (M) is very high, the realized variance in Eq. (2.3.3) could be written as:

$$RV_t = \sum_{j=1}^M r_{t,j}^2 \quad (2.3.4)$$

Where  $r_t$  is intradaily return and  $j = 1, 2, 3, \dots, M$

In reality, market microstructure noise such as bid-ask spread influences the realized volatility. There are two ways to reduce this impact:

- Realized Kernel introduced by Barndorff-Nielsen (2009) could be used. Compared with Realized Volatility, Realized Kernel considers the effect of microstructure noise.
- The impact of microstructure noise could be minimized by choosing an appropriate data frequency (M). In general, realized volatility increases with the decline of frequency and this tendency becomes stable when 5-minute data is used.

The resolution of data used will be discussed later in this paper.

### 2.3.2 The Leverage Function

The leverage function can measure the leverage effect of the negative correlation between today's return and tomorrow's volatility. A leverage function can be constructed in this way:

$$\tau(z_t) = \tau_1 a_1(z_t) + \dots + \tau_k a_k(z_t), \text{ where } E(a_k z_k) = 0, \forall k \quad (2.3.5)$$

In this formula, the parameters  $\tau_1$  and  $\tau_2$  give an indication of how dependent the volatility is upon the changes in return.

When using hermite polynomials, the leverage function could be represented as follow:

$$\tau(z) = \tau_1 z + \tau_2 (z^2 - 1) + \tau_3 (z^3 - 3z) + \dots \quad (2.3.6)$$

However, it is proved that terms after the first two are insignificant. (Hansen et al. 2011). Hence, we consider the leverage function as a simple quadratic form:

$$\tau(z) = \tau_1 z + \tau_2 (z^2 - 1) \quad (2.3.7)$$

### 2.3.3 Structure of Realized GARCH model

The Realized GARCH model relates the observed realized measure to the latent volatility via a measurement equation, which also includes the asymmetric reaction to shocks, making for a very flexible and rich representation. For the volatility specification, the model is as follows:

$$\log h_t = \omega + \sum_{i=1}^p \beta_i \log h_{t-i} + \sum_{i=1}^q \alpha_i \log x_{t-i} \quad (2.3.8)$$

$$\log x_t = \xi + \delta \log h_t + \tau(z_t) + \mu_t, \mu_t \sim N(0, \sigma_u^2) \quad (2.3.9)$$

where the log of the conditional variance ( $h_t$ ) and the log of the realized measure ( $x_t$ ) are used. The asymmetric reaction to shocks comes via the  $\tau(\cdot)$  function. The function is based on the Hermite polynomials and could be written as a simple quadratic form:

$$\tau(z_t) = \eta_1 z_t + \eta_2 (z_t^2 - 1) \quad (2.3.10)$$

While Standard GARCH models specify  $h_t$  as a function of the past values of  $h_t$  and  $z_t$ , the Realized GARCH model specifies it as a function of the past values of  $h_t$  and  $x_t$ . Equation (2.3.9), called measurement equation, relates the realized volatility to the true volatility.

We estimate the Realized GARCH models of (1,2), (2,1) and (2,2), but the performance does not show large difference. Therefore, we just explain the simplest version of realized GARCH (1,1):

$$\log h_t = \omega + \beta \log h_{t-1} + \alpha \log x_{t-1} \quad (2.3.11)$$

$$\log x_t = \xi + \delta \log h_t + \eta_1 z_t + \eta_2 (z_t^2 - 1) + \mu_t, \mu_t \sim N(0, \sigma_u^2) \quad (2.3.12)$$

If  $\eta_1 < 0$ ,  $\log x_t$  will be larger when  $z_t < 0$ , which will make the  $h_t$  larger through equation (2.3.12) if  $\alpha > 0$ . This is consistent with the fact that there is a negative correlation between today's return and tomorrow's volatility.

Compared with GARCH and EGARCH model, the Realized GARCH model has an advantage that it enables us to estimate the parameters of return and volatility equations simultaneously.

## 2.4 Distribution

All models described above initially assume a Gaussian distribution for  $z_t$ . However, both heavy tail and skewness should be considered in real financial data. The first density function we could use is the generalization of the student's t distribution with normalized unit variance (Bollerslev 1987):

$$g(z|v) = \frac{\Gamma(\frac{v+1}{2})}{\sqrt{\pi(v-2)}\Gamma(\frac{v}{2})} \left(1 + \frac{z^2}{(v-2)}\right)^{-(v+1)/2} \quad (2.4.1)$$

Compared with standard normal distribution, quantiles for the t-distributions lie further from zero and the tails decrease more slowly. The t-distributions are more spread out than the normal.

In addition, skewed student's t distribution might be a natural extension to the regular student's t distribution. Density function of skewed student's distribution is introduced as follow (Hansen, 1994):

$$g(z|v, \epsilon) = \begin{cases} bc(1 + \frac{1}{v-2} (\frac{bz+a}{1-\epsilon})^2)^{-(v+2)/2} & \text{if } z < -a/b \\ bc(1 + \frac{1}{v-2} (\frac{bz+a}{1+\epsilon})^2)^{-(v+2)/2} & \text{if } z \geq -a/b \end{cases} \quad (2.4.2)$$

where  $2 < v < \infty$  and  $-1 < \epsilon < 1$ . The constants a, b and c are given by:

$$a = 4\epsilon c \left(\frac{v-2}{v-1}\right), \quad b^2 = 1 + 3\epsilon^2 - a^2, \quad c = \frac{\Gamma(\frac{v+1}{2})}{\sqrt{\pi(v-2)}\Gamma(v/2)} \quad (2.4.3)$$

The skewed student's t distribution considers not only the fat tail, but also skewness of financial data.

In this paper, we will compare the result fitted by realized GARCH model under normal distribution, student's t distribution as well as skewed student's t distribution.

## III. Empirical Result

### 3.1 Data and Basic Analysis

#### 3.1.1 Basic Analysis

The CSI 300 index is a capitalization-weighted stock market index designed to replicate the performance of 300 stocks traded in the Shanghai and Shenzhen stock exchanges. Thus, the performance of the index could reflect the condition of Chinese stock market.

In the second half of 2015, Chinese stock market experienced a debacle. The Shanghai stock market had fallen 30 percent over three weeks by 9 July. In the meanwhile, values of Chinese stock markets continued to drop despite efforts by the government to reduce the fall. After three stable weeks, the Shanghai index fell again on the 24th of August by more than 8 percent.

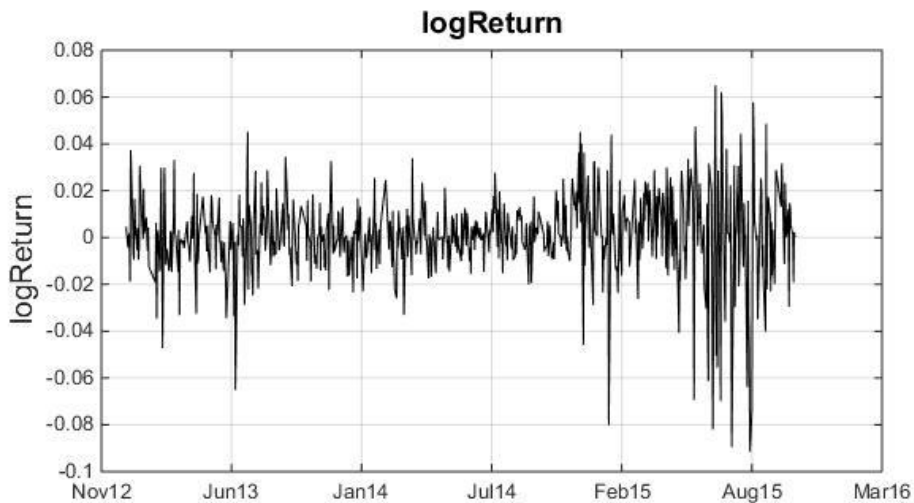
The intraday data on returns and realized volatilities of the CSI 300 stock index are used in this paper. The sample period is from January 2015 to October 2015. The data is obtained from the Bloomberg terminal.

In order to reduce trends in volatility and mean of return, we calculate log return over the period:

$$r_t = \ln P_t - \ln P_{t-1} \quad (3.1.1)$$

The log return of SCI 300 from January 2013 to October 2015 is showed in chart:





As can be seen from the chart, the log return exhibits an unusually high volatility during from January 2015 to October 2015.

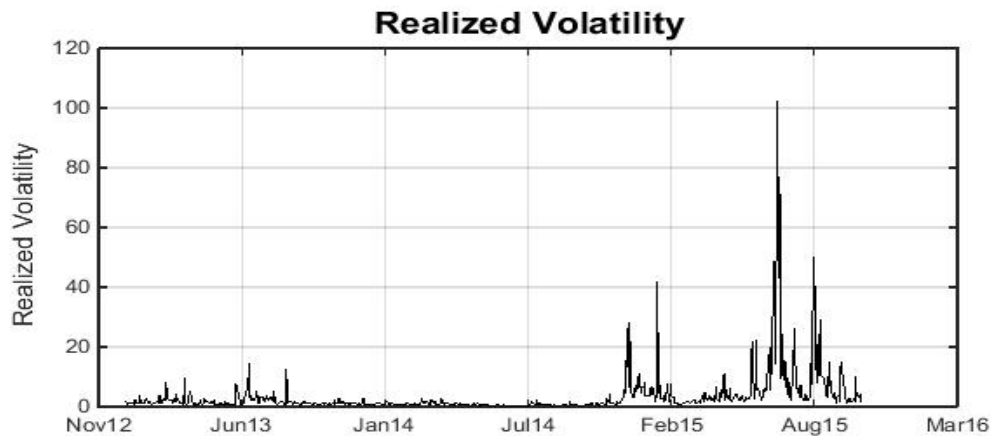
### 3.1.2 Realized Volatility

In reality, market microstructure noise such as bid-ask spread will influence realized volatility. Thus, adding the squares of overnight returns may make realized volatility noisy. In theory, realized kernel proposed by Barndorff-Nielsen (2008) could eliminate this noise. In practice, however, the Realized GARCH model can adjust the bias of RV caused by microstructure noise correctly. (Toshiaki Watanabe 2011)

Thus, we use the sum of intraday returns as realized volatility in this paper. The RV is calculated as follows:

$$RV_t = \sum_{j=1}^M r_{t,j}^2 \quad (3.1.2)$$

The chart below shows the realized volatility of CSI 300 Index from January 2013 to October 2015:



As we can see from the chart, the realized volatility is extremely high from January 2015 to October 2015. Actually, China stock market experienced a debacle during this period.

### 3.2 Fitting Result

In this paper, we compare the uses of GARCH, EGARCH, and Realized GARCH models to analyze the Shanghai Shenzhen CSI 300 index from January 2015 to October 2015.

For the conditional mean, ARMA model is used. After trial and error, we obtain the optimal order ARMA (2,2).

In Table 1, it shows the results of GARCH (1,1) with normal distribution, EGARCH(1,1) with normal distribution, and Realized GARCH(1,1) with 3 different distributions: the normal, student's t and skewed student's t distributions. The statistic used was the likelihood ratio statistic, the log-likelihood.

From Table 1, we can find out that the  $\beta$  are quite close to each other in all three

Realized GARCH models with different distribution and smaller than EGARCH's. This is because the volatility estimated in Realized GARCH model is affected by the latent volatility as well as the measure of realized volatility. Furthermore, a key feature of the realized GARCH framework is the measurement equation that relates the observed realized measure to latent volatility. In all three Realized GARCH models, the realized measure parameters,  $\delta$ , are also close to each other, which are around 1.3.

The leverage effect is a well-known phenomenon in stock markets of a negative correlation between today's return and tomorrow's volatility. Because  $\eta_1 < 0$  and  $\alpha > 0$ ,  $\log x_t$  will be larger when  $z_t$  is negative, which will make the  $h_t$  larger. This demonstrates the fact that negative return would have a bigger impact on volatility.

In measurement equation, the effects from last volatility are quite similar in different distributions. As a result, no matter what the distributions of standardized error term are, the volatility, realized measure, and error term of realized measures are quite close to each other in all three Realized GARCH models. This explains that these parameters are relatively stable in different distribution assumption.

When we compare the log likelihood in different models, the analysis needs to be based on the same data during the same period. It is necessary that each models actually fits on the real distribution of the data we used. As the likelihood function in Realized GARCH models includes two different data sets, one is the residual of return

and the other is the error term of measurement equation, the log-likelihood we obtain for these models cannot be compared to those of the GARCH models.

$\zeta$  and  $\nu$  determine the skewness and kurtosis respectively. In the Realized GARCH model with skewed student's  $t$  distribution,  $\zeta$  is significantly below 1, indicating a negative skewness of  $z_t$ . The degree of freedom of student's  $t$  and skewed student's  $t$  distribution are 7.68 and 7.21 respectively, indicating a large kurtosis.

Even though we could compare the partial log-likelihood in Realized GARCH models with that of a standard GARCH or EGARCH models, the partial log-likelihood is not the most optimizing function. Therefore, we choose not to compare the log-likelihood in Realized GARCH models with standard GARCH's and EGARCH's.

However, we can compare the log-likelihood in three Realized GARCH models with different distribution assumptions. From Table 3, it shows that the Realized GARCH with skewed student's  $t$  distribution model has the largest log-likelihood. In the log-likelihood ratio test between student's  $t$  distribution and skewed student's  $t$  distribution in Realized GARCH model, the null hypothesis is rejected with a  $p$  value in 0.007. This suggests the unrestricted model, Realized GARCH with skewed student's  $t$  distribution, fits the data better than the restricted model, Realized GARCH with student's  $t$  distribution. Thus, we could draw the conclusion that the Realized GARCH model with skewed student's  $t$  distribution would lead to a better fit than Realized GARCH models with student's  $t$  distribution and normal distribution models for the data and period we chose.

Table 1: Empirical Results in Each GARCH Specification

Model	$\omega$	$\beta$	$\alpha$	$\xi$	$\delta$	$\eta_1$	$\eta_2$	$\nu$	$\zeta$	Log L
GARCH (Norm)	0.000	0.409	0.299	/	/	/	/	/	/	465.08
	(0.000)	(0.150)	(0.115)	/	/	/	/	/	/	
EGARCH (Norm)	-1.617	0.786	/	/	/	-0.243	0.055	/	/	468.49
	(0.145)	(0.020)	/	/	/	(0.047)	(0.089)	/	/	
RGARCH (Norm)	0.215	0.543	0.287	-0.362	1.318	-0.198	0.098	/	/	- 599.93
	(0.101)	(0.067)	(0.070)	(0.384)	(0.250)	(0.047)	(0.029)	/	/	
RGARCH (T)	0.213	0.540	0.289	-0.357	1.322	-0.187	0.109	7.678	/	- 596.18
	(0.112)	(0.069)	(0.074)	(0.409)	(0.266)	(0.047)	(0.030)	(4.151)	/	
RGARCH (ST)	0.212	0.539	0.304	-0.325	1.263	-0.190	0.111	7.208	0.773	- 592.54
	(0.114)	(0.069)	(0.076)	(0.390)	(0.244)	(0.048)	(0.032)	(4.193)	(0.073)	

\* RGARCH (T) and GARCH (ST) represent the RGARCH with student's t and skewed student's t distribution respectively.

## IV. VaR Forecast

Value at Risk (VaR) is defined as the upper limit of the left tail of the assumed distribution. For a given portfolio, time horizon, and probability  $p$ , the  $p$  VaR is defined as a threshold loss value, such that the probability that the loss on the portfolio over the given time horizon exceeds this value is  $p$ . A violation is said to occur when the daily loss is larger than the VaR. In a perfectly specified model, this violation should occur with percent probability. The observed probability of a violation is called the empirical failure size.

$$Prob(\Delta V > VaR) = 1 - \alpha$$

Where  $\Delta V$  means the expected loss of the portfolio. In addition, the difference in accumulated distribution function would result in a different value of VaR.

In this paper, we used Shanghai Shenzhen CSI 300 index from January, 2015 to October, 2015 to estimate the parameters in each model. Then, we use these data sets to implement the VaR forecast. Each model is estimated using a sample size of 199 observations, the estimation window. Each model is estimated 99 times each, moving the estimation window one step forward each time.

The implement method we used to forecast the VaR is rolling estimation and forecasts. The ‘rugarch’ package in RStudio allows for the generation of 1step ahead rolling forecasts and periodic re-estimation of the model. The resulting object contains the forecast conditional density, namely the conditional mean, sigma, skew, shape, and the realized data for the period under consideration. The violations, empirical failure

rate, are summarized, and the Kupiec score (likelihood ratio) is calculated to compare the different models.

The forecasts are evaluated using the Kupiec test with a 5% significance level. VaR is evaluated using a likelihood ratio test developed by Kupiec (1995). Because of the usage of 5% significance level in this paper, if the Kupiec score (Likelihood Ratio) is larger than 3.84, the null hypothesis will be rejected. If the null hypothesis is rejected, this mean the specific model is not a suitable specification to estimate the VaR.

However, there are some flaws in the Kupiec test. Firstly, the test does not take the sequence of violations into account. Secondly, the Kupiec score is not affected by how large the violation is. This means that a 1% violation or a 3% violation will have the same weight (Lehar et al., 2002).

Table 2: Empirical Failure Rate (EFR)

$\alpha$	10%	5%	1%
RG (ST)	5.050	2.020	1.010
RG (T)	7.070	4.040	1.010
RG (Norm)	10.101	9.090	2.020
EG (Norm)	18.182	13.131	7.070
SG (Norm)	17.172	12.121	6.060

Table 3: Likelihood Ratio (LR)

$\alpha$	10%	5%	1%
RG (ST)	3.230	2.370	0.000
RG (T)	1.040	0.205	0.000
RG (Norm)	0.001	2.840	0.803
EG (Norm)	6.080*	9.710*	15.700*
SG (Norm)	4.760*	7.690*	11.900*

H0 is rejected at a 5% significance level if the Kupiec score is larger than 3.84

Table 4: p-value from LR test

$\alpha$	10%	5%	1%
RG (ST)	0.072	0.124	0.992
RG (T)	0.308	0.651	0.992
RG (Norm)	0.973	0.092	0.370
EG (Norm)	0.014*	0.002*	7.27e-05*
SG (Norm)	0.029*	0.006*	0.001*

The numbers in the table above are p-values from the LR test.

\* indicates that the null hypothesis is rejected at a 5% significance level.

\* RG (T) and RG (ST) represent the RGARCH with student's t and skewed student's t distribution respectively.



Tables above show the EFR, LR and the p-value for the Kurpiec LR test for  $\alpha = 1\%$ ,  $5\%$  and  $10\%$ . As we can see from Table 3, the null hypothesis is rejected for each Standard GARCH and EGARCH models with different level of  $\alpha$ . Thus, we could conclude that compared to Realized GARCH model, Standard GARCH and EGARCH are not suitable to estimate the VaR for Shanghai Shenzhen CSI 300 index. However, we could not conclude which distribution is better in Realized GARCH model because the null hypothesis is not rejected for all three different distributions.

## V. Conclusion

In this article we described the use of Realized GARCH model to analyze the CSI 300 index during a high volatility period using high frequency data. We also applied GARCH models to forecast VaR to compare GARCH, EGARCH and Realized GARCH models.

From the empirical result of log likelihood, we conclude that the Realized GARCH model with skewed student's  $t$  distribution is better than other distribution assumptions to model volatility during a high volatility period. In addition, Realized GARCH model performs better in the forecast of VaR where GARCH and EGARCH with normal distribution are suggested not suitable model specifications for this given period. The results suggest that use of high frequency data improve the modeling of conditional volatility and the realized measures incorporate more relevant information during the given period for Shanghai Shenzhen CSI 300 index. Further analysis is needed to support this conclusion in different market during other time period.

Several extensions are possible. First, it is worthwhile using other distributions that have recently been applied to financial returns, like the normal inverse Gaussian (NIG) and generalized hyperbolic (GN) skew student's  $t$  distribution. However, it is difficult to estimate the parameters by the maximum likelihood method. Second, other realized measures of volatility such as the realized range (Christensen and Podolskij, 2007) could be used to improve the model.

## VI. References

Andersen, Torben G., and Tim Bollerslev. "Answering the skeptics: Yes, standard volatility models do provide accurate forecasts." *International economic review* (1998): 885-905.

Andersen, Torben G., et al. "The distribution of realized stock return volatility." *Journal of financial economics* 61.1 (2001): 43-76.

Andersen, Torben G., Tim Bollerslev, and Francis X. Diebold. "Roughing it up: Including jump components in the measurement, modeling, and forecasting of return volatility." *The Review of Economics and Statistics* 89.4 (2007): 701-720.

Barndorff-Nielsen, Ole E., and Neil Shephard. "Non-Gaussian Ornstein–Uhlenbeck-based models and some of their uses in financial economics." *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* 63.2 (2001): 167-241.

Barndorff-Nielsen, Ole E., and Neil Shephard. "Power and bipower variation with stochastic volatility and jumps." *Journal of financial econometrics* 2.1 (2004): 1-37.

Barndorff-Nielsen, Ole E., et al. "Designing realized kernels to measure the ex post variation of equity prices in the presence of noise." *Econometrica* (2008): 1481-1536.

Barndorff-Nielsen, Ole E., et al. "Realized kernels in practice: Trades and quotes." *The Econometrics Journal* 12.3 (2009): C1-C32.

Bollerslev, Tim. "Generalized autoregressive conditional heteroskedasticity." *Journal*

*of econometrics* 31.3 (1986): 307-327.

Bollerslev, Tim. "A conditionally heteroskedastic time series model for speculative prices and rates of return." *The review of economics and statistics*(1987): 542-547.

Comte, Fabienne, and Eric Renault. "Long memory in continuous-time stochastic volatility models." *Mathematical Finance* 8.4 (1998): 291-323.

Engle, Robert F. "Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation." *Econometrica: Journal of the Econometric Society* (1982): 987-1007.

Gencay, Ramazan, et al. *An introduction to high-frequency finance*. Academic press, 2001.

Hansen, Bruce E. "Autoregressive conditional density estimation." *International Economic Review* (1994): 705-730.

Hansen, Peter Reinhard, Zhuo Huang, and Howard Howan Shek. "Realized garch: a joint model for returns and realized measures of volatility." *Journal of Applied Econometrics* 27.6 (2012): 877-906.

Nelson, Daniel B. "Conditional heteroskedasticity in asset returns: A new approach." *Econometrica: Journal of the Econometric Society* (1991): 347-370.

Watanabe, Toshiaki. "Quantile forecasts of financial returns using realized garch models\*." *Japanese Economic Review* 63.1 (2012): 68-80.